GAMMA RAY EMISSION TOMOGRAPHY AND ANGULAR CORRELATION MEASUREMENTS TO STUDY THE DISTRIBUTION AND BINDING SITE OF SELENIUM

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ABSTRACT

The distribution of $^{75}\text{Se}$ in tissue equivalent materials was investigated employing Gamma ray Emission Tomography with a rectilinear scanner utilizing NaI(Tl) and BGO detectors. The reconstructed images, using Filtered Back Projection and Iterative techniques were presented in 2D colour and 3D representations. Using a lead collimator of aperture 1.5x20 mm and 70 length, the distribution of selenium with variation of volume and concentration was examined and clearly seen. Several corrections such as background, scattering, attenuation compensation and X-ray characteristic suppression, were performed to improve the quality of the images which was evaluated in terms of the fidelity factor. The possibility of quantifying an image was considered with regard to spatial resolution and least detectable concentration. The spatial resolution was measured using two small vials containing the same concentration of selenium, the value obtained was the same as the width of the collimator aperture. The value of the least detectable concentration of selenium however, was difficult to find, due to the many ambiguous factors involved.

The binding site of selenium which is based on quadrupole interaction with the surrounding electric field, was investigated employing Perturbed Angular Correlation (PAC) experiments using NaI(Tl) and BaF$_2$ detectors. Using NaI(Tl) detectors, it was difficult to observe the perturbation, due to the poor time resolution.
The BaF$_2$ detector according to the literatures has a shorter light emission decay time constant (0.6 ns), suggested that a better time resolution than that found with the NaI(Tl) detectors could be obtained. A Perturbed Angular Correlation experiment employing BaF$_2$ detectors and a fast-slow coincidence system was set up. The time differential PAC of selenium in solution showed an unperturbed angular correlation pattern. The main problem is the very short half life of the intermediate state of $^{75}$Se (0.3 ns), making it difficult to observe the perturbation effect. The time resolution of the system (5.4 ns) will need to be improved by an order of magnitude or more for the investigations to continue.
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Untuk Isteri dan anak-anak yang sangat kucintai,
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### CHAPTER 2  GAMMA RAY EMISSION TOMOGRAPHY

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CHAPTER 1
INTRODUCTION

During the past two decades, interest in trace elements has grown tremendously and nowadays many analytical techniques are available for the determination of trace element concentration in biomedical and environmental materials. The awareness that trace elements play a very important role either beneficial or harmful in human health has been stimulated by the rising concern in industrialised nations of man's impact on his environment and its biological effect. Some elements are being considered in research with regard to their essentiality and toxicity to the body.

In order to study the role of essential and toxic elements in biological systems, it is important to know the nature and the properties of the chemical form in which the elements exist. When selecting a speciation method, two major requirements should, where possible be met:

1. the method must not have any influence on the chemical composition or on the structure of the medium and should preferably allow for measurement in situ.

2. the method must be sensitive and selective.

This is especially essential when studying very dilute systems as frequently occurs in biological systems. As a result, physical speciation methods will often be preferred to chemical techniques.
Gamma ray Emission Tomography is a method for determining the distribution of radiation emitting substances introduced into the system based on the reconstruction of the image of the radionuclide distribution.

Experiments in Perturbed Angular Correlation of gamma rays provide a method to observe the perturbation acting in the nuclear site based on nuclear interactions with atomic and molecular electric field surroundings. Thus the nature of the binding site in compounds can be studied by observing this perturbation.

These two nuclear methods, are both noninvasive techniques without any influence on the chemical composition or the structure of the medium. The sensitivity of these methods depends on the photon detection efficiency of the system. $^{75}$Se which has long half life ($120$ d) and emits gamma ray of several energies, can be used as the radionuclide tracer in a Gamma ray Emission Tomography and Perturbed Angular Correlation experiments to study the distribution and binding site of selenium compounds in a biological system.

Selenium is an essential trace element and as a vital substance along with the other minerals which the body needs in very small quantities, is notable for its very high biological activity (Pra 82). It occurs in minute amounts in mostly all substances and was found to be an integral part of protein and was a strong and powerful link with vitamin E. The isolation and characterisation of a selenium compound in the living organism is made very difficult by the factor of low concentration and high lability. Very little is known about the structure mechanism of action of organoselenium compounds concerned with the essential role of selenium.
Selenium has the ability to function as a biological antioxidant reacting with free radicals formed in the initial stage of lipid oxidation. The use of selenium in many industrial processes (electronics, rubber, glasses etc.) carries with it an increased risk of exposure to toxic amounts especially for those workers directly involved (Zin 74).

The distribution of selenium in body organs differs widely from one individual to another and it is difficult to establish selenium levels for body organs and fluids which might be considered as normal. The biochemical behaviour of selenium is complex and not fully understood. Very little information was available on any facet of selenium in waste disposal systems. The range of selenium in the body is not clear and it is not known where the cut-off marks might come between subclinical and clinical deficiency and toxic effect.

Recently the essentiality of selenium to the human has been receiving considerably more attention because of the possibility of a relationship between selenium and cancer and the claim that selenium may possess cancer protecting properties. Some investigators have therefore advocated increased selenium intake as a means of combatting human cancer and other diseases such as heart disease, arthritis and rheumatism (Lew 85). So work in the study of selenium is continuing, even getting more intense, in many laboratories to determine what the interacting factors might be.

The commonly used method to determine selenium concentration in biological tissues is Neutron Activation Analysis. It is known that selenium concentrates in all the cells and tissues of the body at normal trace amounts ranging from 0.05 to 0.7 ppm (Iye 78, Aka 87). In biological reference
materials such as bovine liver, Bowen's kale and animal blood, Egan showed that bovine liver contains the highest amount of Se (1 ppm) while blood contains only about 0.3 ppm (Ega 77). Using an isotopic neutron source $18.5 \times 10^{10}$ Bq (5 Ci) Am(Be) and a combination of cyclic, prompt and capture activation analysis for in-vivo measurements of Se and Cd, the concentration of selenium in pig's liver was also found to be of the same order of concentration as in bovine liver and verified by in vitro activation analysis (Nic 83). In the experiment to determine variation of selenium concentration in human blood after selenium supplementation, it was found that plasma rather than red blood cells is more readily influenced by the dietary intake of the element. The concentration of selenium in the blood of subjects suffering from senile dementia was found to be significantly lower than that of controls (Aka 87).

The isotope detected in Cyclic Activation Analysis is $^{77}$Se$m$ with a half life of 17.6s (Spy 81). The target nuclide $^{76}$Se is suitable in terms of natural abundance and activation cross-section. The commonly method used until that time for determining selenium concentration was through the long lived isotope $^{75}$Se in Neutron Activation Analysis and is still used in neutron irradiation facilities where the transfer time of the sample is long compared to the short-lived product $^{77}$Se$m$. $^{75}$Se, however, is mostly used in radiotracer experiments to study selenium distribution and its function in biological systems. It has low beta absorbed dose, high usable photon yield and suitable long half life (120 d) for labelling and scanning.

The first use of the radionuclide $^{75}$Se for labelling was employed by McConnel to study the absorption, distribution and route of excretion in laboratory animals (McC 41). Using a
labelled radionuclide in vivo in rats, Rotruck studied the nutritional importance of selenium and its relation to vitamin E (Rot 73). He produced evidence for the essentiality of biochemical function of selenium in animals and provided an explanation for its relation to vitamin E. Selenium was identified as the essential component of a dietary factor that protected animals (rats) from severe necrotic degeneration of the liver and several important live-stock diseases are actually due to selenium deficiency syndrome (Sta 74).

The technique of isotope scanning in vivo using $^{75}\text{Se}$ was first performed by Blan and Blender. It showed that the human pancreas could be visualized externally after $^{75}\text{Se}$ selenomethionin administration (Bla 62). Sarkar et al used $^{75}\text{Se}$ in selenocholesterol (NP-65) to study the distribution of selenium in animal adrenal. The results showed high adrenal concentration and good adrenal images (Sar 76). $^{75}\text{Se}$ selenomethyl cholesterol had been used for scanning the human adrenal by Hawkin et al employing a rectilinear scanner. It resulted in quite blurred image, due to the high background (Haw 80). Another label very commonly used is 23-Selena-25-homocholyl taurine ($^{75}\text{SeHCAT}$). This label was first used by Meller to study the dynamics of the enterohepatic circulation of bile acids using a gamma camera. $^{75}\text{Se}$ of $9.25 \times 10^5$Bq (25 $\mu$Ci) was taken orally and after several hours, $^{75}\text{Se}$ activity in the liver and the gall bladder were measured and determined by static abdominal scintigrams (Mel 81). Using the same label, Eastwood et al measured the absorption of conjugated bile acid. After administration of SeHCAT orally, the whole body retention of SeHCAT was measured using a shadow-shield whole body counter. This label appeared as a promising method to study ileal function.
A more intensive study of ileal disfunction was carried out by Sciaretta et al. Abdominal gamma counting was carried out after administration of the label containing $^{75}\text{SeHCAT}$ on some patients suffering from ileum disfunction by means of a gamma camera using the energy window of (260-280) keV (Sci 86). Monks and Boyd showed that the SeHCAT label was more resistant than other commonly used labels (taurocholate-$^{14}\text{C}$). This label was used to examine the biological stability of SeHCAT towards fecal bacterial enzymes in the rabbit (Mon 88).

In this work, in order to study the distribution of selenium in tissue equivalent materials, a rectilinear scanner in Gamma ray Emission Tomography mode using NaI(Tl) and BGO detectors was employed. The NaI(Tl) detector is less efficient compared to the BGO detector, but the energy resolution is superior. The reconstruction algorithms used to obtain the selenium distribution were based on Filtered Back Projection and Iterative techniques. The selenium distribution is presented in a 2D colour images or 3D graphical representations. The quality of the image is evaluated in terms of fidelity factor. The main problem in Gamma ray Emission Tomography is improvement of the image to obtain the best quality but also to obtain quantifiable values for the radionuclide concentration. The quantifiable properties of the image are considered with regard to spatial resolution and least detectable concentration.

The use of Angular Correlation of gamma rays to study the binding site of selenium in a material has not been extensive. The technique was first used by Kaplan to study the binding site of selenium in a resin compound. In this experiment one investigates the alteration of the angular correlation pattern which may be perturbed by the binding site of the Se nucleus with
its surroundings. The results however, were unsatisfactory and did not show significant alteration of the angular correlation pattern (Kap 74).

The main problems in the application of Angular Correlation experiments are time resolution of the coincidence system and binding site factor. The binding site of the atomic Se in compounds is based on the quadrupole interaction of the Se nucleus with the surrounding electric field. To observe this perturbation by using Angular Correlation experiments, the time resolution should be less than the half life of the intermediate state of the Se nucleus (0.3 ns). If the binding is not strong (or significant) and the time resolution is greater than the half life of the intermediate state, the perturbation in Angular Correlation can not be observed. The time resolution in Kaplan's experiment using a combination of NaI(Tl) and Ge(Li) detectors was much larger than the half life of the $^{75}\text{Se}$ intermediate state.

The time resolution of the system can be improved by using fast timing detectors and a fast timing coincidence system. The time resolution of NaI(Tl) detector is better than that of Ge(Li) detector. Recently new scintillation detectors of BaF$_2$ have been used in Angular Correlation applications, due to their very good timing resolution. Using a sophisticated fast slow coincidence system, the time resolution that can be achieved is --- 0.2 ns (Wis 86, Vri 87, Lav 82, Cau 85).

The BaF$_2$ scintillator detector is therefore a promising detector for use in applications of Angular Correlation to study the binding site of selenium in compounds.
CHAPTER 2
GAMMA-RAY EMISSION TOMOGRAPHY

2.1 INTRODUCTION

Gamma-ray Emission Tomography is a method used to produce an image of radionuclide quantity and its distribution in a plane of an object, utilizing reconstruction techniques. Photons coming from the object are collected by the detector to produce projection data. Using a reconstruction technique, projection data from a large number of projection angles are processed to obtain the image.

The reconstruction techniques in general can be classified into analytical and empirical methods. The analytical methods process the data directly employing a mathematical formula to produce the reconstructed data. The empirical methods need initialisation as rough reconstructed data, then improvement is given in steps repeatedly. This method is called the iterative technique. The image of a radionuclide distribution from this reconstructed data can be displayed in 2D colour or 3D projection plot. The Inverse Radon Transform, Inverse Fourier Transform and Filtered Back Projection utilize the analytical method. The reconstruction methods both empirical and analytical however still lack the capability of processing satisfactorily the wide variety of projection measurement geometries and quantities and precision of the data collected which occur in practical applications.

The reconstruction technique used and the reliability of data will affect the quality of the image. The reliability of data is related to the detection and scanning system used.
The reconstructed image usually incorporates the noise fluctuation, boundary degradation and attenuation effect. The attenuation effect due to the absorption of gamma-rays in the object depends on photon energy, atomic number and density of the object. It will distort the image and becomes serious for low energy photons or objects with high density and high atomic number. Attenuation compensation may be introduced in processing the data to improve the image. In analytical reconstruction techniques, the compensation is performed before or after reconstruction. When Iterative techniques are used, attenuation compensation can be carried out during the process of iteration. Some attenuation compensation methods will be described in the next sections.

Image quality can be represented quantitatively by means of fidelity factor. The fidelity of an image is a factor related to the mean square difference between the spatial intensity of the actual (reconstructed) image and the spatial intensity of the expected (ideal) image. This fidelity factor will be used to assess the effect of correction methods used to improve the quality of an image and in quantifying the radionuclide distribution.

2.2 BASIC THEORY

In a Gamma-ray Emission Tomography experiment, one uses a scanning system consisting of single (or multiple detectors) and a scanner to translate and rotate the detector or the object. The detector opposes the object and scans it at many angles of projection. If one uses a single detector, first a linear scan along the object is obtained, then scanning is repeated for other projection angles. The scanning system using a single detector
is shown in fig. 2.1. From this figure, one can define two coplanar reference frames, i.e. the \( X-Y \) frame, which is fixed with respect to the object and the \( X_r-Y_r \) frame as the rotating frame in which the \( Y_r \) direction is the direction of the detector.

![Diagram](image)

**Fig 2.1.** Scanning system using one detector. The object fixed in the \( X-Y \) frame. The detector scans from position 1 to N repeatedly from projection angle 1 to M.

A beam of gamma rays emitted by the object is detected by the collimated detector. The detector scans linearly from position 1 to N to generate projection data \( D(i,1) \), \( i=1 \) to N. Scanning is repeated at many projection angles to generate one complete set of projection data \( D(i,j) \), \( j=1 \) to M. The fixed frame \( (X,Y) \) and the rotating frame \( (X_r,Y_r) \) in which the \( Y_r \) is the direction of the incident beam and parallel to the direction of the detector are illustrated in figure 2.2. The origin of both frames is located at the centre of rotation of the scanning gantry. The radionuclide concentration in the object may be represented by \( A(x,y) \) and the attenuation coefficient is \( \mu(x,y) \).
Fig. 2.2. The fixed frame \((X,Y)\) and the rotating frame \((X_r,Y_r)\). The rotation angle is \(\phi\). Only the beam along \(l\) can reach the detector at that position.

From both of the frame systems, one can write the following equivalences,

\[
A(x,y) = A(R) = A_r(X_r,Y_r) = A_r(R_r,\phi_r)
\]

with \(R_r = R\) \(\phi_r = \phi - \phi\)

\(x = R \cos \theta\) \(y = R \sin \theta\)

\(x_r = R_r \cos \theta_r = R_r \cos(\theta - \phi)\)

\(= R_r \cos \phi \cos \phi - R_r \sin \theta \sin \phi\)

\(= x \cos \phi - y \sin \phi\)

\(y_r = R_r \sin \theta_r = R_r \sin (\theta - \phi)\)

\(= y \cos \phi - x \sin \phi\).

The number of counts in one projection depends on the position \(x_r\) and rotation angle \(\phi\). It represents the integration of radionuclide concentration along the line \(l\) in the object,
attenuated by the intervening medium, so it can be written as follows,

\[ D(x_r, \phi) = \int I A(x,y) \left\{ \exp \int -\mu(x',y') \, ds' \right\} \, ds. \quad (2.2) \]

The length of \( l \) depends on lateral position \( x_r \) and angle \( \phi \).

From this equation, if the function of attenuation coefficient \( \mu(x,y) \) is known, one can derive the line integral of attenuation,

\[ G(x,y) = \exp \int -\mu(x',y') \, ds'. \quad (2.3) \]

The projection, from equation (1,2), will be

\[ D(x_r, \phi) = \int I A(x,y) G(x,y) \, ds. \quad (2.4) \]

If the attenuation coefficient \( \mu \) can be neglected, so \( G = 1 \), the projection equation (2,4) becomes simpler and thus the reconstruction to obtain the image of radionuclide distribution can be performed. The reconstruction of the image with the assumption that the attenuation is neglected will be described in the next section.

2.3 IMAGE RECONSTRUCTION

The main problem in Emission Tomography is how to obtain the radionuclide concentration \( A(x,y) \) reconstructed from the experimental data \( D(x_r, \phi) \). The analytical method of reconstruction based on solving the problem of reconstruction directly from the equation of projection data. Three methods are described, 1) Inverse Radon Transform, 2) Fourier Transform and 3) Simple Back Projection. The empirical methods are based on iteration techniques using algebraic algorithms and gradient conjugate techniques will also be described briefly.
2.3.1 INVERSE RADON TRANSFORM RECONSTRUCTION

The Radon Transform is the transformation of coordinates or the numerical space of a physical quantity. The implementation of Radon Transform in Emission Tomography involves the transformation of projection data \( D(x_r, \phi) \) into the radionuclide concentration \( A(x,y) \). The projection data, from eq. (2.4) can be written as,

\[
D(x_r, \phi) = \int_A(x,y) G(\mu) \, d\mu
\]

where \( G(\mu) = \exp \int_{s'} - \mu(x',y') \, ds' \).

The Radon transform can be written as follows,

\[
D(x_r, \phi) = R \, A(x,y), \quad (2.6)
\]

where \( R \) is the transformation operator.

If the inverse Radon transform \( R^{-1} \) can be derived, the reconstruction of the radionuclide concentration can be obtained by this technique, so

\[
A(x,y) = R^{-1} \, D(x_r, \phi). \quad (2.7)
\]

The derivation of the inverse Radon transform is complicated, because in general the function of the radionuclide concentration \( A(x,y) \), the coefficient \( \mu(x,y) \) and the mathematical function of projection data \( D(x_r, \phi) \) are not known.

To simplify the problem, as a first approximation, one can neglect the attenuation factor \( \mu \), so with \( \mu = 0 \).
D(xr,\phi) = \int_A(x,y) \, d_1 = R \cdot A(x,y). \hspace{1cm} \text{(2.8)}

The uniqueness theorem requires infinite sampling and state that $A(x,y)$ is uniquely determined when its Radon transform is known everywhere, that is when its line integral $D(xr,\phi)$ is known for all $(l,\phi)$. The Inverse Radon Transform may be expressed as follows (Tod 84),

$$A(x,y) = \frac{1}{2 \pi} \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} d_1 \frac{\partial}{\partial l} \frac{1}{R \cos(\theta - \phi) - x_r} \hspace{1cm} \text{(2.9)}$$

where $\tan \theta = y/x.$

To derive the distribution $A(x,y)$, it requires complex mathematical manipulation, because the inner integral contains a singularity. If $A(x,y)$ can be derived, the reconstruction can be performed to obtain the distribution of the radionuclide concentration.

To remove the singularity, one may use the convolving method as the second approximation. The eq. (2.9) can then be written as

$$A(x,y) = \frac{1}{2 \pi} \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} d_1 \quad \text{D}(x_r,\phi) \quad q(x,l), \hspace{1cm} \text{(2.10)}$$

where $q(x,y,l)$ is the convolving function.

The convolving function can be chosen, so that the approximation provided by (2.10) results in the best image of the reconstructed projection.

In implementation, the integration is replaced with discrete summation i.e. by summing over all the projections and all projection angles, so (2.10) is written as,

$$A(i,j) = \sum_n \delta \phi \sum_m d \cdot \text{D}(n \quad d_m \delta \phi) \quad q(i,j,n \quad d) \hspace{1cm} \text{(2.11)}$$

where $d$ is the length step of each ray sum.
The successful transition from mathematical theory, however demands the condition of infinite sampling, so the degradation of accuracy depends on the sampling interval and $\delta \phi$. This approximate reconstruction never results in the true distribution image, despite the introduction of the attenuation compensation as a weighted function; however it leads to a concise interpretation of the reconstruction process consistent with intuition (Kou 82).

2.3.2 INVERSE FOURIER TRANSFORM RECONSTRUCTION

The reconstruction of the image from projection data is based on the Fourier Transform and the Inverse Fourier Transform method. The projection data of radionuclide distribution, neglecting the attenuation factor, from eq.(2.2) is,

$$D(x_r, \phi) = \int A(x,y) \, dx.$$  \hspace{1cm} (2.12)

The radionuclide concentration $A(x,y)$ as a function of 2 variables $(x,y)$, can be decomposed into sine and cosine functions, i.e. a two dimensional Fourier integral, so

$$A(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(k_x,k_y) \exp 2\pi i(k_x x + k_y y) \, dk_x \, dk_y \hspace{1cm} (2.13)$$

where $k_x$ and $k_y$ are the wave numbers in the $x$ and $y$ directions, i.e. number of nodes of sine and cosine per space interval.

$F(k_x,k_y)$ is the Fourier coefficient.

From the Fourier transformation (2.13) one can derive the Inverse Fourier Transformation, which can be written as follows,
By rotating the coordinates from \((X,Y)\) to the new coordinates \((X_r,Y_r)\) (see fig. 2.3), this equation will become simpler. From this figure, the parameters \(k_x\) and \(k_y\) can be written as follows,

\[
k_x = k \cos \phi \quad k_y = k \sin \phi
\]  
(2.15)

The projection \(R\) in the new coordinate is,

\[
R \cos(\theta - \phi) = x_r.
\]

From this figure, \(x_r\) can be represented in terms of \(\phi\),

\[
x_r = x \cos \phi + y \sin \phi.
\]  
(2.15a)

Substituting equations (2.15) and (2.15a) into (2.14), one obtains,

\[
F(k_x,k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A(x,y) \exp \left(-2\pi i (k_x x + k_y y)\right) dx \, dy.
\]  
(2.16)
The element of integration \( dx\,dy \) is a unit area, which in the rotating coordinates can be written as \( dx\,dl \), so the equation may be written as,

\[
F(k_x, k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A(x, y) \exp(-2\pi i k_x r) \, dx \, dy.
\]

(2.17)

The integration of \( A(x, y) \) with respect to \( dl \) is in fact the projection data \( D(x_r, \phi) \) in eq.(2.12), so one obtains the equation

\[
F(k_x, k_y) = \int D(x_r, \phi) \exp(-2\pi i k_x r) \, dx_r.
\]

(2.18)

This equation is a one dimensional Fourier transform of the projection data \( D(x_r, \phi) \). From equation (2.18), given the projection data \( D(x_r, \phi) \), one can evaluate the coefficient of Fourier transform \( F(k_x, k_y) \), then using (2.13) the radionuclide concentration \( A(x, y) \) can be obtained.

In the implementation, the integration is replaced with discrete summations. The number of summations depends on the number of detector steps (ray sums) and projection angles. Because this technique is based on a Fourier transform algorithm, it needs a number of certain value of the summations (2.18) to obtain the degree of a good approximation (Bro 76).

In discrete summation, eq. (2.18) can be written as follows,

\[
F(k, m) = \frac{1}{K} \sum_{j=0}^{K-1} D(x_r, \phi) \exp(-2\pi i k j / K) \]

(2.19)

where \( K = n \log_2 n \).

\( n \) is the number of projection angles.

In the discrete Fourier transformation with a series of \( m \) terms, approximately \( n \log_2 n \) multiplications are required to obtain a completely accurate of the Fourier coefficient transform,
provided the function is spatially limited, so that the Fourier series can be truncated. The discrete inverse transform of \( F(k,m) \) then gives the convolved radionuclide concentration,

\[
A(k,m) = \sum_{j=0}^{K-1} F(j,m) \exp(i2\pi kj/K).
\]

(2.20)

The reconstructed \( A(k,m) \) still needs a weighting factor to obtain a better approximation, since the Fourier coefficient resulting from the projection does not fall on point \((k,m)\) in the fixed frames \((x,y)\). The reconstructed projection becomes,

\[
A^*(i,j) = \sum_{i,j} W_{ij}^{km} A(k,m)
\]

(2.21)

where \( W_{ij}^{km} \) are the weighting factors.

A number of alternative approaches of the weighting factors have been developed as described by Bracewell (Bra 78).

This Fourier transform reconstruction still requires interpolation, because the Fourier coefficients obtained from projection do not fall on a rectangular matrix, as required for the inverse two-dimensional transform. This interpolation, however requires a large amount of computer time (Bro 76).

2.3.3 SIMPLE AND FILTERED BACK PROJECTION RECONSTRUCTION

The simple Back Projection is based on the summation of projection data. The projection of \( D(x_r,\phi) \) on the coordinate \((x,y)\) which consists of a set of line integrals of radionuclide distribution in the object will smear uniformly over all space in this fixed frame. At a point \((x_1,y_1)\) in this fixed frame, there are many line integrals of \( L(x_r,\phi) \) crossing this point. One line \( L \) represents one line integral or raysum. Fig. 4 shows 3 lines with different angles \( \phi \), all crossing at point \((x_1,y_1)\).
If the attenuation coefficient $\mu$ can be neglected, the projection data (eq. 2.2) can be written in digital variation as follows,

$$D(x_r, \phi) = \sum_{i,j} A(x_{i}, y_{j}) \Delta_1$$

where $\Delta_1, N_i = 1$

$I$ is the photon path in the object (see fig. 2.2).

Fig. 2.4. Line integral of radionuclide (raysum L) with different $\phi$, all crossing the point $(x_1, y_1)$. The summation of these 3 line raysums represents the radionuclide intensity at the point $(x_1, y_1)$.

The summation of all raysums $L(x_r, \phi)$ crossing at the point $(x, y)$ gives the representation of radionuclide concentration $A(x, y)$, i.e.

$$A(x, y) = \sum_{j=1}^{M} \sum_{i=1}^{N} D(x_r, \phi_j) \Delta \phi.$$  \hspace{1cm} (2.23)

$N$ is number of detector steps = number of raysums per projection angle $\phi$, $M$ is number of proj. angles.

From the fig. 2.3 and fig. 2.4, the relation of $(x_r, \phi)$ and $(R, \theta)$ is,

$$x_r = R \cos(\theta - \phi) \quad \text{tg} \ \theta = y/x.$$  \hspace{1cm} (2.24)
For a fixed point \((x_1, y_1)\) or \((R, \phi)\), there are many combinations of \(x_r\) and \(\phi\). In the fixed coordinate system, the positions of \(x_r\) show a circular trajectory with diameter \(R\) (Fig. 2.5). So from the eq. (2.23) one can obtain the distribution of the radionuclide concentration in the object by summing all raysums crossing the point of interest in the fixed frame.

Fig. 2.5. The positions of \(x_r\) in the fixed coordinate system \((x,y)\) show a circular trajectory.

The image of a radionuclide distribution which is obtained by this summation technique directly from the projection data, is called Simple Back Projection reconstruction. This image does not give the true distribution, because one has used line summation instead of point summation, which will result in distortion of the image around the object. It shows as a star effect or artifact (Fig. 2.6). This star effect degradation depends on the spatial resolution of the raysum step and the number of projection angles.
To reduce the star effect, one uses a filtering technique. It can be introduced before or after reconstruction. The projection data are multiplied by a factor that will result in reducing the starring. Each projection is multiplied by the inverse Fourier transform derived from a filter function. The filter functions that are commonly used are Ramp, Han and Parzen filters. In this reconstruction, the Ramp filter was used, because despite being the simplest filter to employ, it gives the best resolution.

![Star effect or artifact in image reconstruction using Simple Back Projection technique from 4 projections.](image)

The Ramp filter uses a continuous line, i.e. a linear function of frequency \( f \),

\[
F(f) = \begin{cases} 
|f| & \text{for } f < f_m \\
0 & \text{for } f > f_m 
\end{cases} \tag{2.25}
\]

where \( f_m \) is the maximum frequency as the band limiting for frequency space which depends on the spatial resolution of the steps.

The inverse Fourier transform of this function is,
The integration can be split into two parts, so

\[
F_f = \int_{-f_m}^{0} f \exp(-2\pi f x) \, dx + \int_{0}^{f_m} f \exp(-2\pi f x) \, dx
\]

\[
= 2i \int_{0}^{f_m} f \cos(2\pi f x) \, dx
\]

(2.27)

If one takes the real value and integrates by parts, the function becomes,

\[
F_f = \frac{f_m \sin(2\pi f_m x)}{\pi x} - \frac{1}{\pi x} \int_{0}^{f_m} \sin(2\pi f_m x) \, dx
\]

\[
= \frac{f_m \sin(2\pi f_m x)}{\pi x} - \frac{2}{(2\pi x)} [\cos(2\pi f_m x) - 1]
\]

\[
= \frac{f_m \sin(2\pi f_m x)}{\pi x} - \frac{\sin^2(f_m \pi x)}{(\pi x)^2}
\]

(2.28)

This Ramp filter function is illustrated in fig. 2.7.

Fig. 2.7. The filter function.
a. Ramp filter      b. The inverse filter.
The projection data $D(x_r,\phi)$ are multiplied by this filter function to obtain the filtered projection data,

$$D^*(x_r,\phi) = D(x_r,\phi) \int_{-\infty}^{\infty} \left[ \frac{f_m \sin(2\pi f_m x)}{\pi x} - \frac{\sin(\pi f_m x)}{(\pi x)^2} \right] dx. \quad (2.29)$$

In the computer implementation, the variation of spatial $x$ is presented in discrete, so

$$x = d \cdot i \quad i = 1,2,\ldots,N$$

$d$ is the spatial interval.

The maximum of the spatial frequency is,

$$f_m = 1/(2d).$$

The filtered projection data in discrete variation can be written as follows,

$$D^*(i,\phi) = D(i,\phi) \left\{ \frac{\pi}{2d^2} - \sum_{i=1}^{N} \frac{\sin \pi / 2(i-i')}{d^2 \pi^2 (i-i')^2} \right\}. \quad (2.30)$$

The sine function has the value 0 or 1,

$$\sin \pi (i-i')/2 = 1 \text{ if } i - i' \text{ odd}$$

$$= 0 \text{ if } i - i' \text{ even}.$$ 

So the equation of $D^*$ becomes,

$$D^*(i,\phi) = \frac{\pi}{2d^2} D(i,\phi) - \sum_{i=\text{odd}}^{N} \frac{D(i,\phi)}{d^2 \pi^2 (i-i')^2}. \quad (2.31)$$

This Ramp filter will give the best resolution in the reconstruction for perfect projection data compared with the other filters, but amplifies noise for data with statistical fluctuations (Bud 79). The sharp cut-off gives rise to intensity oscillation in the region of sharp contrast and still generates artifacts in the reconstructed image. The other filters are superior in suppressing the noise, but at a loss in
resolution. The reconstruction using the Simple Back Projection incorporating the filtering is called Filtered Back Projection reconstruction. This method is very commonly used in reconstruction of the image in Emission Tomography, because it is the simplest reconstruction technique which does not need a large amount of memory.

2.3.4 ITERATIVE METHODS

Iterative methods are the techniques to solve the problem of image reconstruction empirically, based on approximating the reconstructed image by iteration and incorporating correction with respect to the projection data. This is an attempt to find a two-dimensional distribution that matches all the projections. An initial distribution is assumed and it is compared with the measured projection. Using one of a variety of iterative algorithms, the initial distribution is successively modified. The Iterative reconstruction as illustrated in Fig. 2.8 is based on the general premise that the resultant reconstruction should match the measured projection.

Fig. 2.8. Cellular array in iterative reconstruction. Number of steps N, number of pixels = N x N. The solid cell is the weighting factor W.
The iterative process is started with all reconstruction elements $A_{ij}$ set to a constant such as the mean $\overline{A}$ or zero. In each iteration, the difference between the measured data for projection $D(k,m)$ and sum of the reconstructed elements along that ray $\sum A_{ij}$ is calculated. $A_{ij}$ in the fixed frame represents an element along the $j$th line forming the projection ray $D(k,m)$ in the rotated frame. For $\phi = 90^\circ$, the area of $A_{ij}$ elements (pixels) is the same as the area of $D(k,m)$ cells. This difference is evenly divided among the N reconstruction elements.

The iterative algorithm is defined as,

$$A_{ij}^{p+1} = A_{ij}^p + \left[ D_{km} - \sum_{i=1}^{N} A_{ij} \right] \frac{1}{N}$$  \hspace{1cm} (2.32)

for a value of $j$.

The superscript $p$ indicates the iteration number. The algorithm recursively relates the values of the elements to those of the previous iteration.

If the projection angle $\phi \neq 90^\circ$, the direction steps vary according to the variation of $\phi$ and the cell area of $D_{km}$ in the rotated frame is $\gamma$ pixel area of $A_{ij}$ in the fixed frame. The difference in area is called the weighting factor $W$. For $\phi = 90^\circ$, the weighting factor $W = 1$.

The difference between the reconstructed $A_{ij}$ and the measured projection $D_{km}$, in general can be obtained from the least square method of minimum norm. The projection data $D_{km}$ as the summation of pixel $A_{ij}$ incorporating the weighting factor $W_{ij}$, can be written as,

$$D_{km} = \sum_{j=1}^{N} \sum_{i=1}^{N} A_{ij}^{km} W_{ij}^{km}$$  \hspace{1cm} (2.33)

where $W_{ij}^{km}$ is the weighting factor that represents the contribution of the $i$th cell of the object to the raysum.
This factor depends on the models of distribution within each cell. If the weighting factor $W$ has been known, one may solve the eq. (2.33) as $M$ equations in $N$ unknowns of $A$ using matrix inversion. This method, however can be performed only if the dimension of matrix $N$ is not large and the number of $M$ should be $< N$.

Hence, instead of the matrix inversion method, one employs the iterative techniques. The basic procedure is the same as before, applying the correction to arbitrary initial density in an attempt to match the measured projection data. The procedure is repeated until the calculated result agrees with the measured one within the desired accuracy.

The general equation of Chi-squared that represents the difference between the measured projection data and the reconstructed distribution in the object, from eq. (2.33) is written as follows,

$$
\Psi^2(A) = \sum_{k=1}^{N} \left[ \sum_{j} W_{kj} A_{ij} - D_{km} \right]^2 \frac{1}{\sigma_{km}^2} .
$$

(2.34)

$\sigma_{km}$ are the uncertainties with which $D_{km}$ were measured.

From this equation, one takes the least square method of minimum norm. The variation of $A_{ij}$ can be represented iteratively using step length,

$$
A_{ij}^{n+1} = A_{ij}^n + c_n \Delta_{ij}^n
$$

(2.35)

$\Delta_{ij}$ is the direction of step.

$c_n$ is a factor to be calculated using a least square method.

Substituting (2.35) into (2.34) and by taking the derivative with respect to $c_n$,
\[ \frac{\delta}{\delta c_n} \left( A_{ij}^{n+1} \right) = 0, \]  

(2.36)

one can derive the factor \( c_n \).

The final equation of the variation \( A_{ij} \), is

\[
A_{ij}^{n+1} = A_{ij}^n + \frac{\Delta_{ij}^n}{\Delta_{ij}^n} \left[ \sum_{km} W_{ij}^{km} \frac{D_{ij}^{km}}{\sigma_{ij}^{km}^2} - \sum_{km} W_{ik}^{km} W_{jk}^{km} A_{ij}^n \right]. 
\]  

(2.37)

The values of \( A_{ij} \) are back projected and they are added to the previous values. The iteration is continued until some criterion is satisfied e.g. the error or rate of decrease of the error is below a given level. The next problem is how to determine the direction step \( \Delta^n \).

The commonly used methods for these algorithms are the Algebraic Reconstruction Technique (ART) and Gradient methods. In the ART, the \( A_{ij} \) are considered as scaler, so the direction step \( \Delta \) can be set to unity. The corrections can be performed ray by ray. A given set of ray projections are calculated and corresponding correction applied to all points. These updated density values are then used for calculating the next projection. This variation is the most efficient, as a large amount of updating is incorporated during each iteration without any increase in computation time, however this method is less able to handle noisy data (Bro 76).

In the gradient method, \( A_{ij} \) are considered as vectors. The step direction is chosen so that its direction is opposite to the gradient of \( \nabla^2 \)

\[
\Delta_{ij}^n = -\nabla \nabla^2 (A_{ij}). 
\]  

(2.38)
This gradient method decreases the values of $\psi^2$ rapidly. The most rapid method is the conjugate gradient, where the step direction is made orthogonal to all previous steps (Bud 79).

The three important considerations of the most appropriate iterative techniques involve how well the technique handles noise, speed of convergence and the ease with which a priori information can be incorporated such as the attenuation compensation. Of the iterative techniques, however no single method has yet evolved as superior for all cases and a tested general method has yet not been presented for incorporation of a priori information. The large number of iterations performed does not guarantee obtaining the better result, since after a number of iterations, the value of $\psi^2$ becomes saturated.

2.4 THE ATTENUATION PROBLEM IN GAMMA RAY EMISSION TOMOGRAPHY

In analytical methods of reconstruction, one has neglected the attenuation effect. The attenuation of photons by the medium will reduce the projection data. It will affect the reconstructed image by distorting it. This distortion depends on the attenuation factor and the geometrical size of the object. If the projection angles covered $2\pi$, the reduction of intensity suffers most around the centre of the object. To improve the distorted image, one needs compensation of this attenuation effect.

The projection data undergo attenuation by the medium as given in (2.2),

$$D(x_r,\phi) = \int A(x,y) \exp \int_{s} -\mu(x',y') \, ds' \, ds \, .$$

(2.39)
The attenuation depends on the length $l$ of the line integration and the attenuation coefficient $\mu(x',y')$. The attenuation coefficient $\mu$ depends on photon energy and composition of the medium and object material. If one does not know the function $\mu(x,y)$, we do not know how the attenuation influences the projection data. This is not really a simple problem in Emission Tomography. The reconstructed image may be seriously affected unless some compensation is made.

In some cases, the correction to the projection data needs some modification to meet the practical requirements. The attenuation compensation is mainly important in the reconstruction if one uses the analytical methods. It can be introduced before or after the reconstruction. The introduction of attenuation compensation in the pixel by pixel summation, Average operation and Geometrical mean methods, are performed before reconstruction. The Post exponential attenuation compensation is performed after reconstruction. The following section describes these attenuation compensations. In the iterative techniques, the attenuation compensation is incorporated during the reconstruction, which can be involved in the weighting factor.

2.4.1 PIXEL BY PIXEL SUMMATION

The projection data with attenuation factor $\mu \neq 0$, from eq.(2.2), is

$$D(x,\phi) = \int_1 A(x,y) \exp \int_s -\mu(x',y') \, ds'ds.$$  \hspace{1cm} (2.40)

Instead of integration, the projection data $D$ can be expressed as a summation of elements of $A$ and $\mu$. 

29
\[ D(x, \phi) = \sum_{i=1}^{n} A(i) \left\{ \exp \left[ - \sum_{k=1}^{n(i)} n(i) \mu(k) a k \right] \right\} a \]

(2.41)

where \( n(i) = s(i)/\Delta s \)

\[ n = 1/\Delta s \]

\( a = \Delta s d \), the unit area of element.

\( \mu(k) \) is attenuation per unit length at element \( k \).

\( \Delta s \) is step length or pixel size.

In the reconstructions to obtain the image of \( A \), one has neglected the attenuation coefficient by putting \( \mu = 0 \). So the projection data,

\[ D^0(x, \phi) = \sum_{i=1}^{n} A(i) a \]

(2.42)

The attenuation compensation should be given to this \( D^0 \) to obtain the true value. If one does not know the variation of \( A \) and \( \mu \) in the object, it is not possible to give the attenuation compensation, by substituting \( A(i) \) from (2.41) to (2.42).

If the variation of \( A \) in the object is not very large, one may set an average concentration \( A_m \) constant.
The projection data $D$ and $D^0$ can be written as,

$$D(x_j, \phi) = A_m \sum_{i=1}^{n} \exp \left[ - \sum_{k=1}^{n^{(i)}} \mu_j(k) a k \right] a$$

$$D^0(x_j, \phi) = A_m \cdot n \cdot a$$

Substituting $A_m$ from these equations, one obtains,

$$D^0(x_j, \phi) = \frac{D(x_j, \phi) \cdot l(j)}{\sum_{i=1}^{n} \exp \left[ - \sum_{k=1}^{n^{(i)}} \mu(k) a k \right] a}$$

$$= \frac{D(x_j, \phi) \cdot n(j)}{\sum_{i=1}^{n} \exp \left[ - \sum_{k=1}^{n^{(i)}} \mu(k) a k \right] a}.$$  \hfill (2.44)

If the attenuation $\mu$ is constant, the corrected projection data have the simple equation,

$$D^0(x_j, \phi) = \frac{D(x_j, \phi) \cdot n(j)}{\sum_{i=1}^{n^{(j)}} \exp \left[ - \mu \cdot n(i) a \right]}.$$  \hfill (2.45)

In the implementation of computer programming, it can be written as,

$$D^0(j,m) = \frac{D(j,m, \cdot n(i))}{\sum_{i=1}^{n^{(j)}} \exp \left[ - \mu \cdot n(i) a \right]}.$$  \hfill (2.46)

$j = 1, 2, \ldots \ldots , N,$

$m = 1, 2, \ldots \ldots , M.$

$N$ is the number of steps

$M$ is the number of projection angles.

$n(j)$ depend on the geometrical size of object.

If the attenuation is not constant, a map of $\mu_{jk}$ as seen in eq. (2.44) is required. By performing this attenuation compensation, the corrected projection data $D^0$ become larger than the uncorrected $D$. This correction depends on the attenuation coefficient $\mu$. 

31
This method is based on the average of two opposing projection data with the assumption that the local variation of the radionuclide distribution and the attenuation coefficient are not so large. So in the integration, one may set the average concentration of \( A_m \) and the average of attenuation factor \( \mu \) constant. The projection data \( D \) can be derived as follows,

\[
D(x_r, \phi) = \int_0^1 A_m \exp - \int_0^\phi \mu ds' ds
\]

\[
= \frac{A_m}{\mu} \left[ 1 - \exp(-\mu l) \right].
\]

In the reconstruction, the attenuation coefficient \( \mu \) is set to 0, where the projection data, from (2.43),

\[
D^0 = A_m l.
\]

Substituting \( A_m \) from these two equations, one obtains

\[
D^0(x_r, \phi) = \frac{D(x_r, \phi) l \mu}{[\exp(-\mu l) - 1]}.
\]

The opposing projection data, with the same length \( l \), can be written as,

\[
D^0(-x_r, \phi + \pi) = \frac{D(-x_r, \phi + \pi) l \mu}{[\exp(-\mu l) - 1]}.
\]

By taking the average of these two opposing projection data, one can write,

\[
\bar{D}^0(x_r, \phi) = \frac{D^0(x_r, \phi) + D^0(-x_r, \phi + \pi)}{2} \frac{1}{\mu}
\]

\[
\left[ D(x_r, \phi) + D(-x_r, \phi + \pi) \right] \frac{1}{2} \left[ \exp(-\mu l) - 1 \right] l \mu.
\]

This equation can also be written as a sinh function,
\[
\overline{D^0}(x_r, \phi) = \frac{D(x_r, \phi) + D(-x_r, \phi+\pi)}{4 \sinh(\mu \frac{1}{2})} \mu \exp(\mu \frac{1}{2}). \tag{2.52}
\]

In the implemented computer programming, the equation is written as follows,

\[
\overline{D^0}(i,j) = \frac{[D(i,i)+D(N-i,k)] \mu I(i) \exp[I(i)/2]}{\exp[-\mu I(i) - 1]} \tag{2.53}
\]

where \( k = j + M/2 \) if \( j < M/2 \) and \( = j - M/2 \) if \( j > M/2 \).

\( i = 1 \) to \( N \); \( N \) is number of steps

\( j = 1 \) to \( M \); \( M \) is number of projection angles.

\( l \) depends on the geometrical object.

In the operation, since it is derived by integration, the number of sampling \( N \) should be as large as possible, otherwise it will undergo degradation for \( l \approx 0 \).

2.4.3 GEOMETRICAL MEAN

This method is almost similar to the average operation, just instead of the average, one takes the geometrical mean. The geometrical mean of two opposing projection data, from (2.49) and (2.50) is written as follows,

\[
\overline{D^0}(x_r,\phi) = [D^0(x_r,\phi)D^0(-x_r,\phi+\pi)]^{1/2} \tag{2.54}
\]

\[= \frac{[D(x_r,\phi),D(-x_r,\phi+\pi)]^{1/2}}{2 \left[ \exp(-\mu I) - 1 \right]} \mu I. \]

In the implementation for computer programming, this equation can be written as,

\[
\overline{D^0}(i,j) = \frac{[D(i,i),D(N-i,k)]^{1/2}}{2 \left[ \exp(-\mu I(i)) \right]} \mu I(i). \tag{2.55}
\]
This equation is almost the same as the average operation, but instead of addition, here use multiplication operation.

2.4.4 POST EXPONENTIAL ATTENUATION

This method is based on linear summation of two opposing differentials of projection data.

From the projection data (2.2),

\[ D(x_r, \phi) = \int I A(x,y) \exp \left[ \int_{-\mu(x',y')} ds' \right] ds, \]

the differential of this equation is,

\[ \frac{\partial}{\partial s} D(x_r, \phi) = A(s) \exp \left[ \int_{s}^{-\mu(x,y)} ds \right]. \] (2.56)

In the same way, one can write the differential of the opposing projection,

\[ \frac{\partial}{\partial s} D(-x_r, \phi+\pi) = A(L-s) \exp \left[ \int_{L-s}^{-\mu(x,y)} ds \right]. \] (2.57)

L is the scan length.

The average of these two differential opposing projection data, is

\[ \frac{\partial}{\partial s} \overline{D(x_r, \phi)} = \left[ \frac{\partial}{\partial s} D(x_r, \phi) + \frac{\partial}{\partial s} D(-x_r, \phi+\pi) \right] \frac{1}{2}. \] (2.58)

If the average attenuation coefficient is given in this equation, by substituting (2.57) and (2.56), one obtains,

\[ \frac{\partial}{\partial s} \overline{D(x_r, \phi)} = \left[ A(s) \exp(-\mu \cdot s) + A(L-s) \exp(-\mu (L-s)) \right] \frac{1}{2}. \] (2.59)

The graphics of these differential projections are illustrated in fig. 2.10. The average of two opposing projection data shows symmetry with respect to the centre, so the result of the reconstruction image will show a reduction in the intensity.
around the centre of the object. From fig. 2.10, it can be seen that the reduction is exponential. The attenuation compensation is performed after the reconstruction by introducing an exponential attenuation function.

\[
\frac{\delta D}{\delta S}
\]

Fig. 2.10. The differential of projection data (1), the opposing of this differential (2), and the average of these two differentials of projections (3).

If the object has circular symmetry at the centre, the exponential attenuation is simple,

\[ f(r) = \exp(-\mu r) \quad (2.60) \]

In the digital computing, the radius \( r \) can be written as,

\[ r = \left[ (I - N/2)^2 + (J - N/2)^2 \right]^{1/2} \quad (2.61) \]

\( I = 1 \) to \( N \), \( J = 1 \) to \( N \).

where \( N \) is number of raysums.

The reconstructed image is \( N \times N \) matrix, or \( N^2 \) pixels. All pixels are multiplied by this compensation factor \( f \), so the corrected reconstruction is,
If the distribution of the radionuclide concentration $A$ is not symmetrical, the exponential compensation (2.60) is also not symmetrical.

2.4.5 **DUAL ENERGY CORRECTION METHOD**

If the radionuclide emits photons of more than one energy and two peaks in the energy spectrum are well separated, the attenuation compensation can be performed using the dual energy correction method. This method is based on the ratio of detected counts of the two gamma ray energies.

Consider the attenuation intensity equations of two gamma rays passing through the homogeneous medium, with attenuation coefficients $\mu_1$ and $\mu_2$.

\[ I_1 = I_0 f_1 e^{-\mu_1 x} \]
\[ I_2 = I_0 f_2 e^{-\mu_2 x} \]

where $I_0$ is the emitted intensity,
$f_1$ and $f_2$ are fractions of gamma ray emission,
$I_1$ and $I_2$ are the detected counts,
$x$ is the average distance of photons in the medium.

The ratio of these two intensities is,

\[ \frac{I_1}{I_2} = \frac{f_1}{f_2} e^{-(\mu_1-\mu_2) x} \] (2.64)

From this equation, one can obtain the length $x$,

\[ x = \frac{1}{(\mu_1-\mu_2)} \ln \left[ \frac{I_2}{I_1} \cdot \frac{f_1}{f_2} \right]. \] (2.65)
Substituting $x$ into eq. (2.63), one obtains the corrected intensity,

$$I_0 f_1 = I_1 \exp \left\{ \frac{\mu_1}{\mu_1 - \mu_2} \ln \left[ \frac{I_2}{I_1} \frac{f_1}{f_2} \right] \right\}.$$  \hspace{1cm} (2.66)

The ratio of $f_1$ and $f_2$ can be obtained from the table of isotopes and the attenuation coefficients $\mu_1$ and $\mu_2$ can be obtained from measurement or from tables of attenuation coefficients (Hub 82). This equation is used only for constant attenuation or for an homogeneous medium.

If the medium is not homogeneous with variable attenuation coefficient, eq. (2.66) can be written as follows

$$(I_0 f_1) = (I_1) \exp \left\{ \sum_{i=1}^{2} \frac{\mu_{1,i}}{\mu_{1,i} - \mu_{2,i}} \ln \left[ \frac{I_{2,i}}{I_{1,i}} \frac{f_{1,i}}{f_{2,i}} \right] \right\}.$$ \hspace{1cm} (2.67)

$j = 1,2 \ldots \ldots \ldots \ldots \ldots \ldots N$

$i = 1,2 \ldots \ldots \ldots \ldots \ldots \ldots N$

$N$ is the number of raysums

In the implementation, it needs maps of the attenuation coefficients of these gamma rays in the medium. This map can be obtained by performing an experiment of Gamma ray Transmission Tomography. The Emission and Transmission Tomography can be performed simultaneously, but in Transmission it needs another source which has energies a little apart from the gamma ray energies selected in Emission Tomography. This dual energy method is performed before reconstruction.
2.5 IMAGE QUALITY

The data from Gamma ray Emission Tomography is reconstructed using the reconstruction technique to obtain the image representing the distribution of the radionuclide concentration in an object. The image in fact suffers from many degradations such as, artifact, attenuation, noise etc. The image quality may be defined as how clearly and faithfully structures in the image can be visualised.

The artifact may be reduced by selecting a good filter in a Filtered Back Projection and good correction factor in iterative techniques. But there is no single method that may overcome the artifact satisfactorily. In experiment, this artifact is dependent also on the collimator hole used.

The methods of attenuation compensation that have been discussed in the previous section still lack capability in implementation for practical use with general geometrical objects. In Iterative techniques, the attenuation compensation is performed during the process of reconstruction, but in realization always gets overestimated, since it is based on an approximation of the reconstructed image by the iteration process.

If the activity of the object is not sufficiently detectable and has a high background or is a fast decaying activity during the scanning, this will result in a very noisy image. The noise may also come from the instrumentation, such as the efficiency and energy resolution of the detector, the collimator, linearity of the scanner, stability of the electronic system and in handling the scattering problem.
Image quality in general is related qualitatively to sharpness and contrast. The sharpness refers to the spatial extent of the image representation of a sharp boundary between the two different regions of object. The contrast is expressed in terms of the difference in the image density levels of two objects which are physically different as regards the properties of interest.

The image as a square matrix $A(i,j)$, can be presented as a two dimensional image in colour or a three dimensional projection in variation of intensity per pixel. The image quality can be expressed quantitatively by comparing the obtained or actual image with the expected or ideal image. The noise is represented as the deviation between these two images.

The deviation of the two images can be formulated as follows,

$$
\sigma = -\frac{1}{N} \left\{ \sum_{i,j} \left[ A(i,j) - A^*(i,j) \right]^2 \right\}^{1/2}, \tag{2.68}
$$

$A(i,j)$ is the actual image,

$A^*(i,j)$ is the expected/ideal image,

$N$ is the matrix/pixel size.

This deviation can be related to more quantitative values of image quality e.g. the fidelity and correlation factors. The fidelity factor is defined as (Eva 82),

$$
Q_f = 1 - \sum_{i,j} \frac{[A(i,j) - A^*(i,j)]^2}{[A^*(i,j)]^2} \tag{2.69}
$$

and the correlation factor is defined as,

$$
Q_c = \sum_{i,j} \frac{A(i,j) \times A^*(i,j)}{[A^*(i,j)]^2} \tag{2.70}
$$

So the goodness or quality of an image can be represented by the
factors $Q_f$ and $Q_c$ with values between 0 and 1.

Another parameter of image quality is spatial resolution i.e. the minimum distance between two just distinguishable points of the object. It depends mainly on the step size and collimator hole as well as on the degree of noise.

The spatial resolution is expressed in line width or number of linepairs per unit length. Methods of quantifying spatial resolution include the point spread function (PSF), the line spread function (LSF) and modular transfer function (MTF).

The PSF describes the unsharpness that results when a point in the object is not reproduced as a true point in the image. The LSF also describes the unsharpness of the image, when a line or slit object is not reproduced as a line or slit image but is rather spread out over a measurable distance. The MTF can be derived from the LSF or the PSF through Fourier transformation. It is used to measure the resolution capabilities of a system by breaking down an object into frequency components. The value of MTF is between 0, which means that there is no transfer image and 1 which means that the imaging system has exactly reproduced the object. This is in fact similar to the fidelity factor.
CHAPTER 3

EXPERIMENTS IN GAMMA RAY EMISSION TOMOGRAPHY

3.1 INSTRUMENT AND DETECTION SYSTEM

The instrument used in this experiment was a simple one, employing one detector in a rectilinear scanning system. In the theoretical basis of emission tomography, it is assumed that the detecting system scans the object linearly, while the object is fixed at the centre of rotation. In our experiments, the detector was set at a fixed position facing the object. The object was moved linearly in steps perpendicular to the detector, then rotated for another linear scan to be performed. The data was collected for each step to produce the projection data.

The scanning system and data acquisition were controlled by a BBC microcomputer. The object was put at the centre of a turntable. In this experiment, NaI(Tl) and BGO detectors were used. The set up of equipment is shown in fig 3.1. The scanner system was built by J. M. Sanders and F. Balogun (San 82, Bal 86). The detector was shielded with a lead collimator. The Single Channel Analyzer (SCA, CA 2030), by selecting the energy from the Amplifier (CA 2011) sent the output signals to the counter (CA 2072). The counter has two input signals, so it can be used for double energy scanning simultaneously. The time of measurement per step was preset with a timer (CA 2070). Its total measurement time depends on the number of steps and projection angles. The NaI(Tl) detector used was 50 mm diameter and length 50 mm with energy resolution 7.5% at 662 keV and the BGO detector was 16 mm diameter and length 16 mm with energy resolution of 13% at the same energy.
Fig. 3.1. The set up of Gamma-ray Emission Tomography employing one detector in a rectilinear system.

The data from the counter was sent to the BBC micro computer via the IEEE interface and stored on floppy discs. The BBC also controlled the two motors, one for rotation and the other for linear scanning. The parameters for running the scanning programme on the BBC are time of measurement per step, number of steps and number of projection angles. The total time of measurement depends on these parameters, the source activity and the width of the collimator hole. Slit collimators of area, 1.5x20 mm and length 70 mm have been used for reducing the total time of measurement instead of circular hole collimator since the activity of the source used was low. This collimator however, only can be used for an homogeneous object in the longitudinal plane. Before carrying out the experiment of Gamma-ray Emission Tomography with $^{75}\text{Se}$, the object containing $^{125}\text{I}$ was
used to study the attenuation effect and attenuation compensation in the medium.

3.2 ATTENUATION STUDY WITH Iodin

The radioisotope $^{125}\text{I}$ has only single gamma ray energy 35 keV, so there is no interference with other energies and with characteristic X-rays from the lead collimator. The sample was sodium iodide dissolved in water, in a polythene bottle of diameter 40 mm. The low energy gamma ray has a high attenuation factor in water, which may represent soft tissue equivalent material. The tomographic image representing the iodine concentration using Filtered Back reconstruction, step length 2 mm and number of projection angles 20 is shown in fig. 3.2. From this 3D image projection, the attenuation effect appears clearly i.e hollow in the middle of the object image.

Attenuation compensation was performed using various analytical methods, before and after reconstruction. The 3D representations, after performing the attenuation compensation are given in fig. 3.3. The attenuation compensation methods used were pixel by pixel summation, average operation, geometrical mean and post exponential attenuation. In pixel by pixel attenuation compensation, some small spikes still appear around the middle. The geometrical mean and average operation methods gave worse improvement and produced more noise around the object. In the tomographic image this will appear as more artifacts. The noise and artifact become less in compensation using the post exponential method. This method is the simplest and fastest in terms of computer operation, but it works well only for homogeneous attenuation and symmetrical objects.
Fig. 3.2. a. 3D tomogram of Iodine distribution in a bottle of 40 mm diameter, Scan 60/2 mm, No correction. b. Line scan of cross-section.

Fig. 3.3 Analytical attenuation compensation on Iodine image with $\mu = 0.03$/mm. a. Pixel by pixel correction. b. Geometrical mean. c. Average Operation. d. Post Exponential compensation.
For a non homogeneous attenuation and nonsymmetrycal object, the correction function \( f(r, \mu) \) will be very complicated. The quality of the attenuation compensations are presented in fidelity factor term as given in table 3.1. From this table, the pixel by pixel summation and post exponential exhibit better compensation compared with average operation and geometrical mean methods.

<table>
<thead>
<tr>
<th>Attenuation Comp method</th>
<th>Fidelity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. No correction</td>
<td>0.59</td>
</tr>
<tr>
<td>2. Pixel by pixel sum.</td>
<td>0.70</td>
</tr>
<tr>
<td>3. Average operation</td>
<td>0.66</td>
</tr>
<tr>
<td>4. Geometrical mean</td>
<td>0.66</td>
</tr>
<tr>
<td>5. Post exponent comp.</td>
<td>0.69</td>
</tr>
</tbody>
</table>

The average operation and geometrical mean methods are more flexible operations. They can be used for more general conditions of the object, but need a large number of pixels or steps to get less noise. The use of these methods in fact produces more noise compared with the other methods. However, all the attenuation compensations may be used for the improvement of image, depending on the condition of the object under test.

In reconstruction using iterative techniques, the attenuation compensation is performed empirically, in which the attenuation compensation is incorporated during the iterative operations. The analytical methods can be used also in the
iterative techniques, but the attenuation compensation is performed after the reconstruction. All these methods have disadvantages; so there is no superiority analytically as well as empirically in this attenuation compensation.

The problem of attenuation compensation in gamma ray tomography is known to be complex. Many methods have been proposed using analytical or empirical techniques, combinations of both as well. Hybrid methods which combine the iterative with simple multiplicative corrections have been evaluated by Faber et.al (Fab 84). The problem is, this method needs a map of attenuation coefficients of the object. Guldberg et.al (Gul 85) used the combination of emission and transmission tomography experiments to perform the attenuation compensation using the iterative technique. It produced good attenuation correction, however the algorithm required many projections and back projections in operations. A twice attenuation correction method was also given by Axelsson et.al using two methods (Axe 87). The primary uncorrected image is used to estimate the effect of attenuation on measured projection data, and the corrected projection is finally reconstructed using the iterative technique. This algebraic technique, however consumes more computation time.

Attenuation corrections employing simultaneous emission and transmission tomography experiments have also been performed by Barly et.al (Bar 87). Improvement of quantifiable in emission tomography was shown with a variable attenuation coefficient. They used dual isotopes with the energy of both only a little apart. From the transmission tomography experiments, a map of attenuation coefficients was produced to be used in the attenuation compensation. The use of dual energies, infact
produces high scattering, which is still a problem to overcome. So the problem of attenuation compensation in general is in fact still a complex subject in gamma ray emission tomography. Of the attenuation compensation methods, there is no single one that results satisfactorily in practical use.

3.3 EXPERIMENT OF GAMMA RAY EMISSION TOMOGRAPHY
TO STUDY THE DISTRIBUTION OF SELENIUM

3.3.1 ENERGY SELECTION

The radionuclide used in this experiment was $^{75}$Se bound in Na$_2$SO$_3$ and dissolved in water. This isotope undergoes electron capture followed by multiple gamma ray transitions to the ground state of $^{75}$As. The initial activity delivered by Amersham was 1.27 mCi ($4.7 \times 10^7$ Bq). The transition and decay mode of $^{75}$Se is shown in fig. 3.4. The energy spectrum of $^{75}$Se obtained by NaI(Tl) and BGO detectors is shown in fig. 3.5.

![Gamma ray transitions in the decay of $^{75}$Se.](image)

Fig. 3.4. Gamma ray transitions in the decay of $^{75}$Se.
Fig. 3.5. Energy spectrum in decay of $^{75}$Se using NaI(Tl) (a) and BGO detector (b), with and without lead collimator.

The spectra show multiple full energy photopeaks overlapping one another, because the energy resolution of the detectors is not sufficient to resolve gamma-lines of neighbouring energies e.g. the energies at 121 and 136 keV and then at 265 and 280 keV. The spectra also exhibit lower energy components due to scattering and the characteristic X-rays from the lead collimator.

In multiple Gamma ray Emission Tomography experiments, some criteria should be considered in the selection of a suitable gamma ray energy for data acquisition. The best full energy photopeak for scanning is one which has high intensity with low background and scattering and no interference from other full energy photopeaks. From fig. 3.5, it can be seen that there is no single photopeak that fulfils these criteria. The full energy photopeak at 400 keV which is well separated from other peaks is of very weak intensity. The lower energies of (120+136) keV, when using NaI(Tl) detector are strongly distorted by the X-ray characteristic and scattering from the collimator.
The X-ray disturbance appears less in the spectrum using the BGO detector due to the small size of the crystal (16 mm) and low efficiency at the lower energies. So in an experiment in gamma ray tomography employing NaI(Tl) detector, the coupled energies of 265 and 280 keV, which can not be separated can be used for scanning. The setting of other energy peaks i.e. 400 and 120+136 keV will result in a more noisy and blurred image as shown in the next experiments. Either of the coupled energy peaks (120+136 keV) and (265+280 keV) may be selected in gamma ray emission tomography employing a BGO detector, although they have high background and scattering contributions, since the detector energy resolution is less than that of the NaI(Tl) detector used. The 400 keV energy peak may be set for scanning employing the NaI(Tl) as well as the BGO detector if the activity of the source is high, so that the noise in the image due to the statistical fluctuation can be made suitably low with reference to image quality.

3.3.2 IMAGING THE DISTRIBUTION OF SELENIUM

The source of selenium (1.2 mCi/4.4.10^7 Bq. $^{75}$Se) delivered by Amersham was diluted in 10 ml of water in a glass vial of 22 mm diameter. Since it was considered as an homogeneous medium, the slit collimator of area 1.5x20 mm$^2$ and length 70 mm has been used in Gamma ray Emission Tomography employing NaI(Tl) and BGO detectors. Using this collimator, the step length may be set fixed at 1.5 mm. The number of steps depends on the scanning length, which should be larger than the size or diameter of the object. The number of projection angles in general was less then the number of steps. Filtered Back Projection and
Iterative techniques have been used in the reconstructions to obtain the image of selenium distribution. The selenium distributions are presented in 2D colour and 3D plots.

In a study comparing the distribution of Se obtained employing NaI(Tl) and BGO detectors, the 10 ml of selenium in the glass vial with air as the medium outside the object has been used. The energy selection for scanning was set on the unresolved peaks (265+280) keV and (120+136) keV. The line scan of one projection angle with 30 steps, before performing the reconstruction is shown in fig. 3.6. From this line scan, the base line outside the hot object is not zero line. This base line arising from the back ground and scattering, should be removed before the reconstruction. To remove this, the base line suppression and Compton scattering subtraction techniques have been used. The second method requires selection of two energy windows of the same width i.e. the full energy photopeak and scattering peak adjacent to it in the valley of the spectrum which are defined by two counters.

Fig. 3.6. Line scan of projection data. Det NaI(Tl). Scan 45/1.5 mm. a. Scan on setting energy window at (265+280) keV. b. Scan on setting energy window at (120+136) keV.

The base line suppression technique is much simpler, since it only used the counts in the full energy peak sitting on the
back ground and the correction was performed by suppressing the data so that the base line becomes the zero line. The two energies in fact produced almost the same quality of image. A more quantitative study concerning the quality will be discussed in the next section.

In fig 3.7 are shown the reconstructed images in colour of the selenium distribution employing NaI(Tl) and BGO detectors with the setting energy window at (265+280) keV and the data was corrected by baseline suppression. In the two images, an artifact appears as a ring around the hot object. The image employing the NaI(Tl) detector looks better, i.e. there is evidence of less artifact than in the one obtained with the BGO detector. So better energy resolution will result in less artifact in the image due to less scattering.

Employing the BGO detector, however a better image is obtained compared with the NaI(Tl) detector, if the energy selection is set on the low energy (120+136) keV window, while employing the NaI(Tl) detector the image appears worse due to the X-ray contamination. The quality of image employing NaI(Tl) and BGO detectors is not so much different if the setting energy window is on (265 + 280) keV window.

Reconstruction without correction using the baseline suppression will result in a high intensity ring around the edge of the scan area besides the artifact ring around the hot object. The high base line when setting the energy at (120+136) keV is due to the X-ray characteristic contribution. Correction for X-ray contribution by subtraction using two SCA and two counters, also gave the same blurred image. If the subtractor is of comparable magnitude with the subtracted data, this correction method will result in a large statistical error.
Fig. 3.7. Image of Se distribution, setting energy at (265+280) keV. a. Detector NaI(Tl). b. Detector BGO.

Fig. 3.8. Image of Se distribution, setting energy at (120+136) keV. a. Detector NaI(Tl). b. Detector BGO.
Fig. 3.7. Image of Se distribution, setting energy at (265+280) keV. a. Detector NaI(Tl). b. Detector BGO.

Fig. 3.8. Image of Se distribution, setting energy at (120+136) keV. a. Detector NaI(Tl). b. Detector BGO.
The 3D image representations of the Se distribution employing NaI(Tl) and BGO detectors are given in fig. 3.9. The height is related to the concentration of Se in the object. Since the concentration is homogeneous, it looks flat and going down at the edge, which appears as an artifact in the 2D colour image.

The decreasing intensity around the edge is due to the number of photons received by the detector which is not only coming from the volume in the object covered by the solid angle of the collimator hole, but also from volume covered by the opening solid angle seen by the detector. The gamma rays coming from the collimator wall can also reach the detector. So the number of counts coming from the hot area will be greater than the true value i.e. the line integral of concentration along the collimator direction, and at the edge will get a smaller number of counts, since only half of the opening solid angle contributes to the number of counts detected. This effect reduces the image intensity around the edge. It can be minimized if one employs a detector with a high resolution such as a Ge(Li) and a
long collimator with a small hole, however it will need a stronger source or longer counting times.

The next experiments to study selenium distribution in tissue equivalent material with variation of volume and concentration were performed employing only the NaI(Tl) detector. The glass vial was replaced by a polyethylene vial of the same diameter and the activity was subdivided into smaller volumes i.e. three vials of 5.6 mm diameter each. Two vials contained the same concentration as the host vial and one vial contained half the concentration, making up the rest of the volume by the addition of more water. The volume of solution in each of these vials was 0.2 ml. The vials were then placed in the tissue equivalent material made of artificial rubber which the attenuation coefficient is equivalence with soft tissue. The artificial rubber is made from a fluid rubber compound based on depolymerised natural rubber which can be considered chemically as pollyisoperine i.e. (CsH₈)n and is prepared from the raw rubber by subjecting it to mechanical and thermal processes (Cho 87). The diameter of the tissue equivalent material was 40 mm.

The image of Se distribution with the same concentration but different volume is shown in fig. 3.10a. A line scan through the reconstructed data for the big volume and the smaller volume is given in fig.3.10b. It shows that the colour of the big volume and small volume is different, also the intensity of the line scan. The small volume appears lower in intensity than the big volume. This is because in the reconstruction, it uses the projection data as the integration of concentration rather than the radionuclide intensity per pixel (eq. 2.2). So the colour of the image with different volume is not related linearly to the concentration.
Fig. 3.10. Tomogram of Se distribution using NaI(Tl) detect. Setting energy at (265+280) keV. Two objects of 22 mm and 5.6 mm diameter of the same radionuclide concentr. in tissue eqv. material. b. Line scan through the rec. objecgt.

Fig. 3.11. Tomogram of Se distribution using NaI(Tl) detect. Setting energy at (265+280) keV. Three objects of same volume, diameter 5.6 mm, in tissue eqv. material. Ratio of concentration is 1:0.5:1.
Fig. 3.10. Tomogram of Se distribution using NaI(Tl) detect. Setting energy at (265+280) keV. Two objects of 22 mm and 5.6 mm diameter of the same radionuclide concentr. in tissue eqv. material. b. Line scan through the rec. objecgt.

Fig. 3.11. Tomogram of Se distribution using NaI(Tl) detect. Setting energy at (265+280) keV. Three objects of same volume, diameter 5.6 mm, in tissue eqv. material. Ratio of concentration is 1:0.5:1.
The tomographic representation of Se distribution in 3 vials of the same volume, 5.6 mm diameter but different Se concentrations in the ratio 1:0.5:1 is given in fig 3.11. The colour scale shows the low concentration as much weaker than the other concentration, the intensity of the 0.5 concentration is 20% lower than half the peak of the full concentration. So the colour in 2D image for objects which are different in size and concentration does not depend linearly on the concentration.

Fig. 3.12: 3D representation of Se distribution with various reconstruction techniques. a. Filtered Back Projection. b. ART with 3 iterations. c. Iterative with Conjugate Gradient method, 20 iterations. Three objects of the same concentration, diameter 22 and 5.6 mm.

In fig 3.12 are given in 3D projection the Se distributions reconstructed using Filtered Back Projection and Iteration techniques. The iteration techniques used were ART and gradient
conjugate methods. From these, it can be seen that the reconstruction techniques did not result in significantly different distributions. Use of Iterative techniques with a large number of iterations did not result in improvement of the reconstructed distribution. In ART, 4 iterations is sufficient to obtain a good image. By performing more iterations, it does not resulted in significant improvement of the image.

All the images shown in 2D and 3D projections, do not show significant attenuation effect i.e. darkening of the colour in the middle of the 2D image or hollowing in the 3D projections. The attenuation effect is relatively small since the attenuation coefficient is low and the diameter of the object was not sufficiently large. The attenuation coefficient of this tissue equivalent material at energy 265 keV, is 0.018/mm (Hub 82).

3.3.3 QUALITY OF THE IMAGE

According to the definition quality of the image i.e. how clearly and faithfully structures in the image can be visualized, one should make comparison between the true/expected image and the resulting image. The 2D colour image of an homogeneous radionuclide distribution should show a one colour image and in 3D projection a flat surface.

Comparison of the colour variations in two images can give directly the quality of the image, however it is difficult to represent in quantitative terms, and it may be influenced by subjective reference, which depends on psychological conditions. In 3D projection, the intensity of colour is represented as height of the projection. So the quality of the image can be deduced by comparing the height of the distribution. It will give
A method for obtaining a quantitative value to represent image quality is by making use of the 'fidelity factor' as given in eq. (2.64),

$$Q_F = 1 - \frac{1}{N} \sum_{i,j=1}^{N} \left[ \frac{(A(i,j) - A^*(i,j))^2}{[A^*(i,j)]^2} \right].$$

The problem is how to obtain the ideal or the expected distribution image $A^*(i,j)$. This ideal distribution may be obtained by reconstruction of a model theoretically or by making a model from the reconstructed image $A(i,j)$ based on the geometrical object, assuming there is no degradation in the image, such as noise, artifact and attenuation effect. The first method can be found in the RECLAB package programme (Hue 77). This programme is a simulation model based on the geometrical object and written in FORTRAN. For a simple geometrical object such as a homogeneous distribution in a cylindrical container, the second method is simple to apply by constructing a model.

From the distribution images of Se in a cylindrical container placed at the centre of the turn table, the second method has been used. Fig. 3.13 shows the Se distribution image reconstructed using Filtered Back projection and the expected image. The expected image is not an ideal image with smooth boundaries, since it was constructed from pixels which depend on its size and the step length used. The colour is fully white, which means that the Se distribution is really homogeneous. The variation of colour in the actual image depends on the variation of pixel intensity of the reconstructed distribution $A(i,j)$. Since there is no variation intensity in the expected image, it appears only the maximum colour intensity i.e. white.
The difference between the actual and the expected image is quite constrasting, which makes it difficult to evaluate the quality of the image by comparison of colour alone. From these two images, however one can show the boundary of the artifact, the noise, fluctuation of distribution and attenuation if it is seriously affecting the distribution.

![Image of Se distribution](image)

Fig. 3.13. The Image of Se distribution. a. The actual image from the experiment  b. The expected image.

The 3D projection of Se in the actual and expected distribution is given in fig. 3.14. The height of the expected projection is the average intensity of the actual projection of the hot distribution. From these figures, the image quality may be evaluated by comparing the z intensity. The two reconstruction methods used, Filter Back Projection and ART show no significant difference in quality. The quantitative value of the fidelity factor is evaluated using eq. (2.64). Table 3.2 presents the fidelity factors and the correlation of the Se distribution image.
The difference between the actual and the expected image is quite constrasting, which makes it difficult to evaluate the quality of the image by comparison of colour alone. From these two images, however one can show the boundary of the artifact, the noise, fluctuation of distribution and attenuation if it is seriously affecting the distribution.

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Fig. 3.14. 3D projection of Se distribution images. Det NaI(Tl) with setting energy window at (265+280) keV. a. Filtered Back Proj. b. ART, 3 iterations c. The expected projection.

Table 3.2.
The fidelity factors $Q_f$ and Correlation $Q_c$ of Se distribution. Detector NaI(Tl), setting energy at (265+280) keV, Scan 45/1.5 mm, no. of projections 20.

<table>
<thead>
<tr>
<th></th>
<th>$Q_f$</th>
<th>$Q_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Full photopeak, FBP</td>
<td>0.62</td>
</tr>
<tr>
<td>2</td>
<td>Bacgr suppress, FBP</td>
<td>0.69</td>
</tr>
<tr>
<td>3</td>
<td>Bacgr suppress, ART</td>
<td>0.67</td>
</tr>
</tbody>
</table>

Full photopeak means there was no correction due to background or scattering.
After background suppression, the Fidelity and the Correlation factors increase. The Filtered Back Projection reconstruction gives a slightly better fidelity factor than ART. In table 3.3 is presented the fidelity factors of the Se distribution image with number of projections 24, but the activity was about half that used before.

Table 3.3. The fidelity and Correlation factors of Se distribution images. Detector NaI(Tl), setting energy window at (265+280) keV, Scan 45/1.5 mm, No. project. 24.

<table>
<thead>
<tr>
<th></th>
<th>$Q_f$</th>
<th>$Q_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Full photopeak, FBP</td>
<td>0.61</td>
<td>0.86</td>
</tr>
<tr>
<td>2. Backgr correct, FBP</td>
<td>0.66</td>
<td>0.95</td>
</tr>
<tr>
<td>3. Scatter correct, FBP</td>
<td>0.65</td>
<td>0.92</td>
</tr>
<tr>
<td>4. Backgr+att correct, FBP</td>
<td>0.66</td>
<td>0.91</td>
</tr>
<tr>
<td>5. Backgr correct, ART</td>
<td>0.65</td>
<td>1.00</td>
</tr>
</tbody>
</table>

All the fidelity factors except no. 1 are almost the same. No significant improvement of the image fidelity obtained by introducing the attenuation compensation. The average fidelity factor is slightly lower than before, due to the lower radionuclide activity.

The normal value of fidelity factor is between 0 and 1. From the definition of fidelity (2.69), if $A(i,j) = 2 \times A^*(i,j)$, the value of Fidelity $Q_f = 0$. This does not mean that there is no image transfer at all. If the background $\gg$ number of counts within the photopeak, so that $A(i,j) > 2 \times A^*(i,j)$, the value of the Fidelity factor becomes negative.
This situation is illustrated in fig. 3.15. Using a NaI(Tl) detector with the setting energy window at (120+136) keV, where it is strongly contaminated by X-ray contribution, the number of counts within the photopeak is $\ll$ the X-ray intensity.

![3D projection of Se distribution Det NaI(Tl)](image)

Fig. 3.15. 3D projection of Se distribution Det NaI(Tl).
Setting energy window at (120+136) keV. a. Without correct. b. With X-rays correct. c. With total backgr suppression. d. The same as a. but with ring and base suppression.

The 3D representation of the Se distribution without correction is shown in fig. 3.15. It appears that the high range of hills forming a ring around the edge of the scan area has peaks higher than the hot area in the middle. The fidelity factor of this projection $Q_f = -1.64$. This abnormal value of fidelity is due to the high background from the X-rays contamination. Fig. 3.15b and 3.15c are 3D projections after correction for X-ray
contribution and total background contribution using the base line suppression method. Fig. 15d is the same as fig. 3.15a, but the hill and base line has been suppressed using a manipulation technique i.e eliminating the hill as noise outside the hot object after reconstruction. It looks a much better image than the others. The fidelity factors of the images are given in table 3.4

Table 3.4 The Fidelity factors of Se distribution images.
Detector NaI(Tl) of setting energy window at (120+136) keV, scan 45/1.5 mm, No. of projections 20.

<table>
<thead>
<tr>
<th></th>
<th>Qf</th>
<th>Qc</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Full photopeak</td>
<td>-1.64</td>
<td>1.12</td>
</tr>
<tr>
<td>2. X-ray correction</td>
<td>0.48</td>
<td>0.79</td>
</tr>
<tr>
<td>3. Base line suppress (before reconstr)</td>
<td>0.50</td>
<td>0.78</td>
</tr>
<tr>
<td>4. Edge ring suppress (after reconstr)</td>
<td>0.68</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Full photopeak means there is no correction. From the table, it shows that if the background under the photopeak is much larger than the counts within the full photopeak window, the Fidelity factor becomes negative and correlation factor Qc > 1.

The correction for X-ray and background suppression give a positive value of fidelity, but still result in a low value and are shown to be very noisy in the distribution representation. By eliminating the edge ring as noise outside the hot object, one may obtain a good image with a high fidelity factor, but this technique will be ineffective if the object is placed close to the edge of the scanning area. If the object is not simple symmetry or for multiple objects with variation of volume and
concentration, it will be difficult to eliminate the edge ring, mainly if the ring overlaps with the hot object.

3.3.4 *SPATIAL RESOLUTION AND MINIMUM DETECTABLE CONCENTRATION*

The spatial resolution and minimum detectable concentration are involved in the image quality description. Good image quality and high fidelity means also high spatial resolution and low minimum detectable concentration. To measure the spatial resolution, small point or line sources of the same concentration are required. Spatial resolution of the system is the minimum distance between two objects that still look separate on the image. It can be expressed as the fwhm of the image of a point source or valley of half maximum (vhm) between the image of the two objects.

Since the source activity was not strong enough, the small vials with diameter 5.6 mm and 8 mm, inner and outer, respectively and slit collimator of area 1.5x20 mm², have been used. The two vials contain the same activity and were placed in adjacent positions. The image and line scan through the reconstructed image crossing the two objects are given in fig. 3.16. The two objects can be separated clearly. The image shows that the hot spot in the middle is only 2 steps or two pixels wide, which means 3 mm width, while the width of the objects are 5.6 mm or 4 pixels. From the line scan fig. 3.16b, if the width of the objects is 4 pixels, the gap between the two hot objects is 2 pixels or 3 mm. This 3 mm gap results in a valley to peak ratio of 15%.
Fig. 3.16. The image of Se distribution in two vials with the same concentration and diameter (5.6 mm), closely spaced.

If this gap is narrowed to be 1 pixel, the valley to peak ratio will be still < half maximum, which means that the two objects are still clearly separated. If the gap is < 1 pixel, the valley to peak ratio becomes > half maximum, which means that the objects can not really be separated. So the spatial resolution of this system is 1 pixel or 1.5 mm. The main reason of the non zero intensity seen as the valley between the peaks in this minimum distance, are the acceptance solid angle of the collimator hole, photon scattering and background.

The minimum distance in fact depends on the object position with respect to the collimator and the source activities or the background. If the distance to the collimator is larger or the background is higher, it will result in larger the minimum distance or the spatial resolution. As seen in fig. 3.11, since the concentration of the middle object is only half of the others, it is not clearly separable. The spatial resolution can be improved by narrowing the collimator hole and using a detector.
If this gap is narrowed to be 1 pixel, the valley to peak ratio will be still < half maximum, which means that the two objects are still clearly separated. If the gap is < 1 pixel, the valley to peak ratio becomes > half maximum, which means that the objects can not really be separated. So the spatial resolution of this system is 1 pixel or 1.5 mm. The main reason of the non zero intensity seen as the valley between the peaks in this minimum distance, are the acceptance solid angle of the collimator hole, photon scattering and background.

The minimum distance in fact depends on the object position with respect to the collimator and the source activities or the background. If the distance to the collimator is larger or the background is higher, it will result in larger the minimum distance or the spatial resolution. As seen in fig. 3.11, since the concentration of the middle object is only half of the others, it is not clearly separable. The spatial resolution can be improved by narrowing the collimator hole and using a detector.
with a higher energy resolution capable of resolving the two full energy peaks of the Se energy spectrum, however it will need a much stronger source, due to the lower detection efficiency.

The minimum detectable concentration description is really a complex situation in Gamma ray Emission Tomography. If it is only based on judgment of colour in the image, it is not possible to present the minimum detectable concentration quantitatively. The minimum detectable concentration mainly depends on the collimator hole and detection efficiency. It is related to the signal to noise ratio and time of measurement.

From the basic formula of projection data eq (2.2),

$$D(\Phi, x, r) = \int A(x, y) \exp \left\{ - \int \mu(x', y') ds' \right\} ds,$$

for uniform concentration distribution and no attenuation effect, the projection data becomes,

$$D = A l.$$  \hspace{1cm} (3.1)

$A$ is the radionuclide concentration and $l$ is the length of the radionuclide along the collimator direction.

The number of counts under the photopeak detected is,

$$N_0 = \varepsilon A l a T.$$  \hspace{1cm} (3.2)

where $\varepsilon$ is the total efficiency of the detection, $a$ is the area of the collimator hole and $T$ is the measurement time.

The radionuclide concentration $A$ from this equation is,

$$A = \frac{N_0}{\varepsilon l a T}.$$  \hspace{1cm} (3.3)
What is the minimum concentration of A to obtain the number of counts under the photopeak (N) after reconstruction resulting in an image that will be still a visible image representing the radionuclide concentration of the object clearly? This term is related to the quality of the image. The quantification of the image quality may be represented in terms of the fidelity factor, the normal value of which is between 0 and 1.

What is the minimum fidelity factor to be correlated with the minimum detectable concentration? The fidelity factor 0 does not mean that there is no transfer image and even if it is negative, as shown in fig. 3.15a, it will still show the image of an object, but not very clearly due to the high value of the noise.

It is difficult to make a criterion of the image quality or the fidelity factor to be correlated with the minimum detectable concentration. The image quality in general depends on the background or signal to noise ratio, variation of volume and concentration of objects, collimator hole, number of projection angles, the reconstruction technique used and on handling the scattering and background suppression.

Assuming that the object is a simple uniform distribution of $^{75}$Se in a cylindrical vial. Using a NaI(Tl) detector, with the energy setting at (265+280) keV, the image will be as shown in fig. 3.7. The concentration was 40 μCi/ml (1.5x10^6 Bq/ml).

The fidelity factor is 0.69. By setting the energy window at (120+136) keV, the image becomes blurred and quite noisy (fig. 3.15). The fidelity factor is -1.64 (table 3.4). The signal to noise (background) ratios at the area, from the line scan of the reconstructed data (fig.3.15), were 50 and 0.5 for each high and low energy setting. So to produce an image with a normal value of the fidelity factor > 0, the signal to background ratio at the
If the background counts per unit time = \( N_b \), the total counts per unit time is,

\[ N_t = N + N_b. \]

From the definition of statistical error of counts \( N \),

\[ \sigma_N = \frac{d N}{N} \approx \frac{\sqrt{N}}{N}, \quad (3.4) \]

one can derive the average statistical error of \( N \),

\[ \sigma_N' = \left[ \frac{N_t + N_b}{(N_t - N_b)^2} \right]^{1/2}. \quad (3.5) \]

If the time of measurement is \( T \), the equation becomes,

\[ \sigma_N^2 = \frac{1}{T} \frac{(N_t + N_b)}{(N_t - N_b)^2} \]

\[ = \frac{1}{T} \frac{(N + 2N_b)}{N^2}. \quad (3.6) \]

By substituting \( T \) from (3.6) into (3.3), the concentration \( A \) can be written as,

\[ A = \frac{N \sigma_N^2 N_0}{\varepsilon 1 a (1 + 2N_b/N)}. \quad (3.7) \]

So the minimum concentration is dependent on the signal to background ratio and the statistical error. If these values can be determined, the minimum concentration can be evaluated. Another factor that should be considered is fidelity. This factor is dependent also on the signal to background ratio.
The efficiency of the detection system $e$ can be determined empirically from eq. (3.2) using a standard source. The value of $e$ in this system using $^{75}\text{Se}$ ($7.4 \times 10^4$ Bq), detector NaI(Tl) and collimator 1.5x20 mm is $1.5 \times 10^{-4}$. The background was 20 /sec.

For the signal to background ratio = 3, the diameter of the object $L = 10$ mm and by introducing the statistical error $= 3\%$, from (3.7) the radionuclide concentration is,

$$A_{\text{min}} = \frac{(3 \times 20) \ (3 \times 10^{-2})^2 (3 \times 20) \times 30}{1.5 \times 10^{-4} \times 1.5 \times 20 \times (1 + 2/3)}$$

$$= 1.3 \times 3 \times 10^{-6} \ \mu\text{Ci/ml}$$

$$= 3.5 \mu\text{Ci/ml} (1.3 \times 10^6 \text{ Bq/ml}).$$

With the same factors as above, for signal to background = 4, the concentration $A_{\text{min}} = 48 \mu\text{Ci/ml} (1.8 \times 10^6 \text{ Bq/ml}).$

The fidelity factor of these models was determined empirically using $^{75}\text{Se}$ of activity/volume of 20 $\mu\text{Ci/ml}$ ($7.4 \times 10^5$ Bq/ml), by introducing variation of signal to background ratio with the reference of fig. 3.6. The fidelity factor as a function of signal to background ratio is given in fig. 3.17. The value of concentration $A_{\text{min}}$ and the fidelity factors as function of the signal to background ratio is given in table 3.5.

If the signal to background ratio is a large enough, the statistical error can be reduced. For the signal to background ratio = 10, with the statistical error $= 1\%$, the minimum concentration $A_{\text{min}} = 36 \mu\text{Ci/ml} (1.3 \times 10^6 \text{ Bq/ml})$, however the measurement will take quite a long time with this system. So the factor of measurement time should be considered also to determine the minimum detectable concentration.
Table 3.5. The minimum concentration of radionuclide $A_{\text{min}}$ and the fidelity factors ($Q_f$) as function of signal to background with statistical error 3%.

<table>
<thead>
<tr>
<th>Signal/Backgr</th>
<th>$A_{\text{min}}$</th>
<th>$Q_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$35\muCi/ml$</td>
<td>0.26</td>
</tr>
<tr>
<td>4</td>
<td>48</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Fig. 3.17 Fidelity factor as a function of background to signal ratio. Filtered Back Proj. rec. Scan 45/1.5 mm No. projections 24.

In assigning the minimum fidelity factor, it might be influenced by the psychology factor if it is just related to the appearance of the image on a monitor and even the time of measurement is dependent on the condition of the system used.
So determination of minimum detectable concentration quantitatively is complex due to the many ambiguous factors involved.

2.4 DISCUSSION

The study of the distribution of $^{75}\text{Se}$ in phantoms of tissue equivalent material has been performed using Gamma-ray Emission Tomography employing NaI(Tl) and BGO detectors. Because these detectors are unable to resolve the $\gamma$ - ray lines in the decay of $^{75}\text{Se}$, the full peak of energy window (265+280) keV and (120+136) keV, were chosen in scanning of the objects. Employing a NaI(Tl) detector and a lead collimator however, only the (265+280) keV window could be used to obtain a good image. The low energy setting at (120+136) keV produced a poor image due to the high X-ray interference from the collimator.

In employing the BGO detector, both energy settings can be used but the images show more artifact due to the poor energy resolution. The advantage of using the BGO detector is its small size and better efficiency compared with the NaI(Tl) detector. So if a large number of detectors are used for scanning in Gamma-ray Emission Tomography, employing a BGO detector will exhibit more resolution in the image i.e. smaller pixels because it depends mainly on the size of the detector.

The collimator with a slit of area (1.5x20) mm$^2$ and length 70 mm has been used instead of a circular hole, since the distribution of Se in the object was uniform and the activity used was low.
If a collimator hole with diameter the same as the slit width were used, the measurement would take about 10 times longer, and would produce more noise due to higher background. To exhibit a comparable image quality of $^{75}$Se distribution using a collimator with the circular hole, it will need a source with about 5 times stronger activity than the source used.

The most frequently used reconstruction technique was the Filtered Back Projection method, yet the Iterative technique with ART and Gradient Conjugate methods have been used for comparison. The results in fact did not show much difference in image quality.

The 2D colour images of $^{75}$Se distribution always appear to contain artifacts i.e. weakening the colour on the boundary of the hot distribution. This artifact is caused by the solid angle of acceptance of the collimator hole, the scattering contribution and the energy resolution of the detector. The use of a collimator with a smaller diameter hole and a detector with better energy resolution, able to resolve all the $\gamma$-ray lines, will reduce the artifact, however it will need quite a strong source due to the low detection efficiency of the system.

The attenuation effect in the Se distribution image did not show significant distortion as seen in the Iodine distribution image. The attenuation coefficient in the Se object was in fact very low compared with the attenuation in Iodine and the volume or diameter only half that of the Iodine sample, so the effect of attenuation was small and did not exhibit the hollow expected in the middle of the image. Therefore the correction of attenuation compensation in the Se distribution did not give significant improvement in the image quality.
The image quality was represented quantitatively by a factor of fidelity. To obtain the fidelity factor, one compares the reconstructed image with the expected or ideal image. The ideal image can be produced by construction of a model from the reconstructed image using manipulation of data and constructing the model theoretically. The effect of some correction techniques to improve the image were represented in terms of the fidelity factor.

Correction by background suppression gave significant improvement, comparable with scattering correction using the dual energy setting technique. The first technique was simple to perform, since it did not require dual energy in energy selection. So the baseline suppression technique has been used for correction of scattering and background.

The variation of concentration and volume of the Se distribution could be shown clearly in 2D images and 3D representations, however the intensity of the image does not depend linearly on the concentration distribution of the objects. The same concentration but a smaller volume object suffer due to a weakening intensity of the image. So the intensity or colour of the image in variation of volume does not depend on the concentration of the source linearly since the projection data infact not only depend on the concentration but also on volume of the object. To obtain a more quantitative distribution of Se in multiple objects with variation of volume, correction of volume should be introduced. It is really difficult to perform in practice for general geometrical objects. The study of Se distribution employing Gamma-ray Emission Tomography is more qualitative rather than quantitative due to this effect, if such a volume effect is not introduced.
The spatial resolution was measured using two small objects with the same concentration of Se. With the slit collimator of area $1.5 \times 20 \text{ mm}^2$ and length 70 mm, the spatial resolution or the minimum distance between two objects with the same concentration was 1.5 mm, the same as the collimator width. This is the optimum value of spatial resolution in Gamma-ray Emission Tomography that the minimum distance is the width of the collimator hole used. If the collimator is not sufficiently long and the distance between collimator and object is large, the spatial resolution becomes broader due to the acceptance solid angle of the collimator. It will be degraded also by variation of volume and concentration for multiple objects.

The minimum detectable concentration is a complex description in Gamma-ray Emission Tomography. It is dependent on many ambiguous factors which are difficult to make into fixed values related to the minimum detectable concentration. In 2D black and white or colour images, it is difficult to select which is the minimum intensity of colour that still can be seen clearly. The colour intensity of the image in fact depends on volume. The low value of the signal to background ratio will produce noisy background and result in a blurred image that makes it difficult to distinguish the hot and cold areas. The signal to background ratio in the data does not depend linearly on colour intensity to background ratio in the image. It is influenced by the time of measurement, handling of the scattering and attenuation and the reconstruction technique used. Using the fidelity factor as a comparison between the obtained and expected images, with a slit collimator (1.5 mm x 20 mm x 70 mm) and Filtered Back reconstruction, the minimum detectable concentration in this system is about $40 \text{ nCi/ml (1.5 x 10}^6$
Bq/ml) for objects of 10 mm diameter. This value in fact is still ambiguous, since many assumptions have been introduced in the evaluation and for multiple objects with variation of volume it still needs correction for volume effect.
4.1 INTRODUCTION

The probability of emission of a gamma quantum by a radioactive nucleus depends in general on the angle between the nuclear spin axis and the direction of emission. Under ordinary circumstances, the total radiation from a source is isotropic. An anisotropic radiation pattern can be observed from an ensemble of nuclei that are not randomly oriented.

One distinguishes the meanings of angular and directional correlation. In directional correlation, one observes the direction of two radiations from nuclei which decay through successive emission. In angular correlation, not only the direction but also the linear or circular polarisation of the gamma emission is observed. The directional or angular correlation probability as a function of the angle between the two radiations depends on the nuclear parameters such as nuclear spin, parity, multipolarity and the extra nuclear field acting on the nucleus.

If the nucleus has magnetic dipole moment or electric quadrupole $Q \neq 0$, there is an interaction between the nucleus and the electromagnetic surround or extra magnetic field that may affect the pattern of the angular correlation. The interaction between the nuclear quadrupole and the surrounding electric field depends on the atomic binding site.

By observing the alteration of the angular correlation pattern due to the quadrupole interaction, one may study the
nature of atomic or molecular binding sites in a material of interest. The experiments based on such conditions are called Perturbed Directional/Angular Correlation. In the next section Perturbed as well as Unperturbed Directional Correlation are briefly described.

4.2 BASIC THEORY

The main problem in Angular Correlation theory is how to derive the probability of two gamma rays from a nucleus emitted in successive transition as a function of the angle between the two emissions. The first section which follows, describes the Unperturbed Angular Correlation, the second describes the Perturbed Angular Correlation and some applications of Angular Correlations are introduced in the third section.

4.2.1 UNPERTURBED ANGULAR CORRELATION

Consider a nucleus which emits two gamma rays in successive transition. The mode of decay of transitions in Fig.4.1 shows the nuclear parameters incorporated i.e. nuclear spin I, parity $\pi$, multipolarity L and energy E.

\[ I_1 \quad \pi_1 \quad \gamma_1, L_1 \quad E_1 \]
\[ I_e \quad \pi_e \quad \gamma, L_e \quad E_e \]
\[ I_f \quad \pi_f \quad \gamma_2, L_2 \quad E_f \]

Fig. 1. Successive transition of two gamma rays with the angle between their direction $\theta$. $\tau_e$ is life time of the intermediate state.

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The probability transition of two gamma rays in successive transitions $W(\theta)$ depends on the angle $\theta$, and the nuclear parameter $I$, $\pi$ and $L$. These gamma transitions are governed by the conservation laws, e.g.

\begin{align*}
E_{\gamma 1} &= E_i - E_e \\
E_{\gamma 2} &= E_e - E_f \\
I_i + L_1 &= I_e \\
I_e + L_2 &= I_f \\
\pi_i, \pi_e &= \pi_f
\end{align*} \tag{4.1}

The probability of emission of $\gamma_1$ depends on the parameters $I_1, L_1, \pi_i, I_e, \pi_e$ and its quantized orientation to the fixed space, and for $\gamma_2$ depends on $I_e, L_2, \pi_e, I_f$ and $\pi_f$. The angular correlation function of these two successive transitions is the multiplication of these two probabilities and the angle $\theta$, so it can be written as,

$$W(\theta) = W_1(I_1, L_1, \pi_i, I_e, \pi_e).W_2(I_e, L_2, \pi_e, I_f, \pi_f)$$ \tag{4.2}

The probabilities $W_1$ and $W_2$ can be derived using the Fermi transition rule in quantum mechanics. The derivation of $W(\theta)$ can be found in many references (Fra 79, Bie 64). The final equation of the angular correlation function can be written in simple terms as follows,

$$W(\theta) = 1 + A_{22} P_2(\cos \theta) + A_{44} P_4(\cos \theta) + \ldots$$

$$= \sum_{k=0,2,\ldots} A_{kk} P_k(\cos \theta)$$ \tag{4.3}

where $P_k(\cos \theta)$ are Legendre polynomials and $A_{kk}$ the coefficients which depends on the transition parameters.

$$A_{kk} = A_k(I_1, L_1, I_e, \pi_i, \pi_e).A_k(I_e, L_2, I_f, \pi_e, \pi_f)$$ \tag{4.3a}
k are quantum numbers which depend on the transition polarity and selection rules of angular momenta.

The values of k are always even following from the conservation of parity. The coefficients $A_k$ which depend on nuclear parameters I and L have been tabulated by Ferentz and Rosenweig (Fer 79).

The correlation function $W(\theta)$ as a function of angle $\theta$ between the two gamma rays is the probability transition that these gamma rays are emitted from the nucleus in the same time, within the life time of the intermediate state. It can be measured using two detectors e.g. by measuring the number of coincidences between these two gamma rays. The number of coincidence counts depends on the probability $W(\theta)$, so

$$N(\theta) = N_0 \, W(\theta)$$

$$= N_0 \left[ 1 + A_{22} \, P_2(\cos \theta) + A_{44} \, P_4(\cos \theta) + \ldots \right]$$

$$= N_0 \sum_{k=0,2} A_{kk} \, P_k(\cos \theta) \quad (4.4)$$

where $N_0$ is a normality factor.

For $\gamma$ radiation with quadrupole polarity, $k = 0, 2, 4$; the angular correlation equation will become,

$$N(\theta) = N_0 \left[ 1 + A_{22} \, P_2(\cos \theta) + A_{44} \, P_4(\cos \theta) \right]$$

$$= N_0 \left[ 1 + A_{22} \, (3\cos^2 \theta - 1) \, 1/2 + A_{44} \, (35 \cos^4 \theta - 30 \cos^2 + 3) \, 1/8 \right], \quad (4.5)$$

From measurements of coincidence counts as a function of $\theta$, using this equation, one can evaluate the normality factor $N_0$, and the coefficients $A_{22}$ and $A_{44}$.

From the table of $A_k(I,L,\pi)$ one may deduce the value of nuclear parameters $I,L,\pi$ in which the coefficients of $A_k$ will fit.
with coefficients $A_{22}$ and $A_{44}$ according to the equation (4.3a). This is the main subject in nuclear spectroscopy experiments. If we are interested in the application of the Angular Correlation technique, the perturbation between the nuclear parameters and the surrounding field must be taken into account as the Perturbed Angular Correlation.

**4.2.2 PERTURBED ANGULAR CORRELATION (PAC)**

The perturbation acting on the nucleus is due to electromagnetic fields e.g. static magnetic or electrostatic fields. If the magnetic moment of the nucleus $M \neq 0$, it will interact with the static magnetic field $B_o$. The magnetic moment will precess around the axis of the magnetic field with angular frequency $\omega = M.B_o/\hbar$ (Larmor frequency). This precession will result in altering the angular correlation pattern. The angular correlation $W(\theta)$ now is a function of time,

$$W(\theta,t,B_o) = e^{-t/\tau} \sum_{k=0, 2, \ldots} A_{kk} \{\cos(\theta - \omega t)\}$$

where $\tau = \hbar/(B_o M)$.

The pattern of $W(\theta,t,B_o)$ as a function of time can be measured by employing a delay technique, i.e variation of the delay between the two gamma ray signals in the coincidence system. To be able to observe the alteration of the angular correlation pattern as a function of time $t$, the value of $\tau$ should be $< \text{life time of the excited state } \tau_e$. The angle $\theta$ can be set fixed. This experiment is called a Time Differential Perturbed Angular Correlation experiment (TDPAC). From the graph of $W(B_o,t)$ as a function of $t$, one can evaluate the Larmor frequency $\omega$ and the magnetic moment $M$. 

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It is also possible to measure the Larmor frequency using the time integral technique i.e. by varying the angle $\varphi$ and setting the interval time $> \tau_0$.

The angular correlation $W$, therefore is independent of time, so

$$W(\varphi, B_0) = \sum_{k=0,2} A_{kk} P_k \{ \cos(\varphi - \omega t) \}$$

By evaluating the shifting of the angular correlation pattern, before and after setting the magnetic field $B_0$, the Larmor frequency $\omega$ can be calculated. Such an experiment is called a Time Integral Perturbed Angular Correlation experiment (TIPAC).

Experiments of a Perturbed Angular Correlation using magnetic field obtain the nuclear parameter values i.e. the magnetic moment and related parameters in nuclear spectroscopy. Very little information related to the environment site can be deduced because the perturbation is based on the magnetic interaction. The information about the nature of the atomic site can be obtained if the extra perturbation is an electric field which interacts with the nuclear quadrupole $Q$. The description is the same as with the magnetic field perturbation.

If the quadrupole $Q \neq 0$, it will interact with the electric field surround $V$. The interaction energy is,

$$E_Q = \frac{e Q}{4} \int \frac{\partial^2 \mathcal{V}}{\partial r \partial r}$$

where $$Q = \frac{1}{e} \int (3z^2 - r^2) \rho \, \delta V$$

$\rho$ is the nuclear charge density.

If the nucleus absorbs the energy $\Delta E$, the energy level will be excited according to the selection rule $\Delta m = 1$, where $m = -I, -I+1, ..., I$. The quadrupole moment in this excited state is,
\[ Q_m = \frac{1}{2} (3 \cos^2 \beta - 1) Q \]  

(4.9)

where \( \beta \) is related to \( m \) and \( I \), so that

\[ \cos \beta = \frac{m}{\sqrt{I(2I-1)}} . \]

If the electric \( V \) is in axial symmetry with respect to the \( z \) axis, the interaction energy, is

\[ E_0 = \frac{e \ Q}{4} \frac{\partial^2 V}{\partial z^2} . \]  

(4.10)

The energy level of the excited state,

\[ E_m = \frac{e \ Q_m}{4} \frac{\partial^2 V}{\partial z^2} = \frac{e \ Q \left(3 \ m^2 - I(I+1)\right)}{4 \ I(2I - 1)} \frac{\partial^2 V}{\partial z^2} . \]  

(4.11)

The absorption quadrupole energy from this energy level, is

\[ \Delta = E_{m'} - E_m = \frac{3 \left( m^2 - m'^2 \right)}{4 \ I(2I - 1)} \frac{e \ Q}{\partial z^2} \]  

(4.12)

\[ = 3 \ n \ \omega \ Q \]  

where \( \omega = \frac{e \ Q}{4 \ I(2I-1)h} \), \( n = m'^2 - m^2. \)

If the intermediate state with a spin \( I \) undergoes transition due to this interaction, it will reduce the population state, and therefore will result in attenuating the angular correlation distribution probability. The attenuation of the Perturbed Angular Correlation probability, in the simple equation can be written as follows, (Fra 79)

\[ W(\theta, t) = \sum_k G_{kk}(t) A_{kk} P_k(\cos \theta) \]  

(4.13)

where \( G_{kk}(t) = \sum C(n,k) \cos(n \omega_q t) \)

\( C(I,k) \) is the perturbation coefficient.
From the table of $C(n,k)$ for $n = 0$ and $k = 0$, $C(0,0) = 1$.

In Time Differential Perturbed Angular Correlation (TDPAC) experiments, one measures the Angular Correlation $W(\theta,t)$ or $N(\theta,t)$ i.e. the number of coincidences as a function of the delay time with the angle $\theta$ fixed. From the graph of $W(\theta,t)$, the frequency $\omega_q$ can be evaluated. This $\omega_q$ represents the atomic binding site according to the quadrupole interaction (4.12).

The experiment can be performed also by measuring the asymmetry as a function of the delay time or differential time.

The asymmetry is defined as follows,

$$A_s(t) = \frac{W(180,t) - W(90,t)}{W(90,t)} = \frac{N(180,t) - N(90,t)}{N(90,t)}.$$  

From the fitting graph of theory and experiment, $\omega_q$ can be evaluated and the nature of the atomic binding site may be deduced.

In Integral Perturbed Angular Correlation experiments, the Angular Correlation is measured over a long time interval $\Delta t \gg \tau_e$.

The total integral time of $W(\theta)$ is,

$$W(\theta) = \frac{1}{\tau_r} \int_0^{\tau_r} G_{kk}(t) A_{kk} P_k(\cos \theta) \, dt$$ 

where $\tau_r$ is the relaxation time.

Substituting $G_{kk}(t)$ from (4.13), one obtains,

$$W(\theta) = \sum_k A_{kk} P_k(\cos \theta) C(n,k) \frac{1}{(1 + n \omega_q \tau_r)^2}$$ 

where $G_{kk} = \sum_k C(n,k) \frac{1}{1 + (n \omega_q \tau_r)^2}.$

The experiment can be performed by measuring $W(\theta)$ or $G_{kk}$ as a function of $\omega_q$. The frequency $\omega_q$ which is factors related to the
atomic binding site may be influenced by such as viscosity, temperature and molecular binding type in a crystal.

If the electric potential $V$ is not in axial symmetry, the quadrupole interaction energy becomes,

$$
E_q = \frac{1}{4} e \int \left[ \left( 3 \frac{\delta^2 V}{\delta z^2} - \frac{1}{2} \delta^2 V \right) - \frac{1}{2} \left( x^2 - y^2 \right) \right] \rho \, dz
$$

where $\eta = \left[ \frac{\delta^2 V}{\delta x^2} \frac{\delta^2 V}{\delta y^2} \right] / \left[ \frac{\delta^2 V}{\delta x^2} + \frac{\delta^2 V}{\delta y^2} \right] \quad (4.17)$

is the potential asymmetric factor.

The frequency $\omega_0$ will depend on $\eta$.

The coefficient $G_{kk}$ from (4.16) as a function of $\eta$ can be written as,

$$
G_{kk}(t) = \sum_n C_{nn}(\eta) \cos \{\omega_n(\eta)t\}, \quad (4.18)
$$

This equation is based on the assumption that nuclei or molecules are randomly oriented as in a liquid. The movement or rotation of molecules will change the axes of the principle coordinate system as a function of time. By observing the behaviour of $G_{kk}(t)$ as a function of differential time $t$, the value of $\eta$ may be obtained. This $\eta$ is related to the asymmetry factor of the molecular binding site and this factor can be influenced by viscosity, temperature and pH.

3.2 REVIEW OF SOME APPLICATIONS OF PAC

The applications of the Perturbed Angular Correlation (PAC) are mainly based on the interaction of nuclear quadrupole moment with the extra nuclear electric field. This electric field gradient can not be produced externally and can not be calculated with accuracy as well. The radionuclide isotopes that can be
used in PAC applications are few because they should have a cascade transition and high anisotropy, and the life time should be longer than the measurement time and the life time of the intermediate state is of the order of the time resolution. Particularly in Time Differential measurements, the life time of the intermediate state should be much longer than the time resolution of the coincidence system. All these conditions make the restrictions of PAC applications.

Although so many restrictions have to be overcome, extensive applications have long been performed using certain radionuclides in material sciences and biological problems, as well. The commonly used radionuclides are $^{111}$Cd, $^{111}$In and $^{181}$Hf. The first application of PAC to study polycrystalline In in metal was performed by Lehman and Miller (Leh 56). The polycrystalline Hf metal was studied by Ouseph and Canavan (Ous 64) and the influence of temperature was investigated by Solomon et al (Sal 64). PAC experiments of $^{117}$In and $^{111}$Cd in various Cd compounds were performed by Raghavan et al, who showed the evidence of the ionic nature of Cd and In bonds (Rag 73). A comprehensive study was performed by Haas and Shirley in the determination of quadrupole interactions in metals and insulating solids and further the dynamic effects were studied in solution and gases (Haa 73). A review of the manifold problems in chemical effects to study the influence of temperature and pressure on metallic and ionic crystals was presented by Matthias in 1974 (Mat 74).

The most intensive application of PAC in material science until recently is to study the impurity defects in metals, semiconductors, insulators and intermetallic compounds, as well, in surfaces and interfaces. By melting, diffusion or
implantation $^{111}\text{In}$ or $^{181}\text{Hf}$ as probe atoms, were introduced on substitutional sites of metals. Then the interaction of defects is studied by investigating the quadrupole interaction frequency $\omega$ and asymmetry parameter $\eta$ of the electric field gradient (efg). The application of this method to various fields of impurity defect interaction in metals has proven that it can provide unique information of the probe (Via 88). The study of Nitrogen and Oxygen impurities trapped in Hf, Nb and Ta was performed by Mendes et al (Men 88) and Hydrogen in metals using $^{181}\text{Hf}$ probe was performed by Wiedenger (Wie 88). Schatz carried out a study of the interface compound of the thin film system Cu-In (Sch 88).

The application of PAC for studying the binding site of Se in a crystal and bound tightly in resin using $^{75}\text{Se}$ isotope by Kaplan did not give significant perturbation (Kap 74). He performed the experiments only employing the integral time because the resolution of the coincidence system using Ge(Li)-NaI(Tl) was very poor (30 ns) while the life time of the intermediate state is 0.3 ns. So time differential PAC has not been performed yet.

The research in biophysical problems employing PAC is less intensive compared with applications in material sciences because the biomaterial interests in general are non crystalline coordination which makes the electric field gradient low. The activities of radionuclide probes are sometimes insufficient to obtain good statistics, particularly in the measurements in vivo/vitro when the measurements take longer than the radionuclide life time.

Three lines of biophysical research employing PAC methods may be classified i.e. coordinating geometry of metal sites in zinc enzymes, conformation and dynamics of biophysical macromolecules and identification of binding sites in vivo/vitro.
of either metals or labelled biomolecules.

The first application of the PAC method in biophysics was performed by Leipert and Baldescshwieler (Lei 68) to observe the effects of molecular conformation on the Angular Correlation pattern of gamma rays following the decay of $^{111}$In when this isotope interacts with binding sites of biological macromolecules in solution. The $^{111}$In was added to aqueous solution containing bovine serum albumin with varying the pH. The Angular Correlation pattern was strongly perturbed by variation of pH, which may reflect a change in the effective binding constant.

The dynamics of biomolecules i.e. the rotational diffusion of macromolecules in enzymes was studied by Rich et al (Rich 74). The enzymes of bovine carbonic anhydrase (BCA) were implanted with $^{133}$Ba to study the structural and motional information about macromolecules. These enzymes have assumed a major role in studies because of their physiological functions (controlling function in respiration).

The application of PAC to study the protein - metal interaction, and the motional and structural information about protein molecules using $^{111}$Cd isotopes have been reviewed briefly by Graf (Gra 74). All these studies were mainly based on measurement of quadrupole frequency $\omega_q$ and asymmetry parameter $\eta$. Vis and Idzenga investigated the influence of temperature on the nature of the binding of Cd with DNA molecules using DPAC and IPAC as well (Vis 79). The study of binding type of Cd in tomato plants was performed by Bode and Bruin (Bod 84). It was confirmed that Cd$^{2+}$ most probably is not entering the cell cytoplasm, but is bound to the cell walls. The results however still lack confidence due to the large statistical error since the measurements were performed in vivo.
The applications of PAC to protein identification, conformation and dynamics are concentrated using the zinc enzymes. In zinc enzymes, the divalent zinc ion may often be substituted by the corresponding divalent cadmium ion. The PAC method is capable of discriminating between the coordination geometry in different zinc enzymes. In many zinc enzymes, the metal atom strongly bound in the near vicinity of the substrate binding site is lost when zinc is removed (Bau 74, Bau 85, Vas 82). The biophysical materials that have been investigated were human carbonic anhydrase dissolving in sucrose, carboxyptidase from bovines with variation of pH, the superoxida dismutase from yeast, and alcohol dehydrogenase from horse liver. All the studies used $^{111}$Cd is preference to $^{111}$In because $^{111}$In decays via electron capture which may disturb the electric gradient in molecules. Unfortunately the life time of $^{111}$Cd is only 50 minutes, so the measurement should be as close as possible to the Reactor or Accelerator producing the isotopes.

The detector mostly used in PAC applications is the NaI(Tl) scintillator because of high efficiency and good time resolution. In a well set coincidence system using these detectors, it has been reported that one can obtain time resolution up to 1.5 ns. The radioisotopes mostly used as the nuclear probe are $^{111m}$Cd, $^{111}$In, $^{181}$Hf and $^{133}$Ba.
Table 4.1. The radioisotopes mostly used as nuclear probes in application of PAC.

<table>
<thead>
<tr>
<th>Isotopes</th>
<th>Half life</th>
<th>Gamma trans</th>
<th>$\Gamma_{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{111m}$Cd</td>
<td>48 min</td>
<td>150-245 keV</td>
<td>85 ns</td>
</tr>
<tr>
<td>$^{111}$In</td>
<td>2.8 d</td>
<td>173-245</td>
<td>85 ns</td>
</tr>
<tr>
<td>$^{181}$Hf</td>
<td>46 d</td>
<td>133-482</td>
<td>16 ns</td>
</tr>
<tr>
<td>$^{133}$Ba</td>
<td>10.5 y</td>
<td>365-80</td>
<td>6.4 ns</td>
</tr>
</tbody>
</table>

$\Gamma_{1/2}$ is half-life of the intermediate state.

Recently the new BaF$_2$ detector is preferred in PAC applications because its efficiency is at least comparable with the NaI(Tl) detector and so is the energy resolution. But the most attractive feature is the time resolution, where one can obtain up to 0.2 ns in a coincidence system (Lav 82, Spe 86, Cha 86). For simple PAC measurements, two detectors are needed, while for sophisticated equipment, 4 detectors with a combination of fast and slow coincidence systems are required. If the time resolution employing these detectors can reach < 0.3 ns, one may perform PAC experiments using radionuclides with life times of the intermediate states down to 0.3 ns such as $^{75}$Se and $^{199m}$Hg (2.4 ns).
CHAPTER 5
ANGULAR CORRELATION EXPERIMENTS IN THE DECAY OF $^{75}\text{Se}$

5.1 APPARATUS FOR ANGULAR CORRELATION EXPERIMENTS

The main problem in Angular Correlation experiments is setting up the coincidence system to obtain the number of coincidences between the two gamma-ray energies of interest. The experiments of Angular Correlation in this work mean experiments in Directional Correlation, and so do not involve polarity measurements, but only the direction of the gamma rays.

The coincidence unit is characterized as slow and fast coincidence, which depends on the range of the time resolution, from ns to $\mu$s. The coincidence events can also be obtained by utilizing a Time to Amplitude Converter (TAC). The interval time between two gamma ray signals coming to the TAC will be converted to a pulse height. The number of output pulses as the number of coincidences can then be obtained using a Single or Multi Channel Analyzer (SCA or MCA). The TAC can be used in fast coincidence mode and slow coincidence mode as well. The time spectrum of a Time Differential Angular Correlation can be obtained on the MCA. The schematic diagram of a coincidence system using TAC is shown in fig. 5.1.

If a conventional coincidence is used, the TAC is replaced by a coincidence unit and it is not necessary to use SCA and MCA. The Time Single Channel Analyzer (TSCA) selects the energy required for the coincidence. The delay time between the two signals can be adjusted on the TSCA. In an Integral Angular Correlation experiment, one measures the coincidence number as a
function of angle \( \theta \). In a Differential Angular Correlation measurement, one measures the coincidence number as a function of delay time with a fixed \( \theta \). The pattern of the time spectrum can be obtained directly on the MCA.

Fig. 5.1 The schematic diagram of a simple Angular Correlation experiment using TAC.

The schematic diagram in fig. 5.1 is one of the slow coincidence type, since the amplifier is unable to pickup fast timing pulses. The fast timing can be obtained utilising a time pick-off controller or a Constant Fraction Discriminator (CFD) circuit. The constant fraction timing involves inversion, delaying and recombination of the signals with a fractional part of the second signal to create a zero crossing mark of the rise time. The CFD usually is provided with a low level discriminator. If an upper level discriminator is also provided, the energy
selection can be performed on the CFD. The schematic diagram is the same as fig. 5.1 only the amplifier and TSCA are replaced by the CFD. The more common CFD is not provided with an upper level discriminator. The energy selection is performed using another coincidence circuit. These fast and slow coincidence circuits can be combined as a fast-slow coincidence system. This system is commonly used in Integral or Differential Perturbed Angular Correlation experiments (IPAC and DPAC). The schematic diagram is shown in fig. 5.2.

![Diagram](image)

Fig. 5.2. Fast and slow coincidence circuit for IPAC AND DPAC experiments.

In an IPAC experiment, the number of coincidences \( N(\theta) \) as a function of \( \theta \), is obtained by sorting the pulses from the TAC using the SCA after being gated with energy selection from the slow coincidence. The Time DPAC pattern is obtained directly on the MCA. The Time IPAC can also be obtained directly from the
slow coincidence using a triple coincidence technique i.e. coincidence of three inputs, two from the TSCA as the energy selection and one from the SCA following the TAC as the fast timing input.

In differential asymmetry measurements, two patterns of differential time spectra are measured at two angles $\theta$, 90° and 180°. From the asymmetry definition eq. (4.14),

$$\text{As} = \frac{N(90,t) - N(180,t)}{N(180,t)} ,$$  \hspace{1cm} (5.1)

one asymmetry pattern will be obtained after measurement of DPAC twice using two detectors, first at $\theta = 90°$ and second at $\theta = 180°$. In practice, in order to save time of measurements, more than two detectors are used. By employing 3 detectors, it is not necessary to rotate the detectors, two detectors are placed at fixed positions $\theta = 90°$ and 180° with respect to the third. So two coincidences can be measured simultaneously using two sets of coincidence circuits. Employing 4 detectors and 6 coincidence systems will be more time saving and better statistics can be obtained, since 6 combinations of coincidence spectra can be collected simultaneously.

5.2 EXPERIMENTS EMPLOYING NaI(Tl) DETECTORS

The detectors used were the same as in the Gamma ray Emission Tomography experiments. NaI(Tl) detectors are commonly used in Angular Correlation experiments with regard to their efficiency and timing resolution. With an energy resolution of 7.5 % at 662 keV, infact this detector can not resolve the gamma ray lines in the energy spectrum arising from the decay of $^{75}$Se, i.e (120-136) keV and (265-280) keV (see fig. 3.5).
So the Angular Correlation experiment of the (120-280) keV, could not be performed with isolated energy peaks. The experiments were carried out using the two groups of gamma ray energies (120+136) keV and (265+280) keV. In order to obtain the Angular Correlation pattern of the (120-280) keV combination from the interference contaminated coincidence, a special technique to analyse the data had to be employed.

These detectors have a single output signal only from the anode, so experiments of Angular Correlation employing fast and slow coincidence systems could not be performed. The system used in this experiment was the slow coincidence with the TAC as given in fig. 5.1. The schematic diagram of the electronic modules is shown in fig. 5.3.

Fig. 5.3. Angular Correlation experiment of (120-280) keV in the decay of $^{75}\text{Se}$. The numbers in the blocks are the modules codes Canberra (C) and ORTEC (O).
The signals from the detectors after amplification (C2011) were selected to obtain the energy of interest by the TSCA (O. 455). The (120+136) keV signals were sent to the start and the (265+280) keV signals to the stop input in the TAC (C 1443). From the TAC, one obtains the time spectrum which represents the coincidence between the two signal groups. The analog pulses from the TAC are then sorted by the SCA built into the TAC and sent to the Counter (C 2072). The number of coincidences N(θ) as a function of angle θ were obtained from the counter. The typical time spectrum from the MCA is shown in fig. 5.4.

![Time Spectrum Image]

Fig. 5.4. Typical time spectrum of 120-280 keV in the decay of $^{75}$Se using NaI(Tl) detectors without fast timing selection.

The fwhm of the time spectrum was 33 ± 2 ns. It is too large compared with the half life of the intermediate-state of 120 keV (0.3 ns). So only Integral Time Angular Correlation was performed. From the data N(θ), one evaluates the coefficients A_{22}, A_{44} and the normalisation factor N_0 using the angular correlation eq. (4.4). The equation can be written as follows,
\[ N(I) = N_0 \left[ 1 + A_{22} P_2(x(I)) + A_{44} P_4(x(I)) \right] \]

\[ = A(1) + A(2) P(I,2) + A(3) P(I,3) \]

\[ = \sum_{J=1}^{3} A(J) P(I,J) \quad (5.2) \]

where \( x = \cos \theta \), \( P(I,1) = 1 \), \( P(I,2) = P_2(\cos \theta) \), \( P(I,3) = P_4(\cos \theta) \), \( I = 1,2, \ldots, N; \ N \) is the number of angles \( \theta \).

This homogeneous equation of \( N \) equations with 3 unknown parameters can be solved using a matrix inversion technique (Bev 69). In matrix notation, eq. (5.2) can be written as,

\[ N(I) = A(J) P(I,J) \quad (5.3) \]

The inverse of this matrix equation is,

\[ A(J) = \sum_{I=1}^{N} \sum_{K=1}^{N} \frac{N(I) P(I,K)}{P(I,J) P(I,K)} \quad (5.4) \]

where \( Q(J,K) = \sum_{I} \frac{N(I)}{P(I,J) P(I,K)} \).

By solving eq. (5.4), one can evaluate \( A(J) \) and then the coefficients \( A_{22}, A_{44} \) and \( N_0 \). The errors of \( \sigma_{A11} = Q(I,I) \) (Bev 69). The sample of \(^{75}\text{Se}\) was \( 7.4 \times 10^5 \) Bq (20 \( \mu \)Ci) in 0.2 ml.

The time of measurement was typically 1000 s per angle. From the experimental result \( N(\theta) \) as a function of \( \theta \), shown in fig. 5.5, the calculated values of \( A_{22} \) and \( A_{44} \) were,

\[ A_{22} = 0.095 \pm 0.007 \]

\[ A_{44} = 0.002 \pm 0.006. \]

These values are in fact too far from the expected ones.
Fig. 5.5a. Angular correlation experiment of 120-280 keV in decay of $^{75}$Se. No correction.

Fig. 5.5b. Angular correlation experiment of 120-280 keV in decay of $^{75}$Se. Corrected by the interference coincidence from 136-265 keV.
From the experiments performed by Prasad and Segal (Pra 68), Speidel et al. (Spe 68) and Kaplan (Kap 74), the average value of $A_{22}$ is $-0.40$ (table 5.1).

Table 5.1. Experimental data of Angular Correlation coefficient $A_{22}$ in the decay of $^{75}$Se (120-280 keV).

<table>
<thead>
<tr>
<th></th>
<th>$A_{22}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prasad</td>
<td>$-0.395 \pm 0.020$</td>
</tr>
<tr>
<td>Speidel</td>
<td>$-0.430 \pm 0.006$</td>
</tr>
<tr>
<td>Kaplan</td>
<td>$-0.411 \pm 0.012$</td>
</tr>
<tr>
<td>This work*</td>
<td>$-0.395 \pm 0.029$</td>
</tr>
</tbody>
</table>

*) after the correction from the interference coincidence.

In these experiments they used combinations of NaI(Tl) and Ge(Li) detectors, so the interference coincidence could be removed by employing the high resolution of the semiconductor detector. The disagreement between the result before correction and the expected value was due to the strong interference coincidence from the gamma rays of energy 136 and 265 keV.

From the experiments performed by others, it was shown that the angular correlation anisotropy of 136-265 keV is very weak. The value of $A_{22} \neq 0.02$ and $A_{44} \neq 0$ (Pra 68, Spe 68). It means that this transition is nearly isotropic and if there is perturbation to this transition, it will very difficult to observe. The number of coincidences of 136-265 keV is, infact much larger than those from 120-280 keV, because the fraction ratio of the 136 keV and 120 keV transitions is 64:19. Using the value of $A_{22}$ and the fraction ratio f of these two gamma rays, the corrected N(e) of 120-280 keV can be written as,
\[ N_{c}(\theta) = N(\theta) - N_{0} f W(\theta, 1.36-2.65) \]
\[ = N(\theta) - N_{0} \left[ 1 - 0.04 (1 - 3 \cos^{2} \theta) \right] 0.64/83. \quad (5.5) \]

The angular correlation after correction is shown in fig. 5.5b. The corrected value of \( A_{22} \) and \( A_{44} \) are,

\[ A_{22} = -0.395 \pm 0.029 \]
\[ A_{44} = 0.016 \pm 0.024. \]

The coefficient of \( A_{22} \) is now in good agreement with the expected value. The coefficient of \( A_{44} \) is of less significance, because this factor has only a small influence on the angular correlation pattern.

The errors on the measurements of \( A_{22} \) were large due to the interference and chance coincidences. The interference coincidence can be removed experimentally if one employs high energy resolution such as that of Ge(Li) detector and the chance coincidence can be minimized by using a better time resolution coincidence system.

The large value of \( A_{22} \) in the angular correlation of the 120-280 keV gamma rays shows that this transition is highly anisotropic. If there is significant perturbation from the environment of the Se nuclear site, it will alter the coefficient of \( A_{22} \) and the pattern of the angular correlation. However in this Integral PAC, if the effect of perturbation is small, it is difficult to observe compared with Differential PAC experimentally.
5.3 EXPERIMENT EMPLOYING BaF₂ DETECTORS

The BaF₂ detector is similar to the NaI(Tl) detector in the way of processing photon radiation to light emission in the crystal based on the scintillation process and in employing a PMT to produce the electric output signal. This scintillator detector has turned out to be highly attractive over the recent past associated with gamma ray fast timing measurements. It offers a five times better time resolution compared with the NaI(Tl) detector at the expense of only slightly reduced energy resolution. The specific characteristic of the detector is the emission of fast component radiation peaking at 220 nm with a decay constant of 600 ps, incorporated with a slow component peaking at 310 nm with a decay constant of 600 ns. It has particular advantages over alkali iodide scintillators by the factor of a low solubility nonhygroscopic nature with comparable efficiency for gamma ray detection, but much better time resolution.

The commonly used PMTs for this detector are XP2020Q and RCA8575 which have fast time response and efficiency for ultraviolet radiation. Two cylindrical BaF₂ crystals of 50 mm diameter and 25 mm length used in this work were obtained from Harshaw Chemie BV, as were the two PMTs (Philips XP2020Q) and two Voltage dividers.

The detectors were made by wrapping the BaF₂ crystal in teflon tape and coupling to the PMT with silicon grease in between the surfaces. The detectors were then skirted with black cellotape and shielded by a mumetal can. High voltage operation,
according to the manufacturers recommendations, is between -2000 V and -3000 V with an output current of 1 to 2 mA.

Before employing the detectors in the Angular Correlation experiments, one performs the testing of energy and timing resolution. The energy spectrum of $^{137}$Cs is shown in fig. 5.6. The energy resolution of the 662 keV gamma ray of $^{137}$Cs with HV = -2500 V was found to be $11.8 \pm 0.3\%$. It is somewhat larger than the energy resolution obtained by Schaeffer (Sch 87) i.e. 10.6\%, but with a smaller size (44 x 25 mm). The optimum energy resolution of the BaF$_2$ detector is 9.5\% (Wis 86, Zhu 85).

![Energy spectrum of $^{137}$Cs using BaF$_2$ detector (50 x 25 mm), HV = -2500 V.](image)

The time resolution measurement was performed utilizing the fast-slow coincidence technique with a $^{22}$Na source. The schematic diagram of the apparatus was the same as given in fig. 5.2. The fast timing spectrum on TAC was gated for the energy selection at 511 keV from the TSCA. A typical time spectra of $^{22}$Na with a
certain delay is shown in fig. 5.7. The delay time between the two peaks was used as the calibration time. The time resolution obtained was $5.4 \pm 0.2$ ns. It is in fact much larger than the expected value.

![Fig. 5.7 Typical time spectrum of $^{22}$Na employing BaF$_2$ detector using a fast and slow coincidence system.](image)

The expected time resolution of a BaF$_2$ detector is 0.3 ns (Van 85, Cha 86). Attempts to improve the time resolution have been made by varying the high voltage, by replacing the voltage divider with the same as used by Schaeffer (Sch 87) which consumes high current (15 - 20 mA), and by altering the amplifier and shaping time, however no significant improvement was recorded.

The Angular Correlation experiments in the decay of $^{76}$Se employing BaF$_2$ detectors were performed using a fast and slow coincidence system. The schematic diagram of the equipment is given in fig. 5.8. This set up can be used for Integral and Differential Perturbed Angular Correlations as well (IPAC,DPAC). The energy selection (120+136) keV and (265+280) keV was performed using the TSCA and output coincidence of these energies
gated the time spectrum in the TSCA. The integral time coincidence was obtained from the SCA following the TAC by setting the interval time 10 ns covering to the peak time spectrum (fig. 5.10). The procedure of data acquisition was the same as the Angular Correlation experiment employing NaI(Tl) detectors.

![Diagram of IPAC and DPAC experiments in the decay of $^{75}$Se employing BaF$_2$ detectors. The numbers in the blocks are Canberra (C) and ORTEC (O) code series.](image)

Fig. 5.8. The schematic diagram of IPAC and DPAC experiments in the decay of $^{75}$Se employing BaF$_2$ detectors. The numbers in the blocks are Canberra (C) and ORTEC (O) code series.

The data were corrected of the interference coincidence (136-265) keV according to the eq. (5.5). The typical Integral Angular Correlation of (120-280) keV in the decay of $^{75}$Se employing the BaF$_2$ detectors after correction for the interference coincidence, is shown in fig. 5.9. The coefficients $A_{22}$ and $A_{44}$ obtained were,
A_{22} = -0.46 \pm 0.03
A_{44} = -0.01 \pm 0.02

Fig. 5.9. Angular Correlation experiment of 120-280 keV in the decay of \(^{75}\)Se employing BaF\(_2\) detectors and a fast-slow coincidence system. The data have been corrected for the interference coincidence of 136-265 keV.

The value of the coefficient \(A_{22}\) is larger than the expected average value (-0.40). The distance between the source and detectors was too small because the source was getting too weak (0.5 \(\mu\)Ci/1.8x \(10^4\) Bq). The coefficient of \(A_{22}\) obtained should be corrected by the the finite solid angle. With correction factor for the distance of 10 cm \(Q = 0.97\) (Yat 79), the coefficient of \(A_{22}\) was,

\[ A_{22} = -0.44 \pm 0.04. \]

This value although closer, is still larger than the expected value.
In the Integral Perturbed Angular Correlation (IPAC) experiment, the procedure is the same as in the Unperturbed Angular Correlation experiment. The sample is just replaced with another one to be investigated. By comparing between the unperturbed and perturbed coefficients of $A_{22}$, one deduces the typical perturbation and the nature of the binding site.

The $A_{22}$ value obtained should not be considered as the perturbed coefficient, since the sample was diluted in water, so the perturbation from the environment of the nuclear site can be neglected. The disagreement between the obtained $A_{22}$ and the expected could be due to the instability of the HV power supply, the unsuitably low activity of the source and the timing resolution of the system that still required improvement.

The Time Differential PAC experiment was observed in the same experiment as the Time Integral, but the angle was set fixed. The time spectrum from TAC after being gated with the energy selection was sent directly to the MCA. A typical time differential spectrum of 120-280 keV in decay of $^{75}$Se with $\theta = 180^0$ is shown in fig. 5.10.

To evaluate the differential asymmetry factor, time differential spectra at angles $\theta = 90^0$ and $180^0$ were measured. From these two spectra, the differential asymmetry was obtained according to eq. (5.1). The differential asymmetry $A_s(t)$ spectrum is shown in fig. 5.11.
Fig. 5.10. Typical time spectrum of 120-280 keV in the decay of $^{75}$Se employing BaF2 detectors at $\theta = 180^0$.

Fig. 5.11. Time Differential PAC (asymmetry) of 120-280 keV in decay of $^{75}$Se in Na2SeO3 diluted in water.

The spectrum shows statistical fluctuations around a straight line. It seems that there is no perturbation at the nuclear site from the environment. If there is a significant observable perturbation, the curve of asymmetry will be seen to increase or decrease near the zero time, then to fluctuate around the base line with a pattern according to the perturbation type based on the quadrupole interaction (eq. 4.13 and 4.14).
5.4 DISCUSSION

The experiment of Angular Correlation employing the NaI(Tl) detectors that has been performed was only based on Integral time using a simple coincidence system. The system was a slow coincidence circuit utilising TSCAs and TAC. The Fast timing coincidence has not been performed because the detectors only have one output from the anode.

The experiment of Angular Correlation on the (120-280) keV in the decay of $^{75}$Se, despite utilising such a simple coincidence system, obtained a value for the angular correlation coefficient that was in good agreement with the expected value. A special technique in analysing the data has been performed in order to remove the interference coincidence, since the detectors were unable to resolve the closely lying energy peaks of 120 and 136 keV, and of 265 and 280 keV. To set up a fast-slow coincidence system, the detectors have to be provided with dynode outputs. By using such a system, the time resolution employing NaI(Tl) detectors will be around 2 ns (Van 85).

The Time Differential PAC experiment can be performed if the time resolution of the system is less than the life time of the intermediate state. The nuclear probes that are commonly used in the application of Angular Correlation employing NaI(Tl) detectors are $^{111}$Cd, $^{111}$In, $^{133}$Ba and $^{181}$Hf. The half life of the intermediate-state 120 keV in $^{75}$Se is 0.3 ns, so it is not possible to perform a Time Differential PAC experiment employing NaI(Tl) detectors. Time Integral PAC can be performed, but it will not give specific information about the perturbation, since the perturbation factors such as $Q, \omega$ and $\eta$ can not be deduced.
only from the static coefficient $A_{22}$.

The first experiment of the Time Integral PAC in decay of $^{75}$Se has been done by Kaplan using a Na$_2$SO$_3$ source but bound tightly to a resin (Kap 74). The results showed lack of perturbation, and did not record significant alteration of the angular correlation pattern that was represented by the coefficient $A_{22}$. Time Integral PAC is in fact not commonly used in the application of Angular Correlation experiments compared with Time Differential PAC.

The BaF$_2$ detector offers a possibility to be used in Differential PAC for very short life times of nuclear intermediate states such as in the decay of $^{75}$Se, due to its special characteristic in fast timing which is much better than that of the NaI(Tl) detector. From the experimental testing of the BaF$_2$ detectors in this work, the energy resolution 11.8% obtained, was comparable with the expected value, but the time resolution was unsatisfactory. The energy resolution be obtained by the others are 10 % (Sch 87), and 9.6 % (Zhu 85) and the optimum value 8 % can be obtained by cooling the crystal (Wis 87).

The energy resolution of BaF$_2$ detector is dependent on many factors i.e. dimension of crystal, light reflector material, PMT base voltage and stability of the HV power supply. Two types of voltage divider have been used, Philips PMT base with low current 1 to 2 mA and Valvo PMT base with high current 15 to 20 mA, the same as used by Schaeffer (Sch 87). Both PMT bases gave almost the same energy resolution. The focussing and amplification in the PMT base should be well adjusted to obtain the optimum energy resolution.

The time resolution measured utilizing the fast-slow coincidence circuit and a $^{22}$Na source was 5.4 ns, while the
expected value is 0.5 ns. The typical fast-slow coincidence system in this experiment is commonly used in time resolution measurement and in application of Angular Correlation experiments. The fast time spectrum derived from the CFDs is gated by energy selection from the TSCA.

The time resolution is dependent on the same factors as in the energy resolution description. The most importance one is picking up the fast timing from the output signal. It seems that the slow component which has decay time of 600 ns was still involved in the timing measurement. This is still a problem in this experiment, how to be rid of the slow component in order to obtain better time resolution.

In the Integral Angular Correlation experiment of (120-280) keV in the decay of $^{75}$Se, a fast-slow coincidence system was used. Since the BaF$_2$ detectors are also unable to resolve the closely lying energy peaks of 120 and 136 keV, and of 265 and 280 keV, the interference coincidence was removed using the same procedure as employed with the NaI(Tl) detectors, which had been proven to be successful. The result for the $A_{22}$ coefficient was a little larger than the expected value. This difference is not due to a perturbation factor, but came from the high background coincidence and systematic error, since the source was getting very weak and the power supply was not quite stable. The HV high current supply is very old, still uses valve components and the long periods of counting employed was a source of error.

The result of the Time Differential PAC experiment with a Na$_2$SO$_3$ solution in this work shows a pattern of statistical fluctuation around a straight line from which it may be concluded that there is no perturbation at the Se nuclear site from the surroundings. This conclusion, however was really not justifiable
only from the straight line shown, since the interval time of the spectrum is quite broad, despite the fact that there is no significant perturbation, because the selenium was in water solution. The fwhm of the time differential asymmetry is 3.5 ns, while the half life of the intermediate state of $^{75}$Se (120 keV) is only 0.3 ns. To observe the true perturbation effect in $^{75}$Se, the time resolution should be < 0.3 ns. So with the time resolution obtained, it is very difficult to observe the perturbation and study the binding site of Se.

If there is a significant perturbation, the pattern of asymmetry will show a high fluctuation at zero time then decrease to the base line. The pattern will show the perturbation type which is based on quadrupole interaction.
CHAPTER 6

CONCLUSION AND SUGGESTIONS FOR FURTHER WORK

6.1 STUDY OF SELENIUM DISTRIBUTION EMPLOYING GAMMA RAY EMISSION TOMOGRAPHY

The literature studies concerning selenium show that selenium as an essential trace element and naturally occurring toxicological agent with its high biological activities, has been investigated broadly in biophysics as well as in material physics for more than two decades. However, isolation and characterisation of compound selenium produced in the living organism is made very difficult by the factor of low concentration and high lability and wide distribution in the body organ. The clinical effect of deficiency and excess are still not quite clear, although countless associations have been suggested, many of them by analogy from animal experiments.

Selenium is recognized as an essential nutrition for prevention of several deficiency disease syndromes in animals. The isotope $^{75}\text{Se}$ has been commonly used as a label to study the function of selenium in animals and humans due to the lower beta absorbed dose and higher usable photon yield and suitable long life time. The label test is easy to perform in any hospital with gamma counting facilities and has negligible radiation risk and is more resistance against the deconjugation than other commonly used labels.
Gamma ray Emission Tomography can be used to study the selenium distribution in body organs employing NaI(Tl) or BGO detectors and other gamma-ray detectors if the activity of $^{75}$Se is sufficient to detect in a suitable time of measurement. Employing NaI(Tl) and BGO detectors, however one can not set a single energy window for the scanning, since these detectors are unable to resolve the energy peaks at (120+136) keV and (265+280) keV. Employing a NaI(Tl) detector, only the (265+280) keV energy window can produce good images of the Se distribution. The low energy (120+136) keV is strongly contaminated by the characteristic X-rays from the lead collimator. Employing the BGO detector, the X-ray contamination is low, so both the energy windows can be used. The Se distribution image employing the NaI(Tl) detector exhibits better quality compared with the BGO detector due to its better energy resolution. If the distribution of Se in objects is uniform, a slit collimator can be used to increase the detection efficiency, particularly for low activity sources.

The fidelity factor can be used to represent the image quality and its improvement after some corrections for scattering, back ground, X-ray contamination, attenuation compensation etc. This fidelity factor can be used as a reference to establishing the optimum setting of a scanner system to obtain the best quality of image.

The commonly used reconstruction techniques are Filtered Back Projection and Iterative techniques. Filtered Back Projection is simpler in operation, it does not require big memory in computing operation, and small computers can be used to process the image reconstruction. If attenuation compensation should be incorporated during the reconstruction however, it can
be performed only by the iterative techniques.

In Filtered Back Projection, the attenuation compensation can be given before or after reconstruction. Many attenuation compensation techniques have been introduced, but there is no single one that results satisfactory in practical use. The attenuation effect in the image of Se distribution in tissue equivalent material did not show significant distortion, because the attenuation coefficient is very low.

The variation of concentration and volume of objects containing a Se distribution can be shown clearly in 2D colour images, or 3D representations. For multiple objects with different volumes, however it seems that the small volume undergoes weakening of intensity in comparison to a large volume. This effect still needs more testing. If this volume effect does really exist, it should be taken into account in the quantitative study of Se and other radionuclide distributions.

The spatial resolution was evaluated using two small vials containing Se solution of the same volume and concentration. The result obtained was found to be the optimum spatial resolution of the system i.e. the same as the width of the collimator aperture. The spatial resolution can be improved by narrowing the collimator aperture.

The minimum detectable concentration is a complex description in Emission Tomography due to many ambiguous factors involved which are difficult to put into fixed values. In two dimensional colour images, it is difficult to select the minimum colour intensity which still can be visualised clearly. The colour intensity to background ratio in the image does not linearly depend on the signal to background ratio in the data before reconstruction, because it is dependent on many factors.
such as the time of measurement, the handling of scattering and attenuation effects, the reconstruction technique and the scanner system used. The minimum detectable concentration evaluated in this experiment was even less reliable, since it is based on many assumptions in the assigning of factors related to the minimum detectable concentration.

6.2 STUDY THE BINDING SITE OF SELENIUM EMPLOYING PAC

From the literature study, the applications of Perturbed Angular Correlation (PAC) have been intensively used in material sciences since about 1960. These investigations have demonstrated that PAC provides an excellent non-invasive technique for obtaining structural and motional information about crystals and macromolecules. In material sciences, PAC is mainly used to study the structure and binding site of crystals and alloys containing the impurities In or Cd.

The application of PAC to study biological problems was begun around 1970. Nearly all the experimental work in this area has involved protein solutions and has been directed towards the observation of quadrupole interactions and the use of probe nuclei to follow the dynamic behaviour of solvated macromolecules. Recently, the application of PAC in material sciences and biological problems has become more extensive after the introduction of fast timing BaF₂ detectors.

All the experiments in PAC applications are performed by means of fast-slow coincidence systems with multiple coincidence circuits. So more than two detectors are required. It requires probe nuclei, which have a special nature able to produce the perturbation effect in the Angular Correlation pattern.
The Angular Correlation experiments in the decay of $^{75}$Se in this work were considered as preliminary experiments in application of the Perturbed Angular Correlation. The available sample was in water solution, so the perturbation is very weak. To observe the perturbation in Selenium compounds, in fact needs a sophisticated system for Angular Correlation experiments with regards to the very short half life of the intermediate state of $^{75}$Se.

Employing two NaI(Tl) detectors and a simple coincidence system, only Integral Time Angular Correlation has been carried out. The result obtained values of the coefficients of Angular Correlation $A_{22}$ and $A_{44}$, which were in good agreement with expected values. In analysing the data, a special technique was needed to remove the interference coincidence, because the detectors are unable to resolve the close lying energy lines. A Time Differential PAC experiment has not been performed yet using the NaI(Tl) detectors because these detectors did not have a fast timing output.

The fast and slow coincidence in Angular Correlation experiments has been performed employing BaF$_2$ detectors. The parts of the detectors, i.e. BaF$_2$ crystals, PMTs XP2020Q and Philips PMT bases were supplied by Harshaw ltd. The energy resolution of 11.8 % obtained is reasonable but the measurement of time resolution was not satisfactory. The expected value should be less then 0.5 ns, but the result was 5.4 ns.

The Integral Angular Correlation experiment in the decay of $^{75}$Se was performed using a fast and slow coincidence circuit. The measured $A_{22}$ and $A_{44}$ coefficients were slightly higher than other results. The discrepancy mainly arose from the high background/chance coincidence due to not quite stable power supplies and the
unsuitable activity i.e. low of the source used. The activity had become quite weak by the end of the experiments (0.02 μCi/7x10^2 Bq in 0.2 ml).

The Time Differential PAC experimental results were spectra of angular correlation as a function of time delay. The results showed a statistical fluctuation around a straight line i.e. the unperturbed line spectrum. It means that there is no perturbation acting in the Se nuclear site. This conclusion, however, was not really justified, since the interval of time delay was much larger than the lifetime of the intermediate state of the $^{75}$Se nucleus. The perturbation pattern can only be observed if the time resolution is less than the half life (0.3 ns).

Attempts to improve the time resolution have been made but the results are still far from expected value (< 0.5 ns). It seems that the slow rise time component of the BaF$_2$ emission is still involved in the timing resolution. So the main problem in the work that follows is how to get rid of this slow component.

6.3 SUGGESTIONS FOR FURTHER WORK

The study of the distribution of selenium using Gamma-ray Emission Tomography still needs further experiments in order to obtain more quantitative results. If a stronger source and a $^{75}$Se source of larger volume can be made available, one may carry out experiments with more variation of volume and concentration. By using a large volume and varying the medium inside the hot source as well as outside it, the effect of attenuation may be observed more closely and various attenuation compensations can be performed. The dual energy correction method may be performed if
one employs high energy resolution, such as that obtained with a Ge detector.

From the image of Se distribution with variation of volume, the small volume or diameter of the object suffered from intensity degradation. This volume effect requires still more testing. There is not enough available data concerning the variation of intensity as a function of volume with the same concentration. To perform the experiments, a phantom of large volume and several other objects with smaller volumes are needed. It is better to use a circular collimator aperture rather than a slit collimator to eliminate edge effects. The phantom can contain more than two objects, one fixed large volume and the other small variable volumes. By variation of the small volume diameter and line scan through the objects, one measures the image intensity of the small volume as a function of diameter. The volume effect can be deduced from this function. This volume effect correction is required in a more quantitative study of Se or any radionuclide distribution.

The minimum detectable concentration is still a complex problem in Gamma-ray Emission Tomography due to many factors involved that are difficult to fix. It is related to the minimum intensity of image that can still be shown to be visible, clearly. It will be simpler to derive the minimum detectable concentration empirically rather than theoretically, by measuring the intensity of the image as a function of concentration.

The minimum detectable concentration is dependent on many factors i.e. width of collimator aperture, number of pixels, measurement time, photon energy, handling the background, scattering and attenuation effects. If the minimum detectable concentration can be determined, it can be applied in practice to
evaluate the suitable minimum activity of the source label to be administered to the patients that can still produce reliable images of radionuclide distribution.

In Angular Correlation experiments to study the binding site of selenium, the results are not yet satisfactory due to many problems, mainly related to the time resolution of the coincidence system. The main problem is to achieve a resolution time that is less than the half life of the intermediate state of $^{75}\text{Se}$ (0.3 ns). Time Integral PAC experiments could be performed, but they would only exhibit very rough perturbation factors compared with Time Differential PAC experiments. Employing NaI(Tl) detectors with sophisticated fast and slow coincidence systems, the resolution time may be reduced to about 1.5 ns; that is still much larger than the half life of the intermediate excited state of $^{75}\text{Se}$. So the study of the Se binding site cannot be performed by using NaI(Tl) detectors. They can be used to carry out the PAC experiment employing probe nuclei with long half-lives of intermediate states such as $^{111}\text{In}$, $^{111}\text{Cd}$, $^{151}\text{Hf}$ and $^{133}\text{Ba}$.

The setting up of an Angular Correlation employing BaF$_2$ detectors demonstrated that the system worked, despite the results which were found to be not entirely satisfactory. The time resolution of the system still needs improvement. The expected value is < 0.5 ns, while the resolution time obtained was 5.4 ns. The electronic equipment used in fact was out of date. The system needs good quality electronic components and stable HV power supply.

To build a system for Angular Correlation experiments, which is suitable for use in applications, the system should have high time resolution with a multiple coincidence circuit. Such a
system should consist of 3 or 4 detectors. The simple system employing 3 detectors with 2 sets of coincidence circuits is shown in fig. 6.1. The schematic diagram of more sophisticated systems using 4 detectors with more combinations of coincidence circuits, by utilising AND and OR exists as shown in fig 6.2 (Via 88).

![Schematic diagram of experimental electronic system](image)

Fig. 6.1. Schematic diagram of experimental electronic system in PAC measurements employing 3 detectors and 2 fast-slow coincidence circuits.

In a slow circuit, the gamma energies are analysed (Amp TSCA) and only if the right photons ($\gamma_1, \gamma_2$) have been registered, the fast (routing) unit determines which coincidence event and generates a routing signal which ensures the storage of the event in the memory section of a MCA assigned to this combination. Such a system is really very costly.
Fig. 6.2. Electronic components of a Perturbed Angular Correlation apparatus consisting of 4 fixed detectors and utilizing AND and OR circuits; a large number of combinations of coincidence systems can be obtained.

In Angular Correlation experiments, the sample is considered as a point or line source, since the angle $\theta$ between the two detectors is defined as the subtended angle from a point. If in Emission Tomography experiments the object is small and isotropic, experiments of Angular Correlation and Emission Tomography may be performed simultaneously. If the radionuclide distribution is larger than a point source and is anisotropic or the radiation emitted is attenuated inhomogeneously, then methods to correct for these effects must be introduced.
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