Multiuser Detection for Mobile CDMA Systems

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Summary

The goal for the third generation (3G) of mobile communications system is to seamlessly integrate a wide variety of communication services such as high-speed data, video and multimedia traffic as well as voice signals for transmission on a Wideband Code Division Multiple Access (WCDMA) air interface.

CDMA suffers from interference and in this thesis multiuser detection for the mobile uplink has been considered. A thorough comparative study for different multiuser detection methods is done. RAKE-IC as an architecture for mixing the ideas of RAKE receiver, and parallel Interference Cancellation, are introduced. The basic concept is to maximize the signal to noise ratio of all users in the system by using adaptive algorithms. The structure of RAKE-IC has been extended to multi-stages and several adaptive algorithms are implemented.

An iterative method for interference cancellation has been considered and its convergence issue has been analytically studied. An improvement in convergence using the Rayleigh-Ritz theorem is proposed which in consequence increases the convergence speed in synchronous scenarios. Using analytical methods another improvement using the Gershgorin theorem has been proposed which does not impose a great complexity in the system, yet works well even in asynchronous environments.

A suboptimum search algorithm for correcting the reliable detected information has been introduced with the property that its structure can be combined well with the iterative detectors. This combination achieves a better performance than partial parallel interference cancellation method even in rather low interference regions of operation. The structure of the sub-optimum search algorithm has been extended to multiple stages and its performance in terms of bit error rate has been analytically derived in closed form that shows good agreement with the simulation results.

Considering the power profile of the users and by sacrificing a little performance, the suboptimum search structure has been further simplified.
Dedicated To

My family
Acknowledgments

I thank my supervisors without whose help this thesis could not have been finished.
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<th>Abbreviation</th>
<th>Definition</th>
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<tbody>
<tr>
<td>AMD</td>
<td>Adaptive Multiuser Detection</td>
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<tr>
<td>AMPS</td>
<td>Advanced Mobile Phone System</td>
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<tr>
<td>ARIB</td>
<td>Japan’s Association of Radio Industry and Business</td>
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<tr>
<td>ATIS</td>
<td>Alliance for Telecommunications Industry Solutions</td>
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<td>AWGN</td>
<td>additive white Gaussian noise channel</td>
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<td>BAMUD</td>
<td>Blind adaptive multiuser detection</td>
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<tr>
<td>BER</td>
<td>Bit Error Ratio</td>
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<td>BPSK</td>
<td>Binary Phase Shift Keying</td>
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<td>BS</td>
<td>Base Station</td>
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<tr>
<td>CD</td>
<td>Conventional Detector</td>
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<tr>
<td>CDMA</td>
<td>Code Division Multiple Access</td>
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<tr>
<td>CSMA</td>
<td>Carrier Sense Multiple Access</td>
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<tr>
<td>CWTS</td>
<td>China Wireless Telecommunication Standard Group</td>
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<tr>
<td>DCH</td>
<td>Dedicated Channel, which is mapped into Dedicated Physical Channel</td>
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<tr>
<td>Dec</td>
<td>Decorrelator</td>
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<tr>
<td>DL</td>
<td>Down Link (forward link)</td>
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<tr>
<td>Dm</td>
<td>Decorrelating Multipath</td>
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<td>DPCCH</td>
<td>Dedicated Physical Control Channel</td>
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<td>DPCH</td>
<td>Dedicated Physical Channel</td>
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<tr>
<td>DPDCH</td>
<td>Dedicated Physical Data Channel</td>
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<tr>
<td>DS-CDMA</td>
<td>Direct-Sequence Code Division Multiple Access</td>
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<td>DSP</td>
<td>Digital Signal Processor</td>
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<td>EGC</td>
<td>Equal Gain Combining</td>
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<td>ETSI</td>
<td>European Telecommunications Standards Institute</td>
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<tr>
<td>FBI</td>
<td>Feedback Information</td>
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<td>FDD</td>
<td>Frequency Division Duplex</td>
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<td>FDMA</td>
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<td>FEC</td>
<td>Forward Error Correction</td>
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<td>FGLMS</td>
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<tr>
<td>FM</td>
<td>Frequency Modulation</td>
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<td>GSM</td>
<td>Global System for Mobile Communications</td>
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<tr>
<td>IC</td>
<td>Interference Cancellation</td>
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<tr>
<td>IF</td>
<td>Intermediate Frequency</td>
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<td>IMT-2000</td>
<td>International Mobile Telecommunication-2000</td>
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<tr>
<td>ISI</td>
<td>Inter symbol interference</td>
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<td>ITU</td>
<td>International Telecommunication Union</td>
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<tr>
<td>LFSR</td>
<td>Linear Feedback Shift Register</td>
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<tr>
<td>LMS</td>
<td>Least Mean Squares</td>
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<tr>
<td>LOS</td>
<td>Line of Sight</td>
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<tr>
<td>MAI</td>
<td>Multi Access Interference</td>
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<tr>
<td>Mcps</td>
<td>Mega Chip Per Second</td>
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<tr>
<td>mD</td>
<td>Multipath Decorrelating detector</td>
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<tr>
<td>MIMO</td>
<td>Multiple input multiple output</td>
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<tr>
<td>ML</td>
<td>Maximum Likelihood</td>
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<td>MLSR</td>
<td>Maximum Length Shift Register</td>
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<tr>
<td>MMSE</td>
<td>Minimum Mean Squared Error</td>
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<td>MOE</td>
<td>Minimum Output Energy</td>
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<td>MRC</td>
<td>Maximal Ratio Combining</td>
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<td>MS</td>
<td>Mobile Station</td>
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<tr>
<td>MSE</td>
<td>Mean Squared Error</td>
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<td>MUD</td>
<td>Multi User Detection</td>
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<tr>
<td>NAMPS</td>
<td>Narrowband AMPS</td>
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<tr>
<td>NLOS</td>
<td>Non Line of Sight</td>
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<tr>
<td>NMT-900</td>
<td>Nordic Mobile Telephone System</td>
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<tr>
<td>OVSF</td>
<td>Orthogonal Variable Spreading Factor (codes)</td>
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<tr>
<td>PCN</td>
<td>Public Communication Network</td>
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<tr>
<td>PDC</td>
<td>Pacific Digital Cellular</td>
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<tr>
<td>PE</td>
<td>Polynomial Expansion</td>
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<tr>
<td>PFGLMS</td>
<td>Partially Filtered Gradient LMS</td>
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<td>PIC</td>
<td>Parallel Interference Cancellation</td>
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<td>PPIC</td>
<td>Partial Parallel Interference Cancellation</td>
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<tr>
<td>psd</td>
<td>Power spectral density</td>
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<tr>
<td>PSTN</td>
<td>Public Switched Telephone Network</td>
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<tr>
<td>QoS</td>
<td>Quality of Service</td>
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<tr>
<td>QPSK</td>
<td>Quadrature Phase Shift Keying</td>
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<tr>
<td>RACH</td>
<td>Random Access Channel</td>
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<tr>
<td>RF</td>
<td>Radio Frequency</td>
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<td>RLS</td>
<td>Recursive Least Squares</td>
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<tr>
<td>RRC</td>
<td>Root-Raised Cosine</td>
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<tr>
<td>RX</td>
<td>Receive</td>
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<td>SDR</td>
<td>Software Defined Radio</td>
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<td>SF</td>
<td>Spreading Factor</td>
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<td>SIC</td>
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</tr>
<tr>
<td>SIMO</td>
<td>Single input single output</td>
<td></td>
</tr>
<tr>
<td>SIR</td>
<td>Signal to Interference ratio</td>
<td></td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
<td></td>
</tr>
<tr>
<td>S-PCN</td>
<td>Satellite PCN</td>
<td></td>
</tr>
<tr>
<td>TACS</td>
<td>Total Access Cellular System</td>
<td></td>
</tr>
<tr>
<td>TDD</td>
<td>Time Division Duplexing</td>
<td></td>
</tr>
<tr>
<td>TDMA</td>
<td>Time Division Multiple Access</td>
<td></td>
</tr>
<tr>
<td>TFCI</td>
<td>Transport format combination indicator</td>
<td></td>
</tr>
<tr>
<td>TPC</td>
<td>Transmit power control</td>
<td></td>
</tr>
<tr>
<td>TTI</td>
<td>Transmission Time Interval</td>
<td></td>
</tr>
<tr>
<td>TX</td>
<td>Transmit</td>
<td></td>
</tr>
<tr>
<td>UE</td>
<td>User Equipment</td>
<td></td>
</tr>
<tr>
<td>UL</td>
<td>Up Link (reverse link)</td>
<td></td>
</tr>
<tr>
<td>UMTS</td>
<td>Universal Mobile Telecommunication System</td>
<td></td>
</tr>
<tr>
<td>USDC</td>
<td>United States Digital Cellular</td>
<td></td>
</tr>
<tr>
<td>UT</td>
<td>User Terminal</td>
<td></td>
</tr>
<tr>
<td>UTRA</td>
<td>Universal Terrestrial Radio Access</td>
<td></td>
</tr>
<tr>
<td>UTRAN</td>
<td>UMTS Terrestrial Radio Access Network</td>
<td></td>
</tr>
<tr>
<td>WARC</td>
<td>World Administrative Radio Conference</td>
<td></td>
</tr>
<tr>
<td>WCDMA</td>
<td>Wideband Code Division Multiple Access</td>
<td></td>
</tr>
<tr>
<td>WGN</td>
<td>White Gaussian noise</td>
<td></td>
</tr>
<tr>
<td>ZF-DF</td>
<td>Zero-Forcing Decision-Feedback</td>
<td></td>
</tr>
</tbody>
</table>
ZF-LE Zero-Forcing Linear Equalizer
List of Mathematical Symbols

A  A matrix containing the amplitude of users

$A_k$  Amplitude of user $k^{th}$

$\hat{A}_k$  Estimated amplitude of user $k^{th}$

b  Transmitted vector of bits

$(m)B_n$  A sub-matrix inside $(m)U$

$b_{c(k)}$  Control bits transmitted by user $k$ in WCDMA uplink

$b_{d(k)}$  Data bits transmitted by user $k$ in WCDMA uplink

$\hat{e}_{k,i}^{(s)}$  $i^{th}$ bit estimate for user $k$ at stage $s$ (of processing in multistage PPIC)

C  Set of complex numbers

c_{(MMSE),k}  Spreading sequence of MMSE detector for user $k$

c_c  Spreading code of control channel

c_{ch, SF, m}  Channelisation codes, with spreading factor SF and code number m

c_d  Spreading code of data channel

c_{dn}  Coefficients of sinusoid terms for generating Rayleigh process

c_{i(k)}^{(s)}  Upsampled $i^{th}$ Walsh code for user $k$

$C_k^{(s)}$  Partial coefficient of user $k$ in stage $s$ of PPIC detector

c_n  A vector representing the filter coefficients at the $n^{th}$ iteration in LMS

c_{sc(k)}  Scrambling sequence for user $k$

d_k  Effective spreading code for user $k$ for decorrelator

E_b  Average energy per bit

E_c  Average energy per PN chip

$e_k(\sigma)$  Effective energy of user $k^{th}$

$f_{d, max}$  Maximum Doppler frequency

$f_{dn}$  Frequencies of sinusoids for generating Rayleigh process

G  Iteration matrix

g  Gradient vector

$(m)h_{n,(k)}$  Channel impulse response during $n^{th}$ control bit at antenna-$m$ for user $k$

I  Identity matrix

$I_0(.)$  Modified Bessel function of first kind and zeroth order
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_{\text{min}}$</td>
<td>MSE for the optimal filter,</td>
</tr>
<tr>
<td>$J_n$</td>
<td>The RLS error metric</td>
</tr>
<tr>
<td>$J_n^{\text{excess}}$</td>
<td>Excess MSE that results from non-optimal correlator coefficients</td>
</tr>
<tr>
<td>$K$</td>
<td>Number of users</td>
</tr>
<tr>
<td>$k$</td>
<td>User index</td>
</tr>
<tr>
<td>$k_R$</td>
<td>Rice factor</td>
</tr>
<tr>
<td>$l$</td>
<td>Path index (in multipath fading)</td>
</tr>
<tr>
<td>$L_k$</td>
<td>Number of resolvable multipath components for user $k$</td>
</tr>
<tr>
<td>$M_{(k)}^{(k)}$</td>
<td>Number of codes transmitted by user $k^{th}$ (WCDMA uplink)</td>
</tr>
<tr>
<td>$\mathbf{n}$</td>
<td>Noise vector</td>
</tr>
<tr>
<td>$N$</td>
<td>Spreading factor of users in the system, when only one spreading code is used by each user</td>
</tr>
<tr>
<td>$^{(m)}\mathbf{n}$</td>
<td>Noise vector in the antenna $m$</td>
</tr>
<tr>
<td>$n(t)$</td>
<td>Noise signal</td>
</tr>
<tr>
<td>$N_{c(k)}$</td>
<td>Number of bits in one slot, transmitted in control channel for user $k$</td>
</tr>
<tr>
<td>$N_{\text{Cor}}$</td>
<td>Correlation length</td>
</tr>
<tr>
<td>$N_{d(k)}$</td>
<td>Number of bits in one slot, transmitted in data channel for user $k$</td>
</tr>
<tr>
<td>$N_R$</td>
<td>Number of sinusoid terms for generating Rayleigh process</td>
</tr>
<tr>
<td>$N_0$</td>
<td>One-side power spectral density of noise</td>
</tr>
<tr>
<td>$p$</td>
<td>The probability of error in one bit for matched filter</td>
</tr>
<tr>
<td>$P(\sigma)$</td>
<td>Probability of error</td>
</tr>
<tr>
<td>$P_{\text{Optimum}}(\sigma)$</td>
<td>The probability of error for optimum detector</td>
</tr>
<tr>
<td>$P_k$</td>
<td>The power of user $k$</td>
</tr>
<tr>
<td>$P_k(\sigma)$</td>
<td>Probability of error for user $k$</td>
</tr>
<tr>
<td>$P_{k,m}$</td>
<td>The power of user $k$ at antenna $m$</td>
</tr>
<tr>
<td>$P_{\text{MF}}(\sigma)$</td>
<td>Error probability of matched filtering.</td>
</tr>
<tr>
<td>$P_{\text{PIC,}(s)}(\sigma)$</td>
<td>Error probability of PIC detector at stage $s$</td>
</tr>
<tr>
<td>$P_r$</td>
<td>Power of the input signal</td>
</tr>
<tr>
<td>$p_{\text{Ray}}(r)$</td>
<td>Rayleigh pdf</td>
</tr>
<tr>
<td>$p_{\text{Rice}}(r)$</td>
<td>Rice pdf</td>
</tr>
<tr>
<td>$P_{\text{SO}}(\sigma)$</td>
<td>Error probability of Suboptimum search algorithm</td>
</tr>
<tr>
<td>$p_T(t)$</td>
<td>Unit pulse function of duration $T_b$ equal to the bit period</td>
</tr>
<tr>
<td>$Q(\cdot)$</td>
<td>$Q$ function</td>
</tr>
</tbody>
</table>
List of Mathematical Symbols

\( Q_c^{(k)} \)  
Spreading factor of control channel for user \( k \), (WCDMA uplink)

\( Q_d^{(k)} \)  
Spreading factor of data channel for user \( k \), (WCDMA uplink)

\( Q_i^{(k)} \)  
Spreading factor of user \( k \), for code of \( i \)th, (WCDMA uplink)

\( \mathbf{R} \)  
Correlation matrix

\( R \)  
Set of real numbers

\( (\mathbf{R}^{-1})_{ij} \)  
Element \((i,j)\) of matrix \( \mathbf{R}^{-1} \)

\( r(t) \)  
Received signal in receiver side

\( \hat{r}_{k,l}^{(s)}(t) \)  
Estimated received signal for \( t \)th path of user \( k \) at stage \( s \)

\( (m)r \)  
Received vector in the antenna \( m \)

\( \text{Re}(.) \)  
Real part of a complex number

\( R_{ij} \)  
Element \((i,j)\) of matrix \( \mathbf{R} \)

\( \text{sc}_k \)  
Scrambling code of user \( k \)

\( S_k \)  
Transmitted signal of user \( k \)

\( s_k \)  
Spreading sequence of user \( k \)

\( s_k(t) \)  
Signature waveform of user \( k \)

\( \hat{s}_{j,\lambda}^{(s)}(t) \)  
Estimated signal for path \( \lambda \) of user \( j \) at stage \( s \)

\( T \)  
Linear transformation to be applied at the output of matched filter

\( T \)  
Continues-time index

\( T_b \)  
Bit duration

\( T_c \)  
Chip duration

\( T_{\text{dec}} \)  
Linear transformation as an estimate for decorrelator

\( T_f \)  
Frame time (WCDMA uplink)

\( T_{\text{slot}} \)  
Slot time (WCDMA uplink)

\( \mathbf{U} \)  
Convolutional matrix

\( (m)\mathbf{U} \)  
Convolution matrix in the antenna \( m \)

\( w \)  
Vector of coefficients used in Taylor series expansion

\( W \)  
Length of the channel impulse response

\( w_i \)  
Coefficients of polynomial expansion detector

\( W_P \)  
Length of the window of observation to carry out the power estimation

\( y \)  
Output of matched filters

\( y_k \)  
Output of matched filter for user \( k \)

\( y_{k,l,i}^{(s+1)} \)  
Decision statistics of the \((s+1)\)th stage of the PIC for user \( k \), path \( l \), bit \( i \)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{k,l,i}$</td>
<td>Complex gain of the Rayleigh fading (user $k$, path $l$, bit $i$)</td>
</tr>
<tr>
<td>$\hat{\alpha}_{k,l,i}$</td>
<td>Estimate of the multipath attenuation coefficients</td>
</tr>
<tr>
<td>$\sigma_{\text{opt}}$</td>
<td>Optimum value of coefficient in Taylor series expansion</td>
</tr>
<tr>
<td>$\beta_c$ and $\beta_d$</td>
<td>Values for controlling the gain of data and control channels</td>
</tr>
<tr>
<td>$\beta_L$</td>
<td>Loading factor</td>
</tr>
<tr>
<td>$\delta(t)$</td>
<td>Dirac’s delta function</td>
</tr>
<tr>
<td>$\phi_i$</td>
<td>Orthonormal basis</td>
</tr>
<tr>
<td>$\eta_k$</td>
<td>Asymptotic efficiency</td>
</tr>
<tr>
<td>$\bar{\eta}_k$</td>
<td>Near-far resistance</td>
</tr>
<tr>
<td>$\lambda_{\text{max}}$</td>
<td>Maximum value of eigen value of a matrix</td>
</tr>
<tr>
<td>$\lambda_{\text{min}}$</td>
<td>Minimum eigen value of a matrix</td>
</tr>
<tr>
<td>$\theta_d$</td>
<td>Phases of sinusoids for generating Rayleigh process</td>
</tr>
<tr>
<td>$\theta_k$</td>
<td>Received phase of user $k$</td>
</tr>
<tr>
<td>$\theta_{k,\lambda}$</td>
<td>Carrier phase of user $k$ and multipath $\lambda$</td>
</tr>
<tr>
<td>$\rho_{ij}$</td>
<td>Partial cross-correlations between the spreading factors of users $i$ and $j$</td>
</tr>
<tr>
<td>$\rho_s(G)$</td>
<td>Spectral radius of matrix $G$</td>
</tr>
<tr>
<td>$\rho_{\text{sw}}$</td>
<td>Optimum value of spectral radius in Taylor series expansion</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Standard deviation of Gaussian noise</td>
</tr>
<tr>
<td>$\eta_k$</td>
<td>Delay of user $k$</td>
</tr>
<tr>
<td>$\eta_{k,l}$</td>
<td>Delay of $l^{th}$ multipath component of user $k$</td>
</tr>
<tr>
<td>$\eta_{k,l,m}$</td>
<td>Delay of $l^{th}$ multipath component of user $k$ for antenna $m$</td>
</tr>
<tr>
<td>$\tau_l$</td>
<td>Delay of path $l$ (in multipath fading)</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Number of samples per chip</td>
</tr>
<tr>
<td>$(\cdot)^H$</td>
<td>Transpose-conjugate operation on a matrix</td>
</tr>
<tr>
<td>$(\cdot)^T$</td>
<td>Transpose operation on a matrix</td>
</tr>
<tr>
<td>$\Omega(.)$</td>
<td>Optimum detector’s criteria</td>
</tr>
<tr>
<td>$\nabla$</td>
<td>Gradient operator</td>
</tr>
</tbody>
</table>
Chapter 1  Introduction

1.1 Motivation

CDMA is already in use or is being proposed for a large number of communication systems. The third generation mobile systems are designed to provide voice or low to medium data rate services (in vehicular environments up to 144 kbps, outdoor to indoor and pedestrian environments up to 384 kbps, and in indoor offices up to 2 Mbps) [HT02]. On the other hand, future mobile satellite broadband systems aim at providing multimedia services with data rates up to a few Mbps to a large number of users. The requirements of these systems are therefore significantly different from those of already existing systems and a number of issues have to be investigated in order to derive the best possible air interface performance.

In CDMA communication systems, all the users share the same time and frequency channel. Every user is assigned a particular waveform to modulate the information data. Since the different signature waveforms are never fully orthogonal, CDMA systems will suffer from multiple access interference (MAI) caused by the users transmitting in the system. The level of MAI is an important factor in the determination of the system capacity. Assuming a given (Quality of Service) QoS, the reduction of the MAI level will result in an increase in the system capacity. Moreover, it is important to note that the capacity of future broadband satellite systems will need to be significantly higher than that
of current S-PCN. Hence, keeping control of the MAI is of utmost importance for future systems.

In this study, a technique aimed at reducing the MAI is considered. This approach, named (Multi User Detection) MUD, is based on complex signal processing algorithms at the receiver that use information of the MAI characteristics.

1.2 Thesis Organization

This thesis is organized in the following manner:

• Chapter two provides a broad overview of CDMA systems and addresses the interference issue. Channel characteristics and the structure of conventional detectors are discussed.

• Chapter three reviews previous work in multiuser detection and considers a comparative view of different schemes. The primary goal is to highlight the gains that can be achieved by using more sophisticated signal processing algorithms at the receiver in a wideband CDMA application.

• Chapter four looks at Partial Parallel Interference Cancellation (PPIC) and introduces an architecture for mixing the ideas of the RAKE receiver and PPIC. The basic concept – RAKE-IC – is aimed at maximizing the signal to noise ratio of all users in the system by using adaptive algorithms. Advantages and limitations of this approach are considered.

• Chapter five looks at iterative linear detectors. An iterative algorithm was introduced and its convergence issues have been considered. By mathematical analysis a more accurate, yet simple condition has been derived which performs well in synchronous scenarios. However its performance is not acceptable in asynchronous systems. By further mathematical analysis and using the Gershgorin theorem in linear algebra, an acceptable performance for CDMA systems close to the decorrelator and better than the Rayleigh-Ritz theorem has been achieved.

• Chapter six focuses on improving the performance of iterative algorithms. The idea is to use the optimum criteria from a reliable starting point. Starting from reliable detected information can reduce the number of search steps. A suboptimum search algorithm has been introduced and its performance in a simple scenario has been analytically investigated. Its structure is well suited to the iterative algorithm introduced in chapter five and has a moderate complexity. By sacrificing some performance, its complexity can be further reduced. Using the information of users’ powers leads to a less complex architecture.
• In chapter 7, the main findings of the thesis are summarised and conclusions are drawn. Areas of future work are also proposed.

1.3 Novel Works undertaken and major achievements

The list of original work presented in this thesis is:

1. RAKE-IC as an architecture for reducing interference is introduced. The basic concept is to maximize the signal to noise ratio of all users in the system by using adaptive algorithms. RAKE-IC has an architecture that makes it possible to operate in multiple stages. Several adaptive algorithms have been considered, as well.

2. An iterative method for Interference cancellation has been considered and its convergence issues have been analytically considered. An improvement using Rayleigh-Ritz theorem is proposed and the performance of the system is satisfying in the synchronous scenarios.

3. Using an analytical approach another improvement using Gershgorin theorem has been implemented. This method does not impose a great complexity in the system yet improves the performance even in asynchronous environments.

4. A suboptimum search algorithm for correcting the detected information has been introduced with its structure being well suited with the detector introduced in item 3 and collaboratively achieves a performance better than the partial parallel interference cancellation even in low Eb/N0 regions.

5. The structure of the suboptimum search algorithm has been extended to multiple steps.

6. The performance of this new suboptimum search algorithm has been analytically derived in closed form and is in agreement with the simulation results.

7. Considering the power profile of the users and by sacrificing a small amount of performance, its structure has been further simplified. Different combinations of mixing the users’ power information and search algorithm are considered.
Chapter 2 CDMA Systems and Interference

2.1 Introduction

The goal for the third generation of mobile communications system is to seamlessly provide a wide variety of communication services to anybody, anywhere, anytime. The intended provision for next generation mobile phone users includes services such as high-speed data, video and multimedia traffic as well as voice signals. The technology needed to tackle the challenges to make these services available is popularly known as the Third Generation (3G) Cellular Systems. The first generation systems are represented by the analog mobile systems designed to carry voice application traffic. Their subsequent digital counterparts are known as second-generation cellular systems. Third generation systems mark a significant leap, both in applications and capacity, from the current second generation standards. Whereas the current digital mobile phone systems are optimized for voice communications, 3G communicators are oriented towards multimedia message capability.

Code Division Multiple Access (CDMA) has been chosen as the air interface access scheme for the third generation standards of mobile telecommunications systems. CDMA is an attractive solution for wireless communications and there is a significant interest in the design of wireless CDMA networks to enable the users to have access to different data rates. However, CDMA systems suffer from the presence of MAI. In fact all users in a way
interfere with each other [Ver98]. In this chapter there will be a brief review of different
generations of the mobile cellular systems and of course the main emphasis will be on the
3rd generation. Communication channels and fading environments with the mathematical
generation will be presented.
Then the features of CDMA systems will be considered and the causes of MAI will be
analytically examined. This will be through examination of the conventional detector in the
both synchronous and asynchronous scenarios.

2.2 Generations of Cellular Systems

2.2.1 First Generation Cellular Systems

The first generation cellular systems generally employed analog Frequency Modulation
(FM) techniques. The Advanced Mobile Phone System (AMPS) is the most notable of the
first generation systems. AMPS was developed by the Bell Telephone System. It uses FM
technology for voice transmission and digital signaling for control information. Other first
generation systems include:
• Narrowband AMPS (NAMPS)
• Total Access Cellular System (TACS)
• Nordic Mobile Telephone System (NMT-900)
All the first generation cellular systems employ Frequency Division Multiple Access
(FDMA) with each channel assigned to a unique frequency band within a cluster of cells.

2.2.2 Second Generation Cellular Systems

The rapid growth in the number of subscribers and the proliferation of many incompatible
first generation systems were the main reason behind the evolution towards second
generation cellular systems. Second generation systems take advantage of compression and
coding techniques associated with digital technology. All the second generation systems
employ digital modulation schemes. Multiple access techniques like Time Division
Multiple Access (TDMA) and Code Division Multiple Access (CDMA) are used along
with FDMA in the second-generation systems. Second generation cellular systems include:
• United States Digital Cellular (USDC) standards IS-54 (TDMA) and IS-136 (TDMA)
• Global System for Mobile communications (GSM)
• Pacific Digital Cellular (PDC)
• cdmaOne (IS-95).

2.2.3 Third Generation Cellular Systems

Work to develop third generation mobile systems started when the World Administrative Radio Conference (WARC) of the ITU (International Telecommunications union), at its 1992 meeting, identified the frequencies around 2 GHz that were available for use by future third generation mobile systems, both terrestrial and satellite. Within the ITU these third generation systems are called International Mobile Telephony 2000 (IMT-2000). Within the IMT-2000 framework, several different air interfaces are defined for third generation systems, based on either CDMA or TDMA technology. The UTRA FDD (WCDMA) and cdma2000 are part of the CDMA interface, as CDMA Direct Spread and CDMA Multi-Carrier respectively. UWC-136 and DECT are part of the TDMA-based interface in the concept, as TDMA Single Carrier and TDMA Multi-Carrier respectively. The TDD part in CDMA consists of UTRA TDD from 3GPP and TD-SCDMA from CWTS (the China Wireless Telecommunication Standard Group). For the FDD part in CDMA interface, the harmonization has been completed, and the harmonization process for the CDMA TDD modes within 3GPP resulted for the 1.28Mcps TDD completed 2001.

Third generation cellular systems are being designed to support wideband services such as high-speed Internet access, video and high quality image transmission with the same quality as the fixed networks. The primary requirements of the next generation cellular systems are [HT02]:

• Voice quality comparable to Public Switched Telephone Network (PSTN)
• Support of high data rate (Table 2-1 shows the data rate requirement of the 3G systems)
• Support of both packet-switched and circuit-switched data services
• More efficient usage of the available radio spectrum
• Support of a wide variety of mobile equipment
• Backward compatibility with pre-existing networks and flexible introduction of new services and technology
• An adaptive radio interface suited to the highly asymmetric nature of most Internet communications: i.e., a much greater bandwidth for the downlink than the uplink. Research efforts have been underway for more than a decade to introduce multimedia capabilities into mobile communications.
2.3 The Multiple access Channel

Multiple access is the basis of how the common transmission medium is shared between users. In multi-access communications, several transmitters share a common channel. Usually, the superposition of signals sent by different transmitters occurs unintentionally.

The basic multiple access schemes are: frequency division multiple access (FDMA), time division multiple access (TDMA) and code division multiple access (CDMA).

In FDMA, the total available frequency is divided into frequency channels that are allocated to users. In TDMA, each frequency channel is divided into time slots and each user is allocated a time slot. In CDMA, multiple access is done by assigning each user a pseudo-random code with good correlation properties. These codes change the original narrow band signal to wideband spread spectrum signal. These three schemes are shown in Figure 2-1 [OP98, PO98].

![Figure 2-1 Multiple access methods](image-url)
2.3.1 FDMA and TDMA Systems

Frequency Division Multiple Access (FDMA) assigns a different carrier frequency to each user so that the resulting spectra do not overlap. Band-pass filtering enables separate demodulation of each channel.

In Time Division Multiple Access (TDMA), time is partitioned into slots assigned to each incoming digital stream in round-robin fashion [Pra96]. Demultiplexing is carried out by simply switching on to the received signal at the appropriate epochs. Time division can be used not only to multiplex collocated message sources but also can be used by geographically separated users who have the ability to maintain time-synchronization, in what is commonly referred to as TDMA. It should be noted that FDMA allows completely uncoordinated transmissions in the time domain, and in consequence there is no need to establish time-synchronization among the users. This advantage is not shared by TDMA where all transmitters and receivers must have access to a common clock.

The important feature of frequency-division and time-division multiple access techniques is that, the various users are operating in separate non-interfering channels. It will be paraphrased in the signal-space language of digital communications by saying that: those multi-access techniques operate by ensuring that the signals transmitted by the various users are mutually orthogonal. Channel or receiver non-ideal effects may require the insertion of guard times in TDMA and spectral guard bands to avoid co-channel interference.

Using multiaccess methods that adhere to the principle of dividing the channel into independent noninterfering subchannels can waste channel resources when the number of potential users is much greater than the number of simultaneously active users at any given time. If each user were assigned a fixed radio frequency channel, only a tiny fraction of the spectrum would be utilized at any given time. Analogously, in TDMA most of the time slots would be empty, at any given time.

2.3.2 Random Multiple access

Random multiaccess communications is one of the approaches to dynamic channel sharing. When a user has a message to transmit he goes ahead and transmits it as if it were the sole user of the channel. If indeed nobody else is transmitting simultaneously, then the message is received successfully. However, the users are uncoordinated and the possibility always exists that the message will interfere (in time and frequency) with another transmission. In
such a case, it is typically assumed that the receiver cannot reliably demodulate several simultaneous messages. The only alternative is to notify the transmitters that a collision has happened and, thus, their messages have to be retransmitted. Collisions would reoccur forever if, upon notification of a collision, the transmitters involved were to retransmit immediately (or after a similar delay). To overcome this, users wait a random period of time before retransmitting.

The first random multiaccess communication system was the ALOHA system proposed for a radio channel in 1969. Some coaxial cable local area networks, typified by the widely used Ethernet, employ a polite version of ALOHA, called Carrier Sense Multiple Access (CSMA), where users listen to the channel before transmitting so as not to collide with an ongoing transmission.

2.3.3 CDMA Systems

In Code Division Multiple Access (CDMA) systems, different codes or “signatures” are assigned to different users. These signature waveforms must have special correlation properties. In real systems, by removing the restriction of being orthogonal from the signature waveforms, it enables the CDMA to be an attractive solution for many multiuser communication systems:

- The users can be asynchronous, that is, their time epochs need not be aligned.
- Sharing of channel resources is inherently dynamic: reliability depends on the number of simultaneous users, rather than on the number of potential users of the system. Thus, unlike orthogonal multiaccess, it is possible to trade off reception quality for increased capacity. The capacity in TDMA and FDMA systems is hard but in CDMA systems this capacity is soft and it is possible to allow one extra user by sacrificing some of this quality for all users.

2.3.3.1 Features of CDMA

- **Lower sensitivity to interference and jamming:** Since the signals transmitted by the different users are spread, it is possible for CDMA systems to operate with a $C/I$ ratio much lower than narrowband systems. Hence, CDMA systems will have a lower sensitivity to interference. Spreading also means that CDMA systems are inherently more robust against jamming.
• **Frequency reuse:** Due to their lower sensitivity to interference, CDMA systems can be implemented with a frequency reuse factor of one. The use of the same frequency in each cell creates interference, but thanks to the spreading operation the random-like self-noise from the adjacent beams can be maintained below an acceptable level. By so doing, the overall frequency resource management is eased.

• **Soft handover:** A CDMA system supports several types of handover, including hard handover, soft handover and softer handover. A mobile station in hard handover switches from one base station to another base station via a brief interruption of the traffic channel. Soft handover is a technique in which a mobile station, while moving between one cell and its neighbouring cells, simultaneously transmits and receives the same signal from several base stations. On the uplink, the mobile switching centre can decide which base station is receiving the strongest signal. In softer handover, neighbouring sectors of the same cell support the mobile station's call. Proper use of soft and softer handover can enhance call quality, improving cell coverage and capacity.

• **Diversity:** In terrestrial mobile communication systems, the signals corresponding to the different multipath echoes can be combined in order to mitigate the fading effect. In the satellite environment, the delay spread is usually lower than the signalling duration and the multipath fading components cannot be resolved at the receiver. However, when more than one satellite is visible from the user terminal (UT), it is possible to receive replicas of the transmitted signals through different propagation paths. This helps combat the effect of both shadowing and multipath fading.

• **Soft Capacity:** For TDMA systems, the capacity directly depends on the number of time slots. On the other hand, in CDMA systems, the capacity is dictated primarily by the level of interference that can be supported while providing the required QoS. The system requirements are therefore calculated for a given traffic distribution over the coverage area. CDMA is flexible in accommodating variations in the traffic distribution which corresponds to a given interference distribution scenario. Moreover, when the number of users in the system increases, the level of interference increases gracefully and it is possible to accept more users than the nominal capacity by trading-off the QoS. This can prove interesting when the traffic requirements peak for a very short period and one does not want to drop calls.
Finally, it should be stressed that there still is a hard limit on the capacity of CDMA systems, which is set by the number of available spreading sequences. Hence the extent to which soft capacity can prove useful depends on the set of spreading sequences that is chosen.

2.4 WCDMA, Air Interface for 3G

Looking back to the recent history of 3G, one can see that the approach used for 3G was to combine a Wideband CDMA (WCDMA) air interface with the fixed network of GSM. Several proposals supporting WCDMA were submitted to the International Telecommunication Union (ITU) and its International Mobile Telecommunications for the year 2000 (IMT2000) initiative for 3G. Among several organizations trying to merge their various WCDMA proposals were:

- Japan’s Association of Radio Industry and Business (ARIB)
- Alliance for Telecommunications Industry Solutions (ATIS)
- TIP1
- European Telecommunications Standards Institute (ETSI) through its Special Mobile Group (SMG)

All of these schemes tried to take advantage of the WCDMA radio techniques without ignoring the numerous advantages of the already existing GSM networks. The standard that has emerged is based on ETSI’s Universal Mobile Telecommunication System (UMTS) and is commonly known as UMTS Terrestrial Radio Access (UTRA) [HT02]. The access scheme for UTRA is Direct Sequence Code Division Multiple Access (DS-CDMA). The information is spread over a band of approximately 5 MHz. This wide bandwidth has given rise to the name Wideband CDMA or WCDMA. There are two different modes namely

- Frequency Division Duplex (FDD)
- Time Division Duplex (TDD)

Since different regions have different frequency allocation schemes, the capability to operate in either FDD or TDD mode allows for efficient utilization of the available spectrum. A brief definition of FDD and TDD modes is given next.

FDD: The uplink and downlink transmissions employ two separated frequency bands for this duplex method. A pair of frequency bands with specified separation is assigned for a connection.
TDD: In this duplex method, uplink and downlink transmissions are carried over the same frequency band by using synchronized time intervals, thus time slots in a physical channel are divided into transmission and reception part. We have developed a simulator for a WCDMA system operating in the FDD mode. Therefore the system description provided in the forthcoming chapters holds for the FDD mode only.

2.4.1 WCDMA Key Features

The key operational features of the WCDMA radio interface are listed below [PO98]:

- Support of high data rate transmission: 384 kbps with wide area coverage, 2 Mbps with local coverage.
- High service flexibility: support of multiple parallel variable rate services on each connection.
- Both Frequency Division Duplex (FDD) and Time Division Duplex (TDD)
- Built in support for future capacity and coverage enhancing technologies such as adaptive antennas, advanced receiver structures and transmitter diversity
- Support of inter frequency hand over and hand over to other systems, including hand over to GSM.
- Efficient packet access

2.4.2 WCDMA Key Technical Characteristics

The table 2.2 shows the key technical features of the WCDMA radio interface:

The chip rate may be extended to two or three times the standard 3.84 Mcps to accommodate for data rates higher than 2 Mbps. The 200 kHz carrier raster has been chosen to facilitate coexistence and interoperability with GSM.

<table>
<thead>
<tr>
<th>Multiple Access Scheme</th>
<th>DS-CDMA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duplex Scheme</td>
<td>FDD/TDD</td>
</tr>
<tr>
<td>Packet Access</td>
<td>Dual mode (Combined and dedicated channel)</td>
</tr>
<tr>
<td>Multirate/Variable rate scheme</td>
<td>Variable spreading factor and multi-code</td>
</tr>
<tr>
<td>Chip Rate</td>
<td>3.84 Mcps</td>
</tr>
<tr>
<td>Carrier Spacing</td>
<td>4.4-5.2 MHz (200 kHz carrier raster)</td>
</tr>
<tr>
<td>Frame Length</td>
<td>10 ms</td>
</tr>
</tbody>
</table>
Table 2-2 WCDMA Key Technical Characteristics

Apart from the features of CDMA discussed above, it should be mentioned that as with other CDMA systems it suffers from MAI. In the next section, based on a mathematical model, this problem is addressed.

## 2.5 CDMA Systems model

Transmitters in a CDMA system can either transmit information simultaneously or without any time alignment. Being synchronous or asynchronous changes the equations governing the system.

### 2.5.1 Synchronous CDMA Systems

In synchronous systems, it is assumed that different users send their data in a synchronous way, which means there is no relative delay between them. Using this assumption a basic synchronous CDMA system model for K users using BPSK modulation and sharing the same AWGN channel is as follows [Ver98]:

$$r(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + \sigma n(t), \quad t \in [0,T]$$

(2-1)

The notations introduced in the above formula are:

- $r(t)$ is the received signal at the input of the receiver.
- $T$ is the inverse of the data rate.
- $s_k(t)$ is the signature waveform assigned to the $k^{th}$ user, normalized to have unit energy:

$$\|s_k\|^2 = \int_0^T s_k^2(t) dt = 1.$$  

(2-2)

- The signature waveforms are assumed to be zero outside the interval $[0, T]$.
- $A_k$ is the received amplitude of the $k^{th}$ user’s signal. $A_k^2$ is referred to as the energy of the $k^{th}$ user.
• $b_k \in \{-1, +1\}$ is the bit transmitted by the user $k$th.
• $n(t)$ is white Gaussian noise with unit power spectral density.

Different demodulation strategies' performance depends on the signal-to-noise ratio $(A_k / \sigma)^2$ and on the similarities between the signature waveforms, quantified by their cross-correlations defined as

$$
\rho_{i,j} = \langle s_i, s_j \rangle = \int_0^T s_i(t) s_j(t) dt
$$

and according to the Cauchy-Schwartz inequality:

$$
|\rho_{i,j}| = |\langle s_i, s_j \rangle| \leq \|s_i\| \|s_j\| = 1
$$

The cross-correlation matrix $R = \{\rho_{i,j}\}$ has diagonal elements equal to 1 and is symmetric nonnegative definite, because for any $K$-vector $x = (x_1, x_2, \ldots, x_K)^T$

$$
x^T Rx = \left( \sum_{k=1}^K x_k s_k^* \right)^2 \geq 0
$$

Therefore the cross-correlation matrix $R$ is positive definite if and only if the signature waveforms are linearly independent.

### 2.5.2 Asynchronous CDMA Systems

In asynchronous systems [Ver98] the bit epochs are not aligned to each other, thus it is necessary to introduce a delay for each user in the receiver. In this case the received signal is:

$$
r(t) = \sum_{k=1}^K \sum_{i=-M_k}^{M_k} A_k b_k[i] s_k(t - iT_k - \tau_k) + \sigma n(t), \quad \tau_k \in [0, T]
$$

It is assumed that each user sends $N_b = 2^*M_k + 1$ bits. Also, a new definition of the cross-correlations is needed. Considering users $k$ and $l$ in the system and assuming $(k < l)$, partial cross-correlations are defined as follows:
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\[ \rho_{k,l}(\tau) = \int_{-\infty}^{\infty} s_k(t) s_l(t - \tau) \, dt \]  \hspace{1cm} (2-7a)  

\[ \rho_{l,k}(\tau) = \int_{0}^{T} s_k(t) s_l(t + T - \tau) \, dt \]  \hspace{1cm} (2-7b)  

For example in the case of two users \((K=2)\) that each sends only 3 bits \((N_b=3)\), the correlation matrix has a size of \(KN_b \times KN_b\) (here \(6 \times 6\)). We can assume that there are \(KN_b\) (here 6) users are active in the system. Sorting all bits \((KN_b\) bits) according to their delays and considering each bit as a user, then according to their delays and using Eqs.(2-7a, 2-7b), the correlation matrix becomes [Mos96]:

\[
R(\tau) = \begin{pmatrix}
1 & \rho_{2,1} & 0 & 0 & 0 & 0 \\
\rho_{1,2} & 1 & \rho_{3,2} & 0 & 0 & 0 \\
0 & \rho_{2,3} & 1 & \rho_{4,3} & 0 & 0 \\
0 & 0 & \rho_{3,4} & 1 & \rho_{5,4} & 0 \\
0 & 0 & 0 & \rho_{4,5} & 1 & \rho_{6,5} \\
0 & 0 & 0 & 0 & \rho_{5,6} & 1 \\
\end{pmatrix}  \hspace{1cm} (2-8)  
\]

A more general model for Eq.(2-8) will be given in chapter 5.

Asynchronous channel design is much simpler than synchronous, because it doesn’t need an external synchronization system. However, it degrades the performance of the system and from the viewpoint of multiuser detection of signals, it becomes much more complex. For example size of the cross-correlation matrix is \(M\) times bigger than the synchronous case.

2.6 Signature Waveforms

Different spreading sequences can be used in CDMA systems [LM98, Taf02]:

**Walsh Codes**- Walsh codes are orthogonal codes and can be obtained from the following formulas [LM98]:

\[
H_{2^n} = \begin{bmatrix} H_{2^{n-1}} & H_{2^{n-1}} \\
H_{2^{n-1}} & \overline{H}_{2^{n-1}} \end{bmatrix}, \quad H_1 = [0]  \hspace{1cm} (2-9)  
\]

However, in asynchronous systems they no longer remain orthogonal.

**Pseudo-Noise Codes (PN Codes)**- PN codes are Pseudorandom sequences with the following features:

- Noise-like characteristics
  - Sharp Autocorrelation
  - Very small Cross-correlation
- Easy to implement
- Periodic and long

M-sequences and Gold codes are two examples of these codes. M-sequences are called Maximal Length Sequences because they have the maximum possible period (N). They can be produced by an n-stage Linear Feedback Shift Register (LFSR) circuit. M-sequences with very long periods are used in CDMA Mobile systems. In these cases, a different portion of the same M-sequence spreads each bit. Figure 2.2 shows the generation of m-sequences and Gold codes [Taf02].

An example of a short code is a maximal length sequence of length $2^{15}-1$, which is generated using a shift register of length 15. A long code of length $2^{42}-1$ is generated by a shift register of length 42. To have a better cross-correlation for the code sequences, Gold codes could be used. According to the Gold theorem, sequences that are combined by adding bit-by-bit modulo-2 of two pseudorandom sequences of the same length but generated by two distinct primitive polynomials, give cross-correlation peaks that are no greater than minimum possible cross-correlation peaks between any pair of maximal length sequences of the same length. The circuit shown in Figure 2.2 generates Gold codes.

![Figure 2-2 Gold codes generation](image-url)
2.7 Communication channels

2.7.1 AWGN Channel

The simplest practical case of a mobile radio channel is an additive white Gaussian noise channel (AWGN). In this case, the signal is perturbed only by the addition of some noise and some fixed path loss. It also assumes that the mobiles and the surrounding objects are not in motion.

2.7.2 The Narrowband Fading Channel

For the most part, mobile radio performance will not be as good as the pure AWGN case. The detailed characteristics of the propagation environment result in fading, which shows itself as a multiplicative, time-variant process applied to the channel. The channel is narrowband because the fading affects all frequencies in the modulated signal equally, so it can be modelled as a single multiplicative process.

In practice a transmitter and receiver are surrounded by objects which reflect and scatter the transmitted energy, causing several waves to arrive at the receiver via different routes. This is multipath propagation. Each of the waves has a different phase and this phase can be considered as an independent uniform distribution, with the phase associated with each wave being equally likely to take on any value.

No attempt is usually made to predict the exact value of the signal strength arising from multipath fading as this would require a very exact knowledge of the positions and electromagnetic characteristics of all scatters. Instead a statistical description is used.

2.7.2.1 The Rayleigh Distribution

The central limit theorem shows that, under certain conditions, the sum of a large number of independent random variables, approaches very closely to a normal distribution. In the non-line-of-sight (NLOS) case, the real and imaginary parts of the multipath components fulfil these conditions since they are composed of a sum of a large number of waves. Considering in-phase and quadrature, or $I$ and $Q$ components of a complex baseband signal:

$$\alpha = x + jy$$

The absolute value of $\alpha$, represents the fading amplitude:
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\[ r = |\alpha| = \sqrt{x^2 + y^2} \]  \hspace{1cm} (2-11)

The pdf of \( r \) is the Rayleigh distribution:

\[ p_{\text{Ray}}(r) = \left(\frac{r}{\sigma_r^2}\right) e^{-r^2/2\sigma_r^2} \]  \hspace{1cm} (2-12)

Here \( \sigma_r \) is the standard deviation of either the real or imaginary parts of \( \alpha \).

### 2.7.2.2 The Rice Distribution

In the Line of Sight (LOS) situation, the received signal is composed of a random multipath component, whose amplitude is described by the Rayleigh distribution, plus a coherent line-of-sight component, which has essentially constant power. The power of this component will usually be greater than the total multipath power before it needs to be considered as affecting the Rayleigh distribution significantly.

The Rice distribution is given theoretically by:

\[ p_{\text{Rice}}(r) = \frac{r}{\sigma_R^2} e^{-r^2 + s_R^2/2} \frac{1}{\sigma_R^2} I_0 \left( \frac{rs_R}{\sigma_R^2} \right) \]  \hspace{1cm} (2-13)

where \( \sigma_R^2 \) is the variance of either of the real or imaginary components of the multipath part alone and \( s_R \) is the magnitude of the LOS component. The function \( I_0 \) is the modified Bessel function of the first kind and zeroth-order.

The Rice pdf is often expressed in terms of another parameter, \( k_R \), usually known as the Rice factor and defined as

\[ k_R = \frac{s_R^2}{2\sigma_R^2} \]  \hspace{1cm} (2-14)

The Rice pdf can then be written in either of these forms:

\[ p_{\text{Rice}}(r) = \frac{2k_R r}{s_R^2} e^{-kr^2/2s_R^2} e^{-ks_R} I_0 \left( \frac{2k_R r}{s_R^2} \right) = \frac{r}{\sigma_R^2} e^{-r^2/(2\sigma_R^2)} e^{-ks_R} I_0 \left( \frac{r\sqrt{2k_R}}{\sigma_R^2} \right) \]  \hspace{1cm} (2-15)

For very large values of \( k_R \), the line-of-sight component dominates completely, very little fading is encountered, and the channel reverts to AWGN behaviour.
2.8 The Wideband Fading Channel

It is important to mention at this stage that the signal is travelling through space, causing several waves to arrive at the receiver via different routes. Due to multipath reflections the received signal contains delayed, distorted replicas of the original transmitted signal. This is the nature of the mobile channels and will be considered in the forthcoming sections.

2.9 Simulation Model for radio channels

In general, simulation models for mobile radio channels are realized by employing at least two or more coloured Gaussian noise processes. For instance, for the realization of a Rayleigh or Rice process, two real coloured Gaussian noise processes are required, whereas the realization of a Suzuki process [PKL94], which is the product of a Rayleigh and a lognormal process, is based on three real coloured Gaussian noise processes. Such processes (Rayleigh, Rice, and Suzuki) are often used as appropriate stochastical models for describing the fading behaviour of the envelope of the signal received mobile channels. Another case is given by modelling an $n$-path frequency selective mobile radio propagation channel by using the $n$-tap delay line model [PKL94]. This requires the realization of $2n$ real coloured Gaussian noise processes. All these examples show that an efficient design method for the realization of coloured Gaussian noise processes is of particular importance in the area of mobile radio channel modelling. A well-known method for the design of a coloured Gaussian noise process is to shape a white Gaussian noise (WGN) process by means of a filter that has a transfer function equal to the square root of the Doppler power spectral density (psd) of the fading process. Another method is based on Rice’s sum of sinusoids [Sau02j. In this case a coloured Gaussian noise process is approximated by a finite sum of weighted and properly designed sinusoids.

Diverse methods have been developed for the derivation of the relevant model parameters (Doppler coefficients $c_n$ and discrete Doppler frequencies $f_n$), for example, the method of equal distances and the mean square-error method [PKL94].

For a Rayleigh process all the scattered components in the received signal are represented by a zero-mean complex Gaussian noise process:

$$X(t) = x_1(t) + jx_2(t)$$  \hspace{1cm} (2-16)
With uncorrelated real components $x_i(t)$, $i=1, 2$, and variance $\text{Var}(X(t)) = 2\sigma_i^2$.

$$|X(t)| = \sqrt{x_1^2(t) + x_2^2(t)}$$

$$x_1(t), x_2(t) \sim N(0, \sigma_i)$$

\[ \text{Figure 2-3 The Shape of equation (2-18)} \]

A typical and often-assumed shape for the Doppler psd of the complex Gaussian noise process $X(t)$, $S_R(f)$, is for mobile fading channel given by the Jakes psd

$$S_R(f) = \begin{cases} 
\pi f_{d_{\text{max}}} \sqrt{1 - \left( \frac{f}{f_{d_{\text{max}}}} \right)^2} & |f| < f_{d_{\text{max}}} \\
0 & |f| > f_{d_{\text{max}}} 
\end{cases}$$

(2-18)

where $f_{d_{\text{max}}}$ is the maximum Doppler frequency. The shape of the above equation is given on Figure 2.3.

$x_1(t)$ and $x_2(t)$ are approximated by a sum of sinusoids as follows:

\[ \begin{align*}
\tilde{x}_1(t) &= \sum_{n=1}^{N_1} c_{d_n} \cos(2\pi f_{d_n} t + \theta_{d_n}) \\
\tilde{x}_2(t) &= \sum_{n=1}^{N_1} c_{d_n} \sin(2\pi f_{d_n} t + \theta_{d_n})
\end{align*} \] 

(2-19)
According to [PKL94], the coefficients in the above formula can be calculated as:

\[
\begin{align*}
    f_{dn} &= f_{d_{\text{max}}} \sin\left(\frac{\pi n}{2N_R}\right); \quad n = 1, 2, ..., N_R \\
    c_{dn} &= \frac{1}{\sqrt{N_R}}
\end{align*}
\]

(2-20) (2-21)

2.10 Conventional Detector for CDMA Systems

A conventional DS-CDMA system treats each user separately as a signal, with other users considered as either interference or noise. Conventional detectors suffer from various problems:

First, when a large number of users are active in the system, the effect of interference becomes large. Second, even when the number of users is relatively low but the signals of some users have higher powers, the low power users see a huge interference. This is the so-called near-far effect: the users near to the receiver are received at higher powers than those far away. Third, even if users are at the same distance to the receiver, some of them can be in deep fade and this causes an "effective near-far effect" and degrades the performance. Two important limits to current conventional detectors are:

- All users interfere with all users and this interference degrades the performance of the system.
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- The near-far effect is a serious problem and tight power control is needed to combat it [Ver98, OP98, PO98].

In this section after representing the behaviour of the conventional detector in synchronous and asynchronous systems, the statistics of interference terms in its output will be formulated.

2.10.1 Conventional Detector in Synchronous Scenarios

The conventional detector consists of a bank of matched filters (Figure 2.4). The behaviour of the conventional detector in synchronous and asynchronous systems is as follows:

In synchronous scenarios, assuming in AWGN channel and BPSK modulation, the output of the matched filter for the \( i \)th user is:

\[
y_i = \int_{0}^{t} r(t)s_i(t)dt \quad (2-22)
\]

The above expression can be rewritten as:

\[
y_k = A_i b_k + \sum_{j \neq k} A_j b_j \rho_{jk} + n_k \quad (2-23)
\]

In matrix form this is:

\[
y = RA_b + n \quad (2-24)
\]

where \( y \) is the output of the matched filters, \( R \) is the cross-correlation matrix, \( A \) is a diagonal matrix which its diagonal elements are the amplitude of the users, \( b \) is the vector of transmitted bits by all users and \( n \) is vector of noise added in transmitter. Assuming \( K \) active users in system each transmitting \( N_b \) bits, then in synchronous scenarios and in AWGN channels, the size of the above matrices and vector are: \( y_{K \times 1}, R_{K \times K}, A_{K \times K}, b_{K \times 1} \) and \( n_{K \times 1} \). In asynchronous scenarios, the Eq.(2-24) is still valid, however the sizes are: \( y_{KN_b \times 1}, R_{KN_b \times KN_b}, A_{KN_b \times KN_b}, b_{KN_b \times 1} \) and \( n_{KN_b \times 1} \).

The above statements for the sizes of vectors and matrices are valid for narrowband fading channels, as well. A more general model for the WCDMA channels is given in chapter 5, and yet the canonical Eq.(2-24) remains valid.
2.10.2 Conventional Detector in Asynchronous Scenarios

If we assume the delays are sorted as \( \tau_1 < \tau_2 < \ldots < \tau_k \), then the output is:

\[
y_k[i] = A_k b_k[i] + \sum_{j<k} A_j b_j[i+1] \rho_{kj} + \sum_{j<k} A_j b_j[i] \rho_{jk} + \sum_{j>k} A_j b_j[i] \rho_{kj} + \sum_{j>k} A_j b_j[i-1] \rho_{jk} + n_k[i]
\]

The above formula can be written in matrix form as Eq.(2-24) but with extended \( R \) matrix that is defined in section 2.5.2 for the asynchronous system.

2.11 Performance of Conventional Detector

2.11.1 Synchronous Users

As was shown earlier, in the AWGN channel and a synchronous system, the \( k^{th} \) user matched filter output is equal to

\[
y_k = A_k b_k + \sum_{j \neq k} A_j b_j \rho_{jk} + n_k
\]

where

\[
n_k = \sigma \int_0^\tau n(t) s_k(t) dt
\]

is a Gaussian random variable with zero mean and variance equal to \( \sigma^2 \). If the signature waveform of the \( k^{th} \) user is orthogonal to the other signature waveforms, then \( \rho_{jk} = 0, j \neq k \) and the matched filter output reduces to that obtained in the single-user problem:

\[
y_k = A_k b_k + n_k
\]

Using BPSK modulation and in AWGN channel, the probability of error of a threshold comparison of \( y_k \) is

\[
P_k(\sigma) = \frac{A_k}{\sigma}
\]

which is the same error probability one would obtain in the absence of other users.
In the nonorthogonal CDMA channel and considering only two users, the probability of error of user 1 occurs when the transmitted bit \(b_1\) is not equal to the detected bit \(\hat{b}_1\). Because the conventional detector uses the sign of the matched filter’s output \(y_i\) for detection purpose, for calculating the error probability we should seek the occasions that one bit is transmitted by user 1, however the output of the matched filter has different sign. As we have used the BPSK modulation, the transmitted bit can be either \(-1\) or \(+1\). Now we can write the following equations for calculating the probability of bit error rate [Ver98]:

\[
P_1(\sigma) = P[b_1 = \hat{b}_1] = P[b_1=+1]P[y_i<0 \mid b_1=+1] + P[b_1=-1]P[y_i>0 \mid b_1=-1]
\]

(2-30)

Yet, \(y_i\) is not Gaussian conditioned on \(b_1\), so another condition on \(b_2\) is needed:

\[
P[y_i>0 \mid b_1=-1] = P[y_i>0 \mid b_1=-1, b_2=+1]. P[b_2=+1] + P[y_i>0 \mid b_1=-1, b_2=-1]. P[b_2=-1]
\]

(2-31a)

\[
= P[n_i>A_1-A_2\rho]. P[b_2=+1] + P[n_i>A_1+A_2\rho]. P[b_2=-1]
\]

(2-31b)

\[
=0.5Q((A_1-A_2\rho)/\sigma) + 0.5Q((A_1+A_2\rho)/\sigma)
\]

(2-31c)

We have used the independence of \((b_1, b_2, n_i)\) and the simplified notation \(\rho_{12}=\rho\). Eq.(2-31b) has been achieved by combination of Eq.(2-26) and Eq.(2-31a). By symmetry, we get the same expression for \(P[y_i<0 \mid b_1=+1]\). Therefore, the bit-error-rate of the conventional detector for user 1 in the presence of one interfering user is

\[
P_1(\sigma) = 0.5Q((A_1-A_2\rho)/\sigma) + 0.5Q((A_1+A_2\rho)/\sigma)
\]

(2-32)

The generalization of the BER of the single-user matched filter from two users to an arbitrary number of users is straightforward. Following the same reasoning as before, we can write the BER of the \(k^{th}\) user as [Ver98]

\[
P_k(\sigma) = P[b_k=+1]P[y_k<0 \mid b_k=+1] + P[b_k=-1]P[y_k>0 \mid b_k=-1]
\]

(2-33)

\[
= P \left[ \sum \frac{1}{2} \left[ n_k > A_k - \sum_{j \neq k} A_j b_j \rho_{jk} \right] \right] + P \left[ \sum \frac{1}{2} \left[ n_k < -A_k - \sum_{j \neq k} A_j b_j \rho_{jk} \right] \right]
\]

(2-34)

\[
= P \left[ \sum \frac{1}{2^{k-1}} \sum \cdots \sum \frac{1}{2^{k-1}} \sum \cdots \sum \frac{1}{2^{k-1}} Q \left( \frac{A_k}{\sigma} + \sum_{j \neq k} \frac{A_j}{\sigma} \rho_{jk} \right) \right]
\]

(2-35)
where Eq.(2-34) follows by symmetry and Eq.(2-35) is obtained by conditioning on all the interfering bits. Error probability of the single user matched filter in the CDMA Gaussian channel depends on the shape of the signature waveforms only through their crosscorrelations. Moreover, the error probability depends on the received amplitudes and the noise level only through the ratios $A_i/\sigma$, as decisions are invariant to scaling of the received waveform.

The average of the Q-functions in (2-34) is upper-bounded by

$$P_k(\sigma) \leq Q\left(\frac{A_k}{\sigma} - \sum_{j \neq k} \frac{A_j}{\sigma} |\rho_{jk}|\right)$$

(2-36)

The number of operations required for computation of the Eq.(2-35) grows exponentially as the number of users. Using an approximation for the binomial random variable $\sum_{j \neq k} A_j b_j \rho_{jk}$ by a Gaussian random variable with identical variance, the BER would be

$$P_k(\sigma) = Q\left(\kappa \sqrt{\sigma^2 + \sum_{j \neq k} A_j^2 \rho_{jk}^2}\right)$$

(2-37)

Whereas at low signal-to-noise ratios the approximation is generally good, for high signal-to-noise ratios it may be unreliable.

### 2.11.2 Asynchronous Users

The analysis in the asynchronous case is entirely similar to the synchronous one. The main difference is that now each bit is affected by $2K-2$ interfering bits [Ver98, TY95]. This doubles the number of terms in Eq(2-35):

$$P_k(\sigma) = \frac{1}{4^{K-1}} \sum_{(e_i, d_i) \neq (1, 1)}^{} \sum_{(e_j, d_j) \neq (1, 1)}^{} \sum_{(e_k, d_k) \neq (1, 1)}^{} Q\left(\frac{A_k}{\sigma} + \sum_{j \neq k} \frac{A_j}{\sigma} (e_j \rho_{jk} + d_j \rho_{kj})\right)$$

(2-38)

In [Hol92] a simple and accurate method for calculating the BER is addressed. Assuming that there are $K$ active and asynchronous users in the system, and they are using random spreading sequences and the chip waveforms are rectangular, the final BER is:
Chapter 2. CDMA Systems and Interference

\[
P(\sigma) = \frac{2}{3} Q \left[ \left( \frac{K-1}{3N} + \frac{N_0}{2E_b} \right)^{-0.5} \right] + \frac{1}{6} Q \left[ \left( \frac{(K-1)(N/3) + \sqrt{3}\zeta}{N^2} + \frac{N_0}{2E_b} \right)^{-0.5} \right]
\]

where \( \zeta \) is defined as

\[
\zeta^2 = (K-1) \left[ N^2 \frac{23}{360} + N \left( \frac{1}{20} + \frac{K-2}{36} \right) - \frac{1}{20} \frac{K-2}{36} \right]
\]

Figure 2-5 Decision regions in the two-dimensional space of matched filter outputs

2.12 Decision Regions of the Conventional Detector

It is useful to visualize the operation of the conventional detector (or, any detector, for that matter) in a signal space diagram. The conventional demodulator bases its decisions on the \( K \)-dimensional vector

\[
(y_1, \ldots, y_K) = \left( \int_0^T r(t)s_1(t)dt, \ldots, \int_0^T r(t)s_K(t)dt \right)
\]

computed from the original observations. Therefore, we can actually view the observations as the \( K \)-vector \((y_1, \ldots, y_K)\), instead of the original received waveform. Considering \( K=2 \), is
of particular interest. In this case, conditioned on \((b_1, b_2), (y_1, y_2)\) is a Gaussian vector with mean
\[
(A_1 b_1 + A_2 b_2 \rho, A_2 b_2 + A_1 b_1 \rho)
\]
and covariance matrix [Ver98]
\[
\text{cov}(y_1, y_2) = \sigma^2 \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}
\]

In Figure 2.5, the mean-vectors for each of the four hypotheses in the \((y_1, y_2)\) space have been depicted. The assumptions are \(A_1 = A_2 = 1\) and \(\rho = 0.2\). The received vector is as the sum of a transmitted vector and a zero-mean Gaussian vector \((n_1, n_2)\).

The decision regions in the \((y_1, y_2)\) space of the single-user matched filter detector are simply the four quadrants. A shortcoming of the \((y_1, y_2)\) diagram in Figure 2.5 is that the noise components \(n_1\) and \(n_2\), added to the transmitted vector are correlated [Ver98]:

\[
E[n_1 n_2] = \sigma^2 \rho
\]

consequently, the distribution of the noise vector is not circularly symmetric and the norm of the noise vector does not determine the likelihood of that realization. This impairs the intuition one would expect to gain from the comparison of the respective decision regions (in \((y_1, y_2)\) space) of different detectors. A better choice than the \((y_1, y_2)\) diagram is a signal space diagram whose components \((\tilde{y}_1, \tilde{y}_2)\) are equal to the correlations of the received waveform with an (arbitrary) orthonormal basis \((\phi_1, \phi_2)\) that spans the linear space generated by the signal \((s_1, s_2)\). A choice for this orthonormal basis is

\[
\phi_1 = s_1 \\
\phi_2 = \frac{1}{\sqrt{1 - \rho^2}} s_2 - \frac{\rho}{\sqrt{1 - \rho^2}} s_1
\]

Conditioned on \((b_1, b_2), (\tilde{y}_1, \tilde{y}_2)\) is Gaussian with mean
\[
(A_1 b_1 < s_1, \phi_1 > + A_2 b_2 < s_2, \phi_1 >, A_1 b_1 < s_1, \phi_2 > + A_2 b_2 < s_2, \phi_2 >)
\]
\[
= (A_1 b_1 + A_2 b_2 \rho, A_2 b_2 \sqrt{1 - \rho^2})
\]

and covariance matrix equal to
The counterpart to Figure 2.5 in the alternative orthogonal representation \((\tilde{y}_1, \tilde{y}_2)\) using the basis in Eq.(2-44) is shown in Figure 2.6 where the decision regions are defined by the lines orthogonal to \(s_1\) and \(s_2\) respectively.

\[
\text{cov}(\tilde{y}_1, \tilde{y}_2) = \sigma^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]  

\text{(2-46)}
Even though \((\tilde{y}_1, \tilde{y}_2)\) are not computed by the demodulator, it is useful to visualize the received vector as belonging to the two-dimensional space depicted on Figure 2.6. Deriving decision regions for the conventional detector was straightforward. For complex receivers this may be very difficult and it will be easier to find them by simulation. Using computer simulations, the decision regions for the matched filter for two equal power users \((A_1 = A_2)\) are obtained by simulations and depicted in Figures 2.7 and 2.8.

For simulation, a pair of random bits (one bit per user) was generated and they were passed through the AGWN channel. After matched filtering, two values were obtained \((y_1, y_2)\). These values using Eq.(2-43) and Eq.(2-44) were transformed to \((\tilde{y}_1, \tilde{y}_2)\); then based on their sign, a decision were performed to indicate which region they belong to. In Figures 2.7 and 2.8, the horizontal and vertical axes are values of \(\tilde{y}_1\) and \(\tilde{y}_2\), respectively. This method will be used in the following chapters when we deal with more advanced receivers.

### 2.13 Conclusion

Code Division Multiple Access (CDMA) is used as the access method for the third generation mobile systems. In this chapter the features of CDMA systems was considered. This access scheme suffers greatly from the MAI, as all the transmitters transmit in the same frequency and time. By assigning different spreading codes to different users, this scheme tries to differentiate the users. The orthogonality of spreading codes in the fading environment both for uplink and downlink and asynchronous nature of the uplink, are severely damaged. The interference causing the MAI was also analytically examined. Having a CDMA system model and an insight about the structure of the MAI is necessary for applying the more complex Multiuser detection methods to be studied. These methods are the major topic of this thesis.
Chapter 3 Multiuser detection Methods for CDMA Systems

3.1 Introduction

Multiuser detection is a technique that jointly demodulates jointly interfering digital information. Communication systems subject to multiaccess interference are: mobile cellular, satellite communications, high-speed data transmission lines, digital video and audio broadcasting.

The third generation mobile communication systems use Code Division Multiple Access (CDMA) as an air interface access scheme. CDMA is an attractive solution for wireless communications and there is a significant interest in the design of wireless CDMA networks, which enables the users to have access to different data rates. However, CDMA systems suffer from the presence of multiple access interference (MAI). In fact all users in a way interfere with each other [Ver98]. Multiuser detection is a method that tries to extract useful information from the actual interference.

Although multiuser detection is expected to play a major role in enabling high performance, most existing systems do not yet incorporate these advanced techniques. Three primary explanations are offered for this present situation:

- Developments in the field are relatively recent in origin.
Questions persist about the complexity and robustness of multiuser algorithms, especially for portable applications [Mos96, ORH96]. Energy-efficient implementations have only recently become feasible.

This chapter reviews previous work in multiuser detection as a basis for the system design and analysis presented in subsequent chapters. Performance metrics and receiver optimality are introduced. Two important classes of multiuser detectors are described that achieve significant performance gains over conventional, single-user receivers. The principal goal is to establish the claim that significant performance improvements can be achieved in wireless CDMA systems with the use of more sophisticated receiver signal processing.

3.2 Optimality, Complexity, and Performance Metrics

3.2.1 System Model

Figure 3-1 Block diagrams for the transmitter and receiver front-end in a synchronous, Direct sequence CDMA
This section introduces a mathematical model for a base station providing wireless access to \( K \) simultaneous users as shown in Figure 3.1. For simplicity, direct sequence CDMA data modulation is assumed. Individual user data is aggregated at the base station, allowing symbol-synchronous signal transmission. Although apparently restrictive, this system model is justified on the following grounds:

1. It simplifies notation, but remains sufficiently general to illustrate the principles of multiuser detection.
2. It accurately represents many base stations to mobile links.
3. Extension of this model to asynchronous systems is (at least conceptually) straightforward.

Each user \( k \in \{ 1, K \} \) is assigned a discrete-time signature waveform \( s_k \) that has unit energy. Independent, binary antipodal data streams \( b_k \in \{ 1, -1 \} \) modulate these signature waveforms, with a power level \( A_k^2 \) that remains constant over a symbol period. User signals are linearly combined to yield an equivalent baseband transmit signal:

\[
S = \sum_{k=1}^{K} A_k b_k s_k \quad (3-1)
\]

The discrete-time signal is converted into a continuous-time representation, shaped by a transmit filter with frequency response \( F(j\omega) \), and modulated to the appropriate carrier frequency prior to signal transmission.

For the moment, the signal is assumed to pass through an additive, white Gaussian noise (AWGN) channel with single sided power spectral density \( N_0 \) prior to amplification, demodulation, and filtering in the receiver. (A matched filter \( F^*(j\omega) \) is used to perform the necessary front-end receive filtering). The resulting signal is passed through a chip-rate sampler, yielding the discrete time baseband received signal:

\[
r = \sum_{k=1}^{K} A_k b_k s_k + n \quad (3-2)
\]

where \( n \) represents the discrete-time, filtered random Gaussian noise process.
3.2.2 Optimality and Complexity

First, consider optimal signal detection in a multiuser communication system. A classical result from single-user detection theory states that for an AWGN channel, a (symbol-rate sampled) matched filter generates a sufficient statistic for signal detection [Ver98]. The multiuser version of this principle requires not one, but an entire bank of (symbol-rate sampled) matched filters, one for each active user as illustrated in Figure 3.2 [Ver98]. Accordingly, the optimal multiuser detector passes the received signal through a bank of correlators whose outputs, sampled at the symbol rate, are given by:

$$ y_k = \sum_{m=1}^{N} s_k^*[m] r[m] $$  \hspace{1cm} (3-3)

The resulting set of $K$ output samples may be conveniently expressed as:

$$ y = R A b + n $$  \hspace{1cm} (3-4)

where
is the cross-correlation matrix of signatures \((R_{ij})\) is the \((i, j)\) element of matrix \(R\), \(A=\text{diag}(\sqrt{A_1}, \ldots, \sqrt{A_K})\) is the user transmit amplitude matrix, \(b\) is the vector of user data bits, and \(n\) is a zero-mean Gaussian random vector with covariance matrix \(\sigma^2 R\). For any perfectly orthogonal signal set, \(R\) is the identity matrix, and the multiuser channel can be decomposed into \(K\) independent, single-user AWGN channels. The sizes of the matrixes and vectors in Eq.(3-4) have been described in section 2.10.1. Non-orthogonal signal sets, in contrast, lead to coupled equations that describe the signal correlation between users.

In both the orthogonal and non-orthogonal cases, the optimal multiuser detector chooses the data estimate \(b_{\text{opt}}\) as follows [Ver98]:

\[
 b_{\text{opt}} = \arg \max_{b \in [-1,1]^K} (2b^T y - b^T A R A b) 
\]  

(3-6)

The method of derivation of Eq.(3-6) is given in [Ver98] and we also refer to this in chapter 6, section 6.2.1. However, herein we point to the following issues about the optimum detector:

1. Optimal detection requires knowledge of signature waveforms and transmit-power levels for all users in the system.
2. Optimal detection requires a complexity that is exponential in the number of users. Consequently, systems with large user populations require suboptimum schemes for practical implementation.
3. Sufficient statistic generation alone requires a complexity that grows linearly with the number of active users. Hence, increased user densities require increased receiver processing to maintain comparable levels of performance.

### 3.2.3 Performance Metrics

Characterization of multiuser detectors, both optimal and suboptimum, requires suitable performance metrics. For obvious reasons, bit error rate (BER) constitutes the primary measure of interest in these systems. For a single-user AWGN channel, the optimal detector has probability of error [Pro01]:

\[
 P_b(\sigma) = Q\left(\frac{A_k}{\sigma}\right) 
\]  

(3-7)
Figure 3-3 Plot of the bit error rate for a single user, assuming BPSK data modulation over an AWGN channel

where

\[ Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt \]  

This error probability function (plotted in Figure 3–3) serves as the baseline metric for multiuser receiver evaluation. Because the presence of other users in the channel cannot improve the error rate, losses due to interference are conveniently expressed relative to this single-user performance limit. Following [Ver98], the “effective energy” of user \( e_k(\sigma) \), is defined as the energy required by user \( k \) to achieve a bit error rate equal to \( P_k(\sigma) \) in the absence of interfering users:

\[ P_k(\sigma) = Q\left(\frac{\sqrt{e_k(\sigma)}}{\sigma}\right) \]
The "efficiency" of user $k$ is defined as the ratio of effective to actual signal energies ($v_k$)

$$\frac{e_k(\sigma)}{v_k}$$  

(3-10)

and is restricted to the interval $[0, 1]$. "Asymptotic efficiency" is defined as

$$\eta_k = \lim_{\sigma \to 0} \frac{e_k(\sigma)}{v_k}$$  

(3-11)

and represents the performance loss due to the presence of interfering users as the background noise level tends toward zero. A closely related receiver metric is near-far resistance, which represents the minimum asymptotic efficiency over all users when transmit power levels are unconstrained [Ver98],

$$\bar{\eta}_k = \inf_{\forall \sigma > 0} \eta_k$$  

(3-12)

Intuitively, near-far resistance provides an indication of the worst-case performance loss due to interference for any individual user over all possible other-user transmit power profiles. In practice, of course, no system ever permits entirely arbitrary transmit power levels.

### 3.3 Sub-optimum Multiuser Detection Methods

Because optimal multiuser detection requires a complexity that is exponential in the number of users, current research focuses on suboptimum architectures that achieve comparable performance gains yet require substantially less complexity [Mos96]. This section considers two basic classes of suboptimum multiuser detectors: linear and nonlinear.
3.3.1 Linear Multiuser Detectors

3.3.1.1 Single User Correlator

The simplest linear detector is the single user correlation receiver illustrated in Figure 3.4. Received symbol estimates for individual users are based solely on the corresponding matched filter output:

$$\hat{b}_k = \text{sgn}(y_k)$$  \hspace{1cm} (3-13)

![Figure 3-4 Block diagram of a single-user correlation receiver](image)

Although this receiver is extremely simple to implement, its performance is strongly dependent on signal correlations between users and on other-user transmit power levels. Using Eq.(2-3), the decision statistic for this receiver is given by:

$$y_k = A_k b_k + \sum_{j \neq k} A_j b_j \rho_{jk} + n_k$$  \hspace{1cm} (3-14)

Of particular note is the second term, which indicates that any non-orthogonal user can drive the error probability to one half by transmitting at a sufficiently large power level. This situation gives rise to the well known near-far problem in which a strong interfering signal can completely overwhelm the desired user’s signal unless power control is provided.
The degradation due to multiple access interference (MAI) is illustrated graphically in Figure 3-5 for a synchronous CDMA system with a spreading factor of 128. In this simulation, users spreading codes are constructed using cyclic shifts of a single maximum length shift register (MLSR) sequence. A simple, additive white Gaussian noise channel is inserted between the transmitter and receiver. For comparison purposes, the single user performance bound is also provided. Clearly, the presence of multiple access interference can severely degrade the conventional detector, rendering it virtually unusable. Because of the severe degradation caused by MAI, systems that rely on simple correlation receivers must implement sophisticated power control and forward error correction (FEC) techniques in order to improve receiver performance. The asymptotic efficiency and near-far resistance of the single user correlator receiver are given by

$$
\eta_k = \max\left[0, \ 1 - \sum_{j \neq k} \frac{A_j}{A_k} |\rho_{jk}| \right]
$$

(3-15)
respectively [Ver98]. It is worth noting that the near-far resistance for this detector is equal to zero unless all user signals are perfectly orthogonal.

### 3.3.1.2 Decorrelating Detector

As was shown in the previous sections, the output of the bank of matched filters can be written:

\[
y = RAb + n
\]  \hspace{1cm} (3-17)

The vectors and matrices in this canonical equation are described in section 2.10.1.

In the conventional detector, decision is performed based on \( y \). The decorrelating detector, illustrated in Figure 3–6, uses a modified matched filter bank output for detection [LV89, LV90, XSR90]:

\[
\hat{b} = \text{sgn}(R^{-1}y) = \text{sgn}(Ab + R^{-1}n)
\]  \hspace{1cm} (3-18)

As the name suggests, this detector decorrelates the received signal so that each output from the decorrelating block is composed of only two components: the desired user’s signal, and background noise. The probability of error for this detector is easily calculated to be [Ver98]:

\[
P_k(\sigma) = Q\left(\frac{A_k}{\sigma \sqrt{(R^{-1})_{kk}}}\right)
\]  \hspace{1cm} (3-19)

where \((R^{-1})_{kk}\) stands for the element \((k,k)\) of the \(R^{-1}\) matrix.
Note that these losses are independent of the transmit power levels of other users in the system. The efficiency, asymptotic efficiency, and near-far resistance are all identical, having value

$$\frac{1}{(R^{-1})_{kk}}$$

which is independent of background noise level and user transmit-power levels.

The decorrelating detector eliminates interference at the expense of noise enhancement since

$$(R^{-1})_{kk} \geq 1$$

If user transmit levels are assumed unknown, the decorrelating detector also generates the maximum likelihood (ML) received signal estimate [Ver98]. The principal drawbacks of the decorrelating detector are noise enhancement and the need for (perfect) knowledge of all users spreading codes in order to implement this detector. Also, because the channel between transmitter and receiver is generally unknown, an approximation to the decorrelator is usually required. Figure 3–7 shows an alternative implementation of the decorrelating detector when data recovery for only one user is required. This

---

Figure 3-6 Block diagram of the decorrelating detector for K users
implementation exploits the linearity of the decorrelator, resulting in a structure that is only slightly more complicated than the conventional single-user correlation receiver. The key difference is that the despreading code for the decorrelator is a (linear) function of all users spreading codes (this despreading code is shown in Figure 3-7 as $d_1$), not just the desired user's code as in the case of the single-user correlation receiver.

![Figure 3-7 Block diagram of the decorrelating detector for a single user](image)

$$d_1 = \sum_{m=1}^{K} (R^{-1})_{m,n} s_k$$

3.3.1.3 MMSE Detector

![Figure 3-8 Block diagram of the minimum mean squared error detector for K users](image)
The minimum mean-squared error (MMSE) detector, shown in Figure 3-8, is closely related to both the single-user correlation receiver and the decorrelating detector. When data recovery for only a single user is desired, the MMSE detector in Figure 3-9 chooses the $c_{(MMSE),k}$ of duration $T$ that can achieve [MH94]:

$$c_{(MMSE),k} = \min_{s_k} E[(b_k - c_{(MMSE),k}^T r)^2]$$

and outputs the decision:

$$\hat{b}_k = \text{sgn}(c_{(MMSE),k}^T r)$$

Like the MMSE linear equalizer, the MMSE multiuser detector allows some residual interference to appear at the correlator output in order to reduce background noise enhancement. The net result is an output with a mean squared error that is a minimum over all linear detectors. Note that as $\sigma \to \infty$ the MMSE detector approaches the single user correlation receiver, whereas for $\sigma \to 0$ it approaches the decorrelating detector. It follows that the asymptotic multiuser efficiency and near-far resistance for the MMSE detector are equivalent to that of the decorrelating detector. Whenever background noise levels are negligible relative to other-user interference the MMSE detector and the decorrelating detector will have identical performance. However, the MMSE detector retains one crucial advantage over the decorrelator: ease of adaptive implementation. Because the mean squared error is a convex function of the spreading code coefficients, global convergence can be achieved using well-known iterative techniques. Specific methods for adaptive multiuser detection are discussed in greater detail below. As with the decorrelating detector, when data recovery for only a single user is desired, the MMSE detector can be implemented as shown in Figure 3.9 where

$$c_{(MMSE),k} = (1 + s_k^T M_k^{-1} s_k)^{-1} M_k^{-1} s_k$$

and

$$M_k = \sum_{m=1 \atop m \neq k}^{K} s_m s_m^T + \sigma^2$$

The minimum mean squared value at the output for this detector is given by:
\[ MMSE_k = \left(1 + s_k^T M_k^{-1} s_k \right)^{-1} \] (3-26)

And the maximum achievable signal to interference is given by [MH94]:

\[ SIR_k = s_k^T M_k^{-1} s_k \] (3-27)

**Figure 3-9 MMSE multiuser detector for a single user.**

### 3.3.1.4 Polynomial expansion detector

To simplify the matrix inversion operation, a polynomial expansion of the inverted matrix can be used:

\[ T = \sum_{i=1}^{N} w_i R_i \] (3-28)

which \( w_i \) are a set of coefficients and their values impact the performance the detector and their calculation is the major problem of these detectors.

### 3.3.1.5 Comparison of different linear multiuser detectors

A comparison of linear multiuser detectors in terms of their advantages and disadvantages is given in Table 3.1.

The logical relation of different linear multiuser detects is shown in Figure 3-10.
### Table 3-1. Advantages and Disadvantages of Linear Detectors

<table>
<thead>
<tr>
<th>Detector</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decorrelator</td>
<td>• Eliminating the MAI completely</td>
<td>• Enhancing the noise</td>
</tr>
<tr>
<td></td>
<td>• More Efficient over Conventional detector</td>
<td>• Inverting of $R$ matrix (which has an order of $KN$ in asynchronous mode) is needed</td>
</tr>
<tr>
<td></td>
<td>• More less complex than maximum likelihood sequence detector</td>
<td>• Difficult for Real time implementation</td>
</tr>
<tr>
<td></td>
<td>• Decorrelating each bit of data</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Energy of signals doesn’t affect the BER</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Estimation of received amplitudes is not needed</td>
<td></td>
</tr>
<tr>
<td>Minimum Mean-Squared Error (MMSE)</td>
<td>• Taking into account the background noise</td>
<td>• Estimation of users’ powers are essential</td>
</tr>
<tr>
<td></td>
<td>• Doing a balance between noise and MAI</td>
<td>• The performance depends on the power of interfering users</td>
</tr>
<tr>
<td></td>
<td>• Generally have a better BER than decorrelator</td>
<td>• The near-far resistance is worse than with the decorrelator</td>
</tr>
<tr>
<td>Polynomial Expansion (PE)</td>
<td>• Less Complex than decorrelator and MMSE</td>
<td>• Matrix inversion is needed</td>
</tr>
<tr>
<td></td>
<td>• Can behave like decorrelator and MMSE approximately</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Estimation of received amplitudes is not needed</td>
<td>• Coefficients must be updated</td>
</tr>
<tr>
<td></td>
<td>• Is applicable to both short and long codes</td>
<td>• Computing the Cross-correlations are still needed</td>
</tr>
<tr>
<td></td>
<td>• Its coefficients work in a large range of system parameters</td>
<td></td>
</tr>
</tbody>
</table>
3.3.2 Subtractive Multiuser Detectors

Another important class of Multiuser detectors is subtractive interference cancellation detectors. The basic idea is the separate estimation of interference at the receiver. This interference rises from the contribution of each user and the detector subtracts out some or all of the interference seen by each user. Usually these detectors are implemented in several stages, hopefully to improve the estimates through the stages. The same method is used for mitigating the ISI in feedback equalizers. For estimating the interference, either soft or hard decisions can be used. In the situation that a good amplitude estimation of data is available, hard decision has a better performance than soft decision. There are two important classes in the IC family: Serial Interference Cancellation (SIC) and Parallel Interference Cancellation (PIC). In the next sections, these IC methods are described further.
3.3.2.1 Successive Interference Cancellation

Successive interference cancellation (SIC) was one of the earliest proposals for implementing multiuser detection using nonlinear processing [Vit90, PH94, Mos96, DHZ95]. This detector requires knowledge of both relative amplitudes and signature sequences for all active users at the receiver. Figure 3-11 illustrates the basic concept of a SIC detector.

This detector takes a serial approach to cancelling interference. In each stage of this detector, decision, regeneration, and cancelling out the interference for one user take place. In the next step, remaining users see less MAI. To achieve this, users are ranked according to their powers. The first step of detection aims to extract the strongest user and subtract its contribution to MAI. This process consists of the following stages:

1. Detection of the strongest user with conventional detector.
2. Making a hard decision on the signal.
3. Regeneration of an estimation for the received signal for user one, \( \hat{S}_1 \), using:
   a. Data decision from step 2.
   b. Knowledge of its PN sequence.
   c. Estimate of its timing and amplitude and phase.
4. Subtract out \( \hat{S}_1 \) from the total received signal to yield a partially cleaned version of the received signal.

![Figure 3-11 A successive interference cancellation (SIC) detector](image-url)
Assuming that the signal estimation for the first user is correct, the outputs of the first step are as follow:

- A data decision on the strongest user.
- An improved version of the received signal without the MAI caused by the first user.

This process repeats for the other users and the $k^{th}$ stage's output are the data decision for the $k^{th}$ user and a cleaner version of the received signal without the contribution of the $(k-1)$ users in MAI.

Decision statistics of symbol $i$ and for this detector are as follow:

$$y_{k,i} = y_{k,i} \cos(\theta_k) + y_{Qk,i} \sin(\theta_k)$$

where $\theta_k$ is the received phase of the user $k$ and also the followings hold:

$$y_{k,i} = \int_{(i-1)T+\tau_k}^{iT+\tau_k} \hat{r}_{l}^{(k)}(t) s_k(t-\tau_k) dt$$

$$y_{Qk,i} = \int_{(i-1)T+\tau_k}^{iT+\tau_k} \hat{r}_{Q}^{(k)}(t) s_k(t-\tau_k) dt$$

and

$$\hat{r}_{l}^{(k)}(t) = r_l(t) - \sum_{i=1}^{N_k} \sum_{j=1}^{k-1} \frac{y_{j,i}}{T} s_j(t-\tau_k) \cos(\theta_j)$$

$$\hat{r}_{Q}^{(k)}(t) = r_Q(t) - \sum_{i=1}^{N_k} \sum_{j=1}^{k-1} \frac{y_{j,i}}{T} s_j(t-\tau_k) \sin(\theta_j)$$

$\hat{r}_{l}^{(k)}(t)$ is the $k^{th}$ user's received signal after users 1 through $k-1$ have been estimated and cancelled. Ordering the users according to their powers is essential in the algorithm because it gives the most benefit to the weaker users. However, the strongest user does not see any reduction in MAI.

This detector can give a good performance compared with the conventional detector and has a relatively simple structure. However it suffers from two major problems: first, one additional bit delay is required per stage of cancellation and this causes a long delay. Second, power re-arrangement is needed when the power profile changes.

A potential problem in the SIC detector occurs when the reliability of the conventional detector is poor. In such cases even if perfect amplitude, timing and phase estimation were
available, wrong decisions in first stages can propagate through the cancellation process. Advantages and disadvantages of this detector are briefly as follow [PH94-1, PH94-2, BCW96, OPH98, BCW00]:

**Advantages**

- Has a good performance when the power of the users becomes more diverse.
- Viterbi showed that SIC can approach the channel capacity for aggregate Gaussian noise, so the method is not limited by MAI.
- Robust to strong interferers.
- Low complexity.

**Disadvantages**

- In each stage one additional bit delay is needed which yields longer delay.
- Reordering the users according to their signal power is needed when the powers change.
- Performance degrades if the initial data estimates are not correct.
- Performance depends on the spread of users’ powers.
- Performance is significantly worse than the Decorrelator, MMSE and PIC in perfect power control.

3.3.2.2 Parallel Interference Cancellation

![Block diagram of a parallel interference cancellation (PIC) detector](image)

**Figure 3-12** Block diagram of a parallel interference cancellation (PIC) detector
A straightforward modification of the SIC detector is the parallel interference canceller (PIC) shown in Figure 3–12 [VA90, DHZ95, Mos96, BCW96, BKSW96, BCW00]. This detector computes preliminary estimates of each user’s transmit symbol using a front-end decision device (typically, a matched filter bank or decorrelator). Symbol estimates are then scaled by the corresponding amplitude estimate, and re-spread using signature sequences for each user. A partial summer block adds together all signals except the desired user’s signal, and subtracts the result from the original received signal. If symbol and amplitude estimates are correct, then each user is detected in the absence of multiple access interference. Incorrect estimates have a similar effect on performance as previously noted for the SIC detector. Detailed analyses of PIC detector performance can be found in [PH94, DS95]. From an implementation perspective, the PIC detector has reduced latency relative to the SIC detector, but requires multiple, parallel signal processing blocks. A more detailed discussion of PIC is given in chapter 4.

3.4 Multiuser Detectors for Multipath Channels

The simplified system model presented previously can be extended to include fading. The signature of the user $k$ undergoes a linear time-varying transformation fully characterized by the complex-valued (baseband) impulse response:

$$h_k(t,\tau)$$

which denotes the response of the system at time $t$ due to a delta function at time $\tau$, $\delta(t-\tau)$.

In the special case of a time invariant system, the dependence of $h_k(t,\tau)$ on its arguments is only through $t-\tau$. The effect of frequency selective fading on the basic CDMA model is that the signature waveform seen at the receiver is not $s_k(t)$ and the received signal will be:

$$r(t) = \sum_{k=1}^{K} A_k b_k s'_k(t) + n(t)$$

(3-33)

where $n(t)$ is noise, and effective signature waveform is the convolution:

$$s'_k(t) = \int_{-\infty}^{\infty} h_k(t,\lambda)s_k(t-\lambda)d\lambda$$

(3-34)
All of the detectors previously discussed, both linear and non-linear, can be extended to multipath channels by substituting $s_k$ with $s'_k$ and $R$ with $R'$ (where elements of $R'$ can be calculated using $s'_k$ and Eq.(2-7)) [Ver98, HV98].

The presence of a multipath channel substantially increases the need for multiuser detection. In the synchronous scenarios and without multipath, the system designer can carefully select spreading codes in order to minimize the cross-correlation between user signals. In fact, in the absence of multipath, the optimum code choice is an orthogonal code set that eliminates MAI completely. A single-user correlation receiver then achieves optimal performance. Once multipath effects are introduced, however, the system designer loses the ability to guarantee signal orthogonality at the receiver. Since the effective spreading codes become random and time varying, the cross-correlations between user signals also become channel-dependent. The presence of a multipath channel increases the complexity of a multiuser detector.

The original bank of front-end matched filters is now replaced by a bank of channel-dependent RAKE matched filters, one for each user. A RAKE matched filter for a single user is shown in Figure 3-13 [PG58]. Implementation of the RAKE filter requires channel coefficient estimation, with estimation errors directly degrading receiver performance. Subsequent signal processing blocks also become more complicated because of their channel dependence. Non-linear detectors (e.g. SIC and PIC) that rely on accurate gain estimates become difficult to implement under fading conditions.

![Figure 3-13 Block diagram of a RAKE matched filter for multipath channels.](image-url)
3.4.1 Diversity Methods and Receiver Robustness

Several techniques have been proposed to improve multiuser detection performance over fading multipath channels. Two topics receive particular attention here: diversity techniques and receiver robustness to estimation errors.

3.4.1.1 Diversity for Fading Environments

Diversity is a powerful technique for combating the effects of fading environments. Diversity methods rely on multiple, independent signal paths between transmitter and receiver to improve detector performance. Common forms of diversity reception include spatial (using multiple antennas), temporal (data interleaving with coding), and frequency (DS or FH spread spectrum) diversity.

![Figure 3-14 The impact of diversity order in a single-user fading environment.](image)

The benefits of diversity in a single user environment are illustrated in Figure 3.14. The top curve shows the average BER when the channel gain varies randomly with a Rayleigh amplitude distribution. The other curves illustrate the performance of diversity receivers.
Chapter 3 Multiuser Detection Methods for CDMA Systems

...over the same Rayleigh faded link, assuming maximal ratio combining with perfect channel estimates. Wideband spread spectrum signals possess an inherent form of frequency diversity [Pro01].

Antenna diversity is another way to improve the performance. It has a great impact on performance as is depicted in Figure 3.15.

Considering one user in AWGN channel, having two received antennas will provide a 3dB gain (antenna gain) in compare with a receiver having only one antenna. In the fading channel, the extra antenna will provide the diversity gain, which makes the BER curve to become sharper.

In most cases, each multipath signal component can be assumed to provide an independent signal observation. The total number of distinguishable multipath arrivals therefore determines the diversity order for the receiver. Increased spreading provides greater immunity to signal fading (assuming, of course, the receiver takes advantage of these additional diversity paths). “Combining diversity” and “selection diversity” are two

![Figure 3-15 Antenna and Path diversity in Vehicular Channel A [3GPP]](image)

(with different orders of diversity) over the same Rayleigh faded link, assuming maximal ratio combining with perfect channel estimates. Wideband spread spectrum signals possess an inherent form of frequency diversity [Pro01].

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techniques for exploiting diversity in multipath signal reception. The following subsections describe the performance and implementation tradeoffs associated with these techniques.

### 3.4.1.2 Combining Diversity

Maximal ratio combining (MRC) weights each multipath component according to its relative SNR, and phase shifts each component to allow coherent signal combining. The coherent RAKE receiver of Figure 3-16 provides a classic example of maximal ratio combining [PG58]. This technique is most appropriate in situations where phase changes in individual multipath components vary slowly enough that an absolute phase reference can be maintained throughout the channel estimation interval. In terms of implementation complexity, MRC requires datapath “fingers” for each multipath component, channel estimation blocks, and signal combining blocks. Equal gain combining (EGC) is a simpler, though suboptimal combining alternative.

**Figure 3-16** Block diagram of a coherent RAKE matched filter receiver

### 3.4.1.3 Selection Diversity

In a selection diversity system, the receiver simply selects the strongest multipath component, and uses it for signal detection. A significant advantage of this approach is that multiple data paths are not required as the diversity order increases (although additional...
hardware is required to distinguish the strongest path). However, the gains are also not as significant. For small diversity orders, performance differences are virtually indistinguishable.

### 3.4.2 Robustness Considerations in Multiuser Detection

![Block diagram of the decorrelating multipath (Dm) detector](image)

Figure 3-17 Block diagram of the decorrelating multipath (Dm) detector

One of the earliest proposals for multiuser detection in a multipath environment suggested interchanging the order of operations between signal combining (RAKE matched filtering) and decorrelation to increase receiver robustness to channel estimation errors [ZB95]. The resulting receiver, later called the decorrelating multipath (Dm) detector, is shown in Figure 3–17. For this receiver, every multipath signal component for each user is separately decorrelated. Individual components from each user are then optimally recombined in a manner that depends on the transmitter encoding method. The Dm detector – unlike the RAKE multipath decorrelating (mD) detector of Figure 3–18 – remains near-far resistant even in the presence of imperfect channel estimates. Unfortunately, the price paid for near-far resistance is an increase in noise enhancement and a reduction (by a factor of $KL$) in the number of users the system can support [HS94, HV98].
A detailed comparison of the Dm and mD detectors is presented in [HS94] and extended in [HV98] to include the concept of a Dm* detector. A conceptual illustration of the Dm* detector is shown in Figure 3-19. The primary difference between the Dm and Dm* detectors is that the Dm* detector only decorrelates the multipath components of the desired user's signal from the composite signature sequences of all other users. As described in [HV98], the Dm* detector is attractive because it achieves the near-far resistance of the Dm detector with a smaller noise enhancement penalty. In fact, noise enhancement for the Dm* detector is only slightly larger than for the mD detector.
3.5 Adaptive Multiuser Detection

Adaptive multiuser detection (AMD) is an area of active research, and one that has particular relevance to wireless portable system designers [Ver94, PW97, SW96, GPB94]. Receiver adaptation provides a powerful technique for reducing losses associated with time variations on the wireless link. Most AMD proposals borrow heavily from single user adaptive filtering theory and include multiuser equivalents of the gradient, least squares, and lattice adaptive algorithms [NC96, MH94, Lee93, Lee96]. In addition, there exist algorithms in the multiuser space that have no counterpart in single user theory. An excellent example is the blind adaptive multiuser detector discussed below [HMV95].
Figure 3.20 illustrates the basic concept of an adaptive multiuser detector. The receiver’s task is to continuously update the despreading code in order to minimize a specific error metric between the output $y_k$ and the desired signal $\hat{b}_k$. The most popular error metric is the minimum mean squared error (MMSE), which has the following attractive characteristics:

- **Global convergence**: the MMSE criterion results in a convex cost function, which ensures global convergence for appropriate iterative algorithms.

- **Approximates minimum BER receiver**: for signals degraded by additive white Gaussian noise, the MMSE receiver is identical to the minimum BER receiver. Since multiuser interference is neither white nor Gaussian, the MMSE receiver is sub-optimal. However, in many instances the MMSE receiver provides an excellent approximation to the optimal receiver.

- **Reduced complexity**: well-established algorithms have been developed to solve the MMSE problem in polynomial time including variations of the LMS, RLS, and lattice stochastic algorithms. Optimized hardware implementations exist for each of these algorithmic alternatives.

Together with these advantages, most adaptive MMSE algorithms possess one important limitation: the need for training sequences. In many communication systems, training sequences do not pose a problem since they can be provided as part of an initialization process. Training sequences are especially useful in situations where the channel and environment remain static for long periods of time. High data rate wireline modems, for example, use training sequences during initial “handshaking” to learn the channel and adapt the receiver accordingly. Since the telephone channel tends to change slowly with
time, this training sequence is a one-time overhead expense. If further adaptation is required, decision-directed techniques can be applied because of the slow rate of variation. The situation is fundamentally different in mobile DS-CDMA applications. In these systems, the channel does not remain static, but is susceptible to large changes in phase and amplitude over relatively short periods of time. Interference levels at the receiver can also change quite dramatically. Since users in a DS-CDMA system simultaneously share the channel in both time and frequency, all users are affected each time a user enters the system, exits the system, changes his data rate or adjusts his transmit power level. Such events precipitate discontinuous changes in interference levels at the portable receiver, since the correlation matrix $R$ also changes discontinuously. In many instances, decision-directed algorithms may not be sufficient to track these changes.

### 3.5.1 Blind Adaptive Multiuser Detection

![Block diagram of a blind adaptive multiuser detector](image)

**Figure 3-21** Block diagram of a blind adaptive multiuser detector, designed to minimize the mean output energy (MOE)

Blind adaptive multiuser detection (BAMUD) is one variant of adaptive MMSE detection that eliminates the need for training sequences [HMV95]. The only information required at the receiver is knowledge of the desired user’s spreading sequence. Instead of minimizing
the mean squared error directly, the blind detector minimizes the mean output energy (MOE) error metric given by:

\[ MOE(x) = E(<r, s + x>^2) \]  

(3-35)

where \(<.>\) is defined as Eq.(2-3). Here, \(s(t)\) is the desired user’s normalized spreading waveform and \(x(t)\) is a signal constrained to be orthogonal to \(s(t)\). It can be shown that minimizing the MOE is equivalent to minimizing the MSE. The key difference is that MOE minimization requires no training sequence; the principal drawback is a “noisier” adaptation. Figure 3–21 provides one interpretation of the BAMUD detector. The adaptive correlator is decomposed into two orthogonal components: a fixed (anchored) component \(s\) corresponding to the desired user’s signature sequence, and an adaptive component \(x\) that attempts to minimize the residual error by suppressing interference that is not in the direction of the desired signal. A detailed analysis of the BAMUD is provided in [HMV95].

A previous section described two architectures for linear multipath combining multiuser detectors: the mD, and Dm* receivers. Both can be approximated using detectors that implement blind adaptive techniques [HV98]. The mD detector is potentially the simplest to realize: if channel coefficient estimates are available, the receiver simply constructs the RAKE matched filter for user \(m\):

\[ s_m(t) = \sum_{i=1}^{L} c_i s_m(t-\tau_i) \]

A normalized version of this signal serves as the anchored component in the conventional BAMUD algorithm, and a straightforward implementation can be realized. The only ambiguity arises when the channel coefficients change, and the anchored component needs to be updated. Since the BAMUD algorithm requires that \(x(t)\) always remains orthogonal to \(s(t)\), \(x(t)\) must be adjusted appropriately whenever the direction of \(s(t)\) changes. Two possibilities would be to reset \(x(t)\) to zero, or to subtract any portion of \(x(t)\) which lies in the new direction of \(s(t)\).

A blind adaptive implementation of the Dm* detector requires additional modifications. For this detector, the multipath signature sequence is assumed to be unknown. The only information required at the receiver is the set of sequences that span the subspace of the desired user’s signal. As derived in [HV98], the blind adaptive implementation for this detector requires that the adaptive orthogonal sequence be constrained such that it remains orthogonal to the entire subspace spanned by the desired user’s multipath signal. Blind
adaptation without this constraint results in cancellation of the desired signal. Each multipath component of the desired user's signal requires a separate adaptive receiver (referred to here as an adaptive branch correlator for obvious reasons). Figure 3–22 illustrates an adaptive branch correlator, while Figure 3–23 shows a complete representation of an adaptive Dm* detector implementing $L^{th}$ order diversity.

![Adaptive branch correlator for $i^{th}$ multipath component](image)

**Figure 3-22** Adaptive branch correlator for $i^{th}$ multipath component

The anchored sequence is the $i^{th}$ multi-path component of the desired user's signal.

![An adaptive implementation of the Dm* detector](image)

**Figure 3-23** An adaptive implementation of the Dm* detector
3.5.2 Adaptation Algorithms

Implementations of adaptive MMSE multiuser detection require the use of algorithms that converge toward an optimal MMSE criterion. Fortunately, several such algorithms have been developed previously in the context of adaptive filtering [Cla93, Hay96]. Because the MMSE adaptive correlator problem can be formulated identically to an MMSE adaptive filtering problem [MH94], adaptive multiuser detectors can make use of these existing algorithms. The following sections describe two of the most popular MMSE adaptive algorithms: the least mean squares (LMS) and recursive least squares (RLS). Four primary issues are considered: speed of convergence, stability, robustness, and implementation complexity.

3.5.2.1 Least Mean Squares (LMS)

The LMS algorithm is one of the most popular techniques for filter adaptation to an MMSE criterion. The basic approach uses the method of steepest descent to iteratively adapt filter coefficients toward the optimal solution. The LMS adaptive correlator chooses its coefficients to minimize the quantity $E[|e_n|^2]$ as illustrated in Figure 3.24 [Cla93, Hay96]. The ideal LMS correlator update equation is given by

$$c_n = c_{n-1} - \mu g_{n-1}$$

(3-36)
where $c_n$ is a vector representing the filter coefficients at the $n^{th}$ iteration, $\mu$ is a positive number chosen small enough to guarantee convergence of the iterative algorithm, and $g_n$ represents the gradient direction on the MSE surface which is given by $g_n = E(e_n^*r_n)$. Since the true gradient vector is not observable at the receiver, an approximation to this term is normally required. A standard approximation is to replace the gradient with its instantaneous value $\hat{g}_n = e_n^*r_n$. This estimate has the desirable property of being unbiased since $E(\hat{g}_n) = g_n$; it is also easily generated at the receiver. The modified coefficient update equation for the basic LMS algorithm thus becomes:

$$c_n = c_{n-1} - \mu e_n^*r_n$$ (3-37)

or equivalently

$$c_n = c_{n-1} - \mu(y_n - d_n)^*r_n$$ (3-38)

The parameter $\mu$ is known as the step size parameter, and must be chosen small enough for the iterative process to converge.

The choice of $\mu$ directly impacts the convergence speed, stability, and accuracy of the LMS algorithm. A larger value of $\mu$ speeds convergence by taking larger steps in the direction of the stochastic gradient. If $\mu$ is chosen too large, however, the adaptation process can become unstable. A standard rule of thumb is to choose $\mu$ such that

$$0 < \mu < \frac{2}{N_{Cor}P_r}$$ (3-39)

where $N_{Cor}$ is the correlation length, and $P_r$ is the power of the input signal $r_n$. This range is adequate to ensure stability under most circumstances. Given this constraint on $\mu$, an obvious choice for the step size parameter would then appear to be $\mu = \frac{2}{N_{Cor}P_r}$ since this will guarantee the fastest convergence. However, there is also a tradeoff between the value of $\mu$ and the optimality of the final solution. Large values of $\mu$ can result in "ping-ponging" of the filter coefficients in the vicinity of the optimal solution. The net effect is an increase in the mean squared error (MSE) value at the correlator output after convergence. In general, the total MSE at time $n$ can be written as
Chapter 3 Multiuser Detection Methods for CDMA Systems

\[ J_n = E[|e_n|^2] = J_{\text{min}} + J_n^{\text{excess}} \]  

(3-40)

where \( J_{\text{min}} \) is the MSE for the optimal filter, and \( J_n^{\text{excess}} \) is the excess MSE that results from non-optimal correlator coefficients. As \( n \to \infty \), it can be shown that

\[ J_n^{\text{excess}} = \frac{\mu_N \cdot P}{2} \]  

(3-41)

Because of this proportional dependence on \( \mu \), smaller step size values result in reduced MSE.

Convergence speed is directly related to the eigenvalue spread of the correlation matrix \( \mathbf{R} \). (Eigenvalue spread is defined as the absolute value of the ratio of the largest to smallest eigenvalue \( \frac{\lambda_{\text{max}}}{\lambda_{\text{min}}} \)). In general, larger eigenvalue spreads result in slower convergence rates.

Unfortunately, this means that convergence time becomes a strong function of both the channel response and the users’ transmit power levels (since both directly affect the correlation matrix \( \mathbf{R} \)). Two principal reasons account for the broad popularity of the LMS algorithm: simplicity of implementation and robustness. The complexity of the LMS algorithm is relatively low: coefficient updates require only \( N_{\text{Cor}} + 1 \) multiplications and \( N_{\text{Cor}} + 1 \) additions every symbol time. As for the robustness of the algorithm, a significant body of research has previously examined this subject. The general consensus is that the LMS algorithm is quite robust.

3.5.2.2 Recursive Least Squares (RLS)

The slow convergence rate of the LMS algorithm can be overcome using recursive least squares (RLS) techniques [Cla93, Hay96]. In addition to its faster convergence, the RLS algorithm also has the desirable property that convergence time is essentially independent of the correlation matrix \( \mathbf{R} \). This means that the algorithm convergence time becomes channel-independent. The principal drawbacks of the RLS algorithm are substantially increased complexity and reduced algorithmic robustness. An important point to note is that the RLS algorithm differs from the LMS in that it minimizes a different error metric. The RLS error metric is given by:
Note that this expression does not include any statistical quantities. Unlike the LMS, the RLS minimization is deterministic, not stochastic. The RLS update equation can be written as

\[
c_n = c_{n-1} + \alpha_n R_n^{-1} r_n e_n^* \tag{3-43}
\]

where

\[
\alpha_n = \frac{1}{1 + r_n^T R_n^{-1} r_n} \tag{3-44}
\]

is the adaptation coefficient, and

\[
R_n^{-1} = R_{n-1}^{-1} \cdot \alpha_n R_{n-1}^{-1} r_n r_n^T R_{n-1}^{-1} \tag{3-45}
\]

is the inverse correlation matrix \( R_n^{-1} \) estimate at the \( n \)th symbol iteration. The RLS algorithm provides \( N_{\text{cor}} \) degrees of freedom in the update direction through the \( N_{\text{cor}} \times N_{\text{cor}} \) dimensional matrix \( \alpha_n R_n^{-1} \) (unlike the LMS algorithm that provides only one degree of freedom). This difference accounts for the faster convergence relative to the LMS. Faster convergence speed is offset by increased implementation complexity for this algorithm. The number of operations required to implement the RLS is proportional to \( N_{\text{cor}}^2 \) for large \( N_{\text{cor}} \). Most of these operations are related to the update of the inverse correlation matrix \( R_n^{-1} \). Like most algorithms that require computation of an inverse, the RLS is also susceptible to stability and robustness problems. Sensitivity to roundoff errors and finite precision effects are other key limitations of the algorithm. Square root versions of the algorithm have been developed to address some of these numerical issues. The square-root Kalman algorithm provides one excellent example that exhibits good numerical properties. However, it still requires a computational complexity of

\[
1.5N_{\text{cor}}^2 + 6.5 N_{\text{cor}} \tag{3-46}
\]

complex-valued multiplications and divisions per output symbol [Pro01].
3.6 Other classes of Multiuser Detectors

There are several other multiuser detection structures that mix different basic ideas to achieve a better one. For example using the decorrelating detector as the first stage is used to improve the first estimation of the multistage detector, because the estimation of the bits in the first stages has a big effect on the performance of the system.

Another example is Zero-Forcing Decision-Feedback (ZF-DF) [Due93], which performs two operations: linear preprocessing followed by a form of SIC detection. The linear operation partially decorrelates the users without enhancing the noise and, by SIC operations, interference is cancelled out from users in descending order from the strongest to weakest one. The ZF-DF needs to decompose the cross-correlation matrix as a multiplication of two triangular matrixes and also estimation of the received signal amplitudes is essential. The performance of the ZF-DF detector depends on the reliability of amplitude estimation. If the soft outputs of the decorrelating detector are used to estimate the amplitudes, the ZF-DF detector is equivalent to the decorrelator detector. If the amplitude estimates are more reliable than those produced by decorrelator detector, the performance of the ZF-DF is better than the decorrelator. If less reliable, the ZF-DF detector performs worse than the decorrelator.

Blind detectors, Neural Networks and Genetic algorithms are other techniques for MUD's that are currently active research topics in CCSR [QAET03].

3.7 Conclusion

Multiuser detection (MUD) is a powerful technique for improving receiver performance in the presence of MAI. Multiuser detectors exploit the structure of the interference in order to improve the desired signal estimate.

A review of the available methods addressed in the literature was given in this chapter concentrating mainly on their advantages and disadvantages. Performance metrics and receiver optimality were introduced.

Linear multiuser detectors are another family that have a reasonable performance. However they suffer from a complexity due to a need to matrix inversion. Their proper implementation is worth investigating. This issue will be considered in chapter 5.

Subtractive interference cancellation methods were another family of multiuser detectors considered in this chapter. The Successive (SIC) and Parallel (PIC) were addressed. The
SIC suffers from a long delay as it processes serially. The PIC had some attractive features such as low delay of processing and simple structure with good performance in the systems that use power control, that make it a powerful candidate for applications.

We also considered the adaptive and blind multiuser detectors. Blind detectors do not need knowledge from the other users, and this enables them to be good candidates for the downlink. However, in the work of this thesis we concentrate on the uplink and hence application in the base station in which case the information of the other users is available.

We also considered other multiuser detection schemes such as genetic algorithm and neural networks in this chapter. However the complexity versus performance issue still remains even for the base-station applications. One of the techniques as far as the complexity-performance trade-off is concerned in the literature is PIC and hence we will develop this further in the next chapter.
Chapter 4  Parallel Interference Cancellation (PIC) and RAKE-IC

4.1 Introduction

Amongst the various interference cancellation methods, parallel interference cancellation (PIC) benefits from several desirable features. In this chapter we will first focus on this method and will consider it in more detail. As this detector has favourable characteristics, it is justifiable to compare our new multiuser detection methods with PIC. The basic idea of PIC is extended to a multiple antennas version and is applied to the WCDMA uplink. The realistic parameters for the WCDMA uplink are considered for this purpose and these parameters are described briefly in this chapter.

Based on the structure of an adaptive partial PIC detector, an adaptive RAKE-IC receiver for CDMA systems is proposed. The basic concept is to maximize the signal to noise ratio of all users in the system by using adaptive algorithms. Performance improvements of the method and the effect of modified combining factors are compared with the standard RAKE receiver. The basic method was improved in some areas: In terms of adaptive
algorithms, NLMS and new reported PFGLMS algorithms were used in simulations and in terms of structure, the idea was extended to operate in multistages.

4.2 Parallel Interference cancellation (PIC) and Partial PIC (PPIC)

4.2.1 Parallel Interference cancellation (PIC)

In the PIC method, the MAI is estimated and cancelled out for all users in a parallel way. The basic multistage PIC detector is shown in Figure 4.1. The first estimates of data bits are obtained from the conventional detector. In the next step, interference caused by the other users is estimated for each user separately in parallel. In Figure 4.1, just one stage of the structure is shown and these stages can be repeated to improve the estimation process. The estimated bits from the conventional detector are then scaled by the amplitude estimates and re-spread by the codes, which produces a delayed estimate of the signal for each user. The summer folds up all but one input signal at each of the outputs, and consequently creates estimation for the MAI and then removes it from the signal. The process can be done in several stages [BN99, BON98]. Several improvements are performed on the basic structure, and some of them are discussed in following sections.

PIC has its own advantages and disadvantages as follow

**Advantages**

- Low delay, at most few bits.
- In some approaches, its structure becomes simple and can be implemented.
- Has good performance for perfect power control.

**Disadvantages**

- Not very robust without power control. Subtractive interference cancellation methods are attractive for the practical implementation of multiuser detection, because of the lower complexity. For practical implementation, the multistage PIC has good computational complexity performance, and is robust to moderate timing errors.

In the rest of this section we examine analytically the default structure. Decision statistics of the \((s+1)^{th}\) stage of the basic parallel IC structure is given as [BW96, CBW99, ESB98]:
Figure 4-1 Basic Structure of Parallel Multistage Detector

\[ y_{k,l}^{(s+1)}(t) = \int_{T+s_{k,l}}^{T+s_{k,l}+T} r_{k,l}^{(s)}(t) s_{k}(t - \tau_{k,l}) \cos(\omega_{l} t + \theta_{k,l}) dt \]  
(4-1)

where the received signal \( r_{k,l}^{(s)}(t) \) for the \( l \)-th path of user \( k \) at stage \( s \) is estimated:

\[ r_{k,l}^{(s)}(t) = r(t) - \sum_{j=1}^{K} \sum_{\lambda=1}^{L_{k}} s_{j,l}^{(s)}(t) \]
(4-2)

And the signal \( \hat{S}_{j,l}^{(s)}(t) \) corresponds to the estimated signal for path \( \lambda \) of user \( j \) at stage \( s \). This signal is reconstructed according to

\[ \hat{S}_{k,\lambda}^{(s)}(t) = \frac{2}{T_{b}} s_{k}(t - \tau_{k,\lambda}) \cos(\omega_{l} t + \theta_{k,\lambda}) \sum_{l=-\infty}^{\infty} |\hat{b}_{k,l}^{(s)}| p_{T}(t - iT_{b} - \tau_{k,\lambda}) \]  
(4-3)

where \( \hat{b}_{k,l}^{(s)} \) is the \( l \)-th bit estimate for user \( k \) at stage \( s \), and \( p_{T}(t) \) is a unit pulse function of duration \( T_{b} \) equal to the bit period. The bit estimates \( \hat{b}_{k,l}^{(s)} \) are compared via maximal ratio combining of the decision metrics from the \( L_{k} \) resolvable multipath components.
where $\hat{\alpha}_{k,j,l}$ is an estimate of the multipath attenuation coefficients $\alpha_{k,j,l}$.

### 4.2.2 Partial PIC

The performance of the PIC detector is greatly dependent on the estimation of the first stages. In heavy loaded systems the error propagates through the stages and the performance degrades and becomes even worse than conventional detectors. One way to mitigate this effect is by reducing the effect of the first estimated values by multiplying them with a set of coefficients less than 1 [DS94, DS95, DSR98]. In this case Eq.(4-2) changes to:

$$\tilde{r}^{(s)}_{k,j}(t) = r(t) - C^{(s)}_k \sum_{j=1}^{K} \sum_{\Delta \neq \Delta_{ij}} \hat{\alpha}^{(s)}_{j,\Delta} s^{(s)}_{j,\Delta}(t)$$  \hspace{1cm} (4-5)

For the first stages this number is smaller than the same coefficients for the final stages. Because in the final stages the estimated values are more reliable than the ones obtained in the early stages. This method is called Partial PIC because in each stage a part of the interference is cancelled [DS94, DS95, DSR98, BN99].

### 4.2.3 Simplified PIC

By considering the basic formulas of the parallel IC, and rewriting them in an appropriate form, its structure can be simplified [CBW99]. The output of this structure can be written as:

$$y^{(s+1)}_{k,j} = C^{(s)}_k y^{(s)}_{k,j} + \text{Re} \left[ \int_{(i-1)T+\tau_k}^{iT+\tau_k} \tilde{r}^{(s)}(t) s_k(t - \tau_k)e^{-j\theta_k} dt \right]$$  \hspace{1cm} (4-6)

The reduction in complexity over the straightforward full-cancellation comes from the fact that the residual signal is identical for all users, and thus needs to be generated only once per stage. This residual signal is added to the decision statistics of the initial stage. The block diagram of the simplified structure is shown in Figure 4.2.
Based on the idea shown in Figure 4.2 we have introduced another simplified structure for PIC as shown on Figure 4.3. This structure performs the same task as PIC, however it calculates cumulative regenerated signals only once in its operation. For processing each individual user, the regenerated signal of that user is injected before matched filtering.

![Figure 4-2 Block Diagram of the coherent PPIC with simplified structure](image)

![Figure 4-3 Another simplified structure for PIC](image)
4.2.4 Performance of PIC

Based on analytical analysis, BER performance of the PIC detector operating in AWGN channel can be expressed as [CBW99]:

\[
p_{\text{PIC}}(\sigma) = Q\left[\frac{1}{2E_b/N_0} \left(1 - \frac{K - 1}{3N}\right)^t + \frac{1}{(3N)^t} \left(\frac{(K-1)^t}{J} - (-1)^t \sum_{j=1}^{J} \frac{P_j}{P_k} + 1 \right)\right]^{-1/2}
\]

where \(K\) stands for the number of users, \(N\) is the processing gain, and \(P_k\) is the transmitted power of user \(k\).

For the case of equal signal powers (\(P=P_1=P_2=\ldots=P_k\)), the expression simplifies to:

\[
p_{\text{PIC}}(\sigma) = Q\left[\frac{1}{2E_b/N_0} \left(1 - \frac{K - 1}{3N}\right)^t + \left(\frac{K-1}{3N}\right)^t\right]^{-1/2}
\]

Based on Eq.(4-8), it is possible that by increasing the number of stages, the BER degrades. Under the following conditions this divergence occurs:

\[
\left\{\frac{K - 1}{3N} > 1\right\} \cup \left\{\frac{E_b}{N_0} \leq \frac{1}{2\left(1 - \frac{K - 1}{3N}\right)}\right\} \Rightarrow p_{\text{PIC}}^{(s+1)}(\sigma) > p_{\text{PIC}}^{(s)}(\sigma)
\]

which means that the system operates either in a heavy loaded scenario or in low signal to noise ratio regimes. Other conclusions for PIC are:

1) The main performance can be achieved in early stages of operation.
2) Even by using infinite stages, interference effects do not cancel. Figure 4.4 shows the performance of PIC operating in multiple stages and in AWGN channel. All users have \(E_b/N_0=8\,\text{dB}\) and their processing gain is equal to 256. As can be seen, the main gain is achieved in the early stages of operation and it completely outperforms the matched filter.
Chapter 4. Parallel Interference Cancellation (PIC) and RAKE-IC

Figure 4-4 Performance of multistage PIC in AWGN channel.

Provided the conditions of Eq.(4-9) do not hold, the number of stages can be allowed to approach infinity. In this case we have:

\[
\lim_{s \to \infty} P_{\text{pic},(s)}(\sigma) = Q \left[ \frac{1}{2} \left( \frac{E_b}{N_0} \left( \frac{1}{K-1} \right) \right)^{-1/2} \right]
\]  

(4-10)

According to this formula, by increasing the number of stages, PIC is not capable of completely removing the interference.

Figure 4.5 shows the performance of PIC operating in multiple stages and in AWGN channel. All users have $E_b/N_0=12$dB and their processing gain is equal to 32. The performance of the PIC saturates in heavy loaded scenarios and by increasing the number of stages cannot be resolved.
4.2.5 Adaptive PPIC

One of the problems of the basic PIC structure is that in each stage interference caused by the other users must be calculated for each user and this process repeated in each stage. The value of the coefficients can be optimised via adaptive algorithms. Figure 4.6 shows the structure of adaptive PPIC.

We will now go on to examine the PIC performance in real WCDMA channels.

4.3 WCDMA Physical Layer

This section provides a layer 1 (also termed as physical layer) description of the radio access network of a WCDMA system operating in the FDD mode. The spreading and modulation operation for the Dedicated Physical Channels (DPCH) for uplink is illustrated and the spreading and scrambling codes are investigated.
4.3.1 Physical Channel Structure

WCDMA defines two dedicated physical channels for the uplink:

- Dedicated Physical Data Channel (DPDCH): to carry dedicated data generated at layer 2 and above.
- Dedicated Physical Control Channel (DPCCH): to carry layer 1 control information.

Each connection is allocated one DPCCH and zero, one or several DPDCHs.

The spreading and modulation for the DPDCH and the DPCCH for the uplink are described in the following two subsections.

4.3.2 Uplink Frame Structure

Figure 4.7 shows the principal frame structure of the uplink dedicated physical channels. Each frame of 10 ms is split into 15 slots. Each slot is of length 2560 chips, corresponding to one power control period. The super frame length is 720 ms; i.e. a super frame corresponds to 72 frames.

Pilot bits assist coherent demodulation and channel estimation. TFCI stands for transport format combination indicator and is used to indicate and identify several simultaneous services.
Feedback Information (FBI) bits are to be used to support techniques requiring feedback. TPC which stands for transmit power control is used for power control purposes. The exact number of bits of these different uplink DPCCH fields is given in [3GPP].

The parameter \( k \) in Figure 4.7 determines the number of bits in each slot. It is related to the spreading factor (SF) of the physical channel. The spreading factor ranges from 256 down to 4 and is selected according to the data rate.

### 4.3.3 Uplink Spreading and Modulation

Referring to Figure 4.8 as a baseband model for a mobile terminal transmitting in the uplink, each mobile transmits its information through both data and control channels (DPDCH and DPCCH). Data information is spread by OVSF codes \((c_d)\) with suitable spreading factor. The signal transmitted by the \( k \)th mobile consists of time-slots of duration 0.666ms and every 15 time-slots make a frame. The chip rate for both data and control channel is fixed and is equal to 3.84Mcps. In each time-slot there are 10 control bits that are spread with OVSF codes \((c_c)\) with spreading factor of 256. Depending on the service type [30 to 960 Kbps], in each time-slot, [20 to 640] data bits are transmitted. The real-valued spread signals are weighted by gain factors, \( \beta_c \) for DPCCH, \( \beta_d \) for all DPDCHs. The \( \beta_c \) and \( \beta_d \) values are signalled by higher layers or calculated as described in [3GPP]. At every instant in time, at least one of the values \( \beta_c \) and \( \beta_d \) has the amplitude 1.0. After the weighting, the stream of real-valued chips on the I- and Q-branches are then summed and treated as a complex-valued stream of chips.
This complex-valued signal is then scrambled by the complex-valued scrambling code $s_{ck}$. Here, short scramble codes, that are complex numbers, are considered. Before transmitting, the chips are shaped with a filter whose frequency response is a square root raised cosine with roll-off factor of 0.22. The scheme of modulation is BPSK.

### 4.3.3.1 Channelisation codes

The spreading code, as the name suggests, spreads the data to the chip rate of 3.84 Mcps. The most important purpose of the spreading codes is to help preserve orthogonality amongst different physical channels of the uplink user. OVSF codes are employed as uplink spreading codes. The OVSF codes can be defined using the code tree of Figure 4.9.

In Figure 4.9, the channelisation codes are uniquely described as $c_{ch, SF, m}$, where SF is the spreading factor of the code and $m$ is the code number, $0 \leq m \leq SF-1$.

In allocating codes for DPCCH and DPDCH the following applies:

- The DPCCH always uses $c_{ch, 256,0}$.
- When only one DPDCH is to be transmitted, DPDCH$_1$ is spread by code $c_{d,1} = c_{ch, SF, m}$ where SF is the spreading factor of DPDCH$_1$ and $m = SF / 4$. 

![Figure 4-8 Spreading for uplink DPCCH and DPDCHs](image)
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4.3.3.2 Scrambling codes

All uplink physical channels are subjected to scrambling with a complex-valued scrambling code. The DPCCH/DPDCH may be scrambled by either long or short scrambling codes. WCDMA systems are designed to provide reasonable service quality without using complex receivers that use joint detection of multiple user signals. However, if required, short scrambling codes can be used at the uplink to implement multiuser receivers of moderate complexity. The scrambling codes are designed so that they have very low cross-correlation that ensures good MAI rejection capability. The long and short scrambling codes are built from constituent sequences. In the next section, the short scrambling codes that make the implementation of multiuser detectors easier and used in this thesis, are described.

4.3.3.2.1 Short scrambling sequence

The short scrambling sequences $c_{\text{short,1},n}(i)$ and $c_{\text{short,2},n}(i)$ are defined from a sequence from the family of periodically extended S(2) codes.

Let $n_{23}n_{22}...n_0$ be the 24 bit binary representation of the code number $n$.

The $n^{th}$ quaternary S(2) sequence $z_n(i)$, $0 \leq n \leq 16777215$, is obtained by modulo 4 addition of three sequences, a quaternary sequence $a(i)$ and two binary sequences $b(i)$ and $d(i)$,
where the initial loading of the three sequences is determined from the code number $n$. The sequence $z_n(i)$ of length 255 is generated according to the following relation:

$$z_n(i) = a(i) + 2b(i) + 2d(i) \mod 4, \quad i = 0, 1, \ldots, 254; \quad (4-11)$$

where the quaternary sequence $a(i)$ is generated recursively by the polynomial $g_0(x) = x^8 + x^5 + 3x^3 + x^2 + 2x + 1$ as:

- $a(0) = 2n_0 + 1 \mod 4$; \hspace{1cm} (4-12a)
- $a(i) = 2n_i \mod 4, \quad i = 1, 2, \ldots, 7$; \hspace{1cm} (4-12b)
- $a(i) = 3a(i-3) + a(i-5) + 3a(i-6) + 2a(i-7) + 3a(i-8) \mod 4, \quad i = 8, 9, \ldots, 254; \hspace{1cm} (4-12c)$

and the binary sequence $b(i)$ is generated recursively by the polynomial $g_1(x) = x^8 + x^7 + x^5 + x + 1$ as:

$$b(i) = n_{8i} \mod 2, \quad i = 0, 1, \ldots, 7, \quad (4-13a)$$

$$b(i) = b(i-1) + b(i-3) + b(i-7) + b(i-8) \mod 2, \quad i = 8, 9, \ldots, 254, \quad (4-13b)$$

and the binary sequence $d(i)$ is generated recursively by the polynomial $g_2(x) = x^8 + x^7 + x^5 + x^4 + 1$ as:

$$d(i) = n_{16i} \mod 2, \quad i = 0, 1, \ldots, 7; \quad (4-14)$$

$$d(i) = d(i-1) + d(i-3) + d(i-4) + d(i-8) \mod 2, \quad i = 8, 9, \ldots, 254. \quad (4-14)$$

The sequence $z_n(i)$ is extended to length 256 chips by setting $z_n(255) = z_n(0)$. The mapping from $z_n(i)$ to the real-valued binary sequences $c_{\text{short},1,n}(i)$ and $c_{\text{short},2,n}(i)$, $i = 0, 1, \ldots, 255$ is defined in Table (4.1).

<table>
<thead>
<tr>
<th>$z_n(i)$</th>
<th>$c_{\text{short},1,n}(i)$</th>
<th>$c_{\text{short},2,n}(i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>+1</td>
<td>+1</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
<td>+1</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>+1</td>
<td>-1</td>
</tr>
</tbody>
</table>

**Table 4-1**: Mapping from $z_n(i)$ to $c_{\text{short},1,n}(i)$ and $c_{\text{short},2,n}(i)$, $i = 0, 1, \ldots, 255$

Finally, the complex-valued short scrambling sequence $c_{\text{short},n}$ is defined as:
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4.4 PPIC structure considering multiple antennas

In this section, a version of PPIC, which utilizes multiple antennas, is considered and applied to uplink. For a wideband CDMA system in an L-path fading environment with $M$ different antennas and having $K$ active users, the received signal by the $m^{th}$ antenna in a window of $i \in [1, N_b]$ can be expressed as:

$$r_m(t) = \sum_{k=1}^{K} \sum_{l=1}^{N_b} b_k^{(l)} \sqrt{P_{k,m}} \sum_{i=1}^{L} \alpha_{k,l,m}^{(i)} s_k(t - iT - \tau_{k,l,m}) + n_m(t)$$

(4-16)

where $b_k^{(l)}$ is the transmitted data symbol at symbol interval $n$, $P_{k,m}$ is the energy per symbol of the corresponding real bandpass signal, $\tau_{k,l,m}$ is the delay of $l^{th}$ multipath component of $k^{th}$ user for antenna $m$. The $\alpha_{k,l,m}^{(i)}$ is the complex gain representing the effect of the Rayleigh fading, $n_m(t)$ complex zero mean additive white Gaussian noise in antenna $m$ and $s_k(t)$ is signature waveform of user $k$. 

Figure 4-10: Uplink short scrambling sequence generator for 255 chip sequence

$$C_{\text{short,n}}(i) = c_{\text{short,1,n}}(i \mod 256)(1 + (-1)^i c_{\text{short,2,n}}(2(i \mod 256)/2))$$

(4-15)

where $i = 0, 1, 2, \ldots$ and $\lfloor \cdot \rfloor$ denotes rounding to nearest lower integer. An implementation of the short scrambling sequence generator for the 255 chips sequence to be extended by one chip is shown in Figure 4-10.
In the uplink, each user transmits data and control information with different rates and codes that will affect the received signal. In this case the received signal of each user is composed of two signals similar to those in Eq.(4-16) with little differences in $b_k^{(i)}$, $P_{k,m}$, and $s_k$.

$$r_m(t) = \sum_{k=1}^{K} \sum_{n=1}^{N_k} b_k^{(n)} \beta_d \sqrt{P_{k,m}} \sum_{l=1}^{L} \alpha_{k,l,m} c_{d,k} (t-n_d T_d - \tau_{k,l,m}) S_{c,d}(t-n_d T_d + n_m(t)) +$$

$$\sum_{k=1}^{K} \sum_{n=1}^{N_k} b_k^{(n)} \beta_c \sqrt{P_{k,m}} \sum_{l=1}^{L} \alpha_{k,l,m} c_{c} (t-n_c T_c - \tau_{k,l,m}) S_{c,c}(t-n_c T_c + n_m(t)) + n_m(t) \quad (4-17)$$

where $S_{c,d}(t)$ and $S_{c,c}(t)$ indicates the scrambling codes for data and control channels for user $k$. The outputs of matched filters of all antennas for all users and multipath components produce sufficient statistics for the detection of data symbols. The sampled output of the matched filter in interval $n$ for user $k$, path $l$ and antenna $m$ is:
Based on the matched filter outputs, a structure for multi-antenna PPIC is used (Figure. 4.11). The channel estimation uses the pilot bits in the control channel and consists of a bank of matched filters and uses a basic block-by-block average algorithm. For channel estimation other sophisticated algorithms like weighted-multislot or Wiener filtering could be used. The channel estimation values are used in RAKE combining and regeneration of users. By little modification to Eq.(4-18) the structure of MF which does the De-Scrambling and De-Spreading at the same time and an PPIC structure can be written as follows:

\[
y_{k,j,m,n}(t) = \int_{nT_m}^{(n+1)T_m} r_m(t) c_k(t - nT - \tau_{k,j,m}) dt
\]  

(4-18)

where in this equation \( n \) could be either \( n_d \) or \( n_c \) and \( T \) could be \( T_c \) or \( T_d \). Received signal of antenna \( m \) at stage \( s \) for user \( k \), \( \hat{r}_{k,m}(t) \), is estimated according to:

\[
\hat{r}_{k,m}(t) = r_m(t) - C_k^{(s)} \sum_{j=1}^{K} \sum_{l=1}^{L} \hat{s}_{j,l,m}^{(s)}(t)
\]  

(4-20)

and the signal \( \hat{s}_{j,l,m}^{(s)}(t) \) corresponds to the estimated signal for antenna \( m \), path \( l \) of user \( j \) at stage \( s \). The partial coefficients are represented by \( C_k^{(s)} \). This signal can be constructed based on \( \hat{b}_{k,l}^{(s)} \) which are the estimated bits at stage \( s \). \( \hat{b}_{k,l}^{(s)} \) are produced from maximum-ratio combining of decision statistics of all paths and antennas:

\[
\hat{b}_{k,l}^{(s)} = \text{sgn} \left[ \sum_{i=1}^{M} \sum_{m=1}^{N} \hat{c}_{k,j,m}^{(s)} * y_{k,j,m}^{(s)}(i) \right]
\]  

(4-21)

where \( \hat{c}_{k,j,m}^{(s)} \) is the estimated value of multipath attenuation coefficient \( c_{k,j,m}^{(s)} \) for symbol \( i \), path \( l \), antenna \( m \) and user \( k \). These coefficients can be extracted from the channel estimation block. The idea of partial coefficients is based on the fact that errors in the first stages of estimation can propagate through the stages and decrease the performance. The second reason arises from a bias in the decision statistics in the early stages [CBW99]. Simply by introducing coefficients between stages this effect could be diminished.
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### Figure 4-12 Effect of partial coefficients on the performance

The values of partial coefficients depend on the loading. For example in the situations where estimations in early stages are reliable enough, applying small coefficients could decrease the performance of succeeding stages since they reduce the functionality of each stage. This is the case that occurs in light-loaded systems or the systems working in high SNR. Hence after doing a simulation and finding the best value for one antenna, it is applied for other succeeding simulations described in next section.

#### 4.4.1 Performance of PPIC for uplink

A scenario involving 20 users with equal powers is considered. Each mobile transmits control and data symbols with spreading factors of 256 and 128 respectively. Short scrambling is assumed. The radio channel model is Vehicular-A at 110 km/h and fading from one antenna to another antenna is independent. Three fingers per antenna are assumed for combining in RAKE reception. Figure 4.12 illustrates the effect of partial coefficients on the performance of the system for one antenna. Having found the best value for coefficient, it is applied for another simulation. In Figure 4.13, the complete simulation for PPIC with one and two antennas is shown.
4.5 RAKE-IC

In this section an adaptive receiver that attempts to improve the standard RAKE is presented. The receiver uses adaptive algorithms and does not treat the interference just as unpredictable Gaussian noise. Based on this criterion, some modifying coefficients are generated that outperforms the Maximum Ration Combiner (MRC). The idea is extended for a multistage structure. Its adaptive criteria uses Least Mean Square (LMS) and Partially Filtered Gradient LMS (PFGLMS) algorithm [Lim99]. The system is simulated and results are compared with standard RAKE.

The standard RAKE receiver is an optimum receiver (in terms of maximizing the signal to noise ratio) for capturing different paths of received signal in multipath fading environments considering the fact that the noise is white Gaussian. However, this receiver treats the interference of the other users as Gaussian noise and does not use any kind of Interference Cancellation (IC) scheme. As a result it is an interference-limited detector. This problem can be seen from another point of view that standard RAKE is not optimised for the whole system, as it is optimised per each user. The scheme presented in forthcoming sections considers the whole system as an entity and to improve the total
signal to noise ratio. Unlike the RAKE whose structure is derived from analytical expressions, due to complexities that arises in multiuser scenarios, RAKE-IC uses adaptive algorithms to find a better substitute for the RAKE.

### 4.5.1 RAKE-IC Structure

The proposed structure is shown in Figure 4-14. Instead of using Maximum Ratio Combiners (MRC), another set of coefficients are used.

### 4.5.2 RAKE-IC using LMS

We define an optimal cost function in terms of the squared Euclidean distance between the received signal $r(t)$ and the weighted sum of the estimates of all users’ signals. In AWGN channel we have:

$$
\varepsilon = \int_0^T \left| r(t) - \sum_{i=1}^K \lambda_i \sqrt{P_i} \hat{b}_i s_i(t) \right|^2 dt
$$

(4-22)

where $\lambda_i$ is the weight for the $i^{th}$ user. $\hat{b}_i$ denotes the estimate for $b_i$. We try to minimize $\varepsilon$ with a set of optimal weights $\{\lambda_i\}$

- If all $\lambda_i = 1$, then searching for all $\hat{b}_i$, will lead to the maximum-likelihood detector.

- Without noise, the optimum set of $\lambda_i$'s are $-1$ or $+1$, depending on whether the estimate $\hat{b}_i$ is correct or not. With the optimal weights, the weighted sum of all estimated signals is exactly the received signal, and hence, $\varepsilon$ is zero. These new factors are calculated using an adaptive algorithm and the criteria for adaptation is Minimum Square Error (MSE) per chip:

$$
\psi(m) = \frac{1}{2} E(e_m^2)
$$

(4-23)

where $e(m)$ stands for error per chip:

$$
e(m) = r(m) - \hat{r}(m)
$$

(4-24)
By using this criterion, the receiver makes the regenerated signal as close as possible to the received signal. Regeneration of the signal is based on re-spreading and a new set of combiner coefficients:

\[
\hat{r}(m) = \sum_{i=1}^{K} \sum_{l=1}^{L} \hat{S}_i(m,l) \lambda_{ni} 
\]

(4-25)

that:

\[
\hat{S}_i(m,l) = s_i(m) \hat{b}_l 
\]

(4-26)

According to the LMS algorithm, the desired information (RAKE-IC coefficients) are updated by the following equation:

\[
\lambda(m + 1) = \lambda(m) + \mu \cdot g_m 
\]

(4-27)

\(g(m)\) is an estimation for gradient in LMS and can be calculated by:

\[
g_m = \frac{\mu}{\|\hat{s}(m)\|^2} [\hat{s}(m) e(m)]^* 
\]

(4-28)

and finally, decision for each bit is based on a new combination scheme. The sign of the new decision variable is the output of the detector.
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\[ y_i = \text{Re} \left\{ \sum_{l=1}^{L} \hat{b}_l \cdot \mathcal{A}_l \right\} \]  \hspace{1cm} (4-29)

and

\[ \hat{b}_i = \text{sgn}[y_i] \]  \hspace{1cm} (4-30)

It is worth mentioning that the idea of RAKE-IC is also applicable to the AWGN channels. Because by introducing a set of coefficients (like the ones used in a fading multipath environment), there will be a chance to find proper coefficients that change the sign of the wrongly detected bits in BPSK modulation scheme. In this case the coefficients are not used as combining factors. We are going to find better estimates for detected bits by matched filters and suppress the interference effect, indirectly.

Figure 4.15 shows the performance of the RAKE-IC structure in comparison with a conventional matched filter for 15, 20 and 30 users having spreading factors of 32. The RAKE-IC uses the normalized LMS for its adaptive algorithm.

Figure 4.16 shows the performance of the RAKE-IC structure in comparison with the conventional RAKE in a fading environment.

This good idea can be further improved in two ways. The first is using a better algorithm instead of LMS. The second approach is using multistage processing. These two schemes will be discussed in the following sections.

4.5.3 RAKE-IC using filtered gradient LMS (FGLMS) and partial FGLMS (PFGLMS)

In the LMS algorithm, the MSE is defined in Eq.(4-23). As an accurate estimate of the MSE, a time averaged operation with exponentially weighted least-square errors can be defined by [Lim99]:

\[ \psi(m) = \frac{1}{2} \sum_{i=0}^{m} \beta^{m-i} \epsilon_i^2 \]  \hspace{1cm} (4-31)
where $\beta$ is a forgetting factor ($0 < \beta < 1$). The MSE estimate of Eq.(4-31) can be recursively represented as:

$$\psi(m) = \beta \psi(m-1) + \frac{1}{2} e_m^2$$  \hspace{1cm} (4-32)

and its negative gradient vector becomes:

$$\tilde{g}_m = -\nabla \psi(m) = \beta \tilde{g}_{m-1} + g_m$$  \hspace{1cm} (4-33)

g_m is defined in Eq.(4-28) and $\tilde{g}_m$ is the new gradient. This modification constitutes Filtered Gradient LMS (FGLMS) algorithm.

A more effective estimate of the MSE can be obtained by modifying the weighted least-square errors as follows:
Figure 4-16 Performance in fading environment, 10 users, PG=32

\[
\psi(m) = \frac{1}{2} \left( e_m^2 + \sum_{i=0}^{m} \beta^{m-i} \delta e_i^2 \right) = \frac{1}{2} e_m^2 + \hat{\psi}(m)
\]

where

\[
\hat{\psi}(m) = \frac{1}{2} \left( \sum_{i=0}^{m} \beta^{m-i} \delta e_i^2 \right)
\]

and \( \delta \) is a scaling factor \( 0<\delta<1 \) required to take a fractional portion of square errors.

The negative gradient vector is then given by:

\[
\bar{g}_m = -\nabla \psi(m) = g_m + \hat{g}_m
\]

with

\[
\hat{g}_m = \beta \hat{g}_{m-1} + \delta g_m
\]
Figure 4-17 Performance comparison of PFGLMS and NLMS. 10 users, PG=32

Now the negative gradient vector consists of the instantaneous gradient vector and the filtered gradient vector given by Eq(4-36).

Figure 4-17 compares the performance of NLMS and PFGLMS algorithms for a scenario containing 15 users using spreading factor of 32.

4.5.4 Multi-stage RAKE-IC receiver:

The idea of RAKE-IC can be easily extended to a multistage structure. The only difference is that regeneration of information is based on the output of the first stage of the RAKE-IC rather than the output of the RAKE. Intuitively if the first stage of the processing has been able to improve the decisions, the second stage (which does the same kind of processing) would make better decisions, as its input is further improved. Figure 4-18 shows the performance of RAKE-IC using NLMS algorithms implemented in one stage and two stages for a scenario containing 15 users using spreading factor of 32.

Figure 4-19 shows the performance of RAKE-IC using different combinations of adaptive algorithms implemented in multiple stages for a scenario containing 15 users using spreading factor of 32.
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**Figure 4-18** Performance of multistage RAKE-IC using NLMS, 10 users, PG=32, AWGN channel

**Figure 4-19** Comparison between different combinations of RAKE-IC
4.6 Conclusions

Amongst the various interference cancellation methods, PIC benefits from several desirable features. In this chapter this method was considered in more detail. Since this detector has good characteristics, it was acceptable as a comparator for the new multiuser detection methods. Extension of the PIC structure to multiple antennas partial PIC and application to the WCDMA uplink was carried out. The realistic parameters for WCDMA uplink were considered for this purpose and these parameters were also briefly described. Performance improvements due to the new method and the effect of real channel estimation were simulated and compared with the standard RAKE receiver.

Based on the structure of an adaptive partial PIC detector, an adaptive RAKE-IC receiver for CDMA systems was proposed. Its structure combines the ideas of the RAKE receiver and the PIC. RAKE-IC uses the concept of maximizing the signal to noise ratio of all users in the system by using adaptive algorithms. Performance improvements of the new method and the effect of modified combining factors are compared with the standard RAKE receiver. The basic method was improved in two areas: Firstly in terms of the adaptive algorithms, NLMS and newly reported PFGLMS algorithms and secondly in terms of the structure, with the idea of extension to operate in multistages.

The simulation results showed that the RAKE-IC performs better than the standard RAKE receiver. In our simulations, coding schemes were not considered. In realistic systems that use coding, the RAKE-IC will perform well in region of $E_b/N_0$ that are commonly found in practical systems.

The partial PIC can achieve a better performance than the PIC and uses the same structure as PIC, however calculation of the partial coefficients should be carried out via trial and error or adaptive algorithms.

In the following chapters we will consider the other methods of multiuser detection and use the performance of the partial PIC as a baseline in our comparisons.

Having considered and evaluated the performance of the RAKE-IC we will now turn attention to other class of multiuser detection –linear multiuser detectors- and assess their performance. The aim is to compare their performance with the IC schemes.
Chapter 5 Iterative linear multiuser detection

5.1 Introduction

The family of linear detectors for multiuser detection in CDMA are good candidates for suppressing interference levels. However, implementation of these detectors suffers from computational complexity due to a need for inverting large matrices. Some methods for simplifying the inversion process have been proposed that are mainly based on a series expansion. Use of Taylor series expansion is addressed in the literature [KHI83, Ver93]. One of the main obstacles is that convergence of this series relies on parameters that depend on the values of eigenvalue of the correlation matrix. Calculation of eigenvalues is a large number crunching process that in practice renders it unusable. Some authors have tried to tackle this issue, for example [LL98] proposes a condition for convergence of the series and in consequence uses a simplified structure. However, this method is not accurate and in consequence ends up with a slow convergence rate. Using convergence results for weight optimisation is also addressed in the literature [MV01].

In this chapter the main focus is on the iterative implementation of linear multiuser detectors. Firstly, a canonical model for the received signal and the linear detectors will be presented. This model enables us to develop the idea of linear multiuser detectors and is
different from the one presented in previous chapters, as it represents the system in matrix format.

Then we will focus on the Taylor expansion of the inverted matrix and its convergence rate. The parameters involved will be investigated and by analytical analysis using the Rayleigh-Ritz theorem an improved condition to increase the convergence rate for synchronous systems will be derived. This idea, however, for asynchronous systems does not perform as well as expected. We thus tackled the problem with another method in linear algebra know as the Gershgorin theorem to seek improved performance.

The method was implemented and its performance found promising. Using the Gershgorin theorem, a more accurate, yet simple, condition for convergence is derived. The results are simulated and compared. We also investigate the signal to noise ratio and the weight optimisation method, addressed in the literature, and will compare our method with theirs in terms of BER.

Matrix inversion using the Fourier algorithm that has recently been reported [VHG01], is also addressed and investigated. An analysis for complexity of our method is also presented and is compared with the complexity of the Fourier algorithm.

5.2 Mobile Uplink Modelling for linear multiuser detection

As the main purpose of this chapter is iterative implementation of linear multiuser detectors, a suitable model for both transmitter and receiver is needed.

The continuous signal transmitted by the $k^{th}$ mobile consists of time-slots with every 15 time-slots constituting a frame. Each time-slot carries a stream of $N_c^{(k)} = 10$ control bits (15 kbits/s) and (at least) one of $N_c^{(k)} \in \{20,40,80,160,320,640\}$ information bits. The modulation scheme is BPSK where, for the most basic service, data bits (representing speech) are mapped on to antipodal in-phase symbols $b_d^{(k)} \in \mathbb{R}^{N_d}$ and control bits (representing pilot symbols, power control and rate information) are mapped on to antipodal quadrature symbols $b_c^{(k)} \in \mathbb{R}^{N_c}$. Control and data symbol sequences are spread by real orthogonal Walsh codes of lengths $Q_c^{(k)} = Q_c = 256$ and $Q_d^{(k)} \in \{128,64,32,16,8,4\}$ chips respectively, depending on the service type. In any time-slot $N_c^{(k)}Q_c^{(k)} = N_d^{(k)}Q_d^{(k)} = 2560$ for all spreading factors and data rates, implying a fixed chip-rate of 3.84 Mchips/s. Note that even for the most basic speech service, the signal transmitted by a mobile carries symbol sequences at two different rates. The spread (and complex) time-slot signal is scrambled by a mobile-specific complex scrambling code. Both short and long scrambling
are defined. In short scrambling, \( c_e \in C^{Q_e} \), while in long scrambling, \( c_e \in C^{16Q_c N_c} \). Short scrambling allows significant reductions in the complexity of detection and is assumed in this chapter. The modulated, spread and scrambled signal is distorted by a composite channel impulse response of length \( W \) chips. The transmit filter is specified to have a square root raised cosine frequency response with a roll-off factor of \( \alpha = 0.22 \). Extraction of the transmitted data may be performed at the base-station receiver through the use of single-user coherent RAKE detection along with pilot-assisted channel estimation. The use of multi-user detection is an attempt to overcome the deficiencies of the RAKE receiver in dealing with multiple-access interference.

5.2.1 SIGNAL MODEL

5.2.1.1 Single Antenna

Consider an M-antenna base station receiver. The received signal, \( (m)r \), corresponding to a time-slot sampled at \( \xi \) times the chip-rate at the output of the \( m^{th} \) antenna receive filter can be represented as [SROL99]

\[
(m)r = (m)U b + (m)n
\]  

where \( (m)U \in C^{[g(N_c Q_c + W - 1) + 1] \times N} \) is the convolution matrix at antenna-\( m \), \( b \in R^{N_b} \) is the vector of \( N_b = \sum_k N(k) \) \( k \) (data and control) symbols transmitted by all \( K \) users within one time-slot, \( 0 < \tau_1 < \tau_2 < ... < \tau_K < \xi Q_c - 1 \) are the user asynchronous delays and \( (m)n \) represents cellular interference and thermal noise. Efficient implementation of multiuser detection relies heavily on the exploitation of the structure of \( (m)U \). As shown in Figure 5.1, when representing a block-length of one time-slot, \( (m)U \) can be completely defined via sub-matrices \( (m)B_1 \) to \( (m)B_{N_c} \) each containing \( Q_c \) columns and a maximum of \( \xi(2Q_c + W - 1) - 1 \) rows such that if \( M^{(k)} \) is the number of codes transmitted by user-\( k \) and \( Q_{(k)} = Q_c \), then

\[
K_c = \sum_{k=1}^{K} \sum_{i=1}^{M^{(k)}} \frac{Q_c}{Q_{(i)}}
\]  

A possible (but not unique) structure for \( (m)B_n \) can be described by considering an example where \( K = 2 \) users are active on the uplink. User-1 transmits \( M^{(1)} = 2 \) codes with spreading factors of \( Q_1^{(1)} = 256 \) and \( Q_2^{(1)} = 128 \), while user-2 transmits \( M^{(2)} = 3 \) codes with spreading...
factors of \( Q_1^{(2)} = 256, \ Q_2^{(2)} = 128 \) and \( Q_3^{(2)} = 64 \). The detailed structure of a submatrix \((m)B_n\) is illustrated graphically in Figure 5.2.

**Figure 5-1** Structure of the convolution matrix

**Figure 5-2** Structure of the sub-matrixes shown in Figure 5.1
The non-zero elements of the columns of $^{(m)}B_n$ corresponding to the $v^{th}$ symbol transmitted on the $i^{th}$ code of the $k^{th}$ user are given by

$$f^cG_i^{(k)}\left\{ [c_{sc}^{(k)}]_{j(v-1)Q^{(k)}+1:jQ^{(k)}} \cdot c_i^{(k)} \right\} = ^{(m)}h_n^{(k)}$$

(5-3)

where operators $\circ$ and $\ast$ denote elemental multiplication and sequence convolution respectively, $\alpha=0$ or 1 depending on whether the symbol is transmitted in phase or in quadrature, $G_i^{(k)}$ is a scale factor and

$c_i^{(k)} \in \mathbb{C}^{5Q(k)} = \text{Upsampled } i^{th} \text{ Walsh code.}$

$[c_{sc}^{(k)}]_{1,1} \in \mathbb{C}^{5Q(k)} = \text{Upsampled scrambling sequence segment.}$

$^{(m)}h_n^{(k)} \in \mathbb{C}^{(w-1)+i} = \text{Oversampled channel impulse response during } n^{th} \text{ control epoch at antenna-}m.$

Under circumstances where short scrambling is used and the channel can be assumed fixed over a time-slot period, $^{(m)}B_1 = ^{(m)}B_2 \ldots = ^{(m)}B_{Nc} = ^{(m)}B$, and so matrix $^{(m)}U$ can be completely defined via a single sub-matrix. The implementation of MUD algorithms invariably involves the correlation matrix

$^{(m)}R = ^{(m)}U^H \times ^{(m)}U \in \mathbb{C}^{NcKc \times NcKc}$

(5-4)

Given the above structure of $^{(m)}U$, the Hermitian product $^{(m)}R$ is not only block-banded and block multi-diagonal, but is also block-Toeplitz. In general, $^{(m)}R$ has a block $(2\rho_T +1)$-diagonal structure where $\rho_T$ depends on the degree of overlap between successive sub-matrices of $^{(m)}U$.

5.2.1.2 Multiple Antennas

The signal $r$ at the output of an array of $M$ antennas may be modeled as

$$r = \begin{bmatrix} r^{(1)} \\ \vdots \\ r^{(M)} \end{bmatrix} = Ub + n = \begin{bmatrix} r^{(1)} \\ \vdots \\ r^{(M)} \end{bmatrix} = \begin{bmatrix} U^{(1)} \\ \vdots \\ U^{(M)} \end{bmatrix} b + \begin{bmatrix} n^{(1)} \\ \vdots \\ n^{(M)} \end{bmatrix}$$

(5-5)

The antenna-array correlation matrix $R=U^H U$ retains the block-banded, block multi-diagonal and block-Toeplitz structure of $^{(m)}R$ since $R^{(1)} + R^{(2)} + \ldots + R^{(M)}$. 

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5.3 Detection by Polynomial expansion method

Suboptimal linear multiuser detection is an effective technique for combating the effects of multiple access interference in CDMA communication systems. Its implementation, however, is accompanied by a prohibitive computational complexity when supporting large numbers of asynchronous users and channels with long impulse responses.

The RAKE receiver performs as following:

\[ b = \text{sgn}(U^Hr) \]  \hspace{1cm} (5-6)

Suboptimal block-based multiuser detection can be expressed as the solution to the following set of linear equations [Kar99]:

\[ b = \text{sgn}(L^U + \gamma I)^{-1} U^Hr) \]  \hspace{1cm} (5-7)

where it has been assumed that \( E\{nn^H\} = \sigma^2 I \). The operation \( U^Hr \) represents the RAKE. For \( \gamma = 0 \), the solution results in decorrelator detection whilst for \( \gamma = \sigma^2 \) this corresponds to minimum mean square error detection (MMSE) [JL00, Jun00].

The decorrelator completely eliminates the effects of MAI, but at the expense of a degree of noise enhancement. Although the MMSE does not completely eliminate MAI, it suffers less noise enhancement and generally improves on the decorrelator.

Solving Eq.(5-7) needs matrix inversion which is a number crunching process. In order to avoid matrix inversions, an approximate decorrelator was proposed in [KHI83, Ver93]. It is based on the first-order Taylor approximation \((1+x)^{-1} = 1 - x + o(x), |x| < 1\). Applying the Taylor series for only one term yields:

\[ T_{dec} = (2I - R) r \]  \hspace{1cm} (5-8)

The idea of the polynomial expansion is to achieve a formula for inversion of the \( R \) matrix based on a series expansion. Eq.(5-8) can be generalized to an arbitrary \( L_T \)th order:

\[ T_{dec} = \sum_{i=0}^{L_T} w_i R^i \]  \hspace{1cm} (5-9)

where \( w_{LR} = [w_0, w_1, \ldots, w_L]^T \) are a set of coefficients that control the convergence rate of the series. Calculation of these coefficients is the main issue in the polynomial expansion.
method. The weighted polynomial detector can be shown to implement the decorrelator and MMSE detector, if the weights are selected to depend on the eigenvalues of the matrix $R$ [MKS96].

The polynomial approximations to the decorrelator and the MMSE detector are only helpful in practice if the weights can be calculated more easily than applying the exact solutions, i.e., performing matrix inversion. As the optimum weights depend on the eigenvalues of $R$, which are also not easily calculated, [MKS96] suggested to calculate them in advance and store them in tables. This method, however, seems troublesome, as the eigenvalues depend on various changing parameters and it is not clear what advantage is gained over storing the inverse correlation matrix.

In the next section we will investigate mathematical aspects of the polynomial expansion method and will derive approximations to achieve a reasonable performance without encountering a high degree of complexity.

### 5.4 Decorrelator implementation by Taylor series expansion

By using the Taylor series, the expansion of $R^{-1}$ can be written as:

$$\alpha^{-1}R^{-1} = \sum_{i=0}^{m} (I - \alpha R)^i$$  \hspace{1cm} (5-10)

If and only if the eigen-values of $R$ satisfy the conditions:

$$|1 - \lambda_i(\alpha R)| < 1$$  \hspace{1cm} (5-11)

For a positive semi-definite matrix $R$, the above condition can be written as:

$$0 < \alpha < \frac{2}{\lambda_{\text{max}}(R)}$$  \hspace{1cm} (5-12)

[LL98, Bou99] use this fact that $R$ is positive and semi-definite because it is the auto-correlation matrix. In this case all of the eigen-values are positive so $\alpha$ can be freely chosen from the following:
Because:
\[
\sum_i \lambda_i = \sum_i R_{ii} = \text{trace}(\mathbf{R}) > \lambda_{\text{max}}(\mathbf{R})
\]  

As \( \alpha \) becomes smaller, the convergence rate of the series gets smaller, which is not desirable. Choosing the value of \( \alpha \) is important for the above reasons. For this, we introduce a new definition for \( \alpha \) that leads to a good balance between convergence rate and stability.

### 5.5 Convergence improvement by mathematical analysis

We will commence our analysis from a scenario containing synchronous users with equal powers. The diagonal elements of \( \mathbf{R} \) are all 1s and the non-diagonal elements are in the range of, \((-\beta, +\beta), 0 < \beta < 1\), (\( \beta \) is the maximum value of the cross-correlations). \( \mathbf{R} \) is symmetric and its structure is as follows:

\[
\mathbf{R} = \begin{bmatrix}
1 & R_{1,2} & R_{1,3} & \cdots & R_{1,K} \\
R_{1,2} & 1 & & & \\
R_{1,3} & \ddots & \ddots & \\
\vdots & & \ddots & \ddots & \\
R_{1,K} & \cdots & R_{K,K-1} & 1
\end{bmatrix}
\]  

The condition for convergence of Eq.(5-10) for every matrix is given in Eq.(5-12). This condition must be satisfied for all of the eigen-values of the matrices. Here, we examine Eq.(5-12) for symmetric correlation matrix \( \mathbf{R} \). For symmetric matrices, maximum and minimum of the eigen-values can be obtained by the Rayleigh-Ritz theorem [Lut96]:

\[
\lambda_{\text{max}}(\mathbf{R}) = \max_{\mathbf{x}(K \times 1) \text{real}, x \neq 0} \frac{x^T \mathbf{R} x}{x^T x} \\
\lambda_{\text{min}}(\mathbf{R}) = \min_{\mathbf{x}(K \times 1) \text{real}, x \neq 0} \frac{x^T \mathbf{R} x}{x^T x}
\]

For our purpose, combination of Eq.(5-16, 5-17) and Eq.(5-15) yields the following equation:
\[ \sum_{i=1}^{K} R_{i,j} x_i x_j \] (5-18)

Minimum and maximum of this equation gives the \( \lambda_{\text{min}}(R) \) and \( \lambda_{\text{max}}(R) \). When \(-\beta < R_{ij} < \beta\), it is easy to show that:

\[
\lambda_{\text{min}} > 1 - \beta (K - 1) \quad (5-19)
\]

\[
\lambda_{\text{max}} < 1 + \beta (K - 1) \quad (5-20)
\]

By considering Eqs. (5-19, 5-20, and 5-11), the following equations were obtained:

\[
1 + \beta (K - 1) < 2/\alpha \quad (5-21)
\]

\[
1 - \beta (K - 1) > 0 \quad (5-22)
\]

Since \( R \) is a positive, semi-definite matrix, Eq.(5-22) is not needed as the condition in Eq.(5-22) ensures that the eigen-values are positive. (Note that we have solved the problem in the general case). According to Eq.(5-21), \( \alpha \) can be chosen as:

\[
\alpha = 2/(1 + \beta (K - 1)) \quad (5-23)
\]

In the situation when the cross-correlation values are large, \( \beta \) takes the maximum value of (\( \beta = 1 \)) and Eq.(5-23) becomes equal to Eq.(5-13), because:

\[
\text{trace} (R) = \sum_{i=1}^{K} R_{ii} = K \quad (5-24)
\]

Eq.(5-23) has a better accuracy than Eq.(5-13) and this leads to a faster convergence rate. The performance of these methods is compared in the next section.

5.6 Performance of iterative linear multiuser detector

5.6.1 Performance in synchronous systems

The algorithm described in the previous section was implemented for 10 synchronous users using short spreading codes having random cross-correlations less than 0.1 in the AWGN.
Figure 5-3 Decorrelator with 10 Synchronous users, 5 stages with two conditions: simple condition Eq.(5-13) and modified condition Eq.(5-23)

channel. The performance is shown in Figure 5.3 and is also compared with the precise decorrelator and approximate-decorrelator addressed in [LL98]. Implementing in 5 stages accurately follows the performance of the decorrelator.

To visualize the operation of our detector, we obtained its decision regions via simulations. Yet again, the transformations introduced in Eq.(2-43, 2-44) are used. For a two-user case, the decision regions of the iterative linear detector (implemented in one, three, five and ten stages) in comparison with the conventional detector and precise decorrelator are shown in Figures 5.4-5.9. As can be seen in these figures, decision regions are getting closer to the decision regions of the precise decorrelator (Figure 5.8, 5.9). In these simulations a range of 0-12 (dB) are considered for the $E_b/N_0$. 
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Figure 5-4 Decision regions for Conventional Detector

Figure 5-5 Decision regions for one stage of iterative Decorrelator

Figure 5-6 Decision regions for three stages of iterative Decorrelator

Figure 5-7 Decision regions for five stages of iterative Decorrelator

Figure 5-8 Decision regions for ten stages of iterative Decorrelator

Figure 5-9 Decision regions for exact Decorrelator
5.6.2 Performance in asynchronous systems

Of particular interest is the performance of the iterative methods in the asynchronous environments, as in the uplink users are transmitting at random times and the channels are subject to multipath fading. At first we consider a simple condition having only two users in a one-path fading model. The methods described in the previous sections are implemented and their performances are depicted on Figures (5.10, 5.11 and 5.12). In Figure 5.10, the number of iterations is 3 and the simulation is carried out for two asynchronous users having partial cross-correlation of 0.1. As can be seen in the plots, the performance has considerably degraded in the asynchronous environments. Increasing the partial cross-correlation to 0.3 worsens the performance, as can be seen in Figure 5.11. Even implementing in 8 stages (shown in Figure 5.12), the performance does not improve much.

Figure 5-10 Performance comparison of different iterative Decorrelators with 3 stages in one path fading. Two asynchronous users with partial cross-correlation between the codes=0.1. The simple condition is Eq.(5-13) and the modified condition is Eq.(5-23)
Figure 5-11 Performance comparison of different iterative decorrelators with 3 stages in one path fading. Two asynchronous users with partial cross-correlation between the codes=0.3. The simple condition is Eq.(5-13) and the modified condition is Eq.(5-23).

Figure 5-12 Performance comparison of different iterative decorrelators with 8 stages in one path fading. Two asynchronous users with partial cross-correlation between the codes=0.3. The simple condition is Eq.(5-13) and the modified condition is Eq.(5-23).
The reason for this behaviour is that the matrices in the asynchronous scenarios are very sparse and as a result, the estimated eigen-values are inaccurate.
To improve the performance in the asynchronous system we need to consider the convergence issues more accurately. The following sections will focus on this matter.

5.7 Convergence issue in iterative linear systems

To complete our investigation on the convergence of the Taylor series we need to refer to some convergence issues for iterative methods in linear algebra. The information in this section will be used in the forthcoming parts of the thesis.

For solving a set of linear equations, most of the iterative methods in linear algebra define a sequence of iterations of the form:

\[ x^{(s+1)} = G x^{(s)} + f \]  \hspace{1cm} (5-25)

In which \( G \) is a certain iteration matrix and \( x \) is the desired vector. The questions of interest are:
a) If the iterations for solving a linear system converge, is the limit indeed a solution of the original system?
b) Under which conditions does the iteration converge?
c) When the iteration does converge, how fast is it?

If the above iteration converges, its limit \( x \) satisfies:

\[ x = G x + f \]  \hspace{1cm} (5-26)

If \( I - G \) is non-singular then there is a solution \( x^\ast \) to equation (26). Subtracting (26) from (25) yields:

\[ x^{(s+1)} - x^\ast = G ( x^{(s)} - x^\ast ) = \ldots = G^{s+1} ( x^{(0)} - x^\ast ) \]  \hspace{1cm} (5-27)

Standard results in [Sad96] imply that if the spectral radius of the iteration matrix \( G \) is less than unity, then \( x^{(s)} - x^\ast \) converges to zero and the iteration Eq.(5-25) converges toward the solution defined by Eq. (5-26). Conversely, the relation

\[ x^{(s+1)} - x^{(s)} = G ( x^{(s)} - x_{s-1} ) = G^2 ( x^{(s-1)} - x^{(s-2)} ) = \ldots = G^s ( f - (I - G) x^{(0)} ) \]  \hspace{1cm} (5-28)
shows that if the iteration converges for any \(x^{(0)}\) and \(f\) then \(G^tv\) converges to zero for any vector \(v\). As a result, \(\rho_l(G)\) (spectral radius of \(G\), which is defined as the maximum modulus of the eigen-values of matrix \(G\)) must be less than unity and the following theorem is proven:

**Theorem 5.1** Let \(G\) be a square matrix such that \(\rho_l(G) < 1\). Then \(I - G\) is non-singular and the iteration Eq.(5-25) converges for any \(f\) and \(x^{(0)}\). Conversely, if the iteration Eq.(5-26) converges for any \(f\) and \(x^{(0)}\), then \(\rho_l(G) < 1\).

Since it is difficult to compute the spectral radius of a matrix, sufficient conditions that guarantee such can be useful in practice. One such sufficient condition could be obtained by utilizing the inequality, \(\rho_l(G) < \|G\|\), for any matrix norm.

**Corollary 5.1** Let \(G\) be a square matrix such that \(\|G\| < 1\) for some matrix norm \(\|\cdot\|\). Then \(I - G\) is non-singular and the iteration Eq.(5-26) converges for any initial vector \(x^{(0)}\).

Apart from knowing that condition Eq.(5-26) converges, it is also desirable to know how fast it converges. In [Sad96] it is shown that the convergence rate of the sequence is equal to its spectral radius of \(G\).

### 5.8 Taylor series expansion and Iterative Detection

The ultimate goal of the Decorrelator is solving a set of linear equations and achieving a reliable vector of information (Detected bits). Taylor series expansion (using Eq.(5-10)) was an attempt to reduce the complexity of matrix inversion. Implementation of the Taylor series can be viewed as an iterative detector that in each step tries to update the detected bits:

\[
\hat{b}^{(n+1)} = \hat{b}^{(n)} + \alpha (y_0 - R \hat{b}^{(n)})
\]

which \(y_0\) is the output vector of the conventional detector and is subject to interference cancellation. To make the similarity of Eq.(5-10) and Eq.(5-29), one can easily verify the following equations:

\[
b^{(0)} = \alpha y_0 \\
b^{(1)} = \alpha [2I - R] y_0 \\
\cdots
\]
After observing such a nice relation between the Taylor series and an iterative method, now we can use the information in the previous section and derive the optimum value for \( \alpha \).

Eq. (5-29) can be rewritten as:

\[
\hat{b}^{(i+1)} = (I - \alpha R) \hat{b}^{(i)} + \alpha y_0
\]

(5-32)

Thus the iteration matrix is:

\[
G_\alpha = I - \alpha R
\]

(5-33)

And the convergence factor is \( \rho(I - \alpha R) \). Assuming that the eigenvalues \( \lambda_i, i=1, \ldots, n \), are all real and such that:

\[
\lambda_{\text{min}} \leq \lambda_i \leq \lambda_{\text{max}}
\]

(5-34)

Then the eigenvalues \( \mu_i \) of \( G_\alpha \) are such that:

\[
1 - \alpha \lambda_{\text{max}} \leq \mu_i \leq 1 - \alpha \lambda_{\text{min}}
\]

(5-35)

In particular, if \( \lambda_{\text{min}} < 0 \) and \( \lambda_{\text{max}} > 0 \), at least one eigenvalue is > 1, and so \( \rho(G_\alpha) > 1 \) for any \( \alpha \). In this case the method will always diverge for some initial guess. Assuming that all eigenvalues are positive, i.e., \( \lambda_{\text{min}} > 0 \), then the following conditions must be satisfied in order for the method to converge:

\[
1 - \alpha \lambda_{\text{min}} < 1
\]

\[
1 - \alpha \lambda_{\text{max}} > -1
\]

(5-36)

(5-37)

We see that the above conditions are the same as the one in Eq. (5-11).

The main question here is: What is the best value \( \alpha_{\text{opt}} \) for the parameter \( \alpha \), i.e., the value of \( \alpha \) which minimizes \( \rho_\alpha(G_\alpha) \)? The spectral radius of \( G_\alpha \) is:

\[
\rho_\alpha(G_\alpha) = \max \{ |1 - \alpha \lambda_{\text{max}}|, |1 - \alpha \lambda_{\text{min}}| \}.
\]

(5-38)

This function of \( \alpha \) is depicted in Figure 5-13. As the plot shows, the best possible \( \alpha \) is reached at the point where the curve \(|1 - \alpha \lambda_{\text{max}}|\) with positive slope crosses the curve \(|1 - \alpha \lambda_{\text{min}}|\) with negative slope, i.e., when:
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Figure 5-13 The curve $\rho(G_\alpha)$ as a function of $\alpha$.

\[-1 + \alpha \lambda_{\text{max}} = 1 - \alpha \lambda_{\text{min}} \]  \hspace{1cm} (5-39)

This gives:

\[ \alpha_{\text{opt}} = \frac{2}{\lambda_{\text{max}} + \lambda_{\text{min}}} \]  \hspace{1cm} (5-40)

Replacing this in one of the two curves gives the corresponding optimal spectral radius:

\[ \rho_{\text{opt}} = \frac{\lambda_{\text{max}} - \lambda_{\text{min}}}{\lambda_{\text{max}} + \lambda_{\text{min}}} \]  \hspace{1cm} (5-41)

To achieve good convergence, eigenvalue estimates are required. In the next section we will consider the problem in asynchronous systems. Estimation of eigenvalues to achieve the optimum convergence rate in such systems will also be addressed.
5.9 Gershgorin Algorithm

In asynchronous systems most of the elements of $R$ are zero while Eq.(5-23) considers only the maximum value of non-diagonal elements of $R$ and ignores the majority of elements that are zero. This ignorance decreases the accuracy and shows its effect in the convergence rate and the performance. Another point about Eq.(5-23) is that it does not consider $\lambda_{\text{min}}$. To overcome these problems, we refer to the Gershgorin theorem in linear algebra [Sad96]. According to this theorem, any eigenvalue of a matrix is located in one of the closed discs of the complex plane centred at $a_{ii}$ and having radius $\sqrt{\sum_{i,j \neq i} R_{ij}^2}$ (which $R_{ij}$ is the $(i,j)$ element of the matrix $R$). In other words,

$$|\lambda_i - R_{ii}| \leq \sum_{i,j \neq i} |R_{ij}|$$  \hspace{1cm} (5-42)

By a simple calculation on the elements of $R$, two approximate values can be derived for $\lambda_{\text{max}}$ and $\lambda_{\text{min}}$:

$$\lambda_{\max} \leq \max \{ R_{ii} + \sum_{i,j \neq i} |R_{ij}| \} ; i,j=1,2,\ldots,KN_b$$ \hspace{1cm} (5-43)

$$\lambda_{\min} \leq \min \{ R_{ii} + \sum_{i,j \neq i} |R_{ij}| \}$$ \hspace{1cm} (5-44)

Using Eq.(5-40), Eq.(5-43) and Eq.(5-44), a proper estimate for $\alpha$ could be derived. The main benefit of the Gershgorin algorithm is that it introduces almost no complexity overhead to the detector. Its performance is also acceptable as shown in the next section.

5.9.1 Performance of the asynchronous system using Gershgorin Algorithm

Of most practical interest is the performance for the asynchronous users in fading environments. This new modification is simulated for an asynchronous CDMA system consisting of two heavily interfering users, $|\rho_{12}|=0.3$ and $|\rho_{21}|=0.35$ in a Rayleigh fading channel. The result is shown in Figure 5.14. In Figure 5.14, methods discussed in the previous section are simulated and compared with the conventional detector and the exact decorrelator. These methods are implemented in 3 iterations. As can be seen, the method using Eq.(5-13) does not have a particularly good performance and is comparable to the conventional detector. The method using Eq.(5-23) also does not perform well.
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Figure 5-14 Comparison of the performance for different iterative decorrelators. 1 Path fading, 2 users, Rho= 0.3, 3 stages of iterations. The comparison is between simple condition Eq.(5-13), Rayleigh-Ritz Theorem Eq.(5-23) and the new modification by Gershgorin Eq.(5-40, 5-43) [Ignoring $\lambda_{\min}$]

Figure 5-15 Decorrelator with 2 asynchronous users, Crosscorrelations= 0.3, 0.35, and 5 stages. The comparison is between simple condition Eq.(5-13), Rayleigh-Ritz Theorem Eq.(5-23) and the Gershgorin Eq.(5-40, 5-43) [Ignoring $\lambda_{\min}$]
Chapter 5. Iterative Linear Multiuser Detection

![Figure 5-16 Comparison of the performance for different iterative decorrelators. 1 Path fading, 2 users, Rho= 0.3, 5 stages of iterations. The comparison is between simple condition Eq.(5-13), Rayleigh-Ritz Theorem Eq.(5-23) and the Gershgorin Eq.(5-40, 5-43, 5-44) [Considering $\lambda_{\min}$]

However, the third method that uses the Gershgorin algorithm (Eq.(5-40, 5-43)) performs much better and copes well with the asynchronous environments. It should also be mentioned that in this figure for simplicity, we have ignored the $\lambda_{\min}$ in comparison with $\lambda_{\max}$. Figure 5.15 shows the performance of iterative linear detectors implemented in 5 stages. In Figure 5-16 $\lambda_{\min}$ is also considered and it is obvious that using this, a better performance for the Gershgorin algorithm can be achieved.

5.9.2 Performance in multi-stages and comparison with multistage PPIC

So far our main focus has been on the performance in simple scenarios. However simple scenarios have benefit in developing the basic ideas and provide useful systems insight, of particular interest, of course, is the performance of the algorithm in multiuser and realistic scenarios. Of interest also is to compare our algorithm with other known interference cancellation methods in the literature.
PPIC is a well-known method with good performance and has been of recent interest to the research community. It also uses a multistage structure and makes it even more suitable to our purpose, as our detector incorporates multiple stages.

Before commencing the comparison between these two detectors, we will investigate the performance enhancements of our detector via different number of the stages. It is known that PPIC achieves most of its performance in the first 3 stages of its operation. If our detector achieves its performance in its early stages, then these two detectors are more comparable.

A scenario involving \( K=10 \) equi-powered mobiles is considered. Each mobile transmits data symbols with spreading factors of \( Q_s=16 \) respectively. The transmitter and receiver are simulated at the chip-rate. Short scrambling is assumed. Mobile transmissions are asynchronous with relative delays of up to one symbol period. The radio channel model is Vehicular-A at 120 km/h. The channel estimation is assumed perfect. \( L=3 \) fingers are derived by the channel estimator with spacings at multiples of the chip period. The BER values presented are averaged across all users. \( E_b \) refers to the average energy per bit.
Figure 5.18 Performance of the iterative linear multiuser detector in compare with the PPIC detector operating in 3 stages, 10 asynchronous users, Random spreading codes of length 16, Vehicular A fading channel, $E_b/N_0=14$ dB

Figure 5.17 illustrates the convergence of the iterative algorithms at an $E_b/N_0$ of 14dB with exact channel estimates. The algorithm appears to achieve a suitable performance in 3 iterations and, for practical purposes, provides sufficiently good performance after only two iterations.

Our detector achieves its main performance in 3 stages and so is comparable with the PPIC. With the same parameters as in the previous simulation, the comparison results are shown in Figure 5.18. The values of the partial coefficients in PPIC were obtained by simulation. The performance of the algorithm is slightly better than the PPIC and the main benefit of the method is that the coefficients are calculated automatically without encountering a high degree of complexity.
5.10 Iterative linear multiuser detection and signal to noise ratio

Another approach taken by other authors [MV01], is to optimize the coefficients via signal to noise ratio aspects. In this approach one of the key assumptions is randomness of the cross-correlation matrix elements. This assumption does not use any more information about the elements and hence leads to inaccuracies. Since the approach uses random matrix theory and this theory does not include sparse matrices, their second assumption is that the users are synchronous. These two assumptions confine the applicability of their method.

Based on this method, the optimum weighting coefficients are obtained in closed form, as shown in Table (5-1).

| $w_1 = \begin{bmatrix} w_0 \\ w_1 \end{bmatrix}$ | $w_0 = -N_0w_1 + 2 + 2\beta_l$  
| | $w_1 = -1$ |
| $w_2 = \begin{bmatrix} w_0 \\ w_1 \\ w_2 \end{bmatrix}$ | $w_0 = -N_0w_1 + 3 + 4\beta_l + 3\beta_l^2$  
| | $w_1 = -N_0w_2 - 3 - 3\beta_l$  
| | $w_2 = 1$ |
| $w_3 = \begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ w_3 \end{bmatrix}$ | $w_0 = -N_0w_1 + 4 + 6\beta_l + 6\beta_l^2 + 4\beta_l^3$  
| | $w_1 = -N_0w_2 - 6 - 9\beta_l - 6\beta_l^2$  
| | $w_2 = -N_0w_3 + 4 + 4\beta_l$  
| | $w_3 = -1$ |
| $w_4 = \begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}$ | $w_0 = -N_0w_1 + 5 + 8\beta_l + 9\beta_l^2 + 8\beta_l^3 + 5\beta_l^4$  
| | $w_1 = -N_0w_2 - 10 - 18\beta_l - 18\beta_l^2 - 10\beta_l^3$  
| | $w_2 = -N_0w_3 + 10 + 16\beta_l + 10\beta_l^2$  
| | $w_3 = -N_0w_4 - 5 - 5\beta_l$  
| | $w_4 = 1$ |

**Table 5-1** The optimised weights for Eq.(5-9) by maximizing the signal to noise ratio [MV01]

In Table 5.1, the $\beta_l$ is the loading factor and can be calculated by: $\beta_l = K/N$ (which $K$ indicates the number of active users and $N$ stands for the spreading factor). We were interested to compare our method with this approach. For this purpose, a simulation was carried out containing 10 synchronous users having random cross-correlations of less than 0.1 in the AWGN channel. The performance is depicted in Figure 5-19.
Figure 5-19 Comparison between iterative linear detectors implemented in three stages, 10 synchronous users, AWGN channel. Three methods for calculation of the coefficients: a) Simple method Eq.(5-13), b) Rayleigh-Ritz Theorem Eq.(5-23), c) Asymptotic SNR optimization

5.11 Multiuser Detection in Frequency Domain

There are other methods that have recently been reported in the literature concerning the inversion of the matrices in the decorrelating process. [VHG01] has introduced a method that performs the matrix inversion in the frequency domain. They also compare their method with three other methods (which are based on Cholesky factorisation, the Levinston algorithm and the Schur algorithm) in terms of performance and complexity. The authors of [VHG01] have also provided a program [Vol01] that calculates complexity of their algorithms. Their main conclusion was a reduction in terms of complexity via the Fourier algorithm, yet achieving the same performance as the exact Cholesky algorithm.
However, there are some comments that we will make on the Fourier algorithm and we will also compare our algorithm in terms of complexity with the Fourier method using the program in [Vol01].

5.11.1 Comments on the Fourier algorithm

In the following we will address some comments on the methods presented in [VHG01]:

- The Fourier algorithm and the other three methods introduced in [VHG01] are all conditioned on being applied to a block-Toeplitz matrix. Although, the system matrices in WCDMA environments are in block-Toeplitz, for simple scenarios such as synchronous users, this assumption is not generally valid. Our algorithm however supports both synchronous and asynchronous environments.

- The important point in the Fourier algorithm is that at some stage it needs to extend the convolutional matrix (shown by $T$ in their paper) to be block-circulant. They present a simulation in which the distribution introduced by this extension is insignificant, as it will mainly affect the guard periods between the bursts. This assumption is valid for TD-CDMA systems but not for FDD-WCDMA. In this thesis we considered the FDD-WCDMA uplink and there is no guard time provided in these systems. Implementing the Fourier algorithm in this system will not work as well as in TD-CDMA and is thus very limited in its application.

- Throughout [VHG01], it has been assumed that the spreading factor of the users are equal to 16 and all the complexity evaluations have been carried out based on this value. In the FDD-WCDMA uplink, the spreading factor of the users is in the range of 4-256. We will see in the next section that for high values of the spreading factors, the Fourier algorithm will suffer in complexity comparisons.

5.11.2 Complexity evaluation of the Fourier algorithm and the iterative linear multiuser detector

For calculating the operations involved in the Fourier algorithm we have used the program presented in [Vol01]. In the Fourier algorithm the FFT length of $D=32$ was considered and the prelap and postlaps were also 3 and 5 respectively.

For computing the operations in our algorithm for comparison we need to investigate carefully the structure of $R$. As stated in section 5.2.1.1, $^{(m)}R$ has a block $(2\rho+1)$-diagonal
structure where $\rho_T$ depends on the degree of overlap between successive sub-matrices of $(m)U$. $\rho_T$ can be calculated as from [Kar99] as:

$$\rho_T = \text{floor} \left(1 + \frac{(W-1)}{JV} \right)$$  \hspace{1cm} (5-45)

We use the following parameters (as used in [Vol01]) for complexity evaluations:

- $K$: Number of users. (12 users)
- $Q1$: Spreading factor of each user.
- $N_b$: Number of symbols transmitted per user in one slot of FDD-Uplink. Depends on the spreading factor of users. However in one slot we have: $Q1.N_b=2560$.
- $W$: Maximum channel impulse response, (57 chips).
- $M$: Number of antennas.
- $B_i$: Number of blocks transmitted per burst. (Here equal to 1)

According to [Vol01], the following symbols are defined as well:

- $P=M*(Q1+W-1)$
- $Q=M*Q1$;
- $\nu=\text{ceil}(P/Q)$;

Our algorithm is composed of four stages as shown in Table 5.2. The complexity of each stage, in terms of real multiplications, is given in Table 5.2.

The real multiplications needed to calculate $R=U^HU$ and matched filtering ($y=U^Hr$) is same as the calculations presented in [Vol01] and we have used them for our purpose.

Because in the Gershgorin algorithm we need to find the radius of the disk that the eigenvalues are located in them, we need to find the absolute values of the complex numbers (non-zero elements) that are in the matrix $R$. There are

$$\left[2\rho_T + 1\right] (N - 2\rho_T) K^2 + 2K^2 \sum_{i=\rho_T+1}^{2\rho_T}$$

non-zero elements in the $R$ matrix and calculating each absolute value needs 2 real multiplications.

Implementing the iterative linear detection method in $I_{\text{max}}$ iterations, will need dealing with non-zero elements of matrix $R$, as well. Each complex multiplication needs 4 real multiplications.

It should be pointed-out that in the actual systems the matched filtering process would have already been done by other subsystems and we can apply our iterative algorithm to the output of matched filters ($y$).
Chapter 5. Iterative Linear Multiuser Detection

Calculating $R = U^H U$

$2KP + 2KP(K-1) + 4K^2 \sum (P-(1:v)Q)$; [Vol01]

Matched Filtering: $y = U^H r$

$4B_L N_b KP$; [Vol01]; ($B_L = 1$)

The Gershgorin Algorithm.

$$2 \left( 2\rho_T + 1 \right) (N - 2\rho_T) K^2 + 2K^2 \sum_{i=\rho_T+1}^{2\rho_T} i$$

<table>
<thead>
<tr>
<th>Iterative Linear detection</th>
<th>$I_{\text{max}} \cdot 4 \left( 2\rho_T + 1 \right) (N - 2\rho_T) K^2 + 2K^2 \sum_{i=\rho_T+1}^{2\rho_T} i$</th>
</tr>
</thead>
</table>

Table 5-2 Complexity of linear iterative algorithm in terms of real multiplications

In our complexity evaluations we have considered two cases: 1) Performing the calculations considering the complexity of matched filtering (with $y$).

2) Performing the calculations without considering the complexity of matched filtering (without $y$ [wo/y]).

Figure 5-20 shows the comparison of two methods and the effect of the number of antennas on the complexities. In this simulation a large value for the spreading factor is considered (128). As can be seen, for large values of spreading factor, the Fourier algorithm suffers from the complexity.

The effect of the spreading factor is shown in Figure 5-21. As the spreading factor varies, the number of bits that each user transmits ($N_b$), varies, because in each slot (Spreading factor $Q1) \times$ (Number of transmitted bits $N_b$)$=2560 remains the same. It can easily be seen that for large values of the spreading factor, the complexity of the Fourier algorithm is high, however in low spreading factors its complexity is much better than other schemes. In this simulation, one antenna is considered.

As can be seen from the entries in Table 5.2, the first and second entries are related to building the correlation matrix and matched filtering and hence are dependent on the value of spreading factor. The third and fourth entries, however, are the complexity values of the iterative linear MUD operations. As these MUD operations are performed on the output of matched filters ($y$), there is no dependency on the spreading factor anymore. In the situations that large values of spreading factor are considered, this will avoid the complexity to increase a lot.

(In the above simulations for evaluating the Fourier algorithm, we used the function “four_mult_FY” in [Vol01], as it was used in its manual as default function).
**Figure 5-20** Effect of number of antennas on complexity. Spreading factor of users is 128. Number of symbols transmitted by each user is 10, $l_{max}=3$.

**Figure 5-21** Effect of spreading factor on the complexity of algorithms. One antenna is considered.
5.12 Conclusions

In this chapter the iterative implementation of linear multiuser detectors were addressed and analysed. Although, the family of linear detectors for multiuser detection in CDMA are a good candidate for suppressing the interference level, precise implementation of these detectors suffers from computation complexity due to a need for inverting the crosscorrelation matrix.

After presenting a suitable model for the received signal, we focused on the Taylor expansion of the inverted matrix and its convergence rate. The parameters involved in this series were investigated and by analytical analysis using the Rayleigh-Ritz theorem a more accurate condition to increase the convergence rate for synchronous systems was derived. This idea, however, in asynchronous systems did not perform as well as expected. This issue was tackled with another method in linear algebra know as the Gershgorin theorem. The method was implemented and its performance found promising. Using this method, a more accurate, yet simple, condition for convergence was derived. The performance of the iterative linear multiuser detector, that uses the Gershgorin theorem, is also compared with the partial parallel interference cancellation method and performs almost as well without any need to calculate the parameters with trial and error or encountering a high degree of complexity.

We have also investigated the signal to noise ratio and the weight optimisation method [MV01], addressed in the literature, and compared our method with these in terms of BER. Matrix inversion using the Fourier algorithm, which has recently been reported [VHG01], was also considered and investigated. An analysis for complexity of our proposed method was also presented and was compared with the complexity of the Fourier algorithm.

The Fourier algorithm had low complexity with small values of spreading factors, whereas in scenarios that high values of spreading factors are used, it suffers from complexity increases.
Chapter 6 Suboptimum Search Algorithm

6.1 Introduction

Optimum multiuser detection needs a huge number of searches and as a result suffers from a high degree of complexity that makes it useless in practice. On the other hand studying its structure provides good insight for suboptimum detectors. An approach to fill the gap between the performances of already known suboptimum detectors and the optimum detector is to study the optimum detector and seek a suboptimum search strategy. This chapter first considers the optimum detector and then introduces a suboptimum search algorithm. The idea is to use the optimum criteria and to start off the search process from a reliable starting point. Starting from reliable detected information can reduce the number of search steps, as one would intuitively expect that the optimum sequence of data is located in a close neighborhood of the starting point. Analytical evaluation of the algorithm in a simple scenario is also carried out in AWGN environments. Another aspect of the algorithm that can be cited as an advantage is that it is well suited to the iterative linear detector discussed in chapter 5. These two detectors use almost the same set of information and this fact can reduce unnecessary recalculations. The basic idea of the search algorithm has been extended in the multistage operations. Another modification to the basic idea is to consider the power profile of the users. Sacrificing some performance and using the power
profile of the users leads to a complexity reduction. The performances are also compared with the PPIC method.

### 6.2 Optimum Multiuser detector

The optimum multiuser detector yields the minimum achievable probability of error in CDMA channels. In the derivation of an optimum receiver, the receiver knows the signature waveform and timing of every active user and also the received amplitudes of all users as well as the noise levels. In this section we briefly consider the optimum detector, as some parameters and concepts will be used in forthcoming sections.

#### 6.2.1 Optimum detector in synchronous system

One important issue about the conventional detector's output is that it has sufficient statistics for all users. In this section we will briefly investigate a simple synchronous scenario consisted of K users in AWGN channel. Considering \( y = [y_1, y_2, \ldots, y_K]^T \) is the output of matched filters in a synchronous scenario containing \( K \) users, although \( (y_1, y_2, \ldots, y_K) \) is a sufficient statistic for the data \( b = (b_1, b_2, \ldots, b_K) \), it is not true that \( y_k \) is sufficient for \( b_k \). In the jointly optimum decision, the problem is to find the \( b_k \) in such a way as to maximize the \( P\{b \mid y\} \) [Ver98]. According to the detection theory, this maximization for a system with \( K \) users in synchronous channels is equivalent to finding the \( b \) to maximize the following formula:

\[
\exp \left( -\frac{1}{2\sigma^2} \int_0^T \left[ r(t) - \sum_{k=1}^K b_k A_k s_k(t) \right]^2 dt \right) 
\]

or, maximizing the following:

\[
\Omega(b) = 2 \left[ \int_0^T \left[ \sum_{k=1}^K A_k b_k s_k(t) \right] r(t) dt - \int_0^T \sum_{k=1}^K A_k b_k s_k(t) \right]^2 dt 
\]

\[
= 2b^T A y - b^T H b 
\]

where \( A = \text{diag}\{A_1, A_2, \ldots, A_K\} \) is \( K \) by \( K \) diagonal matrix of received amplitudes and \( H \) is un-normalized cross-correlation matrix: \( H=ARA \).
For maximizing Eq.(6-3), a search algorithm can be used. For finding the optimum $b$, $O(2^K)$ operations is needed, so the time complexity per bit is $O(2^K/K)$.

### 6.2.2 Optimum detector in asynchronous system

In the asynchronous case the problem is maximising the following equation:

$$\Omega(b) = 2b^T A_M y - b^T R b$$

(6-4)

where $A_M$, is a $KN_b \times KN_b$ diagonal matrix whose $k + iK$ diagonal element is equal to $A_k$. $R$ is the extended cross-correlation matrix for the asynchronous case and $N_b$ bits per user have been considered for the search process. If a similar method as in the synchronous case were to be used, the complexity would be exponential in the product $KN_b$. However, by using the Viterbi algorithm, a simplified method is accessible. There is a huge gap in performance between the conventional and optimum detector. For example in a fifteen-user channel with equal powers, and cross-correlations equal to 0.09, when the signal to noise ratio reaches 15dB, the BER of conventional detector is about $2 \times 10^{-4}$ while the BER of the optimum detector is $10^{-8}$, Of course at the expense of higher complexity [Ver98].
This huge difference between performance and complexity motivates researchers to seek suitable sub-optimum detectors. Decision regions of the optimum detector for a two user case is depicted in Figure 6.1.

### 6.3 Suboptimum search algorithm (Basic Idea)

To avoid the complexity of the optimum search algorithm, we investigate the use of a suboptimum search algorithm. If the search process starts off from a reliable starting point, then the optimum sequence will be in the close neighbourhood of the starting point and only small number of bits in the observation window will be in error. Our strategy is to pinpoint these bits by an algorithm shown in Figure 6.2. In this method the detection process is performed in two stages. The first stage provides a reliable set of bits and the second stage performs the search algorithm. The search algorithm works as follows:

Considering the sequence \((b_1, b_2, ..., b_K)\) to be the output of the initial stage, based on Eq.(6-4), two values for \(\Omega\) related to \((b_1, b_2, ..., b_K)\) and \((-b_1, b_2, ..., b_K)\) are calculated. Of these two sequences, the one with greater value for \(\Omega\) will be kept as the more likely sequence and the process continued the same way for the next bit. For the second bit, two sequences of \((b_1', b_2, ..., b_K)\) and \((b_1', -b_2, ..., b_K)\) will be in consideration, where \(b_1'\) is the detected bit after the first step of the search algorithm. This process continues for all other bits. The length of this process is only dependent on the observation window and the number of users.

In synchronous environments, \(K\) stages of search are needed to detect the first bits of \(K\) users and in each stage there are two values for \(\Omega\) calculated. In other words the complexity is \(O(1+1/K)\) per bit in synchronous scenarios. This process by comparison was \(O(2^K/K)\) stages per bit in the optimum detector case.

In asynchronous environments the same idea extends for \(N_b\) bits per user. In this case, total bits are \(N_bK\). So \(N_bK+1\) stages are needed for calculating \(\Omega_s\). Then the complexity will also be \(O(1+1/N_bK)\) per bit. As shown in the previous section, the complexity for the optimum detector per bit is \(O(2^{N_bK}/N_bK)\).

Through simulations the decision regions of the suboptimum detector for the two-user case is also obtained and is shown in Figure 6-3.
Figure 6-2 The basic idea of suboptimum search algorithm

Figure 6-3 Decision regions of suboptimum search detector for two users
Figure 6.4 Performance of the Suboptimum search algorithm in compare with the 3 stages of PPIC, The search algorithm is applied to the output of conventional detector.

6.4 Performance of the suboptimum search algorithm

A CDMA system, containing 10 users using processing gain of 16, in Vehicular-A is considered. Figure 6.4 shows the performance of the suboptimum search algorithm operating on the output of the conventional detector. For comparison, the performance of the 3 stages PPIC detector with convenient coefficients is also shown (The coefficients are derived by simulations).
6.5 Suboptimum search algorithm in conjunction with iterative Multiuser detection

As seen in the previous chapter, both the iterative linear multiuser detector and the suboptimum search algorithm use the correlation matrix. Calculating this matrix is not an easy task. However, using the same matrix for both stages will save the processing power and this leads to less complexity. This consistency makes these two stages work very well in collaboration. As can be seen in Figure 6.5, a reasonable gain can be achieved in the operating range of practical systems, i.e. (2-3) dB gain in the operating points of (5-10) dBs.

Figure 6-5 Performance of the Suboptimum search algorithm for 10 Users, Multipath fading environment, PG=16, 3 stages of Iterative detection at first stage using Gershgorin algorithm, Vehicular A Channel
6.6 Multistage Implementation of suboptimum search Algorithm

The idea of the suboptimum search algorithm is based on starting the algorithm from a reliable initial point. After finishing one stage of the search algorithm and detecting all the bits, these new detected bits can themselves be used as starting points for another round of the search algorithm. This process can be repeated several times, which in consequence implies operation in multistages.

Figure 6.6 shows the performance for a two-user case having cross-correlations of 0.3 between their spreading codes operating in a one-path fading environment. Performance of the conventional detector (CD) and decorrelator (Dec) are also depicted. Multistage search algorithm that uses the decorrelator as its initial stage outperforms the rest of detectors.
6.7 On the analytic performance of the algorithm

In this section an analytic analysis is considered for a simple case consisting of 2 users. The analysis is divided into several steps:

Step 1. Formulation of the problem

Using the method given in [Ver98], the error probability that the detector makes on $b_1$ is:

$$P_1(\sigma) = \frac{1}{4} P(++)+ \frac{1}{4} P(+-)+ \frac{1}{4} P(-+)+ \frac{1}{4} P(--)+ \frac{1}{4} P(++)+ \frac{1}{4} P(+-)+ \frac{1}{4} P(-+)+ \frac{1}{4} P(--)+ \frac{1}{4} P(++)$$

where we have employed the notation $P(b_1 b_2 \rightarrow \hat{b}_1 \hat{b}_2)$ to denote that the observations fall in the region of $(\hat{b}_1, \hat{b}_2)$ conditioned on $(b_1, b_2)$ being transmitted. Because of the symmetry in the system, the following equations hold:

$$P(++) = P(--), \quad (6-6a)$$
$$P(+-) = P(-+), \quad (6-6b)$$
$$P(-+) = P(+-), \quad (6-6c)$$
$$P(--)= P(++) \quad (6-6d)$$

Combining these equations the final formula is:

$$P_1(\sigma) = \frac{1}{2} P(--)+ \frac{1}{2} P(++)= \frac{1}{2} P(+-)+ \frac{1}{2} P(--)+ \frac{1}{2} P(++)$$

Step 2. Expanding individual terms of Eq.(6-7)

As seen before the detection is carried out in two steps: At first initial detection is performed on the received signal and in the second stage a search algorithm is applied to the detected data provided by the first stage. In this analysis the first stage is assumed on just matched filtering. Since detection is done in two stages, the final BER will depend on the performance of both stages. Before continuing the analysis it should be mentioned that the following notations have been used:

$P_{MF} = $ Error probability of the first stage, which is matched filtering.

$p = $ The probability of error in one bit for matched filter.

$P_{SO} = $ Error probability of the second stage, which is the Suboptimum search algorithm.
The first term in Eq.(6-7) is:

\[
P(- - \rightarrow + -) = \\
P_{MF} (- - \rightarrow + -) \cdot P_s [\Omega(- -) < \Omega(+ -)] (---) Transmitted \cdot P_s [\Omega(- +) < \Omega(+ -)] (---) Transmitted + \\
P_{MF} (- - \rightarrow - -) \cdot P_s [\Omega(- -) < \Omega(+ -)] (---) Transmitted \cdot P_s [\Omega(+ +) < \Omega(+ -)] (---) Transmitted + \\
P_{MF} (- - \rightarrow - +) \cdot P_s [\Omega(- +) < \Omega(+ -)] (---) Transmitted \cdot P_s [\Omega(+ +) < \Omega(+ -)] (---) Transmitted + \\
P_{MF} (- - \rightarrow + +) \cdot P_s [\Omega(- +) < \Omega(+ +)] (---) Transmitted \cdot P_s [\Omega(- +) < \Omega(+ +)] (---) Transmitted \\
\]

(6-8)

In the above formula, (- -) has been sent by two users with occurrence probability of (+ -), (that is the final state of the detector) as desired.

It is noted that the detected bits at the output of the first stage can take four possible situations each with a given probability.

However, there are two situations possible for the second stage:

- Desired bits have been correctly detected in the first stage, and second stage does not change the sequence any more. The first term in Eq.(6-8)

- Desired bits have not been correctly detected by the first stage, but the second stage can correct them. Second and third terms in Eq.(6-8).

And also the following equations hold for the matched filter:

\[
P_{MF} (- - \rightarrow + -) = p(1-p) \text{ (6-9a)}
\]

\[
P_{MF} (- - \rightarrow - -) = (1-p)(1-p) \text{ (6-9b)}
\]

\[
P_{MF} (- - \rightarrow - +) = p(1-p) \text{ (6-9c)}
\]

\[
P_{MF} (- - \rightarrow + +) = p(1-p) \text{ (6-9d)}
\]

For example in the transition from (- - \rightarrow - +) only one bit is in error, with a probability of \(p\), and the other bit is correct, with a probability of (1-p). The other terms follow the same logic.

For the rest of the terms in Eq.(6-7) the same method can used:

Second term:

\[
P(- - \rightarrow + +) = \\
P_{MF} (- - \rightarrow + +) \cdot P_s [\Omega(- +) < \Omega(+ +)] (---) Transmitted \cdot P_s [\Omega(- +) < \Omega(+ +)] (---) Transmitted + \\
P_{MF} (- - \rightarrow - +) \cdot P_s [\Omega(- +) < \Omega(+ +)] (---) Transmitted \cdot P_s [\Omega(+ +) < \Omega(+ +)] (---) Transmitted + \\
P_{MF} (- - \rightarrow - +) \cdot P_s [\Omega(- +) < \Omega(+ +)] (---) Transmitted \cdot P_s [\Omega(+ +) < \Omega(+ +)] (---) Transmitted + \\
P_{MF} (- - \rightarrow + +) \cdot P_s [\Omega(- +) < \Omega(+ +)] (---) Transmitted \cdot P_s [\Omega(- +) < \Omega(+ +)] (---) Transmitted \\
\]

(6-10)
Third term in Eq.(6-7):

\[ P(-+\rightarrow +-) = \]

\[ P_{MF}(-+\rightarrow -+)[P_{SO}[\Omega(+-) < \Omega(+-) | (-+) \text{Transmitted}][P_{SO}[\Omega(-+) < \Omega(+-) | (-+) \text{Transmitted}]] + \]

\[ P_{MF}(-+\rightarrow --)[P_{SO}[\Omega(--|--) < \Omega(--|--) | (++) \text{Transmitted}][P_{SO}[\Omega(--|--) < \Omega(--|--) | (++) \text{Transmitted}]] + \]

\[ P_{MF}(-+\rightarrow ++)[P_{SO}[\Omega(+-) < \Omega(+-) | (+-) \text{Transmitted}][P_{SO}[\Omega(+-) < \Omega(+-) | (+-) \text{Transmitted}]] + \]

\[ P_{MF}(-+\rightarrow ++)[P_{SO}[\Omega(--|--) < \Omega(--|--) | (++) \text{Transmitted}][P_{SO}[\Omega(--|--) < \Omega(--|--) | (++) \text{Transmitted}]] + \]

And finally the forth term in Eq.(6-8):

\[ P(--\rightarrow +++) = \]

\[ P_{MF}(--\rightarrow --)[P_{SO}[\Omega(--|--) < \Omega(--|--) | (++) \text{Transmitted}][P_{SO}[\Omega(--|--) < \Omega(--|--) | (++) \text{Transmitted}]] + \]

\[ P_{MF}(--\rightarrow --)[P_{SO}[\Omega(--|--) < \Omega(--|--) | (++) \text{Transmitted}][P_{SO}[\Omega(--|--) < \Omega(--|--) | (++) \text{Transmitted}]] + \]

\[ P_{MF}(-+\rightarrow ++)[P_{SO}[\Omega(+-) < \Omega(+-) | (+-) \text{Transmitted}][P_{SO}[\Omega(+-) < \Omega(+-) | (+-) \text{Transmitted}]] + \]

\[ P_{MF}(-+\rightarrow ++)[P_{SO}[\Omega(--|--) < \Omega(--|--) | (++) \text{Transmitted}][P_{SO}[\Omega(--|--) < \Omega(--|--) | (++) \text{Transmitted}]] + \]

\[ (6-11) \]

**Step 3. Calculating the expanded terms of Eq.(6-7)**

Now the above formulas can be expanded and calculated to achieve the final BER:

Since the first stage of detection is Match filtering, the outputs of two matched filters can be written as:

\[ y_1 = A_1 b_1 + A_2 b_2 \rho + n_1 \]

\[ y_2 = A_2 b_2 + A_1 b_1 \rho + n_2 \]

\[ (6-13a) \]

\[ (6-13b) \]

\[ \Omega(d_1,d_2) \] is the criterion and for two synchronous users is:

\[ \Omega(d_1,d_2) = d_1 A_1 y_1 + d_2 A_2 y_2 - d_1 d_2 A_1 A_2 \rho \]

\[ (6-14) \]

Using these equations, for calculating the terms in Eq.(6-8), we will have:

\[ b_1 = -1, b_2 = -1 \]

\[ (6-15a) \]

\[ y_1 = -A_1 - A_2 \rho + n_1 < 0, \]

\[ (6-15b) \]

\[ y_2 = -A_2 - A_1 \rho + n_2 > 0 \]

\[ (6-15c) \]

\[ \Omega(--|--) = -A_1 y_1 - A_2 y_2 - A_1 A_2 \rho \]

\[ (6-15d) \]

\[ \Omega(+--+|--) = A_1 y_1 - A_2 y_2 + A_1 A_2 \rho \]

\[ (6-15e) \]

and
Chapter 6. Suboptimum Search Algorithm

\[ P^{so}[\Omega(-) < \Omega(+) | (-)\text{ Transmitted}] = \]
\[ P^{so}[-A_1y_1 - A_2y_2 - A_1A_2\rho < (A_1y_1 - A_2y_2 + A_1A_2\rho)] = P^{so}[A_1y_1 + A_1A_2\rho > 0] \]  
(6-16)

Using Eq.(6-15b) we end up with:

\[ P^{so}[A_1y_1 + A_1A_2\rho > 0] = P^{so}[-A_1 - A_2\rho + n_1 + A_2\rho > 0] = P^{so}[n_1 > A_1] = Q\left(\frac{A_1}{\sigma}\right) \]  
(6-17)

The same method can be used to calculate the other terms in the Eq.(6-8):

For the second term we can write:

\[ P^{so}[\Omega(-) < \Omega(+) | (-)\text{ Transmitted}] = \]
\[ P^{so}[-A_1y_1 + A_2y_2 - A_1A_2\rho < A_1y_1 - A_2y_2 - A_1A_2\rho] = P^{so}[A_1y_1 - A_2y_2 > 0] \]  
(6-18)

We also have:

\[ y_1 = -A_1 - A_2\rho + n_1 \]  
(6-19a)

\[ y_2 = -A_2 - A_1\rho + n_2 \]  
(6-19b)

Combining Eq.(6-18) and Eq.(6-19):

\[ P^{so}[\Omega(-) < \Omega(+) | (-)\text{ Transmitted}] = P^{so}[A_1n_1 - A_2n_2 > A_1^2 - A_2^2] = \]
\[ Q\left(\frac{A_1^2 - A_2^2}{\sigma\sqrt{A_1^2 + A_2^2 + 2A_1A_2\rho}}\right) \]  
(6-20)

where Eq.(6-20) follows because the random variable \( A_1n_1 - A_2n_2 \) is a zero mean Gaussian with variance \( \sigma^2(A_1^2 + A_2^2 + 2A_1A_2\rho) \).

Analogously we have:

\[ P^{so}[\Omega(+) < \Omega(+) | (-)\text{ Transmitted}] = \]
\[ P^{so}[A_1y_1 + A_2y_2 - A_1A_2\rho < A_1y_1 - A_2y_2 + A_1A_2\rho] = P^{so}[y_2 < A_1\rho] = \]
\[ P^{so}[-A_2 - A_1\rho + n_2 < A_1\rho] = P^{so}[n_2 < A_2 + 2A_1\rho] = 1 - Q\left(\frac{A_2 + 2A_1\rho}{\sigma}\right) \]  
(6-21)

Similarly:
\[ P^{SO}[\Omega(++) < \Omega(+-) \mid (--) Transmitted] = P^{SO}[n_1 > A_1 + 2A_2\rho] = \left( \frac{A_1 + 2A_2\rho}{\sigma} \right) \quad (6-22) \]

Using equations (6-17, 6-20, 6-21 and 6-22) and substitution into Eq.(6-8), we end up with the following result:

\[
P(- \rightarrow + -) = p(1-p)Q\left( \frac{A_1}{\sigma} \right)Q\left( \frac{A_1^2 - A_2^2}{\sigma \sqrt{A_1^2 + A_2^2 + 2A_1A_2\rho}} \right) + \\
(1-p)(1-p)Q\left( \frac{A_1}{\sigma} \right)\left[ 1 - Q\left( \frac{A_2 + 2A_1\rho}{\sigma} \right) \right] + pQ\left( \frac{A_1 + 2A_2\rho}{\sigma} \right)\left[ 1 - Q\left( \frac{A_2 + 2A_1\rho}{\sigma} \right) \right] \quad (6-23)\]

Following the same calculations the other terms of Eq.(6-7) can be calculated as follows:

\[
P(- \rightarrow + +) = (1-p)\left[ 1 - Q\left( \frac{A_1 + A_2\rho}{\sigma} \right) \right]Q\left( \frac{A_2 + 2A_1\rho}{\sigma} \right) + \\
p(1-p)Q\left( \frac{A_1 + 2A_2\rho}{\sigma} \right)Q\left( \frac{A_2 + 2A_1\rho}{\sigma} \right) + p^2Q\left( \frac{A_1 + 2A_2\rho}{\sigma} \right)Q\left( \frac{A_1^2 + A_2^2}{\sigma \sqrt{A_1^2 + A_2^2 + 2A_1A_2\rho}} \right) \quad (6-24)\]

and

\[
P(\rightarrow \rightarrow + +) = p^2Q\left( \frac{A_1 - A_2\rho}{\sigma} \right)Q\left( \frac{A_1^2 + A_2^2 - 2A_1A_2\rho}{\sigma \sqrt{A_1^2 + A_2^2 + 2A_1A_2\rho}} \right) + \\
p(1-p)Q\left( \frac{A_1 - A_2\rho}{\sigma} \right)\left[ 1 - Q\left( \frac{2A_1\rho - A_2}{\sigma} \right) \right] + (1-p)Q\left( \frac{A_1}{\sigma} \right)\left[ 1 - Q\left( \frac{2A_1\rho - A_2}{\sigma} \right) \right] \quad (6-25)\]

and finally:

\[
P(- \rightarrow ++) = pQ\left( \frac{A_1 - 2A_2\rho}{\sigma} \right)Q\left( \frac{2A_1\rho - A_2}{\sigma} \right) + \\
(1-p)(1-p)Q\left( \frac{A_1}{\sigma} \right)Q\left( \frac{2A_1\rho - A_2}{\sigma} \right) + p(1-p)Q\left( \frac{A_1}{\sigma} \right)Q\left( \frac{A_1^2 - A_2^2}{\sigma \sqrt{A_1^2 + A_2^2 + 2A_1A_2\rho}} \right) \quad (6-26)\]

Using Eq.(6-7) the final Probability of error can be obtained.

To have some insight into the formulation so far, it is worthy considering a simple case when the both users have the same power: \( A_1 = A_2 = 1 \).

In this case we will have:
\[
T_1 = \frac{1}{2}p(1-p)Q\left(\frac{1}{\sigma}\right) + \left[1-Q\left(\frac{1+2\rho}{\sigma}\right)\right]p(1-p)Q\left(\frac{1+2\rho}{\sigma}\right) + (1-p)Q\left(\frac{1}{\sigma}\right) \tag{6-27}
\]

\[
T_2 = (1-p)\left[1-Q\left(\frac{1+\rho}{\sigma}\right)\right]Q\left(\frac{1+2\rho}{\sigma}\right) + p(1-p)Q\left(\frac{1+2\rho}{\sigma}\right)Q\left(\frac{1+2\rho}{\sigma}\right) + \nonumber \]
\[
p^2Q\left(\frac{1+2\rho}{\sigma}\right)Q\left(\frac{\sqrt{2}}{\sigma\sqrt{1+\rho}}\right) \tag{6-28}
\]

\[
T_3 = p^2\left[Q\left(\frac{1-\rho}{\sigma}\right)Q\left(\frac{2-2\rho}{\sigma\sqrt{2+2\rho}}\right)\right] + p(1-p)\left[Q\left(\frac{1-\rho}{\sigma}\right)\left\{1-Q\left(\frac{2\rho-1}{\sigma}\right)\right\}\right] + \nonumber \]
\[
(1-p)\left[Q\left(\frac{1}{\sigma}\right)\left\{1-Q\left(\frac{2\rho-1}{\sigma}\right)\right\}\right] \tag{6-29}
\]

\[
T_4 = p\left[Q\left(\frac{1-2\rho}{\sigma}\right)Q\left(\frac{2\rho-1}{\sigma}\right)\right] + (1-p)(1-p)\left[Q\left(\frac{1}{\sigma}\right)Q\left(\frac{2\rho-1}{\sigma}\right)\right] + \nonumber \]
\[
p(1-p)\left[0.5Q\left(\frac{1}{\sigma}\right)\right] \tag{6-30}
\]

Where the following equation should be considered for the matched filter:

\[
p = \frac{1}{2}Q\left(\frac{1-\rho}{\sigma}\right) + \frac{1}{2}Q\left(\frac{1+\rho}{\sigma}\right) \tag{6-31}
\]

And for our proposed detector:

\[
P_i(\sigma) = \frac{1}{2}(T_1 + T_2 + T_3 + T_4) \tag{6-32}
\]

The upper-bound BER for the optimum detector could be written as follows [Ver98]:

\[
P_i^{\text{Optimum}}(\sigma) \leq Q\left(\frac{A_1}{\sigma}\right) + \frac{1}{2}Q\left(\frac{\sqrt{A_1^2 + A_2^2 - 2A_1A_2\rho}}{\sigma}\right) \tag{6-33}
\]

The above equations assuming the \(A_1=A_2=I\), have been pictured in Figure 6.7.
Figure 6-7 Analytical performance of the Conventional detector, Suboptimum search detector and optimum detector. Two synchronous users having cross-correlation of 0.4 between spreading codes. Users have equal powers.

We have simulated the results as before and they are seen to agree with the analytical expressions defined above as shown in Figure 6.8.
6.8 Suboptimum search in conjunction with users’ power profile

The main idea of the search algorithm is to find the bits in error detected by an initial stage of detection. Intuitively it can be observed that weaker users are more prone to errors than others. In this section we will use this fact to investigate the suboptimum search algorithm in conjunction with users’ power profiles. We hope that by sacrificing some performance, the complexity of the algorithm can be reduced. Our main purpose here is to sort the users based on their power levels.

First of all, an algorithm is needed for estimating the users’ SNRs. In the next section we describe this algorithm.
6.8.1 Power estimation algorithm

The power control algorithm used can be formulated as follows:

\[ a_v = \frac{1}{W_p} \sum_{n=1}^{W_p} |y(n)| \]  
\[ \sigma = \frac{1}{W_p} \sum_{n=1}^{W_p} |y(n)|^2 - \left( \frac{1}{W_p} \sum_{n=1}^{W_p} |y(n)| \right)^2 \]  
\[ \frac{E_b}{N_0} = \frac{1}{2} \frac{a_v^2}{\sigma} \]

where \( W_p \) is the window of observation for carrying out the power estimation process. In our simulations we used one slot as the observation window. \( y(n) \) indicates the value of the decision variables.

The SNR estimation and sorting the users according to their power profiles can be performed before or after any interference cancellation. The expectation is that sorting after interference cancellation will be more accurate and provide a better performance than the one that sorts the users based on their SNR estimated before IC. We will address this issue in the forthcoming sections.

6.8.2 Suboptimum search considering only the weakest user

Intuitively, among the bits in a window of observation, the weakest user is more prone to errors. Instead of searching for every bit of every user, we can confine our suboptimum search only to the weakest user. This will reduce the complexity by a factor of \( K \) but some performance will be lost. This particular scenario is of interest and its performance is shown in Figure 6.9. The estimation of SNR is carried out according to Eq.(6-34, 6-35, 6-36).

The simulation was carried out for 10 users in the Vehicular-A channel. Processing gain of the users is 16. The Performance of the basic suboptimum search algorithm (that carries out the searching process for all users) has been compared with a method that confines the search process to the weakest user. The initial stage of detection process is either conventional detector (RAKE) or an iterative linear MUD implemented in 3 stages using the Gershgorin algorithm.
6.8.3 Sub optimum search considering a group of the weakest users

Although confining the suboptimum search to the weakest user achieves a reasonable performance considering the reduction in complexity by a factor of $K$, however the performance sacrificed is not negligible. To achieve a balance between the complexity and performance, we can confine the sub optimum search to a percentage of the users. In this case, after an initial pre-processing of the users and sorting them based on their power profile, the search will be carried out on the weakest users in the system.
To observe qualitatively the effect of a group of weakest users, consideration on the performance of the suboptimum search algorithm, a simulation was carried out and the results are shown in Figure 6.10. This simulation considers the users operating with \( E_b/N_0 = 9 \) dB.

As can be seen from the figure, only 40%-50% of the users can achieve a performance almost the same as the complete suboptimum search. This means that by considering the users' power profiles, the complexity can be reduced to 40%-50%.

Figure 6.11 is another simulation that also compares the performances of three suboptimum search algorithms:

a) Complete suboptimum search algorithm for all users
b) Suboptimum search algorithm only for the weakest user in system
c) Suboptimum search algorithm for a group of 4 weakest user in system

As shown in this figure, in the region of low \( E_b/N_0 \) (less than 9 dB), considering only 40% of the active users in the search process, almost the same performance as the full suboptimum search can be achieved and performance is 2-3 dB better than the PPIC.
Figure 6-11 Effect of a group of the weakest users (here 4 users) in performance and comparison with all-users search and weakest-user search, Users' SNR obtained based on the output of conventional detector

As shown in Figure 6-11, for high values of $E_b/N_0$ the curve doesn’t follow the performance of the full suboptimum search algorithm. In the following sections, a modification will be presented to resolve this issue.

### 6.8.4 Power control procedure after interference cancellation

The power profile calculation can be carried out in two separate places in the structure. It could be either before the initial stage of the iterative linear MUD or after the initial stage, and before the suboptimum search stage. Power profile extraction at the output of the conventional detector (RAKE) has the benefit that in the real systems there already exists a power control unit and SNR values are already available without introducing any extra overhead to the system. However, they are not as accurate as the one that calculates the power profile based on the output of the iterative linear detector.
Figure 6-12 Effect of number of weakest users considered in search process in suboptimum algorithm, first stage is iterative linear MUD (Gershgorin), SNR obtained at the output of MUD, PG=16, 10 users, $E_b/N_0=12$dB, Vehicular-A Channel

These two different approaches and their comparison are the concern of this section. Similarly in the SNR estimation at the output of the conventional detector, confining the suboptimum search algorithm to a group of the weakest users (SNR derived from the output of the initial stage of MUD operation) can achieve as good a performance as the complete suboptimum search.

Figure 6-12 shows the behaviour of the system. In this simulation the SNR of the users are extracted from the output of the initial MUD.

Figure 6.13 compares the performance of the 4 weakest users obtained in two different stages:

a) SINR estimated at the output of the RAKE (input to initial stage of iterative MUD)

b) SINR estimated at the output of the initial stage of the iterative MUD (input to suboptimum search algorithm)

It is shown well that the estimation of the powers using scheme (b) outperforms the one estimated by (a). This behaviour becomes clearer in the high interference regimes.
Different numbers of weakest users are considered in the search algorithm. Reduction of the complexity of the suboptimum search algorithm by around 60% is achieved with a performance which is almost unaltered.

6.9 Conclusions

Optimum multiuser detection needs a huge number of searches and as a result suffers from a high degree of complexity that makes it useless in practice. In this chapter after looking at the optimum detector performance, a suboptimum search algorithm was proposed. The idea is to use the optimum criteria and to commence the search process from a reliable starting point. Starting from the reliable detected information reduces the number of search
steps. The performance of this method in a simple scenario was also derived analytically for AWGN channels. The analytical expressions showed good comparison with the simulation results. The basic idea uses a structure that is well suited to the iterative algorithm introduced in chapter five. They both use the same correlation matrix and merging these two ideas lead to a final detector with a moderate degree of complexity.

Extending the suboptimum search algorithm and implementing in multistages was also investigated.

Considering the power profile of the users was another contribution given in this chapter. Sacrificing some performance and focusing only on the weakest users in the system, leads to a reduction in its complexity. Users' power profiles were obtained from two different places in the system: Before and after the initial stage of MUD. Carrying out the process at the output of the initial stage of the MUD was achieved a better performance without introducing any more complexity.

The detector proposed in chapter 5 calculates its optimal coefficients without any need for adaptive algorithms and had a comparable complexity to the Fourier algorithm. The detector proposed in this chapter was mainly designed to reduce the complexity further. Joining these two detectors (that fortunately use similar information) will lead to a final detection method with a modest complexity.
Chapter 7 Conclusions

7.1 Research Summary

Multiuser detection for WCDMA systems constitutes the primary focus of this research. The goal for the third generation (3G) mobile communication system is to seamlessly integrate a wide variety of communication services such as high-speed data, video and multimedia traffic as well as voice signals. The air interface used in 3G is Wideband Code Division Multiple Access (WCDMA). CDMA suffers from interference and multiuser detection is a sophisticated signal processing method to tackle this problem.

The main conclusions of research carried out in this thesis are as follows:

- As an interference cancellation method, PIC benefits from many desirable features. This method was considered in more detail than had been presented in the literature so far. Since the detector has good characteristics, it was considered justifiable to compare the new multiuser detection methods designed herein with the PIC. Extension of PIC structures to multiple antennas partial PIC and application to the WCDMA uplink was carried out. Realistic parameters for the WCDMA uplink were considered for this purpose and these parameters were described. Performance improvements of the method were simulated and compared with the standard WCDMA RAKE receiver.
• Based on the structure of an adaptive partial PIC detector, an adaptive RAKE-IC receiver for CDMA systems was proposed. Its structure combines the ideas of the RAKE receiver and PIC. RAKE-IC as it was named uses the concept of maximizing the signal to noise ratio of all users in the system by using adaptive algorithms. Performance improvements of the method and the effect of modified combining factors are compared with the standard RAKE receiver. The basic method was improved via some extensions. In terms of adaptive algorithms, NLMS and the newly reported PFGLMS algorithms were used in simulations and in terms of receiver structure, which used a multistage configuration.

• Another topic covered in this thesis was iterative implementation of linear multiuser detectors. Although, the family of linear detectors for multiuser detection in CDMA are a good candidate for suppressing the interference level, the precise implementation of these detectors suffers from computational complexity due to a need to invert the crosscorrelation matrix. After presenting a suitable model for the received signal, we focused on the Taylor expansion of the inverted matrix and its convergence rate. The parameters involved in this series were investigated and by analytical analysis using the Rayleigh-Ritz theorem, an accurate condition to increase the convergence rate for synchronous systems was derived. This idea, however, in the asynchronous system did not perform as well as one would have expected. Because the matrices were sparse and the method did not have accuracy. This issue was tackled via another method in linear algebra know as the Gershgorin theorem. The method was implemented and its performance was found to be promising. Using this method, a more accurate, yet simple, condition for convergence was derived. The performance of the iterative linear multiuser detector, that uses the Gershgorin theorem, was also compared with partial PIC and shown to perform better without any need to calculate the parameters via trial and error or encountering a high degree of complexity. This detector also led to a suitable structure that enabled it to work well with the ideas proposed in the previous work of the thesis.

• Optimum multiuser detection needs a huge number of searches and as a result suffers from a high degree of complexity that makes it impractical. In the last chapter after looking at the optimum detector a suboptimum search algorithm was proposed. The idea was to use the optimum criteria and start off the search process from a reliable starting point. Starting from reliable detected information reduces the number of search steps. The performance of this method in a simple scenario was also investigated analytically. The analytical
expressions were also compared with simulation results. The basic idea uses a structure that is well suited to the iterative algorithm introduced in chapter five. They both use the same correlation matrix and merging these two ideas leads to a final detector with a moderate degree of complexity. Extending the suboptimum search algorithm and implementing in multistages was also investigated.

- Considering the power profile of the users was another contribution of this chapter. Sacrificing some performance and focusing only on the weakest users in the system lead to a reduction of its complexity. Users’ power profiles were obtained from two different places in the system: before and after the initial stage of MUD. Carrying out the process at the output of the initial stage of the MUD achieved a better performance without introducing any more complexity.

Based on this thesis the following papers are published:


And another paper is also submitted to IEEE Transactions on Wireless Communications: M. Mozaffaripour, R. Tafazolli, “Collaborative Operation of Suboptimum search algorithm and polynomial expanded linear multiuser detector for interference suppression”, June 2003

7.2 Future Work

Additional work is needed to extend the results of this research. This work has identified at least three major directions for future research:
1) Systems beyond the third generation: There has been a great interest in the research community for systems beyond 3G. The extension of the methods introduced in this thesis for multi carrier CDMA (MC-CDMA) would be a very suitable issue to consider. In CCSR work on this issue has recently been started. Essential features of the detectors considered in this thesis were their canonical formulations. This enables them to be extendable to other systems such as systems using multiple antennas, as it is still possible to have a canonical formulation for multiuser scenarios that have several transmit/receive antennas. For example MIMO (Multiple Input Multiple Output) and SIMO (Single Input Multiple Output) are among the methods that can be used in multiuser scenarios.

2) Analytical trend:
Random matrix theory and asymptotic analysis of the CDMA and MC-CDMA is a very new field that facilitates the evaluation of such systems [SV01, GVR02, Ver03]. Unfortunately, these methods are confined to synchronous users. This thesis provided some tools for evaluation in asynchronous systems. These tools appear to be applicable in the asymptotic evaluation methods in asynchronous environments. Another priority in the analytical approach would be to evaluate the detectors proposed in chapters 4, 5, 6 in terms of channel capacity, as there has been new interest in the research community in this field [GVR02]. Extension of the evaluation method for the suboptimum search detector in chapter 6 for asynchronous environments and scenarios containing more than two users will also have academic value.

3) Joint decoding and detection:
Combining decoding and multiuser detection schemes has also been considered. Turbo multiuser detection uses exchanges of information between these two powerful methods. Detectors proposed in this thesis could be implemented in multistages. Some modifications of these methods to make them work in conjunction with iterative decoders would be a logical research extension.
References


References


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