Spectral Reconstruction by Scatter Analysis for Accelerator Photon Beam

by

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Abstract

Pre-existing methods for photon beam energy spectral reconstruction are reviewed. Spectral reconstruction by attenuation analysis is discussed and the Schiff analytical spectral model is introduced.

An alternative reconstruction method by "scatter analysis" is proposed. The method consists in irradiating a phantom and collecting beam spectral information by measuring scatter around the phantom as a function of scatter angle. A Monte Carlo code is used to simulate the irradiation setup and determine scatter behavior as a function of angle for mono-energetic beams. Based on the mono-energetic beam data and using a parametric spectral Schiff model, the spectrum is unfolded by optimization.

The method is applied to a 6MV photon beam accelerator, and the reconstructed spectrum matches the Monte Carlo calculated spectrum for the same accelerator to within 6.2% (direct comparison of spectral shapes). Depth dose values derived from the reconstructed spectrum agree with the physically measured depth dose values within 1% for depths up to 20 cm.

This new method is intended as a practical one requiring few measurements under standard 100cm SSD broad beam geometry, feasible in any radiotherapy department equipped with a scattering phantom and a conventional Farmer chamber.
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Chapter 1

Introduction

The need for a practical method for the determination of a medical linear accelerator’s photon beam energy spectrum has been frequently reported [3,H1,I4,N1,P2,T1]. Such a method would provide the means for increased dosimetric accuracy, periodic quality assurance and for optimization of accelerators’ electronic portal imagers. Indeed, when a photon beam’s energy spectrum is known, one can derive most of its relevant dosimetric quantities with a high degree of accuracy.

The American and European standards for high energy photon beam calibration (AAPM TG-21 A2 1983, AAPM TG-51 A4 1999, IAEA TRS277 12 1997 and IPSM 11 1990) have recommended specifying photon beam quality with a Quality Index, a ratio of doses measured at two different specific depths in a specific medium. The most commonly used Quality Index is the TPR 20/10, the ratio in water of a detector’s reading at 20cm to that at 10cm, with constant source to detector distance. Once the TPR 20/10 is measured for a beam, a dosimetrically ‘equivalent’ mono-energetic beam is inferred (the nominal accelerating potential) which has the same TPR 20/10. This ‘equivalent’ energy is then used to look up all the needed dosimetric quantities. This practice is not entirely accurate since clinical photon beams comprise a continuum of different energies; and the fact that a poly-energetic linear accelerator beam shares the same TPR 20/10 with another mono-energetic beam does not guarantee that other dosimetric characteristics will also be identical, like stopping power ratios, energy absorption coefficients, scatter properties etc... In water for example, the assimilation of an accelerator’s 15MV beam to a monoenergetic average energy of 5MeV, will wrongly preclude the pair production interactions contributed by photons in the 10 to 15MeV range of the spectrum. Yet historically, in most applications, errors resulting from these over-simplifications have been rather tolerable.

Recently, three dimensional treatment planning systems have appeared that use powerful convolution or Monte Carlo based algorithms which require accurate knowledge of the complete beam spectrum to adequately carry on sophisticated dose calculations in the presence of heterogeneities, interfaces and all kinds of beam...
modifying devices. And with the widespread use of portal imagers whose design and optimization require an accurate knowledge of the beam energy spectrum, a renewed interest in photon spectrum reconstruction techniques was recently triggered\textsuperscript{P1,B3}.

The direct measurement of a medical linear accelerator's energy spectrum with a spectrometer is very difficult\textsuperscript{B1,L1,L3,L4}. Dose rates from accelerators are very high, detector pile-up and high dead times occur, thus signals overlap and cannot be individually resolved. In the past, both NaI(Tl) and Ge(Li) scintillators have been used to detect and resolve scattered radiation rather than primary to take advantage of the lower dose rate of the scattered beam, and primary spectral information was inferred from the measured scattered spectrum. Such methods require very careful and tedious calibration of the detectors as well as heavy shielding from unwanted scattered photons in the accelerator room and overall very expensive equipment.

As an alternative to direct spectrometry, several authors developed methods for spectral reconstruction based on the early work of Silberstein in 1933\textsuperscript{S5}, who first proposed that a diagnostic x-ray beam spectrum could be inferred from a series of transmission measurements through several different thicknesses of an attenuator. Since then, the method was refined and its applicability extended tentatively into the megavoltage energy range. However this method requires the achievement of strict 'narrow beam geometry', which is generally difficult to accomplish in a typical linear accelerator room with limited space. It also requires the availability of a high purity attenuator, numerous transmission measurements (usually 25 measurements\textsuperscript{H4,W1}), an accurate determination of room scatter and head leakage conditions\textsuperscript{C1,B4} as well as the use of sometimes impractically large attenuators. Huang \textit{et al}\textsuperscript{H3} measured 25 point attenuation curves in lead (99.99% purity) and aluminium (97.9% purity) to unfold 4MV photon spectra for three Varian Clinac 4 accelerators. They shielded the detector by placing it inside a lead box with an opening, and further collimated their 3x3 cm\textsuperscript{2} beam (measured at detector) by using a thick lead bloc (thickness not reported) with a hole drilled at the centre. The attenuation coefficients used were corrected for the presence of impurities in the aluminium attenuators. They used an iterative least square technique with constraints placed on the output spectral shapes. Their method yielded approximate but physically sound spectra\textsuperscript{P2}, in agreement with Monte Carlo derived spectra for the same accelerator. Piermattei \textit{et al}\textsuperscript{P2} applied a comparable
method in an attempt to unfold spectra for higher energy beams. They used larger source to detector distances (3 to 3.5 meters) and collimated the beam with lead blocks in addition to the standard accelerator collimators. Some reconstructions were simulated rather than physically carried out. They obtained acceptable results with beams up to 10MV but concluded that the method was inapplicable for photon beams with energies above that limit.

This thesis attempts to elaborate an alternate spectral reconstruction method related to the pre-existing methods described above, but feasible in broad beam geometry and requiring fewer measurements. Such a method may then be used in any radiotherapy department for the monitoring of beam energy spectrum, as part of a quality assurance programme.

In chapter 2, the different established methods for spectral reconstruction are reviewed briefly. Chapter 3 will provide a general tool for comparing energy spectra. Chapter 4 will focus on the Schiff bremsstrahlung model while in chapter 5 scatter analysis is introduced as this work's alternative reconstruction method. Chapter 6 will then describe the method and materials used in scatter analysis, and chapter 7 will explain the Monte Carlo simulations used. Chapter 8 will report on the method's reconstruction results and validation. In Chapter 9, the stability of the reconstruction method is examined; and finally, Chapter 10 will conclude with relevant future work which may follow after this thesis.
Chapter 2

Methods in Spectral Reconstruction

2.1 Spectrometry
The use of spectrometry for determining a linear accelerator’s photon beam energy spectrum is fraught with difficulties. The beam’s very high particle fluence within each very short pulse of radiation causes pile-up in the detector and, due to limited space in a typical clinical accelerator room, the inverse square rule cannot be exploited enough to measure a decreased dose rate. On the other hand, the photons’ high energy dictates the use of a very large bulky detector, whose accurate calibration can be complex (usually achieved by Monte Carlo modelling) and subject to numerous approximations. Also, expensive equipment as well as extensive shielding are required. In an effort to overcome problems due to high dose rates and photon energies, several workers have preferred to first measure the beam’s Compton scattered spectrum, then determine the original beam’s spectrum by calculation using the Klein-Nishina equations $^{S2,B6,L5,J1}$. Other researchers have used spectrometry directly with the primary beams of experimental non-clinical accelerators, where sophisticated electronic means were already in place which allowed reducing the primary photon beam dose rate by drastically reducing the electron beam current $^{F1}$. These means are not available in the usual clinical settings of a radiotherapy department.

2.2 Photo-activation
In 1974, Sherman et al $^{S3}$ unfolded the energy spectrum of a 25MV photon beam by irradiating a deuterium target and measuring the photo-neutron yield. The photo-neutrons were detected at a 90 degree angle from the incident photon beam direction using a large plastic scintillator located some 32 meters away from the deuterium target. No validation of the resulting spectra was reported in their paper. The implementation of such a method would be prohibitively expensive and impractical in general in a radiotherapy department. Therefore this type of spectral reconstruction will not be discussed any further in this work.
2.3 Monte Carlo Simulation

This spectral reconstruction technique consists in using a Monte Carlo computer program to model the accelerator's head geometry accurately. Typically, a large number N of particle histories are simulated and followed from the instant the electron beam hits the target to the instant when the resulting photons exit through the collimation system, after passing through (and being scattered by) target, primary collimators, flattening filter, head shielding etc...

At present, Monte Carlo simulation methods are gaining widespread use thanks to the development of faster computer hardware and more performing computer codes. Computational speed problems aside, Monte Carlo simulation constitutes an accurate spectral reconstruction tool, in theory at least.

In practice, the method's weakness is that the returned accuracy of the results depends completely on the accuracy with which the head geometry is modelled into the Monte Carlo code, on the user's ability to choose the appropriate physics options available with the code and suited to the problem's nature, as well as on the accuracy of the cross sections used by the code.

Unfortunately, the exact head geometry details are usually not found, as this information is typically well guarded by the manufacturer and not readily available in the literature. Another minor limitation of this spectral reconstruction technique is that it inherently assumes that the accelerator's photon beam energy determining parameters (for example initial electron beam energy and its angle of incidence onto the target) are all running exactly to manufacturer specification; which is an approximation. Nevertheless, efficient feedback servo systems are widely included nowadays into all medical linear accelerators, thus only small swings in critical photon beam energy determining parameters are possible. Consequently, photon beam spectra reconstructed by this method constitute one of the very few available references against which other spectra obtained through other reconstruction methods can be validated.
2.4 Attenuation Analysis

In 1933, Silberstein \(^8\) showed that the spectrum of a diagnostic x-ray beam could be reconstructed from a set of transmission data for that beam measured in an appropriate material. Since then, the method has been modified and refined by several workers in an effort to extend its coverage into the range of energies encountered in medical linear accelerators \(^1,3\). The basic idea behind spectral reconstruction by attenuation analysis is to measure a beam’s "narrow beam transmission" through several thicknesses of a material and work back from this data to the spectral energy distribution of the beam, as follows:

Let \(\Phi(k)\) represent a photon beam’s energy fluence per unit energy interval as a function of photon energy \(k\), for example in units of \(\text{MeV} \cdot \text{cm}^{-2} \cdot \text{MeV}^{-1}\). \(\Phi(k)\) is then called the energy spectrum for this photon beam \(^3\). This is the quantity which this work will seek to ultimately determine.

The quantity \(\Phi(k)dk\) represents the amount of energy fluence carried by all the photons in the beam having energy between \(k\) and \(k + dk\); and the total energy fluence carried by the beam is

\[
\int_0^{k_{\text{max}}} \Phi(k)dk 
\]

where \(k_{\text{max}}\) is the photon beam spectrum’s maximum energy often taken equal to the manufacturer’s quoted average energy of the electron pencil beam incident on the target in the accelerator’s head.

Suppose this photon beam is used to irradiate a detector placed at some distance in air at point \(P\). Then a signal \(S(0)\) will be recorded at the detector,

\[
S(0) = \int_0^{k_{\text{max}}} F(k)dk 
\]

where \(F(k)\) represents the amount of detector signal generated per unit energy interval as a function of energy \(k\). Thus \(F(k)\) expresses [Signal/MeV].

Now if \(C(k)\) (expressing [Dose/Signal]) is the detector’s energy dependence in terms of dose to air per unit detector signal, then the product \(F(k)C(k)\) expresses
[Signal/MeV][Dose/Signal] = [Dose/MeV], which is the dose to air at P per unit energy interval as a function of k.

Furthermore, we know that \( \Phi (k) \left( \frac{\mu_{\text{en}}(k)}{\rho} \right) \) is the dose to air at point P (assuming prevalence of charged particle equilibrium) per unit energy interval, expressed as [Dose/MeV]; where \( \left( \frac{\mu_{\text{en}}(k)}{\rho} \right) \) is the mass energy absorption coefficient for air at energy k.

Thus we can write

\[
\Phi (k) \left( \frac{\mu_{\text{en}}(k)}{\rho} \right) = F(k)C(k) \tag{2.1}
\]

and,

\[
\Phi(k) = \frac{F(k)C(k)}{\left( \frac{\mu_{\text{en}}(k)}{\rho} \right)_{\text{air}}} \tag{2.2}
\]

Therefore one only needs to solve for \( F(k) \) in order to ultimately obtain \( \Phi (k) \) (assuming of course that \( C(k) \), the detector's energy dependence is known).

Now assume an attenuator of thickness x is interposed in the beam between the source and the detector, in narrow beam geometry. The signal recorded in the detector per unit energy interval as a function of energy k becomes \( F(k)e^{-\mu(k)x} \), rather than \( F(k) \). \( \mu(k) \) represents the total linear attenuation coefficient for the attenuator at energy k.

The total signal in the detector becomes a function of x,

\[
S(x) = \int_0^{k_{\text{max}}} F(k)e^{-\mu(k)x} \, dk
\]

The transmission \( T(x) \) for this attenuator of thickness x can be defined as the ratio of the detector's signal in the attenuated beam to that in the unattenuated beam,

\[
T(x) = \frac{S(x)}{S(0)} = \frac{1}{S(0)} \int_0^{k_{\text{max}}} F(k)e^{-\mu(k)x} \, dk
\]
And normalizing the transmission to $1/S(0)$, we get

$$T(x) = \int_{0}^{k_{\text{max}}} F(k) e^{-\mu[k] x} \, dk$$

(2.3)

In equation 2.3, $T(x)$ is termed the "transmission curve" and is determined experimentally, point by point by measuring the beam's transmission under narrow beam geometry for various consecutive thicknesses $x$ of attenuator. $k_{\text{max}}$ is usually approximated as mentioned above and $e^{-\mu[k] x}$ is calculated by looking up $\mu(k)$ in total linear attenuation coefficient tables. Once $F(k)$ is determined from equation 2.3, $\Phi(k)$ is determined next from equation 2.2.

While many authors have attempted with difficulties to solve equation 2.3 directly, most did not explain the underlying fundamental difficulty in solving for $F(k)$. Equation 2.3 is mathematically known as a "linear Fredholm integral equation of the first kind" in which $F(k)$ is the unknown function we are seeking, $e^{-\mu[k] x}$ is called the "kernel" function, and $T(x)$ is termed the "data function" usually a measured function. A detailed mathematical treatment of this subject is beyond this text's scope. However it is important to emphasize that such equations seldom have solutions for $F(k)$, and when they do, the solutions are usually not unique. That's assuming that $T(x)$ is entirely known, that is $T(x)$ is known at every single value of $x$ from minus infinity to plus infinity. This of course is impossible since in practice $T(x)$ is measured and is therefore known only at a few points, and there, with unavoidable measurement errors.

Linear Fredholm integral equations of the first kind are mathematically referred to as "ill-posed", in a sense that minute errors in $T(x)$ result in very large swings in the solution $F(k)$. This unpleasant phenomenon has been experienced by many researchers who attempted attenuation analysis and in 1981 Baird concluded "We now state a general principle: Attenuation errors render the estimated spectra unreliable if the only input information consists of finitely many measured values for $T(x)$". It is therefore necessary to use special means to partially reduce the solution's instability in order to obtain acceptable, physical solutions.
One of these means is to turn equation 2.3 into a "parametric optimization scheme": First, an analytical model is adopted for $F(k)$ which contains some unknown parameters. For example, and for the only purpose of making this example simpler to understand, let us assume that it is known that $F(k)$ is quadratic in $k$, (in reality $F(k)$ is not quadratic at all) then one could write the following model:

$$F(k) = ak^2 + bk + c$$

and the problem becomes that of determining the correct values for parameters $a$, $b$ and $c$. Second, a corresponding set of theoretical parametric transmission values $[T_1, T_2, \ldots, T_n]$ is calculated for the model (as a function of $a$, $b$ and $c$) for each one of the attenuator thicknesses $[x_1, x_2, \ldots, x_n]$ using equation 2.3:

$$T_1 = \int_0^{k_{\text{max}}} \left(ak^2 + bk + c\right) \exp(-\mu(k)x_1)dk$$

$$T_2 = \int_0^{k_{\text{max}}} \left(ak^2 + bk + c\right) \exp(-\mu(k)x_2)dk$$

$$\ldots$$

$$\ldots$$

$$T_n = \int_0^{k_{\text{max}}} \left(ak^2 + bk + c\right) \exp(-\mu(k)x_n)dk$$

(2.4a)

Next, the same attenuator thicknesses $[x_1, x_2, \ldots, x_n]$ are used to physically measure the actual beam's transmission, and the measured transmission values are recorded: $[M_1, M_2, \ldots, M_n]$.

To match the model's transmission values with the measured transmission values, an objective function is written of the form

$$O(a,b,c) = \sum_{i=1}^{n} \left(1 - \frac{T_i}{M_i}\right)^2$$

(2.4b)

$O(a,b,c)$ is then minimized by computer algorithm, yielding optimal values for $a$, $b$ and $c$, which determines $F(k)$.

Note that by binning the spectrum into several successive energy bins and discretizing the integral, we can rewrite system 2.4 in terms of matrices $F^2$,

$$M = AF$$

(2.5)
where italic boldface characters denote matrix notation. In equation 2.5, \( M \) is a vector containing the measured transmission values for each attenuator thickness, \( A \) the attenuation matrix or kernel containing calculated transmission values \( e^{-u(k)x} \) for each combination of attenuator thickness and energy bin, and \( F \) a vector containing the unknown spectral values. Other workers have preferred mathematically manipulating equation 2.5 rather than 2.4a and 2.4b to make use of the "regularization" methods available in matrix algebra. Regularization methods typically deal with the ill-posedness of the problem by subjecting the solution to certain constraints, thus adding some stability to the reconstruction problem.\(^{w2}\)
Chapter 3

A basis for the quantitative comparison of spectra

In order to adequately compare two different photon energy fluence spectra A and B smoothed and similarly normalized (having the same area under their spectral curves), we propose to quantify the degree to which they match each other in spectral shape by a quantity we will term "spectral difference", obtained as follows: We first calculate the absolute value of the difference in energy fluence between respective individual bins of A and B, add up these individual differences and divide the sum by the total amount of energy fluence carried in either spectrum (A or B). In the case of fine binning, the geometrical interpretation of the "spectral difference" thus defined, would be the total sum of all areas which do not fall simultaneously (cross-hatched areas, figure 3.1) under both spectral lines A and B, divided by the total area under spectrum A or B. This is illustrated in figure 3.1.
Figure 3.1 – "Spectral difference" is the ratio of the sum of all cross-hatched areas to the total area under spectrum A or B. Both A and B are normalized such that the area under the spectral line is 100.

Figures 3.2 to 3.9 are intended to convey a qualitative idea of spectral difference. Arbitrary spectra S1 and S2 are plotted with different magnitudes of spectral differences ranging from 1 to 50%.
Figure 3.2 – Spectral difference between S1 and S2 is 1.0%.

Figure 3.3 – Spectral difference between S1 and S2 is 2.0%.
Figure 3.4 – Spectral difference between S1 and S2 is 3.0%.

Figure 3.5 – Spectral difference between S1 and S2 is 5.0%.
Figure 3.6 – Spectral difference between S1 and S2 is 10.0%.

Figure 3.7 – Spectral difference between S1 and S2 is 20.0%.
In the case where spectral lines are not smooth due to statistical noise, spectral differences can only be computed after smoothing the spectral lines. Otherwise the result is overestimated due to artifactual excursions of the line above and below its true value. The following shows the calculated spectral difference when one of the spectra is noisy (figure 3.9) and then smoothed. The spectral differences thus calculated differ measurably, and in this work noisy spectra will be smoothed before any spectral difference is calculated. Of course, smoothing a noisy spectral line never returns its true value, but as figure 3.1 shows, it contributes in calculating a more accurate value for the spectral difference. The smoothing technique used in this work is a piecewise cubic spline function (called csaps) provided by Matlab M1.
In extreme cases, smoothing of spectra which are too noisy and/or evaluated at very few spectral points (large bin width) will give unreliable results. The noisy spectra we smoothed in this work (and used in the calculation of spectral difference) had all a bin width within 0.25 MeV and a pre-smoothing average statistical error less than 10% (calculated as the spectral difference between the original noisy spectrum and the smoothed one). In these cases, it was visually evident that the obtained smooth and continuous spectrum made sense and offered a better approximation to the "true" spectrum than the original noisy curve did, especially that we know (as will be seen in the next chapter) that the "true" underlying spectrum is indeed continuous and smooth. (An exception is spectrum S1 in chapter 8, whose average pre-smoothing statistical error is higher than 10%).
Chapter 4

The Schiff Bremsstrahlung Model

In the previous chapter a quadratic model for the photon beam's energy spectrum was chosen arbitrarily, as an example, for purely illustrative reasons. Neglecting head scatter, the real spectral form of an accelerator's primary photon beam is that of a bremsstrahlung spectrum, since the beam is produced by electrons hitting a target and gradually slowing down in it. The following is a summary of the history and derivation of a current compact analytical form for the bremsstrahlung cross section, known as the Schiff formula. The information is derived from references $^{S4,K2,D1}$ to which the reader is referred for a complete discussion of the subject.

In 1934, Bethe and Heitler $^{B7}$ provided the following expression for the bremsstrahlung differential cross section with respect to photon energy, photon angle and outgoing electron angle (using Kosh and Motz $^{K2}$ notation, formula 1BS)

$$\frac{d\sigma}{dk\,d\Omega_{\gamma\gamma}\,d\Omega_{el}} = \frac{Z^2}{137} \frac{(1-f)^2}{2\pi} \frac{p}{k} \frac{p^2 \sin^2 \theta \left(4E_0^2 - q^2\right) + p_0^2 \sin^2 \theta_0 \left(4E^2 - q^2\right)}{(E-p \cos \theta)^2 \left(E_0 - p_0 \cos \theta_0\right)^2}$$

$$- \frac{2pp_0 \sin \theta \sin \theta_0 \cos \phi}{(E-p \cos \theta)(E_0 - p_0 \cos \theta_0)} \left(4E_0 - q^2\right)$$

$$+ \frac{2k^2 \left(p^2 \sin^2 \theta + p_0^2 \sin^2 \theta_0 - 2pp_0 \sin \theta \sin \theta_0 \cos \phi\right)}{(E-p \cos \theta)(E_0 - p_0 \cos \theta_0)}$$

(4.1)

where

$$q^2 = p^2 + p_0^2 + k^2 - 2p_0k \cos \theta_0 + 2pk \cos \theta - 2pp_0 \cos \theta \cos \theta_0 + \sin \theta \sin \theta_0 \cos \phi$$

(4.2)
The geometry pertaining to equation 4.1 is given in figure 4.1, and the parameters used are:

\[
\begin{align*}
Z &= \text{atomic number of target where electron is interacting} \\
\alpha^2/m_0c^2 &= 2.82 \times 10^{-13} \text{ cm, classical electron radius} \\
e &= 1.602 \times 10^{-19} \text{ C} \\
m_0c^2 &= 0.511 \text{ MeV, electron rest mass} \\
E_0, E &= \text{incident and outgoing total energy of the electron in collision, in } m_0c^2 \text{ units.} \\
p_0, p &= \text{incident and outgoing momentum magnitude of the electron in collision, in } m_0c \text{ units.} \\
k &= \text{energy of emitted bremsstrahlung photon, in } m_0c^2 \text{ units.} \\
\theta_0, \theta &= \text{angles of } \mathbf{p}_0 \text{ and } \mathbf{p} \text{ with respect to emitted photon direction } \mathbf{z}. \\
\phi &= \text{angle between the two planes defined by vector pairs } (\mathbf{p}_0, \mathbf{z}) \text{ and } (\mathbf{p}, \mathbf{z}). \\
d\Omega_{\text{ph}} &= 2\pi \sin \theta_0 \sin \theta_0 \sin \phi \, \sin \phi, \text{ the ph subscript denoting differentiation with respect to photon angle since } \theta_0 \text{ is the angle between photon and incident electron} \\
d\Omega_{\text{el}} &= \sin \theta \sin \phi \, \sin \phi \, \sin \phi, \text{ the el subscript denoting differentiation with respect to outgoing electron angles } (\theta, \phi), \\
q &= \text{momentum transferred to the nucleus, in } m_0c \text{ units.}
\end{align*}
\]

and \(\frac{(1-F)^2}{q^4}\) is a term that accounts for the screening of the Coulomb nuclear field by the atomic electrons.
Outgoing bremsstrahlung photon direction

Figure 4.1 - Geometry for the differential bremsstrahlung cross section given by equation 4.1. $\vec{p}_0$ is the incident electron momentum; $\vec{p}$ is the outgoing electron momentum; $\vec{z}$ is the emitted bremsstrahlung photon direction.

Equation 4.1 is based on the Born approximation, which assumes that:

$$Z/137 \ll 1 \quad \text{and} \quad (E_0, E \text{ in } m_0 c^2 \text{ units}) \gg 1$$

This implies that the Bethe-Heitler formula is supposed to break down for high atomic number targets, low incident electron energy and at the high energy end of the produced photon's energy spectrum where the outgoing electron energy is small. Looking into the validity of the approximations used above, Koch and Motz noted that "...even when there is a breakdown of the Born approximation, the accuracy of the related cross section formulae is still reasonably good..."

Continuing therefore with the analysis, and considering forward directed bremsstrahlung only, we set $\theta_0=0$ in equation 4.1 and obtain:
\[
\frac{d\sigma}{dk \, dQ_b \, dQ_d} = \frac{Z^2 (\frac{r_0}{2\pi})^2 (1-F)^2 \, p^3 \sin^2 \theta}{q^4 \, k \, p_0} \left[ \frac{(4E_0^2 - q^2)}{(E - p \cos \theta)^2} + \frac{2k^2}{(E - p \cos \theta)(E_0 - p_0)} \right]
\]

(4.3)

and removing the constant terms, we get the Bethe-Heitler differential cross section for the forward bremsstrahlung case, based on the Born approximation:

\[
\frac{d\sigma}{dk \, dQ_b \, dQ_d} = \frac{(1-F)^2 \, p^3 \sin^2 \theta}{q^4 \, k \, p_0} \left[ \frac{(4E_0^2 - q^2)}{(E - p \cos \theta)^2} + \frac{2k^2}{(E - p \cos \theta)(E_0 - p_0)} \right]
\]

(4.4)

with,

\[
q^2 = p^2 + p_0^2 + k^2 - 2p_0k + 2pk \cos \theta - 2pp_0 \cos \theta
\]

(4.5)

The applicability of a forward bremsstrahlung formula to linear accelerators is discussed next where a typical medical linear accelerator treatment geometry is depicted (figure 4.2).
With a typical 100cm treatment distance from target to patient, a classic 10x10 cm\(^2\) square field corresponds to a full emission angle at the target of around 5.7\(^\circ\) (\(2\tan^{-1}(5/100)\)) or bremsstrahlung production angle of 5.7\(^\circ\)/2 = 2.8\(^\circ\). Such emission angles are small enough to justify the use of forward directed bremsstrahlung formulae\(^{D1,59}\). Other larger fields are often used in clinical treatment. A 40x40 cm\(^2\) field (measured at 100 cm from the source) is the largest achievable field for most current accelerators. The diagonal of such a field is 56.6 cm, and some points in the field will therefore be exposed to photons produced at angles of up to 15.8\(^\circ\).
( \tan^{-1}(56.6/2/100) ). At such off axis points, the spectrum differs measurably from that of forward directed bremsstrahlung. Sheikh-Bagheri and Rogers \textsuperscript{58} reported a variation in average photon energy from 2.03 MeV to 1.50 MeV between central axis and 20 cm off axis at 100 cm from target for an Elekta 6MV beam. The reported change in average energy from central axis to 5cm off axis (10x10 cm\textsuperscript{2} field size) was from 2.03 MeV to 1.96 MeV. Thus for fields larger than 10x10, the cross section in equation 4.4 would need corrections before it is applied to off-axis points.

In 1951, Schiff introduced to the derivation an exponential approximation to the Coulomb screening factor. He then integrated the expression over all photon and electron angles, and the following final form for the bremsstrahlung energy spectrum was obtained:

\[
\Phi(k) = \left(1 - \frac{k}{E_0}\right)(\ln \eta - 1) + \left(\frac{k}{E_0}\right)^2(\ln \eta - 0.5)
\] (4.6)

where again,

- \(k\) = energy of emitted bremsstrahlung photon,
- \(\Phi(k)\) = relative energy fluence at photon energy \(k\),
- \(E_0\) = total energy of electron incident on the target,

\[
\eta = \left[\left(\frac{k}{2E_0E}\right)^2 + \left(\frac{Z^2}{111}\right)^2\right]^{-1/2}
\] (4.7)

111 is a constant calculated by Schiff.

In equation 4.7, the nominator of the \(\frac{k}{2E_0E}\) term is \(1\times k\) where 1 is \(\mu\) (the electron rest mass) expressed in \(m_0c^2\) units. Thus the \(\frac{k}{2E_0E}\) term is unit less.

Schiff's formula applied only to thin-targets since it expressed the cross section in energy for one individual interaction at a time per electron. It did not apply readily to
accelerators where the target is usually thick enough that usage of more complicated thick-target formulae is instead required.

In 1991, Desobry and Boyer \textsuperscript{D1} reported that while a thick target formula is more accurate for accelerators, the forward directed spectrum produced in a thick target may be adequately approximated by the Schiff thin-target formula (\textit{despite the different approximations used in deriving it}), if the latter is modified to include an appropriate term which accounts for the additional attenuation through the target's deeper layers, the accelerator's flattening filter and any other filters used.

The terms they included were a series of \( n \) simple exponential attenuation terms:

\[
\prod_{i=1}^{n} \exp(-\mu_i(k)t_i)
\]

where \( n \) is the number of filters, \( t_i \) is the thickness of the \( i^{th} \) filter and \( \mu_i(k) \) the total linear attenuation coefficient for the \( i^{th} \) filter at energy \( k \). With this additional term, the Schiff formula provided a practical expression for the forward directed photon bremsstrahlung energy spectra in thick targets, applicable to accelerators:

\[
\Phi(k) = \left[ 1 - \frac{k}{E_0} (\ln \eta - 1) + \left( \frac{k}{E_0} \right)^2 (\ln \eta - 0.5) \right] \prod_{i=1}^{n} \exp(-\mu_i(k)t_i) 
\]  

\text{(4.8)}

Baker \textsuperscript{B2} and Partridge \textsuperscript{P1} observed that the model in equation 4.8 suffered no significant loss in accuracy if all \( n \) filters were lumped into a single filter of thickness \( T \) (usually of target material) with total linear attenuation coefficient \( \mu_T(k) \). Hence the model becomes

\[
\Phi(k) = \left[ 1 - \frac{k}{E_0} (\ln \eta - 1) + \left( \frac{k}{E_0} \right)^2 (\ln \eta - 0.5) \right] \exp(-\mu_T(k)T)
\]

Noting that this expression would underestimate the number of low energy photons produced by the electrons interacting in the deeper target layers, Baker \textsuperscript{B2} added an additional parameter \( \alpha \) intended to reduce the overestimated number of high energy photons in the expression by reducing the \( k^2 \) term. He also added a second parameter \( \beta \) to provide more flexibility in accounting for the overall attenuation phenomenon. He used a positive \( \beta \) when the filtration is predominantly due to the target, and
negative when the filtration is predominantly due to the flattening filter. The final parametric Schiff model becomes

\[
\Phi(k, E_0, T, \alpha, \beta) = \left[1 - \frac{k}{E_0} \right] \left(\ln \tau - 1\right) + \alpha \left(\frac{k}{E_0}\right)^2 \left(\ln \tau - 0.5\right) \exp\left(\beta/k - \mu_\tau(k)T\right) \quad (4.9)
\]

The adequacy of this latter model can be measured by the extent to which it can accurately represent actual accelerators' photon energy spectra while simultaneously resist taking unrealistic (unphysical) spectral shapes. This means that given any accelerator with a known photon energy spectrum, if the model in equation 4.9 is accurate, we should be able to find proper values for \(E_0\), \(T\), \(\alpha\) and \(\beta\) such that \(\Phi(k)\) as given by equation 4.9 produces a curve that very closely matches the actual spectrum.

This fitting test was done by Baker \(^{32}\) who fitted equation 4.9 to some twenty-one validated accelerator photon spectra taken from diverse sources, some spectra measured experimentally while others calculated by Monte Carlo simulation. These spectra were for several different electron energies \(E_0\), target materials and thicknesses and they served as a benchmark to test the versatility of the parametric Schiff expression to fit accelerator spectra. The match between the fitted spectra and the initial benchmark spectra was evaluated by calculating transmission values for the fitted spectra and the benchmark spectra and comparing these transmission values. The calculated transmission values agreed to within 0.03% on the average. The returned optimal values for \(\alpha\), \(\beta\) and \(T\) were physically sound, ie \(\alpha\) assumed in general values less than unity while \(T\) assumed values of a few mm of tungsten. The values of \(T\) increased steadily as the hardness of the spectra analysed increased.

The following two figures 4.3 and 4.4 show the versatility of the parametric Schiff model whereby its shape can be modified in many different ways by varying the free parameters to allow modelling of very different spectra. The intention is to give a more practical "feel" as to how each parameter can affect the spectrum. Figure 4.3 illustrates how gradual changes in \(\alpha\) affect the spectrum while figure 4.4 how changes in \(T\) affect the spectrum.
Figure 4.3 - The full line represents an arbitrarily chosen baseline spectrum with $\alpha=1.0$, $\beta=0$ and $T=1.0$ cm. The dashed line shows the parametric Schiff model changing shape as $\alpha$ is progressively varied while $\beta$ and $T$ are kept constant. The $\alpha$, $\beta$ and $T$ values in each case are shown on the graphs.
Figure 4.4 - The full line represents an arbitrarily chosen baseline spectrum with $\alpha=1.0$, $\beta=0$ and $T=1.0$ cm. The dashed line shows the parametric Schiff model changing shape as $T$ is progressively varied while $\alpha$ and $\beta$ are kept constant. The $\alpha$, $\beta$ and $T$ values in each case are shown on the graphs.
Chapter 5

Spectral Reconstruction by Scatter Analysis

In general, the difficulties associated with spectral reconstruction by attenuation analysis are:

• 1- The problem is fundamentally ill-posed, therefore oversensitive to small errors and difficult to solve. The term ill-posed must not be misconstrued to indicate that the particular approach used in solving the problem is inadequate, but rather that one is trying to solve a problem which, by definition, seldom accepts stable solutions. As explained earlier in chapter 2, any problem that mathematically takes the form of equation 2.3 belongs automatically to the well known class of fundamentally unstable problems called "Linear Friedholm Integral equation of the First Kind" \(^{w2}\). Therefore the difficulty here is inherent to the problem's nature and is not related to the particular choice of transmission as a quantity to analyse; the problem would remain ill-posed if an alternate quantity is chosen for reconstruction. For example the use of depth dose data rather than transmission in spectral reconstruction does not remedy the ill-posedness of the problem. However, there are degrees of ill-posedness, and a better behaviour can be expected of the problem when the quantity chosen for analysis is a steeper function of energy; as Wing \(^{w2}\) indicates, the smoother (or flatter) the kernel matrix, the more ill-posed the problem.

• 2- The method's practicality is not optimal:

  - It requires ideally narrow beam geometry, and not all treatment rooms are large enough to accommodate a large source to detector distance.
  - It requires the non-trivial determination of the magnitude of background (room scatter) contribution to the detector for each different attenuator thickness used \(^{B4,C1}\).
  - It requires the availability of an attenuator of high purity or accurately known atomic composition for the accurate determination of the linear attenuation coefficients. This attenuator can be impractically large at high energies to provide low enough transmission values.
- A relatively large number of measurements have to be taken (usually 25 \(H_3, H_4, W_1, P_2\)) to gather enough spectral information.

- The total linear attenuation coefficient of the attenuator varies slowly with energy:
  In order to choose an attenuating material with a linear attenuation coefficient monotonically decreasing over the radiotherapy energy range \(^{H_1, H_2, H_3, B_2}\), a low \(Z\) material must be used. This can be seen in figure 5.1 below, which shows that the only materials offering a monotonously decreasing attenuation coefficient over the energy range 1 to 20MeV are aluminium, water and carbon (among other possible low \(Z\) materials not represented in the figure). But the problem with low \(Z\) materials is that their total mass attenuation coefficient is a slow (flat) function of energy after about 4MeV. This is undesirable because the reconstruction method relies on exploiting the differences that exist in the amount of attenuation which photons experience in different energy bins of the spectrum. It is therefore desirable to have a large value for the slope \(d\mu/dk\) in order to enhance the differentiation between energy bins and improve on the resolving of successive energy bins within the spectrum.

![Figure 5.1](wasy04em10d1802fig10.fig)

Figure 5.1 - Total mass attenuation coefficient \(\mu/\rho\) in units of \(\text{cm}^2/\text{g}\) for tungsten, lead, copper, iron, aluminium, water and carbon. Data from NIST (based on original data from Hubbell 1982).
Thus it may be useful finding an alternate measurable physical quantity which avoids, at least partially, difficulties 2 and 3 above (for example, a quantity which varies steeper with energy) and analyse this alternate quantity (rather than attenuation) to derive spectral information for spectral reconstruction.

This chapter introduces the idea of analyzing beam scatter for energy spectral reconstruction. Consider figure 5.2 below which represents a standard treatment room with a linear accelerator producing a conventional diverging photon beam impinging onto a scattering phantom placed at 100cm from the target in standard broad beam geometry. An ionization chamber is placed at a specific position outside of the primary beam with the intention of measuring the ionization to air, due to the secondary photons produced in the scatterer at a specific angle. The measurement of this quantity is done by taking two readings for the same amount of monitor units and subtracting one from the other: the first reading is taken with the scatterer removed from the beam (this measures the background signal arising from jaw transmission, scatter off the room walls and floor and accelerator head leakage). For the second reading, the scatterer is placed in the beam, the same amount of monitor units is delivered and the reading recorded. Subtracting the first reading from the second one should yield the pure contribution to the chamber from secondary photons produced in the scatterer.

In the rest of this paper the term "scatter signal" will refer to the amount of ionization in the chamber due uniquely to the secondary photons produced in the scatterer, a physical quantity measured by the subtraction technique described above. It is therefore important to keep in mind that one measurement of scatter signal is in reality the result of the subtraction of two measurements.

The scatter angle pictured in figure 5.2 is geometrically defined by the beam's central axis and a line joining the scatterer volume's centre to the chamber sensitive volume's centre. Dosimetrically, this of course is not an exact scatter angle but a representative average angle in a sense that, owing to the finite size of the scatterer and chamber, not all photons reaching the chamber in position 1 will have scattered by the same exact angle. In addition, head scatter (from flattening filter, primary collimators and collimating jaws) reaching the scatterer will undergo scatter towards the detector with a different angle than that shown in figure 5.2. But these photons are of negligible contribution to the detector: in their paper, Sheikh-Bagheri et al used Monte Carlo
simulations to determine the photon contributions to a 10x10 cm² field for an Elekta SL25 6MV (same geometry as our Elekta SL18 accelerator used here) photon beam. They found that, out of 100 photons reaching the treatment field, 97.0 originated directly from the target, 2.1 directly from the flattening filter, 0.6 directly from the primary collimator and 0.3 directly from the collimating jaws. Thus the very vast majority of the photons scoring in our detector will originate directly from the target, and the scatter angle depicted in figure 5.2 is a valid representative average scatter angle. These considerations are fully accounted for in our Monte Carlo simulations which do not assume one single exact angle, but instead model the scattering geometry as is, in its actual finite scatterer and flattening filter sizes. In what follows, and whenever in the context of the setup used, referring to a specific scatter angle is meant geometrically as an exact angle but dosimetrically as a representative average scatter angle.
Figure 5.2 - Top view of setup for measuring scatter signal as a function of angle. Not drawn to scale.

It was questioned whether the presence of the scatterer in the beam increased the background contribution to the chamber by scattering [jaw/head leakage + room scatter] from the scatterer back onto the chamber, which would undermine the soundness of the subtractive technique. To test this, we moved the scatterer laterally out of the beam, placed it under jaw transmission where it is only exposed to background, i.e. [jaw/head leakage + room scatter]. We kept the same scatterer to
chamber geometry. We took two measurements, one with the scatterer totally out of
the experimental setup, and one with the scatterer under jaw transmission but out of
the primary beam. The subtraction in this case ought to yield an estimate of how much
additional background the scatterer scatters towards the chamber. The experiment is
described in figure 5.3 and 5.4 below. Figure 5.3 shows the setup for measuring direct
[head/jaw leakage + room scatter] only. In figure 5.4, the chamber is exposed to direct
[head/jaw leakage + room scatter] as well as background scattering from scatterer to
chamber. The difference in readings between the setups of figure 5.3 and 5.4 was less
than 0.1% of the background reading, which is negligible.
Figure 5.3 - Top view of setup for measuring background, i.e [head/jaw leakage + room scatter]. Not drawn to scale.
Figure 5.4 - Top view of setup for measuring [head/jaw leakage + room scatter] + [background scattering from scatterer to chamber]. Not drawn to scale.

It was thus concluded that the presence of the scatterer in the beam did not increase the background contribution to the chamber, and that it was therefore sound to consider that the background contribution to the chamber is the same with or without the scatterer in the beam and thus will cancel out fully with the subtraction technique.

Figure 5.5 below shows the relative probabilities of all three main photon interactions in the scatterer (photoelectric, Compton and pair production) as a function of
scatterer's atomic number Z and photon energy. The figure is divided in three main regions, region one to region three from left to right. The dashed curve separating region one from region two represents the combinations of Z values and photon energy values for which photoelectric and Compton effect are equally probable. The solid curve separating region two from region three represents the combinations of Z values and photon energy values for which Compton effect and pair production are equally probable. Thus in region 1 photoelectric effect is predominant, in region 2 Compton effect is predominant while in region 3 pair production is the most probable interaction.

In figure 5.5, the region which corresponds to scatter analysis (using a low Z scatterer at radiotherapy energies) is represented by the narrow horizontal rectangular area determined by the inequalities:

- \( 5 \leq Z \leq 10 \) (low Z scatterer: water, plastic etc...) and
- \( 0 \leq \text{Photon energy} \leq 30 \text{ MeV} \) (radiotherapy photon energy range)

This rectangle falls largely in the Compton effect region and the interaction which contributes most to *scatter signal* is therefore Compton scatter.

![Figure 5.5 - Relative preponderance of photoelectric, Compton and pair production interactions in scatter analysis as a function of scatterer material and incident photon energy. From Attix \(^1\), modified.](wasy04m06d2701fig1.fig)
Even when photoelectric effect or pair production are predominant, as can be inferred from figure 5.5 at respectively very low and very high energies, these two interactions do not contribute much to the measured scatter signal. In the photoelectric case, the reason is obvious: once the primary photon interacts, it disappears and subsequent characteristic X-rays produced in the scatterer are too soft (a few keV) to make it through the 1.5 cm acrylic build-up (attenuation is of the order of $10^{-5}$)\textsuperscript{A1}. Therefore little or no signal is registered in the ionization chamber due to the photoelectric interactions in the scatterer. In the pair production case, when a high energy primary photon interacts in the scatterer it produces a high energy (positron, electron) pair. If the scatterer's dimensions are small enough in the directions perpendicular to the central axis (narrow scatterer), the positron undergoing tortuous interactions will most likely escape laterally from the scatterer before it has lost all of its kinetic energy and annihilated. Thus the two 0.511MeV annihilation photons typical of pair production are most likely to be produced in the room's walls (rather than in the scatterer) far from the ionization chamber. For this reason, pair production will contribute little to the scatter signal even when it is the predominant interaction as it is the case at higher energies. Therefore Compton scatter should prove to be the dominant contributor to scatter signal in a scatter analysis situation where the scatterer is narrow and made of a low Z material.

Based on the previous analysis, and since the cross section for Compton scatter is steeply dependent on primary photon energy and scatter angle, scatter signal was chosen as an energy dependent quantity which may efficiently convey energy spectral information.

Consider the differential Klein-Nishina cross section as a function of incident photon energy and scatter angle given by\textsuperscript{A1}:

$$\sigma = C \left( \frac{h\nu'}{h\nu} \right)^2 \left( \frac{h\nu}{h\nu'} + \frac{h\nu'}{h\nu} \sin^2 \varphi \right)$$

(5.1)

with

$$h\nu' = \frac{h\nu}{1 + \left( \frac{h\nu}{m_0c^2} \right)(1 - \cos \varphi)}$$

Where,

C is a constant of no relevance to this discussion

$h\nu$ = incident photon energy
\( h\nu' \) = scattered photon energy
\( \phi \) = scatter angle of photon measured with respect to primary photon direction

Equation 5.1 is plotted in figures 5.6 and 5.7 below. The steep dependence of Compton scatter on photon energy can be best appreciated by comparing the slopes in figure 5.6 to those in figure 5.8 which is a plot of total linear attenuation coefficient for aluminium and water as a function of energy.

---

Figure 5.6 – Plot of equation 5.1: Compton scatter cross section as a function of incident photon energy, drawn for scatter angles of 40, 60, 80, 100 and 180 degrees. Ordinate is in relative units because the aim is to show only the curve's steepness as a function of incident photon energy.
Figure 5.7- Plot of equation 5.1: Compton scatter cross section as a function of scatter angle, drawn for energies 0.5, 1, 5, 10 and 30 MeV.
To illustrate how scatter measurements may convey spectral information about an accelerator's photon beam, consider the following non-realistic symbolic example of an unknown photon spectrum X as it exits the accelerator's head, containing only three different energies of photons (figure 5.9): 1MeV photons (in energy bin number 1), 2MeV photons (in energy bin number 2), and 3MeV photons (in energy bin number 3).
Let $x_1$, $x_2$ and $x_3$ represent the fractional number of photons in each bin of $X$; these are the problem's unknowns which need to be determined. Let $N$ represent the total number of photons emitted at the source for 100 monitor units of radiation; then the number of photons in each bin in 100 monitor units of radiation becomes $N x_1$, $N x_2$ and $N x_3$ respectively in bin number one, two and three. At any specific angle of scatter measurement, the scatter signal from such a beam will be the sum of the scatter signals contributed by each individual bin. Using the subtractive technique described above, suppose we take measurements of scatter signal corresponding, for example, to 100 monitor units at three different scatter angles: reading $s_1$ is scatter signal measured for 100 monitor units with the chamber at position 1 (figure 5.2), reading $s_2$ at position 2 and reading $s_3$ at position 3. $s_1$, $s_2$ and $s_3$ thus measured constitute a three point scatter curve.
Now suppose we use Monte Carlo simulations (chapter 7) to estimate the *scatter signal* per single photon exiting the accelerator's head for mono-energetic beams of energy 1, 2 and 3 MeV:

Let \( a_1, a_2, a_3 \) be respectively the Monte Carlo calculated relative *scatter signal* per single photon exiting the head for a mono-energetic beam of energy 1 MeV, 2 MeV and 3 MeV when the chamber is in position 1.

Let \( b_1, b_2, b_3 \) be respectively the Monte Carlo calculated relative *scatter signal* per single photon exiting the head for a mono-energetic beam of energy 1 MeV, 2 MeV and 3 MeV when the chamber is in position 2.

Let \( c_1, c_2, c_3 \) be respectively the Monte Carlo calculated relative *scatter signal* per single photon exiting the head for a mono-energetic beam of energy 1 MeV, 2 MeV and 3 MeV when the chamber is in position 3.

Thus \( a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2 \) and \( c_3 \) are the "bin-specific mono-energetic scatter signals per photon" and we can write using the proportionality sign \( \propto \),

\[
\begin{align*}
s_1 &= a_1x_1 + a_2x_2 + a_3x_3 \\ s_2 &= b_1x_1 + b_2x_2 + b_3x_3 \\ s_3 &= c_1x_1 + c_2x_2 + c_3x_3
\end{align*}
\]

or

\[
\begin{align*}
s_1 &\propto a_1x_1 + a_2x_2 + a_3x_3 \\ s_2 &\propto b_1x_1 + b_2x_2 + b_3x_3 \\ s_3 &\propto c_1x_1 + c_2x_2 + c_3x_3
\end{align*}
\]

System 5.3 is equivalent to the statement that when we irradiate the scatterer with 100 monitor units with the chamber being in position 1 (figure 5.2), the measured *scatter signal* (represented by \( s_1 \) in system 5.3) is proportional to the sum of three contributions. The first contribution has a magnitude of \( a_1x_1 \) and is due to \( x_1 \) fractional photons present in bin number 1 of the spectrum \( X \), the second having a magnitude of \( a_2x_2 \) and is due to \( x_2 \) fractional photons present in bin 2, and the third having a magnitude of \( a_3x_3 \) and is due to \( x_3 \) fractional photons present in bin 3. The previous
statement applies again when the chamber is moved to position 2 and then 3. System 5.3 can then be resolved by optimization, as will be discussed in chapter 6.

Using matrix algebra, system 5.3 can be written as:

\[ S \propto AF \quad (5.4) \]

where italic boldface characters denote matrix notation. \( S \) is a vector (single column matrix) containing the scatter signal measurements at position 1, 2 and 3, \( A \) is the scatter matrix or kernel containing Monte Carlo calculated \textit{scatter signal} values for each combination of chamber position (or measurement angle) and energy bin, and \( F \) a vector (single column matrix) containing the unknown spectral values \( x_1, x_2 \) and \( x_3 \). Thus,

\[
S = \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix}; \quad A = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}; \quad F = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}
\]

Equation 5.4 is similar to equation 2.5, thus we have at hand a discrete version of an ill-posed Fredholm integral of the first kind which we propose to solve by optimization, like others have attempted in attenuation analysis problems. In reality, spectra are binned into much smaller bins and so the number of columns in \( A \) and rows in \( F \) are much larger than 3 (we will be using about 681 energy bins for a 6 MV spectrum). Note that while the number of columns in \( A \) represents the number of bins used, the number of rows in \( A \) (and in \( S \)) represents the number of \textit{scatter signal} measurements taken, each one at a different scatter angle (we will be using five \textit{scatter signal} measurements per reconstruction).
In attenuation analysis, the physical quantity which is relied upon for spectral reconstruction is attenuation $e^{-\mu(k)x}$, where measurements are taken for different values of attenuator thickness $x$. The idea behind measuring beam transmission for several different attenuator thicknesses is that each new thickness used brings along additional spectral information, since the shape of the transmission vs. energy curve $e^{-\mu(k)x}$ is different with each attenuator thickness used.

![Graph showing normalized transmission curves for 10 and 30 cm Al thicknesses.](wasy04m10d1802fig11.fig)

**Figure 5.10** – Transmission analysis: Normalized transmission curves for 10 and 30 cm Al thicknesses. Curves are normalized to 1 at 3MeV.

This is shown in figure 5.10 where normalized transmission is plotted vs. photon energy for 10 and 30 cm thickness of aluminium attenuator. Had the shape of the transmission vs. energy curve been the same for all attenuator thicknesses, it would have been sufficient to measure attenuation for one single thickness. Clearly then, the more the transmission vs. energy curve $e^{-\mu(k)x}$ changes shape with attenuator thickness, the more the additional spectral information acquired with each new
attenuator thickness. Thus it would be to the reconstruction's advantage if the two curves in figure 5.10 were completely different (perhaps one of them sharply decreasing with energy with the other sharply increasing with energy).

This is just the case with scatter analysis where, depending on what angle is being sampled, the scatter signal vs. energy curve changes from decreasing to increasing within the radiotherapy energy range. This is illustrated in figure 5.11.

![Graph of scatter signal vs. energy in an acrylic scatterer for small and large scatter angles.](wasy04m06d1101 fig5.fig)

Figure 5.11 – Scatter analysis: Variation of scatter signal with energy in an acrylic scatterer for small and large scatter angles. Both curves are normalized to 1 at 3MeV. Calculated by Monte Carlo simulation.

In scatter analysis, the physical quantity which is relied upon for spectral reconstruction is scatter signal or, "Relative energy imparted to the chamber due to the secondary photons produced in the scatterer", where measurements are taken at different scatter angles. This quantity is a function not only of the probability that a
photon will scatter at a specific angle into the chamber (nearly equal to Compton scatter probability and thus decreasing with energy for low Z scatterer, figure 5.6) but also a function of the amount of energy imparted to the chamber per scattered photon, a quantity which is increasing with photon energy.

At small scatter angles (10° curve in figure 5.11, ≈ forward scatter), the scatter signal increases with energy because while the Compton scatter probability decreases with energy (figure 5.6), the ionization imparted to the chamber per scatter event increases more rapidly with energy, thus the overall scatter signal increases with energy. Conversely, at large scatter angles (160° curve in figure 5.11, ≈ backscatter) the scatter probability again decreases with energy (figure 5.6), but this time however, the ionization imparted to the chamber per scatter event remains constant with energy (a backscattered photon always has around 0.255 MeV energy, whether it results from a 1 MeV primary or a 15 MeV primary); and thus the overall scatter signal at large scatter angles decreases with energy. As a result, sampling scatter signal at small scatter angles conveys brand new information compared to that conveyed during sampling at large scatter angles. Thus there is the potential to gather more spectral information in scatter analysis owing to the large difference in slope between the two curves of figure 5.11, a difference which is smaller in figure 5.10.

The above argument is presented quantitatively in table 5.1:

Rows 1 to 4 of this table show the angle of the slopes at energy points 0.5, 1.5, 2.5, 3.5, 4.5 and 5.5 MeV for each of the four curves in figure 5.10 and 5.11: this is the angle between the tangent to the curve and the abscissa axis. Row 5 shows the difference in slopes between the two attenuation curves of figure 5.10; and row 6 shows the difference in slopes between the two scatter analysis curves of figure 5.11. At a specific energy point, 1.5 MeV for example, the difference in slopes between the two attenuation curves is 4.9°, compared to a larger difference in slopes between the two scatter analysis curves of 43.1°. This advantage of the scatter analysis curves over the attenuation curves is seen to diminish with increasing energy without however disappearing.
Table 5.1 — Quantitative comparison between:
- Row5 giving the difference existing between the two curves of figure 5.10 (attenuation curves) and
- Row6 giving the difference existing between the two curves of figure 5.11 (scatter analysis curves).

Figure 5.11 was obtained by Monte Carlo simulation of monoenergetic photon beams with setup characteristics identical to the scatter signal measurement setup in an accelerator room. This will be explained in Section 7.1 "Monte Carlo Calculation of Mono-energetic Scatter Signals".
Chapter 6

Materials and Method

The rationale behind the method for scatter signal measurement was described in chapter 5. The measurements were taken with a conventional PTW Farmer chamber with nominal 0.6cm³ collecting volume, model M30001 connected to a PTW electrometer of the UNIDOS model. The reconstruction method was applied to an SL18 6MV Elekta linear accelerator in our centre for radiotherapy. The scatter signal measurement setup is shown in figure 5.2. Standard broad beam geometry is used with a 100 cm SSD from source to scatterer surface.

As indicated earlier, in order to reconstruct the energy spectrum of a photon beam, our method requires first the measurement of a "scatter curve" (figure 5.2). The latter consists of a series of measurements of scatter signal (chamber ionization measurements in the order of several hundred picoCoulomb with an average error in precision of 0.84% (one standard deviation) combined for positional and reading precision) taken at specific chamber positions (figure 5.2), each position intended to sample a different scatter angle (the angle being defined as mentioned in chapter 5). Clearly, the more the sampled angles, the more and the finer the spectral information collected for the reconstruction. But more sampled angles means more time consuming measurements. The reconstruction results were consistently unstable with 1 to 3 measurement angles per reconstruction. Results became stable and reproducible at 5 measurement angles (the reconstruction precision and accuracy is addressed in chapters 7 and 8); thus our scatter curve consisted of five scatter signal measurements taken at five different angles from central axis; one large (160 degrees) angle to sample the backscatter behaviour of the primary beam, one small (10 degrees) angle to sample the forward behaviour of the primary beam and three other intermediate positions to sample intermediate scatter angles.

The scatterer consisted of a rectangular shaped acrylic block (acrylic is also known as Polymethyl Methacrylate, PMMA, Perspex, lucite or plexiglas). The scatterer's length parallel to central axis was 20cm. The block's square cross-section was originally 2cm x 2cm to avoid annihilation interactions within the scatterer as indicated in chapter 5.
But with these small dimensions, the scatter provided too faint a signal to be measured with our 0.6cm³ Farmer chamber. So the square's cross-section was then increased to 10x10 cm² for a better signal to noise ratio. The signal to noise ratio was not calculated directly, it was simply considered "acceptable" for our purpose when we started getting somewhat stable reconstructions with only 4 points scatter curves. The gantry was rotated to 90 degrees to produce a horizontal beam shaped by the accelerator's secondary collimators into a 12x12 cm² field (measured at isocenter) in order to fully cover the scatterer. The useful part of the beam was still 10x10 cm² as the additional 1cm coverage of the 12x12 cm² field beyond each 10cm side of the scatterer only served to ensure that the scatterer stays well within the beam, unexposed to the penumbra. Hence, the effective scatter-contributing field size was 10x10 cm².

The scatterer and chamber system were placed on top of the couch, lying on a low density wooden stand parallel to the couch (rather than directly on the couch) to minimize scatter from metallic components in the couch, thus increasing the overall signal-to-noise ratio. The low density wooden stand was shaped like a flat table with a rectangular 40x100 cm² surface (parallel to the couch). The stand was placed on the couch, standing like a table on four 35cm high legs. Its undesired contribution to scatter signal was considered as a part of the background room scatter, as was the contribution from treatment couch and room floor and walls. Therefore the stand's contribution was considered to be cancelling out in as much as the entire background's contribution did, owing to the subtractive measurement technique analysed previously.

The ionization chamber was inserted with its acrylic cap on into a rectangular shaped 4cm x 5cm x 6cm acrylic build-up block shown in figure 6.1. With the scatterer and the build-up block (housing the chamber) both lying on the horizontal wooden stand's surface, the build-up bloc was made to slide on the stand's surface up and down the scatterer (but always outside of the primary beam) on a straight line parallel to the central axis and located at 11.5cm away from central axis as shown in figure 5.2 which is not drawn to scale.
Readings at each sampled angle were taken by irradiating the scatterer with 300 monitor units twice (once with the scatterer in the beam and once with the scatterer removed as described in chapter 5) and then subtracting them. After subtraction, the scatter signal was around several hundred pC for the 300MU readings. The highest scatter signal readings were recorded with the chamber in positions 3 and 4, per figure 5.2. The lowest scatter signal readings were obtained both at small scatter angles (position 5), and large angles (position 1).

A sample measured scatter signal curve is shown in figure 6.2, one curve for the open non-wedged beam and the other for the 40° wedged beam. The corresponding data are shown in table 6.1 and table 6.2. For each detector position, a set of scatter signals measured during several consecutive setups (on different days) was gathered, and a mean scatter signal and associated standard deviation were calculated. This standard deviation is reported in table 6.1 and table 6.2 in row 4 as precision error.
<table>
<thead>
<tr>
<th>Chamber position number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row1, scatter in beam, 300MU reading, pC</td>
<td>724.1</td>
<td>1009.0</td>
<td>1501.0</td>
<td>1664.1</td>
<td>1446.0</td>
</tr>
<tr>
<td>Row2, scatter out of beam, 300MU reading, pC</td>
<td>638.8</td>
<td>725.4</td>
<td>812.6</td>
<td>885.3</td>
<td>934.0</td>
</tr>
<tr>
<td>Row3=Row1-Row2, scatter signal, 300MU, pC</td>
<td>85.3</td>
<td>283.6</td>
<td>688.4</td>
<td>778.8</td>
<td>512.0</td>
</tr>
<tr>
<td>Precision error associated with measurement of scatter signal in row3, 1 std dev., %</td>
<td>0.83</td>
<td>0.41</td>
<td>0.82</td>
<td>1.03</td>
<td>1.11</td>
</tr>
</tbody>
</table>

Table 6.1 – Measured scatter signal as a function of chamber's position number. (per figure 5.2). Open beam, non-wedged.

<table>
<thead>
<tr>
<th>Chamber position number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row1, scatter in beam, 300MU reading, pC</td>
<td>569.3</td>
<td>702.6</td>
<td>932.6</td>
<td>1008.0</td>
<td>879.6</td>
</tr>
<tr>
<td>Row2, scatter out of beam, 300MU reading, pC</td>
<td>528.1</td>
<td>568.1</td>
<td>594.4</td>
<td>610.8</td>
<td>608.3</td>
</tr>
<tr>
<td>Row3=Row1-Row2, scatter signal, 300MU, pC</td>
<td>41.2</td>
<td>134.5</td>
<td>338.2</td>
<td>397.2</td>
<td>271.3</td>
</tr>
<tr>
<td>Precision error associated with measurement of scatter signal in row3, 1 std dev., %</td>
<td>0.86</td>
<td>0.22</td>
<td>0.76</td>
<td>1.18</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Table 6.2 – Measured scatter signal as a function of chamber's position number. (per figure 5.2). 40° wedged beam.
Figure 6.2 - Measured scatter signal for 6MV beam as a function of chamber position number as indicated in figure 5.2. Upper curve is for open beam, no wedge used; lower curve is for 40° wedged beam.

To illustrate visually how a scatter curve can reflect beam quality, figure 6.3 shows three scatter curves: one measured for an open non-wedged beam, the second for a 20° wedged beam and the third for a 40° wedged beam. To compare the curves correctly, they are normalized all three to 1 at chamber position 1. The normalization is required because it is the change in shape of the scatter curve which reflects the beam quality change, and not the absolute value of the scatter signals collected. We cannot rely on absolute ionization values because, while the monoenergetic MCNP obtained scatter signals are calculated on a "per emitted photon" basis, we do not know how many individual photons are emitted in 300MU. Therefore we can only use these measurements as "relative" scatter signal readings. It is this difference in scatter curves depicted in figure 6.3 which scatter analysis exploits to reconstruct the spectrum.
Once a scatter curve is measured and a parametric model is chosen for the spectrum, the model's parametric scatter curve is calculated next and forced through optimization to match the actual measured scatter curve.

System 5.3 presented symbolically earlier as an example will now be re-written in its actual form used in the reconstruction. The actual system used in the reconstruction consists of five measurements (versus three in the example) for the scatter curve ($s_1$ to $s_5$) and of an energy bin width of 0.01MeV (versus 1MeV in the example of chapter 5). For the 6 MV beam, 681 bins were used from 0.2MeV to 7MeV, each bin being
0.01 MeV wide; further decrease in bin width did not seem to bring any further improvements to the reconstruction. Contrary to intuition, too much decrease in bin width can destabilize the solution; that's because when the bin size is too small one approaches the situation where an integral operator is used, as if equation 2.3 was being solved directly without replacing the integral by a discretization \(^w2\).

Let \([A, B, C, D\) and \(F]\) represent the relative model's parametric scatter curve, and \([s_1, s_2, s_3, s_4\) and \(s_5]\) the actual measured scatter curve. System 5.3 then becomes:

\[
\begin{align*}
A & \propto a_1x_1 + a_2x_2 + \ldots + a_{681}x_{681} \\
B & \propto b_1x_1 + b_2x_2 + \ldots + b_{681}x_{681} \\
C & \propto c_1x_1 + c_2x_2 + \ldots + c_{681}x_{681} \\
D & \propto d_1x_1 + d_2x_2 + \ldots + d_{681}x_{681} \\
F & \propto e_1x_1 + e_2x_2 + \ldots + e_{681}x_{681}
\end{align*}
\]

To save time, "the bin-specific mono-energetic scatter signals per photon" \([a_1, a_2, \ldots, a_{681}]\), \([b_1, b_2, \ldots, b_{681}]\), \([c_1, c_2, \ldots, c_{681}]\), \([d_1, d_2, \ldots, d_{681}]\) and \([e_1, e_2, \ldots, e_{681}]\) were obtained by interpolating within a coarser matrix of Monte Carlo calculated values from 0.2 MeV to 7 MeV in steps of 0.2 MeV (thirty-five bins). Thus, for each sampled angle only thirty-five mono-energetic responses were actually calculated by Monte Carlo; finer values at 0.01 MeV intervals were obtained by a smoothing interpolation technique. Without interpolation, the time it would have taken to simulate all 0.01 MeV bins would have been about

\[
5 \text{ (positions)} \times 681 \text{ (simulations/position)} \times 1 \text{ (hour/simulation)} \text{ (as an average)} = 4.7 \text{ months.}
\]

For the parametric spectral model we used

\[
\Phi(k, E, T) = (1/k) \left[ \left(1 - \frac{k}{E} \right)(\ln \eta - 1) + \left(\frac{k}{E} \right)^2 (\ln \eta - 0.5) \right] \exp(-\mu_r(k)T) \tag{6.2}
\]

and changed the notation for the incident electron energy from \(E_0\) to \(E\) while \(T\) is still the equivalent tungsten thickness of target and all subsequent filters, and \(k\) the produced bremsstrahlung photon energy. The \(1/k\) factor converts the energy fluence
into particle fluence. Only two free parameters \( e \) and \( t \) were used in order not to overburden the optimization algorithm, since the problem at hand is ill-posed. With an ill-posed problem, stability in the solution is always a concern, and the lesser the free parameters the more restrained and stable the solution.

System 6.1 then becomes:

\[
\begin{align*}
A &= a_1 \Phi(k_{1,E,T}) + a_2 \Phi(k_{2,E,T}) + \ldots + a_{681} \Phi(k_{681,E,T}) \\
B &= b_1 \Phi(k_{1,E,T}) + b_2 \Phi(k_{2,E,T}) + \ldots + b_{681} \Phi(k_{681,E,T}) \\
C &= c_1 \Phi(k_{1,E,T}) + c_2 \Phi(k_{2,E,T}) + \ldots + c_{681} \Phi(k_{681,E,T}) \\
D &= d_1 \Phi(k_{1,E,T}) + d_2 \Phi(k_{2,E,T}) + \ldots + d_{681} \Phi(k_{681,E,T}) \\
F &= e_1 \Phi(k_{1,E,T}) + e_2 \Phi(k_{2,E,T}) + \ldots + e_{681} \Phi(k_{681,E,T})
\end{align*}
\] (6.3)

In system 6.3 above, 
\([A, B, C, D \text{ and } F]\) are the relative parametric model's scatter signals for chamber positions 1, 2, 3, 4 and 5,
\([k_{1}, k_{2}, \ldots, k_{681}] = [0.20, 0.21, 0.22, \ldots, 6.69, 7.00] \text{ MeV},\)
\([a_1, a_2, \ldots, a_{681}], [b_1, b_2, \ldots, b_{681}], [c_1, c_2, \ldots, c_{681}], [d_1, d_2, \ldots, d_{681}] \text{ and } [e_1, e_2, \ldots, e_{681}]\) are already calculated with Monte Carlo as described previously.

Since \([s_1,s_2, s_3, s_4 \text{ and } s_5]\) are the actual measured relative scatter signal readings for chamber's positions 1, 2, 3, 4 and 5, we should be able to write

\[
\begin{align*}
A/F &= s_1/s_5 \\
B/F &= s_2/s_5 \\
C/F &= s_3/s_5 \\
D/F &= s_4/s_5
\end{align*}
\] (6.4)

Next we use a simple sum of squares minimization to solve for system (6.4). Thus the following objective function is written and minimized

\[
O(e,t) = \sum_{k=0.2}^{7} \left[ \frac{A/F}{s_1/s_5} - 1 \right]^2 + \left[ \frac{B/F}{s_2/s_5} - 1 \right]^2 + \left[ \frac{C/F}{s_3/s_5} - 1 \right]^2 + \left[ \frac{D/F}{s_4/s_5} - 1 \right]^2
\] (6.5)
Minimizing was first attempted by using a least squares minimization algorithm called "lsqnonlin" from the Matlab® computational package which unfortunately converged sometimes toward local minima. We found that the most efficient method to guarantee the attainment of a global minimum in our search is to simply evaluate $O(e,t)$ over an entire matrix of values for $e$ and $t$ of finite dimensions, then choose the optimal $(e,t)$ pair which gives the smallest value for the objective function.
Chapter 7

Monte Carlo Simulations

7.1 Monte Carlo Calculation of Mono-energetic Scatter Signals

This work used the Monte Carlo N-Particle code (MCNP) version 4C2. The code was originally written at the Los Alamos National Laboratory; new and improved versions are continuously being produced.

Essentially, the code allows solving for radiation transport problems through three steps. In a first step, the user describes the radiation problem: s/he defines in MCNP coded language its geometry, defining the radiation source, the nature, energy, direction of flight and number of particles which will be emitted by the source; the shape, atomic composition and coordinates of all materials exposed to the radiation. All shielding or scattering objects are included in the geometry.

In a second step, the user specifies the quantity which s/he wishes to be evaluated by the code (called tally), say for example "photon fluence crossing surface S" located at a certain depth inside a water phantom exposed to the source.

Once the problem has been defined and the quantity sought has been specified, the code executes: this is the third step. In this step, the code generates particles at the source in the direction of flight predefined in step one. It then follows each particle through its "history" of interactions until the particle has left the "universe", the user defined geometrical space of interest. As long as it is in the universe, the particle is thoroughly tracked from one interaction to the next. At each interaction, the code stores in memory the secondary, tertiary, etc. particles which are generated by the interaction. Once the primary has left the universe, each of the generated particles (and all of their progenies...) are tracked in turn until they too have left the universe.

The nature and outcome of each particle interaction is determined statistically, i.e. it is sampled from a probability distribution based on the cross section for the particle's interaction in that particular material. For example, if one hundred 1MeV photons impinge on a water phantom, the code will assign some photons to undergo Compton interactions, some others to undergo photoelectric interactions and some others pair
production or Rayleigh scattering or no interaction at all; with higher probabilities of
being selected assigned to the processes with higher cross sections, and vice versa.
Thus, in this particular example, the probability that the code will choose Compton
over photoelectric for the primary 1MeV photons scales like the ratio of the respective
cross sections, i.e. Compton will be more frequently selected than photoelectric.
Once an interaction, say Compton, has been selected for the primary particle, another
sampling takes place to select the energy and scatter direction of the secondary photon
and electron. Here again, energy and scatter angle are both sampled from a table of
relative cross sections for energy and scatter angles. And so on, until all primary
particles as well as all secondaries and progenies have left the universe.

Since the user has specified a tally of photon fluence crossing surface S, each time a
photon crosses S it is recorded in the tally as a score. (Note that by choosing S with a
small surface area, one can approximate the fluence at a point). Simultaneously, the
energy of the scoring particle is also recorded. Having thus tracked all particle
histories, the code finally outputs the tally, which would be "photon fluence at S" in
this example. Since the energy of each scoring particle was also recorded, the tally
can also be obtained distributed over different photon energy bins: the result is photon
particle fluence spectrum at S. Other tallies are available in MCNP, like "energy
deposited in a volume" which we used extensively in this work. It is important to note
that all the tallies given by MCNP are normalized per starting particle. Thus in our
example, if the photon fluence was sought in a situation where one million photons
are initially emitted by the source, then the tally returned by the code would have to be
multiplied by one million to obtain the correct photon fluence at S. Along with every
tally, MCNP provides an estimate of the tally's precision (not accuracy) by calculating
the standard deviation associated with the tally produced. This quantity is as
important as the tally itself, because if the reported error is large, then the tally may be
grossly in error. Therefore a tally result is acceptable only if the associated error is
small enough. How small the error should be, depends on the problem itself and
should be decided on a case by case basis. Obviously the higher the number of
particles started at the source, the higher the statistical precision. However, increasing
the number of particles started at the source in an attempt to increase the result's
precision, can only be achieved at the cost of higher computing time.
Accuracy, on the other hand, depends on:

- How well appropriate a choice the user has made of the physics options that dictate the flow of the simulations and their possible bias.
- How authentically the code succeeds in reproducing the real world physics interactions, and
- How accurately the user has defined the problem from a geometrical stand point.

Regarding the appropriateness of the physics options chosen by the user, this is a non trivial task requiring much familiarity with the code and its intricate details. The simplest radiation transport problem can produce inaccurate results if the user turns ON (or OFF) the wrong options in the problem definition. An example out of many others would be the inaccuracy in the code's estimated dose to a material if the user did not turn ON bremsstrahlung production in a problem involving high energy electrons incident on a nearby high Z material.

The code itself has been proven to accurately model interactions involved in radiation transport, in some situations however more than in others. The interaction cross section library the code uses has been extensively validated by many users over time. In this work, we have attempted to spot check the accuracy of the code's calculations by simulating elementary geometries where the tallies can be accurately reproduced by hand calculations. In a simple test problem configuration where an incident photon beam is attenuated by a metallic slab, our calculations (based on cross section data from XCOM B9) agreed within 2% with the code's estimate of the transmission through the slab. This tested all combined photoelectric, Compton and pair production cross sections stored in the code's library. The test problem was chosen of the narrow beam type, to allow for the use of the well validated photon interaction cross sections which would have otherwise had to be approximated in a broad beam situation. The test was repeated for various photon beam energies and attenuator material.

As to the accurate definition of the problem's geometry, this is an obvious condition, and the MCNP code provides several tools for checking that the geometry of the problem has been encoded correctly by the user before starting the simulations. One of these tools we found most useful is the code's ability to plot a two dimensional view of the geometry of the problem, revealing in one picture what the user has formulated in many lines of coded language.
The number of histories run in this work for each different scatter angle and each energy bin was in the order of 10 million, and the time it took for each simulation to complete depended on the problem's configuration. A very approximate average (with a large standard deviation) would be 1 hour of simulation time per energy bin. For a configuration where particles have to go through much material before exiting the universe, the time to follow 10 million histories is obviously much larger, since the progeny of each starting particle is larger and the total number of particles thus followed is larger. The most expensive transport mode in terms of time is electron transport because of the tremendously large number of interactions an electron and its progeny typically undergo, versus an uncharged particle like a photon (for the same thickness of material). The ionization track of an electron is far denser than that of a photon at therapy energies, and the system is considerably slowed down when electron simulation is turned on. For this reason, in the scatterer for example, a lower electron energy limit of 0.1MeV was specified below which the electron is killed since it can no longer contribute to the tally. The exact number of histories simulated was adjusted to yield in each case a statistical error in the evaluated quantity smaller than 1%.

Variance reduction techniques are often used in Monte Carlo simulations to reduce the result's associated error or variance. These techniques allow the user to increase the result's precision (not accuracy) without necessarily increasing the number of tracked histories. For example, one variance reduction technique that was used in this work consists in forcing every photon that enters the scatterer to interact in the scatterer thus increasing the number of scattered photons depositing dose in the chamber for the same number of starting photons. This technique is called "Forced Collision". To keep the calculations correct, the weight of the scattered particles originating from a forced collision is decreased to account for the fact that in reality less particles would be scoring. Thus instead of having the scatterer generate N scattered particles each with a weight of 1, the scatterer is made to generate 2N particles each with a corrected weight of 1/2, such that the energy deposited in the chamber (the tally) remains unchanged, yet its error is reduced as more particles are contributing to the tally (with a smaller contribution per particle).

The MCNP simulations were run at the American University of Beirut's Centre for Advanced Mathematics (CAMS) on an IBM p630 dual processor, POWER4, 4 nodes.
using AIX5.1L under PVM (Parallel Virtual Machine) mode. PVM reduces considerably the computation time as it splits the simulation job into several subtasks which are then run in parallel on the 8 CPU's.

The acrylic scatterer's atomic composition was obtained from NIST and entered in MCNP as 8.05% Hydrogen, 59.99% Carbon, 31.96% Oxygen by weight; the density used was 1.19 g/cm³. Air in the ionization chamber was assumed to be at standard temperature and pressure (density 0.001205 g/cm³) with an atomic composition of 0.0124% Carbon, 75.53% Nitrogen, 23.18% Oxygen and 1.28% Argon by weight.

The tally used to score energy deposited in a volume is labelled *F8 in MCNP4C2.

Based on the previous considerations, the "bin-specific mono-energetic scatter signals per photon" were calculated. The configuration simulated with MCNP is shown in figure 7.1.
Figure 7.1 – Configuration used for Monte Carlo calculation of the "bin-specific mono-energetic scatter signals per photon".

7.2 Monte Carlo Simulation of Accelerator Head

One of the few tools available nowadays to evaluate the accuracy of any spectral reconstruction method, is to compare it to the spectrum obtained when the accelerator's head is modelled with a Monte Carlo code. This requires accurate knowledge of head geometry and the materials used in the different components in the head. In this work, this information was obtained partly from the accelerator's manual.
but mostly from direct correspondence with the manufacturer (Elekta). Referring to figure 7.2, the components of interest are:

- The Tungsten target: shaped like a penny, with a 5.40mm diameter and 1mm thickness.
- The Copper cooling bloc: in contact with the tungsten target, 10.00mm thickness.
- The Primary jaws: 101mm (nominal) thick, lead.
- The Flattening Filter: triangular shape, height is 24.10mm, material is stainless steel.
- Secondary collimators: about 100mm (nominal) thick, lead.

The object of the simulation in this section is to reconstruct the beam's energy spectrum with MCNP by letting a tightly bundled mono-directional electron beam collide with the accelerator's target and let the code calculate the bremsstrahlung yield resulting from the target as well as from any other head component where the electrons may have interacted. The collimation system was not included in the simulation because of indications of negligible effect on the spectrum's shape from other researchers like Mohan et al\textsuperscript{M3}, Faddegon et al\textsuperscript{F1}, Sixel et al\textsuperscript{S9} and Sheikh-Bagheri et al\textsuperscript{S8} (already discussed in chapter 5). However in this work's simulations, the surface over which photons were scored to determine the beam's energy spectrum (photon collection surface) was a 10x10 cm\textsuperscript{2} square located and centred at isocenter.

In these simulations, 640 million electrons were started impinging vertically onto the target. The photon fluence at the isocenter was tallied and binned in 0.2MeV wide bins. The time for simulation was about 5 hours.
Figure 7.2 - Basic head components
Chapter 8

Spectral Reconstruction Results and Validation

8.1 Introduction

In general, two different methods are possible for validating a reconstructed spectrum S for an accelerator photon beam. The first method is a direct one: using spectral difference (discussed in chapter 3), S is compared directly (i.e., bin by bin) to another already validated spectrum. The second indirect method relies on calculations to derive spectrum dependent quantities from S (depth dose and stopping power data for example), and match them with the same quantities physically measured for that accelerator.

8.2 Results and Validation of Monte Carlo Spectra

A research of the literature for Monte Carlo calculated 6MV photon spectra yielded four specimens depicted in figure 8.1:

Spectrum S1, provided by Mohan et al. [3] (1985) who used the EGS 3.0 (Electron Gamma Shower) Monte Carlo package to calculate the 6MV spectrum from a Varian Clinac 6 accelerator. In his simulations, he used 0.25MeV wide energy bins at lower energies and 1MeV at higher energies; he used a photon cutoff energy of 0.01 MeV and an electron cutoff energy of 1MeV. He observed that the collimation system had very minor influence on the resulting photon spectral shape. The photon collection surface used was a 6cm diameter disk located at isocenter.

Spectrum S2, provided by Sixel and Faddegon [9] (1995) who used the EGS4 Monte Carlo package to calculate the 6MV spectrum from a Therac-6 accelerator. In their simulations, they used .1MeV wide energy bins, a photon and electron cutoff energies of respectively 0.01 and 1 MeV. Their simulations also excluded the whole collimation system. The photon collection surface used was a 10cm diameter disk located at isocenter.

Spectrum S3 is provided by Jong Oh Kim et al. [12] (2001) who used a combination of both Monte Carlo codes, the MCNP4B and the EGS4 code, to calculate the 6MV
spectrum from a Varian Clinac 2100 accelerator with a 10x10 cm$^2$ field size with multileaf collimators.
Spectrum S4 is provided by Sheikh-Bagheri and Rogers $^{88}$ (2002) who used the BEAM Monte Carlo code to calculate the 6MV spectrum (with collimator modelling) from an Elekta SL25 accelerator using a 10x10 cm$^2$ field size and a .25 MeV bin width.

Finally, we will refer to this work's Monte Carlo calculated spectrum for our Elekta SL18 6MV photon beam as S5 where the MCNP4C code was used without collimator modelling (but with a 10x10 cm$^2$ photon collection square at isocenter) and with a bin width of 0.2 MeV.

Spectra S1 to S4 are depicted in figure 8.1 below.

Figure 8.1 – Four Monte Carlo calculated 6MV photon beam spectra taken from the literature.
In the direct comparison method, in order to calculate spectral differences for all present spectra, we must first smooth the spectral lines to get rid as much as possible of the statistical fluctuations (such smoothing is subject to the limitations stated earlier in chapter 3). It is assumed that the actual underlying noise-free spectral curves are smooth because the Schiff analytic expression (equation 4.9) confirms this, as can be seen from the spectral figures of chapter 4.

Thus figure 8.2 below is a reproduction of figure 8.1 with the spectra smoothed and with the addition of this work's Monte Carlo 6MV photon spectrum (S5).

![Figure 8.2](wasy04m10d1201.fig08.fig)

**Figure 8.2** – This work’s Monte Carlo spectrum S5 shown with the S1 to S4 spectra smoothed.

To provide a quantitative estimate of the differences between the previous five spectra of figure 8.2, table 8.1 below reports on the calculated spectral differences.
Table 8.1 – Spectral differences between five Monte Carlo spectra of figure 8.2.

For a better visual comparison of S5 (this work's Monte Carlo spectrum) with S4 (obtained also by Monte Carlo simulation) these two spectra are drawn alone in figure 8.3. A direct comparison (in terms of spectral difference) is more pertinent in this case where both spectra are for the same accelerator as explained below.
Figure 8.3 - Monte Carlo calculated spectrum S5 for our Elekta/SL18 6MV photon beam shown with another Elekta 6MV Monte Carlo calculated spectrum S4. Spectral difference is 6.3%.

The 6MV beams produced by the SL25 and SL18 are identical because of identical target, cooling plate and flattening filter used in both accelerators. In fact these dual energy accelerators are similar, except for minor nominal differences: the accelerator’s model number is changed only to reflect the higher energy chosen by the customer and provided by the manufacturer \( E^1 \). Thus the SL25 accelerator produces 6 and 25MV photon beams while our SL18 produces 6 and 15MV (not 18MV) photon beams. But both 6MV beams are identical.

From table 8.1, the quantitative agreement between our Monte Carlo spectrum S5 and S4 (which involves the same nominal beam) is 6.3 %. The disagreement at the curve’s tail is due to this work’s approximation that the electron beam impinging on the target is mono-energetic at 6.2MeV, based on average photon energy data from Nisbet \( N^1 \)
(6.2MeV) and Sheikh-Bagheri $^{S3}$ (6.3MeV); while S4 took into account the energy spread in the electron beam; thus some photons with energies above 7.0MeV will show in the spectrum. Krmar $^{K1}$ for example, reports the manufacturer's quote for the Siemens Mevatron MD7445 operating at 6MV, that the energy spread for the electron beam incident on the target is Gaussian with a FWHM of 14%.

As for the Varian accelerators, the agreement between S1 and S3 (involving the same manufacturer, thus presumably the same nominal beam) is 9.8 %.

The previous results pertained all to open beams, without the use of a wedge. Another simulation was done with the accelerator's 60 degrees wedge in the beam (the wedge was arbitrarily oriented with the heel-to-toe axis pointing towards the detector side of the couch, per figure 5.2). The corresponding result is plotted in figure 8.4 as spectrum S6, along with the unwedged spectrum S5. Unfortunately we could not find comparative spectra in the literature for the 6 MV Elekta wedged photon beam S6, and the validation of this spectrum could only be very partial, done by a logical comparison with the unwedged spectrum. The wedge-induced beam hardening is evident through the increase in average energy (from 1.9MeV for S5 to 2.5MeV for S6) and through the reduction in overall spectral spread as a large part of the low energy photons are preferentially filtered through the high Z lead wedge. A partial quantitative validation is reported in section 8.4. Table 8.2 below reports on the average and most probable (modal) energies for spectra S5 and S6.

<table>
<thead>
<tr>
<th></th>
<th>S5, Monte Carlo, this work</th>
<th>S6, 60 deg. wedge, Monte Carlo, this work</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average energy</td>
<td>1.9 MeV</td>
<td>2.5 MeV</td>
</tr>
<tr>
<td>Most probable energy</td>
<td>2.2 MeV</td>
<td>2.5 MeV</td>
</tr>
</tbody>
</table>

Table 8.2 – Average and most probable (modal) energies for spectra S5 and S6
Note that the difference in modal energy between S5 and S6 is smaller than the difference in average energy, which confirms the appropriateness of the use of the average energy as a representative single energy for the whole spectrum rather than modal energy. For S6, average and modal energies are equal owing to a more symmetrical spectral shape about the mode than for S5.

Figure 8.4 – Monte Carlo calculated spectra for both open and wedged beams for our Elekta SL18 6MV photon beam.

8.3 Results of Scatter Analysis Reconstruction and Validation

The spectra obtained with our scatter analysis method using two free parameters $\epsilon$ (incident electron energy) and $\tau$ (equivalent tungsten thickness of filters) are shown in figure 8.5 and 8.6 below. As mentioned in chapter 6, the reported reconstructions were done by measuring five-point scatter curves. Figure 8.5 shows the reconstructed spectrum in comparison with S4 and S5, the two Monte Carlo spectra taken from figure 8.3.
In the case of figure 8.5, the degree to which our scatter analysis reconstructed spectrum matches the Monte Carlo calculated spectra is expressed by the spectral differences reported in table 8.2. It can be noted that the scatter analysis spectrum's agreement with the Monte Carlo spectra is comparable to the agreement of the Monte Carlo spectra between themselves reported in table 8.1 (6.3% for S5 versus S4, and 9.8% for S1 versus S3); suggesting that scatter analysis may be used as a reconstruction method with comparable accuracy to the Monte Carlo calculation method.
Monte Carlo calculated spectrum, this work, S5
Monte Carlo calculated spectrum, Sheikh-Bagheri (2002), S4

<table>
<thead>
<tr>
<th>Spectral difference for scatter analysis spectrum</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6.2 %</td>
<td>8.8 %</td>
</tr>
</tbody>
</table>

Table 8.2 – Spectral difference as a measure of goodness of match between the scatter analysis spectrum and the Monte Carlo spectra.

Overall, the scatter analysis reconstructed spectrum appears to be softer than predicted by the Monte Carlo methods. This softness was observed in most of our scatter analysis reconstructions and may be due to some inaccuracies in the Monte Carlo estimation of the "bin-specific mono-energetic scatter signals per photon". A correction vector was thus incorporated into the reconstruction matrix to harden the output spectra. The correction vector was calculated by considering a variety of arbitrary input spectra, simulating the reconstruction method by Monte Carlo and comparing the reconstructed spectrum to the input spectrum. The multiplicative hardening vector was thus extracted as the average ratio of input to reconstructed spectra. This procedure served as a 'calibration' for the reconstruction system and needs to be done once only.

In order to assess the scatter analysis method's ability to correctly reflect small changes in beam hardness, we tested the reconstruction method on three different beam qualities obtained with the 6MV beam by using a different wedge factor for each beam (Figure 8.6). Our accelerator uses one single physical 60 degrees lead wedge moveable in and out of the beam's path inside the accelerator's head. Any beam with a wedge angle below 60 degrees is obtained by combining, in appropriate monitor unit proportions, two sub-beams, one open (using no wedge) and the other fully wedged. The three spectra shown in figure 8.6 pertain to one open beam (no wedge), one beam with a 20 degree wedge and another with a 40 degree wedge. For each one of these three beams, two scatter curves were measured resulting in two distinct reconstructed spectra for each beam quality. The spectral differences between
any two spectra reconstructed for one beam quality are included in figure 8.6's captions, and the spectral difference is below 5% for each pair of spectra. The 20 and 40 degrees wedged beams reflect noticeable hardening of the beam, but the overall accuracy needs to be improved upon since the maximum photon energy obtained for the 20 and 40 degrees spectrum is higher than that for the non-wedged beam, which is incorrect since hardening a beam cannot create photons of higher energy than the initial beam.

Figure 8.6 - Spectra for 0, 20 and 40 degrees wedging. Two sets of scatter signal measurements were taken for each beam quality resulting in two unfolded spectra for each beam quality. Spectral differences are reported for each pair of unfolded spectra.

Next, we consider reconstructions done by anchoring the electron energy $E$ at 6.2MeV and varying only $T$ in the scatter analysis optimization algorithm. Thus only one free parameter is used in this optimization. When this is done, figure 8.7 is obtained. In this case, the reconstruction's precision is very good as the two measured scatter curves for each beam quality have yielded two indistinguishable overlapping reconstructed
spectra. Also, the accuracy seems acceptable since the beam hardening is reflected in the gradual shift of average energy without a change in maximum energy (table 8.3).

Figure 8.7 - Spectra for 0, 20 and 40 deg. wedging obtained with one free parameter used in the optimization algorithm. Two sets of scatter signal measurements were taken for each beam resulting in two spectra (indistinguishable, superimposed) for each beam quality. Average and maximum spectral energies are also shown.

<table>
<thead>
<tr>
<th></th>
<th>S5, Monte Carlo, this work</th>
<th>0 deg. wedge, scatter analysis, 1 free parameter</th>
<th>20 deg. wedge, scatter analysis, 1 free parameter</th>
<th>40 deg. wedge, scatter analysis, 1 free parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculated average energy</td>
<td>1.9 MeV</td>
<td>1.9 MeV</td>
<td>2.0 MeV</td>
<td>2.1 MeV</td>
</tr>
</tbody>
</table>

Table 8.3 – Shift of average energies for the spectra in figure 8.7.
The final two scatter analysis reconstructions (with hardening vector embedded in the reconstruction system) are shown in figure 8.8 below; they pertain both to our Elekta SL18 6MV photon beam fully wedged. Each reconstruction is based on a different measured scatter curve. The reconstruction's precision and accuracy are relatively good (spectral differences in table 8.4) because both reconstructed spectra agree between themselves as well as with S6, the Monte Carlo wedged spectrum already presented in figure 8.4.

Figure 8.8 – Elekta 6MV 60 degree wedged spectra: Spectra from two measured scatter curves plotted against Monte Carlo simulation S6.
8.4 The Depth Dose Test

As other workers have done \cite{B, P}, we will use Monte Carlo simulation to calculate depth dose values at 10 and 20 cm for both open beam (no wedging) reconstructed spectra which we obtained: one open beam spectrum we calculated using Monte Carlo simulation \cite{S} and the second one obtained via scatter analysis with two free parameters, both spectra shown in figure 8.5. These calculated depth dose values will then be compared to actual measured values, to further validate our reconstructed spectra. Assuming that the Monte Carlo calculation of the depth dose values is accurate, an eventual disagreement between measured and calculated depth dose values would reflect a lack of accuracy in the reconstructed spectra. The opposite, however, is not true; a close agreement between measured and calculated depth dose values does not necessarily imply a close agreement between reconstructed and actual spectrum: Partridge \cite{P}, reported on two very different spectra having nearly equal transmission curves (within 2%). He concluded that quantities like transmission and depth dose curves were "not very sensitive to small changes in the spectrum". Nevertheless, as mentioned previously, if our spectra are accurate then they should pass the depth dose test; otherwise they are inaccurate. It is in this "one way" sense only that the depth dose test can be used as a partial validation of the reconstructed spectra. Table 8.5 shows the result of the depth dose test. The agreements are very

<table>
<thead>
<tr>
<th>S6,60 deg. wedge, Monte Carlo, this work</th>
<th>60 deg. wedge, scatter analysis, curve 1</th>
<th>60 deg. wedge, scatter analysis, curve 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0%</td>
<td>4.0%</td>
<td>3.6%</td>
</tr>
<tr>
<td>60 deg. wedge, scatter analysis, curve 1</td>
<td>0.0%</td>
<td>3.6%</td>
</tr>
<tr>
<td>60 deg. wedge, scatter analysis, curve 2</td>
<td></td>
<td>0.0%</td>
</tr>
</tbody>
</table>

Table 8.4 – Spectral differences for the wedged spectra of figure 8.8.

77
good. Figure 8.9 shows the geometry simulated with Monte Carlo in calculating the depth dose values. The same geometry was used for the depth dose measurements.

![Diagram](image)

**Figure 8.9** - Configuration simulated with Monte Carlo in calculating depth doses. The same geometry was used for the physical depth dose measurements.

<table>
<thead>
<tr>
<th>Measured depth dose values in %</th>
<th>Depth dose for Spectrum S5 reconstructed by Monte Carlo in % (Deviation from measured values)</th>
<th>Depth dose for Spectrum reconstructed by scatter analysis in % (Deviation from measured values)</th>
</tr>
</thead>
<tbody>
<tr>
<td>at 10cm</td>
<td>67.4</td>
<td>67.6 (0.3%)</td>
</tr>
<tr>
<td></td>
<td>67.5 (0.2%)</td>
<td></td>
</tr>
<tr>
<td>at 20cm</td>
<td>39.4</td>
<td>39.2 (1.0%)</td>
</tr>
<tr>
<td></td>
<td>39.0 (0.5%)</td>
<td></td>
</tr>
</tbody>
</table>

**Table 8.5** - Results of depth dose test with deviation from measured depth dose value in brackets.
Chapter 9

Stability of Reconstruction by Scatter Analysis

In this chapter, we study the robustness of the scatter analysis reconstruction method against measurement errors. An arbitrary but realistic 6MV photon energy spectrum \( F \) is chosen by selecting appropriate values for \( e \) and \( \tau \) in equation 6.2. A scatter curve \( S \) is calculated which will serve as a starting point for recovering the original spectrum \( F \) by the scatter analysis method. Next, the scatter curve \( S \) is intentionally perturbed by adding white noise to it and a second attempt is made to recover \( F \). The deviation of the obtained spectrum \( F' \) with respect to \( F \) is recorded. This deviation is reported as a measure of the reconstruction method's stability versus statistical measurement errors.

We first consider the case of spectral reconstruction using five-point scatter curves and a spectrum with 2 free parameters, \( e \) and \( \tau \). Thus our spectrum is of the parametric form

\[
\Phi(k,E,T) = \left(1 - \frac{k}{E}\right)(\ln \eta - 1) + \left(\frac{k}{E}\right)^2 (\ln \eta - 0.5) \exp(-\mu(k)\tau)
\]  \hspace{0.5cm} (9.1)

with \( \Phi(k,E,T) \) binned into 0.01MeV wide energy intervals, \( 0.2 < k < 7 \) MeV. Thus \( \Phi(k,E,T) \) is a 681x1 column vector.

By arbitrarily choosing \( \tau=1 \) cm and \( e=6 \) MeV, we obtain the starting energy spectrum \( F \) (size=681x1) shown in figure 9.1. A five-point scatter curve \( S \) (size=5x1) is calculated using equation 5.4 (the proportionality sign is replaced by an equality sign for error analysis):

\( S = AF \)

where \( A \) (size=5x681) is the scatter matrix containing Monte Carlo calculated scatter signal values for each combination of chamber position and energy bin.

Next \( S' \) (size=5x1) is calculated

\( S' = S + w \)
where vector \( \mathbf{w} \) (size=5x1) represents intentionally added white noise expressed as one standard deviation, sigma. Based on scatter curve \( S' \), we reconstruct the corresponding spectrum \( F' \) by minimizing

\[
O(E,T) = \sum_{k=0.2}^{7} \left( \frac{A F'(k,E,T)}{S'} - 1 \right)^2
\]  

(9.2)

Values of \( E \) and \( T \) obtained by minimizing equation 9.2 are shown in table 9.1 as a function of sigma. The corresponding recovered spectra are plotted in figure 9.2. When sigma=0, \( F \) is equal to \( F' \) and the original 6MeV and 1 cm values for parameters \( E \) and \( T \) are recovered correctly.

<table>
<thead>
<tr>
<th>sigma, ( % )</th>
<th>E, MeV</th>
<th>T, cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.0</td>
<td>1.00</td>
</tr>
<tr>
<td>1</td>
<td>6.1</td>
<td>0.99</td>
</tr>
<tr>
<td>2</td>
<td>6.0</td>
<td>1.09</td>
</tr>
<tr>
<td>3</td>
<td>6.2</td>
<td>0.92</td>
</tr>
<tr>
<td>4</td>
<td>5.5</td>
<td>1.29</td>
</tr>
<tr>
<td>5</td>
<td>5.4</td>
<td>1.55</td>
</tr>
<tr>
<td>10</td>
<td>7.4</td>
<td>1.78</td>
</tr>
<tr>
<td>20</td>
<td>8.3</td>
<td>2.83</td>
</tr>
</tbody>
</table>

Table 9.1 – Returned optimal values for \( T \) and \( E \). \( T \) is reported to the 10th of the mm because 0.1mm of tungsten attenuates by 0.9% only (at 2MeV ≈ average spectral energy for 6MV beam), while 1mm attenuates by as much as 8% and would therefore not provide enough accuracy.
Figure 9.1 - Initial spectrum F to be reconstructed
Figure 9.2 – Recovered spectra corresponding to errors expressed as 1 standard deviation (1%<sigma<20%) for the case of two free parameter reconstruction. Corresponding values for $e$ and $\tau$ are given in table 9.1.

Next we consider the case of spectral reconstruction using a three-point only scatter curve and a spectrum with 1 free parameter only, $\tau$ ($e$ is fixed at 6MeV). Thus our spectrum is of the form

$$\Phi(k,\tau) = \left[ 1 - \frac{k}{6.511} \right] (\ln \eta - 1) + \left( \frac{k}{6.511} \right)^2 (\ln \eta - 0.5) \exp(-\mu_i(k)\tau)$$

By arbitrarily choosing $\tau=1$ cm in equation 9.3 above, we obtain the same starting energy spectrum $F$ (size=681x1) shown above in fig.9.1. A three-point scatter curve $S$ (size=3x1) is calculated as before:

$S=AF$

where $A$ (size=3x681) is the scatter matrix containing Monte Carlo calculated scatter signal values for each combination of chamber position and energy bin.
Next $S'$ (size=3x1) is calculated

$$S' = S + w$$

where vector $w$ (size=3x1) represents intentionally added white noise expressed as one standard deviation, sigma. Again, based on scatter curve $S'$, we reconstruct the corresponding spectrum $F'$ by minimizing

$$O(\tau) = \sum_{k=0.2}^{2} \left( \frac{AF'(k, \tau)}{S'} - 1 \right)^{2} \quad (9.4)$$

Values of $\tau$ obtained by minimizing equation 9.4 are shown in table 9.2 as a function of noise's standard deviation sigma. The corresponding recovered spectra are plotted in figure 9.3.

<table>
<thead>
<tr>
<th>sigma, %</th>
<th>$\tau$, cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.00</td>
</tr>
<tr>
<td>1</td>
<td>0.94</td>
</tr>
<tr>
<td>2</td>
<td>1.19</td>
</tr>
<tr>
<td>3</td>
<td>0.71</td>
</tr>
<tr>
<td>4</td>
<td>0.83</td>
</tr>
<tr>
<td>5</td>
<td>0.75</td>
</tr>
<tr>
<td>10</td>
<td>1.42</td>
</tr>
<tr>
<td>20</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Table 9.2 - Returned optimal values for $\tau$. 
Recover spectra for values of $\sigma$ from 1 to 20% in the case of one free parameter reconstruction. Corresponding values for $\tau$ are given in table 9.2.

Thus it appears from figure 9.2, where 2 free parameters $\varepsilon$ and $\tau$ are used in the reconstruction, that the reconstructed spectra agree well with the original spectrum as long as the error $\sigma$ remains below 3%. Thus, our reconstruction method requires a precision in the scatter signal measurements of about 3%. Higher measurement errors may lead to incorrect spectra (not matching the original spectrum) without necessarily leading to unphysical (negative spectral values, discontinuities...) spectral shapes.

On the other hand, from figure 9.3, the single free parameter reconstruction technique requires less precision: it can work with errors up to 10% in scatter signal measurements. 20% errors still yield physical solutions, in disagreement however with the original spectrum.

In figure 8.7 of the previous chapter, spectral reconstruction by scatter analysis with one free parameter was done twice for each beam quality, each time using a different measured scatter curve. The two spectra produced for each beam quality are in very close agreement with each other. For the open beam (no wedge) the average deviation between the two measured scatter curves was 0.45% ($<10\%$); for the 20% wedged
beam it was 0.67% (<<10%) and for the 40% wedged beam it was 0.82% (<<10%). Since the required precision is 10% only for the success of the one free parameter reconstruction (as discussed above, figure 9.3), this explains the close agreement between the two reconstructed spectra for each beam quality of figure 8.7.
Chapter 10

Conclusion and Further Work

Scatter analysis is a new method for spectral reconstruction. The results in this work are promising but the method needs further development and refinements to improve on its present overall accuracy and stability. Tables 6.1 and 6.2 show a consistently higher degree of precision associated with scatter signals measured at detector position 2. The reasons behind this should be investigated in order to generalize this higher degree of precision to the other detector positions. Reconstruction with 6 or 7 point scatter curves may provide improved accuracy and stability and should therefore be investigated. In the field, the choice of the adequate number of measurement points per scatter curve may then be determined as a trade off between practicality and accuracy.

The reconstruction method's feasibility with photon beams other than 6MV can be explored. For such higher energy beams, new energy bins above 6MeV are introduced that require Monte Carlo calculations of the bin-specific mono-energetic scatter signal values. These simulations are time consuming: the higher the incident electron energy, the more abundant the progeny, the more abundant the particle histories to follow and the longer it takes for each simulation. We were unable to find safe, yet efficient enough, variance reduction techniques which would permit reducing the duration of these high energy simulations enough to allow making them part of this first study. Thus beams higher than 6MV were excluded from the present work.

The method used here to validate the spectra reconstructed by scatter analysis is more relevant than the classic depth dose test prone to false positives: we first reconstructed the spectrum by Monte Carlo simulation, validated that against similar Monte Carlo results obtained by other workers; then compared the scatter analysis spectrum to the Monte Carlo spectra.

The use of only one free parameter in the Schiff spectral expression adds stability to the reconstruction, while two free parameters can offer more flexibility and thus the potential for improved accuracy in the reconstructed spectrum. Further work should make it possible to use three free parameters in the model (E,T,α) without
compromising the reconstruction's accuracy which is currently about 6.2% expressed as spectral difference, with this work's Monte Carlo calculated spectrum taken as reference. This would allow the solution to take advantage of the parametric Schiff model's versatility shown in figures 4.3 and 4.4. The current lack of accuracy with our results using two free parameters may be related to possible inaccuracies in the Monte Carlo calculation of the bin-specific mono-energetic scatter signals, and Monte Carlo experts may shed some light on the subject. There may also be room for improvement on the accuracy (not precision) of the subtractive method we used to measure scatter signal.

Consideration may be given to using a long band of film laying aside the scatterer and parallel to central axis, rather than an ion chamber which can only offer point by point scatter signal measurements. Once exposed, developed and scanned, such a film would provide scatter signal information at very high resolution, a single film taken bringing as much spectral information as may be collected by the simultaneous use of a line up of hundreds of small ionization chambers (100-point scatter curves...).

Consideration can also be given to using a narrower scatterer and compensating for the low signal intensity by using a chamber with a larger sensitive volume. A larger sensitive volume would increase the measured scatter signal but may lead to a possible loss in spatial accuracy.

In conclusion, this work establishes the basis for scatter analysis and demonstrates the potential for professionally designing a portable, scatter analysis based, "scatter-spectrometer" which consists in a small (few decimeters) case-carried device housing a relatively inexpensive scatterer-detector pair. The scatterer vs. detector geometry would typically be rigid (but variable by the user) for precision and accuracy sake. The design and calibration of the device would rely on Monte Carlo simulations which need only be performed once by the designer. Indeed the bin-specific mono-energetic scatter signals calculated with MCNP4C are applicable to any other 6MV photon beam. Furthermore, if we succeed in freeing the reconstruction from the present necessity of using the previously discussed "correction vector", then such a system would become readily applicable to photon beams of other energies. The system could then be used in any radiotherapy department for photon beam spectral reconstruction and beam energy constancy QA; the procedure requiring only a "quick" non-cumbersome irradiation at standard patient treatment setup geometry.
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Personal correspondence, Local Elekta's representative company: Intermedics, SAL. Beirut- Lebanon.


M2- *MCNP4C2 June 2001*, RSICC Computer code collection, code CCC-701, Oak Ridge National Laboratory.


Bibliography


