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APPLICATION OF FERROELECTRICITY
TO
PARAMETRIC AMPLIFICATION

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PREFACE

The author is extremely grateful to the authorities of the Battersea College of Technology and specially to Dr. L.R.B. Elton, D.Sc., F.Inst.P., Head of the Physics Department for employing him as a Grade B Lecturer for nearly two years and for the latter's constant encouragement.

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ABSTRACT

The object of the present project was to investigate some of the problems of using ferro-electric material for permittic amplification.

The material was used in the form of a cylindrical rod. Analysis shows that a percentage change of dielectric constant do not necessarily give the same percentage change of reactance of a dielectric rod. For a particular value of frequency and $k'$, the relative dielectric constant, the diameter of the rod which gives a large percentage change of reactance is fixed. With very large dielectric constant ($\approx 5000$), this value of the diameter becomes very small.

The required condition can be expected to be satisfied, in practice, by varying the dielectric constant, keeping the other parameters constant. With this in view, samples of different percentage compositions were made with barium titanate and polythene.

Several methods have been discussed for measuring the non-linearity of the samples. The microwave bridge is a quick and sensitive method for the detection of the non-linearity. The primary interest being in the shift of the resonant frequency of a cavity, detail measurements have
been carried out in the cavity.

A cavity was designed in which the frequency condition for parametric amplification, i.e., \( \omega_p = \omega_s + \omega_i \), was satisfied. The output at the signal frequency was observed while the cavity was subjected to high microwave power at the pump frequency. With high average pump power, the cavity tuning at \( f_s \), the signal frequency changed and on retuning the output at \( f_s \) was larger than before. The effect was not instantaneous but slow indicating heating effect. This effect was also noticed with a sample which did not satisfy the above frequency condition.

Perturbation measurements showed that the field at the sample, inside the cavity, was small. This is one of the possible factors which contributed to the absence of the parametric amplification.

An attempt has been made to measure the quality of these ferroelectric samples in a way similar to that used for finding the quality of the variable capacitance diode. A value of 2 for change of susceptance can be obtained with some of the compositions with a d.c. bias field of 9.6 kV/cm.

Finally, the complex permittivity of some of the
compositions have been measured in a cylindrical cavity operating on \( E_{010} \) - mode at 3800 Mc/s. It is found that the dielectric constant of the compositions vary from 48 to 11 with 93 to 78 per cent of barium titanate in the composition. The measured low frequency value of the dielectric constant of barium titanate is 6470. Small signal measurements at microwave frequencies show that the incremental dielectric constant of the compositions is dependent on their previous history.
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Fig. 1 (a) Resonant circuit with capacitance varied by changing the spacing of the plates, (c) full-line shows variation of voltage across fixed capacitor with time and dotted line shows the same when the capacitance is varied as shown in (b).
1.1 BASIC PRINCIPLES OF A PARAMETRIC AMPLIFIER.

The conventional use of electron beam for amplification at microwave frequencies by converting d.c. energy of the beam to a.c. is currently being supplanted by the use of variable reactance. One of the new devices is commonly known as parametric amplifier or variable reactance amplifier, the main advantage of which is its low noise. The principle of parametric amplification can be traced back to Lord Rayleigh\(^1\) who treated this problem in a mechanical system.

The basic principle of parametric amplification can be explained by considering a simple resonant circuit of Fig.1(a). It is supposed that the spacing of the capacitor plates can be varied at will. When the voltage across the capacitor plates is maximum, either positive or negative, the condenser plates are pulled apart. As a result, the capacitance of the condenser is decreased and its voltage increased. This is shown in Figs. 1 (b) and 1(c). Work is done by an external source to separate the plates. The plates are brought back to their original
Fig. 2. A two tank variable parameter system.

Fig. 3. Equivalent circuit of the parametric amplifier at resonance.
position when the voltage across the condenser is zero, without any work being done. If the process is repeated, we get voltage amplification, the energy being supplied by an external source, called the pump, which is varying the magnitude of the capacitor at twice the frequency of the resonant circuit. In this case, the relative phase of the pump and the signal is extremely important.

It can be shown that the problem of phasing is solved by using two resonant circuits known as signal and idler circuits. Fig. 2 shows a two-tank variable parameter system. The variable capacitor, driven by the pump at a frequency \( \omega_3 \), couples the two different tank circuits of resonant frequencies \( \omega_1 \) and \( \omega_2 \) such that

\[
\omega_3 = \omega_1 + \omega_2
\]

\( C_3 \) is the maximum amplitude of the capacitance variation and the subscripts 1, 2, 3 refer to signal, idler and pump frequencies respectively.

The power gain and bandwidth of such a device can be calculated by assuming that the resonant circuits have high Q's, so that they present a short-circuit at frequencies to which they are not tuned. The effective impedance presented to the signal circuit by the combination of the
variable capacitor and the idler circuit is

\[ Y(\omega) = -\frac{\omega_1 c_3^2}{4Y_2^*} \]  

where \(Y_2^*\) is the complex conjugate of the admittance of the idler circuit.

A signal generator of internal impedance \(G_g\) and tuned to \(\omega_1\) and a load \(G_L\) are connected to the signal circuit. The signal generator produces a voltage across tank 1. The mixing action of the variable capacitor will, then, produce a voltage across tank 2, which will, in turn, produce a voltage across tank 1 in phase with the existing voltage giving amplification.

At resonance, a pure negative conductance \(-G\) is presented to the amplifier as shown in Fig. 3; the magnitude of which is given by

\[ Y(\omega) = -G = -\frac{\omega_1 c_3^2}{4G_2} \]  

The power gain is

\[ = \frac{4G_g G_L}{(G_{T1} - G)^2} \]  

where \(G_{T1} = G_g + G_1 + G_L\)

for large gains \(G \gg G_{T1}\)
Assuming the Q of the idler circuit to be considerably higher than the loaded signal circuit, it can be shown that

\[
\text{(power gain)}^{\frac{1}{2}} \times \text{(band-width)} = \frac{1}{\frac{Q_1^2}{\omega_1^2} + 2} \left( \frac{G_g}{G} \cdot \frac{G_L}{G} \right)^{\frac{1}{2}}
\]

This expression shows that as the gain is increased the bandwidth of the device goes down. Larger bandwidth can be obtained by increasing the ratio of the idler to signal frequency.

The basic arguments presented above hold good irrespective of the nature of the variable reactance. The variable reactance can be either a capacitance or an inductance. The type of variable reactance which has been extensively investigated \(^4,5,6\) is a variable capacitor in the form of a back biased variable capacitance diode. The problem of using ferrites as a variable inductance has been discussed theoretically by Suhl\(^7\) and some ferromagnetic parametric amplifiers have been built by Weiss\(^8\) and others\(^9\).

These devices are essentially negative resistance type. As such they are not very stable. They have a small bandwidth. More stable amplifiers having larger bandwidth on the travelling-wave principle have been proposed by
Tien & Suhl\textsuperscript{10}, and others \textsuperscript{11,12,13,14} and constructed by Engelbrecht\textsuperscript{15} and others\textsuperscript{16}.

Electron beams have been used to produce a variable reactance. An electron beam, modulated by pump power, produces a variable susceptance\textsuperscript{17} across a "floating drift tube" cavity having two equal gaps separated by a field free drift space. If the frequency of the pump is twice the resonant frequency of the cavity, then the signal fed to the cavity is amplified. In the Adler\textsuperscript{18} tube, the noise-free fast wave, produced in a Cuccia\textsuperscript{19} coupler, is amplified by pumping at twice the signal frequency by means of the Quadrupole coupler.

The power relations in a nonlinear reactor have been worked out by Manley and Rowe\textsuperscript{20}. If a pump and signal of frequencies \(f_p\) and \(f_s\) respectively are presented on a nonlinear reactor, the output contains, among others, frequencies \(f_p + s\) and \(f_p - s\).

Manley Rowe relations become

\[
\frac{P_s}{f_s} + \frac{P_{p+s}}{f_{p+s}} - \frac{P_{p-s}}{f_{p-s}} = 0 \quad \text{1.8}
\]

where \(P\) is the power at a frequency denoted by the subscript. Positive \(P\) indicates generation of power by
the amplifier and negative $P$ indicates absorption of power by the non-linear element.

In the negative resistance case, the amplifier is tuned to signal and idler frequencies, then

$$\frac{P_s}{f_s} = \frac{P_{p-s}}{f_{p-s}} \quad \ldots \ldots \ldots \quad 1.9$$

In this case, one gets output at both signal and idler frequencies.

In the up-converter case, the amplifier is tuned to signal and upper side-band frequencies, then

$$\frac{P_s}{f_s} = - \frac{P_{p+s}}{f_{p+s}} \quad \ldots \ldots \ldots \quad 1.10$$

To get output at the upper side-band $f_{p+s}$ power must be absorbed at signal frequency $f_s$. The power gain is equal to

$$\frac{P_{p+s}}{P_s} = \frac{f_{p+s}}{f_s} \quad \ldots \ldots \ldots \quad 1.11$$

The present work investigates the possibility of the use of a different property, i.e., ferroelectricity, for the design of a parametric amplifier. The use of ferroelectric
materials for parametric amplification has been suggested theoretically by Cassedy\textsuperscript{21} and others\textsuperscript{22}. A letter\textsuperscript{23} gives a very brief report about a parametric amplifier being built with this type of material. But to the best knowledge of the author this is the first detail report explaining some of the problems of using these materials for parametric amplification at microwave frequencies.

Apart from academic interest, ferroelectric parametric amplifier has some advantage over back-biased variable capacitance diode and ferromagnetic parametric amplifiers. Some of the ferroelectric materials are easier to manufacture in any shape and size. In the case of ferromagnetic type of amplifiers an additional item i.e., a magnet is required. This makes the design of a tunable ferromagnetic amplifier more difficult as it will be necessary to vary the magnetic field for changing the ferromagnetic resonance frequency. Ferroelectric materials can be used for uniformly loading a system for obtaining a wide-band device.
Fig. 4 (a) Capacitance of a diffused silicon nonlinear capacitor as a function of voltage.

Fig. 4 (b) Dielectric constant versus field strength
D.c. biasing is a common feature of these three types of parametric amplifiers working at microwave frequencies. Different reasons govern the necessity of biasing these non-linear elements.

The capacitance of a semi-conductor diode increases with positive bias and decreases with negative bias (Fig. 4a). The quality factor $Q$ of a diode of capacitance $C$ is given by

$$Q = \frac{1}{2\pi f C r}$$

where $f$ is the operating frequency and $r$ is the spreading resistance.

It is desirable, for optimum performance of the amplifier, to make $Q$ as large as possible. This is achieved partly by reducing the operating capacitance of the diode. This reduction is limited by the amount of negative voltage that could be safely applied to the diode short of breakdown.

The dielectric constant of a nonlinear dielectric,
on the other hand, decreases equally with both positive and negative applied field (Fig 4b). When this nonlinear element is used with zero bias, output will be obtained at the second harmonic and not at the fundamental frequency. To avoid this frequency doubling, it is essential to use d.c. biasing field the magnitude of which should be larger than the peak a.c. voltage.

For a ferromagnetic amplifier, a d.c. magnetic field is necessary to obtain ferromagnetic resonance. The amount of d.c. field required is determined, for a particular ferrite, by the ferromagnetic resonant frequency.

1.2 FERROELECTRICITY

In Progress in Dielectrics, Franklin explains ferroelectricity, briefly and clearly, and gives a list of ferroelectric materials.
The first material to have exhibited ferroelectric properties is Rochelle salt. Subsequently ferroelectricity was discovered in potassium dihydrogen phosphate (KDP) in 1935 by Busch & Scherrer, in Barium titanate in 1943-44 by von Hippel and co-workers. In recent years, a number of ferroelectric materials have been discovered.

Ferroelectric materials are characterised by the presence of a "spontaneous polarisation". It means that polarisation has a finite non-zero value even in the absence of any external field. Application of suitable electric field changes the direction and magnitude of this polarisation. Polarisation changes non-linearly with the applied field. Increasing and decreasing field of same magnitude give different values of polarisation. Ferroelectric materials display loops similar to those displayed by magnetic materials. Those loops are also called hysteresis loops.

Another important property of this type of materials is the presence of a Curie Temperature, at which a transition in crystal structure takes place and many other physical properties undergo sharp changes. Spontaneous polarisation decreases with increasing temperature and becomes zero at Curie point.
Thirdly, the crystal structure becomes distorted when it becomes ferroelectric, giving it a spontaneous strain. An applied field alters spontaneous strain along with polarisation.

Ferroelectric materials can be classified into several groups depending on some common property. Double salts of tartaric acid, specially, NaK[C\(_4\)H\(_4\)O\(_6\)] \(_4\)H\(_2\)O (Rochelle Salt) and LiNH\(_4\)C\(_4\)H\(_6\)O\(_6\), H\(_2\)O belong to the first group. Their crystal structures are extremely complicated. Possibly O-H-O bond is responsible for ferroelectricity.

O-H-O bond is also important in the second group which contains the alkali dihydrogen phosphates and arsenates.

In the third group, which include materials of perovskite structure like BaTiO\(_3\) & KNbO\(_3\), trioxide like WO\(_3\) and the pyrochlores, the basic unit is an octahedron.

In the fourth class of materials ferroelectricity is due to a bond either N-H- group or N-H-O. This category contains Guanidine Aluminium Sulphate Hexahydrate (GASH) and a number of other compounds the important constituent of which is ammonium ion.

Colemanite, CaB\(_2\)O\(_4\) (OH.) \(_3\)H\(_2\)O and Ca\(_2\)Sr (CH\(_3\)CH\(_2\)COO)\(_6\)
**Fig. 5a. Cubic unit cell of BaTiO₃**

**Fig. 5b. Projection of BaTiO₃ unit cell at room temp. on the (010) plane.**
are two isolated ferroelectric materials.

BaTiO$_3$ has been extensively investigated because of its simple crystal structure. Moreover, as a ceramic, it is very rugged and it can easily be formed in any desired shape. To understand ferroelectricity we shall take ferroelectric single-crystal BaTiO$_3$ at room temperature as an example.

At 120° C, the Curie temperature, BaTiO$_3$ passes from Cubic to ferroelectric tetragonal structure. There are two other ferroelectric structural changes at 5°C & -70°C to respectively orthorhombic and possibly to monoclinic phase.

In the cubic phase, the titanium ion is in the centre of six oxygen ions in an octahedral structure. The TiO$_6$ constellation is the cause of high dielectric constants of all crystals which contain it. Above 120° C, BaTiO$_3$ has a centre of symmetry and as such, its permanent dipole moments cancel out. The Cubic unit cell is shown in Fig. 5a. In the ferroelectric phase, the Ti$^{4+}$, Ba$^{2+}$ and O$^{2-}$ ions are shifted relative to their positions in the antiferroelectric phase. The crystal structure at room temperature is represented in Fig. 5b which shows a projection on the (010) plane. The horizontal line
Fig. 6. Hysteresis loop of (a) single crystal and (b) polycrystalline BaTiO₃.
joining $O^{2-}_{11}$ ions is the centre line of the cell. The shifts from their original positions are respectively 0.06 Å & 0.12 Å for the $Ba^{2+}$ & $Ti^{4+}$ ions in the upward direction and 0.03 Å for each of the $O^{2-}$ ions in the downward direction. The resultant dimensions of the cell are 4.031 Å in the direction of the shifts, and 3.092 Å at right angles to it.

The unit cell acquires a dipole moment because the centres of gravity of positive and negative charges do not coincide. This is spontaneous polarisation whose value is $26.02 \times 10^{-2} \text{ coul/m}^2$. Multiplying the polarisation by unit cell volume of $64 \times 10^{-8} \text{ m}^3$ gives a displacement of

$$\mu = 16.65 \times 10^{-30} \text{ (coul-m)} = 5 \text{ Debye} \ldots \ldots 1.12$$

If this moment is caused by the shift of the $Ti^{4+}$ ion from its original central position, a displacement of

$$d = \frac{\mu}{4e} = 0.26\text{Å} \ldots \ldots 1.13$$

would be expected.

Ferroelectric properties of a crystal can be discussed with the help of a hysteresis loop, a schematic diagram of which is shown in Fig.6. For high positive
voltage at A, the polarisation is high. Polarisation reduces in magnitude with decreasing field attaining a value $P_r$, known as remanent polarisation, at zero field. The top linear portion of the loop extrapolates to spontaneous polarisation $P_s$, at zero field. $P_s$ and $P_r$ are nearly the same for good single crystals which have square loops, but for poor crystals and ceramics they are different.

With increasing negative field, the polarisation reduces in magnitude up to a point B, where it suddenly reverses in sign and comes to the point C. The polarisation is zero at a field $E_c$, called the coercive field. By applying suitable voltages, one comes back to A. With high a.c. fields, this loop is traversed at each cycle.

The entire process is irreversible and there is a transference of energy from the field to the material. The area of the hysteresis loop gives the energy loss per cycle.

The complex relative permittivity is

$$k = \frac{\varepsilon' - j \varepsilon''}{\varepsilon_0} = k' - jk'' \quad \text{(1.14)}$$

and the loss tangent is

$$\tan \delta = \frac{\varepsilon''}{\varepsilon'} = \frac{k''}{k'} \quad \text{(1.15)}$$
Fig. 7. Antiparallel C-Domains Of A Barium Titanate Crystal Plate

Fig. 8. Both C-And A-Domains Of A Barium Titanate Crystal
For polycrystalline materials, the loop is rounded (Fig. 6B.) and polarisation changes relatively gradually with field.

Some properties of the crystal, like coercive field, etc., are dependent on extraneous factors. As such they are called dynamic properties. Other properties, like spontaneous polarisation etc., are fundamental properties of the crystal. They are referred to as the static properties.

In a crystal, polarisation is not uniform over any appreciable area except in the presence of high fields. Normally, polarisation is constant over a very small area called domains.

The dipoles of BaTiO$_3$ crystals can lie in three directions i.e., perpendicular to the major surface, called the c-domain and along either of the two axes in the plane of the crystal, called a-domains.

In the absence of high fields, many anti-parallel domains, as shown in Fig. 7, are observed in a c-domain crystal. Very thin walls of thicknesses of few unit cells separate these domains whose polarisations are in opposite directions. Hence these walls are known as
180° walls.

Fig. 8. shows the domain structure of a crystal in which both c- and a-domains are present.

When an external field is applied two things can happen. Either the polarisation and strain of each domain can change or the domain walls can move. Properties, which depend on the polarisation of each domain are static. On the other hand, properties which involve the motion of domain walls or creation of new domains, are dynamic.

The object of the present work is to investigate the possibility of using the nonlinear characteristic of ferroelectric materials for parametric amplification. We are not primarily interested in measuring the properties of these materials.
Fig. 2.1 Dielectric constant of BaTiO₃ versus d.c. field strength at 33.5°C (after Roberts²⁵).
2.1 LOW FREQUENCY MEASUREMENTS

At low frequencies, the nonlinear relation between the dielectric constant and the applied electric field of a ferroelectric material can be measured with an impedance bridge. A capacitor is made with the ferroelectric material. The capacity, as measured on an impedance bridge, changes in magnitude when a d.c. bias is applied to it. The magnitude of the permittivity and its variation can be calculated from the measured values of the capacitance and the dimensions of the capacitor. Fig. 2.1 shows a typical curve of variation of dielectric constant of barium titanate with the applied d.c. field. This curve in itself does not give sufficient evidence of ferroelectricity and similar curve can be obtained even above the Curie point.

Existence of hysteresis loop is a definite indication of ferroelectricity. In the Sawyer & Tower circuit, a large a.c. signal is applied to a capacitor made of a ferroelectric material. Signals proportional to the input voltage and the charge on the capacitor are fed to the vertical and the horizontal plates of a cathode ray tube. The resultant picture obtained is a
hysteresis loop.

The measurements at low frequencies, reported later, were taken on a Marconi Q-meter type TF329G. The material was shaped into a circular disc, opposite surfaces of which were coated with either silver or aluminium on an evaporating unit. Electrodes of copper foil were placed on each surface. This formed a capacitor, the value of which was measured in the following way.

A standard coil was connected across the coil terminals of the Q-meter which was tuned to resonance by varying its variable capacitor. The magnitudes of the capacitance and Q factor \( C_1, Q_1 \) and \( C_2, Q_2 \) at resonance, without and with the test capacitor connected across the capacitor terminals were noted.

The capacitance \( C \) of the sample is given by

\[
C = C_1 - C_2 = \frac{10^{-9}}{36\pi} \times \frac{k' A}{d} \quad \text{Farad} \quad 2.1
\]

or

\[
k' = \frac{d C}{A} \times 36\pi \times 10^9 \quad 2.2
\]

Where \( A \) is the area of the disc in square meters, \( d \) is the thickness of the disc in meters and \( k' \) is the relative dielectric constant.

Assuming that the ferroelectric sample can be represented by a capacitance in parallel with a conductance \( G \),
the loss factor $\tan \delta$ is given by

$$\tan \delta = \frac{1}{Q} = \frac{G}{\omega C} = \frac{k''}{k} \quad \text{--- 2.3}$$

where $\omega$ is the angular frequency of measurement.

It can be shown that

$$Q = \frac{Q_1 - Q_2}{Q_1} \left[ -\frac{Q_1 Q_2}{Q_1 - Q_2} \right] = \frac{1}{\tan \delta} \quad \text{--- 2.4}$$

To prevent the biasing d.c. voltages from going to the Q-meter, the test capacitor was connected in series with a 0.05 $\mu$Fd capacitor as shown in Fig. 2.2.

![Fig. 2.2 Circuit arrangement for low frequency measurement of dielectric constant.](image)

In actual measurements, the values of the capacitance and the Q factor, without and with the test capacitor connected to the point M, were noted. The complex permittivity is, then, calculated with the help of the equations 2.2 and 2.4.
2.2.1 MEASUREMENTS AT MICROWAVE FREQUENCIES.

At low frequencies, the measurement of dielectric constant and its variation is a problem of measuring lossy capacitance. As the dielectric constant becomes higher and higher, one has to be more and more careful to make sure that the electrodes are in good contact with the materials without having an air-gap in between them.

At microwave frequencies, a large portion of the incident energy is reflected from the high permittivity material. The transmitted energy, which is small, is further reduced when the material is highly lossy.

The variation of dielectric constant of high permittivity material with applied d.c. fields has been measured by Davis & Rubin, Benedict & Durand and others.

Powles and Jackson made measurements on polycrystalline barium titanate specimens at 1.5, 9450 and 24000 Mc/s. Methods of microwave measurements depended on the dielectric constant and the loss tangent of the sample. The specimen was in the shape of a rectangular slab filling the entire cross-section of the X-band wave-guide. For high-loss high permittivity material, impedance
Fig. 2.3 Behaviour of BaTiO$_3$ at 1.5 and 9450 Mc/s (after Powles and Jackson).
presented at the front face of the specimen was determined by standing-wave measurements. The complex permittivity of the specimen was calculated from this measured impedance assuming the reflection from the back face to be negligible. This was adequately attained by a combination of the effects of partially correct termination by a quarter-wave slab and high specimen attenuation. To reduce the standing wave ratio in front of the specimen, a quarter-wave slab was used at the front face also. With BaTiO$_3$, the values obtained by them for the dielectric constant and loss tangent are 1500 & 0.015 and 300 & 0.53 at 1.5 and 9450 Mc/s respectively. Fig 2,3 shows some of their results.

Davis & Rubin$^{27}$ made measurements on barium-strontium titanate ceramics at 10 Kc/s, 500 Kc/s and 3000 Mc/s. The samples were fabricated in the form of cylinders of 2.1 cm o.d., 1.6 cm i.d., and having lengths ranging from about 0.5 cm to 2 cm. To match properly high dielectric material to the co-axial line, and thus avoid large standing-wave ratios, quarter wave-length sections were used on both sides of the material. These sections were made from mixtures of strontium titanate and magnesium titanate with dielectric constants in the range of 40 - 60. Measurements were taken on a 3000 Mc/s bridge.
Fig. 2.4 Temperature and electric field dependence of dielectric constant at 3000 Mc/s of 73 % BaTiO$_3$ : 27 % SrTiO$_3$ (after Davis & Rubin$^{27}$).
Some results of their measurements are shown in Fig. 24. No frequency dependence was apparent at temperatures above the Curie point.

Benedict and Durand measured the dielectric constant of single domain crystals of BaTiO$_3$ at 24 kMc/s. The crystals of thickness $\approx 0.040$ cm were shaped to fit the cross-section of a standard K-band waveguide. Measurements were made on a microwave bridge. An unambiguous value of dielectric constant and loss tangent of the sample was obtained by using more than one sample thickness. At $20^\circ$ C, they obtained a value of 2000 and 0.15 for the dielectric constant and the loss tangent. The dielectric constant was about one-half that observed at low-frequency below the Curie point, but the low and high frequency value agreed quite well above the Curie point.

The shape of the sample chosen for the present investigation is quite different from that used by previous investigators using high permittivity materials. Three factors governed the choice of using the sample as a cylindrical rod i.e., difficulty of getting suitable samples, convenience in the preparation of sample and its possible application to parametric amplification.

A cylindrical rod, of about 1/16" in diameter, of a ferroelectric material, placed in the centre of a waveguide, will present a reactance. From the measurement of the reactance
of the rod and its change with the applied d.c. field, the
dielectric constant and its variation can be calculated.

2.2.2 MEASUREMENTS WITH A STANDING-WAVE DETECTOR.

A cylindrical sample of diameter $d \approx 0.2$ cm was placed
at the centre of the waveguide. A thin wire for carrying
the d.c. bias passed through a small insulated hole at the
narrow side of the waveguide and was wrapped round the central
portion of the sample as shown in Fig.2.5.

![Sample in Waveguide](image)

**FIG.2.5 SAMPLE IN WAVEGUIDE**

The sample was backed by a matched load and the admittance
of the combination was measured with a standing-wave
detector by von Hippel's method. By referring the measure­
ments to the plane of the sample, the admittance of the sample
can be determined. Variation of the admittance of the sample
on the application of d.c. bias will change the position of
the standing-wave minimum and/or the standing-wave ratio.

A sample of polycrystalline barium titanate was
received from Stanford University. It was stated$^\text{29}$ that
the material has a loss tangent of 0.02 at S-band and a dielectric constant of 5000 and a 2:1 change in the dielectric constant can be obtained by the application of a d.c. field of 20 kV/cm.

The rod was placed in the guide as shown in Fig. 2.5 and its admittance was measured. No measurable change in the position of the voltage minimum or in the magnitude of the standing-wave ratio was noted with a d.c. bias of 4 kV. To avoid the presence of any minute air-gap between the sample and the wire carrying the d.c. voltage, silver paste was applied at the place where the wire was wrapped round the sample.

The d.c. field decreases in magnitude from the outer periphery of the sample towards the centre, being smallest at the centre.

The sensitivity of the system in measuring a change in susceptance is dependent on the percentage change of susceptance. If the original susceptance of the sample is very high and the susceptance changes by a small amount on the application of d.c. bias, then changes in the position of voltage minimum and/or standing-wave ratio will not be noticeable.

With a field of 8 kV/cm, this sample is expected to
show appreciable change of dielectric constant. As yet, the relation between the change of the dielectric constant and the change of susceptance have not been analysed.

In the following section, the reactance presented by a rod of dielectric material and its change with the change of the dielectric constant will be discussed.

2.3 **REACTANCE PRESENTED BY A DIELECTRIC ROD.**

A cylindrical loss-free dielectric rod placed in the centre of a waveguide propagating $H_{01}$ mode can be represented by the equivalent circuit shown in Fig. 2.6.

![Diagram of dielectric rod and equivalent circuit](attachment:image.png)

(a) Cross-sectional view  (b) Top view

(c) Equivalent circuit.

**FIG 2.6 (a) and (b) DIELECTRIC ROD IN A GUIDE AND (c) ITS EQUIVALENT CIRCUIT.**
The reactance of the rod is given by Marcuvitz\textsuperscript{32} by

\[
\frac{X_a}{Z_0} - \frac{X_b}{2Z_0} = \frac{a}{2\lambda g} \cos \theta \sin^2 \frac{\pi}{2} \left[ \frac{J_0(\beta)}{J_0(\alpha)} \frac{1}{\beta J_0(\alpha) J_1(\beta) - \alpha J_0(\beta) J_1(\alpha)} - s_0 + \frac{\alpha^2}{4} \right] = 2.5
\]

and

\[
\frac{X_b}{Z_0} = \frac{2a}{\lambda g} \left( \frac{\pi d}{a} \right)^2 \sin^2 \frac{\pi}{2}
\]

\[
\alpha = \left[ \frac{2}{\beta J_1(\beta)} \frac{1}{\beta J_0(\alpha) J_1(\beta) - \alpha J_0(\beta) J_1(\alpha)} \right] - 2
\]

where

\[
\alpha = \frac{\pi d}{\lambda}
\]

\[
\beta = \frac{\pi d}{\lambda} \sqrt{k}
\]

and

\[
s_0 = \log_e \left( \frac{4a}{\pi d} \sin \frac{\pi}{2} \right) - 2 \sin^2 \frac{\pi}{2} + 2 \sum_{n=2}^{\infty} \sin^2 \frac{\pi n}{2} \left[ \frac{1}{\sqrt{n^2 - \frac{2a^2}{\lambda}}} - \frac{1}{n} \right]
\]
Assuming that these formulae are also valid for high permittivity materials, one can obtain a general picture of the problem.

Consider a case in which \( \frac{d}{a} \) is very small, so that we can neglect the series reactance and can represent the rod by a shunt reactance alone. Then, we have,

\[
\frac{X_a}{Z_0} = \frac{a}{2 \lambda'} \left[ \frac{J_0(\beta)}{J_0(\alpha)} - \frac{1}{\beta J_0(\alpha) J_1(\beta) - \alpha J_0(\beta) J_1(\alpha)} \right] S_0 + \frac{\alpha^2}{4}
\]

Taking some typical figures,

\[
f = 9375 \text{ Mc/s}; \quad \lambda = 3.197 \text{ cm}; \quad \lambda_g = 4.473 \text{ cm}.
\]

\[
k' = 5000; \quad \tan \delta = 0.02 \text{ and } d = 0.2 \text{ cm}.
\]

we have

\[
\alpha \approx 0.197, \quad \beta = 13.90
\]

and

\[
S_0 = +0.665
\]

For this value of \( \alpha \), we have

\[
J_0(\alpha) = 0.99 \quad \text{ and } J_1(\alpha) = 0.099
\]

\[
\approx 1.0 \quad \approx 0.1
\]

Under these assumptions the negative term in the denominator of equation 2.10 is negligible compared to the positive term except when \( J_1(\beta) \) approaches zero. Equation 2.10, then reduces to
Fig. 2.7 Variation of $\frac{J_0(\beta)}{\beta J_1(\beta)}$ with $\beta$. 
Let us put \[ \frac{J_0(\beta)}{J_1(\beta)} = f(\beta) \]  

\[ \frac{X_a}{Z_0} = \frac{a}{2\lambda_g} \left[ \frac{J_0(\beta)}{\beta J_1(\beta)} - S_0 \right] \]  

\[ f(\beta) \] passes through a series of resonances and anti-resonances as the value of \( \beta \) is changed. The variation of \( f(\beta) \) against \( \beta \) is plotted in Fig. 2.7. When \( J_1(\beta) \) approaches zero, i.e., at anti-resonance, the magnitude of \( X_a \) is higher as given by equation (2.1) than by equation (2.11), because in the denominator of the former equation there is a subtraction of two small nearly equal quantities.

The value of \( \beta \) at which anti-resonance takes place is not affected by the presence of \( S_0 \) which modifies the magnitude of \( f(\beta) \).

In the neighbourhood of anti-resonance a small change in \( \beta \) gives rise to a large variation in the magnitude of \( f(\beta) \). Consequently the percentage change of reactance is considerable.

But for values of \( \beta \) far away from anti-resonance, the value of \( f(\beta) \) and its change are small compared to the
constant term $S_0$. As a result, large change in the value of $\beta$ will not give any appreciable percentage change of reactance.

At a constant frequency, the value of $\beta$ is dependent on the permittivity and the diameter of the sample.

Above analysis shows that small changes in permittivity will give large variation in the reactance only when $\beta$ and hence the diameter of the sample is suitably chosen. The values of $\beta$ for which $f(\beta)$ attains a high value are $\pi 0$, $3.83$, $7.01$, $10.17$ .... etc., The corresponding value of $d$, the diameter of the sample, for the previously specified values of $k$ and $f$ are $\approx 0$, $0.055$ cm, $0.101$ cm and $0.146$ cm .... etc.

Examination of Fig. 2.7 reveals that the sharpness of anti-resonance increases with increasing value of $\beta$.

In a practical case BaTiO$_3$ is not loss-free. Previous equations will be modified to take the loss into account.
When a material has a complex permittivity

\[ k = k' - jk'' \]

equation (2.8) is modified to

\[ \beta' = \frac{\pi d}{\lambda} \sqrt{k' - jk''} \quad --- \quad 2.13 \]

For \( \tan \delta = 0.02 = \frac{k''}{k'} \) equation 2.13 reduces to

\[ \beta' \approx \frac{\pi d}{\lambda} \sqrt{k''(1 - j \frac{k''}{2k'})} \quad --- \quad 2.14 \]

Let us put \( \beta' = \beta e^{-j\theta} \)

\[ \quad --- \quad 2.15 \]

For small values of \( \theta \), we have,

\[ \beta' \approx \beta(1 - j \theta) \quad --- \quad 2.16 \]

Comparing equations (2.14) and (2.16) we have

\[ \frac{k''}{2k'} = \theta \quad --- \quad 2.17 \]

Substituting the value of \( \beta' \) in equation 2.11, we have

\[ \frac{jR_a + X_a}{Z_0} = \frac{a}{2\lambda_g} \left[ \frac{J_0(\beta e^{-j\theta})}{\beta (i-j\theta) J_1(\beta e^{-j\theta})} - S_0 \right] \quad --- \quad 2.18 \]
Fig. 2.8 Variation of $f(\beta)$ with $\beta$ for $\theta = 0.0873$. 
Putting \( J_0 = e^{-j\theta} = A - jB \) \[\text{--- 2.19}\]

\( J_1 = e^{-j\theta} = C - jD \) \[\text{--- 2.20}\]

we have

\[
\begin{align*}
\frac{jR_a + X_a}{Z_o} &= \frac{a}{2\lambda_g} \left[ \frac{A - jB}{(C - jD)(1 - j\theta)} - S_o \right] \quad \text{--- 2.21}
\end{align*}
\]

\[
\begin{align*}
\frac{1}{\lambda} &= \frac{a}{2\lambda_g} \left[ \frac{(A - jB)(C + jD)(1 + j\theta)}{\beta(c^2 + D^2)} - S_o \right] \quad \text{--- 2.22}
\end{align*}
\]

\[
\begin{align*}
X_a &= \frac{a}{2\lambda_g} \left[ \frac{(AC + BD) + (BC - AD)\theta}{\beta(c^2 + D^2)} - S_o \right] \quad \text{--- 2.23}
\end{align*}
\]

\[
\begin{align*}
R_a &= \frac{a}{2\lambda_g} \left[ \frac{(AD - BC) + (AC + BD)\theta}{\beta(c^2 + D^2)} \right] \quad \text{--- 2.24}
\end{align*}
\]

and

Let us put equation 2.24 in the following form.

\[
\begin{align*}
\frac{X_a}{Z_o} &= \frac{a}{2\lambda_g} \left[ f(\beta') - S_o \right] \quad \text{--- 2.24(a)}
\end{align*}
\]

For \( \theta = 5^\circ = 0.0873 \) radian, i.e., \( \tan \delta = 0.175 \),
the values of \( f(\beta') \) are plotted in Fig. 2.8 for
\( \beta \) ranging from 0.7 to 5. The value of \( f(\beta') \) is small for
values of \( \beta \) above first resonance. For the specified
values given above, the peak value of \( f(\beta) \) for 1st and higher anti-resonance is less than the constant term \( S_0 \).

Therefore, to obtain the largest change in reactance for a small change in the dielectric constant one has to choose the diameter of the sample corresponding to the smallest value of \( \beta \). The sample diameter is to be fairly uniform, otherwise the theoretical sharpness of resonance will not be obtained in practice.

The diameter of the sample was reduced by grinding it with a Carborundum stone. But even for a reduced diameter of about 0.5 mm, the change in reactance on the application of d.c. bias is hardly measurable. It is extremely difficult to make a very thin rod of fairly uniform diameter. Moreover, one has to be extremely careful in handling a rod of diameter of 0.5 mm as it becomes very fragile.

The smallest value of \( \beta \) corresponding to a large value of \( f(\beta) \) can be obtained by varying the permittivity of the sample for a particular frequency and diameter of the sample. If \( \text{BaTiO}_3 \) is mixed with a nonpolar binder as suggested by Cassedy, \( J_r \), the permittivity of the composite material can be varied and one can hope to obtain the required value of \( \beta \) by trial and error.
Fig. 2.9. (a) Admittances and (b) Impedances of samples A, B and C.
<table>
<thead>
<tr>
<th>Variation of dia. of the sample</th>
<th>Frequency (Mc/s)</th>
<th>V.S.W.R.</th>
<th>TOTAL Shift of Voltage (λg)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>11,648</td>
<td>17.9</td>
<td>-0.0016</td>
</tr>
<tr>
<td></td>
<td>11,477</td>
<td>18.1</td>
<td>0.0022</td>
</tr>
<tr>
<td></td>
<td>11,359</td>
<td>16.7</td>
<td>0.0052</td>
</tr>
<tr>
<td></td>
<td>11,200</td>
<td>16.5</td>
<td>0.0057</td>
</tr>
<tr>
<td></td>
<td>11,054</td>
<td>22.2</td>
<td>0.0094</td>
</tr>
<tr>
<td></td>
<td>10,747</td>
<td>16.2</td>
<td>0.0107</td>
</tr>
<tr>
<td></td>
<td>10,430</td>
<td>14.9</td>
<td>0.0116</td>
</tr>
<tr>
<td></td>
<td>10,303</td>
<td>14.5</td>
<td>0.0128</td>
</tr>
<tr>
<td></td>
<td>10,090</td>
<td>11.6</td>
<td>0.0124</td>
</tr>
<tr>
<td></td>
<td>9,985</td>
<td>12.8</td>
<td>0.0132</td>
</tr>
<tr>
<td></td>
<td>9,949</td>
<td>10.9</td>
<td>0.0134</td>
</tr>
<tr>
<td></td>
<td>9,751</td>
<td>10.7</td>
<td>0.0154</td>
</tr>
<tr>
<td></td>
<td>9,175</td>
<td>9.4</td>
<td>0.0194</td>
</tr>
<tr>
<td></td>
<td>8,582</td>
<td>9.1</td>
<td>0.0214</td>
</tr>
<tr>
<td></td>
<td>8,382</td>
<td>8.6</td>
<td>0.0414</td>
</tr>
<tr>
<td></td>
<td>8,275</td>
<td>8.1</td>
<td>0.0425</td>
</tr>
</tbody>
</table>

**Sample C**: a mixture of BaTiO₃ and polythene having an average diameter of 0.192 cm.
Fig. 2.10 Variation of Normalised reactance of sample C with frequency.
### TABLE 2.2

Admittances and Impedances of Samples A and B. Both Stanford University sample having average diameters of 0.199 cm and 0.054 cm respectively.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.95 to 2.01 mm Av dia = 1.99 mm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>10208</td>
<td>8.0</td>
<td>0.0344</td>
<td>-3.38</td>
<td>1.0</td>
<td>0.273</td>
<td>0.077</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9329</td>
<td>10.34</td>
<td>0.0288</td>
<td>-4.25</td>
<td>1.3</td>
<td>0.216</td>
<td>0.065</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9050</td>
<td>11.7</td>
<td>0.0274</td>
<td>-4.6</td>
<td>1.4</td>
<td>0.196</td>
<td>0.058</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8693</td>
<td>12.95</td>
<td>0.0226</td>
<td>-5.3</td>
<td>2.0</td>
<td>0.159</td>
<td>0.058</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8350</td>
<td>14.2</td>
<td>0.0215</td>
<td>-5.8</td>
<td>2.1</td>
<td>0.148</td>
<td>0.056</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8025</td>
<td>14.6</td>
<td>0.0169</td>
<td>-6.6</td>
<td>3.3</td>
<td>0.120</td>
<td>0.057</td>
</tr>
<tr>
<td>B</td>
<td>0.44 to 0.59 mm Av dia = 0.54 mm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>10198</td>
<td>5.03</td>
<td>0.059</td>
<td>-1.95</td>
<td>1.0</td>
<td>0.510</td>
<td>0.023</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9718</td>
<td>5.7</td>
<td>0.0519</td>
<td>-2.25</td>
<td>0.35</td>
<td>0.433</td>
<td>0.030</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9027</td>
<td>7.43</td>
<td>0.0458</td>
<td>-2.76</td>
<td>0.39</td>
<td>0.360</td>
<td>0.024</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8727</td>
<td>8.27</td>
<td>0.0409</td>
<td>-3.05</td>
<td>0.50</td>
<td>0.316</td>
<td>0.050</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8528</td>
<td>8.95</td>
<td>0.0383</td>
<td>-3.28</td>
<td>0.60</td>
<td>0.296</td>
<td>0.050</td>
</tr>
</tbody>
</table>
Fig. 2.11 Variation of normalised reactance of samples A and B with frequency.
With this in view, a sample was made up of uncertain composition of BaTiO$_3$ and polythene. The average diameter of this sample, denoted hereafter as C was 0.192 cm. Two other samples of the solid dielectric were taken for comparison with the mixture. One sample, referred to as A, had an average diameter of 0.199 cm; the second one, denoted as B, had a diameter of 0.054 cm.

The impedance of these three samples were measured with a standing wave detector as explained in 2.2.1 over a large frequency range of not less than 1600 Mc/s.

In Fig. 2.9, the admittance and impedance of all these three samples are given in Smith's Chart.

In Table 2.1, the results of measurement of sample C are given. The reactance of the sample C is plotted against frequency in Fig. 2.10.

In Table 2.2, the results of measurements of samples A and B are given. The reactances of both these samples are plotted against frequency in Fig. 2.11.

The normalised reactances of these three samples at 8550 Mc/s, 9350 Mc/s and 10,150 Mc/s are taken from
Figs. 2.10 and 2.11 and are given in Table 2.3

### Table 2.3

<table>
<thead>
<tr>
<th>Sample</th>
<th>Normalised Reactances at Frequencies Mc/s</th>
<th>Change in normalised Reactance (a)-(c)</th>
<th>Change in Reactance Reactance at 9350(b) x100</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8550 (a)</td>
<td>9350 (b)</td>
<td>10,150 (c)</td>
</tr>
<tr>
<td>A</td>
<td>+0.158</td>
<td>+0.214</td>
<td>+0.269</td>
</tr>
<tr>
<td>B</td>
<td>+0.297</td>
<td>+0.399</td>
<td>+0.501</td>
</tr>
<tr>
<td>C</td>
<td>-0.317</td>
<td>-0.260</td>
<td>-0.184</td>
</tr>
</tbody>
</table>

In column (e) of Table 2.3 Change of reactance is given for each sample shows a considerable change of reactance with frequency.

Before discussing these results, it is to be noted that variation of frequency varies not only $\beta$ but also $\alpha$ and $S_0$. On the other hand, variation of permittivity affects $\beta$ only. As such, the variations of reactances of these samples with frequency as shown in Fig 2.9 do not
necessarily give the variations of reactances with $\beta$ alone or with the square root of the dielectric constant. The reactances of these three samples are not directly comparable.

Samples A and C have nearly the same diameters and as such, the same $\alpha$ and $S_0$ at any particular frequency. Their permittivities and $\beta$s are different. Hence, their reactances are different. Sample B, on the other hand, has a different diameter. This gives it a different value not only of $\beta$ but also of $\alpha$ and $S_0$ from those of samples A and C. Consequently, while comparing the reactance figures, it is to be remembered that the different samples have different reactances not only because they have different $\beta$ but also because they have different $\alpha$ and $S_0$.

Considering all these, it will not necessarily be correct to say from the percentage change of reactance figures given in column (e) of Table 2.3, that all these three samples will give similar percentage change of reactance for the same percentage change of dielectric constant.

From the measured reactances given in Table 2.1, the dielectric constant $k'$ of sample C is calculated at
several frequencies with the help of equation 2.11 as well as with the following approximate equation given by Marcuvitz\textsuperscript{32}.

\[
\frac{X_0}{Z_0} \approx \frac{\varepsilon}{2\lambda g} \left[ \frac{2}{(k' - 1)\lambda^2} - \frac{1}{4} \frac{k' - 3}{k' - 1} \right]
\]

Equation 2.26 agrees with Equation 2.11 to within a few per cent for \( k' < 16 \).

The results of these calculations are given in Table 2.4.

**TABLE 2.4**

Dielectric Constant \( k' \) of Sample C at different frequencies.

<table>
<thead>
<tr>
<th>Frequency Mc/s</th>
<th>8273</th>
<th>8582</th>
<th>9090</th>
<th>9151</th>
<th>9985</th>
<th>11,359</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k' ) using eqn. 2.26</td>
<td>27.29</td>
<td>27.35</td>
<td>27.62</td>
<td>27.46</td>
<td>28.78</td>
<td>31.44</td>
</tr>
<tr>
<td>( k' ) using eqn. 2.11</td>
<td>24.87</td>
<td>24.80</td>
<td>26.50</td>
<td>26.62</td>
<td>27.35</td>
<td>30.48</td>
</tr>
<tr>
<td>( k' ) using eqn. 2.26</td>
<td>563.21</td>
<td>523.22</td>
<td>475.07</td>
<td>469.6</td>
<td>396.6</td>
<td>320.2</td>
</tr>
<tr>
<td>( k' ) using eqn. 2.11</td>
<td>1784.0</td>
<td>1680.9</td>
<td>1507.6</td>
<td>1482.8</td>
<td>1249.5</td>
<td>978.6</td>
</tr>
<tr>
<td>( k' ) using eqn. 2.26</td>
<td>3755.6</td>
<td>3495.8</td>
<td>3122.5</td>
<td>3080.9</td>
<td>2599.1</td>
<td>2018.9</td>
</tr>
</tbody>
</table>
At each frequency, solution of Equation 2.11, gives several values of $\beta$ for the same value of $\frac{J_0(\beta)}{\beta J_1(\beta)}$. Values of $k'$ for four values of $\beta$ starting from the minimum one are given in Table 2.4.

Analysis of the results show that the probable value of $k'$ is 25.

Neglect of loss will partly account for the small variation of the dielectric constant with frequency. 11359 Mc/s is very close to resonance frequency and as such the calculated $k'$ at this frequency is of questionable accuracy.

No attempt is made to calculate the complex permittivity of the solid samples because the only table of Bessel functions for the complex arguments known to the author gives values of functions for $\theta$ varying in steps of $5^0$ from $0^0$ to $90^0$. The value of $\theta$ we are interested in is much less than 5.

To detect and measure small changes of reactances, a more sensitive arrangement is necessary. This is described in the next section.
2.4 MICROWAVE BRIDGE MEASUREMENTS

Several bridges with minor variations were tried. The arrangement found most sensitive is described below. The sample, mounted in the guide, was placed in one of the side arms, the test arm, of a magic-T as shown in Fig 2.12.

It was backed by a calibrated variable short-circuit which was placed at any odd multiple of quarter-wavelength behind the centre of the sample and as such presents an open
circuit at the plane of the sample. On the other arm a variable calibrated attenuator was connected. It was followed by a calibrated short circuit.

The E-arm was connected to a crystal detector followed by a sensitive galvanometer. H-arm was connected to the source. The bridge is balanced by adjusting the variable attenuator and the short-circuit at the reference arm till the deflection in the galvanometer is zero.

It can be shown that $P_D$, the power reaching the detector is given by\(^3\)\(^4\).

$$P_D = \frac{1}{4} \left| \Gamma_t - \Gamma_r \right|^2 P_i$$

Where $\Gamma_t$ and $\Gamma_r$ are the complex reflection coefficients of the test and reference arms respectively and $P_i$ is the power fed to the bridge at the H-arm. At balance, the two complex reflection co-efficients $\Gamma_t$ and $\Gamma_r$ are equal in magnitude and phase and voltages reflected from the test and reference arms cancel each other in the E-arm.

When a d.c. bias is applied to the sample, the bridge balance is upset due to a change of reactance of the sample and an out of balance current is obtained in the galvanometer.
Let the new complex reflection coefficient of the test arm be \( \Gamma'_t \), then the power in the detector is

\[
P_D = \frac{1}{4} \left| \Gamma'_t - \Gamma_r \right|^2 \quad \text{......} \quad 2.28
\]

Let the normalised impedance of the sample be \( r + j x \). At balance, with d.c. bias,

\[
\Gamma'_t - \Gamma_r = \frac{r + jx - 1}{r + jx + 1} \quad \text{......} \quad 2.29
\]

Let \( \Delta x \) be the change of the capacitance of the sample on the application of d.c. bias. It is assumed that the resistive part is unchanged. Fig.2.9 indicates this to be a fair supposition. It is shown in Chapter VI that the resistive part remains constant.

Then,

\[
\Gamma'_t = \frac{r + j (x + \Delta x) - 1}{r + j (x + \Delta x) + 1} \quad \text{......} \quad 2.30
\]

\[
\Gamma'_t - \Gamma_r = \frac{r + j (x + \Delta x) - 1}{r + j (x + \Delta x) + 1} - \frac{r + jx - 1}{r + jx + 1} \quad 2.31
\]

\[
= \frac{2j \Delta x}{\left[ r + j (x + \Delta x) + 1 \right] (r + jx + 1)} \quad 2.32
\]

Substituting equation 2.32 in equation 2.28, we have

\[
P_D = \frac{1}{4} \left[ \left| \frac{2j \Delta x}{\left[ r + j (x + \Delta x) + 1 \right] (r + jx + 1)} \right|^2 \quad 2.33
\]
<table>
<thead>
<tr>
<th>SAMPLE</th>
<th>FREQUENCY Mc/s</th>
<th>POSITION AT BALANCE OF REFERENCE ARM</th>
<th>ATTENUATION dB</th>
<th>BIAS FIELD kV/cm</th>
<th>OUT OF BALANCE CURRENT IN THE GALVANOMETER, cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>9675</td>
<td>Short Cnt. m.m. 15.115</td>
<td>0.389</td>
<td>0.35</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Attenuator m.m. 0.389</td>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
<td>15.10</td>
<td>0.389</td>
<td>0.35</td>
<td>2.56</td>
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<td>15.075</td>
<td>0.381</td>
<td>0.34</td>
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<td>0.378</td>
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<td></td>
<td></td>
<td>15.029</td>
<td>0.373</td>
<td>0.32</td>
<td>4.68</td>
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<td></td>
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<td>0.369</td>
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</tr>
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<td></td>
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<td>14.980</td>
<td>0.365</td>
<td>0.30</td>
<td>6.72</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14.935</td>
<td>0.355</td>
<td>0.29</td>
<td>8.00</td>
</tr>
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<td>B</td>
<td>9552</td>
<td>2.28</td>
<td>0.43</td>
<td>0.42</td>
<td>0</td>
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<tr>
<td></td>
<td></td>
<td>2.28</td>
<td>0.43</td>
<td>0.42</td>
<td>3.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.272</td>
<td>0.432</td>
<td>0.43</td>
<td>4.0</td>
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<td>2.26</td>
<td>0.438</td>
<td>0.43</td>
<td>5.6</td>
</tr>
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<td>2.245</td>
<td>0.44</td>
<td>0.43</td>
<td>6.4</td>
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<td></td>
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<td>2.215</td>
<td>0.441</td>
<td>0.44</td>
<td>8.0</td>
</tr>
<tr>
<td>A</td>
<td>8942</td>
<td>21.46</td>
<td>0.372</td>
<td>0.32</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>21.45</td>
<td>0.373</td>
<td>0.32</td>
<td>8.0</td>
</tr>
</tbody>
</table>
Let us calculate an approximate value of the out of balance current when \( x \) changes from 0.1 to 0.105 and let \( r = 0.05 \) (Fig. 2.9). Let the incident power be 40 milliwatt.

\[
P_D = \frac{1}{4} \left| \frac{j 0.01}{(1.05 + j 0.1)(1.05 + j 0.105)} \right|^2 \text{ mW}
\]

\[\approx 0.8 \text{ W}\]

This power will be completely absorbed by a matched crystal detector. In the 3-cm region the current sensitivity, defined as the ratio of short-circuit rectified current to absorbed r-f power, ranges from 0.5 to about 1.5 microamp per microwatt. Taking the current sensitivity to be 1 microamp per microwatt, the current through the crystal detector is 0.8 microamp. This will be the value of the current flowing through a low resistance galvanometer connected in series with the crystal detector. The sensitivity of the galvanometer was 200 mm per microamp, the out of balance current in the galvanometer will be 0.8 x 200 mm = 16 cm.

Samples A, B and C were measured on the bridge at several frequencies, some typical results are shown in Table 2.5.
Sample C gives by far the largest amount of out of balance current.

The admittances of the reference short-circuit attenuator combination for each balance position can be measured with a standing wave detector. These give admittances at the flange of the test arm where the sample is mounted.

Though the method is sensitive to detect non-linearity, the accuracy of the change of admittance measurement is no better than that described earlier in section 2.2.1.

It is useful to know the equivalent circuit of a dielectric rod. The following experiment was carried out to find this.

The sample has a finite diameter. By placing a short circuit at a suitable position behind the sample, it is possible to short it out if it can be represented by a shunt element alone.

Normally some attenuation is necessary in the reference arm to balance out the loss of the sample. When the attenuation in the reference arm is zero, it may be possible to balance the bridge by adjusting the two short
circuits one at the reference arm and the other behind the sample. This condition indicates that the sample reactance is shorted out by the short circuit behind it. Now application of d.c. bias will change the sample reactance, but if it is shorted out by the short circuit, no out of balance current will be obtained in the galvanometer. This can happen only if the equivalent circuit of the sample is a shunt element.

With zero attenuation in the reference arm, such a balance is obtained with sample C and this balance is not upset even when a d.c. bias of 8KV/cm is applied to it. This indicates that the assumption made in section 2.3, that the dielectric rod can be represented by shunt elements alone, is valid for sample C, a mixture of BaTiO$_3$ and polythene.

Neither of the two experimental methods described in this chapter give a definite value of the dielectric constant of the sample at microwave frequencies and its change on the application of a d.c. bias.

An experimental method by which the effect produced by the change of reactance can be measured more accurately is described in the next chapter.
FIG 3. (a) & (b). EXPERIMENTAL SET-UP FOR CAVITY MEASUREMENTS
Fig 3.2 Transmission wave-guide cavity with sample mounted inside it.
CHAPTER III

WAVEGUIDE TYPE CAVITY MEASUREMENTS

If a dielectric rod is placed in a cavity at a place having a non-zero electric field, a change of its reactance on the application of a d.c. bias will change the resonant frequency of the cavity. When the sample is at a maximum electric field, the frequency shift is maximum.

3.1 EXPERIMENTAL SET-UP

The experimental arrangement for making these measurements is shown in Fig 3.1 (a) and (b).

A cavity was made up with two pieces of waveguide connected together. Thin irises were fitted one at each end of this combination. The sample was placed at the junction of the two waveguides (Fig 3.2). The biasing system used was the same as explained in Fig 2.5.

A high voltage klystron, CV 323, was used throughout this project. In the early stages of the project C.W. operation of the klystron was used. The detected output was monitored in a sensitive galvanometer (Tinsley, resistance 133.5Ω, sensitivity 200 mm per micro-amp).
At a later stage, the reflector voltage of the klystron was modulated by square wave and the detected output was monitored in a selective amplifier.

The resonance curve of the loaded cavity, without any d.c. bias applied to the sample, was plotted point by point. The output of the klystron was monitored and kept constant all the time.

The resonance curve enables one to find out $f_0$, the resonant frequency of the cavity and $\Delta f$, the difference in frequency for the half-power points. From these two figures the $Q$ of the loaded cavity is calculated.

$$Q = \frac{f_0}{\Delta f} \quad ------ \quad 3.1$$

The procedure is repeated with the application of a d.c. bias to the sample. Let the new resonant frequency be $f'_0$ and let

$$f'_0 - f_0 = \Delta f' \quad ------ \quad 3.2$$

The ratio of $\Delta f'/\Delta f$ is a measure of the quality of the material, defined in Chapter V.

Preliminary measurements were made in a cavity having a total length of 13.82 cm. One section of the guide had
Fig 3.3 Resonance curve of the cavity (length 13.82 cm) loaded with sample C (a) without bias (o) and (b) with a d.c. bias field of 8 kV/cm (x).
a length of 7.48 cm. Sample C was placed in this guide as shown in Fig 3.2, so that the centre of the sample was 7.38 cm from one end of the cavity which was terminated at each end by irisirs having diameters of 0.25" and 0.125".

The results of the measurements with sample C are shown in Fig. 3.3. Application of a d.c. bias of 4 kV, i.e., a d.c. field of 8 kV/cm to the sample changed the resonant frequency of the cavity by 7.5 Mc/s.

The accuracy of determining one particular frequency depends, to a large extent, on how accurately the wavemeter calibration curve can be read. But the accuracy of the determination of small frequency differences can be considerably increased by taking the slope of the calibration curve of the wavemeter which is fairly linear over a small frequency range. The frequency difference for two wavemeter readings is found by multiplying the difference of the wavemeter readings by the previously determined slope.

Reaction wavemeters were used for measuring frequency. Sanders wavemeter type WM 16/1 was used towards the latter part of the project. One division of the micrometer is equal to 0.35, 1.17 and 1.85 Mc/s at the low, middle and high-frequency end of the wavemeter. A tenth of one
division can, with care, be estimated.

From the resonance curves plotted in Fig. 3.3, the resonant frequencies of the loaded cavity without and with d.c. bias are obtained. The output power of the cavity is directly proportional to the galvanometer reading as the crystal detector was working over the square law portion of its characteristic. The frequency difference for the half-power points is, therefore, obtained by reading from the resonance curve the frequencies at which the galvanometer is reduced to half its maximum value. The Q of the loaded cavity is calculated with Equation 3.1. The results are given in Table 3.1.

**TABLE 3.1.**

Resonant frequency and Q of the waveguide cavity loaded with Sample G (Composition)

<table>
<thead>
<tr>
<th>WITHOUT BIAS</th>
<th>WITH A D.C. BIAS OF 4 KV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resonant frequency $f_0$</td>
<td></td>
</tr>
<tr>
<td>Bandwidth $\Delta f$</td>
<td>Q</td>
</tr>
<tr>
<td>Mo/s.</td>
<td></td>
</tr>
</tbody>
</table>

| 9037.5 | 28.4 | 318 | 9045 | 22.8 | 397 |

The cavity showed resonance at a number of other frequencies,
but at no other frequency was the resonant frequency of the cavity changed on the application of a d.c. bias to sample 0 (composition).

Sample D, which is similar to A (solid) but has an average diameter of 0.171 cm was placed in the waveguide cavity in place of sample 0. The centre of the sample was 7.05 cm from one end of the cavity. Resonance was obtained at several frequencies but at no frequency the resonant frequency of the cavity changed on the application of a d.c. bias of 4kV to the centre of the sample. The results of the resonance curve measurements are given in Table 3.2.

Sample A was then tried in the cavity described above. The centre of the sample was 7.38 cm from one end of the cavity. The loaded cavity was resonant at several frequencies, but no trace of any shift of the resonant frequency was obtained on the application of a d.c. bias to the sample. The results of the resonance curve measurements are given in Table 3.2.

<table>
<thead>
<tr>
<th>SAMPLE</th>
<th>RESONANT FREQUENCY Mo/s</th>
<th>BANDWIDTH Mo/s</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>9238.0</td>
<td>14.3</td>
<td>646</td>
</tr>
<tr>
<td>A</td>
<td>8857.4</td>
<td>46.8</td>
<td>189</td>
</tr>
<tr>
<td>EMPTY CAVITY</td>
<td>9222.1</td>
<td>3.75</td>
<td>2460</td>
</tr>
</tbody>
</table>
Both samples A and C are believed to be operating in the same mode corresponding to the empty cavity resonant frequency of 9222 Mc/s.

The Q of the cavity loaded with Sample A is lower than that with Sample C. This is an indication of the greater loss of Sample A (solid) compared to that of Sample C (composition). The resonant frequency of the cavity loaded with Sample D is 16 Mc/s higher than that of the unperturbed mode, mentioned previously. As such, it is likely that the operating mode of the cavity loaded with Sample D is different from that with Samples A & C. Moreover, the position of Sample D in the waveguide was different from that of Samples A & C.

This is an explanation of the higher measured Q with Sample D compared to those of Samples A and C.

Out of the three samples, only Sample C (composition) gave a shift of the resonant frequency of the cavity on the application of a d.c. bias to the sample. This shows that Sample C exhibits a marked change of reactance. It is a further indication of the fact that $\beta$ for Sample C is more favourable than those of Samples A and D.

It is likely that if the dielectric constants
Fig. 3.4 Material (a) D4A1, (b) Technical Ceramics Ltd., (c) Single Crystal barium titanate and (d) cylindrical composition.
of the samples are varied, a value of $\beta$ can be obtained by trial and error, which will give the largest value of frequency shift of the cavity on the application of a d.c. bias. When $\text{BaTiO}_3$ is suspended in a non-polar binder, the dielectric constant of the composite mixture can be varied by varying the percentage composition of the constituents. Following this a number of samples were prepared with various percentage composition.

3.2 MATERIALS AND PREPARATION OF COMPOSITIONS.

Some ferroelectric materials were obtained during this time both in single crystal and polycrystalline form.

(1) Single Crystal barium titanate. They were thin irregularly shaped small crystals and they were not uniformly polarised in any particular direction.

(2) G.E.C. ceramic material mentioned as D4Al. This was a disc of 1" diameter and thickness 2.23 mm. This has a Curie temperature of 30°C.

(3) Barium titanate, prepared by Technical Ceramics Ltd., having a Curie temperature of 20°C. The materials were in the form of circular discs.

The materials D4Al, Technical Ceramics, single crystal $\text{BaTiO}_3$ and the cylindrical composition are shown in Fig 3.4 (a), (b), (c) and (d) respectively.
Fig. 3.5 Variation of dielectric constant with bias of (a) Technical Ceramics' sample (o) and G.E.C. sample D4A1 (x).
Compositions of single crystal, D4Al and Technical Ceramics material are mentioned hereafter as Bl, B2, Dl, D2, and El, E2, respectively.

Measurements of the dielectric constant and its variation with d.c. bias of some of these samples were taken at a low frequency (850 Kc/s). The results of a typical measurement on Technical Ceramics material, at room temperature, are shown in Fig 3.5.

The dielectric constant of the sample, at room temperature, is 6470.

Variation of the capacity of the material is directly proportional to the variation of the dielectric constant. Technical Ceramics material gives a large variation of the dielectric constant i.e. 48.8% with a field of 8.5 kV/cm.

The results of a low frequency (1000 Kc/s) measurements on material D4Al are shown in Fig 3.5.

The dielectric constant of D4Al has not been accurately calculated, because the sample used was irregular in shape. But it is estimated to be of similar magnitude as the Technical Ceramics material.

Variation of the dielectric constant of D4Al is
Fig. 3.6 Powdered barium titanate.
(Magnification 24)
less than that of the Technical Ceramics material.

Barium titanate was crushed in a mortar by pestle manually. The crushed powder is shown in Fig 3.6. A weighed quantity of this powder was mixed thoroughly with a known quantity of fine polythene powder of average diameter 250 microns.

The mixed powder was put into a small piece of glass tube of 2 mm nominal internal diameter and heated on a bunsen burner till it became a paste. It was pressed on two sides while it was allowed to cool and solidify. The composition was then taken out of the tube and cut to the required length. For the application of a d.c. bias, a small, thin copper electrode was placed at the centre of the sample during its preparation. A thin wire was soldered to this copper electrode. Towards the later stage, slightly folded copper wire was placed inside the composition in place of the copper electrode.

From the known weights of the materials used, the percentage composition of the sample is calculated. Taking different weights of the materials, samples of different percentage compositions were prepared.

Glass tube of nearly the same diameter was used
throughout the preparation of the samples, making them all of the same diameter. \( \beta \) of the samples vary primarily because they have different dielectric constants.

Initially, the presence of the nonlinearity of the samples was detected in a microwave bridge. The microwave bridge takes considerably less time compared with other experimental arrangements described for the detection of the nonlinearity and it is very sensitive.

Since the primary interest was in the change of the resonant frequency of the cavity, more detailed measurements were taken in the cavity.

### 3.3.1 Measurement in Single-Mode Cavity

The general and most convenient method of measuring the properties of a dielectric is a cylindrical \( E_{010} \) - mode cavity technique. To find out whether this technique could be used conveniently for these compositions, initial experiment was made in an almost square cavity the field pattern in which is not much different from \( E_{010} \) - mode. A rectangular waveguide cavity 1.8 cm long was made up. Each end of the cavity was closed by an iris of diameter 3/16". Sample B5 was placed near the iris so that the loaded resonant frequency of the cavity was very close to the unloaded resonant frequency. The sample was slowly shifted towards the centre of the cavity, each time finding the new resonant frequency, until the sample was at the centre of the cavity. The resonance curves of the loaded and empty cavity were measured point by point. The results of the measurements are given in Table 3.5.
TABLE 3.5

<table>
<thead>
<tr>
<th>SAMPLE</th>
<th>RESONANT FREQUENCY</th>
<th>BANDWIDTH</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mc/s</td>
<td>Mc/s</td>
<td></td>
</tr>
<tr>
<td>EMPTY</td>
<td>10.568</td>
<td>53</td>
<td>199</td>
</tr>
<tr>
<td>B5</td>
<td>8.360</td>
<td>113.4</td>
<td>73.7</td>
</tr>
</tbody>
</table>

The results show that loading a single-mode cavity with sample B5, one of the compositions, the resonant frequency of the cavity is reduced by 2208 Mc/s.

An approximate value of the dielectric constant of the sample can be obtained by assuming that the above single-mode cavity is equivalent to a cylindrical cavity of diameter 2 cm and operating in E₀¹⁰ mode. The value of the dielectric constant is given by the approximate formula:

\[ k = 1 + 0.539 \left( \frac{a}{b} \right)^2 \left( \frac{\Delta f}{f} \right) \]  

where,
- \( a \) = radius of the cylindrical cavity
- \( b \) = radius of the sample
- \( f \) = resonant frequency of the empty cavity
- \( \Delta f \) = shift of the resonant frequency of the cavity on loading

Substituting the figures in equation 3.1, the dielectric constant of sample B5 is

\[ k' \approx 1 + 0.539 \left( \frac{1}{0.1} \right)^2 \times \frac{2208}{10.568} \]

\[ \approx 12 \]
With increasing percentage content of BaTiO$_3^3$, samples of higher dielectric constant would be obtained. The shift in the resonant frequency of the cavity when loaded with B$_5^5$, was large and this sets a limit to this technique even using a properly designed cavity. The limitation at X-band is primarily due to the large radius of the sample which could not be appreciably reduced. Measurements at S-band are possible because the ratio of the radii are more favourable. The compositions were later measured in a cylindrical cavity at S-band and the results are reported in Chapter VI.

It was of interest to find out the behaviour of the compositions at X-band in a cavity. As such, a large waveguide cavity, several wavelengths long, was chosen for the purpose. The compositions will have comparatively small effect on a large cavity than on a single-mode cavity.

3.3.2 MEASUREMENT IN THE SECOND WAVEGUIDE CAVITY

A cavity, 12.21 cm long, was made up with three pieces of waveguide, the two outer sections were of the same length.

Fig. 3.7. Section in the middle of the central waveguide.
Fig. 3.8 Resonance curve of the cavity (length 12.21 cm) loaded with sample E1'(a) without bias (solid) and (b) with a bias field of 12 kV/cm (dotted).
The samples were made 0.2" long and were held in position by two rods screwed into the middle of the central wave-guide as shown in Fig 3.7.

This arrangement enabled the sample to be placed in the centre of the cavity. Secondly a high d.c. bias field could be obtained with comparatively smaller d.c. voltages.

The resonance curves of the cavity unloaded and loaded with a number of samples were measured point by point without and with d.c. bias applied to the samples. Results of a typical measurement with sample E1 in the cavity are shown in Fig 3.8.

The indication of the non-linearity of the samples, in this or future cavities, in general is as follows. The klystron frequency is varied till the output of the cavity is a maximum, indicating that it is in resonance. On the application of a d.c. bias to the sample, the output changes. The output sometimes goes up initially and then drops below its original value, indicating some change taking place inside the cavity. On retuning, maximum output is obtained at a different frequency. The output of the cavity with d.c. bias is, in general, larger than that obtained without bias. It is found from the measurement
Fig. 3.9 Display on oscilloscope of the klystron output (top) and the resonance curve of the loaded cavity (bottom)
of the resonance curve that $Q$ of the cavity is slightly higher when a d.c. bias is applied to the sample.

To observe visually all these the following arrangement was made. The klystron was frequency modulated by feeding to the reflector of the klystron a voltage of sawtooth waveform from a Cossor double-beam oscilloscope. The detected output of the cavity was fed to one beam of the oscilloscope. The detected output from the forward arm of the auxiliary line of the directional coupler was fed to the second beam which displayed the output of the klystron over the operating mode as shown in Fig 3.9 (top). The tuning of the wavemeter in this region is manifested in a sharp dip in the above curve. The sharpness of the dip depends on the $Q$ of the wavemeter. The first beam displays a curve which is a product of the resonance curve of the cavity and the operating mode of the klystron.

The two beams and the wavemeter position were adjusted so that at the tuned condition of the cavity, the peak of the resonance curve of the cavity touched the dip due to the wavemeter whose reading gives the resonant frequency of the cavity. On the application of a d.c. bias to the sample, the resonance curve was tilted and its peak was shifted towards a higher frequency. The klystron frequency was
changed till resonance was obtained again. The dip of the wavemeter and the peak of the resonance curve were now separated. The wavemeter position was, then, changed till they touched again. From the difference of the two readings of the wavemeter, the shift in the resonant frequency of the cavity is obtained.

The visual display on the oscillograph is a quick and convenient method of determining the frequency shift.

But there are severe limitations of the method. Firstly, it is not possible to measure the Q of the loaded cavity because the frequency difference of the cavity between its 3 dB points is often comparable to the total frequency sweep of the klystron. Secondly, the output of the klystron is not constant over the swept frequency range. Thirdly, the output of the cavity was often small and an extra transistor pre-amplifier was necessary to amplify the output before feeding to the oscilloscope. Sometimes, the output was so small that a reasonable display was difficult to obtain.

Measurement of the resonance curve point by point takes considerable time. Instead, the cavity was tuned to resonance, the output being adjusted to a suitable value. The klystron frequency was then varied on either side
<table>
<thead>
<tr>
<th>SAMPLE &amp; ITS % BaTiO₃ CONTENT</th>
<th>RESONANT FREQUENCY $f_0$ Mc/s</th>
<th>$Q$</th>
<th>FIELD $kV/cm$</th>
<th>SHIFT IN $f_0$ WITH BIAS $\Delta f'$</th>
<th>$\frac{\Delta f'}{\Delta f} \times 100$</th>
<th>SHIFT IN $f_0$ FROM EMPTY CAVITY Mc/s</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>EMPTY CAVITY</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(seven $\frac{1}{2}$)</td>
<td>10882</td>
<td>1450</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>(five $\frac{1}{2}$)</td>
<td>8964.5</td>
<td>2360</td>
<td></td>
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<td></td>
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<tr>
<td><strong>EMPTY CAVITY WITH RODS FOR HOLDING SAMPLES</strong></td>
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<tr>
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<td>8906</td>
<td>1164</td>
<td></td>
<td></td>
<td></td>
<td>58.5</td>
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<tr>
<td><strong>E1' 90.3</strong></td>
<td>9644.7</td>
<td>574</td>
<td>613</td>
<td>11.84</td>
<td>11.8</td>
<td>70.2</td>
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<td>564</td>
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<td>71</td>
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<td>8172.5</td>
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<tr>
<td><strong>E11 89.9</strong></td>
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<td>566</td>
<td>11.0</td>
<td>8.1</td>
<td>49</td>
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<td></td>
<td>8152.7</td>
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<tr>
<td><strong>E8 89.4</strong></td>
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<td>8153</td>
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</tr>
<tr>
<td><strong>E10 89.4</strong></td>
<td>9657.5</td>
<td>531</td>
<td>677</td>
<td>11.2</td>
<td>11.88</td>
<td>65.3</td>
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<td>-</td>
<td>7.6</td>
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<td>6.8</td>
<td>-</td>
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</table>
of the resonance curve till the output of the cavity, as indicated in the amplifier, dropped by half. The frequencies at both half-power points and at resonance were measured by the wavemeter. The power on the cavity was monitored and kept constant all the time. From these measurements the Q of the cavity is calculated.

Measurements, mentioned in Chapter VI, showed that the dielectric constant of the compositions is dependent on their previous history. As such, the compositions were "depoled", as described in Chapter VI, before taking any measurement.

A summary of the results of measurements of different samples in the second waveguide cavity is given in Table 3.3.

It is clear from these measurements that a change of reactance of the sample with d.c. bias can definitely be obtained if it has a suitable composition. A value of 50% and above for $\Delta f / \Delta f$ is not difficult to obtain with a d.c. bias field of about 12 kV/cm. This, for a sample length of 0.5 cm requires a voltage of 3 kV.

The possible resonant frequencies of the empty cavity and its length in terms of half wave-lengths
TABLE 3.4

EMPTY CAVITY RESONANT FREQUENCIES

<table>
<thead>
<tr>
<th>FREQUENCY $f_0$ Mc/s</th>
<th>GUIDE WAVELENGTH $\lambda_g$ cm</th>
<th>LENGTH OF CAVITY IN HALF WAVELENGTHS</th>
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</thead>
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<tr>
<td>11,812</td>
<td>3.051</td>
<td>Eight</td>
</tr>
<tr>
<td>10,810</td>
<td>3.488</td>
<td>Seven</td>
</tr>
<tr>
<td>9,863</td>
<td>4.069</td>
<td>Six</td>
</tr>
<tr>
<td>8,983</td>
<td>4.882</td>
<td>Five</td>
</tr>
<tr>
<td>8,192.5</td>
<td>6.103</td>
<td>Four</td>
</tr>
</tbody>
</table>

To get maximum electric field at the centre of the cavity, the length of the cavity is required to be an odd multiple of half-wavelengths i.e., five, seven, etc., Maximum effect with the application of bias to the sample was obtained at two frequencies which are believed to correspond to seven and five half-wavelengths of the empty cavity resonant frequencies.
Fig. 3.10 Shift of the resonant frequency of the wave-guide cavity (length 12.21 cm) on loading with samples of different percentage content of BaTiO₃ (Technical Ceramics sample).
In Fig. 3.10, the shift of the resonant frequency of the loaded cavity from that of the unloaded cavity is plotted against the percentage composition of the Technical Ceramics samples. The shift of the loaded resonant frequency, from the unloaded one, increases with increasing percentage content of barium titanate in the composition.

These experiments show that by varying the composition of the samples, nonlinearity can be obtained at X-band.

Before the samples could be utilised for obtaining parametric amplification, the following problems are to be solved.

(1) A cavity, loaded with sample, is to be designed satisfying the frequency condition given by equation 1.1.

(2) Will switching take place at microwave frequencies when the sample is subjected to high microwave field?

(3) What should be the value of $\Delta f'/\Delta f$ for obtaining parametric amplification?

The first problem is discussed in the next chapter, the other two will be dealt with in Chapter V.
4.1 DESIGN OF RECTANGULAR CAVITY

For a parametric amplifier, it is necessary to have the cavity tuned to three frequencies satisfying equation 1.1.

High microwave power, necessary for the pump, can be obtained from a pulsed magnetron which normally operates at a fixed frequency.

So, out of the three frequencies, one frequency is fixed. The cavity will have to be tuned to this frequency before any measurement at high microwave field can be undertaken. The other two frequencies can be varied within the limits of the available klystrons. It was decided that the signal frequency be in the S-band and the idler frequency be in the 6 kMc/s band.

In designing this cavity attention has to be paid to the following things:

(1) The field patterns of all the three modes should be such that the electric field is maximum for all of them at the centre of the cavity where the sample is placed.
(2) The height of the cavity is to be kept as minimum as possible to prevent any variation of the field in the vertical direction due to the setting up of unwanted modes.

(3) The cavity is to be tuned to the lowest possible modes, to prevent unwanted modes being set up destroying the desired field pattern.

Conditions (1) and (3) are satisfied if the three modes to which the cavity is tuned are $T_{101}$, $T_{301}$ and $T_{103}$ corresponding to the signal, idler and pump frequencies respectively.

The magnetic field patterns for these three modes are shown in Fig. 4.1

**Fig. 4.1** MAGNETIC FIELD PATTERNS OF $T_{101}$, $T_{301}$ and $T_{103}$ - modes.
The solid line, the dashed lines and the broken lines represent the magnetic field patterns of the signal, idler and pump frequencies respectively.

Let $\lambda_s$, $\lambda_i$, $\lambda_p$ be the signal, idler and pump wavelengths respectively, then,

\[
\left( \frac{f_s}{c} \right)^2 = \frac{1}{4a^2} + \frac{1}{4L^2} \tag{4.1}
\]

\[
\left( \frac{f_i}{c} \right)^2 = \frac{1}{4a^2} + \frac{9}{4L^2} \tag{4.2}
\]

and

\[
\left( \frac{f_p}{c} \right)^2 = \frac{9}{4a^2} + \frac{1}{4L^2} \tag{4.3}
\]

where $a$ is the width of the cavity and $L$ is its length and $c$ is the velocity of light.

Let

\[
f_p = yf_s \tag{4.4}
\]

and

\[
f_i = xf_s \tag{4.5}
\]

Substituting equations 4.4 and 4.5 in equation 1.1, we have

\[
y = x + 1 \tag{4.6}
\]

and

\[
f_p : f_i : f_s = y : x : 1 \tag{4.7}
\]
With the help of equations 4.4, 4.1 and 4.3, one obtains,

\[
y^2 \left[ \frac{1}{4a^2} + \frac{1}{4l^2} \right] = \frac{9}{4a^2} + \frac{1}{4l^2} \quad \text{------ (4.8)}
\]

or

\[
\frac{l^2}{a^2} = \frac{y^2 - \frac{1}{y^2}}{9 - y^2} \quad \text{------ (4.9)}
\]

Similarly, combining equations 4.1, 4.2 and 4.5, one obtains

\[
x^2 \left[ \frac{1}{4a^2} + \frac{1}{4l^2} \right] = \frac{1}{4a^2} + \frac{9}{4l^2} \quad \text{------ (4.10)}
\]

or

\[
\frac{l^2}{a^2} = \frac{9 - x^2}{x^2 - 1} \quad \text{------ (4.11)}
\]

Therefore, from equations 4.9 and 4.11, we have

\[
\frac{l^2}{a^2} = \frac{9 - x^2}{x^2 - 1} = \frac{y^2 - \frac{1}{y^2}}{9 - y^2} \quad \text{------ (4.12)}
\]

or

\[
\frac{x^2 - 9}{x^2 - 1} = \frac{x^2 + 2x}{x^2 + 2x - 8} \quad \text{------ (4.13)}
\]

Two values of \( x \) are obtained by solving the quadratic equation 4.13. Taking the positive root,

\[
x = 1.6794 \quad \text{------ (4.14)}
\]
Substitution of this value of $x$ in equation 4.11, gives

$$l = 1.8428a$$  

(4.15)

Equation 4.15 determines the ratio of $l$ to $a$, the absolute values of them being determined by the designed frequency of the cavity. Substituting this value of $l$ from equation 4.15 in equation 4.3, one obtains

$$\left(\frac{f_p}{c}\right)^2 = \frac{9}{4a^2} + \frac{1}{4 \times 3.396a^2}$$  

(4.16)

The first rectangular box type cavity was designed with dimensions believed to be reasonable. But it did not work satisfactorily. A very brief account of it will now be given.

The designed resonant frequencies of the cavity are 10,675, 6695 and 3980 Mc/s. The cavity was 7.90 cm. long, 4.28 cm wide and 1.25 cm in height. Measurements were taken at X-band with a number of samples. The average resonant frequency of the cavity with the sample was 10,300 Mc/s, giving an average shift of the resonant frequency of 375 Mc/s, which is small compared to those obtained in the waveguide cavity. Only a small fraction of all the samples tested gave detectable shift of the resonant frequency of the
Cavity on the application of a d.c. bias field of 9.6 kV/cm. One of the maximum shift is 3.5 Mc/s corresponding to a \( \Delta f / \Delta f \) of 26.9%. To find an explanation for this unexpectedly low shift in the resonant frequencies, extensive measurements of the field pattern inside the cavity were taken. A detailed description of the method of these measurements is given later in this chapter.

These measurements were taken in the empty cavity and with a number of samples inside the cavity. The field patterns of the loaded cavity were distorted which was quite expected, but very much asymmetrical. The field in the vertical direction of the cavity should be constant, but experimentally it was found to vary indicating the presence of unwanted modes. The conclusion derived from these series of measurements was that the height of the cavity was too long and as a result unwanted E-modes were set-up. The total field inside the cavity is a combination of the desired and undesired modes resulting in peculiar asymmetrical patterns.

4.2 MEASUREMENTS IN 2ND RECTANGULAR BOX TYPE CAVITY.

As a result of the measurements taken in the first rectangular box type cavity, attention was drawn to two points. The height of the cavity is to be made lower
and a figure of 0.7 cm was taken. This figure was mainly decided by the dimension of the socket of the pick-up plug. The second point is the shift of the resonant frequency of the cavity on loading with samples. This was found to be smaller than those of the waveguide cavity. The X-band resonant frequency of the empty cavity was taken, to start with, as 9560 Mc/s, 120 Mc/s higher than the magnetron frequency. For increasing the resonant frequencies of the cavity, it was made up in such a fashion that small reduction of the dimensions could be carried out without much difficulty.

Substituting the above value of $f_p$ in equations 4.16, the value of $l$ is 8.816 cm and $a$ is 4.784 cm. Substituting these values of $l$ and $a$ in equation 4.1 and 4.2, the idler and the signal frequencies are 5990.8 and 3569.2 Mc/s respectively.

Section at the centre of the rectangular cavity made up to these dimensions is shown in Fig. 4.2.
The box was made up of 1/8" brass plates screwed together. The two side plates are stepped as shown in Fig. 4.2. To feed pump power, a 1/4" diameter coupling hole was made on the centre of the front plate on top of which a short piece of No.16 waveguide terminated by a flange was mounted. On the centre of the rear plate a hole of 1/8" diameter was drilled for the application of d.c. bias. This was filled with a piece of polystyrene with a small hole about 1/64" in diameter to allow the wire carrying high d.c. voltage to pass through it.

For feeding in and taking out signal power from the
Fig. 4.3 (a) Rectangular box type cavity and (b) its interior with the sample mounted inside it.
cavity, sockets for the pickup plug were fitted one on each of the side plates midway between ends. One socket is placed at the centre of the side piece and the second one is placed at one third the distance from one edge. The second socket was placed in this position where the pickup due to the pump is expected to be small if not negligible.

To hold the samples in position two 9 B.A. screws were fixed centrally one each on the top and bottom plates. The portions of the screws protruding inside the cavity was machined to a diameter of 0.12 cm.

Two 9 B.A. and four 6 B.A. screws were suitably fitted on the top plate for using them as tuning screws to make small adjustments of resonant frequencies of the cavity.

The completed box and its interior with the sample mounted inside is shown in Fig. 4.3 (a) and (b).

The resonance curve of the cavity at 9514.5 and 10,656.4 Mc/s were measured point by point. These frequencies correspond to $T_{103}$ and $T_{303}$ modes.

Samples were placed in the cavity and the resonance curve was measured with a small number of samples.
<table>
<thead>
<tr>
<th>SAMPLE &amp; ITS % BaTiO₃ CONTENT</th>
<th>RESONANT FREQUENCY f₀ Mc/s</th>
<th>Q</th>
<th>D.C. BIAS FIELD kV/cm</th>
<th>SHIFT IN f₀ WITH BIAS Δf Mc/s</th>
<th>Δf' Δf x 100</th>
<th>SHIFT IN f₀ FROM EMPTY CAVITY Mc/s</th>
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<td>8.0</td>
<td>20.5</td>
<td>365.4</td>
</tr>
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</table>
Fig. 4.4 Wayne-kerr S-band test oscillator type S 281
But with all the others, the Q of the cavity and the shift of the resonant frequency on the application of a d.c. bias of 9.6 kV/cm were measured.

Application of bias to the samples shifted the resonant frequency of the cavity at two sets of frequencies. These frequencies were believed to be perturbed $T_{103}$ and $T_{303}$ modes.

A summary of the results of measurements on a number of samples are given in Table 4.1. A second batch of samples were received from Technical Ceramics Limited. Compositions from E12 onwards were made from this second batch which is believed to be similar to the first batch.

Measurements at S-band were carried out with Wayne Kerr S-band Test Oscillator type S281, shown in Fig. 4.4. This covers a frequency range of 2700 – 4000 Mc/s and has an output of approximately 500 mw. The oscillator is provided with two output probes. The frequency of the oscillator was measured with a built-in co-axial resonator having a Q of 500 – 1000. The smallest scale division reads 0.01 cm and a fifth of this division can, with care, be estimated. At 10 cm, this corresponds to a frequency of 0.6 Mc/s. The oscillator
Fig. 4.5 The Idler (6 kMc/s) bench.
frequency is varied by means of a micrometer head driving a tuning plunger in the resonant cavity of the Heil tube. The frequency of the oscillator was measured with the built-in wavemeter at different positions of the micrometer head of the oscillator and a calibration curve was plotted with the oscillator frequency against the position of the micrometer. This curve is fairly linear up to 3500 Mc/s. From this calibration curve it is found that over the linear portion, one small division of the micrometer head corresponds to 3.8 Mc/s. This division is subdivided into half and a fifth of this half division can be estimated i.e., a frequency change of 0.38 Mc/s can be measured. The oscillator can be operated at C.W. or modulated with internal square-wave of frequencies 400 - 4000 c/s.

Measurements at the idler frequency was taken in the following way. A KS7 - 85 klystron operating in this frequency region was obtained from Mullards Limited. It was mounted on No. 14 waveguide bench as shown in Fig. 4.5.

The reaction type Mid-Century Wavemeter operates from 5850 Mc/s to 8200 Mc/s. The frequency can be
measured down to 5830 Mc/s. One small division of
the wavemeter corresponds to 0.35, 0.93 and 1.36 Mc/s
at the low, middle and high frequency range respectively.
Frequency changes of a tenth of a small division can be
estimated. The output at the idler frequency was taken
out of the No. 14 waveguide by a co-axial cable and was
fed to the cavity by a loop. Measurements on the empty
cavity at the signal and idler frequencies were taken.
The results are shown in Table 4.2.

**TABLE 4.2**

**EMPTY CAVITY RESONANCES**

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<thead>
<tr>
<th>Resonant frequency</th>
<th>Bandwidth</th>
<th>Q</th>
</tr>
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<td>Mc/s</td>
<td></td>
</tr>
<tr>
<td>3564</td>
<td>2.92</td>
<td>1220</td>
</tr>
<tr>
<td>5960</td>
<td>4.69</td>
<td>1270</td>
</tr>
</tbody>
</table>

Measurements at S-band were taken with samples
inside the cavity. Shifts of the resonant frequency
of the cavity, on the application of d.c. bias to the
sample was obtained at an average frequency of 3300 Mc/s.

Indication of any non-linearity of the samples at the idler frequency was not obtained, to the lowest frequency given by the klystron. The lowest frequency of the klystron cannot be easily measured. This is well below 5800 Mc/s.

The main problem was to tune the Cavity to the magnetron frequency. From Table 4.1 it is evident that one set of frequencies is far too high to bring down by tuning screws. The other set of frequencies is below the magnetron frequency. The second set of frequencies can be increased by reducing the dimensions of the Cavity. But before this is done a knowledge of the working mode is necessary. To find out the mode, the field pattern of the cavity was measured by perturbation method.

4.5. MEASUREMENT OF FIELD PATTERN INSIDE THE CAVITY BY PERTURBATION METHOD.

Perturbation by bead has been applied by various authors for finding different properties of cavity resonators.

Among others, Slater and Maier used this technique for finding the field strength of resonant cavities. They discussed spherical, disc and needle shaped perturbing
bodies. They also used this technique for the determination of the Field Strength in a Linear Accelerator cavity.

This method has been used by Dekleva and Robinson in the measurement of the shunt impedance of a cavity. Mullet discusses both metallic and dielectric bodies of various shapes.

The resonant frequency $f$ of a cavity is reduced by the introduction of a dielectric sphere of radius $a$. The frequency shift is dependent on the electric field $E_0$ at the point and is given by

$$\frac{\Delta f}{f} = -\pi a^3 \left[ \frac{k' - 1}{k' + 2} \right] \varepsilon_0 \frac{E_0^2}{U}$$

Where $U$ is the stored energy.

A glass bead of nominal 2.5 mm diameter was taken as the perturbing element. This will perturb the electric field only. The bead was mounted symmetrically on cotton thread which passed through two tiny holes, about $1/64"$ in diameter, drilled opposite to each other on the plates of the cavity. The thread was kept stretched by hanging small weights at each end. The position of the bead was noted
against a metre scale by a marker on the thread.
Slight rotation of the thread does not affect the
measurement as the glass bead is spherical.

The cavity was tuned to resonance with the glass
bead at one end. The position of the bead was slightly
changed and the cavity was retuned to resonance. The
process was repeated until the bead travelled from one
end of the cavity to the other. At each position of the
bead, the resonant frequency of the cavity was measured
with the wavemeter.

The resonant frequency of the cavity with the bead
placed at one of its end is taken as reference. As the
bead travels through higher fields, the shift of the
frequency increases becoming maximum at maximum field.

The shift of the resonant frequency of the cavity
is plotted against the position of the bead. The distance
travelled by the bead between two consecutive points of
minimum field corresponds to one loop. From the curve
mentioned above, the number of loops in any particular
direction perpendicular to each other give the working mode
of the cavity.

Perturbation measurements were carried out along the
Fig. 4.7 (a) and (b) Perturbation measurement of rectangular box type cavity loaded with sample E7 at 9212 Mc/s.
Fig. 4.3 Perturbation measurement of rectangular box type cavity loaded with sample E7 at 9990 Mc/s.

Fig. 4.9 Perturbation measurement of rectangular box type cavity loaded with sample E5 at 10,309 Mc/s.
length and the breadth of the cavity as shown in Fig. 4.6

These measurements were carried out at two sets of cavity frequencies, with empty and with a number of samples inside it.

Results of typical measurements with sample E7 in the cavity are shown in Figures 4.7 (a) and (b) and 4.8 and with sample E5 in Figure 4.9. The position of the sample in these figures is shown by a circle.

Combining Figures 4.8 and 4.9, it is clear that this set of frequencies ($\sim 10,000$ Mc/s) corresponds to $T_{303}$ mode.

Combining Figures 4.7 (a) and (b), it appears that this
### TABLE 4.3

**SUMMARY OF TESTS IN SECOND RECTANGULAR CAVITY**

**DIMENSIONS: 8.562 cm x 4.574 cm x 0.684 cm. IRIS DIAMETER = 0.25"**

<table>
<thead>
<tr>
<th>SAMPLE &amp; ITS % BaTiO₃ CONTENT</th>
<th>RESONANT FREQUENCY $f'_o$ Mc/s</th>
<th>D.C. BIAS FIELD $E$ kV/cm</th>
<th>SHIFT IN $f'_o$ WITH BIAS $\Delta f$ Mc/s</th>
<th>$\Delta f_{x 100}$</th>
<th>$f'_o + \Delta f$ Mc/s</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>E22 85.23</strong></td>
<td>5900</td>
<td>7.5</td>
<td>9.9</td>
<td>45.4</td>
<td>9479</td>
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<tr>
<td>3579</td>
<td>7.5</td>
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<td>65.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9649.6</td>
<td>7.5</td>
<td>5.16</td>
<td>21.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9409.5</td>
<td>7.5</td>
<td>6.8</td>
<td>37.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>E21 85.10</strong></td>
<td>5930</td>
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<td>18.0</td>
<td>26.9</td>
<td>9494</td>
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<tr>
<td>3564</td>
<td>7.6</td>
<td>5.32</td>
<td>46.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9652.6</td>
<td>7.6</td>
<td>5.0</td>
<td>24.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9413</td>
<td>7.6</td>
<td>6.4</td>
<td>35.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>D6 85.02</strong></td>
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<td>9.0</td>
<td>25.7</td>
<td>47.4</td>
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<tr>
<td>3606</td>
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<tr>
<td>9605.3</td>
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<td>9.2</td>
<td>27.5</td>
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</tr>
<tr>
<td>9412</td>
<td>9.0</td>
<td>8.75</td>
<td>27.0</td>
<td></td>
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<td></td>
</tr>
<tr>
<td><strong>D5 81.97</strong></td>
<td>5966</td>
<td>9.0</td>
<td>17.4</td>
<td>28.8</td>
<td>9593</td>
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<tr>
<td>3632</td>
<td>9.0</td>
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<td>31.7</td>
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</tr>
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<td>9700</td>
<td>9.0</td>
<td>9.5</td>
<td>19.6</td>
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</tr>
<tr>
<td><strong>E29 82.45</strong></td>
<td>5948</td>
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<td>3.1</td>
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<td>46.1</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>9669</td>
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<td>4.7</td>
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<td>45.7</td>
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</tr>
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<td>-</td>
<td></td>
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</tr>
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<td>77.6</td>
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<td>29.9</td>
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<tr>
<td><strong>E33 92.93</strong></td>
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<td>9.4</td>
<td>40.4</td>
<td>75</td>
<td></td>
<td></td>
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<tr>
<td>3727</td>
<td>9.4</td>
<td>14.8</td>
<td>95</td>
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<tr>
<td>9554.5</td>
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<td>2.8</td>
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<td>5.2</td>
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<td><strong>E11 89.9</strong></td>
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<td>9.6</td>
<td>-</td>
<td>-</td>
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<td><strong>E8 89.4</strong></td>
<td>3613</td>
<td>9.6</td>
<td>14.8</td>
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<td>573</td>
<td>9.6</td>
<td>4.3</td>
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<td><strong>E37 90.21</strong></td>
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<td>5.6</td>
<td>18</td>
<td>Tuned with T.S.</td>
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</tr>
<tr>
<td>9414</td>
<td>7.6</td>
<td>2.9</td>
<td>19</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
set of frequencies (≈ 9200 Mc/s) corresponds to $T_{301}$ mode. This is anomalous as $T_{301}$ mode resonant frequency of the empty cavity is 5990.8 Mc/s. This anomaly has not been resolved.

The dimensions of the cavity were reduced to increase the resonant frequency of the loaded cavity to 9420 Mc/s. After the third reduction, the final measured dimensions of the cavity are 8.562 cm x 4.574 cm x 0.684 cm.

Measurements with all the samples inside the cavity were taken at X-band, S-band and at idler frequency. Results of useful measurements on a selected number of compositions are given in Table 4.3.

A number of interesting things are found from these measurements. Firstly, there are samples with which $\Delta f'/\Delta f$ equal to or greater than unity can be obtained at the signal and idler frequencies. Compositions such as E2 and E33, which give large values of $\Delta f'/\Delta f$ at these frequencies have a higher percentage content of BaTiO$_3$ than other compositions. But, unfortunately, the amount of $\Delta f'/\Delta f$ obtained at the pump frequency is rather small compared to those obtained previously.

Secondly, non-linearity is found at the idler
frequency of about 5900 Mc/s. But some samples, those who have a higher percentage content of BaTiO₃, show non-linearity at a frequency of about 7300 Mc/s. This is far above the unperturbed $T_{301}$ mode resonant frequency of the cavity. Most of the samples show non-linearity at frequencies much lower than 5800 Mc/s.

It is useful to know the field patterns inside the cavity specially the field at the sample at the working frequencies.

The frequency condition for parametric amplification given by equation 1.1 can be satisfied with a number of samples. But before proceeding further, field patterns inside the cavity were found out by perturbation measurements taken in two directions perpendicular to each other and going through the centre of the cavity as shown in Fig 4.10.

**FIG. 4.10.** PERTURBATION MEASUREMENT WITH THE BEAD PASSING THROUGH THE CENTRE OF THE CAVITY.
Fig. 4.11 (a) & (b) Perturbation measurement of empty cavity at 9963.8 Mc/s, Q = 1795.
Fig. 4.12 (a) and (b) Perturbation measurement of empty cavity at 6180 Mc/s. Q ∼ 924.
Fig. 4.13 Perturbation measurement of cavity loaded with sample E21 at 9420 Mc/s. Frequency lowered with tuning screws.
Fig. 4.14 Perturbation measurement of the cavity loaded with sample E21 with a glass bead of diameter 1.25 mm at (b) 9650 Mc/s & (a) 9420 Mc/s, the freq. being lowered with T.S.
A third socket, for a pickup loop, was fitted on the rear plate of the cavity at one third distance from one end as shown in Fig 4.10.

Results of perturbation measurements of the empty cavity at 9963.8 Mc/s are shown in Figures 4.11 (a) and (b). The field patterns correspond to $T_{103}^q$ - mode. Results of measurements of the empty cavity at the idler frequency are shown in Figures 4.12 (a) and (b). The field in this case is due to $T_{301}^q$ - mode. Measurements at S-band show that the field pattern is due to $T_{101}^q$ - mode.

Results of measurements of the cavity loaded with sample E21 at 9420 Mc/s are given in Fig.4.13. The maximum frequency deviation was rather large. The diameter of the perturbing glass bead used was 2.5 mm.

Measurements of the loaded cavity with a glass bead of 1.25 mm diameter was taken by Warren in a more elaborate set-up devised for making perturbation measurements only. The results of measurements are shown in Figures 4.14 (a) and (b).

Figures 4.13 and 4.14 (a) bear a qualitative similarity to each other. This indicates that the larger diameter bead used had not upset the original field pattern.
In the ideal case, the diameter of the bead should be very small so that the field on its entire surface could be assumed to be approximately constant. But this is limited by the practical difficulties of measuring small frequency shifts.

Figures 4.13, 4.14 (a) and (b) show that the field at the sample and its neighbourhood is rather small at X-band.

Some calculated and measured resonant frequencies of the empty cavity at different modes are given in Table 4.4.

<table>
<thead>
<tr>
<th>MODE</th>
<th>RESONANT FREQUENCY</th>
<th>MEASURED Q</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calculated Mc/s</td>
<td>Measured Mc/s</td>
</tr>
<tr>
<td>T(_{101})</td>
<td>3716.3</td>
<td>3726</td>
</tr>
<tr>
<td>T(_{301})</td>
<td>6191.9</td>
<td>6180</td>
</tr>
<tr>
<td>T(_{501})</td>
<td>9349.3</td>
<td>9325.3</td>
</tr>
<tr>
<td>T(_{103})</td>
<td>9988.8</td>
<td>9963.8</td>
</tr>
<tr>
<td>T(_{303})</td>
<td>11149</td>
<td>-</td>
</tr>
</tbody>
</table>
It has been mentioned in Chapter 1 that to avoid frequency doubling the nonlinear dielectric is to be biased.

The central biasing used so far is convenient for measurements with low level signals but unsuitable for parametric amplification.

Let the sample be centrally biased so that the working point is at P (Fig. 4.15b). With signal, at any instant, the a.c. field increases the polarisation of one half of the sample and reduces the polarisation of the other half. The changes in the dielectric constant, due to a.c. signal, in the two sections of the sample are opposite in sign. Assuming these changes to be of equal magnitudes, they will cancel each other. For low level measurements, where one is primarily interested in $k'$ at the working point and not in its change due to the a.c. signal, central biasing does not affect the results.

But for application to parametric amplification, where the change in the dielectric constant due to the signal is also important, central biasing is not useful. The output using
### TABLE 4.5

**TESTS IN SECOND RECTANGULAR CAVITY**

(a) Central biasing

<table>
<thead>
<tr>
<th>Sample &amp; its % BaTiO$_3$ content</th>
<th>Resonant frequency $f_0$ Mc/s</th>
<th>$Q$</th>
<th>D.C. Bias Field kV/cm</th>
<th>Shift in $f_0$ with Bias $\Delta f$ Mc/s</th>
<th>$\frac{\Delta f}{\Delta f} \times 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>E24 84.22</td>
<td>5921</td>
<td>366</td>
<td>8.0</td>
<td>10.6</td>
<td>65</td>
</tr>
<tr>
<td></td>
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<td>8.0</td>
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<td>23</td>
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<td>310</td>
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<td>6.5</td>
<td>57</td>
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<td>9616</td>
<td>404</td>
<td>7.4</td>
<td>5.4</td>
<td>22.6</td>
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</table>

(b) Biasing at end, Length of sample = 0.3 cm

<table>
<thead>
<tr>
<th>Sample &amp; its BaTiO$_3$ content</th>
<th>Resonant frequency $f_0$ Mc/s</th>
<th>$Q$</th>
<th>D.C. Bias Field kV/cm</th>
<th>Shift in $f_0$ with Bias $\Delta f$ Mc/s</th>
<th>$\frac{\Delta f}{\Delta f} \times 100$</th>
<th>$f_1 + f_s$ Mc/s</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>E24' 84.22</td>
<td>5987</td>
<td>554</td>
<td>8.3</td>
<td>2.3</td>
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<td>636</td>
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<td>2.0</td>
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<td></td>
<td>9410</td>
<td>555</td>
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<td>3.4</td>
<td>20</td>
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<td></td>
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<td>8.3</td>
<td>4.6</td>
<td>15.8</td>
<td>9506</td>
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<td>476</td>
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<td>2.9</td>
<td>14.7</td>
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</tbody>
</table>
central biasing system, will be primarily at the second harmonic frequency. To avoid this, the sample is to biased at one end.

Measurements were taken with two samples with central biasing. The results are given in Table 4.5 (a). These two samples were then reduced in length and biased at one end. The electrode carrying the d.c. bias was insulated from the metal rod by layers of mica stuck together with araldite. (Fig.4.16). Measurements were taken at the pump, signal and idler frequencies with the above-mentioned samples. The results are given in Table 4.5 (b).

Fig. 4.16 Biasing the sample at one end.

The field patterns were also plotted with these samples in two directions perpendicular to each other along A'B and C'D (Fig.4.10) by perturbation method. The patterns were similar to those shown in Fig.4.14.

In the next chapter, the behaviour of some of these samples subjected to high microwave field will be discussed.
5.1.1 DESCRIPTION OF THE HIGH-POWER BENCH

Measurements at high microwave power were taken on a 725A magnetron bench (Fig 5.1)

![Diagram](image)

Fig 5.1 MAGNETRON BENCH

The magnetron is pulsed by a Marconi modulator type 239. The unit is triggered by positive pulses of not less than 7.5 volts and a duration of 5 μs. The unit operates for input pulses having a repetition frequency of 250 to 500 and 500 to 800 p.p.s. and delivers output from 250 to 400 p.p.s. When operated from pulses in the higher frequency range, a frequency halving circuit is switched into operation.

The measured peak output of the magnetron is 10.6 kW in 3 μs pulses.
Fig 5.2 (a) and (b). Power Sharing Device.
Fig 5.3 Power (solid line) going to load B, the best V.S.W.R (dotted line) obtained and the position of short-circuit (broken line) C against the position of short-circuit D.
It is necessary to control the amount of microwave power fed to the cavity providing simultaneously a good match looking towards the magnetron.

The power sharing device used is shown in Figs. 5.2 (a) and (b).

Power is fed to the H-plane of the Magic-T and its E-plane is covered with a short-circuit. The power going to the matched load B can be controlled by the position of the short-circuit D. For each position of the short-circuit D, the short-circuit C was adjusted till the best possible match, looking towards the generator, was obtained. A matched thermistor waveguide mount was connected at the position of load B and the power going to it was measured by a Polarad Power meter.

The results of these measurements, taken on a klystron bench, are shown in Fig 5.3.

With this arrangement, the input V.S.W.R. can be kept between 1.5 and 1.0 when the power fed to load B is varied from zero to maximum input power.

5.1.2. EXPERIMENTAL ARRANGEMENT.

The cavity loaded with a sample was tuned first to 9420 Mc/s, the magnetron frequency, with the help of the tuning
screws 4 and 5 (Fig 5.4).

They reduce the signal frequency to a small extent but have practically no effect on the idler frequency.

The tuning screws 3 and 6, which had no effect on the pump frequency, were then adjusted to reduce both the signal and idler frequency to satisfy the frequency condition given by equation 1.1.

The cavity was matched perfectly on a klystron bench at 9420 Mc/s with a variable short-circuited stub and then with a three-screw tuner (Fig 5.5).
The variable short-circuit was used to reduce the bad mismatch. Subsequent matching was done by the three-screw tuner. This enabled (a) the use of lower depth of penetration of the tuning screws preventing spark-over at high power and (b) a more flexible arrangement to be obtained.
As shown in Fig 5.6, the properly matched cavity was connected to the position of lead B (Fig 5.2a) in series with a 30 dB directional coupler for measuring the forward and reflected power. The thermistor mount and power meter mentioned before were used for this purpose.

The signal was fed in at Q (Fig 5.7)

![Diagram](image)

and taken out of R.

It is necessary to observe the output at the signal frequency alone. Otherwise, the presence of the increasing pump signal will make it difficult to identify any parametric amplification. The two low-pass filters, obtained on loan from Decca Radar Limited, reduced the output at the pump frequency by more than 50 dB. As a result, no output was obtained at the pump frequency out of the low pass filters while the signal was kept at a level convenient for measurement on the oscilloscope.
Combined pump and signal power was taken out at P and was fed to a co-axial thermistor mount. The power was monitored by a Sullivan microwatt bridge.

The magnetron power fed to the cavity was gradually increased to the highest value the cavity could withstand without sparking inside. By adjusting slightly the tuning screw 1, it was made sure that the cavity was tuned to the pump frequency all the time.

The signal was successively modulated by (1) internal square wave (2) 25 μs pulse from the modulator feeding the magnetron and (3) 3 μs pulse used for pulsing the magnetron. Observation was also made with unmodulated signal.

The penetrations of the tuning screws 3 and 6 were adjusted to avoid the possibility of missing the frequency condition of equation 1.1.

Preliminary measurements were taken with samples using central biasing arrangement. Measurements were carried out with zero bias and a d.c. field of 8 kV/cm. No parametric amplification was noticed. With high average pump power, working with measured pulse length of 3 μs and 250 p.r.f., the tuning and the output of the cavity at the signal frequency changed. This change was instantaneous but took place gradually indicating
heating effect. With samples D8 and E21, the resonant frequency of the cavity at the signal frequency increased by 3.8 and 7.6 Mc/s respectively for net peak power of 6.0 and 4.7 kW fed to the cavity. The net power going to the cavity was found by measuring the forward and reflected power by the directional coupler (Fig. 5.6).

Measurements were then carried out with samples E24' and E23' biased at one end (Fig. 4.16). These samples had earlier been tested at low power with central biasing system and then with bias at one end. The d.c. bias field was varied in steps from 8.0 to 10 kV/cm. In this case also, no parametric amplification was found, but the heating effect was observed. Results of observations are given in Table 5.1.

<table>
<thead>
<tr>
<th>Sample</th>
<th>D.C. bias Field kV/cm</th>
<th>Net Peak Power fed to the cavity kW</th>
<th>Increase in signal frequency Mc/s.</th>
</tr>
</thead>
<tbody>
<tr>
<td>E24'</td>
<td>8.4</td>
<td>2.1</td>
<td>2.0</td>
</tr>
<tr>
<td>E23'</td>
<td>8.4</td>
<td>2.4</td>
<td>2.5</td>
</tr>
</tbody>
</table>

This heating effect was absent when the p.r.f. was reduced from 250 to 50 c/s.

To find the temperature of the sample, a thermocouple was made up with 42 s.w.g. enamelled copper and constantan wires. One junction of the thermocouple was embedded in the centre of the sample in place of the central biasing wire. The wires of the thermocouple were twisted and taken out of the box in the same fashion as the central biasing wire.
The second junction was kept at the room temperature. The two copper wires of the thermocouple were connected to a calibrated Cambridge potentiometer designed to measure the thermo-e.m.f. set-up due to the difference of temperature of the two junctions. With sample E37 and with 3.5 kW net peak power fed to the cavity, the resonant frequency of the cavity at the signal frequency increased by 13 Mc/s, the potentiometer indicated 1.6 millivolt. This corresponds to a rise of temperature of the sample of 40° C above the room temperature (20° C).

Before any conclusion could be drawn, it had to be ascertained that this rise of temperature was not due to any pick-up. The thermocouple wires were removed from the sample. One junction of a similar thermocouple was placed in the centre of a small piece of polyfoam (k' nearly one). This was kept close to the sample and moved first towards D' along C'D' (Fig.4.10 p.120) and then towards B' along A'B'. It was also placed in several positions along CD (Fig.4.6 p.117). All the time, the net peak power fed to the cavity was kept constant and the output at the signal frequency was observed. When the polyfoam was close to the sample the indicated temperature rise was 10° C. This temperature rise reduced in magnitude as the polyfoam was moved away from the sample. For places well away from the specimen and where the field was strong, the pick-up was small when the wires of the thermocouple were not coupled to the fields inside the cavity. By deliberately increasing the coupling, the potentiometer reading could be considerably increased.
The indicated temperature rise of the junction when it was close to the specimen is presumably due to the direct radiation of heat from the sample.

To find out the order of magnitude of the frequency shift of the cavity effected by the change of the dielectric constant of the sample due to the variation in its temperature, the box was first cooled below and then heated above the room temperature. With sample E35 in the cavity, a rise of temperature of 40°C and 22°C above the room temperature (21°C) increased the resonant frequency of the cavity at the signal frequency by 33 and 12 Mc/s respectively. This increase in the resonant frequency of the cavity (loaded with sample E35) by heating the box is about 2 1/2 times greater than that obtained by heating sample E37 with microwave energy. The different k' of the samples used in the two cases will partly account for the difference. But the heat distribution is quite different in the two cases.

With microwave power the centre of the sample was at the highest temperature, either side being in contact with the metal box will be at the room temperature. As a result, the average temperature of the sample will be smaller than the measured figure. For an order of magnitude, the increase in the resonant frequency of the cavity when pumped can be explained by the change of the dielectric constant of the sample due to its rise of temperature.
An approximate theoretical expression will now be derived for the amount of frequency shift required at the pump frequency for obtaining large gains.

5.2 THEORETICAL VALUE OF $\Delta f' / \Delta f$ REQUIRED AT PUMP FREQUENCY FOR LARGE GAINS.

For large gains, the negative conductance presented to the signal circuit is equal to its conductance $G$. Combining equations 1.3 and 1.6,

$$G = \frac{\omega_1 \omega_2 C_0}{4G} \quad \ldots \quad 5.1$$

Let $C_0$ be the capacitance of the cavity at the pump frequency $f_p$ and let $\Delta C$ be the capacity swing for the half-power points. Then,

$$\frac{C}{\Delta C} \cdot \frac{\Delta C}{C_0} = \frac{1}{C_0} \sqrt{\frac{4G_2 G_{T1}}{\omega_1 \omega_2}} \quad \ldots \quad 5.2$$

or,

$$\frac{C}{\Delta C} = \frac{Q_p}{C_0} \sqrt{\frac{G_2 G_{T1}}{\omega_1 \omega_2}} \quad \ldots \quad 5.3$$

where $Q_p$ is the $Q$ of the cavity at the pump frequency.

It is not an easy task to derive a simple expression for the capacitance of a cavity. This has been treated by Beringer$^{142}$. He considers the equivalent circuit of a lossless, unperturbed cavity to be a L-C circuit.
From a calculation of the electric and magnetic energies of the system, he derives the following expressions for the equivalent capacitance $C_a$ and inductance $L_a$ of the cavity:

$$C_a = \frac{\varepsilon}{k_a V} \quad \ldots \ldots \, 5.4$$

and,

$$L_a = \mu k_a V \quad \ldots \ldots \, 5.5$$

where $V$ = volume of the cavity

and

$$k_a = \omega_a \sqrt{\mu / \varepsilon} \quad \ldots \ldots \, 5.6$$

The cavity has a number of normal modes which are the periodic solutions of Maxwell's equations. Each of the modes is characterised by a resonant frequency $\omega_a$. The expressions for $L_a$ and $C_a$ are consistent in so far as

$$\omega_a^2 L_a C_a = 1$$

Let $f_p : f_1 : f_s$ be equal to $3 : 2 : 1 \quad \ldots \ldots \, 5.7$

Substitution of equation 5.7 in equation 5.6 results in

$$C_o = \frac{4}{81} \sqrt{C_1 C_2} \quad \ldots \ldots \, 5.8$$

Substituting this value of $C_o$ from equation 5.8 in
equation 5.3,

\[ \frac{Q_3}{\Delta \theta} = 20.25 \frac{Q_p}{\sqrt{\frac{Q_1}{Q_2}}} \]  

\[ \text{or, } \frac{\Delta f'}{\Delta f} = 20.25 \frac{Q_p}{\sqrt{Q_1 Q_2}} \]  

consider a case, when

\[ Q_p = \sqrt{Q_1 Q_2} \]  

then,

\[ \frac{\Delta f'}{\Delta f} = 20.25 \]  

Equation 5.10 gives the value of \( \Delta f' / \Delta f \) for very large gains approaching oscillation, but some gain is to be expected when the ratio of \( \Delta f' / \Delta f \) is lower than 20.

5.3 DISCUSSION.

The reactance of a dielectric post changes with the change of the dielectric constant. It is shown in Chapter 2, that a certain percentage change of dielectric constant do not necessarily give the same percentage change.
of reactance. If the value of $\beta$ is not suitably chosen, large change of dielectric constant will not produce any appreciable change of reactance. The value of $\beta$ in those experiments was varied by varying the dielectric constant of the specimens.

Care was taken in finding accurately the percentage content of the composition. The amount of materials used are very small. Typical weight of materials are given below:

Sample E32.

\[
\begin{align*}
\text{BaTiO}_3 & = 74.34 \text{ mgm.} \\
\text{Polythene powder} & = 6.00 \text{ mgm.} \\
\text{T O T A L} & = 80.34 \text{ mgm.}
\end{align*}
\]

Percentage content of $\text{BaTiO}_3 = \frac{74.34}{80.34} \times 100 = 92.53\%$

Three balances were used altogether. The smallest scale division of the first one is 0.2 mgm. The smallest scale division of the second and third balance is 0.1 mgm and a fifth of this (0.02 mgm) can be estimated in the third balance which was used towards the latter part of the experiment.

The particles of $\text{BaTiO}_3$ are not uniform in size (Fig 3.6). They vary within very wide limits.
Polythene is a good binder for making these compositions in a simple set up. But, as a result, the composition cannot withstand much heating. Alumina is a better binder from this aspect. Alumina can be powdered and mixed with powdered BaTiO₃ before sintering.

When the percentage content of polythene in the composition is reduced, it is found increasingly difficult to make specimens.

No parametric amplification was obtained with these compositions. The following three factors are involved.

(1) Firstly, the frequency condition (equation 1.1) might not have been satisfied.

(2) The dipoles may not be switching at the microwave frequencies.

(3) The microwave field might not have been large enough to cause the switching of the dipoles.

The tuning of the cavity at the signal and idler frequencies was varied by the tuning screws 50 Mc/s on either side of the required value (equation 1.1). Accepting the wavemeters to be reasonably accurate, it can be accepted that the frequency condition was satisfied.

The samples got heated up when large amount of average
pump power was incident on them. The expected rise of temperature for sample E37 during one pulse is calculated below.

Energy supplied

\[ = 3.5 \text{ kW} \times 3 \text{ ms} \]
\[ = 10.5 \times 10^{-3} \text{ joules.} \]
\[ = 2.5 \times 10^{-3} \text{ calories.} \]

Volume of the sample

\[ = \frac{\pi}{4} \times (0.2)^2 \times 0.7 \]
\[ = 2.2 \times 10^{-2} \text{ c.c.} \]

density \times volume \times specific heat \times rise of temperature

\[ = 6 \times 2.2 \times 10^{-2} \times 0.1 \times t \]
\[ = 2.5 \times 10^{-3} \text{ calories.} \]

or,

\[ t = \frac{0.25}{1.32} \approx 0.2^0 \text{ C.} \]

Because of the poor thermal conductivity of the ceramics the temperature of the sample can be expected to go up with the number of pulses per second. The equilibrium temperature of the sample is reached when the heat lost by conduction, radiation and convection is equal to the heat supplied.
In order to reduce the frequency drift in a practical ferroelectric parametric amplifier, the total heat supplied should be reduced. The amount of average pump power fed to the cavity can be reduced by reducing both the p.r.f. and the pulse length. The heating will be negligibly small if a pulse of 50 p.r.f. is used. The duty ratio in this case is 20,000.

No definite conclusion could be reached about microwave switching from the high power measurements described here. One possible reason for the absence of parametric amplification could be that the microwave field at the sample was not strong enough. An approximate value of the field at the sample will now be calculated.

\[ Q = \frac{\omega W}{P} \]

or,

\[ P = \frac{\omega W}{Q} = \omega W \tan \delta \]

where

\( P = \) Power supplied

\( W = \) Energy stored in the sample

\[ = \frac{1}{2} \varepsilon E^2 V \]

where \( V = \) Volume of the sample

and \( E = \) Electric field at the sample.
For sample E37, with $P = 3.5\, \text{kW}$

$Q = 36, \quad k' = 35.8$ (Table 6.4).

\[
W_{PQ} = \frac{3.5 \times 10^3 \times 36}{2 \times 9420 \times 10^6} = 21.2 \times 10^{-7} \text{ joules}
\]

\[
W = \frac{1}{2} \times 8.854 \times 10^{-12} \times 35.8 \times E^2 \times \frac{T}{E} \times 4 \times 10^{-6} \times 7 \times 10^{-3} = 3.5 \times 10^{-18} E^2 \text{ joules.}
\]

Therefore,

\[
E^2 = \frac{21.2 \times 10^{-7}}{3.5 \times 10^{-18}} = 60.6 \times 10^{10}
\]

or, $E = 7.75 \times 10^5 \text{ volts/m.}$

\[
= 7.75 \text{ kV/cm.}
\]

When the loss of the cavity is taken into account, the actual field at the sample will be less than that calculated above.

This microwave field of 8 kV/cm is of the same order of magnitude as the d.c. bias field used for low signal measurement. With sample E37 and a d.c. bias field of 7.6 kV/cm, a value of 0.2 for $\Delta f'/\Delta f$ was obtained. This is much lower than that required for oscillation.
Fig. 5.8 Diagram of Goldstein's ferroelectric parametric amplifier.
The low field at the sample is contributing to the absence of parametric amplification. It is interesting, in this connection, to review the field recommended by other people.

Goldstein\(^2\) gave the following figures (Table 5.2) for the performance of a ferroelectric parametric amplifier.

**TABLE 5.2**

PERFORMANCE OF FERROELECTRIC PARAMETRIC AMPLIFIER

<table>
<thead>
<tr>
<th>(f_s) Mc/s</th>
<th>(f_p) Mc/s</th>
<th>RELATIVE AMPLITUDE OF IDLER TO SIGNAL FREQUENCY</th>
<th>PUMP POWER</th>
</tr>
</thead>
<tbody>
<tr>
<td>2600</td>
<td>5200</td>
<td>-31 dB</td>
<td>1 watt</td>
</tr>
<tr>
<td>1200</td>
<td>2400</td>
<td>-6 dB</td>
<td>150 mW</td>
</tr>
<tr>
<td>820</td>
<td>1640</td>
<td>-6 dB</td>
<td>150 mW</td>
</tr>
</tbody>
</table>

A small chip of Aerovox body 90 of 0.005 inch thickness and 0.10 inch on a side was used on a modified crystal holder.

The cavity (Fig 5.8) was a simple TEM co-axial type, operating in the \(\frac{3\lambda}{4}\) mode for the signal frequency, and the pump was operated in a number of different modes: for
Fig. 5.9 Characteristic of the material used in Goldstein's ferroelectric parametric amplifier.
example \( \text{TE}_{11} \) in one case. The loaded \( Q \) of the cavity at the signal frequency was noted as 240 and the insertion loss was 7.5 dB. The ferroelectric sample did not appear to load the cavity down in too great a manner.

The material used is polycrystalline BaTiO\(_3\). The non-linear characteristic of the material used is shown in Fig 5.9. The non-linear characteristic of this material is similar to that supplied by Technical Caramics Limited (Fig 3.5).

The field at the sample, in this device, is not known, but the C.W. pump power is surprisingly small.

Cassedy\(^{21}\) discusses the pump field in his proposed surface wave parametric amplifier. The structure is the "H-guide" with a slab of ferroelectric material placed between the two conducting planes. The line is operated in the dominant \( \text{TE}_{10} \) mode.

The following is a tabulation\(^{21}\) of the estimated performance of a parametric H-guide line, calculated from published data on this type of material:

Material:

Polycrystalline BaTiO\(_3\) - SrTiO\(_3\)

\[
\frac{\varepsilon'}{\varepsilon_0} = 2000 \quad \text{assuming d.c. bias field of } 12.8 \text{ kV/cm}\]
\[ \frac{\chi}{E} \text{ (pump)} = 100 \text{ (per kV/cm of pumping field)} \]

where \( \chi = \text{Electrical susceptibility of the material} \)

\[ E \text{ (pump)} = 1.18 \text{ kV/cm}, \quad \chi = 118 \]

Net gain \( \sim 19.3 \text{ dB} \).

These operating characteristics could be attained only by the use of a pulsed X-band magnetron, with a one-kW peak power, as a pumping oscillator.

A much more practical device would be possible if polycrystalline titanate could be suspended in a non-polar binder, such that the following would hold:

Material:

\[
\begin{align*}
\frac{\varepsilon'}{\varepsilon_0} &= 100 \quad \text{d.c. biasing again used}, \\
\tan \delta &= 0.01
\end{align*}
\]

\[ \frac{\chi}{E} = 5.0 \text{ (per kV/cm of pump field)}. \]

\[ E \text{ (pump)} = 1.14 \text{ kV/cm}, \quad \chi = 5.7 \]

Net gain = 19.3 dB.

This device could be operated with a 200 watt (CW) pump, with pump frequencies in excess of 10 kMc/s.
According to Cassedy\textsuperscript{21}, it is of considerable advantage to suspend polycrystalline titanates in a non-polar binder. His calculation shows that such compositions require much less pump power. The \( \tan \delta \) of the composition is smaller than the parent material. This is certainly the case if \( \tan \delta \) of the binder, such as polythene, is negligible compared to that of the titanates.

In the next chapter the results of measurements of some of the properties of the compositions used in this project will be given.
6.1 EVALUATION OF THE QUALITY OF FERROELECTRIC MATERIAL

Adopting the method, described below, Pearson and Trevena assessed the quality of a variable capacitance diode. This method has been used here for measuring the quality of the ferroelectric samples.

A parametric device has losses when unpumped. The usefulness of the device as an amplifier depends on the ratio of the negative conductance produced to the unpumped losses of the device.

The negative conductance produced in a parametric amplifier is given by equation 1.3

\[ G = \frac{\omega_1 \cdot \omega_2 \cdot \mathcal{C}^2_3}{4G_2} \]

\( G_1 \) is the unpumped conductance at the signal frequency. Then, one is interested in

\[ \frac{G}{G_1} = \frac{\omega_1 \cdot \omega_2 \cdot \mathcal{C}^2_3}{4G_1G_2} \]

\[ = \frac{1}{4} \left[ \frac{\text{change of susceptance at } \omega_1}{\text{conductance at } \omega_1} \right] \times \left[ \frac{\text{change of susceptance at } \omega_2}{\text{conductance at } \omega_2} \right] \]

\[ \text{--- 6.1} \]

\[ \text{--- 6.2} \]
At any frequency, one is primarily interested in the ratio

\[
\text{the change in susceptance available} \quad \frac{\text{change in susceptance}}{\text{conductance}}
\]

Let the following diagram (Fig 6.1) be the equivalent circuit of the ferroelectric material.

![Equivalent circuit of ferroelectric material](image)

Fig 6.1 Equivalent circuit of ferroelectric material

R is the resistance of the substance, \( C_0 \) is the zero-voltage capacitance and \( C_3 \) is the maximum amplitude of the change in capacitance.

When this is placed in a waveguide it looks, after a quarter-wave shift, like an admittance given by

\[
Y = Y_0^2 \left[ R + \frac{1}{j\omega C_0} \right]
\]

where \( Y_0 \) is the characteristic admittance of the waveguide.

Let \( C_v \) be the capacity of the specimen at a bias of \( V \) volts/cm.
Then, the change in susceptance conductance

\[
A = \frac{1}{\omega C_v} - \frac{1}{\omega C_o} \quad \text{(6.4)}
\]

\[
= \frac{C_0 - \frac{1}{\omega C_v}}{C_0} \quad \text{(6.5)}
\]

When \( Y \) is plotted on a Smith chart it will lie on a circle of constant conductance. It is possible to transform this circle to unit conductance circle by the addition of lossless elements alone. Let this be explained with the help of a Smith chart (Fig 6.2).

Consider, for example, that the ferroelectric material has a normalised conductance of 2 which remains constant with the application of bias. Let the zero-bias position of the specimen be at \( P \) corresponding to a normalised susceptance of 1. Let us travel along the guide till \( P' \) reaches the point \( P' (1,1) \) on the Smith chart corresponding to the plane \( M \) (Fig 6.3). Two other points \( Q \) and \( R \)
Fig. 6.3 EXPERIMENTAL ARRANGEMENT FOR MEASURING
CHANGE OF SUSCEPTANCE

are rotated by the same angle to $Q' \& R'$.

The new circle $P'Q'R'$ represents the admittance of
the sample alone at the plane $M$. Now let us introduce
a normalised susceptance of $-1$ at $M$ with the help of
a tuning screw. The points $P', Q'$ and $R'$ are shifted to
$P'', Q''$ and $R''$. This is a circle of radius unity and it touches both the centre and the circumference of the
Smith chart. The zero-bias point of the ferroelectric
sample is transferred to $P''$ having unity conductance and
zero susceptance. As a result of this looking at $N$
towards the sample (Fig 6.3) a perfect match is obtained.
The circle $P''Q''R''$ represents the transformed admittance
of the sample at the plane of the tuning screw.

By suitable choice of the reference plane, the
circle P Q R can be brought in coincidence with the constant conductance circle \( G = 1 \). It is to be noted that this reference plane is neither at the position of the tuning screw nor at the position of the sample.

The ratio of \( \frac{\text{change of susceptance}}{\text{conductance}} \) for two points P and Q at the plane of the sample and for corresponding points P' & Q' at the unity conductance circle is the same.

Thus by matching the sample to the waveguide by the use of lossless elements, the change in susceptance can be measured for any applied d.c. bias.

It can be shown that the change of susceptance on the unity conductance circle

\[
A = S^{1/2} - \frac{1}{S^{1/2}}
\]

--- 6.6

where S is the V.S.W.R.

By measuring the V.S.W.R, the change of susceptance is directly obtained from equation 6.6. The advantage of the method is that a knowledge of the absolute values of the susceptance and conductance of the sample is not necessary. It is assumed that the conductance does not change.
A plot may now be obtained of
\[
\frac{\text{change in susceptance}}{\text{conductance}} \quad \text{against bias.}
\]

A possible figure of merit of the sample is, therefore,
\[
\text{maximum available change in susceptance over conductance.}
\]

The position of the variable short circuit (Fig 6.3) behind the sample was adjusted to reduce the mismatch. Subsequently complete matching was obtained by the tuning screw (Fig 6.3). Different magnitudes of d.c. bias was, then applied to the sample. The V.S.W.R. and the position of the voltage minimum was noted at each bias.

The position of the voltage minimum shifted towards the generator with increasing bias voltage. The effect of the bias is to move the susceptance of the sample in an anti-clockwise direction (towards load) on the Smith chart. This means that the susceptance of a sample, which is capacitive, reduces in magnitude with increasing bias. This agrees with the cavity measurements which showed that the resonant frequency of the cavity increased with increasing bias applied to the sample.

The change of admittance of the sample is plotted on a Smith chart taking any arbitrary reference plane. One point is rotated till it comes on the unit conductance
Fig. 6.4 Change of susceptance of sample E1 with different d.c. bias (kV/cm) at 9415 Mc/s.
### TABLE 6.1

Results of Measurements of susceptance change of samples.

**Frequency - 9415 Mc/s.**

<table>
<thead>
<tr>
<th>SAMPLE &amp; ITS BaTiO$_3$ CONTENT %</th>
<th>BIAS FIELD (kV/cm)</th>
<th>CHANGE OF SUSCEPTANCE CONDUCTANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>E1</strong> 90.3</td>
<td>4.48</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>6.4</td>
<td>1.21</td>
</tr>
<tr>
<td></td>
<td>8.0</td>
<td>1.52</td>
</tr>
<tr>
<td></td>
<td>9.6</td>
<td>1.79</td>
</tr>
<tr>
<td><strong>E33</strong> 92.9</td>
<td>3.7</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>5.3</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>6.7</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td>8.0</td>
<td>1.22</td>
</tr>
<tr>
<td><strong>E2</strong> 92.8</td>
<td>4.48</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>6.4</td>
<td>1.22</td>
</tr>
<tr>
<td></td>
<td>8.0</td>
<td>1.52</td>
</tr>
<tr>
<td></td>
<td>9.6</td>
<td>1.76</td>
</tr>
<tr>
<td><strong>E21</strong> 85.10</td>
<td>3.4</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>4.86</td>
<td>0.25</td>
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<td></td>
<td>6.08</td>
<td>0.30</td>
</tr>
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<td></td>
<td>7.3</td>
<td>0.37</td>
</tr>
<tr>
<td><strong>E7</strong> 90.1</td>
<td>4.48</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td>6.4</td>
<td>1.52</td>
</tr>
<tr>
<td></td>
<td>8.0</td>
<td>1.79</td>
</tr>
<tr>
<td></td>
<td>9.6</td>
<td>2.05</td>
</tr>
</tbody>
</table>
Fig. 6-5 Plot of diode (mesa) admittance versus bias volts. [After Pearson 44] at 9375 Mc/s.
circle. All other points are shifted by the same angle. They are found to lie reasonably on the unit conductance circle. A typical example for $E_1$ is shown in Fig 6.4. Results of measurements on some samples are given in Table 6.1.

It is important to ascertain that, with zero-bias, all the incident microwave power is absorbed by the sample. Adsorption of power by any circuit element will reduce the quality of the samples.

It is useful to compare the quality of the ferroelectric compositions with that of the variable capacitance diode. Results of measurements on a typical mesa diode are given in Fig 6.5 and Table 6.2.

**TABLE 6.2**

QUALITY OF MESA DIODE AT 9375 Mc/s

<table>
<thead>
<tr>
<th>Bias Voltage (Reverse) volts</th>
<th>Change of susceptance conductance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.83</td>
</tr>
<tr>
<td>0.50</td>
<td>1.54</td>
</tr>
<tr>
<td>1.0</td>
<td>2.9</td>
</tr>
<tr>
<td>2.0</td>
<td>4.3</td>
</tr>
<tr>
<td>4.0</td>
<td>7.2</td>
</tr>
<tr>
<td>6.0</td>
<td>10.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bias Voltage (Forward) volts</th>
<th>Change of susceptance conductance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.49</td>
</tr>
<tr>
<td>0.2</td>
<td>0.91</td>
</tr>
<tr>
<td>0.4</td>
<td>2.2</td>
</tr>
<tr>
<td>0.5</td>
<td>3.05</td>
</tr>
<tr>
<td>0.6</td>
<td>4.1</td>
</tr>
<tr>
<td>0.7</td>
<td>5.3</td>
</tr>
</tbody>
</table>
The mesa diode gives considerably greater susceptance change compared to the ferroelectric compositions.

6.2 MEASUREMENT OF THE COMPLEX PERMITTIVITY OF THE COMPOSITIONS AT S-BAND

It is useful to know the complex permittivity of these compositions.

The permittivity of some of the compositions were measured at S-band using cylindrical cavity method, developed by Professor Willis Jackson and co-workers.

A cylindrical cavity of height 0.7 cm and diameter 6.02 cm was machined from brass and fitted with flat lids secured with six bolts. Two holes were made at diametrically opposite points on the curved surface of the cavity. The feeder and detector plugs couple into the cavity through these holes.

The resonant frequency and the quality factor of the cavity without and with the sample were noted. For \( E_{010} \) - mode, the real part of the relative permittivity is given by the following equation.

\[
k' = \frac{\beta_1}{\beta_0} \cdot \frac{J_0(\beta_1 b) J_1(\beta_0 b)}{J_1(\beta_1 b) J_0(\beta_0 b)} \left[ \frac{Y_0(\beta_0 a) - Y_1(\beta_0 b)}{J_0(\beta_0 a) - J_1(\beta_0 b)} \right] \left[ \frac{Y_0(\beta_0 a) - Y_0(\beta_0 b)}{J_0(\beta_0 a) - J_0(\beta_0 b)} \right]
\]

--- 6.7
where, \( \beta_o = \frac{2\pi}{\lambda_o} \)

\( \lambda_o \) is the free-space resonant wave-length of the cavity loaded with the sample and

\[ \beta_1 = \beta_o \sqrt{k'} \]

--- 6.9

\( a = \) radius of the cavity

\( b = \) radius of the sample.

An approximate value of the loss factor for very thin specimens is given \(^3\text{6}\) by

\[ \tan \delta = \frac{0.269}{k'} \frac{a^2}{b^2} \left[ \frac{1}{Q} - \frac{1}{Q'} \right] \]

--- 6.10

where \( Q \) is the quality factor of the cavity with the lossy sample and \( Q' \) that of the same cavity filled with a loss-free dielectric of the same permittivity.

Equation 6.7 is an exact relation for a loss-free system. Since \( \beta_1 \) contains \( k' \), this relation is not in a convenient form for the calculation of relative permittivity. Substitution of the value of \( k' = \beta_1^2 / \beta_o^2 \) in equation 6.7 gives
Fig. 6.6 (a) Variation of $\beta_0 f(\beta_0)$ with $\beta_0$ and (b) variation of $\beta_1 f(\beta_1)$ with $\beta_1$ for $a = 3.01$ cm and $b = 0.102$ cm.
Equation 6.11 can be rewritten in a simplified form

\[
\beta_1 f(\beta_1) = \beta_0 f(\beta_0) \quad \text{--- 6.12}
\]

\( \beta_0 f(\beta_0) \) is plotted against \( \beta_0 \) in Fig 6.6 (a) for the required values of \( \beta_0 \). From the measured values of \( A_0 \), the corresponding value of \( \beta_0 f(\beta_0) \) is found from Fig 6.6(a).

The value of \( \beta_1 f(\beta_1) \) are plotted against \( \beta_1 \) in Fig 6.6 (b) for the required values of \( \beta_1 \). For the above known value of \( \beta_1 f(\beta_1) \), the corresponding value of \( \beta_1 \) and hence \( k' \) is obtained from Fig 6.6(b).

---

**Fig 6.7** SECTION OF THE CYLINDRICAL CAVITY

\( \ell = 7 \text{ mm} \)
\( a = 3.01 \text{ cm} \)
For the application of d.c. bias, the wire attached to the sample was taken through a very small insulated hole, (Fig 6.7) on the wall of the cavity.

The hole for feeding H.T. was blocked with a copper rod. The resonant frequency of the empty cavity and its quality factor were measured. With the available equipment it was found difficult to plot the resonance curve of the empty cavity with accuracy. The measured $Q$ (1750) of the empty cavity is much lower than the theoretical value. The interior of the cavity and the lids were silverplated, but the improvement, if any, was not measurable. The input and output coupling was very small.

A small reduction of the coupling did not increase the $Q$ of the cavity.

The method used by Professor Jackson and co-workers is used for calculating the loss factor of the sample. The theoretical loss factor of a cylindrical cavity is given by

$$Q' = \frac{2}{\Delta} \left[ \frac{a \ell}{a + \ell} \right]$$

Where $\ell$ is the height of the cavity and $\Delta$ is the depth of current penetration in the walls of the cavity at the resonant frequency. From the measured value of $Q'$
<table>
<thead>
<tr>
<th>COMPOSITION</th>
<th>RESONANT FREQUENCY Mc/s</th>
<th>Q</th>
<th>DECREASE IN $k$ (approximate)</th>
<th>BIAS FIELD kV/cm</th>
<th>Mc/s INCREASE OF RESONANT FREQUENCY WITH BIAS</th>
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</thead>
<tbody>
<tr>
<td>D8</td>
<td>3375</td>
<td>80</td>
<td>44.0</td>
<td>7.5</td>
<td>28</td>
</tr>
<tr>
<td>E22</td>
<td>3605</td>
<td>243</td>
<td>26.1</td>
<td>7.2</td>
<td>6</td>
</tr>
<tr>
<td>E26</td>
<td>3726</td>
<td>-</td>
<td>11.1</td>
<td>7.1</td>
<td>1</td>
</tr>
<tr>
<td>E34</td>
<td>3625</td>
<td>-</td>
<td>21.9</td>
<td>6.9</td>
<td>2</td>
</tr>
<tr>
<td>E31</td>
<td>3673</td>
<td>342</td>
<td>16.5</td>
<td>7.1</td>
<td>5</td>
</tr>
<tr>
<td>E16</td>
<td>3576</td>
<td>362</td>
<td>26.8</td>
<td>6.9</td>
<td>8.7</td>
</tr>
<tr>
<td>E37</td>
<td>3488</td>
<td>208</td>
<td>35.8</td>
<td>6.9</td>
<td>5.3</td>
</tr>
</tbody>
</table>
of the empty cavity, an effective value of $\Delta$ is computed. The value of the quality factor at any other frequency is calculated by using this value of $\Delta$ after taking into account the fact that the skin depth is a function of frequency.

To start with, the dielectric constant of Nylon "66" ($2b = 2.38$ mm) was measured in this cavity.

The sample was placed inside the cavity and the decrease in the resonant frequency of the cavity on loading was measured. A d.c. bias was then applied to the sample. The increased resonant frequency of the cavity was measured. From these measurements, the values of dielectric constant, without and with bias, are calculated. The results of the measurements are given in Table 6.3.

These values of permittivity are approximate for the following reasons. The wire connected to the sample for the application of d.c. bias was found to carry microwave energy. When the H.T. source was connected through $130\ \Omega$ isolating resistance to this wire, the resonant frequency & $Q$ of the cavity changed. Secondly, due to the process of preparation, a groove of small depth runs along half the length of the sample.
Fig. 6.8 (a) Resonance curve of the cylindrical cavity loaded with sample E16, $f_o = 3579$ Mc/s.

Fig. 6.8(b) Variation of the dielectric constant with percentage content of BaTiO$_3$. 
TABLE 6.4

Empty Cavity Resonant frequency = 3816 Mc/s.

Measured Q = 1750
Theoretical Q = 11100

(Δ for silver = 10.2 x 10^-7 meters at 3816 Mc/s)
Diameter of the samples = 2.04 mm

<table>
<thead>
<tr>
<th>COMPOSITION &amp; ITS % BaTiO₃ CONTENT</th>
<th>RESONANT FREQUENCY Mc/s</th>
<th>Q</th>
<th>k'</th>
<th>1/\tan δ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nylon &quot;66&quot;</td>
<td>3792</td>
<td>1100</td>
<td>3.01</td>
<td>52.5</td>
</tr>
<tr>
<td>E26 78.62%</td>
<td>3720</td>
<td>955</td>
<td>11.7</td>
<td>107</td>
</tr>
<tr>
<td>E31 82.16%</td>
<td>3700</td>
<td>770</td>
<td>13.9</td>
<td>82.5</td>
</tr>
<tr>
<td>E34 85.19%</td>
<td>3627</td>
<td>507</td>
<td>21.8</td>
<td>67</td>
</tr>
<tr>
<td>E16 88.77%</td>
<td>3579</td>
<td>410</td>
<td>26.8</td>
<td>62</td>
</tr>
<tr>
<td>E37 90.21%</td>
<td>3489</td>
<td>207</td>
<td>35.8</td>
<td>36</td>
</tr>
<tr>
<td>E35 93.31%</td>
<td>3365</td>
<td>147</td>
<td>47.7</td>
<td>33</td>
</tr>
<tr>
<td>E36</td>
<td>3168</td>
<td>99.4</td>
<td>67.0</td>
<td>30</td>
</tr>
</tbody>
</table>
To get an absolute value of the permittivity of the sample, the wire was taken out of it. It was, then, prepared again by heating in a glass tube and remeasured in the cavity. The hole in the cavity for feeding H.T. was blocked with a copper rod and the resonance curve of the loaded cavity was plotted. A typical resonance curve with sample E16 inside the cavity is shown in Fig 6.8 (a). The relative dielectric constant and the loss factor of the samples are calculated as indicated earlier.

The results of measurements with a few samples are given in Table 6.4 and plotted in 6.8 (b).

Analysis assumed that the cavity was working in $E_{010}$ - mode. This is true when the dielectric constant is low and the perturbation is small. But it is necessary to show that the cavity was working in $E_{010}$ - mode when it was loaded with a sample having a large dielectric constant.

The hole for feeding H.T. was blocked with a copper rod. Two small holes (≈ 1/64") were drilled diametrically opposite to each other on the wall of the cavity, midway between the ends. Ends of cotton thread, which carried a glass bead of ≈ 2.5 mm diameter, passed through these holes. Perturbation measurements were taken in the way
Fig. 6.9 Perturbation measurement of cylindrical cavity loaded with sample B35.
explained in Chapter IV. The result of a typical measurement with sample E35 in cavity is shown in Fig 6.9.

The maximum shift of the resonant frequency of the cavity with this bead was small (~2Mc/s). As such, the variation of the output of the cavity was measured, keeping the oscillator frequency constant. With samples of lower dielectric constant and loss, the variation in output was considerably larger. But with sample E36, which has the highest dielectric constant, the variation in output was extremely small due to low Q and, as a result, a correct picture of the operating mode could not be obtained with this sample. It can be stated that for a dielectric constant of 48, the mode in the cavity was the perturbed E_{010} mode.

The measured dielectric constant (3.01) of Nylon "66" compares favourably with the published figure (3.03).

Samples E22 and D8 had earlier been subjected to high microwave field from the magnetron. During the calculation of the dielectric constant of samples E22 and D8, it was assumed that they have a length of 0.7 cm, while their actual lengths (appendix) are less.

Samples E34 to E37 were made up for making measurements
It is to be noted that the dielectric constant of the composition is reduced considerably by the addition of 5% polythene to barium titanate. A fairly smooth variation in dielectric constant can be obtained by varying the percentage content of the compositions.

6.3 DETERMINATION OF HYSTERESIS OF THE COMPOSITIONS.

To determine the effect of the previous history on the non-linear property of the samples, the following measurements were taken. The sample, to start with, was subjected to the highest voltage up to which the test was to be carried out. It was, then, depoled by subjecting it to alternately positive and negative voltages of lower magnitudes till the voltage was reduced to zero. Prior to switching off the voltage was - 150 V, which was the minimum voltage that could be obtained from the power supply available.

The second waveguide cavity (Chapter III), loaded with a depoled sample, was tuned to resonance. A small positive voltage was, then, applied to the sample and the shift of the resonant frequency of the cavity was measured. The bias voltage was gradually increased in magnitude till the
Fig. 6.10 Shift of resonant frequency of the cavity (length 12.21 cm) loaded with sample B8 versus bias applied to the sample.

With the particular range of the biasing power supply used, the minimum voltage gave a biasing field of ±3.6 kV/cm. As such, the portion of the curve between 0 and ±3.6 kV/cm is shown dotted.
maximum bias was reached. It was then gradually reduced till it became zero. It was then reversed in polarity and gradually increased to the maximum negative value and then reduced to zero in slow steps. The resonant frequency of the cavity was determined at each voltage.

Results of a typical measurement with sample E8 in the cavity is shown in Fig 6.10. In this figure, the shift in the resonant frequency of the cavity with bias applied to the sample is shown against d.c. bias field, the resonant frequency of the cavity loaded with depoled sample at zero bias is taken as reference.

The results show that the resonant frequencies of the cavity for a bias of same magnitude but of increasing and decreasing sense are different i.e. the shift of the resonant frequency of the cavity is dependent on the previous history of the sample. On reversing the bias, the descending position of the curves are obtained. Measurements were taken with a large number of samples. They all show the same general characteristic.

The effect of the application of varying d.c. field is to change the polarisation of the sample around the hysteresis loop. When a small a.c. signal is applied to
Fig. 6.11 Incremental Dielectric Constant

\[ \text{Incremental dielectric constant} = \frac{\Delta P}{\Delta E} \]
Fig. 6. Variation of dielectric constant with biasing field (a) theoretical, (c) experimental (after Matthias and von Hippel) and (b) the expected variation of the resonant frequency of a cavity loaded with a material of above characteristic (a).
the sample, biased to a point M (Fig 6.11), a small subsidiary loop is traversed. The incremental dielectric constant of the sample is given by the slope of the subsidiary loop and is equal to \( \Delta P / \Delta E \) where \( \Delta P \) is the small change in polarisation due to a small change in the electric field, \( \Delta E \). The microwave signal is expected to measure the incremental dielectric constant of the sample. For very small a.c. signal and for high permittivity materials the incremental dielectric constant is equal to the slope of the hysteresis loop at the operating point.

The slope at each point, plotted in Fig 6.12 (a), of a hysteresis loop, gives the incremental dielectric constant at the corresponding biasing field. The resonant frequency of the cavity, loaded with a sample having above characteristics, will vary as shown in Fig 6.12 (b). Upon reversing the direction of the biasing field, a decreased value of the incremental dielectric constant is obtained and consequently, the resonant frequency of the cavity will follow the ascending portion of the curve (Fig 6.12(b)). This is opposite to the experimental observation. (Fig 5.10).

Matthias and von Hippel \(^{45}\) measured the dielectric constant of single crystal barium titanate with a small
low-frequency a.c. signal applied perpendicular to the polar axis (Fig 6.12 (c)). Then a biasing field was applied in the same direction and gradually increased in magnitude, thus rotating the polar axis by 90° in the parallel position. From here on, a decrease and reversal of the polarity of the bias voltage changed only the sign of the polar axis but not its angular position. Hence the a.c. field measured the slope of the hysteresis loop for the parallel orientation. The dielectric constant parallel to the polar axis amounts nearly to 550 and perpendicular to about 1600 - 1800, at room temperature. Neglecting the initial portion of the curve, which is due to rotating polar axis, Fig 6.12 (c) resembles Fig 6.12(a).

The hysteresis loop is normally traced out in anticlockwise direction. It is useful to know the direction of the hysteresis loop of the ferroelectric samples. To find out more information by visual observation, the following experiment was performed. This also eliminates any error due to the heating effect which is partially responsible for the irregular shape of Fig 6.10.

The klystron feeding the loaded cavity was operated C.W. and a 50 c/s a.c. bias was applied to the sample. The detected output of the cavity and a portion of the bias
Fig. 6.13 Butterfly loops obtained with klystron set at (a) 9401 (b) 9405.8 and 9409.4 Mc/s with sample E7 in a 3-piece waveguide cavity of total length 17.12 cm, the centre of the sample being 8.38 cm from one end. $Q = 653$; $\Delta f'/\Delta f = 70\%$ for a d.c. bias field of 9.6 kV/cm. A.C. (50c/s) bias 900 V (R.M.S.).
voltage were applied to the Y - amplifier and X-plates respectively, of an oscilloscope. This resulted in butterfly loops, a selection of which are shown in Fig 6.13. The three curves of Fig 6.13(a), (b) and (c) which represent the output of the cavity versus bias field, correspond to the klystron frequency $f_1$, $f_2$ and $f_3$ (Fig 6.12 (b) ) respectively.

The hysteresis loop of the ferro-electric sample is traced out at the low frequency (50 c/s). The microwave is effectively measuring the incremental dielectric constant at each point of this low frequency hysteresis loop. The resonant frequency of the cavity is changing due to the a.c. bias applied to the sample. Whenever the resonant frequency of the loaded cavity is equal to the klystron frequency maximum output is obtained from the cavity. At other points of the bias cycle the output is less. When the klystron is at $f_1$ (Fig 6.12 (b) ) two loops are obtained, because, twice in each cycle of the bias field, the resonant frequency of the loaded cavity and klystron frequency are the same. When the klystron frequency is at $f_2$ (Fig 6.12 (b) ) four loops are obtained, as the klystron frequency is equal to the resonant frequency of the cavity four times in each
cycle of the biasing field. For similar reasons two loops are obtained when the klystron frequency is \( f_3 \) (Fig 6.12 (b)). The shape and size of the loops depend on (a) the effective dielectric constant of the material at each point of the low frequency hysteresis loop as measured by the microwaves, (b) the magnitude of the applied a.c. bias, (c) the loaded Q of the cavity and (d) the setting of the klystron frequency.

To determine the direction of the loops, a 2 micro-second pulse of 50 p.r.f. was superimposed on the butterfly curves (Fig 6.13). The pulse was manually delayed from 0 to 4 milliseconds. The direction of the travel of the pulse was noted. This gave the direction of the biasing voltage and the butterfly loops. It was observed that this direction, marked by arrow, in each of the curves of Fig 6.13 correspond to Fig 6.10 and not to Fig 6.12 (b).

The apparent direction of the loop depends on (a) the direction of deflection of the electron beam of the oscilloscope with increasing output of the cavity and (b) the delay characteristic of the pulse. By feeding a frequency modulated signal to the cavity it was observed that an output of the cavity gave an upward deflection on
Fig. 6.14 Butterfly loops obtained with standing wave detector at the position of (a) the voltage minimum, (b) between minimum and maximum and (c) maximum. The sample being matched at zero bias position. Peak value of a.c. (50 c/s ) bias 2500 V.

Frequency 9415 Mc/.
the oscilloscope. Secondly, by superimposing the 2 microsecond pulse on a 50 c/s sinusoidal voltage displayed on the oscilloscope, it was ascertained that the pulse was actually delayed when it was delayed by turning the knob on the pulse generator.

Measurements taken in the cylindrical cavity at S-band with 50 c/s bias applied to the sample also gave the same results.

To get further information, the arrangement (Fig 6.3) for measuring the susceptance change was used; the sample being biased by 50 c/s a.c. voltage. The output of the standing-wave detector and a fraction of the bias voltage were fed to the Y-amplifier and X-plate of a d.c. oscilloscope. Again butterfly loops were obtained some of which are shown in Fig 6.14 which are plots of the microwave voltage reflected from the sample at each position of the low frequency hysteresis loop. The matching elements were adjusted to obtain a perfect match at the zero-bias point. The standing-wave detector was then moved and different pictures obtained. The output of the crystal detector in this case gave a downward deflection.

Figs 6.14 (a), (b) and (c) are obtained when the
Fig. 6.15 Butterfly loops obtained with standing wave detector at the position of the voltage (a) minimum and (b) maximum. The sample was perfectly matched at the maximum swing (right hand side) of the bias voltage. Peak value of a.c. (50 c/s) bias 2500 V.

Frequency 9415 Mc/s.
standing-wave detector is at the position of the voltage minimum, slightly towards and at the position of the voltage maximum respectively. The direction of the loops (shown by arrow) was determined by the 2 microsecond pulse, as explained before.

When the standing-wave detector is set at the position of voltage minimum, points of increasing bias voltage gave lower output compared to that at the decreasing voltage. Correspondingly when the standing wave detector is set at the position of voltage maximum, point of increasing bias voltage gave larger output compared to that at the decreasing voltage. It means that a larger change of V.S.W.R. and correspondingly of susceptance is obtained at increasing voltage.

It was also observed that the position of the voltage minimum travelled towards the generator with increasing bias voltage. This means that for a capacitive sample the susceptance is comparatively lower for increasing voltage.

Matching elements were adjusted to obtain a perfect match at the maximum (R.H.S.) value of the biasing voltage. Figs 6.15 (a) and (b) show photographs of the butterfly
loops with standing-wave detector placed at the position of voltage minimum and maximum respectively. The zero level is indicated by the second beam on top (Fig 6.15). To get a better picture of the shape of the loops, the d.c. level was not included in the earlier photographs (Fig 6.14). The spike on the curves (Fig 6.15) is due to the marker signal. The direction of the loop is shown by the arrow.

Analysis of the photographs (Fig 6.15) show that with reference to the maximum biasing voltage position a smaller change of susceptance is obtained for increasing bias voltage compared to that for the decreasing bias. It means that the apparent slope of the hysteresis loop is greater for decreasing voltage compared to that of the increasing voltage.

All these measurements consistently show that the effective dielectric constant of the sample, for the same biasing field is lower for increasing field.

The unexpected behaviour of the samples at the microwave frequencies leads one to either of the two following conclusions.
Firstly, if it is accepted that the effective dielectric constant measured at microwave frequencies corresponds to the incremental dielectric constant measured at low frequencies, then it follows that the hysteresis loop is traced out in a clockwise direction. This is not possible, otherwise the sample will deliver energy instead of absorbing it.

Secondly, accepting the fact that the hysteresis loop is traced out in a counter-clockwise direction, it follows that the effective dielectric constant measured at microwave frequencies is not equivalent to the incremental dielectric constant measured at low frequencies.

This work shows the necessity of re-thinking and more investigation to determine the precise property of a dielectric material measured at microwave frequencies.
APPENDIX

SAMPLE HISTORY

Samples were not made all at the same time. They were made up at different times as and when necessary.

Samples E12 onwards were made from second batch of materials, received from Technical Ceramics Limited, which are similar to the first batch of barium titanate.

TECHNICAL CERAMICS COMPOSITIONS

<table>
<thead>
<tr>
<th>DATE OF PREPARATION</th>
<th>SAMPLE NUMBER</th>
<th>PERCENTAGE CONTENT OF BaTiO₃</th>
<th>SAMPLE LENGTH</th>
<th>SAMPLE DIAMETER</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.7.60.</td>
<td>E2, E2</td>
<td>92.8</td>
<td>10.1 (9.17)</td>
<td>1.96</td>
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<tr>
<td>18.7.60.</td>
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<td>91.8</td>
<td>10.1 (3.24)</td>
<td>1.96</td>
</tr>
<tr>
<td>11.7.60.</td>
<td>E1, E1</td>
<td>90.3</td>
<td>10.1 (5.16)</td>
<td>1.97</td>
</tr>
<tr>
<td>5.8.60.</td>
<td>E7</td>
<td>90.1</td>
<td>5.20</td>
<td>2.01</td>
</tr>
<tr>
<td>10.8.60.</td>
<td>E11</td>
<td>89.9</td>
<td>5.32</td>
<td>1.97</td>
</tr>
<tr>
<td>8.8.60.</td>
<td>E8</td>
<td>89.4</td>
<td>5.19</td>
<td>2.01</td>
</tr>
<tr>
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<td>E10</td>
<td>89.4</td>
<td>5.17</td>
<td>1.99</td>
</tr>
<tr>
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<td>86.5</td>
<td>5.90</td>
<td>1.95</td>
</tr>
<tr>
<td>2.8.60.</td>
<td>E4, E4</td>
<td>84.3</td>
<td>10.1 (5.30)</td>
<td>1.95</td>
</tr>
<tr>
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### TECHNICAL CERAMICS COMPOSITIONS

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Though some compositions with higher percentage content of barium titanate had been prepared, it was generally found extremely difficult to prepare samples with barium titanate more than about 93%.
REFERENCES

1. Lord Rayleigh, Phil Mag. S 5 (1883).
REFERENCES

(continued)


33. Table of Bessel functions \( J_0(Z) \) & \( J_1(Z) \) for complex arguments, MTP, NBS, Columbia University Press 1947.
34. Technique of Microwave Measurements, Edited by C.G. Montgomery. p. 521.
42. Principles of Microwave Circuits, Edited by C.G. Montgomery and others. p. 221.
44. J.D. Pearson and D.H. Trevena, Ferranti Limited, Manchester, Private communication.