Gamma-Ray Spectroscopy and Shell Model Description of High Spin States in $\beta$-Stable $^{91,92}$Zr

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Submitted for the degree of Doctor of Philosophy

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March 2008
Abstract

Identification of near-yrast states in the stable $^{91,92}$Zr isotopes has been carried out using the fusion evaporation reactions $^{82}$Se($^{13}$C,xn)$^{95-97}$Zr at an incident beam energy of 50 MeV from the Yale ESTU tandem accelerator. Gamma-ray spectroscopy of more than fifty previously unreported states up to $\sim 20\hbar$ and to excitation energies of more than 13 MeV are identified for the first time above the previously observed $\tau = 6\ \mu$s, $I^\pi = \frac{31}{2}^+$ isomer in $^{91}$Zr. Angular Correlation and Linear Polarisation analyses were performed on these transitions. Ten new transitions were also observed in $^{92}$Zr. A modified effective interaction has been produced for the shell model code OXBASH which embeds the empirical matrix elements of the Ji-Wildenthal $n50j$ interaction into a new interaction developed by Hjorth-Jensen for N=50 nuclei. This interaction together with newly derived single particle energies allows calculations in the basis $\pi[1f_{5/2}, 2p_{3/2}, 2p_{1/2}, (1g_{9/2})], \nu[2p_{1/2}, 1g_{9/2}, 2d_{5/2}, 3s_{1/2}, 1g_{7/2}, 2d_{3/2}, 1h_{11/2}]$ limited only by the matrix sizes possible with the code. Results from the calculations performed aided in the assignment of spin and parity to the lower lying yrast levels above the $I^\pi = \frac{21}{2}^+$ isomer which were shown to result from the alignment of a single neutron in the $\nu h_{11/2}$ orbit coupled to two maximally aligned protons in $g_{9/2}$. 
It is not what you look at that matters, it is what you see.

Henry David Thoreau
Acknowledgements

Throughout the last three years there have been many unexpected highs. Unexpected because they arose out of unheard of new activities, such as the joy of finding my first new gamma rays, or because they came from unexpected experiences such as learning to sail around the beautiful blue Connecticut coastline on a yacht. There have also been many simpler but as wonderful times such as epic yet thoroughly enjoyable sessions working on the Yrastball array with colleagues in the Wright Nuclear Structure Laboratory at Yale or just being on the beach with Dr. Paddy Regan and his family. However, all of the activities I have undertaken during this PhD have relied on the wisdom, knowledge and ability of other people to realise them.

Firstly, a huge thankyou to Paddy, whose unrelenting confidence that things can be made to happen, combined with a work capacity beyond my comprehension enabled this PhD. The vivacity with which Paddy attacks all things in life and the humanity with which it is accomplished will remain with me as an example always.

A big thank you to my sponsors, Nexia Solutions (a subsidiary of BNFL) who made these endeavours financially possible with funding from the Nuclear Decommissioning Authority and to Dr. Robert Mills, who was responsible for many arrangements behind the scenes and for providing varied and interesting project work at Sellafield.

I have spent many days in laboratories in America, Italy and Germany, to all the people that made me welcome and took the time to teach me things they have probably taught to new visitors over and over, I extend my thanks.

I would like to thank Prof.D.r R. F. Casten for allowing me to spend a year at Yale and providing some of the paradoxically most complicated yet comprehensible lectures I have had the pleasure to attend. The staff and students there have my gratitude for involving me in their lives and work. The lending of cars, late night coffee and good times is hugely appreciated. Thanks to Ryan ‘Sprinkler’ Winkler for all the sports, introduction to local colour and getting bitten by a bat - I will return to climb the Whitney-Gilman ridge in its entirety.

My peers and colleagues at Surrey University deserve my thanks and respect. I am privileged to know such interesting, funny and kind people. There are far too many to mention here, but a few standout individuals cannot go unmentioned; Peter
'Stinky' Shaw for the most bizarre times and making me understand where our national insurance contribution goes, Dr. Adam Garnsworthy for keeping me sane on trips abroad and providing many an amusing story involving paving slabs, Steven Steer for outrageously shameless dancing, Dr. Gareth Jones for humour and largesse combined, Simon Pitts for unassuming wit, Dr 'Dangerous' Dave Garrity for explosive knees and Dr. Scott Williams for Linux genius plus rather insensible eating contests which took some of us months to recover from, Sam Thompson for the ‘Sammy fact’ and humour arising from the consumption of Haribo, all of the people from the University of Surrey Mountaineering Club and No Wave who were the source of so much easy cheer and engaging diversion.

Any undertaking like this requires a huge amount of support. I could have had none better than my parents who were infinitely patient, encouraging and generous, just as they have been for all my 27 years. They have my unending thanks.
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Chapter 1

Introduction

1.1 Introduction to nuclear structure and $^{91}\text{Zr}$

Philosophical ruminations upon the nature of matter date back to the ancient Greeks and Indians of the fifth and sixth centuries BCE. It was Democritus who perhaps during such ruminations coined the name atom which comes from the Greek $\dot{a}r\omicron\omicron\omicron\sigma$ or $\dot{a}tomos$ meaning indivisible [1]. The idea was all but lost until the Renaissance period. During the 19\textsuperscript{th} century John Dalton an English Chemist and Physicist was interested in why substances could be broken down into their constituents. He put forward the idea of there being differing elements defined by weight. It was near the 20\textsuperscript{th} century when finally J.J. Thomson through his investigations on cathode rays derived that the atom was composed partly of light negatively charged particles. He had discovered the electron cloud surrounding the nucleus [2]. Soon after in 1911, Ernest Rutherford, through experimentation involving the scattering of $\alpha$ particles on gold foil [3] realised that the atom consisted of an extremely small positively charged nucleus. It was not until 1932 however that James Chadwick was able to identify other previously seen radiation as being due to the neutron. With some further theoretical explanation of the nuclear force and the neutrino by Yukawa [4] in the same year, the modern model of the nucleus was born.

Nuclear structure as we understand it today, particularly the shell model (the idea of which was borrowed from the successful electron shell model), has as one of its major precepts the idea of groups of nucleons occupying successive ‘orbits’ within
1.1 INTRODUCTION TO NUCLEAR STRUCTURE AND $^{91}$Zr

Figure 1.1: Location of singly and doubly magic nuclei on the Segre chart. (from ref [6]). Nuclei shown as black squares are stable. Red are proton rich nuclei unstable to $\beta^+$ decay. Blue are neutron rich nuclei unstable to $\beta^-$ decay. Nuclei highlighted in yellow are unstable to $\alpha$ decay. The Nuclei in green are unstable to spontaneous fission. Horizontal and vertical lines marking out magic nuclei at $Z,N = 2,8,20,28,50,82,128$ and also predicted $Z=114$ and $N=162,184$.

The nucleus. It also includes the concept of ‘magic’ proton and neutron numbers which correspond to nuclei which have large proton or neutron ($\pi$ or $\nu$) separation energies when compared to a nucleus of $Z + 1$ or $N + 1$ respectively [5]. These magic numbers correspond to when an orbit has the maximum number of nucleons it can contain and the next orbit above is significantly less bound. There are a few doubly magic nuclei (nuclei with both magic $N$ and $Z$) which sit on the line of stability on the Segre chart; $^{16}$O, $^{40}$Ca, $^{48}$Ca, $^{82}$Pb, and $^{208}$Pb. The lines of magicity are shown in the Segre chart of figure 1.1. Line intersections represent the doubly magic nuclei [6].

The lines shown are the major closed shells, but there are also some minor closed shells, known as sub-shell closures. These occur within a major shell but are where there is still a significant energy gap between orbits. One of these is the gap at $N,Z=40$ which results from the splitting of the $1g_{9/2}$ and $2p_{1/2}$ orbits.

Many attempts were made to reproduce theoretically the experimentally observed
1.1 INTRODUCTION TO NUCLEAR STRUCTURE AND $^{91}$Zr

nuclear shell gaps. After several attempts in the 1930s and 1940s, M. Goeppert Mayer [7] as well as Haxel, Jensen and Suess [8] recognized that the inclusion of a strong spin-orbit coupling interaction in the nuclear mean field was necessary to reproduce the shell ordering and magic numbers which appear at the shell gaps. Using a spin-orbit force which binds the $j = l + \frac{1}{2}$ orbital stronger than the $j = l - \frac{1}{2}$ orbital, almost all the then known nuclear ground-state spins and magnetic moments of spherical nuclei could be explained within what became known as the Independent Single-Particle Model. This model assumes that the intrinsic properties such as spins and magnetic moments of odd-$A$ nuclei are determined by the orbit of the final, unpaired nucleon in the system. Figure 1.2 shows the level ordering created by differing potentials, these are explained further in chapter 3.

This leads to the statement that in nuclei near closed shells, single-particle structure tends to dominate over rotational and vibrational collective effects at low excitation energy [10, 11].

The nucleus $^{91}_{40}$Zr$_{51}$ has a sub-shell closure of 40 protons and is one neutron out from a closed shell at $N = 50$. An understanding of the structure of $^{91}$Zr should follow logically from the understanding of $^{90}$Zr. The magic nucleus, $^{90}$Zr belongs to a very interesting region of the nuclear chart encompassing the Zr isotopes between mass numbers 90 and 100. The lightest stable isotope $^{90}$Zr lies at the $N=50$ shell closure whilst the heaviest stable isotope $^{96}$Zr lies close to the known region of deformation at $A=100$ with $^{100}_{40}$Zr$_{60}$ being reported as strongly deformed [12]. The Zr isotopes with $A=90-100$ therefore span from nearly spherical to strongly deformed with a clear transition from spherical to deformed ground state shapes occurring at $N=60$. Shape coexistence, whereby two shape energy minima are present is also reported around $^{100}$Zr [13]. A similar transition from spherical to deformed is visible in neighboring Strontium (Sr, $Z=38$) isotopes at $N=60$ [14].

Nuclei in the region $A=90$ display some of the least collective motion of the entire Segre chart [10, 11]. That is, many of the lower lying states are formed predominantly by the excitation of single nucleons (admixtures of different configurations are usually still present but to a small degree). This lack of collectivity has been shown to be a function of the competing shell gaps between the single particle levels [15]. As such $^{91}$Zr
Figure 1.2: The nuclear shell model with differing terms in the potential. See Chapter 3 for more detail. Modified from [9].
resides in the least collective, spherical region of the nuclear chart of greater mass than $^{56}\text{Ni}$ except for the spherical Pb isotopes [16]. To demonstrate this, figure 1.3 shows the $2^+_1$ energies and reduced matrix elements for the Zr isotopes. The $2^+_1$ energies vary with neutron number in a fashion attributable to the filling of first the $\nu g_{9/2}$ (complete at $^{90}\text{Zr}_{50}$) and then the $\nu d_{5/2}$ (complete at $^{96}\text{Zr}_{56}$). A detailed knowledge of the medium and high-spin states in these nuclei is necessary as it sheds light on the understanding of the transition between spherical and deformed nuclear shapes at N=60 as the higher lying orbitals are filled.

The spherical sub-shell closures at Z=38 and Z=40 [20, 21] allow for simple shell model descriptions for Zr isotopes to be constructed, particularly at low spin. The core nucleons within the shell closures are paired in time-reversed orbits [22] which effectively render them inert to the valence nucleons outside. The effect of being one nucleon outside a closed core is that most low-lying excitations are due to the excitation of that single nucleon as this is energetically favourable to breaking a pair of lower-lying
1.2 Previous studies of $^{91}\text{Zr}$

The nucleus $^{91}\text{Zr}$ is a well-studied system at low spins. The experimental studies which are currently evaluated and considered in the ENSDF database [17] are listed in Table 1.1 together with the highest lying state observed for that study and its spin/parity ($E_{\text{max}}, \Gamma_{\text{max}}$) or the largest value of $l$ measured ($l_{\text{max}}$).
1.2 PREVIOUS STUDIES OF $^{91}\text{Zr}$

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<th>$I_{\text{max}}^\gamma$</th>
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<td>[24, 25]</td>
<td>3085.1</td>
<td>$\frac{13}{2}$</td>
<td>-</td>
</tr>
<tr>
<td>$^{88}\text{Sr}(^6\text{Li}, p2n\gamma)$</td>
<td>[20]</td>
<td>3167</td>
<td>$\frac{21}{2}$</td>
<td>-</td>
</tr>
<tr>
<td>$^{90}\text{Zr}(n, \gamma)$ $E=\text{thermal}$</td>
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<td>7192.4</td>
<td>$\frac{7}{2}$</td>
<td>-</td>
</tr>
<tr>
<td>$^{91}\text{Zr}(\gamma, \gamma)$</td>
<td>[27]</td>
<td>4704</td>
<td>$\frac{9}{2}$</td>
<td>-</td>
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<td>$^{91}\text{Zr}(n, n'\gamma)$</td>
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<td>3331</td>
<td>$\frac{11}{2}$</td>
<td>-</td>
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<tr>
<td>$^{90}\text{Zr}(\alpha, ^3\text{He})$</td>
<td>[29]</td>
<td>4254</td>
<td>$\frac{11}{2}$</td>
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<td>5</td>
<td>-</td>
</tr>
<tr>
<td>$^{90}\text{Zr}(\text{pol d}, p)$</td>
<td>[31]</td>
<td>3908</td>
<td>$\frac{11}{2}$</td>
<td>-</td>
</tr>
<tr>
<td>$^{91}\text{Zr}(\alpha, \alpha')$</td>
<td>[32]</td>
<td>5600</td>
<td>5</td>
<td>-</td>
</tr>
<tr>
<td>$^{91}\text{Zr}(d, d')$</td>
<td>[33]</td>
<td>4790</td>
<td>5</td>
<td>-</td>
</tr>
<tr>
<td>$^{91}\text{Zr}(p, p')$</td>
<td>[30]</td>
<td>4459</td>
<td>5</td>
<td>-</td>
</tr>
<tr>
<td>$^{92}\text{Zr}(d, t)$</td>
<td>[34]</td>
<td>4005</td>
<td>$\frac{7}{2}$</td>
<td>-</td>
</tr>
<tr>
<td>$^{92}\text{Zr}(p, d)$</td>
<td>[30]</td>
<td>4820</td>
<td>4.5</td>
<td>-</td>
</tr>
<tr>
<td>$^{93}\text{Nb}(d, \alpha)$</td>
<td>[35]</td>
<td>5682</td>
<td>3</td>
<td>-</td>
</tr>
<tr>
<td>Coulomb Excitation</td>
<td>[36]</td>
<td>1210</td>
<td>$\frac{5}{2}$</td>
<td>-</td>
</tr>
<tr>
<td>$^{91}\text{Zr}(n, n')$</td>
<td>[37]</td>
<td>2170</td>
<td>$\frac{11}{2}$</td>
<td>-</td>
</tr>
<tr>
<td>$^{90}\text{Zr}(^{12}\text{C}, ^{11}\text{C})$</td>
<td>[38, 39]</td>
<td>3466</td>
<td>$\frac{7}{2}$</td>
<td>-</td>
</tr>
<tr>
<td>$^{90}\text{Zr}(^{13}\text{C}, ^{12}\text{C})$</td>
<td>[40]</td>
<td>1205</td>
<td>$\frac{5}{2}$</td>
<td>-</td>
</tr>
<tr>
<td>$^{90}\text{Zr}(^{13}\text{C}, ^{12}\text{C}\gamma)$</td>
<td>[41]</td>
<td>2558.2</td>
<td>$\frac{11}{2}$</td>
<td>-</td>
</tr>
<tr>
<td>$^{90}\text{Zr}(^{18}\text{O}, ^{19}\text{O})$</td>
<td>[42]</td>
<td>5000</td>
<td>$\frac{11}{2}$</td>
<td>-</td>
</tr>
<tr>
<td>$^{90}\text{Zr}(^7\text{Li}, ^6\text{Li})$</td>
<td>[43]</td>
<td>2030</td>
<td>$\frac{3}{2}$</td>
<td>-</td>
</tr>
<tr>
<td>$^{91}\text{Nb} \epsilon \text{Decay (680 y)}$</td>
<td>[44, 45]</td>
<td>0</td>
<td>$\frac{5}{2}$</td>
<td>-</td>
</tr>
<tr>
<td>$^{91}\text{Nb} \epsilon \text{Decay (60.86 d)}$</td>
<td>[44, 46, 47]</td>
<td>1204</td>
<td>$\frac{5}{2}$</td>
<td>-</td>
</tr>
<tr>
<td>$^{91}\text{Y} \beta^- \text{Decay}$</td>
<td>[48, 49, 50]</td>
<td>1204</td>
<td>$\frac{5}{2}$</td>
<td>-</td>
</tr>
<tr>
<td>$^{91}\text{Zr}(^{18}\text{O}, ^{19}\text{O}')$</td>
<td>[51]</td>
<td>1205</td>
<td>$\frac{1}{2}$</td>
<td>-</td>
</tr>
<tr>
<td>$^{91}\text{Zr}(^3\text{He}, ^3\text{He}')$, (pol $^3\text{He}, ^3\text{He}$)</td>
<td>[52, 53]</td>
<td>2770</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>$^{92}\text{Zr}(\text{pol p}, d)$</td>
<td>[54]</td>
<td>2040</td>
<td>$\frac{7}{2}$</td>
<td>-</td>
</tr>
<tr>
<td>$^{92}\text{Zr}(^3\text{He}, \alpha)$</td>
<td>[55]</td>
<td>2940</td>
<td>$\frac{9}{2}$</td>
<td>-</td>
</tr>
<tr>
<td>$^{93}\text{Nb}(\mu^-, 2\nu\gamma)$</td>
<td>[56]</td>
<td>1463.6</td>
<td>$\frac{5}{2}$</td>
<td>-</td>
</tr>
<tr>
<td>$^{88}\text{Sr}(^{12}\text{C}, ^9\text{Be})$</td>
<td>[57]</td>
<td>2000</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{90}\text{Zr}(^{20}\text{Ne}, ^{10}\text{Ne})$</td>
<td>[58]</td>
<td>2500</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

Table 1.1: Previous studies of $^{91}\text{Zr}$. 
1.2 PREVIOUS STUDIES OF $^{91}$Zr

Figure 1.5: Previously known level scheme for $^{91}$Zr as reported in ref [20]. Only decays from the yrast $\frac{21}{2}^+$ isomeric state at 3167 keV are shown. The excitation energies of non-yrast levels identified by other studies listed in table 1.1 are indicated to the left of the figure.

The study by Brown et al [20] is that which reported the highest in spins in $^{91}$Zr prior to the current work, with an isomeric state ($t_\frac{1}{2} = 6.3(2)\mu s$) of spin/parity $I^m = \frac{21}{2}^+$ at $E_{ex} = 3.167$ MeV. The single-particle configuration of this isomer is mainly $\langle \nu d_\frac{3}{2} \otimes (\pi g_{\frac{9}{2}})^2 \rangle$ as determined through a g-factor measurement [59]. This isomer is formed by the decay of the two transitions which de-excite the state being an 880 keV E3 (branching ratio (br) = 3.8%) or a highly internally converted 21 keV E2 transition (br = 96.2%).

No transitions identified as yrast and feeding the $\frac{21}{2}^+$ isomer in $^{91}$Zr had been identified prior to this study. This lack of knowledge has hindered the expansion of theory and in particular, restricted basis shell model calculations in this region [60]. It is the aim of the current work to make spectroscopic measurements at spins and excitation energies above the $\frac{21}{2}^+$ isomer in $^{91}$Zr and compare the results with new shell model calculations which include the breaking of the N=50 shell closure and/or the inclusion of the negative parity high-$j$ $h_{11/2}$ neutron intruder orbital. A simple restricted-basis single particle valence space can be considered to form near-yrast states
1.2 PREVIOUS STUDIES OF \(^{91}\text{Zr}\)

The valence space required to reach the yrast \(^{21}_{2}^{+}\) state in \(^{91}\text{Zr}\) is 3 nucleons outside a \(^{88}\text{Sr}\) closed core. The single particle energies relative to \(^{88}\text{Sr}\) are in MeV and were calculated from binding energy differences (see chapter 7).

Figure 1.6: The valence space required to reach the yrast \(^{21}_{2}^{+}\) state in \(^{91}\text{Zr}\) is 3 nucleons outside a \(^{88}\text{Sr}\) closed core. The single particle energies relative to \(^{88}\text{Sr}\) are in MeV and were calculated from binding energy differences (see chapter 7).

The expansion of this space will be a requirement in order to generate higher angular momentum states within the shell model basis.

A basic tenet of the independent particle motion in the shell model and in particular shell model calculations is that lower-lying and fully-filled shells have no influence over the relative spacings of single particle levels within valence shells. One of the consequences of this is the ability to use effectively inert, ‘closed cores’ in calculations which means that only valence particles are considered in the computation (e.g. a \(^{88}\text{Sr}\) inert core in earlier \(^{90},^{91}\text{Zr}\) calculations). The filled shells do however impact the valence single particle energies for a nucleus. A major shell usually consists of several sub-shells defined by the angular momentum quantum number \(j\). Each \(j\) shell from within a closed shell can have a differing spherically symmetric effect on the valence \(j\) orbits. For this reason, single particle energies of any given major shell depend specifically upon the filled and lower lying shells [9].

The relative shifts in energy of single particle levels in the \(N=51\) isotones are plotted in figure 1.7. As the movement towards midshell in the \(Z=40-50\) shell occurs the first excited state migrates lower and at \(^{97}\text{Pd}\) swaps with the \(g_{7/2}\) sub-shell which is undergoing a large monopole migration down in energy driven by the filling of its proton spin orbit partner \(l_{9/2}\) orbit. Of note is that the size of the single particle energy gaps can also be related to the stability of the isotones [61].

The \(N=50\) shell closure represents a magic number resulting microscopically from
1.2 PREVIOUS STUDIES OF $^{91}$Zr

Figure 1.7: Single particle energies for the N=51 isotones. Coloured lines link levels of similar single-neutron character across the isotones. Data are taken from the ENSDF database [17].

the spin-orbit splitting of the $1g_{9/2}$ and $1g_{7/2}$ orbits [7]. The energy gap between the $\nu 1g_{9/2}$ and $\nu 2d_{5/2}$ orbits as found experimentally is \( \sim 4.5 \, \text{MeV} \) (see figure 7.7). The single-particle energies for neutron-states in N=51 $^{91}$Zr suggest that the $3s_{1/2}$ single-particle level lies 1.2 MeV above the predominantly $d_{5/2}$ ground-state configuration [62]. This shell gap should enable a truncation of the neutron orbits involved in a simple shell model calculation, limiting the calculation to a single valence neutron.

This thesis aims therefore to extend current knowledge of medium to high spin states in $^{91,92}$Zr by presenting new data on these nuclei from the experiments considered within. It also aims to place some theoretical consideration on the underlying nuclear structure using the nuclear shell model.

States of higher spin than $^{21+}$ and positive parity may result from an increase of seniority (which describes the number of particles not paired to 0 angular momentum) to 5 unpaired particles and thus breaking the simple $^{88}$Sr core (for example by promoting 4 protons into the $g_{9/2}$ orbital to form a $[(\pi g_{9/2})^412^+ (\nu d_{5/2})_2^+] = \frac{29}{2}^+$ configuration. Another explanation may involve breaking of the neutron N=50 core (as discussed in ref. [63]). In order to investigate high-spin states theoretically various truncated model
spaces can be used. Table 1.2 shows the maximum spin/parity configurations $I_{max}$ in $^{91}$Zr assuming different numbers of valence particles and model spaces.
1.2 PREVIOUS STUDIES OF $^{91}$Zr

<table>
<thead>
<tr>
<th>Valence space</th>
<th># of unpaired particles</th>
<th>$I_{max}^+$</th>
<th>$I_{max}^{-ve}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu 2d_{5/2}$</td>
<td>1</td>
<td>$\frac{5}{2}$</td>
<td>$-$</td>
</tr>
<tr>
<td>$\nu 2d_{5/2} \otimes \pi (1g_{9/2})^2$</td>
<td>3</td>
<td>$\frac{21}{2}$</td>
<td>$-$</td>
</tr>
<tr>
<td>$\nu 2d_{5/2}, 1g_{7/2}, 3s_{1/2}, 2d_{3/2}, 1h_{11/2} \otimes \pi (1g_{9/2})^2$</td>
<td>3</td>
<td>$\frac{23}{2}$</td>
<td>$\frac{27}{2}$</td>
</tr>
<tr>
<td>$\nu 2d_{5/2}, 1g_{7/2}, 3s_{1/2}, 2d_{3/2}, 1h_{11/2} \otimes \pi (1g_{9/2})^3 \otimes 2p_{3/2}$</td>
<td>5</td>
<td>$\frac{35}{2}$</td>
<td>$\frac{31}{2}$</td>
</tr>
<tr>
<td>$\nu 2d_{5/2}, 1g_{7/2}, 3s_{1/2}, 2d_{3/2}, 1h_{11/2} \otimes \pi (1g_{9/2})^3 \otimes 1f_{5/2}$</td>
<td>5</td>
<td>$\frac{37}{2}$</td>
<td>$\frac{33}{2}$</td>
</tr>
<tr>
<td>$\nu 1g_{9/2}^{-1} \otimes (2d_{5/2})^2 \otimes \pi 1g_{9/2} \otimes 2p_{1/2}$</td>
<td>5</td>
<td>$-$</td>
<td>$\frac{27}{2}$</td>
</tr>
<tr>
<td>$\nu 1g_{9/2}^{-1} \otimes (2d_{5/2})^2 \otimes \pi (1g_{9/2})^2$</td>
<td>5</td>
<td>$\frac{33}{2}$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

Table 1.2: Differing model spaces together with the corresponding seniority and maximum angular momentum available ($I_{max}$).

The following chapters in this thesis split neatly into groups.

Chapters two and three describe nuclear spectroscopy and nuclear theory respectively. Both requisite for the study of nuclei. Chapters four and five detail the experimental equipment used together with the experiments designed for that equipment to obtain the spectroscopic information deemed necessary. Chapters six and seven are the results obtained and the discussion of those results. Chapter eight contains an explanation of the shell model calculation performed and the results produced by that study to provide some theoretical insight to the structures responsible for the experimental information obtained.
Chapter 2

Nuclear spectroscopy

The structure of the atomic nucleus, corresponding to configurations of protons and neutrons can be investigated through the examination of γ rays emitted during the de-excitation of excited states [2]. The aim of spectroscopy is to build up a complete picture of these excited states, represented by energy levels in a level scheme. Transitions between these energy levels take on a particular character, having a multipole order and electric or magnetic character. With the information obtained directly from γ rays, the spin and parity ($I^\pi$), together with an associated excitation energy $E$ (relative to the ground state) and a lifetime $\tau$ of an excited state may be deduced.

The techniques used to measure these attributes in the current work are discussed in this chapter.

2.1 Energy conservation during nuclear transitions

The decay of a state $E_i$ to a lower lying one $E_f$ must follow the law of conservation of energy in that the energy $E_i - E_f$ must be transferred. In the case of transitions within the nucleus this happens mostly through the emission of a γ ray. This γ ray will be almost equal to $E_i - E_f$, with the small difference accounted for by the recoil of the nucleus from which the γ ray eminates. This difference however is small enough (a factor of $\sim 10^{-5}$) that it is insignificant when compared to the statistical errors inherent in the experimental energy measurement possible with current HPGe arrays [2].
Figure 2.1: Decay modes and associated movement about the Segré chart. The possible decay modes include the emission of $\alpha$ and $\beta^{+/-}$ particles, protons (p), neutrons (n) and the electron capture process ($\epsilon$). Fission is another possibility not shown here.

Nuclear excitation energy may also be carried away through the emission of a particle or collection of particles e.g. proton, neutron, $\beta^-$, $\beta^+$, $\alpha$ or even a larger nuclear fragment. Particle emission however, changes the isotopic nature of the nucleus (with the exception of internal conversion). Figure 2.1 shows decay modes together with their corresponding movement about the Segré chart.

2.2 Internal conversion

Internal conversion is whereby the excitation energy, $E_{ex}$ (being above the ionisation energy for an atomic electron) is passed directly to a bound atomic electron. The electron is emitted from the atom and the remaining electrons re-order themselves, leading to the emission of characteristic X-rays.

Internal conversion is a competing process to $\gamma$ ray emission in the decay of an excited nuclear state [64, 65]. For some states $\gamma$ ray emission may be inhibited (e.g. by high multipolarity transition or a low $\gamma$ ray energy) and the competing internal
conversion decay branch becomes significant. The internal conversion electron will have a kinetic energy given by [66]:

\[
E_{e^-} = E_{ex} - E_b
\]

where \(E_b\) is the binding energy of the electron. Conversion electron spectra may be complicated by the fact that electrons from different shells may be converted giving rise to more than one peak in the recorded spectra.

The internal conversion coefficient \(\alpha\) is defined as,

\[
\alpha = \frac{I_e}{I_\gamma}
\]

where \(I_e\) and \(I_\gamma\) are the relative electron and \(\gamma\)-ray intensities respectively.

Using the internal conversion coefficient calculator ICCK in the Kantele suite of programs, described in ref [67], figure 2.2 was produced showing total conversion (sum of \(K_a + K_{...}\) ) coefficients from 50 to 500 keV for \(Z=40\).

### 2.3 Parity & angular momentum in nuclear transitions

Conservation laws dictate what properties a \(\gamma\) ray will have. Photons themselves have an intrinsic spin of \(s=1\) which may be shown as a component \(s_\pm=\pm 1\) and can also carry a relative orbital angular momentum \(l\). A photon may therefore carry with it a total angular momentum of

\[
L = l + s.
\]

Due to having an intrinsic spin \(s = 1\), an individual photon can never carry away \(0\ h\) of spin from the nucleus. This is embodied in the triangular rule for spins of initial and final states \(I_i\) and \(I_f\)

\[
I_i = I_f + L
\]
where $|L|$ denoted $L$, can have any non-zero integer value which satisfies the relation:

$$|I_i - I_f| \leq L \leq |I_i + I_f|.$$  

Transitions in which $L$ takes on the maximum value ($L = I_i + I_f$) are referred to as stretched, whereas others are known as folded transitions.

The transition multipolarity may be determined when both knowledge of the value of $L$ and also the electric (E) or magnetic (M) character ($\sigma$) are known. For example, classically $\sigma$ represents the means by which the radiation was formed.

### 2.3.1 Magnetic dipole radiation

This is formed classically from the variation in current $j$ in a circular loop which encloses an area $A$, such that the magnetic dipole moment oscillates according to:
2.3 PARITY & ANGULAR MOMENTUM IN NUCLEAR TRANSITIONS

\[ \mu(t) = jA\cos\omega t. \]

### 2.3.2 Electric dipole radiation

This is formed by the relative motion of two charges \(+q\) and \(-q\) which are separated by a distance \(d\) which oscillates with a frequency \(\omega\), \([2]\). This classical picture translates to the quantum system of the nucleus by considering the electric and magnetic radiations to arise from the dynamics of charge and current distributions respectively resulting from movement of the nucleons.

### 2.3.3 Transition selection rules

When depopulating a state with spin \(I_i\) to a state \(I_f\), the lowest two radiation multipolarities given by equation 2.4 are \(L\) and \(L' = L + 1\). The transition is generally made up of these two multipolarities, mixed with a ratio \(\delta_i\). This mixing ratio is related simply to the relative probabilities of these multipolarities \(L\) and \(L'\) by \([22]\)

\[
\delta = \frac{\langle I_f | \sigma L' | I_i \rangle}{\langle I_f | \sigma L | I_i \rangle}
\]

which leads directly to the the relationship,

\[
\delta^2 = \frac{\text{intensity of } \sigma L'}{\text{intensity of } \sigma L}\]

Multipolarities are deduced experimentally to infer spin and parity changes between states. This process can be complicated by the fact that not all transitions are stretched and may include a degree of mixing.

A related rule to angular momentum selection is a rule regarding parity selection for a transition and determines electric or magnetic nature. Sign changes under a change in parity \((\Delta \pi)\) between initial and final states can be determined by \([68]\)

\[
\pi(EL) = (-1)^L
\]
2.4 Transition Probabilities

<table>
<thead>
<tr>
<th>Multipolarity</th>
<th>Dipole</th>
<th>Quadrupole</th>
<th>Octupole</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of Radiation</td>
<td>E1 M1</td>
<td>E2 M2</td>
<td>E3 M3</td>
</tr>
<tr>
<td>L</td>
<td>1 1</td>
<td>2 2</td>
<td>3 3</td>
</tr>
<tr>
<td>Δπ</td>
<td>Yes No</td>
<td>No Yes</td>
<td>Yes No</td>
</tr>
</tbody>
</table>

Table 2.1: Summary of EM transition rules. E = Electric, M = Magnetic.

\[
\pi(ML) = (-1)^{L+1}.
\]

These rules are summarised in table 2.3.3.

2.4 Transition probabilities

The lifetime of a nuclear state \( \tau \) is related to its intrinsic width by the Heisenberg uncertainty principle such that

\[
\Gamma \tau \geq \hbar.
\]

The decay probability from an initial state \( |\psi_i\rangle \) to a final state \( |\psi_f\rangle \) is proportional to the energy width \( \Gamma \) and depends entirely on the matrix element between \( |\psi_i\rangle \) and \( |\psi_f\rangle \) and the operator which governs the decay between them as

\[
\Gamma \alpha |\langle \psi_f | M | \psi_i \rangle|^2
\]

where \( M \) is known as the multipole operator. The transition rate \( T(\sigma L) \) is then given by [22]

\[
T_{\Gamma \alpha}(\sigma L) = \frac{8\pi (L + 1)}{\hbar L[(2L + 1)!!]^2} \left[ \frac{E_\gamma}{\hbar c} \right]^{2L+1} B(\sigma L)
\]

where \( B(\sigma L) \) is the reduced matrix element of the transition,
2.4 TRANSITION PROBABILITIES

<table>
<thead>
<tr>
<th>Multipolarity</th>
<th>Electric Transition Rate (s(^{-1}))</th>
<th>Magnetic Transition Rate (s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(1.587 \times 10^{15} E_\gamma^3 \ B(E1))</td>
<td>(1.779 \times 10^{13} E_\gamma^3 \ B(M1))</td>
</tr>
<tr>
<td>2</td>
<td>(1.223 \times 10^9 E_\gamma^5 \ B(E2))</td>
<td>(1.371 \times 10^7 E_\gamma^5 \ B(M2))</td>
</tr>
<tr>
<td>3</td>
<td>(5.689 \times 10^2 E_\gamma^7 \ B(E3))</td>
<td>(6.387 \times 10^9 E_\gamma^7 \ B(M3))</td>
</tr>
<tr>
<td>4</td>
<td>(1.649 \times 10^{-4} E_\gamma^9 \ B(E4))</td>
<td>(1.889 \times 10^{-6} E_\gamma^9 \ B(M4))</td>
</tr>
<tr>
<td>5</td>
<td>(3.451 \times 10^{-11} E_\gamma^{11} \ B(E5))</td>
<td>(3.868 \times 10^{-13} E_\gamma^{11} \ B(M5))</td>
</tr>
</tbody>
</table>

Table 2.2: Transition probabilities \(T(s^{-1})\) for \(L = 1\) to \(5\) expressed by \(B(EL)\) in \((e^2(fm)^{2L})\) and \(B(ML)\) in \((\frac{eh}{2mc}(fm)^{2L-2})\). \(E_\gamma\) is the \(\gamma\)-ray energy, in MeV. (Taken from ref [69]).

\[
B(\sigma L) = \frac{2j_f + 1}{2j_i + 1} |\langle J_f | M(\sigma L) | J_i \rangle|^2.
\]

In equation 2.14, the \(M(\sigma L)\) operator corresponds to the \(2^L\)-pole charge distribution and the \(M(\sigma L)\) operator includes the orbital motion of a charged particle and the intrinsic magnetisation. Note that the reduced transition probability has no \(E^{2L+1}\) energy dependence as is present in \(T(\sigma L)\). Therefore, the \(B(\sigma L)\) value is often quoted to compare decay rates between different nuclei. Table 2.2 shows the lifetimes \(\tau\) for various multipolarities, where the transition probability per unit second \(T = \frac{1}{\tau}\).
2.4.1 Weisskopf estimates

When examining and constructing nuclear energy level schemes it is often appropriate to compare lifetimes obtained to any measurements of multipolarity to see if they are consistent. Comparisons to Wiesskopf estimates of lifetimes [70] which are a theoretical lifetime for a single particle type system, often yield useful comparisons. Theoretical estimates for lifetimes of states which decay by single particle-like transitions with proposed multipoles may be obtained by making an estimate for the reduced matrix element [71]. The estimates for the reduced matrix elements of electric and magnetic transitions are [72]: if expand

\[ B_{sp}(EL) = \frac{1.2^{2L}}{4\pi} \left( \frac{3}{L + 3} \right)^2 A^{\frac{2L}{3}} e^2 f m^{2L} \]

and

\[ B_{sp}(ML) = \frac{10}{\pi} 1.2^{2L-2} \left( \frac{3}{L + 3} \right)^2 A^{2L-2} \left( \frac{e\hbar}{2Mc} \right)^2 f m^{2L-2}. \]

Table 2.3 shows example Weisskopf estimates for \( \sigma L \) transitions.

Good agreement between the experimentally deduced \( B(\sigma L) \) and the Weisskopf single particle estimates hints that the de-excitation within the nucleus comes from a shell model based transition, as opposed to enhanced values which suggest collective excitations.
### 2.4 Transition Probabilities

<table>
<thead>
<tr>
<th>Multipolarity</th>
<th>Electric Transition Rate (s(^{-1}))</th>
<th>Magnetic Transition Rate (s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(1.0 \times 10^{14} A^{2/3} E_\gamma^3)</td>
<td>(3.1 \times 10^{13} E_\gamma^3)</td>
</tr>
<tr>
<td>2</td>
<td>(7.3 \times 10^7 A^{4/3} E_\gamma^5)</td>
<td>(2.2 \times 10^7 A^{2/3} E_\gamma^5)</td>
</tr>
<tr>
<td>3</td>
<td>(3.4 \times 10^1 A^2 E_\gamma^7)</td>
<td>(1.0 \times 10^1 A^{4/3} E_\gamma^7)</td>
</tr>
<tr>
<td>4</td>
<td>(1.1 \times 10^{-5} A^{8/3} E_\gamma^9)</td>
<td>(3.3 \times 10^{-6} A^2 E_\gamma^9)</td>
</tr>
<tr>
<td>5</td>
<td>(2.4 \times 10^{-12} A^{10/3} E_\gamma^{11})</td>
<td>(7.4 \times 10^{-13} A^{8/3} E_\gamma^{11})</td>
</tr>
</tbody>
</table>

Table 2.3: Single-particle Weisskopf transition probability estimates for \(L = 1\) to 5 as a function of the atomic mass measured in atomic mass units \(\mu\) and the \(\gamma\)-ray energy \(E_\gamma\) in MeV.
2.5 Nuclear isomers

Excited nuclear states generally are short lived ($t_{1/2} \sim 10^{-12}$s). States with significantly longer lives are being hindered from decay by one or a combination of mechanisms. These transitions are called Isomeric transitions with their respective long lived states being Isomeric states, Isomers or metastable states.

Metastable or 'Isomeric' states are named by analogy with chemical isomers whereby two sets of the same constituent particles may have differing configurations, this is the same for nuclear isomers. Nuclear isomers however can have energy differences of several MeV between the two configurations, whereas the chemical isomers tend to be of comparable energies.

The energy width of a state $\Gamma$ (which is, by Heisenberg's uncertainty principal inversely proportional to the half life) is therefore much narrower than that for a non-isomeric state.

The key to knowing where to find and how to account for isomers is to understand what controls their decay rates e.g. high multipolarity transitions / low energy transitions are slower than low multipolarity high energy transitions.

2.6 Gamma-ray angular distributions

The spatial distribution of emitted $\gamma$ rays may be used to infer the multipole order of transitions emanating from the decay of an excited nuclear state. To do this requires a defined axis with respect to which the distribution of emitted $\gamma$ rays can be measured. In the case being studied in this thesis, fusion evaporation reactions are employed and so the axis taken as a reference is that of the beam.

If one takes a perfect classical example of such a reaction, the angular momentum input to the compound nucleus, $L$ would be described by the vector cross product,

$$L = r \times p$$

This product will result in the angular momentum, $L$ being perpendicular to both the direction of the projectile momentum ($p$) and the vector $r$ which describes
the position of the projectile with relation to the target. The compound system angular momentum is therefore aligned in the plane normal to the beam axis.

In this perfect classical system the population of magnetic substates $P(m)$, may then be written as,

\begin{align}
P(m) &= \begin{cases} 
1, & m = 0 \\
0, & m \neq 0 
\end{cases} 
\end{align}

which is known as full alignment. The decay of compound nuclei and ensuing particle (neutron, proton, alpha) emission reduces the alignment and populates $m \neq 0$ sub-states. The relative sub-state population can be described by a Gaussian distribution centred about $m = 0$ and with width $\sigma$ [73],

\begin{align}
P(m) &= \frac{\exp\left(-\frac{m^2}{2\sigma^2}\right)}{\sum_{m'=-I}^{+I} \exp\left(-\frac{m'^2}{2\sigma^2}\right)} 
\end{align}

The angular distribution of an ensemble of $\gamma$ rays depends upon the initial and final magnetic substates which a nucleus decays from and to respectively. As an example, consider a dipole transition from $I_i = 1$ to $I_f = 0$. The initial spin $I_i$ has three sublevels defined by $m_i = +1, 0, -1$ whereas the final state has only one sublevel, $m_f = 0$. In the case of $m_i = 0, m_f = 0$ the $\gamma$ emission probability varies as $\sin^2 \theta$ (where $\theta$ is defined with respect to the Z-axis used to measure the components of $I_i$). The transitions from $m_i = \pm 1$ to $m_f = 0$ have angular distributions that vary as $\frac{1}{2}(1 + \cos^2 \theta)$. Angular distributions for dipole ($m = 0$, $m = \pm 1$) and quadrupole ($m = 0$ only) radiations are shown in figure 2.3. These have the form of Legendre polynomials.

In order to measure the multipole of a transition one must isolate a component of the transition or introduce an anisotropy into the angular distribution by creating an unequal population of magnetic substates so that the total distribution does not sum to unity. The following technique is one of several methods available and is described here for completeness. Further reading may be found in reference [2].

Assuming a cascade of two transitions where the initial and final states have $I_{o,f} = 0$ and the intermediate state has $I_i = 1$ as in figure 2.4. The first $\gamma$ ray is observed in detector 1. This direction is now defined as the z-axis. The second transition is
2.6 GAMMA-RAY ANGULAR DISTRIBUTIONS

Figure 2.3: Angular distributions for dipole and quadrupole radiations. Note only the $m=0$ substate is shown for quadrupole radiation (modified from [74]).

Figure 2.4: Angular correlation measurements. In this example a cascade of two radiations assumed to have spins of $0 \rightarrow 1 \rightarrow 0$.

The first transition decays with a dipolar angular distribution. For $m_o=0$ to $m_1=0$ it is proportional to $\sin^2 \theta$ (where $\theta$ = the angle between the measured gamma rays), whilst for $m_o=0$ to $m_1=\pm 1$ it is proportional to $\frac{1}{2}(1+\cos^2 \theta)$. Since $\theta_1=0$ (so $\sin^2 \theta_1 = 0$), we can remove the $m_1=0$ substate population and so the angular distribution of $\gamma_2$ with respect to $\gamma_1$ becomes [2]

$$W(\theta) \propto \frac{1}{2} \left[ \frac{1}{2}(1+\cos^2 \theta) \right] + 0(\sin^2 \theta) + \frac{1}{2} \left[ \frac{1}{2}(1+\cos^2 \theta) \right] \propto 1 + \cos^2 \theta.$$ 

The above is for simple dipole radiation. The general angular distribution func-
2.6 GAMMA-RAY ANGULAR DISTRIBUTIONS

Figure 2.5: A schematic of angular correlation measurements. The angular distribution of $\gamma_2$ is measured relative to the direction of $\gamma_1$.

The angular distribution for correlations of different multipole radiations is of the form [2]

$$W(\theta) = A_0(1 + A_{22}P_2(cos\theta) + A_{44}P_4(cos\theta) + ...)$$  \hspace{1cm} (2.21)

Where $P_2$ and $P_4$ are standard Legendre polynomials.

$$P_2(cos\theta) = \frac{1}{2}(3cos^2\theta - 1)$$  \hspace{1cm} (2.22)

$$P_4(cos\theta) = \frac{1}{8}(35cos^4\theta - 30cos^2\theta + 3).$$  \hspace{1cm} (2.23)

The coefficients $P_2$ and $P_4$ are calculated with programs such as [67, 75] or taken from tabulated data [76]. If the angular distribution of a transition is measured then the Legendre polynomials can be fitted to the data and information extracted. There are many variables however: $\delta, I_n, I_f$ and also variables associated with the population term, $P(m)$. The mixing ratio, $\delta$, is usually restricted to that between dipole and quadrupole radiations exclusively on the premise that multipole with orders $L > 2$ have insignificant contributions to the radiation field. The distribution of $P(m)$ is taken to be Gaussian and depends upon the reaction model. It is possible however to make quantitative assumptions for these variables allowing the extraction of $P_2$ and $P_4$ from fits to the data.

At spin of $\sim 8\hbar$ and above the $P_2$ and $P_4$ coefficients reach an asymptotic limit on their values. This means that for spins greater than this only the spin changes resulting
from a transition may be deduced. However, angular correlations can aid in building arguments for spins and parities, mixing ratios and relative substate populations.

2.7 DCO ratio analysis

As shown previously, the angular correlation of coincident $\gamma$ rays depends upon their multipolarity. Information about the $\gamma$-ray multipolarity can therefore be obtained by observing the angular distribution of emitted radiation with respect to the beam axis [2]. One method for determining these multipolarities is the Directional Correlation of Oriented states (DCO) method using pairs of coincident $\gamma$ rays [77]. Here the angular correlation is taken between pairs of coincident transitions. By setting a software gate on a transition energy of known multipolarity within a gamma-gamma coincidence matrix where the axes correspond to differing detector angles relative to the beam axis, it is possible to distinguish transition multipolarities by comparing the intensity of the projected transition for given angles. In the case of this work, two angles are present corresponding to two rings of detectors. This information is then used to produce a DCO ratio as described in chapter 5.11.

2.8 Linear polarisation of $\gamma$ rays

A DCO analysis can help determine the transition multipolarity but no information on the electric or magnetic nature of the radiation under study is directly given. For this purpose it can be useful to investigate Linear Polarisation [78] measurements.

The basis for linear polarisation measurements is the use of the clover detectors as Compton polarimeters [78]. This method utilises the direction of scattered radiation to determine electric or magnetic properties [79, 80]. This is because the direction of Compton scatter is dependent upon linear polarisation.

A $\gamma - \gamma$ matrix can therefore be constructed of all $\gamma$ rays versus scattered $\gamma$ rays in a direction perpendicular to the beam direction (pol-perp) and another of all versus parallel scatters (pol-para). In this way spectra can be produced which are gated on the general axis to remove contamination and on the projected axis there will be peaks whose intensity is a function of polarisation. By removing random background and
2.8 LINEAR POLARISATION OF $\gamma$ RAYS

Figure 2.6: A schematic diagram of the crystals in a clover detector relative to the beam, target and reaction product $\gamma$ rays.

subtracting one of the projected spectra from the other, differing polarisations will appear as positive or negative peaks. The polarisation asymmetry can be defined as [79],

$A = \frac{(N_\perp - N_\parallel)}{(N_\perp + N_\parallel)}$  \hspace{1cm} (2.24)

where $N_\perp$ is the number of counts in a peak that have scattered perpendicular to the beam axis and $N_\parallel$ is the number that have scattered parallel to the beam axis.

Since the angular momentum vector of an electric transition is polarised perpendicular to the beam axis, and that of a magnetic transition is polarised parallel, a positive value of asymmetry $A$, will show the transition to be predominantly electric in character.
2.9 Heavy-ion fusion evaporation reactions

Heavy-ion fusion followed by nucleon evaporation is the primary means for production of nuclei at high spin and excitation energy allowing spectroscopic studies with γ rays. By careful selection of beam, beam energy and target a broad selection of nuclei can be manufactured with considerable selectivity.

The general technique proceeds as follows. A beam of ionized nuclei produced by an ion source is accelerated onto a (generally metallic) foil of another isotope. Should two nuclei collide with a small enough impact parameter and a relative energy large enough to overcome their mutual Coulomb repulsion, then their nuclear potentials overlap and they can fuse to form a single compound nucleus. The impact parameter, \( b \), is central to the type of reaction produced as shown in figure 2.7.

The Coulomb potential between a proton and a nucleus with atomic number, \( Z \), is described by

\[
V_c = \frac{Z e^2}{4 \pi \varepsilon_0 r}
\]
where $r$ is the separation of the nucleus and proton and $e$ is the charge associated with an electron.

The excitation energy $E_x$ of the final compound nucleus can be calculated by the following [77],

\[
E_x = \frac{M_t}{M_b + M_t} E_b + Q
\]

where $M_t$ and $M_b$ are the mass of the target and beam respectively, $E_b$ is the energy of the beam in the laboratory reference frame and the reaction $Q$ value (the difference in binding energy of the nucleus in the entrance channel and the compound nucleus) = $(M_b + M_t - M_{CN})c^2$, where $M_{CN}$ is the mass of the compound nucleus.

Upon formation, the internal excitation energy of the compound nucleus is usually greater than the particle emission threshold ($S_p$ and/or $S_n$). Assuming the nucleus does not fission, the decay from a hot to a cold nucleus is dominated initially by the statistical process of particle emission. Each proton or neutron (or occasional $\alpha$-particle) taking away the order of $5\text{ MeV}$ each and typically $1\ h$ of spin. Neutrons are commonly the ejectile in near stable compound nuclei as they have no Coulomb barrier to overcome. An $\alpha$ will typically take away the order of $10\text{ MeV}$. The nucleus can also decay by $E1$ giant dipole resonances which take place at an excitation energy also of order $10\text{ MeV}$ [81].

In order to conserve linear momentum as particles boil off from the nucleus, the residual nucleus must recoil. The evaporated particles are emitted approximately isotropically in the centre of mass frame, however the nucleus is moving in the lab frame of reference in the direction of the beam. This results in a forward focussing cone of recoils in the lab frame known as the ‘recoil cone’.

By conservation of linear momentum (neglecting mass loss from evaporated particles), the velocity of the recoiling nucleus $v_{cm}$ from a heavy ion fusion evaporation reaction is given by,

\[
v_{cm} = \frac{m_b}{m_b + m_t} \sqrt{\frac{2E_b}{m_b}}.
\]
Figure 2.8: The stages in a heavy-ion fusion-evaporation reaction following formation of the compound nucleus. In the first $10^{-19}$s nucleons are emitted leaving the final excited nucleus. This excited nucleus decays by the emission of gamma rays over typically $10^{-15}$s which tend to follow an yrast path to the ground state due to the tendency for nucleon evaporation to take away energy in the order of MeV but relatively little angular momentum. (Modified from [83]).

At high excitation energy, the nuclear level density can be very large (up to $10^{12}$ MeV$^{-1}$) with level widths typically of the order of eV, resulting in a large degree of overlap, with discrete levels effectively forming a continuum. Particle emission (i.e. proton, neutron and $\alpha$) cools the nucleus considerably in terms of energy yet retains much of the angular momentum input to the system. This means fusion evaporation is excellent for populating near-yrast states to high spin (e.g. [82]).

Figure 2.8 shows schematically the stages of a heavy-ion fusion evaporation reaction during its entire lifetime of typically $10^{-9}$ seconds.

The maximum angular momentum ($l_{\text{max}}$) that may be imparted to the compound
2.10 DOPPLER SHIFT

After a nucleus of interest has been created following a fusion-evaporation reaction the compound system has the total linear momentum of the two individual original systems minus any linear momentum carried away by evaporating nucleons. Any gamma rays emitted from moving nuclei will have their energy Doppler shifted in the nucleus in the direction of travel of the $\gamma$ ray.

For transitions whose lifetime causes the resulting $\gamma$ ray to be emitted during the time the compound nucleus is coming to rest, e.g. in a thick backed target the resulting distribution of source velocities of very prompt (typically of the order of picoseconds) $\gamma$ rays can cause spectral lines to appear broadened, with a lineshape ranging from the full Doppler shifted energy ($E_{s\_max}$) to the stopped, unshifted energy, $E_0$.

In terms of the Yrastball array (see section 4.2) which has two rings of detectors at $\theta = 90^\circ$ and $140^\circ$, the $90^\circ$ ring of detectors will remain unaffected (with the caveat that most compound nuclei will have some small motion perpendicular to the original beam direction). The $140^\circ$ ring of detectors will need to have an average Doppler correction applied as follows [84]:

\begin{equation}
E_0 = E_s \left\{ \frac{\sqrt{1-\beta^2}}{1-\beta \cos 140} \right\}
\end{equation}

where $E_s$ is the Doppler shifted energy. For $\beta \leq 5\%$, $\beta^2$ is negligible and equation 2.29 reduces to

\begin{equation}
E_0 = E_s(1 + \beta \cos 140).
\end{equation}
Chapter 3

Nuclear Theory

3.1 The nuclear force

One can deduce the existence of the nuclear force from the fact that nuclei comprised of protons and neutrons exist. The rather more easily measured Coulomb force is repulsive between positively charged protons and yet the nucleus is still held together. Therefore the nuclear force can be seen to be strong and attractive for all nucleons. Experimental evidence such as that of anomalous Rutherford scattering suggests nuclei are of the order $10^{-15}$ m in diameter and as such the nuclear force is of almost no concern for molecular or atomic interactions which occur on a much larger scale of $\sim 10^{-10}$ m. Electron scattering measurements indicate the nuclear density to be roughly constant. This indicates the nuclear force acts only upon its nearest neighbours, therefore the nuclear force becomes saturated. If each nucleon interacted with every other then the binding energy would increase roughly as: $A(A-1)/2$ and so correspondingly would the nuclear density increase [9].

3.2 The nuclear shell model

This model was originally adopted when an attempt to describe the nuclear system with the atomic shell structure proved to be successful. The model describes the filling of orbits and completing shell of nucleons with increasing energy within the nuclear potential. Shells are filled in a manner consistent with the Pauli exclusion
3.2 THE NUCLEAR SHELL MODEL

Each nucleon is treated individually as an independent orbiting particle in a central potential despite the existence of strong interactions between nucleons [85]. The motion of each nucleon is therefore governed by this central potential which is designed to approximate the bulk of individual interactions between nucleons. Each nucleon retains however an individual set of quantum numbers and wavefunction.

The aim of the model potential therefore is to allow the construction of a suitable Hamiltonian capable of reproducing the sequence of nuclear shell closures or magic numbers. Early attempts to achieve this used the infinite square well and infinite harmonic oscillator potentials (equations ??, 3.1 and 3.5) but did not reproduce the correct set of magic numbers (See ref [86]). The effect of various differing potentials can be seen in figure 1.2.

The simple harmonic oscillator potential is often used due to its simplicity and ease of manipulation allowing many results to be determined analytically. The form of the potential can be written as

\[ U_i(r) = -U_i(0) + \frac{1}{2}m\omega^2(r^2). \]

Modifications to the harmonic oscillator potential may be suggested by considering the short range nature of the nuclear force. If a nucleus is considered whose dimensions are much larger than the effective range of the nuclear force and sufficiently far within the nuclear volume to be screened from the asymmetric distribution of nucleons, it should feel no net force. This would make the nuclear potential for that nucleus constant and may point to a square well potential being a better description than the harmonic oscillator.

It is also possible to add an \( l^2 \) term to the potential which will flatten out the radial shape (resulting in a potential well with rounded edges). This also means that higher angular momentum orbits will feel an increasing attractive potential thus lowering their energies more than the lower-\( l \) orbits.

All of these potentials are approximations to the "exact" potential with further refinements possible. One of the most commonly used potentials is the Woods-Saxon potential which has the form
3.2 THE NUCLEAR SHELL MODEL

\[ V_{ws} = \frac{-V_0}{1 + \exp(r - R_0/\alpha)} \]

where \( V_0 \sim 50 \text{ MeV} \), \( R_0 \sim 1.2 \text{ } A^{1/3} \text{ fm} \), and \( \alpha \sim 0.6 \text{ fm} \), \( r \) is the inter-nucleon separation and \( A \) the nucleon number.

Returning briefly to the harmonic oscillator, for a 3-dimensional harmonic oscillator, the eigenvalues \( E_{nt} \) can be written as [9]

\[ E_{nt} = (2n + l - 1/2)\hbar \omega \]

Where \( n \) is the harmonic oscillator shell number and \( l \) is the angular momentum quantum number, with the convention that the lowest value of \( n \) is \( n = 1 \). From equation 3.3 we can see degenerate energies characterise two orbits with

\[ \Delta l = -2\Delta n \]

Where \( \Delta \) is an integer number. For example \( 2s \) or \( 1d \), and larger groupings such as \( 1h, 2f, 3p \). This grouping of orbits begins to define the shells for which any nuclear model must aim.

A Hamiltonian which does describe the system is given by

\[ H = [-\frac{\hbar^2}{2m} \Delta + U_i(r)] \]

where

\[ \Delta = \frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} - \frac{\hat{p}^2}{\hbar^2 r^2} \]

The degeneracy of each angular momentum orbital, \( l \), left by the simple potential was broken by with the introduction of a phonomenological spin-orbit interaction [85]. The full potential \( U_{i,total} \) now becomes
3.2 THE NUCLEAR SHELL MODEL

\[ U_{i \text{ total}}(r) = U_i(r) + U_{ls}(r) \]

The spin orbit term is a surface effect due to nucleons within the nucleus having equal interactions with nucleons surrounding all sides. Only nucleons at the surface of the nucleus feel this imbalance and therefore the force can be written as a radial function [68].

\[ U_{ls}(r) = -U_{ls} \frac{\delta U(r)}{\delta r} \cdot l \cdot s \]

where \( U(r) \) is the chosen function for the central potential and \( U_{ls} \) is the strength constant [9].

Since all Fermions have intrinsic spin \( s = \frac{1}{2} \), allowed values for the total angular momentum \( j \), where \( j = l + s \) are:

\[ \langle l \cdot s \rangle = \frac{\hbar^2}{2} [j(j + 1) - l(l + 1) - s(s + 1)] = l/2 \quad \text{for} \quad j = l + 1/2 \]

\[ -(l + 1)/2 \quad \text{for} \quad j = l - 1/2 \]

The energy splitting between two 'spin-orbit' sub-states may then be written as:

\[ \Delta E_{ls}(j = l - \frac{1}{2}) - \Delta E_{ls}(j = l + \frac{1}{2}) = \frac{2l + 1}{2} \hbar^2 \int d^3r \ | \Psi_{ls}(r) |^2 U_{ls}(r) \]

The resulting eigenfunctions \( \psi_{jm} \) (with \( m \) being the z component of total angular momentum) result from the coupling between the orbital angular momentum \( l \) and the spin angular momentum \( s \).

\[ \Psi_{jm} = R_j(r) \sum_{k\mu} <l^1k\mu | jm > Y_{l\kappa}(\theta, \phi) \chi_\mu \]

where \( Y_{l\kappa}(\theta, \phi) \) are the spherical harmonics and \( R_j(r) \) is the radial component of the wavefunction.
This potential $U(r)$ has been shown experimentally to be negative [68] with the result that the spin-orbit splitting is larger for higher values of orbital angular momentum. This produces the experimentally observed level crossings of intruder orbitals thus reproducing the correct magic number sequence as shown in figure 1.2.

Simple inspection of the orbit energies shows that the magnitude of the spin-orbit force is nearly the same size as that between neighbouring multiplets of orbits. For the highest $l$-values this splitting can move the orbit into an entirely different multiplet. This orbit will have a differing parity to the other orbits and is therefore called the unique-parity orbit. An example would be the $1i_{13/2}$ positive parity orbit appearing in the 82 to 126 shell alongside the negative parity orbits $2f_{7/2}, 1h_{9/2}, 2f_{5/2}, 3p_{3/2}$ and $3p_{1/2}$. This unique parity orbit is important because it does not mix with the other orbits in the multiplet. It is also sufficiently far from any same parity orbits that any mixing with these is extremely weak, therefore configurations arising from this orbit are very pure. This makes them useful experimentally as the wavefunctions are simple and easily understood. Such orbits are also inherently of high angular momentum and can have a significant influence on the structure of near yrast states.

3.2.1 Pauli principle

This states that no two identical nucleons may occupy the same space at the same time which is embodied in the fact that no two nucleons may have identical quantum numbers. This may be formally expressed with the mathematical concept that the nuclear wavefunction must be antisymmetric with respect to the interchange of two nucleons. That is, all of the components must be antisymmetric; spatial, spin and isospin. So if these coordinates are interchanged with those of another particle then the sign of the whole wavefunction must change. It follows therefore that,

\begin{equation}
\Psi_{ab}(r_{12}) = [\Psi_a(r_1)\Psi_b(r_2) - \Psi_a(r_2)\Psi_b(r_1)] = -[\Psi_a(r_2)\Psi_b(r_1) - \Psi_a(r_1)\Psi_b(r_2)] = \Psi_{ba}(r_{12})
\end{equation}

where $\Psi_{ab}(r_{12})$ is a wavefunction of two identical particles with orbits labelled as $a$ and $b$, and $r_{12}$ is the radial distance between the two particles.
3.2.2 Independent particle motion

A fundamental precept of the shell model pertains directly to the filling of these “shells” within the nuclear potential. That is the idea of independent particle motion. The nuclear radii (the distance at which the nuclear potential is comparable to that of the Coulomb potential) empirically can be described by $R = 1.2A^{1/3}$, found from the anomalous scattering of $\alpha$ particles. Therefore assuming the nucleon radius is $\sim 1$ fm the ratio of the nuclear volume to that of a single nucleon is approximately $1.23^3 A$. Why therefore are there not many collisions which would possibly raise a nucleon to a higher lying state?

The solution to this apparent anomaly lies with the Pauli principle. If two low lying nucleons were to collide, then it seems sensible one may gain energy and be promoted to a higher lying orbit. However, the Pauli principle dictates that the following higher lying orbits are filled sequentially according to energy. Therefore, for a nucleon to be promoted to a higher energy state it would have to gain enough energy to be excited above the Fermi energy level (the energy of the highest occupied quantum state). Therefore the interaction has no effect on the motion of these two nucleons. Motion in the nucleus can be though of as “transparent” to the surrounding nuclear medium.

3.3 Multiparticle configurations

The spherical shell model gives us an understanding of the observed magic numbers in atomic nuclei. Odd-mass spherical nuclei can also be considered within this regime and the ground state spins predicted by inspection of the Fermi level under the independent particle model such as in figure 1.2. These results however, only afford information pertaining to the ground state of these nuclei. Excited states are often complex linear combinations of multi-particle wavefunctions. This is a large topic with profound implications and the author directs the reader to the text by Casten [9], for details. Semi-classically, two particles coupling with angular momenta $j_1$ and $j_2$ must obey a ‘vector triangle rule’ in the formation of the final state with integer values of angular momentum $J$ such that,
3.4 TWO-STATE MIXING

(3.14) \[ |j_1 - j_2| \leq J \leq j_1 + j_2. \]

The Pauli principle however requires the total wavefunction to be anti-symmetric and therefore restricts allowed values of $J$. It can be shown with arguments of symmetry [9] that for identical nucleons in equivalent orbits (i.e. $j^2$ configurations) the coupled angular momentum has only even values of $J^z = 0^+, 2^+, 4^+...(2J-1)^+$. For two non-identical nucleons isospin can be either $T=1$ or $0$. For $T=1$, the proton-neutron isospin wave function is symmetric and is synonymous with the aforementioned n-n and p-p instances where $J$ couples to even spin.

3.4 Two-state mixing

Rarely does one encounter pure nuclear states which are not mixed and can be described when the Hamiltonian and single-particle basis mesh precisely. Nuclear states are often complex admixtures of many components for which a detailed calculation would involve the diagonalisation of a large Hamiltonian matrix. A series of two state mixing calculations can however often approximate this calculation well enough to garner an understanding of the underlying physics [9].

Here a mixing of two states with an interaction $V$ will be considered. The original wavefunctions of these states are denoted by $\phi_1$ and $\phi_2$ as in figure 3.1, with energies $E_1$ and $E_2$ respectively, where both of the states have the same spin and parity $I^z$. The wavefunctions of the mixed states can be written in terms of the original wavefunctions as follows,

(3.15) \[ \Psi_1 = \alpha \phi_1 + \beta \phi_2 \]

(3.16) \[ \Psi_2 = \beta \phi_1 - \alpha \phi_2 \]

where $\alpha$ and $\beta$ are the mixing amplitudes $\alpha^2 + \beta^2 = 1$. 

3.4 TWO-STATE MIXING

The new perturbed energies of the mixed states $E_1$ and $E_2$ are obtained by diagonalising the Hamiltonian that takes into account the interaction $V$. The resulting energies are given by [9],

$$E_{\parallel} = \frac{1}{2} \left( (E_1 + E_2) \pm \sqrt{(E_2 - E_1)^2 + 4V^2} \right).$$

Where $E_{\parallel}$ are the eigenvalues of $\Psi_{\parallel}$ respectively and $V$ is the mixing matrix element $V = \langle \phi_1 | V | \phi_2 \rangle$, i.e. $V$ is the strength of the interaction.

The mixing amplitude $\beta$ can be written in terms of the mixing matrix element $V$ [9],

$$\beta = \frac{1}{\sqrt{1 + \frac{(E_2 - E_1)^2}{4V^2}} \sqrt{1 + \frac{(E_2 - E_1)^2}{4V^2}}}}.$$

Equations 3.17 and 3.18 are universal and not dependent upon the nature of the interaction, $V$. 

Figure 3.1: Schematic representation of a two state mixing calculation. $E_1$ and $E_2$ are unshifted energies, $E_s$ is the energy shift, $\Delta E_u$ is the original splitting between the levels. Other values are defined in the text. Modified from [9].
3.5 Residual interactions

The mean field potential, whilst reproducing adequately many nuclear properties in the limit of the single particle model (explicitly a single particle or hole outside a closed shell) fails to adequately describe the experimental data involved with excitation. For instance, if the limit of the independent particle model were true then if three particles were available to couple to differing angular momenta \( J = j_1 + j_2 + j_3 \) all of the 3 particle configurations would be degenerate. This is not the case in nature and so we must consider extra, or a ‘residual’ interactions \( H_{\text{residual}} \). These are added to the basic independent particle model Hamiltonian to make theoretical calculations a closer approximation to what is observed experimentally. Residual interactions are responsible for all configuration mixing, collectivity and other correlations in the shell model.

3.5.1 The pairing interaction

This simulates the strong attractive force felt by two identical nucleons in the \( J = 0 \) configuration. The effect is shown to lift the degeneracy only of the \( 0^+ \) state in figure 3.2 whilst leaving the other \( 2j - 1 \) states unaffected. The pairing interaction is useful as it allows the build-up of pairing correlations (represented in this formalism by non-diagonal scatterings) and as such can reproduce the pairing gap seen experimentally in even-even nuclei.

3.5.2 The \( \delta \)-function residual interaction

The \( \delta \) interaction is a radial interaction which is defined as being equal to zero unless the nucleons occupy the same spatial position. An intuitive explanation of this arises from the nuclear force being short range (although the \( \delta \) interacion does contain a long range component which makes it more suitable for representing collective effects than the pairing interaction). In this respect the \( \delta \) interaction will be largest for \( J = 0 \) and \( J = J_{\text{max}} \) configurations in which the nucleon angular momentum vectors are alligned parallel or anti-parallel. For a fuller conceptual discussion of this, including antisymmetrisation which completes the above argument, the reader is referred to ref
3.5 RESIDUAL INTERACTIONS

The important results of this interaction can be summarised as:

(i) An antisymmetric spatial wavefunction vanishes at the point where the $\delta$ interaction is effective, i.e. $r_1 = r_2$. To agree with the Pauli principal the spin or isospin parts of the wavefunction must therefore be antisymmetric.

(ii) For identical nucleons (those with symmetric isospin components of the wavefunction) in equivalent orbits, the energy degeneracy of the $J = 0, 2, 4...(2j - 1)$ states is lifted. The effects of the interaction increase with increasing $j$.

Following from a further mathematical derivation of the effects of a delta interaction (available in reference [9]) the analytic result of the angular dependence of the $\delta$ interaction is seen in figures 3.3 and 3.4. The reason for two curves corresponding shapes lie with the Pauli exclusion principle and what can be semi-classically described as the overlap between the orbits of the two particles. At an angle between the orbital planes of $\theta = 90^\circ$ the interaction should be small for both $T=0$ and $T=1$ as the perpendicular orbits rarely bring the particles close enough to interact. For a $T=1$ configuration (identical particle argument) the interaction is strong when the particles orbit in opposite directions ($J=0, \theta = 180^\circ$). If they orbit in the same direction ($J = J_{\text{max}}, \theta = 0^\circ$) the interaction strength disappears as the particles are forced be spatially antisymmetric and so do not come into contact. This results in a small interaction for small $\theta$ but allows a larger magnitude for greater angles. The $T=0$ case has by definition, distinct particles and so for both the small and large $\theta$ corresponding to smallest and largest $J$ the orbits are coplanar without the need for the orbits to be out of phase as required by anti-symmetrisation. This produces a strong interaction for both small and large $\theta$. 
3.5 RESIDUAL INTERACTIONS

0^+, 2^+, 4^+, 6^+  2^+, 4^+, 6^+  6^+  4^+  2^+

No residual interaction  Pairing Interaction  Delta Interaction

Figure 3.2: The effects of Pairing and δ interactions on level degeneracy (modified from [9]).

Figure 3.3: Angular dependence of the δ-function residual interaction strength (larger values are defined as more attractive) for two particles in equivalent orbits. T=1 (J even) states. The analytical expression for this dependence is $\tan^{\theta}_2$. Modified from [9].
3.5 RESIDUAL INTERACTIONS

Figure 3.4: Angular dependence of the $\delta$-function residual interaction strength (lower values are defined as more attractive) for two particles in equivalent orbits. $T=0$ ($J$ odd) states. The analytical expression for this dependence is $\cot^2 \theta \left[ 1 + \frac{1}{\cos^2 \frac{\theta}{2}} \right]$. Modified from [9].
Chapter 4

Details of the Yale experiments

4.1 The ESTU tandem Van de Graaff

The EST-U (Extended Stretched Trans-Uranium) Ion accelerator at Yale is a horizontally mounted Tandem Van De Graaff accelerator [87, 88]. This type of accelerator works by an ion source producing a specified isotope of positively charged ions, which are accelerated to a between 50 and 150 keV and focused onto the beam tube object plane by a cylindrical electrostatic einzel lens. This beam passes through a region containing a gas at low pressure, where some of the ions are converted to negative ions by the addition of electrons. As the mixture of charged particles moves through a magnetic field, those with negative charge are deflected into the accelerator tube (see figure 4.2), and those with positive charge are deflected away. The beam of negative ions is then accelerated toward the positive high-voltage terminal. In this terminal, the particles pass through a thin single or double layer of carbon foil at the centre of the terminal that strips off electrons, changing many of the negative ions into positive ions. These are now repelled by the positive terminal and are further accelerated through the second part of the tube where the voltage gradient is reversed (hence the name Tandem). Transmission efficiency through the accelerator in the terminal voltage range of 10-20 MeV, with single foil stripping, is around 30-40% for ions from $^{12}$C to $^{58}$Ni [89].

The ESTU tandem has a maximum working voltage of approximately 20 MV [87]. This is dependent upon the amount of SF$_6$ gas within the pressure vessel. The
4.1 THE ESTU TANDEM VAN DE GRAAFF

Figure 4.1: Schematic of the Yale ESTU-1 Tandem Van De Graaff showing the layout of Ion source injecting into the tandem chamber. The Stripper can be seen at the center of the chamber (at position labelled camac). The ion beam then passes through a magnetic dipole to direct it into a variety of target rooms. Reproduced from [87]

optimum value is ~5 atmospheres but due to its expense and the high leakage rate at this pressure the amount used is constrained to ~3 atmospheres. The maximum beam energy is then dependent upon the charge state achieved after stripping. This is found to first order as follows,

\[ E_{\text{beam}} = V_T (1 + Q)e \]

where \( V_T \) is the terminal voltage, \( Q \) the charge state and \( e \) the charge of the electron. Corrections to the energy should be made for the ion source injection energy (around 200 keV) and energy loss within the carbon foil stripper.
Figure 4.2: The ESTU Tandem Accelerator pressure vessel. Included to give the reader an idea of scale.
4.2 THE MODIFIED YRASTBALL (Y') ARRAY

The modified YRASTBALL [90] array at WNSL Yale as used in this experiment was placed at the focal plane of the SASSYER [91] recoil separator. This configuration comprised 10 LN$_2$ cooled clover detectors and 2 Low Energy Photoelectron Spectrometer (LEPS) detectors. These were placed as shown in figure 4.3. In this configuration the array has a total photopeak efficiency of ~3% at 1.3 MeV [90].

The clover detectors are Compton suppressed with the use of Bismuth-Germanate (BGO) shields. These are used to veto $\gamma$ rays that scatter out of the HPGe crystals and therefore deposit only a fraction of their energy, adding a large Compton continuum to the spectra.
4.3 Detectors

4.3.1 Gamma-ray interactions in matter

In order to detect $\gamma$ rays an indirect process must be used involving an interaction of the $\gamma$ ray in a detector medium and the resultant conversion of the photon energy into one or more electrons through the photoelectric effect. The charge produced in this way is then converted to a voltage in the detector pre-amp, this can then be measured with standard ‘NIM’ electronics.

It is desirable that the total incoming photon energy be converted linearly into electron kinetic energy, and that no energy escapes from the detector volume in the form of scattered photons or secondary electrons. Three processes dominate over the energy range appropriate to $\gamma$-ray spectroscopy [66].

Photoelectric effect

This is the dominant interaction mechanism between electromagnetic waves and matter below 100 keV. In this process the total energy of an incident photon is absorbed in the material and an atomic electron is released. The electron energy $E_e$ is equal to the incident photon energy $E_\gamma$ minus the binding energy $E_b$ (also called the Work Function) of the atomic electron.

\[ E_e = E_\gamma - E_b. \]  

(4.2)

The re-ordering of electrons to fill the vacancy now created within the shell structure of the atom results in the production of characteristic X-rays. These are usually absorbed within a very short distance in the detector medium. The total signal therefore corresponds to the total conversion of the original photon energy into electron kinetic energy.

The interaction probability is highly dependent on the atomic number of the material upon which the photon is incident. The interaction cross section can be approximated by equation 4.3 [66].
where $k_{pe}$ is a proportionality constant and $\sigma_{pe}$ is the probability of a photon of energy $E$ interacting with an electron in a material with atomic number $Z$. This illuminates why dense materials are often employed as high efficiency radiation detectors.

**Compton effect**

A Compton scatter event is whereby an incoming $\gamma$ ray strikes an atomic electron and deposits only a fraction of its total energy. The energy of the scattered photon is given by [66].

\[
E_\gamma = \frac{E_\gamma^0}{1 + \frac{m_0c^2}{E_\gamma^0(1 - \cos \theta)}}
\]

where $E_\gamma^0$ is the initial photon energy, $m_0$ is the electron rest mass, and $c$ the speed of light. It can be seen that for small scattering angles, little energy is transferred. The maximum kinetic energy $E_{\text{max}}$ given to the electron in a head-on collision is:

\[
E_{\text{max}} = \frac{E_\gamma^0}{1 + m_0c^2/2e_\gamma^0}
\]

In practice, Compton scatter produces a distribution of energies known as the *Compton Continuum* up to the energy given by equation 4.5, which is known as the *Compton Edge*.

The differential cross section for Compton scattering at an angle $\theta$ is predicted by the *Klein-Nishina formula* for the differential cross section per electron [66].

\[
\frac{d\sigma}{d\Omega} = \frac{1}{2}r_0^2 \left[ \frac{1}{[1 + \alpha (1 - \cos \theta)]^2} \left[ 1 + \cos^2 \theta + \frac{\alpha^2(1 - \cos \theta)^2}{[1 + \alpha (1 - \cos \theta)]^2} \right] \right]
\]

where $\alpha$ is the photon energy in units of the electron rest energy ($\alpha = \frac{E_\gamma}{m_0c^2}$) and $r_0$ is the *classical electron radius* $= \frac{e^2}{4\pi\varepsilon_0 m_0 c^2} = 2.818 \text{fm}$, (this is a convenient parameter only and has no physical significance) figure 4.4 shows the scattering angle probabilities for differing $\gamma$-ray energies.
4.3 DETECTORS

Figure 4.4: Differential cross section $\frac{d\sigma(\theta)}{d\Omega}$ for the production of secondary photons from Compton scattering of incident photons at differing incoming angles and energies $\alpha$. Reproduced from [2].

**Pair production**

This process can take place when a photon carries more than twice the rest mass energy of two electrons; 1.022 MeV. The process actually creates an electron-positron pair with the excess energy going to the kinetic energy of the two bodies. This pair can then cause ionisation along their respective paths of travel. The positron will annihilate with an electron from the surrounding material to produce two 511 keV photons which are emitted back-to-back in the center of mass frame in order to conserve momentum. A full energy peak is produced in the spectrum when both 511 keV annihilation photons are re-absorbed in the detector volume. A secondary peak 511 keV below the full energy peak is observed when one of the annihilation photons escapes the detector volume (single escape peak). A third may also be observed 1.022 MeV below the full energy peak when both 511 keV photons escape (double escape peak).

The dominance of these interaction mechanisms can be seen for germanium in figure 4.5.
Germanium

Figure 4.5: The dominance of three interaction mechanisms; Incoherent scattering, Photoelectric absorption and Pair production at differing photon energies in Ge. Taken from [92].
4.3 DETECTORS

4.3.2 Detector material

In order to detect highly penetrating γ rays it is necessary to use a detector made from a material with sufficient stopping power. A solid detection medium is preferable to an equivalent gas filled detector as solid densities are (pressure depending) many times greater than that of a gas.

A solid scintillator such as NaI(Tl) may therefore be chosen for this task. However, in applications of high-spin spectroscopy where the energy resolution of a detector is paramount, the relatively small number of charge carriers produced together with limitations introduced by the small number of electrons produced in the first stage of the photomultiplier tube means statistical fluctuations place an inherent limit on the resolution possible. This leaves Si and Ge semiconductor detectors. Ge detectors are by far the most suitable due to their high energy resolution of 2-3 keV and their higher detection efficiency due to their larger atomic number (Z(Ge)=32, Z(Si)=14).

4.3.3 Semiconductor band structure

In a semiconductor detector the resulting crystalline lattice of the material is responsible for establishing allowed energy bands for electrons within the crystal. The gaps between these bands are energies which are forbidden to the electron. To be excited from one band to the next requires the input of a precise, lower limited, amount of energy.

The bands of interest within a semiconductor detector are the valence band and the conduction band (see figure 4.6). The valence band is the lower lying of the two and corresponds to electrons involved in covalent bonding of the lattice atoms. The higher-lying band is the conduction band. This corresponds to electrons which are free to migrate about the crystal lattice and contribute to the electrical conduction of the material. The bandgap between the two determines if the material is a semiconductor [66].
4.3 DETECTORS

![Figure 4.6: The band structure in semiconductors.](image)

4.3.4 Thermal excitation

There is a probability that at any time that electrons will be thermally excited into the conduction band. These electrons will be collected along with the charge of interest from incident $\gamma$ rays and contribute to unwanted noise. The probability of this happening $p(T)$, is given by equation 4.7 [66]:

\[
p(T) = CT^{3/2} \exp \left( -\frac{E_g}{2kT} \right).
\]

Any excitation of an electron into a higher band leaves a hole or 'vacancy' in the valence band. This hole is also free to migrate throughout the crystal but due to its movement involving the re-ordering of other electrons within the band its mobility is $\sim \frac{1}{4}$ that of the electron [66]. The collection of the electron-hole pairs is essential to the measurement of the energy deposited within the crystal volume by a radiation event. The number of pairs created is directly proportional to the energy of the incident radiation and is experimentally observed to be largely independent of the incident radiation type [66].

In order to collect the charge produced, a bias voltage is applied across the detector volume. Generally this voltage is a large as possible (without causing dielectric breakdown) in order to collect as much charge as possible as quickly as possible. With most modern detectors a voltage sufficient to saturate the velocity of the charge carriers can be achieved. This velocity is of the order $10^7 \text{cm/s}$ in Ge and results in charge collection times of around 50 ns in an average clover type detector as used in the present
4.4 HPGE DETECTORS

work.

4.3.5 Leakage current

The large bias voltage applied to collect charge also results in a continual flow of current through the crystal. This 'dark current' if not controlled can obscure the small current produced by a radiation event. A typical number of charge carriers (\(\sim 10^5\)) resulting from a radiation event represents a current of \(\sim 10^{-6}\) A and so leakage current must be kept to around \(10^{-9}\) A to avoid significant interference with the signal [66]. For this reason 'blocking contacts' must be used to collect charge. These contact do not allow the injection of charge back into the detector at the opposite contact to which they were collected thus depleting charge carriers from the volume. In this way, the leakage current can be reduced sufficiently to allow the detection of the extra charge added by the creation of electron-hole pairs.

The output from an HPGe detector is usually split into two separate streams, one for timing and one for energy. This is explained further in section 4.7.

4.4 HPGe detectors

4.5 HPGe clover detectors

The HPGe detectors in the WNSL array are clover detectors [93]. This means that each 'clover' consists of four HPGe diode crystals of 50x50 mm areafacing the target and 70 mm length mounted together into a common cryostat.

This arrangement allows for a large detector volume which alleviates some of the adverse effects of Compton scattering by increasing the volume to surface area ratio thus reducing scatter out of the detector volume [93]. The segmented nature of the detector reduces charge collection problems associated with single large detector volumes whereby the probability of charge running into a trapping site is increased with the greater distance it must travel to a collection contact. Another advantage over a single large crystal is that in this case, four sets of electronics share the events that would otherwise be handled by one set, therefore allowing higher count rates to
be used. If an event Compton scatters from one crystal to the next then the two gammas will be added back together in offline sorting algorithm known as addback (see figure 4.8). The clover group is mounted within a set of BGO shields. These detectors function as a veto for gammas which scatter out of the HPGe volume and therefore have contributed to the Compton background. The high density of BGO means a relatively thin slab can be used and so does not interfere too greatly with the way the clover groups fit together.

4.6 BGO detectors

Bismuth Germanate ($Bi_4Ge_3O_{12}$) detectors are inorganic scintillators with a large atomic number ($Z(Bi)=83$) and therefore a high radiation stopping power. It has a comparatively low light output when compared to other scintillators and also a high refractive index which makes light collection challenging. For these reasons it is well suited to use as a veto detector for compton scattered events which are leaving the HPGe detector volume without having deposited the full $\gamma$-ray energy.

Light photons are collected on the photoemissive photocathode of a photomultiplier and through the photoelectric effect electrons are emitted. The number of electrons is amplified in the photomultiplier by sequentially attracting electrons to dynodes at a higher and higher voltage. Upon impinging the next dynode, secondary electrons are produced, thus multiplying the signal logarithmically. This allows for large amounts of gain which is important when considering the low light output of the scintillator material.

4.7 Electronics

The signal from most detectors are similar in functional form, being a group of electrons collected at a terminal attached to the detector volume. Initially many electrons are measured (rise time to peak value typically $\sim$50 ns [94]), with this charge decaying exponentially as the radiation deposits energy within the detector.

The two detector systems; HPGe semiconductors and BGO scintillators have their output signals treated differently, with the BGO scintillators simply used as Compton
scatter veto detectors, detector energy threshold and timing information are the only considerations. For the HPGe detectors both timing and high-resolution energy information is needed and so a more complex series of electronics is involved. The signal directly from a Ge detector is usually small, a 662 keV $\gamma$ ray will typically create within a semiconductor detector $\sim 2.2 \times 10^5$ electron-hole pairs as the energy required to create one pair is $\sim$ 3 eV. For this reason the signal is passed directly to a pre-amplifier which is placed as close to the detector as possible to reduce interference and attenuation introduced by long cables. Usually the pre-amp is attached to the LN$_2$ cooling system for the detector to keep it cold, again to reduce noise. The job of the pre-amp is to convert the charge pulse to a voltage whose magnitude is proportional to the total integrated charge.

4.7.1 HPGe signal

The signal from the HPGe detectors is now split into two as mentioned earlier.

HPGe Energy Signal

The first signal is fed straight to a spectroscopy amplifier. The aim of this amplifier is to magnify the amplitude from the mV regime present after the pre-amp to the 0.1-10 V range whilst shaping the pulse to optimise energy resolution and reduce overlap of successive pulses by restoring the baseline voltage as quickly as possible. The output produced is a unipolar (Gaussian) pulse with the amplitude proportional to the energy deposited in the detector volume by the incident radiation. Here the gain of the detector is also set. There are 4096 channels available with the Yale Analogue to Digital Converters and it is important to achieve the best resolution possible to ease data analysis. A study of transition energies in the region surrounding $^{91}$Zr suggested 3 MeV was the highest $\gamma$-ray energy likely. Gains were therefore set to 0.7 keV/channel rather than the usual 1 keV/channel to provide the required energy detection range.

The ‘Spec’ amp has a variable shaping time (at Yale the Ortec 855 Dual spec amp is used and can be altered via jumpers within the module). Longer shaping times have less noise associated with them but cause more pile-up events whereby two signals overlap and must be discarded. An experimentally determined shaping time
appropriate for Ge detectors is $\sim 3 \mu s$ (as used in this experimental study).

The shaping time is related to the output Gaussian pulse by the output Full Width at Half Maximum being 2.35 times the shaping time.

**Pole-zero cancellation**

Occasionally the baseline is not restored quickly after a signal pulse, over or undershooting the baseline with a long decay time. If a subsequent signal arrives before the baseline is restored then the pulse height recorded will be incorrect and therefore the energy will be incorrect. The Pole-Zero is again adjusted for each amplifier during calibration by looking at the unipolar output to reduce this effect.

After the Spectroscopy Amplifier the signal is passed to the analog-to-digital converter (ADC) which measures accurately the pulse height and converts this to a digital signal to be recorded on a computer (IOTWO at Yale) with the other event information.

**Timing signal**

The germanium timing signal goes directly from the pre-amplifier output into a Timing Filter Amplifier (TFA). This is a very fast amplifier to magnify further the signal from the pre-amp. Some shaping (either differentiating or integrating) may be introduced to reduce noise.

After the basic amplification the signal is passed to a Constant Fraction Discriminator (CFD). The CFD splits the timing signal in two. One part is attenuated to a fraction of the original and the other delayed and inverted. The delay is set so that the optimum fraction point on the leading edge of the delayed pulse lines up with the peak amplitude of the attenuated pulse. These two summed signals therefore produce a bipolar signal with a zero-crossing that corresponds to the point of optimal fraction on the delayed signal [66, 94]. This manufactured zero crossing is easy to discriminate electronically, and less dependent on the pulse amplitude than other methods (for example: Leading edge discriminator which has more time walk problems. Time walk is where the time identified as the beginning of an event deviates from the correct value in relation to a parameter such as peak amplitude.) and so provides accurate timing.
4.7 ELECTRONICS

The delay is set to 110 ns in the YRASTBALL array and the zero-crossing (discriminator) threshold can be fine tuned on the module to minimize time walk and ensure that all zero-crossings on all detector channels occur at the same point regardless of signal amplitude.

Noise events are also rejected at the CFD. The threshold is adjusted so that low energy events are visible (they are viewed on an oscilloscope triggered by the CFD output) but little noise can be seen. The output of the discriminator is a simple logic pulse which is transferred directly to the Time-to-digital-converter (TDC). This pulse is delayed however to ensure that it arrives just after the master trigger signal which triggers the data acquisition.

**Compton veto**

If an event is recorded within a HPGe volume and at the same time an event is recorded in an associated BGO shield then a veto is passed to the time gate and delay generator (GDG), which means that the time signal is not passed to the TDC and therefore not recorded. When the data is checked for consistency in software sorting, this event will be flagged as incomplete and rejected. This effectively removes a significant number of unwanted Compton scattered events from the data stream thus reducing off-line data sorting times.

**Master trigger**

A master trigger condition can be set on the Yale electronics if needed. The master trigger sets a condition on the multiplicity of the event varying from 1 up to the number of clover detectors available. A master trigger is used to help remove random events which may swamp the data set. With the trigger set to two or more, coincident events only will be passed to the data acquisition. This generally selects cascades of events which cleans the data set. For this experiment the master trigger was set to a multiplicity of two i.e. two separate suppressed clovers were required to fire in coincidence. With the master trigger included the overall electronics setup is shown in schematic form in figure 4.7.
Figure 4.7: Schematic overview of electronics setup for one clover detector. Only channels corresponding to one full clover detector and shield are shown for simplicity. The discriminator (Disc) after the Timing Filter Amplifier is a Constant Fraction Discriminator type which feeds into a Fan In Fan Out module which acts as an OR gate for multiple inputs. GDG is a Gate and Delay Generator. Modified from [94].
Figure 4.8: The effect of addback on the 1408 keV transition from the $^{152}$Eu calibration data. Addback sums the energies of two simultaneous events in two separate segments of an individual clover, producing one event only. This reduces the Compton background and increases peak intensity as shown.
Chapter 5

Experimental details and results

The investigation of $^{91}\text{Zr}$ naturally fell into two separate experiments. The first, preliminary study was to identify states immediately above the $^{21+}_{12}$ 3.167 MeV isomer using a thin-target, Doppler shift method. The second was a thick-target, statistics gathering, high multiplicity experiment to allow inspection of high spin states using $\gamma$-ray gating techniques. Details of these experiment are given below:

5.1 Experiment 1 - June 2005

$^{82}\text{Se}(^{13}\text{C},xn\gamma)^{95-\nu}\text{Zr} @ 50 \text{ MeV Thin target}$

Identification of states immediately above $^{21+}_{12}$ isomer through Doppler shift. A thin target of 200$\mu$g cm$^{-2}$ $^{82}\text{Se}$ with 60$\mu$g cm$^{-2}$ Au flash (thin coating) was used which allowed the recoiling $^{91}\text{Zr}$ residual nuclei to leave the target after formation. The $^{181}\text{Ta}$ Faraday cup was situated 5cm behind the target and acted as a stopper for both the reaction products and any $^{13}\text{C}$ that passed through the target. Any prompt decays above the 3.167 MeV isomer were Doppler shifted due to their decay in-flight. At 50 MeV the compound $^{91}\text{Zr}$ nucleus travels 5cm in $\sim$13 ns. Any delayed components (lifetimes $\geq$ 13 ns) were recorded without Doppler shift due to being stopped on the Faraday cup (but still in view of the $\gamma$-ray detectors).

Three days of beam time were allocated to this experiment in June 2005, producing $1.2 \times 10^6 \gamma - \gamma$ coincidence events. ADC gains were set to the Yale default of 1
5.2 EXPERIMENT 2 - DECEMBER 2005

keV per channel which allows an energies ranging up to 4 MeV to be examined.

5.1.1 Target for experiment 1

In order to identify prompt decays as being Doppler shifted it was necessary to use a thin target. This allowed the recoiling nucleus to escape the target volume and decay in flight.

The $^{197}\text{Au}$ flash on both sides of the target was to prevent sublimation of the target and the aggressive production of SeO$_2$ which occurs in air. At 3.1$\mu$m thick it did not stop a significant fraction of the beam as the range of $^{13}\text{C}$ at 50 MeV in $^{197}\text{Au}$ is 18.7 $\mu$m (calculated with Trim [95]).

5.2 Experiment 2 - December 2005

$^{82}\text{Se}(^{13}\text{C},xn\gamma)^{95-97}\text{Zr}$ @ 50 MeV Thick target

High statistics production of $^{91}\text{Zr}$ to high spin. As the initial transitions above the isomer were identified in the earlier experiment, a high statistics $\gamma - \gamma$ data set was required in order to examine the level scheme to high spin.

Two weeks beam time were allocated in December 2005, producing $3\times10^9 \gamma - \gamma$ coincident events. A search of literature on neigbouring nuclei revealed no $\gamma$ rays with an energy over 3 MeV, therefore ADC gains were set to 0.7 keV per channel to increase energy resolution in what was demonstrated in the first experiment to be a relatively complex total projection spectra. With the available 4096 channels this produced a total energy range of 3 MeV in the ADCs.

5.2.1 Target for experiment 2

Dedicated targets were produced at Daresbury consisting of a 70$\mu$g/cm$^2$ $^{197}\text{Au}$ flash on the front, 950$\mu$g/cm$^2$ of $^{82}\text{Se}$ and a 5mg/cm$^2$ $^{197}\text{Au}$ backing corresponding to thicknesses of 3.6, 197.5 and 259 $\mu$m respectively. These targets can be seen in figure 5.1. The target was significantly thicker than the first experiment stopping all of $^{91}\text{Zr}$ ions which have a calculated range of 10 $\mu$m in $^{82}\text{Se}$ (Calculated with Trim [95]).
5.3 $^{13}\text{C} \text{ BEAM SELECTION}$

Figure 5.1: $^{197}\text{Au}$ backed $^{82}\text{Se}$ targets prepared for the second experiment. These were kept in a sealed, Argon filled container during transit to prevent oxidation and contamination.

The backing was to prevent any recoils leaving the target and forming any significant Doppler shifted component to the data. Using gold as a backing did not present any significant contamination of the results as the Coulomb barrier for $^{13}\text{C} + ^{197}\text{Au}$ is 58 MeV, 8 MeV higher than the beam energy used. There was a small amount of Coulomb excitation to the first excited state in $^{197}\text{Au}$ at 77.4 keV visible in the spectra, no other contamination from $^{197}\text{Au}$ was observed.

5.3 $^{13}\text{C} \text{ Beam selection}$

Reaching stable elements can be challenging with fusion evaporation reactions readily available in the laboratory. Due to the line of stability curving away from the $N=Z$ line towards $N>Z$ for $A>40$, most combinations of stable beams and targets will trend towards a neutron deficient compound nucleus. In order to reach $\beta$-stable nuclei then, both beam and target should be as neutron rich as possible and the Q-value kept reasonably low to prevent boiling off too many neutrons after fusion (see section 2.10).

At first inspection it may appear sensible to use $^{12}\text{C}$ which has a much larger natural abundance at 98.9% compared to 1.1% for $^{13}\text{C}$ isotope. However, its lower natural abundance has a negligible effect on the overall experiment as it is still readily produced by the WNSL ion source in amounts approaching the limit at which heating of the target may become problematical. Favourably, $^{13}\text{C}$ represents the 4n channel to $^{91}\text{Zr}$ rather than the 3n, which means a higher beam energy can be used, imparting
more angular momentum input to the compound nucleus. The $^{13}$C beam also allowed more angular momentum to be imparted compared to $^{12}$C due to its higher mass.

5.4 Beam energy selection - PACE calculations

During the fusion evaporation reaction $^{13}$C underwent fusion with the $^{82}$Se target to form $^{95}$Zr. Neutrons boiled off due to excess energy leaving a distribution of Zr isotopes. The reaction was tuned in energy using the program PACE4 [96] so that a large cross section for $^{91}$Zr was present in the reaction. The PACE4 estimate also produces values for $l_{\text{max}}$, total cross section and $\gamma_c$ for the compound nuclei. Figure 5.2 shows the relative yields for $^{91,92}$Zr and $^{88,89}$Sr with a $^{13}$C beam on a $^{82}$Se target.

A beam energy of 50 MeV was selected from these simulations as a suitable compromise between angular momentum input and cross section. This gave a predicted cross-section of approximately 600 mb for the $^{91}$Zr+4n evaporation channel, with the other significantly populated residual channels being $^{90}$Zr+5n, $^{92}$Zr+3n and $^{88}$Sr+αn. The beam energy was also kept at 50 MeV to keep below the Coulomb barrier of $^{181}$Ta which is $\sim$55 MeV (necessary for the first experiment which employed a Ta Faraday cup). Calculated using the LISE suite [67]). This beam energy provided a calculated $L_{\text{max}}$ of $\sim$26 $\hbar$ for the reaction.

5.5 Data sorting

Data recorded from both experiments were sorted offline using the Yale university developed program CSCAN [97]. This allowed standard 1, 2 and 3D spectra to be sorted. These were examined using the RADWARE [98] suite of programs which allow general spectrum manipulation and for single and double gating whilst building a level scheme graphically using coincidences and intensities.

Data were also sorted into rings of detectors (equal $\theta$ in polar coordinates) to allow measurement of Directional Correlations from Oriented states (DCO ratios) which identified the transition multipolarity.

Since this study is primarily interested in the prompt decays from $^{91}$Zr the Doppler shift of $\gamma$ rays emitted by nucleons in flight or while slowing down in the target
Figure 5.2: Cross sections for various beam energies in MeV calculated with PACE4 [96]
5.6 CALIBRATION

Energy calibrations were performed initially by using $^{133}$Ba and $^{152}$Eu with x-ray lines being used for calibrating low energies [66]. The calibration spectra as used in experiment 2 is shown in figure 5.3.

After the initial calibration, an internal calibration using a gamma ray of known energy in $^{91}$Zr (2170 keV) was performed above the highest source $\gamma$-ray energy. This was shown to be consistent with the known energy and as such the energy calibration is thought to be good within this range.

The energy calibration was performed internally using known gammas in the data

Figure 5.3: Example energy calibration spectra for experiment 2 using $^{133}$Ba and $^{152}$Eu. Insert shows weak transitions in expanded area between 450 and 750 keV.

must be accounted for. Using the equations discussed in section 2.11 this equates to a possible $\gamma = \beta$ of 0.0124 assuming the target nucleus was stationary at impact.

Linear polarisation matrices were sorted which, as discussed in section 2.9 are matrices containing only events scattered between crystals in one plane. This allowed identification of electric or magnetic $\gamma$-ray character.

Some data sorting using SSORT [99] was also carried out in the calibration stages as this software features excellent multiple spectra building capabilities.
Figure 5.4: Gain drift of detector on adc channel 113 measured with 2170 keV peak from $^{91}$Zr

set, at every run for the whole data set. A large amount of gain drift was observed for 11 of the 40 individual crystals, possibly due to the recent movement of all clovers from a different target position and some movement introduced into the array from a newly fitted autofill system. A plot of this drift for one of the affected crystals is shown in figure 5.4.

Relative energy efficiency is measured in singles mode using known absolute intensities from $^{133}$Ba and $^{152}$Eu sources with addback enabled. The array is calibrated as a whole and also in its two rings of detectors, allowing calibration for total summed spectra and DCO analysis. The efficiency curves are shown in fig 5.5.
Figure 5.5: Top Panel: The efficiency curves for the total array. Middle Panel: Efficiency curve for 90° detector ring. Lower Panel: Efficiency curve for 140° detector ring. The curves are fitted using a least squares method to a coupled pair of exponential terms.
5.7 Experiment 1: Initial identification of $^{91,92}$Zr

Primary identification of $^{91}$Zr was achieved online in the test experiment by confirming known transitions below the $21/2^+$ isomer. These are visible in the total projection spectra 5.6 (lower panel). By looking at a gate on the 859 keV with no Doppler correction, $^{91}$Zr transitions below the $21/2^+$ isomer alone are visible (figure 5.6 middle panel).

In order to estimate how much spin the reaction had managed to create, the spectra for $^{92}$Zr were examined in detail. A gated spectra for $^{92}$Zr is presented in figure 6.4. Previously known transitions up to $I^\pi = (18^+)$ [21] are clearly visible. Consequently it is likely that mid-high lying states in $^{91}$Zr have also been populated.

With the largest reaction product predicted from PACE calculations (see section 5.4) being $^{91}$Zr, it was sensible to suggest that most of the new transitions arise from $^{91}$Zr. To confirm that they lie above the isomer timing information must be used. A matrix of early vs. delayed events was constructed using the timing information recorded within the $\gamma - \gamma$ data. By gating on the suspected new prompt transitions (early axis) $\gamma$-rays below the isomer on the projected (delayed) axis should be returned.

During the experiment two ready-made $\gamma$-ray coincidence cubes were analysed from a previous experiment [100] which contained $^{91}$Zr data. These had axes of prompt-prompt-delayed and delayed-delayed-prompt, allowing double gating and greater certainty that for the transitions being correctly placed.

Figure 5.7 shows three gates from these cubes taken from the $^{27}$Al + $^{178}$Hf fusion-fission data set. The upper panel shows delayed events (below the isomer) gated on the previously reported 2170 keV transition. This returns in the projection the 89, 289 and 850 keV transitions from $^{91}$Zr along with a weakly populated, unknown 880 keV transition. The middle panel shows a sum of gates on 89 and 859 keV $^{91}$Zr transitions demanded in coincidence with the 2170 keV $^{91}$Zr transition (delayed axis) projected onto a prompt axis. This effectively looks 'across' the isomer from the low lying delayed events to higher lying prompt events. Lines at energies of 128, 901 and 1545 keV are clearly visible. These match with high statistics spectral lines seen in the total projection.

Since the 128, 901 and 1545 keV lines are now thought to be in coincidence with
Figure 5.6: Top panel: A gate placed on the 1099 keV transition shows $^{92}$Zr is populated to the current highest known spin of $I^\pi = 18^+$. Middle panel: With Doppler correction turned off (ie assuming $\beta = 0$), a gate on the 859 keV transition returns transitions below the 3167 keV $^{31}_2$ isomer in $^{91}$Zr. Also weakly visible are unknown transitions suspected to be above the isomer (128, 305, 901 keV). Lower panel: A total projection of the data set, with an inset from 400-950 keV showing weaker transitions. Well populated, unknown gammas are suspected to be in $^{91}$Zr (128, 305, 901 and 1545 keV).
$^{91}$Zr transitions below the isomer it is prudent to 'gate back' by demanding coincidences between these new transitions and a transition below the isomer. Again placing the 128, 901 and 1545 keV transitions thought to be energetically above the isomer on prompt axes and the 2170 keV transition on a delayed axis, these project onto a prompt axis and again display the 128, 901 and 1545 keV transitions. Consistently showing them to be in $^{91}$Zr. A transition at 305 keV is also present and should be investigated as possible belonging to $^{91}$Zr.
5.7 EXPERIMENT 1: INITIAL IDENTIFICATION OF $^{91,92}$Zr

Figure 5.7: $^{178}$Hf on $^{27}$Al fusion-fission data. Top panel: Gating on the delayed 2170 keV transition in $^{91}$Zr and projecting a delayed axis shows transitions below the isomer in $^{91}$Zr. Middle panel: Gating on a delayed axis (2170 keV) and a sum of two gates (859 and 90 keV) on another delayed axis and projecting a prompt axis returns prompt events in $^{91}$Zr (128,901,1545). Lower panel: To confirm the new transitions they are included now as part of the gating conditions to see if they return themselves. The 128 and 1545 are clearly present. A 305 keV transition is also suggested.
5.8 Timing

In order to clean the spectra of unwanted events a number of techniques involving the timing information present with the data can be employed. The first involves the fact that this experiment is concerned primarily with prompt events. By constructing a matrix of γ-ray energy vs. time, the structure of spectra with differing timing gates may be explored. To this end the spectra displayed in figures 5.8 and 5.9 were constructed showing the transitions present in the differing timing regimes observed in the top panel of figure 5.8. The 'prompt peak' is expected to be the narrow region containing most of the counts and labelled as section 6. This is shown to be correct by simple inspection of gate 6 in figure 5.9 (the reader is directed to look particularly at the ratio of the delayed 860 keV and prompt 901 keV transitions from $^{91}$Zr).

The two regions labelled as 5 and 7 either side and representing the pre-prompt and 'early-delayed' still contain an abundance of prompt events however. One explanation is that these are true random events, the effects of which are visible due to the sheer volume of data available. Another explanation comes from L. Williams [74] who conducted further development work on the Yale array. It was discovered that individual leaf times had become inconsistent since they were last checked. This means that if a single pulser signal is taken and patched into two leaf inputs from a particular clover two distinct (NOT simultaneous) signals would be seen at the CFD level for those two leaves. When the timing for the Yrastball array was overhauled it was also discovered that the coincidence conditions for many detectors were inconsistent. This means that the logic pulses produced by the OR unit within each clover which feed into the master trigger multiplicity unit are slightly different lengths. With the production of a master trigger depending upon the overlap of two of these pulses this will cause a preferential trigger condition for detectors with longer pulses. Due to the possible random nature of these events in sections 5 and 7 it was decided not to include them in a gate to select prompt events.

In order to reduce random events and enhance transitions which are part of a consecutive chain of decays (with no isomers) a condition on requiring the coincident γ rays to be coincident within a smaller window than the main event window of 3.2 μs can be set. Since cascades of γ rays will often decay in the order of 100 ns this is the
Figure 5.8: Top panel: timing spectrum total projection showing the event-spread about the prompt peak (centered at channel 1500). Differing sections of note are labelled and have timing cuts made of them - see panels labelled 1-4 and also 5-9 on the following page for the corresponding γ-ray projections.
Figure 5.9: $\gamma$-ray projections made from timing cuts placed on sections labelled in figure 5.8.
coincidence region to be investigated. Figure 5.10 shows the resulting total projection spectra for no timing conditions, then three timing regimes which demand coincidences between any pair of $\gamma$ rays to be incremented into a matrix to be 80, 120, or 160 ns. The peak to background ratio for all three timing settings is about the same, displaying an average 2% increase compared to the data with no timing conditions imposed. Overall statistics however are sharply affected with the 80, 120 and 160 ns settings containing 49, 67 and 80% of the total statistics respectively. Random peaks such as the 596 keV $^{74}$Ge n-n' peak do not appear significantly affected and their intensity decreases only proportionally with the total statistics. The ratio of prompt / delayed counts measured between the 901 and 290 keV transitions from $^{91}$Zr increases by 17% over the total projection for each of the time difference conditions. Due to the increase in peak to background combined with relatively little loss in statistics, the 160 ns condition was chosen as a standard to be used.
Figure 5.10: Top panels: Right hand side is a total projection of the $\gamma - \gamma$ data with no conditions imposed. The left hand side shows the region of 550 to 750 keV, a convoluted section of the total projection. The three lower panels in descending order show the total projections with 80, 120 and 160 ns time difference restrictions imposed upon coincidences between any two $\gamma$ rays.
5.9 Experiment 2: $^{91}$Zr level scheme

The total projection spectra of figure 5.11 shows the success of the second experiment. During two weeks of beam time the beam current was kept stable for the first week at $\sim$15 enA which at a charge state of $4^+$ translates to 0.375 pnA. Once it was clear the experiment had already produced enough statistics to be successful the beam current was increased towards 0.5 pnA at which point the master trigger was reaching 4 kHz which is approaching the limit of the electronics handling ability. Therefore the master trigger was switched to a $\gamma - \gamma - \gamma$ coincidence gate (returning the master trigger to $\sim$800 Hz) and the beam current increased further to 1 pnA. Two data sets were therefore obtained. One with a doubles trigger and one with a triples master trigger. For the purposes of placing gates the two data sets can be summed and made into cubes. However, if the data is sorted into lower order constructs (1D,2D) the higher order triples events deconstruct to give incorrect intensities. The data sets must therefore be separated when making these measurements. As a result, the total projection contains $\sim 1.1 \times 10^9 \gamma - \gamma$ events.

The near yrast level scheme for $^{91}$Zr [20] has been greatly extended over previous knowledge (see figure 1.5 for previously known yrast scheme). With the aid of the RADWARE [98] suite of programs, particularly the 2D and 3D data manipulation packages XMESC and XMLEV the level scheme above the $^{91}$Zr $^{31}_{2}$ 3167 keV isomer shown in figure 5.12 has been built up.

Transitions identified in experiment 1 are confirmed by the spectra shown in figure 5.13 which are made from a prompt-delayed matrix. These show a total projection of the delayed axis and a prompt 90 keV gated projection in the upper and lower panels respectively. The lower panel therefore highlights transitions in $^{91}$Zr.

To achieve a consistent level ordering for newly discovered transitions, many gates were placed and the projected spectra analysed for coincidences and null-coincidences. Intensity balances were used to ensure as much feeding as possible was identified. Previously identified gammas were eliminated by importing level schemes for known nuclei populated in the current reaction.

As proof of the robustness of the level scheme, selected coincidence spectra are shown here in figures 5.14, 5.15, 5.16. The first few transitions above the isomer (1545,
Figure 5.11: Top panel: The total projection of all 2 weeks of data from the $^{13}$C($^{82}$Se,xn$\gamma$)$^{95-n}$Zr thick target experiment. Middle panel: $\gamma - \gamma$ coincidence data only, total projection. Lower panel: $\gamma - \gamma - \gamma$ coincidence data only, total projection.
Figure 5.12: Level scheme for $^{91}$Zr constructed in the current work. All transitions above the $^{21+}$ 3167 keV level are new and result from this study.
Figure 5.13: Prompt-delayed matrix spectra: Using a prompt-delayed matrix corresponding to events within the prompt peak labelled as section 6 in the top panel of figure 5.8 and a delayed axis corresponding to sections labelled 8 and 9 in the top panel of figure 5.8. Top panel: a total projection of the delayed axis with $^{91}$Zr transitions labelled. Transitions below the $^{91}$Zr isomer are clearly visible. Lower panel: gating on the 90 keV transition below the $^{1/2}^+$ isomer returns transitions above the isomer in $^{91}$Zr.
901, 2141, 305, 128) were placed according to intensity as in [101] and were used as a starting point.

Figure 5.11 shows the total projection with most prominent peaks ascribed to known transitions in well populated nuclei from the reaction. transitions and leaving many that are later placed as $^{91}$Zr transitions. An example is the top panel which shows a double gate on both 128 and 1905 keV lines. The main known $^{91}$Zr transitions are still present, confirming the 1905 (7645 $\rightarrow$ 5741 keV) to be in $^{91}$Zr. From other double gates made in combination with the 128 keV transition it is also likely that the 707, 935, 1243, 1401, 1545, 1160 keV transitions are present in $^{91}$Zr.

Having found many of the transitions around the 128 keV (5741 $\rightarrow$ 5613 keV) and placed them according to coincidences, null coincidences and intensity, double gating is used to select $\gamma$ cascades and look high in spin, up the various decay branches. Once a sensible conclusion was reached for a section of the scheme, a double gate was placed on the two highest lying transitions. The resultant spectra should contain, within the limits of statistics, all of the transitions which are fed by the two gated transitions.

A double gate on 404 and 1710 keV transitions (12773 $\rightarrow$ 12369 and 12369 $\rightarrow$ 10659 keV respectively) shown in figure 5.15 (top panel) yields the transitions heading back toward the isomer.
Figure 5.14: Top panel: A double gate on 305 and 1545 keV to see transitions feeding into the 5613 keV level of $^{91}$Zr, Middle panel: A double gate on 773 and 935 keV to show high spin levels on the left hand side of the level scheme shown in figure 5.13. Lower panel: A double gate on 418 and 1547 keV transitions to highlight transitions on the right hand side of figure 5.13.
Figure 5.15: Top Panel: Double gate on high lying 404 and 1710 keV transitions to show the likely yrast $\gamma$-cascade down to the $\frac{31}{2}^+$ isomer in $^{91}$Zr. Middle Panel: A double gate on the 2575 and 1904 keV transitions shows only the higher lying 1244 keV $\gamma$ ray, confirming the placement of the 2574 keV transition. Lower Panel: A double gate on the 452 and 694 keV transitions displays the large number of transitions that the 7213 keV level feeds.
Figure 5.16: Single gates shown in the top panel: 374 keV, middle panel: 418 keV and lower panel: 2501 keV showing less restricted decay paths than the double gated spectra. In the upper two spectra particularly points where two decay branches split and converge respectively.
5.10 Experiment 2: $^{92}$Zr level scheme

The nucleus $^{92}$Zr has previously been investigated to a spin of $(18^+)$ [21] through the fission of $^{197}$Pb formed in a $^{24}$Mg + $^{173}$Yb reaction at $E(^{24}$Mg$)=134.5$ MeV. The level scheme from this study was used as the basis for the analysis of this nucleus. The investigation of $^{92}$Zr using the same methodology as for $^{91}$Zr produces the level scheme shown in figure 5.17.

The level scheme has been extended higher in spin and side feeding branches have been discovered. Previous tenuous levels are confirmed. Figures 5.18 and 5.19 are offered as proof for the current level scheme.
Figure 5.17: Level scheme for $^{92}$Zr as deduced in the current work, built upon the level scheme of Fotaides et al. All non-Yrast levels above $I^* = 14^+$ were discovered in the current work. The 10276 keV $I^* = 20^+$ level is also newly found.
Figure 5.18: Top panel: A gate on the high lying 2004 keV transition shows the transitions to the ground state. Middle panel: A double gate on the two lowest lying transitions displaying all the main yrast transitions. Lower panel: A double gate on high lying 1326 and 1683 keV transitions illuminating the fact that the 1326 and 1145 keV gammas are not in series due to a null coincidence.
Figure 5.19: Top panel: A double gate on 1145 and 1099 keV transitions shows clearly the yrast decay branch. Middle panel: A double gate on 1401 and 1845 keV transitions confirms the level ordering in transitions feeding the 7448 keV level. Lower panel: A double gate on 1401 and 1007 keV transitions to show transitions feeding in parallel to the 1845 keV transition.
5.11 Directional Correlations from Oriented States (DCO) measurements

Experimental data was sorted into coincidence matrices with axes that corresponded to the 90° and 140° rings of detectors. The DCO ratio for a particular transition was obtained through the following equation,

\[
R_{DCO} = \frac{I(140°) \text{ gated at } 90° \times \varepsilon_g(140°) \times \varepsilon_p(90°)}{I(90°) \text{ gated at } 140° \times \varepsilon_g(90°) \times \varepsilon_p(140°)}
\]

where \( I \) is the number of counts in a particular peak and \( \varepsilon_g \) is the efficiency of the gate at that energy and \( \varepsilon_p \) is the efficiency of the projection at that energy.

Equation 5.1 shows that the \( R_{DCO} \) depends explicitly upon the multipolarity of the gating transition such that [77],

\[
R_{DCO}[\text{quadrupole \leftrightarrow quadrupole}] = R_{DCO}[\text{dipole \leftrightarrow dipole}]
\]

\[
R_{DCO}[\text{quadrupole \leftrightarrow dipole}] = R_{DCO}[\text{dipole \leftrightarrow quadrupole}]
\]

when the transitions involved are pure (\( \delta_1 = \delta_2 = 0 \)) and stretched (\( J_1 - J_2 = \lambda_1, J_2 - J_3 = \lambda_2 \)) in character.

Therefore, gating on a known transition, the differing intensities of the projected intensities provides information on the nature of the multipolarities of other transitions.

In order to test the method worked it was sensible to look at a well populated and well known nucleus in the current data set. For this \(^{88}\text{Sr}\) was used. It also has the benefit of three different multipolarity transitions in its lower lying states (figure 5.20) which allowed for a good example of DCO values when the gating and projected transitions are of differing multipolarity. Fig 5.21 shows the 851 keV E2 and 898 keV E1 gated by the 1837 keV E2 transition. The DCO ratio is clearly almost unity and therefore the 851 keV transition is also E2, which is correct [102, 61]. figure 5.22 then shows the same test carried out on transitions in \(^{91}\text{Zr}\).
Figure 5.20: A truncated level scheme for $^{88}\text{Sr}$ (from ref [103]). The differing multipole of the low lying transitions allow its use as a good testing ground for DCO ratios.
Figure 5.21: Example of gated DCO ratio spectra as proof of method for $^{88}$Sr. The red line is provided as an aid to observe the change in intensity of the 898 keV E1 transition when compared to the 851 keV E2 transition. It can be seen in this instance that if the 898 keV transition was of unknown multipolarity, the DCO would not be equal to unity and an E2 could therefore be ruled out.
Figure 5.22: Example of gated DCO ratio spectra as proof of method for $^{92}$Zr.
5.12 Linear polarisation measurements

Polarisation matrices were made of Unpolarised Vs. Perpendicular (to beam axis) gammas and Unpolarised Vs. Parallel as described in section 2.8. The asymmetry in the scattering data describing electric or magnetic nature will be greatest at 90° to the beam axis. Therefore only the ring of detectors at 90° were included in the analysis to make the polarisation results as clear as possible.

A test was performed on the well studied nucleus $^{88}$Sr to be sure the method worked (figure 5.23). As can be seen the known electric transition at 851 keV is a positive peak in the spectrum and the known magnetic transition at 435 keV is negative as expected.
A polarisation plot for $^{91}$Zr is also shown in fig 5.24. The plot is used purely for demonstration to the reader. To obtain the real asymmetry term $A$, the individual spectral peaks are fitted and integrated over area. Errors arise mainly from efficiency calibrations, statistical errors and the fit obtained to the peak.

### 5.12.1 Experimental limits for linear polarisation measurements

An interesting aside on linear polarisation measurements is the experimental limit imposed by the predominance of scattering only at medium $\gamma$-ray energies ($\sim$400-900 keV). Lower than this and photoelectric absorption dominates, higher and pair-production dominates [66]. These effects can be seen in the current data, fig 5.25.
Figure 5.25: Experimentally observed limits of polarisation measurements. It can be seen that there are too few statistics to usefully make any measurements below $\sim 400$ keV.
5.13 $^{91}$Zr tabulated results

See table 5.1 showing for all newly discovered transitions: $E_{\gamma}$ (to nearest keV), $I_l(h)$, $I_R(h)$, $E_i$ (initial level energy in keV), $E_f$ (final level energy in keV), $I_{rel.}$ (relative intensity), Polarisation result, DCO result and DCO$_{gate}$ used.

5.14 $^{92}$Zr tabulated results

See table 5.2 showing for all newly discovered transitions: $E_{\gamma}$ (keV), $I_l(h)$, $I_R(h)$, $E_i$ (initial level energy in keV), $E_f$ (final level energy in keV), $I_{rel.}$ (relative intensity), Polarisation result, DCO result and DCO$_{gate}$ used.
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Table 5.1: Experimentally determined transitions in $^{91}$Zr from the current work, part A of table. Figures in brackets are uncertainties.
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Table 5.2: Experimentally determined results for $^{91}$Zr cont'd (part B). Figures in brackets are uncertainties.
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<th>( E_{\gamma} ) (keV)</th>
<th>( I_I (h) )</th>
<th>( I_F (h) )</th>
<th>( E_I ) (keV)</th>
<th>( E_F ) (keV)</th>
<th>( I_{rel.} )</th>
<th>( P )</th>
<th>( R_{DCO} )</th>
<th>( DCO_{gate} )</th>
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<td>1601</td>
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<td>( \frac{25^-}{2} )</td>
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<td>( \frac{27^-}{2} )</td>
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<td>3167</td>
<td>7.5 (7)</td>
<td>--</td>
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Table 5.3: Experimentally determined results for \(^{91}\text{Zr}\) cont'd (part C). Figures in brackets are uncertainties.
Table 5.4: Experimentally determined transitions in $^{92}$Zr from the current work. Figures in brackets are uncertainties.
Chapter 6

Discussion

6.1 Assignment of spins and parities in $^{91}$Zr

6.1.1 A suitable place to start

In order to assign spins and parities to the newly discovered transitions above the $I^w = \frac{21}{2}^+$ at $E_{ex} = 3.167$ MeV it is preferable to have a starting point. To begin with, it is sensible to suggest that 4 of the levels of interest, marked 1-4 in figure 7.5 arise from the alignment of a single neutron in the low lying $1h_{11/2}$ orbital coupled to two maximally aligned protons in the $1g_{9/2}$. This would allow for spins of $\frac{23}{2}^-, \frac{25}{2}^-, \text{or} \frac{27}{2}^-$ (neutron core breaking may be important also but this is explored later). The preliminary assignment of the level (labelled 4) at 5741 keV as a possible $\frac{27}{2}^-$ is supported by the 2574 keV ($E_{ex} = 5741 \rightarrow 3141$ keV) transition which spans 3 neighbouring transitions to decay directly to the $\frac{21}{2}^+$ isomer. Together with the fact that yrast transitions are usually stretched in nature this would make the 2574 keV ($E_{ex} = 5741 \rightarrow 3141$ keV) $\gamma$ ray a candidate to be of electric octupole character. An E3 transition of this nature would have a Weisskopf single particle lifetime of 3.3 ns. In order to assess the plausability of this line of reasoning the multipolarity of the 128 keV ($E_{ex} = 5741 \rightarrow 5613$ keV) must be established.
6.1 ASSIGNMENT OF SPINS AND PARITIES IN $^{91}\text{Zr}$

6.1.2 The multipolarity of the 128 keV ($E_{ex}=5741 \rightarrow 5613$ keV) transition

For a transition of energy 128 keV a lower order multipole is suspected. If the assumption that yrast transitions are stretched is true then the multipole order is likely to equal one as there are three strong and therefore probably yrast transitions spanning the same $\Delta I$ as the suspected E3. From part B of appendix table A the most suitable competing lifetime belongs to an M1 transition (11 ps).

A simple experimental procedure to narrow the search would be to perform a linear polarisation measurement which would provide the E or M character. However, at this low energy photon absorption dominates and no statistically significant scattering events are recorded. The low energy of the transition, whilst being too low to allow linear polarisation measurements is low enough to give the transition a substantial electron conversion component. Future investigation may look specifically at internal conversion coefficients on this nucleus to aid in spin/parity assignment.

From the DCO ratios another useful piece of information can be gained in this matter. By gating on the 901 keV ($5613 \rightarrow 4712$ keV) transition the DCO ratio for the 128 keV transition is obtained to be $1.03 \pm 0.01$. This confirms that the sequence of $\gamma$ rays $128 \rightarrow 901 \rightarrow 1545$ are likely stretched dipole radiation.

In a recoil doppler shift (RDS) experiment performed recently at WNSL Yale at the time of writing [104] the lifetime for the decay of the 5741 keV level was deduced to be $46.1 \pm 5$ ps. This results is less than a factor of two different from the Weisskopf single particle lifetime estimate of $1.05 \times 10^{-11}$ s (see table A confirming the assignment of the 128 keV transition as M1.

6.1.3 2574 keV transition and consequences for levels fed by the 5613 keV level

Having established the 128 keV ($E_{ex}=5741 \rightarrow 5613$ keV) transition to be M1 and the 2574 keV ($E_{ex}=5741 \rightarrow 3141$ keV) to likely be E3, first tentative spin/parity assignments can be made for the 5613 and 5741 keV levels as $\frac{25}{2}^-$ and $\frac{27}{2}^-$ respectively.

Following from the above arguments, linear polarisation measurements show the
305 keV ($E_{ex}=5613 \rightarrow 5308$ keV) transition to be of predominantly magnetic character $P=-0.083 \pm 0.002$. A DCO measurement using the M1 128 keV transitions as a gate produces a ratio of $1.00 \pm 0.01$. This rather unambiguously makes the 305 an M1 and the 5308 keV level correspondingly $^{23/2}$. 

In a similar simple manner to the 305 keV transition, the 901 keV ($E_{ex}=5613 \rightarrow 4712$ keV) transition has a DCO ratio of $0.95 \pm 0.03$ (gate on 128 keV M1) and a strongly electric character. The assignment is therefore E1 and allows the 4712 keV level to be labelled as $^{33/2}$. 

6.1.4 1545 keV ($E_{ex}=4712 \rightarrow 3167$ keV) and 2141 keV ($E_{ex}=5308 \rightarrow 3167$ keV) transitions

Having established the 128 keV transition as of M1 character it is straightforward to use DCO ratios on the transitions below to establish the spin of the 5741 keV level. This of course is only true if they are pure dipole transitions. The 2141 keV transition is found to be of electric ($P=0.030 \pm 0.001$) and also dipolar ($R_{DCO} = 0.93 \pm 0.06$), confirming the assignments made so far.

The 1545 keV transition has a DCO ratio of $2.61 \pm 0.04$ which indicate the possibility of mixed multipolarity. The strong magnetic polarisation result ($-0.099 \pm 0.004$) may suggest an M1/E2 admixture when examined in conjunction with the spin change ($\Delta I=1$). However, folded higher order multipolarities cannot be ruled out. More data is currently being analysed from the recent RDS experiment at Yale to elucidate this [104].

In this region of the level scheme the only unknown is the 596 keV ($E_{ex}=5308 \rightarrow 5471$ keV) transition. Because it is convoluted with the 596 $^{74}$Ge n-n' peak neither DCO nor linear polarisations are possible. The initial and final levels are already established so this is not a problem. $I^{\pi}_{initial} \rightarrow I^{\pi}_{initial} = ^{23}_{2}^{-} \rightarrow ^{23}_{2}^{+}$ making the 596 keV transition a likely candidate for a folded E1.
6.1 ASSIGNMENT OF SPINS AND PARITIES IN $^{91}$Zr

6.1.5 Transitions between 5613 and 7645 keV (right hand side)

Determining the multipolarity of the 1904 keV ($E_{ex}= 7645 \rightarrow 5714$ keV) transition at first seems problematic. The $R_{DCO}$ is $1.88 \pm 0.04$ using a dipole gate and no currently identified E2 is available within the same gate to assess a possible quadrupole nature. The parallel 872 keV ($E_{ex}= 7645 \rightarrow 6773$ keV) also has a similar $R_{DCO}$ of $1.74 \pm 0.2$. Both these transitions have electric character with $P=0.064 \pm 0.004$ and $0.100 \pm 0.005$ for the 1904 and 872 respectively. The 1904 keV transition from consideration of $\Delta I$ in the region which is $\geq 1$ and Weisskopf lifetimes which limit the multipolarity to $\leq 3$ for an electric transition must be either E1, E2 or E3. Comparing Weisskopf single particle lifetimes between the 1904 and 872 keV transitions does not aid greatly as even though the DCO ratios suggest they have the same multipolarity E1, E2 and E3 are all shown to be competitive to the same degree.

The assignment decision comes from the three weak transitions in parallel (the 261, 1260 and 1626 keV) which span between 8889 and 5741 keV. These are all dipolar (table 5.13) fixing the 8889 keV level to $I=\frac{33}{2}$. The 1244 keV ($E_{ex}= 8889 \rightarrow 7645$ keV) transition is an E1 with an $R_{DCO}$ of $1.18 \pm 0.23$ (This DCO value is 18% from unity but along with the lack of transitions which branch out from the 889 keV level and back into the 7645 level as might be expected if the 1244 keV transition was an E2 the E1 assignment is considered safe). The 7645 keV level may now be assigned as $I=\frac{31}{2}$, making the 1904 keV an electric quadrupole transition and so setting the parity for the $I=\frac{31}{2}$ level at 7645 keV as negative and the $I=\frac{33}{2}$ level at 8889 keV as positive.

The rest of this region of the level scheme is more difficult to make spin/parity assignments for due to the convoluted nature of the region in which some of these transitions lie resulting in particular, limited DCO information. Two transitions for which the available gates produce clean spectra with good statistics are the 1273 keV ($E_{ex}= 7014 \rightarrow 5741$ keV) and 241 keV ($E_{ex}= 7014 \rightarrow 6773$ keV) transitions which both appear as dipoles ($R_{DCO} = 0.98 \pm 0.08$ and $1.11 \pm 0.13$ respectively). The 1273 keV transition has a P value of $0.029 \pm 0.002$ which makes it an M1 and also shows the 7014 level to be $I^x = \frac{29}{2}^-$. The 1401 keV ($E_{ex}= 7014 \rightarrow 5613$ keV) transition has an electric character with $P=0.118 \pm 0.014$. The $R_{DCO}$ for this transition is $2.06 \pm 0.13$. As there
is no known higher order multipole to gate on for DCO analysis of the 1401 keV $\gamma$ ray all that can be said directly is that it is not a dipole. However this transition spans a region of $\Delta I = 2$ and so is suspected as an E2. For the 1273 and 1401 keV transitions to compete and produce the similar relative intensities displayed ($I_{el} = 14.4 \pm 0.6$ and $17.3 \pm 0.7$ respectively) they should have comparable lifetimes. The Weisskopf estimates put the 1273 keV M1 transition an order of magnitude quicker than the 1401 keV E2. This is thought to be acceptable considering that at $I = \frac{29}{2}$ the single particle ideal is unlikely to be perfectly applicable.

The 241 keV transition being dipole would infer that the 6773 keV level is $I = \frac{27}{2}$. 241 keV is too low in energy to obtain a linear polarisation measurement so the E or M nature must be determined later. The 872 keV ($E_{ex} = 7645 \rightarrow 6773$ keV) transition is now a likely E2 candidate because of its strong electric nature ($P = 0.100 \pm 0.015$) and ‘non-dipole’ $R_{DCO}$ of 1.74 $\pm 0.19$ along with the $\Delta I$ between the 7645 keV $\frac{31}{2}^-$ and 6773 keV, $I = \frac{29}{2}$ levels. This would also infer that the 6773 keV, $I = \frac{29}{2}$ level is negative parity and so the 241 keV transition is magnetic. The $E_{ex} = 6773 \rightarrow 5613$ keV transition is shown to be magnetic with $P = -0.97 \pm 0.014$, $\Delta I = 1$ and so 1160 keV is a likely M1. The 1032 keV ($E_{ex} = 6773 \rightarrow 5741$ keV) transition is left to be assigned as a folded transition with electric nature and non-dipole $R_{DCO}$, M1/E2 is suggested.

As already mentioned in the discussion of the assignment for the 7645 keV $I^r = \frac{31}{2}^-$ level, the three transitions on the far right of the level scheme in figure 5.9 261, 1260 and 1626 keV are all dipole. The 261 and 1626 keV both have electric character ($P = 1.05 \pm 0.25$ and $1.05 \pm 0.19$ respectively). This leaves the 1260 keV ($E_{ex} = 8628 \rightarrow 7367$ keV) to be assigned as an E1 to conserve parity.

The three remaining transitions in the group are the 418 keV ($E_{ex} = 8889 \rightarrow 8471$ keV) which feeds both the 1104 keV ($E_{ex} = 8471 \rightarrow 7367$ keV) and 1457 keV ($E_{ex} = 8471 \rightarrow 7014$ keV). These transitions have $R_{DCO}$ ratios of 0.80 $\pm 0.2$ 0.87 $\pm 0.4$ 0.72 $\pm 0.4$ respectively. Each has within errors the the possibility of being dipole and all are significantly lower in value than the current known quadrupole values. Together with the $\Delta I = 2$ for these three transitions arranged in a fork it is highly likely they are all dipolar. The 418 and 1457 keV transitions are M1 and E1 respectively when combined with the DCO ratios. The 8471 keV is therefore assigned $I^r = \frac{31}{2}^-$ and the 1104 keV
transition for which no polarisation information is available is therefore of magnetic nature.

One more transition which is tentatively placed in this region, the 246 keV \( \left(E_{ex} = 9135 \rightarrow 8889 \text{ keV}\right) \) has too few statistics to make any reliable spin/parity assignment.

### 6.1.6 Transitions between 3617 and 9465 keV (left hand side)

Starting with the 1601 keV \( \left(E_{ex} = 7213 \rightarrow 5613 \text{ keV}\right) \) transition, which from an \( R_{DCO} \) of 1.00 ±0.17 and a polarisation result of -0.25 ±0.1 is shown to be an M1 it is possible to assign the 7213 keV level as \( I^\pi = \frac{37^-}{2} \). The neighbouring 1924 keV \( \left(E_{ex} = 7665 \rightarrow 5741 \text{ keV}\right) \) transition is also dipole \( \left(R_{DCO} = 0.86 \pm 0.08\right) \) and electric \( \left(P = 0.17 \pm 0.05\right) \) which infers the 7665 keV level to be \( I^\pi = \frac{39^+}{2} \). The 452 keV transition has errors of more than 50% on both the \( R_{DCO} \) and polarisation measurements but can now be deduced as an E1.

The 694 keV \( \left(E_{ex} = 8359 \rightarrow 7665 \text{ keV}\right) \) transition is convoluted with the random events of the \(^{72}\text{Ge n-n'}\) peak and so its properties must be deduced from establishing the \( I^\pi \) for the feeding level at 8359 keV. This can be done by determining the characteristics belonging to the 530 keV \( \left(E_{ex} = 8889 \rightarrow 8359 \text{ keV}\right) \) transition which ties the band to the left hand side of the level scheme. A suitable DCO gating transition with sufficient statistics is found in the 576 keV \( \left(E_{ex} = 9465 \rightarrow 8889 \text{ keV}\right) \) transition which itself has an \( R_{DCO} \) of 0.95 ±0.10 when examined with a 901 keV dipole gate. The 530 keV transition by these means is found to have an \( R_{DCO} \) of 1.10 ±0.05 and \( P = -0.051 \pm 0.008 \) showing it to be an M1. This leads to assignment of the 8359 keV level as \( I^\pi = \frac{31^+}{2} \) which also allows the 694 keV transition to be labelled as M1.

The remaining transitions below the 7213 keV \( I^\pi = \frac{37^-}{2} \) level do not have any DCO results. This is due to the limited statistics of these weak transitions, a problem compounded by the higher-lying and therefore weak, gates which have to be used to determine them. The polarisation measurements are unrestrained by the choice of gate affecting the polarisation result and so some success is achieved with the 2502 keV \( \left(E_{ex} = 7213 \rightarrow 4712 \text{ keV}\right) \) determined to be magnetic.
6.1 ASSIGNMENT OF SPINS AND PARITIES IN $^{91}$ZR

6.1.7 Transitions between 8359 and 12135 keV (left hand side)

Moving to the 880 keV ($E_{ex} = 10068 \rightarrow 9188$ keV) and 829 keV ($E_{ex} = 9188 \rightarrow 8359$ keV). These two transitions were ordered according to their relative intensity. It can be seen that both transitions are electric, with linear polarisation values of 0.155 ±0.089 and 0.027 ±0.007 respectively. The DCO value for the 880 keV transition is 0.80 ±0.34 and 0.61 ±0.64 for the 829 keV. From this the most likely multipolarity for both transitions is E1, and so the 9188 keV level is tentatively assigned $I^\pi = \frac{37}{2}^+$. For the 936 keV ($E_{ex} = 10068 \rightarrow 9132$ keV) transition an $R_{DCO}$ of 1.08 ±0.12 is recorded, conforming the spin assignment for the 10068 keV level. The polarisation result is inconclusive (0.002 ±0.002) and so the parity of the 10068 keV $I^\pi = \frac{35}{2}^-$ is decided by the result of the 880 → 829 keV branch and must therefore be tentative.

Transitions feeding the 10068 keV $I^\pi = \frac{35}{2}^+$ level are ordered according to relative intensity. The 374 keV ($E_{ex} = 10441 \rightarrow 10068$ keV) transition has a clear magnetic dipole nature ($R_{DCO} = 0.94 \pm 0.07$ and $P = 0.041 \pm 0.003$) which leads to the 10441 keV level being labelled as $I^\pi = \frac{37}{2}^+$. The 723 keV ($E_{ex} = 11164 \rightarrow 10441$ keV) transition does not yield an $R_{DCO}$ value with a small enough error to be meaningful. However the polarisation result of $P = -0.047 \pm 0.008$ suggests a magnetic nature. No polarisation or DCO results were able to be obtained for the 971 keV ($E_{ex} = 12135 \rightarrow 11164$ keV) or 474 keV ($E_{ex} = 11638 \rightarrow 11164$ keV) transitions due to limited statistics.

6.1.8 Remaining transitions between 9465 and 13872 keV

The decay chain of 1710 → 404 → 1194 → 546 → 552 → 576 has been ordered entirely on the basis of relative intensities. The lower lying three transitions from 8889 keV to 10564 keV are all found to be dipoles. The 1194 keV ($E_{ex} = 11758 \rightarrow 10564$ keV) transition has an $R_{DCO}$ of 0.96 ±0.11 when gating on the 1904 keV E2. Only the 576 keV dipole has a measurable polarisation of $P = 0.117 \pm 0.006$. This enables spin and parity assignment for the 9465 keV level of $I^\pi = \frac{35}{2}^+$ and spin assignments for the 10018, 10564, 11758 and 12162 keV levels to be made of $I = \frac{37}{2}$, $I = \frac{39}{2}$, $I = \frac{43}{2}$ and $I = \frac{45}{2}$ respectively. No results were obtainable for the highest lying 1710 keV ($E_{ex} = 12162 \rightarrow 113872$ keV) transition in the current work.
Two transitions remain to be described. The weak branch of 935 keV ($E_{ex}= 10531 \rightarrow 9596$ keV) and 707 keV ($E_{ex}= 9596 \rightarrow 8889$ keV). These are ordered from the measured relative intensities. However it should be noted that the intensity was convoluted with that of the 936 keV transition in a neighbouring branch and so has large uncertainties associated with it. With this in mind, a gate on the 1244 keV transition shows the intensity of the 707 keV to be three times that of the 935 keV providing confirmation of level order. Both the 707 keV and 935 keV transitions are relatively weak and only the 707 keV transition yields a DCO ratio ($0.70 \pm 0.3$). The uncertainty on this measurement is large and with no supporting information a spin assignment for the 9596 keV and hence the 10531 keV levels is not possible. Both transitions do record electric character however with $P = 0.037 \pm 0.004$ and $0.013 \pm 0.002$ for the 707 and 935 keV transitions respectively.

6.2 Assignment of spins and parities in $^{92}$Zr

6.2.1 $I^\pi$ confirmation for previously observed transitions with tentative assignments

Previous study of $^{92}$Zr [105, 106] has yielded mostly yrast transitions up to $I^\pi = (18^+)$. The assignments are tentative from $12^+$ to $18^+$ but in this current work the spin assignments have been confirmed along with the parity for the 1099 and 1401 keV transitions also. The 1099, 1401 and 1683 keV transitions have $R_{DCO}$ values of $1.04 \pm 0.07$, $1.06\pm0.07$ and $0.98 \pm 0.3$ respectively when examined with a 561 keV E2 gate and the 1099 plus 1401 keV have $P = 0.081 \pm 0.024$ and $0.010 \pm 0.002$.

6.2.2 New transitions feeding into the $I^\pi = 14^+$ and $16^+$ levels

Beginning with the lower lying section of the newly observed structure feeding into the yrast band sets a starting point of the 1361 keV ($E_{ex}= 7408 \rightarrow 6047$ keV) transition. An electric polarisation result ($P = 0.086 \pm 0.045$) together with an $R_{DCO}$ of $1.01 \pm 0.2$ suggests an E2 transition and assignment of the 7408 keV level as $I^\pi=16^+$. The feeding transition of 1047 keV ($E_{ex}= 8455 \rightarrow 7408$ keV) is clearly of quadrupole
6.2 ASSIGNMENT OF SPINS AND PARITIES IN $^{92}$Zr

nature ($R_{DCO} = 1.05 \pm 0.11$) and the polarisation result is positive ($P = 0.118 \pm 0.051$) revealing it to be an E2. This solid assignment of the 8455 keV level as $I^\pi = 18^+$ is useful for deciding upon the multipolarity of the 1007 keV ($E_{ex} = 8455 \rightarrow 7448$ keV) transition. This is because the polarisation result has poor statistics and resides in an area of the spectra with complex background. The error on this measurement is therefore large ($P = 0.105 \pm 0.8$). It does however agree with the 8455 keV level being $I^\pi = 18^+$ as the DCO ratio indicates a quadrupole nature ($R_{DCO} = 1.00 \pm 0.06$).

New transitions higher in spin for $^{92}$Zr now become relatively weak, indeed the reaction cross section for $^{92}$Zr is \( \sim 20 \times \) less than that for $^{91}$Zr and so is to be expected (see chapter 5.4). Indeed the only transition remaining for which a multipolarity can be determined is the 1165 keV ($E_{ex} = 10453 \rightarrow 9293$ keV). This is likely an E2 due to an $R_{DCO}$ of 0.86 \( \pm 0.2 \) and $P = 0.262 \pm 0.04$.

DCO results which might confirm the spin for the highest lying level (10458 keV) by using the the 1326 and 2004 keV transitions. The results obtained are both small in value at 0.70 \( \pm 0.09 \) and 0.68 \( \pm 0.3 \). Thus the data points only to the unlikelihood of the 10458 keV level being $20^+$ as might be first suspected. A spin of 21 or 19 may result from these transitions being hexadecapole or dipole respectively. Weisskopf estimates (see appendix A A) shown the dipole transitions to be orders of magnitude more favourable suggesting the $I=19$ result for the 10458 keV level. No polarisation results were obtained for either transition due to lack of statistics. The final spin will have to be determined from consistency with other parallel transitions.

Since the 867 keV ($E_{ex} = 10458 \rightarrow 9592$ keV) and 299 keV ($E_{ex} = 9592 \rightarrow 9293$ keV) straddle the space spanned by the 1165 keV E2 transition it is likely that they are both E1 or M1 transitions. If they were both M1 this would make the 9592 keV level $18^-$, E1 transitions would make the 9592 keV level $18^+$. With the 10448 keV level suspected as $I=19$ and the 1165 keV ($E_{ex} = 10453 \rightarrow 9293$ keV) transition as E2, the 1845 keV ($E_{ex} = 9293 \rightarrow 7448$ keV) is therefore dipolar in nature.

The 1137 keV ($E_{ex} = 9592 \rightarrow 8455$ keV) transition has an $R_{DCO}$ of 0.84 \( \pm 0.1 \). No polarisation result is available due to lack of statistics. The DCO ratio is inconclusive for deciding multipolarity. The most likely outcome for the group of 299, 867, and 1137
keV transitions therefore is as all dipole, with the 1137 keV being a folded transition. No definitive decision as to the parity of the 9592 level can be made on this basis since the 867 keV γ ray lies in a convoluted region of the spectra and the 299 keV transition is too low in energy to obtain a polarisation result. The group of three transitions may therefore be reasoned in two ways. The 299 and 867 keV transitions as E1 and the 1137 keV transition as M1/E2 or, 299 and 867 keV transitions as M1/E2 and the 1137 keV transition as E1.

When comparing lifetimes of the 1165 and 867 keV transitions which are both of the order $1 \times 10^{-11}$ s if the 867 keV transition is an E1. This would fit with the similar measured relative intensities of $4.7 \pm 1.8$ and $4.6 \pm 0.6$ for the 1165 and 867 keV branches respectively. No direct experimental observation can confirm the 867 and 299 keV transitions as E1 because of insufficient statistics and particularly for the 867 keV γ ray, a convoluted spectra.

In order to determine the parity of the 10458 keV $I=19$ level, the choice of electric or magnetic transition type for the 1325, 1845 and 2004 keV dipole transitions is necessary. For consistency it is sensible to believe that they are the same multipolarity. DCO ratios for each are $0.7 \pm 0.09$, $0.93 \pm 0.19$ and $0.68, \pm 0.31$, all of which could be dipole but not all of which can be quadrupole (E3 transitions may also make a consistent argument but are unfavourable compared to lower multipolarities). Since no linear polarisation data is available for these transitions it seems sensible to assign dipole (E1) character in the basis that Weisskopf estimates suggest them to be two orders of magnitude more likely than M1/E2. The spin/parity of the 10458 keV is therefore assigned as $I^\pi = 19^(-)$. 

In order to proceed, the most reliable result which is for the 1137 keV ($E_{ex}= 9592 \rightarrow 8455$ keV) is used to further the next spin assignment. It has an $R_{DCO}$ of $0.84 \pm 0.1$. No polarisation result is available due to lack of statistics. This allows a tentative assignment of $I=20$ to be made for the 9592 keV level.
Chapter 7

Shell model calculations

7.1 Introduction

The validity of simple shell model descriptions to describe several nuclei in the mass-90 region has been reported by several authors [10, 107, 108]. Since the full many-body problem is currently impossible to solve computationally due to the enormous complexity of such a calculation involving 91 mutually interacting nucleons, it is essential to choose a restricted basis space in which to work. However, by choosing a limited basis it possible to cut out some potentially important degree of freedom, particularly when wishing to look at high excitation energy where excitations from lower lying orbitals may become important. What follows then is an essential review of the previous efforts to study nuclei relevent to $^{91,92}$Zr via shell model calculations.

7.1.1 First treatment of $\pi p_{1/2} - g_{9/2}$

Talmi and Unna - Nucl.Phys. 19, 225 (1960)

The first shell model treatment of N=50 nuclei was performed by Talmi and Unna [109]. For the $1g_{9/2} 2p_{1/2}$ model space they determined the interaction matrix elements using a least squares fit to the limited available data. Despite limited experimental information and imprecise binding energies, they were able to infer that seniority was a good quantum number (that it is conserved) in the identical-nucleon $1g_{9/2} - 2p_{1/2}$ configurations $0^+$ and $2^+$. 
7.1 INTRODUCTION

Cohen et al. - Phys.Lett. 10, 195 (1964)

The following N=50 study by S.Cohen et al [107] used the same proton \( 1g_{\frac{3}{2}} - 2p_{\frac{1}{2}} \) model space as Talmi and Unna to describe states up to \( 8^+ \) in \(^{90}\)Zr. A set of interaction parameters was produced that conserved seniority to a high degree. The two body matrix elements describing the seniority mixing between \( \nu = 1 \) and \( \nu = 3 \) states were around -30 keV, which is small compared to the typical two-body matrix elements of ten times this. The parameters in this case were determined by a least-squares fit to 20 data - excited-state energies in \(^{89}\)Y, \(^{90}\)Zr, \(^{91}\)Nb, \(^{92}\)Mo and \(^{93}\)Tc.

Ball et al. - Phys.Lett. 41B, 581 (1972)

In 1972 a review of the N=50 nuclei was made by Ball et al [110] to see if a better choice of effective interactions could be made over the previous set derived by Cohen et al. The reason was that at high spin in \(^{92}\)Mo and \(^{94}\)Ru both energies and lifetimes predicted by the model deviated from the experimental data.

Starting with a modified surface delta interaction using four parameters, 31 experimental energy levels from \(^{89}\)Y to \(^{94}\)Ru were fitted. With this interaction used as a starting point the 11 one and two-body matrix elements of the \( 1g_{\frac{3}{2}} - 2p_{\frac{1}{2}} \) valence space were fitted to a more complete set of 44 energy levels. Calculated energy levels in N=50 nuclei had an rms deviation of 53 keV, many new data, previously deviant from the theory were now accurately reproduced.


Following experimental research by Jaklevic et al. [111] in which some E2 lifetimes were measured it was found by Gloekner et al. that most of the new experimental data agreed well with predictions made by the older theoretical models. However, in \(^{94}\)Ru the newly measured \( 8^+ \rightarrow 6^+ \) transition rate was found to be overestimated by a factor of \( \geq 25 \) in the theory [112]. In an effort to understand this, four fits were made to 45 experimental data in the \( 1g_{\frac{3}{2}} - 2p_{\frac{1}{2}} \) space to produce four sets of interactions [113]. The interactions were individually aimed to reproduce either transition rates, total energy, excitation energy or seniority conservation. The transition rate fit was constructed so that the components of the \( 8^+ \) and \( 6^+ \) wavefucions cancel, allowing the E2 transition...
to proceed only via small components of the initial and final states. However, in order to produce this cancellation, the two-body interaction was built with a small amount of seniority violation. The authors proceeded to calculate many energy levels with all four fits. The rates fit had an rms energy deviation of 80 keV whilst the other three had a deviation of 55 keV.

Gloekner in 1975 [60] went on to increase the model space used to both the original proton $1g_{\frac{3}{2}} - 2p_{\frac{1}{2}}$ and also neutron $2d_{\frac{3}{2}} - 3s_{\frac{1}{2}}$ in order to study shell model systematics in the Zr and Nb region. Good agreement was reached at low spins. Interestingly the $3^{-}$ state in $^{90}$Zr along with many $7^{+}$ and $3^{-}$ states in the Zr isotopes were interpreted as containing much of the $g_{\frac{3}{2}}$ and $d_{\frac{1}{2}}$ single neutron strength as these states have no model analogues.


Ji and Wildenthal expanded the now commonly used $\pi 1g_{\frac{3}{2}} - 2p_{\frac{1}{2}}$ space to $\pi 1f_{\frac{3}{2}}, 2p_{\frac{3}{2}}, 2p_{\frac{1}{2}}, 1g_{\frac{1}{2}}$. This was due to the shortcomings of the currently available interaction in being limited to nuclei with $\geq$38 protons, whilst experimental data of the time had caught up with the theory and now calculations for recently measured nuclei such as $^{88}$Sr, $^{87}$Rb, $^{86}$Kr and $^{85}$Br were required. For nuclei close to $^{88}$Sr which has a relatively small sub-shell closure at Z=38, particle-hole effects which are ignored in the $1g_{\frac{3}{2}} - 2p_{\frac{1}{2}}$ space will effect transition strengths in nuclei close to $^{88}$Sr and the assessment of these effects was to be undertaken. Matrices were diagonalised using the shell-model code RITSSCHIL [114].

Ji and Wildenthal were subsequently able to treat nuclei from $^{78}$Ni to $^{100}$Sn with this new interaction. The effective interaction derived for this region was found by iteratively fitting the two body matrix elements and single particle energies to ~170 experimental levels in nuclei from $^{82}$Ge to $^{96}$Pd.

Of interest here is that in the fit performed, the single particle energy of the $1f_{\frac{3}{2}}$ is the lowest, with the energies being -14.386, -13.233, -11.831 and -7.121 MeV for the $1f_{\frac{3}{2}}, 2p_{\frac{3}{2}}, 2p_{\frac{1}{2}}, 1g_{\frac{1}{2}}$ orbits respectively. This order is in contradiction to the traditional ordering of single particle energies and is a consequence of the neutron mean field changing due to the filling of the neutron orbits $1f_{\frac{3}{2}}$ to $1g_{\frac{3}{2}}$. 
The following study of \( N=50 \) nuclei using this interaction [115] showed little deviation for nuclei above \( ^{90}\text{Zr} \) compared to previous interactions in the \( 1g_{\frac{5}{2}} - 2p_{\frac{3}{2}} \) space. Indeed most of the matrix elements available for study were dominated by the \( 1g_{\frac{5}{2}} - 2p_{\frac{3}{2}} \) orbits. Indeed excellent reproduction of \( ^{88}\text{Sr} \) levels was achieved and in \( ^{90}\text{Zr} \) spins up to \( 11^+ \) and \( 10^- \) are in good agreement up to an excitation energy of \( \sim 7\text{MeV} \). The limited experimental data of lighter nuclei at the time precluded a definitive conclusion about the lesser studied \( \pi 1f_{\frac{5}{2}} - 2p_{\frac{3}{2}} \) structures. However, more complicated transitions in \( ^{90}\text{Zr} \) in which mixing between competing \( \pi 1g_{\frac{3}{2}} \) and \( 2p_{\frac{1}{2}} \) and the onset of dominance of the \( 1f_{\frac{5}{2}} \) and \( 2p_{\frac{3}{2}} \) were reproduced.

2d5/2 Talmi - Phys. Rev. 126, 2116 (1962)

Talmi in 1962 went on to analyse energy levels of neutron \( 2d_{\frac{5}{2}} \) configurations based on previous \((d, p)\) and \((d, t)\) evidence that ground states in of \( Zr^{91-96} \) and some lower lying excitations in these nuclei belonged predominantly to this orbital [116]. The reasoning the wavefunctions of these states are so pure lies with the fact that the \( \nu d_{\frac{5}{2}} \) orbital lies 1 MeV below the next orbital \((g_{\frac{7}{2}})\). The next two neutron orbits in this shell are found to lie even higher; over 1.5 MeV above the \( g_{\frac{7}{2}} \) for the \( 3s_{\frac{1}{2}} \) and \( 2d_{\frac{3}{2}} \) respectively. The 40 protons were considered to be in closed shells. This is not strictly true as 75% of the time the protons occupy the \( 2p_{\frac{3}{2}} \) orbit whilst the other 25% are in the \( 1g_{\frac{3}{2}} \) [109].

7.2 The need for a larger model space

Core polarisation

For nuclei close to \( ^{100}\text{Sn} \) the assumption of a closed \( ^{88}\text{Sr} \) core is better than that for lighter nuclei \((^{91}\text{Zr} \) for instance). This is due to core polarisation whereby some states may contain large components of particle-hole excitations between \( 1g_{\frac{3}{2}} - 2p_{\frac{1}{2}} \) and \( 1p_{\frac{3}{2}} - 0f_{\frac{5}{2}} \) subspaces [117]. When particle-hole excitations from the previously closed core appear, the core no longer comprises nucleons paired in time-reversed orbits, has angular momentum, and is therefore polarised. The previous interactions derived will therefore have distorted two-body matrix elements which were fitted for the \( 1g_{\frac{3}{2}} - 2p_{\frac{1}{2}} \)
space. Core polarisation effects may come into play at higher spins with $^{91}\text{Zr}$ as protons get promoted from lower lying orbits. The inclusion of these lower lying orbits therefore may alleviate this.

7.2.1 Necessary valence space

$^{91}\text{Zr}_{51}$ can be considered to consist of a $^{90}\text{Zr}$ core plus an extra neutron outside of this. When looked at with a shell model perspective $^{90}\text{Zr}$ is a magic nucleus having a closed shell of neutrons at N=50. This is attested to by neutron pick up experiments on $^{90,91}\text{Zr}$ [118]. Most of the states below 3.6 MeV were described by Bayman et al. as far back as 1956 [119] as having simple configurations involving the last two protons in the $1g_{\frac{7}{2}, 2p_{\frac{1}{2}}}$ space as in figure 7.1. The last two protons can be assumed to work alone due to the large N=50 closure from which neutron excitations are unlikely to affect levels below $\sim$4 MeV. This is due to the experimentally observed single particle gap between the $\nu_{g_{\frac{9}{2}}}$ and $d_{\frac{5}{2}}$ of the order 4 MeV (see table 8.1).

The spacing between the $\pi 2p_{\frac{3}{2}}$ and $1g_{\frac{7}{2}}$ is smaller than that for the $\pi 1f_{\frac{5}{2}}$ or $2p_{\frac{3}{2}}$ to $2p_{\frac{1}{2}}$ (see table 8.1). Hence it is sensible to expect that excitations involving the $1f_{\frac{5}{2}}$ and $2p_{\frac{3}{2}}$ lie higher in the spectrum.

It is not complicated to describe the level scheme therefore to 3.6 MeV excitation energy by coupling a neutron in one of the N > 50 orbitals ($2d_{\frac{5}{2}, 3s_{\frac{1}{2}}, 2d_{\frac{3}{2}}, 1g_{\frac{7}{2}}, 1h_{\frac{11}{2}}}$) to the ground state or an excited state of the $^{90}\text{Zr}$ core.

7.2.2 Breaking the N=50 core

In order to decide if the breaking of the N=50 shell is important a simple comparison between a shell model calculation and experimental data can be performed.

The simplified case of $^{90}\text{Zr}$ is examined using the Ji-Wildenthal $n50j$ model space. This comprises the $\pi 2p_{\frac{3}{2}}, 1f_{\frac{5}{2}}, 2p_{\frac{1}{2}}, 1g_{\frac{7}{2}}$ orbits with no configuration constraints. The $^{78}\text{Ni}$ core is closed, therefore output from this calculation will not contain any levels resulting from N=50 shell breaking. When this output is compared to the experimental data the energy at which states are missing from the calculation are likely to arise from neutron excitations and the breaking of the N=50 neutron closed shell. This comparison can be seen in figure 7.2.
Figure 7.1: Low lying excitations in $^{90}\text{Zr}$ together with proposed simple shell model configurations. Reproduced from [30].
7.2 THE NEED FOR A LARGER MODEL SPACE

Figure 7.2: A comparison of a $^{90}$Zr shell model calculation using the $gl$ and $n50j$ interactions [10, 117] with orbits limited to $2p_\frac{1}{2}, 1g_{\frac{9}{2}}$ and $2p_\frac{3}{2}, 1f_{\frac{3}{2}}, 2p_{\frac{1}{2}}, 1g_{\frac{3}{2}}$ respectively alongside the evaluated available experimental data [17]. It should be noted that none of these interactions consider breaking of the N=50 core.
As can be seen from figure 7.2, some level density is obviously missing from the calculation at \( \geq 3.5 \text{ MeV} \). Therefore at energies of 3.5 MeV and above the N=50 core breaking must be considered. This is supported by evidence from \(^{91}\text{Zr} \ (d,p)\) and \(^{92}\text{Zr} \ (p,t)\) reactions \([118, 110]\) which display a low transfer cross section below 4 MeV.

Negative parity states

In many cases, when looking at a shell model description for observed states the opposite parity intruder orbital can be useful. The reasoning behind this is that the states made from this orbital tend to be fairly pure due to the limited mixing encountered allowing them to be easily identified. In the case of \(^{91}\text{Zr} \) the \( h_{11/2} \) intruder orbit may well be dominant in the construction of negative parity states due to how lowered it appears when the single particle energies are normalised to the newly constructed interaction (see figure 7.8).

7.3 Spin available and energy considerations for valence spaces

Assuming two protons maximally aligned in the \( 1g_{7/2} \) orbit a maximum spin of \( 8^+ \) is possible. Coupled to the \( d5/2 \) neutron gives a maximally aligned spin of \( 21/2^+ \). This is the confirmed configuration of the \( 21/2^+ \) isomer in \(^{91}\text{Zr} \) as discussed in chapter 1. In order to study higher spin and consider more complex configurations more orbits must be included.

The first and possibly most obvious extension to make to the model space is to include the higher \( j \) neutron orbits \( g_{7/2} \) and \( h_{11/2} \). With \( \pi(1g_{9/2})^2 \otimes \nu h_{11/2} \) the maximum spin is \( 27^-/2 \). This configuration when compared to the level scheme obtained from the experiment would sit well with level 4 in figure 8.4 which is a likely candidate to be \( 27^-/2 \) due to the probability of the 2574 keV transition being an E3 (see chapter ?? for discussion on this). This is also energetically likely as the \( 27^-/2 \) state is of a similar order in energy above the \( 21/2^+ \) state to the difference in single particle energies of the \( d_{5/2} \) and the \( h_{11/2} \), at around 3 MeV.

Allowing excitations from the lower lying \( f_{5/2} \) and \( p_{3/2} \) proton orbits will also
generate spin with 1 particle 1 hole excitations producing the most spin per unit energy. The configuration \((f_{5/2})^{-1} \otimes (g_{9/2})^3 \otimes d_{5/2}\) has a maximally aligned spin of \(\frac{39}{2}^+\) which in an estimate ignoring residual interactions that will take place at an excitation energy of the order 6 MeV (\(2 \times 0.8\) MeV from \((g_{9/2})^2\) + 4.5 MeV from exciting the \(p_{3/2}\) proton). If the \(p_{3/2}\) orbit is included however, the \(f_{5/2}\) should also be included due to the narrow separation of these orbits.

The other possibility for spin generation is to break the \(N=50\) core. It would be simple enough to make a state such as \(\frac{27}{2}^-\) with the configuration \(\pi p_{1/2} \otimes g_{9/2} \otimes \nu(d_{5/2})^2 \otimes g_{9/2}\) which would have an estimated energy of \(\sim 5\) MeV again from extremely simplified energy arguments.

It is clear from these considerations that the likely configuration for the \(\frac{27}{2}^-\) state involves the \(h_{11/2}\) neutron. Without the effect of residual interactions however these arguments must be considered a starting point only. An interaction which allows all of the above excitation regimes to be studied is therefore deemed necessary.

**7.4 Repeat of B. Alex Brown’s calculation**

The first step to producing a larger scale calculation of \(^{91}\text{Zr}\) is to reproduce the last calculation of B. A. Brown [20] successfully. The gl interaction and model space [60] are available as standard with OXBASH. This includes the \(\pi 1g_{\frac{9}{2}} - 2p_{\frac{1}{2}}\) and \(\nu 3s_{\frac{1}{2}}, 2d_{\frac{5}{2}}\) orbits. The purpose of this exercise is to familiarise the author with the OXBASH program and also to show what level of accuracy is possible with an interaction well fitted to the experimental data (figure 7.3).

**7.5 New \(^{91}\text{Zr}\) calculation**

Selections of model space and interaction

Calculations within OXBASH require the selection of an appropriate model space and interaction for that model space.

From the DCO results (section 5.11) together with some sensible initial thoughts (section 7.3), the states of interest have up to spin \(\frac{27}{2}^-\) and \(E \geq 4\) MeV.
Figure 7.3: $^{91}\text{Zr}$ spectrum calculated with the OXBASH $gl$ interaction and $\pi 2p_{1/2}, 1g_{9/2} \nu 3s_{1/2}, 2d_{5/2}$ model space as a repeat of original calculation by B.A. Brown [20]. Extremely good agreement between theory and experiment has been achieved which can be seen by direct visual comparison as many of the calculated levels are within 200 keV of their experimental counterparts.
The model space necessary for determining which configurations are responsible is therefore: $\pi f_{5/2}, 2p_{3/2}, 2p_{1/2}, (1g_{9/2}), \nu 1g_{9/2}, 2d_{5/2}, 3s_{1/2}, 1g_{7/2}, 2d_{3/2}, 1h_{11/2}$.

A model space this large is at the frontier of current research in the shell model due to the matrix dimensions required within the calculation. A region such as the SD shell has an interaction called the Universal SD interaction. This interaction covers all nuclei with valence particles in the SD region (see figure 7.5) and provides reliable results within this region. The next shell model region is the PF shell for which many interactions exist e.g. $f5puh$ and $ho$ [120, 121]. Each interaction covers a large region of nuclei in this shell well, yet no one interaction has been derived to cover all nuclei. The PFD shell as it is known (see fig. 7.5) has currently several interactions defined for small regions around its edges (close to closed shells) such as the sn132 interaction which is one of the most successful in the region yet covers only a few nuclei north of $^{132}\text{Sn}$.

It is clear from the literature review that no interaction is freely available to cover the region of interest. One must therefore be constructed.

Following discussion with B.A. Brown and M. Hjörth-Jensen, it was decided to use a new interaction recently developed by M. Hjörth-Jensen [122] for $N=51$ nuclei. This interaction, called the $uni78$ interaction, uses a $^{56}\text{Ni}$ closed core. The empirical two-body matrix elements of the Ji-Wildenthal interaction would be used to replace the corresponding ones in M. Hjörth-Jensen's $^{78}\text{Ni}$ interaction and the whole interaction...
Figure 7.5: The region of the nuclear chart covered by differing orbital groupings. Pink regions are places where successful interactions exist and blue the current theoretical frontier, other colours follow the key in the figure and represent average deviations in energy of the calculated levels from the experimental data. Reproduced from [123].

used to renormalise the corresponding new experimental singles particle energies.

### 7.5.1 Single particle energies

An essential ingredient for a successful shell model calculation is accurate single particle energies. These are obtained by taking the differences in binding energies of nuclei which differ in mass by one nucleon. The first nuclei should have a closed shell and the second should have one nucleon in the next shell, thus allowing the difference in energy of the shells to be calculated as equation 7.1

\[
\Delta BE = BE(A + 1) - BE(A)
\]

\(^{88}\text{Sr}\) was chosen as the nucleus around which to base the single particle energies due to the wealth of experimental information available. Figure 7.6 shows the nuclei involved and which levels each provided.
Figure 7.6: Nuclei used in generating single particle energies for the new shell model calculation (with $^{88}\text{Sr}$ used as the core). Each nucleus has the orbits shown in green for which the total binding energy was used.

The experimental S.P.E.s obtained in this way using the program Toi which is part of a suite of programs provided along with OXBASH and searches the Audi-Wapstra-Thibault 2003 atomic mass compilation [124] are shown in table 7.1.

These values are useful to show schematically as in figure 7.7 to give the reader a more intuitive format from which to work.

The single particle energies determined experimentally must be renormalised to the new interaction that has been built. This is because the new interaction is based on a $^{56}\text{Ni}$ core and so the mean field is rather larger for a mass 88 nucleus. This renormalisation is done by entering the experimental S.P.Es into the new interaction as a starting point. This interaction is then used to calculate the same binding energies used in the experimental determination of the S.P.Es. The difference between the experimental values and those obtained theoretically are the values by which the S.P.Es in the interaction must be altered. The interaction will now roughly reproduce the experimental values. The final values obtained are shown in table 7.2 and figure 7.8.

Again shown schematically as in figure 7.5.1.
Table 7.1: Experimentally determined single-particle energies from binding energy data in the 2003 atomic mass compilation.

<table>
<thead>
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<th>Orbit</th>
<th>Energy $E(\pi)$</th>
<th>Energy $E(\nu)$</th>
</tr>
</thead>
<tbody>
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<td></td>
</tr>
<tr>
<td>$2d_{5/2}$</td>
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<td></td>
</tr>
<tr>
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<td></td>
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</tr>
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<td>-11.50</td>
</tr>
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<td>$2p_{3/2}$</td>
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<td>$1f_{5/2}$</td>
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<td>-12.37</td>
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</table>

Figure 7.7: Schematic valence space for $^{91}$Zr. The single particle energy of the orbitals is not fixed and may vary according to occupancy.
Table 7.2: Experimentally determined, re-normalised single-particle energies.

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<tr>
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<td>1(g_{\frac{7}{2}})</td>
<td>-</td>
<td>-0.27</td>
</tr>
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<td>1(f_{\frac{5}{2}})</td>
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Figure 7.8: Schematic valence space for \(^{91}\)Zr showing renormalised single particle energies as used in the calculation.
Chapter 8

Shell model results, summary and future work

8.1 Shell model calculation

After significant trials with different valence spaces it was observed that many of the higher spin states within $^{91}$Zr were constructed with proton particle-hole excitations involving the $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$ and $g_{9/2}$ orbitals. This would allow a truncation in the calculation. Keeping the basis for the calculation minimal where possible was a main goal in order to allow necessary degrees of freedom to be opened up within the restricted capability of OXBASH.

The necessary matrix sizes to allow both proton excitations and neutron N=50 core breaking are too large for OXBASH to handle even if truncated. The calculation therefore included all of the proton degrees of freedom and allowed one neutron to move from $\nu 2d_{5/2}$ up to $\nu h_{11/2}$ as shown in table 8.1.

Explicit results for the configurations giving rise to the first three eigenvalues of each spin $I$ are listed in appendix B.

Plotting these results yields figure 8.1. The standout features are the negative parity states at $I=\frac{25}{2}^-$ and $\frac{27}{2}^+$ which are situated $\sim 2$ MeV below the next state of the same spin. These features have a high confidence of being observed experimentally due to their very yrast nature.
### Table 8.1: The restrictions imposed upon the OXBASH calculation which allows all necessary proton degrees of freedom limits neutron excitations to one valence particle.

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<td>$N_{1h_{11/2}}$</td>
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</table>
Figure 8.1: The first three eigenvalues of each parity plotted versus spin from $I=\frac{21}{2}$ upward. It is easy to observe the low lying yrast negative parity states at $I=\frac{25}{2}^-$ and $\frac{27}{2}^+$. Experimental energies are plotted and, where possible, joined to the respective theoretical level.
8.2 Summary and future work

8.2.1 $^{91}\text{Zr}$

In $^{91}\text{Zr}$ 52 new transitions have been identified as feeding into the 3.167 MeV $^{21+}_{2}$ isomer. The known level scheme has been extended up to an excitation energy of 13.872 MeV. This is $\sim 7$ MeV higher than the previous highest observed state of 6.457 MeV.

With the use of DCO ratios and linear polarisation measurements together with information from the shell model calculations and Weisskopf estimates, spin and parity assignments in were able to be made up to a spin of $\frac{49}{2}h$. This is $12h$ higher than the previously highest-spin level observed [20].

A lifetime measurement for the 5741 keV level performed at WNSL Yale in 2006 [104] improved confidence in the assignment of the 128 keV transition as M1, therefore improving confidence in the other assignments made locally to this transition.

It has been shown that the shell model can be used to study high-spin states effectively even though a high level density may be expected. A defining feature is required to facilitate this however. For $^{91}\text{Zr}$ this feature is the low lying $h_{\frac{11}{2}}$ orbit which produces very yrast states.

The configurations responsible for the states from $^{23-}_{2}$ to $^{27-}_{2}$ appear from the shell model calculation to involve a neutron in the $h_{\frac{11}{2}}$ orbit. This turns out as predicted from a consideration of the extremely low lying nature of this intruder orbital. The only suprise comes from the $^{27-}_{2}$ leading configuration ($> 80\%$) being of a more complex proton nature than originally thought. The calculation predicts a configuration of $\pi[(f_{5/2})^{-2} \otimes (g_{9/2})^4] \otimes \nu h_{11/2}$ rather than the simple $\pi(g_{9/2})^2 \otimes \nu h_{11/2}$ as first thought.

8.2.2 $^{92}\text{Zr}$

In $^{92}\text{Zr}$ 12 new transitions have been identified which feed into at or above the $I^r = 14^+ 6047$ keV level. Notable is the $18^+ 8455$ keV level which appears to be yrast. Previously observed transitions have been confirmed, along with their respective spin/parity assignments. The overall spin regime has been extended to $I^r = 20^+$. 
8.2 SUMMARY AND FUTURE WORK

8.2.3 Further work

In order to study this nucleus to higher spin, a different reaction would need to be used. A radioactive beam may be employed to ameliorate the problem of boiling off too many neutrons. The collection of further statistics at higher spin would also allow greater confidence particularly for the DCO and linear polarisation measurements.

For a more complete description of the new states observed in this work it would be necessary to establish B(E2) values for transitions. The use of inverse kinematics, e.g. $^{13}\text{C}(^{82}\text{Se},xn)^{96-7}\text{Zr}$, would allow the large recoil velocity appropriate for a recoil distance method or Doppler shift attenuation method experiment to be carried out for lifetime measurements of high spin states $^{91}\text{Zr}$. To fully evaluate the configurations involved, g-factor experiments may also be employed.

At the end of the current work a new shell model code entitled NuShell [125] was released for beta testing. The interaction $ni78gl4$ developed for this work can be used with this code. The possibility of greatly increased matrix sizes ($>10^8$) should make a calculation using the valence space necessary for investigating neutron core breaking possible. This should provide further insight into possible yrast configurations at high spin (I = $\geq \frac{27}{2}$).
Bibliography


BIBLIOGRAPHY


BIBLIOGRAPHY


Appendix A

Weisskopf single particle lifetime estimates

Tables A and A give tabulated values of Weisskopf single particle estimates for $^{91,92}\text{Zr}$ respectively.
<table>
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Table A1: Part A. Weisskopf single particle lifetime estimates for $^{91}$Zr.
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Table A.2: Part B of table A. Weisskopf single particle lifetime estimates for $^{91}\text{Zr}$. 
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Table A.3: Part C of table A. Weisskopf single particle lifetime estimates for $^{91}$Zr.
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<td>$1.11 \times 10^{-01}$</td>
<td>$1.47 \times 10^{-14}$</td>
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Table A.4: Weisskopf single particle lifetime estimates for $^{92}$Zr.
Appendix B

Shell model output
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<th>Spin $I(h)$</th>
<th>Parity ($\pi$)</th>
<th>Yrast, 1st, 2nd</th>
<th>Energy eigenvalue (MeV)</th>
<th>Leading Configuration</th>
<th>% of w.f.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{21}_{2}$</td>
<td>+Ve</td>
<td>Yrast</td>
<td>-391.7</td>
<td>$(g_{9/2})^2 \otimes d_{5/2}$</td>
<td>85.7</td>
</tr>
<tr>
<td>$^{21}_{2}$</td>
<td>+Ve</td>
<td>1st</td>
<td>-389.9</td>
<td>$p_{1/2} \otimes g_{9/2} \otimes h_{11/2}$</td>
<td>83.6</td>
</tr>
<tr>
<td>$^{21}_{2}$</td>
<td>+Ve</td>
<td>2nd</td>
<td>-389.4</td>
<td>$(g_{9/2})^2 \otimes g_{7/2}$</td>
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<td>Yrast</td>
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<td>84.0</td>
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<td>1st</td>
<td>-388.7</td>
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<td>$(g_{9/2})^2 \otimes g_{7/2}$</td>
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<td>1st</td>
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Table B.1: Part A. Leading configurations of three lowest eigenvalues for given spin. Calculated as in section 7. N.B. First two protons are assumed to be excited into $1g_{9/2}$ orbit unless stated.
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<th>Spin I ((\hbar))</th>
<th>Parity ((\pi))</th>
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<th>Energy eigenvalue (MeV)</th>
<th>Leading Configuration</th>
<th>% of w.f.</th>
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Table B.2: Part B of table B.
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<th>Parity (π)</th>
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<th>Energy eigenvalue (MeV)</th>
<th>Leading Configuration</th>
<th>% of w.f.</th>
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<td>Yrast</td>
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<td>1&lt;sup&gt;st&lt;/sup&gt;</td>
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<td>2&lt;sup&gt;nd&lt;/sup&gt;</td>
<td>-381.2</td>
<td>$(f_{5/2})^{-1} \otimes (p_{3/2})^{-1} \otimes (p_{1/2})^{-1} \otimes (g_{9/2})^3 \otimes d_{5/2}$</td>
<td>89.3</td>
</tr>
<tr>
<td>35/2</td>
<td>+Ve</td>
<td>Yrast</td>
<td>-382.4</td>
<td>$(f_{5/2})^{-1} \otimes (g_{9/2})^3 \otimes h_{11/2}$</td>
<td>61.6</td>
</tr>
<tr>
<td>35/2</td>
<td>+Ve</td>
<td>1&lt;sup&gt;st&lt;/sup&gt;</td>
<td>-381.7</td>
<td>$(f_{5/2})^{-1} \otimes (g_{9/2})^3 \otimes h_{11/2}$</td>
<td>62.4</td>
</tr>
<tr>
<td>35/2</td>
<td>+Ve</td>
<td>2&lt;sup&gt;nd&lt;/sup&gt;</td>
<td>-381.2</td>
<td>$(f_{5/2})^{-1} \otimes (g_{9/2})^3 \otimes h_{11/2}$</td>
<td>55.4</td>
</tr>
<tr>
<td>35/2</td>
<td>-Ve</td>
<td>Yrast</td>
<td>-380.5</td>
<td>$(f_{5/2})^{-1} \otimes (p_{3/2})^{-1} \otimes (p_{1/2})^{-1} \otimes (g_{9/2})^2 \otimes d_{5/2}$</td>
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<tr>
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<td>-Ve</td>
<td>1&lt;sup&gt;st&lt;/sup&gt;</td>
<td>-380.1</td>
<td>$(f_{5/2})^{-2} \otimes (p_{1/2})^{-1} \otimes (g_{9/2})^3 \otimes d_{5/2}$</td>
<td>96.1</td>
</tr>
<tr>
<td>35/2</td>
<td>-Ve</td>
<td>2&lt;sup&gt;nd&lt;/sup&gt;</td>
<td>-379.0</td>
<td>$(f_{5/2})^{-2} \otimes (p_{1/2})^{-1} \otimes (p_{3/2})^{-1} \otimes (g_{9/2})^3 \otimes g_{7/2}$</td>
<td>39.7</td>
</tr>
</tbody>
</table>

Table B.3: Part C of table B.
Appendix C

Publications

First author publications

N.J. Thompson et al., Spectroscopy of $^{91}\text{Zr}_{51}$ at Medium to High Spins,
Proceedings of the 2006 Zakopane Conference on Nuclear Physics,

Peer reviewed publications

A.Bhagwat, N.J.Thompson, J.K.Tuli
Nuclear Data Sheets for $A = 254$
Nucl. Data Sheets 105, 959 (2005)

A.B. Garnsworthy, N.J. Thompson et al.,
Spectroscopy of $^{212}\text{Po}$ and $^{213}\text{At}$ using a $^{8}\text{He}$ Beam and EXOGAM,
Proceedings of NUSTAR 2005,

G.A. Jones ... N.J. Thompson et al.
Microsecond and nanosecond isomers populated in fission reactions,

P. Boutachtov ... N.J. Thompson et al.
Measurement of the g factor for the 41+ state in 70Ge,

G.A. Jones ... N.J. Thompson et al.
High-Spin, Multi-particle Isomers in 121,123Sb,

T.R. Saito ... N.J. Thompson et al.
Yrast and Non-Yrast 2+ States of 134Ce and 136Nd Populated in Relativistic Coulomb Excitation

A. Wolf ... N.J. Thompson et al.
g factor of the 2+ state of 170Hf,

G.A. Jones ... N.J. Thompson et al.
Identification of a High-Spin Isomer in 99Mo,

V. Werner ... N.J. Thompson et al.
Evidence for configurational isospin polarization of mixed-symmetry states from magnetic moment measurements,

Other publications

P.H. Regan, N.J. Thompson et al.
Isomer and In-Beam Spectroscopy of $^{91,92}$Zr,