Computer Vision Techniques for the Automatic Interpretation of Thermochromic Paint

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UniS

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In memory of Grandma
Summary

The aim of this study is to provide an automatic method for the interpretation of images of objects that are coated with thermal paint.

Thermal paint changes colour permanently according to the temperature to which it is heated and can be employed as a temperature gauge where more cumbersome measurement apparatus may not be suitable. Such a gauge requires a means to convert the manifestation of the measurement to the corresponding numerical values. In our case this involves the grouping of ranges of colour together into temperature bands and the extraction of the temperature contours between these bands, a task currently performed by a human operator.

This study will demonstrate some success in the automatic interpretation of thermal paints through computer vision approaches. In summary the main contributions of this work are:

- The demonstration that edge detection is not a useful step. Human operators tend to interpret thermochromic paint not simply by colour matching, but by locating prominent colour change points. We demonstrate why in our opinion this in not necessarily the best step through an exploration of colour edge detection
- The development of a feature space model of the paint colour formation based on B-splines and the employment of this within a maximum likelihood estimation scheme [GKWG96],[CSGW97]
- The development of a paint interpretation method based on a Markov Random Field and Simulated Annealing [GSW+98]

Our methods are applicable to cases of ideal data. We highlight some troublesome paint artifacts that occur in real cases and that hinder interpretation. We discuss possible solutions. Finally we draw conclusions and point to directions for possible future work.

Key words: thermochromic paint, maximum likelihood estimate, simulated annealing
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Chapter 1

Introduction

Thermochromic or thermal paints are a class of paint, developed for use in the aerospace industry, for which the colour of the paint is related to the highest temperature that the paint has experienced. Such paint provides a coarse means of temperature measurement in mechanical components in situations where the use of conventional sensors is not practical, for example on rotating components or components that lie in the combustion gas stream of a jet engine. Temperature measurement would be impossible in these situations using conventional equipment.

The subject of this investigation is the interpretation of Multi-Change Thermochromic Paints. These contain dyes and pigments whose chemical compositions exhibit several distinct reactive changes at certain empirically calibrated temperatures. These irreversible changes give rise to pronounced changes in the colour of the pigment, thus, the colour change boundaries on the surface of engine components after exposure to elevated temperature correspond to isotherms of maximal temperature during the heating process.

Thermal paints are at present interpreted by human operators who rather than explicitly match colour, locate the colour change boundaries that correspond to the boundaries
of coarsely quantised temperature ranges. This is a non trivial task and often a great deal of high-level interpretation is involved; colour changes may be between subtle shades, the boundaries may be diffuse, the paint surface may have become damaged or the physical or chemical conditions that the paint experienced may have caused undesirable colour changes. As a result the positioning of the isotherms can be fairly subjective and is subject to error.

1.1 Aims and Objectives

It is beyond the scope of this project to model all the factors involved in the interpretation of the paints. Firstly, a human operator uses a great deal of heuristic and high level thought as part of the interpretation process and secondly the paint colours observed after a test may not match the calibrated paint response due to carbon deposits, paint damage or colour bleaching.

Whilst we do not believe it is possible to solve the thermal paint interpretation problem in its entirety, we do believe that computer vision techniques can be developed that would aid and augment the role of human interpreter. We will instead concentrate on interpreting the paints under ideal circumstances. That is, we assume that the paints are heated under conditions that cause the colour change progression to follow those colours observed in the calibration set and that we suffer neither from carbon deposits on the paint surface, nor from paint damage, although in chapter 7 we will consider how to extend our methods theoretically to cope with many of the real-world artifacts that hinder paint interpretation.
1.2 Statement of the problem

Our ultimate goal is to find methods that can provide a means to interpret thermo­chromic paint automatically or with as little human intervention as possible. The following statements summarise the main problem areas.

- Thermal paints provide a mapping from temperature to colour
- Colour changes with respect to temperature are perceptively negligible apart from at calibrated change points at which the colour changes are distinct
- The colour changes always follow a known sequence, predetermined by the chemical composition of the paint
- Thermal paint may be interpreted by locating the prominent colour changes and classifying the colours on either side
- Thermal paint may be interpreted by matching colours to a standard colour response and classifying the prominent changes
- In homogeneous regions, the same colour corresponds to many temperatures, it is the prominent colour changes that provide calibrated temperatures
- Aside from the homogeneous regions, the same colour may occur at several points in the colour response. It is the context of the colour within the colour progression that disambiguates the temperature

1.3 Approaches

There are two main classes of approach to the problem that we shall employ. Firstly we consider image space based methods such as edge detection, our intent being to simulate the interpretation by locating colour boundaries. Results of our investigations
1.4 Data Set

Unfortunately because we are dealing with a very specialised subject data was fairly scarce. We have been provided with examples of four paints which will be denoted tp5, tp6, tp8 and tp11. We believe that these paints are representative and that any methods that provide a successful interpretation may be adapted to work on other thermal paints in a straightforward manner.

When thermal paints are designed, a set of "coupons" are produced, whereby small pieces of metal are coated with thermal paint and each heated to one of a number of temperature values quantised within the useful temperature range of the paint. This gives a standard colour response of each paint, which may be used for colour matching during the interpretation process. These calibration coupon sets were unavailable to us during the course of our research. Instead, we took as our calibration set a set of colours chosen along an axis of increasing temperature from a bow-tie piece similar to...
1.5. Summary and Overview of the Thesis

The aim of this study is to address whether it is possible to achieve an automatic interpretation of thermochromic paints through the use of Image Processing and Computer Vision techniques. Not only would this reduce the effort required in interpretation, but should also improve accuracy and consistency. An overview of thermal paints and the current methodology of interpretation is given in chapter 2, along with pre-requisite background information of fundamental colorimetry. Chapter 3 explores colour edge detection. The manual interpretation of thermal paint relies primarily on edge location, however we demonstrate that edge detection is not a useful step for a computer vision interpretation system, the edges being diffuse. Colour classification is addressed firstly in chapter 4 where we form the basis of a model of the range of colours that a thermal paint may take. In chapter 5 we extend our feature space model and estimate the mapping from temperature to colour for a thermal paint. Chapter 6 draws the methodology from chapters 4 and 5 together and provides a means for calculating the maximum likelihood estimate of temperature given colour. We then provide results from a "simulated annealing" optimiser that maximises the likelihood of the resulting interpretation given a thermal paint image. We will demonstrate experimentally that this gives a good interpretation of thermal paint through results on synthetic and real data. Chapter 7 will look towards working with real test pieces and the problems that

those shown in figure 1.1. The paint is heated by passing an electric current through the metal "bow-tie". Obviously the resistance is least at the widest point and highest at the narrowest point, therefore the temperature is hottest at the narrowest point of the bow-tie. Whilst not knowing the exact mapping from bow-tie width to temperature, we believe it is safe to assume both that this gives a smooth gradient of temperatures and also that this gives us a true indication of the thermochromic response under calibration-like conditions.
1.5. Summary and Overview of the Thesis

this would introduce. Finally chapter 8 will provide a summary of the investigation and draw conclusions.

In brief our contributions are as follows.

- The demonstration that edge detection is not a useful step for a computer vision implementation of paint interpretation
- The production of a model of thermochromic paint in terms of homogeneous colour regions joined by transition regions that form a straight line in feature space
- The extension of the thermochromic model to a continuous process mapping temperature to colour
- The maximum likelihood estimate of temperature from colour
- The derivation of a cost function for the classification of thermochromic paint from colour data
- The development of an optimisation process that minimises the total cost of interpretation over the image to give the solution with greatest Maximum Aposteriori Probability

We believe that this goes some way to providing a solution to the problems of automatic thermochromic paint interpretation.
Chapter 2

Background Review

The interpretation of thermochromic paint is a specialised subject area and has not been greatly explored previously. Information is therefore not generally found in the literature. To understand the problems of automatic interpretation we look at design considerations of thermochromic paint and review elements of colour theory and colour computer vision, specifically colour segmentation.

2.1 Thermochromic Paints

Thermal paints have been introduced primarily as an aid to the design and development of aircraft engines. Engine efficiency increases with increased temperature; however, there is obviously an upper bound at which high temperature may cause undesirable effects such as the failure of internal parts. During the design process it is therefore necessary to be able to monitor somehow the internal temperature of the engine during tests runs. Obviously with a complex mechanical object such as an engine it is impossible to use conventional sensors such as thermocouples and the like as these would be too cumbersome and would be liable to being damaged by moving machine parts.
Thermochromic (thermal) paints have been introduced as a solution to this problem.

Thermochromic paints have been designed such that at various temperatures the chemical constituents of the paints will react with each other causing a change in the colour of the paint. As the interpretation of thermochromic paints has traditionally been performed by trained human operators the colour changes of the paints have been designed with this in mind. Colour change points are intended to be distinct as perceived by the human eye, and colours between the change points are intended to be roughly constant. The colour changes are permanent (although are subject to further change at higher temperatures), so that upon cooling, the colour of the paint will indicate the highest temperature that the paint has reached, and thus indicates a greatest temperature profile of the surface upon which the paint was applied.

Prior to an engine test, all components may be sprayed with thermochromic paint. The engine is then constructed and subjected to a test run. The engine is deconstructed and examined. The components are then inspected and the paint interpreted. At each of the distinct change points an isotherm will be marked, thus providing a full maximal-temperature profile of the engine during the test. This along with other measurements taken during the test allows the engineer to make decisions on the design process.

2.1.1 The Design of Thermochromic Paint

Paint is a mixture of finely ground pigments, extenders, binders and solvents which after application to a surface dries to provide a film-like covering that is both decorative and resistant to the environment. Pigments provide for different colours, whereas extenders increase the chemical stability and durability of the paints. The liquid binder is the transport medium for the other paint constituents; as the paint dries this slowly cures and hardens to form the paint surface [Mor85]. The binder comprises the majority of the paint and is generally approximately transparent [Kli93]; it is the pigments that
provide the colour through light absorption, reflection and refraction.

For thermal paints [Alt95],[RR97] the temperature related colour changes are provided primarily through the choice of the pigments, specifically by their interactions both with each other and with their environment during heating. Other constituents of thermochromic paint include resins, amongst which silicone resin is used primarily to preserve the paint in the presence of the extreme conditions in which the paint is intended to be used, and volatile organic solvents which readily evaporate to produce an air dried finish. Organo-metallic dyes are occasionally used, but are unstable at higher temperatures, and thus are only used for detecting lower temperature changes, typically at less than 500°C.

Types of Thermochromic Paint

Thermochromic paints fall into one of two categories, multi-change paints and single-change paints. Single-change paints are useful for lower temperature ranges and exhibit few colour changes. The focus of this research has been multi-change paints. These contain binders specially designed to give high resistance to the forces present in gas turbine engines, and are typically useful to approximately 1000°C. The mixtures of pigments are chosen such that at several (5 or more) temperatures or rather between several temperature ranges, the pigments exhibit distinct chemical or physical changes that cause a change in the colour of the paint.

As mentioned previously, the colour changes are irreversible, although they are subject to further change as the environmental conditions change. As such the colour of the paint provides a lasting record of the highest temperature that each point of the paint surface has reached, with isotherms corresponding to the colour change boundaries.

Examples of the temperature-colour response of multi-change thermochromic paints are shown in figure 2.1. A bow-tie shaped strip of metal has
been coated with thermochromic paint and heated by passing an electrical current between the two ends. Obviously, at the narrowest point of the metal strip, the electrical resistance is lowest, and as such the temperature is highest at this point, the temperature is lowest at the widest points of the strip and increases as the width decreases. Colour change boundaries are clearly visible.
2.1. Thermochromic Paints

Physical and Chemical Factors Affecting Paint Colour

Thermal paints are based on well understood chemical phenomena which are exhibited at certain temperatures or thermal energies. Given suitable ambient conditions, the elevated temperatures provide enough energy for a chemical reaction to occur. These may occur through interactions between pigments, or through the chemical breakdown of the pigments into sub-compounds that are stable in the surrounding conditions [Alt95],[Nas83] causing an overall change in the colour of the pigment layer.

In practice, the temperature-colour response of thermochromic paint is affected by many factors, relating from the composition of the paint, the way it is applied through to the environment in which it is used and the length of time it is exposed to the conditions. In brief these include:

- Paint composition
  The amount of pigment of each type within the paint
  Particle size within the paint
  Distribution of pigments and other constituents within the paint
  Chemical impurities
  The thickness of paint surface (after application)

- Physical and Environmental factors
  Ambient temperature
  Gas pressure
  Chemical composition of the operating environment

- Chemical factors
  Exposure time
  Colour leaching
2.1. Thermochromic Paints

Resin breakdown

Paint glazing

Given the wide number of factors involved in the thermochromic response, for each batch of paint that is made there will be a slightly different reaction to temperature. To ensure the reliability of the paint a full calibration is taken. Several square metallic coupons are coated with thermochromic paint and each heated to one of several calibrated temperatures for a specified time. This provides a set of colours that is representative of the thermal paint response in a standard environmental condition which is assumed representative of test conditions.

Aside from issues related to the chemical changes within the paint the colours of the paint may have suffered from paint damage or flaking. In engine test conditions it is highly likely that the paint surface will become coated with a layer of carbon. This needs to be removed prior to interpretation. After cleaning, the paint surface is often varnished to prevent post-test damage.

2.1.2 Interpreting Thermal Paint Colour Changes

Given the existence of a calibration set of colours for each thermal paint, one might expect that thermal paints could be interpreted by matching colours observed on test pieces to those on calibration coupons under some standard illuminant, however as stated previously undesirable effects often come into play. At high temperature the paint may glaze or burn, at lower temperatures “resin burn” may occur which appears as a (non-pigment based) colour change. With some paints the combustion gases of the engine may cause the paints to appear bleached after prolonged running. In all practical applications, the combustion gas stream of the engine will lead to a layer of carbon being deposited on the surface of the paint. Whilst this may be cleaned, any remaining deposits would lead to false colour matches.
To aid human interpretation, the paints have been designed so that at various temperatures the colour changes sharply. This helps avoid the problems of carbon deposits obscuring colour matches and the fundamental problem itself of having to match close colours. Rather than colour matching, the task becomes one of locating the colour change points. The reality is still not straightforward though; occasionally the ideal situation as described above may not apply exactly. This forces the interpreters to use high level knowledge such as knowing the order in which the colours should appear. If a change point is missing (or heavily obscured) they will make a best guess as to where it should be placed. They can also ignore unexpected changes such as those induced by resin burn. In cases of paint glaze they may even resort to other analysis techniques such as, for example, testing the hardness with a scalpel to further split the temperature range.

To the outsider, thermochromic paint interpretation seems very much a black art. Whilst the interpreters have many years of experience, given all the obstacles described, it is possible that there is a fair amount of variation between operators in borderline interpretation case. It is hoped that a computer based system will supply a tool that will help improve the accuracy of interpretation and also the consistency between interpretations.

2.2 Colour Fundamentals

Colour is a perceptual response to light of different wavelengths. Of the electromagnetic spectrum only a small part can be sensed by the human eye. Within this small range, the visible spectrum, the varying wavelengths and combinations of wavelengths are perceived differently. These different perceptions are colour. Colour is not only pleasing to the eye, but within an image processing environment it can also be used to provide extra information about the scene being studied. Humans can perceive only a few dozen different grey-scales, but can discriminate between thousands of colours [Jain89].
2.2. Colour Fundamentals

2.2.1 The Perception of Colour

The perception of colour arises due to the construction of the eye. The eye contains three types of colour sensor\(^1\) that give ‘greyscale’ responses to light of three linearly independent, but overlapping, subsections of the visual spectrum. It is the interpretation in the brain of the combination of the responses within these three subsections of the spectrum that gives the sensation of the various colours.

The responses of the sensors are centred around three primary colours, red, green and blue. Let \(C(\lambda)\) denote the spectrum of the light incident on the eye, and let the \(S_{\text{RGB}}\) be the spectral responses of the three sensors. The sensors provide response to the incident light as follows.

\[
R = \int_{-\infty}^{\infty} C(\lambda)S_R(\lambda)d\lambda \tag{2.1}
\]

\[
G = \int_{-\infty}^{\infty} C(\lambda)S_G(\lambda)d\lambda \tag{2.2}
\]

\[
B = \int_{-\infty}^{\infty} C(\lambda)S_B(\lambda)d\lambda \tag{2.3}
\]

As the response of the red, green and blue sensors are formed by the integral over a range of frequencies it is possible that more than one spectral composition of light can give rise to the same tristimulus response, that is, it is the response within the three sub-bands that gives the impression of the specific colour, not the specific spectral composition. For this reason the spectral composition of light for any given colour cannot be inferred from the measurements of the sensors.

2.2.2 Digital Colour Cameras

Modern electronic cameras are based on CCD (charge coupled device) technology [Tru92]. A CCD array contains a matrix of photodiodes each of which accumulates \(^1\)These sensors, called cones are used when the level of light is good. There are also other sensors, rods, which are used when the level of light is low, and are unsuitable for colour vision
2.2. Colour Fundamentals

charge in an amount proportional to the amount of light falling on it. The amount of charge that has built up over each frame period is measured and stored in a frame store; it is this amount of charge that forms the basis of the image.

The resolution of a CCD captured image is determined by the number of CCD elements, their size and their density. With each element having some non-zero surface area they measure the total light falling on the surface, and therefore the field of view is quantised into small blocks. The charge measurement corresponds to the mean amount of light falling on each CCD element.

To provide selectivity to the charge accumulation, coloured lenses are employed to provide measures of the red, green and blue components. In commercial digital cameras one of two strategies for providing colour capture are employed, single CCD cameras and three-CCD cameras. A three CCD camera provides colour capture by placing 3 coloured filters in front of three separate CCDs. These filter through red, green or blue light to the CCD surface which then measures the light response for that colour. This gives tristimulus measurement at every pixel. A cheaper alternative that is often utilised in lower budget video cameras and in commercial stills cameras is the single chip colour CCD. Obviously each element of the CCD can only be filtered to measure one colour response, whether it is red, green, blue or greyscale. In a single chip CCD, each element on the CCD array is filtered to measure red, green or blue, the arrangement of the different colour filters allows colour interpolation to estimate values for the other colour planes. A typical layout of such a single chip CCD is shown in figure 2.2.

As can be seen from the diagram, there are two green elements for every red and blue element in a $2 \times 2$ square\(^2\). Wherever the the true value of the red, green or blue pixels is not measured they are estimated using an interpolation algorithm (for an example of such an algorithm see [Kod95]), for example, if at a pixel we measure

\(^2\)As green light occupies the central region of the visible part of the electromagnetic spectrum, the green values capture luminance levels which more readily translate across to the red and blue image planes.
2.2. Colour Fundamentals

say the green component, then we estimate the red, and blue components. The colour interpolation for a single chip CCD is based on smoothing. At any pixel, only one of the tristimulus values is measured, missing values are obtained by averaging neighbouring pixels. Although this gives a visually pleasing result when employing the data within a computer vision environment we would desire to use as accurate data as possible. We require the discrimination of possibly very close colours. To avoid potential problems due to smoothing we use a three-CCD camera that provides truly measured date in each colour band. That is, full colour at the full resolution of the CCD.

2.2.3 Colorimetry

The tristimulus values can be combined in one of two different ways. Subtractive colour mixing relates to reflection. When white incident light is reflected, such as on a printed page, or in paint, light of various wavelengths is absorbed by the dye. The perceived colour is then the perceived colour of the illuminant light less the absorbed spectrum due to the dye (or filter). Additive colour mixing relates to illumination. Starting from no illumination (black) the intensities of the three primary illuminants are combined additively to give a colour match.
Grassmann provided the following laws of colour mixing [Bar73]:

- Colours are characterised by three independent measurements.
- If one colour of a two colour mixture is changed, the colour produced changes in a corresponding manner.
- Primaries evoking the same colour response give identical effects in additive mixtures, regardless of their spectral compositions.
- When two stimuli evoking the same colour are added to or subtracted from two other stimuli evoking the same colour, the resulting mixture are matches.
- Increasing the intensity of two stimuli that give a colour match does not destroy the match provided the ratios of intensities between them are maintained.

These combined with Abney's law that the luminance of an additive mixture is equal to the sum of the luminances of its components form the basis of colorimetry.

2.2.4 Chromaticity

Under the laws of colorimetry, a colour may be represented by any linear or non-linear invertible function of the sensed RGB values, mapping from the RGB colourspace to another colourspace with different properties. This has benefits for many applications for example the mappings to YUV, and YIQ as utilised in television broadcasting ([Tan93]), or CIE-L*a*b* ([WS82]) to provide perceptual uniformity. For our application a potentially important mapping is from RGB to an intensity/chromaticity representation. The RGB values obtained from the colour camera are based on the integral spectral power distributions of 3 filters based roughly in the red, green and blue areas of the electromagnetic spectrum. As such, they contain a measure of the brightness of the scene. Let now,
2.2. Colour Fundamentals

\[
R = \int_{-\infty}^{\infty} kC(\lambda)S_R(\lambda) d\lambda \\
G = \int_{-\infty}^{\infty} kC(\lambda)S_G(\lambda) d\lambda \\
B = \int_{-\infty}^{\infty} kC(\lambda)S_B(\lambda) d\lambda
\]  

(2.4)

(2.5)

(2.6)

with \( k \) being a brightness factor. As the illumination present within the scene varies, accordingly each of our tristimulus values is scaled by our brightness factor \( k \). When dealing with colour image processing, and in particular colour discrimination and classification it seems intuitive to attempt to remove in some way the dependency on brightness, and deal with a pure chromatic representation.

Let \( R' \), \( G' \), and \( B' \) be the tristimulus values for some arbitrary "unity" brightness, these then define our chromatic representation. The chromatic values are clearly linked to the measured tristimulus values:

\[
R = kR' \hspace{1em} G = kG' \hspace{1em} B = kB'
\]  

(2.7)

It follows therefore that for \( r, g \) and \( b \) defined as

\[
\begin{align*}
    r &= \frac{kR'}{k(R' + G' + B')} \\
g &= \frac{kG'}{k(R' + G' + B')} \\
b &= \frac{kB'}{k(R' + G' + B')}
\end{align*}
\]  

(2.8)

\( r, g \) and \( b \) are independent of the value of \( k \). Whilst not dependent on the brightness factor, the ratios of \( R, G \) and \( B \) remain important, and thus \( r, g, \) and \( b \) define chromatic values independent of the amount of illumination present in the scene. These are obtained straightforwardly from the pixel values as:

\[
\begin{align*}
    r &= \frac{R}{(R + G + B)} \\
g &= \frac{G}{(R + G + B)} \\
b &= \frac{B}{(R + G + B)}
\end{align*}
\]  

(2.9)
2.2. Colour Fundamentals

It is clear that $r + g + b = 1$ and that $b = 1 - r - g$. It therefore follows that a colour may be specified by two chromatic values (co-ordinates on the chromatic plane) and a luminance value. As the equations for obtaining $r$, $g$ and $b$ each contain a term of the reciprocal of $(R + G + B)$, the equations suffer from a singularity at $(R + G + B) = 0$. Accordingly chromatic values are unstable in the presence of dim light.

**Intensity, Hue and Saturation**

When talking of chromatic colour specification in terms of computer vision, one is generally referring to the $r, g, b$ triplet as described above. All we have done however is transform the $R, G, B$ triplet into an intensity and a co-ordinate on a 2-d plane - the chromatic plane. Any colourspace that similarly produces an intensity and two other values is similarly chromatic, although the "chromatic planes" will obviously be different and have different properties.

One such similar transformation is that from red, green and blue to intensity, hue and saturation. As described in [Jai89] the perceptual aspects of colour are brightness, hue and saturation. The hue refers to the fundamental colour (for example red, orange and so on), saturation refers to the amount of 'white light' in the colour whilst brightness relates to the perceived intensity of light. Hue (as defined in computer vision terms) does not relate numerically to the wavelength of the coloured light, but the above description compares to the description of coloured light as light of a spectral frequency, an intensity and a saturation.

$I,H,S$ is however of limited use in computer vision; being angular, there are hue discontinuities at $2\pi$ radian intervals. The calculation of $I,H,S$ is often calculated in $\pi/3$ segments within the hue circle. This introduces yet further discontinuities in the colour space. Colour mixing is non-trivial in polar co-ordinates.

With regards to our problem, although chromatic colour specification would appear to
be a useful tool to reduce dependence on lighting specification we shall demonstrate that many of the colour changes of the paint occur as lightness changes rather than chromatic changes. This is discussed in later chapters.

2.3 Colour Segmentation

The solution of the thermochromic paint interpretation problem requires the partitioning of images into homogeneous regions based upon colour. Image segmentation\(^3\) [HS85] is the task of splitting an image into regions of homogeneity in respect of some features of the input values. In colour terms these features may be the RGB values, or any combination of them. Once a similarity or dissimilarity measure has been defined on the features, spatial segmentation (grouping nearby pixels with similar measure, not grouping nearby pixels with dissimilar measure; including region growing or region splitting) or measurement-space clustering (grouping similar values in the measurement-space and thus inferring homogeneous regions in the image) [HK69] may be performed. Typical examples of spatial colour image segmentation include [HK69],[PR93] (region growing) and [OKS80] (region splitting).

Measurement-space clustering was used (amongst others) by Ali et al [AMA79] where the user defines decision surfaces in Yrg space, Ferri and Vidal [FV92], Haralick and Kelly [HK69] who starting from ‘center sets’ grows in the feature space until the values become too dissimilar, Schettini [Sch93] who analyses the 1-d histograms of each feature and looks for distinct peaks which are then used to threshold the image and Syeda-Mahmood [Say92],[SM92] who uses colour segmentation prior to object selection. In his work, the \(2^{24}\) colours represented by 8 bit/band tristimulus values were quantised into 7200 colours. By performing psychophysical experiments, from these 7200 colours he determined around 220 colour categories that he used to describe the colourspace. The image pixel values were mapped to their respective category and pixels that belonged

\(^3\)Segmentation by means of chromatic edge detection will be reviewed in the next chapter
to the same category were grouped. The approach of representing the colourspace by a small number of colours is explored by Dekker [Dek94] who driven by the problem of displaying colour images on a terminal with only 256 colours (chosen from $2^{24}$) available analyses the entire image using a Kohonen network. This idea of colour quantisation is often referred to as colour indexing.

Ohta [OKS80] performed an analysis of colour features and sets of colour features to determine which gave the most discriminability of colours. Obviously when performing sementation it is desirable to use as few features as possible to reduce the amount of computation involved. Ohta also derives by experimentation three new colour features $I_1, I_2, I_3$ that approximate the colour features obtained by the Karhunen Loève transformation [Pra78]. He then goes on to demonstrate that these are effective for colour image segmentation. These features are calculated as follows:

\[ X_1 \approx I_1 = \frac{(R + G + B)}{3} \quad (2.10) \]

\[ X_2 \approx I_2 = \frac{(R - B)}{2} \quad (2.11) \]

\[ X_3 \approx I_3 = \frac{2G - R - B}{4} \quad (2.12) \]

$X_1$ is the feature with the largest variance (ie. it has the 'largest discriminant pow-er') and it is approximated by $\frac{(R+G+B)}{3}$ which is the grey-level intensity. Ohta says therefore, that intensity is the most important feature in colour image processing.
2.3. Colour Segmentation

2.3.1 Colour Discrimination

As part of any colour based segmentation scheme we need to be able to quantify the difference between two colours. A simple metric of colour difference is to take the change in colour between two colours as the Euclidean distance between them in a colourspace. The main problem is that the colourspace is not perceptually uniform, that is, a constant Euclidean distance between two arbitrary colours leads to a perceived colour difference that depends on the specific colours involved. There exist no shortage of equations for quantifying colour difference in terms of tri-stimulus values [WS82]. In 1976 the CIE recommended the CIE L*a*b* system [Nob86] as an attempt to provide uniformity of perception, where L* represents the lightness, a* represents the redness/greenness and b* represents yellowness/blueness of the colour. The colour difference is then quantified as,

\[
\Delta E = (\Delta L^* + \Delta a^* + \Delta b^*)^{\frac{1}{2}}
\]  

(2.13)

in terms of lightness, chroma and hue, this is,

\[
\Delta E = (\Delta L^* + \Delta C^* + \Delta H^*)^{\frac{1}{2}}
\]  

(2.14)

and the difference in hue is given by

\[
\Delta H^* = (\Delta E^2 + \Delta L^* + \Delta C^*)^{\frac{1}{2}}
\]  

(2.15)

For small colour difference work, it is accepted that in fact the CIE L*a*b* colourspace is not uniform [AR86]. To this end, new formulas have been suggested including the CMC(1:1:1) formula [AR86] which was tested experimentally and found to perform better than the CIE L*a*b* formula.
2.3. Colour Segmentation

Healey [HB88],[Hea89] tries to quantify colour difference for colour as sensed by a computer vision system by recovering an approximation to the spectral power distribution of the light incident on the sensor as a linear combination of a set of basis functions. In short this is achieved using the equation

$$\hat{I}(\lambda) = B^T(\lambda)A$$

(2.16)

where $\hat{I}(\lambda)$ is the approximation to the SPD, $B^T$ is the vector of basis functions $[b_0(\lambda)\ldots b_{n-1}(\lambda)]$ and A is calculated from the sensor measurements, the basis functions and the bandpass characteristics of the colour filters of the sensor.

Normalised colour is defined as

$$\overline{I}(\lambda) = \frac{I(\lambda)}{\int_{\lambda_1}^{\lambda_2} I(\lambda) d\lambda}$$

(2.17)

Given that $\hat{I}(\lambda) = B^T(\lambda)A$, where $A$ is calculated from the sensor measurements, $A$ is affected by noise. $A$ has mean $\overline{A}$ and covariance matrix $\Sigma_A$. Healey defines a colour metric between two normalised colours $\overline{A}_1, \overline{A}_2$ as the Mahalanobis distance given by

$$d(\overline{A}_1, \overline{A}_2) = \sqrt{(\overline{A}_1 - \overline{A}_2)^T \Sigma_A^{-1} (\overline{A}_1 - \overline{A}_2)}$$

(2.18)

Vriesenga [VHPS92] describes how to control the colour of illumination to optimise the discriminability of objects in an image.
2.4 Summary

Thermochromic paints can provide a rough indication of temperature where other sensors would fail due to the harshness of the environment in which measurements need to be taken.

In this chapter we have explored the relevant background of thermochromic paint, and colorimetry in general.

Isotherms are indicated by the colour change boundaries which occur as a direct result of the chemical and physical interactions of the pigments and the surrounding conditions. The temperatures at which the chemical reactions occur are statistically stable, and thus the change points are calibrated empirically. The effect of the engine environment however is less predictable: carbon deposits may partially obscure colours, combustion gasses may cause bleaching and the high temperatures can cause paint glazing. Because of this, a human interpreter employs many high-level heuristics in the interpretation of the paint and the location of the isotherms.

Our task is to locate the calibrated change points. These colour change points are expected to appear as colour edges on the thermochromic paint image. The first method explored is chromatic edge detection. This is detailed in the next chapter.
Chapter 3

Colour Edge Detection

Colour image segmentation methods fall into one of two categories, feature space methods and image space methods. Feature space methods consider the clustering of colour values over the whole image and partition the image by partitioning the colour cube. Spatial techniques on the other hand operate purely on the relationship between pixels local to each other within an image; measurements such as image gradient, or relative pixel value are compared between neighbouring pixels without reference to the wider distribution pixel values within the greater image. An example of such a technique is edge detection.

Edge detection locates sharp discontinuities in pixel grey-value, or colour by comparing the magnitude of the image gradient or some other edge response function at each pixel to a threshold. A high gradient value indicates a pixel that lies on an intensity or colour edge. These are collected and linked to other edge pixels to form the edge.

With reference to chapter 1, the current manual interpretation method of thermal paint is akin to colour edge location. That is, rather than matching colours (due to problems of colour occlusion) human operators find the location of the transitions between colour bands to be more stable. In this chapter we demonstrate the apparent inadequacy of
standard edge-detection approaches for the automatic interpretation of colour images derived from thermochromic paints.

3.1 Monochromatic Edge Detection

Accounts of monochromatic edge detection may be found in many text books on image processing ([Jai89],[Pra78] and [Low91] for example): A survey of edge detection algorithms may be found in [Pet94].

3.1.1 Simple Edge Detector

A simple monochromatic edge detector generally operates as follows. Firstly the image is convolved with local edge operators (for example the Sobel operators in figure 3.1) to provide two gradient images $\partial I/\partial x$ and $\partial I/\partial y$ corresponding to the image gradient in the two image directions $x$, and $y$. The overall image gradient is then calculated as having magnitude $G$ and orientation $\theta$.

$$G = \sqrt{g_x^2 + g_y^2}, \quad \theta = \arctan \frac{g_y}{g_x} \quad (3.1)$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \quad (3.2)$$

Figure 3.1: Sobel edge operators

3.1.2 Harris Edge Detector

A more sophisticated edge detector is the Harris feature detector [HS88]. Obviously the Canny edge detector as mentioned above will provide a response not only to step
3.1. Monochromatic Edge Detection

edges, but also to corners and image noise; the sub-process that links these pixels to form edges would be required to reject these isolated points. The Harris corner detector incorporates as part of its edge or corner location strategy a decision as to whether a pixel actually lies on an edge, or is in fact a corner.

Let \( A, B, \) and \( C \) be as follows:

\[
A = \frac{\partial I}{\partial x} \frac{\partial I}{\partial x} \otimes w \quad (3.3)
\]

\[
B = \frac{\partial I}{\partial x} \frac{\partial I}{\partial y} \otimes w \quad (3.4)
\]

\[
C = \frac{\partial I}{\partial y} \frac{\partial I}{\partial y} \otimes w \quad (3.5)
\]

where \( \frac{\partial I}{\partial x} \) and \( \frac{\partial I}{\partial y} \) denote the image gradients and \( w \) is an arbitrary image window. Let \( \alpha \) and \( \beta \) be the eigenvalues of the \( 2 \times 2 \) matrix \( M \):

\[
M = \begin{bmatrix}
A & C \\
C & B
\end{bmatrix}
\]

\( \alpha \) and \( \beta \) will be proportional to the principal curvatures of the local autocorrelation function. Should both \( \alpha \) and \( \beta \) be small then this indicates a region of little image texture, and no features. Two large eigenvalues indicates the presence of a corner, whereas only one large eigenvalue (and one small) indicates a locally ridge-shaped autocorrelation function, and thus an edge.

To avoid an explicit eigenvalue decomposition of \( M \) at each pixel, the following formulation is used. Let:

\[
Tr(M) = \alpha + \beta = A + B \quad (3.7)
\]

\[
Det(M) = \alpha \beta = AB - C^2 \quad (3.8)
\]

Then let the edge response \( E \) be as follows:

\[
E = Det(M) - kTr(M)^2 \quad (3.9)
\]

\( E \) takes values that are small in areas of low image texture (ie. in areas containing no image features), large and positive in corner regions, and large and negative in edge
3.2 Chromatic Edge Detection

The problem is that in colour processing we are not dealing with single images, nor are we (necessarily) dealing with sets of entirely unconnected images. The problem is one of combining the values of each red, green and blue image to give a meaningful measure of colour gradient.

DiZenzo [DiZ86] suggests that a simple method is to consider colour images as an array of monochromatic images and then either take the gradient as the vector sum of the gradients in each images or as the RMS of the gradient magnitudes; this is equivalent to taking the gradients as the Euclidean distance in colourspace between the colours assumed by adjacent pixels in the x and y directions.
3.2. Chromatic Edge Detection

\[
\frac{\partial \text{RGB}}{\partial x} = \left[ (R_{x,y} - R_{(x-1),y})^2 + (G_{x,y} - G_{(x-1),y})^2 + (B_{x,y} - B_{(x-1),y})^2 \right]^{\frac{1}{2}} \tag{3.10}
\]

\[
\frac{\partial \text{RGB}}{\partial y} = \left[ (R_{x,y} - R_{x,(y-1)})^2 + (G_{x,y} - G_{x,(y-1)})^2 + (B_{x,y} - B_{x,(y-1)})^2 \right]^{\frac{1}{2}} \tag{3.11}
\]

Nevatia [Nev77] suggests that three possible directions exist for chromatic edge detection:

- Use a distance metric in a colour space that models human perception.
- Compute edges separately in each band, allow these to be independent and form some function of them to give the overall edge image.
- Constrain edges to not be independent in each band, and thus utilise the gradients of the three component images concurrently.

Nevatia uses the final method, constraining that the orientations of the edges detected in each band all have the same orientation. By using edge detection in an IHS colourspace he then goes on to demonstrate that most of the colour edges that are detected also exist as luminance edges.

3.2.1 Differential-Geometric Approaches

Machuca and Phillips [MP83] and then DiZenzo [DiZ86] took a differential-geometric approach to the subject of forming the gradient of a multi-image. Taking the image as a function \( f(x, y) = (R(x, y), G(x, y), B(x, y)) \), the vectors \( \mathbf{u} \) and \( \mathbf{v} \) form a basis for the tangent of the image at a given pixel, and are given by

\[
\mathbf{u} = \frac{\partial R}{\partial x} \mathbf{L} + \frac{\partial G}{\partial x} \mathbf{G} + \frac{\partial B}{\partial x} \mathbf{B} \tag{3.12}
\]

\[
\mathbf{v} = \frac{\partial R}{\partial y} \mathbf{L} + \frac{\partial G}{\partial y} \mathbf{G} + \frac{\partial B}{\partial y} \mathbf{B} \tag{3.13}
\]
3.2. Chromatic Edge Detection

where \( \mathbf{r}, \mathbf{g}, \) and \( \mathbf{b} \) are unit vectors in the direction of the RGB axes.

The problem is then one of finding the direction \( \theta \) in which the gradient \( F(\theta) \) is maximum.

\[
F(\theta) = g_{xx} \cos^2 \theta + 2g_{xy} \cos \theta \sin \theta + g_{yy} \sin^2 \theta
\]  

(3.14)

where \( g_{xx}, g_{xy} \) and \( g_{yy} \) are the coefficients of the tensor field \( g(x,y) \). These may be approximated as follows:

\[
\hat{g}_{xx} = \mathbf{u} \cdot \mathbf{u} = \left. \frac{\partial R}{\partial x} \right|_{x}^{} + \left. \frac{\partial G}{\partial x} \right|_{x}^{} + \left. \frac{\partial B}{\partial x} \right|_{x}^{} \tag{3.15}
\]

\[
\hat{g}_{yy} = \mathbf{u} \cdot \mathbf{u} = \left. \frac{\partial R}{\partial y} \right|_{y}^{} + \left. \frac{\partial G}{\partial y} \right|_{y}^{} + \left. \frac{\partial B}{\partial y} \right|_{y}^{} \tag{3.16}
\]

\[
\hat{g}_{xy} = \mathbf{u} \cdot \mathbf{u} = \left. \frac{\partial R}{\partial x} \right|_{y}^{} + \left. \frac{\partial G}{\partial y} \right|_{y}^{} + \left. \frac{\partial B}{\partial y} \right|_{y}^{} \tag{3.17}
\]

The equation for maximal \( \theta \) can be rewritten as,

\[
\theta_1 = \frac{1}{2} \arctan \frac{2g_{xy}}{g_{xx} - g_{yy}}
\]

\[
\theta_2 = \theta_1 \pm \frac{\pi}{2}
\]

These two orthogonal angles correspond to the extremal values of the gradient, one maximum and the other minimum. The values of \( F(\theta) \) are thus compared to decide on which of \( \theta_1, \theta_2 \) corresponds to the maximum. In approximating the gradients, DiZenko uses the digital approximations obtained by fitting a plane \( z = ax + by + c \) to the grey levels in a \( 2 \times 2 \) neighbourhood to give edge operators

\[
\frac{1}{2} \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \quad \frac{1}{2} \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}
\]  

(3.18)

Chapron [Cha92] uses the Canny-Deriche gradient within DiZenko's framework, and Cumani [Cum91] uses surface fitting on a \( 3 \times 3 \) neighbourhood.
3.3 "Watershed" Image Segmentation

Watershed based image segmentation methods\(^1\), whilst not true edge detectors are also image space techniques, and may be used to locate edges within images. In the segmentation technique considered below [SPK97], one starts with an image that is oversegmented such that we consider at the start of the process that an edge lies between every pair of horizontally or vertically adjacent pixels. We then decide to discount some of these edges. This repeats until some termination criterion is met. The remaining edges are the image edges. Each iteration proceeds as follows: Each border is assigned the Euclidean distance between the colours of the image patches it separates. The distances are sorted into increasing order and the regions separated by the border with the least distance are merged.

3.4 Application to Thermochromic Paint

There are two major issues in the interpretation of thermochromic paint. The first is the location of the calibrated colour transition boundaries, the second is the classification of those boundaries. There are two possible directions.

- Locate colour boundaries using edge detection techniques, classify the boundaries through matching of colours on either side using feature based techniques
- Match the observed paint colours to a standard paint response. The boundaries are extracted through matching transition colours

\(^1\)The term watershed relates to a topological analogy. Consider a gradient-magnitude image as a topographic relief, with "holes" at each minimum. Immersing the relief in water will cause lower lying areas (corresponding to areas of low image contrast) to become flooded, higher lying areas remaining dry. As the relief is immersed further, only infinitely tall "dams" placed at the watershed lines remain. The resulting network of dams defines the watershed [SPK97], and in our case the edges
3.5. Experimentation

Through discussions with human interpreters, it appears that the first approach is the one that is adopted in practical situations. A computer based simulation of this method would have two facets. Firstly edge detection would extract the boundaries, and then a subsequent decision based on a clustering of colours would map the located boundaries to the temperature which they indicate. The following experimentation looks at the feasibility of applying colour edge detection to thermochromic paint transition extraction.

3.5 Experimentation

We considered three different edge detection methods. The first used DiZenz’s method for finding the gradient of a multi-image (section 3.2.1). Secondly we took a simple Harris edge detector using Euclidean distance in RGB colourspace to calculate image gradients (equations 3.10, 3.11). Finally we looked at the method based on Shafarenko’s work with watershed segmentation (section 3.3).

We make no attempt at this stage to link edge pixels. We consider this current experimentation to be little more than a feasibility study, we are purely looking for an indication that edge pixels are being located correctly on colour boundaries and not incorrectly within colour bands.

3.5.1 DiZenz’s Multi-Image Gradient

Implementing DiZenz’s method for finding the gradient of a multi-image [DiZ86] yields the result shown in Figure 3.4. The gradients were approximated using Prewitt operators (as in [Cum91] - see figure 3.3). The result shown with the original image indicates the pixels with a gradient magnitude greater than 1. Boundaries appear fairly distinct, but not all have been revealed, and noise is increasing with lowering threshold. If an
3.5. Experimentation

An attempt was made to link the edge pixels shown in this result then it is unlikely that all of the boundaries would be closed. Reducing the value of the threshold would only lead to more false contours being introduced. In the area at the top of the image, it appears that virtually every pixel is a candidate edge pixel. It ought to be noted that the paint the result is shown for has the highest contrast edges (by eye) of all the paints currently studied.

\[
\begin{bmatrix}
-1 & 0 & 1 \\
-1 & 0 & 1 \\
-1 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
-1 & -1 & -1 \\
0 & 0 & 0 \\
1 & 1 & 1 \\
\end{bmatrix}
\]

Figure 3.3: Prewitt edge operators

3.5.2 Harris Edge Detection

Harris edge detection results for a typical RGB image of thermal paint are shown in Figure 3.5. The orthogonal gradient values $\partial I/\partial x$ and $\partial I/\partial y$ were found using the equations for $\partial RGB/\partial x$ and $\partial RGB/\partial y$ from section 3.2. Values for $E$ were found corresponding to various sizes of window $w$ and value $k$. It was thought that the failure of the DiZenzo based edge detector may be due to the small size of the filters that were used. Larger window sizes were used in an attempt to counteract the effect of diffuse colour transitions which often have greater width than the filter size. The results obtained were typical of those obtained using the DiZenzo gradient based edge detector. Again, not all boundaries have been revealed. The result shown is typical of all results attained irrespective of window size or (suitable for edge detection) value $k$. 

3.5. Experimentation

Figure 3.4: DiZenko's gradient of a multi-image applied to paint-tp8

Figure 3.5: Harris edge detection results on thermal paint (tp8)
3.5.3 Watershed Image Segmentation

Figure 3.6 shows results from applying Shafarenko's watershed method to a number of paints. Whilst these results are encouraging (the edges are located roughly horizontally as would be expected given the nature of the preparation of the samples) the termination-threshold value needs to be set very low to bring out all the edges. This leads in some parts of the image to oversegmentation. Some of the weaker edges located are not particularly smooth; this indicates that the method may be reaching its limits with respect to noise at these points.

3.6 Discussion

The results for all the methods considered were disappointing. We believe that these methods are typical of many colour edge detection methods, and as such the results
3.6. Discussion

Figure 3.7: Gradient image $\partial R G B / \partial y$ for paint-tp8

will be generally representative of the edge detection approach. The problem appears to be in all cases that between the stable colour bands the transition regions are diffuse. This is illustrated in figure 3.7. This is an image of $\partial R G B / \partial y$ for tp8. The image was oriented so that all colour edges should be roughly horizontal. About two thirds of the way down the image (from the top) a colour change from creme to pink is observed, however in the gradient image there is no evidence that an edge should be detected even from an optimally oriented gradient image. This is because even though there is a colour change, it is a gradual change rather than a step edge. One shouldn’t expect an edge detector to cope with the situation of gradient of such small magnitude. Multi-resolution strategies may be able to answer the question as to whether an edge lies between colour bands, but without suitable gradient information, any edge placement would be subject to inaccuracy.
3.7 Summary

In this chapter we have explored basic colour edge detection methods based upon edge detection on single colour bands, simple combinations of edges in three colour bands, and methods that approximate the three-dimensional vector gradient of the colour. Whilst human interpretation methods rely heavily on the location of perceptual colour boundaries, the results presented here suggest that the process uses higher level interpretation; there is little in the data to suggest that edge detection alone can provide a solution. Even if one could provide linked edges that formed closed contours then the colours within the regions would still need to be classified to provide the sought after temperature information (this may be non trivial). Whilst at some stage a decision needs to be reached regarding the location of the boundaries, we pass over edge detection and methods based purely in the image space and explore feature based methods. We believe that colour edge detection does not appear to be a useful step in the interpretation of thermochromic paint.
Chapter 4

Modelling the Feature Space

In this chapter we cast the problem from one of locating boundaries between distinct colour changes and then deciding to which class the colours each side of the boundary belong (which is the human based interpretation method) into the dual problem of deciding which colours are either side of each colour boundary and then classifying each pixel of an image of a test piece to one of the colour classes. From edge based spatial methods, we move to feature based methods. Elements of this chapter have appeared in [GKWG96].

4.1 Nearest Neighbour Classification

We start with the assumption that the colours that the paint takes belong to a set of distinct colours. Each of these colours corresponds to the colour that the paint takes when heated to a temperature between two calibrated temperature values, Outside these calibrated temperatures the paint takes a different colour (to that between the calibrated temperature values). At this stage we assume the classes are distinct and the colours homogeneous (apart from the influence of image noise). In this chapter we refer
to transition regions as the areas of the image where the paint colour changes at the calibrated colour change points and to homogeneous regions as the areas between the transition regions where the colour is stable.

Given that we do not know a priori the statistical distribution of the paint colours other than that we expect the colours to form distinct clusters in feature space (that is the colour is assumed at this stage to be constant between colour boundaries) it appears that a solution may lie in nearest neighbour classification based on colour distance from the centroids of the colour clusters.

### 4.1.1 Nearest Neighbour Classifiers

A nearest neighbour classifier ([DK82]) is based on the principle that features that fall together in feature space generally belong to the same class. A set of class examples are chosen and samples classified based on their distance from members of the example set under some metric.

The simplest form of nearest neighbour is the 1-nearest-neighbour classifier (1-NNR). This assigns each observed feature vector to the class that is represented by the single nearest example feature vector under some arbitrary distance measure. In our example:

\[
\hat{w}(C_{RGB}) = j \iff \|C_{RGBC_j}\| = \min_{i=1..n} \|C_{RGBC_i}\| \tag{4.1}
\]

that is, the distance \(\|C_{RGBC_j}\|\) between the observed RGB value and the RGB value of the class example \(j\) implies that the class \(\hat{w}(C_{RGB})\) to which pixel \(C_{RGB}\) should be assigned is class \(j\) if the distance \(\|C_{RGBC_j}\|\) is the least of all distances \(\|C_{RGBC_i}\|\) for all class examples \(i\). Colour difference is quantified by Euclidean distance

\[
\|C_{RGBC_i}\| = \sqrt{(R_{RGB} - R_t)^2 + (G_{RGB} - G_t)^2 + (B_{RGB} - B_t)^2} \tag{4.2}
\]
4.2 Thermal Paint Colour Boundaries

Whilst we assume that the paint colour is roughly constant in the homogeneous regions, the behaviour at the interface between colours is less well defined. In an attempt to model this behaviour, we use an analogy with boiling water. In a similar way that boiling water does not become steam at exactly 100°C, some water molecules change state at a higher temperature, some at a lower temperature, it is our assumption that some paint particles change colour at a lower temperature than the calibrated boundary temperatures and some at a higher temperature. This appears a reasonable analogy. We assume that situations such as the boiling water example may be modelled by an equation adapted from the Fermi-Dirac distribution function of semiconductor physics.

In our particular application, we assume that the Fermi-Dirac distribution function ([Par94]) may be used to describe the probability that the paint takes either of the colours on either side of a transition when heated to a temperature corresponding to part of a colour transition region. The relationships between probability and temperature for the two classes are illustrated in figure 4.1 and are given as follows:

\[
\begin{align*}
P_{\text{classA}}(T) &= \frac{1}{1 + e^{(T - T_c)/k}} \\
P_{\text{classB}}(T) &= 1 - P_{\text{classA}}(T) \\
&= 1 - \frac{1}{1 + e^{(T - T_c)/k}}
\end{align*}
\]

Figure 4.1 shows the relationship between temperature and probability at a transition region. At lower temperature the probability that the pixels belong to class A is equal to one - this corresponds to the edge of the homogeneous colour region. As temperature increases, the probability drops across the colour transition region until the probability is zero - the edge of a second homogeneous colour region. In this binary decision example, the pixels can only belong to one of the two classes A and B, and as such the probability of a pixel belonging to class B is simply \(1 - P_{\text{classA}}(T)\). We may take \(T_c\), the decision boundary between the classes as the \(P_{\text{classA}}(T) = 0.5\) midpoint of the transition from class A to class B.
4.3 Colour Mixing and Boundary Classification

The paint molecules are obviously sufficiently small and the camera resolution sufficiently low that each pixel of the digital image is an image of many coloured paint particles. For each pixel \((x, y)\) the image is of \(k_i(x, y)\) particles of each possible coloured chemical compound \(i\) that exists within the paint. We shall assume at the moment that each separate homogeneous paint colour only contains contributants that are of at most one single colour of particles, although this may not necessarily be the case in real examples.

The number of particles that occupy the image at any pixel is unknown, let us call this value \(N_p\). We shall call the number of particles of class 1 \(N_1\) and the number of particles of class 2 \(N_2\). Therefore:

\[
N_p = N_1 + N_2
\] (4.3)

Furthermore the colour of particles of class 1 is \([R_1, G_1, B_1]^T\), and the colour of particles of class 2 is \([R_2, G_2, B_2]^T\). It therefore follows that the observed colour of the pixel
4.4. Extended Nearest Neighbour Classifier

\[
[R_{\text{Obs}}, G_{\text{Obs}}, B_{\text{Obs}}]^T \text{ is:}
\]

\[
\begin{bmatrix}
R_{\text{Obs}} \\
G_{\text{Obs}} \\
B_{\text{Obs}}
\end{bmatrix}
= \begin{bmatrix}
N_1 \\
G_1 \\
B_1
\end{bmatrix}
+ \begin{bmatrix}
N_2 \\
G_2 \\
B_2
\end{bmatrix}
= \begin{bmatrix}
R_1 \\
G_1 \\
B_1
\end{bmatrix}
+ (N_p - N_1)
\begin{bmatrix}
G_2 \\
B_2
\end{bmatrix}
\]

Thus the colour of each pixel lies on the straight line in RGB colourspace from the basic colour of class 1 to the basic colour from class 2. Under the assumption that colour is homogeneous except at the diffuse colour boundaries, the points on the line between the endpoints constitute the entire range of observable colours across the boundary, and each endpoint of the line constitutes the colour of a homogeneous region.

4.4 Extended Nearest Neighbour Classifier

Figure 4.2 highlights problems a simple nearest neighbour classifier would have at transition regions. From the above analysis, the transitions are modelled as straight lines, or as vectors from one homogeneous colour class to another. In figure 4.2 it is clear that \(C_{R,G,B}\) lies closest to \(C_7\), but it is more likely to lie on the transition between \(C_3\) and \(C_4\).

\[\begin{array}{c}
\times \quad C_{R,G,B} \quad \rightarrow \quad C_4 \\
\bullet \quad C_3 \\
\bullet \quad C_7
\end{array}\]

Figure 4.2: Problems with Nearest Neighbour method

Initially the transitions were modelled using vectors (figure 4.3) from the class centre of each homogeneous colour band to the centre of the adjacent homogeneous colour bands.
The direction of the vectors is determined by the relative temperatures of the two colour bands, the vector is directed from the class centroid corresponding to the cooler temperature to the centroid corresponding to the hotter temperature. By modelling the transition as a vector we imply extra members of the 'nearest neighbour' design set along the transition corresponding to the colour path of the transition through RGB space. A practical application would not map to the nearest class centre as in a traditional nearest neighbour classifier, but to the nearest transition between class centres and then to the most probable class centre, depending to which side of the midpoint of the transition is made. This is illustrated as follows. Figure 4.4 shows the possible cases of mappings from position in feature space to transition. For points exterior to an obtuse angle between transitions there is a single mapping to the endpoint (corresponding to a homogeneous region). For feature points that map to positions between the endpoints there are often many possible mappings, although we may discount those mappings which are comparatively distant. We draw our extended nearest neighbour classifier design set from the colours of the homogeneous regions. Following this we then calculate the straight line segments that model the transitions between each pair of colours for adjacent temperature bands (figure 4.4). Each line segment (Figure 4.6) can be thought of as infinite in extent. Looking at figure 4.6, an RGB pixel value projects onto each line segment by the following equations.

\[
\cos \alpha = \frac{\vec{AP} \cdot \vec{AB}}{|\vec{AP}| |\vec{AB}|}
\]  

(4.5)
4.4. Extended Nearest Neighbour Classifier

Figure 4.4: Colour distribution versus temperature

Figure 4.5: Straight line transitions between homogeneous regions
4.4. Extended Nearest Neighbour Classifier

Figure 4.6: Projecting points onto a line segment

The perpendicular distance from the pixel to the line segment is given by

\[ d_p = |\overrightarrow{AP}| \sin \alpha \]  \hfill (4.6)

and the distance along the line segment to the normal of \( \overrightarrow{AB} \) which passes through \( P \) by

\[ d_m = \frac{|\overrightarrow{AP}| \cos \alpha}{|\overrightarrow{AB}|} \]  \hfill (4.7)

Thus if the pixel projects to a point on the line segment between its endpoints \( d_m \) will lie in the interval \([0..1]\).

In this manner the pixel is projected onto each possible transition. Projections with \( d_m < 0 \) are discarded unless it is a projection onto the first line segment, and projections with \( d_m > 1 \) are discarded unless it is a projection onto the last line segment. Should a feature point lie on the outside of two transitions forming an obtuse angle, it is projected to that corner \((d_m = 0/1)\) and \( d_p \) is simply the Euclidean distance from the point to that corner. After mapping feature points to each transition, the closest mapping is taken, and the pixel assigned to the class at the closest endpoint.
4.5. Experimentation

With practical consideration in mind, we endeavoured to counteract any effects of sensor noise or any slight variation of colour within the homogeneous regions. To obtain the members of our nearest neighbour design set we took a number of pixel values within each homogeneous region from an image of a thermal paint bowtie and calculated the mean. Results for a simple nearest neighbour classifier are shown in figures 4.7 and 4.8. After estimating the mean colour of the homogenous regions each pixel in the thermal paint image was classified to the nearest homogeneous colour class in RGB space using equation 4.1, and distance measure from equation 4.2. The results show the mappings of pixels to different classes which are represented by shades of grey. We would expect the classes to be distinct and for the result to show a (discretised) lightening from black to white from the bottom of the resulting image shown to the top.

Figures 4.9 and 4.10 show the results obtained using the extended nearest neighbour classifier. We took the mean colours of each homogeneous region from the previous experimentation with the standard nearest neighbour classifier and calculated the vector
transitions between all "adjacent in temperature" colour classes. This time we mapped each pixel to the nearest transition. Using equations 4.6 and 4.7, for each pixel in the image we calculated $d_p$ and $d_m$ for the mapping to each transition. We then chose the nearest transition using the following decision rule.

\[
i(C_{RGB}) = t(C_{j-1}C_j) \iff d_p(C_{j-1}C_j) = \min_{i=1\ldots n} d_p(C_{i-1}C_i)
\]  

(4.8)

That is, the RGB pixel is assigned to the transition $t(C_{j-1}C_j)$ if for that transition $d_p(C_{j-1}C_j)$ is the least for all $j$. After finding the nearest transition we considered the value of $d_m$ for that transition. If $d_m(C_{j-1}C_j) < 0.5$ then the pixel was assigned to class $j - 1$ otherwise it was assigned to class $j$. Again the results show the mapping from RGB to class through different shades of grey. Once more we would expect a lightening from black to white from the bottom of the resulting images to the top.

4.6 Discussion

Figures 4.7 and 4.8 were obtained using a standard nearest neighbour classifier. These highlight the problems that a simple classification scheme has at a transition region.
Figure 4.9: Extended Nearest Neighbour Result for paint-tp5

Figure 4.10: Extended Nearest Neighbour Result for paint-tp8
Given that the temperature on the bowtie test-piece of which the image was taken increased monotonically from the bottom (coolest) of the image to the top we would expect that the greyscale progression would be smooth from black to white, and that all pixels within homogeneous regions should be assigned to a single class. The results show that this has not been the case, homogeneous regions are divided into multiple classes and "hot" pixels have been assigned to "cool" classes and \textit{vice versa}. The model of the transition regions as vector transitions appears to work more successfully. The classes are much more distinct (although errors remain), the greyscale progression is on the whole correct and the boundaries are located where perceptively they appear on the image.

4.7 Summary

In this chapter we have laid the foundations of a feature space model of the colour formation of thermochromic paint. We have presented the problem as one of colour matching to one of the set of colours of the homogeneous regions. Hypothesising about the formation of transition regions we have formed a model of the feature space in these image areas and extended the nearest neighbour classifier to incorporate this model. The results are pleasing, misclassification has been reduced and class boundaries occur at perceived colour boundaries on the thermal paint image.

Whilst we have an apparently successful model of the transition regions, problem classification areas still remain. These are due to a combination of the influence of noise and transitions that cross each other in feature space. Although the model is good, it needs to be extended to cope with these artifacts. This will be discussed in the next chapter.
Chapter 5

Modelling the Colour Formation

The previous chapter laid the foundations of a feature space model of the colours that occur in thermochromic paints. This model was formed from a description of the problem as being one of classifying homogeneous colour regions and diffuse transitions. It is clear that there is an underlying chemical process that drives the colour changes. This chapter attempts to model the colour formation as being a continuous function of temperature. This is an extension of the model from the previous chapter, but with a continuous mapping, rather than discrete, and with a higher order fit across the transition regions. We include derivations of the temperature-colour model and equations and consider the role of noise and uncertainty in the image measurements. This work has been presented in [CSGW97].

5.1 Re-statement of aims

To find the mapping from colour to temperature we aim to find the inverse of the mapping from temperature to colour. As thermal paint is heated, the mapping from temperature to colour occurs as a chemical process. On an image of thermal paint this
becomes

\[
t_{x,y} \rightarrow \begin{bmatrix} R_{x,y} \\ G_{x,y} \\ B_{x,y} \end{bmatrix}
\]  

(5.1)

where \( t_{x,y} \) is the temperature to which the object was heated at the position \((x, y)\) on the image and \( R_{x,y}, G_{x,y} \) and \( B_{x,y} \) are the tristimulus values at position \((x, y)\) on the image.

Our task is to find the mapping:

\[
\begin{bmatrix} R_{x,y} \\ G_{x,y} \\ B_{x,y} \end{bmatrix} \rightarrow t_{x,y}
\]  

(5.2)

5.2 Thermochromic Curves

Observation of the RGB histogram for each object shows that the RGB values of the pixels form a curve (see figure 5.1 for example). We assume that this curve is continuous. Taking a painted (and heated) "bow-tie" as before, we sample RGB values from the image in such a way that we take pixels corresponding to an axis of increasing temperature. It can be demonstrated that the position on the curve for each pixel relates directly to temperature and that the mapping is apparently continuous. This curve occupies 3-dimensions (the RGB values), and thus can be represented as the function \( f(R, G, B) \). It is also parametric in terms of temperature and thus can be written \( g(t) \). The curve is not unbounded. For each particular paint under analysis there is a starting colour, the colour that the paint takes before it is heated, and an end colour, the colour that the paint takes before it turns to glaze. We may find the mapping
5.2. Thermochromic Curves

Figure 5.1: Example temperature curve in RGB space with orthogonal views
where $r_{x,y}$ is a measure of temperature based on the position along the temperature axis of the image from which pixels are drawn. $r_{x,y}$ relates in some manner to true temperature, although the relationship is not necessarily linear.

5.3 Linear Transitions and the Thermochromic Curve

The Extended Nearest Neighbour approach of the previous chapter modelled the transition regions with vectors between the colours corresponding to the homogeneous regions. This ordering of the vectors was decided by the order of the homogeneous region colours at the endpoints within the thermochromic colour progression. Under this formulation the set of vector transitions forms a piecewise linear approximation of the thermochromic curve as illustrated in figure 5.2. As mentioned previously a piecewise linear approximation still leads to errors in classification. As part of an improved solution we look at a higher order of fit to the thermochromic curve.
5.4 Statistically Fitting the Thermochromic Curve

Temperature $t$ maps to colour $c = \{R, G, B\}^T$ as:

$$c(t) : \mathbb{R} \rightarrow \mathbb{R}^3$$  \hspace{1cm} (5.4)

The function $c(t)$ is defined by the unknown chemical composition and chemical response of the paint and is itself unknown. We may, however, estimate a discrete version of the function from “bow-tie” pieces by sampling an image of thermochromic paint along a temperature axis. Let us call this discretised function $r(t)$ which is constructed for $K$ evenly spaced “temperature” values.

At each point on the temperature axis, the colour measurements on a thermochromic image are corrupted by sensor noise and the uncertainty of the paint state (as before we assume that at any given temperature the particular colour that occurs is to some degree probabilistic). To counter the effects of noise we take a number of measurements $\hat{r}$ of similar temperature from a bow-tie. We assume that any lateral variation of colour due to the shape of the bowtie is minimal. At each discrete temperature $t_k = k\Delta, k = 1...K$ we have $N_k$ measurements of colour, $\hat{r}_i(t_k), i = 1...N_k$. From these samples we may estimate the statistical properties of the discretised curve $\hat{r}_i(t_k)$.

$$\bar{r}(t_k) = \frac{1}{N_k} \sum_{i=1}^{N_k} \hat{r}_i(t_k)$$ \hspace{1cm} (5.5)

$$W(t_k) = \frac{1}{N_k} (\hat{r}_i(t_k) - \bar{r}(t_k))(\hat{r}_i(t_k) - \bar{r}(t_k))^T$$ \hspace{1cm} (5.6)

$\bar{r}(t_k)$ is the temperature dependent colour mean and $W(t_k)$ the covariance.

The above representations are discrete, that is they are valid at the discrete temperature values corresponding to our sample points. We desire, however, a continuous function between colour and temperature. To achieve this we fit a B-spline to our discrete version of the the thermochromic curve.
5.5 B-spline approximated thermochromic curves

We choose B-splines for our curve approximation for one main reason. As opposed to most other curve-fitting techniques, such as polynomial regression or Bezier curves, B-splines are fitted locally to data. For most other techniques a small change in the positioning of a control or curve defining point is propagated throughout the curve. With the noise in the data set this would cause any outliers to have influence over the entire curve fit. To obtain a close fit using a polynomial expression would require a polynomial of very high order. This would be highly unstable if presented with noisy data. Conversely the B-spline approximates the curve using a set of blending functions that have only local influence. In our experiments we have used a standard quadratic B-spline. This curve is influenced locally by the positions of just 3 control points. The quadratic B-spline curve is continuous up to its first derivative.

The standard form for our B-spline curves are as follows:

\[ r(t) = \sum_{i=0}^{P} \tilde{r}_i \phi_i(t) \]  
\[ W(t) = \sum_{i=0}^{P} W_i \phi_i(t) \]

The \( \phi_i \) are the quadratic B-spline blending functions and are defined by:

\[ N_{i,1}(t) = \begin{cases} 1 & \text{if } t_i \leq t < t_{i+1} \\ 0 & \text{otherwise} \end{cases} \]

\[ N_{i,k}(t) = \frac{(t - t_i)N_{i,k-1}(t)}{t_{i+k-1} - t_i} + \frac{(t_{i+k} - t)N_{i+1,k-1}(t)}{t_{i+k} - t_{i+1}} \]

In this notation we take \( P = 2 \) and \( \phi_i(t) = N_{i,3}(t) \). The \( t_i \) are knot values that relate \( 1 \) Incidentally, the piecewise linear curve formed by the model in the previous chapter may be provided...
the temperature parameter \( t \) to the control points. If \( i < k \) then \( t_i = 0 \), if \( k \leq i \leq n \) then \( t_i = i - k + 1 \) otherwise \( i > n \) and \( t_i = n - k + 2 \) with \( i \) in the range \( 0 \leq i \leq n + k \) and \( t \) in the range \( 0 \leq t \leq n - k + 2 \).

Whilst not necessarily providing the closest possible fit we use our set of evenly spaced colour samples from the thermochromic curve. The B-spline curve will pass through the mid-point of the straight line between any two adjacent control points. With increasing number of control points this will give us a close fit, with fewer control points it may be more desirable to perform some sort of minimisation based on the positioning of our control points though this seems unnecessary as we are able to design the curve with many sample points.

We fit B-spline curves to our three thermochromic functions of temperature relating to the red, green and blue components of the colour.

5.6 Obtaining the Maximum Likelihood Estimate

Having obtained an approximation of the thermochromic curve, we seek the probabilistic mapping from colour to temperature. Due to the thermochromic curve looping and passing close to itself, and due to the influence of noise, we cannot simply calculate the inverse function \( r^{-1}(t) \), there are possibly multiple solutions to the inverse. Instead we take the maximum likelihood estimate of temperature given colour, that is we decide probabilistically the most likely interpretation of the colour.

We assume that a Gaussian noise model is plausible for the conditional probability of measuring a particular colour \( r \) given \( t \). by \( \phi_r(t) = N_{r,2}(t) \)
5.6. Obtaining the Maximum Likelihood Estimate

\[ p(r|t) = \frac{1}{(2\pi)^{n/2} \sqrt{W(t)}} e^{-\frac{(r-r(t))^\top W(t)^{-1}(r-r(t))}{2}} \]  

We apply Bayes rule to obtain

\[ p(t|r) \propto e^{-\frac{(r-r(t))^\top W(t)^{-1}(r-r(t))}{2}} = e^{-E(t)/2} \]

The term \( E(t) \) in the above expression is the Mahalanobis distance \( E(t) = (r - r(t))^\top W(t)^{-1}(r - r(t)) \). In this form, we are dependent on the unknown thermochromic function \( r(t) \). Now, with the added information about the covariance of the colour data with respect to temperature we may take the maximum likelihood estimate of temperature, \( \hat{t} \) as the value of \( t \) for which the Mahalanobis distance from the colour to our curve approximation is least. When searching for minima of the Mahalanobis distance we may suffer from there being local yet non-global minima. These obviously occur at points on the curve where it loops close to itself. The further a pixel colour is away from the temperature estimate \( \hat{t} \), the less likely it is to be correct, even though it may be the most likely projection. For this reason we threshold the Mahalanobis distance as part of the classification process. Pixels for which the distance of projection is comparatively large compared to the spread from the curve are treated as outliers. A typical value of Mahalanobis distance threshold in this case is of the order of 10. Figure 5.3, shows two possible projections within the threshold. In this case the uncertainty of the mapping to the closest point may be modelled in terms of the relative distances of the two (or more projections).

The uncertainty of \( \hat{t} \) in the direction of the curve may be expressed as follows. We assume that the colour measurement \( r \) is close to the thermochromic curve relative to the radius of curvature. Under this assumption we may approximate the curve locally by a straight line. We linearise about the maximum likelihood estimate \( \hat{t} \).
5.6. Obtaining the Maximum Likelihood Estimate

\[ r(t) \approx A + B(t - \hat{t}) \]  

(5.13)

where

\[ A = r(\hat{t}) \]  

(5.14)

\[ B = \left. \frac{dr(t)}{dt} \right|_{\hat{t}} \]  

(5.15)

We also assume that the variance \( W(t) \) may be approximated locally by the constant value \( W(\hat{t}) \). The Mahalanobis distance thus becomes

\[ E(t) = (r - A - B(t - \hat{t}))^T W^{-1}(\hat{t})(r - A - B(t - \hat{t})) \]  

(5.16)

This is quadratic in \( t \) and, ignoring constants, may thus be written \( E(t) = (t - t')^2/\sigma_t^2 \).

Absorbing the constants into the normalisation we obtain:

\[ p(t|\tau) = \frac{1}{\sqrt{2\pi}\sigma_t} e^{-(t-t')^2/2\sigma_t^2} \]  

(5.17)

\( t' \) may be obtained from

\[ (t - t') = \frac{\sigma_t^2}{\sigma_T^2} \]  

(5.18)
5.7 On the role of the choice of colourspace

\[ t' - \hat{t} = \frac{B^T W^{-1} (r - u)}{B^T W^{-1} B} \]  

(5.18)

We expect that \( t' - \hat{t} \approx 0 \). If \( t' - \hat{t} \) becomes large relative to the variance \( \sigma \) then the linear approximation has broken down. Variance is obtained from:

\[ \frac{1}{\sigma^2} = B^T W^{-1} B \]  

(5.19)

5.7 On the role of the choice of colourspace

It seems necessary to make comment on our choice of colourspace at this point. As mentioned previously there are many different colourspace each with their own properties. We may have expected to model the feature space of the thermal paints in some perceptually uniform colourspace so that the matching would follow more closely the matching of human operators. Having demonstrated that the thermal paint's colour response forms a curve in the RGB colourspace, we feel that it is quite reasonable to operate entirely in the RGB colourspace. Not only does this provide ease of matching, we guarantee that no undesirable warping or breaking of the curve (for example due to discontinuities in the IHS colourspace etc.). We are no longer matching colours as such, we are matching feature points to a curve and assume that our model of uncertainty counters any direct advantage of other colourspace.

5.8 Experimentation

We took a number of paints and performed our B-spline fitting procedure. Figure 5.4 shows a typical result. The image on the left is of a thermochromic curve for the paint tp8, on the right is the curve approximation. The fit appears good to the eye.
The quality of fit of the B-spline obviously increases with the number of design points. Figures 5.5 and 5.6 show the quality of fit to a typical thermochromic curve using differing numbers of design points. In figure 5.5 the black curve is obtained using 10 design points, the red curve 30 design points and the green curve using 60 design points. The original curve is shown in blue. Adding extra control points from 30 to 60 has not yielded a closer fit. The true curve is quite noisy it would require a substantially higher number of design points to model accurately, although it would be undesirable to fit to the noise in this way. Figure 5.6 shows the sum of square errors between the fitted curve and the original thermochromic curve. It can be seen that 30 design points will give a close fit.

Figures 5.7 and 5.8 highlight the problems of ambiguity. Figure 5.7 shows uncertainty $\sigma_t$ as a function of the temperature $t$ for two thermochromic paints. In homogeneous colour regions the rate of change of colour with respect to temperature is very low, the position of interpretation along the curve moves slowly. This phenomenon corresponds to peaks in the graphs of uncertainty. Figure 5.8 demonstrates problems at the curve self intersections. For two thermochromic paints we took the square root of the Mahalanobis distance between the observed data $\bar{r}(t_2)$ and the fitted curve $r(t_1)$. Black regions indicate Mahalanobis distance within the rejection threshold, white regions correspond to Mahalanobis distances greater than our outlier rejection threshold. In the ideal case we would expect these graphs to form a single line, however the spread at several points along the curve show that we are suffering from curve self-intersections both real and induced by noise.

5.9 Summary

In this chapter we have modelled the mapping between thermal paint colour and temperature as a curve in RGB colourspace. This curve is different for each paint. The thermochromic curve is estimated using quadratic B-splines fitted to the mean values of
Figure 5.4: A thermochromic curve and its B-spline approximation

the red, green and blue components as sampled from the true thermochromic curve and we have also fitted the covariance of the data. Highlighting the causes of interpretation ambiguity we have modelled the uncertainty of the classification due to the projection distance. We have also modelled the uncertainty of the classification in the direction of the curve and have shown experimentally that the B-spline gives a good fit to our data set given a modest number of design points.
Figure 5.5: Graphs showing the quality of fit in each of the colour bands
5.9. Summary

Figure 5.6: Square error of fit vs number of control points

Figure 5.7: Uncertainty $\sigma_t$ as a function of $t$ for two different paints [CSGW97]
5.9. Summary

Figure 5.8: Graphs highlighting the problems of ambiguity [CSGW97]
Chapter 6

Countering Ambiguity

In the previous chapter we presented a feature space model of the thermochromic process. Presenting a method for estimating the thermochromic curve by fitting a B-spline to statistically modelled colour data we highlighted the expected result that ambiguity may arise in interpretation due to the combined problems of sensor noise and the thermochromic curve intersecting itself.

This chapter looks at overcoming the problem of ambiguity. The colour order of the paints is known. We overcome the ambiguity problem using the assumption that the colour change is smooth, that is, in ambiguous image regions we guide the classification by disallowing rapid changes in the mapped temperature, disqualifying jumps between distant temperatures at the self-intersections of the curve. The image regions that may be classified un-ambiguously provide the context for the interpretation of the ambiguous regions.

We derive a cost function for the interpretation based on our statistical estimation procedure from the previous chapter and on the requirement that our interpretation be continuous. We then utilise this cost function in a Simulated Annealing framework and show that the method is successful in overcoming ambiguities. This work was
6.1 Methodology

As before, we assume an unknown relationship between the temperature $t$, which we need to recover, and the pixel color $r(t) = \{R, G, B\}^T$.

In a similar way to the last chapter we take $k$ colour measurements at evenly spaced temperatures, calculate the mean $\bar{r}(t_k)$ and covariance $W(t_k)$ and interpolate the distribution with a least squares B-spline curve.

$$r(t) = \sum_{n=1}^{P} \phi_n(t)r_n, \quad W(t) = \sum_{n=1}^{P} \phi_n(t)W_n$$  \hspace{1cm} (6.1)

where $\phi_n(t)$ are the standard quadratic B-spline basis functions and $r_n$ are our $P$ control points and $W_n$ our covariance. $r(t)$ is our estimate of $c(t)$.

We assume Gaussian sensor noise and model the conditional probability of interpreting a particular colour $r$ as temperature $t$ by

$$p(t|r) \propto e^{-\frac{(r-r(t))^TW(t)^{-1}(r-r(t))}{2}}$$ \hspace{1cm} (6.2)

$$\propto e^{-E(t)/2}$$ \hspace{1cm} (6.3)

where $E(t)$ is the Mahalanobis distance of pixel $r$ from $r(t)$.

Ambiguities in interpretation typically occur in regions of the image where the temperature curve crosses itself, or where the curve passes close to itself in the presence of noise. This leads to local minima in the Mahalanobis distance. In applications
we may assume that modest spatial variations of temperature occur, the context of a pixel can resolve such ambiguities. Specifically we need to discount sharp jumps in the temperature map which correspond to positions of the image where the classification is alternating between the correct solution and an incorrect solution in adjacent pixels. We propose the following approach for the solution of this problem.

Let \( p(t|r) \) be the probability that we assign temperature \( t \) to position \( (x,y) \) given the RGB vector \( r \) at \( (x,y) \). We need to maximise this probability over the whole image. Assuming we have \( N \) colour pixels \( \{r_{1..N}\} \) we need to infer a set of temperatures \( \{t_{1..N}\}_{MAP} \) such that

\[
\{t_{1..N}\}_{MAP} = \max_{t_{1..N}} p(t_{1..N}|r_{1..N})
\]

\[
= \max_{t_{1..N}} p(r_{1..N}|t_{1..N})p(t_{1..N})
\]

A simple model of temperature variation is assumed where \( p(t_{1..N}) \) factorises.

\[
p(r_{1..N}|t_{1..N})p(t_{1..N}) = \prod_{i}^{N} p(r_{i}|t_{i}) \left( \prod_{i\neq j \in S} p(t_{i}, t_{j}) \right)^{1/2}
\]

where \( p(t_{i}, t_{j}) \) is the probability that the two temperatures \( t_{i} \) and \( t_{j} \) will occur jointly at positions \( i \) and \( j \) within the image. \( S \) is the set of all pixels within the image. We assume an 4 neighbourhood such that for \( i, j \) not neighbouring each other \( p(t_{i}, t_{j}) = 0 \) and for \( i = j \) \( p(t_{i}, t_{j}) = 1 \). In all cases of \( i \) neighbouring \( j \), \( p(t_{i}, t_{j}) \) the temperature difference between \( t_{i} \) and \( t_{j} \) reflects the smoothness of the temperature profile. The square-root term in the \( p(t_{1..N}) \) factorisation occurs as a consequence of considering each \( i, j \) pairing twice.

Taking a MAP-MRF (Maximum A-Posteriori-Markov Random Field) approach [Li95] we classify by minimising the posterior energy
6.1. Methodology

\[ E(\{t_1..t_N\}_{MAP}) = \min_{t_1..t_N} E(t_1..t_N) \]  
(6.7)

\[ = \min_{t_1..t_N} [-\sum_i \ln p(r_i|t_i) - \frac{1}{2} \sum_{ij \in S} \ln p(t_i, t_j)] \]  
(6.8)

The expression within the brackets is of the form of the sum of a measure of the energy due to the pixel distance from \( r(t) \) and the energy due to a lack of smoothness within the image,

\[ E(t_1..t_N) = \sum_i E_{data}(t_i) + \sum_{i,j} E_{smooth}(t_i, t_j) \]  
(6.9)

We take \( E_{data} \) (the energy imparted by the classification of a particular colour to a particular point on the thermochromic curve) as the Mahalanobis distance between the position on the curve of each \( t \) from our set of discrete temperatures and the RGB vector,

\[ E_{data}(t_i) = \frac{1}{2} (r_i - r(t_i))^TW^{-1}(r_i - r(t_i)) \]  
(6.10)

and to penalise a large variation between \( t_i \) and \( t_j \) where \( i, j \) are neighbours we take \( E_{smooth} \) (the energy imparted by classifying two particular temperature values to neighbouring pixel positions) as simply the squared magnitude of the temperature difference.

\[ E_{smooth}(t_i, t_j) = (t_i - t_j)^2 \]  
(6.11)

The \( E_{data} \) term has the effect of biasing the classification towards following the MLE, whereas the \( E_{smooth} \) term imposes smoothness in the ambiguous areas of the image. To allow the smoothness term to have more influence in the presence of noisy data, or less
6.2 Simulated Annealing

Influence in the presence of un-ambiguous data we introduce the following formulation of energy with the manually selected parameter $q$.

$$E(t_1..t_N) = \sum_i E_{data}(t_i) + q \sum_{i,j} E_{smooth}(t_i, t_j)$$  \hspace{1cm} (6.12)

The choice of the value of $q$ should be made experimentally. As $q \to 0$, $E$ is dominated by the $E_{data}$ term, and the classification approximates the MLE. As $q \to \infty$, $E$ is dominated by the $E_{smooth}$ term. In this case, the classification would approximate a straight line of gradient 0. Each pixel across the entire image would take the same temperature value.

We employ a Simulated Annealing [KGV83],[GG84] scheme to minimise the total energy $E(t_1..t_N)$ within the image.

6.2 Simulated Annealing

Simulated annealing is an extension of a Monte Carlo method for optimising some quantity, in our case, minimizing the posterior energy of our classification. The concept is based on an analogy with which liquids freeze or metals crystalise in the process of annealing. The start of the simulated annealing process is analogous to “melting” the system. At the initially high temperature, the state of our system is free and takes a random classification. This disordered system is then “cooled” and becomes more ordered, finally reaching a “frozen” state when the annealing temperature reaches $T = 0$.

The original proposal of Simulated Annealing by Metropolis et al [Met53] is based around an energy $E$ and temperature $T$. At each iteration, the current state is perturbed by a random amount based on the temperature $T$ and the change in energy
is calculated. A drop in energy is accepted unconditionally. An increase in energy is accepted with probability $e^{\Delta E/T}$. This process is repeated several times for each annealing temperature $T$ to ensure that any statistically extreme perturbation does not overly compromise the current state of the solution. The annealing temperature is then lowered, and the entire process repeated until the final state is achieved at $T = 0$.

6.2.1 Implementation

Our optimisation strategy is as follows. Firstly an initial random classification is chosen. At each pass for each temperature value we calculate the combined energies of $E_{data}$ and $E_{smooth}$ required to classify any pixel to take each particular temperature value. For each pixel, we may then convert these energies to a discrete probability density function which relates the probability that we assign that pixel to temperature $t$ with the classification of this iteration. For this we use using the following distribution.

$$p(t_i|r) = \frac{1}{Z} e^{-(E_{data}(t_i) + \sum_j E_{smooth}(t_i,t_j))/T}$$

(6.13)

with $j$ ranging over an 8-neighbourhood. At each iteration, a new classification for each pixel is chosen probabilistically using this set of distributions. This is achieved at each pixel by comparing the value of a random variable to the discrete integrals of these distributions with respect to $t$. At ambiguous regions the $E_{data}$ term will have two or more local maxima (with respect to temperature), the $E_{smooth}$ term serves to suppress the peaks corresponding to classifications that would promote discontinuity. Our strategy combines the random perturbation of the data and the acceptance/rejection of an energy change (increase or decrease) into one step. In summary:
6.2. Simulated Annealing

Choose an initial random classification

Set annealing parameter $T = T_{\text{max}}$

Loop over trials {

  Loop over image {

    Calculate probability density function $p(t|r)$ based on $E_{\text{data}}$ and $E_{\text{smooth}}$

    Calculate the discrete integrals of $p(t|r)$ w.r.t $t$ between 0 and $t$ for all $t$

    Choose $t_{\text{new}}(x,y)$ by comparing a random variable to these integrals

  } 

  $T = k \times T$ - where multiplication by $k < 1$ relates a cooling the annealing parameter

} until $T = T_{\text{min}}$ or no reclassifications

To avoid entrapment in local, rather than global, minima of the energy function we must ensure that the initial temperature of the system is sufficiently high, we must also ensure that the cooling rate is slow. This appears simple enough to achieve, but at the expense of processing time. However, major speed improvements may be made by constraining the range of allowable temperature values in image regions for which there is no ambiguity. We still, however, calculate the energy function over the whole image.
6.3 Experimental Results

Experiments were performed initially on a set of synthetic images which were intended to represent the range of image space and feature space artifacts encountered in real thermal paint images. These may lead to misclassifications when using current approaches and include crossings of the curve and positions where the curve passes close to itself at the image borders, the latter leading to a lack of support when attempting to iterate towards the correct solution. For this we chose a lemniscate shape as illustrated in figures 6.1 and 6.2.

The synthetic images were produced with known temperature gradients, and the resulting images corrupted by Gaussian noise. An arbitrary training subset of an image was then used to fit our temperature model to the mean (temperature curve) and covariance (noise) of the data. After an initial random classification, simulated annealing was employed to find the global minimum of $E$ across the image. Encountered problems
Figure 6.2: Histogram of Synthetic Paint Image
6.3. Experimental Results

of premature convergence were overcome by assuming that as the data itself imparts smoothness to the result, we may choose a lower weighting for the $E_{\text{smooth}}$ variance than we may initially have chosen given the level of smoothness we require. That is, although we require smoothness in the result, the data is itself smooth. The smoothness term of the cost function relates to disqualifying jumps in the recovered temperature parameter at points where the curve self-intersects. In general we are not penalised by this provided we are mindful of the parametric distance between ambiguous points of our temperature model.

Figure 6.5 shows a temperature gradient obtained when using the MLE as described in previous chapters without using contextual information. It is clear that a successful classification in problem areas is not possible without contextual information. Figure 6.6 shows the result obtained using our new framework, indicating an improvement over the result of figure 6.5 by achieving our twin goals of smoothness across the recovered temperature map, and a monotonic relationship between the true and recovered temperatures. The fact that the method has recovered a fair approximation of the linear temperature gradient that we provided when producing the images suggests that it is working well. Figure 6.8 shows a typical result obtained using our approach on real data. This result was obtained for paint tp6 as illustrated in figures 6.3 and 6.4. As can be seen there are many loops and intersections in the thermochromic curve. We would expect this image to be a stern test of the method. It should be noted that the ground truth of the classification is linear whereas from figure 6.7 the MLE appears non linear. On closer inspection however, the general trend is in fact linear, non-linearities are due to ambiguity. In these areas we seek a smooth gradient consistent with the unambiguous areas which we achieve though the use of a heavier weighting of the smoothness term $q$ of equation 6.12. It should be noted though that the classification in these areas is perhaps quite arbitrary. We need to ensure that the $E_{\text{data}}$ weighting is strong enough that we are not purely fitting to a smooth transition.

In figure 6.8 we have again recovered a smooth result with a monotonic relationship
with temperature. Whilst the result appears linear, the amount of variation from linearity (appearing as noise) demonstrates that our smoothness term $q$ is unlikely to be over-weighted (the noise indicates we are still fitting to the noisy $E_{data}$), we haven’t discounted temperature gradients between adjacent temperature values that are significantly higher than the true gradient. The relationship between the recovered parameter and the true temperature may be obtained as part of a calibration procedure during the design of the paint.

As mentioned previously the relative weighing $q$ of $E_{data}$ and $E_{smooth}$ in the expression of total energy $E$ operates as follows. If $E = E_{data}$ then we have classification biased towards the MLE whereas if $E = E_{smooth}$ the method finds the smoothest classification. Without the influence of $E_{data}$ we would expect it to choose all temperatures as having the same value ($E_{smooth} = 0$). If we have constrained our classification in unambiguous areas then the behaviour given an over weighting of $E_{smooth}$ is less defined. We hypothesise that depending on the weighting and the data we may either see linearity of classification in ambiguous regions, or we may see regions dominated by temperature gradients lower than the true gradient which are separated by sharp jumps. These cases are illustrated in figure 6.9.

Given that it is possible that an over weighting of the smoothness term may produce linearity, figure 6.10 shows the method applied to data with a non-linear temperature gradient. For this experiment we took a largely unambiguous section of thermochromic paint data and fitted it to a curved temperature profile. To further test the method we synthesised an ambiguous region roughly centered at a temperature of around 75 as shown in the figure. We also have an outlying value at a temperature of approximately 150. The black curve shows the result obtained using our method, the red curve shows the curved temperature profile and the green curve the classification by discarding smoothness and calculating energy from the $E_{data}$ term alone. Our method has fitted the true temperature profile well, and has not compromised the non-linearity.
6.4 Conclusion

In this chapter we have presented an approach for overcoming the ambiguity in the interpretation of thermal paint images. After fitting a temperature model to a training set taken from the image we employ simulated annealing utilising an energy function based on a statistical description of the thermal paint data. We have tested the method on images generated that model the specific feature space artifacts that cause ambiguity in interpretation and on images of real paint data. Results demonstrate that this energy measure enables us to successfully interpret the ambiguous regions within the image whilst recovering a good approximation of the known temperature gradient.

Figure 6.3: Paint Colour Response of tp6
Figure 6.4: Colour Histogram of the Colour Response of tp6

Figure 6.5: Temperature obtained from MLE on synthetic image
6.4. Conclusion

Figure 6.6: MAP-MRF recovered temperature for synthetic image

Figure 6.7: Temperature obtained from MLE for tp6
6.4. Conclusion

Figure 6.8: MAP-MRF recovered temperature for tp6

Figure 6.9: Oversmoothing and constrained unambiguous temperatures (recovered vs true)
Figure 6.10: MAP-MRF recovered temperature for a non-linear temperature profile
Chapter 7

Towards A Practical Application

The previous chapters have demonstrated a possible solution to the thermochromic interpretation problem through an exploration of feature space models of the thermochromic process. The methods used in the solutions have relied on assumptions that will not necessarily apply in real situations. This chapter looks at the problems associated with looking at real test pieces.

Due in the largest part to difficulties in obtaining data for engine (due to reasons of commercial sensitivity) we have not explored in detail the application of our methods to real pieces such as turbine blades and so on, however in this chapter we address the likely problems and hypothesise about directions that successful solutions may take to overcome any failure of the methods.

7.1 Paint interpretation methods applied to real data

When working with real pieces we are no longer dealing with ideal, unobscured flat paint surfaces, we are now looking at objects with possibly complex shape and that are
7.1. Paint interpretation methods applied to real data

Figure 7.1: Simulating the effect of paint coating by combustion products possibly covered with opaque combustion products.

The paints are designed to be used in combustion engines, causing the side effect of coating the pieces with deposits. This is illustrated in figure 7.1. To simulate the effect of the combustion gas stream two metal plates were coated with thermochromic paint. One plate was heated from behind with a blow-torch. In this configuration, the paint surface is not directly in the combustion gas stream and the surface is unobscured. The other plate was heated by a blow-torch applied directly to the paint surface. This has two effects. Firstly, some of the paint appears to be “damaged”, as if burnt, these areas cannot be interpreted. Secondly the paint surface has been obscured by the carbon deposits. Both of these effects cause the colours to not follow the ideal colour progression required by the previous methods.

Figure 7.2 shows part of the front and back surfaces of an actual test object. As can be seen on both images, there is little colour variation on the surface compared to the calibration bow-tie pieces. Observation of the RGB histogram (figure 7.3) for the “back” (the right hand image) paint surface does not show the ideal “linear” form of the thermochromic curve but instead shows a three-dimensional cluster. This is likely to be due largely to the influence of combustion products. Observation of the RGB histogram for the “front” (the left hand image) paint surface, does show a linear cluster, however the positioning of this cluster has possibly been affected by shading.
7.1. Paint interpretation methods applied to real data

Figure 7.2: Part of the front and back surfaces of a real test object
due to curvature and by combustion deposits. We would be required to map these
clusters to their positions on the thermochromic curve. Whereas our previous methods
were self calibrating, to self calibrate in this case we would require a greater amount of
colour variation and an a-prori knowledge of the general temperature gradient (to allow
us to map colour to a image-space based parameter). To map to a standard calibration
would require a standardised lighting arrangement. The combined effect of the shading
due to curvature and of the combustion products warps the ideal colour progression in
an unpredictable manner, it is non-trivial to deduce the inverse mapping from these
clusters to their correct positions on the thermochromic curve, especially given areas
of little colour variation.

Any method that successfully interprets thermochromic paint under these conditions
must successfully overcome these two main problem areas. A full interpretation scheme
does not simply depend on the grouping of similar colours, these contiguous areas must
also be classified to the set of colours corresponding to the calibrated temperatures.

It is clear that the methods presented previously will not work directly with real pieces,
the methods being based on assumptions that do not hold in the general case. Solutions to the problem based on our methods must look at ways of overcoming these inadequacies, or ways of adapting the data so that the assumptions hold. In summary the key assumptions that have been made in the previous chapters are:

- A sample is self calibrating, that is, we have the full thermochromic curve available to build our model
- A test-piece is flat and suffers no problems associated with shading to curvature
- The paint surface is undamaged
- The paint surface is not obscured by carbon deposits from the heating process
- The colour progression is ideal, and follows a standard colour progression, that is the combustion gas stream does not cause bleaching
7.2 Definition of and Mapping to Thermochromic Curves

In our experiments we were fortunate enough that we were able to derive an approximation of the thermochromic curve from a bow-tie piece. The curve was fitted to the data obtained from one side of the bow-tie piece, the classification was performed on the other. On for example a turbine blade it is possible that the full range of colours for the thermochromic curve are not presented. We may form a model of the thermochromic curve from the calibration coupon set, but when it comes to matching colours this may cause problems. The bow-tie presents the full range of colours for a paint, the context of the classification is guaranteed at least for part of the image, and from here the interpretation may be grown. On a turbine blade, it may be the case that there is little colour variation across the surface. Should the colour lie in an unambiguous area or the noise be controlled such that an unambiguous interpretation may be made then there is little problem. Should the colour lie in an ambiguous area then the matching process may suffer from a lack of context. This problem would also hinder a human operator.

7.3 Possible Curvature of Test Pieces

In the experiments that comprise part of this study we examine flat test pieces. This ensures that we do not suffer from the shading associated with piece curvature. For real test pieces there is likely to be a high degree of curvature. As light passes in a straight line parts of the surface not under direct illumination from the light source are less illuminated and thus appear darker. The methods presented in previous chapters assume that illumination is controlled and that we may match to colour in 3-dimensions. Different shades of the same colour may have the same basic chromaticity but are different colours as far as our methods are concerned, that is, we are not robust to shading.
7.3. Possible Curvature of Test Pieces

7.3.1 Overcoming shading with lighting and piece unwrapping

A seemingly simple method of overcoming problems with shading is to ensure that the lighting is sufficient such that there is no shading. This in itself is non trivial, multiple lights may be used to light shaded regions, but in regions that were originally unshaded the multiple light arrangement still causes an unevenness of illumination. A better solution suited to curvature around a single axis is to use an arrangement with a turntable and single lamp placed near a line camera. As the test piece rotates concave surfaces receive the same illumination as convex surfaces. The line camera takes a single line image of the intersection of the surface and a plane formed by the focal point of the camera and the axis of rotation. This produces an unwrapped image of the surface with even illumination. This method is unsuitable when the curvature is around multiple axes of curvature.

7.3.2 Chromatic representations

As mentioned previously the RGB values obtained from the camera contain a removable illumination factor. Dividing out the brightness $R + G + B$ any two of the chromatic values $r, g, b$ specify the brightness invariant chromaticity of the colour. Figure 7.4 demonstrates the implications for the thermochromic curve when working in chromatic space rather than the 3-dimensional RGB colourspace. The dimensionality reduction of the curve from 3 dimensions to 2 causes more curve self-intersections. It appears from figure 7.4 that much of the colour variation for the paint is in a variation of lightness. When projecting to the chromatic plane all the colours have been compacted, the distribution of chromatic values is much more dense than for the curve in RGB space. Working in this representation would severely challenge our methods.
Figure 7.4: A thermochromic curve on the chromatic plane and the original curve
7.3.3 Reflection model based interpretation

A dried paint surface consists of coloured particles within a transparent binding medium. Of the light that approaches the paint surface a percentage will penetrate the binder, become scattered, reflected and absorbed at various wavelengths by the pigment particles and then re-emerge from the paint surface with colour that is largely determined by the colour of the pigment. Conversely as the paint and the surrounding air have different refractive indices a percentage of the light does not pass through the paint surface but instead is reflected at the interface between the surface and the surrounding air, as illustrated in figure 7.5 ([Wri58]). This gives rise to two colours of light reflected from the paint - one corresponding to the interface reflection (or “Fresnel reflection” [Hea89]) at the paint surface and one corresponding to the light that penetrates the surface to become affected by the pigment.

Shafer’s dichromatic reflection model [Klin93] expresses the reflection of light from dielectrics mathematically. Assuming one light source, no ambient light and no interference between objects then the light reflected from an object is comprised of the sum of light reflected from the material surface and light reflected from the material
7.3. Possible Curvature of Test Pieces

Figure 7.6: “Skewed T” clusters in feature space

\[ R(g, \lambda) = M_S(g)C_S(\lambda) + M_B(g)C_B(\lambda) \] (7.1)

\( R(g, \lambda) \) is the light reflected by the surface parameterised by wavelength \( \lambda \) and viewing geometry \( g \). The subscripts ‘S’ and ‘B’ denote the surface reflection and body reflection respectively. \( M_S \) and \( M_B \) describe the viewing geometry and \( C_S \) and \( C_B \) denote the spectral response of the light corresponding to the two reflections. That is, each component imparts a different spectral shaping to the incident light, and each is dependent on the viewing geometry, specifically the angles between the light rays’ approach to the surface and their reflection to the eye or light sensor.

In pioneering work using Shafer’s dichromatic reflection model, Klinker et al performed colour image segmentation and extraction of highlights from images. It was shown that clusters from single objects (with shading and highlights) formed a plane in the feature space defined by two vectors relating to the body reflection and the highlight colour. These take the form of a skewed 'T' shape in feature space (figure 7.6). The body reflection vector will point roughly towards the (0,0,0) black point of the histogram and is formed by the colour of the object undergoing various amounts of shading due to, for example, curvature. The highlight is formed by specular reflection and will
point in the direction of the illuminant light. The directions the the two vectors may be estimated via principal component analysis of an image patch. Obviously this will work most successfully in image regions of homogeneous colour, in shaded transition regions without highlights we would expect the colours to form a plane.

In our example we may assume that any observed colour lies on the straight line formed by the true colour and the (0,0,0) black point. We may then use a mapping based on the nearest approaches of the thermochromic curve to the straight line from (0,0,0) to the observed colour. Unlike when matching simply to RGB where we only consider mappings within a certain projection distance, in this case we would need to consider all possible mappings. Care should be taken to avoid highlights through judicious lighting arrangements.

7.4 Possible carbon deposits on the paint surface

In the engine conditions that test pieces encounter, the combustion gas stream causes carbon deposits to be formed on the paint surface. These may be cleaned prior to interpretation but are unlikely to be removed completely. We assume that any remaining carbon deposit has a constant colour across the test piece and that the carbon has been removed at least partially so that the colour below the deposit is visible.

In a similar manner to the multiple colours at transition regions we assume that the resolution of the camera is much lower than the "density" of the surface deposit. If the colour of the carbon deposit can be estimated for the given lighting configuration then we assume that all observed colours lie on the straight lines formed by the true colour (unobscured by the carbon deposit) and the colour of the carbon. Instead of taking the mapping to the curve with smallest distance we may take the nearest intersection between the line formed by the colour of the carbon and the observed colour to the thermochromic curve.
Another approach would be to reduce dimensionality by a colour conversion similar to the projection to the chromatic plane, but that counters the influence of the carbon deposit. Consider an alternative chromatic representation. Instead of dividing each colour component by the brightness \( R + G + B \) we may treat the "brightness" as the Euclidean distance of the RGB colour value from the \((0,0,0)\) black point. Dividing each RGB component by this distance causes the thermochromic curve to form an image on the surface of a sphere of unit radius centred around \((0,0,0)\). On the sphere surface the curve is still parameterised by temperature and the 3-dimensional RGB colourspace, but we may also parameterise by temperature and the 2-dimensional latitude and longitude of the curve points on the sphere surface. This representation is clearly invariant to shading. Similarly one may centre the sphere around any RGB colour point. If we centre around the colour of the carbon deposit then for any true paint colour covered with a carbon deposit we assume the observed colour will lie on the straight line from the true colour to the colour of the carbon deposit. By dividing each observed red, green and blue component by the Euclidean distance to the colour of the carbon deposit the curve will form an image on the unit sphere centred on the colour of the carbon deposit. This is illustrated in figures 7.7, 7.8 and 7.9. The left hand image of figure 7.7 shows an thermochromic image, the right hand image shows the same image but with a linearly added colour shift to the grey value \((128,128,128)\) with increasing grey from left to right of the image. Figure 7.8 shows the colour histograms of these two images. The image modelling carbon deposit forms a histogram for which there is no obvious thermochromic curve. Figure 7.8 shows the projection of both histograms onto the unit sphere centred around \((128,128,128)\), the thermochromic curve is now clearly visible. The noise speckles are caused by numerical instability at the right hand of the image where the colours have been shifted almost entirely to the grey position. Depending on the colour of the carbon deposit this transformation will provide a carbon invariant representation of the thermochromic curve that is less compacted than the chromatic representation. This approach will obviously increase the mathematical complexity of the methods and is dependent on the carbon colour being single, being constant and
7.4. Possible carbon deposits on the paint surface

Figure 7.7: A thermochromic curve on the chromatic plane and the original curve being accurately estimatable. It is unknown whether this will be the case in practical situations.

Obviously if we suffer from both shading and carbon deposits then this approach becomes too simplistic, we have a two dimensional problem involving an unknown shading and unknown influence of carbon. In terms of the thermochromic curve, instead of forming a line, the corresponding image data from the test piece will fall within a 3-dimensional solid depending on the amount of shading or carbon. Should the thermochromic curve, for any paint, form a suitably "spread" curve in the chromatic plane, we may reduce this dimensionality by conversion to chromatic values and we would expect the image data to sweep out a strip (rather than the desired single line). If we know the chromatic coordinate of the carbon colour then we would expect to be able to use the "closest approach" idea of this section. We may expect the observed colours when mapped to the chromatic plane to lie on the line from the carbon chromatic point to the chromatic co-ordinate of the observed colour. This line would intersect the thermochromic curve roughly at the true position (subject to image noise). As the chromatic representation suffers from instability as \( R + G + B \) approaches zero, and
7.4. Possible carbon deposits on the paint surface

Figure 7.8: The influence of carbon deposits inhibits the formation of the thermochromic curve

Figure 7.9: The additive colour invariant projection onto a unit sphere
7.5 The paint surface may be damaged

The colour of carbon is likely to be nearly black, then it is unknown how accurate (or feasible) this approach would be in practice.

7.5 The paint surface may be damaged

At points on the image corresponding to areas of paint damage, the pixels should be rejected as outliers from the uncertainty checker. The classification may then proceed without being compromised.

7.6 The colours suffer from bleaching

For some paints, the bleaching of colours may occur as a consequence of a non-thermal based reaction of the paint constituents. We do not know the true cause of this, and whilst we could model it as another colour progression, we are unsure whether the inferred temperature would be accurate. As the colour change to white is not a standard thermochromic colour change, without further knowledge we cannot be sure that the bleaching would occur at a calibrated temperature.

7.7 Summary

This chapter has looked at the possible problems that may occur when interpreting thermochromic paint on real, three-dimensional test pieces that suffer from a variety of interpretation problems. We have attempted to theorise about the solution of these problems. The methods presented in previous chapters cannot be applied directly to these test pieces as the mappings from observed data to the corresponding ideal colour progressions are unknown.Whilst inroads into the associated problems of interpreting curved, carbon deposited, bleached or damaged paint surfaces have not been made
experimentally, it is hoped that the discussion contained within this chapter will provide some benefit. Obviously the most satisfactory approach is to take care to minimise the impact of the causes of interpretation problems, such as by cleaning carbon deposits prior to interpretation or ensuring adequate lighting.
Chapter 8

Conclusions

In this thesis we have presented the results of our experimentation into the problem of the automatic interpretation of thermochromic paint. The key problem areas that have been addressed in this study have been firstly in exploring chromatic edge detection methods and then secondly in modelling the thermochromic response of the paints. We achieved some success with our thermochromic feature space model by employing this within a simulated annealing optimising framework.

8.1 Summary

Chapter 3 dealt with chromatic edge detection. Human paint interpreters work by locating colour edges on the test piece. This avoids problems of explicit colour matching under uneven lighting conditions or with deposits on the paint surface. We demonstrated that under ideal conditions the edges are weak and that consequently for computer vision purposes edge detection is not a particularly useful step. The success of human interpretation based on edge location is likely to involve high level decision processes and whilst we cannot dismiss it, with the more simple reasoning available to us with-
8.2 Conclusion and Scope for future work

in software implementations we see no indication that at present it is worth further investigation.

Chapters 4 and 5 took the converse approach of looking at discriminability in feature space. We formed a model of the colour formation firstly in terms of homogeneous colours linked by transition regions (from the description of thermochromic paint colour being constant except at calibrated change points) and then as a continuous process of temperature. By fitting B-spline curves to the thermochromic response we were able to define the maximum likelihood estimate of temperature given colour.

Ambiguity arises in interpretation due to the mapping from colour to temperature being many-to-one at self-intersections of the thermochromic curve. This problem is exacerbated by image noise. The context of the colours allows us to disambiguate the interpretation mapping. Chapter 6 presented a method based upon simulated annealing that employed a temperature smoothness criterion to adversely affect the probability of choosing incorrect classifications in ambiguous regions. This method was demonstrated to successfully interpret "synthetic" and real thermochromic data.

8.2 Conclusion and Scope for future work

We have presented methods that successfully interpret thermochromic paint. The methods may not be useful in all practical situations, however they will provide a useful tool to aid human interpretation.

As discussed in chapter 7, the methods that we have developed work most successfully with ideal data. The implementation of these methods in practical situations is unlikely to be successful unless there are strict controls with regards to, for example, the lighting configuration or pre-interpretation cleaning (to remove carbon deposits). For small test pieces the implementation of such a controlled environment is likely to be feasible, but
for some, larger, engine parts it is hard to see how such conditions may be ensured. It is therefore desirable to extend the methods to cope with these interpretation hindering artifacts. Due to the lack of real data for analysis we have not greatly explored these ideas, however the discussion that forms part of chapter 7 highlights possible avenues for exploration.
Bibliography


Bibliography


