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<tr>
<th>DATE</th>
<th>SIGNATURE</th>
<th>ADDRESS</th>
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</thead>
<tbody>
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</tr>
</tbody>
</table>
Preamble

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RELATIONS BETWEEN THE GEOMETRY AND ACoustics of Brass Instruments

John Christopher Goodwin

A thesis submitted to the Faculty of Mathematical and Physical Sciences of the University of Surrey in requirement for the degree of Doctor of Philosophy

February, 1981
This thesis examines the role of the bore shape in brass instruments. Difficulties in finding the precise nature of the bore shape in an assembled instrument are discussed. Several measurement methods employed to overcome these are developed, and the overall bore shape found. A model is developed whereby input impedance and pressure transfer function of an instrument may be predicted from the bore shape, and the musically important effect of temperature investigated. Good agreement with experimental measurements of instruments under similar conditions is found.

Time domain measurements using a computer controlled impulse source and digital signal acquisition are given which illustrate the effect of small discontinuities of the bore upon the impulse response. Synthesis of bore shape from acoustic data is discussed with reference to reflections from such features.

The methods currently employed by manufacturers to design new instruments are discussed, and an iterative process combining the prediction of acoustic properties from bore shape, synthesis, and psychometric techniques postulated.
Le Piston
Hochschnitt von Félix Vallotton (Lausanne 1865-1925 Paris), aus der Folge "Instruments de Musique" 1897
I would like to thank my supervisor, Dr. J. M. Bowsher for his advice and help during the course of study.

My fellow research students, in particular Mr. P. S. Watkinson and Dr. S. J. Elliott have contributed many hours of useful, occasionally heated discussion which I have found invaluable. The assistance of Dr. R. Shepherd in the design and execution of player’s reactions to instruments is also worthy of much thanks.

Much insight into the craft of instrument design and the attitudes of the professional musician has been given by Mr. R. Merewether of Paxmans Ltd., whose industrial collaboration via the SRC/CASE studentship included the provision of several specially constructed instruments.

Many personal contacts in the world of professional music have also influenced the progress of this work, albeit sometimes unwittingly.

The departmental workshop staff have been helpful throughout in the construction of apparatus, and particular thanks must go to Mr. E. Worpe for the design and construction of the impulse device.

Computing assistance has always been forthcoming, in particular from Mr. D. Munro whose ability to find rapid solutions to software problems was invaluable.

Finally I should like to thank my wife Philippa for her continued support and understanding, especially in the final stages of the study.
<table>
<thead>
<tr>
<th>Figure no.</th>
<th>Caption</th>
<th>After page no.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1.1</td>
<td>The component parts of a trombone</td>
<td>3</td>
</tr>
<tr>
<td>2.1.0</td>
<td>Analogy between the 2-port and the cylindrical tube</td>
<td>14</td>
</tr>
<tr>
<td>2.2.2</td>
<td>Benade's equations</td>
<td>20</td>
</tr>
<tr>
<td>2.3.2</td>
<td>Comparison between measured and predicted input impedance of a trombone</td>
<td>26</td>
</tr>
<tr>
<td>2.3.3</td>
<td>Input impedance evaluated at different temperatures</td>
<td>26</td>
</tr>
<tr>
<td>2.4.2</td>
<td>The step discontinuity</td>
<td>27</td>
</tr>
<tr>
<td>2.4.2</td>
<td>Equivalent circuit</td>
<td>27</td>
</tr>
<tr>
<td>2.4.3</td>
<td>Karal's discontinuity correction factor</td>
<td>32</td>
</tr>
<tr>
<td>2.5.0</td>
<td>Impulse response of a French horn</td>
<td>33</td>
</tr>
<tr>
<td>2.5.1</td>
<td>Impulse response multiplied by a linear gain function</td>
<td>34</td>
</tr>
<tr>
<td>2.5.2</td>
<td>Reflection and transmission coefficients</td>
<td>35</td>
</tr>
<tr>
<td>2.5.3</td>
<td>The general manifold</td>
<td>35</td>
</tr>
<tr>
<td>2.5.4</td>
<td>Comparison of measured and synthesised trombone bore shape</td>
<td>34</td>
</tr>
<tr>
<td>2.6.1</td>
<td>Linear causal system</td>
<td>44</td>
</tr>
<tr>
<td>2.6.2</td>
<td>Input reflection response</td>
<td>44</td>
</tr>
<tr>
<td>2.6.3</td>
<td>Domain relations</td>
<td>45</td>
</tr>
<tr>
<td>2.6.4</td>
<td>A Hilbert pair illustrating the acausal nature of the transform</td>
<td>46</td>
</tr>
<tr>
<td>2.6.5</td>
<td>Hilbert transform of real part of input impedance</td>
<td>47</td>
</tr>
<tr>
<td>2.6.6</td>
<td>The same data in modulus and phase form</td>
<td>47</td>
</tr>
<tr>
<td>2.7.1</td>
<td>Computation procedure for autocorrelation and cepstra</td>
<td>49</td>
</tr>
<tr>
<td>2.7.2</td>
<td>Homomorphic deconvolution</td>
<td>49</td>
</tr>
<tr>
<td>3.1.1</td>
<td>Internal bore of French horn leadpipe</td>
<td>54</td>
</tr>
<tr>
<td>3.1.2</td>
<td>Shadowgraph of trombone bell</td>
<td>55</td>
</tr>
</tbody>
</table>
Table of figures

<table>
<thead>
<tr>
<th>Section</th>
<th>Figure Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1.3</td>
<td>Composite measurement of trombone bore shape</td>
<td>58</td>
</tr>
<tr>
<td>3.1.4</td>
<td>Internal bore of a trombone mouthpiece</td>
<td>59</td>
</tr>
<tr>
<td>3.3.1</td>
<td>Simultaneous impedance and transfer function measuring device</td>
<td>65</td>
</tr>
<tr>
<td>3.3.2</td>
<td>Impulse response measuring device</td>
<td>69</td>
</tr>
<tr>
<td>3.3.3</td>
<td>Typical impulse generated by the device</td>
<td>69</td>
</tr>
<tr>
<td>4.4.1</td>
<td>Flowchart of programme IMPULSE</td>
<td>79</td>
</tr>
<tr>
<td>4.3.1</td>
<td>Flowchart of programme JIMPTRAN</td>
<td>80</td>
</tr>
<tr>
<td>5.1.2</td>
<td>Intonation at different temperatures</td>
<td>85</td>
</tr>
<tr>
<td>5.2.1</td>
<td>Sum function equations</td>
<td>88</td>
</tr>
<tr>
<td>5.2.2</td>
<td>Sum functions</td>
<td>88</td>
</tr>
<tr>
<td>5.3.1</td>
<td>Diagram of mouthpiece</td>
<td>89</td>
</tr>
<tr>
<td>5.3.2</td>
<td>Mouthpiece equivalent circuit</td>
<td>89</td>
</tr>
<tr>
<td>5.3.3</td>
<td>Input impedance of a trumpet with mouthpiece</td>
<td>92</td>
</tr>
<tr>
<td>5.3.4</td>
<td>Input impedance of a trombone with mouthpiece</td>
<td>92</td>
</tr>
<tr>
<td>5.3.5</td>
<td>Input impedance of a French horn with mouthpiece</td>
<td>92</td>
</tr>
<tr>
<td>5.4.1</td>
<td>Results of subjective experiment (Freeblowing scale)</td>
<td>98</td>
</tr>
<tr>
<td>5.6.1</td>
<td>Traditional design process</td>
<td>105</td>
</tr>
<tr>
<td>5.6.2</td>
<td>Modification suggested by Pratt</td>
<td>105</td>
</tr>
<tr>
<td>5.6.3</td>
<td>Revised design process</td>
<td>106</td>
</tr>
<tr>
<td>Symbol</td>
<td>Quantity</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>----------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Propagation coefficient</td>
<td></td>
</tr>
<tr>
<td>$Z_c$</td>
<td>Characteristic impedance</td>
<td></td>
</tr>
<tr>
<td>$Z_l$</td>
<td>Load impedance</td>
<td></td>
</tr>
<tr>
<td>$Z_{in}$</td>
<td>Input impedance</td>
<td></td>
</tr>
<tr>
<td>$a, b$</td>
<td>General tube radii</td>
<td></td>
</tr>
<tr>
<td>$l$</td>
<td>Tube length</td>
<td></td>
</tr>
<tr>
<td>$\Gamma_o$</td>
<td>Output reflection coefficient</td>
<td></td>
</tr>
<tr>
<td>$\Gamma_i$</td>
<td>Input reflection coefficient</td>
<td></td>
</tr>
<tr>
<td>$p(0)$</td>
<td>Pressure at input</td>
<td></td>
</tr>
<tr>
<td>$p(l)$</td>
<td>Pressure at output</td>
<td></td>
</tr>
<tr>
<td>$Z(x)$</td>
<td>Series impedance / unit length at position $x$</td>
<td></td>
</tr>
<tr>
<td>$Y(x)$</td>
<td>Shunt admittance</td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>Fluid density</td>
<td></td>
</tr>
<tr>
<td>$c$</td>
<td>Velocity of sound (Phase velocity)</td>
<td></td>
</tr>
<tr>
<td>$S$</td>
<td>General cross-sectional area</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Attenuation coefficient</td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>Wavenumber</td>
<td></td>
</tr>
<tr>
<td>$V_i$</td>
<td>Incident volume velocity</td>
<td></td>
</tr>
<tr>
<td>$P_i$</td>
<td>&quot; pressure</td>
<td></td>
</tr>
<tr>
<td>$V_r$</td>
<td>Reflected volume velocity</td>
<td></td>
</tr>
<tr>
<td>$P_r$</td>
<td>&quot; pressure</td>
<td></td>
</tr>
<tr>
<td>$V_t$</td>
<td>Transmitted volume velocity</td>
<td></td>
</tr>
<tr>
<td>$P_t$</td>
<td>&quot; pressure</td>
<td></td>
</tr>
<tr>
<td>$w$</td>
<td>time periodic wavenumber (frequency constant)</td>
<td></td>
</tr>
<tr>
<td>$k$</td>
<td>space &quot; ( phase &quot; )</td>
<td></td>
</tr>
<tr>
<td>$z$</td>
<td>General spatial coordinate</td>
<td></td>
</tr>
<tr>
<td>$R$</td>
<td>Pressure reflection coefficient</td>
<td></td>
</tr>
<tr>
<td>$M$</td>
<td>&quot; transmission coefficient</td>
<td></td>
</tr>
</tbody>
</table>
List of symbols

\[ \xi = \text{Ratio of tube radii} \]
\[ \omega = \text{General angular frequency} \]
\[ m, k = \text{Manifold junction number} \]
\[ \ast = \text{Symbol for convolution} \]
\[ F[\ ] = \text{Fourier transform} \]
\[ F^{-1}[\ ] = \text{Inverse Fourier transform} \]
\[ y(t) = \text{general time series} \]
\[ z(t) = \text{Time series with superimposed echoes} \]
\[ a = \text{Echo amplitude} \]
\[ d = \text{Echo delay} \]
# Table of Contents

**CHAPTER 1 : INTRODUCTION**

1.1 STATEMENT OF PROBLEM.................................1
1.2 APPROACHES TO PROBLEM................................3
  1.2.1 Possible important factors..........................3
1.3 MEASUREMENT TECHNIQUES....................................6
1.4 MANUFACTURERS AND PLAYERS VIEWS........................8

**CHAPTER 2 : THEORY**

2.1 THE CYLINDRICAL TUBE AS A TWO-PORT NETWORK............13
  2.1.1 Input impedance and transfer function................13
  2.1.2 The cylindrical tube..................................14
2.2 CHARACTERISTIC IMPEDANCE AND PROPAGATION COEFFICIENT....17
  2.2.1 The characteristic impedance..........................17
    2.2.1.1 Ideal gas........................................17
    2.2.1.2 Real gas........................................19
  2.2.2 Practical considerations of models used.................22
2.3 THE LOOK-BACK METHOD.....................................24
  2.3.1 The look-back method and the two-port...............24
2.4 MODELLING OF THE STEP DISCONTINUITY......................27
  2.4.1 Plane wave model......................................27
  2.4.2 Higher-order mode model...............................29
2.5 REFLECTION METHODS.......................................33
  2.5.1 Simple reflection model...............................33
2.5.2 Multiple reflection model .............................................. 34
2.5.3 Multiple reflection contribution .................................. 37
2.5.4 Termination impedances and reflection .......................... 38
2.5.5 Non-ideal pulse excitation and calibration .................... 40
2.6 IMPULSE RESPONSE AND THE F.F.T ................................ 43
  2.6.1 Definitions .......................................................... 43
  2.6.2 The Hilbert transform ............................................ 46
2.7 ECHO DETECTION USING CEPSTRAL TECHNIQUES ..................... 48
  2.7.1 Definition of the Cepstrum ...................................... 48
  2.7.2 Definitions of cepstra and associated processing ............ 51

CHAPTER 3 : EXPERIMENTAL METHODS ..................................... 53
  3.1 MEASUREMENT OF BORE SHAPE ...................................... 54
    3.1.1 Manometric method ............................................ 54
    3.1.2 Shadowgraph method .......................................... 55
    3.1.3 X-Ray shadowgraph ........................................... 56
    3.1.4 Ball of known radius ......................................... 56
    3.1.5 Disk gauges .................................................. 57
    3.1.6 Radius of curvature gauge method ......................... 58
    3.1.7 Microscopic method ........................................... 59
  3.2 MEASUREMENT OF TEMPERATURE GRADIENT ........................... 61
    3.2.1 Temperature and playing conditions ........................ 61
  3.3 IMPEDANCE AND TRANSFER FUNCTION MEASUREMENTS ................ 64
    3.3.1 Apparatus .................................................. 64
    3.3.2 Programmes .................................................. 66
  3.4 MEASUREMENT OF IMPULSE RESPONSE ................................ 67
    3.4.1 Criteria for measurement ................................... 67
1.1 STATEMENT OF PROBLEM

Musical instruments are complex systems. The shape of the internal bore is the major factor influencing the acoustics. Little accurate work has been done to establish the relations between the bore profile and the acoustic behavior, mainly because the bore shape can only rarely be modeled analytically. Temperature gradients during playing further complicate models of sound propagation.

Many other secondary effects are also present, for example the nonlinearity of the player's lips and vibration of the wall material, but are not considered in this study as they have recently been dealt with by others.

The input impedance is readily found from the bore shape (Young, 1960), but the inverse problem, that of reconstructing the bore shape from acoustic data is far more complex. At present, it is only possible for systems which may be reduced to one-dimensional propagation. If a time response synthesis is attempted, multiple reflections complicate the extraction of the bore profile (Stansfield and Bogner, 1973). Alternative methods in the frequency domain employ the transmission line analogy and signal flow theory (Holte and Lambert, 1961).

Features such as bore discontinuities have long been known to influence both the subjective and objective acoustics of musical instruments but have never been examined in detail, possibly due to the difficulty of the detection of such small features in an assembled instrument.
At present, instrument design is mainly empirical, though manufacturers have employed scientific methods to investigate instruments (KENT, 1956), presumably to try to rationalise the design process and/or quality control in manufacture.

Recent work has demonstrated the possibilities of assessment of brass instrument quality by scientific means, and though some basic correlations between subjectively perceived quality and acoustic properties have been derived (PRATT, 1978), it remains for the final link between bore geometry and instrument quality to be forged.

This will enable the design process to be placed on a more rational footing, and eliminate the long and often counterproductive alterations made by the designer in the present development methods for new instruments.
1.2 APPROACHES TO PROBLEM

The deduction of the relations between the geometry and acoustics of brass instruments can be broken down into two broad areas, analysis and synthesis. Analysis may be experimental or theoretical, while the inverse problem of synthesis must be considered only as a theoretical estimate of the physical dimensions of the system. As in most inverse problems, many a priori assumptions must be made before a solution is attained, and it is in this area where consideration of the problem from both directions simultaneously is helpful.

On the experimental side, before any measurements are made, decisions must be made as to the type of data to be collected, and under what conditions. A good experiment is one in which these decisions are shown to be reasonable and which indicates the direction(s) for future progress.

1.2.1 Possible important factors

The properties of the air column confined in the instrument are of great importance. The air is confined by the tubing of the instrument, which is usually made in several distinct sections and joined by soft soldering (Fig.1.1.1). The trombone (shown in the figure) is unique in the contemporary brass family of instruments in that is the only instrument not furnished with valves, which makes initial investigation of the acoustics easier. It is of considerable assistance if the propagation inside may be considered linear, though for example in actual playing situations it has been argued (incorrectly) that the high sound pressure levels inside the instrument cause harmonic distortion. Another relevant area is the nature of the propagation in
The component parts of a trombone
the flaring parts of the instrument.

Much discussion of the theory of sound propagation in horns has been published, starting with WEBSTER (1918), for many different kinds of horn shape ranging from conical, exponential and hyperbolic to much more exotic shapes. However, opinion is divided as to the real nature of the wavefront in such horns, but it must be said that the bulk of the theoretical speculations have not been followed by experiments. A simple measurement of the iso-phase contour would considerably assist in this area.

In any duct acoustic problem, the nature of the wall impedance is an important issue especially as far as dissipative mechanisms are concerned. If the wall impedance is small enough, coupling between the acoustic wave and the confining structure is likely and the possibility of a system of transmission paths which extract energy from one part of the system and reintroduce it elsewhere must not be overlooked. This problem is further complicated if the confining structure (i.e. the tube) is itself resonant. Absorption is always of interest in acoustics, and the effect of the boundary layer losses has been shown to be significant. Under playing conditions, a steady airflow is superimposed upon the acoustic wave, the possibility of turbulence or other effects must be examined, (for example, vortex shedding at a bore discontinuity).

In a playing situation, the internal humidity and gas content are different from the normal composition of air. More important, the strong dependence of the velocity of sound on temperature is likely to be relevant.

While not a topic in these discussions, the coupling between the player's lips and the instrument is of great interest in processes of harmonic generation, when the internal and external spectra of the played note are related to the time-varying impedance of the players
lips and the input impedance of the instrument.

Radiation of sound from the open end (and possibly the tube walls also), the directionality and efficiency in transferring the wave from the lips to the outside air are factors which are doubtless related to the bore shape, and are of interest to the musician.

Most important of all, the impedance seen by the player looking into the instrument (the input impedance) has been shown to be a fairly reliable parameter to use when correlations are made between the subjectively perceived and objectively measured intonation, and to some extent timbre of the played instrument. The input impedance has been shown by many to be related to the bore shape, and as a consequence of the Fourier Transform the impulse response also, another important factor in the assessment of acoustical quality in music.

The foregoing discussion has established that the understanding of the effects of bore geometry and the propagation within is of paramount importance in the investigation of brass instruments.
1.3 MEASUREMENT TECHNIQUES

In order to categorise a system, normal practice is to introduce a test signal and examine the effect the system has upon it. To minimise problems in subsequent analysis, test signals in common use are chosen so only one parameter of the system is varied at a time. A variety of signals used in acoustics are listed below.

Some acoustic test signals

a. Swept sine
b. Wideband noise
c. Narrowband noise
d. Step function
e. Impulse
f. Chirp

Problems are encountered with some of these in practical situations where both real and imaginary parts of the system response are required. Another difficulty is that of signal to noise ratio, where the combination of swept sine with matching filter has traditionally excelled. Wideband noise of considerable power must be used to obtain equivalent power per unit bandwidth. The technical achievement of truly flat noise source over acoustic bandwidths is virtually impossible, especially at high power. Leakage and spurious signal complicate accurate measurements with narrow band noise. Ringing, echoes and the possible excitation of nonlinearities are difficult to avoid with impulsive signals, which also suffer from poor signal to noise ratio as all the input energy is confined in a very short time. To obtain a useful signal to noise ratio large pulse amplitudes are usually needed.

However, with the advent of computers many inherent defects of
acoustic sources may be compensated for and accurate calibrated results obtained.

A system which measures calibrated values of input impedance at discrete frequencies has been developed at Surrey University. It is described in full in theses by PRATT, ELLIOTT, and WATKINSON, and is described briefly in Chapter 4. While this data is of enormous value, it suffers from the limitation of the acoustic driving loudspeaker, which becomes inefficient at frequencies above 2kHz. Recalling the effectiveness of radar systems in the detection of objects by the use of echoes from a narrow emitted pulse, and bearing in mind the Fourier transform, an impulse source was postulated where a repeatable impulse was used in conjunction with suitable averaging to improve the signal to noise ratio problem. If a narrow enough impulse is used, the spectral components will extend well beyond 2kHz and extend the measurement bandwidth. While normal playing modes are not above this frequency, the impulse response found by this method shows the effect of small features in the instrument bore.

Another measurement introduced by ELLIOTT is the transfer function, which is the complex ratio of the pressures at the output and input of the instrument. The discrete-sine method has been proven in this field, but there is no reason why the impulse method should not also be employed, though in practice the lack of anechoic facilities made this measurement unrealisable.
Considering the important role the instrument plays in the livelihood of the musician and manufacturer, it is surprising that the general attitude to the present work is one of studied ignorance and even disdain.

A great proportion of those who subscribe to this attitude no doubt have been presented with scientific approaches to music in a poor light, or, in the case of players, are unwilling to admit that music is more than an art form where personal skills reign.

However, in fairness it must be said that many members of both groups are becoming aware of the possibilities of musical instrument research, though are sometimes unwilling to admit the fact for commercial reasons.

A great mystique surrounds the design of instruments and mouthpieces, and again, manufacturers are unwilling to elucidate for fear of losing potential customers who have fixations about a particular aspect of instrument design. The author has seen this in operation in numerous establishments. To his knowledge in one firm it is management policy not to persuade players against their currently-held views. Sales staff are instructed not to express any definite opinions.

The point about current views is important, as these change as a player becomes more proficient and experienced. For example, many beginners choose mouthpieces with small rim diameters and cup volumes, because this tends to facilitate production of high notes. While hampering development of the embouchure (DALE, 1965), this also damages intonation (WOGRAM, 1972) and timbre. This is a classic example of an inexperienced player who does not consider all aspects of instrument design simultaneously, but is willing to sacrifice all for one
Current fashion in music also dictates the choice of instrument to some extent, an example being the general change from small-bore instruments to larger-bore over the last fifty years in this country.

Material has always been a hotly disputed issue, dating to baroque times and before. Many players attribute certain quality to a particular alloy or even finish. Others hold that certain parts of the instrument should be made of a particular alloy. This is particularly true in the bell section, where controversy has long raged about the relative merits and demerits of red and gold brasses, nickel silver and other materials. One manufacturer has adopted a strong sales pitch based on the metal he uses for bells, (a beryllium bronze alloy), claiming improved tone quality and ‘projection’. Again, to some extent this may have a scientific explanation (recent research by WATKINSON (1981) shows small differences in the vibrational modes of bells of different materials), but much of the differences due to material, if any, have long been attributed by the manufacturers to the differing bore shape that results when metals of different hardnesses and elastic constants are shaped on the bell or leadpipe mandrel. It is widely acknowledged that a series of instruments which are nominally identical, coming from the same production line, will be distinguishably different in some cases. The small differences in bore shape mentioned can also be caused by the same material being in different states of annealing, a process essential in manufacture to ensure easy working of the brass and avoid splitting.

Attention is paid to the gradations in thickness of the bell by some manufacturers, as it is claimed this has bearing on the vibrational properties, a view corroborated to some extent by WATKINSON.

How much effect vibration has on the acoustic performance is yet
unknown, as is its contribution to the players subjective assessment of the instrument.

Mechanical properties of the instrument are obviously important to the player, an example being the trombone slide friction, a factor which has been shown sometimes to outweigh acoustic quality (PRATT, 1976).

Manufacturers have long recognised the significance of leaks around slides and valves, but sometimes not been very diligent in reducing these to a minimum, possibly because this and the above requirement are conflicting. It is far more likely that a player will be put off purchasing a trombone with a stiff slide than one which leaks slightly. Current engineering enables an acceptable compromise to be made between the above two restrictions. One manufacturer believes the reduction of leaks in the valve section to a minimum is vital in the manufacture of a good French horn (MEREWETHER, private communication).

Mouthpieces are without doubt the most controversial issue of all, and will be dealt with in the discussion section. Suffice to say that most players claim to notice very small changes of dimension in this region, and some will spend large amounts of money to find what they consider to be the correct shape for them.

The bore of the instrument seems to be low on the list of priorities, and selection is usually based on whether the instrument is 'small', 'medium', 'medium large', or 'large'. However, many players do believe that the leadpipe is important. Some horn players modify their instruments by replacing the leadpipe with a 'favourite', in the belief that it alone possesses an advantageous quality. This is obviously done in ignorance of the fact that the balance between leadpipe geometry and bell shape is crucial in establishing correct intonation (SMITH et al., 1976), and in fact such a practice is more likely to ruin a previously good instrument. Others attribute
qualities to the internal finish, long known to be important in flutes. A popular opinion is that an instrument is not at its best until a layer of detritus has accumulated inside. This view may have some scientific credence as small discontinuities could be smoothed by such a layer, so reducing undesirable reflections. Some players, however ensure that the bore of the instrument is kept to a good finish. One manufacturer claims to polish the internal bore even though it is general practice to 'plug' an instrument, i.e. force ball bearings down the cylindrical sections during finishing to smooth out small dents and other imperfections.

In conclusion, it seems that the scientific community and the players and manufacturers often misunderstand each other. Some of this is due to deliberate actions, advertising and trade secrecy, while some is undoubtedly due to communication problems in two fields so widely separated. Irresponsible reporting of total success by some members of the scientific fraternity destroys trust and credence also.

However, history shows that what at first seemed to be an incredible claim made by either party often has proved to have solid foundation.
CHAPTER 2 : THEORY

The theory employed in the analysis of brass instruments is presented in this chapter.

The use and application of the transmission line analogy is discussed. Boundary effects, characteristic impedance, propagation coefficient, and the limitations of these models are examined. The effect of a discontinuity in a tube of circular cross section is considered.

The Fourier transform is employed to relate the frequency and temporal behaviour of the acoustic system, and the problems of multiple reflections in such an extended structure are discussed.

Use of modern techniques such as the cepstrum method to overcome these difficulties, and the problems involved in recovering the geometry of the duct from acoustic data are considered.
2.1 THE CYLINDRICAL TUBE AS A TWO-PORT NETWORK.

This section discusses the use and applicability of electrical analogies in the treatment of duct acoustics. Expressions for the plane wave propagation condition in a cylindrical tube element are derived.

2.1.1 Input impedance and transfer function

The acoustic equations used in many methods of impedance calculation are reproduced in this section. Expressions for the pressure transfer function are also given. These equations are fundamental to the application of the Look-Back method.

Using standard transmission line theory, it is possible to calculate the input impedance, transfer function, and propagation coefficient of a cylindrical (straight) tube, if the characteristic and load impedances are known.

The input impedance and pressure transfer function are defined as:

\[
\text{Input impedance} \quad Z_{in} = \frac{P_{in}}{U_{in}} \quad -(A)
\]

\[
\text{Transfer function} \quad T' = 20 \log \left( \frac{\text{Mod}(P_{out})}{\text{Mod}(P_{in})} \right) \quad -(B)
\]

(Log form)

where:

\[ U_{in} = \text{input volume velocity} \]
2.1.2 The cylindrical tube

Setting up the co-ordinate system around the tube (Fig. 2.1.0), adding the load impedance \( Z_l \), and defining:

- \( \gamma \) = Propagation coefficient
- \( Z_c \) = Characteristic impedance
- \( Z_l \) = Load impedance
- \( Z_{in} \) = Input impedance
- \( a \) = Internal radius of tube
- \( l \) = Length of tube
- \( \Gamma_o \) = Output reflection coefficient
- \( \Gamma_i \) = Input reflection coefficient
- \( p(0) \) = Pressure at input
- \( p(x) \) = Pressure at distance \( x \) from input

Where:

\[
\Gamma_o = \frac{(Z_l - Z_c)}{(Z_l + Z_c)}
\]
\[ \tag{C} \]

\[
\Gamma_i = \frac{(Z_{in} - Z_c)}{(Z_{in} + Z_c)}
\]

and starting with the transmission line equations, as given by WALKER and WAX (1946):

\[
\frac{dp}{dU} = - Z(x) \cdot U
\]
\[
\frac{dU}{dp} = - Y(x) \cdot p
\]

where:

- \( Z(x) \) = series impedance per unit length
- \( Y(x) \) = shunt admittance per unit length

where:

- \( p \) = generalised pressure
- \( U \) = generalised volume velocity
Fig. 2.1.0 ANALOGY BETWEEN THE 2-PORT AND THE CYLINDRICAL TUBE

\[ Z_{\text{in}}(f) = \frac{\text{pin}(f)}{U_{\text{in}}(f)} \]
\[ T(f) = \frac{p_{\text{out}}(f)}{\text{pin}(f)} \]
\[ Z_l(f) = \frac{p_{\text{out}}(f)}{U_{\text{out}}(f)} = Z_{\text{out}} \]
\[ \Gamma_{\text{out}}(f) = \frac{Z_{\text{out}}(f) - Z_c(f)}{Z_{\text{out}}(f) + Z_c(f)} \]
it can be shown that:

\[
p(x) = p(0) \frac{(\exp(-\gamma x) + \exp(\gamma[x - 2l]))}{(1 + \exp[-2\gamma l])} \tag{D}
\]

Given the complex form of the transfer function, which includes the phase:

\[
T = \frac{p(1)}{p(0)} \tag{E}
\]

it can be shown that:

\[
\Gamma_l = \Gamma_0 \exp(-2\gamma l).
\]

Substituting (E) into (D):

\[
T = \frac{(\exp(-2\gamma l))}{(1 + \exp(-2\gamma l))} \tag{F}
\]

This gives the transfer function in terms of the output reflection coefficient. In terms of the input reflection coefficient, substituting from (E) we get:

\[
T = \frac{(\exp(-\gamma l) + \exp(\gamma l))}{(1 + \Gamma_0^2)} \tag{G}
\]

Similarly, the more well-known expression for the input impedance is:
(1 + \exp(-2 \gamma_1))
Zin = Zc \frac{------------}{(1 - \exp(-2 \gamma_1))} \quad -(H)

or:

\frac{(Zt \cdot \cosh(\gamma_1) + Zc \cdot \sinh(\gamma_1))}{(Zc \cdot \sinh(\gamma_1) + Zt \cdot \cosh(\gamma_1))} \quad -(I)

This may be reduced by division to give an expression in terms of
\tanh(\gamma_1).

\frac{(Zt + Zc \cdot \tanh(\gamma_1))}{(Zc + Zt \cdot \tanh(\gamma_1))} \quad -(J)
2.2 CHARACTERISTIC IMPEDANCE AND PROPAGATION COEFFICIENT

Two models of the characteristic impedance are presented, and the difference between them examined. The boundary layer and its effect on the attenuation and propagation coefficient is discussed.

2.2.1 The characteristic impedance

This is an acoustic property intrinsic to a duct-like system, and is different from the specific acoustic impedance, which is defined for materials in unconstrained situations. It is defined entirely by the dimensions and nature of the duct, the fluid enclosed in it, the frequency, and is independent of position. Using the transmission line analogy of OLSON (1957), the characteristic impedance is defined as the impedance seen at any point of an infinite homogeneous transmission line.

Many workers have discussed the nature of the characteristic impedance from several viewpoints. In the elementary model, the characteristic impedance is derived from the simplified models of fluid behaviour in the tube.

2.2.1.1 Ideal gas

Since:

\[
Z = \frac{\text{Pressure}}{\text{Volume Velocity}} = \frac{p}{U} = -(A)
\]
or:

\[
\frac{\text{Pressure}}{\text{Area} \times \text{velocity}} = \frac{\text{Spec. ac. impedance}}{\text{Area}}
\]

From consideration of a plane wave propagating in a tube of infinite length, (i.e. no reflections) of the tube, the characteristic impedance is, assuming the following:

Zero attenuation in the bulk of the fluid
Zero fluid viscosity
Infinitely rigid tube walls
Perfectly compressible fluid
Zero heat exchange between walls and wave,

\[
Z = \frac{\rho}{c} \frac{c}{S} \quad \text{(B)}
\]

(See KINSLER and FREY, (1950) for a derivation.)

where:

\( \rho \) = fluid density (kg m\(^{-3}\))
\( c \) = phase velocity of wave in tube (ms\(^{-1}\))
\( S \) = cross sectional area of tube (m\(^2\))
For any real gas enclosed in a tube, the previous assumptions are significant, especially those of viscosity and thermal conductivity.

Many workers have discussed these effects, starting with Stokes (1845), (viscosity only), and Kirchhoff (1868), who derived the fundamental equation of sound propagation in a tube, including both viscous and thermal effects.

Their work was reviewed by Rayleigh (1894), and later a re-analysis was undertaken by Henry (1931).

Henry shows that for a tube of reasonable rigidity, the basic boundary conditions of:

a. zero axial particle velocity at the tube walls
b. the gas at the tube wall being at the same temperature as the tube wall.

are such that the result is little different from the above for the ideal gas, though his computations are somewhat inaccurate, considering only the first few terms of the Bessel series in the Kirchhoff equation of propagation. However he discusses deviations from ideal gas behaviour, such as the effect of molecular slip at the tube walls which was later shown to be important.

Weston (1953) continues with good, though again approximate algebraic solutions to the fundamental equation. He approaches the problem from the point of view of the relative size of the tube and sound wavelength, and solves by using "large" and "small" tube approximations to compensate for computation inaccuracies. He also gives the form of the boundary layers of amplitude, velocity, temperature, and the shape of the isophase surface. One important
point he noted was the difficulty of comparing theory with experiment, as even small roughnesses and corrugations in the tube walls can change propagation conditions significantly.

SHEILDS et al (1965) form an iterative numerical solution including molecular slip at the tube walls, and introduce thermal relaxation, though this effect is not significant at room temperatures and pressures. These are employed in a form of the Kirchhoff equation, though with somewhat modified boundary conditions. Some comparisons between theory and careful experiment are given, which show excellent agreement even at elevated temperatures and reduced pressures, so it can be concluded that the result for room temperature is acceptable.

Another approach is to consider propagation of an ideal gas in a tube with yielding walls. This was originated by LAMB (1925), and developed further by MALECKI (1969). The modelling depends strongly on the wall impedance, which is not generally known, and attempts to include attenuation by inventing higher-order modes. The actual physics of this method is somewhat dubious, and was not pursued.

BENADE (1963) gives a set of equations based on the work of CRANDALL (1927), which deals with the real effects of viscosity and heat transfer by means of the introduction of boundary layers near the tube wall. The equations give solutions directly in terms of the transmission line parameters, and as such are directly applicable to the present work (Fig.2.2.2).
Table 2.2.2  BENADE’S EQUATIONS

\[
Z = \frac{j(\omega \rho / \pi a^2)}{(1 - F_v e^{j\phi_v})^{-1}}
\]

\[
Y = j(\omega \pi a^2 / \rho c^2)(1 + \gamma t_1 F_t e^{j\phi_t})
\]

where:

\[
F_v e^{j\phi_v} = \frac{2}{r_v \sqrt{(-j)}} \frac{J_1[r_v(\sqrt{-j})]}{J_0[\sqrt{-j}]}
\]

and:

\[
r_v = \sqrt{\omega \rho / \eta} \cdot a \quad \text{(Viscous boundary layer)}
\]

\[
r_t = \sqrt{\omega \rho C_p / \kappa} \cdot a \quad \text{(Thermal)}
\]

\(\omega = \text{angular frequency (rad s}^{-1}\))

\(\eta = \text{viscosity of gas (Ns m}^{-2}\))

\(\rho = \text{density of gas (kg m}^{-3}\))

\(\kappa = \text{thermal conductivity of gas (Wm}^{-1} \text{K}^{-1}\))

\(\gamma = \text{ratio of principal specific heats of gas}

\(C_p = \text{specific heat of gas at constant pressure (J kg}^{-1} \text{K}^{-1})\)

\(a = \text{radius of tube (m)}

\(c = \text{velocity of sound in tube (ms}^{-1}\))
Given the standard transmission line equations (e.g. SLATER (1942), CONNOR (1975)),

\[
\frac{1}{2} Z_c = \left( \frac{Z}{Y} \right)^{1/2}
\]

\[
\gamma = \left( \frac{Z \cdot Y}{Z_c^2} \right)^{1/2}
\]

The characteristic impedance and propagation coefficients may be calculated.

Using the numerical approximations to \( J_1/J_0 \), as found in ABRAMOWITZ and STEGUN (1965), \( Z_c \) has been calculated (By P. Watkinson) and compared to the simple value \( (\rho c/S) \) for a particular tube. It was found that the Bessel function approximation was within 1% of \( (\rho c/S) \) above 200Hz., and deviated more below this frequency.

If the value for \( \gamma \) is split into attenuation \( (\alpha) \) and wavenumber \( (\beta) \) coefficients, it is found that \( \beta \) approximates to the simple formula \( \beta = w/c \) to better than 1% in the frequency region 350 to 2000 Hz.

The attenuation vs. frequency fits to a power law very close to

\[
\alpha = (K/a) \cdot f^{0.48}
\]

where \( K \) is close to 3.0x10^{-5}

This value had been verified experimentally by the author (GOODWIN, 1979), where similar values were obtained using a method to extract the values of attenuation coefficients from input impedance data of a straight tube.
2.2.2 Practical considerations of models used

A simple consideration of the steps used in the method will graphically illustrate the reasons for the choice of approximation. If only 128 sections are employed to model the bore shape of an duct, then the time taken to calculate the input impedance at just one frequency is approximately 17 seconds, using the model below:

Equations used in model

\[
\alpha = (3.0 \times 10^{-5} /a) \times f^{0.48}
\]

\[
\beta = \frac{w/c}{c/s}
\]

\[
\gamma = \alpha + j\beta
\]

\[
Z_c = \frac{j \sigma}{2\pi}
\]

For every value of impedance, it can be seen that \( \gamma \) must be re-calculated for every section. Alternatively, using the Bessel function approximation and transmission line model given by BENADE (1968), the calculation time for each section, (if 80 terms are used in the Bessel series) took approximately 2 seconds.

Therefore, the calculation for 128 sections of \( Z_c \) and \( \gamma \) for one frequency alone would take over two minutes, without other operations being carried out such as the connecting of the elements in the model, when the figure will be nearer 4 minutes.

Using the simple approximation, the overall time for calculation of 1024 impedance values is over 3 hours, whereas the full expression would take an order of magnitude longer. It was felt that an increase in magnitude was not justifiable bearing in mind the negligible difference in accuracy. The errors due to inaccuracy in bore measurement were known to be of this order (1%) anyway, for example.

This approach seems to be justified bearing in mind the
limitations of the experimental procedures. The good agreement between experimental measurements of instrument impedance and the predictions, with only a rough model of the bore shape, in both the input impedance and transfer function domains, at least as far as the correlation between impedance peak frequencies is concerned, is significant.
2.3 THE LOOK-BACK METHOD

This section deals with the concepts involved in the method called by the author the look-back method. The types of element which may be used are discussed.

2.3.1 The look-back method and the two-port

The method is named the look-back simply because it involves taking a complex shape, approximating it by an ensemble of simple shapes and, starting with the load end of the structure progressively calculating the input impedance and transfer function of each element looking back towards the source, until the source is reached.

Referring to the section on two-ports, it can be seen that any linear system may be represented by a two-port network. The internal representation of the network in lumped impedance terms may be exceedingly complex. If the two-port is viewed as a succession of two-ports of simpler internal representation, the complete system may in turn be described in terms of the parts. This is schematically shown in Fig.2.5.3.

The input impedance of one section is employed to load the next until the source is reached. Similarly, the transfer functions of the elements are multiplied.

In musical instrument work, WOGRAM (1972) gives a highly simplified representation of the instrument bore shape in terms of an exponential horn, and cylindrical tubes. The solutions of the wave equations are amenable (e.g. OLSON, 1957) and the input impedance, without losses, is calculated accordingly. Due to the crude nature of
the approximation to real bell shapes, this model is of limited use. YOUNG (1950), employs a slightly different method, using many sections of exponential shape to approximate to the horn shape. He too assumes an unrealistic bore profile, entirely neglecting the leadpipe and the mouthpiece backbore. Absorption effects are also neglected.

In the present work, cylindrical elements are employed for modelling, for several reasons:

A large proportion of a brass instrument's bore is cylindrical, so only one element need be employed in these regions.

Boundary layer losses are more easily calculated than in other shapes.

Temperature gradients may be easily included to model the effect of the instrument actually being played. This is not so easy with other shapes.

The effect of a discontinuity between cylindrical sections of different radii has been extensively discussed (MILES, 1944), (KARAL, 1953), and may be modelled easily, cf. Chapter 2.4. The effect on propagation of a discontinuity between other shapes is less clear.

Both input impedance and transfer function of the cylindrical elements are predictable to a high degree of accuracy. The similar expressions for other shapes are not useful due to lack of completeness, or neglect of losses, etc.
In a real instrument, there are discontinuities in the bore, for example at the valves or at a junction between a slide and the main bore. The effect of this may be conveniently modelled using cylindrical sections.

By using many sections, it can be shown (YOUNG and YOUNG, 1961) that the approximation reduces to the exact solution of the Riccati equation for a wave in the continuous structure.

YOUNG (1960), states that 50,000 cylindrical elements are needed to approximate the bore shape of an exponential horn, to give a result within 1 per.cent. of the exact equation. It is not clear what parameter this tolerance applies to, but in terms of impedance peak frequencies, far fewer elements are in fact needed (of the order of 100-200) to give a result within the specified accuracy. In any comparison between the look-back method and experimental measurement (Fig.2.3.2), the effects of a small temperature deviation are of far greater effect than poor modelling of the bore. Thus, temperature as well as shape need be taken into account if an accurate representation of the instrument is to be achieved.

This may be illustrated by the graph of input impedance for the same instrument shape, but evaluated at different temperatures (Fig.2.3.3). The percentage error in the frequency shift of the impedance peaks is reasonably constant with linear temperature gradient, and changes more when a temperature gradient more representative of an real instrument is employed.
Comparison of measured and predicted (dashed) input impedance of a trombone
Comparison of measured and predicted (dashed) transfer function of a Trombone

20 \log \left| \frac{p(\text{out})}{p(\text{in})} \right| \quad \angle \left( \frac{p(\text{out})}{p(\text{in})} \right)
Input impedance of a trombone predicted for three temperatures.
Input impedance of a straight tube evaluated at different temperatures.
2.4 MODELLING OF THE STEP DISCONTINUITY

This section deals with the modelling of the step discontinuity of the acoustic transmission system formed when two circular tubes of differing cross-section are joined together.

2.4.1 Plane wave model

The plane wave model of the propagation across the discontinuity is given in many texts on acoustics (e.g. SKUDZRYK), and so only the general result is quoted here (Fig.2.4.1). MALECKI gives the simple expressions for the pressure and volume velocity at each side of the discontinuity. This model assumes a boundary condition that the volume velocity is conserved across the discontinuity at z=0, and that the sum totals of the acoustic pressures on each side of the discontinuities are equal.

This boundary condition may be expressed as:

\[(V_i+V_r) \pi a^2 = V_t \pi b^2\]  
\[P_i + P_r = P_t\]  

-(A)

If the incident, reflected and transmitted plane waves are expressed by:

\[P_i = A_i \exp j(\omega t-kz)\]  
\[P_r = A_r \exp j(\omega t+kz)\]  
\[P_t = B \exp j(\omega t-kz)\]  

-(B)
Fig. 2.4.1  THE STEP DISCONTINUITY

---

Fig. 2.4.2  EQUIVALENT CIRCUIT

---
and defining the pressure reflection and transmission coefficients as:

\[ R = \frac{A_r}{A_i} \]  

\[ M = \frac{B}{A_i} \]  

Then it can be shown that the reflection and transmission coefficients, in terms of the characteristic impedances of the two sections, are:

\[ R = \frac{Z_b - Z_a}{Z_b + Z_a} \]  

\[ M = \frac{2Z_a}{Z_a + Z_b} \]  

These simple expressions give an adequate description of the basic nature of the wave propagation in the discontinuity region if only the approximate values are required. The reason for the approximation lies in the assumed boundary condition, (A). The equation for conservation of volume velocity in the axial (z) direction does not adequately describe the physical situation in the plane z=0.

The element of volume velocity \( V_t \) is assumed to exist equally over the cross-section in region B. This is not the case, as at z=0, in the region \( a<r<b \), the axial volume velocity is in fact zero. To allow for this more accurate description of the propagation, the boundary condition must be altered and higher order modes considered to account for the non-uniform velocity distribution in the region adjoining z=0.

In most cases in the present work, the duct radius is much smaller than the plane wavelength in the axial direction, so the higher order modes may be considered evanescent, and of importance only in the immediate vicinity of the discontinuity.
2.4.2 Higher-order mode model

The theory of propagation in discontinuity regions has been discussed extensively by MILES (1953), KARAL (1953), and ALFREDSON (1972). The first two authors are concerned with transmission and reflection only, while the last utilises the theory to calculate radiation directivity from duct ends. The starting point in all cases is the general solution of the wave equation for a circular duct:

\[
p = B_0 \exp(\pm jkz) + B_1 \exp(-jkz) + B_n \text{Jo}(\{Kn\}.[r/a]).\exp(Y_n z)\]

Where the factor \(\exp(jwt)\) has been omitted for clarity, and:

- \(B_0, B_1\) = Amplitudes of plane modes in each axial direction
- \(k\) = plane wavenumber
- \(n\) = order of mode
- \(B_n\) = amplitudes of the higher order modes
- \(Kn\) = higher order wavenumbers
- \(Y_n\) = Higher-order propagation coefficients expressed in the z direction
- \(r\) = radial distance from tube axis
- \(a\) = tube radius.

The radial wavenumbers are derived from the solutions to the radial wave equation:
Which assumes that the radial component of velocity at the walls is zero, i.e. that the duct walls are perfectly rigid. Other equations exist for yielding walls, (e.g. MALECKI, though his notation is confusing) which may be substituted if the wall face impedance is known. In the present case, the wall face impedance is extremely high, and as this in any case a third-order correction, the inclusion of the equation for yielding walls was felt to be unjustified. Two equations of form (E) are set up in regions A & B, and solved simultaneously using the higher-order boundary conditions:

\[ \text{Pa} \bigg|_{z=0} = \text{Pb} \bigg|_{z=0} \quad 0 < r < b \]

\[ \frac{1}{j\omega} \frac{\partial \text{Pa}}{\partial z} \bigg|_{z=0} = \frac{1}{j\omega} \frac{\partial \text{Pb}}{\partial z} \bigg|_{z=0} \quad 0 < r < b \]  

\[ \frac{1}{j\omega} \frac{\partial \text{Pb}}{\partial z} \bigg|_{z=0} = 0 \quad a < r < b \]

The resultant mathematics is not trivial and is quoted in both MILES and KARAL so the result only is given here. One important result is that the contribution to the total volume velocity by the higher order modes is zero, i.e. the total volume flow at any place is identical to the plane mode flow. This result is of paramount importance, as the propagation may be modelled in terms of plane modes only, and the impedance approach is not rendered invalid by the consideration of higher-order modes. This is followed by the
consideration that the effect of the higher-order modes at the discontinuity may be expressed in terms of an acoustic series inductance. The equivalent circuit diagram is given in Figure 2.4.2. The magnitude of the discontinuity inductance $Z_d$ is proportional to the size of the ratio of the tube radii.

The accuracy of the determination of $Z_d$ may be adjusted by the number of higher-order modes considered. Using KARAL's approximation to the exact solution for the inductance:

$$L(\xi) = \frac{8 \rho}{3 \pi^2 a}, \quad H(\xi)$$

given that:

$$H(\xi) = \frac{3 \pi}{2} \sum_{m=1}^{\infty} \frac{J_1(x_m \xi)}{(x_m \xi)^2} \left[ x_m J_0(x_m) \right]$$

where:

$$x_m = \text{solutions of } (F)$$

$$\xi = a/b \quad (0<\xi<1)$$

The values of $H(\xi)$ were calculated for values between $0<\xi<1$, using the first fifteen modes and a series solution for the Bessel functions. The eigenvalues of the modes are given in the tables by ABRAMOWITZ and STEGUN. Karal also gives a graph of $H(\xi)$, reproduced in Fig.2.4.3. As in the present work, the values of $\xi<0.1$ were encountered (a small discontinuity ratio) and the convergence of the Bessel series is slow in this range, a piecewise linear approximation was employed to keep calculation of the correction factor down to manageable limits. Three regions of approximation were chosen to give a good fit (Pearson's $r>0.98$) and as this is a second order correction, this was felt to be sufficient. The effect of including this inductance is that the duct is increased slightly in acoustic length. This is conceptually easy to understand if one considers the result for $\xi=0$ (open end), and compares
it with NEDERVEEN's result for the termination impedance of an unflanged pipe. In turn, this is analogous to the "End-correction" discussed in the literature. Thus, the discontinuity inductance can be considered as a form of "internal end-correction".

Comparison between Nederveen's and Karal's results:

Karal:

Solution for open end.

\[ a = a, \quad b = \infty \]

Therefore \( \xi = 1 \)

\[
L(\xi) = \left( \frac{8 \rho}{3 \pi r^2} \right) = (0.327)(1/r)
\]

Nederveen:

\[
L = (0.613)(\frac{\rho}{\pi r}) = (0.385)(1/r)
\]

Where \( r \) = tube radius.
2.5 REFLECTION METHODS

The division of a continuous structure into many elements has already been discussed in terms of impedance in Chapter 2.3. The analysis is developed further in the time domain in this section, which deals with the consideration of reflections and multiple reflections and the methods used to manipulate them.

The detail of how the reflection at each junction of the bore-shape model affects the input reflection response is considered.

An organic method for the recovery of the area coefficients of the model from measured acoustic impulse data is discussed. This may be further extended to generate such a response, if the bore shape is known. The effect of different terminations of the model are examined, together with the effects of absorption and non-ideal test pulses.

2.5.1 Simple reflection model

The instrument bore may be modelled as a manifold of cylindrical sections, as has been shown (in the frequency domain) in Chapter 2.3. Extending the concept to the time domain, we again assume these cylindrical elements have a reflecting plane at the interface between sections (Fig.2.5.3). If sections of equal length are presumed, subsequent processing with an equally-sampled signal is facilitated. A typical experimental impulse response of a French horn is given in Fig.2.5.0.

In the present problem, reflection coefficients are generally much less than unity. The amplitudes of the multiply reflected components go as the power of the number of reflections, so if the reflections are small, these may be neglected.
Impulse response of a keyboard

Reflection from bell

Reflection from valve

Reflection from slide (3rd valve)

Reflection from valves

Reflection from valves

Reflection from slide

Reflection from waterkey

Reflection prior to slide

Initial impulse
A simple model is developed using this result, with the additional restrictions that the source is nonreflecting (i.e. no reflection back into the structure from the wave returning to the source), and that the attenuation is zero. The first condition may be satisfied experimentally by suitable choice of source termination (Table 2.5.3), the second may be simulated with real data by multiplying the impulse response with a gain function (Fig.2.5.1), which amplifies progressively so that the first reflection from the bell is the same magnitude as the input pulse.

We know that the reflection coefficients inside the structure may be defined in terms of the internal geometry. Similarly, the transmission coefficients from one manifold element to another are known, and a general term for the pressure at the input at all times may be derived (Fig.2.5.2).

From inversion of this solution, the reflection coefficients may be found from knowledge of the pressure at the input, and hence the areas of each manifold element found recursively. Only the area at the source end of the structure and the pressure-time record need be known. A result of this approach is compared to measurement of the bore shape in Fig.2.5.4.

2.5.2 Multiple reflection model

The simple model, while giving good results when the reflections are small, breaks down in regions of large reflection coefficient, or when many small reflections are concentrated in a small region, thus making the identification of the particular reflection difficult due to phantoms.

A suitable model is needed to enable consideration of multiple
Fig. 2.5.1

Impulse response of a French horn multiplied by a linear gain function.

MAX. AMPLITUDE = 63.1 Pa
RECORD LENGTH = 33.333 mSec.
LENGTH = 2.537 (synthesised, +/- 0.2m.)

LENGTH = 2.76 (measured)

MAX. RADIUS = 0.12 (measured)

Comparison of measured and synthesised bore shapes of a Trombone

(Simple reflection model)
We start with the fundamental equation for pressure reflection coefficient at the junction between the ith and (i+1)th sections, in a structure of N elements (Fig.2.5.2 (A)). In this case Z(i) and A(i) are respectively the characteristic impedance and area of the ith section. Assuming that the sections are lossless transmission lines, then Z(i) is real, and hence R(i) is also real and independent of frequency. It should be noted that this is unrealistic for real data, but this criterion is necessary in the model to avoid signal distortion due to differing attenuation of different frequency components of the signal, (i.e. dispersion) though it may be relaxed slightly as will be shown later. Considering the waves travelling in both directions at a junction, below:

\[
\begin{align*}
R(i) \\
\downarrow \quad (i+1) \\
\hline \\
f(i) \to f(i+1) \to \\
b(i) \leftarrow b(i+1) \leftarrow
\end{align*}
\]

where f are forward-travelling waves, and b are backward. The pressure transmission coefficients at the junction in both directions are given in Fig.2.5.2 (B).

Modelling the bore shape as a general manifold of N sections (Fig.2.5.3), and starting at a time before the test impulse is put in, it is possible to invoke causality to model the subsequent pressure changes at any point in the structure due to an input pulse of either
REFLECTION AND TRANSMISSION COEFFICIENTS

\[ R(i) = \frac{Z(i+1) - Z(i)}{Z(i) + Z(i+1)} = \frac{A(i) - A(i+1)}{A(i) + A(i+1)} \]  

\[ A(i+1) = A(i) \times \frac{1 - R(i)}{1 + R(i)} \]

Forward wave transmission coefficient = \(1 + R(i)\)
Backward wave transmission coefficient = \(1 - R(i)\)

GENERAL TERM FOR PRESSURE

\[ P(n) = \text{pressure in first element at time } t \]

\[ P(1) = p(1) \]
\[ P(2) = p(1) \times R(1) \]
\[ P(3) = p(1) \times (1 + R(1)) \times R(2) \times (1 + R(1)) \]
\[ P(4) = p(1) \times (1 + R(1)) \times (1 + R(2)) \times R(3) \times (1 - R(2)) \times (1 - R(1)) \]

\[ \cdots \]

\[ P(n) = p(1) \prod_{k=1}^{n-1} \left( 1 - R^2(k) \right) \]  

(assuming no multiple reflections)

THE GENERAL MANIFOLD
pressure or volume velocity, (depending on the input termination). The symbols used in the analysis are to be found in Glossary 2.5.

The total reflected pressure signal seen at the input end of the manifold can be considered as the sum of two pressure signals; the direct reflected signal from the farthest junction inside the causality limit, and the signals due to multiple reflections at all intermediate junctions.

Starting with the signal seen at the input at elapsed time $T$, $p(2kT)$, the direct reflected signal is evaluated. By causality, the junctions for $m, m > k$ cannot give rise to any signal as the time needed for a signal to be returned from such a junction is $2mT > 2kT$. The $k$th junction only makes one contribution by direct reflection to the signal $p(2kT)$. This is the result of the impulse being transmitted by $1-R(m)$ in the forward direction, at each junction on the way to the $k$th junction, being reflected by $R(k)$, and returning via the transmission coefficients $1+R(m)$ on the backward journey to the input.

Hence:

$$p(2kT) = R(k) \prod_{m=1}^{(k-1)} (1 + R(m)) \left(1 - R(m)\right)$$

or:

$$p(2kT) = R(k) \prod_{m=1}^{(k-1)} (1 - R(m)^2)$$

The contribution to the signal seen at the input due to multiple reflections in the sections for $m, m < (k - 1)$ is more complex, and is denoted $p'(2kT)$. Then:

Total signal = direct contribution + multiple reflections
or:

\[
p'(2kT) = R(k) \sum_{m=1}^{(k-1)} (1 - R(m)) + p(2kT) \quad -(D)
\]

From this, an equation for \( R(k) \) may be deduced:

\[
R(k) = \frac{[p(2kT) - p'(2kT)]}{\sum_{m=1}^{(k-1)} (1 - R(m))} \quad -(E)
\]

Starting with \( k=1 \) and a known pressure signal, this equation may be applied recursively with increasing \( k \) to obtain \( R(k) \). From this data, and using equation (A), it is possible to recover the area coefficients.

2.5.3 Multiple reflection contribution

Taking two arbitrary junctions in the structure, with forward and backward waves existing at time \( t \), it is possible to calculate the amplitudes of the waves after one traverse time, \( T \).
Bearing in mind the results of the previous section for the transmission coefficients, the result after one transit is:

\[
\begin{align*}
 f_{[t+T]} &= f_{[t]}(1 - R)_{m-1} + b_{[t]}(-R)_{m-1} \quad \text{--- (F 1&2)} \\
 b_{[t+T]} &= f_{[t]}R_{m} + b_{[t]}(1 + R)_{m+1} 
\end{align*}
\]

These equations may be applied recursively to determine the multiple reflection contribution to the input reflection response, directly from measured data, and hence the simply reflected contribution may be found using equation (E). From this, the area coefficients may be found.

2.5.4 Termination impedances and reflection

Several different termination impedances are feasible at either end of the structure. The effects of these are briefly evaluated and tabulated. The differences between the effects at the input and output are examined.
<table>
<thead>
<tr>
<th>TERMINATION IMPEDANCE</th>
<th>INPUT</th>
<th>OUTPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>zero</td>
<td>Pr = -Pi</td>
<td>not practically possible</td>
</tr>
<tr>
<td>(open circuit)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>infinite</td>
<td>Pr = Pi</td>
<td>Pr = Pi</td>
</tr>
<tr>
<td>(short circuit)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zc (characteristic)</td>
<td>Pr = 0</td>
<td>Pr = 0</td>
</tr>
<tr>
<td>Zr (radiation)</td>
<td>not practically possible</td>
<td>Pr = ( \frac{(Zr - Zc)}{(Zr + Zc)} ) Pi</td>
</tr>
</tbody>
</table>

At the input:

Case 1.

\[ Z_{source} = Z_{characteristic \ of \ structure} \]

\[ R = \frac{(Zc - Zc)}{(Zc + Zc)} = 0 \]

hence no reflection

Case 2.

\[ Z_{source} = 0, \ open \ circuit \ input. \]

\[ R = \frac{(0 - Zc)}{(0 + Zc)} = -1 \]

Pressure totally reflected antiphase.

Case 3.

\[ Z_{source} = \text{infinite, short circuit input} \]

\[ R = 1 \]

Pressure totally reflected in phase
At the output: Case 1.

\[ Z_{\text{load}} = Z_{\text{characteristic}} \]
\[ R = 0, \text{ no reflection} \]

Case 2.

\[ Z_{\text{load}} = \text{infinite, short circuit output} \]
\[ R = 1 \]
Pressure reflected totally, in phase

Case 3.

\[ Z_{\text{load}} = Z_{\text{radiation}} \]
\[ R = \frac{Z_r - Z_c}{Z_r + Z_c} \]
R may be complex, hence pressure reflected with change of amplitude and phase.

2.5.5 Non-ideal pulse excitation and calibration

For analysis of experimentally-obtained data, some consideration must be given to the fact that no real pulse source is ideal, i.e. generates a delta function. Typical departures from the ideal are finite pulse width, secondary spikes and ringing. All of these features will cause serious loss of accuracy in the multiple reflection methods, if not allowed for.

One approach to the solution is to record the pulse alone, with no system of any kind attached, to measure the pulse shape. This can be then introduced into the model, progressively in time, and a kind of deconvolution results. There are severe difficulties in measuring the pulse shape experimentally, however. The source is in general coupled to the system in some way, and in general the coupler will have its own
characteristics, which change drastically when the system is removed, as the termination impedance will be altered. Another method which may be employed to remove this effect is to terminate the coupler in a perfectly matched load to eliminate reflections. This again is difficult, as in general the characteristic impedance of the coupler is not known. A perfectly absorbing load also eliminates reflections, however, and this is far easier to approximate experimentally.

Yet another approach is to employ the cepstral methods of Homomorphic Deconvolution, as discussed in Chapter 2.7, where the initial impulse is separated from the subsequent response. If this procedure is followed, there is no need for the calibration whatsoever, as (in theory at least), it is possible to extract the true impulse response with this method also. In this case, the deconvoluted data may be applied directly to the multiple reflection process.
<table>
<thead>
<tr>
<th>Source impedance</th>
<th>Reflected pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero (short circuit)</td>
<td>( b(1) = -f(1) )</td>
</tr>
<tr>
<td>Infinite (open circuit)</td>
<td>( b(1) = f(1) )</td>
</tr>
<tr>
<td>Characteristic impedance of first section</td>
<td>( f(1) = 0 ) for delta function excitation ( f(1) = \text{calibration pulse amplitude} ) for real excitation</td>
</tr>
</tbody>
</table>
2.6 IMPULSE RESPONSE AND THE F.F.T.

Many time signals are erroneously classified as impulse responses. To clarify the matter, a detailed view of the response of linear systems is needed, in order not to confuse the responses at input and output, reflections, and the response of the system itself. Furthermore, the real and imaginary parts of the Fourier transformed signal are related by the Hilbert transform, given that the signal is real and causal. The effect of this property on the methods used to process experimental data is discussed.

2.6.1 Definitions

For a linear causal system, with transfer response $H(w)$, the impulse response $h(t)$ is defined by the inverse Fourier transform. This impulse response is exactly the same as the signal seen at the output of the system, providing the input is a Dirac delta function.

These responses are related by:

$$G(w) = F(w) \times H(w)$$

$$g(t) = f(t) * h(t)$$

where $*$ denotes convolution.

By causality, we have the relations:

if $f(t) = 0$ for $t < t'$

then $g(t) = 0$ for $t < t'$
In the present case, the signal is measured at the input end, not the output, so the quantity measured is not the impulse response. This may be clarified by Fig. 2.6.1 & 2., where \( f'(t) \) is the signal seen at the input end after the delta function has been introduced. This quantity is denoted the input reflection response. The distinction between the boundary conditions for the two cases must now be made clear. For the impulse response, the input signal must only be a delta function, so the input condition is that of a short-circuit input after the introduction of the delta function, for the case \( g(t) = h(t) \).

In the case of the input reflection response, the input must be open-circuit, or no signal will be seen. Hence the boundary conditions are different, i.e. 'closed pipe' and 'open pipe' respectively.

In previous work on musical acoustics by PRATT(1978) and ELLIOTT(1979), the impulse response has been defined as either the inverse Fourier transform of the transfer response (more nearly \( g(t) \)) or, more frequently, the inverse Fourier transform of the input impedance.

KRUGER(1979) measures a quantity more nearly the input reflection response by means of the STL-Ionophone, as described by FRANSSON and JANSSON (1975). He places the Ionophone in the mouthpiece cup, but does not discuss the effects of possible cavity resonance or the effects of the compliance of the cup volume upon the measurement.

Clearly, then, the inverse Fourier transform of the input impedance is nothing like either the true impulse response \( h(t) \), measured at the output end with the input short-circuited, or the input reflection response measured at the input end with the input open circuit. What this quantity actually is can be deduced from the properties of the Fourier Duality Theorem.

Starting from the fact that the quantity is the inverse Fourier transform of the input impedance, defined at a plane in the input of
Fig. 2.6.1 LINEAR CAUSAL SYSTEM
Domain       Input       System       Output
Time         f(t)        h(t)         g(t)
Frequency    F(\omega)   H(\omega)   G(\omega)

Fig. 2.6.2 IMPULSE REFLECTION RESPONSE
f(t)        h(t)        g(t)
f'(t)
the system, which in turn is defined as:

\[ Z(w) = \frac{p(w)}{U(w)} , \]

\[ p(w) = Z(w) \times U(w) , \]

then as a consequence of the Duality theorem,

\[ z(t) = F^{-1} \{ Z(w) \} \]

and so

\[ p(t) = z(t) \ast u(t) \quad (\ast = \text{convolution}) \]

Thus, the inverse Fourier transform of the input impedance is the pressure signal seen at the input plane when a delta function of volume velocity is applied, with the input open circuited.
2.6.2 The Hilbert transform

The Hilbert transform is related to the Fourier transform, but differs in respects which make the application of both transforms together more useful than either alone (Fig. 2.6.3).

The Hilbert transform is especially useful in the case of signals which are real and causal (OPPENHEIM, 1975). Strangely, the Hilbert transform of such a signal is itself non-causal (Fig. 2.6.4). It also is domain-preserving, i.e. a signal Hilbert transformed in the frequency domain remains in that domain, which is not true of the Fourier transform. The most useful property of the Hilbert transform is that it relates the real and imaginary parts of the frequency response of a real causal signal. This result may be extended to include the modulus and phase, but a stricter condition applies that the system must be minimum phase for the modulus and phase to be a Hilbert pair. This implies that the inverse Fourier transform of a function with its Hilbert transform is one-sided, i.e. the inverse transform has either the real or imaginary part uniquely equal to zero. This may be applied in many situations where access to a system is limited to only one measurement, and the whole of the system response (in time and frequency domains via the Fourier transform) is needed.

As an illustration, Fig. 2.6.5 shows a comparison between experimental real and imaginary parts of a trombone input impedance, and the real part of the impedance and its Hilbert transform. Fig. 2.6.6 shows this data converted to modulus and phase form.

The change in overall gradient of the two imaginary portions illustrates a physical property explored by WATKINSON (1981), that of the inertance of the narrow mouthpiece throat. This is superimposed on measured impedance data but is not a property of the real part of the
A Hilbert pair illustrating the acausal nature of the transform
Comparison between Real and imaginary parts of the input impedance of a trombone, and the Hilbert transform of the Real part of the input impedance (dashed).
The data in Fig.2.6.5. converted to modulus and phase form, illustrating the minimum phase condition.
impedance, being purely imaginary. Hence it is not 'seen' by the real part, and as such effectively does not exist when the Hilbert transform is taken. Hence the effect of capacitances or inductances in series or parallel with the input port of a network are removed. This is illustrated by the schematic of the network representation of a musical instrument with, for example, a constriction in the mouthpiece end.

Another useful property of the Hilbert transform is that the modulus and phase of the response are related via the natural logarithm. This has considerable application in fields where one of these is not available. The phase is found simply by taking the Hilbert transform of the log of the modulus (Fig. 2.6.7).

There are several ways of computing the Hilbert transform, and some methods rely on the Fourier transform of the signal being available.

Basically, in the original domain, the procedure is to convolve \( f(x) \) with \( 1/(-x) \), or alternatively, multiply the Fourier transforms of these two quantities, and inverse transform. This second operation is better suited to computation, for the reasons of speed and roundoff error, both of which are inherently poor in numerical convolution operations, with respect to the FFT.

The Fourier transform of the second term is simply \(-j\) for positive frequencies and \(+j\) for negative. Thus the Hilbert transform corresponds to filtering in the other domain, where the spectral components are unchanged in amplitude and their phases changed by \( +/- \pi/2\), depending on the sign of the frequency, (BRACEWELL, 1978). Other useful properties are found by applying the concept of the analytical signal and the properties of odd and even sequences, but will not be enlarged on here.
Log modulus of impedance

The phase found by taking the Hilbert transform of the log of the modulus of the input impedance

Derived phase (radians)
This section is concerned with the applications of signal processing to the recovery of the impulse response from measured pulse data. The recovery of bore shape from this data is discussed, together with the combinations of various methods. The powerful method of the cepstrum is employed in seismology, where neither the form of the exciting pulse, nor the echo path are known \textit{a priori}, to recover both pieces of information.

2.7.1 Definition of the Cepstrum

The term 'Cepstrum' was first coined by BOGERT et.al.(1963), and the analysis had recently been vastly simplified by the introduction of the Fast Fourier Transform by COOLEY et.al. in the early 1960's.

The general problem in seismology is the separation of the arrival of one or more echoes, possibly distorted and/or superimposed upon the original pulse (often called the wavelet in the literature). The distortions and superimpositions, together with noise, in general make the determination of the arrival time difficult, if not impossible, by inspection of the time record. The standard technique of autocorrelation, which compares the signal with a time-shifted version of itself, can be useful if the distortion of the echo is not great, as then the echo shape is similar to the original pulse, and hence the autocorrelation function is high at the echo arrival time (the echo epoch). However, in the presence of noise and distortion, this technique rapidly become unable to detect the echo. Similar limitations exist in the case of the autocovariance. BOGERT et.al.(1963) analysed the effect of a simple echo superimposed on a
time series of form:

\[ z(t) = y(t) + a_y(t - d) \]  \( \text{-(A)} \)

where:

\[ y(t) = \text{original time series} \]
\[ z(t) = \text{time series with echo} \]
\[ a = \text{echo amplitude} \]
\[ d = \text{echo delay} \]

to evaluate the effect on the spectrum of the original time series. Now if \( Y(f) \) is the power spectrum of the original time series \( y(t) \), then by application of the Fourier Transform to (A), we get:

\[ Z(f) = Y(f) \{1 + 2a\cos(2\pi fd) + a^2\} \]  \( \text{-(B)} \)

If the echo amplitude is small, which it usually is, then the logarithm of the modified power spectrum becomes:

\[ \log \{Z(f)\} = \log \{Y(f)\} + 2a\cos(2\pi fd) \]  \( \text{-(C)} \)

ignoring the terms in \( a \).

The effect of taking the logarithm is immediately apparent, as the effect of the echo in the time domain is simply to add a ripple in the power spectrum of nearly cosinusoidal form, with the ripple 'frequency' and amplitude are related to the echo parameters \( a \) and \( d \). Clearly, confusion is going to arise between the terms used to describe the ripple in the frequency domain and those used in the frequency domain.

BOGERT et.al. overcame this neatly by shuffling the letters in
the words commonly in use, to coin a new word applicable to a similar quantity in the new domain. This way, the new domain is the Cepstrum (cf. spectrum), and so on. A full catalogue of the new terms is given in Glossary 2.7. The problem of multiple echoes then arises. If several echoes of differing amplitudes and delays are present, then the overall effect is to superimpose many different ripples on to the original spectrum, (which is probably highly irregular already). The identification of these ripples is then impossible by eye, and further analysis is therefore needed. The conceptual leap which makes this technique so useful is the divorce from the real and physical. The log spectrum of the time series is itself treated as a time series, and Fourier Transformed to the new domain (the Quefrency Domain) (Fig.2.7.1). The transform effectively identifies the ripples in the log power spectrum and converts them to spikes in the quefrency domain. The position and size of the spikes then correspond to the echo epoch and amplitude.

The usual practice of windowing in the time domain also applies to the transform from spectrum to cepstrum, although the effect of such a window is not clear, it is needed to eliminate discontinuity effects of the transform. (B & K., 1977).

Another advantage of cepstral analysis is that not only are the echoes identified, but the original wavelet cepstrum is also present at a low quefrency, which corresponds to a short arrival time. This means that, if suitable care is taken with phase (and saphe), then it is possible to separate the effects of the echo and the original signal by use of a lifter (filter), and transform back to the spectrum and waveform of either the echo(s) or the original wavelet. As is to be expected, the power cepstrum (based on the power spectrum), is of no use in reconstruction as the phase information is lost in the squaring process.
COMPUTATION PROCEDURE FOR AUTOCORRELATION AND CEPSTRA

Fig. 2.7.1
KEMERAIT and CHILDERS (1972) improved the technique of cepstrum analysis with the introduction of the distinction between the power and complex cepstrum.

ULRYCH (1971) is concerned with Homomorphic Deconvolution: that is, the deconvolution of a signal with itself, after cepstrum processing, to identify the transfer characteristics of the system, and the original wavelet (Fig.2.7.2).

In the present case of musical instruments, it can be seen that these techniques have application in the recovery of the echo, and hence the bore shape from acoustic data, either of time series or spectrum form. As discussed in Chapter 4, the spatial resolution gained by the consideration of an experimentally-derived time series is better than that possible from that obtainable from input impedance measurement, due to bandwidth criteria.

Examples of autocorrelation, power and complex cepstra are given in the appendix.


2.7.2 Definitions of cepstra and associated processing

POWER CEPSTRUM

is the F.T. of the log of the modulus of the spectrum, with the phase set to zero.

COMPLEX CEPSTRUM

is the F.T. of the complex log of the spectrum, with the linear component of the imaginary part removed. (Phase unwrapping). The linear component must be stored if reconstruction is done.
HOMOMORPHIC DECONVOLUTION

COMPLEX CEPSTRUM
Lifter
F
Re
Im
Restore linear component
Complex
Exp
F⁻¹
DECONVOLVED SIGNAL

Fig. 2.7.2
AUTOCORRELATION

is the inverse F.T. of the log of the modulus of the spectrum.

DECONVOLUTION

is the inverse F.T. of the complex exponential of the F.T. of the liftered complex cepstrum with the linear phase component restored.
CHAPTER 3: EXPERIMENTAL METHODS

The experimental methods employed in the measurement of mechanical and acoustic properties of brass instruments are reviewed. The problems of measuring internal bore shape are examined, together with apparatus designed to overcome some of the difficulties. The apparatus employed in the assessment of acoustic properties is described, together with the methods used to investigate the effect of temperature gradients.

The impedance measuring device is briefly described, and the design and construction of two versions of the impulse source given.
3.1 MEASUREMENT OF BORE SHAPE

This section deals with the various methods developed to measure the radius of the bore of a brass instrument along its length. Applicability and accuracy of methods are described.

3.1.1 Manometric method

This method is most useful in tubes of small radius and which are without bends. One arm of the manometer is a glass tube, with a scale to facilitate accurate readings of the meniscus level, and the other is the tube under test. The internal radius of the glass tube must be known. This may be measured to a high degree of accuracy, as tube manufacturers work to extremely close tolerances (typically 0.01 %) and the degree of concentricity is excellent. The glass tube is connected to the tube under test by a thick rubber tube. This eliminates error due to the hydrostatic pressure in the rubber tube deforming the tube walls. A burette is employed to dispense known quantities of fluid into the manometer. The fluid used was water, plus a small amount of detergent to aid wetting and minimise the meniscus. Other, better fluids are available. A starting level is set, at a known radius of the tube under test (this may be measured by means of a vernier caliper), the fluid level noted, and a known amount of fluid inserted. The new level is noted. Assuming the increment in volume of the tube under test to be a truncated cone, a simple relation exists which was used to calculate the radius of the tube. In several checks over many increments on three different tubes, the accuracy obtainable was found to be more than sufficient for present purposes, and repeatable to better than +/- 0.001 cm. The measured shape of a typical French horn
Internal bore shape of French Horn leadpipe measured by the manometric method.

Fig. 3.1.1

Distance from mouthpiece throat

Radius, cm.
leadpipe, medium-large bore, is shown in Fig.3.1.1. A curve fitting procedure was used to approximate the equation governing the radius of the tube. For French horn and Trumpet leadpipes, the best fit was obtained assuming a Bessel-type flare. This is slightly different from that suggested by PYLE (1975), and completely different from that claimed in the patents of CARDWELL (1967), who states that leadpipe shapes are predominantly conical.

3.1.2 Shadowgraph method

This was developed by the Audio-Visual Aids Unit of the University of Surrey. It is a photographic method utilising a pseudo-parallel light source. Full-size, scaled prints of the silhouette of the object, which may be measured manually or otherwise to give the outer radius of the object. A smaller scale print is shown in Fig.3.1.2. The wall thickness may then be subtracted to give the internal radius. Owing to imperfections, the prints displayed a slight parallax error, and as a result scaling varied by about 1-2 % over the print area. This was overcome by using a separate scale factor for different areas of the print. This method is convenient for rapid measurements of sections of large dimensions, where ordinary calipers are difficult to use due to the problems in locating the mid-point of the section.
3.1.3 X-Ray shadowgraph

This is another photographic method, more applicable to the measurement of the internal radius of the tube directly in regions where stays, joints etc. render the ordinary shadowgraph method useless. It is also applicable in regions of bent tubes, where the manometer will give an erroneous result. In particular, the valve part of the instrument may be examined after assembly. Features such as blobs of solder, ill-fitting joints etc., may be readily identified and used in the modelling process. The tube wall thickness may also be estimated in this way. The main drawback to achieving good results is the gaining of adequate contrast on the X-ray negative. If this is not done, the walls appear blurred and an accurate estimation of any dimension is difficult. Further, the process of 'seaming' with solder and the assembly of the component parts of instruments with soft solder means that the high lead content, being more opaque than the surrounding areas to X-Rays, obscures detail further. Scaling is unreliable in the negatives as most X-Ray tubes have a small anode area which produces a point source, hence introducing parallax error. Careful calculation of scales on all regions of the print is required.

3.1.4 Ball of known radius

This method is used widely by manufacturers both as a measuring tool, and to ensure that a tube of constant radius is produced, (after a bending process for example, by forcing ball bearings of a known radius down the tube. Repairmen also use a set of graduated 'Plugs' to remove dents in instruments. The ball bearings may also be used to measure the radius of the tube, by dropping them into the tube and
measuring their position. There are several inherent disadvantages to this technique:

Non-circular tubes or dents will cause error by stopping the ball at the point where the smallest dimension of the tube is equal to the ball diameter.

Re-entrant bore shapes may be difficult to measure, even if both ends of the tube are accessible. If there are several constrictions, no measurement may be possible in some regions by this method.

Small-radius bends are in general difficult to measure without disassembly of the instrument.

The set of bearings must be finely graduated in size if an accurate representation of the bore shape is to be achieved. This is costly, as many balls are needed.

3.1.5 Disc gauges

A set of discs were made up in the mechanical engineering workshop of the University physics department, specifically for the purpose of measuring bell sections. The discs were calibrated in 0.1 cm. steps in a range from 15 cm. to 1 cm. radius. These were thought to be sufficient to measure most of the brass instrument family to a good degree of accuracy (+/- 1%), with the possible exception of the piccolo trumpet. A smaller set of discs could be made for this purpose. The discs were mounted on a rod with a movable crosspiece, which was
arranged to rest on the rim of the bell while the disc was inserted until it fouled against the bell wall. The material of the discs was chosen as melinex-paper laminate as this material has a relatively high coefficient of friction and hence would not slide around the bore easily. More important, the lamination stiffens the material considerably. Other materials tried tended to deform when a little force was used, which could lead to an erroneous result and also damage the instrument. Moisture absorption of the laminate was also considered as a possible source of error, as it well known that some plastics change their dimensions by several percent when saturated. This problem could well arise in the case of an instrument being measured shortly after it had been played. Melinex laminate has very low moisture absorption. The same disadvantages also apply to the discs as the ball bearings, as far as measurements on re-entrant, dented or non-circular bore shapes are concerned. However, if the location of a dent is known, the disc may be moved past it obliquely if the dent is not too serious. Of course, this method may not be used on the parts of the bell which are bent.

The trombone is particularly easy to measure with this device, as large parts of the bore are straight. A typical result combining the disc, manometric methods and including results found with calipers, with interpolation between inaccessible points is shown in Fig.3.1.3.
Hybrid measurement of a Trombone bore shape

Length (m) | Radius (m) | Temperature (°C)
---|---|---
2.76 | 0.12 | 34.99
3.1.6 Radius of curvature gauge method

The radius of curvature gauge was originally used in optics to measure the radius of curvature of a lens accurately. Essentially the process assumes the surface to be spheroidal, and by measuring the degree of displacement of a point midway between two points on a datum, the radius of the surface may be determined. A large-scale version of the gauge was made up by the mechanical engineering workshop, in order to test the viability of using this method on brass instrument bells, and other regions of curvature. The measuring instrument used was a dial gauge, calibrated in .001 cm. graduations. The gauge was found to be most useful in regions of large radius, where the accuracy was larger. Small dents almost invisible to the naked eye impaired accuracy severely, however. Due to the finite thickness of the probe ends (although pointed to some degree, which damaged instrument lacquer), the position of the point where the datum line was could not be estimated accurately. As the position of contact of the point changed with the surface curvature, this gave rise to an error of unknown magnitude. The limited range of use of this method on the widely-varying curvatures of a typical brass instrument, together with the inaccuracy, led to this method being abandoned.

3.1.7 Microscopic method

The main problem in measuring internal dimensions of small tubes is due to the difficulty of getting the measuring device into the tube. The device must be of necessity small and therefore flimsy. This limits the accuracy, as the device will itself distort if long tubes are measured.
Measurement of the internal shape of a trombone mouthpiece by the microscopic method.

Fig. 3.1.4.

Length (m)

Radius (m)

LENGTH = 0.08 (m)
MAX. RADIUS = 0.01 (m)
One solution developed by the Mechanical Engineering workshop overcame most of these limitations, where an impression of the bore was made by spraying the inside of the tube with a silicone releasent and filling it with a non-shrink epoxy resin. When cured, the impression was removed and measured optically using a travelling microscope fixed to a precision milling machine. The accuracy of this device was typically $\pm 0.01$ mm. Of course, re-entrant shapes could not be measured in this way unless two impressions were taken from each end of the tube. This method was applied to the measurement of mouthpiece contour, where two impressions were made, one of the cup and one of the backbore.

The result of one such measurement (on a Denis Wick 9BS trombone mouthpiece) is given in Fig. 3.1.4.

It can be seen that the throat section is clearly defined, and that the back bore is not simply conical but flares slightly. This is visible if one looks up the backbore, but was previously unmeasurable.
3.2 MEASUREMENT OF TEMPERATURE GRADIENT

Several different methods for the measurement of the internal air temperature gradient are discussed, and results given.

3.2.1 Temperature and playing conditions

The measurement of the internal temperature gradient is of importance if accurate predictions of the instrument's characteristics under playing conditions are to be made.

The position of the impedance peaks has been shown to have a strong effect on the timbre and "feel" of an instrument (Smith, 1978) and it has long been known that the intonation of an instrument varies with temperature.

As it is known that the velocity of sound varies as the square root of the temperature, the careful examination of the bore shape without the consideration of the temperature gradient also is rather surprising, especially if results intended to be comparable to the playing situation are desired.

Little systematic research has been done on the topic however, apart from an empirical review based on a doctoral thesis by Pottle (1943), measurements by Kent (1956) using a thermocouple inside the instrument, and a measurement by Wogram (1972), this time using a thermistor as a detector.

Kent positioned the thermocouple inside the instrument, but unfortunately no results are given, nor estimate made for the perturbation in bore shape by the thermocouple and wires.

Wogram did not state whether the thermistor was positioned inside the instrument to measure the air temperature directly, or was placed
in contact with the tube walls, so the results quoted are somewhat suspect.

He found that the air expelled from the lips was at approximately 36 deg.C., which is to be expected as the air in the lungs must be close to blood temperature, 37 deg.C.

Using a trombone blown by a musician, he plotted the temperature at five points along the bore, and found that the temperature decay was to a first approximation exponential. It was also proportional to ambient temperature, but mostly to the duration of the played note. When there was a pause between played notes, the excellent thermal conductivity of the instrument walls rapidly reduced the internal temperature to nearly ambient.

Wogram concluded that the temperature (and hence velocity of sound) varied more in a normal playing situation than when long notes were being played. To test this, he asked the musician to play a short study which had rests, and recorded the internal temperature.

The result is given as an average of 27 deg.C., with no figure for scatter.

An experiment was made to verify the above results, using a series of thermocouples soldered to the outer wall of the tube. A cylindrical tube of brass was used, with the source being the air velocity regulator and heater employed with the impedance apparatus. In this way, both temperature and airflow could be varied, though these were not totally independent. The results showed that given time to reach equilibrium, the temperature gradient was indeed exponential, though many difficulties with the accurate calibration of the thermocouples, together with ageing, made an accurate conclusion difficult. Statistical curve-fitting was employed to identify the general trend of the data. One interesting fact emerged, that the temperature distribution was not only proportional to the input and ambient
temperatures, but also to the air velocity within the tube. Unfortunately due to poor results a functional relation could not be reliably established, but this is undoubtedly of interest when more accurate models of the transient thermal situation inside a played instrument are made.
3.3 IMPEDANCE AND TRANSFER FUNCTION MEASUREMENTS

The apparatus and programmes used to measure the input impedance and transfer function are briefly described.

3.3.1 Apparatus

The impedance measuring device was designed and built by PRATT (1975), and later refined and extended by ELLIOTT (1979), who added the transfer function measurement. Work by WATKINSON (1981) showed that it was desirable to make both input impedance and transfer function measurements simultaneously, and a further modification was made to allow this.

The apparatus is centred around a Data General Nova 4 computer system. It is used to control frequency and excitation level, and is also used for data acquisition, calibration and subsequent processing. Control of the experiment is effected by means of feedback loops implemented in the programme structure, which produce signals which are interfaced to the equipment digitally.

The exciting device for the musical instrument is an 18 inch loudspeaker, (Cetec 'Gauss', type 5940), rated at 200 watts, enclosed in a substantial damped cabinet. Coupling to the instrument is via an inverted horn, which is terminated in a universal adaptor system which fits all types of brass instruments. The adaptors accept the ordinary mouthpiece of that instrument.

The horn-coupled microphone (B & K type 4170) and hot-wire anemometer (DISA type 55KP11) are placed in the throat of an appropriate mouthpiece, which has been cut in half to enable easy positioning of the anemometer in the centre of the throat. A standard
1/2 inch microphone (B & K type 4134) is placed in the plane of the bell rim of the instrument to measure the external pressure for the transfer function measurement.

The output from the hot-wire anemometer is fed into a bridge (DISA type 55K10) which maintains the wire at a constant temperature, the out-of-balance signal being proportional to the velocity.

The frequency source is a programmable oscillator (Adret Codasyn 201), working into a frequency divider which outputs the oscillator frequency divided by 480, used for excitation, and also 24 pulses per period of the excitation frequency, employed as a clock for the ADC's.

The divided output is fed into a programmable attenuator, which controls the excitation level. Finally, a 100W. power amplifier (H-H type TPA-100D), is employed to drive the loudspeaker.

A two-channel ADC (Micro Consultants type DW 3258) is used for sampling. The channels sample simultaneously, via a sample-and-hold circuit, (to preserve phase information), and so 12 samples per channel per period are taken. An additional switch run from a DAC channel (Micro Consultants type DW 3504) is employed to swap the input to the ADC from the anemometer to the external 1/2" microphone. This is needed for the runs in which both input impedance and transfer function are measured, where pressure and velocity at the input and pressures at the input and output are compared, respectively.

To avoid rectification of the signal from the anemometer, which is sensitive only to flow magnitude and not direction, a D.C. bias in the form of a steady air supply is needed. This is provided by a needle valve and pressure reducing regulator system (PRATT, 1976), which is run from a 200psi. air supply. It incorporates a heater which may be used to simulate some kind of temperature distribution inside the instrument, though the thermal losses from the horn coupler walls increase the thermal settling time to several hours.
SIMULTANEOUS IMPEDANCE AND TRANSFER FUNCTION MEASURING DEVICE

FREQUENCY DIVIDER

PROGRAMMABLE
OSCILLATOR

PROGRAMMABLE
ATTENUATOR

f

f/480

f/12

(CLOCK)

PROBE MICROPHONE

1/2" MICROPHONE

BCD Control
output lines

NOVA 4
MINICOMPUTER

DAC

ADC

CHANNEL
SWITCH

LOUDSPEAKER
DRIVER

HOT-WIRE
ANEMOMETER

BRIDGE
The velocity of air flow is variable by means of the needle valve from zero to the onset of turbulence in the bore, after which no useful measurement can be made. The D.C. air supply is introduced via the horn coupling the loudspeaker to the instrument. A foam filter is employed at the end of the tube introducing the air to reduce turbulence and noise. The horn coupler is damped by means of cork strips glued to the outer surface. This reduces spurious acoustic radiation which may otherwise affect the transfer function measurements, and vibration.

The interconnection of these units is best appreciated by means of a block diagram, Fig.3.3.1.

3.3.2 Programmes

The experimental apparatus was controlled by a programme which had many functions. Feedback loops control the excitation level and maintain the level within the narrow linear operation region of the anemometer. One loop examines the previous few levels in an attempt to predict the new level, in order to reduce settling time. Noise rejection is accomplished by forming a Fourier series from the signal and retaining only the first term, which is identical in frequency to the excitation signal.

Calibration is achieved by reading files containing the relevant corrections, and incorporating this data in the calculation.

The measured, averaged and calibrated data is then stored in a file of standard form which is then easily accessible for further processing.
3.4 MEASUREMENT OF IMPULSE RESPONSE

This section deals with the practical aspects of the measurement of impulse response. Limitations of various methods are discussed, and the arrangement used for the measurement of brass instruments is described.

3.4.1 Criteria for measurement

Several options are available for the measurement of impulse response. The choice of the particular method adopted is largely governed by the result desired, and the restrictions existing in the experimental situation. In the case of the measurement of the impulse response of brass instruments, these are:

The pulse must be appreciably shorter than the round-trip time of the pulse from the mouthpiece to bell and back, in order to distinguish small internal detail.

The pulse must be free from secondary spikes, have a fast recovery, and ringing should be kept to a minimum. This entails careful design of the impulse source and adequate damping of the coupler used to introduce the impulse into the test object.

In order to increase the signal to noise ratio, standard practice is to repeat measurements and take a running average, so the source must be capable of repeated operation with precise timing of the pulse epoch.
These criteria narrow the choice of available techniques: for example, the exploding wire method, while offering high peak pulse amplitudes, and hence good signal to noise ratios, is not amenable to repetition. The same disadvantages apply to the shock-tube. The STL ionophone described by FRANSSON and JANSSON (1953) was also considered as pulse shapes from this device are very good. However, the construction of this device is not easy, requiring highly stable D.C. EHT supplies and an EHT modulator. For these reasons, a spark source was designed to meet the criteria.

The experimental device is completely controlled by computer, with averaging to increase the signal to noise ratio performed in the software.

The averaging process assumes the noise to be random, uncorrelated, ergodic, and stationary. By averaging many measurements, the noise is inherently self-cancelling, while the deterministic part of the signal is increased cumulatively. To ensure that the wanted signal is not distorted or cancelled, it is essential that the recording and pulse excitation are time-locked so that the signal always appears in the same place in the data record.

If the pulse itself is measurable, (without the test object) then the effects of non-ideal pulse shape may be removed by means of deconvolution of the impulse and the measured 'impulse response', to obtain the impulse response as if the object were excited with a pulse of infinitesimal width. The theory of this method is given in Chapters 2.5 & 2.7. Alternative methods such as cross- and autocorrelation may be employed to identify pulse epochs.
3.4.1 Description of experimental apparatus (Mark 1)

The acoustic impulse was generated by a spark discharge across an modified car spark plug, driven by an EHT pulse generator, which in turn was triggered by a pulse from the DAC connected to the computer. The device and control system is shown schematically in Fig. 3.4.1.

The modifications to the spark plug involved the removal of the curved electrode, to increase the spark length and hence acoustic output, incidentally reducing the pulse width also, and the filling of the cavity between the central electrode and body with insulating sealant. This eliminated unwanted cavity reflections which tended to degrade the pulse shape.

The Mark 1 EHT generator was simply an automotive electronic ignition system with a variable pulse delay and width unit built by the electronics workshop, as described by GOODWIN (1980). The hot ionised air produced by the breakdown produced the impulse, which could be varied in width by means of a control on the EHT generator, in the range 0.5 to 3 milliseconds. The impulse was then sensed by means of a probe microphone (B & K type 4170) or solid state capacitive transducer (C-Tape) placed in the throat of the coupler to the instrument. In experiments, this was a trombone, horn or other mouthpiece modified to enable fitting of the transducer by cutting the mouthpiece in half normal to its axis at the narrowest bore dimension (thus the highest pressure region), and inserting washers with suitable holes to enable insertion of the transducer.

In all cases, the coupler was packed with absorbing material. This reduced the effects of cavity resonances in the coupler, by the inherent damping of the substance, as well as eliminating any possible non-linearity. It can be seen from the calibration section that the pulse was in fact nowhere near the nonlinear region of
Fig. 3.4.1.

Schematic of impulse response measuring device

Fig. 3.4.2.

Typical pulse produced by the apparatus (Mark 1).
amplitude. The other reason for filling the cup of the mouthpiece is to preserve a correlation between the impulse response measurements and the impedance measurements. The impedance measurements were so arranged so as to give a high-impedance source. The effect of filling the cavity with absorber is to introduce a large resistance in series with the input of the instrument.

The high resistance is equivalent to an absorbing termination. This corresponds roughly to a high impedance source, in frequency terms. In this way, Fourier transforms of impedance measurements could be compared with the direct measurement. (Fig.3.4.3) The effect of multiple reflections from a reflecting source were also reduced. The resonances and early reflections due to the reverberant source would cause added, superimposed pulses on the pressure signal which were not caused by reflections from features of the instrument's internal geometry. These would prove to be difficult to remove as the wide bandwidth of the pulses and small dimensions of the cavity would provide many echoes of high amplitude and short epoch (arrival time), which would introduce error in the early part of the time signal.

This early epoch region is the most useful for the recovery of the radius function of the instrument, as multiple reflections have not become too complex to deal with theoretically, regardless of the pulse width generated by the source. (Ch.2.5)

After amplification, the signal was lowpass filtered at a cutoff frequency chosen suitably to avoid aliasing with the particular sample rate employed. (Reference - B. and K. Ltd. Applications to Frequency Analysis).

The sampled data was then stored, normalised, and a running average taken of repeated measurements, to increase the signal to noise ratio. This was essential as brass instruments, being horn-shaped, are highly effective in coupling ambient noise back into the probe
Comparison of input impedances measured by the swept sine method (dashed) and the Fourier transform of the impulse response (solid).
microphone. Fortunately, high frequency noise is coupled far more efficiently than low, (as can be seen from the Transfer Function curve, Fig.2.3.2), so problems with D.C. type shifts of level were not major. The delay between the trigger signal and the triggering of the EHT pulse was necessary to enable the capture of the entire pulse. A extra time-shift was available in the software (after averaging) to start the data record at the any desired position in the original record, should the delay be set too long.

3.4.2 The Mark 2 system

The Mark 1 impulse system improved the resolution available in the time domain by an order of magnitude over the Fourier transform of the input impedance as described by ELLIOTT (1979). The bandwidth employed was increased from 1kHz. to 10kHz.

However, still better resolution was needed if small detail (2mm.) in the bore was to be seen, and hence a yet narrower pulse was required.

Although a reasonable (only 100-200 averages required) signal to noise ratio was acheived by the Mark 1 apparatus, it was hoped to improve on this figure. One problem that was anticipated was due to the increased bandwidth to be used, as the coupling of high frequency noise to the transducer by the horn would cause severe degradation of the time record if precautions were not taken.

The pulse produced by the Mark 1 system was limited in that the maximum height and width were not independent due to the internal circuitry used in the ignition system. At minimum, the pulse width was limited to 0.2 millisec. with the peak pulse amplitude decreasing with decreasing pulse width.
These conditions were exactly opposite to the desired test signal, as input energy decreased rapidly with decreasing pulse width, thus degrading the signal to noise ratio.

A new system was designed around the old, using the same basic principles but incorporating different triggering circuitry. By varying the size of the capacitor across the coil, it was possible to change the resonant frequency of the LC combination, and hence the pulse width. The peak amplitude was independently variable by means of altering the voltage applied across the capacitor.

The triggering was improved with the use of an SCR, which has a switching time of the order of nanoseconds.

The spectral components of the resultant 20 microsecond pulse extended well beyond the limits of the probe microphone (cutoff frequency 10kHz.) so another microphone (B & K type 4138) which is 1/8" in diameter was substituted. The response of this unit extended to 100kHz (+/- 1.5 dB.), and was -10 dB at 200kHz. The wavelength at 200kHz is approximately 1.7mm, so detail of the above mentioned size could be resolved (based on a crude half-wavelength criteria).

The 1/3" microphone could also be placed directly in the bore of the instrument, so the large linear phase shift due to the length of the horn probe coupler was eliminated, which considerably simplified subsequent processing.

An important point which must be made here concerns the relation between the wavelength of the higher spectral components and the dimensions of the tube.

The smallest wavelength being about 2mm, and the diameter of the bore being at least three times this dimension at all points means that the possibility of excitation of higher-order modes must be considered. If these modes are excited significantly, then problems arise in the interpretation of results.
Fortunately, such modes are strongly evanescent in the frequency region under discussion, due to high attenuation and poor reflection even from a reasonably 'hard' surface like the tube wall (SKUDRYZK, 1971). The added factor of a tube of varying cross section will ensure these waves are highly dispersive, and hence it was felt that, even if such waves were excited, they would be attenuated strongly and be insignificant relative to the plane mode.

3.4.3 Calibration of measuring systems

Microphones were calibrated with the laboratory standard calibrated sound source. In the present case, this was a B&K Pistonphone (Type 4220), which gives a long-term stable calibration level of 31 Pascals (123.8 dB). This was used to calibrate the microphone and amplifier together, by means of a subroutine (MIKECAL) in the program which sampled the waveform at the input of the ADC (which is accurately calibrated to +/- 0.1 microvolt) at an externally selected clock rate. The data record thus obtained was examined for zero crossings, and the average period calculated.

This information was employed in another subroutine (FUNDN), also used in the impedance run programme, which calculates the amplitude of the first harmonic of a time series, given the period. This process removes the effects of harmonic distortion and noise.

The effect of static atmospheric pressure on the microphone response was compensated for in the subroutine by employing the reading from the calibrated barometer supplied with the pistonphone. This correction was typically of the order +/- 0.1 dB.

In this way, the complete measurement chain was calibrated and providing no gain controls were altered, accurate calibration of the
spark source was possible by recording the pulse, averaging out noise and plotting the result (Fig. 3.4.2). The result of the calibration was also be stored in a file (CAL).

In this way, the characteristics of the Mark 1 & 2 sources under various operating conditions were investigated.

A typical measurement of an instrument using this method is given in Fig. 2.5.0.
Typical pulse parameters

<table>
<thead>
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<th>Mark 1 system</th>
<th>Pulse Width</th>
<th>Amplitude</th>
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</thead>
<tbody>
<tr>
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<td>Millisec.</td>
<td>Pascals</td>
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<td>18.83</td>
<td>119.5</td>
</tr>
<tr>
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<td>23.0</td>
<td>121.1</td>
</tr>
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<td>3.0</td>
<td>23.0</td>
<td>121.1</td>
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<table>
<thead>
<tr>
<th>Mark 2 system</th>
<th>Pulse Width</th>
<th>Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.02</td>
<td>57.4</td>
</tr>
</tbody>
</table>
CHAPTER 4 : COMPUTING

This chapter is concerned with the computing aspect of the study, both theoretical and experimental. The control of experiments by computer is discussed, as is data acquisition and subsequent processing.

Computing equipment and programming methods are examined, and example flowcharts and programmes given.
4.1 COMPUTING FACILITIES

Most computing in the study was done on a Data General NOVA 4 minicomputer. This was used for control of experiments and data collection as well as subsequent storage, processing and theoretical work. This machine has 128k bytes of core, and is supported by several peripheral devices. Main storage was on a 2.5M byte moving head disk with additional 150k bytes of floppy disk.

Normal operation was via a VDU and lineprinter, while graphical output was available on three devices, an oscilloscope, XY analogue plotter and a programmable digital plotter for high quality output.

Analogue data channels were provided by a two-channel ADC and a two-channel DAC, of 14 and 15 bits respectively. In conjunction with a multiplexer and parallel sample-and-hold circuits, these were capable of 25000 conversions per channel per second.

Two BCD output channels were also available for digital control of apparatus.

The NOVA 4 while not being particularly fast, had the advantage of foreground and background running. This was of use in the case of a large theoretical programme which had a long run time, where other work (e.g. programme development) was in progress. The theoretical programme could be run in the foreground, as long as it was designed not to need operator interaction (i.e. data input), while the operator could be, for example, editing a programme in the background.

Some computing, mainly theoretical, was carried out on the University PRIME mainframe machine.

All programmes were written in FORTRAN, though some assembler routines (written by the Physics department computing staff) were employed to access the analogue data channels and perform the fast Fourier transform.
4.2 PROGRAMME DESIGN

A general philosophy of programme design based around a standard data file was used, partly for historical reasons. Wherever possible, programmes were written in modular form. In a process involving several operations on a set of data, several programmes were ordinarily written, one for each section, and the intermediate results stored. This procedure gave great flexibility, as one programme could be employed in several different fields (for example, the Fourier Transform programmes).

A library of subroutines useful to acoustics was also maintained, such as one which calculated the properties of a cylindrical tube, given certain boundary conditions.

The standard file format was based on the data produced by the impedance experiments, which produces 1024 modulus and 1024 phase values. Transfer function runs were stored in the same format, and time records in a file of equivalent length (2048 samples). The data was stored on disk in binary rather than ASCII format, so although it could not be directly read, the resultant saving in space was considerable.

In this way rapid programme development, easy access to data and exchange of results between members of the research group was facilitated.
The main problem in control of experiments is the correct data acquisition and immediate processing (for feedback), and subsequent output.

Usually calibration is trivial, as this data is held in a file created previously and may be added as the experiment progresses.

4.3.1 Impulse response measurement programme

An example is given by the flowchart (Fig.4.3.1) and listing (in the appendix) of IMPULSE, the programme used to obtain calibrated, averaged impulse data.

In this programme the system connected to the ADC is first calibrated by sampling a test signal and calculating the RMS signal voltage. The trigger signals to the impulse device are then sent, and a number of recordings made. A check is made to ensure that the ADC was not overloaded at the pulse maximum, and that the pulse maximum was above a certain threshold. This was to cover the eventuality of the spark source not firing, in which case the programme would still attempt to normalise the record. This would increase the relative gain of the particular record, which only contained noise. If this happened, the signal to noise ratio would be severely degraded by averaging, not improved. The data records are averaged, by finding the maximum value, normalising and taking a running average.

Overall correction for the system was made by taking a running average of the data maxima and multiplying with the calibration constant. The processed data is then stored in the standard file.
Flowchart of IMPULSE

Read no. of averages

Calibrate

MIKECAL

FUNDN

Send pulse to DAC

Record 2048 data points

Find max.

Is max in limit?

N

Y

Normalise, Running av. of max.

Running av. of data

Completed

N

Y

Store averaged calibrated result

Multiply data by cal. and av. max.
4.4 THEORETICAL COMPUTATION

Programming to find the solution to an equation was not difficult in most problems encountered in the present study. Familiarity with the manipulation of complex algebra terms in programming was perhaps the most essential step, as this is not ordinarily used in more general problems. Wherever possible, both real and imaginary parts of expressions were always calculated since little more effort in programming is needed to gain much more information.

4.4.1 Programme for theoretical input impedance

One example is the programme JIMPTRAN, used to calculate the input impedance and transfer function of a manifold of cylindrical tubes of known length, radius and temperature. The effect of differing terminations could be easily included. As the computation time for a 100-element manifold was of the order of 20 minutes, the software was written to enable several calculations to be made consecutively. This removed the need for operator intervention, and the programme was run overnight. Two subroutines were employed for the calculation of the termination (ZNEDER), and cylindrical element (ELEMENT). These are listed in the Appendix.

The sequence of operations is given in the flowchart Fig.4.4.1., with a listing in the appendix.
FLOWCHART OF PROGRAMME "JIMPTRAN"

1. Read frequency step, radius file name, boundary conditions
2. Read radius data
3. Increment frequency
4. Calculate termination impedance
5. Calculate input impedance and transfer function of next element
6. Store results
7. Check: Done 1024 steps?
   - Yes: End of manifold?
     - Yes: End of program
     - No: Go to step 3
   - No: Go to step 2
4.4.2 Fast Fourier Transform programme

Another computation often used in acoustics is the Fast Fourier Transform (FFT). A machine code algorithm has been developed (by R.A. Bacon) to execute the FFT, with computation times of about 2 seconds for a 4096 point transform. This time is long compared to other FFT processors currently available, but it must be remembered that these employ dedicated hardware to improve speed.

The FFT routine is too long to be reproduced here but a programme which employs it twice to take the Hilbert transform (HILBERT) is listed in the Appendix.

One point requiring care is the packing of the array data prior to taking the FFT. The algorithm, like most FFTs, required the real and imaginary parts of the response to be interleaved (a time signal being considered purely real). The restriction on the data for an inverse FFT to produce a real, causal signal is that the real part is even and the imaginary part odd in the positive and negative frequency regions (BRACEWELL, 1978).

A useful shortcut in FORTRAN is to consider a complex array length N to be a real array length 2N, and some simplifications of the code can then be made.

Programme HILBERT simply takes the FFT of the input, multiplies the positive frequency portion by \(-j\) and the negative frequency portion by \(+j\), and then takes the inverse FFT. This method is faster than trapezoidal integration (STANSFIELD et al, 1973) or convolutional (BRACEWELL, 1978) methods, and due to the relatively small number of arithmetic operations required by the FFT more accurate as round-off error is reduced.
4.4.3 Computation procedure for recovery of area function

The FORTRAN program employed to implement the theory given in Chapter 2.5 is listed in the Appendix. The sequence of operations is given here.

At time $t=0$, all values of A, $f_b$, r, etc. were set to 0. The data for the impulse reflection response was read from the data file, as was the calibration data. Source and termination conditions were read in from the terminal.

In the case of the delta function type of excitation, the signal $f(1) = 1.0$ at time $t=0$ was injected into the input. At a time $t=T$ (one transit time of a section), equations F1 and F2 (Chapter 2.5.2) were used to calculate the terms for $f(T)$ and $b(T)$ in the first section. Due to causality, the wave had not reached any other section by this time, so calculation was not carried out for such sections of the manifold. Incrementing time by $T$, the process is repeated, and the terms for the wave in the first two sections found. This is continued, bearing in mind the termination conditions of the structure, until the end of the manifold is reached.

At this point, the calculation may be stopped, as all the information regarding the area function has been recovered. The area coefficients may be recovered by recursive application of equation (2.5.2 A), assuming an initial area. The data was converted to radius, for convenience, and written to a file for further analysis and plotting.
The overall conclusions and results are presented in this chapter. The effect of the bore shape and temperature is discussed, as is the effect of the mouthpiece. Synthesis of bore shape from acoustic data is given as a possible solution to some problems in instrument design. This is discussed in detail, in conjunction with some subjective experimental results which show that a scientifically-based design may not be far off.
5.1 GEOMETRY

It has been shown that the consideration of the bore shape of a brass instrument by means of a manifold of cylindrical sections can be made, and that results are comparable to experiment. Questions of accuracy are then raised.

A simple comparison of measured and predicted results does not always show good agreement, as the temperature can vary considerably over the 3-hour measurement period, thus displacing resonance peaks. The magnitudes, however, seem to be little affected by temperature. One way of assessing the model is to compare the characteristics of the peaks, and a programme which does this has been written, which considers the impedance peak to be Lorentzian i.e. the resonance maps to a circle on a Nyquist plot (ELLIOTT, 1979). This property is exploited in a circle fitting programme which can interpolate between impedance values of each peak and so give a better estimate of the resonance frequency and peak magnitude. (Programme SQPDRAW, author D.Munro). The accuracy is thus increased (Fig.5.1.1).

At the moment, the best procedure if a fair comparison is needed seems to be to measure the temperature closely during an experiment, rejecting the results if this varies by more than a degree or so, and modelling using this temperature.

The effect of the D.C. airflow has been discussed by TRIMMER (1937) and dealt with in the present situation by PRATT (1979). He found that the difference in input impedance was so small it could only just be discerned experimentally by a slight change in phase angle, with an unmeasurable change in peak amplitude, when airflows comparable to the playing situation are used. It should be noted that the anemometer instability was typically greater than the value found, so not much credence can be attached to the result. It is concluded that
EXAMPLE OF PROGRAMME "SQPDRAW"

Fig. 5.1.1

Imaginary part of input impedance

Original and fitted curve in relation to angle round circle.

Error

Difference between original and fitted curve x 20
this factor is relatively unimportant relative to errors introduced elsewhere, for example in bore measurement.

5.1.1 Temperature

The effect of uniform temperature distribution and temperature gradients was modelled using the Look-back method. Accurate values for the resonance frequencies were found, employing the circle-fitting programme as before. One way of assessing the effect of small changes in instrument intonation is to plot the normalised mode frequency, (essentially the mode frequency divided by the mode number) against the mode number. Thus, if an instrument is perfectly harmonic, the series of points formed will be a horizontal straight line. Figure 5.1.2 shows such a plot, where the same instrument was theoretically modelled at four different temperatures and also with a temperature gradient. It can be seen that the graph is far from linear. This is unimportant in the present discussion, as only the relative effect of temperature is of interest. The deviations from a straight line are partly due to the mouthpiece being omitted (WGRAM, 1972).

One approach adopted by PRATT (1979) to attempt to establish a simple single-figure criteria for the intonation of an instrument was to find the mean and standard deviation of the normalised mode frequencies. A programme (JBANAL) has been written by J.M.B. to facilitate calculation of these figures from impedance data after processing by the circle-fitting programme.

The figures given in the glossary (5.1) shows that the use of such figures in attempting to find a correlation with subjective reports of intonation is rather dubious, as the 'Harmonic standard deviation' does change more for different temperatures than for different instruments,
INTONATION AT DIFFERENT TEMPERATURES

NORMALISED MODE FREQUENCY, Hz

- = 20°C.
X = 25°C.
+ = 30°C.
□ = 35°C.
◯ = 35–20°C. Gradient
It can be shown that the increase in harmonic frequency is proportional to the square root of the absolute temperature for uniform distributions, as is to be expected. The result for the temperature gradient is therefore very interesting, as it too deviates little from the overall pattern. Perhaps one expectation is that the overall intonation pattern would change, as at higher mode numbers, more wavelengths are confined in the higher temperature regions. This would lead one to expect a significant increase in resonant frequency in this region, whereas in fact no such change is found. In fact, the effect of the temperature distribution such as one that can exist under playing conditions seems merely to raise the overall intonation in a manner very similar to a uniform temperature rise.

This result is important in musical acoustics. One factor in the measurement of acoustic properties frequently overlooked is temperature, and this result shows that the interpretations made by many from the resonance frequencies have been extremely fortuitous. Until this result was derived, it was not proven that measurements made at steady temperatures could be extrapolated to the playing situation.

It is concluded that the consideration of bore shape alone is not sufficient to categorise the intonation of an instrument. If the Harmonic standard deviation is used as an overall measure of the intonation, then it is seen that this varies and that the instrument displays lowest deviation when a temperature gradient is applied. It remains that most measurements of intonation of musical instruments have disregarded temperature which makes subsequent conclusions rather dubious. Much of the careful analysis of PRATT (1979) for example, can only be taken as a rough guide bearing in mind this result, though in fairness it must be said that he was aware of this limitation.
5.2 THE SUM FUNCTION

This quantity was defined by VOGRAM (1972) in an attempt to correlate the input impedance data measured at single frequencies with the played note, with all its spectral components. The process involves taking a sum of the values of the real part of the input impedance at integral multiples of the desired frequency. The general equation is given in Fig.5.2.1, together with the modification made by later workers to attempt to include the effect of the decreased amplitude of the higher harmonics in the internal spectrum at low playing levels.

The concept has been proved useful in simple deductions of the approximate intonation of an instrument (PRATT et al., 1979). However it is known that players do not always play at the frequency of the sum function peak exactly. This has been reported by the players themselves, who often 'lip' a note out of tune slightly to obtain improvements in the timbre of a played note.

The transfer function may provide a clue to why they do this. It too has a number of harmonically related peaks, which lie close to the peaks of the input impedance frequencies. There is no reason why a sum function should not be formed from the transfer function also. In a manner similar to the impedance sum function, it relates the overall amplitudes of the spectra at the input and output of the instrument. One important distinction exists here between the impedance and transfer sum functions.

This concerns phase audibility, a problem which has not yet been satisfactorily solved. In the present case, it is taken that the ear is insensitive to phase. If this is so, it is reasonable to form the sum function from the modulus of the transfer function, i.e. the ratio of pressure amplitudes alone is important.
SUM FUNCTION FORMULAE

Impedance sum function

Formed from the real part of the input impedance:

\[
SF[Z(f)] = \frac{1}{n} \sum_{i=f}^{n_{f1}} \text{Re}[Z(i)]n^{-k} \quad n=1,2,3,\ldots
\]

Transfer sum function

Formed from the modulus of the transfer function (linear form)

\[
SF[T(f)] = \frac{1}{n} \sum_{i=f}^{n_{f1}} T(i)n^{-k} \quad n=1,2,3,\ldots
\]

\(n_{f1}\) = highest measured frequency.

\(k\) is the index extending the sum function concept to include the changes in playing frequency caused by the internal harmonic spectra changing with playing level.

\(k = 0\) corresponds to fortissimo playing

Large \(k\) reduces the sum function to the case where the first harmonic predominates, similar to piano playing.
Impedance sum function

Transfer sum function
If this is done, it is seen that the peaks of transfer sum function do not always coincide with the impedance sum function (Fig. 5.2.2). Furthermore, the relative position of the sum functions peaks changes with temperature. This result could help to explain why, although a player would get maximum regeneration at an impedance peak, he prefers to play at another slightly different frequency to optimise the transmission of sound from his lips to the outside air.

The relative positions of the sum function peaks also depend on the characteristics of the mouthpiece. As will be shown in the next section, the position of the impedance peaks is modified by the mouthpiece, and the additional lengthening of the instrument also changes the transfer function.

It should be noted that, if the input impedance and transfer function are known then the transfer impedance is also known by complex multiplication.

\[
Z(\text{transfer}) = Z(\text{in}) \times T = \frac{p(\text{in})}{U(\text{in})} \cdot \frac{p(\text{out})}{p(\text{in})} = \frac{p(\text{out})}{U(\text{in})}
\]
5.3 THE MOUTHPIECE

The function of the mouthpiece has become a disputed issue among musicians and physicists alike. Players and manufacturers have built up a body of conflicting information on the subject, regarding the effects of cup diameter, depth, and shape, backbore dimensions, the size of the throat, and so on (Fig.5.3.1). Some players even contend that the material has some bearing on the acoustic properties.

In some instrumental tutors, reference is made to the shape of the cup and relative direction of the airstream issuing from the lips as being important.

WICK, (1973) states 'the air flow bounces off the bottom of the mouthpiece cup, finally making a vortex into the bore'. In a similar vein, SCHULLER (1962) says '... this concept of air direction is based on an acoustical principle of wave direction', though which particular one is left for the reader to decide. More specifically, 'the higher notes are produced by the air stream hitting close to the inside edge of the rim of the mouthpiece, whereas the lower notes will be produced by blowing straight in to the mouthpiece' (DALE, 1965). Dale also has much to say about the role of the various parts of the mouthpiece.

From a more scientific viewpoint, BACKUS (1970) states 'such representations are meaningless and should not be taken seriously'. His argument, and it is one reflected in the work of BENADE (1976), WOGRAM (1972), and many others is that the dimensions of the mouthpiece are small enough relative to the wavelengths being considered to render the actual contour unimportant. If this is true, the mouthpiece may be considered as a combination of lumped acoustic elements, a representation now in common use. To clarify this point, the equivalent-circuit of the player-mouthpiece-instrument combination is given in Fig.5.3.2.
Mouthpiece equivalent circuit

\[ Z_A = \left[ \frac{j\omega C + \frac{1}{j\omega L + Z_B}}{j\omega L + Z_B} \right]^{-1} = \frac{j\omega L + Z_B}{1 - \omega^2 LC + j\omega CZ_B} \]

The parts of the mouthpiece
The compliance is associated with the mouthpiece cup, not forgetting that the players lips protrude into it and reduce the volume somewhat. The resistive loss is more general but predominates in the throat, while the inertance predominates in the backbore region. The resistance is generally less than 5% of the reactance at playing frequencies, so is neglected. It must be stressed that these quantities are difficult to measure for real mouthpieces due to the curved nature of the bore.

It can be seen that the mouthpiece alone is a resonant system. In practice, the resonant frequency of the mouthpiece with the rim closed and the backbore open is in about the middle of the playing range of the instrument. Work by BENADE (1976) shows that this frequency has musical significance. He measured it by the simple expedient of slapping the rim with the palm of the hand and comparing the fundamental frequency of the 'pop' produced with a tone generated by an oscillator. The system as described is a simple Helmholtz resonator.

Unfortunately work by BOWSHER (unpublished) shows that the resonance frequency measured in this way is dependent on the shape of the palm, and also dependent on the radiation impedance of the open backbore, hence a small change in the dimensions of the open end causes large changes in the acoustics. The radiation impedance is of course not seen in normal operation of the instrument, so the popping frequency measurement can only be taken as a rough guide to the mouthpiece properties. Furthermore, the Helmholtz resonant frequency appears to change by as much as 100Hz when measured experimentally with a solid backplate instead of the palm.

However, another experiment by Benade to determine the effect of the contour, if any, is interesting. Three mouthpieces of identical Helmholtz resonance frequency, but differing internal contour were supplied to a professional musician for tests. Benade reports that no
difference in either subjectively perceived intonation or timbre were detected by the player or listeners, though this result must be viewed with some suspicion bearing in mind the lack of experimental control and the ability of brass players to vary the above qualities within large ranges (PRATT, 1976), (ELLIOTT, 1979).

Another approach was taken by KENT (1961), and CARDWELL (1966), who studied the effect of a mouthpiece on a straight tube. A 'mouthpiece equivalent length' was derived, defined as the length of cylindrical tube that had to be added to the mouthpiece/tube combination to produce identical resonance frequencies to those of a cylindrical tube of the same length. This length increased with frequency, lowering the frequencies of the higher resonances.

So far, it can be seen that the mouthpiece has several functions:

a. To support the embouchure,

b. To establish correct intonation,

c. To match the input impedance of the instrument to the lips.

This last condition is interesting in light of the regeneration theory developed by ELLIOTT (1979).

A trombone alone is a minimum-phase system. This may be proved by taking the Hilbert transform of one side of the input impedance and comparing it to the other measured half (Glossary 2.6). If the system is minimum phase, these will be identical, as is the case. For an example, see Fig.2.6.1.

From simple theory of driven oscillators, a suitable condition for regeneration can be found to be that, for maximum regeneration, the impedance is maximum and the phase is close to zero. This implies that, as the trombone is a minimum phase system, then the mouthpiece too must be adjusted so that the impedance seen by the player's lips is
also minimum phase. This in turn imposes a simple condition on the mouthpiece:

\[ jwC = \frac{1}{jwL} \]

Unfortunately, this is not the case. The lips are not a rigid driving system, and their impedance is nonlinear in playing situations. The regeneration condition in this case is that the reactance seen at the lips is always negative (ELLIOTT, 1979), so in fact the regeneration condition is one of nonminimum phase. Even more unfortunately for the above hypothesis, it seems that the regeneration condition is rather insensitive to the phase, mainly due to the players ability to vary the lip parameters over a large range to maintain oscillation. While this is interesting in itself, a more rigid criterion is probably needed for musical instrument work where ease of sound production and timbre are important.

Perhaps surprisingly, the effect of the mouthpiece is considerably different in the cases of the trumpet and the trombone (Fig.5.3.3 & 4). The French horn is different again (Fig.5.3.5). It has been noted that trombone mouthpieces have a relatively larger cup volume than trumpet mouthpieces, (ELLIOTT, 1979).

PYLE (1975), discussing the mouthpiece in terms of effective length and popping frequency, states that the reason cup volumes are different in the case of the French horn and the trumpet is due to the normal playing range of the instrument in relation to the mouthpiece resonant frequency. A horn player (and trombonist) is rarely called upon to play notes higher in frequency than the Helmoltz resonance of the mouthpiece, whereas the trumpeter frequently has to.

Another property of the mouthpiece which is important to the musician is the d.c. flow condition. Very little work, if any, has
Fig. 5.3.3

Input impedance of a Trumpet with mouthpiece

Modulus of impedance

Frequency (Hz)

Phase of impedance
Modulus of impedance

Input impedance of a Trombone with mouthpiece

Frequency (Hz)

Phase of impedance
Modulus of impedance

Input impedance of French horn with mouthpiece

Phase of impedance

Frequency (Hz)
been done on the topic, with the exception of the d.c. resistance measurements of ELLIOTT (1979). He measured the resistance of the mouthpiece with and without the instrument. It was found that the resistance was dominated by the mouthpiece. For trumpets and trombones, the resistance was of the order of 0.3 Megohms, and roughly proportional to the flow velocity above 2m/sec. In this region, the flow is turbulent (INGARD et.al, 1967), and below this velocity the resistance was roughly constant, implying laminar flow.

In normal playing, the steady airflow ranges from a small value to greater than 20m/sec. in the mouthpiece throat so it can be presumed that turbulence has some effect. Furthermore, airflow velocity is periodic with playing frequency, so periodic generation of turbulence is possible.

As with other fluid mechanics problems, the onset of turbulence is strongly related to the contour of the confining medium, so some work in this field could prove fruitful. Some insight may be gained from musicians, who claim that the dimensions of the mouthpiece throat are critical, though an experiment by PRATT et.al. (1979) showed that players are not able to discern differences of 4.3% which they had previously stated they were able to. However the experiment, like all subjective experiments, limited the musician to unrealistic playing conditions, so the conclusion may be erroneous.

The general conclusion is that the mouthpiece performs a somewhat complex function acoustically, simultaneously adjusting the intonation and adjusting the reflected wave to the appropriate phase for regeneration. There has been much discussion in the literature about turbulence and tone production, most of it misdirected, but it would seem that this effect does have some bearing on the playing properties and could repay careful study.
Some collaborative work has been done with Dr. R. Shepherd in the field of the players ability to detect early reflections in the impulse response. Use was made of standard psychometric methods employed previously in the area, such as pair comparison and the Semantic Differential scale. As extensive work by Pratt (1976) using impedance data in an attempt to find a correlation between this and such factors as timbre and intonation had produced useful insight, it was decided to extend this to the time domain.

It was postulated that premature reflections affected the transient properties of the instrument in a musically significant way. Work by Plomp (1976) and many others has established the importance of the transient in the perception of music, and it has been shown that musical quality is to some extent characterised by this factor. Recent work by loudspeaker manufacturers (Fincham, KEF Ltd.) has shown that early reflections can be detected by a listener to recorded music, and that the reflections degrade perceived quality. This experiment was designed to establish whether a player could detect such reflections. A complete assessment of the effects of these reflections on the regeneration conditions of the transient is not at present possible as the problem is exceedingly complex, involving transient solutions of coupled nonlinear equations, and was not attempted. Rather, a qualitative study was undertaken to find if future work in this area would be profitable.

The industrial collaborator also expressed interest as he felt that this was an unknown area in the manufacturing process. He supplied a test instrument for the purpose, a B-flat/F double French horn. This was equipped with two sets of slides, one 'standard' where the tube ends were left square, and a set where the ends were chamfered.
to a knife edge, so the discontinuity between inner and outer slides was reduced.

The arrangement of the air passages in a French horn are complicated in that the B-flat 'side' uses tubing common to the F 'side'. From the point of view of a subjective experiment this was fortuitous, as it enabled the two sets of slides to be used simultaneously, for example the standard in the F side and the chamfered in the B-flat. In this way, the player was able to change from one set of slides to the other by using valve combinations in common use. The slides were made as identical in outward appearance as possible, so the player had no idea of which particular set was being employed. It must be remembered that, in normal playing, these slides are usually pulled out to some extent to correct small deficiencies in intonation and so a pair of discontinuities are formed. In impedance measurements made previously, the slides were left 'home' so some degree of experimental consistency could be achieved, at the expense of a more realistic measurement of the instrument under playing conditions.

An empirical investigation of the effect of the slides in normal playing position made using the impulse device was revealing, as reflections could clearly be seen from the discontinuities. Furthermore, when a gain function was applied, it appeared that the reflections were of the same order of magnitude. This is to be expected, as in general the discontinuities are also.

One simple way to assess the effect of the discontinuities is simply to subtract the normalised impulse records of the standard and test slides. Any common features should then disappear (although not totally due to the differing attenuation in the two cases caused by the discontinuities themselves) leaving the effect of the slides alone. In practice, this only worked for the first few reflections, as noise degraded the measured responses to a similar order of magnitude as the
Cross correlation of the impulse record with the pulse itself was also employed in an attempt to identify reflections, with little more success, as the general effect is of a 'time-smear' and removes fine detail which is precisely what is wanted. Autocorrelation is even worse in this respect.

Eventually it was found that inspection of the gained time record was most useful.

5.4.1 Experimental procedure

The procedures and results presented in this section are taken from an internal report written by Dr. R. Shepherd, who was in a large part responsible for the design of the experiment and its execution and who also performed the statistical computations.

Three players took part in the experiment. All were proficient horn players and one was a professional musician. One subject took part in two sessions, separated by about a week.

The experiment was divided into two sections. In the first, individual notes were tested, and the subject was asked to play one of eight notes and rate the instruments performance on these notes alone. The eight notes were, as written, on the F side C, Bb, A and C sharp, and on the Bb side, F, Eb, D and F sharp. The notes were chosen to lie in the mid-range of the instrument, i.e. being in the first octave above middle C (written). These notes correspond to equal fingerings on the two sides, but more importantly equal proportions of tube being added to each side. The fingerings were open, 1st valve, 1st and 2nd valve, and all three depressed respectively. Thus for each successive
note, an additional slide section is brought into use, and, if a difference between slides is present, it should accumulate. The subject was asked to rate the performance of the instrument on several Semantic differential scales, namely:

a. Intonation  
b. Responsiveness  
c. Pleasantness of timbre  
d. Degree of freeblowing

Each rating scale was from 1 to 7, and the subject was allowed to play the note until he felt that he could judge the score adequately. He was also allowed to play other notes with which to compare it.

The second part of the experiment was similar in that there were four presentations (one with each combination of slides) and the same rating scales were used. This time the player was allowed to play the instrument freely for as long as he wished before giving scores.

5.4.2 Results

First part - individual notes.

The data from the three subjects were analysed as though there were four subjects by including both sessions with the player who participated twice. The data for each of the four rating scales, and for the two sides of the instrument, were analysed seperately using a two-way analysis of variance method (SPSS).

The factors used were:

No. of valves pressed (4 levels)  
Type of slides (2 levels)

Repeated measures on each factor were used, and the results are given in the Glossary (5.4).
Fig. 5.4.1

Results for freeblowing scale on Bb side of instrument
The significant differences seem to be related to the number of valves used rather than the different sets of slides. Thus intonation becomes progressively worse with increasing number of valves pressed on both sides of the instrument. This result has physical basis, as the slide lengths are chosen to be a compromise, which deteriorates with increasingly larger added length of tube (CARDEWELL, 1966). Also on the F side of the instrument, the responsiveness is decreased, the stuffiness increased, and there is a nonsignificant tendency for the timbre to become less pleasant with increasing number of valves pressed. In terms of differences between the test and standard slides, the responsiveness of the Bb side is better with the test slides (mean ratings for standard = 4.81, test = 5.20). Also on the Bb side with the freeblowing scale, the interaction between the type of slides and the number of valves used approached significance. Whilst not much importance can be attached to this result the nature of this interaction is shown in Fig. 5.4.1. The effect is not simply one of the stuffiness increasing more with more valves pressed for the standard slides as might have been predicted.

None of the other effects were significant.

In the second part of the experiment, the data was analysed using t-tests. Each side of the instrument and each set of slides were analysed separately. The results are given in the Glossary (5.4), but none of these results showed a significant difference between the standard and test slides.
5.5 SYNTHESIS

The results of the synthesis methods were not as good as at first hoped. Major problems included instability of the equations used, and particularly experimental data where the initial pulse was wide compared to the desired delta function input. No easy solution to the problems was found, and the synthesis method only was of use when a few (40) sections were modelled, without multiple reflections.

One computation difficulty was the inclusion of attenuation. This was particularly difficult in sections where the radius flared rapidly, and hence the attenuation changed rapidly. The effect of the progressive rounding of the pulse due to the high-frequency attenuation along the manifold progressively degraded the resolution obtainable. This defect was to a certain amount compensated for by pre-gaining the impulse record, but this introduced further errors as the noise was also gained.

The attenuation was of crucial importance in the determination of the multiply-reflected components. If attenuation were omitted, these would persist for all time with relatively undiminished amplitude, so causing major error in the estimation of the direct reflection component. If too large, the multiply reflected components would become negligible relatively rapidly and the situation would revert to the simple reflection case.

Finite impulse widths also caused problems. While the initial impulse could be estimated by placing the impulse source in a damped cavity, or alternatively by Homomorphic deconvolution, in practice the results were not accurate enough for the multiple reflection algorithm to remain stable.

These and other difficulties severely restricted the possibility of a useful reconstruction of the bore shape where features could be
reliably distinguished. A more robust algorithm is needed for this purpose, together with better experimental data. One solution may be to employ modern signal processing methods to the deconvolution of the impulse record, when the problem becomes considerably easier.

Another is to adopt a different approach to the collection of experimental data, for example the methods of BERKHOUT et.al. (1980), which use a swept sine signal and matched filtering, together with Hilbert transform methods to obtain deconvoluted impulse responses of 3-D spaces.
5.6 THE DESIGN PROCESS AND POSSIBLE IMPROVEMENTS

This section examines the methods instrument designers have used in the past, and are using today, in their development of new instruments. Attitudes towards the design process are discussed, and a possible new method of design given.

5.6.1 History

The manner in which instruments have been designed and evaluated has always been empirical, ever since the first animal horn was blown. Not until modern psychometric techniques have become available has it been possible to get close to an objective view of instrument timbre (PLOMP, 1976, GREY, 1976), and quality (PRATT, 1978).

Changing musical demands, the development of music and technology, the advent of mechanisation and new materials have all made their mark.

However, despite centuries of apprenticeship and teaching, the underlying processes of the design of instruments has always been unclear, for many reasons. Commercial secrecy is one, but bearing in mind that most instruments are designed and put into production without even the most rudimentary testing, (and often by one person) then hunch, hearsay and intuition seem to take precedence over acoustical performance.

This view until very recently was justified in that designers did know more about instrument technology than scientists, as the equipment used to 'evaluate' instruments was so crude as to be practically useless, and so no reliable scientific judgements could be made. However, designers seem to be out of date with regard to progress in this field, surprisingly enough, so communication is the first priority.
before practical advances can be made.

The methods sometimes used by a manufacturer in the past involved professional players being invited to give their opinions of a new instrument, before production was started.

This is often exploited by manufacturers in advertising material, for example, the associations between Ifor James and Paxmans for the XL bore horn, Denis Wick and Boosey and Hawkes for the Sovereign range of instruments, Selmers and Maurice Andre, and Vincent Bach who combines being a player, designer and manufacturer in one.

This immediately gives rise to all kinds of problems, both acoustical and semantic.

5.6.1.1 Semantics

The player in the assessment panel may express his opinions perfectly well in his own words. What these terms may mean to the designer or builder is a matter of their respective backgrounds and training. A jazz player and a classically trained player have completely different conceptions of the word 'bright' as far as timbre is concerned, for example.

Not until the advent of modern psychometric tests, such as the semantic differential scale developed by SHEPARD (1972), and employed by EDWARDS (1977) in the present field, has this area of confusion been partially cleared.
Even if the designer has understood the comments of the player correctly, he is left with the problem of what to physically change to produce an instrument closer to the desired characteristics, and eliminate the undesirable ones.

While experience is of assistance, it has been shown by SMITH (1978) and others that while a change in bore dimension in one section of the instrument may correct a deficiency in intonation, other intonation problems may be created.

Manufacturers are aware of the importance of the correct bore proportions, and that the flare of the bell and leadpipe sections must be balanced to produce correct intonation (e.g. advertising material of Holton, U.S.A. referring to new trombones, designed in collaboration with Jay Freedman of the Chicago Symphony, and Horns with Philip Farkas. Also private communication with Richard Merewether of Paxmans.)

Unpublished work by WATKINSON (1980) shows that the dimensions, material and construction of the bell have significant effects on the vibrational modes of this section of the instrument. The changes needed to correct some deficiency in this field are extremely complex.

It is therefore highly unlikely that a designer would be able to correct any such fault in a few iterations of the testing process, by the use of empirical methods alone.

In the past trial and error, evolution and craftsmanship have sufficed to produce acceptable instruments. From the demands of modern music and the players, and the effects of competition from the more scientifically oriented manufacturers such as Schilke and Yamaha, it appears that this is no longer enough.
5.6.1.3 Criterion of 'satisfactory'

Assuming that the testing is done with the aim of improving the instrument, then some criterion has to be applied to terminate the test procedure at some point. Again, this decision is the tester's, and depends on a number of factors.

a. His training.
b. His present instrument, is it 'better' or 'worse'
c. His present field of employment.
d. His present set of preconceptions.
e. Fashion
f. Finish, weight, accessories and price of instrument.

to name but a few. For a more complete discussion, see PRATT (1976).

5.6.1.4 Technology.

A precise copy of a prototype in a production situation is difficult to achieve, especially when all instruments are made by hand (though individual piece parts may be machine formed). Some tolerance criteria is therefore required to ensure that the final product is recognisably close to the prototype, or the testing will have been pointless. An objective test method is needed to fulfil this criterion, preferably implemented on the production line.

At present, most manufacturers employ a full-time tester who plays every instrument as it leaves the production area. While every instrument is tested, it may be that the tester is not expert on a particular instrument, and hence cannot make any kind of qualitative
judgement other than that outlined by finish, mechanical operation etc.

However, manufacturers do claim that a skilled tester is able to detect small inconsistencies in large batches of instruments.

5.6.2 The new design process.

This method of instrument design has been developed to overcome the shortcomings of the traditional method (Fig.5.6.1). By making use of techniques described elsewhere in this thesis, the process may be made less subjective, repeatable, and it is hoped more economical in terms of time and money expended in the prototype stage.

The limitations of the design process proposed by PRATT (1979), shown in Fig.5.6.2 are, it is hoped, largely overcome.

a.

There are two separate evaluation processes, scientific and musical. Each process interlinks with the other, so that, for example, the correlation between measured features and musical effects become better understood as the evaluation progresses.

b.

The method is amenable to the changes and different demands made by different players without the designer having to change his entire conceptual framework and semantic structure.

All that is needed is a revision of the rating scales.

c.

A large part of the design is done 'on paper', or on a computer.
The traditional design process
The modified design process suggested by Pratt

1. Start
2. Construct prototypes
3. Measure prototypes complex acoustic impedance
4. Submit prototypes to a panel of players for subjective assessment using SDS and MDS
5. Players satisfied with quality?
   - NO: Modify instrument using Perturbation Theory
   - YES: Enter next stage
If a suitable suite of programmes is designed and written, then the resultant saving in money, time and materials may well offset the investment.

The necessary computer time may be hired, and the evaluation time by the players is cut due to the increased efficiency. No mandrels need be cut for 'one-off' prototypes which will not be used again (an expensive part of prototype design). The new process is shown schematically in Fig.5.6.3.
MODIFIED DESIGN PROCESS

Design new bore shape

Modify bore shape

Calculate input impedance, etc.

Good intonation?

Y

Submit to objective test methods based on existing SDS and MDS scales.

Satisfactory?

N

Construct Prototype to spec.

Modify rating scales for objective tests.

N

Satisfactory?

Y

Submit to panel of players for subjective assessment using SDS and MDS.

Start production
Symbols used in multiple reflection analysis

A(i) = cross-sectional area of ith section of manifold

b(t) = backward travelling impulse in mth section of manifold model

R = pressure reflection coefficient at junction between ith and (i+1)th section of manifold

c = velocity of sound in free air at S.T.P.

f = frequency, Hz.

f(t) = forward-travelling impulse in mth section of manifold at time t

k = index referring to time and distance

L = total length of manifold

m = index referring to mth physical section of manifold

N = total number of sections in manifold

n = index referring to time, distance and order of iteration on recursion equations

r(t) = input impulse reflection response

r'(t) = multiple reflection contribution to r(t)

T = round-trip time of impulse across one section of manifold (simple reflection model)

T = transit time of impulse across one section of manifold (multiple reflection model)

t = time

U = volume velocity

Z = load impedance of manifold

Z = source impedance of manifold

Z = input impedance as a function of frequency
Properties of minimum and maximum phase systems

A minimum phase system is defined as one whose real and imaginary responses are related via the Hilbert transform. This is generally true of real causal systems. However there is no a priori reason why a sequence should be minimum phase. A system may have minimum and maximum phase components, i.e. the system is mixed-phase.

An alternative approach is given by TRIBOLET (1979) and others, who give the same result in terms of the z-transform of the sequence. In this form, the z-transform of the minimum phase sequence has all poles and zeros inside the unit circle, while the z-transform of the maximum phase sequence has all poles and zeros outside.
Terms used in Cepstral analysis

Grammitude

Repiod

Rahmonic

Quefreny (s)

Grammitude

Short-pass lifter

Long-pass lifter

Quefreny (s)
## Terms used in Cepstrum analysis

<table>
<thead>
<tr>
<th>TIME</th>
<th>FREQUENCY</th>
<th>QUEFRENCY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (s)</td>
<td>Frequency (s⁻¹)</td>
<td>Quefrency (s)</td>
</tr>
<tr>
<td>Amplitude</td>
<td>Amplitude</td>
<td></td>
</tr>
<tr>
<td>Magnitude</td>
<td>Gamplitude (Log amplitude)</td>
<td></td>
</tr>
<tr>
<td>Phase (rad)</td>
<td>Saphe (s)</td>
<td></td>
</tr>
<tr>
<td>Filter</td>
<td>Lifter</td>
<td></td>
</tr>
<tr>
<td>High-pass filter</td>
<td>Long-pass lifter</td>
<td></td>
</tr>
<tr>
<td>Low-pass filter</td>
<td>Short-pass lifter</td>
<td></td>
</tr>
<tr>
<td>Harmonic</td>
<td>Rahmonic</td>
<td></td>
</tr>
<tr>
<td>Period (s)</td>
<td>Repiod (s)</td>
<td></td>
</tr>
<tr>
<td>Analysis</td>
<td>Alanysis</td>
<td></td>
</tr>
<tr>
<td>Radius</td>
<td>Darius</td>
<td></td>
</tr>
<tr>
<td>Polar (plot)</td>
<td>Lopar</td>
<td></td>
</tr>
</tbody>
</table>

As the quefrency domain is the inverse of the log frequency domain, the dimensions of quefrency remain the same as time and amplitude is related to the log amplitude of the time domain variable, and has log (same dimensions).

The terms defined in the cepstral domain are loosely borrowed from both time and frequency domains and are best described by the diagrams opposite.
Glossary to Chapter 5.1

Statistical calculations from impedance data

The table gives results for harmonic mean frequency, harmonic standard deviation, the gradient and slope of a straight line fitted to the impedance peak frequencies, together with the associated error in the fitting. The first 15 impedance peaks are used in the analysis. Calculations are based on theoretically predicted input impedance data for a bore shape close to a Boosey and Hawkes 'Sovereign' bass trombone, evaluated at four different ambient temperatures and also for an approximate exponential gradient.

Data given by PRATT for seven different medium bore trombones is also given, though it is noted that these were almost certainly measured under differing temperature conditions.

<table>
<thead>
<tr>
<th>Temp, deg.C.</th>
<th>M.H.F, H.S.D., (Hz)</th>
<th>H.S.D., (Hz)</th>
<th>Slope, Hz/n</th>
<th>Slope error</th>
<th>Intercept, Hz</th>
<th>Intercept error</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>60.96 0.6834</td>
<td>61.976</td>
<td>0.3548</td>
<td>-6.60</td>
<td>3.3375</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>61.48 0.6958</td>
<td>62.509</td>
<td>0.3684</td>
<td>-6.68</td>
<td>3.4658</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>62.01 0.7218</td>
<td>63.115</td>
<td>0.3911</td>
<td>-7.22</td>
<td>3.6790</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>62.50 0.7085</td>
<td>63.562</td>
<td>0.3760</td>
<td>-6.89</td>
<td>3.5374</td>
<td></td>
</tr>
<tr>
<td>Gradient</td>
<td>61.53 0.5406</td>
<td>62.128</td>
<td>0.1814</td>
<td>-3.42</td>
<td>1.6020</td>
<td></td>
</tr>
</tbody>
</table>

Data given by Pratt

H.S.D. ranges from 0.700 to 1.29 for 7 medium bore tenor trombones.
### Results of factor analysis
(for first part of experiment)

<table>
<thead>
<tr>
<th></th>
<th>F side</th>
<th>Bb side</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Intonation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>12.02 **</td>
<td>10.31 **</td>
</tr>
<tr>
<td>B</td>
<td>1.62</td>
<td>1.65</td>
</tr>
<tr>
<td>AxB</td>
<td>0.30</td>
<td>0.71</td>
</tr>
<tr>
<td><strong>Responsiveness</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>8.49 **</td>
<td>0.66</td>
</tr>
<tr>
<td>B</td>
<td>1.90</td>
<td>3.88 *</td>
</tr>
<tr>
<td>AxB</td>
<td>1.67</td>
<td>2.93</td>
</tr>
<tr>
<td><strong>Timbre</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Pleasantness)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>3.56 (*)</td>
<td>2.59</td>
</tr>
<tr>
<td>B</td>
<td>1.85</td>
<td>0.20</td>
</tr>
<tr>
<td>AxB</td>
<td>0.69</td>
<td>0.04</td>
</tr>
<tr>
<td><strong>Freeblowing</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>133.42 **</td>
<td>1.38</td>
</tr>
<tr>
<td>B</td>
<td>1.79</td>
<td>2.05</td>
</tr>
<tr>
<td>AxB</td>
<td>2.34</td>
<td>3.48 (*)</td>
</tr>
</tbody>
</table>

A = number of valves pressed  
B = type of slide

Significance levels

** p < 0.01  
* p < 0.05  
(*) 0.01 < p < 0.05

### Results from second part of experiment

Values of t for the comparisons of test and standard slides in the second part of the experiment (general playing).

<table>
<thead>
<tr>
<th></th>
<th>F side</th>
<th>Bb side</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Intonation</strong></td>
<td>0.14</td>
<td>1.0</td>
</tr>
<tr>
<td>Responsiveness</td>
<td>1.73</td>
<td>1.0</td>
</tr>
<tr>
<td>Timbre</td>
<td>0.77</td>
<td>1.58</td>
</tr>
<tr>
<td>Freeblowing</td>
<td>1.0</td>
<td>0.77</td>
</tr>
</tbody>
</table>
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APPENDIX

Programme listings.
IMPU* SE
ON 21/01/81 AT 16:14

COMPILER NOSTACK
DIMENSION AV(2048), NAME(3)
DIMENSION IDATA(2048), DUMMY(2048)
DIMENSION NAME(20)

IMPU* SE
SEND A SV. 5V. PULSE FROM THE DAC AND ENABLES
THE ADC (CH 0) TO RECORD 2048 SAMPLES
SAMPLE RATE IS SELECTED EXTERNALLY TYP. 25kHz.
ANTI-ALIASING FILTERS ARE NEEDED BEFORE INPUT
INTO CH 0.

16 BLOCKS OF DATA ARE WRITTEN AWAY
CALIBRATION OF MEASURING CHAIN USING SUBROUTINE MIKECAL
RUNNING AVERAGE OF PEAK VALUE
GIVES FINAL PEAK CALIBRATION

TYPE "********** IMPULSE RESPONSE PROGRAMME**********"
TYPE "IMPULSE RESPONSE PROGRAMME**********"
TYPE "SAVE FILE NAME?"
READ *** NAME(1)

11 FORMAT (280)
ACCEPT "NUMBER OF AVERAGES?", NAV
ACCEPT "MICROPHONE CALIBRATION 1=YES", ISKP
IF( ISKP) GO TO 567
CALL MIKECAL(MDATE,CAL)
567 IF( ISKP).EQ.1 ACCEPT "MICROPHONE CALIBRATION: HA/V?", CAL
VOL=+6
PAUSE IS EVERYTHING SET UP?
PULSeq=0.0
DO 100 NUMBER = 1, NAV
1 TYPE "RUN ", NUMBER " OF ", NAV
CALL BIU(0.0, VOL, 1.0)
CALL SET(0, IDATA(1), 2048)
CALL BIU(0.0, VOL, 1.0)
FMAX=-16.3

FIND MAX AND MIN INPUT VOLTAGE OF THIS RUN:
RMIN=1.43
DO TO 1=1, 2048
DUMMY=FLOAT(IDATA(1))/1638.4
RMAX=MAX( RMAX, DUMMY(1))
RMIN=MIN( RMIN, DUMMY(1))
CONTINUE
59 IF PULSE AMPLITUDE LESS THAN UNDER REPEAT RUN
59 OVERLOAD DETECTION - IF SO, REPEAT RUN
OVER=SMAX( RMAX, ABS(RMIN))
UNDER=7.0
IF( OVER .GT. 9.9) TYPE "********** ADC OVERLOAD **********"
IMPULSE (2)

IF(RMAX .LT. UNDER) TYPE = PULSE TOO SMALL PULSE
IF(ROVER .LT. 9.9 .OR. RMAX .LT. UNDER) CALL FDEL(0.99)
IF(ROVER .LT. 9.9 .OR. RMAX .LT. UNDER) GO TO 1
IF(NUMBER .EQ. 1 .AND. RMAX=RMAX) GO TO 1
IF(NUMBER .LT. 1) RNUMAX=RNUMAX*NUMBER-1 + RMAX
IF(NUMBER .LT. 1) RNUMAX=RNUMAX/NUMBER
TYPE = MAXIMUM INPUT VOLTS = *RMAX

CCCC NORMALISE DATA TO 1
DO 5 I=1,2048
DUMMY(I)=DUMMY(I)/RMAX
5 CONTINUE

CCCC RUNNING AVERAGE TO REDUCE NOISE
AV(I)=AV(I)*(NUMBER-1) + DUMMY(I)*FLOAT(NUMBER)
55 CONTINUE

100 CONTINUE

CCCC CALIBRATE AVERAGED DATA
DO 777 I=1,2048
AV(I)=AV(I)*RNUMAX*CAL
777 CONTINUE

CCCC AVERAGED MAXIMUM RECORDED PRESSURE
PAMAX=CAL*RNUMAX
TYPE = MAXIMUM RECORDED PRESSURE = *PAMAX, * PASCALS
CALL FOPEN(1, NAME(1,1))
CALL WRBLK(1, 0, AV, 16, IER)
CALL FCLOS(1)
END
HILBERT TRANSFORM

TAKES A SIGNAL, FOURIER TRANSFORMS IT, FILTERS WITH +/- I
ACCORDING TO WHETHER +/- FREQ. AND INVERSE TRANSFORMS

FORMAT(520)
TYPE 'FILE TO BE HILBERT TRANSFORMED?'
READ (11,1,1) NAME(1)
TYPE 'TO BE PUT IN FILE CALLED....'
READ (11,1,1) NAME (1)
CALL FILED (1, NAME )
CALL ROBLK (1, 0, 5, 8, IER)
CALL ROBLK(1, 0, 5, 8, IER)
CALL FCLOS(1)

SET FFT FLAG
JFLAG=0
DO 10 I=1, 1024
T( I ) = COMPLEX(R( I ), 0, 0)
I( 2048+I )=CONJG(T(I))
10 CONTINUE

FIRST FFT

CALL NDF4(T, 2048, JFLAG)
TYPE 'END FFT 1'
DO 20 I=1, 1024
T(I)=T(I) * COMPLEX(0, 0, -1, 0)
T(I+1024) =T(I+1024)*COMPLEX(0, 0, 1, 0)
20 CONTINUE
RESET FFT FLAG
JFLAG=1

CALL NDF4(T, 2048, JFLAG)
TYPE 'END FFT 2'
DO 30 I=1, 1024
F(I) = REAL(T(I))/2048.
30 CONTINUE

STOP
END
DIMENSION NAME(20), ZMOD(1024), ZPH(1024), TPH(1024)
DIMENSION NAME(20), NAMOUT(20)
INTEGER DAY(3), STARTIME(3), FINITIME(3)
INTEGER FSTEP(10), IFRAD(10), KSTEP(10), NELEMENT(10)
COMPLEX ZIN, ZT, TRANS, ZTEMP
REAL L(128), R(128), INC, TEPH(128)

INPUT IMPEDANCE & TRANSFER FUNCTION OF AN
INSTRUMENT USING STRAIGHT ELEMENTS
AND "LOOK BACK" METHOD. REAL CHARACTERISTIC IMPEDANCE
THE TEMPERATURE GRADIENT OPTIONAL
& BLOCK (1024 IMPEDANCE VALUES PER RUN)
UP TO 128 SECTIONS PER INSTRUMENT
KARAL DISCONTINUITY INDUCTANCE OPTION ALSO AVAILABLE

ACCEPT NUMBER OF RUNS (1 TO 10) ?? " , NRUN
DO 110 I = 1, NRUN
TYPE "HORN SHAPE FILE NAME FOR RUN " , I
READ(11,11) NAME(I,1)
TYPE "FILENAME FOR IMPEDANCE & TRANSFER FUNCTION ? "
IP4 = 1+10
READ(11,11) NAME(1,IP4)
ACCEPT "FREQUENCY Step? " , FSTEP(I)
ACCEPT "NUMBER OF ELEMENTS TO BE USED ? " , NELEMENT(I)
ACCEPT "TEMPERATURE GRADEuent 1 = YES, OTHER = AMBIENT" , TGRAD(I)
ACCEPT "KARAL DISCONTINU INDUCTANCE INDUCTANCE CORRECTION ? 1 = YES", KTEST(I)
CONTINUE
110 FORMAT(20)

START SEQUENCE
DO 111 IRUN = 1, NRUN
FIND START TIME OF RUN
CALL TIME(STARITME, IEP)
SORT OUT THE NAMES FOR FILE OPEN/CLOSE
DO 222 J = 1, 20
NAME( J) = NAME(J, IRUN)
IRUNP4 = IRUN+10
NAMOUT(J) = NAME(J, IRUNP4)
CONTINUE
222 FORMAT(20)

READ IN RADIUS AND TEMPERATURE GRADIENT DATA
CALL KPB(1, NAME1)
CALL RBK(1, N, IER, IBLK)
CALL RBK(1, B, IER, IBLK)
IF ERR(1, N, 1) .EQ. 0 THEN NAME(D, 2, TEMP, 1, IER, IER, IM)
CALL IFILM(1)
IF (IROL(1)) = 40 THEN DO 33

READ FREQUENCY FROM 1 TO 1024 TIMES STEP VALUE
DO 160 I = 1, 1024
FTEMP = FSTEP(I)
TYPE "RUN " , IER , " OF " , IRUN , " ITERATION " , I , " OF 1424" , IER , " TEMP " , FTEMP
ZTEMP = CMPLX(1, 0, 0)

CALCULATE IMPEDANCE & TRANSFER FUNCTION PROGRESSIVELY
NELE = NELEMENT(IER)
DO 280 J=1,NELE

CONTINUE
160 CONTINUE
33 CONTINUE
JIMPTRAN (2)

100 CONTINUE
CALL FUPEN(2,NAMCUT)
CALL WRBLK(2,2,TMOD,S,IER)
CALL WRBLK(2,16,TFH,S,IER)
CALL WRBLK(2,24,TFH,S,IER)
CALL FCLOS(2)
FINISH TIME
CALL DATE(DAY,IER)
WRITE(12,14)
WRITE(12,15)IRUN
WRITE(12,16)DAY(1),1=1,3)
WRITE(12,17)STARTIME(I),I=1,3)
WRITE(12,18)FINITIME(I),I=1,3)
WRITE(12,12)NAME(I,IRUN),NAME(I,IRUNP4)
WRITE(12,13)TSTEP(IRUN),NEME(M),TERAD(IRUN),KTEST(IRUN)
12 FORMAT(1H,"RADIUS FILE NAME ",S20/* IMPEDANCE FILE NAME ",S20/
13 FORMAT(1H,"NUMBER OF ELEMENTS USED IN RUN ",14/
14 /* 1 FOR TEMP GRAD ",I2/
15 /* 1 FOR KARAL DISCONTINUITY OPERATOR ",I3/)
16 FORMAT(1H,"RUN NUMBER ",I3," "
17 FORMAT(1H,"RUN ON ",14)
18 FORMAT(1H,"START TIME ",3(12,1X)/)
19 FORMAT(1H,"FINISH TIME ",3(12,1X)/)
20 CONTINUE
WRITE(12,14)
TYPE* "RUN COMPLETE. "
TYPE
TYPE
TYPE
111 CONTINUE
WRITE(12,14)
111 CONTINUE
WRITE(12,14)
STOP
REPLACE TEMPERATURE ARRAY WITH AMBIENT
22 DO 300 I = 1,128
TEMP(I) = TERA(I)
300 CONTINUE
GO TO 33
STOP
END
SUBROUTINE ZHEMI( TRAD, FREQ, ZT )
REAL K
COMPLEX ZT

RETURN
END

SUBROUTINE ELEM EN T( FREQ,L,RAD,ZT,ZIN,TRANS,TEMP )
COMPLEX ZIN,ZT,RHO,ZTEMP,ZC,GAMMA,TRANS
REAL 

CALCULATES THE INPUT IMPEDANCE AND TRANSFER FUNCTION OF A CYLINDRICAL TUBE.
GIVEN THE: TERMINATION, RADIUS, TEMPERATURE, LENGTH

C=331.6*SQRT(1.0+TEMP/273.0)
PI=4. *ATAN(1.)
A=3E-5*SQRT(FREQ)/TRAD
B=2. *PI*FREQ/C
GAMMA=CMPLX(A,B)
ZC=CMPLX(1.21*C)/(PI*TRAD*TRAD),0.0)
RHO=(ZT-ZC)/(ZT+ZC)
ZTEMP=RHO*CEXP(-2.0*GAMMA*L)
ZIN=ZC*(1.0+ZTEMP)/(1.0-ZTEMP)
TRANS=(CEXP(-1.0*GAMMA*L)*(1.0*RHO))/(1.0+ZTEMP)
RETURN
END
MULTIPLE REFLECTION RADIUS RECOVERY PROGRAMME

11 FORMAT (528)
READ (11, 11) MICAL (1)
CALL FOPEN (1, NAME )
CALL FOPEN (1, 0, REF, L, IER)
CALL FOPEN (1, 1)
CALL FOPEN (2, MICAL )
CALL FOPEN (2, 0, CAL, IER )
CALL FOPEN (2)
TEMPER = 290
ACCEPT * INITIAL RADIUS?*, RADIUS
ACCEPT TOTAL LENGTH OF INSTRUMENT?, L
ACCEPT * SAMPLING FREQUENCY, Hz?*, SAMRA
SAMRA = SAMRA * 180/

SET UP CONSTANTS AND INITIAL CONDITIONS

ACCEPT * INPUT REFLECTION COEFFICIENT FOR FIRST SECTION?*, SAM (1)
NORMALISE DATA TO 1

DO 111 I=1, 1024
READ (11, 11) MICAL (1,0)
TYPE MAX, MAX, REF, L, ILEN
111 CONTINUE
DO 343 I=1, 1024
READ (11, 11) MAX (1, MAX, CAL (1))
TYPE FIRST MAXIMA FOUND AT 1=*, ILEN
MAX = ILEN / SAMRA
TYPE FIRST REFLECTION ARRIVES AT *, AKTIM
TYPE ELEMENTAL LENGTH=*, SEGL
DO 222 I=1, 1024
READ (11, 11) CAL (1, L, CAL )
TYPE L=REF, L IZOMAY
222 CONTINUE

CALIBRATION IMPULSE INPUT
CAUSALITY 1000

SAMRAPHD = 1.0
DO 1 K=1, ILEN
TRANSIT TIME LOOP

DO 2 NTRANS=1,2

END

MULTIPLE REFLECTION COEFFICIENT

RA=1.2

BEGIN

STOP

END