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RESTRAINED THERMAL MOVEMENT IN
CONCRETE ROADBASES

G.D.TAYLOR

A thesis submitted for the degree of Doctor of Philosophy
in the Faculty of Engineering
University of Surrey
1977
ABSTRACT

Lean concrete has, on account of its high stiffness, excellent load-spreading properties when used as a roadbase material in flexible pavements, although on the same account, appreciable stresses arise when movements are restrained by the subgrade or sub-base. Consequently, owing to the low tensile strength of the material, temperature falls result in the formation of cracks which, if sufficiently wide, may be transmitted through the superimposed bituminous surfacing.

Developments in the design of flexible pavements and in the specification and use of lean concrete roadbases are reviewed, followed by a consideration of the causes of cracking.

Expressions for the spacing and width of cracks resulting from restrained thermal contraction of a cementitious roadbase are derived in terms of the temperature fall, material properties and a constant frictional coefficient, in order to identify the critical parameters involved in each case.

Thermal movement measurements were carried out on a number of cement stabilised materials in various moisture conditions and an apparatus which would cause cracking of lean concrete by restrained thermal movement was designed and built. A programme of tests was devised principally to determine the susceptibility of the material to cracking as a function of age, the results indicating that cracking in lean concrete roadbases is likely to occur within one day of placing.

Additional tests were carried out to investigate the effects of inadequate compaction, the use of mixes of higher water content and cement content and the incorporation of steel fibres. It was found that mixes of increased tensile strength were more resistant to cracking, especially at early ages, but that they ultimately give rise to wider cracks which would be more likely to reflect through the bituminous surfacing.

Experimental results and published values of subgrade restraint were combined to give a more accurate estimate of crack spacing and crack width in a lean concrete roadbase at a given age and for a given temperature fall. Temperatures in a newly constructed roadbase were measured and the results used for the prediction of cracking patterns which were found to be in broad agreement with those detected.

The thesis concludes with a review of methods for controlling reflection cracking in flexible roads with cementitious bases.
ACKNOWLEDGEMENTS

The author wishes to extend his sincere thanks and gratitude to Mr. R.I.T. Williams, Reader in the Department of Civil Engineering, for the opportunity to undertake this course of study and for continuous help and guidance throughout the period of research. Thanks are also extended to Dr. D.J. Hannant for helpful criticism and to other members of staff of the Department for assistance in the manufacture of apparatus.

The experimental work was carried out at the North East Surrey College of Technology and the author is indebted to Mr. R.J. Towler, Head of the Department of Construction Studies for the provision of facilities, to internal supervisors, Mr. R.C. Slater and Dr. B.J. Smith and to technical staff for much practical assistance over a number of years.

Last but by no means least, the author would like to thank his wife for her patience and encouragement during the period of study and for typing the thesis.
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NOTATION

1. Symbols
Where symbols are used without subscripts, they denote:

\( f \) tensile stress
\( \varepsilon \) tensile strain
\( E \) tensile static modulus
\( T \) temperature change
\( \sigma \) Poisson's ratio
\( \gamma \) unit weight
\( \alpha \) coefficient of thermal movement
\( d \) slab depth
\( t \) slab displacement
\( \mu \) coefficient of friction or subgrade restraint
\( L \) crack spacing
\( \delta \) crack width
\( W \) density

Subscripts are reserved for specific aspects of properties and are defined in the appropriate section of use. S.I. units are used throughout.

2. Sign convention
For the purposes of analysis of stresses in restrained movement tests, forces, stresses, strains, movements or temperature changes which correspond to increases in dimensions are denoted positive and vice versa. Hence, for example, tensile stress, thermal expansion and temperature rise are normally denoted positive. In certain sections, however, as well as in most published information, the terms 'temperature fall', 'thermal contraction' and 'compressive stress' are employed in conjunction with one another and in these instances the negative sign is omitted since it is implied in the term.

3. Mix proportions
Mix proportions are by weight unless otherwise stated. Ratios for aggregates correspond to oven dry weights. Percentage water contents are expressed in terms of the total weight of dry material.
1. INTRODUCTION

The thesis is concerned with the performance of lean concrete roadbases used in the construction of flexible roads with particular emphasis on the effects of temperature reductions at early ages.

The introduction reviews briefly the historical development of road construction in the United Kingdom and the evolution of design standards and material specifications. Developments in the design of flexible pavements and in the use of cement-bound bases are reviewed with particular emphasis on lean concrete.

1.1. Historical background

The earliest well-documented form of road construction took place in Roman times and Roman roads were indeed built to last; for example, the formation of the Via Appia, which connects Rome to Brindisi, is still largely intact today. This high degree of durability was achieved by the intelligent use of natural materials, gravels forming the roadbase and stone slabs the wearing surface. The system of construction depended, however, on the intensive use of slave labour and was not therefore employed on a large scale once the Roman Empire had collapsed.

Road construction was not systematically studied further until the 18th century, when the French engineer, Tresaguet (1716-96), identified two essentials of any lasting road - a firm footing protected by an impervious surface. Telford (1757-1834) recognised these requirements and the need for the control of pavement gradients and alignment, and provided the first rate road which runs from Shrewsbury to Holyhead, completed in 1819 and now part of the A5 trunk road. The method of construction involved the use of a heavy stone base with successive layers of progressively smaller stones and a cambered surface which assisted in the shedding of rainwater.

McAdam (1756-1836) understood that the weight of vehicles must ultimately be borne by the soil and designed roads on the assumption that the soil, if dry, could carry substantial loads without deformation. He built road pavements consisting of layers of 50 mm
stones on a compacted cambered footing, the surface becoming smooth and watertight owing to the action of metal-tyred vehicles. These roads, though in some respects inferior to those of Telford, were much cheaper to build and were used extensively until the beginning of the 20th century. They formed the basis of what is now described as the flexible system of road pavement construction in which wheel load stresses are progressively reduced by particle interaction in successive layers of material.

The advent of rubber-tyred mechanised transport in the early 1900s tended to cause much dusting and eventual disintegration of unbound stone surfaces, creating the need for a binder which would resist the severe wear and tear that resulted. This requirement was fulfilled by the introduction of natural tar and bituminous binders which were first employed as early as 1840 and increasingly used in the cities, particularly in the City of London in the latter half of the 19th century. The opening decades of the 20th century were devoted by highway engineers to providing what were predominantly water bound pavements with some form of bituminous surfacing. In areas in which gravel aggregates were more readily available than roadstones, concrete roads were constructed, and these were early examples of the 'rigid' type of pavement construction. Design was, in each case, based on the experience of highway engineers in relation to local conditions and materials available. Between the First and Second World Wars, the number of motor vehicles in use more than trebled and the need for a rationalised road system was recognised. The Road Research Laboratory, formed in 1933, undertook the role of developing design codes for road pavements and for a motorway network which was proposed in 1938.

1.2. Introduction of design standards for new road pavements in the United Kingdom

The function of the design process is to produce, at reasonable cost, a durable road pavement of sufficient strength and stiffness to withstand anticipated traffic loadings, having regard to materials and techniques available. Design procedure may be approached from two quite different standpoints; either by the application of
knowledge gained from experience to date of the construction and subsequent performance of roads, or from theoretical considerations in which the interaction of the components of a pavement is studied in relation to measured properties of the materials used.

In common with many technological disciplines, the structural design of roads is traditionally based on the former approach since, although by the Second World War the bearing capacity of soils was defined in terms of California Bearing Ratios (C.B.R.)\(^6\), the interaction of components in a layered pavement proved most difficult to analyse and detailed material properties were not available. In the early 1950s, the Road Research Laboratory commenced an extensive programme of full scale experimental road construction and in the meantime produced design curves based on the performance of existing roads. These indicated the total depth of pavement required for a given commercial vehicle loading and C.B.R. of the subgrade and were first published in 1955 in the form of Road Research Note No. 19, 'The design thickness of concrete roads'\(^7\) and Road Research Note No.20, 'Construction of housing estate roads using granular base and sub-base materials'\(^8\). These were later combined to form Road Research Note No.29, 'A guide to the structural design of flexible and rigid pavements for new roads'\(^9\) published in 1960. Consideration is restricted in this thesis to flexible pavements, which are essentially multilayered with material of higher quality and higher cost at the surface and material of progressively lower quality in underlying layers. A description follows of the essential elements of a flexible pavement referred to in the guide.

(a) Surfacing. This provides an impervious wearing surface and contributes to the structural performance of the pavement while being sufficiently flexible to withstand movements of underlying layers and resistant to the long-term effects of traffic loads and environmental influences. The functions are most effectively fulfilled by the use of materials incorporating a bituminous binder, the surfacing usually consisting of a base-course containing aggregate particles of relatively large size and low binder content, and a wearing course containing aggregate particles of smaller average size and higher binder content.
(b) Roadbase. This performs the chief structural function of the road pavement in reducing the vertical stresses caused by wheel loads to levels which the underlying layers can withstand. Four alternative groups of material may be employed: rolled asphalt, dense-coated macadam, unbound stone and cement-bound materials. The bituminous binder used in the first two categories provides lubrication to assist in compaction during construction and contributes to strength in the completed base. The load spreading action of an unbound stone base is provided chiefly by particle interlock. In cement-bound bases, some flexural strength is developed due to the presence of the cement matrix, the strength level depending on the grading and quality of aggregate used. Two materials were initially defined: soil-cement, implying a material of fine particle grading and possibly low quality, and lean concrete, sometimes referred to as 'dry lean concrete', which employed aggregates of concreting quality and resulted in bases of substantially higher strength than could be obtained using soil-cements. More recently, a third category, cement-bound granular material was designated in which the material to be stabilised was of particle grading near the coarser limit of the soil-cement spectrum.

(c) Sub-base. The sub-base has some load spreading action though it may have the prime function of providing a stable, frost-resistant material to a specified depth in the road pavement. Sub-base materials generally comprise unbound stones of lower quality than those employed in roadbases, though cementitious sub-bases have been employed on account of their suitability to use by site construction traffic.

(d) Subgrade. This is normally the soil which prevails at the required depth of excavation, though it may be replaced by a more suitable material if, for example, it is frost-susceptible. When the road is constructed on an embankment, the fill is then regarded as the subgrade.

The 1960 edition of Road Note 29 specified by means of simple charts, thicknesses of surfacing, roadbase and sub-base dependent on the C.B.R. value of the subgrade and the commercial vehicle
loading, a commercial vehicle being defined as one having an unladen weight in excess of 1½ tons or 1530 kg. Loadings were based on anticipated commercial traffic intensities at an age of 20 years and, in the absence of other data, an assumed annual growth rate of 4 per cent. No allowance was made for differences in the various surfacing or roadbase materials which might be employed.

1.3. Developments in design standards for flexible roads

In the second edition of Road Note 29, published in 1965, additional information on the relative performance of bituminous and non-bituminous bases resulted in a reduction in design thickness of the former, while, for very heavily trafficked roads employing lean concrete bases, a composite construction was adopted comprising a 75 mm bituminous layer laid on the cement-bound base but of the same total thickness as specified in the 1960 edition.

During the 1960s, the advent of motorways and the curtailment of the railway system together with increased industrial outputs caused road traffic to increase considerably. Heavy vehicular traffic was no exception and results of the A.A.S.H.O. test\textsuperscript{11} provided vital data regarding the relative effects of various axle loads on pavement deterioration. For example, the passage of one vehicle of axle load 8200 kg was shown to be equivalent to the passage of 5000 vehicles of axle load 910 kg which is rather in excess of that corresponding to the average family car. Equivalence factors were therefore introduced so that, with the measured spectral distribution of axle loads, the traffic intensity of any one road could be estimated in terms of standard axles of load 8200 kg. The total number of standard axles is then calculated over a defined design life for a given traffic growth rate and figures so produced range from less than 30 000 standard axles for minor residential roads to over 10 million for motorways. This information, combined with the performance results of experimental roads, formed the basis of the third and current edition of Road Note 29\textsuperscript{12}, published in 1970. Sub-base thicknesses are determined from the cumulative number of standard axles loading and the C.B.R. of the underlying subgrade,
the roadbase and surfacing thickness being indicated by curves shown in Figure 1.1 when cement-bound bases are employed. The use of soil-cement in roadbases is restricted in this edition to roads designed to carry less than 1.5 million standard axles while cement-bound granular bases are restricted to use in roads designed to carry not more than 5 million standard axles. For cumulative traffic on roads having lean concrete or unbound stone bases, progressively increased thicknesses of surfacing are required, though for loads of over 11 million standard axles, any thickness over 100 mm may be provided by use of a bituminous surfacing or roadbase material forming a composite base with the wet mix, dry-bound macadam or lean concrete roadbase material. The use of unbound stone bases in motorways and dual carriageway trunk roads is now precluded for cumulative traffic loads of over this level so that lean concrete is now the only non-bituminous roadbase material which may be employed for very heavily trafficked roads. It is on this basis that the cracking performance of lean concrete was considered to merit more detailed study.

1.4. Use of cement-bound roadbase materials
Descriptions here are confined to soil-cement and lean concrete; soil-cement being the first widely used cement stabilised roadbase material and lean concrete the most extensively used cementitious roadbase material in the United Kingdom at the present time.

1.4.1. Soil-cement
The earliest recorded use of cement stabilised roads was in 1917 when Brooke-Bradley used Portland cement to impart stability into muddy tracks on Salisbury Plain in order to render them capable of carrying traffic, mostly in the form of steel-tyred horse-drawn vehicles. Cement stabilised roadbases were used on a substantial scale in South Carolina in 1932, the cement being used chiefly to help the soil retain its fully compacted state under prevailing traffic and weather conditions. It was in the United States of America that the most rapid development of soil-cement roads took place, Andrews reporting that
Figure 1.1. Thicknesses of cementitious roadbase and surfacing for flexible roads as a function of design life (Road Note 29).
by 1955 over 100 million square metres of road and runway pavement had been constructed in this way. The material is now employed worldwide on account of its cheapness and convenience though in the United Kingdom, rigorous testing requirements on the one hand and the ready availability of roadstone and concreting aggregates on the other, have been such that soil-cement has not become a material of major importance in road construction, its use being confined chiefly to sub-bases and to pavements having low loadbearing requirements such as housing estate roads, car parks and playgrounds.

1.4.2. Lean concrete

One of the earliest references to lean concrete was in 1942 when Markwick and Keep described the use of 'lean-mix rolled concrete' which they referred to as a material intermediate between soil-cement and normal concrete and it became evident at that time that substantial concrete cube strengths, for example, of over 30 MN/m², could be obtained if aggregates of concreting quality were used, in contrast to soil-cements for which strengths of approximately 2 MN/m² were more appropriate. In the United Kingdom, the use of lean concrete increased in post-war years and the Ministry of Health, in conjunction with the Institution of Municipal Engineers, published in 1944 a specification for the use of lean concrete in housing estate roads. The first extensive use of the material in roadbases was undertaken at Crawley New Town commencing in 1952 and the material was also employed in the bases of airfield runways and taxiways, being controlled by Air Ministry Specifications.

An indication of the extent of use of lean concrete in recent years in major road construction can be obtained from an analysis by James of 60 successful tenders for roads between 1969 and 1972. Of the 60 contracts, 56 were for flexible roads, lean concrete being used in 26 cases and forming 58 per cent of the total mileage constructed. The
relative use of flexible and rigid road pavements is based on the comparative cost of tenders, as well as subsequent maintenance costs and has fluctuated in the past ten years; for example, in 1973, over 20 per cent of motorway contracts were let using concrete paving, although according to information recorded by the Department of the Environment, the average proportion between 1968 and the present is approximately 15 per cent. Nevertheless, flexible pavements are currently the predominant form of road construction and lean concrete in this context remains an extensively used roadbase material in the United Kingdom.

1.5. Nature and specification of lean concrete

Lean concrete mixes have in essence a low cement content and involve the use of washed aggregate of concreting quality. The water content is chosen so as to give extremely low workability to suit compaction by rolling, though on account of the low cement content, water/cement ratios are generally high compared to those of normal concreting mixes and typically in the region of 1.0. The material can be transported in covered tipping lorries and bases are laid without joints other than construction joints, permitting rapid construction to take place. After a 7 day curing period, lean concrete bases may be used by construction traffic, providing a convenient working platform.

In the immediate post-war years, mixes of aggregate/cement ratio generally in the range of 8 to 14 were used, though many roads developed surface cracks which led to the adoption of mixes of lower cement content on the basis that lower strength material would crack more frequently to yield finer cracks with less risk of propagation into the surfacing. This philosophy was reflected in the 1957 edition of the Ministry of Transport specification which required the use of aggregate/cement ratios in the range 15 to 20 for lean concretes employing gravel aggregates and 18 to 24 for those with crushed rock aggregates since the angular shape of the latter tends to result in a concrete of increased strength for given mix proportions. A water content of 5 per cent by weight...
on the basis of saturated surface-dry aggregates was specified and requirements in respect of aggregate grading, quality and cleanliness were included, though no strength specification was laid down at that time.

The specification was modified in the 1963 and 1969 editions to include aspects of performance, a complex requirement being introduced regarding the compressive strengths at ages of 7 and 28 days of concrete compacted to refusal which would be generally satisfied by cubes having an average 28 day strength not lower than 14 MN/m². A limit to the range of strengths was also introduced and the density of compacted concrete obtained in the field was required to be not less than 95 per cent of the theoretical density at zero air voids. Aggregate/cement ratios in the range 15 to 20 were specified for all types of aggregate.

In the current edition of the specification²⁵ published in 1976, the former aggregate/cement ratio limits were withdrawn, though aggregate/cement ratios richer than 12 are not permitted and the 'Notes for Guidance'²⁶ include the following clause concerning very lean mixes:

"To conform with the strength requirements the normal aggregate to cement ratio should be between 15 and 20. However, some limestones with a high proportion of fines can become cementitious and when mixed as lean concrete provide higher strengths. In such cases the Engineer may permit an aggregate to cement ratio greater than 20, provided the method of mixing and quality control are adequate to produce a homogeneous material. There are difficulties in producing a consistent material with low cement contents, so an increase in the rate of testing may be necessary if aggregate to cement ratios are greater than 20. Aggregate to cement ratios higher than 24 are unlikely to result in an acceptably homogeneous mix".

The lower strength limit of the 1969 edition is retained and an upper limit expressed in similar terms introduced, which effectively requires an average strength of not greater than approximately 16 MN/m², 'in order to prevent regular cracking which inevitably
The current specification underlines the widespread belief that the strength of lean concrete roadbases should be very closely controlled, a minimum strength being necessary to ensure sufficient load spreading action and a maximum strength to guard against sympathetic surface cracking. As a result, however, only a narrow range of concrete strengths is acceptable so that difficulties may arise in designing mixes, particularly in view of the constraints applied to aggregate/cement ratios at the lean end of the range. Furthermore, the introduction of the upper strength limit may in some cases reduce the margin of protection of lean concrete against overstressing. It was in order to clarify the effects of strength on the performance of cementitious roadbases and to attempt to understand more fully the effects and mechanism of cracking generally that the project was undertaken.
2. EFFECTS OF CRACKING IN LEAN CONCRETE ROADBASES

It is widely accepted that some cracking will occur in cementitious roadbases laid in large lengths without movement joints, cracks often forming at early ages and becoming visible within the 7 day curing period\(^{27}\). However, pavement deterioration does not necessarily occur as a direct result of cracking in the base; it depends on the spacing and width of cracks and on the design and construction of the composite pavement. A consideration of the predicted and observed effects of cracking of lean concrete roadbases is first given in order to clarify the problem and this is followed in the next section by an assessment of the possible causes and their likely contributions to cracking in roadbases constructed in the United Kingdom.

Cracking in lean concrete roadbases may affect the performance of the pavement by causing discontinuities in the surfacing and it may also affect the structural performance of the pavement as a whole.

2.1. Surface cracking

Cracking in the surfacing caused by transmission of a crack originating in the roadbase is known as reflection cracking. A properly designed surfacing has considerable flexibility but if laid on a roadbase in which cracking has occurred, it would become subject to substantial strains in the region of cracks since thermal movements of the pavement tend to be concentrated at these positions. Reflection cracking may therefore occur in the course of time due to the effects of fatigue. If, in addition, a roadbase in a cracked condition shrinks after the surfacing is applied, tensile stresses may arise above discontinuities in the base, adding to fatigue effects.

If cracks occur in the base after surfacing, reflection cracking may follow relatively rapidly, subsequently becoming more extensive on account of fatigue. Each type of cracking will be influenced by the thickness and nature of the surfacing, the properties of the binder used and environmental conditions.

Once reflection cracking has occurred, water is admitted to the
road foundation leading in some cases to pumping at cracks and in addition the passage of traffic tends to cause gradual deterioration of the surfacing at the edge of cracks.

Significant transverse cracking has been widely observed in flexible roads constructed with lean concrete roadbases\textsuperscript{28}; Wright, for example, in a survey of 35 sites in England and Wales\textsuperscript{29,30} found transverse cracks in the surfacing in 12 cases, the intensity of cracking increasing with the age of the road. In a survey of county highway authorities by Brewer and Williams\textsuperscript{31} transverse cracking was reported in 42 per cent of replies though this did not appear to be regarded as a serious defect and the great majority of authorities reported that they were satisfied with the performance of lean concrete roadbases. In contrast, a similar proportion reported that local failure had occurred and this was considered to have a more serious effect on performance. A number of authors\textsuperscript{27,32,33} have observed that reflection cracking is associated with the use of relatively thin surfacings which would account for the fact that reflection cracking is particularly widespread in minor roads such as those serving housing estates.

2.2. Structural performance of the pavement

The load spreading action of a lean concrete base depends to some extent on flexural action, so that if cracking continued until slabs of less than perhaps 2 m were formed, the bearing properties of the pavement would be impaired, resulting in overstressing of underlying layers. The measured spacings of cracks vary widely, Williams reporting, for example,\textsuperscript{27} that many roads employing roadbases of aggregate/cement ratio 12 or 14 with some 80 mm of surfacing, constructed in the early 1950s, developed cracks of spacing 6 to 10 m, (figures were not based on detailed measurements\textsuperscript{34}) though some roads were relatively free of cracks. Lister\textsuperscript{35} reported values of between less than 2 m and 15 m with an average spacing of approximately 3.5 m for a road comprising a 60 mm surface layer of rolled asphalt on 100-200 mm of lean concrete after 13 years traffic.

The effect of cracking of the roadbase on load spreading performance
may be measured in terms of the change of effective elastic modulus which ensues and Jones suggested, as a result of surface wave propagation tests\textsuperscript{36}, that the modulus of an extensively cracked, weak, thin roadbase could become comparable to that of a compacted crushed stone base. This would lead in turn to marked pavement deterioration, though the general satisfaction of highway engineers with lean concrete roadbases and the excellent performance of the 230 mm thickness roadbases in the Alconbury Hill experimental road after 15 years service\textsuperscript{37}, would appear to indicate that cracking of this intensity does not normally occur.

In reporting the results of the Alconbury Hill experiment, Thompson et al\textsuperscript{37} suggest, however, that cracking in lean concrete roadbases may cause more rapid structural deterioration than when other roadbase materials are employed, probably due to the loss of flexural action on which the load spreading properties of the pavement depend. They found that by selecting an elastic modulus for lean concrete of 13.8 GN/m\textsuperscript{2}, about half that normally measured in the laboratory on uncracked test specimens, pavement deflections obtained by a purely elastic analysis were similar to those measured in a deflection (Benkelman) beam test. This suggested that the elastic modulus selected was not seriously in error, though further work is considered necessary to determine the causes of the substantial reduction in elastic modulus which appeared to be appropriate. Wang\textsuperscript{38}, for example, carried out deflection measurements on an experimental road with a soil-cement base in the U.S.A. and reported that, although progressive cracking occurred, there was no conclusive evidence that it could cause a reduction of pavement stiffness, though the effects would clearly depend on the degree of conservatism exercised in design in relation to the loading intensity actually occurring.

2.3. Discussion

If the statement of Thompson et al regarding the structural deterioration of pavements employing lean concrete roadbases is taken to apply only in cases of extended cracking, then overriding current opinion would appear to be that the performance of a well designed roadbase should not be seriously impaired by cracking
of the intensities normally observed in existing roads and that reflection cracking is not generally regarded as a major problem in the United Kingdom. This view is echoed by other countries in which cementitious roadbases are employed. The current climate of opinion is probably based on the fact that the treatment of reflection cracking is relatively easily achieved by sealing or by surface dressing; indeed, Earle reported in 1974 that 65 percent of all bituminous roads in England and Wales are dressed. In contrast, the structural failure of roads is likely to be more serious in terms of both expense and inconvenience to the road user, since affected areas must either be removed or covered with overlays of substantial thickness.

It is unfortunate that avoidance of the two effects places conflicting demands upon the roadbase material since a very low strength material would be highly unlikely to result in reflection cracking as previously defined while a concrete of high strength would be more satisfactory from a structural standpoint. Clearly the upper strength limit for lean concrete in the current Department of Transport specification together with the requirement for a composite roadbase and substantial surfacing thickness for heavily trafficked roads represent a safeguard against reflection cracking from the design standpoint, though it is considered that a careful balance must be maintained in guarding against reflection cracking on the one hand and structural failure on the other. In order to achieve this balance, it is essential to investigate the detailed causes of cracking, to identify those properties on which cracking depends and to provide information regarding these properties for lean concrete. Further discussion on this important topic is, therefore delayed until Section 19 when each aspect has been more fully investigated.
3. CAUSES OF CRACKING IN LEAN CONCRETE ROADBASES

The stresses in a roadbase at any time may be due to the combined effects of loads on the road pavement and restrained movements within the roadbase material. Movements occur principally in the form of shrinkage or as a result of heat of hydration and external temperature changes. Figure 3.1 indicates schematically the interplay of these constraints in producing stress and subsequent deterioration in the base. They will be considered in turn and, on account of the low tensile strength of lean concrete relative to its compressive strength, particular emphasis will be given to those influences which result in tensile stress. Lean concrete of aggregate/cement ratio 18 would, for example, be expected to have a tensile strength in the region of 1.4 MN/m$^2$ and a static tensile elastic modulus of 35 GN/m$^2$ at an age of 28 days. These would correspond to a failure strain of $40 \times 10^{-6}$, assuming a linear stress-strain relationship although, in practice strains about 20 per cent higher than this would normally occur at failure due to non-linearity.

3.1. Stresses due to loading

These will depend on the extent to which traffic loads are dispersed by the surfacing and also on the effective stiffness of underlying layers, as well as on the properties of the base itself. The bituminous surfacing and sub-base or subgrade have elastic moduli of approximately 10 per cent and 1 per cent of the value for lean concrete respectively so that a lean concrete roadbase would be expected to attract a substantial flexural stress when wheel loads are applied to the pavement surface.

A number of difficulties arise in attempting to calculate the maximum flexural stress caused by wheel loading in roadbases designed to current standards. For example, the strength requirements of the present specification for lean concrete are based on compression tests and the relationship between compressive strength and flexural strength is not unique, so that materials complying with the strength specification may have a range of flexural strengths. In addition, pavement performance depends on strengths and elastic moduli obtained in the field rather than
Figure 3.1. Schematic illustration of the mechanism of deterioration in flexible roads with cementitious bases due to loading and environmental influences.
those measured on control specimens though Lister\textsuperscript{35} has recorded good agreement between values measured on site and those measured in the laboratory where lean concrete was laid at moisture contents slightly wetter than optimum. Current field density requirements also guard against excessive density shortfall of the in-situ material.

Subject to such limitations, Lister computed the maximum tensile stresses to be expected in lean concrete bases employed in pavements designed according to Road Note 29\textsuperscript{12} and found that stresses occurring in the bases of very heavily trafficked roads should be significantly lower than those in less heavily trafficked situations on account of the load spreading action of the thicker surfacings which are required. Having made a correction for warping stresses arising from the maximum likely temperature gradient through the pavement, Lister found that combined stresses would probably cause cracking in pavements having design lives of less than 10 to 20 million standard axles. Thicker pavements, though stable in the short term, would form cracks owing to fatigue over longer periods and even those having the greatest design lives may also crack under repeated application of the very heaviest wheel loads. Lister added that although a prediction of the intensity of cracking of roadbases is not amenable to numerical analysis, it is reasonable to suppose that when lean concrete bases are designed according to Road Note 29, extended cracking which is a major cause of pavement deterioration would be avoided. The analysis also showed that maximum stresses in the base due to traffic would generally be at least four times greater than those due to maximum temperature gradients occurring in the base during the daytime.

Thompson et al\textsuperscript{37} carried out a theoretical elastic analysis of the stresses occurring in various types of pavement caused by passages of a 4100 kg wheel load and predicted that cracking in a 75 mm lean concrete base would occur quite rapidly owing to fatigue effects, while a base of 230 mm thickness would not be expected to fail. The performance of bases of 150 mm thickness would depend
on the long-term effects of fatigue and the authors emphasised the importance of having an accurate knowledge of fatigue data for lean concrete as laid on the road.

Otte, while accepting that some transverse cracking due to shrinkage is inevitable, affirmed that extensive cracking, in which the effective elastic modulus of the roadbase is reduced to that of an unbound material, only occurs when the base is overstressed as a result of inadequate design. He used the description 'secondary cracking' to distinguish severe cracking from that caused by shrinkage and suggested that, by using present knowledge, it is possible to utilise fully the high stiffness of cement-treated layers in road construction.

In summarising the effects of loading, it is evident that some cracking in a previously uncracked roadbase may arise, particularly if the base or the superimposed surfacing is of relatively low thickness. However, it should be recalled at this stage that some cracking in the roadbase often develops during the 7 day curing period in which not even site traffic is permitted access. In order to predict whether additional cracking would be caused by loading it is therefore necessary to determine the frequency of cracks already in existence prior to the application of traffic loads. Hence, it is apparent that loading effects cannot be considered in isolation, they must be assessed in conjunction with the other possible mechanisms of cracking still to be investigated.

3.2. Stresses due to drying shrinkage

Drying shrinkage, abbreviated subsequently as 'shrinkage', is the initial contraction caused by drying out of the concrete and it originates from the reduction in volume of cement gel which occurs on removal of adsorbed water, the contraction being restrained by the aggregate to a degree dependent upon its stiffness and volume fraction. Lean concrete should, on this basis, exhibit low shrinkage compared to normal concrete mixes, on account of its low cement content and high aggregate content.
There is little published information on the shrinkage of lean concretes, though Marais measured that of concretes containing aggregates of average particle size 5.7 mm with a cement content of 7 per cent and an initial moisture content of 7.4 per cent. Samples were cured for 7 days and then dried to constant length in air of relative humidity 90 per cent and temperature 25°C. A shrinkage strain of $200 \times 10^{-6}$ was obtained. Nakayama and Handy measured a shrinkage strain of $220 \times 10^{-6}$ in a pure silica sand stabilised with 10 per cent cement and dried in air after 7 days moist curing. A limited series of tests undertaken as a part of this project and carried out in accordance with BS 1881 Part V yielded an average shrinkage strain of $200 \times 10^{-6}$ after a one week curing period and two weeks drying, there being no significant difference between concretes cured in the sealed or saturated states prior to testing.

Although strains of this magnitude would cause tensile failure in a restrained lean concrete roadbase, it is doubtful whether a substantial fraction of potential shrinkage could occur within a few days of placing since it is usual to apply an impervious membrane in the form of a sprayed bitumen emulsion shortly after placing. The resultant humidity would be likely to approach 100 per cent under these conditions and drying out would be slow, though dependent on the effectiveness of the curing membrane and the prevailing weather conditions. A number of authors have nevertheless attributed reflection cracking to shrinkage of cement-treated bases, though in most instances this may be linked to aspects of construction methods or materials used. For example, Otte reported extensive reflection cracking caused by shrinkage of cementitious bases in roads in South Africa but he indicated that the base curing membrane was in some cases not applied until three weeks after construction and the relatively high temperatures encountered in South Africa compared to the United Kingdom could also reduce the effectiveness of the curing membrane once applied. It is considered significant that shrinkage cracking took one month to become visible in this case, compared to the much smaller periods which elapse before cracking becomes visible in the United Kingdom. Bofinger, commenting on the high shrinkages observed
in soil-cements, suggested that large quantities of moisture could be drawn downwards into the subgrade, especially in the case of roads constructed in the summer season in tropical countries, and this could significantly increase shrinkage even at early ages. Substantial shrinkage has been observed in cement stabilised soils in the sealed state and this is considered to be an indication of the inherent differences between lean concrete and soil-cement; for example, increasing the cement content of the latter within certain limits has been observed to decrease shrinkage, in contrast to the well-known effect of increasing the shrinkage of normal concretes.

There is, therefore little theoretical or experimental evidence to suggest that shrinkage is a major cause of cracking in lean concrete roadbases in the United Kingdom, the phenomenon probably rather increasing the width of cracks once formed and possibly contributing to further cracking in the material at greater ages.

3.3. Stresses due to heat of hydration

Some heat of hydration is generated as soon as mixing water is added to the concrete and the heat input to the concrete rises subsequently to a maximum value which depends on the type and fineness of the cement and the temperature of the cement paste. The maximum rate usually occurs at an age of 10 to 15 hours, approximately 50 per cent of the total heat having been liberated by the latter time. The concrete does not harden significantly during the earlier part of this period so that some movement caused by the consequent temperature rise would result in plastic rather than elastic distortion, though by the time the temperature returns to ambient level, the concrete has normally stiffened and in conditions of restraint, would therefore be subject to tensile stress.

An indication of the likely temperature rise of a lean concrete roadbase can be obtained from a knowledge of the heat produced by hydration and the thermal capacity of the material provided the rate of heat loss to the atmosphere can be estimated.
The heat output of a typical ordinary Portland cement during the first day is approximately 200 kJ/kg of cement which would correspond to 10 kJ/kg of concrete having an aggregate/cement ratio of 18 or to 3.4 MJ/m² for concrete in the form of a slab of density 2300 kg/m² and thickness 150 mm. Assuming a specific heat capacity for lean concrete of 960 J/kg°C, a temperature rise of 10°C would occur if the heat of hydration were applied adiabatically, though in practice, a rise of this magnitude would be most unlikely since heat would always be lost from the slab upwards and, to some extent, downwards as soon as the temperature rose above ambient level.

A simple prediction of the temperature rise to be expected on account of heat of hydration in a lean concrete roadbase is given in Appendix A. A temperature rise of approximately 4°C was estimated for constant ambient temperature, and although the approach considered only an averaged heat input to the concrete over a period of one day, it is considered to give an adequate indication that temperature rises significantly in excess of 4°C on account of heat of hydration are unlikely. Results of work by Hunt who computed temperature changes in concrete pavements of 250 mm thickness at early ages would appear to be in broad agreement with this view, heat of hydration effects being masked by environmental temperature changes. It was found that for concrete placed at 08.00 hours on a hot sunny day in June, the maximum surface temperature on the same day would be 2°C higher than that on the second day and that a similar decrease of 2°C would occur on the third day, assuming identical daily weather patterns (Figure 3.2). The results suggest that the cumulative effect of heat of hydration would reach a maximum during the first night after placing, the predicted maximum temperatures on the first and second days being before and after this peak respectively.

It is considered similarly that if lean concrete roadbases are constructed during daylight hours, the main effect of heat of hydration would be to compensate to some extent for the reduction in environmental temperature during the first night after placing. Further consideration will be given to the effects of heat of hydration once environmental effects have been investigated.
Figure 3.2. Predicted temperatures at the upper and lower surfaces of a 250 mm thick concrete slab cast on a hot sunny day in June.

Figure 3.3. Temperature records of the top and bottom of a 225 mm thick concrete road slab together with air temperature.
3.4. Stresses due to external temperature changes

A description of weather patterns in the United Kingdom is first given, followed by details of predicted and measured pavement temperatures and finally by a consideration of the stresses likely to arise.

3.4.1. Environmental temperatures and solar radiation levels in the United Kingdom

Daily and seasonal temperature changes are fundamental to the system of planetary motion of which the earth is part and the superposition of related climatic effects caused by atmospheric pressure changes results in weather patterns which are enormously complex and often unpredictable. Weather forecasting, even in the short term, is therefore often tentative and inaccurate and these situations pose extensive problems both for the engineer in allowing for the effects of environmental changes in design of the pavement and for the construction team in carrying out work programmes in frequently changing weather conditions.

Although average air temperatures, together with diurnal variations at a given location and time of the year are well documented, the most detrimental effects of weather probably correspond to the most extreme temperature variations. It is therefore more important from a design standpoint to present the information in statistical terms such that the probability of a given temperature or temperature range occurring in a certain period can be predicted. Unfortunately, little information has been expressed in these terms and therefore design must be based on the restricted data available. On account of the heavy dependence of climatic conditions on geographic location, consideration here is restricted chiefly to temperatures predicted and measured in the United Kingdom, where the influence of the Gulf Stream results in relatively small diurnal and seasonal variations compared to those of most other countries.
Average daily maximum and minimum air temperatures at Kew at various times of the year are shown in Table 3.1, it being evident that greatest diurnal variations occur in the Spring. In Summer months, air temperature variations are slightly smaller than during the Spring and in the Winter months they reach a minimum value.

Table 3.1. Average daily maximum and minimum air temperatures at Kew

<table>
<thead>
<tr>
<th>Month</th>
<th>Average daily temperature, °C</th>
<th>Average temperature variation, °C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max.</td>
<td>Min.</td>
</tr>
<tr>
<td>January</td>
<td>6.3</td>
<td>2.2</td>
</tr>
<tr>
<td>March</td>
<td>10.1</td>
<td>3.3</td>
</tr>
<tr>
<td>May</td>
<td>16.7</td>
<td>8.2</td>
</tr>
<tr>
<td>July</td>
<td>21.8</td>
<td>13.5</td>
</tr>
<tr>
<td>September</td>
<td>18.5</td>
<td>11.3</td>
</tr>
<tr>
<td>November</td>
<td>10.1</td>
<td>5.3</td>
</tr>
</tbody>
</table>

During daylight hours, there is a tendency for substantial solar heat inputs to occur in horizontal surfaces, especially during the Summer months. For example, the maximum solar radiation on a clear June day in Southern England would be approximately 800 W/m², while in December, it would be approximately 200 W/m², assuming the ground reflects 20 per cent of the radiation. The total daily heat input due to solar radiation varies within much wider limits between Summer and Winter than the solar intensity and therefore has an important effect on pavement temperatures. For example, on a clear June day in Southern England, the total radiant energy input to a horizontal surface would be typically in the region of 15 MJ/m², whereas a similar day in December would result in an accumulated heat input of only 1.5 MJ/m². These figures, if compared to the heat of hydration input of 3.4 MJ/m² for a 150 mm lean concrete roadbase referred to earlier, indicate that in the
Summer, significant temperature rises can be expected, particularly in view of the fact that a large proportion of the solar heat input occurs within a period of about eight hours.

The principal mechanisms by which the temperature of a road pavement may respond to environmental changes are convection, which takes place when there is a temperature difference between the air and pavement surface, and radiation due to solar effects and modified by the presence of clouds. These two effects are often interdependent and in addition, temperature changes may be caused by precipitation, particularly in hot weather, though the occurrence of these situations is considered insufficiently frequent to justify detailed examination.

Changes in the environmental temperature or radiation level directly affect only the surface material of the pavement, underlying layers responding subsequently according to their thermal conductivity, specific heat capacity and density. Since heat can only flow in a solid material as a result of a temperature gradient, it follows that inputs of heat to the surface of a slab will result in temperature differentials within the slab and that there will be a time delay between the occurrence of maximum and minimum temperatures at the surface and at lower levels.

Although temperature distributions in completed flexible road pavements have been measured, there is little published information relating to temperatures occurring in the roadbase at early ages prior to surfacing and attention will therefore be focussed mainly upon the effects of environmental changes in concrete road pavements, since, from a thermal point of view, their behaviour is likely to approximate most closely to that of lean concrete roadbases.
3.4.2. Temperature variations in concrete roadbases and pavements

The theoretical temperature distribution in a pavement to which surface temperature fluctuations of a sinusoidal nature are applied has been analysed by Thomlinson, though it is only more recently and with the aid of computers that the response of pavements to more realistic heat exchanges, including those due to convection and heat of hydration, have been predicted.

Results obtained by Hunt who computed temperatures in concrete road slabs at early ages on the basis of these exchanges have already been considered from the point of view of heat of hydration, though Figure 3.2 indicates that on a clear June day, environmental effects are dominant. The temperature of concrete placed at $20^\circ C$ rose by over $15^\circ C$ with maximum temperature differentials of approximately $10^\circ C$ occurring between the underside and surface of a 250 mm slab at about dawn on each day. Hunt also predicted pavement temperatures at other times of the year and found that temperature variations would be much less marked in the Spring and Autumn. Heat gains by hydration almost balanced radiation and convection losses of concrete placed at a temperature of $20^\circ C$ on a clear day in October. Predicted temperatures in each case were based on a surface absorptivity of 0.4 corresponding to the level which would be expected for a slab covered with a curing membrane, containing an aluminium pigment.

Sparkes, reporting on experimental work to determine the effect of environmental changes on concrete pavements, found that temperature variations of over $10^\circ C$ could occur in the concrete between day and night and that temperature differentials of the same magnitude could arise between the surface and underside of slabs during the daytime owing to the heating of surface layers by radiation. Figure 3.3 shows a record, described as typical, of temperatures of the air, and at the surface and underside of a 225 mm concrete slab. Details of weather
or season were not included but the air temperatures of Figure 3.3 would suggest fine Summer weather at the time of measurement.

Work by Franklin on reinforced concrete slabs at early ages is of interest since average, rather than extreme, weather conditions were prevalent at the time of placing. Figure 3.4 shows temperatures recorded at a depth of 50 mm in a 254 mm thick reinforced concrete slab constructed in the month of June, together with air temperatures for the first 26 hours after placing. Franklin presented information on temperature differentials in the form of temperature gradients at a depth of 50 mm, the largest value obtained in the test illustrated being \(-0.057^\circ\text{C/mm}\) 12 hours after placing, the negative sign indicating that the surface was cooler. If assumed to apply to the entire slab section, this temperature gradient would correspond to a temperature differential of over \(14^\circ\text{C}\) between the underside and the surface, though the validity of such an assumption is questionable, since temperature gradients in the slab would be expected to vary, being greatest near the main heat source or sink which is normally the surface of the slab. A similar test carried out in November and shown in Figure 3.5 resulted in very much reduced temperature variations and the maximum differential in this case was \(-0.028^\circ\text{C/mm}\) 18 hours after placing.

3.4.3. Temperature variations in bituminous pavements

Prior to discussion of the effects of temperature variations in concrete roadbases, brief consideration is given to two further papers which, although relating to bituminous pavements, include aspects of temperature changes not covered in detail in the previous section.

Wilson made temperature measurements in bituminous pavements which were adjacent to the experimental road at Alconbury by-pass. Although none of the pavements
Figure 3.4. Temperatures recorded in a newly placed reinforced concrete slab of thickness 254 mm constructed in the month of June, together with air temperatures\textsuperscript{57}.

Figure 3.5. Temperatures recorded in a newly placed reinforced concrete slab of thickness 254 mm constructed in the month of November, together with air temperatures\textsuperscript{57}.
employed lean concrete roadbases, temperature distributions in a variety of bituminous and non-bituminous bases were found to be very similar and it is considered therefore that a lean concrete roadbase would also behave in much the same manner. Figures 3.6 and 3.7 show temperature distributions in a pavement comprising a 100 mm rolled asphalt surfacing on a 250 mm granite aggregate tar macadam base in hot and cold weather respectively. During hot weather, temperatures varied greatly and temperature differentials in the region of $10^\circ\text{C}$ arose between the surface and a depth of 100 mm, the maximum differential occurring at about midday. At night, differentials were reduced to approximately $2.5^\circ\text{C}$. In Winter, maximum temperature differentials were much lower than in Summer, values varying between $2^\circ\text{C}$ and $4^\circ\text{C}$ depending on weather. At night, temperature differentials were very small except during frosty nights, Figure 3.7 indicating a value of $3^\circ\text{C}$ on two occasions. In both Figures 3.6 and 3.7 the effects of changing weather patterns on pavement temperatures are apparent.

Galloway$^5$ measured temperature durations at various depths in bituminous roads with lean concrete and asphalt bases over a period of several years, providing information as to the likely average temperature which would occur in the roadbase at a given time of the year, together with the observed scatter of values. Figure 3.8, which is extracted from measurements taken by Galloway over a one year period, shows average temperatures at the upper surface of the 250 mm lean concrete base which was covered by a 100 mm rolled asphalt surfacing. The graph indicates that Summer temperatures of the base were broadly in the range 20-25°C with Winter temperatures in the region of $10^\circ\text{C}$, variations due to weather changes being apparent. From May to September temperatures of the base were relatively evenly distributed between $15^\circ\text{C}$ and $30^\circ\text{C}$ probably due to diurnal changes, while in Winter they
Figure 3.6. Temperatures recorded at different depths in a bituminous road pavement during hot weather.$^8$
Figure 3.7. Temperatures recorded at different depths in a bituminous road pavement during cold weather."
Figure 3.8. Recorded average temperatures at the upper surface of a lean concrete roadbase under a 100 mm rolled asphalt surfacing. 
varied comparatively little. It is unfortunate that Galloway, who was concerned chiefly with the effect of temperatures on the bituminous surfacing, recorded totals of temperature durations on a weekly rather than on an hourly basis, since the latter would provide valuable information on the average and scatter of diurnal temperature changes.

Predictions of temperature gradients in road pavements have been made by Williamson and Hunt and measurements on a pavement comprising a 100 mm rolled asphalt surfacing on a 250 mm granite aggregate tar macadam base were obtained by Wilson. Figure 3.9 shows temperature profiles recorded at two hourly intervals on a hot Summer's day. The differential is seen to be greatest at 14.00 hours, the lowest values being during the latter part of the night. In almost all cases, there were marked variations in temperature gradient with depth, gradients generally being greatest at the surface. The graph also indicates that diurnal temperature variations below a depth of 350 mm in the pavement were relatively small, being less than 5°C in spite of very large variations in surface temperature in the period of measurement. Differentials within the 350 mm depth were much reduced in Winter, rarely exceeding 5°C.

3.4.4. Stresses arising in concrete roadbases due to temperature change

These, for the purpose of analysis, are conveniently broken down into the components resulting from:
uniform temperature change
uniform temperature gradient
non-uniform temperature gradient.

Uniform temperature change. This would cause a strain in the roadbase and consequently a stress of magnitude dependent on the degree of subgrade restraint, the maximum value in a situation of absolute uniaxial
Figure 3.9. Temperature profiles in a bituminous road pavement during hot weather at stated times. 

Temperature, °C

Depth of thermocouple, mm

06 08 10 12 14 16 18 20 22 24 26 28

55 50 45 40 35 30 25 20 15

350 300 250 200 150 100 50
restraint being equal to the product of the thermal strain in the material and its elastic modulus. For a material of elastic modulus $E$ and coefficient of linear expansion $\alpha$, having undergone a temperature change $T$, the stress would be $E\alpha T$. In the case of biaxial restraint in the horizontal plane which would apply to a slab of great width and length, the stress would be increased to

$$\frac{E\alpha T}{1 - \sigma}$$

where $\sigma$ is the Poisson's ratio of the material, and this would increase the stress in the concrete by between 15 and 25 per cent, depending on its maturity. The earlier expression would apply near the edges of such a slab.

Subgrade restraint acts at the underside of the slab, which would therefore tend to warp, a temperature rise resulting in a convex surface profile and a temperature fall in a concave surface profile. In the case of large slabs, warping would be balanced by stresses due to the self-weight of those parts of the slab that tend to lift, except near the edges where some warping would occur.

**Uniform temperature gradient.** Timoshenko\textsuperscript{60} has shown that a uniform temperature gradient in a completely unrestrained slab would not result in any stress in the material, though warping of the slab would occur. However, warping would, in practice, always be restrained by the self-weight of the slab and possibly also by traffic loading, so that significant stresses may arise. Westergaard\textsuperscript{61} showed that the resulting flexural stress for a differential $T$ between the surface and underside of central regions in a very large slab is

$$\frac{E\alpha T}{2(1 - \sigma)}$$

which is half the stress occurring due to a uniform temperature change of the same value as given above. It follows that, if a slab has cracked, thereby allowing some
freedom of longitudinal movement, warping stresses could be of greater magnitude than those due to longitudinal restraint. Warping stresses are lower near the edge of slabs owing to reduced self-weight in these regions; for example, Westergaard found that the stresses in a 150 mm slab progressively reduced from the maximum value at a distance of five to six metres from the edge, to zero at that edge. Slabs of dimensions in excess of double these distances would be subject to a constant maximum warping stress in central areas and slabs of smaller dimensions would be subject to a lower and variable warping stress, increasing with distance from each edge.

Non-uniform temperature gradient. The effect of any non-linearity in the temperature gradient is to cause internal stress, whether or not the slab is restrained by its self-weight. The type and magnitude of the internal stress can be determined by a consideration of the equilibrium of a slab section having a temperature distribution of the form illustrated in Figure 3.10 which is similar to that occurring at 10.00 hours in Figure 3.9. If the slab were completely restrained, the strain in it at any depth $x$ would be proportional to the difference between the actual temperature at that depth and the initial zero stress or reference temperature. The equilibrium of the slab could then be established by means of internal forces $F_x$ which will also be proportional to this temperature difference (Figure 3.10). The equilibrium of the whole section could be obtained by application of a single force $F$ equal to the sum of elemental forces $F_x$. The magnitude of this force would be determined by the difference between the average slab temperature and the reference temperature, and its eccentricity by the magnitude of the temperature gradient across the slab. The internal stress at any point is then equal to the difference between the stress due to the imaginary external force $F$ and the stress caused by the thermal strain at that point (Figure 3.11). Figure 3.12
Figure 3.10. Equilibrium of a concrete slab under stress due to non-uniform temperature gradient.

Figure 3.11. Stresses due to elemental forces $F_x$ and the resultant force $F$.

Figure 3.12. Warping stresses due to the temperature differential of Figure 3.10.
shows the internal stress distribution for a 150 mm slab during the daytime predicted by Sparkes, together with the stress distribution which would be obtained if the temperature gradient were assumed to be uniform. For simplicity, the stress arising due to the uniform temperature component has been omitted. Additional compressive stress occurs in surface layers with a region of tensile stress in central regions of the slab and a substantial reduction in tensile stress at the underside compared to that which would arise due to a uniform temperature gradient. At night, the effect of internal stress would be reversed, an additional tensile component occurring at the surface.

3.4.5. Discussion
Comments at this stage are intended to focus attention onto those aspects of environmental changes which are of chief importance in determining cracking patterns in lean concrete roadbases at early ages. Although much of the information given is based on the results of research carried out into concrete roads or completed flexible pavements, the principal arguments can also be applied to lean concrete roadbases provided allowance is made for differences in material properties and the method of construction employed.

It is evident that hot Summer weather represents the most adverse conditions from a thermal stress point of view in concrete roadbases since diurnal temperature variations of over 20°C are by no means uncommon and these may be accompanied by temperature gradients in excess of 10°C. Taking an elastic modulus of 35 GN/m² which would correspond to a mature concrete, together with a coefficient of thermal expansion of $12 \times 10^{-6}$/°C and a Poisson's ratio of 0.2, a temperature differential of 10°C through a concrete slab during the daytime would result in a flexural stress of 2.6 MN/m² ignoring end restraint, though the value would be reduced by the effect of internal stress, if a surfacing was used over the material and by restraint of the subgrade to overall expansion of the base. Lister, for example,
assumed a maximum thermal stress of 0.82 MN/m² when a cementitious base was employed under a surfacing of approximately 60 mm thickness and this was reduced to 0.31 MN/m² when the surfacing thickness was increased to over 200 mm.

In Winter, temperature variations and differentials were much lower although an additional overall tensile stress would arise in the pavement on account of the seasonal reduction in average temperature of between 10°C and 15°C suggested by Figure 3.8. if thermal contraction were restrained by the subgrade.

In making an assessment of thermal stresses in roadbases, it is most important to consider the extent to which movements are restrained by the subgrade. For instance, in calculating the stresses given above Lister assumed that transverse cracking had occurred and that stresses due to subgrade restraint were therefore negligible. In contrast Otte calculated stresses likely to arise in uncracked roadbases and found that the maximum tensile stress would occur in the period 06.00 hours to 09.00 hours when the average temperature of the base, which was in this case under a 150 mm crusher-run layer as well as a bituminous surfacing, reached a minimum. Subgrade restraint would be largely responsible for such stresses and at early ages in particular, failure may occur rapidly if a lean concrete roadbase is subjected to restrained thermal contraction since the elastic modulus of the material rises rapidly after stiffening but tensile strengths are at this stage extremely low. Kolias, for example, measured a uniaxial tensile strength of 0.49 MN/m² in a typical lean concrete at the age of 2 days which is only 35 per cent of the 28 day value, whereas the elastic modulus at this age was approximately 57 per cent of its later value. The tensile strain capacity of the material is evidently much reduced at early ages so that the effect of tensile stress
would be much greater at this stage. The temperature patterns described are now therefore applied to a lean concrete roadbase from the time of construction.

Road construction is carried out almost exclusively during daylight hours, so that volume changes owing to temperature rises occurring in a lean concrete roadbase on the day of placing would be unlikely to result in stresses in the material, the chief effect of any temperature rise being to accelerate hydration, particularly during fine Summer weather. The temperature to be used as a reference for the calculation of stresses in a roadbase would be that prevailing at the time of stiffening, though this process is progressive and it is likely that extensive stress relief by creep occurs in concrete in which some degree of stiffening has taken place. Franklin detected the time of stiffening in pavement quality concrete by means of a discontinuity in the response of vibrating wire strain gauges and found that the time of stiffening, so defined, ranged from 4½ to 8 hours, though it would probably be greater for lean concrete owing to the smaller volume fraction of cement employed. It is possible that, for lean concrete roadbases laid in the morning in Summer weather, some stiffening could occur by the time its temperature reaches a maximum, usually at about 15.00 hours, though in Winter, the time of the effective 'zero stress' temperature would be later. On this basis, Summer conditions would result in the greatest stress; for example, the temperatures in the roadbase at the point of stiffening could well be as high as 25°C, while by dawn of the following morning, a temperature fall in excess of 10°C could occur in the material. Significant temperature gradients could also occur at this time although the absence of construction joints in lean concrete roadbases is considered to have the most decisive effect on their behaviour, since the consequently high degree of end restraint would result in a tensile
stress due to the overall temperature fall. At the surface of the roadbase the stress would be supplemented by tensile components of internal and warping stresses and, on account of the low tensile strength of lean concrete at this stage, there would be a high probability of cracking even in the case of quite modest temperature falls. It is possible that the effects of warm Summer weather could increase still further this tensile stress at the surface of the roadbase, since solar heat would cause concrete at or near the surface to hydrate more rapidly than that in underlying layers, causing more rapid stiffening and a higher elastic modulus than in the latter. The subsequent reversal of the temperature differential during the first night would then exacerbate the effect of the tensile surface stress, accelerating failure.

Stresses arising shortly after construction would be much reduced at other times of the year on account of lower diurnal variations, smaller temperature differentials and the tendency for the point of stiffening of concrete to be delayed until the night after placing, thereby resulting in a lower zero stress temperature. Expansion failures could however occur subsequently if temperature rose substantially after setting\textsuperscript{23}. Results such as those given in Figures 3.6 and 3.7 indicate clearly that weather would always be an important factor in determining the magnitude of stresses.

### 3.5. Experimental observations of restrained thermal cracking

The location of cracks in concrete pavements at early ages by visual inspection is often difficult and in the case of lean concrete roadbases the coarse surface texture and presence of a bituminous curing membrane do not facilitate visual detection. There is however considerable experimental evidence to suggest that restrained thermal movement is an important factor in causing cracking. Rhodes\textsuperscript{63}, for example, found that cracking in concrete pavements constructed in Spring and Summer occurred predominantly in slabs
laid in the morning, while pavements constructed in the Autumn did not exhibit this tendency. Williamson observed that reflection cracking in a flexible road with a cement-bound base was completely absent in a shaded area of pavement beneath a wide bridge, other parts of the pavement being extensively cracked. Wright, in a full-scale experiment on roads incorporating lean concrete roadbases, found that significantly less reflection cracking occurred when limestone aggregate was used, resulting in a reduced coefficient of thermal expansion of the base.

3.6 Overall comment

In concluding generally this review of the causes of cracking, it is evident that the combined effects of load and temperature must be determined in assessing the structural stability of the pavement in service. It is likely however that temperature changes will cause significant cracking in cementitious roadbases within a few days of construction, and that susceptibility to further cracking and reflection cracking will depend on the degree of cracking occuring at this stage. Whereas warping stresses caused by temperature differentials through the roadbase probably represent the most adverse effect of temperature changes in pavements in service, overall temperature reductions are the most likely cause of cracking at early ages since thermal contraction of the material is restrained by the subgrade at a stage when its tensile strength is low compared to its stiffness.
4. THEORY OF CRACKING CAUSED BY RESTRAINED THERMAL CONTRACTION

The following analysis is confined to the effect of temperature reductions in a cementitious roadbase and is based on the assumptions that
(a) creep effects are negligible (The effects of stress relief by creep are discussed later)
(b) temperature gradients in the slab are negligible
(c) restraint due to subgrade is confined to the longitudinal direction
   (In cases where the theory predicts crack spacings of the same order of magnitude as the width of a base, predictions of cracking intensity will be conservative.)
(d) subgrade restraint produces uniform stress in the base.

4.1. Derivation of expressions for crack spacing and width

A simple prediction of the minimum temperature fall $T_1$ required to cause cracking in a very long slab can be made by supposing that no contraction owing to thermal movement is possible. The tensile strain due to thermal contraction would be $\alpha T_1$ and this must be equal to the ratio of the tensile strength of the material $f$, to its elastic modulus in tension $E$ if the slab is on the point of failure:

$$\alpha T_1 = \frac{f}{E}$$

or

$$T_1 = \frac{f}{\alpha E} \quad -----(4.1)$$

Cracks so produced would be widely spaced, since they would result in a small, but critical, reduction in stress to below the previous value at failure. On further temperature reduction of total magnitude $T$, increased frictional restraint would however occur, resulting in a further increase of tensile stress. The spacing of cracks, so produced, can be calculated by considering the consequent build up of stress in each slab. If the frictional coefficient a distance $x$ from the free end of a slab is $\mu_x$ and the slab is on the point of cracking, the force per unit width at the centre due to frictional restraint will be:

$$\int_0^{L/2} \mu_x \gamma \, dx = f d \quad -----(4.2)$$

where $L$ is the length, $d$ the depth and $\gamma$ the unit weight of the slab.
The total elastic extension of the slab is:

\[ 2 \int_0^{L/2} \frac{f_x}{E} \, dx = 2 \int_0^{L/2} \mu \gamma \, dx \, dx \quad \text{(4.3.)} \]

The crack width \( \delta \) is equal to the difference between the thermal contraction of the slab and the elastic extension:

\[ \delta = \alpha LT - \frac{1}{E} \int_0^{L/2} \mu \gamma \, dx \, dx \quad \text{(4.4.)} \]

The evaluation of equations 4.2., 4.3. and 4.4. requires a knowledge of material properties and of the coefficient of frictional restraint, \( \mu \), which is influenced by a number of factors including slab displacement. The equations therefore become implicit, though it may be appropriate to suppose that \( \mu \) is constant in order to obtain a simple, if tenuous, analysis of the chief factors involved in determining crack spacings and widths.

On this basis, equation 4.2. becomes

\[ L = \frac{2f}{\mu \gamma} \quad \text{(4.5.)} \]

and integration of equation 4.4. results in a crack width of

\[ \delta = \alpha LT - \frac{\mu \gamma L^2}{4E} \quad \text{(4.6.)} \]

4.2. Preliminary discussion

Equation 4.1. indicates that the temperature fall necessary to cause cracking in a long slab depends on the ratio of tensile strength to elastic modulus and on the coefficient of thermal expansion of the material, being proportional to the former and inversely proportional to the latter.

It is now established that the ratio of tensile strength to elastic modulus for lean concrete increases significantly with age, the approximate range of values in strain units being from \( 15 \times 10^{-6} \) at 1 day of age to \( 50 \times 10^{-6} \) at 28 days. Taking a thermal coefficient of \( 12 \times 10^{-6}/^\circ \text{C} \), initial cracking should occur as a result of
temperature reductions of 1.25°C and 4.2°C at these ages respectively. At ages of less than 1 day, it is possible that cracking could occur as a result of even smaller temperature falls since the ratio of tensile strength to modulus is likely to be lower at this stage.

The coefficient of thermal movement for concrete is known to vary with the type of aggregate used. Limestone aggregate concrete, for example, which has a coefficient of thermal expansion of approximately $8 \times 10^{-6}/°C$ would, if the same strength/modulus ratios given above are used, crack on temperature reductions of approximately 1.9°C and 6.3°C at 1 day and 28 days respectively.

Equation 4.5 indicates that providing the frictional coefficient is independent of slab movement, the slab length depends only on the frictional restraint and unit weight of the base and on the tensile strength of the material, being independent of its elastic modulus and coefficient of thermal movement. Taking, for example, a coefficient of friction of 1.5 and unit weight of concrete equal to 23 kN/m³, crack spacings for various values of the tensile strength are given in Table 4.1. The spacing of cracks is, on this basis, directly proportional to the tensile strength of the lean concrete.

Table 4.1. Crack spacings for concrete roadbases (constant frictional coefficient).

<table>
<thead>
<tr>
<th>Tensile strength, MN/m²</th>
<th>0.1</th>
<th>0.2</th>
<th>0.5</th>
<th>1.0</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crack spacing, m</td>
<td>5.8</td>
<td>11.6</td>
<td>29.0</td>
<td>58.0</td>
<td>116</td>
</tr>
</tbody>
</table>

The significance of each part of equation 4.6 becomes clearer if values of crack spacings from equation 4.5 are inserted, giving

\[
\text{Crack width} = \frac{2\alpha fT}{\mu Y} - \frac{f^2}{\mu \gamma E}\]  

(4.7)

At the point of cracking, $T = \frac{f}{E\alpha}$ and therefore

\[
\text{Crack width} = \frac{2f^2}{\mu \gamma E} - \frac{f^2}{\mu \gamma E}
\]
The elastic extension of the slab which is represented by the second part of the expression is in this case numerically equal to the crack width, but as temperature reductions become greater, the first part of the expression tends to become more important since it is proportional to the temperature fall. For temperature falls well in excess of those required to cause cracking, crack widths are approximately proportional to \( f/\mu_y \) which is also the ratio governing crack spacings. In addition, however, and unlike crack spacings, they are also proportional to the coefficient of thermal movement and the temperature fall.

Taking the values of tensile strength, coefficient of friction and unit weight used in the evaluation of equation 4.5, together with appropriate corresponding values of the elastic modulus and coefficient of thermal expansion for concretes using gravel and limestone aggregates respectively, the relationship between crack width and temperature reduction is indicated in Table 4.2.

It is noticeable that crack widths increase as temperature reductions become larger. For a given temperature reduction, crack widths also increase substantially in the case of stronger concretes since these result in larger crack spacings. The increases are not, however, in proportion to the strength increases causing them since the strain which is accommodated by elastic extension of slabs also increases, though it is independent of the temperature reduction and the coefficient of thermal expansion. Elastic extensions corresponding to each tensile strength/elastic modulus value are indicated at the base of the table.

Though approximate, the figures given in Tables 4.1. and 4.2. indicate the chief factors determining crack spacings and widths while equations 4.1 to 4.4 emphasise the importance of obtaining information concerning the physical properties of lean concrete and its restraint on given subgrades.
Table 4.2. Crack widths for concrete slabs using gravel and limestone aggregates as a function of temperature reduction. Constant frictional coefficient of 1.5. Assumed coefficient of thermal expansion of gravel aggregate concrete $12 \times 10^{-6}/°C$ (denoted G) assumed coefficient of thermal expansion of limestone aggregate concrete $8 \times 10^{-6}/°C$ (denoted L).

<table>
<thead>
<tr>
<th>Tensile strength MN/m²</th>
<th>0.1</th>
<th>0.2</th>
<th>0.5</th>
<th>1.0</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus GN/m²</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>20</td>
<td></td>
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<tr>
<td>25</td>
<td></td>
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<tr>
<td>30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Temp. fall °C</th>
<th>Crack width mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>G.</td>
</tr>
<tr>
<td>1</td>
<td>0.04</td>
</tr>
<tr>
<td>2</td>
<td>0.11 0.06</td>
</tr>
<tr>
<td>3</td>
<td>0.18 0.11</td>
</tr>
<tr>
<td>4</td>
<td>0.25 0.16</td>
</tr>
<tr>
<td>5</td>
<td>0.32 0.20</td>
</tr>
<tr>
<td>6</td>
<td>0.39 0.25</td>
</tr>
</tbody>
</table>

| Elastic ext. mm | 0.03 | 0.08 | 0.36 | 1.16 | 3.86 |
| Slab length m   | 5.8  | 11.6 | 29.0 | 58.0 | 116  |
5. SCOPE OF INVESTIGATION

It has been established that:
(a) the structural performance and durability of a flexible road employing a cementitious roadbase may be affected by the spacing and width of cracks in the roadbase since frequent cracking impairs the load-spreading properties of the pavement and wide cracks may propagate into the bituminous surfacing.
(b) Environmental temperature changes and, in particular, temperature reductions at early ages, are an important contributory cause of cracking.
(c) Cracking patterns depend in addition on certain properties of the material used; chiefly its tensile strength, elastic modulus and coefficient of thermal expansion. They also depend on the degree of frictional restraint occurring between the roadbase and the sub-base or subgrade.

The programme of study was designed to provide information on those properties of lean concrete which affect its susceptibility to cracking in order that likely cracking patterns resulting from temperature reductions may be predicted. Particular attention was paid to the effect of tensile strength on resistance to thermal cracking.

The detailed programme of investigation was as follows:

5.1. Experimental work

5.1.1. Preliminary thermal movement tests
There is little published information on the thermal movement properties of cement stabilised materials generally, so that a number of preliminary tests were conducted to determine coefficients of thermal expansion for a wide range of cementitious materials which might be used in the construction of roadbases or sub-bases. Additional tests were conducted to detect any irreversibility of movement which might occur when cement stabilised materials undergo cyclic temperature changes.
For cycled tests, three materials having a spectrum of aggregate gradings were employed; lean concrete, a cement stabilised clay and a synthetic mixture, comprising equal volumes of the two materials. It was hoped in this way to provide information on the effect of particle grading on thermal movement. An age of 7 days was selected for the commencement of all tests.

5.1.2. Restrained thermal movement tests

In order to investigate the interdependence of the relevant properties of lean concrete in causing cracking by restrained thermal contraction, an apparatus was devised which would cause cracking of cementitious materials in this way in the laboratory. Temperature reductions at the point of cracking were measured and, by means of thick cylinder theory, the results were modified according to the relative stiffness of test specimens and the restraint apparatus to give the temperature fall under which cracking would occur in a situation of absolute restraint. The tensile strength, elastic modulus and coefficient of thermal movement of each material were also measured at the time of restrained thermal movement tests. The following materials were employed:

Lean concrete. Since attention was chiefly centred on this material, tests were carried out at a number of ages including 16 hours, 1, 2, 4, 7, 14 and 28 days. The ages of 2 and 7 days were selected for repeated measurements, at least three tests being carried out in each case,

Partially compacted lean concrete. It is rarely possible to achieve in the field the levels of compaction obtainable in laboratory-made specimens and although the effect of undercompaction on strength is well known, it was considered desirable to investigate its effect on the elastic modulus, thermal movement and resistance to restrained thermal cracking of lean concrete. Specimens with a density shortfall of 5 per cent compared to the
fully compacted lean concrete were manufactured and tested at the ages of 1 day, 2 days and 7 days.

Wet lean concrete. This material has, in the hardened state, similar properties to (dry) lean concrete but being more workable in the fresh state, would be suited to placing using a slip-form paver, though its use is at present restricted to sub-bases for flexible roads. The change in properties was brought about by proportionate increases in cement and water contents. Tests were carried out at ages of 2 and 7 days.

Conventional and high strength concrete. Strengths were increased progressively by increase of cement content with workability remaining approximately constant, in order to compare the cracking performance of stronger concretes with that of lean concrete. Aggregate/cement ratios of 6 and 3.4 were selected and tests were carried out at ages of 2 and 7 days.

Steel fibre reinforced concrete. The use of steel fibres has been considered both as a means of controlling cracking in roadbases and for the reinforcement of concrete used in overlays. A single test was therefore carried out using a specially designed concrete mix to determine the effect of fibres on the resistance to thermal cracking and on the mode of failure of steel fibre reinforced concrete. The test was carried out at an age of 2 days.

The detection of stress relief by creep. Little information has been published on the effects of creep in cement stabilised materials under tensile stress and at the same time the simple nature of the apparatus adopted for restrained thermal movement tests appeared to commend it for this type of use. A single test was therefore undertaken using a concrete mix of medium strength and workability in order to ascertain the suitability of the apparatus for tensile creep testing and to obtain, if possible, some indication of the extent of stress relief which would occur at early ages, an age of 2 days being selected for test.
5.1.3. Temperature measurements in a lean concrete roadbase

Temperatures of a lean concrete roadbase were monitored continuously for a period of one week from the time of construction and the spacings of cracks which developed during this period were recorded.

5.2. Analytical studies

5.2.1. Frictional restraint with the subgrade

Published information on the frictional restraint between concrete road slabs and various subgrades is reviewed and a mathematical model suggested which, in the case of defined subgrades, would aid computation of cracking spacings and widths.

5.2.2. Prediction of cracking patterns

The information obtained from tests is combined with the mathematical model for subgrade restraint to solve equations 4.2. to 4.4. giving crack spacings and widths for differing concretes on defined subgrades. The significance of resulting values is considered in relation to their effect on the performance and durability of flexible pavements employing concrete bases.

5.3. Control of reflection cracking

The thesis concludes with a consideration of the overall implications of test results and conclusions, in relation to the design and performance of flexible pavements with cementitious bases and a review of methods for reducing the incidence of reflection cracking.
6. DESIGN AND PREPARATION OF MIXES

6.1. Preliminary thermal movement tests

Emphasis was placed initially on testing a wide range of cement stabilised materials and subsequently on repeated tests using three materials with a range of particle gradings.

6.1.1. Materials used

Concreting aggregates. A locally obtained Thames Valley natural gravel coarse aggregate of maximum size 20 mm was employed and in addition, samples of crushed coarse aggregates belonging to the chief petrological divisions were selected. These comprised a limestone from Chipping Sodbury, Gloucestershire, a granite from Mount Sorrel, Leicestershire and a gritstone from Bolton, Lancashire. The fine aggregate employed in each case was a Thames Valley natural sand of grading zone 2/3.

Soils. Two samples of clay were tested, one obtained during excavations for an M4 access road at Virginia Water in Surrey and the other, a brick earth, was obtained from Littlehampton in Sussex. A sample of chalk excavated during earthworks at Sutton in Surrey was also used for measurements.

Synthetic material. Pulverised fuel ash was included in tests on account of its widespread use in many forms of construction, a sample being obtained from Croydon Power Station.

Cement. In view of the fact that the purpose of strength testing was to give only a broad indication of compliance of mixes with specification requirements, no special precautions were taken in the use of cements in preliminary thermal movement tests, a typical ordinary Portland cement being used throughout.

6.1.2. Design of mixes

An aggregate/cement ratio of 16 was adopted for all
aggregates which, together with a total water content of 6 per cent was expected to give compressive strengths in compliance with the prevailing Department of the Environment requirements. A fine aggregate content of 37 per cent of the total aggregate content was employed in each case.

A cement content of 14 per cent was selected for the chalk in order to meet strength requirements for soil-cements and an optimum moisture content of 29 per cent was adopted based on compaction tests using a vibrating hammer.

In view of the relatively high cement contents which would have been necessary if the clay samples were to produce strengths conforming to Department of the Environment soil-cement requirements, the former aggregate/cement ratio of 16 was adopted. Maximum density was obtained by using these materials at moisture contents of approximately 15 per cent.

The pulverised fuel ash sample showed considerable instability, a moisture content of 40 per cent being required for optimum compaction and a relatively high cement content of 10 per cent was therefore selected for thermal movement tests.

In the additional tests carried out to detect irreversibility in movement, the range of materials was restricted to the lean concrete mix employing Thames Valley gravel aggregate, the stabilised clay from Littlehampton and a 50:50 combination by volume of these two materials. A cement content of 5 per cent by volume, corresponding to an aggregate/cement ratio of approximately 11 was selected with optimum water content in each case.

6.1.3. Preparation of materials for use

Concreting aggregates for preliminary thermal movement tests were used in the as-received condition.
Clay samples were allowed to air dry for 7 days and then crushed until they passed a 2.36 mm B.S. sieve. Chalk samples were similarly treated except that a 20 mm sieve was employed.

Pulverised fuel ash was stored in the slightly damp condition in which it was received.

The moisture contents of all materials were measured immediately prior to use.

6.2. Main test series

Attention was focussed chiefly upon lean concrete mixes, though testing was also carried out on mixes of progressively higher water and cement contents. Details of mixes are summarised in Table 6.1.

6.2.1. Materials used

**Aggregates.** The aggregate employed in all tests was the Thames Valley gravel used in the preliminary thermal movement tests and the series of tests on lean concrete was carried out using a single batch of material in order to minimise variations in the concrete.

**Cements.** The length of the period of research was such that it was impracticable to use cement from a single batch. However, by use of ordinary Portland cement from the same works throughout, variations in quality were minimised. As a further precaution, ages of test for lean concrete mixes were selected at random. A test certificate relating to cement used in the latter stages of the project is shown in Figure 6.1.

6.2.2. Design of mixes

**Lean concrete.** An aggregate/cement ratio of 18 and total water content of 6 per cent by weight of dry materials were adopted as in preliminary thermal movement tests. The theoretical dry density of the mix was calculated in
Table 6.1. Summary of the proportions of concrete mixes used in the main test series.

<table>
<thead>
<tr>
<th>Material</th>
<th>A/C ratio</th>
<th>Fine agg. %</th>
<th>Coarse agg.</th>
<th>Water absd.</th>
<th>Fine agg.</th>
<th>Water absd.</th>
<th>Total water content</th>
<th>Total water %</th>
<th>Net water</th>
<th>Net water %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lean concrete</td>
<td>18</td>
<td>40</td>
<td>10.8</td>
<td>0.13</td>
<td>7.2</td>
<td>0.14</td>
<td>1.14</td>
<td>6.0</td>
<td>0.87</td>
<td>4.6</td>
</tr>
<tr>
<td>Wet lean concrete</td>
<td>12</td>
<td>35</td>
<td>7.8</td>
<td>0.10</td>
<td>4.2</td>
<td>0.08</td>
<td>1.11</td>
<td>8.5</td>
<td>0.93</td>
<td>7.2</td>
</tr>
<tr>
<td>Conventional concrete</td>
<td>6</td>
<td>35</td>
<td>3.9</td>
<td>0.05</td>
<td>2.1</td>
<td>0.04</td>
<td>0.60</td>
<td>8.5</td>
<td>0.51</td>
<td>7.3</td>
</tr>
<tr>
<td>High strength concrete</td>
<td>3.4</td>
<td>60</td>
<td>1.36</td>
<td>0.02</td>
<td>2.04</td>
<td>0.04</td>
<td>0.48</td>
<td>10.9</td>
<td>0.42</td>
<td>9.6</td>
</tr>
<tr>
<td>Mix used with steel fibres</td>
<td>4.5</td>
<td>60</td>
<td>1.8</td>
<td>0.02</td>
<td>2.7</td>
<td>0.05</td>
<td>0.63</td>
<td>11.4</td>
<td>0.56</td>
<td>10.2</td>
</tr>
</tbody>
</table>
The Cement Marketing Company Limited

North East Surrey College of Technology
Dept of Construction Studies
Reigate Hill
Ewell
Surrey

For the attention of Mr G D Taylor

Dear Sirs

With reference to the sample of Northfleet OPC recently obtained from you our laboratory have completed their examination and reported as follows.

| Surface Area | 347 m²/kg |
| Setting Time | Initial 170 mins | Final 205 mins |
| Expansion    | 0.5 mm |

BS12 Concrete Compressive Strength

@ 3 days 24.7 MN/m²
@ 7 days 33.0 MN/m²

We trust this information will be of assistance to you.

Yours faithfully
THE CEMENT MARKETING CO LIMITED

B J Davies
Technical Department

Figure 6.1. Test certificate relating to cement used in the later stages of the project.
order to give an indication of the void content of concrete compacted to refusal and using apparent relative densities of 2.74 and 2.61 for fine and coarse aggregates respectively together with a cement relative density of 3.12, a value of 2310 kg/m$^3$ was obtained. Table 6.2. shows volume and mass proportions of the mix used.

Table 6.2. Volume and mass proportions of lean concrete used in main test series.

<table>
<thead>
<tr>
<th>Material</th>
<th>Cement</th>
<th>Fine agg</th>
<th>Coarse agg</th>
<th>Water</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass, kg</td>
<td>1</td>
<td>7.2</td>
<td>10.8</td>
<td>(1.14)</td>
<td>19</td>
</tr>
<tr>
<td>Apparent rel. den.</td>
<td>3.12</td>
<td>2.74</td>
<td>2.61</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>Volume, litres</td>
<td>0.32</td>
<td>2.63</td>
<td>4.14</td>
<td>1.14</td>
<td>8.23</td>
</tr>
<tr>
<td>Volume % of total</td>
<td>3.9</td>
<td>32.0</td>
<td>50.3</td>
<td>13.8</td>
<td>100</td>
</tr>
</tbody>
</table>

Partially compacted lean concrete. For these mixes, the density of lean concrete compacted to refusal was measured and then an additional 5 per cent of air voids was introduced.

Wet lean concrete. An aggregate/cement ratio of 12 was selected, which, together with a total water content of 8.5 per cent, resulted in a mix of similar strength to that of the dry lean concrete mix. To avoid harshness in the fresh state, the fine aggregate content was reduced from 40 to 35 per cent and 5 per cent of limestone dust was included, the dust being of the type used in asphalts, having 85 per cent of material passing a 75 μm sieve. The resultant mix was of good cohesion and low workability, having zero slump and a V.B. time of six seconds.

Conventional concrete. An aggregate/cement ratio of 6 was chosen which, together with a total water content of 8.5 per cent as used in wet lean mixes resulted in a mix of free water/cement ratio 0.51, having low workability and medium strength. Adequate cohesion was obtained by using
a fine aggregate content of 35 per cent.

**High strength concrete.** An aggregate/cement ratio of 3.4 was selected and the water content was increased to 10.9 per cent in order to improve workability so that this mix could also be used for the incorporation of steel fibres. To this end, a Thames Valley aggregate of maximum size 10 mm was employed with a fine aggregate content of 60 per cent which resulted in a very high cohesion. This concrete was of free water/cement ratio 0.42 with a slump of approximately 25 mm.

**Mixes incorporating steel fibres and used in creep testing.** Although it was originally envisaged that the mix of aggregate/cement ratio 3.4 would be used in conjunction with steel fibres, the magnitude of strengths obtained was such that the restrained thermal movement apparatus, which was designed chiefly with lean concretes in mind, was barely able to cause cracking using the range of temperatures obtainable. A further mix was therefore designed which would result in a similar strength to that of the conventional concrete employed but of the same aggregate size and grading as used in the high strength mix. Plain high tensile steel fibres of length 32 mm and diameter 400 μm were employed, the proportion used being 1.5 per cent by volume corresponding to 4.9 per cent by weight of the concrete which was of aggregate/cement ratio 4.5. The same mix proportions, without fibres were used in the test to detect stress relief by creep.

6.2.3. Preparation of materials for use

After drying to constant weight at a temperature of 105°C for 16 to 24 hours, fine and coarse aggregates were sieved into the size ranges of 20-10 mm, 10-5 mm, 5-2.36 mm, 2.36 mm - 600 μm and passing 600 μm.

When used in the manufacture of lean concrete, they were blended to conform to Zone B defined by aggregate grading
curves for 20 mm aggregate given in Road Research Note No.4.\textsuperscript{68}

The total mixing water was added to aggregates 24 hours before addition of cement in order to minimise workability changes caused by absorption soon after mixing. Damp aggregates were then sealed and stored at a temperature of approximately 20°C until required for use.

When used for concrete mixes other than lean concrete, fine and coarse aggregates were stored separately in an oven-dry state until required for use.
7. APPARATUS USED IN THE MEASUREMENT OF MATERIAL PROPERTIES

7.1. Thermal movement

Initial measurements were carried out using a form of the BS 1881\textsuperscript{44} drying shrinkage apparatus having a steel frame and designed for use with 250 x 100 x 100 mm prisms. Reference pieces at the end of each prism were in the form of stainless steel balls. A dial gauge calibrated to 0.002 mm was used for measurements, resulting in a strain resolution of $8 \times 10^{-6}$ and corrections were made as necessary to compensate for movements of the steel frame which occurred as a result of small ambient temperature changes.

In the main series of tests, the above method of measurement was supplemented by independent readings obtained on the same prisms using a demountable mechanical strain gauge. There was good agreement between results and the latter method was therefore used exclusively in later tests since reference studs were found to be easier to attach to prisms than the steel balls used initially.

For heating specimens in air or in the sealed state, a conditioning oven conforming to BS 1881 requirements for shrinkage measurements was employed, temperatures being controlled by contact thermometer to an accuracy of 0.5°C. Specimens to be tested under water were heated by means of a bath equipped with an electric stirrer and similar temperature control. Sealed prisms in the second series of cycled tests were also heated under water.

7.2. Strength

In view of the fact that cooling of concrete always results in a tensile stress, the main programme of strength testing was orientated towards tensile strength measurements, cube tests only being carried out for control purposes and in preliminary thermal movement tests to check compliance of materials with the then Department of the Environment specification\textsuperscript{10}.

Tensile tests were carried out on 500 x 100 x 100 mm prisms using
friction grips developed by Johnston and Sidwell. These held the concrete by a double scissor action, load transfer being obtained by means of frictional forces between the concrete surface and the serrated surface of bearing plates. The equipment was purpose built to a design which has been in use for a number of years at the University of Surrey, tests having shown that reliable results are obtainable even when, as occasionally happens, the concrete fails within, rather than clear of the grips.

The Avery 500 kN universal testing machine type 7110 DCJ was used for all tensile tests, enabling a resolution of 0.05 kN to be obtained when operating on the 25 kN range. The machine was calibrated regularly during the period of testing and proved eminently suitable for use with the friction grips described.

7.3. Elastic modulus

There are a number of techniques which might be used for the determination of elastic modulus, the most direct method being to measure static modulus by means of some form of extensometer during tensile testing of prisms. While this would be likely to give an elastic modulus closely approximating to that which is relevant to thermal contraction in roadbases, it was found to be impracticable when concretes of very low strength were tested, since failure could occur at strains of as low as \(15 \times 10^{-6}\) and no extensometer was available with sufficient sensitivity to measure strains of this magnitude. In addition, the surface of lean concretes at early ages tends to be friable so that it is difficult to obtain a firm bond between reference pieces and the concrete.

The use of an ultrasonic tester was also considered since ultrasonic pulse velocities in concrete may be conveniently measured from the time of casting. However, the calculation of elastic modulus from pulse velocities requires a knowledge of the Poisson's ratio of the material for which accurate values, especially in the case of lean concretes at early ages, are not available and this method is not therefore generally recommended for the determination of elastic modulus.
As a further alternative, BS 1881 Part V describes an electrodynamic method of measurement of elastic modulus suitable for use with 500 x 100 x 100 mm prisms which can be employed as soon as units are sufficiently mature to handle without damage. Since Poisson's ratios are not involved in the determination of the dynamic modulus, the method was used for this purpose throughout the period of research. Dynamic moduli are appreciably higher than those obtained by static tests and results were therefore modified to correspond to static moduli by use of correlations obtained by Kolias for lean concrete.
8. THERMAL MOVEMENT IN CEMENTITIOUS MATERIALS

8.1. Published information

While the subject of thermal movements in conventional concrete mixes is well documented, there is relatively little published information concerning mixes of low cement content which were used chiefly in this project. There follows, however, a review of measurements carried out on concretes of various types which may allow some tentative conclusions to be drawn regarding the thermal movement of lean concrete.

Bonnell and Harper measured dimensional changes in 150 x 75 mm cylinders which they subjected to temperature cycles in the sequence: 20 - 0 - 20 - 40 - 20°C, after curing in water, or in air at various humidities which were maintained at constant values by use of salts in equilibrium with their saturated solutions.

Relationships between temperature and length were linear and final lengths were found to be very close to initial lengths, suggesting no loss of water during temperature cycles. Coefficients of thermal movement for neat cement pastes were found to be highest, a value of $22.6 \times 10^{-6}/°C$ being obtained for ordinary Portland cement cured in air at 65 per cent relative humidity. Coefficients for concretes depended on the type of aggregate used, gravels resulting in the highest values and limestones in the lowest values with granite aggregates being intermediate. For example, at 65 per cent relative humidity the thermal coefficients for gravel, limestone and granite aggregate concretes were $13.1 \times 10^{-6}/°C$, $7.4 \times 10^{-6}/°C$ and $9.5 \times 10^{-6}/°C$ respectively. Bonnell and Harper also compared results for concretes of aggregate/cement ratio 6 with measured coefficients of thermal expansion of corresponding aggregates. They found that there was good correlation, though the expansion of each concrete was always greater than that of the aggregate used and the difference in percentage terms increased in the case of aggregates having very low thermal movement. Thermal movements were found to be greater for rich mixes or if concretes were cured in air rather than in
water. The relative humidity during curing appeared to affect coefficients only slightly; for example, an increase from 32 per cent to 52 per cent caused an increase in the coefficient for gravel aggregate concrete from $14.0 \times 10^{-6}/\degree C$ to $14.6 \times 10^{-6}/\degree C$ and a further increase to 100 per cent reduced the value to $12.2 \times 10^{-6}/\degree C$. Wet cured concretes also gave the latter value, as did concretes which were oven-dried after a period of wet curing. No conclusive effect was obtained by varying water/cement ratio or by testing at different ages.

Loubser and Bryden measured coefficients of thermal expansion of 3:1 sand-cement mortars by heating oven-dried and water saturated specimens 100 mm in length in the temperature range 20 - 80°C. They obtained values of $12.7 \times 10^{-6}$ and $13.5 \times 10^{-6}/\degree C$ in the two states respectively.

Meyers carried out measurements on 25 x 25 x 250 mm prisms cured by a similar method to that used by Bonnell and Harper, the humidity during curing being measured by a cell contained within each specimen. Though details are not given, samples appear to have been heated until a steady length was obtained. Meyers found that neat cement pastes stored at between 60 per cent and 70 per cent relative humidity gave highest thermal coefficients of expansion and values were considerably reduced by an increase or decrease of humidity. Typical results at 30, 65 and 100 per cent relative humidity were $11.7 \times 10^{-6}/\degree C$, $18.9 \times 10^{-6}/\degree C$ and $9.9 \times 10^{-6}/\degree C$ respectively. The corresponding coefficients for concretes were much reduced compared to those of neat cement pastes as were the susceptibilities to varying humidities. For example, the thermal coefficients for a quartz aggregate concrete at relative humidities of 33, 66 and 100 per cent were $10.3 \times 10^{-6}/\degree C$, $10.9 \times 10^{-6}/\degree C$ and $9.36 \times 10^{-6}/\degree C$ respectively while for a limestone aggregate concrete they were $5.9 \times 10^{-6}/\degree C$, $6.5 \times 10^{-6}/\degree C$ and $4.1 \times 10^{-6}/\degree C$. The variation was almost completely eliminated when cement prisms were autoclaved, Meyers suggesting that this behaviour could therefore be attributed to the cement gel fraction of the concrete, since cement gel is not formed in
the autoclaving process. The thermal coefficient of neat cement pastes decreased with age, though in another paper\textsuperscript{76}, Meyers reported that the expansion coefficient of a sealed concrete rose to a maximum value at an age of between 6 months and one year, decreasing slightly thereafter.

Browne\textsuperscript{77} cycled gravel and limestone aggregate concretes between 4°C and 70°C and found that the first heating produced an expansive set while subsequent cycles resulted in slight decreases in the coefficient of expansion. Coefficients of thermal movement varied with age, increasing to a maximum at two months, Browne attributing this to a progressive decrease of humidity within the concrete caused by self-dessication of the cement by hydration.

Helmuth\textsuperscript{78} measured length changes of hollow 25 mm cement paste cylinders by subjecting them to temperature cycles while immersed first in water and then in mercury. A temperature range of 10°C was employed, specimens being first cooled from 25°C at the rate of 0.5°C per minute. Helmuth also obtained hysteresis effects, though specimens tended to expand if held at the lower temperature and contract if maintained at the higher temperature. The slope of strain/temperature graphs varied between 25 to 27 x 10\textsuperscript{-6}/°C at the start of the tests and 13.6 x 10\textsuperscript{-6}/°C between equilibrium points. Results in the two fluids were very similar from which Helmuth concluded that the hysteresis was due to internal rather than to surface effects. The behaviour was attributed to a tendency for cement gel to absorb water by diffusion if held at lower temperatures and to expel water if held at higher temperatures and was observed to decrease if the rate of temperature change was reduced or if more porous cement pastes were employed. Helmuth found that if hardened cement pastes were maintained in a completely saturated state during temperature changes, a thermal expansion coefficient of only 11.6 x 10\textsuperscript{-6}/°C was obtained. He also obtained a permanent contraction as a result of the first cooling/warming cycle of water saturated pastes, which was considered to be related to the irreversibility of drying shrinkage.
Work by Mehra and Uppal carried out in India is of interest since they measured the thermal expansion of compacted soils and soil-cement mixtures. Cylindrical specimens of dimensions 375 x 20 mm were employed, being cured for one week and then dried 'in the shade' before being finally oven dried. The temperature range over which expansion was measured is not indicated, though it is inferred from the magnitude of length changes recorded, to be approximately 100°C. Coefficients of thermal expansion of unstabilised materials varied according to soil type and density from $6.4 \times 10^{-6}/°C$ to $9.6 \times 10^{-6}/°C$. Stabilisation of soils with cement in quantities up to 10 per cent by weight progressively increased the thermal coefficient by between 12 and 75 per cent depending on soil type.

Williamson measured the thermal expansion and contraction of granular roadbase materials stabilised with 4 per cent of blast furnace cement, cut from roads in South Africa. Measurements were carried out on samples of size 200 x 75 x 75 mm during heating and cooling in the range 20 - 60°C. Generally small thermal movements were obtained, expansion coefficients being, for example, $5.1 \times 10^{-6}/°C$ and $8.5 \times 10^{-6}/°C$ for concretes employing granite and reef quartzite aggregates respectively. Coefficients of thermal contraction were not significantly different. Reductions in moisture content averaging approximately 1.3 per cent were obtained during tests, though a high degree of repeatability in length measurements was reported.

8.2. Discussion and suggested mechanisms

The results referred to were based on a variety of experimental techniques employed in testing many different materials and it is not surprising therefore that the detailed findings of many of the experimenters are also different. However, it is considered that a number of conclusions may be drawn on the basis of the results presented and these are summarised as follows:
8.2.1. Effects of aggregate type and cement content

(a) The coefficient of thermal expansion of neat cement paste is higher than that of concretes employing conventional aggregates.

(b) The thermal expansion of concrete depends on the type of aggregate used, gravel aggregate concretes having, in general, the highest values and limestone aggregate concretes, the lowest values, granites being intermediate.

(c) The coefficient of thermal expansion of concrete increases with increase of cement content.

Properties (b) and (c) reflect the differences between the coefficients of thermal expansion of neat cement paste and those of conventional aggregates, the latter having a range of values normally well below that of cement paste so that the thermal movement of concretes is generally restrained by the aggregate fractions.

8.2.2. Effects of curing conditions

(a) The coefficient of thermal expansion of cement paste depends on its moisture condition prior to heating, coefficients being greatest when humidities are in the region of 60 per cent. Coefficients corresponding to the water saturated condition are similar to those obtained in the dessicated state.

(b) The coefficient of thermal expansion of concrete is similarly affected though to a lesser extent.

Explanations of these effects would appear to involve gel water in view of Meyers' observation regarding autoclaved cement. Bazant80, for example, attributed the increase in expansion at intermediate humidities to a rise of internal humidity in the concrete on heating. This causes an increase in vapour pressure and consequently an expansion, with a further gradual expansion as water diffuses into pores to fill the extra space created. The
rise of internal humidity on heating and consequently additional swelling decrease progressively to zero at lower or higher humidities. Powers suggested that an increase in specific free energy of adsorbed water which occurs as temperature rises, would cause a decrease in tensile stress in the adsorbed water film and therefore result in swelling. The opposite effect occurs within capillary water, the two effects therefore balancing if large quantities of capillary water are present as would be the case at high humidities. At low humidities, the swelling is again reduced since there is less adsorbed water.

These two suggested mechanisms result from a consideration of the vapour and liquid states of water in the cement paste respectively and, although it is not clear whether they are equivalent from a theoretical standpoint, they each account adequately for observed behaviour and confirm that there is ample theoretical justification for changes in the coefficient of thermal expansion to occur in differing moisture states. However, in view of the observed comparative behaviour of concrete and cement paste in relation to humidity, it would be expected that when volume fractions of cement are further reduced, as is the case in cement stabilised materials, the effects of humidity would be similarly less pronounced than in normal concrete mixes.

8.2.3. Delayed movements; hysteresis effects

It is not possible to give firm conclusions on the basis of results presented other than to recall the work of Helmuth in which delayed movements occurred particularly in the case of low water/cement ratio concretes or if changes of temperature were rapid. Such effects would be expected in concretes in which the thermal expansion coefficient is dependent on moisture effects; that is, in
states other than the dessicated or saturated condition and could be readily interpreted in terms of moisture diffusion which would occur in conditions of changing internal humidity.

Hysteresis effects would naturally be produced in conjunction with delayed movements, though they do not appear to have been obtained as a result of tests carried out on normal concretes as, for example, by Bonnell and Harper. It is interesting also that Brown obtained expansive sets as a result of heating while Helmut reported a net contraction, possibly associated with shrinkage during a temperature cycle which commenced with cooling. In most cases, however, hysteresis effects other than those caused by rapid temperature changes were small compared to overall thermal movements.

8.2.4. Effect of age

Bonnell and Harper obtained no clear change in thermal expansion with age, though Meyers recorded a maximum at ages of between six months and one year for both neat cement paste and concrete while Browne recorded a maximum at approximately two months for concrete. If, as suggested by Browne, the effect is due to self-dessication, the effect would be expected to depend on water/cement ratio and, in the case of higher water/cement ratio concretes, on curing conditions, though insufficient results are available to evaluate fully this explanation. In the case of cement stabilised materials used in roadbases it is unlikely, however, that self-dessication could occur since water/cement ratios are normally relatively high and normal curing procedures should result in high internal humidities, especially at early ages.

8.3. Programme of tests

Two series of tests were undertaken.
The first test series was designed to provide additional information as to the effect of aggregate type and variations in curing conditions. Materials were subject to a single heating cycle in the air dry, sealed or saturated moisture condition after curing in the same state. A temperature range of 20°C to 65°C was employed.

In planning the second series of tests using cementitious materials of different particle gradings, it was hoped to clarify those aspects of thermal movement which were not well defined in the literature review and particularly the possibility of irreversibility in temperature cycles. As a further precaution, temperature cycles were commenced in each case by cooling in order to simulate the likely initial temperature change in roadbases and an overall temperature range of 4°C to 40°C was chosen to correspond approximately with a range which might typically occur in practice. All tests in this series were carried out in the sealed state since this condition was considered to approximate most closely to that of a lean concrete roadbase.

In both series of tests, limitations of space in the conditioning oven and water bath were such that tests were confined to single samples of each material used.

8.4. Experimental procedure

8.4.1. First series of tests

Optimum water contents of materials were first determined and then, having measured the moisture content of aggregates or soils as prepared, the balance of water was added. Mixing was carried out by hand, three 250 x 100 x 100 mm prisms and two 100 mm cubes being manufactured from each material and compacted in 50 mm layers using an electric vibrating hammer applied direct to the material surface. Test specimens were covered by polyethylene sheeting for three days and then demoulded.
A number of methods for attaching reference pieces to the ends of prisms was considered. BS 1881 Part V \(4^{th}\) describes a method in which studs are located in the casting mould, though this was considered inappropriate for very low workability concretes in view of the difficulty of ensuring full compaction of concrete around the studs and also the possibility of their being damaged during compaction by the foot of the vibrating hammer. In early tests 6 mm stainless steel reference balls were cemented to the ends of prisms using either ordinary Portland or high alumina cement. In the case of air cured prisms, however, the cements tended to dry due to absorption of moisture by the concrete before hydration could occur and in consequence, reference balls were insecurely held. In later tests, therefore, the reference balls were soldered to 10 mm length sheradised steel screws which were embedded into the concrete after hardening using a rapid setting mixture of Portland and high alumina cements. These performed much more reliably and the error incurred by the incorporation of the steel at each end of the concrete was ignored on account of the small difference between the expansion coefficients of steel and concretes generally, combined with the small proportion of the total prism length which was occupied by the screws.

Cubes were cured under water with the exception of stabilised clays which were cured in the sealed state to avoid the risk of disintegration. Air cured prisms were stored at a relative humidity of approximately 65 per cent. Sealed specimens were wrapped initially in polyethylene sheeting, though weight measurements showed this to be unsatisfactory and electrodeposited copper foil was therefore used in later tests, seals at joints being effected by soldering. A curing temperature of 20°C was adopted for specimens in each moisture condition.
At the commencement of each test, readings of the concrete and ambient temperature were recorded, prisms were weighed and length readings obtained. The orientation of mounting of each prism in the measurement frame was standardised and in this way it was found possible to obtain a high degree of repeatability when taking readings. Prisms were heated to 65°C, air cured and sealed specimens being heated in the conditioning oven and water cured prisms using a water bath. Length measurements were taken periodically during tests until constant readings were obtained, the time taken for this to occur being approximately 24 hours for prisms heated using the oven and 12 hours for prisms heated under water. In early tests length/time graphs were plotted on withdrawal of prisms from the oven for measurement in order to detect by extrapolation any contraction which might occur in the period which elapsed between removing a prism and taking the dial gauge reading. The contraction of prisms during this period which was in the region of 15 seconds was found to be negligible on account of their relatively high thermal inertia. Some air cured prisms showed noticeable shrinkage at the steady high temperature and in these cases, maximum length readings were recorded. On cooling, further length and mass readings were obtained. Cubes were in most cases tested at the ages of 7 days or 28 days appropriate to the Department of the Environment requirements\(^{10}\) for soil-cement and lean concrete respectively.

8.4.2. Second series of tests

Further prisms were manufactured from lean concrete, stabilised clay and an equal mixture by volume of the two materials. On demoulding at an age of 7 days, reference pieces were attached and each prism was sealed in copper foil. Prisms were then immersed in water, initially at a temperature of 17°C and subjected to the temperatures in the sequence 4 - 22 - 40 - 22 - 4°C, each change of 18°C
being made weekly. Measurements of length and mass were recorded near the middle and at the end of each weekly period and checks for leaks were made by pressurising prisms with air under water. The prism comprising the mixture of clay and lean concrete unfortunately developed a leak which became unacceptably large after seven weeks but tests on the other materials were continued until length changes over $2\frac{1}{2}$ complete cycles were recorded.

8.5. Experimental results

8.5.1. First series of tests

Coefficients of thermal expansion for the materials tested are indicated in Table 8.1, and these will be considered first, in terms of the effect of material type and second in terms of the effect of moisture conditions of test specimens.

Coefficients of thermal expansion of lean concrete varied with the type of aggregate used, gritstone aggregate resulting in the highest values and limestone, the lowest values, with Thames Valley gravel and granite aggregates being intermediate.

Thermal coefficients of the stabilised chalk were slightly greater than those for lean concrete containing the limestone aggregate.

The stabilised clay and brick earth gave somewhat variable coefficients probably on account of large weight changes which occurred on heating. The figure indicated in brackets for Littlehampton brick earth in the air cured state was the result of a second test on that prism. It was not possible to carry out measurements on these materials in the water cured state since progressive disintegration occurred during immersion in water.

Stabilised pulverised fuel ash gave generally low thermal coefficients, the air cured value corresponding to the
Table 8.1. Coefficients of thermal expansion for cement stabilised materials.

<table>
<thead>
<tr>
<th>Material</th>
<th>Coefficient of thermal expansion, $x 10^{-6}/^\circ$</th>
<th>Sealed</th>
<th>Air</th>
<th>Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>Siliceous gravel</td>
<td>11.0 11.1 12.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Granite</td>
<td>7.5 8.0 9.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Limestone</td>
<td>6.3 7.6 8.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gritstone</td>
<td>14.2 15.2 15.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chalk</td>
<td>8.4 8.7 8.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clay</td>
<td>10.9 11.6 -</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brick earth</td>
<td>14.1 22.0 (15.2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pulverised fuel ash</td>
<td>7.9 12.0 7.2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 8.2. Cube strengths for materials used in preliminary thermal movement tests

<table>
<thead>
<tr>
<th>Material</th>
<th>Cement content, % of agg.cont.</th>
<th>Age, Days</th>
<th>Average strength, MN/m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Siliceous gravel</td>
<td>6.25</td>
<td>28</td>
<td>13.0</td>
</tr>
<tr>
<td>Granite</td>
<td>6.25</td>
<td>28</td>
<td>18.6</td>
</tr>
<tr>
<td>Limestone</td>
<td>6.25</td>
<td>28</td>
<td>36.0</td>
</tr>
<tr>
<td>Gritstone</td>
<td>6.25</td>
<td>28</td>
<td>13.7</td>
</tr>
<tr>
<td>Chalk</td>
<td>14.0</td>
<td>7</td>
<td>5.15</td>
</tr>
<tr>
<td>Clay</td>
<td>6.25</td>
<td>28</td>
<td>2.25</td>
</tr>
<tr>
<td>Brick earth</td>
<td>6.25</td>
<td>17</td>
<td>2.25</td>
</tr>
<tr>
<td>Pulverised fuel ash</td>
<td>10.0</td>
<td>7</td>
<td>0.38</td>
</tr>
</tbody>
</table>
maximum expansion which was followed by a marked contraction probably associated with a substantial moisture loss recorded.

In the case of the concreting aggregates and chalk, highest values of the thermal coefficient were obtained by curing and heating prisms under water, those in the air cured state resulting in intermediate values and those in the sealed state, in the lowest values. In the three other materials, there is no clear pattern, though coefficients corresponding to air curing were consistently greater than those obtained in the sealed state.

Cube strengths are indicated in Table 8.2, together with the corresponding cement content and age of test for each sample and these may be compared to the requirements of the Department of the Environment specification at the time of the test which effectively necessitated that the average cube strength of lean concrete should not be less than approximately 14 MN/m$^2$ at 28 days, while that of soil-cement should not be less than approximately 5 MN/m$^2$ at 7 days.

The Thames Valley and gritstone aggregate concretes were slightly below the required strength while the granite complied adequately and the limestone aggregate concrete was of very high strength comparatively.

In the soils group of materials, the stabilised chalk was the only material to conform to the 7 day strength requirement and the pulverised fuel ash resulted in a very low strength, though this may have been caused in part by a degree of bonding between the material and the cube mould surfaces which made demoulding difficult.

8.5.2. Second series of tests

Results are indicated graphically in Figure 8.1, in the form of length change as a function of time in weeks, the
Figure 8.1. Length changes of prisms subjected to the temperature cycle indicated.
Figure 8.2. Coefficients of thermal movement obtained in cycled tests.
readings obtained at the end of each weekly period being joined by straight lines for convenience. Corresponding coefficients of thermal expansion are presented graphically in Figure 8.2.

The contraction of the stabilised brickearth during the first cooling stage was much less than that of the lean concrete while, in the subsequent heating stages, the brickearth showed a marked increase of expansion. Nevertheless, the thermal coefficient was still substantially lower than that of the lean concrete in spite of a considerable decrease in the expansion coefficient of the latter in the two heating stages which followed the initial cooling. During repeated cycles, thermal movement became progressively more uniform, that of the lean concrete always being greater than that of the stabilised brickearth. As far as tests were conducted, the thermal movement of the mixture was of intermediate magnitude. There was generally no clear dependence of thermal coefficients on the direction or range of the temperature changes, though coefficients for lean concrete appeared to decrease slightly with age while those for stabilised brickearth remained approximately constant.

8.6. Discussion

For the purposes of discussion it is convenient to consider the results of both series of tests together, taking lean concretes and the other materials separately.

8.6.1. Lean concretes

The effect of aggregate type on the thermal movement of lean concrete is similar to that reported by other workers for normal concretes, coefficients of thermal expansion increasing in the order: limestone, granite and gravel. Substantially greater than all of these, however, was the coefficient for lean concrete employing the gritstone
aggregate, though no published information relating to concretes of this type has been traced.

It is interesting to note that the limestone aggregate concrete, in addition to exhibiting the lowest thermal movement also gave the highest cube strength, suggesting that resistance to thermal cracking should be substantially greater than that of other types of lean concrete. The performance of the granite aggregate concrete was also good on this basis and that of the gritstone concrete would on account of its high thermal movement be poor and below that of the Thames Valley aggregate concrete even though the material exhibited a slightly higher strength than the latter. (Rather poor coarse aggregate gradings in each of these concretes is considered to have been partially responsible for low strengths obtained). Williamson also observed a marked correlation between strength and thermal movement in stabilised granular materials, those having the highest strength again exhibiting the lowest thermal movement.

The decrease of coefficients obtained in each case in the sealed moisture state compared to the air cured condition is in agreement with results reported by Meyers, since in materials of this type, the sealed state is likely to correspond to a relative humidity of 100 per cent. In quantitative terms, the difference between results ranged from less than 1 per cent for the Thames Valley gravel concrete to over 20 per cent for the limestone aggregate concrete compared to differences of 16.5 per cent and 58.5 per cent obtained by Meyers for the quartz and limestone aggregate concretes at relative humidities of 66 and 100 per cent respectively. These figures suggest a reduction in susceptibility of the thermal movement of lean concretes to relative humidity changes compared to conventional concrete mixes, though such a comparison can only be tentative in view of the different sources of aggregates used in the two cases.
The relatively high values of thermal coefficient obtained from prisms cured and heated under water was unexpected since these conditions were presumed to correspond to the saturated state in which, according to Bazant\textsuperscript{80} and Powers\textsuperscript{81}, additional swelling due to moisture effects should be absent. The coefficient of thermal expansion should in this state correspond to the increase of kinetic energy of the solid component and it is therefore surprising that coefficients should be greater than those obtained in the air cured state, which should represent the most marked effect of the presence of moisture. In attempting to account for this apparent difference in behaviour, it is worthwhile to compare both the materials employed and the method of test to those which formed the basis of published data. It may, for example be significant that free water/cement ratios employed in the test series were approximately 0.9 while the maximum value used by Bonnell and Harper was 0.75 (for which the corresponding concrete was reported to segregate during manufacture). It is also possible that the structure of the hydrated cement gel may be modified at high water/cement ratios and that moisture effects may be influenced by the highly porous cement matrix which results. Nevertheless, it seems unlikely that changes of this type could reverse the effects observed at lower water/cement ratios, especially since behaviour in the sealed and air cured states was in accord with published results. In considering possible effects of the method of test, it may be significant that prisms were heated under water as well as being cured in that state whereas published results for concretes were in relation to heating in air. While in relatively rich, non-porous concrete mixes there may be little difference between heating in these two states, it is possible that in high water/cement ratio concretes, the effects of substantial rises in the vapour pressure of water which occur on heating or the relatively great expansion of water itself might contribute to an increased thermal coefficient in the condition of complete saturation compared to the sealed state. A
further possible influence may also be the effect of saturation on the hydration rate since a larger proportion of cement gel would be expected to form in the condition of total saturation. Although a clear pattern of comparative results in differing moisture states was established for the range of lean concretes employed further testing would be recommended to confirm this behaviour in view of the fact that results were based on only one measurement for each material.

In cycled tests, the lean concrete prism exhibited a very large thermal contraction on being cooled initially from 17°C to 4°C, the material appearing to undergo a permanent contraction at the low temperature, since a negative strain of $30 \times 10^{-6}$ occurred between the fifth and seventh days at the steady value of 4°C. This may be related to the permanent set observed as a result of cooling by Helmuth and it is noticeable that on heating to 22°C the concrete regained the length which formerly corresponded to a curing temperature of 17°C. The somewhat random fluctuations in the thermal coefficient on subsequent changes of temperature suggest that experimental tolerances may have been in excess of the value of $0.4 \times 10^{-6}/°C$ based on tolerances in strain and temperature measurements, although after the sixth week there is evidence of a slight overall reduction in the thermal coefficient with continued testing, values being in the region of $11 \times 10^{-6}/°C$ which was the value obtained from the single heating cycle.

8.6.2. Other materials

In spite of its high moisture content, the stabilised chalk recorded the most consistent thermal expansion coefficients and the highest strength, although the cement content of the material was substantially higher than that of the others in this group. Of the two fine grained soils the clay sample from Virginia Water exhibited the more uniform thermal movements in the single heating cycle,
although in each case substantial moisture losses occurred during the cycle. Higher initial thermal coefficients may have resulted from internal pressure caused by the increase of water vapour pressure. On cooling to the original temperature, significant shrinkage strains were in each case observed and the subsequent heating cycle applied to the brick-earth probably resulted in lower movement because its moisture content was, by this time, lower. Results for the pulverised fuel ash were probably also substantially affected by moisture changes, the value given for heating in air corresponding to the maximum length obtained. The substantial contraction which followed would have resulted in a net expansion coefficient similar to those in the sealed or water cured states, suggesting that the moisture in the material may, prior to being lost by evaporation, have produced a similar effect to that reported earlier on concretes heated at intermediate humidities.

In cycled tests on the stabilised brick-earth, the remarkably small initial contraction may also be linked to moisture changes since an expansive strain of $30 \times 10^{-6}$ occurred between the fifth and seventh days at the steady temperature of $4^\circ C$ probably due to progressive swelling of the brick-earth fractions caused by condensation of moisture during cooling. The thermal coefficient obtained was only half that resulting from the heating test on the same material in the sealed state. It is evident from Figure 8.1. that an expansive set occurred in the first half cycle in contrast to the net contraction recorded in the lean concrete. Subsequent smaller fluctuations in the thermal coefficient may again be caused in part by experimental tolerances, there being no further irreversibility or overall trend in values. Coefficients were however well below those obtained from the single heating cycle perhaps since the cement content was higher and possibly linked also to the lower maximum temperature employed in the cycled tests. The intermediate results of the 50:50 mixture of brick-earth and Thames Valley gravel are considered to validate to some
degree, the results obtained, especially during the first cooling stage, though further work would be necessary to supplement the measurements obtained and to clarify the effects of the type and range of temperature changes on thermal movements.

The considerable differences in behaviour of the fine grained soils comparative to lean concretes are a reflection of the fact that whereas in lean concretes the aggregates tend to restrain movements of the cement caused by thermal changes and concomitant moisture effects, in the soils category, the cement is the less susceptible of the two materials such that its role is reversed. Increases of cement content would therefore be expected to produce opposite effects on the two groups of materials and this has already been commented on in the context of shrinkage in Section 3.2.

8.7. Conclusions on thermal movement

The broad scope of the overall project did not permit the amplification of the test programme which would have been desirable in order to obtain more detailed results but a number of patterns of behaviour nevertheless appear to emerge from the results of tests conducted.

(1) The effect of aggregate type on the coefficient of thermal expansion of lean concrete is similar to that in conventional concretes. Although a direct comparison of results based on aggregates from different sources is not possible there is no indication that reductions in thermal movement are caused by the relatively low cement contents employed.

(2) Of the lean concretes tested, it is evident that the limestone aggregate concrete would be by far the most resistant to cracking caused by thermal contraction since the material exhibited both the lowest thermal movement and the highest strength of this range of materials.

(3) Comparative values of the thermal coefficient of lean concrete in the sealed and air cured states are similar to those of conventional concrete, although there is some evidence that the coefficient is less dependent on humidity than in richer concrete
mixes. When lean concretes were cured and heated under water significant increases in thermal movement were obtained, in contrast to the effects of heating in air after curing in the saturated state reported in literature.

(4) Cement stabilised chalk behaved in a similar manner to limestone concrete when subjected to thermal changes, though the finer grained materials underwent large moisture losses on heating resulting in widely variable coefficients of expansion.

(5) In cycled tests, irreversible length changes occurred in both the lean concrete and stabilised soil when the temperature was maintained at the steady value of 4°C for the first time, the lean concrete exhibiting a negative strain and the stabilised clay a positive strain. These changes may be related to moisture effects or movements in respective materials. No further changes occurred during subsequent cycles, the average thermal coefficient of lean concrete decreasing slightly and that of the stabilised soil remaining approximately constant.
9. MEASUREMENT OF RESTRAINED MOVEMENT IN THE LABORATORY

The section gives details of published work on restrained movements followed by an assessment of the requirements of a restrained thermal movement apparatus which would be suited to use with the materials under study in this project and finally a discussion leading to the choice of thermal restraint system employed.

9.1. Previous work on restrained movements in concrete

The papers to be referred to are concerned with the restraint of either thermal movement or of shrinkage, the former being of interest both from the standpoint of the equipment used and on account of results obtained, the latter chiefly on account of the method of restraint employed.

Springenschmid\textsuperscript{82,83} employed a linear system in which both expansion and contraction of a 1 m length prism were restrained by the use of twin 70 mm diameter alloy steel rods, the ends of prisms being held by notched, tapered end pieces. While details of the alloy steel were not given, it was reported to have a coefficient of thermal expansion of only $0.8 \times 10^{-6}/^\circ\text{C}$. The sides of the mould were constructed from wood and were insulated so that cooling could occur only from the upper and lower surfaces of prisms. Figure 9.1. shows longitudinal and transverse sections through the apparatus. Springenschmid cast test specimens at a temperature of 30°C, cooling them after a predetermined time by decreasing the temperature of the surrounding chamber at the rate of 2°C per hour until failure occurred. Stresses in the concrete were measured to an accuracy of 0.05 MN/m\textsuperscript{2} using strain gauges bonded to the alloy steel restraining bars. Temperatures in the concrete were recorded by thermocouples mounted within test specimens.

Tests were undertaken on concretes containing an ordinary Portland cement and a rapid hardening cement at various water/cement ratios and with a cement content of 380 kg/m\textsuperscript{3}. Cooling was commenced at various ages in the range 2 hours to 48 hours.
Figure 9.1. Linear restraint system employed by Springenschmid\textsuperscript{82}.

Figure 9.2. Test specimen used by Dinh in restrained thermal contraction studies\textsuperscript{84}. 

\begin{itemize}
\item[\textbf{1 m constant length}]
\item[Expanded polystyrene]
\item[Test prism Wooden sides]
\item[70 mm alloy steel restraining bars]
\end{itemize}

\begin{itemize}
\item[160 mm]
\item[140 mm]
\item[320 mm]
\end{itemize}
In all cases, the comparatively high cement contents in conjunction with insulated test specimens, resulted in significant temperature rises within a few hours of casting. Since the concrete was plastic during the earlier part of the period in which its temperature increased, but had stiffened either before or during the subsequent cooling period, substantial tensile stresses were induced by the time its temperature had returned to 30°C. Figure 9.3. shows temperature/time and stress/time relationships for a typical concrete in which cooling was commenced at an age of 24 hours. Following an earlier state of compressive stress, the concrete returned to zero stress at a temperature of 34°C and an age of 12 hours. Cracking occurred at an age of 32 hours and a temperature of 21°C, representing a reduction of 9°C relative to the starting temperature or 15°C relative to the maximum concrete temperature.

Figure 9.4. shows how the zero stress temperature, failure temperature and failure stress of concrete were related to the time at which cooling commenced and the time of failure, Springenschmid suggesting that there is a critical age at which concretes have a maximum susceptibility to cracking. This was considered to be between 8 and 14 hours for concretes manufactured at 30°C and between 12 and 20 hours for concretes manufactured at 20°C. Rapid hardening cement was found to result in higher zero stress temperatures and therefore higher stresses on cooling, failure in all cases occurring by the time the temperature had fallen to 25°C.

Figure 9.5. shows results obtained for differing aggregates and water/cement ratios, indicating that concretes produced from aggregates having a low coefficient of thermal movement were less susceptible to cracking and that changes in water/cement ratio and therefore strength did not produce any marked effect.

Dinh employed a linear restraint system in which 140 mm diameter cylinders with tapered end pieces were attached by means
Figure 9.3. Temperature/time and stress/time relationships for concrete subjected to restrained thermal contraction at the age of 24 hours."
Figure 9.4. Zero stress temperature, failure temperature and failure stress of concrete as a function of time of failure, Springenschmid\textsuperscript{82}.

Figure 9.5. Effect of water/cement ratio and aggregate type on failure temperature and failure stress\textsuperscript{82}.
of glued end plates to a tensile testing rig housed in a thermostatically controlled enclosure. Details of a test specimen are shown in Figure 9.2. The apparatus was designed so that negative strains in the concrete could be reversed as soon as they occurred by means of a servo-system controlled by an extensometer mounted on the centre section of the cylinder. The servo-system operated a hydraulic loading arrangement whenever a strain of $2 \times 10^{-9}$ was detected in the 300 mm gauge length employed. Temperatures in the chamber were measured by resistance thermometer. The system was designed to be used either for direct tension testing, for thermal movement measurements, or for restrained thermal movement tests.

Dinh used materials described as a gravel-cement which contained 3.5 per cent of Portland cement, and a gravel-slag, in which pozzolanic action occurred due to the inclusion of 15 to 20 per cent of granulated blast furnace slag and 1 per cent lime. In addition, the performance of each of these materials when modified by treatment with bituminous emulsions was investigated. Two methods of incorporating the emulsions were employed; in the first, the 6 - 20 mm size fraction of aggregate was precoated with a slow breaking emulsion and left for 48 hours before mixing, while in the second, a rapid breaking emulsion was added at the same time as the cement. Hence, six different materials were tested in total, and ages of test were 28 and 90 days for gravel-cements and 60, 180 and 360 days for gravel-slags, the latter ages reflecting the lower rate of strength development of these materials. Curing temperatures were not stated, but the temperature of specimens at the commencement of tests was $35^\circ C$, a period of 24 hours being allowed before the commencement of each test to allow them to stabilise. The air was then cooled at a rate of $0.5^\circ C$ per hour until failure occurred.

Summaries of restrained thermal movement results for the two groups of materials are indicated graphically in Figures 9.6. and 9.7. The resistance to thermal cracking of gravel-cements increased with age in the untreated or treated state, the in situ mixing
Figure 9.6. Relationship between tensile stress and temperature for gravel-cement.

Figure 9.7. Relationship between tensile stress and temperature for gravel-slag.
of the bitumen emulsion producing a marginal improvement in cracking resistance compared to the untreated material at the age of 90 days. In situ mixing of bitumen impaired performance at the age of 28 days as did precoating the aggregate at each age. In the case of untreated gravel-slags, the temperature fall to cause cracking decreased as the age of the material increased indicating a fundamental difference between the properties of gravel-slags and those of gravel-cements. The behaviour of treated gravel-slags was less straightforward, though greatest thermal cracking resistance was obtained at an age of 60 days using precoated coarse aggregate. The in situ mixing of bitumen emulsion was less successful in combating thermal cracking at the ages of 60 and 180 days than precoating the aggregate, though at the age of 360 days each method resulted in a significant increase in resistance to thermal cracking. In all cases, it is evident from the gradients of graphs that treatment of the aggregate with bitumen emulsion decreased the elastic moduli of materials, though in most cases cracking resistance was not improved since even greater reductions in failure stress occurred.

9.2. Published work on restrained shrinkage

Although restrained shrinkage measurements have been carried out using linear restraint systems, for example, by Ohno, they are not described here since the principles of operation are in general similar to those of the linear thermal movement restraint systems described above. Descriptions are therefore confined to annular restraint systems which have been employed on a number of occasions on account of their simplicity.

Probably the earliest work carried out using ring-shaped specimens was by L'Hermite and Grieu who cast small cement paste rings around a solid steel disc which subsequently restrained shrinkage of the cement, causing cracking. There was no provision for measurement of stresses occurring during restraint but dimensional changes were monitored and the point of cracking was detected by means of a break in electrical contact of a metallic strip fixed
around the circumference of the ring. Figure 9.8. shows sections through and dimensions of the apparatus. In order to investigate the effect of specimen size, L'Hermitte and Grieu used a second, larger version of the apparatus of internal diameter 127 mm and specimen thickness 40 mm though each version was suitable only for cement pastes on account of the small dimensions employed.

Coutinho\textsuperscript{87} employed a modified version of the apparatus in which a 12 mm thick steel ring was substituted for the solid disc and stresses in the steel were measured by means of vibrating wires fixed diametrically across the ring. He employed rings with an internal diameter of 150 mm and specimen thickness of 25 mm for cement pastes and mortars while for concretes these dimensions were increased to 650 mm and 80 mm respectively in order that 19 mm maximum size aggregates could be employed. The paper referred to did not indicate the steel thickness in the latter case though inspection of a photograph suggested that it was approximately 40 mm. The basic ring of Coutinho was further developed by Bennet and Loat\textsuperscript{88} who measured drying shrinkage by means of rings of dimensions indicated in Figure 9.9. Stresses in the steel restraining ring were monitored by means of semiconductor gauges having gauge factors of about 100 which enabled a thickness of 32 or 44 mm of steel to be used while still permitting measureable strains to be obtained. In this way, the movement of concretes was reduced to about 10 per cent of unrestrained shrinkage. Average stresses in the concrete were obtained by calculation from the measured strain in the steel.

Fujimatsu\textsuperscript{89} referred to a number of annular systems of varying dimensions which were used for shrinkage testing, concrete being cast around a steel annulus such that shrinkage was largely prevented and cracking therefore occurred. Internal diameters of the steel restraining rings varied between the 54 mm and 239 mm. However, he drew the general conclusion which is translated:" It is impossible to analyse crack and creep property in regard to free shrinkage and the breadth of the crack because of its ring shape."
Figure 9.8. Annular restrained shrinkage apparatus employed by L'Hermite and Grieu\textsuperscript{86}, (actual size).
Figure 9.9. Annular restrained shrinkage apparatus employed by Bennet and Loat. Dimensions are in mm.

Support
9.3. Requirements of a restraint system for the materials under study

The fundamental requirement of the apparatus was to restrain the concrete from contracting on lowering of temperature, although the additional question arose as to whether possible expansions of the concrete due to temperature rise should also be restrained, since the temperature of a concrete roadbase may rise shortly after placing and, in these conditions, long sections would be subject to compressive stress due to the high degree of restraint which would prevail. The temperature rise occurring due to the variation of external temperature is likely, however to be substantially greater, in general, than that due to heat of hydration. The latter would also not normally be expected to produce any observable change of temperature until at least eight hours after placing of the concrete by which time its temperature would normally be falling in sympathy with the decrease of external temperature which occurs at night. On this basis, there would be little point in restraining thermal expansion of lean concrete caused by heat of hydration in laboratory tests, since in practice heat of hydration only would be unlikely to cause a rise of temperature in absolute terms but rather a reduced temperature fall. An alternative system might therefore be to apply or remove heat to give a predetermined temperature/time function to represent environmental changes, and to restrain expansion and contraction, but although such an investigation might provide additional information on the effects of an increase of concrete temperature due to environmental change, it was considered preferable to adopt a simpler experimental approach initially, maintaining constant concrete temperature prior to cooling.

9.4. Discussion and choice of restraint system

Subject to overcoming the practical difficulties of producing effective end restraint, some form of linear system would have been most effective for the restraint of thermal contraction of roadbase materials since the specimen shape is more representative of a roadbase slab and the prediction of stresses in the material during test is more simple and therefore probably more accurate than for an annular system (which prompted the general conclusion
of Fujimatsu). A linear system of the type employed by Springenschmid might therefore be considered to be an ideal restraint system, provided it could be used in conjunction with roadbase materials rather than conventional concrete mixes and it is perhaps unfortunate that published details of the system were not obtained until the present project was underway. The chief uncertainty of the apparatus is in relation to its suitability to very low workability concrete mixes of the type used in roadbase construction, on account of the difficulty which might be experienced in compacting concrete into the notched, tapered end sections of the mould. It is likely however that with modifications to the type and shape of mould, the apparatus could be effectively used in restrained thermal movement measurements on roadbase materials if restraint to both thermal expansion and contraction was required.

Although the apparatus employed by Dinh appeared to have been used successfully with mature low strength materials, there was some doubt as to whether it would be suited to testing at early ages since handling of specimens would be involved in attaching of end pieces and mounting in the test frame. In addition, any stratification of material which occurred in cylinders during compaction would be in a plane normal to the direction of tensile stress and may therefore have a different and possibly more serious effect on performance than if stratification is parallel to the stress direction, as would be the case in roadbases in practice. There was also some doubt as to whether the construction of an elaborate servo-operated hydraulic mechanism was within the scope of the available resources.

A system of restraint considered at an early stage in the project was that of Bennet and Loat, with the inner annulus in the form of an invar steel ring such that thermal contraction would be restrained. A particularly attractive feature of this system was the simplicity and rugged nature of the basic apparatus which should therefore be resistant to damage during compaction and a further important advantage compared to the apparatus of Dinh
is that a high degree of mechanical restraint could be obtained with excellent long term stability. These advantages combined with the fact that only thermal contraction would be restrained, were considered to outweigh the disadvantages of the specimen shape and the more complex stress pattern and it was therefore decided to employ such an annular system with an invar steel restraining ring.
10. DESIGN OF AN ANNULAR RESTRAINT SYSTEM

Design is based on an analysis of the stresses obtained by cooling of a composite invar steel/concrete annular ring, the effects of bonding in a direction parallel to the ring axis also being considered. Ancillary equipment and instrumentation are then described.

10.1. Stress distribution in a concrete/steel annulus

The following analysis is given in full in the main text because a number of assumptions and principles upon which it is based have a direct bearing on the use of the apparatus and the interpretation of results. For the purpose of this analysis, the following notation is employed:

- \( \alpha \) coefficient of expansion of concrete relative to invar steel
- \( E_s \) modulus of elasticity of invar steel in tension
- \( E_m \) modulus of elasticity of concrete in tension
- \( \sigma_s \) Poisson's ratio for invar steel
- \( \sigma_m \) Poisson's ratio for concrete

Three suffixes are used for stresses and strains:
- The first denotes direction of stress; radial (r), tangential (t), axial (a)
- The second denotes material; invar steel (s), concrete (m)
- The third denotes position as indicated in Figure 10.1.

For example, \( f_{ts} \) denotes tangential stress in the invar steel at the inner circumference of the restraining ring. Consider a concrete annulus of dimensions shown in Figure 10.1, which is restrained by an invar ring in contact with the inner face of the concrete. According to Timoshenko\(^9\), the radial and tangential stresses in a thick cylinder under an externally applied stress \( f_{rs} \) as would apply to the invar steel, are given by

\[
f_{rs} = \frac{f_{rsb} b^2}{(b^2-a^2)} \left[ 1 - \frac{a^2}{x^2} \right]
\]

and

\[
f_{ts} = \frac{f_{rsb} b^2}{(b^2-a^2)} \left[ 1 + \frac{a^2}{x^2} \right]
\]
Figure 10.1. Section of annular ring used for restrained thermal contraction tests.
Both \( f_{rsx} \) and \( f_{tsx} \) may be expressed in terms of \( f_{tsa} \), the tangential stress in the steel at radius \( a \), since

\[
f_{tsa} = \frac{2 f_{rsb} b^2}{(b^2-a^2)}
\]  

(\[10.3\])

Substituting in equations 10.1 and 10.2

\[
f_{rsx} = \frac{f_{tsa}}{2} \left[ 1 - \frac{a^2}{x^2} \right]
\]  

(\[10.4\])

\[
f_{tsx} = \frac{f_{tsa}}{2} \left[ 1 + \frac{a^2}{x^2} \right]
\]  

(\[10.5\])

Also from Timoshenko, the stresses in the concrete under uniform internal pressure \( f_{rmb} \), which is the same as the external pressure on the steel \( f_{rsb} \), are given by

\[
f_{rmx} = \frac{f_{rsb} b^2}{(c^2-b^2)} \left[ \frac{c^2}{x^2} - 1 \right]
\]  

(\[10.6\])

\[
f_{tmx} = \frac{-f_{rsb} b^2}{(c^2-b^2)} \left[ \frac{c^2}{x^2} + 1 \right]
\]  

(\[10.7\])

Expressing these in terms of \( f_{tsa} \) gives, from equation \[10.3\]

\[
f_{rmx} = \frac{f_{tsa}}{2} \left[ \frac{b^2-a^2}{c^2-b^2} \right] \left[ \frac{c^2}{x^2} - 1 \right]
\]  

(\[10.8\])

and

\[
f_{tmx} = \frac{-f_{tsa}}{2} \left[ \frac{b^2-a^2}{c^2-b^2} \right] \left[ \frac{c^2}{x^2} + 1 \right]
\]  

(\[10.9\])

Since \( f_{tsa} \) is negative under the conditions of tests, \( f_{rmx} \) represents a compressive stress in the concrete radially and \( f_{tmx} \) a tensile stress in the concrete tangentially.

The total force across any section through the invar steel/concrete...
composite may be obtained by integration and is equal to:

\[ \int_a^b f_{tsa} \, dx + \int_b^c f_{tmc} \, dx \]

On evaluation, the integrals give numerically equal quantities of opposite sign, confirming that mechanical equilibrium would exist across the section.

The tangential stresses in the concrete at the inner and outer circumference of the annulus are:

\[
f_{tmb} = -\frac{f_{tsa}}{2} \left[ \frac{b^2 - a^2}{c^2 - b^2} \right] \left[ \frac{b^2 + c^2}{b^2} \right]
\]

and

\[
f_{tmc} = -f_{tsa} \left[ \frac{b^2 - a^2}{c^2 - b^2} \right]
\]

The ratio of these is

\[
\frac{f_{tmb}}{f_{tmc}} = \frac{b^2 + c^2}{2b^2}
\]

10.2 Choice of suitable dimensions for the concrete annulus and invar steel restraining ring

A number of requirements must be considered in the selection of suitable values for a, b and c. The ratio \( \frac{f_{tmb}}{f_{tmc}} \), which should be minimised, increases with the difference between b and c. Furthermore, if 20 mm aggregates are to be employed, \( c - b \) should be at least 100 mm. The ratio \( \frac{f_{tmb}}{f_{tmc}} \) decreases as b increases, though large values of b would result in difficulty in handling the apparatus. A value of 300 mm was therefore considered to be a suitable optimum value of b, with c equal to 400 mm. The value of b - a which is equal to the thickness of the invar steel should be large enough to cause substantial restraint but small enough to result in measureable strains to assist in crack detection. A thickness of 25 mm was selected by trial and error, resulting in \( a = 275 \) mm. These values of a, b and c give:
The actual values of $f_{tmb}$ and $f_{tmc}$, determined from equations 10.10 and 10.11 are

$$f_{tmb} = -0.285 f_{tsa} \quad \text{(10.13)}$$

$$f_{tmc} = -0.205 f_{tsa} \quad \text{(10.14)}$$

10.3. Derivation of stresses in the steel and concrete for a given temperature fall

The tangential strain in the invar steel restraining ring at the interface with the concrete results from the combined effects of the tangential stress $f_{tmb}$ and the Poisson's ratio effect of the radial stress $f_{rsb}$. Hence, the total strain at this point is

$$\varepsilon_{tsb} = \frac{f_{tsb}}{E_s} - \frac{\sigma_m f_{rsb}}{E_s} \quad \text{(10.15)}$$

The tangential strain in the concrete at the interface with the invar steel results from the combined effects of its thermal contraction, its tangential stress $f_{tmb}$ and the Poisson's ratio effect of the radial stress $f_{tmb}$. Hence, the total strain is

$$\varepsilon_{tmb} = \alpha_T - \frac{\sigma_m f_{tmb}}{E_m} + \frac{f_{tmb}}{E_m} \quad \text{(10.16)}$$

In each case the negative signs are used for Poisson's ratio strains because they are opposite in sign to the stresses causing them. Since the invar steel and concrete are in contact, the tangential strains at the interface must be equal. Combination of equations 10.15 and 10.16 and insertion of their respective values derived above gives:

$$f_{tsa} = \frac{2\alpha_T}{\left[ \frac{b^2 - a^2}{b^2} \left( \frac{\sigma_m}{E_m} - \frac{\sigma_a}{E_s} \right) + \left( \frac{b^2 - a^2}{b^2} \right) \frac{b^2 + c^2}{E_s} + \left( \frac{b^2 - a^2}{c^2} \right) \frac{b^2 + c^2}{E_m} \right]} \quad \text{(10.17)}$$
Taking $a = 275$ mm, $b = 300$ mm and $c = 400$ mm, gives

\[
f_{tsa} = \frac{2\alpha_r T}{0.16 \left( \frac{\sigma_m}{E_m} - \frac{\sigma_s}{E_s} \right) + 1.84 \frac{\sigma_s}{E_s} + 0.57 \frac{E_s}{E_m}}
\]

\[\text{----------(10.18)}\]

The orders of magnitude involved can be determined by insertion of typical values, for example:

- $\sigma_m = 0.15$,
- $\sigma_s = 0.26$,
- $E_m = 30$ GN/m$^2$,
- $E_s = 144$ GN/m$^2$,

which gives

\[f_{tsa} = 0.620 \alpha_r T \times 10^5 \text{ MN/m}^2\]

For a temperature fall of 15$^\circ$C in a concrete for which

- $\alpha_r = 10 \times 10^{-6}/^\circ$C;
- $\alpha_r T = -150 \times 10^{-6}$, and therefore

\[f_{tsa} = -9.3 \text{ MN/m}^2\]

From equations 10.13 and 10.14 the tangential stresses in the concrete at its inner and outer circumferences are

- $f_{tmc} = 2.65 \text{ MN/m}^2$ and
- $f_{tmc} = 1.91 \text{ MN/m}^2$

The strain at the inner surface of the invar steel ring would be

\[\varepsilon_{tsa} = \frac{f_{tsa}}{E_s} = -64.6 \times 10^{-6}\]

The strain contributions at the invar steel/concrete interface are given by equations 10.15 and 10.16 and illustrated in Figure 10.2. It should be noted that the measured compressive strain in the invar steel at the interface is not equal to the tensile strain in the concrete, the latter being approximately equal to the temperature induced strain in the concrete minus the compressive strain in the ring, though Poisson's ratio effects are also involved. The tensile strain in the concrete is in this case approximately 60 per cent of the negative thermal strain causing it, the
Figure 10.2. Representation of strains at the interface when the invar steel/concrete composite is cooled by 15°C.

The tensile strain in the concrete would be modified by the effects of creep, should it occur.
remaining strain occurring in the invar steel. This would imply that to simulate a temperature fall of, for example, 6°C in practice would require a temperature fall of 10°C using this restraint system. A stiffer system would have the advantage that stronger concrete mixes could be tested to destruction using normal environmental temperatures for casting without reaching freezing point, but the dimensions given above were employed in this project in order that measurable strains be obtainable in the restraining ring when lean concrete was tested at early ages.

A graph of stresses in the invar steel/concrete composite is shown in Figure 10.3., the total tensile area being equal to the total compressive area and the stress at the inner perimeter of the concrete being 39 per cent higher than that at the outer perimeter. The annular arrangement is such that the change of stress at the invar steel/concrete interface gives rise to a radial stress rather than a shear stress at this surface.

10.4. Effects of bonding between the concrete and invar steel in an axial direction

Although it has been shown that bonding between the concrete and invar steel is of no direct consequence as far as tangential stresses are concerned, it is possible that bonding in the axial direction may cause substantial stresses in this direction and that it may also modify tangential stresses due to Poisson's ratio effects. A simple analysis carried out in Appendix B showed that failure in the concrete could theoretically occur axially if there was sufficient bonding in this direction. At the same time, it was considered undesirable to apply a release agent in the normal manner or to employ a slip membrane at the invar steel/concrete interface owing to the possibility that some cushioning action might occur during tests either by retardation of hydration or due to the thickness of the film itself. No reference has been made to the possibility of bonding by previous users of annular restraint systems and therefore particular attention was paid to this in trial tests which were carried out. Stress reductions at failure in these tests were
Figure 10.3. Stress distribution in invar steel/concrete composite for a temperature fall of 15°C.
normally substantial and there was little sign of bonding between concrete and the invar steel, rings being easily broken off once cracking had occurred. Stress reductions in concretes at early ages were however smaller in proportion to the maximum stress, indicating that some bonding may have occurred in these cases, although it is considered very unlikely that the extent of bonding actually occurring would result in other than a small fraction of stresses calculated in Appendix B. Further consideration is therefore restricted to a qualitative description of the effects of partial bonding between concrete and invar steel, the various stresses involved being indicated in Figure 10.4.

(a) Concrete. At the outer circumference of the concrete the average tensile stress in a tangential direction \( f_{tmc} \) would be increased by the Poisson's ratio effect of the axial compressive stress \( f_{amc} \). At the concrete/invar steel interface, the axial tensile stress would, on the same account, induce a compressive stress in the tangential direction reducing the existing stress \( f_{tmb} \). Axial stresses therefore act against the previous tendency for the stress at the inner circumference of the concrete to be greater than that at the outer circumference, though it is uncertain whether the former stress gradient would be reduced, cancelled or reversed.

(b) Invar steel. The average compressive stress in an axial direction would tend to induce a Poisson's ratio tensile strain in a tangential direction thereby stiffening the ring and increasing its effective restraint. At the internal face of the ring the tensile stress axially would partially increase the Poisson's ratio tensile stress already present, increasing the sensitivity of the resistance strain gauges. Again, the exact extent of such effects would be very difficult to estimate.

Notwithstanding the difficulty in estimating the overall
Figure 10.4. Stresses applied to concrete ring section as a result of cooling.
effect of bonding between the concrete and invar steel, it was decided to carry out the full test programme with no special debonding arrangement at the interface other than a thin film of release agent, any excess being removed with tissue paper before casting of the concrete. The calculations given earlier were also assumed to be valid, although further consideration was given to the possibility that bonding might affect the behaviour of the system at the time of analysis of results.

10.5. Strain measurement

Strains were measured in order to provide an indication of the point of failure in annular rings and also as a means of checking stresses in the concrete.

Particular care was necessary in the selection at early ages of a strain measurement system since, when testing at early ages, the stresses in the concrete at failure were likely to be substantially smaller than those measured for example by Springenschmid \(^8\) and Dinh \(^8\). In section 10.3., a concrete stress of 2.65 MN/m\(^2\) was shown to correspond to a strain of \(-64.6 \times 10^{-6}\) at the inner surface of the invar steel ring. In view of the fact that tensile stresses of perhaps less than 20 per cent of this value might cause failure at early ages, it was considered essential that the strain gauges have a sensitivity of at least \(1 \times 10^{-6}\) with good long term and temperature stability.

The semi-conductor gauges used by Bennet and Loat \(^8\) for high sensitivity measurements during shrinkage testing are unfortunately rather susceptible to temperature change so that electrical resistance strain gauges were employed instead, foil gauges of 1000 \(\Omega\) nominal resistance being used in a full bridge network. In order to permit the use of a high sensitivity voltage recorder to measure strain gauge outputs, the resistance of gauges already supplied as matched
pairs was measured to 0.1 Ω accuracy and gauges were then wired so as to produce voltage outputs corresponding to zero stress of as close to zero as practicable. Two complete bridges were employed, the opposite sides of each bridge being at 90° to one another on the circumference of the invar steel restraining ring so as to cancel out bending effects. The two dummy gauges in each bridge were bonded to the ring in an axial direction. The resolution of strain is given by:

\[
\text{Voltage} = \frac{\text{voltage} \times \text{gauge factor} \times \text{smallest resolvable output}}{\text{input strain}}
\]

Although the maximum recommended input voltage was 50 Volts, a value of 8 Volts was used to minimise the heat input and produce maximum long term stability. For a gauge factor of 2.14, and 10 microVolts sensitivity in recording equipment the smallest resolvable strain should be \(1.17 \times 10^{-6}\). The dummy gauges were bonded to the invar steel in a direction at right angles to the live gauges and would therefore respond to the Poisson's ratio strain in this direction, increasing the effective sensitivity of the gauges. The true strain per 10 microVolts output, taking a Poisson's ratio of 0.26 would then be \(0.93 \times 10^{-6}\). The sensitivity could be increased, if necessary, by use of higher bridge voltages, though for the purpose of initial tests, it was considered satisfactory.

10.6. General comments on the design adopted

Unless an annular ring of extremely large diameter was employed, there would inevitably be a variation of stress across the concrete section, though the possibility that this may be modified by the effect of bonding in an axial direction has already been discussed. Nevertheless, for the purpose of calculation of stresses at failure, equations 10.13 and 10.18 were employed on the basis that cracking in the concrete would be initiated at the more highly stressed inner circumference and propagate rapidly through the
remaining ring section. Although the evaluation of equation 10.18 requires a knowledge of the Poisson's ratio for concrete, this is not considered to be a disadvantage of the annular system since errors in \( \sigma_m \) would result in very small errors in the stress so obtained, as illustrated in Figure 10.2.

The design also represented a compromise between the requirements of stiffness and sensitivity and although the system was intended to result in measurable strains in low strength concrete while having sufficient stiffness to cause failure in stronger mixes, it is accepted that ideally two or more designs of various stiffnesses might be employed with advantage. However, in carrying out all tests with a single restraint system the apparatus requirements were simplified and, in addition, any difficulty which might arise in equating results from systems of different stiffnesses was avoided.

10.7. Measured properties of the invar steel ring

An annular ring was manufactured by rolling from a 100 mm x 40 mm x 1500 mm invar steel strip, the welded joint being formed from the same metal. The ring was then annealed and machined to the required dimensions. Routine measurements carried out on the ring gave the following results:

Outside diameter 600 ± 1 mm
Thickness 25.2 ± 0.2 mm
Depth 97 ± 0.5 mm
Elastic modulus, measured by loading across a diameter 144 ± 2 KN/mm²
Coefficient of thermal expansion measured across a diameter over the temperature range 5-30°C 2.5 ± 0.5 x 10⁻⁶/°C
Coefficient of thermal expansion of invar steel bar machined from offcut (Temperature range 20-60°C) 2.6 ± 0.2 x 10⁻⁶/°C
10.8. Ancillary equipment used in restrained thermal movement tests

This equipment comprised instrumentation used for recording strains and temperatures in the steel/concrete annulus, moulding apparatus and an environmental chamber for the controlled cooling of concrete rings.

**Strain gauges.** The two independent strain gauge bridges were bonded to the invar steel restraining ring using epoxy resin adhesive and wired to an X-t chart recorder with a maximum voltage resolution of 10 microVolts and operated continuously at a chart speed of 7.5 mm per hour. Wires were fixed inside the ring using a proprietary silicone rubber sealer adhesive and in order to minimise moisture penetration, additional waterproofing was carried out using "Di-jell" silicone grease which was renewed periodically. All electrical connections were soldered. The direct current input to strain gauges was supplied by a constant voltage generator.

**Thermocouples.** Copper-constantin thermocouples having an output of 38.8 µV per °C were used in all tests. These were wired to a data logger having a maximum resolution of 10 microVolts enabling temperature changes of 0.25°C to be detected reliably. Cold junctions were situated in an isothermal unit which generated a variable voltage dependent on ambient temperature, such that the recorded total output voltage corresponded to a cold junction temperature of 0°C.

**Moulding apparatus.** The outer mould consisted of a 25 mm thick steel ring in three sections, held together with angle brackets and bolts. The base-plate contained a groove to locate inner and outer rings. The inner ring and concrete could be lifted by four feet, connected to form a rigid frame and fitting into recesses in the base-plate in order that the concrete section remained constant. Movement of the inner and outer rings in the base-plate grooves was avoided by tight fitting lugs on the external perimeter of the two rings. Plate 10.1. shows the moulding assembly and lifting frame.

**Environmental chamber and cooling unit.** A galvanised steel
Plate 10.1. Moulding assembly and lifting frame used in restrained thermal movement tests.
The chamber was designed and built, enabling the temperature of annular rings and control prisms to be decreased quickly and conveniently. Heat was exchanged by means of water recirculated in the sides and base of the chamber which formed a jacket. The total capacity of this jacket was 19 litres and it was found that a model heat pump having an extraction rate of 300 Watts could reduce the water temperature from 20°C to 4°C in approximately 1 hour if required. The chamber was lagged all round with expanded polyurethane, the upper layer being removable so that the contents could be inspected through a close fitting perspex lid. The sides and lid of the chamber were, in addition, protected by an aluminium cover which also reduced radiant heat exchanges with the laboratory. The water temperature in the chamber was controlled by contact thermometer operating a heat pump by means of a low-voltage relay. Cooling rates could be also controlled by means of a time switch with 15 minute on/off selections. Figure 10.5. shows a section through the environmental chamber and Plate 10.2. a general view of the chamber and ancillary equipment.

10.9. Selection of cooling rate

It was desired to cool the concrete as quickly as possible, but to avoid excessive temperature gradients across the concrete/invar steel section. In order to detect such temperature gradients and to check the functioning of the equipment generally, a series of preliminary tests was carried out with a prefabricated thermocouple insert in the concrete. This comprised 16 thermocouples sandwiched between two 10 mm thick 100 mm square fine aggregate-cement mortar slabs sawn from a 100 mm cube (Plate 10.3.). The thermocouples were mounted on a 20 mm grid on one slab, insulated from one another using polyester resin and then the other slab was bonded to the first, also using a polyester resin. The composite slab, wired back to the data logger was mounted in the annular mould prior to casting the concrete,
Figure 10.5. Section through environmental chamber and ring assembly.
Plate 10.2. General view of restrained thermal movement apparatus and ancillary equipment.
Plate 10.3. Thermocouple bank used in preliminary tests to determine temperature gradients in the concrete during cooling.
which was compacted carefully at each side of the slab. It could reasonably be assumed therefore that the thermocouple assembly behaved, for thermal purposes, as part of the concrete ring.

In a first trial test, no circulating fan was employed and the cooler was operated at its maximum power. One of the concrete interfaces with the thermocouple slab failed, as would be expected, at a relatively low temperature reduction though the subsequently plotted temperature/time curves showed that the crack did not affect the ring thermally. Temperatures at diagonally opposite extremities of the ring section are shown in Figure 10.6, the lower, outer perimeter of the concrete ring cooling more quickly than the upper, inner perimeter. This was due to the fact that, in the absence of forced air movement, cooling was probably caused largely by radiant heat exchanges with the sides and base of the environmental chamber in which the cooling water circulated. Isotherms in the concrete obtained 75 minutes after the commencement of cooling are shown in Figure 10.7, and these represent the greatest differentials obtained during the test. The maximum and minimum temperatures recorded in the section at this time were 19.25°C and 16.25°C, the isotherms suggesting that the temperature at the lower, outer corner of the section might be below 16°C.

It was considered necessary to reduce this maximum differential to perhaps 10 per cent of the minimum temperature fall likely to cause failure in order that possible systematic errors involved did not exceed experimental tolerances. Supposing that a temperature fall of 5°C might cause cracking in low strength materials at early ages, a temperature differential of 0.5°C maximum was selected as a target value. At the same time, it was essential, from a practical standpoint, that temperature falls of 15°C, which might be required to crack stronger more mature concretes, be obtainable within a period of eight or ten hours. A circulating fan
Figure 10.6. Variation of temperature with time in preliminary cooling tests.
Figure 10.7. Isotherms in the concrete section during the first cooling test in which no fan was employed.

Figure 10.8. Isotherms in the concrete section during the second test in which a circulating fan was employed.
was therefore included in an attempt to reduce temperature differentials and increase the rate of heat exchange and a second test was conducted with the cooler again operating at the maximum rate. A 240 Volt 24 Watt fan was employed, though by the application of only 120 Volts to the fan, the power consumption was reduced to 6 Watts, thereby reducing the heat input to the fan itself while still providing adequate air movement in the chamber. The results of this test are shown in Figures 10.6. and 10.8. The maximum differential was not significantly reduced though the upper surface of the concrete in this case cooled in a similar manner to the lower surface, the principal cause of the differential now being the low rate of radiation exchange and high thermal inertia of the invar steel restraining ring. However, the overall heat exchange in the early period was improved, the second fan-assisted test resulting in an average temperature fall of 6°C in the initial 90 minute period, whereas, in the earlier test, an average fall of 4°C occurred in the same period. The starting temperature in the second test was 3°C higher which would in any case result in a slightly greater initial cooling rate but the time taken for a 10°C fall of temperature was considerably reduced, being 2 hours 30 minutes, compared to 4 hours without the fan. In order to preserve the relatively high cooling rate provided by the fan and, at the same time, reduce the maximum temperature differential, it was decided to retain the fan but to operate the cooler intermittently. The cooling rate obtained with the cooler switching on and off at 15 minute intervals and monitored by thermocouples attached to the inner invar steel surface is shown in Figure 10.9. The maximum cooling rate was just over 2°C per hour; which on the basis of gradients measured in Figure 10.6. is slightly in excess of that which would produce a maximum temperature differential of 0.5°C. However, more intermittent operation of the cooler would have resulted in much longer times being necessary to reach
Figure 10.9. Temperature/time curve employed in restrained thermal movement tests.
the relatively low temperature required to cause failure in richer more mature concretes and the temperature function of Figure 10.9 was therefore adopted for all tests. Once the cooling water temperature was near to the value of 4°C set on the contact thermometer, the time switch was reset to give continuous operation, cooling being interrupted only by the operation of the thermometer. Concrete temperatures were normally measured by thermocouples attached to the inner circumference of the restraining ring since good thermal contact with the metal could be obtained at this position and the risk of damage to thermocouples was minimised. Readings obtained would then correspond to the highest temperature in the concrete section such that temperature falls to cause cracking would be conservative by a maximum of approximately 0.5°C.
11. EXPERIMENTAL PROCEDURE

The procedure adopted for dry lean concrete mixes is described as these formed the major part of the testing programme, any modifications which were required for other materials being detailed subsequently.

11.1. Manufacture of test specimens

For the purpose of casting concrete rings, the mould assembly was mounted on a solid concrete floor so that sufficient compactive effort could be obtained when using the electric vibrating hammer. To prevent heat being lost downwards, the apparatus was insulated from the floor by a 75 mm thick expanded polyurethane foam cushion. Prior to casting, all joints in the mould were sealed with adhesive tape and surfaces were coated with a neat oil release agent. The outer surface of the invar ring was then wiped with absorbent tissue paper so that only a very thin film of oil remained, minimising any cushioning effect or retardation of hydration of the concrete due to the oil.

Concrete was thoroughly mixed by hand and the annular mould was filled in 50 mm layers, the concrete being compacted using the vibrating hammer under pressure with a specially shaped foot attachment which fitted accurately the shape of the mould, though care was necessary to avoid damaging the invar steel ring and the outer moulding ring during compaction. The surface of concrete was finished by vibration in conjunction with firm trowelling by hand, after which, the concrete was covered with polyethylene sheeting. From the same concrete mix, one 250 x 100 x 100 mm prism and one 500 x 100 x 100 mm beam were made for thermal movement and strength tests respectively. The three specimens were stored in the same laboratory, the air temperature being controlled thermostatically by means of a fan heater and contact thermometer set to a temperature of approximately 21°C.

After curing for a period of 24 hours, the concrete ring assembly was demoulded by lifting clear of the base plate and locating lugs using a differential pulley, the outer moulding ring then
being removed by releasing the three securing brackets. The concrete ring was then transferred by means of an overhead track into the curing chamber and the airtight lid fitted into place. Care was taken during transfer to avoid impact stresses, though there was generally no evidence of damage either visually or from strain gauge readings at this stage. The thermal movement reference studs or ball bearings were attached to the 250 mm prism which was also cured in the environmental chamber, temperatures being monitored by thermocouples fixed to the upper and lower faces. On account of the relatively high evaporable water content of the lean concrete mixes, the air in the chamber became saturated soon after rings were inserted. Temperatures of the concrete in the chamber were monitored continuously during curing, cooling or heating being applied as necessary to maintain a steady level, although the thermal inertia of the system, together with the thermostatic control of the laboratory, were such that once heat of hydration had dissipated, changes of temperature of the lean concrete mixes were minimal, not usually exceeding 0.5°C.

Changes of temperature occurring in the concrete during the first few hours after casting, though small for lean concretes, introduced the need to adopt some criterion as to the effective casting temperature for the purpose of determining the fall of temperature to cause cracking in each case. This reference temperature was taken to be that corresponding to an age of 10 hours, measured from the time of contact between water and cement since the change from the plastic to solid state was assumed to occur at about this time. In each test, the temperature in the environmental chamber was adjusted therefore to be as near as possible to the temperature of the concrete 10 hours after mixing. In the case of lean concretes, this was approximately 0.5°C higher than the casting temperature.

11.2. Test procedure and detection of cracks

Prior to the commencement of cooling, temperatures were checked
and the 'Demec' gauge readings on opposite faces of the 250 mm prism were noted. Cooling was then carried out and temperatures were recorded at 10 minute intervals until failure. Immediately failure was indicated by discontinuities in the strain gauge outputs, the temperature of the concrete and 'Demec' gauge readings on the 250 mm prism were recorded. The strain change at failure tended to vary according to the position of a crack relative to the strain gauge mounting positions, though in almost all cases, a clear discontinuity was recorded by both pairs. Figure 11.1 shows the strain/time relationship for lean concrete tested at 28 days of age. Cooling was normally continued for a period of one or two hours after cracking had occurred in order to allow crack widths to increase, thereby facilitating visual detection. Although in most cases, cracks were located visually, there were a number of occasions, chiefly in the case of young or partially compacted concretes, on which crack positions could not be located. In many tests, two cracks occurred shortly after one another as indicated, for example, by the strain gauge readings of Figure 11.1. The second crack probably occurred due to an expansion of the invar steel in the region of the first crack after the compressive stress in that position decreased, resulting in a flexural stress in the concrete on the opposite side with a corresponding tensile stress at the inner circumference. When two cracks occurred, they were always diametrically opposed with an approximate tolerance of 10 degrees. In early tests, crack positions were noted using an arbitrary scale on the circumference of the invar ring with a view to the detection of any tendency towards systematic cracking caused, for example, by ring eccentricity or the welded joint, but none was observed. Plate 11.1 shows a crack obtained in a 4.5:1 concrete mix at 2 days of age, photographed after the crack width had been increased by shrinkage.

In total, 13 tests were carried out on dry lean concrete over a period of 13 months, this relatively long testing period being necessary because large, uncontrollable solar heat inputs
Figure 11.1. Stress developed in lean concrete during restrained thermal contraction at 28 days of age.

This was an excellent profile on a mature specimen. Strain patterns were less well defined in lean concrete specimens tested at earlier ages.
occurred in the laboratory during the months of June to August so that testing could not take place in this period. The supply of cement was changed on several occasions during this time, though the effect of variations was minimised by selecting ages for test at random.

11.3. Measurement of elastic modulus and tensile strength

The 500 x 100 x 100 mm beams used for these measurements were left in their moulds, covered with an impermeable sheet until required for testing in order that their curing condition approximated to that of the sealed state. Dynamic elastic modulus was then measured using the method described in BS 1881.".

Immediately after the measurement of elastic modulus, the uniaxial tensile strengths of prisms were measured using scissor grips. The loading rate was as given in BS 1881" for the indirect tension test though, on account of the rather friable surface of lean concretes tested at early ages, some degree of slipping at the bearing plate surfaces often occurred, causing loading rates to deviate slightly from the correct value.

Both elastic modulus and direct tension tests were carried out at the approximate time of failure of the corresponding concrete rings subjected to restrained thermal contraction.

11.4. Testing details for materials other than lean concrete

11.4.1. Partially compacted lean concrete

The masses of concrete which would be required to fill the three moulds on the basis of a 5 per cent shortfall of the average density obtained in previous tests were calculated. Concretes were then placed and compacted in several layers using slight pressure only from the electric hammer, until the moulds contained the calculated quantity of concrete. No attempt was made to finish the surface of test specimens. Examination of failed
samples showed that a good degree of uniformity across the section of specimens appeared to have been obtained.

11.4.2. Wet lean concrete; conventional and high strength concrete

Procedure followed was as for lean concretes except that water was added at the time of mixing and compaction was carried out by application of the foot of the vibrating hammer to the outside edge of the outer steel moulding ring, care being taken not to allow the hammer to contact inner edges of the mould. After a number of tests, some distortion of the outside edge of the outer moulding ring occurred but this in no way affected the dimensions of the concrete ring section or the performance of the apparatus. Prisms for routine tests were compacted using a vibrating table.

To a small extent with the 6:1 mix and to a larger extent with the 4.5:1 and the 3.4:1 mixes, difficulty was experienced during the first 24 hours in controlling the temperature of the concrete due to the effects of heat of hydration. However, the earlier procedure of curing the concrete at the temperature which occurred 10 hours after mixing was continued although in some instances, this resulted in a curing temperature of approximately 22°C. In the case of the mix of aggregate/cement ratio 3.4, cooling using a small electric fan in the centre part of the equipment was carried out prior to transfer of concrete to the environmental chamber. Figure 11.2. shows typical temperature/time curves for three mixes of differing cement content. Strain gauge readings obtained from the high strength concrete mix indicated a tensile stress in the concrete at the steady curing temperature probably similar to those obtained by Springenschmid, and indicating that the 10 hour criterion may have been inappropriate for this material. The recorded stresses were in all cases very small compared to failure
Figure 11.2. Temperature rises above ambient of different types of concrete due to heat of hydration.
stresses, however, and were not allowed for in stated
temperature falls to cause cracking.

11.4.3. Concrete incorporating steel fibres

The procedure for the manufacture of richer mixes was
adopted, fibres being scattered slowly by hand into the
concrete in the latter stages of mixing. Two tests were
undertaken, one on concrete containing fibres and, for
comparison, one test on the same concrete mix but
without fibres.

11.4.4. Detection of stress relief by creep

The concrete mix employed was the same as that incorporating
steel fibres. The concrete ring was transferred in
the normal way to the environmental chamber at an age
of 24 hours and then cured for a further 24 hours before
being cooled at the standard rate. Four hours after
cooling was commenced, the water temperature was reset
such that the temperature recorded at the time was
maintained and then the prevailing concrete temperature
which was approximately 11°C was held for a period of
24 hours. During a first test, stress relief apparently
occurred quite quickly and it was found to be due to an
error in resetting the cooling water temperature. The
steady temperature selected had been based on thermocouple
readings obtained from the inner face of the invar steel
ring and was therefore slightly higher than the average
concrete temperature in the non-steady state, so that,
on setting the concrete to this temperature, some parts
became slightly warmer. In the subsequent test, therefore,
a thermocouple was attached to the concrete itself and
care was taken to ensure that the concrete temperature
increased at no point of the test.
12. RESULTS FOR LEAN CONCRETE

The results obtained for lean concrete are presented and discussed in relation to their likely effect on the thermal cracking properties of roadbases. Table 12.1. summarises the results of the main programme of tests, including statistical variations at the ages of 2 and 7 days.

12.1. Coefficient of thermal expansion

Prior to discussion of thermal movement results, the conditions under which measurements were made are considered since these have a bearing on the accuracy and magnitude of thermal coefficients obtained.

In order to determine thermal coefficients over a temperature range as near as possible to that which caused cracking in corresponding restrained thermal movement tests, length measurements of prisms were taken at the approximate time of cracking of concrete rings. Prisms were not, therefore, in a state of thermal equilibrium, though differentials of temperature within the prism cross-section were generally expected to be small compared to the 0.5°C target value for the rings in which cooling of concrete in the region of the invar steel interface was impeded by the low emissivity of the inner metal annulus. Nevertheless, the effects of the non-steady state might be considered and these are two-fold: first, the surface at which measurements were taken would tend to cool and contract more than inner parts and second, the inner, warmer sections would, in consequence, restrain the surface movements. The two effects would oppose one another but lead, on balance, to a low estimate of the coefficient of thermal movement since the surface temperatures measured would give the maximum range occurring while the movement measured would be less than that which would theoretically correspond due to restraint by warmer, inner parts of the prism. The largest error resulting would occur in tests carried out at early ages and, in particular, at the ages of 16 hours and 1 day since corresponding rings fractured at a time at which relatively large temperature differentials existed. In
Table 12.1. Results of restrained thermal contraction tests and control tests on lean concrete.

<table>
<thead>
<tr>
<th>Age</th>
<th>Coeff. thermal movement x $10^{-7}$°C</th>
<th>Dynamic modulus E GN/m²</th>
<th>Static modulus % of 28 days</th>
<th>Static modulus E GN/m²</th>
<th>Static modulus % of 28 days</th>
<th>Measured tensile strength $f$, NN/m²</th>
<th>Measured tensile strength % of 28 days</th>
<th>$f/E$ x $10^{-6}$</th>
<th>Apparent temp. fall o°C</th>
<th>Total calc. strain x $10^{-6}$</th>
<th>Predicted temp. fall abs. restr o°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 hours</td>
<td>12.0</td>
<td>14.0</td>
<td>9.0</td>
<td>26</td>
<td>0.12</td>
<td>8</td>
<td>13.3</td>
<td>3.5</td>
<td>28.0</td>
<td>2.3</td>
<td></td>
</tr>
<tr>
<td>1 day</td>
<td>11.8</td>
<td>23.4</td>
<td>18.4</td>
<td>54</td>
<td>0.31</td>
<td>21</td>
<td>16.8</td>
<td>3.75</td>
<td>25.2</td>
<td>2.1</td>
<td></td>
</tr>
<tr>
<td>2 days</td>
<td>13.4</td>
<td>(0.9)</td>
<td>28.1</td>
<td>(1.0)</td>
<td>23.1</td>
<td>68</td>
<td>0.56</td>
<td>(0.03)</td>
<td>39</td>
<td>5.25</td>
<td>(0.42)</td>
</tr>
<tr>
<td>4 days</td>
<td>13.1</td>
<td>32.3</td>
<td>27.3</td>
<td>80</td>
<td>0.93</td>
<td>64</td>
<td>34.1</td>
<td>6.25</td>
<td>42.3</td>
<td>3.2</td>
<td></td>
</tr>
<tr>
<td>7 days</td>
<td>12.0</td>
<td>(0.7)</td>
<td>35.3</td>
<td>(1.3)</td>
<td>30.3</td>
<td>89</td>
<td>1.12</td>
<td>(0.12)</td>
<td>77</td>
<td>7.4</td>
<td>(0.26)</td>
</tr>
<tr>
<td>14 days</td>
<td>12.1</td>
<td>37.1</td>
<td>32.1</td>
<td>94</td>
<td>1.39</td>
<td>96</td>
<td>43.3</td>
<td>8.9</td>
<td>51.2</td>
<td>4.2</td>
<td></td>
</tr>
<tr>
<td>28 days</td>
<td>13.4</td>
<td>39.1</td>
<td>34.1</td>
<td>100</td>
<td>1.45</td>
<td>100</td>
<td>42.5</td>
<td>8.6</td>
<td>55.0</td>
<td>4.1</td>
<td></td>
</tr>
</tbody>
</table>
the case of the 16 hour test, for example, the temperature fall of the prism at the time of measurement was 7°C which might result in a maximum percentage error of 7 per cent if the temperature differential was 0.5°C, though the likely error would be smaller due to increased thermal contraction at the surface. In addition, the experimental tolerances involved in measurement of the small temperature and length changes would result in relatively large uncertainties at early ages.

Values of the thermal coefficient were generally rather in excess of the value $11.0 \times 10^{-6}/°C$ obtained in the sealed state in the preliminary series of thermal movement tests and similar to values obtained on the cycled tests for the same material (Figure 8.2.), although the first test of that series resulted in the higher value of $17.5 \times 10^{-6}/°C$. The results are plotted graphically against age in Figure 12.1. which does not seem to indicate any overall significant trend, the total range being only $1.6 \times 10^{-6}/°C$. Most significance should be attached to the 2 and 7 day results since they were each obtained from at least three tests, but even these cannot be regarded as indicating conclusively any possible maximum value of the thermal coefficient. The results nevertheless suggest that, if an overall thermal coefficient of perhaps $12.5 \times 10^{-6}/°C$ were taken at all ages for the purpose of predicting cracking patterns, the consequent errors incurred would be small compared to other random uncertainties considered later.

12.2. Elastic modulus and uniaxial tensile strength

Electrodynamic moduli, given in Table 12.1., increased from 14.0 GN/m² at an age of 16 hours to 39.1 GN/m² at 28 days, and a high degree of repeatability was obtained as reflected by the low standard deviations resulting from measurements at 2 and 7 days of age. Since dynamic elastic moduli are effectively measured at very low stresses, they are appreciably higher than those measured in static tests and, in order to obtain an estimate of corresponding static values, results of Kolias\textsuperscript{40} were referred to. Figure 12.2.
Figure 12.1. Coefficients of thermal movement for lean concrete as a function of age.
Figure 12.2. Relationship between dynamic and static tensile elastic modulus obtained by Kolias for lean concrete together with line of equality.
shows the relationship between dynamic modulus and the static secant modulus for a lean concrete of aggregate/cement ratio 18, measured at half the ultimate strength at ages of 2, 7, 28 and 100 days. It is evident that over the range of values covered, there is good correlation between static and dynamic moduli with the latter being 5 GN/m² in excess of static values. In view of the fact that relationships between the two moduli usually depend on the type of concrete employed and on strength properties as well as age, it was considered advantageous to use this relationship which was derived for lean concrete tested in tension, rather than more general relationships which might be of doubtful validity in this context. Results of Felton for a cement stabilised fine grained soil are broadly in agreement with those of Kolias over the range of values of Figure 12.2. although they indicate that the numerical difference between moduli may decrease to 4 GN/m² or less when dynamic moduli are in the region of 10 GN/m². However, in the absence of more detailed information for lean concrete, the constant difference of 5 GN/m² was also applied to results obtained at early ages.

Uniaxial tensile strengths, indicated in Table 12.1., increased from 0.12 MN/m² at an age of 16 hours to 1.45 MN/m² at 28 days, the smooth progression of results suggesting that a reasonable degree of accuracy was obtained even in respect of tests at early ages in which specimens tended to slip in the grips during testing.

The relative development of static elastic modulus and strength may be compared by plotting values at intermediate ages as a percentage of 28 day values. Figure 12.3. shows the relationship and it is noticeable that the elastic modulus is almost always greater, as a percentage of its 28 day value, than the tensile strength. The ratio of tensile strength to static elastic modulus which, according to equation 4.1. is of fundamental significance in determining cracking properties, is plotted graphically in Figure 12.4. together with results by Kolias for comparison (age is plotted logarithmically in all graphs).

12.3. Observed temperature fall to cause cracking

Strain gauges gave clear indications of the point of cracking of concrete rings, except in the case of one test at 16 hours
Figure 12.3. Static elastic modulus and tensile strength plotted as a percentage of their 28 day values at intermediate ages.
Figure 12.4. The ratio of tensile strength to static elastic modulus for lean concrete as a function of age together with results by Koliás for comparison.
of age in which there was no visible crack and no conclusive sign of stress reduction in the concrete. It was inferred in this case either that the concrete was already damaged perhaps by impact during transfer to the environmental chamber or, more likely, that bonding with the invar steel restraining ring masked the point of cracking.

Observed temperature falls to cause cracking increased from 3.5°C for lean concrete at 16 hours of age to 8.6°C at 28 days. The increases were progressive with age with the exception that at 14 days of age, the temperature fall was slightly in excess of that at 28 days, although this was not necessarily considered significant since each was the result of a single test. The rate of increase of temperature fall to cause cracking with age was much greater at early ages, for example, between the ages of 16 hours and 7 days the increase was 3.9°C compared to an increase of 1.2°C obtained between 7 days and 28 days of age.

12.4. Cracking properties in condition of absolute restraint

In order to predict the temperature fall which would cause cracking when restraint is absolute, values of the observed temperature fall in Table 12.1. were inserted together with thermal movement and elastic properties into the equation 10.18, giving the tangential stress at the inner surface of the restraining ring. Strain components of equation 10.16 were then calculated. The total tensile strain in the concrete at the interface including the Poisson's ratio contribution was divided by the coefficient of thermal movement of the concrete to estimate the temperature fall to cause cracking in a condition of absolute restraint. Values of the elastic properties of the invar steel were as used in the earlier analysis and a value of 2.5 x 10^-6/°C was used for the coefficient of thermal expansion of the material in calculating the relative coefficient αΤ. Static secant modulus values were used for the concrete. Poisson's ratios of concrete tend to vary in the range 0.25 to 0.15 and Kolias obtained a value for lean concrete of 0.175
measured at 1/3 of the ultimate load when tested in tension at an age of 7 days. However, since variations of Poisson's ratio in the above range did not affect the second significant figure in calculation of the total strain in the concrete at failure, the value of 0.18 was used in the evaluation of equation 10.16 at all ages.

The temperature falls calculated in this way are plotted graphically in Figure 12.5., values ranging from 2.1 °C at an age of 1 day to 4.2 °C at an age of 14 days. With the exception of each of the single 1 day and 14 day test results, the graph might be considered to be approximately linear when plotted on the logarithmic time scale, though the smooth progression of points is considered to provide some justification for the curve drawn at earlier ages. Values of the ratio \( f/aE \) determined from experimental results are plotted on the same logarithmic time scale in Figure 12.6., the ratio being a prediction of the temperature fall required to cause cracking as given by equation 4.1.

12.5. Discussion

The discussion is focussed on three main aspects of the results; first, briefly, on general implications in the context of roadbases, second on an apparent discrepancy between performance as predicted by the results of restrained thermal movement tests and that predicted by the results of strength, modulus and thermal movement measurements and finally, on a comparison of results with those of other researchers on related materials.

General implications

Figure 12.5. confirms the suggestion made in Section 4.2. that tensile cracking in lean concrete roadbases may occur as a result of small temperature reductions at early ages. The graph indicates that resistance to cracking may reach a minimum at the age of 24 hours, subsequently increasing rapidly in the first 7 days. By the age of one month the temperature fall required to cause cracking was almost double the lowest value measured, although even at this age the temperature fall required would be well
Figure 12.5. Temperature fall required to cause cracking in lean concrete subject to absolute restraint.

Figure 12.6. Temperature fall required to cause cracking in lean concrete as given by the ratio $f/aE$. 
Comparison between predicted and obtained performance

The general pattern of the graph of Figure 12.5. is similar to that of the ratio $f/E$ as given in Figure 12.6., though the temperature falls to cause cracking based on properties measured in this way are substantially different from those given in Figure 12.5., particularly at early ages when the possible reversal of the gradient of Figure 12.5. is absent. The ratio of corresponding temperatures as given in the two graphs varies from 2.1 at 16 hours of age to 1.2 at 14 days. The apparent deviation between measured performance and that predicted from strength and modulus measurements could alternatively be represented by comparing values of the tensile components of strain in equation 10.16. with the ratio $f/E$ obtained from measurements on prisms. The relationship is shown in Figure 12.7. together with a line of equality. It is evident that the failure strain calculated from restrained thermal movement test results is approximately $10 \times 10^{-6}$ greater than the ratio $f/E$ which would suggest that the strain capacity of the concrete is increased by an approximately constant margin in the range of ages measured and that the temperature fall to cause cracking in restrained thermal tests was approximately $0.8^\circ$C in excess of that predicted from strength and modulus tests. It may be argued that discrepancies of this numerical size are not necessarily significant and that they are only large in percentage terms at early ages at which experimental tolerances would also be proportionately larger. However, the form of the graphs of Figures 12.5. and 12.6. would appear to indicate a systematic trend and a consideration is therefore given of possible mechanisms which may in any case be of some general value in interpreting test results of this kind.

Possible origins of the observed differences may be classified broadly into those involving the method of test and those
Figure 12.7. Failure strain calculated from restrained thermal movement results compared to the ratio of tensile strength to elastic modulus.
involving deviations of material behaviour from that which formed the basis of calculations.

Figure 12.5 suggests that the cracking resistance of lean concrete is greater at early ages than that predicted by the measured properties tensile strength \( f \), coefficient of thermal movement \( \alpha \) and elastic modulus \( E \), and therefore that \( f \) might have been underestimated or that \( \alpha \) and \( E \) may have been overestimated in respective tests. It is possible that the tensile stress causing failure in restrained thermal movement tests may be in excess of that obtained by uniaxial tension tests, since there is some small degree of flexure in concrete in an annular restraint system and it is well known that flexure results in higher apparent failure stresses. Kolias\(^{40}\) for example obtained increases of 68 per cent and 85 per cent in flexural tests on lean concrete at the ages of 7 and 28 days compared to uniaxial tension tests. However, only a very small proportion of this increase would be expected to apply to the annular system employed since the stress variation across the concrete ring section was 39 per cent, compared with the 100 per cent reversal theoretically occurring in a flexural test. It is also considered unlikely, in view of comments made regarding thermal movements in Section 12.1., that the coefficient of thermal movement could have been overestimated, particularly at early ages when the differences between predicted and observed cracking properties were greatest. Perhaps the most uncertain of the measurements of material properties was elastic modulus, for two reasons - first, since static-dynamic relationships have not been established at early ages and second, since the static modulus value depends on the stress level at which it is measured. Results of Kolias given in Figure 12.2. give no indication that static modulus values at early ages might fall very much below dynamic moduli and it would seem unlikely that a dynamic modulus of 14.0 GN/m\(^2\) at an age of 16 hours could correspond to a static modulus as low as the value of 4 GN/m\(^2\) which would be needed to account for observed differences at this age. However, significant differences in the static secant modulus are obtained when the stress reference level is
altered; for example, results of Kollas indicate that secant moduli based on strains of 95 per cent of the ultimate value would be approximately 15.5, 25.8 and 28.8 GN/m² at the ages of 2, 7 and 28 days, compared with secant moduli based on half of the failure stress of 20.0, 30.75 and 34.95 GN/m² at these ages respectively. Since static moduli used in the analysis of results were based on the latter values, the observed differences at greater ages could be explained in this way.

Related to these effects is the possibility that when tests are carried out over a period of several hours, time dependent relief of stress may occur, indeed, it has been suggested by Ward and Cook⁹ that the microcracking mechanism which is normally used to explain reductions in elastic modulus in the advanced stages of destructive tests may itself be time dependent since it has been shown that microcracks can be propagated by time dependent movements of adsorbed water which is present in cementitious materials⁸. This is, of course, not to the exclusion of other creep processes. A full and detailed study of the effects of creep in lean concrete at early ages could itself provide the basis for a research programme of some length since, although in recent years considerable attention has been given to the creep of concrete in tension as well as in compression, this has been chiefly in respect of conventional concrete mixes having aggregate/cement ratios in the range of 3 to 8 and water/cement ratios in the range 0.4 to 0.7. Furthermore, ages at the commencement of loading have normally been much greater than 1 day and the effects of creep have been measured over relatively large periods of time compared to those over which stress and subsequent failure arose in restrained thermal movement tests. Clearly, an accurate prediction of the rate of stress relief by creep in lean concrete at early ages would, on the basis of published results to date, be most difficult, though some results are considered briefly here in order to permit a qualitative assessment of the likely effect to be made. Table 12.2 indicates selected results of uniaxial creep tests obtained by
Domone\textsuperscript{95}, Ward and Cook\textsuperscript{93}, and Illston\textsuperscript{96} which illustrate some important aspects of creep properties.

Table 12.2. Some results of creep tests carried out in tension on conventional concretes.

<table>
<thead>
<tr>
<th>Author</th>
<th>W/C ratio</th>
<th>A/C ratio</th>
<th>Age at loading days</th>
<th>Stress MN/m(^2)</th>
<th>Period of load days</th>
<th>Moisture condition</th>
<th>Spec. creep (x 10^6) MN/m(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domone</td>
<td>0.45</td>
<td>4.5</td>
<td>28</td>
<td>0.8</td>
<td>14</td>
<td>sealed</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>4.5</td>
<td>28</td>
<td>0.65</td>
<td>14</td>
<td>sealed</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>7.5</td>
<td>28</td>
<td>0.65</td>
<td>14</td>
<td>sealed</td>
<td>17</td>
</tr>
<tr>
<td>Illston</td>
<td>0.4</td>
<td>3.2</td>
<td>7</td>
<td>0.9</td>
<td>40</td>
<td>R.H.63% average</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>3.2</td>
<td>49</td>
<td>0.9</td>
<td>40</td>
<td>R.H.63% average</td>
<td>11</td>
</tr>
<tr>
<td>Ward &amp; Cook</td>
<td>0.55</td>
<td>4.5</td>
<td>7</td>
<td>1.34</td>
<td>5</td>
<td>R.H.100%</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>0.55</td>
<td>4.5</td>
<td>7</td>
<td>1.34</td>
<td>40</td>
<td>R.H.100%</td>
<td>70</td>
</tr>
</tbody>
</table>

Results of Domone illustrate the effects of changes in concrete mix proportions, an increase of water/cement ratio from 0.45 to 0.6 resulting in a marked increase of specific creep in sealed concrete, while an increase of aggregate/cement ratio produced an opposite though much lower change. The results of Ward and Cook indicate that a substantial fraction of the long term creep strain occurs relatively early during the loading period, 70 per cent of the creep measured at 40 days after loading having occurred within 5 days. Results of Illston indicate the influence of the age of concrete at loading, much greater creep strains being obtained when concretes were subjected to stress at earlier ages. Creep was, in general, found to be proportional to the applied stress for stresses of up to 50 per cent of the failure.
level, increasing thereafter. The moisture condition of concrete also affected creep, though effects were generally small compared with those already considered.

In order to obtain a tentative assessment of the extent of stress relief by creep in lean concrete at early ages, the effects of the changes of mix proportions and time scale are now considered. The free water/cement ratio of the lean concrete used in the main test series was 0.87 which, on the basis of Domone's results, would correspond to a specific creep substantially greater than the value of $23 \times 10^{-6}$ given in Table 12.2. for a 0.6 water/cement ratio mix. Subsequently increasing the aggregate/cement ratio would have the converse effect, possibly reducing specific creep and cancelling the effect of the increased water/cement ratio. Results presented in the papers of Domone and Illston suggest that proportionate reductions in the age of loading and the period of loading would not greatly affect the specific creep of the concrete mixes tested and although the extrapolation of time periods to this degree might give rise to substantial deviations in behaviour, it is possible on this basis that loads of duration as short as one day or less might produce creep strains in lean concrete at early ages comparable with those observed in more mature concretes over longer periods. This view is possibly supported by results of Illston and Hirst et al. who observed significant creep strains within one hour of loading concrete in compression.

While it is not possible on the basis of information reviewed to make firm predictions as to the contribution of creep to the apparently increased cracking resistance of lean concrete tested at early ages, there would appear to be some justification in principle for this theory, especially in view of the fact that stress relief was very much larger at early ages when creep effects would be expected to be more important.

The scope of the project did not permit a detailed study of stress relief but it was decided to attempt to obtain, if possible, more experimental results for lean concretes in the period of 12 to 24 hours after mixing in order to confirm the trends already obtained.
However, although the apparatus was designed to result in measureable strains in lean concrete at ages of as low as 1 day, the strain gauges were insufficiently sensitive to obtain a high degree of reliability in crack detection when testing lean concretes having an age of only a few hours and the age of 16 hours at which a test result was obtained after two attempts was considered the earliest practicable age using the existing form of the apparatus. In the circumstances, it was therefore decided to attempt to obtain further elastic modulus and tensile strength values at ages prior to 1 day. Difficulty was unfortunately experienced in demoulding the 500 mm concrete prisms at ages of under 13 hours since the use of a vibrating hammer in compacting the concrete tended to displace the release agent from the sides of the moulds, causing sticking. Consequently, the very friable concrete tended to disintegrate if demoulding was attempted. A single prism was however demoulded successfully at this age after several attempts and electrodynamic tests carried out. It was found that resonance peaks at ages prior to about 16 hours were extremely faint so that there was some uncertainty in their detection, resonance peaks in some cases being masked by spurious resonances from the mounting frame and surrounding apparatus. Thereafter, the intensity of resonances increased rapidly, this in itself being an indication of the rate at which the properties of the concrete were changing. The results of this series of tests are indicated in Figure 12.8. and the graph suggests that at an age of 12 hours, the elastic modulus of the concrete would be almost zero though the validity of extrapolation is questionable.

Attempts were made to obtain additional tensile strength data at very early ages but it was not found possible to mount specimens in the tensile test grips at ages of earlier than 15 hours and even at this age the weight of the lower grip mechanism itself accounted for over 30 per cent of the total failure load. Unfortunately, therefore, additional results could not be obtained, although a single result at 15 hours of age confirmed the positive gradient of $f/E$, obtained at very early ages in Figure 12.6. It is considered, nevertheless, that the form of the graph of
Figure 12.8. Electrodynamic modulus of lean concrete measured at very early ages.
Figure 12.5. at early ages may still be justified since tensile tests conducted at these ages lead to failure very rapidly, the test period in some cases being less than 30 seconds so that creep effects would be minimised.

In summarising this comparison of the results obtained by the two methods of test, it would seem reasonable to conclude that over very short term loading periods, temperature falls required to cause cracking in lean concrete roadbases at very early ages may be as low as 1°C, but that in practice greater temperature falls, possibly in the region of 2°C, would be required, stress relief being the most likely cause of this apparent difference.

12.6. Comparison with published results

While there is little published material which relates specifically to laboratory tests on restrained thermal cracking in lean concretes, there are a number of papers in which information relevant to these tests is given.

In work referred to in Section 9.1, Dinh carried out restrained cooling tests on a granular material having an aggregate grading similar to that of concreting aggregates and stabilised with 3.5 per cent of cement. He obtained temperature falls (T) to cause cracking of 5.0°C and 5.8°C at the ages of 28 days and 90 days respectively, no corrections to absolute restraint being necessary on account of the nature of the restraint system. In addition, tensile strengths and elastic moduli were measured from tensile tests with loading rates of 0.01 MN/m² per second and 0.04 MN/m² per hour. Coefficients of thermal movement, α, were measured and although results were not explicitly stated, the values of αT were given. Table 12.3. indicates 28 day results together with the 28 day result obtained for lean concrete in this project.

The tensile strength(f) of the gravel-cement was approximately half that of the lean concrete and this would be expected on account of its relatively low cement content. It is interesting that the tensile strength of the material measured using a rapid rate
Table 12.3. Test results for lean concrete compared to those obtained using a gravel-cement by Dinh.

<table>
<thead>
<tr>
<th>Material</th>
<th>Stress at failure at stated loading rate</th>
<th>Tensile static modulus</th>
<th>Temp. fall °C</th>
<th>αT x 10^{-6}</th>
<th>f/E x 10^{-6}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fast MN/m²</td>
<td>Thermal MN/m²</td>
<td>Slow MN/m²</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gravel cement (Dinh)</td>
<td>0.63</td>
<td>0.65</td>
<td>0.73</td>
<td>25.2</td>
<td>5.0</td>
</tr>
<tr>
<td>Lean concrete</td>
<td>1.45</td>
<td></td>
<td></td>
<td>34.1</td>
<td>4.1</td>
</tr>
</tbody>
</table>

Figure 12.9. Suggested relationship between the strain capacity of neat cement and age, Alexandre.

Figure 12.9. Suggested relationship between the strain capacity of neat cement and age, Alexandre.
of loading corresponding to a time to failure of 63 seconds was lower than that measured by a low rate of loading in which failure occurred after approximately 18 hours. The elastic modulus of the gravel-cement (E) was, as would be expected, slightly lower than that for lean concrete of the same age, though the water content of the material was not stated so that a close comparison is not possible. The temperature fall to cause cracking of the gravel-cement was higher than that for lean concrete which is surprising in view of its much reduced tensile strength.

The product αT should theoretically correspond to the ratio f/E and an inspection of Table 12.3. shows that, in the case of lean concretes, the values are $55.0 \times 10^{-6}$ and $42.5 \times 10^{-6}$ respectively. However, for the gravel-cement, the respective values are $58 \times 10^{-6}$ and $26 \times 10^{-6}$ (based on the stress of 0.65 MN/m$^2$ measured in the cooling test), the difference between these figures being substantially greater than apparent differences in failure strains given in Table 12.1. and in percentage terms approximately equal to the ratio obtained at the age of 16 hours. Dinh obtained a similar difference at the age of 90 days and it is unclear as to their origin in view of the fact that the static modulus quoted was measured over a longer period than that of the duration of the restrained contraction test. A possible contributory cause may be that temperatures appeared to have been measured in a container of sand in the thermostatic chamber rather than on the test specimen itself. However, some elucidation of the details of test would be essential to determine more fully the reasons for these important differences in order that the most accurate prediction of performance of these materials in the field be made.

Although the work of Springenschmid described earlier was in reference to relatively strong concrete mixes, it may be appropriate to recall his suggestion that the age at which cracking resistance would be a minimum would be between 12 and 20 hours, the latter value being in approximate agreement with the results of
restrained tests on lean concrete. Springenschmid's results are discussed further in Section 13 which reports on the test results of richer mixes.

Alexandre cast cement paste, mortar or concrete rings around a metal annulus and subsequently subjected them to tensile stress leading to failure by means of hydraulic pressure applied in the centre enclosure of the annulus. Little information is given regarding the performance of cement stabilised materials but Alexandre suggested as a result of tests on neat cements that there will be an age at which the capacity of cement paste to withstand tensile stress is a minimum. Figure 12.9. shows the suggested relationship between strain capacity and age with a minimum value at an age of slightly less than 6 hours. The graph is dotted prior to this age which is the earliest age at which results were obtained, Alexandre indicating that strain capacities may be very small indeed at this stage. The relatively early age suggested here probably reflects the fact that tests were carried out using neat cement paste.

12.7. Partially compacted lean concrete

Dry densities of lean concrete compacted to refusal averaged 2245 kg/m³ compared to the theoretical value at zero air voids of 2310 kg/m³, calculated in Section 6.2.2., indicating that 97.2 per cent of the theoretical dry density had been achieved in tests on this material. The 5 per cent density shortfall introduced would therefore represent a total void content of 7.7 per cent.

Results of tests are indicated in Table 12.4., the values at 1 day of age being based on a single test and those at 2 and 7 days based on the average of three tests at each age. Standard deviations are included at the latter ages.

In commenting generally on tests of this type in which air voids are included in the material, it is to be expected that variability in results would be increased since the uniform
Table 12.4. Test results for partially compacted lean concrete together with those for the fully compacted material, for comparison.

<table>
<thead>
<tr>
<th>Property</th>
<th>1 day Fully Compacted per cent</th>
<th>2 days Fully Compacted per cent</th>
<th>7 days Fully Compacted per cent</th>
<th>Partially Compact. A/B per cent</th>
<th>Partially Compact. C/D per cent</th>
<th>Partially Compact. E/F per cent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coeff. $E/Nm^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dynamic mod. $E/Nm^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Static mod. $E/Nm^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tensile $strain \times 10^{-6}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Apparent temp $\times 10^{-6}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Temp. fall $\times 10^{-6}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Abs. resid $\times 10^{-6}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 day</td>
<td>11.2</td>
<td>14.1</td>
<td>13.4</td>
<td>13.4</td>
<td>12.5</td>
<td>12.0</td>
</tr>
<tr>
<td>2 days</td>
<td>11.8</td>
<td>14.9</td>
<td>13.4</td>
<td>13.4</td>
<td>12.5</td>
<td>12.0</td>
</tr>
<tr>
<td>7 days</td>
<td>12.5</td>
<td>13.4</td>
<td>13.4</td>
<td>13.4</td>
<td>12.5</td>
<td>12.0</td>
</tr>
</tbody>
</table>
distribution of air voids in the material, even with a standardised method of compaction, is difficult to achieve. It is considered, therefore, that some caution may be necessary in their interpretation even though coefficients of variation obtained in tests were similar to those for the fully compacted material.

Thermal movement. With the exception of the single 1 day result, thermal movements of the partially compacted material were slightly higher than those of fully compacted lean concrete, though the significance of differences of this magnitude is questionable. It is possible however that in the light of the effects of aggregate content and moisture condition described in Section 8.2, small differences in thermal movement may arise in the partially compacted material, caused by, for example, its lower stiffness or the increased migration of moisture between cement gel and the more numerous voids.

Tensile strength. Tensile strengths were significantly decreased at all ages, the more accurate 2 and 7 day results suggesting that decreases increased with age in this range. Most published results on the effect of undercompaction on strength have been in relation to compression tests. Williams\textsuperscript{100}, for example, measured the effect of dry density on 100 mm cube strengths of a 16:1 concrete mix at 7 days and obtained the relationship:

\[
\frac{f_c}{f_{\text{max}}} = \left(\frac{W}{W_{\text{max}}}\right)^{10.67}
\]

\text{------------------(12.1.)}

where \(f_c\) is the measured 100 mm cube strength
\(f_{\text{max}}\) is the cube strength corresponding to a calculated dry density based on zero air voids
\(W\) is the estimated dry density of the cube tested
\(W_{\text{max}}\) is the calculated dry density based on zero air voids.

Taking density ratios of 0.972 and 0.923 for the fully and partially compacted materials respectively, the ratio \(f_c/f_{\text{max}}\) would be equal to 0.58 compared to values of 0.53, 0.70 and 0.56 obtained at the ages of 1, 2 and 7 days respectively, thus indicating broad agreement with the relationship given above. Williams found that the value of the index was reduced to 10.56 for concretes of aggregate/cement ratio 24 and this may be due to
the corresponding increase of water/cement ratio of the latter mix, such that if capillary pores are regarded as voids, the effective void content of the mix in both the fully and partially compacted states would be increased, thereby reducing the effect in percentage terms of a given increase in void content in the partially compacted concrete. Williams in another paper obtained a value of 13.5 for the index for 16:1 concrete mixes tested at 28 days of age which suggests that age may also affect the index, though differences in test procedures or materials may have been partially responsible for the increase. Extrapolation of this effect to earlier ages would also support the higher strength ratio of 0.70 obtained at 2 days of age which corresponds to an index of 7.0 in the density-strength relationship. The value of 0.53 obtained at 1 day of age is not considered to carry equal weight to the 2 and 7 day results in view of the increased experimental uncertainties at this age and the fact that only a single test was conducted.

Little information has been traced on the effect of undercompaction on tensile strength although Kolias recorded an average strength reduction of 28 per cent for a lean concrete having a density shortfall of 2.5 per cent compared to the fully compacted state when tested in uniaxial tension at the age of 28 days. Assuming that the latter state corresponded to zero air voids, the index of equation 12.1. which would fit in this case is approximately 13, suggesting broad agreement with the 28 day results of Williams in respect of compression tests on a similar material.

Reductions of strength due to undercompaction in lean concretes generally appear to be greater than those in conventional concrete mixes in which, for example, compressive strength reductions of approximately 30 per cent are normally found to occur with an air void content of 5 per cent102,103, equivalent to an index of approximately 7 in equation 12.1. Elastic modulus. Reductions in the dynamic modulus of the partially compacted lean concrete were 28 per cent, 21 per cent and 19 per cent respectively at the ages of 1 day, 2 days and 7 days, though corresponding reductions in the static modulus,
estimated in the same way as for the fully compacted material were 35 per cent, 26 per cent and 28 per cent. These reductions were nevertheless significantly lower than those of strength, indicating that elastic modulus is determined by different properties of the concrete mix. Although detailed results on the effect of incomplete compaction on the dynamic modulus of lean concretes have not been located, results by Kaplan on low workability concrete mixes having aggregate/cement ratios in the range 3 to 9 indicate that reductions in the dynamic modulus would be approximately half corresponding compressive strength reductions for void contents up to 7.5 per cent, the ratio increasing at higher void contents. Although tests were conducted at ages of between 7 days and 91 days, no comment was made regarding their effect, probably indicating that age did not significantly affect behaviour. There is broad agreement between the results of Kaplan and those given in Table 12.4. at the age of 7 days, reductions of 44 per cent and 19 per cent in strength and elastic modulus respectively being obtained at this age. The lower strength reductions at the age of 2 days, combined with the higher apparent modulus reductions would appear to indicate that the effect of undercompaction of lean concrete during the first few days may be substantially different from that at greater ages.

Temperature fall to cause cracking. Reductions in the ratio of tensile strength to static elastic modulus varied from 28 per cent at 7 days of age to only 6 per cent at 2 days, the reduction at the age of 1 day being 17 per cent. The values at the ages of 2 and 7 days are a reflection of the above trends in modulus and strength and the 2 day result indicates that resistance to thermal cracking should not be greatly reduced at this age. Apparent temperature falls to cause cracking, given in Table 12.4., underline this suggestion while at the age of 1 day the temperature fall at failure was slightly higher than that in the fully compacted material. Correcting values to conditions of absolute restraint as described in Section 12.4., it is found that the behaviour of the undercompacted concrete at 2 days of age was unchanged while a slight increase and decrease
in resistance to cracking occurred at the ages of 1 day and 7 days respectively. In view of the generally similar thermal movements in the undercompacted and fully compacted states, the results would seem to indicate an additional margin between predicted and observed performance in the undercompacted material, particularly at the age of 1 day although experimental errors resulting from this single test may be considerable. It is tempting to explain differences of this type in terms of modified creep behaviour although no information on the creep of partially compacted concrete has been located, but it is possible in addition that the effective restraint provided by the annular ring during cooling might be reduced, particularly if a density decrease in the concrete occurred near to the concrete/invar steel interface, although there was no visible sign of this.

In concluding the discussion on the effect of undercompaction it would appear that the consequent resistance of concrete to thermal cracking is unlikely to be significantly altered if cracking occurs at ages of 1 or 2 days but that significant reductions in cracking resistance would be obtained at the age of 7 days and at greater ages if the observed trend continued at greater ages. The results may, however, be influenced by the nature of the restraint system and possibly also by the method employed in manufacturing and testing specimens so that there is some uncertainty in their application to the performance of lean concrete roadbases in situ.

12.8. Overall comment

Taking results for lean concretes as a whole, it is evident that the most critical period from a thermal cracking point of view is the first day after placing so that there is a need for further experimental work to supplement results obtained in this period, perhaps using a variety of curing temperatures to simulate the effect of varying weather conditions in roadbases. However, although a relatively small number of tests was carried out at very early ages, the main test programme is considered to have been successful in focussing attention onto this period,
in providing a tentative indication of the resistance of lean concrete to thermal cracking at this stage and in identifying certain areas which require more detailed and careful study. These include the measurement of very early strengths and stiffnesses together with tests to determine the extent to which time dependent relief of stress occurs in recently stiffened concrete.
13. RESULTS FOR MATERIALS OTHER THAN LEAN CONCRETE

Whereas the object of the main series of tests was to provide information on the development of the thermal cracking resistance of lean concrete with age, subsequent tests were designed to compare the performance of different materials at given ages. For each material therefore, comparisons with the results for lean concrete or other materials are given. Results of the single tests on steel fibre reinforced concrete and to detect stress relief by creep are considered separately and the section concludes with a consideration of published results on restrained movements and the implications of results from the point of view of the performance of roadbases. Table 13.1. summarises test results.

13.1. Coefficient of thermal movement

It is noticeable that all thermal coefficients were in the range $12.3 \times 10^{-6}/\degree C$ to $13.0 \times 10^{-6}/\degree C$, there being no clear trend in values as age increased. Averaging results at 2 and 7 days, for each material, gives thermal coefficients of 12.65, 12.4, 12.65 (2 days only) and 12.85 $\times 10^{-6}/\degree C$ for mixes in order of increasing cement content compared to the average value of $12.7 \times 10^{-6}/\degree C$ obtained for lean concrete at these ages. It would appear therefore that any tendency for thermal coefficients to increase with cement content is very small and probably masked in these tests by experimental tolerances. In view of the very small variations obtained, the errors incurred by taking, as for lean concrete, an overall thermal coefficient of $12.5 \times 10^{-6}/\degree C$ for the materials tested, in this moisture condition and age range would be small.

13.2. Uniaxial tensile strength

The standard deviations of results indicate that a high degree of consistency was obtained.* There was found to be good correlation between uniaxial tensile strength and free water/cement ratio at each age as shown in Figure 13.1. which includes the relationship between indirect tensile strength at 7 days of age and water/cement ratio to be found in the current Department of the Environment design method for normal concrete. It is perhaps surprising that the design curve

*The majority of specimens failed clear of or just within the grips.
Table 13.1. Test results for other materials together with those for lean concrete for comparison.

<table>
<thead>
<tr>
<th>Material</th>
<th>Age days</th>
<th>Coefficient of thermal movement x 10^{-6} /°C</th>
<th>Dynamic modulus GN/m²</th>
<th>Density kg/m²</th>
<th>Static modulus E GN/m²</th>
<th>Tensile strength f MN/m²</th>
<th>f/E x 10^{-6}</th>
<th>Apparent temp. fall °C</th>
<th>Total calculated strain x 10^{-6}</th>
<th>Predicted temp. fall abs. restr °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wet lean concrete</td>
<td>2</td>
<td>12.6 (0.4)</td>
<td>24.2 (0.75)</td>
<td>2370</td>
<td>15.6</td>
<td>0.56 (0.02)</td>
<td>35.9</td>
<td>5.45 (0.17)</td>
<td>41.4</td>
<td>3.3</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>12.7 (0.3)</td>
<td>33.9 (1.5)</td>
<td></td>
<td>25.1</td>
<td>1.03 (0.04)</td>
<td>41.0</td>
<td>7.25 (0.6)</td>
<td>48.5</td>
<td>3.8</td>
</tr>
<tr>
<td>Aggregate/cement ratio 6</td>
<td>2</td>
<td>12.5 (0.45)</td>
<td>35.2 (0.6)</td>
<td>2400</td>
<td>26.1</td>
<td>1.45 (0.03)</td>
<td>55.5</td>
<td>11.4 (0.8)</td>
<td>73.8</td>
<td>5.9</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>12.3 (0.06)</td>
<td>42.6 (0.1)</td>
<td></td>
<td>34.1</td>
<td>2.42 (0.04)</td>
<td>71.0</td>
<td>11.8 (0.9)</td>
<td>67.5</td>
<td>5.5</td>
</tr>
<tr>
<td>Aggregate/cement ratio 3.4</td>
<td>2</td>
<td>12.7</td>
<td>29.1</td>
<td>2300</td>
<td>20.9</td>
<td>1.59</td>
<td>76.1</td>
<td>17.0</td>
<td>120.9</td>
<td>9.5</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>13.0</td>
<td>38.8</td>
<td></td>
<td>31.2</td>
<td>3.0</td>
<td>96.1</td>
<td>17.25</td>
<td>109.9</td>
<td>8.5</td>
</tr>
<tr>
<td>A/C 4.5</td>
<td>plain</td>
<td>2</td>
<td>12.65</td>
<td>2290</td>
<td>14.9</td>
<td>1.15</td>
<td>77.2</td>
<td>9.75</td>
<td>75.6</td>
<td>6.0</td>
</tr>
<tr>
<td></td>
<td>with fibres</td>
<td>2</td>
<td>11.4</td>
<td>2400</td>
<td>15.3</td>
<td>1.55</td>
<td>101.3</td>
<td>13.5</td>
<td>91.0</td>
<td>8.0</td>
</tr>
<tr>
<td>Lean concrete</td>
<td>2</td>
<td>13.4 (0.9)</td>
<td>28.1 (1.0)</td>
<td>2380</td>
<td>23.1</td>
<td>0.56 (0.03)</td>
<td>24.2</td>
<td>5.25 (0.42)</td>
<td>38.7</td>
<td>2.9</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>12.0 (0.7)</td>
<td>35.3 (1.3)</td>
<td></td>
<td>30.3</td>
<td>1.12 (0.12)</td>
<td>37.0</td>
<td>7.4 (0.26)</td>
<td>43.1</td>
<td>3.6</td>
</tr>
</tbody>
</table>
Figure 13.1. Relationship between uniaxial tensile strength and free water/cement ratio for the range of mixes tested, together with 7 day cylinder splitting strengths given in the D.O.E. mix design method\textsuperscript{104}. 

\begin{itemize}
\item Wet lean concrete
\item Concrete of A/C 6
\item Concrete of A/C 3.4
\item Concrete of A/C 4.5
\item Dry lean concrete
\end{itemize}

\begin{itemize}
\item Dept. of Env. mix design curve
\end{itemize}
which is based on uncrushed aggregates and ordinary Portland cement should lie slightly below the direct tension curve obtained at the same age since cylinder splitting tests which form the basis of the curve normally give results in excess of direct tension values\textsuperscript{105}. However, it is considered that the relatively high direct tension results can probably be attributed to a somewhat higher fineness of the cement used compared to that which would form the basis of design curves. Figure 6.1. indicates, for example, a fineness of 347 m\textsuperscript{2}/kg for a typical ordinary Portland cement used, compared to the British Standard minimum value of 225 m\textsuperscript{2}/kg\textsuperscript{106} for this cement. Such variations of fineness would affect strength more greatly at 7 days of age than at later ages. In addition, Brooks and Neville\textsuperscript{107} found that direct tensile strengths ($f_t$) are more nearly equal to those obtained from cylinder splitting tests ($f_{st}$) at lower strength values. They have given the relationship

$$f_t = 1.19f_{st}^{0.75}$$

for $f_{st}$ values of greater than 2MN/m\textsuperscript{2}. Cylinder splitting strengths of 2.1 MN/m\textsuperscript{2} and 3.0 MN/m\textsuperscript{2} would, on this basis correspond to direct tensile strengths of 2.1 MN/m\textsuperscript{2} and 2.7 MN/m\textsuperscript{2} indicating much smaller differences than would be obtained at higher strengths.

13.3. Elastic modulus

The elastic modulus of concrete depends on the elastic moduli and volume fractions of the constituent components and on the degree of structural continuity between them. The modulus of the aggregate which normally forms the bulk of the material, has an important influence on the modulus of the composite, concretes with high volume fractions of stiff aggregates and low water contents generally exhibiting a higher elastic modulus than wetter mixes employing lower volume fractions of aggregates or aggregates of lower stiffness. Increases in modulus occur with age as the stiffness of the cement paste fraction rises and rich, dry mixes may have high elastic moduli due to the influence of the unhydrated cement component.

Dynamic moduli, given in Table 13.1., generally reflect these effects, moduli for wet lean concrete being lower than those of lean concrete on account of the higher volume fraction of water and lower volume fraction of aggregate employed. The volume
fraction of water in the 6:1 mix remained approximately equal to that of the wet lean material and the aggregate volume fraction was further reduced, though increases of modulus were obtained at each age, probably since the aggregate was effectively replaced by cement, resulting in a stiffer matrix. Moduli of the high strength concrete were lower than those of the 6:1 mix on account of the increased volume fraction of water in this case. Values for the 4.5:1 mix were reduced further as a result of the still higher water content, the water in this mix occupying approximately 23 per cent of the total bulk volume of concrete. Increases of dynamic modulus between the ages of 2 and 7 days varied considerably with concrete type, values of 26, 40, 21 and 33 per cent being obtained for the 18:1, 12:1, 6:1 and 3.4:1 mixes respectively. Corresponding strength increases were 100, 84, 67 and 89 per cent respectively, suggesting that with the exception of the wet lean concrete, increases of modulus with age may be linked to increases of strength. The reason for the comparatively large increase in the dynamic modulus for wet lean concrete is unclear. It may, however be related to chemical action involving the limestone filler used in the mix to increase cohesion, although if such action was involved, a proportionately greater increase in tensile strength would be expected to occur.

Consideration is now given to the conversion of dynamic elastic moduli to static values in view of the fact that it was not possible to obtain static moduli directly. There was some doubt as to whether the correlation obtained by Kolias for lean concrete of aggregate/cement ratio 18 could be applied to much richer mixes since volume fractions of cement paste and aggregate in these mixes were substantially altered. A more general relationship was therefore sought and this was found in the form of results by Takabayashi who correlated the static/dynamic modulus ratio to compressive strength, and Popovics who obtained an empirical relationship between dynamic modulus, density and static modulus using the results of other workers. In the absence of detailed compression test results, the relation of Popovics was adopted,
although the stress level on which static moduli were based was unfortunately not stated. The relationship, converted to metric units is

$$ E = \frac{428 E_D^{1.4}}{W} $$

where $E_D$ is the dynamic modulus of the concrete in GN/m$^2$

$E$ is the corresponding static modulus of the concrete in GN/m$^2$

$W$ is the density of the concrete in kg/m$^3$

Differences between $E$ and $E_D$ so obtained varied between 7.6 and 9.1 GN/m$^2$ and averaged 8.5 GN/m$^2$ compared to the difference of 5 GN/m$^2$ employed for lean concrete. By taking a compressive strength equal to 12 times the measured tensile strength, ratios of moduli obtained using Takabayashi's data were generally in good agreement with those calculated using Popovics' relationship, with the exception of the 4.5:1 mix in which Popovics' expression resulted in surprisingly low values.

13.4. Temperature fall required to cause cracking

Measured temperature reductions at failure, indicated in Table 13.1 increased greatly as the cement content of mixes increased and in the case of the concrete of aggregate/cement ratio 3.4, the temperature at failure was barely within the range of temperatures obtainable using the apparatus in its existing form since ice tended to form on cooling coils if the water temperature was set to below approximately 3°C. In addition, the effective stiffness of the restraint system was reduced when stiffer concretes were employed so that a greater proportion of the thermal strain was, in these cases, accomodated by radial compression in the Invar steel.

Temperature falls which would be required to cause failure in conditions of absolute restraint were calculated using the same method as described in Section 12.4. employing a Poisson's ratio of 0.18 as before in each case. Resulting values which are also given in Table 13.1. varied between 3.3°C for wet lean concrete at 2 days of age to 9.5°C for the 3.4:1 mix at 2 days of age.
Results are plotted against water/cement ratio in Figure 13.2., values for lean concrete at the ages of 2 and 7 days being included for comparison.

13.5. Discussion

Restrained thermal movement test results are first considered and these are then compared to those of control tests, with suggestions as to the cause of observed differences. The results indicate that the thermal cracking properties of wet lean concrete are broadly similar to those of the dry lean material at the ages of 2 and 7 days, restrained thermal tests indicating that slightly greater temperature falls would be necessary to cause cracking at each age. Temperature falls at failure of the 6:1 mix were increased at each age compared to the wet lean concrete, though the increase at 2 days of age was 2.6°C compared to an increase of only 1.7°C at 7 days with the result that the resistance to cracking of the material at 7 days of age, as predicted by restrained thermal movement tests was lower than that at 2 days. Increasing the cement content further resulted in a similar effect, the temperature fall to cause cracking of the 3.4:1 mix being 1°C greater at 2 days of age than at 7 days. The resistance to cracking of the 4.5:1 mix was similar to that of the 6:1 mix at the same age, even though the water/cement ratio was, in this case, slightly higher than that of the latter mix.

In summarising the effects of increased cement contents generally it is evident that very substantial increases in cracking resistance can be obtained compared to lean concrete but that these are greater at the age of 2 days than at 7 days.

It is interesting to compare Figure 13.2. with Figure 13.1. in which tensile strength is related to water/cement ratio. Approximately three-fold increases in strength were obtained between mixes of extreme cement content at each age while temperature falls at failure increased by factors of 2.9 and 2.2 at the ages of 2 and 7 days respectively. In view of the fact that the richer mixes were generally of higher modulus, it is surprising
Figure 13.2. Relationship between temperature fall required to cause cracking and free water/cement ratio for the range of materials tested.
that at the age of 2 days the increase of temperature fall was almost equal to the increase of strength measured, since a rise of modulus would offset a rise in strength according to the ratio \( f/E \). In order to examine these differences more closely, the ratio \( f/E \), calculated from uniaxial tensile strengths and static modulus values, is plotted against the predicted tensile strain in the concrete based on equation 10.16. Figure 13.3 shows the relationship together with a line of equality and corresponding strains for lean concrete which are included for comparison. Some scatter of points is apparent and it should be recalled that most results for the 4.5:1 and 3.4:1 mixes were based on single tests and that a generalised relationship between dynamic and elastic modulus was employed other than for lean concrete, in the absence of more detailed data. It is noticeable however that the strain calculated from restrained test results differs from the ratio \( f/E \) by increasing amounts both at earlier ages and in the case of richer mixes. The differences are relatively small in the lean concrete and wet lean mixes, being on average, in the region of \( 10 \times 10^{-6} \). However, in the case of the 6:1 and 3.4:1 mixes, the calculated strain at 2 days of age was substantially in excess of the ratio \( f/E \). The point corresponding to the 4.5:1 concrete tested at 2 days of age is well separated from other points on the 2 day curve and this is regarded as being due to the very low static modulus of elasticity predicted by Popovics' relation. If the relationship of Takabayashi were employed in this case, better conformity to the 2 day curve of Figure 13.3. would be obtained and no further comment is made in relation to this mix in view of the uncertainty in the dynamic/static modulus relationship and the fact that the results were based on single determination of each property. The data presented in Figure 13.3. is considered to be in support of the view expressed in the context of lean concrete that effective elastic moduli in restrained tests are substantially lower than static moduli measured in short term tests, probably on account of creep. The effect of age is similar to that obtained earlier, apparent failure strains in restrained tests being much greater at the age of 2 days than the strength/modulus ratio would suggest.
Figure 13.3. Relationship between strain at failure calculated from the results of restrained thermal movement tests and the ratio of tensile strength to elastic modulus.
In order to examine the effect of water/cement ratio on time dependent strains, published results are again referred to. Table 12.2 indicates that the specific creep of concrete is reduced by decreasing the water/cement ratio although the effect is counterbalanced by decreases of aggregate/cement ratio which often accompany such changes and if allowance is included for the increased stresses at failure of these concrete mixes, it is found that the product of specific creep and strength is similar for a range of mixes. Such results were obtained, for example, when 0.40 and 0.50 water/cement ratio concretes were tested in tension at 7 days by Ward and Cook\textsuperscript{93}, and Neville\textsuperscript{110} has reported the same effect in compression. Domone\textsuperscript{95} also showed that the curvature of stress/strain graphs for a range of concrete mixes tested to failure in tension at an age of 28 days was similar, and this would suggest that creep strains at failure may also be similar for different mixes tested in similar conditions. Test results at the age of 7 days are in general support of this effect, predicted strains in restrained thermal movement tests being in excess of the ratio $f/E$ by between $10 \times 10^{-6}$ and $20 \times 10^{-6}$ with the exception of the 7 day result for the 6:1 mix. However, at the age of 2 days there is clear evidence that non-elastic strains increased substantially in the case of richer mixes, the effect of increasing cement content apparently outweighing the effect of reduced water content. No published experimental evidence has been traced which would support this finding though there may be some justification in theoretical terms since quantities of free water in the cement paste would, in the early stages of hydration, be greater, perhaps resulting in creep properties similar to those of wetter, more mature mixes.

In view of the limited number of tests carried out, only tentative conclusions can be drawn though it is evident that non-elastic strains in concretes tested at early ages increase considerably with increase of cement content if workability is maintained approximately constant. At greater ages, non-elastic strains do not appear to be greatly affected by change of cement content. In order to permit a more accurate comparison of strains calculated
by these two methods, however, it would be recommended in further testing to measure the static tensile modulus directly rather than to estimate the value from dynamic results. It is likely that uncertainties in this correlation would account for much of the observed scatter of points in Figure 13.3.

13.6. Steel fibre reinforced concrete

Results of the test are included in Table 13.1. in order to assist in comparison with other types of concrete.

Some reduction in thermal movement of the steel fibre reinforced concrete would be expected since the coefficient of expansion of steel, being \( 11.0 \times 10^{-6}/^\circ\text{C} \), is lower than that of concretes employed in the main test series, such that the fibres should restrain thermal volumetric changes. The value of \( 11.4 \times 10^{-6}/^\circ\text{C} \) recorded is nevertheless surprisingly low in view of the relatively small volume of fibres employed, even if the low stiffness of the concrete at this age is taken into consideration. A similar reduction was however obtained by Hanna for a concrete of 0.40 water/cement ratio, suggesting that steel fibres may have a significant effect on the thermal movement of concretes in spite of the small proportions normally employed.

Dynamic moduli were weighted to give static moduli using Popovics' relation, the resulting value for the mix incorporating steel fibres being 3 per cent greater than that of the control mix. The increase is broadly consistent with the increase predicted by applying the law of mixtures, similar rises having been reported by Edgington et al. In contrast, Hanna reported that for fibre contents in excess of 1 per cent by volume, decreases in modulus were obtained though no suggested mechanism for this effect was given.

The uniaxial tensile strength of the concrete appeared to have been substantially improved by the inclusion of fibres, an increase of 35 per cent being recorded, compared to increases in the region of 10 to 15 per cent more commonly obtained and
suggested by graphs of Edgington et al\textsuperscript{111}. It is possible, however, that the difference in this case may be partially due to the early age at which tests were conducted. Experimental tolerances may also have contributed since fibre reinforced and control tension tests were carried out on separate batches of concrete.

The restrained thermal movement test resulted in a well defined end-point with a substantial stress reduction and the formation of cracks of similar width to those in the unreinforced material, suggesting that the concrete fraction was supporting the greater proportion of stress prior to failure and that some debonding of steel fibres occurred as they became subject to higher stresses at the point of failure. Inspection of failed surfaces after restrained movement and uniaxial tension tests showed that all fibres, which were of plain, circular section, had pulled out, there being no evidence of fibre failure.

The strength/modulus ratio was increased by 31 per cent by the incorporation of fibres and this increase of strain capacity of the material was confirmed by the result of the restrained thermal movement test, the temperature fall at failure in a condition of absolute restraint being 8.0\textdegree C compared to 6.0\textdegree C in the control mix. As a result of reinforcement with steel fibres, the concrete therefore behaved in a manner corresponding to a mix of lower water/cement ratio and similar workability, Figure 13.3, suggesting that a temperature fall of 8\textdegree C at failure would be equivalent to a free water/cement ratio of approximately 0.45 at 2 days of age.

13.7. Test to measure the effect of creep.

It was intended in the creep test to reduce the temperature of concrete to a point near to the known failure temperature in order that any time dependent strains over a period of 1 day be detected at a stress level comparable to that causing failure. During a first test however the concrete temperature rose slightly
in this period and in ensuring that the concrete temperature rose at no time in the second test, a temperature fall slightly in excess of 10°C was produced, which was approximately 0.5°C greater than the previously observed temperature fall at failure. Nevertheless, failure did not occur and this may have been due to the lower rate of temperature fall as the temperature approached 11°C, allowing greater stress relief by creep than would be possible in the control test, though experimental tolerances may also have contributed. A maximum creep rate would be expected in the conditions of test since the concrete would have been at a stress near to failure. Figure 13.4. indicates the relationship between the voltage output of strain gauges and time, together with temperatures recorded at the upper surface of the concrete. It is unfortunate that the sensitivity of strain gauges at this stage was impaired by partial debonding which occurred in one of the tests on a concrete of aggregate/cement ratio 3.4 so that the strain gauge calibration derived in Section 10.5. could not be employed. Proportionality between voltage output and strain was nevertheless assumed to hold, Figure 13.4. indicating a reduction of stress in the steel of approximately 6 per cent over a time period of 14 hours from the point of maximum stress, though a very slight temperature reduction was recorded in this period. The proportionate reduction in concrete stress would be similar and there was initially some surprise that such a small percentage reduction in stress should occur in view of the apparent magnitude of time dependent strains suggested by test results at the age of 2 days. However, it is considered, on reflection, that a substantial proportion of any time dependent strain would probably have occurred during the 10 hour period between the commencement of cooling and the development of maximum stress, this strain not being easily discernible from a measured strain/time relationship of the form given in Figure 13.4. Similarly, it would be difficult to separate creep recovery from elastic recovery on return of the concrete to normal temperatures since the establishment of thermal equilibrium in response to temperature changes of concrete sections of the size employed involves a time period of several hours.
Figure 13.4. Results of the test to determine the extent of stress relief by creep in a 4.5:1 concrete mix.
This effect reveals a shortcoming of the apparatus employed in measuring the creep properties of concretes generally in that, if stress is induced by thermal contraction rather than by applied load, time dependent strains cannot be measured until thermal equilibrium has been established. However, in recognising that the object of tests was to determine the degree of stress relief by creep during thermal contraction, it would seem that perhaps the most profitable future course of action might be to conduct further tests to destruction, employing if possible a range of cooling rates in order to produce differing creep strains at failure and aiming to provide information regarding relative cracking performance at different rates of temperature reduction rather than to obtain creep data in conventional terms. To conduct such tests, it would probably be necessary to reduce the size of the apparatus considerably in order that more rapid temperature changes be easily obtainable without excessive temperature gradients arising in the concrete. It is considered that the annular ring system employed would be eminently suitable for such tests, though concrete mixes would need to be adapted to suit much smaller ring dimensions recommended.

13.8. Published results on restrained movements

Springenschmid\textsuperscript{92} carried out restrained thermal movement tests at various ages on concrete mixes with a cement content of 380 kg/m$^3$ and a water/cement ratio, assumed to be total, of 0.50. He found that the temperature fall to cause cracking, measured from the zero stress temperature as indicated in Figure 9.4., decreased from 12°C for concretes which failed at an age of 13 hours to a minimum value of approximately 9°C when failure occurred at an age of 14 hours, increasing subsequently to 15°C for failure at 58 hours. Table 13.2. indicates results extracted from graphs together with the time taken for failure to occur and failure stresses in each case. The mix used in this project which corresponded most nearly to that of Springenschmid was the 6:1 mix tested at 2 days of age for which the predicted temperature fall to cause cracking was 5.9°C, the concrete having a uniaxial
tensile strength of 1.45 MN/m². Springenschmid obtained a corresponding temperature fall of 15°C for concrete which failed at an age of 58 hours, a stress of 1.6 MN/m² being recorded.

Table 13.2. Restrained thermal movement test results obtained by Springenschmid².

<table>
<thead>
<tr>
<th>Age at commencement of test, hrs</th>
<th>Time taken to failure, hours</th>
<th>Temp. fall based on zero stress temp., °C</th>
<th>Failure stress, MN/m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>11</td>
<td>12</td>
<td>0.20</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>14</td>
<td>0.45</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>10</td>
<td>0.55</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>9</td>
<td>0.70</td>
</tr>
<tr>
<td>12</td>
<td>7</td>
<td>11</td>
<td>0.90</td>
</tr>
<tr>
<td>24</td>
<td>8</td>
<td>13</td>
<td>1.30</td>
</tr>
<tr>
<td>48</td>
<td>10</td>
<td>15</td>
<td>1.60</td>
</tr>
</tbody>
</table>

Although the failure stresses are similar, there is a considerable difference between temperature falls at the point of cracking. The difference might be explained in part if the rate of creep at the higher initial temperature employed by Springenschmid was increased or if the elastic modulus of the material was lower, though values were unfortunately not indicated. However, the product $E \Delta T$ for the 6:1 mix used in this project was 1.3 times the value of $E \Delta T$ determined by tensile tests which should theoretically correspond, while taking similar modulus and thermal movement values for Springenschmid's concrete, the corresponding ratio would be approximately 3.4. The reasons for such a large discrepancy are not clear and further details of test procedure, effective apparatus stiffness and material properties would be required before attempting to determine the origin of this relative behaviour in apparently similar concrete mixes.

It is also interesting that Springenschmid's tests did not indicate a dependence of the failure temperature of concrete on water/cement ratio for values in the range 0.4 to 0.7, though this was
probably due to the fact that cement contents in mixes of lower water/cement ratio were higher, resulting in increased heat outputs and higher zero stress temperatures. Subsequent cooling would then result in failure at temperatures similar to those of weaker concretes of lower strain capacity. These results indicate a fundamental difference between the work of Springenschmid in which concrete was insulated such that temperatures increased considerably after casting, and work reported in this thesis in which temperatures were maintained at constant values as far as practicable. The differences obtained are indicative of the substantial changes in the cracking performance of the concrete which may be caused by variations in the temperature regime during the first day after placing.

Work by Bennet and Loat\textsuperscript{88} on restrained movements referred to in Section 9.2. was in the context of shrinkage, though it may be appropriate to refer to results in order to compare theoretical stresses with those actually measured in restrained testing. At the point of cracking, the free shrinkage of samples was measured, together with the static elastic modulus in compression. The results for a 3:1 concrete using ordinary Portland cement at water/cement ratios of 0.3 and 0.375 are indicated in Table 13.3., together with the product of free shrinkage and elastic modulus, the products being 2.4 and 4.1 times the average stress measured in the steel at failure respectively.

<table>
<thead>
<tr>
<th>W/C ratio</th>
<th>Age at cracking, days</th>
<th>Max. stress, MN/m²</th>
<th>Unrestrained shrinkage, S x 10⁶</th>
<th>Modulus of elasticity, E GN/m²</th>
<th>Product (SE), MN/m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>68</td>
<td>4.05</td>
<td>260</td>
<td>37.9</td>
<td>9.85</td>
</tr>
<tr>
<td>0.375</td>
<td>62</td>
<td>2.80</td>
<td>360</td>
<td>31.7</td>
<td>11.4</td>
</tr>
</tbody>
</table>

The differences are probably due to the effects of creep, though it should be noted that relatively long periods of 68 and 62 days were required to cause failure and that tests were carried out.
using very rich mixes. In the same paper, Bennet and Loat described creep tests on concrete which showed that considerable creep strains could occur at early ages in rich mixes, particularly if water contents were high. Their results are taken as a further indication that the differences between the theoretical strain capacity of concrete as determined by the ratio \( f/E \) and strains calculated from restrained thermal movement tests could be caused at least in part by time dependent strains.

13.9. General implications in relation to cracking in roadbases

Some observations are now made regarding the use of richer concrete mixes in roadbases from the point of view of cracking performance, although a number of other considerations such as construction procedure and economic viability would be involved in addition in an overall assessment of the suitability of these concretes for use as roadbase materials.

The resistance to thermal cracking of wet lean concrete was slightly greater than that of the dry lean material at 2 days of age although, if greater creep strains arise in the wetter, richer material at the critical age of 1 day, significantly greater resistance to cracking might be obtained at this stage. It is likely therefore that both crack spacings and crack widths in a wet lean base would on average be in excess of those in a dry lean concrete base, since for a given strength and subgrade restraint, fewer cracks would develop. Assuming that no further cracking occurs subsequently, the width of cracks would, in the long term, increase further due to drying shrinkage which is likely to be greater than that of lean concrete owing to higher water and cement contents. Consequently, the possibility of reflection cracking in flexible pavements would appear to be greater when wet lean concrete is employed. It is perhaps these properties of the material which result in its restriction in the current Department of Transport specification\(^{25}\) to sub-bases, although the overall quality of wet lean concrete as defined in the specification may be considerably lower than that employed in the project, since the aggregate is required to conform only to
the 'granular' material particle grading range, in contrast to the much narrower range of gradings specified for lean concreting aggregates. In addition, aggregate/cement ratios as low as 20 are also permitted subject to the average 7 day strengths of groups of five cubes not being less than 3.5MN/m². Nevertheless, it is apparent that in terms of performance, the wet lean concrete tested would offer no advantage over dry lean concrete, although from the constructional viewpoint the quite different properties of the material in the fresh state may in some circumstances be in its favour, permitting it to be placed, for example, using a slip form paver.

The use of concrete mixes having further increases in cement content may modify their behaviour very substantially at early ages in view of their increased strength and greater creep potential. If comparative behaviour of richer mixes at 7 days and 2 days is extrapolated to the formerly critical age of 1 day, for example, it is possible that temperature falls in excess of 5°C might be required to cause failure, such that cracking may not occur within 1 day after placing except when comparatively large temperature decreases occur in this period. Heat of hydration of richer mixes would also offset environmental temperature falls on the first night after placing and in these circumstances cracking may not be induced until much greater ages, at which failure may be due to shrinkage, to differentials of temperature or to traffic stresses. Additionally, there is the possibility that if a substantial fraction of the tensile stress arising during the first night after placing were relieved by creep, significant compressive stress might occur on the following day, especially in fine Summer weather. Clearly the use of richer mixes as roadbase materials would require a reappraisal of environmental effects, as well as other possible modes of failure, if the occurrence of cracks of excessive width is to be avoided.
14. PERFORMANCE OF THE RESTRAINT SYSTEM

A general assessment of performance of the annular restraint system in relation to the materials tested is followed by details of the response and stability of strain gauges together with possible implications of results.

14.1. Invar steel annular ring

By far the most attractive feature of the annular ring assembly was its rugged nature, no observable deterioration in the invar steel ring, mould or base-plate having occurred over the period of testing in spite of the use of an electric hammer for compaction purposes. There was also little evidence to suggest that the annular form of the apparatus constituted a major disadvantage in regard to the evaluation of stresses since results were, after making allowances for possible reductions in the effective elastic modulus during restrained thermal movement tests, in broad agreement at greater ages with those obtained from measured elastic properties of the material used. This would also indicate that the effects of bonding between the concrete and invar steel were slight. Perhaps the most stringent requirement of the apparatus was in relation to the range of materials investigated, reasonable sensitivity to stresses in lean concretes at early ages being necessary as well as adequate restraint to contraction of high strength concretes at greater ages. The overall range of temperature falls producing cracking, together with corresponding strains at the inner circumference of the invar steel ring, based on equation 10.18, is indicated in Figure 14.1, strains in the steel varying between $6 \times 10^{-6}$ and $79 \times 10^{-6}$. The points suggest a straight line relationship, though this would have little significance since the results correspond to a considerable range of elastic modulus values. The gradient of the graph is equal to $f_{	ext{t,sa}}/E_s T$ which from equation 10.18, is equal to

$$2\alpha = 0.16 \left[ \frac{\sigma_m - \sigma_s}{E_m} \right] + 1.84 + 0.57 \frac{E_s}{E_m}$$
Calculated strain, inner circumference, invar steel $\times 10^{-6}$

Lean concrete

Partially compacted lean concrete

Wet lean concrete

Aggregate/cement: 6

Aggregate/cement: 3.4

Temperature fall, °C

- $E = 30 \text{ GN/m}^2$
- $E = 20 \text{ GN/m}^2$
- $E = 10 \text{ GN/m}^2$

Figure 14.1. Strain calculated at the inner circumference of the invar steel restraining ring as a function of the measured temperature fall at failure.
The results may therefore be classified according to elastic modulus as well as temperature fall to cause cracking, by indicating gradients on Figure 14.1. Taking an average value of the coefficient of thermal movement of $12.5 \times 10^{-6}/^\circ\text{C}$, $\alpha_T$ would be $10 \times 10^{-6}/^\circ\text{C}$ and straight lines representing elastic moduli of 10, 20 and 30 GN/m² have been drawn. These illustrate the progression of static modulus from the lowest value, in the region of 10 GN/m² at early ages, to values in excess of 30 GN/m² in the more mature concretes. It is evident that if points representing test results of individual materials are joined to the origin, curved graphs are obtained, representing the much increased strains necessary in the invar steel to cause cracking of concretes as their stiffness increases on ageing. The high strength concrete mix effectively illustrates this, an increase in strain of over $20 \times 10^{-6}$ occurring in the invar steel for an increase in failure temperature of $0.25^\circ\text{C}$, obtained between 2 and 7 day tests.

It was in this general area of testing that the chief limitation of the particular design employed was considered to lie, temperatures near to freezing point being required to cause cracking of concretes manufactured at $20^\circ\text{C}$, which in situations of absolute restraint would fail on cooling to approximately $10^\circ\text{C}$. An increased ring thickness, perhaps of 40 mm, would be more suited to high strength concretes, though such a design would probably be unsuitable for low strength concretes at early ages. It is, indeed, considered an advantage of the system used that even at early ages, there was significant compression in the invar steel annulus, thereby magnifying the temperature reduction to cause cracking and permitting somewhat greater accuracy in the measurement of the temperature changes.

14.2. Strain gauges

Although the main function of strain gauges was to detect the point of cracking, strain recordings were made throughout the testing programme as an additional check on the operation of the annular restraint system. Gauges performed satisfactorily in

*The sensitivity of the strain gauges was not known with sufficient accuracy to justify calculation of the failure stress in the concrete by this means.
general with very little zero shift over a two year period and checks made periodically on their response to unrestrained temperature changes of the invar steel annulus alone showed that wandering of zero readings over the temperature range employed was also small. It is unfortunate however that the sensitivity of gauges was seriously reduced after the point of failure of the first test of the concrete of aggregate/cement ratio 3.4, carried out at 7 days. The reduction in sensitivity was attributed to partial failure of the adhesive in this test caused by the relatively high failure stress at a temperature near to freezing point, at which some embrittlement of the adhesive may have occurred. It was not possible to replace gauges at this stage and although they continued to give a clear indication of failure in subsequent tests, performance was not completely satisfactory. Where importance was attached to strain readings, as in the creep test, the reduced sensitivity was a particular disadvantage. In future testing of high strength concretes, it would be recommended that adhesives suited to large strains or to low temperatures be employed although it is unlikely that those in use would have failed if a stiffer restraint system, as has been recommended for these concretes, were used.

Consideration is now given to the relationships between strains measured in the invar steel, the measured temperature fall to cause cracking and strains calculated by means of equation 10.18., attention being confined to those results obtained prior to the gauges being damaged. Strains at the inner circumference of the invar steel were calculated from the difference between output voltages at failure and those at reference temperatures defined in Section 11.1. The strain corresponding to a voltage output of 10 microVolts was taken to be 0.93 $\times 10^{-6}$ as derived in Section 10.5. Figure 14.2, shows the relationship between measured strains at the inner surface of the invar steel and the observed temperature fall to cause cracking. There is good correlation between these two parameters at lower temperature falls for the dry lean concrete, partially compacted lean concrete and wet lean concrete, the graph suggesting that for a given
Figure 14.2. Strain measured at the inner circumference of the invar steel restraining ring as a function of the measured temperature fall at failure.
temperature fall, the wet lean concrete which was of lower stiffness underwent larger thermal strains, thereby causing similar stresses in the steel. This is exemplified by the lean concrete and wet lean concrete results at 7 days of age, in which similar temperature falls slightly in excess of 7°C resulted in similar strains in the invar steel, the lower modulus of the wet lean material being compensated to some extent by its higher thermal movement. The reason for the apparently higher strain in the invar steel in the case of wet lean concrete at 2 days of age is not clear since the recorded thermal movement at this age was lower than that of the dry lean material. The discrepancy is apparent on comparison with Figure 14.1, which should, theoretically, correspond closely, the only difference being that in Figure 14.1, strains were calculated from equation 10.18., rather than measured. The strain calculated at failure for wet lean concrete at 2 days of age was substantially lower than that obtained for the dry lean material and this raises the possibility that some experimental error may be involved.

A closer comparison of measured and calculated strains generally can be made from Figure 14.3, in which they are plotted against one another. It is evident that measured strains in the invar steel were in good agreement with those calculated at early ages, while at greater ages the measured strains exceeded the latter by an increasing margin. The most likely cause of this difference is considered to be an overestimate of the effective gauge factor, since a lower value would bring measured and calculated strains at greater ages into closer agreement and result in measured strains being lower than those calculated at earlier ages at which creep effects should be more significant. It is unlikely for example, that the strain of 6 x 10⁻⁶ in the steel, calculated at 16 hours of age could have arisen in practice because this could be shown to correspond to a stress in the concrete of 0.24 MN/m² and it is most doubtful even when stresses are relieved by creep that this value would be sustained when a tensile strength of only 0.12 MN/m² was measured. The strain measurement
Figure 14.3. Comparison of strains measured and calculated at the inner surface of the invar steel. A line of equality is indicated.
system employed may, in this context, perform a useful function in that, correctly calibrated to give the stress in the invar steel, it should provide, according to equation 10.10., an accurate prediction of the stress in the concrete with only limited dependence on the elastic modulus of, or creep effects in, the concrete in as much as they affect the stiffness of the system. In future tests of this type, it would on this basis be worthwhile to calibrate strain gauges in situ by application of external pressure to the ring though no suitable method has been found to date of carrying out such a calibration on an annular system of this size. In considering the causes of a possible reduction in gauge factor, it may be questioned whether the value should have been increased by 26 per cent appropriate to tensile Poisson's ratio strains registered by dummy gauges mounted parallel to the ring axis. Some Poisson's ratio effect would nevertheless be judged to occur in this direction due to the compressive strain in the invar steel in the tangential direction, though such uncertainties would increase further the value of calibration if a method could be devised.
15. RESTRAINT OF ROADBASES DUE TO FRICTION WITH THE SUBGRADE

The underlying causes of subgrade restraint are first considered, followed by a survey of published information. Finally, a simple mathematical model for subgrade restraint between a lean concrete roadbase and two different types of subgrade is suggested.

15.1. Nature of subgrade restraint

The process of compaction of lean concrete roadbases tends to cause aggregate particles to form small depressions in a granular sub-base or subgrade which results in some degree of mechanical interlock between the two layers of material. Subsequent application of a horizontal force of increasing magnitude to the base tends to result in an increasing degree of resistance to movement as the aggregate particles are pulled sideways in depressions until, at a certain displacement, the frictional force no longer rises. The use of a coefficient of friction defined as the horizontal force required to cause movement of the slab divided by its weight is therefore inappropriate since it is normally used to describe limiting friction in which the force does not depend on the extent of movement of a slab. Instead, the term 'coefficient of subgrade restraint' defined as the force required to cause a given slab displacement divided by the weight of the slab will be used. The value of the coefficient of subgrade restraint, so defined, would depend on the nature of the underlying material; if there is a distribution of fine granular material at the interface between base and subgrade, the value might be very small since particles would to some extent, behave as ball bearings. A smooth membrane between base and subgrade would reduce frictional forces at large as well as small displacements. Very large slab displacements or repeated movements may cause reductions in frictional restraint due to the breaking down of irregularities at the interface.

15.2. Published results on subgrade restraint

Teller and Sutherland measured force-displacement relationships of 1200 mm square slabs of concrete on a rolled level silty
Loam soil by means of a calibrated jack. Slab thicknesses of between 50 and 200 mm were used and Figure 15.1. shows results obtained for a thickness of 150 mm. The figure includes, in addition, results obtained by Sparkes\textsuperscript{56} for 150 mm concrete slabs constructed on a 75 mm layer of rolled clinker on brick earth and for the same material with a layer of waterproof paper between the concrete and the clinker. The subgrade restraint was in each case practically zero at zero displacement, increasing to a value dependent on the type of subgrade, the clinker resulting in the greatest restraint of maximum value approximately 13.6 kN/m\textsuperscript{2} while inclusion of paper reduced this to 6.7 kN/m\textsuperscript{2}, the silty loam subgrade being intermediate. Subgrade restraint increased with the thickness of slabs, though not in proportion to their weight, so that on evaluation of stresses due to movement, thin slabs would be subject to higher values. If the concrete used in these tests is assumed to have a density of 2300 kg/m\textsuperscript{3}, the limiting coefficients of subgrade restraint would be 4.0, 2.0 and 2.5 for the three sub-bases respectively.

Bofinger and Sullivan\textsuperscript{47}, carried out subgrade restraint tests on 150 mm soil-cement slabs on rolled unstabilised clay and obtained a value, assumed to be for limiting friction, of 6.2 kN/m\textsuperscript{2}. When a layer of polythene was included between the subgrade and concrete, the value was reduced to 3.4 kN/m\textsuperscript{2}. The lower values obtained from these tests are probably due to the finer nature of both base and subgrade used.

Kelley\textsuperscript{112} obtained coefficients of subgrade resistance for a variety of materials at different displacements, and investigated in detail the variations of subgrade restraint with thickness of concrete and slab displacement for concrete slabs on a silty loam soil, Table 15.1. indicating results. For a slab thickness of 150 mm a maximum coefficient of approximately 2.5 was obtained which is equal to the value obtained by Teller and Sutherland for a similar subgrade and good agreement is obtained at other displacements also. Kelley noted that coefficients of subgrade restraint decreased with cyclic movements, eventually reaching a uniform value.
Figure 15.1. Subgrade restraint as a function of slab displacement for various types of subgrade.

- rolled clinker
- silty loam soil
- clinker and paper
Table 15.1. Coefficients of subgrade restraint for silty loam soil after Kelley.

<table>
<thead>
<tr>
<th>Slab depth, d, mm</th>
<th>Slab displacement, mm.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.25</td>
</tr>
<tr>
<td>200</td>
<td>0.8</td>
</tr>
<tr>
<td>150</td>
<td>0.9</td>
</tr>
<tr>
<td>100</td>
<td>1.1</td>
</tr>
<tr>
<td>50</td>
<td>1.3</td>
</tr>
</tbody>
</table>

15.3. A mathematical expression for subgrade restraint

In order to assist in arriving at a general expression for crack spacings and widths, it was considered desirable to represent subgrade restraint data by a simple mathematical formula. An inspection of the curves given in Figure 15.1 suggests a logarithmic relationship between subgrade restraint and displacement and this is confirmed in the case of the clinker subgrade by Figure 15.2, in which coefficients of subgrade restraint $\mu_t$, calculated from Figure 15.1 are plotted against $\log(1 + 8000t)$ where $t$ is the slab displacement in metres. Points representing displacements of as low as 0.1 mm which may occur in short slabs as a result of small temperature reductions are seen to satisfy the relationship. The equation of the straight line drawn through points for the clinker subgrade is

$$\mu_t = 3.03 \log(1 + 8000t) \quad \text{(15.1.)}$$

Data for the silty loam subgrade obey the logarithmic law less well, though it was considered that, for simplicity, it would be an advantage to represent the coefficient of subgrade restraint again by the same type of relationship. The equation of the straight line drawn is

$$\mu_t = 2.0 \log(1 + 8000t) \quad \text{(15.2.)}$$

For small slab displacements the equation would overestimate the subgrade restraint though better conformity is obtained at
Figure 15.2. Relationship between coefficient of subgrade restraint and $\log (1 + 8000t)$, where $t$ is the slab displacement in metres.
displacements of 0.7 mm and above. The third case in which waterproof paper is included is not given in Figure 15.2, since it does not conform well to the type of expression of equations 15.1 and 15.2., reflecting the fact that frictional properties are significantly affected by such an intermediate membrane. Although the use of paper under lean concrete roadbases is uncommon, it should be possible to derive a suitable expression for the corresponding coefficient of subgrade restraint if so required.
16. PREDICTION OF CRACKING PATTERNS IN CONCRETE ROADBASES CAUSED BY RESTRAINED THERMAL CONTRACTION

The equations 4.5. and 4.6. for crack spacing and width given in Section 4 cannot be relied upon to give an accurate indication of cracking properties because the coefficient of subgrade restraint in any slab would vary considerably with the displacement of the ends of the slab, the coefficient also decreasing towards the slab centre where it would be zero. Clearly, restraint to movement only occurs when movement occurs, this being illustrated by the data given in the previous section. An attempt is now made, therefore, to obtain a more realistic solution to equations given in Section 4 and results are then applied to various types of concrete which might be employed as roadbase materials.

16.1. Calculation of crack spacings and widths

The evaluation of equations 4.2. to 4.3. requires an expression relating the coefficient of subgrade restraint \( \mu_x \) a distance \( x \) from the free end of a slab to the distance \( x \). So far, \( \mu_x \) has been related to slab displacement, \( t \), at that point, so it is now necessary to determine how \( t \) depends on \( x \). The relationship is however extremely complex and cannot be easily solved mathematically since evaluation of the slab displacement requires a solution of equation 4.3. and this cannot be achieved without a prior knowledge of the dependence of the coefficient of subgrade restraint \( \mu_x \) on \( x \). In an attempt to obtain a simplified solution to this problem, stresses and movements in a slab having undergone thermal contraction in a situation of constant coefficient of subgrade restraint, \( \mu \), will be considered and these will then be modified qualitatively to accommodate the effect of varying \( \mu \).

Continuous lines in Figure 16.1. represent the tensile stress \( f_x \), tensile strain, \( \epsilon_x \), thermal strain, \( \epsilon_{th} \), net strain, \( \epsilon_n \), and slab displacement, \( t \), for constant \( \mu \), and a brief explanation of these diagrams is now given.

The gradient of the stress diagram is proportional to \( \mu_x \), so that if \( \mu_x \) is constant, a straight line relationship is obtained. The tensile strain, \( \epsilon_x \), is proportional to \( f_x \). The thermal strain, \( \epsilon_{th} \), is constant throughout the slab. The net strain is equal to the difference between \( \epsilon_{th} \) and \( \epsilon_x \).
Figure 16.1. Stresses, strains and displacements in a concrete roadbase slab subjected to cooling on defined subgrades.
In the case of a heavily restrained slab; for example, a long section of roadbase, the net strain at the centre would be zero, indicated by continuous line (1) in Figure 16.1. (e) and (f). If however, the slab were short or already in a cracked state, the thermal strain may be greater than the maximum tensile strain as in continuous line (2). The displacement of the slab, indicated in Figure 16.1. (g) is obtained by integrating net strain with respect to distance from the slab centre. It is evident that in the short or cracked slab, the displacement is approximately proportional to the distance from the centre as shown in curve (2). When the slab is longer, however, the displacement is proportional to the square of the distance from the slab centre.

The diagrams are now reconsidered for \( u_x \) decreasing as \( x \) increases. Diagrams are in each case normalised to give the same values at the end and centre of the slab and lines are dotted. The stress \( f_x \) increases more rapidly initially but becomes constant at the slab centre as \( u_x \) reduces to zero. The strain \( e_x \) is proportional to \( f_x \). The thermal strains are as before, two cases again being considered. The net strain is now non-linear with respect to distance from the centre in each case.

On integration of strain to give Figure 16.1. (g), it is found that provided the thermal strain is large compared to the maximum tensile strain, the graph is again approximately linear as in curve (2). However, when restraint is large such that the strain at failure is no larger than the thermal strain, the displacement deviates considerably from linearity, though the curvature would depend on the slab displacement since, for large displacements, \( u_x \) would be approximately constant over a large part of the slab.

In these circumstances, the only apparent course of action was to assume linearity between displacement and distance from the centre of the slab, accepting that in heavily restrained slabs at the point of failure errors would be incurred. Thus the
theory would become increasingly accurate when temperature falls are large compared to those required to cause failure. This method would also, in practice, be most useful since long roadbase sections can be considered to be fully restrained initially, cracking therefore being given by the equation

\[ T = \frac{f}{\alpha E} \]

and assuming that cracking patterns develop at early ages, commonly occurring diurnal temperature variations would be well in excess of values of the ratio \( f/\alpha E \) determined from experimental tests at these ages. In the theory which follows, the displacement \( t \), a distance \( x \) from the free end of the slab is therefore assumed to be given by

\[ t = k(L/2 - x) \]

If equations 15.1 and 15.2 are represented by the general equation

\[ y^2 = a \log (1 + bt) \]

the relationship in terms of \( x \) would be

\[ y_x = a \log (1 + b(L/2 - x)) \]

constant \( b \) in this case taking a different value to that in equation 16.2.

A further difficulty arises with this expression, however, since the evaluation of the equation 4.4. with the logarithmic function inside the double integral sign becomes extremely complex. A simpler expression was therefore sought to relate the coefficient of subgrade restraint \( \mu_x \) a distance \( x \) from the free end of the slab to the distance \( x \) in terms of the maximum coefficient of subgrade restraint \( \mu_0 \) corresponding to the coefficient at the free ends of a slab with a crack width \( \delta \) and given by

\[ \mu_0 = a \log (1 + b\delta) \]

(b taking half the previous value of that in equation 16.2, since \( \delta = 2t \)).
It was found that the relationship

$$\mu_x = \mu_0 \left( \frac{L}{2} - \frac{x}{L} \right)^\frac{1}{2}$$

would give a reasonable fit to the curves of Figure 15.1. assuming the linear relationship of equation 16.1. between slab displacement and x. Curves of this type are drawn on the subgrade restraint graphs for clinker and silty loam subgrades taking maximum displacements of 1.0 mm and 2.0 mm as shown in Figure 16.2. It is noticeable that at greater slab displacements equation 16.5. underestimates $\mu_x$ while at lower slab displacements $\mu_x$ is overestimated. The equation was nevertheless used in the hope that if the maximum subgrade restraint was determined by the more accurate logarithmic relationship, the errors incurred by inaccuracies in the estimation of restraint within the slab would be small.

The stress $f_x$ at a distance x from the free end of the slab is therefore, according to equations 4.2. and 16.5.

$$f_x = \mu_0 \gamma \int_0^x \left( \frac{L}{2} - \frac{x}{L} \right)^\frac{1}{2} \, dx$$

which gives

$$f_x = \frac{2}{3} \sqrt{L} \mu_0 \gamma \left[ \left( \frac{L}{2} \right)^2 - \left( \frac{L}{2} - x \right)^2 \right]$$

The maximum stress occurs when $x = L/2$ and is given by

$$f_{L/2} = \frac{1}{3} \mu_0 \gamma L$$

From equations 4.3 and 16.6. the total extension of the slab is

$$2 \cdot \frac{2}{3} \sqrt{L} \mu_0 \gamma \frac{E}{2} \int_0^{L/2} \left[ \left( \frac{L}{2} \right)^3 - \left( \frac{L}{2} - x \right)^3 \right] \, dx = \frac{\mu_0 \gamma L^2}{3E}$$

The crack width $\delta$ is equal to the difference between the thermal contraction and the elastic extension of the slab:

$$\delta = \alpha L T - \frac{\mu_0 \gamma L^2}{3E}$$
Figure 16.2. Relationship between subgrade restraint and slab displacement together with values predicted by equation 16.5. for maximum slab displacements of 1 and 2 mm.
This expression is very similar to equation 4.6, except that \( \mu_\delta \) in this case is the maximum coefficient of subgrade restraint in the slab.

In order to determine the value of the crack width \( \delta \) and crack spacing \( L \) in a situation corresponding to the maximum tensile stress \( f \) in the roadbase, \( f_L \) in equation 16.7, is made equal to \( \frac{L}{2} \) \( f \) and the value of \( L \) substituted into equation 16.9:

\[
\delta = \frac{3f\alpha T}{\mu_\delta Y} - \frac{9f^2}{5\mu_\delta YE}
\]

\[
= \frac{3f}{\mu_\delta Y} \left[ \alpha T - \frac{3}{5} \frac{f}{E} \right]
\]

Inserting the expression for \( \mu_\delta \) from equation 16.4, gives

\[
\delta = \frac{3f}{aY \log(1 + b\delta)} \left[ \alpha T - \frac{3}{5} \frac{f}{E} \right]
\]

The equation would only be valid for temperature falls of a magnitude which could result in the maximum stress \( f \) in the roadbase and in practice, if failure occurred at early ages in roadbases of considerable length, this temperature fall is easily calculated from the tensile failure strain in a condition of absolute restraint. In a base of length 50 m, for example, it is found that the end displacement required to cause maximum stress at early ages is very small compared to the thermal movement so that restraint could be considered to be absolute.

The error in the crack width \( \delta \) would decrease as the temperature reduction becomes greater, probably becoming negligible for temperature falls in excess of double those required to crack heavily restrained slabs. Stronger concrete slabs of a given length would effectively be less heavily restrained, the thermal strain at failure therefore being greater than the tensile strain such that the errors incurred in \( \delta \) by inserting temperature falls near to failure temperatures into equation 16.11 should be relatively small. Similarly, the errors in crack
widths should be reduced as crack widths increase, resulting in more uniform restraint across the greater part of the slab length. Nevertheless, it should be recalled that in the absence of a generally applicable relationship, the theory was designed to predict cracking patterns accurately when temperature falls are well in excess of those causing initial failure and on this basis, crack widths, given later for stronger concretes should only be taken as being approximate. Crack spacings can be estimated from equation 16.7. by calculating μδ from δ and inserting the maximum stress f, the same arguments applying. When errors occur, these would be in the form of underestimates of L and δ since the restraint would in practice decrease more rapidly towards the centre of slabs than assumed in the theory, resulting in lower stresses than those calculated.

16.2. Crack spacings and widths in defined situations

These have been calculated from equations 16.7. and 16.11., the latter being evaluated first, by a graphical method, though it should be possible to solve the equation by computer using Newton's approximation if required.

Crack spacings and widths are based on strengths and elastic moduli determined from control tests and in using the properties measured in this way, values will correspond to the most extreme environmental conditions, that is, rapid cooling, though without temperature gradients, since creep effects are largely ignored. Ages of 16 hours, 1 day, 2 days, 7 days, and 28 days were selected for the purpose of calculations together with temperature falls in the range 1°C to 6°C. It was considered worthwhile to include results for richer mixes in addition to lean concrete data although on inspection of values of the ratio of tensile strength to static modulus in Table 13.1., it is found that for a temperature fall of 6°C the only concrete which would crack other than the wet lean material is the 6:1 mix at the age of 2 days. These results have therefore been included for comparison. Following observations made earlier on the range of thermal movements of concretes tested, a single
coefficient of thermal movement of $12.5 \times 10^{-5}/^\circ C$ was adopted. A slab thickness of 150 mm is assumed throughout but the relationships given would apply to other thicknesses provided the value of constant $a$ in equation 16.3. is modified to take account of differing coefficients of subgrade restraint. Some indication of appropriate weightings can be obtained from Kelley's data in Table 15.1.

In order to predict cracking properties of a limestone aggregate concrete, equations 16.7. and 16.11. were also solved, employing a thermal coefficient of $8 \times 10^{-6}/^\circ C$ first, on the assumption that the strength and elastic properties of the concrete are similar to those of lean concrete at the same age, and second, assuming a 50 per cent increase of tensile strength at each age in order to take account of observed strength increases in this material compared to lean concrete of similar proportions.

Results for uniform temperature reductions of up to $6^\circ C$ in the concrete are indicated in Tables 16.1. to 16.4. and where figures are not indicated, cracking would not be expected to occur, since the strength/modulus ratio $f/E$ is, in these cases, greater than the thermal strain. Values in Table 16.1. correspond to a silty loam subgrade as defined by equation 15.2., remaining tables being based on the higher subgrade restraint defined by equation 15.1. Poisson's ratio effects have not been included, so that where crack spacings of less than the width of a roadbase are predicted, restraint in the transverse direction would reduce longitudinal crack spacings and widths to below the values indicated.

Crack spacings given would be the maximum values to be found in that situation since they correspond to maximum tensile stress in the concrete. If, for example, a slab of slightly greater length than any value indicated were subject to the corresponding temperature fall, it would theoretically crack into two slabs of equal length and crack spacings would in practice therefore
Table 16.1. Calculated crack spacings and widths for gravel aggregate lean concrete roadbase on silty loam subgrade as a function of temperature fall.

<table>
<thead>
<tr>
<th>Age hours/days</th>
<th>Tensile strength MN/m²</th>
<th>Elastic modulus GN/m²</th>
<th>Crack spacing in metres for stated temperature fall in °C</th>
<th>Crack width in millimetres for stated temperature fall in °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>0.12</td>
<td>9.0</td>
<td>20.9 16.9 14.4 13.2 12.3</td>
<td>0.35 0.49 0.64 0.75 0.86</td>
</tr>
<tr>
<td>1</td>
<td>0.31</td>
<td>18.4</td>
<td>38.9 31.4 27.8 25.6 23.9</td>
<td>0.60 0.88 1.12 1.35 1.57</td>
</tr>
<tr>
<td>2</td>
<td>0.56</td>
<td>23.1</td>
<td>49.6 43.6 40.1 37.4</td>
<td>1.16 1.54 1.89 2.23</td>
</tr>
<tr>
<td>7</td>
<td>1.12</td>
<td>30.3</td>
<td>90.8 76.4 68.9 63.9</td>
<td>1.40 2.10 2.76 3.40</td>
</tr>
<tr>
<td>28</td>
<td>1.45</td>
<td>34.1</td>
<td>93.0 85.3 79.0</td>
<td>2.33 3.13 3.90</td>
</tr>
<tr>
<td>6:1 A/C 2</td>
<td>1.45</td>
<td>26.1</td>
<td>90.9 82.7</td>
<td>2.63 3.41</td>
</tr>
</tbody>
</table>
Table 16.2. Calculated crack spacings and widths for gravel aggregate lean concrete roadbase on clinker subgrade as a function of temperature fall.

<table>
<thead>
<tr>
<th>Age hours/days</th>
<th>Tensile strength MN/m²</th>
<th>Elastic modulus GN/m²</th>
<th>Crack spacing in metres for stated temperature fall in °C</th>
<th>Crack width in millimetres for stated temperature fall in °C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>16</td>
<td>0.12</td>
<td>9.0</td>
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</tr>
<tr>
<td>1</td>
<td>0.31</td>
<td>18.4</td>
<td>-</td>
<td>30.5</td>
</tr>
<tr>
<td>2</td>
<td>0.56</td>
<td>23.1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>1.12</td>
<td>30.3</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>28</td>
<td>1.45</td>
<td>34.1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>6:1 A/C</td>
<td>1.45</td>
<td>26.1</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Table 16.3. Calculated crack spacings and widths for limestone aggregate lean concrete roadbase on clinker subgrade as a function of temperature fall.

<table>
<thead>
<tr>
<th>Age hours/days</th>
<th>Tensile strength MN/m²</th>
<th>Elastic modulus GPa/m²</th>
<th>Crack spacing in metres for stated temperature fall in °C</th>
<th>Crack width in millimetres for stated temperature fall in °C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>16</td>
<td>0.12</td>
<td>9.0</td>
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</tr>
<tr>
<td>1</td>
<td>0.31</td>
<td>18.4</td>
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<tr>
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</tr>
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<td>28</td>
<td>1.45</td>
<td>34.1</td>
<td>-</td>
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</tr>
<tr>
<td>6:1 A/C 2</td>
<td>1.45</td>
<td>26.1</td>
<td>-</td>
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</table>
Table 16.4. Calculated crack spacings and widths for higher strength limestone aggregate lean concrete roadbase on clinker subgrade as a function of temperature fall.

<table>
<thead>
<tr>
<th>Age hours/days</th>
<th>Tensile strength MN/m²</th>
<th>Elastic modulus GN/m²</th>
<th>Crack spacing in metres for stated temperature fall in °C</th>
<th>Crack width in millimetres for stated temperature fall in °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>0.18</td>
<td>9.0</td>
<td>- - 23.6 19.5 16.9 15.2 - - 0.29 0.38 0.48 0.57</td>
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<tr>
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<td>- - - 37.0 32.4 29.6 - - - 0.64 0.82 0.98</td>
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<td>26.1</td>
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</tbody>
</table>
be distributed at random between the values given in tables and half these values. Conversely, if stress relief by creep occurred at early ages to the extent which was apparent in restrained thermal movement tests, crack spacings might be greater than those shown. Some indication as to approximate values might be inferred by supposing that creep reduced the effective thermal contraction of the roadbase. For example, in a material having a thermal coefficient of $12.5 \times 10^{-6}/^\circ C$ which is known to exhibit an average creep strain of $25 \times 10^{-6}$ in the prevailing conditions, the crack spacing and width could be obtained by reading values corresponding to a temperature fall $2^\circ C$ smaller than that actually occurring. In making corrections of this type, it should be noted that the average creep strain in a slab at the point of failure would be substantially below the value corresponding to maximum stress since this only occurs at the centre of the slab, decreasing to zero at the ends.

Interest in crack spacings will be centred chiefly on the lowest values which probably occur at ages in the region of 1 day. It is possible, however, that the most vulnerable age from the point of view of cracking may not be represented by values in tables such that, subject to the effects of creep already considered, maximum crack spacings and widths lower than the smallest values indicated could be obtained.

16.3. Discussion of results

Results are considered first according to the effects of temperature fall, age, subgrade type, material type and stress relief by creep on crack spacing. Attention is then focussed on factors determining the width of cracks with particular reference to their likely influence on reflection cracking.

16.3.1. Effect of temperature fall

Crack spacings in all cases decrease as temperature falls become greater since slab displacements rise on continued cooling, resulting in greater coefficients of friction and consequently increased tensile stress in a slab of a
given length. This is in contrast to values given in Table 4.2. in which temperature falls did not affect crack spacings since a constant frictional coefficient was assumed. At lower temperature falls, T, the crack spacings increase more rapidly as T reduces since an infinite slab length would be stable when T is less than \( f/aE \). It has been suggested in addition that crack spacings at the lowest temperature falls indicated may in practice be higher than those given.

16.3.2. Effect of age

The age at which cooling occurs has by far the most pronounced effect on crack spacings. For example, Table 16.2. indicates that spacings may be as low as 4.7 m if a temperature fall of 6°C occurs in a lean concrete roadbase at an age of 16 hours. If, on the other hand, the concrete were protected from temperature change until the age of 28 days, the minimum crack spacing would, for the same temperature fall, be 29.1 m. The critical factor in determining crack spacings as governed by temperature changes would therefore appear to be the age at which cracking first occurs, since spacings are not affected to the same extent by the magnitude of the temperature fall once cracking has occurred.

16.3.3. Effect of subgrade

A comparison of Tables 16.1. and 16.2. indicates that a subgrade of coarser texture results in substantially reduced crack spacings, values corresponding to the clinker subgrade being approximately 75 per cent of those corresponding to the silty loam subgrade. Corresponding subgrade restraints given in Figure 15.1. differed by approximately 66 per cent, suggesting that crack spacings are approximately inversely proportional to subgrade restraint.
16.3.4. Effect of material type

Results of 6:1 mixes at 2 days of age, indicated at the foot of each table show that resistance to cracking is much greater than that of lean concrete at the same age such that, if significant stress relief by creep occurred in practice, cracking due to thermal contraction may well not occur over the range of temperature falls given. Lower values of the ratio $f/E$ at earlier ages would in turn be offset by higher creep capacities, though crack spacings would be difficult to estimate on the basis of work carried out to date. It is evident that cracking in mixes of this type will be more likely to arise from other influences such as shrinkage or loading.

Crack spacings and widths in Table 16.3. are based on the same information as those of Table 16.2. except that a coefficient of thermal movement of $8 \times 10^{-6}/\degree C$ has been employed in order to predict the comparative performance of limestone aggregate concretes having the same strength and elastic modulus as gravel aggregate concretes at the same age. The temperature falls at which cracking occurs are, at early ages, approximately $1\degree C$ greater than those corresponding to the gravel aggregate concrete since thermal strains on cooling would be reduced. Nevertheless, a temperature fall of $2\degree C$ during the first night after placing would, subject to creep effects, cause cracking in the material such that there is still considered to be a high probability of failure at this stage. The table indicates that once cracking occurs, crack spacings for a limestone aggregate concrete roadbase would be only slightly greater than those in a gravel aggregate concrete roadbase of similar strength and modulus. This is due to the fact that crack spacings are only governed by thermal movement in as much as it affects slab displacements and therefore the subgrade restraint on the roadbase. The point is further
illustrated by the simplified equation 4.5. which indicates that crack spacings depend on the coefficient of subgrade restraint but not on thermal movement.

Table 16.4. shows that if the strength of limestone aggregate concretes is increased by 50 per cent, significant increases in crack spacing arise and these are greatest at temperatures which only just cause failure of the limestone concrete since slab lengths increase rapidly as temperature falls decrease at this stage. Increases in crack spacings at greater temperature falls are not in proportion to the strength increases causing them since effective frictional restraint is increased due to greater crack widths which correspond to longer slabs.

16.3.5. Effect of stress relief by creep

It has been suggested that the simplest way of accommodating the effect of creep in predicting crack spacings and widths is to make a correction to the observed thermal strain in any situation by an amount which is equal to the expected creep strain. Thus, creep can be considered to reduce the effective temperature fall and the phenomenon should therefore not affect crack spacings greatly provided the temperature fall is still sufficient to cause cracking. Supposing, for example, that a temperature fall of 4°C occurs in a gravel aggregate concrete at the age of 16 hours (Table 16.2.). If the effective average creep strain at failure is 12.5. x 10^-6, then the corresponding maximum crack spacing would be increased from 11.4 m to 13.0 m, corresponding to a temperature fall of 3°C. There may, however, be instances in which creep increases the strain capacity of the material by the critical margin that prevents tensile failure and in this case substantially greater crack spacings might be obtained if cracking does not subsequently occur until the tensile strength of the material is increased.
16.3.6. Factors determining the width of cracks and the onset of reflection cracking

It is evident from Tables 16.1. to 16.4. that crack widths increase generally with increase of strength, age or thermal movement of the material, and increased temperature falls, values varying greatly over the range of ages and materials selected. For a given material the principal factors affecting crack width are the temperature reduction and the crack spacing, the latter being determined principally by the age of the material when cracking first occurs. As crack spacings become smaller with increasing temperature reduction, crack widths become closer to the thermal contraction of the slab since smaller elastic extensions exist within each slab.

The susceptibility of a flexible pavement to reflection cracking is considered to be determined by both the magnitude and variation with time of crack widths in the base, each of which is directly affected by the crack spacing.

If the surfacing is applied before shrinkage has occurred in the roadbase, tensile strains would occur in the surfacing in the neighbourhood of cracks due to subsequent shrinkage strains, corresponding slab movements tending to be concentrated at cracks. Taking, for example, a shrinkage strain of $200 \times 10^{-6}$ for lean concrete which was suggested in Section 3.2., each 1 m length of slab would contract by 0.2 mm which is considerably in excess of any of the crack widths per unit length indicated in Tables 16.1. to 16.4. Displacements due to shrinkage of slabs of greater length would be proportionately greater and these would therefore be more likely to give rise to reflection cracking.

If the surfacing is applied after the bulk of shrinkage
has occurred the principal cause of surface deterioration would be fatigue resulting from variations in crack widths in sympathy with temperature changes. These would again be directly influenced by slab length for a given material. Some indication of the fluctuation of crack width of a given slab length with temperature is given by equation 16.9: and the first part of the expression, $\Delta L_T$, would be dominant when stresses in slabs are relatively small compared to failure stresses, or after repeated movements since subgrade restraint was reported to decrease progressively during temperature cycles. Taking a slab length of 10 m, a temperature change of 10°C would result in a movement of 1.25 mm for a gravel aggregate concrete which would be largely concentrated at cracks. Clearly, crack spacings will therefore have a decisive influence on fatigue effects and on this basis the use of excessively strong concretes should be avoided. Similarly, reflection cracking will be more likely in pavements constructed on roadbases in which environmental conditions are such that cracking does not occur during the night after placing.

A further important consideration arises in terms of the use of limestone aggregate concretes. In so far as crack spacings are not greatly influenced by the use of a limestone aggregate concrete of strength equal to that of a gravel aggregate concrete, the effect of shrinkage would be similar to that of the latter, assuming that shrinkage properties are similar. Fatigue effects in the surfacing should however be reduced since movements for a given temperature change would be approximately two thirds those of a gravel aggregate concrete. If the concrete strength were increased due to chemical action involving the limestone aggregates, crack spacings would be increased such that if the surfacing were applied before shrinkage had occurred in the base, susceptibility to reflection cracking would
be increased. If, on the other hand, shrinkage were allowed to occur before the surfacing is applied, the greater length of slabs would be approximately compensated by their lower coefficient of thermal movement such that susceptibility to fatigue cracking in the surface should not be increased. It would appear therefore that provided shrinkage takes place before the surfacing is applied, stronger limestone aggregate concretes would be no more likely to cause reflection cracking than gravel aggregate lean concretes cured in similar conditions. The increase of aggregate/cement ratio to values in excess of 20 which may be required by the Department of Transport Specification in order to avoid high strengths, would therefore only be necessary when substantial shrinkage occurs after the surfacing is applied.

16.4. Conclusions regarding thermal cracking in roadbases

On account of their importance, the chief points established in this section are summarised prior to drawing more general conclusions in Section 18. The theory given, in conjunction with experimental results and published information, is considered to provide ample evidence that normal daily variations in environmental temperature will cause cracking in lean concrete roadbases. If, as is most likely, cracking occurs at early ages, spacings may be as low as those reported by Williams, though crack spacings of below 4 m would, on the basis of experimental results, appear to be unlikely in normal circumstances. If cracking does not occur during the first night after placing, crack spacings would probably be much greater. The magnitude of subgrade restraint affects crack spacings, though not to the same extent as the age at which cracking first occurs.

Shrinkage strains are expected to make a greater contribution to crack widths than temperature reductions in the range normally occurring in the United Kingdom, though they would not normally affect crack spacings. If the surfacing is applied before shrinkage has taken place, the probability of reflection cracking
will be much greater. The effect would be exacerbated by large crack spacings obtained principally if cracking in the roadbase is delayed but also when stronger materials are employed since these generally exhibit higher shrinkage and form slabs of greater length. If surfacing is delayed until shrinkage has occurred, susceptibility to reflection cracking would be reduced and would depend chiefly on crack spacings and the coefficient of thermal movement of the material. In this context, the use of limestone aggregate concretes would decrease significantly the risk of reflection cracking, since fluctuations in crack width due to temperature change, which are probably the principal cause of fatigue cracking in the surfacing, would be reduced. Increased strengths which are sometimes obtained in limestone aggregate concrete would offset this advantage if they resulted in greater crack spacings.

Qualifying many of these conclusions is the fact that the most significant results indicated in tables are based on experimental results which are somewhat tentative on account of difficulties experienced in obtaining measurements at early ages and it is therefore considered that prior to further application of this method of prediction of cracking patterns, further and more detailed experimental measurements should be carried out; especially on limestone aggregate concretes which were not included in the main experimental programme.

16.5. Overall comment on the method of analysis

The method of analysis is considered to represent a significant improvement compared to that employed in Section 4 in which a constant coefficient of subgrade restraint \( \mu \) was employed, since the estimation of an appropriate average value for \( \mu \) in the latter method is most difficult. It is evident that at early ages the simple method considerably underestimates crack spacings since coefficients of subgrade restraint at the small displacements occurring are well below the overall value of 1.5 assumed earlier. A further improvement that could be made might
be to develop a more widely applicable relationship between frictional restraint and slab displacement than that given in equation 16.5. although repeated attempts to achieve this have resulted in expressions of great mathematical complexity. A simpler modification was considered in terms of omitting the fraction $\frac{3}{5}$ qualifying the ratio $f/E$ in equation 16.11. since the ratio increases towards unity when the slab displacement falls off more rapidly with distance from cracks and the change would only affect crack widths and spacings significantly at temperature changes around failure values, resulting then in expected increases. Similarly, increasing the numerical factor 3 in equations 16.7. and 16.11. would take account of a more rapid reduction in subgrade restraint at increasing distance from a crack at temperature falls in the region of failure of a heavily restrained slab. In this case, however, substantial errors would arise when temperature falls are well in excess of the value $f/aE$. Results for these modified equations could easily be obtained although values have not been given since the situations to which they correspond are not easily defined and since the method already given is considered sufficient to provide an adequate description of likely cracking patterns.
At a late stage in the project, a unique opportunity presented itself to monitor temperatures in a lean concrete roadbase immediately after construction. The base was part of an almost straight service road on the campus of the North East Surrey College of Technology, of length approximately 100 m and width 3.9 m, laid on level ground and passing within 2 m of a building which could be used to house instrumentation. Plate 17.1 shows an overall view of the road which lay in an approximately north east/south west direction.

17.1. Construction and experimental procedure

A sub-base consisting of a 150 mm layer of crushed granular material was placed, levelled and compacted on a clay soil subgrade and the lean concrete base was laid directly onto this material, no blinding material being used. The lean concrete was of aggregate/cement ratio 18, employing Thames Valley aggregate of maximum size 37.5 mm. The design thickness of the base was 150 mm. The main 84 m section was placed in one day between the times 10.00 and 14.00 hours. After compaction by vibrating roller, a sprayed bitumen emulsion was applied and the base was then closed to all traffic for 10 days. The final finish of the base was generally smooth, though in some areas an excess of larger aggregate fractions prevented a sealed surface from being obtained.

On a section of base which had been placed at 12.00 hours, holes of 10 mm diameter were formed to depths of 50, 100 and 150 mm in the concrete at a distance of 400 mm from the steel edge forms. At 16.30 hours, copper-constantin thermocouples were inserted into each hole. Leads were protected from dampness by p.v.c. sheathing and the actual junctions were coated with silicone grease. Thermocouples were fixed in position by pouring a cement grout of water/cement ratio 0.6 into the holes until they were completely full. A further thermocouple was fixed to the surface of the concrete using a bitumen emulsion, which was also applied to the small disturbed areas of concrete around each hole. The reference thermocouple junctions were situated on adjacent ground approximately 1.5 m away from those in the roadbase,
Plate 17.1. Overall view of the roadbase in which temperature measurements were made.
attached to a small plastic plate at ground level and shielded from solar radiation. Thermocouple outputs were monitored on a chart recorder giving a temperature sensitivity of 0.25°C. Air shade temperature and humidity were also recorded using a thermohygrometer near to the reference junctions of thermocouples. Temperature and humidity recordings were made for a period of one week, the zero setting of the recorders being checked daily.

17.2. Results of control tests

It was not possible to carry out a comprehensive series of tensile strength and elastic modulus tests on the lean concrete used in the roadbase since moulds suited to a 37.5 mm aggregate size were not available. Cube strengths were therefore measured and the average result of three tests carried out at an age of 7 days was 11.1 MN/m². In the absence of cube test results from lean concretes tested in the project, the tensile strength was estimated from correlations obtained for similar aggregates by Kolias, who obtained an average equivalent cube strength of 15.2 MN/m² for a similar material at the same age. The uniaxial tensile strength of this material was 1.17 MN/m² and on the basis that equivalent cube strengths are approximately 5 per cent greater than normal cube strengths, the uniaxial tensile strength of the roadbase concrete would be approximately 0.90 MN/m² at 7 days of age assuming a linear relationship between the two parameters. This value was below the value of 1.12 MN/m² obtained at 7 days for the lean concrete used in this project but it was decided to estimate crack spacings from detailed results given in Section 12 in order to give at least tentative information in this situation.

17.3. Results of temperature measurements

Figure 17.1. shows details of recorded air and concrete temperatures.

Air temperatures were slightly lower than those which would normally be expected in early Spring in Southern England and the average daily variation in temperature was 6.0°C compared with the value of approximately 7.5°C indicated by interpolation from
Figure 17.1. Temperatures measured in a lean concrete roadbase during the 7 day period after construction.
Table 3.1. On days 2, 4, 5 and 7, skies were clear during the late afternoon and solar radiation caused air temperatures to rise to a maximum at approximately 18.00 hours although these rather late times for maxima were probably also caused in part by the fact that neighbouring buildings to the south side of the road reduced solar heat gains at the point of measurement until relatively late in the day. On the other days, maximum air temperatures occurred at about 15.00 hours. Minimum temperatures were normally reached at about 06.00 hours.

The surface temperature of the concrete was at all times greater than the air temperature, probably on account of heat released from the concrete during the night time hours, and also indicating that large radiation losses associated with still clear nights did not occur in the period of measurement. During daylight hours and particularly during sunny spells, the surface temperature of the concrete, which was not protected from solar heat gain, rose rapidly to as much as $8^\circ\text{C}$ above the air temperature. Fluctuations in surface temperature were generally far greater than those of the air at greater depths in the concrete.

There was no clear heat of hydration peak during days 1 and 2, but it is considered significant that in the period 06.00 to 18.00 hours on day 2 in which the air temperature rose by $5^\circ\text{C}$, the concrete at a depth of 50 mm was for the bulk of the period nevertheless warmer than that at the surface. This effect was not observed in any other period of rising air temperature and was probably therefore caused by heat of hydration since the period corresponded to concrete ages in the range 19 to 31 hours. There is, however, no evidence of a substantial temperature rise having occurred due to heat of hydration.

Temperature patterns in the roadbase section were generally rather similar to those obtained by Wilson$^{58}$ in a bituminous pavement in cold weather, shown in Figure 3.7, though the average temperature over the period of measurement was higher. To permit a comparison with the results of Wilson differentials of temperature between the depths of 50 mm and 100 mm in the lean
concrete roadbase are considered. These reached a maximum of approximately 3°C during the morning of day 2, maximum differentials on cooling not exceeding 2°C. Temperature differentials between the depths of 40 mm and 100 mm in the bituminous pavement reached a maximum of approximately 3°C during heating and 1.5°C on cooling. In each case time delays between the occurrence of maximum and minimum temperatures at increasing depths were small.

Differentials through the complete lean concrete section varied considerably, depending chiefly on the intensity of solar radiation. For example, on day 4, the differential reached a maximum of 4°C at approximately 14.00 hours, whereas on day 5, a maximum of almost 6°C occurred when solar radiation reached a peak at 18.00 hours. It would appear that when the solar radiation level is high, maximum temperature differentials occur at the time of maximum solar radiation, whereas when changes of air temperature are responsible for changes in pavement temperature, maximum differentials occur during the heating period.

Since temperature differentials through the base are reversed twice daily, it follows that they must pass through minimum values with the same frequency. This is particularly well illustrated on the mornings of days 4, 5 and 7 when curves cross at the times of 08.00 to 09.00 hours, minimum temperature differentials generally occurring slightly later than average minimum temperatures. A similar effect is evident in Figure 3.6. and Figure 3.7. though reversals of temperature differentials during cooling periods are in all cases less well marked.

17.4. Prediction of stresses and cracking patterns

Although it was suggested earlier that overall temperature reductions in roadbases at early ages have the most critical effect on their cracking performance, the possibility that cracking may be influenced by temperature differentials will also be considered in the prediction now given of stresses developed in the roadbase during the first 7 days.
The base was constructed between 10.00 hours to 14.00 hours but for simplicity the point of stiffening is taken to be 9 hours after the centre of this period which is assumed to be 10 hours after mixing. On this basis, its average effective zero stress temperature would have been approximately 12.5°C according to Figure 17.1. During the first night after placing the average concrete temperature fell to approximately 9.75°C although the surface temperature was reduced further to 9°C and in consequence cracks would probably initiate at the surface and then propagate through the more highly stressed remaining section. The most adverse condition occurred at 06.00 hours when the concrete was 19 hours of age, though this might correspond to an earlier age of concrete cured at temperatures employed during laboratory tests. Taking, for example, the 16 hour results given in Table 16.2., for a clinker subgrade, the observed temperature fall would result in a maximum crack spacing of between 13 m and 16 m, though the uncertainty in the tensile strength of the in-situ material combined with other experimental tolerances would clearly make accurate predictions very difficult. For crack spacings of this magnitude, transverse restraint in the 3.9 m width roadbase would be small compared to longitudinal restraint so that Poisson's ratio corrections are considered unnecessary.

Inspection of Figure 17.1, shows that on day 2 the average temperature increased to slightly over 16°C and that a temperature differential of 4°C arose in mid afternoon. Shrinkage at this time would be negligible and therefore an overall compressive stress corresponding to a temperature rise of 3.5°C would occur in the concrete. Assuming an elastic modulus of 20 GN/m² at the age slightly in excess of 1 day, this temperature rise would result in an average compressive stress of 0.87 MN/m² in the material. The stress would be increased at the surface and decreased at the underside by the effect of the temperature gradient. For a differential of 4°C, the flexural stress would, according to the relationship given in Section 3.4.4., be equal to 0.5 MN/m² (ignoring the Poisson's ratio component), increasing the surface stress to 1.37 MN/m² and decreasing the stress at the underside.
to 0.37 MN/m². The internal stress would increase the total stress at the surface of the slab and decrease the stress in the lower and internal regions, though compression failure would at this stage be unlikely since the concrete should have a compressive strength of approximately 3 MN/m² at the age of 1 day, based on tensile strengths measured in the laboratory at this age. On day 5, the temperature of the roadbase again rose to 16°C with a rather larger differential than on day 2 but by this stage the compressive strength of the material would have been well above likely compressive stresses calculated as above.

During the night following day 2, temperatures followed a similar pattern to that of the first night, but by this time the concrete would have increased in strength sufficiently to resist further cracking. The lowest temperatures of the week were recorded on the night following day 3, the temperature differential also being greater. It has been predicted however that by this time slabs of maximum length of approximately 15 m would be formed such that according to Table 16.2., longitudinal restraint alone would not cause further failure. The maximum temperature differential during this night-time period was 3°C which would result in a flexural stress of 0.47 MN/m² assuming an elastic modulus of 25 GN/m² if the slabs were completely restrained from warping. The tensile surface stress would be further increased by subgrade restraint in the longitudinal direction and by the internal stress arising from the non-linear temperature distribution. The stress arising from subgrade restraint can be calculated from equations 16.7. and 16.9. If a maximum temperature fall of 7°C on the third night is assumed, the resultant tensile stress would be 0.27 MN/m² for slabs of 15 m length. Calculation of the internal stress is more difficult, though with an overall temperature differential of only 3°C this would also be expected to be small. It is apparent that in this situation the flexural stress at the surface of the concrete would reach a maximum tensile value of approximately 0.74 MN/m² if warping were completely restrained which, in practice, would be the case at the centre of 150 mm depth slabs of dimensions in excess of 10 m to 12 m according
to Westergaard's prediction given in Section 3.4.4. The uniaxial tensile strength of concrete measured in the laboratory at 2 days of age was 0.56 MN/m² and a flexural stress of 0.74 MN/m² at the age of 2.75 days might therefore be expected to cause further cracking of slabs of length 15 m. Whether or not additional cracking actually occurred at this stage would depend on the strengths obtained in the field relative to results obtained in the laboratory and also on the magnitude of permanent strains caused by creep during initial failure at the earlier age. A further substantial temperature fall occurred during night 6, though the concrete by this time would be of substantially increased strength compared to that of night 3 such that further cracking would not be expected.

In summarising the effects of the temperature patterns observed it is evident that, assuming stiffening occurred 9 hours after placing, the most critical period from a cracking point of view was the early morning of the day following construction of the roadbase even though the air temperature range during the first day was below 5°C. With the possible exception of night 3, much greater temperature changes occurring subsequently during the week would have been generally unlikely to cause further failure since the tensile strength of the concrete increased progressively over the period. Relatively large temperature differentials occurring during the daytime would not cause failure because prior to shrinkage an overall compressive stress was generated in the concrete by restraint in the longitudinal direction as the temperature increased.

17.5. Observed cracking patterns

The bituminous curing membrane applied to the roadbase was, on inspection during the second day, still tacky and further inspection was therefore, perhaps unwisely, not made until the morning of day 5. At this age, five cracks were detected and in each case these could be traced across the complete width of roadbase, the overall line of cracks being approximately at right angles to the direction of the road at that point. A
typical crack, photographed when the concrete was 5 days of age, is shown in Plate 17, some indication of the scale being obtainable from loose 37.5 mm aggregate particles visible at the top of the photograph. Distances of cracks from the south west end of the road were measured and found to be 13, 25, 40, 61 and 84 m. They probably occurred in stages, the 84 m section originally cracking into slabs of length 40 m and 44 m, subdivision then occurring into lengths of 25, 15, 21 and 23 m and finally into lengths of 13, 12, 15, 21 and 23 m on progressive reduction of temperature. Further inspection after a period of some weeks revealed an additional crack 72 m from the south west end of the road and it is considered likely that this formed at an early age but that it was not detected on account of some surface roughness in this area of the roadbase. The crack divided the largest remaining section into lengths of 11 m and 12 m.

If cracking occurred as suggested then it would appear that the maximum stable slab length was in the region of 21 m since failure occurred in slabs of length 23 m and 25 m. This may be compared to crack spacings of 16.1 m and 13.0 m for temperature falls of 2°C and 3°C given in Table 16.2, for a clinker subgrade or to values of 20.9 m and 16.9 m given in Table 16.1, for the same temperature falls in the case of a silty loam subgrade each at the age of 16 hours. The higher subgrade restraint is considered more appropriate for the roadbase in consideration and for a temperature fall approaching 3°C, a crack spacing in the region of 14 m is suggested from values given. However, there are several ways in which this apparent difference could be accounted for at this age. Perhaps the most important of these would be the effects of creep which might be such that the effective temperature reduction in the slab was perhaps 0.5°C lower than that recorded. If this were the case, it has already been indicated that at temperatures in the region of failure, estimates of crack spacings would be low and application of modifications to equation 16.11, which were suggested at the end of Section 16 would result in crack spacings in the region of 20 m at this stage. Alternatively, if the concrete effectively stiffened at a temperature below that supposed, a
Plate 17.2. Typical crack in the lean concrete roadbase, photographed at 5 days of age.
similar increase in crack spacing would result. Clearly, an accurate prediction of crack spacings is difficult in the circumstances, especially as the temperature fall in the concrete during the first night was unfortunately rather small, but the observed values are considered to be in broad agreement with those predicted at this stage.

The next critical period was the third night which was prior to any observations having been made so that cracking could conceivably have occurred at this time. Crack spacings given in Table 16.2. would not in this situation be fully appropriate since a substantial proportion of stresses induced during the third night arose on account of temperature differentials, such that crack spacings could be well below values indicated for a uniform temperature fall. It was suggested in Section 17.4. that a slab of length 15 m might fail at this stage due to the combined effects of thermal contraction and flexure and therefore it cannot be ruled out that if, for example, stiffening of the concrete was slow during the first day, or if stresses were relieved by creep, the much greater temperature fall on the third night might either have caused failure or increased the number of cracks in the base. It is surprising however that the 21 m length slab appeared to remain intact at this stage since longitudinal restraint of a slab of this length would, under a temperature fall of 7°C, result in a uniform tensile stress in the region of 0.40 MN/m² in addition to the flexural stress of 0.47 MN/m², the total tensile surface stress therefore being well in excess of the direct tensile strength of 0.56 MN/m² measured at 2 days. There is no obvious reason why this single slab which is almost 50 per cent longer than the next greatest length should have remained intact in this situation. It is possible however that effective flexural strengths may be substantially in excess of direct tensile values measured in the laboratory and also that some form of stress relief may have occurred locally near the surface of the concrete such that overall failure of the slab was avoided.

It is perhaps unfortunate that on two occasions during the first
three days, stresses in the region of failure values were predicted, since this has resulted in some difficulty in predicting on which of the occasions failure actually occurred. However, it is considered on balance that failure would have been most likely at the earlier stage since on the basis of properties measured in the laboratory at the age of 16 hours, which did not necessarily correspond to the lowest ratio of tensile strength to elastic modulus, the observed temperature reduction of 2.75°C would have been approximately 1°C greater than the temperature fall required to cause cracking.

In conclusion, it can be stated that crack spacings predicted as a result of temperature conditions occurring during the first night after placing are in broad agreement with those observed. In the absence of detailed flexural strength data accurate predictions of crack spacings resulting from the non-uniform temperature reductions occurring on the third night are more difficult to arrive at. Application of the theory given in Section 16 suggests however that, if flexural strengths are substantially in excess of direct tensile strengths, predicted crack spacings would again be in the region of those observed.

Surfacing of the roadbase was not carried out until the concrete was approximately three months of age by which time the width of existing cracks had increased greatly due to shrinkage. With the exception of the additional crack observed 72 m from the south west end of the road, no new cracks were detected in this period. Plate 17.3. shows one of the cracks photographed immediately prior to surfacing. The average width of the crack shown was approximately 2 mm at this stage which in a slab of length 12 m would correspond to a shrinkage strain in the region of $170 \times 10^{-6}$. The surfacing comprised an average thickness of 50 mm of rolled asphalt and at the time of writing which was three months after surfacing no reflection cracking had been observed, although temperature changes in this period were unusually low and no cold spells were recorded.
Plate 17.3. Typical crack in the lean concrete roadbase, photographed at the age of 3 months.
18. CONCLUSIONS

The programme of study was directed towards clarifying the effect of temperature changes on the cracking properties of concrete roadbases with particular emphasis on behaviour at early ages. The principal conclusions of each area of study are now summarised.

1. Environmental temperature changes cause significant stresses in concrete roadbases laid in large lengths without joints, overall temperature reductions at early ages being most likely to cause failure since the tensile strength of concrete is very low, comparative to its stiffness at this time. The uniform tensile stress occurring due to an overall temperature reduction at night time may be supplemented by warping and internal stresses.

2. Coefficients of thermal expansion of lean concrete mixes are similar to those of normal concrete mixes employing the same aggregates. Coefficients for gravel aggregate concretes are in general at least 50 per cent greater than those for limestone aggregate concretes. Thermal movements in soil-cements are generally less predictable than those in concrete and they depend on moisture changes occurring during heating or cooling. There may be some irreversibility of thermal movement in cementitious materials, particularly when temperature is reduced for the first time.

3. The compressive strength of lean concrete of given mix proportions depends on the type of aggregate employed, some limestone aggregates resulting in substantially higher strengths than gravel aggregate concretes.

4. The restraint apparatus incorporating an invar steel annulus provides a satisfactory means of inducing cracking in lean concrete by cooling, being of a robust nature and well suited to use with low workability concretes. The strain sensitivity of the design employed was, however, rather low for testing lean concrete at very early ages while for stronger more mature concretes, a stiffer restraint system would be recommended.

5. Restrained thermal contraction tests indicate that the temperature reduction required to induce cracking in a gravel aggregate lean concrete decreases to a minimum value of approximately 2°C at ages
in the region of 1 day, increasing considerably thereafter. The
temperature reduction required to induce cracking in mature lean
concrete is approximately twice that of the lowest value measured
in this way.

6. If the temperature reduction required to induce cracking in lean
concrete is based on short term tensile strength, static elastic
modulus and thermal movement measurements, values obtained are
significantly lower at early ages than those based on restrained
thermal movement tests. Stress relief by creep may account for this
difference.

7. Restrained thermal movement tests indicate that lean concrete having
a density shortfall of 5 per cent is more susceptible to thermal
cracking than the fully compacted material at the age of 7 days
but that resistance is not significantly affected at the ages of 1 day
and 2 days.

8. The resistance to thermal cracking of richer concrete mixes is
greater at all ages than that of the lean concrete mixes tested.
Although tensile strength and elastic modulus measurements suggest
that cracking resistance should increase with age, restrained
thermal movement tests indicate that resistance at 2 days of age is
marginally greater than at 7 days, probably due to the effect of
stress relief by creep at the earlier age.

9. The addition of 1.5 per cent by volume of 40 mm plain steel fibres to
a concrete mix of aggregate/cement ratio 4.5 results in an increase
in the ratio of tensile strength to elastic modulus, a slight
decrease in thermal movement and an increase of 2°C in the temperature
reduction required to induce cracking at the age of 2 days.

10. The annular ring apparatus used is not suited to the measurement of
creep strains occurring due to a sustained tensile stress induced by
reduction of temperature, since early creep strains are masked by
those due to variations in tensile stress in the concrete prior to
the establishment of thermal equilibrium. Some indication of the
extent of time dependent relief of stress could nevertheless be
obtained by testing concrete rings to failure at varying cooling
rates.
11. Crack spacings, predicted using an approximate expression for subgrade restraint combined with tensile strength, elastic modulus and thermal movement properties, may be as low as 5 m if the temperature of a lean concrete roadbase is reduced by 5°C during the first night after placing. If a very small temperature reduction occurred on the first night such that cracking was not induced at this time, the consequent crack spacings may be substantially increased. The spacings of cracks are therefore greatly influenced by the weather conditions prevailing immediately after placing of the roadbase.

12. Stress relief by creep in a lean concrete roadbase would be unlikely to affect crack spacings greatly except on occasions in which cracking is in consequence delayed until the maturity of the material is increased.

13. It is possible that if concretes of increased strength are employed in roadbases, cracking may not occur at very early ages since stronger concretes have a substantially increased strain capacity and stress relief by creep is, in addition, likely to be greater than in lean concrete. Cracking might therefore be induced by shrinkage which would result in much greater spacings, or by loading.

14. Limestone aggregate lean concrete of similar strength to gravel aggregate lean concrete would result in only slightly increased crack spacings. If, as is commonly observed however, strengths are also significantly increased, increases in crack spacings may be substantially greater.

15. Cracks of large spacing in cementitious roadbases are most likely to result in reflection cracking since corresponding movements at cracks will be greater. Particularly wide cracks will develop if shrinkage in cementitious bases occurs after surfacing. Susceptibility to reflection cracking would therefore be increased if stronger concretes were employed. The use of limestone aggregate lean concrete would only increase the risk of reflection cracking if substantial shrinkage occurred after surfacing, increased crack spacings resulting from higher strengths otherwise being approximately compensated by lower thermal movement.

16. The spacings of cracks observed at the age of 5 days in a lean concrete roadbase were in broad agreement with spacings predicted on the basis of work undertaken in the project.
19. APPLICATION OF RESULTS TO THE DESIGN AND CONSTRUCTION OF FLEXIBLE PAVEMENTS WITH CEMENTITIOUS BASES

19.1. General implications

The work undertaken in the thesis is considered to provide a clear indication that commonly occurring temperature changes will result in the formation of cracks in lean concrete roadbases at early ages, the spacings of cracks depending principally on the age at which cracking first occurs and on the material properties. According to published information reviewed in Section 3 further cracking may be caused by combinations of loading stresses and temperature gradients, particularly in roads designed only for moderate cumulative traffic loads. Some fatigue cracking may also occur even in pavements having the greatest design lives. It is likely however that where regular cracking patterns already exist in a roadbase due to temperature changes, further cracking caused by these influences would be reduced since existing cracks would serve to some extent in reducing stresses in the base.

There is sound theoretical evidence that the likelihood of reflection cracking will be increased when relatively large crack spacings which are associated with stronger concretes occur. There would in consequence appear to be ample justification for a maximum strength limit for lean concrete used in roadbases, in so far as cracking is induced by changes of environmental temperature and traffic loading, since spacings would be reduced by use of a lower strength material in either case. Inevitably, however, there is the risk that unacceptably large crack spacings may nevertheless result if certain environmental conditions prevail after construction or when traffic loads are light, but also that very heavy traffic loads may lead to accelerated structural failure of the base. Both problems have in fact arisen in recent years in motorways constructed in the United Kingdom. Reflection cracking has been repeatedly transmitted through renewed surfacings applied to flexible pavements with cement-bound granular bases and since underbridge clearances have been reduced to near minimum values in some cases, major
reconstruction of pavements is now required. In certain areas, the cementitious base itself has also failed although the overriding cause of deterioration over the past 15 years has been the unexpected increase in traffic loading, traffic on the M6, for instance, having, by July 1977, exceeded 14 million standard axles compared to the 5 million standard axles design level. It is apparent that if present trends in traffic loading continue, the structural demands placed on pavements will, in the future, be considerable. Indeed, since recent work on the effect of repeated heavy traffic loads on pavements with cementitious bases has indicated that deterioration may occur more rapidly than the results of the A.A.S.H.O. test suggest, it is possible that increases in either design thickness or strength of cementitious bases may be necessary to resist the long term effects of traffic comprising very heavy axle loads.

Higher tensile strengths which are easily obtainable in concretes of the type already employed in roadbases could make a significant contribution to increased structural performance in flexible pavements. However, since existing bases of comparatively low strength have resulted in reflection cracking, attention must clearly be paid to alleviating this problem prior to the use of stronger cementitious roadbase materials. Reflection cracking has been shown to be determined chiefly by crack spacings so that it may be considered necessary to attempt to control these by artificial methods rather than to leave their determination to the uncertain influences of weather and traffic loading. Alternatively, however, it may be possible to design the pavement in such a way that cracks in the base are not transmitted through the surfacing, or to allow cracks to form naturally as at present and to develop an effective remedial treatment for use as and when necessary.

There follows a brief consideration of these three methods by which reflection cracking may be controlled.
19.2. Control of cracking in the roadbase

Suggested methods for the control of cracking within the roadbase itself have included modification of the properties of the roadbase material, reinforcement of the base and the induction of cracks by artificial means.

By reducing the elastic modulus of the roadbase material it may be possible to increase its strain capacity and therefore avoid the well defined cracks associated with high modulus, low strength materials. A number of workers have experimented with bituminous emulsions to this end. Patankar and Williams\textsuperscript{118}, in a laboratory investigation, found however that replacing either 10 per cent or 20 per cent of the mixing water with bitumen emulsion is unlikely to change crack susceptibility significantly. Precoating the coarse aggregate increased the strain capacity of the material and reduced the tensile strength of the material, but the authors concluded that appreciable load spreading characteristics should remain and recommended further trials under field conditions. Bonnot\textsuperscript{119}, as a result of work undertaken at about the same time in France, suggested that precoating gravel with asphalt would lead to increases of 50 per cent to 100 per cent in the temperature drop producing cracking in cementitious materials, though these findings were not confirmed by later, more detailed results for the same materials obtained by Dinh\textsuperscript{84}, already considered in the thesis. Boussion\textsuperscript{120} and Rakovskij\textsuperscript{121}, in independent field trials, used a bitumen emulsion to facilitate the stabilisation of sand with cements for use in roadbases. Boussion reported satisfactory performance and Rakovskij found that roadbases so constructed had sufficient bearing capacity to allow use by construction traffic immediately after compaction. No thermal cracking was evident in the latter case.

Reinforcement has been incorporated into cementitious roadbases on a number of occasions. Dobrinskii\textsuperscript{122}, for example, included a relatively low weight of reinforcement in soil-cement bases and reported a high degree of resistance to thermal cracking as well
as greatly increased bearing capacity which allowed the thickness of the base to be decreased. Dobrinskii considered that the method could also prove competitive on economic grounds. Williams reported excellent crack control in a road in which the lean concrete roadbase was reinforced though the use of a bituminous paver for laying the lean concrete was in consequence precluded. A further disadvantage was also considered to be the added complexity of construction and the risk that a plane of weakness may be formed at the level of the reinforcement. Following a study of the properties of steel fibre reinforced concrete, Williams reported encouraging results for a wet lean concrete mix which may allow the use of a slipform paver for placing the concrete, flexural strength being significantly improved and the material exhibiting considerable ductility after cracking. Substantially increased material costs compared to those of lean concrete may be offset by reduced roadbase and surfacing thicknesses.

A further means of crack control in the roadbase itself has been to preload the base to produce a relatively large number of comparatively small cracks, although an additional motivation for studies of this type has been the possibility that site construction traffic might use the base immediately after construction, thereby expediting progress on subsequent sections. Bofinger and Sullivan cast 28 m lengths of soil-cement base and preloaded different sections at the ages of 7 days and 11 days. The base loaded at 7 days showed smaller eventual crack spacings than that loaded at 11 days and the authors concluded that the application of controlled wheel loads after a short curing period may help to regulate crack spacings. Yamanouchi and Ishido measured stresses due to wheel loads at the base of a cement stabilised sandy gravel soil. They found no significant difference in performance when bases were opened to traffic immediately after compaction, though the magnitude of traffic loads was not stated. Hair-line cracks were found in a section in which a 7 day curing period was allowed but not in the section which was subjected immediately to traffic. Sherman et al., investigating the use
by construction traffic of newly constructed cement treated bases found that loads of 100 kN per axle would not cause serious deterioration provided that the period of loading did not exceed 3 to 6 days. They concluded however that fatigue damage could occur if loads of this magnitude were applied to bases on wet subgrades during the first 7 days after construction and therefore recommended that such traffic be excluded for this period.

In the case of lean concrete roadbases, Williams has been more in favour of producing regular cracking by means of crack inducers positioned at perhaps 5 m intervals in the sub-base prior to laying the roadbase to avoid the possible loss of load spreading properties which may be associated with preloading. If cracking could be controlled in this way, stronger concretes could be employed and some form of debonding of the surfacing in the region of cracks would then reduce the likelihood of reflection cracking.

19.3. Prevention of transmission of cracks into the surfacing

The most adverse conditions from the point of view of the surfacing occur in cold weather since the tensile strain capacity of bitumens is reduced at lower temperatures, repeated temperature fluctuations resulting in progressive cracking through the material. Thicker surfacings are more resistant to cracking since temperature variations in the underlying roadbase are reduced and the average stress in the surfacing is lower for a given movement at a crack. Nevertheless, reflection cracking may eventually occur even in surfacings of substantial thickness. Considerable effort has been put into reducing the temperature susceptibility of bituminous binders, though Hanson has reported that of over 3000 patents taken out on methods of modifying bitumen properties, none has been completely successful. It would seem therefore that the practice of employing thick surfacings is the only present means of reducing reflection cracking so long as surfacings are laid directly onto cementitious roadbases in which cracking has given rise to slabs of substantial length.
A number of attempts have however been made to control reflection cracking by means of an intermediate layer between the base and the surfacing. Oerbom\textsuperscript{126} in a review of Swedish experience found that a thin layer of unbound gravel between the stabilised subgrade and the pavement reduced reflection cracking and Norling\textsuperscript{127}, reporting on the extensive use of 100 to 150 mm gravel layers between the bituminous surfacing and the cement-treated base in roads in the United States of America, indicated that reflective cracking was minimised and delayed. Marais\textsuperscript{128}, however, commenting on methods of overcoming severe reflection cracking in roads with cement-treated crushed rock bases in South Africa, reported that a 100 mm crushed rock layer between a 30 mm asphalt surfacing and the cementitious base did not in general prevent surface cracking. Relatively poor performance in this case may have been due to the small surfacing thickness employed.

An alternative type of intermediate layer has been tried in the form of ground scrap rubber combined with a mineral filler and bitumen emulsion. Norling\textsuperscript{127} reported on the successful use of a 3 to 6 mm layer of the material in previously cracked pavements prior to overlaying but suggested that it might also be applied direct to the cement treated base before the surfacing at the time of the original construction. Norling also recommended that surfacing should be delayed until shrinkage had occurred in order to avoid additional tensile strains in the material in the region of cracks.

19.4. Remedial treatment in cases of reflection cracking

In the absence of any effective means of preventing reflection cracking in flexible pavements in the past, some effort has been directed into providing methods of treating cracks once they occur and there would indeed be every justification for this type of approach if an effective method could be found of preventing the deterioration of an otherwise sound pavement in which reflection cracking of only slight or moderate intensity has arisen.
Hanna conducted tests on steel fibre reinforced concrete laid over joints in existing concrete slabs and found that the presence of the fibres retarded early cracking. Davis reported on the use of tensioned wire mesh reinforcement in bituminous overlays used as a remedial measure in badly cracked flexible pavements. After a number of practical problems including a tendency for the fabric to curl in the transverse direction were overcome, good results were obtained. Davis also described the use of metal plates over cracks to allow some degree of debonding when renewed surfacings were applied, though adhesion of the surface to these plates was poor. A further possible treatment to cracks prior to resurfacing is the use of a thin glass reinforced cement mat covering a length of 300 mm each side of the crack. Satisfactory performance was reported after two to three years' service under a 40 mm bituminous surfacing, movements at underlying cracks being absorbed by microcracking in the mats.

19.5. Concluding comments

It is evident that many techniques have been employed in attempting to reduce reflection cracking and that the method most suited to overcoming the problem in any one situation would depend on the materials and techniques available and on the traffic loads imposed on the pavement. Focussing attention on the motorway system in the United Kingdom, in which performance standards as well as traffic loads are high, the use of an unbound gravel layer above the cementitious base would, for example, probably be inappropriate since large axle loads may result in unacceptably high radial stresses in the surfacing unless a very substantial surfacing thickness were employed. In the case of lean concrete roadbases, which are the only cementitious bases permitted at the present for very heavily trafficked roads, it would seem highly desirable to retain as far as possible the admirable stiffness properties of the material while at the same time controlling reflection cracking. The ideal solution may therefore be considered to be the prevention of cracking within the base itself, although the total prevention of cracks in cementitious materials subject to restraint is normally considered unattainable on
account of the effects of shrinkage and thermal movements.
Perhaps the next most effective method would be reinforcement of
the base resulting in a network of distributed microcracks.
Clearly, practical considerations such as those given earlier by
Williams may override advantages in performance but there is
considered to be some justification for further field trials
since the use of reinforcement would appear to be the only
practical means of effectively resisting cracking within the
base itself without sacrificing strength and stiffness properties.
In carrying out trials of this type, it would be recommended
that concretes of strengths higher than those currently permitted
in roadbases should be employed and that low weights of
reinforcement be considered since, if cracking patterns are
established at early ages, the use of greater weights would be
unnecessary and possibly deleterious if a plane of weakness were
created in the concrete. Meshes of spacing between 0.5 m and 1 m
may, for instance, prove sufficient. If the problem of cracking
could be overcome in this way, substantial economies could be
effected by reductions in both the roadbase and surfacing thickness
and the promise of significantly longer life and reduced
maintenance costs.

The use of crack inducers is also considered to merit further
attention especially in view of the fact that methods are now
available for preventing the transmission of cracks into the
surfacing. Field trials would however again be necessary in
order to determine the most efficient design of inducer and to
ensure that cracks could be produced at the desired spacings.
Similarly, field trials on the effect of controlled preloading of
concrete roadbases, which have not to the author's knowledge,
been carried out, would also provide valuable information as to
this comparatively simple method of crack control.

In concluding this review of methods of controlling reflection
cracking, it is considered that in the case of heavily trafficked
roads with concrete roadbases, attention should be directed
towards the artificial control of cracking in the base since
considerable variations in cracking patterns and therefore in long-term performance are likely to arise as long as crack spacings are determined by combinations of environmental and loading influences. Field trials are probably the most effective way of assessing relative performance of the possible methods described. With continuously increasing traffic loads, however, the benefits would be judged to repay amply the work involved, allowing greater exploitation of the excellent load spreading properties of concrete roadbases in alliance with the first class riding qualities of bituminous surfacings.
20. RECOMMENDATIONS FOR FURTHER WORK

In addition to field trials suggested in the previous section in order to investigate the effectiveness of methods of controlling cracking in roadbases, there were a number of areas of experimental work encountered in the project which commended themselves to further study. The main subjects were as follows:

1. Irreversibility in the thermal movement of lean concrete may influence the spacing and width of cracks arising in roadbases. There is a need for further testing to determine the extent and importance of these effects.

2. There would be considerable value in designing an apparatus which would permit the measurement of the tensile strength and static elastic modulus of concrete from the time of casting, thereby permitting the prediction of strain capacities over the most critical 24 hour period.

3. Further restrained thermal movement tests conducted at very early ages would lead to more accurate predictions of the performance of concrete roadbases under environmental influences. For the purpose of such tests, a restraint apparatus of lower stiffness or greater strain sensitivity combined with a more accurate means of temperature control would be advantageous. Provision may also have to be made for testing specimens in the position of manufacture to avoid damage during handling.

4. Additional information on the relief of stress by creep during restrained thermal movement tests could be obtained by the use of a smaller version of the restraint apparatus which would allow the effect of much more rapid temperature reductions to be investigated.
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APPENDIX A

TEMPERATURE RISE OCCURRING IN A LEAN CONCRETE ROADBASE DUE TO HEAT OF HYDRATION

This brief analysis is designed to give some indication of how the heat of hydration generated during the first day in a lean concrete roadbase is distributed and to give an approximate assessment of the temperature rise above ambient level which would occur. More detailed and accurate predictions have been made by others in relation to concretes of normal strength employed in pavements and bridge decks, where the larger heat outputs of concrete require a more careful analysis.

A roadbase of thickness 150 mm is considered and properties of the concrete are assumed to be:

- Thermal conductivity = 1.4 W/m°C
- Density = 2300 kg/m³
- Specific heat capacity = 960 J/kg°C

The heat is assumed to be generated at the centre of the slab. Consider a temperature rise of 1°C occurring over a period of 1 day in a section of base of area 1 square metre. The corresponding heat input would be distributed in the following ways:

**Specific heat input.** The heat required for a 1°C temperature rise would be

\[
\text{mass} \times \text{specific heat} \times \text{temperature rise} = 0.33 \text{ MJ/m}^2
\]

**Heat lost upwards.** The effective resistance to heat flow upwards is normally described by the 'heat transfer coefficient' and values measured for horizontal concrete surfaces are reported to vary between 14 W/m²°C and 57 W/m²°C, according to the degree of exposure. An intermediate value of 25 W/m²°C is assumed here. Since heat is, in this case, assumed to be generated at the centre of the 150 mm slab, a correction corresponding to the resistance to heat flow through the slab itself is necessary and using the method described in Building Research Digest 108, the former value of 25 W/m²°C is reduced to approximately 11 W/m²°C. For an average temperature rise of 0.5°C over the 1 day period, the heat flowing upwards is numerically equal to half this value or 5.5 W/m². The total heat flow upwards over the period would therefore be 0.48 MJ/m².

**Heat flowing downwards.** The temperature at a depth of 300 mm below the centre of the slab is assumed to be approximately unchanged since results such as
those given in Figure 3.9, indicate that temperature changes at this depth are very small compared to short term temperature changes occurring at the surface. The heat flowing downwards is obtained from the average temperature gradient, which is \(0.5/0.3 = 0.17 \, ^\circ C/m\). The rate of heat flow is therefore

\[
\text{thermal conductivity} \times \text{temperature gradient} = 0.24 \, W/m^2
\]

This corresponds to a total heat flow of 0.02 MJ/m\(^2\) over the period.

**Combined effects.** Evidently, the heat flowing downwards is likely to be smaller than the specific heat and upward flowing components, although in practice the value would be increased due to the specific heat requirements of underlying layers and a non-linear temperature variation with depth in the non-steady state.

The total heat lost from the slab is 0.48 + 0.02 = 0.50 MJ/m\(^2\) which, combined with the specific heat requirement of 0.33 MJ/m\(^2\) corresponds to a total heat input of 0.83 MJ/m\(^2\). The total heat input over the first day of 3.4 MJ/m\(^2\) suggested in Section 3.3 would therefore correspond to a temperature rise of 3.4/0.83 = 4.1\(^\circ C\). It is noticeable that since upward heat losses form the largest fraction of the total heat input, the temperature of the slab would be substantially affected by changes in ambient temperature.
APPENDIX B

EFFECT OF BONDING BETWEEN STEEL AND CONCRETE IN AN AXIAL DIRECTION

If bonding of this type occurs, cooling would result in a shear force at the invar steel/concrete interface, varying from zero at the centre circumference to a maximum at some circumference above and below. The position and value of the maximum shear force would depend on the degree of bonding with the restraining ring. The shear force would, in turn, cause a tensile stress in the concrete increasing at a rate proportional to the value of the shear force at any point. The extreme case which would imply no slip in the axial direction at the interface between steel and concrete will help to establish stress profiles. A very high shear force would occur at upper and lower circumferences, decreasing rapidly to zero and resulting in a uniform tensile stress in the remaining section of the concrete.

Using the notation of Section 10, the axial strain arising in the concrete relative to the steel as a result of a temperature change $T$ must be $\alpha_r T$ since no movement at the interface is possible. If, therefore, the net strain in the steel as a result of this change is $\varepsilon_{asb}$, that in the concrete would be $(\varepsilon_{asb} - \alpha_r T)$. The corresponding stresses in the steel and the concrete at the interface would be $E_s \varepsilon_{asb}$ and $E_m (\varepsilon_{asb} - \alpha_r T)$ respectively. If, in each case, the force is treated as an eccentric load applied at the edge of the section and simple bending theory is applied, the interface stress would be four times the average stress. Also, since the total axial shear force in the steel is equal to the total axial shear force in the concrete:

$$\frac{\text{average stress} \times \text{thickness of steel}}{\text{in steel}} + \frac{\text{average stress} \times \text{thickness of concrete}}{\text{in concrete}} = 0$$

This gives

$$\frac{E_s \varepsilon_{asb} \times 25}{4} + \frac{E_m \times 100}{4} (\varepsilon_{asb} - \alpha_r T) = 0$$

or,

$$E_s \varepsilon_{asb} (E_s + 4E_m) = 4 \alpha_r T E_m$$

so that $\varepsilon_{asb}$ is given by

$$\varepsilon_{asb} = \frac{4 \alpha_r T E_m}{E_s + 4E_m}$$

---------- (B.2)
Inserting values of the elastic modulus used in Section 10, and \( \alpha_T = -150 \times 10^{-6} \) gives \( \epsilon_{asb} = -68.2 \times 10^{-6} \). The stress pattern across a radial section would be as shown in Figure B.1. It is noticeable that the average stresses in the invar steel and concrete are not as high as would be obtained in the tangential direction for a given temperature fall, this being due to the fact that the eccentricity of stress in each results chiefly in a bending effect. However, the tensile stress at the inner surface of the concrete is almost as great as that due to radial restraint so that failure could, on this basis, occur axially as well as tangentially. The inside surface of the restraining ring is, according to this analysis, subject to a tensile stress in spite of the compressive effect of the concrete.
Figure B.1. Stresses in invar steel/concrete section due to bonding in an axial direction. Temperature fall of 15°C.