AN EXPERIMENTAL STUDY OF THE SIGNALS
TO BE EXPECTED OF A LASER RADAR SYSTEM
AND THEIR OPTIMISATION

by

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A thesis submitted for the
Degree of Doctor of Philosophy
at the University of Surrey
ABSTRACT

An experimental study of the effect that the target, transmitter aperture, and receiver apertures have on the performance of a laser doppler radar is presented in this thesis. Empirical design equations are presented and methods of using these equations to predict system performance for a particular target material are described.

Two important parameters are defined and have been measured together with their statistical properties. They are: $\beta$, the 'scattering loss factor' which accounts for the power lost due to scatter at the radar target, and $\gamma$, the 'detector loss factor' which accounts for signal loss at the detector due to inefficient mixing of the scattered light. Knowledge of these two parameters for a given target enables one to determine the signal level at the output of the detector.

$\beta$ is shown to be a function of the surface material and a method of obtaining its value is described. The statistics of $\beta$ are shown to be a function of the speckle diameter/receiver diameter ratio, but appears to contradict theoretical predictions.

It is found that $\gamma$ is a function of the ratio of the speckle size to the receiver size and has been determined for ratios from .01 to 12. The statistics of $\gamma$ are presented and agree with the theoretical description.
Doppler broadening, which limits the sensitivity of a Doppler system, has been measured for rotational and translational velocities. From these results the optimum optical configuration can be predicted.

A general design problem is discussed to show how the parameters described can be used to predict the system performance and to determine the operational parameters.
ACKNOWLEDGEMENTS

My thanks to Professor D.R. Chick, Head of the Department of Electrical and Electronic Engineering, University of Surrey, for his encouragement and support during my term of research.

I am indebted to Mr. Q.V. Davis, my Supervisor, for his help and guidance through my three years of research. Thanks are also due to him for his many useful suggestions and comments during the preparation of this thesis.

Many thanks, too, to Mr. P. Simms for the design, and to Mr. F. Keitch and his staff for the construction, of many of the mechanical components used in the experimentation.

My grateful thanks are also extended to Mrs. E. Phillips for the onerous task of typing this thesis.

This research was supported by a grant from the Royal Society's Paul Instrument Fund.
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<td>Area</td>
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<td>A_T</td>
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<tr>
<td>A_R</td>
<td>Receiver aperture area</td>
</tr>
<tr>
<td>a</td>
<td>Constant</td>
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<td>B</td>
<td>Electronic bandwidth</td>
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<td>b</td>
<td>Ratio of $\phi_R / \phi_S$</td>
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<td>C(\tau)</td>
<td>Correlation function</td>
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<td>D_S</td>
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<td>D_R</td>
<td>Receiver aperture diameter</td>
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<td>D_T</td>
<td>Transmitter aperture diameter</td>
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<td>d_R</td>
<td>Virtual receiver spot diameter</td>
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<td>d_T</td>
<td>Transmitter spot diameter</td>
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<td>E</td>
<td>Electric field strength</td>
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<td>F_R</td>
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N Integer variable

$P_T, P_S, P_{LO}$ Optical Power

$P_{AC}, P_{DC}$ Effective optical power

$P(f)$ Spectrum function

$P(\cdot)$ Probability density function

$p$ constant

$R$ Range

$r$ Radius

$T$ Correlation length

$t$ time

$u$ Dummy aperture variable

$v$ Velocity

$v_x, v_y, v_z, v_{xy}$ Scattering vectors

$x$ Discrete variable

$\beta$ Scattering loss factor

$\gamma$ Detector coherence loss factor

$\gamma_T, \gamma_D, \gamma_P$ Coherence loss factors

$\Delta F$ Minimum measurable frequency

$\zeta$ Loss factor

$n$ Impedance of free space

$\theta$ Angle

$\theta_1$ Incident angle

$\theta_2, \theta_3$ Scattering angles

$\lambda$ Wavelength

$\mu$ Mean value

$\rho$ Scattering coefficient
\( \sigma \)  
Standard deviation, rms roughness of surface profile

\( \sigma_B \)  
Radar cross section

\( \tau \)  
time function

\( \phi(t) \)  
Phase angle (time dependant)

\( \phi \)  
Angle

\( \phi_T \)  
Transmitter angular aperture

\( \phi_R \)  
Receiver angular aperture

\( \phi_S \)  
Speckle angle

\( \omega \)  
Angular frequency, angular velocity.

**ABBREVIATIONS**

S/N  
Signal to noise ratio

eqn.  
Equation

rms  
Root mean square

P.M.T.  
Photo-multiplier tube

C.N.R.  
Carrier signal to noise ratio

S.N.R.  
Discriminator output signal to noise ratio

i.f.  
Intermediate frequency

inf.  
Information

Fig.  
Figure
Lasers, because of their short wavelength and narrow beam width are, in many senses, ideal sources of coherent radiation for the doppler radar field. The short wavelength, giving five orders of magnitude of doppler shift over microwave systems, makes it possible to measure low velocities and the narrow beam width provides excellent spatial resolution. It was therefore inevitable that the laser doppler radar would begin to make an impact in the field of velocity and vibration measurement.

For the early experiments retro-reflectors were fixed to the target to give ample signal; however, it was soon realized that the device should work with unco-operative targets if commercial instruments were to be viable.

Some experiments were performed (26 - 28) to investigate the effect this sort of target had on the signal levels of a doppler radar, but these were by no means conclusive. W.Kulczyk (1) found that there was something like a 6 db loss in his system that could only be explained by introducing a speckle coherence loss. A comprehensive survey of the literature of the time made it clear that useful information for the design of, and the prediction of the performance of, the optical system of a laser doppler radar was lacking.
When work started at the University of Surrey to develop a turbine blade vibration measuring instrument, the sponsors (The Paul Instrument Fund), took the same view. They quoted as one of the objectives of the work: "3) Studies of laser speckle patterns and their effect on Doppler signal (especially with rotating or sideways translating target). Methods of overcoming this problem."

The work described in this thesis was carried out in part to realise these objectives directly and also in part to examine more closely the effects of the target surface, with a view to obtaining design equations for predicting the system performance for a particular target material.

The research described in this thesis is essentially experimental in nature, and represents the results of some quarter-million measurements. The nature of the problem has encompassed the disciplines of electronics, computing, optics, mechanical design, and statistical mathematics, and in a thesis of this length the majority of problems can only be briefly discussed. Wherever possible theoretical verifications of the results are presented; however, due to the complexity of the situation, in some circumstances the results shown are of a purely experimental nature, and some in fact appear to contradict what theoretical predictions are available. The results presented provide sufficient information for a complete analysis of a particular system response, and a brief description of a typical design is given.
A survey of current knowledge on the subject of doppler radar systems, using diffuse targets, is presented in Chapter 2.

2.1. highlights the problem of a specific doppler radar system. The scatter of coherent radiation from rough surfaces is described in sections 2.2. and 2.3, while the detection methods and signal to noise ratio are summarised in section 2.4.

The theoretical aspects of the measurements presented in this thesis are described in Chapter 3, starting with incoherent detection (3.2). Here the first important system parameter \( \beta \) is defined and an expression for its value is derived. In 3.3. the second parameter \( \gamma \) is defined and methods for measuring it are described. Also included are details of a simulation program used to calculate expected values of \( \gamma \). The problems of measuring \( \beta \) and \( \gamma \) with respect to slowly moving targets are outlined in 3.4. The two final sections describe a typical radar problem and the limitations imposed by \( S/N \) ratio and doppler broadening, and the effects of depolarization and coherence losses.

In sections 4.2, 4.3, 4.4, the experimental design of the interferometer, target holder, and electronic circuitry is discussed. A brief description of the calibration procedures and measurements is presented in 4.5, followed by the details of sample preparation and surface roughness measurements in 4.6. This chapter is concluded by a summary of the computational algorithms used during the analysis stage of the study.
The results of the detailed measurements are presented in Chapter 5, beginning with those for incoherent detection and the measurement of $1/\beta$. The results of $1/\gamma\beta$ are presented in 5.3, together with the spectrum results for a rotating and a sideways translating surface. Since eight surfaces, including scotchlit, were studied, a clear knowledge of the performance of scotchlit with respect to other target materials has been obtained. The Chapter is concluded with a discussion of the important results and a general system design optimisation technique is presented. This is followed by the Appendices.

1.3. CONCLUSIONS

The results presented in this thesis provide sufficient experimental data for a scientific design of a laser doppler system to be possible. The parameters of interest have been defined in a useful and meaningful way, and verified experimentally, so that a systematic optimisation can be performed.

The performance of the system ultimately depends on the property of the scattering surface of the target, and it has been found that it is possible so to modify the surface that one can substantially improve the signal to noise ratio.

One extremely interesting and possibly surprising result is that, for backscatter normal incidence, scotchlit does not improve the signal to noise ratio compared to typical metallic surfaces, although at other angles of backscatter the improvement is marked.
When the signal to noise ratio at the detector output is more than 10 db, the dominant factor in limiting system accuracy is the Doppler broadening introduced by undesirable velocity components at the target. Both translational and rotational velocity broadening depend on the system optical configuration and an optimum can be found. Results also show that transverse velocity broadening is independent of the transmitter focusing. A more detailed study of this is needed.

The standard alignment equations, which apply to two uniform fields, were found to be incorrect for a uniform field and a speckle field, and it has been shown that alignment conditions could be relaxed for certain optical configurations.

Some experimental measurements of surface roughness of the targets were made in an attempt to predict, theoretically, the scatter from them. The results, however, were disappointing, and it is suggested that the most convenient method is to measure the scatter using a laser and an incoherent detection method, and use this result to predict the signal for a coherent system.

From an investigation of the general design equations presented in this thesis, it appears that a system with the transmitter and receiver apertures equal is optimum for most situations, but the actual size of the aperture depends on the many other system parameters.
2.0. REVIEW

2.1. LASER DOPPLER INTERFEROMETER

Fig. 1a shows a typical laser Doppler radar system (from (1)). It consists of a beam splitter creating two beams from the single laser beam. One, a reference beam, (the "local oscillator") is reflected at a stationary mirror to the photo detector D, and the other, the signal beam, is transmitted to the target.

2.1.1. Doppler Shift

Upon reaching the moving target the radiation suffers a Doppler shift. If the target moves with a velocity vector \( \mathbf{v} \), the scattered light suffers a shift in frequency \( f_D \) given by (2)

\[
f_D = \mathbf{v} \cdot (\mathbf{k}_i - \mathbf{k}_s) / 2\pi
\]

2.1.1.

where \( \mathbf{k}_i \) and \( \mathbf{k}_s \) are the propagation vectors of the incident and scattered beams respectively, and

\[
|k| = \frac{2\pi}{\lambda}
\]

2.1.2. Doppler Broadening

In radar systems, absolutely parallel beams are non-existent so there is always a finite angular spread of the beam. This spreading of the \( \mathbf{k} \) vectors generates a spectrum \( P(f) \) centred on \( f_D \) composed of the sum of all the Doppler products present in the incident and scattered beams. The accuracy of the determination of \( f_D \) and hence \( \mathbf{v} \) is reduced by this broadening. A general Doppler analysis is not helpful at this stage, but one is presented later.
a) Basic Laser Interferometer using a Polarising Beam Splitter.

b) Scattering Geometry.

FIG. I. INTERFEROMETER AND SCATTERING GEOMETRY.
2.1.3. Coherent Signal Analysis

The Doppler shifted and broadened light from the target is scattered back to the receiving lens L, thence passing through the optics to the photo detector. If the two beams are correctly aligned (2.1.7), the total light intensity $I_D$ is proportional to $(E_1 + E_2)^2$, where $E_1$ and $E_2$ are the local oscillator and signal beam amplitude functions respectively. If $E_1$ and $E_2$ are harmonic, of the same frequency, and have a relative phase difference $\phi(t)$ ($\phi(t)$ is a function of time and accounts for any phase or frequency modulation present in either beam), then writing

$$E_1 = E_{L0} e^{j \omega t} \text{ and } E_2 = E_s e^{j [\omega t + \phi(t)]}$$

and expanding, gives

$$I_D = \frac{1}{2\pi} \left[ E_{L0}^2 e^{j 2\omega t} + E_s^2 e^{j [2\omega t + \phi(t)]} + 2 E_{L0} E_s e^{j [2\omega t + \phi(t)]} \cos \left( \phi(t) \right) \right]$$

where $n$ is the impedance of free space.

If the response time, $\tau$, of the detector is

$$\frac{2\pi}{\omega} < \tau < 2\pi / \frac{d\phi(t)}{dt}$$

then, ignoring all high frequency components,

$$I_D = \frac{E_{L0}^2}{2\pi n} + \frac{E_s^2}{2\pi n} + \frac{E_s E_{L0}}{n} e^{j \phi(t)}$$

In terms of optical power, and assuming 100% mixing efficiency, eqn. 2.1.3 becomes

$$P_D = P_{L0} + P_s + \frac{2 \sqrt{P_{L0} P_s}}{\pi} \cos \left[ \phi(t) \right]$$

for a receiver of area $A_R$ such that $P_D = I_D A_R$. This eqn. is the basic coherent mixing equation used by many authors.

In analysing his system, Kulczyk (1) introduces a "coherent
reflectivity loss factor" $\zeta$, which is used to equate $P_s$ to the power $P_T$ incident on the target. Hence, by definition,

$$P_s = P_T / \zeta.$$  

Then, if the beam-splitting ratio is $m$ to $(1 - m)$ (See Fig. 1a),

$$P_s = P(1 - m) / \zeta$$  

and $P_{LO} = P_m$ where $P$ is the total optical power emitted by the laser. By substituting these quantities in eqtn.2.1.4, Kulczyk shows that there are optimum conditions for the signal to noise ratio (S/N) and beam-splitting ratio of a doppler system, in terms of $\zeta$. He considers three cases.

2.1.4. Photo Multiplier as Detector

From (1)

$$(S/N)_{\text{max}} = \frac{K_p P}{B \zeta}$$  

where $K_p$ is the photo-multiplier constant and $B$ the electronic bandwidth. The value of $m$ for such an optimum is

$$m_{\text{opt}} = \sqrt{\frac{1}{\zeta}}$$  

2.1.5. Pin Photodiode as Detector

Amplifier noise predominates in a pin diode detector system and in this case $m_{\text{opt}}$ is independent of $\zeta$ and if $\zeta$ is large

$$S/N = \frac{K_p P}{\zeta F_n}$$  

where $F_n$ is the noise of the amplifier.

2.1.6. Avalanche Photodiode as Detector

In this case $m_{\text{opt}}$ does depend on $\zeta$ but only marginally, and for values of $\zeta > 10^4$, $m_{\text{opt}}$ is very nearly constant. $S/N$ is again proportional to $1/\zeta$.

In all three cases $S/N$ is inversely proportional to $\zeta$ therefore to obtain a low $\zeta$ is the ultimate aim of any optimisation
of optical system performance provided it is not done at the expense of mixing efficiency.

2.1.7. **Beam Alignment**

In the above calculations it was assumed that there is perfect mixing between $E_1$ and $E_2$. This is only true if the waves are both plane and parallel. In (7) and (8) it is shown that for two slightly misaligned plane waves, at a circular aperture, the mixing ratio is given by

$$\frac{P}{P_{\text{max}}} = J_0(\Delta) + J_2(\Delta)$$

where $J_0$ and $J_2$ are Bessel functions of the first kind and

$$\Delta = \frac{2\pi r \theta}{\lambda}$$

where $\theta$ is the angular misalignment, $r$ the radius of the apertures, and $\lambda$ the wavelength of the radiation.

It must be stressed that these equations apply only to two plane parallel waves, and to date there are no publications providing any indication of the results to be expected when misalignment exists while mixing scattered light with a uniform wave.

A trivial alignment requirement is that the beams overlap, or that they are parallel and can overlap when brought to the same focus. A fuller treatment of the subject can be found in Warden (8) and Siegman (49).
2.2. SCATTERING FROM A ROUGH SURFACE I.

MEAN SCATTERED POWER

2.2.1. Introduction

The study of the scattering of E.M. waves from a random rough surface has been extensive and active since the appearance of radar (10). More recently, the activity increased when the arrival of the laser provided a suitable source of coherent radiation for modelling experiments in the laboratory (11-14). These experiments were concerned with the mean value of the scattered field, and various analytical methods of corroborating the experimental results were proposed.

Beckmann and Spizzichino (9) have tried to provide a concise treatment of the theory to explain the experimental results on radar scattering from the earth and moon. Arguments still reign as recent papers by Barrick (15,50), and Barrick and Peake (16) show. Some of the interesting details of the previous studies are summarised below.

2.2.2. A Random Rough Surface Model

A very complete treatment of the scattering of radiation from surfaces is provided by Beckmann (9).

The derivation of an expression for the solution of the scattered power as a function of angle of incidence, \( \theta_1 \) and angles of scatter, \( \theta_2 \) and \( \theta_3 \) (Fig. 16), is based on the calculation of the Helmholtz integral for the boundary conditions imposed by the surface.

The result has certain limitations. These are :-
a) No shadowing or multiple scattering is accounted for.
b) There should be no small radii on the surface.
c) Area of spot on the surface $A >> \lambda^2$.
d) The surface is a conducting one.

In general, the limitation in (b) above means that for optical illumination the solution only applies to surfaces with a roughness period of many microns. As most surfaces are of a spiky nature, one cannot expect the solution to be very accurate.

Beckmann considers the case of a random rough surface with a roughness function

$$ Z = f(x, y) \text{ with a normal distribution of height,} $$

$$ W(z) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{z^2}{2\sigma^2}\right) $$

and zero mean. Also a correlation function along the surface

$$ c(\tau) = \exp\left(-\frac{\tau^2}{T^2}\right) $$

where $T$ is the correlation length and $\sigma$ the rms roughness.

He shows that the ensemble averaged scattering coefficient $<\rho\rho^*>$ is given by

$$ <\rho\rho^*> = \frac{\pi}{A} \frac{J^2}{v_z^2 \sigma^2} \exp\left(-\frac{v_{xy}^2 T^2}{4 v_z^2 \sigma^2}\right) $$

for $\sigma > \lambda / 10$, where

$$ J = \frac{1 + \cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2 \cos \theta_3}{\cos \theta_1 (\cos \theta_1 + \cos \theta_2)} $$

$$ v_z = 2 \pi (\cos \theta_1 + \cos \theta_2) / \lambda $$
$$ v_{xy} = \sqrt{v_x^2 + v_y^2} $$
$$ v_x = 2 \pi (\sin \theta_1 - \sin \theta_2 \cos \theta_3) / \lambda $$
$$ v_y = 2 \pi \sin \theta_2 \sin \theta_3 / \lambda $$
Clearly, the parameter \( T/\sigma \) plays an important role in determining the scattering coefficient of the particular surface. Eqn. 2.2.3 only applies to a very rough surface satisfying the conditions set out earlier.

2.2.3. A More Realistic Model of Scatter

The result proposed by Beckmann (and many other authors, albeit in a different form), is ineffective in providing the correct results at large angles of incidence for backscatter, although the discrepancy is not so large for scatter at other angles. Barrick & Peake have proposed that the scattering can be more correctly predicted if two roughness scales are considered to be present. One scale consists of long correlation roughness of large r.m.s. value \( T/\sigma \gg 100 \). For such a case, the Beckmann type solution is valid. The second roughness scale is one of small \( \sigma \) and small \( T \) \( T/\sigma \ll 10 \), so that there is considerable scatter at large angles. A solution can be obtained for this situation by using the perturbation approach (17).

If the backscatter patterns for the two solutions are combined (simply adding the two scattering coeff.), the comparison with experimental results is remarkably close.

2.2.4. Conclusions on Scattering Models

The analysis briefly discussed above has been developed to explain scattering of radio waves from the moon and the sea. While the results are valid at optical frequencies, the serious limitations imposed by the Kirchoff tangent plane approximation must be borne in mind (2.2.2.b). Also the more realistic model of Barrick & Peake is of little practical use in the optical region since it is
virtually impossible to measure the small scale roughness.
Alternatively, if the theory can be assumed established from
measurements at radar wavelengths, then this provides a useful
method of measuring small scale roughness at optical frequencies.

2.3. SCATTERING FROM A ROUGH SURFACE II

DISTRIBUTION OF SCATTERED POWER

2.3.1. Introduction

It is shown in the previous section that the mean power
scattered from a surface depends on the nature of the surface. The
statistics of the scattered field about the mean do not, in general,
depend on the surface properties but on the transmitter beam geometry (19).
The mathematical description of the statistics is presented in the
following sections together with a description of the physical nature
of the scatter pattern as it appears visually.

2.3.2. Amplitude and Intensity Statistics of the
Scattered Field

Goodman (18) has shown that the distribution of the amplitude
of the scattered field from a coherently illuminated rough surface is
the Rayleigh function of the form:

\[ P \{ |E_F| \} = \frac{E}{\sigma^2} \exp \left( - \frac{E^2}{2\sigma^2} \right) \]  

\[ E \geq 0. \]  

The intensity distribution is therefore

\[ P \{ |E_F|^2 \} = \frac{1}{2\sigma^2} \exp \left( - \frac{1}{2\sigma^2} \right), \]  

the negative exponential distribution. A complete, but rather
involved analysis can also be found in Beckmann \(^9\). Combining the results of 2.3.1. and 2.3.2. with the Beckmann solution, allows one to predict the full statistical properties of the scattered light (subject to the limitations already described for the Beckmann results).

2.3.3. **Speckle patterns and their Physical Appearance**

The physical manifestation of the mathematical description of the scattered intensity is called a speckle pattern. It consists of many needle-like lobes, of random peak intensity, spread over the full field of scatter of the target. Goldfishe \(^{19}\) has shown that the lobe size or correlation distance \(D_s\) for a speckle field at range \(R\) is

\[
D_s = 2 \frac{\lambda R}{d_t}
\]

where \(d_t\) is the spot size at the target. Typically, using an \(H_e N_e\) laser, the speckle diameter at 100 cm for a spot of 100 \(\mu m\) is 12.6 mm.

2.3.3.1. **Rotation of the Target.**

Consider an illuminated area on a rotating target (Fig. 2a). After a time \(t\) the new position of sources A, B, C, D, is \(A^1, B^1, C^1, D^1\). For small angles of rotation the well defined phase change will produce a rotation of the whole speckle pattern, at twice the rate of the target rotation. The pattern tends to break up after rotating more than \(5^0 - 10^0\) \(^{21}\).

2.3.3.2. **Transverse Movement of the Target.**

Consider the target moving across the beam (Fig. 2b). The
Fig. 2. EFFECT OF SURFACE MOVEMENT

(a) Rotation

(b) Translation
point sources A, B, C, and D become A\textsuperscript{1}, B\textsuperscript{1}, C\textsuperscript{1} and D\textsuperscript{1}. The phase relationship between A and A\textsuperscript{1}, B and B\textsuperscript{1}, etc. is defined by the surface profile. As this is a random variable, the phases of the secondary radiators will be randomly modulated by the movement of the surface. This modulation of the point scatterers results in an intensity modulation, or twinkling of the far field pattern.

If the motion is fast and the detector observing the speckle pattern responds slowly, then speckles will not be observed, since the detector will average out the fluctuations (22-25).

2.3.3.3. Imaging Speckle Patterns

If the scattered light from a coherently illuminated object is imaged onto a screen, the size of the speckles is set by the f-number of the optical system (20, 22, 23, 51, 52). This situation does not usually arise in laser Doppler systems, but is mentioned for completeness.

Summary

The scattered field has been shown to have a very irregular structure, distributed about the mean value predicted by eqn. 2.2.3. This structure changes as the target rotates, or when the target moves across the beam of light. Ennos et al (40) have shown that a useful instrument can be constructed to determine the position of stationary parts of a vibrating surface, using this fact.

To conclude, the scattered field is far from uniform, and the intensity is very sensitive to surface movement.
2.4. DETECTION OF SCATTERED LASER LIGHT

2.4.1. Introduction

Basically, there are two detection methods used in laser radar systems:

a) Energy or incoherent detection.

b) Heterodyne or coherent detection.

Both are studied here, and their important parameters are discussed in the following sections. A complete description of a wide range of laser receivers can be found in Ross (6).

2.4.2. Incoherent Detection

The incoherent detector consists of an optical receiver collecting and focusing the optical energy onto a small photo-detector. For a receiver of area $A_R$ at a range $R$ from an isotropic scatterer, and situated normally to the surface, the average received power is

$$P_R = \frac{A_R}{2\pi R^2} P_T$$

where $P_T$ is the power incident on the target. $P_R$ will in fact fluctuate, since the scattered field will not be uniform. Provided $A_R > D_s$ there is every likelihood of receiving a signal at all times. However, the fluctuations in the signal can be considerable (18), except if $A_R >> D_s$. A typical example of such a receiving technique is in range-finding.

2.4.3. Coherent Detection of Scattered Light

A description of the coherent receiver is presented in section (2.4). The analysis presented there is typical of the descriptions given in papers on the subject (4,5,6,8).
Various experiments have been performed to investigate the effects of coherently detecting scattered light (2.6 - 28).

Gould (26) and Kroeger (28) confirm that the amplitude of the scattered field is Rayleigh distributed (2.3.2), while Warden (27) suggests that there is a saturation value for the detected power as the receiver aperture is opened.

To date, a complete mathematical description of the coherent detection of scattered light has not been presented. Empirical laws are applied to the design of experimental systems. A survey of the literature shows a need for a comprehensive and accurate experimental study of the subject, and to present clear and concise criteria for the design of such systems.

Coherent receivers are used in the measurement of velocity, displacement, and vibration analysis where the laser radar Dopplerometer provides a non-contact method of measurement (1, 3, 31, 39 - 44).

2.4.3.1. Determination of Signal and Noise in Coherent Systems.

One of the most important design criteria is the signal to noise ratio of the system. Kroeger (25) measured signal to noise ratios for the coherent system, using a selection of surfaces, but the results are of limited use.

Warden's (24) work gives relative signal levels and suggests that there is a maximum signal obtainable for any system, but no absolute figures of scattered power or S/N ratio are presented.
2.5. **THE LASER DOPPLER VELOCIMETER**

The Laser Doppler Velocimeter was initially developed as an instrument to measure the velocity of gases or fluids in enclosed spaces. The first instrument was demonstrated by Yeh and Cummins (60) in 1964 and since then there has been considerable research expended in developing this technique (41 - 43, 53 - 55). Rudd (56) proposed a new model for the velocimeter and this has since been exploited by many workers in developing simpler velocimeters (57, 58, 59). Created (44) further modified the instrument to measure the velocity of moving solid surfaces.

In all cases, except the latter, the scattering occurs from small particles moving in a random manner with a mean drift in the direction of flow. The velocity measured is the component normal to the beam direction; hence the sensitivity diminishes significantly as the velocity becomes more parallel to the beam direction.

As this research is concerned with studying the measurement of the velocity of solid targets moving in the beam direction, the results presented in the many publications on laser velocimeters are only of limited use.

The very interesting paper by Created (44), utilising such an instrument on solid targets, is far from comprehensive enough since it does not provide any useful data on different target samples.
Investigation indicates that there is a serious lack of useful data on coherent detection systems. Some method of determining the expected signal in a doppler system is desirable, since the detector system and post-detector electronics depend on the S/N ratio of the optical system.
3.0. THEORETICAL ASPECTS OF MEASUREMENTS

3.1. INTRODUCTION

The present state of knowledge in the design of laser doppler radar has been shown (Chapter 2) to be lacking in the data and understanding required to predict the expected performance of the optical system. The originality claimed for this thesis is that it provides such information.

Laser Doppler devices are now appearing on the measurement benches and it is desirable that the optimum design and performance predictions can be scientifically determined before construction begins. The proposed design philosophy is based on the following four simple steps.

Assuming a given transmitter power:

1) Determine the total optical power that can be received from the target.
2) Multiply this by the detector and coherence losses of the system and calculate the expected signal.
3) Calculate the noise induced by the optical system.
4) Calculate the signal to noise ratio at the detector output.

The study presented here is aimed at providing the information to perform the first three steps, since this is the area in which data are lacking. The necessary equations are calculated using the parameters defined in this chapter. Optical power, rather than field strength or intensity, is used as it has been found to be more useful in the understanding and calculation of the system parameters. Various
scattering geometries are discussed and their relative merits considered.

The effects of Doppler broadening on the measurement of these parameters is described. A typical radar problem is presented to show how broadening affects system performance, and a theoretical study of translational and rotational Doppler broadening is presented.

3.2. INCOHERENT RECEPTION

3.2.1. Mean Power and Scattering Loss Factor $\beta$

In spite of its limitations, Beckmann's equation will be used to provide an indication of the type of results to be expected. The equation can be modified to yield the mean power scattered to a receiver of diameter $D_R$. In eqn. 2.2.3. $<\rho_\rho^*>$ is defined as the mean power density for scatter from a rough surface divided by the power density when a correctly aligned specular reflector replaces the rough surface. Thus,

$$<\rho_\rho^*> = \frac{<I_s>}{I_R}$$

where $<I_s>$ is the mean scattered field intensity and $I_R$ is the specular reflected intensity. Referring to Fig.3a, in which the main geometric parameters are defined, from simple geometric considerations

$$I_R = P_T \left( \frac{2 F_T}{D_T} \right)^2 \frac{1}{\pi R^2}$$

assuming $P_T$ is uniformly distributed over the transmitter aperture $A_T$. Also the received optical power $P_S$ is given by

$$<P_S> = \left( \frac{D_R}{2} \right)^2 \pi . <I_s>$$

for $D_R$ small.
Fig. 3a. TRANSMITTER/RECEIVER GEOMETRY

Fig. 3b. BACKSCATTER PLOTS FROM EOTN. 3.2.2.
combining these three equations gives an expression for $P_s$ in terms of $P_T$:

$$\frac{P_s}{P_T} = \langle \rho \rho^* \rangle = \left( \frac{D_R F_T}{D_T F_R} \right)^2 = \frac{1}{\beta} \quad \text{3.2.1.}$$

This equation introduces the "scattering loss factor" $\beta$ and also provides a definition for it. The factor $1/\beta$ will be used frequently in the text from henceforth. It is similar to the back-scatter cross-section $\sigma_B$ as used by Renau (13 and 14) and radar engineers.

Then $1/\beta = \frac{\sigma_B A_R}{4\pi F_R^2 A}$ where $A$ is the area of the spot.

Substituting for $\langle \rho \rho^* \rangle$ from eqn. 2.2.3. and putting

$$A = \left( \frac{2.44 F_T \lambda}{D_T} \right)^2 \frac{\pi}{4}$$

gives

$$1/\beta = \frac{D_R^2 \lambda^2}{4 \nu_z} \frac{4}{(2.44 \lambda)^2} \frac{A}{\sigma^2} \exp \left( -\frac{\eta^2 \nu_x \nu_z^2}{\sigma^2 4 \nu_z^2} \right) \left( \frac{D_R}{F_R} \right)^2 \quad \text{3.2.2.}$$

$1/\beta$ is independent of the spot size, and $\lambda$, but is a square law function of $\frac{D_R}{F_R}$. These only apply to a rough surface and for $D_R/F_R < \cdot 1$ since clearly, if the surface has a narrow lobe of scatter, then increasing the receiver area beyond this width will not improve the signal level. (In the experiments performed the maximum value for $D_R/F_R$ was .025. It is extremely unlikely that a value greater than this should be used in practice.)

Measurement of $1/\beta$ at optical frequencies is relatively simple and has been performed by various authors (13, 30). One important point not made clear by these authors is that the mathematical description above describes the ensemble averaged value, whereas they used a spatial averaging technique, using a large spot on the surface.
3.2.1.1. Ensemble Average of $1/\beta$.

Since $<\rho \rho^*>$ is the ensemble average for a statistically stationary rough surface, it is found by averaging a large number of measurements made at different points on the surface. To do this, the signal can be detected through a small receiver aperture, as the surface is translated across the beam. By sampling at time intervals $\tau$ such that

$$\tau > \frac{d_1}{v}$$

where $v$ is the surface velocity, an ensemble average can be computed.

3.2.1.2. Spatial Average of $1/\beta$.

The measurement of the ensemble average is a time consuming and unnecessarily involved process. An alternative, and simpler, solution is to perform a spatial average. In this case a large beam of light is directed onto the target, and the receiver allowed to collect many lobes of the speckle pattern. Then, as the surface moves, there is less fluctuation in the signal; hence a measurement with the surface stationary is sufficient. Under these conditions plots of back-scatter and scatter are simply performed.

A series of experiments were run to verify that ensemble and spatial averaging produce the same results.

3.2.2. R M S Noise Power

In essence, the difference between the ensemble average and the spatial average is that the noise power has been removed in the latter case, by averaging at the aperture.
In the case when the receiver encloses $M$ speckles, say, Goodman (18) shows that the probability density function of the detected signal (integrated intensity) is

$$p(p_s) = \frac{a^M p_s^{M-1} \exp\left(-\frac{p_s}{\beta}\right)}{R(M)}$$

where \(\frac{M}{a} = \overline{p}_s = \frac{P_T}{P_0}\).

Goodman suggests $M$ and $a$ are adjusted to fit the mean and variance of the measured distribution

$$a = \frac{\overline{p}_s}{\sigma_s} \quad M = \left(\frac{\overline{p}_s}{\sigma_s}\right)^2$$

where $\sigma_s$ is the optical r.m.s. noise power. Eqn. 3.2.3. is derived by assuming that the intensity of each cell $M$ is constant, but independent of the neighbouring areas. Using this result Dainty (20) calculated the value of $\sigma$ for various ratios of receiver to speckle size and shows that

$$\sigma_s^2 = \frac{4}{\pi^3} \frac{\overline{p}_s^2}{b^2(0.305)^2} \int_0^1 \left[\cos^{-1}u - u \sqrt{1-u^2}\right]$$

$$\times J_1^2 \left[\frac{4\pi b u (0.305)}{u}\right] \, du$$

where $b = \phi R / \phi_s = \frac{D_R}{D_s}$

and $u$ is the aperture variable.

3.2.3. Back-scattered Power

Eqn. 3.2.2. shows that $1/\beta$ depends on the factors $J, v_x, v_y, v_z$ and $v_y$. They, in turn, depend on the geometrical configuration of the incident and scattered beams. Calculation of $1/\beta$ for backscatter
with various values of $T/\sigma$ have been made and are shown in Fig. 3b. The fast fall in signal level can be explained simply by the fact that as the angle of incidence increases, so does the angle of scatter, and for any particular surface the scatter predominates in the direction in which the slope of the surface facets produce a specular reflection, (i.e. away from the incident direction, except for normal incidence.) The probability of obtaining a high density of facets normal to the incident direction decreases as the angle increases. This, of course, assumes no multiple reflections.

The back-scatter configuration is the most popular for doppler systems since it simplifies the alignment requirements. For this reason it figures prominently in the measurements described later.

3.2.4. Scattered Power

In this instance $\theta_1$ is kept constant, while $\theta_2$ the scattering angle ($\theta_3 = 0$) is varied. Predictions, using Beckmann's equations, are easily performed and a substantial range of plots for various values of $T/\sigma$ and $\theta_1$ can be found in his publication (9).

Summary

The scattering loss $g$ has been shown to be easily derived from Beckmann's results, and consequent upon its definition, is an easy parameter to measure. It can be recorded either by ensemble averaging or spatial averaging. The latter technique allows one to obtain continuous scatter patterns of $1/g$ versus $\theta_1$ and $\theta_2$. Alternatively, the ensemble averaged results yield, after manipulation,
another parameter of interest, namely $\sigma_s$ the r.m.s optical power. When this is combined with $P_{T/B}$ the S/N of the optical signal is obtained. Such a measurement is easily performed.

3.3. COHERENT RECEPTION

To calculate the signal expected for a coherent receiver, a start is made with eqn. 2.1.3. suitably modified. Introducing space dependent terms for $E_s$ and $\phi(t)$, since the scattered field is non-uniform, this equation becomes

$$I_D = \frac{1}{2n} \left( \frac{E_{LO}^2}{2} + \frac{E_s(r,t)^2}{2} + E_L E_s(r,t) e^{j\phi(r,t)} \right)$$

where $E_s$ the peak amplitude of the scattered field has a time dependance as well. The optical power passing the receiver aperture $A_R$ is

$$P_D = \iint_{A_R} I_D \, da = P_{LO} + P_{DC} + P_{AC}$$

where

$$P_{LO} = \frac{1}{2n} \iint_{A_R} \frac{E_{LO}^2}{2} \, da$$

$$P_{DC} = \frac{1}{2n} \iint_{A_R} \frac{E_s(r,t)^2}{2} \, da,$$  

and

$$P_{AC} = \frac{1}{2n} \iint_{A_R} E_{LO} E_s(r,t) e^{j\phi(r,t)} \, da$$

Now for all the cases considered here the local oscillator field is constant, hence:

$$P_{LO} = \frac{E_{LO}^2}{4n} A_R$$
Also from eqn. 2.1.4, it is found that for a uniform signal field the peak value of $P_{AC}$ is

$$|P_{AC_{\text{max}}}| = 2 \sqrt{\frac{P_{LO} P_{DC}}{2}}$$

the result used by many workers. This, of course, is incorrect when considering a speckle field since it is far from uniform.

In general terms, a 'detector loss factor' $\gamma$ can be defined which accounts for any difference between the maximum signal expected, as defined by the above equation, and the signal $P_{AC}$ actually detected. (Assuming zero coherence loss elsewhere in the system).

The equations presented above will now be discussed in detail.

3.3.1. Solution for Non-uniform Field

Since, in practice, the speckle field is a random variable, only an indication of the nature of the expected signal can be presented. Referring to equations 3.3.1, 3.3.2, and 3.3.3, it can be seen that the problem amounts to solving the integral for $P_{AC}$ when $E_s(r, t)$ and $\phi(r, t)$ are undefined. To provide a suitable indication in the first instance some simplifying assumptions are made:

a) Ignore the time dependance of $E_s$ and $\phi$.

b) Assume $\phi(r)$ is constant.

Then $P_{AC} = \frac{1}{2\pi} E_{LO} \bar{E}_s A_R$ and $P_{DC} = \frac{1}{2\pi} \frac{E_s}{2} A_R$

so, using eqn. 3.3.4., it is found that

$$\frac{|P_{AC}|}{|P_{AC_{\text{max}}}|} = \frac{E_s}{(E_s^2)^{\frac{1}{2}}}$$
The numerator is the mean, and the denominator the root mean square, of $E_S$ and $\overline{E_S} = \sqrt{\overline{E_S^2}}$ (the equality arises when $E_S(r)$ is constant).

Thus, for a uniform field mixing with a non-uniform field the expected signal will always be less than that predicted by eqn. 3.3.5. even if the phase term $\phi(r)$ is constant.

By removing condition (b), the integration becomes a phasor summation over the aperture, and since

$$\left| \sum_{n=1}^{N} E_n \angle \phi_n \right| \leq N E_n$$

the total effective power $P_{AC}$ must be reduced.

Inclusion of the phase dependance $\phi(r)$ deteriorates the expected signal even more. Mathematically then, $\gamma$ can be defined from

$$\frac{1}{\gamma} = \frac{\langle |P_{AC}|^2 \rangle}{\langle |P_{AC,\text{MAX}}|^2 \rangle}$$

which is consistent with the general definition presented earlier.

The brackets denote ensemble average. Using eqn. 3.3.5. and noting that $\langle P_{DC} \rangle = \frac{P_T}{\beta}$

then

$$\langle \| P_{AC} \|^2 \rangle = \frac{4 P_T P_{LO}}{\gamma \beta}$$

Since $P_T$ and $P_{LO}$ are optical powers $P_{AC}$ can be imagined to be the 'effective' coherent optical signal power.

For any system, where $P_T$ and $P_{LO}$ will be known, $P_{AC}$ can be determined from the knowledge of $\beta$ and $\gamma$.

The product $1/\gamma \beta$ is identical in its effect to the factor $1/\xi$ introduced by Kulczyk to express the loss in signal strength due to the target being non-specular. However, the introduction of two factors enables one to identify separately two different effects.
and, in fact, it is later shown that $\gamma$ is independent of $\beta$.

It must be stressed that $1/\gamma \beta$ is the average value to be expected and for any particular measurement the actual value depends on the statistics of the distribution of $1/\gamma \beta$.

3.3.2. Speckle Simulation

One possible way of obtaining a realistic theoretical value for $1/\gamma$ is to simulate the integration of the speckle field on a digital computer.

For simplicity, a one-dimensional model was taken and a program written which allowed for adjustment of the following parameters:

1) Speckle shape.
2) Speckle peak value.
3) Adjacent speckle phases (phase constant inside a speckle).
4) Averaging Aperture $D_R/D_S$.

Full details and a copy of the program can be found in Appendix I. Fig. 4 shows the mean value of $1/\gamma$ plotted against $D_R/D_S$ and the phase angle of adjacent speckles was altered to produce the family of curves.

The limitations in the use of a deterministic model of a random process did not warrant the problems involved in writing a program for a two-dimensional model.

Stetson (31) mentions that adjacent speckles appear to have phases in opposition, i.e. $180^0$ apart. If this is so for a Gaussian (one-dimensional) speckle shape, a 4 db decrease in $1/\gamma$ can be expected when the receiver aperture increases from a small value
Fig. 4. SIMULATION OF $1/\gamma$ VERSUS $D_R/D_S$

The phase of adjacent speckles is a) $0^0$ b) $90^0$ c) $180^0$. 

Gaussian Speckles of constant peak value

- Modulated peak value
to approximately $D_S$, and thereafter it will increase still further.

3.3.3. R.M.S Noise Power

$1/\gamma$ defines the mean signal loss expected when coherently detecting a speckle field. Combined with $P_T/\beta$ the mean effective optical power can be calculated. It is also useful to obtain the r.m.s deviation of the signal about this mean.

Since the field distribution is Rayleigh (2.3.1.) and the detected power is proportional to the field, the distribution of the coherent optical power might be expected to be Rayleigh. Hence $|P_{AC}|^2$ will have a negative exponential distribution for very small receiver apertures. In fact, an additional factor (see section 3.4.2,2) also contrives to make this the case, even for large values of $D_R/D_S$. Hence, there is no smoothing of the signal by aperture averaging, as is the case with incoherent detection.

Two simple results for the distributions mentioned above are noted:

Rayleigh

$$\sigma = \text{mean}^{-1/2}$$

Negative Exponential

$$\sigma = \text{mean}$$

This suggests that knowledge of the mean should be sufficient to determine the r.m.s noise present in the optical power.

3.3.4. Coherent Scattered Power

It was shown earlier (eqn. 3.2.2.) that $1/\beta$ was independent of the spot size at the target (subject to certain limitations), and
that spatial aperture averaging is equivalent to ensemble averaging (see Chapter 5). Hence it becomes a simple matter to obtain the scatter patterns of various surfaces, in terms of $1/\beta$.

The expression

\[ \langle |P_{AC}|^2 \rangle = \frac{4 PT}{\beta \gamma} \]

is only useful provided $\beta$ and $\gamma$ are independent. $\beta$ depends on the surface properties of the target but not on the size of the target spot. $\gamma$ on the other hand, is due to the inter-action of the speckle and the receiving aperture. A number of workers have observed the speckle and its variance to be independent of the surface properties of the target (within limits of roughness) and to depend only on the optical system, in particular on spot size and receiver aperture. It is therefore postulated here that $\beta$ and $\gamma$ are, in fact, independent and this is indeed corroborated by experiments.

This being the case, measurements of coherent scatter patterns prove unnecessary. Having determined $1/\beta$ for a range of angles of incidence and angles of scatter (3.2.3, 3.2.4), to find the coherent detection scatter pattern (i.e. $\langle |P_{AC}|^2 \rangle$ versus $\theta_1$ and $\theta_2$), it is sufficient merely to modify the variation in $1/\beta$ by the factor $1/\gamma$. The value of $1/\gamma$ is determined entirely from the ratio of $D_R/D_S$ for the system.

Measurements to prove the validity of these statements were made by comparing the curves for $1/\gamma$ for different surfaces (i.e. different $1/\beta$).

There are two points to note about determining the speckle size $D_s$: --
1) for back-scatter the angle the spot subtends at the receiver is independent of the angle of incidence (the actual spot size increases with θ₁) so that the speckles are always the same size for back-scatter.

2) For scatter at other angles the angle Φₓ subtended by the spot at the receiver is given by

\[ \Phi_x = \frac{d_r}{R} \frac{\cos \theta_2}{\cos \theta_1} \]

The speckle size does then depend on the angle of incidence and the angle of scatter.

**Summary**

By introducing a detector loss factor γ, and performing measurements to determine its value, it is possible to predict coherent detection signal levels for specific optical systems. Since 1/γ is expected to depend only on this ratio, and since the speckle diameter can be determined from the resolved spot size in the receiver direction, coherent scatter patterns can be determined from their incoherent counterparts using the product 1/γβ. Simulation experiments suggest that 1/γ can be expected to be small, even for a unity receiver/speckle ratio.

3.4. **DETECTION**

Calculations so far, and measurements described later, are in terms of either the optical power or 'effective' optical power that reaches the detector. By using this description the results can be used for assessing the performance of any photodetector system.
The calculations of signal current for a photomultiplier situated adjacent to the receiver aperture (see Fig. 3a) are now presented for both coherent and incoherent detection systems. It is shown that for both systems it is possible to determine the instantaneous value and mean value of $1/\beta$ and $1/\gamma$ from the photo-multiplier current.

3.4.1. Output current: Incoherent Detection

It is well known that the output current of a photodetector is proportional to the incident optical power, subject to frequency response limitations. Therefore, using the relationship

$$i_D = K_p P_D$$

where $i_D$ is the photodetector output current, and eqn. 3.2.1, gives:

$$\frac{1}{\beta} = \frac{i_S}{i_T}$$

where $i_T$ is the measured current when a mirror is inserted at the target, and all the incident power passes onto the photodetector.

To increase the sensitivity and dynamic range in the measurement of $i_S$ and $i_T$, the signal beam is modulated by a rotating disc, and the peak value of the signal recorded. In practice, this has been found to give sufficient sensitivity although performance could be improved still further, using phase detection techniques.

3.4.2. Output Current: Coherent Detection

By combining equations 3.4.1, 3.3.1, 3.3.2, and 3.3.3, the output current $i_D$ is given by:

$$i_D = i_{LO} + i_{DC} + i_{AC}$$
where $i_{LO}$ is the direct current due to the local oscillator, $i_{DC}$ is the slowly varying "d.c." of the signal field, $i_{AC}$ is the "a.c." component due to mixing. The purpose of this analysis is to investigate the time varying nature of $i_{AC}$ of the output current $i_D$.

Thus, from eqn. 3.3.3.

$$i_{AC} = \frac{K_B}{2\pi} \int \int_{A_R} E_{LO} E_S (r, t) e^{j\phi(r, t)} da$$

3.4.4.

In general $r$ and $t$ are not independent in the functions $E_S (r, t)$ and $\phi(r, t)$, and therefore the time factor cannot be removed from the integration. Remembering that $\phi(r, t)$ includes all the relative phasor terms in the two beams, they can be separated into the individual components, such that

$$\phi(r, t) = \phi_R (t) + \phi_S (r, t) + \phi_{LO} (t)$$

3.4.5.

where $\phi_R (t)$ is a random phase modulation due to optical component vibration, $\phi_S (r, t)$ is the phase modulation of the scattered field, $\phi_{LO} (t)$ is an applied phase modulation of the local oscillator (the term "modulation" is used since only the phase changes are of interest and not the absolute phase).

3.4.2.1. Stationary Surface:

Consider now the effects of each of these terms with the surface stationary (i.e. $\phi_S$ is time independent). Then equation 3.4.4. becomes after integration

$$i_{AC} = 2 \sqrt{\frac{i_{LO}}{Y_i}} i_{DC} e^{j [\phi_R (t) + \phi_{LO} (t)]}$$

where $Y_i$ is some instantaneous $\gamma$. 
The maximum and minimum values of the exponential term are 1 and -1 respectively, and provided \( \phi_R(t) \) and \( \phi_{LO}(t) \) fluctuate with large enough amplitude and frequently enough to be able to detect the peak to peak value,

\[
i_{pp} = \sqrt{\frac{i_{DC} \cdot i_{LO}}{\gamma}}
\]

An actual method of modulation is discussed in Chapter 4. It can be shown that

\[
\frac{1}{\gamma} = \frac{i_{pp}^2}{4 <i_{DC}> i_{LO}}
\]

Hence \( \gamma \) can be determined from the output current of a photomultiplier provided sufficient samples of \( i_{pp} \) and \( i_{DC} \) are obtained to get an accurate average. To achieve this, a target surface is moved across the beam at a constant velocity, samples of the signal being taken at equal time intervals.

3.4.2.2. Sideways Moving Surface:

The result above was derived for a stationary surface, but to perform the measurements, the surface is moved. The effect this has on the output current must therefore be investigated. Consider the transmitter configuration of Fig. 5a. A source point at A, a distance, from the centre of the aperture, radiates to the target spot. From this spot, A subtends an angle \( \phi/2 \) from the normal to the velocity vector. Hence light from A incident at the target undergoes a phase shift of \( \lambda \left| \frac{\text{v} \cdot \text{t} \cdot \sin \left( \frac{\phi}{2} \right)}{\lambda} \right| \). It is well known that a linear phase modulation \( \omega_m t \) of a wave \( E e^{j\omega_c t} \) is equivalent to a constant frequency modulation, i.e. a frequency
The vibrational velocity to be measured is $v_B(t)$. 

**Fig. 5a.** TRANSMITTER DOPPLER BROADENING

**Fig. 5b.** SYSTEM CONFIGURATION FOR MEASURING VELOCITY
shift of $\omega_m$. Hence each point in the transmitter contributes
light of a slightly different frequency to the spot at the target.
A similar effect occurs at the receiver. The result is that the
photo-detector current is composed of the sum of many signals at
different frequencies, each with a random amplitude.

The output current will show amplitude modulation due to the
beating of these frequency components, as well as modulation due to
fluctuations of $E_s(r, t)$.

Provided many samples of the signal are recorded, so averaging
out the beating modulation, the measurement technique described
in 3.4.3.1. should be valid.

3.4.2.3. Rotating Target:

Rotation of the surface will tend to rotate the speckle pattern
(see 2.3.3.1). The resulting fluctuation of $E_s(r, t)$ at the
receiver aperture modulates the current $i_{AC}$. At the same time,
a phase modulation, similar to that for a sideways translating
surface, will be present, although the physical process involved will
be somewhat different. (A full analysis is presented in 3.5.2.2.)
Measurements of $1/\gamma$ for a rotating target can be made, although
difficulty arises since $1/\beta$ changes as the target is rotated.

Summary

The above section contains a description of how the output
current from a photo-detector contains the required information to
determine $1/\beta$ and $1/\gamma$ and to measure the distribution of these
values. The signal from the coherent detector also contains
additional information which can be used to find the Doppler
broadening due to movement of the surface.
To measure $1/\gamma$ and the spectral content of the coherent signal, it is only necessary to have information on the envelope of the signal.

3.5. **A DOPPLER RADAR PROBLEM**

The Doppler broadening of the scattered light has a more drastic effect on the system performance than that mentioned earlier (3.4.2.2, 3.4.2.3.) A typical Doppler radar problem is now presented, so that the effect of Doppler broadening can be clearly defined. This is followed by the theoretical predictions of the broadening expected for a rotating and translating target.

A theoretical description of the Doppler shift mechanism was introduced in 2.1.1., and a qualitative representation of its effects on the detector output current $i_D$ was discussed in 3.4.3.2. and 3.4.3.3.

In the previous section it was shown that sideways translation or angular rotation of the target introduces Doppler broadening into the detected signal.

An analysis of a particular Doppler system is now presented. Consider the horizontal vibration of the rotating shaft represented in Fig.5b. The velocity vector, due to the rotation at the centre of a spot of light on the circumference, is $\omega r$ at an angle $(90 - \theta)$ to the beam. The velocity can be resolved into two mutually perpendicular components and a rotational component $\omega$, all acting at or about the spot centre.

The axial velocity $v_\phi$ is given by

$$v_\phi = \omega r \sin \theta + v_B(t)$$

3.5.1.
and the transverse, or sideways, velocity \( v_t \) is

\[ v_t = \omega r \cos \theta \]

Since the motion can be resolved into three separate components, the analysis and measurement of the effect of each can be treated separately.

Assume the surface material to have a particular backscatter pattern (say one of Fig. 3b), then clearly, as the beam is moved vertically from the centre of the shaft, the back-scattered signal decreases. However, for \( \theta = 0 \) the broadening due to translational velocity is a maximum, so it is likely that the angle \( \theta \) for optimum resolution is not zero. Thus there is, presumably, some optimum angle \( \theta \) at which the resolution of the system is a maximum.

In order to calculate performance equations for the system, analytical functions for the broadening are required.

3.5.1. The Doppler Signal

From eqn. 2.1.1, the doppler shift of the signal is

\[ f_D = \frac{2 v_B}{\lambda} = \frac{2}{\lambda} \left[ \omega r \sin \theta + v_B(t) \right] \]

To obtain the information required, the signal must be fed to a frequency discriminator centred at a frequency of \( 2 \omega r \sin \theta / \lambda \) and then detect the shift \( \frac{2 v_B(t)}{\lambda} \) about this frequency. However, there are two factors which affect the accuracy of such a measurement:

1) Signal to noise of the input.
2) Doppler broadening of the input.
3.5.2. Resolution of the Discriminator I  S/N of Input

It has been shown in earlier sections that the signal at the output of the detector can be determined once \( \gamma \) and \( \beta \) are known. This allows one to predict the S/N ratio at the input of the discriminator. Kulczyk \(^{(1)}\) discusses this problem and provides an equation that gives the minimum measurable frequency \( \Delta F \) at the discriminator output, in terms of the input signal to noise \( C/N R \) as

\[
\Delta F = \left[ \frac{2}{3} \frac{B_{in}}{B_{if}} \frac{3}{C/N} \right]^{1/3}
\]

where \( B_{if} \) is the i.f. bandwidth, \( B_{in} \) is the output bandwidth, and \( S/N \) is the output signal to noise ratio.

A 9db S/N has been shown \(^{(34)}\) to be sufficient to distinguish a signal in noise, and under these conditions, and with \( C/N = 10 \text{ db} \), \( B_{if} = 1 \text{ MHz} \) and \( B_{in} = 50 \text{ KHz} \)

\[
\Delta F = 8500 \text{ Hz}
\]

which is equivalent to a peak velocity of

\[
= 3 \text{ mm/sec}.
\]

3.5.3. Resolution of Discriminator II  : Doppler Broadening

Many experimenters working with laser doppler instruments have found that the spectrum of the signal is not a narrow spike of the type predicted in equation 3.5.3, but a much wider function of either a Gaussian (Foreman \(^{(41)}\) and Pike \(^{(43)}\) type instrument) or triangular, (Greats \(^{(44)}\) ) shape. Spreading of the spectrum of this sort reduces the resolution of the system and, as such, represents a significant problem to the designer.
3.5.3.1. Transverse Velocity Broadening:

a) Transmitter broadening.

For a uniform illumination of the transmitter aperture the normalized frequency spectrum is given by (see Appendix 2)

\[ p(f) = \frac{D_T}{2} \sqrt{1 - \left(\frac{2 F T \lambda}{V_T D_T}\right)^2} \]

The effective full bandwidth is given by

\[ f_{TT} = \frac{V_T D_T}{F T \lambda} \left(\frac{3}{32}\right) \]

Thus the broadening of the spectrum is directly proportional to \( D_T \), the transmitter aperture diameter, and \( V_T \) the target velocity. A more detailed analysis is given by Estes et al. who give the result

\[ f_{TT} = 2.34 \frac{V_T}{\lambda} \frac{\sigma}{F_T} \]

where \( \sigma \) is the radius of the Gaussian beam. The difference is probably due to the different beam intensity distributions assumed.

An important point to consider is that both results suggest that the spectrum is independent of the target position, i.e. in or out of focus.

b) Receiver broadening.

The result above applies to transmitter broadening only and similar conclusions can be drawn concerning the receiver aperture. Broadening due to the receiver aperture has been ignored by some other authors (1 and 5). The combined spectral broadening is

\[ f_T = \frac{V_T}{\lambda} \sqrt{97 + \left(\frac{\phi_T}{\phi_R}\right)^2} \]
from the fact that the total spectral width is the root of the sum of the squares of the individual widths. This function is plotted together with experimental results in Fig. 26.

3.5.3.2. Rotational Velocity Broadening:
If the target rotates at an angular frequency $\omega$ about the spot centre, then the point sources of light at some distance $x$ from the axis will be moving at a velocity $x \omega$. These sources suffer a Doppler shift of magnitude

$$ f = 2 \omega \frac{x}{\lambda}. $$

a) Transmitter Broadening:
The intensity distribution at the focus of a uniformly illuminated lens is of the form (35)

$$ I_{sp} = \left[ \frac{2 F_T}{k D_T} \right] \frac{J_1 \left( \frac{D_T}{F_T} \sqrt{\frac{x^2 + y^2}{x^2 + y^2}} \right)}{\sqrt{x^2 + y^2}} \right] I_0 $$

The normalized spectrum of the light scattered becomes:

$$ P(f) = \frac{2 \omega}{\lambda} \int_{-\infty}^{\infty} I_{sp} \, dy $$

when the axis of rotation is the $y$ axis. The effective width of this spectrum is found to be (see Appendix 3: eqn. A.3.1.)

$$ f_{RT} = \frac{3.8 \omega}{\phi_T} $$

Having substituted for $F_T/D_T$ it must be stressed that this only applies when the target is at the focus of the beam. A completely general solution would depend on the position of the target. This result is similar to the simplified and approximate solutions used by Teich (5) and Kulczyk (1).
b) Receiver Broadening:

As in the case of transverse movement of the target, the receiver contributes broadening of an identical nature. The spot is in this case a virtual one, from which the local oscillator appears to originate. Thus

\[ f_{RR} = 1.9 \cdot 2 \pi \frac{F_R}{D_R} = \frac{3.8w}{\phi_R} \] 3.5.8.

Combining the receiver and transmitter broadening is analytically more involved than for the transverse case. To overcome this, it is sufficient to consider the results of equations 3.5.7 and 3.5.8. as the asymptotes of a curve, showing the spectral width \( f_R \) versus \( \phi_T \) say for a particular value of \( \phi_R \). This is the technique used in the results section for the case of angular rotation (see Fig. 27).

---

**Summary**

For a particular radar problem the optimum system of optics can be determined by consideration of the resolution of the instrument. The S/N of the i.f. signal, which depends on the optical signal level \( I_{sc} \), will set one resolution level. Alternatively, the spectral broadening due to target motion must set some other limit. However, to adjust one limit, will affect the other in some way.
The spectral broadening must be balanced between the transverse broadening and the rotational broadening by altering $\Phi_T$ and $\Phi_R$. But changing both of these will inevitably result in a change in both $\beta$ and $\gamma$, thus altering $I_{ac}$. By using computational techniques, the system design can be optimised quite quickly from a general equation for the minimum measurable frequency. Other factors may affect the choice of the optical system, i.e. depth of focus, area of target, and these could be incorporated into the general equation (at the expense of simplicity, though).

3.6. COHERENCE AND POLARIZATION LOSSES

In a practical doppler system additional coherence losses will reduce the signal at the photo-detector. These can be attributed to:

a) Turbulence loss $\gamma_T$

This results from phase distortion of the wavefront by refractive index changes in the medium.

b) Path difference loss $\gamma_D$

As the laser output consists of many longitudinal modes separated in frequency by $c/2L\text{Hz}$. ($L$ is the length of the laser, $c$ the velocity of light), spatial beats exist in the beam. For high mixing efficiency the path difference must be such that the
two beams interfere at an antinode of the beat envelope. The path difference must be integer multiples of $2L$. This problem is fully discussed by Foreman (29).

In this research both effects are reduced to a minimum, by keeping temperature variations to a minimum and carefully adjusting the path lengths to be equal.

Depolarization of the beam can be accounted for by a loss factor $\gamma_p$. The details of the depolarization of many different surface materials can be found in references (37, 14, 15, 36). A polarising beam-splitter has been used for all back-scatter measurements, so that the depolarized component does not reach the detector.

**Summary**

This Chapter introduces most of the important parameters and design equations applicable to the research covered by this thesis. The two loss factors $\beta$ and $\gamma$ have been defined and equations presented indicating how they can be measured. Determining these two factors experimentally provides the necessary data to predict the expected signal for a coherent receiver system. Theoretical predictions have been presented for $1/\beta$ and $1/\gamma$ and the rms noise in the signal. The measurement of $1/\gamma$ may be affected by the Doppler broadening beats in the signal, but these beats also provide a means of measuring the spectral width, using an envelope detection technique.

When considering a specific system, it was shown that the S/N of the signal current and the spectral broadening of the Doppler signal set a lower limit on the minimum measurable frequency, and
it is the determination of this lower limit that provides a performance prediction of the system.

The measurement of the parameters introduced in this chapter is of prime importance, for the scientific design of any system to be possible. The following chapters describe how these measurements were performed.
4.0. EXPERIMENTAL DESIGN AND MEASUREMENT PROCEDURES

4.1. INTRODUCTION

In Chapters 2 and 3 the parameters, important to this study, have been defined and discussed in their theoretical context. Listed below are each of the variables, together with their design requirements:

a) $\beta$, the scattering loss factor

b) $\gamma$, the detector coherence loss factor.

$\beta$ was measured for many transmitter receiver configurations, but $\gamma$ was only recorded for back-scatter geometry.

The majority of the measurements of $1/\gamma$ and $1/\beta$ were performed independently, but the system was designed with enough flexibility so that both functions could be measured at the same time.

c) Ensemble averaging was performed by moving the target at a constant velocity across the beam. The same system provides results for transverse spectra measurements.

d) To obtain back-scatter records, the target was rotated at a slow constant speed. Rotational spectra were recorded, using the same system.

e) The receiver, mounted on a pivoted arm, was free to rotate about the spot at the target for incoherent scatter measurements.

f) The transmitter/receiver focal lengths and apertures could be altered simply, without recourse to an involved re-alignment procedure.
The system was stable enough for a measurement sequence to be performed automatically for periods up to 10 minutes.

4.2. INTERFEROMETER

The interferometer constructed provided these facilities with a minimum amount of re-alignment and setting up after each change of configuration. A view of the optical system and the associated electronics is shown in Fig. 6. A block diagram of the interferometer is shown in Fig. 7.

A Spectra Physics 124 A laser operating at 6328 Å producing 15 mW of power, was the source of light. The length of the laser (80 cm) was such that it produces approximately 7 longitudinal modes. It was rigidly mounted to a large steel table by 1½" diameter 'dural' rods, fixed to magnetic clamps. The latter were used frequently to facilitate the easy positioning of components anywhere on the table, and to provide a firm clamp to reduce vibration.

4.2.1. Attenuator and Telescope

The beam first passed through a compensated adjustable wedge filter, whose attenuation characteristic is shown in Fig. 8a. Attenuation of the beam was required when the larger receiver apertures were used. The filter was not completely homogeneous and caused some distortion of the beam, but by situating it before the telescope and spatial filter, the defects were removed and a clean uniform beam was obtained for all settings of the attenuator.

The telescope consists of a 2 cm eyepiece focusing the light through a 50μ pinhole. Both components were adjustable in
a) Full view of optical apparatus with the beam path illuminated

b) Instrumentation showing Data logger on the right

Fig. 6
Fig. 7. DIAGRAM OF OPTICAL SYSTEM
Fig. 8. EQUIPMENT CALIBRATION CURVES

a) Neutral density attenuator
b) Detector / log amplifier combination
vertical and horizontal planes, and the eyepiece could also be moved along the beam axis. This combination provided for beam alignment onto the aperture at the objective of the telescope. A maximum beam radius of 4 mm was used, and at this distance the intensity had dropped by ≈ 15% compared to the centre. This drop was considered tolerable as the only alternative was to expand the beam still further, thus losing power. A thick aluminium-coated glass beam splitter B1 separated the signal and local oscillator beams at the output of the telescope. These components were all mounted on a heavy optical bench situated on the steel table.

4.2.2. Signal Beam and Polarizing Beam Splitter

The signal beam was accurately aligned over a 1 metre optical bench, so that components located on this bench were always centred on the beam. The length allowed for lenses of focal length from 15 cm to 100 cm to be used for the transmitter. Using this arrangement alignment, after a change of lens, was achieved in a very short time. The transmitter aperture limited the beam to the required size.

The focused beam then passed through a \( \lambda/2 \) plate, polarizing beam-splitter and \( \lambda/4 \) plate onto the target. The \( \lambda/2 \) plate was rotated so as to provide 100% transmission at the polarizing prism. The \( \lambda/4 \) plate rotated the polarization vector of the scattered light 90° so that 100% reflection occurs at the prism. The table below compares the advantages and disadvantages of the conventional metal film beamsplitter and the polarizing beamsplitter:
The only disadvantage of the polarizing splitter was No. 5, but this was far less noticeable than, say No. 3, for a metal film splitter. Because they must be thick (to separate the reflections from the front and back faces), there was very bad defocusing when short focal length lenses were used. The polarizing prism must have the associated $\lambda/2$ and $\lambda/4$ plates, to align the polarization vector, and for an inexpensive system the cost of these components may be prohibitive.

4.2.3. Receiver and Local Oscillator Beam

The light scattered by the target was collected by the receiver lens and passed to the photo-multiplier (PMT), through the receiver aperture.

The local oscillator reaches the PMT by way of an adjustable mirror, M 1, and beam-splitter, B 3. Fig. 9 shows a photograph of the transmitter / receiver optics.

The mirror M 1 was adjusted by micrometer screws on a 3" lever and was sensitive enough for aligning apertures of up to 4 mm diameter. A photograph of the mount is shown in Fig. 9b. Notice the $\frac{1}{2}$" duralumin rod that supports this item so as to obtain the necessary rigidity in the structure.
Apparatus for isotropic scatter measurement

Local oscillator beam alignment jig

Transmitter/Receiver optics and sample table drive

Fig. 9
Because of the long path lengths involved, and the height of the beam above the table, the system was susceptible to vibration. Careful design and choice of layout reduced this to an acceptable minimum, and on a quiet day the vibrations were less than .5\(\mu\)m in amplitude. The photographs of Fig. 10 show typical short-term signals. In three cases the vibrations were random, while the fourth shows the output when an energised mains transformer was placed on the bench.

The path length of each arm of the interferometer from B1 to the detector was 59", and the beam was 11\(\frac{1}{2}\)" above the table surface, yet the two beams remained correctly aligned to within a few seconds of arc for periods up to a week.

4.2.4. Scatter Measurement System

For the scattering measurements, the system remained very much the same, only in this case the receiver optics and the P M T were mounted on an arm that was free to rotate about the spot centre at the target. The system is shown in Fig. 9a.

4.2.5. Transmitter and Receiver Optical Components

Table I shows the full complement of receiver / transmitter focal lengths and aperture diameters. Aperture diameters were measured on a travelling microscope, and the manufacturer's quotations of focal length were checked, using the light from the laser.
Fig. 10. SIGNALS FROM THE INTERFEROMETER

a) Table excited by energised mains transformer located on its surface (b, c, d) random vibrations of the table near the minimum, maximum, and mid range of the interference pattern. (Vert. 0.058 µm/cm. Horiz. 20 mS/cm.)
<table>
<thead>
<tr>
<th>TRANSMITTER</th>
<th>RECEIVER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focal Length ($F_T$)</td>
<td>Focal Length ($F_R$)</td>
</tr>
<tr>
<td>Aperture Diameter ($D_T$)</td>
<td>Aperture Diameter ($D_R$)</td>
</tr>
<tr>
<td>mm</td>
<td>mm</td>
</tr>
<tr>
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<td>1.92</td>
</tr>
<tr>
<td>1000</td>
<td>7.96</td>
</tr>
<tr>
<td></td>
<td>3.95</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Accuracy</td>
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</tr>
<tr>
<td>Focal length $\pm .5$ mm</td>
<td>1.62</td>
</tr>
<tr>
<td>Aperture $\pm .01$ mm</td>
<td>2.00</td>
</tr>
<tr>
<td></td>
<td>4.00</td>
</tr>
</tbody>
</table>

**TABLE I**

Transmitter / Receiver Apertures and Focal Lengths.

The early experiments were performed with a 16 cm receiver lens, but the later series used a better quality achromat of 20 cm focal length. The transmitter lenses were of a high quality, though unbloomed. Tests on the transmitter lenses were performed by measuring the diffraction limited spot produced by the apertures and lenses used in the experiments.

A microscope was focused onto the focal plane of the lens in question and a photograph taken of the image. The spot profile was recorded on a microdensitometer and the width of the main lobe, or 'airy disc' measured. Calibration was checked by photographing a 50 $\mu$ wire observed by the microscope with the same magnification,
thus eliminating the need to calibrate both the microscope and the microdensitometer. Accuracy was set at ±5%, being due to diffraction and scatter in the microscope. The measurements are recorded together with the theoretical values in Table 2.

<table>
<thead>
<tr>
<th>Diameter</th>
<th>Theoretical</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>38.6</td>
<td>52.2</td>
<td></td>
</tr>
<tr>
<td>77.2</td>
<td>74.8</td>
<td></td>
</tr>
<tr>
<td>154.1</td>
<td>153.5</td>
<td></td>
</tr>
<tr>
<td>193.0</td>
<td>175.0</td>
<td></td>
</tr>
<tr>
<td>386.0</td>
<td>346.0</td>
<td></td>
</tr>
<tr>
<td>772.0</td>
<td>705.0</td>
<td></td>
</tr>
</tbody>
</table>

**Table 2**

Diameters of Transmitted Spots

(Accuracy of Experimental results ±5%)

For the smallest spot, distortion due to the microscope was very severe. Another possible cause of error, in this case, could have been saturation of the photographic emulsion.

4.2.6. Incoherent Detection

The interferometer described is by definition the arrangement for coherent detection. The modifications required to obtain the incoherent results for back-scatter are:

(a) insert modulator (shown in Fig. 7) into the signal beam
(b) block off the local oscillator beam.

Tests were carried out to find the best position for the modulator. When placed before the polarizing prism, any light scattered by the prism and the λ/4 plate was detected. The alternative was to
place it between the prism and the target. In this case the light scattered from the back of the modulator (when the beam was blocked out) was detected, since this was also modulated at the same frequency (but in antiphase to the true modulation). The latter position was found to be 8db worse than the former and this was selected (see Fig.7).

4.3. THE TARGET HOLDER

To satisfy the requirements set out in the introduction (4.1), the target holder was designed to be translated across the beam, rotated about the beam, and have the facility to accept an arm, holding the receiver components, that could be rotated about the spot centre. A spring-loaded plate, on the holder, located the target surface flush against the holder, so that the surface was always situated on the axis of rotation. Surfaces could be changed rapidly and located accurately, using this technique. The finished product is shown in Fig. 9a and Fig. 9 c.

4.3.1. Rotation and Translation Drive

A variable speed servo motor drove an adjustable ratio reduction gear that could be coupled to either the rotation or translation drive. This provided a slow constant speed of traverse or rotation that was adjusted to suit the optical configuration, and so obtain the optimum signal bandwidth, compatible with the highest sample speed possible.
4.3.2. Rotation of Receiver Optics

The motion that rotated the target was also designed to rotate the arm supporting the receiver components. However, much trouble was encountered in obtaining a sufficiently free bearing for the arm to rotate, but a tight enough joint, so that vertical movement of the arm was small. Automation, in this instance, had to be abandoned and a tight bearing was built so that the slack was small, and manual force used to rotate the components. Using the coupling already provided by the drive system, a 0.1% linear potentiometer was coupled to the motion to provide an analogue voltage proportional to the angular position of the arm. This voltage was used to drive either the X axis of an X-Y recorder, or the reference channel (channel 2) of the data logger described in Appendix 4.

4.3.3. Modulation of the Coherent Signal

It was shown in 3.4.2.1. that phase modulation of the reference beam must be applied to make the signal reach its maximum and minimum value frequently. A vibrating mirror, using a piezo-electric crystal, was used, but when the motor on the sample table was energised, sufficient vibration existed in the table for this to be discarded.

4.4. ELECTRONICS AND DATA LOGGER

The electronics designed for this study fulfil the requirements set out in Chapter 3. They were:-

(a) peak to peak detection of the a.c. component \( i_{AC} \) of \( i_D \)

(b) isolation of the slowly varying 'd.c.' component \( i_{DC} \) from \( i_D \)
Fig. 11.

A.C. V/V/M

A.F. oscillator

VR1 sets signal level

D.C. V/V/M

Fig. 11.
A block diagram of the electronic system is shown in Fig. 11 and a photograph of the equipment in Fig. 6. Wherever possible, commercial equipment was used, and only the detector, logarithmic amplifier and data logger were built specifically for this research. Since its commissioning, the data logger has been used on many other projects, and copies of it are being built in the Department for other research groups.

4.4.1. Description of Overall System

The signal from the photo-multiplier was fed to a Brookdeal low noise amplifier via the attenuator VR1. The bandwidth of the amplifier was adjusted to give the maximum S/N ratio. Because the Brookdeal amplifier removes the low frequency signals, it had to be by-passed when these components of the signal were needed. The signal then entered a peak to peak detector, and the output of this was fed to a logarithmic amplifier and then to channel 1 of the data logger. The low frequency component from the detector was fed directly to channel 3. The data logger records, on paper tape, the voltages present at the inputs 1 and 3. The other equipment depicted in the diagram was used for calibration purposes, and will be described in a later section.

4.4.2. Detector

The detector circuit was built, using integrated circuits (see Fig. 12). Briefly, the circuit operates as follows. The signal $i_{AC}$ was separated from $i_{LO}$ and $i_{DC}$ by a high pass filter and the peak to peak of this detected, using two peak detectors

(c) some method of recording many samples of the signals $i_{AC}$ and $i_{DC}$.
driven with antiphase voltages. The outputs were summed, using a low pass summing amplifier. The signals $i_{DC}$ and $i_{LO}$ were separated from the signal $i_{AC}$ by low pass filtering. The component $i_{LO}$ was then offset, using a potentiometer and summing amplifier; thus the output of this stage was $i_{DC}$.

The final stage in the circuit was an inverting amplifier with a low level voltage offset used to trim the characteristic of the logarithmic amplifier to the correct place.

4.4.3. Logarithmic Amplifier

This circuit, shown in Fig. 13, was taken from the S G S Handbook (46) and adapted to suit the operating conditions required. The circuit operates on the exponential characteristic of the forward biased base-emitter junction. This controls the current in the feedback loop of an operational amplifier. A matched transistor $T_2$ was used to compensate for temperature changes and drift.

The performance of the overall system, i.e. detector and logarithmic amplifier, was measured and the results shown in Fig. 8b.

The linearity of the characteristic over 60 db of signal depends mainly on the exceptional linearity of the peak detectors.

The reasons for using a logarithmic amplifier are threefold:

1. It provides a large unswitched dynamic range (a necessity where automatic recording is needed).

2. When coupled with a digital data recording system such as the data logger used here, in which accuracy is limited by digitization (thus reducing with signal level), the combination provides a system with optimum operational accuracy (provided the logarithmic amplifier is stable and is calibrated frequently).
LOGARITHMIC AMPLIFIER

Fig. 13.
(3) In a system where the signal is of a random nature, the safety margins for a large signal can be increased (so there is no overload) without setting the average signal level in the inaccurate region of the converter.

To achieve these advantages, a careful check is needed of the logarithmic amplifier's characteristic. It was found that after setting up and leaving for one hour to stabilize, the drift was insignificant over two hours of operation.

4.4.4. Data Logger

A complete description of the data logger can be found in Appendix 4; only a brief summary is presented here.

After performing some preliminary experiments, it was found that unless some automatic recording system was utilised, the task of gathering statistically meaningful data would be enormous. Thus an automatic recording system was designed that would provide the data ready for easy analysis. The system chosen was based on a paper tape punch providing data in a computer compatible form, ready for analysing on the University's ICL 1905 computer. The logger converts a voltage (in the range 0-9.99 V) at its input into a number from 000 to 999 onto the tape. Variable gain amplifiers were provided to allow for a range of signal levels, and conversion rates were adjustable. At the time of writing, the system has performed some \(1/4\) million conversions (four punch operations / conversion), and errors have been very infrequent.
4.5. CALIBRATION PROCEDURES AND MEASUREMENTS

4.5.1. Alignment of the Optical System

The alignment of the telescope and its associated components has been described earlier. The alignment of the local oscillator was very critical, and was checked for every measurement or experimental run. The position of the target material was not critical since the spring arrangement used to hold the samples ensured that the front surface was in the correct plane.

4.5.2. Calibration of Electronic Circuits

After measuring the characteristic of the logarithmic amplifier, repeated checks were made to make sure that the system operated on this characteristic. A small d.c. offset (VR 2 in Fig. 12) was provided to adjust the lower operating point. A 2 mV rms signal was applied to the input of the detector from the A.F. oscillator (Fig. 11). The output was then adjusted to -4V, as on the characteristic. To set the offset potentiometers in the data logger amplifier, a signal of 541 mV rms was fed into the detector and the data logger offset adjusted until 850 was punched on the paper tape. These calibrations remained stable through a working day if set after an initial warm-up period.

4.5.3. Photo-multiplier Supply Voltage

The supply to the photomultiplier was fixed at 1000 V by using a stabilised power supply. This was checked periodically to ensure consistent operation. It was found that it was better to work with a low tube voltage, therefore keeping the excess noise to a minimum.
4.5.4. Measurements

A detailed account of the problems and difficulties associated with the experimental measurement of scattering loss \( \beta \) and detector coherence loss \( \gamma \) is considered inappropriate for a work of this nature, so only the basic outline will be described here. In essence, the method was as follows. The signal current was measured for some particular optical arrangement first with a mirror as the target, then with the required target material present. The ratio of these two quantities gives \( 1/\beta \) for the incoherent case, and \( 1/\beta \gamma \) for the coherent system.

A complication arises because a correction for laser fluctuations was essential. Recording the local oscillator current for both measurements and taking the ratio of the two, provided the correction factor (true for coherent and incoherent measurements). The local oscillator now served two purposes:

1. It provided the reference beam for coherent detection;
2. It provided a simple means of recording the relative laser power.

A requirement of the experimental system as specified in section 4.1 (f) and (g) was that modifications of transmitter / receiver configurations should be uninvolved. By using the method described above, every change of configuration requires a separate measurement with a mirror present. Such a technique was clumsy and experimentally inconvenient.

To overcome this, a normalization procedure was devised that eliminated the need to use the mirror more than once. A theoretical description of the two procedures, one for coherent, the other for incoherent, can be found in Appendix 5. By using this technique, after one setting up procedure involving some twenty or so measurements, the only data required for each run was:
Normalization was then performed by a computer program, using the above information. The additional information that was recorded for particular experiments is now outlined.

4.5.4.1. Incoherent Back-scatter ($\beta$):

(a) Angle of incidence $\theta_1$ fixed at $0^\circ$: no additional data was required for such a measurement. The computational requirements were:

(1) average value of $1/\beta$

(2) standard deviation of $1/\beta$

These were calculated for a selection of surfaces as the target was translated across the beam.

(b) Angle of incidence $\theta_1$ variable.

In this case the surface was rotated at a constant rate, the aperture averaging the speckles. The additional data required was:

(1) starting angle

(2) speed of rotation

(3) the sampling rate

From this data and that listed earlier, the computer calculated and drew an angular plot of $1/\beta$ for the particular surface material.

Footnote

* The voltages produced by the currents $i_{LO}$, $i_{AC}$, $i_{DC}$ will be used from now on.
4.5.4.2. Incoherent Scatter:

As described earlier (4.3.2), the procedure for this experiment was somewhat different to that for back-scatter. When the receiver optics were manually rotated, a facility on the data logger commanded the logger to record samples at fixed angular increments. A simple computational operation generated an angular plot of $1/\beta$ versus $\theta_2$. The experiment was repeated for a range of incident angles $\theta_1$.

4.5.4.3. Coherent Detection ($1/\gamma \beta$):

$1/\gamma \beta$ for coherent detection was recorded in a similar way to $1/\beta$ for incoherent detection. From the experimental results obtained for $1/\beta$, $1/\gamma$ can be deduced. Again, the average value and the standard deviation were computed. The signal was recorded as the surface moved across the beam.

4.5.4.4. Coherent Detection: Angular Alignment:

Some experiments were performed to check the alignment equations shown in section 2.1.4. This was done by coupling the vertical adjustment micrometer (Fig. 9b) to a moving table, and recording the signal as the table rotated the micrometer. The data recorded was:

1. speed of rotation
2. sampling speed of data logger
3. the starting angle with respect to the peak of the signal.

From this data plots of signal power versus angular misalignment were obtained. This was performed for various speckle / receiver ratios.
4.5.4.5. Coherent Detection: Transverse Spectra.

To obtain the spectrum of the signal, additional data was required as well as that for computing $\gamma$. The sampling frequency was arranged to be above twice the highest frequency content of the signal. This was achieved by adjusting the speed of traverse and the sampling rate of the data logger. These two factors were also recorded so that spectra could be calculated either in terms of frequency or in reciprocal displacement ($\text{m m}^{-1}$). To obtain reliable spectra, the sample trace must be long (38). Traces of 1000 samples were long enough to give a raw spectrum accuracy of $\approx 16\%$. By hanning (or time filtering), a more stable representation of the spectrum of the random process was obtained.

4.5.4.6. Coherent Detection: Rotational Spectra.

The same considerations apply to the study of rotational spectra, but here the angular velocity and the starting angle $\theta_1$ were recorded. Traces of $1/\gamma \beta$ versus angle $\theta_1$ were drawn by the computer.

4.5.4.7. Coherent Detection ($\gamma$):

It was shown in equation 3.3.6. that $1/\gamma$ can be obtained independently of $\beta$ by recording $P_{LO}$, $P_{DC}$, and $|P_{AC}|$. A detector was constructed that separates these three terms, and hence a separate recording of each can be made simultaneously. This could only be achieved satisfactorily for certain transmitter/receiver configurations. The limitations were set by the minimum direct current the converter could detect, and the maximum local oscillator aperture that did not overload the PMT or amplifier. The complete
possible range of measurements were made and a program written to perform the necessary calculations.

4.6. PREPARATION, ROUGHNESS MEASUREMENT AND ANALYSIS OF TARGET SAMPLES

The surfaces used in this study originate from two sources:

(a) Metallic surfaces whose roughness was modified by abrading
(b) Unadulterated samples, typical of those expected in doppler systems.

The most used specimens were the former as, by measuring their roughness, some prediction of scattering performance could be made. The latter samples were usually too rough to be measured, and as they were of the dielectric type, Beckmann's predictions did not apply.

4.6.1. Sample Preparation

A large variety of techniques were tried in an attempt to obtain a conducting random rough surface, from sand-blasting glass and coating with aluminium or gold, to the simple approach of abrading with a fine grain carborundum. The latter technique was, in fact, the method chosen to roughen two samples, one of rolled aluminium, and the other a mild steel plate. For the aluminium, an attempt was made to get an isotropic surface roughness, but the results suggested that this was not quite achieved. The steel was abraded by using parallel strokes to obtain a random corrugated type of surface. One object of the experiment was to discover a way of quickly modifying a surface to produce a satisfactory scatter pattern. So the surfaces were cleaned, using standard techniques, and carefully sanded in the required fashion. The loose particles were then removed by buffing
Fig. 14. MICROGRAPHS OF FOUR SURFACES

a) Aluminium oxide.  b) Aluminium.

(c) Scotchlite.   d) Steel.

(3" or 1.3 cm = 50 μ)
with a fine diamond paste, and then washed again. These two samples and the samples of aluminium oxide and scotchlite are shown in Fig.14.

4.6.2. Measurement and Analysis

Measurement of the roughness of the surfaces was performed by the kind permission of Rank Precision Industries, on the taly step with an ultra-fine diamond needle. The needle shape is a 65° truncated pyramid with a flat point of dimensions 0.12 μm x 0.1 μm. The direction of traverse is normal to the shorter edge. A typical trace is shown in Fig.15. Table 3 contains a list of the rms roughness σ and correlation lengths T obtained from measurement. The correlation lengths in two perpendicular directions for the aluminium are different, suggesting that the surface was not quite isotropic. The measurements of T and σ were used to predict the scatter patterns for the surfaces and the results can be found in the next Chapter.

<table>
<thead>
<tr>
<th>Surface material and orientation (1)</th>
<th>σ (μm)</th>
<th>T (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium (</td>
<td></td>
<td>)</td>
</tr>
<tr>
<td>Aluminium (⊥)</td>
<td>0.47 ± 0.043</td>
<td>16.4</td>
</tr>
<tr>
<td>Steel (⊥)</td>
<td>0.202 ± 0.008</td>
<td>7.75</td>
</tr>
</tbody>
</table>

TABLE 3

Surface Properties of Aluminium and Steel.

Notes: (1) (||) and (⊥) indicates that the measurements were taken parallel and perpendicular to the grain direction.

(2) T was measured using the spectral width. The standard error was then 16%.

Footnote: * The taly step is an instrument for measuring surface roughness. A diamond stylus, coupled to a variable inductance, traverses the surface of the sample. The deflections of the needle cause an imbalance in a bridge circuit and generates an e.m.f., which is then recorded. (49)
Fig. 15. TYPICAL SURFACE PROFILE.
Surface material: Aluminium.
A probability density analysis of the surface roughness function shows the distribution of the surface heights to be asymmetrical about the mean. This could be the result of smoothing the surface with the fine grinding paste. As the distribution was not far from the Gaussian, it was assumed to be valid for the Beckmann analysis.

4.6.3. Additional Target Samples

A selection of samples, which represent typical targets expected in practice, completed the complement of those used. They are:

(3) A scotchlite sample: a semi-co-operative target material.
(4) Aluminium oxide: a thick layer of white oxide (a dielectric).
(5) Cartridge Paper: one sample plain, another sprayed black.
(6) A selection of other white papers.
(7) Concrete: a sample obtained from a piece of pavingstone.
(8) Aluminium-coated resin with sinusoidal corrugation impressed into the surface.

Scotchlite is included as it is frequently used in Doppler systems because of its high back-scatter ratio at large angles of incidence. The Aluminium Oxide represents that class of dielectrics supposed to give typical lambertian scatter. Concrete is a material of general interest.

The scatter results for these surfaces are presented empirically; no detailed theoretical analysis being presented to confirm findings.
4.6.4. Conclusions on Roughness Measurement

Many preliminary measurements of surface roughness, using a standard needle, indicated that useful results would only be obtained by having an ultra-fine point on the diamond stylus. Such a device is expensive and extremely fragile. The stylus used at Rank was the best available in the world at the time the measurements were taken, and since its dimensions at the tip were $0.1\mu \times 0.12\mu$, it cannot resolve the fine scale roughness, which appears to be the cause of scatter at large angles. Thus this technique has limited applications in the field of optical scatter pattern prediction. In a practical system, it is also unlikely that the target will be small enough or mobile enough to get near a taly step head.

In conclusion, it appears that this approach, though apparently an ideal solution, does not come up to expectations. A more convenient method of obtaining the scatter pattern must be found. A possible solution is suggested in the next Chapter. The results in the next Chapter also show how selective roughening of the surface can improve the scatter pattern.

4.7. Programming

The programming was performed in I C L Algol, using block programming techniques. Each block contains a specific operation, e.g. calculation of mean, and was called by a single program statement. Thus any sequence of operations could be performed by inserting or removing a few cards from the main pack.

The operations supplied were:

(a) Plot of data string.

(b) Calculation and plot of probability density histogram of a data string.
(c) Calculation of the mean and standard deviation of a data string.

(d) Autocorrelation.

(e) Power spectral analysis.

(f) Effective width of data string.

A flow diagram of the full program is presented in Fig. 16. Most of the operational calls were optional and different equations are used for the different experimental configurations.

A brief mathematical description of each block of Fig. 16 is given below:

1. The calculation of $1/\beta$ and $1/\gamma\beta$ was performed using the data recorded during the experiments; the equations used are presented in Appendix 5. The calculation of $1/\gamma$ as described in 4.5.2.7 was more involved, but still only required the addition of 3 extra cards.

2. Calculation of the mean $\mu_d$ was as given by Benda and Piersol (38)

$$\mu_d = \frac{1}{N} \sum_{n=1}^{N} x_d(n)$$

where $x_d(n)$ is the data string and $N$ the number of samples.

The standard deviation $\sigma_d$ was computed using an equation of the form

$$\sigma_d = \left( \frac{1}{N} \sum_{n=1}^{N} \left( \mu_d - x_d(n) \right)^2 \right)^{1/2}$$

the error due to the bias (38) was less than 1% for $N$ greater than 250.

3. The auto-correlation function $A(\tau)$ was computed as given in (38, 33)

$$A(\tau) = \frac{C(\tau)}{C(0)}$$

where

$$C(\tau) = \frac{1}{N-1} \sum_{n=1}^{N} (x_d(n) - \mu_d)(x_d(n + \tau) - \mu_d)$$

and

$$C(0) = \sigma_d^2$$
Details of the blocks numbered are given in the text.

Fig. 16. FLOW CHART OF COMPUTER PROGRAM
Each block is a separate entity; therefore allowing for any combination of operations.
where \( \tau = t_s, 2t_s, \ldots, m t_s \), \( t_s \) is the sampling time interval and \( m \) is the maximum lag value.

\( m \) was selected as 25 to give an accuracy of \( \pm 16\% \) in the power spectrum computed from \( A(\tau) \).

(4) The raw power spectrum \( \hat{P}_d(f) \) was computed from the following equation (33)

\[
\hat{P}_d(f) = 2t_s \cdot \sigma_d^2 \left[ 1 + \sum_{\tau = t_s}^{m t_s} A(\tau) \cos \left( \frac{\pi \tau f}{p m} \right) \right]
\]

where \( f = 0, f_a, 2f_a, \ldots, \ldots, p m f_a, \) and \( f_a = 1/2 p.m.t_s \).

\( p \) determines the number of uncorrelated frequency points in \( \hat{P}_d(f) \), and was taken as 2 for all the calculations.

To smooth the spectrum still further, hanning was applied to the raw spectrum. This effectively time-filters the data.

Mathematically the smoothed spectrum \( P_d(f) \) is defined as

\[
P_d(0) = 1/2 \hat{P}_d(0) + 1/2 \hat{P}_d(f_1)
\]

\[
P_d(f_n) = 1/4 \hat{P}_d(f_{n-1}) + 1/2 \hat{P}_d(f_n) + 1/4 \hat{P}_d(f_{n+1})
\]

\[
P_d(f_{pm}) = 1/2 \hat{P}_d(f_{pm-1}) + 1/2 \hat{P}_d(f_{pm})
\]

(5) The effective width \( f_{\text{eff}} \) of the spectrum is defined as

\[
f_{\text{eff}} = \frac{\sqrt{p.m.\sum_{n=0}^{p.m.f_a} P_d(f_n)}}{2 p.m.t.s. \sum_{n=0}^{p.m.f_a} P_d(f_n)} \left[ P_d(f_n) \right]^2
\]

(6) The probability density histogram was computed by counting the number of times the data string \( x_n \) fell in a class interval.
of width $\Delta z = x_n$. The program determined the maximum and minimum of $x_n$ and divided this interval into 34 class intervals. 34 is the optimum given by Bendat and Piersol for a data string of 1000 samples.

All the programs used here are completely original. This ensured that:

(a) their working was understood;
(b) that they were tailor-made to fit in the overall scheme as described earlier;
(c) that transfer of data from one procedure to another was easier (I C L packages require data in non-standard Algol format); and
(d) that modifications could be made readily (I C L packages cannot be changed).

A plotting program was written to produce graphs of A 4 size, scaling the data accordingly (see Fig. 15). Hundreds of graphs were produced this way, but condensed copies appear in this thesis. The rest of the results were obtained from computer print out.

**Summary**

This Chapter contains a description of the experimental system, calibrating and setting-up procedures, and measurement routines. Since these are in most cases somewhat detailed, only the bare outline was presented. The results of surface measurements were presented together with the list of surfaces used. Mathematical details of the programs used to analyse the experimental data were given and a basic flow chart of the program was shown.

The results obtained from the system described, the discussion of them and conclusions are presented in the next Chapter.
5.0. RESULTS

5.1. INTRODUCTION

The results presented in the following text represent something in the region of 200,000 individual measurements. In most cases, points on a graph have been obtained by averaging, in some way, from 250 to 1000 samples. Such a large quantity of data could not have been obtained without the automation and computation described earlier. On the other hand, so many measurements have been taken that the results should represent a sound estimation of the ensemble of results possible.

The results are presented individually, and the discussion on each given. The general discussion follows the presentation of all the results and the conclusions which follow complete this Chapter.

5.2. INCOHERENT MEASUREMENTS

5.2.1. $1/\beta$ versus Receiver / Speckle Size

Fig. 17 shows plots of $1/\beta$ versus receiver angle, $\phi_R$, for various speckle angles, $\phi_s$, (note: for a focused spot $\phi_T = \phi_s$ and spot size $d_T \propto 1/\phi_T$). The plots represent $1/\beta$ for aluminium at back-scatter normal incidence and the theoretical results, for the same configuration, using the roughness and correlation length obtained for the aluminium surface. Since there were two roughness scales recorded, both are included on the graph. This experiment verifies the form of eqn. 3.2.2. versus $\phi_R$ and the equation is accurate in predicting the absolute
Fig. 17. PLOT OF $1/\beta$ VERSUS RECEIVER APERTURE ANGLE at backscatter normal incidence.

a) Aluminium - Experimental
b. & c) Aluminium - Theoretical
value of $1/\beta$ (assuming the surface is isotropic). The results prove that for the range of apertures used the value of $1/\beta$ is independent of the speckle size or spot size.

Similar results were obtained for other surfaces, and from these a table of $1/\beta$ for a particular value of $\phi_R$ is presented below.

<table>
<thead>
<tr>
<th>Target Material</th>
<th>$1/\beta$ at Backscatter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium (1)</td>
<td>$4.24 \times 10^{-4} \pm 1.5 \times 10^{-4}$</td>
</tr>
<tr>
<td>Steel (2)</td>
<td>$.98 \times 10^{-3} \pm .19 \times 10^{-3}$</td>
</tr>
<tr>
<td>Scotchlite (3)</td>
<td>$1.25 \times 10^{-3} \pm .12 \times 10^{-3}$</td>
</tr>
<tr>
<td>Aluminium oxide (4)</td>
<td>$8.2 \times 10^{-6} \pm .1 \times 10^{-6}$</td>
</tr>
<tr>
<td>White paper (5)</td>
<td>$6.09 \times 10^{-6} \pm .23 \times 10^{-6}$</td>
</tr>
<tr>
<td>Concrete (7)</td>
<td>$2.58 \times 10^{-6} \pm .11 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

**TABLE 4**

Back-scatter Coefficients of various Materials at normal incidence ($\phi_R = .00625$ radians)

It is interesting to note that the isotropic surfaces (paper, scotchlite) have a low spread in the value of $1/\beta$. Concrete appears to be the worst surface of the group.

5.2.2. Ensemble Averaging and Spatial Averaging

An ensemble average of $1/\beta$ was recorded at various angles of incidence (back-scatter) for the aluminium surface and the aluminium oxide surface. This was performed using a spot of 193 $\mu$. A continuous plot of $1/\beta$ versus $\theta_1$ for back-scatter, using a large aperture, thus performing spatial averaging, was recorded for both surfaces.
Fig. 18. BACKSCATTER RESULTS, SPATIAL AND ENSEMBLE AVERAGING

a) Aluminium  b) Aluminium Oxide

c & d) Beckmann's Result for Aluminium
The results are presented in Fig. 18. Also shown are the theoretical results for aluminium, using the Beckmann equation. The agreement is quite satisfactory for $\theta_1$ less than $10^\circ$.

5.2.3. Back-scatter $1/\beta$ versus angle of incidence $\theta_1$

Having shown that spatial averaging and ensemble averaging produce identical results, plots of $1/\beta$ can be made using the former technique. The results for a few surfaces are shown in Fig. 19. The receiver angle was $0.0125$ radians.

Graph (d) represents the result for a turbine blade that was available at the time of the measurements. It is interesting that the scatter does not decrease to an insignificant value. This agrees with visual observation of the blade that showed it to be of a smooth, but matt, finish (i.e. very small scale roughness present). In (a) and (b) the scatter from steel is depicted. In (a) the corrugations are parallel to the plane of incidence, while at (b) they are perpendicular. The difference is significant and important, since by abrading the surface in this way, an improved scattering performance can be achieved in one plane only.

The result for Scotchlite (c), is as expected, showing a high scattering coefficient at large angles of back-scatter incidence. The small peak at $\theta_1 = 0^\circ$ is due to the small specular component from the smooth surface.

The results for aluminium oxide (e), white paper (f), black paper (h), and concrete (g), are all very similar. The blackening of the paper has the expected result, but the change in shape is interesting. The concrete again appears as a poor loser, although
Fig. 19. BACKSCATTER PLOTS OF VARIOUS SURFACES.

Noise level below -60 db.

1/β db

---

a Steel (2) (corrugation $\parallel$ to plane of incidence)
b Steel (2) 
c Scotchlite (3)
d Turbine blade
e Aluminium oxide (4)
f White paper (5a)
g Concrete (7)
h White paper sprayed black (5b)

$\phi_R = 1/\beta$ radians for both graphs.

Angle of incidence $\theta$, degrees.
its significant attraction is that the back-scatter coefficient is virtually independent of angle.

Other plots taken of the different samples of white paper were, in all but one case, within 1 dB of each other. The exception was a sample with a gloss finish, which had a 5 dB peak of width ± 10° in the specular direction. Outside this region the scattering loss factor $1/\beta$ was virtually the same as all the other paper samples. Since $1/\beta$ for paper is well below the theoretical Lambertian scatterer, this suggests that there is significant absorption at the surface, this absorption being the same for all white papers at 6328 Å.

5.2.4. $1/\beta$ for other Angles of Scatter

The plots in Fig. 20 show the response of $1/\beta$ versus $\theta_2$ for angles of incidence of $10^\circ$, $40^\circ$, $70^\circ$. The receiver angle is $0.0125$ radians. Only a few of the results are shown as there appeared to be nothing of a startling nature in those obtained. The steel surface has a notable change in character for $\theta_1 = 70^\circ$, in the appearance of a specular component. However, the scattered component does not appear to change significantly.

In some early preliminary tests, the sinusoidal surfaces were used in an attempt to confirm the results of Beckmann. These were reasonably successful, but in time the resin seemed to become pitted, and the fine scale roughness started to overshadow the effect of the sine wave. The scatter for one of the coarser sine waves is presented in these plots. Comparison with Beckmann's result is virtually impossible because of the effect of the fine scale roughness.
FIG. 20. SCATTER PATTERNS FOR VARIOUS SURFACES
Noise level below -60 db.
Theoretically the pattern should consist of a lobe structure similar to the line structure of the spectrum of a carrier, frequency modulated with a sine wave, but the scatter from the pits and peaks completely smooths this out. (This fine scale roughness could not be detected on the taly step). In simple terms, the three peaks are a result of the three largest specular areas. They are the two regions around zero in the sine wave, and the peak and valley of the sine wave. Viewing the sine wave under a microscope showed a flattening of the peaks and the valleys, hence the centre peak. This would not normally exist.

5.2.5. RMS Fluctuations of Incoherent Signal

Using the results of section 5.2.1., the mean value $1/\beta$ and the standard deviation associated with it, $\sigma_s$, were calculated. Also a probability density histogram was obtained and an attempt made to find a fit with eqn. 3.2.3., without success. The ratio of mean to standard deviation ($S/N$) was plotted against the ratio $\phi_R/\phi_S$. This is shown in Fig. 21. Again, an attempt was made to fit this to eqn. 3.2.4. However, this proved to be impossible since the experimental curve only increases in proportion to $\sqrt{\phi_R/\phi_T}$ whereas eqn. 3.2.4. is directly proportional to $\phi_R/\phi_T$ in the limit. By using a curve fitting technique, the experimental curve was found to fit the equation.

$$S/N = (2.16 \phi_R/\phi_S + 1)^{5.2.1.}$$

to within $1\% \pm 7\%$. The fit can be seen in Fig. 21.

Such a discrepancy can only be explained by assuming the basic assumptions, on which the analysis for equations 3.2.3. and 3.2.4. rests, do not in fact describe this situation. These assumptions
Fig. 21. SIGNAL/NOISE RATIO OF THE INCOHERENT OPTICAL POWER $V_{FR}/V_{FS}$

Empirical eqtn. of the form \[ Y = (2.16 \frac{\phi_R}{\phi_S} + 1)^{\frac{3}{2}} \]
Fits data to within $1\% \pm 7\%$
fail because as the surface is moved, the speckles do not remain constant. They are in a state of flux at all times; thus the situation is far more chaotic than suggested. The equations may apply at some ratio very much larger than the range investigated here.

Angular rotation of the target may more nearly represent the analytic situation, but the non-stationarity of the output prevents a valid experimental investigation being carried out.

5.3. COHERENT MEASUREMENTS

As mentioned earlier, all the coherent measurements were performed in the back-scatter mode, and all but the angular rotational spectra results were obtained at normal incidence. Measurements of $1/\gamma$ and $1/\gamma \beta$ were found to have a large spread of values, and consistent results were difficult to obtain. By recording many results, some degree of reliability was achieved.

5.3.1. $1/\gamma \beta$ versus the Ratio of Receiver to Speckle Size

To date there has been no theoretical description of the way $1/\gamma \beta$ varies with the ratio of the receiver to speckle size. Only intuitive guesses could be made at what result to expect. For these reasons many experimental results were taken for this situation, in the hope that they would produce a clear answer to the problem.

The surfaces used were:

(a) A highly roughened piece of aluminium.
(b) The specially prepared aluminium surface.
(c) Scotchlite (3).
The range of $\phi_R/\phi_S$ ratios used covered all of those of Table I. As the range of ratios for any particular speckle size $\phi_S$ were limited, the results were normalized to one value of $\phi_S$.

5.3.1.1. Normalizing of Results for $1/\gamma\beta$

It was shown in earlier results that $1/\beta$ was independent of $\phi_S$, and a square law function of $\phi_R$. If the plot of $1/\gamma\beta$ showed any deviation from this characteristic, then it could be attributed to $1/\gamma$ only. It was suggested earlier, and this had to be confirmed by experimental results, that $1/\gamma$ would only depend on the ratio $\phi_R/\phi_S$. Assume, for example, that $1/\gamma$ is of the form

$$1/\gamma = \frac{1}{1 + (\phi_R/\phi_S)^2}$$

and it has been shown that $1/\beta = K \phi_R^2$ (see earlier results).

Normalizing $1/\gamma$ to some fixed $\phi_S$ is merely a matter of shifting the abscissa value by the factor $\phi_S/\phi_S^n$ where $\phi_S^n$ is the normalizing value. But since the variable plotted is $1/\gamma\beta$ a correction must now be made for the $1/\beta$ term, and since the value of $\phi_R$ has changed by the factor $\phi_S/\phi_S^n$ in normalizing $1/\gamma$

$$1/\beta \text{ becomes } K \phi_R^2 \left(\frac{\phi_S^n}{\phi_S}\right)^2$$

For a measurement of $1/\gamma\beta$ at values of $\phi_R$ and $\phi_S$ the normalized value is

$$\frac{1}{1/\gamma\beta} = \frac{\phi_S}{\phi_S^1}$$

at a receiver aperture angle of $\phi_S^1$. This normalization does not implicitly rely on the form of the function $1/\gamma$; however, $1/\gamma$ must only be dependent on the ratio...
for this to be valid. Should the normalized values then
fit a single curve, then this dependance can be assumed proven.

The normalized results are shown in Figs. 22 and 23. As
can be seen, many results were obtained for the special aluminium
surface (Fig. 22), but there is a significant spread in these
values of $1/\gamma^3$ for ratios of $\phi_R/\phi_S$ in the region 1 to 10.

Included on these curves is $1/\gamma$ as plotted in Fig. 17.
The difference between the two curves gives the factor $1/\gamma$.
These curves require some explanation.

Two factors could be identified that
could account for the spread of the results. Assuming the
experimental system was reliable, they were:
(a) Poor alignment.
(b) Non-uniform (stationary) surface.

The first variable was checked by very careful adjustment
of the local oscillator beam before each run. However, there was
no significant difference in the results. Investigation into the
effect of the second variable factor, by using the same area of
target for each traverse while carefully checking the alignment,
produced results with a much lower spread (see Fig. 22). Thus it
appears that the spread in the results shown on graphs 22 and 23
is due in some way to the random nature of the surface. A discussion
on this will appear later.

The break point of the results in Figs. 22 and 23 is in the
region of $\phi_R/\phi_S = 1$. Since the results shown represent many
values of $\phi_S$ which have been normalized mathematically to obtain
one continuous curve, the dependance of $1/\gamma$ on $\phi_R/\phi_S$
can be accepted as proven. By fitting the asymptote of $1/\gamma^3$ versus $\phi_R/\phi_S$ for
Fig. 22. NORMALISED PLOT OF $1/\gamma B$ FOR ALUMINIUM
a) 1st series.  b) 2nd series.
c) repeat of b) using same area of surface.

$\phi_s = .004$ radians

$1/\beta$ from Fig. 17.

$\phi_R = \phi_s$

Function of form shown in Fig. 23.
The broken line represents the equation of the form

\[ \frac{1}{\gamma \beta} = \frac{1}{\beta'} \left( \frac{\phi_R^2}{\phi_R^2 + \phi_S^2} \right) \]

where \( \beta' \) is the value of \( \beta \) at \( \phi_S = 0.004 \) radians.

Fig. 23. PLOT OF \( 1/\gamma \beta \) FOR TWO MATERIALS

a) Aluminium  b) Scotchlite

- \( \times \) Rough Aluminium
- \( \circ \) Scotchlite
large values of the latter, a theoretical curve of $1/\gamma \beta$ for different speckle sizes is obtained. The equation of this curve is

$$\frac{1}{\gamma \beta} = \frac{1}{\beta^1 \left( \phi \right)^2} \frac{\phi^2 R^2 S^2}{\phi S^2 + \phi R^2},$$

where $\frac{1}{\beta^1}$ is the value of $1/\beta$ for receiver aperture $\phi R^2$, an empirical relationship that fits the results reasonably well.

Fig. 23 shows the equation above plotted together with the results obtained using the rough aluminium target and scotchlite. Eqn. 5.3.2. does, in fact, include $1/\gamma$ in the form assumed in eqn. 5.3.1.

These results were obtained with the transmitter in focus and to change the ratio of $\phi R/\phi S$ either of the apertures $A_T$ or $A_R$ had to be altered. An alternative method is to move the transmitter out of focus, thereby increasing the spot size, or decreasing the speckle size. The experiment was performed for many combinations of $A_T$, $A_R$ and $F_T$. The normalized results are presented in Fig. 24.

These results confirm the statement above that the detector coherence loss factor $1/\gamma$ is solely a function of $\phi R/\phi S$, although only for the range of spot sizes used in the experiments.

5.3.2. RMS Fluctuation about $1/\gamma \beta$.

The results used to plot $1/\gamma \beta$ versus $\phi R/\phi S$ were also used to investigate the RMS fluctuation of $|P_{AC}|$ and $|P_{AC}|^2$.

In section 3.3.3. it was mentioned that

(a) for the Rayleigh distribution : $\sigma = \mu/1.9$

(b) for the negative exponential distribution : $\sigma = \mu$. 
\[ \phi_R = 0.004 \text{ radians} \]

Broken line represents equation of the form

\[ \frac{1}{\gamma \beta} = \frac{1}{\beta^1} \cdot \frac{\phi S^2}{\phi S^2 + \phi R^2} \]

where \( \beta^1 \) is the value of \( \beta \) for \( \phi_R = 0.004 \text{ radians} \)

Fig. 24. PLOT OF \( 1/\gamma \beta \) AS TRANSMITTER IS MOVED OUT OF FOCUS
Fig. 25. PROBABILITY DENSITY DISTRIBUTION OF $1/\gamma \beta$

$1/\gamma \beta = 1.68 \times 10^{-5}$

$\sigma_s = 1.62 \times 10^{-5}$

$D_T = 4\text{mm}$  $F_T = 20\text{mm}$

$D_R = 1\text{mm}$  $F_R = 16\text{mm}$
The ratio of $\mu/\sigma$ for the distributions (a) and (b) above were obtained from experimental results and plotted against $\phi_R/\phi_S$.

For the distribution of $|P_{AC}|$ the results were

$$\text{mean} = 1.92 \pm 0.164 \times \text{standard deviation.}$$

For the distribution of $|P_{AC}|^2$

$$\text{mean} = 1.02 \pm 0.08 \times \text{standard deviation.}$$

The ratio for scotchlite was within the error above for the aluminium surface. A typical distribution of $|P_{AC}|^2$ is presented in Fig. 25. When fitted to a negative exponential such that the $1/e$ point equals the mean value it has $\chi^2 = 11$ with 30 degrees of freedom, giving a level of significance of 0.995.

Having shown that the probability density of $|P_{AC}|^2$ is negative exponential, we can now obtain the cumulative probability distribution $P_C(Z)$ from the relationship

$$P_C(Z) = \int_0^Z \frac{1}{2 \left( \frac{1}{\gamma \beta} \right)^2} e^{-\frac{Z}{\left( \frac{1}{\gamma \beta} \right)^2}} dZ$$

where $Z = \frac{|P_{AC}|^2}{P_T P_{LO}}$ and

$$P_C(Z) = 1 - e^{-Z^2 / \left( \frac{1}{\gamma \beta} \right)^2}$$

This equation is needed in determining how frequently the signal drops below some threshold level $Z$. 5.3.1.
5.3.3. Transverse Power Spectra

A discussion of Doppler broadening was presented in section 3.5.2. The equation for the effective spectral width was found to be \( f_{T} = \frac{V}{\lambda} \cdot 97 \left( \phi_{T}^{2} + \phi_{R}^{2} \right)^{\frac{1}{2}} \).

By dividing both sides of the above equation by \( v_{T} \), the spectral width \( f_{T}/v_{T} \) has units of \((\text{mm}^{-1})\). The results are presented in this form. As the spectrum obtained by experiment was only half the full spectrum, the theoretical width is divided by two. The theoretical curve is shown in Fig. 26 together with the experimental results for aluminium and scotchlite. The results for aluminium are in error by 18.0% ± 10.8%. The scotchlite results are slightly worse. This deviation is to be expected since the technique of using the spectrum of the envelope of a signal to predict the frequency content of the signal is subject to error.

With this information the spectral width of the doppler signal due to the transverse movement of the surface can be predicted.

The results used for obtaining a plot of \( 1/\gamma \beta \) for an out-of-focus spot can also be used in transverse broadening experiments. Fig. 26 also includes the spectral width versus \( \phi_{S} \) as the transmitter was moved out of focus. In this case the change in the spectrum is insignificant.

A discussion of these results will be included in a later section.
Fig. 26. SPECTRAL BROADENING DUE TO TRANSVERSE MOVEMENT OF THE TARGET.
The measurement of the rotational spectrum of the signal poses more problems than the other measurements. There are two reasons for this:

(a) the nonstationary nature of the back-scatter signal
(b) the limited length of trace available (\(-180^\circ\) to \(180^\circ\)).

The results are presented in Fig. 27 together with the two asymptotes calculated from equations 3.5.6. and 3.5.7. Considering the associated difficulties, these results are as good as can be expected and prove, though somewhat loosely, the form of the spectrum as expressed in section 3.5.2.2.

5.3.5. Angular Alignment

Fig. 29 shows the results for some of the angular alignment experiments performed. The broken line is the result for a uniform signal field (i.e. mirror at the target); the remaining lines show a selection of plots for the same receiver aperture but with a speckle field. The alignment restrictions are more relaxed but there are many peaks on which to align the local oscillator.

Table 5 shows the widths of the peaks for all the measurements recorded, and compares them to the theoretical values.

<table>
<thead>
<tr>
<th>(\phi_R/\phi_S)</th>
<th>D_R</th>
<th>Angular Misalignment</th>
<th>Target Material</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Experimental</td>
<td>Theoretical(1)</td>
</tr>
<tr>
<td></td>
<td>mm</td>
<td>secs arc</td>
<td>secs arc</td>
</tr>
<tr>
<td>0.1</td>
<td>0.16</td>
<td>1950 ± 35</td>
<td>1980</td>
</tr>
<tr>
<td>0.5</td>
<td>2.0</td>
<td>148 ± 20</td>
<td>159</td>
</tr>
<tr>
<td>0.5</td>
<td>2.0</td>
<td>168 ± 14</td>
<td>159</td>
</tr>
<tr>
<td>1.6</td>
<td>0.64</td>
<td>506 ± 98</td>
<td>500</td>
</tr>
<tr>
<td>2.9</td>
<td>1.0</td>
<td>640 ± 45</td>
<td>318</td>
</tr>
<tr>
<td>5.0</td>
<td>2.0</td>
<td>572 ± 32</td>
<td>159</td>
</tr>
</tbody>
</table>

TABLE 5. Angular Misalignment Data
Fig. 27: Spectral broadening due to rotation of the target.

\[ \frac{1}{\phi_T} = 0.0125 \text{ rads.} \]
Angular rotation of LO mirror - .secs. arc

Fig. 28. SIGNAL VERSUS ALIGNMENT

for a) Uniform field; b) & c) Speckle field.
Notes:

(1) Based on equation 2.1.7. presented earlier from Warden (7).

(2) With a mirror at the target a uniform signal field is generated, and reproduces the conditions under which 2.1.7. is true.

For $\phi_R/\phi_S < 1$ equation 2.1.7. gives similar results, but for $\phi_R/\phi_S > 1$ there is a significant discrepancy. This can be explained by considering the virtual spot at the target produced by the local oscillator. If $\phi_R > \phi_S$, then the virtual spot will be smaller than the transmitter spot, and to obtain a mixing signal, the virtual spot can be located anywhere inside the transmitter spot. Thus the alignment conditions depend on the transmitter as well as the receiver.

5.3.6. $1/\gamma$ versus $\phi_R/\phi_S$.

$1/\gamma$ was measured using the electronic circuit described in Chapter 4. The results are tabulated below in Table 6.

<table>
<thead>
<tr>
<th>$D_R / D_S$</th>
<th>Mean Signal</th>
<th>Signal to Noise</th>
<th>1/3</th>
<th>coh</th>
<th>in coh</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>coh</td>
<td>in coh</td>
<td>coh</td>
<td>%</td>
<td>%</td>
</tr>
<tr>
<td></td>
<td>$1/\gamma \times 10^{-4}$</td>
<td>$1/\beta \times 10^{-4}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td>1.9</td>
<td>15.0</td>
<td>1.05</td>
<td>2.9</td>
<td>12.5</td>
</tr>
<tr>
<td>2.5</td>
<td>2.5</td>
<td>16.0</td>
<td>.9</td>
<td>2.9</td>
<td>15.5</td>
</tr>
<tr>
<td>1.8</td>
<td>1.7</td>
<td>6.8</td>
<td>.95</td>
<td>2.4</td>
<td>25.0</td>
</tr>
<tr>
<td>1.8</td>
<td>1.9</td>
<td>7.6</td>
<td>.91</td>
<td>2.45</td>
<td>25.0</td>
</tr>
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<td>1.25</td>
<td>1.1</td>
<td>3.0</td>
<td>1.0</td>
<td>1.75</td>
<td>37.00</td>
</tr>
<tr>
<td>.625</td>
<td>1.1</td>
<td>2.2</td>
<td>1.0</td>
<td>1.55</td>
<td>50.00</td>
</tr>
<tr>
<td>.41</td>
<td>0.8</td>
<td>1.1</td>
<td>.99</td>
<td>1.3</td>
<td>72.00</td>
</tr>
</tbody>
</table>

**TABLE 6**

Results of Direct Measurement of $1/\gamma$. 
Difficulty was experienced in obtaining a large enough signal to overcome the local oscillator drift, so only a restricted number of results have been recorded. They show a significant drop in $1/\gamma$ as $\phi_R/\phi_S$ increases, and again corroborate the theoretical and experimental results already presented.

5.4. DISCUSSION

Some of the results described earlier require explanation in the light of present knowledge of the subject. This is especially true of the coherent detection results.

5.4.1. Incoherent Measurements

Although the theoretical equation presented by Beckmann and Spizzichino is accurate within experimental error in describing the measured results over a small range of angles ($\pm 10^\circ$), the limitations mentioned earlier of the surface measuring process restrict the possibility of obtaining a more complete model of the scattering process at optical frequencies. On the other hand, laboratory experiments of the form described in 4.5. and the results of 5.2., suggest that a far simpler solution to the problem of determining $1/\beta$ for any particular system is to perform a measurement in the laboratory or use the results published in the literature over the past three to four years. Such an experiment can be carried out quite simply, and if need be, the surface could be modified to obtain what appears to be a satisfactory scatter pattern.

5.4.1.1. Wavelength.

No experiments were performed to investigate the effect of wavelength on the value of $1/\beta$. Theoretical analysis for a very rough surface predicts that the scattering profile is independent
of the wavelength. This is true for small changes of $\lambda$, but as $\lambda$ is increased to lengths of the order of 10 $\mu$m, even a surface rough at .6 $\mu$m may look quite smooth at the former wavelength. This will have the effect of making the target more specular and hence $1/\beta$ will decrease quite rapidly out of the specular direction. An effect like this could be an inconvenience, and may cancel the advantage the higher powered $\text{CO}_2$ laser provides at this wavelength.

There is the possibility, with the appearance of the high power argon lasers, to operate at a shorter wavelength. The change in $\lambda$ from .63 $\mu$m to .4 $\mu$m would have an insignificant effect on $1/\beta$; thus the full potential of the increased power available may be realized in terms of an incoherent system.

5.4.2. Coherent Measurements

The results depicted in Figs. 22, 23, and 24 require some explanation. It is abundantly clear that the results are not consistent, and the analytical function shown represents a maximum value for $1/\gamma\beta$ rather than a mean.

Experiments were performed to investigate the reason for the spread in the results, and it appears that using the same area of the target for each traverse of the surface, does in fact decrease the spread of $1/\gamma\beta$. The wide deviation in results suggests they may be due to surface anisotropy. However, this seems unlikely since the widest divergence coincides with the experimental condition of a large spot at the target, and the surface was traversed some 3 - 4 cm to obtain sufficient uncorrelated measurements of $1/\gamma\beta$. 
An alternative explanation is that the frequency beats due to
doppler broadening have a greater effect on the value of $\frac{1}{\gamma \beta}$
than expected. This may be so since the results for Fig.22 are
the average of 250 samples for each point, and those of Fig.23 of
1000 samples. This four-fold increase would reduce the spread
introduced by frequency beats.

5.4.2.1. An Explanation of the Shape of the Curve.

A quite simple explanation of the results can be presented
that satisfies the measurement of $\frac{1}{\gamma \beta}$ and the misalignment results.
An interpretation of this form may also lead to a development of a
different form of instrument.

The local oscillator beam consists of a uniform optical
field of aperture $A_R$. This field appears to originate from a virtual
target spot of diameter

$$ d_R = 2.44 \frac{F_R \lambda}{D_R} \quad \text{(See Fig. 29)}, $$

and angle of view $\phi_R = \frac{D_R}{F_R}$.

Similarly, for the transmitter beam

$$ d_T = 2.44 \frac{F_T \lambda}{D_T} \quad \text{and} \quad \phi_T = \frac{D_T}{F_T}. $$

Mixing of the local oscillator and the signal beam can be considered
by referring everything to the target position rather than the
photodetector surface (or receiver aperture). There are then
three conditions to investigate.

1. $d_R > d_T$

All the transmitted light is enclosed by the virtual spot $d_R$;
therefore all of the signal beam mixes with the local oscillator.
Fig. 29. TRANSMITTER AND RECEIVER CONFIGURATION

d_T transmitter spot.  d_R virtual receiver spot.
However, $\phi_R$ is small and so only a small portion of the mixed light actually reaches the detector. $1/\gamma \beta$ is proportional to $(\phi_R)^2$ in this region.

\[(2)\quad d_R = d_T.\]

In this situation all the signal mixes with the local oscillator and the angle $\phi_R$, equal to $\phi_T$, is greater than in condition (1); hence the signal is larger.

\[(3)\quad d_R < d_T.\]

Under this condition only the light that coincides with $d_R$ can participate in the mixing; thus as $d_R$ is reduced ($\phi_R$ increasing) the active area of mixing is diminishing $\propto d_R^2$. However, at the same time, $\phi_R$ is increasing at the same rate; therefore although the number of scattering centres participating in the mixing is decreasing, the angle into which the mixed signal scatters is increasing at the same rate; thus in the limit the signal level remains constant.

This argument also lends insight into the alignment problem, since the requirement for good alignment is that the two spots should overlap. Now in (1) and (3) above there is a margin of error, whereas in (2) this is not so. Of course the absolute angular stability does decrease as $\phi_R$ increases, but only for $\phi_R < \phi_T$. Beyond this the angular stability of the system remains constant. Thus in some environments where vibration may affect alignment, it could be advantageous to use a ratio of $\phi_R/\phi_T$ greater than unity.

5.4.2.2. Spectral Broadening.

The important feature of the experiments to measure spectral broadening is that they show (Fig.26) that the broadening does not depend on the speckle size directly but on the transmitter angular
aperture $\phi_1$ thus proving the form of equation 3.5.6. Estes et al (45) experienced difficulty in establishing this dependance using an out of focus spot, probably because they used a self-beating technique (no local oscillator) instead of true coherent detection. Out of focus coherent mixing was not investigated in detail, so no definite conclusions can be drawn as to the actual physical mechanism creating such an effect or the implications of the results. It appears unlikely that a system would operate at any significant distance out of focus since the signal level drops so quickly (Fig. 24) under this condition.

5.4.2.3. Scotchlite.

Apart from the obvious differences in the backscatter $(1/\beta)$ results, one of the most interesting results of this research is the remarkable similarity of results between scotchlite and rough aluminium. The only time that there was a significant difference was for the situation when the spot diameter $d_1$ was less than the glass bead diameter $(50\mu)$. Here the signal was noticeably different, consisting of high amplitude, but narrow, current spikes, separated by an unusually low signal. A signal of this nature would make intolerable demands on the design of suitable detecting electronics and this situation should be avoided if at all possible.

For a spot diameter of $1 \frac{3}{4}$ times the bean size, the signal reverted to the more normal speckle modulated signal, and the coherent detection results for this spot diameter and all larger spot sizes were identical to those for Aluminium, within the experimental error. It is this similarity, especially in the
measurement of \( 1/\gamma \), that helps to establish the independence of \( 1/\gamma \) on the property of the scattering material.

5.4.3. **System Design Optimisation**

A complete system design cannot be treated here since each problem has its own peculiarities and characteristics and a purely general treatment is too cumbersome to be able to provide any meaningful or useful results. However, a design method is described that provides a secure grounding on which to base an analysis of a particular problem.

It is assumed that the problem is of the Doppler system type, where the aim is to measure velocity by detecting and measuring the frequency of the Doppler shift.

Errors in the measurement of the frequency arise in two areas:

(a) Signal to noise ratio of the signal at the discriminator (noise is Johnson or shot).

(b) Spectral broadening due to undesirable velocity components.

These will be treated separately for the present.

5.4.3.1. **Signal to Noise Ratio.**

All the information required to predict the signal to noise ratio has been presented. The most important parameter is the value of \( 1/\beta \). If the system must operate at a fixed angle of incidence, then \( 1/\beta \) must be determined at that angle, otherwise a plot of \( 1/\beta \) is made for the specific conditions envisaged for the instrument. An alternative solution is to use the results published
in the literature (13, 14, 30), although it may be necessary to convert the results to the correct form.

An approximate analytic function for $1/\beta$ is then required, so that a design equation can be produced which includes the geometric parameters ($\theta_1, \theta_2, \theta_3$).

Thus, assuming

$$1/\beta = f(\theta_1, \theta_2, \phi_R)$$

then from equations 3.3.8, 5.3.1.

$$< 1 P_{Ac} >^2 = \frac{m(1 - m) p^2 4\Omega_1 \Omega_2}{[1 + (\phi_R / \phi_T)^2]} \cdot f(\theta_1 \theta_2) \quad \text{..... 5.4.1.}$$

where $m/(1 - m)$ is the beam splitting ratio, and $\Omega_1, \Omega_2$ are the power losses in the signal and local oscillator beams, respectively, and the target is assumed to be in the focal plane of the transmitter.

From equations 3.4.1 and 3.4.3, the signal power $W_S$ at the output of the photo-multiplier is given by

$$W_S = K_p^2 R_L < 1 P_{Ac} >^2 \quad \text{..... 5.4.2.}$$

and the $S/N$ is given by

$$S/N = W_S / W_n \quad \text{..... 5.4.3.}$$

where $W_n$ is the thermal noise and shot noise.

By combining equations 3.5.4, 5.4.1, 5.4.2 and 5.4.3, an equation giving the minimum measurable frequency in terms of the system parameters can then be obtained.
5.4.3.2. Spectral Broadening.

It was shown in previous chapters that broadening of the doppler signal is created by transverse and rotational velocities at the target. A theoretical description of the broadening in terms of the system parameters has been proven experimentally and can be used to predict the performance of the system.

The broadening due to a transverse velocity $V_T$ is given by equation 3.5.5.

$$f_T = \frac{V_T}{\lambda} \cdot 0.97 \sqrt{\frac{\phi_T^2 + \phi_R^2}{\phi_T^2 + \phi_R^2}} \quad \ldots \; 5.4.4.$$ 

and for a rotational velocity $\omega$

$$f_R = \frac{3.8 \omega}{\sqrt{\phi_T^2 + \phi_R^2}} \quad \ldots \; 5.4.5.$$ 

The equation 5.4.5. is an empirical relationship that fits the results and has asymptotes as shown in Fig.27. The combined spectrum, to a first approximation, is given by

$$f_B = \sqrt{f_R^2 + f_T^2} \quad \ldots \; 5.4.6.$$ 

and adopting $f_B/2$ as a reasonable criterion for the minimum detectable frequency shift, equations 5.4.4, 5.4.5, 5.4.6. provide an expression giving the minimum detectable frequency in terms of the optical system.

5.4.3.3. System Optimisation.

The overall system performance depends on the relative size of the two error producing terms. An ideal system
optimisation would result in the two terms (a) and (b) being equal. However, in some situations where the transverse or rotational velocities are high, the $S/N$ ratio would be so poor under this condition that the system could not in fact work. A completely general design equation could be solved on a digital computer, but the sensible designer could make some simplifying assumptions and greatly reduce the complexity of the problem.

Losses due to an 'out of focus' transmitter can be included by using the results shown in Fig. 24. However, as no results were recorded for an 'out of focus' receiver, the actual loss may be higher if the target moves some distance from the focus.

In conclusion, the results of this study allow the designer to produce performance predictions of a laser doppler radar system, but each system requires special treatment to account for its own peculiarities and merits.
# REFERENCES


11. MIROVITSKIY, D.I. et al. 'Optical Simulation of Reflections and Scatter of Microwaves'.

   IEEE. Trans.1968, AP-16, p.612.

13. RENAU, J., COLLINSON, J.A. 'Measurements of Electromagnetic Backscattering from Known Rough Surfaces'.

14. RENAU, J., CHEO, P.K., COOPER, H.G. 'Depolarization of Linearly Polarized E.M. Waves Backscattered from Rough Metals and Inhomogeneous Dielectrics'.

15. BARRICK, D.E. 'Unacceptable Heigh Correlation Coefficients and the Quasi-Specular Component in Rough Surface Scattering'.


17. PEAKE, W.H. 'Theory of Radar Return from Terrain'.
   IRE Intern. Conv. Record., 1959, 7, p.27.

18. GOODMAN, J.W. 'Some Effects of Target-Induced Scintillation on Optical Radar Performance'.

19. GOLDFISCHER, L.I. 'Autocorrelation Function and Power, Spectral Density of Laser Produced Speckle Patterns'.

20. DAINTY, J.C. 'Some Statistical Properties of Random Speckle Patterns in Coherent and Partially Coherent Illumination'.


32. Private Communication - (M.S. Hodgart, University of Surrey).


47. Private Communication - (M.S. Hodgart, University of Surrey).


To obtain some insight into the nature of the variations of $\gamma$ as the receiver aperture is increased in size, so enclosing more speckles, a program was written to calculate $\gamma$ as a function of $D_R/D_S$. A one-dimensional speckle field is created, the shape of the speckles being arbitrary, and a one-dimensional aperture of varying size scanned across it. The phase of a speckle is constant, but can be arbitrarily set with respect to its neighbour. $\gamma$ is calculated as

$$\left(\frac{\sum E_n \Delta\phi}{\sum E_n^2}\right)^2$$

where the summation is performed over the receiver aperture and the average taken over the scanning positions.

A copy of the program is shown overleaf.

The format for the input data is as follows:

<table>
<thead>
<tr>
<th>Symbol in Program</th>
<th>Number of Data Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>NP</td>
<td>Number of peaks in speckle field.</td>
</tr>
<tr>
<td>DPS</td>
<td>Number of discrete points inside half speckle.</td>
</tr>
<tr>
<td>A (I)</td>
<td>Sample values of half speckle shape (0 - 1).</td>
</tr>
<tr>
<td>PKS</td>
<td>Peak value of 1st speckle (0 - 100)</td>
</tr>
<tr>
<td>PHS</td>
<td>Phase of 1st speckle (Repeat last two for each speckle NP times).</td>
</tr>
<tr>
<td>NAP</td>
<td>Number of aperture values to be used.</td>
</tr>
<tr>
<td>APM</td>
<td>Max. aperture (given as the ratio of true aperture/speckle size)</td>
</tr>
<tr>
<td>AP</td>
<td>Aperture values (given as number of discrete points of speckle field inside half the true aperture)</td>
</tr>
</tbody>
</table>
Program then prints out a table of the instantaneous values of:

- Real part of field vector $E$
- Imaginary part of field vector $E$
- Intensity ($\alpha E^2$)
- Coherent signal power $P_{ac}$
- Incoherent signal power $P_{dc}$
- $1/\gamma$ (ratio $P_{ac}/P_{dc}$)

Printed at the foot of the table are the mean values of these quantities.
11/10/71     COMPILED BY XALE MK. 5C

1. LIST (LP, 47)
2. SENDTO (ED, QA COMP, ZZZZ)
3. WORK (ED, QA WORK)
4. COMPACT DATA

5. PROGRAM USER
6. INPUT 3=CR0
7. OUTPUT 0=LP0, 45
8. OUTPUT 2=LP2, 45
9. SPACE 100
10. TRACE 2
11. BEGIN SELECTINPUT(3); SELECTOUTPUT(0);
12. "COMMENT" **** INPUT FROM DOC SOURCE **** ;
13. "BEGIN" REAL PI, CS, SN, PKS, D1, C1, C2; INTEGER NP, DPS, PKS, I, J, K, AP, NAP, APM;
14. NP=READ; DPS=READ; DPS=DPS*2;
15. "BEGIN" ARRAY B[0]; DPS=NP, 1:6, 1:1; A[1:DPS];
16. PI=4*ARCTAN (1);
17. "FOR" I=1 STEP 1 'UNTIL' DPS/2' DO' "BEGIN" A[I]:=READ;
18. A[(DPS+1-I)]:=A[I];
19. "END' OF SPECKLE SHAPE CALC;
20. "FOR" I=0 STEP 1 'UNTIL' NP-1' DO'
21. "BEGIN" PKS:=READ; PKS:=PKS/100; PKS:=READ; CS:=COS(PKS*PI/180);
22. SN:=SIN(PKS*PI/180);
23. "PRINT" (PKS, 0, 2); "PRINT" (PKS, 3, 0);
24. "FOR" J=1 STEP 1 'UNTIL' DPS 'DO'
25. "BEGIN" B[J+DPS, 2, 1]:=CS*A[I]*PKS;
28. "END'
29. "NEWLINE" (2);
30. "END' OF SPECKLE FIELD CALC;
31. NAP:=READ; APM:=READ; APM:=(APM/2+0.5)*DPS;
32. "FOR" K=1 STEP 1 'UNTIL' DPS 'DO' "BEGIN" B[K, 5, 1]:=B[K, 6, 1]:=0; "END'';
33. "FOR" K=NP*DPS-APM 'STEP' 1 'UNTIL' NP*DPS 'DO' "BEGIN" B[K, 5, 1]=;
34. B[K, 6, 1]:=0; "END'';
35. "FOR" I=1 STEP 1 'UNTIL' NAP 'DO'';
36. "END".
```
'BEGIN' AP:=READ;'FOR' K:=2*STEP 1'UNTIL'6'DO' B[0,K,1]:=0;
'FOR' K:=AP+1*STEP 1'UNTIL'NP*DPS=APM'DO'
'BEGIN' D1:=C1:=C2:=0;
'FOR' J:=AP*STEP 1'UNTIL' AP-1'DO'
'BEGIN' C1:=C1+B[0,K,2];C2:=C2+B[0,K,4];
D1:=D1+B[0,K,6];

'END'OF APERTURE INTEGRATION;
B[K,5,1]:=(C1+C1+C2+C2)/AP/AP/4;
B[K,6,1]:=D1/AP/2;
'FOR' J:=2*STEP 1'UNTIL'6'DO' B[0,J,1]:=B[0,J,1]+B[K,1];
'END'OF CALC FOR ONE APERTURE;
WRITETEXT('COHERENT%CALC%APERTURE=1');PRINT(2*AP/DPS,1); r2);
NEWLINE(2);
WRITETEXT('%REAL%EX%IMAG%INTENS%SIG%DC%1/GAMMA%1');
NEULINE(1);
'FOR' K:=1*STEP 1'UNTIL'DPS=NP'DO'
'BEGIN' 'FOR' J:=2*STEP 1'UNTIL'6'DO' PRINT(B[K,J,1]);
'END'OF PRINTING DATA;
'FOR' J:=2*STEP 1'UNTIL'6'DO' 'BEGIN' B[0,J,1]:=B[0,J,1]+100/
(NP*DPS=CAPM);
'PRINT(B[0,J,1]);
'END'OF PRINTING MEANS;
PRINT(B[0,5,1]/B[0,6,1]=100,3,1);

PAPERTHRUSH;
'END'OF ONE SCAN;
'END';
'END'OF PROGRAM;
'END';
```

```
NO OF BUCKETS USED 30
COMPILED #USER EC
```
The doppler broadening introduced by transverse movement of the target can be considered to be due to the finite angular spread of the converging transmitter beam or diverging scattered beam. Consider an example as shown in Fig. 5a. A transmitter point A is focused onto the target at C. A plane through A and C, parallel to the vertical OB, makes an angle $\phi/2$ with the plane normal to the velocity vector (plane BOC). Thus the doppler shift of the light from A reaching the target is

$$\frac{v \sin(\phi/2)}{\lambda}$$

All the other points on the intersection of the plane ACD and the transmitter aperture will suffer the same doppler shift on reaching the target. Therefore the spectrum $P(f)$ of the light at the target is given by

$$P(f) = \int_{-\infty}^{\infty} F(x, y) \, dy$$

where $F(x, y)$ is the intensity distribution over the transmitter aperture, and $x = \frac{F_T \lambda f_v}{v}$.

Assuming $\phi/2$ is small.

For a uniformly illuminated aperture

$$F(x, y) = A \quad \sqrt{x^2 + y^2} < D_{T/2}$$

$$= 0 \quad \sqrt{x^2 + y^2} > D_{T/2}$$

where $A$ is the intensity.
The spectrum then becomes

\[ P(f) = 2A \sqrt{\frac{D_T^2}{4} - \left(\frac{F_T \lambda f}{v_T}\right)^2} \]

for \( f < \frac{v_T D_T}{2F_T \lambda} \)

\[ f > \frac{v_T D_T}{2F_T \lambda} \]

The effective width of this spectrum is

\[ f_{TT} = \frac{v_T D_T}{F_T \lambda} \frac{\pi^3}{32} \]

This analysis has been kept simple to provide a clearer understanding of the mechanism involved. Estes et al.\(^{45}\) give a more detailed analysis for a Gaussian beam profile. However, for the uniform profile considered, a similar analysis would be unduly involved.

The analysis above describes the effect of the transmitter only, and assumes that the receiver angular aperture is much smaller than that of the transmitter. For a situation where the transmitter is a nearly parallel beam, and the receiver has a large angle of view, then a similar result is obtained for receiver broadening. For a complete solution, involving both apertures, a two-dimensional convolution of the apertures must be performed. A simpler approximate solution is to take the root of the sum of the squares of the two spectral widths.

Thus

\[ f_T = \frac{V_T}{\lambda} .97 \sqrt{\left(\frac{D_T}{F_T}\right)^2 + \left(\frac{D_R}{F_R}\right)^2} \]

describes the spectrum of the signal for all the conditions of receiver and transmitter aperture.
ROTATIONAL BROADENING

By resolving the motion of a target into three basic components, linear motion in the direction of the beam, linear motion normal to the beam, and rotation, the rotational component can be considered to act about an axis through the centre of the spot at the target.

The doppler broadening due to the rotational component is therefore a function of the amount of light in a particular element of the spot, and the distance this element is from the axis of rotation (See Fig.30).

If the spot intensity distribution is of the form \( F(x, y) \) and the rotational axis is the \( y \) axis, then the spectrum \( P(f) \) is given by

\[
P(f) = \int_{-\infty}^{\infty} F \left( \frac{f \lambda}{2 \omega}, y \right) dy.
\]

where \( \omega \) is the rotational velocity.

Thus the magnitude of the component at a frequency \( f \) is the sum of the components parallel to the \( y \) axis at a distance \( \left( \frac{f \lambda}{2 \omega} \right) \) from the axis.

For a uniformly illuminated lens of aperture diameter \( D_t \) the function is of the form:

\[
f(r) = 4 \frac{J_1^2(a \cdot r)}{a^2 r^2}
\]
Fig. 30. **ROTATIONAL BROADENING**

Rings show the minima of the spot intensity function. Total power incident on surface element with velocity $\omega x_1$ is given by

$$\int_{-\infty}^{\infty} I(x, y) \, dy$$

where $I(x, y)$ is the spot intensity function.
Where \( r \) is a radius vector,
\[
    r = \sqrt{x^2 + y^2},
\]
\[
    a = \frac{2 \pi D_T}{\lambda F_T}
\]
and \( F_T \) is the focal length of the lens.

To find the spectrum the function
\[
    \int_{-\infty}^{\infty} 4 J_1^2 (a r) \frac{dy}{a^2 r^2}
\]
must be evaluated. However, this is somewhat tedious and as the effective width of the spectrum is all that is needed, a more convenient solution, using the Hankel transform, is used (47).

From a table of Hankel transforms it can be found that
\[
    2 \frac{J_1^2 (a r)}{r^2} \leftrightarrow \text{h.t.} \quad S(\omega)
\]
and from the definition of the Hankel transform
\[
    P(f) = \int_{-\infty}^{\infty} \frac{J_1^2 (a r)}{r^2} dy \quad \text{f.t.} \quad S(\omega)
\]

Therefore if \( S(\omega) \) can be evaluated, its fourier transform yields the result required. However, since the width of the function is all that is required, this can be simplified still further.

The effective width \( f_{RT} \) of \( P(f) \) is given by
\[
    f_{RT} = \left[ \frac{\int P(f) df}{\int P(f)^2 df} \right]^2
\]
and by simple transform theory and Parsevals' theorem
\[
    f_{RT} = \frac{S(0)}{\int S(\omega)^2 d\omega}
\]
From tables of transforms $S(w)$ is given as

$$S(w) = 2 \cos^{-1} \left( \frac{\omega}{2a} \right) - \frac{\omega}{a} \sqrt{1 - \frac{\omega^2}{4a^2}} \quad \omega < 2a$$

$$S(w) = 0 \quad \omega > 2a.$$

The effective width of $P(f)$ was evaluated using this method and found to be

$$f_{RT} = \frac{3.8 \, \omega}{\phi_T} \quad \ldots \text{A3.1}$$

$f_{RT}$ is the doppler broadening of the transmitter spot due to rotation of the target.

A similar result exists for the virtual receiver spot. Thus as $D_R/D_S$ is increased the receiver spot contracts, causing a drop in the broadening until $D_R = D_S$. The broadening then remains constant. Thus we have two asymptotes to the rotational broadening function $f_R$

$$f_R = \frac{3.8 \, \omega}{\phi_R} \quad D_R << D_S$$

$$f_R = \frac{3.8 \, \omega}{\phi_T} \quad D_R >> D_S.$$
DATA LOGGER
DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

University of Surrey

A/D Converter and Recorder (MK1) Manual

Design    E Dagless
Construction G Dore
October 1971
A/D Converter and Recorder (MK1) Manual

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Appendix

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Component list
1.0 INTRODUCTION

The initial requirement for a simple and cheap data logging system arose from my research into the properties of laser speckle patterns. No system on the market offered single or two channel working at any reasonable speed or price. The completed system provides computer (ICL 1905) compatible paper tape at a total cost (excluding technicians' time) of around £500.

In the three months or so from the completion date to the present, enthusiastic interest in the unit, and requests for detailed information on the unit, has prompted me to write this report. To date there have been enquiries as to the feasibility of using this system on the following projects:-

1) Doppler signal data acquisition.
2) Submarine control system analysis.
3) Thermally stimulated current experiments.
4) Hall rig measurements.
5) System analysis for MSc projects.

many other possibilities also spring to mind.

This report includes relevant information for the particular requirements of the described systems as decided after consultation with the respective parties.

Chapter 2 describes basic design and tape format,
Chapter 3 the operating procedures,
Chapter 4 gives a detailed description of the operation.

The last section provides some hints to users and modifications for specific systems.

The Appendix describes the Channel 2 operation. This is put here because I feel that only the very few will want to read this section.
2.0 DESCRIPTION OF COMPUTER SYSTEMS

2.1 Computer data handling

The ICL 1900 series computer has four modes of access for data, each with their idiosyncracies. These are summarised in Table 1.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Capital cost</th>
<th>Speed Ch/sec</th>
<th>Storage density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Card</td>
<td>£1000+</td>
<td>50 - 500</td>
<td>V low</td>
</tr>
<tr>
<td>Paper tape</td>
<td>250 - 1000</td>
<td>20 - 300</td>
<td>low</td>
</tr>
<tr>
<td>Magnetic tape</td>
<td>£2000+</td>
<td>20 K</td>
<td>high</td>
</tr>
<tr>
<td>Direct link</td>
<td>£8000+</td>
<td>100 K+</td>
<td>Not applicable</td>
</tr>
</tbody>
</table>

Table 1 data transfer modes for 1900 computers

Paper tape provides a low cost link but at the expense of speed of acquisition. The magnetic tape system is preferable, and less time consuming at the computer terminal, but the cost is prohibitive. With these considerations in mind the logger was designed to provide computer compatible paper tape, with a view to converting to magnetic tape if the possibility arose.

2.2 Sampling rate and accuracy of samples

The maximum sampling rate of a data logging system is limited by the hardware storing the data (paper tape punch, magnetic deck). To maintain the standard format one has to trade off accuracy for speed. As sampling speed was of secondary importance an accuracy of 1:1000 was selected, the 40 ch/sec rate of the punch being reduced to a maximum sampling rate of 10 samples/sec.

(Note:- the sampling rate can reach the maximum rate of the punch but only at the expense of accuracy and loss of standard format. See 2.3 on formatting.)

2.3 ICL 1900 Standard Format (ASCII)

The paper tape format for all digits 0 - 9 is the standard BCD code and parity bit.

Characters are combined in sequence (MSB first) to form numbers, these numbers being separated by a standard character (newline in this system). To represent a 3 digit decimal number four characters must be used.
Table 2a Standard ICL characters.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>0000</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0000</td>
</tr>
<tr>
<td>2</td>
<td>0000</td>
</tr>
<tr>
<td>3</td>
<td>0000</td>
</tr>
<tr>
<td>4</td>
<td>0000</td>
</tr>
<tr>
<td>5</td>
<td>0000</td>
</tr>
<tr>
<td>6</td>
<td>0000</td>
</tr>
<tr>
<td>7</td>
<td>0000</td>
</tr>
<tr>
<td>8</td>
<td>0000</td>
</tr>
<tr>
<td>9</td>
<td>0000</td>
</tr>
<tr>
<td>newline</td>
<td></td>
</tr>
<tr>
<td>TC4</td>
<td></td>
</tr>
<tr>
<td>erase</td>
<td>0000</td>
</tr>
</tbody>
</table>

Table 2b A typical section of tape.

Channels 7, 6, 5, are always the same for digits.

All the characters available on the unit are shown on Table 2a columns 1 - 4 form the BCD word and column 8 the parity bit. The other symbols shown will be described later, (see 5.3 - 5.6). A typical section of a tape is shown on Table 2. This indicates two numbers punched as 596 followed by 311.

An improvement in sampling rate could be obtained by using a format below.

```
87654321
(00) (3rd BCD word) (00) (2nd BCD word)
(00) (1st BCD word)
```

Alternative format for higher speeds shows 356 punched to use only two punch characters. This is an inconvenient method because a special unscrambling program is required.

For use and arranging of paper tape with an Algol program see the Algol Manual.

- 5 -
2.4 Basic description of operation

2.4.1 Single channel operation automatic sampling at either 9, 6, 3, 1.5/sec. See Figure 1.

The signal to be sampled is applied at channel 1 and amplified, if necessary, and fed to the ADC input. Operation is then as follows:

1) On command the signal is sampled and converted to
   3 BCD words on 12 parallel outputs of the ADC
2) the most significant word (hundreds) is punched
   on tape
3) the next word (tens) is punched
4) the least significant word (units) is punched
5) the newline is punched
   then repeated from 1.

To speed up the cycle operation 1 is performed at the same time as operation 5. There is a delay between the word being selected at the ADC and the punch command to give time to generate the parity bit and to let the digits ripple through.

At operation number 5 a pulse is applied to a counter and when this reaches either 1000, 500, or 250 the sequence is halted and the machine stops.

2.4.2 Two channel operation

Signals are applied at inputs 1 and 3. On command channel 1 is converted, as before, and channel 3 is sampled and held. When channel 1 has been punched channel 3 is then converted and punched. This sequence is repeated, but as punch rate has not been changed the sample rate is now half that for mode 2.4.1.

2.4.3 Asynchronous operation

For the operation described in 2.4.1 and 2.4.2 the sequence 1 - 5 is automatically repeated until the required number of samples are obtained. In the asynchronous mode an external sample command can be given which will start the sequence. When 5 has been performed the cycle stops and waits for the next command. The counter still operates and halts the cycle at the specified time.
If operating on two channels the two pulses must be applied for every sample required.¹

¹ These pulses need not be equally spaced and indeed if the time interval between samples is large two pulses together may be better (see 4.5).
3.0 OPERATING PROCEDURES

The operation of channel two will not be described here so as to avoid cluttering the important details with information that only the few will want (see appendix).

Refer to front panel photographs Figures 2 and 3.

1 Switch on

An automatic delay prevents the machine starting up after switch on (reset delay figure).

2 and 3 Gain control (Channel 1 and Channel 3)

A four position gain control in each channel has gain values 1, 3, 10, 30, 100; note At high gain the input impedance is low.

<table>
<thead>
<tr>
<th>Channel 1</th>
<th>Channel 3</th>
<th>Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>30</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>

4 Input offset (Channel 1)

This gives an offset of ±10 V at the output ie for gain of 10 offset is ±1 V at the input. This can be modified to give ±10 V at input if required.

5a Channel test switch

Selects outputs of the amplifiers to check that 'gain is correct': ie input to ADC is in the range 0 - 10 V.

1) Comparator (Channel 2)

2) Channel 3

3) Channel 1

4) Channel 2

5b Voltage range

Alters voltage range on meter for setting input offset accurately.

1) 1 V FSD

2) 3 V FSD

3) 10 V FSD
5c Test output

The voltage at the voltmeter appears here for visual check on oscilloscope etc.

6 Start

Initiates the clock pulses on all modes. If operating under asynchronous sampling then this opens gate ready for the first sample command.

7 Stop

Inhibits the clock pulses and stops the basic sequence immediately.

8 Clear

Clears the two registers controlling the main sequence.

9 Reset

Sets the registers to the starting position.

Note: the sequence 7 - 8 - 9 should always be performed to ensure the registers are correct for starting and before using the manual controls.

10 Feed

Feeds paper tape without punching.

11a Clock speed

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9/sec</td>
</tr>
<tr>
<td>2</td>
<td>6/sec</td>
</tr>
<tr>
<td>3</td>
<td>3/sec</td>
</tr>
<tr>
<td>4</td>
<td>1.5/sec</td>
</tr>
</tbody>
</table>

Note these must be divided by 2 if using channel 3.

11b Sample counter

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000</td>
</tr>
<tr>
<td>2</td>
<td>500</td>
</tr>
<tr>
<td>3</td>
<td>250</td>
</tr>
</tbody>
</table>

On new machines an inhibit counter switch will be incorporated.

---

1 Later models will have an automatic start stop facility as well as manual.

2 On original the sequence 7 - 8 - 9 must always follow the use of number 10. This will be modified on later machines.
12 **Multiplex and sample hold**

Switches in the logic for the multiplexer and sample-hold when using channel 3.

13 **Synchronous - asynchronous mode**

Selects either the automatic sampling rate set by switch 11 or allows asynchronous sampling under external control via socket number 4.

1) Asynchronous
2) Synchronous

The following operations are all manual functions for editing tape and closing off the tape. The stop-clear-reset sequence must be performed before using any manual button after using the automatic buttons.

14 - 17 **Manual data and selected symbols**

By using these switches data can be punched onto tape for numbering tapes or adding multiplying factors etc. The colon shown earlier in the format description is obtained by switching 15 and 17 on and punching\(^3\).

18 **Manual punch**

When the numbers have been set by switches 14 - 17 this button is pushed to activate the punch.

19 **Newline**

Depressing this button punches a newline.

20 **End of tape character or TC\(^4\)**

If button 20 is held down and 19 pressed the E.O.T character is punched.

21 **Erase**

Should an error be made, on a tape, the character can be erased by manually backspacing the punch (knurled disc on the side of the feed mechanism) holding down button 21 and depressing the manual punch button (18).

\(^3\) This symbol has a special use (see 5.3).
Sockets

1) Channel 1 input
2) Channel 2 input
3) Channel 3 input
4) External sample input
5) Punch signals output.

22a and b

Channel 2 gain and input offset respectively. For gain figures see 2.
4.0 **DETAILED DESCRIPTION**

The control unit is the heart of the system and controls all the various functions. However it aids comprehension of operating procedures if the peripheral equipment is described first.

'O' indicates logical 0  'l' logical 1 + means the signal changes to the state shown next.

**4.1 Punch unit**

To operate the punch 9 signals must be applied. Eight are the coding signals for each channel, a negative going edge indicates a punched hole. No change leaves the tape blank. The punch signal, applied at the same time, (also a negative edge), initiates the punch and transport sequence in the punch unit.

**4.2 ADC (Figure 4)**

A transition from 'l' -> 'O' starts conversion $(S_A)$ and while this is happening $E_A$ is at 'l' and + 'O' when conversion is complete. The potentiometer is an input offset to correct for unbalance in the input amplifier, adjust to give 000 when no signal applied. The input signal must be in the range $0 \rightarrow -10 \text{ V}$

**4.3 Drive Logic (Figure 5)**

The ADC output appears at the 12 lines on the left of the diagram. When TI + 'l' the four lines are transferred to the bank of lamps and 2 input nand gates. Parity is generated by the mass of 4 input gates in the centre and the signal passes to line 8. When strobe + 'l' there is a negative transition on all those lines with a 'l' present and these are then coded.\[\text{Similarly for T2 and T3 each selecting their respective words. When newline + 'O' channel 2 and 4 are coded and if TC4 is depressed and newline + 'O' then channels 3 and 8 are coded. If E is depressed all the lines + 'l'. When any of the manual switches M - M4 are energised the respective lines + 'l'. The diodes between line 4 and lines 2 and 3 inhibit signals on either 2 and 3 if 4 is at a 'l' ie it cannot punch anything above 1001 in a BCD word.}\]

---

\[\text{See punch description.}\]
Every sample cycle a pulse arrives at Di and enters the 10 bit counter at a point determined by switch Sz. When the correct combination turns up the 8 input nand gate flips and end = 'O'.

The DAC generates a ramp voltage as the counter increments, (used on channel 2).

The amplifier and multiplier (Figure 7)

The amplifier circuits are common and will not be described here.

The multiplier signals are derived from the outputs of 1 J - K. The J - K is switched at the sampling rate, the reed relays R1 and R2 alternating, selecting channel 1 then channel 3 respectively.

The sample and hold is a large low leakage capacitor and a FET input Amp.

The reed relay R3 opens when channel (1) is converted. The voltage is then held until sampled at the output of the FET amplifier. As channel 3 is held by the Sample/Hold circuit and only released when the next sample pulse arrives, this interval should not be too large. On synchronous operation this is no problem as the largest interval is 1.5 secs, but with slow sampling speeds on the asynchronous mode the capacitor may discharge a significant amount, producing errors in sampling. As both channels are actually sampled together it is immaterial when channel 3 is converted. Under such conditions it is better to provide a double pulse say 2 secs apart when the sample interval is greater than 10 secs.

Control logic (Figure 8)

This is described under two operating headings.

Synchronous (automatic sampling) T1 and T2 in position 2

All other switches are at rest in position 1. At rest the line from J - K number 1 to gate (2) is at 'O' and inhibiting the clock. When 'start' is pressed the clock passes through gates (2), (3), (4), (5) to the 3 bit shift register(1) previously cleared and reset with a '1' at terminal 13. As the clock operates this '1' cycles the register. Every time a '1' appears at 15 the register (2) is advanced. This also resets with '1' at terminal 11. When the '1' appears at T1 the MSB of the ADC is transferred to the output (see 4.3). On the next clock pulse
the '1' in register (1) appears at pin 14 which passes through gates 10, and 13 to code the punch, and through MS2 and P as the punch signal. This is repeated every cycle of register (1) sequentially selecting T1, T2, T3. Then on the fourth, gate (13) is inhibited and the signal passes as the newline signal this time punching the newline character. This cycle takes 12 clock pulses. When the '1' appears at pin 11 of register (2) it also passes through switch T1 and gate (17) to start conversion. The converter then generates 3 new BCD words and the punching cycle is repeated.

When stop is depressed to position 2 the '0' at gate (2) stops the clock and triggers the J - K (1) to set 14 to '0' holding gate (2) off. Depressing the start button sets (14) of J - K (1) to '1' opening gate (2) for the clock and starting the converter.

Every cycle the '1' appearing at 11 passes through gate (11) (when pin 15 of register (1) is at '1') through (14) to the counter. End is normally at '1' and when the counter reaches its preset value 1000, 500, 250, end + '0'. When pin 11 of register (2), pin 13 of register (1) and the clock at pin 1 register (1), all go to '1' the clock is stopped at the end of the last punching sequence. This also activates the self-holding J - K (1).

4.6.2 Asynchronous mode TI and T2 position 1

The only difference in operation is that the convert signal S_A now comes from the sample signal Do which is fed from the external source. At the same time the clock is inhibited by pin 3 of J - K (3) being set to '0' by reset line. The sequence is as follows:

Start depressed. Do + '0' and the ADC converts holding E_A at '1' until it has converted. E_A then + '0' letting the clock through gate (7) to the J - K (3) thus setting pin 15 to '1'.

The clock then passes through (3)-(4)-(5) and the basic sequence starts. Only this time when pins 11 of register (2), 13 of register (1) and 1 of register (1) all go to '1' the clock is inhibited by clearing J - K (3) through gates (8) + (9). The sequence repeats when Do + '0'.

This assumes that Do + '1' sometime before.

Note:

The applied sample pulse, Do, must be long enough for at least one clock pulse to pass to J - K (3) to start the sequence.

- 14 -
4.7 **Minor functions**

All other operations are common to both modes. C, a clear switch, clears register (1) and (2). MS1 and MS2 applies a pulse to the punch to manually punch a signal. MN applies a punch signal and newline coding for a newline character, (the registers must have been reset for both these operations).

F allows the clock through to the registers but inhibits any coding signals to pass to the punch (gate 10) therefore only punching sprocket holes. It is more convenient if this stopped in the reset position, a modification that can be made easily, (see 5.1).

The reset lines \( R_B \) and \( R_A \) come from circuits shown in Figure (9) when first switched on C in circuit is uncharged and charges through the resistor. \( R_B \) is '1' and reset line at '1', holding all register and J - K's. After a few seconds \( R_B + '0' \) and all logic is released but in the reset position, preventing a runaway after switch on. When \( R_A '0' \) all are reset.

Timing wave forms are shown in Figure 13.
5.0 MODIFICATIONS AND OPERATING HINTS

5.1 Modifications

The various modifications are shown on Figure 14. All are really self explanatory. The feed modification is to leave the registers in the reset position and therefore reduce the number of stop-clear depressions the operator has to perform.

5.2 Construction notes

The machines for S Williamson will not have channel 2 present or the DAC on the counter circuit. All the modifications should be included.

If a unit is constructed for the Implantation Group it will be of a similar design.

5.3 Use of the colon

As the counter is not totally immune to noise the number of samples taken is not always equal to that set by the counter switch. For this reason it is best to allow for the program to take 99% of the data for calculation and ignore the other 1%. If a colon is the last symbol on the tape the program can look for this knowing it is the last character. The program can then look for a new tape after finding the colon.

5.4 Operating notes

To prevent the computer unit messing the tapes up, always manually number a tape so it can be checked by the program. The number can be ignored when computation is started.

When using the machine with channel 2 present and providing external sample pulses, before starting always make sure the comparator output (position 1 on switch 5a) is +ve ≈ 5 V. To do this adjust the input offset control (22b) until channel 2 is negative. The comparator is then +ve.

5.5 Data format

As the data logger does not put a decimal point onto paper tape, this must be performed in the program. The number on tape is a three digit number i.e. 000 - 999 and to get this to the correct voltage it must be divided by 100.

5.6 End of tape mark

The end of every tape must be marked with an 'end of tape' character.
The sequence is

TC4 (EOT)

This can be performed very easily using buttons 19 and 20 (see 3.0).
APPENDIX

Channel 2

When using channel 2 in conjunction with channel 1 the data logger is converted to a digital equivalent of an X - Y plotter the X signal is applied at channel 2 and the Y signal at channel 1.

The X signal must only increase and it should do this slowly. The maximum rate of increase is shown below for the fastest clock speed.

<table>
<thead>
<tr>
<th>Samples</th>
<th>Rate (mV/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>90</td>
</tr>
<tr>
<td>500</td>
<td>180</td>
</tr>
<tr>
<td>250</td>
<td>360</td>
</tr>
</tbody>
</table>

If slower clock speeds are used then the numbers must be multiplied by the ratio of the clock speeds i.e. at a clock speed of 3/sec not 9/sec the rate of increase for 1000 drops to 30 mV/sec.

Basic description

Channel 2 must be offset to be below zero, then as it passes through zero, after pressing the start button, it is greater than DAC out. A sample is then taken of channel 1 and DAC out advanced one increment. When the signal at channel 2 becomes greater than DAC out again, another sample is taken. This is repeated at equal voltage increments from 0 - 10 V in steps of either 10 - 20 - 40 mV.

The data on the tape then represents the signal at channel 1 sampled at equal voltage increments of the signal applied to channel 2.

This facility is useful where one has a linear transducer, i.e. $V_o \propto T$, but $T$, the temperature, does not rise uniformly. Samples taken at equal time intervals are not linearly related to temperature but samples taken at equal voltage increments $\Delta V_o$ are directly related and can be calibrated as such.
Figure 1

Block diagram of data logger.
FIG. 2 FRONT PANEL
FIG. 3 SIDE PANEL
CHANNEL 3 AND MULTIPLEXER

Input
ADC

Relays

ON/OF
Multiplexer

+5V

Polyethylene

556

Buffer Brown

10 Ω

10 Ω

100 Ω

3.3 Ω

3.3 Ω

100 Ω

R 1,2,3: Read Relays
FIG 11

POWER SUPPLY (5V)
inhibit counter

ASYCHRONOUS SAMPLE

RESET MODIFICATION

gate 4

FEED MODIFICATION

AUTOMATIC STOP

AUTOMATIC START
COMPONENTS

Burr Brown ADC No ADC 30-12N-BCD
Burr Brown FET AMP
Daytec Punch FL82211

Logic gates

<table>
<thead>
<tr>
<th></th>
<th>7400N</th>
<th></th>
<th>7430N</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td></td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>4</td>
<td>7404N</td>
</tr>
<tr>
<td>4</td>
<td>7410N</td>
<td>2</td>
<td>7493N</td>
</tr>
<tr>
<td>4</td>
<td>7420N</td>
<td>2</td>
<td>7496N</td>
</tr>
</tbody>
</table>

Switches

7 push changeover 2 2 pole push changeover
2 2 pole 2 way switch
5 switch changeover
2 2 pole 5 way Mbb
1 1 pole 4 way bbM
1 3 pole 3 way bbM

Miscellaneous

1 R/S meter MR215 - AMP
1 R/S relay type 40
11 lamps and lamp holders
3 reed relays (clare electronics)
12 BC107
12 ZTX 302
2 OA7
4 BC109
12 ZTX 502

Resistors 1%

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>100 K</td>
<td>2</td>
<td>33 K</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3.3 K</td>
<td>2</td>
<td>1 K</td>
<td></td>
</tr>
</tbody>
</table>

Capacitor 1 0.5µF Polystyrene

Power supply components as in cet diag.
CALCULATION OF $1/\gamma_B$ AND $1/\beta$

It has been shown in chapter 3 that it is possible to determine the value of $1/\gamma_B$ and $1/\beta$ from the photodetector current (3.4). In this section the details involved were not discussed, these are now presented.

Consider an interferometer with laser power $P$ split into the ratio $m/(1-m)$ (see fig. 1). The signal beam then passes through the transmitter aperture with a power loss of $V_n$ so that the power $P_n$ focused onto the target is given by:

$$P_n = V_n (1-m) P, \quad \text{where } n \text{ signifies the } n\text{th. experimental measurement.}$$

If the local oscillator passes through the receiver aperture with loss $Q_n$ then $P_{LO}$ is given by:

$$P_{LO} = Q_n m P_n$$

From eqns. 3.3.7 and 3.4.1 the signal $I_{AC_n}$ can be found to be

$$I_{AC_n} = (4 K_p Q_n V_n (1-m) m P_n^2 / (\gamma_B)_n )^{1/3} \quad \text{A.5.1}$$

Thus provided the parameter in eqnA5.1 can be measured $\gamma_B$ can be determined from the measurement of $I_{AC_n}$. However the direct measurement of $P_n$ proved difficult and an indirect method, using the local oscillator, was devised.

The local oscillator photodetector current $I_{LO}$ is given by

$$I_{LO_n} = K_p Q_n m P_n \quad \text{A.5.2}$$

and substituting this in eqn. A5.1 gives
\[
\left( \frac{1}{\gamma^B} \right)_n = \frac{K_p}{(1-m)} \left( \frac{I_{AC}^n}{I_{LO}^n} \right)^2 \frac{m Q_n}{4(1-m) V_n}
\]

Now if \( I_{AC} \) and \( I_{LO} \) are measured with a mirror as the target \( 1/\gamma^B \) is unity. Thus rearranging eqn. A.5.3 and inserting a suffix, \( m_i \), for this measurement it can be found that

\[
\frac{m K_p}{(1-m)} = \left( \frac{I_{LO}^{m_i}}{I_{AC}^{m_i}} \right)^2 \frac{4 V_{m_i}}{Q_{m_i}}
\]

\( V_{m_i} \) and \( Q_{m_i} \) are the transmitter and receiver apertures for this configuration. Substituting eqn. A.5.4 in A.5.3 gives the result

\[
\left( \frac{1}{\gamma^B} \right)_n = \left( \frac{I_{LO}^{m_i}}{I_{AC}^{m_i}} \right)^2 \frac{Q_n V_{m_i}}{Q_{m_i} V_n}
\]

This expression shows that it is possible to determine \( 1/\gamma^B \), by measuring four photodetector currents and two aperture ratios. Determining the aperture ratios is a simple exercise and \( I_{LO}^{m_i} \) and \( I_{AC}^{m_i} \) need only be recorded once.

The computer program calculated \( 1/\gamma^B \) from eqn. A.5.5 when the various parameters, measured during the calibration procedures, and experimentation, were presented to it on cards or papertape.

For the calculation of \( 1/\beta \) a similar equation can be obtained. The signal current \( I_{sn} \) is given by

\[
I_{sn} = \frac{K_p V_n (1-m) P_n}{\beta}
\]

and the local oscillator current \( I_{LO} \) by eqn. A.5.2 (The signal and local oscillator are not present, at the photocathode, at the
same time). Thus

\[ \frac{1}{\bar{\rho}} = \frac{I_{s_n} \cdot m \cdot Q_n}{I_{LO_n} \cdot (1-m) \cdot V_n} \]

and after recording \( I_s \) and \( I_{LO} \) with a mirror present,

\[ \frac{1}{\bar{\rho}} = \frac{I_{s_n} \cdot I_{LO_{mi}} \cdot Q_n \cdot V_{mi}}{I_{LO_{n}} \cdot I_{s_{mi}} \cdot Q_{mi} \cdot V_{n}} \]

For the incoherent case the local oscillator serves as a reference for the laser power. \( 1/\bar{\rho} \) was computed using the eqn. A.5.8. after presenting the various measured variables to the computer.

The method, presented above, for calculating \( 1/\sigma^A \) and \( 1/\bar{\rho} \) has three important advantages. The measurement is independent of the absolute laser power, it does not depend on the photodetector responsivity and finally, the most important point, it only depends on the aperture ratio. The last condition means that all the combinations of \( D_T \) and \( D_R \) can be accounted for by either measuring or calculating the ratios of \( D_T \) and \( D_R \).