Sea State Monitoring by Radar Altimeter
from A Microsatellite

Yiping Sun

Submitted for the Degree of
Doctor of Philosophy
from the
University of Surrey

UniS

Surrey Space Centre
School of Electronic Engineering, Information Technology and Mathematics
University of Surrey
Guildford, Surrey GU2 5XH, UK

January 2001

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Abstract

This thesis constitutes a general survey and a study of significant extensions to the usual conventional satellite radar altimetry. Historically radar altimeter has been configured to the measurement of mean sea level. It is well known that other statistics such as Significant Wave Height (SWH) and wind speed are in principle recoverable from the radar echo and these are currently of great interest.

It has been the aim in this thesis to optimize such measurements, for a general meteorological application, with less interest shown in absolute measurement of sea level. Current technology makes possible a total Earth survey using a constellation of small satellites, covering the entire Earth sea surface with short revisit time. Such solutions necessitate less cost, lower power, and less precise attitude control than the scientific satellites used hitherto.

The purpose of this thesis is to present a novel two mode radar altimeter for sea state monitoring. SWH is still measured by conventional high-resolution mode, which is not sensitive to off-nadir pointing. An additional novel low-resolution mode is proposed for wind speed measurement. By using this mode, wind speed measurement is much more robust to pointing error than by using conventional high-resolution mode. An improved wind speed measurement can be achieved by using a cost effective small satellite.

Some considerable time was also spent on incorporating SAR (Synthetic Aperture Radar) into altimetry techniques to improve the signal to noise ratio. For sea state monitoring the improvements are relatively disappointing, although greater improvement are expected for ice sheet monitoring.
First, I would like to take the opportunity to thank my supervisor, Dr. Stephen Hodgart, for his considerable and enthusiastic guidance throughout my study. His academic suggestion and criticism is very helpful to complete this thesis. I would also like to thank Prof. Martin Sweeting, whose continuous support and encouragement is invaluable.

I would like to thank Mr. Alex da Silva Curiel, who is the team leader responsible for radar altimeter project in SSTL, has provided me valuable material and suggestion during my study in Surrey Space Centre.

I would like to extend my appreciation to many radar altimeter experts, like Prof. Hugh Griffiths from UCL, Dr. Graham Quartly from University of Southampton, Dr. Keith Raney from APL, Dr. Stefano Sorge from University of Roma, for their kind discussion and suggestion.

Most important, I would like to thank my wife Zhe Liu, for her general support and lasting patience during these three years.

Finally, a very special word of appreciation is due my parents, parents in law, my daughter Niannian and my sisters, for their generosity and tolerance.
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List of Acronyms

ADC (A/D Converter): Analogue to Digital Converter
ADCS: Attitude Determination and Control System
AGC: Automatic Gain Control
ASIC: Application Specified Integrated Circuit
BW: Bandwidth
CW: Continue Wave
DAC: Digital Analogy Converter
DBS: Doppler Beam Sharpened
DCG: Digital Chirp Generator
DDS: Direct Digital Synthesiser
DFT: Discrete Fourier Transform
DSP: Digital Signal Processor
ECMWF: European Centre for Medium Range Weather Forecasts
EM: Electromagnetic
ERS: European Remote-sensing Satellite
FET: Field Effect Transistor
FFT: Fast Fourier Transform
FM: Frequency Modulation
FNOC: Fleet Numerical Oceanography Centre
GPS: Global Positioning System
IF: Intermediate Frequency
LEO: Low Earth Orbit
LNA: Low Noise Amplifier
LO: Local Oscillator
LPF: Low Pass Filter
LSB: Least Significant Bit
MLE: Maximum Likelihood Estimation
MMIC: Monolithic Microwave Integrated Circuit
MSB: Most Significant Bit
MTI: Moving Target Indicator
MWR: Microwave Radiometer
NASA: National Aerospace Agency
OBC: On Board Computer
OBDH: On Board Data Handling
PDF: Probability Density Function
PM: Phase Modulation
PRF: Pulse Repetition Frequency
RA: Radar Altimeter
RA-2: Radar Altimeter 2, the radar altimeter for ENVISAT
RAM: Random Access Memory
RCS: Radar Cross Section
ROM: Read Only Memory
S/N (SNR): Signal to Noise Ratio
SAR: Synthetic Aperture Radar
SAW: Surface Acoustic Wave
SMLE: Suboptimal Maximum Likelihood Estimation
SSC: Surrey Space Centre
SSTL: Surrey Satellite Technology Limited
SWH: Significant Wave Height
TR module: Transmitter Receiver module
TT&C: Telemetry Tracking and Commanding
UoSAT: University of Surrey SATellite
VCO: Voltage Control Oscillator
VLSI: Very Large Scale Integrated circuit
List of Symbols

Chapter 1

$\sigma^0$ Normalized radar cross-section

Chapter 2

$dA$ illuminated area of target

$B$ the bandwidth of Chirp signal

$d$ the decorrelation distance

$E_i$ the incident electronic field at target

$E_r$ the reflected electronic field at radar

$f_0$ the carrier frequency

$L$ the loss of the remote sensing system

$P_p$ peak transmitted power

$P_r$ received power

$P_{fs}(\tau)$ the flat surface impulse response function

$P_r(\tau)$ the mean received echo

$P_r(\tau)$ the radar system point target response function

$q(\tau)$ the sea surface specular point distribution function

$r$ the radius of the first pulse limited circle

$R$ the distance from radar to target

$R_e$ the radius of the earth

$R(0)$ the Fresnel reflection coefficient of the surface at normal incidence

$S_r^2$ the filtered mean square slope

$T_c$ the time duration of Chirp signal

$V$ the amplitude of return signal vector

$\Delta t$ the time resolution of Chirp signal

$\Lambda$ the sea wavelength

$\alpha$ the linear FM rate

$\delta_\theta$ the azimuth resolution
\( \phi \) phase of the Chirp signal
\( \phi_0 \) the phase of output signal
\( \phi_r \) phase of reference Chirp signal
\( \varphi \) the phase of return signal vector
\( \varphi_i \) the phase of each elementary vector
\( \theta \) the angular distance between two target
\( \theta_i \) the incident angle
\( \sigma \) Radar Cross-Section
\( \sigma_0 \) Normalised radar cross-section
\( \tau_0 \) the delay time of a returned echo
\( \tau_d \) the estimated time delay

Chapter 3

\( A(n) \) the surface area limited by the nth Doppler cell
\( d(n) \) the delay time of nth Doppler cell
\( f_{dl} \) one of the observed Doppler frequency
\( f_{DD}(t) \) flat surface response function of Delay Doppler radar altimeter at zero Doppler cell
\( \delta f_d \) the Doppler resolution
\( H \) the altitude of the satellite
\( k_x \) wavenumber at along track direction
\( k_R \) wavenumber at range direction
\( L_a \) the antenna aperture in azimuth direction
\( L_p \) the two way propagation loss
\( n \) the number of Doppler cell
\( P_r(t, n) \) the mean received echo of nth Doppler cell
\( P_i(t, n) \) the flat surface impulse response function of nth Doppler cell
\( R_0 \) the range from satellite to a target on ground at closest approach
\( \Delta R(x_a) \) the extra delay of each point along flight path relative to range \( R_0 \)
\( S_r(t) \) the radar system point target response function
\( V \) the speed of the satellite
\( \alpha_i \) the azimuth angle of one target on the ground
\((x_a, y_a, z_a)\) the coordinates of satellite
\((x_t, y_t, z_t)\) the coordinates of target on ground
\( \Delta \varphi(t) \) the relative phase shift as a function of along track time
\( \delta_x \) the azimuth resolution in meter
\( \lambda \) the wavelength of the carrier
\( \theta \) the azimuth angle of the first target relative to the radar
\( \theta_a \) the antenna beam width in radian
\( \rho \) the radius of a certain point within the footprint
\( \sigma_s \) the standard deviation of the sea surface height distribution function
\( \tau_c \) the compressed pulse width

**Chapter 4**

\( A(\omega) \) the amplitude distortion
\( a_0 \) the ideal amplitude of system response
\( a_1 \) the amplitude of the sinusoid amplitude distortion
\( B(\omega) \) the phase distortion
\( b_0 \) the constant of ideal linear phase term
\( b_1 \) the amplitude of the sinusoid phase distortion
\( C_1 \) is the number of cycles of the amplitude distortion per Hertz in the frequency domain
\( C_2 \) is the number of cycles of the phase distortion per Hertz in the frequency domain
\( E(i) \) the mean echo waveform got from Brown model plus mean noise power
\( g_i \) \( i \)th sample value of the theoretic return echo
\( \bar{g}_i \) \( i \)th sample value of the real echo
\( G_0 \) the peak antenna gain
\( G(\theta, \omega) \) the antenna’s gain described by the angles relative to the boresight
\( h_i \) the slope of leading edge
\( H \) is the satellite’s height
\( l \) the specular point displacement relative to the mean sea level.
$L$ the likelihood function

$L_\nu$ is the two way propagation loss;

$LL$ the log likelihood function

$M$ the number of the samples within the range window

$M_{AGC}$ the average value derive from the AGC gate

$N_x$ the number of samples within leading edge estimated by SMLE

$P_r$ the peak transmitted power;

$P_r(\tau)$ the mean received echo

$P_r(\tau)$ the radar system point target response function

$P_{rs}(\tau)$ the flat surface impulse response function

$q(\tau)$ the sea surface specular point distribution function

$r1$ the value of noise pedestal evaluated from the first 8 samples

$r2$ the value that is equal to $M_{AGC}$

$r3$ the value of summation of $V''(i)$ in the leading edge

$r4$ the summation of $V''(i)$ of the fractional part in the leading edge

$r5$ the value of summation of $V''(i)$ in the first half of leading edge

$r6$ the value of summation of $V''(i)$ in the second half of leading edge

$R$ is the range from the radar to elemental scattering area $dA$ on the surface.

$T_1$ the half leading edge width measured in units of time

$T_2$ the half leading edge width measured in number of range samples

$V(i)$ the ith averaged waveform sample

$V'(i)$ the ith averaged waveform sample after the noise pedestal being removed

$V''(i)$ the ith sample of $V'(i) - M_{AGC}$

$\tau$ the time delay relative to the first nadir return;

$w$ the half leading edge width measured in number of range sample

$\alpha$ the coefficient of optimal linearized $erf(\cdot)$ function

$\epsilon_i$ the difference between input waveform and estimated waveform for each specific sample point

$\epsilon_t$ the average difference within the leading edge

$\epsilon_h$ the slope error signal
\[ \gamma = 2.895 \cdot \sin^2(\zeta / 2) \]
\( \eta \) the pulse compression ratio; \( \lambda \) the radio wavelength
\( \mu \) the normalized frequency
\( \sigma^0(\nu) \) the normalized radar cross section
\( \sigma_s \) the rms height of the specular points relative to the mean sea level.
\( \sigma_p \) Gaussian shape pulse parameter
\( \tau_l \) the range misalignment
\( \psi \) the off-nadir looking angle
\( \xi \) antenna boresight off-nadir looking angle
\( \zeta \) antenna 3dB beam width
\( \Delta \tau \) is estimated range error
\( \Delta h_l \) the slope error

**Chapter 5**

\( G_0 \) the antenna gain at boresight axis
\( H \) the satellite's height
\( L_p \) the two way propagation loss
\( P_r(\tau) \) the mean received echo
\( P_{FS}(\tau) \) the flat surface impulse response function;
\( P_T(\tau) \) the radar system point target response function.
\( q(\tau) \) sea surface specular point distribution function, which is a Gaussian function
\[ \gamma = 2.895 \cdot \sin^2(\theta / 2), \] where \( \theta \) is antenna 3dB beam width
\( \lambda \) the radio wavelength
\( \tau \) the time delay relative to the first nadir return
\( \sigma^0(\nu) \) the normalized radar cross section, it is the function of radar look angle
\( \xi \) the antenna off nadir angle
\( \psi \) the off-nadir-looking angle

**Chapter 6**

\( A_r \) the effective antenna footprint on the ocean surface
\( B_f \) filter bandwidth
$D(n)$ the discrete amplitude of FM signal

$E_r$ the quantization noise power

$f_s$ the sampling frequency

$F$ receiver noise figure

$F_{out}$ the output frequency

$F_{clk}$ the clock frequency of DDS

$F_0$ the start frequency of Chirp signal

$F(n)$ the discrete frequency of FM signal

$G$ antenna gain

$H$ the satellite altitude

$L$ overall loss

$m$ the subsequent frequency multiplication factor

$N$ the number of sampling point

$P_p$ the peak transmitted power

$P_r$ the peak received power

$R_e$ the earth radius

$S_r(t)$ the received echo signal

$S_{dr}(t)$ the de-ramp output signal

$\alpha$ the linear FM rate of Chirp signal

$\phi_p$ the peak phase error of one term expanded by Fourier transform

$\varphi(n)$ the discrete phase of FM signal

$\lambda$ the wavelength of carrier

$\theta$ the off-nadir angle

$\sigma^0$ the normalized radar cross section

$\tau_0$ the compressed pulse width

$\omega$ the angular rate of the dominant phase error distortion

$\Delta f$ the frequency resolution of DDS

$\Delta T$ the sampling period

$\Delta \phi(t)$ the phase distortion function

$\Delta \varphi$ the phase increment value
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Chapter 1 Introduction

1.1 Development of Radar Altimeter

The conventional pulse limited radar altimeter is a mature technique with more than 20 years operational experience. This altimeter is designed to measure with high accuracy the mean sea surface level, typically achieving a few centimeters error. It is also capable of measuring or monitoring the actual sea state: the wave height and wind speed. Some other oceanographic information can also be derived from these measurements, such as tides and ocean currents.

Up to now remote sensing radar altimeters have been optimized primarily for operating over ocean, which is relatively flat compared to the land surface. Recently scientists are also interested in measuring the topographic surface such as land and ice sheets. One of the major problems associated with land and ice altimetry is that the existing remote sensing altimeters work in a broad-beam pulse-limited mode. For a flat surface like the sea, the first return always comes from the nadir. But when this radar altimeter measures the topographic surface, the range measurement is made not necessarily to the nadir point, but to point which is nearest to the altimeter. This causes an interpretation error and misregistration of the measured range. So the conventional pulse-limited radar altimeter is not suited to operate over topographic terrain. Therefore a narrow beam radar altimeter is needed to localize the footprint, so that the location of the first backscattered signal does not depend on the topography of the terrain. Such a narrow beam can possibly be achieved by synthesis, accordingly there have been studies, adapting the technique of synthetic aperture radar (SAR) to this radio altimetry problem [Raney, 1998][Phalippou, 1998].

A description of the early history of radar altimeter is written by McGoogan, et. al.. The NASA activity in radar altimetry from space platforms derives from early efforts in satellite geodesy beginning in 1962. The National Geodetic Satellite Program was established in 1964 as a joint activity of the Department of Defense, the department of Commerce and NASA. The goals of this program were to develop a world datum accurate to ±10cm and to refine the description of the earth's gravity field. An ad hoc
Scientific Advisory Group was formed in 1966 to advise the Geodetic Satellite Office of NASA on the potential application to geoscience of making geodetic measurements to an accuracy of the ±1m and ±10cm. The report of that group led to funding support for applicable measurement systems, including the radar altimeter. The National Academy of Sciences Space Applications Seminar Study at Woods Hole in 1967 also underscored the importance of altimetry to oceanography. Oceanographers emphasized the need for the ability to measure dynamic features of the earth's surface such as the departure of the sea surface from the perfect geoid due to tidal action, ocean currents, and wind forces. In 1968 it became apparent that the measurement accuracy required by oceanographers and geodisists could be achieved from space, and a multidisciplinary group was organized in 1969 to study the application of space and astronomic techniques to the terrestrial environment. The study group explored the scientific value and expanded applications of satellite altimetry and published a report further emphasizing the long-term need for altimeters capable of ±10cm accuracy. For the short-term NASA Earth Survey's Program proposed the development of the altimeter technology to provide a sea-surface height measurement accuracy of ±10cm by 1980s [McGoogan, et. al, 1974].

The first satellite altimeter on Skylab flew in May, 1973; The second was on GEOS3 in March, 1975 and third was onboard SEASAT that was launched in June 1978. The ability of spaceborne pulse-limited radar altimeters to contribute to geodesy and earth physics and to measure ocean wave height and wind speed has been demonstrated. After the short period of SEASAT radar altimeter, US Navy developed GEOSAT series altimeters, while NASA/CNES developed TOPEX/POSEIDON series radar altimeters, and ESA developed ERS-radar-altimeter series. All of these altimeters are mainly for sea surface monitoring, except that ESA's missions have some ability to measure the land topography using a new on board tracker. The brief characteristics of each altimeter is given as follow:

**Skylab**

The primary object of the Skylab altimeter was to serve as a source of experimental data to be used in the design of future satellite radar altimeter systems. The altimeter system is designed to operate in five modes: (1) waveform or impulse-response measurement and altitude determination; (2) radar cross-section experiment; (3)
signal correlation experiment; (4) 10ns pulse-compression evaluation; (5) nadir-seeker experiment. It also served to demonstrate the ability of satellite-borne radar altimeter in the acquisition of geodetic contour and oceanographic information over selected earth-surface areas. Although a considerable body of theoretical literature existed concerning planetary and ocean backscatter [Davies, 1954][Barrick, 1968][Barrick, 1974][Berger, 1972], this program represents the first instance in which such experimental short pulse radar data had been collected from earth-orbit geometry. The performance of Skylab was relatively poorer than the follow-up altimeter missions, but its experimental result founded the theoretical and practical basis for future altimeter missions.

**GEOS-3**

The NASA's Geophysical Satellite-3 (GEOS-3) carried the first instrument to yield useful measurements of sea level and its variability with time. It was a precision satellite radar developed primarily to measure ocean surface topography and sea state. The range accuracy of GEOS-3 was around 0.5m. It obtained the first estimation of surface wind speed from altimeter data.

**SEASAT**

SEASAT radar altimeter utilized precise satellite surface range determination to study the phenomena relating to the detailed shape of the marine geoid and departures resulting from phenomena such as ocean currents, storm surges and tides. It was the first altimeter to achieve ±10cm ranging accuracy on 1-second average. Pulse signal processing yielded significant wave height estimates in the range from 1 to 20m to an accuracy of 0.5m or 10 percent. A determination of the ocean backscatter coefficient beneath the satellite to within ±1dB was obtained via ground processing. SEASAT was the first altimeter using 300MHz RF bandwidth, which is used almost as a standard for all successive radar altimeters to achieve better than 10cm range accuracy [Allen, 1983][Robinson, 1995].

**GEOSAT**

GEOSAT was launched on March 12, 1985. Its primary mission was to provide the dense global grid of altimeter data required to improve the determination of the earth's gravitational field. The secondary mission was to detect mesoscale oceanographic features in a timely manner. It was an US Navy mission. The
GEOSAT altimeter was similar to the SEASAT altimeter in its mechanical, thermal, and electrical interfaces. Different hardware included a 20-watt long life traveling wave tube amplifier, an 8085 microprocessor for improved radiation tolerance, a first digitally synthesized transmit waveform generator and GaAs FET preamplifier for the receiver front end. These changes resulted in an improved measurement precision approaching 3cm for 2m SWH. Comparing with previous altimeter missions, these hardware improvement resulted in a power-efficient light weight radar altimeter

TOPEX/POSEIDON

TOPEX/POSEIDON is a NASA/CNES joint Earth observation mission, launched on August 10, 1992. TOPEX is regarded as the SEASAT successor mission. The mission object is to measure the sea level in way that allows the study of ocean dynamics, including the calculation of the mean and variable surface geostrophic currents and the tides of ocean dynamics, climatology, and meteorology. The TOPEX was the first dual frequency altimeter. This dual frequency design allows for height measurement corrections due to the ionospheric effect on the signal. The TOPEX radar measures the local height to precision of about 2cm over 3s average of data. Poseidon was a first radar altimeter to use solid-state power amplifier. The transmitted peak power reduced to 5W. The total power consumption reduced to 50W. [Zieger, et. al., 1991][Marth, et. al., 1993][Abardie, et. al., 1987]

GEOSAT-Follow-on (GFO-1)

The last launched altimeter was for the US Navy GEOSAT follow-on altimeter mission. It was launched on 10 Feb., 1998. The GEOSAT Follow-on program is the Navy's initiative to develop an operational series of radar altimeter satellites to maintain continuous ocean observation from the GEOSAT Exact Repeat Orbit. The altimetry data from GFO program will be used to obtain ocean topography measurements which can be used to derive the location of fronts, eddies and the current data [Finkelstein, 2000]

ERS-1 Radar Altimeter (RA)

Parallel to US altimeter mission, ESA initiated the European Remote Sensing Satellite program in 1981. ERS-1 satellite was launched on July 17, 1991. The radar altimeter was just one of the payloads on board the satellite. It was the first radar altimeter to
include operation over non-ocean surface as one of its design requirements [Duchossois, et. al., 1995].

**Envisat-1 Radar Altimeter (RA-2)**

The radar altimeter on board Envisat is ESA’s second generation radar altimeter. It is a fully redundant, nadir-pointing pulse-limited radar, operating via a single antenna dish at 13.5GHz and 3.2GHz. Its design is based on that of the ERS-1 radar altimeter, but new features have been added in order to measure echoes from ocean, ice and land masses with improved accuracy and without interruption. The consequence of this design is that the estimation of the range has to be performed off-line using the echo waveforms. This increased demand on the ground processing is compensated by the significant increase in the robustness of the tracker. There is a clear separation between tracking and estimation in RA-2. Four resolutions are available corresponding to the transmitted pulse bandwidth of 320MHz, 80MHz 20MHz and CW signal.

**JASON Radar Altimeter**

After the success of on-going joint NASA/CNES TOPEX/POSEIDON altimeter mission started in August 1992. The follow-on mission JASON is under preparation. The satellite will be composed of a low weight (50kg redundant), low power (75W), low data rate (21kbps) and dual frequency altimeter. The spaceborne radar altimeter will be used to measure three oceanographic parameters of scientific interest: the height of ocean surface with centimeter accuracy, significant wave height and backscattering coefficient. It will be the first full solid state radar altimeter, both for Ku-band and C-band. It employs digital signal generation, digital demodulation and digital signal processing. It will use ASIC’s and MMIC’s intensively and has an improved calibration procedures, all of which features make it the most advanced radar altimeter up to date [Mavrocordatos, et. al., 1994].

**Delay Doppler Radar Altimeter**

Current interest in global climate change has resulted in increased interest in monitoring the continental ice sheet mass balance. This is a task that is well suited to satellite radar altimetry because of the radar’s ability to make measurements that represent the mean ice elevation over a large area and to make this measurement in the presence of cloud cover. Unfortunately, the utility of conventional ocean
altimeters such as TOPEX, ERS-1 and GEOSAT for this measurement is limited because these altimeters operate in a pulse-limited mode. This means that the satellite measures the shortest distance to the ground, which will not necessarily be the actual height above the ground at nadir, whenever there is a high sloping terrain. To overcome this problem, a synthetic aperture radar altimeter has been proposed that can improve the along track resolution by using the synthetic aperture technique and hopefully cope with the along track slope. The effect of across-track slope can be removed in principle through the use of a phase-monopulse approach. With such a technique the angle to the point of first return from the ice surface is measured along with the range, by displacing two antennas in the across track direction [Raney, 1995][Jensen, 1995].

1.2 Object of This Research

The primary purpose of radar altimetry has been to determine the ocean surface topography, i.e. mean sea level. But with high range resolution and highly stable radar system, in principle one can measure the Significant Wave Height (SWH) and wind speed. Sea state information (SWH and wind speed) would be of obvious benefit to all marine users requiring such meteorological data. To cover the Earth sea surface with a ‘revisit time’ in the order of hours rather than days or weeks, a constellation of low Earth orbiting satellite is needed. Economics dictates that these should be cost effective microsatellites.

A cost-effective microsatellite means a microsatellite having limited power supply and limited attitude control accuracy. Any method that can improve the power efficiency and relax the requirement of attitude control accuracy will benefit such a radar altimeter mission.

Because a synthetic aperture radar altimeter can make use of more energy returned from footprint, in principle it can improve the signal to noise ratio [Raney, 1998]. This result is attractive to people who want to operate radar altimeter in a power limited condition. I have investigated the signal-to-noise ratio improvement by synthetic aperture processing for varying SWH conditions. The results show for the condition of low SWH the improvement is about 7dB. The improvement will be only about 4dB at 8m SWH and will be even less at higher SWH. These results show that synthetic aperture processing can benefit flat surface measurements such as ice sheet
and sea surface which have a low SWH. But signal to noise ratio improvement is very limited in a high SWH condition.

This thesis goes on to investigate sea states monitoring by using a two-mode radar altimeter. The attitude control error of a typical cost effective microsatellite is assumed to be about 0.5°. With this order of nadir pointing error a conventional radar altimeter with 1 meter antenna can achieve a satisfactory SWH measurement, in a meteorological application. But measurement of wind speed, which is derived from $\sigma^o$ (normalized radar cross-section), is much more sensitive to off-nadir pointing error. We argue that $\sigma^o$ estimation error would be no more than 0.4dB. We show that to achieve this we need a nadir-pointing error no greater than 0.2° using a 1m antenna. This accuracy is very critical for a cost effective microsatellite. A smaller antenna may mitigate the sensitivity to pointing error but requires a much higher peak transmitted power to compensate this link budget loss.

It seems that to improve wind speed measurement with limited power supply, we still need an antenna having 1m aperture. Such an antenna conventionally forces an unwanted narrow beam width and therefore a high accuracy pointing requirement. In this research a new additional low-resolution mode is proposed to measure normalized radar cross section. By using this mode, still using 1m dish with its narrow beam width, we can get much better wind speed measurement, even when pointing error is as large as 0.5°. Finally a two-mode radar altimeter is proposed for improved sea state monitoring on board a cost effective microsatellite. Details of the performance of SWH and wind speed measurement are evaluated by simulation. Some preliminary system design has been considered.

1.3 Structure of Thesis

In chapter 2, an introduction to the principle of radar altimeter is given. Also in order to support this introduction a brief explanation of radar technology and some concept of oceanography are given.

In chapter 3, I explain the principle of synthetic aperture radar altimeter, including the algorithm of synthetic aperture processing, the derivation of flat surface impulse response function for different Doppler cell and the simulation of synthetic aperture processing. Also in this chapter, I evaluate, for what seems to be the first time in open
literature, the signal to noise ratio improvement of synthetic aperture processing at different SWH conditions.

In chapter 4, the principle of signal processing of a conventional radar altimeter is given. Signal processing simulation is carried out by software. The echoes are generated by using the Brown model [Brown, 1977]; the on board signal processing is based on Sub-optimal Maximum Likelihood Estimation. In order to evaluate the system distortion effect on the signal processing, the antenna off-nadir pointing error, signal amplitude and phase distortion are studied by this simulation.

In chapter 5, a novel low-resolution mode radar altimeter is proposed to measure the radar cross-section. As we have mentioned, this new mode is much more robust to pointing error than a conventional radar altimeter when it is used to measure the radar cross-section. But this low-resolution mode requires an accurate sampling time that can be derived from the high-resolution mode radar altimeter. Thus a combined two-mode radar altimeter operating in different modes alternately can give satisfactory sea state monitoring.

In chapter 6, the link budget of the two-mode radar is studied. It shows that a 1m antenna is needed to operate the altimeter in a power-limited condition. The digital Chirp generation is studied based on a direct digital synthesizer (DDS). By using a high speed DDS the Chirp signal is generated in IF so that single side band modulation can be avoided. This will greatly improve the quality of the generated Chirp signal. The effects of phase and amplitude distortion are also discussed in this chapter.

In chapter 7, a summary and conclusion of this study is given. The main achievement of this study is outlined.
A radar altimeter is a highly accurate ranging instrument. It looks down towards the ground, and the delay time of the returned pulse can be used to determine the distance from the radar to the ground. The philosophy of radar altimetry is shown below:

An altimeter can measure the round-trip delay from satellite to the nearest target on the ground. If we know the propagation speed of microwaves, we can determine the distance from the satellite to the ground. As shown in Fig. 2-1, there are extra delays when an electromagnetic wave propagates through ionosphere and troposphere. Using a two-frequency altimeter or using a global ionospheric model one can correct the ionospheric delay. The tropospheric delay correction is based on operational forecast analyses of surface pressure and temperature from different organizations (the Fleet Numerical Oceanography Center (FNOC), European Center for Medium Range Weather Forecasts (ECMWF)) and the on board radiometer. A detailed description of the correction algorithm can be found in [Cudlip, 1994].
Chapter 2 Principle of Radar Altimeter Remote Sensing

Beside the sea surface topography measurement, by investigating the radar cross section we can know the roughness of the sea surface, from which the wind speed can be determined. Then by studying the slope of the leading edge of the mean returned pulse from the sea surface, the significant wave height can be determined. All these will be analyzed in next section.

2.1 Basic Radar Technology

RADAR is the acronym of Radio Detection And Ranging. It is a powerful microwave sensor that can provide its own illumination and penetrate cloud. Radar has become the most important remote sensing tool for the past 20 years. The radar altimeter and Synthetic Aperture Imaging Radar are the most successful examples of radar remote sensing. In this section, I will give some basic radar theory that is useful to understand radar altimeter. Detailed information can be found in some reference books. [Skolnik, 1970][Raymond, 1965][Cook, 1967]

Radar Cross Section (RCS) of Target

Radar Cross Section $\sigma$ (dimension of area) is defined as $4\pi$ times the ratio of the power per unit solid angle scattered back toward the transmitter, to the power per unit density (power per unit area) in the wave incident on the target. In other words, if at the target the power incident on an area $\sigma$ placed normally to the beam were to be scattered uniformly in all directions, the intensity of the signal received back at the radar set would be just what it is in the case of the actual target. [Ridenour, 1965]

$$
\sigma = 4\pi R^2 \left| \frac{E_r}{E_i} \right|^2
$$

(2.1)

$R$ is the distance from radar to target;

$E_r$ is the reflected electronic field at radar;

$E_i$ is the incident electronic field at target.

The radar cross-section is proportional to the illuminated area if other characteristics of the target are the same, The normalized radar cross section is defined

$$
\sigma_0 = \sigma / dA
$$

(2.2)
\( dA \) is the illuminated area.

The normalized radar cross-section (RCS) is a very useful concept when discussing distributed targets. Mapping of the characteristics of distributed targets are related to the normalized RCS, which can be derived from the radar equation:

\[
P_r = \left( \frac{P_p G}{4\pi R^2} \right) \sigma \left( \frac{1}{4\pi R^2} \right) \frac{G\lambda^2}{4\pi} \frac{1}{L} \tag{2.3}
\]

- \( P_r \) is the received power;
- \( P_p \) is the peak transmitted power;
- \( G \) is the antenna gain;
- \( L \) is the loss of the remote sensing system.

In order to determine the radar cross-section \( \sigma \), we have to make an accurate calibration of \( P_r, P_p, G, L \). This is one of the most important and difficult tasks of remote sensing.

![Fig. 2-2 Vectorial summation of returns from scatterers within one pixel](image)

Because at every instant of time the radar pulse illuminates a certain surface area that consists of many scattering points, the returned echo is the coherent addition of the elementary echoes from this large number of points. The returns from these points add vectorially and result in a single vector that represents the amplitude \( V \) and \( \varphi \) of the total echo Fig. 2.2. The phase \( \phi_i \) of each elementary vector is proportional to the distance between the sensor and the corresponding scattering point. If the sensor moves over some distance, all the phase \( \phi_i \) will change leading to a change in the
composite amplitude \( V \). Thus, successive observations of the same surface area as the sensor moves by will result in different values of \( V \). This variation of \( V \) is called **fading**. In order to characterize the backscatter properties of the surface area, many observations need to be acquired and then averaged. Similarly, if we take two neighbouring areas that have the same backscatter cross section \( \sigma \) but have somewhat different fine details, the returned signals from the two areas will be different. Thus an image of homogeneous surface with a constant backscatter cross-section will show brightness variations from one resolution element to the next. This is called **speckle**. Speckle noise follows a Rayleigh distribution if it is detected by linear detector; and follows an exponential distribution if it is detected by square law detector. In order to measure the backscatter cross section of the surface, the returns from many neighbouring pixels will have to be averaged [Elachi, 1988].

**Resolution of Radar System**

Radar resolution is the ability of radar to distinguish two targets. It can be classified in terms of range resolution, angular resolution and Doppler resolution. Range resolution is the radar’s ability to distinguish two targets in range direction or in time. It is usually determined by the transmitted pulse width [Woodward, 1953]. This is easy to understand. But with the technique of pulse compression, the final compressed pulse width is proportional to the reciprocal of signal bandwidth. So fundamentally range resolution is measured by the signal bandwidth. The wider the bandwidth, the higher the range resolution.

Angular resolution is referred to radar’s ability to distinguish two targets in a plane that is orthogonal to the line of sight of the radar. Usually it is defined by radar’s beam width and the range from radar to target. High angular resolution requires narrow beam and requires a large antenna. This can be costly and difficult to manufacture, especially for the space borne radar. The angular resolution is defined as;

\[
\delta_\theta = R \times \theta \tag{2.4}
\]

\( \delta_\theta \) is the azimuth resolution;

\( R \) is the distance from radar to target;
\( \theta \) is the angular distance between these two targets.

When two targets are at the same range and within the same radar beam, it is difficult for conventional radar to distinguish them. But if these two targets move at different speeds with respect to the radar, they will generate different Doppler shifts. Modern radar systems can make use of this feature to distinguish different targets. The Doppler resolution is determined by the time during which radar echoes are processed coherently. Moving target detection radar (MTI) [Skolnik, 1970], pulse Doppler radar and synthetic aperture radar (SAR) [Curlander, 1991] are good examples of radar that makes use of Doppler resolution.

**Chirp Radar**

![Chirp Radar Diagram](image)

In a Chirp radar, a pulse is synthesised having a high bandwidth-time product, typically in the form of a pulse having linear frequency modulation. In a system, the effective compressed pulse width is a small fraction of the width of the transmitted linear FM pulse. In the receiver, some method is required to compress the pulse return, and the de-ramping technique is one such method. As shown in Fig. 2-3, the Chirp signal bandwidth is \( B \), the time duration is \( T_c \). The reflected signal from a point target is first mixed with a local generated replica Chirp signal. The mixing process removes the linear frequency modulation of the received signal and gives an output.
that has a constant frequency proportional to the range of the target, which is \( \frac{B}{T_c} \Delta t \), where \( \Delta t \) is the delay time relative to the local Chirp signal. After passing this output through a spectrum analyser the range information is mapped to the frequency domain and the range resolution becomes frequency resolution. As shown in Fig. 2-3, after de-ramp processing, the wideband signal transforms into a very narrow bandwidth output signal. Effectively high range resolution is achieved in the frequency domain.

The effective width of this frequency-compressed signal, which ideally has a \( \frac{\sin x}{x} \) form, is equal to the reciprocal of the overlap time \( \Delta f = \frac{1}{T_c} \) (because \( \Delta t \) is very small comparing with the Chirp pulse length \( T_c \), the overlap time is assumed to be equal to Chirp pulse length \( T_c \)). The corresponding time resolution is

\[
\Delta t = \Delta f \cdot \frac{T_c}{B}
\]

(2.5)

Application of the de-ramp pulse compression requires that the total time span of the signals being processed should be much less than the transmitted pulse width, otherwise truncation will result in a loss of power and reduction of the range resolution. Typically the time extent of altimeter echoes is within about 300ns and the Chirp pulse length usually is 100\( \mu \)s, so the loss of power and resolution can be ignored.

2.2 Brief Theory of Altimeter Oceanography

Three main parameters are measured by radar altimeter: mean sea surface height, significant wave height and wind speed. In order to understand radar altimetry well, it is necessary to understand the physical process of radar measurement. In this section, I will first introduce some basic theory of microwave reflection from a rough sea surface and the factors that control the roughness, I then discuss the space and time scale of oceanographic phenomena that is to be measured.

2.2.1 Microwave scattering from rough surface

The radar cross section \( \sigma \) was defined in the last section. It is a function of viewing angle, frequency and electromagnetic properties of the propagation medium. It is the
dependence of $\sigma$ on the environmental properties of the surface that enables the microwave sensor to be used as a remote-sensing instrument.

**Specular Reflection**

Radar can only receive backscattered energy. For near-nadir looking, specular reflection is the principle process for returning energy to the sensor. The magnitude of $\sigma$ is proportional to the area of surface whose inclination is such as to directly reflect energy back to the sensor [Robinson, 1995]. For a flat calm surface, the surface as a whole will reflect energy back to a sensor emitting and viewing at nadir, but any other incident angles will result in no reflection back to the sensor. As it is roughened, the surface will present many facets, as shown in Fig. 2-4, reflecting energy in different directions. So that return energy to a nadir sensor will reduce. Conversely, off-nadir sensors will now receive some energy. But since even the roughest sea surface is unlikely to have slopes tilted at more than $20^\circ$-$25^\circ$ from the horizontal, specular reflection is important only for viewing angles between $0^\circ$-$15^\circ$.

![Fig. 2-4 Decomposition of natural surface into facets and slightly rough component](image)

A natural surface can be mathematically described as a series of large facets on which the small-scale roughness is superimposed, as shown in Fig. 2-4. The facets correspond to long wave length roughness, and small roughness correspond to short wave length roughness, which is described by surface height spectrum [Durden, 1985]. The radar backscatter from a collection of facets is fairly straightforward.

![Fig. 2-5 Radiation pattern from a set of facet](image)
The radiation pattern from a set of facet are shown in Fig. 2-5, assuming that each facet is perfectly smooth. When rough component is added, the only effect is that the strength of each pattern is reduced [Elachi]. The theoretical analysis can be found in [Barrick, 1968]. The normalized radar cross section for specular reflection is:

\[
\sigma^0(\theta_i) = \frac{|R(0)|^2}{S_f^2} \sec^4 \theta_i \exp\left[-\frac{\tan(\theta_i)}{S_f^2}\right] \tag{2.6}
\]

\(R(0)\) is the Fresnel reflection coefficient of the surface at normal incidence;

\(S_f^2\) is the filtered mean square slope, representing the portion of surface roughness elements with length scale greater than diffraction limit. The long wavelength sea wave elements will contribute to the facet slopes.

\(\theta_i\) is radar incident angle.

For nadir-looking radar altimeter, the \(\theta_i\) is zero. This leads to

\[
\sigma^0(\theta_i) = \frac{|R(0)|^2}{S_f^2} \tag{2.7}
\]

Because \(S_f^2\) is related to wind speed and it can be derived from normalized radar cross-section, so the wind speed can be derived from the normalized radar cross section.

**Bragg Scattering**

![Subdivided rough surface into its Fourier spectrum components](image)

At large incident angles, the surface scattering is dominated by the effect of the small-scale roughness. For this kind of scattering, the Bragg model is commonly used. As shown in Fig. 2-6, a random surface can first be divided into its spectral components;
then the assumption is that the backscattered return is mainly due to the spectral component $\lambda$ that leads to Bragg resonance with the incident wave. This occurs when

$$\Lambda = \frac{n\lambda}{2\sin \theta_i}, \quad n = 1, 2, \ldots$$  \hspace{1cm} (2.7)

$\Lambda$ is the sea wavelength;
$\lambda$ is the radar wavelength;
$\theta_i$ is the incident angle.

The first term ($n=1$) leads to the strongest scattering. Only Bragg scattering can be seen by imaging SAR, which is much weaker than specular scattering. From (2.7), because $\lambda$ is only few centimeters, only small wavelength $\Lambda$ can contribute to Bragg scattering. Long wavelength will modulate the incident angle $\theta_i$. So this modulation can also be seen in SAR image as wave feature.

2.2.2 Space and Time Scales of Oceanographic Phenomena in Relation to Altimeter Remote Sensing

In last section, we found that the radar’s return is greatly affected by the sea surface wave shape. Surface waves are found in the ocean with periods ranging from a few tenths of a second to around twenty seconds. There are of course wavelike motions occurring in the sea on much longer timescales of minutes, hours, or days, but these are associated with the dynamical phenomena such as tides or mesoscale eddies. All these phenomena have different space scales. Within the range of surface waves, it is convenient to make the distinction between capillary waves at the short millimetre wavelength end of the spectrum, and swell, which has waves of several hundred metres length. The middle range of wavelengths between centimetres and tens of meters are generally termed wind-waves. The measured local wind speed is usually related to capillary waves and wind-waves, but also affected by swells that are generated far away. These waves are closely related to altimeter wind speed algorithm. The length and time scales of ocean surface waves are listed below [Robinson, 1995]:
Table 2-1 Length and time scales of ocean surface wave

<table>
<thead>
<tr>
<th></th>
<th>Wind wave</th>
<th>Swell</th>
<th>Capillary wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height from trough to crest</td>
<td>5mm to 10m</td>
<td>10 to 20m</td>
<td>a few mm</td>
</tr>
<tr>
<td>Wavelength</td>
<td>0.05 to 100m</td>
<td>100 to 500m</td>
<td>1 to 50 mm</td>
</tr>
<tr>
<td>Periods</td>
<td>1 to 10s</td>
<td>10 to 20s</td>
<td>&lt;1 s</td>
</tr>
<tr>
<td>Variability length scale</td>
<td>100km</td>
<td>1000 km</td>
<td>10 m</td>
</tr>
<tr>
<td>Variability time scale</td>
<td>1 hr</td>
<td>10 hr</td>
<td>10 s</td>
</tr>
</tbody>
</table>

Besides sea surface wind, a very important content of satellite oceanography is to study the dynamic ocean phenomena -- as shown below [Robinson, 1995]:

Table 2-2 Length and time scale of dynamical ocean phenomena

<table>
<thead>
<tr>
<th></th>
<th>amplitude m</th>
<th>water velocity m/s</th>
<th>length scales km</th>
<th>Time scale</th>
<th>surface feature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equatorial currents</td>
<td>0.3</td>
<td>0.01</td>
<td>5000</td>
<td>month to year</td>
<td>surface slope</td>
</tr>
<tr>
<td>Large ocean gyres</td>
<td>0.5</td>
<td>0.01</td>
<td>3000</td>
<td>one to many years</td>
<td>surface slope</td>
</tr>
<tr>
<td>Western boundary current (e.g. Gulf stream)</td>
<td>1.5</td>
<td>1</td>
<td>100</td>
<td>days to years</td>
<td>surface slope, surface roughness</td>
</tr>
<tr>
<td>Eastern boundary current</td>
<td>0.3</td>
<td>0.1</td>
<td>100</td>
<td>days to years</td>
<td>surface slope</td>
</tr>
<tr>
<td>Rings (e.g. from Gulf stream meanders)</td>
<td>1</td>
<td>0.1</td>
<td>100</td>
<td>week to years</td>
<td>surface slope</td>
</tr>
<tr>
<td>Mesoscale eddies</td>
<td>0.25</td>
<td></td>
<td>100</td>
<td>100 days</td>
<td>surface slope</td>
</tr>
<tr>
<td>Ocean fronts</td>
<td>0.05</td>
<td></td>
<td>10</td>
<td>10 days</td>
<td>surface roughness</td>
</tr>
<tr>
<td>Ocean tides</td>
<td>1</td>
<td>0.1</td>
<td>1000</td>
<td>12hr to 1year</td>
<td>surface slope</td>
</tr>
<tr>
<td>Shelf sea tides</td>
<td>5</td>
<td>1</td>
<td>100</td>
<td>4hrs to 1year</td>
<td>and roughness</td>
</tr>
<tr>
<td>Internal waves</td>
<td>0.10 cm</td>
<td>0.1-100 secs</td>
<td>surface roughness</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Storm surges</td>
<td>1</td>
<td>1</td>
<td>100</td>
<td>hours to days</td>
<td>surface slope</td>
</tr>
<tr>
<td>Tsunami</td>
<td>0.1-1.0</td>
<td>1-100 mins</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It is obvious that with an instrument like radar altimeter that has a range accuracy of a few centimeters and spatial resolution about 20km, most of the sea features can be measured. But it should be pointed out that in any observational science, the length and time scales of the observations must match those of the phenomenon under investigation. For example, most of the tidal amplitude lies in the frequency band of one or two cycles per day, and satellite in a repeat orbit has a regular return time of a few days. Some aliasing is inevitable and we must therefore consider whether the length and time scales of ocean phenomena are capable of being sampled adequately by the techniques of remote sensing from space. This requirement is especially
important for oceanographic research. Wide coverage and short revisit time are always the contradictory requirements in satellite remote sensing. With satellite constellations, this problem will be relaxed, hence the current interest in such constellations.

2.3 Principle of Radar Altimeter measurement

2.3.1 Pulse-limited Radar Altimeter and Beam-limited radar Altimeter

There are two kinds of altimeters, one is called beam-limited altimeter and the other is called pulse-limited altimeter. The advantage of a beam-limited radar altimeter is its power efficiency and high spatial resolution, because the transmitted power will be limited within the footprint and all backscattered power from the footprint will be received by radar altimeter. The disadvantage is that it needs a very big antenna to achieve high spatial resolution and is very sensitive to attitude stability. On the contrary, a pulse-limited radar altimeter is less power efficient and spatial resolution is sensitive to target topography, because the footprint is much larger than the area from which the power will be used by the radar altimeter. For the pulse-limited radar altimeter a spherical shell defined by the radar pulse wavefront intersects the ocean surface and defines regions where the lateral extent is small compared with that defined by the antenna beam width. That is, the surface area corresponding to the range resolution of the altimeter is much smaller than that encompassed by the antenna beam. This feature prevents it from application over land and ice sheets; but it only requires a small antenna and is less sensitive to attitude control. The beam-limited radar is generally inapplicable to altimetry. Detailed analysis can be found in [Chelton, 1989][Elachi]. Because the main application requirement is for oceanography, where the sea surface slope is generally small, it is reasonable to choose a pulse-limited radar altimeter to measure the sea surface. Below, I will focus on the pulse limited radar altimeter.

2.3.2 Waveform of Pulse-limited Radar Altimeter

Fig. 2-7 shows the footprint of conventional radar altimeter on a rough sea surface. Under illumination from a rectangular pulse, one can define a region of sea at any time instant that is currently under illumination from that pulse. This region starts as a point then expands to a circle. The circular footprint increases linearly with time until the trailing edge of the pulse reaches the wave troughs at satellite nadir. Thereafter the
footprint becomes an annulus with constant area [Alex, 1998].

When the pulse scatters from a rough surface, the returned echo has a shape that reflects the statistical properties of the surface. In the case of the ocean, where the surface is homogeneous, the height statistics are the main factor in determining the echo’s shape. Usually the Gaussian probability density function is assumed to apply to the sea surface height distribution. From this echo shape, an important ocean parameter, Significant Wave Height (SWH) can be derived. The significant wave height (SWH) is defined as 4 times the standard deviation of \( q(r) \). Brown has shown that the mean sea surface echo can be expressed by a convolution of three terms as shown in Fig. 2-8 [Brown 1977]:

\[
P_r(t) = P_T(t) \ast P_{FS}(t) \ast q(t)
\]  

\( P_r(t) \) is the mean received echo;

Fig. 2-8 Decomposition of mean received echo
Chapter 2 Principle of Radar Altimeter Remote Sensing

$P_T(t)$ is the radar system point target response function;
$P_{FS}(r)$ is the flat surface impulse response;
$q(r)$ is the sea surface specular point distribution function.

Ideally $P_{FS}(r)$ should be a step function. But due to the antenna weighting, the trailing edge decreases gradually. In practice, antenna off-nadir looking will relax the decreasing of the trailing edge of $P_{FS}(r)$. Usually this information can be used to estimate the off-nadir angle based on ground processing [Brown, 1977]. Because the late samples at the trailing edge are more sensitive to off-nadir pointing error, new radar altimeters generally use a wider echo window (128 sample points) instead of that of earlier radar altimeters (64 samples points), to estimate the off-nadir pointing angle. But they still use about 64 early samples to estimate the sea surface parameters (mean sea surface height, SWH and wind speed).

The specular point probability density function $q(r)$ will distribute the energy from the same area to different time delays. Its effect is shown by convolution with the flat surface response function. The leading edge of $P_s(t)$ reflects the shape of $q(r)$. The shape of leading edge of $P_s(t)$ can be used to estimate the SWH. In fact, the specular point probability density function (PDF) can be different from the PDF of the physical surface elevation. This difference is referred to as the EM bias. Generally the surface elevation is not exactly Gaussian, as is assumed in Brown’s model. All these will affect the accuracy of the mean sea surface height and the SWH estimations [Dudley, 1989][Rodriguez, 1988]. For simplicity, I will not discuss them in my thesis.

2.3.3 PRF Selection of Radar Altimeter

The smooth mean waveform $P_s(t)$, as shown in Fig.2.8, only exists as the average of many individual echoes. Within the altimeter footprint on the sea surface, there will always be many wave facets specularly reflecting the incident signal at a given range. The returned signal from a single wave facet can be expressed as an amplitude and a phase, or equivalently, in terms of real and imaginary components. Since the radar wavelength is short (approximately 2.2cm for 13.6GHz signal), the phase relationship between signals reflected from the various facets is random. The total signal received by the altimeter is the vector summation of the real and imaginary components from
all individual specularly oriented wave facets in the altimeter footprint. By the Central
Limit Theorem, the real and imaginary components of the total signal are both
approximately Gaussian distributed. The amplitude of the total returned signal is
therefore Rayleigh-distributed and the returned power (square law detection) is an
exponentially distributed random variable. Fig. 2.9 shows radar altimeter waveforms
after different amount of averaging.

Fig. 2-9 Individual echo (left) and echo after 50 times averaging (right)

These graphs show that the greater the averaged time the smoother the averaged
waveform and more accurate the estimated waveform parameter. So by improving the
pulse repetition frequency (PRF), more echoes may be obtained and more echoes can
be averaged. Echoes being averaged are assumed to be independent of each other.
According to Citter-Zernike theory the decorrelation distance for a uniformly
illuminated circular area of radius $r$ is [Walsh, 1982]:

$$d = \frac{0.30577}{r} \left( \frac{\lambda}{2.877} \right)$$  \hspace{1cm} (2.8)

$H$ is the altimeter altitude;

$\lambda$ is the radar wavelength.

As the radius of the first pulse limited circle is determined by

$$r = \sqrt{c \tau_c H}$$  \hspace{1cm} (2.9)

$\tau_c$ is the compressed pulse width. The decorrelation distance is then expressed as:
\[ d = 0.305 \lambda \frac{H}{\sqrt{c \tau_c}} \]  \hspace{1cm} (2.10)

The decorrelation PRF is obtained by dividing the satellite velocity \( V \) by \( d \) and correcting \( H \) for earth curvature effects, using the radius of the earth \( R_e \):

\[ PRF = \frac{\sqrt{c \tau_c}}{0.305 \lambda \sqrt{H} \left( \frac{R_e + H}{R_e} \right)} \]  \hspace{1cm} (2.11)

For an altimeter such as that on ERS-1 the decorrelation PRF is about 1074Hz.

The decorrelation distance of (2.10) is derived from the flat sea surface. At high SWH condition the circular footprint corresponding to the leading edge has a larger radius. So the decorrelation PRF is higher than that of the low SWH condition. The adaptive PRF is adopted by modern radar altimeter, such as TOPEX, to improve the accuracy of the measurement.

2.3.4 Block Diagram of Radar Altimeter

Radar altimeter is a wide bandwidth ranging radar. A typical diagram is shown in Fig. 2-9.

![Fig. 2-10 Principle diagram of a typical radar altimeter](image)

The altimeter is an active microwave instrument that transmits pulses modulated onto a fixed carrier frequency. These pulses are linearly frequency modulated. Each transmitted pulse is reflected by the sea and received at the input of the altimeter.
about 5.3ms after the transmission (800 km orbit). At that time, a second pulse is generated in the altimeter receiver for mixing with the received pulse. The phase of the transmitted pulse can be expressed as

$$\phi(t) = \pi \alpha t^2 + 2nf_d t$$

(2.12)

$\alpha$ is linear FM rate;

$f_0$ is the carrier frequency.

The phase of the pulse reflected from a point at a delay time $\tau_0$ is (see appendix I):

$$\phi(t - \tau_0) = \pi \alpha (t - \tau_0)^2 + 2nf_d t + 2f_0\tau_0 + 2nf_0 t$$

(2.13)

$f_d$ is Doppler frequency shift.

The phase of frequency shifted reference pulse is (corresponding to an estimated delay time $\tau_d$):

$$\phi_r(t) = \pi \alpha (t - \tau_d)^2 + 2nf_d t$$

(2.14)

$\tau_d$ is a estimated time delay.

After mixing, the phase of the output is:

$$\phi_o(t) = \pi \alpha [(\tau_0^2 - \tau_d^2) - (\tau_0 - \tau_d)t] + 2nf_d t + 2\pi(f_0 - f_i)t + 2nf_0 \tau_0$$

(2.15)

Because $\tau_d$ and $\tau_0$ are small and their difference is even smaller. So $(\tau_0^2 - \tau_d^2)$ can be ignored.

- The first term can be expressed as $\pi \alpha (\tau_0 - \tau_d)t$. It shows a frequency shift proportional to different time delay $\tau_0$. After Fourier transform, different frequencies in the spectrum will correspond to backscattered signal from different time delay.
- The second term is Doppler induced range error as mentioned in appendix I;
- The third term is the intermediate frequency after mixing and can be cancelled after second mixing with LO2;
- The fourth term becomes Doppler phase shift due to distance change between satellite and target. This term will be used in azimuth resolution of SAR altimeter. It can be ignored in the conventional radar altimeter.
The signal passes through the IF receiver and is then demodulated to a base band in an amplitude & phase module. Then the orthogonal I & Q output signal is A/D converted and sent to the DSP module. Finally the digital signal is Fourier transformed. To repeat, different lines correspond to echoes at different delay times. This process is called “de-ramp” pulse compression.

The principle of de-ramp is shown in Fig.2-8 and also can be found in [Walsh, et al, 1989]:

(a) is the frequency variation of the transmitted pulse;

(b) the thin lines of (b) are the backscattered signals corresponding to different delayed targets. The middle thick line stands for the local reference signal.

(c) is the output signal frequency from (b). Each constant frequency line stands for the signal from a different delay target.

(d) is the spectrum of (c), where different spectrum lines stand for different signals from different delay targets.

After de-ramp pulse compression, the echo waveform is formed. The echoes are averaged to form an averaged waveform. Then three parameters are estimated and filtered based on the averaged waveform.

2.3.5 Principle of On board Digital Signal Processing

The main tasks of on board digital signal processing is to track the mean sea surface delay time, estimate the significant wave height and estimate the normalized radar cross section.

To be able to implement the de-ramp technique properly, the Chirp generator must be triggered at the exact instant when the echo is expected to return, so delay tracking is needed. For the purpose of maintaining a constant output level and to ensure the
operation within the linear region of all the receiver stages, an AGC loop is implemented in the altimeter. SWH, as a valuable parameter, is estimated in order to chose the proper tracking gate for the range tracking.

The range tracking function must be able to keep the return pulses within the range window. When this function fails, tracking is lost and the echo signal must be acquired again. Tracking involves the derivation of an error signal proportional to the offset of the echo from the tracked point of the range window. Different estimation algorithms have been proposed for the generation of the error signal [Townsend, 1980] [Bucciarelli, et. al., 1988][Wingham, 1986]. The basic Split Gate algorithm was invented for the SEASAT altimeter mission [Townsend, 1980], and is still used by the TOPEX altimeter [Marth, et. al., 1993]. It is an effective and computationally efficient algorithm specialized for echoes from the sea surface. In principle the optimum estimation algorithm is Maximum Likelihood Estimation (MLE) algorithm. It has been proposed during the SEASAT mission [Brooks, 1975]. But due to computational inefficiency, it has not been used for on board signal processing. A Sub-optimal Maximum Likelihood Estimation (SMLE) algorithm is employed for on board signal processing of ERS-1 and RA-2. Although it is not as good as MLE, it is computationally efficient and slightly better than split gate estimation. Another method suggested for radar altimeter parameter estimation is to estimate the parameters from the specular point pdf obtained by deconvolution. This is used for ground processing to estimate more parameter from conventional radar altimeter waveform [Rodriguez, 1988]. Because SMLE will be explained in detail in chapter 4, here I just give a brief introduction to the split-gate algorithm.

The averaged echo waveform of TOPEX radar altimeter is shown in Fig. 2-12. To minimise the noise, the echoes are averaged many times to form a ‘tracker waveform’. Usually the data rate of the tracking loop is 20Hz. For TOPEX about 228 pulses can be used for averaging. For other radar altimeters, usually about 50 echoes can be averaged. The TOPEX radar altimeter has a set of 128 waveform samplers uniformly spaced at 3.125ns separation as shown in Fig. 2-12. The E, M and L designate Early, Middle, and Late Gates respectively. There are in total 6 sets of these gates changing with SWH. And there are also a Noise gate and an AGC gate designed to measure the noise floor and signal strength. The AGC gate can be used to adjust the receiver gain to maintain the receiver working in linear area. All these gates have a
fixed position with respect to pulse samplers. We find only samplers from 5 to 56 are used in tracking. The 128 samples are regrouped into 64 samples as shown in Fig. 2-12 in order to be telemetered to the ground.

Fig. 2-12 Waveform and tracking gates of TOPEX radar altimeter [Marth, et al, 1993]

The AGC gate is the widest fixed gate. It measures the average value from sample 17 to sample 40. These samples are centered around the waveform sample number 32.5, which is the desired track point. This value represents the mean level of the waveform. The Middle gate measures the average value of the central part of the leading edge. The Middle gate changes its width as SWH changes. Values derived from the AGC gate and Middle gate should be equal if the waveform has been tracked at the right position. The difference between them is the range error signal that is used to update the timing of range tracking. The Early gate and Late gate are used to measure the slope of the leading edge, which is related to the SWH. The estimated SWH is then used to select the proper middle gate.
Chapter 3 Synthetic Aperture Radar Altimeter

The synthetic aperture radar altimeter is not a new concept. It is easy to think of using synthetic aperture technique to improve the along track spatial resolution, which is required by coastal area and ice sheet topographic mapping. Besides, it can also give an improved radiometric response by using more energy within footprint [Raney, 1998], which may result in improved signal to noise ratio. The improvement of signal to noise ratio may save transmitted power, and allow operation of the radar altimeter in a power limited condition. In this chapter I shall first introduce the basic theory of synthetic aperture radar, then explain the signal processing of synthetic aperture radar altimeter; finally I will discuss the signal to noise ratio improvement by using synthetic aperture processing.

3.1 Introduction of Synthetic Aperture Radar Altimeter

Prof. Griffiths suggested a method of unfocused SAR processing for radar altimeter in 1988 [Griffiths, 1988]. It can synthesize an unfocused aperture when radar flies over the target. Other authors have proposed a Doppler Beam Sharpening (DBS) radar altimeter as a way to reduce the along track footprint by using Doppler frequency discrimination. But different Doppler beams, corresponding to different off-nadir angles, will induce varying and different extra delay times. Dr. Raney has improved the idea by proposing a delay compensation method, which can compensate extra delay for echo from each synthesized beam [Raney, 1998]. This delay compensation is very similar to range curvature correction in SAR. After delay compensation, for a flat surface such as the sea, each look within one burst or in different bursts can be averaged. Although the high resolution is sacrificed, more looks can be averaged and higher accuracy estimation will be achieved.

3.1.1 Principle of Synthetic Aperture Radar

The synthetic aperture technique can be explained in different ways. It can be explained in terms of an antenna array, which is familiar to an antenna engineer or physicist who have a similar topic in optics. It can also be explained in terms of Doppler resolution, which will be explained as follows.
Two targets on the ground separated by an amount of $\delta_x$ in the azimuth direction and at the same slant range $R$ can be resolved only if they are not both in the radar beam at the same time. Thus we have

$$\delta_x = R \theta_a = R \frac{\lambda}{L_a}$$

(3.1)

$\lambda$ is the wavelength of the carrier;

$L_a$ is the antenna aperture in azimuth direction.

$\theta_a$ is the antenna beam width in radians.

This quantity is the resolution limit of conventional side look real aperture radar in the azimuth direction.

To improve the along track resolution $\delta_x$ at some specified slant range $R$ and wavelength $\lambda$, it is necessary to increase the antenna aperture in the along track direction. The mechanical problems include constructing an antenna with a surface precision accurate to within a fraction of wavelength, and the difficulty in maintaining that level of precision in an operational environment. This problem has been solved by the synthetic aperture technique.

![Fig. 3-1 Principle of azimuth resolution](image_url)

As shown in Fig. 3-1, there are two point targets, at slightly different angles with respect to the track of the moving radar. This will induce two different component
velocities at any instant relative to the platform. Therefore, the radar pulse reflected from the two targets will have two distinct Doppler frequency shifts. The Doppler shift relative to the transmitted frequency is

\[ f_{d1} = \frac{2V \sin \theta}{\lambda} \approx \frac{2V(x_{n1} - x_a)}{\lambda R} \]  \hspace{1cm} (3.2)

\( f_{d1} \) is the observed Doppler frequency;
\( V \) is the speed of satellite. Here I assume satellite flies in a straight line;
\( x_{n1} \) is the azimuth co-ordinate of the first point target on the ground;
\( \theta \) is the azimuth angle of the first point target relative to the radar.

Therefore, if the received signal, received at the instant shown in Fig. 3-1, is frequency analysed, any energy observed in the return at time corresponding to range \( R \) and at Doppler frequency \( f_{d1} \) will be associated with a target at co-ordinate

\[ x_{n1} = \frac{\lambda R f_{d1}}{2V} + x_a \]  \hspace{1cm} (3.3)

Similarly, energy at a different frequency \( f_{d2} \) will be assigned to a corresponding co-ordinate \( x_{n2} \). Even though the targets are at the same range and in the same beam at the same time, they can be discriminated by analyzing of the Doppler frequency spectrum of the return signal. This is called ‘Doppler beam sharpening’ in early studies. Because during the period, \( R \) and \( x_t - x_a \) can be regarded as a constant, \( f_d \) is a constant. So just by Doppler filtering different targets with different Doppler frequency can be discriminated. The usage of Doppler frequency effectively provides a second co-ordinate for the use in distinguishing targets. The two co-ordinates are cross-track range \( Y \) and along-track \( X \) relative to a point directly beneath the radar platform as shown in Fig. 3-1.

With the use of Doppler analysis of the radar returns, the resolution \( \delta_x \) of the targets in the along-track co-ordinate is related to the resolution \( \delta f_d \) of measurement of the Doppler frequency. The antenna beamwidth in the azimuth direction no longer enters directly as a limiting factor. From (3.3)

\[ \delta x = \left( \frac{\lambda R}{2V} \right) \delta f_d \]  \hspace{1cm} (3.4)
The measurement resolution in the frequency domain is normally the inverse of the
time span $T$ of the waveform being analyzed ($\delta f_p = \frac{1}{T}$). Since the time is potentially
the time during which any particular target is in view of the radar (i.e. the time during
which the target remains in the beam) from Fig. 3-1 we can derive:

$$T = \frac{R \theta_a}{V} = \frac{R \lambda}{L_a V}$$  \hspace{1cm} (3.5)

Here $\theta_a$ is the real antenna beamwidth in radian.

This leads to

$$\delta x = \left(\frac{\lambda R}{2V}\right) \left(\frac{L_a V}{R \lambda}\right) = \frac{L_a}{2}$$  \hspace{1cm} (3.6)

This means the ultimate resolution is half the real antenna aperture. This is somehow
a counter-intuitive result. But the shorter the antenna aperture, the longer the
illumination time and higher is the Doppler resolution, giving a higher azimuth spatial
resolution.

In fact there is a restriction in the derivation leading to the azimuth resolution
expression of (3.6). If a target is to be positioned along track (relative to the platform)
in accord with its observed frequency, it must produce a constant Doppler frequency
over the observation interval. However if this interval is the entire time that the target
is within the radar footprint, as was assumed for (3.5), then the corresponding Doppler
signal will have a frequency which sweeps over the entire Doppler bandwidth as the
platform passes by the target. The actual analysis interval available using a frequency
filtering technique may be much less than the interval in (3.5), because it is restricted
to the time span over which any particular point target has essentially a constant
Doppler frequency. I will investigate this point as follows:

As shown in Fig. 3-2, suppose the nearest range from target to radar during the period
when target is in radar footprint is $R_0$. Then the range difference between any
distance and the nearest distance is:

$$\Delta R(x_a) = \left[ R_0^2 + (x_a - x_t)^2 \right]^{1/2} - R_0 \approx \frac{(x_a - x_t)^2}{2R_0}$$  \hspace{1cm} (3.7)

$(x_a, y_a, z_a)$ is the position of satellite;
(x_i, y_i, z_i) is the position of target on the ground.

\[ (xt, yt, zt) \] is the position of target on the ground.

**Fig. 3-2 Radar target and quadratic relation between range and time**

\( \Delta R(x_a) \) is the extra delay of each point along flight path relative to the range \( R_0 \) at the point of the closest approach.

Suppose the radar platform starts from \( x = 0 \), then

\[ x_a = Vt \] \hspace{1cm} (3.8)

Substitute (3.8) into (3.7), we have

\[ \Delta R(t) = \frac{V^2}{2R_0} t^2 - \frac{Vx_i}{R_0} t + \frac{x_i^2}{2R_0} \] \hspace{1cm} (3.9)

The relative delay distance is a quadratic function of the along-track time \( t \). Then the relative phase shift is:

\[ \Delta \phi(t) = -\frac{4\pi \Delta R(t)}{\lambda} = \frac{4\pi}{\lambda} \left( \frac{V^2 t^2}{2R_0} - \frac{Vx_i t}{R_0} + \frac{x_i^2}{2R_0} \right) \] \hspace{1cm} (3.10)

Obviously the relative phase shift is also a quadratic function of the along-track time. It is easy to understand that the quadratic function can be approximated by a series of
short linear functions. This means if we only process the signal during a short period, the quadratic phase can be approximated by linear phase, so that the simple Doppler filtering technique can be applied to the processing. This kind of signal processing is used in unfocused SAR, which will result in a low azimuth resolution. If we want to process all pulses collected, we have to compensate the quadratic phase before using Doppler filter. This requires a matched filter that can cancel the quadratic phase. This is used in focused SAR processing.

3.1.2 Synthetic Aperture Radar Altimeter

Synthetic aperture radar altimeter can be defined as a nadir-looking unfocused SAR system. Synthetic aperture radar altimeter usually operates in burst mode. Because the round-way delay time is about 5.3ms if the satellite's altitude is about 800km above the earth, the pulse transmission should be finished within 5.3ms. This means the maximum integration time is about 5.3ms. For an altimeter like TOPEX the Doppler bandwidth is about 9940Hz if it is at an altitude of 800km. The PRF required for the synthetic aperture radar altimeter should be higher than that. If we choose the integration time to be 4.8ms (this is related to azimuth resolution), there will be more than 48 pulses that should be transmitted. Since the along track processing strategy is based on an FFT, it makes sense to select the number of transmitted pulse to be a power of two, which in this case would be 64. Thus the PRF is actually about 13kHz. There are about 80 bursts that can be transmitted per second and totally about 5120 independent looks can be obtained per second. A greater number of looks generally means more accurate measurement.

Fig. 3-3 Normalized flat surface response function of SAR altimeter
The synthesized antenna has a narrow beamwidth in the along-track direction and the same beam width in across-track direction as the conventional radar altimeter. The flat surface response function of synthetic aperture radar altimeter at zero Doppler cell is as follows [Raney, 1998]:

\[ f_{D0}(t) = 0 \]

\[ \frac{1}{\tau_c} \left[ t - \frac{2h}{c} \right] \leq 0 \]

\[ 0 < \frac{1}{\tau_c} \left[ t - \frac{2h}{c} \right] \leq 1 \]

\[ 1 < \frac{1}{\tau_c} \left[ t - \frac{2h}{c} \right] \]

(3.11)

Its trailing edge decreases very fast.

A complete expression of flat surface response functions in different Doppler cells is given in Picardi’s paper [Picardi, 1998]. In that expression it is assumed that the back scattering coefficient is homogeneous and the synthesized antenna pattern is rectangular with constant gain.

Fig. 3-3 shows the flat surface response function of the central Doppler cell according to (3.11). The solid curve stands for half-beam width off-nadir pointing in cross track direction. The dotted curve stands for zero off nadir angle.

Fig. 3-4 Typical flat surface response for different Doppler cell
But in the off-nadir direction, the interaction between the pulse limited concentric annuli and the Doppler limited areas (when signal returns from off nadir direction, it will take more than one compressed pulse duration time to propagate through one Doppler cell width) causes a Doppler dependent rising time and peak loss on the flat surface response function as shown in Fig 3.4 [Picardi, 1998]. Also the looks at the high Doppler frequency correspond to low antenna gain, which will result in a low signal-to-noise ratio. In order to keep to a high signal to noise ratio and sharp flat surface response within each burst there are only about 30 looks can be obtained after processing. Without these loss the peak point of flat surface response can improve about 10dB [Raney, 1998]. But due to the above reasons, only about 8dB improvement may be achieved. This radiometric response improvement is not equal to final signal to noise ratio improvement. When flat surface response function convolves with specular point distribution function, the final peak point value will change with SWH. I will define the signal to thermal noise ratio for simulation in the next section.

The pulse-limited footprint of a conventional radar altimeter, such as the ERS-1 radar altimeter, is about 1.8km in diameter. According to the Cittert-Zernike theorem the decorrelation distance is about 4.9m [Walsh, 1982]. This means only about 1100 independent looks can be obtained by a conventional radar altimeter, supposing the satellite's speed is 7km/s. From the above explanation, with 30 looks per burst times 80 bursts, a total of about 2400 independent looks can be obtained by the synthetic aperture radar altimeter.

### 3.2 Signal Processing for Synthetic Aperture Processing

Signal processing of synthetic aperture radar altimeter can be separated in two steps:

1. Synthetic aperture processing;
2. On board waveform tracking.

The waveform of SAR altimeter is much like the waveform reflected from the ice sheet. So the tracking algorithm used for ice sheet radar altimeter echo tracking can be used by the SAR altimeter. It will not be discussed here.

#### 3.2.1 Synthetic Aperture Processing

The synthetic aperture processing is separated into two steps:
Using along track Fourier transform to synthesize different off nadir antenna beam;

Using delay compensation to compensate extra delay of echoes got from different beams;

The detailed algorithm can be found in Appendix II. The diagram of synthetic aperture radar altimeter signal processing is shown in Fig. 3.5.

Dr. Raney has proposed a delay compensation method analogous to range curvature correction in burst mode synthetic aperture radar, which supposes a zero Doppler centroid. In fact because the satellite has a relative motion to the earth, the central Doppler frequency is not zero. For conventional radar altimeter, the range error caused by radial speed can be removed by using the speed information derived from the orbit parameters after on ground processing. For synthetic aperture radar altimeter this compensation must be done on board the satellite. We here call it Doppler centroid compensation. After this compensation, the satellite can be thought of as flying in a straight line.
For different along track positions there are different range delays caused by Doppler speed that can be removed according to different Doppler frequencies. These different delays must be removed before azimuth averaging; otherwise they will smear the leading edge.

3.2.2 Simulation of Synthetic Aperture Processing for Radar Altimeter

The simulation of the algorithm as described in last section and also in Appendix II starts with the standard transmitted Chirp pulse of radar altimeter. The usual assumptions of plane Earth geometry is used, where a regular Chirp pulse is transmitted to an idealized point target on the ground. The simulation computes each return pulse, which is cross-multiplied by a reference pulse to give the standard de-ramped pulse of the conventional altimeter.

Following the techniques of SAR and Raney [Raney, 1998], an along track DFT is computed. It gives a two dimensional function in what is now the standard $(k_x, k_R)$ space as defined in SAR theory [Sun, et al., 1999]. Following Raney, each point in this data space is multiplied by a complex exponential with a phase term embodying two forms of compensation

1. the effect of the increasing time delay with an effective beam direction in each $k_x$ cell (phase shift $\propto k_x^2 k_R$) – delay Doppler compensation.

2. the effect of actual Doppler shift with the increasing $k_x$ requiring phase shift $\propto k_x k_R$. The compensated data space is then inverse Fourier transformed in $k_R$ to give a data space in $(k_x, r)$ space $-S_3(k_x, r)$.

The simulation conditions are as follows (The simulation parameters can be refer to section 3.1.2 or [Raney, 1998]):

A point target is supposed to be located on the ground, which is about 5km displaced from the center of the synthetic aperture, $(x_t = 5000, y_t = 0)$;

Satellite flies in a straight line parallel to the ground;

Satellite’s altitude is 800km;

Satellite’s speed is 7.3km/s.
Chapter 3 Synthetic Aperture Radar Altimeter

Radar PRF is 13000Hz;
The pulse length of transmitted chirp signal is 60μs;
The chirp signal bandwidth is 300MHz;
64 pulses are transmitted in one burst.
The point target simulation result of SAR altimeter is shown in Fig. 3-6 and Fig.3-7.

![Point target image without delay compensation](image1)

![Point target image with delay compensation](image2)

Fig. 3- 6 Point target image without delay compensation

Because the target is located about 5km displaced from the nadir, there is about 31m extra round trip distance with respect to nadir distance. From Fig. 3.6 we find that there is about 31 pixels shift from the across track center. The right hand picture is a zoomed version of the left one.

![Point target image with delay compensation](image3)

Fig. 3- 7 Point target image with delay compensation
After doing delay Doppler compensation and Doppler induced range error compensation, the point target image is now located in the correct position as shown in Fig. 3-7. The right hand image is a zoomed image of the left one.

3.3 Performance Improvement by Synthetic Aperture Processing

In this section I will first give the analytical expression of the mean waveform of synthetic aperture radar altimeter. Then I will evaluate the signal to noise ratio improvement by comparing the mean waveform of the SAR altimeter with the waveform of the conventional radar altimeter numerically.

3.3.1 Waveform of SAR altimeter

Because synthesized antenna looks at different off nadir angle, each individual beam experiences a different flat surface impulse response function. The individual flat surface impulse response function of synthetic aperture radar altimeter can be expressed as [Picardi, 1998][Brown, 1977]:

\[
P_I(t,n) = \frac{\lambda^2 G_0^2 \sigma^0}{(4\pi)^3 L_p} \int_{\delta(n)} \frac{\delta_0\left(t - \frac{2\sqrt{H^2 + \rho^2}}{c}\right)}{(H^2 + \rho^2)^2} \rho d\rho d\phi
\]  

(3.12)

Here I assume an uniform normalized radar cross-section \( \sigma^0 \) and rectangular antenna pattern with gain \( G_0 \).

- \( n \) is the number of Doppler cells;
- \( \lambda \) is the wavelength of the altimeter;
- \( L_p \) is the two way propagation loss;
- \( A(n) \) is the surface area limited by the nth Doppler cell;
- \( H \) is the altitude of the satellite;
- \( \rho \) is the radius of a certain point within the footprint.
The SAR altimeter footprint is shown in Fig. 3-8. It has been shown that each different Doppler cell has different impulse response function [Picardi, 1998]. The mean received echo from Doppler cell $n$ can be expressed as:

$$P_r(t, n) = S_r(t) \otimes P_f(t, n) \otimes q(t)$$

$$= S_r(t) \otimes \left[ \frac{\lambda^2 G_0^2 \sigma_0}{(4\pi)^3 L_p} \right] \int_{\Lambda(n)} \delta_0(t - \frac{2\sqrt{H^2 + \rho^2}}{c}) \rho \, d\rho \, d\phi \otimes \left\{ \frac{1}{\sqrt{2\pi(\frac{2\sigma_s}{c})}} \exp\left[-\frac{(t - \tau)^2}{2(\frac{2\sigma_s}{c})^2}\right] \right\}$$

$$= \frac{\lambda^2 G_0^2 \sigma_0 c}{2\sqrt{2\pi(4\pi)^3 \sigma_s L_p}} \cdot S_r(t) \otimes \left[ \int_{\Lambda(n)} \delta_0(t - \frac{2\sqrt{H^2 + \rho^2}}{c}) \rho \, d\rho \, d\phi \otimes \exp\left[-\frac{(t - \tau)^2}{2(\frac{2\sigma_s}{c})^2}\right] \right]$$

(3.13)

$P_r(t, n)$ is the mean received echo;

$S_r(t, n)$ is the radar system point target response function;

$P_f(t, n)$ is the flat surface impulse response;

$q(t) = \frac{1}{\sqrt{2\pi(\frac{2\sigma_s}{c})}} \exp\left[-\frac{(t - \tau)^2}{2(\frac{2\sigma_s}{c})^2}\right]$ is the specular point probability density function in time.

$\sigma_s$ is the standard deviation of the sea surface height distribution function.
From (3.12), if we suppose a rectangular pulse is used by altimeter, the flat surface response function can be expressed as [Picardi, 1998]: (in following simulation the rectangular pulse is used)

\[
P_{Ps}(t,n) = \frac{\lambda^2 G_0^2 \sigma_0}{(4\pi)^3 H^3} \int_{\rho_0(n)}^{\rho_b(n)} \int_{-\cos^{-1} \rho_0(n)}^{\cos^{-1} \rho_b(n)} \int_0^\infty \rho d\rho \int_0^{\pi} d\phi \frac{\text{rect}_r(t-T/2 - 2\sqrt{H^2 + \rho^2}/c)}{(H^2 + \rho^2)} \delta_\sigma(t-t_\sigma(n))
\]

\[
- \int_{\rho_0(n)}^{\rho_b(n)} \int_{-\cos^{-1} \rho_0(n)}^{\cos^{-1} \rho_b(n)} \int_0^{\pi} d\phi \frac{\text{rect}_r(t-T/2 - 2\sqrt{H^2 + \rho^2}/c)}{(H^2 + \rho^2)} \delta_\tau(t-t_\tau(n))
\]

\[
= \frac{2^2 G_0^2 \sigma_0}{(4\pi)^3 H^3} [w(t,n) - w(t-T,n)]
\]

Here

\[
w(t,n) = \frac{1}{2} \cos^{-1} \left( \frac{2t_\sigma(n)}{t} - 1 \right) - 2 \sqrt{t_\sigma(n)(t-t_\sigma(n))} \delta_\sigma(t-t_\sigma(n))
\]

\[
- \cos^{-1} \left( \frac{2t_\tau(n)}{t} - 1 \right) - 2 \sqrt{t_\tau(n)(t-t_\tau(n))} \delta_\tau(t-t_\tau(n))
\]

\(\delta_\sigma(t)\) is the Dirac function. \(\delta_\tau(t)\) is the unit step function, \(t_\sigma(n)\) and \(t_\tau(n)\) are defined by:

\[
t_\sigma(n) = \frac{2}{c} (\sqrt{H^2 + \rho^2} - H) \approx \frac{\rho^2}{cH}, \quad t_\tau(n) = \frac{2}{c} (\sqrt{H^2 + \rho^2} - H) \approx \frac{\rho^2}{cH}
\]

After delay compensation, the mean received echoes from different Doppler cells are then averaged to get a average mean received echo of SAR altimeter which is expressed as follows:

\[
p_r(t) = \sum p_r(t,n)
\]

\[
= \sum \left\{ \frac{\lambda^2 G_0^2 \sigma_0}{\sqrt{2\pi}(4\pi)^3(\frac{2\sigma_\sigma}{c})L_\rho} S_r(t) \otimes \int_{\mathcal{A}(n)} \delta_\sigma(t - \frac{2\sqrt{H^2 + \rho^2}}{c} + d(n)) \rho d\rho d\phi \exp[- \frac{(t - z)^2}{2(\frac{2\sigma_\sigma}{c})^2}]\right\}
\]

\[
= \sum \left\{ \frac{\lambda^2 G_0^2 \sigma_0}{\sqrt{2\pi}(4\pi)^3(\frac{2\sigma_\sigma}{c})L_\rho} S_r(t) \otimes \int_{\mathcal{A}(n)} \delta_\sigma(t - \frac{2\sqrt{H^2 + \rho^2}}{c} + d(n)) \rho d\rho d\phi \exp[- \frac{(t - z)^2}{2(\frac{2\sigma_\sigma}{c})^2}]\right\}
\]

\[
= \sum \left\{ \frac{\lambda^2 G_0^2 \sigma_0}{\sqrt{2\pi}(4\pi)^3(\frac{2\sigma_\sigma}{c})L_\rho} S_r(t) \otimes \int_{\mathcal{A}(n)} \delta_\sigma(t - \frac{2\sqrt{H^2 + \rho^2}}{c} + d(n)) \rho d\rho d\phi \exp[- \frac{(t - z)^2}{2(\frac{2\sigma_\sigma}{c})^2}]\right\}
\]
\[ X = \frac{\lambda^2 \sigma_0 G_0^2}{(4\pi)^3 \sqrt{2\pi (\frac{2\sigma_L}{c})^2} L_p} \exp\left(\frac{(t-r)^2}{2(\frac{2\sigma_L}{c})^2}\right) \sum \{ S_r(t) \otimes \left[ \delta_0 \left( t - \frac{2\sqrt{H^2 + \rho^2}}{c} + d(n) \right) \right] \rho \, d\rho \, d\phi \} \]

\[ = \frac{A}{\sqrt{2\pi (\frac{2\sigma_L}{c})^2}} \exp\left(\frac{(t-r)^2}{2(\frac{2\sigma_L}{c})^2}\right) \sum \{ S_r(t) \otimes \left[ \delta_0 \left( t - \frac{2\sqrt{H^2 + \rho^2}}{c} + d(n) \right) \right] \rho \, d\rho \, d\phi \} \]

\[ d(n) = \frac{P_{\text{a}}(n)}{cH} \]

is the delay time corresponding to \( n \)th Doppler cell.

\[ A = \frac{\lambda^2 \sigma_0 G_0^2}{(4\pi)^3 L_p} \]

(3.16)

From (3.15), we can see that the mean received echo of the SAR altimeter is the convolution of the specular point distribution function and the mean flat surface response function expressed as the average of flat surface response function of different Doppler cells.

3.3.2 Evaluation of Signal to Noise Ratio Improvement

In this section I will evaluate the signal to noise ratio improvement at different SWH conditions. The antenna gain variation has not been considered in this evaluation. The range resolution is 0.5m. I generate different mean waveforms according to equation (3-15) at different SWH.

It is known that a SAR altimeter can improve the radiometric response [Raney, 1998]. Dr. Raney gave the radiometric response improvement by calculating the flat surface response function. For an altimeter such as Topex, the peak signal strength from the zero Doppler cell could be about 10dB higher than the signal derived from the conventional radar altimeter [2]. As shown in section 3.1.2, the actual flat surface responses vary with different Doppler cells. If we suppose the peak radiometric response improvement at zero Doppler cell is 10dB[Raney, 1998], the peak of averaged flat surface response (averaged over 30 central Doppler cells) is about 7.7dB, as shown in Fig.3-9. Suppose these two different altimeters have the same thermal noise level (same receiver), the different peak signal values will result in different peak signal to thermal noise ratios.
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Fig. 3-9 Averaged flat surface response function

When the mean flat surface response function of the SAR altimeter convolves with the normalized Gaussian specular point probability density function of different SWH, it will result in different peak signal values as shown from Fig. 3-10 to Fig. 3-12. This means if the sea surface reflectivity is the same, the peak output will vary inversely with SWH, which is quite unlike the conventional radar altimeter. According to the above definition, for same thermal noise level this means different signal to thermal noise ratios. Comparing to the conventional radar altimeter, the signal to noise ratio improvement (shown in Fig. 3-10 to 3-12) is about 7.3dB, 5.3dB and 4dB respectively under different SWH conditions. This means when conventional radar altimeter operates at 0dB, the SAR altimeter will operate at 7.3dB, 5.3dB and 4dB respectively.

Fig. 3-10 Mean received echo of conventional radar altimeter and SAR altimeter at 1m SWH
3.4 Conclusion

As shown in section 3.2, the signal processing required for a SAR altimeter is much more complex than the conventional radar altimeter, but the signal to thermal noise ratio improvement for sea surface monitoring is a function of SWH. In high SWH condition this improvement is very limited. And also in the above simulation, I suppose an antenna with a uniform gain. But actually the echo got from the high Doppler cell is weighted by the lower antenna gain than the central Doppler cell. Therefore the actual signal to noise improvement is even less. Therefore sea surface monitoring by radar altimeter will not benefit much from SAR processing. But it can be predicted that ice sheet monitoring will benefit by about 7.7dB signal to noise ratio improvement from SAR processing.
4.1 Brown Model for Sea Surface Echo

The on-board signal processing of a radar altimeter is to estimate the delay time of the echo, normal incident radar cross-section and the slope of leading edge of the echo. All this signal processing is based on a good understanding of echo model of sea surface derived by G. S. Brown [Brown 1977]. This model was used by SEASAT and ERS-1 radar altimeters as the base for signal processing.

Brown showed in his paper that the mean received echo of sea surface can be expressed as a triple convolution as follow:

\[ P_r(t) = P_T(t) \otimes q(t) \otimes P_{FS}(t) \]  

(4.1)

\( P_r(t) \) is the rough surface response function;
\( P_T(t) \) is the radar system point target response function;
\( q(t) \) is the specular point distribution function;
\( P_{FS}(t) \) is the flat surface impulse response function;

I will derive each term in equation (4.1) as follows.

4.1.1 Flat Surface Impulse Response Function

Fig.4-1 shows the geometry of radar altimeter measurement. When a flat surface with a small scale of roughness is illuminated by an impulse signal, the power of the returned echo can be expressed as superposition integration according to the radar equation:

\[
p_{FS}(t) = \frac{\lambda^2}{(4\pi)^3 L_p} \int \frac{\delta(t - \frac{2R}{c})G^2(\theta, \omega)\sigma^0(\psi, \phi)}{R^4} dA
\]  

(4.2)

\((\xi, \Phi)\) is the antenna boresight;
\(G(\theta, \omega)\) is the antenna’s gain described by the angles relative to the boresight;
\(L_p\) is the two way propagation loss;
\(R\) is the range from the radar to elemental scattering area \(dA\) on the surface.
Suppose $\sigma^0(\psi, \phi)$ is independent of $\phi$, antenna gain is independent of $\omega$ and is of the form $G(\theta) = G_0 e^{-(2/\gamma)\sin^2 \theta}$, $\theta$ is the angle from the antenna axis. Let $\tau = t - 2H/c$, and substitute $\sigma^0(\psi)$ and $G(\theta)$ into equation (4.2), which can be simplified as:

$$P_{FS}(r) \approx \frac{G_0^2 \lambda^2 \sigma^0(\psi)}{4(4\pi)^2 \lambda^2 H^3} \exp\left[-\frac{4}{\gamma} \sin^2 \xi - \frac{4c}{\gamma H} \tau \cos 2\xi\right] \cdot I_0\left(\frac{4}{\gamma} \sqrt{\frac{c\tau}{H}} \sin 2\xi\right)$$

Equation (4.3) shows that the flat surface response function depends on the normalized radar cross-section $\sigma^0(\psi)$, antenna off-nadir look angle $\xi$, the parameter $\gamma$ describing the antenna gain as a function of angle from the antenna axis, and the range from the radar to the surface which is expressed by $\tau$. 

\[\text{(4.3)}\]
4.1.2 The Specular Points Distribution Function

In Brown’s model the specular point density as a function of height is assumed to be Gaussian, the mean value is the mean surface height that is assumed to be zero in the Gaussian distribution function. Thus it can be expressed as:

\[ q(l) = \frac{1}{\sqrt{2\pi\sigma_s^2}} \exp\left[ -\frac{l^2}{2\sigma_s^2} \right] \]  

(4.4)

\( l \) is the specular point displacement relative to the mean sea level.

\( \sigma_s \) is the rms height of the specular points relative to the mean sea level.

But usually the distribution function is expressed as a function of delay time relative to the delay time of mean sea level. It is easy to show that it can be expressed as follows as a function of delay time \( \tau \):

\[ q(\tau) = \left( \frac{c}{2} \right) \frac{1}{\sqrt{2\pi(\frac{2\sigma_s}{c})^2}} \exp\left[ -\frac{\tau^2}{2(\frac{2\sigma_s}{c})^2} \right] \]  

(4.5)

Here \( \tau = \frac{2l}{c} \).

The parameter \( \sigma_s \) is related to sea surface SWH:

\[ \text{SWH} = 4\sigma_s \]  

(4.6)

4.1.3 Radar System Point Target Response Function

The point target response is the echo in power returned from a point target. For simplicity the point target response can be adequately represented by a Gaussian function, which is:

\[ P_T(\tau) = P_T \eta \cdot \exp\left[ -\frac{\tau^2}{2\sigma_p^2} \right] \]  

(4.7)

\( P_T \) is the peak transmitted power;

\( \eta \) is the pulse compression ratio;

\( \sigma_p \) is related to the point target 3dB width \( T \) by \( \sigma_p = 0.425T \).

4.1.4 Mean Received Echo

The mean received echo is a triple convolution of equation (4.3), (4.5), (4.7). It is easy to find that the convolution of (4.5) and (4.7) results in another Gaussian function;
\[ u(\tau) = P_r(\tau) \otimes q(\tau) = \left( \frac{c}{2} \right) \frac{\sqrt{2\pi \sigma_p^2} P_r \eta}{\sqrt{2\pi \sigma_c^2}} \exp \left[ -\frac{\tau^2}{2\sigma_c^2} \right] \] (4.8)

\[ \sigma_c \approx \sqrt{\sigma_c^2 + (2\sigma_c / c)^2} \] (4.9)

So the triple convolution becomes:

\[ P_r(\tau) = u(\tau) \otimes P_{FS}(\tau) = \left( \frac{c}{2} \right) \frac{\sqrt{2\pi \sigma_p^2} P_r \eta}{\sqrt{2\pi \sigma_c^2}} \int_{-\infty}^{+\infty} \exp \left[ -\frac{(\tau - \hat{\tau})^2}{2\sigma_c^2} \right] P_{FS}(\hat{\tau}) d\hat{\tau} \] (4.10)

The equivalent width of \( g(\tau) \) is small relative to the time scale over which \( P_{FS}(\tau) \) exhibits appreciable variation, thus

\[
P_r(\tau) \approx \begin{cases} 
P_{FS}(0) \frac{\sqrt{2\pi \sigma_p^2} P_r \eta}{\sqrt{2\pi \sigma_c^2}} \int_{0}^{+\infty} \frac{1}{\sqrt{2\pi \sigma_c^2}} \exp \left[ -\frac{(\tau - \hat{\tau})^2}{2\sigma_c^2} \right] d\hat{\tau} & \tau < 0 \\
+\infty \frac{\sqrt{2\pi \sigma_p^2} P_r \eta}{\sqrt{2\pi \sigma_c^2}} \int_{0}^{+\infty} \frac{1}{\sqrt{2\pi \sigma_c^2}} \exp \left[ -\frac{(\tau - \hat{\tau})^2}{2\sigma_c^2} \right] d\hat{\tau} & \tau \geq 0 
\end{cases}
\] (4.11)

The integration of the Gaussian function results in the Error function:

\[
P_r(\tau) \approx \begin{cases} 
P_{FS}(0) \frac{\sqrt{2\pi \sigma_p^2} P_r \eta}{\sqrt{2\pi \sigma_c^2}} \left[ 1 + \text{erf} \left( -\frac{\tau}{\sqrt{2\sigma_c^2}} \right) \right] / 2 & \tau < 0 \\
P_{FS}(\tau) \frac{\sqrt{2\pi \sigma_p^2} P_r \eta}{\sqrt{2\pi \sigma_c^2}} \left[ 1 + \text{erf} \left( \frac{\tau}{\sqrt{2\sigma_c^2}} \right) \right] / 2 & \tau \geq 0 
\end{cases}
\] (4.12)

\[ \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-y^2} dy \]

Equation (4.12) shows that mean received echo is an \( \text{erf}(\) function weighted by the flat surface impulse response function. When antenna off-nadir look angle is 0° the flat surface impulse response function can simplified as [Wingham, 1988]:

\[
P_{FS}(\tau) \approx \frac{C_0^2 \lambda^2 c \sigma_c^0 (\psi)}{4(4\pi)^2 L_p h^2} \exp \left[ -\frac{4c}{\gamma h} \right] \] (4.13)

A typical mean received echo is shown in Fig. 4-2. The delay time is expressed as the number of samples, as the sample interval is equal to the compressed pulse width. In this example the antenna diameter is chosen as 1 meter; off-nadir look angle is 0°. The
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decay in trailing edge of the mean received signal is caused by the antenna pattern, which is described by the last exponential term in equation (4.13).

This decay can be corrected by multiplying an inverse term \(\exp\left(\frac{4c}{\gamma h} r\right)\) by the mean received echo that is given by equation (4.12). After this correction the mean received echo is changed to a \(\text{erf}(\cdot)\) function as shown in Fig. 4-3.

![Fig. 4-2 Mean received echo](image1)

![Fig. 4-3 Corrected mean received echo](image2)

The corrected mean received echo can be expressed as:

\[
P_r(\tau) = \Lambda \sigma^0 [1 + \text{erf}\left(\frac{\tau}{\sqrt{2\sigma_c}}\right)]
\]

\[\Lambda \sigma^0 = P_{rs}(0) \eta P \gamma \sqrt{2\pi\sigma_p} / 2\]

\(\Lambda\) is a constant determined by the radar equation;
$\sigma^0$ is the normalized radar cross section, which is included in $P_{rs}(0)$.

On board signal processing will be based on the corrected mean waveform. When the off-nadir angle is not equal to 0°, the correction term should include the off-nadir pointing angle $\xi$, otherwise the estimation will generate a bias resulting from the pointing error.

4.2 Maximum Likelihood Estimation for Radar Altimeter Signal Processing

In this section, I will explain the method of parameter estimation based on Maximum Likelihood Estimation (MLE) [Brooks 1975].

The mean received echo of equation (4.14) is rewritten as follow:

$$P_r(\tau) = A\sigma^0[1 + erf(\frac{\tau}{\sqrt{2}\sigma_e})]$$

$A\sigma^0$, the magnitude of the waveform, is proportional to the normalized radar cross section $\sigma^0$ that is related to wind speed. The slope of the leading edge of the waveform is determined by $\sigma_e$, which is related to SWH. The middle point $\tau_0$ in the leading edge corresponds to the mean sea surface height. Estimating $\sigma^0$, $\sigma_e$ and middle point $\tau_0$ in the leading edge is the purpose of altimeter signal processing.

What is shown in Fig. 4-3 is the mean or average received echo. An individual echo will look quite different from the average echo as the returns from the various scatterers may combine as phasors in such a way as to produce a wide range of amplitudes for individual points in individual pulses, which is shown in Fig. 4-4.

![Individual echo waveform](image-url)
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The probability density function of each sample $g_i$ of the return echo, measured at the $i$th gate, has a negative exponential distribution with mean equal to the theoretical return power $g_i$ expressed by $P_r(\tau)$. Sample $g_i$ depends on three parameters $\sigma_z$, $\tau_0$, $\sigma^0$. Thus the pdf of each sample is

$$f(g_i) = \frac{1}{g_i} e^{-\frac{g_i}{g_i}}$$ (4.15)

In practice $N$ pulses are averaged together before estimation. A waveform derived from averaging 50 individual echoes is shown in Fig. 4-5.

![Averaged echo waveform](image_url)

Fig. 4-5 Averaged echo waveform

The average of $N$ pulses will have a gamma, or chi-squared distribution [Challenor 1989]:

$$f(\bar{g}_i) = \frac{N^{N-1}g_i^{N-1}}{N!g_i^N} e^{-\frac{N\bar{g}_i}{g_i}}$$ (4.16)

So the likelihood of an averaged pulse can be defined as

$$L = \prod_{i=1}^{M} \frac{N^{N-1}g_i^{N-1}}{N!g_i^N} \exp\left(-\frac{Ng_i}{g_i}\right)$$ (4.17)

$M$ is the number of samples within range window.

The log likelihood is given by

$$LL = \sum_{i=1}^{M} \left\{ (N-1)\ln N + (N-1)\ln \bar{g}_i - Ng_i - Ng_i/g_i - N\ln g_i - \ln N! \right\}$$ (4.18)
To derive the maximum likelihood estimators, the derivative of (4.18) is taken with respect to the three parameters \( \theta_j \) (\( j=1,2,3 \), \( \theta_j \) corresponds to \( \tau_0, \sigma_r, \text{ and } \sigma_0 \)) and the results are set to zero. Thus

\[
\frac{\partial L}{\partial \theta_j} = N \sum_{i=1}^{M} \left( \frac{g_i}{g_i^2} \right) \frac{\partial g_i}{\partial \theta_j} - N \sum_{i=1}^{M} \frac{1}{g_i} \frac{\partial g_i}{\partial \theta_j} = N \sum_{i=1}^{M} \left( \frac{g_i - g_i^2}{g_i^2} \right) \frac{\partial g_i}{\partial \theta_j} = 0
\]  

(4.19)

Solving the three simultaneous equations will result in the MLE of \( \theta_j \).

![Fig. 4-6 Weighting function of linear model](image)

According to (4.19), first the estimated waveform is compared with the received waveform to obtain a difference signal \( g_i - g_i \). Second the difference is normalized by the estimated variance of the return \( g_i^2 \). Then the normalized difference signal is weighted by \( \frac{\partial g_i}{\partial \theta_j} \) to emphasize those range bins which are most effected by variations in the parameters of interest. Finally the resultant signal is summed over the whole range window to obtain the error signal \( \varepsilon_{\theta_j} \), and this error can be used to obtain a new estimate \( \theta_j \). Fig. 4-6 shows \( \frac{\partial g_i}{\partial \theta_j} \) as dotted lines with \( \theta_j \) equal to \( \tau_0, \sigma_r \), and \( \sigma_0 \).

The whole process of MLE is shown in Fig 4-7. The error signals are used by three tracking loops to modify the estimates. The estimated delay is used to start the de-ramp signal. The estimated \( \sigma_r \) and \( \sigma_0 \) are used to generate the estimated waveform \( g_i \) and the partial derivatives of the three parameters \( \frac{\partial g_i}{\partial \theta_j} \). Although the diagram seems
simple, the computation required to derive $\frac{\partial g_i}{\partial \theta_j}$ is enormous. In sub-optimal Likelihood Estimation (SMLE), the weighting function $\frac{\partial g_i}{\partial \theta_j}$ is replaced by a rectangular function in each case, which is shown in Fig. 4-6 as solid lines. The simplification greatly reduces the computation requirement, which will be described in the next section.

4.3 SMLE for Radar Altimeter Signal Processing

In MLE the division by $g_i^2$ gives different weights to the various differences according to the Brown model, that increase with distance. This normalization loses the importance when the thermal noise is high, i.e. at low signal to noise ratio. To avoid this, in Sub-optimal Maximum Likelihood Estimation (SMLE), the normalization is set to a constant proportional to $\sigma^0$ [Levrini]. Then (4.19) becomes:

$$\sum_{i=1}^{M}(g_i - g) / \sigma^0 \frac{\partial g_i}{\partial \theta_j} = 0$$  \hspace{1cm} (4.20)

In order to reduce the quantity of computation, three rectangular functions replace $\frac{\partial g_i}{\partial \theta_j}$ to give a simple weighting on $g_i - g$. The key point in SMLE is the
determination of leading edge width that defines the rectangular weighting function and slope of leading edge. It is realized by using a linearized waveform model, which is shown in Fig. 4-8.

4.3.1 Linearized Waveform Model

Sub-optimal Maximum Likelihood Estimation (SMLE) is based on a linearized waveform model. In this section I introduce such a model.

An optimal linear approximation of equation (4.14) can be obtained by minimizing the root mean square deviation between the \( \text{erf}(\cdot) \) function and its linear approximation, see Fig. 4-7. If \( \alpha_o \) is a coefficient for optimal \( \text{erf}(\cdot) \) linearization, the resulting linear function for the leading edge itself is given by:

\[
P_r(\tau) = \frac{D}{2} + \frac{\alpha_o D}{\sigma_e} \tau
\]

(4.21)

Here \( D \) is the peak to peak value of the waveform.

\( \sigma_e \) is defined in equation (4.9).

The optimal \( \alpha_o \) value is 0.3227 by minimizing the root mean square deviation of the \( \text{erf}(\cdot) \) function and its linear approximation [Somma 1982].

The width of leading edge is thus \( \sigma_e / \alpha \) in time, as shown in Fig. 4-8.
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We know that after de-ramp processing, the delay time \( r \) transfer to frequency difference \( \Delta f \) and the following equation exists:

\[
\Delta f = \frac{B}{T_c} r
\]  
(4.22)

\( T_c \) is the chirp pulse duration.

\( B \) is the signal bandwidth.

Equation (4.21) can also be expressed as:

\[
P_r(\Delta f) = \frac{D}{2} + \frac{\alpha_o D T_c}{\sigma_c} \Delta f = \frac{D}{2} + \frac{\alpha_o D}{\sigma_c B} \mu = \frac{D}{2} + \frac{\alpha_o D}{\sigma_c} r_c \mu
\]  
(4.23)

\[ r_c = \frac{1}{B}, \] is the compressed pulse width.

In equation (4.23) a normalized frequency \( \mu \) has been introduced. It is given by

\[
\mu = \Delta f \cdot T_c = B r_c, \quad \frac{\sigma_c B}{2\alpha_o} \leq \mu \leq \frac{\sigma_c B}{2\alpha_o}
\]  
(4.24)

According to the meaning of \( \mu \), a discrete integer \( i \) can be used for filter identification, so that equation (4.23) becomes

\[
P_r(i) = \frac{D}{2} + \frac{\alpha_o D r_c}{\sigma_c} i = \frac{D}{2} + h_i i
\]  
(4.25)

where \( h_i \) is the slope of the leading edge of linearized model, which is given by

\[
h_i = \frac{\alpha_o D r_c}{\sigma_c} \approx \frac{\alpha_o D c}{\sigma_c r_c}
\]  
(4.26)

The approximation in equation (4.26) is obtained by neglecting the contribution of \( \sigma_p \) to \( \sigma_c \).

From (4.21) we know that the width of the leading edge can be expressed as a time interval that is \( \frac{\sigma_c}{\alpha_o} \) second, or expressed in range as \( \frac{c\sigma_c}{2\alpha_o} \) meter. From (4.23) the width of the leading edge can also be expressed as a normalized frequency \( \frac{\sigma_c}{\alpha_o r_c} \) or its approximation \( \frac{2\sigma_c}{c\alpha_o r_c} \). These results are shown in Fig. 4-8.

It is more practical to use a number of filters, or the number of samples as the x-axis of Fig. 4-8. From equation (4.24) and (4.25), the number of samples corresponding to the echo leading edge can be determined as follows [Somma 1982]:

55
If we know the SWH and compressed pulse width, we can determine the width of the leading edge of the linearized waveform as an integer number of samples from equation (4.27). Supposing $\sigma_s = 5m$, which corresponds to a 20m SWH, and using ERS-1 compressed pulse width that is 3.2ns, from (4.27) the maximum leading edge width is 33 samples.

4.3.2 Sub-optimal Maximum Likelihood Estimation [Somma 1982] [Levrini]

Sub-optimal Maximum Likelihood Estimation (SMLE) is based on the linearized echo waveform and simplified $\frac{\partial g_i}{\partial \theta_j}$ as given before. There are 64 samples for each waveform, which facilitates the FFT processing. But actually only the first 63 samples are used in SMLE processing. In order to facilitate my description I shall define some of the variables first. We define the samples used in SMLE processing as $LV(i)$ $i = 1, \cdots, 63$. The set of samples is divided into five parts: AA, BB, CC, DD and EE as shown in Fig. 4-9. BB, CC, DD forms the leading edge of the echo. CC is the middle
of the leading edge and also the middle of the echo. In SMLE, CC is always located at the 32nd range bin. The width of the leading edge \( N_e \) is determined by equation (4.27) varying from 3 to 33 according to SWH. The half leading edge width is calculated as:

\[
\begin{cases}
\text{Int} \left( \frac{\sigma_i}{\alpha c \tau_c} \right) = \text{Int} \left( \frac{D}{2h_i} \right), & \text{if } \text{Int} \left( \frac{\sigma_i}{\alpha c \tau_c} \right) \geq 1 \\
1, & \text{if } \text{Int} \left( \frac{\sigma_i}{\alpha c \tau_c} \right) < 1
\end{cases}
\]  

(4.28)

\( N_e = 2w + 1 \)  

(4.29)

Here we also define some names for some specific filter numbers:

- \( MI = 32 - (N_e - 1) / 2 \)
- \( MA = MI - 1 \)
- \( MS = 32 + (N_e - 1) / 2 \)
- \( ME = MS + 1 \)

The five parts of the range samples are defined as follow:

- \( AA = \sum_{i=MI}^{MA} LV(i) \)
- \( BB = \sum_{i=32}^{31} LV(i) \)
- \( CC = LV(32) \)
- \( DD = \sum_{i=33}^{MS} LV(i) \)
- \( EE = \sum_{i=ME}^{63} LV(i) \)

\( M_{AGC} = (AA + BB + CC + DD + EE) / 63 = \frac{\sum_{i=1}^{63} LV(i)}{63} \)  

(4.30)

4.3.2.1 AGC Tracking Loop

As described before, SMLE needs to estimate \( \sigma_0 \) that determines the signal strength. By using a constant weighting function as shown in Fig. 4-6, we can estimate the signal strength by minimizing the difference between the summation of all samples within the echo window of the received echo and the summation of that of the estimated signal. When the difference approaches zero, the incoming signal strength should be the same as the estimated signal strength. Because we have defined a
reference of signal strength, by using an AGC system the strength of the incoming echo can be adjusted to match the reference. In this way the $\sigma_0$ can be derived from

![Diagram of AGC loop](image)

**Fig. 4-10 Principle of AGC loop**

the AGC value. This is the principle of $\sigma_0$ estimation. Fig. 4-10 shows a diagram of the AGC loop.

However due to the thermal noise, the signal after square law detection is not the $\text{erf}(\cdot)$ function as mentioned before. The noise power makes a pedestal to the ideal waveform as shown in Fig. 4-11. This pedestal must be removed before SMLE is applied. The noise pedestal can be estimated from the first few samples that are purely noise. The actual AGC processing constitutes of the following steps:

1. Estimate the level of the noise pedestal from the first few samples;
2. Remove the noise pedestal from the waveform;

![Effect of noise on linearised echo](image)

**Fig. 4-11 Effect of noise on linearised echo**
3. Compensate the decay of the trailing edge by using an exponential correction
4. Estimate $\sigma_0$ by using the SMLE method.

### 4.3.2.2 Range Tracking Loop

According to SMLE’s delay time weighting function, range error estimation is achieved by using $N_E$ samples of the leading edge. The range tracking system acts by comparing the position of the echo signal with that of an estimated reference signal.

This echo position is corrected by acting on the generation of the local oscillator chirp and on fine adjustment in the filter bank. As indicated in Fig. 4-12, the assumed range error signal is the shaded area that is estimated within the leading edge. The

![Input waveform](image)

**Fig. 4-12 Schematic of range error signal**

width of the leading edge is derived from equation (4.27), which is $N_E$ as shown in Fig. 4-12. The difference between the input waveform and the estimated waveform is then $e_i$ for each specific sample point. So the average difference within the leading edge is:

$$
\varepsilon_r = \frac{\sum_{i=M1}^{MS} e_i}{N_E} = \frac{\sum_{i=M1}^{MS} (\hat{g}_i - g_i)}{N_E} = \frac{\sum_{i=M1}^{MS} \hat{g}_i - \sum_{i=M1}^{MS} g_i}{N_E} - \frac{\sum_{i=M1}^{MS} \hat{g}_i - M_{AGC}}{N_E} \quad (4.31)
$$

Because the mean waveform and its leading edge are symmetric about the half-power point, the average of the samples of the echo should be equal to the average of the samples of the leading edge and equal to the middle point of the leading edge. In practical application $M_{AGC}$ that is derived from the AGC loop is used in (4.31). The diagram of the range tracking loop is shown in Fig. 4-13.
Fig. 4-13 Principle of range tracking loop

Because the slope of the leading edge is \( \frac{D\alpha_e}{\sigma_e} \), the range error (in centimetres) can be derived from the error signal (shaded area in Fig. 4-12) as follow:

\[
\Delta R = \frac{\sum_i \varepsilon_i \sigma_e}{N_e D \alpha_o} = \frac{D \tau_1 - D \tau_1 \frac{\tau_1 \alpha_e}{c \sigma_e \sigma_e}}{\frac{c \sigma_e^2}{2 \alpha}} = \frac{D \tau_1 (1 - \frac{\tau_1 \alpha_e}{c \sigma_e})}{\sigma_e \alpha_o}
\]

\( \Delta R = \tau_1 (1 - \frac{\tau_1 \alpha_e}{2 \sigma_i}) \) if \( |\tau_i| \leq \frac{c \sigma_e}{2 \alpha_o \alpha_o} \approx \frac{\sigma_c}{\alpha_o} \) (4.32)

Here

\( \Delta R \) is estimated range error;

\( \sigma_e \) is defined in equation (4.9);

\( \sigma_i \) is defined as \( \sigma_e \cdot \frac{c}{2} \);

\( \tau_1 \) is the range misalignment defined in Fig. 4-12;

\( \varepsilon_i \) is the \( i \)th sample difference between input waveform and estimated waveform within leading edge.

Then the characteristic of the range discriminator is:
\[\Delta \tau = \begin{cases} \tau_1(1 - \frac{\tau_1}{\alpha_o}) & \text{if } |\tau_1| \leq \frac{\sigma_h}{\alpha_o} \\ \frac{\sigma_h}{2\alpha_o} & \text{if } \tau_1 \geq \frac{\sigma_h}{\alpha_o} \\ -\frac{\sigma_h}{2\alpha_o} & \text{if } \tau_1 \leq \frac{\sigma_h}{\alpha_o} \end{cases} \quad (4.33)\]

\(\Delta \tau\) is discriminator output (cm), which is shown in Fig. 4-33.

4.3.2.3 Slope tracking loop [Griffiths, et al., 1985]

The main purpose of the slope tracking loop is to derive the width of leading edge. It depends on the following assumptions: first, the form of the radar echo is given by the Brown model. Second, the SMLE tracker correctly positions the return so that gate 32 corresponds to the mean level. Third, the SMLE tracker applies a correction to the return waveform to remove the trailing edge decay due to the antenna beam pattern and mispointing. Clearly violation of any of these assumptions can lead to bias in the estimation of SWH.

Under the assumptions listed above the mean echo waveform can be expressed as

\[p_r(\tau) = [1 + \text{erf}(\frac{\tau}{\sqrt{2\sigma_s}})]/2 \quad (4.34)\]
A linearized waveform model is achieved by equalizing the shaded areas shown in the schematic representation of the return in Fig. 4-15. We may represent this procedure analytically by equating the area under \( P_r(\tau) \) between A and B with the area of the trapezium ABCO (see Fig. 4-15), thus

\[
\int_{-T_i}^{T_i} P_r(\tau) d\tau = 3T_i / 4
\]  \hspace{1cm} (4.35)

where \( T_i \) is the half leading edge width, measured in units of time. (The SMLE tracker half-width is measured in range gates). Alternatively, because of the anti-symmetry of \( \text{erf}(\cdot) \), the area equalization may be represented by

\[
\int_{-T_i}^{T_i} P_r(\tau) d\tau = \int_{-T_i}^{0} P_r(\tau) d\tau + T_i / 2
\]  \hspace{1cm} (4.36)

By using equation (4.34) and (4.35), equation (4.36) may be reduced to:

\[
\int_{0}^{T_i} \text{erf}(\frac{\tau}{\sqrt{2}\sigma_c}) d\tau = T_i / 2
\]  \hspace{1cm} (4.37)

or

\[
\int_{0}^{y} \text{erf}(x) dx = y / 2
\]  \hspace{1cm} (4.38)

where \( y = T_i / (\sqrt{2}\sigma_c) \)  \hspace{1cm} (4.39)

As \( \text{erf}(x) \) is a standard function, equation (4.38) can be solved numerically in a straightforward manner to obtain the solution
Combining equations (4.9), (4.39), and (4.40) yields the following relationship between $\sigma_c$ and $T_i$

$$\sigma_c = \frac{c}{2} \sqrt{(0.6801T_i)^2 - \sigma_p^2}$$  \hspace{1cm} (4.41)

Thus given the half leading edge width $T_i$ from the SMLE slope tracker we may obtain the significant wave height from the equation above. In terms of the SMLE tracker, variables $T_i$ is given by the half-width in range gates multiplied by the range bin width.

By using the linearized model, we can derive the error signal of the slope-tracking loop as follow:

![Fig. 4-16 Schematic explanation of slope error derivation](Image)

As shown in Fig. 4-16, suppose the leading edge of the real echo is AC, and the estimated leading edge of the echo is A'C'. The estimated leading edge width is $2T_i$ as shown above. The slope-tracking algorithm acts to minimize the area of OMC and OMC'C' which will minimize the slope difference. It is undertaken as follow:

1) Calculate the MOFB and M'OFC' and then derive the difference between MOFB and M'OFC';

2) Subtract the above difference from OFBD and derive the slope error signal $\varepsilon_i$;

3) It is easy to prove that this slope error signal $\varepsilon_i$ is equal to the area summation of OMA' and OMC';

4) Suppose the slope difference is small so that we can derive the slope error $\Delta h_i$:
\[ \Delta h = \frac{\delta h}{T_1^2} \]  

(4.42)

The above analysis assumes that the return is a continuous function of the delay time \( \tau \) given by (4.34). In practice, the SMLE tracker obtains estimates of the return power at 63 range gates and this discrete form of the return will lead to a change in the SWH estimation.

![Discrete version of slope error algorithm](image)

Fig. 4-17 Discrete version of slope error algorithm

The discrete version of the slope error algorithm is shown in Fig. 4-17. Let \( T_2 \) be the half width, measured in range gates. It has following relationship with leading edge sample numbers given in equation (4.27) and (4.28):

\[ \frac{N_s - 1}{2} = Int(T_2) = w \]  

(4.43)

\( Int(T_2) \) stands for the integer part of the \( T_2 \).

So the discrete algorithm will result in an error shown in Fig. 4.17 as \( \Delta BB \) and \( \Delta DD \) which is the fractional part of \( T_2 \). This error must be compensated in calculation. Thus the slope error signal is calculated as

\[ \varepsilon_h = DD + \Delta DD - BB - \Delta BB - T_2 \]
Chapter 4 Simulation of Radar Altimeter Signal Processing

\[ \sum_{i=32}^{32+[T_2-0.5]} LV(i) + \{T_2 - 0.5 - [T_2 - 0.5]\} LV(32 + [T_2 - 0.5] + 1) \]

\[ - \sum_{i=32-[T_2-0.5]}^{31} LV(i) + \{T_2 - 0.5 - [T_2 - 0.5]\} LV(32 - [h_i - 0.5]) - T_2 \]  \hspace{1cm} (4.44)

\( T_2 \) also stands for the area of OFBD as shown in Fig. 4-16. Then by using equation (4.42) the slope error can be derived.

4.4 Simulation of Altimeter Signal Processing

This simulation is to investigate the system distortion effects on parameter estimation, especially on SWH estimation. The distortions can be classified as:

1) Antenna off nadir looking;
2) Radar system phase and amplitude distortions that come from the signal generator, transmitter and receiver.

The phase distortion due to the signal generator might be very large because the digital generated Chirp signal is multiplied before we get the final Chirp signal. Because the de-ramp technique can cancel most of this kind of phase error [Griffiths 1992], the residual phase error will be small. The general phase and amplitude errors come from whole system will be modelled as a sinusoid error with amplitude \( A_p \) and \( A_a \). These phase and amplitude error will generate paired echoes in the compressed pulse.

In the following I will give the simulation of signal processing for radar altimeter. The echo generation is based on the Brown model and the on board signal processing is based on the SMLE method.

4.4.1 Simulation diagram

The simulation is composed of following simulation modules:

- Signal generation module
- AGC tracking module
- Range tracking module
- SWH tracking module
Their relationships are shown in Fig. 4-18.

4.4.2 Signal generator module

The signal generation module generates the sea surface echo according to the sea surface wave height, antenna pointing error, antenna diameter, radar system phase and amplitude distortions. The first three effects are presented by the flat surface response function. The last one is presented by the point target response function. Triple convolution will result in the mean received echo.

The flat surface impulse response function is given in equation (4.3),

$$P_{fs}(\tau) = \frac{G_o^2 \lambda^2 c \sigma_0(\nu)}{4(4\pi)^2 L_p H^3} \exp\left[-\frac{4}{\gamma} \sin^2 \xi - \frac{4c}{\gamma H} \tau \cos 2\xi\right] \cdot I_0\left(\frac{4}{\gamma} \sqrt{\frac{c\tau}{H}} \sin 2\xi\right) \quad (4.45)$$

By changing the $\xi$ in equation (4.45), we can get the flat surface impulse response function at different off-nadir angles.

The Gaussian specular point density function is

$$q(\tau - \tau_0) = \frac{1}{\sqrt{2\pi(2\sigma_z/c)^2}} \exp\left[-\frac{(\tau - \tau_0)^2}{2\left(\frac{2\sigma_z}{c}\right)}\right] \quad (4.46)$$
\( \sigma_r \) is the rms height of the specular points relative to the mean sea level. While generating the Gaussian specular point density function, a delay time \( r_0 \) accounts for the difference between echo delay and estimated echo delay. This delayed Gaussian specular point density function convolves with the radar system impulse response function and the flat surface impulse response and generates a delayed mean received echo.

The Brown model uses a Gaussian point target response function for simplicity. When we study the effect of system distortion on parameter estimation, we have to use the
Chapter 4 Simulation of Radar Altimeter Signal Processing

distorted \( sinc^2(x) \) function instead of the Gaussian point target response function.

The distortion function is expressed in the frequency domain [Cook 1967]:

\[
H(\omega) = A(\omega) \exp[-jB(\omega)]
\]

(4.48)

here

\[
A(\omega) = a_0 + a_1 \cos C_1 \omega
\]

(4.49)

\[
B(\omega) = b_0 \omega - b_1 \cos C_2 \omega
\]

(4.50)

\( A(\omega) \) is the amplitude distortion.
\( a_0 \) is the ideal amplitude of system response.
\( a_1 \) is the amplitude of the sinusoid amplitude distortion. It will generate a paired echoes with amplitude of \( 20 \log \frac{2a_0}{a_1} \).

\( C_1 \) is the number of cycles of the amplitude distortion per Hertz in frequency domain.
\( B(\omega) \) is the phase distortion.
\( b_0 \) is the constant for linear phase term.
\( b_1 \) is the amplitude of the sinusoid phase distortion. It will generate a paired echo with amplitude of \( 20 \log \frac{2}{b_1} \).

\( C_2 \) is the number of cycles of the phase distortion per Hertz in frequency domain.

After Fourier transform, we derive the distorted impulse response with paired echo as mentioned above.

By convolving (4.45) with (4.47), we can get the mean received echo.

Depending on the assigned input SNR, we can add a proper noise level to the mean received echo. Suppose the mean received echo now is \( E(i) \), including the noise. We then introduce the ocean statistics as follows:

The statistics of each sample of single received echo is an exponential distribution.

The random signal can be generated according to

\[
\hat{g}(i) = -\ln(1 - n(i)) \cdot E(i)
\]

(4.51)

\( n(i) \) is uniformly distributed random number between [0 1].

68
\( g(i) \) is the exponentially distributed random number, which is what we want. There are about \( n \) echoes that are averaged to get a final averaged one \( V(i) \), which are then sent to AGC tracking loop.

### 4.4.3 AGC tracking module

The signal from the Signal generator module is first attenuated by an attenuator. Then the first 8 range samples are averaged to evaluate the noise pedestal \( r_1 \). The noise is removed from the waveform \( V(i) \). Then the noise-free waveform is corrected for the antenna shape weighting. We now have a signal with mean value of an \( \text{erf}(\cdot) \) function.

![Flow diagram of AGC loop](image)

Fig. 4-20 Flow diagram of AGC loop
This corrected signal is then used to calculate the average signal power by summing the 63 range samples. The resulting value $M_{AGC}$ is then compared with the predefined reference value to derive the AGC error signal. This error signal is then filtered by a $\alpha-\beta$ filter. The output is the filtered AGC value and is sent to the digital attenuator to control the receiver gain.

The mean amplitude $M_{AGC}$ is subtracted from the corrected signal and then sent to the range tracking loop.

### 4.4.4 Range tracking loop

![Flow diagram of range tracking loop](image)

The range tracking loop closes after the AGC has locked. The leading edge width is initialized corresponding to 20m SWH according to equation (4.27). After running the SWH tracking loop, the new leading edge width will be derived. The range error signal is derived from the summation of the signal within the leading edge. Then the error divided by the slope of the leading edge will result in the range error, according to equation (4.32).
4.4.5 SWH tracking loop

The SWH tracking loop closes after the range tracking loop has closed. It uses the data processed by AGC loop. The principle of processing is described in 4.2.3. The output is the width of the leading edge that can be used by the range tracking loop. The SWH can also be derived from equation (4.41).

4.4.6 Simulation results of SWH estimation

The purpose of the simulation is to study the estimation bias due to different system distortion as described before. All the three tracking loops use $\alpha$-$\beta$ trackers with $\alpha = \frac{1}{4}$ and $\beta = \frac{1}{64}$. The transients of these loops have not been paid much attention. Usually each loop starts with a wide loop bandwidth to reduce the transit.
Chapter 4 Simulation of Radar Altimeter Signal Processing

time. When the loop comes into the steady state, a narrow bandwidth is used to reduce the output noise. In this simulation, the loop bandwidth is about 3Hz. Various errors due to system distortion are simulated and the results are shown as follows.

**Discrete model induced error**

As described in 4.2.3, the SMLE uses a discrete summation to approximate the relation of equation (4.36), which is given in (4.44). This will induce some error in SWH estimation. In this simulation, the SWH value derived by using the SMLE algorithm is compared with the real SWH which is used in echo generation. The difference between the estimated SWH by SMLE and the true WH is shown in Fig. 4-23.

![Fig. 4-23 The difference between SMLE SWH and true SWH](image)

This result shows that at low SWH condition there is a bias, about 0.12m. When SWH increases, this bias will become small and can be ignored.

**Phase and Amplitude Distortion**

Radar system phase and amplitude distortion effects on range accuracy have been given more attention than those on SWH estimation. Range estimation error caused by phase distortion can be expressed analytically [Sheehan 1992]. Amplitude distortion will increase the side lobe and widen the main lobe. Phase error will shift the main lobe and widen the main lobe. All these will affect the final estimation of
leading edge slope. In this research, phase and amplitude distortion effect on SWH estimation are studied by simulation.

![Fig. 4-24 SWH error induced by distorted $\sin^2$ response](image)

Fig. 4-24 shows the SWH error when the phase ripple is about 20° and the amplitude ripple is 0.5 dB. These distortions given above are relatively large for a radar system. But the induced SWH error is relatively small. This means that the SWH is not sensitive to amplitude and phase distortion.

**Pointing Error Induced SWH Error**

![Fig. 4-25 SWH error due to 0.5 deg. off-nadir pointing](image)

Fig. 4-25 SWH error due to 0.5 deg. off-nadir pointing
Antenna pointing error will reduce the overall signal strength, and change the echo shape by raising the trailing edge. The first effect will give rise to an error of wind speed estimation that will be discussed in next chapter. The latter effect will induce range error. Although the slope of leading edge does not change much, the error of range tracking point will result in an error in the SWH estimation. The SWH error due to 0.5° off-nadir pointing is shown in Fig.4-25. Although a small antenna will reduce the off-nadir pointing effect it will reduce antenna gain proportionally. A 1m antenna and a 13.5GHz carrier frequency are used in the simulation.

The simulation shows that SWH error mainly comes from off-nadir pointing. It is not sensitive to amplitude and phase distortion. Under 0.5° pointing error a radar altimeter with a 1m antenna still can give a satisfactory SWH measurement.

4.5 Conclusion
In this chapter, the on-board signal processing has been simulated, based on the Brown model and SMLE. In particular the errors of SWH estimation have been evaluated. The results show that when the magnitude of amplitude distortion is 0.5dB and the amplitude of phase distortion ripple is 20°, the resulting SWH error is less than 0.2m. The main contribution of SWH error is from the antenna off-nadir pointing. But when the off-nadir pointing angle is smaller than 0.5°, the SWH error will be smaller than 0.5m or 10% of the real SWH, whichever is larger.
From Chapter 1 to Chapter 3, I have given the general background of radar altimeter remote sensing. In Chapter 4, I focused on the antenna off-nadir looking effect on the SWH estimation. The result showed that even with 0.5° off-nadir pointing the SWH estimation is still satisfactory. But wind speed measurement is much more sensitive to pointing error. In Chapter 5, I am going to describe the original work of mine, which is to measure the wind speed by using a low-resolution mode radar altimeter. This mode can cope with off-nadir pointing error as large as 0.5°.[Sun, et al., 2000]

Wind speed is derived from normalized radar cross section (\(\sigma^0\)). Off-nadir pointing results in varying antenna gain in the nadir direction, which will induce an error in the \(\sigma^0\) measurement. In order to reduce \(\sigma^0\) bias to less than 0.4dB, the pointing error for a conventional radar altimeter with 1m antenna should be smaller than 0.2°. In this chapter, the principle of \(\sigma^0\) measurement will first be introduced. Then some methods that can improve the \(\sigma^0\) measurement will be compared. Finally a new low-resolution mode is proposed to improve the wind speed measurement.

5.1 Principle of Wind Speed Measurement

Wind speed is derived from the normalized radar cross section. In this part of study, I am going to describe how to derive the normalized radar cross section from the mean received echo.

The mean received echo is expressed as usual by the triple convolution:

\[
P_r(\tau) = P_{FS}(\tau) \otimes q(\tau) \otimes P_T(\tau)
\]

\(\otimes\) is the convolution.

\(P_{FS}(\tau)\) is the flat surface impulse response function;

\(q(\tau)\) is the sea surface specular point distribution function,

\(P_T(\tau)\) is the radar system point target response function.

\[
P_{FS}(\tau) = \frac{G_0^2 \lambda^2 c \sigma^0(\psi_0)}{4(4\pi)^2 I_p H^2} \exp[-\frac{4}{\gamma} \sin^2 \xi - \frac{4c}{\gamma H} \tau \cos 2\xi] I_0\left(\frac{4}{\gamma} \sqrt{\frac{c\tau}{H}} \sin 2\xi\right)
\]

\(= \sigma^0(\psi_0) \cdot P_{FS}(\tau)\)  \hspace{1cm} (5.2)

where
Chapter 5 Low Resolution Mode for Wind Speed Measurement

\[ P_{FS}(\tau) = \frac{G_0^2 \lambda^2 c \sigma^0}{4(4\pi)^2 L_p H^3} \exp\left[-\frac{4}{\gamma} \sin^2 \xi - \frac{4c}{\gamma H} \tau \cos 2\xi \right] I_0 \left(-\frac{4c}{\gamma H} \sin 2\xi \right) \]  

(5.3)

\( \tau \) is the time delay relative to the first nadir return;

\( G_0 \) is the antenna gain at boresight axis;

\( \lambda \) is the radio wavelength;

\( \sigma^0(\psi) \) is the normalized radar cross section, it is a function of radar look angle;

\( L_p \) is the two-way propagation loss;

\( H \) is the height of satellite;

\( \gamma = 2.895 \cdot \sin^2(\theta/2) \), where \( \theta \) is the 3dB beam width of the antenna;

\( \xi \) is the antenna off nadir angle;

So (5.1) can be expressed as:

\[ P_r(\psi) = \sigma^0(\psi) \cdot q(\psi) \otimes P_T(\psi) \otimes p_{FS}(\psi) \]

(5.4)

Within small look angle \( \psi \), \( \sigma^0(\psi) \) can be considered to be a constant \( \sigma^0 \).

Also \( q(\psi) \) and \( P_{TT}(\psi) \) are the fixed time functions. The received power signal \( P_r(\psi) \) will change only when the flat surface impulse response function changes. From (5.2) we can see that the flat surface impulse response function \( P_{FS}(\psi) \) is a function of radar off nadir look angle. By using (5.4), we can derive the mean received echo \( P_r(\psi) \) at different off-nadir look angles as shown in Fig.5-1. Off nadir looking will change the amplitude of the mean waveform and reduce the slope of the trailing edge.

![Fig. 5-1 Mean received echo under different antenna off-nadir pointing angles](image)
From (5.4) we can derive radar cross section as follow:

For \( \sigma^0(\psi) \) within all reasonable values of \( \psi \), \( \sigma^0(\psi) \) can be regarded as a constant \( \sigma^0 \). Integrating \( P_r(t) \) will result in the energy received by the altimeter

\[
E = \int P_r(t)dt = \sigma^0 \int [q(t) \otimes P_{pr}(t) \otimes P_{fs}(t)]dt
\]

(5.5)

\[
\sigma^0 = \frac{E}{\int [q(t) \otimes P_{pr}(t) \otimes P_{fs}(t)]dt}
\]

(5.6)

The integration of \( \int [q(t) \otimes P_{pr}(t) \otimes P_{fs}(t)]dt \) can be determined by knowing variables in (5.3). The variables in (5.3) are determined from pre-launch and real time calibrations. The off nadir angle \( \xi \) is very critical in the calculation of the integration. Without the knowledge of the off-nadir angle, an error of \( \sigma^0 \) will be created, which can be seen in Fig. 5-2. (The diameter of the antenna is 1 metre in this simulation.)

![Fig. 5-2 Sigma0 error with respect to off-nadir look angles](image)

At an indicated pointing angle of 0.5°, a 0.2° error of attitude knowledge results in about 1.5dB error in \( \sigma^0 \) estimation.[Townsend 1980] In high wind speed condition this \( \sigma^0 \) error translates to about 6m/s in wind speed measurement [Wingham 1988]. Fig. 5-2 shows that at large off-nadir look angle \( \sigma^0 \) is more sensitive to pointing error than in a small off-nadir angle condition.
Chapter 5 Low Resolution Mode for Wind Speed Measurement

The wind speed as a function of \( \sigma^0 \) is inferred from fitting an analytical formula to the experiment data. By choosing a different set of experimental data, different empirical formulas can be derived [Donna 1991]. The ‘modified Chelton and Wentz wind speed model function’ relating Geosat measurements of \( \sigma^0 \) to wind speed at 10m above the sea surface is shown in Fig 5-3. It shows in large wind speed conditions, that the wind speed is more sensitive to \( \sigma^0 \) error.

![Wind speed model function relating sigma0 to wind speed](image)

For conventional high-resolution radar altimeter with 1m antenna, in order to reduce the \( \sigma^0 \) estimation error to less than 0.4dB, the pointing error should be smaller than 0.2°. At larger off-nadir pointing conditions attitude knowledge should be much better than 0.2° in order to achieve \( \sigma^0 \) error smaller than 0.4dB. This requirement is relatively difficult for a cost effective microsatellite.

5.2 Origination of \( \sigma^0 \) Error

A conventional radar altimeter operates in pulse limited mode. As shown in Fig. 5-4, a radar altimeter footprint can be classified as:

- Equal distance contour footprint
- Equal power density contour footprint

Power returns in time according to the equal distance contour. Power is distributed in space according to the equal power intensity contour that is determined by the radar look angle.
Only power returning from the equal distance contour footprint can be received by the radar altimeter. When these two footprints overlap with each other, maximum energy will be received by the radar altimeter. This is the nadir looking condition as shown in left side of Fig. 5-4. When these two footprints do not overlap, which is shown in right side of Fig. 5-4, only the part of the energy coming from equal distance contour footprint will be received by radar altimeter. The remaining energy outside the equal distance contour footprint, but within the equal power intensity contour footprint, cannot be received by radar altimeter. The energy received in these two different conditions is different. This difference results in the error of $\sigma^0$ measurement.

5.3 Some Methods to Reduce the $\sigma^0$ Error

5.3.1 Reduce Antenna Gain Variation by Using A Small Antenna

As being shown in last section, the energy difference is a result of the antenna gain variation with nadir direction. If we use a small antenna that has a flatter antenna gain pattern, the variation of antenna gain will be smaller than that using a large antenna.
Chapter 5 Low Resolution Mode for Wind Speed Measurement

Fig. 5-5 Sigma0 error as a function of pointing error with 0.4m antenna

Fig. 5-5 shows the $\sigma^0$ error as a function of off-nadir look angle with a 0.4m antenna. This time the $\sigma^0$ error is smaller than 0.4dB, even at 0.5° off-nadir look angle. However, compared to the 1m antenna, the antenna gain reduction is about 8dB. Two-way antenna gain loss will be about 16dB. This means the transmitted power must increase about 40 times. This is an unacceptable trade off.

5.3.2 Correct the Sigma0 Error by Accurate Attitude Determination

It is obvious that if we can measure the off-nadir pointing angle, according to Fig. 5-2 we can correct the corresponding $\sigma^0$ error. There are several ways to achieve accurate attitude determination:

- Using highly accurate sensors, such as star sensor, horizon sensor, sun sensor.
- Deriving the pointing error from the radar echo waveform itself.

The first method is used by modern radar altimeters. By using high accurate attitude sensor the satellite attitude can be determined and controlled with high accuracy. For ERS-1 radar altimeter the attitude control is better than 0.2°. TOPEX achieve even more accurate attitude control, better than 0.1°. Under these conditions there is no need to do further correction.

Estimates of pointing errors for the Skylab radar altimeter were obtained[McGoogan, 1974] from the decay rate of the trailing edge of the average return waveform. At that time, pointing-angle determination had been evaluated by fitting the theoretical waveform to the experimental data. The best fit was obtained by using the theoretical
waveform with 0.3° off-nadir pointing angle. Because the theoretical waveform was calculated in 0.05° increments, the 0.05° accuracy figure was empirically determined to be the accuracy obtainable.

A feasibility study of onboard correction to antenna mispointing was done by Dr. Wingham [Wingham, 1988]. It showed with a loop response time of about a minute it is possible to estimate the off-nadir pointing angle and correct the $c^0$ bias caused by off-nadir pointing. But this correction requires very accurate calibration of the range bin samplers. The example shows that for an altimeter such as that on ERS-1, to achieve 0.2dB accuracy after correction the sampler calibration error should be smaller than 0.01dB. This requirement is really very stringent.

5.4 Low Resolution Mode for Wind Speed Measurement

Conventional high-resolution radar altimeter only receives energy backscattered from the small nadir area. Actually collecting most of the energy received by the antenna beam can reduce the variation of energy due to small off-nadir looking. The normalized flat surface impulse response function is shown in Fig. 5-6. It is normalized by the maximum value of the nadir look impulse response function.

![Normalized Flat Surface Impulse Response Function](image)

Fig. 5-6 Flat surface impulse response function at different off-nadir look angle
I integrated each of the flat surface impulse response function and normalized the energy by the maximum value obtained under nadir looking condition. The derived curve is shown in Fig. 5-7.

![Fig. 5-7 Normalized integrated energy under different off-nadir look angles](image)

The difference of the normalized integrated energy between nadir look and 0.5° off-nadir look as a function of integration time is shown in Fig. 5-8.

![Fig. 5-8 Difference of normalized integrated energy between 0 deg. and 0.5 deg. off-nadir look angles](image)
The relative error can be calculated by dividing the value in Fig. 5-8 by the value of 0° off-nadir in Fig. 5-7. The result is shown in Fig. 5-9.

![Relative integrated energy error](image)

**Fig. 5-9 Relative integrated energy error**

From Fig. 5-9 we can see that if we integrate the energy until 1000ns the relative energy error is less than 0.4dB.

The mean received echo is the convolution between the flat surface impulse response function and a function which is itself the convolution of the radar system point target response function and the sea surface specular point distribution function. During convolution the radar system point target response function acts as a shifting integration window. By using a low-resolution pulse we can obtain a wide radar system point target response function. Thus each output point of the mean received echo from the convolution is a weighted integration of the flat surface impulse response function over a time defined by the width of radar system point target response function. In order to facilitate simulation we assume a 2μs Gaussian shape point target response function. The SWH is supposed to be 8m. The simulated mean received echoes are shown in Fig. 5-10. The waveform in Fig.5-10 corresponds to nadir pointing and 0.5° off-nadir pointing conditions. The amplitude error at the peak point is about 0.3dB. This result shows that low-resolution mode for $\sigma^0$ estimation is very robust to off-nadir pointing error.
Chapter 5 Low Resolution Mode for Wind Speed Measurement

Fig. 5-10 Normalized echo of low resolution pulse at different off-nadir angles

The $\sigma^0$ error as a function of off-nadir look angle is shown in Fig. 5-11.

Fig. 5-11 Sigma0 error measured by new method as a function of off-nadir angles

The above simulation assumes that the low-resolution mode radar altimeter can always track the echo waveform point corresponding to mean sea surface. Because the radar altimeter is operated at a low-resolution mode, the peak of the echo waveform is very flat. The simulation shows even with 50ns delay time tracking error the resulting amplitude error is still small enough to be ignored. For a conventional high-resolution radar altimeter the delay time tracking error is less than 1ns. So by operating the radar altimeter at high-resolution mode and low-resolution mode alternatively we can measure wind speed with an improved accuracy. Also high-
resolution mode offers high accuracy delay time tracking which can satisfy the requirement of low-resolution waveform sampling.

### 5.5 \( \sigma^0 \) Variation Effect

As mentioned before, if we integrate the power of the flat surface impulse response function up to 1\( \mu \)s, the relative energy error is smaller than 0.4dB. Thus this new method will use a low-resolution pulse with a pulse width about 2\( \mu \)s. The sample point is about 2\( \mu \)s later than the first arriving point of the echo. This means that the echo from 1.7° off-nadir angle will be used in \( \sigma^0 \) estimation. The above simulation supposes \( \sigma^0 \) is a constant within \( \pm 1.7^\circ \). But actually the \( \sigma^0 \) is a function of radar look angle as given in (5.7)[Robinson, 1995].

\[
\sigma_0(\psi) = \frac{|R(0)|^2}{S_f^2} \sec^3 \psi \exp \left[ -\tan^2 \frac{\psi}{S_f^2} \right]
\] (5.7)

\( R(0) \) is the Fresnel reflectance of the air-sea interface at normal incidence. \( \psi \) is radar look angle and \( S_f^2 \) is the mean square slope of sea surface as given below [6].

\[
S_f^2 = 5.44 \times 10^{-3} U_{10} - 1.09 \times 10^{-3}
\] (5.8)

\( U_{10} \) is wind speed at 10m above the sea surface.

The flat surface impulse response function including \( \sigma^0 \) variation is different from (5.2) and is expressed as follows [Brown, 1977]:

\[
P_{fs}(t) = \frac{G_0^2 c \sigma^0(0)}{4(4\pi)^2 L_p h^3} \exp\left[ -\frac{4}{\gamma} \sin^2 \xi - c t \left( -\tau \cos 2\xi + \frac{1}{s^2} \right) \right] I_0 \left( \frac{c t}{\gamma h} \sin 2\xi \right)
\] (5.9)

\( \sigma^0(0) = \frac{|R(0)|^2}{S_f^2} \) is the normalized radar cross section at the nadir pointing condition.

From (5.7) and (5.8) it is obvious that in low wind conditions the \( \sigma^0 \) variation has a larger effect on the flat surface impulse response function than in high wind condition and will affect the final output mean waveform. The variation of \( \sigma^0 \) as a function of off-nadir look angle under different wind speed conditions is shown in Fig. 5-12. For wind speed larger than 2m/s this variation is smaller than 0.4dB even if the off-nadir pointing angle is 1.7°. Although at large incident angle the \( \sigma^0 \) will reduce
significantly, as shown in Fig. 5-12, the antenna gain at large incident angle will be small. Therefore, the reduction of $\sigma^0$ at large incident angle will have a small effect on general $\sigma^0$ measurement.

By using equation (5.9) instead of equation (5.2), we can simulate a low resolution echo waveform that includes the $\sigma^0$ variation effect. The worst case of the lowest wind speed of only 1m/s is used in this simulation. The echo is normalized by the maximum value of the echo excluding the $\sigma^0$ variation. Both echoes are received at 0° off-nadir look angle. It shows that the peak point value of the waveform changes less than 0.1 dB. (see Fig. 5-13)
Chapter 5 Low Resolution Mode for Wind Speed Measurement

Because at low wind speed condition, wind speed measurement is not sensitive to \( \sigma^0 \) error, small \( \sigma^0 \) measurement error at low wind condition due to incident angle change will not effect wind speed estimation. Thus by using a low resolution mode we can improve the wind speed measurement with pointing error up to about 0.5°.

5.6 Conclusion

Wind speed measurement is much more sensitive than SWH to pointing error. The simple way to reduce off-nadir pointing effect is to use a small antenna, but the transmitted power will increase dramatically. The other way is to compensate the \( \sigma^0 \) error by having a knowledge of pointing angle. This will require an accurate sensor. If the control error is 0.5°, the knowledge of pointing angle should be better than 0.1°. The accuracy of the knowledge of pointing angle will increase when the controlled error increases. In this chapter a low-resolution mode is proposed to improve the wind speed measurement. It can cope with a pointing error as large as 0.5°. This mode should work with the high resolution, which can provide the sampling time reference for the new operation mode. The new mode only requires a fractional part of the power used by the high-resolution mode. Thus a two-mode radar altimeter can achieve a satisfactory sea state monitoring with pointing error as large as 0.5°.
Chapter 6 System Study of Two Mode Radar Altimeter

For any radar system study the link budget estimation is the first step towards system design. It directly influences the system parameter choice. In this chapter I will first calculate the link budget for this two-mode radar. This calculation will effect the orbit selection based on limited power supply onboard the UOSAT platform. Of the whole radar altimeter the wide band chirp signal generator is the most critical subsystem. With the development of modern digital electronic technology, digital Chirp signal generation becomes more and more popular. This is because it is more flexible and gives better performance. Section 2 will discuss the digital chirp generation based on DDS (Direct Digital Synthesizer). The phase and amplitude distortion of the digital chirp generator will be discussed as well.

6.1 Link Budget Estimation for High Resolution Mode

As discussed in Chapter 4 and Chapter 5, in high resolution mode a small antenna will induce only small estimation bias due to off-nadir looking. But in order to operate the radar altimeter in a power-limited environment we still need a proper antenna aperture that can offer enough gain. The radar equation is expressed as follow [Levrini, 1984]:

\[
P_r = \frac{P_p G}{4\pi H^2} \pi H c \tau_0 \sigma_0^\circ \left[ \frac{1}{4\pi H^2} \right] G \lambda^2 \frac{R_e}{R_e + H} \]

\[
\text{SNR} = \frac{P_p G^2 \lambda^2 \pi H c \sigma_0^\circ \tau_0}{(4\pi)^3 H^4 k T B_f F L} \frac{R_e}{R_e + H} \]

Where

- \( P_p \) is the peak transmitted power;
- \( G \) is antenna gain;
- \( H \) is the satellite altitude;
- \( \tau_0 \) is the compressed pulse width;
- \( \sigma_0^\circ \) is the normalized radar cross section;
- \( \lambda \) is the wavelength of carrier;
- \( L \) is overall loss;
Chapter 6 System Study of Two Mode Radar Altimeter

\[ B_f \] is filter bandwidth;
\[ F \] is receiver noise figure;
\[ R_e \] is the earth radius.

The overall loss can be from the radar system, rain and cloud attenuation. We assume a 2.2dB loss for the radar system [Rey, 1999][Zelli, 1998], and leave 9dB loss margin for rain and cloud attenuation at Ku band. The cloud attenuation is relatively small. When rain rate is 20mm/hr, which is a very rare situation (much less than 0.2% of the time) [Chelton, 1989], the two-way attenuation is about 1.8dB/km [Monaldo, 1986]. Usually the thickness of a rain cell is less than 5km. So 9dB margin is usually enough for a Ku band radar altimeter. In addition, in heavy rain conditions, the rain will perturb the sea surface, which will induce \( \sigma^0 \) error.

Antenna gain is assumed to be 40.5dB for a 1m antenna [Townsend, 1980].

The state of the art of microwave receiver technology can achieve system loss plus noise figure about 4.7dB at Ku band [Rey, 1999][Zelli, 1998].

For a solid-state radar altimeter we can use a long pulse length with a low peak transmitted power. This will reduce electromagnetic interference. Also, a solid-state amplifier has a small volume and a light weight, allowing it to be integrated with the LNA front end. This is called the TR module, that can be placed very near to the antenna in order to reduce the overall system loss. A 7W peak power Ku band solid-state power amplifier has been developed through Jason radar altimeter mission. This is a good candidate for our altimeter power amplifier. The pulse length is assumed here to be 100\( \mu \)s, which is quite common.

The normalized radar cross section \( \sigma^0 \) varies from 7dB at 20m/s wind speed to about 20dB at 0.05m/s [Witter, 1991]. The worst case will be used to calculate the radar altimeter link budget.

The choice of microwave wavelength is determined by the link budget. Within a confined antenna aperture, a short wavelength will give a high antenna gain. But a short wave length will suffer a large rain and atmospheric attenuation. Also due to the calibration requirement it is better to use a frequency that has been used before. Thus Ku band is still the best choice for a sea state monitoring radar altimeter.
The orbit altitude is determined by link budget and satellite lifetime. High altitude will have a small atmospheric drag and a stable orbit, which is very critical for scientific altimetry mission. This is the reason why TOPEX use 1300km high altitude orbit. For our sea state monitoring radar altimeter mission the orbit stability is not important. Usually a low altitude will benefit the link budget. However, the atmospheric drag will increase significantly when altitude is below 600km. We choose 650km as the baseline of orbit altitude. Table 6-1 shows the link budget of radar altimeter based on the parameters chosen above:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_p$: peak transmitted power (W)</td>
<td>7</td>
</tr>
<tr>
<td>$G_0$: antenna gain (dB)</td>
<td>40.5</td>
</tr>
<tr>
<td>$\sigma^0$: normalized radar cross section (dB)</td>
<td>7</td>
</tr>
<tr>
<td>$\lambda$: radar wavelength (m)</td>
<td>0.022</td>
</tr>
<tr>
<td>$R_e$: earth radius (m)</td>
<td>6378000</td>
</tr>
<tr>
<td>$T_c$: pulse duration (s)</td>
<td>0.0001</td>
</tr>
<tr>
<td>$c$: light speed (m/s)</td>
<td>3.00E+08</td>
</tr>
<tr>
<td>$\tau_c$: compressed radar pulse length (s)</td>
<td>3.03E-09</td>
</tr>
<tr>
<td>$H$: satellite height above nadir (m)</td>
<td>650000</td>
</tr>
<tr>
<td>$L$: system losses + F (noise figure) (dB)</td>
<td>4.7</td>
</tr>
<tr>
<td>$k$: Boltzmann constant (J/K)</td>
<td>1.38E-23</td>
</tr>
<tr>
<td>$T_s$: Source noise temperature (K)</td>
<td>290</td>
</tr>
<tr>
<td>$R_e + H$: (m)</td>
<td>7028000</td>
</tr>
<tr>
<td>$L_r$: loss for rain attenuation (dB)</td>
<td>9</td>
</tr>
</tbody>
</table>

Usually when the signal to thermal noise ratio is larger than 10dB, the thermal noise effect can be ignored [Bucciarelli, 1989]. The signal to noise ratio derived from Table 6-1 is about 10.3dB, which is quite satisfactory.

### 6.2 Link Budget Estimation for Low Resolution Mode

In a low-resolution mode, the illuminated area is defined by the antenna pattern, which is a beam-limited mode. Because the transmitted and received power are weighted by different antenna gains at different off-nadir angles, the effective antenna footprint should be calculated carefully. The normalized radar cross section is expressed by the following radar equation [Jones, 1977]:

$$
\sigma^0 = \frac{P_r (4\pi)^3 H^4}{P_p G^2 \lambda^2 A_T}
$$

(6.3)
where,

- \( P_p \) is the peak transmitted power;
- \( P_r \) is the peak received power;
- \( G \) is antenna gain;
- \( H \) is the distance from the satellite to the sea surface;
- \( \sigma^0 \) is the normalized radar cross section;
- \( \lambda \) is the wavelength of carrier;
- \( A_r \) is the effective antenna footprint on the ocean surface. It can be expressed as:

\[
A_r = \frac{\pi}{4} (\beta_{eq} H)^2 \quad \tag{6.4}
\]

where \( \beta_{eq} \) is the effective antenna width (approximately equal to the half power antenna beam width). We derive (6.3) as follow:

\[
P_r = \int_0^{1/2} \frac{P_p G(\theta)}{(4\pi H^2)} \sigma^0 (Hd\theta 2\pi H) \frac{1}{4\pi H^2} \frac{G(\theta) \lambda^2}{4\pi}
\approx \frac{P_p \sigma^0 G_0^2}{(4\pi H^2)^2} \frac{H^2 \lambda^2}{2} \int_0^{\pi/2} \frac{\gamma}{\sin^2(\theta)} \theta \, d\theta
\approx \frac{P_p \sigma^0 G_0^2}{(4\pi H^2)^2} \frac{H^2 \lambda^2}{2} \int_0^{\pi/2} \exp\left(-\frac{4\theta^2}{\gamma}\right) \theta \, d\theta
\approx \frac{P_p \sigma^0 G_0^2}{(4\pi H^2)^2} \frac{H^2 \lambda^2}{2} \frac{\pi}{4 \gamma}
\approx \frac{P_p \sigma^0 G_0^2}{(4\pi H^2)^2} \frac{\lambda^2}{2} \frac{\pi}{4} H^2 2.98 \sin^2(\theta_{sd}/2)
\approx \frac{P_p \sigma^0 G_0^2}{(4\pi H^2)^2} \frac{\lambda^2}{2} \frac{\pi}{4 \gamma} (0.85 \theta_{sd} H)^2 \quad \tag{6.6}
\]

If we substitute the Gaussian antenna pattern into (6.5), which is:

\[
G(\theta) = G_0 e^{-2\theta^2} \quad \gamma = 2.98 \sin^2(\theta_{sd}/2)
\]

is the 3dB antenna beam width. We have

\[
P_r \approx \frac{P_p \sigma^0 G_0^2}{(4\pi H^2)^2} \frac{H^2 \lambda^2}{2} \frac{\pi}{4 \gamma} (0.85 \theta_{sd} H)^2
\]

\[
\approx \frac{P_p \sigma^0 G_0^2}{(4\pi H^2)^2} \frac{\lambda^2}{2} \frac{\pi}{4} H^2 2.98 \sin^2(\theta_{sd}/2)
\]

\[
\approx \frac{P_p \sigma^0 G_0^2}{(4\pi H^2)^2} \frac{\lambda^2}{2} \frac{\pi}{4 \gamma} (0.85 \theta_{sd} H)^2
\]
Chapter 6 System Study of Two Mode Radar Altimeter

This is a proof of (6.3) and (6.4). For Gaussian antenna pattern the effective antenna width is about 0.85 of the 3dB antenna width.

The signal to noise ratio of low-resolution mode can be expressed as:

\[
SNR = \frac{P G_0^2 \lambda^2 \sigma^0 A_r}{(4\pi)^3 H^4 kT_s B_f F L}
\] (6.7)

Now we transmit a short 2\(\mu\)s pulse and use the new effective antenna footprint as derived above, we can calculate the link budget for the low-resolution mode:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_p): the peak transmitted power (W)</td>
<td>7</td>
</tr>
<tr>
<td>(G_0): antenna gain (dB)</td>
<td>40.5</td>
</tr>
<tr>
<td>(\sigma^0): normalized radar cross section (dB)</td>
<td>7</td>
</tr>
<tr>
<td>(\lambda): radar wavelength (m)</td>
<td>0.022</td>
</tr>
<tr>
<td>(A_r): the effective footprint (m(^2))</td>
<td>82.7</td>
</tr>
<tr>
<td>(T_c): pulse duration (s)</td>
<td>0.000002</td>
</tr>
<tr>
<td>(H): satellite height above nadir (m)</td>
<td>650000</td>
</tr>
<tr>
<td>(L) (system losses)+F(noise figure) (dB)</td>
<td>4.7</td>
</tr>
<tr>
<td>(k): Boltzmann constant (J/K)</td>
<td>1.38E-23</td>
</tr>
<tr>
<td>(T_s): Source noise temperature (K)</td>
<td>290</td>
</tr>
<tr>
<td>(L_r): loss for rain attenuation (dB)</td>
<td>9</td>
</tr>
</tbody>
</table>

See Table 6-2, the result shows that even using a very short pulse length, the final signal to noise ratio is about 53dB, which is much higher than high-resolution mode. This is because in low resolution-mode the effective antenna footprint is much larger than that in the high-resolution mode.

6.3 Digital Chirp Generator

There are several ways to generate a Chirp signal. A voltage controlled oscillator (VCO) fed with a linear voltage ramp can generate a Chirp signal. But the linearity is greatly affected by the nonlinearity of the VCO. Although the control voltage can compensate the nonlinearity, the VCO is very sensitive to temperature variation and input noise. Thus the reproducibility is poor. The most commonly used technique is a dispersive delay line using a surface acoustic wave (SAW) device. However a SAW device is very sensitive to temperature variation. Another disadvantage of this method is that the SAW device reduces the power level of the signal, particularly if the time-
bandwidth product is large. And also it is not convenient to change the parameters of
the waveform, which is very critical for radar applications.

With the development of modern digital technology, digital Chirp signal generation
becomes feasible and practical. It is precisely repeatable, easy to change the
parameters of the waveform and robust to temperature variation. The disadvantages of
digital Chirp generation are the limited signal bandwidth and relative large power
consumption. But advances in high speed CMOS technology will solve these
problems.

6.3.1 Principle of Digital Chirp Generation

Digital Chirp signal generation is an inverse process of signal digitizing. The
sequence of waveform samples are first computed according to the required
waveform parameters, and stored in the RAM. During Chirp generation the samples
are clocked out of memory. Then the digital waveform is converted into an analogue
waveform by an A/D converter. The analogue waveform is filtered by a band pass
filter to remove the harmonic spectrum. The principle diagram is shown in Fig. 6-1

![Fig. 6-1 Principle diagram of digital Chirp generation](image)

There are two ways to address the RAM lookup table. The first method is quite
straightforward. We can calculate the waveform amplitude at each sampling point,
after assuming that the sampling rate is same as the clock rate used to generate the
waveform digitally. These amplitudes are stored in the fast RAM sequentially. For
waveform generation, these amplitudes are read out sequentially by using a digital
counter. This method obviously requires storing all waveform samples. The memory
requirement for a narrow bandwidth short pulse width system is modest. For example
a $100\,\mu s \times 2\,MHz$ pulse generator requires only 400 words RAM if we generate the
pulse in base band. And also because this method is relatively simple, it is widely
used in digital pulse generation for radar application.

But if we want to generate a wide bandwidth long pulse-length signal the situation
will be different. For example if we want to generate a $100\,\mu s \times 40\,MHz$ pulse in base
band, we need about 8K words. If we want to generate the pulse in IF, we will need even larger memory. So another waveform addressing method is proposed here, based on phase information, because there is a defined relationship between signal phase and amplitude of the sine function. If we can calculate the phase as a function of time in the form of a series of digital numbers, then these phases can be used as addresses to cyclically access the contents of the waveform memory. Because the sine function is a periodic function, the samples in one period are enough to be used by samples in other periods. This will greatly reduce the memory, especially when the signal has a long time duration. This is the principle of the Direct Digital Synthesis (DDS). DDS is a cheap and powerful method in modern frequency and signal synthesis. Following I will first introduce the principle of DDS and then discuss the digital Chirp generation based on DDS.

6.3.2 Principle of DDS [Qualcomm, 1996][Goldberg, 1996]

6.3.2.1 General DDS Operation

The block diagram of DDS is very similar to the Fig. 6-1, except the address generator is a phase accumulator. A typical functional block diagram of DDS (Q22401-1N) is shown in Fig. 6-2.

![Fig. 6-2 Q22401-1N functional block diagram](image)

The phase value stored in an input frequency register is added to the value in the phase accumulator once during each period of the system clock. The resulting phase
value (from 0 to 2π) is then applied to the sine lookup table once during each clock cycle. The lookup table converts the phase information to its corresponding sine amplitude, as illustrated in Fig. 6-3. The digital word is then output from the DDS.

![Diagram of phase output from phase accumulator and sine output from sine computation function]

**Fig. 6-3 Sample sine waveform**

To output a particular frequency, the associated phase increment value must be loaded into the input frequency register. The output frequency ($F_{OUT}$) and clock frequency ($F_{CLK}$) are related to the phase increment value ($\Delta \phi$) by the following equation:

$$F_{OUT} = \frac{F_{CLK} \cdot \Delta \phi}{2^N}$$  \hspace{1cm} (6.7)

Where $N$ equals the number of bits in the phase accumulator.

Any frequency can be generated by programming the phase change within the bit resolution of the phase accumulator. The frequency resolution can be determined by the following formula:

$$\Delta f = \frac{F_{CLK}}{2^N}$$  \hspace{1cm} (6.8)

Where $\Delta f$ is the frequency resolution.

For example, using Q2368I-1s1 (a model of Qualcomm DDS) where $F_{CLK}$ is 20MHz and $N$ is 32, the frequency resolution will be 0.00465Hz. Even a DDS operating at a clock rate of 500MHz, for $N=32$, the resolution is still 0.125Hz. Therefore the size of the accumulator controls the frequency resolution, and increasing the accumulator
size is quite simple and adds little to the cost and complexity of the design. Compared to the cost and complexity of resolution in PLL techniques, this is a remarkable advantage.

The maximum generated frequency is limited by the Nyquist sampling theorem, which is half of the clock rate. Practical limitation in the realization of the LPF sets the maximum output frequency at about 0.40\(F_{CLK}\), or 40% of the clock frequency.

In DDS all parameters are generated digitally and are therefore easy to manipulate. This means that very accurate and fast modulation is easy to implement.

A complete descriptive block diagram is shown in Fig. 6-4. The accumulator input is the frequency control port. Since the direct digital synthesizer can be switched very fast and is phase-continuous, it is easy to change the frequency and generate frequency modulation (FM). An interesting application is to put one accumulator in front of another accumulator, as shown in Fig. 6-4.

Since the accumulator performs an integration function, the phase output will result in a quadratic function and the result will be a linear FM or Chirp signal. This is the basic structure of the DDS Chirp generator.
A more general block diagram of DDS is shown in Fig. 6-5. As the output of the phase accumulator is the phase of the generated signal, it is rather simple to include phase control. By putting an adder between the accumulator and the ROM we can produce phase modulation. This function is very useful when we want to compensate the phase distortion of the generated Chirp signal, which is caused by the filters and power amplifier in the RF and microwave system. Since the output of the ROM represents the amplitude of the sine, amplitude modulation is possible if a multiplier is inserted between the ROM and the DAC. This function may be used to compensate the amplitude distortion in Chirp generation.

![DDS block diagram and functionality](image)

6.3.2.2 Spectral Purity of DDS

The spectral quality of a DDS system is obviously very important in frequency synthesizer application. It is also very important in pulse waveform generation. A spurious signal of a strong echo will be regarded as a small target in radar operation. The spurious signal of DDS is dependent upon a number of factors including the phase noise of the clock source, the number of phase bits applied to the sine lookup table (i.e. phase truncation, which is an internal operation of the DDS and cannot be externally influenced), and the number of bits output from the lookup table (i.e. amplitude truncation).

The specifications of the DAC, LPF design, and circuit card design also affect the quality of the converted sine wave. The linearity and glitch energy specifications of the DAC are especially important to the generation of pure sine wave signals. Careful attention to layout of the printed circuit design is important for limiting the noise of the synthesizer. Digital switching and power supply noise must be limited from coupling with clock and analogue signals.

- **Clock Source**
The clock source input to a DDS system is the major contributor to the phase noise of the system, even though its effect is reduced by the frequency division process of the DDS. The phase noise of the DDS output will show an improvement over phase noise of the clock source itself of \(20 \log (F_{CLK}/F_{OUT})\), where \(F_{OUT}\) is the generated frequency. By using a high stability crystal oscillator we can synthesize frequencies with very low phase noise by DDS.

- **Phase Truncation**

Given that a DDS accumulates \(N\) bits of phase information, only a portion of the Most Significant Bits (MSB) are input to the sine lookup algorithm, suppose \(K\) bits, \(K<N\). This reduced number of phase bits input to the sine lookup function is called phase truncation. Phase truncation is an internal operation of DDS and cannot be externally influenced. The truncation of the Least Significant Bits (LSB) is a loss of phase information and contributes errors. Only when the truncated bits in the control word are zero will the frequency generation not have phase quantization error. Thus the number that will not have phase quantization error is only \(2^{K-1}\). An example will give a demonstration of phase quantization error.

Let us suppose that \(N = 4\), \(W = 2\), which is the phase increment at the clock rate. In this case after 8 clocks the accumulator will reach the exact state \(2^{N} \cdot 2^K\). Thus the output frequency will be \(F_{CLK}/8\) and the accumulator states will be

\[0, 2, 4, 6, 8, 10, 12, 14, 0, \ldots\]

However, if \(W = 6\), then the cycle will be

\[0, 6, 12, 2, 8, 14, 4, 10, 0, \ldots\]

In this case, 3 cycles of the fundamental output frequency (of frequency \(6/16 F_{CLK}\)) were needed to come back to the original state. This will create a spurious signal at one-third of the output frequency given by \(6/16 F_{CLK}\) and its harmonics (this is due to the phase error is 3 cycles in period and this phase error modulation will result in harmonics).

A general simple approximation for phase error modulation spurious signal is given by [Goldberg, 1996]:

\[\text{PM spurious signals} \approx 10 \log \left( \frac{\pi \cdot 2^{-K}}{3} \right)^2 \text{ dBc}\]
Chapter 6 System Study of Two Mode Radar Altimeter

The spurious signal approximates to -6KdBc. The phase resolution is determined by the bits used to address the lookup table. The more bits used to address the lookup table the smaller is the phase quantization error. But the phase is finally represented by the bits of the output from the lookup table, which represents the amplitude of the sine waveform mapped from the input phase. By using a 12bit DAC we can achieve a spurious signal of phase quantization smaller than -72dB. Most DDS chips have a phase randomization function that can further reduce the spurious signal caused by phase quantization. Qualcomm DDS with 12bits DAC can achieve phase quantization spurious signal smaller than -76dBc.

- **Amplitude Quantization**

Amplitude quantization occurs in the sine lookup process. The lookup takes in a fixed number of bits of phase information and converts it to the equivalent sine amplitude. Since an ideal sine representation would require an infinite number of bits for most values, the value must be truncated. Here the amplitude quantization error starts by assuming that there are no errors introduced by any of the elements and that the only source of error in the quantization process is the DAC. In this case, if the DAC is represented by $D$ bits, the integrated signal to quantization noise is easily calculated as follows: the sine wave has a peak-to-peak amplitude of $2^D$, and therefore its power is given by

$$\left(\frac{2^D}{2\sqrt{2}}\right)^2 = 2^{2D-3} \quad (6.9)$$

The quantization error is assumed to be random and equally distributed from -0.5 to 0.5. The quantization noise power is thus given by

$$E_r = \int_{-0.5}^{0.5} x^2 p(x) \, dx \quad (6.10)$$

$p(x)$ is the probability density function of the error and is assumed to be uniformly distributed from -0.5 to 0.5. The error power is therefore $E_r = 1/12$. Thus the integrated signal to noise ratio (SNR) of an ideal direct digital synthesizer would be

$$\frac{2^{D-3}}{E_r} = 1.5 \cdot 2^{2D} = 6D + 1.78 \text{ dB} \quad (6.11)$$
which is the well known quantization noise in DSP theory. There is an assumption that the noise is random and evenly distributed. A further approximation is made that the quantization is white noise, equally distributed across the operational BW of the direct digital synthesizer. One should expect a $\frac{\sin x}{x}$ noise spectrum, but once again as an approximation we can assume that the noise is white and occupies a BW of $1/T$, where $T$ is the DDS clock period. Thus the noise power density $N_0$ can therefore be approximated as

$$\frac{C}{N_0} = 6D + 1.78 + 10\log \frac{1}{T}$$

(6.12)

The noise floor performance for a few DDS parameters is shown in Fig. 6-6.

![Fig. 6-6 Quantization noise floor: -(6D+1.78+10logFclk) dBC/Hz](image)

It is also of interest, for a practical application, $K$ (the number of bits at ROM input) should be bigger than or equal to $D+2$ ($D$ is the number of ROM output bits). As a rule of thumb, and in order to provide good performance and economics, generally designers use $W = D + 2$ or $W = D + 3$. We can gain an intuitive insight for this requirement by just looking at some numbers. Bear in mind that the sine is a nonlinear function, and as a consequence the dynamic range is lost in the mapping. The price paid is the increase in the number of input bits to the ROM. For example, for 8-bit by 8-bit ROM, the mapping will be given by

$$\sin(n) = \text{int}(127 \sin \frac{n \cdot 2\pi}{256} + 0.5) + 128$$

(6.13)
As shown in Table 6-3, the output of the first 64 values (the first quadrant) shows that many addresses of the DAC will never be accessed. This obviously increases the quantization error.

Table 6-3 Digital samples of sine wave for mapping 8 bits to 8 bits

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However if we change to 10 bits by 8 bits the ROM will give a mapping as

\[
\sin(n) = \text{int}\left(127\sin\left(\frac{n \cdot 2\pi}{1024}\right) + 0.5\right) + 128
\]  

(6.14)

The result is shown in Table 6-4.

Table 6-4 Digital samples of sine wave for mapping 10 bits to 8 bits

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Now every address of the DAC is accessed and every possible output is being generated without skipping values.

- **Spurious effects Due to Sampling and DAC Non-linearities**
The frequency and the amplitude of the discrete spurious signal are dependent on the ratio of the generated frequency \( F_{\text{OUT}} \) to the clock frequency \( F_{\text{CLK}} \). The 12-bit digitized sine output theoretically will provide a broadband signal-to-spurious ratio of 72dB, again depending on the quality of the DAC and LPF design as well as the printed circuit characteristics and the phase noise of the clock frequency.

The DDS is able to generate frequencies over a range from 0Hz to \( \frac{1}{2} \) the frequency of the clock. However, limitations on the slope of the roll off of the LPF determine the practical upper limit of the output frequency to about 40% of the clock frequency. Let \( F_{\text{CLK}} \) indicate the frequency of the system clock and \( F_{\text{OUT}} \) indicate the generated frequency. Discrete alias images due to sampling frequency and its multiples will be produced at frequencies of \( F_{\text{CLK}} - F_{\text{OUT}}, \ F_{\text{CLK}} + F_{\text{OUT}}, \ 2F_{\text{CLK}} - F_{\text{OUT}}, \ 2F_{\text{CLK}} + F_{\text{OUT}} \), and so forth, unless the LPF filters these images to acceptable levels. Also due to the non-linearities of the ADC, lots of harmonics will be generated. Fig. 6-7 shows the spectrum of a DDS when the clock is 100MHz and the output frequency is 24MHz. The harmonics are usually smaller than -40dBc. By choosing a proper clock rate and filtering most of the image and harmonics will be attenuated to a satisfactory level. General speaking the characteristic of the DAC is very critical in the DDS application. High resolution, fast speed, high linear DAC is very important for DDS design.

![Fig. 6-7 DDS output showing effect of DAC non-linearities](image-url)
6.3.3 Design of Digital Chirp Generator (DCG)

The digital Chirp generator presented here is based on DDS. The Chirp signal can be generated in either base band or IF frequency. Chirp generation at base band can give a larger bandwidth than generation at IF frequency. But base-band Chirp generation needs a signal sideband modulation. In order to cancel one sideband the two arms of the mixer should be in precise balance. The input and output should have a good isolation to prevent the break-through of the carrier. All these are main factors that prevent the building of high quality Chirp signals [Posteam, 1987]. Although generating the Chirp signal at IF will remove this problem it requires a much higher speed of digital circuit than generating at base band. Thus the bandwidth of Chirp will be limited. This is why until about ten years ago wide band Chirp signals were still generated in base band [Griffiths, 1992] [Eber, 1975]. Now DDS can readily handle clock rates up to 400MHz. Here a digital Chirp generation at IF is proposed based on the 400MHz DDS technology. The schematic diagram of Chirp generation is shown in Fig. 6-8.

Fig. 6-8 Schematic diagram of Chirp signal generation

The Chirp signal is first digitally generated at 140MHz with a bandwidth of 40MHz. Then it is DA converted and filtered to remove the image and harmonics frequencies. The output of the filter is mixed with a 680MHz carrier to up convert to 820MHz. Then this signal is 4 times multiplied to up convert to 3280MHz with a bandwidth of
160MHz. This signal is then mixed with a 3600MHz carrier to up convert to 6880MHz. Finally this signal is 2 times multiplied to 13.76GHz with a bandwidth of 320MHz, which is the transmitted Chirp signal.

Here I focus on the generation of first IF Chirp signal, which is the first two parts of DDS and band pass filter.

6.3.3.1 Spectrum of Digital Chirp Signal generated at IF frequency

For a Chirp signal with carrier frequency $f_c$, Chirp bandwidth $BW$, time duration $T$, the frequency expression is (refer to Fig. 6-4):

$$f(t) = (f_c - \frac{BW}{2}) + \frac{BW}{T} t = F_0 + \frac{BW}{T} t, \ t \in [0,T]$$  \hspace{1cm} (6.15)

Because $\varphi(t) = \int_0^t 2\pi f(t) dt = 2\pi [F_0 + \frac{BW}{2T} t], \ t \in [0,T]$, the time domain expression of the Chirp signal can be expressed as:

$$F(t) = A \sin(2\pi [F_0 + \frac{BW}{2T} t]), \ t \in [0,T]$$  \hspace{1cm} (6.16)

$A$ is the amplitude of the Chirp signal.

According to sampling theorem, we use following sampling frequency, so that the signal can be reconstituted by low pass filtering:

$$f_s \geq 2(F_0 + BW)$$  \hspace{1cm} (6.17)

The sampling period is $\Delta T = \frac{1}{f_s}$, the number of sampling pointing is: $N = \frac{T}{\Delta T} = T \cdot f_s$

Thus the discrete expression of linear FM signal is

$$F(n) = A \sin[2\pi (\frac{F_0}{f_s} + \frac{BW}{2Nf_s} n)n] = A \sin[2\pi \frac{F_0}{f_s} n + 2\pi \frac{BW}{2Nf_s} n^2], \ n \in [0,N].$$  \hspace{1cm} (6.18)

$F(n)$ is the $n$th sample, $N$ is the number of samples.

The corresponding phase of $F(n)$ is

$$\varphi(n) = 2\pi \frac{F_0}{f_s} n + 2\pi \frac{BW}{2Nf_s} n^2, \ n \in [0,N]$$  \hspace{1cm} (6.19)

Because the sample resolution is limited by the number of the bits of DAC, the samples stored in the lookup table are:

$$D(n) = (2^{D-1} - 1)\{1 + \text{int}[\sin(2\pi \frac{F_0}{f_s} + 2\pi \frac{BW}{2Nf_s} n^2)]\}, \ n \in [0,N]$$  \hspace{1cm} (6.20)
int() is the quantization function to get the closest integer and generate a quantization error of not more than ±1/2 LSB. Then the DDS-based Chirp generator can be built as shown in Fig. 6-9. The carrier frequency \( 2\pi \frac{F_0}{f_s} \) is input as the frequency modulation, the ramp rate of the Chirp signal \( 2\pi \frac{BW}{2Nf_s} \) is input from the first frequency accumulator. The output from the phase accumulator will be \( \phi(n) \), which will be used to address the ROM. The output from ROM will be \( D(n) \).

Fig. 6-9 Schematic diagram of digital Chirp generator

Suppose the clock rate of DCG is 400MHz, the generated Chirp is at IF with a 140MHz carrier and bandwidth is 40MHz. Because of sampling, the spectrum has been shifted repeated by the sampling frequency and its integer multiples. The spectrum of the generated digital Chirp signal is shown in Fig. 6-10. (Only the spectrum within the main lobe of the sample pulse is given.)

Fig. 6-10 Spectrum of digital Chirp signal
From Fig. 6-10 we find that in order to select the generated Chirp signal and reduce the phase distortion a linear-phase band-pass filter is needed. In order to reduce the amplitude distortion the 3dB cutoff frequency should be larger than the signal bandwidth. The attenuation of signal at the edge of the signal spectrum should be smaller than 1dB. The attenuation at 240MHz should be larger than 30dB, which is the spurious signal level requirement of the radar altimeter. A 6-pole linear-phase with equiripple error (phase error=$0.5^\circ$) band-pass filter can be used to achieve this [Williams, 1981]. The 3dB cutoff frequency is set to be $140\pm33$MHz, which should realize a signal attenuation of less than 1dB. The attenuation at 240MHz is now larger than 35dB.

The frequency response of the DAC depends on the sample-and-hold action of the DAC. It will also shape the signal spectrum of the generated Chirp as shown in Fig. 6-10. The power spectrum of the shaping function is:

\[
F_{DAC}(f) = \left[ \frac{\sin(\pi \frac{f}{f_s})}{\pi \frac{f}{f_s}} \right]^2
\]  

(6-20)

This causes a different attenuation at the edges of the signal spectrum, the difference being about 1dB. The shaping function also gives a larger attenuation of the first image spectrum than that of the wanted Chirp signal, by about 3.5dB. So the alias signal at 240MHz should be about 40dB lower than the generated Chirp signal. This distortion caused by filter and DAC can be corrected by a pre-distortion. This pre-distortion can be applied as an amplitude modulation at the output of the lookup table.

6.3.3.2 Phase and Amplitude Distortion

From the above discussion the phase and amplitude distortion of the Chirp signal can be classified as two categories: errors from the digital parts of the DCG and errors caused by the whole signal path (filters, transmitter, signal multiplication, etc.). As mentioned before, the DDS will generate phase quantization errors and amplitude quantization errors. When the number of input bits to the ROM is about 2bits more than the number of output bits from the ROM, the phase quantization error will be determined by the number of bits of the output of the ROM, which are the bits of the
DAC. Phase distortion generated in DDS usually is a short-wavelength phase distortion, while phase distortion generated from filter, transmitter and amplifier usually is long-wavelength phase distortion. A thorough discussion is given by Prof. Griffiths [Griffiths, 1992].

**Long-Wavelength Phase Distortion**

Now consider a Chirp signal with phase distortion present. Let the target echo be given by

\[ S_e(t) = \cos[\omega_0 + \frac{\alpha t^2}{2} + \Delta \phi(t)] \]  (6-21)

\( \Delta \phi(t) \) is the phase distortion, \( \alpha \) is the linear FM rate. As the transmitted signal and the local oscillator are generated from the same unit, the distortion will be similar (except for the phase distortion generated within the transmitter, which we temporally ignore). A typical long-wavelength phase distortion is shown in Fig. 6-11.

![Fig. 6-11 Long wavelength phase error of Chirp signal](image)

The local oscillator signal is then:

\[ S_{LO}(t) = \cos[\omega_0 (t + \tau) + \frac{\alpha (t + \tau)^2}{2} + \Delta \phi(t + \tau)] \]  (6-22)

The de-ramped signal is:

\[ S_d(t) = \cos[\alpha \tau + (\omega_0 \tau + \frac{\alpha \tau^2}{2}) + \Delta \phi(t + \tau) - \Delta \phi(t)] \]  (6.23)
The first term in (6.23) is a linear phase. The second term is a constant phase. The third is the difference of long wavelength phase distortion. Suppose the phase distortion $\Delta \phi(t)$ is expanded as a Fourier series, the dominant term (usually the first sinusoid term) can be expressed as:

$$\Delta \phi(t) = m\phi_p \cos \omega t$$  \hspace{1cm} (6.24)

where

$\phi_p$ is the peak phase error of this term.

$m$ is the subsequent frequency multiplication factor.

$\omega$ is the angular rate of the dominant phase error distortion, it can be expressed as:

$$\omega = \frac{2\pi}{\text{period}} = \frac{2\pi}{\Delta f \frac{df}{dt}}$$ \hspace{1cm} (6.24)

The value of $\tau$ is usually a few hundred nanoseconds for typical case, thus $\omega \tau \ll 2\pi$. The phase distortion may approximate as:

$$\Delta \phi(t + \tau) - \Delta \phi(t) \approx \tau \omega \phi_p \sin \omega t = \tau \frac{2\pi}{\Delta f \frac{df}{dt}} \phi_p \sin \omega t$$  \hspace{1cm} (6.25)

Let’s suppose $\tau = 300\text{ns}$, $\Delta f = 10\text{MHz}$, $\frac{df}{dt} = \frac{320\text{MHz}}{100\mu\text{s}}$, $\phi_p = 2^\circ$, the final residual phase distortion is about 0.6°. If the period of phase distortion is even larger, more phase error will be cancelled. As pointed out in the literature [Griffiths, 1992], if only 5 cycles of phase variation occur over the chirp bandwidth the residual phase error will be less than 4° even if the original phase error is as large as 6 rad. This is a distinct characteristic of de-ramp processing of linear FM radar.

**Short-Wavelength Phase Distortion**

The short-wavelength phase distortion usually comes from the digital quantization. The period is much less than $\tau$. The approximation of (6.25) is not valid. This phase distortion can be treated as a sinusoid phase modulation, at a frequency equal to the clock frequency $f_{clk}$. Thus

$$\Delta \phi(t) = \phi_p \sin 2\pi f_{clk} t = \phi_p \sin \omega_{clk} t$$  \hspace{1cm} (6-26)
\[ \Delta \phi(t + \tau) - \Delta \phi(t) = \phi_p [\sin \omega_{ck}(t + \tau) - \sin \omega_{ck} t] \]

The resulting phase error is still a sinusoid function, but the amplitude varies with \( \tau \). It has a maximum value:

\[ \Delta \phi(t + \tau) - \Delta \phi(t) = 2\phi_p \sin \omega_{ck} t, \quad \text{when} \quad \tau = (n + 0.5) \frac{2\pi}{\omega_{ck}} \]

Taking the worst case and ignoring the constant phase term, (6.23) can be express as

\[ S_{dx}(t) = \cos(\alpha t + 2\phi_p \sin \omega_{ck} t) \]

which is a typical phase modulation function. It can be expanded as

\[ S_{dx}(t) = J_0(2\phi_p) - J_1(2\phi_p)[\cos(\alpha t - \omega_{ck})t - \cos(\alpha t + \omega_{ck})t] \]

\[ + J_2(2\phi_p)[\cos(\alpha t - 2\omega_{ck})t - \cos(\alpha t + 2\omega_{ck})t] \]

\[ - J_3(2\phi_p)[\cos(\alpha t - 3\omega_{ck})t - \cos(\alpha t + 3\omega_{ck})t] \]

\[ + \ldots \quad (6.27) \]

where \( J_n(\cdot) \) are Bessel functions of the first kind of order \( n \). The sideband spectrum will be at \( \pm n\omega_{ck} \). For \( f_{ck} = 400 \text{MHz} \), this sideband will be outside the range window. The relative amplitude of the first pair of sidebands to the carrier is

\[ \frac{J_1(2\phi_p)}{J_0(2\phi_p)} \quad (6.28) \]

For an 8 bit DAC the quantization phase distortion is about 0.22°. Because 8 times frequency multiplication is applied to the digitally generated Chirp signal to get a final 320MHz bandwidth, the final phase distortion will be \( \phi_p = 0.22^\circ \times 8 = 1.76^\circ \).

Substituting \( \phi_p = 1.76^\circ \) into (6.28), the sideband will be 30dB lower than the carrier.

**Amplitude Distortion**

The amplitude distortion usually comes from the filter distortion and power amplifier distortion. It is usually slow variation distortion. The distortion present on the transmitted signal and local oscillator can also be expanded in a Fourier series representation. The important thing is that when the echo and locally generated Chirp multiply with each other the amplitude distortion will add. We let the amplitude
modulation of echo and local oscillator due to one term of the Fourier series be expressed in terms of a modulation index $s$. Then

$$S_r(t) = (1 + s \cos \omega_d t) \cos[\alpha + \frac{a t^2}{2}]$$  \hspace{1cm} (6.29)

The locally generated Chirp signal is:

$$S_{\text{LO}}(t) = [1 + s \cos \omega_d (t + \tau)] \cos[\omega_0 (t + \tau) + \frac{\alpha (t + \tau)^2}{2}]$$  \hspace{1cm} (6.30)

When these two signals are coincident, i.e. $\tau = 0$, it is easy to show that their product is amplitude modulated by $(1 + 2s \cos \omega_d t)$. The term $s^2 + \cos^2 \omega_d t$ is ignored because $s$ is very small (for 0.5dB amplitude ripple, $s = 0.12$). Thus a given value of amplitude ripple on the two signals will result in twice the amplitude modulation of the de-ramped echo. According to amplitude modulation theory this will result in a pair of AM spectrum, which will be a spurious signal in the final demodulated waveform. For $s = 0.024$, which corresponds to a 0.2dB amplitude ripple, the system will generate a $-32$dB spurious signal.

6.4 Conclusion

The link budget of the two-mode radar altimeter has been evaluated. It shows that in order to operate a radar altimeter in a power limited condition, a 1m antenna is needed. At high-resolution-mode, it leaves about 9dB margin for rain attenuation, which is a common figure used in radar altimeter design. Actually, by increasing the pulse length to 300µs we can get another 4.7dB link budget margin. For low resolution mode, there is a big margin in link budget because it has a large beam-limited footprint, which means it has a much larger radar cross-section than the pulse-limited case.

A detailed discussion is given about digital Chirp signal generation based on DDS. A digital Chirp signal generated at IF frequency is proposed so that can remove the single sideband modulation. This can avoid carrier leakage and unwanted spurious signals, which are the main limitations of high quality Chirp signal generation. Usually frequency multiplication will produce a signal with much greater spurious content. The greater the multiplication the higher the spurious signal. By using high
speed DDS a wide bandwidth digital Chirp signal can be generated. Thus only 8 times multiplication is needed to generate a 320MHz Chirp signal.

The analysis of phase and amplitude distortion shows that most of the long wavelength phase distortion can be cancelled by the de-ramping processing. Short wavelength phase distortions generate a spurious response that is far away from the echo window. They have little effect on the radar altimeter. But the amplitude distortion tends to add when the signals are aligned. Thus the amplitude distortion will be increased. Some methods such as amplitude limiters may be used to cut the amplitude ripple and reduce the amplitude distortion.

In above discussion, I assume the echo and locally generated Chirp signal experience the same electronic path, so that they have the same distortions. But actually the echo will go through the transmitter and front end, while locally generated Chirp does not. This will result in different phase distortion that cannot be cancelled. One way to overcome this problem is to reduce the distortion generated in these parts. Another way is to make a pre-distortion at the digital chirp generator as long as the distortion is known and stable. By using DDS, this compensation is not very difficult.
Chapter 7 Summary And Future Work

7.1 Summary

The aim of this research is to investigate a possible way to build a radar altimeter for sea state monitoring on board a microsatellite. Several methods that can save power and reduce the requirement of accurate ADCS have been studied. Both the conventional pulse-limited radar altimeter and the synthetic aperture radar altimeter have been evaluated to try to find their advantages and disadvantages. Finally a combined two-mode radar altimeter is proposed for sea state monitoring. The principle ideas generated in the research are summarized as follows:

- Conventional radar altimetry is a well-established technology. Its main purpose is to determine the mean sea surface height with an accuracy of a few centimeters. This application requires accurate orbit determination and accurate estimation of propagation delay caused by ionosphere and troposphere. Even more accurate measurement is required to correct the error caused by EM bias and skewness bias. But for sea state (SWH and wind speed) monitoring, the operation may become relatively simple. For SWH measurement, only the shape of the echo is important, which means the above delay correction is not necessary. But a high range resolution radar is demanded. For wind speed estimation, an accurate calibration of power link is required. The attenuation due to rain and cloud must be accounted for before the wind speed can be derived, which means an on board radiometer may be wanted. Also due to the system ageing and instability, the transmitted power and receiver gain should be monitored routinely. Comparing to usual scientific radar altimeter mission, most of the work of sea state monitoring relies on the radar altimeter itself.

- Because the radar altimeter will be on board a microsatellite, any method that can improve the power efficiency is valuable. It was stated that synthetic aperture processing could improve the signal-to-noise ratio by making use of more power reflected from the footprint. The signal processing method of the
synthetic aperture radar altimeter and its effect on the final signal-to-noise ratio improvement has been evaluated. We showed that the signal to noise ratio improvement reduces as the SWH increases. Compared to the conventional radar altimeter, the signal processing of synthetic aperture radar altimeter is more complex. So for sea state monitoring, synthetic aperture processing will not provide much benefit.

- While we conclude that the conventional radar altimeter will remain the best choice for sea state monitoring, we need to know the factors that will affect the performance of SWH and wind speed measurement. In chapter 4, the performance evaluation is done based on computer simulation. The Brown model is used for sea echoes simulation. SMLE is used for on board signal processing simulation. The system distortions, including antenna off-nadir pointing, signal amplitude and phase distortions were studied in detail. The conclusion is that antenna off-nadir pointing is the main contributor to the SWH error. But if antenna off-nadir pointing angle is smaller than 0.5°, which is a normal performance of a microsatellite, the SWH estimation error will be within 0.5m or 10%, whichever is larger. SWH error caused by reasonable amplitude and phase distortions are very small and can be ignored.

- Normalized radar cross-section measurement is more sensitive to pointing error. Even 0.5° off-nadir pointing induces a big error in normalized radar cross-section measurement. Normally 0.2° control accuracy is required in order to have a satisfactory wind speed measurement. Although the error can be removed by having knowledge of pointing angle, at 0.5° off-nadir pointing condition, the pointing knowledge should be better than 0.1° in order to make the normalized radar cross-section error smaller than 0.4dB. A novel low-resolution mode radar altimeter is proposed in order to give a satisfactory measurement of normalized radar cross-section at 0.5° pointing error. This has been discussed and simulated in detail in chapter 5.

- In chapter 6 the link budgets of the two-mode radar altimeter have been studied. They show that the low-resolution mode has a big link budget margin. But for conventional high-resolution mode, it is necessary to have a 1m
antenna and satellite altitude should be around 650km in order to give a satisfactory link budget.

In this chapter, the digital Chirp generator is also studied, based on high speed DDS. The Chirp signal will be generated at IF instead of generated at base band. This will avoid carrier leakage and unconcealed sideband, which dominate the system performance in terms of spurious signals. By using de-ramp pulse compression, the long wavelength phase error can be cancelled. Short wavelength phase distortions generate a spurious response that is far away from the echo window. They have little effect on the radar altimeter performance. But amplitude distortion tends to add when the signals are aligned. Thus the amplitude distortion will be increased. Some methods such as amplitude limiters may be used to cut the amplitude ripple so as to reduce the amplitude distortion.

7.2 Conclusion

The proposed two-mode radar will operate at conventional high-resolution mode and low-resolution mode alternately. The low-resolution mode is actually a nadir-looking wind scatterometer, which is specialized for radar cross-section measurement. This two mode operation will share the same front-end electronic part, but has separate IF parts. As we mentioned before, the low-resolution mode requires a reference sampling time that will be derived from the high-resolution mode. The system delay of the two modes must be calibrated well before launch. Because the delay time calibration requirement is relatively coarse (about 50ns), it may be maintained after launch. Thus on board delay time calibration may not be necessary or may be required very loosely. But this relies on detailed testing after building the radar altimeter.

By operating the radar altimeter in two modes alternately, we can cope with a pointing error as large as 0.5°. By using a long Chirp pulse we can transmit the pulse in a low power, about 7W. Thus a solid-state, lightweight radar altimeter may be built on board a microsatellite. The requirement of system distortion is not very high, in terms of amplitude and phase distortion. It is therefore possible to build a cost effective radar altimeter in the future for the sea state monitoring.
7.3 Future Work

The two-mode radar altimeter will operate in high-resolution mode and low-resolution mode alternately. They will share the same front end but have different IF sections. The low-resolution mode operation needs a time reference to sample the peak point of the echo. The requirement of time reference accuracy is about 50ns if we use a 2µs pulse. The accuracy of the time reference is affected by the IF receiver delay. This different delay can be calibrated before launching the altimeter. But the delay time will change due to the temperature and system ageing. Therefore the delay time variation must be monitored routinely. Thus an internal calibration system is required.

The next step of the work is to build two IF receivers for each resolution mode respectively. For the high-resolution mode, the processing starts from the first de-ramping. For the low-resolution mode, it starts from the first down converter. Both are ended before the AD converter.

It will be necessary to build an internal delay time calibration system to measure the delay time difference. We then change the delay time between the input signals and test the accuracy of the calibration system. The delay time accuracy after calibration should be about 50ns.

Finally the low-resolution mode IF system can be integrated into the conventional high-resolution radar altimeter ready for further flying tests.
Appendix I Delay and Doppler effect on returned signal

![Diagram showing the relation between time and distance from target to radar](image)

Fig. AI-1 Relation between time and distance from target to radar

Suppose a target is moving in a straight line with a constant speed \( v \). The distance from radar to target is \( z_0 \) when a pulse is transmitted towards the target. The pulse touch the target at time \( t_1 \) and reflected back at that time when the distance from target to radar is \( z_1 \). The radar received the echo at time \( t_2 \) when the distance is \( z_2 \).

We can get following relations:

\[
\begin{align*}
  t_1 &= t_0 + \frac{z_1}{c} \quad \text{(AI.1)} \\
  z_1 &= z_0 + vt \quad \text{(AI.2)} \\
  z_1 &= z_0 + vt_0 + \frac{vz_0}{c} \quad \text{(AI.3)} \\
  z_1(1 - \frac{v}{c}) &= z_0 + vt_0 \quad \text{(AI.4)} \\

t_2 - t_1 &= \frac{2z_1}{c} = \frac{2z_0}{c-v} + \frac{2v}{c-v}t_0 \quad \text{(AI.5)}
\end{align*}
\]
Appendix I

\[ t_2 = \frac{2z_0}{c-v} + \left(1 + \frac{2v}{c-v}\right)t_0 = \frac{c+v}{c-v}t_0 + \frac{2z_0}{c-v} \quad (AI.6) \]

We see therefore a time dilation factor - responsible for the Doppler shift and delay term which is slightly larger than time it would take to get to the distance \( z_0 \).

Now a pulse received as \( q(t) \) resembles what was transmitted earlier as \( p(t) \). If we are at time \( t = t_2 \) and looking at \( q(t) \) that would be the same as looking at \( p(t) \) at an earlier time \( t = t_0 \).

\[ p(t_0) = q(t_2) \quad (AI.7) \]

and dropping subscript

\[ q(t) = p((\frac{c+v}{c-v})t - \frac{2z_0}{c+v}) \quad (AI.8) \]

To an adequate approximation we have

\[ q(t) = p((1 - \frac{2v}{c})t - \frac{2z_0}{c(1 - \frac{v}{c})) \quad (AI.9) \]

and if we substitute for a carrier frequency \( \omega_c \)

\[ p(t) = \exp(j \omega_c t) \quad (AI.10) \]

\[ q(t) = \exp(j \omega_c t - j \frac{2\omega_c v}{c} t - j \frac{2\omega_c z_0}{c} + \frac{2\omega_c vz_0}{c^2}) \quad (AI.11) \]

Identifying terms then

\[ q(t) = \exp(j \omega_c t - j \omega_p t - j \omega_c \tau_0 + j \frac{\omega_p \tau_0}{2}) = \exp j \omega_c (t - \tau_0) \cdot \exp j \omega_p t \cdot \exp j \frac{\omega_p \tau_0}{2} \quad (AI.12) \]

The last term in (AI.12) can be ignored because it is almost a constant.
Dr. Raney proposed a delay compensation method for synthetic aperture radar altimeter [Raney, 1998], but only a simple result was given. In fact it can be derived from wavenumber domain algorithm.

The mapping relation between conventional radar altimeter and synthetic aperture radar altimeter is shown in Fig. AII-1, which is from left side to right side. From this mapping, we can easily derive the flat surface response function. The mapping relation can be proved as follow according to \( \omega-k \) algorithm. Because radar altimeter is de-ramp on receiving, I will derive it from the beginning.

- **Signal receiving**
  As shown in Fig. AII-2, satellite flies in space in a straight line with a speed of \( V \). The distance from satellite to ground is \( R_s \). A co-ordinate system is built as follow: the \( X \) co-ordinate is on the ground and parallel to the satellite’s path, \( Z \) co-ordinate is perpendicular to the ground and cross the satellite’s path through the center of a certain aperture. A point target on the ground \((x_t, y_t, z_t)\), at certain time the satellite is at \((x_a, y_a, z_a)\).
Appendix II

Suppose we transmit a linear FM signal:

$$S_r(n,t) = a_r \text{rect} \left( \frac{\hat{t}}{T_p} \right) e^{j2\pi f_d \hat{t}} e^{j\pi f t^2} \quad (AII.1)$$

Here $\hat{t} = t - nT$ represents fast time, that is time within a signal received from a specific transmitted pulse and reference to the time of individual pulse generation.

A backscattered signal from a point $(X, Y, Z)$ is:

$$S_r(n,t) = a_r \text{rect} \left( \frac{\hat{t} - 2R/t}{T_p} \right) e^{j2\pi f_d \left( \frac{t - 2R/t}{c} \right)} e^{j\pi f t^2} e^{j\pi f_d \hat{t}} \quad (AII.2)$$

$R$ is the distance from satellite to the target at a certain time. Frequency $f_d$ is the Doppler shift of the echo returned from that point target, which is always been ignored by SAR designers. But this term is widely used in tracking radar for ranging accuracy evaluation. The origin of equation (5-2) is given in section 2.1. Here,

$$R = \sqrt{(X_a - X_r)^2 + (Y_a - Y_r)^2 + (Z_a - Z_r)^2} = \sqrt{(X_a - X_r)^2 + R_s^2} \quad (AII.3)$$
where, \( R_B = \sqrt{(Y_a - Y_t)^2 + (Z_a - Z_t)^2} \), is the minimum range from radar altimeter to the scatterer occurring when \( x_a = x_t \).

The reference signal used to de-ramp the echo from \((x_n, y_n, z_n)\) is

\[
S_{ref}(n,t) = a_r \text{rect} \left( \frac{\hat{t} - 2R_z/c}{T_p} \right) e^{j2\pi f_\gamma \left( \frac{\hat{t} - 2R_z/c}{c} \right)} e^{j\phi_{out}(x_a, \hat{t})} \quad \text{(AII.4)}
\]

Here \( R_z = z_a \) is distance from satellite to ground. And this distance is tracked by on-board \( \alpha-\beta \) tracker. We set the delay time corresponding to \( \frac{2R_z}{c} \), and \( S_{ref} \) mixed with \( S_r \) and low-pass filtered to get the output of

\[
S_{out}(x_a, \hat{t}) = a_r \text{rect}(\frac{x_a - X_{ac}}{L}) \text{rect}(\frac{\hat{t} - 2R_z/c}{T_p}) e^{j\phi_{out}(x_a, \hat{t})} \quad \text{(AII.5)}
\]

Here \( \text{rect}(\frac{x_a - X_{ac}}{L}) \) is a function for the signal collection range in along track direction at a certain squint angle.

\( x_{ac} \) is the central position of a certain aperture;

\( L \) is the aperture length;

\[
\frac{1}{2} \leq \frac{x_a - X_{ac}}{L} \leq \frac{1}{2}.
\]

\[
\phi_{out}(x_a, \hat{t}) = -\frac{4\pi f_\gamma}{c} (\frac{\hat{t}}{\gamma} + \hat{t} - \frac{2R_z}{c}) (R_t - R_z) + \frac{4\pi f_\gamma}{c^2} (R_t - R_z)^2 + 2\pi f_d \hat{t} \quad \text{(AII.6)}
\]

From (AII.5) we use \( x_a \) instead of \( n \) to stand for the along track position, because \( n = \frac{x_a}{\gamma} \). The second term in (AII.6) is an unwanted phase effect known as residual video phase (RVP). It is specific to chirp systems that de-ramp in the time domain as the above model does. If the variation of this term is small over a synthetic aperture, we can safely ignore it. In SAR altimeter application we can safely ignore it. So (AII.6) can be written as

\[
\phi_{out}(x_a, \hat{t}) = -\frac{4\pi f_\gamma}{c} \left( \frac{\hat{t}}{\gamma} + \hat{t} - \frac{2R_z}{c} \right) (R_t - R_z) + 2\pi f_d \hat{t} \quad \text{(AII.7)}
\]
We define

\[ K_R = \frac{4\pi}{c} \left( f + \frac{2R_s}{c} \right) = \frac{4\pi}{c} \left( t - \frac{2R_s}{c} \right) = \frac{4\pi}{\lambda} \] (AII.8)

\[ t' = t - nT - \frac{2R_s}{c} = t - \frac{2R_s}{c} \] (AII.9)

\[ K_x = \frac{2\pi f_d}{V} = \frac{2\pi f d V \cos(\alpha_d)}{c} = -\frac{4\pi \cos(\alpha_d)}{\lambda} = K_R \cos(\alpha_d) \] (AII.10)

From (5-8) we find \( K_R \) is the wavenumber (something like Doppler but divided by speed, which is the phase change per unit length) changed with time.

So from (5-5), (5-7), (5-8), (5-9) we can get:

\[ S_{ou}(X_a, K_R) = a_r \text{Ant} \left( \frac{X_a - X_{ac}}{L} \right) \text{rect} \left( \frac{t'}{T_p} \right) e^{-jK_R (R_1 - R_s) + 2\pi f_d t' + 2\pi f_d \frac{2R_s}{c}} \] (AII.11)

Now signal is in azimuth-range wavenumber domain, or you can say in the azimuth-range frequency domain after range de-ramp. A more obvious explanation is given to a two dimensional geometry in [10] where you can find de-ramp is a range Fourier transform to a two dimensional scattering function.

Following I will deal DD altimeter signal as given by Dr. Raney but with a modification on range-skew compensation and range Doppler ambiguity compensation.

- On board signal processing for synthetic aperture radar altimeter

As already given in my report, derivation given here will be as simple as possible.

Along track FFT

From (5-11), using Principle of Stationary Phase, we get:

\[ S(K_x, K_R) = \int_{X_{ac}}^{X_{ac+L}} S_{ou}(X_a, K_R) e^{-jK_x X_a} dX_a \]

\[ = \int_{X_{ac}}^{X_{ac+L}} a_r \text{Ant} \left( \frac{X_a - X_{ac}}{L} \right) \text{rect} \left( \frac{t'}{T_p} \right) e^{-jK_R (R_1 - R_s) + 2\pi f_d t' + 2\pi f_d \frac{2R_s}{c}} e^{-jK_x X_a} dX_a \]

\[ = a_r \text{rect} \left( \frac{t'}{T_p} \right) e^{-j2\pi f_d t' \frac{2R_s}{c}} \int_{X_{ac}}^{X_{ac+L}} \left[ \text{Ant} \left( \frac{X_a - X_{ac}}{L} \right) e^{-jK_R (R_1 - R_s)} \right] e^{-jK_x X_a} dX_a \]
\[ \Phi(K_x, K_R) = -K_x X_t + K_R(R_s - R_B) - R_B K_R \frac{K_x^2}{2K_R^2} \]

with

\[ \Phi(K_x, K_R) = -K_x X_t + K_R(R_s - R_B) - R_B K_R \frac{K_x^2}{2K_R^2} \]  \hspace{1cm} (AII-13)

\[ |S(K_x, K_R)| = \text{rect}(\frac{K_R - 4\pi f_c}{c}) \text{rect}(\frac{K_x - 2}{L(K_R - K_x^2)}) \]  \hspace{1cm} (AII-14)

using the substitution \( t' = \frac{cK_R - f_c}{4\pi \gamma} \) in the \text{rect} function.

(AII.14) is the amplitude of the spectrum of the de-ramped data in two dimensional wavenumber domain. Its maximum occurs when

\[ \frac{K_R R_B}{L(K_R - K_x^2)} + \frac{X_{ac} - X_t}{L} = 0 \]  \hspace{1cm} (AII.15)

Using geometric relationship (AII.15) condition can be expressed as:

\[ K_x = K_R \cos(\alpha_d) \]  \hspace{1cm} (AII.-16)

For narrow signal bandwidth, (AII.16) can be simplified to

\[ K_x \approx K_{R_0} \cos(\alpha_d) \] \( K_{R_0} \) corresponds to carrier frequency.

The second \text{rect}(\cdot) function in (AII-14) then can be simplified to:

\[ \text{Sinc}(K_x - K_{R_0} \cos(\alpha_d)) = \text{Sinc}(K_x - K_{x_0}) \]  \hspace{1cm} (AII.17)

Equation (AII.13) becomes:

\[ \Phi(K_x, K_R) \approx -K_x X_t + K_R(R_s - R_B) - R_B K_R \frac{K_x^2}{2K_{R_0}^2} \]

\[ = -K_x X_t + K_R(R_s - R_B) - R_B \Delta K_R \frac{K_x^2}{2K_{R_0}^2} - R_B K_{R_0} \frac{K_x^2}{2K_{R_0}^2} \]  \hspace{1cm} (AII.18)
The last term in (All-18) is a range-dependent phase. If $R_b$ changes very little within the imaging area (which is the case of synthetic aperture radar altimeter), this term is effectively a constant. Otherwise, range dependent phase compensation should be applied to focusing, which is the case of imaging SAR. For synthetic aperture radar altimeter, (All-18) can simplified to be:

$$
\Phi(K_x, K_R) = -K_x X_t + K_R (R_s - R_b) - R_b \Delta K_R \frac{K_x^2}{2K_{R0}^2}
$$  \hspace{1cm} \text{(AII.19)}

The last term is the delay compensation proposed by Dr. Raney.

**Doppler induced range error compensation**

From (AII-11) we find there is a phase term $e^{j2\pi\alpha (t' + \frac{2R_b}{c})}$. The constant phase $e^{j2\pi\alpha (\frac{2R_b}{c})}$ can be put aside. But the linear Doppler phase $e^{j2\pi\alpha t'}$ should be compensated before range FFT, otherwise it will induce extra delay.

After along-track Fourier transform, the Doppler frequency of the data lying within each azimuth cell is known. So just by multiplying the phase term $e^{j2\pi\alpha t'}$ in each azimuth cell in the two dimensional wavenumber domain, we can compensate the Doppler induced range error easily. So the signal becomes,

$$
S_2(K_x, K_R) = \text{rect}(\frac{K_x - 4\pi\alpha T_p / c}{4\pi\alpha T_p / c}) \text{Sinc}(K_x - K_{x0}) e^{-jK_R (R_s - R_b)} e^{j2\pi\alpha t'} (AII.20)
$$

**Delay Doppler compensation**

The last term in (AII.20) can also be expressed as

$$
e^{\frac{-jR_b}{2}(K_{R0} + \Delta K_R) - \frac{K_x^2}{2K_{R0}^2}} = e^{\frac{-jR_b}{2} K_{R0} - \frac{K_x^2}{2K_{R0}^2}} e^{\frac{-jR_b}{2} \Delta K_R} (K_{R0})^2 (AII.21)
$$

The first term in (AII.21) is a constant phase term that can be put aside. The second term is related to what Dr. Raney has suggested for delay compensation.

Because the delay is sorted by Doppler frequency, we can multiply the phase term

$$
e^{\frac{-jR_b}{2} \Delta K_R} (K_{R0})^2
$$

to each Doppler cell of (AII.21).
So it is easy to do delay compensation according to Doppler frequency. After delay compensation the signal in two dimensional frequency domain can be expressed as

\[
S_3(K_X, K_R) = \text{rect} \left( \frac{K_R - 4\pi f_c / c}{4\pi T_p / c} \right) \text{Sinc} \left( K_X - K_{X0} \right) e^{-jK_R(R_B - R_S)} e^{jK_{X0} / 2K_{R0}} \quad (AII.22)
\]

The only residual phase error in (AII.22) is

\[
\Delta \varphi(K_X, K_R) = \frac{1}{2} \Delta K_B \frac{X_0}{K_{R0}^2} (R_B - R_S),
\]

which will induce an extra few millimetres range migration at the edge of the antenna beam. So it can be completely ignored.

Because

\[
K_X = -\frac{4\pi \cos(\alpha_d)}{\lambda} = -\frac{2V}{\lambda} \frac{X_0}{R_B} \quad (AII.23)
\]

(5-27) can also expressed as

\[
S_3(X_r, K_R) = \text{rect} \left( \frac{K_R - 4\pi f_c / c}{4\pi T_p / c} \right) \text{Sinc} \left[ \frac{2V}{\lambda R_B} (X_{r0} - X_r) \right] e^{-jK_R(R_B - R_S)} \quad (AII.24)
\]

After range FFT, the energy distributed in the two dimensional frequency domain from a point target located in \((x_{r0}, (R_B - R_S))\) will be mapped back to \((x_{r0}, (R_B - R_S))\).

The range Doppler ambiguity compensation and delay compensation can combined together in practical compensation.
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