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AN INVESTIGATION INTO THE TEMPERATURE-CREEP EFFECTS ON THE DESIGN OF CONCRETE BRIDGE DECKS

By

Sabah E. Khudairi

A Thesis Submitted For the Degree Of Doctor of Philosophy

University of Surrey
Department of Civil Engineering

April 1982
SUMMARY

The aim of the work in this thesis is to investigate the combined effects of differential temperature and creep on the behaviour of concrete bridge decks. A method of analysis is developed which can be used to investigate the effect of the creep-temperature mechanism on the redistribution of stresses within a prestressed concrete deck. The method is based on past experimental work and existing theories which are reviewed in the thesis.

The developed method of analysis is then used to study the effect of the creep-temperature mechanism on the following:

1. Two span continuous prestressed concrete deck under the action of a constant temperature gradient. Three types of deck are investigated: solid slab, cellular slab and T-section.

2. Two span continuous prestressed solid slab deck under the action of cyclic temperature gradient.

3. A skew simply supported prestressed solid slab deck under the action of a constant temperature gradient.

Finally, general conclusions and recommendations for design are given in the last Chapter.
ACKNOWLEDGEMENTS

My sincere gratitude is due to Dr. B. Richmond for his dedicated and encouraging guidance throughout the development of my research work. I am also indebted to Mr. P. Lindsell for his help in the presentation of the Thesis.

I would also like to thank Mrs. P. Wadsworth for typing the Thesis and Mrs. S.J. Rudman for drawing the Figures.
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CHAPTER 1: INTRODUCTION
It has been recognised for a long time that bridge superstructures exposed to environmental conditions exhibit considerable structural response. Temperature effects in bridges are influenced by both daily and seasonal temperature changes. For a statically determinate structure, the seasonal change will not lead to temperature induced stresses. This temperature change, however, causes large overall expansion and contraction. Daily fluctuations of temperature, on the other hand, result in temperature gradients through the depth of the bridge deck which in turn induce high internal stresses.

It is well known that no induced stresses are produced in a single span statically determinate bridge superstructure if the temperature distribution has either a uniform or linear form. However, evidence from field measurements in various types of bridges (1,2,3) indicate that temperature distributions over the depth of concrete bridge decks are nonlinear. The nonlinear temperature distribution is a source of induced stresses even if the bridge superstructure is statically determinate.

For indeterminate superstructure, the structural response under temperature differentials is believed to be more severe. This is true since additional internal stresses are induced due to the flexural restraint caused by interior supports. Hence, in general, the temperature stresses are attributed to two principal factors. These are the nonlinear form of the temperature gradient through the depth of the deck and to the form of statical indeterminancy of the superstructure.

Research which has been performed on the subject of temperature distributions in concrete bridge decks(4) indicate that the magnitude and form of the temperature gradient are mainly functions of the intensity of the solar radiation, ambient air temperature and wind speed. The shape and depth of the bridge deck cross-section and its material properties are also significant factors. For example, due to the low
thermal conductivity of concrete, the nonlinearity of the temperature
distribution in concrete superstructures is found to be considerably
greater than that experienced in steel superstructures. Consequently,
high stresses can be induced in deep concrete decks. These stresses,
under particular conditions, are additive to stresses caused by dead
load, live load and prestressing forces thus increasing the magnitude
of the final stresses.

In the past, most bridge designers did not recognize the importance
of thermally induced stresses. Consequently, they were often neglected
in the design process. This problem may have been due partly to the
lack of information and methods of analysis and partly to the fact that
few bridge failures had been attributed to thermal stresses. However,
thermal cracking and problems associated with thermally induced movements
have been observed. Leonhardt and Lippoth\(^5\) reported crack damage
in two span continuous beams at the bottom surface over the interior support.
They attributed this cracking to thermal effects.

Other investigators have noted the significance of thermal movements
and strain in concrete slab bridges. In 1973, Willems\(^6\) tested a
continuous three-span reinforced concrete slab deck for the effects of dead
load, live load and temperature. Weldable strain gauges were attached
to the reinforcing steel to evaluate the effects of the various loadings.
The bridge was skewed with the slab thickness varying from the free ends
of the superstructure to the interior supports. Willems found as a result
of his study that the effect of temperature was not secondary but in many
cases equalled or exceeded the effects of dead loadings and overshadowed
completely the effect of test live loading.

In the past 10 years, several international design codes have required
bridge engineers to design for thermal stresses in bridge superstructures
Different codes of practice \((7,8,9,10)\) specified different thermal gradients
to be allowed for in the design; depending on location of the bridge,
material properties of the concrete, shape of the deck's section, thickness
of surfacing material, intensity of solar radiation, daily ambient
temperature range and site wind speed.
Despite the theoretical and experimental evidence\(^{(11)}\) indicating the importance of diurnal thermal stress, it is tempting to under-rate the problem of thermal loading on the grounds that, as previously mentioned, a large number of prestressed and conventionally reinforced concrete bridge decks have been built over the past 50 years without any consideration of thermal loading in the design. The general record of performance has been good. Smith \(^{(12)}\) in reviewing the causes of 143 bridge failures does not attribute any failure to effects of thermal loading. Therefore, it may be concluded that thermal loading is unlikely to significantly affect the ultimate load capacity of a bridge superstructure, but may seriously affect the performance at service load levels, as several cases of cracking of prestressed decks have been attributed to thermal load. This being one of the reasons for the work in this thesis to be limited to prestressed concrete superstructures under service conditions.

Creep is defined as the increase in strain of a structure under a constant stress. Experimental investigations \(^{(13,14)}\) have shown that the rate of creep strain increases with an increase in the temperature of the concrete. The object of this study is to investigate the effect of the temperature creep interaction on the relief of part of the stresses due to long-term loadings on the bridge superstructure, such as dead load, prestressing forces and thermal load.

This study is based on a two-span post-tensioned continuous concrete deck and also a post-tensioned simply supported skew deck. Different sections are considered for continuous right decks, while only a solid slab type of deck is considered for the simply supported skew deck for reasons given later in this study. Constant thermal gradient and cyclic thermal gradient are both studied, and the results compared, for the continuous superstructure. As a result of this comparison, only constant thermal gradient is assumed for the simply supported skew slab.

The study is limited to prestressed concrete superstructures. The reason being that the developed method of analysis of the effect of temperature creep interaction on the stress redistribution in a concrete
The deck is based on two main assumptions. The first assumption is that the section is uncracked. While the second assumption is that the whole section consists of concrete only. Both assumptions cannot be applied to a reinforced concrete deck, but they may be applied to a prestressed concrete deck. The reason is that in prestressed superstructures, it is possible to keep the whole deck in compression under long term loading, and so satisfying the first assumption. While the second assumption is satisfied by the fact that the cross sectional area of the prestressing tendons is a very small percentage of the cross sectional area of the deck and can be ignored.

The main reason for limiting this study to prestressed concrete superstructures is that, in general, they are more sensitive to elastic stress levels than reinforced concrete structures.

Bridges designed before the introduction of sophisticated analytical techniques such as computer grid analysis, finite element, and folded plate theories, were typically designed on a conservative basis. Thus unexpected stresses were accommodated for by the large factors of safety. With relatively recent improvements in analytical and computing capacity, there has been a tendency to refine the design process. This resulted in better definition of the stresses and deformations induced by specific load cases, that in turn resulted in a corresponding reduction in factors of safety in the serviceability and ultimate limit states of the structure. In this study as mentioned earlier, an attempt is made to understand the effect of temperature creep interaction on stress redistribution in prestressed concrete bridge decks. It is hoped that this may lead to an improvement in the design process of bridge superstructures, particularly those built in hot climates where thermal stresses are more significant.
CHAPTER 2: SURVEY OF EXPERIMENTAL WORK ON TEMPERATURE AND CREEP IN CONCRETE
2.1 Introduction

A survey was made of laboratory experiments and field tests that have been carried out over the years to investigate the effect of temperature and creep on concrete. This thesis is particularly concerned with the combined effect of temperature and creep on the stress redistribution in prestressed concrete bridge decks. The range of concrete temperatures that is of interest is between 30-60°C. However, the effects of lower and higher temperature are briefly discussed. Most researchers were found to prefer laboratory experiments to field testing. This may be due to the better and cheaper control on the experiment in the laboratory.

The problem with field testing is that the environmental variables, such as temperature and humidity, could vary so much within a day or a month. This makes it difficult to relate results obtained from field tests with those obtained from simulated laboratory experiments. Although surprisingly good agreement was sometimes found between the two types of testing, but this is not always the case.

This Chapter deals first with the prediction of temperature distribution within a concrete bridge deck, based on laboratory and field testing of concrete slabs subjected to temperature gradients. The effect of temperature on the heat flow properties of concrete is then studied. This latter study is based on laboratory testing of the effect of temperature on thermal conductivity, thermal diffusivity, specific heat and surface conductance of concrete.

The study is then continued with laboratory investigations of the effect of temperature on the deformation properties of concrete, (that is, modulus of elasticity, coefficient of thermal expansion, Poisson's ratio, and creep) and compressive strength. Finally, the effect of creep on the redistribution of stresses in a prestressed concrete deck is investigated, based on field and laboratory testing carried out over a period of about 14 years, at the University of Illinois, U.S.A.
2.2 Temperature distribution in concrete decks

A considerable amount of research\(^{(1,15)}\) has been carried out concerning the temperature distribution in concrete bridge decks. The research carried out at the University of Texas at Austin\(^{(15)}\), U.S.A., was focused on the development of a computer program for the prediction of temperature distribution in a concrete deck. The temperature distribution is assumed to be constant along the centre line of the bridge, but can vary arbitrarily over its cross section.

It should be noted that the actual flow of heat is a result of all three mechanisms of heat transfer, which are conduction, convection and radiation. In general, convection and radiation will govern the flow of heat at the boundaries of the deck, while conduction governs the heat flow within the deck. The purpose of the computer program was to couple these basic mechanisms of heat transfer in an effort to arrive at a systematic way of predicting the bridge temperatures as a function of time.

The research team\(^{(15)}\) at the University of Texas used a two-dimensional mathematical model for predicting the temperature distribution in a concrete deck. This type of model is useful for the study of bridge decks with an irregular shape of cross section. For a known time-dependent boundary temperature distribution, the interior temperatures in a homogeneous isotropic body with no internal heat source is governed by the classical two-dimensional heat conduction equation,

\[
\frac{\partial T}{\partial t} = \frac{k}{\rho c} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad \ldots \ldots (2.1)
\]

where \(x, y\) = directions in Cartesian coordinates, m,
\(t\) = time, hours h,
\(T\) = temperature at any point \((x, y)\) at time \(t\), °C,
\(k\) = thermal conductivity, W/h/m/°C,
\(\rho\) = density, Kg/m\(^3\)
\(c\) = specific heat, W/Kg/°C.

The developed computer program, based on Equation (2.1), was checked by measuring the temperature distribution within a bridge deck. The selected bridge was a simply supported structure with a span of 14.4m,
and its cross section consisted of reinforced concrete T-beams. The bridge is situated in Texas, where high temperature variations are common.

Temperature effects in the bridge were investigated by Wah and Kirksey\(^{(2)}\). A total of 390 thermocouples were installed in the bridge to record temperatures. Thirteen thermocouples, six in the slab portion and the remaining seven in the beam portion, were used to measure temperatures over the depth of each T-beam of the deck.

August is usually considered as the hottest summer month in Texas, and during that month, temperatures were recorded on the bridge from 6.00 a.m. to 8.00 p.m.\(^{(2)}\). It was apparent that the bridge temperature distribution was almost uniform at 9.30 a.m., as the heating phase began to equalize the temperature gradient developed during the night. Hence, for simplicity of input data, it was assumed that the starting time for the computer program is at 9.30 a.m., and the temperature is uniform over the cross-section at that time.

Plots of predicted and measured temperature distributions are shown in Figure 2.1. Comparisons of temperature distributions are shown to be satisfactory. Good correlations are obtained at all time increments. Therefore, the two-dimensional heat flow computer program can predict the temperature distribution within a concrete deck quite accurately.

It was necessary to use a two-dimensional heat transfer mathematical model because the temperature distribution over the cross section has been found to vary nonlinearly in both vertical and horizontal directions. This is due to the fact that the temperature in the beam is a result of the heat conducting from the top surface and that of the surrounding air temperature exposed to the exterior surfaces of the beam. The distribution of temperature, over the depth of the T-beam deck, computed from a one-dimensional heat flow model gave values that disagreed with the measured values of temperatures. However, for a solid slab deck, both the one-dimensional and two-dimensional heat flow models gave temperature distributions that agreed well with the measured values of temperatures.

The study, at the University of Texas, demonstrated the feasibility and validity of analytically predicting the thermal response of a bridge structure subjected to daily atmospheric variations. The computer
Fig. 2.1 Measured and predicted temperature distributions.
The program developed was shown (2) to be quite general, as it can be used to treat various conditions of the environment and types of highway bridge cross sections. Although the study was focused on the climatic conditions that have existed in Texas, U.S.A., other locations can also be studied if the relevant weather conditions were used as input to the computer program.

It has been found (16) that the important weather parameters influencing the bridge temperature distributions are radiation, air temperature and wind speed. Incoming solar radiation is the primary source that increases the top surface temperature of the deck during the day. Outgoing radiation, on the other hand, results in decreasing the deck temperature during the night. The important material thermal properties are surface conductance, thermal diffusivity and conductivity. Shape, size and thickness of the deck also affects the temperature variations over the cross section of the deck.

Extreme environmental conditions have been found (15) to take place on a clear night followed by a clear day with a large range of air temperature. In general, on a clear sunny day in Austin, Texas, the maximum incoming solar radiation intensity occurs at about noon, the maximum shade temperature occurs at 4.00 p.m., and yet the top surface temperature is found to be a maximum at 2.00 p.m.

The main disadvantage (17) of the two-dimensional heat flow computer program is the significant time involved in preparing the data, and also the significant computer time required for the analysis. Therefore, this computer program should only be used when absolutely necessary.

The Transport and Road Research Laboratory, England, have carried out an extensive research programme (18,19,20) on the temperature distribution in concrete bridge decks. This research has included both experimental and theoretical work. The theoretical work included the development of a computer program based on the one-dimensional heat conduction equation,

\[
\frac{\partial T}{\partial t} = \frac{k}{\rho c} \left( \frac{\partial^2 T}{\partial x^2} \right) \quad \ldots \ldots \quad (2.2)
\]
where $T$, $t$, $k$, $P$, $C$ and $x$ are as defined for Equation (2.1). The computer program is suitable for evaluating the temperature distribution within a solid slab or a cellular slab deck. Since the program is based on linear heat flow, therefore, it cannot be applied to conditions where the heat flow is non-linear, that is, close to the edges of a deck slab. However, the edge strips where the heat flow is two-dimensional are comparatively narrow, and have no significant effect on the temperature distribution in the rest of the slab deck. Therefore, when using the computer program to calculate the temperatures in a slab deck, the edge effects can be neglected.

The Laboratory experiments and field measurements carried out by the Transport and Road Research Laboratory confirmed that their one-dimensional heat flow computer program could predict temperature distribution in concrete slab decks very accurately. However, better agreement was found between predicted and measured values of temperatures in slab decks where the thickness was 0.8m or more. The disagreement between the predicted and measured values increases with the reduction in thickness of the slab deck, but it is not of practical significance until the slab thickness is 0.2m or less.

2.3 Effect of temperature on heat flow properties of concrete

2.3.1 Thermal Conductivity ($k$)

The thermal conductivity of a material measures the ability of that material to conduct heat, and is defined as the ratio of the flux of heat to temperature gradient. The thermal conductivity of concrete is essentially determined by the following factors:

(1) The thermal conductivity of the aggregate (sand + coarse aggregate) which occupies a volume between 60 to 80% of the concrete$^{(21)}$.

(2) The water content (degree of saturation) at the time of heating$^{(22)}$.

Hardened saturated cement paste made with Ordinary Portland type cement will have a thermal conductivity between 1.1 and 1.6 W/m°C at temperatures between 5 and 15°C$^{(23)}$. 

The thermal conductivity of aggregates varies widely between less than 1.0 W/m°C to greater than 4.5 W/m°C \(^{(21,24)}\). However, three groups of rock types can be isolated as follows:

1. Siliceous rocks, \(k = 3.3\) to \(4.2\) W/m°C.
2. Igneous Crystalline, \(k = 2.3\) to \(2.8\) W/m°C.
3. Igneous Amorphous, \(k = 1.0\) to \(1.7\) W/m°C.

Concrete manufactured from the above groups of aggregates can be likewise grouped as follows:

1. Concrete made with Siliceous type of aggregate, \(k = 2.4\) to \(3.6\) W/m°C.
2. Concrete made with Igneous Crystalline type of aggregate, \(k = 1.9\) to \(2.8\) W/m°C.
3. Concrete made with Igneous Amorphous type of aggregate, \(k = 1.0\) to \(1.6\) W/m°C.

The above values of thermal conductivity are for saturated concrete as obtained at temperatures between 5 and 25°C.

The extent to which increasing temperature affects the thermal conductivity of concrete \(^{(25,26,27)}\) depends upon both its initial moisture content and the variation in moisture content with heating. When the concrete is predried, Figure 2.2, then increasing the temperature will lead to a linear reduction in thermal conductivity. However, the behaviour of an initially wet concrete is different and is demonstrated in Figure 2.3. In this latter condition, it can be seen that heating to approximately 60°C causes an increase of about 10% in thermal conductivity. This increase is due to a combination of water migration and the substantial increase in the thermal conductivity of water. Heating to above 60°C causes a reduction in the thermal conductivity of concrete.

Blundell, Dimond and Brown \(^{(28)}\) concluded the following remarks concerning the choice of a value for thermal conductivity for a particular concrete:
Fig. 2.2 The effect of temperature on the thermal conductivity of predried concrete.
Fig. 2.3. Effect of temperature on thermal conductivity of concrete (initially saturated concrete)
(1) For wet concrete, at normal air temperature, the following minimum values of \( k \) should be used:

<table>
<thead>
<tr>
<th>Aggregate type</th>
<th>( k (\text{W/m}^0\text{C}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Siliceous rocks</td>
<td>2.4</td>
</tr>
<tr>
<td>Igneous Crystalline</td>
<td>1.9</td>
</tr>
<tr>
<td>Igneous Amorphous</td>
<td>1.0</td>
</tr>
</tbody>
</table>

(2) Where the proposed aggregate type is not known during design, the minimum value of \( k = 1.0 \text{ W/m}^0\text{C} \) should be used.

(3) For those structures in which drying might occur at elevated temperature, use can be made of Figure 2.3 to evaluate an appropriate value of \( k \).

2.3.2 Thermal Diffusivity (D)

The thermal diffusivity represents the rate at which temperature changes within a mass can take place, and is thus an index of the facility with which concrete can undergo temperature changes. Thermal diffusivity can be measured either directly or by evaluation of the other properties of the concrete given in the following equation,

\[
D = \frac{k}{C.P} \quad \ldots \quad (2.3)
\]

where

- \( D \) = thermal diffusivity,
- \( k \) = thermal conductivity,
- \( C \) = specific heat,
- \( P \) = density.

The thermal diffusivity of concrete is essentially governed by the thermal diffusivity of the aggregate used. The thermal diffusivity of the aggregate varies widely between \( 0.69 \times 10^{-6} \) to \( 2.8 \times 10^{-6} \text{ m}^2/\text{S} \) \((29,30)\).

The thermal diffusivity of concrete varies widely depending on the type of aggregate used, the value ranges from \( 0.69 \times 10^{-6} \text{ m}^2/\text{S} \) for basalt type of aggregate to \( 1.89 \times 10^{-6} \text{ m}^2/\text{S} \) for quartz. If the type of aggregate is not known, a mean value of \( 1.25 \times 10^{-6} \text{ m}^2/\text{S} \) should be used \((28)\).
It was found (28) that the thermal diffusivity of dry concrete decreases with increase in temperature. The decrease in the value of (D) is not significant up to about 70°C, as raising the temperature of dry concrete from 20°C to 70°C will only reduce the value of (D) by about 10%. Even if the temperature of concrete is increased to 200°C, the value of (D) will be reduced by about 30%.

It has been concluded, as a result of experimental investigation, that the value of (D) is not significantly affected by the moisture content (24), and that the effect of age at heating is negligible (28).

2.3.3 Specific Heat

The specific heat of concrete, which represents its heat capacity is governed mainly by the water content at the time of heating, due to the high specific heat of water. A typical value for saturated concrete at 20°C would be 1.0 KJ/Kg°C. However, the value can vary between 0.7 and 1.5 KJ/Kg°C (24). A reduction in water content of approximately 5% will reduce the specific heat by about 20%. Changes in aggregate content or type have little effect on the specific heat of concrete (29,31).

The extent to which increasing temperature affects the specific heat of concrete also depends on its initial moisture state. Where the concrete is predried, heating causes a linear increase in specific heat of 75% up to 550°C (32). However, an increase in temperature from 20°C to 70°C will increase the specific heat by about 10%.

Where the concrete is initially wet, heating to temperatures of approximately 90°C causes a rapid, but temporary, rise in specific heat of up to 280%. This peak is due to the rapid release of free water from the concrete (33).

The specific heat of concrete is a property of particular importance in calculating the thermal diffusivity of a concrete structure.
2.3.4 Surface Conductance

The severity of the temperature gradient, through a concrete wall in contact with fluid on both sides, is to a large extent determined by the temperature change which occurs at the concrete to fluid surface on both sides of the wall. This interface change in temperature is governed by the magnitude of the surface conductance\(^{(34)}\), together with the thermal conductivity of the concrete.

The factors governing the magnitude of surface conductance are:\(^{(34)}\)

1. The surface texture and colour of the solid interface material.
2. The density, thermal conductivity and specific heat of the solid material.
3. The velocity, flow characteristics and viscosity of the fluid.

For a concrete surface in contact with a relatively warmer fluid, the value of surface conductance increases linearly with increases in the velocity of the fluid. However, the value of surface conductance is completely independent of temperature and is mainly affected by the factors listed above.

2.4 Effect of temperature on deformation properties of concrete

2.4.1 Modulus of Elasticity (E):

The modulus of elasticity of concrete is determined by the quantity and type of aggregate used, and the water-cement ratio of the mix. However, it is frequently the case that for design purposes the modulus of elasticity is related to the compressive strength of the concrete. Table 1 in CP110, Part 1, 1972\(^{(35)}\), presents such a relationship for use in design. The CP110 relationship was found to be valid\(^{(28)}\), giving an indication of the average value of E to be expected. However, the actual value of modulus of elasticity for any one concrete may vary by $\pm$ 20% from the values given in CP110\(^{(28)}\).
The modulus of elasticity of concrete increases approximately with the square root of its strength. This relationship is unaffected by temperature of storage up to about $230^\circ C$ or possibly higher, since both properties vary with temperature in approximately the same manner (36). However, for the same strength, the modulus is somewhat higher the lower the early curing temperature; thus steam cured concrete has a lower modulus than moist cured concrete of the same strength, but the difference is under 10 per cent.

Heating concrete causes a reduction in its modulus of elasticity. A considerable variation occurs, depending on the type of aggregate used, in the magnitude of $E$ value reduction when concrete is heated to temperatures of up to approximately $80^\circ C$. Above this temperature, further reduction in the value of $E$ is similar for all concrete types(28). The variation in $E$ up to temperatures of approximately $80^\circ C$ is caused by the different types of aggregate used. Concretes containing siliceous aggregates, such as gravels and sandstones, show a smaller reduction in the value of $E$ than concretes containing limestones or dolomite. This is due to the greater thermal dislocation which occurs when concrete manufactured from the latter materials is heated, due to the aggregate's lower thermal expansion (37).

The modulus of elasticity of concrete manufactured from gravel aggregate is reduced by about 15% when the temperature of the concrete is increased from 20 to $80^\circ C$. The $E$ value for the same concrete is further reduced by 55% if the temperature of the concrete is increased to $350^\circ C$, making a total reduction in the value of $E$ at $20^\circ C$ of about 70%. This behaviour can be compared with that of concrete manufactured from limestone aggregate, where the reduction in the value of $E$ will be about 30% when the temperature of the concrete is increased from 20 to $80^\circ C$. However, the total reduction in the value of $E$, if the limestone concrete is increased to $350^\circ C$, will be about 70%, which is similar to the percentage reduction in the value of $E$ for a gravel concrete (28).

It can be concluded that, in the absence of better information, for normal weight concrete the short term elastic modulus may be taken from Table 1, CP110, Part 1 (35). The 28 days $E$ value is usually referred to as the short term value. The elastic modulus of concrete increases rapidly during the first month after casting and usually reaches a value
which is over 70% of the one year value of \( E \). In the period between one month and three months after casting, the rate of increase in the value of \( E \) is much slower than that during the first month, and the \( E \) value may reach 90% of the one year value. Therefore, when choosing a value for \( E \), the short term value must first be multiplied by a factor, as a correction for the age of concrete, and then multiplied by another factor as a correction for temperature increase in the concrete.

### 2.4.2 Coefficient of Thermal Expansion (\( \alpha \))

Like most engineering materials, concrete has a positive coefficient of thermal expansion. The \( \alpha \) value of concrete is an important property in determining thermal gradient stresses in structural components subjected to differential heating. The stresses obtained are directly related to \( \alpha \) value of concrete which can vary between \( 5 \times 10^{-6} \) to \( 15 \times 10^{-6} \) per °C.\(^{23,24}\)

The influence of the mix proportions, on the \( \alpha \) value of concrete, arises from the fact that the two main constituents of the concrete, cement paste and aggregate, have dissimilar thermal coefficients and the coefficient for concrete is a resultant of the two values. The coefficient of thermal expansion of cement paste varies between about \( 11 \times 10^{-6} \) and \( 20 \times 10^{-6} \) per °C\(^{39}\), and is higher than the coefficient of ordinary aggregates. In general terms, the \( \alpha \) value of concrete is a function of the quantity of aggregate in the mix and of the coefficient of thermal expansion of the aggregate by itself\(^{40}\).

The \( \alpha \) value of aggregate increases with silica content\(^{41}\), siliceous gravel having the highest values \( (12 \times 10^{-6} \) per °C) and limestones having the lowest values \( (5.5 \times 10^{-6} \) per °C). Concrete manufactured from siliceous gravel will have a coefficient of thermal expansion of about \( 13 \times 10^{-6} \) per °C, while concrete manufactured from limestone aggregate has a coefficient of thermal expansion of about \( 8.5 \times 10^{-6} \) per °C.

The coefficient of thermal expansion of concrete up to a temperature of 90°C increases approximately linearly with temperature rise\(^{28}\). However, the rate of increase in the value of \( \alpha \) for concrete with temperature rise is very low, and therefore a constant value of \( \alpha \) can
be taken for concrete where the temperature varies between, say, 30 and 60°C. When the temperature of concrete greatly exceeds 90°C, the rate of increase in the value of $\alpha$ with temperature rise is then substantially increased \(^{(27)}\). This is due to the failure of the aggregate-cement paste bond and the resultant loss of restraint to expansion of the paste. This non-linearity causes an effect, such that when the concrete cools down, a permanent strain remains. Subsequent thermal cycling increases the permanent strain, but to a diminishing extent \(^{(28)}\). It is not possible to be precise on the magnitude of this permanent thermal strain as it is dependent upon the aggregate type, moisture state and maximum temperature achieved. However, for one set of tests on saturated concrete heated to 75°C, no permanent strain occurred with siliceous gravel concrete, while there was a permanent strain of $50 \times 10^{-6}$ when a limestone aggregate was used \(^{(24)}\).

When the type of aggregate is unknown during design, the value of $\alpha$ for concrete should be taken as $10 \times 10^{-6}$ per °C. Reinforcing steel will increase the $\alpha$ value of limestone concrete member, which in turn affect thermal movements within the structure. When there is 2% of reinforcement in the member, the value of $\alpha$ should be increased by 10% of the value of plain limestone concrete. The coefficient of thermal expansion of gravel concrete having a value similar to the value of $\alpha$ for steel ($12 \times 10^{-6}$ per °C) is not affected by the presence of the reinforcing steel.

### 2.4.3 Poisson's Ratio ($\nu$)

The ratio between the lateral strain accompanying an applied axial strain and the latter strain is used in the design and analysis of many types of concrete structures. Poisson's ratio of concrete varies in the range 0.11 to 0.21 when determined from strain measurements.

No reliable information on the variation in Poisson's ratio with age or strength of concrete is available, but it is generally believed that Poisson's ratio is lower in high strength concrete. Under high loads, Poisson's ratio increases rapidly owing to cracking with the specimen, with an apparent Poisson's ratio as the specimen is no longer a continuous body.
Values for Poisson's ratio may be obtained from the ratio of strains initiated by many different combinations of stress\(^{(42)}\). Since even small errors in a creep value will lead to large errors in the Poisson's ratio, one can expect a large scatter in the results. This is especially so since adjustments are needed for shrinkage at both elevated and normal air temperatures, as well as for coefficient of thermal expansion during a temperature change.

The ratio has been determined for both the elastic condition and the creep condition for sealed concrete at room temperature\(^{(43)}\). The values varied between 0.1 and 0.2, but were most frequent within the range 0.15 to 0.20, though the values were slightly higher for cement paste:

A similar conclusion has been reached for the creep Poisson's ratio at high temperatures\(^{(44)}\), the creep Poisson's ratio being defined as the ratio of the total accumulated strains.

Sanders\(^{(42)}\) concluded that a value of 0.18 would be a reasonable estimate for the creep and elastic Poisson's ratio for ordinary concretes at normal temperatures. However, Blundell, Dimond and Brown\(^{(28)}\) concluded that the average value of Poisson's ratio for concrete at both normal and elevated temperatures is 0.18.

2.4.4 Creep

Considerable amount of research has been directed towards the influence of temperature on concrete creep. Different investigators have employed different intrinsic variables, which makes it difficult to compare the relevant results. Workers have studied cement paste, mortar or concrete, made with different aggregate types, various water-cement ratios and aggregate-cement ratios. The size and shape of specimens have varied enormously from slabs to beams to cubes or cylinders. These tests have been conducted under a range of conditions to study the influence of various factors on the creep of concrete.
2.4.4.1 Magnitude and range of temperature

Most researchers (42) agree that creep increases, almost linearly, with temperature up to at least 70°C. The magnitude of the creep at 70°C is about 3.5 times the creep at room temperature.

Even above 70°C, many researchers have found increased creep with higher temperatures. For instance, Arthanari and Yu (45) recorded the creep at 80°C to be 3 times the creep at 20°C, with the creep almost linear with temperature. The tests were carried out on sealed slabs tested in either uniaxial or biaxial compression, following only 12-18 hours at the test temperature.

Other researchers (42) found that after heating the test specimens one day before loading, creep was observed to be linear in the range 27°C to 77°C. The creep being 4 to 4.8 times greater at 77°C than at 27°C. At temperatures of 95°C, the creep rate was found to be greater than linearity would predict. Specimen, heated to 70°C or 90°C 7 days after loading, obeyed a linear relationship between creep and the logarithm of time in days.

The most important conclusion to be drawn from the tests of Nasser and Neville (46) is that creep is not a monotonic function of temperature, but passes a maximum in the vicinity of 71°C. However, Arthanari and Yu (45) have found no creep rate maxima for tests of up to 80°C, though the rate of creep increase curve was flattening off.

2.4.4.2 Cyclic temperature

Little research has been undertaken on the effect of changing temperature on the deformation of concrete under sustained loading. However, a series of tests have been undertaken (47) in which temperature was increased and decreased on concrete samples which had been kept under sustained load at temperatures of 20, 65, or 95°C. Two series of tests were undertaken. In the first series, loaded samples were subjected to a change in temperature occurring over a period of approximately 5°C per week.
Results from the above tests showed (28), as might be expected, that raising the temperature caused the creep to increase, while lowering the temperature resulted in an initial decrease in creep, followed by continued deformation but at a reduced rate.

Analysis of the data indicated that the changing creep rate could be predicted with an acceptable degree of accuracy. This can be done by using a modified strain superposition method (47). Two versions of the method need to be applied depending upon whether the temperature is being increased or decreased. Where the temperature is increased, the difference between the total strain needs to be taken into account due to an increase in both the elastic and creep strain with increased temperature. Where the temperature is reduced, only changes in the creep rate need to be taken into account.

2.4.4.3 Stress-strength ratio and age of concrete

The relation between creep and stress-strength ratio at any higher temperature is linear, just as at room temperature (46). The range of stress-strength ratios was extended from 50 per cent in previous tests to 70 per cent in the tests by Nasser and Neville (46).

The age of concrete at loading has a considerable effect on the amount of creep. Results of tests can be used to compare the creep of concretes loaded at 14 days and at one year of age of concrete. After 91 days under load, at a stress-strength ratio of 0.45 and a temperature of 21°C, the creep of the older concrete is 0.41 of the younger.

At higher temperatures the ratio is somewhat higher, indicating that the relative influence of temperature on creep is smaller in older concrete. Taylor and Williams (48) also found that the sensitivity of creep to the age at loading decreases with an increase in temperature. For instance, at 20°C the ratio of creep strains for loading at 180 days and at 7 days age of concrete is about 0.5, but at 93°C the ratio is 0.84. This behaviour applies both to sealed and unsealed concrete specimens.
2.4.4.4 Preheating and predrying of concrete

Tests on concrete creep include some on specimens which have been desiccated by heating, usually at $105^\circ\text{C}$ (42). All tests produce the same result, that is when the specimens are loaded at a temperature below that of drying, the rate of creep is extremely low. Results of tests showed that the magnitude of creep was independent of subsequent loading temperatures over the range of $5^\circ\text{C}$ to $58^\circ\text{C}$.

Another method of preheating is to steam-cure the specimen. Low-pressure steam curing reduces creep (49). For instance, steam curing at $65^\circ\text{C}$ for 13 hours reduces creep by 30 to 50 per cent, partly because of the accelerated hydration of cement, and partly because of the moisture loss occurring when the hot specimens are moved to a drier and cooler atmosphere.

Reutz desiccated specimens at $105^\circ\text{C}$ prior to loading at temperatures in the range $20^\circ\text{C}$ to $80^\circ\text{C}$, and he found that the creep after six days increased monotonically with temperature. This finding agrees with results of other tests (42), where specimens were desiccated at $105^\circ\text{C}$ to eliminate evaporable water and to effect any chemical changes which occur in the concrete by heating to $105^\circ\text{C}$. The specimens were then heated in the range $20^\circ\text{C}$ to $400^\circ\text{C}$, and the creep measured under a constant stress. The creep rate increased monotonically throughout the range, also creep rate was proportional to the logarithm of time. This type of creep is probably due to the viscous flow within the crystalline structure of the concrete.

Therefore, it can be concluded that, (42) concrete creeps even following drying at a higher temperature than that at which it is tested. This shows that there is a component of creep which is not related to the movement of moisture within the cement gel, a component of creep which is due to the above mentioned viscous flow or movement. It may be a stable component, with its rate proportional to temperature, but this is difficult to determine, since its magnitude is small compared with creep initiated by moisture movement.
2.4.4.5 Order of loading and heating

Raising the temperature of concrete while under load produces greater creep than raising the temperature prior to loading. Therefore there must be a component of creep which occurs only during the change of temperature while under load. This component of creep requires investigation and is discussed in section 3.2.2.

2.5 The variation of compressive strength of concrete with temperature

Considerable work has been carried out over recent years which has shown that when concrete is heated, the compressive strength is initially reduced \(^{(28)}\). More recent research has shown that a similar reduction occurs in both the tensile strength and bond strength.

However, the variation in strength with temperature is complicated by the interaction of two factors. The first factor being the thermal incompatibility between the aggregate and the cement paste. While the second factor is the amount of drying which can occur in the cement paste during heating.

Heating concrete to temperatures of up to approximately \(85^\circ C\) causes a reduction in strength, the magnitude of which is determined by the coarse aggregate used \(^{(51)}\).

When concrete is heated, stresses are generated at the cement paste aggregate interface, causing micro-cracking resulting in concrete strength reduction \(^{(52)}\). The magnitude of these stresses depends upon the modulus of elasticity and coefficient of thermal expansion of the coarse aggregate. Aggregates with low coefficient of thermal expansion such as limestone, will result in a higher degree of bond dislocation than will occur with the use of higher thermal expansion gravel.

The large increase in compressive strength at temperatures below \(0^\circ C\) is due to freezing of the water within the concrete \(^{(28)}\). This
increase, however, only occurs whilst the low temperature prevails; subsequent thawing may reduce the strength to below the original 5°C value. When concrete is heated to above 100°C, rapid drying normally occurs. This drying increases the compressive strength of concrete. Heating to above 250°C, at which point drying is complete, causes a further almost linear reduction in strength.

Compressive strength tests (52) were conducted on cylinder specimens, heated for short duration to temperatures of 93°C to 871°C. Variables included aggregate type and test procedure (heated without load and tested hot, heated with load and tested hot, and tested cool after heating). The following conclusions were derived from these tests:

(1) Specimens made of the carbonate aggregate concrete retained more than 75% of their original strength at temperatures of up to 649°C, when heated without load and tested hot.

(2) Strengths of specimens stressed in compression during heating were generally 5 to 25% higher than those of companion specimens which were not stressed during heating.

(3) Unstressed residual strengths (that is, specimens heated, cooled and then tested) were somewhat lower than the strengths of companion specimens tested at high temperatures.

(4) Strengths of specimens stressed in compression during heating were not significantly affected by the applied stress level, which ranged from 25 to 55% of the original strength.

(5) Original strength of the concrete had little effect on the percentage of strength retained at test temperatures.

More series of tests were carried out to investigate the influence of high temperature on the compressive strength of concrete (53). The following conclusions were derived from the results of these tests:
When concretes of portland and blast furnace cement with quartz and baryta aggregates are exposed, unloaded, to temperatures of up to $300^\circ C$, a strength decrease of about 20% is to be expected. The above strength decrease is reduced (quartz aggregate) or prevented (baryta aggregate) when the concrete is loaded during the temperature rise.

Other researchers found that for unsealed concrete specimens, an increase of temperature to $100^\circ C$ does not affect the short-term strength significantly. Severe loss in strength at higher temperatures is associated with increase in internal dislocation from differential expansion between the hardened cement paste and the aggregate particles, and from increase in loss of non-evaporable water above $105^\circ C$. The effect of temperature on sealed concrete has also been investigated, the tests recorded a negligible loss at $50^\circ C$, with a loss in compressive strength of 30% at $100^\circ C$ for a limestone concrete. A sustained temperature of $100^\circ C$ for 3 months for a high strength concrete does not increase the 20% loss found immediately after heating, tested in both the hot and cold state.

Prolonged exposure to elevated temperature has a different effect on concrete, particularly where siliceous aggregates have been used. It will cause a marked increase in strength, such that any losses that may have occurred upon initial heating are fully recovered. Where limestones or other non-siliceous aggregates are used, this improvement in strength does not always occur. When the temperature of a prestressed concrete element is varied cyclically, an increase in strength is unlikely even for siliceous aggregate concrete. The reason being the periodic internal restressing of the concrete resulting from temperature variation.

Although there are some contradictions between the work of several researchers, the following conclusions can be drawn that particularly concern this study.
Raising the temperature of concrete from 25°C to 50°C has an almost negligible effect on the compressive strength of concrete.

When the concrete temperature rises to about 60°C, the compressive strength loss could be as little as 8% of the 25°C value.

2.6 Effect of creep on prestressed concrete bridge decks:

The University of Illinois at Urbana, U.S.A., started project IHR-93 in July 1965, with the objective of investigating the long term behaviour of prestressed concrete highway bridges under service loads. The project goals were broad, and it was also envisaged that design methods would be evaluated, and the relationship between the structure that was designed and the as-built structure would be examined.

As the project developed over its 14 years life, three major tasks were examined. The first task was the installation of instrumentation in three structures, the gathering of the deformation data, and the interpretation of that data. The interpretation of the data required the development of theoretical analyses for the prediction of long-term deformations and changes in stress in multispans bridge structures.

A second major part of the programme was the construction of two 1/8th scale models of a three span line of beams from one of the structures instrumented in the field. The initial purpose of this work was to see whether small structures could be used to provide valid information about the long-term behaviour of prestressed concrete members. This objective was satisfied, and the results of the small model studies formed an important part of the validation process for the analytical procedures developed in support of the field measurement programme.

The third major area of study was an analytical investigation of the effects of span diaphragms on the load distribution behaviour of slab-girder bridge decks, made with precast pretensioned concrete I-section
girders. The significant variable governing load distribution were studied, and it was concluded that span diaphragms are generally inefficient in improving load distribution characteristics\(^{(57)}\). As a result, this study was confined to bridge decks without in-span diaphragms.

For the purpose of my study the first part of the research programme at the University of Illinois is of interest, that is, the long-term deformations and changes in stress in mult spanning bridge decks. The test structure used for this investigation was a three span highway bridge in Champaign County, Illinois \(^{(55)}\). Each span contained six precast prestressed I-section girders, spaced at 2.44m centres. All girders were 14.07m long and 1.07m deep. The bridge had no skew.

The structure was designed to meet the requirements of the 1969 AASHO Bridge Specification \(^{(58)}\), except that no span diaphragms were used in the structure. The structure may be described as being a three simply supported spans for dead load, but at the same time as a three span continuous structure for live loads, because of the manner in which it was constructed.

Concrete test specimens were made with samples of the concrete used in the girders and deck. They were used for determination of the compressive strength and modulus of elasticity of concrete, for shrinkage measurements, and in creep tests.

The strains which occurred in the instrumented girders of the structure were measured at appropriate time intervals, so that a picture of the time-dependent response could be obtained. All data which was plotted versus time was referenced to the time of transfer of the prestressing force. At this time, the girder concrete was 3 days old, and the deck concrete was cast 57 days later, that is when the girder concrete was 60 days old.

The creep strains were computed using a simple definition of creep, as follows:

\[
\varepsilon_c = \varepsilon_t - \varepsilon_i - \varepsilon_s \quad \ldots (2.4)
\]
where \( \epsilon_c \) = component of the total strain which is attributed to the creep of concrete,
\( \epsilon_t \) = total strain measured in loaded specimen, starting from time immediately before loading,
\( \epsilon_i \) = initial strain occurring as an immediate effect of loading,
and \( \epsilon_s \) = strain in companion specimen, identical to creep specimen except that it is not loaded, starting from immediately before loading creep specimen.

The several components of the total strain were assumed to be independent quantities. All computations were done at particular time intervals, with both total and shrinkage strains measured at the same time. These definitions of creep and shrinkage may not be adequate for the material scientist, but they are convenient and satisfactory definitions for structural engineering uses.

Theoretical concrete strains were plotted versus time for midspan sections in Figure 2.4. These strains were computed using the field creep and shrinkage data. The computed strains were found to be in general agreement with the measured strains\(^{(55)}\), although the measured strains were slightly larger than the computed values. Both the computed and measured strains demonstrated that casting of the deck caused only a temporary reduction in the compressive strain. The strains continued to increase after the deck was cast, Figure 2.4.

Creep strains versus time for deck and girder concrete are shown in Figures 2.5 and 2.6 respectively. From the graphs, it can be seen that there is good agreement between field and laboratory tests.

There is a substantial change in the stress distribution over the depth of the girder after the deck is cast, Figure 2.7. During this time the deck shrinks and creeps to a very limited extent, while the top fibre of the girder creeps rather extensively as a result of the
Fig. 2.4. Theoretical strain-time curves for bottom fibre of girder at midspan
Fig. 2.5 Creep strains versus time for concrete under a compressive stress of $7.0 \text{ N/mm}^2$. 
Fig. 2.6. Creep strain versus time for concrete under a compressive stress of 7.0 N/mm²
Fig. 2.7. Distributions of stress in girder at midspan sections at various times.
increase in stress accompanying the casting of the deck. The top of the girder also shrinks during this period, but this would normally be expected to be a small movement because of the greater age of the girder. If the compressive strains, with time, in the girder exceed those in the deck, compressive stresses will be induced in the deck. As the stress in the deck increases, the stress in the top fibres of the girder decrease, and some of the compression force in the top of the girder may be thought of as being transferred to the deck. The deck stresses never become large, but there is a large area of concrete available so that the force in the deck is a significant part of the total compression force in the section.

The University of Illinois project IHR-93 was ended in May 1980. The final reports (59, 60) on the above project produced many important conclusions for prestressed concrete beams. A number of these were relevant to this study and are discussed below, as they influenced the variables considered in the proposed design procedure developed in Chapter 4.

(1) The net initial concrete stress in a prestressed beam is an important factor in the estimation of creep, as increasing the initial stress increases the creep. However, additional permanent loads on the beam will have a significant effect on the creep strains as they cause a stress reduction at the centre of gravity of the strands, and thus lead to a decrease in creep. The time of placement of an additional permanent load after prestressing makes some difference in the creep strains, as the earlier the time, the smaller the creep.

(2) Age of concrete at transfer of prestress makes some difference in the values of creep strains. Younger concretes, which have most of their potential creep still remaining, will creep more than older concretes. This age factor is less significant for post-tensioned than for pretensioned members.
(3) Environmental conditions, especially variable humidity and temperature in the field, have a significant influence on creep. Creep increases with the increase in the temperature of the concrete and also with the decrease in the relative humidity of the environment.

(4) The estimated values of modulus of elasticity of concrete at transfer and at time of major stress changes, based on the specified minimum concrete strength are important. However, the normal increase with time in these estimated values produces only minor changes in the creep behaviour.

(5) The results of long-term creep measurements on concrete subjected to both constant laboratory environment and to field exposure, indicate that the creep strains were comparable in spite of the continuous variation in temperature and relative humidity in the field.
CHAPTER 3: THE EFFECT OF TEMPERATURE AND CREEP ON CONCRETE BRIDGE DECKS
3.1 Temperature

3.1.1 Introduction

The problem of thermal effects in various types of highway bridges has been of major interest to bridge design engineers for many years. Past research (61) which has been done in this area indicated that a temperature difference between the top and bottom of a bridge can result in high temperature induced stresses. However, there still exists uncertainties concerning the magnitudes and effects of these stresses caused by daily variations of the environment.

Ekberg and Emanuel (62) reported that temperature effects have been considered more frequently for steel bridges than for concrete bridges. This is perhaps attributed to the relative lack of both theoretical and experimental work on the thermal behaviour in concrete structures. As concrete bridges become more frequently designed to behave continuously under live load, the temperature effects become more significant than those designed with simple spans. In addition, for deep concrete sections which are commonly found in long span bridges, the temperature distribution over the depth of the section will be highly nonlinear, thus resulting in high internal stresses. For example, Van (65) found that under certain conditions, thermal stresses could cause serious cracking in reinforced concrete structures.

The magnitude of temperature induced stresses (64) principally depends on the nonlinear form of the temperature gradient over the depth of the section. Bridge temperatures (65) can be determined using recorded weather data, but the method is limited only to the unidirectional heat flow. The shape of the section will influence the temperature distributions inside the concrete deck. For structures with a complicated cross section, the temperature will be nonlinear both vertically and horizontally, which may require the use of a two-dimensional heat flow theory.
3.1.2 The heating of a concrete deck

3.1.2.1 Environmental variables

The temperature in a highway bridge is caused by both short-term (daily) and long-term (seasonal) environmental changes. Seasonal environmental fluctuations from winter to summer, or vice versa, will cause large overall expansion and contraction. If the deck is free to expand longitudinally, the seasonal change will not lead to temperature-induced stresses. However, daily changes of the environment result in a temperature gradient over the deck cross section that may cause temperature induced stresses, depending on the nonlinearity of the temperature gradient and the shape of the deck cross section.

The most significant environmental variables which influence the temperature distribution in a concrete deck are solar radiation, ambient air temperature and wind speed. The significance of these variables will be discussed below.

3.1.2.1.1 Solar radiation

Solar radiation is the principal cause of temperature changes over the depth of highway bridges. Solar radiation is a maximum on a clear day, as there is no obstruction to sun rays. The sun's rays are absorbed directly by the top surface and cause it to be heated more rapidly than the interior region, thus resulting in a temperature gradient over the bridge cross section. The amount of solar radiation actually received by the surface depends on its orientation with respect to the sun's rays. The intensity is a maximum if the surface is perpendicular to the rays and is zero if the rays become parallel to the surface. Therefore, the solar radiation intensity received by a horizontal surface varies from zero just before sunrise to maximum at about noon and decreases to zero right after sunset.
In order to predict daily bridge temperature distribution, the variation of the solar radiation intensity during the day must be known. This can be accomplished by field measurements. Several types of pyranometers have been developed for this purpose. Another approach is to use data published in the weather bureau reports. Unfortunately, this data is recorded as a daily integral, i.e. the total radiation received in a day. Since it is desirable to use data which has been recorded to predict the bridge temperature distribution, several approximate procedures have been developed (2) to estimate the variation of solar radiation intensity during the day using the daily radiation data.

It has been confirmed from field measurements (1) that the variation of daily solar radiation intensity on a horizontal surface is approximately sinusoidal squared that is, \((\sin)^2\). The variation of solar radiation with time may be calculated from the equation:

\[
I(t) = \frac{2S}{T} \sin^2 a. \quad \text{..... (3.1)}
\]

where \(I(t)\) = intensity of solar radiation at time, \(t\), (W/m\(^2\)h)
\(S\) = total radiation for the day, (W/m\(^2\)),
\(T\) = length of solar day, (h)
and \(a = \frac{\pi t}{T}\).

### 3.1.2.1.2 Air temperature

Air temperature varies enormously with locations on earth and with the seasons of the year. The manner in which daily air temperature varies with time must be known in order to predict temperature effects in bridges. The maximum and minimum values of air temperature in a day are regularly recorded at weather stations in most countries. The hourly temperature variation, however, can only be obtained from local weather reports if available.

On clear days with little change in atmospheric conditions, the air temperature generally follows two cycles. The normal minimum temperature is reached at or shortly before sunrise, followed by a
steady increase in temperature due to the sun's heating effect. This increase continues until the peak temperature is reached during the afternoon. Then the temperature decreases until the minimum reading is reached again the next morning. However, clouds and rain can change the cyclic form of temperature variation. Clouds, for example, form a blanket so that much of the sun's radiation fails to reach the earth, this results in lowering the air temperature during the day. At night, back radiation from the clouds where some heat was stored during solar hours, cause a slight increase in air temperature.

It is worth noting that the times of high and low ambient air temperature do not coincide with the times of maximum and minimum solar radiation intensity \(^{(1)}\). This is true for both the daily and seasonal conditions. In Austin, Texas, the month of August is generally considered as the hottest month of the year, January the coldest; yet the greatest intensity of radiation occurs in June and the lowest in December \(^{(66)}\).

3.1.2.1.3 Wind Speed

Wind is known to cause an exchange of heat between surfaces of the bridge and the environment. The speed of the wind has an effect in increasing and lowering surface temperatures. It has been found\(^{(67)}\) that maximum temperature gradients over the bridge deck depth are reached on a still day. On a sunny afternoon, wind decreases temperatures on the top surface and increase temperatures at the bottom surface of the deck. This effect results in lowering the temperature gradient during the day. At night, maximum reversed temperature gradient are also decreased by the presence of the wind.

3.1.2.2 Heat flow conditions

The prediction of time varying bridge temperature distributions involves the solution of the heat flow equations governing the flow of heat at the bridge deck boundaries and within the bridge. In general heat is transferred between the bridge boundaries and the environment by radiation and convection. Heat is then transferred within the bridge
deck by conduction. In order to estimate the temperatures in bridge decks, the relationship between atmospheric conditions and the heat transfer from the deck boundaries to the atmosphere must be known. The development of this relationship is discussed below.

3.1.2.2.1 Heat flow by radiation

Radiation is the primary mode of heat transfer which results in warming and cooling of bridge deck surfaces. The top surface gains heat by absorbing solar radiation during the day and loses heat by emitting out-going radiation at night. The amount of heat exchange between the environment and the bridge deck boundaries depends on the absorptivity and emissivity of the surface, the surface temperature, the surrounding air temperature and the presence of clouds. Values of absorptivity of a plain concrete surface depends on its surface colour. In general, its value lies between 0.5 to 0.8. Concrete with an asphaltic surface has higher absorptivity and published values (15) are between 0.85 to 0.98. The emissivity, on the other hand, is independent of the surface colour. Its values lie between 0.85 to 0.95.

The heat transferred by radiation is carried by both short-wave and long-wave radiation. Heat energy absorbed by the surface from the short wave radiation can be estimated from:

\[ Q_s = rI \]  \hspace{1cm} ... (3.2)

where \( Q_s \) = heat gained by short wave radiation \( \left( \text{W/m}^2\text{h} \right) \),
\( r \) = absorptivity of the surface,
and \( I \) = solar radiation intensity, \( \left( \text{W/m}^2\text{h} \right) \).

Values of \( I \) on a cloudless day can be obtained either by field measurement or by using Equation 3.1. If Equation 3.1 is used, the total total radiation for the day \( S \) must be known.
It can be shown theoretically\(^{(68)}\) that heat loss by long wave radiation from bridge surfaces to the environment can be approximated by:\(^{(15)}\)

\[
Q_L = e \sigma (\theta_s^4 - \theta_a^4) \quad \ldots \ldots \text{(3.3)}
\]

where \(Q_L\) = heat loss by long wave radiation, (W/m\(^2\)h),
\(e\) = emissivity,
\(\sigma\) = Stefan-Boltzmann Constant\(^{(5)}\), (W/m\(^2\)h \(\circ\)K\(^4\)),
\(\theta_s\) = surface temperature (\(\circ\)K),
and \(\theta_a\) = air temperature, (\(\circ\)K), \(k = 273 + \circ\)C.

It has been confirmed from field measurements\(^{(67)}\) that Equation 3.3 yields a reasonable estimate of radiation loss between the earth's surface and the environment under a cloudy sky condition. Equation 3.3, however, is found to underestimate the net heat loss from the surface when the sky is clear. This is due to the fact that the clouds, which can be regarded as a black body, absorb solar radiation during the day and emit it back to the earth during the night. According to Swinbank\(^{(69)}\), the incoming long wave radiation \(R\), can be estimated from:

\[
R = \epsilon \theta_a^4 \quad \ldots \ldots \text{(3.4)}
\]

where \(R\) = incoming long wave radiation under clear sky, (W/m\(^2\)h),
\(\epsilon\) = constant \(^{(69)}\), (W/m\(^2\) h \(\circ\)K\(^6\)),
and \(\theta_a\) = air temperature, (\(\circ\)K),

Equation 3.4 has been shown to represent the data from a number of sites in the U.S.A. with a high accuracy. Therefore, net radiation loss at the top surface of a bridge can be estimated from:

\[
Q_{LC} = e \sigma \theta_s^4 - \epsilon \epsilon \theta_a^6 \quad \ldots \ldots \text{(3.5)}
\]

where \(Q_{LC}\) = net heat loss by long wave radiation under clear sky condition, (W/m\(^2\) h),
3.1.2.2.2 Heat flow by convection

The two types of heat exchange by convection between bridge boundaries and the environment are termed free and forced convection. In the absence of wind, heat is transferred from the heated surface by air motion caused by density differences within the air. This process is known as free convection.

It has been shown from field measurements\(^\text{(67)}\) that the heat loss by convection at bridge surfaces can be approximately estimated by assuming heat loss to be proportional to the first power of the temperature difference. Therefore, in general, heat loss by convection from a dry bridge deck is given by:\(^\text{(67)}\)

\[
Q_c = h_c(T_s - T_a)
\]  

(3.6)

where \(Q_c\) = heat loss by convection (W/m\(^2\)h),  
\(T_s\) = surface temperature, (°C),  
\(T_a\) = air temperature, (°C),  
and \(h_c\) = convection film coefficient which is proportional to wind speed.

3.1.2.2.3 Heat flow by conduction

Heat is transferred within the bridge boundaries by conduction. For a known time dependent boundary temperature, the interior temperature distribution is governed by the classical transient heat-conduction equation, based upon three dimensional heat flow,

\[
\frac{\partial T}{\partial t} = \frac{k}{\rho C} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right)
\]

(3.7)

where \(T\) = temperature of mass as a function of \(t, x, y\) and \(z\),  
\(t\) = time,  
\(x, y, z\) = directions in rectangular coordinates,  
k = thermal conductivity,  
\(\rho\) = density,  
and \(C\) = specific heat.
The thermal conductivity, $k$, is a specific characteristic of the material. It indicates the capacity of a material for transferring heat. Experiments (70) have shown that its magnitude increases as the density of the material increases. The minimum value of thermal conductivity of ordinary concrete is $1.0 \text{ W/m} \cdot \text{°C}$. Whether the material is wet or dry affects thermal conductivity. It has been shown (28) that a wet concrete has a higher thermal conductivity than a dry concrete.

Shapes of temperature distribution over the depth of a bridge deck depend mainly on its conduction properties. A steel beam, for example, because of its high thermal conductivity, will quickly reach the temperature of the surrounding air temperature. However, this is not true for a concrete beam. Nonlinear temperature distribution is usually found in the concrete structure as a result of its low thermal conductivity.

3.1.2.3 Heat of hydration

The reaction of cement with water is exothermic and liberates a considerable quantity of heat over a curing period of approximately 28 days (71). Thin structural members with a high surface area to volume ratio are able to dissipate the heat by conduction and convection from the surfaces, and thus avoid significant temperature rise. The heat released by hydration in thick concrete members cannot be conducted away at a rate sufficient to avoid temperature build up. Temperature rises, at the centre of large concrete members, of up to $50^\circ\text{C}$ have been measured (72).

For practical purposes, it is not necessarily the total heat of hydration that matters, but the rate of heat evolution. The same total heat produced over a longer period can be dissipated to a greater degree with a consequent smaller rise in temperature. The rate of heat development can easily be measured by an adiabatic calorimeter, and typical time-temperature curves have been obtained (73). For the usual range of Portland cements, Bogue (74) observed that about one-half of the total heat is liberated between 1 and 3 days after casting, and about
three-quarters of the total heat is liberated in the first 7 days after casting. Therefore, it can be concluded that the heat of hydration will only have an effect on the structure during its very early life.

In general, the actual temperature rise is not particularly significant in itself, as mechanical material properties are effectively independent of temperature within this range. However, deformations induced by the temperature rise and subsequent cooling need to be considered. Restraint of the free thermal expansion and subsequent contraction can induce thermal stresses that may be of sufficient magnitude to cause cracking. Structural elements where such thermal cracking is possible and significant include, among others, thick solid slab decks and bridge diaphragms.

Thurston, Priestley and Cooke (75) developed a simple method, based on linear heat-flow analysis, for estimating temperatures and thermally induced stresses resulting from heat of hydration. Comparison with experimental data established that accurate estimates of the time history of temperature, for large placements, during hydration could be made using this method. Despite difficulties in measuring stress in young concrete, satisfactory agreement was obtained between measured and predicted stresses for hydrating concrete.

### 3.1.3 Temperature distribution in bridge decks

A temperature distribution is the variation of temperature through any vertical section of a bridge deck at any instant in time, Figure 3.1.1. The environmental conditions which control the shape of the distribution of temperature through the depth of the deck of a bridge are complex. Figure 3.1.1 shows that, for a concrete bridge, the temperatures within approximately the top 0.5m of the deck are controlled by the solar radiation (76). From a depth of about 0.5m to within approximately 0.3m of the soffit the temperatures are controlled by the weather conditions of the previous one or two days, and the temperatures within the bottom 0.3m are controlled by the shade temperature and the heat reflected or re-radiated from the ground beneath the bridge.
Fig. 3.1.1  Temperature distribution measured in the Abur Bridge slip road at 1630 hrs. on 1st. June 1971.\textsuperscript{18}

Fig. 3.1.2  Temperature difference distribution derived from Fig.3.3.1

Fig 3.1  Temperature distribution, temperature difference and temperature gradient.
A temperature difference $^{(77)}$ at a given depth is defined as the difference between the temperature of the surface of the deck and the temperature of the deck at the specified depth. A temperature difference is said to be positive if the temperature of the surface of the deck is higher than the temperature of the deck at the specified depth. Conversely, a temperature difference is said to be reversed if the temperature of the surface of the deck is lower than the temperature of the deck at the specified surface. A temperature difference distribution is the variation of temperature difference through any vertical section of a deck at any instant in time. It is derived from a temperature distribution by calculating the temperature difference at all levels through the depth of the deck, using the temperature of the surface of the deck as the datum temperature. Figure 3.1.2 shows the temperature difference distribution which has been derived from the temperature distribution shown in Figure 3.1.1. Distributions of temperature differences can consist wholly of positive temperature differences, wholly of reversed temperature differences, or of a combination of both.

A maximum temperature difference, whether positive or reversed, is obtained from a temperature difference distribution, and is the maximum value of the difference in temperature between the deck surface and the coldest, for maximum positive differences, or hottest, for maximum reversed differences, area of the deck, irrespective of the depth of the coldest, or hottest, area of the deck. Temperature differences are considered in a vertical direction only, that is through the depth of the deck, and do not include surfacing temperatures. In other directions, such as in the transverse direction, a slightly different analytical approach is required.

As temperature differences are derived from temperature distributions, it therefore follows that all temperature distributions are also considered in a vertical direction only, and do not include surfacing temperatures.

Except during fairly prolonged periods of heavily overcast or wet weather, temperature differences will always exist within the deck of a bridge. Their magnitude will depend on such factors as the type
of construction, the time of day, the time of year, the depth of construction, the depth of surfacing and in the case of some concrete structures such as solid slab decks, the weather conditions of the previous one or two days.

The combinations of environmental conditions necessary to cause large positive or reversed temperature differences are difficult to define. For example, consider a solid concrete deck 1.0m deep with a vertical distribution of temperature differences as shown in Figure 3.1.2. The temperature differences within the top 0.5m (approximately) of the deck are controlled mainly by the incident solar radiation, the greater the amount of radiation, the larger the resultant temperature differences. The depth of surfacing on the deck will also have an effect. As would be expected, the greater the depth of surfacing, the smaller the temperature difference within the upper half of the concrete deck.

Another term that will be used throughout the course of this work is the temperature gradient. The definition of a gradient is a rate of change in a quantity with distance; thus a temperature gradient is a rate of change in temperature with distance, the distance in this case being the depth of the deck. As the variation of temperature with depth, that is the temperature distribution, is usually non-linear, the temperature gradient is not constant, as shown in Figure 3.1.2.

The temperature gradients in BS5400 (7) are based upon research carried out at the Transport and Road Research Laboratory (TRRL). They are expressed as temperature values at particular depths in a member for both the heating and cooling periods and are referred to as positive and reverse gradients respectively. For convenience and simplification the gradients are straight lines and are based upon a surfacing thickness of 100mm. It is possible to derive temperature gradients using TRRL research reports (1,77) for any depth of surfacing and member depth.
There are two problems not covered explicitly by BS5400 which have to be resolved when preparing the data for grillage or finite element analysis of the deck:

1. **The gradient in the edge member:** At the present time there is a dearth of published information on measured temperatures and hence likely gradients in edge members. It is suggested that the gradient should be based upon the average surfacing thickness over the width of the idealised edge member. Where there is no substantial edge stiffening, this gradient can be applied to the normal edge member. When the edge stiffening is significant, there is no information concerning the section to which the gradient should be applied. From analysis of slab decks it has been observed that the edge member gradient has negligible effect on the forces in the inner members and for convenience could even be given the same gradient as inner members.

2. **The gradient in the transverse member:** In the solid slab decks the gradient for transverse members is the same as for the longitudinal members. However, in beam and slab decks, there are two possibilities for transverse members. Firstly, a gradient based upon the slab thickness in its own right. Secondly, a gradient based upon part of the adjacent longitudinal gradient, i.e., the temperature of the top of the top slab and the temperature at the bottom of the top slab. For beams at centres, the first method seems logical, but for beams placed side by side the correct solution probably lies somewhere between the two methods. To assess the significance, the methods have been considered on an M-beam deck with beams side by side. The difference in the resulting forces and stresses was insignificant longitunally and it was recommended that the first solution is used, that is, a gradient in the transverse members in their own right.

3.1.4 The use of a computer program to estimate temperature distribution.

The evaluation of expected temperatures and temperature gradients in concrete structures is of great importance in the design of bridges. Thermal movements and stresses should be considered in bridge design.
calculations. The iterative heat flow equations \(^{(1)}\) may be used to calculate temperature gradients in concrete bridge decks, for given conditions of shade temperature, radiation and wind speed. However, these calculations are lengthy and this section describes a method for calculating bridge temperatures in a concrete bridge deck using a computer program\(^{(79)}\). The program was based on the single dimensional heat flow equations.

The output of the program, consists of values of the temperatures at the levels of the deck specified in the input data. These values are calculated for every 15 minutes beginning with the starting time, and continuing for the length of day. For each interval of time, that is, 15 minutes, the value of the effective bridge temperature is calculated.

The computer program has a number of limitations, these are:

1. The program cannot be used to calculate temperatures over a continuous 24 hours cycle. To overcome this, the day is divided into daytime and nighttime cycles, with each part being analysed separately.

2. The program only calculates temperature distributions in simple box and solid slab decks.

3. Since the program is based on one dimensional heat-flow equations, it cannot accurately calculate temperatures at or near the edge of a deck. However, because areas of a bridge deck subjected to edge effects are only a small proportion of the total area, these edge effects are ignored in this analysis.

4. The temperature distributions calculated, using the program, are not accurate for thin slabs when the surfacing material represents a large percentage of the deck thickness \(^{(1)}\).
(5) When calculating temperature distributions in a deck, all
temperatures in the deck are assumed equal to each other at the
starting time, and equal to the shade temperature at that time.
This type of condition has been observed in concrete bridge decks,
two or three hours after sunrise \(^{(15)}\), depending on location of
the bridge and the month of the year.

3.1.5. Evaluation of thermal stresses

3.1.5.1 Introduction

The most significant aspect of daily thermal response of bridges
is the restraint of free thermal movement. This is of direct concern
to bridge designers because it results in induced forces of substantial
magnitude.\(^{(80)}\) In particular, longitudinal tension stresses
induced at the soffit of continuous bridges can be isolated as the
single most troublesome effect.\(^{(81)}\) In special cases, thermally
induced reactions can result in overstress of bearings, and the
associated shear force variations can result in shear cracking.
During construction, thermally induced deflections may cause tolerance
difficulties, particularly when construction in cantilever proceeds
inwards from adjacent piers towards a final joining pour or segment
placement.\(^{(82)}\)

A simple method \(^{(83)}\) of thermal stress analysis is chosen. The
application of the method to continuous solid slab and box section
type of decks is discussed in some detail. The assumptions, that
will be made, enable the calculations for most designs to be carried
out manually. However, a computer \(^{(84)}\) will be used for all design
involving considerable variations in moment of inertia, while for small
variations in cross sectional properties, average values may be used.

The following method \(^{(85)}\) of thermal stress analysis can be
applied to solid slab and hollow box section designs in reinforced
and prestressed concrete, for both statically determinate and
indeterminate span arrangements.
3.1.5.2 Assumptions

The following assumptions were made in developing this method of thermal stress analysis:

1. Material properties are independent of temperature.
2. Sections are prismatic throughout, that is, moment of inertia is constant.
3. Temperature gradient induces circular bending, that is, radius of curvature is constant.
4. The Navier-Bernoulli hypothesis that initially plane sections remain plain after bending.
5. In solid rectangular cross-sections, the temperature gradient is linear between the top and bottom faces of the concrete deck. The gradient is determined from the temperatures measured in the concrete at the top and bottom faces.
6. In hollow box sections, the temperature differential is contained wholly within the top slab, with the webs and bottom slab at a different constant temperature. The top slab temperature is that measured in the middle of the slab thickness. The web and bottom slab temperature is the average of temperatures measured along the depth of the web and bottom slab.

3.1.5.3 Solid slab on a continuous span

Consider the rectangular solid slab, Figure 3.2.1, subjected to a linear temperature gradient, on a continuous span of total length L, Figure 3.2.4. The following steps can be taken to calculate thermal stresses in the section due to the applied temperature gradient, Figure 3.2.2.

1. Evaluate T, which is the difference in temperature between the top and the bottom faces of the section, Figure 3.2.2.
2. The intermediate support provides a restraint against free hogging, Figure 3.2.6. If the deck is released from the intermediate support, temperature gradient causes circular hogging with a radius of curvature R, Figure 3.2.5.
3.2.1. Cross section

3.2.2. Temperature gradient

3.2.3. Hogging of deck

3.2.4. Two-span deck

3.2.5. Free hogging

3.2.6. Induced forces

3.2.7. Induced moments

Fig. 3.2. Solid slab on a continuous span.
where \( \alpha \) = coefficient of thermal expansion
\( d \) = depth of the section.

(3) From the properties of a circular arc, calculate the vertical offset \( \Delta_1 \), Figure 3.2.5.

(4) From the flexural properties of the section, calculate force \( W_1 \), Figure 3.2.6, which when applied, gives a deflection equal and opposite to \( \Delta_1 \), Figure 3.2.5.

(5) As \( W_1 \) is an induced force arising from the application of the temperature gradient, then the stresses arising from the induced bending moment \( M_1 \), Figure 3.2.7 are those induced by the temperature gradient.

3.1.5.4 Box section on a continuous span

Consider the box section in Figure 3.3.1 on a continuous span \( L \), Figure 3.2.4, subjected to a temperature differential \( T \), Figure 3.3.2.

If the slab is freed from the webs, then there is a temperature strain of \( \alpha T \), Figure 3.3.3. Since there is a composite action between the slab and the rest of the section, the free strain in the slab is reduced. Strains are then induced in the webs and soffit so that at the interface on the underside of the slab, the nett total strain is the same in both the slab and the webs.

From the fourth assumption in Section (3.1.5.2) the total strain is linear, Figure 3.3.4. The induced strains due to temperature differential, Figure 3.3.5, are obtained by subtracting the free strain from the total strain. The stresses corresponding to the induced strains, are those due to temperature differential. Stresses due to continuity forces must also be added to give total stresses caused by the temperature gradient.
Fig. 3.3. Box section on a continuous slab.
This method \(^{(85)}\) can be summarised as follows:

(1) Composite action induces a compressive force \(P_1\), in the slab, and an equal tensile force \(P_2\) in the webs and soffit. The two forces create a couple \(M\) which causes hogging of the composite beam, Figure 3.3.7.

(2) There are no external forces or moments acting on the beam. Therefore \(P\) and \(M\) are constant throughout the span.

(3) The following equations were derived \(^{(85)}\), Figure 3.3.7;

\[
P = P_1 = P_2
\]
\[
M = M_1 + M_2 = P_1 h = P_2 h = Ph.
\]
\[
M_1 = \frac{EI_1}{R} \quad \ldots \ldots \quad (3.8)
\]
\[
M_2 = \frac{EI_2}{R} \quad \ldots \ldots \quad (3.9)
\]
\[
P = \frac{E}{4R} [I_1 + I_2] \quad \ldots \ldots \quad (3.10)
\]
\[
R = \frac{[\frac{1}{R}(I_1+I_2)(\frac{1}{I_1} + \frac{1}{I_2}) + h]}{\alpha T} \quad \ldots \ldots \quad (3.11)
\]

where \(I_1\) = moment of inertia of slab about its neutral axis,

\(I_2\) = moment of inertia of webs and soffit about their neutral axis,

\(A_1\) = area of slab,

\(A_2\) = area of webs and soffit,

\(E\) = modulus of elasticity,

\(\alpha\) = coefficient of thermal expansion,

and \(R\) = radius of curvature of composite beam.

(4) Calculate \(P\) and \(R\) from Equations (3.10) and (3.11).

(5) Calculate \(M_1\) and \(M_2\) from Equations (3.8) and (3.9).
(6) Calculate the stresses due to $P$, $M_1$, and $M_2$, which are induced by temperature differential.

(7) Stresses due to continuity forces are found by following steps (3), (4) and (5) in Section (3.1.5.3).

(8) The total stress due to the temperature differential in the algebraic sum of all stresses due to $P$, $M_1$, $M_2$ (step 6), and those due to continuity forces (step 7).

3.1.5.5 Improvements in thermal stress analysis

Priestley (14, 86) improved the methods of thermal stress analysis by assuming a temperature distribution based on a fifth power parabola, Figure 3.4.2. The longitudinal thermal stress is separated into two components, Figure 3.6, these are:

3.1.5.5.1 Primary thermal stresses

The structure to be analysed is made statically determinate by the removal of sufficient internal redundancies, Figure 3.5, and the stresses due to the nonlinearity of the temperature profile are calculated. These are known as the primary thermal stresses.

Consider the section in Figure 3.4.1 subjected to a temperature difference $t(y)$, Figure 3.4.2, where $y$ is measured relative to some convenient datum. For totally unrestrained expansion at all heights, the free strain profile would be,

$$\varepsilon_f = \alpha t(y) \quad \ldots \ldots \quad (3.12)$$

where $\alpha$ is the linear coefficient of thermal expansion. However, plane sections are assumed to remain plane. Thus the final strain profile $\varepsilon(y)$, Figure 3.4.3, should be linear. The difference between the free strain and the final strain gives the restraint strain, Figure 3.4.3. Therefore, the restraint stress equals to,
3.4.1. Section

3.4.2. Temperature Change

3.4.3. Strains

Fig. 3.4. Primary thermal strains

3.5.1. Restraint of hogging

3.5.2. Induced moments due to restraint

Fig. 3.5. Continuity moments

3.6.1. Section

3.6.2. Primary stress

3.6.3. Secondary stress

3.6.4. Total stress

Fig. 3.6. Longitudinal thermal stress
\[
fp(y) = E \left[ \varepsilon(y) - \alpha t(y) \right] \quad \ldots \quad (3.13)
\]

where \( E \) is the concrete modulus of elasticity, which is a constant as material properties are assumed to be independent of temperature. Although the temperature varies with depth, Figure 3.4.2, but it is assumed constant at all points of equal elevation.

Integration of Equation (3.13) over the depth of the section, \( d \), yields the axial force,

\[
P = E \int \left[ \varepsilon(y) - \alpha t(y) \right] \cdot b(y) \cdot dy \quad \ldots \quad (3.14)
\]

where \( b(y) \) is the section width at height \( y \). Taking moments about the neutral axis, the internal moment induced by \( t(y) \) will be,

\[
M = E \int \left[ \varepsilon(y) - \alpha t(y) \right] \cdot b(y) \cdot (y-n) \cdot dy \quad \ldots \quad (3.15)
\]

Now the linear final strain distribution, \( \varepsilon(y) \), may be expressed as,

\[
\varepsilon(y) = \varepsilon_0 + \psi y \quad \ldots \quad (3.16)
\]

where \( \varepsilon_0 \) is the final strain at \( y=0 \) and \( \psi \) is the curvature of bending.

For a simply supported span without moment or axial force restraints, we have \( P = M = 0 \). Substituting Equation (3.16) into Equations (3.14) and (3.15), setting both to zero, and noting that,

\[
\int y \cdot b(y) \cdot dy = n.A. \quad \ldots \quad (3.17)
\]

and

\[
\int y^2 \cdot b(y) \cdot dy = I + A.n^2 \quad \ldots \quad (3.18)
\]

yields (14) the following equations:

\[
\psi = \frac{a}{I} \int t(y) \cdot b(y) \cdot (y-n) \cdot dy \quad \ldots \quad (3.19)
\]

and
where \( I \) = the moment of inertia of the section about its neutral axis,
\( n \) = the distance from the neutral axis of the section to the datum line,
and \( A \) = area of the section.

The primary thermal stress given by Equation (3.13) can be rewritten as follows,

\[
f_p(y) = E \left[ \varepsilon_0 + \psi \cdot y - \alpha \cdot t(y) \right] \quad \ldots \ldots (3.21)
\]

Although the section shown, Figure 3.4.1, represents a box girder, the method is general and can be applied to all sections. The only limitation of this method is that it can be used for prestressed and uncracked reinforced concrete decks only, that is, homogeneous isotropic behaviour is assumed. Thus thermal stresses can be considered independently of stresses imposed by other loading conditions, and the principle of superposition is applicable.

3.1.5.5.2 Secondary thermal stresses:

For continuous decks, the hogging curvature \( \psi \), Figure 3.5.1, resulting from the primary thermal response is restrained by internal supports. Thus inducing secondary thermal stresses caused by the restraining moment \( M \), Figure 3.5.2. The stresses induced by \( M \) will be,

\[
f_s(y) = \frac{M(y-n)}{I} \quad \ldots \ldots (3.22)
\]

The total thermal stress, Figure 3.6.4, at any point along the depth of the section is the algebraic sum of the primary thermal stress, Figure 3.6.2, and the secondary thermal stress, Figure 3.6.3. From equation (3.21) and (3.22), the total thermal stress is given by the following equation,

\[
f_t(y) = E \left[ \varepsilon_0 + \psi \cdot y - \alpha \cdot t(y) \right] + \frac{M(y-n)}{I} \quad \ldots (3.23)
\]
It can be seen that the primary thermal stresses are caused by the nonlinearity of the temperature distribution in the deck. While the secondary thermal stresses are caused by the continuity of the deck.

3.2 Creep

3.2.1 Introduction

Concrete, when kept under sustained load, deforms slowly with time, the movement being commonly known as creep. Engineers have been aware of the existence of this property for many years, but only comparatively recently that its harmful attributes have become apparent. For example, the deflection and the stress losses in prestressed concrete beams increase with time, and are more serious in large span beams.

Considerable research and testing have been and are still being undertaken on the subject of creep throughout the world. This work ranges from using the electron-microscope to postulate the cause of creep in the microstructure of the creep-inducing material of concrete, namely the hardened cement gel, to actual observations of structures including prestressed bridges, mass concrete dams and reinforced concrete framed buildings. The knowledge available concerning creep is fragmented and the suggested methods for predicting the effect of creep range from simple to complicated methods. In the simple methods, creep is estimated by a single multiplier of the reciprocal of the elastic modulus. While complicated methods require information as to the likely concrete mix and the environment to which the structure will be exposed throughout its life.

It is the designer who ultimately has to assess the importance of concrete creep for his design. Whether the creep effect is upon beam deflection, over all structural movements, or modifications to the state of stress distribution. The designer has to make allowance for these effects by, for instance, the inclusion of adequate horizontal movement joints in bridge decks or the use of adequate initial prestress forces to allow for relaxation in the tendons' stresses.
The designer has three major interests in creep:

1. its influence upon the behaviour of the concrete itself as a structural material under sustained loading;
2. the effect of creep in the concrete in relation to its composite action with other structural materials;
3. the effect of creep upon the behaviour of the total structure.

Creep is one of a number of factors which influence design, and its importance will vary. In some cases, the effects of creep will be marginal in the total design and a coarse assessment is all that is required. In other cases, creep is an important consideration and a more careful and accurate assessment of its magnitude and effect will be necessary such as in the case of large span prestressed beams. In general, the designer is interested not so much in the creep itself, but in its effect upon the structure. The ways in which creep may affect the behaviour of a total structure, both beneficially and harmfully, may have to be considered by the designer.

In those cases where the designer recognizes that an accurate assessment of creep is necessary, then the designer needs to be provided with a detailed account of the various factors that affect creep, the magnitude of these effects and their relative importance.

3.2.2 Components of creep

When concrete which has been subjected to a sustained stress is unloaded, the recovery of strain is of two types, Figure 3.7. The first is the instantaneous recovery, which represents the elastic strain corresponding to the stress removed, and to the modulus of elasticity at the time of the removal of the load. This immediate recovery is followed by an additional gradual recovery, which is called creep recovery by analogy to the creep under load.

The creep recovery is smaller than the preceding creep, that is, creep appears to be a partly recoverable phenomenon. Information about creep recovery is of importance in predicting behaviour of concrete
Fig. 3.7. Components of creep (Constant temperature)

Fig. 3.8. Transitional thermal creep component
under variable stress, and also in elucidating the mechanism of creep. Many theories postulating a reversible or a partly reversible mechanism of creep hinge on the relation between creep and the subsequent creep recovery.

A theory of reversibility of creep was proposed by McHenry[^95], and is generally referred to as the principle of superposition. In this theory, creep is considered as a delayed elastic phenomenon in which full recovery is impeded only by further hydration of cement. Thus, removal of load is treated as a negative load which induces a creep equal and opposite to that which would be caused by a positive load of the same magnitude applied at the same time.

Davies[^96] found that creep recovery in prestressed concrete is smaller than the value predicted by the principle of superposition. Tests of Kimishima and Kitahara[^97] show that even under mass concrete conditions, the principle of superposition overestimates the creep recovery. Despite this, the principle of superposition, which assumes that creep recovery is a negative creep and is smaller than the preceding creep only because of a change in the creep potential of concrete, is a valuable tool in design[^91].

For a concrete specimen under a constant temperature, the two components of creep are the recoverable and the irrecoverable creep. Figure 3.7. Ishai[^98] found that the irrecoverable creep increases with each cycle of loading and unloading, but at a reduced rate for each successive cycle. He ascribed this behaviour to the assumption that a substantial part of water lost during drying creep is not recovered, so that the irrecoverable deformation increases with an increase in the period under load. The recoverable creep initially increases with time, but then reaches a constant value[^98]. Illston[^100] considers the recoverable creep as a delayed elastic strain with a limiting magnitude proportional to stress. This strain is smaller in saturated concrete than in drying concrete, in which the recoverable creep remains almost constant with age.
The results of extensive tests on creep recovery of mortar suggest that the magnitude of creep recovery cannot be related to the strength of mortar or to the properties of cement used. Likewise, in Theuer's tests on concrete loaded for 3 days and then allowed to recover for 3 days, there seems to be no systematic relation between the ratio 'creep recovery/creep' and the mix proportions or the moisture conditions of concrete. Other tests suggest that the ratio 'creep recovery/creep' maybe somewhat higher at a higher water/cement ratio.

Lyse's tested concrete specimens stored at a relative humidity of 50 per cent, loaded at 7 days for 115 days and then unloaded. These tests are not adequate for general conclusions to be drawn, but there are some indications that the creep recovery is a linear function of the 'stress/strength' ratio up to a value of about 0.65. Roll also found that the ratio 'creep recovery/creep' decreases with an increase in the 'stress/strength' ratio. This was attributed to the fact that at high 'stress/strength' ratios, a large part of the creep is due to microcracking which is irrecoverable.

Although stresses at the aggregate matrix interface may influence the creep recovery, the aggregate presence is not necessary for creep recovery to take place. This was demonstrated by Mamillan who found a measurable creep recovery of neat cement paste unloaded at the age of 210 days. The creep recovery of neat cement paste was found to be considerably slower than that of concrete. This indicates that the elastic compression of the aggregate may influence the phenomena involved. Counto found that the magnitude of the creep recovery increased with a decrease in the modulus of elasticity of the aggregate.

Storage conditions probably influence the magnitude of the ratio "creep recovery/creep", since little of the drying creep is recoverable. This was proved by Mamillan, who found that drying cement paste exhibits little creep recovery. Davis and Davis also found lesser recovery of creep under drying conditions.

Tests at temperatures between 21 and 96°C have shown that the
creep recovery of concrete to be independent of temperature. The independence of creep recovery from temperature is apparent also from tests by Hannant\(^{(111)}\). On the other hand, Serafim and Guerreiro\(^{(112)}\) found that the rate of creep recovery of mass concrete at a higher temperature is greater during the first 5 days after unloading, but thereafter is not affected.

The components of creep that have been discussed so far in this section, recoverable and irrecoverable, are for concrete under constant temperature. It has been found\(^{(113)}\) that a temperature rise induces creep, known as transitional thermal creep (TTC), which is in excess of that which would be expected due to constant temperature creep. Bamforth\(^{(114)}\) defined the TTC as a component of creep which occurs when concrete is heated, while under load, to a temperature to which it was never previously subjected and which cannot be attributed to changes in either the elastic strain, recoverable or irrecoverable components of creep strain, Figure 3.8.

Within the temperature ranges examined by Bamforth\(^{(114)}\), 20 to \(95^{\circ}\text{C}\), it was found that the magnitude of the TTC component increased with an increase in temperature. Furthermore, for a given change in temperature, the magnitude of the TTC appears to increase with the initial concrete temperature. However, the development of the TTC is not instantaneous in mature concrete, but appears to increase linearly on a logarithmic time scale, at least over the first month after heating as the results of longer tests are still not published. When the temperature is reduced after a short period of heating, there is a strain recovery in the TTC component equal in magnitude to approximately half the difference in elastic strains resulting from the temperature change.

Tests carried out in this country\(^{(115)}\) and Sweden\(^{(116)}\) to observe the TTC effect indicated that the magnitude of this component of creep is significantly large in relation to the total load deformation. Therefore, it should be considered in any analysis where concrete under load is to be subjected to a variation in temperature.
3.2.3 Mechanism of creep

A number of theories have been proposed, over the years, which attempted to explain the mechanism of creep. However, none of those theories are capable of accounting for all the observed phenomena of creep. Yet each theory explains a number of observations. It is likely that the actual creep involves two or more mechanisms. Nevile suggested that creep of concrete may be accounted for by combining the following three theories.

3.2.3.1 Plastic theory

It has been suggested that the creep of concrete may be in the nature of crystalline flow, that is, a result of slipping along planes within the crystal lattice. A partial acceptance of the crystalline flow theory was suggested by Glanville and Thomas, who thought that creep at low stresses may be viscous and at high stresses in the form of crystalline slip. At very high stresses, the deformation of concrete resembles plasticity as a basis for ultimate strength design.

Thus, it appears that some form of plasticity may enter the deformational behaviour of concrete at stresses nearing failure. However, this is not the case for the creep behaviour of concrete under service loads.

3.2.3.2 Viscous theory

The viscous flow theory of creep is one of the more important theories. There are strong reasons to believe that viscous flow contributes in some measure to the creep of concrete. The basic argument is that hydrated cement paste is a highly viscous liquid, whose viscosity increases with time as a result of chemical changes within the structure.

Viscous flow as a mechanism of creep was first postulated by Thomas who considered concrete to consist of two parts. The first being cementitious material which behaves in a viscous manner when loaded.
While the second part being the inert aggregate which does not flow under load. When the concrete is loaded, the cement flow is resisted by the presence of the aggregate, and as a result of this resistance, the aggregate becomes more highly stressed while the stress on the cement paste decreases with time. Now, since the creep of cement paste is proportional to the applied stress, the rate of creep will be progressively reduced as the load is transferred from the viscous to the inert material.

Arnstein and Reiner concluded that the rate of creep should depend on the nature of the cement paste, and not on the properties of the aggregate. However, this was shown not to be the case. Furthermore, the viscous flow requires a constant volume, while axial creep of concrete results in a lateral creep.

3.2.3.3 Seepage theory

The seepage theory of creep was first postulated by Lynam. The theory arises from the observation that hydrated cement paste is a rigid gel, and in such gels generally, load causes an expulsion of the viscous component from the voids in the elastic skeleton. This results in a redistribution of stresses from the viscous component to the elastic skeleton. Thus creep in concrete is taken to be due to seepage of gel water under pressure.

The seepage theory of creep was supported and elaborated by Seed, and by Lea and Lee. Their explanation of seepage is that the application of an external stress to concrete causes a change in the internal vapour pressure, and hence in the gel water content, with accompanying volume change. Now, since hydrated cement paste is essentially a rigid gel, equilibrium exists between the swelling pressure of the gel and the solid framework, and disturbance of this equilibrium causes a change in the gel water content.

The rate of seepage depends on the moisture gradient. Moreover, as water is squeezed out, the stress on the solid increases while the pressure on the water correspondingly decreases, with a resulting
reduction in the rate of expulsion of the water. Creep is a manifestation of the delay in re-establishing the equilibrium between the gel and its surroundings.

The creep recovery is simply the tendency to re-establish the original state of equilibrium, when the external load has been removed. However, full recovery is prevented by the formation of new bonds when the gel particles are allowed to come closer to one another as a result of creep. Thus a new stable position of the gel particles is established. It should be emphasized that it is only the gel water that is involved in the seepage movement, and not the capillary water or the chemically combined water.

Although the seepage theory has had considerable support, there are several objections\(^{(126, 127)}\). The major objection arises from the fact that the measured loss of water from concrete under a compressive stress is insufficient to account for the quantity supposed to be lost by seepage.\(^{(126)}\)

The general principle of creep\(^{(117)}\) established from the above theories is that viscous flow of the cement paste occurs. This results in load being transferred from the cement to the aggregate, which then exhibit plastic flow under the action of increased stresses. Due to the applied load some seepage also takes place. This is due to some of the absorbed gel water being squeezed out.

3.2.4 Rheological models for creep studies

The study of the relation between stresses and strains in the most general sense is called rheology. Theoretical ideal bodies with strictly defined rheological properties have been postulated,\(^{(128)}\) and these can be combined so as to result in rheological behaviour similar to that of real materials.

As far as concrete is concerned, a number of attempts have been made to simulate creep deformation by imaginary rheological models consisting of elements\(^{(128)}\) each of which represents a specific
deformational characteristic of a given component or phase of concrete. This approach is largely empirical and its success depends upon the ability to assign a specific part of the creep deformation of concrete to a given element of the model. In other attempts, a number of rheological elements are combined simply to approximate the observed overall deformational pattern, without regard to their physical significance.

The idealized deformations which are used to build up real behaviour of concrete are elastic and viscous\(^{(129)}\) and are represented by a spring and a dashpot respectively, Figures 3.9 and 3.10. The bodies with these ideal linear properties are referred to as a Hookean solid and a Newtonian liquid respectively. It should be emphasised that these mechanical devices do no more than represent the deformational behaviour. It is not suggested that there is any behavioural similarity between a rheological element and the mechanism of deformation of a real material.

A perfectly elastic body, Figure 3.9, is one that exhibits completely reversible deformation. If the load-deformation relation is linear, the body is represented by a linear spring with the following equation,

\[
\chi = \alpha P \quad \ldots \ldots \quad (3.24)
\]

where \(\chi\) = extension of the spring, 
\(\alpha\) = the spring compliance, 
and \(P\) = the applied load.

As Hooke's law applies to perfectly elastic body, then,

\[
\sigma = E \varepsilon \quad \ldots \ldots \quad (3.25)
\]

where \(\sigma\) = stress, 
\(E\) = elastic modulus, 
and \(\varepsilon\) = strain.
Fig. 3.9. Hookean elasticity

\[ \sigma = E \varepsilon \]

Fig. 3.10. Newtonian viscosity

\[ \sigma = \eta \varepsilon \]

Fig. 3.11. Kelvin model

Fig. 3.12. Maxwell model

\[ \text{Creep } \varepsilon = \frac{\sigma}{\eta} t \]

\[ \text{Total strain } \varepsilon = \frac{\sigma}{E} + \frac{\sigma}{\eta} t \]

\[ \sigma \text{ constant} \]
An ideal viscous body, Figure 3.10, undergoes a shearing deformation at a rate which is a function of the applied shearing stress. The mechanical device representing an ideal viscous body is a dashpot, with a piston moving through a fluid of viscosity $\nu$

Under a sustained load $P$, the piston moves with a velocity $dx/dt$ such that,

$$P = \nu \frac{dx}{dt} \quad \ldots \quad (3.26)$$

As the body is assumed to exhibit Newtonian viscous behaviour, then the following equation also applies,$^28$,

$$\sigma = \eta \dot{\varepsilon} \quad \ldots \quad (3.27)$$

where $\eta$ = viscous modulus,

and $\dot{\varepsilon}$ = rate of strain.

The basic elements, elastic and viscous, can be built up into rheological models of varying complexity. There are two basic models,$^29$ known as a Kelvin model, Figure 3.11, and a Maxwell model, Figure 3.12.

In the Kelvin model the spring and the dashpot are in parallel so that they undergo the same displacement. The total stress in this case is the sum of the stresses on the individual components, such that,

$$\sigma = E \varepsilon + \eta \dot{\varepsilon} \quad \ldots \quad (3.28)$$

where $\sigma$ = total stress on the model.

In the Maxwell model, the spring and the dashpot are in series so that they take the same load. The total displacement being the sum of the displacements of the two components of the model. The total strain is given by the following equation,

$$\varepsilon = \frac{\sigma}{E} + \frac{\sigma}{\eta} \cdot t \quad \ldots \quad (3.29)$$

where $\varepsilon$ = total strain at time $t$. 

Use can be made of rheological models to study the time-dependent deformations of concrete. A combination of elements, spring and dashpot, or models, Kelvin and Maxwell, can be made to form a rheological model that will simulate the creep behaviour of concrete.\(^{(130)}\)

3.2.5 Factors affecting creep in prestressed concrete bridge decks.

The creep behaviour of prestressed concrete structures can be affected in a variety of ways by a relatively large number of significant factors. Creep affects the time dependent deformations in two ways. The first by directly increasing the compressive strains in concrete with time, and second by indirectly causing a decrease in prestressing force. The increase in compressive strains may produce, in addition to axial shortening, curvature changes reaching as high as two or three times the initial values, and hence cause a substantial increase in camber. The loss of prestress causes an elastic relief in stresses, strains and curvature, and hence affects the rate of creep. This will lead to creep strains significantly less than those under a constant stress equal to the initial stress.

Normally, in bridge members, the stresses induced by prestressing will be higher than those due to the sustained load, since prestressing must also offset the stresses due to live loads. Hence immediately after prestressing, the initial stress gradients over the depth of the cross-sections are usually large, and creep will cause the deflection to increase. Thus overestimation of creep will lead to overestimation of deflection values.

Fadl and Gamble\(^{(131)}\) studied the factors that affect creep in prestressed concrete bridge decks. The main factors are listed below.

(1) Effect of environmental conditions

Humidity and temperature of the surrounding environment are known to have significant influence on the rates and magnitudes of creep. It has been observed that more creep takes place in prestressed concrete girders in hot weather than in cold weather. Therefore, creep increases
with temperature, the effect being less for dry than for wet concretes. It has also been observed that the higher the relative humidity, the smaller the creep.

(2) Effect of age of concrete at time of post-tensioning

Depending on construction schedules and design specifications, post-tensioning could occur as early as 7 days after casting or as late as 2 or 3 months, and in some cases it might even take much longer. However, for practical reasons, post-tensioning of bridge members does not usually occur before the deck is about 28 days old. Thus at the time of prestressing, the deck concrete might be relatively young or quite old. It has been observed that, under the same sustained load and level of prestress, concrete members loaded at early ages undergo more creep than similar members loaded at later ages. It has also been noted that the age at loading affects both the magnitude and rate of creep.

(3) Effect of variations in the modulus of elasticity of concrete

Mossiessian and Gamble\(^{(132)}\) concluded that theoretical changes in the creep behaviour due to variations in the elastic modulus of concrete are not very significant. These changes in creep strains are expected to be less pronounced in case of post-tensioned members, since the concrete is prestressed at a relatively much older age, and any variations in the concrete modulus thereafter are expected to be small. According to the 1970 C.E.B.\(^{(133)}\) recommendations, creep strains are not affected by variations in the concrete modulus, as the latter is given only as a function of the 28 days modulus. Therefore, it may be concluded that the normal increase with time of the elastic modulus of concrete produces only minor changes in the creep behaviour, for a concrete older than 28 days.

(4) Effect of stresses in the deck's sections

The initial stresses, in prestressed concrete, immediately after anchorage are important to the creep behaviour of the deck. The
higher the initial concrete stress, in compression at the centre of gravity of the strands' level, the greater the creep strain. The relationship between the long-term creep strains and the initial concrete stress was found to be linear, indicating proportionality. However, there are other factors that were found to affect the creep behaviour of prestressed concrete bridge decks but their effects are minor. Those factors include the type of curing, size and shape of the cross section, type of cement and continuity of the deck's spans.

3.3 Temperature-creep interaction

3.3.1 Introduction

The characteristics of temperature creep interaction are illustrated in the following methods of stress analysis. The first method, steady state solution, does not require a detailed formulation of the mechanisms of creep. While the second method, the rate of creep, is based on a more detailed study of the interaction of creep and temperature. The third method, effective modulus solution, is a more approximate method for estimating the effect of creep and has been used over a longer period of time.

3.3.2 Steady-state solution

The work of several investigators was discussed in Section 2.4.4.3 where it was shown that for concrete stressed within the working range, the rate of creep is directly proportional to the applied stress. There is general agreement also that the rate of creep is increased at elevated temperatures, and that within the range $20^\circ C$ to $70^\circ C$ it may be represented approximately as being directly proportional to temperature in degrees centigrade as mentioned in Section 2.4.4.1. Thus a quantity $'c'$ may be defined by the following equation:
The specific creep is thus assumed to be a basic property of the material, and varies with time only.

Creep gives rise to changes in length. When the stress is not uniform over the cross-section both rotations in addition to displacements will result from creep in a concrete structure in which plane sections remain plane. Such stresses may arise from applied external loading or from restrained thermal movements. When non-uniform temperature and stress conditions act simultaneously, creep which is stress and temperature dependent, causes redistribution of stress to take place. If the temperature and load conditions are sustained, redistribution will continue until stresses reach steady values.

For simplicity, the redistribution process is described below for a section which is subjected to sustained constant applied loading, and both the initial stress and temperature distributions are linear over the cross-section.

Initially, an elastic calculation will yield the linear stress and strain distributions corresponding to the combined effects of external loads and temperatures. The stresses will undergo changes due to creep, and the new distributions will at all times satisfy the equilibrium conditions of the section. It is convenient to consider short time intervals during which the stresses may be assumed to remain constant. During the first interval the unrestrained creep strains, which are proportional to the product of the linear distributions of stress and temperature, will be non-linear over the section. The total strain distribution, elastic plus creep, must be linear due to the assumption that plane sections remain plane. Adjustment is therefore made to the
strains, with consequent alteration to the stress distributions, at the end of the interval, thus making these stress distributions non-linear also. Further changes of stress distribution occur in subsequent time intervals.

These changes become progressively smaller, indicating that stresses tend towards steady values. A limit is reached when no further changes of stress occur. These stresses are called limiting steady-state stresses.

G.L. England\(^{134}\) developed the following equations for the calculation of steady-state stresses for a symmetrical two span continuous beam of rectangular cross-section, Figure 3.13. The beam is subjected to a sustained temperature gradient and a prestress force.

\[
M = M_p - \frac{3y}{2L} \left( M_p + N \frac{b_2}{a_2} \right) \quad \ldots \ldots (3.31)
\]

where \(M\) = steady-state bending moment at any section,
\(M_p\) = moment due to prestressing force,
\(y\) = distance from the end support to any section up to the interior support,
\(L\) = length of each span,
\(N\) = constant compressive force,
\[
a_2 = -\frac{\beta}{b'.D} \frac{\log(T_1/T_2)}{D\beta - \alpha \log(T_1/T_2)}
\]
\[
b_2 = \frac{\beta}{b'.D}
\]
\(T_1, T_2\) = temperatures at the top and bottom faces of the beam respectively,
\(\alpha, \beta, b'\) and \(D\) are defined in Figure 3.13,
and \[
\sigma = \frac{A + \beta x}{\alpha + \beta x} \quad \ldots \ldots (3.32)
\]
Fig. 3.13. Continuous beam subjected to sustained prestress loading and temperature gradient

Fig. 3.14. Elastic and creep strain
where \( \sigma = \) steady-state longitudinal stress,
\[
A = a_1 M + b_1 N,
\]
\[
a_1 = \frac{\beta}{b'.D},
\]
\[
b_1 = \frac{\alpha}{b'.D},
\]
\[
B = a_2 M + b_2 N,
\]
and \( x \) = distance from the centroid of the section to any point along the depth of the beam.

Experiments\(^{(135)}\) carried out at King's College, London, on prestressed continuous concrete beams have indicated that stress redistribution takes place under non-uniform temperature and stress conditions. Eventually a steady-state will be reached if stress redistribution is allowed to become complete, that is, it is not restricted by lack of creep. By adopting simple creep laws for temperature and stress, the above steady-state equations were developed\(^{(134)}\) for cases of plane sections. Two comparisons with experimental evidence provided support to the steady-state solution.

The steady-state stresses are of importance from the designer's point of view, since together with initial stresses they provide the designer with upper and lower bounds between which the stresses will lie for constant temperature states.

The basic concept of the steady-state solution is that if infinite creep has occurred, then the elastic behaviour is no longer significant, and the response of the structure is more like a dynamic one, where the flow is proportional to temperature and stress, but without mechanical effects of mass of material. This results in a quasi-elastic solution.

3.3.3 Rate of creep

At working stress levels, that is, less than 50% of the ultimate load, creep is proportional to the applied stress, as was shown in Section 2.4.4.3. Temperature influences creep behaviour and under
sustained constant stress and temperature, the creep rate in uniaxial compression may be represented by,

\[ \varepsilon = \sigma \cdot T \cdot \delta(t) \]  \( \text{... (3.33)} \)

where \( \varepsilon \) = strain as a function of time,
\( \sigma \) = applied stress,
\( T \) = temperature, (°C),
and \( \delta(t) \) = specific creep per unit temperature as a function of time.

Tests on models and results from operating prestressed concrete pressure vessels and other experimental correlation\(^{137}\) suggest that the above rate of creep equation is satisfactory. However, Equation 3.33 may not be absolutely valid, but it seems to give a reasonable approximation for estimating the response of the structure.

### 3.3.4 Effective modulus solution

The creep deformation of concrete is frequently handled in design by expressing it in terms of a reduced modulus of elasticity that is, the effective modulus, Figure 3.14,

\[ E_{\text{eff}} = \frac{\sigma}{\varepsilon_e + \varepsilon_c} \]  \( \text{... (3.34)} \)

Where \( E_{\text{eff}} \) = effective modulus,
\( \sigma \) = applied stress,
\( \varepsilon_e \) = elastic strain,
\( \varepsilon_c \) = creep strain.

The effective modulus approach is an approximate method for estimating the effect of creep on deflections, and has been used over a long period.

If the stresses are constant, the elastic strain and creep strain are given by Equation 3.33 and the following equation,
\[ \varepsilon_e = \frac{\sigma}{E} \quad \ldots \ldots (3.35) \]

where \( E \) = elastic modulus.

Using Equations 3.33, 3.34 and 3.35, the effective modulus can then be expressed for time \( t \) as,

\[ E_{\text{eff}} = \frac{\sigma}{\sigma + \sigma \cdot \phi(t) \cdot T} \]

or

\[ E_{\text{eff}} = \frac{E}{1 + E \cdot \phi(t) \cdot T} \quad \ldots \ldots (3.36) \]

In a structure not at a uniform temperature, but in which the stress distribution remains unchanged by creep, the value of the effective modulus could be varied according to the particular value of temperature. Such conditions are relatively rare, as the stress distribution is usually changed by creep which invalidates the effective modulus method. The extreme limits of the effective modulus method are the simple initial elastic modulus and the quasi-elastic modulus in the steady-state condition. Thus, the effective modulus approach permits a method of calculation for a structure at any stage between these two limits.

The effective modulus method as applied to an element subjected to a thermal gradient was studied by Richmond\(^{136}\), Figure 3.15. In a concrete structure, if sufficient small elements are used, the temperature can be assumed constant over each area. If the temperature is the same on both faces, the value of \( E_{\text{eff}} \) given in Equation (3.36) is sufficient. When there is a temperature gradient across the wall, the effective elastic modulus of the wall must include this variation. Numerical integration is a convenient method of doing so. Simpson's rule can be used to find both the membrane stiffness and bending stiffness. This is described below, using the minimum two-strip division of the wall, which is likely to be of sufficient accuracy for practical use.
Fig 3.15. Effective modulus solution for element with thermal gradient
For the purpose of numerical calculations, the transformed section in Figure 3.15 can be used, where 'K' is the ratio of 'E_{eff}' to 'E', and the coefficients $K_0$, $K_1$ and $K_2$ are as follows:

\[
K_0 = \frac{E_{\text{eff}}}{E} = \frac{1}{1 + E \phi(t) T_1}
\]

\[
K_1 = \frac{1}{1 + E \phi(t) (T_1 + T_2)/2}
\]

and \[
K_2 = \frac{1}{1 + E \phi(t) T_2}
\]

The area of the transformed section, $A_{\text{eff}}$, is given by,

\[
A_{\text{eff}} = \frac{hb}{6} (K_0 + 4K_1 + K_2)
\]

The effective thickness can be defined by the relationship,

\[
b_{teff} = A_{\text{eff}}
\]

or \[
t_{eff} = \frac{A_{\text{eff}}}{b} = \frac{h}{6} (K_0 + 4K_1 + K_2)
\]

The flexural stiffness depends on the second moment of area of the transformed section about its centroid.

Taking moments about the right hand face gives, using Simpson's rule,

\[
\bar{x} = \frac{K_0 + 2K_1}{K_0 + 4K_1 + K_2} \cdot h \quad \text{...... (3.37)}
\]

The second moment of area about the right hand face is, using Simpson's rule,

\[
I_{2-2} = \frac{h^3 b}{6} (K_0 + K_1) \quad \text{...... (3.38)}
\]
The second moment of area about the centroid is,

\[ I_c = \frac{h^3 b}{6} \cdot \frac{(K_0 K_0 + K_0 K_2 + K_1 K_2)}{(K_0 + 4K_1 + K_2)} \quad \ldots (3.39) \]

This can be expressed as an equivalent thickness for flexural behaviour, \( t'_\text{eff} \), by,

\[ t'_\text{eff} = h \left( \frac{2(K_0 K_0 + K_1 K_2 + K_2 K_0)}{(K_0 + 4K_1 + K_2)} \right)^{1/3} \quad \ldots (3.40) \]
CHAPTER 4: DEVELOPMENT OF THE CREEP-TEMPERATURE METHOD OF ANALYSIS
4.1 Introduction

The effect of creep on the stresses in a prestressed concrete section was shown, in Section 2.6, to be significant. It can be seen in Figure 2.7 that there is almost a 50% relief in stress at the top of the prestressed section in a period of 458 days. This relief is due to the creep of concrete under the action of the compressive stresses that are the result of a permanent loading on the prestressed beam. However, this relief in stress should be even more if the concrete is heated, as it was shown in Section 2.4.4.1 that creep increases with temperature.

In this Chapter, a method of analysis will be developed for predicting the combined effect of creep and temperature on the redistribution of stresses in a prestressed concrete beam. Use will be made of the extensive amounts of experimental data that are available which concern the effect of temperature and creep on concrete.

A time scale will be chosen for the study of the creep behaviour of concrete. It will be shown that it is more convenient to use this time scale, pseudo time, than real time in creep analysis.

4.2 Time scale used for creep analysis

To define the time scale used in creep calculations, it is better to define the term 'specific strain' first, as this term forms the basis of all creep time measurements.

4.2.1 Specific strain

The total specific strain, of a concrete, at any time under load is given by (28):

\[ \varepsilon_s = \frac{\varepsilon_t}{\sigma} = \frac{1}{E_{\text{eff}}} \quad \ldots \quad (4.1) \]
where \( \varepsilon_s = \text{total specific strain}, \)
\( \varepsilon_t = \text{total deformation (elastic + creep)}, \)
\( \sigma = \text{applied stress}, \)
and \( E_{\text{eff}} = \text{effective modulus of elasticity}. \)

Extensive creep testing of a number of high strength concretes whose minimum cube strengths at 28 days are 40N/mm\(^2\), has shown that providing the aggregate has a modulus of elasticity of more than 70KN/mm\(^2\), the specific strain behaviour is similar\(^{47}\). Hence, it is possible to express the behaviour of the concrete likely to be used in bridge decks by one single set of specific strain curves, together with a factor assigned to take into account the effect of elevated temperatures.

The influence of concrete age at the time of loading and temperature on specific strain can be expressed mathematically by the following relationship\(^{28}\):

\[
\text{Specific strain} = a t^n \quad \ldots \ldots (4.2)
\]

where \( a = \text{total specific strain at } t = 1 \text{ day}, \)
\( n = \text{slope of the specific strain curve plotted on log-log axis}, \)
and \( t = \text{time from loading} +1, \text{in days}. \)

Equation 4.2 can be used with the help of Figures 4.1 and 4.2. These figures are based upon the results of a series of creep tests over a range of temperatures between 20°C and 95°C, and at ages of concrete at loading of 7 days to 12½ years \(^{24,47}\).

Figure 4.1 shows the variation in parameter 'a' over the temperature range 5°C to 95°C, at different ages of concrete at loading from 7 days to 30 years. While Figure 4.2 shows the variation in parameter 'n' with parameter 'a'. It can be seen that the logarithmic relationship
Fig. 4.1. Variation in 'a' with loading age and temperature \(^{(28)}\)

Fig. 4.2. Variation in 'n' with 'a' \(^{(28)}\)
that exists between the two parameters 'a' and 'n' is independent of both temperature and age of concrete at loading.

Therefore, it is possible to determine the creep characteristics of a particular concrete under any combination of age of concrete at loading, temperature and time under load by using Equation 4.2. Values of the parameter 'a' can be obtained from Figure 4.1, while the corresponding values of parameter 'n' can be obtained from Figure 4.2.

4.2.2 Pseudo time

The value of specific strain, obtained using Equation 4.2 and Figures 4.1 and 4.2, in units of strain per unit stress per unit temperature, can be used as a measure of time because it corresponds to a unique point on the time scale for the concrete element under consideration.

Figure 4.3 shows the relationship between specific strain $\phi(t)$, which is sometimes called specific creep, and real time (t). The curve in the figure is shown intercepting the specific strain axis in order to allow for the transitional thermal creep which is discussed in Section 3.2.2. This component of creep is produced when the temperature of a concrete element is increased for the first time while already under stress.

The discontinuity in the creep curve in Figure 4.3, can be overcome by using an equivalent relationship shown in Figure 4.4. This linear relationship is between specific strain and a new time scale called pseudo time.

From Figures 4.3 and 4.4, a relationship between real time and pseudo time can be obtained for a particular concrete. The main reason for using pseudo time is to generalise the solution so that it is applicable to any creep curve. Thus, the same results can be used for different creep data by changing the relationship between pseudo time (c) and real time (t), via the curve of Figure 4.3.
Fig. 4.3. Variation of specific creep with real time + 1 days

Fig. 4.4. Linear variation of specific creep with pseudo time

\( c_i = \phi (t) \)
Therefore, this new time scale will be used throughout the course of the study, which is sometimes called creep time or effective time in addition to the more common name of pseudo time.

4.3 Development of the rate of creep equation for creep-temperature analysis

Tests on models and results from operating prestressed concrete pressure vessels and other experimental correlations suggest that the rate of creep formulation, which was discussed in Section 3.3.3, is satisfactory. The rate of creep equation can be written in the following form:

\[ \frac{d\varepsilon}{dt} = \sigma . T . \phi(t) \]  \hspace{1cm} .... (4.3)

where 
- \( \varepsilon \) = strain,
- \( t \) = real time,
- \( \sigma \) = stress,
- \( T \) = temperature,

and \( \phi(t) \) = specific creep in units of strain per unit stress per unit temperature as a function of real time,

\[ \phi(t) = \frac{d}{dt} \left[ \int \phi(t) \right] \]

It was discussed in Section 4.2 that pseudo time can be used as a substitute for real time in creep problems. The reason being to simplify the process of analysis. By substituting pseudo time instead of real time in Equation 4.3, the following equation is obtained,

\[ \frac{d\varepsilon}{dc} = \sigma . T \]  \hspace{1cm} .... (4.4)

since \( \frac{d\phi(t)}{dc} = 1 \) for \( \phi(t) = c \), from Figure 4.4,

where \( c = \) pseudo time.

Therefore, Equation 4.4 can be written in the following form,
that is, the change in strain due to creep is proportional to the applied stress, temperature and the increment in pseudo time. However, for an elastic material

\[ \sigma = E \cdot \varepsilon \]

therefore, Equation 4.5 can be written in terms of change in stress as follows,

\[ d\sigma = \sigma \cdot T \cdot E \cdot dc \quad ..... (4.6) \]

Equation 4.6 will be used, for the creep temperature analysis, to find the change in stress due to creep \( (d\sigma) \), for an increment in pseudo time \( (dc) \), at a stress \( (\sigma) \) and a temperature \( (T) \).

### 4.4 Incremental solution for creep-temperature analysis

This method (136) is based on the rate of creep solution discussed in Sections 3.3.3 and 4.3. Equation 4.5 will be used to find the change in strain for an increment in pseudo time. The incremental solution, which is sometimes called the step-by-step approach, will be described in terms of a finite element analysis. For a uniform stress field, the following steps will be followed:

1. The pseudo time scale, Figure 4.4, is divided into a series of steps.

2. The finite element shown in Figure 4.5.1 is at a uniform temperature \( T \). During the first pseudo time interval \( \Delta c \), the change in strains due to creep are derived from Equation 4.5 and are as follows:

\[ \Delta \varepsilon_y = (\sigma y - \nu oz) \cdot T \cdot \Delta c \]

\[ \Delta \varepsilon_z = (\sigma z - \nu oy) \cdot T \cdot \Delta c \quad ..... (4.7) \]

where \( \Delta \varepsilon_y, \Delta \varepsilon_z \) = change in strains in the \( y \) and \( z \) directions, respectively,
4.5.1 Increase in creep strain

\[ \Delta \varepsilon_z = (\sigma_z - v\sigma_y) T \Delta c \]

4.5.2 Restraining stresses

4.5.3Loads applied to produce zero external load

Fig. 4.5 Incremental method
\( \sigma_y, \sigma_z = \text{uniform stresses on the element in the } y \\
\text{and } z \text{ directions respectively,} \\
\text{and } v = \text{creep Poisson's ratio.} \)

The creep Poisson's ratio can be taken to be equal to the elastic Poisson's ratio as discussed in Section 2.4.3.

3. For each finite element of the structure, such as the element shown in Figure 4.5.1, the stresses required to nullify the creep strains \( \Delta e_y \) and \( \Delta e_z \) by elastic response are evaluated and are shown in Figure 4.5.2. The corresponding increments in nodal forces for the specified element are then determined.

4. Loads are then applied to the nodes of the specified finite element system so as to produce zero external load. The values of those applied nodal loads are shown in Figure 4.5.3.

5. The stresses in step 2, starting stresses, step 3, nullifying stresses, and step 4, due to applied nodal loads are added to give the stress system after creep interval, \( \Delta c \).

6. The process is then repeated for another increment in pseudo time, using the modified system of stresses as the starting stresses for the next interval of pseudo time.

4.5 Analysis of beam and slab structures

It has been recognised during the course of this study that creep and temperature affect the stress distribution within the sections of prestressed concrete beams. A method will be developed in this section to analyse the effect of creep and temperature on a two span continuous prestressed beam. The method will be described for a beam structure, but it can be used for slab decks by simulating the slab with a set of longitudinal and transverse beams. Beam theory can then be used to analyse the stresses in the structure as a grillage.

The method of creep-temperature analysis is based on the rate of
creep equation, Section 4.3, and the incremental solution, Section 4.4. The method is described in the following steps:

**Step 1:** Figure 4.6 shows a typical two span prestressed concrete beam. The beam is divided into segments of equal lengths. The numbering system of segments and sections of the beam is as shown in Figure 4.6.

**Step 2:** Sub-divide each segment of the beam into equal horizontal layers (Figure 4.7). The reason for doing so is that in concrete bridge decks there is a non-uniform temperature distribution, as discussed in Section 3.1.3. Therefore, each layer of the deck will creep by an amount depending on the stress and temperature in that layer.

**Step 3:** Analyse the beam structure for the effects of long-term loadings on the bridge deck, which are dead load, prestressing load and thermal gradient. Live loading will not be included in the creep analysis as it is only a temporary load, and it was shown in Section 3.2 that only permanent load affects the magnitude of creep in a concrete structure.

Calculate the stresses due to long term loadings at sections (n) and (n+1) of the beam shown in Figure 4.6. The two sets of stresses are then averaged and the averaged stresses are assumed to act upon segment (n), Figure 4.8.

It can be seen from Figure 4.8 that each strip of segment (n) is subject to the action of a different magnitude of stress and a different value of temperature. For each strip of the segment, calculate the change in strain ($\Delta \varepsilon$) due to creep and temperature. To do this, Equation 4.5 can be used and may be written in the following form,

$$\Delta \varepsilon = \sigma \cdot T \cdot \Delta c.$$  \hspace{1cm} (4.8)

**Step 4:** Apply a restraining elastic stress system to nullify the strain ($\Delta c$) due to creep for each segment of the beam structure.
4.6. Numbering of Sections and Segments

Fig. 4.6. Numbering of Sections and Segments

4.7. Sub-division of Segment 'n'

Fig. 4.7. Sub-division of Segment 'n'
Fig. 4.8. Segment 'n' under the action of varying stresses \( \sigma \) and temperatures \( T \)

Fig. 4.9. Nullifying stresses acting on Segment 'n'
This is done by multiplying the values of the creep strains for each strip of the segments by the elastic modulus of the concrete. The resulting systems of stresses are then applied in an opposite direction to the creep strains so as to nullify them. Equation 4.6 can be used to obtain the above systems of stresses.

Figure 4.9 shows segment (n) of the beam which lies between sections (n) and (n+1). The stress system which is required to nullify the creep strains is shown to apply in an opposite direction to the creep strains.

**Step 5:** Calculate by Simpson's Rule, the bending moment ($\Delta M$) about the centroid of the segment and the axial force ($\Delta N$) produced by the nullifying system of stresses for each segment of the beam structure.

Figure 4.9 shows the bending moment ($\Delta M_n$) and axial force ($\Delta N_n$) produced by the nullifying system of stresses for segment (n) of the beam. The direction of the above bending moment and axial force for each segment is important, as all segments will be combined in later steps to produce the net effect on the whole structure.

**Step 6:** The bending moment ($\Delta M$) and axial force ($\Delta N$) for each segment of the beam will give rise to resultant bending moment and axial force for each section of the beam. Those resultant moments and forces should not exist as discussed in Section 4.5. Therefore, apply opposite moments and axial forces at each section of the beam so as to complete the loading of the structure.

To clarify this step of the analysis, consider segments (n-1) and (n) which are contained between sections (n-1), (n) and (n+1). The bending moments and axial forces in segments (n-1) and (n), due to the nullifying system of stresses, are ($\Delta M_{n-1}$), ($\Delta M_n$), and ($\Delta N_{n-1}$), ($\Delta N_n$), respectively. The resulting moment and axial force at section (n) are

$$[\Delta M_{n-1} + \Delta M_n] \quad \text{and} \quad [\Delta N_{n-1} + \Delta N_n],$$

respectively,
However, care must be taken while carrying out this step of the analysis, as the directions of the moments and axial forces are very important. The above resultant moment and axial force at section (n) should not exist, therefore, equal moment and axial force but opposite in direction should be applied at section (n) so as to complete the structure. The stresses due to the latter moments and axial forces can now be evaluated.

Step 7: - As the beam structure is now complete, all the stresses that were obtained in steps 3, 4 and 6 can be summed. This will give the stresses in the beam structure after the first interval of pseudo time.

Step 8: - The above process for further increments in pseudo time can now be repeated. However, the starting stresses will be those that were obtained at the end of the previous interval of pseudo time.
CHAPTER 5: CREEP BEHAVIOUR OF SOLID, CELLULAR AND T-SECTION DECKS WITH A CONSTANT TEMPERATURE DISTRIBUTION
5.1 Introduction

The effect of temperature-creep interaction on the stresses in continuous prestressed concrete decks can be investigated by using the method of analysis developed in the previous Chapter. The method will be used for three types of bridge deck sections: solid slab, cellular slab and T-section. The study in this Chapter will be limited to a constant temperature distribution. In addition there are a number of assumptions and approximations that had to be made to simplify the process of analysis.

5.2 Assumptions

5.2.1 Temperature distribution

The temperature distribution in concrete slab decks is non-linear, as described in Sections 2.2 and 3.1.3. To obtain the actual non-linear temperature distribution in a bridge deck, field measurements will have to be carried out by installing thermocouples in the concrete deck to record temperatures. An alternative to the above is the use of a computer program to predict the temperatures within a concrete deck. Such a program was described in Section 3.1.4, and it was shown that the computer program gives temperature distributions that are very close to actual distributions obtained by field measurements.

For the bridge designer, the above two methods for predicting temperature distribution are not always practical. The reason is that it is not possible to carry out field measurements prior to the design of the deck, unless a simulated structure is built which makes the design costly, but it may be necessary for very complicated deck sections. Therefore, it is more practical to use the computer program mentioned above, but unfortunately such a facility is seldom available to bridge designers. In these circumstances, a constant temperature distribution over the depth of the slab deck is commonly assumed. This simplified approach is based on a top surface temperature, bottom surface temperature, and a linear distribution between the two surfaces.
5.2.2 Modulus of elasticity (E)

It is a well known fact that the modulus of elasticity of concrete increases with time after casting, and that this value decreases with an increase in the temperature of the concrete, as shown in Section 2.4.1.

The basic equation used in the previous Chapter to develop the method of creep-temperature analysis can be written in the following form,

\[ \Delta \sigma = (\sigma \Delta T \Delta c) E. \] 

where \( \Delta \sigma \), \( \sigma \), \( T \), \( \Delta c \) and \( E \) are as defined earlier. It can be seen that the change in stress (\( \Delta \sigma \)) due to creep is directly proportional to the value of \( E \). The analysis will be considerably simplified by taking a constant value of \( E \), which will then be the only non-variable factor in the above equation.

As discussed in Section 2.4.1, the short term elastic modulus may be taken from Table 1, CPI110, Part 1.\(^{(35)}\) The 28 days \( E \) value will be taken as 34KN/mm\(^2\) for high strength concrete. This value of \( E \) will be considered constant in the analysis, as an approximation, for the following reasons:

1. The value of \( E \) for concrete increases rapidly during the first 28 days after casting, when it reaches a value which is over 70% of the one year value of \( E \), then the rate of increase becomes progressively smaller.
2. The value of \( E \) for concrete will be reduced with an increase in the temperature of the concrete.

By combining the above two factors \(^{(28)}\), for a 28 days old concrete, it can be concluded that it is a reasonable approximation to take a constant value of \( E \), based on the 28 days strength, for the creep-temperature analysis.

5.2.3 Applied stress (\( \sigma \))

In the method of analysis developed in the previous Chapter, the depth of deck is divided into layers, with each layer being under a different value of stress. This stress (\( \sigma \)), for each layer, is assumed to be constant for the duration of the pseudo
time increment (Δc). In fact, this is not true as the stress changes continuously with pseudo time and does not remain constant over any interval of time during the early life of the structure. However, if the pseudo time increment (Δc) is very small, then the change in stress (σ) during this interval is very small too, and therefore can be taken to be constant and equal to the value of stress at the start of the increment in pseudo time (Δc).

By considering Equation 5.1, it can be seen that the smaller the value of Δc that is taken, the more this assumption is valid. However, there is a practical limit on the size of Δc that can be taken in the analysis since the smaller it becomes the longer the analysis will be. The bridge designer will have to choose a practical value of Δc, so that the analysis can be done in a reasonable amount of time. Acceptable results are obtained if the pseudo time interval Δc is taken to be to $0.1 \times 10^{-6}$.

5.2.4 Pseudo time (c)

There are several factors that affect the creep characteristics of a concrete structure. Those factors were discussed in Sections 2.4.4 and 3.2.5. Considering Equation 5.1, it is clear that the change in stress due to creep according to the equation is proportional to the applied stress (σ), temperature (T), modulus of elasticity (E), and pseudo time interval (Δc). It should be noted that there are other factors, that affect creep, which are left out of the above equation, such as stress history and temperature history.

However, the main factors that affect creep are σ, T and E, and are included in Equation 5.1. All other factors that are related to the creep characteristics of the material are assumed to be combined in the remaining factor in the above equation, which is the pseudo time interval (Δc).

Therefore, pseudo time combines factors such as age of concrete, type of cement, type of curing, and other factors that affect the creep characteristics of the material, that is, type and quantity of aggregate, aggregate/cement ratio and water/cement ratio.
5.2.5 Coefficient of thermal expansion (\(\alpha\))

The coefficient of thermal expansion of concrete is needed to calculate thermal stresses in the deck due to the applied temperature gradient. It was discussed in Section 2.4.2 that although the value of \(\alpha\) increases with temperature, the rate of increase is very low. It was also concluded that a constant value of \(\alpha\) can be taken for concrete where the temperature varies between 30°C and 60°C, which is the range of temperatures used in the analysis. Therefore, a constant value of \(\alpha\), \(12 \times 10^{-6}\) per °C, was taken.

5.2.6 Heat flow properties

The computer program discussed in Section 3.1.4 is based on the heat flow properties of the concrete to predict the temperature distribution in the deck. It was discussed in Section 2.3 that heat flow properties, such as thermal conductivity (\(k\)), thermal diffusivity (\(D\)) and specific heat either increase or decrease with change in temperature. However, in the range of temperatures used in the analysis, the change in the values of \(k\), \(D\) and specific heat is very small and can be ignored. Therefore, constant values were adopted for the heat flow properties in the computer analysis.

5.3 Creep-temperature analysis

5.3.1 Solid slab deck

The method of creep-temperature analysis described in Section 4.5 will now be used to analyse the two-span continuous prestressed solid slab deck shown in Figure 5.1. A one metre wide strip of the deck will be considered for the analysis.

5.3.1.1 Step (1)

The idealized beam shown in Figure 5.1 was divided into 8 equal segments for each span. The length of each segment is 2.5m, and the depth is 0.8m which is the depth of the solid slab deck.

5.3.1.2 Step (2)

Each segment of the beam was sub-divided into 4 equal layers, with each layer being 0.2m in depth.
Fig. 5.1.1  Plan of deck

Fig. 5.1.2  Elevation of idealized beam

Fig. 5.1.3  Eccentricity of tendon (m)

Fig 5.1  Solid slab deck
5.3.1.3. Step (3)

The beam structure was analysed for long-term loadings on the bridge deck, using design loads that are specified by BS5400\(^7\), Part 2. The prestressing force calculated\(^{138}\) for the beam was 5000 KN, with the eccentricity of the strand varying along the length of the beam as shown in Figure 5.1.3. The stresses due to the constant linear temperature gradient acting on the bridge deck, were calculated using the method listed in Section 3.1.5.3.

The constant linear temperature gradient used for the analysis was 60°C at the top surface of the concrete deck, and 30°C at the bottom surface. Such a severe temperature difference between the top and bottom surfaces of the slab decks were found to occur on summer days in hot climate locations such as in Texas, U.S.A.\(^{16,17}\) However, the top and bottom surface temperatures of a slab deck can be evaluated using the computer program discussed in Section 3.1.4.

The elastic stresses due to dead load of the deck, prestressing force in the strand and temperature gradient were calculated at sections 1 to 9. The stresses in adjacent sections were averaged and were assumed to act on the relevant segment of the beam. These averaged elastic stresses were the starting stresses for the creep-temperature analysis, and are listed in Table 5.1, where a negative sign indicates a tensile stress.

For each strip in segments 1 to 8, the change in strains (\(\Delta e\)) due to creep and temperature were calculated using Equation 4.8, for pseudo time interval (\(\Delta c\)) of \(0.1 \times 10^{-6}\) (from \(c'=0\) to \(c'=0.1 \times 10^{-6}\)), where \(c'=0\) represents the moment of application of load to the structure.
Table 5.1 - Elastic stresses (N/mm²)

<table>
<thead>
<tr>
<th>Segment Number</th>
<th>Strip Number</th>
<th>5 - 5</th>
<th>4 - 4</th>
<th>3 - 3</th>
<th>2 - 2</th>
<th>1 - 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.059</td>
<td>4.529</td>
<td>5.000</td>
<td>5.471</td>
<td>5.941</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3.364</td>
<td>4.182</td>
<td>5.000</td>
<td>5.818</td>
<td>6.637</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3.588</td>
<td>4.294</td>
<td>5.000</td>
<td>5.706</td>
<td>6.413</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4.528</td>
<td>4.764</td>
<td>5.000</td>
<td>5.236</td>
<td>5.473</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>3.930</td>
<td>4.465</td>
<td>5.000</td>
<td>5.535</td>
<td>6.070</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-0.784</td>
<td>2.108</td>
<td>5.000</td>
<td>7.892</td>
<td>10.784</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>-6.266</td>
<td>-0.633</td>
<td>5.000</td>
<td>10.633</td>
<td>16.266</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>-7.662</td>
<td>-1.331</td>
<td>5.000</td>
<td>11.331</td>
<td>17.662</td>
<td></td>
</tr>
</tbody>
</table>

5.3.1.4 Step (4)

The restraining elastic stress system required to nullify the strains produced in Step (3) were then calculated. These nullifying stresses are given in Table 5.2.

Table 5.2 - Nullifying Stresses (N/mm²)

<table>
<thead>
<tr>
<th>Strip Number</th>
<th>Temperature °C</th>
<th>5 - 5</th>
<th>4 - 4</th>
<th>3 - 3</th>
<th>2 - 2</th>
<th>1 - 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Segment Number</td>
<td>30</td>
<td>37.5</td>
<td>45</td>
<td>52.5</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-0.414</td>
<td>-0.577</td>
<td>-0.765</td>
<td>-0.977</td>
<td>-1.212</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-0.343</td>
<td>-0.533</td>
<td>-0.765</td>
<td>-1.039</td>
<td>-1.354</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-0.366</td>
<td>-0.548</td>
<td>-0.765</td>
<td>-1.019</td>
<td>-1.308</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-0.462</td>
<td>-0.607</td>
<td>-0.765</td>
<td>-0.935</td>
<td>-1.117</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-0.398</td>
<td>-0.569</td>
<td>-0.765</td>
<td>-0.988</td>
<td>-1.238</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>+0.080</td>
<td>-0.269</td>
<td>-0.765</td>
<td>-1.409</td>
<td>-2.200</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>+0.639</td>
<td>+0.080</td>
<td>-0.765</td>
<td>-1.898</td>
<td>-3.318</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>+0.782</td>
<td>+0.170</td>
<td>-0.765</td>
<td>-2.023</td>
<td>-3.603</td>
<td></td>
</tr>
</tbody>
</table>
5.3.1.5. Step (5)

The nullifying stress system obtained in Step (4) is shown in Figure 5.2.1, for segment 1. For each segment, this system of stresses will produce a bending moment ($\Delta M$) about the centroid of the section, and an axial force ($\Delta N$). For any segment $n$, Figure 5.2.2, $\Delta M_n$ and $\Delta N_n$ can be calculated using Simpson's Rule as follows:

$$\Delta N_n = \frac{200}{3} \left[ \sigma_1 + 4 \sigma_2 + 2 \sigma_3 + 4 \sigma_4 + \sigma_5 \right]$$

and

$$\Delta M_n = \frac{200}{3} \left[ (\sigma_1 \times 400) + 4(\sigma_2 \times 200) + 0 \\
- 4(\sigma_4 \times 200) - (\sigma_5 \times 400) \right]$$

The values of $\Delta N$ and $\Delta M$ are for 1mm width of the beam. Table 5.3 gives the values of $\Delta N$ and $\Delta M$ for segments 1 to 8.

Table 5.3 - Values of $\Delta N$ and $\Delta M$

<table>
<thead>
<tr>
<th>Segment Number</th>
<th>$\Delta N$ (N)</th>
<th>$\Delta M \times 10^3$ N.mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>625</td>
<td>42.613</td>
</tr>
<tr>
<td>2</td>
<td>634</td>
<td>53.947</td>
</tr>
<tr>
<td>3</td>
<td>632</td>
<td>50.240</td>
</tr>
<tr>
<td>4</td>
<td>619</td>
<td>34.960</td>
</tr>
<tr>
<td>5</td>
<td>626</td>
<td>44.747</td>
</tr>
<tr>
<td>6</td>
<td>702</td>
<td>117.333</td>
</tr>
<tr>
<td>7</td>
<td>808</td>
<td>211.013</td>
</tr>
<tr>
<td>8</td>
<td>784</td>
<td>233.893</td>
</tr>
</tbody>
</table>

Figure 5.3 shows the magnitudes and directions of $\Delta M$ for all segments of the beam, with the resultant moments at each section.

5.3.1.6. Step (6)

For each segment, an axial force which is equal in magnitude but opposite in direction to those obtained in Step (5) was applied. At each section, a moment which is equal in magnitude but opposite in direction
Fig. 5.2.1 $\Delta M$ and $\Delta N$ for segment 1

Fig. 5.2.2 $\Delta M$ and $\Delta N$ for segment n

Fig. 5.2 $\Delta M$ and $\Delta N$ produced by nullifying stress system
Resultant $\Delta M$ at each section

$\Delta M$ within each segment

Fig. 5.3  Values of $\Delta M$ (x $10^3$ Nmm)

Fig. 5.4  Application of opposite moments to $\Delta M$ produced by nullifying stress system (x $10^3$ Nmm)
to those obtained in Step (5) was applied, as shown in Figure 5.4. The loading of the structure is now complete for the first pseudo time interval. Due to the moments shown in Figure 5.4, the reaction at support A was found to be $0.01 \times 10^3$N. Figure 5.5 shows the bending moment diagram due to the moments applied in this step. From the figure, the bending moment at the middle of each segment was evaluated, and was assumed to be constant along each segment. Table 5.4 gives the stresses due to the axial forces and bending moments applied in this step.

Table 5.4 - Stresses due to loading of Step (5)

<table>
<thead>
<tr>
<th>Segment Number</th>
<th>Stress due to axial force (N/mm²)</th>
<th>Moments</th>
<th>Top fibre bending stress (N/mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Magnitude N.mm (x 10³)</td>
<td>Direction</td>
</tr>
<tr>
<td>1</td>
<td>0.781</td>
<td>30.113</td>
<td>S</td>
</tr>
<tr>
<td>2</td>
<td>0.793</td>
<td>16.447</td>
<td>S</td>
</tr>
<tr>
<td>3</td>
<td>0.790</td>
<td>12.260</td>
<td>H</td>
</tr>
<tr>
<td>4</td>
<td>0.774</td>
<td>52.540</td>
<td>H</td>
</tr>
<tr>
<td>5</td>
<td>0.783</td>
<td>67.753</td>
<td>H</td>
</tr>
<tr>
<td>6</td>
<td>0.878</td>
<td>20.167</td>
<td>H</td>
</tr>
<tr>
<td>7</td>
<td>1.010</td>
<td>48.513</td>
<td>S</td>
</tr>
<tr>
<td>8</td>
<td>0.980</td>
<td>46.393</td>
<td>S</td>
</tr>
</tbody>
</table>

5.3.1.7. Step (7)

The loading of the beam structure is now complete. The stresses obtained in Steps (3), (4) and (6) were then summed, that is, the stresses in Tables 5.1, 5.2 and 5.4 respectively. The resulting stresses at the end of the first pseudo time interval are listed in Table 5.5.
Fig 5.5 Bending moment diagram due to moments applied in Step 6 ($\times 10^3$ N.mm)
Fig. 5-6-1  Redistribution of stress in segment 2  (Solid slab)

- $a. c^- = 0$
- $b. c^- = 0.1 \times 10^{-6}$
- $c. c^- = 0.2 \times 10^{-6}$
- $d. c^- = 0.3 \times 10^{-6}$
- $e. c^- = 0.4 \times 10^{-6}$
- $f. c^- = 0.5 \times 10^{-6}$
- $g. =$ steady state stresses
Redistribution of stress in segment 4 (Solid slab)

Steady state stresses

- $C = 0.1 \times 10^{-6}$
- $C = 0.2 \times 10^{-6}$
- $C = 0.3 \times 10^{-6}$
- $C = 0.4 \times 10^{-6}$
- $C = 0.5 \times 10^{-6}$

Fig. 5.6.2
Fig. 5.6.3 Redistribution of stress in segment 6 (Solid slab)

a. C = 0
b. C = 0.1 \times 10^{-6}
c. C = 0.2 \times 10^{-6}
d. C = 0.3 \times 10^{-6}
e. C = 0.4 \times 10^{-6}
f. C = 0.5 \times 10^{-6}
g. = Steady state stresses

Stress N/ mm^2

Depth of deck 800 mm
Fig. 5.6.4  Redistribution of stress in segment 8  (Solid slab)  Stress N/mm²

a. \( c' = 0 \)
b. \( c' = 0.1 \times 10^{-6} \)
c. \( c' = 0.2 \times 10^{-6} \)
d. \( c' = 0.3 \times 10^{-6} \)
e. \( c' = 0.4 \times 10^{-6} \)
f. \( c' = 0.5 \times 10^{-6} \)
g.  = Steady state stresses
Table 5.5 - Stresses at pseudo time c=0.1x10^{-6}
(N/mm²)

<table>
<thead>
<tr>
<th>Segment Number</th>
<th>Strip Number</th>
<th>5 - 5</th>
<th>4 - 4</th>
<th>3 - 3</th>
<th>2 - 2</th>
<th>1 - 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.143</td>
<td>4.592</td>
<td>5.017</td>
<td>5.418</td>
<td>5.793</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.660</td>
<td>4.365</td>
<td>5.028</td>
<td>5.649</td>
<td>6.230</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.127</td>
<td>4.594</td>
<td>5.025</td>
<td>5.420</td>
<td>5.780</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.303</td>
<td>5.148</td>
<td>4.979</td>
<td>4.799</td>
<td>4.607</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.951</td>
<td>4.997</td>
<td>5.018</td>
<td>5.012</td>
<td>4.979</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.363</td>
<td>2.812</td>
<td>5.113</td>
<td>7.267</td>
<td>9.273</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-5.072</td>
<td>0.230</td>
<td>5.245</td>
<td>9.973</td>
<td>14.413</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-6.335</td>
<td>-0.398</td>
<td>5.215</td>
<td>10.506</td>
<td>15.474</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5.3.1.8. Step (8)

The above process was then repeated for further increments in pseudo time. However, the starting stresses for the next pseudo time interval were those obtained in the previous step and are listed in Table 5.5.

Figures 5.6.1-4 show the redistribution in stresses in the deck slab, caused by creep, for different segments along span AB, at increasing pseudo time.

5.3.2 Cellular deck

The same method of creep-temperature analysis that was applied to the solid slab deck in Section 5.3.1 will now be applied to a two span continuous prestressed cellular deck shown in Figure 5.7. One cell of the deck will be considered for the analysis, Figure 5.7.3.

A constant linear temperature gradient was used for the analysis similar to that used for the solid slab deck in Section 5.3.1.3. However, the method of calculating thermal stresses for the cellular deck is different from that used for the solid slab deck and is given in Section 3.1.5.4.
5.7.1 Two span continuous cellular deck

5.7.2 Tendon profile (mm)

5.7.3 One cell of deck

5.7.4 Temperature distribution

Fig. 5.7 Cellular deck
For this type of deck, a typical nullifying stress system obtained in Step (4) of the creep-temperature analysis is shown in Figure 5.8. For each segment of the deck, the bending moment ($\Delta M$) and axial force ($\Delta N$) produced by the above system of stresses can be calculated using Simpson's Rule as follows:

top flange,

$$\Delta N_1 = \frac{h_1}{3} (\sigma_1 + 4 \times \sigma_2 + \sigma_3) (b_1)$$

web,

$$\Delta N_2 = \frac{h_2}{3} (\sigma_4 + 4 \times \sigma_5 + 2 \times \sigma_6 + 4 \times \sigma_7 + \sigma_8) (b_2)$$

bottom flange,

$$\Delta N_3 = \frac{h_3}{3} (\sigma_8 + 4 \times \sigma_9 + \sigma_{10}) (b_3)$$

therefore $\Delta N = \Delta N_1 + \Delta N_2 + \Delta N_3$

and top flange,

$$\Delta M_1 = \frac{h_1}{3} \left[ (\sigma_1 \times L_1) + 4(\sigma_2 \times L_2) + (\sigma_3 \times L_3) \right] \times (b_1)$$

web,

$$\Delta M_2 = \frac{h_2}{3} \left[ (\sigma_4 \times L_4) + 4(\sigma_5 \times L_5) - 4(\sigma_7 \times L_7) - (\sigma_8 \times L_8) \right] \times (b_2)$$

bottom flange,

$$\Delta M_3 = \frac{h_3}{3} \left[ (\sigma_8 \times L_8) + 4(\sigma_9 \times L_9) + (\sigma_{10} \times L_{10}) \right] \times (b_3)$$

where $L_m = \text{distance from } \sigma_m \text{ to centroid of section}.$

therefore $\Delta M = \Delta M_1 + \Delta M_2 + \Delta M_3$

The process of creep-temperature was then carried out for successive pseudo time intervals in a similar manner to that of the solid slab deck in Section 5.3.1. The redistribution of stresses in the cellular deck, caused by creep, is shown in Figure 5.9.
Fig 5.8 Nullifying stress system
Fig. 5.9.1 Segment 4 Stress N/mm²

- a. \( c' = 0 \)
- b. \( c' = 0.1 \times 10^{-6} \)
- c. \( c' = 0.2 \times 10^{-6} \)
- d. \( c' = 0.3 \times 10^{-6} \)
- e. \( c' = 0.4 \times 10^{-6} \)
- f. \( c' = 0.5 \times 10^{-6} \)

Fig. 5.9.2 Segment 8 Stress N/mm²

- a. \( c' = 0 \)
- b. \( c' = 0.1 \times 10^{-6} \)
- c. \( c' = 0.2 \times 10^{-6} \)
- d. \( c' = 0.3 \times 10^{-6} \)
- e. \( c' = 0.4 \times 10^{-6} \)
- f. \( c' = 0.5 \times 10^{-6} \)

Fig. 5.9 Redistribution of stress in cellular deck
5.3.3 T-section deck

The T-section shown in Figure 5.10 was analysed for creep-temperature effects in almost exactly the same process of analysis as that used for the cellular deck in Section 5.3.2. The only difference was that the temperature distribution assumed for the T-section, Figure 5.10.4, was not linear as used for the solid slab and cellular decks. The reason for the above assumption is based on work carried out at the Transport and Road Research Laboratory (1, 19, 20, 65, 79), U.K. The webs of the T-sections of the deck are exposed to the environment on their both sides and bottom surface, but they are protected from the solar radiation by the top slab. Therefore, the temperature of the web is mainly controlled by the shade temperature, and can be assumed constant due to the small thickness of the web which is subjected to almost similar temperatures on all its exposed surface.

The temperature distribution in the top slab is assumed to be linear and is affected by several factors. The temperature of the top surface of the slab is mainly affected by the solar radiation. While the temperature of the bottom surface of the slab is mainly affected by the thickness of the slab, the spacing and thickness of the webs and shade temperature.

The redistribution of the stresses caused by creep for the T-section deck is shown in Figure 5.11.

5.4 Discussion of results

5.4.1 Solid slab deck

The starting stresses for the first pseudo time interval are the elastic stresses due to long term loading. For the subsequent pseudo time intervals, the starting stresses are those obtained as a result of the previous interval. For each interval, Steps 5 and 6, Section 4.5, of the creep-temperature analysis need careful consideration due to their relative complexity. In Step 5, the axial force ($\Delta N$) and bending moment ($\Delta M$) produced by the nullifying stress system are calculated for each segment of the beam. The usual directions of $\Delta N$ and $\Delta M$ are shown in Figure 4.9.
Fig. 5.10-1 Two span continuous T-section deck

Fig. 5.10-2 Tendon profile (mm)

Fig. 5.10-3 One cell of deck

Fig. 5.10-4 Temperature distribution

Fig. 5.10 T-section deck
Fig. 5-11 Redistribution of stress in T-section deck
Values of stresses ($\Delta N_\sigma$) due to the axial force ($\Delta N$) are listed in Table 5.6 together with the values of bending moments ($\Delta M$) for successive pseudo time intervals. The values given in the table are for 1 mm widths of the beam structure.

From Table 5.6, it can be seen that the values of $\Delta N_\sigma$ and $\Delta M$ decrease with each successive pseudo time interval. This can be explained by examining Figures 5.6.1-3 which show that for each segment of the beam structure, the top fibre stresses decrease and the bottom fibre stresses increase for each successive pseudo time interval. Considering Equation 5.1, which is used for calculating the nullifying stress system, it can be noted that nullifying stresses ($\Delta a$) are dependent on the magnitude of the stress ($\sigma$) and the temperature ($T$), for equal pseudo time intervals ($\Delta c$). Therefore, the values of $\Delta N$ and $\Delta M$ produced by the nullifying system of stresses are reduced after each $\Delta c$, due to the fact that the value of $\Delta a$ at the top fibre is reduced as the $\sigma$ is decreased, while $\Delta a$ at the bottom fibre is increased as the $\sigma$ there is increased. The temperature at the top fibre is twice that at the bottom fibre, thus the loss in $\Delta a$ in the upper half of the section will exceed the gain in $\Delta a$ in the lower half of the section which will lead to a reduction in the values of $\Delta N$ and $\Delta M$.

The values of $\Delta M$ listed in Table 5.6 are plotted for five intervals of pseudo time in Figure 5.12, showing the resulting bending moment at each section of the beam structure. Step 6 of the creep-temperature analysis requires the application of moments equal in magnitude but opposite in direction to the resulting moments shown in Figure 5.12. These moments will produce the bending moment diagrams shown in Figure 5.13 for five pseudo time intervals. It can be seen from Figure 5.13 that the bending moment diagram for the five intervals are very close together, and the moments at the middle of each segment of the beam are very slightly changed for successive $\Delta c$. The bending stresses due to the above moments are then combined with the axial stresses, due to the application of an opposite $\Delta N$ to that evaluated in Step 5, to produce the third and last system of stresses for any $\Delta c$; the first two stress systems are the starting stresses and the creep nullifying stresses. For the third system of stresses, the change in axial stresses between successive pseudo time intervals is more significant than the change in bending stresses.
Table 5.6
Values of $\Delta N_\sigma$ (N/mm$^2$) and $\Delta M$ ($\times 10^3$-N.mm) for 1 mm width of solid slab.

<table>
<thead>
<tr>
<th>$c'$ ($\times 10^{-6}$)</th>
<th>Segment (1)</th>
<th>Segment (2)</th>
<th>Segment (3)</th>
<th>Segment (4)</th>
<th>Segment (5)</th>
<th>Segment (6)</th>
<th>Segment (7)</th>
<th>Segment (8)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta N_\sigma$</td>
<td>$\Delta M$</td>
<td>$\Delta N_\sigma$</td>
<td>$\Delta M$</td>
<td>$\Delta N_\sigma$</td>
<td>$\Delta M$</td>
<td>$\Delta N_\sigma$</td>
<td>$\Delta M$</td>
</tr>
<tr>
<td>1</td>
<td>0.781</td>
<td>42.613</td>
<td>0.793</td>
<td>53.947</td>
<td>0.790</td>
<td>50.240</td>
<td>0.774</td>
<td>34.960</td>
</tr>
<tr>
<td>2</td>
<td>0.780</td>
<td>40.560</td>
<td>0.788</td>
<td>48.000</td>
<td>0.780</td>
<td>40.587</td>
<td>0.755</td>
<td>21.360</td>
</tr>
<tr>
<td>3</td>
<td>0.778</td>
<td>38.960</td>
<td>0.783</td>
<td>42.987</td>
<td>0.771</td>
<td>32.320</td>
<td>0.743</td>
<td>9.840</td>
</tr>
<tr>
<td>4</td>
<td>0.776</td>
<td>37.493</td>
<td>0.778</td>
<td>38.773</td>
<td>0.764</td>
<td>25.333</td>
<td>0.733</td>
<td>0.773</td>
</tr>
<tr>
<td>5</td>
<td>0.775</td>
<td>36.293</td>
<td>0.775</td>
<td>35.253</td>
<td>0.758</td>
<td>19.440</td>
<td>0.725</td>
<td>-7.840</td>
</tr>
<tr>
<td></td>
<td>$\Delta N_\sigma$</td>
<td>$\Delta M$</td>
<td>$\Delta N_\sigma$</td>
<td>$\Delta M$</td>
<td>$\Delta N_\sigma$</td>
<td>$\Delta M$</td>
<td>$\Delta N_\sigma$</td>
<td>$\Delta M$</td>
</tr>
<tr>
<td>1</td>
<td>0.783</td>
<td>44.747</td>
<td>0.878</td>
<td>117.333</td>
<td>1.010</td>
<td>211.013</td>
<td>0.980</td>
<td>233.893</td>
</tr>
<tr>
<td>2</td>
<td>0.765</td>
<td>27.387</td>
<td>0.844</td>
<td>99.493</td>
<td>0.939</td>
<td>185.573</td>
<td>0.950</td>
<td>204.133</td>
</tr>
<tr>
<td>3</td>
<td>0.750</td>
<td>12.533</td>
<td>0.824</td>
<td>81.093</td>
<td>0.916</td>
<td>163.547</td>
<td>0.925</td>
<td>178.720</td>
</tr>
<tr>
<td>4</td>
<td>0.738</td>
<td>6.693</td>
<td>0.809</td>
<td>65.627</td>
<td>0.898</td>
<td>145.013</td>
<td>0.904</td>
<td>157.333</td>
</tr>
<tr>
<td>5</td>
<td>0.726</td>
<td>-10.267</td>
<td>0.795</td>
<td>52.667</td>
<td>0.883</td>
<td>129.440</td>
<td>0.885</td>
<td>139.360</td>
</tr>
</tbody>
</table>
Fig. 5.12 Net values of $\Delta M$ (x$10^3$ N/mm) at the beam sections for the five pseudo time intervals. ($\Delta C$)
Fig. 5.13 Bending moment diagram due to \((-\Delta M)\)
The process of redistribution of stresses within the section of the deck will continue until there is a balance between the above second and third systems of stresses. This state of balance will be reached with time. Figures 5.6.1 - 4 show that the stresses are approaching such a balanced state, as the redistribution in stresses is becoming smaller with each successive $\Delta c$, until ultimately the redistribution will have become so small such that it can be assumed to have stopped.

The steady state solution (134) discussed in Section 3.3.2, was used as a comparison with the results obtained by the creep-temperature method of analysis developed in Chapter 4. Figures 5.6.1 to 5.6.4 show that the redistributed stresses in the section are approaching such a state. Therefore it can be concluded that the amount of creep assumed is sufficient to produce almost all the redistribution of stress that is possible by this mechanism.

The initial moment in the beam structure due to long term loading (dead load, prestress and temperature gradient) is shown in Figure 5.14; From the figure, it can be seen that a considerable change had occurred in the moment during a pseudo time interval of $0.5 \times 10^{-6}$, in which the effect of bending moments generated by the temperature gradient had almost disappeared. In the limiting condition of infinite creep time the steady state solution is reached. The bending moments are then almost identical with the initial dead load plus prestress solution without a temperature gradient. The stresses do not however tend to the elastic dead load plus prestress values as the stress distributions over the cross sections are nonlinear.

5.4.2 Cellular and T-section decks

The discussion of Section 5.4.1 for solid slab deck is also applicable to cellular and T-section decks. However, in the later type of decks, there is differential stress between the bottom surface of the top flange and the top of the web, which is due to the assumed severe temperature gradient. This differential stress diminishes with time, Figures 5.9 and 5.11, as a result of the stress redistribution process.
Fig. 5.14 Moments in span

- Initial moment due to dead load + prestress + temperature gradient
- Moment at $C = 0.5 \times 10^{-6}$
- Moment due to dead load + prestress
- Steady state moment

Moments per 1 mm width ($\times 10^3$ N mm)

Sections of span
The second system of stresses mentioned in Section 5.4.1 decreased in the top flange with each successive $\Delta c$, while increased from the top of the web and downwards. While the third system of stresses decreased continuously with pseudo time for the whole depth of the deck. However, Figures 5.9 and 5.11 show that the stress redistributions in the decks are approaching a state where the second and third systems of stresses will balance each other, leaving the first or starting system of stresses almost unchanged during that pseudo time interval.

5.4.3 Choice of pseudo time interval ($\Delta c$)

The method of creep-temperature analysis, developed in Chapter 4, suggested that the smaller the magnitude of $\Delta c$, the more accurate the analysis would be according to the theory assumed. Figure 5.15 shows the difference in the stress redistribution obtained using two different pseudo time intervals. Using one large $\Delta c$ will give more stress redistribution than when using smaller intervals, as can be seen in Figure 5.15. The reason is that in choosing large $\Delta c$, it is being assumed that the starting stresses are not changed during the chosen $\Delta c$. In fact the starting stresses are continuously changing with each finite interval. Therefore a practical value of $\Delta c$ was found to be $0.1 \times 10^{-6}$.

5.5 Conclusions

Using the temperature distribution calculated in accordance with Section 5.2.1, and assuming constant values for Youngs' modulus and the coefficient of thermal expansion, the following conclusions can be made.

1. The stress changes in the section of the deck are significant due to the stress redistribution occurring during a pseudo time interval of $0.5 \times 10^{-6}$, Figures 5.6.1 - 4.

2. The moment in the beam structure, due to long term loadings, changes considerably with pseudo time, until at $c' = 0.5 \times 10^{-6}$ it reaches a state in which the effect of the temperature gradient almost disappears.

3. Choosing successive pseudo time intervals of $0.1 \times 10^{-6}$ each is reasonable, as can be seen from Figures 5.6, 5.9 and 5.11. Taking
**5.15.1 Segment 1**

**5.15.2 Segment 3**

**5.15.3 Segment 6**

- Stress at $C = 0$
- Stress at $C = 0.5 \times 10^{-6}$ using 5 intervals of $\Delta C = 0.1 \times 10^{-6}$
- Stress at $C = 0.5 \times 10^{-6}$ using 1 interval of $\Delta C = 0.5 \times 10^{-6}$

Fig. 5.15 Stress redistribution obtained using different pseudo time intervals. (solid slab)
one large pseudo time interval, in the creep temperature analysis, will give bigger redistribution of stresses than when using smaller successive intervals, as shown in Figure 5.15. According to the theory developed in the thesis, the smaller the $\Delta c$, the more accurate the stress redistribution obtained. The design engineer will have to decide on the magnitude of $\Delta c$ to be used, depending on his particular problem.

4. The redistributed stresses seem to be approaching the steady state stresses in a period of $0.5 \times 10^{-6}$ pseudo time, as can be seen in Figures 5.6.1 - 4.

5. The redistribution in stresses become smaller with every successive pseudo time interval, as can be seen from Figure 5.6, 5.9 and 5.11.

6. The creep temperature mechanism for a constant temperature distribution can be an advantage as it eliminates the small tensile stresses that may be present in the initial elastic state of the structure, as can be seen in Figure 5.6.3, segment 6. It may also minimise a substantial tensile stress, Figure 5.6.4, segment 8.

7. For cellular and T-section decks, the creep temperature mechanism helps to minimize the initial differential stress at the interface between the lower surface of the top flange and the upper surface of the web, as shown in Figures 5.9 and 5.11.
CHAPTER 6: CREEP BEHAVIOUR OF SOLID SLAB DECK UNDER CYCLIC TEMPERATURE DISTRIBUTION
6.1 Introduction

The temperature distribution in a concrete deck changes continuously during the day, due to the variation of the intensity of solar radiation, wind speed and air temperature. In this Chapter, the temperature of the concrete deck is represented by daily temperature cycles, which is a better theoretical representation than the constant temperature distribution assumed in the previous Chapter.

Weather data, from several locations in the United States\[^{15,55}\] and the Middle East\[^{138,139}\], were analysed, in order to choose extreme summer conditions that will cause high temperature distributions within a concrete deck. In the region of Baghdad, Iraq, the months of June and July represented such summer conditions. Available weather data from this region \[^{139,140,141}\] was used to work out the temperature distribution within the deck at various times of the day, using the computer program discussed in Section 3.1.4.

The two span continuous prestressed solid slab deck, Figure 5.1, investigated in the previous Chapter, will be used for the study of cyclic temperature distribution in this Chapter. The bridge deck is assumed to be under the effect of long-term loading, that is, dead load, prestressing effects and thermal stresses.

6.2 Cyclic temperature:

The variations in temperature of the top and bottom surfaces of the solid slab deck are shown in Figures 6.1.1 and 6.1.2. These curves were evaluated using weather data \[^{139,140,141}\] for a hot summer day in the Baghdad region of Iraq.

For the purpose of a cyclic temperature study, the temperature of the concrete deck during one day will be represented by one temperature cycle. This daily cycle can be divided into two half cycles. Each half cycle will represent 12 hours. These half cycles will be called the hot half cycle and the cold half cycle, being the equivalent of the deck temperatures shown in Figures 6.1.1 and 6.1.2 respectively.
Fig. 6.11: Hot half cycle

Concrete temperature (°C)

Top surface of deck

Bottom surface of deck

Hours of day

0900 1100 1300 1500 1700 1900 2100

0 5 10 15 20 25 30 35 40 45 50
6.3 The evaluation of pseudo time for cyclic temperature analysis

The evaluation of pseudo time for different ages of concrete at loading and at different temperatures requires the use of Equation 4.2 and Figures 4.1 and 4.2.

Table 6.1 illustrates the values of pseudo time intervals (Δt) for a concrete whose temperature is 5°C. The following procedure was used to work out the various values in the table:

1. The first column in the table, t, represents the time from loading plus one, in days. Therefore the value of 1 in this column represents the moment of application of the load.

2. The creep strains per unit stress in the second column are calculated using Equation 4.2, with the help of Figures 4.1 and 4.2 to obtain the values of 'a' and 'n' in the above equation, for the particular temperature and age of concrete at loading. However, by using the above equation, the specific strain per unit stress at t=1 works out to be $38 \times 10^{-6}$. This value represents the elastic strain, as there could not be any creep strain at t=1, which is the moment the load is applied. Therefore, this value of elastic strain will be used as a datum, and will be subtracted from the values of specific strain for t=1 to t=10, to obtain the creep strain values in the second column of the table.

3. The values of creep strain per unit stress per °C, in the third column, are obtained by dividing the values of creep strain per unit stress by the temperature of the concrete, which is 5°C in this case. The values in this column represent the pseudo time.

From Table 6.1, and similar tables for concrete temperatures (T) equal to 20°C, 40°C and 65°C, Figures 6.2 and 6.3 can be drawn. These figures show the linear variation of creep per unit stress and pseudo time with the value t.
Table 6.1. Pseudo time, T=5°C

\[
a = 38 \times 10^{-6} \quad n = 0.086
\]

Age at loading = 1 month

<table>
<thead>
<tr>
<th>( t ) (days)</th>
<th>( (x10^{-6}) ) Creep Strain per unit stress</th>
<th>Pseudo time, ( \Delta c ) (x 10^{-6})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1.5</td>
<td>1.35</td>
<td>0.27</td>
</tr>
<tr>
<td>2</td>
<td>2.33</td>
<td>0.47</td>
</tr>
<tr>
<td>2.5</td>
<td>3.12</td>
<td>0.62</td>
</tr>
<tr>
<td>3</td>
<td>3.77</td>
<td>0.75</td>
</tr>
<tr>
<td>3.5</td>
<td>4.32</td>
<td>0.87</td>
</tr>
<tr>
<td>4</td>
<td>4.81</td>
<td>0.96</td>
</tr>
<tr>
<td>4.5</td>
<td>5.25</td>
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<td>5</td>
<td>5.64</td>
<td>1.13</td>
</tr>
<tr>
<td>5.5</td>
<td>6.00</td>
<td>1.20</td>
</tr>
<tr>
<td>6</td>
<td>6.33</td>
<td>1.27</td>
</tr>
<tr>
<td>6.5</td>
<td>6.64</td>
<td>1.33</td>
</tr>
<tr>
<td>7</td>
<td>6.92</td>
<td>1.38</td>
</tr>
<tr>
<td>7.5</td>
<td>7.19</td>
<td>1.44</td>
</tr>
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<td>8</td>
<td>7.44</td>
<td>1.49</td>
</tr>
<tr>
<td>8.5</td>
<td>7.68</td>
<td>1.54</td>
</tr>
<tr>
<td>9</td>
<td>7.90</td>
<td>1.58</td>
</tr>
<tr>
<td>9.5</td>
<td>8.12</td>
<td>1.62</td>
</tr>
<tr>
<td>10</td>
<td>8.3</td>
<td>1.66</td>
</tr>
</tbody>
</table>
Fig. 6.2 Creep strain / $\sigma$ versus (days + 1)
Fig. 6.3 Creep strain $\sigma / T$ versus (days +1)

Creep strain per unit stress per unit $^\circ$C \times 10^{-6}
6.4 The equivalent pseudo time cycle (Fe)

The daily temperature variation within the concrete deck can be represented by one temperature cycle, as stated in Section 6.2. Figure 6.4.1 shows 10 such cycles, representing the concrete temperature variations for 10 days. For each cycle, the hot half \( (T_2) \) is equivalent to the temperature variation shown in Figure 6.1.1, while the cold half \( (T_1) \) is equivalent to the temperature variation shown in Figure 6.1.2.

The pseudo time equivalent to real time is evaluated for each half cycle, with the help of Figure 6.3, or more accurately by using Table 6.2, which is constructed in the same manner as Table 6.1, using the same datum of \( 38 \times 10^{-6} \) when calculating the creep strain per unit stress.

However, the pseudo time intervals obtained in Table 6.2 are for concrete temperatures of \( 40^\circ C \). The reason that this temperature was chosen for the pseudo time calculations is that, by examining the creep data available in Figure 4.1, the mean temperature of the deck estimated from Figures 6.1.1 and 6.1.2 was found to be closest to \( 40^\circ C \).

In Figure 6.4.1, the pseudo time intervals \( \delta c_1, \delta c_2, \delta c_3, \ldots, \delta c_{10} \) are equivalent to the real time intervals of the hot half cycles for the 10 cycles shown in the figure. While \( \delta' c_1, \delta' c_2, \delta' c_3, \ldots, \delta' c_{10} \) are the pseudo time intervals that are equivalent to the real time intervals of the cold half cycles.

The real time cycles shown in Figure 6.4.1 are equal to each other, with each cycle being equal to one day. While the pseudo time cycles do not have to be equal to each other. From Table 6.2, it can be noted that the first pseudo time cycle \( (\delta c_1 + \delta' c_1) \) is more than the second \( (\delta c_2 + \delta' c_2) \), and this is self-evident from Figure 6.3.2.

From Figure 6.4.2 and Table 6.2, an equivalent pseudo time cycle can be formed, which is shown in Figure 6.4.3. This cycle represents the 10 pseudo time cycles shown in Figure 6.4.2, who in turn represent the 10 real time cycles in Figure 6.4.1. The following equations can then be derived:
Table 6.2 - Pseudo time, T=40°C

\[ a = 52 \times 10^{-6} \quad n = 0.112 \]

Age at Loading = 1 month

<table>
<thead>
<tr>
<th>t days</th>
<th>Creep Strain per unit stress (x 10^{-6})</th>
<th>Pseudo time ( c ) (x 10^{-6})</th>
<th>( \Delta c ) (x 10^{-6})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14.0</td>
<td>0.35</td>
<td>0</td>
</tr>
<tr>
<td>1.5</td>
<td>16.4</td>
<td>0.41</td>
<td>0.06</td>
</tr>
<tr>
<td>2</td>
<td>18.2</td>
<td>0.46</td>
<td>0.05</td>
</tr>
<tr>
<td>2.5</td>
<td>19.6</td>
<td>0.49</td>
<td>0.03</td>
</tr>
<tr>
<td>3</td>
<td>20.8</td>
<td>0.52</td>
<td>0.03</td>
</tr>
<tr>
<td>3.5</td>
<td>21.8</td>
<td>0.55</td>
<td>0.03</td>
</tr>
<tr>
<td>4</td>
<td>22.7</td>
<td>0.57</td>
<td>0.02</td>
</tr>
<tr>
<td>4.5</td>
<td>23.5</td>
<td>0.59</td>
<td>0.02</td>
</tr>
<tr>
<td>5</td>
<td>24.3</td>
<td>0.61</td>
<td>0.02</td>
</tr>
<tr>
<td>5.5</td>
<td>24.9</td>
<td>0.62</td>
<td>0.01</td>
</tr>
<tr>
<td>6</td>
<td>25.6</td>
<td>0.64</td>
<td>0.02</td>
</tr>
<tr>
<td>6.5</td>
<td>26.1</td>
<td>0.65</td>
<td>0.01</td>
</tr>
<tr>
<td>7</td>
<td>26.7</td>
<td>0.67</td>
<td>0.02</td>
</tr>
<tr>
<td>7.5</td>
<td>27.2</td>
<td>0.68</td>
<td>0.01</td>
</tr>
<tr>
<td>8</td>
<td>27.6</td>
<td>0.69</td>
<td>0.01</td>
</tr>
<tr>
<td>8.5</td>
<td>28.1</td>
<td>0.70</td>
<td>0.01</td>
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<tr>
<td>9</td>
<td>28.5</td>
<td>0.71</td>
<td>0.01</td>
</tr>
<tr>
<td>9.5</td>
<td>28.9</td>
<td>0.73</td>
<td>0.01</td>
</tr>
<tr>
<td>10</td>
<td>29.3</td>
<td>0.73</td>
<td>0.01</td>
</tr>
<tr>
<td>10.5</td>
<td>29.7</td>
<td>0.74</td>
<td>0.01</td>
</tr>
<tr>
<td>11</td>
<td>30.0</td>
<td>0.75</td>
<td>0.01</td>
</tr>
<tr>
<td>20</td>
<td>34.7</td>
<td>0.87</td>
<td>0.12</td>
</tr>
<tr>
<td>30</td>
<td>38.11</td>
<td>0.95</td>
<td>0.08</td>
</tr>
</tbody>
</table>
Fig. 6.4.1

Fig. 6.4.2

Fig. 6.4.3

Fig. 6.4

Evaluation of an equivalent pseudo time cycle
\[ \Delta c_1 = \Sigma \delta c = \delta c_1 + \delta c_2 + \ldots + \delta c_{10} \quad \ldots \quad (6.1) \]

\[ \Delta' c_1 = \Sigma \delta' c = \delta' c_1 + \delta' c_2 + \ldots + \delta' c_{10} \quad \ldots \quad (6.2) \]

and \[ \Delta c_1 \neq \Delta' c_1 \quad \ldots \quad (6.3) \]

where \( \Delta c_1 \) represents the hot part of the equivalent pseudo time cycle, and \( \Delta' c_1 \) represents the cold part.

6.5 Creep temperature analysis for the hot half cycle

6.5.1 Introduction

It was decided, earlier in the Chapter, that the hot half part of the daily temperature cycles, Figure 6.4.1, represents the variable temperatures within the deck, from 9.00 to 21.00 hours, Figure 6.1.1. This simulation of the variable temperatures through the deck by a constant temperature distribution is studied in this section. The object of the investigation described in this section is to investigate whether using either temperature distribution for the creep temperature analysis will produce similar redistributed stresses in the prestressed concrete deck section.

6.5.2 Temperature distribution

The temperature distribution through the deck, for each 2 hourly interval during the hot half cycle, Figure 6.1.1, are given in Table 6.3 and shown in Figure 6.5.1. These values were obtained by using the computer program discussed in Section 3.1.4. to calculate the temperatures of the deck concrete for every 15 minutes, then taking the mean value of temperatures over a period of 120 minutes to obtain the values in Table 6.3.

The last column in Table 6.3 represents the constant temperature distribution which is assumed to act through the duration of the hot half cycle. These values are obtained by taking the mean values of temperatures for the 2 hourly interval, Figure 6.5.2.
Table 6.3 - Temperature Distribution

<table>
<thead>
<tr>
<th>Distance from Top Surface of Deck (mm)</th>
<th>9.00-11.00</th>
<th>11.00-13.00</th>
<th>13.00-15.00</th>
<th>15.00-17.00</th>
<th>17.00-19.00</th>
<th>19.00-21.00</th>
<th>9.00-21.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>46.9</td>
<td>60.0</td>
<td>62.2</td>
<td>53.4</td>
<td>42.9</td>
<td>37.0</td>
<td>50.4</td>
</tr>
<tr>
<td>200</td>
<td>26.6</td>
<td>28.8</td>
<td>32.1</td>
<td>35.3</td>
<td>37.0</td>
<td>33.0</td>
<td>32.1</td>
</tr>
<tr>
<td>400</td>
<td>26.0</td>
<td>26.1</td>
<td>26.4</td>
<td>27.1</td>
<td>28.1</td>
<td>30.6</td>
<td>27.4</td>
</tr>
<tr>
<td>600</td>
<td>26.0</td>
<td>26.3</td>
<td>26.7</td>
<td>27.3</td>
<td>28.1</td>
<td>30.6</td>
<td>27.5</td>
</tr>
<tr>
<td>800</td>
<td>28.3</td>
<td>30.2</td>
<td>32.2</td>
<td>33.7</td>
<td>33.7</td>
<td>33.4</td>
<td>31.9</td>
</tr>
</tbody>
</table>

Table 6.4 - Pseudo time intervals

\[ a = 52 \times 10^{-6}, \quad n = 0.112, \quad \text{datum} = 38 \times 10^{-6}, \quad c = 0.0601 \times 10^{-6} \]

<table>
<thead>
<tr>
<th>Real time (hours)</th>
<th>Real time (days)</th>
<th>( t ) (days +1)</th>
<th>Creep strain per unit stress (x 10^{-6})</th>
<th>( C ) (x10^{-6})</th>
<th>( \Delta c ) (x 10^{-6})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>14.00</td>
<td>0.3500</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.0833</td>
<td>1.0833</td>
<td>14.50</td>
<td>0.3620</td>
<td>0.0117</td>
</tr>
<tr>
<td>4</td>
<td>0.1666</td>
<td>1.1666</td>
<td>14.91</td>
<td>0.3726</td>
<td>0.0106</td>
</tr>
<tr>
<td>6</td>
<td>0.2499</td>
<td>1.2499</td>
<td>15.32</td>
<td>0.3829</td>
<td>0.0103</td>
</tr>
<tr>
<td>8</td>
<td>0.3332</td>
<td>1.3332</td>
<td>15.70</td>
<td>0.3926</td>
<td>0.0097</td>
</tr>
<tr>
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<td>0.4165</td>
<td>1.4165</td>
<td>16.07</td>
<td>0.4017</td>
<td>0.0091</td>
</tr>
<tr>
<td>12</td>
<td>0.5000</td>
<td>1.5000</td>
<td>16.42</td>
<td>0.4104</td>
<td>0.0087</td>
</tr>
</tbody>
</table>
Fig 6.5.1 Temperature distribution for 2 hours intervals

<table>
<thead>
<tr>
<th>Hours</th>
<th>9:00 - 11:00</th>
<th>11:00 - 13:00</th>
<th>13:00 - 15:00</th>
<th>15:00 - 17:00</th>
<th>17:00 - 19:00</th>
<th>19:00 - 21:00</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>d</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>e</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>f</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>a</td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>e</td>
<td>b</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>f</td>
<td>c</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fig. 6.5.2 Temperature distribution for 12 hours interval
6.5.3 Method of investigation

The solid slab deck structure of the previous Chapter was used for this study. The initial stresses, due to long-term loading of the structure were used as the starting stresses for the following two stages of the investigation:

1. The method of creep-temperature analysis, developed in Chapter 4, was used to evaluate the redistributed stresses in the section at the end of each 2 hourly interval. The pseudo time intervals given in Table 6.4, and the temperature distributions shown in Figure 6.5.1 were used for the analysis. Finally, the redistributed stresses, for each segment of the deck, were evaluated for six intervals of 2 hours each, thus completing the duration of the hot half cycle.

2. In this stage of the study, the initial stresses were analysed for the creep-temperature effects over one creep time interval which is the equivalent of 12 hours of real time. The temperature distribution used in this stage of analysis is shown in Figure 6.5.2, and the value of pseudo time interval over a period of 12 hours is given in Table 6.4.

The results obtained from the above stages of the investigation are discussed in the following section.

6.5.4 Results of the investigation

The amount of stress redistribution varied for each segment of the solid slab deck shown in Figure 5. Tables 6.5, 6.6, 6.7 and 6.8 show the stress redistributions in the deck at the end of the first hot half cycle for segments 2, 4, 6 and 8, respectively. The tables give the following information concerning the stresses in each segment of the deck.

1. The initial or starting stress distribution, $\sigma_o$.
2. The redistributed stresses calculated using 6 intervals of pseudo times, each equivalent to 2 hours of real time intervals, $\sigma_{s.1}$.
### Table 6.5 Stress redistribution in Segment 2

<table>
<thead>
<tr>
<th>Distance from Top Surface of Deck (mm)</th>
<th>( \sigma_0 ) ( \text{N/mm}^2 )</th>
<th>( \sigma_{s.1} ) ( \text{N/mm}^2 )</th>
<th>( \frac{\sigma_{s.1} \times 100}{\sigma_0} )</th>
<th>( \sigma_{s.2} ) ( \text{N/mm}^2 )</th>
<th>( \frac{\sigma_{s.2} \times 100}{\sigma_0} )</th>
<th>( \frac{\sigma_{s.2} - \sigma_{s.1} \times 100}{\sigma_0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.637</td>
<td>6.322</td>
<td>95.25</td>
<td>6.315</td>
<td>95.15</td>
<td>-0.10</td>
</tr>
<tr>
<td>200</td>
<td>5.818</td>
<td>5.782</td>
<td>99.38</td>
<td>5.781</td>
<td>99.36</td>
<td>-0.02</td>
</tr>
<tr>
<td>400</td>
<td>5.000</td>
<td>5.045</td>
<td>100.90</td>
<td>5.048</td>
<td>100.96</td>
<td>0.06</td>
</tr>
<tr>
<td>600</td>
<td>4.182</td>
<td>4.254</td>
<td>101.72</td>
<td>4.258</td>
<td>101.82</td>
<td>0.10</td>
</tr>
<tr>
<td>800</td>
<td>3.364</td>
<td>3.433</td>
<td>102.05</td>
<td>3.439</td>
<td>102.23</td>
<td>0.18</td>
</tr>
</tbody>
</table>

### Table 6.6 Stress redistribution in Segment 4

<table>
<thead>
<tr>
<th>Distance from Top Surface of Deck (mm)</th>
<th>( \sigma_0 ) ( \text{N/mm}^2 )</th>
<th>( \sigma_{s.1} ) ( \text{N/mm}^2 )</th>
<th>( \frac{\sigma_{s.1} \times 100}{\sigma_0} )</th>
<th>( \sigma_{s.2} ) ( \text{N/mm}^2 )</th>
<th>( \frac{\sigma_{s.2} \times 100}{\sigma_0} )</th>
<th>( \frac{\sigma_{s.2} - \sigma_{s.1} \times 100}{\sigma_0} )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>91.12</td>
<td>4.973</td>
<td>90.86</td>
<td>-0.26</td>
</tr>
<tr>
<td>200</td>
<td>5.236</td>
<td>5.092</td>
<td>97.25</td>
<td>5.085</td>
<td>97.12</td>
<td>-0.13</td>
</tr>
<tr>
<td>400</td>
<td>5.000</td>
<td>5.041</td>
<td>100.82</td>
<td>5.042</td>
<td>100.84</td>
<td>0.02</td>
</tr>
<tr>
<td>600</td>
<td>4.764</td>
<td>4.944</td>
<td>103.78</td>
<td>4.947</td>
<td>103.84</td>
<td>0.06</td>
</tr>
<tr>
<td>800</td>
<td>4.528</td>
<td>4.806</td>
<td>106.14</td>
<td>4.813</td>
<td>106.29</td>
<td>0.15</td>
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</table>
### Table 6.7 Stress redistribution in Segment 6.

<table>
<thead>
<tr>
<th>Distance from Top Surface of Deck (mm)</th>
<th>$\sigma_o$ N/mm²</th>
<th>$\sigma_{s.1}$ N/mm²</th>
<th>$\frac{\sigma_{s.1} \times 100}{\sigma_o}$</th>
<th>$\sigma_{s.2}$ N/mm²</th>
<th>$\frac{\sigma_{s.2} \times 100}{\sigma_o}$</th>
<th>$\frac{\sigma_{s.2} - \sigma_{s.1} \times 100}{\sigma_o}$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>10.784</td>
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<td>92.41</td>
<td>9.939</td>
<td>92.16</td>
<td>-0.25</td>
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<td>7.892</td>
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<td>97.43</td>
<td>7.681</td>
<td>97.33</td>
<td>-0.10</td>
</tr>
<tr>
<td>400</td>
<td>5.000</td>
<td>5.064</td>
<td>101.28</td>
<td>5.067</td>
<td>101.34</td>
<td>0.06</td>
</tr>
<tr>
<td>600</td>
<td>2.108</td>
<td>2.369</td>
<td>112.38</td>
<td>2.377</td>
<td>112.76</td>
<td>0.38</td>
</tr>
<tr>
<td>800</td>
<td>-0.784</td>
<td>-0.322</td>
<td>41.07</td>
<td>-0.305</td>
<td>38.90</td>
<td>-2.17</td>
</tr>
</tbody>
</table>

### Table 6.8 Stress redistribution in Segment 8.

<table>
<thead>
<tr>
<th>Distance from Top Surface of Deck (mm)</th>
<th>$\sigma_o$ N/mm²</th>
<th>$\sigma_{s.1}$ N/mm²</th>
<th>$\frac{\sigma_{s.1} \times 100}{\sigma_o}$</th>
<th>$\sigma_{s.2}$ N/mm²</th>
<th>$\frac{\sigma_{s.2} \times 100}{\sigma_o}$</th>
<th>$\frac{\sigma_{s.2} - \sigma_{s.1} \times 100}{\sigma_o}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>17.662</td>
<td>16.468</td>
<td>93.24</td>
<td>16.433</td>
<td>93.04</td>
<td>-0.20</td>
</tr>
<tr>
<td>200</td>
<td>11.331</td>
<td>11.080</td>
<td>97.78</td>
<td>11.072</td>
<td>97.71</td>
<td>-0.07</td>
</tr>
<tr>
<td>400</td>
<td>5.000</td>
<td>5.096</td>
<td>101.92</td>
<td>5.099</td>
<td>101.98</td>
<td>0.06</td>
</tr>
<tr>
<td>600</td>
<td>-1.331</td>
<td>-0.995</td>
<td>74.76</td>
<td>-0.982</td>
<td>73.85</td>
<td>-0.91</td>
</tr>
<tr>
<td>800</td>
<td>-7.662</td>
<td>-7.018</td>
<td>91.59</td>
<td>-6.994</td>
<td>91.28</td>
<td>-0.31</td>
</tr>
</tbody>
</table>
3. The redistributed stresses calculated using one interval of pseudo time which is equivalent to 12 hours of real time, $\sigma_{s.2}$.

4. For the purpose of comparison between the results obtained by the above two methods, the following values are shown in the tables; $\sigma_{s.1}/\sigma_0$, $\sigma_{s.2}/\sigma_0$ and $(\sigma_{s.2}-\sigma_{s.1})100/\sigma_0$.

From Tables 6.5 to 6.8, it can be seen that the values of $\sigma_{s.1}$ and $\sigma_{s.2}$ are very close to each other. Therefore, there is no need to use relatively pseudo time intervals, which are equivalent to 2 hours of real time, for the creep-temperature analysis. Using one pseudo time interval for the duration of the half cycle reduces the calculations.

If the above is true for the first hot half cycle of 12 hours, it is reasonable to assume it will be true for subsequent cold and hot half cycles. The first reason is that the pseudo time interval which is equivalent to the duration of the first hot half cycle is more than that for any subsequent cold or hot half cycles, as can be seen from Figure 6.3. The second reason is that the cold half cycles have lesser effect on the redistribution of stresses, due to their lower temperatures than the hot half cycles.

6.5.5 Conclusions

The following conclusions can be drawn from the investigations of the effects of the first hot half cycle on the stress distribution in a solid slab deck.

1. The creep temperature analysis can be carried out for half daily cycle by using one pseudo time interval which is equivalent to 12 hours of real time.

2. The temperature distribution within the deck, for the duration of the hot half cycle, can be taken as the mean value of the temperatures during the same period.
6.6 The creep-temperature analysis for 10 daily cycles:

6.6.1 Introduction

The creep-temperature method of analysis, developed in Chapter 4, is a time consuming method when the pseudo time intervals are taken to be equivalent to half daily cycles. Therefore, the equivalent pseudo time cycle, discussed in Section 6.4, was devised. The later cycle combines the effects of 10 daily cycles, or more, in one cycle which minimises the creep-temperature calculations considerably.

The use of the equivalent pseudo time cycles will be investigated in this section. The object of the investigation is to find out whether the use of these cycles, in the creep-temperature analysis, will produce the same results as those obtained by using several smaller pseudo time intervals, each equivalent to a one daily cycle.

6.6.2 Temperature distribution

The temperature distribution shown in Figure 6.5.2 will be used for the creep-temperature analysis for the hot portion of the equivalent pseudo time cycle, Figure 6.4.2, and will also be used for the hot half daily cycles, Figure 6.4.1. While a constant temperature through the deck of 30°C will be assumed to act for the duration of the cold portion of the equivalent pseudo time interval and for the cold half daily cycles. The later assumption is based on the temperature variations through the deck during the cold half daily cycle, Figure 6.1.2.

6.6.3 Method of investigation

The solid slab deck of Section 6.5 will be used for this investigation which can be divided into the following two stages:

1. Starting with the initial stresses due to long term long, \( \sigma_0 \), the creep temperature analysis was carried out for 10 daily pseudo time cycles. Beginning with pseudo time interval \( \sigma c_1 \), and ending with \( \sigma'_{10} \) which are shown in Figure 6.4.1. Therefore, the analysis had to be repeated 20 times, once for each pseudo time interval.
Each time the starting stresses used are those obtained as a result of the previous interval. Table 6.9 gives the $\Delta c$ values equivalent to each of the 20 real time half daily cycles.

2. Starting with the same initial stresses, $\sigma_0$, the creep-temperature analysis was carried out for 2 pseudo time intervals, $\sigma c_1$ and $\sigma c_2$, shown in Figure 6.4.2 and given in Table 6.9. The two intervals, when combined constitute the equivalent pseudo time cycle for the 10 daily cycles.

The results from the above stages of investigation are discussed in the following section.

6.6.4 Results of investigation

The stress redistribution shown in Figure 6.6 compares the stresses obtained by using 10 daily cycles, $\sigma_{10}$, with those obtained using the equivalent pseudo time interval, $\sigma_1$, in the creep-temperature analysis, for different segments along the deck. The initial stresses, $\sigma_0$, are also shown in the figures.

From Figures 6.6.1-3 it can be seen that the redistributed stress curves $\sigma_1$ and $\sigma_{10}$ are very close together. The differences between the two curves being so small that they can be neglected.

Figures 6.7.1-4 show the change in stress for different segments along the structure, at various depths within the deck. It can be noted that the comparatively large change in stress occurring within the first day of loading is followed by successively smaller changes occurring in the following 9 days. This is an expected phenomena of the exponential decay type, as creep diminishes continuously with time. However, the rate of change of stress due to creep reduces dramatically within the first few days of loading, as can be seen from Figures 6.7, then it becomes successively slower in the following days.
Table 6.9 Pseudo time intervals, \( \Delta c \).
\( (T= 40^\circ C) \)

\[
a = 52 \times 10^{-6} \quad n = 0.112
\]

\[
\Delta c_1 = \varepsilon \delta c = 0.2156 \times 10^{-6}
\]

\[
\Delta'c_1 = \varepsilon \delta'c = 0.1848 \times 10^{-6}
\]

<table>
<thead>
<tr>
<th>Time (days +1)</th>
<th>Creep Strain per unit stress ( (x \times 10^{-6}) )</th>
<th>Pseudo time, ( c_1 ) ( (x \times 10^{-6}) )</th>
<th>( \Delta c ) ( (x \times 10^{-6}) )</th>
<th>HOT - H COLD- C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14.0</td>
<td>0.35</td>
<td>0</td>
<td>H</td>
</tr>
<tr>
<td>1.5</td>
<td>16.416</td>
<td>0.4104</td>
<td>0.0604</td>
<td>H</td>
</tr>
<tr>
<td>2</td>
<td>18.198</td>
<td>0.4550</td>
<td>0.0445</td>
<td>C</td>
</tr>
<tr>
<td>2.5</td>
<td>19.620</td>
<td>0.4905</td>
<td>0.0355</td>
<td>H</td>
</tr>
<tr>
<td>3</td>
<td>20.809</td>
<td>0.5202</td>
<td>0.0297</td>
<td>C</td>
</tr>
<tr>
<td>3.5</td>
<td>21.833</td>
<td>0.5458</td>
<td>0.0256</td>
<td>H</td>
</tr>
<tr>
<td>4</td>
<td>22.734</td>
<td>0.5684</td>
<td>0.0226</td>
<td>C</td>
</tr>
<tr>
<td>4.5</td>
<td>23.541</td>
<td>0.5885</td>
<td>0.0201</td>
<td>H</td>
</tr>
<tr>
<td>5</td>
<td>24.271</td>
<td>0.6068</td>
<td>0.0183</td>
<td>C</td>
</tr>
<tr>
<td>5.5</td>
<td>24.940</td>
<td>0.6235</td>
<td>0.0167</td>
<td>H</td>
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<tr>
<td>6</td>
<td>25.556</td>
<td>0.6389</td>
<td>0.0154</td>
<td>C</td>
</tr>
<tr>
<td>6.5</td>
<td>26.128</td>
<td>0.6532</td>
<td>0.0143</td>
<td>H</td>
</tr>
<tr>
<td>7</td>
<td>26.663</td>
<td>0.6666</td>
<td>0.0134</td>
<td>C</td>
</tr>
<tr>
<td>7.5</td>
<td>27.164</td>
<td>0.6791</td>
<td>0.0125</td>
<td>H</td>
</tr>
<tr>
<td>8</td>
<td>27.637</td>
<td>0.6909</td>
<td>0.0118</td>
<td>C</td>
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<tr>
<td>8.5</td>
<td>28.084</td>
<td>0.7021</td>
<td>0.0112</td>
<td>H</td>
</tr>
<tr>
<td>9</td>
<td>28.509</td>
<td>0.7127</td>
<td>0.0106</td>
<td>C</td>
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<td>28.913</td>
<td>0.7228</td>
<td>0.0101</td>
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<td>0.7325</td>
<td>0.0097</td>
<td>C</td>
</tr>
<tr>
<td>10.5</td>
<td>29.667</td>
<td>0.7417</td>
<td>0.0092</td>
<td>H</td>
</tr>
<tr>
<td>11</td>
<td>30.020</td>
<td>0.7505</td>
<td>0.0088</td>
<td>C</td>
</tr>
</tbody>
</table>
Fig. 6.6.1 Redistribution of stresses in Segment 2
Fig. 6.6.2 Redistribution of stresses in Segment 4
Fig. 6.6.3 Redistribution of stresses in Segment 6
6.6.5 Conclusions

6.6.5.1 The equivalent pseudo time cycle (Fe)

The following conclusions can be drawn from the investigation concerning the use of the equivalent pseudo time cycle (Fe):

1. The use of Fe, in the creep-temperature analysis, is valid, Figure 6.6.
2. The study was based on an Fe whose frequency was one cycle per 10 days, Figure 6.4.2. If the use of such a frequency is valid, Figure 6.6, then due to the discussion in Section 6.6.4, the use of an Fe whose frequencies are 1 cycle /100 days and 1 cycle /1000 days are also valid.
3. The use of Fe, Figure 6.4.2, reduces the creep temperature analysis considerably, if compared with the use of the pseudo time daily cycle, Figure 6.4.1.

6.6.5.2 Stress redistribution in the first 10 days of loading

The following conclusions can be drawn from Figure 6.7.1-4 which show the change in stresses during the first 10 days of loading.

1. The stress history over the 10 day period is shown in the above figures for a mid-span segment and a support segment. The periods of high and low temperature of the upper surface produce the rectangular wave form of the stress-time curves.

2. The rate of creep at both temperature distributions is shown to be significant by the slope of the curve at the upper and lower sections of the rectangular wave. Both slopes are considerably reduced over the 10 day period.
Fig. 6.7.1 Redistribution of stress with time in top fibre of mid-span segment of deck
Fig. 6.7.2 Redistribution of stress with time in top fibre of an outer support segment of deck

- Value obtained using constant temperature distribution (Chapter 5)
Fig. 6.7.3  Redistribution of stress with time in bottom fibre of mid-span segment of deck

Value obtained using constant temperature distribution (Chapter 5)
Fig. 6.7.4 Redistribution of stress with time in bottom fibre of an outer support segment of deck

Value obtained using constant temperature distribution (Chapter 5)
CHAPTER 7: THE CREEP-TEMPERATURE EFFECTS ON SKEW DECKS
7.1 Introduction

In the previous two chapters, the effect of the creep temperature mechanism was investigated for beam structures. The deck was considered to consist of a number of independent longitudinal beams, with no regard to the transverse stiffness of the deck. In this chapter, a prestressed skew solid slab deck was investigated for the effects of creep and temperature.

7.2 Grillage analysis of the deck

The investigation was carried out on a 45° skew deck shown in Figure 7.1. The geometric layout (142) of the grillage beams that was used to simulate the behaviour of the skew slab is shown in Figure 7.2.

The deck is represented by longitudinal and transverse members. Figure 7.3 shows the cross-sections of all members. The longitudinal members consist of two types. The edge members are 2.5m wide while the internal members are 5.0m wide. The transverse members are all of 5.0m width. Figure 7.2 shows the 25 joints of the grillage and the global axis of the structure.

A computer program was used to analyse the grillage. As in the previous two chapters, the skew deck was subjected to long term loading only, that is, the dead load of the deck, prestressing effects and a constant linear temperature gradient shown in Figure 7.4. Both the longitudinal and the transverse members were prestressed, and so the deck was kept in a state of elastic compression. However, the longitudinal members were prestressed eccentrically, thus producing an axial stress of 5N/mm² in the section in addition to a hogging moment that will counter the sagging moment due to the dead load. The transverse members were prestressed concentrically thus producing only axial stresses of 1.0N/mm².

The magnitude of the prestressing force in the longitudinal members was much higher than that of the transverse members. The reason being that the dead load moments in the longitudinal members were higher than the transverse members according to the grillage analysis. The prestress was calculated as if the longitudinal grillage members were actual discrete members. The above also applies to the effect of the temperature gradient as it was assumed to apply to the longitudinal members only.
Fig. 7.1 Skew deck

Fig. 7.2 Grillage structure
Fig. 7.3 Cross-section of members

Fig. 7.4 Temperature distribution

Fig. 7.5 Temperature gradient

Fig. 7.6 Torsion of a thin strip

Fig. 7.7 Directions of shearing stresses in the torsion of a thin strip
7.3 Long term loading of the grillage

The dead load carried by the edge longitudinal members, for a 1m deep solid slab, was 87.5kN/m. While the internal longitudinal members, whose cross sectional widths were twice that of the edge members, carried a dead load of 175kN/m. To counter the effect of the above dead load, a prestressing force of 25000 kN was applied to each longitudinal internal member at an eccentricity of 50mm below the neutral axis of the cross-section. The longitudinal edge members were subjected to half the above value at the same eccentricity. A comparatively smaller prestressing force of 5000 kN was applied to each of the transverse members at their neutral axis, and thus producing no bending moment.

The effect of the temperature gradient was applied to the longitudinal members in the following manner.

1. Restraining moments to prevent curvature due to temperature gradients are assumed.
2. The temperature gradient in Figure 7.5 was assumed to act on the longitudinal members.
3. The above gradient produces an average thermal strain in the section which equals $\frac{1}{2} \alpha T$, where $\alpha$ is the coefficient of thermal expansion of the concrete used in the deck.
4. The above strain produces a stress of $E \frac{1}{2} \alpha T$, if the concrete is assumed elastic, where $E$ is the elastic modulus of the concrete.
5. From the theory of beam bending, the above stress will induce a moment in the section which is equal to $EI \alpha T$, where $I$ is the second moment of area of the section and $h$ is the depth of the section.

7.4 Torsion of a flat rectangular plate

The effect of torsion in skew decks is quite significant. The effect on the deck increases with the increase in the angle of skew until it reaches a maximum at 45°. In this section, torsion of flat plates will be discussed and later used in the analysis of the deck.

Consider a long flat strip of rectangular cross-section with a width of $b$, thickness $t$, and length $L$. For uniform torsion about the centroid of the cross-section, the strip may be treated as a set of concentric thin hollow tubes, all twisted by the same amount. Consider such an elemental
tube which is rectangular in shape, with the longer sides being a distance \( y \) from the central axis of the strip; the thickness of the tube is \( \delta y \), Figure 7.6.

If \( \delta T \) is the torque carried by this elemental tube, then the shearing stress in the longer sides of the tube is (143)

\[
\tau = \frac{\delta T}{4by\delta y} \quad \cdots \cdots \text{(7.1)}
\]

where \( b \) is assumed to be much greater than \( t \).

The above relation gives,

\[
\frac{dT}{dy} = 4by\tau \quad \cdots \cdots \text{(7.2)}
\]

For the angle of twist of the elemental tube (143)

\[
\theta = \frac{L\delta T}{8by^2G\delta y} \quad \cdots \cdots \text{(7.3)}
\]

On comparing Equations 7.2 and 7.4, we have,

\[
\frac{dT}{dy} = 8by^2G \frac{\theta}{L} \quad \cdots \cdots \text{(7.4)}
\]

On comparing Equations 7.2 and 7.4, we have,

\[
\tau = 2yG \left( \frac{\theta}{L} \right) \quad \cdots \cdots \text{(7.5)}
\]

This shows that the shearing stress \( \tau \) varies linearly throughout the thickness of the strip, having a maximum value at the surface of the strip,

\[
\tau_{\text{max}} = Gt \left( \frac{\theta}{L} \right) \quad \cdots \cdots \text{(7.6)}
\]

An important feature is that the shearing stresses \( \tau \) act parallel to the longer side \( b \) of the strip, and that their directions reverse over the thickness of the strip. This approximate solution gives an inexact picture of the shearing stresses near the corners of the cross-section. Therefore, it is better not to consider rectangular elemental tubes but flat tubes with curved ends. The contours of constant shearing stress are then continuous curves, Figure 7.7.
The total torque on the cross-section is

\[ T = \int_{0}^{2t} 8 y^2 \, G(\frac{y}{L}) \, dy = \frac{1}{3} \, bt^3 \, G(\frac{y}{L}) \] ........ (7.7)

The polar second moment of area of the cross-section about its centre is,

\[ J = \frac{1}{12} \, (bt^3 + b^3t) \] ........ (7.8)

If \( b \) is much greater than \( t \), then approximately,

\[ J = \frac{1}{12} \, b^3t \] ........ (7.9)

The geometrical constant occurring in Equation 7.7 is \( \frac{1}{3} \, bt^3 \); thus, in the torsion of a thin strip, we cannot use the polar second moment of area for \( J \) in the relation

\[ \frac{T}{J} = G\left(\frac{y}{L}\right) \] ........ (7.10)

Instead, we must use,

\[ J = \frac{1}{3} \, bt^3 \] ........ (7.11)

From equations 7.6 and 7.10,

\[ \tau_{\text{max}} = \frac{Tt}{J} \] ........ (7.12)

Therefore, Equation 7.11 was used to evaluate the torsional inertia of the grillage members, but only half of the value obtained by the above equation was given to each longitudinal and transverse member as they are part of a slab. Equation 7.12 was used for the evaluation of maximum torsional shear stresses or the torsional moment due to a given torsional stresses.
7.5 Method of creep-temperature analysis

Using the long term loading stated in Section 7.3, a grillage computer program produced a set of bending and torsional moments for each joint in the members of the grillage shown in Figure 7.2. The properties of the members are given in Table 7.1. The bending moments acting on the members of the grillage due to long term loading which includes the dead load, prestress and temperature gradient, are shown in Figure 7.8, while the net torsional moments acting on the joints of the grillage due to long term loading are shown in Figure 7.9.

The method of creep temperature analysis developed in Chapter 4 was used to analyse the grillage for the creep temperature effects as shown in the following sections but with torsional strains and torques included as well as bending strains and moments.

7.5.1 Bending effects

For each member of the grillage, the bending due to long term loading, shown in Figure 7.8, at the middle of the member was evaluated. This moment was assumed to be constant over the full length of the member.

The depth of the member was then divided into several layers and the bending stresses calculated for each layer. These stresses are the initial stresses \( \sigma_0 \) shown in Table 7.2 for a random member 6 in the grillage.

These stresses will induce creep strains at each layer of the member. These strains were evaluated by using Equation 4.8 which is

\[
\Delta \varepsilon = \sigma \cdot T \cdot \Delta c
\]
Table 7.1 Properties of members

<table>
<thead>
<tr>
<th>Number of members</th>
<th>Cross-sectional area (m²)</th>
<th>Bending inertia (m⁴)</th>
<th>Torsional inertia (m⁴)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-16 and 21-36</td>
<td>5.0</td>
<td>0.416</td>
<td>0.834</td>
</tr>
<tr>
<td>1-4 and 17-20</td>
<td>2.5</td>
<td>0.208</td>
<td>0.417</td>
</tr>
</tbody>
</table>

Table 7.2 - Bending stresses in member 6 (N/mm²)

<table>
<thead>
<tr>
<th>Type of stress</th>
<th>Distance from top surface of deck (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1000</td>
</tr>
<tr>
<td>σ₀</td>
<td>4.63</td>
</tr>
<tr>
<td>σ₁</td>
<td>-2.36</td>
</tr>
<tr>
<td>σ₂</td>
<td>2.79</td>
</tr>
<tr>
<td>σ₃</td>
<td>5.06</td>
</tr>
</tbody>
</table>

Table 7.3 - Torsional stress at joint B (N/mm²) (about the x-axis)

<table>
<thead>
<tr>
<th>Type of stress</th>
<th>Distance from top surface of deck (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1000</td>
</tr>
<tr>
<td>τ₀</td>
<td>+0.26</td>
</tr>
<tr>
<td>τ₁</td>
<td>-0.13</td>
</tr>
<tr>
<td>τ₂</td>
<td>+0.13</td>
</tr>
<tr>
<td>τ₃</td>
<td>+0.26</td>
</tr>
</tbody>
</table>

Table 7.4 - Torsional stresses at joint B (N/mm²) (about the y-axis)

<table>
<thead>
<tr>
<th>Type of stress</th>
<th>Distance from top surface of deck (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1000</td>
</tr>
<tr>
<td>τ₀</td>
<td>+0.32</td>
</tr>
<tr>
<td>τ₁</td>
<td>-0.16</td>
</tr>
<tr>
<td>τ₂</td>
<td>+0.29</td>
</tr>
<tr>
<td>τ₃</td>
<td>+0.45</td>
</tr>
</tbody>
</table>
Fig. 7-8 Bending moment at middle of longitudinal members (KN.m)
Fig. 7-9   Initial torsional moments at joints  (KN.m)
As stated in Section 4.5, a restraining elastic stress system was applied to nullify the strains due to creep, by using Equation 4.6 which can be written in the following form,

$$\Delta \sigma = (\sigma \cdot T\Delta c)E$$

These stresses form the second set of stresses in the system of the structure and are shown as $\sigma_1$ in Table 7.2 for $\Delta c = 0.5 \times 10^{-6}$. The negative sign indicates that they are tensile stresses.

These stresses will induce nullifying moments in each member of the grillage in addition to an axial force $\Delta N$ within the member, thus giving rise to net moments at each joint of the structure. To complete the loading of the structure, as explained in Section 4.5, moments were applied at each joint which were equal in magnitude but opposite in direction to the net nullifying moments at all joints of the grillage. These latter moments are shown in Figure 7.8. In addition to the moments, an axial force $\Delta N$ is applied in each member, in an opposite direction to that due to the nullifying effects.

### 7.5.2 Torsional effects

For each joint of the grillage, there are two net torsional moments, one for each axis of the structure. These torsional moments, due to long term loading of the structure, are shown in Figure 7.9.

Repeating the same method stated in Section 7.5.1, we obtain the shearing stresses $\tau_0$ and $\tau_1$ shown in Tables 7.3 and 7.4 for a random joint 8 for a net torsion about the x and y axis of the structure, where

$$\tau_0 = \text{shearing stresses due to long term loading of the structure}$$

and

$$\tau_1 = \text{nullifying shearing stresses}.$$ 

The loading of the structure was then completed for torsional effects, by applying torsional moments that were equal in magnitude but opposite in direction to those due to the nullifying shearing stresses, $\tau_1$, about both axes of the structure, at each joint of the grillage. These latter torsional moments are shown in Figure 7.10.
Fig. 7.10  Torsional moments opposing to nullifying effects (KN.m)
7.5.3 Combined effect of bending and torsion

The grillage was finally loaded with the bending moments shown in Figure 7.8, and the torsional moments shown in Figure 7.10. Due to these loading and by using a grillage computer program, a new set of bending and torsional moments were obtained.

Using the above new results, the following steps of analysis were carried out:

1. The bending moments at the middle of each member were evaluated, then the bending stresses were calculated for each layer of the member, these stresses are $\sigma_2$ in Table 7.2.

2. The net torsional moments, about both axis of the structure, for each joint of the grillage were evaluated, then the shearing stresses due to the above torsional moments were calculated for each layer of the member at that joint. These shearing stresses $\tau_2$ are shown in Tables 7.3 and 7.4, for torsional effects about the x and y axis respectively.

The stress resultants in the structure can now be determined for a pseudo time interval of $0.5 \times 10^{-6}$. Adding $\sigma_0$, $\sigma_1$ and $\sigma_2$ in Table 7.2, and $\tau_0$, $\tau_1$ and $\tau_2$ in Tables 7.3 and 7.4, will produce the set of bending and torsional stresses, $\sigma_3$ and $\tau_3$ respectively. This can be repeated for as many pseudo time intervals as necessary.

7.6 Results of investigation

The redistribution of bending stresses for a number of the longitudinal members during a pseudo time interval of $0.5 \times 10^{-6}$ are shown in Figure 7.11. It can be seen from the above figure that a significant amount of redistribution of bending stresses had occurred in each of the longitudinal members. However, the bending stresses at the end of the above pseudo time interval seems to approach the initial bending stresses due to dead load and prestressing effects only (without the temperature gradient effects). Thus it seems that the effect of the temperature gradient had almost disappeared during the above period of pseudo time. Therefore it can be concluded that the creep temperature mechanism diminishes the effect of the temperature gradient with an increase in pseudo time, at a rate which is approximately equal to that found in the single continuous beam of Chapter 5.
Fig. 7-11 Redistribution of bending stresses in longitudinal members
Transverse members of the grillage structure were under a much lower state of stress when compared with the longitudinal members. Figure 7.12 shows the redistribution of bending stresses for some transverse members during a pseudo time period of $0.5 \times 10^{-6}$. It can be seen that although the stresses are small the redistribution in bending stresses is significant. However, the redistribution of stresses in the transverse members has a small effect on the behaviour of the grillage structure because of their comparatively smaller magnitudes when compared with the bending stresses in the longitudinal members.

The net torsional moments acting on the joints of the grillage structure, about the x and y global axes, are quite significant and are comparable to the magnitudes of the bending moments on the longitudinal members of the grillage. The redistribution in torsional stresses during a pseudo time period of $0.5 \times 10^{-6}$ are shown for some joints of the grillage structure in Figures 7.13 and 7.14 for torsional moments about the y and x global axes, respectively. It can be seen from the above figures that a considerable amount of redistribution in torsional stresses had occurred during a pseudo time interval of $0.5 \times 10^{-6}$. The change in torsional stresses with pseudo time has a significant effect on the behaviour of the grillage structure that may equal the effect of change in bending stresses with pseudo time in the longitudinal members.

The initial moment in the longitudinal members, 5, 6 and 7 and 8 due to long-term loading (dead load, prestress and temperature gradient) is shown in Figure 7.15. From the figure, it can be seen that a considerable change had occurred in the moment during a pseudo time interval of $0.5 \times 10^{-6}$, in which the effect of bending moment due to the temperature gradient had almost disappeared. This phenomena was noticed in the study of the effect of creep-temperature mechanism on the variation in bending moment with pseudo time for beam structures, as shown in Figure 5.14.

The geometry of a skew slab supported on opposite sides is such that both uniformly distributed loads and a temperature gradient that is constant over the whole structure will produce complex interaction of bending and torsion. This will lead to a complex system of reactions at the supported sides. Figure 7.16 shows the skew deck with the contours of curvature that the deck is subjected to, as a result of a loading on the deck. The reactions at the supported joints are shown in Figure 7.17.
Fig. 7.12-1 Member 21  
Fig. 7.12-2 Member 24

Fig. 7.12-3 Member 25  
Fig. 7.12-4 Member 28

Fig. 7.12 Distribution of bending stresses in transverse members
Fig. 7.13.1 Joint 4

Fig. 7.13.2 Joint 6

Fig. 7.13.3 Joint 10

Fig. 7.13.4 Joint 11

Fig. 7.13 Redistribution of torsional stresses at joints (about y axis)
Fig. 7.14 Redistribution of torsional stresses at joints (about x axis)
Moments due to dead load + prestress

Moments due to dead load + prestress + temperature gradient after $\Delta C = 0.5 \times 10^{-6}$

Fig. 7.15 Moments in longitudinal members
Fig. 7.16 Contours of curvature

- Reactions due to dead load + prestress
- Reactions due to dead load + prestress + temperature gradient
- Reactions due to dead load + prestress + temperature gradient after $\Delta C = 0.5 \times 10^{-6}$

Fig. 7.17 Reactions at supported joints
From the figure, it can be seen that the reaction at joint 21, due to dead load and prestress, is much higher than that at joint 1. This seems obvious by examining Figure 7.16. In fact, the reactions increase continuously from joint 1 to joint 21. Adding the effect of the temperature gradient will reduce the differences between the reactions, as can be noted from Figure 7.17. The effect of the temperature gradient on the reactions almost disappeared during $\Delta c = 0.5 \times 10^{-6}$.

7.7 Conclusions

From the investigation of this Chapter, the following conclusions can be made.

1. A considerable amount of stress redistribution occurred, during a pseudo time interval of $0.5 \times 10^{-6}$, in bending and torsional stresses in the grillage structure.

2. The effect of the temperature gradient on bending stresses and reactions almost disappears during a pseudo time interval of $0.5 \times 10^{-6}$.

3. Transverse members have a comparatively minor effect on the grillage structure due to the system of loading used in the investigation.

4. Torsional moments at the joints of the grillage have a significant effect on the behaviour of the grillage which may equal that of the bending moments in the longitudinal members. Therefore, the redistribution in the torsional stresses has a significant effect on the structure.

5. There is a significant variation in the reactions at the supported joints for dead load and prestress, due to the severity of the angle of skew. However, the magnitude of variation reduces when the temperature gradient effect is added.
CHAPTER 8: GENERAL CONCLUSIONS AND RECOMMENDATIONS FOR CREEP-TEMPERATURE DESIGN
Prestressed concrete structures creep under normal temperatures. However, creep is enhanced by an increase in the temperature of the concrete, as proved by experimental work reviewed in Chapter 2. Other major factors that were found to affect creep in prestressed concrete bridge decks were the net initial concrete stress and the age of concrete at transfer of prestress.

In this Chapter, general conclusions based on the work in Chapters 4 to 7 will be presented as an aid to designing for the effects of creep and temperature in prestressed concrete continuous decks and simply supported prestressed skew decks. The conclusions are as follows:

1. The method of creep-temperature analysis developed in Chapter 4 enables the bridge designer to investigate the combined effect of creep and temperature on the stress redistribution in a prestressed deck section at different periods in the life of the structure.

2. The moments in prestressed concrete continuous deck changes significantly with time as evident from Figure 5.14. Therefore, the designer of such decks will have to consider the possible changes of moments in the span.

3. The choice of the pseudo time interval that will be used to study the effect of the creep-temperature mechanism is left to the designer. The above intervals should be very small (Table 6.9) if the designer is interested in the early stages after loading of the structure. A one large pseudo time interval of $0.5 \times 10^{-6}$ can be used to study the long-term effects of creep and temperature, as long as the above interval can be well related to real time.

4. The stress changes in prestressed sections due to creep and temperature reduces with each successive pseudo time interval (Figures 5.6.1 to 5.6.4). This is due to the diminishing effects of creep with time. The above phenomena will contribute to the choice by the designer of the magnitude of the pseudo time interval to be used for his particular problem. It will also help the designer to decide on the number of intervals that should be used in the analysis.
5. In the analysis of the two span continuous beam in Chapter 5, it was found that the creep-temperature mechanism caused a reduction in the compressive stresses at the top of the deck while increasing or inducing compressive stresses at the sofit (Figures 5.6.1 to 5.6.4). The above phenomena is advantageous for the part of deck from segment 1 to 6 as in this portion of the deck the live load causes compression in the top surface and tension at the sofit. The opposite is true for segments 7 and 8 where live load causes tension in the top surface and compression at the sofit. However, a continuous prestressed beam can be designed in such a way that the creep-temperature mechanism will cause changes in the stresses at the top and bottom surface of the deck which will relieve part of the induced stresses due to live load.

6. Although assuming a constant temperature distribution (Chapter 5) for the creep-temperature study is a practical assumption as it reduces the time needed for the analysis but a cyclic temperature variation (Chapter 6) is a better representation of the actual action of temperature within a concrete bridge deck. Using cyclic temperature variation for creep temperature analysis is a time consuming process.

7. The equivalent pseudo time cycle, Fe (Section 6.4) used in the creep-temperature analysis produces results that are very close to those using daily cyclic temperature variation (Section 6.2). Therefore, the Fe is recommended to be used for the creep-temperature analysis of prestressed concrete decks as it produces more accurate results, according to the method of analysis, than the constant temperature distribution (Chapter 5). However, the use of the constant temperature distribution gives reasonable results. However, assuming that the temperature remains constant over the same period of time that the cycling is assumed to take place, still gives a reasonable approximation to the more refined solution.

8. A simply supported prestressed skew deck behaves in a manner similar to continuous prestressed decks with respect to the creep-temperature mechanism.
The effect of creep on the temperature induced stresses produces a reverse effect when the temperature gradient is removed as will occur during the daily cycle. Thus, since the above results indicate a large proportion of the temperature stresses are nullified by the creep process, the reversal effect will approximate to a reversal of the temperature gradient. If it is assumed that the creep takes place over relatively short periods of time, during the construction period for example, it can be concluded that the stress effects of a temperature gradient are opposite in sign and approximately equal in magnitude to the stresses calculated by conventional methods.

The effect of the above conclusion on the design philosophy of prestressed concrete bridges will be considered below.

It could be suggested that the conventional theory provides a tried and tested means of providing satisfactory designs even if the stresses are being calculated incorrectly. There is some validity in this view but if it is adopted there is then no possibility of advance in design methods and significant improvement in design. It also overlooks the fact that it cannot be assumed that severe maintenance problems will not develop in the present generation of designs.

A reasonable compromise would be to apply the results of creep-temperature theory to all major bridge structures which would require an independent (category 3) check by the Department of Transport but not to simpler structures.

In doing so however, it would be logical to review the limiting tensile and compressive stresses since existing designs are based on incorrect stresses.

The future work needed in this field requires investigations on actual highway structures to obtain quantitative values of stresses, crack sizes and fatigue effects although the latter can probably only be determined from specially simulated test structures. It must be emphasised that progressing understanding and applying the creep-temperature phenomenon further requires consideration of all other aspects of the behaviour of concrete structures at the same time.
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