The Interaction between Soil and Large Diameter Rigid Pipe

by

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Abstract

The aim of this thesis is to use numerical modelling to evaluate practical guidelines for soil and large diameter rigid pipe interaction problems. A detailed critique is also given on current design methods, followed by an explanation of the alternative general criteria for field performance. Conventional design methods are generally based on Marston-Spangler theory developed for small diameter concrete non-prestressed pipe in the 1920's, which take no account of the actual material properties, true geometry and field construction procedure. Numerical analyses have been performed using the geotechnical finite element package CRISP, together with data from full scale site installation trials and three-edge bearing load tests.

Practical guidance is given for various construction conditions. A brief summary of the main factors influencing the behaviour of buried rigid pipe are:

- the elastic properties of both the pipe and the surrounding soil,
- the placement, compaction, and any subsequent volumetric changes in the soils placed around the pipe,
- the variation of earth cover, geometry and bedding conditions
- the weight of water, internal pressures, and prestressing effects from helically-wound steel wires.

From this study, it was found that the prestressing and internal water pressures significantly affect the behaviour of buried pipe, leading to the change of bending stresses in the pipe wall. The improvement of construction techniques is also very important in terms of bedding condition and compaction of side fill, etc. In conclusion, current design practice is highly deficient, causing very uneconomical designs for buried large diameter rigid pipe. The finite element method has provided useful insights into an important practical engineering problem, and has indicated the critical design factor for large diameter buried rigid pipe.
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CHAPTER 1 - INTRODUCTION

Hitherto, little has been written about the interaction between soil and large diameter rigid pipe. It is therefore not surprising that no information is available on the interaction between soil and large diameter prestressed pipe in particular. In 1996, an opportunity arose to make a systematic study of this subject following about 15 years of experience (on the part of the writer) associated with the manufacturing and installation of large diameter prestressed and non-prestressed pipes.

From this study, it has been recognised that the current design practice is inappropriate for soil and pipe interaction problems. This is because it is not only impossible to elaborate a sensitive response of the pipe to the surrounding soils, but also rarely possible to account for the sophisticated deformation characteristics of both the pipe and the surrounding soils caused by the relative stiffness between the two media and the construction procedure. The conventional design method is generally based on the early development of rational methods for concrete pipe design in the 1920's (termed Marston-Spangler theory), which does not take into consideration the true geometry and actual material properties. It is acknowledged that it is difficult to keep step with the variety of pipes (both in size and constituent materials) developed over time, but the dominant viewpoint regarding current design method for the pipelines is that they are unacceptable. Moreover, improved field compaction techniques, which result in an increase in the lateral earth pressures, are usually neglected.

The aim of this study is to use numerical techniques to analyse the influence of the various properties of surrounding soils on the structural behaviour of the pipe wall: especially the reaction of bedding and subgrade. Various bedding conditions are appraised, together with changes in material stiffness. A critical assessment of the deformation characteristics of the prestressed and non-prestressed pipes, which
affect the flexural bending stresses in the pipe wall, is the main theme of the study. Practical guidance to the rigid pipeline industry will be given for various construction conditions. No analytical solution has been found to date to solve these complicated problems, necessitating the use of numerical technique.

Through finite element analysis using the CRISP program, it is confirmed that the current design method is too conservative in general. For the period of service, the existence of cracks in the core leads to the worst situation due to the effect of internal water pressures being neglected in traditional design practice. For instance, the trend of the relative movement between the pipe and surrounding soils causes pre-cracks near the support area to be propagated more seriously. For this reason, the influence of the reaction of the bedding and subgrade will be focussed on, although the stiffness of the soil appears to be insignificant compared to the pipe. Due to the paucity of studies in this subject, most of comments stem from either the writer’s observations during his professional practice, or the writer’s understanding of the structural mechanics of prestressed and non-prestressed rigid pipes.

The results of three-edge bearing tests for prestressed concrete cylinder pipe in the range of 1600 mm to 4000 mm diameter are evaluated and included in this thesis, concerning the flexural rigidity of the pipe structure. The input data for concrete properties used in the numerical analyses was based on the experimental data obtained from the site in Libya. The concept of the shaped bedding installation was verified by full scale site trials conducted in the field, but the data are briefly described in Chapter 6, since the prerequisite of the site trials is fully explained in the results section dealing with analysis.

The structure of the thesis is divided into six parts:

- Review of past work on buried pipelines
- Introduction to CRISP and some interpretation techniques
- Numerical modelling of rigid pipe
The composition of the six chapters of this thesis is given below.

Chapter 2 begins with a review of the current design practice employed by most engineers. This includes a historical development of the Marston-Spangler theory together with relevant experimental work. Through a wide range of the geotechnical problems, the basic concept of the active and passive modes is reviewed and discussed in detail.

Chapter 3 describes a brief introduction to both the CRISP finite element program and the constitutive models used for the numerical modelling. The interpretation techniques are also presented in terms of the key work required for post processing the analysis. This is divided into two categories; firstly, the least squares smoothing method for the discontinuous finite element functions, and secondly the general bending theory for the computation of the normal and shear stresses in soil-pipe interaction problems, and the flexural bending moment and stresses generated around the pipe wall. A detailed critique of the program in the context of compaction-induced stresses in soil-structure interaction problems is also given in this chapter.

Concerns about the validity and reliability of numerical modelling are reviewed and discussed in Chapter 4. The numerical modelling effect is linked to both the prestressed and the non-prestressed rigid pipes and a framework for assessment is constructed by comparing numerical analysis with a pure analytical solution for a closed circular ring. The agreement is excellent for non-prestressed pipe. The stiffness and strength parameters for the constituent materials are evaluated as a function of confining pressure. The non-linear nature of pipe behaviour is also examined by using the data obtained from the full scale three-edge bearing tests performed at the site.
Following this appraisal of numerical modelling, Chapter 5 underlines some serious shortcomings in existing design methods, which mostly arise from interaction between soil and pipe. Numerous investigations are conducted using combinations of various installation conditions and material properties, including the prestressing effect and internal water pressures.

Chapter 6 provides a summary and discussion of the numerical research work presented herein, and the results of full scale site installation trials and three-edge bearing tests carried out at the GMRP (Great Man-Made River Project) site in Libya. Some experimental data concerning the elastic modulus of concrete are also provided.

Finally, Chapter 7 restates the problems/issues presented at the outset of the thesis, together with the main conclusions and practical implications of the main findings. Recommendations for further research are also given.
CHAPTER 2 - REVIEW OF PAST WORK ON BURIED PIPELINES

2.1 Introduction

The magnitude of earth pressure imposed on the top of buried structures has been developed by Marston et al.(1913 and 1917), Marston(1930), Schlick et al.(1926), Schlick(1920, 1931, 1932 and 1952), Spangler et al.(1926), Spangler(1933 and 1950), and associates at Iowa State University. Terzaghi also contributed to the engineering of underground structures, based on extensive experiments in the same period; for instance, he introduced the arching effect of granular materials above a yielding trapdoor(1936). Later, the application of arching effect was widely extended to any sand layer on a locally yielding strip at depth such as a rectangular culvert or a tunnel(1943). Since Terzaghi’s(1936) approach was given a detailed critique followed by site investigation and laboratory observation for both the pressures imposed on a trap door or buried objects, numerous analytical studies (including numerical modelling) have been presented by Jaky(1948), Caquot(1957), Jakobson(1958), Balla(1961), Comerre et al.(1961), Finn(1963), Sutherland(1965), Vesic(1971), Braja et al.(1975), Laesen, H. (1977), Vardoulakis et al.(1981), McVay(1982), Gumbel, J.E. (1983), Koutsabeloulis(1985), Oak(1988), Koutsabeloulis et al.(1989), Fujita(1994), etc.. The behaviour of soils above underground structures and that of granular materials above a trapdoor appeared to be similar. It is therefore not surprising that the theory of earth pressure on underground structures introduced by Terzaghi is fundamentally the same as Marston’s earth pressure formula for trench conditions which assumes that the structure is constructed or installed within the trench and then backfilled, since the background to both theories proposed by Marston and Terzaghi were fundamentally based on the arching theory.
in a silo provided by Janssen's active mode in 1895.

In contrast, the problems of breakout forces of objects embedded in sediments subjected to upward forces at the ocean bottom were encountered in many marine operations. These objects require certain normal outward forces to remove them from the ground, acting similar to an earth anchor in a retaining structure. This condition requires a different approach for estimating the magnitude and distribution of the stresses, compared to the trap door problem, as it is more of a passive mode. Here, the breakout force implies the magnitude of force required for the complete withdrawal of the object when the ocean bottom sediment is mobilised. The passive mode can also be idealised by the classical bin theory when the displacement of the yielding strip occurs upwards.

The concept of pressure distributions on buried structures is therefore classified into two main categories; namely, active and passive conditions, which will be discussed and reviewed in the following sections.

2.2 Earth Pressure with Local Yield at Depth

2.2.1 Active mode

The stress distribution on an underground structure is highly dependent on its geometry and the relative stiffness of the buried structure and surrounding soils. As has been introduced, the experimental studies using a narrow and long trapdoor in the base of a soil bin with respect to the arching action were carried out by Terzaghi(1936), where the trapdoor was relatively flexible compared to the base platform. On
filling, the trapdoor starts deflecting downward, whereas the surrounding floor platform remains relatively fixed. With the deformation of trapdoor, a large degree of pressure on the door is transferred to the adjoining parts of the fixed base platform. This phenomenon is totally due to the mobilisation of shearing resistance along the surfaces of the soil prism directly over the trapdoor, arching in an upward direction.

Figure 2.1 presents a typical example of Terzaghi's experimental work on a trap door 7.3 cm wide and 46.3 cm long, using a sand in a dense state. The pressures were measured by the steel tape method. Figures 2.1(a) and (b) illustrate the pressure distributions induced before yielding the trap door and when the trap door has been deformed downward by 1 mm, respectively. The maximum lateral pressure ratio $K$, along a vertical section directly above the trap door was about 1.6 at depth brought about by critical arching effect. The minimum pressure on the trapdoor was observed when the ratio of vertical downward movement to the trapdoor width reaches around 0.8% for dense sand and 2.0% for loose sand, but the ultimate loads for both samples occurred at approximately 8-11% of downward movement. The results of Terzaghi's experimental work are summarised in Table 2.1.

In a similar approach to trap door problems, the effects of local yield of the horizontal strip $ab$ (which is a part of the solid base supporting the sand layer) are shown in Figure 2.2(a). Lowering the strip $ab$ causes the sand layer to be deformed downward. Here, it is assumed that the extent of the vertical movement is enough to generate failure of the sand layer on a rupture surface $bd$ that passes through the edge of fixed end 'b' extending to the free surface of the sand layer. Accordingly, the trend of displacement will be opposed to frictional resistance along the slip surfaces $ac$ and $bd$ leading to boundaries forming between the deformed and stationary mass of soil. This frictional resistance results in the total vertical pressure on the yielding strip being decreased by the same amount; conversely the pressures on
parts adjacent to the yielding strip will be increased.

As can be seen, the failure lines of the deformed body are non-linear. The greatest angle of the slope of the failure line would occur near the ground surface at around $90^\circ$, and the slope near the yielding strip becomes approximately $45^\circ + \phi/2$ (Terzaghi, 1943). However, as yet there is no method to estimate the slip surfaces $ac$ and $bd$ due to the difficulties in finding a comprehensive solution. For this reason, the pressure distributions on underground structures including buried pipes are usually determined by Marston’s theory, assuming the linear failure lines $aa'$ and $bb'$ as shown in Figure 2.2(a). The difference between areas $abdc$ and $abb'a'$ gives rise to an error when calculating the pressure on the yielding strip.

2.2.2 Passive mode

In a similar manner to the active mode, Figure 2.2(b) illustrates an idealisation of a slip surface for objects embedded in the seabed, suggested by Balla in 1961. In his proposal, the circle intercepts the surface of the ground at an angle of $45^\circ - \phi/2$ and the plate edge at an angle of $90^\circ$. It was confirmed that the assumption was in good agreement with the shape of the earth mass breaking out in the laboratory model tests in wet sand.

For shallow earth cover in Figure 2.2(b) the difference between the areas $abdc$ and a vertical slip surface $abb'a'$ is relatively insignificant compared to the deep earth cover. Therefore, Spangler’s formula for embankment conditions seems satisfactory for computing the vertical load imposed on a buried object, as discussed in the subsequent section.

Vesic (1971) made observations from small scale tests performed at Duke University and stated that this failure shape occurs only in the case of
shallow objects in dry soils, and the shape in wet soils at shallow depth
could be a vertical cylindrical around the perimeter of the object. In a
passive condition for deep foundations, the shape of the yielding surface
is unlike the general shear failure shown in Figure 2.2(b), irrespective of
the properties of the soil. The slip surface progresses vertically for
considerable distances, with failure patterns similar to that of punching
shear failure..

Moreover, for an ideal granular mass in plane strain a series of trap
door model tests were carried out by Ladanyi et al. (1969) using
Schneebeli’s rod model at Montreal University, with a stationary camera
during lowering and raising of the model structure. It was confirmed
that the displacement trajectories are almost identical with those
assumed in Figures 2.2(a) and (b).

2.2.3 Earth pressures on yielding strip

2.2.3.1 Janssen’s arching analysis

The arching theory in plane strain was initiated by Janssen in 1895.
With reference to Figure 2.3, the earth pressure at a depth can be taken
as the difference between the weight of the prism abb’ a’ and the
frictional resistance exerted along the sides of the prism aa’ and bb’.

The theory was derived based on the following assumptions (Bulson,
1985):

(i) the coefficient of horizontal pressure to vertical pressure
    at any depth is a constant.
(ii) the coefficient of friction tan ϕ, is constant along the
    whole depth of the boundary of yielding and unyielding
    parts.
(iii) a unit length is assumed in the longitudinal direction and earth cover \( H \) is considered from the ground surface.

(iv) the shear resistance of cohesive materials is determined by the Mohr–Coulomb equation \( \tau = c + \sigma \tan \phi \).

Now, consider a horizontal element of infinitesimal thickness at a depth \( z \) from the ground surface. The horizontal pressure \( \sigma_h \), acting along the slip surface at depth \( z \), will be a product of the lateral earth pressure coefficient \( K \), and the vertical pressure \( \sigma_v \), imposed on the horizontal element. The shear resistance on the vertical slip surface is therefore \( c + \sigma_v K \tan \phi \). The weight of the thin element is given by \( B_t \gamma dz \), where \( \gamma \) is the unit weight of the soil. Therefore, the vertical equilibrium equation for the element per unit length is,

\[
B_t \left( \sigma_v + d \sigma_v \right) = B_t \sigma_v + B_t \gamma dz - 2c dz - 2K \sigma_v dz \tan \phi
\]

To simplify the above equation, the term \( \tan \phi \) is replaced with \( \mu \). In addition, it is assumed that a uniform surcharge pressure \( \sigma_t \), on the ground surface exists. The equilibrium equation at depth \( z \), can be hence expressed as follows:

\[
\sigma_v = \frac{B_t (\gamma - 2c/B_t)}{2K \mu} \left[ 1 - e^{-2K\mu z/B_t} \right] + \sigma_t e^{-2K\mu z/B_t} \tag{2.1}
\]

For non-cohesive materials without surcharge loading Eqn 2.1 becomes,

\[
\sigma_v = \frac{B_t \gamma}{2K \mu} \left[ 1 - e^{-2K\mu z/B_t} \right] \tag{2.2}
\]

As can be seen, the magnitude of pressure is proportional to the value of \( (1 - e^{-2K\mu z/B_t})/2K \mu \), termed a coefficient of load in this thesis, and the generated earth pressure can therefore be expressed by \( \sigma_v = f(\phi, K, H/B_t) \).
In the above, the shear resistance due to the cohesion of the fill material should not be considered, because newly worked soil has no cohesion. To regain bond strength after a failure event due to flowing and dilation, soil needs a long time for ageing and creep. The peak strengths are caused by inter-particle interlocking and the strength depends on the packing geometry, not on the chemistry of bonds. For this reason, the factors affecting peak strength of dense soils are interlocking and friction among the soil particles (Schofield, 1998). According to the Rowe’s (1954) experiments to find the degree of mobilisation of the friction angle in shear tests, a rapid loading caused the value of $\phi$ to be 1–1.5\(^\circ\) higher, whereas samples left stressed for 24-hour periods resulted in the value to be 2–3\(^\circ\) lower. These might be due to the dissipation of an interlocking component. Under these circumstances, Eqn 2.2 seems to be more appropriate for determining the vertical pressure.

2.2.3.2 Theory of elasticity considering boundary conditions

Finn (1963) attempted to develop a coherent theory for the vertical and lateral pressures in a soil mass with a yielding strip, based on the theory of elasticity and considering it as a general boundary value problem in soil mechanics. His analytical expressions may provide the best theoretical basis for analysis, giving detailed pressure distribution very close to those observed, compared to the writer’s studies. For instance, although the theories of the lateral earth pressure against retaining structures have been investigated by many researchers, they are restricted primarily to an assessment of lateral pressures induced by surface loads only, and the boundary conditions of the problem were ignored.

Finn’s analytical expressions were based on a two-dimensional soil mass with a yielding strip similar to Terzaghi’s expression (1943), and
considered two types of displacement of the yielding base; i.e. pure translation downward and pure rotation. When the displacement of the soil base occurs in the vertical direction, the soil mass exerts some drag on the base in reality. However, a small displacement does not develop the maximum angle of wall friction, as shown by many researchers, such as Terzaghi (1934), Rowe (1962, 1969), Rowe and Peaker (1965), Wroth and Bassett (1965), Bolton (1986, 1991), etc. Thus, it can be assumed that no slip occurs between the base and the soil when a small displacement takes place, implying the strains at the base in the horizontal direction to be zero. Pure translation of the base with conditions of both slip and no slip on the rigid base are therefore reviewed and summarised in the following.

No friction on base

Figure 2.4 presents the boundary conditions of the soil mass for the yielding strip with a frictionless base, which provides no shear stress $\tau_{xy(b)}$ on the base of the soil mass. Here, it is assumed that a small prescribed displacement $d$, of the yielding base is insufficient to generate slip surfaces. Therefore, the vertical stress on the surface of soil mass at an infinite distance ($y = \infty$) will be zero and the vertical displacement is, subsequently, assumed to have been dissipated. The complete stress distribution for pressures imposed on a smooth, horizontal yielding strip of width $(2b)$ in a mechanism of elasticity is expressed by the sum of stresses due to displacement and to the unit weight of the material $\gamma$:

$$\sigma_x = \sigma_{xd} + \sigma_{x\gamma} \quad (2.3a)$$
$$\sigma_y = \sigma_{yd} + \sigma_{y\gamma} \quad (2.3b)$$

and

$$\tau_{xy(b)} = \tau_{xyd(b)} \quad (2.3c)$$
where, the stresses due to displacement of the soil mass are,

\[
\sigma_{xd} = \frac{d}{2\beta\pi} \left[ \frac{x+b}{(x+b)^2 + y^2} - \frac{x-b}{(x-b)^2 + y^2} \right] - \frac{2(x+b)v^2}{[(x+b)^2 + y^2]^2} + \frac{2(x-b)v^2}{[(x-b)^2 + y^2]^2}
\] (2.4a)

\[
\sigma_{yd} = \frac{d}{2\beta\pi} \left[ \frac{x+b}{(x+b)^2 + y^2} - \frac{x-b}{(x-b)^2 + y^2} \right] + \frac{2(x+b)v^2}{[(x+b)^2 + y^2]^2} - \frac{2(x-b)v^2}{[(x-b)^2 + y^2]^2}
\] (2.4b)

and

\[
\tau_{xyd(b)} = \frac{yd}{2\beta\pi} \left[ \frac{y^2 - (x-b)^2}{[(x+b)^2 + y^2]^2} - \frac{y^2 - (x+b)^2}{[(x-b)^2 + y^2]^2} \right]
\] (2.4c)

where, \( \beta \) is the reciprocal of Young’s modulus of the soil mass in plain strain, assuming the displacement in the horizontal direction to be zero. From Hooke’s law, \( \beta = (1 - \nu^2)/E_s \).

The stresses due to the self-weight of soil are,

\[
\sigma_{x\gamma} = \frac{\nu}{1-\nu} \gamma(H-y)
\] (2.5a)

\[
\sigma_{y\gamma} = -\gamma(H-y)
\] (2.5b)

and

\[
\tau_{xy\gamma(b)} = 0
\] (2.6c)

To find the horizontal pressure at the yielding strip where \( y = 0 \), Eqn 2.3a (=2.4a+2.5a) is modified to,

\[
\sigma_{h,y=0} = \frac{\nu}{1-\nu} \gamma H - \frac{dB_1}{2\beta\pi} \left[ \frac{1}{\left( \frac{B_1}{2} \right)^2 - x^2} \right]
\] (2.6a1)
where \( B_t \) corresponds to \( 2b \). The horizontal pressure through the centre line of the yielding strip \((x = 0)\) is,

\[
\sigma_{h,x=0} = \frac{\nu}{1-\nu} \gamma(H-y) - \frac{d B_t}{2\beta\pi} \left| \frac{(-\frac{B_t}{2})^2 - y^2}{\left\{ (-\frac{B_t}{2})^2 + y^2 \right\}^2} \right|
\] (2.6a2)

For total vertical pressure on the yielding strip \((y = 0)\), Eqn 2.3b \((= 2.4b + 2.4b)\) can be,

\[
\sigma_{v,y=0} = \gamma H - \frac{d B_t}{2\beta\pi} \left| \frac{1}{(-\frac{B_t}{2})^2 - x^2} \right|
\] (2.6b1)

where, downward pressure is considered as positive sign. The pressure at the centre of the yielding strip is now obtained by putting \( x = 0 \) in Eqn 2.6b1, that is

\[
\sigma_{v,y=0,x=0} = \gamma H - \frac{2d}{\beta\pi B_t}
\] (2.6b2)

Since the pressure at the edge of yielding strip \((x = B_t/2)\) approaches negative infinity, the pressure (theoretically) can be assumed to be zero.

**Adhesion and Friction on Base**

Where no slip occurs, the strain at the rigid base in the x direction will be zero. Irrespective of the existence of strain, shear stress occurs along the rigid base contacting overlying yield materials due to either adhesion, friction or a combination of both. The horizontal, vertical and shear stress distributions in the soil mass due to displacement and adhesion can be given by Eqns 2.7a, 2.7b and 2.7c, respectively, which
includes the influence of self-weight of soil obtained from Eqn 2.5 for the complete solution.

\[ \sigma_{xd} = \frac{d}{\pi(3\beta-\rho)} \left[ \frac{2\rho}{\beta+\rho} \left\{ \frac{x+b}{(x+b)^2+y^2} - \frac{x-b}{(x-b)^2+y^2} \right\} - \frac{2(x+b)v^2}{((x+b)^2+y^2)^2} + \frac{2(x-b)v^2}{((x-b)^2+y^2)^2} \right] \]  (2.7a)

\[ \sigma_{yd} = \frac{d}{\pi(3\beta-\rho)} \left[ \frac{2\beta}{\beta+\rho} \left\{ \frac{x+b}{(x+b)^2+y^2} - \frac{x-b}{(x-b)^2+y^2} \right\} + \frac{2(x+b)v^2}{((x+b)^2+y^2)^2} - \frac{2(x-b)v^2}{((x-b)^2+y^2)^2} \right] \]  (2.7b)

and,

\[ \tau_{xyd(b)} = \frac{d}{\pi(3\beta-\rho)} \left[ \frac{\beta-\rho}{\beta+\rho} \left\{ \frac{v}{(x-b)^2+y^2} - \frac{v}{(x-b)^2+y^2} \right\} + \frac{v}{((x-b)^2+y^2)^2} - \frac{\{v^2-(x+b)^2\}^2}{((x+b)^2+y^2)^2} \right] \]  (2.7c)

where, \( \rho = \frac{\nu(1+\nu)}{E_s} \).

The horizontal pressure at the yielding strip \( (y=0) \) can be therefore,

\[ \sigma_{h,y=0} = \frac{\nu}{1-\nu} \gamma H \left\{ \frac{d \rho B_t}{\pi(3\beta-\rho)(\beta+\rho)} \left[ \frac{1}{(\frac{B_t}{2})^2-x^2} \right] \right\} \]  (2.8a1)

which is the sum of Eqns 2.7a and 2.5a. The horizontal pressure through the centre line of the yielding strip \( (x=0) \) becomes,
In a same way, the total vertical pressure on the yielding strip \((y = 0)\) can be expressed by

\[
\sigma_{v,y=0} = \gamma H - \frac{2d\beta}{\pi(3\beta+\rho)(\beta+\rho)} \left[ \frac{B_t}{(\frac{B_t}{2})^2 - x^2} \right] 
\]

(2.8b1)

Here, the pressure at the centre of the yielding strip can be obtained by putting \(x = 0\):

\[
\sigma_{v,y=0,x=0} = \gamma H - \frac{8d\beta}{\pi(3\beta+\rho)(\beta+\rho)B_t} 
\]

(2.8b2)

Figure 2.5(a) exhibits the pressure distributions computed by using Eqn 2.4a for the entire base \((y = 0)\), and Figure 2.5(b) represents the pressure distribution considering both the displacement of the yielding base and the self-weight of soil \(\gamma H\). They are in good agreement with the essential features of arching demonstrated by Terzaghi and Peck(1967) and the results obtained from the finite element method using CRISP performed by writer (which is not shown in this thesis). With increasing distance from the yielding strip, the vertical stresses due to the displacement dissipates rapidly regardless of the existence of friction between rigid base and soil mass in Figure 2.4. The decrease of the vertical stress due to displacement at a distance of \(2.5B_t\) above the centreline of the yielding strip is slightly less than 4% of that at the base. This agrees with Terzaghi's(1936) experiment, indicating that the arching effect due to displacement of a yielding trapdoor at distance of \(2.5B_t\) from the trapdoor was negligible. Finn also pointed out that the
stress induced for a small displacement \(d\) is in proportion to its magnitude.

In general, Finn’s expression is similar to Janssen’s analysis, comprising both the hydrostatic pressure due to the weight of the soil above the strip, and the negative pressure due to the downward displacement of yielding strip. However, Finn’s expressions provide more realistic results, since his theory considers both the magnitude of displacement of the yielding base and the elastic properties of soil such as Poisson’s ratio and Young’s modulus.

2.3 Current Practice on Underground Pipelines

The use of underground pipes was recorded as long ago as 3000 years. Despite accumulating experience in parallel with extensive research work leading to improvements in the techniques for design and construction, there were many failures caused by poor construction, poor engineering or a combination of both. On the other hand, the majority of pipelines were constructed using excessively strong pipes. There were ambiguous ideas for determining the earth loads acting on buried structures until the theory of loading was introduced by Anson Marston in 1913. One simple approach was to consider the load on a buried pipe to be in proportion to the height of the earth cover. Two other ideas assumed certain conditions, hence, the load might be either smaller or greater than the geostatic load, as illustrated in Figure 2.6. (Spindler, 1955) With regard to the engineering impact of ignoring the influence of the interaction between pipe and surrounding soils, it is timely to investigate the current practices commonly used in the pipe manufacturing and installation industries.
2.3.1 External loads

Following Marston’s contribution to the theory of earth loads in 1908 (published in 1913), his study was vastly extended to a range of installation conditions by Spangler during 1926 and 1952. As a result, underground pipe installations have been classified into several groups, as described in Figure 2.7.

The basic principles of arching and inverted arching action correspond to active and passive modes discussed previously, which are directly based on the trend of relative movement (or settlement) of the top soils placed above the pipe and the side fills. Based on this rational principle, the installation condition of the buried pipe can be identified by two main conditions: "trench condition" and "embankment condition".

2.3.1.1 Trench condition

Trench condition is defined as "a state of pipe installation in a narrow trench, where the natural ground is in relatively passive and undisturbed soil; when the pipe is installed, the trench is then completely covered with relatively compressible material compared to the adjacent natural ground". Since the backfill tends to settle down, an appreciable frictional resistance along the side of the trench occurs, causing the vertical earth load to be reduced.

No difference in either the development of theory or the basic assumption can be found between Marston’s(1913) theory and Janssen’s. However, Marston assumed that the pipe supports the whole vertical load exerted with the trench, and that the load is uniformly distributed over its breadth. These assumptions both consider that the fills on each side of the pipe are relatively compressible compared to the buried pipe.

The mathematical process for the development of the formula is
fundamentally identical to that of Janssens’ analysis in 1895, but the cohesion between the natural ground and backfill was ignored. The solution is therefore given by,

\[ V = \frac{1-e^{-2K \mu_s \frac{H}{B_d}}}{2K \mu_s} \gamma B_d \]  

(2.9)

where \( B_d \) is the horizontal width of trench at the top of the pipe.

Putting \( W_d = B_d V \), hence,

\[ W_d = \frac{1-e^{-2K \mu_s \frac{H}{B_d}}}{2K \mu_s} \gamma B_d^2 \]  

(2.10a)

For convenience Eqn 2.10a is simplified as follows:

\[ W_d = C_d \gamma B_d^2 \]  

(2.10b)

where,

\( W_d \) = load on pipe per unit length in longitudinal direction

\( C_d \) = coefficient of load

Unlike the other, Marston’s formula uses the active Rankine’s earth pressure coefficient \( K = \tan^2(45^\circ - \phi/2) \) and the term "coefficient of friction between fill material and sides of trench" \( \mu_s = \tan \phi_s \), giving an upper bound.

Even though Marston’s formula has great merit in use for the case of rigid pipes with relatively compressible side fills, there is, however, a large difficulty in selecting the proper values of frictional properties between fill material and the sides of trench. Therefore, a considerable number of friction measurements have been made by the use of the
simple apparatus. From early tests, it was found that the consistency of filling material is greatly affected by the characteristics of the particles of material and especially by the degree of saturation with water, which results in wide variations of loads on pipes.

From Marston's early experiments comparing the measured and calculated loads on pipes, it is clear that the cohesive resistance can not be considered in the effect of load diminishment. In particular, the increase of load on the pipes was recorded during the work stops for night and sunday. It seems to be due to the dissipation of an interlocking component between two materials. Marston et al. (1913) suggested that allowing for such lag, the difference between the weight of filling material and the actual loads measured on the pipe is fully accounted for in every case by the shear resistance alone.

2.3.1.2 Embankment condition

Embarkment conditions are defined as those in which the pipe is layed on the natural ground or in a very wide trench, and then covered by fill materials up to the planned elevation. The embarkment condition is mainly classified into three groups (i.e. positive projection condition, negative projection condition and induced trench condition) according to the field installation conditions detailed by Spangler and Handy (1982).

For the positive projection condition (which occurs when the pipe is installed in a shallow bedding with its top projecting above the natural ground surface, and the pipe is then covered with fill material), the majority of field cases are divided further into four subgroups (See Figure 2.8); i.e.

(i) complete projection condition: \( s_m + s_g > s_f + d_c \) and \( H_e > H \)
(ii) incomplete projection condition: \( s_m + s_g > s_f + d_c \) and \( H_e < H \)
(iii) complete trench condition: \[ s_m + s_g < s_f + d_c \text{ and } H_e > H \]
(iv) incomplete trench condition: \[ s_m + s_g < s_f + d_c \text{ and } H_e < H \]

The terms of each condition are due to both the relative movement between the interior prism (which is a part of embankment directly over the pipe) and the exterior prisms (which are two masses of embankment on each side of the pipe adjacent to the tangent vertical planes) and the location of plane of equal settlement \( H_e \).

where,
- \( s_f \): displacement of the bottom of the pipe into its foundation
- \( d_c \): decrease of vertical height of the pipe
- \( s_g \): settlement of the ground surface in the exterior prism
- \( s_m \): settlement of the soil in the projection height \( p_{B_c} \)
- \( p_{B_c} \): vertical distance from the ground to the top of the pipe
- \( p \): projection ratio
- \( B_c \): outside diameter of the pipe

The term "critical plane" is defined as the horizontal plane through the top of pipe \( (H = 0) \), and "plane of equal settlement" is the horizontal plane where the frictional resistance is zero at some height above the pipe (or where the differential settlement between the interior prism and the exterior prism above the pipe becomes theoretically zero). The vertical shear plane, therefore, forms between the critical plane and the plane of equal settlement.

By a process similar to the trench condition, the formula becomes:

\[ W_c = C_c \gamma B_c^2 \quad (2.11) \]
For the complete trench or projection condition \((H_e > H)\), where \(H_e\) is imaginary, the coefficient of load in Eqn 2.11 corresponds to:

\[
C_c = \frac{e^{\pm 2K\mu (\frac{H_e}{B_e})}}{\pm 2K\mu} - 1
\]  

(2.12)

where the plus and minus signs apply to the complete projection condition and the complete trench condition, respectively.

For the incomplete trench or projection condition \((H_e < H)\), the coefficient of load in Eqn 2.11 is then expressed by

\[
C_c = \frac{e^{\pm 2K\mu (\frac{H_e}{B_e})}}{\pm 2K\mu} - 1 + \left( \frac{H}{B_c} - \frac{H_e}{B_e} \right) e^{\pm 2K\mu (\frac{H_e}{B_e})}
\]  

(2.13)

where the plus signs are applicable to the incomplete projection condition and the minus signs for the incomplete trench condition.

In order to determine the earth load in Eqn 2.11 it is necessary to classify the installation condition in advance, including the height of the plane of equal settlement \(H_e\). This plane was first identified by Marston in 1922, who then went on to derive the formula in 1930 on the basis of the following assumptions (Spangler, 1950):

(i) That the internal friction in the embankment materials distributes the infinitely small increments or decrements of pressure from shear into the interior prism below the plane of equal settlement in such manner that the effect on settlement may be assumed to be substantially the same as for uniform vertical pressure.

(ii) That the internal friction in the embankment materials
distributes the infinitely small decrements or increments of pressure from shear into the exterior prisms so completely that their effect on settlement may be neglected.

Marston considered the settlements of the interior prism and exterior masses of soil below the plane of equal settlement due to the fill material placed above the plane of equal settlement only (called plane of equal additional settlement). Later, his formula was slightly modified by Spangler (1948) based on the settlements of the interior prism and the exterior prism below the plane of equal settlement due to the full height of the fill material placed above the top of the pipe (called the plane of equal total settlement). Spangler's alternative solution would seem to be a more logical approach (Young and Trott, 1984). The following is the foundation of the modification (Spangler, 1950):

\(\text{(ii) That the internal friction in the embankment materials distributes the infinitely small decrements or increments of pressure from shear into each of the exterior masses of soil, below the plane of equal settlement, in such a manner that the effect on settlement is substantially the same as though the pressure were distributed uniformly over the width equal to some ratio times the width of the conduit (named } j B_c\).

The modified formula is:

\[
\frac{1}{2K\mu} \pm \left( \frac{H}{B_c} - \frac{H_e}{B_c} \right) \pm \frac{\gamma_{sd} D}{1 + 2j} e^{\pm 2K\mu(H_e/B_c)} - 1 \pm \frac{1}{2} \left( \frac{H}{B_c} \right)^2 \\
\pm \frac{\gamma_{sd} D}{1 + 2j} \left( \frac{H}{B_c} - \frac{H_e}{B_c} \right) e^{\pm 2K\mu(H_e/B_c)} - 1 \pm \frac{1}{2K\mu} \left( \frac{H_e}{B_c} \right)^2 + \frac{H}{B_c} + \frac{H_e}{B_c} = \pm \gamma_{sd} D \frac{H}{B_c} \quad (2.14)
\]

Eqn 2.14 concerns the compression of elements of the interior prism and exterior prism considering the mean elastic modulus, assuming that the
width of the exterior prism can be taken as equal to the interior prism \((jB_c = B_c)\), which is an empirical value based on experiments because there is no rational method for determining the value of \(j\).

In Eqn 2.14, the upper signs are used for the incomplete projection condition, and the lower signs apply to the incomplete trench condition, and the settlement ratio \(\gamma_{sd}\), is given by:

\[
\gamma_{sd} = \frac{(s_m + s_g) - (s_t + d_c)}{s_m} \quad (2.15)
\]

Thus, in the positive projection condition the major factors determining the coefficient of load will be the ratio of the soil cover and the outside diameter of pipe \(H/B_c\), and the product of settlement ratio and the projection ratio \(\gamma_{sd}\). Unlike the trench condition, this settlement ratio in the positive projection condition considerably affects the load on the pipe, but its magnitude can not be readily determined for a particular installation condition. Spangler (1950) therefore advised that the settlement ratio can be standardised by the accumulation of a large body of information. Table 2.2 provides design values of settlement ratio computed on the basis of the series of field measurements obtained from some highway culverts in Iowa, which are usually referenced by field engineers.

### 2.3.2 Supporting strength of rigid pipe installation

This section reviews the theories of the field supporting strength based on both Spangler’s contribution in 1933, and the working papers of the pipe manufacturing industry. A rigid circular pipe is usually installed on bedding material, forming the foundations which affect the design and behaviour of the pipe in service. Increasing the circumferential contact length between the bedding material and the pipe results in a lower
peak bending moment in the pipe wall; especially at the invert area. In the performance of the pipe manufacturing and installation, an essential prerequisite is therefore how to estimate the field supporting strength and how to achieve the bedding satisfactorily to meet the specifications so that they would safely and economically perform their function. For this reason, the investigation was commenced by Spangler in 1915 in order to define the natural laws governing their structural behaviour (under the direction of Anson Marston in Iowa Engineering Experiment Station). The first extensive scale experimental work was conducted in 1924 (published initially in 1926) and further experiments in 1930 and 1931. As a result, the methods of determining supporting strength of any buried structure, termed semi-empirical design method, were suggested by Spangler in 1933. The aim of his studies was mainly to develop the bedding factor expressed by \( B_f = \frac{W_c}{P_{teb}} \), which is the ratio of the supporting strength to the three-edge bearing (teb) strength of any similar pipe in the plant. This method applies to the rigid pipe installation only, takes account of the site conditions, and is used the most by field engineers.

On the other hand, purely theoretical analyses for circular underground structures (called stress analysis design method here) proposed by Paris(1921) and Olander(1950) are also widely used by some pipe manufacturing industries. One difference between the two approaches is that Paris assumed the vertical and horizontal pressures to be of uniform distribution, while Olander idealised the pressures to be acting normal to the pipe surface and varying as a trigonometric function. Determining the bedding angle supporting the pipe is an essential prerequisite to the stress analysis. This is a major disadvantage, because it can not be readily obtained from either a given field condition or published standards.
2.3.2.1 Elastic theory (Spangler, 1933)

The external load carrying capacity of the buried pipe depends upon the shape and intensity of the soil reaction against some area under the bottom of the pipe caused by the vertical downward load on top of the pipe. This capacity is usually determined by performing the three-edge bearing test in the laboratory.

A general description of the three-edge bearing test is given by several standards, such as ASTM C497 and BS 4625. Figure 2.9 illustrates the essential feature of three-edge bearing test apparatus, together with the application of an external line loading at the top and bottom of the pipe. A view of the field test is provided in Figure 2.10. ASTM C497 calls for recording the 0.01 inch crack load, which is the limiting design load used for non-pressure applications such as reinforced concrete sewer or culvert pipe. AWWA C301-84 Appendix A defines the limiting value for external load W₀, which for a prestressed concrete pressure pipe is defined as being nine tenths of the three-edge load producing incipient cracking in the core, where no internal pressure is considered. This is a more restrictive limit than the requirement of ASTM C497.

Returning to the theory of bedding factor, Spangler(1933) noted that the application of the elastic theory of flexure for thin rings to field installations was impractical because it led to an inconsistent hypothesis on the stress distribution in the pipe. For this reason, he carried out experimental work accompanying a theoretical approach to establish the method of determining the supporting strength of a buried pipe. In this research, although the normal pressures were of a continuously varying intensity, the pressures were measured at 16 points equally spaced around the periphery of the pipe for the consideration of simplicity. Figure 2.11 is a basic assumption of radial forces acting on a pipe for the determination of bending moment. The moment at any point on the ring may be obtained from general bending theory considering a portion of half circle as a free body as shown in Figure 2.11(a). For example,
to estimate moments about C in Figure 2.11(b) the free body must be in an equilibrium state by the summation of the moment generated by the external forces at C. Since the equation derived from the bending theory contains an unknown value of $T_b$, the displacement theory of arches must be employed. From this equation, the value of $T_b$ is given by

$$T_b = 0.335P_8 + 0.530P_7 + 0.577P_6 + 0.500P_5$$

$$+ 0.346P_4 + 0.177P_3 + 0.048P_2$$

(2.16)

Finally, the solution of the bending moment at the bottom of the pipe becomes:

$$M_b = r (0.318P_9 + 0.277P_8 + 0.013P_7 - 0.137P_6$$

$$- 0.182P_5 - 0.150P_4 - 0.084P_3 - 0.024P_2)$$

(2.17)

Even so, an error exists in Eqn 2.17 due to the assumption of the concentrated loads equally spaced at 16 points, which indicates $M_b = 0.031Pr$ and not zero. In order to reduce this error to be the same as that for the loads at 32 points, the individual value in Eqn 2.17 was corrected as follows:

$$M_b = r (0.318P_9 + 0.260P_8 + 0.012P_7 - 0.150P_6$$

$$- 0.182P_5 - 0.150P_4 - 0.086P_3 - 0.022P_2)$$

(2.18)

As a result, the moment with equal pressures approaches 0.003Pr, indicating that this error is negligible. It should be noted that the self-weight of the pipe was not taken into account throughout the above discussion.
2.3.2.2 Determination of bedding factor

Further to the elastic theory for a circular rigid pipe, Spangler attempted to measure the distribution of radial pressures on a buried concrete pipe of 36 inches internal diameter and 4-inch wall thickness in the field. The height of fill was gradually increased until a crack appeared in the pipe wall.

The measured radial pressures at a fill of height of 8 ft (where the crack first appeared) were not symmetrical about a vertical axis, therefore they were adjusted by averaging the measured pressures on corresponding segments of each half of the pipe for convenience. Figure 2.12 exhibits the measured and symmetrized pressure distributions, and the relevant data are provided in Table 2.3. From this diagram, replacing the values of symmetrised pressures, P₁, P₂, P₃, etc., in Eqn 2.16 and Eqn 2.18, the thrust and the bending moment at the bottom of the pipe were \( R_b = 0.221W_c \) and \( M_b = 0.153rW_c \), respectively, where \( W_c \) is the total vertical load on the pipe.

By setting all values of \( P \) (except \( P₉ \)) equal to zero in Eqn 2.18, the bending moment corresponding to the three-edge bearing test will be:

\[
M_{teb} = 0.318rP_{teb}
\]  

(2.19)

The ultimate fibre stress produced by the three-edge bearing test can therefore be written as

\[
f_{teb} = \frac{0.75M_{teb}y}{I} = \frac{1.431rP_{teb}}{t^2}
\]  

(2.20)

where the value of 0.75 is a correction, considering the stress may go beyond the elastic limit in some cases, and the materials may have unequal elastic properties in tension and compression. In a similar way,
the bending stress of the pipe in the field (which is usually governed by tension stress incurred at the invert area in case of a rigid pipe) is therefore combined with the stress due to the thrust, thus giving:

\[
f_t = \frac{0.75 M_b y}{I} - \frac{R_b}{t} = \frac{0.689 r W_c}{t^2} - \frac{0.221 W_c}{t}
\] (2.21)

Putting \( f_{eb} = f_c \), and since \( B_t = W_c/P_{eb} \),

\[
B_t = \frac{1.431 r}{0.689 r - 0.221 t}
\] (2.22)

Substituting \( r = 20 \) inches and \( t = 4 \) inches, the bedding factor becomes 2.22. This theoretical bedding factor could be compared directly with the experimental bedding factor of 2.06 obtained from an average of eight tests with similar pipes (Refer to Table 12 of Spangler, 1933) where a group of four pipes was used to determine the first cracking loads in the field and the other group for three-edge bearing load at first crack. The results are reasonable in consideration of the variations in the field.

2.3.2.3 General expression for bedding factor

Since Eqn 2.22 is restricted to a particular pipe, a general expression for the bedding factor — a traditional system of "equivalent uniformly distributed loads on the top, bottom and two sides of the pipe" — was therefore conceived by Spangler (1933). Figure 2.14 illustrates a uniform loading system, comprising vertical and horizontal pressures, corresponding to the field installations of pipe shown in Figure 2.13 which has been employed by most engineers to date.

Figure 2.14(b) details the conventional uniformly distributed loading system for the ordinary bedding condition, consisting of the vertical load above the horizontal midpoint of the pipe (180 degrees); the vertical
reaction to the bottom of the pipe (60 degrees); and the lateral pressure over the projected portion of the pipe above the natural ground ($m B_c$).

Employing a similar approach to the radial load case, the general expression of the bedding factor for a positive projecting circular pipe was finally derived as the following (Spangler and Handy 1982):

$$B_r = \frac{1.431}{N - xq}$$  \hspace{1cm} (2.23)

where,

- $N$ = a parameter related to the distribution of the vertical reaction
- $x$ = a parameter related to the area of the vertical projection of the pipe on which active lateral pressure of the fill material acts
- $m$ = the ratio of the vertical projection above the subgrade to the vertical dimension of the pipe (for a circular pipe, $m = p$)
- $p$ = the projection ratio
- $q$ = ratio of the total lateral pressure to the total vertical load

$$q = \frac{m K}{C_c} \left( \frac{H}{B_c} + \frac{m}{2} \right)$$

- $K$ = Rankine's lateral earth pressure coefficient
- $C_c$ = load coefficient for embankment condition
- $H$ = height of the fill above top of pipe
- $B_c$ = horizontal outside width of pipe

This equation is only applicable to a pipe installation where the first crack occurs on the inner surface at the bottom of the pipe. In contrast, when the maximum outer fibre stress occurs at the top (in the case of a concrete cradle particularly) the values of $x'$ shall be substituted for $x$ of Eqn 2.23. These parameters are provided in Table 2.4 for a
circular pipe and Table 2.5 for an elliptical pipe, where a constant corresponding to the shape of the pipe of 1.431 shall be substituted by 1.337 for horizontal elliptical, and 1.021 for vertical elliptical. The working values of the bedding factor for circular pipe were also provided by the graphical representation as a function of the value of $H/B_c$ and the settlement ratio $\gamma_{sd}$, based on the experiments in Spangler’s paper. Later, it was expanded to accommodate the different shape of pipe, such as vertical and horizontal elliptical pipes or precast concrete box culverts with various installation conditions.

2.4 Discussion

As has been reviewed, due to the high degree of interaction between soil and rigid pipe, many complexities and uncertainties still remain in current practice. It might be that their work (Marston-Spangler theory) contributed greatly to the pipeline engineering and construction, but several critical factors were either ignored or equalised for simplicity.

These include relative stiffness of the buried pipe and the surrounding soil, the effect of compaction leading to a change in lateral pressure ratio, the compressibility characteristics of the fill materials, the configuration of the trench, etc. This section will continue the discussion on the most important factors in soil-pipe interaction problems. Several limitations are summarised in the following.

♦ A higher stiffness of backfill followed by compaction results in a higher ratio of relative stiffness. The stiffness of soil is mainly related to type of soil and degree of compaction. The variation is extremely large. Unlike the assumption of Marston-Spangler theory, a certain part of the load must be carried by the side backfill, irrespective of installation conditions.

- 31 -
From typical ranges of system behaviour for different pipe materials (as a function of the relative stiffness of the pipe and the surrounding soil) Gumble (1983) claimed that when a value of $Y$ is less than 10, more than 90% of the backfill load will be supported by the ring bending action of the pipe; when $Y$ increases to a range of 10 to 1000, a part of the load supported by the pipe reduces from 90% to 10%; and when $Y$ is greater than 1000, more than 90% of the load will be carried by the surrounding soils. The general expression of the relative stiffness $Y$, is given by the ratio of the plane strain elastic modulus of the soil to the flexural stiffness of a circular pipe.

- In more detail, the performance of bedding highly depends on the condition of the contact area under the bottom of the pipe. The different stiffness of every section along the contact area causes a variation of bending moment in the pipe wall. This nature cannot be identified with current standard practice.

Although the works in the field satisfy the requirements for a given bedding condition, its performance will not be identical according to the different construction sequence or construction method.

- The current design philosophy does not take account of the characteristics of the pipe. For instance, the rigidity of the structure varies with several factors such as the pipe size, the wall thickness, the constituent materials, etc.

The rigidity of the pipe wall greatly influences the magnitude of the bending moment or flexural bending stresses, since the deformation of the pipe is controlled by both its rigidity and the stiffness of the surrounding soils.

- Moreover, the prestressing wires which are helically wrapped onto the pipe wall results in a different behaviour of the pipe,
compared with the non-prestressed rigid pipe. Can the same earth pressures be applied around the pipe, regardless of the existence of the prestressing effect?

In reality, the governing factor of design is the magnitude of maximum bending stress in the pipe wall, rather than the intensity of radial earth pressures. Not only is it nearly impossible to anticipate the induced bending stress with the current design practice, but, even if the induced bending stress is calculated through a complicated process, it is almost certain that there will be an error. Therefore, under these circumstances, no analytical methods to forecast the real behaviour of the buried pipe exist. However, as an alternative approach, the finite element method for soil-structure interaction problems has potential to take into account all the above factors with intelligent simplifications of reality (Gunn et al., 1993). This will be considered in subsequent chapters.
Table 2.1 Parameters for Terzaghi's experiments on a yielding trap-door (Terzaghi, 1936)

<table>
<thead>
<tr>
<th>Material Type</th>
<th>Total vertical load on the yielding strip</th>
<th>Settlement ratio ((\delta/Bt))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Computed</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Minimum ((N))</td>
<td>Maximum ((N))</td>
</tr>
<tr>
<td>Dense sand</td>
<td>22.85</td>
<td>10.40</td>
</tr>
<tr>
<td>Loose sand</td>
<td>37.46</td>
<td>16.48</td>
</tr>
</tbody>
</table>

Table 2.2 Design values of settlement ratio (Spangler and Handy, 1982)

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Settlement Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rigid culvert on foundation of rock or unyielding soil</td>
<td>+1.0</td>
</tr>
<tr>
<td>Rigid culvert on foundation of ordinary soil</td>
<td>+0.5 to +0.8</td>
</tr>
<tr>
<td>Rigid culvert on foundation of material that yields with respect to</td>
<td>0 to +0.5</td>
</tr>
<tr>
<td>adjacent natural ground</td>
<td></td>
</tr>
<tr>
<td>Flexible culvert with poorly compacted side fills</td>
<td>-0.4 to 0</td>
</tr>
<tr>
<td>Flexible culvert with well-compacted side fills</td>
<td>-0.2 to +0.2</td>
</tr>
</tbody>
</table>
Table 2.3 Distribution of pressure around the periphery of a 36-inch concrete pipe under an embankment (Spangler, 1933)

<table>
<thead>
<tr>
<th>Location from top of pipe (equally spaced)</th>
<th>Measured pressure (kPa)</th>
<th>Symmetrised pressure (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50.036</td>
<td>50.036</td>
</tr>
<tr>
<td>2</td>
<td>81.556</td>
<td>54.823</td>
</tr>
<tr>
<td>3</td>
<td>36.070</td>
<td>31.282</td>
</tr>
<tr>
<td>4</td>
<td>31.840</td>
<td>21.386</td>
</tr>
<tr>
<td>5</td>
<td>22.424</td>
<td>19.950</td>
</tr>
<tr>
<td>6</td>
<td>14.444</td>
<td>16.279</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>1.915</td>
</tr>
<tr>
<td>8</td>
<td>68.389</td>
<td>77.246</td>
</tr>
<tr>
<td>9</td>
<td>112.199</td>
<td>112.199</td>
</tr>
<tr>
<td>10</td>
<td>86.104</td>
<td>77.246</td>
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<tr>
<td>11</td>
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<td>1.915</td>
</tr>
<tr>
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<td>16.279</td>
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<td>17.476</td>
<td>19.950</td>
</tr>
<tr>
<td>14</td>
<td>11.012</td>
<td>21.386</td>
</tr>
<tr>
<td>15</td>
<td>28.573</td>
<td>31.282</td>
</tr>
<tr>
<td>16</td>
<td>28.090</td>
<td>54.823</td>
</tr>
</tbody>
</table>
Table 2.4 Parameters for bedding factor: Circular pipe (Spangler and Handy, 1982)

<table>
<thead>
<tr>
<th>p = m</th>
<th>x</th>
<th>x'</th>
<th>Type of bedding</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.217</td>
<td>0.743</td>
<td>Class D</td>
<td>1.310</td>
</tr>
<tr>
<td>0.5</td>
<td>0.423</td>
<td>0.856</td>
<td>Class C</td>
<td>0.840</td>
</tr>
<tr>
<td>0.7</td>
<td>0.594</td>
<td>0.811</td>
<td>Class B</td>
<td>0.707</td>
</tr>
<tr>
<td>0.9</td>
<td>0.655</td>
<td>0.678</td>
<td>Class A</td>
<td>0.505</td>
</tr>
<tr>
<td>1.0</td>
<td>0.638</td>
<td>0.638</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.5 Parameters for bedding factor: Elliptical pipe (ACPA, 1988)

<table>
<thead>
<tr>
<th>Type of pipe</th>
<th>Type of bedding</th>
<th>N</th>
<th>m</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal</td>
<td>Class C</td>
<td>0.763</td>
<td>0.3</td>
<td>0.146</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.5</td>
<td>0.268</td>
</tr>
<tr>
<td></td>
<td>Class B</td>
<td>0.630</td>
<td>0.7</td>
<td>0.369</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.9</td>
<td>0.421</td>
</tr>
<tr>
<td>Vertical</td>
<td>Class C</td>
<td>0.615</td>
<td>0.3</td>
<td>0.238</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.5</td>
<td>0.457</td>
</tr>
<tr>
<td></td>
<td>Class B</td>
<td>0.516</td>
<td>0.7</td>
<td>0.639</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.9</td>
<td>0.718</td>
</tr>
</tbody>
</table>
Figure 2.1 Arching above yielding trap door in compacted state (Terzaghi, 1936)
Figure 2.2 Failure mechanism of yielding strip
Figure 2.3 Janssen's arching analysis  
(after Bulson, 1985)

Figure 2.4 Boundary conditions for soil mass  
with yielding strip (Finn, 1963)
Figure 2.5 Vertical pressure distribution on yielding strip
(Finn, 1963)
Figure 2.6 Three theories of loads (After Spindler, 1955)

Figure 2.7 Classification of installation conditions
Figure 2.8 Settlements that influence loads on embankment pipe (After Spangler, 1950)
Figure 2.9 Three-edge bearing test mechanism (ACPA, 1990)
Figure 2.10 View of full scale three-edge bearing test for 4000mm diameter prestressed concrete cylinder pipe at Sarir pipe manufacturing plant in Libya
Figure 2.11 Diagram of radial forces acting on a circular pipe (Spangler, 1933)
Figure 2.12 Radial earth pressure on concrete pipe as measured by pressure ribbons; ID = 36 inches, $tw = 4$ inches, $EC = 8$ feet (Spangler, 1933)
Concrete Cradle Bedding

First Class Beddings

Ordinary Beddings

Impermissible Bedding

Figure 2.13 Four types of projection bedding for circular pipe
(Spangler, 1933 and ACPA, 1990)
\[ V = \frac{W_e}{2r} \]

(a) Impermissible bedding

(b) Ordinary bedding

(c) First class bedding

Note:
- \( W_e \) = total earth load on top of pipe
- \( B_c \) = outside diameter of pipe
- \( m \) = ratio of projected part of pipe where lateral pressure is effectively induced
- \( h = \frac{W_e}{r(1+\cos a)} \)
- \( q = \frac{\text{ratio of total lateral pressure to total vertical load}}{\text{angle to distinguish location of natural ground}} \)
- \( r \) = mean radius of pipe
- \( t \) = pipe wall thickness

Figure 2.14 Equivalent uniformly distributed earth pressures
(Spangler, 1933)
3.1 Introduction

In 1946, the innovation of the high speed digital computer contributed to the development of the finite element method which stems from matrix methods of structural analysis. Turner et al. (1956) were the first to present the basis of the finite element method for a continuum structure, to compute the stresses and displacements of the aeroplane wings. Since then, increasing accessibilities and computational powers have stimulated the development of the finite element method throughout all engineering disciplines. The finite element method in geotechnical engineering is fundamentally the same as in other fields. The general description of finite element method will not hence be detailed in this chapter.

The CRISP program is briefly introduced in the following section, including the historical development, program structure, program limitations, etc.. The available constitutive soil models within the current CRISP program are also reviewed. Some interpretation techniques for the smoothing of discontinuous finite element functions using a least squares method will also be reviewed and discussed in this chapter, using examples extracted from finite element analyses conducted by the writer. The procedures for the determination of radial earth pressures, flexural stresses and bending moments from the output of the program are explained in detail in the end of this chapter. The simulation of compaction effects in the CRISP program, together with the special interface elements, are considered at the end of this chapter also.
3.2 Basic Principles

3.2.1 Historical development of CRISP program

The purpose written geotechnical finite element package CRISP (Critical State soil mechanics Program) was developed at Cambridge University, which is now incorporated along with state of the art interfaces into the SAGE CRISP package. The first version of the computer program was initiated by Simpson in 1973, and Thompson and Zytynski in 1976, then a considerable number of enhancements and modifications to the program were made by Gunn (since 1977) and Britto (since 1980). Since it was offered to the public domain in 1982, the first PC-based version was released in 1990. Following CRISP90, versions were released in 1992, 1993 and 1994 according to the development of the graphical interface. The analysis engine is now owned by The CRISP Consortium, who fund its ongoing development. Critical state soil mechanics (CSSM) provides a complete theoretical framework for describing the mechanical behaviour of soil. Finite element techniques allow CSSM theories to be applied to predicting ground movements associated with foundations, excavations, and other situations in geotechnical engineering.

The commercial program, with a completely new user friendly CAD-style Windows interface, was launched in 1996, called SAGE CRISP owned by SAGE Engineering Limited. However, until very recently, there has been no difference between the two program capabilities such as element types, constitutive models, etc. The analyses carried out in the subsequent chapters made use of the DOS based version, CRISP94.

A full description is given by Britto and Gunn (1987), and programing manuals are provided by Britto and Gunn (1990 and 1992). CSSM is described fully in a number of texts (Schofield and Wroth, 1968; Atkinson and Bransby, 1978; Bolton, 1979; Atkinson, 1981 and 1993; Wood, 1991).
3.2.2 Program structure

The CRISP program contains two distinct components; i.e. the geometry program (GP) and the main program (MP). The geometry program deals with mesh processing and checking based on the input data specified by the user, including the vertex node coordinates, element connectivity and element types. The main roles of the geometry program are the generation of mid-side and interior nodes, the derivation of the optimised solution order for the frontal solver and the creation of a permanent link file of geometric data to be subsequently read by the main program. The main program solves the finite element equations, such as the displacements, stresses and pore pressures based on the boundary conditions of the analysis. The general flow chart of the structure of the analysis is described in Figure 3.1.

The input data required is classified into the following three categories:

(i) Information for the formation of the finite element mesh associated with the coordinates of nodal points for each element, which are subdivided into domain information, maximum value of user vertex node and element numbers, curved sides, nodes and element Information, etc.

(ii) Material properties and in-situ stresses for each element

(iii) Boundary conditions and construction sequences for the analysis; for instance, imposed displacements, applied loads, geometry changes, etc.

The CRISP engine is written in ANSI (American National Standards Institute) standard FORTRAN 77; a programing language which is still widely used by the engineering and science research community. It is suitable for mounting on many different computers; the size of the problem which can be modelled by CRISP only depends on the size of the memory and the processing power of the user’s computer.
3.2.3 Program limitations and equilibrium

The program allows for drained, undrained and time dependent analysis of static problems under single loading and/or unloading conditions. Since the CRISP engine is based on the small strain/small displacement, the analysis associated with large deformation may be partially covered by using the geometry updating option (IUPD in Record C1 of main program input data). CRISP makes possible to model the performance of excavation and backfilling according to the construction sequences in 3D, plane strain and axisymmetry.

The analysis of an extremely large deformation caused by the compaction effect or stress cycling is not possible in the current form. Both the principle of compaction effect and the result of the finite element analysis will be detailed in the final section of this chapter. The driving of a pile with or without torsion can not be adequately modelled using the axisymmetric option, since it is a dynamic event.

Critical state based models in the CRISP program use an incremental approach without any stress correction. This results in the response to be far away from the true solution in the case of analysis using an insufficient number of increments. The stress state for elastic perfectly plastic models (model no. 5) is, however, corrected back to the yield surface at the end of every increment when yielding occurs. Figure 3.2(a) illustrates the stress state correction for a single integration point. If the stress state moves outside the yield locus/failure surface \((P_1)\), the stresses will be adjusted back to the yield surface \((P_2)\). The out-of-balance load for a set of equivalent nodal loads \(\Delta P_{c1}\), is then computed by the integration of a single correction. Figure 3.2(b) represents an iterative technique, where the correction for further yielding is repeated until the resulting incremental displacements or \(\Delta P_{cn}\) is less than a preset tolerance. The imbalanced load arising from this stress correction is reapplied in the following increment. Moreover, the increase in strain is subdivided into ten equal steps routinely and then the stress state is re-evaluated at the end of each step. The stress state is therefore closer to the true solution. Unlike elastic perfectly plastic model, for CSSM models no correction is made due
to the difficulty of trying to correct back to a yield surface which is changing in size (Britto and Gunn, 1987).

3.2.4 Constitutive soil models

The constitutive soil models available in CRISP94 includes:

(i) Linear elastic
- Homogeneous anisotropic
- Homogeneous isotropic
- Non-homogeneous isotropic

(ii) Linear elastic perfectly-plastic
- von Mises failure criterion
- Tresca failure criterion
- Drucker-Prager failure criterion
- Mohr-Coulomb failure criterion

(iii) Critical state
- Cam Clay (CC)
- Modified Cam Clay (MCC)
- Schofield soil model (SCHO)

3.2.4.1 Linear elastic model

CRISP94 allows for linear elasticity with anisotropy or non-homogeneity in modelling the elastic response of soil. For isotropic materials there are only two independent elastic constants. However, most geomaterials are anisotropic. A completely general description of an anisotropic elastic material requires 21 elastic constants. The depositional history of many soils introduces symmetries which may reduce the number of constants considerably. The cross-anisotropy (or transverse isotropic) linear elastic model in CRISP, which assumes that, during deposition, soils undergo one-dimensional compression, requires the five independent elastic constants
to be specified, which are

\[ E_v : \text{Young's modulus in the vertical direction} \]
\[ E_h : \text{Young's modulus in the horizontal direction} \]
\[ \nu_{hh} : \text{Poisson's ratio for a stress increment in the vertical direction} \]
\[ \nu_{vh} : \text{Poisson's ratio for a stress increment in the horizontal direction} \]
\[ G_{hv} : \text{Shear modulus of anisotropic soil} \]

This model is usually limited for use in over-consolidated clays which shows \( E_h \neq E_v \), since difficulties arise in determining the constants either from field or laboratory testing. Isotropic model is also covered, being a special case of anisotropy.

On the other hand, the non-homogeneous elastic (isotropic) model requires only 4 independent parameters as follows:

\[ E_o : \text{Young's modulus at datum elevation} \]
\[ y_o : \text{Datum elevation at which } E = E_o \]
\[ m : \text{Rate of increase of Young's modulus with depth} \]
\[ \nu : \text{Poisson's ratio} \]

The governing equation for elastic Young's modulus at an elevation is given by \( E = E_o + m (y_o - y) \). Here, Poisson's ratio is invariable, therefore, the shear modulus is directly related to Young's modulus; i.e. \( G = E/2(1 + \nu) \). This model is widely used due to its simplicity, and the problems relating to excavations in stiff clays, in particular. Moreover, the most of construction materials such as concrete, reinforcement, etc. are modelled using the homogeneous elastic model with \( m = 0 \).
3.2.4.2 Elastic perfectly-plastic model

Elastic perfectly-plastic models are fairly widely used in the analysis of geotechnical engineering problems. The response of soil is broadly linear elastic until it yields. Once the stress state passes the yield point, simultaneous elastic and plastic strains occur and stiffness decreases. This nature makes it possible to limit a shear strength. Unlike the critical state soil models, the yield surface neither expands nor contracts during plastic yielding. Facilities for the linear variation of both the Young's modulus and the shear strength with depth (i.e. nonhomogeneity of elastic stiffness and strength) are provided in a similar way to nonhomogeneity elastic model (by using the soil parameters $E_0, c_0, m_E, m_C$), however, the current package permits only fully isotropic behaviour.

CRISP provides a large variety of yield criteria so that user can select an appropriate model to characterise the stress-strain and failure behaviour of the construction materials, which are

(i) Tresca or von Mises
    for total stress, undrained ($E_u, c_u, \phi_u = 0$)

(ii) Mohr-Coulomb or Drucker-Prager
     for effective stress, drained and consolidation ($E, \nu', c', \phi'$)

The above four different criteria have both advantages and limitations, depending on the particular application. It is assumed that the plastic potential is the same as the failure envelope in stress (or strain) space, implying an associated flow rule. It is known that Tresca and von Mises yield criteria are well suited to model metals. The Tresca yield criterion is represented by a hexagonal cross-section, while the von Mises yield criterion defined by cylindrical surface centred on the principal stress space with three orthogonal Cartesian coordinate. The Mohr-Coulomb criterion forms an irregular hexagonal pyramid, which is probably most widely accepted as the more accurate of all of these solutions for the general geotechnical engineering problems (Bishop, 1966).
Concerning real soil behaviour, the shortcomings of the Tresca and von Mises yield criterion exist because of independency on change in mean stress, wherein the stress state reaches yielding point the soil deforms at a constant yield stress in a stress-strain curve. While on the other hand, the yield stress for the Mohr–Coulomb or Drucker–Prager criteria tends to increase or decrease after the yield point, according to the variation of the mean stress. From the idea of a cylindrical surface (due to von Mises (1913)), the conical surface (which coincides with the Mohr–Coulomb surface on the edges) was proposed by Druker–Prager (1952) as it is rather more useful to 'round-off' the Mohr–Coulomb yield surface. However, a common feature is that when the stress state for soil is located inside the yield surface, the behaviour of the material is elastic. Similarly, yielding is defined by the stress state being placed on the yield surface; however, no assumption can be made that it is located outside the surface.

A major disadvantage of the above four criteria is due to the open-ended yield surfaces in the direction of the hydrostatic compressive axis, assuming linear elastic soil response for predominantly compressive stress states. Since the real soil behaviour is non-linear under most stress conditions, there exists a severe physical shortcoming. In order to overcome this deficiency, a closed yield surface with a strain hardening cap is required (Oettl et al., 1998). However, the use of critical state models or cap models are beyond the scope of the present study. In keeping with many of the previous investigations using the finite element method for soil-pipe interaction problems, the Mohr–Coulomb criterion will be adopted for the soil elements.

3.3 Interpretation of CRISP

This section deals with the interpretation techniques used in post-processing the CRISP analyses carried out by the writer. The techniques for smoothing and extrapolation of the eight-noded quadrilateral element (LSQ) with full 3x3 integration are described. A free body diagram of a small square element and Mohr's circle of stress are employed for the determination of radial earth pressure on the pipe together with the flexural
bending stresses in the pipe wall. Finally, a simple bending theory to compute the bending moment induced around the pipe wall from the combination of loading conditions will be discussed.

3.3.1 Least squares smoothing of discontinuous finite element functions

In general, four different \textit{a posteriori} local stress smoothing methods have been identified, together with a fifth method with respect to soil-wall interface problems implemented by Woods(1999).

(i) simple averaging
(ii) best-fit line
(iii) best-fit plane
(iv) 2x2 interpolation and smoothing
(v) translocal best-fit line/curve

The translocal best-fit line/curve method is based on the concept of the best-fit line method, but extended to a larger zone of elements adjacent to the wall.

Past experiences with numerically integrated elements indicate that the error-minimal points are the Gauss points. The worst results are shown at the edge of the element, but this location is generally the place for the interpretation of the output (Hinton and Campbell:1974, Barlow:1976). It is therefore a prerequisite that the extrapolation of the output obtained from the interior of the element (especially for the finite element models based on the displacement approximations) is done before the calculation of the stresses. A global smoothing or local smoothing method (involving the whole of the finite element domain or each individual element, respectively) is usually carried out by a least squares smoothing method, as proposed by Hinton and Campbell(1974) for the case of 2x2 rectangular element.

An eight-noded quadrilateral element for plane strain has much improved characteristics compared to a lower order element, such as a constant strain
triangular element or four-noded quadrilateral element. The displacement functions for these latter elements are based on a bilinear displacement field only. Obviously, improvement can be achieved by the addition of a number of nodal points along the sides of such elements, in order to allow a smaller number of variables to be used for the solution of practical problems with a given degree of accuracy (Ergatoudis et al., 1968). Non-dimensional (auxiliary) coordinates measured from the centre of the element \( (\xi = (x - x_c)/a \) and \( \eta = (y - y_c)/b ) \) can overcome incompatibility of displacements between adjacent elements due to parabolic variation of displacement along the edge, and make it possible to have nonrectangular elements. In Figure 3.3, \( \xi \) and \( \eta \) vary between -1 and +1, and \( x_c \) and \( y_c \) are the global coordinates at the centre of the element. The most popular numerical integration procedure is the Gauss–Legendre quadrature method (often termed simply Gauss quadrature or Gauss method), which can be used to approximate the various integrals of matrix products. Figure 3.4(a) represents a bilinear element (2x2) with \( \xi = \eta = \pm 1/\sqrt{3} \). Figure 3.4(b) illustrates the optimal stress locations for a quadratic element (3x3) with \( \xi = \eta = \pm \sqrt{3/5} \) and weighting factors of 5/9 at corner nodes and 8/9 at central node. The addition of a central node with the coordinates of \( \xi = \eta = 0 \) transforms the 8-node quadratic element (sometimes called a "serendipity" element) into a 9-node Lagrange element, leading to more accurate results.

The generalised form of shape functions can be constructed directly by taking products of the linear Lagrange polynomials for the 4-node rectangular element (Cheung and Yeo, 1979). Similarly the 8-node quadratic element and 9 node Lagrangian element have been introduced by some researchers such as Hinton and Owen (1979) and Cook et al.(1989). For a 4-node rectangular element, all of the shape functions can be expressed by the same form, while the shape functions for a 8-node quadratic element must be distinguished by two general forms: corner nodes \( (i = 1,2,3,4) \), and midside nodes \( (i = 5,6,7,8) \). The axes of a quadratic element can be either straight or curved.

The general form of shape functions for a Lagrange element with 9 Gauss points are given by the following (Hinton and Owen, 1979):
(i) for corner nodes (i = 1, 2, 3, 4)
\[ N_i = \frac{1}{4} (\xi_i^2 + \xi_i \xi_j)(\eta_i^2 + \eta_i \eta_j) \] (3.1a)

(ii) for midside nodes (i = 5, 6, 7, 8)
\[ N_i = \frac{1}{2} \eta_i^2 (\eta_i^2 - \eta_j)(1 - \xi_i^2) + \frac{1}{2} \xi_i^2 (\xi_i^2 - \xi_j)(1 - \eta_i^2) \] (3.1b)

(iii) for the central node (i = 9)
\[ N_i = (1 - \xi_i^2)(1 - \eta_i^2) \] (3.1c)

For example, midside node with \( \xi_i = 0 \) and \( \eta_i = -1 \) leads to
\[ N_i = -\frac{1}{2}(1 - \xi_i^2)(1 - \eta_i)/2 \]. The boundaries are defined by both \( \xi = \pm 1 \) and \( \eta = \pm 1 \), and the coordinates for \( \xi_i \) and \( \eta_i \) are taken from their global values converted to the local system.

The smoothing shape function of each node can be then calculated by using Eqn 3.1. Alternatively, the shape functions for the 8-node quadrilateral element can be derived immediately by inspection (Ball, 1980), and for the 9-node Lagrangian element as follows: (Cook et al., 1989)

\[ N_1 = \frac{1}{4} (1 - \xi)(1 - \eta) - \left( \frac{1}{2} N_6 + \frac{1}{2} N_8 + \frac{1}{4} N_9 \right) \]
\[ N_2 = \frac{1}{4} (1 + \xi)(1 - \eta) - \left( \frac{1}{2} N_6 + \frac{1}{2} N_8 + \frac{1}{4} N_9 \right) \]
\[ N_3 = \frac{1}{4} (1 + \xi)(1 + \eta) - \left( \frac{1}{2} N_6 + \frac{1}{2} N_7 + \frac{1}{4} N_9 \right) \]
\[ N_4 = \frac{1}{4} (1 - \xi)(1 + \eta) - \left( \frac{1}{2} N_7 + \frac{1}{2} N_8 + \frac{1}{4} N_9 \right) \]
\[ N_5 = \frac{1}{2} \left( 1 - \xi^2 \right)(1 - \eta) - \frac{1}{2} N_9 \]
\[ N_6 = \frac{1}{2} \left( 1 + \xi^2 \right)(1 - \eta) - \frac{1}{2} N_9 \]
\[ N_7 = \frac{1}{2} \left( 1 - \xi^2 \right)(1 + \eta) - \frac{1}{2} N_9 \]
\[ N_8 = \frac{1}{2} \left( 1 + \xi^2 \right)(1 + \eta) - \frac{1}{2} N_9 \]
\[ N_9 = (1 - \xi^2)(1 - \eta^2) \] (3.2)
For the 8-noded quadrilateral element (LSQ) with full 3x3 integration, such as used in CRISP, the locally-smoothed nine nodal stresses \((\sigma_1, \sigma_2, \cdots, \sigma_9)\) can be derived from the Gauss point stresses \((\sigma_i, \sigma_{ii}, \cdots, \sigma_{ix})\) by using the following expression:

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\sigma_4 \\
\sigma_5 \\
\sigma_6 \\
\sigma_7 \\
\sigma_8 \\
\sigma_9 \\
\end{bmatrix} =
\begin{bmatrix}
a & b & c & b & d & e & d & f & \\
b & a & b & c & d & e & e & f & \\
c & b & a & b & e & d & e & f & \\
b & c & b & a & c & e & d & f & \\
0 & 0 & 0 & 0 & g & 0 & h & 0 & i \\
0 & 0 & 0 & 0 & 0 & g & 0 & h & i \\
0 & 0 & 0 & 0 & h & 0 & g & 0 & i \\
0 & 0 & 0 & 0 & h & 0 & g & 0 & j \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & j \\
\end{bmatrix}
\begin{bmatrix}
\sigma_i \\
\sigma_{ii} \\
\sigma_{iii} \\
\sigma_{iv} \\
\sigma_v \\
\sigma_q \\
\sigma_{qii} \\
\sigma_{qiii} \\
\sigma_{ix} \\
\end{bmatrix}
\]

(3.3)

where, from either Eqn 3.1 or Eqn 3.2, the shape function values \(N_i\), of \((a, b, c, d, e, f, g, h, i, j)\) can be substituted by \((2.1869, 0.2778, 0.0353, -0.9859, -0.1252, 0.4444, 1.4788, 0.1878, -0.6667, 1)\), respectively.

Although the local smoothing work has now been completed, it does not produce unique values at the nodal points, and therefore, it is necessary to apply further nodal averaging. Figure 3.5 depicts the local smoothing and averaging method by using the normal stresses generated along a part of inner fibre of pipe wall. As can be seen, after the application of smoothing and extrapolation, the oscillatory nature of finite element stresses has been removed. It is natural that, even though the smoothing has been completed, the stresses are not continuous between two neighbouring elements due to fundamental assumptions in the finite element method. This phenomenon can be mitigated by a nodal averaging technique, as described elsewhere (Hinton et al., 1974 and 1975).

3.3.2 Normal and shear stresses in soil-pipe interaction problems

Figure 3.6(a) presents a small square element of unit thickness oriented in direction \(x, y\) for the general case of plane stress considering both normal and shear stresses. The horizontal and vertical stresses \(\sigma_x, \sigma_y\) act on the edges of the element. The shear stresses \(\tau_{xy}, \tau_{yx}\) act on
planes in directions parallel to the y and x axes respectively, which are in turn perpendicular to the direction of the normal stresses. The sign conventions for stress will be the same in this thesis, irrespective of material types employed. Compressive stress and anticlockwise shear stress are assumed to be positive, whereas tensile stress and clockwise shear stress are negative.

By the principle of complementary shear to satisfy static equilibrium of the element abcd for moments taken about a point such as "a", the shear stresses acting on both planes $bc$, $cd$ are equal in magnitude but opposite in sign ($-r_{xy} = r_{yx}$).

A free body diagram of a wedge taken from the small square element is shown in Figure 3.6(b). The resultant normal stress $\sigma_{n\theta}$, acting on a plane inclined at an angle $\theta$ to the x direction is given by:

$$\sigma_{n\theta} = \sigma_y \cos^2 \theta + \sigma_x \sin^2 \theta + r_{yx} \sin 2\theta$$  \hspace{1cm} (3.4)

The corresponding shear stress $r_\theta$, is:

$$r_\theta = \frac{(\sigma_x - \sigma_y)}{2} \sin 2\theta + r_{yx} \cos 2\theta$$  \hspace{1cm} (3.5)

Figure 3.7 is a graphical representation of Mohr’s stress circle for the foregoing stress relationships, which was discovered by Culmann in 1866 and developed in detail by Mohr in 1882. The Mohr circle provides a useful graphical method for determining the normal and shear stresses on any plane through a point in a stressed body when the major and minor principal stresses, $\sigma_1$ and $\sigma_2$, are known. The circle can be drawn in $\tau-\sigma$ space, using a boundary conditions given in Figure 3.7. The stresses $\sigma_{n\theta}$ and $r_\theta$ in the direction of inclination $\theta$ anticlockwise in Figure 3.6(b) can be determined by rotating the stress point on the Mohr circle with an angle of $2\theta$ clockwise.
From this figure, the resulting normal and shear stresses imposed on an inclined plane can be expressed by

\[
\sigma_n = \frac{(\sigma_x + \sigma_y)}{2} + \frac{(\sigma_y - \sigma_x)}{2} \cos 2\theta + r_y \sin 2\theta \quad (3.6)
\]

\[
\tau_\theta = r \sin (2\alpha - 2\theta) \quad (3.7)
\]

where, \(\tan 2\alpha = 2 \tau_y / (\sigma_y - \sigma_x)\). The resulting stresses obtained from the above equations are identical to Eqns 3.4 and 3.5 as required. (Parry, 1995)

Eqns 3.4 and 3.5 for normal and shear stresses acting on the inclined plane of a small element will be used to compute both the normal and shear stresses occurring in soil in contact with the pipe (corresponding to radial earth pressures) and flexural bending stresses at the extremities (inner and outer fibres) of pipe wall in the subsequent chapters.

### 3.3.3 flexural bending moment in pipe wall

Internal forces occurring at any cross section around the pipe wall can be resolved into two components, normal and tangential, to the section. The component normal to the section is known as the bending stress distribution, which consists of compressive stress on one side of the neutral axis and tensile stress on the other. These components can be used for the determination of bending moment using simple bending theory. The other component tangential to the section is the shear stress, which will not be discussed in this section since either the prestressing effect reduces diagonal tension in the pipe wall (tending to cause ovalisation of the pipe) to a negligible level, or a rigid pipe with a large section of steel plate is assumed to be sufficiently safe in shear.

The stress–strain relationship is shown in Figure 3.8 under the following fundamental assumptions; the pipe wall behaves as a linear
elastic homogeneous material. More simply, the maximum strain at any extremity of the section is assumed to be within the linear elastic limit, thus the stress components normal to the section (compression and tension on either side of the axis) are in proportion to the distance from the neutral axis, according to Hooke's law.

From the fibre stresses induced at inner and outer fibres, the bending moment is given by

$$ M = \frac{f_1}{y_1} I_y = \frac{f_2}{y_2} I_y $$

(3.8)

where, $f_1$ is the stress at the inner and outer fibres, and $y_1$ is the corresponding distance from the neutral axis. $I_y$ is the second moment of inertia of the section of the beam with respect to the neutral axis.

As can be seen in Eqn 3.8, the ratio of $f_i$ to $y_i$ is invariable in a given cross-section of beam, hence, the distance $y_1$ in Figure 3.8(c) is derived as:

$$ y_1 = \frac{f_1}{f_1 - f_2} t_w $$

(3.9)

where, $t_w$ is a total wall thickness ($= y_1 + y_2$). Now, if $y_1$ in Eqn 3.8 is replaced by Eqn 3.9, the bending moment in the wall can readily be obtained as:

$$ M = \frac{t_w^2}{12} (f_1 - f_2) $$

(3.10)

This equation is based on the stresses at the inner and outer fibres of the pipe wall computed by post processing the F.E. results. In a similar way, Woods(1999) has introduced a simple method for the estimation of bending moment directly from the integration point of LSQ elements.
3.4 Modelling Compaction Effects in CRISP

Due to the complexity of the soil-pipe installation geometry, a simple rigid retaining wall has been used to confirm the possibility of analyses which incorporate the compaction effects in CRISP. This section provides a comparison between the results obtained from numerical analysis by the writer and an approximate analytical solution suggested by Ingold(1979a).


Compaction loading is defined as "a transient moving surficial load of finite lateral extent" (Seed and Duncan, 1986), causing a temporary increase in the vertical stresses within the fill. In compression and shear tests, a removal of loading leads to a relatively small strain reversal. Such a phenomenon can be applied to retaining structures whereas, after application of compaction plant to the fill behind a wall, the lateral pressure increases nearly to a value indicating a state of overconsolidation (Rowe, 1954). In other words, the compaction of the fill (representing a process of loading and unloading) results in a significant increases in residual lateral pressures which approach the value corresponding to passive pressure (Duncan and Seed, 1986).
A brief introduction to compaction is given in the following sub-sections. No attempt is made here to describe either the compaction effect in complete detail or the relevant finite element techniques.

3.4.1 Stress due to vibration effect

Compaction results in the improvement of the engineering properties of a fill, which is usually represented by strength and compressibility as a function of soil density. The physical mechanisms of vibration are generally adopted in controlling the sand densification to meet the requirements for use. However, there are many uncertainties about compaction work in the field. The variables can be classified into two categories; firstly, details about the compaction plant including its size and weight, operating frequency, forward speed, and proceeding orientation; secondly, properties of the fill materials being compacted such as particle size, gradation, void ratio, water content, existence of cohesion, etc.. Moreover, the height of an individual lift of fill and the number of passes also affects the degree of compaction. Since no qualitative method for estimating the dynamic stresses and strains due to compaction exists in the current practice, qualitative experimental data from published reports are used to clarify our understanding of the principle features associated with the compaction effect.

The zones influenced by dynamic compaction plant can be distinguished into three parts as shown in Figure 3.9. The first region is called “free fall zone” where great dynamic stresses and accelerations occur due to repetition of intervals of free-fall by impact. The second is the “large stress fluctuation zone” where compaction is caused by stress fluctuations within the soil away from free fall state. Finally, the “shear failure zone” where the relaxation of stress occurs by overcompaction or overvibration near to the compaction plant. (D’Appolonia et al., 1969 and Youd, 1972)

For developing compaction criteria and techniques including the proper
selection of compaction plant, D'Appolonia et al.(1969) carried out extensive tests by using a "Washington" in-situ densometer in test pits. Figure 3.10(a) shows typical density-depth curves with respect to the number of roller passes. A 55.6 kN (12.5 kip) vibratory roller was used on a single soil layer of 2.4 metre (8 foot) total thickness. The operating frequency and forward speed of the roller were 27.5 cps and about 0.6 metre per second, respectively. From this, it can be found that with increasing the number of the roller passes the maximum density and effective compaction depth increase, but the compaction effect of the upper part of the ground diminishes, initiating from the critical depth $z_c$, due to overcompaction.

A detail of density profiles for two consecutive layers placed and compacted in lifts of 0.6 metre thickness is provided in Figure 3.10(b). The solid curve represents the density profile after completion of compaction on the second lift, while the dashed curve corresponds to the first lift. As has been expected, the density of the shear failure zone for the first lift is slightly increased by consecutive compaction on the second lift, thus the final density of the first lift becomes the total sum of the initial density of the first lift, the increase due to the compaction on that and the increase by the compaction on the second lift (refer to the bold dashed line). To put it more simply, the density curve of the first layer tends to deflect slightly upward resulting from the consecutive compaction of the second lift, which could be expressed by $\gamma_1 = \gamma_0 + \Delta \gamma_1 + \Delta \gamma_2$. For this reason, the height of each layer should not be markedly greater than the critical depth $z_c$, in order to prevent a loose layer becoming stratified near the interface between the two layers.

For vertical stress induced by dynamic compaction plant, no qualitative solution taking account of the dynamic effect of vibratory roller has been provided, although studies on this subject have been made by many researchers in both ways; the experimental approach by Whiffen(1954), D'Appolonia et al.(1969), Toombs(1972), and the theoretical approach by Selig(1963), Yoo et al.(1979), Ingold (1979a), etc.. Hence, the total force generated by dynamic compaction is normally
calculated as twice the static load based on limited experimental work carried out by Whiffen (1954) and D’Appolonia et al. (1969). However, if the centrifugal force is known, the line load can be obtained by the summation of static load and centrifugal vibrator force. (Ingold, 1979a and Clayton et al., 1986).

3.4.2 Lateral pressures in a granular fill

The stress path for compaction pressures for a granular soil can be modelled using the results of a K₀ triaxial test for normally consolidated loading, where the cell pressure is controlled to maintain the initial diameter of the specimen during the increase and/or decrease of the vertical pressure. Figure 3.11 shows a typical single cycle of the non-linear stress path model for a granular soil.

Andrawes et al. (1973) defines the coefficient of earth pressure at rest as “the ratio of the increment in the minor principal effective stress to the corresponding increment in the major principal effective stress when no strain occurs in the direction of the minor principal stress”. Thus, increasing the vertical stress on the specimen gradually at first loading, the cell pressure should also be increased to restrain the sample from barrelling. This means that the ratio of horizontal to vertical effective principal stress K₀ remains constant, with σ₉/₀ = K₀σᵥ. For the modelling of a triaxial test idealising the stress path followed by a granular soil undergoing compaction, the coefficient of lateral earth pressure at rest for primary loading can be represented by Jaky’s (1944) equation for normally consolidated soil, which is probably the simplest and most reliable, (Carder et al., 1977; Seed & Duncan, 1984; Clayton, 1986; Ou, 1988). That is K₀ = 1 − sin φ', where φ' is the effective stress friction angle of soil evaluated experimentally by conventional drained triaxial or direct shear tests. Mayne et al. (1982) confirmed that Jaky’s simple relationship (considering only the friction angle) is generally reasonable for primary loading with respect to the laboratory data compiled from over 170 different soils tested and reported by many researchers. They also performed a statistical analysis to establish the individual equations
for cohesive and cohesionless soils based on Jaky's equation. Collecting all available data corresponding to 121 clays and sands, a best fit line was derived from a linear regression analysis, \( K_0 = 1 - 1.003 \sin \phi' \).

Andrawes et al. (1973) performed an investigation of the factors affecting the value of \( K_0 \), using triaxial test results from 4 in. by 4 in. loose and dense sand samples with free-end platens. The factors were classified into two main categories, concerning the effect of the applied strain condition and the physical properties of the soil. Table 3.1 summarizes the properties of four different materials on the loose and dense states. According to the test results, it was concluded that the coefficient of earth pressure at-rest is mainly governed by the true angle of friction between particles (\( \phi_p \)) identified by Rowe (1962). With increasing values of porosity, crushing, and elastic modulus of the mineral particles, the value of \( K_0 \) increases. Moreover, it was emphasised that the cause of an increase in the sample diameter during loading is due to plastic strain whereas elastic deformation results in a decrease in the diameter. As a result, loose sand tends to deform more significantly compared with dense sand at a given condition, since the magnitude of irrecoverable deformation of loose sand is much higher than that of dense sand. Therefore, sand material in a loose state causes a higher value of \( K_0 \) to be induced. From the above, Jaky's equation correlating only to the effective stress friction angle of soil as a function of stress history seems to be generally reasonable for most engineering purposes, and has the attraction of simplicity.

Returning to Figure 3.11, decreasing the vertical stress gradually from the peak point A, little reduction in the cell pressure is initially required in order to maintain zero lateral strain conditions. However, when the vertical stress approaches the horizontal stress, increasing reduction of horizontal stress becomes necessary. As the vertical stress is further reduced, the passive failure state will be approached, and eventually the unloading curve moves down to point B. The stress path between points B and C with an approximately linear path is expressed as \( K_r = \sigma'_h / \sigma'_v \).
Ingold (1979b) suggested that $K_a$ and $K_p$ based on the Mohr–Coulomb failure criteria can be used for the lateral pressure ratios for loading and unloading curves under the consideration of limiting passive failure. This assumption was partially or fully agreed by some researchers (Pruska, 1973; Mayne et al., 1982.). However, from the view point of experiments at pilot scale carried out by Carder et al. (1977), it is more reasonable to use $K_o$ and $K_r = 1/K_o$, rather than $K_a$ and $K_p$, particularly in the case of a rigid structure, where the value of $K_o$ is based on the Jaky’s (1944) equation for granular soils. (Broms, 1971; Clayton et al., 1991; Murray, 1980; Lambe et al., 1969; Symons et al., 1989 & 1992)

3.4.3 Simplified bi-linear stress path model

A simplified analytical procedure was developed by Broms (1971), which takes into account the hysteretic loading and unloading behaviour of granular soils. Figures 3.12(a) details a simple stress path for a shallow soil element and Figure 3.12(b) for a deeper soil element, during loading and unloading. Point A indicates the initial state of stress at rest; i.e. the state before compaction takes place on the surface of the fill. The lateral earth pressure can therefore be expressed by $\sigma_{hi} = K_o \sigma_{vi}$, where $\sigma_{hi}$, $\sigma_{vi}$ and $K_o$ are the initial lateral effective stress, initial vertical effective stress (the overburden pressure), and coefficient of lateral earth pressure at rest during loading (assuming $K_o = 1 - \sin \phi'$), respectively.

When the compaction plant is positioned above the soil element the vertical stress will be increased, whence the stress path follows from A to B. The equation for lateral earth pressure is then replaced by $\sigma_{hm} = K_o \sigma_{vm}$, where $\sigma_{hm}$ is the lateral effective stress and $\sigma_{vm}$ the vertical effective stress. The vertical effective stress $\sigma_{vm}$, is obtained by adding the increase in vertical stress during loading, $\Delta \sigma_v$, to the geostatic pressure of fill material (i.e. $\gamma z + \Delta \sigma$). Once the compaction plant moves away from the fill, the vertical stress $\sigma_{vm}$, at a shallow depth decreases until the original vertical stress $\sigma_{vi}$ is regained.

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Although vertical stress is reduced after first loading, a large proportion of the increased lateral stress remains. Hence, it is assumed that the stress path follows line \( BC \) until the vertical stress reaches point C, after which it follows the \( K_r \) line (i.e. \( \sigma'_v = K_r \sigma_v \) where \( K_r \) is the coefficient of limiting earth pressure for unloading at rest). On the other hand, in Figure 3.12(b) the maximum lateral effective stress at depth upon unloading \( \sigma'_{hm} \), is retained at point C'. However, the rate of increase of lateral pressure is insignificant compared to the shallow soil element.

A critical depth \( z_c \), where the stress state following unloading returns to C, is roughly estimated by \( z_c = K_0^2 \sigma'_{vm} / \gamma \) (Clayton et al., 1993). At shallow depth, where the increase of vertical pressure due to compaction is much higher than the self weight of the fill (\( \Delta \sigma \gg \gamma z \)), the critical depth is further simplified by assuming \( \gamma z \) to be negligible; i.e. \( z_c \approx K_0^2 \Delta \sigma_v / \gamma \). Eventually, once the compaction plant is removed, material deeper than the critical depth retains its residual lateral stress, (since the lateral strains are largely plastic and therefore irrecoverable) while the lateral stress shallower than the critical depth approaches \( K_r \sigma_v \).

### 3.4.4 Application of elastic theory to practice

Ingold (1979a) has made a useful approximation for lateral earth pressures, using Holl’s (1941) analytical method to evaluate the stresses below the ground surface induced by an infinite line load on an elastic half space. The vertical stress increase at depth \( z \), immediately below the line load \( p \), is given by \( \Delta \sigma_v = 2p / \pi z \). Ingold (1980b) pointed out that the resulting error due to the limited roller width is generally insignificant except for very narrow roller widths.

Figure 3.13(a) depicts the lateral earth pressure distribution during compaction at the surface of a fill. Dotted lines “1” and “3” correspond to a conventional active lateral pressure due to the earth fill and a
passive state, respectively. Curve "2" represents an increase of the lateral earth pressure due to compaction. Figure 3.13(b) highlights a locus of the maximum compaction pressure at rest, indicating the reduction of lateral stress above the critical depth $z_c$, whereas the increased lateral stress below the critical depth remains. However, it is noted that from the critical height $h_c$ the active lateral earth pressure tends to be greater than the pressure due to compaction.

It is claimed that, under most circumstances, the lateral pressures due to compaction will not be as high as $K_o \sigma'_{vm}$, since the fill is at least partially able to yield in the horizontal direction during compaction. Besides, Spangler (1936) reported that, in his experiments, the pressure distributions on the retaining wall were considerably greater than those expected from the Boussinesq solution for a point load. In response, Mindlin (1936) introduced the "method of images" resulting in double stress on a smooth unyielding wall compared with the general solution. In conclusion, difficulties exist in making elastic predictions adjacent to retaining walls, as classical solutions utilise loads on the surface of an elastic half space.

3.4.5 Numerical analysis of stress on a retaining wall with/without interface elements

Numerical analyses have been performed by the writer to examine the compaction effect in CRISP, by comparison with the trend of stress distribution shown in Figure 3.13. As stated earlier, a simple rigid retaining wall was used for the preliminary analyses. Since CRISP has no functions to model static and dynamic line loads (normal to the wall face) imposed on the surface of each fill layer, various loading cases have been examined, including the distributed load and line load running parallel with the wall face, but limited data will be included in this thesis. It is nearly impossible to identify the magnitude of the load imposed on the surface of each layer as an input data, although this appears to be insignificant in the results of the analysis.
The inclusion of interface elements greatly affects the modelled performance of underground structures in some situations. This section therefore considers both cases; with and without interface elements between two media.

3.4.5.1 Without interface elements

In conventional implementations of numerical analyses, the constraint of no overlapping between adjacent elements, combined with compatibility requirements, requires the interface to be fully bonded. Figure 3.14 illustrates the distribution of lateral pressure induced by backfill at each successive layer (300 mm thickness) accompanied by compaction with a static roller from the lower ground surface. Strictly speaking, it depicts the horizontal stresses at the corner nodes along the edge of the soil (See node numbers 1 and 4 in Figure 3.3.) contacting the wall face. As was expected, neither the elastic perfectly plastic soil model nor the elastic soil model provided useful information throughout the analyses. The observations are summarised as follows:

♦ When unloading, no dissipation of input energy in the soils above the critical depth (the shear failure zone) occurs, which is a major deficiency of the current CRISP program.

♦ The magnitude of lateral pressures tends to increase more highly than expected due to an excessive expansion of the soil during consolidation. For instance, Appendix A1-1 represents a compression state when the boundary of a current backfill layer is loaded in 20 equal increments. A1-2 corresponds to the state when the load is completely removed from the surface of the soil mass in 20 equal increments. Finally, in A1-3, consolidation takes place for a period of time (50 increments) with no external loads imposed.

♦ In general, loading leads to volume change in a soil mass, consisting of elastic and plastic deformation. Following unloading,
elastic deformation should be recovered, while the plastic deformation is permanent. This recovery results in an increase of lateral pressure, which is normally greater than the geostatic lateral pressure (at zero lateral strain conditions) within the boundary of critical height ($h_c$).

- The lateral pressures should be almost identical with each other, irrespective of the magnitude of the external loads, since the resultant stress is totally dependent on the strength properties (Refer to Figure 3.11, where the final stress reaches point C, however, it tends to slightly increase with repetition of loading and unloading.).

Nevertheless, the trend of pressure patterns are almost identical to the previous experimental works carried out by Sims et al.(1970) and Broms and Ingelson (1971), in respect of the pressure distribution produced along the wall face. Moreover, the stresses acting on the middle third of the wall were always higher than those on the lower part, forming a trapezoidal shape.

3.4.5.2 With interface elements (LSQ)

Interface (slip) elements are widely used to model the junction of two media (structure and surrounding soils), which allows relative displacement; for example, analysis of soil-reinforcement interaction (Gens et al.;1988), rock joints (Goodman et al.;1968), and soil-structure interaction (Ng et al.;1998), etc. The methods for modelling of interfaces/joints fall into one of two categories. The first model uses discrete springs for the discontinuous behaviour at the interface as shown in Figure 3.15(a), and the second is a quasi-continuum of small thickness. The latter is subdivided into; (i) the conventional finite element with thin layer shown in Figure 3.15(b), where the material type is usually identical with the main structure or soil elements (Griffiths;1985, and Desai et al.;1988), and (ii) the zero thickness interface element shown in Figure 3.15(c), with restricted tension.
function in which the relative displacement between soil and structure is the main variables (Goodman et al.;1968, Beer;1985, Gens et al.;1988 and Ng et al.;1998). This zero thickness interface element implies that two adjacent nodes along the larger dimension can be given the same co-ordinates as the corresponding element of the main structure. This results in possible overlapping of the interface elements on the surrounding structural elements.

Normal and shear stress-strain behaviour of the interface element in direct shear testing exhibit closure of the interface in compression state, debonding at even a small tensile stress, and slip at a limiting shear stress. The idealisation of the debonding state is assumed to commence at the transition from compressive to tensile at the normal stress. The modes of deformation at an interface due to stress cycling are distinguished as non-slip, slip, unloading, debonding and rebonding, which eventually results in the increase or decrease of normal and shear stresses of the interface element. (Ng et al.;1998 and Desai et al.;1988)

**Interface elements in CRISP**

The current interface element in CRISP94 has been described by Britto and Gunn(1990) and is limited for plane strain analysis at present (as the validity of the program is not yet confirmed for axisymmetric analysis yet). This element can only be used with linear strain triangle or linear strain quadrilateral element types.

Figure 3.16(a) represents typical interface elements used in a CRISP analysis with a flat 8 nodal points composed of 6 nodes and 2 dummy nodes midway along each of the narrow sides, thus being compatible with the 8-noded quadrilateral element. The narrow sides of the interface element can not be attached to either structure or soil elements, only with the narrow side of another interface element, exiting on a free boundary, or joining a fixed boundary. (Britto and Gunn, 1990)
The interface element in CRISP consists of six material properties as follows (with typical SI units):

- $C$: cohesion (kN/m²)
- $\phi$: angle of friction (°)
- $k_n$: normal stiffness (kN/m²)
- $k_s$: shear stiffness (kN/m²)
- $k_{sres}$: residual shear stiffness (kN/m²)
- $t$: thickness of slip element (m)

The modulus of the interface element for normal or shear behaviour can be determined by the corresponding stiffness, $k_n$ or $k_s$ multiplied by the chosen thickness of interface $t$. The real moduli of the interface element can be therefore expressed by

\begin{align*}
K_n &= k_n t & \text{(3.11)} \\
K_s &= k_s t & \text{(3.12)}
\end{align*}

As shown in Eqns 3.11 and 3.12, the moduli of the interface element are a function of the thickness $t$, which implies that with increase of thickness the normal and shear stiffnesses increase.

The shear stress–strain curve (when very small load increments are being used and when the normal stress is compressive) is modelled as an elastic perfectly plastic behaviour. Therefore, when the shear stress reaches $0.99 \tau_{ult}$, the stiffness $k_s$, is transformed to the residual shear stiffness $k_{sres}$, (this effect is insignificant in the program), then, flows without further stress increase (corresponding to plastic deformation). With decreasing shear stress from the ultimate stress state it follows the lines with same gradients of residual shear stiffness $k_{sres}$, and then elastic shear stiffness $k_s$. When the normal stress is tensile, the normal stiffness is reduced by a factor of 100, which leads to a small negative value in order to prevent the stress drifting away to a large negative
value. If any subsequent compressive normal stress is applied, the program restores the normal stiffness to $k_n$ with the reverse displacement. Therefore, a gap may physically exist although the program assumes that the interface element is closed (Ng et al., 1998).

Returning to the Figure 3.14, the result of numerical analysis including the interface elements exhibits some energy dissipation near the ground elevation, unlike the others. However, no major differences in the stress distribution can be found compared with the case without interface elements. The distributions of displacement with depth on both sides of the interface elements are plotted in Figure 3.16(b) (Refer to Appendix A2). The displacement for the edge of the interface elements contacting the wall face could be assumed to be zero. For the edges contacting the soil, the vertical displacement may be reasonable, but an excessive horizontal displacement seems to be unrealistic. In this respect, Figure 3.17 illustrates some discrepancy between the internal stresses occurring in the wall face and in the adjacent soils in the horizontal direction. It would appear that excessive displacement of the interface elements in the horizontal direction causes internal stresses to be "lost" during transmission from soil to wall, which may produce invalid estimates of wall stresses.

As shown in Figures 3.12(b) and 3.13(b), with increasing earth depth the compaction effect becomes insignificant, and there is no influence on the horizontal stress at depth. Under these circumstances, it would appear that, as an alternative approach, it would be safer to use the coefficient of lateral earth pressure $K_o$, instead of applying the external load (loading and unloading).

Here, the coefficient of lateral earth pressure $K_o$, could be deduced from Holl’s(1941) linear elastic stress analysis for determining the stresses and surface displacements in a semi-infinite elastic condition. His assumptions were made similar to the Boussinesq’s solution; firstly, the materials of both supporting and stressed bodies behave as a plane strain condition, therefore, the stresses and deformations are the same in all transverse sections; secondly, the supporting element has uniform elastic properties unless otherwise specified; and finally, the stress distribution is entirely due
to surface loads excluding the body forces and weight of the supporting element. Figure 3.18 illustrates a principle feature of stress distribution and the basic symbols and components of infinite line loads and stresses on the supporting element, in rectangular coordinates. The stresses due entirely to an infinite line load were derived by integration of the Boussineq solution for a concentrated point load in rectangular coordinates, where the horizontal stress is expressed by:

$$\sigma_x(=\sigma_h) = \int_{-\infty}^{\infty} [\sigma_y]_h \, dy = \frac{2\nu \rho z}{\pi \rho^2}$$  \hspace{1cm} (3.13)

Here, setting $\rho = z$, Eqn 3.13 becomes

$$\sigma_y(=\sigma_h) = \frac{2\nu \rho}{\pi z} = \nu \sigma_z(=\nu\sigma_x).$$  \hspace{1cm} (3.14)

Here, it can be seen that Eqn 3.14 is exactly the same as the general form of conventional horizontal pressure $\sigma_h = K_0 \sigma_v$, leading to $\nu = K_0$. In conclusion, the compaction effect in the finite element analysis would indirectly be simulated by using Poisson’s ratio based on the Jaky’s equation.
Table 3.1 Values of $K_0$ in loose and dense states (Andrawes et al., 1973)

<table>
<thead>
<tr>
<th>Material</th>
<th>Mineral composition</th>
<th>Particle shape</th>
<th>Particle size (mm)</th>
<th>True angle of friction $\phi_\mu$ (degree)</th>
<th>Porosity</th>
<th>Value of $K_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>loose dense</td>
</tr>
<tr>
<td>Glass Ballotini</td>
<td>mostly quartz</td>
<td>spherical</td>
<td>0.1</td>
<td>17</td>
<td>0.411</td>
<td>0.364</td>
</tr>
<tr>
<td>Silver Sand</td>
<td>mostly quartz</td>
<td>rounded</td>
<td>0.08-0.6</td>
<td>27</td>
<td>0.411</td>
<td>0.352</td>
</tr>
<tr>
<td>Crushed Feldspar</td>
<td>feldspar</td>
<td>very angular</td>
<td>0.15-0.3</td>
<td>36</td>
<td>0.495</td>
<td>0.407</td>
</tr>
<tr>
<td>Copper Particles</td>
<td>copper</td>
<td>spherical</td>
<td>0.15-0.3</td>
<td>-</td>
<td>0.439</td>
<td>0.376</td>
</tr>
</tbody>
</table>
Figure 3.1 General flow chart of CRISP program (Hillier, 1992)
Figure 3.2 Out-of balance load for elastic-perfectly plastic model in CRISP (Britto, 1992)
Figure 3.3 Global and natural coordinates for a quadratic element
Figure 3.4 Integration points in a quadrilateral element
Normal bending stress, kPa

Angular position from pipe crown, degree

Unsmoothed normal stress

Locally smoothed/ extrapolated and averaged normal stress

Locally smoothed/ extrapolated normal stress

Figure 3.5 Typical bending stress distribution around inner fibre of pipe wall
ID = 4000mm, tw = 280mm (data obtained from Chapter 5)
Figure 3.6 Biaxial stresses at a small square element for a plane stress analysis
Figure 3.7 Mohr stress circle for normal and shear stresses (Parry, 1995)
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Figure 3.13 Stress distribution in a granular soil
(Broms, 1971)
Figure 3.14 Variation of horizontal stress of soils contacting to wall face as a function of various working conditions
(a) Spring model

(b) Continuum model

(c) Special model

Figure 3.15 Interface elements (after Barkhordari-Bafghi, 1998)
(a) Interface elements between two contacting media

(b) Displacement of nodes of interface elements contacting two media

Figure 3.16 Element type and displacement of special interface element in CRISP program
Figure 3.17 Comparison of horizontal stresses generated in wall and adjacent soils due to compaction and soil weight
(a) Stress distribution due to an infinite line load

(b) Infinite line load acting normal to the surface

Figure 3.18 Holl's linear elastic stress analysis (Holl, 1941)
CHAPTER 4 - NUMERICAL MODELLING OF RIGID PIPE

4.1 Introduction

The aim of this chapter is to evaluate the major properties of materials used in soil-pipe interaction problems, such as the stiffness and strength parameters for both the pipe and the surrounding soils. The work is based on the plane strain condition, since the length of pipe along the centroidal axis is considerably larger than its diameter.

The flexural rigidity of pipe will be discussed in detail, using some experimental data regarding the full scale three-edge bearing tests together with concrete properties obtained from reliable laboratories. Then, the procedure for the evaluation of the pipe wall stiffness is introduced in relation to the prestressed and non-prestressed pipe. The study includes the work carried out to prove the validity of CRISP by means of comparison with the results computed from the rational solution. Modelling issues will also be discussed in the end of this chapter in connection with the adequacy of the number of elements used in the mesh and the flexural rigidy of the pipe wall, which both influence bending moment distribution in the pipe wall.

4.2 Soil Properties

The soil input parameters for finite element analysis can be deduced from the relevant experiments with respect to soil-structure interaction problems. During the past decade, two important parallel developments have taken place which have resulted in the measurement of stiffness being considered more important than that of strength in geotechnical design, particularly for sensitive structures. These developments are:
(Clayton et al., 1995)

(i) methods of measuring strain locally on laboratory test specimens have shown that the stress–strain behaviour of many soils and weak rocks is significantly non-linear, with very high stiffness at the small operational strains around most engineering structures, and

(ii) certain features of field measurements of ground deformation around full-scale structures, which could be modelled using linear elastic theory, are resolved when non-linear formulations are used incorporating very high initial stiffness.

The establishment of appropriate input parameters to adequately characterise the surrounding soils is very difficult. In other words, it is rarely possible to predict real soil properties, since the soil is very complex material which exhibits nonlinear stress–strain behaviour, highly dependent on the previous stress history and current stress level. In the soil–structure system, the stiffness and strength parameters of newly worked soil vary with the confining stress level and density, which is governed by the degree of compaction.

4.2.1 Young’s modulus and Poisson’s ratio

To determine the soil parameters for the analysis, it is necessary to conduct a consistent set of tests representing the actual soils. However, it is impractical to perform extensive laboratory tests for a single design. As an alternative, reliable data obtained from past work can be used for the analysis of buried pipelines. For instance, the hyperbolic model for the non-linear stress–strain behaviour of soils (Duncan et al.:1980, Selig:1988, and Haggag:1989) is useful for the determination of initial input data for linear or bilinear soil models such as elastic or elastic–perfectly plastic in consideration of the compacted soil behaviour of the structural backfill. The stress–strain curves in a triaxial test
with a constant confining pressure can be approximated by a hyperbola as shown in Figure 4.1, with the expression (Kondner, 1963 and Kondner et al., 1963):

\[
\frac{(\sigma_1 - \sigma_3)}{E_i} = \frac{\varepsilon}{(\sigma_1 - \sigma_3)_u}
\]  

(4.1)

where,

\[
\begin{align*}
(\sigma_1 - \sigma_3) &= \text{deviator stress} \\
\varepsilon &= \text{axial strain} \\
E_i &= \text{initial tangent modulus}
\end{align*}
\]

In Eqn 4.1, a subscript \( u \), indicates the deviator stress at ultimate state, and the initial tangent modulus \( E_i \), varying with confining pressure \( \sigma_3 \), is substituted by the following equation obtained from the experimental studies by Janbu (1963).

\[
E_i = KP_a\left(\frac{\sigma_3}{P_a}\right)^n
\]

(4.2)

where \( P_a \), is the atmospheric pressure used to nondimensionalise the soil parameters for hyperbolic model; i.e. the modulus number \( K \), (not to be confused with lateral earth pressure coefficient) and the modulus exponent \( n \). The variation of dimensionless initial tangent modulus is shown in Figure 4.2 as a function of the confining pressure \( \sigma_3 \) in a logarithmic scale. From the above equations, the tangent modulus \( E_t \), for any stress state can be obtained by differentiating Eqn 4.1 with regard to the axial strain \( \varepsilon \), which is:

\[
E_t = \left[ 1 - \frac{R_1(1 - \sin \phi)(\sigma_1 - \sigma_3)}{2C \cos \phi + 2\sigma_3 \sin \phi} \right]^2 KP_a\left(\frac{\sigma_3}{P_a}\right)^n
\]

(4.3)

where \( R_1 \), \( C \) and \( \phi \) are the failure ratio (shown in Figure 4.1), the cohesion and the internal friction angle, respectively. These parameters were provided by collecting data from the available literature (Duncan
et al., 1980) and by experiments in triaxial tests (Selig, 1988). The elastic Young's modulus $E_s$, can be determined from Eqn 4.3 by putting the failure ratio $R_f$, and modulus exponent $n$, both to be equal to zero, that is:

$$E_s = KP_a$$  \hspace{1cm} (4.4)

Soil types being considered can be placed into one of the principal soil types classified by the Unified Soil Classification System shown in Table 4.1. Most constructed soils will be composed of a variety of different particle sizes, some of which may be cohesive. Selig's soil parameters were obtained from the triaxial test results conducted by Yang (1987) in which three soil types were chosen; i.e. SW, ML and CL, representing each group in Table 4.1. Plastic clay CH (group 4) was not considered for use in constructed (fill) soils. Table 4.2 provides soil parameters for determining the tangent Youngs' modulus (Eqn 4.3) for the hyperbolic model in a finite element analysis to predict the buried pipe behaviour.

As shown in Table 4.2, to obtain the values of soil parameters, $K$, $n$, $R_f$, $C$, $\phi$, and $\Delta\phi$, each sample was tested at a different compaction degree as very loose state, 80%, 85%, 90% and 95% of the AASHTO T-99 maximum dry density (Standard Proctor density) with the range of confining pressures 35 to 315 kPa. The secant Young's modulus at 50% of the failure stress (Figure 4.3) for various confining pressures (in a range of 5 to 200 kPa) and degrees of compaction are provided in Figures 4.4, 4.5 and 4.6 for soil types of SW, ML, CL, respectively. From the figures, it is clear that the stiffness of soil is highly dependent on the compaction degree, confining pressure and soil type. The stiffness of coarse materials is generally higher than that of fine materials. The coarse materials are more susceptible to confining pressure compared with the fine materials. For the CL soil, Figure 4.6 illustrates that the stiffness tends to increase by a certain level with increasing confining pressure, and then it decreases with further increase of confining pressure.
Poisson’s ratio $\nu_s$, varies with vertical strain until the stress level reaches a peak state in the stress–strain relation. For a well graded calcareous sand from Libya, an increase of the vertical strain results in an increase of Poisson’s ratio, indicating a nonlinear response. Poisson’s ratio at the initial stage (first loading) shows a value of 0.1 to 0.2. During cyclic loading, the typical Poisson’s ratio becomes more of a constant ranging between 0.3 to 0.4. (Lambe and Whitman, 1979)

From Hooke’s law, Young’s modulus $E_s$, and Poisson’s ratio $\nu_s$ can be converted to $E_{sp}$ and $\nu_{sp}$ in the plane strain conditions where $\varepsilon_y = 0$. They are

$$E_{sp} = \frac{E_s}{(1 - \nu_s^2)} \quad (4.5)$$

$$\nu_{sp} = \frac{\nu_s}{1 - \nu_s} \quad (4.6)$$

It is claimed that The Eqns 4.5 and 4.6 can be used for an analytical solution only.

### 4.2.2 Evaluation of shear strength

The strength parameters $\phi$ and $C$ are not intrinsic material properties but rather are parameters dependent on the relative density, applied stresses, degree of consolidation under those stresses, and the drainage conditions during shear. Accordingly, their values should be based on laboratory test data wherein the conditions replicate those expected in the field. However, the friction angle $\phi$, is usually inferred from in-situ tests or is conservatively assumed to be based on material type, due to the difficulty of obtaining undisturbed samples.

In Table 4.2, a silty soil (ML) and a clayey soil (CL) at compacted states require all three strength parameters ($C$, $\phi$, and $\Delta\phi$) to represent the failure envelope, unlike a coarse-grained gravelly sand soil
Figure 4.7 (a) shows the curved failure envelope for the majority of soils. The wide range of pressure results in significant curvature. This suggests that the internal friction angle for a given material varies with the confining pressure, leading to difficulties in the selection of a single value of $\phi$ for a construction material. By using the friction angle estimated from triaxial tests, where $\sin \phi = (\sigma_1 - \sigma_3)/(\sigma_1 + \sigma_3)$, the actual non-linear failure envelope can be described by the Duncan model, varying with logarithm of the confining pressure $\sigma_3$, as shown in Figure 4.7 (b):

$$\phi = \phi_o - \Delta \phi \log_{10}(\sigma_3/P_o)$$ (4.7)

where $\phi_o$ is a peak value of $\phi$, assuming the confining pressure is very low, i.e. for $\sigma_3 = P_o$, and $\Delta \phi$ is the reduction in $\phi$ for a ten-fold increase in $\sigma_3$. The values shown in Table 4.2 are assumed to be maxima, and Eqn 4.7 represents the lower part of Figure 4.7 (b). It seems that Eqn 4.7 is of practical use based on the experimental data obtained from the laboratory test.

On the other hand, the "saw blades" model of dilatancy on the rupture surface ($\phi' = \phi'_cs + \phi$, where $\phi'_cs$ and $\phi$ are the critical state angle of shearing resistance and the angle of dilation, respectively.) clarifies the concepts of friction and dilatancy with respect to the strength parameters for design, which provides a firm theoretical basis. More recently, this formula has been slightly modified as $\phi' = \phi'_cs + 0.8\phi$ in order to reflect both the failure mechanism optimised and the correlation between the direction of principal stress and strain increments in the original concept of an elementary friction-dilatancy relation. (Bolton, 1986) The concept of the "saw blades" model starts from the friction angle in a critical state with constant volume, and the excess of the friction angle is totally dependent on the volumetric expansion in a simple shear test, wherein Eqn 4.7 begins at the peak value with very low confining pressure and the reduction is made due to the increase of normal stress. The basic theory for both equations seems to be similar, but the approach is quite different.
4.2.3 Some observations for Libyan soils

The pipeline route in Libya generally crosses Quaternary deposits such as Proluvial and Wadi deposits. These deposits overlie mostly the carbonate rocks of the Zimam and Hurfah formations. The Wadi deposits appear generally to be gravelly, silty sand, which are loose to dense sand, subangular or subrounded. The Proluvial deposits appear to be silty, sandy gravel and cobbles, medium dense and of angular shape. The predominant material within the superficial deposits is silty to clayey sand (SM to SC-SM). It occurs at most locations along the route, usually layered with material from the other classification bands and occasionally with calcrete. Sands and gravels within the GW to GP and SW-SM to SP-SC bands are also present throughout the route.

Figure 4.8 presents the friction angles of the samples obtained from the direct shear test under drained conditions as a function of relative density in the range of 55-87%. It also indicates a difference between the peak states and the critical states for the samples corresponding to each relative density. The degree of compaction in-situ affects mainly the friction angle at peak state. It can be seen that with increasing degree of compaction, the friction angle increases mainly at the peak state. The difference of friction angles observed at peak state and critical state is constant at about 10 degrees. The mobilised angles at the peak state are between 42 and 48 degrees and those at the critical state are between 34 and 41 degrees. The wet density of soil is described in Table 4.2 separately as an input data of the numerical analysis.

4.3 Mechanical Reponse of Prestressed Concrete Cylinder Pipe

Prestressed Concrete Cylinder Pipe (PCCP) is classified into two general types of construction: (1) pipe with a steel cylinder lined with a concrete core (termed Lined Cylinder Pipe; LCP), where the concrete is
spun onto a steel cylinder by a centrifugal process, and (2) pipe with a steel cylinder embedded in a concrete core (termed Embedded Cylinder Pipe; ECP), in which concrete is poured around the steel cylinder between inner and outer moulds by vertical casting. Detail of the pipe section is provided in Figures 4.9.

A welded steel cylinder and joint ring assembly provides watertightness, adequate longitudinal tensile strength, and an increase in beam strength. The core is cast in steel moulds, providing a smooth interior wall for optimum flow. After the concrete core has attained the required compressive strength, it is helically wrapped with prestressing wire under controlled tension. The prestressing wires are protected by a sprayed mortar coating applied to the outer surface of the pipe. The prestressed concrete core provides the pipe with the structural strength and rigidity necessary to withstand heavy external loads and high internal pressures.

The mechanical response of a buried concrete pipe is accompanied by an intricate soil-structure interaction problem because of the nonlinear inelastic behaviour of constituent components (Corotis and Krizek, 1977). In this respect, it is necessary to investigate the response of the whole structure against the external loads. In other words, it is less important to understand the behaviour of each constituent material of the pipe.

4.3.1 Properties of concrete

Due to the experimental difficulties associated with establishing the origin of the stress-strain curve of a nonlinearly elastic material, the chord modulus (the slope of the chord drawn between any two specified points on the stress-strain curve which should be designated within the elastic limit of the material) is useful. To overcome significant errors at initial loading, specimen curvature or initial gap alignment, the slope of the stress-strain curve is usually determined in a range of the appropriate preload and the elastic limit. Alternatively, the tangent modulus can be used for the stiffness of concrete under the assumption
of linear elastic behaviour, but the use of initial tangent modulus (i.e. the tangent to the curve at its origin), or secant modulus (i.e. a line drawn from the origin to a point on the curve) is not recommended because of the above-mentioned problems.

Laboratory tests were conducted to determine the stiffness of concrete to be used for the production of PCCP. The value was selected within a range of allowable stress of concrete, ie 0 to 40% of the ultimate concrete strength in practice, according to ASTM C469-83. There is no rational method to determine the secant modulus; it is assumed to be in the stress range between 3 to 14 MPa in some laboratories, or within the stresses up to 15, 25, 33 or 50 percent of the ultimate strength in others; this method is also prescribed in ASTM C469-65, BS 1881: Part 5: 1970 and in ISO/DIS 6784 (Neville, 1994).

Figure 4.10(a) is a graphical representation of the compression stress-strain diagram plotted using data obtained from the 28 days tests of concrete cylinders at Sarir plant in Libya carried out in 1992. To determine the chord modulus of elasticity six samples were tested in Sarir plant and ten samples in Brega plant. Chord modulus was obtained from the increments of load and corresponding contraction by using the following equation:

\[
E_c = \frac{s_2 - s_1}{\varepsilon_2 - 0.000050} \tag{4.8}
\]

where,

\[
s_2 = \text{stress corresponding to 40% of ultimate load}
\]

\[
s_1 = \text{stress corresponding to an axial strain of 50/1,000,000}
\]

\[
\varepsilon_2 = \text{axial strain produced by stress } s_2
\]

In Eqn 4.8, \( s_2 - s_1 \) and \( \varepsilon_2 - 0.000050 \) are load increment and strain increment, respectively. The mean elastic modulus and ultimate strength were recorded at around 33.6GPa and 50.0MPa for Sarir plant, and
34.1GPa and 50.9MPa for Brega plant. From the experiments, it was observed that for high strength concrete there is no significant difference between the initial tangent modulus and secant modulus, and the initial linear part of the stress-strain curve extends to a stress much higher than 40% of ultimate strength employed in the conventional procedure. It may be as high as 85% or even higher. This phenomenon is also confirmed by Neville (1997).

The another series of tests were also conducted in Universite de Sherbrooke in 1996. Table 3 describes the composition and fresh concrete properties of the corrected mix MBL-4 for Brega plant and MS-3 for Sairir plant, which represent mix design for a little lower strength than those used in 1992. The mixing was done with a pan type mixer of 180 L capacity. The dry materials (cement, sand and coarse aggregates) were mixed for about 60 seconds before the beginning of the water addition. The water was continuously added into the mix for about 45 seconds. After that, mixing went on for an additional 60 seconds. According to the AWWA C304-92, 5 cylinders of 150 mm in diameter and 300 mm long were employed to conduct the complete testing program for each test. The moulding of the specimens was done according to the ASTM C192. The cylinders were cast in 3 layers. A vibrating table was used to consolidate each layer of concrete. One day after casting, the specimens were demoulded and put in a moist curing room in order to satisfy the "Standard Curing" described in the ASTM C512. The 28 day compressive strengths and the 28 day elastic modulus of concrete were measured according to ASTM C469 and ASTM C39, respectively (See Table 4.4 and Figure 4.10(b)).

Figure 4.11 presents the relationship between Young's modulus of concrete and its ultimate cylinder strength using the data obtained from the experiments in 1992 and 1996, together with the approximated lines for National Standards. For BS 8110, CP 110 and Eurocode 2, the cube strengths have been converted to cylinder strengths to enable direct comparison with test results. In Figure 4.11, the value of 1.16 is employed as a conversion factor from cubes to cylinders at 28 days according to the site trial results with each 33 specimens, but the cube
strength is taken by product of 1.25 and cylinder strength in normal practice. Subscript numbers 1 and 2 represent the data obtained from the Laboratory of Universite de Sherbrooke in 1996, and numbers 3 and 4 from site laboratories in 1992. Natural aggregates were used for numbers 1 and 3 mixes, and crushed aggregates for numbers 2 and 4 mixes.

In Figure 4.11, the broad relationship between the Young’s modulus of concrete and its compressive strength is not clear, however, the Young’s modulus is generally seen to be larger with a higher strength (under the equivalent test condition). In the case of the same materials, the magnitude of Young’s modulus depends highly on the concrete density and air content (affected by the casting method of the concrete), where the density of concrete generally increases with an increase in the density of aggregate amongst other things. Figure 4.11 in relation to the limited data indicates that (1) the empirical formula for normal weight of concrete recommended by ACI 318 building code, 

$$E_c = 4.73\sqrt{f_{cm}}$$ (GPa) which assumes that the modulus of elasticity of concrete increases with the square root of its ultimate cylinder strength, seems to be a lower bound solution, while (2) Eurocode 2, 

$$9.5(f_{cm} + 8)^{0.33}$$ (GPa), indicates an upper bound of data corrected from two laboratories in Libya and a lower bound for the data obtained from the results conducted by Universite de Sherbrooke in 1996. In general, (3) the formula for BS 8110 based on the limited state design method, 

$$5.5\sqrt{f_{cm}/\gamma_m}$$ (kN/mm²), where \( \gamma_m \) is a partial safety factor for material, and the British code of Practice CP 110 for the Structural Use of Concrete, 

$$E_c = 9.1 f_{cm}^{0.33}$$ (GPa) applicable to the concrete with density greater than 2300 kg/m³, seem to be more appropriate for the underground structures with the high strength concrete in comparison to ACI 318. Eurocode 2 may be sensible for a high performance concrete, satisfying a high modulus of elasticity, high density, low permeability and resistance to some forms of attack. It is noted that the above-mentioned Standard codes use a characteristic strength, while the data obtained from the laboratories corresponds to the mean strength.
To estimate Young’s modulus $E_c$, at different ages of concrete, the value of compressive strength at time $t$ may be taken from the following expressions:

$$f'_c(t) = \frac{f'_c(28)}{0.864 + \frac{3.8}{t}} \quad \text{for vertically cast concrete} \quad (4.9)$$

$$f'_c(t) = \frac{f'_c(28)}{0.959 + \frac{1.40}{t + 6.0}} \quad \text{for spun-cast concrete} \quad (4.10)$$

where $f'_c(28)$ is the design strength obtained from the concrete cylinders at 28 days. (Zarghamee and Heger, 1988)

Poisson’s ratio of concrete for an elastic material is usually in the range of 0.15 to 0.22 for concrete, averaging about 0.17. The concrete for PCCP has a higher density compared with the normal concrete practice, which results in the lower value of Poisson’s ratio.

For a plane strain analysis, the stiffness parameters are also defined by the following formulae:

$$E_{cp} = \frac{E_c}{(1 - \nu_c^2)} \quad (4.11)$$

$$\nu_{cp} = \frac{\nu_c}{1 - \nu_c} \quad (4.12)$$

From Table 4.3, concrete density is fixed at 2400 kg/m³ for all the finite element analyses. The elastic behaviour of mortar coating sprayed onto the surface of the concrete and prestressing wire is assumed to be identical to that of the concrete.
4.3.2 Flexural rigidity of pipe

Flexural rigidity is a major concern in the determination of the elastic deflection of a large diameter rigid pipe. Predicting the behaviour of a four-material composite with different properties such as concrete, cement mortar, steel sheet and prestressing wire is not straightforward; especially in the case of prestressed rigid pipe. Conclusively, in order to evaluate the real behaviour of the pipe, investigation of experimental results seems to be the only reliable way.

Unlike a normal concrete structure, the compression force from the prestressing wire with high tensile stress, wrapped helically on the outer surface of concrete core, results in the increase of virtual flexural rigidity of PCCP in respect to the deformation of the structure. Therefore, it would appear that an evaluation of the flexural rigidity from the correlation of three-edge bearing load and its measure deflection is valuable. This virtual flexural rigidity can be used as an input parameter in order to monitor the behaviour of pipe, including the flexural stresses generated. Generally, elastic deformation is directly proportional to the load imposed on the structure and span, and affected by the support arrangement, but in reciprocal proportion to the flexural rigidity EI. In this chapter, the immediate deformations corresponding to the applied load such as a three-edge bearing load will be considered, excluding time-dependent deformations due to creep and shrinkage of the materials.

4.3.2.1 Three-edge bearing test

In general, the three-edge bearing test is performed to determine the cracking load for a rigid pipe by using the procedure described in ASTM C497. It is also occasionally employed for the evaluation of the structural behaviour of pipe under external load. The test basically involves the application of an external line loading at the top and bottom of the pipe, as shown in Figure 2.19. The specimen of pipe is usually 2.5 metres long for a large diameter PCCP. To ensure the
application of a uniform load along the length of specimen, a fillet of plaster of paris as described in Section 4.3.7 of ASTM C497 is placed on the upper bearing surface. For the bottom bearing surface a similar arrangement is made, but two parallel fillet of plaster of paris are placed with a space not more than 25.4 mm (1 inch) apart for each 300 mm (1 foot) of specimen diameter. The load is applied at a uniform rate based on the length of specimen as described in Section 4.3.1 of C497, and by a single hydraulic jack placed above the pipe which pushes vertically downward on the top of the beam. Occasionally, the jack is placed below the floor to generate an upward force under the bottom of the beam. The self-weight of the pipe is zeroed out on the console dial before starting the test, so that the dial measures only the external load being applied to the test piece of pipe.

The usual three-edge bearing test is run either to establish or confirm a linear design relationship between the resultant prestress in the core $f_{cr}$, and the first visible crack load $W_{ool}$, on an inner fibre of the concrete core at the invert or crown and on the outside coating surface at the springline, where the highest tensile stresses in the pipe wall are generated. The term $W_{ool}$ is defined as the load at which a longitudinal crack appears and reaches a length of 300 mm with a width of 0.025 mm (0.001 inch). ASTM C497 requires the 0.25 mm crack load to be recorded, which is the limiting design load used for non-pressure applications such as reinforced concrete sewer or culvert pipe. (Price Brothers, 1986)

4.3.2.2 Non-linearity of load-deflection relationship

Full scale three-edge bearing tests were conducted at the GMRP site to investigate the load and deformation behaviour of large diameter PCCP. Deflection indicators were attached to the inboard of spring loaded plungers while the outboard ends of the plungers were in contact with the inner surface of the pipe.

A typical nonlinear load-deformation behaviour of two identical 4000
mm diameter PCCP with a 4.88 mm prestressing wire area of 30.90 cm²/m is presented in Figure 4.12. For pipe with identification number 3-12718-B, the mean concrete compressive strength at 28 day was recorded at about 46.7 MPa, indicating a standard deviation of 34.55 from 28 specimens. For 3-14286, the compressive strength was 44.2 MPa with a standard deviation of 32.66 for the same numbers of specimens.

During the test for 3-12718-B, the first visible crack was observed with the unaided eye at the spigot invert of pipe under the applied load of 411 kN per linear metre of pipe length as usual, propagating to a length of 300 mm when the load reached 431.2 kN. After that, another crack was observed at the spigot crown when the load was 451 kN. The load was increased continuously until the coating surface crack appeared on the spring line. When the load reached 510 kN, the coating crack appeared at three o’clock. The observation was made that the load–deformation behaviour of the two pipes was almost identical. Moreover, it can be seen that the slope of the load–deflection curve was gradually increased with an increase in the applied load, and the response of the pipe was apparently nonlinear.

The slope of the load–deformation curve for the 4000 mm PCCP with a large quantity of prestressing wire area (36.42 cm²/m) is less than the pipes with smaller quantity of prestressing wire, as shown in Figure 4.13. The response exhibited are almost identical to each other. The three–edge bearing tests for relatively small diameter PCCPs were also conducted for the same purpose. Regardless of the pipe sizes, a thickness of the concrete core relative to the inside diameter of the pipe was fixed by a minimum value of 1/16 in order to ensure the serviceability. The indication of the deformable characteristics is proportional to the flexural rigidity of the pipe section, where a larger diameter pipe maintains a higher stiffness.

The time of first visible cracks corresponding to $W_{0i}$ seems to be a function of pipe diameter, wall thickness and resultant compressive strength of concrete core relative to the radial confining pressures on
the concrete core toward the centre of the pipe. In this respect, some parts of the industry use the following empirical formulae to determine the allowable design 3-edge load $W_o$, where $W_o$ is defined by 90% of the 3-edge load $W_{00}$, producing incipient cracking in the core, with no internal pressure.

$$W_o = \frac{t_w^2}{R_m} (16.15 f_{cr} + 9.688) - 0.82 W_p$$ for ECP

$$W_o = \frac{t_w^2}{R_m} [ (9.50 + 0.173 D_m) f_{cr} + 5,377]$$ for LCP

where,

$t_w$ = pipe wall thickness

$R_m$ = mean radius of pipe

$W_p$ = pipe weight per unit length

If the minimum design thickness of the embedded cylinder pipe, 1/16 of the pipe inside diameter is used, $W_o$ can be also expressed by

$$W_o = \frac{t_w^2}{R_m} (14.45 f_{cr} + 8,657) - 0.73 W_p$$

Zaghamee and Heger (1988) confirmed that the accuracy of the above formulae has been substantiated for some standard prestressed embedded and lined cylinder pipe.

As has been pointed out, the general trend of the structural behaviour of large diameter rigid pipe is an entirely nonlinear response under three-edge bearing loads, although it can be assumed as a linear elastic material considering a secant flexural stiffness in the initial part of the curve. It can be seen that the hyperbolic model for the non-linear stress-strain behaviour is, in fact, more appropriate for the composite pipe structure.
4.3.2.3 Flexural rigidity

PCCP can be treated as a closed circular ring as a statically indeterminate beam. Castigliano's first theorem can be therefore employed for the structural analysis of pipe. The formulae for bending moments and vertical deflections are introduced to estimate the flexural rigidity of concrete pipe section associated with the three-edge bearing load.

The flexural rigidity of any concrete section is significantly affected by the extent of cracking in the element. Heger(1963) claimed that the magnitude of flexural rigidity of a cracked section is reduced by about 70% of its uncracked section. For the pressure pipe under working loads and pressures, it is necessary to maintain confidence that the concrete core does not develop microcracking at the inner fibre of the core and visible cracks at the outside surface of the core and coating by restricting the tensile strain. Due to the concentrated loads applied to the top and bottom of the ring, shown in Figure 4.14(b), maximum bending moments are generated at both loading points under the assumption of a zero weight of structure, and the diameter changes in the horizontal and vertical directions are given by the following formulae (Roark et al, 1985):

\[
\delta_{hu} = \frac{0.137W_{col}R_m^3}{EL} \\
\delta_{vu} = \frac{0.149W_{col}R_m^3}{EL}
\]

(4.16)  (4.17)

where,

\( \delta_{hu} = \) diameter change in horizontal direction (through the pipe springline at the ultimate state)

\( \delta_{vu} = \) diameter change in vertical direction (between the pipe crown and invert at the ultimate state)

\( W_{col} = \) three-edge bearing test load that produces an incipient core or coating crack
\[ R_m = \text{mean radius of pipe} \]
\[ E = \text{Young's modulus of pipe} \]
\[ I = \text{second moment of inertia of pipe wall section about neutral axis in unit length} \]

In which, the second moment of inertia \( I \), of a pipe section can be replaced by \( \frac{t_w^3}{12}(1 - \nu_c^2) \) in case of plane strain analysis, where \( t_w \) and \( \nu_c \) are the thickness and the Poisson's ratio of the pipe wall, respectively. Since a difference between \( I \) and \( \frac{t_w^3}{12}(1 - \nu_c^2) \) is very small (2–4%), it can be ignored in the analysis.

Displacements due to self-weight of pipe are calculated from the following general form: (Paris, 1921)

\[
\delta_h = \frac{0.4292w R_m^4}{EI} \quad (4.18)
\]
\[
\delta_v = -\frac{0.2798w R_m^4}{EI} \quad (4.19)
\]

where, \( w \) is the weight of the pipe per linear metre (Figure 4.14(a)).

Since the pipe weight is zeroed out on the console dial before starting the test, as explained before, the pure flexural rigidities of pipe for the horizontal and vertical directions can be expressed by the following rational forms:

\[
(EI)_{h,emp} = 0.137 \frac{W R_m^3}{\delta_h} \quad (4.20)
\]
\[
(EI)_{v,emp} = 0.149 \frac{W R_m^3}{\delta_v} \quad (4.21)
\]

where,

\[
(EI)_{h,emp} = \text{flexural rigidity to withstand horizontal}
\]

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deformation, corresponding to the section at the springline of pipe due to a given load $W$

$$\text{(EI)}_{\text{v,emp}} = \frac{\text{flexural rigidity to withstand vertical deformation, corresponding to the sections at the top and bottom of pipe due to a given load } W}{W}$$

Eqns 4.20 and 4.21 are therefore used for the determination of the virtual flexural rigidity of the pipe section. These values were later used for the verification of the CRISP program with respect to soil-pipe interaction problems.

4.4 Multi-Layered Model of Pipe Wall

This section describes a simple multi-layered ring model which will be used for finite element analysis to investigate the change of the flexural stresses and bending moments incurred in a large diameter rigid pipe wall under the various loading conditions in Chapter 5. The validity of the simple ring model is also discussed, comparing the experimental data obtained from the site with a pure analytical solution. This chapter includes a preliminary discussion on the difference of the behaviours between prestressed pipe and non-prestressed pipe in the process of the numerical analyses subjected to the three-edge bearing test.

4.4.1 Geometric configuration and modelling philosophy

Numerical modelling considers only a half segment of pipe, since it is symmetric about the $y$-axis. Figure 4.15 illustrates a basic load-deflection scheme with the half segment pipe for the simulation of the three edge bearing test (but a two edge load system is considered for simplification) in the finite element method, including the
displacement fixities. Each half is assumed to be a curved cantilever beam, free at the top and fixed at the bottom. Since the conditions along the length of the pipe are uniform, it is possible to analyse just a one metre length of pipe based on the plane strain-stress condition. The bottom end of the pipe is restrained both in the x and y directions, to utilise the same magnitude of reaction against the downward force applied to the top of pipe, while the top end section is restrained in x direction only. This allows a downward deflection of the crown, leading to the outward deflection of the springline. For an unyielding fixed edge with no movement in both vertical and horizontal directions, the load \( L^* \), which is the sum of the applied load \( L_{(eb)} \), and the pipe self-weight of a half segment, utilises the imaginary load (or reaction) \( R(=L^*) \), at the bottom of the pipe, producing the two edge load mechanism in the finite element analysis. Appendix B1-1 provides the deformed and undeformed meshes obtained from the CRISP analysis for Case II (TEB21) with a scale in a displacement magnitude factor of 20.00.

A typical multi-layered model of prestressed concrete pipe wall and the plots of the mesh for combined sets of pipe wall is illustrated in Figure 4.16. Element types consist of 8-noded linear strain quadrilateral elements (LSQ) for both concrete and mortar and 3-noded linear strain beam elements for prestressing wire, having a same number of nodes along common element edges as quadrilateral elements. The beam element used is capable of transmitting moments as well as axial forces, and has a displacement degree of freedom at each node. An isotropic linear elastic model has been employed for both constituent materials. Since the deformation of pipe is remarkably small in the field condition, it can be assumed that the pipe exhibits linear load-deformation characteristics (See Figure 4.17). The embedded steel cylinder in the concrete core is ignored, because it is compensated by using the higher flexural rigidity of the core section. The prestressing wire is modelled using a smeared plate steel layer with the equivalent bending stiffness properties. The smeared steel approach can be usually applied to the finite element models for reinforcing bars or prestressing wires in order to avoid a very complicated, fine mesh (Zarghamee, 1988
The model shown in Figure 4.16 is termed "FEM Case I". However, although it was greatly simplified compared to the real configuration of the pipe wall, uncertainties remain; for instance, the interruption between the coarse (concrete) and fine (mortar coating) meshes can not be avoided during the application of prestressing on the outer surface of concrete core in particular, and no validity on the interpretation of flexural bending moment corresponding to the whole section of the pipe can be assumed because it consists of three different elements, as well. A further simplified model (Case II) was therefore introduced, using an LSQ for whole pipe section, with a beam element encased in its outer surface. For a non-prestressed rigid pipe, an extremely simplified model which contains LSQ elements only comprises Case III.

As has been described above, the behaviour of the beam is mostly unpredictable due to the complicated material properties. Although the constitutive relationships for concrete and mortar can be assumed to be trilinear for simplicity of computation, the actual slope of the linear stress-strain relationships are apparently different to each other. Moreover, the onset of microcracking occurs at different strain conditions, and the stress-strain behaviour in the strain-softened state can not be treated as the same material.

 Unlike a conventional diagram of strain and stress distribution for the reinforced concrete structure, Zaghamee and Fok(1990) presented a rational method for the design of PCCP based on certain limit-states criteria. These were (i) serviceability limits for crack control in the concrete core and mortar coating of the pipe, (ii) elastic limits to maintain repeatability, and (iii) strengths limits of the constituent materials. Figure 4.17 is a schematic diagram of strain and stress distributions for a simplified flexural beam of PCCP corresponding to Figure 4.16 (FEM Case I), ignoring the steel cylinder embedded in the concrete core from the original Zarghamee and Fok(1990)'s diagram which is developed for the computation of bending strains and stresses in the pipe wall at the invert and crown. As can be seen, the diagram
is sophisticated in determining a bending moment of pipe wall, and requires a high degree of numerical technique as well. The interpretation method of the results obtained from the conventional finite element analysis are complex, nevertheless, it does not guarantee the validity of the results. However, this type of model has great merit to find the mechanical responses for individual materials, wherein uncertainties remain in the stress of mortar coating in respect to aspect ratios between two edges of each element. Thus, it is concluded that Case II will be used for the analyses of the prestressed pipe, and Case III for a non-prestressed pipe.

4.4.2 Deformation characteristics of prestressed and non-prestressed rigid pipe

Prestressed concrete pipe wall is nonhomogeneous in that it is made of four entirely different materials. From Figure 4.16, section properties of pipe wall can be, however, obtained from a conventional method using parallel axis theorem where \( I = \sum I_x + AC^2 \) at centroid, supposing that (i) a plane cross section remains plane under load and (ii) composite materials behave in a similar manner. The second moment of inertia of pipe wall at the crown or invert and that of the springline should be distinguished because of the different stress states. For instance, at working load, the maximum strain exhibits in the outer fibres of the springline, which is generally assumed beyond the elastic limit (or larger than the modulus of rupture) leading to the development of tension cracks on the surface of mortar coating, while the inner fibre of the concrete section is in compression. Thus the thickness of mortar coating does not usually contribute to the sectional modulus. In contrast, the tensile stress at the inner fibre of the invert (or crown) of the concrete core is kept within the allowable stress limit, thus the whole section of pipe wall can be encountered in the calculation of sectional properties. Fortunately, the pipe design is substantially governed by the section either at invert or crown, hence, the whole section will be considered in this thesis.
The distance of centroid from tension face (inner fibre of concrete core) is given by:

\[ c = \frac{A_c C_1 + A_m C_2 + (n_r - 1) A_s C_3}{A_r + A_m + A_s} \]  

(4.18)

where \( n_r \) is the resultant modula ratio of prestressing wires.

The second moment of inertia for each material about the centroid becomes

\[ I_c' = \frac{(t_c)^2}{12} + A_c (C - \frac{t_c}{2})^2 \]  

(4.19)

\[ I_m' = \frac{t_m^3}{12} + A_m (t_c - C + \frac{t_m}{2})^2 \]  

(4.20)

\[ I_s' = A_s (t_c - C + \frac{d_s}{2})^2 \]  

(4.21)

where,  

- \( I_c' \) = second moment of inertia for concrete core to the centroid  
- \( I_m' \) = second moment of inertia for mortar coating to the centroid  
- \( I_s' \) = second moment of inertia for prestressing wires to the centroid, assuming that \( I_r \approx 0 \)  
- \( t_c \) = concrete core thickness  
- \( t_m \) = mortar coating thickness  
- \( d_s \) = Prestressing wire diameter

By using Eqn 4.17, the empirical elastic modulus of concrete core including steel cylinder due to the prestressing effect can be re-organised by the following equation:

\[ E_{c,emp} = \frac{(EI)_{v,emp} - E_m I_m' + E_w I_s'}{I_c'} \]  

(4.22)
For the pipe having the following geometry and material properties,

\[
\begin{align*}
t_c & = 250 \text{ mm} \\
t_m & = 30 \text{ mm} \\
d_s & = 4.88 \text{ mm} \\
A_s & = 30.90 \text{ cm}^2/\text{m} \\
C & = 152 \text{ mm, assuming } n_r = 6 \\
I_{c'} & = 0.00148 \text{ m}^4/\text{m} \\
I_{m'} & = 0.00039 \text{ m}^4/\text{m} \\
I_{s'} & = 0.00003 \text{ m}^4/\text{m} \\
E_m & = 44 \text{ GPa} \\
E_w & = 200 \text{ GPa}
\end{align*}
\]

A value of elastic modulus of concrete core is estimated by the following simplified equation.

\[
E_{c,\text{emp}} = \frac{(EI)_{v,\text{emp}} - 0.0232}{0.00148} \quad \text{(GPa)}
\]  

Figure 4.18 illustrates how to obtain the stiffness of wall as an input parameter for the numerical analysis. The non-linear curve in Figure 4.18 is adopted from Figure 4.12 (Specimen number 3-12718-B). The straight lines plotted are the results obtained by finite element method using approximated Young’s modulus of concrete by taking a secant gradient of the initial part of curve. In the development of linear elastic behaviour of the system, it may be necessary to re-calculate one or more times if the linearity of the system assumed for that particular step goes on faraway from the actually measured non-linear curve. By trial and error, it is concluded that a design value of wall modulus shall be at least 88 GPa for a non-prestressed pipe to satisfy the deflection control requirement of buried pipe within the working load (generally to be within ±2 mm change in diameter for a rigid large diameter pipe).

Since the working load is usually less than 50% of the upper bound in Figure 4.18, a normal concrete property of 44GPa for a non-prestressed
pipe always results in the higher degree of displacement in vertical and horizontal directions, which seems to be too far from the real behaviour of buried pipe in the aspect of a deflection control. Values even higher than 88 GPa (which supports a larger earth load on the top of the pipe, and results in a large bending moment in the structure) are more desirable for long term stability.

A value of 88 GPa will therefore be used for the analysis of the non-prestressed rigid pipe in Chapter 5. Some analyses will be based on the values of 44 GPa, 66 GPa and/or greater than 88 GPa for comparison purposes.

In addition, Figure 4.19 focusses the different deformation characteristics between prestressed and non-prestressed pipe. As is shown, due to pre-compression, the prestressed pipe is in a state of contraction before starting the three-edge bearing test. Therefore, the starting point of the deformation due to the three-edge bearing loads is different to that for the non-prestressed pipe. The compression force generated by the prestresssing led to the reduction of the pipe diameter by about 1.5 mm, and the self-weight of the pipe caused a further change in diameter of about ±0.5 mm in horizontal and vertical direction. Considering the pre-displacement of the pipe due to prestressing, it was found that the results for the vertical and horizontal curves during the TEB test maintain a similar pattern with the experimental data within a boundary of the working load. From this point of view, the modelling effect of the prestressed pipe (Case II) seems to be reasonable for this aspect of deflection control.

4.4.3 Flexural bending moment in plane of curves

The aim of this section is to verify the validity of the finite element program CRISP together with the properties of pipe materials employed. In Figure 4.20, an idealised circular ring carries concentrated loads W (similar to \( W_{ol} \) in Figure 4.14(b)) acting on the top and bottom of the pipe along the line of AC (Roark and Young, 1976). It is assumed that
the beam is homogeneous leading to the same elastic modulus in tension and compression. \( M_a \) and \( M \) are the internal bending moment at the top of the pipe and a general value at any point between \( \phi = 0 \) and \( \phi = \pi \) with an angle \( \theta \) clockwise to the point C, respectively. \( T_a, T, V_a, V \) are internal forces. A half segment of the pipe is also considered in the analysis. Therefore, a general expression at a radial distance \( R_m \theta \), from the top of pipe is seen to be:

\[
M = M_a + 0.5 W R_m \sin \theta \tag{4.24a}
\]

where \( R_m \) is the mean radius to the centroid of the cross section, and \( M_a \) is the maximum bending moment mobilised at point A. By substituting \( M_a = -0.3183WR_m \), the expression of Eqn 4.24a is then modified as follows:

\[
M = -0.3183WR_m + 0.5 W R_m \sin \theta \tag{4.24b}
\]

Thus, the internal bending moment at springline (B) becomes \( 0.1817WR_m \), and the moment at bottom (C) equals \( M_a \). A graphical representation of bending moment computed by Eqn 4.24b is referenced in Figure 4.21, denoted as "analytical solution".

The numerical analyses were conducted to obtain bending moment distributions along the pipe wall. Appendix B2 presents typical examples of the analyses for Case II (prestressed pipe) with a wall stiffness of 44 MPa and a compression force of 2000 MPa applied to the outer surface of the wall, and Appendix B3 for Case III (non-prestressed pipe) with a wall stiffness of 44 MPa only. Figure 4.21(a) illustrates the increase of bending moment at the pipe invert due to the incremental load of 509.6 kN/m (see Figure 4.15) as a function of the elastic modulus of the pipe wall. From this single graph, it can be noted that the wall stiffness generally influences the magnitude of bending moment. With increasing wall stiffness the bending moment increases until it reaches an ultimate value (refer to Table 4.6 for various wall stiffness). The difference using the conventional wall
stiffness of 44 MPa compared to the analytical solution is about -33% at the pipe top (or bottom) where the load is applied and tolerates with the range of +12% and -39% (-16% in average) throughout the periphery of the wall. Figure 4.22 provides a comparison of the bending moment distributions for Case II and an analytical solution, confirming that the distributions for the increased stiffness take place between two curves. The ratio of the bending moment obtained from the numerical analysis to the bending moment computed from the analytical solution, which is termed bending moment ratio (BMR), along the pipe wall is also provided in Figure 4.21(b), illustrating a variation of BMR.

However, in the case of non-prestressed pipe, no major difference has been observed between the two curves, regardless of the magnitude of the wall stiffness. As shown in Figure 4.21(b), the curve for BMR of the non-prestressed pipe is not distinguishable with the analytical solution (it is observed that an average of BMR throughout the wall is 1.00). From this result, it can be assumed that the number of element used is appropriate for the non-prestressed pipe. The further analyses have been conducted using 16 elements (which is a half of the number in Case III above) to investigate the effect of number of element. The resulting bending moment distribution was very similar to that for the prestressed pipe shown in Figure 4.21(a), indicating the pitfall due to too few elements.

From the above results, the question arises: What is the major factor to influence the change in bending moment in the prestressed pipe wall, for a given condition? In fact, it is difficult to be precise, but the following facts may be considered.

(i) There is no argument that the number of elements is one of the major factors. In order to avoid the pitfall of using both too few elements and too many elements, it should be optimised before implementing the main analysis.

(ii) Unlike the non-prestressed pipe, although the same
number of elements were used for the wall, (except the beam element), the bending moment diagram exhibits a gradual increase with increasing the stiffness of the structure.

(iii) From (i) and (ii) above, it can be seen that, firstly, the mesh requires more elements in regions where the largest stress gradients occur. Secondly, the situation can also be explained by using the general form of elementary theory $M = EI_y/R$, where the bending moment is in proportion to the wall stiffness and in reciprocal proportion to the radius of curvature.

(iv) Accordingly, it is suggested that the effect of pre-compression is one of the most important influence factors. As was explained, when the prestressing wire is wrapped on the outer surface of the concrete core, the pipe diameter is reduced by about 1.5 mm. The compression may cause the behaviour of pipe wall to be changed, compared to the non-prestressed pipe.

For the case of two edge loads applied to the non-prestressed pipe with adequate number of elements and the constant load, the bending moment distribution throughout the wall should be invariable, irrespective of the extent of the stiffness of the pipe wall. This can be explained by applying the general form of elementary theory $M = EI_y/R$, wherein the increase of wall stiffness causes the increase of a radius of curvature internally during operation. In other words, the lower stiffness produces a smaller curvature when external load applies. However, it is a different story for the case where the external loads depend on the deformation of the pipe. For instance, when the pipe is buried in the soil, the bending moment will be variable with either decrease or increase of lateral earth pressure occurred. In another example, with respect to the wall stiffness for a buried structure, the structure with a higher stiffness will not deform as much as a relatively flexible structure. This results in a larger
bending moment due to the absence of the lateral supporting pressure.

In conclusion, it is not easy to explain the reasons why a change in bending moment occurs in the prestressed pipe during a two edge load test, in contrast to the non-prestressed. For the time being, it is suggested that the bending moment safety factor (BMSF) is applied to the design of both prestressed and non-restressed pipe. Since the design is usually governed by the bending moment either at the top or at the bottom of the pipe, BMSFs for both pipes will therefore be at least 1.3 for the prestressed pipe and 1.05 for the non-prestressed pipe.
Table 4.1 Soil group and classification of constructed soils

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<tr>
<th>Soil group</th>
<th>Group symbols</th>
<th>Typical names</th>
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<td>1</td>
<td>SW</td>
<td>Well-graded sands, gravelly sands, little or no fines</td>
</tr>
<tr>
<td></td>
<td>SP</td>
<td>Poorly graded sands or gravelly sands, little or no fines</td>
</tr>
<tr>
<td></td>
<td>GW</td>
<td>Well-graded gravels, gravel-sand mixtures, little or no fines</td>
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<td></td>
<td>GP</td>
<td>Poorly graded gravels or gravel-and mixtures, little or no fines</td>
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<td>Inorganic silts and very fine sands, rock flour, silty or clayey fine sands or clayey silts with slight plasticity</td>
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<td>SM</td>
<td>Silty sands, sand-silt mixtures</td>
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<td>GM</td>
<td>Silty gravels, gravel-and-silt mixture</td>
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<tr>
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<td>GC</td>
<td>Clayey gravels, gravel-sand-clay mixtures</td>
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<td>SC</td>
<td>Clayey sands, sand-clay mixtures</td>
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<tr>
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<td>Inorganic clays of high plasticity, fat clays</td>
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Table 4.2 Selig’s triaxial test parameters (Selig, 1988)

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<th>C</th>
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<th>$\Delta\phi$</th>
<th>$\gamma_m$</th>
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1 psi = 6.9 kPa
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<th></th>
<th>MBL-4 for Brega plan</th>
<th>MS-3 for Sarir plant</th>
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<tr>
<td>Water, kg/m³</td>
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<td>131</td>
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<td>Cement, kg/m³</td>
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<td>742 (Brega)</td>
<td>670 (Sarir)</td>
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<td>1067 (Brega, (d_max = 19 mm)</td>
<td>1191 (Sarir, (natural aggregate)</td>
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<tr>
<td>Water/Cement</td>
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<td>2404</td>
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Table 4.4 Ultimate strength and elastic modulus of concrete for MBL-4 and MS-3 mixes (DAC, 1996a and 1996b)

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<th>Sample No.</th>
<th>MBL-4 for Brega plant</th>
<th>MS-3 for Sarir plant</th>
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<td>2</td>
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<td>Mean</td>
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<td>Std dev</td>
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Table 4.5 Three-edge bearing test results for two 4000 mm diameter prestressed concrete cylinder pipes

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<th>Load (kN)</th>
<th>Deflection 3-12718-B (mm)</th>
<th>Deflection 3-14268 (mm)</th>
</tr>
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<td>17.76</td>
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</table>

(Refer to Figure 4.12)

Prestressing wire area used: 
\[ A_{s1} = 15.45 \ \text{cm}^2/\text{m} \]
\[ A_{s1} = 15.45 \ \text{cm}^2/\text{m} \]

\[ f'_{c} \text{ at 28 day} \]: 
- 46.7 MPa for 3-12718-B
- 44.2 MPa for 3-14269
Table 4.6 Three-edge bearing test results for various size of  
prestressed concrete cylinder pipes

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<th>Load (kN)</th>
<th>4.0m dia.</th>
<th>2.0m dia.</th>
<th>1.6m dia</th>
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<td>(As = 36.98cm²/m)</td>
<td>(As = 17.78cm²/m)</td>
<td>(As = 14.82cm²/m)</td>
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<td>horizontal (mm)</td>
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(Refer to Figure 4.13)
Table 4.7 Bending moment in pipe wall due to two edge loads

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<tr>
<th>Angle from top of pipe</th>
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<th>Finite element method (Moment in kN-m)</th>
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(Refer to Figure 4.21)
Figure 4.1 Comparison of actual stress-strain curve and hyperbola (Selig, 1988)
Figure 4.2 Initial tangent modulus as a function of confining pressure (after Duncan, 1980)

\[ E_i = K P_a (\sigma_3/P_a)^n \]
Figure 4.3 Secant modulus at 50% of the failure stress for a constant confining pressure (after Selig, 1989)
Figure 4.4 Young’s modulus of backfill (SW) as a function of standard Proctor density and confining pressure at minor principal stress/major principal stress = 0.5 based on Selig’s soil parameters.
Figure 4.5 Young’s modulus of backfill (ML) as a function of standard Proctor density and confining pressure at minor principal stress/major principal stress = 0.5 based on Selig’s soil parameters.
Figure 4.6 Young's modulus of backfill (CL) as a function of standard Proctor density and confining pressure at minor principal stress/major principal stress = 0.5 based on Selig's soil parameters
Figure 4.7 Determination of soil parameters for internal friction angle (Selig; 1988 and Duncan et al.; 1980)
Figure 4.8 Typical correlation between friction angle and relative density of sandy material

Results of consolidated-drained direct shear tests on compacted samples SM to SC-SM (Location of sample: Eastern Sahara desert area in Libya)
Figure 4.9  Prestressed concrete cylinder pipe (after PB, 1994)
Figure 4.10(a) Typical stress-strain diagram for concrete mix design No. MS-3, Specimen No. 5, $E_c = 34.7$ GPa (DAC, 1992)
(a) The vibration of the concrete specimens (150 by 300 mm) on a vibrating table

(b) Placement of the demoulded specimens in the testing machine

Figure 4.10(b) View of the preparation of the test specimens for elastic modulus
Note: 1. The numbers 1 to 4 indicate the test series described in Section 4.3.1.
2. The data are based on the mean strength not the characteristic strength used for the international standard.

Figure 4.11 Elastic modulus of concrete as a function of concrete strength (DAC, 1992, 1996a and 1996b)
Figure 4.12 Load-deflection curves for 4000mm diameter prestressed concrete cylinder pipe during three-edge bearing test
Figure 4.13 Load-deflection curves as a function of pipe size and steel area
(Wire size : 4.88mm, Core thickness / Pipe dia. = 1/16)
(a) Pipe supported at bottom and loaded by self-weight

(b) Pipe supported at top and bottom loaded by 3-edge bearing load

Figure 4.14 Diagrams of large horizontal pipe loaded by pipe self-weight and 3-edge bearing test load
Figure 4.15 Load-deflection scheme for two point loads at top and bottom of half section of PCCP
Figure 4.16 Basic modelling of multi-layered wall for prestressed concrete pipe (Case I)
Figure 4.17 Schematic of strain and stress distributions for simplified flexural beam of PCCP (modified from Zarghamee et al., 1990)
Figure 4.18 Load-deflection curves for various stiffness of prestressed and non-prestressed rigid pipe with 4.0m diameter (obtained from FEM and experimental data)
Figure 4.19 Comparison of load-deflection curves for prestressed (Case II) and non-prestressed (Case III) pipes, due to pipe self-weight, prestressing and TEB loads (ID = 4.0 m, $E_c = 44$ GPa, Compression for prestressed pipe = 2000 kPa)
Figure 4.20 Free-body diagram for determination of bending moment $M$ (Roark and Young, 1976)
Elastic modulus of pipe wall, MPa

(a) Max. BM in prestressed pipe as a function of elastic modulus of pipe wall

(b) Bending moment ratio relative to analytical solution

Figure 4.21 Change in bending moment distribution for prestressed and non-prestressed pipe due to two edge loads at pipe top and bottom, ID = 4.0 m, tw = 280 mm, Compression = 2000 kPa, Load = 509.6 kN/m
Figure 4.22 Comparison of bending moment distributions between analytical solution and FEM Case II due to two edge loads at pipe top and bottom ID = 4.0m, tw = 280mm, Load = 509.6kN/m
5.1 Introduction

Finite element techniques are being increasingly used for the analysis of soil-structure interaction problems, since it is possible to perform an integrated analysis of the composite structure. Goodman et al. (1968) claimed that all materials used in the finite element analysis can be treated as a fully integrated part of the total structure in a given domain, and the interaction between different materials can be determined. However, not only are studies of buried pipe problems very rare, they are mainly related to either flexible pipe (Spangler; 1941, Howard; 1977, Gumble; 1983, Watkins; 1995, Kawabata et al.; 1995) or small diameter reinforced concrete pipe (Spangler; 1933, Pettibone et al.; 1967, Davis et al.; 1974a and 1974b, Krizek et al.; 1978, Oak; 1988). In other words, no attempt has been made to clarify the interaction between soil and large diameter rigid pipe because of the complexities of the interrelation between the two media.

As with most underground structures or foundations, the estimation of bending stresses (or moments) is a prerequisite for the design of a buried pipe. Unlike the analytical design methods based on excessive simplification of the loading system, an advanced numerical technique makes it possible to deduce the actual stress-strain response. Nevertheless, the data obtained from the numerical analysis seem not to have been used for the design and construction of buried pipelines, which is probably due to a lack of experience in this field.

In an attempt to clarify the situation, extensive computer simulations have been performed by the writer to model the behaviour of buried pipe, based on site trials and experiments. A threefold approach is adopted herein to emphasise the physical necessities of the finite
element analysis of soil and large diameter rigid pipe interaction problems; The first effort begins with a brief summary of some numerical modelling issues for the constitutive models and finite element discretisation described in previous chapters. Secondly, the influence of the relative stiffness of pipe and surrounding soils is investigated, where the influence of different bedding conditions are also discussed in detail. Installation and construction effects such as arching, internal water pressure and prestressing are examined and discussed at the end of this chapter.

It should also be noted that the main objective of this chapter is to improve the quality of the performance of numerical analysis, obtaining more accurate stress distributions including the radial earth pressure generated around the pipe, and the flexural stresses and bending moment in the pipe wall. Finally, the validity of the current design concept being used in industry will be examined and several points for the improvement of the design will be raised.

5.2 Constitutive Models and Finite Element Discretisation

5.2.1 Constitutive models

The stress system is approximated in two dimensions (plane strain) as are many geotechnical problems, and the constitutive models used for the three different materials in the finite element analyses are briefly summarised in the following.

Rigid pipe is presumed to be composed of homogeneous isotropic linear elastic material, with the assumption that the stiffness is the same in all directions, consisting of Young’s modulus $E_c$, and Poisson’s ratio $\nu$. This model is applicable to soil–pipe interaction problems where the deformation is limited.
The solution presented here assumes that the fill behaves as a linear elastic–perfectly plastic material, which is frequently applied to the analysis of civil engineering materials and the problem related to the soil-pipe interaction (e.g. Oettl et al.;1998, Gerbault;1995, etc.), where it is assumed that the linear elastic nature is applicable up to the yield stress point, after which both elastic and plastic strains occur. The classical Mohr–Coulomb criterion is used in conjunction with an associated flow rule (i.e. \( \psi = \phi \)).

The in-situ soil is considered as a linear elastic medium due to the uncertainties related to the estimation of the in-situ stresses and the soil stiffness, together with inability to take into account time-dependent deformations of the ground (Kyrou;1980, McVay;1982, Chard;1982). This assumption is generally accepted in the field of soil-structure interaction problems where small strains are expected, and it seems to be appropriate in natural ground with a large cementing agent such as silicate and iron oxide, unlike in soft ground where the displacement occurs due to plastic straining and flow.

5.2.2 Finite element discretisation and input parameters

Numerical analyses performed herein are based on the embankment condition (passive mode) and are associated with the cut-and-cover method. The output data are limited both in terms of internal stresses in concrete wall and adjacent soils. Figure 5.1 illustrates a finite element discretisation, including the overall dimensions and commonly used notations. A half cross-section is considered for the model, assuming symmetry about the vertical centreline of the buried pipe.

In this chapter, only prestressed and non-prestressed rigid pipes with an inside diameter (ID) of 4000 mm and wall thickness of 280 mm are considered; the height of earth cover above the top of the pipe (EC) and the half width of trench excavated are assumed to be variable. The aspect ratio of mesh width (L) to outside diameter of PCCP (OD) is
fixed at around 2.5. The depth below the base of the excavation is limited to 1000 mm to minimise computation time (because it has been found that the influence of the depth of ground below the pipe is not significant), and the granular bedding material is placed, allowing at least 150 mm thickness between pipe and base. The in-situ fixities for left and right boundary edges are restrained in the horizontal direction, whilst the bottom boundary is assumed to be fixed in both horizontal and vertical directions.

In Figure 5.2(a), the properties of the backfill materials for each zone are variable so that the influence of different properties can be examined. The elastic moduli of backfill materials, $E_s$, for pipe bedding, haunch and surrounding soils (see Zone 1 and Zone 2 in Figure 5.2(a)) within the trench are assumed to be between $2.0 \times 10^4$ kN/m$^2$ for a loose state and $9.0 \times 10^4$ kN/m$^2$ for a dense state, based upon the results of conventional testing. Since the maximum operational stiffness can be assumed as 50%-80% of the field values obtained from seismic geophysical techniques, which appear to be five or ten times greater than those obtained from laboratory testing, (Clayton and Heymann, in press), the analysis therefore includes materials in an extremely dense state, with $E_s = 3.0 \times 10^5$ kN/m$^2$. Minimum and maximum bulk densities $\gamma_s$, are chosen to be between 18.5 kN/m$^3$ and 22 kN/m$^3$. The properties of top soil above the pipe (Zone 3 in Figure 5.2(a)) are fixed at $E_s = 1.7 \times 10^4$ kN/m$^2$ and $\gamma_s = 18$ kN/m$^3$, considering the ML soil to be in a relatively loose state, unless otherwise specified.

Some of the materials directly beneath the pipe in the bottom and haunch areas (the zones lettered, a-j in Figure 5.2(a)) are considered as a dumped soil (or very loose soil). For example, for the series number S6XX the areas a-d and i-j are voids and the areas e-h belong to the backfill material zone (1). The material zone (4) below the pipe invert area is assumed to be in a relatively denser state than the adjacent material, due to the pipe weight and supporting load.
5.3 Relative Stiffness of Pipe and Side Fill

5.3.1 Concept of construction

In order to improve the quality of the numerical analysis, the modelling of construction activity such as element removal or addition will be kept the same as an actual procedure of construction method in the field. After completion of excavation, a shaped bedding is prepared on the free surface, with the required properties of a granular soil (Material Zones 1 and 4 in Figures 5.2(a) and 5.2(b)). An invert zone (4), expecting a tight contact with the outer surface of the pipe in case of series numbers S1XX, S2XX and S3XX, is assumed to be a little higher in stiffness compared to the outer portion of bedding. The pipe is then installed on the pre-shaped bedding. The layers of side backfill are placed step by step, wherein the degree of compaction shall be controlled to meet the requirements for the supporting system (Material Zone 2, Figure 5.2(c)). The top soil is then placed up to the original ground level. The design density of the soil above the pipe (TSD) is assumed to be lower than side fill, together with lower elastic properties. The general installation procedure is comprised of the following sequence:

(i) Excavate the ground to the required depth, forming a wide trench.

(ii) Place and compact bedding material (Zone 1) to satisfy the design requirements, over a depth extending approximately 0.035 \(D_m\) below the bottom of the pipe, where \(D_m\) is the mean radius of the pipe. (The depth below the pipe is variable according to the type of original ground, for example, if a subgrade is composed of rock or other unyielding material, the replacement of soil below the bottom of pipe shall be greater than 0.035 \(D_m\) to avoid a higher concentrated supporting force at the invert area.)
(iii) This layer is then accurately shaped with a steel template attached to a bulldozer, so that a predetermined profile (coincident to the outside diameter of the pipe) can be achieved.

(iv) After that, the pipe is placed to line and level on the shaped bedding.

(v) Then, the side fill (Zone 2) is placed up to the springline and compacted as required.

(vi) The further side fill and compaction will be continued to 300 mm above crown of the pipe, where the materials used are generally the same as the side fill and thorough compaction is required.

(vii) The soil above the pipe (Zone 3) is also required to undergo a certain degree of compaction, but is more compressible than the side fill. Local materials are usually employed, when the time-dependent deformation is out of consideration.

The numerical analyses are performed based on the following assumptions:

(i) The influence of the natural ground deformation following excavation is negligible in the behaviour of soil and pipe.

(ii) A small strain/or small displacement occurs within the domain.

(iii) The properties of the material are assumed as homogeneous at each incremental block.

(iv) Loading is fully drained.
Figure 5.2 illustrates a detail of bedding and backfilling, including the extent of the void zone which varies with the degree of compaction of the bedding material. This concept covers the construction method with a flat bedding, where the compaction into the lower portion of the pipe can not be achieved with acceptable reliability. For instance,

(i) Series number S1X represents poor construction practice (or flat bedding construction method) with an extremely limited contact area between two media, assuming a large void zone (b-j), whereas

(ii) Series number S4X corresponds to a perfect bedding condition, indicating zero voids in the haunch area.

The third letter "X" denoted in the series number has one of seven digital numbers (1, 2, 3, 4, 5, 6 and 7), corresponding to the stiffness of soil 20, 30, 40, 60, 90, 150, and 300MPa, respectively (see Figure 5.10). The last letter "X" in the series number indicates a various stiffness of pipe wall, showing one digital number among 1 to 8 which are 22, 33, 44, 66, 88, 132, 300 and 600 GPa, respectively (see Figure 5.16).

5.3.2 Conventional installation method

A typical radial earth pressure distribution around the pipe with full contact of surrounding soil is given in Figure 5.4(a). The flexural stresses at the inner and outer fibres of the pipe wall and the corresponding bending moment are presented in Figure 5.4(b) and 5.4(c), respectively. The influence of the bedding type is demonstrated in Figures 5.5 (a), (b) and (c), by considering the case of voids in the haunch area. To enable the wide combinations of parameters used in the analyses to be summarised easily, a coding system has been used. Full details of the numbering structure is provided in Figure 5.3. For example, for the code S445-03, the first letter in the series indicates that the analysis is performed to examine the influence of the soil
stiffness, the subsequent numbers 4, 4 and 5 indicate the bedding type, soil stiffness and pipe wall stiffness, as described in the previous section, and 03 indicates the depth of earth cover.

In Figures 5.4(b) and 5.5(b), compressive stresses are positive and tensile stresses are negative, and the peak tensile stress (occurring at the invert) governs the pipe design. The peak bending moment in Figure 5.5 occurs at the same place where the peak tensile stress occurs. In more detail, the peak bending moment occurs at a place where a distance between two stress diagrams for the inner and outer fibres becomes the largest through the pipe wall. As is to be expected, the locations for zero bending moment coincide with the places where the two stress diagrams intersect.

Returning to Figure 5.5(a), the radial supporting pressure under the bottom of the rigid pipe (with the rare exceptions such as the localised contact bedding condition representing by the series number S1X5) increases from the pipe invert toward the extremities, which is very similar to the case of a perfectly rigid footing explained by Terzaghi(1943) where the shape of the contact pressure varies with a distance from the centre of the base in order to produce uniform settlement of the loaded area of a semi-infinite elasto-plastic solid. The behaviour of subsoil below the pipe bottom is similar to that below a flexible pipe, however the settlement appears in a quite different pattern compared to the flexible pipe. This results in the supporting pressures in the haunch increasing with the settlements, while the supporting pressure and settlements at the invert area decrease.

Figure 5.6 depicts typical vertical and horizontal pressure distributions. The vertical earth load imposed on the top of the pipe is approximately 1.25 times to the weight of the prism of soil directly above the pipe because the pipe system is stiffer than the surrounding soil. The ratio of the total lateral earth pressure (between the top of the pipe and the top surface of the bedding, corresponding to a radial angle of 135 degrees from the pipe crown) to the total vertical earth load imposed on
the top of pipe is recorded at around 0.30. In which, the distribution of vertical supporting pressure in the shaped bedding appears rather close to Terzaghi’s theory, than to Olander’s bulb type used for the conventional design method. (Refer to Figures 5.4(a) and 5.5(a).)

5.3.2.1 Influence of soil stiffness

The relationship between radial earth pressure (or resulting bending moment) and soil stiffness is examined in Figure 5.7 and Figure 5.11, based on the material properties shown in Table 5.1. From the analyses shown in Figure 5.7 and Figure 5.11, it can be seen that the stress response of the pipe is not greatly influenced by the change in the stiffness of soil placed above the pipe, but the decrease of the stiffness of side fill results in an increase of the supporting pressure at the bottom and the decrease of the lateral pressures. The density of fill only slightly affects the magnitude of earth pressure.

Side fill with a lower stiffness produces a bulb-type pressure distribution on the top of the pipe, while the stress distribution for a higher stiffness has a deflected shape toward the extremities, as shown in Figure 5.7(a). This may be due to shearing forces at the (imaginary) vertical planes between the soil mass placed directly above the pipe and the soil mass above the side fill, which produce additional loads on the pipe. The magnitude of shearing force depends on the properties of the fill material, where a higher stiffness results in a higher frictional force. A high shear stress occurs near the boundary of the two masses, and it tends to diminish with increasing distance from the boundary toward the vertical centreline of the pipe. Therefore, for dense granular fill, the pressures for the embankment installation condition will, theoretically, be largest at the extremities of the pipe (the prism load plus the shearing force) and lowest directly above the pipe crown (where the pressure will be the same as the prism load intensity when the pipe size is relatively larger than the earth cover). This phenomenon is, however, slightly modified due to the problems of both the complicated geometry of the soil–pipe system and the intricate deformation characteristics
occurring within the pipe, the fill material and the natural ground.

When the pipe is installed at depth with extremely higher stiffness of soil, the shape of the earth pressure imposed on the top of the pipe and the shape of the supporting pressure under the bottom of the pipe tend to be almost identical. Figure 5.8 demonstrates this situation, where the trend can be seen rather clearly with increasing earth cover depth. Here the soil properties are the same as each other. In other words, the shape of the pressure distribution is almost symmetric about the pipe springline, and the small differences are only due to the pipe self-weight. From this, it is concluded that the shape of the radial earth pressure distribution is greatly influenced by the stiffness of the fill material, and the intensity of pressure at any point along the periphery of the pipe is nearly in proportion to the depth of earth cover.

5.3.2.2 Influence of bedding condition

Figures 5.9 and 5.10 show the difference between two installation conditions (S215 and S415), where the properties of side fill materials are identical. It demonstrates that relatively poor practice (S215) causes bending moments to be increased, as a result of a highly concentrated contact pressure just around the pipe invert, whilst relatively good practice (S415) leads to a decreased bending moment because of well-distributed supporting pressure. Here, the difference in the peak bending moments reached nearly 44%. No difference in the intensity of the radial earth pressure on the upper part of the pipe can be found, although the supporting pressures appear quite different to each other, as shown in Figure 5.9. From this, it can be noted that both the magnitude and the shape of the earth load imposed on the top of the pipe have no relation to the bedding condition.

On the other hand, Figure 5.11 illustrates that the maximum bending moment is significantly affected by the stiffness of the side fill (S215 and S245). By increasing the stiffness of the fill, the reduction of maximum bending moment reached around 32%. Here, the same bedding
condition has been applied, but the stiffnesses of the side fill in S215 and S245 were 20 MPa and 60 MPa, respectively. It would appear that the stiffer side fill makes it possible to share the supporting pressure near the haunch area, restraining the horizontal displacement of the pipe. This constraint seems also to apply to the vertical displacement of the pipe, simultaneously.

From the above, the following preliminarily conclusions are made:

♦ The type of bedding greatly influences the peak bending moment in the pipe wall (Figure 5.10). With increasing stiffness of fill material, the peak bending moment decreases (Figure 5.11).

♦ Alternatively, a similar effect, corresponding to the good quality bedding condition such as S415 (Figure 5.10), can be instantly achieved by using the higher stiffness of the side fill such as S245 (Figure 5.11), resulting from better compaction.

♦ The combination of improved backfilling and bedding condition (i.e. S445 in Figure 5.4(c)) accelerates the reduction in the peak bending moment.

Figure 5.12 summarises the bending moments at the pipe invert (for non-prestressed rigid pipe). Here, the elastic modulus of pipe wall was fixed at 88 GPa, the stiffness of soil above the pipe was assumed to be a relatively soft material having a $E_s = 17$ MPa, and an earth cover of 3.0m was used. In general, even a small increase of stiffness in the fill material at the beginning caused a large reduction of bending moment (around 50%), showing a linear variation up to a stiffness of 60 MPa. After that, the curve shows a non-linear response. A markedly smaller bending moment was observed, when using the installation condition S4X5 representing the full contact of two media. This case seems to be close to shield tunnelling, allowing a very small displacement of the natural ground. This figure provides a guideline for determining the allowable bedding angle out of consideration for a site condition at the initial stage of the design, although still exists still several
uncertainties with respect to the actual material properties such as a material stiffness, Poisson’s ratio, effective friction angle, etc..

The bedding angles shown on the right hand axis of Figure 5.12 are based on Olander’s (1950) analysis. For a given bedding angle, there are many alternatives for the selection of the construction method to allow for the variation in workmanship and field conditions. For example, with reference to Figure 5.12, a design bedding angle of less than 90° can be achieved by using any of the construction conditions of S155, S245, S335 or S415 in the field.

5.3.3 Modified shaped bedding

The essential requirement associated with the soil-pipe installation system is primarily to provide a uniform support under the lower portion of the pipe, to prevent a concentration of stresses at the invert area. Otherwise, it requires more prestressing wire or an additional wall thickness. However, since it is not easy to achieve a desirable supporting system with conventional flat bedding or shaped bedding, an alternative approach, termed the modified shaped bedding, has been suggested by Heger (1988) to satisfy the requirements. The additional procedure for the modified supporting system is comprised of:

(i) Before laying a pipe, a bedding material is placed and compacted over a depth.

(ii) This layer is then accurately shaped with a radius slightly less than the outside diameter of the pipe to ensure the upper parts of the shaped bedding near both extremities are in contact with the pipe wall. This allows an adequate tolerance in shaping the bedding, when the pipe is initially placed into the shaped bedding. (DAC, 1991)
Alternatively, the invert region is loosened to ensure that maximum support can be provided under the pipe haunch. Figure 5.13 shows the concept of the modified supporting system. For instance, when the pipe is supported over a localised area at the invert, the reaction (R) will be almost the same as the sum of the pipe self-weight and the earth loads imposed on the top of the pipe (W). On the other hand, in the modified support system, a half portion of the total weight (R/2) will be supported by a shifted support area in the haunch. This situation results in the magnitude of the maximum bending moment being greatly decreased.

Since the original concept of shaped bedding (here termed standard shaped bedding – assuming full contact around the bottom of the pipe) was suggested by Spangler in 1933, parametric studies for buried pipes were conducted by Herger(1988 and 1990) for the purpose of the establishment of new standard installation criteria (which was adopted by AWWA M9-95 and AWWA C304-92). Heger’s proposal made a great contribution to the construction practice for pipe installation (considered to be the best available option to date), as it takes into account the difficulties of full contact of two different media with the standard shaped bedding (Case S4X5 in Figure 5.12).

It should be noted that a precast concrete pipe is too rigid to reshape over a shaped bedding, although a small vertical deformation is expected. There are two or three possible deficiencies in providing firm contact between two media; for instance,

(i) A construction error to produce the radius of the shaped bedding arc exists.

(ii) A deflection due to the self-weight of the pipe can not be avoided, although the pipe is absolutely round and true during the process of manufacturing, and

(iii) A differential settlement along the surface of the bedding exists during the construction and service period owing
to both the non-homogeneity and inconsistent compaction of the bedding.

Therefore, to improve the supporting system, the suggestion was made that the outer portion of the bedding has to be stiffer than the invert area. Lesser compaction of the bedding material (or soft material) placed in the middle third of the outside diameter of the pipe is recommended.

5.3.3.1 Modified shaped bedding system in practice

For the application of the modified shaped bedding in numerical analysis, it is assumed that voids (or very loose soil) exist at locations "a-d" and "i-j" shown in Figure 5.2 (See Case S6XX.), providing the effective support system at location "e-h". On the other hand, its application to the practice in the field can be idealised by providing a radius of arc less than the pipe outside diameter, which ensures a positive support in the haunch area.

Figure 5.14 illustrates the effect of a modified shaped bedding, producing two symmetric supporting pressure bulbs about the vertical centreline of the pipe and nearly zero contact pressure in the invert region. This has led to the deformed shape of the pipe due to its self-weight to be slightly reshaped. The relevant flexural stress distributions are presented in Figure 5.15, indicating more complex stress distributions compared to the standard shaped bedding condition, resulting from the shifted supporting system. The resultant bending moment at the invert appears much lower than that for the standard shaped bedding. As shown in Figure 5.16, the pipe crown becomes a new critical point in the design. This is mainly due to the reduction in bending moment at the pipe invert, rather than the increase of that at the crown.

Further studies have been performed to compare the influences of the stiffness of the side fill for two different systems. The input parameters are based on Table 5.2. From the resulting bending moments shown in Figure 5.17 for the modified shaped bedding (S645 vs S645m) and
Figure 5.18 for the standard shaped bedding (S245 vs S245m), it can be seen that the influence of the stiffness is less significant in the modified system (24% increase) than in the standard system (36% increase).

Similarly, S745 (voids at a–c and h–j) and S845 (voids at a–b and f–j) have been examined for comparison purpose with the standard shaped bedding approaches. The resulting maximum bending moments are included in Figure 5.12, together with the results for S615, S715 and S815 (having a soil stiffness of 20MN/m$^2$ for bedding and side fills).

5.3.3.2 Improvement of constructability of modified shaped bedding

A further modification to the shaped bedding system, termed the controlled shaped bedding (S645c), which is generally identical with Case S645 except for the stiffness of the void zone at the invert area, is introduced in Figures 5.19, 5.20 and 5.21. Unlike the construction methods for the conventional shaped bedding, the properties of the void zone have been slightly altered for better constructability in the field. The void zone at the invert for the case of S645 is assumed to be filled with soil in a dumped state (1 MN/m$^2$), and that of a counterpart (S645c) is assumed to be filled with soil in a poorly compacted state (40 MN/m$^2$) compared to the outer haunch zone (60 MN/m$^2$). Here, the smoothed pressure distribution along the lower half portion of the pipe is apparently more desirable than two symmetric bulbs, but the induced bending moment is slightly increased, as shown in Figure 5.21(a). However, the long term integrity of the bedding zone is another aspect being considered; for instance, a dumped soil with very lower stiffness (or an absolutely void) could be, according to a point of view, a channel for seepage flow. Highly concentrated pressure under the bottom of the pipe is then likely to happen, resulting from erosion of the support area by flowing water.

Figure 5.21(b) illustrates the relationship between bending moments (generated at pipe invert, springline and crown) and the stiffness of loose soil at the invert. It is notable that the absolutely loosened or void
At the pipe invert region (S645) provides a benefit in the structural design under the normal conditions. In contrast, full contact bedding (S645c) where the stiffness of the invert region is the same as the support in the haunches (i.e. 60 MN/m² in this case) causes higher bending moments at the invert. For the case S645c, when the soil stiffness in the invert region is between 40 MN/m² and 60 MN/m², no further change in bending moment is observed (Figure 5.21(b)). As shown in Figure 5.21(b), the location of maximum bending moment governing the design is variable. The maximum bending moment occurs at pipe spring line for soil stiffness values up to the that loose soil (around 20 MN/m²). Once the soil stiffness in the invert region exceeds 20 MN/m², then the maximum bending moment occurs at the pipe invert.

The influence of material stiffness and bedding condition on the design are herewith summarised:

♦ The supporting system greatly influences the maximum design bending moment and a linear relationship exists between bending moment and stiffness of the side fill up to 60MN/m² (Figure 5.12). Further increase of the stiffness then appears to give a non-linear response. In Figure 5.12, it can be seen that, when the surrounding soil becomes stiffer than 90MN/m², the increase of bending moment is less significant.

♦ A decrease in bending moment at the invert can be promoted by shifting the supporting area to the haunch zone (Modified shaped bedding system).

♦ As has been demonstrated in Figure 5.12, the magnitude of bending moments for the installation conditions S645, S745, S615 and S715 are significantly below the curve for S4X5 representing the full contact bedding condition. Moreover, the bedding conditions for S845 and S815 (which is the worst case for the modified shaped bedding) still provide a similar effect to the full contact bedding condition (S4X5).

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The maximum bedding angle (180°) can be achieved in practice in the field, although it is not a simple matter. This design detail cannot be studied with analytical methods, but the finite element method has been shown to provide useful insights into this important practical engineering problem.

5.3.4 Flexural rigidity of pipe wall

This section deals with the influence of the wall stiffness on its bending moment. Here, the range of the wall stiffness is assumed to be between 22 GN/m² for the flexible state and 600 GN/m² for ideally perfect rigid state, for comparison purposes, and two different soil stiffnesses will be employed, 20 MN/m² for the loose state and 60 MN/m² for the relatively dense state.

Figure 5.22 illustrates the relationship between the stiffness and the deformation of the pipe for various bedding conditions, plotting the results subjected to the bedding conditions 1 and 4 as an upper and lower bound, respectively. The results for the bedding conditions 2 and 3 were confirmed to be somewhere between two curves for the bedding conditions 1 and 4. Typical non-linear response for a given loading condition is observed, indicating a stiff curve at the beginning, becoming more flexible. Generally, the buried pipe in a stiff soil deforms less than that in a loose soil. A poor bedding causes a severe deformation of the pipe, while a good bedding constrains the pipe not to be severely deformed. The effect of a good bedding condition with a lower stiffness of side fill is less than that of a poor condition with a higher stiffness of side fill. It can be seen that the pipe deformation is largely dependent on its own stiffness; for instance, (i) when the stiffness of the wall is over 132 GN/m², not only does the type of bedding have almost no effect on its deformation, but also the influence of the stiffness of backfill is minimal, and (ii) further increase of wall stiffness (likely to be a perfectly rigid state) appears insignificant.
Figure 5.23 illustrates the resulting bending moment as a function of the wall stiffness, the bedding type and the soil stiffness. The four curves with hollow circles correspond to the pipe buried in the relatively low stiffness soil (20 MN/m²), and the other four curves with solid circles in relatively high stiffness soil (60 MN/m²). The importance of the stiffness of the backfill is again substantiated, because the maximum bending moment at the invert is greatly reduced by employing a high stiffness fill. The relationship between S21 and S41 is once more repeated in this figure (compare with Figure 5.10.). The resulting bending moments for both cases are almost identical to each other, regardless of the pipe wall stiffness. Using an extraordinarily high stiffness for the pipe wall, no visual variation in bending moment can be found, irrespective of the bedding condition and the wall stiffness. From this, it appears that the conventional design method seems to be based on the maximum bending moment generated at the perfectly rigid state, assuming that a plane cross-section remains plane under load. Accordingly, the design value of bending moment in conventional engineering practice is close to the value existing at the perfectly rigid state of the pipe wall (such as over 600 GN/m² in Figure 5.23). This therefore overestimates bending moment by at least 25% compared with the real properties of the pipe, especially for poor bedding condition, and in some cases it exceeds almost 100%.

In conclusion, it is rarely possible to predict the complicated behaviour of a buried pipe using analytical design methods. Although a new approach for the prestressed concrete pressure pipe has been developed by American Water Works Association (AWWA C304-92) which adopts the limit state design concept (BS 8110), no difference from the current design method (AWWA C301-89) can generally be found.
5.4 Earth Cover, Water Pressure and Prestressing Force

This section examines the most important factors in the soil-pipe interaction problem which current design methods ignore. Throughout the previous studies, it could be recognised that the behaviour of the buried pipe varies with the construction sequence and method. However, it is not easy to anticipate the influence of the internal water pressure and prestressing force, together with the reaction of surrounding soils associated with the continuity in the material of the foundation. Here, all the above factors will be clarified by means of finite element analysis of both general and specific cases.

Prestressed concrete pipe makes highly efficient use of steel. The prestressing wire is helically wrapped around the concrete core under controlled tension. This process produces a uniform compressive stress in the concrete core (and the steel cylinder embedded in the concrete core). This compressive stress is assumed to offset the tensile stresses caused by internal water pressure and external earth loads. The decompression or zero concrete stress pressure, $P_0$ (which occurs when internal water pressure just balances the compression stress in the core due to the prestressing) is the most important stress condition. The state of stress in the concrete core must be kept within the allowable tensile strength (or modulus of rupture) throughout the service life of the pipe in order to prevent tension cracks.

The study will concentrate on investigation of the variation of the bending moment in the pipe wall and, in the case of prestressed concrete pipe, the bending stresses in the pipe wall, due to:

(i) the increase of the earth cover up to 36 metres
(ii) application of internal water pressure during water transmission
(iii) the effect of prestressing.

The combination of internal and external loads is also considered under the various expected site conditions.
5.4.1 Arching following increase of earth cover

With increasing earth cover over the top of the pipe, many parameters will be changed, namely (i) the properties of the soil around the pipe as a function of confining stress discussed in Section 4.2 and (ii) the earth pressures around the pipe. The main input material properties for backfill zones shown in Figure 5.2 are described in Table 5.3. For this analysis the bedding condition is slightly modified from Series S7XX shown in Figure 5.2, where the loosened soil near the extremities of bedding (h, i and j) is considered as a dense state identical to the adjacent backfilling zone (Zone 1 or Zone 2).

5.4.1.1 Increase of earth cover

Figure 5.24 presents the relationship between bending moment and earth cover. In this analysis, the properties of the surrounding soil vary according to the earth cover. This is denoted C7yX-XX in the coding system. The pipe wall stiffnesses were chosen to be 44 GPa, 88 GPa and 132 GPa. Similar to the previous analyses, the higher wall stiffness generally indicates higher bending moment for a given earth cover. With increasing earth cover the resulting bending moment increases, but the incremental rate decreases. In detail "A" of Figure 5.24, it can be seen that for the modified shaped bedding the maximum bending moment always occurs at the spring line, regardless of the increase of the earth cover. Bending moment at the springline of the pipe with a higher stiffness increases linearly with increasing depth of earth cover, while that for the pipe with a lower stiffness tends to yield due to the relatively large deformation of the pipe.

Figure 5.25 illustrates an important aspect of behaviour of the surrounding soil, which explains why the rate of increase in bending moment diminishes following the increase of earth cover. The effective stresses of the soil are plotted as a function of the height of earth cover. The data were collected from the soils in the vicinity of pipe crown, springline and support.
The curves for pipe crown and support have a similar pattern to each other, showing non-linear behaviour, but the slope of curve at the support is invariably greater than that for the crown.

As can be seen, there exists an arching effect (reduced overburden pressure) even in the embankment installation condition, which is compared with the geostatic stress curve at the pipe crown. The trend of arching indicates that the vertical stress induced at the crown for low earth cover is above the geostatic pressure due to the inverted arching effect, however with increasing depth of earth cover the induced pressure on the pipe crown tends to be less than the geostatic pressure.

On the other hand, the curve for the springline begins with a lower slope than the other, however, it maintains the constant rate of increase of stress (corresponding to lateral earth pressure) with increase of the height of earth cover. (Eventually, the horizontal stress at an extremely great depth will be greater than the vertical stress.) This results in a reduction of the peak bending moment at the crown.

Figure 5.26 demonstrates that the depth of earth cover affects the magnitude of the radial earth pressures. From this, it can be seen that the rate of increase of radial earth pressure around the springline is quite different to that at other places, causing the slope of the bending moment curve to decrease (Figure 5.24). For this reason, the flexural stresses of the pipe wall buried at a depth of 36m are generally less than those at 18 m (Figure 5.27), and the maximum bending moment for 36 m cover depth is, as a result, also less than that of 18m cover depth at the crown (Figure 5.28).

5.4.1.2 Application of internal water pressure

Figures 5.29, 5.30 and 5.31 present the results obtained from the
individual analysis and provide preliminary information on the variation of bending moment when the internal water pressure is applied to the inner surface of non-prestressed pipe. The results imply that, during water transmission, bending moment increases generally at both the support and the crown, but it is stationary (or even decreases slightly) at the springline. With reference to the magnified diagram extracts in Figures 5.29, 5.30 and 5.31, the transition depth of earth cover at which the design bending moment moves to the top from the support is generally less than 2.0 m. Comparing the curves for the case of applied earth pressure only and the case of combined earth pressure and internal water pressure, it is confirmed that the internal water pressure results in an increase of bending moment, therefore the maximum bending moment for the case with internal water pressure is always greater than that with external load only. The degree of increase in bending moment due to the internal water pressure is more susceptible in the case of stiffer pipe, compared with relatively flexible pipe.

Figure 5.32 examines the changes in the vertical and horizontal diameters of the pipe during backfilling as a function of the wall stiffness. A general tendency is that the degrees of change in pipe diameter in the vertical and horizontal directions are almost equal under low earth cover. With increasing cover depth, the change of vertical diameter tends to be slightly larger than the increase of horizontal diameter. It is interesting that at extremely high cover depths the vertical diameter increases with a decrease of the horizontal diameter. This behaviour can be explained by Figure 5.25, where the lateral earth pressure consistently increases with the increase of earth cover.

Figure 5.33 is a graphical representation of diameter changes in vertical and horizontal directions following the application of earth load with or without internal water pressure. Here, an internal water pressure of 10 bar resulted in the pipe diameter increasing by around 0.5 mm in the vertical direction and at around 1.0 mm in the horizontal direction. No specific differences in the trend of the deformation can be found between the three different stiffness of soils. It is evident that the internal water pressure causes the pipe to be expanded outward, and the
pre-deformed shape due to the earth loads is slightly recovered.

Regarding the above situation, the following question arises:

- Is the current design practice valid when it assumes that no changes in bending moment through the pipe wall take place during water transmission, since the hoop stress produces a uniform rectangular shaped tensile stress in the cross-section of the wall. This theory is based on the assumption that; the internal water pressure causes the wall to be expanded outward equally and the rigidity of the wall is considerably higher than the surrounding soils.

To put it more simply, the above assumption is based on the following hypothesis;

- The resultant stresses at the extremities of the core are defined as the sum of the compressive stress produced by prestressing, flexural stresses incurred by earth load and tensile stress due to the internal water pressure. The existing tensile stresses in the inner fibres at the bottom and crown of the pipe and outer fibre at the springline (-) due to the external load increasing by a superimposed tensile stress (-) due to the hoop stress induced by the internal water pressure.

The verification of the current design practice with the difference in the behaviours of non-prestressed and prestressed rigid pipe will be discussed in more detail in subsequent chapters.

5.4.2 Continuity of surrounding soils in working condition

Current design methods for prestressed concrete pipe for determining the earth pressures generated around the pipe are usually based on the Marston-Spangler theory, where the rigidity of the pipe is assumed to be considerably greater than that of the surrounding soil. When
considering soil-structure interaction around large diameter concrete pipe, the current procedure seems to be very crude.

An understanding of the real behaviour of buried pipe is not easy to achieve, since the flexural rigidity of the pipe wall or the elasticity of the foundation is variable. In other words, the relative flexural rigidity of the pipe wall and surrounding soils is variable, and the reaction of the surrounding soils, including the subgrade, is totally different on a case by case basis.

5.4.2.1 Characteristics of pipe deformation

Figure 5.34 presents an overview of the pipe displacement progression at the time of completion of backfilling (3 m of earth cover) and during the water transmission (10 bar and 20 bar of internal water pressure). The total vertical displacement of the pipe comes from the movement of the nodal point (1), and the actual deformation of the pipe in vertical direction is the difference between two transformed nodal points (1) and (17). The horizontal deformation of the pipe is calculated as twice the horizontal movement of nodal point (9), and vertical displacement at the springline is about a half of the total deformation of the pipe in the vertical direction. As shown in Figure 5.34, the displacement of the pipe is mainly due to the earth loads on the top of the pipe, and the influence of the imposed internal water pressure is relatively small. The details of movement for each nodal point at the crown, invert and springline are presented in Figure 5.35.

Figure 5.36 explains the progressive deformation of the pipe under the influence of changing internal and external loads. With water filling (Work Stage 2) the pipe is sinking, and with the application of internal water pressure the whole section of wall then tries to expand outwards. During water transmission, the pre-deformed shape at the invert appears to undergo a minimal recovery, compared to the pipe crown. The trace of vertical movement of the springline is always placed in the middle of the crown and invert, since no excessive deflections at
both top and bottom of the pipe occur. The horizontal expansion at the springline may continue until the pipe bursts.

Figure 5.37 details the relative movement of the nodal points (shown in Figure 5.34) during construction and service. Here, the movement represents an actual displacement between the points on true circle after deformation. A positive sign implies outward movement of the nodal point based on the trace of true circle, while a negative sign implies inward. From this figure, it can be deduced that the internal water pressure influences the peak bending moment, since the extent of displacement varies with angular position of each nodal point. This is due to the different deformation characteristics of the surrounding soils. The elastic modulus of bedding material in the supporting area is always greater than the other parts. Thus, during water pressure loading, the pipe wall contacting the supporting area will not be deformed as much as the other part. This differential deformation of the pipe results in the peak bending moment being increased.

As a consequence, the bending moment generated at any section of the pipe wall is:

- highly dependent on the relative movement of the finite section of the wall compared to the adjacent parts, resulting from the different reaction of the surrounding soils. Although the rigidity of pipe is considerably higher than the surrounding soils, the flexural stresses generated in the extremities of the wall (especially for large diameter rigid pipe) are very sensitive to the variation in the reactions of surrounding soils, and

- in inverse proportion to the radius of curvature, R. It can be assumed that the relative movement of the wall section results in bending moment being increased, based on the general form of the elementary theory of bending \( M = \frac{E I_y}{R} \).
5.4.2.2 Reaction of surrounding soils

Figure 5.38 demonstrates the hypothesis described above, by comparing the two radial earth pressure distributions caused by earth load only and earth load plus internal water pressures. There exists some differences in supporting pressures, while no changes in the earth pressure imposed on the top of the pipe can be observed. From this, it can be seen that the resistance near the support area is rather higher than at other parts, affected by the different stiffness of the surrounding soils. The resulting flexural stresses are compared in Figure 5.39. The stress variation in the pipe wall due to the internal water pressure is not consistent through the pipe wall as assumed in the current design practice, although the outward movement of the pipe wall is reasonably consistent as has been shown in Figure 5.33.

The lower part of Figure 5.40 illustrates the influence of internal water pressure on bending moment. The single stress line obtained from the classical analysis method suggests that there is no further change in bending moment. From normal interpretation techniques, the difference between the flexural stresses generated at the inner and outer fibres results in the change of the bending moment. Unlike the conventional design method, the change in bending moment near the top and bottom of the pipe seems to be considerable, however, around the springline (an angular position of 90°) it appears negligible. It should be noted that the concrete wall is greatly influenced by even a small difference in tension.

♦ In conclusion, the basic concept of the current design practice should be revised so that the influence of the internal water pressure can be reflected in flexural stresses and bending moments.

The influence of the water weight in the flexural stresses of the core has also been simulated in the upper part of Figure 5.40 for comparison purposes. The general trend is similar to the case of the application of internal water pressure.
In Figure 5.41, it can be seen that with increasing pressure, the bending moment increases through the wall gradually. An internal water pressure of 10 bar causes the design bending moment at the crown (where a maximum bending moment of 62.6kN-m/m for C7v5-03 occurs at the spring line) to be increased by about 31%. From this Figure, it is notable that:

- No significant changes in the radial earth pressure on the top half of the pipe appear; nonetheless, the flexural stresses at the top of pipe tend to increase. This is evidence that the stress response is dependent not only on the actual external loads but also on the trend of the relative displacement of the finite section of the wall.

Figure 5.42 shows that this effect occurs for other pipes, regardless of the soil stiffness and the bedding condition. The relevant information for the vertical and horizontal diameter changes is summarised in Figure 5.43.

5.4.2.3 Behaviour of surrounding soil

Figure 5.44 shows the vertical and horizontal stresses in the surrounding soils at the time of completion of backfilling. The vertical soil stresses along the vertical centreline of the pipe exhibit a similar pattern to the geostatic pressure, but are increased a little. As expected, a large stress concentration appears at the bottom of the pipe. In addition, it has been observed that the shear stress of bedding materials increased linearly with increasing normal stress according to the construction sequence, and the stress curve was considerably below the failure line of Mohr–Coulomb criterion.

Figures 5.45, 5.46 and 5.47 provide the vertical and horizontal stress changes caused by the water weight and the hydraulic pressures during the water transmission after the completion of the backfilling. The
vertical stress directly above the pipe crown in Figure 5.45 tends to decrease with the water filling, since the pipe settles into the ground due to the water weight, as experienced in Figure 5.36 and Detail "A" of Figure 5.37. Then, the stress increases with increasing internal pressure, in a similar manner as other locations. In general, the influence of internal water pressure in the upper fill seems to be, however, insignificant compared to the other places. In contrast, as might be expected, the water weight greatly influences the vertical stress of the soils placed directly below the pipe invert, and also the horizontal stress of the soil at the springline, too.

The corresponding displacement diagrams are provided in Figure 5.48. As is expected, the distribution of vertical displacement in the overlying soils is totally unpredictable because of the complex geometry, the different properties between the original ground and backfill material, and the characteristics of the soil-pipe interaction. This is in comparison with the subsurface displacement due to shield tunnelling, which comprises a simple consistent non-linear curve (the trough shape in horizontal direction).

The horizontal displacement of the pipe elevates the increase of lateral earth pressure. Unlike the side wall of the natural ground, due to the weight of the pipe and backfill, the base of the invert tends rather to compress in excess of the recovery of former swelling during excavation, which will elevate the arching effect similar to the complete trench condition shown in Figure 2.8(c). In Figures 5.49, 5.50 and 5.51, the pattern of progressive displacements of the surrounding soils during the backfilling and water transmission support their stress diagrams.

The stress-displacement curves for the soils placed adjacent to the pipe wall are presented in Figure 5.52. In which, the first three points correspond to the backfilling with an earth cover of 1m, 2m and 3m, and the fourth is due to water filling. The other points from the fifth examine the influence of internal water pressures being increased gradually up to 120 bar.
The soil element placed on the top of the pipe settles down during the water filling, and then, it returns to its original place. The transition curve for the vertical stresses both during the backfilling and after water filling is clearly identified in Figure 5.52, i.e., the stress path, $\overline{34}$. During the water transmission at work stage 5 the stress path follows the same trace of $\overline{34}$ due to water filling. The change of stress due to the internal pressure is however immaterial, supposing that the soil remains in an active state. The increase of stress for the soil placed at the bottom of the pipe continues consistently together with the displacement. A rapid increase of stress following the increase of the displacement is due to a different stress state, compared to the top soil. It is supposed that the excessive displacement in the horizontal direction is likely to create a passive state. Moreover, at work stage 11, the stress together with displacement rather decreases based on work stage 10, which seems to be that the supporting soil has yielded before work stage 10.

Returning to the stress-earth cover curve shown in Figure 5.52, the observation was made that the vertical stress in soil above the top of the pipe increases gradually with increasing earth cover, but the rate of the increment decreases. Here, by comparing the two figures, it is deduced that the increase of earth cover influences the soil stress located at the top of the pipe more significantly, compared to the internal water pressure. It is, however, very difficult to define which is critical in pipe design under actual working conditions. Generally, for a lower earth cover, the design seems to be controlled by the internal pressure, whereas in higher earth cover situations the external earth loads will govern the design. Further discussion will be made in subsequent section.

### 5.4.3 Prestressing effects on large diameter rigid pipe

The difference in behaviour between prestressed rigid pipe and non-prestressed rigid pipe are examined in detail in this section. Figure 5.53(a) illustrates the basic concept of equilibrium of forces in the
prestressed concrete pipe. The prestressing force \( T \), results in the pipe wall being compressed. Equilibrium is assured because

\[
A_s f_s = A_c f_c ( + A_y f_y ),
\]

where \( A_s, A_c, (A_y) \) are the area of prestressing wires, concrete core and steel cylinder embedded in the core, respectively, and \( f_s, f_c, (f_y) \) are corresponding stresses. The component of the steel cylinder is not considered in this thesis. The losses due to the elastic and plastic deformations of the constituent materials should be taken into account in the process for determining the compressive stress. They are divided into two major groups: for instance, firstly, the elastic loss due to core shortening, the loss due to wire relaxation and wire embedment occurring in the short term, and then secondly time-dependent losses such as concrete creep and shrinkage.

In Figure 5.53(b), the compressive stress in the core, resulting from the prestressing, is balanced by the tension generated by the sum of hoop stress due to the internal pressure and flexural stresses due to earth load, pipe weight, water weight, etc.. The effect of different reaction of the surrounding soils (which results in an increase of bending moment, as discussed in Figure 5.40) is omitted in the development of the analytical method. Because of this, the resultant stress due to both the internal water pressure and the flexural stresses by the external loads can not be treated as a simple addition. Therefore, new guidelines should be established which consider the distinctive reaction of the surrounding soil along the pipe wall.

### 5.4.3.1 Behaviour of prestressed and non-prestressed rigid pipe

Figure 5.54 presents the resultant bending moment curves generating the peak tensile stress in the wall for both prestressed and non-prestressed concrete pipe. The curves are made according to the construction sequence together with the application of the internal water pressure. The series number with the first letter of \( S \) corresponds to non-prestressed pipe identical with the earlier study, and that of \( P \) assumes that the prestressing force (1MPa of compression) applies to the outer surface of the pipe in addition to the given condition of series.
S. Series numbers Q and R relate to a compression of 2MPa and 3MPa, respectively.

From the results of the analyses, it is concluded that:

♦ The behaviour of prestressed pipe is entirely different compared to non-prestressed pipe. The prestressing force (to prevent a severe deformation of pipe) results in a further reduction of bending moment in the pipe wall.

♦ Regardless of the quantity of the prestressing wires, no change in the pattern of bending moment curves can be found throughout all the construction steps, but the magnitude of bending moment appears proportional to the magnitude of the compression force.

♦ It is also confirmed that the bending moment in the pipe wall is in proportion to the stiffness of the wall, $E$, and in inverse proportion to the magnitude of the compression force.

As has been discussed, the major role of the prestressing is to maintain the pipe in compression and to prevent a severe deformation of the wall during construction and in service. Figure 5.55 illustrates the change in vertical and horizontal diameters of non-prestressed pipe, due to the internal and external loads, while Figure 5.56 shows the rather different behaviour of prestressed pipe, particularly the elastic contraction of the wall caused by prestressing. For instance, the analysis with the series number P743 indicates that the diameter of pipe is reduced by about 1 mm after completion of prestressing of 1MPa. This contraction will improve the performance of the structure during construction and in service.

Figure 5.57 illustrates the different stress response between prestressed and non-prestressed rigid pipes (refer to Table 5.4 for details of the material properties). Figure 5.58 shows the difference of the flexural stresses for various compression forces generated by the prestressing
wires. From these Figures, the following observations can be made:

♦ In Figure 5.57, with increasing compression force the distance between the stress curves for the inner and outer fibre at the invert and crown reduces, while that at the springline tends to increase. Figure 5.58(a) shows an advantage of the compression. The non-prestressed pipe exhibits a significant difference of flexural stresses both at the crown and at the support area, which implies that the design bending moment is inevitably larger than that for the prestressed pipe, as shown in Figure 5.58(b).

♦ Due to prestressing, all the prestressed pipe is in compression during backfilling for a given earth cover, unlike the non-prestressed pipe. With increasing quantity of prestressing wire, the compression in the core increases, which will provide against further increase of earth loads or internal pressures.

As has been explained by Figure 5.54, the prestressing results in a decrease of bending moment, which presumes that the behaviour of prestressed pipe shows very different stress response, compared to the non-prestressed pipe. Although it apparently has a great flexural rigidity due to the prestressing effect, the behaviour of prestressed pipe is not like unprestressed structures with a high stiffness (which was briefly discussed in Chapter 4). Further influence of the internal water pressure in the prestressed pipe is discussed on the subsequent section.

5.4.3.2. Control parameters for structural design

The variation of bending stress and bending moment in the prestressed pipe due to the internal water pressures is shown in Figures 5.59 and 5.60. It is noted that the design considering an internal water pressure of 10 bar (series number Q743-0310 and R743-0310) are of course impractical because they are too conservative. Here, for the series
P743-0310, the maximum tensile stress (-2668kPa) which occurs at the inner fibre of the wall in the support area is assumed to be within the allowable tensile stress of the concrete core.

Regarding the inter-relation between compression force and internal pressure, further studies have been performed. The results are shown in Figures 5.61, 5.62, 5.63 and 5.64. Figure 5.61 was obtained from the analysis using the series number Q743-0320, assuming that the same amount of pressure (2MPa) for both compression and internal pressures are applied to the outside and inside wall. This example can be used for determining the area of prestressing wires required for a given installation condition. The tensile stresses occurring at the invert area seem to be within the tensile strength limit for concrete; therefore, the core will not crack. Figure 5.63 is for pressures of 3MPa applied to both inside and outside surfaces, where the tensile stress is also insignificant.

Now, it is necessary to consider the significance of the earth and the internal water pressure as the controlling parameters in the pipe design for a given condition. In the case of internal pressure of 10 bar and 3 m earth cover, the pipe design does not require any additional provision for the earth load, as shown in Figure 5.59. Accordingly, it may be assumed that a pipe design with a lower earth cover is generally governed by internal water pressure. In contrast, the design for a higher earth cover may be assumed to be governed by the earth load. However, these two design control parameter paradigms do not adequately describe pipe behaviour.

The effects of a higher earth cover (6 m) are provided in Figures 5.65 to 5.68. For the series number starting with a capital letter L (10bar design), M (20bar design) and N (30bar design), the applied compressions are 1.3MPa, 2.3MPa and 3.3MPa, respectively. From the results, it can be deduced that the requirement for additional provision varies with the magnitude of internal water pressure. For instance, the analyses shown in Figures 5.69 and 5.70 for the series number starting with a small letter, m (20bar) and n (30bar) demonstrate that, with
increasing internal pressure, the requirement for additional provision is diminished. This is due to the magnitude of the internal water pressure, where the tensile stress does not exceed the limit of tensile cracking.

Therefore, pipe design in relation to the control parameters is:

♦ Basically, the minimum compression shall be at least equal to or larger than the provision for internal water pressure so that the stress state of concrete core cannot be exceeded the decompression state of the pipe.

♦ For a low earth cover, the internal water pressure generally governs pipe design, except in the case of very low internal pressure.

♦ With increasing earth cover, the earth load becomes a partial or total controlling factor in design, when the magnitude of internal water pressure is low.

♦ When internal pressure is extremely high, the design is generally controlled by its magnitude, regardless of the earth cover,

♦ As has been demonstrated on Figure 5.70, the combined effect of earth load and internal water pressure requires a lesser steel area than the sum of the individual steel areas for both internal and external loading.

5.4.3.3 Reaction of surrounding soil

Finally, the reaction of the surrounding soils to prestressed pipe is reviewed by using the same approach employed in non-prestressed pipe. In Figure 5.71, the results show that the peak tensile stress (in the inner fibre) due to the accumulated effect of water weight and water pressure remains reasonably well within the range of the design criteria (shown by the dashed line in Figure 5.71). However, the bending
moment is slightly increased by the application of water pressure evidenced by the difference in the flexural stresses at the inner and outer fibres. The change of stresses due to the water weight seems to be negligible, as shown in a upper part of Figure 5.71. In general, the effect of the reaction of the surrounding soil on a prestressed pipe is insignificant, compared to non-prestressed pipe (See Figure 5.40.). From Figure 5.40 and 5.71, with respect to the variation of the stresses in non-prestressed and prestressed pipe, the following observations can be made:

- In the classical analysis no account of the different reaction of surrounding soils to the pipe wall is made, which causes the creation of additional bending moment due to the trend of differential movement of each section of the wall. This is similar to the discontinuity in the foundation introduced by E. Winkler in 1867.

- The influence of the water weight in the prestressed pipe is insignificant, compared to the non-prestressed pipe. It is recommended by writer that the criteria used for the influence of the water weight should be reassessed thoroughly, together with the criteria for the accumulation of the water weight and internal water pressure.

- Furthermore, the basic concept of the current design practice, in which it is assumed that there is no change in the bending moment by the internal water pressure, should be reassessed in the future so that the design procedure accommodates the effect of the reaction of surrounding soil subjected to the internal water pressures. In addition, a new design criteria for prestressed rigid pipe should be established in order to optimise the pipe wall, which takes into account the prestressing effect associated with the interaction between soil and pipe.
Table 5.1 Input parameters in FEA for influence of soil stiffness

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Table 5.2. Input parameters used in FEA for influence of side backfill stiffness

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Table 5.4 Flexural stresses in the extremities of the pipe core as a function of pre-compression (Unit: kN/m²)

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Note: TSD = Design density of soil above the top of the pipe

Figure 5.1 Finite element discretization of soil-pipe problem
Figure 5.2(a) Idealisation of parametric installation for embankment condition
Figure 5.2(b) Preparation of shaped bedding
Figure 5.2(c) Backfilling and compaction of side fill
Figure 5.3 Coding structure used for numerical analyses
Figure 5.4(a) Radial earth pressure distributions for full contact between pipe and surrounding soil (S445-03)
Inside diameter of pipe (ID) = 4.0m, Earth cover (EC) = 3.0m, soil density above the top of the pipe (TSD) = 18kN/cu.m, Pipe wall stiffness (Ec) = 88GPa, Surrounding soil stiffness (Es) = 60MPa, Max. earth pressure = 114kPa at pipe haunch
Figure 5.4(b) Bending stress distributions in pipe wall (S445-03)
ID=4.0m, EC=3.0m, TSD=18kN/cu.m, Ec=88GPa, Es=60MPa, Max. tensile stress=5285kPa at pipe invert
Figure 5.4(c) Bending moment distribution (S445-03)
ID=4.0m, EC=3.0m, TSD=18kN/cu.m, 
Ec=88GPa, Es=60MPa, Max. bending moment=-70kN-m/m at pipe invert
Figure 5.5(a) Radial earth pressure distributions (S345-03)
ID=4.0m, EC=3.0m, TSD=18kN/cu.m, Ec=88GPa, Es=60MPa, Max. earth pressure at haunch area = 212kPa
Figure 5.5(b) Bending stress distributions in pipe wall (S345-03)
ID=4.0m, EC=3.0m, TSD=18kN/cu.m, Ec=88GPa, Es=60MPa, Max. tensile stress=-7195kPa at pipe invert
Figure 5.5(c) Bending moment distribution (S345-03)
ID=4.0m, EC=3.0m, TSD=18kN/cu.m,
Ec=88GPa, Es=60MPa, Max. bedding
moment=93kN-m/m at pipe invert
Figure 5.6 Distribution of vertical and horizontal earth pressures induced along pipe surface circumferentially (S345)
ID = 4.0m, EC = 3.0m, TSD = 18 kN/cu.m, Ec = 88 GPa, Es = 60 MPa
Figure 5.7 Comparison of radial earth pressure and bending moment 
distributions due to change of soil stiffness and density
ID=4.0m, EC=3.0m, Ec=88GPa, Es=90MPa (S455m)
Figure 5.8 Comparison of radial earth pressure and bending moment
distributions due to change of earth cover
ID=4.0m, Ec=88GPa, Es=90MPa
Figure 5.9 Comparison of radial earth pressure distributions due to change of bedding condition
ID = 4.0m, EC = 3.0m, TSD = 18kN/cu.m, Ec = 88GPa, Es = 20MPa Max. pressure = 405kPa for S215 & 210kPa for S415
Figure 5.10 Comparison of bending moment distributions due to change of bedding condition
ID = 4.0m, EC = 3.0m, TSD = 18kN/cu.m, Ec = 88GPa, Es = 20MPa
Max. BMs at pipe invert = -157kN-m/m for S215, & -110kN-m/m for S415
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ID = 4.0m, EC = 3.0m, TSD = 18kN/cu.m, Ec = 88GPa, Es = 20 and 60 MPa Max. BMs = -157kN-m/m at pipe invert for S215 & -107kN-m/m at pipe crown for S245
Note: Details of bedding types 6, 7 & 8 are described in Section 5.3.3.

Figure 5.12 Relation of bending moment and elastic modulus of surrounding soil due to pipe weight and earth load, Complete positive projection condition (He>H) (See Figure 2.8(a)), ID = 4.0m, EC = 3.0m, TSD = 18kN/cu.m, Ec = 88GPa
Figure 5.13 Basic concept of modified shaped bedding installation (S6XX, S7XX, S8XX)
Figure 5.14 Radial earth pressure distribution (S645)
ID=4.0m, EC=3.0m, TSD=18kN/cu.m,
Ec = 88GPa, Es = 90MPa
Figure 5.15 Bending stress distributions (S645)

ID = 4.0m, EC = 3.0m, TSD = 18kN/cu.m,
Ec = 88GPa, Es = 60MPa, Max. Tensile
stress at crown = -3763kPa, Max. comp.
stress at springline = +5074kPa
Figure 5.16 Comparison of bending moment distributions as a function of bedding condition
ID=4.0m, EC=3.0m, TSD=18kN/cu.m, Ec = 88GPa, Es = 60MPa, Max. BMs = -49kN-m/m at crown for S645 and -93kN-m/m at invert for S345
Figure 5.17 Comparison of bending moment distributions due to change of side fill stiffness

ID = 4.0m, EC = 3.0m, TSD = 18kN/cu.m, Ec = 88 GPa,
Es = 60 MPa, Max. BMs = 55kN-m/m at spring line for S645 and 72kN-m/m at springline for S645m
Figure 5.18 Comparison of bending moment distributions due to change of side fill stiffness
ID = 4.0m, EC = 3.0m, TSD = 18kN/cu.m, Ec = 88GPa, 
Es = 60MPa, Max. BMs = -107kN-m/m at invert for S245 and -146kN-m/m at invert for S245m
Figure 5.19 Comparison of radial earth pressure distributions due to change of material stiffness of void zone (very loose soil)
ID = 4.0m, EC = 3.0m, TSD = 18kN/cu.m, Ec = 88 GPa, Es = 60 MPa
Figure 5.20 Comparison of bending stress distributions due to change of material stiffness of void zone
ID = 4.0m, EC = 3.0m, TSD = 18kN/cu.m, Ec = 88GPa, Es = 60MPa
Figure 5.21 Change in bending moment as a function of stiffness of loose soil
ID = 4.0m, EC = 3.0m, TSD = 18kN/cu.m, Ec = 88 GPa, Es = 60 MPa
(a) Comparison of bending moment distributions (S645 vs S645c)
(b) Bending moment diagrams at pipe invert, spring line and crown
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Note: A small letter "v" shown in C7vX-XX means that the material properties of surrounding soil are variable according to the earth cover.
Figure 5.25 Vertical and horizontal stresses in soils adjacent to pipe crown, support and spring line as a function of earth cover due to pipe weight and earth loads (C7vX-XX)
ID=4.0 m, TSD=18 kN/cu.m, Ec=88 GPa
Figure 5.26 Comparison of radial earth pressure distributions as a function of earth cover (C7v5-XX)
ID = 4.0m, TSD = 18kN/cu.m, Ec = 88GPa
Figure 5.27 Comparison of bending stress distributions as a function of earth cover (C7v5-XX) 
ID = 4.0m, TSD = 18kN/cu.m, Ec = 88GPa
Figure 5.28 Comparison of bending moment distributions as a function of earth cover (C7v5-XX)
ID = 4.0m, TSD = 18kN/cu.m, Ec = 88GPa
Figure 5.29 Bending moment diagrams at pipe support, crown and spring line as a function of earth cover due to pipe weight, earth loads and internal water pressure (C7v3-XX)

ID = 4.0 m, TSD = 18 kN/cu.m, Ec = 44 GPa
Figure 5.30 Bending moment diagrams at pipe support, crown and spring line as a function of earth cover due to pipe weight, earth loads and internal water pressure (C7v5-XX)

ID = 4.0 m, TSD = 18 kN/cu.m, Ec = 88GPa
Figure 5.31 Bending moment diagrams at pipe support, crown and spring line as a function of earth cover due to pipe weight, earth loads and internal water pressure (C7v6-XX)

ID = 4.0 m, TSD = 18 kN/cu.m, Ec = 132GPa
Figure 5.32 Change in vertical and horizontal diameters of pipe as a function of earth cover due to pipe weight and earth loads (C7vX-XX)
ID = 4.0 m, TSD = 18 kN/cu.m
Figure 5.33 Change in vertical and horizontal diameters of pipe as a function of earth cover due to pipe weight, earth loads and internal water pressure, ID = 4.0 m, TSD = 18 kN/cu.m
Figure 5.34 View of the pipe displacement at the time of backfilling and application of internal water pressure (C7v5-03XX)
ID = 4.0 m, EC = 3.0 m, TSD = 18 kN/cu.m, Ec = 88 GPa, Es = 40 MPa
Figure 5.35 Detail of the pipe displacements at the time of backfilling and application of internal water pressure (C7v5-03XX)
ID = 4.0 m, EC = 3.0 m, TSD = 18 kN/cu.m, Ec = 88 GPa, Es = 40 MPa
Figure 5.36 Vertical and horizontal displacement of pipe at crown, invert and springline at the time of backfilling, water filling and application of internal water pressure (C7v5-03XX), ID = 4.0 m, EC = 3.0 m, TSD = 18 kN/cu.m, Ec = 88 GPa, Es = 35 MPa
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(C7v5-03XX), ID = 4.0 m, EC = 3.0 m, TSD = 18 kN/cu.m, 
Ec = 88 GPa, Es = 35 MPa
Figure 5.38 Comparison of radial earth pressure distributions as a function of internal water pressure.

ID = 4.0m, EC = 3.0m, TSD = 18, Ec = 88GPa
Figure 5.39 Comparison of bending stress distributions for pipes with/without internal water pressure
ID = 4.0m, EC = 3.0m, TSD = 18kN/cu.m, Ec = 88GPa
Figure 5.40 Increase of flexural stresses at inner and outer fibres due to water weight and internal water pressure of 10 bar for non-prestressed rigid pipe (C7v5-03XX), ID = 4.0 m, EC = 3. m, TSD = 18kN/cu.m, Ec = 88 GPa
Design bending moment (for maximum tensile stress)
C7v5-03 : -54.7 kN-m/m at crown
C7v5-0310 : -72.3 kN-m/m at crown
C7v5-0330 : -91.4 kN-m/m at crown

Figure 5.41 Comparison of bending moment distributions for pipes with/without internal water pressure (C7v5-03XX)
ID = 4.0m, EC = 3.0m, TSD = 18kN/cu.m, Ec = 88GPa
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Figure 5.56 Changes in vertical and horizontal diameters of prestressed pipes during construction and service for various stiffness of pipe wall and installation conditions ID = 4.0 m, EC = 3.0 m, TSD = 18 kN/cu.m, Ec = 44 GPa, Es = 60 MPa
Figure 5.57 Comparison of bending stress distributions for prestressed and non-prestressed pipes due to pipe weight and earth loads
ID = 4.0m, EC = 3.0m, TSD = 18kN/cu.m, Ec = 44GPa, Es = 60MPa
Figure 5.58 Studies on variation of flexural bending stresses induced along wall sections as a function of compression force

ID = 4.0 m, EC = 3.0 m, TSD = 18 kN/cu.m, Ec = 44 GPa, Es = 60 MPa
Figure 5.59 Comparison of bending stress distributions for prestressed pipe due to pipe weight, earth loads and internal water pressure 
ID=4.0m, EC=3.0m, TSD=18kN/m/t.m, Ec=44GPa, Es=60MPa, Max. TS at support = -2668kPa (for P743-0310)
Figure 5.60 Comparison of bending moment distributions for prestressed pipe due to pipe weight, earth loads and internal water pressure
ID=4.0m, EC=3.0m, TSD=18kN/cu.m, Ec=44GPa, Es=60MPa,
Design bending moment at support = -44kN-m/m (for P743-0310)
Figure 5.61 Bending stress distributions for prestressed pipe due to pipe self-weight, earth loads and internal water pressure (Q743-0320)

ID=4.0m, EC=3.0m, TSD=18kN/cu.m, Ec=44GPa, Es=60MPa, Max. TS at support = -1616kPa
Figure 5.62 Bending moment distribution for prestressed pipe
due to pipe weight, earth loads and internal water
pressure (Q743-0320)
ID=4.0m, EC=3.0m, TSD=18kN/cu.m, Ec=44GPa,
Es=60MPa, Design BM at support = -43kN-m/m
Figure 5.63 Bending stress distributions for prestressed pipe
due to pipe weight, earth loads and internal water
pressure (R743-0330)
ID=4.0m, EC=3.0m, TSD=18kN/cu.m, Ec=44GPa,
Es=60MPa, Max. TS at support = -566kPa
Figure 5.64 Bending moment distribution for prestressed pipe due to pipe weight, earth loads and internal water pressure (R743-0330)
ID=4.0m, EC=3.0m, TSD=18kN/cu.m, Ec=44GPa, Es=60MPa, Design BM at support = -42kN-m/m
Figure 5.65 Bending stress distributions for prestressed pipe due to pipe weight, earth loads and internal water pressure (L743-0610)
ID=4.0m, EC=6.0m, TSD=18kN/cu.m, Ec=44GPa, Es=60MPa, Compression force = 1300kPa,
Max. TS at crown = -2581kPa
Figure 5.66 Bending moment distribution for prestressed pipe due to pipe weight, earth loads and internal water pressure (L743-0610)

ID = 4.0m, EC = 6.0m, TSD = 18kN/cu.m, Ec = 44GPa, Es = 60MPa, Compression force = 1300kPa, Design BM at crown = -70kN-m/m
Figure 5.67 Bending stress and moment distributions for prestressed pipe due to pipe weight, earth loads and internal water pressure
(M743-0620) ID=4.0m, EC=6.0m, TSD=18kN/cu.m, Ec=44GPa, Es=60MPa, Compression force =2300kPa, Max. TS at crown = -1422kPa, Max. bending moment at crown = -67kN-m/m
Figure 5.68 Bending stress and moment distributions for prestressed pipe due to pipe weight, earth loads and internal water pressure (N743-0630) ID=4.0m, EC=6.0m, TSD=18kN/cu.m, Ec=44GPa, Es=60MPa, Compression force =3300kPa, Max. TS at crown = -268kPa, Max. bending moment at crown = -66kN-m/m
Figure 5.69 Bending stress and moment distributions for prestressed pipe due to pipe weight, earth loads and internal water pressure (m743-0620) ID=4.0m, EC=6.0m, TSD=18kN/cu.m, Ec=44GPa, Es=60MPa, Compression force =2200kPa, Max. TS at crown = -2173kPa, Max. bending moment at crown = -67.4kN-m/m
Figure 5.70 Bending stress and moment distributions for prestressed pipe due to pipe weight, earth loads and internal water pressure

(n743-0630) ID=4.0m, EC=6.0m, TSD=18kN/cu.m, Ec=44GPa, Es=60MPa, Compression force =3200kPa, Max. TS at crown = -2170kPa, Max. bending moment at crown = -68kN-m/m
Figure 5.71 Increase of flexural stresses at inner and outer fibres due to water weight and internal pressure of 10 bar for prestressed rigid pipe: (P743-03XX), ID = 4.0 m, EC = 3.0 m, TSD = 18 kN/cu.m, Ec = 44 GPa
CHAPTER 6 - DISCUSSION

This chapter presents a summary and discussion of the research work provided in the previous chapter. Many interesting findings have been made through the finite element analyses of the installation of the large diameter rigid pipe, which can be used to make general conclusions about installation effects in the field. It is also confirmed that no distinctive differences have been found between the analysis performed and the observations on full scale field installation trials. This chapter is divided into two main categories; (i) numerical modelling and analysis, and (ii) field trials and laboratory tests

6.1 Finite Element Analysis of Installation Effects

6.1.1 Standard and modified supporting systems

Findings from the numerical analyses on the influence of the material stiffness and various bedding conditions appear to indicate that:

♦ The radial earth pressure distribution (normal to surface of the pipe) on the upper half of the pipe is greatly influenced by the stiffness of the side fill. Lower stiffness produces a bulb-shaped pressure distribution, but with increasing stiffness of side fill, the pressure distribution tends to flatten out, becoming nearly uniformly distributed over the span length (See Figure 5.7.).

♦ For the shaped bedding, the radial supporting pressure on the
bottom of the rigid pipe gradually increases from the pipe invert toward the extremities of the shaped bedding as shown in Figure 6.1, with rare exceptions such as the localised contact bedding condition. This behaviour is quite different to that assumed in normal practice, which is based on the assumption of Olander’s bulb shape.

Figure 6.2 depicts the relationship between bending stresses and bending moments generated around the pipe wall. The maximum bending moment is usually thought to be either at the invert where the distance between the tensile stress at the inner fibre and the compressive stress at the outer fibre is the largest through the pipe wall, or at the middle of a zone of supporting pressure. In contrast, in Figure 6.2, the location of zero bending moment coincides with the place where the two stress distributions intersect. From this diagram, the validity of output from the CRISP main program can be partially verified.

In general, the actual peak bending moments that have been obtained from finite element analysis are significantly smaller than those obtained from conventional design methods, because of well-distributed earth pressures. The pressure distribution on the upper half of the pipe tends to be similar to the supporting pressure distribution, and this trend is observed clearly for both increase of the stiffness of the side fill and increase of the earth cover. On the other hand, for lateral earth pressures, no clear influence on the pressure distribution in the pipe can be found, irrespective of the side fill stiffness. However, side fill with a relatively high stiffness causes the vertical earth pressures imposed on both the upper and lower half portion of the pipe to both decrease and become uniformly distributed.

It is important to note that a poor bedding condition (which results in a high concentrated supporting pressure at the invert) can be compensated by a little effort in compaction of the side
fill (which results in a well distributed supporting pressure below the bottom of the pipe).

In conclusion, the bending moment in the pipe wall can easily be controlled by site works such as an increase of fill stiffness, and further improvement can be achieved by changing the construction method. Evaluation of bending moment and design bedding angle is made easy by the use of the computation diagram shown in Figure 6.3, in which values of bending moment for various values of fill material stiffness are plotted for several kinds of bedding conditions.

To improve the shortcomings of conventional shaped bedding (because it is difficult in practice to achieve full contact between the bottom of the pipe and the surface of the bedding) a modified shaped bedding with an intentional void region at the invert has been considered. The basic concept is illustrated numerically in Figure 6.4. Figure 6.4(a) shows that the pipe is placed on a smaller arc in the modified shaped bedding, which makes it possible both (i) to prevent the concentration of supporting pressures at the invert, and (ii) to shift the supporting pressures to the extremities of pre-shaped bedding. By using this system, the following observations are made:

In Figure 6.4(b), the symmetric supporting pressures about the vertical centreline of the pipe have led to some recovery of the pre-deformed shape due to its self-weight before installing the pipe on the shaped bedding. It would appear that this phenomenon contributes to the mobilisation of higher lateral earth pressure during backfilling, and therefore the decrease of bending moment in the pipe wall.

The bending stress distributions are very complex in comparison with the conventional shaped bedding. The maximum bending moment is generally located at the place where the peak supporting pressure occurs. However, the governing tensile
stress is generally shifted to either the springline or the crown. It should be noted that this shift is mainly due to the reduction in bending moment at the invert. The location of the governing tensile stress does not coincide with that of the maximum bending moment.

In conclusion, the modified supporting system results in a great reduction of maximum bending moment, regardless of the locations where the maximum bending moment and peak tensile stress occur. Figure 6.3 highlights the relationship between the bending moment and the stiffness of side fill, as a function of bedding shape. It is very important to note that the modified shaped bedding (i.e. Series Nos. S615, S645, S715 and S745) appears much improved in comparison with the ideal full contact bedding condition. Even S815 and S845 (which correspond to poor practice when using the modified shaped bedding conditions) indicate a similar result with full contact. From Figure 6.3, it has been recognised that the achievement of 180° bedding angle is possible in the field.

6.1.2 Flexural rigidity and flexural stresses

The relationship between the flexural rigidity of the pipe and the resultant bending stresses in the pipe wall are presented according to the results obtained from the extensive analyses reported in Chapter 5.

The field bending moment may be obtained from one of the diagrams shown in Figure 6.5. The pipe wall stiffness and the bedding shape greatly influence the bending moment. Therefore, it can be deduced that conventional design practice has a significant safety margin, since the stiffness of the actual pipe wall is generally far lower than that for a perfectly rigid pipe.
From Figure 6.6, which shows the variation of normal stresses in the soils placed at the pipe top, bottom and springline according to the construction sequence, it can be seen that none of the stress paths can be forecasted prior to carrying out an individual analysis which takes into account the actual field condition. With increasing earth cover, the lateral earth pressure increases linearly, while the changes in vertical earth pressures at the pipe top and bottom are non-linear. Therefore, when earth cover reaches a certain level, the increase of bending moment reaches a plateau, as shown in Figure 6.7. Further increase of earth cover causes a reduction in bending moment. This phenomenon is very similar to the arching action described in Section 2.3.1.

6.1.3 Application of internal water pressure and prestressing effect

The validity of conventional design methods in association with the internal water pressure is examined in Figure 6.8. The principle of the design criteria (i.e. the assumption there is no change in bending moment during water transmission, because internal water pressure produces the uniform tensile stress around the pipe wall) should be criticised for the following reasons.

♦ The interaction problems for the buried circular pipe installed by cut-and-cover method should be distinguished from other classical underground structures; for instance, box culverts with a simple geometry, or shield tunnelling which permits only a very small deformation of the surrounding soil. The reaction of the surrounding soil against the internal water pressure appears to be a very complicated response, caused by different degrees of compaction and confining stress. This phenomenon means that the behaviour of the pipe wall is virtually unpredictable.

♦ In Figure 6.8, the average increase of bending stress in the wall seems to be in reasonably good agreement with the stress
increase predicted by the current design method. However, the tensile stress increase at the inner fibre obtained from the numerical analysis is far higher than that from the conventional analytical method, which may cause the pipe core to crack.

♦ In this respect, new design criteria should be broadly established by back analysis, using the results obtained from the finite element method, taking into account the reaction of the surrounding soils to the internal water pressure.

The normal stresses and displacements of the surrounding soils are examined in Figures 6.9 and 6.10, based on 3 m earth cover. The stress-displacement relation for the surrounding soil is also provided in Figure 6.11.

◆ The stresses in the soils placed around the pipe show a quite different pattern compared to the geostatic pressures. The vertical stresses in the soils placed above the pipe appear rather higher than that for the weight of the soil prism, due to the inverted arching action. A large horizontal stress exists at the springline similar to Rankine’s lateral pressures, unlike the traditional design method. However, it must be made clear that positive deflection of the pipe in the horizontal direction results in the soil achieving a passive state. Rotation of principal stresses would occur in some situations.

◆ Figure 6.10 confirms the complicated behaviour of the soil around the pipe. The side fill at the springline is in a passive state, causing an increase of horizontal stress. No distinctive change in vertical displacement of the natural ground below the bottom of pipe can be found. This implies that the influence of mesh geometry for the natural ground below the bottom of the pipe is insignificant on the stress response of the buried pipe.

◆ The stress-displacement relation for the soils placed adjacent to
the pipe has been reviewed in Figure 6.11, in order to investigate the complete behaviour of the surrounding soil during construction and in-service. Irrespective of the location of the soil elements, with increasing earth cover the stress increases continuously without exception. With water filling (stated in Figure 6.11), it starts sinking down in a whole body movement and expanding outward at the sides, simultaneously. This causes (i) an increase of both vertical stress at the bottom and horizontal stress at the springline, but (ii) a decrease of vertical stress.

With the application of internal water pressure, all stresses increase (See work stages 5 to 11 in Figure 6.11.). The stress path for a soil element at the top of the pipe reverses along the stress path constructed during water filling. Since the induced stresses on the surface of the pipe are not consistent, it can be concluded that internal water pressure influences the magnitude of bending moment.

Further to the influence of internal water pressure, the importance of the prestressing effect should be emphasised in pipe design. No design method considers an entirely different behaviour of prestressed pipe compared to non-prestressed pipe, giving rise to shortcomings in the current design method. It is noted that, although the prestressing produces nearly uniform compressive stresses around the pipe wall, its influence in the behaviour of the pipe is not negligible; especially when the external loads are involved.

The prestressing (which produces a constraining force on the pipe) preserves the critical section of the pipe wall to produce a relatively large radius of curvature when the deformation occurs. Therefore, bending moments in the prestressed pipe wall are always lower than in the non-prestressed pipe. This situation can be explained by the elementary elastic bending theory.
The complete set of analysis results for the prestressed pipe is shown in Figures 6.12 and 6.13 with respect to change in diameter and variation of bending moment during construction and in-service. From these Figures, it can be seen that the behaviour of the pipe is significantly influenced by both the existence of the prestressing force and the quantity of the prestressing wires used.

Figure 6.12 - Detail "A" highlights the effect of prestressing force causing the pipe to contract towards its centre of gravity. For instance, the prestressing of 1MPa (P743) of compression causes a pre-contraction of the pipe (P743) of around 1mm in diameter. The earth load subsequently imposed on the pipe due to backfilling leads to a reduced vertical diameter and increased horizontal diameter, simultaneously.

When internal water pressure is applied to the inner surface of the pipe, the pipe starts to expand outward, reforming the original pipe shape to some degree. However, perfect reshaping of the pipe cannot be achieved in normal working conditions, because changes in the vertical and horizontal diameters are not identical. This inability seems not to be harmful in a structural sense, because the induced lateral earth pressures are kept in a passive state during service.

In the case of prestressed pipe, the induced bending moment is largely dependent on the quantity of prestressing wires provided, as shown in Figure 6.13. In other words, the bending moment in the pipe wall is in inverse proportion to the compression force, with other conditions being equal. In conclusion, the prestressing force results in a significant decrease of bending moment in the pipe wall, compared to non-prestressed pipe (Series No. SXXX).

Finally, the stress responses of prestressed and non-prestressed
pipes are summarised in Figures 6.14, 6.15 and 6.16. No distinctive differences in radial earth pressures can be observed. However, the characteristics of the resulting bending stresses and moments appear quite different due to the compression force.

The combined effect of internal pressure and compression force has also been examined, in order to evaluate the critical factors for the design. It is worth emphasising that no definitive answer concerning these issues was available prior to simulation with finite element analysis, in which actual field conditions and procedures could be modelled.

6.1.4 Constitutive models

The constitutive soil models available in CRISP94 are classified into three major groups; (i) linear elastic, (ii) linear elastic perfectly-plastic, and (iii) critical state. The rigid pipe was assumed to behave as an homogeneous isotropic linear elastic material, requiring only two independent constants. This model was verified by comparing the output of CRISP with the analytical solution obtained using the two-edge loading scheme: the bending moment stress distribution obtained from the finite element analysis was in good agreement with that from the elastic solution. From this, the question arises about the major factor influencing bending moment in the prestressed pipe wall for a given condition. The following facts are considered.

♦ For the model of two-edge loading for non-prestressed rigid pipe, analyses using too few elements indicate that the induced bending moment is (i) generally lower than the analytical solution and (ii) in proportion to the stiffness of the pipe wall. Therefore, the number of elements were one of the major factors, and should be optimised.

♦ It is important to note that;
(i) In Figure 6.17(a) for prestressed pipe, the resulting bending moment distribution around the pipe wall appears generally lower than that for the analytical solution, and it gradually increases with increasing stiffness of pipe wall. From this, it is concluded that the induced bending moment is significantly influenced by the prestressing effect.

(ii) However, in Figure 6.17(b) for non-prestressed pipe, when using an appropriate number of elements no changes in bending moment can be found, regardless of the pipe wall stiffness.

♦ The reason why the compression force results in a decrease of bending moment is unclear to date, but, as explained before, the actual behaviour of prestressed pipe appears to be quite different to non-prestressed pipe due to the compression force.

A linear elastic perfectly-plastic soil model with the Mohr–Coulomb criterion has been used for the fill material. This model is commonly employed in geotechnical engineering problems, since it makes it possible to limit the shear strength. However, the natural ground has been considered as a linear elastic material due to the various uncertainties, such as in-situ stresses, stiffnesses and time-dependent deformation characteristics. The linear elastic response can generally be accepted in soil-structure interaction problems where small strain is expected. Moreover, for sands cemented with silicates and iron oxides, this model seems to be reasonable. It has been reported by many other researchers that the stress distributions for these soil models are in reasonably agreement with Spangler’s early experiments and others using small diameter reinforced concrete pipe, but no published experiments on large diameter pipe are available.
6.2 Laboratory Tests and Full Scale Experiments

The mechanical response of a buried pipe is accompanied by complex soil and pipe interaction, arising from the nonlinear inelastic behaviour of constituent materials. It is therefore impossible to predict the real behaviour of the whole structure precisely. In this respect, the only alternative is to attempt full scale trials in the field, together with laboratory tests for individual materials in detail.

6.2.1 Elastic modulus of concrete

A summary and discussion of the laboratory tests for determining the elastic modulus of concrete used in the prestressed pipe (conducted at the GMRP laboratories in Libya (DAC, 1992) and Universite de Sherbrooke in Canada (DAC, 1996a and 1996b)) are provided in this section. The mean elastic modulus and compressive strength for 20 specimens in Libya were recorded at around 34GPa and 50MPa, respectively. The mean elastic modulus and compressive strength for 10 specimens in Canada were around 40GPa and 47MPa, respectively.

♦ The behaviour of the concrete appears to be a classical linear response up to a certain stress level, and then a non-linear response until cracking occurs. For high strength concrete no distinctive differences between the initial tangent modulus and secant modulus could be found, and the initial linear response was extended far higher than 40% of ultimate strength of concrete.

♦ The mean elastic modulus of concrete using natural aggregates from the Brega plant (37GPa) is considerably lower than that using crushed aggregates from the Sarir plant (44GPa). In contrast, the mean compressive strength of concrete from the Brega plant was a little higher (47.8MPa), compared to the Sarir plant (46.4MPa), but the difference is almost insignificant.
As shown in Figure 6.18, no clear relationship exists between elastic modulus and concrete strength, because of both different raw materials and different laboratories. However, for a given condition, the elastic modulus of concrete is generally seen to increase, with increasing strength.

6.2.2 Three-edge bearing tests

Full scale three-edge bearing tests were carried out to investigate the actual behaviour of large diameter prestressed concrete cylinder pipe (PCCP) at the GMRP site in Libya (See Figure 2.10) according to the procedure described in ASTM C497. The specimen of pipe was 2.5 metres long, irrespective of pipe diameter. The ratio of concrete core thickness to pipe inside diameter was fixed at a minimum value of 1/16 for the aspect of serviceability control. A given test was continued until a longitudinal crack 0.025mm wide reached a length of 300mm. Typical load-deflection curves for various sizes of PCCP are summarised in Figures 6.19 (for two identical PCCPs with a diameter of 4000 mm) and 6.20 (for different pipe size and area of prestressing wires).

In Figure 6.19, the load-deformation behaviour of the two pipes appears identical. Moreover, the slope of the load-deformation curve gradually increases with an increase of the applied load. Therefore, the behaviour of pipe appears to be non-linear.

With an increasing quantity of prestressing wires, the slope of the curve reduces, as shown in Figure 6.20, although no difference in the overall form of the curve is observed.

In general, the character of deformation appears proportional to the flexural rigidity of the pipe section. This results in the deformation of a large diameter pipe being less than that of a small diameter pipe. The first visible cracks corresponding to the
design three-edge bearing load, $W_{3eb}$, employed in the AWWA C301-84 Appendix-A design method occurred when the deformation of pipe reached around 10 to 15mm in the vertical direction. The three-edge bearing loads are in proportion to the area of prestressing wires and the square of wall thickness, and in inverse proportion to the mean diameter of the pipe.

In conclusion, the behaviour of the large diameter rigid pipe exhibits a clearly nonlinear response during the three-edge bearing test. However, it would appear that the response can be assumed linear over a small range of external load, as under working conditions. In reality, the combination of the radial earth pressures makes it possible to imply that the external loads are far smaller than three-edge bearing load used in design.

6.2.3 Pipe installation site trials

The results of the full scale site trials carried out at the GMRP site (DAC, 1991) are briefly reviewed and discussed in this section, based on the pipe installation field trials report. The purpose of this trial was to identify preferred installation procedures, and to confirm the degree of compaction in the shaped bedding and in the different zones of backfilling. In addition, the following aspects were considered:

(i) the effectiveness of shaped bedding
(ii) the construction method of shaped bedding
(iii) alternative bedding materials available in the field

The procedure for the site trials was almost identical with the shaped bedding installation procedure described in Section 5.3.1. Some of the field trials had loosened inverts. A soil density which was lower under the pipe invert than under the haunches was provided by using a serrated teeth arrangement attached to the bulldozer, in order to ensure a special profile of a loosened compacted bed.
The properties and degree of compaction of the bedding and backfilling materials shown in Figure 6.21 are:

**Invert:** The density in the central invert strip (i.e. the middle third of outside diameter of the pipe, \(D_o/3\)) is less than the density at the haunches, and all bedding materials are compacted to a minimum 70% relative density.

**Bedding:** 100% of material should pass the 50.8mm sieve, 95-100% pass the 25.4mm sieve, and 0-12% pass the No.200 sieve. Density to be minimum 70% relative density (95% standard Proctor).

**Zone I:** Backfill - Maximum size 150mm and not more than 10% greater than 75mm. Density between springline and \(D_o/5\) line to be minimum 70% relative density (95% standard Proctor). Density of the material above the \(D_o/5\) line (see Figure 6.21) to be minimum 90% standard Proctor.

**Zone II:** Backfill - Density to be minimum 90% standard Proctor, maximum size 500mm. The finished surface is evenly graded.

**Zone III:** Backfill - Maximum size 75mm. Density to be minimum 90% standard Proctor.

Initially, the pipe was deflected vertically due to its self-weight during storage and transportation. Placing the pipe in the designed shaped bedding configuration with a radius of curvature smaller than the pipe outside diameter (termed modified shaped bedding) ensured positive support in the haunch region. This caused some of the deflected shape...
to be recovered. The general observed behaviour of the pipes are summarised below. Here, the discussion is limited to the pipes installed in the shaped bedding with a loosened invert. Figure 6.22 is a typical representation of the measurements obtained by digital measuring rod (LVDT) and strain gauge at each work stage.

- The settlement of bedding material placed at the middle third points on the pipe invert is about 6mm, resulting from a loosened invert or varying compaction levels. Compared with the results of the CRISP analysis, this value is a little large, which seems to be due to the lower stiffness of the site bedding than that assumed in the analysis.

- The horizontal deflections at stage 5 (completion of surcharge, 13 hours) were recorded at ± 2mm compared to the base measurement at stage 1. No change in the horizontal diameter was observed, implying that the pipes in the field trials were firmly supported at the haunches (In some pipes the horizontal diameter actually reduced.). The vertical deflections were also small, 0 to 2mm for pipes compared to the base measurements at stage 1. From this, it can be seen that the pipes were firmly supported at the haunch areas but not at the invert.

- The Demec gauge measurements of strain at the springline at stage 2 exhibited a large increase in the length gauge, indicating that the pipes were re-rounded due to the firm support of the pipe at the haunches. The mean strains at stage 5 were almost identical with the base measurement at stage 1, implying that the deflection at stage 5 was almost the same as that at stage 1.

- The strains for the invert tended to decrease consistently with the progress of the work, which is further evidence to support that the pipe is re-rounded. The mean strains at stage 5
appeared less than at stage 1 and stage 2. That is, at stage 5 (after 13 hours of total applied earth load), the pipes gained in compressive strain at the invert compared to the base invert measurements.

- As can be seen in Figure 6.22, the results of strain measurements for the haunch indicated that the pipes were firmly supported at the haunches. The Figures showed about the same pattern and the measurements remain practically the same throughout the various backfilling stages from stage 2 to stage 5.

In general, no significant differences in the pipe behaviour can be found when comparing the field trial data with the results obtained from the finite element analyses. Moreover, it has been confirmed that it is practicable to install prestressed concrete pipe over a pre-shaped bedding. The invert can be loosened in order to ensure the peak supporting pressures are shifted toward the extremities of the bedding.
Figure 6.1 Radial earth pressure distributions (S345-03)
Inside diameter of pipe (ID) = 4.0m, Earth cover (EC) = 3.0m, Soil density above the top of the pipe (TSD) = 18kN/cu.m, Pipe wall stiffness (Ec) = 88GPa, Surrounding soil stiffness (Es) = 60MPa, Max. earth pressure at haunch area = 212kPa
Figure 6.2 Bending stress and moment distributions (S345-03)

ID = 4.0m, EC = 3.0m, TSD = 18kN/cu.m, Ec = 88GPa,
Es = 60MPa
Bending moment at pipe invert, \( \frac{Nm}{m} \)

Elastic modulus of fill materials (\( E_s \)), MPa

Note: Standard shaped bedding: S1XX, S2XX, S3XX, S4XX
Modified shaped bedding: S6XX, S7XX, S8XX
Details of bedding condition are provided in Figures 5.2(a) and 5.3.

Figure 6.3 Relation of bending moment and elastic modulus of surrounding soil due to pipe weight and earth load, Complete positive projection condition (\( H_e > H \)) (See Figure 2.8(a)), \( ID = 4.0 \text{m}, EC = 3.0 \text{m}, TSD = 18kN/cu.m, Ec = 88GPa \)
(a) Basic concept of modified shaped bedding

(b) Radial earth pressure distribution (S645-03)

Figure 6.4 Modified shaped bedding installation
S6XX, S7XX, S8XX (See Figure 5.2(a).)
Figure 6.5 Relation between bending moment and elastic modulus of pipe wall due to pipe weight and earth pressures, ID = 4.0 m, EC = 3.0 m, TSD = 18kN/cu.m, Es = 20MPa and 60MPa
Figure 6.6 Vertical and horizontal stresses in soils adjacent to pipe crown, support and springline as a function of earth cover due to pipe weight and earth loads (C7vX-XX)
ID=4.0 m, TSD=18 kN/cu.m, Ec=88 GPa
Bending moment diagrams at pipe support, crown and springline as a function of earth cover due to pipe weight, earth loads and internal water pressure (C7v3-XX)
ID = 4.0 m, TSD = 18 kN/cu.m, Ec = 44 GPa
Figure 6.8 Increase of flexural stresses at inner and outer fibres due to water weight and internal water pressure of 10 bar for non-prestressed rigid pipe (C7v5-03XX), ID = 4.0m, EC = 3.0 m, TSD = 18kN/cu.m, Ec = 88 GPa
Figure 6.9 Distribution of vertical and horizontal stresses in soils around pipe top, bottom and springline after completion of backfilling (D245-03)
Figure 6.10 Distribution of vertical and horizontal displacements of soils around pipe top, bottom and springline after completion of backfilling (D245-03)
Figure 6.11 Stress-displacement curves for soils placed near pipe top, springline and bottom following construction sequence and increase of internal pressure (D245-XXXX), ID = 4.0 m, TSD = 18 kN/cu.m, Ec = 88 GPa, Es = 60 MPa
Figure 6.12 Changes in vertical and horizontal diameters of prestressed pipes during construction and service for various stiffness of pipe wall and installation conditions ID = 4.0 m, EC = 3.0 m, TSD = 18 kN/cu.m, Ec = 44 GPa, Es = 60 MPa
Figure 6.13 Bending moments induced during construction and water transmission for prestressed and non-prestressed pipes
ID = 4.0 m, EC = 3.0 m, TSD = 18 kN/cu.m, Es = 60 MPa
Figure 6.14 Comparison of radial earth pressure distributions between prestressed and non-prestressed pipes (S743-0320 vs Q743-0320), ID = 4.0m, EC = 3.0m, TSD = 18kN/cu.m, Ec = 44GPa, Es = 60MPa
Figure 6.15 Comparison of bending stress distributions between prestressed and non-prestressed pipes (S743-0320 vs Q743-0320), ID = 4.0m, EC = 3.0m, TSD = 18kN/cu.m, Ec= 44GPa, Es= 60MPa
Figure 6.16 Comparison of bending moment distributions between prestressed and non-prestressed pipes (Q743-0320 vs Q743-0320), ID = 4.0m, EC = 3.0m, TSD = 18kN/cu.m, Ec = 44GPa, Es = 60MPa
Bending moment ratio = $\frac{BM_{(FEM)}}{BM_{(Analytical\ solution)}}$

Elastic modulus of pipe wall, MPa

(a) Max. BM in prestressed pipe as a function of elastic modulus of pipe wall

(b) Bending moment ratio relative to analytical solution

Figure 6.17 Change in bending moment distribution for prestressed and non-prestressed pipe due to two edge loads at pipe top and bottom, ID = 4.0 m, $tw = 280$ mm, Compression = 2000 kPa, Load = 509.6 kN/m
Note: 1. The numbers 1 to 4 indicate the test series described in Section 4.3.1.
2. The data are based on the mean strength not the characteristic strength used for the international standard.

Figure 6.18 Elastic modulus of concrete as a function of concrete strength (DAC, 1992, 1996a and 1996b)
Figure 6.19 Load-deflection curves for 4000mm diameter prestressed concrete cylinder pipe during three-edge bearing test
Figure 6.20 Load-deflection curves as a function of pipe size and steel area
(Wire size : 4.88mm, Core thickness / Pipe dia. = 1/16)
Note: Work Stage 1 implies that pipe is placed in temporary storage alongside the trench.

Figure 6.21 Pipe bedding and backfilling
Figure 6.22 Displacement and strain measurements at each work stage
CHAPTER 7 - CONCLUSIONS

This chapter comprises three main sections. It begins with a restatement of the problems/issues presented at the outset of the thesis, and then provides conclusions and practical implications of the main findings. Finally, recommendations regarding the direction of possible future work are given.

The overall intention of this thesis was to use numerical modelling to evaluate practical guidelines for soil and large diameter rigid pipe interaction problems, and to give a detailed critique followed by an explanation of the alternative basic general criteria for assessing field performance. Numerical modelling was carried out to examine the real behaviour of both prestressed pipe and non-prestressed pipe.

Since no specific design methods for large diameter rigid pipe exist, the current design approach is traditionally based on either Marston-Spangler's semi-empirical method (which was originally developed for small-diameter reinforced concrete pipe) or conventional elastic theory suitably generalised. However, these approaches have inevitably led to conservatism in the design, leading to a lack of economy. More recently, modest finite element analyses have been conducted in this field, but these are mainly related to either flexible pipe or small diameter reinforced concrete pipe: no attempt has been made by previous researchers to investigate in detail the interaction problems between the soil and the large diameter rigid (prestressed and non-prestressed) pipe.

Understanding the behaviour of real engineering structures is certainly not a simple matter. For instance, current practice associated with soil-structure interaction problems considers only that the earth pressure and its distribution on an underground structure or the behaviour of structure to the soil are highly dependent upon the relative stiffness of the structure and the surrounding soil (Wu et al.;1985, Heger;1988). The stiffness
relationship between the pipe and soil is very important. However, additional factors such as the modes of prestressing and installation effects are also of great importance in the complete solution of buried pipe problems. Moreover, no account is taken in previous analyses of the influence of internal water pressure together with the complicated reactions between the pipe wall and the surrounding soil. In other words, none of the design methods reflects the inter-relationship, termed here the "rebound stress response", between the two media and its variation during the construction sequence. Therefore, optimisation of the soil-pipe system cannot be anticipated with the current design practice. As an alternative, only the finite element method has the potential to take into account all of the above factors with intelligent simplifications of reality for complicated soil-structure interaction problems. However, some uncertainties remain in determining the input data regarding the actual material properties for use in finite element analyses.

In an attempt to clarify interaction problems using the finite element method, it must be made clear that, for problems in mechanics, a complete solution must satisfy the requirements of equilibrium, compatibility and constitution. Moreover, the modelling of the field situation, including the construction sequence, is of great importance in the numerical analysis, since it significantly affects the results of the analysis; i.e. (i) number of increment blocks, (ii) changes in geometry, (iii) application of loads, (iv) displacement fixities, and (v) excess pore pressure fixities. Finally, the interpretation of output from the CRISP main program (or the results for each specified increment of the finite element analysis) is also of importance, for example (i) the nodal displacements and excess pore pressures in consolidation analyses, (ii) Cartesian stresses, (iii) cumulative strains, (iv) stress invariants (v) out-of-balance loads, and (vi) other parameters such as yield ratios, etc.

Since no published information is available on the interaction between soil and large diameter prestressed pipe, the validity of numerical modelling which takes into account specific field conditions such as the prestressing and installation effects can not be completely verified.
However, simplified conditions were fully verified by comparison with the pure elastic solution for a closed circular ring.

The objective of this study was to attempt to model the complicated interaction between soil and large diameter rigid pipe, using the geotechnical finite element package CRISP. Practical guidance was then given for various construction conditions.

The analysis has taken into account the influences of:

♦ the elastic properties of both pipe and surrounding soil (in-situ and placed/compacted soils) as a function of confining stress varying with the construction procedure,
♦ the placement, compaction, and any subsequent volumetric changes in the soils placed around the pipe,
♦ the variation of earth cover and geometry, replicating the construction process,
♦ the bedding conditions specified in the design; i.e. flat bedding, standard shaped bedding, modified shaped bedding and controlled shaped bedding,
♦ the water weight and internal water pressures applied to the inner surface of the pipe wall, and
♦ the compression forces in the pipe wall, provided by prestressing wires helically wrapped around the concrete core under controlled tension.

Pipes are commonly divided into three broad classifications; i.e. rigid, semi-rigid (or intermediate) and flexible. Although the terms are based on the character of pipe deformation, they are not quantitative. As demonstrated in the results section concerning wall stiffness, it is almost impossible to clarify the actual behaviour of pipe for the range of relative stiffnesses of the pipe and surrounding soil. Therefore, no unique solution can be provided with current analytical methods. From the finite element analyses, it is concluded that the resultant bending stresses (or bending moments) in the pipe wall are greatly influenced
by the material stiffness; specifically, (i) the stiffer the soil, the smaller the bending stresses and (ii) the stiffer the wall the larger the bending stresses. This trend is the opposite to that expected before the finite element analyses were performed.

In numerical experiments, the ability to have a constitutive model for each material zone is an advantage over analytical approaches, since it permits modelling features of real soil–structure interaction behaviour. For fill materials, a linear elastic perfectly-plastic soil model seems to be pertinent in terms of limiting the soil strength. A linear elastic model is adequate for the pipe structure and for the natural ground if it is cemented. Here, it is important to note that the current CRISP program (in common with many others) is unable to simulate compaction effects, since no function in the program permits dissipation of the input energy. Using the proposal from Chapter 3, namely to increase Poisson's ratio to simulate higher lateral stress, the bending stresses in the pipe wall decrease by about 10%, compared to the non-compacted case.

The residual vertical and horizontal stress distributions around the buried pipe are shown to be rather more complex than the linear stress distributions based on Marston–Spangler's earth loads, Coulomb's active and passive earth pressures, or Rankine's soil wedge theory. In soil–pipe interaction problems, the most important aspect is to find a simple and practical method of inducing uniformly distributed support pressures on the bottom half of the pipe. As has been demonstrated, the distribution of vertical reaction on the bottom of the pipe is greatly influenced by the bedding shape. Flat bedding (which requires thorough compaction within the triangular-shaped spaces below the bottom of the pipe) should not be considered for the high degree of bedding angle required. The conventional (or standard) shaped bedding (which requires full contact between the pipe and soil at the bottom of the pipe) seems also to be impractical because it is rarely possible for the pipe and bedding to be formed with an identical radius in the zone of contact. This is mainly due to the deformation characteristics of the two media.
during construction and in-service, and the quality of construction. In practice, the only possibility is to have either a modified shaped bedding in order to ensure the symmetrical shifted supporting system about the vertical centreline of the pipe. This system provides rather better stress response compared even to a conventional shaped bedding formed without any construction errors.

Most design practices based on the concept of "beams on elastic foundation" are generally based on Winkler's simple assumption that the pressure in the foundation at a given point considers only the deflection of the beam at that point, and takes no account of the pressure or deflection occurring in other parts of the foundation. This implies that a complete lack of continuity in the material of the foundation. However, when considering a large diameter rigid pipe, the phenomenon becomes more intricate. Basically, the current pipe design method assumes that internal water pressure induces a uniform reaction from the surrounding soils, providing no additional bending moment in the pipe wall. This is incorrect. The design for any concrete pipe is essentially controlled by the induced tensile stress. Therefore, even a small increase of tensile stress due to the different distribution of the radial reaction on the pipe wall will be critical.

No major differences in the physical properties between prestressed and non-prestressed pipe were noted. The only difference is seemingly the existence of compression stress in the pipe wall induced by prestressing. The prestressing produces a compression force to ensure that the pipe wall is in equilibrium against the tensile stress in the wall arising from hoop stress and flexural stress. However, the major shortcoming of the current design practice is that it takes no account of the prestressing effect, causing a critical section of the pipe wall to have a relatively large radius of curvature (R) when the external load is involved. According to the elementary elastic theory, the induced bending moment is in inverse proportion to R and in proportion to the stiffness of the structure (EI). This implies that the prestressing results in the pipe wall bending moment being decreased.

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In conclusion, since difficulties lie in assessing correctly interaction problems with the current analytical method, excessive safety factors have been applied to the stress–strain behaviours of both the soil and the structure. As an alternative approach, only the numerical technique can solve such complicated problems, in which the the actual field conditions and procedures can be modelled. Moreover, finite element analysis not only takes into account all the fundamental requirements, such as prestressing effect, actual material properties, loading system and construction sequence, but also makes it possible to avoid excessive safety factors for the integrity and long term stability of the structure.

The question remains as to whether the models used for prestressed pipe cover the actual manufacturing and installation situations well enough for engineering purposes. The recommendations with respect to possible future work are:

- Experimental work especially using fully instrumented prestressed pipe (with embedded instruments in the concrete core) in conjunction with laboratory-based material property tests in order to compare the actual stress responses between prestressed concrete pipe and non-prestressed concrete pipe.

- Numerical and field experiments on the behaviour of the pipe and soil for different exterior conditions of the pipe, particularly to investigate the influence of the smoothed dielectric coating surface.

- Field experiments to investigate the radial earth pressure and the bending moment distribution in the pipe wall due to unbalanced load (for example, due to asymmetric construction sequences or live loads) such that the critical point of maximum tensile stress may be between the crown and springline.

- Improved modelling of the interface (slip) in numerical analysis to
take account the real behaviour of soils placed around the pipe wall.

- For use in practice, the development of simplified stress distribution profiles, which encompass the actual (complex) effects of compaction, internal water pressure and prestressing force, is required. This could perhaps be achieved by building up a catalogue of design curves, for particular loading cases, which could be used by practicing engineers.

- Following the field work of Havell and Keeney (1976), investigation of the time dependence of the earth pressure distribution on a large diameter rigid pipes should be conducted.
### NOTATION

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$A_c$</td>
<td>Area of concrete core</td>
</tr>
<tr>
<td>$A_s$</td>
<td>Area of prestressing wire in unit length of pipe</td>
</tr>
<tr>
<td>$A_y$</td>
<td>Area of steel cylinder</td>
</tr>
<tr>
<td>$B_c$</td>
<td>Horizontal external diameter of the pipe</td>
</tr>
<tr>
<td>$B_f$</td>
<td>Bedding factor (ratio of the supporting strength to the three-edge bearing strength of the pipe)</td>
</tr>
<tr>
<td>$C$</td>
<td>Distance from neutral axis</td>
</tr>
<tr>
<td>$C_c$</td>
<td>Load coefficient for embankment condition</td>
</tr>
<tr>
<td>$E$</td>
<td>Elastic Young's modulus of soil (at an elevation $y$)</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Elastic Young's modulus of concrete</td>
</tr>
<tr>
<td>$E_e$</td>
<td>Elastic Young's modulus of pipe wall</td>
</tr>
<tr>
<td>$E_{c_{emp}}$</td>
<td>Empirical elastic modulus of concrete core including steel cylinder due to the prestressing effect</td>
</tr>
<tr>
<td>$E_{cph}$</td>
<td>Elastic Young's modulus of concrete in the plane strain condition</td>
</tr>
<tr>
<td>$E_h$</td>
<td>Elastic Young's modulus of soil in the horizontal direction</td>
</tr>
<tr>
<td>$E_i$</td>
<td>Initial tangent Young's modulus of soil</td>
</tr>
<tr>
<td>$E_m$</td>
<td>Elastic Young's modulus of mortar coating</td>
</tr>
<tr>
<td>$E_o$</td>
<td>Elastic Young's modulus at the datum elevation</td>
</tr>
<tr>
<td>$E_w$</td>
<td>Elastic Young's modulus of prestressing wire</td>
</tr>
<tr>
<td>$E_s$</td>
<td>Elastic Young's modulus of soil</td>
</tr>
<tr>
<td>$E_{sp}$</td>
<td>Elastic Young's modulus of soil in the plane strain condition</td>
</tr>
<tr>
<td>$E_t$</td>
<td>Tangent modulus of soil</td>
</tr>
<tr>
<td>$E_v$</td>
<td>Elastic Young's modulus of soil in the vertical direction</td>
</tr>
<tr>
<td>$E_u$</td>
<td>Elastic Young's modulus of soil for undrained condition</td>
</tr>
<tr>
<td>$G$</td>
<td>Shear modulus at an elevation $y$</td>
</tr>
<tr>
<td>$G_{hv}$</td>
<td>Shear modulus in $v$-$h$ plane</td>
</tr>
<tr>
<td>$H$</td>
<td>Height of fill above top of pipe</td>
</tr>
</tbody>
</table>
\[ h_c \quad : \quad \text{Critical height from ground level} \]
\[ H_e \quad : \quad \text{Height of equal settlement} \]
\[ I_y \quad : \quad \text{Second moment of inertia of the pipe wall section about neutral axis (per unit length)} \]
\[ I_c' \quad : \quad \text{Second moment of inertia for concrete core to the centroid} \]
\[ I_m' \quad : \quad \text{Second moment of inertia for mortar coating to the centroid} \]
\[ I_s' \quad : \quad \text{Second moment of inertia for prestressing wires to the centroid, assuming that} \ I_s \approx 0 \]
\[ K \quad : \quad \text{Rankine's coefficient of earth pressure} \]
\[ K \quad : \quad \text{Modulus number} \]
\[ K_a \quad : \quad \text{Coefficient of active earth pressure} \]
\[ K_o \quad : \quad \text{Coefficient of earth pressure at rest} \]
\[ K_p \quad : \quad \text{Coefficient of passive earth pressure} \]
\[ K_r \quad : \quad \text{Coefficient of residual earth pressure at rest} \]
\[ M \quad : \quad \text{Bending moment in the pipe wall} \]
\[ M_b \quad : \quad \text{Bending moment at the invert of the pipe} \]
\[ M_{teb} \quad : \quad \text{Bending moment produced by the three-edge bearing test} \]
\[ N \quad : \quad \text{Parameter related to the distribution of the vertical reaction} \]
\[ N_i \quad : \quad \text{Shape function} \]
\[ P_a \quad : \quad \text{Atmospheric pressure used to nondimensionalise the soil parameters for hyperbolic model} \]
\[ P_{teb} \quad : \quad \text{Three-edge bearing load} \]
\[ R_b \quad : \quad \text{Thrust at the bottom of the pipe} \]
\[ R_f \quad : \quad \text{Failure ratio,} \quad (\sigma_1 - \sigma_3)_f/(\sigma_1 - \sigma_3)_u \]
\[ R_m \quad : \quad \text{Mean radius of the pipe} \]
\[ W_c \quad : \quad \text{Earth load for the complete trench or projection condition} \quad (H_e > H) \]
\( W_0 \): 90% of the three-edge bearing load \( W_{\text{pol}} \), producing incipient cracking in the core, with no internal pressure.

\( W_{\text{pol}} \): Three-edge bearing test load that produces an incipient core or coating crack.

\( W_p \): Pipe weight per unit length.

\( Y \): Relative stiffness, ratio of the plane strain elastic modulus of the soil to the flexural stiffness of a circular pipe.

\( c \): Cohesion.

\( c' \): Cohesion for drained condition.

\( c_0 \): Cohesion at the datum elevation.

\( c_u \): Cohesion for undrained condition.

\( d_c \): Decrease of vertical height of the pipe.

\( d_a \): Diameter of prestressing wire.

\( f_c \): Bending stress of the pipe.

\( f_c'_{(28)} \): Design strength of the concrete cylinders at 28 days.

\( f_{ct} \): Resultant induced compressive stress in concrete.

\( f_{cu} \): Characteristic cube strength of concrete.

\( f_{cyl} \): Characteristic cylinder strength of concrete at 28 days (for ACI 318).

\( f_i \): Bending stress at the inner and outer fibres.

\( f_{teh} \): Ultimate fibre stress produced by the three-edge bearing test.

\( k_n \): Normal stiffness of slip element (kN/m^2).

\( k_s \): Shear stiffness of slip element (kN/m^2).

\( k_{\text{res}} \): Residual shear stiffness of slip element (kN/m^2).

\( n_r \): Resultant modula ratio of prestressing wires.

\( m \): Projection ratio (for a circular pipe, \( m = p \)).

\( m \): Rate of increase of Young’s modulus with depth.

\( m_{C} \): Rate of increase of cohesion with depth.

\( m_{E} \): Rate of increase of Young’s modulus with depth.
\( n \) : Modulus exponent
\( p \) : Projection ratio of the vertical distance from the top of the pipe to the surface of the natural ground to the outside vertical height of the pipe
\( pB_c \) : Vertical distance from the ground to the top of the pipe
\( q \) : Ratio of the total lateral pressure to the total vertical load on the top of the pipe
\( s_1 \) : Stress corresponding to a axial strain at 50/1,000,000
\( s_2 \) : Stress corresponding to 40% of ultimate load
\( s_f \) : Displacement of the bottom of the pipe into its foundation
\( s_g \) : Settlement of the ground surface in the exterior prism
\( s_m \) : Settlement of the soil in the projection height \( pB_c \)
\( t \) : Thickness of slip element (m)
\( t_c \) : Concrete core thickness
\( t_m \) : Mortar coating thickness
\( t_w \) : Pipe wall thickness
\( x \) : Parameter related to the area of the vertical projection of the pipe on which active lateral pressure of the fill material acts
\( x_c \) : Global coordinate at the centre of the element
\( y_c \) : Global coordinate at the centre of the element
\( y_i \) : Distance between the neutral axis and the extremities
\( y_o \) : Datum elevation at which \( E = E_o \)
\( z_c \) : Critical depth from the ground level
\( \beta \) : Reciprocal of Young's modulus of the soil mass in plain strain
\( \gamma \) : Unit weight of fill
\( \gamma_m \) : Partial safety factor for materials
\( \gamma_{sd} \) : Settlement ratio, a rational quantity in the development of the load formula
\( \delta_{ha} \) : Diameter change in horizontal direction (through the pipe springline at the ultimate state)

\( \delta_{vu} \) : Diameter change in vertical direction (between the pipe crown and invert at the ultimate state)

\( \varepsilon \) : Axial strain

\( \varepsilon_2 \) : Axial strain produced by stress \( s_2 \)

\( \varepsilon_y \) : Strain in longitudinal direction

\( \eta \) : Non-dimensional (auxiliary) coordinate, \( (y - y_c)/b \)

\( \mu \) : Coefficient of internal friction of fill

\( \mu_s \) : Coefficient of friction between fill and sides of trench

\( \nu \) : Poisson's ratio

\( \nu' \) : Poisson's ratio for drained condition

\( \nu_c \) : Poisson's ratio of concrete

\( \nu_{cp} \) : Poisson's ratio of concrete in the plane strain condition

\( \nu_{bh} \) : Poisson's ratio linking both horizontal directions

\( \nu_{vb} \) : Poisson's ratio linking vertical and horizontal directions

\( \nu_s \) : Poisson's ratio of soil

\( \nu_{sp} \) : Poisson's ratio of soil in the plane strain condition

\( \xi \) : Non-dimensional (auxiliary) coordinate, \( (x - x_c)/a \)

\( \rho \) : Constant, \( \nu (1 + \nu)/E_s \)

\( \sigma \) : Compressive stress

\( \sigma_h \) : Horizontal stress

\( \sigma_h' \) : Horizontal effective stress

\( \sigma_{n\theta} \) : Normal stress on plane at an angle \( \theta \) to the plane on which \( \sigma_x \) acts

\( \sigma_u \) : Uniform surcharge pressure

\( \sigma_v \) : Vertical stress

\( \sigma_v' \) : Vertical direct effective stress

\( \sigma_x \) : Normal stress on \( yz \) plane acting in \( x \) direction

\( \sigma_{xd} \) : Horizontal stress due to the displacement of base
\( \sigma_{xy} \) : Horizontal stress due to the weight of fill
\( \sigma_y \) : Normal stress on \( xy \) plane acting in \( y \) direction
\( \sigma_{yd} \) : Vertical stress due to displacement of base
\( \sigma_{y'\gamma} \) : Vertical stress due to weight of fill
\( \tau \) : Shear resistance
\( \tau_{\text{ult}} \) : Ultimate shear strength of slip element
\( \tau_{xy} \) : Shear stress on \( yz \) plane acting in \( y \) direction
\( \tau_{xy(b)} \) : Shear stress on yielding base
\( \tau_{xyd(b)} \) : Shear stress on base due to displacement of yielding base
\( \tau_{xy\gamma(b)} \) : Shear stress on base due to weight of fill
\( \tau_{yx} \) : Shear stress on \( xz \) plane acting in \( x \) direction
\( \tau_{\theta} \) : Shear stress on plane at an angle \( \theta \) to the plane on which \( \sigma_y \) acts
\( \phi \) : Angle of shearing resistance
\( \phi' \) : Angle of shearing resistance for drained condition, or effective stress angle of friction (or shearing resistance)
\( \phi'_{\text{cs}} \) : Critical state angle of shearing resistance
\( \phi_o \) : Value of \( \phi \) for \( \sigma_3 = P_o \)
\( \phi_s \) : Angle of shearing resistance between fill and sides of trench
\( \phi_u \) : Angle of shearing resistance for undrained condition
\( \phi_\mu \) : True angle of friction between particles
\( \phi \) : Angle of dilation

\( \Delta P_{\text{c1}} \) : Out-of-balance load for a set of equivalent nodal load
\( \Delta P_{\text{cn}} \) : Imbalanced load arising from this stress correction
\( \Delta\phi \) : Reduction in \( \phi \) for a ten-fold increase in \( \sigma_3 \)
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APPENDIX A1 - Undeformed/deformed mesh without interface element (Compaction effect)
APPENDIX A2 - Undeformed/deformed mesh with interface element (Compaction effect)
APPENDIX B1 - Typical output from CRISP program
(Three-edge bearing test)
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<th>Label</th>
<th>Value</th>
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<td>J</td>
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<tr>
<td>H</td>
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<tr>
<td>G</td>
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<tr>
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<tr>
<td>E</td>
<td>0.086E+04</td>
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<tr>
<td>D</td>
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<tr>
<td>C</td>
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<tr>
<td>B</td>
<td>0.030E+04</td>
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<tr>
<td>A</td>
<td>0.117E+04</td>
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</table>

**CRISP ANALYSIS**

**EFF VERT STRESS (SIG-YY)**

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<th>Increment</th>
<th>MIN : -690.419</th>
<th>MAX : 27189.100</th>
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<tbody>
<tr>
<td>Total Time</td>
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<td></td>
</tr>
</tbody>
</table>

**PLANE STRAIN**

<table>
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<th>XMIN : -1.975</th>
<th>XMAX : 5.855</th>
<th>YMIN : 1.006</th>
<th>YMAX : 6.239</th>
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</thead>
</table>

**CRISP-93**

**CUED**

**TRL**

**17/09/99**

**CAMBRIDGE UNIVERSITY**

Appendix B1-5
CRISP ANALYSIS

SHEAR STRESS (TYX)

INCREMENT 450 MIN : -9597.300 MAX : 9996.820 NO. OF CONT : 16
TOTAL TIME 0.000

XMIN : -1.975 XMAX : 5.855 YMIN : 1.006 YMAX : 6.239

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Appendix B1-6
APPENDIX B2 - Interpretation of output from CRISP program (Three-edge bearing test for prestressed pipe)
Appendix B2-1 Bending stress distributions for prestressed pipe due to Pipe weight and three-edge bearing load (TEB21), ID = 4.0m, Ec = 44GPa, Compression = 2000kPa, TEB load = 509.6kN/m
Appendix B2-2 Bending moment distributions for prestressed pipe due to Pipe weight and three-edge bearing load (TEB21), ID = 4.0m, Ec = 44GPa, Compression = 2000kPa, TEB load = 509.6kN/m
Appendix B2-3 Bending stress distributions for prestressed pipe due to three-edge bearing load only (TEB22)

ID = 4.0m, $E_c = 44\text{GPa}$, Compression = 2000kPa, TEB load = 509.6kN/m
Appendix B2-4 Bending moment distributions for prestressed pipe due to three-edge bearing load only (TEB22)
ID = 4.0m, Ec = 44GPa, Compression = 2000kPa,
TEB load = 509.6kN/m
APPENDIX B3 - Interpretation of output from CRISP program (Three-edge bearing test for non-prestressed pipe)
Appendix B3-1 Bending stress distributions for non-prestressed pipe due to pipe weight and three-edge bearing load (TEB31)
ID = 4.0m, Ec = 44GPa, TEB load = 509.6kN/m
Appendix B3-2 Bending moment distributions for non-prestressed pipe due to pipe weight and three-edge bearing load (TEB31)
ID = 4.0m, Ec = 44GPa, TEB load = 509.6kN/m
Appendix B3-3 Bending stress distributions for non-prestressed pipe due to three-edge bearing load only (TEB32)
ID = 4.0m, Ec = 44GPa, TEB load = 509.6kN/m
Appendix B3-4 Bending moment distributions for non-prestressed pipe due to three-edge bearing load only (TEB32)
ID = 4.0m, Ec = 44GPa, TEB load = 509.6kN/m