University of Surrey

Effects of Stratification on Flow and Dispersion Around Obstacles in Turbulent Boundary Layers

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Abstract

The effects of stability on the flow and dispersion downstream of a simple obstacle were studied experimentally. Experiments were conducted in the EnFlo atmospheric wind tunnel, where a stably stratified boundary layer is set up by heating the inlet air and cooling the floor of the wind tunnel. Neutral and stable boundary layers with different measures of stability have been successfully modelled and their measured characteristics are reported. The majority of the experiments were then conducted with a neutral boundary layer with a nominal freestream velocity of 2.5 m/s, and two stable boundary layers with nominal freestream velocities of 1.3 m/s and 1.1 m/s. The obstacle used throughout was a 100 mm cube, placed normal to the flow direction and at 45° to the flow direction.

A pressure tapped version of the cube was placed in the neutral and stably stratified boundary layers and the pressure distribution was measured on the surface of the cube for each case. Flow measurements were also made in the wake of the cube for the neutral and two stable boundary layers.

Concentration measurements were made for different release locations and the flow configurations described above. The source was placed at the leading edge, and source heights at ground level, equal to the obstacle height, and 50% higher than the obstacle height were used. The results and statistics reported here are vertical profiles of concentration, concentration fluctuations and concentration flux, vertical and horizontal plume spread, centreline ground level concentrations and probability distributions.

A simple three dimensional wake theory typically used in dispersion modelling was evaluated and the assumptions associated with it reviewed. A Gaussian dispersion model incorporating a building effects model based on the wake theory was then implemented in a computer program. Comparisons were made between the measured and the modelled ground level concentrations. The building effects modelling was then isolated by using the experimental undisturbed plume spread results in the model.
"The wind blows wherever it pleases. You hear its sound, but you cannot tell where it comes from or where it is going. So it is with everyone born of the Spirit."

John 3:8 NIV
Acknowledgements

All honour and praise goes to our Father in heaven who gave me the strength to finish this. My sincere thanks to my wife Annali for her love, encouragement and support. I am also deeply indebted to the following persons and institutions:

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<thead>
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<tr>
<td>$A$</td>
<td>Frontal area</td>
</tr>
<tr>
<td>$\mathcal{A}$</td>
<td>Part of wake solution</td>
</tr>
<tr>
<td>$\vec{A}$</td>
<td>A typical vector</td>
</tr>
<tr>
<td>$A_Y$</td>
<td>Dispersion wake model coefficient</td>
</tr>
<tr>
<td>$A_Z$</td>
<td>Dispersion wake model coefficient</td>
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<tr>
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<td>Frontal area of cube normal to the flow</td>
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<td>$A_{45^\circ}$</td>
<td>Frontal area of cube at $45^\circ$ to flow</td>
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<td>$a_{0\beta}$</td>
<td>Dispersion wake model coefficient</td>
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<td>$b$</td>
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<td>Empirical constant in similarity relations</td>
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<td>$d$</td>
<td>Zero-plane displacement height</td>
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<tr>
<td>$f$</td>
<td>Frequency</td>
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<td>$h$</td>
<td>Boundary layer height or depth</td>
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<td>Average height of roughness elements</td>
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</tr>
<tr>
<td>$L$</td>
<td>Obukhov (buoyancy) length</td>
</tr>
<tr>
<td>$L_y(\xi)$</td>
<td>Lateral extent of wake</td>
</tr>
<tr>
<td>$L_z(\xi)$</td>
<td>Vertical extent of wake</td>
</tr>
<tr>
<td>$\Theta(\xi)$</td>
<td>Length scale in main wake</td>
</tr>
<tr>
<td>$N$</td>
<td>Brunt-Väisälä (buoyancy) frequency</td>
</tr>
<tr>
<td>$n$</td>
<td>Normalised frequency</td>
</tr>
<tr>
<td>$n$</td>
<td>Power-law exponent</td>
</tr>
<tr>
<td>$P$</td>
<td>Pressure upstream of wake</td>
</tr>
<tr>
<td>$Pe$</td>
<td>Péclet number</td>
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<tr>
<td>$p$</td>
<td>Air pressure</td>
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<td>$p$</td>
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<td>$Q$</td>
<td>Transported quantity</td>
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<tr>
<td>$Q$</td>
<td>Source strength</td>
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<tr>
<td>$\overline{Q}$</td>
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<tr>
<td>$Q'$</td>
<td>Fluctuating component of transported quantity</td>
</tr>
<tr>
<td>$Q_R$</td>
<td>Recirculation region source strength</td>
</tr>
<tr>
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</tr>
<tr>
<td>$q_2$</td>
<td>Part of wake solution</td>
</tr>
<tr>
<td>$q_{a\beta\gamma\delta}$</td>
<td>Dispersion wake model coefficient</td>
</tr>
<tr>
<td>$q_{\gamma\delta}$</td>
<td>Dispersion wake model coefficient</td>
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<tr>
<td>$R$</td>
<td>Gas constant</td>
</tr>
<tr>
<td>$R$</td>
<td>Autocorrelation function</td>
</tr>
<tr>
<td>$Re$</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>$Re_{turb}$</td>
<td>Turbulent Reynolds number</td>
</tr>
<tr>
<td>$Ri$</td>
<td>Gradient Richardson number</td>
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</tbody>
</table>
$R_i\delta$  Bulk Richardson number
$R_0$  Rossby number
$R_b$  Bulk Richardson number
$R_f$  Flux Richardson number
$R_L$  Autocorrelation of Lagrangian velocity
$R_{uu}$  Autocorrelation function for $u$ velocity
$R_{ww}$  Autocorrelation function for $w$ velocity
$r$  Radial distance
$S(f)$  Power spectral density
$S_Q$  Source or sink term
$S_{uu}$  Power spectral density for $u$ velocity
$S_{ww}$  Power spectral density for $w$ velocity
$T$  Air temperature
$T_i$  Integral time scale
$T_L$  Lagrangian integral time scale
$T_u$  Total observational time
$T_{Ly}$  Lagrangian integral time scale
$T_r$  Concentration decay time
$T_{xy}$  Shear stress upstream of obstacle
$t$  Time
$t'$  Lag time
$U$  Velocity upstream of obstacle
$U_0$  Characteristic velocity of approach flow
$U_H$  Velocity at obstacle height
$U_\infty$  Freestream velocity
$U_{oo}$  Nominal freestream velocity
$u$  Instantaneous velocity component in $x$ direction
$u'$  Fluctuating velocity component in $x$ direction
$u_*$  Friction velocity
$u_{WW}$  Wake velocity
$V$  Lateral velocity of approach flow
$V_R$  Recirculation region volume
$v$  Instantaneous velocity component in $y$ direction
$v'$  Fluctuating velocity component in $y$ direction
$v_r$  Radial perturbation velocity
$W$  Lagrangian velocity in $z$ direction
$W$  Width of obstacle
$W'$  Lagrangian velocity fluctuation in $z$ direction
$w$  Instantaneous velocity component in $z$ direction
$w'$  Fluctuating velocity component in $z$ direction
$x$  Coordinate position along $x$ axis
$x_m$  Downstream position of maximum ground level concentration
$x_{oy}$  Virtual origin of source
$x_{oz}$  Virtual origin of source
$x_{yy}$  Virtual downstream location
$x_{zz}$  Virtual downstream location
$x_{R_{max}}$  $x$ location of $z_{R_{max}}$
$y$  Lateral wake velocity solution
$y$  Coordinate position along $y$ axis
\( y_p \)  
Plume height

\( Z \)  
Vertical wake velocity solution

\( Z \)  
Particle displacement in \( Z \) direction

\( z \)  
Coordinate position along \( z \) axis

\( z_0 \)  
Roughness length

\( z_p \)  
Plume height

\( z_s \)  
Source height

\( z_{10} \)  
Height of 10 m

\( z_R \)  
Ceiling of recirculating flow region

\( z_{R\text{\tiny max}} \)  
Maximum height of recirculating flow region

**Greek Symbols**

\( \alpha \)  
Dispersion model receptor \( y \) location

\( \alpha_z \)  
Eddy viscosity constant

\( \beta \)  
Dispersion model receptor \( x \) location

\( \beta_h \)  
Stability function coefficient for heat

\( \beta_m \)  
Stability function coefficient for momentum

\( \gamma \)  
Dispersion model plume centre \( x \) location

\( \gamma \)  
Mixing parameter in stable flows

\( \gamma_W \)  
Wake model averaging constant

\( \gamma_z \)  
Wake model averaging constant

\( \chi \)  
Non-dimensional concentration

\( \delta \)  
Dispersion model plume centre \( y \) location

\( \delta \)  
Velocity boundary layer depth

\( \delta_z \)  
Plume half height

\( \delta_{99\%} \)  
Velocity boundary layer depth based on 99\% of freestream velocity

\( \delta_{ij} \)  
Kronecker delta

\( \epsilon \)  
Entrained source fraction

\( \zeta \)  
Similarity variable

\( \eta \)  
Similarity variable

\( \theta \)  
Potential temperature

\( \kappa \)  
Von Karman constant

\( \lambda \)  
Lateral to vertical length scale ratio

\( \lambda \)  
Building effects model scale parameter

\( \mu \)  
Dynamic viscosity

\( \nu \)  
Kinematic viscosity

\( \nu_0 \)  
Eddy viscosity in the undisturbed boundary layer

\( \nu_t \)  
Eddy viscosity

\( \nu_M \)  
Eddy viscosity in the wake

\( \pi \)  
3.14159265358...

\( \rho \)  
Density

\( \sigma_c \)  
Standard deviation of concentration fluctuations

\( \sigma_v \)  
Standard deviation of velocity fluctuations in \( y \) direction

\( \sigma_w \)  
Standard deviation of velocity fluctuations in \( z \) direction

\( \sigma_y \)  
Standard deviation of concentration distribution in \( y \) direction

\( \sigma_z \)  
Standard deviation of concentration distribution in \( z \) direction

\( \sigma_{yi} \)  
Component of \( \sigma_y \) due to turbulence

\( \sigma_{yw} \)  
Component of \( \sigma_y \) due to wind direction instability

xx
\[ \sigma_y \quad \sigma_y \]
\[ \sigma_z \quad \sigma_z \]
\[ \tau \quad \text{Shear stress} \]
\[ \tau \quad \text{Time lag} \]
\[ \phi \quad \text{Stability function} \]
\[ \xi \quad \text{Similarity variable} \]
\[ \omega \quad \text{Frequency} \]

**Other symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>( \diamond )</td>
<td>45° cube</td>
</tr>
<tr>
<td>( \Box )</td>
<td>Normal cube</td>
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<tr>
<td>( \times )</td>
<td>Undisturbed</td>
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**Subscripts**

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<th>Subscript</th>
<th>Description</th>
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<tr>
<td>( h )</td>
<td>Heat</td>
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<tr>
<td>( i, j, k, \beta )</td>
<td>Tensor indices</td>
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<td>( m )</td>
<td>Momentum</td>
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<tr>
<td>( m )</td>
<td>Maximum</td>
</tr>
<tr>
<td>( t )</td>
<td>Turbulent</td>
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<tr>
<td>( WW, EE, EW, WE, E, W )</td>
<td>Wake modelling regions</td>
</tr>
<tr>
<td>( \alpha, \beta, \gamma, \delta )</td>
<td>Variables for wake modelling regions</td>
</tr>
<tr>
<td>( \infty )</td>
<td>Freestream</td>
</tr>
<tr>
<td>( \infty )</td>
<td>Nominal freestream</td>
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</table>

**Superscripts**

<table>
<thead>
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<th>Superscript</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( - )</td>
<td>Time or ensemble average</td>
</tr>
<tr>
<td>( ' )</td>
<td>Fluctuating variable</td>
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1.1 Background

One of the most frequently encountered stratified turbulent boundary layers is the atmospheric boundary layer, which exists in the lower kilometre of the atmosphere. This is also the region where most air pollution is released. This study deals with matters of air pollution, and can therefore be classified under air pollution meteorology. Air pollution meteorology deals with pollutants emitted in the atmosphere (e.g. Lyons & Scott 1990, Arya 1999). These pollutants can be man-made substances or natural substances at a concentration high enough to be harmful to living organisms. The source of these pollutants can be industrial or natural releases. Pollutants in the atmosphere are transported, dispersed, transformed and ultimately removed (Lyons & Scott 1990). Of these four processes only the first two, transport and dispersion, will be considered further.

The word stratified means being arranged in (horizontal) layers. In the context of a flow this implies a vertical gradient of the thermodynamic state variables. The stratification of the state variables is classed as stable when a fluid parcel that is displaced in the vertical will experience a force acting to return the parcel to its original position. The force exerting the ability of such a fluid can only come about from a vertical density gradient. A stably stratified fluid will therefore be density stratified, and the stabilising force a buoyancy force.

Air pollution meteorology does not address the causes of pollution. The only way to remedy some of the harmful effects of pollution on the earth’s environment and its inhabitants will be to cease the emissions of pollutants involved. The economic state in the world renders this solution impossible, and the alternative is to manage air pollution in such a way as to prevent the harmful effects of pollution on earth’s inhabitants.

The atmosphere extends to several hundred kilometres above the earth’s surface. The majority of pollutants are released into the part of the atmosphere adjacent to the earth’s surface. Because of the continuous movement of the atmosphere the classical fluid mechanics concept of a boundary layer (e.g. White 1991) must exist near the earth’s surface. This boundary layer, known as the atmospheric boundary layer, has a typical height of 1 kilometre. Above the atmospheric boundary layer the flow is essentially inviscid where pressure gradients and Coriolis forces balance, resulting in the geostrophic wind. The atmospheric boundary layer
has attracted the attention of a broad field of researchers, including fluid dynamicists, mathematicians, meteorologists and wind engineers, and remains a challenging field of study.

From the top of the atmospheric boundary layer the velocity decreases from the geostrophic wind speed to a value of zero at the earth’s surface. Momentum is thus transferred to the earth surface. Apart from momentum transfer the other fundamental transfer processes, namely energy and mass transfer, are also present and significant at the earth–atmosphere boundary. Mass transfer is important for deposition processes, but can normally be neglected when studying dispersion in the atmosphere. The heat transfer processes largely determine the structure of the planetary boundary layer. The heat flux at the earth’s surface determines the temperature distribution in the lower atmosphere.

The three common states of the atmospheric boundary layer are stably stratified, neutral and convective. Stable density stratification implies a positive potential temperature gradient, and hence negative heat flux at the earth surface. Stable stratification is a very common occurrence in the atmosphere and is the typical flow condition at night, when the earth surface is cooled by long wave radiation to space. A characteristic of stable stratification in a turbulent flow is the damping effect of the stratification on turbulence levels in the flow. The boundary layer with no thermal stratification, usually called neutral, will serve as the reference case for all studies conducted here. The earth surface is continuously being heated by radiation from the sun or being cooled by radiation to space. The conditions for zero heat flux are therefore rarely satisfied, and hence an exactly neutral boundary layer by definition is a rare occurrence in the atmosphere. Strong winds reduce the effect of stratification and boundary layers with strong winds can often be treated as neutral. Neutral stratification is also the limit of stability tending to zero and is therefore an important reference case. The atmospheric boundary layer with unstable stratification is vastly different from the stable case. The unstable atmospheric boundary layer is commonly referred to as the convective boundary because of the buoyancy driven movement of air caused by the surface heating of the earth. The buoyancy force is de-stabilising, and this type of flow is characterised by high turbulence levels generated by the unstable stratification. Because of the fundamental difference between the convective and the stably stratified atmospheric boundary layers they are often considered separately in air pollution meteorology. The convective boundary layer will only be considered briefly in Chapter 2 and ignored further on.

As the heat flux at the earth’s surface affects the state of the atmospheric boundary layer, the geometry of the earth’s surface also has an effect. Topography signifies the geographical features such as hills, valleys and mountains and will have a large influence on the atmospheric boundary layer. The effluents from industrial plants are typically released in the vicinity of buildings and other man-made structures. These smaller features can be expected to influence the flow locally, but will not influence the atmospheric boundary layer as a whole.

### 1.2 Aim

With the current socio-economic emphasis on the environment any study related to air pollution is important. The previous section introduced many of the concepts relevant to this study. The atmospheric boundary layer as a whole is greatly influenced by stratification, but local features may often dominate the flow near the ground. Most studies investigating the flow around obstacles ignore the effects that stratification may have on the flow around the obstacle. The results from these studies are then used to derive general prediction schemes.
that are considered equally applicable for neutral and stable stratification. Robins (1994) highlighted this lack of general prediction techniques specifically aimed at flow around obstacles in stably stratified boundary layers. This study will investigate the effects of stability on the flow and dispersion around obstacles. The differences between the flows under neutral and stably stratification need to be identified to suggest modifications to the current prediction schemes. This study also aims to augment the current observational knowledge of stably stratified flows that can be used for validation of prediction schemes in the future.

Environmental assessment is an essential part of industrial plant feasibility studies and design. The air pollution and emission guidelines are laid down by government and enforced by legislation, in the interest of public health and welfare. The environmental assessment requires modelling of the dispersion of pollutants released by the proposed plant. All methods of modelling are used in air pollution modelling, numerical modelling from simple parametric models through to direct numerical simulation, as well as the full spectrum of physical modelling. Routine calculations are however carried out with so-called 'practical' dispersion models. As will be seen later dispersion modelling is a heavily relied on resource in the planning of industrial complexes, and for environmental assessment studies.

The stably stratified boundary flow over two and three dimensional hills has been the topic of much research, and the pronounced effects that stable stratification can have on the flow are well known. Much of what is known about stably stratified boundary layer flows over bluff obstacles such as elementary cubes and buildings are simply extensions of the neutral boundary layer flow over such obstacles. The near wake region of cubes in stable boundary layers has been studied by several authors, and the conclusion is that stability has little effect on the near wake region. These studies have noted that stable stratification does not have a significant effect on the flow structures in the near wake region because turbulence generated by the building perturbation dominates. Many of these authors have then speculated that stability is likely to have a greater effect further downstream, but little research has gone into studying this region. This study will therefore investigate the effects of stability on the wake structure of simple building shapes beyond the near wake region. Accurate models can account for the downwash effect of obstacles on released substances, but the effect of stable stratification on the wake behind a building has not yet been extensively researched. Stable stratification acts as a sink in the turbulent kinetic energy equation and the turbulence after the recirculation region will decay more rapidly in the very stable case than the neutral case.

The scope of the study is summarised in Table 1.1 and will be discussed briefly. The important variables that define a flow configuration are the characteristics of the boundary layer, the characteristics of the surface, the type of obstacle investigated and for dispersion studies the type of source and the type of release. The types of problems that will be investigated will therefore consist of elevated sources, passive releases, neutral and moderately stable turbulent boundaries, flat terrain and cubical obstacles.

An experimental wind tunnel investigation is the mode of research best suited to the facilities at EnFlo, and the knowledge base acquired over the years. All studies with other approaches to the problem such as full-scale field studies and numerical simulation studies will be relevant, because the end objective of the studies is the same. The aspects of dispersion modelling will also be considered in detail.
Chapter 1. Introduction

Table 1.1: Scope of this study

<table>
<thead>
<tr>
<th></th>
<th>Included</th>
<th>Excluded</th>
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<tbody>
<tr>
<td>Source</td>
<td>Elevated point source; Passive, neutral release; Negligible exit momentum</td>
<td>Line and area sources; Reactive release; Buoyant or dense release;</td>
</tr>
<tr>
<td>Stratification</td>
<td>Neutral and moderately stable</td>
<td>Unstable or Strong stability</td>
</tr>
<tr>
<td>Obstacles</td>
<td>Single cubical building</td>
<td>Every other imaginable obstacle; Obstacle arrays</td>
</tr>
<tr>
<td>Terrain</td>
<td>Flat terrain</td>
<td>Complex terrain</td>
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<td>Applied models</td>
<td>Research models</td>
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<tr>
<td>Research category</td>
<td>Applied</td>
<td>Fundamental</td>
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</table>

1.3 Summary of contents

In what follows a brief description of the chapters and appendices that make up this thesis will be given.

Chapter 2 reviews the relevant literature and introduces the necessary theory concerning turbulence, the atmospheric boundary layer (ABL), dispersion and modelling. Specific attention is awarded to the stably stratified atmospheric boundary layer since that is the state we are most concerned about. Other methods of modelling apart from physical modelling are briefly introduced. The literature on the physical modelling of the stably stratified ABL is reviewed, with specific attention to obstacle affected stably stratified boundary layers.

Chapter 3 discusses the instrumentation and hardware systems that are available at EnFlo and that have been used. The systems are discussed in detail with specific reference to their proper usage.

The experimental results are presented in two chapters. Chapter 4 documents the flow. Four boundary layers were characterised, of which three were selected. Pressure measurements on a cube in neutral and stable boundaries are reported. Flow measurements in the wake of a cube normal to the approach flow are also presented.

Chapter 5 reports results of the dispersion investigation. Mean concentrations, concentration fluctuations and concentration fluxes were measured for different boundary layers, obstacle orientations and source heights.

Chapter 6 discusses the practical implementation of a dispersion model. The dispersion model is largely based on the Cambridge Environmental Research Consultants Ltd. (CERC) ADMS 3 model, with applicability limited to the cases that were investigated experimentally. A comparison is made between the experimental results and the results from the model.

Chapter 7 concludes this study, and lays down suggestions for future work.

Appendix A describes some of the tools that were developed and used in the analysis and presentation of the data.
Chapter 1. Introduction

Appendix B describes the procedure that was used to calculate autocorrelations and spectra from LDA data. Code snippets are also given for the core algorithms.

Appendix C contains detailed specifications of the instrumentation.

Appendix D contains all the detailed dispersion results.

Appendix E contains probability density distribution plots that are discussed in Chapter 5.

Appendix F describes the derivation of the three dimensional wake model.

Appendix G describes the inclusion of a building effects model in a dispersion model.
This chapter summarises the literature survey that was performed for this study. Most of the theory related to the study will be presented here to make this report as self contained as possible. For more detailed discussions of the topics the reader is referred to the cited references. The literature is reviewed in six broad sections. Turbulence is discussed in §2.1. The theory of the atmospheric boundary layer is summarised in §2.2. Aspects of dispersion are discussed in §2.3. Surface obstacles are discussed in §2.4. Physical modelling is reviewed in §2.5, and numerical modelling is briefly introduced in §2.6.

2.1 Turbulence

Hanna (1982) stated that “atmospheric diffusion is a direct result of atmospheric turbulence”, and hence a mathematical description of turbulence is required for dispersion modelling. Bradshaw (1971) reminds the reader that we possess a closed set of accurate equations describing fluid motions fully. The problem is that analytical solutions only exist for a few simple cases, and the computational time requirements for the numerical solution of practical fluid flow problems using these equations in their fundamental form (direct approach) are still too high for everyday use. This led to the vibrant field of turbulence modelling that attempts to model certain aspects of turbulence to simplify the equations such that solutions to practical fluid flow problems can be found for everyday engineering purposes.

There is scope for the direct approach and the turbulence modelling approach in air pollution meteorology, but for routine dispersion modelling a more pragmatic approach to turbulence is required. Turbulence characteristics are often represented by parametric relations (Arya 1984), which, for the atmospheric boundary layer considering all the uncertainties, are often good enough. The fundamentals of turbulence and the current state of turbulence research are beyond the scope of this study and the pragmatic approach to turbulence will also be adopted here.

One concept prevalent in turbulence modelling is the study of the mean values of the turbulent quantities rather than the random fluctuating variable. The averaging procedure and implications thereof will be introduced in terms of a general transport equation describing the turbulent transport of any turbulent quantity $Q$. 
2.1.1 General transport equation

It is convenient to formulate a general transport equation that describes the transport of a general property. With little adaptation the transport equations describing fluid motion can then be derived from the general equation. The concepts of turbulent fluctuations, Reynolds averaging and the closure problem are also best introduced in this context.

Before the general transport equation is stated the tensor notation that will be used throughout this study deserves a brief introduction.

2.1.1 (a) Tensor notation

Tensor notation offers a convenient alternative to conventional vector notation when writing equations involving vector quantities or components of vector quantities. All the transport equations involve components of the velocity vector. The following description of tensor notation is taken from Frederick & Chang (1972).

The Cartesian components \((A_1, A_2, A_3)\) (of a vector \(\vec{A}\)) may be represented by the symbol

\[ A_i \quad (i = 1, 2, 3) \]

where the subscript \(i\) is understood to take on the values \((1, 2, 3)\) in that order. Therefore, the symbol \(A_i\) represents the set of three Cartesian components

\[(A_1, A_2, A_3)\]

Whenever a small Latin letter subscript occurs repeated in a term, it is understood to represent a summation over the range of 1,2,3.

Another important definition is the Kronecker delta, \(\delta_{ij}\), with

\[ \delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \]

For further information on the index notation used extensively in continuum mechanics the reader is referred to Frederick & Chang (1972).

2.1.1 (b) Lagrangian & Eulerian description

Fluid motion can be described either from a fixed location in space or from a moving frame of reference. The former is the Eulerian description and the latter the Lagrangian description.

Measuring instruments can either be fixed in space such as masts and most wind tunnel instrumentation, or fixed to a moving aircraft or balloon. Wind tunnel measurements are therefore best suited to the Eulerian description of fluid motion. Numerical models can be constructed using the Eulerian or Lagrangian description, but Eulerian models are more common.
Chapter 2. Theory & related work

Mathematically the Lagrangian description treats the motion of a particle in a moving frame of reference travelling with the particle. It is therefore clear that the Lagrangian description of fluid motion is unsuitable to general measurements. In the field of theoretical fluid mechanics the Lagrangian description has however proved invaluable to the theoretical understanding of turbulence and dispersion.

2.1.1 (c) The advection–diffusion equation

For a scalar quantity $Q$ the general transport equation can be written as

$$\frac{\partial Q}{\partial t} + \frac{\partial u_i Q}{\partial x_i} = K_Q \frac{\partial}{\partial x_i} \left( \frac{\partial Q}{\partial x_i} \right) + S_Q$$

(2.1)

with $K_Q$ the molecular transport diffusivity for the quantity $Q$ and $S_Q$ the source term. $K_Q$ is assumed constant as is the case for molecular diffusion. Although velocity is a vector the individual velocity components can be considered as scalar quantities and substituted in the equation to form the momentum equations. The term on the left hand side of the equation is the advection term, and on the right hand side apart from $S_Q$ the diffusion term.

2.1.1 (d) Reynolds averaging

Turbulent flows are characterised by random fluctuations of the quantities that describe the flow. By employing an averaging scheme a mean value can be determined for every turbulent quantity. In the analysis of turbulent flows it is common to decompose instantaneous values into mean and fluctuating components. For a quantity with instantaneous value $Q$

$$Q = \bar{Q} + Q'$$

(2.2)

where $\bar{Q}$ is the mean component and $Q'$ the fluctuating component with zero mean. The velocity components in Eq. (2.1) are all turbulent quantities that are decomposed as follows:

$$u_i = \bar{u}_i + u'_i$$

(2.3)

Substitution of Eqs. (2.2) & (2.3) into Eq. (2.1) yields

$$\frac{\partial (\bar{Q} + Q')}{\partial t} + \frac{\partial (\bar{u}_i + u'_i)(\bar{Q} + Q')}{\partial x_i} = K_Q \frac{\partial}{\partial x_i} \left( \frac{\partial (\bar{Q} + Q')}{\partial x_i} \right) + S_Q$$

(2.4)

For unsteady flows the proper averaging technique is ensemble averaging where the results of many successive experiments with identical setups are averaged. After simplification and ensemble averaging Eq. (2.4) becomes

$$\frac{\partial \bar{Q}}{\partial t} + \bar{u}_i \frac{\partial \bar{Q}}{\partial x_i} + u'_i \frac{\partial \bar{Q}'}{\partial x_i} = K_Q \frac{\partial}{\partial x_i} \left( \frac{\partial \bar{Q}}{\partial x_i} \right) + S_Q$$

(2.5)

For steady flows $\frac{\partial \bar{Q}}{\partial t} = 0$ and ensemble and time averaging are therefore equivalent.

The Reynolds decomposition and averaging process has led to the addition of the $\frac{\partial u_i}{\partial x_i}$ terms. Although the equations are now written in terms of average values, the introduction of the new terms leads to the so-called turbulence closure problem.
2.1.1 (e) Closure problem

As stated in the previous section the averaging of the advection term in the general transport equation led to the introduction of new unknowns in the averaged transport equation. The new terms introduced prevent us from having a closed set of equations, thus the term "closure problem". Physical models for the newly introduced terms need to be found before the equations can be solved.

One approximation known as the Boussinesq approximation, (e.g. White 1991), converts the term \( u_i' Q' \) into a gradient diffusion term with the introduction of an eddy diffusivity \( K_{Qi} \), so that

\[
-u_i' Q' = K_{Qi} \frac{\partial Q}{\partial x_i}
\]  

(2.6)

The "kinematic" eddy diffusivities in this form have dimensions \([L]^2[T]^{-1}\). The eddy diffusivities are not fluid properties, but are functions of the turbulence. Bradshaw et al. (1984) state that the eddy viscosity assumption is only valid in turbulent flows with the turbulence close to equilibrium, i.e. where the production and dissipation of turbulent kinetic energy dominates the convection and diffusion of turbulent kinetic energy. In physical terms this requirement follows the molecular transport analogy. Molecular transport only depends on the local gradient of the transported quantity.

The eddy viscosity model is a first order closure model. Another approach is to derive conservation equations for each of the \( \frac{\partial u_i' Q'}{\partial x_i} \) terms. New unknown terms will naturally arise in these second order conservation equations which have to be modelled, but more of the physics of the process is captured by introducing closure models only at the second order stage. The process can be extended by deriving higher order conservation equations, but there will always be higher order terms that have to be modelled. If the process is limited to the second order stage the closure thus achieved is second order closure.

2.1.2 Momentum & continuity equations

The momentum equation for the \( u_j \) velocity component can then be written from Eq. (2.5), with the pressure term derived from an elemental control volume:

\[
\frac{\partial u_j}{\partial t} + \frac{\partial u_i u_j}{\partial x_i} - 1 \frac{\partial p}{\partial x_i} + \nu \frac{\partial}{\partial x_i} \left( \frac{\partial u_j}{\partial x_i} \right)
\]  

(2.7)

The continuity equation can be derived easily for an elemental control volume

\[
\frac{\partial u_i}{\partial x_i} = 0
\]  

(2.8)

The cross products that arose as a result of Reynolds averaging in § 2.1.1 (d) that appear as the \( \overline{u_i u_j} \) terms in Eq. (2.7) are known as turbulent or Reynolds stresses.

2.1.3 Statistical description of turbulence

The concept of Reynolds averaging introduced in § 2.1.1 converts the continuous random turbulent quantities into manageable variables involving averages and co-products. All the
information contained in the continuous time dependent signals can hardly be represented by the averages and products, and some further statistical descriptors are introduced to summarise turbulent signals.

Turbulence is however not just another random variable that can be described completely by statistical descriptors. Tatsumi (1999) ascribed the complexity associated with turbulence to its dual nature - a random appearance with a deterministic nature due to the governing Navier Stokes equations.

2.1.3 (a) Moments & products

The statistical concept of moments, $\mu^n$ for the nth moment, has physical significance for turbulent quantities due to their governing equations. Similar physical significance also applies to the cross products of turbulent quantities. The Reynolds stresses in §2.1.2 can either be second order moments for normal stresses or cross-products for shear stresses.

In general, conservation equations similar to Eq. (2.7) can be defined for the moments and products from their governing equations. For the Reynolds stresses White (1991) describes the procedure as follows:

1. Subtract time averaged momentum equation Eq. (2.7) from its instantaneous value, for both the $i$ and $j$ directions.
2. Multiply the $i$th result by $u_j$ and add to the $j$th result multiplied by $u_i$.
3. Time average to yield Reynolds stress equation

The Reynolds stress equation that results is

$$\frac{\partial u'_i u'_j}{\partial t} + \frac{\partial u'_i u'_j}{\partial x_k} = -\left[ u'_j u'_k \frac{\partial u_i}{\partial x_k} + u'_i u'_k \frac{\partial u_j}{\partial x_k} \right]$$

$$-2\nu \frac{\partial u'_i}{\partial x_k} \frac{\partial u'_j}{\partial x_k} + \frac{\nu'}{\rho} \left( \frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i} \right)$$

$$-\frac{\partial}{\partial x_k} \left[ u'_i u'_j u'_k - \nu \frac{\partial u'_i u'_j}{\partial x_k} + \frac{\nu'}{\rho} (\delta_{jk} u'_i + \delta_{ik} u'_j) \right]$$

Similar equations can be derived for higher order moments and products.

2.1.3 (b) Probability density distributions

The probability density distribution $p(u')$ describes the probability that a continuous function $u'(t)$ will actually attain a value of $u' < u'(t) \leq u' + \Delta u'$. The range $\Delta u'$ is required because the probability of $u'(t)$ exactly equalling a value of $u'$ is always zero. Arya (1999) defines $p(u')$ mathematically as

$$p(u') = \lim_{\Delta u' \to 0} \lim_{T \to \infty} \frac{T}{T \Delta u'}$$

(2.10)
where \( T_u \) is the total observational time that \( u'(t) \) spends in the range \( u' < u'(t) \leq u' + \Delta u' \) and \( T \) is the total observational time.

By definition the following must always hold for \( p(u') \)

\[
\int_{-\infty}^{\infty} p(u') du' = 1 \tag{2.11}
\]

The average of any function \( f \) of \( u' \) can be expressed as follows

\[
\overline{f(u')} = \int_{-\infty}^{\infty} f(u') p(u') du' \tag{2.12}
\]

The moments of \( u'(t) \) can therefore be defined as

\[
\overline{u'^n} = \int_{-\infty}^{\infty} u'^n p(u') du' \tag{2.13}
\]

Many random processes that occur in nature and technology have normal or Gaussian probability density distributions, defined as

\[
p(u') = \frac{1}{\sqrt{2\pi}u'^2} e^{-\frac{1}{2} \frac{u'^2}{u'^2}} \tag{2.14}
\]

2.1.3 (c) Autocorrelation

The autocorrelation of a signal is a measure of the connection between the value of the signal and the preceding or following time history of the signal. The time autocorrelation function in its normalised form for a stationary variable \( u'(t) \) is defined as

\[
R_{uu}(\tau) = \frac{u'(t)u'(t + \tau)}{u'^2} \tag{2.15}
\]

where \( \tau \) is the time lag.

The autocorrelation function of a variable has the following properties

\[
R_{uu}(0) = 1 \tag{2.16}
\]
\[
R_{uu}(\tau) = R_{uu}(-\tau) \tag{2.17}
\]
\[
R_{uu}(\tau) \leq R_{uu}(0) \tag{2.18}
\]
\[
R_{uu}(\tau) \to 0, \text{ as } \tau \to \pm \infty \tag{2.19}
\]

The concept of Lagrangian and Eulerian description was introduced in §2.1.1 (b). The autocorrelation function for a turbulent velocity component can also be defined in terms of the two descriptions, with the Lagrangian autocorrelation function using Lagrangian velocity and the Eulerian autocorrelation function using Eulerian velocity to calculate the autocorrelation. The statistical theory of diffusion discussed in §2.3.2 (c) makes extensive use of the Lagrangian autocorrelation function.
From the autocorrelation function a number of scales can be defined, with \( T_i \), the integral time scale, being the most important to diffusion applications. \( T_i \) is defined as

\[
T_i = \int_0^\infty R_{uu}(\tau) \, d\tau
\]  

(2.20)

The Lagrangian integral time scale defined in Eq. (2.69) uses the same definition of the integral time scale defined in terms of the autocorrelation function of Lagrangian velocities.

### 2.1.3 (d) Frequency spectra

Any continuous function \( f(t) \) can be constructed from an infinite number of elemental sine functions on a continuous range of frequencies \( \omega \). The Fourier transform of a function \( f(t) \) returns a function \( \mathcal{F}(\omega) \) of the amplitudes of the elemental sine waves with frequencies \( \omega \) that make up the original function.

The Fourier transform has found an important application in turbulence. The amplitude and frequency of each of the elemental sine functions computed from the Fourier transform of a turbulent velocity component can be used to calculate the distribution of the total power of the turbulent velocity at different frequencies. The resulting power spectrum, e.g. \( S_{uu}(\omega) \) for the \( u' \) velocity, must satisfy

\[
\int_0^\infty S_{uu}(\omega) \, d\omega = \frac{u'^2}{2}
\]  

(2.21)

A physical interpretation of the frequencies \( \omega \) are the angular velocities of the eddies in turbulent flows, with the larger eddies having lower angular velocities than the smaller eddies.

### 2.1.3 (e) Homogeneous turbulence

Homogeneous turbulence is defined to be statistically independent of position in space. An example of homogeneous turbulence is an unbounded flow with uniform shear. The turbulence in engineering flows may be locally homogeneous, but never fully homogeneous. An extensive statistical theory of turbulence has been developed by Batchelor (1953) for homogeneous turbulence. Many valuable theories of turbulence stem from the assumption of homogeneity, with the Gaussian plume model just one example essential to air pollution meteorology.

### 2.2 The atmospheric boundary layer

Many authors (e.g. Kaimal & Finnigan 1994, Arya 1999, Garrat 1992) have discussed the characteristics of the atmospheric boundary layer (ABL) in considerable detail. This study deals with the atmospheric boundary layer in its stably stratified state, and the characteristics of the atmospheric boundary layer under stable stratification will be repeated here.

#### 2.2.1 Simplifications

To simplify matters only flow over flat uniform terrain will be considered as the assumed approach flow to a building. For flow over changing terrain and flow over hills the reader is
referred to the authors cited above.

Potential temperature $\theta$ is a derived property that is defined as the temperature that an air parcel with absolute temperature $T$ and pressure $p$ would have if it was brought adiabatically to a standard pressure of 1000 mb.

$$\theta = T \left(\frac{1000 \text{ mb}}{p}\right)^{R/c_p} \quad (2.22)$$

In adiabatic atmospheric conditions the absolute temperature decreases with height because of the decreasing air pressure. This temperature gradient is known as the adiabatic lapse rate. The potential temperature is formulated to remain constant with height in adiabatic atmospheric conditions. The potential temperature is therefore more suitable for characterising atmospheric boundary layers than the absolute temperature.

Virtual temperature is commonly used for atmospheric flows to account for the humidity of the air. The virtual temperature is the temperature that dry air would have for its density and pressure to be the same as the actual moist air.

For the purposes of this study temperatures in the atmosphere will always be described in terms of virtual potential temperatures, and laboratory temperatures will always be absolute temperatures even though the symbol $\theta$ will be used to indicate that potential and absolute temperatures are interchangeable in the laboratory.

### 2.2.2 States of the ABL

The wind speed and the magnitude and direction of the heat exchange between the earth's surface and the ABL define the state of the ABL. The surface temperature of the earth is determined by radiation heat transfer from the sun and atmosphere during daytime and radiation heat loss to space and the atmosphere during night time. The sun emits shortwave radiation while the earth and the atmosphere emit longwave radiation. Heat exchange between the earth's surface and the ABL is through forced convection, free convection or a mixture of both.

When the surface air is heated by the earth's surface heat is transferred to the ABL, i.e. positive heat transfer. The warmer packets of air rise naturally and generate turbulence as they move through the surrounding air. This state of the ABL is classified as unstable, and the boundary layer that develops is called the convective boundary layer (CBL).

When the surface air is cooled by the earth's surface, heat is transferred from the ABL, i.e. negative heat transfer. The cooler packets of air are in a stable state, and tend to retain their vertical position. The boundary layer that develops is called the stable boundary layer (SBL).

The third state occurs when there is little or no heat exchange between the earth's surface and the surface air. That can only happen if the surface air temperature is in equilibrium with the earth's surface temperature, i.e. under neutral stratification. Exactly neutral stratification is the least common state of the ABL, usually occurring early in the morning and late evening. Under strong wind conditions stability affects the wind profile and turbulence characteristics of the ABL only slightly, and the laws derived for neutral stratification begin to apply.
2.2.3 Boundary layer depth

For a flow set up in a laboratory the free stream velocity is clearly defined. The driving force in the atmosphere is primarily the geostrophic wind, which results due to the balance of pressure gradients and Coriolis forces. The geostrophic wind is a dynamic phenomenon and stationary flow is seldom observed.

In the laboratory a zero pressure gradient turbulent boundary layer with neutral stratification will continue to grow with downstream distance indefinitely or until a fully developed channel flow state is achieved. A boundary layer depth is therefore dependent on the distance downstream from the point where the boundary layer has started to grow. There are obvious similarities between the surface layers of a simulated boundary layer and the atmospheric boundary layer. One major difference is that the ABL is characterised by the turning of the wind direction with height. Observations suggest that the developed neutral ABL does have a characteristic height independent of the fetch, which has been defined from dimensional analysis as:

\[ h = c \frac{u_s}{f} \]  

(2.23)

The value of the constant \( c \) is subject to the exact definition of \( h \), and considerably different values are found in the literature (e.g. Garrat 1992).

Neutral stratification is however a less commonly observed state of the ABL, and of more importance is the height of the ABL under typical observed states, namely stable and unstable stratification. The height of the stable boundary layer is defined as the top of the surface inversion, where an inversion is the common meteorological term for a stably stratified temperature profile. For unstable stratification the boundary layer is always capped by an inversion layer, and the boundary layer height for unstable stratification is defined as the height of the inversion layer.

\[ z_h = \begin{cases} z_i & \text{(daytime)} \\ h & \text{(nighttime)} \end{cases} \]  

(2.24)

2.2.4 Measures of stability

Parameters are required that classify atmospheric stability. The atmospheric stability classification is an essential input parameter for air quality models. Classification schemes vary from very simple requiring very little input to complex requiring data that are not normally available for routine air quality assessments.

The Richardson number generally represents the ratio of buoyant production or destruction of turbulence to shear production of turbulence. Several forms of the Richardson number exist, depending on the application and the amount of data that is available for a calculation. In all cases a positive Richardson number indicates stably stratified flow, and a negative Richardson number indicates unstably stratified flow. The most common forms will be defined here.

The fundamental form of the ratio of buoyant production/destruction of turbulence to shear production is the Flux Richardson number

\[ R_f = \frac{(g/\theta) w'\theta'}{u'w'(\partial \theta/\partial z)} \]  

(2.25)
The fluxes in Eq. (2.25) can be written in the flux-gradient form, leading to the Gradient Richardson number.

\[ Ri = \frac{(g/\theta)(\partial \theta / \partial z)}{(\partial u / \partial z)^2} \]  \hspace{1cm} (2.26)

The Bulk Richardson number is defined from Eq. (2.26) by estimating the gradients over a height \( z \) of the boundary layer

\[ R_b = \frac{(g/\theta)(\bar{\theta}_z - \bar{\theta}_0)/z}{(\bar{u}_z/z)^2} \]  \hspace{1cm} (2.27)

For laboratory simulations the definition of the bulk Richardson number that is often used is

\[ Ri_\delta = \frac{(g/\theta)\delta(\bar{\theta}_\infty - \bar{\theta}_0)}{U_\infty^2} \]  \hspace{1cm} (2.28)

where \( \bar{\theta}_\infty \) and \( U_\infty \) are evaluated at the top of the boundary layer where \( z = \delta \).

Using the definitions

\[ u_* = \sqrt{-\langle u'w' \rangle_0} , \]
\[ \theta_* = \frac{-\langle \theta'w' \rangle_0}{u_*} \]

and the gradient form of the wind profile for neutral stratification

\[ \partial u / \partial z = u_* / \kappa z \]

which will all be introduced formally in § 2.2.6, Eq. (2.25) can be written as:

\[ R_f = \frac{(g/\theta)\theta_*}{u_*^2 / \kappa z} \]  \hspace{1cm} (2.29)

near the surface.

The vertical position \( z \) where \( R_f \) from Eq. (2.29) is unity can be defined as a convenient length scale \( L \), called the Monin-Obukhov length. The height \( z = L \) does not have particular significance, since the wind profile for neutral stratification was used. For stable conditions \( L > 0 \) and for unstable conditions \( L < 0 \). Put into words \( |L| \) is the height where buoyant production/destruction of turbulence equals shear production of turbulence if the wind profile was not altered by stability.

\[ L = \frac{u_*^2 / \kappa}{(g/\theta)\theta_*} \]  \hspace{1cm} (2.30)

A dimensionless stability parameter can be defined as the ratio of height \( z \) to Monin-Obukhov length \( L \)

\[ \frac{z}{L} = \frac{(g/\theta)\theta_*}{u_*^2 / \kappa z} \]  \hspace{1cm} (2.31)

Pasquill (Lyons & Scott 1990) proposed a stability classification scheme based on routine meteorological observations consisting of wind speed, insolation and cloud cover. The Pasquill stability classes remain popular even today although they have been superseded by more sophisticated and theoretically sound relations.
Typical atmospheric values for the stability parameters defined above under different stability conditions are shown in Table 2.1. The Flux Richardson numbers $R_f$ and the Gradient Richardson numbers $R_i$ are equal near the surface for stable stratification.

The Obstacle Froude number is often used as a stability parameter for the flow around obstacles. In this context the Froude number is the ratio of the potential energy requirements for a fluid parcel to go over an obstacle to the kinetic energy of the parcel. The obstacle Froude number is defined as:

$$F = \frac{U_0}{NH}$$

with $U_0$ the characteristic velocity of the approach flow (usually taken at the top of the obstacle), $H$ is the characteristic height of the obstacle and $N$ is the Brunt-Väisälä or buoyancy frequency of the approach flow.

$$N = \left( \frac{g}{\theta_0} \frac{\partial \theta}{\partial z} \right)^{1/2}$$

Table 2.1: Typical values for the various stability parameters. Modified after Snyder (1981)

<table>
<thead>
<tr>
<th>Qualitative description</th>
<th>Pasquill category</th>
<th>$L$ [m]</th>
<th>$z_{10}/L$</th>
<th>$R_f$</th>
<th>$R_i$</th>
<th>$^{1}R_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Highly unstable</td>
<td>A</td>
<td>-5</td>
<td>-2</td>
<td>-5</td>
<td>-2</td>
<td>-0.03</td>
</tr>
<tr>
<td>Unstable</td>
<td>B</td>
<td>-10</td>
<td>-1</td>
<td>-2</td>
<td>-1</td>
<td>-0.02</td>
</tr>
<tr>
<td>Slightly unstable</td>
<td>C</td>
<td>-20</td>
<td>-0.5</td>
<td>-1</td>
<td>-0.5</td>
<td>-0.01</td>
</tr>
<tr>
<td>Neutral</td>
<td>D</td>
<td>$\infty$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Slightly stable</td>
<td>E</td>
<td>100</td>
<td>0.1</td>
<td>0.07</td>
<td>0.07</td>
<td>0.004</td>
</tr>
<tr>
<td>Stable</td>
<td>F</td>
<td>20</td>
<td>0.5</td>
<td>0.14</td>
<td>0.14</td>
<td>0.05</td>
</tr>
<tr>
<td>Highly stable</td>
<td>G</td>
<td>10</td>
<td>1</td>
<td>0.17</td>
<td>0.17</td>
<td>0.17</td>
</tr>
</tbody>
</table>

$^{1}$The assumed height of the anemometer and upper thermometer was 10 m; The lower thermometer: 2 m.

### 2.2.5 Roughness layer

The roughness layer is the region of three-dimensional flow close to the wall extending above the roughness elements. The velocity cannot be characterised within the roughness layer because of the three-dimensional effects of the wakes of the individual roughness elements. Moving further away from the roughness elements the three-dimensionality of the flow diminishes and the effect of the roughness elements is felt as a uniform shear stress.

### 2.2.6 Surface layer

Unstable stratification will be ignored from here onwards. Many of the laws discussed are equally valid for the convective boundary layer, but the specific features of the stable boundary layer are our main focus. The surface layer is that part of the boundary layer where the earth’s surface is the dominating influence on the flow.
2.2.6 (a) Monin-Obukhov similarity

The Monin-Obukhov theory states that any non-dimensional characteristic of turbulence in the surface layer can depend only on \( u_*, \theta_*, z \) and/or \( \zeta = z/L \), with

\[
    u_* = \left[ -(u'w')_0 \right]^{1/2}
\]

the friction velocity and

\[
    \theta_* = -\frac{(\theta'w')_0}{u_*}
\]

the surface layer temperature scale.

The following stability functions can then be defined

\[
    \frac{\kappa}{u_*} \left( \frac{\partial u}{\partial z} \right) = \phi_m(\zeta)
\]

(2.36)

\[
    \frac{\kappa}{\theta_*} \left( \frac{\partial \theta}{\partial z} \right) = \phi_h(\zeta)
\]

(2.37)

\[
    \sigma_w u_* = \phi_w(\zeta)
\]

(2.38)

\[
    \sigma_\theta |\theta_*| = \phi_\theta(\zeta)
\]

(2.39)

The forms of the functions can only be determined empirically. The following functional forms, known as the Bussinger-Dyer relations have been determined from atmospheric data and are widely used (e.g. Arya 1984).

\[
    \phi_m(\zeta) = 1 + \beta_m \zeta
\]

(2.40)

\[
    \phi_h(\zeta) = 1 + \beta_h \zeta
\]

(2.41)

\[
    \phi_w(\zeta) = 1.25(1 + 0.2\zeta)
\]

(2.42)

\[
    \phi_\theta(\zeta) = 2(1 + 0.5\zeta)^{-1}
\]

(2.43)

Typical values for \( \beta_m \) and \( \beta_h \) are

\[
    \beta_m = \beta_h = 5
\]

(2.44)

The functional forms are however empirical and there is some debate about the actual values of the constants (e.g. Högström 1988).

Substituting Eq. (2.40) in Eq. (2.36), integrating and applying boundary conditions leads to the following wind profile

\[
    \bar{u}(z) = \frac{u_*}{\kappa} \left\{ \ln \left( \frac{z}{z_0} \right) + \beta_m \frac{z}{L} \right\}
\]

(2.45)

where \( z_0 \) is defined as the roughness length of the surface and is theoretically the height \( z \) where \( \bar{u}(z) = 0 \) when \( L \to \infty \). For neutral stratification \( L \to \infty \) and the wind profile reduces to the well known logarithmic profile:

\[
    \bar{u}(z) = \frac{u_*}{\kappa} \ln \left( \frac{z}{z_0} \right)
\]

(2.46)

The temperature profile for stratified flow can be derived similarly, leading to

\[
    \bar{\theta}(z) - \bar{\theta}_0 = \frac{\theta_*}{\kappa} \left\{ \ln \left( \frac{z}{z_0} \right) + \beta_h \frac{z}{L} \right\}
\]

(2.47)
As noted previously the values of the constants in the profile relations are empirical and different values are often quoted (e.g. Garrat 1992).

Hayden (1998) derived an expression for obstacle Froude number directly from the wind and temperature profile equations, and arrived at a limiting Froude number

$$\lim_{L \to 0} F = \frac{\beta_m}{\beta_h}$$

(2.48)

This limit is however only valid within the Monin-Obukhov similarity. There can clearly not be a non-zero limit on the obstacle Froude number as $U_0 \to 0$. Even in the applicable range of the Monin-Obukhov similarity the usefulness of this relation is doubtful due to the uncertainty of the estimates of $\beta_m$ and $\beta_h$ in the laboratory, where $\beta_m$ and $\beta_h$ are not necessarily independent of stability.

### 2.2.7 Outer layer

The outer layer is the region above the surface layer where the surface layer similarity no longer holds (Kaimal & Finnigan 1994). Parametric relations describing the profiles of wind and potential temperature are suggested by Arya (1984). The plumes studied here will be sufficiently close to the ground where surface layer scaling is still valid. The outer layer can therefore be ignored from here onwards.

### 2.3 Dispersion

In air pollution meteorology the volume with a non-zero mean concentration of pollution from a continuous source is collectively referred to as a plume. A plume is a tangible concept, and it is appropriate to define geometrical characteristics of a plume.

#### 2.3.1 Characteristics of the plume

The plume spread parameters $\sigma_y$ and $\sigma_z$ and the mean plume height are important characteristics of the plume.

The formal definition of $\sigma_y$ is

$$\sigma_y^2 = \frac{\int_{-\infty}^{\infty} \int_{0}^{\infty} y^2 C \, dz \, dy}{\int_{-\infty}^{\infty} \int_{0}^{\infty} C \, dz \, dy}$$

(2.49)

In the absence of the ground surface $\sigma_z$ would have a similar definition to Eq. (2.49). The presence of the ground surface virtually folding the plume back into itself invalidates such a definition of $\sigma_z$. $\sigma_z$ is often obtained by fitting the reflected Gaussian plume formula Eq. (2.61) to a vertical concentration profile.

The mean plume height in the absence of inversion layers can be defined as

$$\bar{z}_m(x) = \frac{\int_{-\infty}^{\infty} \int_{0}^{\infty} zC(x,y,z) \, dz \, dy}{\int_{-\infty}^{\infty} \int_{0}^{\infty} C(x,y,z) \, dz \, dy}$$

(2.50)
2.3.2 Dispersion modelling

Weil (1985) reviewed the application of diffusion models, and highlighted the difference between practical dispersion prediction schemes and the achievements of dispersion research. Weil (1985) criticised the widespread use of the Pasquill-Gifford curves to describe stack plumes even though the curves are based on passive ground level releases. The situation is different today, as some prediction schemes keep abreast of current research. The mean and turbulence structure of the atmospheric boundary layer are the dominating factors in plume transport and dispersion, and hence the the similarity framework described in §2.2 forms an integral part of modern dispersion models.

The rest of this section follows the derivation of the Gaussian plume model, with emphasis on the following steps:

1. The diffusion equation is derived from $K$-theory.
2. The assumption of constant eddy diffusivities $K_z$ and $K_y$ leads to the Gaussian plume model.
3. Statistical theory provides $\sigma_y$ and $\sigma_z$ for uniform flow with homogeneous turbulence.
4. Lagrangian similarity theory provides $\sigma_y$ and $\sigma_z$ for non-uniform flow with inhomogeneous turbulence.

2.3.2 (a) Gradient transport ($K$) theory

From the general transport equation Eq. (2.5) the mean diffusion equation for mean steady flow can be written, ignoring molecular diffusion

$$\frac{\partial \bar{c}}{u_i \partial x_i} = -\frac{\partial c'u_i}{\partial x_i}$$

(2.51)

Analogous to molecular diffusion gradient transport theories have been applied to turbulent diffusion. This is to effect closure, as discussed in §2.1.1 (e). The widely used Gaussian dispersion models depend on the theory of gradient transport.

Turbulent or eddy diffusivities $K_\beta$ can then be defined as

$$c'u_i = -K_\beta \frac{\partial \bar{c}}{\partial x_i}$$

(2.52)

where $\beta = i$ without applying the summation rule of §2.1.1 (a). Eq. (2.51) can now be written as

$$\frac{\partial \bar{c}}{u_i \partial x_i} = -\frac{\partial}{\partial x_i} \left( K_\beta \frac{\partial \bar{c}}{\partial x_i} \right)$$

(2.53)

Eq. (2.53) forms the basis of the $K$ type dispersion models. $\bar{c}$ is generally a passive scalar and hence the velocity field can be computed separately. With appropriate boundary conditions and models for $K_\beta$ finite difference solutions of Eq. (2.54) can be computed (e.g. Nieuwstadt & van Ulden 1978).
2.3.2 (b) Gaussian plume models

In a flow with homogeneous turbulence the eddy diffusivities are constants, and Eq. (2.53) can be written as

\[ \frac{u_i}{c} \frac{\partial c}{\partial x_i} = -K_\beta \frac{\partial}{\partial x_i} \left( \frac{\partial c}{\partial x_i} \right) \]  \hspace{1cm} (2.54)

Eq. (2.54) for simple boundary conditions and a simple flow field has an analytical solution. The simplest turbulent flow field is a uniform flow with the concentration equation in a frame of reference moving with the flow. For a uniform mean steady flow with \( \bar{u} = (u, 0, 0) \) Eq. (2.54) reduces to

\[ \frac{\partial c}{\partial x} = -K_x \frac{\partial^2 c}{\partial x^2} - K_y \frac{\partial^2 c}{\partial y^2} - K_z \frac{\partial^2 c}{\partial z^2} \]  \hspace{1cm} (2.55)

The solution to Eq. (2.55) for a continuous point source with strength \( Q \) in a uniform mean flow can be expressed as (Arya 1999)

\[ \overline{c}(x, y, z) = \frac{Q(x*/r_*)}{2\pi u u_y \sigma_z} \exp \left( \frac{-y^2}{2\sigma^2} - \frac{z^2}{2\sigma^2} \right) \]  \hspace{1cm} (2.56)

where

\[ \sigma_x = \left( \frac{2K_x x}{u} \right)^{1/2} ; \quad \sigma_y = \left( \frac{2K_y x}{u} \right)^{1/2} ; \]

\[ \sigma_z = \left( \frac{2K_z x}{u} \right)^{1/2} \]  \hspace{1cm} (2.57)

and

\[ x_0 = \frac{x}{\sigma_x} ; \quad y_0 = \frac{y}{\sigma_y} ; \quad z_0 = \frac{z}{\sigma_z} ; \]

\[ r_0 = (x_0^2 + y_0^2 + z_0^2)^{1/2} \]  \hspace{1cm} (2.58)

The assumption of a slender plume leads to

\[ y_0^2 + z_0^2 \ll x_0^2 \]  \hspace{1cm} (2.59)

Eq. (2.56) can now be written as

\[ \overline{c}(x, y, z) \approx \frac{Q}{2\pi u u_y \sigma_z} \exp \left( -\frac{y^2}{2\sigma^2} - \frac{z^2}{2\sigma^2} \right) \]  \hspace{1cm} (2.60)

which is the Gaussian plume formula.

Rather than estimating the eddy diffusivities \( K_\beta \) air pollution meteorologists develop parametric models for estimating the plume spread parameters \( \sigma_y \) and \( \sigma_z \) directly.

For sources near a solid surface the plume is reflected by the solid surface, and higher concentrations are observed everywhere in the plume. For convenience the origin \( z = 0 \) is taken as the solid surface, and the source is at a height \( z = H \). The effect of the solid surface is
equivalent to the presence of an identical source at \( z = -H \). The reflected slender Gaussian plume formula can be written as

\[
\bar{c}(x, y, z) \simeq \frac{Q}{2\pi \bar{u} \sigma_y \sigma_z} \exp \left( -\frac{y^2}{2\sigma_y^2} \right) \left\{ \exp \left( -\frac{(z + H)^2}{2\sigma_z^2} \right) + \exp \left( -\frac{(z - H)^2}{2\sigma_z^2} \right) \right\} \tag{2.61}
\]

### 2.3.2 (c) Statistical theory

The statistical theory of turbulent diffusion (Lyons & Scott 1990, Venkatram 1988, Arya 1999) can be used to predict the spread parameters \( \sigma_y \) and \( \sigma_z \) in Eq. (2.60) and Eq. (2.61) for a uniform flow with homogeneous turbulence.

In the case of an iso-kinetic passive release with comparable turbulence levels (but not scales) to the fluid the release is not significantly different from the surrounding fluid parcels. The statistical theory therefore makes no distinction whether the particle studied is a local fluid particle or an introduced one.

The statistical theory of diffusion studies the Lagrangian motion of a particle initially at \( y(0) = 0 \)

\[
v'(t) = \frac{dy(t)}{dt} \tag{2.62}
\]

and consequently the displacement of a single particle after time \( t \) is given by

\[
y(t) = \int_0^t v'(t') dt'
\tag{2.63}
\]

Taking the ensemble average

\[
\frac{d\bar{y}^2}{dt} = 2\bar{y}(t) \frac{dy(t)}{dt}
\]

\[
= 2 \left[ \int_0^t v'(t') dt' \right] v'(t)
\]

\[
= 2 \int_0^t v'(t)v'(t') dt'
\tag{2.64}
\]

During the integration \( v'(t) \) remains constant. It would be useful if the ensemble averaging could be done before the integration, and this is allowed because integration is essentially a summation, and the ensemble averaging also involves a summation.

\[
2 \int_0^t v'(t)v'(t') dt' = 2 \int_0^t \bar{v}'(t)\bar{v}'(t') dt'
\tag{2.65}
\]

\( v'(t)v'(t') \) can be replaced by \( \bar{v}'^2 R_L(\xi) \), with \( R_L(\xi) \) the autocorrelation of Lagrangian velocity where \( \xi = t' - t \).

\[
\frac{d\bar{y}^2}{dt} = 2\bar{v}'^2 \int_0^t R_L(\xi) d\xi
\tag{2.66}
\]
After integration

$$\overline{y^2} = 2v^2 \int_0^t \int_0^t R_L(\xi) d\xi dt'$$

Eq. (2.67) is known as Taylor’s theorem after G.I. Taylor. The result reduces the problem of diffusion in homogeneous turbulence to finding the Lagrangian autocorrelation. The Lagrangian autocorrelation is unfortunately a very difficult quantity to measure. Further deductions for small and large diffusion times are possible without determining the Lagrangian autocorrelation.

\[ R(0) = 1 \] and consequently for small diffusion times \( t \to 0 \) Eq. (2.67) reduces to

$$\overline{y^2} = \overline{v^2} t^2$$

i.e. linear spread. The Lagrangian integral time scale can be defined as

$$T_L = \int_0^\infty R_L(\xi) d\xi$$

For times \( t \gg T_L \) the integral in Eq. (2.69) will be a constant proportional to \( T_L \). For large diffusion times \( t \gg T_L \) Eq. (2.67) becomes

$$\overline{y^2} = 2v^2 T_L t$$

as in a simple diffusion process. For homogeneous turbulence the vertical and horizontal spreads can be represented by the following functional forms that match the limits for short and long diffusion times (Venkatram et al. 1984).

\[
\sigma_z = \sigma_{zt} \left( 1 + \frac{t}{2T_{Lz}} \right)^{-\frac{1}{2}}
\]

\[
\sigma_y = \sigma_{yt} \left( 1 + \frac{t}{2T_{Ly}} \right)^{-\frac{1}{2}}
\]

2.3.2 (d) Lagrangian Similarity theory

Statistical theories describe dispersion in uniform flows with homogeneous turbulence quite well. Significant wind shear and stratification calls for modifications to the approach in § 2.3.2 (c). The Lagrangian similarity theory is described by Hunt & Weber (1979) and summarised by Arya (1999). It provides similarity relations for particle displacements, probability density functions, mean concentrations and modifications to the statistical theory to account for wind shear. Particle displacements and the statistical theory will be considered here.

Statistical dispersion relations similar to the statistical theory of § 2.3.2 (c) can be derived for the vertical dispersion of a ground level plume in shear flows. The Lagrangian mean vertical velocity is not zero, and also contributes to the dispersion. The rate of change of the mean square particle displacement \( \overline{Z^2} \) is written as

$$\frac{d\overline{Z^2}}{dt} = 2 \int_0^t W(t)W(\xi)dt'$$
In homogeneous turbulence $\overline{Z^2} = \overline{Z'^2}$, but that is not the case in shear flows where there is a mean vertical component of Lagrangian velocity. $\overline{Z'^2}$ is then written as

$$\frac{d\overline{Z'^2}}{dt} = 2 \int_0^t \overline{W'(t)W'(t')}dt' \tag{2.74}$$

The overbar denotes ensemble averaging, since the mean and fluctuating components of vertical velocity are functions of travel time.

Dimensionless autocorrelation functions can then be defined separately for the mean and fluctuating components of Lagrangian velocity. These autocorrelation functions are functions of lag time $\xi$ and travel time $t$.

$$R_{WW}(\xi, t) = \frac{\overline{W(t)W(t + \xi)}}{\overline{W^2(t)}} \tag{2.75}$$

$$R_{WW'}(\xi, t) = \frac{\overline{W'(t)W'(t + \xi)}}{\overline{W'^2(t)}} \tag{2.76}$$

An integral time scale as a function of travel time is defined as

$$T_{IL}(t) = \int_0^\infty R_{WW'}(\xi, t)d\xi \tag{2.77}$$

By definition $W(t) = \overline{W(t)} + W'(t)$. Substituting $t' = t + \xi$ in Eq. (2.73) and using the definition of the autocorrelation function in Eq. (2.76) results in

$$\frac{d\overline{Z'^2}}{dt} = 2\overline{W(t)} \int_0^t \overline{W(t')}dt' + 2\overline{W'^2(t)} \int_{-t}^0 R_{WW'}(\xi, t)d\xi \tag{2.78}$$

Eq. (2.78) is a general statistical dispersion relation, without making assumptions about the turbulence.

The functional form of the Lagrangian mean vertical velocity can be derived from similarity considerations, with the appropriate velocity scale the friction velocity $u_*$, leading to

$$\overline{W(t)} = \frac{d\overline{Z(t)}}{dt} = bu_* \tag{2.79}$$

In the neutral surface layer the accepted value for $b$ is $b \approx 0.4$. Eq. (2.78) then becomes

$$\frac{d\overline{Z'^2}}{dt} = 2b^2 u_0^2 + 2\overline{W'^2(t)}T_{IL}(t) \tag{2.80}$$

It is assumed that the variance of Lagrangian vertical velocity $\overline{Z'^2}$ is approximately equal to the the variance of Eulerian vertical velocity $\sigma_w$. A relation for $\overline{W'^2(t)}$ can therefore be found
\[ \overline{W'^2} = \sigma_w^2 - W^2 = b'^2 u_*^2 \]  

with \( b' = \sqrt{1.3^2 - 0.4^2} \approx 1.24 \). \( T_{iL} \) is assumed to be proportional to the integral length scale, which increases linearly with height in the neutral surface layer, leading to

\[ T_{iL} \propto \frac{\overline{W'}}{\sigma_w} \propto t \]  

and

\[ T_{iL} = \gamma t \]  

with \( \gamma \approx 0.10 \) the appropriate similarity constant. The mean square particle displacement, or dispersion, can therefore be written after integration with respect to time as

\[ \overline{Z'^2} = b^2 u_*^2 t^2 + b'^2 u_*^2 \overline{t'^2} \approx 0.31 u_*^2 t^2 \]  

2.3.2 (e) Practical models

Practical dispersion models rely heavily on the Gaussian diffusion theory. Additional justification for the Gaussian plume model has been provided by the statistical theory of diffusion and Lagrangian similarity theory.

The ADMS dispersion models (Carruthers et al. 1999) use plume spread formulations of the following form for stable and neutral flow:

\[ \sigma_x = \sigma_w t \left( \frac{1}{b^2} + \frac{N^2 t^2}{1 + 2Nt} \right)^{-\frac{1}{2}} \]  

with

\[ b = \begin{cases} 
\frac{1+0.4u_*/z_s}{1+u_*/z_s} & \text{if } z_s/h \leq 0.05 \\
(1 - \frac{z_s-0.05}{0.1}) \left( \frac{1+0.4u_*/z_s}{1+u_*/z_s} \right) + \frac{z_s-0.05}{0.1} & \text{if } 0.05 < z_s/h < 0.15 \\
1 & \text{if } z_s/h \geq 0.15 
\end{cases} \]  

\[ \sigma_y^2 = \sigma_{y_t}^2 + \sigma_{y_w}^2 \]  

where \( \sigma_{y_t} \) is due to turbulence and \( \sigma_{y_w} \) to wind direction instability. Wind direction instability is due to the large scale variations in the wind direction and is a function of the sampling time. Wind directional instability is not present in the wind tunnel where the wind direction is fixed, and the lateral scales of turbulence are limited by the width of the wind tunnel.

\[ \sigma_{y_t} = \begin{cases} 
\sigma_{yt} (1 + 15.6^{1/3} u_* t L/h^2)^{-1/2} & \text{if } h/L > 1; \\
\sigma_{yt} (1 + 15.6^{1/3} u_* t/h)^{-1/2} & \text{otherwise} \end{cases} \]  

where \( L \) is the Monin-Obukhov length.
Eq. (2.85) is a complex interpolation formula linking the statistical theory of absolute diffusion, the Lagrangian similarity theory and a random walk theory which will be described later. For short diffusion times and \( z_s/h > 0.15 \) Eq. (2.85) reduces to the statistical theory relation for short diffusion times, i.e. \( \sigma_z = \sigma_w t \). For \( z_s/h \) small \( b \) reduces to \( b = 0.4 \) for short diffusion times, and hence \( \sigma_z = 0.4 \sigma_w t \). A common parameterisation of \( \sigma_w \) in the surface layer is \( \sigma_w = 1.3 u_* \). \( \sigma_z \) then becomes \( \sigma_z = 0.52 u_* t \simeq \sqrt{0.31} u_* t \). The Lagrangian similarity theory has therefore been recovered for sources close to the ground at short diffusion times.

For long diffusion times the second term in Eq. (2.85) resembles the random walk model, (e.g. Weil 1985)

\[
\sigma_z = \sigma_w/N \left( \zeta^2 + 2N^2 \tau_{\text{Lz}} \right)^{1/2} \tag{2.89}
\]

with

\[
\tau_{\text{Lz}} = \gamma^2 N^{-1} \tag{2.90}
\]

For long diffusion times Eq. (2.89) becomes

\[
\sigma_z = \gamma \sqrt{2t/N} \sigma_w \tag{2.91}
\]

For long diffusion times the right hand term in Eq. (2.85) becomes

\[
\sigma_z = \sqrt{2t/N} \sigma_w \tag{2.92}
\]

The expected range for \( \gamma \) is \( 0.1 < \gamma < 0.4 \), indicating the degree of mixing between fluid elements in stable flows. The ADMS relation for long diffusion times is therefore similar to the random walk relation apart from the omission of \( \gamma \).

For the lateral spread Eq. (2.88) resembles Eq. (2.72), and hence

\[
15.6^{1/3} u_* L/h^2 \tag{2.93}
\]

and

\[
15.6^{1/3} u_* /h \tag{2.94}
\]

are estimates of \( 1/2T_{\text{Ly}} \).

### 2.3.3 Concentrations statistics

Just as the mean velocity components do not fully represent the turbulent characteristics of a flow, the mean concentrations are also not always adequate in describing plume dispersion. The statistics that will be described briefly are intermittency, probability distribution function and concentration fluctuations.

The fluctuating nature of concentrations is entirely due to the underlying turbulence of the flow. The Turbulent scales larger than the plume dimensions transport the plume, while turbulent scales smaller than the plume dimensions mix and disperse it. It can therefore be expected that the plume at the exit of a relatively small source can undergo large displacements without much dilution. This is the primary cause for the intermittent nature of instantaneous concentrations close to the source. An exponential PDF is an indication of an intermittent or meandering plume (Hanna 1984).
The value of the mean concentration as a toxicity measure has been questioned by Griffiths & Megson (1984). Instantaneous values of concentrations are important for toxic and flammable gasses. Most practical dispersion models exclusively predict mean concentrations. Fackrell & Robins (1982) made detailed measurements of concentration fluctuations and concentration fluxes for passive plumes from an elevated and a ground level source. They found that concentration fluctuations near the source are dependent on the source size. Wilson et al. (1985) found that conditional averaging, which removes the zero concentrations from the time series, largely removed the dependence of the concentration fluctuations on the source size. Weil (1985) described the procedure of conditional sampling as ill-defined, making it hardly suitable for practical modelling, but still a useful concept in the understanding of concentration fluctuations. Sykes et al. (1984) presented a second order closure model describing the diffusion of a passive scalar from a small source in a turbulent flow. Sykes et al. (1986) proposed a Gaussian plume model capable of predicting both mean and fluctuating concentrations. Wilson et al. (1982a, 1982b, 1985) went through several stages of developing a semi-empirical Gaussian plume model capable of predicting intermittency, mean concentration and concentration fluctuations. They also tested their model with experimental data from wind tunnel measurements, and found good agreement.

2.4 Surface obstacles

Obstacles on the surface boundary of a boundary layer flow will have a local effect on the flow near them. The effect of the flow modification is important to humans because buildings are usually present where humans work and live. The effect of obstacles on the flow is especially important when the obstacles are in the vicinity of smoke stacks or other sources of pollution. A pollution source may be safely positioned for dispersion in an undisturbed boundary layer, but a building close to the release may modify the flow such that the pollution is brought down to ground level before being diluted to safe concentration levels. Each individual set-up may have unique characteristics that need to be assessed, but the study of a few simple obstacle configurations can shed light on the fundamental principles involved.

Building wakes in the neutrally stratified boundary layer have been studied in considerable depth, and simple parametric relations have been developed to account for the effect they have on atmospheric dispersion. Obstacle wakes in stably stratified boundary layers are currently treated in the same way as neutrally stratified boundary layers, with the eddy viscosity reduced to account for stability. Little evidence exists to suggest that they should be treated differently.

This study aims to investigate further whether stable stratification has a distinct influence on obstacle wakes. Stable boundary layer flow over smooth obstacles such as hills can vary greatly with the strength of the stability. In the near wake of an obstacle shear generated turbulence dominates, and the wake is not much different from the neutral flow with similar approach turbulence levels. Not much work has been done on the effects of obstacles further downstream. It is felt that beyond the point where turbulence levels have decayed to a level where buoyancy again becomes a significant effect, changes in the wake structure from a neutral equivalent may start to appear.
2.4.1 Nature of the wake

Robins (1994) gave a detailed description of the wake structure behind a cube normal and at incidence to the approach flow in a neutral boundary layer. Fig. 2.1 is taken from Robins (1994) and shows the important features of the flow for the two cases, Figs. 2.1(a) & 2.1(b) normal to the flow and Fig. 2.1(c) at incidence.

It can be seen that the wake forming behind an obstacle can be very different depending on the wind direction. The most persistent vortices present in a wake come from the roof vortex system shown in Fig. 2.1(c). The roof vortex system is not present for the normal cube flow, but the horseshoe vortex can still constitute a significant vortex wake.

2.4.2 Wake theories

Practical dispersion models often employ a simple model to account for the changes in flow and dispersion by obstacles in the path of the flow. Virtual source modelling and integral modelling are the most relevant to dispersion and will be discussed briefly. Robins (1994) noted that building effects modules in dispersion models generally ignore the vortex wake contribution to the main wake, i.e. downstream of the recirculation region. The reason for this is that a satisfactory theory for the vortex wake does not yet exists, whilst the theory for momentum wakes described in §2.4.2 (b) has a sound theoretical foundation. In the recirculation region the strong downwash caused by the vortex wake is often modelled with empirical relations.

2.4.2 (a) Virtual source modelling

The concept of a virtual source location upwind of the actual source is frequently encountered in dispersion modelling, (e.g. Van Ulden 1978). The principle is to determine the source location that will result in equivalent concentrations and plume dimensions at a downstream location. If the flow remains uniform with the same turbulence characteristic, and the source height stays the same the virtual source location will equal the actual source location. If the turbulence characteristics of the flow change the virtual source origin will also change. Different virtual source origins for the lateral and vertical spread terms in the Gaussian plume model may be computed.

The presence of an obstacle in the path of a plume will change the height of the plume and the plume will experience enhanced dispersion in the wake of the obstacle. A virtual origin for the plume can be computed to match the spreading rates and the streamline deflection experienced by the plume.

The simple virtual source modelling, for a fully entrained release, described by Robins (1994) defined virtual origins for each of the vertical and the lateral plume spread formulations such that

\[ \sigma_y(x_{oy}) = b/3 \]  \hspace{1cm} (2.95)

\[ \sigma_z(x_{oz}) = h/3 \]  \hspace{1cm} (2.96)
Figure 2.1: Mean flow patterns around a cube. From Robins (1994)

(a) Mean streamlines on centreline of a cube normal to the approach flow

(b) Mean flow pattern around a cube normal to the approach flow

(c) Mean flow pattern around a cube at incidence to the approach flow
where \( x_{oy} \) and \( x_{oz} \) are the displacements upwind of the actual source location for the lateral and vertical spread terms respectively. The vertical plume deflection can be computed separately.

2.4.2 (b) Integral modelling

The objectives for a three-dimensional wake model are similar to the objectives of Counihan et al. (1974) for their two-dimensional wake model. The model should be inexpensive computationally but still provide a good representation of the flow and turbulence modification due to the obstacle. The wake model should also provide a relation between the velocity deficit and the forces or moments on the obstacle.

The simple analytical theory of Counihan et al. (1974) has been shown to work remarkably well for wakes behind two-dimensional surface obstacles in turbulent boundary layers. With some further simplifications their model can be extended to three-dimensional obstacles quite easily (Apsley 1988, Hayden 1998), and due to its simplicity is ideal for application in practical dispersion models.

Counihan et al. (1974) postulated the existence of three regions in the wake: the Wall region (W), the Mixing region (M) and the External region (E). Their analysis centred around region M, but for the sake of completeness regions W and E were introduced where the assumptions for M could no longer be valid. Region E can also extend upstream of the obstacle, where regions M and W strictly occur downstream of the obstacle. M is the region of the wake where most of the significant streamline deflections and turbulence modifications downstream of the obstacle occur. Their solution in region M exhibited the correct limiting behaviour for \( z = 0 \) and \( z \to \infty \). For dispersion applications the errors associated with extending the solution for region M to \( 0 \leq z < \infty \) are insignificant. Other assumptions are used for the streamline deflections upstream and above the obstacle and its near wake.

The following assumptions apply in region M.

\[
\begin{align*}
\rightarrow & \text{ The approach flow velocity profile can be represented by a power law.} \\
\rightarrow & \text{ The modification to the flow due to the obstacle is a small perturbation on the incident flow.} \\
\rightarrow & \text{ A constant eddy viscosity hypothesis applies in the wake region.} \\
\rightarrow & \text{} z_0 \ll H \ll \delta \text{ where } z_0 \text{ is the roughness length of the surface, } H \text{ is the obstacle height and } \delta \text{ is the boundary layer depth.} \\
\rightarrow & \ell \ll X, \text{ where } \ell \text{ is a scale of the order of the depth of region M, and } X \text{ represents the length of the wake.} \\
\rightarrow & \text{ The perturbation pressure is negligible far enough from the recirculating region.} \\
\rightarrow & \text{ In region M the only relevant length scale of the flow is the height of the body.} \\
\rightarrow & \text{ The boundary layer approximation applies, i.e. } \partial/\partial x \ll \partial/\partial y \text{ and } \partial/\partial x \ll \partial/\partial z \text{ because } \ell \ll X. \\
\rightarrow & \text{ The perturbation velocity profiles in M are self preserving.}
\end{align*}
\]
Appendix F follows the derivation of a three-dimensional wake theory similar to that by Hayden (1998) and Apsley (1988).

The final result of the wake model (Apsley 1988) is the similarity velocity profiles

\[
\begin{align*}
    u &= -\tilde{u}_x \xi^{\frac{n+3}{n+2}} \zeta e^{-\frac{1}{4}\xi^2} e^{-\frac{1}{2}r^2} \\
    v &= \frac{\lambda \tilde{u}}{\xi^2} \left( \frac{n+2}{U_H H} \right)^{\frac{1}{n+2}} \tilde{v} \\
    w &= -\frac{\tilde{u}}{\xi^2} \left( \frac{n+2}{U_H H} \right)^{\frac{1}{n+2}} \tilde{w}
\end{align*}
\]

(2.97)  (2.98)  (2.99)

with \( \xi, \eta \) and \( \zeta \) the similarity parameters in the x, y and z direction respectively, and

\[
\begin{align*}
    \tilde{u} &= \tilde{u}_r \cos \phi \\
    \tilde{v} &= \tilde{u}_r \sin \phi \\
    \phi &= \tan^{-1} \frac{\eta}{\xi} \\
    r &= \sqrt{\eta^2 + \zeta^2} \\
    \tilde{u}_r &= \sin \phi \left\{ (r^2 - 1)e^{-\frac{1}{2}r^2} + \sqrt{\frac{\pi}{2}} \operatorname{erf}(r/\sqrt{2}) \right\}
\end{align*}
\]

The following relation was found for the growth of the wake dimensions

\[
\frac{\ell(\xi)}{H} = \left\{ \left( n + 2 \right) \frac{\nu_M \xi}{H U_H} \right\}^{\frac{1}{n+2}}
\]

(2.100)

2.5 Physical modelling of the ABL

Simulation is the art of obtaining a data set of results for a particular flow configuration by employing techniques other than measuring the actual full-scale case. The advantage of simulation over field measurements is the controlled environment in which we are able to conduct simulations. Numerical simulation is the simplest by far as setting up the geometrical conditions is concerned, but requires accurate mathematical modelling of the behaviour of the fluid. Experimental simulation uses a real fluid in simulating the flow, but a different geometrical scale is used, and the proper conversion of the results to the real case needs to be understood. The accurate mathematical modelling of fluid flow is perhaps the most challenging problem in physics today and is receiving a lot of attention. Just as an inaccurate mathematical model results in the deviation of the solution from the real solution, improper simulation and scaling of experimental results will also lead to an inaccurate solution of the actual results.

2.5.1 Facilities

The wind tunnel is the most realistic simulation device for atmospheric boundary layers in general and for the stratified boundary layer. The first wind tunnel was built by Frank
Wenham in 1871 (Baals & Corliss 1981) for aeronautical research. More than a century later modern aeronautical wind tunnels are capable of remarkable accuracy. Meteorological wind tunnels typically operate at much lower velocities than aeronautical wind tunnels, and are generally susceptible to more inaccuracies than aeronautical wind tunnels.

The working fluid for stably stratified modelling facilities can be either a liquid, such as water, or a gas, such as air. Variations in salinity and temperature are the common methods of achieving density stratification in water. Salinity stratification is more popular due to the high cost of heating water to the point where suitable density gradients are achieved. Salinity stratification is only feasible in water tank facilities where a moving carriage tows the model through the stationary fluid. The approach flow is usually uniform, and the only turbulence present is that generated by obstacles and by sources, (e.g. Snyder 1994, Britter 1985, Liu et al. 2001, Snyder 1985). Thermal stratification has been applied to flowing water in a flume by Komori et al. (1983), thus simulating a stably stratified turbulent shear flow.

Stable stratification in wind tunnels is normally set up by temperature gradients. The most common method is to heat the inlet flow and to cool the floor of the wind tunnel. A inverted stably stratified boundary layer can be set up from the roof of a wind tunnel by simply heating the roof (Grainger & Meroney 1994). Density stratification can also be achieved more directly by mixing air with a dense gas and controlling the density gradient. Ohya et al. (1994) partitioned the whole of their wind tunnel apart from the working section into six horizontal layers, and controlled the density in each layer.

Only a few research centres in the world have wind tunnels capable of simulating stably stratified atmospheric boundary layers. Some of the main facilities with references to published material are listed below.


3. Fluid Mechanics & Acoustics Laboratory, Ecole Centrale de Lyon Morel et al. (1991) and Zegadi et al. (1994)


5. Department of Mechanical Engineering, Monash University Grainger & Meroney (1994)

6. Research Institute for Applied Mechanics, Kyushu University Ohya et al. (1996) and Ohya et al. (1994)

The literature dealing with stably stratified flow relevant to the atmosphere either considers the undisturbed case or deals with disturbances created by building wakes. That will be the classification used for the items of literature that were reviewed. Studies that considered dispersion are mentioned separately.
2.5.2 Undisturbed stable boundary layer

Arya (1975) studied the effects of buoyancy in a horizontal flat-plate boundary layer. Although a smooth wall was used for the experiments some important fundamental observations were made during this study, and the results will be considered here. Boundary layer depths of $\delta > 66$ cm and a ratio of $u_*/U_\infty = 0.024$ were achieved for the boundary layer with the highest stability. Freestream velocities were varied while keeping the temperature difference over the boundary layer depth constant to achieve different degrees of stability. The bulk Richardson number $Ri_b$ was used as the stability parameter. The bulk Richardson numbers of the stable boundary layers investigated ranged from 0 for neutrally stable to 0.09. Stability was shown to affect the velocity profiles significantly in both the inner and outer regions of the logarithmic law of the wall. The coefficients of friction and heat transfer were also shown to decrease with increasing stability. Normalised turbulence intensities and fluxes were greatly suppressed with increasing stability. The horizontal turbulent heat flux was several times the vertical turbulent heat flux and their ratio decreased with increasing distance from the wall. Turbulence spectra are also analysed to some detail which will be considered for future work. The ratio of the eddy exchange coefficients of heat and momentum strongly depended on stability.

Ohya et al. (1994) used a different approach to simulate a density stratified boundary layer in a wind tunnel. The wind tunnel, except for the working section, is divided into six horizontal layers separated by thin aluminium plates. Each layer is filled with a variable air and dense gas mixture to obtain a desired density profile. Flow visualisation results are presented to illustrate the difference between neutral and stable flow past an isolated hill, and also past a two-dimensional ridge.

Ohya, Neff & Meroney (1997) investigated the turbulence structure of stably stratified boundary layers in a thermally stratified wind tunnel. The classes of flows that were investigated were neutral flows, stratified flows with weak stability ($Ri_b = 0.12, 0.20$) and stratified flows with strong stability ($Ri_b = 0.39, 0.47, 1.33$). The streamwise and vertical fluctuations of velocity and temperature were found to decrease with increasing stability. Momentum and heat fluxes were also found to decrease significantly becoming nearly zero close to the wall for strong stability. The different stability regimes were clearly visible when vertical profiles of turbulence quantities and local gradient Richardson number were plotted. The vertical profiles of the turbulent intensities and fluxes showed similar distributions to corresponding observational studies. It was found that the turbulence quantities under stable conditions correlated well with local gradient Richardson number.

Ohya (1999) simulated the stable atmospheric boundary layer in a wind tunnel. Bulk Richardson numbers ranged from 0 to a very stable value of 1.17. Observations were similar to those of Arya (1975), with the addition that the simulated vertical profiles of turbulence statistics were similar to those observed in field studies. For the very stable case with $Ri_b = 1.17$ waves due to the Kelvin-Helmholtz instability were observed by flow visualisation.

Ogawa et al. (1981) reported the use of a new wind tunnel to simulate stably stratified atmospheric boundary layers. A single stable boundary layer with a bulk Richardson number of $Ri_b = 0.25$ is reported. Similar results to Arya (1975) and Ohya (1999) are reported with regard to the damping of turbulence levels.

Heist & Castro (1998) reported one of the first studies investigating stable boundary layers in the EnFlo wind tunnel. Although the emphasis of their work was the heat flux measurement
technique employed, the characteristics of the stable boundary layer are also reported. Heist & Castro (1998) made no mention of the ratio of the eddy exchange coefficients of heat and momentum. Good agreement was reported with the non-dimensional field data of Caughey et al. (1979), and it was concluded that the simulated boundary layer agreed well with the stably stratified atmospheric boundary layer.

Steggel (1999) describes the simulation of stable boundary layers in the EnFlo atmospheric wind tunnel, the same wind tunnel used for the present study. The wind tunnel has an open circuit configuration, and a stable boundary layer is simulated by heating the inlet air and cooling the floor of the wind tunnel. The wind tunnel is described in detail in Chapter 3. A non-uniform inlet profile of temperatures was used by Steggel and Castro as by previous workers (e.g. Heist & Castro 1998). The profile of temperature that was used was essentially uniform except for the wall region where the temperature was increased rapidly towards the wall. Higher shear stresses and heat fluxes were measured and the resulting structure of the flow was judged to be more self consistent than a flow using a constant inlet profile of temperatures. Steggel used the Counihan vorticity generators originally designed for setting up neutral boundary layers with a depth of 1 m. The Counihan vorticity generators have a height of 1 m, whereas the stable boundary layer was only nominally 500 mm deep. The turbulence levels showed a decreasing and then increasing trend with distance from the wall, which may be attributed to the vorticity generators extending above the final boundary layer top. The routine for determining boundary layer characteristics used by Steggel has been adopted in a modified form for the present study.

2.5.2 (a) Dispersion

Ogawa et al. (1985) studied the effects of thermal stratification on the dispersion of a ground level released plume. Five levels of stability were modelled ranging from $R_i = 0$ to $R_i = 0.248$. As stability increased the turbulence levels decreased, with the small eddies being damped first, and under strong stable conditions only long-period wave-like motion remaining. As stability increased plume-spread decreased, and for the strong stable case vertical plume-spread nearly ceased and the plume spread in a thin horizontal layer.

2.5.3 Obstacle affected stable boundary layer

Ogawa & Diosey (1980a) carried out field experiments on the effects of thermal stratification on the flow behind a two-dimensional fence and augmented the results with a wind tunnel study (Ogawa & Diosey 1980b). The turbulent Reynolds number based on the fence height, wind velocity at model height and turbulent diffusivity, and the flux Richardson number were chosen as the characteristic dimensionless parameters for the oncoming flow. Only weak stable stratification was experienced during the field study. It is suggested that the effects of weak stratification are probably felt indirectly by the damping of ambient turbulence. The reattachment length was found to increase with turbulent Reynolds number $R_{\text{turb}} = UH/K_\text{m}$. The reattachment length was also found to decrease rapidly for increasing stability. For the wind tunnel study the wind tunnel described by Ogawa et al. (1985) was used. It is suggested that a zero plane displacement height equal to the roughness element height be used when using large roughness elements compared with the model height. Spectral analysis formed an important part of the analysis of the data. Overall it was found that equality of turbulence intensities was a more appropriate scaling parameter than the turbulent Reynolds
Steggel & Castro (1998) studied the flow behind a two-dimensional fence in both neutral and stably stratified simulated atmospheric boundary layers. An extensive comparison of the neutral and stable cases was done. In the wake cavity, normalised turbulence levels and shear stresses were similar in the stable and neutral case, whereas they were vastly different in the approach flow. The temperature in the wake cavity was relatively constant for the stable case, indicating a well-mixed region inside the cavity. It is also suggested that stability has an indirect effect on the cavity due to the reduced turbulence levels in the approach flow, as also observed by Ogawa & Diosey (1980a). One shortcoming of the comparison, as pointed out by the authors, is that the boundary layer depths were not equal in the stable and neutral simulations, while the same fence height was used.

Kothari et al. (1986) investigated velocity and temperature wakes behind obstacles in slightly stable boundary layers. Wind tunnel measurements were made to validate their mathematical model, and good agreement was found.

2.5.3 (a) Dispersion

Snyder (1994) investigated the influence of stable stratification on diffusion in building wakes by towing a cubical building model through a salt-water stratified tank. The approach flow was therefore a uniform laminar flow and not a turbulent shear layer. Dye was released in the lee of the cube and concentration measurements were made up to 6 building heights downstream. The stable stratification of the flow was characterised by the Froude number, $F = U/NH$. Reynolds number independence of the wake structure was investigated under neutral stratification in the towing tank and in a wind tunnel simulating an atmospheric boundary layer. The critical Reynolds number was significantly lower in the wind tunnel than in the towing tank, which clearly illustrates the effect of the turbulent shear layer approach flow. Stratification was found to have little effect on the cavity length if the Froude number $Fr > 3.0$, and for the dispersion studies concentration patterns in the wake cavity showed no dependence on stratification for $Fr > 2.5$. It is concluded that stable stratification even under full-scale conditions will very rarely have any effect on building downwash. Further downstream in the far field it is possible that stratification will have a significant effect.

Zhang et al. (1996) carried out a numerical investigation of the dispersion behind a cubical building in a stable atmospheric boundary layer. The results were compared with the towing tank data of Snyder (1994) as mentioned above. Some differences were observed between the numerical and experimental results. The numerical results showed a slight decrease in cavity length when the Froude number was increased from 3.0 to 6.0. Above $Fr = 6.0$ the flow structure was independent of stratification, whereas the experimental results showed independence above $Fr = 2.5$. Fair agreement was found between concentration results for the release within the lee of the building for slight stratification, i.e. $Fr > 3.0$, but results agreed poorly for strong stability ($Fr = 1$). The numerical model fails for the strongly stable case because advection rather than turbulent diffusion dominates. The same conclusion is reached as by Snyder that stability conditions that are commonly found in the atmosphere are not stable enough to have an influence in the near wake of a building.

Steggel & Castro (1999a) studied the effects of stable stratification on dispersion characteristics behind a two-dimensional fence. This study is the second stage of the study by the same authors (Steggel & Castro 1998) describing the flow behind a two-dimensional fence under
stratified conditions. A stack was positioned 200 mm upstream of the 100 mm high fence. Stack heights of 10, 50 and 100 mm were tested. It was found that the dispersion characteristics within the recirculation region were similar for neutral and stable stratification. For the stack heights tested it was found that more of the plume was entrained in the separation bubble for the stable case.

Steggel & Castro (1999b) studied the effects of stable stratification on flow and dispersion around a cube in the EnFlo wind tunnel. The cube was rotated at 45° to the approach flow. Stability was found to have little effect on the flow and concentration measurements in the near wake region. Further downstream concentrations are not much higher than the undisturbed values, but stability does seem to have a measurable effect in the cube at 45° case. More detailed measurements were called for.

2.5.4 Relating measurements to the field

2.5.4 (a) Dimensionless parameters

Snyder (1981) laid down guidelines for modelling atmospheric diffusion. When the equations governing fluid motion are non-dimensionalised the dimensionless parameters that need to be matched between experiment and field to obtain dynamically similar flows are typically:

\[
Re = \frac{U_R L}{\nu}
\]

\[
Fr = \frac{U_R}{(g L \Delta T R / T_0)^{1/2}}
\]

\[
P_e = \frac{U_R L}{\kappa}
\]

\[
Ro = \frac{U_R}{L \Omega R}
\]

where \( Re \) is Reynolds number, \( Fr \) is densimetric Froude number, \( P_e \) is Peclét number and \( Ro \) is Rossby number.

It is an impossible task to match all the dimensionless quantities, and in reality above a certain Reynolds number similar flow can be achieved independent of Reynolds number. The appropriate Reynolds number for a rough wall is the roughness Reynolds number (e.g. Snyder 1981), defined as

\[
Re_{z_0} = \frac{\nu \overline{z_0}}{\nu} \geq 2.5
\]  

This requirement in terms of the Reynolds number based on the boundary layer depth becomes:

\[
\frac{U_\infty \delta}{\nu} \geq 2.5 \left( \frac{U_\infty}{U_a} \right) \left( \frac{\delta}{z_0} \right)
\]

The Rossby number is a measure of the local acceleration relative to Coriolis acceleration in the atmosphere, and is impossible to match in a wind tunnel. Snyder (1981) concluded that Rossby number can be ignored when modelling full-scale flows over flat terrain with a length scale smaller than 5 km. Peclét number matching can be ignored, based on similar arguments to Reynolds number independence.
What remains is the Froude or Richardson number, which represents the ratio of inertial forces to buoyancy forces, and is regarded as the most important single parameter that must be matched between full-scale and experiment. The Froude number is in general not difficult to match, but it places restrictions on the wind speeds that can be used in physical modelling.

It is clear that for a steady flow with neutral stratification no dimensionless parameters remain. Length scales determined from experiment can be converted to full-scale by simple multiplication with the geometric scaling factor.

2.5.4 (b) Boundary layer modelling

White (1991) describes similarity as finding the condition where a measurement $A_1$ in a flow about or through a model shape $B_1$ can be scaled by a simple multiplier to yield the measurement $A_2$ about or through a geometrically similar model shape $B_2$.

The roughness Reynolds number requirement Eq. (2.102) has been defined in the previous section as:

$$\frac{U_\infty \delta}{\nu} \geq 2.5 \left( \frac{U_\infty}{u_*} \right) \left( \frac{\delta}{z_0} \right)$$

Scaling between model and full scale is achieved by matching the non-dimensional parameters in parentheses. This also applies for all other length and velocity scales.

It is also important to match the structure of the turbulence between model and full-scale to have dynamically similar flows. For plume dispersal studies Snyder (1981) concludes that not only must the turbulence intensities of each fluctuating component in the fluid be matched between model and full-scale, but also the spectrum of each component.

2.5.4 (c) Dispersion modelling

Britter (1987) discusses the modelling requirements for a number of dispersion cases ranging in complexity. The most complex cases involved significant heat transfer between the plume and the surrounding air. Britter (1987) concluded that cases involving free convection heat transfer between the source and the air cannot be modelled correctly in the wind tunnel. An important point raised was that the dominant mode of heat transfer between full scale and model scale must remain the same. For forced convection heat transfer the dimensionless numbers can be matched theoretically, but the Froude number requirement often changes the mode of heat transfer to a mixed forced and free convection rather than the required forced convection.

For the passive plumes that this study is concerned with the source volume flow rate coefficient must be matched between full scale and model scale, i.e.

$$\frac{Q}{U_\infty L^2} = \text{constant} \quad (2.103)$$

Eq. (2.103) is equivalent to the specification of iso-kinetic releases, i.e. the release velocity of the source matching the local mean flow velocity at the release height, if properly scaled source sizes are used.
Concentration can be normalised as (Snyder 1994)

\[ \chi = \frac{CUH^2}{Q} \]  

(2.104)

if all other similarity conditions are met.

2.5.4 (d) Boundary conditions

Apart from the conditions in the flow, the non-dimensional boundary conditions have to be matched between model and full-scale. Boundary conditions are non-dimensionalised with a characteristic length scale and a characteristic velocity scale.

2.6 Numerical modelling of the ABL & dispersion

For regulatory models a balance is normally struck between complexity and accuracy. This often rules out any finite difference, volume or element numerical solutions of the governing partial differential equations.

Although this study is primarily concerned with the physical modelling of the ABL, the numerical modelling techniques that are available deserve a brief mention. These techniques all rely on a discretisation scheme to reduce the continuous field variables to a discrete set of variables that can be solved by a computer. Greater accuracy is obtained by increasing the resolution of the discretisation. Some methods such as Reynolds Averaged Navier-Stokes (RANS) methods only solve the averaged forms of the transport equations and therefore rely completely on turbulence modelling to account for turbulent motions. Other methods such as Large Eddy Simulation (LES) and Direct Numerical Solution (DNS) solve the time dependent turbulent transport equations requiring little or no modelling of turbulent motions. A brief description of RANS, LES and DNS methods follows.

2.6.1 RANS

As the name implies Reynolds Averaged Navier Stokes methods solve the Reynolds averaged turbulent transport equations introduced in §2.1.1 (d). Different techniques ranging in complexity exist to model the turbulent fluxes that occur in the averaged equations transport equations.

RANS methods are widely used in all fields of fluid modelling and are the least expensive in terms of computing time of the computational fluid dynamics (CFD) methods discussed here. Some examples of the application of RANS methods to atmospheric flows are Zhang et al. (1996), Delaunay (1996) and Meroney et al. (1999). Castro et al. (1999) highlighted the deficiencies of RANS modelling of atmospheric flows by comparing the results of different groups of modellers using the same modelling package. Vastly different results were obtained by the groups for some cases.

Some strengths and weaknesses of RANS methods are:

\[ \leftrightarrow \] Computationally the least expensive of RANS, LES and DNS methods.
\[ \rightarrow \] RANS methods have been sufficiently refined for application to practical engineering problems and are widely used in industry

\[ \rightarrow \] Only mean quantities are solved for

\[ \rightarrow \] Theoretical foundations of most turbulence models are fairly weak

\[ \rightarrow \] Not universally applicable

\[ \rightarrow \] RANS methods are in general not applicable near solid boundaries and other calculation methods are required

2.6.2 LES

Large Eddy Simulation methods attempt to solve the time dependent turbulent transport equations introduced in § 2.1.1 (c). LES models solve most of the turbulent motions directly, and only rely on modelling for the turbulent motions contained in eddies that are too small to be resolved by the chosen discretisation scheme. LES methods are computationally more expensive than RANS methods because the time dependent turbulent motions are solved and because of the greater resolution and therefore greater number of variables that need to be solved.

The application of LES to air pollution meteorology is growing, and the following publications are just a few examples from the literature. Murakami et al. (1987) was the first to use LES to model the flow around an isolated rectangular building. More recently Meeder & Nieuwstadt (2000) studied the dispersion of a reactive plume in a neutral boundary layer, and Smith et al. (2001) considered thermal effects in their simulation of the flow around a cubical building.

Some strengths and weaknesses of LES methods are:

\[ \rightarrow \] Computationally more expensive than RANS, but much less than DNS

\[ \rightarrow \] Mean and turbulent quantities are solved for

\[ \rightarrow \] Application possibilities to engineering problems are limited at present, but commercial applications are emerging

2.6.3 DNS

Direct Numerical Simulation methods solve the fully time dependent turbulent transport equations directly by resolving all scales in the flow up to the smallest energy containing eddy. It is therefore vastly expensive and as yet not applicable to engineering flows. It is however being used extensively for fundamental research of turbulence and stably stratified shear flows.

Some of the recently published works include Galmiche et al. (2000) and Staquet & Godeferd (1998) who studied stably stratified turbulence in the context of infinite shear layers. Ohya, Hashimoto & Ozono (1997) simulated stably stratified boundary layers. Differences between the results of the simulation and experimental results were attributed mainly to insufficient grid resolution.

Some strengths and weaknesses of DNS methods are:
Requires vast amounts of computer processor time

Mean and turbulent quantities are solved for

Application possibilities to engineering problems are limited at present and will remain so for quite some time
The various measurement techniques that need to be mastered to measure the variables of interest in the EnFlo wind tunnel vary in complexity and many studies have been devoted entirely to measurement techniques. It is a daunting task to learn the proper use of all the instrumentation found in a laboratory, let alone master them. One can easily get bogged down by the steep learning curve and feel side tracked from the real goal of research which is the pursuit of novel results from experiments, and not the means of obtaining them. The quality of the results does however depend entirely on the quality of the measurements. No amount of analysis can correct a poorly set-up experiment or incorrect measurement.

The main pieces of hardware and instrumentation used in the EnFlo laboratory will be described in the following sections. Hayden (1998) described the workings of the EnFlo wind tunnel and most of the instrumentation coupled to it in detail.

3.1 EnFlo atmospheric wind tunnel

The main piece of hardware is the wind tunnel itself. A schematic of the EnFlo atmospheric wind tunnel can be seen in Fig. 3.4. Fig. 3.1 shows the inlet of the wind tunnel working section with the various elements that are needed to simulate a realistic atmospheric boundary layer. It is a suck-through open-circuit configuration with a working section 20 m long by 3.5 m wide by 1.5 m high. What follows is a brief description of each element needed to simulate the atmospheric boundary layer.

Surface Roughness  The surface roughness represents the effects of the full-scale ground surface. The drag it creates acts as a momentum sink and establishes a profile of Reynolds stresses through the surface layer which in turn controls the turbulence characteristics and the mean velocity profile.

Barrier wall and vorticity generators  The barrier wall and vorticity generators are artificial boundary layer simulation devices that trick the flow into believing it had a much longer stretch of roughness than it actually had. It is important that the turbulence characteristics eventually established by them are the same as that of a naturally grown boundary layer.

Heater tubes and chilled floor  For a stratified boundary layer the required density profile
is set up by controlling the temperature of the air. For stable stratification in this study the inlet air is typically heated to 58°C and the floor is cooled to about 13°C. The surface heat flux will depend on the inlet air temperature, floor temperature and the surface roughness. Fifteen individual heater tubes are available to set up a tailored profile of inlet temperatures if required.

Models and sources are usually placed between 12 and 16 metres from the entrance to the working section. That allows a sufficiently long stretch of roughness for the local effects of the boundary layer simulation devices to have died down. Between 4 and 8 metres of working section then remains where flow and concentration measurements downstream of sources and models can be taken.

### 3.1.1 Surface roughness

A typical roughness element arrangement can be seen in Fig. 3.2. The roughness elements are typically made from aluminium strips bent 90° to form a corner. When placed on the wind tunnel floor the upright portion is normal to the flow direction. Various roughness arrangements are possible in the EnFlo wind tunnel. A 50 mm wide × 10 mm high element with fixed 200 mm lateral and 180 mm longitudinal spacing has been the standard roughness arrangement for the neutral boundary layer simulations in the EnFlo wind tunnel (Hayden 1998). An 80 mm wide × 20 mm high element with fixed 240 mm lateral and variable longitudinal spacing is becoming the standard roughness arrangement for neutral and stable boundary layer simulations due to the flexibility it provides in adjusting important parameters such as roughness length ($z_0$) and friction velocity ($u_*$). The roughness elements have negligible thickness and are sufficiently spaced that interference between the wakes of individual roughness elements can be ignored. The 80 mm wide × 20 mm high elements were used throughout this study.
An unresolved uncertainty is the influence of individual roughness elements in the wakes of obstacles and sources for dispersion experiments. The roughness elements are sometimes a significant fraction of the height of the source or obstacle. Clearly individual roughness elements may have an effect only slightly smaller than the actual model in extreme cases. Preliminary measurements were made by Maré & Robins (1999) to investigate the effects of individual roughness elements on the dispersion of a dense gas released at ground level. Significant differences in concentration were observed 25 source heights downstream when an individual roughness element was either present or removed close to the source. The boundary layer characteristics change relatively slowly with a change in surface roughness, and small numbers of roughness elements can therefore be safely removed in the wakes of obstacles. The question arises when to remove roughness elements, and when not to. The issue becomes even more important with dense gas releases, where the plume stays close to the ground. Any roughness element arrangement other than a completely uniform arrangement will have areas of high mixing in the wakes of roughness elements that disperse the plume. For a certain plan area, instead of roughening the whole surface, we choose to place a single roughness element to provide the required drag for the whole surface. This thinking is adequate if the scale of obstacles and plumes are of the order of the plan area containing the single roughness element, but not if the scale of the obstacles and plumes are of the order of the single roughness element.

3.1.2 Boundary layer simulation devices

The methods of Counihan (1969) comprising a barrier wall and vorticity generators in addition to the surface roughness, and that of Irwin (1979, 1981) comprising only vorticity generators
Chapter 3. Instrumentation

with the surface roughness, are available in the EnFlo tunnel to accelerate the growth of the boundary layer. Counihan concludes that a working section length of between four and five boundary layer heights is required to produce the simulated atmospheric boundary layer. Robins (1979) suggests a fetch of seven boundary layer heights for the flow to attain relative two-dimensionality. At least 12 boundary layer heights are available for the boundary layer to develop in the EnFlo wind tunnel and any non-equilibrium residual effects of the artificial devices will have decayed by then. The real value of the boundary layer simulation devices is to create the required momentum deficit that would only exist after a very long stretch of roughness in its absence.

3.1.3 Freestream & background velocity measurements

A Gill ultrasonic anemometer is used to measure the freestream velocity in the EnFlo wind tunnel. It is an ideal instrument for this purpose since it does not require calibration, has no moving parts, is temperature insensitive and requires very little maintenance. A second Gill ultrasonic anemometer is also available and can be moved wherever required.

Two propeller anemometers are mounted on the traverse mechanism at a height of 1 m, 500 mm either side of the centreline. The principles of propeller anemometry are described in §3.2.2. The prime function of the propeller anemometers is to indicate deviations from two-dimensional flow.

3.1.4 Traverse mechanism

A computer controlled, 3 axis traverse mechanism is mounted in the wind tunnel. The geometrical reach of the traverse is 1.5 m in the lateral direction, 500 mm in the vertical direction and from 11 m to 18 m from the inlet of the tunnel in the downwind direction. A convenient mounting mechanism allows the mounting of instruments. The traverse can be used for all types of measuring instruments, the mounting of sampling tube rakes for dispersion measurements and for mounting movable sources for dispersion experiments.

3.1.5 Main facility shortcomings

An element that is central to the traditional design of wind tunnels is the settling chamber and contraction. The EnFlo wind tunnel has a rounded inlet without a settling chamber, which leads to inconsistencies in the flow at times. It must be said that it is extremely difficult in general to maintain consistent flows in wind-tunnels running at very low velocities.

3.2 Velocity Measurements

3.2.1 Laser Doppler Anemometer

The LDA measurement technique has been described by many authors including George & Lumley (1973) and more recently Abhil (1995). It is generally accepted that as a particle passes through a laser beam it scatters the light of the incident beam. The frequency of the
scattered light is shifted by an amount proportional to the particle velocity due to the Doppler effect. A detector can be placed at any position theoretically and measure the frequency of the scattered light, and so determine the velocity of the particle.

The LDA system used at EnFlo is a two component fibre-optic system which measures two perpendicular components of velocity simultaneously. Each component operates in a dual beam configuration that can be described by the fringe model. When two laser beams intersect at an angle light and dark planes or fringes are formed due to constructive and destructive interference of the monochromatic laser light. The volume where the beams intersect is regarded as the measuring volume for the dual beam LDA measurement technique. Monochromatic laser light will create fringe planes that are spaced at a fixed separation, and as the particle passes through subsequent planes the time of travel between planes can be recorded by photodetectors. The particle velocity can then be calculated directly from the time of travel between planes and the fringe spacing. The velocity that is being measured is in the direction normal to the fringe planes. It is clear that the principle of operation of the LDA described by the fringe model does not involve the Doppler effect at all (Absil 1995). The principles of Laser Doppler Anemometry can however be described equally well using the Doppler effect.

More important than the principles of operation of the various measurement techniques is the proper application of them. The LDA technique in the present mode of operation appears to be the perfect non-intrusive flow measurement technique that does not require calibration. Absil (1995) warns against this perception. In a laboratory such as EnFlo with a high level of measurement automation and custom developed data acquisition software one can become a "Black Box" user of the underlying measurement technique through the end user software. End users must still strive to be knowledgeable about the underlying measurement technique. The LDA probe and beam expander are sealed units that offer little user intervention and are guaranteed to be accurate by the manufacturer. The signal analysis units validate the velocity measurements and they give a randomly spaced, discrete time trace of validated velocity measurements as output. The possible pitfalls of the LDA measuring technique as described by Absil (1995) that can still occur with a set-up such as the one at EnFlo are:

Seeding the flow The LDA technique does not measure the velocity of the fluid, but rather measures the velocity of small particles that are present in the fluid. The particles need to be sufficiently small to follow the streamlines of the flow exactly. Seeding at EnFlo is achieved by using a commercial “haze generator” operated on sugar solution. Such haze generators are used commonly to generate smoke for stage productions. Banks of ultrasonic transducers are pulsed in a layer of fluid and cause cavitation of the fluid. Bubbles are formed on the surface and when they burst the resulting haze is blown from the machine by a fan. The haze generator is placed in front of the wind tunnel and the haze is transported by the air entering the tunnel. After running the haze generator for a length of time the surrounding air in the laboratory becomes filled with suspended seeding particles. The seeding therefore no longer only comes from a single source, but all the air that is drawn into the wind tunnel contains seeding. This situation is highly advantageous for uniform seeding of the flow. The ability of the particles to remain suspended in the relatively still air in the laboratory is one indication that they are indeed sufficiently small.

Velocity or Sampling bias The time trace of velocity measurements output by the signal analysis units are not equi-spaced, and determining averages from the measurements is not as simple as for equi-spaced data. It is possible that eddies of a certain size are
generally better seeded than eddies of other sizes, and therefore velocity measurement will be biased towards the velocities of the better seeded eddies.

### 3.2.2 Propeller Anemometry

Beckwith & Marangoni (1990) describe the principles of turbine-type flow meters such as propeller anemometers. A permanent magnet is encased in the rotor body, with a corresponding coil pickup in the housing. Electronic circuitry is used to count the number of revolutions of the turbine during a specified period and the angular velocity of the turbine can then be calculated. The output of propeller anemometers is normally a voltage, with a linear relation to angular velocity, and within a limited range also velocity. Propeller anemometers are insensitive to temperature but need to be calibrated for accurate flow measurements. Propeller anemometers are however better suited to qualitative measurements rather than detail flow measurements. For the purpose of mapping out the mean velocity distribution over a large surface they may be ideal.

### 3.3 Scalar Measurements

#### 3.3.1 Cold wire probe

![Figure 3.3: Instrumentation set-up for synchronised LDA and scalar measurements at EnFlo](image)

The cold wire probe is used for instantaneous temperature measurements at a high sampling rate. Thermocouples have a response of a few seconds and are therefore unsuitable to measure turbulent temperature fluctuations in a wind tunnel with high frequency fluctuations.

A normal hot wire can be used to measure temperature by changing its operation from constant temperature mode to constant current mode at a low overheat ratio. The resistance of a hot wire is directly related to the temperature of the wire. For velocity measurements the wire is heated to a temperature well above the ambient temperature of the fluid, and the resistance of the wire is kept constant by changing the current in the wire. For the cold wire operation the wire is virtually kept at the fluid temperature, and by keeping the current constant in the wire, the measured resistance of the wire is directly related to the temperature of the fluid. The low overheat ratio requirement is necessary to prevent the fluid velocity from influencing the temperature measurements.
Bruun (1995) describes the operation of the cold wire probe in detail, while Heist & Castro (1998) describe the measurement of turbulent heat flux by combining LDA and cold wire measurements. The set-up is illustrated in Fig. 3.3. Heist & Castro (1998) found the frequency response of a 5 \( \mu \text{m} \) diameter cold wire adequate for temperature and heat flux measurements in the EnFlo atmospheric wind tunnel where the mean flow speed is low.

Pietri et al. (2000) also made simultaneous temperature and velocity measurements in a slightly heated jet with a cold wire and a Laser Doppler Anemometer. A 0.63 \( \mu \text{m} \) cold wire was used to attain the required frequency response in excess of 5 kHz. A similar triggering mechanism to that used at EnFlo was employed to synchronise LDA and cold wire measurements. A burst detection by the Burst Spectrum Analyser (BSA) triggers the acquisition of a temperature measurement by the analogue to digital converter.

### 3.3.2 Hot wire measurements

A hot wire probe was routinely used to verify the synchronisation between ADC measurements and LDA measurements. A correlation coefficient of > 97% indicated good positioning and synchronisation between ADC measurements and LDA measurements. The same wire was used for both cold and hot wire measurements, and the distinction being the mode they were run in. A cold wire probe is run in constant current mode, and a hot wire in constant temperature, i.e. constant resistance mode.

### 3.3.3 Dispersion measurements

A schematic of the typical set-up used for dispersion measurements is shown in Fig. 3.4.
Two related instruments are used for concentration measurements. Both operate on the principle of the ionisation of carbon atoms. The first instrument, simply known as a Flame Ionisation Detector (FID), can only be used for measuring average concentrations because of its low frequency response. A system with a high frequency response is also available, known as a Fast FID (FFID). The FFID is suitable for measuring concentration fluctuations.

3.3.3 (a) Flame Ionisation Detector

Dispersion measurements at EnFlo are based on the release of an inert carrier gas with the correct thermodynamic state properties required by the experiment, mixed with a known concentration of a hydrocarbon such as propane. Gas is drawn and stored during the experiment using sampling tubes at the spatial positions where concentration measurements are required. After the experiment the stored gas from each measuring location is passed through a FID which measures the concentration of hydrocarbons present in the gas. The known concentration of the release allows the calculation of the dilution that has taken place.

The FID, as described by Lovelock (1961), operates on the principle of the ionisation of carbon atoms during combustion of a hydrocarbon. A fixed proportion of hydrocarbon molecules will temporarily ionise as a carbon cation and an electron. By applying an electrostatic charge over the combustion volume the electrons will be attracted by the positive pole, and an electric current due to the flux of electrons will be created which can be measured. The current will be proportional to the concentration of hydrocarbons in the mixture. The FID has to be regularly calibrated for each type of trace gas, such as propane or ethane. A certified calibration gas with a known parts per million (ppm) concentration is used for each type of trace gas.

The FID is one of many components that are needed for dispersion measurements, shown schematically in Fig. 3.4.

3.3.3 (b) Fast FID

Fackrell (1980) describes the procedure for using a flame ionisation detector to measure fluctuating concentration. The principles of a FFID are exactly the same as that of an ordinary FID. The main difference is that the measurements are done in real time rather than the storage and retrieval method described above. Depending on the tube length and suction rate response times of up to 500 Hz are possible. The real time operation of the FFID implies that the samples are drawn directly into the flame chamber of the FID, thus requiring a vacuum pump. Sample tube lengths of 150 mm and 250 mm have been used at EnFlo. The frequency response of the FFIDs with such long sample tubes is in the order of 100 Hz for the setup used during this study.

Four FFID heads together with their support equipment were available during the course of this study. Simultaneous LDA velocity and FFID concentration measurements were also taken successfully.

Figs. C.1 & C.2 shows the frequency response and transit time calculations for the two different sample tube lengths.
3.3.4 Pressure measurements

A Baratron transducer is used for pressure measurements. The high sensitivity to minute pressure differences makes the Baratron suitable for pressure measurements in a low velocity wind tunnel such as the EnFlo wind tunnel. The Baratron is connected to a computer controlled scanning valve with 24 ports. This allows the automatic measurement of pressure at 24 locations before user intervention is required.

3.4 Measurement software

A powerful suite of software has been developed over the years at EnFlo, and this software was available for use throughout the work reported herein. The software is being enhanced and extended continuously. The software is developed with the National Instruments LabVIEW Integrated Development Environment. The software outputs the averages of the measured quantities, e.g. $\overline{u}$, $\overline{w}$ and $\overline{c}$. The software also outputs the corresponding moments and products, e.g. $\overline{u'^2}$, $\overline{w'^2}$, $\overline{c'^2}$, $\overline{u'w'}$, $\overline{u'c'}$ and $\overline{w'c'}$. The individual measurements, or raw data, are always stored and can be accessed for further processing.
The flow around buildings needs to be measured and understood to pave the way for better building wake models. A further requirement is the investigation of the effects of stratification on the flow around the buildings. There will always be an unlimited number of cases that should be investigated. Time, cost and practical considerations reduce the number of cases that can typically be investigated. The geometry of the model should be simple enough to allow the results to be applicable to models with similar geometries, and for the knowledge in general to be augmented. The simple cube will always be popular with fluid dynamicists because of its simple geometry, but also because it exhibits all the complexities associated with bluff body aerodynamics. More complex geometries can often be reduced to basic shapes such as cubes.

Before the model can be considered, the flow must be simulated correctly. One of the most complex flows that can be simulated is the atmospheric boundary layer. The wind tunnel and the necessary instrumentation have been described in Chapter 3, and where necessary specific details will be repeated in the description of the experiments. It is therefore appropriate that the first experiment discussed is the simulation of the boundary layer. The simpler neutral boundary layer will be discussed first and then the stable boundary layer. Boundary layer characterisation is described in §4.1.

The surface pressure distribution on and around a cube determines the strength of the wake, and is therefore an important parameter characterising the flow and dispersion near obstacles. Pressure measurements were made on the surface of a cube in boundary layers with a range of stabilities, and with the cube normal to the flow and at incidence. The pressure measurements are described in §4.5. The chapter is concluded with §4.6 reporting a set of flow measurements in the wake of the cube.

Ideally the effects of stability would be investigated by comparing a stable boundary layer and a neutral one with all other characteristics similar except the imposed stability. This is generally not possible. Stable stratification reduces turbulence levels and it can be difficult to discern the separate effects of the reduced turbulence levels of the boundary layer and the actual effect of stability itself. Steggel & Castro (1998, 1999b, 1999a) illustrated this side-effect of stability quite effectively.
4.1 Boundary layer characterisation

The most common experiment that needs to be done on a regular basis is the characterisation of boundary layers in the EnFlo wind tunnel. The neutral boundary layer characteristics are influenced by the ambient conditions in the laboratory and these in turn are influenced by the weather. The state of the screens at the inlet of the tunnel also has an effect on the characteristics of the wind tunnel, and the fouling rate of the screens is variable depending on the seeding rate used for LDA measurements. A cost effective way to eliminate the variations in the boundary layer completely has not yet been found, and it is therefore important to document the boundary layer for every set of experiments.

An efficient utility has been developed to quickly assess a boundary layer. It consists of a Microsoft Excel spreadsheet and macros that rapidly analyse profiles of velocities and turbulence characteristics wherever appropriate. Typical results for a neutral and a stable boundary layer are included in Appendix A. Further analysis and discussion will be presented in the following sections. The boundary layer is characterised by calculating the surface layer similarity parameters $u_*$, $z_0$, $\theta_*$ and $L$.

Fig. 4.1 summarises the characteristics of the neutral and stable boundary layers. The boundary layer depth $\delta$ was taken as the $\delta_{99\%}$ value. From the velocity profile for the neutral boundary layer it can be seen that there is virtually no freestream region. The boundary layers will be discussed further in §4.4.

4.2Neutral Boundary Layer

For the simulation of the neutral boundary layers in the EnFlo wind tunnel, 1.2 m high Irwin spires were used that produce a 1 m nominal depth boundary layer according to the design specification of Irwin (1981). The LDA is the principal method of obtaining velocity data and it is mounted on the traversing mechanism.

A profile of 20 points of LDA results usually takes about 2 hours to obtain. During that two hours slow changes in the boundary layer can take place and it is sometimes difficult to get a true representation of a velocity profile. Appendix A gives some criteria for assessing the quality of a measured boundary layer profile.

Fig. 4.2 shows the contours of mean velocity for the 2.5 m/s neutral boundary layer 16 m downstream from the inlet. Because of the open circuit configuration and the lack of a proper contraction at the inlet little can be done to improve the two-dimensionality. Most studies are confined to the bottom 500 mm of the wind tunnel where the flow is best behaved. The undisturbed flow should be documented regularly, and undisturbed profiles should be measured at the location where perturbation quantities due to wakes are to be determined.

4.2.1 Determining $u_*$ and $z_0$

For the neutral boundary layer $L \rightarrow \infty$ and $\theta_* = 0$. The calculation of $u_*$ and $z_0$ for the neutral boundary layer appears relatively straightforward. The shear stress, $\overline{u'w'}$, is measured directly, and in the surface layer it should remain constant. A Least Squares Error Method (LSEM) is then used to determine $z_0$. Table 4.1 summarises the average characteristics of the
Figure 4.1: Neutral and stable boundary layer characteristics, with $U_{\infty}$ the nominal freestream velocity. $^6$ $U_{\infty} = 1.1, 1.3 \& 1.5$ m/s are stable boundary layers; $U_{\infty} = 2.5$ m/s is a neutral boundary layer. (Discussed in § 4.4)

$^1$ $U_\infty$ is the actual freestream velocity.

$^6$ Fewer points were measured for the $v'^2/u_{\kappa}^2$ neutral profile than for the other neutral profiles

$^4$ $U_{\infty}$ is the velocity measured by the fixed ultrasonic anemometer at a height of $z = 1$ m.
neutral boundary layer in the EnFlo wind tunnel. Stull (1988) stated that the aerodynamic roughness length $z_0$ is a property of a particular surface. That implies that the same value should be applicable regardless of stability and wind speed. It was however found that $z_0$ also depended on the flow to some extent, and a better fit on the data could often be achieved by solving for $z_0$.

Table 4.1: Average characteristics of simulated neutral boundary layers

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$u_*/U_{\infty}$</th>
<th>$z_0/\delta$</th>
<th>$d/\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.06 m</td>
<td>0.056</td>
<td>0.0014</td>
<td>0.01</td>
</tr>
</tbody>
</table>

1 Due to the rough surface a zero-plane reference at a displacement height of $d$ can be defined, leading to the modified wind profile $\bar{u}(z) = \frac{u_*}{\kappa} \ln \left( \frac{z-d}{z_0} \right)$

4.2.2 Spectra

Fig. 4.3 shows the measured power spectral densities for $u^2, v^2$ and $uw$ in the neutral boundary layer. The region where $S_{uu}$ exhibit the characteristic $-5/3$ slope as predicted by Kolmogorov for the inertial subrange is quite extensive, spanning more than two decades. The additional requirements for the inertial subrange that ensures local isotropic turbulence as described by Kaimal & Finnigan (1994) are also met in a region of about a decade. These requirements are that $S_{uu} = 4/3 S_{ww}$, and that $S_{ww} \gg S_{uw}$, as shown in Fig. 4.3.
4.3 Stable Boundary Layer

4.3.1 Experimental procedure

Steggel’s (1999) stable boundary layer was set up by heating the inlet air non-uniformly, with an essentially uniform temperature profile above the surface layer rising rapidly towards the surface. This approach ensured higher turbulence levels near the surface and a higher heat flux than a boundary layer with a simple uniform distribution. Essentially a weak convective region is set up near the surface that quickly generates turbulence as it dies away. This inlet profile was fixed empirically to ensure uniform temperature conditions at the beginning of the cooled floor. It is not clear why an already weakly stratified boundary layer at the start of the cooled floor panels is undesirable.

The changes in the current procedure of simulating stable boundary layers to that of Steggel (1999) are numerous. For the present study a uniform temperature profile was chosen to ensure that persistent structures that can appear due to the local unstable stratification are avoided. Although higher heat fluxes were obtained by Steggel, higher shear stresses were also found, resulting in a boundary layer that was less stable than one set up with a uniform temperature profile. Wind tunnel simulation of the atmospheric boundary layer aims to achieve a boundary layer with the greatest stability while still maintaining turbulent flow. This condition was achieved with all the generated boundary layers.

Steggel used the Counihan method to accelerate boundary layer growth. The vorticity generators were 1 m high, although the boundary layer was only 500 mm deep. The turbulence characteristics showed an increasing trend towards the top of the boundary layer that is in disagreement with field observations. This was probably due to the vorticity generator wakes. Irwin spires matching the boundary layer depth were used in all the simulations of the current...
study. Three different boundary layers were characterised for the purposes of the study. The freestream velocity ranges are shown in Table 4.2.

### 4.3.2 Monin-Obukhov similarity

#### 4.3.2 (a) Determining $\theta_*$ and $L$

Heist & Castro (1998) argued that the direct measurement of $\overline{w'\theta'}$ is far superior for determining $\theta_*$ than the curve fitting approach. The results obtained by them justified this argument. However, the technique proved ineffective in the present work, providing inconsistently low estimates for $\overline{w'\theta'}$. Possible causes that were investigated were the balancing of the DISA temperature bridge and interference signals, but to no avail. A different technique was therefore implemented to determine the characteristics of the stable boundary layer without using the directly measured $\overline{w'\theta'}$.

$L$ can be determined from Eq. (2.30) if $\theta_*$ and $u_*$ are known. An iterative procedure was adopted using a least squares error method to fit the Monin-Obukhov similarity profiles Eqs. (2.45) & (2.47) on the measured data. The steps of the procedure are:

1. Estimate $L$
2. Solve for $u_*$
3. Solve for $\theta_*$
4. Calculate $L$ from Eq. (2.30)
5. Go to step 2

When both velocity and temperature profiles are available the preferred method of calculating $L$ is from Eq. (2.30) and not from a least squared error method, thus limiting the number of variables that is being solved by the least squared error method. It is however possible to determine $L$ and $u_*$ employing a least squared error method if the wind profile is the only data available (e.g. Giannini et al. 1997), allowing the calculation of $\theta_*$ from Eq. (2.30).

$\overline{\theta_0}$ is simply a constant of integration (Businger 1973) like $z_0$, but unlike $z_0$, $\overline{\theta_0}$ is not a physical property of the surface. $\overline{\theta_0}$ can also be determined from the LSEM.

To avoid further complication $\beta_m$ and $\beta_h$, were both kept constant with a value of 5.

Table 4.2 summarises the characteristics of the stable boundary layers.

### Table 4.2: Summary of the calculated characteristics of the different stable boundary layers

<table>
<thead>
<tr>
<th>$U_{25}$ [m/s]</th>
<th>$\delta$ [m]</th>
<th>$U_\infty$ [m/s]</th>
<th>$\theta_*$ [K]</th>
<th>$u_*/U_\infty$</th>
<th>$L$ [mm]</th>
<th>$Re_\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>0.6</td>
<td>0.99</td>
<td>0.73</td>
<td>0.027</td>
<td>81</td>
<td>0.91</td>
</tr>
<tr>
<td>1.3</td>
<td>0.6</td>
<td>1.15</td>
<td>0.76</td>
<td>0.031</td>
<td>133</td>
<td>0.69</td>
</tr>
<tr>
<td>1.5</td>
<td>0.6</td>
<td>1.38</td>
<td>0.83</td>
<td>0.034</td>
<td>200</td>
<td>0.47</td>
</tr>
</tbody>
</table>
4.3.3 Spectra

Fig. 4.4 shows the measured power spectral density relations for $u^2$, $w^2$ and $uw$ in the 1.1 m/s stable boundary layer. It is immediately evident that $S_{uw}$ is an order of magnitude smaller at low frequencies than $S_{ww}$, unlike the neutral case. Note however that the profiles are not in a dimensionless form.

The region where $S_{uu}$ exhibit the characteristic $-5/3$ slope as predicted by Kolmogorov for the inertial subrange is smaller than that for the neutral case, extending roughly for one decade. The region of the subrange where $S_{uu} = 4/3S_{ww}$ is also considerably smaller than the neutral case, showing little evidence of an inertial subrange. $S_{uw}$ is however much smaller than $S_{ww}$ throughout the whole frequency region.

4.4 Discussion of neutral and stable boundary layers

Fig. 4.1 summarises the measured characteristics for the four simulated boundary layers. There is little visible difference between the non-dimensional temperature and velocity profiles for the stable boundary layers. Characterisation in terms of the Monin-Obukhov similarity theory, however, reveals the differences in Table 4.2. Comparisons with the full scale measurements reported by Caughey et al. (1979) show that the profiles of $\overline{u'^2}/u_*^2$ and $\overline{v'^2}/u_*^2$ have comparable values near the ground. The turbulent characteristics of the 1.1 m/s stable boundary layer are very different from the other two stable boundary layers. Profiles of $\overline{w'^2}/u_*^2$, $\overline{v'^2}/u_*^2$ and $\overline{w'^2}/u_*^2$ have minimum values at ground level and then increase throughout most of the boundary layer. The profiles of $\overline{u'w'}/u_*^2$ for all three stable boundary layers are very close together, as would be expected. The profiles of $\overline{w'^2}/u_*^2$ are not expected to collapse, because of the different stabilities. The $\overline{u'^2}/u_*^2$ and $\overline{v'^2}/u_*^2$ profiles for the neutral, 1.3 and
1.5 m/s stable boundary layers collapse well enough, considering the circumstances.

4.5 Pressure measurements on a cube

The effects of boundary layer stability on the pressure distribution and drag coefficients on a cube have not been documented before. Castro & Robins (1977) measured the pressure and flow distribution around a cube in a neutral atmospheric boundary layer. The experiments reported here follow the format of Castro & Robins (1977). The drag (or the couple) on an obstacle is an important parameter in predicting the wake characteristics behind the obstacle. It is therefore important to know if stability has an effect, which would call for revised parameters to be used in wake prediction models.

Fig. 4.5 shows the specially made model cube that was used for pressure measurements. The cube is made from aluminium sheet with dimension $H = 100$ mm. 44 pressure taps were arranged on the cube. The Baratron pressure transducer and scanning valve combination as described in §3.3.4 can measure the pressure at 24 ports sequentially. The pressure taps have been arranged in such a way that a full pressure distribution map of all the faces can be obtained by 1 change over of ports connected to the scanning valve, and 4 rotations of the cube for each set of ports.

The neutral boundary layer with a freestream velocity of 2.5 m/s was used for all the neutral flow measurements, giving a ratio of $H/\delta = 0.1$. For stable flow the 1.1 m/s, 1.3 m/s and 1.5 m/s freestream velocity boundary layers were used giving a ratio of $H/\delta = 0.17$. The ratio $H/L$ for the neutral flow is by definition $H/L = 0$, and for the stable flows $H/L = 1.2$, 0.75 and 0.5 for the 1.1, 1.3 and 1.5 m/s boundary layers respectively.

Two types of results will be shown in the graphs that follow. The pressure distribution on a line travelling from the base of the front face of the cube, over the roof and down to the base of the back face of the cube. This type of result will be denoted by $FTB$ (Front–Top–Back); Fig. 4.6 is an example of a $FTB$ plot. The other type of result will show the pressure distribution on a line travelling along the perimeter of the cube on a plane parallel with the
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floor of the wind tunnel. This type of result will be denoted by BLFRB (Back-Left-Front-Right-Back); Fig. 4.9 is an example of a BLFRB plot. The schematic diagram on each graph also shows the direction of the flow in relation to the cube and the pressure profile that is being plotted.

4.5.1 Velocity at roof height

LDA measurements were not taken while carrying out pressure measurements. For the calculation of pressure coefficients on the cube faces the velocity at the cube roof height \( U_r \) was required. Examination of the undisturbed velocity profiles for neutral and stable flow yielded the following relations:

\[
\frac{U_H}{U_\infty} \approx 0.58 \quad \text{for neutral flow and} \\
\frac{U_H}{U_\infty} \approx 0.63 \quad \text{for stable flow to sufficient accuracy, regardless of stability.}
\]

The pressure measurements for each case were sometimes accumulated over several days but still produced smooth pressure coefficient profiles. This serves as an indication that any variability in the boundary layers that may have occurred had little effect on the ratio \( U_H/U_\infty \).

4.5.2 Flow normal to the cube

The first set of measurements was done with the flow direction normal to the cube.

4.5.2 (a) Effect of local roughness elements (NBL)

Fig. 4.6 shows the difference in pressure distribution between a case where the local roughness elements were removed, and a case where the local roughness elements were kept in place. The roughness elements that were removed are the four elements closest to the cube as shown in Fig. 4.5. From Fig. 4.6 it can be seen that this had no effect on the forward face of the cube. It did however influence the pressure distribution on the roof and lee of the cube. Local roughness elements seem to reduce the velocities over the roof of the cube resulting in higher pressure coefficients over the roof and a somewhat weaker wake. The reduced velocities can influence dispersion studies. For the remainder of the pressure measurements the roughness elements were left in place. This is adequate for comparison of pressure distributions, but for dispersion studies local roughness elements may be removed for concentration measurements in the near wake of an obstacle.

4.5.2 (b) Effect of freestream velocity (NBL)

The effect of varying the freestream velocity, and therefore Reynolds number was studied with the results shown in Fig. 4.7. Freestream velocities of 1.5 m/s, 2.0 m/s and 2.5 m/s were used. In terms of Reynolds number \( Re = U_H H/\nu \) the range was \( Re = 9600, 7700 \) and 5800. Apart from some experimental scatter the results show good agreement. The range of Reynolds numbers is however small and no conclusion can be drawn about Reynolds number independence.
Figure 4.6: Effect of local roughness elements on the pressure distribution on a cube.

Figure 4.7: Effect of Reynolds number on the pressure distribution on a cube.
4.5.2 (c) Comparison of neutral & stable flows

Fig. 4.8 compares the distribution of pressure in a neutral and a stable boundary layer. The stable boundary layer is the 1.3 m/s case. Apart from the forward face of the cube there is little difference between the stable and neutral cases for the pressure distribution plotted. The difference on the forward face occurs in the roughness sublayer, and the differences in the velocity profile and turbulence levels between the two can probably account for this. The similar pressure distributions on the roof are surprising since the neutral and stable boundaries have very different $u_*/U_\infty$ ratios, 0.056 and 0.031 respectively (Table 4.2), and reattachment on the roof usually responds to this.

Fig. 4.9 compares horizontal profiles of pressure on the perimeter of the cube for different values of $z/H$ in the neutral boundary layer. On the sides and back of the cube the profiles for all heights are nearly equal. On the front face of the cube differences can be observed as would be expected from Figs. 4.7-4.11. A rising trend can be seen for $z/H$ up to $z = 0.9H$. The next profile at $z = 0.95H$ reports lower pressure coefficients than the profile at $z = 0.1H$ on the front face of the cube.

Fig. 4.10 compares horizontal profiles of pressure on the perimeter of the cube for different values of $z/H$ in the stable boundary layer. The results are similar to the neutral results in Fig. 4.9. The pressure coefficients on the front face start to decrease above $z = 0.9H$ as for the neutral case. The pressure coefficients at $z = 0.95H$ are however still higher than the pressure coefficients at $z = 0.5H$. 

![Figure 4.8: Comparison of Neutral and Stable pressure distributions](image)
Figure 4.9: 2.5 m/s NBL. Horizontal pressure coefficient profiles for constant heights

Figure 4.10: 1.3 m/s SBL. Horizontal pressure coefficient profiles for constant heights
4.5.2 (d) Comparison of stable flows with varying stability

In Fig. 4.11 boundary layers with different measures of stability are compared. All the profiles show similar behaviour over most of the region plotted apart from the leading edge of the roof where the boundary layer with the highest measure of stability shows slightly odd behaviour. The roof separation region also seems to have been weakened significantly by the stability.

4.5.3 Flow at 45° to the cube

The second set of measurements was done with the flow direction at 45° to the cube. The 2.5 m/s neutral case and the 1.3 m/s stable case are used for comparison.

Fig. 4.12 is a FTB comparison of the pressure distribution on a cube with the flow at 45° in stable and neutral boundary layers. Due to symmetry two sets of profiles are available for each case. On the front faces of the cube the pressure distribution for the stable boundary layer first shows lower values than the neutral case up to $z \approx 0.5H$ continuing to rise above the neutral values. This is a similar trend to the normal flow orientation case in Fig. 4.8. The separation region on the roof leading edge is weaker for the stable case. Re-attachment of the flow occurs at the same position on the roof. The stable case exhibits lower relative velocities over the whole of the roof. A weaker wake is also present in the stable case.

Figs. 4.13-4.14 are BLFRB results for neutral and stable flow at 45°. Very similar results for both neutral and stable flow are observed as for the normal flow condition in Figs. 4.9-4.10.
Figure 4.12: Comparison of Neutral and Stable pressure distributions, cube at 45°

Figure 4.13: 2.5 m/s NBL. Horizontal pressure coefficient profiles for constant heights, cube at 45°
4.5.4 Drag coefficients

Table 4.3 shows the result of drag coefficient calculations for neutral and stable flows at normal and 45° flow orientations. Due to a limited number of pressure taps that could be used and restrictions on pressure taps too close to the edges a full map of pressure is not available on the cube. The results in Table 4.3 are from a numerical integration of the available measurements. The numerical integration was performed by dividing the surface area of the cube into small squares, assigning the closest pressure measurement to each square, and performing a summation of the pressure and area products. The drag coefficient $C_D$ is calculated from

$$C_D = \frac{F_D}{1/2 \rho U_H^2 A}$$

where $F_D$ is the drag force and $U_H$ is the characteristic velocity at the cube height. $A$ is the frontal area of the cube, noting that $A_{45°} = \sqrt{2} A_{90°}$. The air density $\rho$ was calculated for the air temperature at cube height.

The stable and neutral drag coefficients for the normal flow case are equal, whereas the drag coefficient in the stable flow is slightly lower than that for the neutral flow with the flow at 45°. The lower drag coefficient is consistent with the higher base pressure shown in Fig. 4.12.

4.5.5 Conclusion from pressure measurements

The differences in the pressure distribution on a cube between neutral and stable boundary layers are small, but still evident. A weaker separation region on the roof and a weaker wake have been observed for the 45° stably stratified case, whereas the normal flow case showed little difference between neutral and stable stratification. This is surprising since the $u_*/U_\infty$ ratios for the neutral and stable boundary layers differ by almost a factor of two, and roof
4.6 Wake measurements behind the cube

The stable boundary layers that were used for the flow measurements behind the cube are the 1.1 m/s and 1.3 m/s ones, with characteristics summarised in Table 4.2 and Fig. 4.1. The 100 mm cube was placed 14 m downstream of the inlet section of the wind tunnel. That left sufficient fetch for the boundary layers to develop upstream and the wake downstream of the model. Measurements were made from 0.25 m to 4 m downstream. The two-component LDA system was used for all velocity measurements. Vertical profiles with 8 data points were made over a vertical range from 34 mm to 300 mm. Following Appendix F the perturbation quantities in Figs. 4.15–4.29 are referred to using plain symbols $u$, $w$, $u^2$, $w^2$ and $uw$. Two sets of measurements were available for each case, and these are shown as outlined and filled markers in the figures.

Hansen (1975) lists one of the important results of the simple wake theory derived in Appendix F as providing the proper non-dimensionalisation for $z$ and the perturbation velocity $u$ to collapse the wake deficit profiles. This method of plotting however accentuates measurement errors as the perturbation velocities become very small. The decay of the perturbation velocity is better illustrated by using the velocity at roof height, $U_H$, for normalising, as shown in Figs. 4.15–4.17.

Figs. 4.15–4.17 show the perturbation longitudinal velocity in the wake of the normal cube.
Figure 4.16: Perturbation velocity $-u/U_H$, stable 1.3 m/s

Figure 4.17: Perturbation velocity $-u/U_H$, stable 1.1 m/s
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Figure 4.18: Perturbation velocity $-w/U_H$, neutral

Figure 4.19: Perturbation velocity $-w/U_H$, stable 1.3 m/s

at different downstream locations for the three stability cases. For the measuring station closest to the cube the neutral case shows higher perturbation velocities than the stable cases. The magnitudes at $x/H = 5$ and further downstream are however similar. The wake model derived in Appendix F shows that the strength of the wake $\bar{u}/U_H$ is proportional to $\bar{u}/U_H \propto (u_*/U_H)^{-3/2}$. This implies that, all else being equal, the magnitudes of the perturbation velocities $u/U_H$ be a factor of 3 higher for the 1.1 m/s stable case than for the neutral case, which is clearly not seen. $u/U_H$ seems nearly independent of $u_*/U_H$.

Figs. 4.18–4.20 show the vertical perturbation velocity $w$. It can be seen that the vertical extent of the perturbation is greater than for the longitudinal perturbation velocity $u$, which is consistent with the wake model in Appendix F. The perturbation velocities have similar magnitudes for the different stabilities.

Figs. 4.21–4.29 show the perturbation Reynolds stresses in the wake of the normal cube at different downstream locations for the three stability cases. The perturbation Reynolds stresses are normalised by $U_H^2$ rather than the Reynolds stress at the obstacle height in the approach flow as used by Hayden (1998). This normalisation is more appropriate to the wake modelling employed in dispersion modelling. The quality of the $w^2$ measurements is better than the $u^2$ and $uw$ measurements. At times the rejection rate of $u$ velocity samples by the burst spectrum analysers was much higher than for $w$ velocity samples. Averages were calculated with the best set of data, and not just where both velocity measurements were
available, the scheme used by the EnFlo suite of software.

The profiles of $u^2/U_H^2$ in Figs. 4.21-4.23 seem unaffected by stability, while the profiles of $uw/U_H^2$ in Figs. 4.27-4.29 show some decrease. The effects of stability are more clearly observed in the profiles of $w^2/U_H^2$ in Figs. 4.24-4.26, with the magnitudes of $w^2/U_H^2$ decreasing slightly with increasing stability. It can be expected that $v^2/U_H^2$ will follow a similar trend to $w^2/U_H^2$. Stability is expected to have the strongest effect on $w^2$, then $uw$, and $v^2$ and $u^2$ will be the least affected.

The perturbation shear stress $uw/U_H^2$ can still be detected at $x = 20H$ for the two stable cases, while the neutral case only experimental scatter can be observed. The other perturbation Reynolds stresses have mostly disappeared at $x = 20H$. 

---

**Figure 4.20:** Perturbation velocity $-w/U_H$, stable 1.1 m/s

**Figure 4.21:** Perturbation normal stress $u^2/U_H^2$, neutral
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Figure 4.22: Perturbation normal stress $u^2/U_H^2$, stable 1.3 m/s

Figure 4.23: Perturbation normal stress $u^2/U_H^2$, stable 1.1 m/s

Figure 4.24: Perturbation normal stress $w^2/U_H^2$, neutral
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Figure 4.25: Perturbation normal stress $w^2/U_H^2$, stable 1.3 m/s

Figure 4.26: Perturbation normal stress $w^2/U_H^2$, stable 1.1 m/s

Figure 4.27: Perturbation shear stress $-uw/U_H^2$, neutral
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Figure 4.28: Perturbation shear stress $-\frac{uw}{U_H^2}$, stable 1.3 m/s

Figure 4.29: Perturbation shear stress $-\frac{uw}{U_H^2}$, stable 1.1 m/s
Appendix B presents autocorrelation measurements in the wake of the cube for stable and neutral flows. The autocorrelation measurements show the interaction of the wake and stability on the integral time scales of the turbulence. In both the neutral and stable boundary layers the $uu$ correlations show similar development with the time scales in the wake being less than in the undisturbed flow, though growing with increasing fetch downstream. The differences are greatest in the neutral case. At the first wake measurement position, $x/H = 5$, the time scales in the two flows are similar; at all other positions time scales in the neutral case are greatest. This picture reflects the interaction between turbulence close to the cube, the time scale of which scales on $H/U(H)$, and in the undisturbed flow, which scales on $\delta/U_\infty$. The situation regarding the $ww$ correlations is different. Behaviour in the neutral flow is similar to that for the $uu$ correlation, except that the time scale in the undisturbed flow, scaling on $z/U(H)$, is much less than for $uu$ and so the relative differences between the wake and undisturbed flow are much reduced. The time scales of the wake in the stable case are actually slightly larger than in the undisturbed flow. Here the vertical scales of the eddies generated by the flow around the cube happen to be greater than those existing in the undisturbed approach flow.
The dispersion of plumes from sources at ground level, \( H_s = 100 \text{ mm} \) and \( H_s = 150 \text{ mm} \) was investigated with and without obstacles. A ground level source is a distinct case from the elevated sources, since the ground influences the plume consistently. All elevated sources exhibit a period where the plume remains unaffected by the ground. Only the undisturbed case was investigated for the ground level source.

Fig. 5.1(a) shows the arrangement for the \( H_s = 100 \text{ mm} \) source with the cube normal to the flow. A source with a 9 mm inner diameter was used. Iso-kinetic release rates were used for all elevated sources. For ground level sources the iso-kinetic release rate at a height of 50 mm was used. It can be seen that the source centreline is actually slightly higher than 100 mm, with the bottom of the source touching the roof of the 100 mm cube.

Velocity and concentration measurements were made simultaneously, which enabled direct measurement of concentration fluxes.\(^1\) The availability of velocity data also enabled the

\(^1\)The concentration flux \( w \bar{c}/u \cdot C_m \) is a non-dimensional form of the mass flux.
condition of the boundary layer to be monitored throughout.

The basic sampling tube (BST) of the FFID was placed 3-5 mm downstream of the LDA measuring volume. Fackrell & Robins (1982) were able to place their BST much closer to their cross wire, because of a much smaller BST arrangement. The BST used in the present case is encased in a calibration tube which allows for the delivery of calibration gas at the BST tip. The setup is similar to the cold wire setup shown in Fig. 3.3.

The operating parameters for the FFID's that were used for the majority of the measurements are shown in Fig. C.1 (p. 133). The BST length was 250 mm with an inside diameter of 0.25 mm. With the selected vacuum pressure, the calculations indicate a transit time of roughly 27 ms. The FFID measurements could not be synchronised directly with the LDA measurements because of the long transit time. When a velocity measurement is made the air packet must first travel up the sampling tube before the equivalent concentration measurement can be made. During the travel time no further velocity measurements can be made, due to the constraints of the electronic circuitry. Direct synchronisation would therefore mean that the maximum sampling rate of the LDA would be around 40 Hz which was unacceptable. A new procedure was developed whereby the FFID measurements were taken at a sampling frequency ≥ 1000 Hz, and the LDA samples were later matched with the corresponding FFID samples to calculate the fluxes. The theoretical value of ≈27 ms corresponds well with the maximum correlation between vertical velocity and concentration shown in Fig. 5.2. The final method of calculating concentration fluxes was to take the maximum correlation for lag times between 20 and 30 ms. The peak in the correlation is not very sharp and the errors are relatively small when varying the lag time by ±5 ms from the maximum correlation value.

![Figure 5.2: Variation of $\overline{w'd'}$ with lag time](image)

Mean concentration profiles are normalised by $C_m$, the maximum concentration at the appropriate downstream location. This makes measurements insensitive to calibration errors, although calibration drift during a vertical profile is not compensated for. This method of normalising also compensates to some extent for plume drift from the centreline when only centreline profiles are measured. It is however not always possible to obtain a good estimate of $C_m$, especially for shallow ground level plumes where measurements close to the ground are restricted. The reflected Gaussian plume distribution predicts a very high concentration at ground level for shallow plumes, which is not always observed for the rough wall boundary layers simulated.
5.1 Ground level source

The source shown in Fig. 5.1 was lowered to the ground to form the ground level source. The volumetric release rate for the source was adjusted to match the boundary layer velocity at an arbitrary height of 50 mm. This corresponds roughly to the top of the roughness sub-layer.

A ground level source in an undisturbed boundary layer is a good first experiment to validate the experimental technique. The ground level acts as a reflecting surface throughout the lifetime of the plume, and a self-similar form is expected. Elevated sources on the other hand remain unaffected by the ground initially, and if sufficiently close to the ground slowly develop to exhibit ground level plume behaviour.

Profiles of \( \overline{w'c'} \) can be made to collapse for a ground level release in neutral flow, but not for ground level releases in stable flows since the eddy transfer coefficients are not linear functions of \( z \). The stability also increased with downstream distance, and \( K_m/K_z \) probably does not stay constant with changing stability as is the case in the atmosphere. In the atmosphere, the ratio of the transfer coefficients decreases with increasing stability before becoming approximately constant once the stability becomes moderate to strong. It is the first part of this behaviour that is relevant here, when the stability is modest.

It is not immediately apparent why the profiles of \( \overline{w'c'} \) should collapse in the neutral boundary layer. It can be shown that this condition only holds in the surface layer similarity region.

The eddy diffusivity of momentum \( K_m \) is defined as

\[
K_m = -\frac{\overline{w'w'}}{\partial u/\partial z}
\]  \hspace{1cm} (5.1)

In the surface layer

\[
K_m = \kappa z u_*,
\]  \hspace{1cm} (5.2)

The eddy diffusivity for vertical dispersion \( K_z \) can be estimated as a ratio of \( K_m \) so that

\[
K_z = \alpha_z K_m
\]  \hspace{1cm} (5.3)

with \( \alpha_z \approx 1.35 \) (Nieuwstadt & van Ulden 1978) in the atmosphere and \( \alpha_z \approx 1.25 \) (Packrell & Robins 1982) in the laboratory.

Therefore

\[
-\overline{w'c'} = \alpha_z \kappa u_* z \frac{\partial C}{\partial z}
\]  \hspace{1cm} (5.4)

The vertical coordinate \( z \) is normalised by the plume half height \( \delta_z \), and to write Eq. (5.4) in terms of the coordinate \( z/\delta_z \) we need

\[
\frac{\partial C(z)}{\partial z} = \frac{1}{\delta_z} \frac{\partial C(z/\delta_z)}{\partial (z/\delta_z)}
\]  \hspace{1cm} (5.5)

leading to

\[
\frac{\overline{w'c'}}{C_m u_*} = \frac{\alpha_z \kappa z \partial C/C_m}{\delta_z} \frac{\partial (z/\delta_z)}{\partial (z/\delta_z)}
\]  \hspace{1cm} (5.6)
Table 5.1: $\sigma_z$ and $\delta^2/C_m^2$ at different downstream locations

<table>
<thead>
<tr>
<th>$x/\delta$</th>
<th>Neutral $\delta_z$ [mm] $\delta^2/C_m^2$</th>
<th>1.3 m/s Stable $\delta_z$ [mm] $\delta^2/C_m^2$</th>
<th>1.1 m/s Stable $\delta_z$ [mm] $\delta^2/C_m^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.33</td>
<td>43</td>
<td>2.0</td>
<td>28</td>
</tr>
<tr>
<td>0.58</td>
<td>70</td>
<td>1.6</td>
<td>34</td>
</tr>
<tr>
<td>1.0</td>
<td>107</td>
<td>1.0</td>
<td>52</td>
</tr>
<tr>
<td>1.8</td>
<td>138</td>
<td>0.79</td>
<td>65</td>
</tr>
<tr>
<td>3.5</td>
<td>270</td>
<td>1.3</td>
<td>78</td>
</tr>
<tr>
<td>6.8</td>
<td>365</td>
<td>0.19</td>
<td>90</td>
</tr>
</tbody>
</table>

Table 5.1 shows the calculated values of $\delta_z$ and $\delta^2/C_m^2$ at the different measuring stations. Estimates of $\delta^2$ and $C_m$ are often inaccurate, especially when $\delta_z$ is small.

Fig. 5.3(a) shows the collapse of the mean concentration profiles at the downstream stations for neutral flow. The vertical coordinate $z$ is normalised by the plume half height $\delta_z$. Both $C_m$ and $\delta_z$ are determined by fitting the reflected Gaussian plume formula to the data. The Gaussian plume formula fits the data well enough and there is little justification to use the modified Gaussian plume formula used by Robins (1978).

In all cases and at all downstream positions the lowest measurement was at 34 mm due to restrictions imposed by the LDA probe diameter. For measurements close to the source the lowest measuring position may be above $\delta_z$. $C_m$ will then be more than double the highest measured concentration. Well mixed conditions exist in the roughness layer, and the plume exhibits a slightly higher spread than above the roughness layer. The fitted value of $C_m$ is therefore probably higher than the actual concentration at ground level. It is then surprising that such a good fit of the reflected Gaussian plume formula can be achieved since the Gaussian formulation relies on the concentration having a value of $C_m$ at ground level, the reflection surface.

Fig. 5.4(a) shows the mean concentrations for the 1.3 m/s stable flow, and Fig. 5.5(a) for 1.1 m/s stable flow. Very few measurements are actually available below $2\delta_z$ for 1.1 m/s stable flow because the plume remains so close to the ground. The Gaussian formula fits the data well enough for both stable cases.

Fig. 5.3(b) shows the concentration fluctuation measurements in neutral flow normalised by $\delta^2$, the maximum concentration fluctuation measurement at each downstream location. The profiles collapse quite well and the solid line represents the best fit through all of the data. The concentration fluctuation results for the stable cases are shown in Figs. 5.4(b) & 5.5(b). The collapse is less satisfactory than the neutral case. The solid lines represent the best fit through the neutral data. The neutral fit is also an acceptable fit for the 1.3 m/s stable data. The 1.1 m/s stable data also follows the neutral trend, given the sparse data and the difficulty in fixing $\delta^2$.

Fig. 5.3(c) shows the profiles of $\omega^2/c^2$ for neutral flow. The normalisation collapses the data quite well. The solid line is the theoretical curve Eq. (5.6) using the eddy viscosity hypothesis with $K_z = K_m$, i.e. $\alpha_z = 1$, and $\partial(C/C_m)/\partial\delta$ is calculated from the fitted Gaussian formula. For
Chapter 5. Experimental results II: Dispersion

$z < \delta_z$ a higher value of $K_z$ is more appropriate, which agrees with Fackrell & Robins (1982). For $z > \delta_z$ Fig. 5.3(c) suggests a lower value of $K_z$. $\delta_z$ is however a characteristic length of the plume and is therefore not appropriate for scaling $K_z$. The mixing length formulation used for the eddy viscosity can only be expected to be valid in the surface layer.

Fig. 5.4(c) shows the $\overline{w'c'}$ results for the 1.3 m/s stable flow and Fig. 5.5(c) the results for the 1.1 m/s stable flow. The solid line represents the calculated values of $\overline{w'c'}$ at $x/\delta = 6.8$. The calculated values use $K_z = K_m$ with $K_m$ derived from the measured values of $\overline{u'w'}$ and $\partial \overline{u'/\partial z}$ using Eq. (5.1). The calculated values follow the same trends as the measured values and are of similar magnitude. The location $x/\delta = 6.8$ was chosen to compare with the theory since it is the location with the most measurements $z/\delta < 3$ and hence useful for comparison. The characteristic nose observed in Fig. 5.3 for the neutral case is also present in the measured values and calculated values of the 1.3 m/s stable case, shown in Fig. 5.4(c). The nose is not apparent in Fig. 5.5(c) for the 1.1 m/s stable case, but from theoretical considerations the curves must return to zero at $z = 0$. The maximum value of $\overline{w'c'}/u_*C_m$ decreases with increasing downstream distance. This can be explained by $K_z(\delta_z)/\delta_z u_*$ decreasing with downstream distance under stable stratification as $\delta_z$ increases. $K_z(\delta_z)/\delta_z u_*$ stays constant in the surface layer for neutral stratification.

5.2 Elevated sources

All the concentration measurements for the elevated sources are collected in Appendix D (page 137). Some representative examples and cases exhibiting unexpected behaviour will be repeated here.
Figure 5.4: Stable 1.3 boundary layer, ground level source.

Figure 5.5: Stable 1.1 boundary layer, ground level source.
5.2.1 Mean concentrations

Figs. D.2–D.5 show the vertical mean concentration profiles for the elevated sources at $H_s = 100$ mm and $H_s = 150$ mm. The solid lines are the best fit reflected Gaussian plume curves.

Figs. D.2–D.4 show the vertical concentration profiles for the 100 mm source.

Figure 5.6: Centreline mean concentration $\overline{C}/C_m$, stable 1.1 m/s, $H_s = 100$ mm, undisturbed.

Fig. 5.6 demonstrates the typical behaviour of the plume released from the $H_s = 100$ mm source in the undisturbed boundary layer. The specific case is the 1.1 m/s stable boundary layer. The vertical spread is significantly less than the neutral case (Fig. D.2(a)) and slightly less than in the 1.3 m/s stable case (Fig. D.1(a)). The plume spread for the 1.1 m/s stable case at $x = 21H_s$ is similar to the plume spread of the neutral case at $x = 6H_s$, and $x = 11H_s$ for the 1.3 m/s stable case.

Figure 5.7: Centreline mean concentration $\overline{C}/C_m$, stable 1.3 m/s, $H_s = 100$ mm, 45° cube.

All the obstacle cases show the enhanced dispersion of the plume due to the wake of the obstacle. Qualitative comparisons of spread parameters can be made for elevated plumes, but for plumes that have reached the ground comparisons can only be made with the derived spread parameters. Fig. 5.7 shows the expected behaviour for the cube at 45° in that the plume is brought to ground level immediately. This is also the behaviour predicted by wake models typically used in dispersion modelling (Robins et al. 1997).
In Fig. 5.8 the initial behaviour of the plume is similar to the neutral flow where the plume is brought immediately to ground. Then it appears as if the plume lifts off the ground between 11.4 and 21.4 source heights downstream. This behaviour would have been attributed to changes in the source or the boundary layer, had it not also happened similarly for the 1.3 m/s stable case shown in Fig. D.1(b). A possible explanation is that the flow may have re-stabilised after the well mixed wake region, and that the plume has risen to its equilibrium height determined by the average density of the plume. Similar effects were observed by Start et al. (1980) during field experiments with smoke tracers released in the lee of a building at 45°. Away from the recirculating region a large portion of the plume appeared to be drawn upwards and then remained at approximately roof height.

Figs. D.3–D.5 show the vertical concentration profiles for the $H_s = 150$ mm source.

Fig. 5.9 demonstrates the typical behaviour of the plume released from the $H_s = 150$ mm source in the undisturbed boundary layer. The specific case is the 1.1 m/s stable boundary layer. The vertical spread is comparable to that for the lower source (Fig. 5.6). The plume remains aloft for both the stable cases for nearly the entire fetch represented by the results shown.
For all the cases investigated the vertical plume deflection resulting from the obstacle is visibly more for the cube at 45° than for the normal cube. For the stable cases the initial downward deflection is however not maintained, and small upward deflections are observed at $x \approx 14H_s$. The net effect of the 45° cube on dispersion in the stable flow with the source $H_s = 150$ mm is much less than in the neutral case. The reduced level of mean downward streamline deflection is particularly relevant as dispersion models, such as ADMS, frequently assume that building effects are not significantly changed by stability in the approach flow. This statement is generally based on the modest levels of obstacle Froude numbers likely in atmospheric flows around buildings. Whilst this is also true of the present work, the results still show significant effects on dispersion around and downstream from the cube.

### 5.2.2 Concentration fluctuations

Figs. D.8–D.11 show the vertical concentration fluctuation profiles for the elevated sources at $H_s = 100$ mm and $H_s = 150$ mm. The solid lines are the best fit curves to the data points, or in some cases, e.g. Fig. 5.12, only data points that followed the expected behaviour were considered for the fit.

The furthest downstream measuring station often displayed significant scatter in the results. The shapes of the concentration fluctuation profiles remain nearly Gaussian as long as the plume stays clear of the ground. This type of behaviour can be seen for the undisturbed 1.1 m/s stable case with $H_s = 100$ mm shown in Fig. 5.11.

It can be expected that the concentration fluctuation profile would start resembling the behaviour observed for the ground level source, shown in Fig. 5.3(b), as either the plume is brought to the ground, or when $\sigma_z \gg H_s$. For the 45° cube and $H_s = 100$ mm the plume is brought immediately to ground in the wake, and the concentration fluctuation profiles in Fig. 5.12 clearly resemble the ground level source profiles shown in Fig. 5.3(b).

### 5.2.3 Concentration fluxes

Results are only presented for the vertical concentration flux $\overline{w'c'}$. When a reference is made to concentration flux in the text it should be taken to mean vertical concentration flux.
Figure 5.11: Centreline concentration fluctuations $c'^2/c'^2$, stable 1.1 m/s, $H_s = 100$ mm, undisturbed.

Figure 5.12: Centreline concentration fluctuations $c'^2/c'^2$, stable 1.1 m/s, $H_s = 100$ mm, 45° cube.
Figs. D.14–D.17 show the vertical concentration flux profiles for the elevated sources at $H_s = 100$ mm and $H_s = 150$ mm. The solid lines are the best fit curves to the data points.

The concentration flux profiles must be interpreted by considering both the concentration gradient and the dependence of eddy diffusivities on height at a point. A number of common, though somewhat obvious, features can be observed for the concentration flux profiles. The vertical concentration flux is zero at the height of maximum concentration, i.e., where the concentration gradient is zero. The concentration fluxes in undisturbed boundary layers are higher above the plume height than below, which can be explained by the higher eddy diffusivities resulting from the greater mixing lengths. This observation and the mixing length argument remains true for the stable boundary layers at both observed heights.

Figure 5.13: Centreline concentration fluxes $\overline{w'd'u^*C_m}$, stable 1.1 m/s, $H_s = 100$ mm, undisturbed.

Figure 5.14: Centreline concentration fluxes $\overline{w'd'u^*C_m}$, neutral, $H_s = 100$ mm, undisturbed.

Fig. 5.13 shows the vertical concentration flux profile for the undisturbed, $H_s = 100$ mm and 1.1 m/s stable boundary layer case. Fig. 5.14 shows the same case in the neutral boundary layer. The magnitude of the fluxes is greatly reduced for the stable case. Take a simple case, on $y = 0$

$$C = C_m e^{-0.5 \left(\frac{x-h}{\sigma_z^2}\right)^2}$$

$$\overline{w'd'} = -K_z \frac{\partial C}{\partial z} = K_z \frac{z - h}{\sigma_z^2} C$$
Assume a constant \( K_z = \alpha u_i h \), then the maximum vertical concentration flux \( \bar{w}'c_m' \) occurs at

\[
\frac{\partial \bar{w}'c_m'}{\partial z} = 0
\]  
(5.9)

It can be shown that \( z - h = \sigma_z \) is the solution to Eq. (5.9). Then

\[
\bar{w}'c_m' = K_z \frac{C_m}{\sigma_z} e^{-1/2}
\]  
(5.10)

and

\[
\frac{\bar{w}'c_m'}{u_i C_m} = K_z e^{-1/2}
\]  
(5.11)

From Eq. (2.57)

\[
\sigma_z^2 = \frac{2K_z}{\bar{u}}
\]  
(5.12)

and hence

\[
\frac{\bar{w}'c_m'}{u_i C_m} = \sqrt{\frac{\alpha h \bar{u}}{2e \bar{x} u_i}}
\]  
(5.13)

It can therefore be seen that whether \( \bar{w}'c_m'/u_i C_m \) increases or decreases depends on how \( \sqrt{\alpha h \bar{u}} \) changes, and the results suggest that for the plume at 100 mm this decreases with stability.

![Graph](image)

**Figure 5.15:** Centreline concentration fluxes \( \bar{w}'c'/u_i C_m \), stable 1.1 m/s, \( H_s = 100 \) mm, normal cube.

Fig. 5.15 shows the vertical concentration flux profile for the \( H_s = 100 \) mm, 1.1 m/s stable boundary layer and normal cube case, and Fig. 5.16 for the 45° cube case. Once the plume is brought to ground by the cube at incidence material can only diffuse upwards, and hence the positive concentration flux so close to the source. There is a definite negative concentration flux at \( x = 21.4H_s \) which coincides with the apparent plume lift-off seen in Fig. 5.8.

Fig. 5.17 is typical of all three cases with \( H_s = 150 \) mm and with the cube normal to the flow. A very high negative concentration flux is observed for all three cases close to the obstacle. The mean concentration profiles, e.g. Fig. D.5(b), show nothing apparently special. The high concentration fluxes occur close to the recirculation region, characterised by rapid mixing and high transfer coefficients, together with a high gradient due to the narrow plume. As expected the conditions are very specific to the region and hence the high fluxes do not persist.
Figure 5.16: Centreline concentration fluxes $\overline{w'd}/u_* C_m$, stable 1.1 m/s, $H_s = 100$ mm, 45° cube.

Figure 5.17: Centreline concentration fluxes $\overline{w'd}/u_* C_m$, stable 1.1 m/s, $H_s = 150$ mm, normal cube.
5.2.4 Spread parameters

The concentration measurements that were used to calculate the horizontal spread parameters $\sigma_y$ were made with the gas sampling and analysis desk described in § 3.3.3 (a). The 16 sampling tubes were mounted on a rake similar to Fig. 3.4. The rake was mounted on the traversing mechanism, and measurements to determine $\sigma_y$ were made at the original source height at each downstream location.

The horizontal spread parameters were estimated by fitting the Gaussian plume formula, Eq. (2.61) to the horizontal concentration profiles. The spread parameters, or in graphical form sigma curves as they are commonly referred to in Air Pollution Meteorology, are presented in dimensional form. In the atmosphere there is little need for dimensionless sigma curves.

Because of the different free stream velocities, the plumes in the different boundary layers will experience different advection speeds, and hence the diffusion times at the downstream locations will differ for each boundary layer. For short diffusion times the statistical theory discussed in § 2.3.2 (c) produces the following relations for the vertical and horizontal spread parameters.

$$\sigma_y = \sigma_y t$$

$$\sigma_z = \sigma_z t$$

$t$ is the diffusion time and the formulation $dt = dx/u$ is used. In a fully developed steady state boundary layer with the plume retaining its release height this simplifies to $t = x/u$. In neutral boundary layers $\sigma_y/u$ and $\sigma_z/u$ remain effectively constant at a fixed height with variation of the freestream velocity, and hence $\sigma_y \approx f(x)$ and similarly $\sigma_z \approx f(x)$. Neutral boundary layers with different freestream velocities should therefore produce similar sigma curves for short diffusion times. For long diffusion times the Lagrangian time scale of the flow also has a linear relationship to the freestream velocity and the sigma curves should also be independent of freestream velocity for long diffusion times in neutral boundary layers. For the same reason the sigma curves in stably stratified boundary layers will reflect the differences in turbulence levels and other effects of stability and not the differences in diffusion times.

Figs. 5.18-5.20 presents horizontal spread data for the flow cases studied. It is immediately apparent that the magnitudes of the sigma curves in the neutral boundary layer are much higher than in the stable boundary layers. That is largely due to the reduced turbulence levels in the stable boundary layers.

The neutral boundary layer results in Fig. 5.18 show that source height has little effect on the horizontal plume spread in the undisturbed boundary layer. The cube normal to the flow also has a relatively small effect, since the plume remains largely at the source height and is not greatly influenced by the wake. With the cube at 45° the plume is carried downwards into the wake by streamline deflection and large effects on lateral spread are seen, particularly for the roof level source.

Both stable boundary layers in Figs. 5.19-5.20 show slightly higher lateral plume spread for the $H_s = 150$ mm source than for the $H_s = 100$ mm source in the undisturbed condition, and the transition from near field behaviour occurs further downstream for $H_s = 150$ mm. This probably suggests larger time scales at $z = 150$ mm which dominate the reduction in lateral turbulence to give greater spread. For the roof level source the two obstacle orientations in the stable boundary layers have a similar effect on the lateral spread to that observed for the
neutral boundary layer, although slightly greater enhancement can be seen for the normal cube. This is probably because wake turbulence is a larger relative perturbation in the stable flow, especially evident for the 1.1 m/s stable boundary layer. The obstacles have no visible influence on the lateral spread for the release at 150 mm, even for the 45° cube. This can be attributed to the lack of streamline deflection, as seen before.

![Graphs](image)

**Figure 5.18:** Horizontal spread, neutral. (×: undisturbed, □: normal cube & ◦: 45° cube)

![Graphs](image)

**Figure 5.19:** Horizontal spread, 1.3 m/s stable. (×: undisturbed, □: normal cube & ◦: 45° cube)

The concentration measurements that were used to calculate vertical spread parameters were made with four FFID heads mounted on the traversing mechanism. The sampling tube tips were spaced ±50 mm apart. Two sets of measurements were made at each height, giving 8 measurements spaced ±25 mm apart, spanning ±90 mm either side of the tunnel centreline. This arrangement was chosen to allow for plume drift off the centreline. The maximum concentration at each height was then used to construct a vertical concentration profile, from which the vertical spread parameters were calculated by fitting the reflected Gaussian plume formula to the data. The advantage of using the FFIDs rather than the FID was that the measurements per vertical position could be completed in just over 10 minutes whereas the FID analysis time is at least 40 minutes. There was no need for the detail of 16 measurements
Figure 5.20: Horizontal spread, 1.1 m/s stable. (×: undisturbed, □: normal cube & ○: 45° cube)

at each vertical height offered by the FID, since the maximum concentration was all that was needed. The horizontal Gaussian plume formula could still be fitted to the FFID data, and the agreement with FID measurements was generally excellent.

Figs. 5.21–5.23 show the vertical spread parameters for all the cases investigated. The magnitudes of the neutral values are visibly higher than the stable values because of the relatively higher turbulence levels.

The vertical spreads for the neutral boundary layer shown in Fig. 5.21 are very similar for both release heights. The obstacle presence results in slightly enhanced spread in the near field, greatest for the lower source height, but otherwise little enhancement. This agrees with Hayden (1998) who found the key factor of building affected dispersion to be the reduction in plume height, and then the increase in lateral spread. Most dispersion models, e.g. ADMS and R91, always increase the vertical spread as well.

Somewhat larger vertical spread can be seen at $H_s = 150$ mm in the far-field for the undisturbed 1.1 m/s stable boundary layer, but little difference otherwise, or at short range. Turbulence levels in the 1.1 m/s stable boundary layer increase with height, and the length scale probably does as well, so these differences are not too surprising. The obstacle affected results in the stable boundary layers follow a similar trend to the horizontal spread, and the same arguments apply.

The statistical theory discussed in §2.3.2 (c) provides relations for plume spread in the near and far field. In terms of $x$ these relations are

$$\sigma_y \propto x$$

$$\sigma_z \propto x$$

in the near field and

$$\sigma_y \propto x^{1/2}$$

$$\sigma_z \propto x^{1/2}$$

in the far field. All the sigma curves for the neutral flow in Figs. 5.18 & 5.21 are predominantly linear, and hence exhibit near field behaviour throughout the region shown. The $\sigma_z$ curves for
the stable flows in Figs. 5.23 & 5.22 clearly show far field behaviour for most of the region investigated. The $\sigma_y$ curves for the stable flows in Figs. 5.20 & 5.19 however show a mixture of near and far field behaviour. The $\sigma_y$ curves for $H_s = 150$ mm and the undisturbed case with $H_s = 100$ mm mostly show near field behaviour, while the presence of the obstacle for $H_s = 100$ mm forces the horizontal plume spread into the far field.

![Graphs](image)

(a) $H_s = 100$ mm  
(b) $H_s = 150$ mm

**Figure 5.21:** Vertical spread, neutral. ($\times$ : undisturbed, $\Box$ : normal cube & $\diamond$ : 45° cube)

![Graphs](image)

(a) $H_s = 100$ mm  
(b) $H_s = 150$ mm

**Figure 5.22:** Vertical spread, 1.3 m/s stable. ($\times$ : undisturbed, $\Box$ : normal cube & $\diamond$ : 45° cube)

### 5.2.5 Centreline GLC's

Ground level concentrations were measured by placing the gas sampling tubes ~10 mm above the ground on the centreline downstream of the source. The FID gas sampling and analysis desk was used for all ground level concentration measurements. It is assumed that the plume has negligible lateral drift. The possible drift increases with downstream distance from the source, but the horizontal spread of the plume also increases and the error associated with making a measurement off the plume probably does not increase.
Ground level concentration measurements are normalised as
\[ \chi = C U H^2/Q \]  \hspace{1cm} (5.20)

where \( H \) is an appropriate length scale for the flow, here taken to be a nominal boundary layer height of 1 m. Ground level concentrations are dependent on the dimensions of the plume and on the height of the plume above the ground. Conclusions must therefore be drawn by considering both these quantities.

Fig. 5.24(a) compares the ground level concentrations of the different obstacle configurations for the neutral boundary layer with \( H_s = 100 \) mm. Both the normal cube and the 45° cube case have reached their maximum ground level concentrations at the first measuring position. The relatively higher concentration for the 45° case is due to the plume centreline experiencing greater deflection by the obstacle. The plume in the undisturbed boundary layer diffused to the ground much later and records a much smaller maximum ground level concentration compared with the other two cases.
Fig. 5.24(b) compares the ground level concentrations of the different obstacle configurations for the neutral boundary layer with $H_s = 150$ mm. Both the normal cube and the $45^\circ$ cube have reached their maximum ground level concentrations at similar positions downstream. The relatively higher concentration for the $45^\circ$ can again be explained on similar grounds as for the $H_s = 100$ mm case. The plume is diluted significantly by the time it reaches the ground, compared with the $H_s = 100$ case.

**Figure 5.25:** Ground level concentrations, 1.3 m/s stable. (x : undisturbed, □ : normal cube & ○ : $45^\circ$ cube)

**Figure 5.26:** Ground level concentrations, 1.1 m/s stable. (x : undisturbed, □ : normal cube & ○ : $45^\circ$ cube)

Fig. 5.25(a) and Fig. 5.26(a) show the ground level concentrations with the source at $H_s = 100$ mm for the 1.1 m/s and the 1.3 m/s stable boundary layers respectively. The behaviour is very similar to the neutral case. However, the ground level concentrations decrease very slowly beyond $x/H_s > 20$ for the two stable cases, unlike in the neutral cases.

Fig. 5.25(b) and Fig. 5.26(b) show the ground level concentrations with the source at $H_s = 150$ mm for the 1.1 m/s and the 1.3 m/s stable boundary layers respectively. The $45^\circ$ cube in the 1.1 m/s stable boundary layer shows an unexpected minimum following the maximum, after which the profile rises again and follows the normal cube profile. The observed increasing
trend may be experimental scatter and a similar behaviour to the 45° cube in the 1.3 m/s stable boundary layer may be more likely. The normal and 45° cubes in the 1.3 m/s stable boundary also have nearly constant ground level concentrations for $x/H_s > 20$.

The effects of stability and release height can be clearly seen with the undisturbed releases in the stable boundary layers. The ground level concentrations will be used for comparison with the dispersion model in Chapter 6.

### 5.2.6 Concentration probability density distribution

Appendix E shows the probability density distributions for 1.1 m/s stable flow and 2.5 m/s neutral flow for all the flow configurations that were simulated. Results are documented at the 2 m downstream station which translates to 13.33 source heights for the $H_s = 150$ mm source and 20 source heights for the $H_s = 100$ mm source.

In the figures shown $c'$ is the fluctuating component of concentration, $p(c')$ is the probability density function as defined in §2.1.3 (b), and $\sigma_c$ is the standard deviation. The calculated values of $\sigma_c/C$ are also shown on the plots where available. Note that $C + c' = 0$ when $c'/\sigma_c = -C/\sigma_c$.

Three representative examples of the common types of probability density distributions for concentration fluctuations (e.g. Fackrell & Robins 1982) are shown in Figs. 5.27–5.29. The corresponding values of $\sigma_c/C$ are shown in the captions. Fig. 5.27 shows an exponential PDF. An exponential-like distribution is expected when the plume is small compared to the local turbulence scales and free to meander vertically and horizontally, or near the plume edge, where the plume is highly intermittent. The parameter that matters is $z_p/\sigma_z$, where $z_p$ is the local plume height. When $z_p/\sigma_z$ is large then highly intermittent plumes, with high levels of concentration fluctuations and exponential PDFs result. When $z_p/\sigma_z$ is small then intermittency is only apparent at the plume edges, fluctuation levels are modest, $c'/\sigma_c < 1$, and the PDF is Gaussian-like. Fig. 5.29 shows a probability density distribution that is approaching a Gaussian distribution. Fig. 5.28 shows a clipped normal distribution, indicating the transition between the scenarios described above.

Fig. E.2 shows the variation with height of the probability density distributions for the undisturbed stable 1.1 m/s stable boundary layer with $H_s = 100$ mm. An exponential probability density distribution can be observed for all the heights, apart from $z/H_s = 0.05$ where the mean concentration is close to zero. The same configuration for the neutral boundary layer in Fig. E.3 shows similar distributions above the source height, but below the source height the probability density distribution gradually assumes a Gaussian form. The values of $\sigma_c/C$ for the stable flow remain $\sigma_c/C \approx 2$ even through the core of the plume, while $\sigma_c/C \approx 1$ for the neutral flow.

For the two obstacle cases with the source at $H_s = 100$ mm the stable, Figs. E.4 & E.6, and neutral, Figs. E.5 & E.7, results are very similar. The plume dimensions are very different, but the probability density distributions do not seem very sensitive to the plume dimensions below roof height. The behaviour of $\sigma_c/C$ is also similar for all the obstacle cases with $H_s = 100$ mm. $\sigma_c/C \approx 0.5$ for $z < H_s$, showing how the building limits the meandering of the plume. The reduced plume dimensions in the stable flow lead to higher values of $\sigma_c/C$ above the roof of the building.

The results are similar for the $H_s = 150$ mm source. Probability density distributions could
Figure 5.27: Concentration probability density distributions for 1.1 m/s stable flow, undisturbed, $H_s = 100$ mm, $z/H_s = 1.5$, $x/H_s = 20$, $\sigma_c/C = 2.6$

Figure 5.28: Concentration probability density distributions for 1.1 m/s stable flow, 45° cube, $H_s = 100$ mm, $z/H_s = 1.15$, $x/H_s = 20$, $\sigma_c/C = 0.78$
not be calculated for the stable undisturbed case at \( z/H_s = 0.033 \) and \( z/H_s = 0.33 \), shown in Fig. E.8, because negligible concentration was measured. The undisturbed neutral flow in this case has values of \( \sigma_c/C \simeq 2 \) through the core of the plume compared to \( \sigma_c/C \simeq 1 \) for the \( H_s = 100 \text{ mm} \) undisturbed neutral case. This is due to the higher \( z_p/\sigma_z \) ratio for \( H_s = 150 \text{ mm} \).

Figs. E.10–E.13 show that the presence of the obstacles tends to even out the differences between the stable and neutral results. A slight difference between the neutral and stable results can be detected at \( z/H_s = 0.33 \), where the probability density distributions for the stable flow have higher peaks than the equivalent neutral distributions. The presence of the cube again leads to lower values of \( \sigma_c/C \), especially below the obstacle height. The values of \( \sigma_c/C \) for the stable obstacle cases are consistently a fraction higher than the neutral cases. The ability of the obstacle to restrict the meandering of the plume above the obstacle height seems much reduced for the stable cases.
The theoretical concepts introduced in §2.3.2, §2.4 and Appendix G were implemented in a dispersion model to test against the wind tunnel data. The advantage of this over using ADMS itself is that all the calculations can be done at wind tunnel scale, and greater control can be exercised over the specification of boundary layer characteristics. The study does however not aim to provide an exhaustive investigation into dispersion modelling. The dispersion model is based on the plume spread and building effects modules in ADMS, described in Carruthers et al. (1999) and Robins et al. (1997). The model implemented here is specifically aimed at the cases investigated experimentally, and hence only stable and neutral stratification are modelled. The building effects model is described in sufficient detail in Appendix G. The current implementation of the building effects model is not an exact replica of the ADMS implementation. A slightly different wake model is used by ADMS, but the dispersion modelling in the wake was implemented according to the ADMS technical specification (Robins & Apsley 2000).

A general observation that can be made from the vertical concentration profiles in Appendix D is that the profiles for the obstacle cases are generally quite well fitted by a Gaussian profile, although the wind shear and eddy diffusivity vary considerably throughout the plume. The validity of the dual plume structure assumed by the building effects model is therefore questionable. The ground level concentrations shown here are however not influenced a great deal by the dual plume assumption, since the plume is on the centreline of the obstacle and remains in the main wake region.

6.1 Results

A comparison is made between the experimental and calculated ground level concentration results. The experimental ground level concentration results are also presented in §5.2.5, Figs. 5.24–5.26. The modelling results were obtained by using the actual measured profiles of wind speed, turbulence intensities and temperature where applicable. The first set of modelling results presented in §6.1.1 use the ADMS plume spread formulations Eqs. (2.88) & (2.85). The model was then adapted to use the experimental plume spread results from the undisturbed cases. The building effects model could therefore be isolated to some extent. The results from the isolated building effects modelling are shown in §6.1.2.
6.1.1 Full modelling

Fig. 6.1 compares the measured and calculated ground level concentrations with the source at \( H_s = 100 \) mm and the flow undisturbed. The positions of the maximum ground level concentration for all the stability cases are predicted to be closer to the source than the measurements show. This can be explained by an over prediction of plume spread by the dispersion model. The solution for 1.3 m/s stable flow matches the experimental results for the neutral case better than the neutral solution does, and the 1.1 m/s stable flow modelling results also fit the the experimental results for the 1.3 m/s case better than the 1.3 m/s stable flow modelling results do. It is therefore clear that either or both of Eqs. (2.85) & (2.88) over-predict \( \sigma_z \) as a function of \( \sigma_w \) and \( \sigma_y \) as a function of \( \sigma_v \) respectively. Factors that influence the ratio of \( \sigma_z \) to \( \sigma_w \) are the magnitude of the constant \( b \) in Eq. (2.79) and the interpolation formula Eq. (2.86). It is possible that wind shear may be influencing dispersion in the wind tunnel above \( z/\delta = 0.15 \), the limit imposed by Eq. (2.86). Eq. (2.85) applied to neutral flows uses the statistical (or Lagrangian similarity) relation for short diffusion times even for long diffusion times. For stable flows it was also shown in Eqs. (2.91) & (2.92) that the factor \( \gamma \) seems to have been ignored in Eq. (2.85).

\[
\frac{C_m \bar{u} H^2}{Q} = \frac{2}{\pi \sigma_z} \left( \frac{H}{H_s} \right)^2 \sigma_x \quad (6.1)
\]

\[
\sigma_z(x_m) = \frac{H_s}{\sqrt{2}} \quad (6.2)
\]

Figure 6.1: Comparison of ground level concentrations between experimental and modelling results. \( H_s = 100 \) mm, undisturbed.

Fig. 6.2 compares the measured and calculated ground level concentrations with the source at \( H_s = 150 \) mm and the flow undisturbed. The more rapid plume spread predicted by the dispersion model can be clearly seen for the two stable cases. The dispersion model also predicted that the neutral plume disperses more quickly than the experimental results show. \( H_s/\delta \geq 0.15 \) for these cases and hence wind shear was ignored.

It can be shown that simple Gaussian plume model predicts that the maximum ground level concentration \( C_m \) and its position, \( x_m \), are given by

\[
\frac{C_m \bar{u} H^2}{Q} = \frac{2}{\pi \sigma_z} \left( \frac{H}{H_s} \right)^2 \sigma_x \quad (6.1)
\]

\[
\sigma_z(x_m) = \frac{H_s}{\sqrt{2}} \quad (6.2)
\]
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So in the undisturbed flow we expect the ratio of $C_m$ for the 100 and 150 mm sources to be $(150/100)^2 \approx 2.25$, whereas we get a value slightly below 2 for the neutral case and probably about 2 for the 1.3 m/s stable boundary layer. Note that the somewhat more sophisticated modelling in ADMS provides a better prediction of these ratios than the simple Gaussian model.

Figs. 6.3 & 6.4 considered the influence of the factor $\gamma$ by modifying Eq. (2.85) to include it. Eq. (2.85) then becomes

$$\sigma_z = \sigma_w t \left\{ \frac{1}{b^2} + \frac{N^2 t^2}{\gamma^2 (1 + 2Nt)} \right\}^{-\frac{1}{2}}$$

A value of $\gamma = 0.58$ was chosen to fit the $H_z = 150$ mm data. The recommended range of $0.1 \leq \gamma \leq 0.4$ underpredicts the plume spread by a considerable margin. The chosen $\gamma$ fits the
data for $H_s = 150$ mm in Fig. 6.4 quite well, but a slightly smaller value for the $H_s = 100$ mm cases in Fig. 6.3 would have been more appropriate. The neutral predictions are unaffected by $\gamma$. $\gamma$ was neglected, i.e. $\gamma = 1$, for the obstacle cases.

The performance of the standard model in neutral conditions is within the expected accuracy (i.e. within a factor of two) for building effects models and is probably just about acceptable as a plume model (i.e. without buildings). However, it is not a convenient starting point for evaluating building effects models. The standard model is poor when applied to stable flows. Clearly the plume spread algorithm does not represent the behaviour in the tunnel. This in part is because it is designed for a certain idealised class of stable boundary layers and the tunnel flows (particularly the 1.1 m/s flow) depart from this ‘model’. However, it is clear from the predictions with $\gamma = 0.58$ that the standard model can be ‘tuned’ for better
performance. This is not an unreasonable value, though higher than the generally given range of 0.1 to 0.4 but better than the value $\gamma = 1$ used by ADMS. Reducing $\gamma$ decreases the vertical plume spread ($\sigma_z \sim \gamma^{1/2}$ in the far field) and reduces ground level concentrations. The whole question is avoided once empirical plume spread data is used, as in the modified predictions in §6.1.2.

Fig. 6.5 compares the measured and calculated ground level concentrations with the source at $H_s = 100$ mm and the cube normal to the flow. For the neutral flow the model predicts the near and far field dispersion to the expected accuracy. In general, the estimate of concentration in the recirculation region is expected to be the least reliable (i.e. has the largest uncertainty attached). The modelling and parameterisations used are a compromise, sometimes leading to over-prediction, and sometimes under-prediction. For the stable flows the over-prediction of the concentrations in the recirculation region is larger than would generally be expected. Predictions downstream of this region are generally acceptable.

Fig. 6.6 shows the measured and calculated ground level concentrations with the source at $H_s = 150$ mm and the cube normal to the flow. The model correctly predicts zero concentration in the recirculation region. The modelled concentrations show the correct behaviour for the neutral case, but the results suffer from the over prediction of plume spread. For the stable flows immediately downwind of the recirculation region the model considerably over-predicts, with the maximum ground level concentrations much too close to the cube. Concentrations in the far field are within a factor of two of the observations but with the wrong trend.

We know that there was little or no mean streamline deflection observed for the normal cube. None is modelled over the building and recirculation region in ADMS, so the comparisons shown in Figs. 6.5 & 6.6 are not affected by this part of the algorithm. Further downwind, streamline deflection is modelled as a result of entrainment into the decaying wake. This is treated in the same way for all stabilities in ADMS, with wake decay proceeding more slowly in stable conditions because of the reduced eddy-diffusivity (proportional to $u_*$) in the flow. Predictions for the neutral and 1.3 m/s stable flows are acceptable (i.e. within the expected accuracy of this type of modelling) for both source heights. They are also acceptable for the 100 mm source in the 1.1 m/s stable flow, but not for the 150 mm source. In the latter
case, the maximum ground level concentration occurs far too early, at around $x/H_s = 5$. The reason for this is not obvious, but is probably a combination of excessive streamline deflection over the initial part of the near wake and the turbulence structure assumed in the wake. In discussing Fig. 5.8 we note that other processes leading to 'plume rise' may also play a role, though it is hard to construct a fully self-consistent story across all the cases studied. We should also remember that the turbulence profiles for the 1.1 m/s stable flow are not 'classical', suggesting that we should pay more attention to the 1.3 m/s stable flow, noting the results for 1.1 m/s as indicators of further work to be carried out.

![Figure 6.7: Comparison of ground level concentrations between experimental and modelling results. $H_s=100$ mm, 45° cube.](image)

**Figure 6.7:** Comparison of ground level concentrations between experimental and modelling results. $H_s=100$ mm, 45° cube.

Fig. 6.7 presents the measured and calculated ground level concentrations with the source at $H_s = 100$ mm and the cube at 45° to the flow. The model performs reasonably well with the neutral flow, except at intermediate range, where it under-predicts by up to a factor of 2 to 3. For the stable flows the model over-predicts recirculation region concentrations by at most a factor of 2 (1.1 m/s stable flow) but performs poorly elsewhere.

Fig. 6.8 compares the measured and calculated ground level concentrations with the source at $H_s = 150$ mm and the cube at 45° to the flow. The modelled maximum ground level concentration for the neutral flow occurs at too short a fetch, which is a general fault of this type of modelling. The stable flow modelling correctly predicts zero concentration in the recirculation region. Immediately downwind the model considerably over-predicts, with the maximum ground level concentrations much too close to the cube. Concentrations in the far field are predicted within a factor of 2 to 3.

### 6.1.2 Isolated building effects modelling

The model was adapted to use the experimental plume spread results from the undisturbed cases. This was done by reading the experimental plume spread results for the undisturbed case into the dispersion model, and using spline interpolation routines to calculate derivatives and values at other locations. The building effects model could therefore be isolated to some extent. Figs. 6.9 & 6.10 show the modelling results for the undisturbed cases. The ground level concentration measurements extend further than the experimental $\sigma$ curves were available,
Figure 6.8: Comparison of ground level concentrations between experimental and modelling results. \( H_s = 150 \text{ mm}, 45^\circ \text{ cube.} \)

and \( \sigma_x \) & \( \sigma_y \) were specified to proceed as \( \sigma_x \propto x^{1/2} \) and \( \sigma_y \propto x^{1/2} \) from the last experimental values of \( \sigma_x \) & \( \sigma_y \). The calculated ground level concentrations are uniformly good, at most over-predicting by a factor of 1.3. Small degrees of plume rise could account for some of the differences between the calculated and measured values. The specification of \( \sigma_x \propto x^{1/2} \) and \( \sigma_y \propto x^{1/2} \) for \( x > 3000 \text{ mm} \) is generally an appropriate assumption, although the transitions are not always smooth. The undisturbed results with the modified model provide a better starting point than the standard model for evaluating the building effects model.

Figure 6.9: Comparison of ground level concentrations between experimental and modelling results. \( H_s = 100 \text{ mm}, \text{ undisturbed with specified } \sigma_y \text{ } \& \text{ } \sigma_x \text{ profiles.} \)

Fig. 6.11 shows the results with the source \( H_s = 100 \text{ mm} \) and the cube normal to the flow. The neutral case shows better agreement with the modified model than was seen with the standard model. The predicted near wake concentrations are too high by a factor of two or more. The 1.3 m/s stable boundary layer shows good agreement between the measured and
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Figure 6.10: Comparison of ground level concentrations between experimental and modelling results. $H_s=150$ mm, undisturbed with specified $\sigma_y$ & $\sigma_z$ profiles.

Figure 6.11: Comparison of ground level concentrations between experimental and modelling results. $H_s=100$ mm, normal cube with specified $\sigma_y$ & $\sigma_z$ profiles.

Fig. 6.12 compares the measured and calculated results for the $H_s = 150$ mm source and the cube normal to the flow. The results for the neutral boundary layer agree quite well, showing some improvement over Fig. 6.6. The stable predictions still suffer from the same faults as the standard model, but to a noticeably lesser degree.

Fig. 6.13 shows the results for $H_s = 100$ mm and the cube at $45^\circ$. The neutral predictions agree well with the experimental values, and are clearly better than the standard model. The concentration in the near wake is also reasonably accurate for the neutral flow, and shows a smooth transition to the ground level predictions downstream of the recirculation region. The near wake concentration estimates for the stable boundary layer are slightly less satisfactory.
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Figure 6.12: Comparison of ground level concentrations between experimental and modelling results. $H_s=150$ mm, normal cube with specified $\sigma_y$ & $\sigma_z$ profiles.

than the standard model. Further downstream the stable results in Fig. 6.13 show that the modified model performs better than the standard model.

Figure 6.13: Comparison of ground level concentrations between experimental and modelling results. $H_s=100$ mm, 45° cube with specified $\sigma_y$ & $\sigma_z$ profiles.

Fig. 6.14 compares the dispersion modelling results for $H_s=150$ mm and the cube at 45° with the measured ground level concentrations. Again the neutral predictions and measurements agree well. The modelled maximum ground level concentration has a sharper peak than the experimental curve shows. The modified model performs much better for the 1.3 m/s stability, but still predicts the maximum ground level concentration too close to the cube. Performance is less satisfactory for 1.1 m/s stable boundary layer, particularly at intermediate range.

The isolation of the building effects model has proved that the building effects model generally works well for neutral flows. The calculation of the concentration in the near wake is very sensitive to the plume spread in the surrounding region, and a suitable model needs to be
found to account for enhanced mixing in this region. The isolated building effects results for the stable flows show the correct trend, but modifications need to be made to the plume spread modelling in the wake to account for stability.

Figure 6.14: Comparison of ground level concentrations between experimental and modelling results. $H_s=150$ mm, $45^\circ$ cube with specified $\sigma_y$ & $\sigma_z$ profiles.
Conclusions & Recommendations

This study's aim was to investigate the effects of stratification on the flow and dispersion around obstacles. The investigation was primarily conducted in a wind tunnel, where the flow and dispersion were investigated for two source configurations, three boundary layers, and three obstacle configurations. The results are intended for the general evaluation of applied dispersion models and specifically for identifying deficiencies in the treatment of building wakes in stratified flows.

7.1 Conclusions

A significant amount of knowledge has been gained on the physical modelling of atmospheric boundary layers. A thorough knowledge of all the instrumentation systems that have been used for the study was acquired. The experimental procedures that were developed or refined will also benefit future workers in the laboratory. The procedure for simulating neutral and stably stratified boundary layers in the EnFlo environmental wind tunnel is now well established. A range of neutral and stably stratified boundary layers with similar characteristics to the atmospheric boundary layer can be set up. The level of automation of experiments with the stably stratified boundary layer is also now comparable to that of the simpler neutral boundary layer. More detailed measurements can now be completed in less time. Three stably stratified boundary layers were characterised with nominal freestream velocities of 1.5, 1.3 and 1.1 m/s, of which the 1.1 and 1.3 m/s ones were selected. The estimates of the Monin Obukhov lengths of the 1.1 & 1.3 m/s boundary layers were respectively lower and higher than the height of the obstacle that was to be investigated. One neutral boundary layer with a freestream velocity of 2.5 m/s was characterised and used throughout.

The effects of stability on the pressure distribution on a 100 mm cube were examined. As expected the differences are small, but still evident. A weaker separation region on the roof and a weaker wake have been observed for the 45° stably stratified case, whereas the normal flow case showed little difference between neutral and stable stratification. Limited pressure measurements have been performed with different stabilities, and it appears as if the weaker roof separation region may exist in the normal flow orientation for stronger stabilities. The moment coefficient is an important parameter characterising the strength of the wake, and is used in building wakes models for dispersion modelling. It can be concluded that the values used for neutral flow are equally valid for the stably stratified flows studied.
A simple three-dimensional wake model has been derived in Appendix F. The longitudinal perturbation velocity has been solved numerically for \( n \), the power law coefficient of the approach velocity profile, greater than zero. Previous workers (e.g. Apsley 1988, Hayden 1998) have only considered \( n = 0 \). It has been confirmed that the errors associated with assuming \( n = 0 \) even for \( n = 0.3 \) are very small. The solution of the vertical perturbation velocity become arduous for \( n > 0 \) and the small gain in accuracy does not warrant the added complexity.

The flow around a 100 mm cube normal to the approach flow was investigated. All the measurements were made downstream of the recirculation region, which is the region where stability is likely to have an appreciable effect. It was difficult to distinguish the wake beyond 10 building heights due to relatively high experimental scatter. Important trends can however be clearly identified. The longitudinal and vertical perturbation velocities normalised by the approach velocity at the obstacle height showed very little dependence on stability. The wake model estimates the eddy viscosity for the perturbation velocities as a factor of the eddy viscosity at the obstacle height in the approach flow. This results in a solution of the perturbation velocities depending on \( u_+ \). \( u_+ \) is strongly dependent on stability, but the experimental perturbation velocity results do not reflect this dependence on \( u_+ \). The perturbation Reynolds stresses for the neutral and the two stable boundaries also have very similar magnitudes when normalised by \( U_f^2 \), apart from \( w^2 / U_f^2 \) which showed a weak dependence on stability. The wake model also predicts the perturbation Reynolds stresses to be dependent on \( u_+ \), which the experiments do not reflect.

The implementation of the routines to calculate autocorrelations and spectra from LDA data has proved an important asset in the analysis of flow measurements. The LDA is the primary instrument for velocity measurements in the EnFlo wind tunnel, and previously spectrum and autocorrelation calculations were not possible with this instrument. In the cases that can be simulated the effects of stratification on the flow are not large, and the relevant data must be analysed as thoroughly as possible. Spectra have been measured for the neutral and for the 1.1 m/s stable flows with and without the cube The results show that for neutral flow the presence of the cube causes the turbulent length scales of the \( u \) and \( w \) velocities measured at the cube height to decrease relative to conditions in the appropriate undisturbed flows. The turbulent length scales for the 1.1 m/s stable case behind the cube only showed a decrease relative to the undisturbed flow for the \( u \) component of turbulence, and a slight increase was detected for the \( w \) component of turbulence. This is due to the reduced vertical turbulent length scales in the stable flow.

Extensive concentration measurements were performed with a conventional flame ionisation detector (FID), and with several fast response FIDs (FFID). The first attempt at measuring LDA velocities and concentration fluctuations concurrently was made. The sugar based seeding for the LDA did not affect the FFIDs significantly if the sampling tubes were cleaned regularly. The technique has proved a viable solution to measuring concentration fluxes.

The first dispersion case that was investigated was a ground level source in the three boundary layers. Mean concentrations, concentration fluctuations and concentration fluxes were calculated. The results for the neutral case showed good collapse of the vertical profiles of mean concentration, concentration fluctuations and concentration fluxes measured at different downstream locations when normalised appropriately. The Gaussian plume model provided a close fit to the mean concentration measurements. The concentration flux measurements also agreed with the theoretical profile calculated with an eddy diffusivity based on a mixing length. The agreement confirmed the validity of the concentration flux measurements, and
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hence the technique of calculating fluxes from simultaneous LDA and FFID measurements. The vertical profiles of mean concentration measurements showed good collapse for the two stable boundary layers when normalised appropriately. The Gaussian plume formula also provided a close fit. Due to the narrow plume in the stable boundary layers very few measurements were available close to the source. The concentration fluctuation profiles for the 1.3 m/s stable flow showed slightly different profile shapes at different downstream locations. The best fit to the neutral profiles did however provide an acceptable fit to the 1.3 m/s stable data. The narrow plume in the 1.1 m/s stable boundary layer resulted in too few data points that could be plotted for comparison. A meaningful conclusion could therefore not be drawn. The concentration flux profiles in the stable boundary layers do not collapse when normalised similarly to the neutral profiles. This is due to the eddy diffusivities in stable flow not being a linear function of $z$. The concentration fluxes were calculated at one downstream location using an eddy diffusivity based on the calculated eddy viscosities from the measured shear stresses. The calculated concentration fluxes showed good agreement with the magnitudes and shapes of the measured concentration flux profiles for both stable cases.

Elevated source plumes released from two heights were investigated. Three obstacle configurations were used: an undisturbed boundary layer, a 100 mm cube normal to the flow and the same cube at 45° to the flow. The obstacle was placed with its leading edge directly below the source. The results were presented as plots of the following: vertical profiles of mean concentration, concentration fluctuations and vertical concentration fluxes; vertical and lateral sigma curves; ground level concentration profiles on the centreline and probability density distributions. The ground level concentrations and the lateral spread were measured with the conventional FID thus giving an independent set of measurements. For the flow with the cube at 45° and the source at 100 mm, i.e. the obstacle height, the initial behaviour is similar to the neutral case for both stable flows. The plume is brought to ground by the strong downwash from the obstacle. After about 10 source heights the plume appears to lift off the ground for both stable flows. This phenomenon can also be observed in the ground level concentration measurements. For the 150 mm source the behaviour is also mostly as expected. The downwash from the 45° cube is however less pronounced for the two stable cases than for the neutral case. The concentration fluctuation profiles have Gaussian like forms for elevated plumes and approach the profile that was measured for the ground level source as the plume behaviour changes to that of a ground based plume. The shapes of the concentration fluctuation profiles are governed by the plume dimensions and the proximity of the plume to the ground, and any direct effects of the obstacles or stability are small. The concentration flux profiles also reflect the gradients of the mean concentration profiles, the proximity of the plume to the ground and the local eddy diffusivities in the boundary layer. The highest absolute value of vertical concentration flux for each of the boundary layers was observed for the 150 mm source with the cube normal to the flow, measured at the first station close to the cube. This region is characterised by high transfer coefficients due to the near wake, and a high mean concentration gradient due to the narrow plume. These conditions do not persist, which can be seen from the measurements at the next measuring station downstream. The influence of obstacle orientation with the source at obstacle height on the horizontal plume spread was found to decrease with increased stability. The $\sigma_y$ curves for the neutral flow showed the normal cube and the undisturbed curves to lie close together, with the 45° cube showing enhanced spreading. With increasing stability the $\sigma_y$ curve for the normal cube case moved closer to the 45° cube curve. For the 100 mm source near constant values were observed for $\sigma_y$ in the 1.1 m/s stable boundary layer from 10 building heights onwards regardless of the obstacle configuration. The trend towards a constant $\sigma_z$ is less clear for the 1.3 m/s stable flow, but still evident. The plume behaviour depicted in the vertical
mean concentration profiles and in the sigma curves can also be observed in the ground level concentration plots. The measurements therefore form a consistent set.

The probability density distributions were also plotted for the combinations of two boundary layers, the two source heights and the three building configurations. The neutral and the 1.1 m/s stable boundary layers were selected. Comparisons were made at a single downstream location. Only qualitative comparisons were drawn. The three typical shapes of the probability distribution function of concentrations were observed, the exponential, the clipped normal and the normal distributions. The presence of the building had a big influence on the probability density distributions, and very similar distributions could be observed between the neutral and the stable boundary layers with obstacles. The distributions for the undisturbed stable flow were governed by the plume dimensions and an exponential distribution was prevalent throughout the plume. The undisturbed neutral flow showed the distributions assumed a more normal distribution closer to the ground.

A dispersion model based on the Cambridge Environmental Research Consultants Ltd. (CERC) ADMS 3 model was implemented in a computer program. Comparisons were made between the modelled and measured ground level concentrations. The model did not predict the ground level concentrations for the undisturbed cases very well, which was attributed to the over prediction of plume spread by the model. It is possible that the effects of wind shear on dispersion in the wind tunnel may extend above 0.15\(\delta\), the limit imposed by the model. For stable flows the factor \(\gamma\) which accounts for the degree of mixing between fluid elements in stable flows has been omitted in the plume spread relations. When \(\gamma\) was included in the dispersion modelling of the undisturbed stable flow, better agreement could be found by 'tuning' the value of \(\gamma\). A single value of \(\gamma\) was however not suitable for both source heights. The modelling of the obstacle cases was also attempted. The neglect of enhanced mixing in the surrounding region by the building effects model often results in too high concentrations in the near wake, especially for the stable boundary layers. The underlying Gaussian dispersion model then suffers from the same factors as the undisturbed modelling described above, resulting in an over-prediction of the plume spread further downstream. The building effects model was then isolated to some extent from the plume spread modelling by using the experimental plume spread relations obtained from the undisturbed cases. The results are encouraging, and good agreement was generally found for the neutral boundary layer. The wake model over predicted the plume spread for the stable flows, suggesting that modifications to the wake modelling are necessary to account for stability.

7.2 Recommendations for future work

The simulated stably stratified boundary layer is a very complex phenomenon and further fundamental investigations are necessary. The measurement of heat fluxes needs to be investigated. A new temperature bridge is available, and the technique used for measuring concentration fluxes can also be applied to measuring heat flux. The dependence of \(K_h/K_m\) and \(K_z/K_m\) on stability in the wind tunnel needs to be investigated further, with the emphasis on the applicability of wind tunnel measurements to the atmosphere and a comparison of the ratios of \(K_h/K_m\).

For this study the probability density distributions and the concentration flux profiles have only been evaluated qualitatively. Further analysis and comparison with the prediction schemes for concentration fluctuations need to be carried out. Closely tied to this are the
probability distribution functions and the intermittency which need to be calculated. Intermittency has not been analysed due to the high levels of noise present in the concentration signal. Further investigation is necessary to find an algorithm to predict intermittency reliably for signals with high noise levels.

The reduced streamline deflections experienced by the plumes released from the 150 mm source in the stable boundary layers need to be investigated closer to the obstacle. Detailed streamline measurements are needed to quantify the effect of stability for incorporation into dispersion models.

Sensitivity studies need to be done for dispersion models that use the simple wake model. The wake model also needs to be revisited to investigate the eddy viscosity formulation in the wake. The enhanced mixing experienced by the plume in the surrounding region needs to be investigated further to obtain a relation suitable for use in a building effects model. The validity of the relations used by ADMS for plume spread in wind tunnel flows also warrants further investigation. The isolation of the building effects model has proved a fruitful exercise, and modifications to the flow and dispersion modelling in the wake to account for stability require further investigation.


URL: [http://www.hq.nasa.gov/office/pao/History/SP-440/contents.htm](http://www.hq.nasa.gov/office/pao/History/SP-440/contents.htm)


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URL: http://www.netlib.org/dierckx/index.html


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Smith, F.B. (1957), 'The diffusion of smoke from a continuous elevated point-source into a turbulent atmosphere', *J. Fluid Mech.* 2, 49-76.


Wilson, D.J., Robins, A.G. & Fackrell, J.E. (1982b), 'Predicting the spatial distribution of concentration fluctuations from a ground level source', Atmos. Envir. 16(3), 497-504.


The computer based analysis tools that have been used during this study are described briefly. This may be of help to other researchers in choosing an appropriate analysis tool. Extensive use have been made of the scripting language Perl (Wall et al. 1996). Perl is particularly well suited to processing text files such as the results files produced by the LabVIEW measurement programs. Perl is however quite slow for mathematical calculations, and where significant mathematical calculations had to be performed programs were written in C (Kernighan & Ritchie 1988).

Boundary Layer Analysis

An efficient utility has been developed to assess and analyse a boundary layer quickly. It consists of a Microsoft Excel spreadsheet and macros that rapidly analyse profiles of velocities and turbulence characteristics wherever appropriate. Separate versions were developed for neutral and stable boundary layers.

Similar spreadsheets with supporting macros that have been developed to assist the analysis of dispersion data will not be shown here.

Apart from the turbulence quantities that are measured there are indicators to determine the quality of the measurement and the quality of the flow during the measurement. These indicators are plotted on the datasheets, and are:

1. **Number of samples.** The number of samples per point is a good indication of whether a good average could be obtained for the turbulence parameters measured. Values $> 10000$ are acceptable, but the higher the better.

2. **Chilled water (CW) temperature.** Because of the open circuit configuration of the wind tunnel, the ambient air temperature in the laboratory can be affected as a result of the wind tunnel running. The air at the exit of the wind tunnel can be cooled by a heat exchanger. The water supply temperature in the heat exchanger loop can be controlled by mixing chilled water with the loop water, and thus maintaining a constant temperature. The basic chilled water supply is however unreliable at times, and this parameter gives an indication of the temperature of the water in the loop. In
the stable configuration this parameter also indicates the temperature of the floor. If
this parameter is different from the setpoint, the velocity profile should be discarded.

3. **Average heater temperature.** There are 15 thermocouples at the inlet of the wind
tunnel, each situated on a horizontal plane with a heater tube. The average of these
thermocouples gives a good indication of the ambient temperature in the working section
during a measurement. In the stable configuration this parameter gives the average of
the air temperature after passing through the heater bank. If this parameter is different
from the setpoint for the stable boundary layer the profile should be discarded.

4. **Ratio of ultrasonic anemometers.** There are two ultrasonic anemometers in the
EnFlo wind tunnel. One is located in the freestream and the other one can be positioned
as required. For all the profiles presented here the second one was mounted to measure
at a position of \( y = 500 \text{mm}, z = 100 \text{mm} \) and \( x = 11 \text{ m} \). Due to the variability of the flow
in the EnFlo wind tunnel the ratio of the two ultrasonics is not always constant. This
ratio can be helpful in assessing the flow quality or when comparing profile differences.

The theoretical profiles for \( \frac{U}{U_{\infty}} \) in Figs. A.1 & A.2 do not use the actual calculated param-
eter for a boundary layer, but standard values of \( u^*/U_{\infty} = 0.055 \) and \( z_0 = 1 \text{ mm} \) for neutral
flows, and \( u^*/U_{\infty} = 0.03, z_0 = 1 \text{ mm} \) and \( L = 100 \text{ mm} \) for stable flows. This is to allow easy
visual comparisons of boundary layers.

Fig. A.1 is an example of a neutral boundary layer analysis, and Fig. A.2 is an example of a
stable boundary analysis.
Neutral Boundary Layer Datasheet

**Setup:**
- Vorticity generators: 5 Irwim 1.2m
- Barrier wall: 150 &190mm
- Roughness: 20mm 4WH12_12
- Sidewall separation: 3m

**2.5 m/s**
- \( x = 14000 \)
- \( y = 0 \)

**Calculated parameters:**
- \( u^* / U_\infty = 0.0486 \)
- \( z_0 = 0.91 \text{ mm} \)

**Notes:**
1. Theoretical profile for \( U / U_\infty \) uses values for \( u^* / U_\infty = 0.055 \)
   and \( z_0 = 1 \text{ mm} \)

Figure A.1: NBL 2.5m/s, \( x=14000\text{mm}, y=0 \) datasheet
### Stable Boundary Layer Data Sheet

**Setup:**
- Vorticity generators: Irwin spires 604 x 94
- Barrier wall: 150 & 190 mm
- Roughness: 20 mm 4WH12_12
- Sidewall separation: 3 m

**Calculated parameters:**
- $u^*/U_\infty$ = 0.031
- $\theta$ = 1.28 K
- $L$ = 120 mm
- $N(z=100)$ = 1.4 Hz

<table>
<thead>
<tr>
<th>$z$ (mm)</th>
<th>$\theta$ (K)</th>
<th>$L$ (mm)</th>
<th>$N(z=100)$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1.28</td>
<td>120</td>
<td>1.4</td>
</tr>
</tbody>
</table>

**Notes:**
1. Theoretical profile for $U/U_\infty$ uses values for $u^*/U_\infty$ = 0.03
   - $z_\theta$ = 1 mm
   - $L$ = 100 mm

**Figure A.2: SBL 1.5 m/s datasheet**
Appendix A. Analysis tools

Perl Modules

Two Perl modules were written, one for processing the raw data files produced by the LabVIEW measurement program, and another for the bulk processing of the results files produced by the LabVIEW measurement program. The module for processing the raw data is the more important of the two since it makes it possible to process the raw data files that contain all the time traces, on machines where LabVIEW is not installed. The documentation for the raw data processing module, EnFlo::Analyse::Raw_Data, is given below.

NAME

EnFlo::Analyse::Raw_Data - Perl interface to LabVIEW generated Raw Data files

SYNOPSIS

use EnFlo::Analyse::Raw_Data;

$raw = EnFlo::Analyse::Raw_Data->new($ARGV[0]);
$results = $raw->readBlock();
foreach (@$results) {
   print join(" ", @$_), "\n";
}

DESCRIPTION

The actual time traces of data taken with the EnFlo Measurement program are stored in Raw Data files in a complex, but efficient format. Access to the time traces is not easy, and involves an intermediate step to create ASCII files from the Raw Data files. The Raw Data files are often stored on CD-ROM, and it is undesirable to first create the ASCII files, and then process the data with scripts or programs. This module was written to provide direct access to the Raw Data files through Perl.

The following methods are defined

new(filename)

Creates a new Raw Data file object using the file specified by filename.

$raw->source(platform)

Specifies the platform the Raw Data was created on. Can be either PC, MAC, or UNIX. MAC is the default.

$raw->readBlock()

Reads a block of Raw Data and returns a reference to an array containing the time traces of all the channels of measured data.
Appendix A. Analysis tools

$raw->hasMoreBlocks()
Determines if another block of data is available.

$raw->getX(n)
Returns the x coordinate of the nth instrument as recorded in the Raw Data file.

$raw->getY(n)
Returns the y coordinate of the nth instrument as recorded in the Raw Data file.

$raw->getZ(n)
Returns the z coordinate of the nth instrument as recorded in the Raw Data file.

$raw->getResultsFile()
Returns the filename of the original results file.

$raw->convertV(n, v)
Converts a voltage value v to the relevant unit of the nth instrument using the calibra-
tion specified by the Raw Data file.

$raw->getADCFreq()
Returns the ADC sampling frequency of the last block of data that was read.

$raw->getADCsamples()
Returns the number of ADC samples per instrument in the last block of data that was
read.

$raw->getLDAamples()
Returns the number of LDA samples per component in the last block of data that was
read.

$raw->getBlockStart()
Returns the absolute start time of the last block of data that was read.

$raw->getBlockEnd()
Returns the absolute end time of the last block of data that was read.

$raw->getSlips()
Returns the number of array slips when matching the arrival times of the two LDA
velocity components.

$raw->startTime()
Returns the relative time of the first LDA sample in the last block of data that was
read.

$raw->endTime()
Returns the relative time of the last LDA sample in the last block of data that was
read.

$raw->getAdam(channel)
Get the average value of the ADAM channel during the whole measurement time.
Appendix A. Analysis tools

$\text{raw} \rightarrow \text{getAdamRE}(\text{channel})$

Similar to $\text{raw} \rightarrow \text{getAdam}(\text{channel})$ but the channel name does not have to be specified in full.

$\text{raw} \rightarrow \text{getAdamChannels}()$

Returns an array containing the names of all the ADAM channels recorded in the Raw Data file.

$\text{raw} \rightarrow \text{samples}()$

Returns the total number of samples processed thus far.

$\text{raw} \rightarrow \text{blocks}()$

Returns the total number of blocks processed thus far.

$\text{raw} \rightarrow \text{instruments}()$

Returns an array containing the names of all the instruments.

$\text{raw} \rightarrow \text{noOfInstrument}()$

Returns the number of instruments.

$\text{raw} \rightarrow \text{reset}()$

Resets the Raw Data file to the beginning, allowing the processing the file from the start.

$\text{raw} \rightarrow \text{dumpoptions}()$

Dumps all the LDA and ADC options contained in the Raw Data file.

Working on a MAC with MacPerl

Scripts on MacPerl can be saved as droplets; files can then be ‘dropped’ onto a droplet to be analysed.

EXAMPLE

This example analyses channels of analog data. Averages and correlations are calculated.

```perl
#!/perl
use EnFlo::Analyse::Raw_Data;

$\text{bins} = 100;
$\text{raw} = \text{EnFlo::Analyse::Raw_Data} \rightarrow \text{new}($\text{ARGV}[0])
\text{or die "Could not analyse Raw Data"};
$\text{block} = 0;
$n = 0;

\text{for (0..$raw\rightarrow{\text{noOfInstruments}}()-1) \{}
  \text{$\text{min}[$_] = 1e99;}
```
$max[\_] = -1e99;
}

$data = $raw->readBlock();

foreach $line (@$data) {
    for $inst1 (0..$#{$line}) {
        $max[$inst1] = $line->[\$inst1] if $max[$inst1] < $line->[\$inst1];
        $min[$inst1] = $line->[\$inst1] if $min[$inst1] > $line->[\$inst1];
    }
}

for (0..$#min) {
    $range = $max[\_] - $min[\_];
    $min[\_] -= $range/20;
    $max[\_] += $range/10;
    $step[\_] = ($max[\_] - $min[\_])/$bins;
}
do {
    $block++;
    print "Block $block\n";
    foreach $line (@$data) {
        for $inst1 (0..$#{$line}) {
            $bin = int((($line->[\$inst1] - $min[$inst1])/$step[$inst1] + 0.5);
            $bin < 0 && print "Oops, one smaller than minimum\n";
            $pdf[$inst1][$bin]++;
        }
    }
    $n++;
}
$data = undef; # Frees up memory
} while ($raw->hasMoreBlocks() and $data = $raw->readBlock());

foreach (0..$#pdf) {
    print ((\$raw->[\texttt{instruments()}])[\$_, "\n"];
    foreach $bin (0..$bins) {
        print $min[\_]+$bin*$step[\_], "\t",
        $pdf[\_][$bin]/$n/$step[\_], "\n";
    }
    print "\n";}
BUGS

Please contact the author if any bugs are discovered.

AUTHOR

Chris Maré <chris@mare.org.uk>

Other utilities

Various other utilities were developed using Microsoft Excel spreadsheets and Visual Basic macros. The CURFIT routine, part of the FITPACK library of routines described by Dierckx (1993) was ported from FORTRAN to Visual Basic to be used for graphs in Microsoft Excel. CURFIT is a general curve fitting and data smoothing routine. The FORTRAN source code is available from http://www.netlib.org/dierckx/index.html.
Calculating autocorrelations and spectra from randomly sampled LDA data

LDA measurements are unlike many other measurements because of the random arrival times of samples. Special calculation techniques are therefore needed to calculate correlations and spectra, where the normal techniques require equi-spaced data. Three distinct paths are possible for spectral analysis of LDA data. Direct Fast Fourier Transform (FFT) methods exist that work on randomly sampled data rather than the conventional uniformly sampled data. Another method calculates spectra directly from the time series by resampling the data as an equi-spaced time series. The third method employs a slotting algorithm to produce an autocorrelation function as an intermediate step. Of these methods the slotting algorithm has shown the most promise and will be considered here.

For the slotting algorithm cross products are formed between all the samples in a time series. These cross products are then grouped, or slotted, based on their lag times. Normalising each slot with the overall variance of the measurement produces an autocorrelation function that is jagged with high variability. A smooth autocorrelation function is necessary for calculating a good spectral estimate. Theoretically an infinite number of zero width slots will produce the theoretical autocorrelation function from an infinite number of samples. The practical implementation of the technique requires a finite number of slots with non-zero width. The technique by Van Maanen et al. (1999) combines two of the most advanced techniques to produce a smooth autocorrelation function that is suitable for spectral estimated. Each slot is normalised with the variance computed only from the samples that fall in that slot. Fuzzy slotting is also employed to assign weights to samples based on their position in a slot. Fuzzy slotting is also employed to assign weights to samples based on their position in a slot. The addition of variable windowing suggested by Tummers & Passchier (1996) had very little effect, and was therefore not used.

The method can easily be extended to include the calculation of cospectra. Instead of using the autocorrelation function as an intermediate step the cross-correlation function is calculated as the intermediate step. The cross-correlation estimator, extended from the autocorrelation formulation given by Van Maanen et al. (1999) is
Appendix B. Autocorrelations and spectra from LDA data

\[ R_{u'u'} = \frac{\sum W(u'_i v'_j)_{(k\Delta T)}}{\sqrt{\sum W(u'_i u'_i)_{(k\Delta T)}} \sqrt{\sum W(v'_j v'_j)_{k\Delta T}}} \]  \hspace{1cm} (B.1)

where the slot number \(k\) is determined by

\[ (k - \frac{1}{2}) \Delta T \leq (t_j - t_i) \leq (k + \frac{1}{2}) \Delta T \]  \hspace{1cm} (B.2)

and the weighing factor \(W\) by

\[ W = 1 - \left| \frac{(t_j - t_i) - k\Delta T}{\Delta T} \right| \]  \hspace{1cm} (B.3)

The average values \( \bar{U} \) and \( \bar{V} \) are known only at the end of processing a time series for a point, and the fluctuating component of a velocity measurement is therefore unknown at the time of processing. It is useful to write \( u' \) as \( u - \bar{U} \) and \( v' \) as \( v - \bar{V} \) where \( u \) and \( v \) are the actual measured velocities. The terms in Eq. (B.1) can be rearranged as follows to allow the efficient processing of a time series without knowing the averages \( \bar{U} \) and \( \bar{V} \) until the end of processing.

\[ \sum W u'_i v'_j = \sum W (u_i - \bar{U})(v_j - \bar{V}) = \sum W (u_i v_j - \bar{U} v_j - \bar{V} u_i + \bar{U} \bar{V}) = \sum W u_i v_j - \bar{U} \sum W v_j - \bar{V} \sum W u_i + \bar{U} \bar{V} \sum W \]

Similarly

\[ \sum W u'_i u'_i = \sum W u_i u_i - 2 \bar{U} \sum W u_i + \bar{U}^2 \sum W \]
\[ \sum W v'_j v'_j = \sum W v_j v_j - 2 \bar{V} \sum W v_j + \bar{V}^2 \sum W \]

The cospectral estimate is then

\[ S_{u'u'} = 4 \bar{u}' \bar{u}' \left( \frac{1}{2} R_{u'u'}(0) + \sum_{k=1}^{M-1} R_{u'u'}(k\Delta T) w(k\Delta T) \cos(k2\pi f \Delta T) \right) \]  \hspace{1cm} (B.4)

with \( w(t) \) a lag-window function, described in detail by Press et al. (1996). A standard cosine FFT routine found in any numerical methods handbook (e.g. Press et al. 1996), can be used to compute the cospectral estimate efficiently. To calculate spectra rather than co-spectra simply set \( v_j = u_j \) and \( \bar{V} = \bar{U} \).

Figs. B.1–B.2 illustrate the program sections to implement the slotting algorithm. Fig. B.1 is the slotting routine, and Fig. B.2 computes the cross correlation.
Appendix B. Autocorrelations and spectra from LDA data

void slotdata(int n) {
    int i, j, k;
    double ti, tj, ui, vi, uiui, vivi, uj, vj, W;

    for (i=0; i < n-1 ; i++) {
        ti = x[i];
        ui = y_1[i];
        vi = y_2[i];
        uiui = ui*ui;
        vivi = vi*vi;

        for (j=i; j<n; j++) {
            tj = x[j];
            vj = y_2[j];
            uj = y_1[j];
            k = (tj - ti)*inv_deltat + 0.5;
            if (k >= binmax) break;
            W = 1. - fabs((tj-ti)*inv_deltat - k);
            u_i[k] += W*(ui+uj);
            v_j[k] += W*(vj+vi);
            Wuivj[k] += W*(ui*vj+uj*vi);
            Wuui[k] += W*(uiui+uj*uj);
            Wvjj[k] += W*(vj*vj+vivi);
            sumW[k] += 2.*W;

            /* Now for other slot I previously forgot about */
            if (((tj - ti)*inv_deltat > k) k++;
            else k--;
            if (k >= binmax) break;
            W = 1. - W;
            u_i[k] += W*(ui+uj);
            v_j[k] += W*(vj+vi);
            Wuivj[k] += W*(ui*vj+uj*vi);
            Wuui[k] += W*(uiui+uj*uj);
            Wvjj[k] += W*(vj*vj+vivi);
            sumW[k] += 2.*W;
        }
    }
}

Figure B.1: Local normalised slotting function
for (k = 0; k < slotmax; k++) {
    rho[k] = (Wu ivj[k] + sumW[k]*U_\*V_ - u_i[k]*V_- v_j[k]*U_)
    /sqrt(
        (Wuiui[k] + (sumW[k]*U_-2.*u_i[k])*U_)*
        (Wvjvj[k] + (sumW[k]*V_-2.*v_j[k])*V_)
    );
    rho[k] *= w((double) k/slotmax);
}
Experimental results

Figs. B.3–B.6 present time domain statistics results for neutral and stable flows in the undisturbed condition and at different downstream positions behind a 100 mm cube. All the measurements are at a height of \(z = H\) and on the symmetry line behind the cube. The undisturbed case is designated by \(x/H = -\infty\) in the legends of Figs. B.3–B.6.

The general trends for the \(R_{uu}\) autocorrelation functions for the neutral and stable cases shown in Figs. B.3 & B.5 are similar. The \(R_{ww}\) autocorrelation functions in Figs. B.4 & B.6 do however differ. The stable \(R_{ww}\) autocorrelation function measured behind the cube shows slightly higher correlation coefficients than the undisturbed case, indicating a higher integral timescale (e.g. Arya 1999) for the \(w\) turbulence and also a corresponding increase in the length scale. The function for the neutral case shows lower correlation coefficients in the wake, compared with the undisturbed case, indicating that the length scale of the \(w\) turbulence decreases. The vertical length scale of the turbulent eddies in the stable case is therefore slightly shorter than the vertical length scale of the eddies shed by the obstacle.

\[ x/H \]
- \(\infty\)
- 5
- 10
- 15

**Figure B.3:** Autocorrelation \(R_{uu}\) profiles (2.5 m/s NBL, \(z/H = 1\))
Appendix B. Autocorrelations and spectra from LDA data

Figure B.4: Autocorrelation $R_{ww}$ profiles (2.5 m/s NBL, $z/H = 1$)

Figure B.5: Autocorrelation $R_{uu}$ profiles (1.1 m/s SBL, $z/H = 1$)
Figure B.6: Autocorrelation $R_{uw}$ profiles (1.1 m/s SBL, $z/H = 1$)
Fast FID Configurations

The Fast FIDs were set up using two configurations. The configuration in Fig. C.2 was used for multiple Fast FID measurements, and the configuration in Fig. C.1 was used for a single Fast FID used in conjunction with the LDA. The calculation package SATFLAP supplied by Cambustion was used for the calculations in Figs. C.1 & C.2.
### Appendix C. Instrumentation

#### Table 1: SATFLAP calculation for 250mm BST length

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample press (mmHg g)</td>
<td>750.0</td>
</tr>
<tr>
<td>FID press (mmHg g)</td>
<td>592.0</td>
</tr>
<tr>
<td>ATM press (mmHg)</td>
<td>750.0</td>
</tr>
<tr>
<td>Tube diam (thou, mm)</td>
<td>10.0 (0.25)</td>
</tr>
<tr>
<td>Tube length (mm)</td>
<td>250</td>
</tr>
<tr>
<td>Tube temp (°C)</td>
<td>25</td>
</tr>
<tr>
<td>Entry press (mmHg g)</td>
<td>750.0</td>
</tr>
<tr>
<td>Exit press (mmHg g)</td>
<td>592.0</td>
</tr>
<tr>
<td>Delta press (mmHg)</td>
<td>158.0</td>
</tr>
<tr>
<td>Mass flow (mg/sec)</td>
<td>0.497</td>
</tr>
<tr>
<td>Reynolds number</td>
<td>137</td>
</tr>
<tr>
<td>Exit mom. flux (mN)</td>
<td>0.005</td>
</tr>
<tr>
<td>Transit time (ms)</td>
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<td>Time constant (ms)</td>
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<tr>
<td>Freq. resp. (Hz)</td>
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</tr>
<tr>
<td>STP flow (l/min)</td>
<td></td>
</tr>
<tr>
<td>Inlet velocity</td>
<td>8.4</td>
</tr>
<tr>
<td>Exit velocity</td>
<td>10.6</td>
</tr>
<tr>
<td>Mach no.</td>
<td>0.024</td>
</tr>
<tr>
<td>Flow rates (cc/sec)</td>
<td>0.4</td>
</tr>
</tbody>
</table>

*Figure C.1: SATFLAP calculation for 250mm BST length*
Appendix C. Instrumentation

<table>
<thead>
<tr>
<th>Sample press (mmHg g)</th>
<th>750.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fid press (mmHg g)</td>
<td>650.0</td>
</tr>
<tr>
<td>Atm press (mmHg)</td>
<td>750.0</td>
</tr>
</tbody>
</table>

| Tube diam (thou, mm) | 10.0 (0.25) |
| Tube length (mm)     | 150 |
| Tube temp (°C)       | 25 |
| Entry press (mmHg g) | 750.0 |
| Exit press (mmHg g)  | 650.0 |
| Delta press (mmHg)   | 100.0 |
| Mass flow (mg/sec)   | 0.547 |
| Reynolds number      | 151 |
| Exit mom. flux (mN)  | 0.006 |
| Transit time (ms)    | 15.215 |
| Time constant (ms)   | 1.688 |
| Freq. resp. (Hz)     | 94 |
| STP flow (l/min)     | 0.03 |

<table>
<thead>
<tr>
<th>velocities (m/s)</th>
<th>Inlet</th>
<th>Exit</th>
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<td>Mach no.</td>
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<td>0.031</td>
</tr>
<tr>
<td>Flow rates (cc/sec)</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Figure C.2: SATFLAP calculation for 150mm BST length
Appendix C. Instrumentation

Instrument identification

The manufacturer and model numbers where available of the instrumentation used during the course of this project are noted for future reference.

**Baratron**
- Manufacturer: MKS
- Model: 120A-13580

**Burst Spectrum Analysers**
- Manufacturer: Dantec
- Model: BSA 57N11

**Laser**
- Manufacturer: Coherent
- Model: Innova 70

**Hot Wire Bridge**
- Manufacturer: DISA
- Model: 56C16

**Cold Wire Bridge**
- Manufacturer: DISA
- Model: 56C01 CTA

**Slow FID**
- Manufacturer: Signal
- Model: Model 3000

**Fast FID**
- Manufacturer: Cambustion
- Model: HFR 400 FFID

**Analogue to Digital Converter**
- Manufacturer: IOtech
- Model: ADC 488/SA
Appendix C. Instrumentation

Ultrasonic anemometer
Manufacturer  :  Gill
Model        :  1172N

Propeller anemometer
Manufacturer  :  Schiltknecht
Model        :  MiniAir2
Detailed dispersion measurements are presented. These measurements are discussed in §5.2.1, §5.2.2 and §5.2.3. The obstacle configurations are abbreviated in the captions of the figures as follows: × for the undisturbed case, □ for the cube normal to the flow and ◊ for the cube at 45° to the flow.
Appendix D. Detailed dispersion measurements

Figure D.1: Centerline mean concentration, $\bar{C}/C_m$, stable 1.3 m/s, $H_s = 100$ mm.

Figure D.2: Centerline mean concentration, $\bar{C}/C_m$, neutral, $H_s = 100$ mm.
Figure D.3: Centerline mean concentration, $C/C_m$, neutral, $H_s = 150$ mm.

Figure D.4: Centerline mean concentration, $C/C_m$, stable $1.1$ m/s, $H_s = 100$ mm.
Appendix D. Detailed dispersion measurements

Figure D.5: Centerline mean concentration, $\bar{C}/C_m$, stable 1.1 m/s, $H_s = 150$ mm.

Figure D.6: Centerline mean concentration, $\bar{C}/C_m$, stable 1.3 m/s, $H_s = 150$ mm.
Appendix D. Detailed dispersion measurements

Figure D.7: Centerline concentration fluctuations $\bar{c}^2/c^2$, stable 1.3 m/s, $H_s = 100$ mm.

Figure D.8: Centerline concentration fluctuations $\bar{c}^2/c^2$, neutral, $H_s = 100$ mm.
Appendix D. Detailed dispersion measurements

Figure D.9: Centerline concentration fluctuations $\overline{c^2}/c^2$, neutral, $H_s = 150$ mm.

Figure D.10: Centerline concentration fluctuations $\overline{c^2}/c^2$, stable 1.1 m/s, $H_s = 100$ mm.
Appendix D. Detailed dispersion measurements

Figure D.11: Centerline concentration fluctuations $\overline{c^2}/c^2$, stable 1.1 m/s, $H_s = 150$ mm.

Figure D.12: Centerline concentration fluctuations $\overline{c^2}/c^2$, stable 1.3 m/s, $H_s = 150$ mm.
Figure D.13: Centerline concentration flux $\bar{w}'c'/u_sC_m$, stable 1.3 m/s, $H_s = 100$ mm.

Figure D.14: Centerline concentration flux $\bar{w}'c'/u_sC_m$, neutral, $H_s = 100$ mm.
Figure D.15: Centerline concentration flux $\overline{w'c}/u_*C_m$, neutral, $H_s = 150$ mm.

Figure D.16: Centerline concentration flux $\overline{w'c}/u_*C_m$, stable 1.1 m/s, $H_s = 100$ mm.
Figure D.17: Centerline concentration flux $\overline{w'c'/u_C}$, stable 1.1 m/s, $H_s = 150$ mm.

Figure D.18: Centerline concentration flux $\overline{w'c'/u_C}$m, stable 1.3 m/s, $H_s = 150$ mm.
Probability distributions of concentrations

Probability density distributions of concentration fluctuations are presented for the 1.1 m/s stable and the 2.5 m/s neutral boundary layer 2 m downstream of the source. Sources are located at heights of 100 mm and 150 mm. The obstacle orientations are a 100 mm cube normal to the flow with the leading edge directly below the source, a 100 mm cube at 45° to the flow with the leading edge directly below the source and the trivial case with the obstacle removed. Fig. E.1 shows an example of the probability density distributions presented with the axis titles shown. Axis titles are absent for all the other graphs in this appendix.

![Graph showing concentration probability density distribution](image)

*Figure E.1: Example concentration probability density distribution showing axis titles*

The obstacle configurations are abbreviated in the captions of the figures as follows: × for the undisturbed case, □ for the cube normal to the flow and ◊ for the cube at 45° to the flow. Empty charts are shown where the concentration is negligible and the probability density distribution only indicates instrument noise. Each graph shows the probability density distributions of the measurements closest to the centreline so that $-50 \text{ mm} < y < 50 \text{ mm}$, if the curves collapsed quite well. That was mostly the case, but some graphs only show the measurement closest to the centreline. Values of $\sigma_c/C$ are also shown on the graphs where available. These results are discussed further in §5.2.6.
Figure E.2: Concentration probability density distributions for 1.1 m/s stable flow \( \times \), \( H_s = 100\text{mm}, \) \( z/H_s = 20 \)
Figure E.3: Concentration probability density distributions for 2.5 m/s neutral flow x, $H_s = 100$mm, $z/H_s = 20$
Appendix E. Probability distributions of concentrations

Figure E.4: Concentration probability density distributions for 1.1 m/s stable flow □, $H_s = 100\text{mm}$, $x/H_s = 20$
Appendix E. Probability distributions of concentrations

Figure E.5: Concentration probability density distributions for 2.5 m/s neutral flow □, \( H_s = 100 \text{mm} \), \( x/H_s = 20 \)

(a) \( z/H_s = 3 \)
(b) \( z/H_s = 3 \)
(c) \( z/H_s = 1.5 \)
(d) \( z/H_s = 1.15 \)
(e) \( z/H_s = 1 \)
(f) \( z/H_s = 0.85 \)
(g) \( z/H_s = 0.5 \)
(h) \( z/H_s = 0.05 \)
Figure E.6: Concentration probability density distributions for 1.1 m/s stable flow \( \Delta \), \( H_s = 100 \text{mm} \), \( z/H_s = 20 \)
Figure E.7: Concentration probability density distributions for 2.5 m/s neutral flow $\phi$, $H_s = 100$mm, $x/H_s = 20$
Appendix E. Probability distributions of concentrations

Figure E.8: Concentration probability density distributions for 1.1 m/s stable flow $x$, $H_s = 150\, \text{mm}$, $x/H_s = 13$
Figure E.9: Concentration probability density distributions for 2.5 m/s neutral flow, $x$, $H_s = 150$mm, $x/H_s = 13$
Appendix E. Probability distributions of concentrations

Figure E.10: Concentration probability density distributions for 1.1 m/s stable flow □, $H_s = 150$mm, $z/H_s = 13$
Figure E.11: Concentration probability density distributions for 2.5 m/s neutral flow □, $H_s = 150 \text{mm}$, $z/H_s = 13$
Figure E.12: Concentration probability density distributions for 1.1 m/s stable flow with \( \phi, H_s = 150\text{mm}, z/H_s = 13 \)
Figure E.13: Concentration probability density distributions for 2.5 m/s neutral flow, $H_s = 150\text{mm}$, $x/H_s = 13$
Appendix F

Wake model derivation

The three dimensional wake model which is an extension of the two dimensional wake model by Counihan et al. (1974) will be derived here. An introduction to the theory is given in § 2.4.2 (b) with the necessary assumptions.

The starting point of the analysis is the time averaged momentum equation in the $x$ direction and the continuity equation.

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \left( \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} \right) \]  
\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \]

where

\[ \tau_{xx} = -\rho u'^2 \]
\[ \tau_{xy} = -\rho u'v' \]
\[ \tau_{xz} = -\rho u'w' \]

In the approach flow the two-dimensional boundary layer approximation applies

\[ U \frac{\partial U}{\partial x} + W \frac{\partial U}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{1}{\rho} \frac{\partial T_{xx}}{\partial z} \]

where $U$, $W$, $P$ and $T_{xy}$ are the quantities in the approach flow.

In the wake

\[ \tau_{xx} = T_{xx} + \tau_{xx} \]
\[ \tau_{xy} = \tau_{xy} \]
\[ \tau_{xz} = T_{xz} + \tau_{xx} \]
\[ u' = \tilde{u} + u \]
\[ v' = v \]
\[ w' = W + w \]
where \( u, v, w, \tau_{xx}, \tau_{xy} \) and \( \tau_{xz} \) are the perturbation quantities in the wake.

Substituting Eq. (F.5) in Eq. (F.1), subtracting Eq. (F.4) and applying the assumptions introduced earlier the momentum equation for the perturbation quantities can be written as

\[
U \frac{\partial u}{\partial x} = \frac{1}{\rho} \frac{\partial \tau_{xy}}{\partial y} + \frac{1}{\rho} \frac{\partial \tau_{xz}}{\partial z} \quad (F.6)
\]

The model does not use the momentum equations for the \( v \) and \( w \) velocities, but employs the continuity equation and an assumption relating \( v \) and \( w \) to calculate \( v \) and \( w \).

The nondimensional coordinates are chosen as

\[
\xi = \frac{x - x_0}{H} \quad (F.7)
\]

\[
\eta = \frac{y}{\lambda \ell(\xi)} \quad (F.8)
\]

\[
\zeta = \frac{z}{\ell(\xi)} \quad (F.9)
\]

with

\[
\lambda = \lambda_0 \frac{W/2}{H} \quad (F.10)
\]

and \( \lambda_0 = \sigma_v/\sigma_w \).

The derivatives are

\[
\frac{\partial \xi}{\partial x} = \frac{1}{H} \quad \frac{\partial \eta}{\partial y} = \frac{1}{\lambda \ell(\xi)} \quad \frac{\partial \zeta}{\partial z} = \frac{1}{\ell(\xi)}
\]

\[
\frac{\partial \eta}{\partial \xi} = -\frac{\eta \, d\ell}{\ell(\xi) \, d\xi} \quad \frac{\partial \zeta}{\partial \xi} = -\frac{\zeta \, d\ell}{\ell(\xi) \, d\xi}
\]

The problem can be closed by relating the perturbation stresses \( \tau_{xy} \) and \( \tau_{xz} \) to the perturbation velocity \( u \). The relation chosen is the eddy viscosity hypothesis

\[
\tau_{xy} = \rho \nu_M \frac{\partial u}{\partial y} \quad (F.11)
\]

\[
\tau_{xz} = \lambda^2 \rho \nu_M \frac{\partial u}{\partial z} \quad (F.12)
\]

where \( \nu_M \) is the eddy viscosity in the main wake region.

The \( \lambda^2 \) term comes from \( a \ priori \) knowledge of the required factor that would result in an analytically solvable differential equation.

Counihan et al. (1974) related \( \nu_M \) to \( \nu_0 \), the eddy viscosity in the undisturbed boundary layer. The well known mixing length concept \( \nu_0 = \kappa u_* H \) is used for the undisturbed flow. In the main wake region the flow is well mixed and a constant eddy viscosity is assumed, based on the upstream eddy viscosity at building height, with a magnification factor to account for the additional rates of strain in the wake. Counihan et al. (1974) arrived at the final result:

\[
\nu_M = 2 \kappa u_* H \quad (F.13)
\]

If

\[
\tau_{xx} = T_{xx} + \tau_{xz}, \quad (F.14)
\]

\[
\tau_{zz} = \rho \nu_M \frac{\partial u}{\partial z} \quad \text{and} \quad (F.15)
\]

\[
T_{xx} = \nu_0 \frac{\partial U}{\partial z} \quad (F.16)
\]
Appendix F. Wake model derivation

then

\[ \tau_{xz} = \rho \nu M (\partial U / \partial z + \partial u / \partial z) - \rho \nu_0 \partial U / \partial z \]
\[ = \rho (\nu_M - \nu_0) \partial U / \partial z + \rho \nu M \partial u / \partial z \]  \hspace{1cm} (F.17)

and not simply

\[ \tau_{xz} = \rho \nu M \partial u / \partial z, \]  \hspace{1cm} (F.18)

the result obtained by Counihan et al. (1974). \( n \) is however small and hence \( \partial U / \partial z \) is small.

Eq. (F.18) is therefore sufficiently accurate.

Eq. (F.6) can then be written as

\[ \frac{U \partial u}{\partial \xi} \frac{\partial \xi}{\partial x} = \nu M \frac{\partial^2 u}{\partial \eta^2} \left( \frac{\partial \eta}{\partial y} \right)^2 + \nu M \frac{\partial^2 u}{\partial \xi^2} \left( \frac{\partial \xi}{\partial z} \right)^2 \]  \hspace{1cm} (F.19)

leading to

\[ \frac{U \partial u}{H \partial \xi} = \frac{\nu M}{\ell(\xi)^2} \left( \frac{\partial^2 u}{\partial \eta^2} + \frac{\partial^2 u}{\partial \xi^2} \right) \]  \hspace{1cm} (F.20)

The \( \lambda \) terms conveniently cancelled out owing to the assumption \( \tau_{xz} = \lambda^2 \rho \nu M \partial u / \partial z \).

With knowledge of other similarity solutions in two and three dimensional wakes (e.g. White 1991) a similarity solution of the form

\[ u = f(\xi)g(\eta, \zeta) \]  \hspace{1cm} (F.21)

can be assumed.

The undisturbed boundary layer flow is approximated by a power law of the form

\[ U = U_H \left( \frac{z}{H} \right)^n \]
\[ = U_H \left( \frac{\ell(\xi)}{H} \right)^n \]  \hspace{1cm} (F.22)

with \( n \) the exponent of the power law.

Hayden (1998) used the power law up to a point in the derivation and then assumed \( n = 0 \) later on. Apsley (1988) performed his derivation with \( U = U_H \) from the onset. For the sake of generality Eq. (F.22) will be used.

Eq. (F.20) then becomes

\[ \frac{U_H}{H} \left( \frac{\ell(\xi)}{H} \right)^n \frac{1}{\zeta^n} \left( f' g - \frac{\eta}{\ell(\xi)} \frac{d\xi}{d\eta} \frac{\partial g}{\partial \xi} - \frac{\xi}{\ell(\xi)} \frac{d\xi}{d\eta} \frac{\partial g}{\partial \xi} \right) \]
\[ - \frac{\nu M}{\ell(\xi)^2} \left( f \frac{\partial^2 g}{\partial \eta^2} + f \frac{\partial^2 g}{\partial \xi^2} \right) = 0 \]  \hspace{1cm} (F.23)

Grouping the terms \{ \} that are functions of \( \xi \)

\[ \left\{ \frac{H U_H}{\nu M} \left( \frac{\ell(\xi)}{H} \right)^{n+2} f' \right\} \zeta^n g - \left\{ \frac{U_H}{\nu M} \left( \frac{\ell(\xi)}{H} \right)^{n+1} \frac{d\xi}{d\zeta} \right\} \zeta^{n+1} \frac{\partial g}{\partial \xi} \]
\[ - \left\{ \frac{U_H}{\nu M} \left( \frac{\ell(\xi)}{H} \right)^{n+1} \frac{d\xi}{d\zeta} \right\} \zeta^{n+1} \frac{\partial g}{\partial \xi} = 0 \]  \hspace{1cm} (F.24)
The similarity solution for $g$ will be independent of $\xi$, and hence all the terms in curly brackets are constants.

\[ \frac{HU_H}{\nu_M} \left( \frac{\ell(\xi)}{H} \right)^{n+2} \frac{f'}{f} = C_{f1} \]  

(F.25)

and

\[ \frac{U_H}{\nu_M} \left( \frac{\ell(\xi)}{H} \right)^{n+1} \frac{d\ell}{d\xi} = C_\ell \]  

(F.26)

$\xi$ was defined to be zero at the virtual origin of the wake where $\ell(0) = 0$. The solution for $\ell(\xi)$ from Eq. (F.26) is then

\[ \ell(\xi)^{n+2} = C_\ell (n+2) \frac{H^{n+1} \nu_M}{U_H} \]  

(F.27)

$\ell(\xi)$ is not a physical length scale, and may as well be defined such that $C_\ell = 1$.

For convenience Eq. (F.27) can be written as

\[ \frac{\ell(\xi)}{H} = \left\{ (n+2) \frac{\nu_M \xi}{HU_H} \right\}^{\frac{1}{n+2}} \]  

(F.28)

Substituting $\ell(\xi)$ in Eq. (F.25)

\[ (n+2)\xi \frac{f'}{f} = C_{f1} \]  

(F.29)

with the solution

\[ f = C_{f2} \xi^{\frac{n}{n+3}} \]  

(F.30)

$C_{f1}$ can be found from a conserved quantity in the wake, the appropriate quantity being the first moment of momentum about the wall (Counihan et al. 1974).

\[ G = -\int_{-\infty}^{\infty} \int_{0}^{\infty} \rho z U(z) u \, dz \, dy \]  

(F.31)

\[ = -C_{f2} \xi^{\frac{n}{n+3}} \frac{\nu_M \lambda \ell(\xi)^{(n+3)}}{H^n} \int_{-\infty}^{\infty} \int_{0}^{\infty} \zeta^{n+1} g(\eta, \zeta) \, d\zeta \, d\eta \]  

(F.32)

From the definition of $\ell(\xi)$ in Eq. (F.27) it can be seen that $G$ can only be constant, i.e. independent of $\xi$, if $C_{f1} = -(n+3)$.

Eq. (F.24) then becomes

\[ (n+3)\zeta^ng + \zeta^n \eta \frac{\partial g}{\partial \eta} + \zeta^{n+1} \frac{\partial g}{\partial \zeta} + \frac{\partial^2 g}{\partial \eta^2} + \frac{\partial^2 g}{\partial \zeta^2} = 0 \]  

(F.33)
For $n = 0$ an analytical solution can be found by assuming a solution of the form

$$g(\eta, \zeta) = \mathcal{Y}(\eta)Z(\zeta)$$  \hspace{1cm} (F.34)

Eq. (F.33) then becomes

$$3\mathcal{Y}Z + \eta\mathcal{Y}'Z + \zeta Z\mathcal{Y}' + \mathcal{Y}''Z + \mathcal{Y}Z'' = 0$$  \hspace{1cm} (F.35)

Following Hayden (1998) this equation can be separated into two independent equations

$$\mathcal{Y} + \eta\mathcal{Y}' + \mathcal{Y}'' = 0$$  \hspace{1cm} (F.36)
$$2Z + \zeta Z' + Z'' = 0$$  \hspace{1cm} (F.37)

The boundary conditions are

$$\mathcal{Y}'(0) = 0$$  \hspace{1cm} (F.38)
$$\mathcal{Y}(-\infty) = \mathcal{Y}(\infty) = 0$$  \hspace{1cm} (F.39)
$$Z(0) = Z(\infty) = 0$$  \hspace{1cm} (F.40)

With solutions

$$\mathcal{Y}(\eta) = C_\eta e^{-\eta^2/2}$$  \hspace{1cm} (F.41)
$$Z(\zeta) = C_\zeta e^{-\zeta^2/2}$$  \hspace{1cm} (F.42)

The final solution will then be

$$u = -u^2_0 \zeta^{n+3} \sqrt{\pi} e^{-0.5(n^2+\zeta^2)}$$  \hspace{1cm} (F.43)

Evaluating the wake constant $G$ introduced in Eq. (F.32) with

$$\int_{-\infty}^{\infty} \int_{0}^{\infty} \zeta e^{-0.5(n^2+\zeta^2)} d\eta d\zeta = \pi$$  \hspace{1cm} (F.44)
$$G = \tilde{u}x_0 \rho U_H \lambda H^3 \left\{ (n + 2) \frac{\nu}{U_H H} \right\}^{\frac{1}{n+3}}$$  \hspace{1cm} (F.45)

For $n \geq 0$ we assume a solution of the form (Kothari et al. 1986, Smith 1957)

$$g(\eta, \zeta) = Z(\zeta) e^{-\frac{\eta^2}{2\lambda^2}}$$  \hspace{1cm} (F.46)

If

$$q_0 = \int_{-\infty}^{\infty} g(\eta, \zeta) d\eta$$  \hspace{1cm} (F.47)

and

$$q_2 = \int_{-\infty}^{\infty} \eta^2 g(\eta, \zeta) d\eta$$  \hspace{1cm} (F.48)
it can be shown that

\[ Z(\zeta) = q_0 \frac{q_0}{2\pi q_2} \]  
(F.49)

\[ A(\zeta) = 2q_2/q_0 \]  
(F.50)

Integrating Eq. (F.33) as follows

\[ \int_{-\infty}^{\infty} \left\{ (n+3)\zeta^n g + \zeta^{n+1} \frac{\partial g}{\partial \eta} + \zeta^{n+2} \frac{\partial^2 g}{\partial \eta^2} + \zeta^{n+3} \frac{\partial^2 g}{\partial \zeta^2} \right\} d\eta = 0 \]  
(F.51)

leads to a differential equation in terms of \( q_0 \)

\[ (n+2)\zeta^n q_0 + \zeta^{n+1} \frac{\partial q_0}{\partial \zeta} + \zeta^{n+2} \frac{\partial^2 q_0}{\partial \zeta^2} = 0 \]  
(F.52)

Multiplying Eq. (F.33) with \( \eta^2 \) and integrating with respect to \( \eta \) from \( -\infty \) to \( \infty \) leads to the differential equation

\[ n\zeta^n q_2 + \zeta^{n+1} \frac{\partial q_2}{\partial \zeta} + \zeta^{n+2} \frac{\partial^2 q_2}{\partial \zeta^2} + 2q_0 = 0 \]  
(F.53)

with boundary conditions

\[ q_0 = q_2 = 0 \text{ at } \zeta = 0 \text{ and } \zeta = \infty \]  
(F.54)

and for convenience in evaluating the wake constant and for comparing profiles with the \( n = 0 \) case we specify

\[ \int_{-\infty}^{\infty} \int_{0}^{\infty} Z(\zeta) e^{-\frac{\eta^2}{2\zeta^2}} d\eta d\zeta = \pi \]  
(F.55)

The final solution will then be of a form

\[ u = -\hat{u} \zeta^{n+3} Z(\zeta) e^{-\frac{\eta^2}{2\zeta^2}} \]  
(F.56)

with \( A(\zeta) = 2 \) and \( Z(\zeta) = e^{-0.5\zeta^2} \) for \( n = 0 \). \( \hat{u} \) combines all the integration constants \( C_{f1}, C_\eta \) and \( C_\zeta \).

Analytical solutions to Eqs. (F.52) & (F.53) are not easily found. Kothari et al. (1986) quotes the method of Smith (1957), but this solution is still an approximate one. Eqs. (F.52) & (F.53) can be solved numerically with a Runge-Kutta method (Gerald & Wheatley 1989). A numerical solution is still applicable to a dispersion model, since the solutions of \( q_0 \) and \( q_2 \) can be calculated and stored in memory before commencing the dispersion calculations. Computing time is negligible. Counihan et al. (1974) found that the solution for the two-dimensional wake is insensitive to \( n \) as long as \( n \ll 1 \). For stable boundary layers \( n \) can be quite large, in the order of \( n \approx 0.35 \).

Figs. F.1–F.3 compare the numerical \( Z(\zeta) \) and \( A(\eta) \) profiles for \( n = 0, n = 0.15 \) and \( n = 0.3 \) with the analytical solution for \( n = 0 \). It can be seen that the differences between the profiles are small even for \( n = 0.3 \), and \( n = 0 \) can be safely assumed for the similarity velocity profiles. \( n \geq 0 \) can be retained to estimate the spread and decay of the wake with downstream distance. It is a trivial matter to use the numerical solution for the velocity profiles in a dispersion model, but as will be shown later the calculation of the vertical and lateral velocities becomes arduous where an elegant analytical solution for \( n = 0 \) can be found.
Appendix F. Wake model derivation

Figure F.1: $n = 0$

Figure F.2: $n = 0.15$
Appendix F. Wake model derivation

Counihan et al. (1974) defined their wake constant $\bar{C}$ as

$$\bar{C} = - \int_0^\infty z U(z) u \, dz \quad (F.57)$$

They could not relate the pressure couple on a two-dimensional body $C$ directly to the wake parameter $\bar{C}$ due to the unknown magnitude of the pressure couple on the surface near the obstacle. For the three-dimensional case this pressure couple is small compared to the pressure couple on the body, and the error in assuming $C = \bar{C}$ is small.

Apsley (1988) and Robins et al. (1997) suggest using the values obtained by Akins et al. (1977) for the moment coefficients. The definition used by Akins et al. (1977) for the moment coefficient was

$$C_{MY} = \frac{M_y}{0.5 \rho U_A^2 L H^2} \quad (F.58)$$

with

$$U_A = \frac{1}{H} \int_0^H U(z) \, dz \quad (F.59)$$

The moment coefficient $C_G$ is defined as

$$C_G = \frac{G}{0.5 \rho U_H^2 W H^2} \quad (F.60)$$

for an assumed power law profile

$$U_A = \frac{U_H}{n+1} \quad (F.61)$$

and

$$C_G = \frac{C_{MY}}{(n+1)^2} \quad (F.62)$$

and not simply $C_G = C_{MY}$ as suggested by Apsley (1988) and Robins et al. (1997).
Eq. (F.45) can be written as

\[ G = \pi \hat{u} \rho \lambda H^3 \left\{ (n + 2) \frac{\nu_m}{HU_H} \right\}^{\frac{n+3}{n+2}} \]  

(F.63)

\( \hat{u} \) can then be calculated as

\[ \hat{u} = \frac{CGU_H}{\pi \lambda_0} \left\{ \frac{HU_H}{(n + 2)\nu_m} \right\}^{\frac{n+3}{n+2}} \]  

(F.64)

To calculate \( w \), we use the continuity equation

\[ \frac{\partial w}{\partial z} + \frac{\partial v}{\partial y} - \frac{\partial u}{\partial x} = -\frac{\hat{u}}{(n + 2)H} \xi^{\frac{2n+5}{n+2}} \left[ (n + 3)g(\eta, \zeta) + \eta \frac{\partial g}{\partial \eta} + \zeta \frac{\partial g}{\partial \zeta} \right] \]  

(F.65)

and

\[ \frac{\partial w}{\partial \zeta} + \frac{1}{\lambda} \frac{\partial v}{\partial \eta} = -\frac{\hat{u}}{\xi^2} \left\{ (n + 2) \frac{\nu_m}{U_H H} \right\}^{\frac{1}{n+2}} \left[ (n + 3)g(\eta, \zeta) + \eta \frac{\partial g}{\partial \eta} + \zeta \frac{\partial g}{\partial \zeta} \right] \]  

(F.66)

\( v \) and \( w \) will then be of the form

\[ v = -\frac{\lambda \hat{u}}{\xi^2} \left\{ (n + 2) \frac{\nu_m}{U_H H} \right\}^{\frac{1}{n+2}} \tilde{v} \]  

(F.67)

and

\[ w = -\frac{\hat{u}}{\xi^2} \left\{ (n + 2) \frac{\nu_m}{U_H H} \right\}^{\frac{1}{n+2}} \tilde{w} \]  

(F.68)

where

\[ \frac{\partial \tilde{w}}{\partial \zeta} + \frac{\partial \tilde{v}}{\partial \eta} = (n + 3)g(\eta, \zeta) + \eta \frac{\partial g}{\partial \eta} + \zeta \frac{\partial g}{\partial \zeta} \]  

(F.69)

Simple solutions for \( \tilde{v} \) and \( \tilde{w} \) in terms of \( g \) cannot be found easily for \( n >= 0 \), and for simplicity the analytical solution for \( g(\eta, \zeta) \) with \( n = 0 \)

\[ g(\eta, \zeta) = \zeta e^{-\frac{1}{2}(\eta^2 + \zeta^2)} \]

will be used.

Eq. (F.69) then becomes

\[ \frac{\partial \tilde{w}}{\partial \zeta} + \frac{\partial \tilde{v}}{\partial \eta} = (4 - \eta^2 - \zeta^2)\zeta e^{-\frac{1}{2}(\eta^2 + \zeta^2)} \]  

(F.70)

An assumption has to be made about the relationship between \( \tilde{w} \) and \( \tilde{v} \) in order to find a solution. Apsley (1988) assumed that the resultant perturbation velocity \( \tilde{v}_r \) in the \( \eta-\zeta \) plane
is always directed at the origin $\eta = 0$, $\zeta = 0$. This was convenient for the $n = 0$ case because Eq. (F.70) could be written in terms of the resultant velocity $\vec{v}$, $r$ and $\phi$ with

$$
\begin{align*}
\vec{v} &= \vec{v}_r \cos \phi \\
\vec{w} &= \vec{v}_r \sin \phi \\
\phi &= \tan^{-1} \frac{\eta}{\zeta} \\
r &= \sqrt{\eta^2 + \zeta^2}
\end{align*}
$$

(F.71)

$$
\begin{align*}
\frac{\partial \vec{v}_r}{\partial \eta} \cos \phi - \frac{1}{r} \frac{\partial \vec{v}_r}{\partial \phi} \cos \phi + \frac{\vec{v}_r}{r} \sin^2 \phi \\
\frac{\partial \vec{w}}{\partial \zeta} &= \frac{\partial \vec{v}_r}{\partial \zeta} \sin \phi + \frac{1}{r} \frac{\partial \vec{v}_r}{\partial \phi} \sin \phi \cos \phi + \frac{\vec{v}_r}{r} \cos^2 \phi
\end{align*}
$$

Substituting Eq. (F.71) in Eq. (F.70)

$$
\frac{\partial \vec{v}_r}{\partial r} + \frac{\vec{v}_r}{r} = r \sin \phi (4 - r^2)e^{-\frac{1}{2}r^2}
$$

(F.72)

The solution to Eq. (F.72) is

$$
\vec{v}_r = \sin \phi \left\{ (r^2 - 1)e^{-\frac{1}{2}r^2} + \sqrt{\frac{\pi}{2}} \frac{\text{erf}(r/\sqrt{2})}{r} \right\}
$$

(F.73)

Figure F.4 show the profile for $\vec{w}$ on the centreline. The $1/r$ asymptotic behaviour is also shown. This asymptotic behaviour results in the profiles for $v$ and $w$ falling off less rapidly with $\eta$ and $\zeta$ than the profile for $u$.

Figure F.4: $\vec{w}$ on the centreline where $\eta = 0$

$\tau_{xz}$ can be solved by evaluating

$$
\tau_{xz} = 2\kappa u_* H \frac{\partial u}{\partial z}
$$

(F.74)

but the dispersion model described in Chapter 6 use a wake averaged $\Delta \tau_{xz}$ which can be derived from a momentum integral.
All that remains for a fully closed solution is the determination of $x_0$. An assumption will be introduced in Appendix G.

**Conclusion**

The main shortcoming of the theory is its reliance on the eddy viscosity hypothesis. The eddy viscosity hypothesis may well be appropriate, but the specification of the eddy viscosity is not a simple matter. Counihan et al.'s (1974) result of $\nu_M = 2\nu_0$ is not entirely convincing. Accurate specification of $\nu_M$ is more important for the three-dimensional wake model where a direct relationship between the couple on the body and the wake constant is assumed. For the two-dimensional wake model the strength of the wake $\tilde{C}$ was chosen to achieve the best agreement with the experimental data.

The solution remains fairly insensitive to $n$ even for $n = 0.3$, and the accuracy gained by allowing $n \geq 0$ may not warrant the added complexity. The $v$ and $w$ perturbation velocities extend far wider than the $u$ perturbation velocity. The magnitude falls off at exactly $x^{-2}$ regardless of the value of $n$. This fall-off rate is higher than the fall-off rate of the $u$ velocity due to the wake expansion in $y-z$ direction.
Building effects model derivation

This section investigates the implementation of the building wakes model in a commercial dispersion model. The dispersion model in question is ADMS, described by Carruthers et al. (1999), Robins et al. (1997) and Robins & Apsley (2000). Only the aspects of the model that are relevant to the present study are described. The model describes the effects of a rectangular obstacle with width $W$, height $H$ and length $L$. Non-rectangular obstacles are converted to equivalent rectangular obstacles. As will be seen in this section the building effects model is a combination of theory, empirical approximations and a practical computational algorithm.

![Figure G.1: Regions for the wake model](image)

In summary the important assumptions of the building wakes model in ADMS are:

1. Divide the space surrounding the obstacle into separate regions (Fig. G.1). Upwind region (U), Surrounding region (S), Near wake region (N), Main wake region (W) and External region (E).

2. The plume dimensions in the vicinity of the obstacle must be similar or smaller than the building dimensions to be influenced by the obstacle, or in practical terms $\sigma_y < W/2$ and $\sigma_z < H/2$.

3. Concentrations in the near wake region are assumed to be constant.

4. For a release in the recirculating region the concentration in the recirculating region $C_R$ is characterised by the concentration decay time scale $T_R$ of the recirculating region.

5. Streamline deflection is applied to the plume trajectory in S only if the obstacle is at an angle to the flow.
The bounding surface of the recirculating region is elliptical in \( x-z \) plane and parallel sided in the \( x-y \) plane.

External and main wake regions are sub-divided.

The lateral and vertical extents of the wake region are assumed to be \( \lambda W \) and \( \lambda H \) respectively, where the scale parameter \( \lambda = 1 + 2 \min(1, W/H) \).

The dispersion characteristics of the plume are only modified in regions W and E, but streamline deflection is modelled in regions S, W and E.

**Flow and turbulence characteristics in the wake**

The flow perturbation due to the obstacle can be calculated separately from the dispersion. Different formulations are also used in the near wake and the main wake.

**Near wake**

The streamline deflection due to the near wake is accounted for by the empirical relations of Fackrell (1984).

The length of the recirculation region is given by

\[
L_R = \frac{1.8(L/H)^{-0.3}}{1 + 0.24W/H}
\]  

(G.1)

The \( z \)-coordinate \( z_R(x) \) of the bounding surface of the near wake region is given by

\[
z_R = z_{Rmax} f(x) = z_{Rmax} \sqrt{1 - \left( \frac{x - x_{Rmax}}{x - x_{Rmax}} \right)^2}
\]  

(G.2)

and

\[
x_R = L_R + L/2
\]  

(G.3)

\( z_{Rmax} \) is the maximum height of the recirculation bubble, and \( x_{Rmax} \) is the \( x \) position at the maximum height. \( z_{Rmax} \) and \( x_{Rmax} \) are determined empirically. For cuboids the flow typically re-attaches on the obstacle roof, and hence \( x_{Rmax} \) is located at the trailing edge of the obstacle. \( z_{Rmax} \) is then equal to the obstacle height.

In the region above the recirculation bubble the plume trajectory is specified as

\[
\frac{dz_{SL}}{dx} = \frac{dz_R}{dx} \left( \frac{z_{Rmax} - z_{SL}}{x_{Rmax} - x_R} \right) \frac{d\theta}{\pi}, \quad 0 \leq \theta \leq \pi/4
\]  

(G.4)

with \( \frac{dz_R}{dx} \) calculated from Eq. (G.2), giving

\[
\frac{dz_R}{dx} = z_{Rmax} \left\{ 1 - \left( \frac{x - x_{Rmax}}{x_R - x_{Rmax}} \right)^2 \right\}^{-\frac{1}{2}} \frac{z - x_{Rmax}}{(x_R - x_{Rmax})^2}
\]  

(G.5)
It can therefore be seen that the plume deflection for a building normal to the flow will be zero, and a maximum for the flow at 45°.

Main wake

$u$, $v$ and $w$ in the main wake are given by a wake model such as the one derived in Appendix F. The wake model used in ADMS differs slightly from the one described in Appendix F, but is not available in the open literature. The wake model in Appendix F will therefore be used here. The Gaussian dispersion formulation assumes bulk spreading rates for the plume. To obtain bulk spreading rates in the wake, average values for the turbulence characteristics in the wake are calculated. The wake averaging is done as follows:

The limits $L_y$ and $L_z$ of the wake region $W$ are defined as

$$L_z = \gamma_z \ell(\xi) \quad \text{and} \quad L_y = \gamma_y \lambda \ell(\xi)$$  \hspace{1cm} (G.6)

$L_y$ and $L_z$ are defined to satisfy the relations in Eqs. (G.8–G.9).

$$L_y L_z \Delta u = \int_{0}^{\infty} \int_{0}^{\infty} u \, dy \, dz = \lambda \ell(\xi)^2 \bar{u} \xi^{-3} \sqrt{\pi/2}$$  \hspace{1cm} (G.8)

$$L_y L_z \Delta u^2 = \int_{0}^{\infty} \int_{0}^{\infty} u^2 \, dy \, dz = \lambda \ell(\xi)^2 \bar{u}^2 \xi^{-3} \pi/8$$  \hspace{1cm} (G.9)

From Eqs. (G.8–G.9) $\Delta u$ can be solved

$$\Delta u = \bar{u} \xi^{-3} \sqrt{2\pi/8}$$  \hspace{1cm} (G.10)

To solve $\gamma_y$ and $\gamma_z$ substitute Eq. (G.10) in Eq. (G.8).

$$L_y L_z = 4 \lambda \ell(\xi)^2$$  \hspace{1cm} (G.11)

If $\gamma_y = \gamma_z$ then $\gamma_y = \gamma_z = 2$.

For the shear stress the following integral relation is used:

$$L_y \Delta \tau = U_H \frac{d}{dx} \left( \int_{0}^{\infty} \int_{0}^{\infty} u \, dy \, dz \right)$$  \hspace{1cm} (G.12)

resulting in

$$\Delta \tau = U_H \bar{u} \left\{ (n + 2) \nu_M \frac{v}{HU_H} \right\}^{n+1/2} \frac{n + 1}{n + 2} \xi^{-2}$$  \hspace{1cm} (G.13)

The excess turbulent stresses in the wake are written as the following ratios

$$\frac{\Delta v'^2}{v'^2} = \frac{\Delta w'^2}{w'^2} = \frac{\Delta \tau}{\bar{u}^2}$$  \hspace{1cm} (G.14)

$x_0$ is determined by setting $u = 0$ at $x = L/2$, a purely pragmatic choice as noted by Robins & Apsley (2000).
Appendix G. Building effects model derivation

Dispersion modelling in the wake

Near wake

The concentration in the near wake is assumed to be uniform. The following relations apply

\[ C_R = \frac{Q_R T_R}{V_R} \]  
\[ V_R = \frac{\pi}{4} z_{\text{max}} W L_R \]  
\[ T_R = \frac{11 H / U_H}{(H / W)^{3/2} + 0.6} \]

where \( C_R \) is the concentration in the recirculation region, \( V_R \) is the volume of the recirculation region and \( T_R \) is the concentration decay time scale.

The procedure adopted is to calculate the average concentration on the surface of the recirculation bubble, considering the plume deflection due to the obstacle considered, but not the effect of the obstacle on the plume spread. This average concentration is then assumed as the concentration \( C_R \) in the recirculation bubble. From Eqs. (G.15–G.17) a source strength for a virtual entrained source in the recirculation bubble can be defined. The fraction of the entrained source strength \( Q_R \) relative to the original source strength is defined as

\[ \epsilon = \frac{Q_R}{Q} \]  

The original source strength must then be reduced by \( \epsilon \) downstream of the recirculation region.

Main wake

For the dispersion model in the main wake the extents \( L_y \) and \( L_z \) are used to subdivide the wake into regions WW, EW, WE and EE, as shown in Fig. G.2.

The plume spread rates in the wake are represented in differential form in Eqs. (G.19) & (G.20). The first term on the right represents the reduction in plume spread due to streamline convergence. The other term accounts for the reduced mean wind speed and excess turbulence in the wake.

\[ \frac{d \sigma_y^{(w)}}{dx} = \frac{1}{2} \sigma_y^{(w)} \frac{d}{dx} \left( \frac{\Delta u}{U_H} \right) + \frac{1 + \Delta \nu^2}{1 - \frac{\Delta \nu}{U_H}} \frac{d \sigma_y^{(E)}}{dx} \bigg|_{\sigma_y^{(E)} = \sigma_y^{(w)}} \]  
\[ \frac{d \sigma_z^{(w)}}{dx} = \frac{1}{2} \sigma_z^{(w)} \frac{d}{dx} \left( \frac{\Delta u}{U_H} \right) + \frac{1 + \Delta \nu^2}{1 - \frac{\Delta \nu}{U_H}} \frac{d \sigma_z^{(E)}}{dx} \bigg|_{\sigma_z^{(E)} = \sigma_z^{(w)}} \]  

\[ \frac{d \sigma_x^{(w)}}{dx} = \frac{1}{2} \sigma_x^{(w)} \frac{d}{dx} \left( \frac{\Delta u}{U_H} \right) + \frac{1 + \Delta \nu^2}{1 - \frac{\Delta \nu}{U_H}} \frac{d \sigma_x^{(E)}}{dx} \bigg|_{\sigma_x^{(E)} = \sigma_x^{(w)}} \]
Appendix G. Building effects model derivation

<table>
<thead>
<tr>
<th></th>
<th>EE</th>
<th>WE</th>
<th>EE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>y</td>
<td>&gt; L_y, z &gt; L_z$</td>
<td>$</td>
</tr>
<tr>
<td>EW</td>
<td>$</td>
<td>y</td>
<td>&gt; L_y, z &lt; L_z$</td>
</tr>
</tbody>
</table>

**Figure G.2:** Sub-regions on a $x = c$ plane in regions W and E

\[
\frac{d\sigma_y(x)}{dx} \quad \text{and} \quad \frac{d\sigma_z(x)}{dx}
\]
are evaluated at virtual downstream locations $x_{vy}$ and $x_{vz}$ such that $\sigma_y(x_{vz}) = \sigma_y^{(E)}(x)$ and $\sigma_z(x_{vy}) = \sigma_z^{(W)}(x)$.

Because of the high wind shear in the wake single values of the plume spreads are inappropriate in the wake. Table G.1 summarises the combinations of plume spread values that are used in the wake model depending on the plume centre $(\gamma, \delta)$ and the receptor $(\alpha, \beta)$ locations.

**Table G.1:** Choice of $\sigma_y$ and $\sigma_z$ based on plume centre and receptor location

<table>
<thead>
<tr>
<th>Receptor location $(\alpha, \beta)$</th>
<th>Plume centre $(\gamma, \delta)$</th>
<th>WW</th>
<th>WE</th>
<th>EW</th>
<th>EE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_y^{(W)}$, $\sigma_z^{(W)}$</td>
<td>$\sigma_y^{(E)}$, $\sigma_z^{(E)}$</td>
<td>$\sigma_y^{(W)}$, $\sigma_z^{(W)}$</td>
<td>$\sigma_y^{(E)}$, $\sigma_z^{(E)}$</td>
<td>$\sigma_y^{(W)}$, $\sigma_z^{(W)}$</td>
</tr>
<tr>
<td>WW</td>
<td>$\sigma_y^{(W)}$, $\sigma_z^{(W)}$</td>
<td>$\sigma_y^{(E)}$, $\sigma_z^{(E)}$</td>
<td>$\sigma_y^{(W)}$, $\sigma_z^{(W)}$</td>
<td>$\sigma_y^{(E)}$, $\sigma_z^{(E)}$</td>
<td>$\sigma_y^{(W)}$, $\sigma_z^{(W)}$</td>
</tr>
<tr>
<td>WE</td>
<td>$\sigma_y^{(W)}$, $\sigma_z^{(W)}$</td>
<td>$\sigma_y^{(E)}$, $\sigma_z^{(E)}$</td>
<td>$\sigma_y^{(W)}$, $\sigma_z^{(W)}$</td>
<td>$\sigma_y^{(E)}$, $\sigma_z^{(E)}$</td>
<td>$\sigma_y^{(W)}$, $\sigma_z^{(W)}$</td>
</tr>
<tr>
<td>EW</td>
<td>$\sigma_y^{(W)}$, $\sigma_z^{(W)}$</td>
<td>$\sigma_y^{(E)}$, $\sigma_z^{(E)}$</td>
<td>$\sigma_y^{(W)}$, $\sigma_z^{(W)}$</td>
<td>$\sigma_y^{(E)}$, $\sigma_z^{(E)}$</td>
<td>$\sigma_y^{(W)}$, $\sigma_z^{(W)}$</td>
</tr>
<tr>
<td>EE</td>
<td>$\sigma_y^{(W)}$, $\sigma_z^{(W)}$</td>
<td>$\sigma_y^{(E)}$, $\sigma_z^{(E)}$</td>
<td>$\sigma_y^{(W)}$, $\sigma_z^{(W)}$</td>
<td>$\sigma_y^{(E)}$, $\sigma_z^{(E)}$</td>
<td>$\sigma_y^{(W)}$, $\sigma_z^{(W)}$</td>
</tr>
</tbody>
</table>

The final concentration formulation Eq. (G.21) introduces the factors $q_{\alpha\beta\gamma\delta}$ to ensure that the concentrations are continuous over the boundaries of the regions in Fig. G.2, and that the conservation of mass is ensured.

\[
C = \left( \frac{q_{\alpha\beta\gamma\delta}Q}{U_H} \right) C_Y(y, y_p, \sigma_y) C_Z(z, z_p, \sigma_z, h) \tag{G.21}
\]

with

\[
C_Y(y, y_p, \sigma_y) = \frac{1}{\sqrt{2\pi\sigma_y}} \exp \left[ -0.5 \frac{(y - y_p)^2}{\sigma_y^2} \right] \tag{G.22}
\]

and

\[
C_Z(z, z_p, \sigma_z, h) = \frac{1}{\sqrt{2\pi\sigma_z}} \left\{ \exp \left[ -0.5 \frac{(z - z_p)^2}{\sigma_z^2} \right] + \exp \left[ -0.5 \frac{(z + z_p)^2}{\sigma_z^2} \right] \right\} \tag{G.23}
\]

$q_{\alpha\beta\gamma\delta}$ is then written as

\[
q_{\alpha\beta\gamma\delta} = a_{\alpha\beta} q_{\gamma\delta} \tag{G.24}
\]
The solutions to $q_{\alpha\beta\gamma}$ and $a_{\alpha\beta}q_{\gamma}$ are then

\[
\frac{1}{q_{ww}} = \frac{u_{ww}}{U_H} \int_W C_Y(y, y_p, \sigma_{yw}) \, dy \int_W C_Z(z, z_p, \sigma_{zw}, h) \, dz \\
+ A_Y \int_W C_Y(y, y_p, \sigma_{yw}) \, dy \int_E C_Z(z, z_p, \sigma_{zE}, h) \, dz \\
+ A_Y A_Z \int_W C_Y(y, y_p, \gamma_E) \, dy \int_E C_Z(z, z_p, \sigma_{zE}, h) \, dz \\
+ A_Y A_Z \int_E C_Y(y, y_p, \sigma_{yE}) \, dy \int_E C_Z(z, z_p, \sigma_{zE}, h) \, dz
\]  
(G.25)

\[
\frac{1}{q_{wE}} = \frac{u_{ww}}{U_H} A_Y \int_W C_Y(y, y_p, \sigma_{yE}) \, dy \int_W C_Z(z, z_p, \sigma_{zw}, h) \, dz \\
+ A_Y A_Z \int_W C_Y(y, y_p, \sigma_{yE}) \, dy \int_E C_Z(z, z_p, \sigma_{zE}, h) \, dz \\
+ A_Y A_Z \int_E C_Y(y, y_p, \sigma_{yE}) \, dy \int_E C_Z(z, z_p, \sigma_{zE}, h) \, dz \\
+ A_Y A_Z \int_E C_Y(y, y_p, \gamma_E) \, dy \int_E C_Z(z, z_p, \sigma_{zE}, h) \, dz
\]  
(G.26)

\[
\frac{1}{q_{EW}} = \frac{u_{ww}}{U_H} A_Z \int_W C_Y(y, y_p, \sigma_{yw}) \, dy \int_W C_Z(z, z_p, \sigma_{zE}, h) \, dz \\
+ A_Z \int_W C_Y(y, y_p, \sigma_{yw}) \, dy \int_E C_Z(z, z_p, \sigma_{zE}, h) \, dz \\
+ A_Y A_Z \int_E C_Y(y, y_p, \gamma_E) \, dy \int_W C_Z(z, z_p, \sigma_{zE}, h) \, dz \\
+ A_Y A_Z \int_E C_Y(y, y_p, \sigma_{yE}) \, dy \int_E C_Z(z, z_p, \sigma_{zE}, h) \, dz
\]  
(G.27)

\[
\frac{1}{q_{EE}} = \frac{u_{ww}}{U_H} A_Y A_Z \int_W C_Y(y, y_p, \sigma_{yE}) \, dy \int_W C_Z(z, z_p, \sigma_{zE}, h) \, dz \\
+ A_Y A_Z \int_W C_Y(y, y_p, \sigma_{yE}) \, dy \int_E C_Z(z, z_p, \sigma_{zE}, h) \, dz \\
+ A_Y A_Z \int_E C_Y(y, y_p, \gamma_E) \, dy \int_W C_Z(z, z_p, \sigma_{zE}, h) \, dz \\
+ A_Y A_Z \int_E C_Y(y, y_p, \sigma_{yE}) \, dy \int_E C_Z(z, z_p, \sigma_{zE}, h) \, dz
\]  
(G.28)

where

\[
A_Y = \frac{C_Y(y, y_p, \sigma_{yw})}{C_Y(y, y_p, \gamma_E)},
\]  
(G.29)
Appendix G. Building effects model derivation

\[ A_Z = \frac{C_2(L_2, z_p, \sigma_{ZW})}{C_2(L_2, z_p, \sigma_{ZE})} \]  \hspace{1cm} (G.30)

and

\[ a_{\alpha\beta} = 1 \text{ if } \sigma_{Y_{a\beta\gamma\delta}} = \sigma_{YW} \text{ and } \sigma_{Z_{a\beta\gamma\delta}} = \sigma_{ZW} \]

\[ a_{\alpha\beta} = A_Z \text{ if } \sigma_{Y_{a\beta\gamma\delta}} = \sigma_{YE} \text{ and } \sigma_{Z_{a\beta\gamma\delta}} = \sigma_{ZW} \]

\[ a_{\alpha\beta} = A_Y \text{ if } \sigma_{Y_{a\beta\gamma\delta}} = \sigma_{YW} \text{ and } \sigma_{Z_{a\beta\gamma\delta}} = \sigma_{ZE} \]

\[ a_{\alpha\beta} = A_Y A_Z \text{ if } \sigma_{Y_{a\beta\gamma\delta}} = \sigma_{YE} \text{ and } \sigma_{Z_{a\beta\gamma\delta}} = \sigma_{ZE} \]  \hspace{1cm} (G.31)

\( \sigma_{Y_{a\beta\gamma\delta}} \) and \( \sigma_{Z_{a\beta\gamma\delta}} \) are obtained from Table G.1

Far wake

There is no downstream limit for the main wake, and it is therefore important that the building effects model reverts to the underlying Gaussian dispersion model for distances far downstream the obstacle.

Implementation

The concepts described above were implemented in a dispersion model, written in C. Fig. G.3 shows an example of an input file used by the program. The wind profile and turbulence characteristics are specified as \( z \) location and value pairs. Chapter 6 compares experimental results obtained for this study with dispersion modelling solutions.
Appendix G. Building effects model derivation

1.1 m/s Stable boundary layer, 100 mm source with cube.

BL

u* = 0.027
z0 = 0.001
H = -22.0
DELTA = 0.6
N = 0.0
SIGMAWPROFILE 21
0.000000000 0.031622777 0.350000000 0.302944969 0.402700005 0.033309908
0.462999992 0.033669214 0.053250000 0.034015379 0.06125 0.034336666
0.070459999 0.034622623 0.081040001 0.038468617 0.093220001 0.039071855
0.107230003 0.035264742 0.123449998 0.035510787 0.141699995 0.035946321
0.163179993 0.036777588 0.187699997 0.038123064 0.215899994 0.040041741
0.248350006 0.04254413 0.285640015 0.046550904 0.328549988 0.048924759
0.377929993 0.052311213 0.446999992 0.055147239 0.500000000 0.056425854
SIGMAVPROFILE 18
0.000000000 0.038729833 0.053250000 0.046589556 0.061250000 0.047684096
0.070459999 0.04878673 0.081040001 0.049881891 0.093220001 0.050964213
0.107230003 0.052034527 0.123449998 0.053100876 0.141699995 0.054197212
0.163179993 0.055390405 0.187699997 0.05679890 0.215899994 0.058330404
0.248350006 0.05972281 0.285640015 0.06046881 0.328549988 0.059960254
0.377929993 0.06876589 0.434699992 0.05826951 0.500000000 0.057374698
UPROFILE 21
0.000000000 0.000000000 0.035000000 0.031939326 0.040270000 0.41629833
0.046299999 0.44971834 0.053250000 0.492285818 0.061250000 0.51925528
0.070459999 0.56143868 0.081040001 0.607201159 0.093220001 0.656076146
0.107230003 0.703793705 0.123449998 0.751461983 0.141699995 0.79518044
0.163179993 0.831599832 0.187699997 0.859616876 0.215899994 0.881347716
0.248350006 0.901798487 0.285640015 0.920266151 0.328549988 0.93867335
0.377929993 0.94561366 0.434699992 0.962218165 0.500000000 0.978626907
TPROFILE 21
0.000000000 13.000000000 0.035000000 19.32606506 0.040270000 20.22813225
0.046299999 21.27300835 0.053250000 22.47579575 0.061250000 23.8303371
0.070459999 25.3073616 0.081040001 26.8456974 0.093220001 28.4071846
0.107230003 29.94966507 0.123449998 31.42490087 0.141699995 32.77852631
0.163179993 33.9271133 0.187699997 34.92141724 0.215899994 35.6191774
0.248350006 36.08302307 0.285640015 36.46772766 0.328549988 37.0526123
0.377929993 37.8496666 0.434699992 38.65368652 0.500000000 39.01019669
BUILDING 1
x=0.05 y=0
H = 0.10
W=0.10
L=0.10
THETA=0.0
GRID 1 # Centreline GLC
x' = 0 x' = 10
y' = 0 y' = 0
z' = 0 z' = 0
nx = 101 ny = 1 nz = 1
SOURCE 1
x=0
y=0
H = 0.1
Q = 1

Figure G.3: Example input file for dispersion model