Capacity and Achievable Sum-Rate of the Cellular Uplink with Global and Clustered Multi-Cell Cooperation

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Summary

With the emergence and continuous growth of wireless data services, the value of wireless networks is not only defined by how many users it can support, but also by its ability to concentrate large amounts of data capacity at localized spots. Information theory provides a mathematical framework which quantifies the maximum achievable data rate over a communication channel. More importantly, information theoretic study for capacity of cellular systems suggests base station cooperation as a mean of overcoming inter-cell interference limitations.

The aim of this thesis is to investigate the information theoretic capacity of the cellular uplink. In order to quantify the performance of the current and future engineering solutions, the designers need to know the rate limits that a particular multi-user cellular system can provide and how far from the limit, lays the efficiency of their design. The gap between the information theoretic limit and the performance of currently known systems will also determine if the current systems are already saturated or have a potential to provide higher rates.

More specifically, the thesis focuses on extending the known formulations for the cellular uplink under BS cooperation by: 1) incorporating, modeling and investigating the various effects of the multi-user channel that correspond to a real-world communication system (e.g. path loss, multipath and shadow fading, thermal noise) to provide a fundamental limit for the capacity of the wireless cellular network, 2) evaluating the effect of various practical parameters of real-world cellular systems on capacity, such as inter site distance, number of UTs per-cell, UT distribution over the cells, UT transmit power, 3) investigating how multiple directional antennas for site sectorization may improve the networks communication rates and, 4) exploring the achieving communication rates of clustered cellular systems as a more feasible strategy to be implemented nowadays to find when clustered cooperation becomes more beneficial.

Key words: Capacity Limits, Multi-cell Joint Processing, Base Station Cooperation, Multi-user Information Theory, Cellular Uplink, Asymptotic Analysis.
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Nomenclature

General Notation

- $x$: Scalar variable or index
- $X$: Constant or System Parameter
- $\mathbf{x}$: Column Vector
- $\mathbf{X}$: Matrix
- $\mathbf{I}$: Identity Matrix
- $\Lambda$: Covariance Matrix
- $\mathcal{X}$: Set
- $\mathcal{CN}$: Complex Gaussian Distribution
- $\log_2(\cdot)$: Logarithm of Base 2
- $\text{det}(\cdot)$: Determinant of a Matrix
- $\text{diag}(\cdot)$: Diagonal Matrix
- $\max$: Maximization or Maximum
- $\min$: Minimization or Minimum
- $E[\cdot]$: Expectation
- $\cdot^*$: Conjugate transpose
- $\cdot^T$: Transpose Matrix
- $\cdot^\dagger$: Hermitian transpose
- $\circ$: Hadamard product
- $\otimes$: Kronecker product
- $\Delta$: defined as
- $\Rightarrow$: converges to

List of Main Symbols

- $C$: Capacity
- $R$: Sum Rate
- $K$: UTs per Cell
- $N$: Total Number of Cells
- $Q$: Number of Cells per Cluster
- $P_{\text{max}}$: UT Transmit Power Constraint
- $x$: Input
- $y$: Output
Nomenclature

\[ z \] Noise
\[ g \] Fading Coefficient
\[ \zeta \] Path Loss Coefficient
\[ \eta \] Path Loss Exponent
\[ H \] System Channel Matrix
\[ \Sigma \] System Path Gain Matrix
\[ G \] System Fading Matrix
\[ r_0 \] Side of Regular Hexagon
\[ r \] Minimal Radius of Regular Hexagon
\[ d_0 \] Radius of Circular Equivalent of Regular Hexagon
\[ S \] Number of Directive Antennas per BS
\[ G_D \] Directivity Gain
\[ M \] Number of Omnidirectional Antennas per BS
\[ \alpha \] Path Attenuation Factor
\[ \rho, \beta \] Truncation Distances depending on UT Distribution
\[ D_{k,l} \] Distance of UT \( k \) in tier \( l \) respectively to BS of Interest
\[ L \] Maximum Number of Tiers of Interference

List of Main Symbols/Acronyms

1D One Dimensional
2D Two Dimensional
AWGN Additive White Gaussian Noise
BC Broadcast Channel
BS Base Station
CC Cell Centre
CE Cell Edge
CSI Channel State Information
HR Hyper Receiver
c.c.s. complex circularly symmetric
i.i.d. independent identically distributed
ISD Inter Site Distance
JP Joint Processor
bps bits per second
LoS Line of Sight
MAC Multiple Access Channel
MCP Multi-Cell Processing
MIMO Multiple Input Multiple Output
QoS Quality of Service
RoT Rise over Thermal
SC Superposition Coding
SISO Single Input Single Output
SNIR Signal to Noise and Interference Ratio
SNR Signal to Noise Ratio
UT User Terminal
Chapter 1

Introduction

In the past, wireless systems were designed to accommodate a large number of voice and/or low data rate users. With the emergence and continuous growth of wireless data services, the value of wireless networks is not only defined by how many users it can support, but also by its ability to concentrate large amounts of data capacity at localized spots. Thus, in the age of wireless data, multi-user data rates surge as an important metric. In that direction, cellular technologies have been evolutionarily improved to provide broadband data services. However, their interference limited nature due to inter-cell and intra-cell interference has not been overcome yet. At the same time, information theory provides a mathematical framework which quantifies the maximum achievable data rate over a communication channel and despite the intensive work on that field, the first important attempt to study the capacity of a cellular system was carried out in the previous decade. Information theory deals only with finding the capacity limits without directly suggesting the approach to achieve these limits. However, the information theoretic studies usually suggest the approach for communication that may be more appropriate to reach the capacity limits. One such approach for cellular systems is Base Station (BS) cooperation as a mean of overcoming important interference limitations and forms a focal point for this thesis.
1.1 Cooperative Wireless Communications

Several cooperative techniques have been proposed in literature apart from BS cooperation, such as User Terminal (UT) cooperation ([1]) and Relaying ([2, 3]). The various UT cooperation techniques may include scheduling, power allocation and virtual antenna diversity, however, it involves many implications in practice due to significant signalling and processing overhead needs. Furthermore, relaying involves communication with multiple hops through relaying elements (either through fixed transponders or through adjacent UTs). In the former case though, there will be an additional cost for the deployment and maintenance of the additional network elements while in the latter case a penalty on the battery life of UTs occurs due to increased processing and transmission needs. On the other hand, BS cooperation may take advantage of the already existing infrastructure, since the required complex processing can be performed either by the existing BSs or by a central processor.

1.1.1 Base Station Cooperation in Uplink

Conventional wireless cellular systems in uplink associate each UT with a BS, usually the one that is most capable to receive the UTs pilot signal. Despite that association, the UTs signal is inevitably received by other BSs as well and thus it causes interference (known as inter-cell or inter-site interference) and loss on performance to these BSs since it cannot be decoded elsewhere apart from its associated BS. In that direction, BS cooperation (or Multi-Cell Processing - MCP) for joint multi-user detection [4, 5] at a central processor (usually referred as "Hyper Receiver" - HR) is a strategy that can tackle the inter-cell interference problem. In the case where BSs are able to exchange messages so as to jointly decode all the signals received, the cooperative network of BSs, as an entity, has the capability of decoding any signal received at any BS and consequently the undesired inter-cell interference can be transformed to desired received information. The MCP pattern presumes high speed delay less and error free interconnection among all the BSs in the system and the HR. Ergo, the HR will be

\[ ^{1}\text{Fiber-optic communications stand for a feasible solution to fulfil the desired interconnection between BSs and HR} \]
able to jointly process all the received signals.

1.2 Information Theory and Rate of Communication

Information theory is a broad and deep mathematical tool which today has fundamental contributions on a wider area of science: Physics, mathematics, computer science, neurobiology and electrical engineering are some of the fields that have progressed through its applications. At its very foundation by Shannon, information theory was immediately related to communication theory and provided the extreme points of the set of all possible communication schemes: The ultimate data compression and the ultimate transmission rate of communication [6]. This was succeeded by introducing the concepts of the entropy $H$ and the channel capacity $C$. The concept of information theory and the basic quantities of information are reviewed in Appendix B.

In the early 1940s it was thought that an increase in the transmission rate over a communication channel increases the probability of error. Shannon changed this concept by proving that as long as the communication rate was below the channel capacity, error free communication was possible.

1.2.1 Channel Capacity

Channel capacity is in general defined as the maximum transmission rate (supremum over all achievable rates) of communication over a channel with arbitrarily low probability of error [7]. According to Shannon, if we consider a memoryless mobile communication channel where the output depends only on its current input, then for input $x$ and output $y$, the capacity $C$ of the channel is defined as the maximum mutual information between the input and the output:

$$C_{ch} \triangleq \max_{p(x)} I(x; y) \quad (1.1)$$

where the maximum is taken over all possible input distributions $p(x)$.

Using the concept of mutual information between the channel input and output, Noisy-channel Coding Theorem [8] proves that a code and a decoding algorithm exists that
1.3 Motivation and Objectives of Research

The aim of this thesis is to investigate the information theoretic capacity of the cellular uplink. Shannon was the first to develop the underlying mathematical concepts to predict the information theoretic capacity of a single communication link but it took half a century of engineering to implement techniques which achieve rates close to this prediction. However, this fact validated the importance of theoretic limits for the capacity of communication systems; in order to quantify the performance of the current and future engineering solutions, the designers need to know the rate limits that a particular multi-user cellular system can provide and how far from the limit, lays the efficiency of their design. Furthermore, the gap between the information theoretic limit and the performance of currently known systems will determine if the current systems are already saturated or have a potential to provide higher rates. Based on this information, the decision makers will be able to make an informed judgement on how much effort is worthwhile making in order to get more capacity from the current systems.

More specifically, the thesis focuses on extending the known formulations for the uplink capacity of wireless cellular systems under BS cooperation by:

- incorporating, modeling and investigating the various effects of the multi-user channel that correspond to a real-world communication system (e.g. path loss, multipath and shadow fading, thermal noise) to provide a fundamental limit for the capacity of the wireless cellular network (although cellular systems are in operation for many years now, the information theoretic treatment of a cellular system with realistic path loss and fading models is still an open area),

- evaluating the effect of various practical parameters of real-world cellular systems on capacity, such as: inter site distance, number of UTs per-cell, UT distribution over the cells, UT transmit power,
1.4 Fundamental Assumptions

- investigating already implemented signal processing techniques that may improve the networks communication rates (e.g. multiple directional antennas for site sectorization),

- exploring the achieving communication rates of cellular systems that may be more feasible to be implemented nowadays (e.g. clustered cellular systems with multi-cell joint processing).

1.4 Fundamental Assumptions

The information theoretic capacity of the cellular uplink study presented in this thesis is based on Shannon’s approach for the single-user channel. In addition, the multi-cell processing scheme implies a specific infrastructure architecture. In that direction, some fundamental assumptions are made for the communication network studied here:

- **Transmission**: The input symbols are assumed to be Gaussian distributed with infinitely long transmitted codeblocks (*Gaussian coding*\(^2\)).

- **Channel**: No frequency selectivity is considered for the fading of the wireless channel. In that case, the coherence bandwidth of the channel is larger that the bandwidth of the transmitted signals (*flat fading*) and thus, the channel fading state can be represented by a single narrowband fading coefficient.

- **Reception**: Either all BSs or the central processor connecting with them are assumed to have perfect knowledge about the channel state of all links among UTs and BSs at any time (*Channel State Information*).

- **Infrastructure**: All the BSs are interconnected through high-speed, delay-less and error-free links either with each other or to a central processor. In any case, infinite processing capabilities are assumed.

\(^2\)Gaussian coding is proved to achieve capacity. In Appendix B.5 of [9] it is discussed how capacity as predicted by the Gaussian coding argument can be achieved in reality by real-world designed codes.
1.5 Major Contributions and Publications

The main contributions of this thesis are summarized as follows:

• Realistic assumptions are considered in order to model accurately the real-world systems. The various channels effects and system parameters that had not been tackled in depth until now in the literature are incorporated into the capacity analysis of the uplink. In that direction:

  - A detailed description of a simplified path loss model and a generic multipath fading model that are suitable for information theoretic analysis is presented.

  - It is shown how shadow fading can be incorporated into that model.

  - The inter-site separation is varied to achieve a variable cell site density for a cellular system.

  - Geometric considerations of the UT spatially distribution over the cells are incorporated and analysed.

  - Multiple tiers of interference for each site are considered.

• A unified system parameter - Rise-over-Thermal (RoT)- is presented which shows the relevance of multi-user information theoretic uplink capacity to the single link information theoretic capacity presented by Shannon. This parameter is defined at each BS receiver end, as the ratio of aggregate desired received signal power to the undesired received power at the receiver. It is shown how capacity is directly proportional to RoT.

• Analytical closed forms are derived for the capacity of the cellular uplink. Typical parameter values for the currently existing cellular systems are used as an example to show how analytical results should be interpreted. The available information theoretic findings are not directly usable to provide realistic estimates for the capacity of practical systems and a usual approach in information theoretic studies
is to provide scaling relations rather than direct numbers for capacity. Here, the use of both tractable and practical propagation models are used to provide a connection between analysis and practice so as to obtain more practical numbers for the capacity of the wireless systems.

- The information theoretic uplink capacity analysis is extended to incorporate the sectorization of sites. Site sectorization is a technique of harnessing the space diversity and despite the wide-spread deployment of sectorized cellular systems it has not been tackled so far in the information theoretic cellular capacity literature. Here, a thorough investigation on the performance of cellular systems with directive antennas at the BSs under the notion of BS cooperation is made. A comparison to a system with omnidirectional antennas at the BSs is also provided.

- A clustered joint processing scheme which decentralizes the joint decoding of all the UTs is investigated to define the uplink capacity of the practical systems of today. The large distances between BSs in most systems make it very hard to achieve lossless cooperation among all BSs. In that direction, the achieving communication rates of clustered cellular systems are analysed. Various cluster isolation schemes are compared with each other and the UT power allocation optimization problem is investigated.

The research carried out during the construction of this thesis has resulted in the following publications:

**Journals**


**Book Chapter**

**Peer reviewed Conferences**


**1.6 Thesis Structure**

The rest of the thesis is organised as follows:

**Chapter 2: “Literature Review”**. In this chapter, the information theoretic channel capacity, as first defined by Shannon, is studied. Subsequently, the extension of the study to the Multiple-Input-Multiple-Output (MIMO) channel is described. Focusing on the multiple access channel, we move on to the definition and the study of the
capacity region of the multi-user system. Furthermore, the concept of BS cooperation in the uplink of a cellular system and its relation to the MIMO multiple access channel is discussed. Finally, the various models in the literature that incorporate that concept are presented.

Chapter 3: “System Description and Capacity Analysis”. In this chapter the basic models and parameters describing a range of wireless communication systems are discussed and it is established how they can be used together to provide the information theoretic uplink capacity when BS cooperation comes into play. An analytical approximation of the propagation model along with a generic fast fading model are introduced first to capture these phenomena in a baseband signal representation. It is also explained how the shadowing effect can be captured in the predefined multiplicative fading processes of the wireless channel. Moreover, the generic system model using a HR is presented and the process of finding its information theoretic capacity is analysed in depth. Finally, the notion of RoT is defined as a unified figure of merit that controls the capacity of HR using communication systems and the relevance of multi-user information theoretic capacity to the single link capacity presented by Shannon is shown.

Chapter 4: “Generic Cellular Uplink with Global Cooperation”. In this chapter the information theoretic capacity of the uplink of the cellular system is investigated in depth by assuming the centralised processing for all BSs in the system. The modified path loss model along with variable cell size are considered and a mean path loss approximation is formulated. Various UT distributions are considered and it is shown how UT distribution can be modelled in the system analysis. Moreover, considering a realistic generic multipath fading environment, the analytical result for the per-cell capacity of the uplink is derived for a large number of UTs distributed over each cell. Furthermore, the general approach is extended to model the uplink of a sectorized cellular system and it is analysed how multiple antennas at BSs can improve the communication system capacity. It is shown how the capacity is increased in comparison to the single receiving antenna system and the asymptotic behaviour is investigated when the number of directive antennas per-BS grows large. Moreover, the capacity when the multiple antennas used for each BS are omnidirectional and uncorrelated is found
and compared to the sectorized case. Finally, numerical results are produced and each parameter affecting the capacity of the system is thoroughly investigated.

Chapter 5: “Extension of the Classic Cellular Uplink to Incorporate Site Sectorization”. In this chapter, the classic multi-cell-multi-user uplink model is extended to incorporate the sectorization of sites. An overview of the problem and the method used to tackle it, is given and the conventional sectorization channel model is presented and analysed in by formulating an analytical form for the per-cell capacity, under various fading environments. The system channel matrix mechanics are exquisitely investigated, the special realistic case of random phase offsets at the specular path is analysed and the limit for the maximum capacity of the system is obtained when the number of site antennas grows large. Finally, analytical and simulation results are presented and the capacity obtained with and without sectorization is compared.

Chapter 6: “Cellular Uplink with Clustered Joint Processing”. In this chapter, coordinated processing is adopted only among the BSs that belong to a Cluster of cells. The analysis begins with a linear cellular system model. In that direction, the importance of UT power allocation on the capacity of the linear clustered cellular uplink is highlighted. Moreover, important insights are obtained on capacity behaviour of that system under a number of clustering schemes. Furthermore, a time-scheduling scheme for cell rate fairness is presented and a discussion on the “coupling” of the linear and the planar models is provided. After the in depth investigation of the linear case, the case of the planar clustered cellular uplink is explored and finally the findings for the capacity of the clustered systems are summarized.

Chapter 7: “Epilogue”. In this chapter, a conclusive summary of the insights and findings acquired by the work presented in the previous chapters is provided. Moreover, future research guidelines are proposed as a step forward to the work presented in the thesis. In that direction, the limiting practical factors of real-world systems of today are discussed in contrast to the fundamental assumptions considered along with possible ways to overcome these assumptions. Furthermore, important issues like Quality of Service (QoS), multiple cooperation strategies and the capacity of the cellular downlink are discussed.
Chapter 2

Literature Review

In this chapter, the information theoretic channel capacity, as first defined by Shannon, is studied. Subsequently, the extension of the study to the MIMO channel is described. Focusing on the multiple access channel, we move on to the definition and the study of the capacity region of the multi-user system. Furthermore, the concept of BS cooperation in the uplink of a cellular system and its relation to the MIMO multiple access channel is discussed. Finally, the various models in the literature that incorporate that concept are presented.

2.1 Introduction

Claude Shannon was the first that developed a mathematical theory for the channel capacity [7, 10] providing the framework for studying performance limits in communication. Shannon's work gave birth to the field of information theory. Since then, communication systems have evolved from a simple transmitter-receiver link to complex ubiquitous communication systems. This revived the original question that Shannon posed for the single link, as a valid question for the communication systems of the new era, developing the network information theory field. This chapter provides the evolution of the study on the information theoretic channel capacity starting from the single-user channel, to the multi-user multiple access channel and reaching finally to the multi-cell case.
2.2 Ergodic Capacity

Throughout this study the *ergodic capacity* of wireless systems of Gaussian\(^1\) fading\(^2\) channels will be investigated. The *ergodic capacity* is defined as the expectation of the channel capacity over a long enough sequence of fading instances \((E_g[\cdot])\) so as the ergodic properties of the fading process to be expressed:

\[
C \triangleq E_g \left[ \max_{p(x)} I(x; y) \right]
\]  

(2.1)

This capacity is defined for a fading channel with long term delay constraints. Here the fading process is assumed to be ergodic with respect to time meaning that all its statistics can be determined from a single time history of that process. Thus the expectation of the mutual information can be achieved by a long enough code when compared to the fading speed of the channel \([11]\). That way a codeword covers all channel states according to the channel probability distribution and the average mutual information is achieved. This average mutual information is defined as ergodic capacity and it follows the usual Shannon theoretic sense and thus any rate above that capacity cannot be achieved "with arbitrarily small error probability" \([7]\). It is noted in \([12]\) that this definition of the capacity is used when it is considered that channel state information is available to the receiver. In the following sections, the ergodic capacity of Gaussian fading channels and multi-user systems that has been studied in the literature is presented and analysed.

2.3 Single-User Gaussian Fading Channel

The Gaussian fading channel model is widely used for theoretical analysis in common communication models. It consists of a transmitter and a receiver of either one or multiple antennas each. Here, both the Single-Input-Single-Output (SISO) and the MIMO channel cases are studied.
2.3. Single-User Gaussian Fading Channel

2.3.1 The SISO channel

The single-user SISO channel is shown in Figure 2.1, and it can be represented as:

\[ y[i] = h[i]x[i] + z[i] \]  \hspace{1cm} (2.2)

where \( y[i] \) and \( x[i] \) stand for the received and transmitted \( i \)th complex symbol respectively and \( h[i] \) stands for the complex channel gain of the \( i \)th fading process. Moreover \( z \sim CN(0, \sigma_z^2) \) where \( \sigma_z^2 \) is the AWGN power. Consider also a uniformly distributed across all fading instances average power constraint on the transmitted signal, i.e. 
\[ \mathbb{E}_i [(x[i])^2] \leq P_{\text{max}}. \]

The ergodic capacity of the channel will be straightforwardly given by (2.1). Based on the information theoretic properties given in the previous section we have:

\[ H(x, y) = H(y) + H(y|x) = H(y) + H(hx + z|x) \]
\[ = H(y) + H(z|x) \overset{(a)}{=} H(y) + H(z) \]  \hspace{1cm} (2.3)

where step (a) follows from the fact that \( z \) is statistically independent of \( x \). The noise entropy is given considering (B.5) as \( H(z) = \log_2 (\pi e \sigma_z^2) \). Thus, to obtain the capacity, the output entropy is needed to be maximized. From [6] we know that the complex Gaussian distribution maximizes the entropy of a variable. According to that, the output signal should follow the Gaussian distribution and it will be given by (considering (B.5)):

\[ H(y) = \log_2 \left( \pi e \sigma_y^2 \right) \]  \hspace{1cm} (2.4)

where

\[ \sigma_y^2 = \mathbb{E} [y^2] = \mathbb{E} [(hx + z)^2] = \mathbb{E} [(hx)^2] + \mathbb{E} [(z)^2] + 2 \mathbb{E} [hx] \mathbb{E} [z] = h^2 P_{\text{max}} + \sigma_0^2 \]  \hspace{1cm} (2.5)

1 The "Gaussian" term refers to the consideration of Additive White Gaussian Noise at the receiver.
2 The "fading" term refers to the complex multiplicative process scaling the transmit signal.
2.3. Single-User Gaussian Fading Channel

Therefore the ergodic capacity of that channel will be derived as:

\[ C_{su,SISO} = E \left( \log_2 \left( 1 + \frac{P_{\text{max}}}{\sigma_0^2} |h[i]|^2 \right) \right) \]  

(2.6)

Note that since the ergodic capacity over a large enough sequence of fading instances is studied and also the channels are assumed to be memoryless, the time index \( i \) and the notion of the expectation over \( i \) may be omitted throughout the rest of the study.

2.3.2 The MIMO channel

Consider now multiple antennas at the transmitter and at the receiver. In that case, we may refer to a single-user MIMO channel (Figure 2.2) which can be represented in a matrix form as:

\[ y = Hx + z \]  

(2.7)

where \( x, y \) denote the \( n_t \times 1 \) input and \( n_r \times 1 \) output vector respectively, the \( n_r \times n_t \) matrix \( H \) contains all the multiplicative fading effects of each sub-channel and \( z \) is the \( n_T \times 1 \) AWGN vector. The entries of the \( H \) matrix are assumed independent identically distributed (i.i.d.) and complex circularly symmetric (c.c.s.). Moreover, \( z \sim \mathcal{CN}(0, \Lambda_z) \) where the covariance matrix of the noise vector is given by \( \Lambda_z = E[zz^H] = \sigma_0^2 I_{n_r \times n_r} \) when the noise elements are assumed independent from each other. Again, each input element has a power constraint, i.e. \( \Lambda_x = E[xx^H] \leq P_{\text{max}} I_{n_t \times n_t} \) where \( \Lambda_x \) is the input covariance matrix.

The ergodic capacity of the channel will be given according to (2.1) by:

\[ C_{su,MIMO} = E_H \left[ \max_{\Lambda_x \leq P_{\text{max}} I_{n_t \times n_t}} I(x;y) \right] \]  

(2.8)

where \( I[(x;y)] = H(y) + H(x) \) similarly to (2.3).

Following the same steps as for the SISO channel we have for a single system time snapshot (non-variant \( H \)):

**AWGN entropy**

\[ H(x) = \log_2 \left( \pi e \sigma_0^2 \right)^{n_r} \]  

(2.9)
2.3. Single-User Gaussian Fading Channel

Maximise the output entropy. From [13] we know that circularly symmetric complex Gaussian distribution maximizes the entropy of a vector. The output vector entropy will be given by

\[ H(y) = \log_2 \left( (\pi e)^{nr} \det \{ A_y \} \right) \]  

(2.10)

where

\[ A_y \triangleq \mathbb{E} \left[ yy^\dagger \right] = \mathbb{E} \left[ (Hx + z)(Hx + z)^\dagger \right] \]

\[ = \mathbb{E} \left[ Hxx^\dagger H^\dagger \right] + \mathbb{E} \left[ Hxz^\dagger + E \left[ z^\dagger H^\dagger \right] + \mathbb{E} \left[ zz^\dagger \right] \right] \]

\[ = HAH^\dagger + \sigma_0^2 I_{nr \times nr} \]  

(2.11)

Therefore, the ergodic capacity of the channel is achieved when all elements of the input matrix have their maximum \( P_{\text{max}} \) available power and it is given by:\(^3\)

\[ C_{su,MIMO} = \mathbb{E}_H \left[ \log_2 \det \left( I_{nr \times nr} + \frac{P_{\text{max}}}{\sigma_0^2} HH^\dagger \right) \right] \]  

(2.12)

Note that if a total average input power constraint \( P_T \) is considered over all input elements and no Channel State Information (CSI) available at the transmitter, the

\(^3\)When comparing to the SISO case it can be seen that the MIMO channel can be used to achieve higher rates due to diversity gain. Furthermore, a MIMO channel can be decomposed into a number of parallel independent channels when cooperation at the receiver is available by performing Singular Value Decomposition to the channel matrix [14]. In this direction, a multiplexing gain on the achievable rate can be achieved compared to the SISO case by multiplexing independent data onto these independent channels.
available power is then uniformly distributed across the input elements\textsuperscript{4} \cite{15, 16} and the ergodic channel capacity will be given again by (2.12) by replacing $P_{\text{max}}$ with $P_T/n_t$.

2.4 Multiple Access Channel

In this section the multi-user channels are reviewed. When considering a single transmitter and a single receiver, also a single real number $C$ can quantify the capacity of that channel. On the other hand, in the case of multiple transmitters and/or receivers in uplink, a vector $(r = [R_1, \ldots, R_N]^T)$ instead of a single value is needed to specify the set of the achievable rates of all $K$ transmitting channels. The set of all possible achievable rates under specific constraints identifies the capacity region of the multi-user channel. Various capacity metrics are employed in the literature to substitute that vector with a single value\textsuperscript{5} for better comprehension and for capacity comparison reasons. In the following, the two most important capacity metrics are presented and then we focus on the Multiple Access Channels (MAC) and the study of the capacity of the SISO and the MIMO channel cases.

2.4.1 Sum Rate Capacity Metric for the MAC

A very widely used metric for the MAC is the sum rate capacity \cite{17, 18, 19}:

\textbf{Weighted Sum Rate Capacity:} Given a vector $w = [w_1, \ldots, w_K]^T$ of relative priorities associated with the rate vector elements, the weighted sum rate capacity is defined as the sum of all weighted rate elements:

$$C_{\text{sum}} = w^T r = \sum_{k=1}^{K} w_k R_k$$  \hfill (2.13)

where $\sum_{k=1}^{K} w_k = 1$. Weighted sum rate capacity metric is used when each channel rate has a priority factor. If a specific rate prioritization does not exist

\textsuperscript{4}If CSI is available at the transmitter the waterfilling technique \cite{9} can be performed for optimizing the spectral allocation for maximal total capacity under an overall power constraint on the transmit signal.

\textsuperscript{5}The single value that substitutes the capacity vector is most commonly characterized as the system spectral efficiency and it is usually measured in bps/Hz per area, per-cell or per-UT.
2.4. Multiple Access Channel

It is noted that in this thesis the capacity investigation is focused on the normalised sum rate capacity and for the rest of the work it is going to be referred simply as sum rate capacity.

2.4.2 The SISO MAC

The SISO MAC as shown in Figure 2.3 stands for the case of multiple transmitters and a single receiver and can be represented as:

\[ y = \sum_{k=1}^{K} h_k x_k + z \]  

(2.14)

where \( y \) stands for the received signal, \( x_k \) stands for the signals from the \( k \)th transmitter, \( z \) is the AWGN at the receiver with \( z \sim \mathcal{CN}(0, \sigma^2) \) and \( h_k \) stands for the complex multiplicative fading effects of the \( k \)th channel. Every transmitter is subject to a power constraint, i.e. \( \mathbb{E} \left[ (x_k)^2 \right] \leq P_{\text{max}}, \forall k = 1, \ldots, K \).

2.4.2.1 Capacity with Superposition Coding and Successive Interference Cancellation

When assuming full CSI at the receiver and the transmitters, the optimal transmission strategy for the channel in question is employment of Superposition Coding (SC) followed by Successive Interference Cancellation (SIC) at the receiver. The SC indicates...
the case where all transmitters share and spread their signal across the entire available spectrum without any kind of orthogonalisation and therefore, the final transmitted signal is the sum of all the individual transmitted signals. Furthermore, SIC (Figure 2.4) indicates a procedure where at every time the signal that is being decoded at the receiver is the one having the highest certainty to be decoded. After its decoding, that signal is considered known to the system and therefore, its effect can be subtracted from the subsequent decodings. SIC is an iterative process that targets to decode all the signals with the specific order explained above rather than treating the interference from all transmitters as noise when decoding a transmitter signal [9].

With an analogous to waterfilling technique for the single-user case, the ergodic capacity of the MAC with full CSI at the receiver and the transmitters can be determined by introducing a set $\mathcal{F}$ of all feasible power allocation policies $\mathcal{P}$ over the instantaneous channels fading states $\mathbf{h} \triangleq [h_1 \ldots h_K]$ that satisfy the individual power constraints:

$$\mathcal{F} = \{ \mathcal{P} : \mathbb{E}_h [P_k(h)] \leq P_{\text{max}}, \forall k = 1, \ldots, K \} \quad (2.15)$$

In that case the ergodic capacity region of the MAC channel will be given as [20]:

$$C = \bigcup_{\mathcal{P} \in \mathcal{F}} C_h(h, \mathcal{P}) \quad (2.16)$$

where for each fading state $\mathbf{h}$ define

$$C_h(h, \mathcal{P}) = \left\{ r : \sum_{k \in \mathcal{S}} R_k \leq \mathbb{E}_h \left[ \log_2 \left( 1 + \frac{\sum_{k \in \mathcal{S}} h_k P_k(h)}{\sigma_0^2} \right) \right], \text{for every } \mathcal{S} \subseteq \{1, \ldots, K\} \right\} \quad (2.17)$$

### 2.4.3 The MIMO MAC

Consider now multiple transmitters, each one communicating with the single receiver through MIMO channel of $n_t$ sub-channels (Figure 2.4) as it was described in section 2.3.2. The received signal can be represented as

$$\mathbf{y} = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{x}_k + \mathbf{z} \quad (2.18)$$

where $\mathbf{x}_k, \mathbf{y}$ denote the $k^{th}$ $n_t \times 1$ input and the $n_r \times 1$ output vector respectively, the $k^{th}$ $n_r \times n_t$ matrix $\mathbf{H}_k$ contains all the multiplicative fading effects of all sub-channels.
from transmitter \( k \) to the receiver and \( z \) is the \( n_r \times 1 \) AWGN vector with \( z \sim \mathcal{CN}(0, \Lambda_z) \) where the covariance matrix of the noise vector of independent elements is given by 
\[
\Lambda_z = \mathbb{E}[zz^T] = \sigma_0^2 \mathbf{I}_{n_r \times n_r}.
\]
Every transmitter is subject to a power constraint, i.e. 
\[
\Lambda_{x,k} \triangleq \mathbb{E}[x_k x_k^T] \leq P_{\text{max}} \mathbf{I}_{n_r \times n_r}, \forall k = 1, \ldots, K.
\]

Similarly to the SISO MAC case, under full CSI at the receiver and the transmitters the optimal transmission strategy is SC followed by SIC at the receiver. The ergodic capacity of the MIMO MAC can be determined by introducing a set \( \mathcal{P}_{\text{MIMO}} \) of all feasible power allocation policies \( \mathcal{P}_{\text{MIMO}} \) over the instantaneous channels fading states 
\[
\mathbf{H} = [\mathbf{H}_1^T \ldots \mathbf{H}_K^T]^T
\]
that satisfy the individual power constraints:
\[
\mathcal{P}_{\text{MIMO}} = \{ \mathcal{P}_{\text{MIMO}} : \mathbb{E}_H [P_k(\mathbf{H})] \leq P_{\text{max}}, \forall k = 1, \ldots, K \} \tag{2.19}
\]
where \( P_k(\mathbf{H}) \) stands for the \( k^{\text{th}} \) channel individual power at an instantaneous fading state \( \mathbf{H} \). The ergodic capacity region of the MIMO MAC channel will be given as [13, 21, 22]:
\[
\mathcal{C}_{\text{MIMO}} = \bigcup_{\mathcal{P}_{\text{MIMO}} \in \mathcal{F}_{\text{MIMO}}} \mathcal{C}_{\mathbf{H}}(\mathbf{H}, \mathcal{P}_{\text{MIMO}}) \tag{2.20}
\]
where for each fading state \( \mathbf{H} \) define
\[
\mathcal{C}_{\mathbf{H}}(\mathbf{H}, \mathcal{P}_{\text{MIMO}}) = \\
\{ \mathbf{r} : \sum_{k \in S} R_k \leq \mathbb{E}_H \left[ \log_2 \det \left( \mathbf{I}_{n_r \times n_r} + \frac{\sum_{k \in S} \mathbf{H}_k \Lambda_{x,k}(\mathbf{H}) \mathbf{H}_k^T}{\sigma_0^2} \right) \right] \},
\]
for every \( S \subseteq \{1, \ldots, K\} \) \tag{2.21}

Note that the MIMO MAC capacity poses a convex optimization problem in the sense that the ordering of transmitters is not straightforward since their channels are characterized by matrices instead scalars. This problem has been shown that can be solved with the implementation of an iterative waterfilling technique [22].

In the case where the only limitation imposed is the maximum transmitted power constraint per channel (constraint on the eigenvalues of the input covariance matrix, \( P_{\text{max}} \)), it can be seen that the sum rate capacity of the MIMO MAC capacity is obtained by maximizing the power for every transmission [23] and can be given by:
\[
\mathcal{C}_{\text{MAC,MIMO}} = \mathbb{E}_H \left[ \log_2 \det \left( \mathbf{I}_{n_r \times n_r} + \frac{P_{\text{max}} \sum_{k \in S} \mathbf{H}_k \mathbf{H}_k^T}{\sigma_0^2} \right) \right] \tag{2.22}
\]
Several works investigated the capacity of the BS cooperation system model to show its benefit over the traditional cellular model [24, 25, 26, 27, 28, 29]. All these are very encouraging results and they show that there is still plenty of capacity available to the system that simply remains to be exploited by more advanced network designs.

2.5.1 Cellular Uplink and MIMO MAC

In a cellular system with no MCP, the communication channels can be viewed as multiple Gaussian interference channels whose capacity has been studied in [30, 31, 32] but it is still in general an open problem [6]. BS cooperation though, enables the interpretation\(^6\) of the communication system channel to a MIMO channel whose capacity has been addressed as discussed in 2.4.3. Specifically in the cellular uplink, the various BSs can be considered as one receiver with multiple antennas. The geographical

\(^6\)In the context of BS cooperation, each UT is associated not only with one but with a number of BSs in the system at any time and thus, the cellular notion ceases to exist.
2.5. Uplink of Cellular Systems with BS cooperation

dispersion though the various BSs differentiates the cellular uplink from MIMO MAC. In a distance dependent path loss cellular uplink environment, a UT will have in general unequal distance, and thus, different power reception from each BS unlike the MIMO case where the UT is considered equidistant from each BS antenna. In the former case the channel statistics become relevant to the BS dispersion, a fact that turns the investigation of the capacity to a challenging problem.

2.5.2 Work on Capacity of Cellular Systems with BS cooperation

Here, the state-of-the-art findings for the capacity formulation of the uplink of cellular communication systems is summarized. In the last few decades, numerous attempts have been made to study the performance of these systems. Later papers attempted to study a realistic environment by bringing multipath fading into the equation. In that direction, Somekh and Shamai extended Wyner's model by introducing fading and they showed that the performance of the system can be improved in the presence of fading when compared with the no-fading case. These findings challenged the usual notion that the interference is always harmful and made clear distinction between the unpredictable noise and the information-containing "interfering" signals. Recently, the assumptions of fixed cell density and "interfering" adjacent cells were tackled by Letzepis. In this study, Wyner's linear model was extended to incorporate distance dependent path loss and multiple-tier "interference".

2.5.2.1 Gaussian Cellular System

The scientific field of network information theory was successfully exploited to provide insight on the uplink performance of cellular systems initially by Wyner [25]. Wyner presented a simple yet tractable model for uplink and he managed to show that capacity can be achieved when the BSs cooperate to jointly decode all the received signals.

Wyner examines two different types of system geometry: 1) an one-dimensional (1D) linear cellular array (Figure 2.5) and 2) a two-dimensional (2D) hexagonal cellular array

\[\text{7In a multi-cell joint processing scheme the notion of interference is used to denote the signals received to a BS from UTs assigned to a different cell. This actually is desired received power.}\]
2.5. Uplink of Cellular Systems with BS cooperation

Figure 2.5: Linear Array of Cells

(Figure 2.6). The capacity for the uplink case of both models is obtained after adopting the following assumptions in addition to the fundamental assumptions of MCP systems which were discussed in the introduction:

- AWGN and channels with no multipath fading effects (Gaussian Cellular System).
- All the UTs in the cell of interest have a channel gain of unity.
- The BS at the cell of interest can receive signal only from UTs in the same cell and in cells that belong to the first tier of "interference" (also known as adjacent or neighbouring cells).
- A single and fixed channel gain factor $\alpha \in [0, 1]$ is used to quantify the amount of desired received power (or inter-cell “interference”) from UTs in neighbouring cells. Note that this assumption essentially stands for the case where all UTs of each cell are collocated with their BS.

Under the above assumptions, considering $K$ UTs per-cell, at a given time the received signal at cell $m$ is given by:

$$y^m = \sum_{k=1}^{K} x_{m,k} + \alpha \sum_{n \in \mathcal{N}_m} \sum_{k=1}^{K} x_{n,k} + z^m$$  \hspace{1cm} (2.23)

where $x_{n,k}$ is the transmitted signal from transmitter $u$ in cell $n$ which must satisfy an average power constraint, e.g. $E[(x_{m,k})^2] \leq P_{\text{max}}$, $\mathcal{N}_m$ stands for the set of the cells that are adjacent to cell $m$ and $z^m$ is an i.i.d. random variable for AWGN with $z \sim CN(0, \sigma_z^2)$. This channel can be considered an instance of the classical multiple access channel presented in [6].

In the following, Wyner’s results on capacity are summarized.

*Linear system.*
Starting from the capacity region definition [6] and using information theoretic quantities Wyner proved that in the linear cellular array model the per-UT ergodic capacity as the number of cells tends to infinity is bounded by:

$$C_{\text{linear}}(\alpha) = \frac{1}{K} \int_0^1 \log_2 \left( 1 + \frac{KP}{\sigma^2} \left( 1 + 2\alpha \cos(2\pi \theta) \right)^2 \right) d\theta$$  (2.24)

where $\theta$ is the frequency domain variable for the 1D transform obtained from the indexing of cells.

Work on the determination of the capacity in cellular networks following the same approach was also done in [24] where it was considered a circular rather than a linear cellular array (as illustrated in Figure 2.7) so as to avoid the edge effects. The capacities for both models are identical while the number of cells tends to infinity.

**Planar system.**

The 2D network model structure of hexagonal cells can be seen in Figure 2.6. The per-UT ergodic capacity in that case, as the number of cells tends to infinity, was found to be:

$$C_{\text{planar}}(\alpha) = \frac{1}{K} \int_0^1 \int_0^1 \log_2 \left( 1 + \frac{KP}{\sigma^2} \left( 1 + 6\alpha F(\theta_1, \theta_2) \right)^2 \right) d\theta_1 d\theta_2$$  (2.25)
where $F(\theta_1, \theta_2) = \cos(2\pi\theta_1) + \cos(2\pi\theta_2) + \cos(2\pi(\theta_1, \theta_2))$ and $\theta_1, \theta_2$ are the frequency domain variables for the 2D transform obtained from the indexing of cells.

Wyner's work showed a promising gain on achievable rates of cellular systems under the notion of BS cooperation and formed the basis for the further exploration on that subject.

### 2.5.2.2 Gaussian Cellular System with Fading

Multipath fading was introduced into Wyner's model by Somekh and Shamai in [28] and [27]. Assuming a flat fading environment, each transmitted signal now from UT $k$ in cell $n$ to BS of cell $m$ is experiencing a multiplicative fast fading effect ($g_{nm,k}^m$) and the received signal at the BS in cell $m$ can be represented as:

$$y_m = \sum_{k=1}^{K} g_{m,k}^m x_{n,k} + \alpha \sum_{n \in N_m} \sum_{k=1}^{K} g_{n,k}^m x_{n,k} + z^m$$  \hspace{1cm} (2.26)

It is considered that each UT experiences a different fading coefficient $g$ and all the fading coefficients when viewed as random complex processes are independent, strictly stationary and ergodic, normalized to unit power, i.e. $g_{n,k}^m \sim \mathcal{CN}(0,1), \forall n, m, k$.

The per-cell ergodic capacity for the linear model, was derived analogous to the Wyner results as:

$$C_{g\text{-linear}}(\alpha) = \int_{0}^{1} \log_2 \left( 1 + \frac{KP}{\sigma_x^2} \left[ (1 + 2\alpha)^2(1 - |m_g|^2) + |m_g|^2(1 + 2\alpha \cos(2\pi\theta))^2 \right] \right) d\theta$$  \hspace{1cm} (2.27)

and for the hexagonal model as:

$$C_{g\text{-planar}}(\alpha) = \int_{0}^{1} \int_{0}^{1} \log_2 \left( 1 + \frac{KP}{\sigma_x^2} \left[ (1 + 6\alpha)^2(1 - |m_g|^2) + |m_g|^2(1 + 2\alpha F(\theta_1, \theta_2))^2 \right] \right) d\theta_1 d\theta_2$$  \hspace{1cm} (2.28)

where $m_g$ is the expected value of an individual complex fading coefficient $g$.

Especially for a Rayleigh flat fading environment (non-LoS channels), which is a zero mean fading environment and thus, $|m_g|^2 = 0$, the per-cell capacity was derived for the linear model as:

$$C_R(\alpha) = \log_2 \left( 1 + \frac{KP}{\sigma_x^2}(1 + 2\alpha)^2 \right)$$  \hspace{1cm} (2.29)
2.5. Uplink of Cellular Systems with BS cooperation

and for the hexagonal model as:

$$G_r(\alpha) = \log_2 \left( 1 + \frac{KP}{\sigma^2} (1 + 6\alpha)^2 \right)$$

(2.30)

The surprising results of [27] demonstrate that for total normalised transmitted power greater than zero (0) dB and a certain range of relatively high inter-cell "interference", the fading improves the system performance as compared to the unfaded case [25]. These interesting results are attributed to the multi-user diversity effect. According to [9], the phenomenon of multi-user diversity in fast fading channels arises from the existence of independently faded signal paths from the multiple UTs in the system. In a fast fading channel, by averaging over the variations of the channel, a high long-term average throughput can be attained. Multi-user diversity improves performance by exploiting the channel fading: there is a high probability at every time that a strong UT will exist and by allocating all the resources to that UT, at this specific time, the system benefits from the strong channel.

Another interesting result in [27] was that the hexagonal model achieves higher capacity than the linear model. This was explained due to the fact that the hexagonal model exhibits larger macrodiversity when compared to the linear model. In the hexagonal model we have seven antennas co-operating to take advantage of the fading at any time (one receiving antenna in the referring cell and a total of six receiving antennas in the six neighbouring cells) while in the linear model we have three antennas (only two neighbouring cells).

2.5.2.3 Gaussian Cellular System with Fading and Path Loss

The model in [29] and later in [33], extends the work on the linear system case of the aforementioned models, by taking into consideration: 1) the received power to a BS from all the UTs of the system and 2) the spatial distribution of UTs. For the UTs of each cell, a path loss coefficient is defined which depends on a path loss model. Although the path loss effect is taken into account, the UTs of each cell have still the same path loss factor and that essentially models the case where all the cell UTs are collocated with the BS.
According to Letzepis’ model, the received signal at cell \( m \in [1, N] \) for the flat fading case is:

\[
y^m = \sum_{j=1}^{N} \sum_{k=1}^{K} \alpha_{n}^{m,j} g_{n,k}^{m} x_{n,k} + z^m
\]  

(2.31)

where the "interference" factors \( \alpha_{n}^{m,j} \) stand for the path gain coefficients of UTs in cell \( n \), calculated according to the power-law path loss model, w.r.t. their distance from receiver in cell \( m \). Moreover, \( g_{n,k}^{m} \) stands for the fading coefficients referring to channel created between UT \( k \) in cell \( n \) (with transmitted signal \( x_{n,k} \)) and receiver in cell \( m \).

The "interference" factors \( \alpha_{j}^{n} \) are given by employing the modified path loss model presented in [34]:

\[
\alpha_{j}^{n} = (1 + |n - m| / \Delta)^{-\eta/2}.
\]

(2.32)

where \( \Delta \) is defined as the cell density, e.g. \( \Delta = \frac{\text{Total span of the linear system}}{N} \). The additional unity factor in (2.32) is used to avoid singularity problems in case of UT and BS collocations. Thus, this model is more detailed than the previously described models, since it decomposes the single "interference" factor \( \alpha \), so that the effect of cell density \( \Delta \) and path loss exponent \( \eta \) on capacity can be studied separately.

The results of this approach can be summarized as:

- For small cell diameters the inter-cell "interference" is not caused only from UTs in the first tier and hence the Wyner's approach gives an overestimate on capacity.

- For low transmitted power, the capacity gap between the two approaches becomes proportionally larger.

- For large cell diameters the two models converge, since the bulk of the inter-cell received power comes from the UTs in the first tier.

2.6 Summary

This chapter has reviewed the fundamental concept of information theory and the results in the literature that are useful for the study of the ergodic capacity of the cellular uplink. Firstly, the channel capacity of the single-user (SISO and MIMO) is
studied and then we move on to the capacity region of the MAC. Finally, the capacity of the cellular uplink under MCP is examined and the most representative studies on the field are presented.
Chapter 3

System Description and Capacity Analysis

In this chapter the basic models and parameters describing a range of wireless communication systems are discussed and it is established how these can be used together to provide the information theoretic uplink capacity when BS cooperation comes into play. An analytical approximation of the propagation model along with a generic fast fading model are introduced first to capture these phenomena in a baseband signal representation. It is also explained how the shadowing effect can be captured in the predefined multiplicative fading processes of the wireless channel. After that, the generic system model using a HR is presented and the process of finding its information theoretic capacity is analysed in depth. Finally, the notion of RoT is defined as a unified figure of merit that controls the capacity of HR using communication systems and the relevance of multi-user information theoretic capacity to the single link capacity presented by Shannon is shown.

3.1 Wireless Communication System Model

A great variety of communication systems nowadays can be combined to provide networks with wide range of services on Earth. The cellular scenario that is widely used to describe a system of UTs and BSs is considered throughout this work. Nevertheless,
the scope of this thesis refers to a wider range of system types since in a cooperative BS system, as explained in the previous chapter, the cellular concept can be boiled down to the more general MIMO system concept.

3.1.1 Planar Cellular System with Hyper Receiver Model

A 2D hexagonal cellular array and a network of cells where the BSs are uniformly distributed over the hexagonal grid is assumed. A BS located at the center of each cell, will receive signals from all the UTs in the system, attenuated according to the path loss and the multipath fading. Assume that all the BSs cooperate to jointly decode the received signals (HR scheme illustrated in Figure 3.1). All the UTs of the system are sharing the same frequency and time resources and they are spatially distributed over the cells. All receiving and transmitting terminals have one antenna each. There exist two approaches to avoid the edge effects introduced by the finite size of the total number of cells and thus, to simplify the analysis and to perform comparable numerical simulations. 1) Consider infinite number of cells or, 2) consider toroidal model (similarly to Hanly’s circular array model for a linear system illustrated in Figure 2.7) to connect the system edges. In that direction, a wrap-around toroidal model can be considered for the planar system which is illustrated in Figure 3.2. In any case, every cell has the same number of surrounding cells. Nevertheless, for large number of cells the edge effects do not significantly affect the results [25, 29].

3.1.2 The Wireless Channel in a Multi-user System

The modeling of the wireless channel is of great importance for the further study of the capacity limits. A representation of the communication system architecture that applies to most systems was given by Shannon in [7]. The wireless channel between the information source and the destination modifies the transmitted signal in various ways. Conventionally, the physical processes included in the channel which modify the transmitted signal can be subdivided into additive (additive noise) and multiplicative (path loss, fast/multipath fading and shadowing/slow fading) effects. In the following
3.1. Wireless Communication System Model

Figure 3.1: The Hyper-Receiver system

Figure 3.2: The wrap-around toroidal model
the modeling of the additive and multiplicative effects that affect the channel in the multi-user cellular system is presented.

### 3.2 Additive Noise

The simplest practical consideration about the noise of a mobile radio channel is to have a flat over frequency power spectral density (white) and a normal distribution (Gaussian) \[35\]. Usually, most of the noise is created within each receiver. Since we consider complex representation for signals, noise will be:

\[
z = z_x + jz_y
\]

where \(z_x, z_y\) are zero mean, independent, real Gaussian processes, each with standard deviation of \(\sigma_z\). The mean power of noise will be given by:

\[
P_z = \mathbb{E}[zz^*] = \mathbb{E}[z_x^2] + \mathbb{E}[z_y^2] = \sigma_z^2
\]

Another way of expressing the noise, of power spectral density \(N_0\), when contained within a bandwidth \(B = \frac{1}{T}\) (when we have signals of symbols with finite duration \(T\)) is:

\[
P_z = \sigma_z^2 = BN_0
\]

It is noted that throughout this work we consider the additive noise to be thermal AWGN with the properties described above.

### 3.3 Path Loss

In this section we summarise the modified path loss\(^1\) model that will be used throughout this study along with its relation to the existing empirical models. That is a similar model to the one introduced in \[34\] and used by the authors \[29, 33\] where collocated

\(^1\)In this work we use the term "path loss" to represent the attenuation that the communication signal suffers as it traverses a distance in the transmission medium. The definition of loss in this context implies a multiplicative factor of less than one (and a negative value in dB). Note that in this context a smaller value implies a larger "loss" or attenuation.
3.3. Path Loss

UTs were assumed. In that thesis, this model was used as a basis and has extended for the case of spatially distributed UTs over the cells.

3.3.1 Modified Path Loss Model

The model that maps the path loss (defined as the ratio of the received $P_r$ over the transmitted $P_t$ power) and the distance in a distance dependent path loss environment, is expressed as:

$$\zeta^2 = \frac{P_r}{P_t} = \frac{L_0}{(D)^\eta}$$  \hspace{1cm} (3.4)

where $\zeta$ is defined as the path loss function describing the path loss environment (more specifically, describing the attenuation of the amplitude of any transmitted signal), $L_0$ is defined as the power received at a reference distance $D_0$ when transmitted power is one unit and $\eta$ is the power-law path loss exponent. The distance $D$ is defined as the actual distance between the transmitter and the receiver. From (3.4) it occurs that:

$$L_0 = \frac{P_r}{1} (D_0)^\eta$$  \hspace{1cm} (3.5)

Hence dimensions of $L_0$ are (distance$^n$) which means that the term $\frac{L_0}{(D)^\eta}$ is dimensionless. In the analysis, the distance of the UTs from the BS of interest ranges in $[0, \infty]$ and as it appears in the denominator of the path loss formula the zero value creates singularities. This can be avoided by considering the modified path loss model, which is explained in the following.

Consider Figure 3.3 where a reference point is located at a distance $D_0$ from the receive antenna. This reference distance is at least equal to the distance from the receiver to the boundary of the near and far field region. The distance from the reference point to the transmit antenna can be defined as $D'$. It is clear that $D = D' + D_0$. Making this substitution in (3.4), we get

$$\zeta^2 = \frac{L_0}{(D_0 + D')^\eta}$$  \hspace{1cm} (3.6)

which can be rearranged to get

$$\zeta^2 = \frac{L_0}{(D_0 (1 + \frac{D'}{D_0}))^\eta} = \frac{L_0}{(1 + D)^\eta}$$  \hspace{1cm} (3.7)
3.3. Path Loss

![Figure 3.3: Definitions of distances for modified path loss model](image)

with $D \triangleq \frac{D'}{D_0}$ defined as the dimensionless normalised distance of the transmitter to the reference point of the receiver. As expected, since in right hand side of (3.7) both the numerator and the denominator are dimensionless, the path loss is verified to be a dimensionless quantity as well.

It should be noted that the modified path loss model is not completely arbitrarily selected but it has a strong one-to-one correspondence to a practical system; A system where a circular exclusion zone has been created around the receive antenna and any transmitter can be placed at a distance from the receiver which is constrained to the range $[D_0, \infty]$ (referring to distance $D'$ in the model described here).

### 3.3.2 Relating the Modified Path Loss Model with Empirical Models

The modified path loss model of (3.7) does not provide a perfect representation of the practical systems, but being a close approximation it can serve as a useful tool for information-theoretic analysis. In the previous section, the physical motivation has been explained for adopting this modified path loss model which attempts to capture two important phenomena in the physical system: actual power (or "envelope"/voltage) attenuation in the physical system at any distance (w.r.t reference distance) and the rate at which this attenuation increases with the increasing distance from the reference point. The parameter $L_0$ captures the actual attenuation at the reference distance and the path loss exponent $\eta$ captures the rate at which the attenuation increases with the distance. In order to provide a one-to-one correspondence between the modified path loss model and the empirical models the values of these parameters need to be determined with the objective of providing a best-fit of the empirical data obtained in
3.3. Path Loss

Table 3.1: Valid Range of Practical Parameter Values for Cellular Uplink.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol</th>
<th>Valid Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency Carrier</td>
<td>$f_c$</td>
<td>1900 MHz</td>
</tr>
<tr>
<td>Channel Bandwidth</td>
<td>$B$</td>
<td>5 MHz</td>
</tr>
<tr>
<td>Thermal Noise Density (in dB)</td>
<td>$[N_0]_{dB}$</td>
<td>$-169 \text{ dBm/Hz}$</td>
</tr>
<tr>
<td>Inter Site Distance</td>
<td>ISD</td>
<td>100 m to 6 km</td>
</tr>
<tr>
<td>Reference Distance</td>
<td>$D_0$</td>
<td>1 m</td>
</tr>
<tr>
<td>Path Loss at Reference Distance (in dB)</td>
<td>$[L_0]_{dB}$</td>
<td>$-34.5 \text{ dB}$</td>
</tr>
<tr>
<td>Path Loss Exponent</td>
<td>$\eta$</td>
<td>{2, 3.5}</td>
</tr>
<tr>
<td>UT Transmit Power Constraint</td>
<td>$P_{max}$</td>
<td>200 mW</td>
</tr>
</tbody>
</table>

the field. A couple of models reported in the literature are summarised below.

3.3.2.1 Empirical Models and Practical Parameters

Two well-known empirical models were selected: 1) Wideband PCS Microcell Model [36] and 2) Urban Macro [37], for micro-cellular and macro-cellular systems respectively using them relevantly to the carrier frequency of the UMTS system (1.9 GHz). Fitting the values of $L_0$ and $\eta$ to these empirical models and considering the typical range of parameters for a real-world scenario summarized in Table 3.1 meaningful practical results can be provided. The micro-cellular model suggests a smaller value of $\eta = 2$ and the macro-cellular model suggests a much larger value of $\eta = 3.5$. Moreover, the path loss in decibels at reference distance $D_0 = 1 \text{ m}$ is $L_0 = -34.5 \text{ dB}$.

It should be noted that if the range of parameters or any values in the empirical models do not fit very well to certain coverage area, field studies can be performed to obtain the path loss data. The values of the constant $L_0$ and $\eta$ (or $\eta_1$, $\eta_2$ etc for multiple slope model) can then be obtained by using the curve fitting to minimise the mean square error between the empirical values and the model values.
3.4 Generic Multipath Fading Model

In the model presented in [14][Section 3.2.2:page 78] and also used in [38], a generic fading environment can be described using Rician distribution fading model. If the channel has a fixed Line-of-Sight (LoS) component the real and imaginary parts of the complex fading coefficients, $g$, are non-zero mean random variables (even when uniform distribution of phase is assumed). A model parameter $\kappa$ is used to define the ratio of the power in the LoS component ($\mu^2$) and the power in the other (non LoS) multipath components ($\sigma^2$):

$$\kappa \triangleq \frac{\mu^2}{\sigma^2}$$

Use the normalisation

$$\mu^2 + \sigma^2 = 1$$

(3.9)

Using this model, the fading coefficients can be generated as:

$$g = g_R + jg_I$$

(3.10)

where $g_R$ and $g_I$ are random variables distributed as $\mathcal{N}(\mu_g/\sqrt{2},\sigma_g^2/2)$ and related to $\kappa$ as

$$\mu_g^2 = \frac{\kappa}{\kappa + 1}$$

(3.11)

$$\sigma_g^2 = \frac{1}{\kappa + 1}$$

(3.12)

In this model, $\kappa \to \infty$ corresponds to the case where all power is concentrated in the specular path. Whereas $\kappa \to 0$ corresponds to the most severe fading i.e. Rayleigh fading with no specular component.

Incorporating the random received phase $\phi$ on the specular path, a generalised model for the fading coefficients can be given by [9][Section 2.4.2, page 36-37],[38, 14]:

$$g = \sqrt{\frac{\kappa}{\kappa + 1}} \sigma_g e^{j\phi} + \sqrt{\frac{1}{\kappa + 1}} \mathcal{CN}(0,\sigma_g^2)$$

(3.13)

where $\mathcal{CN}(0,\sigma_g^2)$ represents a complex Gaussian random variable with independent real and imaginary components each normally distributed with mean zero and variance $\sigma_g^2/2$. Note that due to the normalisation assumed in (3.9) $\sigma_g^2$ is normalised to unity in our case.
Definition 3.1: The expectation of the product between a complex fading coefficient $g$ and the complex conjugate of another complex fading coefficient $\hat{g}$ is defined as:

$$m_{gg} \triangleq E[gg^*]$$

(3.14)

Corollary 3.2: If $g = \hat{g}$ then:

$$m_{gg} = E[gg^*] = E[|g|^2] = 1$$

(3.15)

since the normalized power of the fading coefficients has been assumed to be unity.

3.4.1 Uniform Phase Distribution

As noted from its definition, the received phase $\Phi$ on the specular path is assumed to be random, i.e. in a system with a large number of received signals from independent sources, it would be valid to assume uniform distribution of $\Phi$ over $(0, 2\pi)$. In such a generic fading environment where the received signals have uniformly distributed random phase offsets it can be safely assumed the following to take place: the expected value over time of the product of a fading coefficient with the complex conjugate of another (independent from the first one) fading coefficient is zero.

3.4.2 Rayleigh Fading Environment

In an non-LoS situation, the received signals will be composed of a random multi-path component with its amplitude described by the Rayleigh distribution [35]. The Rayleigh distribution fading environment (no specular component) is, by its nature, a zero mean environment and as already seen, it is a sub-case of the generalised fading model described by (3.13) for $\kappa \to 0$. The expected value of each fading coefficient in this case is zero.

3.5 Shadow Fading environment

The relevance of the aforementioned model in a shadow fading environment is considered here. In the presence of shadow fading, the ratio of transmit to received power
(ψ), for a fixed distance between the transmitter and the receiver, can be modelled as a log-normal random variable with the following distribution [14]:

\[
p(\psi) = \frac{\xi}{\sqrt{2\pi}\sigma_{\psi dB}^{dB}} \exp \left[ -\frac{(10\log_{10} \psi - \mu_{\psi dB})^2}{2\sigma_{\psi dB}^2} \right], \psi > 0
\]  \hspace{1cm} (3.16)

where \( \xi = \frac{10}{\ln 10} \), \( \mu_{\psi dB} \) is the mean value of the variable \( \psi dB = 10 \log_{10} \psi \) and \( \sigma_{\psi dB} \) is the standard deviation of the same variable. The linear average of the random variable \( \psi \) can be found from (3.16) as given below [14]:

\[
\mu_{\psi} = E[\psi] = \exp \left[ \frac{\mu_{\psi dB}}{\xi} + \frac{\sigma_{\psi dB}^2}{2\xi^2} \right]
\]  \hspace{1cm} (3.17)

When distance is also varying, \( \mu_{\psi dB} \) becomes a function of distance,

\[
\mu_{\psi}(d) = \exp \left[ \frac{\mu_{\psi dB}(d)}{\xi} + \frac{\sigma_{\psi dB}^2}{2\xi^2} \right]
\]  \hspace{1cm} (3.18)

where \( \mu_{\psi dB}(d) \) accounts for the propagation loss due to the distance as well as the loss due to blockage caused by shadowing obstacles.

Considering the above model and a large number of independent UTs at each distance (so that the law of large numbers can be invoked), the mean received power can be modelled as a deterministic function of distance and shadow fading standard deviation. But, since random variations in the ratio of transmit to receive power (which define the standard deviation of shadow fading) for any given distance are cancelled out when law of large numbers can be applied, the mean value of the ratio becomes more important for the calculation.

In that direction, considering shadow fading environment, \( \mu_\psi \) is by definition same as the mean of the path loss variable \( \zeta^2 \) in the absence of shadowing. Hence, an alternative approach to study the capacity of the shadow fading case is to capture the mean value of this ratio in the empirical propagation model \(^2\) given in (3.4) using the parameters \( L_0 \) and \( \eta \). Since shadowing is essentially a loss in received power, smaller received power at the reference point and a larger path loss exponent can capture the essence of the shadow fading. These values can be determined using the curve-fitting approach for a

\(^2\)In the rest of the thesis we will provide results on capacity for various values of \( \eta \) since shadow fading can be captured by the variation of the path loss exponent.
3.6 Information Theoretic Capacity Analysis

Consider a network of $N$ cells and $K$ UTs in each cell. Assume for now the HR system model where all BSs in the system cooperate to jointly decode all received signals. Each UT and BS has a single antenna. According to that model, the received signal at the BS antenna of cell $n$ (where $n \in \{1, \ldots, N\}$) will be the sum of the transmitted signals from the UTs 1) within the same cell and also 2) from the rest of the cells in the system (appropriately scaled by the path loss and fast fading coefficients). Hence, the received signal at the BS of a cell $n$ is given by:

$$y^n = \sum_{m=1}^{N} \sum_{k=1}^{K} \left[ e_{m,k}^n g_{m,k}^n x_{m,k} \right] + z^n$$

where $y^n$ and $z^n$ represent the received signal and the AWGN at the receiver of cell $n$ with $z \sim \mathcal{CN}(0, \sigma^2)$. The variable $x_{m,k}$ represents the complex Gaussian inputs for a transmitter $k$ in cell $m$ and $e_{m,k}^n$, $g_{m,k}^n$ represent the path loss coefficients and the fading coefficients between a transmitter $k$ in cell $m$ and the receiver at the BS of cell $n$. All the complex flat fading coefficients are normalized to unit power and when viewed as complex random processes are circularly symmetric i.i.d. Gaussian, strictly stationary and ergodic. It is assumed that a power constraint of $P_{\text{max}}$, i.e. $E[x_{m,k}^n x_{m,k}^*] \leq P_{\text{max}}$ exists for every UT.

The output vector of all the received signals in the system can be given using the channel equation (3.19) in a matrix form, as:

$$y = Hx + z$$

where $y = [y^1, y^2, \ldots, y^N]^T$ is the $N \times 1$ received signal vector, $x = [x_1^T, x_2^T, \ldots, x_N^T]^T$ is the $NK \times 1$ vector of the transmitted signals of all the UTs, with $x_n = [x_{n,1}, \ldots, x_{n,K}]^T$, denoting the concatenation of the transmitted signals from the $K$ UTs in cell $n$, $z$ is
3.6. Information Theoretic Capacity Analysis

the \(N \times 1\) column vector of noise and \(H\) is the overall \(N \times NK\) system gain matrix given by:

\[
H = \Sigma \odot G
\]  

(3.21)

where \(\Sigma\) is a deterministic \(N \times NK\) matrix that contains all the path loss coefficients of the existing channels and \(G\) is the \(N \times NK\) matrix of all the corresponding fading coefficients. In \(H\) matrix, each row corresponds to a specific receiver and each column to a specific transmitter.

For describing the matrices formulated above, consider the representation of the cellular system as a rectangular array, as described by Wyner in [25], a scanning method (e.g. the raster scan method that was used by Somekh and Shamai in [27]) to define the order of the system output vector elements and hence the one-to-one mapping of all 2D index vectors for the cells to a unique 1D index system. Considering the wrap-around toroidal model presented in Figure 3.2, \(\Sigma\) can be considered as a block-circulant matrix, in terms of its row-vector elements. Assume the \(n^{th}\) row corresponding to the receiver of cell \(n\) has the 2D index given as \(v_n = (p, q)\). There are six subsets, the union of which describes the cells that belong to the \(l^{th}\) tier of cells around the cell of interest. Subsequently, they describe the row blocks that contain the appropriate path loss coefficients (see Figure 3.4):

\[
A_1 \triangleq \{ (p-l, q-l), (p-l, q-l+1), \ldots, (p-l, q) \} \\
A_2 \triangleq \{ (p+l, q), (p+l, q+1), \ldots, (p+l, q+l) \} \\
A_3 \triangleq \{ (p-l, q-l), (p-l+1, q-l), \ldots, (p, q-l) \} \\
A_4 \triangleq \{ (p, q+l), (p+1, q+l), \ldots, (p+l, q+l) \} \\
A_5 \triangleq \{ (p, q-l), (p+1, q-l+1), \ldots, (p+l, q) \} \\
A_6 \triangleq \{ (p-l, q), (p-l+1, q+1), \ldots, (p, q+l) \}
\]  

(3.22)

The ergodic capacity of the cellular channel can be given according to (2.1) (omitting the time index) as:

\[
C = \max_{\forall p(x); E[|X|^2] \leq P_{\text{max}}} \mathcal{I}(x; y)
\]  

(3.23)
3.6. Information Theoretic Capacity Analysis

Figure 3.4: The six sets describing which cells belong to the $i^{th}$ tier of cells around a cell $(p, q)$. Two indices describe the position of each cell in the planar system w.r.t. the cell of interest.

Based on the information theory [6] we have that

$$I(x; y) = \mathcal{H}(y) - \mathcal{H}(z) = \log_2 \left( \frac{\det(\Lambda_y)}{\det(\Lambda_x)} \right)$$

(3.24)

where $\mathcal{H}(\cdot)$ stands for the entropy of the respective variable vectors, $\Lambda_x$ is the covariance matrix of the noise vector and $\Lambda_y$ is the covariance matrix of the output vector which for a fixed channel matrix $H$ is described by:

$$\Lambda_y = E[yy^\dagger] = H E[xx^\dagger] H^\dagger + E[zz^\dagger] = H \Lambda_x H^\dagger + \Lambda_z$$

(3.25)

with $\Lambda_x$ defined as the covariance matrix of the input vector, i.e. $\Lambda_x \triangleq E[xx^\dagger]$.

The maximum per-cell rate for a HR uplink system is achieved when all UTs are allowed to transmit all the time at their maximum transmit power (wideband-CDMA scheme.
3.7. Rise over Thermal Definition

presented in [27]), and in this case the per-cell capacity will be given by:

\[
C_{\text{cell}} = \lim_{N \to \infty} E_{\mathbf{H}} \left[ \frac{1}{N} \log_2 \left( \frac{\det(\Lambda_{\mathbf{H}})}{\det(\Lambda_{\mathbf{H}})} \right) \right] = \lim_{N \to \infty} E_{\mathbf{H}} \left[ \frac{1}{N} \log_2 \frac{\det \left( \mathbf{H} \mathbf{A}_{\mathbf{x}} \mathbf{H}^\dagger + \sigma^2_{\mathbf{I}} \mathbf{I}_{N \times N} \right)}{\det(\sigma^2_{\mathbf{I}} \mathbf{I}_{N \times N})} \right]
\]

\[
= \lim_{N \to \infty} E_{\mathbf{H}} \left[ \frac{1}{N} \log_2 \left( \frac{\det \left( P_{\max} \mathbf{H} \mathbf{1}_{N \times K} \mathbf{H}^\dagger + \sigma^2_{\mathbf{I}} \mathbf{I}_{N \times N} \right)}{\det(\sigma^2_{\mathbf{I}} \mathbf{I}_{N \times N})} \right) \right]
\]

\[
= \lim_{N \to \infty} E_{\mathbf{H}} \left[ \frac{1}{N} \log_2 \left( \frac{P_{\max} \mathbf{H} \mathbf{H}^\dagger + \mathbf{I}_{N \times N}}{\sigma^2_{\mathbf{I}} \mathbf{I}_{N \times N}} \right) \right] \tag{3.26}
\]

where the expectation \( E_{\mathbf{H}} [\cdot] \) is taken over all the fading realizations, \( \Lambda_{\mathbf{x}} = E [\mathbf{x} \mathbf{x}^\dagger] \) = \( P_{\max} \mathbf{I}_{N \times K} \) since independent inputs are assumed from the \( NK \) UTs and \( \sigma^2_{\mathbf{I}} \) is the noise power assumed to be the same at every BS. The notion \( N \to \infty \) stands for the case where the number of cells is large enough so as to neglect the edge effects (large or toroidal system).

Consider now the noise normalised to unit power, i.e. \( \sigma^2_{\mathbf{I}} = 1 \). Jensen's inequality provides an upper bound for the per-cell capacity of the system as:

\[
\lim_{N \to \infty} \left( \frac{1}{N} \log_2 \left( \det E_{\mathbf{H}} (\Lambda_{\mathbf{Y}}) \right) \right) \geq \lim_{N \to \infty} E_{\mathbf{H}} \left[ \frac{1}{N} \log_2 \det(\Lambda_{\mathbf{Y}}) \right] \tag{3.27}
\]

Assuming that the number of UTs per-cell is growing large for a fixed number of cells, \( KN \) tends to infinity. In this case, the law of large numbers, that describes the long-term stability of the elements of the covariance matrix when the number of UTs per-cell is large, ensures that the upper bound presented above is tight [27] (see Appendix A).

Under the above consideration, a tight upper bound can be provided for the per-cell capacity of the HR cellular uplink as:

\[
C_{\text{cell}} = \lim_{N \to \infty} \left( \frac{1}{N} \log_2 \left( \det E_{\mathbf{H}} \left[ \frac{P_{\max} \mathbf{H} \mathbf{H}^\dagger + \mathbf{I}_{N \times N}}{\sigma^2_{\mathbf{I}} \mathbf{I}_{N \times N}} \right] \right) \right) \text{ for } K \to \infty \tag{3.28}
\]

### 3.7 Rise over Thermal Definition

In this section the RoT is presented as a parameter that can capture all the parameters that affect the capacity of a cellular uplink system using a HR. In Chapter 5 that notion of RoT is extended to the case of the clustered system. The motivation of considering
the RoT as the unified control parameter for system capacity is first discussed. Then, it is analysed how the multi-user capacity of the cellular system can be interpreted as a function of RoT at each BS of a symmetric cellular system. The well-known results of Wyner [25] and Somekh-Shamai [39] are examined in this context. Finally, some interesting findings of this work are presented, overlaid on the single curve to illustrate the relevance to the single-user information theoretic capacity case and to gain some useful insights.

3.7.1 Motivation for Defining RoT

In the practical engineering design of cellular systems, the main figure of merit that determines the capacity of a channel, is the Signal to Interference-plus-Noise Ratio (SNIR) at the BS receiver, given as

\[ \text{SNIR} \triangleq \frac{P_r}{\sigma^2 + I} \]  (3.29)

where \( P_r \) is the received power at the BS of interest, \( \sigma^2 \) is the thermal AWGN at the receiving BS and \( I \) is the inter-cell and intra-cell “interference” received from other UTs of the system. However, in the information-theoretic analysis of HR cellular uplink systems, the main figure of merit that determines the per-cell capacity (at any BS) will be the RoT:

\[ \text{RoT} \triangleq \frac{\sum_i h_i P_{\text{max}}}{\sigma^2} \]  (3.30)

which is defined as the ratio of the total power received from all UTs to a BS (assuming that all UTs in the system transmit at their maximum allowable power constraint, \( P_{\text{max}} \)) over the thermal noise [40]. The factor \( h_i \) denotes the relative channel attenuation experienced by the transmitted signal of each UT until it reaches the receiver. The numerator term \( \sum_i h_i P_{\text{max}} \) is the total received signal power (signal power received from UTs within the coverage of the BS in consideration and also the power of the signals intended for the other BSs of the system). Splitting the numerator into intra-cell desired, \( P_r \), and inter-cell desired (or conventionally termed “interference”) signal, \( I \), RoT can be expressed as:

\[ \text{RoT} = \frac{P_r + I}{\sigma^2} \]  (3.31)
3.7. Rise over Thermal Definition

which shows that the information theoretic approach of using a central processor that jointly decodes all the received signals has the potential of converting the conventionally harmful interference into a factor that increases the figure of merit RoT by moving the interference term from the denominator to the numerator.

Note that the numerator of (3.30) is a direct function of the transmit power of the UT. Hence we can define the ratio

\[ \gamma \triangleq \frac{P_{\text{max}}}{\sigma^2} \]  

(3.32)

and can also use this ratio as the figure of merit. With this definition incorporated, the RoT is given as:

\[ \text{RoT} = \sum_i h_i \gamma \]  

(3.33)

The main reasons that SNIR does not constitute an appropriate figure of merit for information-theoretic capacity analysis at a HR system are:

- Inter-cell interference is now considered as useful received power as the signals from UTs in the system are not harmful any more and thus the term \( I \) cannot be used in the denominator when a joint decoder is considered.

- Since there is no harmful interference considered, the UTs should constantly transmit with the maximum available power \( P_{\text{max}} \). In this context, the transmit power \( P_{\text{max}} \) remains fixed for all the UTs, whereas the received power at each BS differs for each UT. In this direction, the per-cell capacity can be calculated as a function of \( P_{\text{max}} \), which is a fixed system parameter, common for all the UTs of a cell.

It shall be noted that the problem of finding the per-cell capacity of a cellular system can be greatly simplified by focusing on the single BS receiver and its RoT. Due to the symmetry of the problem (ignoring the system edge effects) all BS receivers are identical and system capacity is simply the per-cell capacity times the number of cells. The mathematical formulation in the next sections backs the heuristic idea described here.
3.7. Rise over Thermal Definition

3.7.2 Uplink Capacity as a Function of RoT

Following the information theoretic approach analysed in Section 3.6, the per-cell uplink capacity is given by introducing $\gamma$ into (3.28):

$$C_{\text{cell}} = \lim_{N \to \infty} \left( \frac{1}{N} \log_2 \left( \det \mathbb{E}_H [I + \gamma H H^\dagger] \right) \right) \quad (3.34)$$

So, the problem reduces to characterize the expected value of expression $I + \gamma HH^\dagger$ for the given statistical properties of fading coefficients and path loss factors.

From (3.21) it is obvious that every element of the overall channel matrix $H$ is a product of a path loss coefficient with a fading coefficient. The coefficients of the path loss function $\varsigma$ defined in 3.3.1 though, are a function of UTs positions. As all these coefficients are dependent on distances between transmitter and receivers whose positions can be considered fixed at any time, path loss matrix $\Sigma$ (presented in (3.21)) is a deterministic matrix for a given snapshot of the system. On the other hand, each received signal experiences an independent fading coefficient, $g$. Hence, the statistical properties of these coefficients determine the capacity.

From (3.21) $HH^\dagger$ can be analysed as

$$HH^\dagger = (\Sigma \odot G) (\Sigma^T \odot G^\dagger) = \sum_{i=1}^{KN} \left( \Sigma \Sigma^T \right)_i \odot (GG^\dagger)_i \quad (3.35)$$

where any matrix $\left( MM^\dagger \right)_i$ is the outer product of the $i^{th}$ column of matrix $M$ with its conjugate transpose.

Considering uniform phase distribution on the specular path as discussed in Section 1.3.1, all the off-diagonal entries of $\mathbb{E}_H [HH^\dagger]$ will be zero while all the diagonal entries will be of the same value when $K \to \infty$ (since $m_{gg}$ is normalised to unity and the system is assumed to be symmetric; the system statistics for each BS point of view are the same over a period of time). Hence, referring to any $j$ diagonal element of $\mathbb{E}_H [HH^\dagger]$, we have that:

$$\mathbb{E}_H [h_j^T h_j] = \sum_{i=1}^{KN} \varsigma_i^2 \quad (3.36)$$

where $h_j^T$ is the $j^{th}$ row of $H$ matrix and $\varsigma_i$ is the path attenuation factor between the BS of interest and the $i^{th}$ UT in the system. As said, all the diagonal entries are same.
and each entry corresponds to a specific BS in the system. This simplifies (3.28) to the following form

\[
C_{\text{cell}} = \lim_{N \to \infty} \frac{1}{N} \log_2 \left( \det \left[ \left(1 + \gamma \sum_{i=1}^{KN} \psi_i^2 \right) I_{N \times N} \right] \right) \tag{3.37}
\]

Noting that the determinant of a matrix is the product of its eigenvalues and that the eigenvalues of a diagonal matrix are its diagonal entries we have the following simplifications:

\[
C_{\text{cell}} = \lim_{N \to \infty} \frac{1}{N} \log_2 \left( \prod_{i=1}^{N} \left(1 + \gamma \sum_{j=1}^{KN} \psi_j^2 \right) \right)
\]

\[
= \lim_{N \to \infty} \log_2 \left(1 + \gamma \sum_{i=1}^{KN} \psi_i^2 \right)
\]

\[
= \lim_{N \to \infty} \log_2 (1 + \text{RoT}) \tag{3.38}
\]

where

\[
\text{RoT} \triangleq \gamma \sum_{i=1}^{KN} \psi_i^2 \tag{3.39}
\]

as discussed in the motivation section. This result suggests that the capacity formulations of the cellular uplink system all fall on the same graph and the different system parameters define the range of operation on this graph by controlling the RoT of each BS in the system.

### 3.7.2.1 Revisiting well-known results under the RoT context

The closed form formula for per-cell capacity, derived by Somekh-Shamai [39], for the uplink of the linear cellular system for a Rayleigh (or in general zero-mean) flat fading environment is given by replacing \( \gamma \) in (2.29):

\[
C_{\text{cell}} = \log_2 (1 + \gamma (1 + 2\alpha)) \tag{3.40}
\]

where \( \alpha \) is the fraction of power from the adjacent cell UTs reaching the BS of interest. Similarly, for the planar system by replacing \( \gamma \) in (2.30) we have:

\[
C_{\text{cell}} = \log_2 (1 + \gamma (1 + 6\alpha)) \tag{3.41}
\]
It is observed that these results also conform to the definition and analysis for RoT. The term $\gamma(1+2\alpha)$ for the linear system and $\gamma(1+6\alpha)$ for the planar system represents the total received power at each BS antenna as there are 2 and 6 "interfering" neighbour cells in linear and planar system respectively.

3.7.3 Parameters Controlling RoT

As capacity is directly proportional to the RoT in a HR uplink system, it is always useful to increase RoT. According to the above, the value of RoT as defined for the HR cellular uplink depends on the following parameters:

Path Loss Exponent: Taking the modified path loss model as an example, the attenuation factor $c_i$ depends on the path loss exponent and with increasing value of the exponent the sum of all $KN$ values of $c_i^2$ will decrease and hence, from (3.39) the total RoT will decrease.

Distribution of UTs: With the same context, different UT distributions over the cells will change the value of $\sum_{i=1}^{KN} c_i^2$ and hence the RoT.

Extent of the System: As the path attenuation factor also inversely depends on the transmitter-receiver separation, putting the same number of UTs on a wider extent increases the transmitter-receiver separations, on the whole, and this results in reduction in the $c_i^2$ values and hence, the total RoT will decrease.

Number of UTs per-cell: Having a larger number of UTs will increase the total power received on the system and hence the RoT.

Transmit Power Constraint: As $\gamma \triangleq \frac{P_{\text{max}}}{\sigma_s^2}$, an increase in $P_{\text{max}}$ will increase the RoT, according to (3.33).

Noise at the Receiver: Similarly, as $\gamma \triangleq \frac{P_{\text{max}}}{\sigma_s^2}$, a decrease in $\sigma_s^2$ will increase the RoT.
3.7.4 RoT Curve

Figure 3.5 illustrates the relation of capacity with the RoT in a logarithmic and a linear scale. It is observed that alike the single-user case (that Shannon described defining the Signal-to-Noise-Ratio, SNR), the multi-user system capacity increases linearly with RoT in a logarithmic scale. The shaded area in the figure represents all the system possible sum rates while the limits of these areas stands for the ergodic sum rate capacity of the respective system.

3.8 Summary

This chapter has described the generic cellular uplink system model using a global HR. Furthermore, the formulations of the important effects of the multi-user channel (path loss, flat fading, shadowing and noise) have been provided along with a fitting example with empirical models to produce meaningful practical numerical results. Subsequently,
the information theoretic capacity analysis of the HR cellular uplink was analysed. In that direction, the notion of RoT was introduced as a figure of merit that captures all the parameters that affect the capacity of such a system. Finally, it is shown how capacity can be interpreted as a function of RoT at each BS and the various practical parameters that are incorporated into the RoT parameter are discussed.
Chapter 4

Generic Cellular Uplink with Global Cooperation

In this chapter, the information theoretic capacity of the uplink of the cellular system by assuming a centralised processing for all BSs is investigated in depth. In Section 4.1, the modified path loss model (presented in Section 3.3) is considered along with variable cell size (variable density of BSs) and a mean path loss approximation method is formulated. Various UT distributions are considered and it is shown how UT distribution can be modelled in the system analysis. In Section 4.2, considering the realistic generic multipath fading environment (Section 3.4), the analytical result for the per-cell capacity of the uplink is derived for a large number of UTs distributed over each cell. Furthermore, the general approach is extended to model the uplink of a sectorized cellular system and it is analysed how multiple antennas at BSs can improve the communication system capacity. To this end, the UTs are assumed to be served by perfectly directional receiver antennas, dividing the cell coverage area into perfectly non-interfering sectors. It is shown how the capacity is increased (due to degrees of freedom and directivity gain) in comparison to the single receiving antenna system and the asymptotic behaviour when the number of directive antennas per-BS grows large is investigated. Moreover, the capacity when the multiple antennas used for each BS are omnidirectional and uncorrelated (power gain on top of degrees of freedom gain) is found and compared to the sectorized case. Finally, Section 4.3 presents the theoretical
and the practical system results. Each parameter affecting the capacity of the sys-
tem is thoroughly investigated. All the numerical solutions are validated with Monte
Carlo simulations for random fading realizations and the results are interpreted for the
real-world system parameters.

4.1 Mean Path Loss Approximation Model

Firstly, the mean path loss approximation approach model is presented. The approach
is analysed based on a conventional cellular system geometry and three different types
of UT distribution over the cells are investigated with respect to their effect on the
path loss coefficients.

4.1.1 Cell and System Geometry

Consider a regular hexagonal cell with its geometry given in Figure 4.1. The side of the
regular hexagon is denoted by \( r_0 \) and the minimal radius (minimum distance from the
centre to the periphery) of the hexagon is \( r = r_0 \cdot \cos (\frac{\pi}{6}) \). Here, the Inter Site Distance
(ISD) can be defined as the distance between two adjacent BSs (ISD \( \triangleq 2r \)), which will
be extensively used later on. In that planar cellular system model, received power from
UTs in multiple tiers of cells around each cell is assumed as shown in Figure 4.2. The
irregular boundary of each tier can be approximated by an equivalent regular hexagon
with the length of its side given by:

\[
\ell_l = \sqrt{[(2l+1)\ell]^2 + \left(\frac{r_0}{2}\right)^2}
\]  

(4.1)

where \( l \) stands for the \( l^{th} \) tier from the cell of interest.

In general, the hexagonal (with side length \( \ell_l \)) boundary of any tier can be approximated
by an equivalent circular boundary, as shown in Figure 4.1. The equivalence is in the
sense that the average distance of all points on the perimeter of the two shapes (circle
and the hexagon) is same. The radius of such an equivalent circular boundary of any
tier, is given by:

\[
d_l = \frac{6}{\pi} \int_0^{\frac{\pi}{6}} \frac{\ell_l}{\cos \theta} \cos \left( \frac{\pi}{6} \right) d\theta
\]  

(4.2)
4.1. Mean Path Loss Approximation Model

Figure 4.1: Geometry of a regular hexagonal cell with side length of $r_0$, the circular equivalent of radius $d_0$ and the distance $D_{l,k}$ of a UT in a cell $m$ from a BS in a cell $n$. Cell $m$ is considered to be at the $l^{th}$ tier of cells w.r.t. cell $n$.

Figure 4.2: Multiple tiers of cells around a cell. The irregular boundary of each tier can be represented by an equivalent regular hexagon and the latter by a circular boundary.
4.1. Mean Path Loss Approximation Model

For evaluating the capacity under any UT distribution with large number of UTs, it is useful to group the UTs in each tier of cells and represent their squared path loss coefficients with an appropriate mean value, denoted by \( \bar{\sigma}^2 \) and defined as the mean squared path loss. This mean value is calculated by focusing on a single cell and averaging the path loss of all UTs in this cell, with reference to the receiver position. This average can be expressed as a function of the distance \( \bar{d}_l \) between the center of the cell in focus and the receiver at a cell in the \( l^{th} \) tier. Given a specific cell and a tier around it, in general, distance \( d_i \) slightly vary when considering different cells in the tier. However, distance \( d_i \) can be approximated from the inner and outer circular boundary of the \( l^{th} \) tier of cells (see Figure 4.2):

\[
\bar{d}_l \approx \frac{d_l + d_{l-1}}{2} \quad (4.3)
\]

4.1.2 UT Distribution and Mean Squared Path Loss

The mean squared path loss for the UTs in a cell will depend on the proximity (which tier the cell belongs to) of the cell to the receiver of interest and also on the UT distribution over the cell. The distance \( D_{l,k} \) of a UT in a cell in the \( l^{th} \) tier of cells from the receiver of interest is defined. With the help of Figure 4.1, one can prove that:

\[
D_{l,k}(\theta, s) = \sqrt{(\bar{d}_l - s \sin \theta)^2 + (s \cos \theta)^2} \quad (4.4)
\]

where \( s \) and \( \theta \) respectively define the radial and angular location of a UT, with respect to a BS as shown in Figure 4.1.

Three different cases of UT distribution are examined here:

1) **Uniform Distribution.** In this case, the UTs are assumed to be uniformly distributed over the planar area. The mean squared path loss for each of the \( K \) UTs in a cell which belongs in the \( l^{th} \) tier of cells from the receiver of interest is given by:

\[
\bar{\sigma}_{l,\text{uni}}^2 = \frac{1}{\pi d_0^2} \int_0^{d_0} \int_{-\pi}^{\pi} \frac{1}{(1 + D_{l,k}(\theta, s))^2} s^2 \, d\theta \, ds \quad (4.5)
\]

Uniform distribution represents a likely distribution in a real-world system when a large number of UTs is considered to be randomly placed over the system.
2) Truncated Cell-Centre Uniform Distribution. Here, the UTs are uniformly distributed around the centre of their cell and thus,

\[ \frac{\sigma^2_{\text{centre}-K}}{\pi} = \frac{1}{\pi \rho^2} \int_0^\rho \int_{-\pi}^{\pi} \frac{1}{(1 + D_{l,k}(\theta, s))^\eta} s d\theta ds \]  

(4.6)

where \( \rho \) (with \( 0 < \rho \leq d_0 \)) is the truncation radius around each BS in which the \( K \) UTs are distributed. Note that for values of \( \rho \) very close to zero (UTs in a cell are co-located with their BS), the mean squared path loss approaches to \( \frac{1}{(1+d_i)^\eta} \).

3) Truncated Cell-Edge Uniform Distribution. In this last case, the UTs are uniformly distributed on an annular segment close to the edge of their cell. We have,

\[ \frac{\sigma^2_{\text{edge}-K}}{\pi} = \frac{1}{\pi (d_0^2 - \rho^2)} \int_\rho^{d_0} \int_{-\pi}^{\pi} \frac{1}{(1 + D_{l,k}(\theta, s))^\eta} s d\theta ds \]  

(4.7)

where \( \rho \) (with \( 0 < \rho < d_0 \)) is the radial distance from the center of the cell to the boundary where the annular section (on which the UTs are distributed) starts. Note that for values of \( \rho \) very close to \( d_0 \) (all UTs are very close to the edge of the cell), the mean squared path loss can be assumed to be given by \( \frac{1}{\pi} \int_{-\pi}^{\pi} \frac{1}{(1 + D_{l,k}(\theta, s))^\eta} d\theta \).

Note that the transmitted signals from the UTs in the cell of interest also follow the same power-law path loss described above. For this case \( d_i = 0 \) and it follows from (4.4) that \( D_{l,k}(\theta, s) \) becomes \( s \) for the path loss calculations using the above analysis.

Note also that in limiting cases of the truncated distributions, Cell Center (CC) distribution occurs when all UTs are co-located with the center of the cell, with \( \rho \to 0 \) and Cell Edge (CE) distribution occurs with \( \rho \to 0 \) when all UTs are located on a circular line at the edge of each cell.

### 4.2 System Models and Capacity Analysis

In the following, the capacity of three different cellular models is analysed and compared. From the generic cellular model where all BSs use one omnidirectional antenna each to receive signals from UTs, we move to the sectorized case where a number of directional antennas at receivers end let the sites to receive from separate directions
providing coverage to the whole system. Moreover, the case where a number of omni-directional antennas is applied to each site is examined to provide a fair comparison with the sectorized case.

### 4.2.1 Generic Cellular Model

In Chapter 3-Section 3.4 it was observed that, for a generic multipath fading environment with uniformly distributed phase assumed at the specular path, the expectation of the covariance matrix of the output vector converges to a diagonal matrix. More specifically, if uniform UT distribution is assumed, a maximum of $L$ tiers of cells contributing to the received power for every receiver and consider that there are always $6 \cdot l$ cells in the $l^\text{th}$ tier of cells, the $N \times N$ expected output covariance matrix over a long enough sequence of system fading snapshots, given in (3.25), is:

$$
\mathbb{E}_H [\mathbf{A}_{\text{uni-K}}] = \left[ \sigma_z^2 + KP_{\text{max}} \left( \frac{2}{\sigma_0^{\text{uni-K}}} + \sum_{i=1}^{L} 6 \kappa_i^{2} \right) \right] \mathbf{I}_{N \times N}
$$

(4.8)

where $\sigma_0^{\text{uni-K}}$ denotes the mean squared path loss for the UTs inside the cell of interest.

Taking into consideration equations (3.26) and (3.28), the per-cell capacity is given by (for $K \to \infty$):

$$
C_{\text{uni-K}} = \lim_{N \to \infty} \frac{1}{N} \log_2 \left( \frac{\det \mathbb{E}_H [\mathbf{A}_{\text{uni-K}}]}{\det \mathbb{E}_H [\mathbf{A}_\text{z}]} \right)
$$

$$
= \log_2 \left[ 1 + \frac{KP_{\text{max}}}{\sigma_z^2} \left( \frac{2}{\sigma_0^{\text{uni-K}}} + \sum_{i=1}^{L} 6 \kappa_i^{2} \right) \right]
$$

(4.9)

Following the same procedure for the other two types of distribution, one can easily reach to similar expressions for the per-cell capacity:

$$
C_{\text{centre-K}} = \log_2 \left[ 1 + \frac{KP_{\text{max}}}{\sigma_z^2} \left( \frac{2}{\sigma_0^{\text{centre-K}}} + \sum_{i=1}^{L} 6 \kappa_i^{2} \right) \right]
$$

(4.10)

and,

$$
C_{\text{edge-K}} = \log_2 \left[ 1 + \frac{KP_{\text{max}}}{\sigma_z^2} \left( \frac{2}{\sigma_0^{\text{edge-K}}} + \sum_{i=1}^{L} 6 \kappa_i^{2} \right) \right]
$$

(4.11)
4.2.2 Sectorized Cellular Model

Consider now a 2D network of $N$, 3-sectoring sites, with $K = K/3$ UTs uniformly distributed into each sector area covered by its BS directive antenna. Label the directive antennas as $A$, $B$ and $C$ as shown in Figure 4.3. The receiver referring to antenna $A$ (sector-$A$-receiver) of a cell $n$ will receive signals from the shaded $120^\circ$ area illustrated in the same figure. Perfect directional antennas are assumed at the BSs which means that each antenna exclusively covers (receives signals from) one third of the system UTs. Hence, sector-$i$-received signal, at cell $n$, is given by:

\[ y_n^i = \sum_{m \in \mathcal{N}^n} \sum_{k \in \mathcal{K}_m} \left[ \left( \sqrt{G_{D,i}(S)} \cdot c_{m,k} \right) g_{m,k}^n \cdot x_{m,k} \right] + z_n^i \quad (4.12) \]

where $G_{D,i}(S)$ is the directivity power gain of each one of the total $S$ receiving antennas used at the BSs and the rest of the variables are defined as for the non-sector case (see Section 3.6). Since a comparison is aimed between the sectorized and the non-sectorized case for reasons of fairness it is considered that $1 \leq G_{D,i}(S) \leq S$ and the treatment of directivity gain is made separately from the other channel effects. In that case, the total power ($G_{D,i}(S) \frac{NK}{S}$) that can be received from each BS antenna at the sectorized model cannot exceed the power ($NK$) that can be received from the BS antenna at the non-sectorized model (i.e., $G_{D,i}(S) \leq S$) and at the same the combined received power ($G_{D,i}(S) NK$) of all $S$ BS antennas at the sectorized case cannot be less than the received power from the BS antenna at the non-sectorized case (i.e., $1 \leq G_{D,i}(S)$). The additional superscript also identifies the specific directive antenna at the receiver end. Set $\mathcal{N}^n$ is the subset of all cells that are in the coverage area of the sector-$i$-receiver of the cell $n$. Moreover, set $\mathcal{K}_m^n$ describes the subset of UTs in any cell $m$ that are in the coverage area of the sector-$i$-receiver of cell $n$ (shaded area in Figure 4.3).

It has to be noted here that directional antennas have been assumed receiving only in the horizontal plane, which is the fairest for comparing with the omnidirectional antenna case, without taking the vertical plane into consideration. Nevertheless, this assumption does not lead to a loss of generality since the antenna gain is defined straightforwardly as the antenna directivity times the factor representing the antenna efficiency and can be easily accessed in the following analysis according to any antenna scenario given. In our
4.2. System Models and Capacity Analysis

Figure 4.3: The sectorized cellular system model. The shaded area denote the area of receiving power for the sector-A-receiver at the cell of interest.

In this case, an efficient directional antenna for a 3-sector system is considered to have a gain equal to its directivity, which is 3 when taking into consideration only the horizontal plane. In the contrary to the previous best case scenario, when the directional antenna is not efficient at all, the gain will be 1. For more details on directive antennas the reader can refer to [41] and references therein.

The system output vector in this case can be written based on (4.12) as:

\[ y = \tilde{H} x + z \]  \hspace{1cm} (4.13)

where \( y = [y_1^A, y_1^B, y_1^C, y_2^A, \ldots, y_N^B, y_N^C]^T \) is the \( 3N \times 1 \) received signal column vector. Vector \( x = [\ldots, x_n^T, \ldots]^T \) is the concatenation of the transmitted signals of all UTs in all \( N \) cells to form a \( 3N K \times 1 \) column vector, with \( x_n^T = [\ldots, x_n, \ldots] \) denoting the row concatenation of the transmitted signals of the all \( 3K \) UTs in each cell \( n \). Finally, \( z \) is the \( 3N \times 1 \) noise column vector and \( \tilde{H} \) is the \( 3N \times 3N K \) overall system gain matrix which is given by:

\[ \tilde{H} = \sqrt{G_{D,(3)}} \tilde{\Sigma} \odot \tilde{G} \]  \hspace{1cm} (4.14)

where \( \tilde{\Sigma} \) is a deterministic \( 3N \times NK \) matrix that contains all the path loss coefficients.
of the system channels and \( \mathbf{G} \) is the \( 3N \times NK \) matrix of all fading coefficients. Based on
the channel definition (4.12) and using same UT ordering for the UTs within each cell
(like the raster scan method used in [27]) so as to specify the ordering of the elements
in \( \mathbf{x} \), the overall path loss matrix \( \mathbf{\Sigma} \) is a block-circulant matrix (in terms of its row-
vector elements) when considering all UTs symmetrically distributed over all cells. In
\( \mathbf{\Sigma} \) matrix, each row corresponds to a specific sector-receiver and the groups of three
rows to a specific cell.

Following the same procedure as in chapter 3 for the generic system, the capacity
will now be given from (3.28) by replacing the appropriate matrices. For the 3-sector
cellular system model presented above, it can be observed that the expectation of the
covariance matrix of the output vector is a block-circulant matrix with \( 3 \times 3 \) blocks.
The capacity of that specific sectorized cellular system will be formulated now.

**3-sector case**

According to the distribution of the UTs over the system, the mean squared path
loss for the \( K \) UTs in each cell belonging in the \( l^{th} \) tier of cells from the receiver of
interest is given by equations (4.5)-(4.7). Assume uniform UT distribution over the cells
and the generic fading environment with uniformly distributed phase at the specular
path. Consider also that each directive antenna at a BS is able to receive signals from
\( \frac{6K}{3} = 6 \cdot l \cdot K \) UTs at the \( l^{th} \) tier of cells. Hence, considering (3.25), the expected
output covariance matrix will be:

\[
\mathbf{E}[\mathbf{y}_{\text{uni}}] = \sigma_y^2 + \mathbf{A}_D(3)\mathbf{P}_{\text{max}} \left( \frac{\sigma_y^2}{\alpha_{\text{uni}}^2} + \sum_{l=1}^{L} \frac{6\epsilon_l^2}{\alpha_{\text{uni}}^2} \right) \mathbf{I}_{3N \times 3N} \quad (4.15)
\]

Consequently, the per-cell capacity is given for the sectorized case by:

\[
\mathcal{C}_{\text{uni},K} = 3\log_2 \left[ 1 + \mathbf{G}_D(3) \frac{\mathbf{P}_{\text{max}}}{\sigma_y^2} \left( \frac{\sigma_y^2}{\alpha_{\text{uni}}^2} + \sum_{l=1}^{L} \frac{6\epsilon_l^2}{\alpha_{\text{uni}}^2} \right) \right] \quad (4.16)
\]

as \( \mathbf{E}[\mathbf{y}_{\text{uni}}] \) has \( 3N \) exactly same eigenvalues. One can easily reach to similar
expressions for the per-cell capacity of the other two types of UT distribution.

**S-sector case**

As the number of directive antennas at each BS, \( S \), tends to infinity, it is considered
the fact that each directive antenna at a BS will be able to receive signals from \( \frac{6K}{S} \).
4.2. System Models and Capacity Analysis

UTs in the \( l^{th} \) tier of cells and that the expectation of covariance matrix of the output vector will have now \( SN \) exactly same eigenvalues. For uniformly distributed UTs, from (4.16), the asymptotic capacity, as \( S \to \infty \), is given by:

\[
\overline{C}_{\text{uni-K}}^* = \lim_{S \to \infty} \overline{C}_{\text{uni-K}}
\]

\[
= \lim_{S \to \infty} S \log_2 \left( 1 + G_D(s) \frac{KP_{\text{max}}}{S\sigma^2} \left( \frac{\overline{2\theta}_{\text{uni-K}}^2}{\sigma^2} + \sum_{l=1}^{L} \frac{6\overline{L}_{\text{uni-K}}^2}{\sigma^2} \right) \right)
\]

(4.17)

The above equation converges to:

\[
\overline{C}_{\text{uni-K}}^* = \frac{G_D(s)K P_{\text{max}}}{\sigma^2 \ln(2)} \left( \frac{\overline{2\theta}_{\text{uni-K}}^2}{\sigma^2} + \sum_{l=1}^{L} \frac{6\overline{L}_{\text{uni-K}}^2}{\sigma^2} \right) \text{ bits/sec/Hz, for } S \to \infty.
\]

(4.18)

and it can be seen that the information theoretic capacity becomes a function of the directivity gain of the receiving antennas at the BSs. It can be safely assumed that, due to hardware limitations (e.g. spatial implementation problems, signal correlation etc.), the directivity gain does not grow linearly with the number of antennas. Thus, the above result indicates that even for infinite number of directive antennas, the system capacity tends always to a finite limit. Note that this limit is of academic interest only as the theoretical physical model of perfectly directional and non-interfering antennas becomes exceptionally questionable past 6 sector-antennas.

4.2.3 MIMO Cellular Model

The analysis above can be readily extended for the case where the BS receive antennas are omnidirectional and uncorrelated. As noted before, under the assumption of multi-cell joint processing, the cellular system can be viewed as a giant MIMO system with distributed antennas. It has been shown in a separate work ([42]) that the capacity for the cellular uplink channel is upper bounded by the total number of transmit/receive antennas and also the number of UTs in the system. To be more specific, the uplink (or MIMO MAC) capacity depends on the smaller of the following two numbers: the total number of receive antennas (number of BS sites times the number of antennas at each site) and the total number of transmit antennas (Number of UT transmitters times the antennas at each transmitter). Thus, the number of antennas at the BS receiver
is more significant in defining the upper bound as usually the total number of receive antennas is the smaller of the two numbers mentioned above. This is due to the fact that we have several transmitting UTs for each receiving BS site. Thus, in the context of joint multi-cell processing, it is unrealistic to expect any substantial improvement in achievable rate by placing more antennas in the transmitters and it is more feasible to introduce more antennas at the BS sites. It shall be noted also here that in [42] equal power allocation is assumed for each of the multiple UT antennas and no beamforming is assumed assuming that CSI is not utilized at the UT end). Due to the “symmetric and large” system nature, it is expected that omnidirectional transmissions and equal power for each antenna will not be far from the capacity achieving approach, though this is an area for further work.

In that direction, $M$ antennas are considered at each BS and assuming uniform UT distribution and the generic multipath fading environment. The expectation of the covariance matrix of the output vector converges to a $MN \times MN$ diagonal matrix with $MN$ exactly same eigenvalues. Hence, the per-cell capacity for a large number of UTs $K$ is given by:

$$\hat{C}_{\text{uni-}K} = M \log_2 \left[ 1 + \frac{KP_{\text{max}}}{\sigma^2} \left( \frac{1}{\lambda_{\text{uni-}K}} + \frac{L}{\sum_{l=1}^{L} \lambda_{\text{uni-}K}} \right) \right]$$

(4.19)

In that case it can be observed that theoretically, for infinite number of antennas at each BS ($M \rightarrow \infty$) the capacity increases unbounded. It is noted that for a more detailed investigation on the capacity of MIMO cellular system the reader may refer to the separate work provided in [42].
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capacity equation (3.26) (keeping the expectation outside the log-det function) as:

\[ C_{\text{sim}} = \mathbb{E}_H \left[ \frac{1}{N} \log_2 \det \left( \frac{P_{\text{max}}}{\sigma_z^2} H_{\text{sim}} H_{\text{sim}}^\dagger + I \right) \right] \]  (4.20)

where \( H_{\text{sim}} \) is the channel matrix simulated for each snapshot. It is noted here that the expectation \( \mathbb{E}[-] \) in simulation is approximated by finding the average over a large number \( J \) (usually of the order of 100) of fading and UT distribution realizations, i.e.:

\[ C_{\text{sim}} = \frac{1}{N} \frac{1}{J} \sum_{j=1}^{J} \log_2 \det \left( \frac{P_{\text{max}}}{\sigma_z^2} H_{\text{sim}} H_{\text{sim}}^\dagger + I \right) \]  (4.21)

For fair comparison the simulation results do not include generation of shadowing. Nevertheless, as discussed in the previous chapter (Section 3.5), valid insights for a shadow fading scenario can still be provided from the modified path loss model by varying \( L_0 \) and \( \eta \).

The small difference between the simulations and the analytical results can be attributed to the fact that for finding the simulation capacity \( C_{\text{sim}} \), a finite system of \( \left[ \left( \sum_{i=1}^{L} 6i \right) + 1 \right] \) cells was considered for each case of \( L \), instead of the wrap-around toric model used for analysis. For \( L > 6 \) the simulations become computational intensive, as the system becomes too large to be simulated, unlike the analytical method which can provide results even for very large \( L \). In any case, as the system grows in size the edge effects should become even more negligible.

Note also that for the UT distribution comparison figures alongside the uniform spatial distribution (which is simulated by the large number of snapshots of random UT positions over the system), only the extreme cases of the truncated cell-center and cell-edge spatial distributions are presented (i.e. CC and CE distributions). For the CC UT distribution, \( \rho \) was considered to be equal to zero (all the UTs at the center of the cells), while for the CE UT distribution, \( \dot{\rho} \) was considered to be equal to \( d_0 \) (all the UTs at the edge of their cells).

4.3.1 Theoretical Results

For the theoretical analysis some normalisations are considered: 1) Normalised ISD, where the normalisation comes from the reference distance \( D_0 \) in equation (3.7) and
2) Normalised transmit power per-cell, where unity noise power is considered at the receiver.

In Figure 4.4 the per-cell capacity of the generic system model is plotted against the normalised ISD. The figure illustrates:

1. The behaviour of the capacity while the number of “interfering” tiers of cells considered changes, for uniformly distributed UTs. It can be seen that for high values of normalised ISD the number of the considered “interfering” tiers has no significant role on the capacity. On the contrary, as the system becomes more dense, the capacity increases with the number of the “interfering” tiers considered, suggesting that in a dense cellular system we can no longer use the model where the “interference” is considered to come only from the adjacent cells.

2. The effect of UT spatial distribution over the cells. For \( L = 5 \), it can be seen that for lower normalised ISD, the capacity for all spatial distributions coincide and reaches the maximum possible value. On the other hand, as the size of the cells grows larger, the capacity decreases and the different spatial distributions correspond to different system capacity. Specifically, for uniform and CE UT spatial distributions, capacity tends to zero. For CC UT distribution, capacity decreases with the size of the cell and reaches to a specific non-zero value. This is due to the fact that the UTs of the cell of interest will always be close to their BSs no matter how large the size of the cell will be. Note that, in all cases, uniform and CE UT distributions provide capacity very close to each other.

The effect of path loss exponent is illustrated in Figure 4.5 where the capacity is plotted against the number of “interfering” tiers considered. Generally, one can say that lower path loss exponent provides higher capacity. Considering uniformly distributed UTs and normalised ISD of unity, for a low path loss exponent \((\eta = 2)\) the capacity increases significantly when the number of “interfering” tiers increases and tends slowly to an asymptotic value. For higher path loss exponent, contrariwise, the capacity reaches close to its asymptotic value even when considering only one “interfering” tier.
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Figure 4.4: Capacity per-cell versus the normalised ISD for the three different UT distributions. The effect of considering different number of tiers of cells \((L = 1 - 6, 20)\) contributing to the total received power for each receiver is illustrated when uniform spatial distribution is assumed. Normalised transmit power = 20dB per-cell, \(\eta = 2\).

Figure 4.5: Capacity per-cell versus the number of “interfering” tiers considered around each cell for different path loss exponents \((\eta = 2, 3, 4)\). Uniformly distributed UTs. Normalised transmit power = 20dB per-cell and Normalised ISD = 1.
Figure 4.6: Capacity per-cell versus normalised transmit power per-cell for different density systems and UT distributions. Five tiers of “interference” for each cell ($L=5$) and a path loss exponent of two ($\eta = 2$) are considered.

In Figure 4.6, the capacity is plotted versus the normalised transmit power per-cell for different system densities and UT distributions. In general, it is noted that capacity increases with the transmit power per-cell. It can be seen also that for dense and normal density systems the capacity does not change significantly for different UT distributions. Especially, for these systems, for the cases of uniform and CC UT distributions the capacity is almost the same. On the other hand, for low density systems and CC UT distribution the capacity is significantly higher than for the other two distributions.

When plotting the capacity against the RoT one can say that a more general view of the capacity behaviour is obtained (Figures 4.7-4.9). From the illustrated graphs it is obvious that the density of the system and the UT distribution do not change the overall behaviour of the capacity but actually the “area of operation” of each specific system. That is an expected result considering the analysis in Chapter 3-Section 3.7 where capacity was shown to be a direct function of RoT (see also equation (3.38)). It is again verified here that the lower the density of the system is, the more crucial becomes the effect of UT distribution on the capacity.
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Figure 4.7: Capacity per-cell versus RoT for different UT distributions in normal density systems. $L = 5$ and $\eta = 2$.

Figure 4.8: Capacity per-cell versus RoT for different UT distributions in low density systems. $L=5$ and $\eta = 2$. 
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4.3.2 Practical Results

Consider now a practical scenario to apply the capacity analysis. The real-world system parameters provided in Table 3.1 (p.35) of Chapter 3-Section 3.3.2 are taken into account here. The effect of some variable system parameters independently on the capacity is first explored and after that it is presented how each system parameters move the system capacity “operational area” on the RoT curve.

4.3.2.1 Independent system parameters effect on capacity

Figure 4.10 presents the effect of UT distribution over the cells. It is observed that for higher capacity, UTs are preferred to be distributed close to the BS. Uniform and CE distributions do not differ much although the former provides always more capacity.

In Figure 4.11 the effect of the cell radius on the average per-UT capacity (averaged over the number of UTs per cell) is presented for low and high path loss exponent. It has to be noted that in reality, the path loss exponent varies with distance and hence,
4.3. Results, Numerical Simulations and Observations

Figure 4.10: Capacity per-cell versus cell radius for different UT distributions (Uniform, CC ($\rho \rightarrow 0$), CE ($\rho \rightarrow d_0$)). $L = 5$, $\eta = 2$, $K = 20$ UTs per-cell with transmit power of 200mW per-UT.

Figure 4.11: Average capacity per-UT versus cell radius for $\eta = 2$ and $\eta = 3.5$. Uniformly distributed UTs, $L = 5$, $K = 20$ UTs per-cell with transmit power of 200mW per-UT.
one can say that Figure 4.11 actually provides an upper and a lower bound for the capacity in that case.

Figures 4.13 and 4.14 present the effect of the number of UTs per-cell on the capacity. In Figure 4.13, where the per-cell capacity is plotted versus the cell radius, one can observe that by increasing the number of UTs per-cell, the capacity increases but with a much lower rate. Figure 4.14 comes to prove that it is preferable, for the average per-UT capacity point of view, to keep a relatively low number of UTs per-cell.

4.3.2.2 General Dependence on Rise over Thermal

As capacity is directly proportional to the RoT, it is always useful as it has already been noted to increase the RoT. One way that RoT can be increased is by increasing the transmitted power for each UT in the system (this is due to the fact that for a joint decoder, interference is not harmful in its effect on capacity). RoT can also be increased if the average loss over the links between the transmitter and the receiver can be reduced by increasing the density of the cells in the cellular systems (i.e. using smaller sized cells). Smaller value of path loss exponent, smaller noise power, larger number of UTs in each cell (effectively this means more transmitted power within each cell), and a distribution of UTs where UTs are closer to the BSs can also increase the RoT.

Figure 4.12 illustrates how the capacity-range for the system changes depending on the cell size (or ISD), the path loss exponent γ, the number of UTs in each cell K and the per-UT transmitted power $P_u$. It is noted that the capacity increases for a relatively low path loss exponent and for small cell size. Furthermore, it is shown that increasing the number of UTs increases the sum-rate capacity (assuming joint decoding of the signals). It can be also observe that a 3dB increase in the transmit power (from 100 to 200mW, represented on the graph with the lower and upper end of the corresponding range) does not have a significant effect on the capacity.

Figures 4.15-4.17 show in more detail the connection between capacity and RoT. Figure 4.15 illustrates how the capacity-range for the system changes depending on the ISD. It can be observed that as ISD decreases (and hence the density of the system
4.3. Results, Numerical Simulations and Observations

Figure 4.12: Capacity per-cell versus per-UT transmit power and versus RoT for different path loss exponents ($\eta = 2, 3.5$) and different ISDs (200m, 2Km, 6Km). Uniformly distributed UTs ($K = 5, 20, 100$) per-cell with transmit power varying between 100 – 200mW. For simulation results (circular points) a system with $L = 5$ is simulated.

Figure 4.13: Capacity per-cell versus cell radius for different number of UTs per-cell ($K = 5, 20, 100$). Uniformly distributed UTs, $L = 5$, $\eta = 2$ and transmit power of 200mW per-UT.
4.3. Results, Numerical Simulations and Observations

Figure 4.14: Average capacity per-UT versus the number of UTs per-cell. Uniformly distributed UTs, $L = 5$, $\eta = 2$, Cell radius = 1Km and transmit power of 200mW per-UT.

Figure 4.15: Per-Cell Capacity versus RoT and ISD for various number of per-cell UTs ($K = 10$, 20, 50), path loss exponents ($\eta = 2.5, 3.5$) and UT transmit power constraints ($P_u = 100$mW). For $\eta = 3.5$ the curve corresponding to $K = 20$ is shown.
4.3. Results, Numerical Simulations and Observations

Figure 4.16: Per-Cell Capacity versus RoT and UT transmit power constraint $P_u$ for various types of UT distributions (Uniform, CC distribution with $\rho \to 0$ and CE distribution with $\rho \to 0$) with $K = 20$ UTs per-cell. $\eta = 2.5$ and ISD = 3km.

increases) the system operates with higher capacity. Moreover, it is shown that low path loss exponent $\eta$, larger number of UTs $K$ per-cell and higher transmitted per-UT $P_u$, increases the per-cell capacity of the system. Figure 4.16 compares the capacity obtained by the three different types of UT distribution and provides a more detailed view of the effect of $P_u$ on capacity. CC UT distribution provides the highest capacity and this has been explained from the fact that the UTs will always be close to at least one BS no matter how large the size of the cells is. Furthermore, uniform UT distribution provides a higher capacity than the CE distribution but both are close to each other. Finally, Figure 4.17 illustrates how the capacity-range for the system may change depending on various system parameters. In that specific case, the cell size, the path loss exponent ($\eta$), the per-UT transmitted power ($P_u$) and the number of UTs in each cell ($K$) are varied to obtain the different capacity ranges.

4.3.2.3 Sectorization and MIMO

The Sectorization and MIMO techniques essentially make use of space diversity and increases the received power at the cell site. This increase is achieved by introducing a larger number of resources (antennas in this case) at each site as compared to the benchmark/generic system: system with single antenna at each cell site. In the model
4.3. Results, Numerical Simulations and Observations

Figure 4.17: Capacity per cell versus per UT Transmit Power and versus RoT for different path loss exponents ($\eta = 2, 3.5$) and different Inter Site Distances (200m, 2Km, 6Km). Uniformly distributed UTs ($K = 5, 20, 100$) per cell with transmit power varying between 100 – 200mW. For simulation results (circular points) a system with received power from five tiers around the cell ($L = 5$) is simulated.

considered here, the only difference between a sectorized and non-sectorized system with multiple antennas at each cell site is the use of directional instead of omnidirectional antennas. In the case of sectorized system the directional antennas have a directivity gain that comes at the cost of reduced angular coverage. Mathematically both approaches—MIMO and Sectorization—provide similar capacity if the directivity gain is accounted for. Figure 4.18 illustrates a theoretical case so as to show how capacity depends both on RoT and the number of the antennas at each BS. It can be observed that the number of BS antennas moves the RoT curve. Various cellular systems may have different capacity-range according to these two basic parameters. It is interesting to point for example that a system implementing three (omnidirectional or perfect directional) antennas at each BS will have similar capacity with another system which is able to achieve triple RoT in dB at each BS. It shall be noted here that the assumption of perfect directivity may be slightly unrealistic as compared to the more realistic nature of multiple antenna systems with omnidirectional antennas.

In a specific case where directivity gain is not considered, it was also observed that
4.3. Results, Numerical Simulations and Observations

Figure 4.18: Capacity dependance on both RoT and number of antennas at each BS.

The capacity can still be increased when the number of antennas increases (spatial diversity gain). However, the analysis suggests that this increase is bounded and in the asymptotic regime, even if we manage to ideally sector the system with a very large number of directive beams for each site, the capacity will converge to a finite number.

Figure 4.19 compares the capacity obtained by the generic, the sectorized and the MIMO system models. When the directivity gain is not accounted for (referred to as the worst case in the figure, e.g. $G_{D,(3)} = 1$), the significant improvement (of the order of number of directive antennas) in the sectorized case can be attributed to increased degrees of freedom due to larger number of receiver antennas (BS antennas in the uplink) in the cellular system. Use of omnidirectional antennas provides a higher capacity that is attributed to the power gain obtained when the antennas receive the signals from all directions. Same power gain can also be obtained when appropriate directivity gains are considered in the case of sectorized cellular system (referred as the “Sectorized System - best case” in the figure, i.e. $G_{D,(3)} = 3$). In the same figure, for the generic system model, the capacity for different UT distributions is illustrated to emphasise on the fact that the UT distribution has an important affect on capacity.
4.3. Results, Numerical Simulations and Observations

Figure 4.19: Capacity per-cell versus the ISD for the Generic, the Sectorized (3 directive antennas per-cell, with worst case diversity gain of 1 and best case diversity gain of 3) and the MIMO system (with 3 antennas at each BS) model. Path loss exponent $\eta = 2$, Uniformly distributed $K = 20$ UTs per-cell with transmit power of $200$ mW per-UT. For simulation results (circular points) a system with $L = 5$ is simulated.

When joint processing of all the receivers is considered for a cellular system, sectorization is not an optimum technique to obtain the highest capacity from the system when the directional antennas do not reach their maximum ideal performance. The reason is that the sectorization is essentially an interference avoidance technique which is not an optimum approach in the presence of a joint decoding receiver. As a result, using a MIMO system with same infrastructure and no sectorization should be preferred as this can provide higher system capacity when joint decoding is in operation. Nevertheless, the wide-spread deployment of sectorized cellular systems makes it an interesting question to quantify the capacity of such a system and in this thesis that question is addressed. Moreover, a starting point is provided for the analysis of the implementation of not omnidirectional antennas at the BSs.
4.4 Summary

The capacity of the planar cellular uplink was investigated. An average path loss approximation model was presented for the analysis of a planar system where every BS receives signals from the same cell and the surrounding cells (arranged in multiple tiers of cells around the receiver of interest). The size of the cells and hence the system density is modelled as a variable. Assuming a joint decoder at the BSs (HR scheme) a tight upper bound for the per-cell sum-rate capacity is provided. The generic cellular system is extended to compare various system scenarios and their effect on capacity. System with single antenna at each BS yields minimum capacity. Spatial degrees of freedom gain provides a higher capacity when a system with multiple receiving antennas is considered where each antenna is perfectly directional and the sites coverage is sectorized. A further gain due to increased received power is obtained when the multiple BS antennas are considered omnidirectional and uncorrelated with each other or when the directivity gain of the directional antennas is considered larger than unity. It is also shown that for the sectorized system, increasing the number of antennas for each site to a very high value, the capacity tends to a finite value which is formulated using asymptotic analysis of the system. Furthermore, various parameters of a practical system that have an effect on the unified parameter of RoT at each BS and hence, on the per-cell capacity are identified and analysed.
Chapter 5

Extension of the Classic Cellular Uplink to Incorporate Site Sectorization

In this chapter, the classic multi-cell-multi-user uplink model (first presented in [25] and later in [27]) is extended to incorporate the sectorization of sites. Based on the modified path loss model analysis it is shown that Somekh-Shamai’s model, although simplistic, can still provide useful insight on the behaviour of the system and thus, it is extended to incorporate sectorization. That model gives us the opportunity to exquisitely investigate the system channel matrix mechanics and in addition to better realize the importance of the multipath fading random phase of the specular component consideration. While studying the capacity of the planar cellular uplink using HR in the previous section, it was observed that various path loss factors $(q_f)$ can be used to approximate the inter-cell attenuated power from UTs in different tier cells from the cell of interest. As stated in the literature review, Wyner’s and Somekh-Shamai’s models were using a single attenuation factor to approximate the first-tier inter-cell received power for a fixed cell size system. Although this crudely simplistic model makes the results a loose upper bound on capacity which does not incorporate the path loss parameters in detail, since it is still in agreement with the model presented above, it could be a useful tool for investigating further the mathematical mechanics that take
place in a cellular system.

In that direction, a planar cellular system is modeled in which UTs are served by three perfect directional antennas at each BS, dividing the site coverage area into perfectly non-interfering sectors and the per-cell capacity is found when the system dimensions (in terms of number of cells) grow large. The UTs are assumed now to be co-located with their BSs. Assuming the global joint decoding of the signals received at the BS antennas (HR scheme), the information theoretic uplink capacity in the presence of multipath fading is found. To find the capacity, a technique is applied in order to obtain the eigenvalues of a block-circulant matrix with non-circulant blocks. It is shown how the capacity is increased in comparison to the non-sectorized case (single omnidirectional antenna system) and the asymptotic behaviour is investigated when the number of directional antennas grows large.

The rest of the section is organised as follows. In Section 5.1 an overview of the problem and the method used to tackle it, is given. In Section 5.2 the conventional sectorization channel model is presented and analysed in Section 5.3 by formulating an analytical form for the per-cell capacity, under various fading environments. In Section 5.4 the special realistic case of uniform phase offsets at the specular path is analysed and the limit for the maximum capacity of the system is obtained when the number of site antennas grows large. In Section 5.5 the analytical and simulation results are presented and capacity obtained with and without sectorization is compared. Finally, in Section 5.6 the findings are summarized.

5.1 Overview of Method to Obtain Capacity

The problem of finding the information theoretic capacity of the cellular system examined in this section is reduced to a problem of finding the eigenvalues of a matrix with large dimensions. This matrix is the result of the multiplication of the channel matrix with its conjugate transpose. The system models without sectorization yield circulant matrices. The asymptotic eigenvalue distribution of these circulant matrices can be found with a straightforward procedure (by simply using discrete Fourier transform of the first row of the matrix) and many books and papers which show and prove this
procedure have been published. There are also known methods that can be used for finding the asymptotic eigenvalue distribution of a block circulant matrix with circulant blocks. The problem with sectorization is slightly more complex. The resultant matrix is an infinite block circulant matrix with non-circulant blocks. Hence, the eigenvalue distribution of such a matrix cannot be determined following a straightforward procedure, but a solution does exist. The large block circulant matrix can be transformed to a low dimension finite Hermitian one. Figure 5.1, provides an overview of how this problem can be solved. After going through the transformations shown in the diagram, it is finally only needed to calculate the real eigenvalues of a finite Hermitian matrix.
5.2 Sectorization Channel Model

A hexagonal cellular array model is assumed. Consider a network of $N$ (with $N \triangleq N^2$) 3-sectoring sites and $K = K/3$ transmitters in each cell area sectored by its BS antennas, where $K$ is the number of UTs per-cell. The model of the sectoring site is illustrated in Figure 5.2(a). The sector-$A$-receiver of cell 0 will receive signals from an 120° shaded area as shown in Figure 5.2(b). Assume that only UTs within one-tier of cells contributing to the received power for each receiver (the BS at the cell of interest can “hear” only UTs in that cell and in its adjacent cells) and perfect directional antennas at the BSs (the antennas at each BS have no correlation with each other). The UTs are assumed to be uniformly distributed around and very close to their corresponding BS (co-location case). A system of large dimensions is considered, i.e. $N \to \infty$, to avoid edge effects at the matrices.

According to the above, the sector-$A$-received signal is the sum of the transmitted signals from same-cell UTs in the area covered by that sector-receiver plus, a constant $\alpha (0 \leq \alpha \leq 1)$ times the sum of the transmitted signals from the UTs at the shaded area of the first tier, as specified in Figure 5.2(b), each one multiplied by the corresponding multipath fading coefficients. Hence, sector-$A$-received signal in a cell $(m, n)$ is given by:

$$y^{(m,n),A} = \sum_{k=1}^{R} x^{(m,n),A,k} + \alpha \sum_{j \in \{A,B,C\}} \sum_{k=1}^{R} x^{(m+1,n),j,k}$$

$$+ \alpha \sum_{j \in \{A,B,C\}} \sum_{k=1}^{R} \bar{a}^{(m,n),A}(m+1,n+1)_{j,k} + \bar{a}^{(m,n),A} \tag{5.1}$$

where $\bar{a}^{(m,n),i}_{(\tilde{m},\tilde{n}),j,k} = \bar{g}^{(m,n),i}_{(\tilde{m},\tilde{n}),j,k} \bar{z}^{(m,n),j,k}$ and subscripts $(\tilde{m},\tilde{n}), j$ identify the cell and its sector-$j$-area in which the transmitting UT are located, $k \in \{1, \ldots, \tilde{K}\}$ identifies the UT index and superscript $(m, n), i$ identifies the sector-receiver $A, B$ or $C$ of cell $(m, n)$.

Furthermore, $g, x, y, z$ stand for fading coefficients, complex Gaussian inputs, outputs and normalised to unit power AWGN with $z \sim \mathcal{CN}(0,1)$ respectively. Note that all the complex fading coefficients are normalized to unit power and when viewed as random.

\[\text{It is noted that in this section, a 2D cell indexing will be used to provide a more thorough investigation on the channel matrices.}\]
5.2. Sectorization Channel Model

Figure 5.2: (a) Model of cell with 3 site directional antennas. (b) A cell with its six neighbouring cells.

Complex processes are circularly symmetric i.i.d. Gaussian, strictly stationary and ergodic. It is also assumed that each UT has a maximum power constraint \( P_{\text{max}} \), i.e. \( \mathbb{E} \left[ |x|^2 \right] \leq P_{\text{max}} \). The same discussion can be held for the other two directive antennas of each site.

The output vector of all the received signals in the system can be written in matrix form as:

\[
y = \mathbf{H} \mathbf{x} + \mathbf{z} \tag{5.2}
\]

where \( y = \ldots, y^{(m,n),A}, y^{(m,n),B}, y^{(m,n),C}, \ldots \) is the \( 3N \times 1 \) received signal column vector, \( \mathbf{x} = \ldots, x^{(m,n),A,k}, x^{(m,n),B,k}, x^{(m,n),C,k}, \ldots \) is the concatenation of the transmitted signals of all the UTs to form a \( 3NK \times 1 \) column vector, \( \mathbf{z} \) is the \( 3N \times 1 \) noise column vector, and \( \mathbf{H} \) is the \( 3NK \times 3NK \) system channel matrix.

The representation of the cellular system as a rectangular array (Figure 5.3) is more tractable. According to Wyner [25] this can be done by scaling and rotating the structure illustrated in Figure 5.2(b). The nodes of the rectangular array are indexed by \((m,n)\), where \( m \) and \( n \) is the row and the column number respectively, and they represent the BSs of the cells. A connecting line in the figure indicates that the UTs in those cells will "interfere" with the sector-A-receiver of cell 0. Similar pattern exists for the other two sector-receivers of cell 0.
5.3. Capacity Analysis with Sectorization

Four $3K \times 1$ vectors are defined as:

$$
\mathbf{g}_{(m,n),i}^{(m,n)} \triangleq \left[ \begin{array}{c}
\mathbf{g}_{(m,n),A}^{(m,n)} \\
\mathbf{g}_{(m,n),B}^{(m,n)} \\
\mathbf{g}_{(m,n),C}^{(m,n)}
\end{array} \right]^T, \\
\mathbf{g}_{(m,n),i}^{(m,n)} \triangleq \left[ \begin{array}{c}
\mathbf{g}_{(m,n),A}^{(m,n)} \\
\mathbf{g}_{(m,n),B}^{(m,n)} \\
\mathbf{g}_{(m,n),C}^{(m,n)}
\end{array} \right]^T, \\
\mathbf{g}_{(m,n),i}^{(m,n)} \triangleq \left[ \begin{array}{c}
\mathbf{g}_{(m,n),A}^{(m,n)} \\
\mathbf{g}_{(m,n),B}^{(m,n)} \\
\mathbf{g}_{(m,n),C}^{(m,n)}
\end{array} \right]^T.
$$

where $\mathbf{g}_{(m,n),j}^{(m,n)}$ is the $K \times 1$ vector, which is the concatenation of the fading coefficients of signals from all $K$ UTs in sector-$j$-area of cell $(m,n)$ received by sector-$(i)$-receiver of cell $(m,n)$, i.e. $\mathbf{g}_{(m,n),j}^{(m,n)} = \left[ \begin{array}{c}
g_{(m,n),j,1}^{(m,n)} \\
\vdots \\
g_{(m,n),j,K}^{(m,n)}
\end{array} \right]^T$ and, $\mathbf{0}_{K \times 1}$ is the $K \times 1$ all-zero column vector. By using these vectors, expression (5.1) can be rewritten as:

$$
\mathbf{y}_{(m,n),i}^{(m,n)} = \left( \mathbf{g}_{(m,n),A}^{(m,n)} \right)^T \mathbf{x}_{m,n} + c \left( \left( \mathbf{g}_{(m,n),A}^{(m,n)} \right)^T \mathbf{x}_{m+1,n} + \left( \mathbf{g}_{(m,n),B}^{(m,n)} \right)^T \mathbf{x}_{m+1,n+1} \right) + \mathbf{z}_{(m,n),A}
$$

where $\mathbf{x}_{m,n}$ is the $3K \times 1$ column vector, which is defined as:

$$
\mathbf{x}_{m,n} \triangleq \left[ \begin{array}{c}
x_{(m,n),A,k} \\
x_{(m,n),B,k} \\
x_{(m,n),C,k} \\
\vdots
\end{array} \right]^T.
$$
5.3. Capacity Analysis with Sectorization

Similar expressions are obtained for the received signals at the other two BS antennas. Based on the channel definition (5.4) and using the raster scan method proposed in [27] the overall channel gain matrix $\mathbf{H}$ which will contain all the path loss and fading coefficients can be created. In $\mathbf{H}$ matrix, each row corresponds to a specific sector-receiver and each group of three rows to a specific cell. For example, each element vector $\mathbf{h}_{3i-2,j,i}$ of the $\mathbf{H}$ matrix indicates the received power to sector-$A$-receiver of cell $i$ from the UTs of cell $j$, where $3i-2$ is the row and $j$ is the column index.

5.3.1 Asymptotic Information Theoretic Capacity

Assuming large number of UTs ($K \to \infty$), the analysis in chapter 3 on the maximum reliable transmitted equal cell rate of a cellular system is considered. In the sectorization case the per-cell capacity of the system will be given again when all UTs are allowed to transmit at their maximum power by:

$$\hat{C} \equiv \lim_{N \to \infty} \left( \frac{1}{N} \log_2 \frac{\det \mathbb{E}_\mathbf{H}[\mathbf{H}^\dagger] \mathbb{E}_\mathbf{H}[\mathbf{H}]}{\det \mathbb{E}_\mathbf{H}[\mathbf{H}]} \right) \text{ for } K \to \infty (5.6)$$

where $\mathbb{E}_\mathbf{H}[\mathbf{H}^\dagger] = \mathbb{E}_\mathbf{H}[P_{\text{max}} \mathbf{H}^\dagger \mathbf{H} + I_{3N \times 3N}]$. To find the capacity, the expectation of $\mathbf{H}^\dagger$ over a large number of system fading realisations needs to be evaluated. When considering the UTs in cells $(m,n)$ and $(u,v)$, and respectively sector-$i$-receiver in cell $(m,n)$ and sector-$j$-receiver in cell $(u,v)$, we have:

$$\mathbb{E}_\mathbf{H}\left[ \left( \mathbf{g}_{(u,v),j}^{(m,n),i} \right)^\dagger \mathbf{g}_{(m,n),i}^{(m,n),i} \right] =$$

$$\mathbb{E}_\mathbf{H}\left[ \sum_{s \in \{A,B,C\}} \mathbf{g}_{(m,n),i}^{(m,n),i} \cdots \mathbf{g}_{(m,n),i}^{(m,n),i} \cdots \mathbf{g}_{(m,n),i}^{(m,n),i} \right] =$$

$$\mathbb{E}_\mathbf{H}\left[ \sum_{s \in \{A,B,C\}} \sum_{k=1}^{K} \mathbf{g}_{(m,n),i}^{(m,n),i} \mathbf{g}_{(u,v),j}^{(m,n),i} \cdots \mathbf{g}_{(u,v),j}^{(m,n),i} \right]$$

(5.7)

where $g_{(m,n),i}^{(m,n),i}$ represents any of the vectors defined in (5.3). As the fading coefficients
are assumed normalized to unit power:

$$E_{\mathbf{h}} \left[ (g_{(m,n),s,k}^{(m,n),i})^* \right] = E_{\mathbf{h}} \left[ g_{(m,n),s,k}^{(m,n),i} \right] = 1. \quad (5.8)$$

Furthermore, considering that two different fading coefficients are i.i.d, we have:

$$E_{\mathbf{H}} \left[ (g_{(m,n),s,k}^{(m,n),i})^* g_{(m,n),s,k}^{(m,n),j} \right] = m_{gg'} \quad (5.9)$$

where $m_{gg'}$ is the expected value of the product of two random different fading coefficients as given in Definition 3.1.

Based on the above assumptions and using the form of $\mathbf{H}$ matrix suggested above, it can be shown that the expectation of the covariance matrix of the output vector $(\mathbf{T} = E_{\mathbf{H}} [\mathbf{A}^*])$ is a Block Circulant matrix with $3 \times 3$ blocks and its first three rows, with $j$ defined as the column index, are given by:

$$\begin{array}{c}
\begin{aligned}
\text{row 1} & : 1 + \bar{\mathbf{K}} P_{\text{max}} (1 + 6\alpha^2) \\
\text{row 2} & : \bar{\mathbf{K}} P_{\text{max}} (3\alpha^2 + 2\alpha) m_{gg'} \\
\text{row 3} & : \bar{\mathbf{K}} P_{\text{max}} (3\alpha^2) m_{gg'}
\end{aligned}
\end{array} \quad (5.10)$$

where $\bar{\mathbf{K}} P_{\text{max}}$ is the total transmitted power by UTs in the cell area received by its
sector-receiver. The sets in (5.10) are defined in the following expression:

\[ A_0 = \{3(N + 1) - 1, 3(N + 1)\} \]
\[ A_1 = \{4, 3\bar{N} - 1, 3(N + 3), 3(2\bar{N} + 1) - 1, 3(2\bar{N} + 2) - 1, 3(2\bar{N} + 2), 3(2\bar{N} + 3), 3N - 2\} \]
\[ A_2 = \{6, 3(\bar{N} + 1) - 2, 3(\bar{N} + 1), 3(N + 2) - 2, 3(N + 2) - 1, 3(N - \bar{N}) - 2, 3[N - (\bar{N} - 1)] - 2, 3N - 1\} \]
\[ B_0 = \{6, 3[N - (\bar{N} - 1)] - 2\} \]
\[ B_1 = \{6, 3(\bar{N} + 2) - 1, 3(\bar{N} + 3), 3(N - 2\bar{N}) - 2, 3[N - (2\bar{N} - 1)] - 2, 3(N - \bar{N}) - 1, 3[N - (\bar{N} - 2)] - 2, 3[N - (\bar{N} - 2)]\} \]
\[ B_2 = \{4, 5, 3(\bar{N} + 1) - 2, 3(\bar{N} + 2), 3(N - \bar{N}) - 2, 3[N - (\bar{N} - 1)] - 1, 3(N - (\bar{N} - 1)], 3N - 1\} \]
\[ C_0 = \{3(N - \bar{N}) - 2, 3N - 1\} \]
\[ C_1 = \{3\bar{N} - 1, 3(\bar{N} + 1), 3[N - (2\bar{N} - 1)] - 2, 3[N - (2\bar{N})] - 2, 3[N - (\bar{N} + 1)] - 2, 3[N - (\bar{N} + 1)] - 1, 3[N - (\bar{N} - 1)], 3(N - \bar{N}) - 1\} \]
\[ C_2 = \{6, 3(\bar{N} + 1) - 1, 3(\bar{N} + 2), 3(N - \bar{N}) - 1, 3(N - \bar{N}), 3[N - (\bar{N} - 1)] - 2, 3N - 2, 3N\} \]

(5.11)

5.3.2 Derivation of the Analytical Capacity using Matrix Theory

The expectation of the covariance matrix is of the form:

\[ \mathcal{T}' = \text{circ}(T_{1,1}, T_{1,2}, \cdots, T_{N,N}) \]

(5.12)

where \( T_{n_1,n_2} \) (\( n_1, n_2 = 1, \cdots, \bar{N} \)) are 3 \times 3 matrices of entries \( t_{n_1,n_2}^{u,v} \) (\( u, v = 1, 2, 3 \) define element-position in the matrix). A methodology similar to the one presented in [43] for Block Toeplitz matrices is used, so as to find the eigenvalues of such a matrix.
Consider the associated matrix

\[
T = \begin{bmatrix}
T(t^{1,1}) & T(t^{1,2}) & T(t^{1,3}) \\
T(t^{2,1}) & T(t^{2,2}) & T(t^{2,3}) \\
T(t^{3,1}) & T(t^{3,2}) & T(t^{3,3})
\end{bmatrix}
\]

(5.13)

where \(T(t^{m,n})\) are the 3 \(\times\) 3 circulant matrices

\[
T(t^{m,n}) = \text{circ}(t^{m,n}_{1,1}, t^{m,n}_{1,2}, \ldots, t^{m,n}_{N,N}).
\]

(5.14)

If we let

\[
T(\theta_1, \theta_2) = \begin{bmatrix}
t^{1,1}(\theta_1, \theta_2) & t^{1,2}(\theta_1, \theta_2) & t^{1,3}(\theta_1, \theta_2) \\
t^{2,1}(\theta_1, \theta_2) & t^{2,2}(\theta_1, \theta_2) & t^{2,3}(\theta_1, \theta_2) \\
t^{3,1}(\theta_1, \theta_2) & t^{3,2}(\theta_1, \theta_2) & t^{3,3}(\theta_1, \theta_2)
\end{bmatrix}
\]

(5.15)

where \(t^{m,n}(\theta_1, \theta_2) = \sum_{m_1, n_2=-\infty}^{\infty} t^{m,n}_{m_1,n_2} e^{i2\pi(\theta_1 m_1 + \theta_2 n_2)}\) is the 2D Fourier transform of each \(T(t^{m,n})\) when \(K \gg 1\), it can be shown that for our case:

\[
T(\theta_1, \theta_2) = \begin{bmatrix}
A_1 & \mathcal{X} & \mathcal{Y} \\
\mathcal{X}^* & A_2 & \mathcal{Z} \\
\mathcal{Y}^* & \mathcal{Z}^* & A_3
\end{bmatrix}
\]

(5.16)

where the elements of the matrix are defined as (using \(w_{1,2} = 2\pi \theta_{1,2}\)):

\[
A_1 \triangleq 1 + 6\alpha^2 + 6\alpha^2 m_{gg} \cos(w_2) + 2\alpha m_{gg} \cos(w_1 + w_2)
\]

\[
A_2 \triangleq 1 + 6\alpha^2 + 6\alpha^2 m_{gg} \cos(w_1 + w_2) + 2\alpha m_{gg} \cos(w_1 + w_2)
\]

\[
A_3 \triangleq 1 + 6\alpha^2 + 6\alpha^2 m_{gg} \cos(w_1) + 2\alpha m_{gg} \cos(w_1 + w_2)
\]

\[
\mathcal{X} \triangleq (3\alpha^2 + 2\alpha) m_{gg} e^{-iw_1} + 3\alpha^2 m_{gg} \left( e^{-i(w_1 + w_2)} + e^{-i2w_1} + e^{i(w_2 - w_1)} \right)
\]

\[
+ \alpha m_{gg} \left( e^{iw_2} + e^{-i(w_1 + w_2)} \right)
\]

\[
\mathcal{Y} \triangleq (3\alpha^2 + 2\alpha) m_{gg} e^{-i(w_1 + w_2)}
\]

\[
+ 3\alpha^2 m_{gg} \left( e^{-i(w_1 + 2w_2)} + e^{-i(2w_1 + w_2)} + e^{-i(2w_1 + 2w_2)} \right) + \alpha m_{gg} \left( e^{-iw_1} + e^{-i2w_2} \right)
\]

\[
\mathcal{Z} \triangleq (3\alpha^2 + 2\alpha) m_{gg} e^{iw_2} + 3\alpha^2 m_{gg} \left( e^{-i(w_1 + 2w_2)} + e^{-i2w_2} + e^{i(w_1 - w_2)} \right)
\]

\[
+ \alpha m_{gg} \left( e^{iw_1} + e^{-i(w_1 + w_2)} \right)
\]

(5.17)
5.4. Capacity in multipath fading environment with uniformly distributed random signal phases

Now, letting $N$ tend to infinity, the 2D extension of Szego's theorem [44],[45] can be used to extend Theorem 3 of [43] so as to obtain an asymptotic expression of the capacity as:

$$\tilde{C} = \int_0^1 \int_0^1 \sum_{u=1}^3 \log_2 \left( \lambda_u(T(\theta_1, \theta_2)) \right) d\theta_1 d\theta_2,$$

where $\lambda_u(T(\theta_1, \theta_2))$ are the eigenvalues of the $3 \times 3$ Hermitian matrix $T(\theta_1, \theta_2)$. Note that 2D transforms are needed here as successive rows are obtained from a scanning of a 2D toric shift of the initial array.

5.4 Capacity in multipath fading environment with uniformly distributed random signal phases

Consider now the generic multipath fading model described in Chapter 3-Section 3.4. As discussed there, the mean value of the product of two independent fading realisations is zero when the received signals on the specular path are assumed to have uniformly distributed random offsets. Furthermore, the Rayleigh fading environment (no specular component) is, as discussed in Section 3.4.2, a zero mean fading environment ($m_{g_y} = 0$). By considering this property, the expectation of $\Lambda_y$ will result to a diagonal matrix.

5.4.1 3-sector case

If 3 directional antennas are considered at every BS that separate the area into 3 sectors, we have:

$$\mathbb{E}_H \left[ \Lambda_y \right] = \left[ 1 + \bar{K} P_{\text{max}} \left( 1 + 6a^2 \right) \right] I_{3N \times 3N}$$

(5.19)

The per-cell capacity can be found using (5.6) where the determinant of $\mathbb{E}_H \left[ \Lambda_y \right]$ is found by the product of the $3N$ eigenvalues. These are simply the diagonal entries of the matrix in (5.19). This gives:

$$\tilde{C}_{3-\text{sec}} = 3 \log_2 \left( 1 + \bar{K} P_{\text{max}} \left( 1 + 6a^2 \right) \right)$$

(5.20)

which is a strict upper bound (due to Jensen's inequality) for the per-cell capacity in our system.
5.4.2 5-sector case

Generalising 3-sector case to 5-sector case (and by keep considering no inter-sector interference), as the number of directive antennas $S$ tends to infinity it can be easily seen that

$$
\tilde{C}^* = \lim_{S \to \infty} S \log_2 \left( 1 + \frac{K P_{\text{max}}}{S} (1 + 6a^2) \right)
$$

where $K P_{\text{max}}$ is the total transmitted power per-cell. By solving the above equation it can be seen that, even for infinite number of antennas at each BS, the information theoretic per-cell capacity of the system tends to a finite limit which is found to be:

$$
\tilde{C}^* \approx \frac{K P_{\text{max}} (1 + 6a^2)}{\ln(2)} \text{ bits/sec/Hz, for } S \to \infty.
$$

5.5 Numerical Results and Observations

In the following figures, the theoretical results are presented and validated by running Monte Carlo numerical simulations to generate the random fading and path loss coefficients so as to create channel matrix ($\tilde{H}_{\text{sim}}$) and finding the long term average (over a large number $J$ of system realizations) capacity, $\tilde{C}_{\text{sim}}$ using:

$$
\tilde{C}_{\text{sim}} = \frac{1}{N J} \sum_{j=1}^{J} \log_2 \det \left( \frac{P_{\text{max}}}{\sigma^2} \tilde{H}_{\text{sim}}^\dagger \tilde{H}_{\text{sim}} + I \right)
$$

These numerical simulation results, that assume 21 UTs per-cell, are plotted as circles in Figure 5.4. The plots show a close match between the analysis and the simulation results a fact that implies that this amount of UTs per-cell is enough for the law of large numbers to hold here. For this plot, relatively low normalised transmit power (10dB per-cell - where the normalisation comes from the unit noise power) is assumed and it can be observed that the capacity increases with the increase in the attenuation factor $\alpha$. The lower curve refers to the case where the signal phases are synchronized at the receiver end ($m_{yy'} = 1$) while the upper curve refers to the cases of Rayleigh distribution fading environment or of a Rician distribution fading environment with random phase considered (unsynchronized signal phases at the receiver end) which, as
5.5. Numerical Results and Observations

Figure 5.4: Per-cell capacity versus the inter-cell attenuation factor $\alpha$. Normalised transmit power of 10 dB per-cell.

has been discussed in chapter 3, is a reasonable assumption for a multi-user practical scenario. Note that in the absence of any adjacent-cell receiving power ($\alpha = 0$) the two curves coincide.

Figure 5.5 compares the capacity obtained with and without sectorization. The improvement in sectorized case as discussed in the previous section can be attributed to the fact that a larger number of received signals corresponding to each transmission, are available at the HR as each cell site has multiple antennas in the sectorized system (e.g. diversity gain). Note that for fair comparison between the two models directivity gain of unity (no gain) for each sector-receiving antenna has been assumed. By this, the received power for both models is equalized and only the effect of diversity gain is investigated.

In Figure 5.6 it is observed that as the number of antennas is increased to a very large value, the capacity asymptotically tends to the value calculated in equation (5.22). It should be noted that in real-world systems, it is very difficult to implement a large number of ideal, non-interfering sector-antennas and the asymptotic behaviour shown in Figure 5.6 just provides an insight that by increasing the number of antennas, even in ideal world scenario, the capacity does not increase un-bounded.

The analysis is applied to the real-world scenario with the system parameters presented
5.5. Numerical Results and Observations

Figure 5.5: Comparing the per-cell capacity with and without sectorization for various values of $\alpha$. Normalised transmit power of 10dB per-cell.

Figure 5.6: Asymptotic behaviour of per-cell capacity with the increase in number of directive receivers at the BSs. $\alpha = 0.1$ and normalised transmit power of 20 dB per-cell.
5.6 Summary

An approach to formulate the information theoretic capacity for the uplink of a sectorized planar cellular system has been presented, in the presence of fading and by assuming joint decoding at the receivers (HR scheme) based on the initial work presented in [25] and [27]. It is shown that the capacity can be in general investigated by evaluating the eigenvalues of the block circulant covariance matrix which has non-circulant blocks. It is shown that the capacity is higher than that of equivalent system with no sectorization. It is also observed that the achieved rate is higher in the presence of non-LoS fast fading environment or when fading environment is Rician (LoS case) and received phases at the specular path are uniformly distributed. Furthermore, for these cases, it is shown that increasing the number of directive antennas in each BS...
to a very high value, the capacity tends to a finite value which is formulated using asymptotic analysis of the system.
Chapter 6

Cellular Uplink with Clustered Cooperation

In this chapter, the assumption of the BS global cooperation is put aside and coordinated processing only among the BSs that belong to a Cluster of cells is adopted. From the information theoretic system achievable sum rate point of view, the joint processing of all received signals in the system is optimal but this scheme is very hard to be implemented in real world nowadays. The large distances between BSs in most systems make it difficult in practice to have a high rate connecting backbone among all BSs. Thus, the investigation of a clustered joint processing scheme which decentralizes the joint decoding of all the UTs in the system becomes essential to define the capacity of the practical systems of today. The gap between the capacities of the two schemes poses a very interesting question to be answered.

The problem of finding the capacity becomes more complex when considering a clustered system than for the global cooperation case. For that reason, the analysis is simplified by starting in Section 6.1 with a more mathematically tractable system model; the 1D (linear) cellular system model. In that direction, in Section 6.1.2 the importance of UT power allocation on the capacity of the linear clustered cellular uplink is investigated. Moreover, in Section 6.1.3 important insights are obtained on capacity behaviour of that system under a number of clustering schemes. Furthermore, in Section 6.1.4 a time-scheduling scheme for cell rate fairness is presented while in Section 6.1.5 a
discussion on the “coupling” of the linear and the planar models is provided. After the in depth investigation of the linear case, in Section 6.2 the case of the planar clustered cellular uplink is explored and finally, in Section 6.3, the findings for the capacity of the clustered systems are summarized.

6.1 Clustered Cooperation in Linear Cellular Systems

The simplest cellular array is the linear or 1D array, that is a linear segment which contains BSs and UTs spaced across a linear grid. Here, instead of a global cooperation scheme (Figure 6.1), a clustered joint processing scheme is considered as illustrated in Figure 6.2. In the following sections, after presenting the system model details, a thorough analysis is provided on the capacity of the linear clustered system under the consideration of inter-cluster interference; transmitted signals from UTs that belong to cells in neighbouring clusters are causing interference to the BSs at cells in the cluster of interest (see Figure 6.3).

6.1.1 Linear Clustered Cellular System Model

A similar system model as the one presented in [46] is investigated focusing on the uplink of a cellular system. Consider a linear system of \( N \) cells, divided into \( N_Q \) smaller networks (clusters of cells) each with \( Q \) cells with \( QN_Q = N \) and \( Q \ll N \). The BSs are uniformly distributed across a linear grid, each one at the centre of each cell. Moreover, the UTs are distributed across the linear cells with \( K \) UTs per-cell. The cooperation among the BSs is limited only to those in cells that belong to the same cluster and hence a Joint Processor (JP) in each cluster of cells jointly decodes all the received signals from the UTs in that cluster. The modified path loss model and the generic multipath fading model with uniform distribution of received phase at the specular path introduced in Chapter 3 are also employed here.
6.1. Clustered Cooperation in Linear Cellular Systems

Figure 6.1: Linear cellular system model with global cooperation.

Figure 6.2: Linear clustered cellular system model.

Figure 6.3: Inter-cluster interference.
6.1.2 Inter-Cluster Interference and UT Power Allocation

The analysis begins without considering any isolation between clusters. In that direction, an inter-cluster interference allowance scheme is assumed where UTs in all clusters are allowed to exploit the full resources allocated to the system. In that case, the cells of every cluster experience the maximum possible inter-cluster interference as their BSs will be interfered by the transmitted signals from all the UTs in the other clusters of the system.

6.1.2.1 Necessity of UT Power Allocation Strategy

When a HR is available, as it has been discussed the optimal case from the sum-rate capacity point of view was to let the UTs transmit at their maximum allowed power. When considering a clustered system though, that strategy ceases to be the optimal one; At any time, a UT in a cluster $m$ may cause strong interference to the neighbouring cluster BSs. At the same time though, if one considers each cluster as a separate and independent cellular system, due to the BS cooperation within the clusters, the stronger the signal of a UT is to its respective BS, the more its contribution on the sum rate (sum of all UT rates in the cluster) will be according to the analysis performed in Chapter 4. Hence, we have a tradeoff between the useful contribution of the UT on its respective cluster sum rate and the harmful effect on the neighbouring cluster(s) (Figure 6.3). The above leads to a realisation that for UTs which at a specific time are in a disadvantageous signal state, it is may needed to allocate less power to maximize the overall cluster sum rate. Thus, a sum rate maximization problem occurs, under the power requirements of the system, that needs to be tackled.

6.1.2.2 Cluster Sum Rate Analysis

To facilitate sum rate analysis, symmetry among all clusters of cells is assumed. In that case, the analysis can be performed only for a cluster and the results will be valid for all the clusters of the cellular system. The received signal $y^{m,q}$ at the BS in cell $q$ of a cluster $m$ is the sum of the transmitted signals $x_{m,q,k}$ of all the UTs in the same
6.1. Clustered Cooperation in Linear Cellular Systems

A cluster of cells $\{q \in \{1, ..., Q\}\}$ appropriately scaled by each channel gain coefficient, plus the sum $z^{m,q}$ of the thermal AWGN ($n^{m,q}$ with $n \sim \mathcal{CN}(0, \sigma_0^2)$) and the interfering complex Gaussian inputs from UTs in cells outside the cluster of interest (inter-cluster interference). Thus, $y^{m,q}$ can be written as:

$$y^{m,q} = \sum_{q=1}^{Q} \sum_{k=1}^{K} c_{m,q,k}^{m,q} g_{m,q,k}^{m,q} x_{m,q,k} + z^{m,q}$$  \hspace{1cm} (6.1)

where $c_{m,q,k}^{m,q}, g_{m,q,k}^{m,q}$ stand respectively for the path loss and multipath fading coefficients between a transmitter $k$ in cell $q$ of cluster $m$ and the BS in cell $q$ of the same cluster.

The undesired signal $z^{m,q}$ can be written as

$$z^{m,q} = n^{m,q} + \sum_{m'} \sum_{q=1}^{Q} \sum_{k=1}^{K} c_{m',q,k}^{m,q} g_{m',q,k}^{m,q} x_{m',q,k}$$  \hspace{1cm} (6.2)

where $m' \neq m$. Inter-cluster interference, since it is a sum of complex Gaussian inputs, can be viewed as additional AWGN component at the BSs and hence, $z^{m,q}$ can still be considered AWGN with power given by

$$Z^{m,q} = \mathbb{E}[z^{m,q} (z^{m,q})^*] = \sigma_0^2 + \sum_{m} \sum_{q=1}^{Q} \sum_{k=1}^{K} \mathbb{E}\left[(c_{m,q,k}^{m,q} g_{m,q,k}^{m,q} x_{m,q,k})^2\right]$$  \hspace{1cm} (6.3)

where $\sigma_0^2$ is the power of the thermal AWGN at the receiver end.

Consider $x = [x_{m,1}^T, x_{m,2}^T, ..., x_{m,Q}^T]^T$ to be the $QK \times 1$ vector of the transmitted signals of all the UTs in cluster $m$, with $x_{m,q} = [x_{m,q,1}, ..., x_{m,q,K}]^T$ denoting the concatenation of the transmitted signals from the $K$ UTs in cell $q$, $z$ to be the $Q \times 1$ column noise vector and $H$ to be the overall $Q \times QK$ cluster channel gain matrix. Following the information theoretic analysis described in Chapter 3-Section 3.6 a tight upper bound for the maximum achievable sum rate of each cluster can be given by:

$$R = \mathbb{E}_H \left[ \log_2 \left( \frac{\det (HA_xH^\dagger + \Lambda_x)}{\det (\Lambda_x)} \right) \right] \approx \log_2 \left( \frac{\det E_H [HA_xH^\dagger + \Lambda_x]}{\det (\Lambda_x)} \right) \text{ for } K \to \infty$$  \hspace{1cm} (6.4)

where the expectation (indicated by subscript $H$) is taken over all the system fading realizations, $\Lambda_x$ stands for the covariance matrix of the respective vectors and the convergence is explained in Appendix A.
6.1.2.3 Variable UT Power

One way to reduce inter-cluster interference is to perform power allocation on the UTs of the system. Since the ergodic capacity of the system is studied, it is assumed that all UT signals during a long enough period of time experience all possible fading states and hence the parameter that defines the strength of a signal over that period of time is in the end the UT position on the cellular system. For that reason, a variable UT power allocation according to the position of each UT on its respective BS and cluster is considered. Since cluster symmetry as well as the same UT distribution at each cell (and hence, at each cluster) is assumed, the power allocation throughout each cluster will be the same as for any cluster. Hence, the cluster index at the power symbolisation can be omitted. The input covariance matrix, since independent inputs are assumed, will be a diagonal matrix of $QK$ elements depending on the power profile that is considered:

$$\Lambda_x = \text{diag} \left( \frac{QK}{P_{1,1} \ldots P_{q,k} \ldots P_{Q,K}} \right)$$

(6.5)

where $P_{q,k}$ denotes the power of UT $k$ in cell $q$ of any cluster. Furthermore, considering equation (6.3) and Corollary 3.4.2, the noise power matrix will be of the form:

$$\Lambda_z = \text{diag} \left( \sigma_0^2 + \sum_{m} \sum_{q=1}^{Q} \sum_{k=1}^{K} P_{q,k} \left( \theta_{m,q} \right)^2 \right)$$

(6.6)

By substituting (6.5) and (6.6) in (6.4) and recalling that the determinant of a matrix is the product of its eigenvalues we have (for $K \to \infty$):

$$R \simeq \log_2 \prod_{q=1}^{Q} \left[ 1 + \sum_{k=1}^{K} P_{q,k} \left( \xi_{m,q} \right)^2 \right]$$

(6.7)

To comprehend the derivation of (6.7) it is reminded that at the diagonal entries of $E_H [H \Lambda_x H^\dagger]$ the product that takes place is $E_g \left[ g_{m,q}^{m,q} \right] = E_g \left[ g_{m,q}^{m,q} \right] = 1$ while for the off-diagonal entries the expectation of the product of two different realisations of the fading coefficients is $E_g \left[ g_{m,q} g_{n,q}^* \right] = 0$ (see Theorem 3.3 - p.41), indicating that $E_H [H \Lambda_x H^\dagger]$ converges to a $Q \times Q$ diagonal matrix.
It shall be noted here that the modeling of the interference from UTs in out of cluster cells as a Gaussian interfering source stands for the worst case scenario from achievable rate point of view. The total AWGN (noise plus interference component) provides a diagonal covariance noise matrix (as given in (6.6)) whose expected value result to the highest possible value at the denominator in (6.7). This is though a fairly valid assumption for practical systems and also simplifies the already complex analysis of our problem. The consideration of a non Gaussian total noise could be the scope of a future extension of this work.

6.1.2.4 Mean Path Loss Approximation for Linear Systems

With Equation (6.7) the exact location of each UT is still needed to be known (and thus generated for the simulations) to calculate the sum rate of the linear system. For a closed form analytical formula on the sum rate of the system a mean approximation is implemented according to the one presented in Chapter 4 for the planar case. Including the variable UT power allocation it is needed to define the mean power and squared path loss \( \bar{\xi} \) entity (similarly to the mean squared path loss \( \bar{\xi}^2 \) defined in Chapter 4) which is given as:

\[
\bar{\xi}_{m,q}^{m,q} = \frac{1}{\rho - \hat{\rho}} \int_{\rho}^{\hat{\rho}} P_{q,k}(s) \left( \xi_{m,q,k}^{m,q}(s) \right)^2 ds
\]

(6.8)

where \( s \) is the variable UT distance from the respective BS, \( \rho, \hat{\rho} \) are the truncation distances depending on the UT distribution over the cells (defined in (4.6) and (4.7)), \( P_{q,k}(s) \) is defined by the UT power allocation profile and \( \xi_{m,q,k}^{m,q}(s) = \left( \frac{l_o}{1+l_o D_h(s)^{1+\theta}} \right)^{\eta} \) with \( l \) defining the tier that cell \( q \) of cluster \( m \) belongs w.r.t to the cell \( q \) of cluster \( m \). With definition (6.8) each one of the \( K \) products \( P_{q,k} \left( \xi_{m,q,k}^{m,q} \right)^2 \) referring to UTs in cell \( q \) of cluster \( m \) is replaced by a single approximating value \( \bar{\xi}_{m,q} \) which captures the path loss environment, the power profile as well as on the UT distribution assumed:

\[
\bar{\xi}_{m,q}^{m,q} \approx P_{q,k} \left( \xi_{m,q,k}^{m,q} \right)^2
\]

(6.9)

In the end, the above are functions of the distance between the BS antenna in cell \( q \) of cluster \( m \) and the BS antenna in cell \( q \) of cluster \( m \).
Distance $D_{l,k}$ of a UT in a cell in the $l^{th}$ tier of "interference" from the receiver of interest is given now by

$$D_{l,k}(s) = 2Rl + s$$  \hspace{1cm} (6.10)

Note also that, since we have turned to a 1D system, all the variable parameters that depend in UT locations will be functions only of distance $s$ of UTs from BSs and not of the angular parameter $\theta$ in that case.

Based on (6.7) and on (6.8) the analytical formula is obtained:

$$R \approx \log_2 \prod_{q=1}^{Q} \left[ 1 + \frac{\sum_{q=1}^{Q} K_{m,q}^{m,q}}{\sigma_0^2 + \sum_m \sum_{q=1}^{Q} K_{m,q}^{m,q}} \right]$$  \hspace{1cm} (6.11)

It is obvious from (6.11) that the sum rate of the cluster will depend on the same parameters affecting the HR systems as studied in Chapter 4 and on the cluster size and the UT power allocation profile considered.

6.1.2.5 BS Contribution on Sum Rate and RoT Distribution

The cluster sum rate given in (6.7) or (6.11) can be written in an equivalent form:

$$R = \sum_{q=1}^{Q} \log_2 \left[ 1 + \frac{P_{m,q}}{P_{m,q}} \right]$$  \hspace{1cm} (6.12)

where $P_{m,q}$ and $P_{m,q}$ stand for the desired and the undesired power received respectively in cell $q$ of cluster $m$. In Chapter 3, for the information-theoretic analysis of the HR cellular uplink, RoT at each BS was introduced as the main figure of merit which determines the per-cell capacity of that system (see (3.30)). It has been also seen how the use of a joint decoder under the information theoretic approach can convert the harmful interference to a desired power increasing RoT.

Accordingly, in the case of clustered cellular systems an alternative definition of RoT can be used. In the clustered system case the cluster edge effects cannot be avoided and thus the RoT referring to a cell will not be the same now for any cell $q$ but it will be defined as:

$$\text{RoT}_q \triangleq \frac{\text{Total Desired Received Power}}{\text{Total Undesired Received Power}} = \frac{P_{m,q}}{P_{m,q}}$$  \hspace{1cm} (6.13)
where $P_{m,q}$ is the intra-cluster received power from UTs within the region of the respective JP and $Z_{m,q}$ is the total undesired power at cell $q$ of cluster $m$ given in (6.3).

Based on the cell RoT definition in (6.13), the contribution of receiver $q$ on the cluster sum rate is defined as:

$$R_q \triangleq \log_2 (1 + \text{RoT}_q) \quad (6.14)$$

and since the RoT$_q$ actually refers to the $q^{th}$ individual eigenvalue from (6.7) we have that:

$$R_q = \log_2 \left( 1 + \frac{\sum_{q=1}^{Q} \sum_{k=1}^{K} P_{q,k}(m,q)^2}{\sigma_0^2 + \sum_m \sum_{q=1}^{Q} \sum_{k=1}^{K} (m,q)^3} \right)$$

$$\approx \log_2 \left( 1 + \frac{\sum_{q=1}^{Q} K Z_{m,q}}{\sigma_0^2 + \sum_m \sum_{q=1}^{Q} K Z_{m,q}} \right) \quad (6.15)$$

As said, in almost all clustering cases in a linear system (apart from the case of $Q = 2$), the BS receivers of a cluster are not all identical in terms of receiving power due to cluster edge effects. Due to that asymmetry, the contribution of a receiving BS on the cluster sum rate depends on the position of the BS respective cell over the cluster.

Considering equation (6.14) a distribution on the contribution rates of each receiver can be obtained according to the individual cell RoT distribution (each one of the $Q$ elements of the summation in (6.7)). In the following along with the cluster sum rate the contributing rates of the BSs in the cluster will be investigated and results will be provided for that case. It is noted that, the distribution of the BS contribution rates does not necessarily provide information about the per-cell sum rate (sum of all user rates in cell) distribution or the cell rate fairness of the system but it will help us on tackling the sum rate optimization problem and in any case it may stand for a useful rate metric for the clustered systems as it shows which BS contributes less or more on the cluster sum rate. That information could be proved useful for the positioning of the BSs in a scenario where the BSs could, by request, be distributed non-uniformly over the clusters and it could be the case for a future work.
6.1.2.6 Sum Rate Optimization

Our aim in this section is to investigate on the optimal power allocation \( p^* = (P_{1,1}^*, P_{1,2}^*, \ldots, P_{Q,K}^*) \) such that the cluster sum rate \( R \) is maximized under the given system power constraints. In the following along with the maximum power constraint \( (P_{\text{max}}) \) applied so far, a minimum power constraint \( (P_{\text{min}}) \) will be assumed to potentially assure the individual UTs with minimum rates for basic services. Determine

\[
p^* = \arg \max_{p \in \{p | P_{\text{min}} \leq P_{q,k} \leq P_{\text{max}}, \forall (q,k)\}} R(p)
\]

where the set \( \{p | P_{\text{min}} \leq P_{q,k} \leq P_{\text{max}}, \forall (q,k)\} \) stands for the feasible set of transmit power vectors under the specific power requirements. Note that (6.16) is in general a non-convex and hence, a difficult optimization problem. Thus, a technique presented in [47] is adopted and extended to narrow down the possible solutions. Following (6.16), given a real factor \( \varepsilon > 1 \) and a power allocation profile \( p \), we have that

\[
R_q(\varepsilon p) = R_q(\varepsilon P_{1,1}, \ldots, \varepsilon P_{Q,K})
\]

\[
= \log_2 \left( 1 + \frac{\sum_{q=1}^{Q} \sum_{k=1}^{K} P_{q,k} \left( \frac{m_{q,k}}{\sigma_{q,k}^2} \right)^2}{\varepsilon + \sum_{n=1}^{\infty} \sum_{q=1}^{Q} \sum_{k=1}^{K} P_{q,k} \left( \frac{m_{q,k}}{\sigma_{q,k}^2} \right)^2} \right) \geq R_q(p)
\]

for all BSs \( q \) and any cluster size \( Q \). Since the total cluster sum rate is \( R(\varepsilon p) = \sum_{q=1}^{Q} R_q(\varepsilon p) \), we have also that

\[
R(\varepsilon p) > R(p)
\]

According to the above the following observations may be constructed.

**Lemma 6.1:** If a UT power constraint exists, at least one element of vector \( p \) must be equal to that constraint to maximize cluster sum rate \( R \).

**Theorem 6.2:** The maximization problem of the cluster sum rate can be split into two subproblems: Maximise cluster sum rate by providing more power to UTs that are

1. closer to their respective BS (defined as \( \text{UT-w.r.t.-cell location dependance} \)) and

2. closer to the center of their respective cluster (defined as \( \text{UT-w.r.t.-cluster location dependance} \)).
There exists an optimal combination of these two separate problems which is the solution to the optimisation problem presented in (6.16).

Proof:

Considering (6.12), the achievable sum rate is increased when the desired received power in the cluster is maximized while the undesired received power is minimized at the same time. In other words, it is needed to maximize the desired received power $\sum_{q=1}^{Q} \sum_{k=1}^{K} P_{q,k} \left( \frac{m_{q,k}}{m_{m,q,k}} \right)^2$ (numerator of (6.7)) while at the same time minimizing the undesired received power $\sum_{m}^{M} \sum_{q=1}^{Q} \sum_{k=1}^{K} P_{q,k} \left( \frac{m_{q,k}}{m_{m,q,k}} \right)^2$ at BS in cell $q$ of cluster $m$ (denominator of (6.7)) for any $q$. In addition to that, the sum of all BS contributing rates in the cluster ($\sum_{q=1}^{Q} R_q = R$) should also maximized. It can be observed that the cluster sum rate is straightforwardly connected to the product of the squared path loss coefficients associated to the UT-BS paths with the respective UT transmitting powers. Moreover:

1. the squared path loss coefficients at the numerator of (6.7) are a function of the UTs' distance from their same cell BS while
2. the squared path loss coefficients at the denominator depend on the distance of the UTs' same cell BS to the BSs of the other clusters.

According to these facts we have that: 1) the UTs that contribute more on the desired power are those close to their respective BSs and at the same time, 2) the UTs that cause less interference to the neighbouring clusters are those close to the center of their respective cluster.

Hence, if an optimal power vector $p^*$ exists, the power element(s) that is(are) equal to the power constraint, so at to maximize sum rate according to Lemma 6.1, should refer to the UT(s) located closer both to its(their) respective BS and to the centre of its(their) respective cluster.

6.1.2.7 Power Allocation Profiles

According to the Theorem 6.2, UTs should be allocated with power according to their current position according to their respective BS and the cluster center. In this direc-
tion, two separate general power allocation profile vectors will be introduced:

- $\mathbf{p}_{\text{cell}}$ for the UT-w.r.t-cell and
- $\mathbf{p}_{\text{cluster}}$ for the UT-w.r.t-cluster location dependance

which can be combined to provide the set of the feasible optimal UT power allocation profiles. An example is illustrated in Figure 6.4. The united power allocation profile vector $\hat{\mathbf{p}}$ will be a weighted combination of profiles $\mathbf{p}_{\text{cluster}}$ and $\mathbf{p}_{\text{cell}}$ controlled by a weighting parameter $\nu$ (with $0 \leq \nu \leq 1$), where higher values of $\nu$ will refer to a power allocation more favoured by the UT-w.r.t.-cluster location dependance power profile and vice-versa:

$$\hat{\mathbf{p}} = \nu \mathbf{p}_{\text{cluster}} + (1 - \nu) \mathbf{p}_{\text{cell}}$$

(6.19)

Furthermore, four parameters ($\alpha_{1,2,3,4}$ for $\mathbf{p}_{\text{cluster}}$ and $\beta_{1,2,3,4}$ for $\mathbf{p}_{\text{cell}}$) are defined to control the curve of each general power profile. We have:

$$0 \leq \alpha_1, \beta_1 \leq 0.5 \text{ - number of edge-UTs allocated with } P_{\min}$$

$$0 \leq \alpha_2, \beta_2 \leq 0.5 \text{ - number of centre-UTs allocated with } P_{\max}$$

$$0 \leq \alpha_3, \beta_3 \leq 1 \text{ - defines } P_{\min} \text{ as: } P_{\min} = \alpha_3 P_{\max} \text{ or } \beta_3 P_{\max}$$

$$\alpha_4, \beta_4 = \{-1, 0, +1\} \text{ - defines the power allocation curve}$$

(6.20)

with $(\alpha_1 + \alpha_2), (\beta_1 + \beta_2) \leq 0.5$. Note also that $\alpha_2, \beta_2 = 0.5$ is equivalent to the case where $\alpha_3, \beta_3 = 1$. The terms edge-,centre- refer to the edge or the center of either the cell or the cluster respectively depending on the general power profile we refer to. Moreover, $\alpha_4, \beta_4 = 0$ refers to a linear power allocation curve while $\alpha_4, \beta_4 = \pm 1$ to sinusoidal curves (respectively increasing or decreasing the average UT power in contrast to the linear one, see Figure 6.5). The sinusoidal curves are considered as they may provide a hint on which is the trend for the preferred UT power allocation (allowing less or more power from the UTs between the edge and the centre of the cluster).
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Figure 6.4: Example of a combination between the two separate UT power allocation profile vectors $p_{\text{cell}}$ and $p_{\text{cluster}}$. $\nu = 0.5$.

Figure 6.5: $p_{\text{cluster}}$ power allocation profile and parameters $\alpha_{1,2,3,4}$ controlling it.
The power of a UT will be a function of distance \( s \) from its respective BS and hence the general power allocation profile vector \( \mathbf{p}_{\text{cluster}} \) will have elements:

\[
\mathbf{p}_{q,k} = \begin{cases} 
\mathbf{p}_\text{min} & Q\text{ISD} \left( \frac{1}{2} - \alpha_1 \right) \leq s \leq \frac{Q\text{ISD}}{2} \\
\mathbf{p}_0(s) & a_4 = 0 \text{ and } \alpha_2 Q\text{ISD} \leq s \leq Q\text{ISD} \left( \frac{1}{2} - \alpha_1 \right) \\
\mathbf{p}_{+1}(s) & a_4 = +1 \text{ and } \alpha_2 Q\text{ISD} \leq s \leq Q\text{ISD} \left( \frac{1}{2} - \alpha_1 \right) \\
\mathbf{p}_{-1}(s) & a_4 = -1 \text{ and } \alpha_2 Q\text{ISD} \leq s \leq Q\text{ISD} \left( \frac{1}{2} - \alpha_1 \right) \\
\mathbf{p}_\text{max} & 0 \leq s \leq \alpha_2 Q\text{ISD} 
\end{cases}
\]  

(6.21)

where the various power functions are defined as

\[
P_0(s) \triangleq \mathbf{p}_\text{min} + \delta P \frac{|s - Q\text{ISD} \left( \frac{1}{2} - \alpha_1 \right)|}{Q\text{ISD} \left( \frac{1}{2} - \alpha_1 \right)} \\
P_{+1}(s) \triangleq \mathbf{p}_\text{min} + \delta P \sin \left( \frac{\alpha_2}{Q\text{ISD} \left( \frac{1}{2} - \alpha_1 \right)} \right) \\
P_{-1}(s) \triangleq 2P_0 - P_{+1}
\]

(6.22)

with \( \delta P \) denoting the difference between \( P_{\text{max}} \) and \( P_{\text{min}} \), i.e. \( \delta P = P_{\text{max}} - P_{\text{min}} \).

An example of the \( \mathbf{p}_{\text{cluster}} \) profile is illustrated in Figure 6.5. Similar expression and representation exists for the \( \mathbf{p}_{\text{cell}} \) general power allocation profile where \( Q\text{ISD} \) and \( \alpha \) expressions in (6.21) and (6.22) will be replaced by \( \text{ISD} \) and \( \beta \) respectively in that case.

As can be seen by Figures 6.4 and 6.5 the combination of the two general power allocations by \( \nu \) and parameters \( (\alpha, \beta)_{1,2,3,4} \) provide a mathematically tractable set of all the feasible power profiles that can maximize the cluster sum rate.

### 6.1.2.8 Numerical Results and Observations

For interpreting the information theoretic results into real-world systems the practical scenario described in Chapter 3 is considered along with the respective system modelling parameters of Table 3.1 (p.35). The results for the cluster sum rate have been produced by generating the corresponding system and applying equations (6.7), (6.11) and they have been verified by running Monte Carlo numerical simulations to generate random fading and path loss coefficients so as to create the desired matrices for various system
snapshots. The per cluster sum rates for the verification are obtained by finding the average over a large number \( J \) of system realizations using

\[
R_{\text{sim}} = \frac{1}{J} \sum_{j=1}^{J} \log_2 \left( \frac{\det (\mathbf{H}_x \mathbf{H}_x^\dagger + \Lambda_2)}{\det (\Lambda_x)} \right)
\]

The simulated linear cellular system contains clusters of cells with 1 to 20 cells each and 100 UTs are uniformly distributed across each cell. Results of the normalized cluster sum rate (averaged cluster sum rate over the cluster size) and of the BS contribution rate distribution (when \( Q = 20 \)) for three different density systems are provided. Various UT power profiles were applied based on the analysis made in the previous subsection and their effect on the sum rate was investigated. Some general observations were:

1. The optimal UT power allocation strategy in any studied case, so as to maximise the cluster sum rate, is to give maximum priority to the UT-w.r.t.-cell (instead of UT-w.r.t.-cluster) location dependence (e.g. \( \nu = 0 \)). In addition, it is preferable to allow only the UTs that are at any time very close to their BS to transmit at their maximum allowed power while limiting all the other UT powers to \( P_{\text{min}} \) (ideally \( P_{\text{min}} = 0 \)). Although that strategy provides a significant increase on the achievable sum rate it is obvious that it is not fair for most of the UTs in the system as in general the UTs cannot be assumed to move uniformly on the cell area over time. For that reason on the following although the maximum achievable sum rate (for \( \nu = 0, \beta_1 \to 0.5 \)) is provided, the behaviour of the achievable sum rate is also examined when UT-w.r.t.-cluster location dependence is considered in which case the rate will be allocated more fairly among the UTs.

Table 6.1 summarizes the various sets of parameters \( \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\} \) providing the different UT-w.r.t.-cluster location dependence power profiles used for the results illustrated in Figures 6.6-6.8.

2. In that direction, when \( \nu \neq 0 \), it is better to restrain the cluster edge UTs to low power while allowing maximum power to cluster centre UTs. It is not optimal though to restrain many cluster edge UTs to the minimum power as the positive effect of reducing their interference to neighbouring clusters is in most
Table 6.1: Various UT power profiles w.r.t.-cluster location dependence.

<table>
<thead>
<tr>
<th>Symbol/Set</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
<th>P6</th>
<th>P7</th>
<th>P8</th>
<th>P9</th>
<th>P10</th>
<th>P11</th>
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</tr>
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<td>0.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
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<tr>
<td>α₂</td>
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<td>0</td>
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<td>0</td>
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<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>α₃</td>
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</tr>
<tr>
<td>α₄</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
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<td>0</td>
<td>+1</td>
<td>-1</td>
</tr>
</tbody>
</table>

cases overlapped by the negative effect of less desired power in the cluster (keep \( \alpha_1 \approx 0 \) in any case). Moreover, although providing marginal improvement, it is usually preferred to use a power profile with \( \alpha_4 = +1 \).

It is first considered a dense system with ISD = 100m and \( \eta = 2 \). It is noted that in such a dense environment the thermal noise power becomes a minor parameter when compared to the other powers and can be neglected. Moreover, when constant power profiles are applied (every UT is allocated with the same power \( P_C \)) equation (6.7) boils down to:

\[
R_{\text{dense}} \approx \log_2 \prod_{q=1}^{Q} \left[ 1 + \frac{\sum_{q=1}^{Q} \sum_{k=1}^{K} \left( \frac{m,q}{m,q,k} \right)^2}{\sum_{q=1}^{Q} \sum_{k=1}^{K} \left( \frac{m,q,k}{m,q,k} \right)^2} \right], \quad \text{for} \ P_{q,k} = P_C \ \forall q, k \quad (6.24)
\]

where it is clear that sum rate is totally independent from the UT powers. Hence, in that case, constant power profiles achieve the same sum rate. Figure 6.6 illustrates results for various UT power allocation profiles. When \( \nu \neq 0 \), it can be observed that allocating \( P_{\text{max}} \) to high percentage of UTs is preferred for maximising sum rate (i.e. \( \alpha_2 = 0.35 \) instead of 0.5, although the latter provides fairer cell RoT distribution). Moreover, it is better to keep the value of \( P_{\text{min}} \) as low as possible (\( \alpha_3 \approx 0 \)) which effectively means to give low power to the edge cluster UTs. It is noted that for \( \nu = 0 \) a significant improvement on the cluster sum rate is observed.

In Figure 6.7, a sparse system with ISD = 6Km and \( \eta = 3.5 \) is considered. In this scenario, as much power as possible should be allocated to the UTs (\( \alpha_2 \approx 0.5 \) or \( \alpha_3 \approx 1 \) when \( \nu = 1 \)). The above means that in a sparse environment the cluster edge effects become negligible and thus, letting UTs to transmit at their maximum power is the best choice. Furthermore, it is observed that constant UT power allocation profiles
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Figure 6.6: Normalised cluster sum rate and BS contribution rate distribution of a dense system for various UT power allocation profiles.

Figure 6.7: Normalised cluster sum rate and BS contribution rate distribution of a sparse system for various UT power allocation profiles.
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Figure 6.8: Normalised cluster sum rate and RoT distribution of a normal density system for various UT power allocation profiles.

provide fairer solutions for cell RoT distribution and in those cases it is valid to state that the normalised cluster sum rate is equivalent to the per-cell sum rate.

Figure 6.8 depicts results for a normal density system with ISD = 2Km and $\eta = 3$. Here, for $\nu \neq 0$, allocating maximum available power is again optimal when the cluster size is large enough. On the other side, for relatively small cluster size, the optimal power allocation set parameters of the dense systems are also preferable here. Various intersection points can be observed which define for which cluster sizes each profile becomes preferable than the other. It is noted that the position of these intersection points on the x-axis (cluster size) depends always on the combination of parameters $\alpha_{2,3,4}$. For $\nu = 0$, achievable cluster sum rate is increased and at the same time the cell RoT distribution becomes fairer.

Note that the maximum achievable per-cell sum rate for a HR-based system is given by:

$$R_{p_{HR}} = \log_2 [1 + \text{RoT}]$$  \hspace{1cm} (6.25)
with RoT as given in (3.31). Hence, based on (6.25), the maximum achievable per-cell sum rate can be provided for each one of the three different density systems presented above when using a HR. Thus, when allocating $P_{\text{max}}$ (here $P_{\text{max}} = 200\,\text{mW}$) to each UT (which is the optimal case in a HR system) the $R_{q\text{-HR}}$ of the 1) dense system reaches approximately to 30.5 bps/Hz/Cell, 2) normal density system to 18.3 bps/Hz/Cell and 3) sparse system to 10.6 bps/Hz/Cell. These results in comparison with the ones illustrated in Figures 6.6, 6.7 and 6.8 indicate the rate differences between the global and the clustered BS cooperation case. It is obvious that the dense system suffer severely from the inter-cluster interference (achieving $< 40\%$ of the maximum capacity even for $Q = 20$ cells per cluster) while the normal density system is proved to be the most viable case for a clustering system design (can achieve $> 80\%$ of the maximum capacity even with less than 5 cells per cluster). It should be noted as well that cooperation between BSs, in general, does not increase the achievable sum rate of very sparse systems.

Finally, in Figures 6.9 and 6.10 a more general view of the cluster sum rate behaviour is obtained. In Figure 6.9, UT-w.r.t.-cluster power allocation ($\nu = 1$) is considered and by choosing the most appropriate UT power allocation profiles for each system density case, the normalised cluster sum rate is plotted versus the cluster size and the ISD for various path loss exponents. It is observed that increasing cluster size significantly increases sum rate only for relatively low ISDs. On the other hand, too low ISD will cause a drop on the sum rate. In general, a combination of relatively low ISD and high $\eta$ may achieve sum rates close to the optimum (HR system) even for small cluster sizes. In Figure 6.10, the beneficial (for the sum rate point of view) UT-w.r.t.-cell power allocation is considered ($\nu = 0$). It is observed that despite a significant increase on the normalised sum rate, the overall behaviour of the cluster sum rate for the various parameters remains similar to the previous case.

6.1.3 Clustering Isolation Schemes

Here, various clustering isolation schemes are investigated and compared with each other. Isolation of cells in a system can be implemented in numerous ways. Frequency, time and space division schemes can be considered so as to separate the clusters from
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Figure 6.9: Normalised cluster sum rate for various system densities and cluster sizes. UT-w.r.t.-cluster power allocation ($\nu = 1$).

Figure 6.10: Normalised cluster sum rate for various system densities and cluster sizes. UT-w.r.t.-cell power allocation ($\nu = 0, \beta_1 = 0.4$).
each other and eliminate inter-cluster interference as much as possible. Moreover, the interference allowance scheme presented in the previous subsection is compared to the various isolation schemes.

On the following, the focus is on the normalised cluster sum rate (or average per-cell sum rate over the cluster size). Note also that the spectrum allocation techniques refer to UTs and BSs. That means that although different BSs can receive signals from the spectrum that has been allocated to them, each JP can receive and process signals from the whole spectrum through a delay and correlation free process.

To simplify the analysis it is assumed that there is a maximum number cell tiers $L$ whose UTs are able to "interfere" with each BS. After that $L$ tiers of cells the received power can be considered to be negligible. The practical validity of this assumption will be justified later on at the chapter. Moreover, the clusters are assumed to be designed in a way so as only cells from adjacent clusters can interfere with each other.

### 6.1.3.1 Normal Spectral Division

The most simple division scheme for clustering is to divide the available frequency or time spectrum and let BSs and UTs in neighbouring clusters to operate in orthogonal frequency/time slots. Due to the cluster symmetry, half of the available spectrum is assigned to each cluster with half of the resources assigned to adjacent clusters (Figure 6.11). According to the system model, the received signal at the BS antenna of cell $q$ in cluster $m$ is the sum of the transmitted signals from the UTs within the same cell and also from its maximum $L$ tier neighbouring cells appropriately scaled by the path loss and multipath fading coefficients:

$$y_q = \sum_{k=1}^{K} \sum_{q'=q-L}^{q+L} r_{q,k} g_{q',k} x_{q',k} + z_q$$

for $|q-q'| \leq L$

where the cluster index is now omitted as inter-cluster interference is not considered. Since the clusters are assumed to be perfectly isolated with each other, the maximum per-cell sum rate is achieved when all UTs are always allowed to transmit at their maximum transmit power constraint (similarly to the global cooperation scheme presented...
in Chapters 3). Thus, based on equation (6.26) and on the general capacity equation for that case (3.28), the average per-cell sum rate is given by:

$$R_{\text{ndt}} = \frac{W_{1,2}}{QW} \log_2 \det \left( E_{\text{HH}} \left[ \frac{P_{\text{max}}}{\sigma_0^2} \text{HH}^\dagger + I_{N \times N} \right] \right) \quad \text{for } K \to \infty \quad (6.27)$$

where $W$ is the total available spectrum to the system, $W_{1,2} = W/2$ stands for either spectrum band $W_1$ or $W_2$ and $\sigma_0^2$ is the AWGN power.

Taking into consideration (6.26) along with the cluster edge effects to create the $\text{HH}^\dagger$ matrix, the average achievable per-cell sum rate becomes:

$$R_{\text{ndt}} = \frac{1}{2Q} \log_2 \prod_{i=0}^{Q-1} \left[ 1 + \frac{K P_{\text{max}}}{\sigma_0^2} \sum_{j=0}^{j \leq L} \frac{\overline{\sigma_j^2}}{\sigma_j^2} + \sum_{j=1}^{Q-1-i} \frac{\overline{\sigma_j^2}}{\sigma_j^2} \right] \quad (6.28)$$

where $\overline{\sigma_j^2}$ stands for the mean squared path loss for the UTs in a cell at the $j$th tier of "interference" from the cell BS of interest. Note that the above analysis presumes that $Q \geq L$.

**6.1.3.2 Efficient Spectral Division**

Although the normal spectral division scheme analysed above is very efficient for isolating the clusters from each other, it has a major disadvantage; Each BS and each UT is allowed to exploit only half of the available spectrum. A more efficient - for achieving rate - division scheme is investigated here. A part of the available spectrum is allocated only to cells close to the edges of every cluster which are able to cause interference to other clusters while letting all other cells to use the full resources. Figure 6.12 depicts such a system where the BSs and UTs at the $L$ cells from each side of the clusters are
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Figure 6.12: Efficient Frequency Division Isolation scheme. When the $2L$ edge cell UTs do not transmit at all, the scheme boils down to the Space Division Isolation one.

Using spectrum $W_{1,2}$ band to respectively receive and transmit. Using that isolation scheme, interference at the edges of each cluster is still avoided and, at the same time, the resources are used with the maximum possible efficiency by the system nodes.

As before, each JP in a cluster may receive all signals and operate at full spectrum without any penalty. In that case, the sum rate at a cluster $m$ can be investigated separately for each spectrum. The maximum average per-cell sum rate will be given by

$$R_{\text{eftd}} = \frac{1}{W} \left( W_{1} R_{\text{eftd}}^{(1)} + W_{2} R_{\text{eftd}}^{(2)} \right)$$  \hspace{1cm} (6.29)

where $R^{(i)}$ is the average per-cell sum rate achieved at spectrum band $W_{i}$.

Following similar analysis as for the normal spectrum division scheme $R_{\text{eftd}}^{(1)}$ and $R_{\text{eftd}}^{(2)}$ for $Q \geq 2L + 1$ can be found as:

$$R_{\text{eftd}}^{(1)} = 2 \log_{2} \prod_{i=1}^{L} \left[ 1 + \frac{K P_{\text{max}}}{N_{0}} \left( \sum_{j=1}^{j \leq L} \frac{\sqrt{s_{j}}}{\sqrt{s_{j}^{2}}} \right) + \frac{1}{Q} \log_{2} \prod_{i=0}^{Q-2L-1} \left[ 1 + \frac{K P_{\text{max}}}{N_{0}} \left( \sum_{j=0}^{j \leq L} \frac{\sqrt{s_{j}}}{\sqrt{s_{j}^{2}}} + \sum_{j=1}^{j \leq L} \frac{\sqrt{s_{j}}}{\sqrt{s_{j}^{2}}} \right) \right] \right]$$ \hspace{1cm} (6.30)

and

$$R_{\text{eftd}}^{(2)} = \frac{W}{W_{2}} R_{\text{eftd}}$$ \hspace{1cm} (6.31)

Hence, the total average achievable per-cell sum rate will be given by replacing (6.30) and (6.31) in equation (6.29).
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6.1.3.3 Cell-based spectrum allocation

A valid question would be if it is possible to use a cell-based spectrum allocation (allocating a part of the total available spectrum to each cell) and narrow down more the inter-cluster interference so as to increase the achievable sum rate. That issue is addressed here. Assume a system of \( Q \) cells per cluster and a total available spectrum of band \( W = W_{\text{max}} - W_{\text{min}} \). By controlling the band of the spectrum allocated to the BS and UTs of each cell we not only decrease the undesired power received to each BS in the neighbouring clusters but also the total desired received power received at BSs in the cluster of interest in a similar way as for the case of UT power allocation that was thoroughly analysed in Section 6.1.2. Our target is to investigate what happens to the achievable sum rate given these two effects of simultaneous increase or decrease of desired and undesired received power respectively. In that direction, the following observation provides the answer to the question posed.

Discussion: In a linear clustered system using a spectrum division isolation scheme, there is only need of a total of two (2) sub-bands to be allocated to the BS and UTs of each cell. To prove the above statement an example will be used; Consider a system (Figure 6.13) with \( Q = 5 \) cells per cluster and a total of \( X = 4 \) equal spectrum sub-bands (whose sum is equal to the total available spectrum) instead of \( X = 2 \) that was considered so far on the analysis. In general, the cluster sum rate can be given by the sum of achievable rates at each spectrum band:

\[
R = \frac{1}{W} \left( W_1 R^{(1)} + W_2 R^{(2)} + W_3 R^{(3)} + W_4 R^{(4)} \right) \tag{6.32}
\]

where \( R^{(i)} \) is the sum rate achieved at spectrum band \( W_i \) and \( W_1 = W_2 = W_3 = W_4 = \frac{W}{4} \). If a part of the spectrum is allocated to each cluster (normal spectral division isolation scheme) the sum rate of a cluster \( m \) will be

\[
R = \frac{1}{W} \left( W_1 R^{(1)} + W_2 R^{(2)} \right) \tag{6.33}
\]

since the BSs of cluster \( m \) do not operate in bands \( W_3 \) and \( W_4 \). Let now allocate band \( W_3 \) to the central cells of all clusters as shown in Figure 6.17 (shaded areas A). The achievable cluster sum rate in that case will be given by:

\[
\hat{R} = \frac{1}{W} \left( W_1 R^{(1)} + W_2 R^{(2)} + W_3 \hat{R}^{(3)} \right) \tag{6.34}
\]
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where $\hat{R}^{(2)} < R^{(2)}$ because of inter-cluster interference and $\hat{R}^{(3)} > R^{(3)}$ due to increased desired power in the cluster. Define the rate differences:

\[
\delta R_{\text{undesired}} \triangleq \hat{R}^{(2)} - R^{(2)} > 0
\]
\[
\delta R_{\text{desired}} \triangleq R^{(3)} - \hat{R}^{(3)} > 0
\]
\[
\delta R \triangleq \delta R_{\text{desired}} - \delta R_{\text{undesired}}
\] (6.35)

with $\delta R$ being either positive or negative if the allocation of spectrum band $W_3$ has an increasing or decreasing respectively effect on achievable cluster sum rate. If now, in addition, spectrum band $W_4$ is allocated to the same central cells of each cluster (shaded areas B) the resulted achievable sum rate will be

\[
\hat{R} = \frac{1}{W} \left( W_1 \hat{R}^{(1)} + W_2 \hat{R}^{(2)} + W_3 \hat{R}^{(3)} + W_4 \hat{R}^{(4)} \right)
\] (6.36)

where $\hat{R}^{(1)} < R^{(1)}$ and $\hat{R}^{(4)} > R^{(4)}$. Note that the additional desired and undesired power in each cluster will be the same as in the previous case (when $W_3$ was allocated). Hence,

\[
\hat{R}^{(1)} - R^{(1)} = \delta R_{\text{undesired}}
\]
\[
R^{(4)} - \hat{R}^{(4)} = \delta R_{\text{desired}}
\] (6.37)

and the effect of allocating $W_4$ to the central cells of each cluster in the system will be exactly the same to the effect observed when allocating $W_3$. Similar discussion can be made for any (and not only for the central) cell in the cluster.
Thus, the allocation of more sub-bands to cells BSs and UTs can only either increase or
decrease the achievable cluster sum rate. In that case, there is no meaning of allocating
sub-bands of spectrum to cells and either half (so as to isolate in case of negative effect
because of inter-cluster interference) or the whole (to gain from the advantageous effect
of cooperation) of the total available spectrum should be allocated to each cell. Note
that although the above example refers to a specific system, the same discussion can be
made for any system of \( Q \) cells per cluster and any \( X \) number of spectrum sub-bands.

6.1.3.4 Spatial Division

Clustering of cells can be also achieved by using spatial instead of spectral division.
Assume a model where the UTs at the \( L + L \) edge cells will not transmit at all during
a time slot (null cells). However, the BSs of these cells continue contributing on the
clustered joint decoding process (Figure 6.12 with \( W_{1,2} = 0 \)).

According to this scheme, following the same steps as above, the maximum achievable
per-cell sum rate will be given, again for \( Q \geq 2L + 1 \), by:

\[
R_{sd} = R_{c_{sd}}^{(1)}
\]  

(6.38)

6.1.3.5 Interference Allowance Scheme

Consider now again the scheme where no isolation is considered between clusters, and
UTs and BSs in all clusters are allowed to exploit the full resources allocated to the
system. In that case, the edge cells of every cluster experience inter-cluster interference
as their BSs will be interfered by transmitted signals from neighbouring cluster UTs.
It is understandable that the additional noise because of inter-cluster interference will
increase the undesirable received power. The advantage on the other hand of that
scheme is that the desired received power at each cluster can be maximized. The
received signal at the BS in cell \( q \) of cluster \( m \) can be represented by:

\[
y^{m,q} = \sum_{k=1}^{K} \sum_{d, k}^{m,q} s_{m,d,k} g_{m,d,k} x_{m,d,k} + z^{m,q}
\]  

(6.39)
where $z^{m,q}$ is in that case the sum of the thermal AWGN and the interfering complex Gaussian inputs from UTs outside the cluster of interest given by

$$z^{m,q} = n^{m,q} + \sum_{m} \sum_{q} \sum_{k=1}^{K} [c_{m,q}^{m,q} G_{m,q}^{m,q} x_{m,q,k}]$$  \hspace{1cm} (6.40)

The noise power matrix will be of the form:

$$\Lambda_z = \text{diag} \left( \frac{L}{Z_{m,q} \cdots Z_{m,q}} \frac{(Q-2L)}{\sigma_0^2 \cdots \sigma_0^2} \frac{L}{Z_{m,q} \cdots Z_{m,q}} \right)$$  \hspace{1cm} (6.41)

with $Z^{m,q}$ given by (6.3) and reproduced here by always considering the restriction implemented by the maximum number $L$ of "interfering" tiers:

$$Z^{m,q} = \mathbb{E} [z^{m,q} (z^{m,q})^*]$$

$$= \sigma_0^2 + \sum_{m} \sum_{q} \sum_{k=1}^{K} \mathbb{E} \left[ (c_{m,q}^{m,q} G_{m,q}^{m,q} x_{m,q,k}^2) (c_{m,q}^{m,q} G_{m,q}^{m,q} x_{m,q,k}^2)^* \right]$$  \hspace{1cm} (6.42)

In agreement to the noise power matrix the following UT power allocation strategy is considered. Assume $P_{\text{min}} \leq P_{\text{max}}$ the power of the $2LK$ edge (from both sides) cell UTs of each cluster. The input covariance matrix will still be a diagonal matrix of unequal diagonal elements:

$$\Lambda_x = \text{diag} \left( \frac{LK}{P_{\text{min}} \cdots P_{\text{min}}} \frac{(Q-2L)K}{P_{\text{max}} \cdots P_{\text{max}}} \frac{LK}{P_{\text{min}} \cdots P_{\text{min}}} \right)$$  \hspace{1cm} (6.43)

It is noted that the specific UT power allocation strategy implemented here is simpler than the one presented in the previous section and it is chosen accordingly to the noise power matrix so as to potentially minimize the inter-cluster interference.

Based on the above, for large number of UTs per-cell and for $Q > 2L + 1$, the average
achievable per-cell sum rate will be given by:

\[
R_I = \frac{2}{Q} \log_2 \prod_{i=1}^{L} \left[ 1 + K \frac{P_{\text{max}} \left( \sum_{j=L+1}^{i} \frac{c_j^2}{\sigma_0^2} \right) + P_{\text{min}} \left( \sum_{j=1}^{L} \frac{c_j^2}{\sigma_0^2} \right)}{\sigma_0^2 + KP_{\text{min}} \left( \sum_{j=1}^{L} \frac{c_j^2}{\sigma_0^2} \right)} \right] + \frac{1}{Q} \log_2 \prod_{i=0}^{Q-2L-1} \left[ 1 + K \frac{P_{\text{max}} \left( \sum_{j=0}^{L-1} \frac{c_j^2}{\sigma_0^2} \right) + P_{\text{min}} \left( \sum_{j=L+1}^{Q-2L-1} \frac{c_j^2}{\sigma_0^2} \right)}{\sigma_0^2 \cdot 2} \right] + \frac{P_{\text{min}} \left( \sum_{j=L+1}^{Q-2L-1} \frac{c_j^2}{\sigma_0^2} \right) + \sum_{j=0}^{L-1} \frac{c_j^2}{\sigma_0^2}}{\sigma_0^2 \cdot 2}
\]

(6.44)

6.1.3.6 Numerical Results and Observations

In the following the validity of the assumption of a finite number of tiers of contribution for the undesired received power at each BS is firstly discussed. After that the results comparing the various clustering schemes with each other on various practical scenarios are provided.

The L tier assumption

According to the practical system model (see Chapter 3-Table 3.1 (p.35)) a very dense system scenario (\(\eta = 2\), ISD= 100m, \(K = 100\) UTs per-cell with \(P_{\text{max}} = 200\text{mW}\) each) is chosen and the cell rate for the case where each cluster is formed by a single cell is investigated. By that, it is intended to estimate the impact of \(L\) on the per-cell rate. Figure 6.14 illustrates the achievable sum rate of the single cell cluster versus the maximum tiers considered on contributing on the received power according to

\[
\tilde{R}_{I,L} = \log_2 \left( 1 + \frac{KP_{\text{max}}c_0^2}{\sigma_0^2 + 2KP_{\text{max}} \sum_{j=1}^{L} \frac{c_j^2}{\sigma_0^2}} \right)
\]

(6.45)

and depicts the percentage loss on the achievable sum rate when increasing the number of \(L\) considered, e.g. \(\%\text{Loss} = \frac{\tilde{R}_{I,L} - \tilde{R}_{I,L+1}}{\tilde{R}_{I,L+1}}\), where \(\tilde{R}_{I,L}\) is the achievable sum rate when considering \(L = l\). It can be seen that in the linear model with these specific parameters there is no point of considering large number of \(L\) as the loss in achievable cell rate because of more tiers considered becomes negligible.
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Figure 6.14: Impact of maximum tiers of cells ($L$) contributing to undesired received power considered on cell sum rate.

**Numerical Results**

According to the above, $L = 3$ (less than 1% loss according to Figure 6.14) was considered to illustrate the results in Figures 6.15 and 6.16. In Figure 6.15 an achievable average per-cell sum rate comparison is provided among the various clustering schemes, the interference allowance (with $P_{\text{min}} = P_{\text{max}}$) and the HR scheme for two different scenarios. In Scenario A, a relatively dense system is considered with $\eta = 2.5$, ISD of 200m, $K = 100$ UTs per-cell with $P_{\text{max}} = 200\text{mW}$ each. It is observed that the efficient spectral isolation scheme is preferable for this case. For a more sparse system (Scenario B: $\eta = 3$, ISD= 2Km, $K = 100$ UTs per-cell with $P_{\text{max}} = 100\text{mW}$ each) the interference allowance scheme becomes optimal when cell clustering is implemented and reaches much closer to the maximum achievable rate of the HR scheme. It is also noted that the normal spectral isolation scheme achieves less than half of the maximum achievable rate while the spatial isolation scheme is always suboptimal to the efficient spectral one. In Figure 6.16 the effect of power allocation strategy presented in this subsection (allocating less power at the edge cell UTs of clusters) on average per-cell
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Figure 6.15: Comparison of the various clustering schemes. Scenario A: $\eta = 2.5$, ISD= 200m, $K = 100$ UTs per-cell with $P_{\text{max}} = 200$ mW each. Scenario B: $\eta = 3$, ISD= 2Km, $K = 100$ UTs per-cell with $P_{\text{max}} = 100$ mW each.

Figure 6.16: The effect of power allocation at the edge cell UTs of clusters on per-cell rate. $P_e$ and $P_k$ stand respectively for $P_{\text{min}}$ and $P_{\text{max}}$. 
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sum rate is investigated for the interference allowance scheme. It is clear that there is no point of allocating different power at UTs in sparse systems while in dense systems a marginal improvement may be obtained by doing so.

In both Figures 6.15 and 6.16 it can be observed that if there is no isolation scheme available, it is preferable to have small clusters when considering a system with large ISDs and high path loss exponent (sparse systems) since inter-cluster interference as well as the gain due to BS cooperation are negligible in that case while larger cluster size $Q$ is needed to achieve higher rates in higher density systems.

6.1.4 Time Scheduling. A Solution for Cell Rate Fairness

In a static linear cellular clustered system, the UTs at the edge cells of each cluster will be able to achieve lower rates than those at the central cells. This is in general true since (considering the geometry of the linear cellular system) the signals of the edge cell UTs will be able to be received with high probability strongly by less BSs in the cluster than the signals of the central cell UTs. Thus, per-cell sum rate will not be the same for each cell and it will depend on each cells' location over its respective cluster. For a more balanced system (with ideally equally served cells, i.e. same sum rate per-cell for every cell), a Time Scheduling scheme may be implemented. In that case, each JP is linked with $Q' = 2Q + 1 (> Q)$ BSs (potential links). At each time though, each JP is allowed to receive signals from UTs in $Q$ cells (active links). Assuming that the active links between the JPs and the BSs that are potentially linked with them change symmetrically and dynamically with time (with a BS in a cell being able to be potentially linked with two JPs at the same time but having only one active link at each time), eventually all the cells will acquire all the possible states within the cluster during a period of time. In that way, it will be possible to achieve fairness in the sense of per-cell sum rate. An example of the time scheduling scheme is illustrated in Figure 6.17 for a system with cluster size of $Q = 3$. It is noted that the number of time slots needed to achieve fairness is exactly $Q$, equal to the active links. It is noted that the time scheduling technique is able to achieve cell rate fairness with the disadvantage of more JPs needed to cover the whole network of cells.
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6.1.5 Discussion on the “Coupling” of Linear-Planar Models

The analysis in that section gives us some important insights on the linear systems uplink performance which can be proved useful for the planar uplink analysis. Of course, in the planar model the 2D plane introduces one more dimension (which is modelled with the introduction of the angular axis) which makes the capacity analysis even more complex. That extra dimension may have also an impact on the capacity analysis in many ways. For example, in 2D plane we have a larger number of cells at each tier (a total of $6l$ “interfering” cells at the planar case instead of the constant 2 “interfering” cells at the linear case from each tier $l$) around a cell. That specific fact is translated to an increased received (both desired and undesired) power for each BS when comparing to the linear case. However, the main observations captured on the capacity behaviour at the linear model should remain similar to the observations of the planar case as have been seen by Wyner and Somekh-Shamai studies on both cellular array models [25, 27].

In our case, the study of the sum rate of the clustered linear model informed us that normal density systems may significantly benefit from a clustered joint-processing scheme while on the other hand very dense systems “suffer” from inter-cluster interference and very sparse systems do not have any gain from BS cooperation anyway. The above should stand for the planar case as well. Moreover, it was seen that in those normal density linear systems the interference allowance scheme in coordination with a UT power allocation strategy was providing higher sum rates than any isolation scheme.

![Figure 6.17: Time scheduling scheme. Example system with $Q = 3$.](image)
For that reason, in the following, where the clustered joint processing of the planar uplink is studied, the interference allowance (no isolation) scheme with UT power control is specifically focused and the cluster sum rate of such a cellular system is studied. Note that the supplementary cases could form the case for a future work to be investigated.

Note that the results based on the linear cellular array are not only of academic interest since the linear model is a valid representation and may describe plenty real-world practical communication scenarios, e.g. Highways, Train lines/Railway networks, Satellite cellular systems and so forth.

6.2 Clustered Cooperation in Planar Cellular Systems

Here, a planar model similar to the one analysed in Chapters 3-5 is studied. A set of hexagonal cells is grouped together to form a cluster on a 2D setting. Due to the restrictions of hexagonal geometry, not all cluster sizes are possible [35]. Feasible values include \( Q = 3, 4, 7, 12, 13, 19, 27 \) as well as the general \( (1 + \sum_{l=1}^{L_Q} 6l) \), where \( L_Q \) stands for the number of the total cluster tiers. In the following analysis the less tractable cases will be omitted and thus, the clusters sizes of \( Q = 3, 4 \) and \( (1 + \sum_{l=1}^{L_Q} 6l) \) will be focused.

6.2.1 Clustered Planar System Description and Considerations

Assume a planar cellular array and a network of cells where the BSs are uniformly distributed in a 2D grid. Unlike the linear case, we have \( N \) cells divided into \( M \) clusters of \( Q \) cells each \( (N = MQ) \). The UTs are again distributed over the cells. A JP in each cluster jointly decodes all the UTs of that cluster. Note that the toroidal array model (see Figure 3.2) can be considered to avoid the system edge effects.

Based on the linear model analysis and observations the research of the sum rate of the planar case is focused on:

1. Interference allowance scheme; At the 1D plane, the isolation schemes may achieve larger sum rates than the interference allowance scheme when cell-based UT power
allocation is not considered and only in dense systems. It has to be noted again here that in the linear case there is a maximum of two “interfering” cells at each tier aside a cell when considering no isolation. On the other hand when performing isolation the band is needed to be splitted to half. When we turn to the 2D case there is a maximum of 6l “interfering” cells for any tier l around a cell which translates to more interference due to clustering. For isolating the clusters though at least 7 sub-bands are needed (instead of 2 in the linear case), a fact that will definitely have as well a vast impact on the achievable cluster sum rate and may be greater than the more interference introduced from the 2D consideration at the interference allowance scheme.

2. Cell-based UT power control. According to the results on the linear model it was seen that to maximise the cluster sum rate, the UT power allocation depending on the UT-w.r.t.-cluster location is of minimal importance when compared to the cell-based UT power allocation.

6.2.2 Clustered Planar System Model Sum Rate Analysis

Again symmetry among all clusters of cells is assumed here and sum rate analysis for only one cluster is performed. The analysis for the planar system will be similar to the one performed for the linear case. Hence the maximum achievable cluster sum rate will again be derived by (6.4). As already mentioned, we now have to consider two variables (including the one extra angular) for defining the location of UTs and BSs on the system. That change will affect only the definition of the mean power and squared path loss (given in (6.8) for the linear case) as:

$$\bar{\xi}_{m,q} \triangleq \frac{1}{\pi (\rho^2 - \rho^2)} \int_0^\rho \int_0^{2\pi} P_q, k(\theta, s) \left( \zeta_{m,q,h}^n(\theta, s) \right)^2 \sin \theta ds$$

(6.46)

For a cell-based UT power allocation the angular parameter may be omitted by considering the circular equivalent of the hexagonal cells and power to UTs will be allocated according only to their distance s from their respective BS. The power allocation profile
vector $\mathbf{p}_{\text{cell}}$ can have now elements:

$$P_{\text{eq}}(s) = \begin{cases} P_{\text{min}} & d_0 (1 - 2\beta_1) \leq s \leq d_0 \\ P(s) & 2\beta_2d_0 \leq s \leq d_0 (1 - 2\beta_1) \\ P_{\text{max}} & 0 \leq s \leq 2\beta_2d_0 \end{cases}$$ (6.47)

where $d_0$, given in (4.2) as $d_0 = \frac{2}{\pi} \int_0^{\pi/2} \sqrt{d_0 \cos(\theta)} \, d\theta$, is the radius of the circular equivalent cells and $P(s) = P_{\text{min}} + \delta P \frac{d_0 (1 - 2\beta_1)}{d_0 (1 - 2\beta_1 - 2\beta_2)}$ with $\beta_1$ and $\beta_2$ as given in (6.20).

Finally, the analytical formula for the achievable sum rate of the cluster will be again given by (6.11) as:

$$\hat{R} \approx \log_2 \prod_{q=1}^{Q} \left[ 1 + \frac{\sum_{q=1}^{Q} K \zeta_{m,q}}{\sigma_0^2 + \sum_m \sum_{q=1}^{Q} K \zeta_{m,q}} \right]$$, for $K \to \infty$ (6.48)

### 6.2.3 Numerical Results and Observations

Here, some interesting analytical results on the sum rate of planar communication systems with clustered joint processing are presented. Our main target is to evaluate the achievable sum rate of a clustered system with and without UT power allocation strategy and investigate on the main differences and the achievable rate gap from the HR optimal achieving scheme. In the following $K = 20$ UTs per-cell with maximum allowed power $P_{\text{max}} = 200\text{mW}$ per-UT, a maximum cluster size of $Q = 37$ cells and the real-world system parameters given in Table 3.1 (p.35) of Chapter 3 are considered.

In Figures 6.18-6.20 the UT power allocation effect for various density systems is depicted. A fixed cluster size of $Q = 37$ and a minimum UT power of $P_{\text{min}} = 0$ ($\beta_3 = 0$) is considered. In general it is preferable to let the UTs very close to their respective BS to transmit at $P_{\text{max}}$ while allocating the minimum allowed power to the rest (high $\beta_2$ and low $\beta_1$ values). Power allocation as has been stated is more meaningful in dense than in sparse systems. As the system becomes very sparse it is better to allow all UTs to transmit at high power as the cluster edge effects become less important than the increase of the desired received power.

Figure 6.21 illustrates two important facts. Firstly, at the bottom graph, it is observed how the normalised (over the cluster size $Q$) cluster sum rate increases with the cluster
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Figure 6.18: Cell-based power allocation on a dense system (ISD = 200m, $\eta = 2$). $Q = 37$, $\beta_3 = 0$.

Figure 6.19: Cell-based power allocation on a normal density system (ISD = 1Km, $\eta = 2.5$). $Q = 37$, $\beta_3 = 0$. 
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Figure 6.20: Cell-based power allocation on a sparse system (ISD = 6Km, \( \eta = 3.5 \)). \( Q = 37 \), \( \beta_3 = 0 \).

Figure 6.21: BS contribution rate distribution over tiers and normalised cluster sum rate versus cluster size. System parameters: ISD = 1Km, \( \eta = 2.5 \). UT cell-based Power profile parameters: \( \beta_1 = 0, \beta_2 = 0.45, \beta_3 = 0 \).
size, tending asymptotically to the capacity of a system with BS global cooperation. Larger cluster sizes will benefit from the higher number of cooperative BS antennas and that is why that increase on the cluster sum rate with the increase of the cluster size is observed. A system with ISD of 1Km and path loss exponent of 2.5 has been selected. The high achieving rates (from Figure 6.19) power profile set of parameters $\beta_1 = 0$ and $\beta_2 = 0.45$ is also considered. Note that for different density systems similar curve is obtained but with different lower and upper rate bounds. Moreover, the upper graph shows the average (over the number of cells per-tier) distribution of BS contribution rate in a cluster of 37 cells according to the tier they belong from the cluster central cell. It can be seen that BSs close to the edge of the cluster contribute less when compared to the ones closer to the centre of the cluster. This is expected, since the cluster edge cell BSs will normally be more interfered by the UTs in neighbouring clusters while at the same time the average distance of the same cluster UTs (and hence the total desired received power) will be smaller than the cluster central BSs.

Figure 6.22 presents the general view of cluster sum rate for different density systems. The normalised cluster sum rate is illustrated for various values of ISDs and path loss exponents for clusters of 37 cells. To obtain the results, the high achieving sum rates UT power allocation strategies for each density system case have been implemented. It is observed that for relatively low (but not too low) ISD and path loss exponents of 3 or 3.5 high sum rates may be achieved while extremely dense systems (low ISD - low $\eta$) and sparse systems achieve lower sum rates.

The two UT power allocation cases will be compared now: 1) UT power allocation which achieves best performance for each system density case, 2) power allocation of $P_{\text{max}}$ to every UT (no-power allocation strategy). Figure 6.23 shows the cluster sum rate gain due to the UT power allocation strategy performed on the former case from the no power allocation latter one. The UT power control gain is defined as

\[
\% \text{ Power Control Gain} = 100 \frac{\text{Power Control Rate} - \text{No Power Control Rate}}{\text{No Power Control Rate}} \% \quad (6.49)
\]

From the results it is observed that UT power allocation strategy may give a tremendous boost (up to 200%) to relatively dense systems. On the other hand, for sparse systems there is no meaning for power allocation strategy as already stated.
6.2. Clustered Cooperation in Planar Cellular Systems

Figure 6.22: Cluster Sum Rate with Power Control. $Q = 37$.

Figure 6.23: Cluster Sum Rate Gain due to UT power allocation strategy. $Q = 37$. 
Finally, the loss in the average per-cell sum rate due to clustering is shown. Two system cases are compared: 1) clustered system with JP (for every $Q = 37$ cells per cluster) and respective UT power allocation which achieves high performance for each system density case and 2) system with a HR jointly decoding all received signals. In Figure 6.24 the clustering loss defined as:

$$\text{% Clustering Loss} = 100 \frac{\text{Hyper Receiver Rate} - \text{Clustering Rate}}{\text{Hyper Receiver Rate}} \%$$

is depicted. The figure illustrates the fact that very dense systems suffer most from introducing clustered joint processing. Clustering has no negative impact for very sparse systems and anyway as already explained in Chapter 4, BS cooperation does not improve the sum rate of very sparse systems. It has to be concluded that for planar systems (as in the linear case), when considering an interference allowance scheme, the use of JPs to introduce clustering is more beneficial for normal density systems as already intuitively expected from the beginning. In that case, the cooperation of BSs in a cluster to bring more desired power in each BS is more important than the undesired power coming from UTs in neighbouring clusters.
6.3 Summary

In this chapter the sum rate for the uplink of a cluster cellular system is investigated. Starting from the linear cellular model the problem is formulated by capturing the inter-cluster interference into the information theoretic sum rate analysis. An interference allowance scheme (no isolation between clusters) is first considered and the necessity of the UT power allocation concept along with the optimization of the cluster sum rate problem under UT power constraints is analysed. After providing a generic UT power allocation profile the effect of power allocation on the no-isolation clustered uplink system is investigated and the results are compared to those of the BS global cooperation case. It is observed that cell-based is preferable to the cluster-based UT power allocation. Moreover, various clustering isolation schemes are presented and compared with each other and to the interference allowance scheme in terms of achievable cluster sum rate. It is shown that normal density systems in an interference allowance environment with UT power allocation strategy significantly benefit from the clustered BS joint-processing and reach closer to the capacity of the respective HR systems. Furthermore, a time scheduling scheme for cell rate fairness is presented. Finally, the results from the analysis of the linear model formulate the basis of the focus for the planar model and thus, we concentrate on the sum rate analysis of the clustered planar cellular uplink with no cluster isolation scheme and UT power allocation. The cluster sum rate gain due to UT cell-based power allocation is investigated. In addition a general overview on the sum rate loss of the clustering scheme when compared with the optimal global BS cooperation scheme is provided.
Chapter 7

Epilogue

7.1 Summary of Insights and Conclusions

The importance of theoretic limits for the capacity of communication systems is great so as to quantify the gap with the current systems and decide if it is worthwhile to invest on further improvements. BS cooperation, with the advantage of the already existing infrastructure and technology to support the strategy, was presented as a mean of overcoming the inter-cell interference limitations of the traditional systems. In that direction, this thesis investigated in depth the information theoretic capacity of the cellular uplink.

In Chapter 2, the fundamental concept of information theory and the results in the literature that are useful for the study of the ergodic capacity of the cellular uplink have been reviewed. Firstly, the channel capacity of the single-user (SISO and MIMO) is studied and then we move on to the capacity region of the MAC channel. Finally, the capacity of the cellular uplink under MCP is examined and the existing studies on the field are presented.

In Chapter 3 the basic cellular uplink system model using a HR has been described. Furthermore, the formulations of the important effects of the multi-user channel (path loss, flat fading, shadowing and noise) have been provided along with a fitting example with empirical models to produce meaningful practical numerical results. Subsequently,
the information theoretic capacity analysis of the HR cellular uplink was analysed. In that direction, the notion of RoT was introduced as a figure of merit that captures all the parameters that affect the capacity of such a system. Finally, it is shown how capacity can be interpreted as a function of RoT at each BS and the various parameters that are incorporated into the RoT parameter are discussed.

Chapter 4 investigates the capacity of the planar cellular uplink. An average path loss approximation model was presented for the analysis of a planar system where every BS receives signals from the same cell and the surrounding cells arranged in multiple tiers of cells around the receiver of interest. The size of the cells and hence the cell density is modelled as a variable. Assuming a joint decoder at the BSs, a tight upper bound for the per-cell sum-rate capacity is provided when the number of users per-cell is large enough. The generic cellular system is extended to compare various system scenarios and their effect on capacity. System with single antenna at each BS yields minimum capacity. Spatial degrees of freedom gain provides a higher capacity when a system with multiple receiving antennas is considered where each antenna is perfectly directional and the sites coverage is sectorized. A further gain due to increased received power is obtained when the multiple BS antennas are considered omnidirectional and uncorrelated with each other or when the directivity gain of the directional antennas is considered larger than unity. It is also shown that for the sectorized system, increasing the number of antennas for each site to a very high value, the capacity tends to a finite value which is formulated using asymptotic analysis of the system. Furthermore, various parameters of a practical system that have an effect on the unified parameter of RoT at each BS and hence, on the per-cell capacity are identified and analysed.

In Chapter 5, a detailed approach to formulate the information theoretic capacity for the uplink of a sectorized classic multi-cell-multi-user planar cellular model is presented, in the presence of fading and assuming joint decoding at the receivers. It is shown that the capacity can be in general investigated by evaluating the eigenvalues of the block circulant covariance matrix which has non-circulant blocks. It is also proved that the capacity is higher than the one for the equivalent system with no sectorization. It is observed that the achieved rate is higher in the presence of non-LoS fast fading environment or when fading environment is Rician (LoS case) and received phases at
the specular path are random. Furthermore, for these cases, it is shown that increasing the number of directive antennas in each BS to a very high value, the capacity tends to a finite value which is formulated using asymptotic analysis of the system.

In Chapter 6 the sum rate for the uplink of a cluster cellular system is investigated. Starting from the linear cellular model the problem is formulated by capturing the inter-cluster interference into the information theoretic sum rate analysis. Firstly, an interference allowance scheme (no isolation between clusters) is considered and the necessity of the UT power allocation concept along with the optimization of the cluster sum rate problem under UT power constraints is analysed. After providing a generic UT power allocation profile, the effect of power allocation on the no-isolation clustered system uplink is investigated and the results to these of the BS global cooperation case are compared. It is observed that cell-based is preferable to the cluster-based UT power allocation. Moreover, various clustering isolation schemes are presented and they are compared with each other and to the interference allowance scheme in terms of achievable cluster sum rate. It is shown that normal density systems in an interference allowance environment with UT power allocation significantly benefit from the clustered BS joint-processing and reach closer to the capacity of the respective HR systems. Furthermore, a time scheduling scheme for cell rate fairness is presented. Finally, the results from the analysis of the linear model formulate the base of the focus for the planar model and thus, we concentrate on the sum rate analysis of the clustered planar cellular uplink with no cluster isolation scheme and UT power allocation. The cluster sum rate gain due to UT power allocation is investigated. In addition a general overview on the sum rate loss of the clustering scheme when compared with the optimal global BS cooperation scheme is provided.

7.2 Future Work

This section proposes future research guidelines as a step forward to the work presented in this thesis. In that direction, the limiting practical factors of real-world systems of today are discussed in contrast to the theoretical assumptions considered on this thesis along with possible ways to implement more realistic considerations. Furthermore,
important issues like Quality of Service (QoS), multiple cooperation strategies and the capacity of the cellular downlink are discussed.

7.2.1 Realistic Considerations

- **Input Modulation.** Ideally, to achieve capacity Gaussian distributed symbols are assumed with infinitely long transmitted codeblocks. In practice though, the assumption of infinitely long codeblocks of Gaussian coding cannot be implemented. This limitation will certainly have an impact on the practical achieving rates since after a certain threshold the rate will not be increasing with increased RoT.

- **Frequency Selectivity.** No frequency selectivity was considered for the fading of the wireless channel. In that direction, a single-tap narrowband fading model was assumed. However, the wireless channels in current systems are usually wideband and consequently frequency selective. It is noted though that the work presented in this thesis may apply straightforwardly to a wideband system if the bandwidth is sufficiently divided so as the fading can be considered flat in each division [9, 14]. In any case, the study of the cellular systems capacity under BS cooperation for wideband channels rises an interesting and important topic to be investigated.

- **Channel State Information.** In order to achieve capacity for the cellular uplink it is considered that the BSs have the uplink channel state information so as to jointly decode the received signals. In practice, only the downlink channel information is available at the receivers. Thus, the imperfect channel estimation has an impact on the achieving rates [48, 49]. However, when the channel reciprocity applies (i.e. no frequency selectivity in frequency-division multiplexing systems, no time selectivity in time-division multiplexing systems) that matter is of minor importance.

- **Infrastructure.** The BS cooperation strategy requires that all the BSs are interconnected through high-speed, delay-less and error-free links either with each
other or to a central processor while very large processing capabilities are assumed. For respectively large systems the computational load will be extremely high considering the processing capabilities of current processors. Clustering though, is a strategy that can tackle that problem by allowing cooperation in smaller clusters of the system. As shown in Chapter 6 of this thesis it is possible to achieve rates close to capacity with proper clustering planning. Moreover, clustering will decrease the cost of the additional infrastructure needed since the cabling and the joint processing will take place locally. Another issue that affects the achieving rates is the limitation of the widespread deployment of optic fibers due to their high cost. In that case, the backhaul can include copper cables or microwave links. These less reliable connections can be viewed as relays and the scheme may be described by a multiple cooperation strategy scheme.

7.2.2 Capacity and Quality of Service

This thesis investigated the sum rate capacity of the cellular uplink. However, the practical systems usually are characterized by the need to provide different priority or to guarantee a certain level of performance to the UTs. In that direction, a number of QoS constraints have to be met. QoS guarantees are important especially for real-time applications. The constraints in general may be minimum rate constraints (e.g. UTs able to establish voice connection at any time) or delay constraints (e.g. real-time streaming multimedia applications). An arousing scenario for future study would be the maximization of the sum rate while the UTs meet specific minimum rate and delay constraints. For that reason, the study of the outage capacity [50] would be a more appropriate metric than the ergodic capacity and resource allocation algorithms [47, 51] would be needed to be designed.

7.2.3 Multiple Cooperation Strategies

Another intriguing scenario for future study would be to quantify the capacity of systems that deploy multiple cooperation strategies at the same time. Although some initial works shown insignificant improvement on capacity when merging relaying and
BS cooperation [52, 53], it is still an open area for investigation which could prove to achieve high capacities when exploited appropriately. For example, clustering could be employed to optimize capacity due to BS cooperation while relaying after a BS positioning planning could provide fairness to the per-user rates.

### 7.2.4 Capacity of Cellular Downlink

The strategy of joint decoding can be extended to the cellular downlink, where the cooperating BSs precancel the interference at the cooperating UTs ("Hyper Transmitter") [54] so as to determine the capacity limits. Recent studies on that area discuss the duality concept ([55]), the need of joint transmission on the downlink ([56]) and the benefits from BS cooperation ([54, 57]). Assuming BS cooperation, the cellular downlink can be viewed as a MIMO Broadcast Channel (BC). In this direction, using the concept of duality the MIMO BC can converted to the dual MIMO MAC as long as the channel gain coefficients at the communication link are preserved and in that case, the downlink channel matrix can be obtained as the Hermitian conjugate of the uplink channel matrix. In contrary to the cellular uplink where a UT power constraint exist, input optimization is possible in the cellular downlink where an overall per-cell (or per-site) power constraint exists.
Consider the $N \times KN$ matrix $H$ with Gaussian i.i.d. entries. Consider also the following multiplication:

$$\Omega = HH^\dagger$$

(A.1)

where $H^\dagger$ is the $KN \times N$ Hermitian transpose matrix of $H$. Each element of the matrix $\Omega$ is the result of the multiplication of a row of matrix $H$ (which is a $KN$ vector) with a column of matrix $H^\dagger$ (again a $KN$ vector). Thus, each element of matrix $\Omega$ is the $KN$ sum of random variables multiplied with the conjugate transpose of other random variables:

$$\omega_{i,j} = \sum_{k=1}^{KN} \left[ h_{i,k} \hat{h}_{j,k}^* \right]$$

(A.2)

where all of the random variables $h, \hat{h}$ are assumed to follow the same distribution.

The matrix $\Omega$ converges to a deterministic matrix equal to $E[\Omega]$ if and only if all its elements converge, which means that the law of large numbers must apply to each element of $\Omega$. For this, it is not sufficient that the dimensions of matrix $H$ grow large. Instead, the horizontal dimension must grow much faster than the vertical dimension.

Hence, when $K \to \infty$ for every fixed $N$, the law of large numbers applies to each element of $HH^\dagger$, i.e. $\omega_{i,j} \equiv KNE \left[ h \hat{h}^* \right] V_i, j$, and thus a deterministic value for the expectation, $E[\Omega]$ and consequently for the capacity can be obtained.
Appendix B

Quantities of Information

This appendix acquaints the reader with the most fundamental quantities of information, i.e. entropy and mutual information, and summarises their basic properties relevant to our concept. In particular, entropy indicates the information (or uncertainty) in a random variable while mutual information provides the amount of information in common between two random variables and can be used to find the communication rate across a channel.

Definition B1. The entropy of a discrete random variable $X$, is a measure of the average uncertainty in $X$. It is the average number of bits (if logarithms to the base 2 are used), nats (if the base of the logarithm is e) or bans (if the base logarithm is 10)$^1$ which are required to describe $X$ (with an alphabet set $\mathcal{X}$, and probability mass function $p(x) = Pr(X = x)$ where $x \in \mathcal{X}$) [6]:

$$H(X) \triangleq - \sum_{x \in \mathcal{X}} p(x) \log_2 p(x) = \sum_{x \in \mathcal{X}} p(x) \log_2 \frac{1}{p(x)}$$  \hspace{1cm} (B.1)

Note that always $H(X) \geq 0$ as $0 \leq p(x) \leq 1$ and consequently $\log_2 (1/p(x)) \geq 0$.

Definition B2. If we consider continuous channels, the entropy of a continuous random variable $x$ with probability density function $f(x)$ in this case is called differential entropy and it is defined as [6]:

$$H(x) \triangleq - \int f(x) \log_2 f(x) \, dx = E_{f(x)} \left[ \log_2 \frac{1}{f(x)} \right] \text{ bits of information}$$  \hspace{1cm} (B.2)

$^1$Throughout the thesis the term -log$_2$- will be used to denote logarithms to base 2.
where the integral is taken over all \( x \in \mathcal{S} \) and where \( \mathbb{E}_{f(x)}[\cdot] \) denotes the expectation taken over \( f(x) \). In the following we focus on continuous random variables.

Take now \( N \) random variables \( x_1, x_2, \ldots, x_N \) (or a \( N \times 1 \) vector \( x \in \mathcal{S}^N \)) with \( x \) being circularly symmetric complex Gaussian with zero mean and \( N \times N \) covariance matrix \( \Lambda_x, x \sim \mathcal{CN}(0, \Lambda_x) \). The probability density function of that distribution is proved to maximize entropy and it is given by [13, 58]:

\[
    f(x) = \frac{1}{\pi^N \det \Lambda_x} \exp \left( -x^\dagger \Lambda_x^{-1} x \right) \tag{B.3}
\]

where superscript \( \dagger \) denotes the conjugate transpose of the respective Hermitian matrix.

According to definition B2 the entropy of \( x \) in that case can be found as:

\[
    H(x) \triangleq - \int_{\mathcal{S}^N} f(x) \log_2 f(x) dx = \int_{\mathcal{S}^N} f(x) \left[ \log_2 \left( \frac{1}{\pi^N \det \Lambda_x} \right) + x^\dagger \Lambda_x^{-1} x \right] dx
    = \log_2 \left( \pi^N \det \Lambda_x \right) + \mathbb{E} \left[ x^\dagger \Lambda_x^{-1} x \right] = \log_2 \left( \pi^N \det \Lambda_x \right) + \text{tr} \left( \mathbb{E} \left[ x x^\dagger \right] \right) \Lambda_x^{-1}
    = \log_2 \left( \pi^N \det \Lambda_x \right) + \text{tr} (I_{N \times N}) = \log_2 \left( \pi^N \det \Lambda_x \right) + N = \log_2 \left( \pi e^N \det \Lambda_x \right) \tag{B.4}
\]

Consequently, the entropy of a single random variable following the complex Gaussian distribution with \( x \sim \mathcal{CN}(0, \sigma_x^2) \) will be:

\[
    H(x) = \log_2 \left( \pi e \sigma_x^2 \right) \tag{B.5}
\]

**Definition B3.** Consider now two continuous random variables \( x \) and \( y \) with joint probability density function \( f(x, y) \) where \( (x, y) \in \mathcal{S}^2 \). Their jointed entropy is defined as [6]:

\[
    H(x, y) \triangleq - \int_{\mathcal{S}^2} f(x, y) \log_2 f(x, y) dx dy = \mathbb{E}_{f(x, y)} \left[ \log_2 \frac{1}{f(x, y)} \right] \tag{B.6}
\]

**Definition B4.** The conditional entropy of \( y \), given another variable \( x \) is defined as [6]:

\[
    H(y|x) \triangleq - \int_{\mathcal{S}} f(x) \left[ H(y|x) \right] dx
    \triangleq - \int_{\mathcal{S}} f(x) \left( \int_{\mathcal{S}} f(y|x) \log_2 f(y|x) dy \right) dx = - \int_{\mathcal{S}^2} f(x, y) \log_2 f(y|x) dx dy \tag{B.7}
\]

Note that the conditional entropy stands for the average uncertainty in \( y \) after observing \( x \).
Chain Rule [6]: The joint entropy of two random variables $x$ and $y$ is equal to the entropy of $x$ plus the conditional entropy of the $y$, given $x$ and vice versa, i.e.:

$$H(x, y) = H(x) + H(y|x) = H(y) + H(x|y) \quad (B.8)$$

The first step (and similarly the second step) of the equation above is proved as:

$$H(x, y) = -\int f(x, y) \log_2 f(x, y) dxdy = -\int f(x, y) \log_2 f(x)f(y|x) dxdy$$

$$= -\int f(x) \log_2 f(x) dxdy - \int f(x, y) \log_2 f(y|x) dxdy$$

$$= H(x) - H(y|x) \quad (B.9)$$

Similarly, it can be shown that for the $x_1, x_2, ..., x_n$ random variables the chain rule can be extended as [6]:

$$H(x_1, x_2, ..., x_n) = \sum_{i=1}^{n} H(x_i|x_{i-1}, ..., x_1) \quad (B.10)$$

Using the above entropy definitions and properties it can be also shown that:

$$H(x, y|z) = H(x|z) + H(y|x, z) \quad (B.11)$$

where $z$ is another continuous random variable.

Definition B5. **Relative entropy** is a measure of the distance between two probability density functions $f(x)$ and $g(x)$. It actually measures the inefficiency of our assumption that the distribution of a random variable $x$ is $g(x)$ when the actual distribution is $f(x)$ and it is defined as [6]:

$$D(f(x)\|g(x)) \triangleq \int f(x) \log_2 \frac{f(x)}{g(x)} dx \quad (B.12)$$

Definition B6. **Mutual Coupling**

**Mutual information** is a measure of the dependence between two random variables $x$ and $y$. It measures the amount of information that the one random variable contains about the other and denotes the reduction in the uncertainty of $y$ because of the knowledge
of $x$ [6]:

$$I(y; x) \triangleq \int_{S^2} f(x, y) \log_2 \frac{f(x, y)}{f(x)f(y)} \, dx \, dy \quad (= D(f(x, y) \| f(x)f(y)))$$

$$= \int_{S^2} f(x, y) \log_2 \frac{f(y|x)}{f(y)} \, dx \, dy$$

$$= \int_{S^2} f(x, y) \log_2 f(y|x) \, dx \, dy - \int_{S^2} f(x, y) \log_2 f(y) \, dx \, dy$$

$$= \mathcal{H}(y) - \mathcal{H}(y|x) \quad (B.13)$$
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