THE EFFECTS OF RADOMES ON THE POINTING ACCURACY OF ANTENNAS

A thesis submitted to the University of Surrey in partial fulfillment of the requirements for the degree of Doctor of Philosophy on a collaborative basis.

by

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The pointing accuracy of the tracking antenna used in a radar guided homing missile is reduced by a streamlined radome. This can have serious effects on the missile's performance. The pointing error of a microwave antenna inside a radome can be predicted with fair accuracy provided that the radome and antenna diameters are ten or more wavelengths. This is done, basically, by determining the modified aperture distribution, taking account of the varying phase shift through different parts of the radome. The polar diagram of the modified aperture distribution is then calculated.

If the radome diameter is only about five wavelengths this procedure (the 'insertion diffraction theory') is not usually adequate. The pointing error can be calculated from known aperture fields, under reasonable conditions, for small antennas. The inadequacy of the theory is therefore mainly due to the inability to calculate the aperture fields with sufficient accuracy. The reasons for the failure of the ray tracing procedure are discussed and it is shown that scattering by the tip of the radome, surface waves guided by the radome and multiple scattering (interaction) between the antenna and the radome would be expected to modify the aperture fields and introduce pointing errors.

The propagation of surface waves on radomes is investigated and a calculation of the pointing error due to the $HE_{12}$ surface wave is carried out. Experiments showed that surface waves can be troublesome on a very lossy radome but for the small ceramic radomes used in this research they are not significant.

A method is described for calculating the pointing error caused by interaction between an antenna and an infinite plane dielectric sheet of uniform thickness. This method is not generally applicable
to radomes and a method for radomes is developed. It is found that interaction is a serious cause of error, especially if the radome has a low dielectric constant. The tip scattering effect is also found to give large pointing errors in small radomes.

The effect of radome diameter (measured in wavelengths) on each of the sources of error is examined and it is shown that, whereas the pointing error due to phase variation effects is inversely proportional to size, the errors due to the other effects vary much more rapidly. This explains why the latter are only of second order importance in large radomes but become predominant in small ones. It is shown that a high dielectric constant is essential for small radomes which are to be used over a narrow frequency band.
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CHAPTER 1 - INTRODUCTION

1.1 The background to the problem

A radome is a protective cover for a microwave aerial, and it is designed to be transparent to electromagnetic radiation in the frequency range of the aerial. This thesis is concerned with radomes for high speed guided missiles of the homing type, for which the radome is typically a conical or ogival shell of dielectric material. Fig.1.1 shows the three shapes of radome which have been used in the present programme. All the radomes are axially symmetric.

A homing missile detects radiation from the target and steers itself so as to collide with the target. The missile may be passive, in which case it homes on radiation originating from the target, for example infra-red radiation from the exhaust of the engines. Alternatively the missile may be of the active type, in which case it carries a radar transmitter to illuminate the target as well as a receiver to detect the radiation scattered from the target. The semi-active missile is a compromise between the active and passive systems, and it carries a radar receiver to detect reflections from the target which is illuminated by an auxiliary transmitter. This transmitter is located on the ground (or on a ship) for surface to air missiles or in the missile's parent aircraft for air to air missiles.

The semi-active principle is the most widely used for long range radar guided missiles. This is because the limited transmitter power and aerial gain available restrict the range of active systems and also because of their increased cost and complexity.

1.2 Guidance methods

There are several ways in which the missile, having computed the target bearing, can steer itself to intercept the target. The most
FIG. 1. RADOME SHAPES

CONE

Generating Rad. = L/4 + D/4

TANGENT OSIVIE

Generating Rad. = 2L/3D + D/2

Optimum Secant Osive

D = 7"

L = 12"
obvious course is to fly straight towards the target, always along the instantaneous line of sight. This is known as a pure pursuit course. It is shown by Clemow\(^1\) that such a course always ends as a tail chase and it has the disadvantage that a high rate of turn is required as the target is approached. A high rate of turn is usually associated with large miss distances. This is undesirable because the probability of destroying the target decreases rapidly as miss distance increases. Another disadvantage of the pure pursuit course is that a target of higher speed than the missile cannot be caught. The pure pursuit course is only useful if a slower, non-maneuvering target is being attacked from near dead astern.

A second type of pursuit course is the deviated pursuit course. In this course the angle between the missile's velocity vector and the line of sight from missile to target (the lead angle) is fixed. If the lead angle is zero then a pure pursuit course results. An analysis\(^2\) of the deviated pursuit course for a target which flies in a straight line shows that, under some circumstances, infinite lateral acceleration is required.

A constant bearing collision course (Fig.1, 2) is one in which the line of sight maintains a constant bearing in space. For a target of constant velocity the missile flies in a straight line to the point of collision. This is a particular case of the deviated pursuit course, and as the missile is not required to turn it can fly at maximum speed. Since no terminal acceleration is required of the missile its accuracy should be high.

In practice the line of sight tends to rotate, and to counter this tendency a method known as proportional navigation is employed. Under ideal circumstances this gives a constant bearing collision course. In this method the rate of change of missile heading angle
\[ V_{M_1} \sin \theta_1 = V_T \sin \theta_T \]

**FIG. 1.2 Constant Bearing Collision Course**
is made proportional to the rate of rotation of the sight line. This
cannot be done exactly, of course, because it would require instan-
taneous measurement of sight line rate and instantaneous response by
the missile. The constant of proportionality (the navigation con-
stant) lies typically in the range 3 to 7.

A radar guided homing missile measures the rate of rotation of
the sight line by means of a homing head, which contains the following
components:

(i) The aerial, its suspension and servo motors.
(ii) The associated microwave circuits and the radar receiver.
(iii) A gyroscope with its torque motors and position pick-offs.
(iv) Computing circuits.

The operation of a typical homing head is described by Clemow\(^1\).

1.3 The effect of the radome on homing missile performance

The aerial is usually placed at the front of the missile so it
must be protected by a radome. The radome has to meet several con-
flicting requirements. It must be strong enough to withstand the
aerodynamic loading and have adequate resistance to erosion by rain
and hail. Since a missile undergoes rapid acceleration to supersonic
speeds considerable heating takes place and the radome must survive a
severe thermal shock. A further undesirable effect of heating is the
change in the electrical properties of the radome which may modify the
radiation pattern of the aerial and increase the transmission loss
through the radome.

The radome must also have a shape which is aerodynamically
suitable. Many early radomes were hemispherical but this shape gives
too much drag for most applications and more streamlined shapes are
usually employed. For high supersonic speeds the drag depends
essentially on the square of the fineness ratio. The fineness ratio
is the ratio of length to base diameter and a high value of fineness ratio is necessary for low drag. Unfortunately a streamlined radome often causes the missile's performance to fall far short of that theoretically obtainable with a perfect radome. This is because the radome distorts the radiation pattern of the aerial resulting in an angular error in the line of sight measured by the homing head. This angular error is a function of the angle between the aerial axis and the missile axis (the angle of look) and of the plane in which the aerial is offset. It also depends on the polarisation of the aerial and the incoming wave and on the frequency. In general the error has a component both in the plane of offset and in the plane perpendicular to both the plane of offset and the aperture plane.

The angular pointing error is termed radome boresight error in the USA. In this country the term radome aberration is widely employed and will be used in this thesis.

To understand the effect of aberration on missile behaviour consider, for simplicity, a missile attacking a target in a single plane engagement (Fig.1.3a). Suppose a small fluctuation occurs in the attitude of the missile. The radome also moves, since it is attached to the missile body, but the aerial is kept locked on to the apparent target position by the servo system. The relative movement changes the angle of look from $\theta_{L1}$ to $\theta_{L2}$ and the aberration from $\theta_1$ to $\theta_2$ (Fig.1.3b). If the target is at long range the change in true sight line is negligible, but an apparent rotation of the sight line is detected, giving rise to a demand for a proportional change in the missile heading angle rate.

It follows that the missile can become unstable if the slope of the aberration curve is negative and sufficiently large and if the phase response of the loop is appropriate. This makes the
FIGURE 1.3(a)

ABERRATION

ANGLE OF LOOK

FIGURE 1.3(b)

FIG13 CONDITIONS LEADING TO INSTABILITY
missile completely ineffective. If, on the other hand, the aberration curve has a large positive slope the missile response becomes sluggish and large miss distances are obtained. It is therefore essential to minimise the slope of the aberration curve.

The adjustment of the control system parameters to suit the aberration characteristics of the aerial-radome system and the effect of aberration characteristics on performance have been investigated.

1.4 The scope and aims of the research

The work described in this thesis formed part of a programme of research carried out by Elliott Brothers (London) Ltd. for the Ministry of Technology. The object of this research programme was to obtain an improved understanding of radome aberration and its effect on the performance of homing missiles, the ultimate purpose being to make it possible to design radomes of better performance than that previously obtained. To make this possible there are two main requirements:

(1) An adequate theory of the electromagnetic aspects of aberration, including correcting devices and the effect on aberration of tolerances in radome construction.

(2) A method of assessing the quality of a radome from its aberration characteristics. (This would also be useful in production testing.)

The author has been concerned mainly with the electromagnetic problems, which become progressively more difficult as the diameters of the aerial and the radome (relative to the wavelength) decrease. The research has led to an improved understanding of radome aberration. This understanding, together with the knowledge of correcting devices which has been obtained and the radome criteria developed by the author's colleagues, made it possible to draw up a radome design
procedure. This procedure is particularly applicable to small radomes (i.e. those with diameters of only a few wavelengths). The radome criteria, the results of the tolerance studies, the work on correcting devices and the design procedure are presented in Ref.3. Only the author's work on the principal electromagnetic problems is described in this thesis.

In Chapter 2 the 'insertion diffraction theory' of radome aberration is described, both in its basic form and in a modified form which allows a simplified model to be used for practical receiving aerial systems. The theory is fairly successful in predicting the aberration of large radomes but it is shown to be inadequate when applied to small radomes. The possible reasons for this are considered. These include the assumption of local planeness for the curved radome wall, scattering at the radome tip, propagation of guided waves on the radome and interaction (multiple scattering) between the aerial and the radome.

Chapter 3 is concerned with waves guided by radomes. A radome is a tapered tube and so the theory of waves guided by dielectric tubes is of interest. These include axial, azimuthal and whispering gallery modes. The theory of axially guided waves is summarised and it is shown by physical reasoning that the other two types of wave are not important from the point of view of aberration. Computer programmes written to solve the characteristic equation for axial surface wave propagation on loss free dielectric tubes reveal interesting features of the behaviour of the waves which are relevant to propagation on radomes. The effect of dielectric losses on the propagation of surface waves on rods and tubes is discussed and it is shown that a fast surface wave propagates on a lossy radome, affecting the aberration.
An unsuccessful attempt to analyse surface wave propagation on a tapered dielectric rod is described. The theory of azimuthal waves on a cylinder is applied to the calculation of losses due to radiation from axial surface waves on ogives. The aberration due to the $HE_{12}^2$ axial surface wave, which is near cut off at 3 cm for the radomes used, is calculated for a relative power of 1% in the surface wave. Two attempts to detect aberration due to surface waves are described.

In Chapter 4 the interaction between a receiving aerial and a plane dielectric sheet is investigated and a theory is developed which is successful in predicting the essential features of the resulting aberration. The theory is found to be unsuitable for application to small radomes but explains experimental results obtained with a small aerial in a large radome. A method for investigating the aberration caused by interaction in small radomes is described and it is shown that it is mainly radiation scattered at wide angles from the receiving aerial which is responsible. The effect is found to be less severe at frequencies below the design frequency and more significant at higher frequencies for the radomes used. A specially designed aerial which has the useful property of scattering only a small fraction of the energy incident upon it substantially reduces the aberration caused by interaction in many cases. The validity of the method of estimating the aberration is examined and this reveals an axial perturbing wave which is not a surface wave and which is attributed to scattering at the tip of the radome. Finally, the effect of radome size and dielectric constant on the aberration caused by interaction is discussed.

Chapter 5 contains a summary and presents the conclusions drawn from the research.
CHAPTER 2 - THEORIES OF RADOME ABERRATION

2.1 Radome types

Radomes consisting of multiple layers of different dielectric materials have been used extensively for aircraft but they are not commonly employed on high speed missiles, and the thesis is concerned with monolithic (single layer) radomes.

The theory of the transmission of a plane electromagnetic wave through a loss free plane dielectric sheet of infinite extent shows that the wave is transmitted without reflection if the thickness, \( d \), of the sheet is related to the free space wavelength, \( \lambda_0 \), in the following way:

\[
\frac{d}{\lambda_0} = \frac{n}{2 \sqrt{K} - \sin^2 i} \tag{2-1}
\]

where \( n \) is a positive integer, \( K \) is the dielectric constant and \( i \) the angle of incidence. The transmission coefficient is unity for polarisation perpendicular or parallel to the plane of incidence (i.e. the plane containing the normal to the surface and the wave normal). The phase of the transmitted wave, relative to that of the incident wave, is the same for both polarisations.

Since any intermediate polarisation can be resolved into parallel and perpendicular components, the transmission properties are the same so long as (2-1) is satisfied. For angles of incidence other than \( i \) the transmission properties depend on polarisation. The transmission coefficient is less than unity except for a wave polarised parallel to the plane of incidence and incident at the Brewster angle, \( \tan^{-1} \sqrt{K} \). If \( n = 1 \) in (2-1) the sheet is said to be a 'first order' half wave sheet, and the radomes used in the present work have thickness near this value.
2.2 The basic insertion diffraction theory

It was known previously that the aberration of large half wave radomes could be predicted with fair accuracy by the so-called 'insertion diffraction' theory. (The term large radomes is used to describe those with diameter greater than about ten wavelengths, i.e. it is the size relative to the wavelength which is important.)

In the simplest form of the insertion diffraction theory a parallel beam of rays is traced from the aperture of a transmitting aerial inside a radome to an imaginary aperture just outside the radome, as shown in Fig. 2.1. The phase and amplitude of the field associated with each ray is modified by the transmission coefficient of the radome at the point of incidence. The transmission coefficient at a point on the radome wall depends on the angle of incidence and the polarisation. Having obtained the modified aperture distribution \( F(\xi, \eta) \), where \( \xi, \eta \) are aperture coordinates, the far field diffraction pattern \( G(\theta, \phi) \) is calculated using the simplified result of scalar diffraction theory given by Silver:

\[
G = \frac{j}{\lambda_0 R} \int_{A} F(\xi, \eta) e^{j k_0 \sin \theta (\xi \cos \phi + \eta \sin \phi)} \, d\xi d\eta \quad (2-2)
\]

where \( R, \theta, \phi \) are the spherical polar coordinates of the observation point and \( k_0 = 2\pi/\lambda_0 \). Numerical integration is used to evaluate the integral because the field is evaluated at a number of discrete points and not as a continuous function. The aberration is then found by determining the direction of maximum intensity with respect to the aerial axis. The above expression (2-2) is usually stated to be valid if:
FIG 21: RAY TRACING FOR THE BASIC INSERTION DIFFRACTION THEORY

FIG 22: CONICAL SCAN RECEIVER

FIG 23: FEED POSITIONS FOR NUTATING AND Rotating SCAN
(1) The phase is nearly constant in the aperture.

(2) The amplitude gradient of the field, measured in the direction normal to the aperture, is small compared with the phase variation expressed in wavelengths. However, in practice these requirements may be unduly restrictive. For example the work of Cornbleet, which was extended by Sellen, and is described in Ref. 3, showed that aberration of various obstacles, such as plane dielectric sheets or discs of moderate insertion phase delay can be calculated with fair accuracy using the above theory.

2.3 The modified insertion diffraction theory

2.3.1 Practical receivers

Instead of regarding the aerial as a transmitter, as in the previous section, it is possible to implement the insertion diffraction theory when the aerial is acting as a receiver with a wave incident upon it from a distant source. This approach has advantages for both theoretical work and for measurements. It allows a simplified theoretical model of the aerial system and receiver to be used. Furthermore the measurement of the fields inside an empty radome, illuminated by a plane wave was necessary for some of the work.

As explained in Chapter 1 a semi-active homing missile detects the radiation scattered from a target aircraft, and from the returns it calculates the bearing of the target. The simplest way to do this would be to use a narrow beam aerial and turn it to maximise the received signal. The angular accuracy of a simple direction finder of this type is of the order of one beamwidth. Since it is necessary to have an accuracy of a few minutes of arc and the beamwidth of an aerial of radius $r_d$ is approximately $\lambda_0/2r_d$, it is clear that
either $\lambda_0$ must be very small or $r_d$ very large to obtain the required accuracy. Missile diameters are usually 20-50 cm, and because of propagation difficulties, the high cost of generating millimetre wave power, etc., wavelengths in X or J band (8-12 and 12-18 GHz) are generally used to track and illuminate the target. The beamwidth is then of the order of $3/30$ radians or 5.7 degrees.

One method of obtaining greater accuracy is to use a conical scan receiver (Fig. 2.2), which has typically a parabolic dish reflector with an offset dipole feed. The feed is rotated so that the axis of the beam describes a cone in space, the axis of the cone coinciding with the dish axis. The split angle (the vertex angle of the cone) is usually made equal to the aerial beamwidth. When radiation from a target is received by the aerial the received signal is amplitude modulated to a depth depending on the angular displacement of the target from the aerial axis. The modulation is at the scan frequency and its harmonics. The motor which spins the feed also drives a two phase reference generator, the voltages from which are applied to two phase sensitive detectors, together with the detected modulation signal from the aerial. The outputs of the phase sensitive detectors consist of error signals roughly proportional to the offset in azimuth and elevation of the target from the aerial axis. These error signals are used in a feedback control system to align the aerial with the target, i.e. the condition is reached where the amplitude modulation of the return from a point target of constant amplitude is zero.

Two types of conical scan aerial have been used in the present work, nutating scan and rotating scan. In the former the polarisation to which the aerial responds remains fixed relative to the
aerial dish. In the rotating scan system the dipole feed always lies parallel to the radius of the dish so that the polarisation rotates with the feed. The two cases are illustrated in Fig. 2.3. The frequency of rotation (scan frequency) of the feed is about 30 Hz.

Sellen\(^7\) performed an analysis of conical scan receivers in which he states that the problem is complicated by the fact that the aberration depends on the type of phase sensitive detector employed. One type responds only to signals at the fundamental of the scan frequency, in which case the servo system positions the aerial to reduce the amplitude of the fundamental frequency in the received signal to zero. An alternative form of detector samples the received signal at the peaks of the waveform from the reference generator.

The following example shows how the type of detector can affect the boresight axis. Suppose the signal applied to the detector is

\[ V_s = a + b \cos \omega_s t + c \cos 2\omega_s t - b \cos 3\omega_s t \]

and let the reference be

\[ V_R = \cos \omega_s t \]

where \(\omega_s\) is the scan angular frequency.

The 'peak sampling' detector compares the received signal at the peaks of the reference, i.e. at \(\omega_s t = 0, \pi, 2\pi \ldots\). At both peaks the signal voltage is

\[ V_s = a + c \]
and no error voltage is obtained. However if the same signal is applied to a detector responding only to the fundamental of the scan frequency (i.e. the harmonics are first filtered out) then an error signal is obtained.

2.3.2 The ideal receiver model

We can define, for the study of aberration, an 'ideal amplitude sensing receiver'. This consists of a single beam aerial (i.e. the beam is not split by a rotating offset feed, or by any other means) which is turned by a servo system to make the output at the aerial terminals a maximum. Although it is not used in practice this system is useful in the analysis of aberration caused by distortion of the wavefront incident on an aerial aperture. Many of the conclusions drawn from a study of the ideal receiver can be applied to practical systems, at least to a first order, and the use of the ideal receiver model greatly reduces the number of calculations required in calculating aberration.

An outline of the analysis, carried out in cartesian axes in the aerial aperture plane, is as follows:

The power polar diagram $G_p(\theta)$ as a function of $\theta$, the angle in the plane containing the x axis and the direction of propagation, is given by

$$ G_p(\theta) = \left| \int_A f(x,y)(E + \alpha x \sin \theta) \exp[jk_0x \sin \theta - \Delta] \, dx \, dy \right|^2 $$

(2-3)

where $f$ is the amplitude weighting function characteristic of the aerial and feed, $E \exp[j(-\Delta)]$ is the field in the aperture plane at $\theta = 0$, and $\alpha = \frac{3E}{2z}$ is the amplitude gradient in the direction of propagation. ($f, \alpha, E$ and $\Delta$ are all functions of position $(x,y)$ in the aperture plane.)
The maximum value of received signal is obtained when

\[ G_p'(0) = 0. \]

The required value of \( \theta \) is assumed to be near zero, so a MacLaurin expansion is used:

\[ G_p'(\theta) = G_p'(0) + \theta G_p''(0) + \frac{\theta^2}{2!} G_p'''(0) + \ldots \]  \hspace{1cm} (2-4)

Let \( G_p'(\theta) = 0 \) when \( \theta = \theta_1 \), hence to a first order

\[ \theta_1 = -\frac{G_p'(0)}{G_p''(0)} \] \hspace{1cm} (2-5)

Now, defining

\[ I(c)m = \int fExm \cos \Delta \, dx\,dy \]

\[ I(s)m = \int fExm \sin \Delta \, dx\,dy \]

\[ I(cax)m = \int f \frac{\alpha}{k_0} x^m \cos \Delta \, dx\,dy \]

\[ I(sax)m = \int f \frac{\alpha}{k_0} x^m \sin \Delta \, dx\,dy \]

the solution becomes

\[ -k_0 \theta_1 = \left[ \frac{I(c)I(s) - I(s)I(cx)}{I^2(cx) + I^2(sx) - I(c)I(cx^2) - I(s)I(sx^2)} + \frac{2[I(c)I(sax) - I(s)I(cax) + I(s)I(cax) - I(cx)I(sax)]]}{I^2(cx) + I^2(sx) - I(c)I(cx^2) - I(s)I(sx^2)} \right]. \] \hspace{1cm} (2-6)

For the special case of zero amplitude gradient

\[ -k_0 \theta_1 = \frac{I(c)I(s) - I(s)I(cx)}{I^2(cx) + I^2(sx) - I(c)I(cx^2) - I(s)I(sx^2)} \] \hspace{1cm} (2-7)

The simple result expressed in equation (2-7) serves as the basis for comparison between the ideal receiver and practical systems.
2.3.3 Nutating scan receiver

We make the assumptions that:

(1) The weighting function of the feed is independent of the angle of rotation $\phi$ (defined with the $x$ axis as datum).

(2) The amplitude gradient is negligible.

(3) The receiver has an overall square law characteristic.

Then the receiver output is given by:

$$G_p(\theta) = \left| \int \int f_E \exp\{jk_0x(\sin \theta + \sin \beta \cos \phi) + k_0y(\sin \beta \sin \phi) - \Delta\} dx dy \right|^2$$

...(2-8)

$2\beta$ is the split angle, i.e. the vertex angle of the cone generated by the beam axis, and $\phi = \omega_0 t$. If the error sensing detector is of the peak sampling type the aberration angle is given by the value of $\theta$ in (2-8) which satisfies

$$G_p(\theta)|_{\phi=0} = G_p(\theta)|_{\phi=\pi}.$$  

...(2-9)

If the detector responds only to the fundamental frequency, $\omega_0$, then the aberration is given by the value of $\theta$ which makes the coefficient of $\cos \phi$ in equation (2-8) zero.

If $\beta$ is sufficiently small that terms containing powers of $\beta$ or $\sin \beta$ greater than the second may be neglected, and if the aberration is small enough for terms of order $\theta^2_1$ and above to be negligible, both approaches give the same solution, namely that given in equation (2-7) for the ideal receiver. A practical aerial is unlikely to be less than five wavelengths in diameter, so the split angle, $2\beta$, which is usually equal to the beamwidth (i.e. about 1/5 radian) is small enough to justify the approximations. Similarly $\theta^2_1$ is unlikely to exceed one or two degrees in practice so $\theta^2_1, \theta^3_1 \ldots$ are negligible.
2.3.4 Rotating scan receiver

The analysis of the rotating scan receiver shows that, making the same approximations as in the previous paragraph, and using a peak sampling detector, the aberration again reduces to the first order solution (equation (2-7)). This is to be expected since this detector effectively converts the receiver to a nutating scan system from the point of view of polarisation characteristics. When a fundamental detector is used it is found that the aberration corresponds to the ideal receiver result only if the field incident on the aperture is circularly polarised.

2.3.5 The use of the ideal receiver model in aberration calculations

In the experimental measurements of aberration made in this research programme both rotating scan and nutating scan aerials have been used, and a peak sampling detector has been employed with both. Experimental evidence to support the view that the ideal receiver is a satisfactory model for practical systems was obtained. An empty radome was illuminated by a plane wave from a remote transmitter (Fig.2.4a) and the amplitude and phase of the field at the plane normally occupied by the aerial were measured using the field measuring equipment described in the appendix. This was done for several values of $\theta_L$ and the aberration as a function of $\theta_L$ was calculated using ideal receiver theory, assuming a 10 inch diameter aerial with $1 - r^2$ aperture illumination law. A computer programme in Elliott 803 Autocode was used to carry out the calculations, based on equation (2-7). The aberration of the radome, at the same frequency and with the same polarisation, was then measured on the aberration test site using a 10 inch diameter rotating scan aerial. (The aberration measuring equipment is described in the appendix.)
FIG 24(b) MEASUREMENT OF FIELDS

FIG 24(h) ABERRATION OF CONICAL RADOME
The two aberration curves were found to be in good agreement (Fig. 2.4b). When the above procedure was repeated using a 6 inch diameter aerial in the same radome reasonably good agreement was again found between the measured aberration curve and the curve calculated from measured fields using ideal receiver theory.

These results so obtained were those from a 4 foot long fibre glass conical radome, of cone angle 19 degrees, and half wave wall thickness at the frequency of measurement (9375 MHz). Thus the ideal receiver model can be used in the present work on small ceramic radomes for the calculation of aberration from the fields in the aperture.

2.4 Other causes of aberration

2.4.1 Limitations of the insertion diffraction theory

The insertion diffraction theory gives fairly satisfactory results when applied to large radomes (i.e. those having base diameters of ten wavelengths or more). The theory is generally less successful in predicting the aberration of small radomes (i.e. those of base diameter of about five wavelengths) particularly if the wall thickness differs from the half wave value. For example in Figs. 2.5a and 2.5b the aberration calculated by insertion diffraction theory is compared with measured aberration for two X band tangent ogive radomes of different sizes. The larger radome had an internal base diameter of 12 inches, the smaller one a diameter of 7 inches. Both were made of alumina $\text{Al}_2\text{O}_3$, a ceramic material of dielectric constant 8.7, and had a wall thickness of 0.225 inch, making the wall 'half wave' at 3.2 cm wavelength for an incidence angle of 70 degrees. The fineness ratio $L/D$, where $L$ is the distance from the centre of the aperture to the radome tip and $D$ the base diameter, was 1.71 in both cases. A 6 inch diameter aerial was used
Fig. 2: Peak to Peak Aberration for Plane Polarisation Perpendicular to Plane of Offset

- 12 Inch Dia. Radome: Insertion Diffraction Theory (■) and Measured (○)
- 7 Inch Dia. Radome: Insertion Diffraction Theory (●) and Measured (□)

Wavelength (cm) ranging from 3.0 to 3.4
FIG. 2.5(e) PEAK TO PEAK ABERRATION, PLANE POLARISATION, PARALLEL TO PLANE OF OFFSET
for the smaller radome and a 10 inch aerial for the larger one, so that the ratio of aerial diameter to radome diameter was very nearly the same for both. The peak to peak value of aberration (as a function of angle of look in the range -20 to +20 degrees) is shown as a function of wavelength. If the slope at 0 degrees angle of look is positive the peak to peak aberration is shown positive, and it is shown negative for negative central slope. It can be seen from the figure that the agreement between theory and experiment is appreciably better for the larger radome, particularly when the wavelength is near the half wave value.

There are two steps in the calculation of aberration by insertion diffraction theory:

(1) Calculation of the aperture field.

(2) Calculation of aberration from the aperture field.

For step (2), using ideal receiver theory, reasonably good results are obtained. It follows that the main cause of the inadequacy of the insertion diffraction theory, when applied to small radomes of nearly half wave wall thickness, must be due to failure to predict the aperture field after transmission of a plane wave through the radome. There are a number of possible causes for this:

(a) The use of plane sheet transmission theory when in fact the radome wall is curved.

(b) Scattering at the tip of the radome, setting up guided waves on the radome.

(c) Interaction (multiple reflections) between the aerial and the radome.

If the wall thickness differs appreciably from the half wave value the scalar diffraction theory used in step (2) becomes less accurate due to the large variations in phase delay with angle of
incidence. However we are primarily interested in radomes with wall thickness near the half wave value and we will therefore devote attention to the study of (a) to (d) above.

2.4.2 The effect of wall curvature

It is known from geometrical optics that the transmission of a ray through a curved dielectric panel (i.e. part of a cylindrical shell) differs from that through a plane panel of the same thickness, except at normal incidence. Fig.2.6 shows how the angle of incidence on the second interface differs from the plane panel case. The ray paths in the dielectric region are longer for the curved panel and the transmitted rays are no longer parallel. Ray trapping can occur (even if \( \theta_0 < 90 \) degrees) if the panel is curved (Fig.2.7). In both Figs.2.6 and 2.7 the plane of incidence is perpendicular to the cylinder axis.

It seems reasonable to expect that some inaccuracy may be caused by assuming that the curved radome wall is locally plane. An attempt was made to apply the results of Barrar\(^9\) who derived the following expression for the transmission coefficient \( T_c \) of a cylindrical dielectric panel in terms of the transmission coefficient \( T_p \) of a plane panel of the same material and thickness:

\[
T_c = 1 + \frac{(T_p - 1) \cos i}{\left[ \cos^2 i - \left( \frac{\sqrt{K - \sin^2 i}}{\cos i} - 1 \right) \frac{0.5d}{a + 0.5d} \right]^{\frac{1}{2}}} \tag{2-10}
\]

where \( K \) is the dielectric constant, \( d \) the panel thickness, \( a \) the inner radius of curvature and \( i \) the angle of incidence. The expression was derived for perpendicular polarisation but Barrar claimed results in agreement with experiment for both perpendicular and parallel polarisation.
**FIG 26 THE EFFECT OF PANEL CURVATURE**

- $i_x$ is the angle of incidence on the inner curved surface.
- $i_x'$ is the angle of incidence on the inner plane surface.

**FIG 27 TRAPPING OF A RAY BY A CURVED PANEL**

- Trapping occurs if $i_x > i_c$.
- ($i_c$ is the critical angle.)
Since Barrar's theory was derived for singly curved sheets and radomes are doubly curved, it was necessary to decide the value of curvature to be used. From the theory of geometrical optics it is known that when a ray is incident on a smooth surface the reflected and refracted rays lie in the plane of incidence, i.e. the plane containing the incident ray and the normal to the surface at the point of incidence. Therefore in applying Barrar's theory it was decided to use the value of curvature in this plane at the point of incidence of the ray.

When the correction formula was applied in the calculation of aberration, using the insertion diffraction theory, for conical radomes of height 12 inches, base diameter 7 inches and dielectric constant 8.7, it was found that the modification produced in the transmission coefficients was negligible. For tangent ogives of similar size the changes in aberration predicted using insertion diffraction theory were about 10%. (This is negligible compared with other sources of error.) For a Newtonian radome, however, (the generating curve of which follows a three fourths power law) the much greater curvatures caused the correction formula to break down, that is, physically unrealistic results were obtained.

It was concluded that where Barrar's theory is applicable, i.e. to radomes with moderate wall curvature, it does not cause significant changes in the aberration predicted by insertion diffraction theory. For radomes with greater wall curvature the theory is inadequate. As it is likely that the error caused by wall curvature may be large for the latter case further work using a different approach is desirable if such radomes are to be used. However for the radomes of most practical interest, the tangent ogive and optimum secant ogive the effect was not considered of sufficient importance to justify further investigation.
2.4.3 Scattering at the radome tip

The tip region of the radome has dimensions comparable with a wavelength, because the wall thickness is \( \lambda_0/(2 \sqrt{K}) \) approximately, (for large \( K \)) and therefore the ray tracing method used in the insertion diffraction theory is unreliable in this region.

The solutions of classical electromagnetic scattering problems can frequently be given a ray optical interpretation if an 'asymptotic expansion' is used, i.e. the ratio of wavelength to scatterer dimensions is allowed to approach zero in the solution. For problems in which the dimensions of the scatterer are comparable with or less than a wavelength Keller\(^1\) showed that ray optical methods may still be applied if a class of rays known as diffracted rays is introduced. These rays are launched when an incident ray strikes an edge, a vertex, at grazing incidence on a curved surface and in several other situations. The fields associated with the diffracted rays augment the fields described by the conventional incident, reflected and refracted rays, and 'diffraction coefficients' are introduced in analogy with reflection and transmission coefficients. The advantage of using ray optics is that it is possible to determine the scattering properties of bodies of complex shape, for which a solution would not be obtainable by exact electromagnetic analysis, by synthesis of the results of simpler (canonical) problems. Unfortunately most of the published work on the application of ray optics to scattering has been concerned with scattering by perfectly conducting bodies, and no published theoretical work on the most useful canonical problem, viz. scattering of a plane wave by a dielectric cone, has been found.

The simple ray formulation of the geometrical theory of diffraction fails in the transition regions surrounding the domain of
existence of a particular ray species, for example at the boundary between the shadow region and the illuminated region when a plane wave is diffracted by a half plane, and at foci and caustics. The more elaborate procedures required for these regions are discussed by Felsen\textsuperscript{11}. The ray theory also requires modification if the scatterer has non-uniform surface impedance. If the surface impedance varies only slightly over a distance of a wavelength then the local character of the scattering process is not greatly disturbed. Since the surface impedance of a radome varies rapidly in the region of the tip, it follows that the results of applying ray optics in this region should be treated with caution until an exact solution for this type of problem has been obtained. However, one may draw qualitative conclusions as to the behaviour at the tip.

Applying ray optics to scattering by the radome tip, and including diffracted rays from the interior and exterior vertices, suggests that the following effects occur (Fig.2.8):

(a) \textit{Trapping of rays}

Rays are trapped and guided by the radome as surface waves.

(b) \textit{Diffracted rays}

Rays are diffracted by the vertices. Some of those from the interior vertex enter the aperture of the aerial. The magnitude of this effect cannot yet be determined because a solution to the canonical problem of diffraction by a dielectric cone has not been found.

(c) \textit{Deviated rays}

Rays incident just beyond the tip undergo refractive shift or deviation (d). The plane wave plane sheet transmission coefficient may not be applicable to these rays because of the discontinuity.
TRAPPED RAY

THIS DIFFRACTED RAY Launches FURTHER DIFFRACTED RAYS FROM THE INTERIOR VERTEX.

FIG. 28  SCATTERING BY THE TIP OF A RADOME
It was thought at first that ray trapping by the tip could be an important feature of tip scattering, with energy trapped at the tip being guided up the radome as surface waves and perturbing the fields in the aerial aperture, thus causing aberration. In other words, it was thought that the radome could act rather like a dielectric tube aerial. It seemed likely that a not insignificant fraction of the incident energy might be trapped by the tip. For example a calculation of the effective cross-section of a typical polyrod aerial of length $4\lambda_0$, maximum diameter $0.4\lambda_0$, minimum diameter $0.3\lambda_0$ and dielectric constant 2.5 was made. It was found that the effective cross-section was about four times the geometrical cross-section. This result suggests that a radome tip having high gain could influence the radiation pattern appreciably.

Evidence will be presented in Chapter 3 to show that surface waves propagate on the body of the radome (above the tip) with very little loss due to radiation. It follows that when a plane wave is incident on a radome the surface waves launched at the tip are guided up the radome without much loss. However it will be shown that, for the radomes of present interest, the surface waves cause very little aberration.

The 'vertex diffracted rays' of Fig. 2.8 indicate that an effective source is created at the tip when a plane wave is scattered by the radome. It is well known that a source near a guiding structure, such as a grounded dielectric slab, gives rise to a total field which contains three components: surface waves, leaky waves and a radiation field. The energy reaching the aerial in the leaky wave and radiation field components is shown later to be a major cause of aberration in small radomes.
The effect of perturbing wave was examined by Lewis and Laite who derived an expression for the aberration caused by a perturbing plane wave in an ideal amplitude sensing direction finding system. In Fig. 2.9 the system is locked on to a primary wave of unit amplitude with a perturbing wave of amplitude $A$ and relative phase $\phi_p$ (measured at the centre of the aerial aperture) propagating at an angle $\psi$ from the main wave as shown. Let $F(0)$ be the amplitude polar diagram of the aerial, where $F(0)$ is assumed real, i.e. the phase of the received signal does not vary with $\theta$.

The received signal is given by

$$V = F(0) + Ae^{j\phi_p}F(0 + \psi) \quad (2-11)$$

The aerial turns to make the signal a maximum, i.e.
This yields the condition
\[ \frac{\partial V}{\partial \theta} = 0. \quad (2-12) \]

provided \(|V| \neq 0\).

By differentiating (2-15) we obtain
\[ \frac{\partial V}{\partial \theta} = F'(0) + Ae^{-j\phi}F'(0 + \psi) . \quad (2-14) \]

Expanding \(F(0)\) in a MacLaurin series, assuming \(\theta\) is small:
\[ F(0) = F(0) + \theta F'(0) = 1 \]
\[ F'(0) = F'(0) + \theta F''(0) = \theta F''(0) . \]

Now using Taylor expansions:
\[ F(\theta + \psi) = F(\psi) + \theta F'(\psi) \]
\[ F'(\theta + \psi) = F'(\psi) + F''(\psi) . \]

Substituting these relations in equations (2-11) and (2-14), we apply condition (2-13), neglecting terms in \(\theta^2, \theta^3\) etc. Since the resultant value of \(\theta\) turns out to be proportional to \(A\), and terms in \(\theta^2\) have been ignored, to be consistent terms in \(A\theta\) and \(A^2\) must also be neglected. The final result is
\[ \theta = \frac{AF'(\psi)}{F''(0)} \cos \phi . \quad (2-15) \]

The 'perturbing plane wave theory' is a special case of a 'multiple target' problem. In general if a radar is tracking (say) two point targets, such as two distant aircraft, the simplifying assumptions made in the above theory, i.e. that the perturbing signal is very small and of fixed relative phase, will not apply. Thus the theory is not very useful in practical multiple target problems. It
does however serve to show the effect of a perturbing signal in causing aberration and will be used extensively in later chapters.

Two interesting points emerge from equation (2-15): (a) The aberration is zero if the perturbing wave is in phase quadrature with respect to the main signal. (b) The aberration is proportional to the slope of the polar diagram \( F'(\psi) \). This shows that perturbing waves are most troublesome when arriving near nulls of the polar diagram, where the slope is greatest. This follows from the nature of the amplitude sensing system. Suppose the system is locked on to a target, and then a perturbing wave is received near a null. In order to make \( 3V/3\theta = 0 \) the aerial must turn through a fairly large angle to make the change in signal due to the main wave compensate for the change caused by the perturbing wave because \( F'(\theta) \) is small near \( \theta = 0 \) and large near the null.

2.4.5 Aerial-radome interaction

It is well known that if a plane wave is incident upon a receiving aerial a large proportion of the incident energy is scattered by the aerial, even if the latter is matched. If the aerial is inside a radome some of the scattered energy is reflected by the radome back into the aperture perturbing the fields and causing aberration. Multiple reflections also take place if the aerial is transmitting.

The term interaction is used here to describe the multiple scattering occurring between aerial and radome. Chapter 4 contains a description of theoretical and experimental work on this subject.
CHAPTER 3 - WAVES GUIDED BY RADOMES

3.1 Dielectric waveguides of uniform cross-section

3.1.1 Axially guided waves

The theory of surface waves guided axially by loss free, homogeneous, isotropic dielectric rods and tubes is well established. Waves guided by tubes are of particular interest because a radome is a tapered tube. An understanding of the characteristics of waves guided by uniform tubes forms a useful starting point for the study of waves guided by radomes.

Source free solutions of Maxwell's equations for both rods and tubes exist in the form of guided waves which propagate axially without attenuation, the phase velocity being less than the free space velocity of light and greater than the TEM wave velocity in the dielectric. Kiely gives a summary of the method used by Astrahan in the latter's analysis of axial surface wave propagation on a loss free dielectric tube of circular cross-section. Starting with Maxwell's equations and assuming zero conductivity, no charges and time dependence \( \exp(j\omega t) \) the wave equations for a homogeneous medium are obtained:

\[
\begin{align*}
\text{Curl} \ E & = -j\omega \mu H \\
\text{Curl} \ H & = j\omega \varepsilon E \\
\text{Div} \ E & = 0 \\
\text{Div} \ H & = 0
\end{align*}
\]  

(3-1)

where \( \mu \) is the permeability and \( \varepsilon \) the permittivity.

Taking the curl of either of the first two equations and substituting from the others the wave equations for a homogeneous medium are obtained:

\[
\begin{align*}
\nabla^2 E & = -\omega^2 \varepsilon \mu E \\
\nabla^2 H & = -\omega^2 \mu \varepsilon H
\end{align*}
\]  

(3-2)
where $\nabla^2$ is the Laplacian operator. It is assumed that the field dependency in $t$ and $z$ is of the form $\exp j(\omega t - \gamma z)$ where $\gamma$ is real and positive. This represents a wave propagating without attenuation in the $+z$ direction. The vector wave equations yield for the $z$ field components the scalar equations

$$\nabla^2 E_z = -\omega^2 \mu e E_z,$$

$$\nabla^2 H_z = -\omega^2 \mu e H_z,$$

i.e. in cylindrical polar coordinates $\rho, \phi, z$

$$\frac{\partial^2 E_z}{\partial \rho^2} + \left(\frac{1}{\rho}\right) \frac{\partial E_z}{\partial \rho} + \left(\frac{1}{\rho^2}\right) \frac{\partial^2 E_z}{\partial \phi^2} = -k^2 E_z,$$

$$\frac{\partial^2 H_z}{\partial \rho^2} + \left(\frac{1}{\rho}\right) \frac{\partial H_z}{\partial \rho} + \left(\frac{1}{\rho^2}\right) \frac{\partial^2 H_z}{\partial \phi^2} = -k^2 H_z,$$

where

$$k^2 = \omega^2 \mu e - \gamma^2.$$ (3-4)

The general solutions of equations (3-3) are

$$E_{zn} = \left[ B_n J_n(k_n \rho) + C_n Y_n(k_n \rho) \right] \exp j(n\phi - \gamma_n z),$$

$$H_{zn} = \left[ D_n J_n(k_n \rho) + E_n Y_n(k_n \rho) \right] \exp j(n\phi - \gamma_n z),$$ (3-5)

for different values of $n$. The time factor $\exp j\omega t$ is omitted for brevity. $B_n, C_n, D_n$ and $E_n$ are arbitrary constants and $J_n, Y_n$ are Bessel functions of the first and second kinds, order $n$.

Physically realisable fields must be the same if $\phi$ is increased by $2\pi$ so $n$ must be a positive or negative integer or zero. It is found that for any given $n$ a solution for guided waves is obtained by considering only combinations of solutions for $+n$ and $-n$ since
the boundary conditions can then be satisfied. Each value of \( n \) corresponds to a type that can be guided by the system of Fig.3.1.

The \( p \) and \( \phi \) field components are now obtained from the expressions for \( E_z \) and \( H_z \) by using the Maxwell curl equations and putting \( \partial / \partial z = -j \gamma_n \) and \( \partial / \partial t = j \omega \). This gives

\[
E_{\phi n} = \frac{1}{k_n} \left\{ \left( \frac{j \gamma_n}{\rho} \right) \frac{\partial E}{\partial \phi} + j \omega \frac{\partial H}{\partial \phi} \right\}
\]

\[
E_{\rho n} = -\frac{1}{k_n} \left\{ j \gamma_n \frac{\partial E}{\partial \rho} + \left( \frac{j \omega}{\rho} \right) \frac{\partial H}{\partial \phi} \right\}
\]

\[
H_{\phi n} = -\frac{1}{k_n} \left\{ j \omega \frac{\partial E}{\partial \rho} + \left( \frac{j \gamma_n}{\rho} \right) \frac{\partial H}{\partial \phi} \right\}
\]

\[
H_{\rho n} = \frac{1}{k_n} \left\{ \left( \frac{j \omega}{\rho} \right) \frac{\partial E}{\partial \phi} - j \gamma_n \frac{\partial H}{\partial \rho} \right\}
\]

The most appropriate form of equations (3-5) is now found for each of the three regions (Fig.3.1). We then match them so that the tangential components of \( E \) and \( H \) are continuous at the boundary surfaces. For region I, writing \( E_{z1} \) for \( E_{zn1} \)

\[
E_{z1} = a_{n1} J_n(k_{n1} \rho) F_n
\]

\[
H_{z1} = b_{n1} J_n(k_{n1} \rho) F_n
\]

where \( F_n = \exp j(n\phi + \omega t - \gamma_n z) \). The \( Y_n \) solution is not used because the fields must remain finite and \( Y_n(x) \) tends to infinity as \( x \) tends to zero. For region II both solutions are used.

\[
E_{z2} = \{ a_{n2} J_n(k_{n2} \rho) + a_{n3} Y_n(k_{n2} \rho) \} F_n
\]

\[
H_{z2} = \{ b_{n2} J_n(k_{n2} \rho) + b_{n3} Y_n(k_{n2} \rho) \} F_n
\]
FIG. 31 TUBE AND COORDINATE SYSTEM
For region III

\[
\begin{align*}
E_{z3} &= a_n H_n^{(1)}(k_{n3}\rho) F_{n3} \\
H_{z3} &= b_n H_n^{(1)}(k_{n3}\rho) F_{n3}
\end{align*}
\]  
(3-9)

where \( H_n^{(1)}(x) = J_n(x) + jY_n(x) \) is the Hankel function of the first kind. This type of solution is appropriate because it represents a decaying field in the \( \rho \) direction, i.e. perpendicular to the guiding surface, if \( k_{n3} \) is positive imaginary. For a hollow dielectric tube

\[
\begin{align*}
k_{n3}^2 &= \omega^2 \mu_0 \varepsilon_0 - \gamma_n^2 \\
&= \left( \frac{\omega}{v_3} \right)^2 - \left( \frac{\omega}{v_{gn}} \right)^2
\end{align*}
\]  
(3-10)

where \( v_3 \) is the TEM wave velocity in medium III and \( v_{gn} \) is the phase velocity of the mode corresponding to \( n \). \( \mu_2 \varepsilon_2 \) is assumed greater than \( \mu_3 \varepsilon_3 \) (as it is if the tube is in air). \( v_2 \) is less than \( v_3 \), and \( v_1 \) is less than or equal to \( v_3 \). Physical reasoning shows \( v_{gn} \) cannot exceed \( v_3 \). Therefore (3-10) shows \( k_{n3} \) is imaginary or zero, and since taking it as positive imaginary gives physically realistic behaviour for large \( \rho \) this restriction is put on \( k_{n3} \).

The \( \rho \) and \( \phi \) components are obtained by substituting equations (3-7) to (3-9) into (3-6) giving the \( \rho \) and \( \phi \) field components. For brevity \( H_n^{(1)} \) is written \( H_n \) and the subscript \( n \) is omitted from \( k_n \) and \( \gamma_n \). For example
\[ E_{\phi_1} = \left( \frac{1}{k_1^2} \right) \left\{ (\gamma_n \frac{a_{n1}}{\rho}) J_n(k_1 \rho) + j \omega \mu k_1 b_{n1} J_n'(k_1 \rho) \right\} F_n \]

\[ H_{\phi_1} = \left( - \frac{1}{k_1^2} \right) \left\{ j \omega \epsilon k_1 a_{n1} J_n'(k_1 \rho) - (\gamma_n \frac{b_{n1}}{\rho}) J_n(k_1 \rho) \right\} F_n \] (3-11)

and the \( E', H', E', \) and \( H' \) components for each region are found similarly. The boundary conditions requiring continuity of tangential (\( \phi \) and \( z \)) components at \( \rho = a \) and \( \rho = b \) are now applied, e.g. \( E_{z1} = E_{z2} \) at \( \rho = a \), \( E_{z2} = E_{z3} \) at \( \rho = b \) and so on. This gives eight equations in the \( a_n \)'s and \( b_n \)'s:

\[ \begin{align*}
\frac{1}{X_1} \left[ \gamma n a_{n1} J_n(X_1) + j \omega \mu_1 b_{n1} J_n'(X_1) \right] \\
= \left[ \frac{1}{cV} \right]^2 \left\{ \gamma n a_{n2} J_n(cV) + a_{n3} Y_n(cV) \right\} \\
+ j \omega \mu_2 cV \left[ b_{n2} J_n'(cV) + b_{n3} Y_n'(cV) \right]
\end{align*} \] (3-12)

\[ \begin{align*}
\frac{1}{X_1} \left[ j \omega \epsilon X_1 a_{n1} J_n(X_1) - \gamma n b_{n1} J_n(X_1) \right] \\
= \left[ \frac{1}{cV} \right]^2 \left\{ j \omega \epsilon cV \left[ a_{n2} J_n'(cV) + a_{n3} Y_n'(cV) \right] \\
- \gamma n b_{n2} J_n'(cV) + b_{n3} Y_n(cV) \right\}
\end{align*} \]

\[ \begin{align*}
a_{n2} J_n(V) + a_{n3} Y_n(V) &= a_{n4} H_n(N) \\
b_{n2} J_n(V) + b_{n3} Y_n(V) &= b_{n4} H_n(N)
\end{align*} \]

\[ \begin{align*}
\frac{1}{V} \left\{ \gamma n a_{n2} J_n(V) + a_{n3} Y_n(V) + j \omega \mu_2 cV \left[ b_{n2} J_n'(V) + b_{n3} Y_n'(V) \right] \right\} \\
= \left[ \frac{1}{N} \right]^2 \left\{ \gamma n a_{n4} H_n(N) + j \omega \mu_3 b_{n4} H_n'(N) \right\}
\end{align*} \]

\[ \begin{align*}
\frac{1}{V} \left\{ j \omega \epsilon cV \left[ a_{n2} J_n'(V) + a_{n3} Y_n'(V) \right] - \gamma n b_{n2} J_n(V) + b_{n3} Y_n(V) \right\} \\
= \left[ \frac{1}{N} \right]^2 \left\{ j \omega \epsilon_3 b_{n2} H_n'(N) - \gamma n b_{n4} H_n(N) \right\}
\end{align*} \]

where \( X_1 = k_1 a, \ cV = k_2 a, \ c = a/b, \ N = k_3 b. \)
These eight equations are then written in the standard form, with corresponding terms in the $a_n$'s and $b_n$'s placed in the same order on the left hand side of each equation, and the right hand sides zero. The equations are consistent only if the determinant of the coefficients of the $a_n$'s and $b_n$'s is zero. In this case seven of the eight unknowns can be found in terms of the eighth, which is determined by the power carried by the wave.

Evaluating the determinant gives the characteristic equation which, for the special case of a hollow tube in air, is

\[
\begin{align*}
&- n^4 \frac{h^2}{2} \left[ \Delta_3 - \Delta_4 \right]^2 \\
&- e^2 k^2 q^2 \left[ \Delta_3 \left( \Delta_1 - \frac{\Delta_9}{K} \right) \left( \Delta_8 - \frac{\Delta_5}{K} \right) - \Delta_4 \left( \Delta_7 - \frac{\Delta_9}{K} \right) \left( \Delta_2 - \frac{\Delta_5}{K} \right) \right]
\\
&\quad \cdot \left[ \Delta_3 (\Delta_1 - \Delta_9) (\Delta_8 - \Delta_5) - \Delta_4 (\Delta_7 - \Delta_9) (\Delta_2 - \Delta_5) \right]
\\
&- 2 n^2 T^2 k q^2 \Delta_3 \Delta_4 (\Delta_1 - \Delta_7) (\Delta_2 - \Delta_8)
\\
&+ n^2 T^2 k q \left[ (\Delta_8 \Delta_3 - \Delta_2 \Delta_4) - \Delta_5 (\Delta_3 - \Delta_4) \right]
\\
&+ n^2 T^2 k q c^2 \left[ (1 - \Delta_3 - \Delta_7 \Delta_4) - \Delta_9 (\Delta_3 - \Delta_4) \right]
\\
&\left[ (\Delta_1 \Delta_3 - \Delta_7 \Delta_4) - \Delta_9 (\Delta_3 - \Delta_4) \right] = 0
\end{align*}
\]

\[\ldots \quad (3-13)\]

where $T = 1/V^2 + 1/W^2$, $jW = k_3 b$, $H_n = H_n^0(jW)/[jW H_n^0(jW)]$, $J_n = J_n^0(jcW)/(jcW J_n^0(jcW))$,

\[
\begin{align*}
Q &= \lambda^2 \lambda^2_0^2 = (V^2 + W^2)/(V^2 + KW^2)
\\
\Delta_5 &= J_n^0(cV)/(cV J_n^0(cV))
\\
\Delta_2 &= J_n^0(V)/(V J_n^0(V))
\\
\Delta_3 &= J_n^0(cV) Y_n^0(cV)
\\
\Delta_7 &= Y_n^0(cV)/[cV Y_n^0(cV)]
\\
\Delta_4 &= J_n^0(V)/Y_n^0(V)
\\
\Delta_8 &= Y_n^0(V)/[V Y_n^0(V)]
\end{align*}
\]
From (3-4)

\[ \gamma^2 = \omega^2 \mu_0 \varepsilon_0 K \left( \frac{V}{b} \right)^2 = \omega^2 \mu_0 \varepsilon_0 + \left( \frac{W}{b} \right)^2. \]  

(3-14)

It is found that, as in the case of the dielectric rod, pure E and pure H modes can only exist in modes possessing radial symmetry of field, i.e. there is no variation of the field components with the coordinate \( \phi \), which is true for \( n = 0 \). Under these conditions the boundary conditions may be satisfied when the \( b_n \)'s are zero and hence \( E_\phi, H_\rho \) and \( H_z \) are zero giving pure E mode. Similarly if the \( a_n \)'s are zero \( E_\rho, H_\phi \) and \( E_z \) are zero giving a pure H mode.

When \( n \neq 0 \) all the modes are hybrid (that is, they have both \( E_z \) and \( H_z \) finite) and are described as HE or EH modes. If the H component wave is stronger the wave is HE, and if there is more power in the E component then it is an EH wave.

The explanation for the non-existence of pure E and pure H waves with \( n > 0 \) is that these waves, which can exist alone in a perfectly conducting hollow tube, are able to do so because longitudinal conduction currents can flow in the perfectly conducting waveguide. These are not present on dielectric rods or tubes and are replaced by the displacement currents set up by an auxiliary wave.

The modes are described as \( HE_{nm}, EH_{nm}, H_{0m} \) or \( E_{0m} \) (\( m = 1, 2, 3 \ldots \)). The subscript \( n \) refers to the field variation with \( \phi \) (\( \sin n\phi \) or \( \cos n\phi \)) and \( m \) refers to the root of the characteristic equation selected. For example \( m = 1 \) corresponds to the lowest root of the characteristic equation (i.e. the smallest value of \( \lambda/\lambda_0 \)), \( m = 2 \) to the next root and so on. The modes are characterised by 'high pass filter' cut off phenomena, as in hollow
conducting tubes. For a given mode, as frequency tends to infinity, all the wave energy is trapped within the dielectric. As the frequency decreases the proportion of energy trapped decreases and as the frequency approaches the cut off value this proportion approaches zero. The fields above the dielectric guiding surface decay in approximately exponential fashion with distance from the surface, the decay constant increasing as frequency increases for a given mode.

3.1.2 Non-axially guided waves on dielectric tubes

In addition to axial surface waves, dielectric tubes can support 'whispering gallery' modes and azimuthal (circumferential) modes. Elliott\textsuperscript{15} studied the propagation of azimuthal surface waves on a dielectric coated conducting cylinder and on a conducting cylinder with axial corrugations. He showed that the propagation constant is complex (even if no resistive losses are present), the attenuation occurring because energy is lost by radiation. This type of wave is not very important in the present work because it can only be launched by a wave propagating in a direction normal to the axis of the tube (radome) and the aberration is only of interest for waves incident up to about 20 degrees from the axis.

The author performed some experiments\textsuperscript{15} on the launching and propagation of azimuthal waves on a radome. These experiments have some relevance to measurements of radome electrical thickness or insertion phase delay. However, as these experiments are not connected with the main theme of radome aberration, they are not included in this thesis.

A dielectric rod of large diameter (relative to a wavelength) can support whispering gallery modes. The propagation of these waves on a dielectric rod is analysed by Wait\textsuperscript{17} whose results show...
that the wave normals follow helical paths, where the axes of the helices coincide with the rod axis.

In view of the similarity between rods and tubes in guiding axial waves it seems reasonable to expect that dielectric tubes of large diameter and suitable wall thickness would also support whispering gallery modes. This view is supported by physical reasoning based on the model of waves trapped in the wall of the tube and propagating by successive reflections at the dielectric-air boundaries. (The reflection is total if the angle of incidence is greater than \( \sin^{-1} \left( \frac{1}{\sqrt{k}} \right) \).) There is no reason why this model should be incorrect when the wave normals follow spiral paths rather than axial or azimuthal paths in the dielectric.

It is thought that axial and azimuthal waves can be regarded as limiting cases of the whispering gallery waves. Let us define the pitch angle of the helical path followed by the wave normal in the wall of the tube as the angle between the wave normal and a line drawn round the circumference of the tube. Thus for an axial surface wave the pitch angle is \( \pi/2 \) and for azimuthal waves it is zero.

Physical reasoning suggests that a plane wave incident on a radome at an angle \( \theta_L \) with the axis launches waves of pitch angle \( \pi/2 - \theta_L \). This is certainly true for \( \theta_L = 0 \) and \( \pi/2 \). The launching of surface waves in a curved sheet can be related to ray optics (Fig.2.7). Applying ray optics to the ogives of Fig.1.1 shows that for \( \theta_L < 20 \) degrees trapping of rays at grazing incidence can only affect a very small proportion of the energy incident on the radome. Further, this trapping occurs high up the radome, unlike axial wave launching which occurs principally at the
FIG 3.2 FLOW DIAGRAM FOR PROGRAMME BAT14
tip of the radome (section 3.4.2). This accords with Wait's statement that a large diameter is necessary for whispering gallery waves to propagate.

It was decided therefore that only axial surface wave propagation would be studied further as the whispering gallery waves appear to be of minor importance for small angles of look and the azimuthal waves are only launched for \( \theta_L = \pi/2 \).

3.2 Axial surface waves on loss free uniform tubes

In order to determine the fields for a given mode on a tube it is first necessary to solve the characteristic equation (equation (3-13)). The author did this for first order (\( n = 1 \)) surface waves by writing an Algol computer programme (code number BAT 14) for the Elliott 920 computer. The left hand side of the equation, which can be regarded as simply \( F(x_1) = 0 \) (where \( x_1 = \lambda/\lambda_0 \)), was calculated and printed together with \( x_1 \). Starting with a suitable value of \( x_1 \) the process was repeated successively, \( x_1 \) being increased by a small amount each time, until the largest value of \( x_1 \) required was reached. Fig.3.2 shows the flow diagram of the programme.

The roots of the equation \( F(x_1) = 0 \) were found either by interpolation or by using smaller \( x_1 \) increments once an approximate root was known. This procedure was adopted in preference to an iterative method because the computer's storage capacity was insufficient for the latter. Polynomial approximations \(^{18} \) were used to calculate the Bessel functions because no library programme for the calculation of the functions was available for this computer. However the accuracy of the polynomial approximations was estimated to be adequate for the purpose. Verification of the correctness of the programme was obtained in two independent ways. First the values of
\( \lambda/\lambda_0 \) calculated by Astrahan\(^4\) were checked for the \( \text{HE}_{11} \) mode on a dielectric tube of relative permittivity 2.5. Then the results of other workers, obtained for the \( \text{HE}_{11} \) mode on dielectric rods of various permittivities, were compared with those obtained from the programme when the tube inner radius was set equal to zero. In both cases it was found that the results agreed with those of the other workers and the programme was considered verified.

A typical print-out of the results obtained with the programme is shown in Fig.3.3. An important result, from the point of view of propagation on radomes, was revealed by the calculations. For tubes of constant wall thickness, with diameters of several wavelengths, the value of \( \lambda/\lambda_0 \) was found to vary only slightly with tube diameter. This suggests that for modes not too close to cut off the taper of a radome forms a very mild perturbation (in the large diameter region away from the tip) and therefore the propagation on the radome near the aerial would be expected to be very similar to that on a uniform tube of the same diameter as the radome in the region of interest. Figs.3.4 and 3.5 show two examples of the variation of \( \lambda/\lambda_0 \) with diameter for tubes of constant wall thickness. Even for the mode close to cut off (Fig.3.5) \( \lambda/\lambda_0 \) varies quite slowly with diameter once this exceeds about six wavelengths. Fig.3.4 also includes results for the \( \text{E}_01 \) mode obtained from Ref.12.

Another interesting result was obtained when a colleague wrote a programme for the Elliott 803 computer to solve the characteristic equation (3-13) with \( n = 1, 2, 3, 4, 5 \). The programme was verified for \( n = 1 \) waves, by comparison with the first programme mentioned. The higher modes could not be checked as
BAT14
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DIELECTRIC CONSTANT = 2.500000
INNER RADIUS = 2.700000
OUTER RADIUS = 3.000000
FREE SPACE WAVELENGTH = 1.000000

\[
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7500000 & 0.000043 \\
7600000 & 0.060469 \\
7700000 & 0.000573 \\
7800000 & 0.000540 \\
7900000 & 0.003632 \\
8000000 & 0.002593 \\
8100000 & 0.000392 \\
8200000 & 0.000218 \\
8300000 & 0.000200 \\
8400000 & 0.000220 \\
8500000 & 0.001753 \\
8600000 & 0.003827 \\
8700000 & 0.003553 \\
8800000 & 0.005413 \\
8900000 & 0.021537 \\
9000000 & 0.4287691 \\
9100000 & 0.0023544 \\
9200000 & 0.009126 \\
9300000 & 0.006709 \\
9400000 & 0.006883 \\
9500000 & 0.009007 \\
9600000 & 0.0014700 \\
9700000 & 0.0030609 \\
9800000 & 0.0089172 \\
9999999 & 0.0050808 \\
9999999 & 5.29 \times 10^{-10} \\
\end{array}
\]

\[HE_{11} \text{ - 1st sign change}\]
\[EH_{11} \text{ - 2nd sign change}\]
(very nearly the same guide wavelength as the \(E_{01}\) mode)

*Note - No further changes of sign, so the \(HE_{12}\) mode is below cut off.*

FINISH

FIG 33 A TYPICAL PRINT OUT OF
PROGRAMME BAT14
FIG. 34 E₀₁ and HE₁₁ MODES on TUBE of CONSTANT WALL THICKNESS
Fig. 3.5 \( \frac{\lambda}{\lambda_0} \) vs Outer Radius

\( \lambda_0 = 3.0 \text{ cm} \)

Wall thickness = 0.577 cm

Dielectric constant = 8.7
Kiely only gives results for the $H_{01}$, $E_{01}$, and $HE_{11}$ waves. A typical print-out is shown in Fig. 3.6 and a graph of $\lambda/\lambda_0$ versus tube radius for $EH_{n1}$ waves ($n = 1, 2, 3, 4, 5$) in Fig. 3.7. Not only do the higher order waves show the previously noted tendency of slowly varying $\lambda/\lambda_0$ for large radius, neighbouring modes have only slightly different values of guide wavelength. This suggests that azimuthal asymmetry in a radome could cause strong coupling between the different waves of a given class such as $EH_{n1}$ waves. This asymmetry could take the form of a departure from circularity of cross-section or an azimuthal variation of wall thickness.

3.3 The effect of dielectric loss on the propagation of axial surface waves

3.3.1 Propagation on lossy dielectric rods

Since practical radomes have finite dielectric loss, it is necessary to enquire into the effects of this on surface wave propagation.

On a loss free dielectric tube or rod the surface impedance $E_z/H_\varphi |_{\rho = \rho_s}$, where $\rho_s$ is the rod radius or tube outer radius, is inductive for the $E_{01}$ mode. It is also inductive for the same mode on a dielectric coated or corrugated conductor of circular cross-section if these structures are loss free. Barlow and Karbowiak showed theoretically and confirmed experimentally that a capacitive surface can also support an $E_{01}$ surface wave over a limited frequency range. The phase velocity in this range is greater than the free space velocity of light. In these experiments the capacitive surface was provided by a lossy dielectric (Perspex) rod of carefully chosen diameter. Below a critical frequency the wave broke up and could no longer be supported.
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DIELECTRIC CONSTANT = 2.70000  
WAVELENGTH = 3.00000

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DIELECTRIC CONSTANT = 2.70000  
WAVELENGTH = 3.00000

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INNER RADIUS = 3.00000  
OUTER RADIUS = 2.17059  
DIELECTRIC CONSTANT = 2.70000  
WAVELENGTH = 3.00000

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FIG. 3.6 HE_{n1}, EH_{n1} & HE_{n2} MODES
\[ \frac{\lambda}{\lambda_0} = 0.96 \]
\[ \lambda_0 = 0.577 \text{ cm} \]

**Fig. 3.7** $E_{n1}$ Modes on a Dielectric Tube

Dielectric Constant = 8.7

Wavelength, $\lambda_0 = 3.0 \text{ cm}$

Free Space Wavelength, $\lambda_0 = 3.0 \text{ cm}$

DIELECTRIC CONSTANT $= 8.7$

WALL THICKNESS $= 0.577 \text{ cm}$

FREE SPACE WAVELENGTH, $\lambda_0 = 3.0 \text{ cm}$
The same conclusion, that a capacitive surface can support an $E_{01}$ surface wave, was also reached by Arnab $^{21}$ who studied the characteristic equation for axial surface wave propagation on a dielectric rod. He did not confine his attention to non-radiating waves but treated the axial and transverse propagation constants as complex variables with (in general) finite real and imaginary parts. He found that leaky waves, i.e. fast radiating waves, can exist on a loss free dielectric rod. These waves can be connected to the surface wave modes of the rod. When the frequency is reduced below cut off for a particular mode a transition to a leaky wave occurs. These waves are improper in that they do not satisfy the radiation condition and therefore can only exist in the neighbourhood of a source. It was also shown that dielectric losses in the rod affect the propagation characteristics strongly. It was confirmed that the surface impedance for the $E_{01}$ mode on a lossy rod can become capacitive and support a non-radiating fast surface wave, as reported by Barlow and Karbowiak. In addition the propagation constants are evaluated for proper and improper $E_{01}$ waves on a dielectric rod of dielectric constant 2.6 and $\tan \delta = 0$ and 0.077. Arnab states that generally similar behaviour is obtained for the more complex $n = 1$ waves. The latter conclusion is of importance because the $HE_{1m}$ modes are of most interest on the radomes used in the present programme.

3.3.2 Propagation on lossy tubes and radomes

Shutt $^{22}$ performed some experiments to examine the effect on aberration of radiation scattered by a receiving aerial inside a radome. Some of this work is more relevant to the contents of
Chapter 4 and will be discussed there, but two experiments suggested that an axially guided wave was propagating on the radome.

First, aberration curves (aberration as a function of angle of look) were measured for a large conical fibre glass X band radome at a wavelength of 3.2 cm, using a 6 inch diameter dish aerial. The aberration curves were measured at various axial positions of the aerial inside the radome. It was found that the peak to peak amplitude of the aberration curves varied periodically with axial distance the period being approximately a wavelength, and this variation was superimposed upon one with a period of about six wavelengths. A similar experiment was performed by Sellen and again a dominant period of about one wavelength was observed. The shorter period variation is accounted for in Chapter 4 in terms of multiple reflections (interaction) between the aerial and the radome.

In a second experiment, which will also be discussed further in Chapter 4, Shutt measured the aberration of a radome using two different aerials, the one mentioned above and a specially designed aerial which scattered very little of the radiation incident upon it. The aberration curves measured with the two aerials differ considerably (Fig.3.8) and it is noteworthy that both curves are asymmetrical. Some of the asymmetry may be caused by imperfect construction of the radome, but the author thought it significant that the polar diagrams of the aerials (Fig.4.23) are also asymmetrical. Then the author 'regularised' the aberration curves, to make the aberration an odd function of look angle $\theta_L$ (as it should be in a perfectly symmetrical system) by averaging the two halves of the curve for positive and negative $\theta_L$. It was found that, for the two aberration plots in question, the mean or regularised curves differed most from the
FIG 3.8 ABERRATION CURVES
measured curves at angles of look where the slopes of the polar diagrams were greatest.

This is seen as evidence for the existence of a perturbing-wave guided axially by the radome, because the perturbing plane wave theory (section 2.4.4) shows that the aberration produced by a perturbing plane wave is proportional to the slope of the polar diagram of the aerial. The wave guided by the radome would not be a uniform plane wave of course, but it is shown in section 3.5 that the \( HE_{12} \) surface wave mode on a radome has a similar effect.

The results of the first experiment may also be interpreted as an indication of the presence of an axially guided wave propagating along the radome. Lewis\(^{24} \) suggested in his 'axial wave theory' that radomes might propagate a complex 'waveguide mode' and/or a surface wave which would cause aberration. He showed that if such a wave exists, of guide wavelength \( \lambda \), and the aberration is measured at various axial positions for a fixed angle of look \( \theta_L \), then cyclic variations in aberration of period \( T_1 \) occur, where \( T_1 \) is given by

\[
T_1 = \frac{\lambda_0}{\frac{\lambda_0}{\lambda} - \cos \theta_L}.
\]

(3-15)

The peak aberration caused by an axial wave should occur at \( \theta_L = 10 \) degrees for \( \lambda_0 = 3.2 \) cm for a 6 inch diameter aerial with \( 1 - r^2 \) illumination, according to the perturbing wave theory, and the maximum value of aberration for the radome used due to insertion phase effects also occurs near this angle of look, so the peak amplitude of the aberration curve is approximately the amplitude at 10 degrees angle of look. Inserting \( \theta_L = 10 \) degrees, \( T = 19.6 \) cm (the measured value of the period) and \( \lambda_0 = 3.2 \) cm in
equation (3-15) and solving for \( \lambda \) we find \( \lambda = 3.25 \text{ cm} \). That is, the axial wave is fast.

To investigate axial propagation further, Shutt performed another experiment in which a guided wave was launched on a radome by means of a transmitting horn at the end of a 7.55 inch diameter, 7 foot long dielectric tube. The other end of the tube was joined to a radome, the material, wall thickness and base diameter of which were the same as those of the tube. The material used was resin bonded glass laminate of dielectric constant 4.0 and loss tangent 0.01. A thin Tufnol rod was inserted through a small hole in the nose of the radome and a 1\(\frac{1}{4} \) inch diameter metal disc was attached to it. The disc was inserted axially into the radome and the power reflected back to the horn was plotted as a function of the insertion of the disc. (The power returned from the disc in the absence of the tube was negligible.) A periodic variation was found, and the guide wavelength was found to be a function of position in the radome, becoming longer near the tip and always greater than \( \lambda_0 \), which was 3.244 cm. This wave was thought by Shutt to be of a similar type to that which propagates in conducting waveguides, since the energy returned from the disc varied with frequency in approximately the same way as the reflection coefficient of the tube wall, and the wavelength was close to that of the \( H_{11} \) mode in circular waveguide of equivalent diameter.

The author has investigated this further and the axial wave detected in the experiment with the tube and radome is thought to be the \( HE_{12} \) surface wave, which is fast at \( \lambda_0 = 3.2 \text{ cm} \). On the radome for \( \theta_L \neq 0 \) the axial wave detected could be a combination of the \( HE_{12} \) and others of the near-degenerate \( HE_{n2} \) group.
An axially incident plane wave or a horn with its axis coincident with the tube axis would only launch $HE_{1m}$ and $EH_{1m}$ waves, but a non-axially incident wave would launch other modes as well. The guide wavelength of the $HE_{12}$ mode was measured using the same tube employed by Shutt. The apparatus used in the measurements is shown in Fig.3.9. The output of the crystal detector was plotted as a function of distance of the disc from the horn. The system was in effect a low $Q$ cavity and by moving the disc through a large number of wavelengths the guide wavelength could be determined accurately, since adjacent minima were spaced by $\lambda/2$. The accuracy of this method is limited by imperfect construction of the tube but it was sufficient to show that fast wave propagation probably occurs for a range of frequencies below $f_c$ and slow wave propagation at higher frequencies, although the exact transition point was not determined. ($f_c$ is the cut off frequency for the $HE_{12}$ mode on a loss free tube of the same dimensions and dielectric constant.)

The results obtained are shown in Fig.3.10, where the quantity $\Delta \lambda/\lambda_0 = (\lambda_0 - \lambda)/\lambda_0$ is plotted against $\lambda_0$. Three calculated values are shown and extrapolated to give the approximate cut off wavelength for the loss free case. It will be noticed that the measured value of $\Delta \lambda/\lambda_0$ for $\lambda_0 = 2.95 \text{ cm}$ lies outside the curve for the loss free tube. Arnbak's results suggest that for a lossy rod the wavelength of the slow wave near cut off is slightly reduced relative to the loss free case, which makes $\Delta \lambda/\lambda_0$ larger.

At a frequency well below the cut off value ($\lambda_0 = 3.6 \text{ cm}$) the wave did not propagate, and instead of the regular pattern observed previously a highly irregular plot was obtained. This behaviour again resembles that of the $E_{01}$ mode on the lossy rod.
FIG. 3.9 APPARATUS USED IN STUDIES OF GUIDED WAVES ON A LOSSY TUBE
FIG 310 PROPAGATION OF HE$_{12}$ WAVE ON A LOSSY TUBE
It is suggested that a designer of half wave radomes should, if other considerations allow, choose the dielectric constant such that the $HE_{1m}$ and $EH_{1m}$ modes which can propagate are not too close to cut off. The reason for this is that modes near cut off carry more energy in the air space inside the tube and hence are more likely to cause aberration. The practical significance of loss is that (apart from attenuation of the direct wave transmitted through the radome wall) the frequency range in which these modes could be troublesome is extended. The designer should also bear in mind that aerodynamic heating of the radome in flight may change the dielectric constant and loss tangent so that the guiding behaviour could differ appreciably from that at normal temperatures. However ceramic materials, which are more likely to be used for missiles, have much lower loss tangent than the resin bonded glass fibre laminate used in the experiments described in this section.

3.4 Tapered dielectric waveguides

3.4.1 Theoretical work on linearly tapered dielectric rods

In order to obtain an improved understanding of surface wave propagation on radomes it was considered advisable to try a theoretical analysis. The radomes shown in Chapter 1 do not lend themselves to simple descriptions in any convenient coordinate system. However, since radomes are tapered tubes and it is known that uniform dielectric tubes have similar guiding properties to uniform rods, it was thought that an analysis of propagation on a tapered dielectric rod would reveal the essential features of propagation on tapered tubes. Also behaviour in the tip region of conical radomes should be clarified.
In an attempt to apply the classical method (the separation of variables) the following procedure was adopted. Using spherical polar coordinates space was divided into two regions (Fig.3.11) by the conical surface $\theta = \theta_0$. Region I consists of the solid, loss free dielectric cone of dielectric constant $K$. Region II lies outside the cone and consists of free space. Separating the wave equation in spherical polar coordinates, as was done in the analysis of conical horns presented by Lamont, and in analogous fashion to the procedure used for the dielectric tube (section 3.1) in cylindrical polar coordinates, the following expressions were obtained for an $E_0$ wave (i.e. $H_r = E_\phi = H_\theta = 0$).

In region I

\[
\begin{align*}
E_{r1} &= \left( \frac{A}{j\omega e_0} \right) \left( \nu_1 \right) \frac{r^{\nu_1 + 1}}{r^{3/2}} J_{\nu_1 + \frac{1}{2}}(k_1 r) P^0_{\nu_1} (-\cos \theta) \\
E_{\theta 1} &= \left( - \frac{A}{j\omega e_0} \right) \frac{d}{dr} \left[ r^{1/2} J_{\nu_1 + \frac{1}{2}}(k_1 r) \right] [P^1_{\nu_1} (-\cos \theta)] \\
H_{\phi 1} &= \left( \frac{A}{r^{1/2}} \right) J_{\nu_1 + \frac{1}{2}}(k_1 r) P^1_{\nu_1} (-\cos \theta)
\end{align*}
\]

(3-16a)

and in region II

\[
\begin{align*}
E_{r2} &= \left( \frac{B}{j\omega e_0} \right) \left( \nu_2 \right) \frac{r^{\nu_2 + 1}}{r^{3/2}} J_{\nu_2 + \frac{1}{2}}(k_2 r) P^0_{\nu_2} (\cos \theta) \\
E_{\theta 2} &= \left( - \frac{B}{j\omega e_0} \right) \frac{d}{dr} \left[ r^{1/2} J_{\nu_2 + \frac{1}{2}}(k_2 r) \right] [P^1_{\nu_2} (\cos \theta)] \\
H_{\phi 2} &= \left( \frac{B}{r^{1/2}} \right) J_{\nu_2 + \frac{1}{2}}(k_2 r) P^1_{\nu_2} (\cos \theta)
\end{align*}
\]

(3-16b)

where $k_1 = \frac{2\pi \sqrt{K}}{\lambda_0}$, $k_2 = \frac{2\pi}{\lambda_0}$, and $P^m_{\nu}(\cos \theta)$ is a solution of Legendre's associated equation, of order $m$ and degree $\nu$. 
FIG 3.11 GEOMETRY FOR ANALYSIS OF GUIDED WAVES ON A DIELECTRIC CONE.
The Bessel function of the first kind is used for both regions since both include the origin. All the other solutions of Bessel's equation become infinite at the origin so cannot describe physical fields there. The Legendre associated function \( P^m_v(\cos \theta) \) becomes infinite at \( \theta = \pi \) unless \( v \) is integral and \( P^m_v(-\cos \theta) = P^m_v \cos (\pi - \theta) \) becomes infinite at \( \theta = 0 \) for non-integral \( v \), so the first one is used for region II and the second for region I.

The next step was to try to match tangential field components at the boundary so that \( v_1 \) and \( v_2 \) can be found. However it is often useful in physical problems to consider limiting cases, and if when we try to match \( E_{r1} \) to \( E_{r2} \) at \( \theta = \theta_0 \) we consider what happens when \( r \) is very large, we find that \( E_{r1} \) is proportional to \( r^{-\frac{1}{2}} \cos (k_1 r) \) and \( E_{r2} \) is proportional to \( r^{-\frac{1}{2}} \cos k_2 r \). Since \( k_1 \neq k_2 \) for \( K > 1 \) it is not possible to match the fields over the whole conical surface.

Since the boundary conditions could not be satisfied by a single solution in each region it was decided to try an infinite series of solutions of the type (3-16) in region II and a single solution in region I. No progress was made in this. The difficulty is resolved by the work of Felsen who showed that in order for the wave equation to be separable on a conical boundary of variable surface impedance, for circularly symmetric fields, it is necessary for the surface impedance \( z_s \) to be proportional to \( 1/r \) for \( E_0 \) waves. For \( H_0 \) waves \( z_s \) should be proportional to \( r \). (For \( E_0 \) waves \( z_s = (E_r/H_\phi)|_{\theta=\theta_0} \) and for \( H_0 \) waves \( z_s = (E_\phi/H_r)|_{\theta=\theta_0} \). A good approximation to the specified form of surface impedance variation could be obtained for \( E_0 \) waves by cutting azimuthal grooves in a conducting cone, the depth of the grooves being varied.
from a quarter wavelength near the tip to zero at larger \( r \). When the surface impedance of a dielectric rod (for \( E_0 \) and \( H_0 \) waves) was calculated as a function of diameter it was found that a non-linear relationship exists. It was concluded that the linear model was not of much use for the present problem.

### 3.4.2 The attenuation of axial surface waves due to radiation

#### 3.4.2.1 Approximate theory

Another approach used in obtaining an improved understanding of axial surface wave propagation on radomes was suggested by the fact that two common radome shapes, the optimum secant ogive and the tangent ogive are both generated by rotating circular arcs about a chord. As already mentioned an exact theory exists for the propagation of azimuthal surface waves when a plane guiding surface is bent into a cylinder. It seems reasonable to expect that the wave guiding behaviour of ogival radomes should be indicated by the above theory, in spite of the double curvature, except near the tip. This expectation is based on the following reasoning. The definition of surface impedance \( z_s \) for the \( H_{01} \) mode on a dielectric tube is

\[
z_s = \left( \frac{\phi_3}{H_3} \right) \mid_{\rho = b} = (j \omega \mu_0 / k_3) (H_0^{(1)}(k_3b) / H_0^{(1)}(k_3b)).
\]

If \( \lambda/\lambda_0 \) for a given mode is nearly constant then \( z_s \) also varies slowly. This condition applies for most modes over most of the radome, except near the tip (Figs. 3.5 and 3.7). An ogival radome is therefore approximately similar to the cylindrical reactive surface in the near-uniformity of its surface impedance. It is well known that discontinuities in surface impedance cause a surface wave to radiate. It follows that radiation from a surface wave mode on a radome, for which the surface impedance is almost constant, should be due primarily to the curvature of the guiding surface.
On a uniform plane sheet or a uniform tube the equiphase planes of the field above the guiding surface are parallel. Curvature of the guiding surface gives rise to divergence of the equiphase planes with an outward flow of energy perpendicular to the guiding surface. The double curvature of an ogival radome obviously causes an increase in the power density within the radome wall as a guided wave approaches the radome tip, but this should not affect the validity of the approximation in the region of constant surface impedance.

Barlow and Brown\textsuperscript{27} derived an expression for the attenuation due to radiation from azimuthal waves on reactive cylindrical surfaces. The field components above the guiding surface in cylindrical polar coordinates $\rho, \phi, z$ of a TM wave with no field variation in the $z$ direction are

\[
\begin{align*}
E_\rho &= -j\nu C_1 \exp(-j\nu\phi) \cdot \frac{H^{(2)}_\nu(k\rho)}{(\rho(\sigma + j\omega))} \\
H_z &= C_1 \exp(-j\nu\phi) H^{(2)}_\nu(k\rho) \\
E_\phi &= \left(\frac{\kappa C_1}{\sigma + j\omega}\right) \exp(-j\nu\phi) H^{(2)}_\nu(k\rho)
\end{align*}
\]

where $k^2 = -j\omega(\sigma + j\omega)$, $\mu$, $\varepsilon$ and $\sigma$ are the permeability, permittivity and conductivity of the medium above the guiding surface, and $C_1$ is a constant. The field varies as $\exp(-j\nu\phi)$ for a wave travelling in the $+\phi$ direction and $\nu$ is a complex number. $\nu$ can be found by matching fields above and below the guiding surface and solving the resulting equation.
The radial power density

\[ P_\rho = \text{re}[E_\rho H^*_\phi] \]

\[ = \frac{2|c_1|^2}{\pi \omega \varepsilon_0 \rho_s} \tag{3-18} \]

\( \rho_s \) is the radius of curvature of the cylinder, \( \varepsilon = \varepsilon_0 \) the permittivity of air, \( \nu \) is assumed real, which is a good approximation for small attenuation. The surface impedance \( z_s = -E_\phi / H_\rho \) at \( \rho = \rho_s \). If \( z_s \) is known this gives a transcendental equation in \( \nu \) which is solved using the Liouville approximations for \( J_\nu(x) \) and \( Y_\nu(x) \). The approximations are valid in the range \( x = |\nu| \).

The azimuthal power outside the surface per unit length in the \( z \) direction is

\[ P_\phi = \text{re} \int_{\rho_s}^\infty (-E_\rho H^*_\phi) d\rho , \]

i.e.

\[ P_\phi = \frac{\nu|c_1|^2 |Y_\nu(x)|^2}{2\pi \omega \varepsilon_0 \rho_s} \tag{3-19} \]

where \( x = k_0 \rho_s \), \( k_0 = \omega \mu_0 \varepsilon_0 \), \( u = (2\pi/\lambda_0) \sqrt{(\lambda_0/\lambda)^2 - 1} \).

Hence \( \alpha = \frac{1}{2} \ln (1 + P_\rho / P_\phi) \).

If \( \alpha \), the attenuation coefficient, is small and \( \beta = 2\pi/\lambda \),

\[ \alpha = \frac{2u}{\left\{ \left( \frac{2\pi}{\lambda} \right) \frac{\rho_s}{\lambda} \right\}^2} \]

\[ \left( Y_{\beta \rho_s} \left( k_0 \rho_s \right) \right)^2 \tag{3-20} \]

For the tangent ogive used in the present work, of generating radius 60 cm, using the average value of \( \lambda/\lambda_0 \). for the \( E_{01} \) mode at \( \lambda_0 = 3.2 \) cm, it was found that \( \alpha \) was negligible. The same therefore applies to the optimum secant ogive which has a larger generating radius and to the cone which has an infinite generating radius.
As an axial surface wave propagating from the base towards the tip of a radome experiences very little loss and the tip of the radome radiates in the fashion of a dielectric rod aerial (section 2.4.3) it follows that a plane wave incident upon a radome launches axial surface waves chiefly in the tip region of the radome and not higher up.

3.4.2.2 Experiments

A coaxial launcher for $E_{01}$ axial waves on radomes was constructed. The inner conductor of the coaxial line was placed in contact with the tip of the radome and the outer conductor contacted the radome a short distance above the tip. The radome and launcher axes were in line. A standing wave pattern was formed by placing a large metal sheet in contact with the base of the radome. The whole apparatus was supported, clear of surrounding objects, by threads.

The fields were measured by a tapered waveguide probe which was moved along different generators of the radome, close to the surface. It was intended to deduce the attenuation of the surface wave from measurements of the VSWR at different positions along the generator. The experiment was thwarted by the presence of a number of modes, in spite of careful construction and alignment of the launcher. The presence of several modes was apparent from the variation in spacing of the minima and the different patterns obtained for different generators. The multiple modes were attributed to asymmetry in the radomes, which can lead to coupling between the $E_{01}$, $EH_{11}$, $EH_{21}$ ... waves, as explained in section 3.2.

A secondary application of this experiment would be to measure the dielectric constant of the radome material, but this is also dependent on single mode propagation. Since one can easily
calculate \( \lambda / \lambda_0 \) as a function of \( K \) for a tube of given dimensions measurement of \( \lambda \) and \( \lambda_0 \) enables \( K \) to be found. The radomes used all had a cylindrical portion, so there would be no error due to taper. The advantage of this method is that it is unnecessary to cut a sample from the radome.

3.5 Calculation of aberration due to surface waves

3.5.1 Outline of method

It has been shown that the taper of the radome has only a small effect on surface wave propagation, except near the radome tip. Therefore, for the purpose of calculating the aberration due to surface waves the radome was assumed to be a loss free uniform tube in the vicinity of the aerial aperture. The assumption of loss free dielectric was made because of the difficulties caused by complex propagation constants in the calculation of the fields in the aperture, and in any case the loss tangent of the alumina used in constructing the radomes was believed to be very small (less than \( 10^{-3} \)).

Aberration was calculated for the \( HE_{12} \) mode since this was the only member of the \( HE_{lm} \) or \( EH_{lm} \) groups which was near cut off in the frequency range of interest (\( \lambda_0 = 3.0 \) to 3.4 cm) for the half wave alumina radomes used. It was thought that a mode near cut off would have greater effect than a tightly bound mode because more of the energy is carried inside the tube. As mentioned above, only the \( HE_{1m} \) and \( EH_{1m} \) modes are launched by a plane wave incident axially on the radome, and because the aberration slope usually has its greatest value for zero angle of look this region is of particular interest. The procedure used in calculating aberration was as follows:
(1) The simultaneous equations (3-12) which resulted from the matching of the field components at the tube surfaces, were solved to give seven of the eight $a_n$ and $b_n$ coefficients in terms of the eighth. The eighth coefficient was then found in terms of the power carried by the surface wave.

(2) The field components in the aperture plane were expressed in cartesian form as a function of angle of look, and to these were added the field of a uniform plane wave incident on the aperture. The direction of propagation of the latter was perpendicular to the aperture plane.

(3) The aberration was calculated as a function of angle of look for an ideal amplitude sensing receiver. Aberration curves were calculated for polarisation perpendicular to the plane of offset and relative power of 1% in the surface wave. This was done for $\lambda_0 = 3.0$ cm. The three steps are described in detail below.

3.5.2 Solution of the equations for the field components and power flow calculation

The first task was to solve the characteristic equation (3-13) and determine $\lambda/\lambda_0$ for the appropriate values of $\lambda_0$, dielectric constant and tube radii. This was done using the programme BAT 14 mentioned above, for the HE$_{12}$ wave at $\lambda_0 = 3.0$ cm.

Another programme, BAT 14B, was written by the author and used to calculate the coefficients of the $a_n$ and $b_n$ terms in equation (3-12). The basic flow diagram is shown in Fig.3.12. The correctness of the programme was verified by calculating the coefficients by hand for one set of data and checking that they agreed with the values determined by the computer.
Read dielectric constant, inner radius, outer radius, $\lambda/\lambda_0$.

Set coefficients for polynomial calculation of Bessel functions.

Calculate coefficients of $a_n$ and $b_n$.

Print coefficients of $a_n$ and $b_n$.

Finish.

FIG 3.2 FLOW DIAGRAM FOR PROGRAMME BAT14B
When equation (3-12) was examined carefully it became apparent that the quantities \( a_{n1}, b_{n2}, b_{n3}, \) and \( b_{n4} \) are imaginary. To get these in real form, and hence make the equation suitable for computer solution, it is necessary to set \( a_{n1} = jA_1, b_{n2} = jB_2 \) (\( A_1 \) and \( B_1 \) real) etc. The first seven equations of (3-12) were then each divided by \( b_{n1} \) so that seven equations in seven unknowns \( A_1/b_{n1}, a_2/b_{n1}, a_3/b_{n1}, a_4/b_{n1}, B_2/b_{n1}, B_3/b_{n1}, B_4/b_{n1} \) were obtained. These seven simultaneous equations were then solved using the standard Elliott 803 M2 computer programme for the solution of \( n \) simultaneous linear equations in \( n \) unknowns.

The total power carried by the surface wave is the sum of the powers \( P_1, P_2 \) and \( P_3 \), the subscripts referring to the inside of the tube, the wall and the region outside the tube respectively.

\[
\begin{align*}
P_1 &= \frac{1}{2} \Re \int \int_{\text{region I}} \mathbf{E} \times \mathbf{H}^\ast dS \\
&= \frac{1}{2} \Re \int_{0}^{2\pi} \int_{0}^{a} \left( \rho \mathbf{H}_1^* - \mathbf{E}_1 \mathbf{H}_1^* \right) \rho d\rho d\phi. \tag{3-21}
\end{align*}
\]

On substituting the expressions from equations (3-6) and (3-7), following Gallett\(^{28}\), we find, after performing the \( \phi \) integration,

\[
P_1 = \frac{1}{2} \Re \left\{ \frac{\pi}{2\kappa_1} \int_{0}^{\beta} \left[ \beta \omega \left( a_1^2 \mu_0 - b_1^2 \right) \left( J_1'(k_1 \rho) \right)^2 + \frac{J_2^2(k_1 \rho)}{k_1^2 \rho^2} \right] \right. \\
- \left. j 2a_1 b_1 (\beta^2 + \omega^2 \mu_0 \epsilon_0) J_1'(k_1 \rho) \frac{J_1(k_1 \rho)}{k_1 \rho} \right\} \rho d\rho. \tag{3-22}
\]

where \( \beta = 2\pi/\lambda. \)
Using the relations

\[ J'_1(k_1 \rho) + \frac{J''_1(k_1 \rho)}{k_1^2 \rho^2} = \frac{1}{2} \left[ J'_0(k_1 \rho) + J''_0(k_1 \rho) \right] \tag{3-23} \]

and

\[ J'_1(k_1 \rho) \frac{J'_1(k_1 \rho)}{k_1 \rho} = \frac{1}{2} \left[ J'_0(k_1 \rho) - J''_0(k_1 \rho) \right] \tag{3-24} \]

and then using the integrals

\[
\int_0^a \left[ \frac{B_n^2(kz)}{n-1} + \frac{B_n^2(kz)}{n+1} \right] zdz = a^2 \left[ \frac{B_n^2(ka)}{n} \left( 1 - \frac{n^2}{2ka^2} \right) \right.
\]

\[ + \left( \frac{B'_n(ka)}{ka} \right)^2 + 2 \frac{B_n(ka)}{ka} \frac{B'_n(ka)}{ka} \] \( \cdots \). \tag{3-25} \]

and

\[
\int_0^a \left[ \frac{B_n^2(kz)}{n-1} - \frac{B_n^2(kz)}{n+1} \right] zdz = \frac{2n\hat{b}_n^2(ka)}{k^2} \tag{3-26} \]

where \( B \) denotes any Bessel function, we find that

\[
P_1 = -\frac{\pi a^2}{4k_1^2} \left\{ J_1^2(k_1 a) \beta \omega (\epsilon_0 a_1^2 - \mu_0 b_1^2) - \frac{1}{k_1^2 a^2} \left[ \beta \omega (\epsilon_0 a_1^2 - \mu_0 b_1^2) \right. \right.
\]

\[ - j2a_1 b_1 (\beta^2 + \omega^2 \mu_0 \epsilon_0) \left. \right\} \]

\[ + \beta \omega (\epsilon_0 a_1^2 - \mu_0 b_1^2) \left\{ (J_1(k_1 a))^2 + 2J_1(k_1 a) \frac{J'_1(k_1 a)}{k_1 a} \right\} \]

\( \cdots \). \tag{3-27} \]

The power flow inside the walls of the tube, \( P_2 \), is calculated in a similar fashion. The integration limits are \( \rho = a \) to \( \rho = b \) and the Bessel function of the second kind is present. This necessitates the use of two more relations:
\[ \int_{0}^{a} \left[ B_{n-1}^{(1)}(kz)B_{n-1}^{(2)}(kz) + B_{n+1}^{(1)}(kz)B_{n+1}^{(2)}(kz) \right] dz \]

\[ = a^{2} \left[ B_{n}^{(1)}(ka)B_{n}^{(2)}(ka) \left( 1 - \frac{n^{2}}{k^{2}a^{2}} \right) + B_{n}^{(1)'}(ka)B_{n}^{(2)'}(ka) \right. \]

\[ + \left. \frac{B_{n}^{(1)}(ka)B_{n}^{(2)'}(ka) + B_{n}^{(1)'}(ka)B_{n}^{(2)}(ka)}{ka} \right] \]

\[ (3-28) \]

and

\[ \int_{0}^{a} \left[ B_{n-1}^{(1)}(kz)B_{n-1}^{(2)}(kz) - B_{n+1}^{(1)}(kz)B_{n+1}^{(2)}(kz) \right] dz = \frac{2nB_{n}^{(1)}(ka)B_{n}^{(2)}(ka)}{k^{2}} \]

\[ \ldots (3-29) \]

where the superfixes (1) and (2) denote different types of Bessel functions.
Hence

\[ P_2 = -\frac{\pi}{4k_2} \left[ \beta \omega (\varepsilon_0 a_2^2 - \nu_0 b_2^2) \left\{ b^2 \left[ J_1^2(k_2b) \left( 1 - \frac{1}{2k_2^2} \right) + (J_1'(k_2b))^2 + \frac{2J_1'(k_2b)J_1^1(k_2b)}{k_2b} \right] \\
- a^2 \left[ J_1^2(k_2a) \left( 1 - \frac{1}{2k_2^2} \right) + (J_1'(k_2a))^2 + \frac{2J_1'(k_2a)J_1^1(k_2a)}{k_2a} \right] \right\} \\
+ j2a_2b_2' (\beta^2 + \omega^2 \nu_0 K \varepsilon_0) \{ J_1^2(k_2b) - J_1^2(k_2a) \} \\
+ \beta \omega (\varepsilon_0 a_3^2 - \nu_0 b_3^2) \left\{ b^2 \left[ J_1^2(k_2b) \left( 1 - \frac{1}{2k_2^2} \right) + (Y_1^1(k_2b))^2 + \frac{2Y_1^1(k_2b)Y_1^1(k_2b)}{k_2b} \right] \\
- a^2 \left[ Y_1^2(k_2a) \left( 1 - \frac{1}{2k_2^2} \right) + (Y_1^1(k_2a))^2 + 2Y_1^1(k_2a)Y_1^1(k_2a) \right] \right\} \\
+ \frac{j2a_3b_3}{k_2^2} (\beta^2 + \omega^2 \nu_0 K \varepsilon_0) \{ Y_1^2(k_2b) - Y_1^2(k_2a) \} \\
+ 2\beta \omega (\varepsilon_0 a_2a_3 - \nu_0 b_2b_3) \left\{ b^2 \left[ J_1(k_2b)Y_1(k_2b) \left( 1 - \frac{1}{2k_2^2} \right) + J_1^1(k_2b)Y_1^1(k_2b) \right] \\
+ \frac{1}{k_2b} (J_1(k_2b)Y_1^1(k_2b) + J_1^1(k_2b)Y_1(k_2b)) \right\} \\
- a^2 \left[ J_1(k_2a)Y_1(k_2a) \left( 1 - \frac{1}{2k_2^2} \right) + J_1^1(k_2a)Y_1^1(k_2a) \right] \\
+ \frac{1}{k_2a} (J_1(k_2a)Y_1^1(k_2a) + J_1^1(k_2a)Y_1(k_2a)) \right\} \\
+ j \frac{2}{k_2^2} (a_2b_3 + a_3b_2) (\beta^2 + \omega^2 \nu_0 K \varepsilon_0) \{ J_1^2(k_2b)Y_1(k_2b) - J_1(k_2a)Y_1^1(k_2a) \} \right\} \right] . \] (3-30)
Using similar procedures $P_3$ is found, the range of integration being $\rho = b$ to $\infty$.

$$ P_3 = \frac{\pi b^2}{4k_3^2} \left\{ H_1^2(k_3b) \left\{ \omega \beta (\varepsilon^0 a^2_4 - \mu^0 b^2_4) \right\} 
- \frac{1}{k_3 b^2} \left\{ \beta \omega (\varepsilon^0 a^2_4 - \mu^0 b^2_4) - j2a^4 b^4 (\beta^2 - \omega^2 \mu^0 \varepsilon^0) \right\} 
+ \beta \omega (\varepsilon^0 a^2_4 - \mu^0 b^2_4) \left\{ (H_1'(k_3b))^2 + 2H_0'(k_3b) \frac{H_0^0(k_3b)}{k_3 b} \right\} \right\} $$

(3-31)

where $H_n$ is the Hankel function of the first kind. Throughout the calculation of $P_1$, $P_2$, and $P_3$ the original notation $\alpha_{n1}$, $\beta_{n1}$ etc. has been replaced by $\alpha_1$, $\beta_1$ etc.

Since the numerical values of $\alpha_1$, $\alpha_2$, $\alpha_3$ etc. are all known in terms of $b_1$ we have $P_1$, $P_2$, and $P_3$ in terms of $b_1$. We now let the power in the plane wave incident on the radome be $S$ W/m$^2$, and let the radius of the dish be $r_d$ metres. Since the aberration is to be calculated for the case when 1% of the power incident on the aperture is converted to the $HE_{12}$ surface wave we have

$$ \frac{P_1 + P_2 + P_3}{b_1^2} = 0.01Sr_d^2. $$

(3-32)

The peak electric field in the incident wave, $E_p$ is given by

$$ \frac{jE_p^2}{120\pi} = S. $$

(3-33)

Hence

$$ \frac{P_1 + P_2 + P_3}{b_1^2} = \frac{0.01\pi r_d^2 E_p^2}{240\pi}. $$

(3-34)

Thus we have $b_1$ in terms of $E_p$ and we can therefore find all the components of the surface wave field in terms of $E_p$. 
3.5.3 Determination of the total field in the aperture

The aerial is offset at an angle of look $\psi$ with the radome axis. The plane of offset is the horizontal plane and the axis of rotation is the $Y$ axis (Fig. 3.13).

It is desired to determine the total fields in the aperture when a plane wave is incident upon the aperture normally (or nearly so) and the surface wave fields are also impressed on the aperture. The total vertical electric field $E_Y$ is required as a function of the cartesian aperture coordinates $X$ and $Y$, and of the angle of look $\psi$.

We convert the cylindrical polar coordinates $\rho, \phi, z$ to cartesian coordinates $x, y, z$ as follows:

\[
\begin{align*}
\begin{cases}
x = \rho \cos \phi \\
y = \rho \sin \phi \\
z = z
\end{cases}
\end{align*}
\] (3-35)

The aperture cartesian coordinates are, in terms of $x, y, z$

\[
\begin{align*}
X &= x \cos \psi - z \sin \psi \\
Z &= z \cos \psi + x \sin \psi \\
Y &= y
\end{align*}
\] (3-36)

Hence

\[
\begin{align*}
X &= \rho \cos \phi \cos \psi - z \sin \psi \\
Y &= \rho \sin \phi \\
Z &= z \cos \psi + \rho \cos \phi \sin \psi
\end{align*}
\] (3-37)

We have, for the surface wave contribution to $E_Y$

\[
E_{Y_s} = (E_\rho \sin \phi + E_\phi \cos \phi) \exp(j\beta X \sin \psi) .
\] (3-38)
OF ABERRATION DUE TO HE'S MODE

FIG. 3:13 GEOMETRY FOR CALCULATION

[Diagram with labeled axes and angles]
The expressions for \( E_\rho \) and \( E_\phi \) given above are substituted in the last equation, and finally to obtain \( E_y \) as a function of \( X, Y \) and \( \psi \) we require the relations derived from the above equations:

\[
\cot \phi = \frac{X}{Y (\cos \psi + \sin \psi \tan \psi)} \quad (3-39)
\]

\[
\rho = \sqrt{\frac{X^2}{(\cos \psi + \sin \psi \tan \psi)^2} + Y^2} \quad (3-40)
\]

A vertically polarised plane wave incident on the aperture such that its direction of propagation makes a small angle \( \theta \) with the aerial axis gives rise to a field in the aperture \( E_{yp} \) where

\[
E_{yp} = E_p \exp j \left( \frac{2\pi X}{\lambda_0} \sin \theta + \phi_p \right) \quad (3-41)
\]

where \( \phi_p \) is the relative phase of the plane wave and the surface wave at the centre of the aperture.

The total field \( E_y \) is therefore

\[
E_y = E_{ys} + E_{yp} \quad (3-42)
\]

### 3.5.4 Calculation of aberration

A computer programme was written by one of the author's colleagues (J.H. Blase) to calculate the aberration, given the total aperture field in \( X, Y \) coordinates. The programme calculated the magnitude of the voltage \( V \) at the terminals of the aerial, as a function of \( \theta \), for \( \theta = 0, \pm \frac{1}{4}, \pm \frac{1}{2}, \pm \frac{3}{4} \) and \( \pm 1 \) degrees, using the equation (2-3). This gave an approximation to the value of \( \theta \) which maximised \( |V| \). An iterative method employing progressively smaller \( \theta \) intervals was then used to find the value of \( \theta \) which maximised \( |V| \), to the nearest minute of arc. This value is the
aberration. The whole procedure was carried out for \( \psi = 0 \) to 20 degrees in 2 degree steps, for relative phase \( \alpha = 0, \pi/2 \) and \( \pi \).

Unfortunately the results of these calculations as first published in Ref. 3 were incorrect as an error was made in deriving the expression for \( P_1 \). However, the shape of the curves is correct, and only the ordinate scale is wrong in Ref. 3, because only the relative amplitude of the surface wave field in region I was wrong. Corrected results are shown in Fig.3.14 for a wavelength \( \lambda_0 \) of 3 cm, using three different dish radii, 2.75, 3.0 and 3.25 inches. The inner radius of the tube was 8.9 cm (3.5 inches) and the figure shows the great effect of small changes in the gap between the dish and the tube. This is not surprising if one examines Fig.3.15 which shows how the electric field \( E_\rho \) in the inside of the tube varies along the diameter at \( \phi = \pi/2 \) (the Y axis). It is worthy of note that aerials are usually designed with an amplitude taper or weighting function (e.g. \( 1 - (r/r_D)^2 \) where \( r_D \) is the aerial radius) in order to reduce side lobe amplitude, and this weighting tends to counteract the surface wave amplitude variation, which is in the opposite sense. This means that conventional aerials inside loss free tubes or radomes would be rather inefficient launchers of surface waves.

From the radome designer's point of view, Fig.3.14 suggests that if the radome diameter, dielectric constant and wall thickness are such that a surface wave mode which is near cut off is able to propagate, it should be possible to obtain a reduction in surface wave aberration by reducing the aerial diameter, thereby increasing the gap, at the expense of a small decrease in aerial gain and an increase in beamwidth.
DISH RADIUS = 2.75 IN.
— — DISH RADIUS = 3.0 IN.
— — DISH RADIUS = 3.25 IN.
FREE SPACE WAVELENGTH = 3 cm.
RELATIVE PHASE = 180 DEG.

ANGLE OF LOOK (DEGREES)

ABERRATION (MINUTES)

FIG. 314 H E12 SURFACE WAVE ABERRATION
FIG 3.15 RELATIVE STRENGTH OF ELECTRIC FIELD ON THE DIAMETER $\phi = \pi/2$
It was found that when the relative phase between the surface wave and the main wave is \( \pi/2 \) or \( 3\pi/2 \) the aberration is zero, if it is \( \pi \) the slope at \( \theta_L = 0 \) is negative, and if it is zero this slope is positive. These results are in accord with the perturbing plane wave theory, which is not unexpected as the transverse electric field lines of the surface wave in region I resemble somewhat those of the \( H_{11} \) mode in circular waveguide, i.e. there are no reversals of field direction and the field is approximately an inhomogeneous plane wave.

Let us consider the effect of \( D/\lambda_0 \) on the aberration due to surface waves. Since axial surface waves are launched mainly in the region of the tip, if the size of a radome of given shape, wall thickness and dielectric constant is increased and \( \lambda_0 \) is unchanged then the power converted to surface waves, \( P_1 + P_2 + P_3 \), is unchanged. However the power directly incident on the aperture is increased by the square of the scaling factor. In addition it is known that \( k_1 \) increases as the tube diameter increases (for a given mode, wall thickness and \( \lambda_0 \)). Hence \( P_1 \), the surface wave power in region I at the aperture plane, is smaller and \( P_1/(P_1 + P_2 + P_3) \) is also smaller.

It follows that the aberration due to surface waves decreases more rapidly than \( 1/D^2 \), the other relevant factors being constant. Further, due to the monotonic increase in field amplitude with radial distance and to the tapered illumination of the aerial, the surface wave field produces less effect in a larger aerial.

3.6 Experiments to detect surface wave aberration

3.6.1 Thickening the radome wall in the vicinity of the aerial

An experiment was devised to determine whether or not surface waves were producing appreciable aberration in a tangent ogive alumina
radome. Fig. 3.16 shows the effect of variation of wall thickness of an alumina tube on the guide wavelength of the $HE_{12}$ surface wave when the wall thickness is such that the mode is near cut off and the free space wavelength and inner radius are constant. The ratio of guide wavelength to free space wavelength was calculated using the programme BAT 14 mentioned above and is plotted as a function of $\Delta$ the increase in wall thickness, where the basic wall thickness is 0.579 cm. It is seen that $\lambda/\lambda_0$ changes rapidly with $\Delta$, so the field intensity inside the radome will also be sensitive to small changes in $\Delta$. This follows because the transverse propagation constant in region I is given by

$$k_1 = \left(\frac{2\pi}{\lambda_0}\right)\sqrt{1 - \left(\frac{\lambda_0}{\lambda}\right)^2}$$

so if $\lambda_0/\lambda \approx 1$, $k_1$ varies rapidly with $\lambda_0/\lambda$.

In this experiment the radome wall was made thicker in the vicinity of the aerial, the object being to modify the aberration caused by the surface wave without affecting the aberration due to other causes. It was not practicable to use alumina to obtain the desired increase in wall thickness as this would have necessitated the construction of another radome. A simpler alternative was used, consisting of a layer of polytetrafluoroethylene (PTFE) tape. PTFE is a low loss dielectric, of dielectric constant about 2.5. To simulate a 0.01 cm increase in thickness of alumina it is necessary to use a layer of PTFE of thickness $0.01 \sqrt{8.7/2.5} = 0.018$ cm, because electrical thickness is proportional to the square root of the dielectric constant. This layer increases the magnitude of $k_1$ by 50%. This choice of layer thickness was based on a compromise between obtaining a reasonable increase in $k_1$ and avoiding a large perturbing effect.
FIG 3.16 \( \frac{\lambda}{\lambda_0} \) vs WALL THICKNESS

FIG 3.17 ASYMMETRY FACTOR \( \alpha_z \) vs FREQUENCY
The tape was wound round the radome, starting 2 cm in front of the aerial aperture plane and continuing for the same distance behind it, sufficient layers being used to provide the desired thickness. Aberration curves were then measured, with \( \lambda_0 = 3 \text{ cm} \), with and without the tape and using plane polarisation perpendicular to the plane of offset. No significant change in aberration was detected, the difference between the two curves being equivalent to the reproducibility of the aberration measurements.

This result shows that either the surface wave power is very small or that the wave is in quadrature with the main wave transmitted through the radome.

3.6.2 Variation of aberration with frequency

The aberration of the tangent ogive was measured at 25 MHz intervals in the frequency range 9.85 to 10.1 GHz, using the rotating scan aerial, for plane polarisation parallel to the plane of offset. The object of this was to try to detect changes in aberration which were expected in the region of the cut off of the HE\(_{12}\) surface wave, since \( k_1 \) varies rapidly with frequency (equation (3-43)) when \( \lambda/\lambda_0 \approx 1 \).

It was found that the aberration curves were highly asymmetrical except at frequencies between 9.95 and 10.05 GHz. In Fig.3.17 the quantity \( \alpha_2 \), which is a measure of the asymmetry of an aberration curve, is plotted against frequency. We define \( \alpha_2 \) by the relation

\[
\alpha_2 = \frac{|A_L| - |A_R|}{|A_L| + |A_R|}
\]  

(3-44)

and \( A_L \), \( A_R \) are the peak values of aberration for negative and positive look angles respectively. It is seen that \( \alpha_2 \) is zero
at 9.975 and 10.025 GHz and is small between 9.95 and 10.05 GHz. It will be recalled that in section 3.3.2 it was shown that asymmetry in aberration curves measured with a practical split beam aerial system may be caused by a perturbing wave travelling axially along the radome. It therefore seems likely that some form of axially travelling wave, which is not necessarily a surface wave (from the previous experiment), is present which is approximately in phase quadrature with the main wave transmitted through the radome wall in the frequency range 9.95 to 10.05 GHz and which has some other phase at frequencies just outside this range. $\alpha_2$ would also be zero if the perturbing wave had zero amplitude, but it seems improbable that the amplitude would drop rapidly in this narrow frequency range. On the other hand a rapid variation of relative phase with frequency is plausible because the relative phase depends on the difference between long, unequal path lengths.

Experiments which will be described in Chapter 4 show that the axial wave is not a surface wave but consists of leaky wave and radiation field contributions from the source created by scattering at the radome tip.
4.1 Interaction between an aerial and a plane dielectric sheet

4.1.1 Theory

If one considers either a receiving aerial or a transmitting aerial inside a radome one is led to suspect that interaction (multiple scattering) takes place between the aerial and the radome, modifying the radiation pattern predicted by the insertion diffraction theory.

Taking a transmitting aerial first, regarding the radiation as an angular spectrum of plane waves \(^{31}\), it is apparent that some modification of the radiation pattern must occur when the aerial is placed inside a radome, because no practical radome can have a reflection coefficient of zero and the same insertion phase delay for all the waves in the spectrum. Since the reflection coefficient is finite for some of the component waves in the spectrum some radiation is reflected into the aperture and then scattered by the aerial. The scattered radiation is again partially reflected by the radome. In short, we have multiple scattering between the aerial and the radome. It follows that the radiation pattern (and the aberration) predicted by the insertion diffraction theory would be perturbed by this effect since multiple scattering is ignored in the theory.

In the case of a receiving aerial, in free space it scatters a high proportion of the incident power when a plane wave is incident upon it, even when perfectly matched. The scattered radiation can also be regarded as an angular spectrum of plane waves, usually different from that which exists when the aerial is transmitting. (In general the scatter pattern of an aerial depends
on the direction of arrival and polarisation of the incident plane wave, but for the present purpose we are concerned with axially incident plane waves with polarisation corresponding to that of the aerial.) Again multiple scattering takes place, making the aperture fields different from the values calculated by insertion diffraction theory. Hence, in general, the radiation pattern of the aerial is modified and aberration results.

In order to gain a better understanding of the interaction between an aerial and a dielectric structure and to develop a method of calculating the magnitude of the aberration caused the interaction between an aerial and a plane uniform dielectric sheet of infinite extent was studied. The plane sheet is the simplest form of dielectric structure which can give useful results in this context.

An approximate method was devised for calculating the aberration caused by reflections from a plane sheet when it is placed in front of an aerial. Consider a receiving aerial (e.g. a rotating scan or nutating scan aerial) with a plane wave incident axially from a remote transmitter and a dielectric sheet placed in front of it (Fig.4.1). The plane of the sheet is vertical and the aerial axis is in the horizontal plane. The incident wave is vertically polarised and the sheet is inclined at an angle $\alpha$ to the aerial axis. The scattered radiation in the horizontal plane containing the aerial axis is represented by an angular spectrum of rays. Fig.4.1 shows that only rays leaving the aerial in the angular range $\theta_1$ to $\theta_2$, near the normal to the sheet, are reflected back to the aperture. As the separation of the aerial and the sheet increases the angular range of the rays reflected into the aperture decreases.
FIG. 4.1 INTERACTION OF SCATTERED RADIATION WITH DIELECTRIC SHEET
For moderately large separation of sheet and aerial the rays reflected into the aperture can be approximated by a plane wave, the field strength of which can be determined by finding the mean value of scattered field strength (relative to that of the incident plane wave) in the range \( \theta_1 \) to \( \theta_2 \) from the measured H plane scatter pattern of the aerial. The relative strength of the scattered field had been measured on an arc of known radius so the amplitude at any other radius in the far field could be determined using the inverse square relation between intensity and radial distance.

The strength of the scattered field at the sheet was then multiplied by the reflection coefficient of the sheet. Second and higher order scattering from the aerial and the sheet was neglected as the reflection coefficient of the sheet used in the experiments was small.

The magnitude of the aberration, \( \theta \), produced was then calculated using the perturbing plane wave theory. Equation (2-15) becomes \(^1\), for the 6 inch diameter conical scan paraboloid used in the experiments.

\[
\theta = \frac{24\lambda_0 A \cos \phi_p}{\pi r_d \cos \psi} \frac{J_3(u)}{u^2}
\]  

(4-1)

assuming an aperture illumination law of the form \( 1 - (r/r_d)^2 \) where \( r_d \) is the aerial dish radius \( \psi \) is the angle of arrival of the perturbing wave and

\[
u = \left(\frac{2\pi r_d}{\lambda_0}\right) \sin \psi
\]

(4-2)

It is clear that if the aerial in Fig.4.1 is displaced along its own axis the relative phase of the perturbing wave, \( \phi_p \), will
vary and produce a sinusoidal variation of aberration with position of period $\lambda_0/(2 \sin \alpha)$. The positive maxima of aberration occurs when $\cos \phi_p = 1$ and the negative maxima when $\cos \phi_p = -1$ in equation (4-1).

The validity of the assumption that the energy reflected from the sheet into the aperture can be treated as a plane wave depends on both the accuracy of the ray approximation and on the path length variation among the rays in the bundle which return to the aperture. Let us examine the path length variation first.

When an observer is in the far field of the scatter pattern the radiation appears to emanate from a point source (the phase centre of the aerial). This point source has an image in the sheet which is the virtual source of the spherical wave reflected back to the aerial. In order to calculate the maximum separation of sheet and aerial for which a plane wave is a reasonable approximation to the actual spherical wave we consider the worst case, which for a given separation of sheet and aerial occurs when the plane of the sheet is perpendicular to the aerial axis. In this case the maximum area of wavefront is intercepted by the aerial so the maximum discrepancy exists between the spherical wave and the plane wave. Fig.4.2 shows that the accuracy of the plane wave approximation, for given values of $R$, $r_d$ and $\lambda_0$ depends on $h$, the maximum path difference between the two wavefronts. It is clear that, for uniform plane and spherical waves, if $h$ is very small compared with the wavelength the aperture fields due to the two wavefronts will be very similar. As $h/\lambda_0$ increases the fields at the centre of the aperture become increasingly unequal, and as $h/\lambda_0$ reaches 1/4 the plane wave field at the centre of
FIG 42 GEOMETRY FOR PLANE AND SPHERICAL WAVEFRONTS
the aperture field will be zero when the spherical wave field is at a positive or negative maximum. If \( h = \lambda_0/2 \) the plane wave and spherical wave fields will be in antiphase at the centre of the aperture. It therefore seems reasonable to expect the plane wave approximation to be quite inadequate if \( h/\lambda_0 > 1/4 \). In Fig.4.2 we have

\[
(R - h)^2 + r_d^2 = R^2
\]

therefore

\[
R = \frac{h^2 + r_d^2}{2h}
\]

Since \( h^2 \ll r_d^2 \)

\[
R \approx \frac{r_d^2}{2h} \tag{4-4}
\]

If the maximum value of \( h \) is to be \( \lambda_0/4 \) we finally obtain for \( R_{\min} \), the minimum value of \( R \) for which we can justify the plane wave approximation

\[
R_{\min} = \frac{2r_d^2}{\lambda_0} \tag{4-5}
\]

Since \( R_{\min} \) is the distance of the virtual source the minimum distance from the phase centre of the aerial to the sheet is \( R_{\min}/2 \), i.e. \( r_d^2/\lambda_0 \).

The second factor which affects the validity of the approximate theory used in calculating the aberration due to scattering is the use of the bundle of rays to represent the scattered radiation. As mentioned above, the rays are weighted in amplitude according to the far field scatter pattern (measured 6 feet from the aerial).

The difficulty lies in deciding how close to the aerial an observer can approach without noticing an appreciable variation in
the shape of the scatter pattern. The scatter pattern and polar diagram of the aerial are shown in Fig. 4.3. It can be seen that the 3 dB beamwidth of the former is 10 degrees while that of the polar diagram is 13 degrees. Also the side lobe level of the scatter pattern is appreciably above that of the polar diagram. It is thought that in the scattering by the aerial of an axially incident plane wave multiple scattering takes place between feed and reflector, but a simplified view is that two basic mechanisms are involved:

(1) The reflector focuses radiation towards the feed which then scatters some of this radiation as well as accepting some. The scatter pattern is probably roughly isotropic. Some of this scattered radiation illuminates the reflector giving a relatively narrow main lobe and a high side lobe level compared with the polar diagram.

(2) Some radiation in the incident wave is scattered directly by the feed without reaching the reflector (the aperture blocking effect).

In addition if the transmission line feeding the aerial is mismatched a reflected wave is set up and re-radiation occurs, following the normal aerial theory.

It seems reasonable to expect that the scattering by mechanism (1) is far more important than that due to the second cause, simply on the basis of the relative areas of wavefront intercepted by the dish and by the feed. Re-radiation due to mismatch is of minor significance.

The shape of the main lobe of the scatter pattern suggests that the aperture is uniformly illuminated by energy scattered by
the feed. On the other hand the first side lobe level is only 11 dB below the main lobe maximum, whereas theory indicates that the first side lobe of a uniformly illuminated circular aperture is 17.6 dB below the main lobe. This difference may be due to the displacement of the feed from the axis or to other non-uniformity in the illumination, but it seems likely that the usual value quoted for the boundary of the Fraunhofer region, $8r_d^2/\lambda_0$, may not be applicable here, especially for the scattered radiation outside the main lobe.

4.1.2 Experiments

The experimental work was carried out using the aberration measuring equipment described in the appendix, with the 15 cm diameter rotating scan receiving aerial. A Perspex sheet 1.2 m by 1 m and 0.793 cm thick was mounted vertically in front of the receiving aerial at an angle $\alpha$ to the axis (Fig. 4.4). The aerial was moved along its own axis and the aberration measured as a function of position. The wavelength used was 3.2 cm. The results for two different values of $\alpha$, 61 and 67 degrees, are shown in Figs. 4.5 and 4.6.

To ensure that there were no errors due to diffraction at the edge of the sheet the measurements were repeated with the sheet displaced $\lambda_0/2$ in its own plane. When these were compared with the original results the difference was found to be negligible. The sheet was also tested for uniformity by turning it through 180 degrees about an axis perpendicular to the plane of the sheet, and again repeating the measurements. The result of this was also satisfactory.

It is standard practice in the measurement of aberration curves for radomes to measure the aberration curve first in the
Fig. 4.4 Apparatus for experiment with plane sheet.
FIG. 45  ABERRATION vs POSITION of ANTENNA  $\alpha = 61^\circ$
FIG. 46. ABERRATION VS POSITION OF ANTENNA $\alpha = 67^\circ$
absence of the radome, so that intrinsic errors due to such causes as reflections from surrounding objects can be detected. This was done in the present experiment and it was found that the aberration in the absence of the sheet varied much more slowly and irregularly than that due to the sheet and with approximately one third the amplitude. The curves in Figs. 4.5 and 4.6 were arrived at by subtracting the 'no sheet' curves from the 'with sheet' curves in an attempt to reduce the irregularities produced by spurious effects.

When the sheet was rotated, reducing the angle \( \alpha \) so that the interaction of the scattered radiation at larger angles to the axis could be studied, it was found that scattering by the edge of the sheet had a severe effect, and this masked the interaction effect. A much larger sheet would be needed to achieve this aim.

4.1.3 Discussion

It is seen that the agreement between the calculated and measured magnitude of aberration, and also the period of the curves, in Figs. 4.5 and 4.6 improves as the separation of the aerial and the sheet increases. This is as expected. The separation of feed and sheet is \( d \sin \alpha \), and the experimental results are in good agreement with theory for \( d \gtrsim 3\lambda_0 \). If the phase centre of the scattered radiation is at the centre of the reflector the phase centre is about six wavelengths from the sheet, since the feed is three wavelengths from the reflector centre. This figure is in good agreement with the calculated value of \( r_d^2/\lambda_0 \) which, for the 15 cm diameter reflector used, is also very nearly six wavelengths. The results confirm that reflection effects give rise to aberration. It also appears that the far field scatter pattern is reasonably well maintained for distances substantially less than
the standard $\theta d^2/\lambda_0$ at least for scattered radiation outside the main lobe.

4.1.4 Application to radomes

A brief attempt was made to apply the above method of calculating the aberration due to multiple scattering to the small radomes used in this programme. Tracing scattered rays showed that only rays back scattered from the tip region or rays travelling almost parallel to the aperture plane can be reflected into the aperture for the range of angles of look of interest.

In investigating the back scattering from the nose, measurements were made of the magnitude of the reflected wave set up when the conical radome was placed over a well matched transmitting horn. These showed that the reflection from the tip would cause maximum aberration of only 5 minutes of arc, approximately. The perturbing plane wave theory was used in calculating the aberration. This magnitude of aberration was thought to be small enough for the multiple scattering between the aerial and the front part of the radome to be considered of secondary importance, for the small radomes used in this work.

It is not possible to use the theory employed in the plane sheet work to calculate the aberration caused by the scattered radiation propagating at large angles to the aerial axis. The radome wall is only 0.5 inch from the edge of the reflector and the method is therefore not applicable.

However, the method was successfully employed to explain the result of the experiment, performed by Shutt, which was described in section 3.3.2. It will be recalled that the peak to peak aberration for a 6 inch diameter aerial inside a large, fibre glass,
conical radome was measured as a function of the axial position of the aerial. It was found that the peak to peak aberration varied periodically with a period of about one wavelength, and this variation was superimposed on a longer period variation. The latter was accounted for in terms of an axially guided wave propagating on the radome, while the shorter period variation was shown to be due to reflections between the aerial and the radome as follows. It was assumed that the wall of the radome behaves in a similar manner to a plane sheet. This is reasonable as the scattered radiation in only a limited angular range, mainly near the plane of offset, is involved. It has been shown that the reflections from a plane sheet cause a periodic variation in aberration as the aerial is moved along its own axis towards the sheet, the period being $\lambda_0/(2 \sin \alpha)$ or $\lambda_0/(2 \cos i)$ where $i$ is the angle of incidence on the sheet. For the conical radome the angle of incidence in the plane of offset for the side of the radome towards which the aerial is turned, is given by

$$i = \frac{\pi}{2} - (\theta_L + \theta_c)$$

(4-6)

where $\theta_c$ is the semi-angle of the cone and $\theta_L$ is the angle of look. The cone semi-angle was 18$\frac{1}{2}$ degrees and since the peak value of aberration occurred at $\theta_L = 10$ degrees the period should be $\lambda_0/(2 \cos 61\frac{1}{2} \text{ degrees})$, i.e. very nearly one wavelength. Also when the magnitude of the aberration due to reflections was calculated using the method developed for the plane sheet it was found to be in good agreement with the experimental result.

As both the calculated amplitude and period of the aberration agreed with the experimental results this lends strong support to
the view that reflections between the aerial and the radome were responsible for the shorter period variation in aberration.

4.2 The aberration due to interaction in radomes

4.2.1 Method

Since the method of calculating aberration used in the plane sheet work was not applicable to the radomes of interest an indirect method was tried. The amplitude and phase of the field inside an empty radome, upon which a plane wave was incident, were measured at the plane normally occupied by the aperture of the aerial. Aberration was then calculated from these measured fields and the resulting aberration curves were compared with those measured on the aberration measuring equipment. In general the two curves were dissimilar and the difference was tentatively attributed to aerial-radome interaction.

In the measurement of the aperture plane fields the radome under test was mounted in exactly the same way as it was on the aberration measuring equipment. Absorbing material was placed behind the radome to prevent radiation scattered from this region from reaching the aperture plane, and the fields were measured by means of a small dipole probe with director and reflector. The fields were measured across the centre of the aperture, and on chords at heights of $\pm r_d/2$, where $r_d$ is the aperture radius, for angles of look in the $-20$ degrees to $+20$ degrees range in 2 degree steps.

The aberration was calculated using a computer programme to evaluate equation (2-7), the expression for the aberration detected by an ideal receiver. This expression is a simplification of the more general form, equation (2-6), and is obtained by assuming that the field amplitude gradient in a direction normal to the aperture
is zero. It is not possible to measure the amplitude gradient (see section 4.2.3). It was mentioned in Chapter 2 that aberration curves calculated by this method were in good agreement with aberration curves measured for a half wave wedge radome and a conical radome using a 10 inch diameter rotating scan aerial with a peak sampling phase sensitive detector. A further experiment using a 6 inch diameter aerial in the same conical radome showed good agreement between the measured curve and the curve calculated from the measured fields.

From these results it appears that the procedure for calculating aberration from known aperture fields is sound. It was reasoned that if this is so, any perturbation of the aperture fields caused by reflection of energy scattered from the aerial would show up as a divergence between the aberration curve calculated from measured fields and the measured aberration curve. It was realised that there is at least one flaw in this approach. If an axially propagating wave exists on the radome the amplitude gradient is not zero so the measured fields curves (henceforth abbreviated to MF curves) would not agree in general with the measured curves, even in the absence of reflections.

Aberration was calculated from measured fields for three different shaped radomes, the cone, tangent ogive and optimum secant ogive described in Chapter 1. Two 6 inch diameter aerials were used in measuring aberration, a rotating scan (CS3) and a nutating scan (NH2). These were positioned with the aperture plane at the junction between the cylindrical and tapered portions of the radome. A peak sampling detector was used so that aerial polarisation differences were minimal.
The results are shown in Figs. 4.7 to 4.21. Plane polarization either perpendicular or parallel to the plane of offset was used. In addition to the in-plane aberration curves calculated from measured fields and those measured directly on the aberration measuring equipment, the in-plane aberration curves calculated using insertion diffraction theory are shown. These were obtained by the method described in section 2.3, assuming an ideal receiver with \(1 - r^2/r_d^2\) illumination.

For the tangent ogive and the optimum secant ogive the aberration curves were also measured with a special aerial designed by Shutt, which he loosely called a 'non-reflecting' aerial. This aerial has the very useful property of scattering only a small proportion of the radiation intercepted when a plane wave is incident axially upon it. The principle of operation can be understood with the aid of Fig. 4.22. A wire grid is placed \(\lambda_0/4\) in front of a parabolic reflector, the wires lying at 45° to the incident polarisation. The incident wave can be resolved into components perpendicular and parallel to the grid. The former is unaffected by the grid, and after reflection at the dish its phase on returning through the grid is \(2\pi\) behind that of the incident wave at the grid. The parallel component, however, is totally reflected at the grid so its phase is changed by \(\pi\). On recombining the two components it is evident that the polarisation of the reflected radiation is turned through 90 degrees with respect to the incident polarisation. A feed is placed at the focus of the reflector, its polarisation being matched to that of the focused radiation, i.e. at 90 degrees to the incident wave. Any of the focused radiation which is not accepted by the feed should be absorbed by the resistive cards placed in front of the feed. The
FIG. 4.7 ABERRATION CURVES
ABERRATION (MINUTES)

TANGENT O G I V E
PARALLEL POLARISATION

$\lambda_0 = 3.2 \text{ cm.}$

--- INSERTION DIFFRACTION THEORY.
--- CALCULATED FROM MEASURED FIELDS.
--- MEASURED WITH NH2 AERIAL.
--- MEASURED WITH NON-REFLECTING AERIAL.

FIG 4.8 ABERRATION CURVES
FIG. 4.9 ABERRATION CURVES
TANGENT OGIVE
PERPENDICULAR POLARISATION
$\lambda_0 = 3.0 \text{ cm.}$

Aberration (minutes)

Angle of Look (Degrees)

- Insertion Diffraction Theory.
- Calculated from Measured Fields.
- Measured with NH2 Aerial.
- Measured with Non-Reflecting Aerial.

FIG. 4.10 ABERRATION CURVES
ABERRATION

$(\text{minutes})$

$-40$ to $+40$

TANGENT OGIVE
PERPENDICULAR POLARISATION

$\lambda_0 = 3.2 \text{ cm.}$

FIG. 4.11 ABERRATION CURVES
ABERRATION (minutes)

TANGENT OGIVE
PERPENDICULAR POLARISATION

\( \lambda_0 = 3.4 \text{ cm} \)

ANGLE OF LOOK (DEGREES)

- --- INSERTION DIFFRACTION THEORY.
- --- CALCULATED FROM MEASURED FIELDS
- --- MEASURED WITH NH2 AERIAL.
- --- MEASURED WITH NON REFLECTING AERIAL.

FIG. 4.12: ABERRATION CURVES
Fig. 4.13 Aberration Curves

- Insertion Diffraction Theory
- Calculated from measured fields
- Measured with CS3 aerial (rotating scan)
- Measured with non-reflecting aerial

Optimum Segant Ogive
Parallel Polarisation $\lambda = 3.0$ cm
ABERRATION (MINUTES)

OPTIMUM SECANT OGIVE PARALLEL POLARISATION

\( \lambda_0 = 3.2 \text{ cm.} \)

FIG. 4.14 ABERRATION CURVES
FIG. 4.15: ABERRATION CURVES
OPTIMUM SECANT OGIVE PERPENDICULAR POLARISATION $\chi_0 = 3.0$ cm.

ABERRATION (MINUTES)

ANGLE OF LOOK (DEG.)

-60 - 60

INSERTION DIFFRACTION THEORY.
--- CALCULATED FROM MEASURED FIELDS
--- MEASURED WITH CS3 AERIAL.
--- MEASURED WITH NON-REFLECTING AERIAL

FIG. 4.16 ABERRATION CURVES
OPTIMUM SECANT OGIVE
PERPENDICULAR POLARISATION
\( \lambda_0 = 3.2 \text{ cm} \)

\[ -40 \quad 0 \quad 30 \quad 20 \]

ANGLE OF LOOK (DEGREES)

\[ -40 \quad -20 \quad 0 \quad 20 \]

ABERRATION (MINUTES)

--- INSERTION DIFFRACTION THEORY
--- CALCULATED FROM MEASURED FIELDS
--- MEASURED WITH CS3 AERIAL
--- MEASURED WITH NON REFLECTING AERIAL

FIG. 4.17 ABERRATION CURVES
OPTIMUM SECANT OGIVE
PERPENDICULAR POLARISATION
λ₀=3-4 cm.

ABERRATION (minutes)

ANGLE OF LOOK (degrees)

--- INSERTION DIFFRACTION THEORY
--- CALCULATED FROM MEASURED
   FIELDS.
--- MEASURED WITH CS3 AERIAL
--- MEASURED WITH NON-REFLECTING
   AERIAL.

**FIG. 4.16 ABERRATION CURVES**
CONE PERPENDICULAR POLARISATION

$\lambda_0 = 3.0\,\text{cm.}$

CALCULATED FROM -20-

MEASURED FIELDS.

---

MEASURED WITH CS3 AERIAL

INSERTION DIFFRACTION THEORY.

---

FIG. 4.19 ABERRATION CURVES
FIG. 4.20 ABERRATION CURVES

CONIC PERPENDICULAR POLARISATION
\( \lambda_0 = 3.2 \text{ cm} \)

- Calculated from measured fields.
- Measured with CS3 aerial.
- Insertion diffraction theory.
FIG. 4.21 ABERRATION CURVES
Fig 4.22 Principle of the 'Non-Reflecting Aerial'

(a) Action of Grid and Reflector, View Along Axis

(b) Side View
cards do not absorb the incident radiation as it is polarised perpen-
dicular to the plane of the cards. The principle is obviously only applicable to a nutating feed. The cards and grid were mounted in a light, low loss dielectric foam holder which could be attached to the reflector. The $H$ plane polar diagram and scatter pattern of the aerial, together with those of the ordinary NH2 nutating scan aerial, are shown in Figs. 4.23 and 4.24. It can be seen that the non-reflecting (NR) aerial reduces the scattered radiation by a substantial amount. It was not possible to measure aberration curves for the cone with this aerial since the clearance between the aerial and the radome was inadequate.

4.2.2 Discussion of results

4.2.2.1 Tangent ogive

Fig. 4.7, $\lambda_0 = 3.0$ cm, parallel polarisation. The ID and MF curves agree quite closely up to 10 degrees angle of look and if the two halves of the MF curve are averaged the agreement is good to 15 degrees. The NR aerial gives much better agreement with the MF curve than does the NH2 aerial, suggesting that interaction between the NH2 aerial and the radome is severe.

Fig. 4.8, $\lambda_0 = 3.2$ cm, parallel polarisation. Neither measured curve shows much correlation with the MF curve. Since the ID and MF curves are of opposite signs through almost the whole range of look angles, it appears that some perturbation, such as an axial wave, is present which contributes more aberration than insertion phase effects.

Fig. 4.9, $\lambda_0 = 3.4$ cm, parallel polarisation. The NR aerial gives better agreement with the MF curve, the NH2 aerial agrees better with ID theory. These results may both be fortuitous (see section 4.2.2.4).
Fig.4.10, $\lambda_0 = 3.0$ cm, perpendicular polarisation. The ID and NR curves coincide over the central region. The MF curve is so distorted in the central region that it seems unlikely that interaction alone is responsible for the difference between MF and NH2 curves and that some other perturbation is present.

Fig.4.11, $\lambda_0 = 3.2$ cm, perpendicular polarisation. There is a striking improvement in the agreement between the measured and MF curves when the NR aerial is used. It was noticed for the other polarisation that the ID and MF curves are of opposite sign, and they are also of opposite sign for this polarisation.

Fig.4.12, $\lambda_0 = 3.4$ cm, perpendicular polarisation. This is the only case where the difference between the MF and measured curves is greater for the NR aerial than for the NH2 aerial. It suggests that interaction is of minor importance in this case. There is also a gross divergence between the MF and ID curves, suggesting large perturbing field or unreliability of the ID theory.

4.2.2.2 Optimum secant ogive

Fig.4.13, $\lambda_0 = 3.0$ cm, parallel polarisation. Agreement between the MF and both measured curves is good as far as measurement is possible (due to the limited range of the aberration measuring equipment). It appears that interaction may be of little relative importance, while the wide divergence between ID and MF curves could be caused by an axially propagating wave. The latter contributes as much aberration as do insertion phase effects.

Fig.4.14, $\lambda_0 = 3.2$ cm, parallel polarisation. This result is difficult to interpret as the aberration is fairly small, and the differences between various curves could as well be due to tolerances in radome construction and measurement errors as to interaction or axial waves.
Fig. 4.15, \( \lambda_0 = 3.4 \) cm, parallel polarisation. Interaction does not seem to be significant here as the MF and both measured curves show similar features. The ID curve differs substantially from the others, again suggesting that an axial wave is present which contributes as much aberration as insertion phase effects.

Fig. 4.16, \( \lambda_0 = 3.0 \) cm, perpendicular polarisation. The various curves differ so greatly and the asymmetry is so marked that it is difficult to reach any conclusion. However, all the curves except the ID curve show negative central slope, suggesting that a severe perturbation is present.

Fig. 4.17, \( \lambda_0 = 3.2 \) cm, perpendicular polarisation. The effect of interaction appears to have been reduced by the use of the NR aerial. It is also noticeable at this wavelength that the ID and MF curves have slopes of opposite sign over much of the angular range. The results again suggest the presence of an axial wave which gives aberration roughly equal in magnitude to that calculated by ID theory.

Fig. 4.18, \( \lambda_0 = 3.4 \) cm, perpendicular polarisation. Again no improvement is brought about by using the NR aerial at this wavelength. The MF and ID curves differ completely, suggesting a major perturbation such as an axially guided wave.

4.2.2.3 Cone

Fig. 4.19, \( \lambda_0 = 3.0 \) cm, perpendicular polarisation. The MF and measured curves are in good agreement for the range \(-8\) degrees to \(+8\) degrees. Interaction does not seem to have much effect here. The poor agreement between ID and MF slopes for \( \theta_t = 10 \) to 20 degrees is attributed partly to the high incidence angles and the fact that this is not the design frequency.
Fig. 4.20, $\lambda_0 = 3.2$ cm, perpendicular polarisation. The MF and ID curves are very similar, indicating that the only minor perturbations in the form of tolerances, axial waves, etc. are present, and the difference between measured and MF curves is due chiefly to interaction. It is noticeable that the latter is appreciable for angles of look near zero and for those greater than 12 degrees.

Fig. 4.21, $\lambda_0 = 3.4$ cm, perpendicular polarisation. There is again only a small difference between the measured and the MF curves, which is consistent with most of the other results at this wavelength in showing little effect due to interaction. There appears to be an axial wave of phase such that it reduces the aberration to a value below that calculated by ID theory.

4.2.2.4 General comments

When $\lambda_0 = 3.4$ cm interaction appears to be small in every case except Fig. 4.9. The other four results at 3.4 cm support the hypothesis (section 4.1.4) that interaction is predominantly due to the wide angle scattered radiation incident on the radome near normal incidence. Fig. 4.25 shows the power transmission coefficient versus wavelength at normal incidence for half wavesheets of dielectric constants 4.0, 9.0 and 2.5 designed for 70 degrees incidence angle and wavelength of 3.2 cm. The graph shows that for the high dielectric constant the reflection coefficient is very small (which minimises interaction) at 3.4 cm wavelength. The dielectric constant of the radomes, 8.7, is sufficiently close to the value shown in the graph, 9.0, not to affect the conclusions. These are that radiation scattered at wide angles is more important than radiation scattered near the axis of the aerial in causing
Half wave sheet designed for 70° incidence angle at 3.2 cm wavelength.

FIG. 4.25 POWER REFLECTION COEFFICIENT AT NORMAL INCIDENCE
aberration, and that interaction is not so significant at frequencies below the centre frequency as at higher frequencies.

The results at 3.2 and 3.0 cm show that in those cases where interaction is clearly present (because of the improved agreement between the MF curves and the measured curves when the NR aerial is used) the aberration caused is greater at the shorter wavelength. For example in Figs. 4.7 and 4.10, the results for the tangent ogive at 3.0 cm, the maximum difference between the measured and MF curves is about 35 minutes while for the same radome at 3.2 cm it is about two thirds as much. Since the amplitude reflection coefficient at normal incidence for 3.2 cm is about half as great as that for 3.0 cm this is roughly the order of change in aberration that one would expect.

The result as a whole show that radiation scattered by the aerial and reflected by the radome into the aperture is an important source of aberration especially at mid band and higher frequencies. The mid band case is of most practical importance because the radome would probably only be required to work over a small bandwidth (typically about 3%) and here interaction causes aberration of similar magnitude to that due to variation of phase delay (ID theory).

From the results obtained it is not possible to say that any one of the three radome shapes used is clearly superior to the others in reducing the aberration due to interaction. All three shapes show large aberrations due to the effect. The tangent ogive is inferior to the optimum secant ogive for parallel polarisation and the cone is better than the ogives for perpendicular polarisation. High aberration slopes can be caused, although in some cases the effect is slightly beneficial to the overall performance (Figs. 4.10, 4.12, 4.14, 4.18, 4.19).
4.3 Errors due to axially propagating waves in the aberration calculated from measured fields

4.3.1 Theory

The validity of the procedure of attributing to interaction the difference between the MF and the measured aberration curves is obviously limited by the accuracy of the calculation of the MF curve. As mentioned above, a possible source of error is an axially propagating wave. There is strong evidence showing that axially propagating waves are often present on radomes. The experiments described in section 3.3.2 showed that an axial surface wave was present on a lossy conical radome, and the theory and experiments in section 3.6 showed that an axial wave, not necessarily a surface wave, was propagating on the ceramic tangent ogive at $\lambda_0 = 3.0$ cm. Tricoles examined the radiation pattern of a horn inside a wedge shaped radome and obtained improved agreement between measured and calculated patterns when the field of a cylindrical wave from the vertex was included in the calculation. It was found empirically that the amplitude of the wave was an order lower than that of the main wave. A 60 degree Perspex wedge, 10 inches long and half wave at X band was used. Tricoles also used this approach for a small axially symmetric radome with an aerial diameter of five wavelengths. Adding the field due to a spherical wave, in antiphase with the main wave and of amplitude 0.07, improved the agreement between ray tracing theory and experiment.

It was shown qualitatively in Chapter 2 that the application of ray tracing to the radome tip, including diffracted rays from the vertices, suggests that radiation is scattered by the tip into the aerial aperture. Another line of reasoning gives a similar
result. The main body of the radome is in effect a transparent sheet with only small variations in phase shift over its surface, but in the tip region the scattering is much more complex. It is known that energy is extracted from the incident wave and guided up the radome as surface waves, most of which have no direct effect in producing aberration. Also it seems likely that there is some backscattering. Therefore the tip acts as relatively opaque small portion of an otherwise transparent sheet. It is thought that the scattered radiation from the tip can be described qualitatively in terms of a multipole source (dipole, quadrupole etc.), the magnitude of the various components depending on angle of look, wavelength, shape and dielectric constant. The difficulty of solving the scattering problem, which would quantify the components of the multipole source, has been discussed in Chapter 2.

Some of the radiation from the tip source reaches the aerial directly, some by reflection from the walls. Alternatively these can be viewed as radiation field and leaky waves due to the source at the tip. As the reflections occur mainly at high incidence angles not much energy is transmitted through the walls. For example Fig.4.26 shows the power reflection coefficient at an incidence angle of 80 degrees for a half wave wall designed for 70 degrees incidence at 3.2 cm, as a function of wavelength, for dielectric constant of 2.5, 4.0 and 9.0. The polarisation is perpendicular to the plane of incidence. For parallel polarisation the reflection coefficient is less (for a given angle of incidence) than it is for perpendicular polarisation. The transmission coefficient for parallel polarisation is unity for a loss free sheet of any thickness when the wave is incident at the Brewster angle. This angle is 71 degrees for alumina.
FIG 4.26  POWER REFLECTION COEFFICIENT AT 80 DEGREES ANGLE OF INCIDENCE FOR PERPENDICULAR POLARISATION.
It therefore follows that the energy reaching the aerial after reflection from the walls is reduced if a high dielectric constant material is used, provided that the wall thickness is half wave. That is, a high dielectric constant is desirable if the radome is to be used over a narrow band of frequencies. If broader band performance is required then a lower dielectric constant is preferable in this respect.

If the radome and aerial are increased in size uniformly in all dimensions, but the wavelength and wall thickness remain constant, then the scattering behaviour at the tip is unchanged. Hence the amplitude of the perturbing wave due to direct (i.e. non-reflected) radiation from the tip source decreases as \(1/r_d^2\) or \(1/D^2\). The same is true for the radiation reaching the aperture after reflection from the walls because scaling the radome does not change the angles of incidence. The power received in the main wave (i.e. by transmission through the radome) is proportional to \(r_d^2\) (or \(D^2\)). The maximum aberration caused by a perturbing plane wave is proportional to its relative amplitude and to \(\lambda_0/r_d\) (equation (4-1)). It follows that the aberration caused by scattering at the tip, for a given shape, \(\lambda_0\) and wall thickness, is proportional to \(1/r_d^3\). It cannot be guaranteed, however, that an increase in size by an arbitrary factor \(n\) will result in a reduction of this aberration by a factor \(n^3\) because the aberration also depends on the phases of the perturbing waves, which will not be the same. Also the angle of arrival which maximises aberration depends on \(r_d/\lambda_0\) because the maximum value of \(J_3(u)/u^3\) occurs at \(u = 2.5\). Thus for \(r_d = 2\lambda_0\) the maximum aberration for a given relative amplitude and phase occurs at \(\psi = 9\) degrees but for \(r_d = 5\lambda_0\) the aberration is a
maximum when $\psi = 45$ degrees. If the size is increased gradually the aberration due to scattering at the tip is expected to vary periodically with decreasing amplitude proportional to $1/r_d^2$.

In order to obtain an estimate of the magnitude of the errors caused by axially propagating waves in the calculation of aberration from measured fields, the aberration due to an axial plane wave was calculated by the perturbing plane wave theory. The total fields in the aperture due to the perturbing wave and the main wave were also calculated and these were used as data in the computer programme employed in the calculation of aberration from measured fields. This was done for relative phases of 0, 45 and 90 degrees at the aperture centre. The aberration curves calculated by the two methods are shown in Figs.4.27 to 4.29. The aerial diameter was 6 inches and $1 - (r/r_d)^2$ aperture illumination was assumed, as before. It is clear from these results that neglecting the amplitude gradient term in equation (2-6), as the 'measured fields' programme does, introduces errors. These are largest when the perturbing plane wave is in quadrature.

As it was expected from the physical reasoning outlined above that the axial wave would be more complex than a uniform plane wave, a further test was made in which the effect of two plane waves travelling at angles of ±5 degrees to the radome axis was examined. The aberration was again calculated by both methods. The waves were given equal amplitude and the same phase at the aperture centre. The object was to simulate, in a very approximate manner, the effect of reflections of the energy from the radome walls.

The results are shown in Figs.4.30 to 4.32. In this case the errors are not so large, at least for angles of look up to about
ABERRATION (minutes)

FIG. No. 4.27

ABERRATION (minutes)

FIG. No. 4.28

ABERRATION (minutes)

FIG. No. 4.29

FIGS. 4.27-4.29 ABERRATION OF PERTURBING PLANE WAVE
Figs. 4.30-4.32

Aberration of Two Perturbing Waves

\( \lambda_0 = 3.0 \text{ cm} \)
\( A = 0.1 \)
\( \beta = 0^\circ \text{ DEG.} \)

Absolute Aberration (minutes)

\( \lambda_0 = 3.0 \text{ cm} \)
\( A = 0.1 \)
\( \beta = 45^\circ \text{ DEG.} \)

Absolute Aberration (minutes)

\( \lambda_0 = 3.0 \text{ cm} \)
\( A = 0.1 \)
\( \beta = 90^\circ \text{ DEG.} \)

Absolute Aberration (minutes)
12 degrees. Referring to Fig. 4.7 it can be seen that the difference between the ID and MF curves for the tangent ogive at 3.0 cm, parallel polarisation, is greatest for angles of look in the range 15 to 20 degrees. Fig. 4.32 shows that the error in calculated aberration due to two perturbing waves in quadrature with the main wave increases rapidly for angles of look greater than 12 degrees. It has already been shown that there is an axial wave in quadrature with the main wave for this case, and these results taken together suggest that an axial wave with crossing components, as in the leaky waves, may exist.

4.3.2 Experiments to detect axial waves

Three attempts were made to detect an axial wave. In the first an indirect approach was adopted and the field measuring equipment (appendix) was used to try to measure the amplitude gradient which such a wave would produce. The amplitude gradient is \( \frac{\partial A}{\partial z} \), where \( A \) is relative amplitude with respect to a reference derived from the transmitter and \( z \) is the distance along the aerial axis. The measurements were performed by moving a measuring probe inside an otherwise empty radome illuminated by an external plane wave. The probe was moved in the plane of offset in a direction transverse to the incident wave normal, first in the aperture plane then in steps of 0.1 inch away from the aperture plane towards the transmitter. The successive amplitude plots were then compared. The expected amplitude gradient was of the order of 0.004 per cm (calculated for an axial plane wave of relative amplitude 0.1). The errors and sensitivity of the equipment were such that no conclusive results could be obtained from these measurements.

The second experiment consisted of calculating the vector 'difference field' between the measured and calculated fields
(Fig.4.33) at a number of points in the aperture plane with the radome illuminated by a plane wave as before. In order to do this it was necessary to establish a phase reference by traversing the aperture diameter in the plane of offset, in the absence of the radome. This was necessary because in the calculation of fields in the ID theory the amplitude and phase are determined with respect to their values in the absence of the radome. The radome was then placed in position and the amplitude and phase measured again. It was found that the shape and average slope of the measured phase plots agreed quite well in some cases with the predicted plots, but that large discrepancies in the distance from the reference phase plot existed. These discrepancies resulted in difference fields of very large amplitude which seemed physically unrealistic.

It was thought at first that the phase errors were due to drift or systematic errors in the measuring equipment or to the use of a wrong value of radome dielectric constant. The predicted fields were recalculated using values of 8.8 and 8.6 instead of 8.7 for the dielectric constant, but the overall agreement was not improved. Various attempts were made to overcome drift and improve the accuracy of the measurements, but without success. It was concluded that the basic procedure is unsound since the errors in establishing the reference levels were comparable in magnitude to the difference field.

The third and final attempt to calculate the difference fields was then made. In this case, instead of trying to establish a phase reference, the calculated and measured phase distributions along the diameter of the aperture were superimposed for best fit. This was thought to be justified because the measured plots were of
FIG 433 DETERMINATION OF DIFFERENCE FIELD
similar shape to the calculated ones when $\lambda_0 = 3$ cm and because the perturbation being sought was expected to have fairly small relative amplitude.

The results for the tangent ogive at 3.0 cm wavelength for both parallel and perpendicular polarisation (with respect to the plane of offset) are shown in Figs. 4.34 and 4.35. The difference fields were calculated for look angles of 0, 4 and 8 degrees at 1 inch intervals along the aperture diameter in the plane of offset. An attempt was made to repeat the process at 3.4 cm wavelength but the results obtained were not mutually consistent. The calculated and measured phase plots differed more than at 3 cm, possibly due to a more severe axial perturbation.

Consider the result for $\lambda_0 = 3.0$ cm, parallel polarisation, Fig. 4.34. It is seen that the difference field has a relative phase of about 100 degrees at the aperture centre, agreeing (within experimental error) with the conclusion of section 3.6.2 that an axial wave propagates on the radome of phase approximately 90 degrees with respect to the main wave at the centre of the aperture. The experiment described in section 3.6 failed to show whether or not the wave was a surface wave. Comparison of the amplitude of the difference field across the aperture diameter (Fig. 4.34, 0 degree angle of look) with the distribution for the \( \text{HE}_{12} \) surface wave (Fig. 3.15) shows conclusively that the wave is not the \( \text{HE}_{12} \) surface wave (which is the only one near cut off set up for $\theta_L = 0$ degree at this frequency) because this wave has decaying field amplitude in a direction perpendicular to the guiding surface. The surface wave may be present, but if so it is of such small relative amplitude that it is masked by the other effect.
FIG. 4.34 DIFFERENCE FIELDS FOR TANGENT OGIVE $\gamma=3\text{cm}$, PARALLEL POLARISATION
FIG. 4.35  DIFFERENCE FIELDS FOR  
TANGENT OGIVE, $\lambda = 3$ cm,  
PERPENDICULAR POLARISATION
It was realised that the procedure of superimposing measured and calculated phase curves to obtain a 'best fit' is somewhat arbitrary. As a check various other fits were tried, by displacing the measured phase ±5, ±10 and ±20 degrees from the 'best fit' position used in deriving Fig.4.34a. It was found that if the displacement exceeded 5 degrees either way physically unrealistic results were obtained in that the difference field had excessive amplitude or showed an asymmetrical phase distribution. For displacements of ±5 degrees the difference field was not very different from Fig.4.34a - it showed a maximum on the axis and a nearly plane wave front with phase of very nearly 100 degrees. It was concluded that an axially guided wave exists which is not a surface wave.

Since the tip acts as a source and the total field near a dielectric guiding structure excited by a source consists of three terms, surface waves, leaky waves and radiation field, it is justifiable to think of the axial wave as, in general, the sum of all three terms. However the surface wave term is of secondary importance for these alumina radomes.

It was intended to repeat the experiment of section 3.6.1 at other wavelengths close to 3.0 cm, as a further check on the nature of the axial wave. Fig.3.18 suggests that the axial wave is not in quadrature if the wavelength is changed by about 1% from 3.0 cm. Unfortunately it was not possible to do this experiment as the radome was destroyed accidentally.

Let us consider now Fig.4.35a, ($\theta_L = 0$ degrees). This represents the same circumstances as Fig.4.34a, in spite of the difference in polarisation, because the radome is axially symmetric and the plane wave is incident axially on the radome. To obtain the results of Fig.4.34a the measuring probe was moved across the diameter parallel to the electric field lines. For Fig.4.35a it was moved perpendicular to the electric field lines. At the centre of the aperture the difference fields should therefore be the same, and in fact the two figures show that both amplitudes and phases agree within a few per cent.
Both the results for $\theta_L = 0$ degrees show that the axial wave has maximum amplitude on the axis, decreasing towards the edges of the aperture. This is consistent with the suggested mechanism of a source at the tip. The radome is axially symmetric, so radiation from a source at the tip is reflected from the walls and travels towards the axis. This may explain the observed maximum on the axis.

The amplitude and phase of the axial wave shown in Figs. 4.34 and 4.35 can be used in calculating the aberration produced if the axial wave is assumed to be a uniform plane wave and the perturbing plane wave theory is applied.

In Fig. 4.35a the mean amplitude is 0.18, and this gives an aberration slope of $-1.5$ minutes/degree at $\theta_L = 0$ degrees. The mean value of the difference between the ID and NR curves (Fig. 4.10) also has a slope of $-1.2$ minutes/degree. At $\theta_L = 4$ degrees the amplitude and phase are such that aberration of $-0.5$ minutes should occur. This is too small to allow meaningful comparison with the experimental results. At $\theta_L = 8$ degrees the amplitude and phase of Fig. 4.35c indicate an aberration of $-13$ minutes but the mean difference between the NR and ID curves is only $-5$ minutes.

For $\theta_L = 0$ degrees the axial wave of Fig. 4.34a gives a central slope of $-1.2$ minutes/degree, and the central slope of the curve obtained by taking the mean difference between NR and ID curves (Fig. 4.7) is $-1.25$ minutes/degree. At $\theta_L = 4$ degrees the calculated axial wave aberration is $-4$ minutes which does not agree with difference between NR and ID curves (+2 minutes mean).

Finally at $\theta_L = 8$ degrees, using the amplitude and phase of Fig. 4.34c, the calculated axial wave aberration is $-2.5$ minutes but the difference between NR and ID curves is $-5.5$ minutes.
The above results show that the axial wave central aberration slope can be predicted from the difference field obtained at \( \theta_L = 0 \) degree. The same approach is less successful for \( \theta_L = 4 \) and 8 degrees, possibly because the asymmetrical excitation of the radome tip gives rise to more reflected energy from the radome walls, making the assumption of a plane axial wave very inaccurate.

4.4 The effect of radome size and dielectric constant on interaction

It is interesting, and relevant to radome design, to consider the effect of radome and aerial size (i.e. \( r_d/\lambda_0 \)) on the aberration produced by interaction. We will deal with only the reflector type of aerial with a front feed, as used in the present work.

Consider the two mechanisms of scattering by the aerial postulated in section 4.1.1. The energy scattered directly by the feed will be a function of the type of feed, but will be reasonably constant irrespective of aerial dimensions. The proportion of the total energy incident on the aerial which is thus scattered therefore decreases as \( r_d/\lambda_0 \) increases.

Some of the focused radiation which is scattered by the feed returns to the reflector and some of it leaves the system in other directions. That proportion which is returned to the reflector is assumed to illuminate the reflector uniformly, because this accounts for the narrow main beam of the scatter pattern (Fig.4.3). Only the scattered radiation leaving the aerial at large angles to the axis contributes to aberration when the aerial is inside a radome. Provided that \( \lambda_0 \) and the focal length to diameter ratio are constant, and the same type of feed is used, the proportion of the incident energy contributing to aberration should be independent of \( r_d/\lambda_0 \). This is so because the angle of the cone of radiation leaving the reflector is constant for a given \( f \) number.
The above argument neglects scattering effects at the edge of the reflector. If these are significant they would have relative importance proportional to $1/r_d$ because the receiving cross-section is proportional to $r_d^2$ and the edge scattering effect varies proportionally to circumference, i.e. to $r_d$.

It follows that the total power in perturbing waves arriving at large angles to the axis decreases slowly as $r_d/\lambda_0$ increases. Let us now examine equation (4-1). Since the effects of interaction are most serious for wide angle scattered radiation $\psi > 30$ degrees, say, and for a practical system $r_d/\lambda_0 > 2.5$. Hence $u > 7.5$. If we use the large argument approximation for $J_3(u)$ the error is only 2% for $u = 10$ and about 10% for $u = 7.5$, so it is not grossly inaccurate. Equation (4-1) becomes

$$\theta = \left(\frac{\lambda_0}{r_d}\right)^{7/2} \frac{6A \cos \phi \cos \psi}{\pi^4 \sin \psi} \cos \left[\left(\frac{2\pi r_d}{\lambda_0}\right) \sin \psi - \frac{7\pi}{4}\right]. \quad (4-7)$$

Since the mean value of $A$ for the perturbing wave decreases as $r_d/\lambda_0$ increases it follows that the aberration due to aerial-radome interaction diminishes with increasing radome and aerial size, at least as rapidly as $(\lambda_0/r_d)^{7/2}$.

It is also of interest to consider the effect of dielectric constant on the aberration caused by reflection of wide angle scattered radiation. This radiation is incident on the radome at incidence angles near zero. The graphs of power reflection coefficient versus wavelength for a plane sheet, Fig. 4.25 (which were discussed in section 4.2.2.4) show clearly that a high dielectric constant is preferable to a small one, for the type of radome considered, except at the high frequency end of the band.
5.1 Summary of the results of Chapters 1 to 4

In Chapter 1 the conflicting requirements which have to be met by radomes for homing missiles were outlined and the harmful effect that excessive radome aberration can have on missile performance was described. The aims of the author's research, viz. to obtain an improved understanding of the electromagnetic aspects of aberration, were stated to be part of a wider programme, the ultimate object of which was to formulate a method of designing small radomes of improved performance. The research has been concentrated mainly on three shapes of small X band alumina radomes in a 12.5% frequency band centred on 9.375 GHz, the frequency at which the wall thickness was half wave for a 70 degrees incidence angle.

In Chapter 2 the insertion diffraction theory was described. The basic method is outlined (section 2.2) for a non-scanning aerial but it could be applied to a scanning aerial by calculating the aerial polar diagram for each position of the feed in the plane of interest and then determining the angle at which the two polar diagrams intersect. However this is a lengthy procedure compared with the use of the ideal receiver model. An analysis was quoted which shows the ideal receiver to be an adequate model for the receiving systems used in the research, the rotating and nutating forms of the conical scan principle. Experimental results were shown which justified the use of the ideal receiver model for a 10 inch diameter aerial in a 4 foot long 12 inch diameter X band half wave radome. A similar result had been obtained for a 6 inch diameter aerial in the same radome. The ideal receiver model was therefore employed in the calculation of aberration from the
aperture fields when the 6 inch diameter aerial was used with the small ceramic radomes.

The expression for the aberration (equation (2-7)) when the amplitude gradient is zero, using an ideal receiver, shows that for a given radome the aberration is proportional to wavelength. This is true if the insertion diffraction theory is applicable, i.e. the aperture field has reasonably small phase variations. These conditions exist for large half wave radomes, especially if the range of incidence angles is not too great (i.e. the fineness ratio is less than 3) and the dielectric constant is high. Clearly the aberration decreases as $\lambda_0/D$ under these conditions. In support of this conclusion aberration calculated by the insertion diffraction theory is shown to be in reasonably good agreement with measured aberration for a 12 inch diameter tangent ogive. The theory is inadequate for a 7 inch diameter tangent ogive of the same fineness ratio and wall thickness over the same frequency band (Fig.2.5).

There are two steps in calculating aberration by the insertion diffraction theory, first the calculation of the fields in the aperture and then determination of aberration using this aperture distribution. Having decided that the procedure used in the second step is reasonably accurate it was decided to investigate possible reasons for the failure correctly to predict the aperture fields.

The assumption of local 'plane wave plane sheet' transmission through the curved radome wall is at first sight of doubtful validity, especially for high incidence angles. The correction formula derived by Barrar was applied to several radome shapes. It was found that the formula gave physically unrealistic results
for a radome of large curvature. For the shapes of most practical interest it indicated only a small change in the aberration calculated by the insertion diffraction theory. It was therefore decided not to investigate the subject further.

The ray tracing used in the insertion diffraction theory is of questionable validity in the region of the tip of the radome as the dimensions are comparable with a wavelength. The more advanced ray theory of Keller's geometrical theory of diffraction, which permits the scattering behaviour of some complex objects to be determined, cannot be used to obtain quantitative results because no solution to the canonical problem (scattering by a dielectric cone) is available. However if one carries out ray tracing at the radome tip and includes diffracted rays at the two vertices (Fig. 2.8) the results suggest that there is back-scattering, trapping of energy (i.e. surface wave launching), a refractive shift and that there is forward-scattering of energy into the aperture.

The results in Chapters 3 and 4 show that surface wave launching does occur at the tip and that radiation scattered by the tip into the aperture is a cause of aberration, so the ray tracing gives a qualitative indication of the scattering process.

Physical reasoning suggested that ray trapping by the tip could be an important feature of scattering by the tip, with the radome acting as a dielectric tube aerial. The effective cross-section of a typical tapered dielectric rod was calculated and found to be four times greater than the physical cross-section of the aerial. This aerial is not greatly different from the tip of a conical radome. It follows that when a plane wave is incident on the radome, near axial incidence, a
significant fraction of the incident power could be trapped in surface waves because the receiving cross-section is $\lambda_0^2/4\pi$ times the gain. If the gain is only 3 dB this means that the cross-section is $2 \text{ cm}^2$, about 1% of the area of the main aperture.

Guided waves, radiation scattered by the tip into the aperture and multiple scattering between the aerial and the radome are all in effect perturbing signals, i.e. they are small amplitude waves arriving from directions other than that of the main or wanted wave. It would therefore be useful to have an understanding of the effect of a basic form of perturbing wave. This is supplied by the perturbing plane wave theory which gives the aberration of a perturbing wave of small amplitude in an ideal amplitude sensing direction finding receiver.

In Chapter 3 the waves guided by radomes were studied and their effect on aberration investigated. Since a radome is a tapered dielectric tube, usually of uniform wall thickness, a start was made by examining the propagation of axial surface waves on uniform dielectric tubes. First the theory of axial surface waves of general order was summarised and the behaviour of azimuthal and whispering gallery surface waves was described.

Azimuthal waves are not significant in the present work because they are only launched when a wave is incident at 90 degrees to the radome axis. The whispering gallery modes were shown by physical reasoning to be of minor importance.

Solution of the characteristic equation for axial surface waves on a loss free uniform tube, first for $n = 1$ and then for $n = 1$ to 5 inclusive, showed two interesting results. It was found that $\lambda/\lambda_0$ for a given mode depends mainly on the wall thickness of the tube and varies slowly with tube diameter, provided this is more than two or
three wavelengths, even for a mode close to cut off. The significance of this result is that, for a gently tapered tube of uniform wall thickness (such as a radome) the taper forms only a small perturbation. The other interesting result was obtained with the second computer programme. This showed that neighbouring modes of a given class, e.g. EH$_{n1}$ ($n = 1, 2, 3, \ldots$) have nearly equal guide wavelength on a given tube at the same frequency, although the higher $n$ the nearer the mode is to cut off. The near degeneracy of neighbouring modes indicates that circumferential perturbations (e.g. a circumferential taper or a departure from circular cross-section) could cause mode conversion. This conclusion was supported by the result of an experiment in which the attempted launching of the E$_{01}$ mode on a radome was frustrated by the presence of several other modes of nearly the same wavelength.

The effect of loss on the propagation of surface waves on radomes is of interest because all real radomes have finite loss. However ceramic radomes are greatly superior in this respect to resin bonded glass fibre radomes. Barlow and Karbowiak$^{20}$ and Arnbak$^{21}$ showed that one effect of loss on the propagation of E$_{01}$ surface waves on dielectric rods is to make surface waves with $\lambda/\lambda_0 > 1$ possible. It has been found that a similar effect occurs for HE modes on a lossy tube. The author suggested an interpretation of the results of Shutt, showing that the axially guided wave detected was the HE$_{12}$ surface wave. Measurement of $\lambda/\lambda_0$ for the mode on a uniform tube of the same wall thickness as the radome indicated that the wave could be fast or slow, depending on whether or not the frequency was above or below the cut off.
frequency for a loss free tube of the same dimensions and dielectric constant.

An analysis of surface propagation on a linearly tapered dielectric rod was attempted as it was thought that this would lead to an improved understanding of the propagation and scattering behaviour in the tip region of a radome. It was later found that the method of separation of variables is not applicable because the boundary conditions are not appropriate.

It was mentioned above that $\lambda/\lambda_0$ (and hence the surface impedance) varies slowly with tube diameter. This was used as justification for the application of the theory of azimuthal surface waves to the calculation of the attenuation due to radiation experienced by axial surface waves on the tangent ogive radome. Having found that it was small for this radome then it was reasoned that the effect must be small for both the optimum secant and the cone since both are ogives of larger generating radius than the tangent ogive. It follows that a plane wave incident on a radome launches axial surface waves mainly in the tip region and not significantly elsewhere.

Having shown that the taper of a radome has only a small effect on propagation except near the tip the radome was assumed (for the purpose of calculating aberration) to be a uniform loss free tube in the vicinity of the aperture. The $HE_{12}$ mode is near cut off, for the radomes used, at a wavelength of 3.0 cm and this mode would be launched by a plane wave incident axially on the radome. The aberration slope of radomes is often greatest at $\theta_L = 0$ degrees so this is an appropriate mode to study.
The procedure used in calculating aberration involved three steps:

(1) Solution of equation (3-12) and evaluation of the power flow to give the field components of the surface wave.

(2) Determination of the total aperture field in cartesian coordinates.

(3) Calculation of the aberration using an iterative procedure which turned the aerial to maximise the received signal.

The aberration was calculated for $\theta_L = 0$ to 20 degrees at 3.0 cm wavelength, for three different aerial diameters. It was found that the size of the gap between the aerial and the radome affected the aberration substantially. The implication of this for radome design is that if a surface wave near cut off is so strongly excited as to cause appreciable aberration then a small reduction should be made in aerial diameter, if possible. This would entail a small reduction in aerial gain and an increase in beamwidth, but the reduction in aberration slope might make this worth doing.

The effect of varying the relative phase of the surface wave was in agreement with the perturbing plane wave theory, that is the aberration was zero when the wave was 90 or 270 degrees out of phase and the central slope was negative for relative phase of 180 degrees. It was shown that the aberration due to surface waves varies more rapidly than $1/D^2$ for a radome of given shape, dielectric constant, $\lambda_0$ and wall thickness.

Two experiments were performed to try to detect aberration due to surface waves at 3.0 cm wavelength using the tangent ogive radome. The first involved modification of the surface wave propagation in the vicinity of the aerial. The measured aberration curve
was then compared with that of the unmodified radome. No change in aberration could be detected and this suggested that either the surface wave power was very small or the surface wave was present which was 90 or 270 degrees out of phase with the main wave.

In the second experiment aberration curves were measured at 25 MHz frequency intervals, in the hope of seeing a marked effect on the aberration due to the rapid variation of $k_1$ with frequency. The hoped for result was not obtained, but it was noticed that the aberration curves were asymmetrical except in a narrow frequency band near 10 GHz. This region of symmetry was attributed to an axially propagating wave in phase quadrature with the main wave. It appeared at first that the first experiment had failed to detect surface wave aberration because the surface wave was in phase quadrature. This interpretation was shown to be incorrect when the field of the axial perturbing wave was examined (section 4.2.3.2) and found to have the wrong radial field variation for a surface wave.

In Chapter 4 it was shown by physical reasoning that multiple scattering is bound to take place between an aerial and any real radome. The simplest structure which can yield any useful information in this respect is an infinite plane dielectric sheet of uniform thickness. A method of calculating the aberration due to interaction between an aerial and a plane sheet was described and it was shown that the magnitude of the aberration, and its periodic variation, could be predicted with fair accuracy for separation greater than about three wavelengths. The mechanism of scattering by the aerial was discussed.
Ray tracing, as used in the above work, was applied to the radiation scattered by the aerial inside a small radome. It was found that only radiation propagating towards the tip of the radome and radiation travelling almost parallel to the aperture plane could be reflected back into the aperture. It was shown experimentally that the former is of secondary importance. The method used in the work on the plane sheet was not applicable to the wide angle scattered radiation for small radomes because the separation of the radome and the aerial was too small. It was possible however to employ the procedure used to calculate aberration due to a plane sheet to explain the results obtained using a 6 inch diameter aerial inside a large conical radome.

In the method used to estimate the aberration caused by interaction in small radomes the fields inside an empty radome illuminated by a plane wave were measured, at the plane normally occupied by the aerial, using a small probe. The aberration was then calculated, using the same method as before, assuming that the measured field distribution was present in the aperture of the aerial. Thus the fields incident on the aperture were exactly the same as in the real radome, except for the absence of perturbing fields due to interaction. (The very small interaction between the probe and the radome was neglected.) The aberration was then measured using a nutating scan or rotating scan aerial. The difference between the measured and calculated aberration can be attributed to interaction provided that the procedure for calculating aberration from the aperture field distribution is reliable. The procedure was thought to be justified, on the basis of the experiment referred to in section 2.3.5, but it could be upset by the presence of an axially propagating wave. This is because the
wave would set up a finite amplitude gradient and in calculating the aberration from the measured fields equation (2-7) is used, in which the amplitude gradient is assumed to be zero. It is difficult to apply the more general expression, equation (2-6), because of the practical difficulty of measuring the amplitude gradient.

The measurements and calculations described above were carried out for the cone, tangent ogive and optimum secant ogive radomes at 3.0, 3.2 and 3.4 cm wavelength for polarisation both perpendicular and parallel to the plane of offset. In addition to the use of the ordinary aerials, aberration curves were also measured for the optimum secant and the tangent ogive using Shutt's 'non-reflecting' aerial. This radome scatters only a small fraction of the total energy when a plane wave is incident upon it.

The results supported the view, derived from tracing rays, that radiation scattered at large angles to the aerial axis and incident upon the radome near normal incidence is of more importance than the radiation scattered towards the tip of the radome. It was also found that the aberration due to interaction is less at frequencies below the centre frequency than at higher frequencies. The aberration at the centre and at the high frequency end of the band is of comparable magnitude to that brought about by variation of insertion phase delay over the radome, and none of the three shapes studied appears superior in minimising interaction.

The errors in the calculation of aberration due to axially propagating waves were investigated. An alternative line of reasoning concerning scattering at the tip of the radome gives the same result as the ray tracing of Chapter 2, i.e. that the tip scatters radiation into the aperture. It is argued that a high dielectric constant minimises the amount of energy from the tip
source which arrives at the aerial after reflection from the radome walls. This is true for a narrow band half wave radome. A lower dielectric constant would be preferable for reducing the reflected energy over a broad band. It is also shown that the aberration caused by scattering at the tip of a radome of given wall thickness and shape at a fixed wavelength is proportional to $1/r_d^2$ or $1/D^2$, provided that most of the scattered radiation propagates directly to the aperture.

The aberration due to an axial plane wave was calculated using the 'measured fields' computer programme and the result compared with the exact value obtained from perturbing plane wave theory. It was found that the error was small when the perturbing wave was in phase with the main wave and large when it was 90 degrees out of phase. The error introduced by the programme was found to be small when the aberration due to two perturbing plane waves at $\pm 5$ degrees to the radome axis was calculated, except for relative phase of 90 degrees and $\theta_L > 12$ degrees. The two perturbing plane waves were thought to give a better approximation than a single plane wave to the axial perturbing wave on a radome. Thus the improved agreement with the exact theory for the two waves gave reassurance about the validity of the programme.

A method was devised to determine the 'difference field', i.e. the field attributed to an axial perturbing wave launched by scattering at the radome tip, for the tangent ogive radome at 3.0 cm wavelength. It was shown that the axial perturbing wave is not a surface wave and that it is about 90 degrees out of phase with the main wave. The latter result agrees with the interpretation of the experiment of section 3.6.2. The central slope of the
aberration due to the axial wave was predicted accurately, but for \( \theta_L > 0 \) degree the simple approach used was not successful.

It was shown, by consideration of the mechanism of scattering by the aerial, that the proportion of the incident radiation which is scattered at large angles to the axis of the aerial decreases as \( \frac{r_d}{\lambda_0} \) increases. That is, the mean amplitude of the perturbing waves reflected into the aperture by the radome decreases. Further, examination of the expression for the aberration due to a perturbing plane wave of given relative amplitude, propagating at a large angle to the aerial axis, showed that the aberration is proportional to \( (\lambda_0/r_d)^{7/2} \). It follows that the aberration due to interaction (for a given shape of radome, \( d/\lambda_0 \) and dielectric constant) decreases more rapidly than \( (\lambda_0/r_d)^{7/2} \).

It was also shown that a high dielectric constant is preferable to a low one for minimising the aberration due to interaction in the type of radome considered, except at the high frequency end of the band.

5.2 Conclusions

It has been demonstrated that the insertion diffraction theory is fairly reliable for the calculation of aberration for large radomes but unsatisfactory for those of small size, i.e. about \( 5\lambda_0 \) diameter. The inadequacy of the theory has been found to be mainly due to the failure to predict the fields in the receiving aerial aperture with sufficient accuracy. The reason for this is the presence of perturbing waves which modify the predicted fields in the aperture. These perturbing waves arise from three causes:

(1) Scattering at the tip of the radome. This gives rise to an effective source at the tip of the radome. Some of the radiation from this source propagates directly to the aperture and some reaches the aperture after reflection from the walls. These two components
can also be regarded as the radiation field and leaky wave constituents of the total field of the source. It was shown that the aberration due to scattering at the tip, for a given wavelength, shape and wall thickness, is proportional to $1/r_d^3$ or $1/D^3$. A high dielectric constant is desirable for a radome which is to be used over a narrow frequency band because the energy reaching the aerial after reflection from the walls is thereby reduced.

(2) Surface waves. These are launched mainly near the tip of the radome and then guided along the radome to the aperture plane. Surface waves are only significant if the radome shape, $d/\lambda_0$, and dielectric constant are such that a mode near cut off is strongly excited. If the dielectric material has appreciable loss, so that a fast surface wave can propagate, experimental evidence suggests that appreciable aberration can be caused. The aberration due to surface waves for a given radome shape dielectric constant, $d/\lambda_0$ and $r_d/D$ decreases more rapidly than $1/d^2$. The aberration (on a loss free radome) is very sensitive to the size of the gap between the edge of the aerial and the radome wall.

(3) Aerial radome interaction (multiple scattering). It was shown that, for the types of radome studied, aberration is caused by radiation scattered at large angles to the receiving aerial's axis. A high dielectric constant minimises the effect for radomes of moderate bandwidth. The aberration caused (for a given radome shape, $K$ and $d/\lambda_0$) decreases at least as rapidly as $(\lambda_0/D)^{7/2}$.

It was found that the axial perturbing wave due to scattering at the tip was the most serious of the three effects and it gave aberration up to approximately twice that caused by insertion phase variation in the small ceramic radomes used in this research.
Interaction gave aberration of similar magnitude to insertion phase effects while surface wave aberration was too small to detect on the ceramic radomes used.

As mentioned in Chapter 2 the aberration caused by variation of insertion phase delay (IPD) over the radome is proportional to $\lambda_0/D$. It is also known that aberration due to IPD effects is approximately proportional to $1/\sqrt{K}$ for a narrow band half wave radome of given size and shape.

It is clear from the above considerations that a high dielectric constant is essential for small radomes for use over a narrow frequency band. It is obvious too why resin bonded glass fibre radomes ($K = 4$) of small size give very poor performance. The reason for the comparatively satisfactory results obtained with insertion diffraction theory for $D/\lambda_0 = 10$ is also apparent. Comparing the aberration from the various effects with those in a five wavelength diameter radome which is otherwise similar, it is seen from the above results that the aberration from IPD effects is halved, that from tip scattering is reduced by a factor of 8 and that due to interaction by $8\sqrt{2}$. In the smaller radome the last two effects give aberration equal to or greater than that from IPD effects, so it is clear that these effects are of major importance for $D/\lambda_0 = 5$ but only of second order significance for $D/\lambda_0 = 10$.

Apart from easing the problem of radome design, other advantages of using a large value of $D/\lambda_0$ include improved discrimination when attacking multiple targets and less tracking error due to the image below the earth's surface of a low flying target. The modulation slope, a measure of the response of a conical scan tracking system to displacement of the target from the aerial axis
is also proportional to $D/\lambda_0$. These advantages may all be out-weighed by other considerations such as small size (which reduces drag and weight and is thus especially important for air to air missiles) and the increasing atmospheric attenuation for electromagnetic waves above J band.

A design procedure was developed which makes use of the improved understanding of the electromagnetic aspects of radome aberration. It also employs the radome quality criteria, the knowledge of the effects of radome tolerances and the ability to design predictable aberration correcting devices, all of which were developed in the main research programme of which the present work formed a part.
A.1 Aberration measuring equipment

The aberration measuring equipment used in the research programme was capable of measuring simultaneously the in-plane and cross-plane components of radome aberration for missile radomes 6 inches to 24 inches in diameter. The frequency band was about 20% in X band and the measuring accuracy was nominally ±1 minute of arc. The transmitter and receiver units were mounted on towers 80 feet apart. The aerials were 15 feet above the level of the flat roof which supported the towers.

The design of the equipment was conventional and a simplified block diagram of the arrangement employed for measurements using rotating or nutating scan receivers is shown in Fig.1. Microwave power from a reflex klystron is fed to the transmitting horn which is servo controlled in the vertical and horizontal directions, i.e. in the plane normal to its axis. Error voltages derived from the receiver are used to control the transmitter position so that it lies on the apparent sight line of the receiver. The displacement of the transmitter from the no-radome position is proportional to aberration for small displacements and is recorded on an X-Y recorder as a function of the angle \( \theta_L \) between the aerial axis and the radome axis. The radome can be yawed, pitched or rolled, while the receiving aerial is normally fixed. However, the receiving aerial can be rotated about its own axis or moved along this axis if required. For the measurements described in this thesis the radome mount was yawed from +20 degrees to -20 degrees. Before carrying out an aberration measurement no-radome plots of aberration versus
FIG. I  BLOCK DIAGRAM ABERRATION MEASURING EQUIPMENT
the yaw angle of the radome mount were performed. Sheets of absorbing material on and near the mount were adjusted in position to cut down spurious reflections from objects near the aerial and make the no-radome plot as flat as possible. With the 6 inch diameter aerials used in the present work the no-radome aberration could usually be kept within the limits of ±5 minutes of arc. The aberration curve would then be measured immediately with the radome in position.

A.2 Field measuring equipment

The measurement of the amplitude and phase of the field distribution inside and around a radome is useful in studying the causes of radome aberration. Two cases are of interest, viz. the field immediately outside the radome resulting from a radiating aerial inside the radome and the field on a plane surface inside the radome when a plane wave is incident from outside. The plane of interest for the second case, at least in the present work, is the plane which normally contains the aperture of the receiving aerial. The aberration can be calculated from the field distribution in the aperture, by using equation (2–7), or the distribution can be compared with calculated values.

The field is measured using a pick up probe consisting of a small dipole with director and a reflector fed from coaxial line. The advantage of this over an open ended waveguide is its smaller physical size, which is important in the small radomes used.

A simplified block diagram of the equipment is shown in Fig.2. The output of the microwave generator, a reflex klystron, is split and part of the output acts as a reference signal at a balanced mixer. The remainder of the output is fed to the transmitting aerial which is situated about 200 feet from the radome.
FIG 2 BLOCK DIAGRAM FIELD MEASURING EQUIPMENT
The receiving probe is driven between preset limits by a drive system and the output of the probe is fed to a balanced mixer. The reference signal is fed to the balanced mixer through a Fox phase shifter in which the central section containing a half wave plate is rotated by a motor at a uniform rate of 110 Hz. The signal coming out of the phase shifter is therefore translated in frequency by 220 Hz. The output of the balanced mixer at 220 Hz retains the amplitude and phase of the RF signal. The amplitude recording is obtained by rectifying and smoothing part of the output of the mixer. The phase recording is made by automatically controlling the phase of an induction generator which is mounted on the shaft of the rotating phase shifter. The phase of this generator is controlled so as to maintain zero output from the phase sensitive detector. The reference for the phase sensitive detector is supplied by the generator and the input to the phase sensitive detector is the 220 Hz signal.

The object of the compensating length of waveguide in the signal path is to equalise the length of the waveguide in the two arms of the bridge. This reduces the effect of changes in ambient temperature which cause expansion or contraction and hence a change in guide wavelength and in the overall electrical length of the arms.

The accuracy of phase measurement is nominally ±1 degree at X band. One advantage claimed for this method of phase measurement is that the measurement is obtained without changing the electrical length of the reference arm of the bridge. This reduces the errors due to standing waves in the arms of the bridge.
PRINCIPAL SYMBOLS

A \quad \text{amplitude of perturbing wave}

a \quad \text{inner radius of dielectric tube}

\begin{align*}
{a_n}^1, {a_n}^2, {a_n}^3, {a_n}^4 \\
{b_n}^1, {b_n}^2, {b_n}^3, {b_n}^4 \\
{B_n}, {C_n}, {D_n}, {E_n}
\end{align*}

\text{field amplitude coefficients}

b \quad \text{outer radius of dielectric tube}

c \quad \text{ratio of radii, } a/b

d \quad \text{wall thickness of radome}

D \quad \text{base diameter of radome}

E \quad \text{electric field}

E_p \quad \text{peak electric field of a plane wave}

E_Y \quad \text{electric field in aerial aperture due to surface wave}

f \quad \text{amplitude weighting in aerial aperture}

F \quad \text{amplitude polar diagram of aerial}

f_c \quad \text{cut off frequency of a surface wave mode}

G \quad \text{far field diffraction pattern}

G_P \quad \text{power radiation pattern of an aerial}

H \quad \text{magnetic field}

\begin{align*}
H_n^{(1)}, H_n^{(2)}
\end{align*}

\text{Hankel functions of order } n, \text{ of the first and second kinds}

i \quad \text{angle of incidence}

j \quad = \sqrt{-1}

J_n \quad \text{Bessel function of the first kind, of order } n

K \quad \text{dielectric constant}

k_i \quad \omega^2 \mu_e - \gamma_i^2

k_0 \quad 2\pi/\lambda_0

L \quad \text{distance from aperture to tip of radome}
m
N
n
P_i
R
r_d
T
T_1
T_C
T_P
t
V
V_s
V_R
v_i
v_gn
W
X
Y
Z
x
y
z
X_1
X_1
Y_n

an integer
= k_3b
an integer
power in region i
radial distance in spherical polar coordinates
aerial radius
= 1/(k_2b)^2 + 1/W^2
period of variation of aberration
transmission coefficient of curved panel
transmission coefficient of plane panel
time
voltage
signal voltage
reference voltage
TEM wave velocity in region i
phase velocity for nth mode
= k_3b/j
cartesian coordinates applicable to the aerial aperture
cartesian coordinates
= k_1a
= \lambda/\lambda_0
Bessel function of second kind, order n
amplitude gradient (Chapter 2) or angle between dielectric sheet and aerial axis (Chapter 4)

\( \beta \) semi-split angle of conical scan aerial

\( \gamma = 2\pi/\lambda \)

\( \Delta \) phase of field in aerial aperture

\( \varepsilon \) permittivity

\( \varepsilon_0 \) permittivity of free space

\( \eta, \xi \) aperture coordinates

\( \theta \) angle defining conical surface in spherical polar coordinates

\( \theta_L \) angle between aerial axis and radome axis

\( \theta_0 \) semi-angle of cone

\( \lambda \) guide wavelength of surface wave

\( \lambda_0 \) free space wavelength

\( \mu \) permeability

\( \mu_0 \) permeability of free space

\( \nu \) complex number describing azimuthal variation of surface wave field

\( \rho \) radial distance in cylindrical polar coordinates

\( \phi \) azimuthal angle in cylindrical or spherical polar coordinates

\( \phi_p \) relative phase of perturbing wave

\( \psi \) angle of arrival of perturbing wave with respect to the aerial axis

\( \omega \) angular frequency

\( \omega_s \) conical scan angular frequency

\( \nabla \) the vector operator del
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