MECHANICAL PROPERTIES OF HYBRID-MATRIX COMPOSITE LAMINATES

M A Leaity

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For my father, my mother and Mary.
SUMMARY

A laboratory scale drum winder has been designed and built for the production of pre-preg. Cross-ply hybrid matrix laminates were made from the pre-preg with glass fibres/epoxy resin in the longitudinal plies and glass fibres/epoxy resin-urethane elastomer in the transverse ply.

The addition of urethane to the matrix in the transverse plies alone increased the applied strains necessary for the initiation and development of transverse cracking during the extension of cross-ply laminates. This resulted in a smaller reduction in laminate stiffness (due to damage) at a prescribed level of strain.

Damage resistance was similar to that in cross-ply laminates with urethane additions to the matrix in both the transverse and longitudinal plies (a uniform matrix laminate). It was found that urethane additions lead to improved damage resistance in cross-ply laminates because they lower the transverse ply modulus and increase the matrix toughness in the transverse ply.

During the extension of cross-ply laminates, stable (constrained) transverse cracking was observed in thin transverse plies and unstable (brittle) transverse cracking in thick transverse plies. The effects of urethane additions on the development of constrained transverse cracking and brittle transverse cracking were modelled with a shear lag stress analysis combined with an energy balance and a statistical expression for the transverse ply strength respectively.
The ultimate properties of hybrid matrix laminates, having improved damage resistance, were expected to be better than uniform matrix laminates with a similar urethane content in the matrix. However, the tensile strength of circular centre-notched (0,90)\textsubscript{s} hybrid matrix laminates was lower than uniform matrix counterparts and the compressive strength of (0\textsubscript{2},90\textsubscript{2})\textsubscript{2s} hybrid matrix laminates was similar to uniform matrix counterparts.
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NOTATION

\((0_\alpha, 90_\nu)^{s \cdot n}\) A lay-up of \(\alpha\) 0° plies, followed by \(\nu\) 90° plies, \(s\) indicates symmetry about the mid-plane and \(e\) and \(n\) specify whether the laminate was prepared by the excess or net resin method

----------

\(a, a_0\) Variable and maximum flaw size in the transverse ply, Wang (1983)

\(a\) Radius of circular notch

\(a_d\) Half size of micro-crack across transverse ply thickness

\(a_d^{\text{crit}}\) Half size of critical micro-crack in thickness direction for first failure

\(a_w\) Half size of micro-crack across transverse ply width

\(\alpha\) Constant relating to the displacement profile across the transverse ply thickness

\(\alpha_l\) Coefficient of thermal expansion of the longitudinal plies along the fibres

\(\alpha_t\) Coefficient of thermal expansion of the transverse plies across the fibres

\(b\) Longitudinal ply thickness

\(\gamma\) Energy release associated with a transverse crack

\(d\) Half transverse ply thickness

\(E\) Reduced elastic modulus of a cross-ply laminate

\(E_0\) Elastic modulus of cross-ply laminate
\(E_f\)  Elastic modulus of fibres
\(E_{\ell}\)  Elastic modulus of a longitudinal laminate
\(E_m\)  Elastic modulus of matrix
\(E_t\)  Elastic modulus of a transverse laminate
\(E_{90^{\circ} \text{ply}}\)  In-situ modulus of the transverse ply
\(\Delta E\)  Change in stored strain energy in cross-ply laminate
\(\varepsilon\)  Strain in cross-ply laminate
\(\varepsilon_0\)  Characteristic failure strain of elements in the transverse ply
\(\varepsilon_{\text{atc}}\)  Applied strain for the initiation of transverse cracking
\(\varepsilon_{\text{deb}}\)  Strain at which fibres debond from the matrix in the transverse ply
\(\varepsilon_f\)  Failure strain of elements in the transverse ply
\(\varepsilon_t^R\)  Residual thermal strain in the longitudinal ply
\(\varepsilon_m\)  Strain in matrix in transverse ply
\(\varepsilon_t\)  Transverse ply strain
\(\varepsilon_{tc}\)  Transverse cracking strain
\(\varepsilon_t^R\)  Residual thermal strain in the transverse ply
\(\varepsilon_v\)  Characteristic failure strain of the transverse ply
\(G\)  Energy release rate per unit area of a transverse crack
\(G_{90^{\circ} \text{ply}}\)  Shear modulus of the 90° ply in the y-z plane
\(G_{\text{IC}}\)  Transverse ply toughness resisting the growth of a matrix crack along the fibres
\( G_{IC}^{\text{int}} \)  
Initiation value of transverse ply toughness from a double cantilever beam test

\( G_{IC}^{\text{plat}} \)  
Plateau value of transverse ply toughness from a double cantilever beam test

\( G_{II} \)  
Toughness of a lamina for Mode II crack growth

\( G_{IIIC} \)  
Toughness of a lamina for Mode III crack growth

\( j \)  
\( j^{th} \) element in the transverse ply

\( K \)  
Stress intensity factor at the tip of a transverse crack

\( K_c \)  
Fracture toughness of the transverse ply

\( L \)  
Gauge length of a tensile test coupon

\( \ell \)  
Length of longitudinal split from notch

\( \ell_0 \)  
Individual length of \( N \) elements in the transverse ply

\( \ell_n \)  
Individual length of \( n \) elements in the transverse ply

\( \lambda \)  
Function of laminate dimensions and moduli

\( m \)  
Weibull shape parameter for elemental failure strain distribution in the transverse ply

\( n \)  
Intermediate number of elements in the transverse ply

\( N \)  
Number of elements in the transverse ply at saturation of transverse cracking

\( \mu_a, \mu_S \)  
Mean of flaw size \( a \), and crack spacing \( S \), distributions, Wang (1983)

\( \nu \)  
Poisson's ratio

\( p(y) \)  
Probability associated with a location of a new transverse crack
<table>
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<td>$P_s(V_0)$</td>
<td>Probability of survival of successive elements in the transverse ply</td>
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<td>$P_s(V)$</td>
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<td>$s$</td>
<td>Half average transverse crack spacing</td>
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<td>$t$</td>
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<td>$\tau$</td>
<td>Shear stress at the $0^\circ/90^\circ$ ply interface</td>
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<td>Angle of fibres in off-axis plies</td>
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<td>$v$</td>
<td>Displacement in transverse ply in $y$-direction</td>
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<td>$V$</td>
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<td>$V_0$</td>
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<td>$V_{f^e}$</td>
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</table>
The net resin method

Transverse ply (and coupon) width

Work done by loading a cross-ply laminate

Distance along the longitudinal fibres in a cross-ply laminate

Distance through the thickness of a laminate
1. INTRODUCTION

1.1 Continuous fibre laminates

A composite is a multiple phase material consisting of a matrix and one or more forms of reinforcement. This work concerns continuous fibre polymer composites in the form of multiple ply laminates. The basic element of a laminate is a lamina or ply (~0.2 mm thick) which contains unidirectional fibres (of glass in this study, ~15 μm diameter) within a polymer matrix.

The design stress state for a laminate may be reduced usually to a set of in-plane stresses. These are principal stresses at 90° to each other, with maximum shear stresses at 45° to them. It is therefore common to construct quasi-isotropic laminates with fibre reinforcement at 0°, ±45° and 90° to the maximum principal stress. A simpler construction is a laminate with fibre reinforcement only at 0° and 90°. This is termed a cross-ply laminate and is the subject of investigation in this work.

1.2 Intrinsic properties of laminae and laminates

The elastic modulus, Poisson's ratio and coefficient of thermal expansion have different values in the direction parallel or perpendicular to the fibre axis in a unidirectional lamina. Typical values of these properties for glass/epoxy are, parallel to the fibres 50 GPa, 0.25 and
3.8 με/°C respectively and perpendicular to the fibres: 15 GPa, 0.075 and 16.7 με/°C respectively.

When 0° (longitudinal) and 90° (transverse) laminae are combined to form a cross-ply laminate to which load is applied, there must be strain compatibility, i.e. the strains in the laminae must be the same. To calculate the overall laminate stiffness properties and the response under loading, a summation must be carried out to account for the thickness and material properties of each lamina (laminated plate theory). Calculations for in-plane material properties of laminae and laminates are given in standard texts on mechanics of composite materials, see for example Jones (1975).

A cross-ply laminate is nearly always made to be symmetrical about the mid-plane. If this is not the case, the mismatch in the coefficient of thermal expansion results in warping of the laminate when cooling down from the cure temperature. A corollary to this is that a symmetric cross-ply laminate, although it does not distort, contains residual thermal stresses in each ply at room temperature.

1.3 Failure processes in cross-ply laminates

When a cross-ply laminate is extended (parallel to the 0° fibres), the strain in the longitudinal and the transverse plies is the same. The transverse plies fail when the strain reaches the fracture strain of the matrix between the fibres (~0.3% for glass/epoxy). On the other hand, the longitudinal plies fail when the strain reaches the failure strain of the
fibres (~2.5% for glass/epoxy). This means that the transverse plies will fail first in a cross-ply laminate under extension.

At a point of failure of the transverse plies, the load which they supported prior to failure must be borne by the unbroken longitudinal plies. However, the interface between the plies surrounding the failure also remains intact. Load is therefore transferred back into the transverse plies with distance from the point of failure by an interfacial shear stress. Almost all of the load in the transverse plies is recovered away from the point of failure. As the cross-ply laminate is further extended, the load supported by the transverse plies increases. Further failures occur in the transverse ply as a result of this. This is known as multiple matrix cracking in the transverse ply, or transverse cracking.

Upon further extension of the laminate, the fibres in the transverse plies oppose Poisson contraction of the longitudinal plies. This causes a tensile strain in the longitudinal plies in the transverse direction. The result is multiple splitting of the longitudinal plies, or longitudinal splitting, in the same manner as multiple cracking in the transverse ply. The cross-ply laminate eventually fails when the strain in the longitudinal plies reaches the failure strain of the 0° fibres.

The sequence of failure in a cross-ply laminate under uniaxial extension therefore begins with transverse cracking followed by longitudinal splitting and finally tensile failure. Design allowables for multiple ply laminates are
based on first cracking in the transverse ply. Therefore in this work, the failure process of interest is transverse cracking. The objective is to increase the critical strain for transverse crack formation and to reduce the density of transverse cracks at higher strain levels.

1.4 Limitation of damage in cross-ply laminates

Transverse cracks are commonly observed to form in cross-ply laminates at low applied strains (~0.3%). Two approaches have been developed to increase the strain for the onset of transverse cracking, so that design allowables for laminates may be increased. The first and most widely used approach is to reduce the transverse ply thickness, see Parvizi, Garrett and Bailey (1978). The second approach is to modify the matrix. Garrett and Bailey (1977a) showed that if a plasticizer was added to the resin in a glass/polyester cross-ply laminate, the transverse cracking strain was raised. By modifying the matrix in this way, the modulus is reduced, the strain to failure is increased, the properties of the fibre/matrix interface are changed and the transverse ply toughness is increased.

1.5 Hybrid matrix laminates for damage limitation

The use of thin transverse plies is effective in delaying transverse cracking and a modified matrix will enhance this effect. However, using a modified matrix to limit damage may
reduce other mechanical properties of the laminate, in particular, matrix dominated properties such as the longitudinal compressive strength. This work therefore evaluates the feasibility of a hybrid matrix laminate\(^1\), in which modified matrices are used only in those plies susceptible to matrix damage. Principal load bearing plies still contain a conventional strong and stiff matrix for optimum laminate properties. In a more general hybrid laminate, the fibre, matrix and fibre/matrix interface in each ply could be tailored to optimise its properties for the design stress state.

1.6 Scope of present work

1.6.1 Manufacture of hybrid matrix laminates

An epoxy resin has been used as the conventional matrix in this work. Modification of the epoxy was achieved by addition of a polyurethane elastomer (urethane). The modified resin is co-curable with the conventional epoxy resin, an important requirement for the preparation of hybrid matrix laminates. To make the laminates, a strand of glass fibres is impregnated with resin and wound onto a drum to form a thin sheet containing unidirectional fibres in the resin matrix (pre-preg). Cross-ply lay-ups were then prepared from the pre-preg, with 0° plies of glass/epoxy and 90° plies of

\(^{1}\) A hybrid matrix laminate differs from a conventional hybrid composite where hybridisation is based on mixing different fibres.


glass/epoxy-urethane. These lay-ups are cured by application of pressure and temperature to yield hybrid matrix laminates.

1.6.2 **Evaluation of hybrid matrix laminates**

A range of \((0_m,90_n)_s\) hybrid matrix laminates\(^2\), with a conventional matrix in the 0° plies and a modified matrix in the 90° plies, have been assessed for damage resistance in simple tensile tests, and ultimate strength in notched tensile and unnotched compression tests. An identical range of \((0_m,90_n)_s\) uniform matrix laminates with a modified matrix throughout the laminate have also been assessed for comparison with the hybrid matrix laminates.

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\(^2\) This notation represents \(m\) 0° plies followed by \(n\) 90° plies, and \(s\) indicates that the laminate is symmetric about the mid-plane.
2. LITERATURE REVIEW

MATRIX CRACKING IN CROSS-PLY LAMINATES

The aim of producing a hybrid matrix laminate is to limit the initiation and growth of low strain damage in the off-axis plies, without reducing the general laminate properties. In the case of a cross-ply laminate under tensile load, this damage takes the form of multiple matrix cracking in the transverse ply. The review of the literature presented here therefore covers experimental observation and theoretical modelling of multiple matrix cracking in the transverse plies of cross-ply laminates.

The dimensions and material co-ordinates for a cross-ply laminate are shown in Figure 2.1. Uniaxial tension is applied parallel to the 0° fibres, i.e. in the y-direction. The geometry of a matrix crack in the transverse ply is also indicated in Figure 2.1. The crack is arrested at the interface with the longitudinal ply. A fully developed transverse ply crack therefore extends across the transverse ply thickness 2d, and the transverse ply width W, in the x-z plane. In order to avoid confusion when discussing crack growth, the term thickness refers to the z-direction and the term width to the x-direction in Figure 2.1.

* Within the literature review various approaches to matrix cracking are outlined in a mainly qualitative way. More quantitative treatments of relevant models are given within the discussion of the results of this study in Chapters 5 and 6.
2.1 Experimental investigation of the mechanism of matrix cracking in cross-ply laminates

A number of studies have been conducted on the mechanism of matrix cracking in cross-ply laminates, e.g. Parvizi, Garrett and Bailey (1978); Bailey, Curtis and Parvizi (1979); Bader, Bailey, Curtis and Parvizi (1979); Bailey and Parvizi (1981) and Smith, Gilbert and Poursartip (1985). These studies have shown that a transverse crack usually initiates at the free edge in the transverse ply and grows simultaneously in the thickness and width directions. The growth rate of the crack in these two directions is a function of the laminate dimensions and properties, as discussed below. In most cases, the crack spans the transverse ply thickness at an early stage and then grows across the width.

2.1.1 The mechanism of crack growth across the transverse ply thickness

Bailey and Parvizi (1981) conducted microstructural studies by scanning electron microscopy (SEM) on a polished edge of glass/epoxy cross-ply laminates under load. This study only provided information concerning crack initiation at the free edge of the coupon and growth across the transverse ply thickness. Therefore, the extent of the developing crack in the width direction was not known.

The authors chose a laminate geometry in which matrix cracks grew in a stable manner across the transverse ply
thickness. They found that at applied strains of 0.1 to 0.3%, some of the fibre/resin bonds in the transverse ply failed. At about 0.4% applied strain, these individual fibre debonds coalesced to form micro-cracks of the order of two or three fibre diameters in length. Further coalescence of fibre debonds and micro-cracks occurred at higher applied strains, leading to cracks spanning the transverse ply thickness.

Debonding of the fibre from the matrix in the transverse ply was explained in earlier work by Garrett and Bailey (1977a) and subsequent work by Bader et al. (1979). They considered the theory of strain magnification developed by Kies (1962), in relation to the matrix surrounding the fibres in the transverse ply. It was suggested that the low modulus of the matrix relative to the fibre results in a higher strain in the matrix. The strain magnification factor (SMF) is taken to be the ratio of strain in the matrix between fibres ($\varepsilon_m$) to the overall strain in the transverse ply ($\varepsilon_t$). In areas of the transverse ply where fibres are virtually touching, the magnification of strain in the matrix can be written as

$$\text{SMF} = \frac{\varepsilon_m}{\varepsilon_t} = \frac{E_f}{E_m} \quad (2.1)$$

For glass/epoxy, the transverse elastic modulus of the fibres is about the same as the longitudinal elastic modulus, $E_f = 70$ GPa and the elastic modulus of the matrix is approximately $E_m = 3.5$ GPa. Therefore, the magnification of strain in the matrix in this case is about twenty times that of the overall transverse ply strain (SMF = 20). This
effectively pulls the matrix away from the fibres.

Investigations by optical microscopy on the mechanism of matrix cracking in cross-ply GFRP\(^3\) and CFRP\(^4\) laminates were also presented by Bailey \textit{et al.} (1979) and Bader \textit{et al.} (1979). Their findings corresponded with those of the SEM study of Bailey and Parvizi (1981), and a similar development of damage was found in the CFRP laminates. CFRP has a stronger bond between fibres and matrix relative to GFRP, which sometimes leads to cracks running through the carbon fibres in the transverse ply. Also, the strain magnification factor for CFRP is less than the value for GFRP because carbon fibres have a lower transverse modulus.

These investigations have shown that transverse cracks originate from fibre debonds, which coalesce to form micro-cracks in the transverse ply. Transverse cracks may propagate from these nuclei either in a stable manner with increasing applied load (stable crack propagation), or having reached a critical size, propagate instantaneously (unstable crack propagation). In general, the stability of crack propagation across the transverse ply thickness has been studied theoretically rather than experimentally.

Ogin and Smith (1985, 1987) studied the propagation of notional micro-cracks in the transverse ply of cross-ply laminates. They found that both the stability of crack growth across the transverse ply \textit{thickness} and the stability of crack growth across the transverse ply \textit{width} are governed by the

\(^3\) Glass fibre reinforced plastic.
\(^4\) Carbon fibre reinforced plastic.
thickness of the transverse ply $2d$ (assuming other material dimensions and properties constant). This is discussed below in relation to crack growth across the transverse ply width.

2.1.2 Crack growth across the transverse ply width

Parvizi et al. (1978) measured the strain for first cracking in the transverse ply of cross-ply glass/epoxy laminates. This was defined as the strain at which the first transverse crack spans the thickness and width of the transverse ply. Cross-ply laminates with a constant longitudinal ply thickness (0.5 mm) and a range of transverse ply thicknesses (0.1 to 4 mm) were studied. Their results are reproduced in Figure 2.2.

For thick transverse plies ($2d > 0.4$ mm), the strain for onset of cracking was independent of ply thickness and the cracks were observed to propagate instantaneously to span the ply. As the transverse ply thickness was reduced, the strain for the onset of cracking rose sharply. Cracks were observed to initiate at a free edge and propagate across the width on further extension.

Cracking was completely suppressed in very thin transverse plies ($2d = 0.1$ mm). As the thickness of the transverse ply is decreased, the residual thermal strain increases, see Chapter 4. This should be added to the first cracking strain when comparing plies of different thickness. However, in GFRP residual thermal strains are small (about 7% of the first cracking strain), and were neglected.
Parvizi et al. (1978) developed a theory to predict the strain at which the first transverse crack occurs, see Figure 2.2. This theory is based on the energy available for crack extension, and is discussed further in Section 2.4.1. They found that theoretical cracking strains agreed well with experimental values for thin transverse plies (2d < 0.4 mm), but underestimated experimental values for thicker transverse plies. Therefore, it was suggested that crack growth was governed by energetics in thin transverse plies, but by other factors in thick transverse plies.

Ogin and Smith (1985, 1987) later established approximate conditions for the stability of crack growth both across and along the fibres in the transverse ply. They considered a micro-crack in the transverse ply resultant from coalesced fibre debonds, see Figure 2.3. Using a fracture mechanics criterion, they calculated the critical micro-crack size for first failure $2a_d^{\text{crit}}$, in relation to the transverse ply thickness 2d. This is discussed with other theories for transverse ply cracking, in Section 2.4.1.

For thick transverse plies, the micro-crack reaches a critical size where $2a_d^{\text{crit}} < 2d$, and grows instantaneously across the ply thickness and width. For thin transverse plies, the transverse ply thickness 2d, is smaller than the critical micro-crack size for unstable crack growth $2a_d^{\text{crit}}$. Therefore, a crack which has grown across the transverse ply thickness is stable, and the applied strain must be increased in order for further crack growth to occur across the width. From this work, Ogin and Smith concluded that the transverse
ply thickness governs the stability of crack growth across and along the fibre direction in the transverse ply. The constraining effect of the longitudinal plies explains the delayed cracking in thin plies (constrained cracking) observed by Parvizi et al. (1978).

Bailey et al. (1979); Bader et al. (1979) and Flaggs and Kural (1982) recorded first cracking strains in the transverse ply of cross-ply CFRP laminates. They found that first cracking strains were higher in thin transverse plies, similar to GFRP. In CFRP, residual thermal strain in the transverse ply is more significant than in GFRP, due to the appreciably higher modulus of the longitudinal plies. The residual thermal strains were added to the first cracking strains for transverse plies of different thickness.

2.2 Multiple matrix cracking in the transverse ply

The mechanisms of initiation and growth of a single transverse crack have been discussed. However, owing to the geometry and material properties of a cross-ply laminate, multiple matrix cracking occurs in the transverse ply.

Garrett and Bailey (1977b) observed multiple matrix cracking in cross-ply glass/polyester laminates with various transverse ply thicknesses under tensile load. After the onset of transverse cracking, the crack density (inverse of the average distance between cracks) increased sharply with increasing applied stress. The occurrence of cracks diminished with increasing applied stress, eventually reaching
a limiting value. They found that the limiting value of crack density decreased for laminates of increasing transverse ply thickness.

Garrett and Bailey compared multiple cracking of a transverse ply in a cross-ply laminate with multiple fracture of a matrix reinforced with unidirectional fibres. This was studied by Aveston and Kelly (1973), who suggested that multiple fracture of a matrix could occur when reinforced with an array of unidirectional fibres of higher failure strain. Load could be transferred to the fibres at the plane of a matrix crack and be shed back into the matrix away from the crack.

A cross-ply laminate with a longitudinal ply of higher failure strain than the transverse ply corresponds, from a theoretical point of view, to the Aveston-Kelly system. At a point of transverse ply failure, load is borne only by the longitudinal plies. Moving away from the transverse crack, load is transferred back into the transverse ply by an interfacial shear stress. At some distance away from the crack plane, the load in the transverse ply is recovered and fracture may occur again at another site.

Subsequent experimental investigations of multiple matrix cracking have confirmed the findings of Garrett and Bailey (1977b) in glass/polyester laminates, for example, Parvizi and Bailey (1978) and Manders, Chou, Jones and Rock (1983) who tested epoxy/glass cross-ply laminates. Bailey et al. (1979), Bader et al. (1979) and Wang, Chou and Lei (1984) have conducted similar work on carbon/epoxy cross-ply laminates.
In addition, Masters and Reifsnider (1982) have presented data for the crack density in the 90° plies with increasing load on (0,±45,90)_s and (0,90,±45)_s carbon/epoxy laminates. They found higher saturation crack densities in the thinner 90° plies of the (0,90,±45)_s laminates. This study therefore extended the above findings for cross-ply laminates to angle-ply laminates.

2.3 Laminate modulus reduction due to transverse cracking

An important consequence of transverse cracking in cross-ply laminates is that the modulus of the laminate decreases as the number of cracks increases. Highsmith and Reifsnider (1982) presented data for the laminate modulus with increasing static load and crack density in (0,90)_s and (90,0)_s glass/epoxy laminates. They found that the laminate modulus decreased with increasing crack density in the 90° ply, in agreement with Hahn and Tsai (1974).

Highsmith and Reifsnider observed the reduction in modulus of the laminates to correlate directly with the crack density in the transverse ply. After large increases at low stress, the crack density and percentage reduction in modulus reached more or less constant values as the stress was further increased. Later work by Ogin, Smith and Beaumont (1985a) on (0,90)_s GFRP laminates confirmed that the reduction in modulus is directly proportional to the density of cracks in the 90° ply.
2.4 Theoretical models of matrix cracking in the transverse ply of cross-ply laminates

Numerous theories have been developed to model transverse ply cracking in cross-ply laminates. They concentrate on three processes which characterise transverse cracking. First, the applied stress required for initiation of transverse cracking. Second, during the progression of transverse cracking, the increase in applied stress to cause an increase in crack density in the transverse ply. Third, the reduction in modulus of the cross-ply laminate as a function of crack density in the transverse ply. In general, they follow one of two approaches.

The first approach is based on the energy available for cracking in the transverse ply. For example, models by Parvizi and Bailey (1978); Wang and Crossman (1980a); Poursartip (1983); Flaggs (1984); Dvorak, Hejazi and Laws (1985); Ogin and Smith (1985, 1987); Dvorak and Laws (1987); Laws and Dvorak (1988); Han, Hahn and Croman (1988); Lim and Hong (1989); Nairn (1989) and Chan and Wang (1990). Some of these models have been modified to account for material variability.

The second approach is based on the strength of the transverse ply. Examples of these are Garrett and Bailey (1977); Parvizi and Bailey (1978); Manders, Chou, Jones and Rock (1983); Fukunaga, Chou, Peters and Schulte (1984 a,b); Peters (1984,1986); Peters and Chou (1987) and Peters and Andersen (1989). In these models, the strength of the
transverse ply is based on either a single value or a statistically determined value in which the transverse ply is taken to have a variable strength.

Some workers, e.g. Highsmith and Reifsnider (1982), Steif (1984), Talreja (1984,1985) have focussed on the reduction in laminate modulus with increasing crack density in the transverse ply. In the main, these models of transverse cracking are based on a form of shear lag stress analysis. This accounts for the redistribution of stress around a transverse crack. An outline of these models is presented below.

2.4.1 Energy based models for transverse cracking

Laws and Dvorak (1988) have presented a model based on the available energy for matrix cracking in the transverse ply. In order to calculate the available energy for progressive transverse cracking, the loads and displacements in each ply around a transverse crack must be known. These may be determined by a one-dimensional shear lag stress analysis. The source of the term "shear lag" originates from the distance from a transverse crack or "lag" over which stress is transferred between plies by a shear stress.

The assumptions necessary for a shear lag analysis of transverse cracking are:

(i) The plies remain linear-elastic during cracking.
(ii) The interface between the plies is linear-elastic.
(iii) The displacement in a longitudinal ply is uniform.
throughout the ply thickness.

(iv) The displacement in a transverse ply is either uniform, or has a profile over the ply thickness.

(v) The interfacial shear stress is proportional to the relative displacement between plies.

In the model of Laws and Dvorak (1988), the displacement in a transverse ply is assumed uniform over the ply thickness. The interfacial shear stress is substituted into the stress equilibrium equation for the transverse ply to find the stresses and displacements around the transverse crack. By applying the boundary condition that the transverse ply stress must be zero at the crack plane, the expression for the transverse ply stress becomes

\[ \sigma_t = \left( \sigma_t^R + \frac{\sigma_s E_t}{E_0} \right) \left( 1 - \frac{\cosh \lambda y}{\cosh \lambda s} \right) \]  

(2.2)

where \( \sigma_t^R \) is the thermal residual stress in the transverse ply, \( \sigma_s \) is the stress applied to the cross-ply laminate, \( E_t \) is the elastic modulus of the transverse ply, \( E_0 \) is the elastic modulus of the cross-ply laminate and \( \lambda \) is a constant involving the laminate moduli and dimensions. Referring to Figure 2.4, \( s \) is half the crack spacing and \( y \) is the distance along the \( y \)-axis.

The transverse ply stress is therefore zero at a transverse crack and recovers most of its original value a distance of about 2d (for glass/epoxy) away from the crack, see Figure 2.4. The stress in the longitudinal ply can be
determined from equation (2.2) combined with a force balance on the laminate. Following this, the displacements around the transverse crack may be determined from the stress-strain relations for the plies.

The available energy for transverse cracking is determined by comparing the work done in loading the laminate to the stored strain energy. These terms are calculated from the stresses and displacements in the plies of the laminate. A matrix crack will grow in the transverse ply if

$$\Delta W > \Delta E + \gamma$$

(2.3)

where $\Delta W$ is the work done by loading, $\Delta E$ is the change in stored strain energy and $\gamma$ is the energy required for transverse cracking. The applied stress for initiation and progression of transverse cracking is determined from the available energy, $\gamma$. In this model, the variability of material properties in the transverse ply is accounted for by including a probability density function, proportional to the transverse ply stress between cracks.

Parvizi et al. (1978) earlier used a similar energy balance to determine the strain for first cracking in the transverse ply. Their experimental and theoretical values for first ply failure strain in glass/epoxy cross-PLY laminates are shown in Figure 2.2. Their theory shows good agreement with experiment for thin transverse plies ($2d < 0.4$ mm) but not for thicker plies. Therefore, it was suggested that transverse cracking is governed by other factors in thick
transverse plies. Reasonable agreement of the Parvizi, Garrett and Bailey model with experimental data for CFRP cross-ply laminates has also been presented by Bailey et al. (1979) and Bader et al. (1979).

Ogin and Smith (1985, 1987) assigned a stress intensity factor $K$, to the tip of a transverse ply crack growing across the laminate width. They took this to be independent of the crack length

$$K = \sigma_t (2d)^{\frac{1}{2}}$$

(2.4)

since this is not a through thickness crack. They set $K = K_c$, where $K_c$ is the fracture toughness of the transverse ply for transverse cracking. From equation (2.4), they determined the strain for first cracking in the transverse ply. This approach has been shown to give very similar predictions to those of Parvizi et al. (1978). Again, there is only good agreement for thin transverse plies.

Therefore, they considered the size of a micro-crack in the transverse ply in relation to the ply thickness, see Figure 2.3. For thick transverse plies, they took the stress intensity factor at the tip of a crack partially spanning the transverse ply thickness to be

$$K = \sigma_t (\pi a_H)^{\frac{1}{2}}$$

(2.5)

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5 The stress intensity factor for a through thickness crack is $K = \sigma(2a)^{\frac{1}{2}}$, where $2a$ is the crack length and $\sigma$ the applied stress.
where $a_d$ is the size of the micro-crack in the thickness direction. By setting $K = K_c$, they determined the critical flaw size $a_d^{\text{crit}}$, for fast propagation of the first transverse crack.

Using values for the epoxy/glass system of Parvizi et al. (1978), they found $a_d^{\text{crit}}$ to be around 0.4 mm which corresponded to the transition found in first cracking strain. This suggested that for cross-ply laminates with a thick transverse ply ($2d > 2a_d^{\text{crit}}$), transverse cracking depends only on the strength of the ply, i.e. the size of existing micro-cracks.

In a series of papers, Wang and Crossman (1980) and Crossman, Warren, Wang and Law (1980) used a finite element based stress analysis, combined with an energy balance, to model the initiation and growth of matrix cracks in the transverse plies of angle-ply laminates. An important difference in crack geometry in this work is that a micro-crack is assumed to extend across the width of the transverse ply. The crack is then considered to grow across the transverse ply thickness. This is in direct contrast to the experimental observations of Bader et al. (1979); Bailey et al. (1979) and Bailey and Parvizi (1981) who noted that micro-cracks in the transverse ply first span the transverse ply thickness and then propagate across the width of the ply.

Wang and co-workers found that the stress for first cracking in the transverse ply depended on $1/(d)^\frac{1}{2}$, which agreed well with experimental results. By performing finite element calculations for a second crack at increasing distances from the first crack, they found a distance which gave the maximum
energy available for cracking. They theorised that this distance defined the minimum crack spacing in the transverse ply. This distance of about 7d (seven times the half-ply thickness) agreed well with experimental results for (±25,90)_s carbon/epoxy laminates.

Minimum transverse crack spacings found by other authors for cross-ply laminates are of the order of the transverse ply thickness, 2d. The discrepancy is due to the reduced constraint exerted by the angle-plies either side of the transverse ply. Wang (1983) and Wang, Chou and Lei (1984) extended the earlier Wang (1980) model to incorporate the effect of variability of the material properties in the transverse ply.

They assigned normal distributions to an inherent flaw size a, and spacing S in the transverse ply. The standard deviations σ_a and σ_s were determined from test data on a large sample of specimens. The distribution means, μ_a and μ_s were determined from the relationship

\[ a_0, S_0 \sim \mu_{a,s} + 3\sigma_{a,s} \]  \hspace{1cm} (2.6)

where a_0 and S_0 are the largest inherent flaw size and spacing respectively. A uniform random number generator was used to define the crack sizes and spacings according to the distributions above and these were ranked in descending order. An energy balance was performed for each sequential crack. In this way, a simulation of progressive transverse cracking was achieved. This compared well with test data for (0,90)_s and
(0,90)\textsubscript{s} carbon/epoxy laminates.

Han et al. (1988) presented a theory for transverse cracking based on an energy balance and a second order polynomial for the crack opening displacement (displacement profile) in the transverse ply. This is an analysis similar to the shear lag models of Steif (1984) and Poursartip (1983) who assumed a parabolic displacement profile in the transverse ply. It gives similar expressions for the applied laminate stress for initiation and multiplication of transverse ply cracks to that of Laws and Dvorak (1988).

Lim and Hong (1989) presented a model for matrix cracking based on the shear lag analysis of Highsmith and Reifsnider (1982). This analysis assumes that all the shear strain in the 90\textdegree ply is concentrated in a thin resinous band at the 0\textdegree/90\textdegree ply interface. A choice of co-ordinate system with origin at the plane of the transverse crack, after Garrett and Bailey (1977b), results in a discontinuous stress distribution between transverse cracks. This is not the case physically, and a smooth stress distribution as predicted by Laws and Dvorak (1988) is preferable. However, they have also predicted the applied stress for the onset of transverse cracking using an energy balance.

Of more use in design, Flaggs (1985) formulated a theory for transverse cracking in laminates under general in plane loading. He combined an energy balance for mixed mode transverse cracking with a two dimensional shear lag analysis. This model has been shown to predict accurately the onset of Mode I matrix failure in the 90\textdegree plies of (±θ,90\textdegree)\textsubscript{s}, and mixed
mode matrix failure in the $\theta^\circ$ ply of $(0_2,\theta)_s$ laminates. It therefore has the capability to predict the onset of matrix failure in off-axis plies for general in plane loading of a laminate.

Nairn (1989) has produced an energy based theory for transverse cracking in conjunction with a variational stress analysis based on earlier work by Hashin (1983). This, alongside the shear lag analysis, is a one dimensional approach. He calculates the available energy for a crack growing along the $90^\circ$ ply fibres, and postulates, in keeping with Laws and Dvorak (1988), that the probability of cracking is proportional to the transverse ply stress there.

Nairn showed this model to predict accurately the applied stress for progressive cracking in $(0,90_3)_s$ glass/epoxy laminates tested by Highsmith and Reifsnider (1982). However, this seems to be in error since a $(0,90_3)_s$ laminate, as discussed earlier, has a "thick" transverse ply. Nairn's energy based model is thought to be more appropriate for "thin" transverse plies, where cracking is energy controlled.

Chan and Wang (1990) have examined the effect of a toughened $90^\circ$ ply\(^6\) on matrix cracking and edge delamination in multiple ply laminates. Cross-ply laminates with transverse plies of carbon/conventional epoxy and carbon/toughened epoxy were evaluated. The longitudinal plies of the laminates were made from carbon/conventional epoxy . Again, contrary to experimental evidence, they assumed a transverse crack first

\(^{6}\) This, in the terminology used here, is a hybrid matrix laminate.
to span the ply width and then grow across the ply thickness in this analysis.

In their experimental work, they could not clearly detect transverse cracks in the cross-ply laminates. They did however calculate the strain for the onset of transverse cracking using the finite element stress analysis and energy balance of Wang and Crossman (1980). In their analysis they assumed the carbon/epoxy systems to have a toughness for transverse cracking of about 227 J m\(^{-2}\).

Chan and Wang attempted to illustrate an increase in the strain for the onset of transverse cracking in the hybrid matrix laminates. The two carbon/epoxy systems should illustrate this since they have similar elastic properties but different toughnesses for transverse cracking. However in their calculations, they assumed the same toughness for transverse cracking in both of these systems. Accordingly, the calculated strain for the onset of transverse cracking was not higher in the carbon/toughened epoxy laminates.

In summary, the models in this section are based on a form of stress analysis and a calculation of the energy available for transverse cracking. It has been established that transverse cracking is energy controlled in thin transverse plies but is dependent on the transverse ply strength in thick plies. The stress for the initiation and multiplication of transverse cracking may be predicted with these energy based models in cross-ply laminates with thin transverse plies.
2.4.2 Strength based models for transverse cracking

For a cross-ply laminate, there is a transitional transverse ply thickness which is greater than the critical size of a micro-crack for instantaneous propagation of transverse cracks. This transitional ply thickness is governed by the material properties of the transverse ply. For cross-ply laminates with transverse plies thicker than this transitional dimension, the transverse ply strength governs failure. Models for transverse cracking based on the strength of the transverse ply are therefore applicable to cross-ply laminates with thick transverse plies.

Garrett and Bailey (1977b) developed a model based on a shear lag analysis and a unique value of strength for the transverse ply. They predicted the applied stress as a function of crack density for the progression of transverse cracking in glass/polyester cross-ply laminates. However, this did not agree with experimental values owing to an incorrect boundary condition in their stress analysis. This was amended by Parvizi and Bailey (1978) and showed reasonable agreement with experimental data for glass/epoxy cross-ply laminates. Bader et al. (1979) and Bailey et al. (1979) used this model for carbon/epoxy cross-ply laminates and found reasonable agreement with experimental data.

There are two features in the shear lag analysis of the Garrett, Bailey and Parvizi (GBP) model which have no physical basis: (i) They assume a linear displacement profile in the transverse ply (ii) Their stress analysis does not predict a
smooth variation of stress between cracks in the transverse ply. Despite these limitations, the model can predict reasonably well the applied stress as a function of transverse crack density for cross-ply laminates with thick transverse plies.

Aside from the stress analysis, the main drawback in the GBP model is the assumption of a unique value for transverse ply strength. There are two sources of variability in the transverse ply which may invalidate this assumption. Firstly, processing defects such as void or micro-cracks from thermal shrinkage are sites for premature failure of the transverse ply. Secondly, variations in fibre packing in the transverse ply may also result in a statistical distribution of strength.

Manders et al. (1983) confirmed this with measurements of transverse crack spacings in glass/epoxy cross-ply laminates. They found irregular crack spacings at low levels of applied strain and deduced that the transverse ply has a variable strength. By considering the statistical variability of transverse ply strength, they could accurately predict the crack density (or spacing) in the transverse ply as a function of the strain applied to the laminate.

Peters (1984, 1986) has developed a method of determining the statistical strength distribution of the transverse ply in cross-ply laminates. He estimates the strength distribution of the transverse ply from a single test piece. This is achieved with the use of a shear lag analysis and the Weibull
An elemental volume $V_0$, or length $\ell_0$, is considered in the transverse ply. Peters defines this length as the distance either side of a transverse crack over which 90% of the stress is recovered in the transverse ply, see Figure 2.4.

A cross-ply test piece is loaded in tension and the fracture strains of the elemental lengths are recorded. The probability of fracture (or survival) of each element is then calculated according to the Weibull theory. The variability of transverse fracture strain may then be characterised by a shape parameter $m$, and a scale parameter $\varepsilon_0$. The scale parameter for transverse fracture strain may simply be converted to that for transverse strength $\sigma_0$, by the relation $\sigma_0 = E_t \varepsilon_{f0}$.

There are two main assumptions on which this method is based. First, it is assumed that the transverse ply stress is uniform outside these elements. Second, the elemental length $\ell_0$, was chosen such that it should only fail once. Therefore, a limit was placed on the applied strain such that the probability of a broken element failing again is only 5%. At high applied strains, transverse cracks are close together resulting in stress non-uniformity in the transverse ply. Also, some elements are more likely to fail twice at these high strain levels. These limitations of the method result in distortion of the transverse strength distribution.

Nonetheless, Peters and Andersen (1989) have shown that a

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7 A statistical theory where the distribution of the probability of an event is characterised by a shape parameter and a scale parameter.
reasonable Weibull distribution may be attained for the transverse strength with this method.

According to the Weibull theory, see for example Fukunaga et al. (1984a,b), the probability of survival of the transverse ply, volume $V$ is

$$P_s(V) = \exp \left\{ \frac{1}{\bar{V}_0} \int_V \left( \frac{\sigma_t}{\sigma_0} \right)^m \, dV \right\} \quad (2.7)$$

where $\bar{V}_0$ is the reference volume from which the shape parameter $m$, and the scale parameter $\sigma_0$, are determined. The transverse ply stress $\sigma_t$, in (2.7) can be expressed in terms of the applied stress $\sigma_a$, see equation (2.2). The applied stress for progressive cracking may then be determined by evaluating the integral in equation (2.7) numerically. This model takes in to account the variability of the transverse ply strength and may be used for modelling progressive transverse cracking in cross-ply laminates with thick transverse plies.

2.4.3 Models for modulus reduction in cross-ply laminates

The reduction in modulus of a cross-ply laminate with cracks in the transverse ply may be calculated as a function of the crack density. This is achieved by summing the increased strain between transverse cracks. Highsmith and Reifsnider (1982) and Steif (1984) developed this prediction from a shear lag analysis. The former showed this to agree
well with experimental data for $(0,90_3)_s$ glass/epoxy laminates. Ogin, Smith and Beaumont (1985a) confirmed these findings by successfully predicting the modulus reduction in $(0,90)_s$ glass/epoxy laminates with this theory.

Talreja (1984,1985) adopts a different approach to modulus reduction in multiple ply laminates. Instead of calculating stress redistribution as a result of transverse cracks, he considers cracks as microstructural features (the same as voids). Talreja clearly demonstrates the effect of constraint in the transverse ply on modulus reduction in cross-ply laminates. This model is shown to agree well with modulus reductions in $(0,90_3)_s$ and $(0,90)_s$ glass/epoxy laminates under extension.
Figure 2.1 A schematic of the dimensions, micro-structure and crack geometry in a cross-ply laminate
Figure 2.2 Experimental and theoretical values of transverse cracking strain for various ply thicknesses in cross-ply glass/epoxy laminates, reproduced from Parvizi et al. (1978)

Figure 2.3 Dimensions of a micro-crack in the transverse ply, reproduced from Ogin and Smith (1987)
Figure 2.4 A schematic showing the distribution of transverse ply stress around a transverse ply crack.
3 EXPERIMENTAL

PREPARATION OF HYBRID MATRIX LAMINATES

A hybrid matrix laminate contains different resins in adjacent plies. This makes wet lay-up techniques impractical. In this work, hybrid matrix laminates have been made from unidirectional pre-preg. This is a sheet of unidirectional fibres in an uncured but coherent resin matrix. The viscosity of the resin has been modified such that the pre-preg has a small amount of tack at room temperature. Sheets of pre-preg could therefore be cut to size and bonded in sequence to make a laminate. This is a convenient method of preparing hybrid matrix laminates since it allows a great deal of flexibility with regard to fibre orientation, ply sequence and the matrix in each ply.

A laboratory scale drum winder, see Figure 3.1, has been designed and built for preparation of the pre-preg. The principle of operation is as follows: A strand of fibres is impregnated with resin, in-situ, and wound onto a drum to form a thin sheet of unidirectional fibres in a resin matrix. The drum winder is a useful research facility since a range of fibres and resins may be used, to yield a variety of pre-preg.

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Techniques where the fibre reinforcement is first placed in the required orientations and then impregnated with resin.
3.1 Matrix and reinforcement

A proprietary resin (SHELL Epikote 8200/9912 DX-6102) has been used for the matrix. This is the conventional resin in this work and is termed epoxy resin. It is a diglycidyl ether of bisphenol-A epoxy resin\(^9\) containing a proportion of novolac epoxy resin\(^{10}\). Novolac epoxy resin is multifunctional which, on curing, results in a tightly cross-linked structure. The curing agent (SHELL Epikure DX-6506) is of the catalytic type (a boron trifluoride complex). The glass transition temperature for the epoxy resin is in the range 160-165°C.

The epoxy resin has been modified by the addition of a polyurethane elastomer (CIL Monothane A80) which is termed urethane in this work. Proportions of 0, 5, 10, and 20% of urethane have been added, by weight of the matrix. The addition of the urethane has the effect of incorporating flexible segments in the polymer chains of the epoxy resin. The urethane is miscible with the epoxy resin in all proportions and has a compatible cure temperature.

The matrix has been reinforced with a glass fibre tow (SILENKA 084-K19-600T). The choice of these fibre and matrix materials resulted in a translucent composite laminate upon fabrication. This means that matrix damage in the laminate may be easily detected by optical techniques.

\(^9\) Formulated from epichlorohydrin and bisphenol A.

\(^{10}\) Formulated from phenol and formaldehyde.
3.2 Manufacture of prepreg

3.2.1 Preparation of the resin

The epoxy resin, curing agent and urethane must be combined to form an homogenous solution. This solution must have a viscosity suitable for wetting of the fibre-tow in the winding process. However, the uncured epoxy resin is solid at room temperature. This is in order that pre-preg made with the resin is coherent, i.e. it may be cut and laminated at room temperature.

The epoxy resin and urethane (which is also very viscous at room temperature) was melted for mixing. In addition, the resin had to be kept liquid throughout the winding process. One way of achieving this was to keep the resin above its melting temperature (a hot-melt method). In this work however, a solvent (dichloromethane) was added to the resin for this purpose.

The solution of resin for winding was therefore prepared as follows: A proportion of epoxy resin and urethane were heated to about 70°C whereupon their viscosity was sufficiently low for mixing. The curing agent (liquid at room temperature) was added to the epoxy resin in the proportion 8g agent to 100g resin. Then 0, 5, 10 or 20% urethane, by weight, was added to the resin.

At this stage (70°C), the dichloromethane was added to the mixture. A proportion of the dichloromethane boiled off since it has a boiling point of 38°C. However, the
temperature of the solution dropped with gradual addition of the solvent whereupon the resin was dissolved. After trials on the winder, an optimum proportion of 60% solvent, by weight, was chosen, see Section 3.2.3.

3.2.2 The process for winding pre-preg

In brief, the mechanics of the drum winder (Figure 3.1) may be described as follows: The drum revolves at 18.75 rpm, driven by a constant speed induction motor via a toothed belt and sprocket. A mobile crosshead which positions the fibre-tow on the drum is driven along a lead screw by a stepping motor. The choice of such a motor is to ensure accuracy of placement of the fibre-tow. A microprocessor has been assembled for the control of the crosshead and the drum. This incorporates variable speed control and dual directionality of the crosshead with limit switches at the ends of travel.

The amount of overlap of the fibre-tow on the drum may be adjusted by changing the crosshead speed. Factors such as out-of-roundness of the drum (a seam-welded stainless steel tube), and the side to side movement of the roving as it unwinds from the reel cause variability in the winding. It is necessary therefore to set the speed of the crosshead for an overlap of 38% of the fibre-tow during winding.

The winding procedure is as follows: A silicone coated backing paper was wrapped round the drum. The fibre-tow was drawn off the outside of a horizontal reel (this avoids twist that would result if the tow were drawn from the inside of the
reel) and passed through a series of alignment and tensioning rollers. It was then passed over a resin bath on the mobile crosshead and affixed to the drum. The dissolved resin was then poured into the resin bath. The drum and crosshead were started simultaneously, causing the fibre-tow to pass through the alignment and tensioning rollers, and then over a revolving wheel in the resin bath. At this point, the fibre-tow was impregnated with resin and placed accurately on the revolving drum. Once the winding was complete, it was cut transverse to the fibre direction and removed from the drum as a sheet of unidirectional pre-preg.

3.2.3 Resin viscosity and pre-preg properties

Wetting of the fibre-tow was achieved by passing it over a wheel which revolves in a bath of resin, see Figure 3.2. Resin was transferred to the fibre-tow as the wheel picked it up from the bath. A profiled lid on the bath limited the amount of resin transferred to the fibre-tow and prevented spillage. Wettability of the fibre-tow depends on a number of variables: speed of winding (line speed), tension of the fibre-tow and the viscosity of the resin. The winding speed has been chosen to be low (0.1 m/s) and the winding tension moderate to aid wetting out of the roving.

The viscosity of the resin must satisfy two requirements: (i) optimum viscosity for the proportion of resin transferred to the fibre-tow (this governs the fibre volume fraction of the laminate) and (ii) minimal viscosity to limit aeration.
resultant from the wheel revolving in the bath and to fully wet out the roving. Proportions of 20 to 80% of dichloromethane, by mass of the resin, have been assessed in winding trials to determine the optimum resin viscosity for the above requirements. As indicated earlier, a proportion of 60% dichloromethane was found to be the most suitable.

When removed from the drum, the pre-preg contained an excess of dichloromethane. This was allowed to evaporate inside a fume cupboard over a period of 24 hours. The pre-preg was then sealed in air-tight polythene bags and placed in a freezer for storage. Sheets of pre-preg, as prepared by this process, are up to 300 mm wide by 1000 mm long, with an overall density of 400 g/m² and a resin content of 0.35, by volume.

3.3 Manufacture of laminates

Pre-preg for a (0,90)ₙ hybrid matrix laminate, for example, is laminated as follows: Two sheets were cut to size from the epoxy matrix pre-preg. Similarly, two sheets were cut to size from the epoxy/urethane pre-preg. The backing paper was peeled from successive sheets which were then laminated in the following sequence: One 0° ply with an epoxy matrix, two 90° plies with epoxy/urethane matrices and then another 0° ply with an epoxy matrix.

The laminated pre-preg was then sealed in a nylon bag and placed in a press-clave (a sealed chamber which is heated between the platens of a hot press), see Figure 3.3. This is
an adaptation of an auto-clave\textsuperscript{11}. The chamber is pressurized with nitrogen, thus exerting a hydrostatic pressure on the laminated pre-preg in the nylon bag. A vacuum may be drawn from the interior of the bag in order that it wraps tightly around the laminated pre-preg. The laminate is consolidated by the differential pressure across the bag and cured by the applied temperature (this is the cure cycle).

The requirements of a cure cycle are to

(i) eliminate air entrapped between plies during lay-up (consolidate the laminate).

(ii) eliminate aeration in the resin resultant from winding the pre-preg.

(iii) eliminate residual solvent in the resin.

(iv) bond but not to mix adjacent matrices in hybrid matrix laminates.

(v) cross-link the resin.

These requirements are common to all auto-clave cure cycles, except for (iv). This requirement is central to the concept of a hybrid matrix laminate. On the basis of this, the best cure cycle is one in which there is little or no flow of resin in the through-thickness direction of the laminate. This may be achieved by a net resin method in which no resin is drawn off the laminate during the cure cycle.

Initially however, poor wetting of the fibre and high void contents were encountered in laminates cured by this method. As an alternative, a preliminary method was assessed where resin is drawn off the laminate (an excess resin

\textsuperscript{11} A heated pressure vessel for curing laminates.
method). Following this, a successful net resin method was developed.

3.3.1 The excess resin method of cure

The laminate pack and bagging for this method are shown in Figure 3.4. A layer of resin-permeable release film was placed on each surface of the laminated pre-preg. Either side of these was placed a layer of absorption cloth. This was covered with a nylon bag which was sealed on the base of the chamber. A cure cycle (Figure 3.5) governing temperature and pressure has been designed for this technique.

Requirements (i) - (iii), generally termed de-bulking, may be met by applying a pressure or drawing a vacuum at low temperature, or both. Browning (1988), in a study on de-bulking of laminated pre-preg, found that applying a pressure alone at a low temperature is the most effective. Following Browning, the laminated pre-preg was kept at a pressure of 100 psi and a temperature of 90°C for one hour before cross-linking the resin at a higher temperature.

Adjacent matrices are likely to mix in hybrid matrix laminates cured by the excess resin method. Nonetheless, the motivation for using this method is that a permeable release film will allow residual gases to escape from the top and bottom of the laminate (limiting voiding). First attempts to cure laminates were at 160°C for two hours, as specified by the manufacturers. Laminates with poor wetting of the fibre and an unacceptably high void content were obtained with these
cure conditions. In successive efforts, the cure temperature was gradually increased until at a temperature of 190°C (Figure 3.5), acceptable quality was attained.

As the temperature of the laminate is increased from 90°C to 190°C during the cure cycle, the viscosity of the resin drops rapidly and resin excess to the laminate flows into the absorption cloths. Tang, Woo and Springer (1987) demonstrated that resin is drawn from the laminate successively from the outside plies, moving inward (Figure 3.6). This means that in a hybrid matrix laminate, modified resin in the inner 90° plies tends to move into the outer 0° plies.

Strictly speaking, flow of the modified resin from the 90° plies into the 0° plies results in something other than a hybrid matrix laminate. However, the modified 90° plies may not be affected in this instance. On the basis of this, comparisons of matrix damage in the 90° ply of these laminates to those in bona fida hybrid matrix laminates may be valid.

3.3.2 The net resin method of cure

Establishing the extent of damage in the 90° plies is necessary but not sufficient for overall assessment of cross-ply hybrid matrix laminates. Matrix dominated properties of the entire laminate, such as compressive strength, must also be evaluated. The material obtained from the excess resin method is not suitable for such tests. Consequently, further trials at the increased temperature of 190°C were conducted with the net resin method.
Browning (1988) conducted a study on resin migration in composite laminates. He inserted a one inch diameter fibre plug, impregnated with brominated epoxy resin, in the centre of an angle-ply laminate. This was cured in an auto-clave by a net resin method. A section of the laminate was examined by a scanning electron microscope. It was revealed that the brominated epoxy had migrated to the periphery in the plane of the laminate but had not fully migrated through the laminate thickness. This suggests that when using a net resin method of cure, adjacent matrices in a hybrid matrix laminate will bond but not necessarily homogenise.

The laminate pack for this method of cure is shown in Figure 3.7. A layer of impermeable release film was placed on either surface of the laminated pre-preg. A flat plate was also placed above the laminated pre-preg to improve the top surface of the laminate. The laminate pack was covered with a nylon bag which was sealed on the base of the chamber.

A longer period of de-bulking is required to eliminate entrapped air and residual gases since there is an impermeable layer either side of the laminated pre-preg. These escape at the periphery of the laminated pre-preg. The laminate was kept at a pressure of 100 psi and a temperature of 90°C for three hours (Figure 3.8).

With an increased cure temperature 190°C, the viscosity of the resin becomes sufficiently low for complete wetting of the fibres. As a result, hybrid matrix laminates of good quality may be prepared with this method of cure. Verification that adjacent matrices in hybrid matrix laminates
prepared by the net resin method are bonded but discrete is achieved by mechanical testing, described at the beginning of Chapter 5.

3.4 Hybrid matrix and uniform matrix laminates

3.4.1 Lay-up and matrices

The general cross-ply lay-up under consideration is \((0_m, 90_n)_s\). In this work, \(m = 1\) so that there is one \(0^\circ\) ply either side of \(2n\ 90^\circ\) plies. The lay-up, matrix, urethane content and cure method for all the laminates prepared are listed in Table 3.1. Unidirectional \((0)_4\) laminates with 0, 5, 10 and 20% urethane additions have also been prepared for the determination of the elastic properties of a unidirectional lamina.

3.4.2 Microstructure of the laminates

The microstructure of laminates prepared by the excess and net resin methods were examined by optical microscopy. Micrographs of polished sections of \((0, 90_2)_s^e\) and \((0, 90_4)_s^n\) laminates\(^{12}\) are shown in Figures 3.9 and 3.10 respectively. The average ply thickness is about 0.19 mm. The average transverse ply thicknesses \(2d\), and laminate thicknesses \(t\), for

\(^{12}\) The superscripts \(e\) and \(n\) refer to the excess and net resin methods of cure respectively. This notation is now adopted and is only omitted where distinction between the methods of cure is not necessary.
the range of laminates prepared are listed in Table 3.2.

There appears to be good fibre packing and little void in laminates prepared by both the excess and the net resin methods. A resin burn-off method has been used to determine the fibre volume fractions of the laminates. These are $V_f^o = 0.72$ and $V_f^n = 0.67$ for laminates prepared by the excess and net resin methods respectively. However, the outer surfaces of the $(0,90_2)_s^o$ laminate are non-uniform (Figure 3.9). This is due to the flow of resin having disturbed fibre tows in the $0^\circ$ plies. The $(0,90_4)_s^n$, on the other hand, has uniform outer surfaces. An important consequence of this is that laminates prepared by the net resin method have improved translucency.
Mechanical tests were carried out on laminates to determine fundamental constants and to monitor damage development in the different materials. In the following sections, the experimental methods are outlined.

3.5 Preparation of tensile testing coupons

Tensile coupons were cut to size from laminates using a diamond circular saw. The nominal dimensions of unidirectional and cross-ply coupons are: width, \( W = 20 \text{ mm} \) and gauge length, \( L = 140 \text{ mm} \) (the overall length is 200 mm). Average values of thickness and width of the coupons were calculated from a number of measurements with a micrometer along the coupon. Cross-sectional areas of the coupons were determined from these average values.

To determine the strain applied to the coupons during testing, polyester backed wire strain gauges were bonded to one surface of the coupon with cyanacrylate adhesive. The axis of the wire element in the strain gauge was aligned with the 0° fibres in the coupon. Gauges with a wire element of length 10 mm were used for \((0)_4\), \((90)_4\) and \((0,90)_3\) coupons and of length 30 mm for \((0,90_2)_s\) and \((0,90_4)_s\) coupons. A length of 30 mm at each end of the coupon was gripped in the tensile testing machine. Emery paper inserts were used in gripping the coupons to avoid slippage.
3.6 Determination of the elastic moduli and tensile strength of a lamina

The longitudinal elastic modulus $E_L$, and the transverse elastic modulus $E_T$, of a lamina could be determined by extension of undirectional coupons. For the measurement of $E_L$, $(0)_4^o$ coupons were extended at a rate of 0.5 mm/min in a tensile testing machine. Load applied to the coupon was measured by a load cell on the tensile testing machine. A chart recorder was connected to the load cell and the strain gauge on the coupon. In this way, a plot of load versus strain could be recorded upon extension of the coupon.

Load could be converted to stress by dividing by the cross-sectional area of the coupon. The stress/strain curve for the coupon, so determined, is linear-elastic almost entirely to failure. $E_L$ was then calculated from the slope of the linear portion of this curve. The tensile strength of the lamina was taken to be the failure load divided by the cross-sectional area of the coupon.

For measurement of $E_T$, $(90)_4^o$ coupons were extended at a rate of 0.2 mm/min. The elastic properties of the lamina are matrix dominated. This means that the stress/strain curve is non-linear in this case. Therefore, $E_T$ changes with applied strain and was determined from a tangent to the stress/strain curve at the point of zero stress and zero strain (Figure 4.1).
3.7 Tensile testing of hybrid matrix and uniform matrix cross-ply laminates

Characterisation of transverse cracking in the laminates was achieved by the extension of plain coupons. Transverse cracking was monitored in two ways during the test. First, acoustic emission from transverse cracks could be monitored with increasing strain applied to the coupon. This gave a quantitative value of strain for the initiation of transverse cracking, and a qualitative description of the progression of transverse cracking. Second, the density of cracks with increasing applied strain could be counted by eye, owing to the translucency of the coupons. Reduction of the longitudinal elastic modulus $E_0$, of the coupon could also be monitored from stress/strain curves for the coupon.

3.7.1 Monitoring the acoustic emission from transverse cracking during the extension of coupons

Acoustic emission (AE) from transverse cracks could be monitored by acoustic transducers affixed to the coupon. The acoustic transducers have a flat surface which was smeared with grease and clipped to the surface of the coupon. This ensured good acoustic contact for the test. There are three acoustic transducers. One is a signal transducer which was placed at the centre of the gauge length of the coupon. The other two were placed near the ends of the coupon. These are guard transducers which filter out unwanted noise from the
These transducers were connected to a processing unit in which the acoustic signal is converted to a voltage. The processing unit was set to count the number of acoustic events in 0.1 s intervals. This was found to be a suitable setting for the frequency of acoustic events (transverse cracking) in the laminates. A chart recorder was connected to the processing unit and the strain gauge on the coupon. The chart recorder could then plot the acoustic counts from the transducers against the strain applied to the coupon.

The output from the load cell of the testing machine was connected to a third input on the chart recorder. Therefore, a load (or stress)/strain curve could be recorded simultaneously to the AE/strain plot (Figure 3.11). In this way, acoustic events could be linked to changes in the stress and strain (the modulus) owing to transverse cracking during extension of the coupon.

It was sometimes possible to establish the applied strain for the initiation of transverse cracking from the AE/strain trace for the coupon (Figure 3.11). However in many cases, the low acoustic output from slow growing transverse cracks could not be detected by the AE equipment. Therefore, the initiation of transverse cracking was determined from the "knee" in the stress/strain curve as a result of the reduction in modulus of the coupon (Figure 3.11).

Nonetheless, AE could be used to qualitatively determine the the extent and nature of transverse cracking. The extent of transverse cracking is described by the number of peaks on
the AE trace while the nature of cracking is described by the amplitude of the peaks. Brittle transverse cracking is noisy and was recognised by high AE while constrained transverse cracking is less so and was recognised by low AE.

3.7.2 Counting transverse cracks during quasi-static extension of coupons

The number of transverse cracks occurring upon extension of a cross-ply coupon could be determined by a quasi-static test. A line was drawn in translucent marker pen along the axis of the coupon. Gauge lengths of \( \sim 10 \) mm for \((0,90)_s\) material; \( \sim 20 \) mm for \((0,90_2)_s\) material; and \( \sim 75 \) mm for \((0,90_4)_s\) material were defined on this line (owing to increasing crack spacing with increasing transverse ply thickness). A diffuse light was placed behind the coupon in the testing machine.

The coupon was extended at a rate of 0.5 mm/min, in increments of 0.1% strain, see Figure 3.12. After each increment, the coupon was allowed to relax completely (the relaxation curve was not recorded). The number of transverse cracks which were seen to intersect the gauge line are then recorded at this point. This was divided by the gauge length to give the average density of transverse cracks corresponding to the previous level of applied strain.
3.7.3 Measuring the reduction in longitudinal elastic modulus during quasi-static extension of coupons

The reduction in longitudinal elastic modulus of the coupon with progressive transverse cracking could be measured in the same quasi-static test as that in which crack counts were made. With incremental extension of the coupon (Figure 3.12), the slopes of progressive stress-strain curves gave the reduced modulus with increasing transverse crack density. In successive stress-strain curves, the coupon exhibits linear-elastic behaviour until the previous level of applied strain is reached. At this point, the modulus of the coupon drops as a result of further transverse cracking. The modulus at a particular level of strain was calculated from the linear-elastic portion of the following stress-strain curve, see Figure 3.12.

3.8 Notched testing of hybrid matrix and uniform matrix cross-ply laminates

Extension of a circular, centre-notched cross-ply coupon results in three types of matrix damage. Firstly, matrix cracks appear in the transverse ply. Secondly, longitudinal splits occur at the width-wise edges of the circular notch. Thirdly, the 0° and 90° plies delaminate underneath the longitudinal splits. Therefore, the extension of notched coupons yields information not only about matrix damage in the 90° plies, but also in the 0° plies and at their interface.
with the 90° plies. As a consequence of this, notched tests are useful for the evaluation of hybrid matrix laminates.

Circular holes of 2, 5 and 10 mm diameter were drilled at the centre of the width and the gauge length of \((0,90)_s\) tensile coupons. Tungsten carbide drill bits were used at a low drilling speed in order to minimize damage around the hole. As for the plain tensile tests, the notched coupons were gripped in the testing machine with emery paper inserts. A diffuse light was placed behind the coupons and a camera was positioned in front of the coupons to photograph matrix damage during the test. The coupons were extended at a rate of 0.2 mm/min, and damage was recorded on camera at regular load intervals. In these tests, the matrix damage processes of interest are *longitudinal splitting* and *ply delamination*. 
<table>
<thead>
<tr>
<th>Lay-up</th>
<th>Matrix</th>
<th>Urethane Content (%)&lt;sup&gt;*&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(0)&lt;sub&gt;_e&lt;/sub&gt;</strong></td>
<td>uniform</td>
<td>0, 5, 10, 20</td>
</tr>
<tr>
<td>(0, 90)&lt;sub&gt;_e&lt;/sub&gt;</td>
<td>uniform</td>
<td>0, 5, 10, 20</td>
</tr>
<tr>
<td></td>
<td>hybrid</td>
<td>5, 10, 20</td>
</tr>
<tr>
<td>(0, 90)&lt;sub&gt;_s&lt;/sub&gt;</td>
<td>uniform</td>
<td>0, 20</td>
</tr>
<tr>
<td></td>
<td>hybrid</td>
<td>5, 10, 20</td>
</tr>
<tr>
<td>*<strong>(0, 90)&lt;sub&gt;_n&lt;/sub&gt;</strong></td>
<td>uniform</td>
<td>0, 5, 10, 20</td>
</tr>
<tr>
<td></td>
<td>hybrid</td>
<td>5, 10, 20</td>
</tr>
<tr>
<td>(0, 90)&lt;sub&gt;_s&lt;/sub&gt;</td>
<td>uniform</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>hybrid</td>
<td>5, 10, 20</td>
</tr>
</tbody>
</table>

* Note that a uniform matrix laminate with a urethane content of 0% may also be considered as a hybrid matrix laminate with a urethane content of 0%.

** The superscript e refers to the excess resin method of cure.

*** The superscript n refers to the net resin method of cure.

Table 3.1 Lay-up and matrices for unidirectional and cross-ply laminates.
<table>
<thead>
<tr>
<th>Lay-up</th>
<th>Transverse ply thickness, $2d$ (mm)</th>
<th>Laminate thickness, $t$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(0)_4$</td>
<td>-</td>
<td>0.73</td>
</tr>
<tr>
<td>$(0,90)_s$</td>
<td>0.38</td>
<td>0.76</td>
</tr>
<tr>
<td>$(0,90_2)_s$</td>
<td>0.74</td>
<td>1.11</td>
</tr>
<tr>
<td>$(0,90_4)_s$</td>
<td>1.44</td>
<td>1.80</td>
</tr>
</tbody>
</table>

**Table 3.2** Average transverse ply and laminate thickness for the prepared lay-ups.
Figure 3.1  A photograph of the laboratory scale drum winder designed and built during this work

Figure 3.2  A photograph of the resin bath on the drum winder used to wet the glass roving
Figure 3.3 A schematic showing a section of the press-clave used for curing laminates
Figure 3.4 A schematic of the laminate pack used for the excess resin method of cure

Figure 3.5 A graph of the change in pressure and temperature in the press-clave, with time, during the excess resin method of cure
Figure 3.6 A schematic showing the flow of resin in a cross-ply laminate during an excess resin cure
Figure 3.7 A schematic of the laminate pack used for the net resin method of cure.

Figure 3.8 A graph of the change in pressure and temperature in the press-clave, with time, during the net resin method of cure.
Figure 3.9  Micrographs of polished longitudinal (above) and cross (below) sections of a $(0,90)_s$ laminate x 55
Figure 3.10 Micrograph of a polished longitudinal section of a \((0,90_4)_s\) laminate \(\times 77\)
Figure 3.11 Graphs of the stress-strain response and the acoustic emission-strain trace during the extension of a $(0,90_2)_s$ epoxy matrix laminate.

Figure 3.12 A schematic showing the stress-strain curves recorded during quasi-static extension of a cross-ply laminate.
4. BASIC MECHANICAL PROPERTIES OF UNNOTCHED UNIDIRECTIONAL
AND CROSS-Ply LAMINATES

The elastic moduli for the laminates listed in Table 3.1 are presented in this chapter. The majority of elastic moduli have been measured on laminates prepared by the excess resin technique with a fibre volume fraction $V_f^e = 0.72$ (the superscript $e$ denotes the excess resin method of cure). Amongst laminates prepared by the net resin method with a fibre volume fraction $V_f^n = 0.67$ (the superscript $n$ denotes the excess resin method of cure), the moduli of only $(0,90)_s^n$ laminates were measured. Therefore, in order to assess the effect of adding urethane to the matrix on the elastic moduli of the laminates, all moduli are normalised for a fibre volume fraction of $V_f = 0.72$. The equations for normalisation of moduli are presented in Appendix 1 to this chapter.
4.1 The longitudinal elastic modulus of unidirectional laminates

The longitudinal elastic modulus $E_e$, has been determined experimentally from (0)\textdegree\textdegree\textdegree\textdegree\textdegree laminates for a range of urethane additions to the matrix (Table 4.1). The experimental results indicate that $E_e$ does not change appreciably with an increasing urethane content. $E_e$ may be expressed as a volume weighted sum of the moduli of the fibres $E_f$, and the matrix $E_m$, in the laminate:

$$E_e = V_f E_f + (1 - V_f) E_m$$  \hspace{1cm} (4.1)

where $V_f$ is the volume fraction of the fibres, see for example Jones (1975). The glass fibres have a modulus of 70 GPa, which is very high in comparison to that of the matrices (~3.5 GPa). Also, there is a high volume fraction of fibres ($V_f = 0.72$) in the unidirectional laminates. From this it may be seen that $E_e$ would not change appreciably with increasing additions of urethane to the matrix. Equation (4.1) therefore confirms the experimental values of $E_e$ for (0)\textdegree\textdegree\textdegree\textdegree\textdegree laminates with urethane contents of 0 - 20\%.

4.2 The transverse elastic modulus of unidirectional laminates

The transverse elastic modulus $E_t$, has been determined from tests on (90)\textdegree\textdegree\textdegree\textdegree\textdegree laminates with increasing proportions of
urethane added to the matrix (Table 4.1). The experimental results indicate that $E_t$ drops with the increased proportion of urethane. $E_t$ may also be expressed in terms of the moduli and volume fractions of the fibres and the matrix:

$$E_t = \frac{1}{V_f/E_f + (1 - V_f)/E_m} \quad (4.2)$$

To obtain this expression, the stress in the fibres and the matrix are assumed equal. Also, no account is taken of the mismatch in Poisson's contractions in the fibres and the matrix. However, equation (4.2) shows reasonable agreement with experimental values for the transverse modulus, see Hull (1981). The term $V_f/E_f$ is small, and therefore $E_t$ is dominated by the matrix. When in equation (4.2), the elastic modulus of the matrix $E_m$, decreases with an increasing proportion of urethane, so too does $E_t$. This confirms the reduction in $E_t$ observed in the experiments on $(90)_4$ laminates.

Also presented in Table 4.1 are calculated elastic moduli for the transverse plies of hybrid matrix cross-ply laminates, $E_{90^\circ \text{ ply}}$ (as opposed to the experimental values which are transverse moduli of unidirectional laminae, $E_t$). These values of $E_{90^\circ \text{ ply}}$ may be determined from the rule of mixtures (ROM) approximation for the modulus of a cross-ply laminate:

$$E_0 = V_\ell E_{\ell} + V_t E_{90^\circ \text{ ply}} \quad (4.3)$$

where $V_\ell$ and $V_t$ are the volume fractions of the longitudinal
and transverse plies respectively. Experimental data for longitudinal moduli of cross-ply laminates $E_0$, are presented in Section 4.4) and experimental data for $E_t$ has been presented above.

Equation (4.3) does not account for the difference in Poisson contraction in the longitudinal and transverse plies. This can be incorporated using laminated plate theory (Jones (1975)) however, the error is small (at most 2%), see Appendix 2 to this chapter. The simpler ROM approximation is therefore considered sufficiently accurate. Values of $E_{90^\circ Ply}$ calculated from equation (4.3) show a reduction as the proportion of urethane in the matrix increases. This is consistent with the experimental results for $E_t$ measured on $(90)_4^\circ$ coupons. Values of $E_{90^\circ Ply}$ calculated from $(0,90)_s^\circ$ laminates are the lowest, increasing as the transverse ply becomes thicker in the $(0,90_2)_s^\circ$ and $(0,90_4)_s^\circ$ laminates. This is further discussed in the following section.

4.3 The effect of thermal strain on transverse modulus

Intuitively, $E_{90^\circ Ply}$ should be the same, regardless of the lay-up. The apparent discrepancy in the results of Table 4.1 may originate from residual thermal strain in the cross-ply laminates. In a glass/epoxy cross-ply laminate, the coefficient of thermal expansion of the transverse ply $\alpha_t$ (typically $16.7 \, \mu e/\circ C$) is greater than that for the longitudinal plies $\alpha_l$ (typically $3.8 \, \mu e/\circ C$). Thus upon cooling from the cure or stress-free temperature (in this case
somewhere near the cure temperature, i.e. 190°C), residual strains $\varepsilon^R_t$ and $\varepsilon^R_\theta$, are generated in the transverse and longitudinal plies respectively. The residual thermal forces in the plies must sum to zero at room temperature to satisfy equilibrium conditions. The following expression for the residual thermal strain in the transverse ply $\varepsilon^R_t$, may be derived by satisfying longitudinal equilibrium

$$
\varepsilon^R_t = \frac{bE_\theta(\alpha_t - \alpha_\theta)\Delta T}{bE_\theta + dE_t}
$$

(4.4)

where $\Delta T$ is the difference between cure temperature and room temperature, see for example Bailey et al. (1979). $\varepsilon^R_t$ is a tensile strain since the transverse ply is prevented from contracting by the higher modulus longitudinal plies.

From equation (4.4) it can be seen that $\varepsilon^R_t$ will decrease for laminates with thicker transverse plies (increasing $2d$). The lower residual thermal strain in thick transverse plies is due to reduced constraint exerted by the longitudinal plies. Data is presented in Table 4.2 from Bailey et al. (1979) for measured thermal strains $\varepsilon^R_t$, in GFRP laminates similar to those used in the present work. They cured their GFRP laminates at a temperature of 100°C. The values of thermal strain have been corrected to allow for the different cure temperature (190°C) used for the laminates in this study.

The stress/strain curves for (90)$^\circ$ laminates were found to be convex (Figure 4.1). Therefore, the effective modulus of thin transverse plies (with large thermal strains built in)
is lower than that for thicker transverse plies. It follows
that the values of \( E_{90^\circ_{ply}} \) calculated for the transverse ply in
\((0,90)_s^e \) laminates should be the lowest (Table 4.1).

The transverse moduli \( E_t \), of a \((90)_s^e \) laminate measured at
increasing values of strain (see Figure 4.1) are listed in
Table 4.2. These values of strain correspond to tensile
thermal strains in the 90° plies of the cross-ply laminates.
There is a reduction in \( E_t \) with increasing thermal strain.
This is comparable to the reduction in \( E_{90^\circ_{ply}} \) as the 90° ply
thickness reduces in the cross-ply laminates (Table 4.2).

This indicates that when there is a high thermal strain
in the 90° ply of a cross-ply laminate, the \textit{in-situ} modulus of
the transverse ply \( E_{90^\circ_{ply}} \), may be significantly different (as
much as 35% in this case) from the value of \( E_t \) measured from
the origin of the stress/strain curve (Figure 4.1).

On this premise, values for \( E_t \) measured from \((90)_s^e \)
laminates (with no residual thermal strain) should form an
upper bound. They are in fact slightly lower than \( E_{90^\circ_{ply}} \)
values calculated from the \((0,90)_s^e \) lay-up (Table 4.1). There
are two sources of error which could cause this discrepancy.
First, non-linearity of the stress/strain curve for transverse
tests of \((90)_s^e \) laminates introduces errors in the measurement
of \( E_t \). Second, the adjustment of moduli for differences in \( V_f \)
introduces further errors.

4.4 The longitudinal elastic modulus of cross-ply laminates

The longitudinal elastic modulus \( E_0 \), of \((0,90)_s^e \), \((0,90_2)_s^e \)
and \((0,90_2)_s^n\) hybrid laminates have been determined experimentally for a range of urethane additions to the matrix, Table 4.1. In all these lay-ups, the reduction in \(E_0\) with an increasing amount of urethane added to the matrix is small (about 10% for a 20% urethane addition), due to the dominance of the fibres in the longitudinal plies \((V_f = 0.72)\).

Uniform and hybrid matrix \((0,90)_s^n\) laminates with the similar level of added urethane have similar moduli (Figure 4.2). The only difference between these laminates is that the uniform matrix laminates have, in addition, modified matrices in the longitudinal plies. This has been shown (Table 4.1) not to affect the elastic moduli since the fibres in the longitudinal plies are dominant.

4.5 The longitudinal tensile strength of unidirectional and cross-ply laminates

The failure strengths of \((0)_4^n\) laminates and the \(0^\circ\) plies of \((0,90)_s^n\) and \((0,90_2)_s^n\) laminates have been measured for a range of urethane additions (Figure 4.3). The \(90^\circ\) plies are discounted in the cross-ply laminates so that the stress at failure in the \(0^\circ\) plies may be compared to that for the unidirectional laminates. Failure strength is predominantly constant (~1100MPa) for all the laminates. In all cases however, the tensile coupons failed near the grips of the testing machine. This means that the stress concentration at the grips constitutes the weakest part of the coupons. It is not known to what applied stress they would survive if stress
concentrations were not present.

4.6 The toughness for Mode I matrix cracking\textsuperscript{13} $G_{IC}$, in unidirectional laminates

In this work, theory based on an energy balance (Section 2.4.1) is used to model the progression of transverse cracking in cross-ply laminates. Therefore, the strain energy per unit crack area required for extension of a matrix crack, or the toughness $G_{IC}$, must be known. This may be determined by monitoring Mode I crack extension in a double-cantilever beam (DCB) coupon (the two opening ligaments behind the crack front are termed "double cantilevers"). In this test, the matrix crack is grown along the fibres in the mid-plane of a $(0)_{24}$ coupon. Messenger (1990) determined $G_{IC}$ for an increasing proportion of urethane added to the matrix.

The toughness $G_{IC}$ is a constant for crack propagation, i.e. it is a material constant. However in the DCB tests, fibre bridging occurred behind the crack front. This caused the measured toughness to increase with crack length (Figure 4.4). In this test, the toughness had an initial value $G_{IC}^{\text{init}}$, then increased with crack length, to approach a plateau value, $G_{IC}^{\text{plat}}$ (Table 4.3). Both these values indicate an increase in toughness as the proportion of urethane in the matrix is increased.

\textsuperscript{13} Mode I matrix cracking occurs when the crack surfaces open only normal to the crack plane.
<table>
<thead>
<tr>
<th>Moduli in GPa</th>
<th>$V_f = 0.72^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Urethane content (%)</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>$E_g$: $(0)_4^e$</td>
<td>$^*$50.2</td>
</tr>
<tr>
<td>$E_t$: $(90)_4^e$</td>
<td>$^+$19.5</td>
</tr>
<tr>
<td>$E_{90^\circ \text{ply}}$:</td>
<td></td>
</tr>
<tr>
<td>$(0,90)_4^s$</td>
<td>$^*$20.3</td>
</tr>
<tr>
<td>$(0,90)_2^s$</td>
<td>$^*$18.1</td>
</tr>
<tr>
<td>$(0,90)_4^e$</td>
<td>$^*$15.1</td>
</tr>
<tr>
<td>$E_0$:</td>
<td></td>
</tr>
<tr>
<td>$(0,90)_4^s$</td>
<td>$^+*26.5$</td>
</tr>
<tr>
<td>$(0,90)_2^s$</td>
<td>$^+*29.2$</td>
</tr>
<tr>
<td>$(0,90)_4^e$</td>
<td>$^+*33.3$</td>
</tr>
</tbody>
</table>

Small corrections of moduli have been made for slight differences in $V_f$, see Appendix 1.

* A mean value of $E_g = 51.4$ GPa is used in calculations for urethane contents of 0 - 20%.
+ Measured for urethane contents of 0 - 20% (shown in bold type).
■ Calculated from ROM, equation (4.3), for urethane contents of 0 - 20%.
○ Adjusted for the difference between $V_f^n = 0.67$ and $V_f^e = 0.72$, for urethane contents of 0 - 20%, see Appendix 1.

Table 4.1 Measured and calculated elastic moduli for unidirectional laminates and hybrid matrix cross-ply laminates as a function of urethane content in the matrix.
<table>
<thead>
<tr>
<th>Lay-up</th>
<th>$(90)_4$</th>
<th>$(0,90)_4$</th>
<th>$(0,90)_2$</th>
<th>$(0,90)_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon^R_t$ (%)</td>
<td>0</td>
<td>0.11</td>
<td>0.16</td>
<td>0.20</td>
</tr>
<tr>
<td>$E_t$ (GPa)</td>
<td>16.9</td>
<td>16.2</td>
<td>14.8</td>
<td>13.4</td>
</tr>
<tr>
<td>$E_{90^\circ\text{ ply}}$ (GPa)</td>
<td>-</td>
<td>17.5</td>
<td>13.9</td>
<td>11.1</td>
</tr>
</tbody>
</table>

* The GFRP laminates of Bailey et al. (1979) have a slightly lower fibre volume fraction and a standard epoxy matrix. However residual thermal strain will not be appreciably affected by either the different fibre volume fraction or urethane additions to the matrix.

- These $E_t$ values are measured from $(90)_4$ laminates (not from the lay-ups in the first row of the table), at strains corresponding to the thermal strains in the cross-ply laminates.

**Table 4.2** Thermal strains $\varepsilon^R_t$, and moduli $E_{90^\circ\text{ ply}}$, in the 90° ply of cross-ply laminates, plus corresponding transverse moduli, $E_t$ (at a urethane level of 20%)
<table>
<thead>
<tr>
<th>Urethane level (%)</th>
<th>$G_{IC}^{init}$ (J/m²)</th>
<th>$G_{IC}^{plat}$ (J/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>220</td>
<td>420</td>
</tr>
<tr>
<td>5</td>
<td>180</td>
<td>525</td>
</tr>
<tr>
<td>10</td>
<td>240</td>
<td>560</td>
</tr>
<tr>
<td>20</td>
<td>420</td>
<td>600</td>
</tr>
</tbody>
</table>

Table 4.3 Measured values of transverse ply toughness from DCB tests, for $(0)_{24}$ coupons with urethane levels of 0 - 20%. 
Figure 4.1 A schematic of a stress-strain curve of a transverse laminate

Figure 4.2 The longitudinal elastic modulus of $(0,90)_s^e$ and $(0,90_2)_s^e$ hybrid and uniform matrix laminates for an increasing urethane content in the matrix
**Figure 4.3** The longitudinal tensile strength of the 0° plies in cross-ply laminates (estimated), and unidirectional laminates (measured experimentally), for an increasing urethane content in the matrix.

**Figure 4.4** The increase in Mode I interlaminar fracture toughness of unidirectional laminates with an increasing crack length in the double cantilever beam test and an increasing urethane content in the matrix.
COMPENSATION OF MODULI TO ALLOW FOR THE DIFFERENT FIBRE VOLUME FRACTIONS IN THE EXCESS RESIN ($V_{fe}$) AND NET RESIN ($V_{fn}$) SYSTEMS

Longitudinal elastic modulus, $E_{l}$

Taking out a common factor $E_{f}$, in equation (4.1), the following relation is obtained

$$E_{l} = \{V_{f} + (1 - V_{f})E_{m}/E_{f}\}E_{f} \quad (A4.1)$$

Taking the ratio of the moduli $E_{l}^{n}$ and $E_{l}^{o}$, for laminates with fibre volume fractions $V_{f}^{o}$ and $V_{f}^{n}$ respectively, $E_{f}$ cancels and the term $(1 - V_{f})E_{m}/E_{f}$ is small, so (A4.1) becomes

$$E_{l}^{n} = E_{l}^{o} \frac{V_{f}^{n}}{V_{f}^{o}} \quad (A4.2)$$

Transverse elastic modulus, $E_{t}$

Taking a common denominator $E_{m}$, and re-writing the right hand side of equation (4.2), the following relation is obtained

$$E_{t} = \frac{E_{m}}{V_{f}E_{m}/E_{f} + (1 - V_{f})} \quad (A4.3)$$
Taking a ratio of moduli $E_t^e$ and $E_t^n$, for laminates with fibre volume fractions $V_f^e$ and $V_f^n$ respectively, the following expression is obtained

$$\frac{E_t^n}{E_t^e} = \frac{(V_f^e)E_m/E_f + (1 - V_f^e)}{(V_f^n)E_m/E_f + (1 - V_f^n)}$$  \hspace{1cm} (A4.4)

Again, the terms containing $E_m/E_f$ are small and therefore (A4.4) becomes

$$E_t^n = E_t^e \frac{1 - V_f^e}{1 - V_f^n}$$  \hspace{1cm} (A4.5)

Cross-ply elastic modulus, $E_0$

The elastic modulus $E_0^n$, for a cross-ply laminate of fibre volume fraction $V_f^n$ may be written, from equation (4.3), as

$$E_0^n = V_f E_g^n + V_t E_t^n$$  \hspace{1cm} (A4.6)

If the elastic moduli $E_g^e$, $E_t^e$, $E_0^e$, are known for a cross-ply laminate of fibre volume fraction $V_f^e$, then from equations (A4.2) and (A4.5), $E_0^n$ becomes

$$E_0^n = V_f E_g^e \frac{V_f^n}{V_f^e} + V_t E_t^e \frac{1 - V_f^e}{1 - V_f^n}$$  \hspace{1cm} (A4.7)
LAMINATED PLATE THEORY FOR THE CALCULATION OF $E_{90^\circ_ply}$

It may be shown from laminated plate theory (LPT), see Jones (1975), that the longitudinal elastic modulus of a $(0,90_n)_s$ laminate with a constant ply thickness can be expressed in terms of its constituent lamina properties as follows

$$E_0 = \frac{E_g + nE_{90^\circ_ply}}{(n + 1)(1 - \nu_{\theta t}\nu_{\phi t})} - \frac{(n + 1)\nu_{\theta t}^2 E_{90^\circ_ply}^2}{(nE_g + E_{90^\circ_ply})(1 - \nu_{\theta t}\nu_{\phi t})}$$

... (A4.8)

where $n$ is half the total number of $90^\circ$ plies, $\nu_{\theta t}$ is the major Poisson's ratio of a longitudinal lamina and $\nu_{\phi t}$ is the minor Poisson's ratio. LPT takes into account the different Poisson's contraction of the longitudinal and transverse plies when a cross-ply laminate is extended.

The major Poisson's ratio $\nu_{\theta t}$, has been measured on $(0)_s$ laminates for urethane additions to the matrix of $0 - 20\%$:

<table>
<thead>
<tr>
<th>urethane (%)</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_{\theta t}$</td>
<td>0.25</td>
<td>0.25</td>
<td>0.26</td>
<td>0.26</td>
</tr>
</tbody>
</table>
Furthermore, the following relation exists for a unidirectional lamina (or a $90^\circ$ ply)

$$v_{lt} = v_{et} \frac{E_{90^\circ \text{ply}}}{E_t} \quad (A4.9)$$

Then, referring to equation (A4.8), $E_{90^\circ \text{ply}}$ is the only unknown. Therefore, values of $E_{90^\circ \text{ply}}$ may be calculated for the $(0,90_n)_s$ laminates

<table>
<thead>
<tr>
<th>$E_{90^\circ \text{ply}}$ (GPa)</th>
<th>Urethane addition (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(0,90_4)_s$</td>
<td>0 5 10 20</td>
</tr>
<tr>
<td>20.2</td>
<td>19.3 18.8 17.4</td>
</tr>
<tr>
<td>$(0,90_2)_s$</td>
<td>17.9 16.5 16.5 13.7</td>
</tr>
<tr>
<td>$(0,90)_s$</td>
<td>14.8 14.4 14.0 10.8</td>
</tr>
</tbody>
</table>

Comparing these values of $E_{90^\circ \text{ply}}$ to those calculated by the ROM prediction, in Table 4.1, there is at most a difference of 2%.
5. DAMAGE DEVELOPMENT IN CROSS-PLY LAMINATES UNDER QUASI-STATIC LOADING

The development of matrix damage in \((0,90)_s^e\) and \((0,90_2)_s^e\) laminates (prepared by the excess resin method) and \((0,90)_s^n\) and \((0,90_4)_s^n\) laminates (prepared by the net resin method) with uniform and hybrid matrices is reported in this chapter.

Laminates prepared by the excess resin method

It has been shown by Tang et al. (1987) that resin moves outwards from the inner plies of a laminate during an excess method of cure. As a result of this, the outer 0° plies of the \((0,90)_s^e\) and \((0,90_2)_s^e\) hybrid matrix laminates contain a proportion of the modified resin from the inner 90° plies. This means that these laminates do not actually have bonded but discrete matrices. Therefore, the term "hybrid matrix" is used for convenience of reference in the case of these laminates. However, it follows that the inner 90° plies of these laminates are not in general affected. On the basis of this, it will be shown that matrix damage in the 90° plies of these hybrid matrix laminates is similar to that in uniform matrix laminates with the same modification (urethane addition) to the matrix.
Laminates prepared by the net resin method

Laminates prepared by the net resin method (as compared to those prepared by the excess resin method) have little or no void, good wetting of the fibres, uniform outer surfaces and a greater degree of translucency. With the above properties being acceptable, uniform matrix laminates prepared by this method are suitable for mechanical testing. There is however an additional requirement for hybrid matrix laminates: that adjacent matrices have bonded but not homogenised during processing. This has been verified by a simple mechanical test.

In this test, a \((0, 90_4)^n\) hybrid matrix laminate and a \((0, 90_4)^n\) uniform matrix laminate, both with a urethane addition of 20% to the matrix have been compared to a \((0, 90_4)^n\) epoxy matrix laminate. These laminates have been extended transverse to the 0° plies (as if they were \((90, 0_4)^n\) laminates) and the density of transverse cracks was recorded in the outer 90° plies. In Section 5.3, it is shown that the density of transverse cracks in a cross-ply laminate under extension depends on the proportion of urethane which has been added to the matrix in the transverse ply (Figure 5.10). This result may be used to verify that adjacent matrices in \((0, 90_4)^n\) hybrid matrix laminates have bonded but not homogenised.

The density of transverse cracks in the outer plies of the \((90, 0_4)^n\) hybrid matrix coupons (taken from the \((0, 90_4)^n\) hybrid matrix laminate) is close to the density of transverse
cracks in the (90,0₄)ₜₑ epoxy matrix coupons (taken from the 
(0,90₄)ₜₑ epoxy matrix laminate), see Figure 5.1. However, 
the density of transverse cracks in the outer plies of the 
(90,0₄)ₜₑ uniform matrix coupons (taken from the (0,90₄)ₜₑ 
uniform matrix laminate) is much lower than the above two 
laminates. This shows that the matrices in the outer plies of 
the (0,90₄)ₜₑ hybrid matrix laminate are almost wholly 
unmodified epoxy resin. A small degree of mixing of the 
matrices has taken place during processing. However this is a 
severe case where there are four plies containing a high proportion of urethane in the resin (20%) for every one ply 
with unmodified epoxy resin.
5.1 The mechanism of transverse cracking in cross-ply laminates

There are two common non-destructive methods of detecting transverse cracking in laminates which have been extended. First, by monitoring the acoustic emission from transverse cracks, see Bailey et al. (1979). Second, by using techniques such as microscopy or X-ray photography, see Bader et al. (1979). In the case of the translucent glass/epoxy laminates used in this work, progressive transverse cracking may be recorded simply by optical methods. Traces of the acoustic emission (AE) from transverse cracking in cross-ply laminates under extension have also been recorded in this work (Figures 5.2, 5.3, 5.4).

5.1.1 (0,90ₙₙ)ₙ hybrid matrix laminates

Transverse cracks in (0,90ₙₙ)ₙ hybrid matrix laminates under extension were observed to initiate at the edges of the coupon and grow instantaneously to span the transverse ply. This is termed brittle transverse cracking. Urethane additions to the matrix of up to 20% were observed not to eliminate this brittle matrix failure mechanism in the transverse ply, although they have been shown to increase the transverse ply toughness by about 40% (Section 4.6).

The above optical observations are corroborated by acoustic emission from transverse cracks in the laminates (Figure 5.2 a-d). Transverse cracking produces a high level
of acoustic output from all these laminates for urethane levels in the matrix of 0 to 20%. This confirms that the matrix in the transverse ply of these laminates failed in a brittle manner, since this emits a high level of noise. Furthermore, each peak on the AE trace can be shown to correspond to a transverse crack, following Peters (1984, 1986). This is discussed further in Chapter 6.

5.1.2 \((0,90_\circ)_s^6\) hybrid matrix laminates

When extending \((0,90_\circ)_s^6\) hybrid matrix laminates with low levels of urethane in the matrix \((0 - 5\%)\), transverse cracks were observed to initiate at the edges of the coupon and grow \textit{instantaneously} to span the transverse ply. This brittle matrix failure in the transverse ply was not seen in laminates having a higher level of urethane in the matrix \((10 - 20\%)\). At these levels of urethane, transverse cracks were observed to initiate in the centre of the coupon width as well as at the edges. The cracks then grew \textit{stably} to span the transverse ply thickness \(2d\), and \textit{sometimes} to span the transverse ply width \(W\). This is termed \textit{constrained transverse cracking}.

These optical observations are reinforced by the AE/strain traces for the laminates (Figure 5.3 a-d). There is a high acoustic output from transverse cracks in laminates with low levels of urethane in the matrix \((0 - 5\%)\), confirming the brittle nature of cracking. There is a much lower acoustic output from transverse cracks in laminates with higher levels of urethane in the matrix \((10 - 20\%)\), since
cracks grew *stably*, emitting less noise.

A $(0,90_2)_s$ *uniform matrix* laminate with a 20% urethane addition has also been tested (Figure 5.3 e). The AE/strain trace for this laminate and that for the corresponding $(0,90_2)_s$ *hybrid matrix* laminate (Figure 5.3 d) are comparable. In both cases, the acoustic emission as a result of transverse cracking is very low. Therefore, matrix failure is very similar (constrained) in these laminates. This indicates that flow of resin during the *excess* resin method of cure has not affected the modified matrix in the transverse plies of the $(0,90_2)_s$ *hybrid matrix* laminates.

5.1.3 $(0,90)_s$ and $(0,90)_s^n$ hybrid and uniform matrix laminates

During the extension of $(0,90)_s$ and $(0,90)_s^n$ hybrid and uniform matrix laminates, transverse cracks were observed to initiate in the centre portion of the coupon width, as well as at the edges of the coupon. They propagated *stably*, to span the transverse ply thickness $2d$, and *sometimes* to span the transverse ply width $W$. The addition of urethane to the matrix had no effect on the *mechanism* of transverse cracking, i.e. cracking remained constrained for all urethane levels. The AE/strain traces for the $(0,90)_s$ laminates are presented in Figure 5.4 a-g. Acoustic emission from transverse cracking in these laminates is low. This confirms the above observations.
5.2 The initiation of transverse cracking in cross-ply laminates

The applied strain for the initiation of transverse cracking in these laminates $\varepsilon_{\text{atc}}$, was determined by two methods. It was possible to determine $\varepsilon_{\text{atc}}$ by optical techniques in laminates prepared by the net resin method ($(0,90)_s^n$ and $(0,90_\alpha)_s^n$ laminates), owing to their translucency. The density of transverse cracks was plotted as a function of the applied strain (Figures 5.10, 5.15). These curves were then extrapolated to a crack density of zero. This intercept with the strain (or stress) axis gave $\varepsilon_{\text{atc}}$ for these laminates.

In the case of the laminates prepared by the excess resin method ($(0,90)_s^\alpha$ and $(0,90_\alpha)_s^\alpha$ laminates), an optical method for determining $\varepsilon_{\text{atc}}$ was more difficult, owing to their poor translucency. Therefore, the applied strain for the initiation of transverse cracking $\varepsilon_{\text{atc}}$, was estimated from the associated knee in the stress/strain curve (Figures 3.11, 5.3 and 5.4).

The residual thermal strain $\varepsilon_t^R$, in the transverse plies of cross-ply laminates changes as the ratio of the transverse ply thickness to the longitudinal ply thickness changes (Tables 3.2 and 4.2). Therefore, the residual thermal strain in these laminates must be taken into account when comparing the strains at which transverse cracking begins. The calculated residual thermal strain $\varepsilon_t^R$, has been added to the applied strain $\varepsilon_{\text{atc}}$, to give the total transverse ply strain at the initiation of transverse cracking, $\varepsilon_{\text{tc}}$ (this is termed the
transverse cracking strain).

$(0,90)_{s}^{n}$ and $(0,90_{4})_{s}^{n}$ laminates

The transverse cracking strain $\varepsilon_{tc}$, for $(0,90)_{s}^{n}$ and $(0,90_{4})_{s}^{n}$ laminates is presented in Figure 5.5 as a function of the level of urethane in the matrix. Transverse cracking strains for $(0,90)_{s}^{n}$ hybrid matrix laminates correspond with those for $(0,90)_{s}^{n}$ uniform matrix laminates at all levels of urethane in the matrix (Figure 5.5), as would be expected. As a consequence of this, only transverse cracking strains for hybrid matrix laminates are discussed further.

The transverse cracking strains in the $(0,90)_{s}^{n}$ laminates (thin transverse ply) are on average 85% higher than those for the $(0,90_{4})_{s}^{n}$ laminates (thick transverse ply), see Figure 5.5. This illustrates the constraining effect of the longitudinal plies on the thinner transverse plies in the $(0,90)_{s}^{n}$ laminates ($2d = 0.38$ mm). Parvizi et al. (1978) reported this effect for GFRP cross-ply laminates with a range of transverse ply thicknesses. The constraining effect of the longitudinal plies may be explained in terms of the size of a micro-crack in the transverse ply, following Ogin and Smith (1985, 1987).

In a cross-ply laminate under an applied stress in the direction of the longitudinal fibres, the matrix in the transverse ply debonds from the fibres at a strain $\varepsilon_{deb}$, of about $0.1 - 0.3\%$ (for the GFRP laminates of Bailey and Parvizi (1981)). At applied strains of about $0.4\%$ and more, these debonded areas link to form micro-cracks of size $2a_{d}$, initially
spanning 2 to 3 fibre diameters in the thickness direction and of size $2a_w$ depending on the fibre packing in in the width direction (Figure 5.7). If the micro-crack has grown in the thickness direction to span the transverse ply thickness and has not exceeded a critical size $2a_d^{crit}$ then, it will subsequently grow in a **stable manner** across the ply width.

Ogin and Smith have attempted to qualify this by considering the stress state at the width-wise front (A in Figure 5.7) of a crack spanning the transverse ply thickness ($2a_w = 2d$). They suggest that over most of the length of the crack ($2a_w$), stress is transferred into the longitudinal plies. There is however an intensification of stress at the crack tip (A), owing to the localized stress disturbance. They estimate the characteristic length of the stress disturbance to be half the transverse ply thickness, $d$. The crack does not span the thickness of the laminate ($2b + 2d$), so the stress intensity factor $K$, at the crack front $^{14}$ *in the width direction* is taken to be

$$K = \sigma_t(2d)^{\frac{1}{2}}$$  \hspace{1cm} (5.1)

Because $K$ is only affected by this local stress disturbance at the crack front, it is independent of the micro-crack size, $2a_w$. This means that once a micro-crack spans the transverse ply thickness, the stress intensity factor $K$, for growth in the width direction, has reached a

$^{14}$ The stress intensity at the tip of a *through thickness* crack, length $2a$, in an infinite plate under a remote stress $\sigma$, is given by $K = \sigma(\pi a)^{\frac{1}{2}}$.  

88
constant, maximum value. If at this point the crack is still stable, it will remain so for the entirety of its growth across the transverse ply width, W. This corresponds to the case of the \((0,90)_s^n\) laminates, with thin transverse plies \((2d = 0.38 \text{ mm})\), which exhibit stable crack growth across the entire transverse ply width, W (Section 5.1.3).

On the other hand, the \((0,90_4)_s^n\) laminates have thick transverse plies \((2d = 1.44 \text{ mm})\). Therefore, a micro-crack in the transverse ply (Figure 5.7) exceeds the critical size\(^{15}\) for fast fracture \(2a_{d_{\text{crit}}}\), before spanning the transverse ply thickness, 2d. This demonstrates the origin of the brittle cracking behaviour observed in the \((0,90_4)_s^n\) laminates (Section 5.1.1). In this case, the stress intensity factor at the crack front *in the thickness direction* (B in Figure 5.7) is taken to be

\[
K = \sigma_c (na_d)\frac{1}{2}
\]

Equations (5.1) and (5.2) may be rewritten to describe the critical condition for crack growth in thin and thick transverse plies respectively

\[
2dE_{90^\circ}\text{ply} \varepsilon_{tc}^2 \geq G_{IC}
\]

\[
na_{d_{\text{crit}}}E_{90^\circ}\text{ply} \varepsilon_{tc}^2 \geq G_{IC}
\]

\(^{15}\) The stress intensity factor at B is higher than that at A in Figure 5.7. This means that the stress state at B governs the critical condition for failure of the transverse ply.
where $G_{IC} = K_{IC}^2/E_{90^\circ \text{ply}}$ for crack growth along the fibres, see Ogin and Smith (1987). The increase in transverse cracking strain $\varepsilon_{tc}$, in $(0,90)_n$ and $(0,90_4)_n$ hybrid matrix laminates with increased levels of urethane in the matrix (Figure 5.5) may now be described in equations (5.3) and (5.4) respectively by increased values of $G_{IC}$ as well as reduced values of $E_{90^\circ \text{ply}}$.

Experimental values of the transverse cracking strain $\varepsilon_{tc}$ for the $(0,90)_n$ laminates have been substituted into equation (5.3) to calculate values of the transverse ply toughness, $G_{IC}$ (Table 5.1). The above values are compared to experimental values of the toughness $G_{IC}^{\text{init}}$ and $G_{IC}^{\text{plat}}$, measured by Messenger (1990) from DCB coupons, for urethane levels in the matrix of 0 - 20%. These two experimental values originate from a changing value of toughness owing to fibre bridging behind the crack front in the DCB test (Section 4.6). All three values for transverse ply toughness ($G_{IC}$, $G_{IC}^{\text{init}}$ and $G_{IC}^{\text{plat}}$) indicate that it increases as the proportion of urethane in the matrix increases, see Table 5.1.

The theoretical values of transverse ply toughness $G_{IC}$ are close to the experimental values $G_{IC}^{\text{init}}$, for an increasing level of urethane in the matrix. This is consistent with the mechanism of cracking in the transverse ply of these laminates. Transverse cracks have a small crack opening owing to the constraining effect of the longitudinal plies, and thus the effect of fibre bridging behind the crack front is small. Similarly, at the initiation of cracking in DCB coupons, there is no fibre bridging. Therefore with the absence of fibre bridging, $G_{IC}$ and $G_{IC}^{\text{init}}$ values are close. However, for a
urethane levels of 0 and 20% in the matrix, there is some discrepancy between the theoretical ($G_{IC}$) and experimental ($G_{IC^{{in}}}$) value. The reason for this is not clear.

The second set of experimental values for toughness $G_{IC^{{plat}}}$ are considerably higher than the theoretical $G_{IC}$ values. With the opening of the two ligaments behind the crack front in the DCB coupon, fibre bridging between the ligaments has the effect of increasing the measured toughness of the coupon. This means that values of toughness measured from the majority of crack growth in the DCB tests ($G_{IC^{{plat}}}$ values) are probably not directly applicable to transverse cracking in these laminates.

The increase in the experimental values of transverse ply toughness $G_{IC^{{in}}}$, with an increase in the level of urethane in the matrix account, to some extent, for the observed increase in transverse cracking strain in the $(0,90)_s^n$ laminates. It is thought that this is because the toughness of the matrix increases with the urethane content, thus impeding growth of the transverse crack along the fibres.

It is also possible that the addition of urethane to the matrix strengthens the interfacial bond between the matrix and the fibres. However, this is more likely to influence first failure of the transverse ply in laminates where cracking is mechanism controlled (brittle). Another contributory factor is that for higher urethane levels in the matrix, the modulus of the transverse ply $E_{90^{{ply}}}$ is lower (Table 5.1). In this case, less energy is stored at a prescribed level of strain. This means that a higher strain must be applied to the
laminate before the transverse ply fails.

There are a number of factors which contribute to an increase in the transverse cracking strain in $(0,90_4)_s^n$ laminates with higher levels of urethane in the matrix (Table 5.2). At the point of failure of the transverse ply, the strain energy (per unit area of the micro-crack) stored around the tip of the micro-crack (B in Figure 5.7) must equal the toughness of the transverse ply $G_{IC}$, see equation (5.4). The process of failure of a transverse ply with a high urethane content in a $(0,90_4)_s^n$ laminate is governed by the following factors:

(i) A lower transverse ply modulus, $E_{99^r,ply}$ (Table 5.2) means that a higher strain must be applied to store an energy comparable to that in an unmodified transverse ply. This has the effect of increasing the transverse cracking strain.

(ii) One probable reason for the observed increase in $G_{IC}$ of the transverse ply (Table 5.2) is that the toughness of the matrix itself is higher. This means that debonded areas and micro-cracks in the transverse ply link (propagate) through the matrix at a higher strain (absorbing more energy). This is supported in some measure by the fractographs of transverse crack surfaces in $(0,90_4)_s^n$ laminates with urethane contents of 0 - 20% (Figure 5.27). The fractographs of higher magnification (Figure 5.27 a,d,g,j) indicate that the fracture surface of the matrix appears more irregular, and by deduction the matrix more ductile, at higher urethane contents.

The plastic deformation involved in the failure of the more ductile matrices means that more energy is expended
during transverse cracking, and hence the transverse ply is
tougher. Fractographs of lower magnification of the same
fields as above (Figure 5.27 c,f,i,l) do not on the whole show
any observable difference in the appearance of the fracture
surfaces with varying urethane content. It is thought that
the fracture surfaces in Figures 5.27 g-l would exhibit more
features typical of ductile failure in the matrix if the fibre
volume fraction of these laminates (Vf^n = 0.67) were lower.

(iii) It is also possible that addition of urethane to
the matrix strengthens the interfacial bond between the matrix
and the fibres. This is another way in which G_{IC} of the
transverse ply could be increased. Thus, a higher strain
energy, and hence strain ε_{deb}, would be needed to debond the
fibres from the matrix. However, on the fracture surfaces of
the transverse cracks in Figure 5.27 a,d,g,j the matrix does
not appear to have adhered to the fibres after failure to any
greater extent at higher urethane contents. This would
suggest that the strength/toughness of the fibre/matrix
interface has not been affected by the addition of urethane to
the matrix.

(iv) A last possibility is that for higher urethane
contents in the matrix, the critical size of a micro-crack for
failure of the transverse ply (Figure 5.7) is larger.
Intuitively, the micro-crack will reach this increased
critical size at a higher strain, i.e. the transverse cracking
strain, ε_{tc}. Bailey and Parvizi (1981) found that debonded
areas in the transverse ply began to coalesce at some strain
ε_{deb}, thought to be intrinsic to the fibre/matrix interface.
The result was micro-cracks of length 2 to 3 fibre diameters. They reported further coalescence of debonded areas and micro-cracks at higher applied strains.

The critical size of a micro-crack, $2a_d^{\text{crit}}$ may be calculated for the first failure in the transverse ply of the $(0,90_4)_s^n$ laminates, using (5.4). Experimental values of $\varepsilon_{tc}$ for the $(0,90_4)_s^n$ laminates (Table 5.2) and the calculated values of $G_{IC}$ (Table 5.1) are used for this purpose. For the entire range of urethane additions to the matrix, $2a_d^{\text{crit}}$ is less than the the transverse ply thickness ($2d = 1.44$ mm). This accounts for the brittle transverse cracking observed in the $(0,90_4)_s^n$ laminates over the entire range of urethane additions to the matrix (Section 5.1.1).

The critical micro-crack sizes ($2a_d^{\text{crit}}$ in Table 5.2) for failure of the transverse ply can be considered to increase from about 35 to 45 fibre diameters for the range of urethane contents in the matrix. This approximate result and the findings of Bailey and Parvizi (1981) lend some support to the idea that in a tougher transverse ply a micro-crack will reach its increased critical size at a higher applied strain. It is thought that this is because the micro-crack must grow longer, following the tortuous path around the fibres.

$(0,90)_s^0$ and $(0,90_4)_s^0$ laminates

Transverse cracking strains for these laminates have been estimated from the knee in their stress/strain curves, (Figures 3.11, 5.3 and 5.4). As before, the transverse
cracking strain is the same in the \((0,90)_{s}^{e}\) hybrid and uniform matrix laminates and it increases with an increasing level of urethane in the matrix (Figure 5.6). Furthermore, the transverse cracking strains in the \((0,90)_{s}^{e}\) laminates are higher than those in the \((0,90_{2})_{s}^{e}\) laminates, as would be expected.

In the case of the \((0,90_{2})_{s}^{e}\) laminates, there is a transition from brittle to constrained cracking behaviour between 5 and 10% urethane levels in the matrix (Section 5.1.2). The transverse cracking strains in these laminates have correspondingly increased from around 0.3% to around 0.4%. The transition from brittle to constrained cracking occurs when the critical size of a micro-crack for fast failure of the transverse ply \(2a_{d}^{crit}\), becomes greater than the transverse ply thickness, \(2d\). This clearly illustrates the effect of an increased transverse ply toughness \(G_{IC}\), at higher urethane levels in the matrix.
5.3 Progressive transverse cracking and associated reduction of modulus in cross-ply laminates

5.3.1 Results

Progressive transverse cracking was observed in cross-ply laminates under extension (Figure 5.8, pg. 109), in keeping with the findings of Garrett and Bailey (1977b). The number of transverse cracks per unit gauge length (the crack density) for an increasing applied strain were determined by optical methods. Also, the reduction in modulus of the laminates as a result of transverse cracking was measured for increasing applied strain, following Highsmith and Reifsnider (1982).

The relationship between the reduction in modulus of the laminates and the crack density in the transverse ply could then be determined from these measurements. In this third part of Chapter 5, the effect of urethane additions to the matrix on the relationships between:

(i) transverse crack density and applied strain
   - Figures 5.10 to 5.17

(ii) modulus reduction and applied strain
   - Figures 5.18 to 5.22

(iii) modulus reduction and transverse crack density
   - Figures 5.23 to 5.26

are assessed for cross-ply hybrid and uniform matrix laminates.
5.3.2 Discussion

The increase in transverse crack density with increasing applied strain in cross-ply hybrid and uniform matrix laminates is illustrated in Figures 5.10 - 5.17 for urethane levels of 0 - 20% in the matrix. At low applied strains, there is a sharp increase in the number of transverse cracks in these laminates. However, their occurrence diminishes, so that the density eventually reaches a limiting value at around 1% applied strain, in keeping with the findings of Garrett and Bailey (1977b).

The effect of adding urethane to the matrix in these laminates is to reduce the density of transverse cracks, owing to the increased transverse ply toughness, $G_{IC}$ and the reduced transverse ply modulus, $E_{90}^{\text{ply}}$. This means that at a prescribed level of applied strain, there are fewer transverse cracks in laminates with urethane added to the transverse ply.

Some of the results for the $(0,90)_{s}$ laminates do not follow this trend (Figures 5.12 and 5.13). It is thought that this is due to errors in the measurement of transverse crack densities (crack counting), owing to diffuse cracking patterns and the poor translucency of these laminates. The effect of increased urethane levels in the matrix on transverse cracking is similar in hybrid and uniform matrix laminates, see for example Figure 5.17. This indicates that hybrid matrix laminates exhibit comparable resistance to the development of transverse cracking, with their uniform matrix counterparts.

Studies on cross-ply laminates by, for example, Garrett
and Bailey (1977b), Parvizi and Bailey (1978), Bader et al. (1979) and Bailey et al. (1979) have shown that the minimum value of transverse crack spacing (inverse of the crack density) at high applied strains is of the order of, but not less than, the transverse ply thickness, 2d. For the \((0,90_4)^n\) hybrid matrix laminates, the minimum transverse crack spacings are about 1.3, 1.6, 1.9 and 1.9 mm for urethane levels in the matrix of 0, 5, 10 and 20% respectively. These are of the order of the transverse ply thickness, 2d = 1.44 mm.

It appears that the minimum transverse crack spacing in the laminate with a 0% urethane addition (an epoxy matrix) is less than the transverse ply thickness, 2d = 1.44 mm. This unlikely result is explained by branching of transverse cracks in this laminate (Figure 5.9 a, pg. 108) which has the effect of increasing the crack count from the front view (Figure 5.9 b, pg. 108). This was also observed to a lesser extent in the laminate with a 5% urethane addition.

For the \((0,90_2)^6\) hybrid matrix laminates, the minimum transverse crack spacing is on average 0.77 mm, which is also of the order of the transverse ply thickness in these laminates (2d = 0.74 mm). Furthermore, for the \((0,90)^6\) hybrid and uniform matrix laminates the transverse crack spacing reached a minimum value of on average 0.46 mm at an applied stress of about 450 MPa (strains were not measured in this case). Again, this value is of the order of the transverse ply thickness (2d = 0.38 mm) in these laminates, in keeping with previous work.

As the density of transverse cracks increases with
increasing applied strain, there is an accompanying reduction of modulus in these laminates (Figures 5.18 - 5.26). With increased levels of urethane in the matrix of these laminates, the reduction in modulus begins at progressively higher strains and is smaller. This is because the moduli of the laminates are proportional to the density of cracks in the transverse ply, of which there are fewer at increased urethane levels. The proportionality between the transverse crack density and the reduction in modulus of the laminates is illustrated in Figure 5.23 - 5.26.

As the density of cracks in the transverse ply increases, the reduction in modulus of these laminates is more or less the same for the entire range of urethane additions to the matrix. This is not surprising, since the occurrence of transverse cracks is on the whole governed by the transverse ply toughness $G_{ic}$, which is affected by the urethane content. However, the effect of transverse cracks on residual stiffness is governed by the elastic properties of the laminates which are not as significantly affected by the urethane content. This is analysed in detail in Chapter 6.

5.3.3 Conclusion

In the first part of this chapter, it was shown that the transverse cracking strain $\varepsilon_{tc}$, is increased in cross-ply laminates with urethane additions to the matrix. Likewise, higher strains are required for the progression of transverse cracking in cross-ply laminates with urethane additions to the
matrix. This also means that the reduction in modulus due to transverse cracking is smaller in laminates which have had urethane added to their matrix.

Whether transverse cracking is brittle (for example in $(0,90_4)_4$ laminates) or constrained (for example in $(0,90)_4$ laminates), it is suppressed by the addition of urethane to the matrix. This confirms earlier work by Garrett and Bailey (1977a) on the uniform addition of a plasticizer to the matrix of glass/polyester cross-ply laminates.

This concept has now been extended to hybrid matrix laminates, where addition of a modifier (in this work, a urethane elastomer) to the matrix of only the $90^\circ$ plies is sufficient for the limitation of transverse cracking. In the following chapter, constrained transverse cracking will be modelled with an energy based theory and brittle transverse cracking with a strength based theory.
<table>
<thead>
<tr>
<th>Urethane level (%)</th>
<th>$E_{90^\circ \text{ply}}$ (GPa)</th>
<th>$\varepsilon_{tc}$ (%)</th>
<th>$G_{IC}^{\text{ad}}$ (J/m$^2$)</th>
<th>$G_{IC}^{\text{init}}$ (J/m$^2$)</th>
<th>$G_{IC}^{\text{plat}}$ (J/m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>12.8</td>
<td>0.56</td>
<td>153</td>
<td>220</td>
<td>420</td>
</tr>
<tr>
<td>5</td>
<td>12.6</td>
<td>0.60</td>
<td>172</td>
<td>180</td>
<td>525</td>
</tr>
<tr>
<td>10</td>
<td>12.2</td>
<td>0.63</td>
<td>184</td>
<td>240</td>
<td>560</td>
</tr>
<tr>
<td>20</td>
<td>9.40</td>
<td>0.79</td>
<td>223</td>
<td>420</td>
<td>600</td>
</tr>
</tbody>
</table>

- These are $E_{90^\circ \text{ply}}$ values for $(0,90)_s^n$ laminates with $V_f = 0.67$.
- Calculated from equation (5.3).
- Experimental values from DCB tests.

Table 5.1 Experimental and calculated values of transverse ply toughness for $(0,90)_s^n$ laminates.

<table>
<thead>
<tr>
<th>Urethane level (%)</th>
<th>$E_{90^\circ \text{ply}}$ (GPa)</th>
<th>$\varepsilon_{tc}$ (%)</th>
<th>$G_{IC}^*$ (J/m$^2$)</th>
<th>$2a_{d}^{\text{crit}}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>17.2</td>
<td>0.31</td>
<td>153</td>
<td>0.59</td>
</tr>
<tr>
<td>5</td>
<td>16.5</td>
<td>0.35</td>
<td>172</td>
<td>0.54</td>
</tr>
<tr>
<td>10</td>
<td>16.0</td>
<td>0.35</td>
<td>184</td>
<td>0.60</td>
</tr>
<tr>
<td>20</td>
<td>14.8</td>
<td>0.39</td>
<td>223</td>
<td>0.63</td>
</tr>
</tbody>
</table>

- These are $E_{90^\circ \text{ply}}$ values for $(0,90)_s^n$ laminates with $V_f = 0.67$.
- Calculated values of $G_{IC}$ from Table 5.1

Table 5.2 Calculated values of the critical flaw size $2a_{d}^{\text{crit}}$, in $(0,90)_s^n$ laminates.
The increase in transverse crack density in the outer 90° plies of (90,0₄)ₙ coupons with applied strain

Figure 5.1
Figure 5.2 Measured stress and acoustic emission as a function of strain applied to (0,90)_{s}^{n} hybrid matrix laminates.
Figure 5.3 Measured stress and acoustic emission as a function of strain applied to \((0,90_2)^s\) hybrid matrix laminates.
Figure 5.3 cont.

(e) 20% urethane (uniform matrix)

(a) epoxy matrix

(b) 5% urethane

(c) 10% urethane

Figure 5.4 Measured stress and acoustic emission as a function of strain applied to (0,90)_s hybrid matrix (b - d) and uniform matrix (e - g).
Figure 5.5 Transverse cracking strains in net resin cross-ply laminates for an increasing urethane content

Figure 5.6 Transverse cracking strains in excess resin cross-ply laminates for an increasing urethane content
Figure 5.7  A schematic of a micro-crack in the transverse ply of a cross-ply laminate

Figure 5.9  Branching of a transverse crack in a \((0,90_4)_s^n\) hybrid matrix laminate
Figure 5.8 Transverse cracks in $(0,90_4)_S^n$ hybrid matrix laminates, with urethane contents of 0 - 20%
Figure 5.10  Transverse crack density as a function of applied strain in \( (0,90_2)_s \) hybrid matrix laminates

Figure 5.11  Transverse crack density as a function of applied strain in \( (0,90_4)_s \) hybrid matrix laminates
Figure 5.12 Transverse crack density as a function of applied strain in $(0,90)_s$ hybrid matrix laminates

Figure 5.13 Transverse crack density as a function of applied strain in $(0,90)_s$ uniform matrix laminates
Figure 5.14  Transverse crack density as a function of applied strain in \((0,90)_{s}^{n}\) hybrid and uniform matrix laminates.

Figure 5.15  Transverse crack density in \((0,90)_{s}^{n}\) hybrid matrix laminates for increasing applied stress.
Figure 5.16 Transverse crack density in \((0,90)_n\) uniform matrix laminates for increasing applied stress

Figure 5.17 Transverse crack density in \((0,90)_n\) hybrid and uniform matrix laminates for increasing applied stress
Figure 5.18 Modulus reduction of $(0,90)_n$ hybrid matrix laminates for increasing applied strain

Figure 5.19 Modulus reduction of $(0,90)_n$ hybrid matrix laminates for increasing applied strain
Figure 5.20 Modulus reduction of \((0,90)\) hybrid matrix laminates for increasing applied strain

Figure 5.21 Modulus reduction of \((0,90)\) uniform matrix laminates for increasing applied strain
Figure 5.22  Modulus reduction of $(0,90)_s^o$ hybrid and uniform matrix laminates for increasing applied strain

Figure 5.23  Modulus reduction of $(0,90)_s^o$ hybrid matrix laminates with increasing transverse crack density
Figure 5.24 Modulus reduction of $(0, 90)_s^e$ hybrid matrix laminates with increasing transverse crack density.

Figure 5.25 Modulus reduction of $(0, 90)_s^e$ hybrid matrix laminates with increasing transverse crack density.
Figure 5.26 Modulus reduction of $(0, 90)_s$ uniform matrix laminates with increasing transverse crack density.
Figure 5.27  Fractographs of transverse crack surfaces in \((0,90_4)_s^n\) hybrid matrix laminates with urethane contents of:
0\% (a - c); 5\% (d - f); 10\% (g - i); 20\% (j - l)
6. THE PREDICTION OF PROGRESSIVE TRANSVERSE CRACKING AND MODULUS REDUCTION IN CROSS-PLY LAMINATES

It has been shown in the previous chapter that the transverse ply toughness (urethane content) and the transverse ply thickness are two important factors which govern transverse ply cracking in cross-ply laminates. For a constant transverse ply toughness, transverse cracking is constrained in thin transverse plies and brittle in thick transverse plies. Similarly, for a constant transverse ply thickness, transverse cracking is constrained in transverse plies with a high toughness and brittle in transverse plies with a low toughness.

It was also found that the extent of transverse ply cracking is the same in hybrid matrix and uniform matrix cross-ply laminates, with the same transverse ply thickness and transverse ply toughness, which is to be expected. In this chapter, the initiation and progression of constrained transverse cracking and brittle transverse cracking in hybrid matrix cross-ply laminates are described by predictive models.

The problem is approached from the point of view of thin transverse plies (constrained cracking) and thick transverse plies (brittle cracking), with a range of transverse ply toughnesses (urethane contents). The flow chart below illustrates the process of failure of thin and thick transverse plies for some value of transverse ply toughness. The properties governing failure are shown in parentheses and strain applied to the laminate increases in the direction of
the arrows:

```
fibres debond from matrix

debonds join to form micro-crack

(fibre packing)

(interfacial strength)

(strength/toughness of matrix)

(fibre packing)
```

In summary, if the application of strain is ceased, a transverse crack spanning a thin transverse ply will stop growing across its width. This means that the growth of such a crack is governed by an energy balance involving the transverse ply modulus, fibre.matrix interfacial strength, toughness of the matrix and the transverse ply thickness. An energy based model (Section 6.3), which incorporates some statistical element for the above properties, is used to describe this.

On the other hand, a transverse crack in a thick
transverse ply reaches a critical stage (a combination of the crack size and applied strain) whereupon it propagates to span the transverse ply *instantaneously*. The stability of such a crack is governed by the transverse ply modulus, fibre/matrix interfacial strength, toughness of the matrix as well as the fibre packing, but is not influenced by the transverse ply thickness. This is described by a model based on a *statistical transverse ply strength*, which accounts for variability in the above laminate properties (Section 6.4).

Both the energy and strength based models include a shear lag stress analysis of transverse cracking. This is described in the following section.

6.1 The shear lag stress analysis

As outlined in the literature review, a number of workers (see for example Garrett and Bailey (1977b), Steif (1984) and Laws and Dvorak (1988)) have developed shear lag analyses as a means of describing, in a simple way, the stress distribution in a cracked cross-ply laminate.

At the plane of a transverse crack, the longitudinal ply supports the load applied to the laminate. With distance $y$ from a transverse crack, load is transferred from the longitudinal ply back into the transverse ply by the interfacial shear stress (Figure 6.1). The shear stress $\tau$, is proportional to the shear strain $\gamma = dv/dz$, in the transverse ply.
\[
\tau = G_{90^\circ \text{ply}} \frac{dv}{dz} \quad (6.1)
\]

where the constant of proportionality, \(G_{90^\circ \text{ply}}\) is the shear modulus of the transverse ply in the \(y\)-\(z\) plane and \(dv\) is an incremental displacement in the \(y\)-direction (Figure 6.1).

The displacement profile across the transverse ply thickness \(v(z)\), is not known directly. Following Steif (1984), it is reasonable to assume that, in the same manner as the crack opening displacement (Figure 6.1), \(v(z)\) is parabolic, as a result of increased constraint near the longitudinal plies. A force balance on an element in the transverse ply (Figure 6.1) gives the relationship between the shear stress \(\tau\), and the transverse ply stress, \(\sigma_t\)

\[
\tau = d \frac{d\sigma_t}{dy} \quad (6.2)
\]

where \(dy\) is an incremental distance in the direction of loading the laminate.

Equations (6.1) and (6.2) may be solved for \(\tau\) and \(\sigma_t\) with the inclusion of the residual thermal stress in the transverse ply \(\sigma_t^R\), and the boundary condition that \(\sigma_t = 0\) at \(y = \pm s\)

\[
\sigma_t = (\sigma_t^R + \sigma_t \frac{E_{90^\circ \text{ply}}}{E_0})(1 - \frac{\cosh \lambda y}{\cosh \lambda s}) \quad (6.3)^{16}
\]

---

\(^{16}\) In this work, the in-situ modulus of the transverse ply \(E_{90^\circ \text{ply}}\) is used, as opposed to the modulus of a transverse laminate \(E_t\), see Section 4.3.
\[ \tau = -d\lambda(\sigma_t^R + \frac{\sigma_a E_{90^\circ \text{ply}}}{E_0}) \frac{\sinh \lambda y}{\cosh \lambda s} \]  
(6.4)

The term \( \lambda \) is a function of laminate moduli and dimensions

\[ \lambda^2 = \frac{\alpha G_{90^\circ \text{ply}}(b + d)E_0}{d^2 b E_a E_{90^\circ \text{ply}}} \]  
(6.5)

In the above equation, \( \alpha \) is a constant relating to the displacement profile \( v(z) \) across the transverse ply thickness. According to the analysis of Garrett and Bailey (1977b) who assume a linear displacement profile, \( \alpha \) has a value of 1 while according to Steif (1984) who assumes a parabolic displacement profile, \( \alpha \) has a value of 3. In the present study, the constant \( \alpha \) is determined by comparing theoretical predictions and experimental data for the reduction in modulus of the laminates (Section 6.2).

Equations (6.3) and (6.4) describe the variation of stress with distance \( y \), between transverse cracks a distance \( 2s \) apart (Figure 6.2). At crack planes, \( \sigma_t \) is zero and \( \tau \) has its maximum value. Away from crack planes, \( \sigma_t \) tends to the value it would have in an uncracked laminate \( (\sigma_a E_{90^\circ \text{ply}}/E_0) \) and \( \tau \) tends to zero. Physically, this is reasonable since the rate of stress transfer between plies with distance \( y \), \( (d\sigma_t/dy \) in equation (6.2)) is high near crack planes but approaches zero far away from them.

The shear lag analysis accounts for the redistribution of stress around transverse cracks in the direction of loading of the laminate (Figure 6.2), i.e. it is one dimensional. It
might seem that this is a limitation of the analysis which may restrict its applicability. In general however, transverse cracks span the transverse ply thickness at an early stage and then grow across the width.

Ogin and Smith (1985, 1987) have suggested that in this case, the stress intensity at the crack tip is independent of the length of the crack in the width-direction. This indicates that consideration of stress gradients in the width-direction are, on the whole, not necessary in cross-ply laminates. Through-the-thickness stresses are also neglected in this analysis. This is reasonable since they cause ply delamination only near failure of the laminate (Bader et al. (1979)).

The shear lag equations for the redistribution of stress in a cross-ply laminate owing to transverse cracking may be used to predict the reduction in modulus of the laminate (Section 6.2) and, in conjunction with an energy balance (Section 6.3) or a strength based criteria (Section 6.4), the applied stress for further cracking.

6.2 Prediction of the reduction in modulus of cross-ply laminates caused by transverse cracking

Highsmith and Reifsnider (1982) and Steif (1984) have calculated the reduction in modulus of a cross-ply laminate as a result of transverse cracking using a shear lag analysis. This was achieved by finding the average increase in strain \( \varepsilon = \sigma / E \) between transverse cracks a distance 2s apart (\( \sigma \) is
calculated from equation (6.3) and a force balance on the laminate). The reduced modulus $E_r$ was found to be

$$\frac{E}{E_0} = (1 + \frac{dE_{90^\circ \text{ply}} \tanh \lambda s}{bE_g} \lambda s)^{-1}$$ (6.6)

Predictions of the reduction in modulus using equation (6.6) are presented in Figures 6.3, 6.4 and 6.5 for $(0,90)_{s}^e$, $(0,90_2)_{s}^a$ and $(0,90_4)_{s}^n$ laminates respectively. The laminate moduli required for these predictions are listed in Tables 6.1 and 6.2, for excess resin laminates ($V_f^e = 0.72$) and net resin laminates ($V_f^n = 0.67$) respectively. The constant $\alpha$ in equation (6.5) was taken to have a value of 3. This corresponds to a parabolic distribution of displacement across the transverse ply thickness, see Steif (1983).

Only the experimental data for the minimum (0%) and maximum (20%) urethane contents in the matrix is shown. Data for the 5 and 10% urethane contents was omitted for clarity because it lies between the above extremes. The agreement between equation (6.6) with $\alpha = 3$ ($v(z)$ parabolic) and experiment is reasonable for all of the above laminates. In comparison, the prediction of equation (6.6) with $\alpha = 1$ ($v(z)$ linear) does not compare well with the experimental data in Figures 6.3, 6.4 and 6.5.

It is concluded that equation (6.6) predicts the reduction in modulus of the hybrid matrix laminates in this work, for the range of urethane contents used in their matrices. Furthermore, a value of $\alpha = 3$ and therefore a...
parabolic displacement profile in the transverse ply, is applicable to all the laminates used in this study. This enables $\lambda$ to be determined explicitly for further theoretical work (Sections 6.3 and 6.4). It is thought that this is preferrable to the determination of $\lambda$ from the first ply failure stress for each laminate, see for example Laws and Dvorak (1988).

6.3 Prediction of progressive constrained transverse cracking in cross-ply laminates

Laws and Dvorak (1988) have formulated a theory based on the energy required for transverse cracking and a probability density function for the location of successive transverse cracks. They determined the stresses and displacements around transverse cracks in a cross-ply laminate from a shear lag analysis. These were used to calculate the work done by loading the laminate $\Delta W$, and the change in stored strain energy in the laminate, $\Delta E$ associated with the formation of a transverse crack at any specified location (Figure 6.6). The energy balance

$$\Delta W > \Delta E + \gamma \quad (6.7)$$

was used to calculate the energy absorbed during transverse cracking, $\gamma$. They could then calculate the energy release rate per unit length $G$, for the growth of a transverse crack spanning the transverse ply thickness, $2d$. 

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Using the shear lag model of the present work, for a crack growing across the ply width $W$, in a laminate with transverse crack spacing $2s$, the expression for $G$ becomes

\[
G = \frac{(b + d)E_0}{\lambda_b E_E E_{90}^{\text{ply}}} \left( \sigma_t^2 + \frac{E_{90}^{\text{ply}}}{E_0} \sigma_a^2 \right) \left\{ \tanh \frac{\lambda s_1}{2} + \tanh \frac{\lambda s_2}{2} - \tanh \lambda s \right\}
\]

...(6.9)

The distances $s_1$ and $s_2$ define the location of a new transverse crack between existing cracks (Figure 6.6). As the location of the new crack is unknown, owing to the material variability of the transverse ply, there is a probability associated with each possible location for the new crack. Based on simple fracture mechanics arguments, Laws and Dvorak (1988) suggest that this probability $p(y)$, may be taken as proportional to the transverse ply stress, $\sigma_t$ (equation (6.3)) at any point between the existing cracks

\[
p(y) \propto \sigma_t(y)
\]

Equation (6.9) may be solved for the applied stress $\sigma_a(s)$, required for a new crack to propagate in the cross-ply laminate with existing crack spacing, $2s$. Now $G$ in equation (6.9) becomes the transverse ply toughness, $G_{IC}$. Then, the expected stress for a new transverse crack is
\[ \sigma_e(s) = \int_0^{2s} p(y) \sigma_a(y) \, dy \] \hspace{1cm} (6.11)

The constant of proportionality in equation (6.10) may be found by setting

\[ \int_0^{2s} p(y) \, dy = 1 \] \hspace{1cm} (6.12)

Then, the expected stress \( \sigma_e \), for a new transverse crack becomes

\[
\sigma_e = \sqrt{ \frac{G_{IC} b E_\perp E_0}{(b + d)E_{90^\circ \perp ply}}} \left( \frac{\cosh \lambda(y - s)}{\cosh \lambda s} - \frac{\cosh \lambda s}{\cosh \lambda(y - s)} \right)^{\frac{1}{2}} \left( \frac{1}{2} \tanh \frac{\lambda y}{2} - \frac{\lambda}{2} \tanh \frac{2s - y}{2} - \tanh \lambda s \right) \frac{\alpha_b E_0}{E_{90^\circ \perp ply}} \, dy 
\]

\[ \frac{dy}{2s} \] \hspace{1cm} \ldots (6.13)

The integral in equation (6.13) must be solved numerically. A copy of the program used for this purpose is presented in Appendix 1 to this chapter. It is noted that in this theory a transverse crack is assumed to span the transverse ply at an early stage and then to propagate stably in the width direction. This is the case for \textit{constrained} transverse cracking, and therefore transverse cracking in \((0,90)_s^n\) hybrid matrix laminates is considered here.

Data for these laminates presented in Chapter 5, has been reproduced in Figures 6.7 to 6.10 for urethane contents in the matrix of 0 - 20%. The predictions of equation (6.13) with calculated values of \( G_{IC} \) from Table 5.1 are also shown in these figures. The laminate moduli required for these predictions are listed in Table 6.1 for \textit{net} resin laminates (\( V_f^n = 0.67 \)).
The agreement between theory and experiment is good for all levels of urethane. A summary of the predictions, Figure 6.11, shows how the theory predicts fewer transverse cracks in laminates with more urethane in the matrix. However, at higher levels of applied stress the theory predicts that the transverse ply will continue to crack whilst the experimental data indicates that a maximum transverse crack density is reached (Figures 6.7 - 6.10). It is thought that this could be due to yielding or failure of the interface between the longitudinal and transverse plies, see for example Peters (1987).

Such failure is likely to occur near transverse cracks where the shear stress at the interface is a maximum (Figure 6.2). Although the strength of the interface is not known, this is a reasonable explanation for the saturation of transverse cracking. In support of this, Pryce (1991) has found a saturation transverse crack spacing consistently less than the transverse ply thickness in cross-ply silicon carbide fibre/glass matrix laminates. In this material the interfacial strength is intuitively higher than glass/epoxy. In contrast, the transverse crack spacing was observed to reach a minimum value of about the transverse ply thickness for the current glass/epoxy laminates.

This energy based theory (equation (6.13)) has also been used to predict the progression of constrained transverse cracking in \((0,90_2)_s^e\) hybrid matrix laminates (Figures 6.12 and 6.13). This is emphasized, since in these laminates transverse cracking is constrained for urethane contents of 10
and 20% in the matrix but brittle at lower urethane contents. The laminate moduli required for these predictions are listed in Table 6.2, for excess resin laminates ($V_f^e = 0.72$). Although there is reasonable agreement, in both cases the theory underestimates the transverse crack density at a prescribed level of applied stress$^{17}$.

This is not surprising since the transverse ply toughness $G_{IC}$, in equation (6.13) was taken to be the same as that for the net resin laminates (Table 5.2). The $(0,90_2)^e_s$ laminates are excess resin laminates which are thought to be of inferior quality to the net resin laminates. Therefore it is anticipated that $G_{IC}$ for the $(0,90_2)^e_s$ laminates is lower and consequently they will exhibit more transverse cracks than the above theory predicts.

It is concluded that this energy based theory is able to predict the progression of constrained transverse cracking in cross-ply hybrid matrix laminates with a range of transverse ply toughnesses reasonably well. As yet however, it does not account for yielding or failure of the interface between the longitudinal and transverse plies which, it is suggested, limits the transverse crack density at higher stresses.

$^{17}$ Data from Figure 5.11 has been replotted as transverse crack density versus applied stress.
6.4 A statistical model for the prediction of brittle transverse cracking in cross-ply laminates

6.4.1 The statistical model

Following Peters (1984), a transverse ply of length L may be considered to consist of N elements of length $\ell_0 = L/N$, each of which will have broken when the saturation transverse crack density is reached. The strengths of these elements are assumed to be independent of each other. Peters (1984) has developed an approximate technique to record the fracture strains of these elements in the transverse ply. This is based on the instantaneous measurement of the drop in load applied to the coupon for each transverse crack. In this work, a similar technique based on the acoustic emission from transverse cracks has been used for this purpose.

On the acoustic traces obtained for the $(0,90_4)_s^n$ laminates in the present work, each peak corresponds to a transverse crack (see Figure 5.2). The applied strains for successive transverse cracks could therefore be determined from these traces. After adding the residual thermal strain in the transverse ply, these strains were taken to be the failure strains $\varepsilon_f$ of the elements, assuming that there is one crack in each element.

However, there is a non-uniform strain (stress) field surrounding each transverse crack (Figure 6.2). This means that for an undistorted measurement of the elemental fracture strains, the number $n$, or corresponding length $\ell_n$, of the
elements must be limited such that say 90% of the strain in
the transverse ply is recovered at their boundaries (Figure
6.14). Then each of the n elements in the transverse ply can
be considered to break at a strain given by the sum of the
nominal applied strain at which it breaks plus the thermal
strain, \( \varepsilon_f \sim \varepsilon + \varepsilon_t^R \).

If further strain is applied to the laminate, a maximum
number of transverse cracks \( N \), is eventually reached. The
latter \( N - n \) cracks occur at unknown strains, other than the
applied strain owing to the non-uniform strain distribution in
the transverse ply. At this stage it is assumed that a
minimum elemental length of \( \varrho_0 = L/N \) has been reached, with one
of the \( N \) transverse cracks in each element. It is further
assumed that the probability of survival of each of the \( N \)
elements may be described by a two-parameter Weibull fracture
strain (or stress) distribution

\[
P_s(V_0) = \exp \left\{ -\left( \frac{\varepsilon_f}{\varepsilon_0} \right)^m \right\} = \exp \left\{ -\left( \frac{\sigma_f}{\sigma_0} \right)^m \right\} \tag{6.14}
\]

where \( V_0 \) is a reference volume, taken as \( V_0 = 2dW\varrho_0 \), the volume
of an element, \( \varepsilon_f \) (or \( \sigma_f \)) the fracture strain (or stress) of an
element and \( \varepsilon_0 \) (or \( \sigma_0 \)) and \( m \) the Weibull scale and shape
parameters respectively. The probability of survival of the
\( j^{th} \) element may be calculated from the Weibull ranking equation
Then if the fracture strains follow a Weibull distribution, a plot of $\ln \ln 1/[P_s(V_0)]$ versus $\ln \varepsilon_f$ will yield a straight line with slope $m$.

Equation (6.14) gives the probability of survival of a small elemental volume $V_0$, in the transverse ply. The probability of survival of the whole transverse ply, volume $V = 2dWL$, is then

$$ P_s(V) = (P_s(V_0))^{V/V_0} \quad (6.16) $$

For a crack spacing of $2s$ in the transverse ply the transverse ply stress, $\sigma_t = E_t\varepsilon_f$, in equation (6.14) is not uniform over the volume $V$, see equation (6.3). Integrating over this volume $V$, the probability of survival of the transverse ply becomes

$$ P_s(V) = \exp \left\{ -\frac{1}{V_0} \int_V (\sigma_t)^m \, dV \right\} \quad (6.17) $$

where $\sigma_0 = E_t\varepsilon_0$. Substituting $dV = 2dWdy$, equation (6.17) becomes

$$ P_s(V) = \exp \left\{ -\frac{1}{\ell_0} \int_{-s}^{s} (\frac{\sigma_t}{\sigma_0})^m \, dy \right\} \quad (6.18) $$

where $\sigma_t$ is given by equation (6.3). Substituting for $\sigma_t$ and
solving for the applied stress $\sigma_a$, equation (6.18) becomes

$$\sigma_a = \left( \sigma_0 \left( \frac{-\ln[P_s(V)]}{N} \int_0^L \frac{1 - \cosh \frac{\lambda y}{s}}{1 - \cosh \frac{\lambda s}{s}} dy \right)^{1/m} - \sigma_t \right) \frac{E_0}{E_{90^\circ \text{ply}}}(6.19)$$

which gives the applied stress $\sigma_a$, for a transverse crack spacing $2s$, based on a statistically distributed transverse ply strength, in the same way as Fukunaga et al. (1984).

Combining equations (6.15) and (6.16) noting that $j = L/2s + 1$ and $V/V_0 = N$, the expression for $P_s(V)$ in equation (6.19) becomes

$$P_s(V) = \left( 1 - \frac{L/2s + 1}{N + 1} \right)^N(6.20)$$

which is the probability for successive failure of the transverse ply as a whole. Equations (6.19) and (6.20) therefore predict $\sigma_a$ as a function of $2s$, based on a decreasing probability of survival of the progressively smaller unbroken ligaments in the transverse ply. This is in contrary to the prediction of Fukunaga et al. (1984) who suggest that during the progression of transverse cracking, remaining unbroken ligaments have a constant probability of survival of 50%.

6.4.2 The distribution of failure strains in the transverse ply

The statistical model developed in the previous section
is applicable to cross-ply laminates in which a distribution of the transverse ply strength can be considered to describe failure of the transverse ply, i.e. laminates in which brittle transverse cracking occurs. In this study, \((0,90_4)_s^n\) hybrid matrix laminates with urethane contents of 0 - 20% in the matrix all exhibit brittle failure in the transverse ply. A restricted number \(n\), of fracture strains for transverse cracks were measured in these laminates, from their acoustic traces (Figure 5.2).

The minimum elemental length \(\ell_n\), or crack spacing \(2s\), allowable for an undistorted fracture strain distribution (Figure 6.14) was calculated from equation (6.3) by setting \(y = 0\) and \(\sigma_t\) to 90% of the undisturbed stress in the transverse ply. This effectively means that within a broken element, \(\sigma_t\) can be considered to be zero and outside of the element boundaries, \(\sigma_t\) tends to the value it would have in an uncracked laminate.

The minimum elemental length, \(\ell_n\) was slightly less than 3 mm for all the urethane contents in the \((0,90_4)_s^n\) laminates. Dividing this distance into the gauge length of the test coupons (90 mm), gave a conservative estimate of \(n = 30\) elements for an undistorted fracture strain distribution.

A maximum number of about 50 transverse cracks have been observed in the \((0,90_4)_s^n\) laminates. This indicates that there are a total of about \(N = 50\) elements in the transverse ply of length \(\ell_0 = 1.8\) mm which will fail. A plot of \(\ln n / [P_s(V_0)]\) versus \(\ln \varepsilon_t\) for the first 30 elements then gives a Weibull distribution of their fracture strains, excluding (to a first
approximation) the distorting effect of stress non-uniformity in the transverse ply.

These distributions of fracture strain for the elements in the transverse ply are presented in Figure 6.15 a - d for (0,904)_s^n laminates with urethane contents of 0 - 20%. One interpretation of the distributions is that for high fracture strains, or high applied strain, the data approximate to a straight line but exhibit an increasing degree of bimodality at low strains as the urethane content increases, see Figure 6.15 e.

Another interpretation is that because these distributions are not linear, it may not be possible to describe transverse cracking by Weibull statistics. However, it is thought that the findings presented below from the fracture strain distributions are valid whether or not transverse cracking is a Weibull process.

Peters and Andersen (1989), working on CFRP cross-ply laminates, suggested that failures of the first few elements in the transverse ply are dominated by the matrix whereas failures of elements at higher strain are dominated by the fibre/matrix interface. This is reasonable since first failures are thought to result from pre-existing micro-cracks (associated with processing defects for example) whereas subsequent failures stem from failure of the fibre/matrix interface, see Bailey and Parvizi (1981).

Peters and Andersen showed that for an increasing matrix

---

18 A bi-modal Weibull distribution exhibits two straight line regions of different slope.
fracture strain in their CFRP laminates, the slope of the Weibull distribution increased at low strains. The same is true for the current laminates, see Figure 6.15, where the Weibull shape parameter (slope) has increased from $m = 7.4$ to $m = 14.7$ as the urethane content in the matrix increases. This would suggest that the matrix fracture strain has increased. The observed increase in the measured transverse cracking strain, $\varepsilon_{tc}$ (Table 5.2) as well as the more ductile appearance of the matrix fracture surface (Figure 5.27 a,d,g,j) at higher urethane contents support this supposition.

Peters and Andersen showed that for an increasing interfacial strength, the slope of the Weibull distribution decreases at high strains. This was based on straight lines fitted to the lower strain portion of the distributions because in some cases the distribution of element fracture strains deviated from a straight line at higher strains, see Figure 6.15 f. They attributed this to the failure of the interface between the longitudinal and transverse plies surrounding the transverse cracks. This, they postulated, inhibited stress transfer between the plies, with the effect of delaying further transverse cracks (elemental failures) to higher stresses (strains).

In this work, it is suggested that the slopes of the distributions at higher strains are not insignificant. It is thought, in agreement with Peters and Andersen (1989), that higher elemental fracture strains identify transverse cracking governed by the failure of the fibre/matrix interface, i.e.
the debonding process observed by Bailey and Parvizi (1981). However, the change in slope of (or transition in) these fracture strain distributions (Figure 6.15 a - d) occur at an approximate strain of at most 0.45%. This is well below the strain for the saturation of transverse cracking in these laminates (about 1%) which is thought to indicate that the interface between the longitudinal and transverse plies has begun to fail around transverse cracks.

On the basis of this, it is suggested that the bimodality of these fracture strain distributions is not as a result of the failure of the interface between the longitudinal and transverse plies. Instead, the observed bimodality (Figure 6.15 g) is attributed to a change in the initiation of transverse cracking from pre-existing micro-cracks to strain induced fibre/matrix debonds.

The majority of data points (about 20 out of 30) at high strain in the fracture strain distributions (Figure 6.15 a to d) approximate to a straight line of slope $m \approx 3$ for each of the urethane contents, see also Figure 6.15 e. In accordance with the discussion above, it is suggested that the fibre/matrix interfacial strength in the transverse ply has not improved with the addition of urethane.

Following Peters and Andersen (1989), the strongest element in the transverse ply may be considered to be almost free of defects and hence, the failure thereof to be representative of the strength of the fibre/matrix interface. Therefore, an approximate measure of the interfacial strength is to calculate the fracture strain of this element, $\varepsilon_{FN}$.
(N = 50) from the slope of the higher strain portion of the distributions.

This may be achieved with the use of equations (6.14) and (6.15) for which j = N = 50 and values of $\varepsilon_0$ and m which have been taken from the Weibull distributions at higher strain (Figure 6.15). These are listed in Table 6.3 together with the calculated failure strains of the strongest elements, $\varepsilon_{ fing}$. There is little change in $\varepsilon_{ fing}$ as the urethane content increases, indicating that the interfacial strength has not been affected. This result lends support to the observation from the fractographs of the transverse crack faces (Figure 5.27) that there is no evidence of the matrix adhering to the fibres at higher urethane contents.

Finally, a measure of the characteristic failure strain of the transverse ply $\varepsilon_v$, may be calculated from the Weibull volume scaling equation

$$\varepsilon_v = \varepsilon_0 \left( \frac{V_0}{V} \right)^{1/m} \quad (6.21)$$

with $\varepsilon_0$ and m from the slopes of the distributions at low strain (Table 6.3). This measure of the characteristic strain of the transverse ply exhibits an increase as the urethane content increases, see $\varepsilon_v$ in Table 6.3. These values are not dissimilar from the transverse cracking strains for these laminates (Table 5.2).
6.4.3 The prediction of brittle transverse cracking in 
\((0,90_4)_s^n\) hybrid matrix laminates

In order to predict the applied stress for progressive transverse cracking up to saturation, the non-uniformity of strain in the transverse ply must be taken into account (Figure 6.14). Equations (6.16) to (6.20) are based on a volume integral of stress in the transverse ply for this purpose. However, the integral in equation (6.19) must be evaluated numerically.

Predictions of the applied stress/crack spacing \((\sigma_a/2s)\) curve have been performed with the aid of a computer program, see Appendix 2 to this chapter. These curves are presented in Figure 6.16 a - d for urethane contents of 0 - 20\%, along with the data reproduced from Figure 5.10 (plotted here as \(2s/\sigma_a\)) for the \((0,90_4)_s^n\) laminates. The laminate moduli required for these predictions are listed in Table 6.1 for net resin laminates \((V_f^n = 0.67)\).

In these predictions, the Weibull shape \((m)\) and scale \((\varepsilon_0 = \sigma_0/E_{90\_p}\_l)\) parameters in equation (6.19) were taken from the low strain portion of the fracture strain distributions until the occurrence of about 10 transverse cracks and thereafter (lower crack spacings), from the high strain portion of the fracture strain distributions, see Table 6.3. In addition to this, these predictions have been calculated with a decreasing probability of survival of unbroken ligaments in the transverse ply as the transverse crack spacing decreases, see equation (6.20).
The predictions in Figure 6.16 a - d follow a similar but distant trend from the experimental data and are not smooth in some cases (Figure 6.16 b and d), owing to the change in Weibull parameters after low strain cracking. Noting that there is some room for error in the estimation of the Weibull parameters at low strain (Figure 6.15 a - d), as well as the point at which the high strain Weibull parameters may be adopted, a second set of predictions has been produced (Figure 6.16 e - h).

In these predictions, the existing Weibull parameters were altered so that the theoretical prediction is closer to the experimental data. It is argued that the difference between the original (Table 6.3) and the adjusted (Table 6.4) Weibull parameters is small and may be accounted for by the variability observed at low strains in the fracture strain distributions (Figure 6.15). The transition from low strain to high strain Weibull parameters was also increased from 10 to 30 transverse cracks.

In addition to the above modifications, it is thought, following Manders et al. (1983), that transverse cracks have little or no effect on each other at high transverse crack spacings (low applied strains). This means that transverse cracking is, on the whole, a statistical process at high crack spacings and therefore the integral for stress non-uniformity in equation (6.19) may be assigned a value of 1.

The stress integral was included after the occurrence of 30 transverse cracks (a crack spacing of 3 mm). This was based on the calculation (Section 6.4.2) of the approximate
minimum transverse crack spacing at which the stress
distributions surrounding adjacent transverse cracks do not
interact. As before, the probability of survival for unbroken
ligaments is given by equation (6.20). The resulting
predictions (Figure 6.16 e - h) show reasonable agreement with
the experimental data but underestimate the minimum transverse
 crack spacing at high stresses. The reason for this is not
clear.

In summary, the assumptions employed in the modified
theory of progressive transverse cracking based on a
statistical distribution of transverse ply strength lead to
the following conclusions:

(i) The progression of transverse cracking in these
laminates may be partially described with the use of a bi-
modal Weibull distribution for the transverse ply strength.
Manders et al. (1983) also observed a bi-modal transverse
fracture strain distribution in glass/epoxy cross-ply
laminates. The above authors however, used only the Weibull
parameters from the first part of the fracture strain
distribution for their model of progressive transverse
cracking.

(ii) In a further departure from the work of Manders
et al. (1983) and Fukunaga et al. (1984), the current theory
assumes that the probability of survival of remaining unbroken
ligaments in the transverse ply decreases with progressive
transverse cracking. This, it is thought, must be so if the
Weibull ranking function (equation (6.15) is used to describe
the distribution of failure strains in the transverse ply.
(iii) Finally it is thought, in agreement with Manders et al. (1983), that the non-uniform stress distributions surrounding adjacent transverse cracks may be ignored at low applied strains but shield the majority of unbroken areas in the transverse ply at high applied strains. This means that in general, at low strains transverse cracking is a statistical process governed by the material properties of the transverse ply, whereas at high strains transverse cracking is governed by the stress distributions in the transverse ply.
<table>
<thead>
<tr>
<th>Moduli in GPa</th>
<th>$V_f^o = 0.72$</th>
<th>Urethane content (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>$E_g$</td>
<td>$\rightarrow 51.4 \rightarrow$</td>
<td></td>
</tr>
<tr>
<td>$E_{90^o , \text{ply}}$</td>
<td>18.1</td>
<td>16.8</td>
</tr>
<tr>
<td>$(0,90_2)^e_s$</td>
<td>15.1</td>
<td>14.8</td>
</tr>
<tr>
<td>$(0,90)^e_s$</td>
<td>29.2</td>
<td>28.3</td>
</tr>
<tr>
<td>$(0,90_2)^e_s$</td>
<td>33.3</td>
<td>33.1</td>
</tr>
<tr>
<td>$(0,90)^e_s$</td>
<td>5.37</td>
<td>5.92</td>
</tr>
<tr>
<td>$(0,90_2)^e_s$</td>
<td>5.32</td>
<td>5.21</td>
</tr>
</tbody>
</table>

$+ G_{90^o \, \text{ply}} = E_{90^o \, \text{ply}}/[2(1 + \nu_{90^o \, \text{ply}})]$ and $\nu_{90^o \, \text{ply}}$

is taken to be 0.42, following Hashin (1988).

**Table 6.1** Moduli for laminates prepared by the excess resin method
<table>
<thead>
<tr>
<th>Moduli in GPa</th>
<th>( V_f^n = 0.67 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Urethane content (%)</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>( E_g )</td>
<td>[\rightarrow 47.8 \rightarrow]</td>
</tr>
<tr>
<td>( E_{90^\circ \text{ply}} )</td>
<td>( (0,90^4)_s^n )</td>
</tr>
<tr>
<td></td>
<td>( (0,90)_s^n )</td>
</tr>
<tr>
<td>( E_0 )</td>
<td>( (0,90^4)_s^n )</td>
</tr>
<tr>
<td></td>
<td>( (0,90)_s^n )</td>
</tr>
<tr>
<td>( ^*G_{90^\circ \text{ply}} )</td>
<td>( (0,90^4)_s^n )</td>
</tr>
<tr>
<td></td>
<td>( (0,90)_s^n )</td>
</tr>
</tbody>
</table>

\( + G_{90^\circ \text{ply}} = \frac{E_{90^\circ \text{ply}}}{2(1 + v_{90^\circ \text{ply}})} \) and \( v_{90^\circ \text{ply}} \)

is taken to be 0.42, following Hashin (1988).

**Table 6.2** Moduli for laminates prepared by the net resin method
Table 6.3 Characteristic ($\varepsilon_0$) and interfacial ($\varepsilon_{IN}$) failure strains for elements in the transverse ply, plus the characteristic failure strain ($\varepsilon_V$) for the whole transverse ply.
Table 6.4 Modified characteristic failure strains (ε₀) for elements in the transverse ply and slope of their fracture strain distribution (m) for low and high strains.
Figure 6.1 A schematic showing a force balance on an element near a crack in the transverse ply

Figure 6.2 A schematic of the variation in the transverse ply stress and interfacial shear stress between two transverse cracks in a cross-ply laminate
Figure 6.3 Experimental and theoretical reduction in modulus of \((0,90)_{a}^{o}\) hybrid matrix laminates with increasing crack density

Figure 6.4 Experimental and theoretical reduction in modulus of \((0,90)_{a}^{o}\) hybrid matrix laminates with increasing crack density
Figure 6.5 Experimental and theoretical reduction in modulus of \((0,90)_s^n\) hybrid matrix laminates with increasing crack density.

Figure 6.6 Location of a new transverse crack between existing cracks a distance \(2s\) apart.
Figure 6.7 Experimental and theoretical increase in transverse crack density with applied stress in (0,90)$_s$ hybrid matrix laminates
- 0% urethane content

Figure 6.8 Experimental and theoretical increase in transverse crack density with applied stress in (0,90)$_s$ hybrid matrix laminates
- 5% urethane content
Figure 6.9 Experimental and theoretical increase in transverse crack density with applied stress in (0,90)\textsuperscript{n} hybrid matrix laminates
- 10% urethane content

Figure 6.10 Experimental and theoretical increase in transverse crack density with applied stress in (0,90)\textsuperscript{n} hybrid matrix laminates
- 20% urethane content
Figure 6.11  Theoretical predictions of transverse crack density for increasing applied strain in $(0,90)_n$ hybrid matrix laminates

Figure 6.12  Experimental and theoretical increase in transverse crack density with applied stress in $(0,90)_n$ hybrid matrix laminates
- 10% urethane content
Figure 6.13 Experimental and theoretical increase in transverse crack density with applied stress in \((0,90_2)^s\) hybrid matrix laminates - 20% urethane content

Figure 6.14 A schematic of \(n\) elements in the transverse ply, approximately each with a uniform stress distribution
Figure 6.15 a - d  Weibull fracture strain distributions for elements in the transverse ply of \((0,90_4)_n\) hybrid matrix laminates
(a) 0% urethane content

(b) 5% urethane content
(c) 10% urethane content

(d) 20% urethane content
A schematic of the typical transverse ply failure strain distribution found by Peters and Andersen (1989) in CFRP cross-ply laminates.
A schematic of the typical transverse ply failure strain distribution found by the author in $(0,90_4)_n$ hybrid matrix laminates
Figure 6.16  Experimental (+) and theoretical (----) transverse crack spacing with applied stress in $(0,90_4)_s^n$ hybrid matrix laminates
(a - d) Weibull parameters from Table 6.3
(e - h) Modified theory with Weibull parameters from Table 6.4

(a) 0% urethane

(e) 0% urethane
(b) 5% urethane

(f) 5% urethane
(c) 10% urethane

(g) 10% urethane
(d) 20% urethane

(h) 20% urethane
APPENDIX 1: PROGRAM FOR THE CALCULATION OF APPLIED STRESS FOR DECREASING TRANSVERSE CRACK SPACING IN CROSS-PLY LAMINATES WITH CONSTRAINED TRANSVERSE CRACKING

40 REM **INPUT LAMINATE PROPERTIES**
60 INPUT "TRANSVERSE PLY SEMI-THICKNESS (D), MM "; D
70 INPUT "LONGITUDINAL PLY THICKNESS (B), MM "; B
75 PRINT
80 INPUT "LONGITUDINAL MODULUS OF A 0 PLY, GPa "; E1
90 INPUT "LONGITUDINAL MODULUS OF A 90 PLY, GPa "; E2
100 INPUT "LONGITUDINAL SHEAR MODULUS OF A 90 PLY, GPa "; GL
110 EO=(B*E1+D*E2)/(B+D)
120 PRINT
130 PRINT "RULE-OF-MIXTURES MODULUS="; EO
140 PRINT
150 LL=SQR((3*GL*(B+D)*EO)/(B*D*D*E1*E2))
160 PRINT "VALUE OF LAMBDA (PARABOLIC SHEAR-LAG)="; LL
170 PRINT
180 INPUT "LAMINATE STRESS AT FIRST PLY FAILURE (FPF), MPa "; SF
190 INPUT "RESIDUAL STRESS IN TRANSVERSE PLY (PRIOR TO LOADING), MPa "; ST
195 PRINT
196 DIM C(100)
200 INPUT "CURRENT VALUE OF CRACK SPACING (2s), MM "; SS
210 S=SS/2
220 REM **NUMERICAL INTEGRATION ROUTINE**
240 TT=S/99
250 Y=0
260 FOR I=1 TO 100
270 IF I>1 GOTO 300
280 C(I)=0
290 GOTO 425
300 X=(LL*S)/10
310 AX=CH:BX=TH
320 X=(LL*(Y-S))/10:GOSUB 520
330 CX=CH
340 PR=1-(CX/AX)/(1-(BX/(LL*S)))
350 X=(LL*Y/2):GOSUB 520
360 DX=TH
370 X=(LL*((2*S)-Y))/2:GOSUB 520
380 EX=TH
390 BK=1/(SQR(DX*EX-BX))
400 T1=(SF+((EO*ST)/E2))*BK
410 T2=(T1-((EO*ST)/E2))*PR
420 C(I)=T2/TT
425 REM **SENT HERE FIRST TIME ROUND LOOP, FROM LINE 290**
427 PRINT I;" ";Y;" ";C(I)
428 Y=Y+TT
430 NEXT
435 REM **ADD UP ALL CONTRIBUTIONS TO THE INTEGRAL ,MULTIPLYING BY dy**
440 SUM=0
450 FOR I=1 TO 100
460 SUM=SUM+(TT*C(I))
470 NEXT
480 SUM=SUM-((C(1)+C(100))*TT*.5)
490 PRINT
500 PRINT"EXPECTED STRESS TO CAUSE NEXT CRACKING=";SUM
505 GOTO 200
510 END
520 REM **SUBROUTINE TO CALCULATE SINH (SH), COSH (CH) AND TANH (TH)**
530 CH=0.5*((EXP(X))+EXP(-1*X))
540 SH=0.5*((EXP(X))-EXP(-1*X))
550 TH=SH/CH
560 RETURN
570 END
APPENDIX 2: PROGRAM FOR THE CALCULATION OF APPLIED STRESS FOR DECREASING TRANSVERSE CRACK SPACING IN CROSS-PLY LAMINATES WITH BRITTLE TRANSVERSE CRACKING

60 DIM C(100)
100 INPUT "TRANSVERSE PLY SEMI-THICKNESS (D), MM "; D
110 INPUT "LONGITUDINAL PLY THICKNESS (B), MM "; B
120 INPUT "LONGITUDINAL MODULUS OF A 0 PLY, GPa "; E1
130 INPUT "LONGITUDINAL MODULUS OF A 90 PLY, GPa "; E2
140 INPUT "LONGITUDINAL SHEAR MODULUS OF A 90 PLY, GPa "; G
150 INPUT "COUPON LENGTH (L), MM "; L
160 INPUT "NUMBER OF CRACKS (N) "; N
170 INPUT "WEIBULL MODULUS (m) "; M
180 INPUT "SECOND WEIBULL MODULUS "; M2
190 INPUT "WEIBULL REFERENCE STRENGTH (SIGMA NOUGHT) (MPa) "; S0
200 INPUT "SECOND REFERENCE STRENGTH "; S02
210 INPUT "CURRENT VALUE OF CRACK SPACING (2s), MM "; SS
220 INPUT "NUMBER OF INCREMENTS"; JK
230 INC= (SS/JK)
240 SS=SS+INC
250 DIM ST(JK); DIM S(JK)
260 FOR G=1 TO JK
270 W=G+(LO*JK)
280 S(W)=SS-INC
290 NUM=L/S(W)
300 IF NUM>30 THEN LET M=M2; LET S0=S02
310 IF NUM<30 THEN GOTO 625
320 TT=S/99
330 X=(LL*S); GOSUB 730
340 AX=CH
350 Y=0
360 FOR I=1 TO 100
370 X=(LL*Y); GOSUB 730
380 BX=CH
390 A=1-(BX/AX)
400 AA=(AA^M)
410 C(I)=(1/S) *AA
420 Y=Y+TT
430 NEXT I
440 SUM=0
450 FOR I=1 TO 100
460 SUM=SUM+(TT*(C(I)))
470 SUM=SUM-((C(1)+C(100))*TT*0.5)
480 MZ=(1/M)
490 ZZ=(2*SUM)
500 ZZ=ZZ^MZ
510 LET ANY=S(W)
520 Z3=(N+1)/(N+1-(L/ANY))
530 IF ANY<1.84 THEN LET Z3=51
540 Z1=(LN(Z3)^MZ
550 IF NUM<30 THEN LET Z2=1
560 ST(W)=(Z1*E0*S0)/(E2*Z2)
570 ST(W)=ST(W)-(TH*EO)/E2
580 SS=S(W)
590 PRINT ST(W), S(W), W
600 NEXT G
610 CH=0.5*((EXP(X))+(EXP(-1*X)))
620 SH=0.5*((EXP(X))-(EXP(-1*X)))
630 THAN=SH/CH
640 RETURN
650 END
The extension of a notched cross-ply coupon (in this case, a circular centre-notch) gives rise to three types of damage:

(i) transverse cracking over most of the coupon area, except for those areas shielded by the notch

(ii) longitudinal splitting from the notch

(iii) delamination of the longitudinal and transverse plies in the region of the longitudinal splits and eventual tensile failure at the notch, see Figure 7.1.

Work by Kortshot and Beaumont (1989 a,b,c) has shown that the strength of a double edge notched (DEN) cross-ply CFRP laminate is directly related to the terminal length of the splits from the notch. Using a finite element analysis (FEA), they calculated the stress distribution around the notch for increasing split length from the notch. From this they found that as the split length increases, the stress concentration factor $K_c$, at the notch decreases. This crack blunting effect means that the stronger specimens show longer splits prior to failure.

Based on the above authors' work, it was thought that a hybrid matrix laminate, having an unmodified matrix in the longitudinal plies, might allow more longitudinal splitting and therefore have a higher notched strength than a modified
uniform matrix laminate.

7.1 Observations of damage growth

In their work on the extension of DEN CFRP cross-ply coupons, Kortshot and Beaumont (1989a) found that the delamination extended to the tip of the split and towards the centre of the coupon as the split grew. Similar damage patterns have been observed around a circular centre-notch in both $(0,90)^n$ uniform and hybrid matrix laminates under extension, see Figure 7.1.

The above authors found that a delamination must accompany split growth to allow relative motion of the $0^\circ$ ply across the split, since the energy required to generate the split is released by this motion. A longitudinal split initiates in the opening mode (mode I) but grows under a combination of opening and shearing mode (mode II), whereas a delamination propagates under a fully three-dimensional mixed mode, opening, shearing and tearing mode (mode III).

7.2 The effect of urethane additions on longitudinal splitting and notched strength

Typical damage pattern development in a $(0,90)^n$ laminate under increasing tensile load is shown in Figure 7.1. The average length of the four splits $l$, from the notch of
diameter 2a, (Figure 7.1) is presented in Figures 7.2 and 7.3 with increasing stress applied to \((0,90)\) hybrid and uniform matrix laminates respectively. The data indicate that the split growth rate \(\frac{d(l)}{d\sigma}\) in these laminates is constant whether or not urethane is added to their matrices uniformly or only to the transverse ply.

Noting that delamination growth must accompany split growth, Kortshot and Beaumont (1989b) suggested that the splitting energy is much less than the delamination energy. In the light of this, it appears that the addition of urethane to these laminates, whether uniform or hybrid matrix has had little or no effect on their resistance to delamination or on their toughnesses \(G_{IIA}\) and \(G_{III}\).

In the mode I DCB tests though (results reported in Chapter 4), it was established that \(G_{IC}\) increases as the proportion of urethane in the matrix is increased. It is thought that the effect of an increased \(G_{IC}\) on split growth is masked by the dominance of \(G_{IIA}\) and \(G_{III}\) for the accompanying delamination. However, the effect of an increased \(G_{IC}\) is evident for split initiation in the longitudinal plies of the uniform matrix laminates.

If the curves in Figure 7.3 are extrapolated\(^{18}\) to a split length of zero, the applied stresses for the initiation of splitting are: 90, 90, 100 and 125 MPa for 0, 5, 10 and 20%.

\(^{18}\) This was necessary because damage was recorded at fixed load intervals (Figure 7.1) from which the initiation of splitting could not be recorded accurately.
urethane in the matrix. Moreover, the initiation stress was 90 MPa for all urethane contents in the hybrid matrix laminates which is as would be expected, providing further evidence that mixing of resin in adjacent plies is negligible.

The notched strengths of the \((0,90)_n\) uniform matrix laminates are slightly higher than those of the hybrid matrix laminates (Figure 7.4). This is contrary to the earlier hypothesis that hybrid matrix laminates might experience more longitudinal splitting at a prescribed level of load and therefore have higher notched strengths. Furthermore, the terminal split length in the uniform matrix laminates are longer (Figure 7.5), confirming that stronger coupons have longer splits.

However, it is no easier (comparing the slopes in Figures 7.2 and 7.3) for a split to grow in a uniform matrix laminate and thus effect more crack blunting. It is concluded therefore that the longer terminal splits in the uniform matrix laminates are as a result of longitudinal plies being stronger and not the reverse.

7.3 The effect of notch size on notched strength

At a level of 5% urethane added to the matrix, the uniform matrix laminate appears to be about 20% stronger than its hybrid matrix counterpart for a 5 mm diameter notch (Figure 7.4). It was thought that it would be useful to
examine this apparent difference in notched strength for two additional notch sizes $2a$, of 2 mm and 10 mm as well (the coupon width $W = 20$ mm was kept constant).

It appears that, as for the 5 mm diameter notch, the growth rate of longitudinal splits from the 2 mm or 10 mm notches are the same whether the laminate has hybrid or uniform matrices (Figures 7.6 and 7.7). By normalising the split length $\ell$, with respect to the notch size $a$ (Figures 7.8 and 7.9), it becomes clear that the notch size alone accounts for the difference in split growth rate. This means that in this case, the size of the notch in relation to the width of the coupon $2a/W$, does not appear to influence the stress state around the notch significantly.

This is in interesting contrast to the work of Kortshot and Beaumont (1989c) who found that $\ell/a$ changes if $2a/W$ changes in DEN CFRP coupons. It is clear that at some notch size greater than 10 mm, the specimen width ($W = 20$ mm) will affect the stress state at the notch. This effect is usually approximated by a finite width correction factor.

There is no conclusive evidence to suggest that at a 5% level of urethane, notched uniform matrix laminates are significantly stronger than hybrid matrix laminates, see Figure 7.10. It appears that the greatest difference is for the 5 mm notch, there is a smaller difference for the 2 mm notch and little or no difference for the 10 mm notch. It might be that the hybrid matrix laminate from which the
coupons containing the 5 mm notch were cut, has a slightly lower strength (Figure 7.4), perhaps due to a fault in processing.

The near-linear relationship between failure stress and normalised terminal split length, $e/a$ (Figure 7.10) suggests that the failure stress is governed by the split length just before failure, in relation to the notch size. The ratio of notch size to specimen width $2a/W$, does not appear to have influenced the notched failure stress for the notch sizes and specimen width used in this study.

7.4 Conclusion

It is concluded from this study of matrix damage around a circular centre-notch in uniform and hybrid matrix cross-ply laminates that

(i) the stress for the initiation of longitudinal splitting from the notch increases with an increasing urethane content in uniform matrix laminates, but not in hybrid matrix laminates.

(ii) the addition of urethane to the matrix, whether uniformly or only to the transverse ply, has little effect on the resistance to delamination and therefore, following Kortshot and Beaumont (1989b), on longitudinal split growth around the notch.

(iii) the notched strength of the uniform matrix
laminates is slightly higher than that of the hybrid matrix laminates, as a result of stronger longitudinal plies.

(iv) the notched strength appears to depend only on the ratio of terminal split length to notch size ($l/a$) and not the ratio of the notch size to the specimen width ($2a/w$).
Figure 7.1 Damage growth around a circular centre-notch (diameter 5mm) in a uniform matrix laminate with a urethane content of 5% in the matrix.
**Figure 7.2** Longitudinal split growth rate with applied stress in $(0,90)_n$ hybrid matrix laminates with a notch size of 5 mm.

**Figure 7.3** Longitudinal split growth rate with applied stress in $(0,90)_n$ uniform matrix laminates with a notch size of 5 mm.
Figure 7.4 Failure stress of circular centre-notched $(0,90)_n$ hybrid and uniform matrix laminates for urethane contents of 0 - 20% and notch size 5 mm

Figure 7.5 Failure stress plotted against terminal length of longitudinal splits for circular centre-notched $(0,90)_n$ hybrid and uniform matrix laminates with urethane contents of 0 - 20% and notch size 5 mm
Figure 7.6 Longitudinal split growth rate with applied stress in (0,90)_n hybrid and uniform matrix laminates with a notch of size of 2 mm and a 5% urethane content.

Figure 7.7 Longitudinal split growth rate with applied stress in (0,90)_n hybrid and uniform matrix laminates with a notch of size of 10 mm and a 5% urethane content.
Figure 7.8 Normalised split growth rate with applied stress in circular centre-notched $(0,90)_s^r$ hybrid matrix laminates with a 5% urethane content

Figure 7.9 Normalised split growth rate with applied stress in circular centre-notched $(0,90)_s^r$ uniform matrix laminates with a 5% urethane content
Figure 7.10  Failure stress as a function of normalised terminal split length in circular centre-notched (0,90)ₜ hybrid and uniform matrix laminates with a 5% urethane content
8. DISCUSSION: DAMAGE RESISTANCE AND ULTIMATE PROPERTIES OF CROSS-PLY HYBRID MATRIX LAMINATES

It was found that transverse cracking in cross-ply hybrid matrix laminates is governed by:

1. the transverse ply thickness
2. the urethane content in the matrix of the transverse plies

Following the studies of Parvizi et al. (1978) and Ogin and Smith (1987), it has been confirmed that (for a similar urethane content) the transverse ply thickness governs the stability of transverse crack propagation in cross-ply laminates. In thin transverse plies, the growth of transverse cracks was found to be stable or constrained, owing to the interaction with the adjacent longitudinal plies. In thick transverse plies, the growth of transverse cracks was found to be unstable or brittle, with instantaneous failure after initiation.

After Garrett and Bailey (1977a) and Chan and Wang (1990) it has been shown that a toughening agent (in this work, a urethane elastomer) improves the resistance to transverse cracking, whether it is added only to the matrix of the transverse ply or uniformly to the matrix of the cross-ply laminate.

This means that the strains for the initiation and development of transverse cracking are higher when urethane is added to the matrix. In \((0,90)_s\) hybrid matrix laminates (with thin transverse plies) the strain for the initiation of
cracking increased by about 40% whereas for \((0,90_4)^n_s\) hybrid matrix laminates (with thick transverse plies), it increased by about 25%, for a 20% urethane addition to the matrix.

It has been shown in Chapters 5 and 6 that the growth of a transverse crack in a thin transverse ply is governed ultimately by an energy balance whereas in a thick transverse ply it is governed by (the statistically distributed) transverse ply strength. Thus, the addition of urethane to the matrix as a toughening agent, whilst having the same effect on the properties of the transverse ply, does not influence the transverse cracking strain in the same way in thin and thick transverse plies.

Fractographs of transverse crack surfaces in \((0,90_4)^n_s\) laminates and a statistical analysis, following Peters and Andersen (1989), of the distribution of failure strains at which each transverse crack occurs have suggested that the addition of urethane to the matrix does not alter the fibre/matrix interfacial strength.

The statistical analysis also suggested that transverse cracking (in thick transverse plies) is a statistical process at low strains (cracks occurring at progressively less severe flaws) but is governed by the stress distribution surrounding adjacent cracks at high strains (cracks form at sites of maximum stress).

Having confirmed an improvement in damage resistance in cross-ply hybrid matrix laminates, it remained to evaluate their mechanical properties. Accordingly, the notched tensile strength and compressive strength of cross-ply hybrid and
Following Kortshot and Beaumont (1989 a,b,c), it became clear from the current investigation that the notched tensile strength of a cross-ply laminate is directly related to the terminal length of longitudinal splits from the notch (in the longitudinal plies). On the basis of this finding, it was thought that cross-ply hybrid matrix laminates, having unmodified longitudinal plies, might develop longer splits from the notch under loading and hence be stronger.

It was found that for both hybrid matrix and uniform matrix laminates, an increasing proportion of urethane added to the matrix did not change the rate of split growth from the notch with applied stress apparently because the dominant energy absorbing process is delamination at the notch root. Accordingly, there was little difference in the notched strength of the cross-ply hybrid matrix and uniform matrix laminates.

Walker (1990), working on a related project, has evaluated the compressive strength of $(0_2, 90_2)_2s$ hybrid matrix and uniform matrix laminates, see Figure 8.1. He found that the addition of up to 20% urethane to the matrix in the transverse plies or throughout the laminates did not reduce the compressive strength.

On increasing the urethane content to 40%, Walker found a significant reduction in compressive strength of both the $(0_2, 90_2)_2s$ hybrid matrix and uniform matrix laminates. It is thought that whether or not the (load bearing) longitudinal plies in these laminates contained urethane, they buckled into
adjacent transverse plies, resulting in no observable difference in compressive strength.
Figure 8.1 The compressive strength of $(0_2, 90_2)_2$ hybrid and uniform matrix laminates for an increasing urethane content in the matrix.
9. CONCLUSIONS

Processing

(i) A laboratory scale drum winder has been designed and built for the production of pre-preg.

(ii) Pre-preg has been made from a basic epoxy resin, a urethane elastomer and a glass fibre roving, using a solvent based winding process.

(iii) $(0,90)_s$, $(0,90^2)_s$ and $(0,90^4)_s$ hybrid matrix laminates have been made from the pre-preg, with glass fibres/epoxy resin in the longitudinal plies and glass fibres/epoxy-urethane resin in the transverse plies. Mechanical tests have shown that adjacent matrices in the longitudinal and transverse plies bonded but did not mix.

(iv) $(0)_4$ and $(0,90)_s$ uniform matrix laminates have been made with glass fibres and epoxy-urethane resin for comparison with hybrid matrix laminates.

Basic mechanical properties of laminates

(v) It was found that the proportion of urethane in the matrix has little effect on the longitudinal tensile modulus $E_l$, of unidirectional laminates, whereas a urethane content of 20% by weight of the matrix reduces the transverse tensile modulus $E_t$, by about 25%.

(iv) The longitudinal tensile modulus $E_0$, of $(0,90_n)_s$ hybrid and uniform matrix laminates alike depends on the
proportion of 90° plies and their urethane content. Owing to
the high fibre volume fraction of the cross-ply laminates
(around 0.7) and the dominance of the stiff fibres in the
longitudinal plies, $E_0$ dropped by only 8% for a reduction of
25% in $E_t$.

(vii) The addition of urethane to the matrix of a
glass/epoxy laminate increases the resistance to crack
propagation along the fibres (the toughness). The Mode I
interlaminar fracture toughness of unidirectional glass/epoxy-
urethane laminates was observed to increase by about 45% for a
20% urethane content in the matrix.

Damage development in cross-ply hybrid and uniform matrix
laminates

(viii) It has been shown that a tougher matrix (in this
work, with a high urethane content) improves the resistance of
a cross-ply laminate to transverse cracking. In $(0,90)_s^6$ hybrid
matrix laminates the strain for the initiation of transverse
cracking increased by about 40% and for $(0,90_4)_s^n$ hybrid matrix
laminates it increased by about 25%, for a 20% urethane
addition to the matrix.

(ix) The addition of urethane to the matrix of the
transverse ply in cross-ply laminates results in fewer
transverse cracks at any prescribed level of strain and
therefore the resulting reduction in modulus of the laminate
is smaller (by about 15% for $(0,90_4)_s^n$ hybrid matrix laminates
and 5% for $(0,90)_s^6$ hybrid matrix laminates).

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(x) An increased resistance to transverse cracking in cross-ply laminates may be achieved by the addition of urethane to the matrix of the transverse ply only.

**Notched tensile strength and compressive strength of cross-ply hybrid and uniform matrix laminates**

(xi) It was found that the tensile strength of a circular centre-notched cross-ply laminate is directly related to the terminal length of longitudinal splits from the notch.

(xii) The addition of urethane to the matrix of only the transverse ply or the entire cross-ply laminate did not alter the rate of split growth from the notch with applied stress. Following from (xi), there was little difference in the notched strength of the cross-ply hybrid matrix and uniform matrix laminates.

(xiii) The addition of up to 20% urethane to the matrix of $(0_2,90_2)_2$ hybrid matrix and uniform matrix laminates did not reduce their compressive strength. An increased urethane content of 40%, however, resulted in a significant reduction (of about 30%) in compressive strength of the uniform matrix laminates. The $(0_2,90_2)_2$ hybrid matrix laminates were no better than the uniform matrix laminates.

(xiv) Cross-ply hybrid matrix laminates, although having comparable damage resistance to uniform matrix laminates with the same urethane content, show no observable improvement in tensile notched strength or unnotched compressive strength.
10. FUTURE WORK

The evaluation of tensile notched strength and compressive strength of cross-ply hybrid matrix laminates has shown that there is no benefit in using hybrid matrices (as opposed to uniform matrices) for these properties. It is thought however, that hybrid matrix laminates might be useful in aggressive environments.

It is suggested that a cross-ply hybrid matrix laminate with a chemically resistant resin in the longitudinal plies and a tough, damage resistant matrix in the transverse plies be tested, in say, hot-wet environments. Such a hybrid matrix laminate may retain resilience to the environment whilst having an improved resistance to transverse cracking and possibly impact damage.

The mathematical theories used to predict the development of constrained and brittle transverse cracking have been partially successful. In the case of constrained transverse cracking, it appears that the energy-based model predicts progressive cracking successfully with the use of an approximation for the variability in the material properties of the transverse ply.

However, failure of the 0°/90° ply interface at high strains inhibits further transverse cracking. This is not accounted for in the current work and is a further modification which will improve predictions of constrained transverse cracking.

In the case of brittle transverse cracking, there is some
doubt as to whether a Weibull fracture strain distribution accounts for all sources of variability in the transverse ply properties. The predictions of progressive brittle transverse cracking show some deviation from experimental observations at high stresses.

The predictive theory used (equation (6.19)) has been modified so that at low strains transverse cracking is entirely statistical (the effect of stress non-uniformity is ignored). At high strains, the stress distribution around transverse cracks has been accounted for. However, this was still based on the statistical model in equation (6.19).

It is suggested that for the prediction of brittle transverse cracking at high strains, the failure of the 0°/90° interface be accounted for and that the statistical element in transverse ply properties be ignored.
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