Internal stresses and the cyclic deformation
of an aluminium matrix composite

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Summary

The development of internal stresses in planar random Saffil fibre reinforced aluminium with a range of fibre volume fractions has been studied theoretically and in monotonic and cyclic deformation (Bauschinger) experiments at room temperature and 77K.

The conventional method of analysing Bauschinger experiments is extended to allow for a separation of the mean and thermal stresses. This analysis is applied to experimental results enabling the mean stress hardening rate and the magnitude of the thermal stress to be measured. The experimental results are compared with predictions of the mean field model, which is based on the Eshelby method of determining internal stresses. For that purpose the Eshelby S tensor for a planar random array of fibres is calculated. Because the aluminium\$Saffil composites are not isotropic in the transverse directions, the plastic strain used in the calculations has to be determined experimentally. A method for quantifying the anisotropic plastic flow of the aluminium\$Saffil composites is proposed and the results are used in calculations of the mean stress hardening rate.

A comparison of predictions for the mean stress hardening rate with results of the experimental analysis proposed here shows that good agreement is obtained for low fibre volume fractions at 77 K. The results also show that relaxation of the mean stress increases with fibre volume fraction and that at 77 K the mean stress hardening rate is about a factor of two larger than at room temperature. The measurements of the thermal stresses obtained in the Bauschinger experiments are in quantitative agreement with results obtained in monotonic tests. The magnitude of the thermal stress at room temperature or 77 K is independent of fibre volume fraction and a comparison with predictions shows that relaxation of the thermal stress increases with fibre volume fraction. Cycling in the Bauschinger experiments reduces the thermal stress and hence the separation of the mean and thermal stresses is essential for a reliable measurement of the mean stress hardening rate.

Matrix hardening contributes considerably to the overall hardening of the composite, both at room temperature and 77 K. The modified Orowan-Wilson model, which enables the plastic friction coefficient to be measured in copper-tungsten composites, has been applied to the aluminium\$Saffil composites. The model requires both the mean stress and the peak stress curves obtained in Bauschinger experiments to be linear in plastic strain.
Most of the peak stress curves for the aluminium\textregistered Saffil composites are non-linear but for the curves which are linear the predictions of the model are not in quantitative accord with experimental results. This may be because relaxation reduces the mean stress and the source shortening stress in different proportions. The diameter of the Saffil fibres is also close to the lower end of applicability of the model.

The temperature dependence of the mean stress hardening rate suggests that relaxation is thermally activated. A model for relaxation of the mean stress is proposed. An equation is derived for the number of Orowan loops per fibre and it is assumed that the rate controlling mechanism of relaxation is cross slip of screw dislocations. The estimated activation energy is independent of fibre volume fraction but the activation volume decreases with increasing fibre volume fraction. The magnitudes of activation energy and activation volume support the assumptions of the model.

A preliminary study on the early stages of fatigue shows that persistent slip bands form in the matrix of the aluminium\textregistered Saffil composites.
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There is
no such thing as a problem
without a gift for you
in its hands.

Richard Bach.
Chapter One
Introduction

In the past thirty years or so, considerable understanding of the properties of metal matrix composites has been gained through experimentation and modelling. Early studies were primarily on continuous boron, carbon and tungsten fibre reinforced systems which can offer significant improvements in, for example, stiffness and strength in the fibre direction. This led to some development exercises in the aerospace industry but application of these new materials did not gain significant momentum at that time because the fibres and the manufacturing processes were expensive and also because the material properties are highly anisotropic. In the beginning of the eighties, the appearance of cheap discontinuous reinforcement renewed the enthusiasm for metal matrix composites, particularly in the automobile industry. Indeed, the first commercial success of a metal matrix composite was the strategic use of discontinuous alumina/silica fibres in a cast aluminium alloy diesel engine piston. The key to this success was that significant improvements in material properties were gained with a smaller degree of mechanical anisotropy than in the continuous fibre systems and the new material was also cost competitive. Many types of discontinuous reinforcement are now being investigated, such as particles, disks, whiskers and fibres. All these are capable of giving improvements in, for example, strength, stiffness, wear properties and the high temperature performance of the matrix.

The material studied in this work is pure aluminium which is reinforced with discontinuous Saffil fibres. Pure aluminium was used for the matrix in order to avoid complications from dispersions and precipitates. Saffil alumina fibres are an example of the discontinuous reinforcement which became available in the beginning of the eighties. The advantages of these fibres over continuous reinforcement are that they are much cheaper and conventional manufacturing methods such as squeeze casting can be used to make near net-shape components. Alloys reinforced with these fibres can therefore compete with conventional materials, both in terms of material properties and cost.
Improvements in the properties of metal matrix composites over those of the matrix are based on the different properties of the phases of the composite. The fibres are usually stiffer than the metallic matrix and hence carry a disproportionate share of any applied load. The yield or fracture strength of the fibres is generally much higher than that of the matrix with the consequence that the work hardening rate of the composite in plastic deformation is greater than that of the matrix alone. The difference in the coefficients of thermal expansion between fibres and matrix is an additional important consideration. A consequence of these differing mechanical and thermal properties is that internal stresses build up in the material as external stimuli, such as stress or temperature, change. The internal stresses can be very large and are an important factor in the performance of the composite. The emphasis in this thesis is on the theoretical and experimental evaluation of these stresses. The predictions are based on the Eshelby method of determining internal stresses and monotonic and cyclic deformation tests are used to measure the response of the material.

The layout of this thesis is as follows. In chapter two the literature relevant to the work presented in this thesis is reviewed and discussed. The theoretical developments of importance to the experimental work are derived and described in chapter three. In chapter four details of the experimental methods for the monotonic and Bauschinger experiments are given. Results of monotonic tests at room temperature and 77 K are described and analysed in chapter 5. In chapter 6, the results of Bauschinger experiments at room temperature and 77 K are outlined and the results compared with theoretical predictions. The experimental results of chapters 5 and 6 are analysed further in chapter 7 in the light of models for matrix hardening and relaxation. The main conclusions of this thesis and some suggestions for further work are discussed in chapter 8.
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Literature review

In this chapter the literature relevant to the work presented in this thesis is reviewed and discussed. The composite studied here is aluminium reinforced with Saffil (alumina) fibres, fabricated by the squeeze infiltration method. Hence, a review of the properties of Saffil fibres is given in section 2.1 and in section 2.2 some parameters of the fabrication method are discussed. Monotonic deformation of metal matrix composites is reviewed in section 2.3, in preparation for the experimental work presented in chapter 5.

The theoretical work of this thesis is based on the Eshelby method of determining internal stresses. In section 2.4 this method is reviewed and its application to metal matrix composites described. This relates particularly to chapters 3, 5, 6, and 7. In section 2.5 the Bauschinger effect is discussed and the developments which led to its use in measurements of internal stresses are reviewed.

Some aspects of matrix hardening and relaxation are discussed in section 2.6.

2.1 Fibre characteristics

The fibre used in this project has specifically been developed for metal matrix composite (MMC) applications. It is known under the trademark of "Saffil" alumina fibre, RF (reinforcement) grade, manufactured by ICI (Data sheet from ICI, 1982; Dinwoodie et al., 1985, 1987; Clyne et al., 1985; Cappleman et al., 1985). The fibre has a polycrystalline structure in which the predominant phase is δ-alumina (Al₂O₃), and is made by a solution route which avoids melting of the ceramic (Birchall, 1983). The melting point of the ceramic is greater than 2000°C (Data sheet ICI, 1982). An aqueous solution of aluminium compounds is first prepared. Centrifugal spinning of droplets of the solution produces a gel-fibre, which is then heat treated to convert it to the ceramic form (Birchall, 1983). The fibre contains about 3-4 wt% silica (SiO₂) which is distributed throughout the fibre, but tends to become slightly concentrated at grain boundaries and free surfaces. The purpose
of the silica is to stabilise the δ-alumina structure against transformation, at high temperatures, to α-alumina. The formation of mullite (3Al₂O₃·2SiO₂) at grain boundaries also inhibits coarsening of the fine (~50nm) grains (Birchall, 1983; Cappleman et al., 1985; Clyne et al., 1985). The Saffil alumina fibres can be taken as elastically isotropic because of the small grain size. Typical distributions of diameter and lengths of fibres are shown in fig. 2.1, giving a fibre aspect ratio of about 100 to 200 (Data sheet ICI, 1982; Birchall, 1983; Clyne et al., 1985). The mean fibre diameter is ~3µm and mean fibre length ~470µm.

In a Saffil fibre preform, the fibres are held together by a silica binder (SiO₂), with a mass equivalent to about 5% of the mass of the fibres (Data sheet ICI, 1982; Cappleman et al., 1985). The binder is in addition to the silica in the fibre and it is a silica enriched fibre surface layer which is in contact with the matrix material during fabrication. This promotes easier wetting by the melt and better bonding to the matrix (Data sheet ICI, 1982). The fibres in the preform lie in the plane of the preform (see fig. 2.2) and their orientation is random in that plane. The orientation distribution of the fibres in a Saffil preform has been measured by van Hille et al. (1989). They polished sections of a composite with f=20% and used the axial ratio of elliptical sections of the fibres to determine the angle (α') between the fibre axis and the plane of the preform (the 2-3 plane). Their results (see fig. 2.3) show that the fibres lie only on the average in the 2-3 plane and that most of the fibres have a misorientation less than 50-60°.

The values quoted in the literature for the ultimate tensile strength (UTS) and Young's modulus of the Saffil fibres are not always consistent. In the data sheet from ICI (1982), the UTS and Young's modulus are given as 2 GPa and 300 GPa respectively, and there is no mention of the associated statistical deviations. However, Dinwoodie and Horsfall (1987) quote the UTS to lie in the range 1-2 GPa and the modulus in the range 250-300 GPa. Birchall (1983) gives the range 1-2 GPa for the UTS and 250-320 GPa for the modulus of Saffil fibres with 3µm diameter and 1mm gauge length.

2.2 Composite fabrication

The principle of squeeze casting of metals is to force a liquid metal into a closed die where it solidifies under high pressures. The major advantages over other metal casting methods are, for example, collapse of gas and shrinkage porosity, elimination of feeders and
risers and a near-net-shape product (Chadwick and Yue, 1989). This principle can also be applied to the production of metal matrix composites, in which case it is called squeeze infiltration (for reviews, see: Clyne, 1987; Girot et al., 1987; Mortensen et al., 1988; Fukunaga, 1988). A brief review of processing parameters of the squeeze infiltration process now follows.

2.2.1 Wetting

Wetting is quantified by Young's equation

$$\gamma_{SA} - \gamma_{SL} = \gamma_{LA} \cos \theta$$

(2.1)

where $\theta$ is the wetting angle and $\gamma$ is the surface/interface energy. S, A and L stand for solid, atmosphere and liquid respectively. $\gamma_{SA}$ is then the solid(fibre)-atmosphere surface energy. Wetting is generally very poor between different forms of alumina and molten aluminium. Reported wetting angles range from $\pi$ (near the melting point) to about $\pi/3$ (at 1800°C) (Clyne et al., 1985).

To improve wetting it is possible to either increase $\gamma_{SA}$ or decrease $\gamma_{SL}$, or both. The solid-fibre/liquid-metal surface energy can by reduced by stimulating chemical reactions between the liquid metal and the fibres by way of fibre coating and alloying additions (Clyne et al., 1985). This can lead to reaction products at the fibre metal interface which often results in too strong a bond between fibre and matrix or in fibre degradation. This can significantly decrease the strength of the composite (Mortensen et al., 1988). The other way is to increase the solid-fibre/atmosphere surface energy by, for example adsorption of oxygen onto the reinforcement (Mortensen et al., 1988). Molten aluminium has an extraordinarily high affinity for oxygen and it might therefore be imagined that a thin oxide layer would be continually reforming at the infiltration front. In that case, the fibres see aluminium oxide rather than molten aluminium (Mortensen et al., 1988). However, an order of magnitude estimate of the availability of atmospheric oxygen shows it to be in surprisingly short supply (Cappleman et al., 1985) and an oxygen containing reaction layer more than a monolayer or so in thickness would not be possible. Ways of increasing $\gamma_{SA}$ are to infiltrate in an atmosphere free of any oxygen (nearly impossible) or to apply a flux that covers the free surface of the metal at the infiltration front (Mortensen et al, 1988).

The problems associated with poor wetting characteristics are mostly circumvented by using the squeeze infiltration method, because infiltration is then assisted by the application of external pressure.
2.2.2 Preform penetration

Resistance to melt penetration at the surface of the preform arises from the increase in curvature of the melt front when pressure is increased. Theoretical expressions for the pressure required for infiltration to commence are quoted by several authors (Clyne and Bader, 1985; Clyne et al, 1985; Clyne and Mason, 1987; Fukunaga, 1988). For preforms of Saffil fibres with fibre volume fraction $f=20\%$, the calculated pressure necessary to initiate (and continue) infiltration is of the order of 1 MPa (Clyne and Bader, 1985).

2.2.3 Melt passage

After sufficient pressure has built up to allow infiltration, the metal starts to flow into the preform. At this stage it is necessary to ensure that air escape paths remain open during infiltration, otherwise some porosity is inevitable (Clyne and Bader, 1985; Clyne and Mason, 1987). Assuming that no pressure build-up occurs in the gas ahead of the front, the only force opposing the passage of melt through the preform arises from viscous resistance to flow through the interstitial channels. An approximate solution is available for this problem. For a 20mm thick preform with fibre volume fraction $f=20\%$ (fibre diameter 3μm, unidirectional fibres), the calculated pressure at the preform surface necessary to ensure that the velocity of the melt front does not fall below 100 mm/s needs to be only about 1 MPa higher than at the melt front (Clyne and Mason, 1987). In practice, the pressures usually applied are 20-50 MPa (Fukunaga, 1988; Clyne and Bader, 1985). The ram velocity used by Clyne and Bader (1985) is 9mm/s but measurements of temperature in the preform show that the infiltration velocity is probably considerably greater than that. In most cases the melt flow between fibres is expected to be laminar and very unlikely to provoke turbulence (except possibly at very low fibre volume fractions) (Fukunaga, 1988; Clyne and Mason, 1987).

2.2.4 Preform deformation and influence of silica binder on infiltration

It is clear that pressures up to a few MPa might be necessary to initiate the preform infiltration and the resulting deformation of typical Saffil preforms has been measured (Clyne and Bader, 1985). Fibre fracture may start at a relatively low strain but at the low
pressures required to infiltrate the preform (~1MPa), the compression of the preform can be largely accommodated by elastic deformation (Clyne and Bader, 1985).

The rate of relaxation of the preform after passage of the infiltration front is expected to have a significant influence on the success of the infiltration (Clyne and Bader, 1985). If the rate is low, the preform may at a given point not be fully relaxed when the solidification front passes. This can lead to variations in local volume fractions of fibre within a single composite billet (Clyne and Mason, 1987).

The effects of silica binder on the infiltration behaviour have been examined (Clyne et al, 1985). It was found that the level of silica binder had no influence on the infiltration behaviour, except very low values (<1%) result in poor handling strength and very high levels (>15%) are expected to start to inhibit flow of the melt.

2.2.5 Heat exchange

Since the preform is at a lower temperature than the melt before infiltration takes place, there will be a heat exchange between the molten metal and the ceramic fibres. The time for the molten metal and fibres to reach the same temperature has been estimated (Fukunaga, 1988; Clyne and Mason, 1987) and in the case of Saffil fibres, that time is less than a few microseconds. In order for the infiltration to be successful, premature solidification has to be avoided. In the case of Saffil fibres of volume fraction 0.2, preheated to 300°C, a calculation by Clyne and Mason (1987) shows the minimum superheat of the aluminium melt to be about 160K. A higher value might be necessary to eliminate the risk of premature solidification.

2.2.6 Nucleation of solidification

Factors such as the particular alloy system used and processing parameters can be expected to influence the nucleation of solidification in the fibre preform. Clyne and Mason (1987) investigated the matrix microstructure of aluminium|Saffil composites. The matrix grain size was observed to be fine within the composite area. A sharp transition was observed from a fine grain size in the lower portion of the composite, to a coarse columnar structure in the top 10-20% of the preform and above. They conclude that this fine grain structure is either caused by the influence of the fibres on the thermal behaviour (see also Clyne, 1987), or, that the chilling effect of the fibres is sufficient to generate a large number
of small crystallites throughout the preform (Clyne and Mason, 1987). Mortensen et al. (1988) conclude that in most cases nucleation does not initiate on the reinforcement surface and the solid phase will avoid the fibres as it grows. Consequently the last portion of the metal to solidify will be located close to or at the fibre matrix interface. Fukunaga (1988) on the other hand argues that a uniform solidified layer initiates around the fibres. Clyne et al. (1985) report that in a matrix containing silicon, the silicon seems to nucleate on the fibre surface.

2.2.7 Matrix structure

The structure of the matrix phase in the composite is found to be different from that in unreinforced material solidified under similar conditions (Clyne et al., 1985; Mortensen et al., 1986). Hardness measurements of the matrix on both the fibre-free and fibre-reinforced regions of the infiltrated billets have been made (Clyne et al., 1985; Mortensen et al., 1988). They show a significant increase in hardness in the fibre reinforced regions, which may be attributed to changes in the matrix structure and constraint from the array of fibres nearby (Clyne et al., 1985). Large stresses can build up in the matrix due to differences in the coefficient of thermal expansion between the matrix and the reinforcement. These stresses result in higher dislocation densities in the matrix and consequently influence heat treatment of the matrix (Mortensen et al., 1988). Heating the preform to 900°C (higher than usual) results in a generation of a very coarse columnar grain structure throughout the preform (Clyne and Mason, 1987). Warren and Li (1990) have studied the composition of high purity aluminium (99.998wt% Al) before and after infiltration of a Saffil fibre preform (f=20%). They found that after infiltration, the amount of Si had increased to 0.1wt%. The amount of other elements, such as Mg, Mn and Fe, increased less.

2.2.8 Interface structure

The Saffil alumina fibres (RF grade) have a polycrystalline structure in which the predominant phase is δ-alumina. As previously mentioned (section 2.1), it is the silica enriched surface layer of the fibre which is in contact with the metal matrix, rather than pure δ-alumina. Warren and Li (1990), using an aluminium-2.3wt%Mg alloy, have detected a penetration of Mg into the alumina fibre (under the binder) to a depth of about 200nm.
The silica in the surface layer does not seem to be chemically changed by being in contact with the liquid metal matrix and elemental silicon only shows up on extracted fibres when it is present in the alloy. Thus, the interface between the matrix and the fibres in aluminium/Saffil composites does not exhibit any clearly identifiable reaction phase (Cappleman et al., 1985). It appears that the idealized interface, composed of a mechanical continuum (coherency on the atomic level) and a chemical discontinuum (absence of any interdiffusion between constituents) is being approached in Saffil fibre/aluminium alloy composites (Cappleman et al., 1985). Warren and Li (1990), on the other hand, conclude that in composites prepared from fibre preforms containing silica binder, Al and Mg in the matrix react with and reduce the silica during casting to form thin interfacial oxide layers, but reaction of Mg with the alumina fibre is suppressed. However, for a matrix of high purity aluminium, very little interaction was observed between the aluminium and fibres and no second phases were observed in the matrix (Warren and Li, 1990). The availability of oxygen during infiltration has been estimated and the results show that formation of a detectable reaction product, requiring the incorporation of oxygen onto the fibre surface, is unlikely. Even the formation of a monolayer may be difficult (Cappleman et al., 1985).

In general, alloying additions to aluminium improve the interfacial bonding in aluminium/Saffil composites (Warren and Li, 1990). This is supported by observation of fracture surfaces by Clegg et al., (1988).

Attention is now turned to mechanical testing of metal matrix composites.

2.3 Monotonic deformation of metal matrix composites

The simplest geometrical configuration of a metal matrix composite is to have continuous fibres aligned parallel to the loading axis. Considerable work has been reported in the literature on the mechanical properties of such composites, and in section 2.3.1 the aspects relevant to this thesis are reviewed. In section 2.3.2 the focus is on discontinuous composites. Estimates of the thermal residual stress in MMCs is the subject of section 2.3.3 and evaluation of the critical fibre aspect ratio is discussed in section 2.3.4.
2.3.1 Aligned continuous fibre composites with copper or aluminium matrices.

Kelly and Lilholt (1969) reinforced copper single crystals with aligned continuous tungsten fibres of 10μm and 20μm diameter, and measured the stress-strain curves in tension. These curves, of which examples are shown in fig. 2.4, display three stages (I to III) before fracture (see also Kelly and Davies, 1965):

I. Both fibres and matrix deform elastically.

II. Fibres continue to deform elastically but the matrix deforms plastically.

III. Both fibres and matrix deform plastically.

The fibres are elastic in stage II, and hence Kelly and Lilholt (1969) derived the stress strain curves of the matrix in the composite, of which examples are shown in fig 2.5. This figure shows that the work hardening of a constrained matrix (in the composite) is much higher than the work hardening of bulk matrix. Hence, the presence of the fibres strongly affects the work hardening of the matrix in stage II. This is known as the constraint effect (Pedersen, 1983, 1990). Brown and Stobbs (1971a) proposed a model in which they calculated the contribution of the mean stress and the source shortening stress to the total work hardening rate of copper-silica composites. The mean stress is induced by plastic deformation and is the stress exerted by the inclusions on the matrix (see sections 2.4 and 2.5). Whereas the mean stress is uniform, the stress near a fibre is non-uniform and causes the effective interfibre spacing to be reduced for glide dislocations (see fig. 2.6). The constrained flow stress of the matrix in the composite is therefore higher than that of bulk matrix (hence the constraint effect). The increase in flow stress of the matrix due to the presence of the fibres is called the source shortening stress (Brown and Stobbs, 1971a).

The theoretical work of Brown and Stobbs was later extended to fibrous composites by Brown and Clarke (1975, 1977) and the source shortening stress was shown to be proportional to the plastic strain of the matrix. Experimental substantiation of this by Lilholt (1977), who carried out cyclic deformation (Bauschinger) experiments (see section 2.5.3) on continuous fibre copper-tungsten composites, shows a dependence on strain to the power of $\frac{1}{2}$, and the theory predicts only the right order of magnitude for the source shortening stress. According to Pedersen (1990), this lack of agreement stems from Lilholt's use of the instantaneous work hardening rate in the analysis, which includes a Bauschinger effect, inherited from the previous half-cycle. The analysis of the source shortening stress was later improved and modified by Pedersen (1985a, 1990), which uses the peak stress hardening rate in Bauschinger experiments as a measure of the overall
composite hardening. His analysis of experimental results from copper-tungsten composites supports the theoretical results of Brown and Clarke (1977), that the source shortening stress is linear in plastic strain. In fact, Pedersen's modified Orowan-Wilson model for analysis of the Bauschinger experiment leads to a quantitative agreement between theoretical and experimental evaluation of the source shortening contribution.

Cheskis and Heckel (1968) measured the internal stress in copper-tungsten composites with an X-ray diffraction technique. Their results accurately show the transition from stage I to stage II. The measurement of constrained work hardening of the matrix confirmed that it is much higher than that of unreinforced copper.

Aluminium and aluminium alloy matrices have also been used in experiments on the behaviour of composites in monotonic deformation. Pinnel and Lawley (1970) reinforced high purity aluminium with continuous stainless steel fibres (150μm diam.), and produced composites with fibre volume fraction in the range 4.1% < f < 32.8%. They tested the material in tension and compression, applying the load parallel to the fibre direction. The microstructure of the matrix was then observed in a TEM microscope, with the main variables being fibre volume fraction, composite strain and distance from the fibre/matrix interface. The authors drew some interesting conclusions from this work. For a given strain, the dislocation densities and configurations were independent of both distance from the fibre/matrix interface and fibre volume fraction, being similar to that found in pure aluminium at an equal level of strain. The authors conclude that the matrix responds to stress as if it were the only phase present, or, in other words, that there is no constraint effect in this composite (Pinnel and Lawley, 1970). This would mean that the work hardening rate of the matrix in the composite is the same at that of bulk aluminium.

The stress strain response of continuous fibre (20μm diam.) reinforced 6061 aluminium was studied by Shetty and Chou (1985), who observed stages I and II to be linear. Isaacs and Mortensen (1992) reinforced high purity aluminium with 20μm diameter continuous alumina fibres (42% < f < 50%) and tested the material in tension and compression. The microstructure of the matrix in the composite was also characterised. The matrix substructure in the as-fabricated material resembles the structure of highly deformed unreinforced aluminium, and no variation in dislocation density with distance from the fibre/matrix interface could be detected. An analysis of the slope of stress-strain curves in tension showed no initial linear region, with the composite tensile yield stress being zero. In compression, stage I was seen to be linear. Between a total strain of about
0.03% and 0.07%, the work hardening rate of the composite was observed to be constant, corresponding to a linear stage II. Contrary to what is observed in copper-tungsten composites (Kelly and Lilholt, 1969), the work hardening in stage II agrees with the contribution of the fibres to the composite modulus, according to the rule of mixtures, \( fE_f \). This indicates that the work hardening of the matrix in the composite is zero in stage II (Isaacs and Mortensen, 1992). These results agree with the findings of Pinnel and Lawley (1970). By comparing the stress-strain response in tension and compression, Isaacs and Mortensen (1992) conclude that the deformation behaviour of the matrix is independent of direction of straining (having accounted for the thermal stresses), and that there is no measurable cyclic hardening of the matrix in the first few cycles.

Cheskis and Heckel (1970) reinforced 2024 aluminium with tungsten and boron fibres (127\( \mu \)m and 100\( \mu \)m diam.) and measured the stress in the matrix and the tungsten fibres by an X-ray technique. They observed linear stress-strain curves in stages I and II, with a transition from stage II to III occurring at the yield strain of the (tungsten) fibres. By comparing measurements of the constrained and bulk matrix, they concluded that the matrix stress-strain curve is the same in both cases. In other words, no constraint effect was observed in this system. These results are thus in accord with the findings of Isaacs and Mortensen (1992) on 20\( \mu \)m diameter alumina fibre reinforced high purity aluminium.

Work has also been reported on continuous fibre reinforced composites in which the fibres are at an angle to the loading axis. Shetty and Chou (1985) measured the stress-strain response of continuous alumina fibre reinforced aluminium-lithium alloy and tungsten fibre reinforced 6061 alloy, with the fibres aligned at angles of \( \theta = 30^\circ, 60^\circ \) and \( 90^\circ \) to the loading axis (fibre volume fraction \( f = 55\% \)). An example of the results is shown in fig. 2.7. For \( \theta = 0^\circ \) (fibres parallel to the loading direction), the composite stress-strain curve is linear in stages I and II. For all other values of \( \theta \) (30\(^\circ\), 60\(^\circ\) and 90\(^\circ\)) it is non-linear in stage II.

Kyono et al. (1988) measured the transverse (\( \theta = 90^\circ \)) flow curve of Boron/1100 aluminium composites. In accord with the results of Shetty and Chou (1985), the hardening in stage II was observed to be non-linear. Kyono and co-workers show that the results can be fitted to a model in which the plastic strain is assumed to be uniform and the volume fraction of debonded fibres is used as an adjustable parameter. Modelling the same experimental results, Pedersen and Withers (1991) use contrary assumptions; that bonding is perfect and the matrix deforms by a non-uniform plastic strain. Their calculations predict a non-linear flow curve in stage II, which is, however, much steeper than experimentally observed.
2.3.2 Discontinuous fibre reinforced aluminium

From the work reviewed in the previous section it is clear that if the fibres are aligned at an angle to the loading axis (θ > 0), the flow curve is non-linear in stage II. This has important implications for aluminium-Saffil composites in which the fibre distribution is random in the 2-3 plane and the load is applied in the 3 direction.

Some experimental work has been reported on the deformation of aluminium and aluminium alloy composites. Harris and Wilks (1986) measured the tensile flow curves of several aluminium alloys which were reinforced with Saffil fibres. They do not analyse the hardening rates, but the flow curves are all non-linear in stage II. The Saffil fibres significantly increased the ultimate tensile strength of composites with 99.85% Al matrices and general increases in Young’s modulus were observed for all alloys used. Non-linear hardening after initial yielding was also observed by Clegg et al. (1988) in high purity aluminium (99.97% Al) reinforced with Saffil fibres. Although considerable data is now available on various systems, no comprehensive study has been published on Saffil fibre reinforced systems with a range of fibre volume fractions.

The stress and strain fields of the matrix in the vicinity of the fibres have been studied experimentally and theoretically by several authors. Barlow (1991) and Barlow and Hansen (1991) report microscopical observations of dislocation structures in Al/SiC whisker composites (99.95% Al), prepared by powder metallurgy. Their observations show that in tensile deformation, dislocation activity is initially concentrated close to the reinforcing phase, particularly near the whisker ends and corners. With increasing strain (3% < ε < 10%), dislocation activity was observed to progress along the whiskers, from the ends. The authors interpreted the results to mean that the dislocations were tending to remain rather close to the whiskers as a result of the inhomogeneous stress field. In this context it is interesting to note the microscopical observations of Clegg et al. (1988) on Al-2.5wt%Mg-Saffil composites (f=18%) which led to a different conclusion. Clegg and coworkers concluded that deformation is uniform throughout the matrix and that there is no apparent concentration of dislocation activity around the fibres.

Christman et al. (1989) use finite element modelling to calculate the stress and strain fields around uniaxially aligned whiskers in tensile loading. These calculations show that plastic deformation commences at the whisker corners and spreads rapidly from there with increasing strain. Withers et al. (1989) applied an analytical model based on the Eshelby method to short fibre MMCs. They conclude that the variable nature of the stress
field in the matrix can give rise to microscopic plastic flow near the fibres before bulk plastic flow occurs.

2.3.3 Thermal stresses in MMCs

The thermal stresses in 6061 aluminium\SiC whisker composites have been evaluated theoretically and experimentally by Arsenault and Taya (1987). They used the Eshelby method to predict the effect of thermal stresses on the yield stress in tension and compression. Monotonic flow curves were measured in tension and compression and the difference in the 0.2% yield stress in tension and compression evaluated. Using the temperature drop $\Delta T$ as an adjustable parameter in the calculations of the thermally induced stress, Arsenault and Taya obtained good agreement between theory and experiment.

Withers et al. (1987) have measured the thermal stresses in aluminium\SiC short fibre MMCs by neutron diffraction. They annealed specimens at different temperatures and quenched them in water, prior to the measurement. The results show the thermal stress to be independent of annealing temperature. By calculating the equivalent temperature drop $\Delta T$ for the thermal strain measurements using the Eshelby method, the authors conclude that $\Delta T$ is a useful representation of thermal stresses, but does not, however, indicate the temperature at which the matrix and fibres were unstrained (Withers et al., 1987).

2.3.4 The critical fibre aspect ratio

In discontinuous fibre reinforced composites, load is transferred from the matrix to the fibres by shear stresses at the fibre matrix interface. Provided the fibres are sufficiently long and the shear stress is large enough, a discontinuous fibre can be loaded to its ultimate stress. According to Kelly and co-workers, the equation relating the controlling parameters is

$$\frac{l_c}{d} = \frac{\sigma_{uf}}{2\tau}$$

(2.2)

where $\tau$ is the shear stress at the fibre/matrix interface, $l_c$ the critical fibre length (if a fibre in a matrix is longer than $l_c$, it can be stressed to failure), $\sigma_{uf}$ the fibre ultimate tensile strength and $d$ the fibre diameter (Kelly and Davies, 1965; Kelly and Macmillan, 1986).
The shear stress $\tau$ in the model is really a measure of the rate at which load increases in the fibre from the fibre end. A larger value of $\tau$ (in other words, the higher the rate at which the load increases in the fibre from the fibre end) gives a smaller critical fibre length. For MMCs in which the bonding is good, the limiting factor is not the shear strength of the fibre/matrix interface, but rather the shear strength of the matrix (Kelly and Davies, 1965). As an approximation, $\tau$ is sometimes taken to be equal to half the tensile yield stress of the matrix (Friend, 1987; Hall, 1991).

The critical fibre length has been evaluated for a Saffil fibre reinforced composite ($f=10\%$) with an Al-1wt%Cu matrix (Hall, 1991). Taking $\tau$ to be equal to half the measured tensile yield stress of the matrix, the author finds a critical aspect ratio of 40 ($d=3\mu$m). This agrees with observed fragment lengths, measured far from the fracture surface in specimens tested to failure (Hall, 1991).

2.4 The Eshelby method and its application to MMCs

In 1957 J.D. Eshelby published a paper on the determination of the elastic field of an ellipsoidal inclusion which has strongly influenced models for metal matrix composites. In this paper he developed a procedure for the calculation of elastic stresses in a misfitting inclusion perfectly bonded to an isotropic matrix with the same elastic constants which is outlined here in section 2.4.1. He solved the same problem for an elastic inhomogeneity by using a similar procedure (section 2.4.2). In section 2.4.3, the derivation for an externally stressed inclusion is described. The application of this method to the work hardening of MMCs has been developed by a number of authors (some key references are: Tanaka and Mori, 1970, 1971; Mori and Tanaka, 1973; Brown and Stobbs, 1971a; Brown, 1973; Brown and Clarke, 1975, 1977; Pedersen, 1979, 1983, 1990; Withers et al., 1989). In particular, the mean field approach developed by Pedersen, allows for calculations of matrix and inclusion stresses, accounting for both elastic and inelastic misfit. The mean field approach is described in section 2.4.4. Calculations of thermally induced mean stresses is the subject of section 2.4.5.

The stress and strain relations in this thesis are written in tensor notation whenever possible (Nye, 1990), using boldface characters to denote tensor quantities. The stiffness

---

1 An elastic inhomogeneity is an inclusion for which the elastic constants are different to those of the matrix. The inclusion itself is elastically homogeneous.
and compliance tensors, which are fourth rank tensors, are therefore reduced to 6x6 matrices. The stress and strain tensors (second rank) are reduced to 6 element column or row vectors. Hooke's law, relating stress (\(\sigma\)), strain (\(e\)) and stiffness (\(C\)), takes the form

\[
\sigma = Ce \quad \text{or} \quad e = C^{-1} \sigma
\]

It should be mentioned that \(\sigma\), \(e\) and \(C\) do not transform according to the rules of tensor transformations, and the contracted variables are, therefore, not tensor quantities.

The description of the Eshelby method given here is largely based on a recent review by Withers et al. (1989).

2.4.1 The transformation problem

The first problem Eshelby considers is to calculate the stress state in a region of an infinite homogeneous isotropic elastic medium which undergoes a change of shape and size. He solves this problem with the help of imaginary cutting and welding operations (see fig. 2.8). The first step is to cut a region (the inclusion) from the infinite matrix and remove it from the hole. A stress free transformation of the inclusion is now allowed to take place, characterised by a transformation strain \(e^T\). Surface tractions are then applied to the inclusion to restore it to its original form, prior to replacing it in the hole from which it was taken and welding across the cut. The stress in the matrix is now zero. When the surface tractions are removed, equilibrium is reached between the matrix and the inclusion at a constrained strain \(e^C\), relative to its initial shape (i.e., \(e^C\) is the strain the hole experiences). The elastic strain in the inclusion at this stage is \((e^C - e^T)\) and the stress is given by Hooke's law as

\[
\sigma_1 = C_M (e^C - e^T) \quad (2.3)
\]

Eshelby (1957) showed that if the inclusion has an ellipsoidal shape, the stress within it is uniform. He also showed that the constrained strain, \(e^C\), can be written in terms of the transformation strain \(e^T\) and a tensor \(S\), which depends only on the Poisson's ratio of the material and the inclusion geometry

\[
e^C = S e^T \quad (2.4)
\]

This expression is only valid for the elastically homogeneous material, but turns out to be the key to the calculation. Equation 2.3 can then be written as

\[
\sigma_1 = C_M (S - I)e^T \quad (2.5)
\]

where \(I\) is the identity matrix. Thus, the stress in the inclusion (which is uniform) can be calculated in terms of the transformation strain, \(e^T\), the Eshelby \(S\) tensor and the elastic
2.4.2 The elastic inhomogeneity

If the elastic constants of the matrix ($C_M$) and inclusion ($C_I$) differ, the procedure to calculate the stress in the inclusion must be modified. The first step is to carry out the same cutting and welding exercise as described in the last section, but this time with the inclusion having the stiffness tensor $C_I$. The inclusion is cut from the matrix and allowed to transform by a strain $e^{T*}$. Surface tractions are now applied to restore it to the shape it had before transformation. Then it is put back into the hole and the cut welded. After releasing the surface tractions, equilibrium is reached at a constrained strain $e^C$. The elastic strain in the inclusion is now $(e^C - e^{T*})$ and the stress, given by Hooke's law, is

$$\sigma_I = C_I(e^C - e^{T*}) \quad (2.6)$$

Now, Brown and Stobbs (1971a) showed that deforming the matrix by a uniform plastic strain of $e_p$ is equivalent to transforming the inclusion by $-e^{T*}$ (i.e., $e_p = -e^{T*}$), in the sense that the stress states are identical after both operations. The aim of the calculation is, therefore, to express the inclusion stress, $\sigma_I$, in terms of $e^{T*}$, as well as known quantities such as $C_M$, $C_I$ and $S$. The problem with eq. 2.6 is that the expression relating $e^C$ and $e^{T*}$ is not known. To circumvent this problem, a second inclusion in an equivalent homogeneous composite is imagined to undergo a stress free transformation, characterised by a transformation strain $e^T$, which is so chosen as to give the same stress state as in the inhomogeneous composite (see fig. 2.8). This means that the constrained strain $e^C$ is the same in both cases and the two inclusions can be interchanged without altering the stress state in the matrix (see fig. 2.8). The inclusion stress in the homogeneous composite is then given by eq. 2.3

$$\sigma_I = C_M(e^C - e^T) \quad (2.3)$$

and in the inhomogeneous composite by eq. 2.6

$$\sigma_I = C_I(e^C - e^{T*}) \quad (2.6)$$

The stress state in both inclusions is identical, so

$$C_I(e^C - e^{T*}) = C_M(e^C - e^T) \quad (2.7)$$

The expression relating $e^C$ and $e^T$ (eq. 2.4) can now be used, since the constrained strain $e^C$ is the same for the elastically homogeneous and inhomogeneous materials. Eq. 2.7 then becomes

$$C_I(Se^T - e^{T*}) = C_M(S - D)e^T \quad (2.8)$$
After rearranging, \( e^T \) can be given in terms of \( e^{T*} \)

\[
e^T = [(C_I - C_M)S + C_M]^{-1} C_I e^{T*}
\]

(2.9)

This expression can now be used in eq. 2.5, giving the inclusion stress in terms of the transformation strain, \( e^{T*} \)

\[
\sigma_I = C_M(S - I)[(C_I - C_M)S + C_M]^{-1} C_I e^{T*}
\]

(2.10)

Thus, by constructing a transformation strain \( e^T \) for the homogeneous composite, giving identical shape and stress state, the inclusion stress in the inhomogeneous composite has been calculated. Eq. 2.9 shows that if \( C_I \to C_M \), then \( e^{T*} \to e^T \), which is a necessary requirement. In the next section it will be shown how the inclusion stress can be calculated for an externally stressed homogeneous composite.

2.4.3 The externally stressed inclusion

The calculation of the inclusion stress for an externally stressed homogeneous inclusion is simple, since the applied stress is evenly distributed throughout the matrix and inclusion (the inclusion and the matrix have the same elastic constants). Therefore, the application of a load \( \sigma^A \) extends both the matrix and inclusion by \( e^A \), and the matrix and inclusion stress increase by \( \sigma^A = C_M e^A \). The total inclusion stress, \( \langle \sigma \rangle_i \), is then given by \( \sigma_I + \sigma^A \), and

\[
\langle \sigma \rangle_i = \sigma_I + \sigma^A = C_M(e^C - e^T) + C_M e^A
\]

or

\[
\langle \sigma \rangle_i = C_M(e^C + e^A - e^T)
\]

(2.11)

When the inclusion has different elastic constants than the matrix, the process of selecting the equivalent transformation strain must be carried out taking the applied stress into account. In parallel with the line of reasoning in section 2.4.2, the stress in the inhomogeneous inclusion, after application of the external stress, can be written as

\[
\sigma_I + \sigma^A = C_M(e^C - e^{T*})
\]

(2.12)

\( e^C \) characterises the shape of the hole (and the inclusion) after the external stress has been applied (see fig. 2.9) and can be described as an effective constrained strain. The inclusion is cut from the matrix in the equivalent homogeneous composite and transformed by \( e^T \), which now depends on the applied stress. Surface tractions are applied to the inclusion before putting it back into the hole, welding across the cut and relaxing the tractions. At
this stage the hole has a strain of \( e^C \), which depends on \( \sigma^A \). After applying the external stress, the inclusion extends by \( e^A \), and the shape and stress state becomes exactly the same as that of the inhomogeneous material. The effective constrained strain of the hole now clearly consists of the applied strain \( e^A \) and the constrained strain \( e^C \), with \( e^C = Se^T \). The stress in the inclusion is then

\[
\begin{align*}
\sigma_1 + \sigma^A &= C_I (e^C - e^T) = C_I (e^C + e^A - e^T) \quad \text{: Inhomogeneity (2.13a)} \\
\sigma_1 + \sigma^A &= C_M (e^C - e^T) = C_M (e^C + e^A - e^T) \quad \text{: Equivalent inclusion (2.13b)}
\end{align*}
\]

\( e^C \) describes the constrained shape of the hole for both composites after application of the stress \( \sigma^A \), and \( e^C \) does not relate to the shape of the hole in the inhomogeneous composite before the application of the external stress.

The analysis presented above is mathematically rigorous for a single inclusion in an infinite matrix. Application of this method to finite composites is the subject of the next section.

2.4.4 The mean field model

When an inclusion is surrounded by an infinite matrix, the average stress in the matrix which balances the inclusion stress is zero. It was in consideration of boundary conditions for a finite composite that Eshelby (1957) introduced the concept of the image stress, without specifying its precise form. When Brown and Stobbs (1971a) found that the matrix stress around a spherical inclusion calculated by the Eshelby method averages to zero in a spherical region of the matrix, they realised that the image stress must balance the stress in the inclusion, if the composite is not stressed externally. The concept of the image stress was later replaced by that of the mean stress (Brown, 1973). For the homogeneous composite it is reasonable to assume that the mean stress is evenly distributed throughout the whole composite, in which case an inclusion samples the mean stress resulting from all the other inclusions. Hence, the stress in the inclusion is \( \langle \sigma \rangle_1 = \sigma_1 + \langle \sigma \rangle_M \) and the stress balance can then be written as

\[
(1 - f) \langle \sigma \rangle_M + f(\sigma_1 + \langle \sigma \rangle_M) = 0 \quad \text{(2.14)}
\]

or

\[
\langle \sigma \rangle_M + f\sigma_1 = 0 \quad \text{(2.15)}
\]

When an external stress, \( \sigma^A \), is applied, the total average stress in the matrix is \( \langle \sigma \rangle_M + \sigma^A \) and in the inclusion it is \( \sigma_1 + \langle \sigma \rangle_M + \sigma^A \).
It is less clear how the mean stress is distributed in the inhomogeneous composite, because the matrix and inclusions have different elastic constants. Pedersen (1979, 1983, 1990) has proposed a mean field approach in which the inclusions are assumed to sample the mean stress in the same way as in the homogeneous composite, so that \( \langle \sigma \rangle_1 = \sigma_1 + \langle \sigma \rangle_M \). The effective constrained strain \( \langle e \rangle^C \) is now composed of the constrained strain \( \langle e \rangle^C \), the applied strain \( \langle e \rangle^A \) and the mean strain \( \langle e \rangle_M \). Hence, eqs. 2.13a and 2.13b for a finite composite with an applied stress \( \sigma^A \) become

Inhomogeneity:
\[
\langle \sigma \rangle_1 + \sigma^A = C_I (\langle e \rangle^C - e^{T*}) = C_I (\langle e \rangle^C + e^A + \langle e \rangle_M - e^{T*}) \quad (2.16a)
\]

Equivalent inclusion:
\[
\langle \sigma \rangle_1 + \sigma^A = C_M (\langle e \rangle^C - e^T) = C_M (\langle e \rangle^C + e^A + \langle e \rangle_M - e^T) \quad (2.16b)
\]

with \( \langle \sigma \rangle_1 = \sigma_1 + \langle \sigma \rangle_M \). Using the expressions
\[
e^C = S e^T, \quad \langle \sigma \rangle_M = C_M \langle e \rangle_M, \quad \text{and} \quad \sigma^A = C_M e^A
\]

and solving eqs. 2.16a and 2.16b for \( e^T \) gives

\[
e^{T*} = -\left[ (C_M - C_I) [S - f(S - I)] - C_M \right]^{-1} \left( C_I e^{T*} - (C_I - C_M) e^A \right)
\]

(2.17)

The mean stress in the matrix can now be calculated. Using eqs. 2.15 and 2.16b, the mean strain can be written as
\[
\langle e \rangle_M = -f(\langle e \rangle^C - e^{T*}) = -f(S - I)e^T
\]

(2.18)

Eq. 2.17 expresses \( e^T \) in terms of \( e^{T*} \), so

\[
\langle e \rangle_M = f(S - I)[ (C_M - C_I) [S - f(S - I)] - C_M ]^{-1} \left( C_I e^{T*} - (C_I - C_M) e^A \right)
\]

(2.19)

The mean stress is \( \langle \sigma \rangle_M = C_M \langle e \rangle_M \), which gives

\[
\langle \sigma \rangle_M = fC_M (S - I)[ (C_M - C_I) [S - f(S - I)] - C_M ]^{-1} \left( C_I e^{T*} - (C_I - C_M) e^A \right)
\]

(2.20a)

or
\[
\langle \sigma \rangle_M = Ae^{T*} + Bo^A
\]

(2.20b)

This is the general form of the mean stress derived by Pedersen (1979). \( A \) and \( B \) are functions of inclusion volume fraction, the elastic constants of the matrix \( (C_M) \) and inclusions \( (C_I) \), and the Eshelby S tensor which depends on the inclusion shape and orientation and is expressed in terms of the matrix Poisson's ratio. The mean stress consists
of an inelastic term which is proportional to the transformation strain \((Ae^T^*)\) and an elastic term proportional to the applied stress \((BoA)\). Eq. 2.20b will be used in chapter 3 where expressions for the permanent softening in Bauschinger experiments are developed. The Eshelby S tensor has been calculated for various inclusion shapes, such as spheres, fibres, disks and ribbons (Brown and Clarke, 1975, 1977), some of which are given in table 2.1. However, the tensor has not yet been calculated for a planar random array of fibres, nor for a completely random array of fibres.

The limitations of the mean field approach are discussed by Pedersen (1990). The mean field approximation may be misleading for systems whose constituents have widely different elastic constants, and for composites with short fibres (i.e., low aspect ratio). The connectivity of fibres at high aspect ratios is also not taken into account (Pedersen, 1990).

2.4.5 Thermally induced mean stresses

When the two constituents of a composite have different coefficients of thermal expansion (CTE), a strain mismatch builds up between the two phases if the temperature changes. The strain mismatch between an inclusion and a matrix material of the same shape can be represented by the transformation strain \(e^T^*\) (Withers et al., 1989)

\[
e^T^* = (\alpha_M - \alpha_I)\Delta T
\]

\(\alpha_M\) and \(\alpha_I\) are the thermal expansion tensors for the matrix and inclusion, respectively, and \(\Delta T\) is the change in temperature. Eq. 2.20b can now be used with eq. 2.21 to calculate the thermally induced mean stress in the matrix. When the applied stress is zero, \(e^A = 0\) and the matrix stress is given by

\[
<\sigma>_M = Ae^T^* = A(\alpha_M - \alpha_I)\Delta T
\]

The expression for the thermally induced mean matrix stress in the aluminium\(\)\textbackslash Saffil composite will be developed in section 3.3.4.

Measurements of internal stresses are considered in the next section.
2.5  The Bauschinger effect and measurements of internal stresses in MMCs

In this section the developments which led to the use of the permanent softening for quantifying internal stresses will be discussed. Alternative methods of evaluation of internal stresses are reviewed briefly.

2.5.1  The Bauschinger effect

In 1886, Bauschinger published work carried out on iron and steel where he reports that when loaded in tension above the elastic limit and reloaded in compression, the elastic limit in compression is lowered. He also observed that a preload in compression lowers the elastic limit in successive tensile loading. This behaviour of metals and alloys was later to be known as the Bauschinger effect. The dislocation mechanisms responsible for this behaviour were first recognised by Orowan (1959) and the basic ideas of his explanation are now generally accepted.

In fig. 2.10, which is borrowed from Orowan's paper, a typical stress - plastic strain curve for a polycrystalline metal is shown. Stress is applied to the specimen in tension and removed at A. If the stress is applied again in tension, the curve follows BAC almost exactly; the only difference being a slightly rounded corner at A, as indicated in the figure. If on the other hand, the stress is removed at A and then reversed, the resulting curve is BD'. In order to compare the curve BD' to BAC (with the slightly rounded corner), it is convenient to plot the absolute value of the stress as a function of the cumulative plastic strain, or, in other words, to mirror the curve BD' through the point B. For both methods the resulting curve is BD. It is now straightforward to compare the forward curve BAC to the reverse curve BD. They differ in two respects. Firstly, the "roundedness" of the reverse curve (BD) at A is much greater than for the forward curve (BAC). This transient softening on stress reversal is indicated by the horizontal shading in fig. 2.10 (this is the Bauschinger effect in Orowan's terminology). Secondly, as the strain increases after stress reversal, the curve BD becomes parallel to BAC but at a lower stress level. The material shows a "permanent softening" on stress reversal, as indicated by the vertical shading in fig. 2.10. Although what has been called the Bauschinger effect is historically confined to the roundedness of the reverse curve, it is now customary to include the permanent softening in that terminology.

At the time when Orowan published his paper, it was usual to attribute what became
known as the Bauschinger effect to backstresses resulting from piling up of dislocations at obstacles such as grain boundaries (Orowan, 1959). Although a simple backstress model like that is capable of explaining permanent softening, it is implied in the model that a significant part of the deformation is reversible upon unloading. In other words, upon unloading from A to zero load after the forward straining, the backstresses will push the dislocations backwards and the plastic strain at zero load would be significantly less than indicated by the point B. As seen in fig. 2.10, such a behaviour is not observed. Another problem is that if the roundedness of the reverse curve were due essentially to internal stresses, a sufficiently thorough annealing should be capable of removing it to a considerable extent (Orowan, 1959). Orowan says that a relatively mild annealing removes the permanent softening but the roundedness is remarkably resistant to such a treatment. These arguments, amongst others, prompted him to propose a model in which the roundedness of the reverse curve is caused by the way in which dislocations statistically sample a randomly positioned array of obstacles (Orowan, 1959; Brown, 1977). In this model, the dislocations may adjust themselves locally when the load is removed, but unless the backstresses are very large, they will stay more or less where they were. On stress reversal, the row of obstacles immediately behind of the dislocations is not as difficult to overcome as that in front, and the material yields at a lower stress. The backstresses will also aid the deformation and the material flows at a lower stress than in the forward direction. In systems where the obstacles are penetrable, the permanent softening will be small, but the roundedness of the reverse curve still remains (Brown, 1977).

2.5.2 Permanent softening and the mean stress

It is appropriate at this stage to consider the evidence relating the backstresses to the permanent softening. Wilson and Konnan (1964) provided direct evidence of backstresses. They used high carbon steel containing a coarse dispersion of spheroidal cementite particles and measured the lattice strains in both phases after plastic deformation, using an X-ray diffraction technique. They concluded that backstresses in the matrix are balanced by larger stresses of the opposite sign in the particles which is directly responsible for the major part of the additional work hardening observed in high carbon steel, as compared to low carbon steel. In a later work, Wilson (1965) set out to test the predictions implied in Orowan's arguments that the permanent softening should be much higher for dispersion hardened systems than for single phase materials. The aim here was to quantify
the correlation between the permanent softening and the backstress. An X-ray technique (Wilson and Konnan, 1964) was used to measure the lattice strains at a given amount of specimen prestrain. The backstresses were then calculated from the measured strains. For evaluation of the permanent softening, a specimen was first strained to a forward strain of 9% and then strained in the reverse direction to a strain of \( e_n \), which is the strain at which the average value of the long range backstresses in the matrix has been reduced to zero. For the materials used, \( e_n \) lies in the range of 1.5%. It is not clear why Wilson chose to measure the permanent softening at \( e_n \). This is because Wilson's flow curves are not parallel at \( e_n \), but for an unambiguous measurement of the permanent softening to be made, the flow curves have to be parallel. The correlation he established between the measured mean matrix stress, \( \langle \sigma \rangle_M \), and the permanent softening, \( \Delta \sigma \), is

\[
\Delta \sigma = (1.9 \pm 0.1) \langle \sigma \rangle_M
\]  

(2.23)

Because of stress relief due to the free surface, the factor of 1.9 is an upper limit (Wilson, 1965). It has been pointed out by Atkinson et al. (1974) that this factor must also be considered an upper limit given that the flow curves are not parallel. Another consequence of the non-parallelism is that it is not clear if the correlation of eq. 2.23 is fortuitous or not. However, Atkinson et al (1974) conclude that their results on copper-silica can be interpreted consistently if the permanent softening is taken to be equal to twice the mean stress in the matrix

\[
\Delta \sigma = 2 \langle \sigma \rangle_M
\]  

(2.24)

With these reservations clearly in mind, eq. 2.24 can be taken to be a calibration of the Bauschinger effect and has frequently been used to evaluate the mean stress and its hardening rate. Such experiments are described in the next section.

2.5.3 Bauschinger experiments

A "Bauschinger experiment" is one in which the forward and reverse flow curves are recorded and the permanent softening is measured using the Orowan-Wilson construction as discussed above. A convenient way of doing this is to record stress - strain hysteresis loops at several plastic strain levels for a given specimen so that the development of the mean stress with plastic strain can be monitored. The first use of this technique was by Lilholt (1974) and Clarke and Lilholt (1975). In 1977, Lilholt published results of comprehensive Bauschinger experiments on continuous copper-tungsten fibre reinforced copper, carried out at room temperature and 77K. This paper was published parallel to a
theoretical paper by Brown and Clarke (1977). The matrix consisted of copper single crystals and was reinforced with continuous tungsten fibres of 20µm diameter and with a volume fraction up to 4%. The fibres were aligned parallel to the loading axis. The permanent softening, \(\Delta\sigma\), was then measured as described above and the values plotted as a function of plastic strain (see fig. 2.11). This allows the mean stress hardening rate to be evaluated from the slope of the initial linear curve (the theory predicts the mean stress to be linear in plastic strain, see chapter 3). At room temperature the prediction for the mean stress hardening rate was about a factor of two higher than the experimental results, but at 77K, the difference was only about 20-30%. Lilholt was the first to recognize that any residual stresses (e.g., of thermal origin, or caused by unintended plastic deformation during mounting for testing) would give asymmetrical hysteresis loops, which again would produce the same offset of all \(\Delta\sigma\) values for a given fibre volume fraction. The slope of \(\Delta\sigma\) with plastic strain will not be affected if the residual stress does not change with cycling. Lilholt observed that the value of \(\Delta\sigma\) at a given plastic strain is independent of the particular cycle used to evaluate it. A value of \(\Delta\sigma\) obtained at a low plastic strain, after a return from a series of cycles at a high plastic strain, fits onto the curve of \(\Delta\sigma\) versus plastic strain. In other words, the hysteresis loop shape, and therefore \(\Delta\sigma\), is independent of loading history, which suggests that the permanent softening is a unique function of the plastic strain (Lilholt, 1977). This condition must be fulfilled for the method used by Lilholt to give reliable estimates of the mean stress hardening rate.

Pedersen (1989, 1990) has carried out similar experiments on copper-tungsten composites but this time with high fibre volume fractions and compared the results with theories for hardening in metal matrix composites. Fig. 2.12, which is borrowed from Pedersen (1989), shows clearly how the permanent softening is evaluated. The general conclusion of the work of Lilholt and Pedersen is that the Orowan-Wilson construction of the permanent softening can be used for quantitative comparison of the experimental mean stress hardening rate with theoretical predictions.

Alternative ways of measuring the internal stresses in composite materials are discussed briefly in the next section.

2.5.4 Measurements of internal stresses by X-ray and neutron diffraction

An X-ray diffraction technique has been used by Cheskis and Heckel (1968, 1970) to measure internal stresses in copper-tungsten composites and aluminium reinforced with
tungsten and boron fibres. This technique enables mean stresses in the individual phases to be monitored. The specimens have to be quite thin because the penetration distance of X-rays in metals is not large (~50 μm in aluminium, Withers et al., 1989). Cheskis and Heckel (1968) used specimens of thickness 0.2 mm to 1.4 mm. A comparison of their results for the copper-tungsten composites with predictions of the mean field model (Pedersen, 1985b) shows that relaxation is linear in stage II.

Neutron diffraction has also been used for measurements of internal stresses (Hutchings, 1991; Withers et al., 1987). The advantage of neutrons is that their attenuation in metals is much lower, giving a penetration depth of ~5 cm in aluminium, but the drawback is the need for very intense neutron beams (Hutchings, 1991). Withers et al. (1987) measured the matrix and fibre strains at room temperature in a short fibre aluminium\SiC composite and concluded that the results are consistent with predictions based on the Eshelby method. The permanent softening in a Bauschinger experiment has not been compared with neutron diffraction measurements of the internal stresses.

2.6 Matrix hardening and relaxation of internal stresses

In section 2.6.1 a model for matrix hardening in MMCs is described. The theory discussed in section 2.4 applies to the hypothetical unrelaxed composite. In practice, relaxation of internal stresses is not uncommon and must be accounted for in any realistic theory of composite materials. Observations of stress relaxation in MMCs are reviewed briefly in section 2.6.2 and in section 2.6.3 some possible mechanisms of relaxation are discussed.

2.6.1 Matrix hardening

In this section, the analysis of the Bauschinger experiment proposed by Pedersen (1985a, 1990) for obtaining the contribution of matrix hardening to the overall flow stress will be discussed.

Pedersen's mean field model (1983, 1990) provides an expression for the mean matrix stress split into two components, plastic and elastic (in matrix notation, see eqs. 2.20a and b)

\[ <\sigma>_M = A e^{Tr} + B \sigma^A \] (2.25)
where $\sigma^A = \sigma^A(0, 0, 1, 0, 0, 0)$ is the applied stress. Since $\sigma^A$ is linearly superposed on the mean matrix stress, the average stress in the matrix can be written as (scalar version)

$$<\sigma_1>_M = A_1 e_p^* + B_1 \sigma^A$$

$$<\sigma_2>_M = A_2 e_p^* + B_2 \sigma^A$$

$$<\sigma_3>_M = A_3 e_p^* + B_3 \sigma^A + \sigma^A$$  \hspace{1cm} (2.26)

Here, $e_p^* = -e_T^*$ is the unrelaxed plastic strain of the matrix and $A_i = -A_i e_{p_i}^* - A_2 e_{p_2}^* - A_3 e_{p_3}^*$ and $B_i = B_i$. Pedersen (1985a) then writes the in situ matrix flow stress as

$$2\tau_M = \alpha e_p^* + \beta \sigma^A + \delta$$  \hspace{1cm} (2.27)

where $\alpha e_p^*$ and $\beta \sigma^A$ are referred to as plastic friction and elastic friction respectively, or, in other words, $\alpha$ and $\beta$ represent the plastic and elastic contributions to the source shortening stress. The term $\delta$ contains all other friction stresses associated with the matrix microstructure. For elastically homogeneous composites ($\beta=0$), eq. 2.27 becomes (Brown and Clarke, 1977; Pedersen, 1990)

$$\tau_M = \frac{\mu b}{L} + \frac{5f\mu e_p^*}{\pi}$$  \hspace{1cm} (2.28)

$L$ is the spacing of the fibre axes (equal to $2(r_1 + r_2)$ in fig. 2.6), $\mu$ the matrix shear modulus, $b$ the Burgers vector and $f$ the fibre volume fraction. Pedersen (1985a, 1990) then derives the composite forward flow stress as

$$\sigma_F = \frac{(A + \alpha)e_p^* + \delta + \sigma^{Th}}{(1 - B - \beta)}$$  \hspace{1cm} (2.29)

with $A = A_1 - A_3$ and $B = B_1 - B_3$ (see eq. 2.26). $\sigma^{Th}$ is the thermal stress in the matrix. He then shows how the peak stress hardening rate, $\theta_p$ (slope of the peak stresses of the hysteresis loops as a function of plastic strain) and the mean stress hardening rate, $\theta_\Delta$, can be used to obtain an estimate of $\alpha/A$, which quantifies the relative proportion of the plastic component of the source shortening stress and the plastic component of the mean stress

$$\frac{\alpha}{A} = \frac{\theta_F}{\theta_\Delta} - 1$$  \hspace{1cm} (2.30)

This result is based on the assumption that the plastic and elastic components of the source shortening stress add linearly. For relaxed composites this result is expected to be valid if the relaxation reduces $\theta_F$ and $\theta_\Delta$ in the same proportion (Pedersen, 1985a).

Experimental measurements of $\theta_F$ and $\theta_\Delta$, and hence of $\alpha/A$, can now be compared
with a theoretical prediction. Pedersen (1985a, 1990) provides a method for predicting the ratio \( \alpha/A \). The equivalent transformation strain \( e^T \) can be written as (see also Withers et al., 1989)

\[
e^T = a \, e^*_p + b \, \sigma^A
\]  

or, in a scalar version, as

\[
e_{11}^T = a_1 \, e^*_p + b_1 \, \sigma^A
\]
\[
e_{22}^T = a_2 \, e^*_p + b_2 \, \sigma^A
\]
\[
e_{33}^T = a_3 \, e^*_p + b_3 \, \sigma^A
\]

\[
(2.32)
\]

\( a \) and \( b \) are 6\times6 matrices, giving the plastic and elastic components, respectively. The calculations for the source shortening stress (eq. 2.28) are performed for shear (Brown and Clarke, 1977) and \( e^T \) in eq. 2.32 is expressed in components along the 1-2-3 axes. Hence, some suitable measure \( |e^T| \) of \( e^T \) has to be substituted for \( e^*_p \) in eq. 2.28. Pedersen (1990) writes

\[
|e^T| = \frac{V}{2} (|e_{22}^T| + |e_{33}^T|)
\]

which gives

\[
\alpha = (5/2\pi)\mu f(|a_2| + |a_3|)
\]

The value of \( A \) can be predicted theoretically (see section 2.4.4 and eq. 2.26) and hence the ratio \( \alpha/A \) can be evaluated (the 1 and 2 directions are identical in the copper-tungsten composites; \( e_{11}^T = e_{22}^T \) and \( a_1 = a_2 \)).

The model of the source shortening stress demonstrates that mean field theories are not sufficient to predict the total work hardening of composites, they must be supplemented by dislocation models which describe how the dislocations are affected by the stress fields which are produced by plastic deformation (Pedersen, 1990). The total energy expenditure in plastic deformation can then be divided into internal energy (work done against the elastic mean stress) and heat (work done against the obstacles and the fluctuating internal stress field, causing the dislocations to bow and dissipate heat as they move; Brown, 1979).

Pedersen (1990) applies this model to hardening results on continuous Cu-W composites tested at room temperature and 77K, enabling him to draw some conclusions about the effect of fibre volume fraction, temperature and scale (fibre diameter) on the matrix hardening and relaxation behaviour in these composites. The conclusion is that the matrix hardening appears to be scale independent, and that the relaxation mechanism is scale dependent in such a way that relaxation increases with fibre diameter. The analysis also indicates that the stress relaxation is independent of fibre volume fraction. Pedersen
(1990) concludes from evidence that the relaxation rate is independent of temperature, that the mechanism of relaxation cannot be diffusion and suggests that whatever the mechanism, it cannot be diffusion and suggests that the relaxation occurs by cross slip of screw dislocations. Pedersen and Withers (1991) suggest that further work is required to identify the relaxation mechanisms in MMCs and incorporate them into overall models for the composites. Indeed, surprisingly little work has been reported in the literature on the identification of the relaxation mechanisms operating in fibre composites.

2.6.2 Observations of relaxation

To begin with it is essential to clarify what is meant by relaxation. As the strain misfit between the two phases in MMCs increases (for example, because of plastic deformation, temperature change etc.), stresses build up in the two constituents. Provided the stresses are high enough, they can be reduced by processes such as cross slip, diffusion, fibre/matrix debonding and fibre failure. Any such processes which bring about a reduction in the strain misfit are classified as relaxation mechanisms. For plastic deformation it is convenient to define a residual elastic strain $\varepsilon_p^*$. If the unrelaxed mean stress hardening rate can be predicted, then $\varepsilon_p^*$ can be obtained from measurements of the (relaxed) mean stress hardening rate. When $\varepsilon_p^* = \varepsilon_p$ there is no relaxation, but if $\varepsilon_p^* < \varepsilon_p$, relaxation has occurred (Brown, 1979).

The tensile flow behaviour of copper and aluminium matrix composites reinforced with aligned continuous tungsten or boron fibres has been studied extensively, both experimentally and theoretically (Cheskis and Heckel, 1968, 1970; Kelly and Lilholt, 1969; Pedersen, 1985a; Withers et al., 1991). An important simplifying feature of these composites is that the hardening in stage II is linear. Pedersen has analysed the results of Cheskis and Heckel (1968) for the Cu-W and Al-B systems and reached the conclusion that stress relaxation occurs throughout stage II (see section 2.3), in such a way that $\varepsilon_p^*$ varies approximately linearly with $\varepsilon_p$. In other words, the matrix hardens linearly in stage II, but at a lower rate than in the hypothetical unrelaxed composite (Pedersen, 1985a, 1985b). The results of Cheskis and Heckel (1970) on 2024 Al-W composites have been analysed by Withers et al. (1991), showing that hardly any relaxation occurs in this system in stage II ($\varepsilon_p^* = \varepsilon_p$).

The internal stress response of commercially pure aluminium reinforced with SiC whiskers ($f=5\%$) has been measured by a neutron diffraction technique, which shows it to
be non-linear. Flow remains unrelaxed below plastic strains of about 0.04%, and the rate of increase of unrelaxed misfit (\(\frac{d\epsilon_p^*}{d\epsilon_p}\)) falls drastically as the strain is increased (Withers et al., 1991).

Lilholt (1977) has carried out cyclic deformation (Bauschinger) experiments on continuous tungsten fibre reinforced copper-tungsten composites and measured the contribution of the mean stress to the work hardening of the composite. Stress relaxation was observed both at room temperature and 77K. The theoretical mean stress hardening rate was generally observed to be about twice the experimental value at room temperature and at 77K about 20-30% higher.

### 2.6.3 Mechanisms of relaxation

Several mechanisms of stress relaxation have been observed or postulated to occur in MMCs. These include:

1. Plastic relaxation (Brown and Stobbs, 1971b)
2. Diffusional relaxation (Stobbs, 1973; Withers, 1988)
3. Voiding at fibre ends (Nutt and Duva, 1986)

The main emphasis in this section will be on plastic relaxation, with particular reference to cross slip, but mechanisms 2 and 3 are discussed briefly.

Relaxation in copper-silica by interface diffusion has been studied by Stobbs (1973) who estimated the activation energy for this mechanism to be 1.7 eV. Stobbs concludes that relaxation in copper by this mechanism becomes possible only above temperatures of 230°C. However, diffusional relaxation in aluminium matrix composites at room temperature cannot be excluded because the activation energies for lattice diffusion and boundary diffusion are lower than for copper and the homologous temperature is higher. Withers (1988) and Withers et al., (1989) have studied the plasticity and relaxation of 2014 aluminium\SiC (particulate and whisker) composites. The data do not confirm any one mechanism, but demonstrate that relaxation by interfacial diffusion is consistent with the observed behaviour.

Christman et al. (1989) demonstrated with numerical calculations that plastic deformation in short fibre MMCs commences at fibre corners (see section 2.3.2). Indeed, void initiation and growth has been observed at corners of whiskers in aluminium\SiC
Composites (Nutt and Duva, 1986; Nutt and Needleman, 1987). A damage of this type will result in a reduction of the internal stresses in the composite.

Cross slip of screw dislocations is known to have a significant effect on the deformation behaviour of metals. Dislocations in aluminium (and in fcc metals) are generally dissociated into two partial dislocations, separated by a stacking fault (Seeger, 1957; Duesbery, 1992). The separation of the two partials, $d_0$, depends on the stacking fault energy $\gamma$ (and the applied stress) in a way that $d_0$ decreases as $\gamma$ increases ($d_0 = \mu b^2/4\pi\gamma$, $\mu$ is the shear modulus and $b$ the Burgers vector; Hull and Bacon, 1986). Cross slip is more difficult to achieve when dissociation occurs because a partial dislocation cannot cross slip. However, it is possible to form a constriction in screw dislocations and then the unit dislocation at the constriction is free to move in other planes. A certain amount of energy is associated with the recombination of the partials and it will occur more readily in metals with high stacking fault energies (with small $d_0$), such as aluminium ($\gamma = 200$ mJ/m$^2$; Ashby and Brown, 1980). Copper is an example of a metal with a low stacking fault energy ($\gamma \approx 40$ mJ/m$^2$; Bonneville and Escaig, 1979). It follows that cross slip will be easier in metals with a high stacking fault energy, and this has significant effects on the deformation behaviour (Hull and Bacon, 1986).

For cross slip to occur the dislocation must recombine in the primary plane over some appreciable length and then bow out to a critical configuration in the cross slip plane before the event can be successfully completed. The details of this mechanism are still being disputed (Duesbery et al., 1992). The contributions to the activation energy of the cross slip event are the energy of the two constrictions (at each end of the recombined segment), the energy of the recombined segment and the increase in line energy of the bowed segment (Schoeck and Seeger, 1955; Seeger, 1957; Duesbery et al., 1992). If the energy barrier of the cross slip event has a magnitude of less than a few electron volts, the cross slip can occur with the aid of thermal activation although the ease of cross slip decreases with decreasing temperature (Schoeck, 1965; Hull and Bacon, 1986). The activation energy for cross slip in aluminium (for a resolved shear stress in the cross slip plane of 1MPa) has been calculated by Schoeck and Seeger (1955) to be about 1.05 eV and this suggests that cross slip can be assisted by thermal activation in aluminium, at or just above room temperature (Seeger, 1957). The same calculations for copper give an activation energy of 10 eV, which would mean that thermal activation of cross slip is only possible at elevated temperatures or high stress concentrations (Schoeck and Seeger, 1955).
More recent calculations of Duesbery et al. (1992) predict the activation energy for cross slip in copper to be between 1.5 and 2.0 eV, which is an order of magnitude lower than the results of Schoeck and Seeger (1955). Duesbery et al. (1992) also calculated the activation volume for cross slip, and predicted it to be between $330b^3$ and $470b^3$, where $b$ is the Burgers vector. Measurements of Bonneville and Escaig (1979) of the activation volume for copper show it to be between $350b^3$ and $400b^3$.

Sherby et al. (1957) and Dorn and Jaffe (1961) measured the temperature dependency of the activation energy for creep of pure aluminium by rapidly changing the temperature in creep tests. Their main conclusion, which is based on metallographic observations and comparison with theoretical calculations (Schoeck and Seeger, 1955), is that the creep rate is controlled by the cross slip mechanism between 275 K and 345 K. The measured activation energy is 1.2 eV (Dorn and Jaffe, 1961). Above about 500 K, the activation energy was measured to be about 1.54 eV, which they attribute to dislocation climb or diffusion (Sherby et al., 1957; Dorn and Jaffe, 1961). Their value is in good agreement with a value for self diffusion in aluminium of 1.47 eV (Frost and Ashby, 1982).

The thermodynamic analysis of glide has been reviewed by Gibbs (1969). The energy barrier which a dislocation has to overcome by thermal activation, in order to pass a local obstacle, can be written as (Gibbs, 1969; Duesbery et al., 1992; Hull and Bacon, 1986)

$$\Delta G = U - V\tau$$  \hspace{1cm} (2.35)

$U$ is the total energy of the barrier, $\tau$ the stress acting in the slip system and $V$ is the activation volume. In some cases it is possible to interpret the activation volume in terms of the geometry of the dislocation configuration. If a dislocation segment of length $2l$ and Burgers vector $b$ is pinned at its midpoint by a local obstacle, the activation volume can be written as (Gibbs, 1969)

$$V = lbd$$  \hspace{1cm} (2.36)

d is the distance the dislocation moves in the activation event. $lb$ can be regarded as the local force acting on the dislocation at the obstacle and $\tau v$ is the total work done by the stress $\tau$ in the activation event. If the dislocation is unextended, $d$ is of the order of $b$, but for extended dislocations $d$ is related to the width of the stacking fault ribbon, $d_0$ (Seeger, 1957). If the width of the extended dislocation in units of the Burgers vector is $N$, then the spacing of obstacles on a dislocation line is (Gibbs, 1969; Conrad et al., 1960)

$$1 = V/Nb^2$$  \hspace{1cm} (2.37)

At zero applied stress, $N \sim 3$ for aluminium and $N \sim 24$ for copper. This estimate of $l$ can be
compared with microscopic studies of the material defect structure and estimates of the dislocation densities (Gibbs, 1969; Conrad et al., 1960). However, great care must be exercised in doing so because the activation volume is generally not related to geometric dislocation quantities (Hirth and Lothe, 1982; Duesbery et al., 1992).

The rate at which cross slip occurs can be calculated by using the theory of statistical mechanics (Gibbs, 1969). If the dislocation is vibrating at a frequency of $v$ (≤ the atomic vibration frequency), the probability, $\Gamma$, that it will overcome an energy barrier $\Delta G$ by thermal activation is given by (Gibbs, 1969; Hull and Bacon, 1986)

$$\Gamma = v \exp \left( - \frac{\Delta G}{kT} \right) = v \exp \left( \frac{-U - V\tau}{kT} \right)$$

(2.38)

Eq. 2.38 has been used by Atkinson et al. (1974) to analyse the kinetics of plastic relaxation in copper-silica. They assume that the rate of loss of Orowan loops from the silica inclusions is given by eq. 2.38, with $v$ equal to the Debye frequency of copper and $\tau$ being the local stress acting on the dislocations (the mean stress). They relate the unrelaxed plastic strain, $e_p^*$, to the number of Orowan loops around a particle, $n$, by the equation

$$e_p^* = \frac{nb}{4r_0}$$

(2.39)

$b$ is the Burgers vector and $r_0$ the particle radius (see also Ashby, 1971). Atkinson and co-workers are then able to write an expression for the rate of loss of Orowan loops, $dn/dt$

$$\frac{dn}{dt} = \frac{4r_0}{b} \frac{de_p^*}{dt} = v \exp \left( \frac{-U - V\tau}{kT} \right)$$

(2.40)

Using measurements of the mean stress at room temperature and 77K they determine the mean stress level at which the deformation deviates from unrelaxed behaviour. By assuming complete relaxation for higher plastic strains (i.e. $de_p^*/dt = 0$) and equating $de_p^*/dt$ to the strain rate imposed on the specimen (which effectively assumes that the Orowan loops are relaxed at the same rate as they are generated), they derive the activation energy and volume for relaxation in the system as

$$U = 0.9 \pm 0.1 \text{ eV} \quad \text{and} \quad V/b^3 = 1.5 \pm 0.3$$

(2.41)

Electron microscopic observations by Brown and Stobbs (1971b) show that the mechanism of relaxation is consistent with cross slip, which is also the conclusion of Atkinson et al. (1974). The activation energy of 0.9 eV is of the same order as that derived by Duesbery
et al. (1992) for cross slip in copper, but the activation volume is more than two orders of magnitude lower than the experimental (Bonneville and Escaig, 1979) and theoretical (Duesbery et al., 1992) values for cross slip in copper. In fact it is similar to that which would be expected for lattice diffusion (Brown and Ashby, 1980).

2.7 Summary of literature review

The literature which forms the basis for the work of this thesis has now been reviewed. The Eshelby method of describing the internal stresses is central to this study and although the Eshelby S tensor has been calculated for various inclusion shapes, it has not been calculated for a planar random or a three dimensional random array of fibres. Calculations of the S tensor for a planar random array of fibres are presented in chapter 3. The review has clearly shown that there is a lack of a comprehensive study on the behaviour of discontinuous fibre reinforced metals in Bauschinger experiments.

The mean field model has been applied extensively by various authors to transversely isotropic MMCs with both aligned continuous and short fibres. Application of the model to aluminium\Saffil composites requires it to be modified to take the anisotropy of transverse plastic flow into account and the theoretical basis of this modification is described in section 3.3 before being applied to experimental results in chapters 5 and 6.

The conventional method of using the permanent softening in measurements of internal stresses is dependent crucially on the constancy of the thermal stresses with cycling. This method has been modified to enable measurements of the mean stress hardening rate to be made even if the thermal stresses change with cycling. The theoretical basis of this modification is described in section 3.3 and the modified analysis is applied to results of Bauschinger experiments on the aluminium\Saffil composites in chapter 6.

Pedersen's modified Orowan-Wilson model for matrix hardening enables the ratio of the plastic components of the source shortening stress and mean stress to be measured. This model is applied to the results of Bauschinger experiments on aluminium\Saffil composites in chapter 7. A model for relaxation in aluminium\Saffil composites is developed in chapter 7, allowing some conclusions to be drawn about the relaxation mechanisms.
Table 2.1 The Eshelby $S$ tensor for various inclusion shapes, as calculated by Brown and Clarke (1975, 1977), and with the correction of Withers et al. (1989). The fibres are cylindrical and aligned parallel to the loading axis, which is the 3 axis. The disks have a circular shape and lie in the 2-3 plane (thickness → 0). $v$ is the Poisson’s ratio of the matrix. The notation adopted here is that of Brown and Clarke.

<table>
<thead>
<tr>
<th></th>
<th>Continuous aligned fibres</th>
<th>Disks</th>
<th>Spheres</th>
<th>Discontinuous aligned fibres</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{1111}$</td>
<td>$\frac{5 - 4v}{8(1 - v)}$</td>
<td>1</td>
<td>$\frac{7 - 5v}{15(1 - v)}$</td>
<td>$Q\pi + Rl_a + \frac{3}{4}T$</td>
</tr>
<tr>
<td>$S_{1122}$</td>
<td>$\frac{4v - 1}{8(1 - v)}$</td>
<td>$\frac{v}{(1 - v)}$</td>
<td>$\frac{5v - 1}{15(1 - v)}$</td>
<td>$\frac{1}{3}Q\pi - Rl_a + \frac{1}{4}T$</td>
</tr>
<tr>
<td>$S_{1133}$</td>
<td>$\frac{v}{2(1 - v)}$</td>
<td>$\frac{v}{(1 - v)}$</td>
<td>$\frac{5v - 1}{15(1 - v)}$</td>
<td>$- Rl_a - p^2T$</td>
</tr>
<tr>
<td>$S_{2211}$</td>
<td>$\frac{4v - 1}{8(1 - v)}$</td>
<td>0</td>
<td>$\frac{5v - 1}{15(1 - v)}$</td>
<td>$\frac{1}{3}Q\pi - Rl_a + \frac{1}{4}T$</td>
</tr>
<tr>
<td>$S_{2222}$</td>
<td>$\frac{5 - 4v}{8(1 - v)}$</td>
<td>0</td>
<td>$\frac{7 - 5v}{15(1 - v)}$</td>
<td>$Q\pi + Rl_a + \frac{3}{4}T$</td>
</tr>
<tr>
<td>$S_{2233}$</td>
<td>$\frac{v}{2(1 - v)}$</td>
<td>0</td>
<td>$\frac{5v - 1}{15(1 - v)}$</td>
<td>$- Rl_a - p^2T$</td>
</tr>
<tr>
<td>$S_{3311}$</td>
<td>0</td>
<td>0</td>
<td>$\frac{5v - 1}{15(1 - v)}$</td>
<td>$- Rl_c - T$</td>
</tr>
<tr>
<td>$S_{3322}$</td>
<td>0</td>
<td>0</td>
<td>$\frac{5v - 1}{15(1 - v)}$</td>
<td>$- Rl_c - T$</td>
</tr>
<tr>
<td>$S_{3333}$</td>
<td>0</td>
<td>0</td>
<td>$\frac{7 - 5v}{15(1 - v)}$</td>
<td>$\frac{4}{3}Q\pi + Rl_c + 2p^2T$</td>
</tr>
<tr>
<td>$S_{3333}$</td>
<td>$\frac{1}{4}$</td>
<td>0</td>
<td>$\frac{4 - 5v}{15(1 - v)}$</td>
<td>$2\pi R - \frac{1}{2}Rl_a - \frac{1 + p^2}{2}T$</td>
</tr>
<tr>
<td>$S_{1313}$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{4 - 5v}{15(1 - v)}$</td>
<td>$2\pi R - \frac{1}{2}Rl_a - \frac{1 + p^2}{2}T$</td>
</tr>
<tr>
<td>$S_{1212}$</td>
<td>$\frac{3 - 4v}{8(1 - v)}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{4 - 5v}{15(1 - v)}$</td>
<td>$\frac{1}{3}Q\pi + Rl_a + \frac{1}{4}T$</td>
</tr>
</tbody>
</table>

$$Q = \frac{3}{8\pi(1 - v)} \quad R = \frac{1 - 2v}{8\pi(1 - v)}$$

$$I_a = \frac{2\pi p}{(p^2 - 1)^{\frac{3}{2}}} \left[ p(p^2 - 1)^{\frac{1}{2}} - \cosh^{-1}p \right]$$

$$I_c = 4\pi - 2I_a \quad T = \frac{Q(4\pi - 3I_a)}{3(p^2 - 1)}$$

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Figure 2.1

Distributions of (a) diameter and (b) length of Saffil δ-alumina fibres used in this project. The mean diameter is 2.9 μm and the mean length is 470μm. From Clyne et al. (1985).

Figure 2.2

A schematic diagram of the fibre preform, showing the coordinate system with respect to the preform. The fibres lie on the average in the 2-3 plane and loading is along the 3 axis.

Figure 2.3

Angular distribution of fibres in a Saffil preform (f=20%) with respect to the horizontal plane (the 2-3 plane). $\alpha'$ is the angle between the fibres and the 2-3 plane. From van Hille et al. (1989).
Figure 2.4

Experimental stress-strain curves of two copper-tungsten composites, showing three stages before fracture. Stages I and II are linear and stage III is non-linear. From Kelly and Lilholt (1969).

Figure 2.5

The stress-strain curve of the matrix in the copper-tungsten composites (fibre diameter 10μm) as derived from the stress-strain curves for the composites. The stress-strain curves of pure copper are also shown for single crystals and polycrystals with 12μm and 30μm grain sizes. From Kelly and Lilholt (1969).
A dislocation bowing between two fibres. The effective interfibre spacing, $2r_2$, is less than the actual spacing between the fibres because of the source shortening stress. From Brown and Clarke (1977).

**Figure 2.6**

Stress - strain curves of continuous $\alpha$-alumina fibre reinforced aluminium in tension. Curves are shown for a loading of the composite at an angle of $0^\circ$, $30^\circ$, $60^\circ$ and $90^\circ$ to the fibre orientation. From Shetty and Chou (1985).

**Figure 2.7**

Stress - strain curves of continuous $\alpha$-alumina fibre reinforced aluminium in tension. Curves are shown for a loading of the composite at an angle of $0^\circ$, $30^\circ$, $60^\circ$ and $90^\circ$ to the fibre orientation. From Shetty and Chou (1985).
Figure 2.8

This figure illustrates the cutting and welding exercises Eshelby (1957) used in the calculation of internal stresses for a misfitting ellipsoidal inhomogeneity (left hand side) and the corresponding equivalent homogeneous inclusion (right hand side). No external stress is applied here. The transformation strain $e^T$ is chosen for the equivalent inclusion such that the stress states in both composites are identical. From Withers et al. (1989).

Figure 2.9

Eshelby's cutting and welding exercise for a composite externally stressed by $\sigma^A$. The transformation strain $e^T$ and the constrained strain $e^C$ now depend on the applied stress $\sigma^A$. From Withers et al. (1989).
Figure 2.10

Typical behaviour of a polycrystalline metal on stress reversal. The horizontal axis shows plastic strain. The permanent softening is the stress difference between curves C and D, after they become parallel. From Orowan (1959).

Figure 2.11

The variation of permanent softening with plastic strain, $e_{\text{per}}$, for continuous fibre copper-tungsten composites (fibre diameter 20\(\mu\)m). The fibre volume fraction, $f$, is shown for each curve. From Lilholt (1977).
Figure 2.12

Figure illustrating how the permanent softening, $\Delta \sigma$, is constructed. The material is a copper-tungsten composite with continuous fibres aligned parallel to the loading axis. From Pedersen (1989).
3 Theoretical developments

3.1 Introduction

3.2 Calculation of the Eshelby S tensor for planar random fibre composites
   3.2.1 Tensors and tensor transformations
   3.2.2 Mean value of a fourth rank tensor over all directions
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3.3 Theoretical evaluation of some measurable properties of a planar random fibre composite
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3.4 Summary

Table 3.1
Figures 3.1 - 3.6
Chapter Three
Theoretical developments

3.1 Introduction

In section 2.4 the theory of internal stresses in composite materials was described. The resulting mean field theory is based on the Eshelby method of calculating the internal stresses and formulated in terms of the Eshelby S tensor and the moduli of the material constituents. The S tensor depends on inclusion geometry as well as the Poisson's ratio of the matrix and must be derived for the particular inclusion geometry used. Brown and Clarke (1975, 1977) calculated the S tensor for many different ellipsoidal shapes and orientations, of which continuous and short fibres orientated parallel to the loading direction are examples.

The fibres used in this work have a high aspect ratio and can, for practical purposes, be considered as continuous fibres. They have a random orientation in one plane, which is the plane from which the specimens are cut. To enable applications of the mean field theory to the planar random fibre composite used in this work, the Eshelby tensor for a planar array of fibres was calculated. The calculations are presented in section 3.2.

It was described in section 2.5 how the permanent softening in a Bauschinger experiment has been used by various authors to obtain information on the mean stress in MMCs. In sections 3.3.1 and 3.3.2 it will be shown how this method of analyzing the Bauschinger experiment can be modified to allow a separation of plastically and thermally induced mean stresses\(^1\). Theoretically this means that both the mean and thermal stress components can be obtained from a single hysteresis loop. In section 3.3.3 it is shown how this modified analysis can be applied to experimental results to obtain the mean stress hardening rate.

\(^1\) For the sake of brevity, the plastically induced mean stress will hereafter be called the mean stress and the thermally induced mean stress will be called the thermal stress.
In the calculations of the mean stress hardening rate the plastic strain of the composite must be known. Most of the theoretical work on this aspect has been on transversely isotropic composites for which the plastic strain has a simple form. The aluminium/Saffil composite is, however, not transversely isotropic. Thus, a method to experimentally evaluate the plastic strain $e_p$ has been devised and is described in section 3.3.3. Finally, in section 3.3.4, calculations of the thermal stress are presented.

This analysis of the Bauschinger experiment has been applied to experimental results and the outcome is described in chapter 6. The method of quantifying the uneven transverse flow of the composite is applied to experimental results in section 5.5 and the calculations of the thermal stress are compared to experiment in section 5.4.

3.2 Calculation of the Eshelby S tensor for planar random fibre composites

3.2.1 Tensors and tensor transformations

In this section a brief introduction to tensors, their definition and properties, is given.

Tensors are mathematical or physical concepts which have certain specific laws of transformation when there is a change in the coordinate system. A tensor is said to be Cartesian when it is expressed in terms of components referring to a rectangular cartesian coordinate system. The specific transformation of axes we are interested in is a rotation about the origin. For such a transformation the relative orientations of the axes are given by the following matrix:

\[
\begin{pmatrix}
    x'_{1} & x'_{2} & x'_{3} \\
    \lambda_{11} & \lambda_{12} & \lambda_{13} \\
    \lambda_{21} & \lambda_{22} & \lambda_{23} \\
    \lambda_{31} & \lambda_{32} & \lambda_{33}
\end{pmatrix}
\]

The $\lambda_{ij}$ are the direction cosines of the transformation and the first subscript refers to the "new" axes (dashed) and the second to the "old" axes (undashed). $\lambda_{ij}$ is the cosine of the angle between axis $i$ in the "new" system and axis $j$ in the "old" system. The nine $\lambda_{ij}$ are not independent of one another and $\lambda_{ij} \neq \lambda_{ji}$ in general (for a complete discussion of the properties of tensors see e.g. Nye, 1990; Jaeger, 1966 and Bourne and Kendall, 1990).
Tensors can be classified according to their "rank". A scalar is a tensor of zero rank, specified by a single component which is not related to any specific axes of reference. Density is one example of a scalar. A vector is a first rank tensor, specified by three components, say \( E_i \), each of which is associated with one of the axes of reference (in this thesis the range of all the indices is 1,2,3). Mechanical force and electric field at a point are examples of vectors. When the coordinate system is rotated about the origin, a vector transforms according to the equation

\[
E'_i = \lambda_{ij} E_j
\]  
(3.1a)

The summation convention applies here: If any index appears twice in any term, a sum is to be taken over all three possible values of that index. Eq. 3.1a has therefore three terms on the right side for each value of \( i \)

\[
E'_i = \lambda_{i1}E_1 + \lambda_{i2}E_2 + \lambda_{i3}E_3
\]  
(3.1b)

The essential point here is that the components of a vector are components of a physical property which does not depend on the coordinate system used to represent it. When the axes of reference are changed, it is only the method of representing that property which changes, the property itself does not change.

A tensor of second rank is specified by nine components, say \( \sigma_{ij} \), each of which is associated with a specific pair of axes. Stress and strain are examples of second rank tensors. They transform according to the following equation

\[
\sigma'_{ij} = \lambda_{ik} \cdot \lambda_{jl} \cdot \sigma_{kl}
\]  
(3.2)

The stiffness tensor \( C_{ijkl} \) relates stress and strain in a body and is an example of a fourth rank tensor. It transforms according to:

\[
C'_{ijkl} = \lambda_{im} \cdot \lambda_{jn} \cdot \lambda_{ko} \cdot \lambda_{lp} \cdot C_{mnop}
\]  
(3.3)

From these examples it emerges that the number of subscripts equals the rank, \( n \), of the tensor. Each tensor of rank \( n \) has \( 3^n \) components, so the stiffness tensor has \( 3^4 = 81 \) components. It should be noted that eq. 3.3 represents 81 equations (one for each set of \( ijk \)) and each of these equations has 81 terms on the right side (one for each combination of \( mnop \)). The total number of terms represented by eq. 3.3 is then \( 81^2 = 6561 \).

The importance of the transformation laws (eqs. 3.1 - 3.3) is that they can be used as definitions of tensors. If \( T \) is a mathematical or physical quantity which can be represented by a set of \( 3^2 = 9 \) scalars, \( T_{ij} \), and transforms according to eq. 3.2 when the coordinate system is rotated about the origin, then \( T \) is a second rank tensor (Nye, 1990).
3.2.2 Mean value of a fourth rank tensor over all directions.

The problem of finding the mean value of a tensor over all orientations in a plane is a part of a more general problem, namely to average a tensor over all directions. The formal solution to this general problem will be derived here from which the more specific solution for the planar average can be found easily.

The starting point is eq. 3.3 which gives the value of a fourth rank tensor when the coordinate system is rotated about the origin as specified by the \( \lambda \) matrix. In this equation, \( C_{ijkl} \) and \( C_{mnop} \) are the stiffness constants and each lambda term is a function of the two angles needed to specify a direction in space (\( \phi \) and \( \theta \)). If the product of the four lambda terms is written as \( f(\phi, \theta) \), the problem becomes that of finding the average of this function \( f(\phi, \theta) \) over all directions or, in other words, of evaluating the integral of the scalar field \( f \) on the surface of the unit sphere.

The surface \( S \) of the unit sphere is described by the vector

\[
\mathbf{r}(\phi, \theta) = (\cos \phi, \cos \theta \sin \phi, \sin \theta \sin \phi)
\]

(3.4)

Fig. 3.1 shows how the angles \( \phi \) and \( \theta \) relate to the 1,2,3 axes. \( \mathbf{r} \) is defined on a region \( T \) in the \( \phi-\theta \) plane with \( 0 \leq \phi \leq \pi \) and \( 0 \leq \theta < 2\pi \) (fig. 3.2). Eq. 3.4 is the vector equation of the mapping and describes how a point \((\phi, \theta)\) in a region \( T \) in the \( \phi-\theta \) plane is carried into an image point \((x_1, x_2, x_3)\) which lies on the surface of the unit sphere in real space (fig.3.2). As \((\phi, \theta)\) runs through the points of \( T \), the endpoint of \( \mathbf{r} \) traces out the surface of the unit sphere. The surface integral of \( f \) over \( S \) is then defined by (Apostol 1969, Bourne and Kendall, 1990)

\[
\iint_{r(T)} f \, dS = \iint_{T} f(\mathbf{r}(\phi, \theta)) \left\| \frac{\partial \mathbf{r}}{\partial \phi} \times \frac{\partial \mathbf{r}}{\partial \theta} \right\| \, d\phi d\theta
\]

(3.5)

The norm of the cross product is the Jacobi determinant of the mapping, \( |J(\phi, \theta)| \) (Apostol, 1969). Geometrically, \( |J(\phi, \theta)| \, d\phi d\theta \) in eq. 3.5 can be interpreted as the area, in spherical coordinates, of a surface element \( dS \) on a sphere. Without performing any calculations it is possible, using fig. 3.3, to make some general statements about the behaviour of the Jacobi determinant. The area of the surface element \( dS \) in this figure must have a maximum at \( \phi = \pi/2 \) and vanish at \( \phi = 0 \) and \( \pi \). The shape of \( dS \) is not dependent on \( \theta \) so \( |J(\phi, \theta)| \) should be independent of \( \theta \). The area of \( dS \) can be calculated with the aid of fig. 3.3 and is found to be \( \pi a^2 \sin \phi \, d\phi d\theta \) for a sphere of radius \( a \). The Jacobi determinant is then \( \sin \phi \) for a unit sphere. As discussed above, this function
has a maximum at $\phi = \pi/2$ and is independent of $\theta$.

The average of the function $f$ over all directions is the integral in eq. 3.5 divided by the surface of the unit sphere, $4\pi$ (the surface of the unit sphere can be calculated by setting $f = 1$ in eq. 3.5 and using $r$ given by eq. 3.4), and is given by

$$<f> = \frac{1}{4\pi} \int f(r(\phi, \theta)) \sin \theta d\phi d\theta$$  \hfill (3.6)

In our particular case the function $f(\phi, \theta)$ is the product of the four lambda terms in eq. 3.3, with each lambda term generally a function of both $\phi$ and $\theta$. However, when the integral is taken in the 2-3 plane, the four lambda terms are independent of $\phi$ (see fig. 3.4) and eq. 3.6 becomes

$$<f> = \frac{1}{2\pi} \int_0^{2\pi} f(\theta) d\theta$$  \hfill (3.7)

The average of the Eshelby S tensor in the 2-3 plane is then

$$S'_{ijl} = \frac{S_{mnp}}{2\pi} \int_0^{2\pi} \lambda_{ip} \lambda_{ko} \lambda_{jo} \lambda_{lp} d\theta$$  \hfill (3.8)

The lambda terms are now only dependent on $\theta$ and the $\lambda$ matrix for an anti-clockwise rotation in the 2-3 plane is

$$\langle \lambda_{ij} \rangle = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix}$$  \hfill (3.9)

To calculate the planar average of the S tensor, eq. 3.8 has to be solved for all 6561 terms.

### 3.2.3 The planar random fibre S tensor.

The method devised here to solve eq. 3.8 is described in Appendix 1 and the resulting Eshelby S tensor for both continuous and short fibres is given in table 3.1. The method is not a general one and is only applicable to the specific problem of finding the planar average of a fourth rank tensor. The first step in finding the solution was to map all possible combinations of $\lambda_{ij}$ which are generally not zero (there are 35) and calculate by hand the value of each integral. Then a computer program was written which identifies the lambda terms in each step and retrieves the corresponding value of the integral from another file. The integral is then printed out, along with the 8 indices (ijkl and mnop, see
eq. 3.8). For a detailed description of the procedure, see Appendix 1. The raw results of the calculation are given in table A1.1, which contains all (177) terms of eq. 3.8 which have a non-zero integral. The transformations given by table A1.1 are then applied to the $S$ tensor components for aligned continuous and short fibres as calculated by Brown and Clarke (1975, 1977) and given here in table 2.1. Examples of this are given at the end of Appendix 1.

From table 3.1 a certain symmetry of the $S$ tensor emerges. Using Nye's notation (Nye, 1990) this symmetry is represented very clearly by the array

$$S = \begin{pmatrix}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot 
\end{pmatrix}$$

Here, a small dot stands for a zero component and a large dot for a non-zero component. A bar connecting two non-zero components means they are equal. The symmetry of the tensor is then $S_{12} = S_{13}$, $S_{21} = S_{23}$, $S_{22} = S_{33}$, $S_{23} = S_{32}$ and $S_{55} = S_{66}$.

The first check on the validity of these results is to observe whether the symmetry of the $S$ tensor is the same as that of disks aligned parallel to the loading axis (table 2.1). For the disks we see that $S_{12} = S_{13}$, $S_{55} = S_{66}$ and that $S_{11}$ is different from all other components. The planar tensor in table 3.1 has the same symmetry conditions. Another, and a more powerful check which can be applied to all components, is to use the transformation calculated here on the Eshelby tensor for spheres calculated by Brown and Clarke (1975). As orientation is irrelevant for spheres, the transformation should leave all the components of the tensor unchanged. Table A1.1 in Appendix 1 contains the prescription for the transformations and the procedure of the check is to carry out the same summations used in the calculation of the planar random fibre tensor. The first term (line 1 in table A1.1) is left unchanged by the transformation. The expression for the second term is

$$S'_{1122} = \frac{1}{2}(S_{1122} + S_{1133})$$
but $S_{1122} = S_{1133}$ for spheres so $S'_{1122}$ does not change. The calculation of $S'_{2222}$ is more elaborate. The expression for this component is

$$S'_{2222} = \frac{1}{9}(S_{2222} + 3S_{3333} + S_{2233} + S_{3322} + 4S_{2323})$$

Inserting the spherical tensor components gives

$$S'_{2222} = \frac{(7 - 5\nu)}{15(1 - \nu)}$$

This is precisely $S_{2222}$ for spheres so this component does not change, as required. The transformation has been applied to all components of the tensor. In all cases the tensor components do not change, which is a necessary condition for the validity of the transformation. A similar check has been carried out using the components for disks aligned parallel to the 3 axis, and again complete agreement is found. The conclusion from these various checks is that the transformation calculated here, and presented in table A1.1, is correct.

Finally, in the limit of infinite aspect ratio, the short fibre $S$ tensor in table 3.1 should become identical to that of the continuous fibre $S$ tensor given in the same table. This was found to be correct for all 12 components.

### 3.3 Theoretical evaluation of some measurable properties of a planar random fibre composite

In section 2.5 of the literature review it was shown how the permanent softening in a Bauschinger experiment can be used to obtain information on the internal stresses in the composite. In section 3.3.1 the expressions relating the mean and thermal stresses to the permanent softening are derived. The resulting equations indicate a way of separating the mean and thermal stresses, and this is shown in section 3.3.2. The expressions for the unrelaxed mean stress hardening rate are then derived in section 3.3.3. In the calculations of the mean stress, all the components of the plastic strain of the composite have to be known. For aligned fibre composites which are transversely isotropic, the plastic strain has a simple form and is given by (Brown and Clarke, 1975)

$$e_p = e_p(-\frac{1}{2}, -\frac{1}{2}, 1, 0, 0, 0)$$

(3.10)

The aluminium/Saffil composite is, however, not transversely isotropic, with the 1 direction being perpendicular to the plane of the fibres, and the 2 direction lying in the plane of the
fibres. Because of the difficulties involved, it seems futile to try to calculate the plastic strain (Brown and Clarke, 1975; Pedersen, 1985b). Hence, a method to experimentally estimate the transverse plastic strain of the composite was devised and is described in section 3.3.3. This method is then applied to experimental results in section 5.5. Finally, the expressions for the thermal stress are derived in section 3.3.4.

3.3.1 The permanent softening

The permanent softening is the difference in the forward and reverse flow curves and hence, any theoretical evaluation of this parameter must include a yield criterion. Brown and Clarke (1975) used the Tresca criterion, which relates the yield stress in shear to the principal stresses

$$\{ \sigma_{3M} - \sigma_{1M} , \sigma_{3M} - \sigma_{2M} \} = 2\tau_{YM}$$

(3.11)

$\sigma_{1M}$ is the average matrix stress in the $i$ direction and $\tau_{YM}$ the matrix yield stress in shear. For composites which are isotropic in the $1-2$ plane, $\sigma_{1M} = \sigma_{2M}$ and eq. 3.11 yields a single expression. However, the planar random fibre composite is not symmetric in the $1-2$ plane (see fig. 3.4) so $\sigma_{1M} \neq \sigma_{2M}$, and eq. 3.11 yields two expressions

$$\sigma_{3M} - \sigma_{1M} = 2\tau_{YM}$$

(3.12a)

$$\sigma_{3M} - \sigma_{2M} = 2\tau_{YM}$$

(3.12b)

It is not clear how the two cases represented by eqs. 3.12a and 3.12b should be used to make a unique estimate of the mean stress hardening rate for this composite. The obvious approach is to calculate the hardening rates for both cases, and one would expect intuitively that the mean stress hardening rate of the composite lies between the two resulting limits. This is the approach used here.

When a stress $\sigma^A$ is applied to the composite, the average stress in the matrix is

$$\sigma_M = \sigma^A + <\sigma>_M + <\sigma>_M^{th}$$

or

$$\sigma_M = \sigma^A + <\sigma>_M^P + <\sigma>_M^A + <\sigma>_M^{th}$$

(3.13)

$<\sigma>_M$ is the mean stress in the matrix, which is split into a plastic ($<\sigma>_M^P$) and elastic ($<\sigma>_M^A$) component and $<\sigma>_M^{th}$ is the thermal stress. The elastic component of the mean stress is (see eq. 2.20b)
so the average matrix stress is

\[ \overline{\sigma}_M = \langle \sigma \rangle^A_M = B \sigma^A \]  \hspace{1cm} (3.14) \]

Inserting this expression into the yield criterion 3.12a and noting that \( \sigma^A = (0,0,\sigma^A,0,0,0) \), the overall forward flow stress becomes

\[ \sigma' = \frac{2\tau_{YM} + (\langle \sigma^p \rangle^A_M - \langle \sigma^T \rangle^A_M) \pm (\langle \sigma^p \rangle^T_M - \langle \sigma^T \rangle^T_M)}{1 + B} \]  \hspace{1cm} (3.16a) \]

with \( B = B_{33} - B_{13} \) (\( B = B_{33} - B_{23} \) if eq. 3.12b is used). For reasons of simplicity, the equations for the flow stress and permanent softening are derived using eq. 3.12a. The consequences of the two flow criterion, 3.12a and 3.12b, are discussed in the following sections. In expression 3.16a, - is for tension forward and + for compression forward. In the reverse direction both the plastic component of the mean stress and the thermal stress component change sign.

\[ \sigma' = \frac{2\tau_{YM} - (\langle \sigma^p \rangle^A_M - \langle \sigma^T \rangle^A_M) \pm (\langle \sigma^p \rangle^T_M - \langle \sigma^T \rangle^T_M)}{1 + B} \]  \hspace{1cm} (3.16b) \]

Now + is for tension forward and - for compression forward. The permanent softening is then

\[ \Delta \sigma = |\sigma'| - |\sigma'| = 2 \frac{|\langle \sigma^p \rangle^A_M - \langle \sigma^T \rangle^A_M| \mp |\langle \sigma^p \rangle^T_M - \langle \sigma^T \rangle^T_M|}{1 + B} \]  \hspace{1cm} (3.17) \]

with \( \pm \) for tension/compression forward. In this expression the plastic and elastic components of the mean stress are taken into account, as well as the thermal stress. It should be noted that this derivation of \( \Delta \sigma \) is, strictly speaking, only valid when the roundedness of the reverse flow curve is absent. An example of a hysteresis loop for which this derivation is valid is given in fig. 3.5.
3.3.2 Separation of the mean stress and the thermal stress

Eq. 3.17 describes how the mean stress and the thermal stress relate to the permanent softening. The equation describes two cases, one for tension and the other for compression as the forward direction. If \( \Delta \sigma^t \) is written for tension forward and \( \Delta \sigma^c \) for compression forward, we obtain two equations

\[
\Delta \sigma^t = 2 \frac{\langle \sigma_3^p >_M - < \sigma_{1,2}^p >_M \rangle - \langle \sigma_3^T >_M - < \sigma_1^T >_M \rangle}{1 + B} \tag{3.18}
\]

\[
\Delta \sigma^c = 2 \frac{\langle \sigma_3^p >_M - < \sigma_{1,2}^p >_M \rangle + \langle \sigma_3^T >_M - < \sigma_1^T >_M \rangle}{1 + B} \tag{3.19}
\]

These equations show that a knowledge of \( \Delta \sigma^t \) and \( \Delta \sigma^c \) enables a separation of the mean and thermal stresses.

\[
\frac{1}{4} (\Delta \sigma^c + \Delta \sigma^t) = \frac{\langle \sigma_3^p >_M - < \sigma_{1,2}^p >_M \rangle}{1 + B} \tag{3.20}
\]

\[
\frac{1}{4} (\Delta \sigma^c - \Delta \sigma^t) = \frac{\langle \sigma_3^T >_M - < \sigma_1^T >_M \rangle}{1 + B} \tag{3.21}
\]

The left hand sides of eqs. 3.20 and 3.21 are experimentally obtainable and the right hand sides show how these quantities can be calculated. \( \Delta \sigma^t \) and \( \Delta \sigma^c \) can be evaluated from a single hysteresis loop in a Bauschinger experiment and their construction is shown in section 6.2. Since \( \langle \sigma_i >_M \) \((i=1,2 \text{ or } 3)\) is proportional to the plastic strain in the 3 direction, \( \varepsilon_{pc} \), eq. 3.20 for the mean stress predicts this parameter to be linear in plastic strain, zero when \( \varepsilon_{pc} = 0 \) and independent of the thermal stress. Eq. 3.21 allows for observations of the thermal stress component for a single hysteresis loop in a Bauschinger experiment. On the right hand side of eq. 3.20, the two flow criteria 3.12a and 3.12b give two limits, as indicated by the two indices, 1 and 2. As the composite is isotropic in the 2-3 plane, the difference between the thermal stress in the 2 and 3 directions is zero. The right hand side of eq. 3.21 is thus expected to be an upper limit to the thermal stress in an unrelaxed composite.
3.3.3 The unrelaxed mean stress hardening rate

In this section it will be show how the theoretical results of the previous section can be compared with experiment.

Eq. 3.20 gives limits for the mean stress in the aluminium\Saffil composite and accounts for both plastically and elastically generated mean stresses. The derivative of eq. 3.20 with respect to the plastic strain of the composite gives theoretical limits to the mean stress hardening rate, which can then be compared to the slope of the experimental curves in which $\frac{1}{4}(\Delta \sigma^c + \Delta \sigma^t)$ is plotted against $e_{pc}$

$$\frac{d}{de_{pc}} \left( \frac{1}{4} (\Delta \sigma^c + \Delta \sigma^t) \right) = \frac{d}{de_{pc}} \left( \frac{\sigma^p_M - \sigma^p_{1,2}}{1 + B} \right)$$

(3.22)

Here, $B = B_{33} - B_{13}$ if $\sigma^p_M$ is used (eq. 3.12a) and $B = B_{33} - B_{23}$ if $\sigma^p_M$ is used (eq. 3.12b). This equation forms the basis for the comparison of theoretical and experimental results on the mean stress hardening rate. All the parameters on the left hand side can be measured in a Bauschinger experiment and the right hand side is obtained as follows. The plastic component of the mean stress $\sigma^p_M$ can be calculated using eq. 2.20b, which gives

$$\sigma^p_M = Ae^{T^*}$$

with

$$A = fC_M(S-I)((C_M-C_I)(S-f(S-I))-C_M)^{-1}C_I$$

(3.23)

At this point, the transformation strain $e^{T^*}$ has to be related to, first the plastic strain of the matrix $e_p$ and then to the plastic strain of the composite $e_{pc}$. If the matrix experiences a uniform plastic deformation $e_p$, the inclusions undergo elastic deformation. The stress field generated is identical to that of an unstrained matrix in which the inclusions undergo a stress free transformation $e^{T^*} = -e_p$ (Brown and Stobbs, 1971a). This relationship is, however, only valid in the absence of stress relaxation. The plastic component of the mean stress can then be written as

$$\sigma^p_M = -Ae_p$$

(3.24)

As $e_p$ is the plastic strain of the matrix, the next step is to find its relation to the plastic strain of the composite $e_{pc}$. One would expect the plastic strain of the composite (in tensile deformation) to be smaller than the plastic strain of the matrix. Consider a composite which contains spherical inclusions which are not stressed. Imagine that all the
inclusions are removed from their holes and the matrix plastically deformed in tension by \( e_p \). The holes have now become ellipsoidal in shape (and the stress in the matrix is zero). We now apply surface tractions to the inclusions so they acquire the shape of the holes from which they came, put them into the holes, weld across the surface and relax the surface tractions. It is now clear that the last step results in a slight elastic contraction of the composite, which means that \( e_{pc} < e_p \). Formally this is expressed by the equation (Brown, 1973)

\[
e_{pc} = e_p + f e^T (e^T)
\]

(3.25)

In this equation the purpose of the bracket is to show that \( e^T \) is a function of \( e^T \). Expressing \( e^T \) in terms of \( e^{T*} \) and using \( e^{T*} = -e_p \), then \( e_{pc} \) can be written as

\[
e_{pc} = \{I - f[(C_I - C_M)(S - f(S - I)) + C_M]^{-1}C_I\} e_p
\]

or,

\[
e_{pc} = D e_p
\]

(3.26)

If the matrix of the composite is polycrystalline, and therefore flows by multiple slip, and if the 1 and 2 directions are identical, the plastic strain for loading in the 3 direction is (Brown and Clarke, 1975)

\[
e_p = e_p \left(\frac{1}{2}, \frac{1}{2}, 1, 0, 0, 0\right)
\]

(3.27)

This equation is valid for composites which are isotropic in the 1-2 plane. The plastic extension in the 3 direction is \( e_p \) and the principle of volume conservation in plastic flow is satisfied because \( \Sigma e_{ip} = 0 \).

However, the 1 and 2 directions in the planar random aluminium\Saffil composite are not identical since the fibres lie in the 2-3 plane (see fig. 3.4) and therefore it is not possible to use \( e_p \) given by eq. 3.27. It would be expected that

\[
e_p = (e_{p1}, e_{p2}, e_{p3}, 0, 0, 0) = e_p(-\eta_1, -\eta_2, 1, 0, 0, 0)
\]

(3.28)

with \( \eta_1 + \eta_2 = 1 \) and \( \eta_1/\eta_2 = e_{p1}/e_{p2} \)

(3.29)

\( \eta_1 \) and \( \eta_2 \) quantify the unequal amounts of plastic flow in the 1 and 2 directions and are analogous to Poisson's ratio in elastic deformation. They can be expressed in terms of \( e_{p1} \) and \( e_{p2} \) as

\[
\eta_1 = \frac{1}{\left(1 + \frac{e_{p2}}{e_{p1}}\right)} \quad \eta_2 = \frac{1}{\left(1 + \frac{e_{p1}}{e_{p2}}\right)}
\]

(3.30)
A cross section of a deformed specimen which is not isotropic in the 1-2 plane is shown schematically in fig. 3.6b. One way of establishing the values of $\eta_1$ and $\eta_2$ is to note that eq. 3.26 can be used to express $e_p$ in terms of $e_{pc}$.

$$e_p = D^{-1}e_{pc}$$

(3.31)

By measuring the composite plastic strains $e_{pc1}$, $e_{pc2}$ and $e_{pc3}$ after a tensile plastic deformation, it is possible to calculate $e_p$ by eq. 3.31 and then use eq. 3.30 to obtain $\eta_1$ and $\eta_2$. This is the method adopted here. The necessary consequence of this approach is that the theoretical evaluation of the mean stress hardening rate will not be entirely free of empirical corrections since $\eta_1$ and $\eta_2$ are derived from measured quantities.

The unrelaxed mean stress hardening rate, as expressed by the right hand side of eq. 3.22, can thus be calculated and compared to experimental results.

3.3.4 Thermal stresses

In most MMCs the matrix and the reinforcing phase have different coefficients of thermal expansion (CTE). The aluminium/Saffil composite is fabricated by the squeeze infiltration method where molten aluminium is squeezed into a fibre preform and many other MMCs are made at elevated temperatures. Considerable internal stresses can therefore develop in the material upon cooling down to room temperature and any temperature changes after fabrication will, of course, also affect the internal stress state. This is particularly important for composites where the CTE mismatch is large, as in Al/Al$_2$O$_3$ composites (Al: $\alpha_M = 23.1 \cdot 10^{-6}/K$; Al$_2$O$_3$: $\alpha_I = 8.3 \cdot 10^{-6}/K$; data from Ashby and Brown, 1980).

In this section it will be shown how the thermal stress in eq. 3.21 can be calculated and compared with experiment. In these calculations it is assumed that there exists a temperature range $\Delta T$ over which no relaxation of internal stresses takes place. This assumption is of course unrealistic; as the composite cools down, the stresses will be able to relax easily at higher temperatures, and at lower temperatures relaxation will also take place, though it will be more difficult. Bearing this in mind, it is, nonetheless, useful when comparing the magnitude of internal stresses to assume that the stresses build up from some stress free temperature which is $\Delta T$ above room temperature.

In plastic deformation, the origin of the strain mismatch $e^{T*}$ (the transformation strain) is the shape change of the matrix. On heating or cooling it is the difference in CTE...
between the matrix and the fibres which is the origin of the strain mismatch \( e^T \) (Withers et al., 1989)

\[
e^T = (\alpha_M - \alpha_F) \Delta T
\]

(3.32)

For isotropic expansion \( \alpha = \alpha(1, 1, 1, 0, 0, 0) \). Eq. 2.20b can now be used to calculate the mean matrix stress

\[
<\sigma>_M = A e^T + B e^A
\]

(2.20b)

As there is no applied stress, \( e^A = 0 \) and

\[
<\sigma>_M = <\sigma>_M^T = A e^T = A(\alpha_M - \alpha_F) \Delta T
\]

\[
<\sigma>_M^T = A(\alpha_M - \alpha_F) \Delta T
\]

(3.33)

This equation shows that the matrix stress resulting from a temperature change of \( \Delta T \), in other words the thermal stress, is proportional to the CTE mismatch and \( \Delta T \). The thermal stress measured in a Bauschinger experiment (see eq. 3.21) is then

\[
\frac{|<\sigma>_M^T - <\sigma>_M|}{1 + B} = \frac{|A(\alpha_M - \alpha_F)|_3 - |A(\alpha_M - \alpha_F)|_1}{1 + B} |\Delta T|
\]

(3.34)

with \( B = B_{33} - B_{13} \). Since the 3 and 2 directions are identical, the difference between the thermal stress in the 3 and 2 directions is zero. Thus, eq. 3.34 is expected to represent an upper limit to the thermal stress in the material. It is not clear how to take the anisotropy in the 1-2 plane into account for making a unique prediction of the thermal stress.

These results will be discussed in the context of the experimental results in chapters 5 and 6.

### 3.4 Summary

1. The Eshelby S tensor for a planar random array of continuous and discontinuous fibres has been calculated.

2. It has been shown, theoretically, how the permanent softening \( \Delta \sigma^I \) and \( \Delta \sigma^o \) can be evaluated from a hysteresis loop and used to separate the thermal stress and the mean stress. This method will be used to analyse Bauschinger experiments on aluminium\-Saffil composites in chapter 6, where both the mean stress and the thermal stress are studied.

3. A theoretical expression for the thermal stress in an aluminium\-Saffil composite has been derived using the Eshelby method.
Table 3.1 The Eshelby $S$ tensor for a planar array of continuous and short fibres with a uniform orientation distribution in the 2-3 plane. Loading is parallel to the 3 axis. $v$ is the Poisson's ratio of the matrix and the terminology for the short fibre tensor is that of Brown and Clarke (1977).

<table>
<thead>
<tr>
<th>Tensor notation</th>
<th>Matrix notation</th>
<th>S tensor Continuous fibres</th>
<th>S tensor Short fibres Aspect ratio $p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{1111}$</td>
<td>$S_{11}$</td>
<td>$\frac{5-4v}{8(1-v)}$</td>
<td>$Q\pi + I_a R + \frac{3}{4}T$</td>
</tr>
<tr>
<td>$S_{1122}$</td>
<td>$S_{12}$</td>
<td>$\frac{8v-1}{16(1-v)}$</td>
<td>$\frac{1}{6}Q\pi - I_a R - \frac{1}{2}(p^2 - \frac{1}{4})T$</td>
</tr>
<tr>
<td>$S_{1133}$</td>
<td>$S_{13}$</td>
<td>$\frac{8v-1}{16(1-v)}$</td>
<td>$\frac{1}{6}Q\pi - I_a R - \frac{1}{2}(p^2 - \frac{1}{4})T$</td>
</tr>
<tr>
<td>$S_{2211}$</td>
<td>$S_{21}$</td>
<td>$\frac{4v-1}{16(1-v)}$</td>
<td>$\frac{1}{6}Q\pi - \frac{1}{2}(I_a + I_c)R - \frac{3}{8}T$</td>
</tr>
<tr>
<td>$S_{2222}$</td>
<td>$S_{22}$</td>
<td>$\frac{23-16v}{64(1-v)}$</td>
<td>$\frac{7}{8}Q\pi + (\pi + \frac{1}{4}I_c)R + \frac{3}{8}(p^2 - \frac{1}{4})T$</td>
</tr>
<tr>
<td>$S_{2233}$</td>
<td>$S_{23}$</td>
<td>$\frac{16v-3}{64(1-v)}$</td>
<td>$\frac{7}{24}Q\pi - (\pi + \frac{1}{4}I_c)R + \frac{1}{8}(p^2 - \frac{1}{4})T$</td>
</tr>
<tr>
<td>$S_{3311}$</td>
<td>$S_{31}$</td>
<td>$\frac{4v-1}{16(1-v)}$</td>
<td>$\frac{1}{6}Q\pi - \frac{1}{2}(I_a + I_c)R - \frac{3}{8}T$</td>
</tr>
<tr>
<td>$S_{3322}$</td>
<td>$S_{32}$</td>
<td>$\frac{16v-3}{64(1-v)}$</td>
<td>$\frac{7}{24}Q\pi - (\pi + \frac{1}{4}I_c)R + \frac{1}{8}(p^2 - \frac{1}{4})T$</td>
</tr>
<tr>
<td>$S_{3333}$</td>
<td>$S_{33}$</td>
<td>$\frac{23-16v}{64(1-v)}$</td>
<td>$\frac{7}{24}Q\pi + (\pi + \frac{1}{4}I_c)R + \frac{3}{8}(p^2 - \frac{1}{4})T$</td>
</tr>
<tr>
<td>$S_{2323}$</td>
<td>$S_{44}$</td>
<td>$\frac{13-16v}{64(1-v)}$</td>
<td>$\frac{7}{24}Q\pi + (\pi + \frac{1}{4}I_c)R + \frac{1}{8}(p^2 - \frac{1}{4})T$</td>
</tr>
<tr>
<td>$S_{1313}$</td>
<td>$S_{55}$</td>
<td>$\frac{5-6v}{16(1-v)}$</td>
<td>$\frac{1}{6}Q\pi + (\pi + \frac{1}{4}I_c)R - \frac{1}{4}(p^2 + \frac{1}{2})T$</td>
</tr>
<tr>
<td>$S_{1212}$</td>
<td>$S_{66}$</td>
<td>$\frac{5-6v}{16(1-v)}$</td>
<td>$\frac{1}{6}Q\pi + (\pi + \frac{1}{4}I_c)R - \frac{1}{4}(p^2 + \frac{1}{2})T$</td>
</tr>
</tbody>
</table>

$$Q = \frac{3}{8\pi(1-v)}; \quad R = \frac{1-2v}{8\pi(1-v)};$$

$$I_a = \frac{2\pi p}{(p^2 - 1)^{\frac{3}{2}}} \left[ p(p^2 - 1)^{\frac{1}{2}} - \cosh^{-1}p \right];$$

$$I_c = 4\pi - 2I_a; \quad T = \frac{Q(4\pi - 3I_a)}{3(p^2 - 1)}.$$
Figure 3.1

The angles $\phi$ and $\theta$ with respect to the 1, 2 and 3 axes.

Figure 3.2

$(\phi, \theta)$ is a point in $T$ with $T = \{\phi, \theta \mid 0 \leq \phi \leq \pi, 0 \leq \theta < 2\pi\}$, and $S$ is the surface of the unit sphere. The one-to-one mapping from $T$ to $S$ is defined by the vector equation

$$r(\phi, \theta) = (x_1, x_2, x_3) = (\cos\phi, \cos\theta\sin\phi, \sin\theta\sin\phi)$$
S is a sphere of radius $a$. The area of the shaded surface element, $dS$, is the product of the length of its sides:

$$dS = (a \sin \phi \, d\theta)(a \, d\phi) = a^2 \sin \phi \, d\phi \, d\theta$$

**Figure 3.3**

A schematic diagram of the fibre preform showing the coordinate system with respect to the preform. The fibres lie in the 2-3 plane and their orientation in that plane is random and independent of $\phi$.

**Figure 3.4**
Figure 3.5

A hysteresis loop for which the derivation of permanent softening is used. The roundedness of the reverse flow curve is omitted. This figure is discussed further in chapter 6 (fig. 6.1).
Figure 3.6

(a) A cross section of a specimen which is isotropic in the 1-2 plane, before and after tensile plastic deformation in the 3 direction (schematic).

(b) A cross section of a specimen in which the fibres lie in the 2-3 plane. In tensile plastic deformation the transverse strain in the 2 direction is less than that in the 1 direction (schematic). $\varepsilon_{pc1}$ and $\varepsilon_{pc2}$ are the plastic strains in the 1 and 2 directions, respectively.

\[
\varepsilon_{pc1} = \frac{a_1 - a_0}{a_0} \quad \varepsilon_{pc2} = \frac{a_2 - a_0}{a_0}
\]
4 Materials and experimental methods

4.1 Materials and elastic properties

4.2 Composite fabrication

4.2.1 Squeeze infiltration
4.2.2 Extraction of fibre preform from composite

4.3 Polishing and etching methods

4.4 Specimen preparation

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4.5.6 Experimental arrangement and procedure for the monotonic tests

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Chapter Four  
Materials and experimental methods

This chapter contains the experimental details and information about the materials used in this work. The elastic moduli of the matrix and fibres are given in section 4.1 and the composite fabrication process is described in section 4.2. The subject of sections 4.3 and 4.4 are methods used for surface treatments and specimen preparation, respectively. The experimental methods of the monotonic and Bauschinger experiments are described in section 4.5.

4.1 Materials and elastic properties

Pure aluminium (99.98% Al) was chosen for the matrix of the composites to avoid complications due, for example, to precipitates and dispersions. The matrix in the composites was observed to be polycrystalline (see section 4.2) and hence the Young's modulus of the matrix was taken to be that of polycrystalline aluminium.

Specifications of the fibres were described in section 2.1. The Young's modulus of the fibres used in calculations in this work was chosen to be 285 GPa which is the average of the upper and lower limits given by Birchall (1983). The moduli are given in table 4.1.

The fibres are arranged in a preform in which they are held together by a silica binder, which constitutes about 5% of the mass of the preform. This gives a rigid network of fibres with a given fibre volume fraction which is strong enough to withstand the pressures during infiltration by metals. The fibres lie, on the average, in the 2-3 plane and have a random orientation in that plane. The preforms used here have a cylindrical shape with a diameter of 10 cm and a thickness of 1.5 cm (see fig. 4.1).
4.2 Composite fabrication

All the material used in this work was made in-house with the squeeze infiltration method. The various parameters of the composite fabrication are described in section 4.2.1. When polished surfaces of composites were inspected in the microscope, it became clear that several fibres were broken into small fragments. An experiment designed to find what causes the fibre breaks is described in section 4.2.2.

4.2.1 Squeeze infiltration

Squeeze infiltration was used to fabricate the composites (see section 2.2) and a schematic diagram of the casting arrangement is shown in fig 4.2. To avoid premature chilling of the melt during infiltration, the die, ram and preform are preheated before infiltration. The temperatures and other casting parameters are given in table 4.2. When preheating is complete, the aluminium is heated and melted with an RF-induction heater. Immediately after the melt has reached the required super-heat, a graphite grease is applied to the ram to avoid sticking and squirting of molten aluminium during infiltration. The preform is then put into the die, the molten aluminium introduced via the launder and the ram lowered at a speed of 9 mm/s. After infiltration is complete, the pre-selected maximum pressure is maintained for about 75 sec to allow for full solidification. The pressure is then taken off and another 75 sec allowed to elapse, before the billet is pulled out of the die (for f=23% the time intervals are longer; see table 4.2). The billet then cools down to room temperature in air. Figs. 4.3-4.7 show photographs of polished composite surfaces, with the plane of the images being (a) perpendicular and (b) parallel to the plane of the fibres (the polishing menu is given in section 4.3). One of the advantages of the squeeze infiltration method is that the level of porosity in the as-fabricated composites is very low. Although some porosity was observed within 1-2 mm of the billet surface closest to the base plate, no porosity was observed in other sections of the billet and specimens were taken well away from this area.

An etching treatment was used to reveal the matrix grain structure of the billets (see section 4.3) and the results are shown in fig. 4.8. Near the bottom surface of the billets, the grains are equiaxed (i.e., dimensions about the same in all directions). Away from the bottom surface, the grains have a columnar structure with a diameter of the order of 1 mm. The composite specimens have a gauge section diameter of 8 mm (section 4.4.1)
and they are machined from the middle of the 1.5 cm thick composite part of the billet (fig. 4.8a). Hence, the grain structure of the matrix in the composite specimens is columnar with the long axis of the grains perpendicular to the loading axis.

4.2.2 Extraction of fibre preform from composite

When polished surfaces of the composites (such as in figs. 4.3b - 4.7b) were inspected at high magnification it became clear that some of the fibres were broken into small fragments. To check whether the fibres had been broken during the squeeze infiltration process, a piece was cut from a composite (f=10%) and the matrix etched away. The etchant used was the same as given in section 4.3. The piece of the as-cast composite was left in the solution for about 30 minutes and the results are shown in fig. 4.9. Fig. 4.9a shows an as-received preform which had not been infiltrated and fig. 4.9b shows the preform which was extracted from the composite billet. These figures clearly show that there is little difference between the two preforms; the fibres in the extracted preform have not been broken up and the preform is in fact more or less intact. Therefore, it was concluded that it was not the infiltration process which causes the observed fibre breaks but the polishing of the surface prior to microscopic inspection.

4.3 Polishing and etching methods

The surfaces of all the composite specimens shown in figs. 4.3 to 4.7 were polished with a menu designed and intended for aluminium (Struers Ltd, 1985). The menu is given in table 4.3.

The process used to reveal the grain structure of the billets was begun by polishing the surfaces by hand with paper of grit 320 to 500 and finishing with a 1000 grit SiC paper. The etchant used to reveal the grain structure was as follows:

- 7 parts HCl
- 3 parts HNO₃

The mixture was used undiluted and immediately after mixing. Cotton wool was wetted in the etchant and used to wipe the surface during etching. The time taken to reveal the grain structure was about 20 sec. Care had to be taken not to over-etch the surface because then the grain structure of the composite became masked by protruding fibres.
4.4 Specimen preparation

Machining and preparation of specimens are described in this section.

4.4.1 Specimen machining

After fabrication of the billets, the first steps in machining the specimens were to cut the billet in half and to polish the surfaces to check for flaws in the composite. If no flaws were found, four composite specimens were machined from the billet. The billet was cut into four sections, each about 15 mm wide, and then the material was turned to the desired specimen shape (fig 4.10). When aluminium specimens were made (f=0%), they were cut from the unreinforced upper half of the billets, as shown in fig. 4.8a. It is clear from fig. 4.8b that the grain structure in the aluminium specimens are not exactly the same as in the composite specimens. Diamond tools were used for turning the specimens and the depth of each cut was 0.5 mm, except for the last cut which was 0.1 mm.

4.4.2 Preparation of specimens for surface inspection

In some specimens, a particularly smooth, well polished surface was required and hence, a polishing treatment had to be developed capable of giving a sufficiently smooth surface. Chemical and electropolishing methods were first attempted, but with no success because of preferential pitting at the fibres. Emphasis was then put on developing mechanical polishing methods and a Morrison polishing machine was used, of which a schematic diagram is shown in fig. 4.11. The polishing range can be controlled and was confined to the gauge section of the specimen. The specimen is rotated about 3 rpm and the rubber wheels traverse backwards and forwards longitudinally at the rate of about 5 Hz. Polishing is thus longitudinal. A polishing menu was developed by trial and error, which gave a sufficiently smooth surface. The abrasive bands used (supplied by Blackson and Kenridge Ltd. of Middlesex) came on rolls 0.5" wide, on which 15μm, 5μm or 3μm alumina particles are embedded. The final polishing menu was as follows:

<table>
<thead>
<tr>
<th>Alumina size</th>
<th>Polishing time</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 μm</td>
<td>0.5 min</td>
</tr>
<tr>
<td>5 μm</td>
<td>0.5 min</td>
</tr>
<tr>
<td>3 μm</td>
<td>1 min</td>
</tr>
</tbody>
</table>
Paraffin oil was used as a lubricant. For the experiment described in Appendix 2 the specimens were annealed for 1 hour in a glass tube at 350°C, after polishing. During annealing, a vacuum of 1.8 mbar was maintained in the tube with a rotary pump. This was done in an attempt to anneal out damage introduced to the surface during polishing and had the additional useful consequence of releasing paraffin oil which had adhered to the surface of the specimen.

4.5 Mechanical testing

The details of the monotonic and cyclic deformation experiments at room temperature and 77 K are described in this section.

4.5.1 Experimental arrangement for the Bauschinger experiments at room temperature

An Instron 1341 fatigue machine, with a 50 kN load cell and hydraulic grips, was used in the cyclic deformation (Bauschinger) experiments. A photograph of the equipment is shown in fig. 4.12. The total strain was measured directly from the specimen gauge section with a clip-on extensometer (range ±5 mm, gauge length 13.0 mm). Fig. 4.13 shows the grips and extensometer details. All the Bauschinger experiments were performed under plastic strain control (the principles are described in section 4.5.4). Details of the experimental procedure are described in section 4.5.5.

4.5.2 Experimental arrangement for the Bauschinger experiments at 77 K

A temperature of 77 K was obtained by immersing the grips, with specimen and extensometer, in a bath of liquid nitrogen. Special grips were designed for that purpose which are shown in fig. 4.14a-e. The extension bar of each grip is locked into the machines jack or crosshead hydraulic grip. Specimens are gripped by hand tightening the four bolts on each grip. Fig. 4.14d shows a specimen, with extensometer, in the grip with the liquid nitrogen container raised. The liquid nitrogen container was then lowered onto its base, the bolts tightened and a polyurethane insulating cover used to surround the nitrogen container. Liquid nitrogen was then poured into the container. Fig. 4.14e shows the arrangement during a test. The level of icing on the grips in this figure is typical.
The relative alignment of the upper and lower grips was checked regularly. This check, together with the gripping procedure, is described in the next section.

4.5.3 Grip alignment and gripping procedure

Part of the preparation work for the Bauschinger experiments was to ensure that the upper and lower grips were accurately aligned. The alignment was checked by gripping onto two straight cylindrical half bars of "silver-steel" (diameter 10 mm) and measuring both the in-plane and angular misalignment with a dial gauge which could be slid around as well as along the bars. The instrument used is shown in fig. 4.15. After quantifying the misalignment, the "pilgrim" nut on top of the crosshead was loosened and proper adjustments made using an alignment collar which is placed in the load train immediately below the crosshead and above the load cell (see fig. 4.14c). After adjustment the misalignment was as follows:

\[
\begin{align*}
\text{In-plane displacement} & : \leq 0.03 \text{ mm} \\
\text{Angular displacement} & : \leq 0.6 \text{ mm/m}
\end{align*}
\]

After the alignment had been checked, and adjusted if necessary, the specimen ends and the grips were cleaned with inhibisol. This was done to avoid slipping of the specimens during the tests.

In general usage, specimens for testing in a fatigue machine when using hydraulic grips should be gripped in position control (as opposed to load or strain control). The problem is, however, that in the process, specimens can be loaded in an uncontrolled manner in compression and the preload can be considerable. This problem was circumvented by switching from displacement control to load control as soon as possible and completing the gripping at zero load. In the room temperature tests, using the hydraulic grips, the procedure was as follows:

1. With the machine in position control, the specimen was gripped at the lower end (at the jack) with a low grip pressure.
2. The jack was moved up to the position where gripping of the other end of the specimen was to occur, the grip pressure was lowered to zero and the upper grip was "closed". The grip pressure was then slowly increased.
3. At the moment the load output became non-zero, the machine was switched to load control and in load control at zero load, the grip pressure was increased to the required value (300-400 psi was required).
This procedure allows gripping of specimens with preloads of less than 20 N. For specimens with a diameter of 8 mm this amounts to about 0.4 MPa.

In the low temperature tests the gripping procedure was easier to carry out because the tightening of the grips was done by hand and could be controlled much more accurately. Gripping could therefore be completed with almost zero preloads.

4.5.4 Extension measurements and plastic strain control

Total extension was measured directly from the specimens with clip-on extensometers (at room temperature: range ±5 mm, gauge length 13.0 mm; at 77 K: range ±2.5 mm, gauge length 12.5 mm and range ±1.0 mm, gauge length 10.0 mm). O-rings made of Nitrile rubber were used to fasten the extensometers to the specimens at room temperature (fig. 4.13a), but at 77 K, metal springs were used (fig. 4.14d). For the tests at room temperature, the extensometer was calibrated using a standard calibration device but for the low temperature test program it was necessary to calibrate the extensometers at 77 K. The arrangement for calibration in liquid nitrogen is shown in fig. 4.16. A bar and a tube connected to the calibrator (with the bar sliding inside the tube) were used to enable the extensometer to be immersed in liquid nitrogen. It was found that the extension was linear at room temperature and 77 K, but a small difference in the calibration was observed. After this had been recorded, it was sufficient to calibrate at room temperature.

In Bauschinger experiments on metals and MMCs it is very useful to be able to control the amount of plastic strain imposed on the specimen in each cycle. For that reason a plastic strain controller (PSC), compatible with the Instron 1341 machine, was commissioned (see fig. 4.17a). It is based on a design by Wilson and Robinson (1977) and a brief description of the basic principles is as follows. The total strain imposed on the specimen is the sum of elastic and plastic strains:

\[ e_t = e_{el} + e_p \]

The elastic strain is the ratio of the stress on the specimen, \( \sigma \), and the Young's modulus, \( E \). The plastic strain is thus given by

\[ e_p = e_t - \frac{1}{E} \sigma \]

or

\[ e_p = e_t - C \sigma \]

where \( C \) is the compliance of the specimen which is found and set at the beginning of a test by straining the specimen within the elastic limit. A schematic diagram of the lead connections of the PSC is shown in fig. 4.17b. The function generator of the Instron is

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bypassed and, instead, the jack is driven by a ramp signal from the PSC. Total extension is measured by the extensometer and load by the load cell. The extensometer and load signals are fed into the PSC which uses them together with the pre-set compliance to subtract electronically elastic strain from the total strain. When the pre-set plastic strain limits are reached in a half cycle, the direction of the ramp signal generated by the PSC is reversed and hence the direction of deformation is reversed.

4.5.5 Experimental procedure for the Bauschinger experiments

The description which follows applies to both the room temperature and the low temperature tests, unless otherwise described in the chapters which follow.

Before testing it was found useful to drive the jack for a few minutes with a low amplitude square wave in position control. This helped to prevent "sticking" of the jack during experiments which could occur because the total displacements are very small.

When the test was the first for a given fibre volume fraction, the compliance setting was adjusted by operating the PSC in self oscillate mode. Very small loads are then imposed on the specimen, but enough to set the compliance. The signals for plastic extension and load were at all times displayed in an oscilloscope for convenience, but the load-plastic extension hysteresis loops were recorded by hand on an X-Y chart recorder.

Two or three cycles were imposed on the specimen at each plastic strain level. At room temperature where cyclic hardening is minimal, only one loop was recorded at each strain. This was because the loops are almost identical and a measurement of the permanent softening would be difficult if all had been recorded. At 77 K, cyclic hardening is greater and 2 or 3 loops were recorded. The plastic strain amplitude was increased from about 0.05% to about 0.4%-0.5%, recording loops every 0.05% and the recording of each loop took about 7-8 seconds. Cycling was always interrupted at zero load when the plastic strain settings were changed. The direction of the initial straining at each plastic strain level was in tension normally.

4.5.6 Experimental arrangement and procedure for the monotonic tests

Monotonic tests at room temperature were performed in an Instron 1175 screw-driven machine with a 100 kN load cell. The same specimen design was used as in the Bauschinger experiments and clip-on extensometers were used to measure the extension.
The grips, fitted with V-grooved inserts, were tightened by hand. The crosshead speed was 0.2 mm/min, giving a strain rate of about $2 \times 10^{-5}$ sec$^{-1}$.

Monotonic tests at 77 K were performed in the Instron 1341 servohydraulic fatigue machine with the low temperature grips, operating in total strain control. As before, a clip-on extensometer was used to measure the extension (range ±2.5 mm, gauge length 12.5 mm) and the strain rate was about $6 \times 10^{-4}$ sec$^{-1}$. 
### Table 4.1  Elastic moduli of polycrystalline aluminium and Saffil fibres.

<table>
<thead>
<tr>
<th></th>
<th>Tensile modulus GPa</th>
<th>Shear modulus GPa</th>
<th>Poisson's ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium</td>
<td>70.0(^{(1)})</td>
<td>26.0(^{(3)})</td>
<td>0.346(^{(1)})</td>
</tr>
<tr>
<td>Saffil fibres</td>
<td>285.0(^{(2)})</td>
<td>115.6(^{(3)})</td>
<td>0.233(^{(1)})</td>
</tr>
</tbody>
</table>

2. Birchall, (1983); see section 4.1.
3. The shear modulus $\mu$ was calculated by assuming the materials are elastically isotropic: $\mu = E/(2(1+v))$. $E$ is the tensile modulus and $v$ the Poisson's ratio.

### Table 4.2  Variables in the squeeze infiltration process. The conditions were changed for the highest volume fraction material because infiltration was significantly more difficult.

<table>
<thead>
<tr>
<th>Fibre volume fraction</th>
<th>7%, 10%</th>
<th>15%, 20%</th>
<th>23%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Melt temperature(^{(1)})</td>
<td>930°C</td>
<td>1000°C</td>
<td></td>
</tr>
<tr>
<td>Preform temperature(^{(2)})</td>
<td>125°C</td>
<td>400°C</td>
<td></td>
</tr>
<tr>
<td>Die and ram temperature(^{(3)})</td>
<td>300°C</td>
<td>300°C</td>
<td></td>
</tr>
<tr>
<td>Melt mass</td>
<td>750 g</td>
<td>750 g</td>
<td></td>
</tr>
<tr>
<td>Final pressure(^{(4)})</td>
<td>31-35 MPa</td>
<td>39 MPa</td>
<td></td>
</tr>
<tr>
<td>Time of max pressure(^{(5)})</td>
<td>1¼ min</td>
<td>1¼ min</td>
<td></td>
</tr>
<tr>
<td>Zero pressure time(^{(6)})</td>
<td>1¼ min</td>
<td>5 min</td>
<td></td>
</tr>
</tbody>
</table>

1. Temperature of the melt at the moment it was poured into the die.
2. Temperature of the preform when it is put into the die.
4. Level of pressure maintained after infiltration. This is the pressure at which the aluminium solidifies.
5. Time at which final pressure is maintained.
6. Time allowed for cooling before extracting the billet out of the die.
Table 4.3 A polishing menu for aluminium which was used for all flat polishing of both aluminium and the composites (from Struers Ltd, 1985). The four grinding stages are followed by three polishing stages. The terminology is that of Struers Ltd. In polishing stages 1 and 2, DP-Mol stands for Diamond Polishing with a cloth of Mol type. In the final polishing stage, the cloth used is that for AP-Ch., which is the Alumina Paste mechanical-chemical polishing method developed by Struers and OP-S is an Oxide Polishing Suspension with 0.05\(\mu\)m abrasive particles.

<table>
<thead>
<tr>
<th></th>
<th>Grinding</th>
<th>Polishing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Base</td>
<td>SiC</td>
<td>SiC</td>
</tr>
<tr>
<td>Grain size</td>
<td>320</td>
<td>500</td>
</tr>
<tr>
<td>Lubricant</td>
<td>Water</td>
<td>Water</td>
</tr>
<tr>
<td>Speed (rpm)</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>Pressure (N)</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Time (min)</td>
<td>Till plane</td>
<td>1</td>
</tr>
</tbody>
</table>
Figure 4.1

Geometry of the Saffil fibre preforms used, showing the relative orientation of coordinate system and preform. The fibres lie on the average in the 2-3 plane and the loading axis is the 3 axis.

Figure 4.2

Schematic diagram of the squeeze infiltration apparatus. The ram has a dove-tail shaped end to facilitate extraction of billet from die after solidification. Adapted from King (1989).
Photographs taken in a light microscope of the Saffil fibre reinforced composites used in this work. The polishing menu is given in table 4.3. The plane of the images is: (a) perpendicular to the plane of the preform (i.e. the 2-3 plane); (b) parallel to the plane of the preform. The scale bar is 100 μm.
Figure 4.8a

This diagram shows a cross section of an as-fabricated billet (scale 1:1, i.e., full size) cut across a diameter. The lower, shaded half is composite and the upper half is unreinforced aluminium. The composite and aluminium specimens are cut from the billet as shown (the specimen shape is shown in fig. 4.10).

Figure 4.8b

Photograph of an as-fabricated billet which has been etched to reveal the grain structure. The fibre volume fraction is 10% and the etchant used is given in section 4.3.

Figure 4.9

(a) An SEM image of an as-received Saffil fibre preform (f=10%).
(b) An SEM image of a Saffil fibre preform which has been extracted from a composite billet (f=10%).
Radius of curvature: 25 mm

Figure 4.10
A diagram of the specimen shape used in all the mechanical tests, with dimensions given in millimetres (scale 2:1). The specimens are cylindrical.

Figure 4.11
A schematic diagram which shows how the specimen polishing machine operates. (a) Specimen end-on. (b) Side view of specimen. The rubber wheels move backwards and forwards longitudinally within the polishing range.
An overview of the apparatus. (a) The frame of the Instron 1341 fatigue machine, (b) hydraulic grips, (c) Instron control units, (d) plastic strain controller, (e) oscilloscope, (f) chart recorder.

Experimental arrangement for the room temperature tests. (a) Specimen with extensometer and (o) rubber O-rings, in the lower grip. (b) A gripped specimen ready to be tested.
Figure 4.14
(a) The grips which were designed and made for low temperature cyclic deformation experiments. The base is locked into the hydraulic grips. Gripping of the specimens is done manually by tightening the four bolts on each grip.
(b) A close-up of the lower grips. The diameter of the circular groove gripping the specimens is 10 mm and the diameter of the grips is 47 mm.

c) The grips in position on the machine. The square plate at the lower grip is the base for the liquid nitrogen container. The purpose of the plastic drip tray is to protect the lower grip from ice and water. a is the alignment collar.
(d) A specimen with extensometer gripped and ready to be tested. Metal springs (s) were used to fasten the extensometer to the specimen. The walls of the container are lowered onto the base and the bolts tightened.
(e) The grips in use. Polyurethane insulation surrounds the container. Liquid nitrogen was in the container for a maximum of about 2 hours and the icing after such a period is similar to that shown in the figure.
Figure 4.15

The equipment used for measuring the misalignment of the grips. The dial just touches the upper half bar and is free to rotate and slide up and down on the lower half bar. Any misalignment is then read directly from the scale. Accuracy is 0.01 mm.

Figure 4.16

The arrangement used for calibration of extensometers in liquid nitrogen.

Figure 4.17

(a) The plastic strain controller.
(b) A schematic diagram showing the lead connections to the plastic strain controller.
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Chapter Five

Monotonic tests

at room temperature and 77 K

5.1 Introduction

To characterize the basic flow properties of the aluminium\Saffil composites and the aluminium matrix, monotonic stress-strain curves were measured at room temperature and 77 K. This provides a basis for the Bauschinger experiments described in chapter 6, as well as giving vital information on both the internal stress state in the material and plastic flow in the transverse directions.

In section 5.2 the flow curves for the matrix and the composites are presented, showing that fibre volume fraction and temperature have a significant effect on the flow properties of the material. In section 5.3 the tensile data obtained from these tests are discussed. The flow curve of aluminium is then used to estimate the critical fibre aspect ratio, the aim being to compare it to the actual aspect ratio of the fibres. An important result of the monotonic tests is that the thermally induced residual matrix stresses can be estimated by comparing the flow curves in tension and compression and this is discussed in section 5.4. The value of the thermal stresses so obtained will then be compared to the results of the Bauschinger analysis in chapter 6.

Plastic flow in the transverse directions can be characterised by the method described in section 3.3.3. This method has been applied to specimens tested in tension at room temperature and 77 K and the results are discussed in section 5.5. The calculation of the unrelaxed mean stress hardening rate in chapter 6 will be based on the plastic strain of the composite evaluated here.

For the experimental details of these tests, see sections 4.5.6.
5.2 Flow curves at room temperature and 77 K

The presentation of these results is divided into two sections, starting with the flow properties at room temperature (section 5.2.1). The stress-strain curves at 77 K are described in section 5.2.2.

5.2.1 Flow curves at room temperature

In figs. 5.1a and b the experimental stress-strain curves at room temperature are shown in tension (fig. 5.1a) and compression (fig. 5.1b), for aluminium ($f=0\%$) and the composite (stress in these figures is load divided by the initial cross sectional area and strain is extension divided by the initial gauge length). The crosshead speed in these tests was 0.2 mm/min. In the room temperature tests, a small tensile preload was unavoidable in the gripping of the specimens. The zero for both load and strain were set before gripping and were not changed subsequently, which causes some of the compressive curves to appear to originate at a non-zero load.

Following Kelly and Lilholt (1969) the stress-strain curves in tension have been divided into three stages. To facilitate the distinction between the three stages, the slope of the tensile curves has been evaluated and is shown in fig. 5.3a. The initial slope of all the curves is high and falls rapidly as the strain is increased. At a total strain of about 0.1% the slope begins to fall at a lower rate and this has been taken as the onset of stage II. The effect of the fibre reinforcement is greatest in stage II ($0.1\% < e_t < 0.7\%$, see figs. 5.1a and 5.3a) where the fibres deform elastically and the matrix plastically. In this stage the composite hardens non-linearly at a rate which increases with fibre volume fraction (a linear work hardening would give a constant slope in fig. 5.3a). At roughly the failure strain of the fibres, 0.7%, the work hardening rate has fallen considerably for all volume fractions and this has been taken as the onset of stage III. It is likely that extensive fibre fracture occurs at strain higher than 0.7% which reduces the strengthening efficiency of the fibres. Void formation (Nutt et al., 1987) and/or fibre/matrix debonding could also contribute to the lowering of the hardening rate. For strains higher than 1.2%, the hardening rates of the composites becomes similar to that of the matrix. The division of the flow curves into three stages is shown for the tension flow curves in fig. 5.1a. No stage III is observed for the compression flow curves in fig 5.1b.

In stage I the aluminium\Saffil composite does not exhibit any clear initial linear
response in tension (fig. 5.1a). Since the thermal expansion of the matrix is about three times that of the fibres, tensile residual stresses build up in the matrix (compressive in the fibres) as the composite cools down from fabrication. This lowers the proportional limit in tension accordingly and raises it in compression (Arsenault and Taya, 1987). From this it would be expected that an estimate of the modulus would be easier to obtain in a compression test. However, even the compression curves (fig. 5.1b) do not show any clear initial linear response. The variable nature of the stress field in the matrix (Brown and Stobbs, 1971a; Withers et al., 1989) is a possible contributor to this effect. As the load on the composite is increased, stresses build up in the matrix and can give rise to considerable microplastic flow, before the mean stresses become high enough to give bulk plastic flow (Withers et al., 1989; Hamann et al., 1990).

Little difference is observed between the tensile and compressive flow curves for pure aluminium. For the composites, at a given plastic strain, the flow stress in tension is lower than in compression, in accord with the expected tensile nature of the thermal stress. The tensile and compressive curves will be used in section 5.4 to estimate the magnitude of the thermal stresses.

5.2.2 Flow curves at 77 K

The flow curves in tension at 77 K for the matrix and the composites are shown in fig. 5.2a (note that the stress scale is different from figs. 5.1a and b). It is noteworthy that the transitions between stages I, II and III are more difficult to detect than at room temperature. As with the room temperature results, the slope of the tensile flow curves was evaluated and is shown in fig. 5.3b. The transition from stage I to stage II is gradual, but can be estimated to be at a strain of 0.1%, which is the same as at room temperature. The transition between stages II and III is approximately at \( e_t = 1.1\% \), which is significantly higher than at room temperature. The reasons for this delay in onset of stage III are not clear. In stage II, the work hardening rate for the composites increases with fibre volume fraction and for each volume fraction the hardening rate is higher than at room temperature. This indicates that relaxation processes operating at room temperature are suppressed to some extent at 77 K. It is interesting to note in figs. 5.3a and 5.3b that both at room temperature and 77 K, the pure aluminium specimens work harden linearly (with a constant slope) above a total strain of about 0.4%.

The flow curves in compression are shown in fig. 5.2b. The transition between
stages I and II takes place at the same strain as in tension, at about $e_t = 0.1\%$. As for the pure aluminium specimens at room temperature, there is little difference between the flow curves in tension and compression at 77 K. For the composites, the stress at $e_t = 0.1\%$ is much higher in compression than in tension, indicating the presence of thermal stresses in the matrix. This is discussed further in section 5.4.

The tensile data extracted from these tests are discussed in the next section.

5.3 Tensile data

This section commences with a discussion of the ultimate tensile strength of the material, followed by a section on how these results can be used to estimate the critical fibre aspect ratio. The strain to failure is then discussed briefly.

5.3.1 Ultimate tensile strength

The ultimate tensile strengths of the matrix and the composites at room temperature and 77 K are shown in fig. 5.4. In the tests at 77 K, the gauge length of the extensometer used set an upper limit to the maximum strain and the specimens with $f=0\%$, $7\%$ and $10\%$ could not be taken to failure (the arrows in fig 5.4 indicate the specimens did not fail and the data points show the maximum stress in the tests). At room temperature the strength increases approximately linearly with fibre volume fraction. In fig. 5.5, the data is compared with published results and the agreement between the different sources is good.

The stress in tension for the different specimens at 2\% total strain (see figs. 5.1a and 5.2a) has been evaluated and is shown in fig. 5.6. From this figure it is apparent that in the room temperature tests, the specimens did not work harden significantly from $e_t = 2\%$ to the failure strain. However, at 77 K the material hardens considerably in most cases between $e_t = 2\%$ and the strain at which the test was either stopped, or the specimen failed.

5.3.2 Critical fibre aspect ratio

The concept of fibre reinforcement is based on a load transfer from the matrix to the fibres, both elastically and by plastic flow of the matrix. It will only be possible to break
a fibre by plastic flow of the matrix, provided the stress in it builds up to the fracture stress. For that to happen, the fibre must be longer than the critical fibre length (Kelly and Davies, 1965). In this section the critical fibre aspect ratio (the critical length divided by the fibre diameter) of Saffil fibres in a pure aluminium matrix is evaluated by using eq. 2.2 (Kelly and Davies, 1965)

\[
\frac{l_c}{d} = \frac{\sigma_{uf}}{2\tau}
\]  

(2.2)

\(\sigma_{uf}\) is the fibre ultimate strength and \(\tau\) is either the shear strength of the matrix adjacent to the interface, or the shear strength of the fibre/matrix interface, whichever is smaller. \(\tau\) is here taken to be the shear yield strength of the bulk matrix, which is assumed to be equal to half the yield stress of the matrix (Friend, 1987). From fig. 5.1a the matrix yield stress is approximately 18 MPa, which gives \(\tau = 9\) MPa. The ultimate strength of the fibres is 1500 ±500 MPa (Birchall, 1983). Eq. 2.2 then gives a critical aspect ratio of

\[
\frac{l_c}{d} = 83 \pm 28
\]

Since the fibres have an average length of 470 ±100\(\mu\)m and a diameter of 3 ±0.5\(\mu\)m (Clyne et al., 1985), their actual aspect ratio is

\[
\frac{l}{d} = 160 \pm 60
\]

Although this result should not be taken literally, particularly in view of the crudity of the assumptions made in its derivation, it is apparent that the aspect ratio of the Saffil fibres is higher than their critical aspect ratio. Given a sufficiently large plastic strain of the matrix, it is then to be expected that the fibres which are parallel to, or close to being parallel to the loading axis, are long enough to be broken. At room temperature the evidence from fig. 5.1a is that at the failure strain of the fibres, \(e_f = 0.7\%\), extensive fibre fracture occurs.

5.3.3 Composite strain to failure

All the specimens tested at room temperature were loaded to failure, except pure aluminium which was extended to a strain of 45% without breaking the specimen. At 77 K the material with \(f=0\%, 7\%\) and 10\% was not taken to failure for the reasons mentioned
above. The strains to failure (or the maximum strains) deduced from these tests are shown in fig. 5.7. Generally, the ultimate strain decreases with fibre volume fraction, except for the material with \( f=23\% \). The material with \( f=23\% \) was manufactured at slightly different conditions compared to the other material, because of the high volume fraction (see section 4.2 for parameters of fabrication). It is possible that the different manufacturing conditions are responsible for the surprisingly high strain to failure of this material, although the precise causes are not known.

We will now turn our attention to the determination of the thermal stresses, using the flow curves of the material.

5.4 Determination of thermal stresses

The difference between the flow stress in tension and compression, \( \Delta \sigma^\text{Th} \), provides an estimate of the thermal stresses (Withers et al., 1989)

\[
\frac{1}{2} \Delta \sigma^\text{Th} = \frac{1}{2}(\sigma_C^T - \sigma_{YC}) = \frac{\langle \sigma_{3>1}^T \rangle_M - \langle \sigma_{1>1}^T \rangle_M}{1 + B} \quad (5.1)
\]

This equation expresses the difference between the composite yield stress in compression and tension and is independent of the matrix yield stress (cf. eq. 3.34). Good agreement between experimental and theoretical evaluation of this quantity has been reported by Arsenault and Taya (1987).

It is not entirely clear how the transverse anisotropy of the aluminium/Saffil composite used here should be taken into account theoretically when making the predictions. The difference between the thermal stresses in the 2 and 3 directions is zero before testing, since the fibres lie in the 2-3 plane and eq. 5.1 is, for that reason, expected to express an upper limit to the experimentally evaluated thermal stress.

A simplified flow curve has been drawn in fig. 5.8 to illustrate the effect of a change in the thermal stress on the flow curves of a linear work hardening composite material. In coordinate system 1 the matrix and the inclusions feel no stress before testing (thermal stress zero) and the resulting flow curve is symmetric in tension and compression. Consequently, when the compressive curve is inverted, it falls exactly on the tension curve, giving a zero stress difference between the flow curves. At the origin of coordinate system 2, the matrix experiences a tensile stress and the inclusions are in compression. The
thermal stress in the matrix, $<\sigma^{\text{Th}}>_2$, is shown on the left. When the compressive flow curve is inverted, it falls above the tensile flow curve, with a stress difference along the elastic line (i.e. at the same value of the plastic strain) of $2<\sigma^{\text{Th}}>_2$. This is because at the origin of coordinate system 2 the matrix is in tension and the thermal stress then aids the deformation. On the other hand, an applied compressive stress first has to overcome the tensile matrix stress before the flow stress in compression is reached. In coordinate system 3, the tensile matrix stress is larger than in system 2, resulting in a larger difference between flow stresses in tension and compression. From this schematic example it is clear that as the thermal stress increases, the tensile yield stress decreases and the compressive yield stress increases.

The method adopted here to estimate the thermal stresses was to measure the stress difference, $\Delta \sigma^{\text{Th}}$, between the tensile and compressive flow curves. For each fibre volume fraction, at least one specimen was tested in tension and another in compression. The two curves thus obtained were then superimposed, with the compressive stress and strain both inverted (in other words, the absolute value of stress and strain were used for the compressive curve). The curves are given in figs. 5.9a-f (room temperature) and 5.10a-f (77 K). As an example of the composite flow curves, consider fig. 5.10b. As the stress is increased from zero, the flow in tension and compression is markedly different, but after a small strain (0.1-0.2%) the flow curves become parallel. The value of the plastic strain at which the stress difference was evaluated was chosen to be $e_p = 0.25\%$. At that strain the flow curves had become parallel. The stress difference was evaluated along the elastic line, where the tension and compression curves had seen the same plastic strain (see fig. 5.10b). The value of the modulus used to draw the elastic line for each composite was calculated with the Eshelby method and since the modulus for aluminium is not greatly temperature dependent (about 12% higher at 77 K than at room temperature), the same value was used at room temperature and 77 K (the difference in Young's modulus at room temperature and 77 K (about 12%) is not significant in the sense that it does not result in a different measurement of the thermal stresses).

The results for the measured values of the thermal stress (i.e. $\frac{1}{2}\Delta \sigma^{\text{Th}}$) are given in table 5.1 and fig. 5.11. For some volume fractions, two specimens were tested in tension and compression and the points in these cases give the maximum and minimum difference between the flow curves. Generally the reproducibility of the flow curves is very good, as can be seen in figs. 5.9d-e and 5.10c. Fig. 5.11 shows clearly that this technique produces an apparent thermal stress of about zero in the pure aluminium specimen. For the
composites (7% ≤ f ≤ 23%) it is interesting to note that the thermal stress appears to be independent of fibre volume fraction and only depends on testing temperature. The results for the composites at room temperature are scattered between 6MPa and 10MPa, and at 77 K between 15MPa and 20MPa.

Theoretically the thermal stress can be expressed by eq. 3.34 (see section 3.3.4)

\[ \frac{1}{2} \Delta \sigma_{th} = \frac{\langle \sigma_3^R M - \sigma_3^H \rangle_1}{1 + B} = \frac{[A(\alpha_M - \alpha_f)]_1}{1 + B} |\Delta T| \]  

As discussed in section 3.3.4, this equation expresses an upper limit to the matrix stress arising from a temperature change of \( \Delta T \) and does not take relaxation into account. The theoretical predictions of eq. 3.34 for the behaviour of the thermal stress are shown in fig. 5.12 for three values of the temperature drop \( \Delta T \); 70K, 150K and 250K. For all \( \Delta T \) values, the predicted thermal stress increases approximately linearly with fibre volume fraction. The theoretical predictions and the experimental results can be combined so the behaviour of \( \Delta T \) can be examined as a function of fibre volume fraction, and this is shown in fig. 5.13. This analysis shows that as the fibre volume fraction increases, the effective temperature drop \( \Delta T \) decreases (\( \Delta T \) is the difference between the "lock-on" temperature for the composite and the testing temperature). This suggests that as the fibre volume fraction increases, the extent of relaxation of the thermal stresses increases as well, because the implication is that the "lock-on" temperature has fallen with increasing fibre volume fraction both at room temperature and 77 K. The difference between \( \Delta T \) at room temperature and 77 K is about 100 K which is much smaller than the 216 K difference between room temperature and 77 K. This suggests that some relaxation of the thermal stresses takes place as the material cools down from room temperature and 77 K.

The experimental results for the thermal stresses will be compared to results obtained by a very different technique in the next chapter, which is on the Bauschinger experiments.

5.5 Determination of transverse flow parameters

Most, if not all of the work reported on the application of the Eshelby method to the development of internal stresses in composites has been carried out on unidirectional transversely isotropic materials (e.g. Brown and Clarke, 1977; Lilholt, 1977; Withers et al.,
The plastic strain for the matrix in such composites can be written as

$$e_p = (e_{p1}, e_{p2}, e_{p3}, 0, 0, 0) = e_p(-\frac{1}{2}, -\frac{1}{2}, 1, 0, 0, 0)$$

e_p is the plastic strain of the matrix in the 3 direction, and the transverse strain in the 1 and 2 directions are identical, with $e_{p1} = e_{p2}$. The aluminium\$Saffil composite is, however, not transversely isotropic, since the fibres lie in the 2-3 plane and load is applied parallel to the 3 direction. The 1 direction is then perpendicular to the plane of the fibres, whereas the 2 direction lies in the plane of the fibres. A calculation of the plastic response of the material for a given applied stress requires the entire deformation to be followed. As it is not clear what principles to use for a calculation like that, it seems futile to try to work out the plastic strain theoretically. This parameter has, therefore, always been obtained by experiment (Brown and Clarke, 1975; Pedersen, 1985b).

A method to account for the uneven transverse flow of the aluminium\$Saffil specimens was described in section 3.3.3, where the flow parameters $\eta_1$ and $\eta_2$ are expressed in terms of the plastic strain of the matrix, $e_{p1}$ and $e_{p2}$ (eq. 3.30). Eq. 3.31 is then used to express $e_p$ in terms of the plastic strain of the composite, $e_{pc}$, which can be obtained by measuring the dimensions of specimens after tensile plastic deformation. This is the method adopted here. The results of these measurements are given in table 5.2 and $\eta_1$ is shown as a function of fibre volume fraction and temperature in fig. 5.14. It is most interesting to note that $\eta_1$ seems to be independent of both fibre volume fraction and temperature (the mean value of $\eta_1$ is 0.858, and $\eta_2$ is then 0.142, see eq. 3.29). This result is somewhat surprising. At $f=0\%$ the material is transversely isotropic and $\eta_1 = \eta_2 = 0.5$. As $f$ decreases from 7\%, $\eta_1$ is then expected to drop quite rapidly from the constant value of 0.858, becoming equal to 0.5 at $f=0$.

Thus, the plastic strain of the matrix in the aluminium\$Saffil composites is given by

$$e_p = e_p(-0.858, -0.142, 1, 0, 0, 0) \quad (5.2)$$

This equation appears to be valid both at room temperature and 77 K, for a fibre volume fraction between 7\% and 23\%. From this equation it is clear that the transverse plastic strain of the matrix in the 1 direction (perpendicular to the plane of the fibres) is about 6 times greater than in the 2 direction (in the plane of the fibres).

Hall (1991) has measured the tensile properties of Al-1\%Cu matrix reinforced with
planar Saffil fibres ($f=10\%$). The observed strain of a composite taken to failure was
\[ e_{pc} = (-0.218, -0.053, 0.209, 0, 0, 0) \]
Using eq. 3.31 to calculate $e_p$ and eq. 3.30 to obtain $\eta_1$ and $\eta_2$, gives
\[ e_p = e_p(-0.809, -0.191, 1, 0, 0, 0) \] (5.3)
Although the matrix used by Hall (1991) is Al-1%Cu, the transverse flow parameters $\eta_1$ and $\eta_2$ are in good agreement with the results obtained here (see eq. 5.2 and fig. 5.14).

It is worth pointing out possible shortcomings of this analysis. Strictly speaking, it can only be used for composites in which no relaxation of internal stresses takes place, for the following reasons. The results of section 5.4 indicate that the internal stresses are able to relax to some extent. If relaxation is such that the relative proportions of $e_{pc1}$, $e_{pc2}$ and $e_{pc3}$ are distorted (for instance if relaxation is more extensive in the 2 direction than in the 1 direction) then the estimate of $\eta_1$ and $\eta_2$ will, presumably, be affected. If the relative proportions of $e_{pc1}$, $e_{pc2}$ and $e_{pc3}$ are not affected by relaxation, the error in $\eta_1$ and $\eta_2$ is expected to be small or negligible. As indicated in section 5.4, the fibre volume fraction will affect the extent of relaxation in the composite. Thermally activated relaxation processes such as diffusion and cross slip are also strongly affected by temperature. It is therefore very encouraging to note in fig. 5.14, that neither fibre volume fraction nor temperature seems to affect the estimate of $\eta_1$ and $\eta_2$. These values of $\eta_1$ and $\eta_2$ will be used in the calculations of the mean stress hardening rate, which are presented in section 6.4 of next chapter.

5.6 Summary

The main conclusions of this chapter are as follows.

1. The stress-strain curves of aluminium and aluminium\$Saffil composites have been measured at room temperature and 77 K. For a given fibre volume fraction (except $f=0\%$), the absolute value of the flow stress at any plastic strain is larger in compression than tension, which shows the existence of tensile thermal stresses in the matrix.

2. The magnitude of the thermal stresses in the composites has been estimated by comparing flow curves in tension and compression for a given fibre volume fraction. The measured thermal stress is independent of fibre volume fraction at room
temperature or 77 K and the thermal stress at 77 K is about a factor of two higher than at room temperature.

3. The measured thermal stress and the theoretical predictions of the magnitude of the thermal stress have been combined to estimate the effective temperature drop, $\Delta T$. The results show that $\Delta T$ decreases with increasing fibre volume fraction, suggesting that the extent of relaxation of the thermal stresses increases with fibre volume fraction.

4. The transverse flow parameters of the material, which describe the plastic deformation in these dimensions, were determined and found to be independent of fibre volume fraction and temperature.
Table 5.1. Thermal stress as measured from difference of flow curves in tension and compression. When more than one specimen was tested for each volume fraction in either tension or compression, two values are given for the thermal stress, representing the resulting minimum and maximum values. f is the fibre volume fraction and the estimated uncertainty in the measurement is ±1 MPa.

<table>
<thead>
<tr>
<th>f %</th>
<th>Room Temperature Thermal stress MPa (± 1 MPa)</th>
<th>77 K Thermal stress MPa (± 1 MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.6</td>
</tr>
<tr>
<td>7</td>
<td>8.5</td>
<td>17.0</td>
</tr>
<tr>
<td>10</td>
<td>6.9</td>
<td>15.1</td>
</tr>
<tr>
<td>15</td>
<td>6.9</td>
<td>18.1</td>
</tr>
<tr>
<td>20</td>
<td>6.2</td>
<td>20.7</td>
</tr>
<tr>
<td>23</td>
<td>8.8</td>
<td>18.5</td>
</tr>
</tbody>
</table>

Table 5.2. Measurements of the transverse flow parameters, \( \eta_1 \) and \( \eta_2 \). As \( \eta_1 + \eta_2 \) is always unity, only \( \eta_1 \) is given in the table. Each measurement is from a tensile test of one specimen. The mean value of all measurements (at room temperature and 77 K) is \( \eta_1 = 0.858 \pm 0.020 \) (error estimated from experimental scatter). These results are shown graphically in fig. 5.14.

<table>
<thead>
<tr>
<th>f %</th>
<th>Room Temperature ( \eta_1 )</th>
<th>77 K ( \eta_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0.859</td>
<td>0.833</td>
</tr>
<tr>
<td>10</td>
<td>0.902</td>
<td>0.854</td>
</tr>
<tr>
<td>10</td>
<td>0.838</td>
<td>0.865</td>
</tr>
<tr>
<td>15</td>
<td>0.853</td>
<td>0.847</td>
</tr>
<tr>
<td>20</td>
<td>0.878</td>
<td>0.865</td>
</tr>
<tr>
<td>23</td>
<td>--</td>
<td>0.843</td>
</tr>
</tbody>
</table>
Experimental flow curves of aluminium and aluminium/Saffil composites at room temperature in (a) tension and (b) compression. The division into stages I, II and III is obtained by using fig. 5.3a which shows the slope of the tensile curves in this figure. The specimen with $f=20\%$ tested in tension at room temperature failed at $1.2\%$ total strain.
Experimental flow curves of aluminium and aluminium\textregistered Saffil composites at 77 K in (a) tension and (b) compression. The division into stages I, II and III is obtained by using fig. 5.3b which shows the slope of the tensile curves in this figure.
Figure 5.3

The slope of the tensile curves at (a) room temperature and (b) 77 K.
Figure 5.4

Measured ultimate tensile strength at room temperature and 77 K. Arrows denote the specimens did not fail and the data-points show the maximum stress for these specimens.

Figure 5.5

Measured ultimate tensile strength shown with literature data.
Figure 5.6

Measured ultimate tensile strength shown with the stress at a total strain of $e_t = 2\%$. The composites with $f=20\%$, tested at room temperature, failed at $e_t = 1.2\%$.

Figure 5.7

Measured composite strain to failure in tension. Arrows denote the specimens did not fail and the points in these cases show the maximum strain.
Figure 5.8

A schematic flow curve of a composite in tension and compression. The thermal stress in coordinate system 1 is zero. In coordinate system 2 the thermal stress is $<\sigma>_{T2}$ and in system 3 it is $<\sigma>_{T3}$. 
Figure 5.9
Experimental stress strain curves at room temperature in tension and compression. Figs. 5.9d-e give an idea of the reproducibility of the flow curves.
Figure 5.10

Experimental stress strain curves at 77 K in tension and compression. Fig. 5.10c gives an idea of the reproducibility of the flow curves. Fig. 5.10b show how the thermal stress is measured.
Figure 5.11
The thermal stress measured from comparison of flow curves in tension and compression.

Figure 5.12
Thermal stress calculated by using eq. 3.34 for three values of the temperature $\Delta T$. These curves are expected to be an upper limit for the thermal stress in unrelaxed aluminium/Saffil composites for the given temperature drop.
The temperature drop $\Delta T$ for each volume fraction, obtained by combining the experimental results of fig. 5.11 and the theoretical prediction of fig. 5.12.

Measurements of the transverse flow parameters $\eta_1$. This parameter appears to be independent of both fibre volume fraction and temperature. The plastic strain of the matrix in the composites at both room temperature and 77 K and for fibre volume fractions between 7% and 23% can then be written as

$$e_p = e_p(-\eta_1, -\eta_2, 1, 0, 0, 0)$$

where $\eta_1$ is the mean value of the measurements, 0.858 ($\eta_2$ is then equal to 0.142).
6 Bauschinger experiments at room temperature and 77 K

6.1 Introduction

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6.2.1 Behaviour of a hypothetical unrelaxed composite
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6.3 The behaviour of the thermal stress at room temperature and 77 K
6.3.1 Experimental procedure and examples of hysteresis loops
6.3.2 The thermal stress at room temperature and 77 K
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6.4 The behaviour of the mean stress at room temperature and 77 K
6.4.1 The mean stress at room temperature and 77 K
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Chapter Six

Bauschinger experiments

at room temperature and 77 K

6.1 Introduction

The monotonic flow properties of the material were characterized in chapter 5, and attention is now turned to cyclic deformation (Bauschinger) experiments and their role in the analysis of internal stresses. The primary aim of the experiments is to measure the mean stress hardening rate of the composites (the rate at which the mean stress increases with plastic strain) and compare experimental results with theoretical predictions.

The analysis of the internal stresses used here is based on the Orowan-Wilson construction of the permanent softening, \( \Delta \sigma \) (Orowan, 1959; Wilson, 1965; Atkinson et al., 1974; Pedersen, 1990). In the early experimental work (Wilson, 1965; Atkinson et al., 1974) the permanent softening was related to the mean stress in the matrix, but Lilholt (1977), working on continuous fibre reinforced copper, showed that the magnitude of \( \Delta \sigma \) is also affected by the thermal stress. Because the analysis for obtaining \( \Delta \sigma \) from the hysteresis loops has always been carried out using tension as forward and compression as reverse (Clarke and Lilholt, 1975; Lilholt, 1977; an example of this construction is given by Pedersen, 1989, 1990), it is appropriate to use \( t \) to denote tension forward, \( \Delta \sigma^t \). The permanent softening can then be written as (cf. Pedersen, 1989, 1990)

\[
\Delta \sigma^t = 2\langle \sigma \rangle^M - 2\langle \sigma \rangle^Th
\]  

\( \langle \sigma \rangle^M \) is the mean stress and \( \langle \sigma \rangle^Th \) the thermal stress (here, the triaxiality of the stress field is ignored for simplicity, cf. eq. 3.18). The procedure used to experimentally evaluate the mean stress hardening rate for a given specimen is to measure the permanent softening \( \Delta \sigma^t \) at several plastic strain levels, plot \( \Delta \sigma^t \) versus plastic strain and use the initial slope of the curve to estimate the hardening rate (Clarke and Lilholt, 1975; Lilholt, 1977; Pedersen, 1989, 1990). The permanent softening (\( \Delta \sigma^t \)) in the copper-tungsten composites is history
independent, i.e., for a given plastic strain, $\Delta \sigma^t$ does not change in the course of the experiment (Lilholt, 1977). This means that the thermal stress, $<\sigma>^{Th}$ in eq. 6.1, is also history independent, and the mean stress hardening rate can be measured unambiguously by evaluating $\Delta \sigma^t$. However, after the first experiments on the aluminium\Sa compozites had been carried out in this work, it became clear that the thermal stress changes with cycling (see section 6.2 and 6.3). This can affect the slope of $\Delta \sigma^t$ versus plastic strain and, therefore, the estimate of the mean stress hardening rate. Hence, the conventional method of using $\Delta \sigma^t$ for experimental evaluation of the hardening rate, cannot be used. At this stage it was realised that the compression part of the hysteresis loops can also be taken as a forward direction, enabling $\Delta \sigma^c$ (c to denote compression forward) to be evaluated and used to extract information about the internal stresses from the hysteresis loops (Johannesson et al., 1991). The theoretical foundation of using both $\Delta \sigma^t$ and $\Delta \sigma^c$ in the analysis of the Bauschinger experiment was discussed in section 3.3, where it was shown how the different dependencies of the plastically and thermally induced mean stresses on the direction of straining enable a separation of these two stress components to be made. This modification of the Orowan-Wilson construction has two important implications for the analysis of internal stresses carried out here. Firstly, it allows for an unambiguous measurement of the mean stress hardening rate, even if the thermal stress changes in the course of the experiment. Secondly, it enables the thermal stress to be evaluated for each hysteresis loop and, therefore, makes it possible to follow its development throughout the experiment. Thus, while the primary aim of this chapter is to measure the mean stress hardening rate, the experiments provide an opportunity for experimental substantiation of the hypothesis put forward in section 3.3 about the separation of the plastically and thermally induced mean stresses.

To clarify the salient features of this proposed analysis of the Bauschinger experiments, the predictions of the theory are compared with the behaviour of a real composite in section 6.2, underlining the advantages of combining measurements of $\Delta \sigma^t$ and $\Delta \sigma^c$. In section 6.3 the analysis of the thermal stress is applied to results of Bauschinger experiments carried out at room temperature and 77 K. The value of the initial thermal stress in the material is estimated and compared to the results of section 5.4 obtained using monotonic tests. This comparison is an important test of the validity of the proposed separation of the plastically and thermally induced mean stresses. The results of the Bauschinger experiment are analysed to obtain the mean stress values in section 6.4, and the experimental mean stress hardening rate is compared to theoretical predictions in that
The parameter $\frac{1}{4}(\Delta \sigma^c + \Delta \sigma^t)$ will hereafter be referred to as the mean stress (and sometimes as the mean stress term) and the parameter $\frac{1}{4}(\Delta \sigma^c - \Delta \sigma^t)$ as the thermal stress (term).

### 6.2 Separation of the mean stress and the thermal stress

In this section the analysis proposed in section 3.3 of the Bauschinger experiment will be demonstrated. The predictions of the theory for a hypothetical unrelaxed composite are discussed in section 6.2.1, and the analysis is then applied to results of Bauschinger experiments on a (real) composite specimen in section 6.2.2.

#### 6.2.1 Behaviour of a hypothetical unrelaxed composite

The idealised hysteresis loop, for which the theoretical derivation of $\Delta \sigma^t$ and $\Delta \sigma^c$ is strictly valid is shown in fig. 6.1. At reversal of loading, the "roundedness" of real loops is omitted (see fig. 6.3 for an example of a real loop), and after the material begins to flow plastically in the reverse direction, it flows at a constant hardening rate which is equal in tension and compression. To evaluate $\Delta \sigma^c$ and $\Delta \sigma^t$, the linear work hardening curves are extended to higher absolute plastic strains (curves 1 and 2). Curves 1 and 2 are then mirrored through the points at $-e_p$ and $+e_p$ respectively, giving curves 1' and 2'. $\Delta \sigma^t$ and $\Delta \sigma^c$ are then evaluated as the stress difference between the extended parallel flow curves, as shown in fig. 6.1. The expressions for $\Delta \sigma^t$ and $\Delta \sigma^c$, are given by eqs. 3.18 and 3.19

\[
\Delta \sigma^t = 2 \left[ \frac{<\sigma_3^P_M - <\sigma_{1,2}^P_M>}{1 + B} - \frac{<\sigma_3^{Th}_M - <\sigma_1^{Th}_M>}{1 + B} \right] 
\]  
(3.18)

\[
\Delta \sigma^c = 2 \left[ \frac{<\sigma_3^P_M - <\sigma_{1,2}^P_M>}{1 + B} + \frac{<\sigma_3^{Th}_M - <\sigma_1^{Th}_M>}{1 + B} \right] 
\]  
(3.19)

Using these expressions to separate the mean stress and the thermal stress gives

\[
\frac{1}{4}(\Delta \sigma^c + \Delta \sigma^t) = \frac{<\sigma_3^P_M - <\sigma_{1,2}^P_M>}{1 + B} 
\]  
(3.20)

\[
\frac{1}{4}(\Delta \sigma^c - \Delta \sigma^t) = \frac{<\sigma_3^{Th}_M - <\sigma_1^{Th}_M>}{1 + B} 
\]  
(3.21)
The effect of the thermal stress on the hysteresis loop now becomes transparent. According to eq. 3.21, $\Delta \sigma^i = \Delta \sigma^o$ for a material with no thermal stress. This means that the centre of the hysteresis loop, $A$, would be exactly at the origin of the coordinate system, $O$, in fig. 6.1. However, if the material has a tensile thermal stress, the whole loop is shifted downwards, with $\Delta \sigma^i < \Delta \sigma^o$, as shown in fig. 6.1 (the shift is upwards if the thermal stress is compressive, and then $\Delta \sigma^i > \Delta \sigma^o$; a residual stress caused by plastic deformation prior to testing will also result in an asymmetric loop).

The functional dependence of the permanent softening on plastic strain for the hypothetical unrelaxed composite is shown in fig. 6.2. Because the thermal stress in the ideal material is independent of plastic strain, the predictions for $\Delta \sigma^i$ and $\Delta \sigma^o$ are linear in plastic strain. If the thermal stress in the matrix, $<\sigma>^{Th}$, is tensile and has the value $\lambda$, then $\Delta \sigma^o > \Delta \sigma^i$ and the intercept for $\Delta \sigma^o$ at zero plastic strain is $+2\lambda$, with $-2\lambda$ for $\Delta \sigma^i$. If, on the other hand, the thermal stress is zero before testing, then $\Delta \sigma^i = \Delta \sigma^o$ and the intercept is zero. The magnitude of the thermal stress can also be evaluated from the difference between $\Delta \sigma^o$ and $\Delta \sigma^i$ (eq. 3.21) and is independent of plastic strain. Should the thermal stress decrease in the course of a series of Bauschinger experiments, then the curves for $\Delta \sigma^i$ and $\Delta \sigma^o$ would approach each other. The mean stress is half the mean value of $\Delta \sigma^i$ and $\Delta \sigma^o$ (eq. 3.20), and is independent of the thermal stress $<\sigma>^{Th}$. In this hypothetical composite, the mean stress is linear in plastic strain and equal to zero at $e_{pc} = 0$.

Having considered the analysis of the hypothetical unrelaxed composite, we now turn to the results of Bauschinger experiments on a real composite specimen.

6.2.2 Behaviour of a real composite

For demonstration purposes, one of the Bauschinger experiments of this chapter will be taken as an example and described in detail. This particular experiment was carried out at room temperature on specimen RT105 with fibre volume fraction $f=10\%$ (RT stands for Room Temperature and 105 means $f=10\%$, specimen number 5). The experiment was divided into three stages. In stage I, hysteresis loops were recorded at eight plastic strain amplitudes between $e_{pc} = 0.05\%$ and $0.5\%$, beginning at $e_{pc} = 0.05\%$ and increasing the plastic strain amplitude to $0.5\%$ (in all the cyclic tests reported in this thesis, the plastic strain amplitude imposed on the specimens was the same in tension and compression). To measure the mean stress hardening rate it is sufficient to carry out stage I of the
experiment. Specimen RT105 was tested further to investigate the effect of cycling on $\Delta \sigma^t$ and $\Delta \sigma^c$. In stage II the specimen was cycled for 465 cycles at the constant plastic strain amplitude of $e_{pe} = 0.23\%$. Finally, in stage III, the loops were recorded again as in stage I, at roughly the same eight plastic strain levels. In these tests the first quarter cycle at each plastic strain amplitude was always in tension, and in stages I and III, three cycles were carried out at each plastic strain level and the second recorded. Fig. 6.3 gives an example of a hysteresis loop and shows how $\Delta \sigma^t$ and $\Delta \sigma^c$ are evaluated. The loop is from stage I of the experiment, with $e_{pe} = 0.23\%$. The figure shows that the real hysteresis loops differ from the ideal loops (see fig. 6.1) in two respects. Firstly, after load reversal, the reverse flow curves are round. Only after a strain in the reverse direction which is similar to the plastic strain amplitude do the flow curves become approximately parallel. The forward and reverse direction cannot be distinguished since each half of the loop (separated by the stress maxima) is at the same time both "forward" and "reverse". Each half inherits a "roundedness" from the previous half cycle. The other difference between the ideal and real hysteresis loops is that the work hardening rate of the real material is always observed to be slightly higher in compression than in tension. Consequently, the two lines drawn to evaluate the permanent softening are not exactly parallel. The method used to obtain $\Delta \sigma^t$ and $\Delta \sigma^c$ is shown in fig. 6.3. The physical mechanism behind this difference in hardening rate is unclear.

The analysis of the results of the Bauschinger experiments on specimen RT105 is given in fig. 6.4 (a-d). The permanent softening $\Delta \sigma^t$ and $\Delta \sigma^c$ from stage I are shown in fig. 6.4a. The similarities of these results with fig. 6.2 are that $\Delta \sigma^t$ and $\Delta \sigma^c$ increase with plastic strain, and that $\Delta \sigma^c$ is larger than $\Delta \sigma^t$, indicating a tensile thermal stress in the matrix. The observed tensile nature of the thermal stress is in accord with the results of the monotonic tests of section 6.4. In contrast to fig. 6.2, the slope of $\Delta \sigma^t$ appears to decrease with increasing plastic strain, and the intercepts of $\Delta \sigma^t$ and $\Delta \sigma^c$ with the vertical axis (extrapolating the curves to zero plastic strain) are not symmetric about the origin. In other words, the absolute values of the intercepts are not equal, with the $\Delta \sigma^c$ intercept being larger. After cycling at an intermediate plastic strain in stage II, the loops were recorded again (stage III). Fig. 6.4b shows the results for $\Delta \sigma^t$ and $\Delta \sigma^c$ for stage I and stage III. The effect of cycling the specimen during stages I and II becomes clear in stage III, where the spacing between the results for $\Delta \sigma^c$ and $\Delta \sigma^t$ is reduced. The absolute values of the 1 Slip bands were observed on the surface of some specimens after cyclic deformation. A preliminary experiment designed to establish whether these slip bands are persistent slip bands or not is described in Appendix 2.
intercepts have also decreased and remain unequal. As discussed in section 6.2.1, the smaller spacing between $\Delta \sigma^c$ and $\Delta \sigma^l$ in stage III means that the magnitude of the thermal stress is smaller in stage III than in stage I (cf. eq. 3.21). Thus, the cycling of the specimen in stages I and II appears to have resulted in a reduction of the thermal stress.

The analysis for the separation of the mean stress and the thermal stress developed in section 3.3 can now be applied to these results. Fig. 6.4c shows the parameter $\frac{1}{4}(\Delta \sigma^c + \Delta \sigma^l)$ for stage I and stage III. This parameter is expected to represent the mean stress and the theory predicts it to be linear in plastic strain, and to be zero at zero plastic strain (eq. 3.20). Fig. 6.4c shows that this parameter increases approximately linearly with plastic strain. What is particularly interesting is that this parameter gives the same results in stages I and III, in spite of the shift in both $\Delta \sigma^l$ and $\Delta \sigma^c$. This indicates that the parameter $\frac{1}{4}(\Delta \sigma^c + \Delta \sigma^l)$ is independent of the thermal stress, as predicted. However, as is evident from fig. 6.4c, extrapolating the results to zero plastic strain gives an intercept of about 4-5 MPa. This is a consequence of the asymmetry of the $\Delta \sigma^l$ and $\Delta \sigma^c$ intercepts. If the mean stress (as represented by the parameter $\frac{1}{4}(\Delta \sigma^c + \Delta \sigma^l)$) is independent of the thermal stress (as eq. 3.20 shows, and as appears to be supported by the experiment), the physical meaning of the non-zero intercept is not clear. The exact relation between the absolute value of the mean stress term ($\frac{1}{4}(\Delta \sigma^c + \Delta \sigma^l)$) and the actual mean stress in the matrix is, therefore, not known. Although desirable, this is not absolutely necessary since a quantitative comparison of theoretical and experimental results will be made in terms of the mean stress hardening rate, which is not affected by uncertainties about the absolute magnitude.

Fig. 6.4d shows the difference $\frac{1}{4}(\Delta \sigma^c - \Delta \sigma^l)$ for stage I and stage III. According to eq. 3.21, this parameter represents the thermal stress in the matrix. Its initial value in the material can be estimated by extrapolating the results for stage I to zero plastic strain. This gives a thermal stress of about 12-13 MPa. It is interesting to note that the term decreases slightly with cycling in stage I, suggesting a continuous reduction in the thermal stress with cycling. In stage III, the thermal stress appears to be constant at about 4-5 MPa. Thus, the cycling of the specimen appears to have resulted in its reduction, from about 12-13 MPa to about 4-5 MPa. The intercepts of $\Delta \sigma^c$ and $\Delta \sigma^l$ in figs. 6.4a,b are not equal in magnitude and hence the correlation of the absolute value of the thermal stress term $\frac{1}{4}(\Delta \sigma^c - \Delta \sigma^l)$ and the thermal stress is questionable. In section 6.3.4 the present analysis of the thermal stress is compared with the earlier measurements of section 5.4.

The continuous decrease in the thermal stress during stage I of the experiment (fig. 6.4d) has serious implications for the estimate of the mean stress hardening rate from $\Delta \sigma^c$.
or $\Delta \sigma^t$ alone (Clarke and Lilholt, 1975; Lilholt, 1977; Pedersen, 1990). Eqs. 3.18 and 3.19 show that a change of the thermal stress during a series of Bauschinger experiments can affect the slope of $\Delta \sigma^t$ (and $\Delta \sigma^r$) versus plastic strain, and thereby, the estimate of the mean stress hardening rate. Hence, the technique developed here to separate the mean stress and the thermal stress is crucial for a reliable measurement of the mean stress hardening rate in the aluminium\Saffil composites.

The above discussion can be summarised in the following way.

1. The parameter $\frac{1}{4}(\Delta \sigma^e + \Delta \sigma^t)$ behaves in a way in which the mean stress is expected to behave. It is independent of the thermal stress and increases approximately linearly with plastic strain, especially at low plastic strains. The non-zero intercept of the mean stress at zero plastic strain casts some doubt on its absolute value.

2. The parameter $\frac{1}{4}(\Delta \sigma^e - \Delta \sigma^t)$ appears to represent the thermal stress and its initial value is about 12-13 MPa. However, the absolute values of the intercepts of $\Delta \sigma^t$ and $\Delta \sigma^e$ are not equal and hence the relation of the magnitude of this term to the thermal stress is not clear.

3. The thermal stress in these composites appears to decrease with cycling during Bauschinger experiments. A separation of the mean and thermal stresses is therefore essential for the measurement of the mean stress hardening rate.

The phrasing of points 1. and 2. is deliberately cautious, since an independent method of measuring the internal stresses is needed in order to verify the analysis beyond doubt.

Comprehensive Bauschinger experiments have been carried out on the aluminium matrix and the aluminium\Saffil composites (7% $\leq f \leq 23\%$) at room temperature and 77 K. The analysis of the thermal stress is presented in the following section and the analysis for the mean stress will be presented in section 6.4.
6.3 The behaviour of the thermal stress at room temperature and 77 K

In this section the behaviour of the parameter $\frac{1}{4}(\Delta \sigma - \Delta \sigma^b)$ in the Bauschinger experiment will be studied. This parameter is expected to represent the thermal stress in the matrix and special attention will be paid to its initial value and its development with cycling. In section 6.3.1 the experimental procedure is described and examples of hysteresis loops are shown. The results are then presented in section 6.3.2. To check if this term behaves as the thermal stress is expected to behave, some experiments are described in section 6.3.3 in which the experimental procedure is varied. In section 6.3.4 the magnitude of this parameter before testing is estimated and the results compared to the thermal stress measured in section 5.4. A summary of the results is given in section 6.3.5.

6.3.1 Experimental procedure and examples of hysteresis loops

The testing procedure in the Bauschinger experiments was as follows. The test was begun at the lowest plastic strain (about 0.05%), with the specimen cycled for three cycles and the third recorded (the second of three cycles was recorded in the experiment described in section 6.2.2; it was found that the results are independent of whether the second or third cycle is recorded). The plastic strain was then successively increased to between 0.4% to 0.5%, with loops recorded at every 0.05% increment (part I). In most of the experiments, loops were then recorded again (part II), but in reverse order compared to part I, ending at the smallest plastic strain amplitude (N.B. with no intermediate cycling between parts I and II). In some cases a third series was recorded (part III) at increasing plastic strains. The procedure used will be indicated for each of the results discussed (note that the procedure described here - parts I, II and III - is not the same as described in section 6.2.2 - stages I, II and III). The first half cycle at each plastic strain amplitude was in tension (except for specimen RT106, see sections 6.3.3 and 6.4.3). In all the Bauschinger experiments described here, the absolute value of the plastic strain amplitude imposed on the specimens was the same in tension and compression.

Examples of the hysteresis loops are given in figs. 6.5a-d. The loops are not symmetric about the horizontal axis and the maximum load was always greater in compression than in tension. The work hardening rate as measured directly from the loops at maximum plastic strain in tension and compression was observed to be slightly greater in compression than in tension.
6.3.2 The thermal stress at room temperature and 77 K

Figs. 6.6a-f and 6.7a-f show examples of the thermal stress, $\frac{1}{2}(\Delta \sigma^0 - \Delta \sigma^1)$, as evaluated from the results of the Bauschinger experiments at room temperature and 77 K respectively. The general features of the results are as follows. For pure aluminium the value can be considered to be close to zero, both at room temperature and 77 K (figs. 6.6a and 6.7a). On the other hand, for the composites at room temperature, the magnitude is much larger than for the aluminium specimen and the term decreases gradually in part I (figs. 6.6b-f). In parts II and III it is roughly constant (figs. 6.6b-f). The net reduction of the thermal stress in the experiments is generally about 5-6 MPa. At 77 K some notable differences in the behaviour of the composites emerge. The most important is that the magnitude of the term at the beginning of the tests is about a factor of two higher than at room temperature. This is a strong indication that the term is related to the thermal stress. The gradual reduction in part I is also smaller at 77 K and the net decrease appears to increase with fibre volume fraction. The experimental scatter is larger at 77 K than at room temperature, which could partly be due to a coarser load scale used at 77 K. Examples of $\Delta \sigma^1$, $\Delta \sigma^0$ and the estimated uncertainties in the measured values are given in table 6.2a and 6.2b.

6.3.3 History dependence of the thermal stress

One way of clarifying how the parameter $\frac{1}{2}(\Delta \sigma^0 - \Delta \sigma^1)$ is related to the thermal stress is to vary the procedure of the Bauschinger experiment, monitor the behaviour of the term and see if it complies with the expected behaviour of the thermal stress. Four such experiments are described below. All the experiments were performed at room temperature on specimens with fibre volume fraction $f=10\%$.

i) Starting the tests at a high plastic strain

It was shown in the last section that the thermal stress term decreases with cycling. Intuitively it would be expected that the rate of decrease of the thermal stress is, to a first approximation, controlled by the cumulative plastic strain imposed on the specimen. This would imply that by reversing the standard procedure of the Bauschinger experiment, that is, beginning the test at the highest plastic strain and successively decreasing it to the lowest
level, the thermal stress would decrease faster than if the standard procedure is used. This experiment has been carried out and the results, which confirm the expectations, are shown in fig 6.8. The first loop was recorded at $e_{pc} = 0.4\%$, giving a thermal stress of about 7 MPa. This is slightly lower than usually observed for the first point when the standard procedure is used. The plastic strain was then decreased to $e_{pc} = 0.35\%$, and already the thermal stress has fallen to the level at which it remains for the rest of the experiment (i.e., the plastic strain amplitude was successively reduced to 0.05\% and then increased to 0.5\%, recording loops at every 0.05\%).

ii) Effect of cycling on the thermal stress

As has been shown above, the thermal stress decreases with cycling, and it might therefore be expected that the reduction of the thermal stress in the material would continue with more extensive cycling. In other words, to what extent can the thermal stress be reduced by cycling? To answer this question, a specimen (RT108) was cycled in stage I by using the standard procedure of beginning at the lowest plastic strain amplitude and proceeding from there to the highest level. The plastic strain was then reduced to an intermediate level (0.3\%) and the specimen cycled for 550 cycles at that level (stage II). Hysteresis loops were recorded at regular intervals in stage II. Finally, the procedure of stage I was repeated (stage III; the three stages of this experiment are the same as in the experiment described in section 6.2.2).

The results for the thermal stress are shown in fig. 6.9. The thermal stress decreases gradually in stage I and for the first cycle at the intermediate stage it is roughly at the same level as at the end of stage I (about 8 MPa). In cycle 105 of the intermediate stage it has decreased to about 5 MPa, in cycle 237 to about 4.5 MPa and finally in cycle 550 to 4 MPa. The crosses show the level of the term in stage III, after the intermediate cycling. The reduction of the thermal stress in stage I and then in the intermediate stage is, qualitatively, in accord with what was expected. The magnitude of the term in the final stage of the test is also lower than generally observed in figs. 6.6 and 6.7 (where there was no intermediate cycling). Thus, this experiment has shown that extensive cycling of the specimen reduces the thermal stress further, but does not reduce it below 2-4 MPa. Withers (1988) and Withers et al. (1989) have suggested that complete relaxation is unlikely to occur as a result of plastic relaxation; the relaxation process stops in a state where the driving and opposing forces (e.g. friction stress) are in equilibrium.
iii) Compression as the forward stroke

One of the predictions of the theory is that the magnitude of the mean stress in the matrix is independent of the direction of straining. This means that the permanent softening, and therefore the thermal stress term, will not be affected by the direction of the initial straining at each plastic strain level. To check this, an experiment was carried out by straining the specimen at each plastic strain level first in compression (as opposed to tension first, as done in all the other tests). As can be seen from fig. 6.10, the term decreases with cycling very much the same way as generally observed. Hence, the conclusion of this experiment is that the direction of the initial straining does not affect the behaviour of the thermal stress.

iv) Annealing experiment

The purpose of the experiment described here was to check whether the cycling in part I of the Bauchinger experiment causes irreversible damage to the specimen, such as voiding, fibre/matrix debonding etc. (parts I and II of the annealing experiment are the same as described in section 6.3.1). Any such irreversible damage is expected to lead to a reduction in the thermal stress, the mean stress and the peak stresses of the hysteresis loops in subsequent testing of the specimen.

A specimen (RT107) was annealed for 1 hour at 350°C and water quenched. Immediately after the quench (i.e., 5-10 mins. later) the specimen was taken to the Instron and stages I and II of the Bauchinger experiment carried out. After this, the specimen was annealed and water quenched, and mechanically tested using the same procedure as before. The results of the experiment are given in fig. 6.11. After the first anneal and water quench, the thermal stress is reduced in stage I to about 5 MPa, and remains roughly at that value at decreasing plastic strain amplitude in part II. After the second heat treatment, the initial value of the thermal stress is recovered and the behaviour of the thermal stress term is the same after first and second annealing treatments. This demonstrates that the reduction in the thermal stress caused by the cycling can be reversed by annealing and quenching. Hence, the reduction is not caused by irreversible damage such as mentioned above. This experiment also reinforces the hypothesis that the thermal stress term represents the thermal stress in the matrix, since it is to be expected that annealing and quenching will restore the thermal stress in the matrix.
If the values in fig. 6.11 are extrapolated to zero plastic strain, an estimate can be made of the initial thermal stress in the matrix (this is done in the next section, 6.3.4, for all the specimens). The initial thermal stress in the annealing experiments of 11-12 MPa is about 2-4 MPa higher than for specimens in other tests. In the annealing experiments the specimen was tested only 5-10 mins. after being annealed and water quenched, compared to a few months in other tests between casting (i.e., manufacture) and testing. This indicates that once the material has cooled down after manufacture, the thermal stress can relax by about 2-4 MPa over a period of a few months, or that gradual cooling enables more relaxation to occur. Withers (1988) has measured relaxation times for the thermal stress in a commercially pure aluminium reinforced with SiC whiskers (0.5μm diameter, 3μm long). At room temperature the time taken to reduce the thermal stress by 63% was 500 mins. and complete relaxation was not achieved, even after several months at room temperature.

As mentioned above, the initial value of the thermal stress can be estimated from the results of the Bauschinger experiment. These results are presented in the next section.

6.3.4 Comparison of thermal stresses measured in Bauschinger and monotonic tests

The initial value of the thermal stress in the Bauschinger experiments is estimated by extrapolating curves such as in figs. 6.6 and 6.7 (part I) to zero plastic strain. The results are given in fig. 6.12 and table 6.1a and 6.1b. For pure aluminium the value obtained is about zero, as expected. For the composites, the thermal stress increases slowly with fibre volume fraction, both at room temperature and 77 K.

By comparing the Bauschinger results of fig. 6.12 to the results obtained by monotonic testing (section 5.4), it is possible to further test the hypothesis that the parameter \( \frac{1}{4}(\Delta \sigma^o - \Delta \sigma^t) \) represents the thermal stress. This comparison is shown in fig. 6.13. For all the specimens, there is reasonable agreement between the two methods. For the higher volume fraction material, the values from the Bauschinger experiments are slightly higher than those obtained in the monotonic tests.

Thus, the conclusion of this comparison is that the parameter \( \frac{1}{4}(\Delta \sigma^o - \Delta \sigma^t) \) is identical to the thermal stress. This conclusion is, however, based on the integrity of the estimates of the thermal stress in section 5.4. The possibility cannot be ruled out that both the monotonic and Bauschinger tests are affected by relaxation during loading (Withers et al., 1989).
6.3.5 Summary

In summary, the main conclusions of this section are as follows.

1. The analysis of the thermal stress proposed in section 3.3 has been verified by application to results of Bauschinger experiments on aluminium/Saffil composites.
2. The thermal stress term represents the thermal stress in the material. This is supported by quantitative agreement with the estimates of the thermal stress obtained in monotonic tests and the qualitative observations of the general behaviour of the thermal stress term in a range of experiments.
3. The thermal stress is reduced by cyclic deformation. The reduction is not caused by irreversible damage, such as voiding or fibre/matrix debonding. Extensive cycling can further reduce the thermal stress, but complete relaxation is not achieved.
4. The thermal stress can be restored to its initial magnitude by annealing and quenching the specimen.

Attention is now turned to analysis of the mean stress.

6.4 The behaviour of the mean stress at room temperature and 77 K

The Bauschinger experiments have been analysed in accordance with the theory that the mean stress is represented by the parameter $\frac{1}{4}(\Delta \sigma^c + \Delta \sigma^d)$ and the results of this analysis are presented in section 6.4.1. The experimental results for the mean stress hardening rate are compared with theoretical predictions in section 6.4.2 and the history dependence of the mean stress is discussed in section 6.4.3. A summary of the results is given in section 6.4.4.

6.4.1 The mean stress at room temperature and 77 K

The results of the analysis for the mean stress as a function of the plastic strain amplitude are given in figs. 6.14a-b (room temperature) and 6.15a-b (77 K). Beginning with the room temperature tests (fig. 6.14a-b), it can be seen that the mean stress in the pure aluminium specimen ($f=0\%$) is independent of plastic strain (zero mean stress hardening rate) and the
mean stress hardening rate for the composites increases with fibre volume fraction. The curves for the lower volume fraction material appear to be linear at low plastic strains (up to 0.2% to 0.3%), but over the complete plastic strain range the curves are generally not linear. The low temperature results are shown in fig. 6.15a-b. The mean stress in the pure aluminium specimens is again independent of plastic strain, with a slightly higher value than at room temperature. The mean stress hardening rate increases with fibre volume fraction, as at room temperature. Generally the curves are again non-linear over the entire plastic strain range, but appear to be linear for \( e_{pc} < 0.2\% \).

It is interesting to note that all the curves in figs. 6.14a-b and 6.15a-b have in common that when extrapolated to zero plastic strain, they originate at a non-zero stress. As discussed in section 6.2.2, the physical meaning of this non-zero intercept is not clear because the mean stress is expected to be zero at zero plastic strain. In an attempt to throw light on this result, the intercepts were plotted as a function of fibre volume fraction (fig. 6.16). The intercepts increase with fibre volume fraction but appear to be independent of temperature. A possible qualitative interpretation of the fibre volume fraction dependency is that on cooling, the mismatch in the coefficients of thermal expansion between the pure aluminium matrix and the alumina fibres causes dislocations to be generated in the matrix to accommodate the strain. A higher volume of fibres would lead to a higher density of dislocations which is present prior to testing.

The consequence of the asymmetry between \( \Delta \sigma^0 \) and \( \Delta \sigma^1 \) is that the precise relationship of \( \Delta \sigma^1 \) and \( \Delta \sigma^0 \) to the mean and thermal stresses is not entirely clear (although it was shown in section 6.3 that the difference between \( \Delta \sigma^0 \) and \( \Delta \sigma^1 \) represents the thermal stress). Nevertheless, an unambiguous measurement of the mean stress hardening rate can be made, enabling a direct comparison of the experimental measurements with the theoretical predictions for this parameter. This comparison is presented in the next section.

### 6.4.2 Comparison of the measured mean stress hardening rate with theoretical predictions

The theoretical analysis of section 3.3 will now be used to obtain the mean stress hardening rate of the aluminium\textbackslash Saffil composite. The basis of the comparison of the experimental and theoretical results is eq. 3.22

\[
\frac{d}{d e_{pc}} \left( \frac{1}{4} (\Delta \sigma^e + \Delta \sigma^f) \right) = \frac{d}{d e_{pc}} \left( \frac{<\sigma_2^p >_M - <\sigma_1^p >_M}{1 + B} \right)
\]

(3.22)
with $B = B_{33} - B_{13}$ if eq. 3.12a is used, and $B = B_{33} - B_{23}$ if eq. 3.12b is used. Eq. 3.22 gives an upper and lower limit to the mean stress hardening rate of the unrelaxed composite. The plastic component of the mean stress is given by eq. 3.24

$$\langle \sigma \rangle_M^p = \frac{-f C_M (S - I) [(C_M - C_p)(S - f(S - I)) - C_M]^{-1} C_M}{e_p}$$

(3.24)

The Eshelby $S$ tensor for a planar array of continuous fibres was calculated in section 3.2, and the stiffness matrices of aluminium and alumina, $C_M$ and $C_p$, are given in Appendix 3. It was assumed that both the matrix and fibres are isotropic. The Young's modulus and Poisson's ratio used are 70 GPa and 0.346 for aluminium (Ashby and Brown, 1980) and 285 GPa (Birchall, 1983) and 0.233 (Ashby and Brown, 1980) for alumina, respectively. In eq. 3.24, the plastic component of the mean stress is proportional to the plastic strain of the matrix, $e_p$. The method described in section 3.3.3 to quantify the transverse plastic strain of the matrix in the composites was applied to experimental results in section 5.5. Plastic flow in the transverse direction appears to be independent of temperature and fibre volume fraction (for $7\% \leq f \leq 23\%$) and the resulting plastic strain, given by eq. 5.2, has been used for all the calculations

$$e_p = e_p(0.858, -0.142, 1, 0, 0, 0)$$

(5.4)

Since the plastic components of the mean stress ($\langle \sigma_i \rangle_M^p$ in eq. 3.24) are proportional to the plastic strain of the matrix in the 3 direction, $e_p$, the derivative $de_p/de_{pc}$ appears on the right hand side of eq. 3.22. This can be calculated using eq. 3.26

$$e_{pc} = D e_p$$

(3.26)

The right hand side of eq. 3.22 can now be calculated and the results are given in fig. 6.17. The upper limit is obtained by using the Tresca criterion given by eq. 3.12a (with $B = B_{33} - B_{13}$) and the lower limit by using eq. 3.12b (with $B = B_{33} - B_{23}$).

Strictly speaking, the prediction of eq 3.22 is only valid for fibre volume fractions in the range $7\% \leq f \leq 23\%$ (and for $f=0\%$) because the transverse plastic strain parameters $\eta_1$ and $\eta_2$ have been evaluated for this range (as discussed in section 5.5, $\eta_1$ and $\eta_2$ are expected to become equal to 0.5 as $f$ goes to zero). However, of particular interest here is the comparison of experiment and theory between $f = 7\%$ and $23\%$.

The experimentally derived mean stress hardening rates at room temperature and 77 K and the theoretical predictions are also given in fig. 6.17. The mean stress hardening rates were taken over the plastic strain range $0.05\% < e_{pc} < 0.2\%$ for each specimen, where the mean stress/plastic strain curves are approximately linear at 77 K and room
Figure 6.20

Hysteresis loops from parts I and II for a specimen of (a) pure aluminium (LT001) and (b) a composite (LT101) with $f=10\%$. The plastic strain is $\epsilon_{pc} = 0.2\%$. Note how the peak stress of the loops in part II is much higher than in part I. In addition, the loops at $\pm \epsilon_p$ are more round in part II than in part I, making the evaluation of permanent softening more difficult.
temperature. In general, two to three specimens were tested at each fibre volume fraction and the results were in good agreement. For f=10%, fig. 6.17 shows the results of 8 specimens from 5 different billets, tested at room temperature, which confirms the small scatter in the results.

The experimentally measured mean stress hardening rates at room temperature and 77 K increase approximately linearly with fibre volume fraction, with the results at 77 K being about a factor of two higher than those at room temperature. This suggests that extensive stress relaxation taking place at room temperature is suppressed considerably at 77 K. For pure aluminium, the prediction coincides with the experimental results (zero mean stress hardening). With increasing fibre volume fraction, the gap between theory and experiment increases, both at room temperature and 77 K. The predictions are for an unrelaxed composite, and hence the results indicate that relaxation of the mean stress increases with fibre volume fraction. At 77 K the predicted lower bound (using eq. 3.12b with $B = B_{33} - B_{23}$) is only 25% higher than the measured values for $f = 7\%$, and only 15% higher for $f = 10\%$. It is interesting in the context of considering the relaxation of the mean stress, that one of the conclusions of section 5.5 was that the relaxation of the thermal stress also increases with fibre volume fraction. A model for the mechanism of relaxation of the mean stress is presented in chapter 7.

A number of reasons can be proposed for the lower values of the measured mean stress hardening rates as compared to the predicted values. Firstly, plastic and diffusional relaxation in the matrix can contribute to a lowering of the mean stress (discussed in chapter 7). Secondly, the fibres are not all perfectly straight and misalignment out of the 2-3 plane (cf. section 2.1) will lower the mean stress hardening rate slightly. Thirdly, the mean field approximation must be used with caution, particularly for very heterogeneous systems, and it becomes less reliable with increasing fibre volume fraction. Fourthly, the connectivity of the fibre phase is not included in the model, but it is exactly the connectivity of the fibres in the Saffil preform which makes the preform rigid and allows for infiltration by molten aluminium during fabrication. Finally, the plastic strain is assumed in the model to be uniform. However, the work of Barlow and Hansen (1991) and Christman et al. (1989) has show that the plastic strain in MMCs is highly non-uniform. All these factors can contribute to the observed discrepancy between the measured and predicted mean stress hardening rates, though plastic and diffusional relaxation are expected to produce the largest effect.

Experiments on the history dependence of the mean stress are described in the next
The procedure for obtaining the hysteresis loops in the Bauschinger experiments was described in section 6.3.1 and the same procedure was used for nearly all the tests, with slight modifications in some. Generally, the specimens were cycled 3 times at each plastic strain level and the third cycle recorded. The initial straining at each plastic strain amplitude was in tension. To check if the measured mean stress hardening rate is affected by the experimental procedure, a few controlled experiments were carried out by varying the procedure. These are described in sections i to iii below. These experiments were performed at room temperature on specimens with a fibre volume fraction \( f = 10\% \).

i) Compression as the forward stroke

The theory of the mean stress predicts that it opposes further forward deformation, irrespective of whether the forward direction is tension or compression. This implies that the mean stress hardening rate measured in a Bauschinger experiment does not depend on whether the first half cycle at each plastic strain level is in tension or compression. To test this prediction, one specimen (RT106, see table 6.1a) was strained first in compression at each plastic strain level (all the other specimens were strained first in tension). Three cycles were recorded at each level and the second recorded. The mean stress/plastic strain curve for this specimen is shown in fig. 6.18. The curve is linear up to about \( e_p = 0.25\% \), with a slope of 4.2 ± 0.4 GPa. This can be compared with the range of 3.9-5.0 GPa observed for the other specimens with \( f = 10\% \) tested at room temperature and the conclusion is that the experimental mean stress hardening rate does not depend on whether the straining is first in tension or compression.

ii) Effect of cycling on the mean stress

The procedure of recording hysteresis loops in the Bauschinger experiment was described in section 6.3.1 - part I (increasing plastic strain amplitude, or psa), part II (decreasing psa) and sometimes part III (with increasing psa repeated) were recorded. Careful comparison of results for parts I, II and III shows that the mean stress is not
completely history independent, especially at 77 K.

Examples of the mean stress from parts I, II and III at room temperature are shown in figs. 6.19a. For the pure aluminium specimen, the mean stress is the same in both series. On the other hand, the mean stress for the composite at low plastic strains in part I (first series, increasing plastic strain) is lower than in parts II and III. This tendency was observed for all the composites.

The mean stress at 77 K for pure aluminium and a composite specimen (f=10%) is shown in fig. 6.19b. For the pure aluminium specimen, the mean stress in part I is independent of plastic strain, but in part II it increases dramatically with decreasing plastic strain. The mean stress in the composite specimen is also shifted upwards in part II compared to part I. Thus the effect which was also observed at room temperature is much more pronounced at 77 K. The peak stresses of the hysteresis loops in part II (decreasing plastic strain amplitude) were much higher than in part I, and figs. 6.20a and b shows examples of loops obtained from specimens LT001 and LT101 (the same specimen as in fig. 6.19b), at a plastic strain amplitude of 0.2%. The evaluation of the permanent softening is more difficult for the loops from part II because the tips of the loops are more rounded. The stability of the peak stresses of the hysteresis loops, with decreasing plastic strain at 77 K suggests that it is the work hardening of the matrix as a consequence of the cyclic deformation which is producing these effects and this is confirmed in the next section.

iii) Annealing experiment

The purpose of the experiment described here was to check whether (a) annealing the specimen can restore the initial value of the mean stress and (b) whether cycling of the specimen during the Bauschinger experiment causes any damage to the specimen, such as voiding and fibre/matrix debonding (see section 6.3.3-iii).

A specimen (RT107) was annealed for 1 hour at 350°C, water quenched and then tested at increasing plastic strain amplitude, from 0.05% to 0.4% (part I), and then at decreasing plastic strain amplitude to 0.05% (part II). The specimen was then removed, given the same heat treatment again and mechanically tested a second time at increasing plastic strains (part I). The mean stress, as evaluated for each part of the experiment, is shown in fig. 6.21. The upward shift in the values of the mean stress at low plastic strains in part II has been removed by the second annealing of the specimen.

The conclusion of this experiment is that annealing the specimen removes the
additional dislocations generated during cyclic straining. In addition, since the virgin mean stress values are restored, it can also be concluded that cycling in the Bauschinger experiment does not cause sufficient damage to the specimen in the form of voiding or fibre/matrix debonding to affect the mean stress hardening rate.

6.4.4 Summary

The conclusions of this section are as follows.

1. The proposed analysis of the mean stress has been applied to results of Bauschinger experiments on aluminium/Saffil composites and has produced unambiguous measurements of the mean stress hardening rates at small plastic strains. Some doubt remains about the absolute value of the mean stress delivered by the analysis.

2. The prediction for the mean stress hardening rate, based on the Eshelby S tensor, was in good agreement with experimental results for low fibre volume fractions (f=7% and 10%) at 77 K, but is less good at higher volume fractions or at room temperature.

3. Extensive relaxation of the mean stress observed at room temperature is suppressed considerably at 77 K and the degree of relaxation increases with fibre volume fraction.

4. The relaxation of the mean stress is not caused by irreversible damage such as voiding, fibre/matrix debonding or fibre failure.

Matrix hardening and mechanisms of relaxation of the mean stress are discussed in the next chapter.
Table 6.1a. Mean stress and thermal stress at room temperature, as evaluated from the Bauschinger experiments. $\theta_A$ is the measured mean stress hardening rate. Specimens RT104 to RT108 were made from a different batch of material than the other specimens. The estimated initial thermal stress in these specimens (in brackets) was higher than generally observed for the other composite specimens with the same fibre volume fraction, and it is not used in figs. 6.12 and 6.13. However, the mean stress hardening rates for these specimens is similar to that found in the other specimens and the values are included in fig. 6.17.

<table>
<thead>
<tr>
<th>Specimen number</th>
<th>$\theta_A$ (GPa)</th>
<th>Mean stress (MPa)</th>
<th>Thermal stress (MPa (±1MPa))</th>
</tr>
</thead>
<tbody>
<tr>
<td>RT001</td>
<td>-0.1 ± 0.1</td>
<td>1.4 ± 0.3</td>
<td>0.5 ± 0.5</td>
</tr>
<tr>
<td>RT071</td>
<td>2.4 ± 0.3</td>
<td>3.8 ± 0.7</td>
<td>8.2</td>
</tr>
<tr>
<td>RT072</td>
<td>3.0 ± 0.3</td>
<td>2.9 ± 0.5</td>
<td>10.2</td>
</tr>
<tr>
<td>RT073</td>
<td>2.4 ± 0.3</td>
<td>4.0 ± 0.6</td>
<td>10.6</td>
</tr>
<tr>
<td>RT101</td>
<td>4.0 ± 0.5</td>
<td>3.7 ± 0.4</td>
<td>7.7</td>
</tr>
<tr>
<td>RT102</td>
<td>4.3 ± 0.5</td>
<td>4.0 ± 0.6</td>
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</tr>
<tr>
<td>RT103</td>
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<td>3.7 ± 0.5</td>
<td>9.4</td>
</tr>
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<td>(11.0)</td>
</tr>
<tr>
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<td>4.7 ± 0.9</td>
<td>(12.5)</td>
</tr>
<tr>
<td>RT106</td>
<td>4.2 ± 0.4</td>
<td>3.7 ± 0.9</td>
<td>(9.5)</td>
</tr>
<tr>
<td>RT107</td>
<td>4.6 ± 0.5</td>
<td>3.8 ± 0.6</td>
<td>(11.3)</td>
</tr>
<tr>
<td>RT108</td>
<td>5.0 ± 0.5</td>
<td>3.2 ± 0.6</td>
<td>(11.7)</td>
</tr>
<tr>
<td>RT151</td>
<td>5.2 ± 0.5</td>
<td>5.6 ± 0.6</td>
<td>10.6</td>
</tr>
<tr>
<td>RT152</td>
<td>6.0 ± 0.4</td>
<td>5.0 ± 0.6</td>
<td>10.0</td>
</tr>
<tr>
<td>RT201</td>
<td>5.9 ± 0.9</td>
<td>10.8 ± 1.2</td>
<td>12.3</td>
</tr>
<tr>
<td>RT202</td>
<td>8.5 ± 0.9</td>
<td>9.5 ± 1.2</td>
<td>9.0</td>
</tr>
<tr>
<td>RT203</td>
<td>7.3 ± 0.5</td>
<td>8.4 ± 0.7</td>
<td>12.0</td>
</tr>
<tr>
<td>RT231</td>
<td>10.0 ± 0.9</td>
<td>10.1 ± 1.3</td>
<td>14.0</td>
</tr>
<tr>
<td>RT232</td>
<td>10.8 ± 0.9</td>
<td>9.3 ± 1.2</td>
<td>12.0</td>
</tr>
</tbody>
</table>

Explanation of symbols is on next page.
Symbols for table 6.1a:

# RT denotes Room Temperature. The first two digits in the specimen number are the fibre volume fraction in % and the last is the specimen number.
· The errors in slope and intercept are derived from a least-square fit to a straight line (Bevington, 1969).
† The initial thermal stress in the specimens, with the estimated error.
+ Initial stroke at each plastic strain amplitude in compression.
Annealing experiment a) After first anneal and water quench, b) After second anneal and water quench.

Table 6.1b. Mean stress and thermal stress at 77 K, as evaluated from the Bauschinger experiments. $\theta_A$ is the measured mean stress hardening rate.

<table>
<thead>
<tr>
<th>Specimen number</th>
<th>f %</th>
<th>Specimen number</th>
<th>$\theta_A$ GPa</th>
<th>Mean stress intercept MPa</th>
<th>Thermal stress MPa (±1MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LT001</td>
<td>0</td>
<td>0.4 ±0.4</td>
<td>2.0 ±0.5</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>LT071</td>
<td>7</td>
<td>4.8 ±1.0</td>
<td>4.1 ±1.2</td>
<td>17.9</td>
<td></td>
</tr>
<tr>
<td>LT072</td>
<td>7</td>
<td>4.9 ±0.6</td>
<td>3.9 ±0.8</td>
<td>16.1</td>
<td></td>
</tr>
<tr>
<td>LT101</td>
<td>10</td>
<td>8.0 ±0.9</td>
<td>6.3 ±1.2</td>
<td>19.0</td>
<td></td>
</tr>
<tr>
<td>LT102</td>
<td>10</td>
<td>7.8 ±0.9</td>
<td>3.3 ±1.2</td>
<td>16.1</td>
<td></td>
</tr>
<tr>
<td>LT151</td>
<td>15</td>
<td>9.9 ±0.7</td>
<td>6.4 ±0.8</td>
<td>18.7</td>
<td></td>
</tr>
<tr>
<td>LT152</td>
<td>15</td>
<td>11.2 ±0.9</td>
<td>6.0 ±1.2</td>
<td>19.6</td>
<td></td>
</tr>
<tr>
<td>LT201</td>
<td>20</td>
<td>13.9 ±0.9</td>
<td>8.9 ±1.2</td>
<td>21.6</td>
<td></td>
</tr>
<tr>
<td>LT202</td>
<td>20</td>
<td>13.8 ±0.9</td>
<td>8.9 ±1.2</td>
<td>23.5</td>
<td></td>
</tr>
<tr>
<td>LT231</td>
<td>23</td>
<td>17.6 ±0.9</td>
<td>10.3 ±1.2</td>
<td>21.7</td>
<td></td>
</tr>
<tr>
<td>LT232</td>
<td>23</td>
<td>16.1 ±0.9</td>
<td>13.2 ±1.2</td>
<td>--</td>
<td></td>
</tr>
</tbody>
</table>

Symbols for table 6.1b:

# LT denotes Low Temperature (77 K). The first two digits in the specimen number are the fibre volume fraction in % and the last is the specimen number.
· The errors in slope and intercept are derived from a least-square fit to a straight line (Bevington, 1969).
† The initial thermal stress in the specimens, with the estimated error.
* This specimen was unintentionally preloaded during gripping.
Table 6.2a. Examples of permanent softening, mean stress and thermal stress (specimen RT152, f=15%). The estimated uncertainty in the measured values of the permanent softening is obtained by the method described in fig. 6.3.

<table>
<thead>
<tr>
<th>e_{pc}</th>
<th>Cycl. No.</th>
<th>\Delta \sigma^f</th>
<th>\Delta \sigma^c</th>
<th>\text{Mean stress } \sigma_v/4(\Delta \sigma^c + \Delta \sigma^f)</th>
<th>\text{Thermal stress } \sigma_v/4(\Delta \sigma^c - \Delta \sigma^f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.05</td>
<td>2.5-3</td>
<td>-3.1 ±1.0</td>
<td>34.0 ±1.0</td>
<td>7.8 ±0.5</td>
<td>9.3 ±0.5</td>
</tr>
<tr>
<td>.10</td>
<td>5.5-6</td>
<td>4.5 ±1.0</td>
<td>40.4 ±1.0</td>
<td>11.2 ±0.5</td>
<td>9.0 ±0.5</td>
</tr>
<tr>
<td>.15</td>
<td>8.5-9</td>
<td>12.7 ±1.0</td>
<td>44.6 ±1.0</td>
<td>14.3 ±0.5</td>
<td>8.0 ±0.5</td>
</tr>
<tr>
<td>.20</td>
<td>11.5-12</td>
<td>18.1 ±1.0</td>
<td>49.6 ±1.0</td>
<td>16.9 ±0.5</td>
<td>7.9 ±0.5</td>
</tr>
<tr>
<td>.25</td>
<td>14.5-15</td>
<td>22.1 ±1.5</td>
<td>51.8 ±1.5</td>
<td>18.5 ±0.7</td>
<td>7.4 ±0.7</td>
</tr>
<tr>
<td>.31</td>
<td>17.5-18</td>
<td>27.3 ±1.5</td>
<td>55.3 ±1.5</td>
<td>20.7 ±0.7</td>
<td>7.0 ±0.7</td>
</tr>
<tr>
<td>.36</td>
<td>20.5-21</td>
<td>32.2 ±2.5</td>
<td>58.0 ±2.5</td>
<td>22.6 ±1.2</td>
<td>6.5 ±1.2</td>
</tr>
<tr>
<td>.41</td>
<td>23.5-24</td>
<td>36.7 ±2.5</td>
<td>63.5 ±2.5</td>
<td>25.1 ±1.2</td>
<td>6.7 ±1.2</td>
</tr>
</tbody>
</table>

Table 6.2b. Examples of permanent softening, mean stress and thermal stress (specimen LT151, f=15%). The estimated uncertainty in the measured values of the permanent softening is obtained by the method described in fig. 6.3.

<table>
<thead>
<tr>
<th>e_{pc}</th>
<th>Cycl. No.</th>
<th>\Delta \sigma^f</th>
<th>\Delta \sigma^c</th>
<th>\text{Mean stress } \sigma_v/4(\Delta \sigma^c + \Delta \sigma^f)</th>
<th>\text{Thermal stress } \sigma_v/4(\Delta \sigma^c - \Delta \sigma^f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.05</td>
<td>2.5-3</td>
<td>-15.4 ±2.0</td>
<td>59.1 ±1.0</td>
<td>10.9 ±1.0</td>
<td>18.6 ±1.0</td>
</tr>
<tr>
<td>.10</td>
<td>5.5-6</td>
<td>-4.5 ±2.0</td>
<td>70.0 ±2.0</td>
<td>16.4 ±1.0</td>
<td>18.6 ±1.0</td>
</tr>
<tr>
<td>.15</td>
<td>8.5-9</td>
<td>6.5 ±3.0</td>
<td>81.0 ±3.0</td>
<td>21.9 ±1.5</td>
<td>18.6 ±1.5</td>
</tr>
<tr>
<td>.20</td>
<td>11.5-12</td>
<td>15.4 ±3.0</td>
<td>89.4 ±3.0</td>
<td>26.2 ±1.5</td>
<td>18.5 ±1.5</td>
</tr>
<tr>
<td>.25</td>
<td>14.5-15</td>
<td>27.8 ±2.0</td>
<td>97.9 ±2.0</td>
<td>31.4 ±1.0</td>
<td>17.5 ±1.0</td>
</tr>
<tr>
<td>.30</td>
<td>17.5-18</td>
<td>31.8 ±2.0</td>
<td>105.3 ±2.0</td>
<td>34.3 ±1.0</td>
<td>18.4 ±1.0</td>
</tr>
<tr>
<td>.35</td>
<td>20.5-21</td>
<td>33.2 ±3.0</td>
<td>113.3 ±3.0</td>
<td>38.1 ±1.5</td>
<td>18.5 ±1.5</td>
</tr>
<tr>
<td>.40</td>
<td>23.5-24</td>
<td>44.7 ±2.0</td>
<td>112.3 ±2.0</td>
<td>39.3 ±1.0</td>
<td>16.9 ±1.0</td>
</tr>
</tbody>
</table>
Figure 6.1

A schematic hysteresis loop for the hypothetical unrelaxed composite, which is used to derive expressions for the permanent softening. Upon stress reversal, the flow stress in the reverse direction is reached after an infinitesimal strain, $\varepsilon_{pc}$. Work hardening is linear and the material has a tensile thermal stress in the matrix which causes the loop to be shifted downwards: O is the centre of the coordinate system and A is the centre of the loop.
Figure 6.2

The functional dependence of $\Delta \sigma^t$ and $\Delta \sigma^c$ on plastic strain for the hypothetical unrelaxed composite (see eqs. 3.18 and 3.19). When the thermal stress is tensile and has the value $\lambda$, the intercepts of $\Delta \sigma^c$ and $\Delta \sigma^t$ are $+2\lambda$ and $-2\lambda$ respectively (symmetric about the origin). When the thermal stress is zero, then $\Delta \sigma^c = \Delta \sigma^t$. 
Figure 6.3

An example of a hysteresis loop obtained from specimen RT105, part I of the experiment, at $e_{pc} = 0.23\%$. The centre of the loop is shifted downwards compared to the origin, indicating a tensile thermal stress in the matrix. Note the rounded reverse flow curves and that the flow curves at $\pm e_{pc}$ are not exactly parallel. The figure shows how $\Delta \sigma^t$ and $\Delta \sigma^e$ are evaluated. The error estimated in $\Delta \sigma^t$ and $\Delta \sigma^e$ is taken to be $\frac{1}{2}(\Delta \sigma_1 - \Delta \sigma_2)$ (which is equal to $\frac{1}{2}(\Delta \sigma_4 - \Delta \sigma_3)$).
Figure 6.4

Analysis of the Bauschinger experiment on a specimen (RT105) with fibre volume fraction $f = 10\%$, tested at room temperature.

a) The permanent softening $\Delta \sigma^c$ and $\Delta \sigma^i$ from stage I of the experiment.

b) $\Delta \sigma^c$ and $\Delta \sigma^i$ from stage I (boxes) and III (crosses). The series in stages I and III are both recorded at increasing plastic strain amplitude.
c) The mean stress from stages I (boxes) and III (crosses), evaluated as $\frac{1}{2}(\Delta \sigma^c + \Delta \sigma^t)$.

d) The thermal stress from stages I (boxes) and III (crosses), evaluated as $\frac{1}{2}(\Delta \sigma^t - \Delta \sigma^i)$. The arrows indicate that both series are recorded at increasing plastic strain amplitude.
Figures 6.5 a-d

Examples of hysteresis loops obtained from specimens with fibre volume fraction $f=15\%$. The loops are from part I of the Bauschinger experiment and are recorded at increasing plastic strain amplitude.

a) Room temperature, specimen RT152. First three loops.
Figure 6.5

b) Room temperature, specimen RT152. Loops four to eight.
Figure 6.5

c) 77 K, specimen LT152. First three loops.
Figure 6.5
d) 77 K, specimen LT152. Loops four to eight.
Figure 6.6

Examples of the thermal stress term, $\frac{1}{4}(\Delta \sigma^c - \Delta \sigma^f)$, obtained from specimens tested at room temperature. Part I is denoted by □ (increasing plastic strain amplitude; psa), part II by + (decreasing psa) and part III by × (increasing psa).

a) $f = 0\%$, specimen RT001.
b) $f = 7\%$, specimen RT072.
c) $f = 10\%$, specimen RT102.
d) $f = 15\%$, specimen RT152.
e) $f = 20\%$, specimen RT203.
f) $f = 23\%$, specimen RT232.
Figure 6.7

Examples of the thermal stress term, \( \frac{1}{4}(\Delta \sigma^e - \Delta \sigma^p) \), obtained from specimens tested at 77 K. Part I is denoted by □ (increasing psa), part II by + (decreasing psa) and part III by * (increasing psa).

a) \( f = 0\% \), specimen LT001.
b) \( f = 7\% \), specimen LT071.
c) \( f = 10\% \), specimen LT101.
d) \( f = 15\% \), specimen LT151.
e) \( f = 20\% \), specimen LT202.
f) \( f = 23\% \), specimen LT231.
Figure 6.8
The thermal stress term in a Bauschinger experiment started at $e_{pc} = 0.4\%$ and ended at $e_{pc} = 0.05\%$ (boxes). The test was then continued from the lowest plastic strain ($e_{pc} = 0.05\%$) up to $e_{pc} = 0.5\%$ (crosses).

Figure 6.9
The change in the thermal stress term for a specimen which was cycled for 550 cycles at $e_{pc} = 0.3\%$ between two evaluations of the thermal stress.
Figure 6.10

The thermal stress term for a specimen which was cycled with the initial strain in compression at each plastic strain level. Both the initial value of the thermal stress term and its reduction with cycling is very much the same as for the other specimens tested.

Figure 6.11

The thermal stress term for a specimen which was annealed for 1 hour at 350°C and water quenched before being tested in parts I and II.

After first anneal and water quench:
- Part I (increasing psa)
- Part II (decreasing psa)

After second anneal and water quench:
- Part I (increasing psa)
- Part II (decreasing psa)
Figure 6.12

The initial values of the thermal stress estimated from the results of the Bauschinger experiments.

- □ Bauschinger experiments at room temperature
- ◯ Bauschinger experiments at 77 K

Figure 6.13

The initial thermal stress in the material as estimated from the Bauschinger experiments and the monotonic tests of chapter 5.

- □ Bauschinger experiments at room temperature
- ◯ Bauschinger experiments at 77 K
- ✭ Monotonic tests at room temperature
- + Monotonic tests at 77 K
Figure 6.14

The mean stress term from part I (increasing psa) of Bauschinger experiments on pure aluminium and composites at room temperature. The fibre volume fraction is given for each curve. a) A typical result at each fibre volume fraction. b) All the results for specimens tested at room temperature.
The mean stress term from part I (increasing psa) of Bauschinger experiments on pure aluminium and composites at 77 K. The fibre volume fraction is given for each curve. a) One example for each fibre volume fraction. b) All the results for specimens tested at 77 K.
Figure 6.16

The value of the mean stress intercepts at 0% plastic strain as a function of fibre volume fraction.
Figure 6.17

Theoretical and experimental mean stress hardening rates for the aluminium\Saffil composites. The prediction is based on the Eshelby S tensor calculated for a planar array of fibres (section 3.2). The upper theoretical curve is calculated using eq. 3.12a and the lower curve using eq. 3.12b. Experimental results are for tests at room temperature and 77 K.
Figure 6.18

The mean stress as a function of plastic strain for a test in which the initial straining at each plastic strain level was in compression. The loops were recorded at increasing plastic strain amplitude.
Figure 6.19

a) The mean stress term as a function of plastic strain for a specimen (pure aluminium) and a composite (f=10%), tested at room temperature. Note the upward shift for the composite specimen at low plastic strains in parts II and III.

b) The mean stress term as a function of plastic strain for a specimen (pure aluminium) and a composite (f=10%), tested at 77K. For the composite the upward shift in the value of the mean stress term at low plastic strains in part II is much greater than at room temperature, and there is also an upward shift for pure aluminium.

Part I □ (psa 0.05% increasing to 0.4%)
Part II + (psa 0.35% decreasing to 0.05%)
Part III * (psa 0.05% increasing to 0.5%)

Figure 6.20

Hysteresis loops from parts I and II for a specimen of (a) pure aluminium (LT001) and (b) a composite (LT101) with $f=10\%$. The plastic strain is $\varepsilon_{pc} = 0.2\%$. Note how the peak stress of the loops in part II is much higher than in part I. In addition, the loops at $\pm \varepsilon_{pc}$ are more round in part II than in part I, making the evaluation of permanent softening more difficult.
Figure 6.21

The mean stress term before and after annealing. The upward shift of the mean stress values at low plastic strains is removed by the annealing treatment.

- After first anneal and water quench: □ Part I (increasing psa) □ Part II (decreasing psa)
- After second anneal and water quench: + Part I (increasing psa)
Matrix hardening and relaxation

7.1 Introduction

7.2 Matrix hardening

7.2.1 Evaluation of the plastic friction coefficient

7.2.2 Summary

7.3 A model for relaxation of the mean stress

7.3.1 Relaxation of the mean stress in aluminium\Saffil composites

7.3.2 Summary

Tables 7.1 - 7.3

Figures 7.1 - 7.6
Chapter Seven
Matrix hardening and relaxation

7.1 Introduction

The subject of this chapter are two competing mechanisms taking place in the matrix. One is matrix hardening which is caused by the presence of the fibres (the constraint effect), and the other is stress relaxation. In section 7.2, an attempt is made to apply Pedersen's modified Orowan-Wilson model (Pedersen, 1990) which was developed for matrix hardening in continuous fibre copper-tungsten composites, to the aluminium\Saffil composites.

It was shown in section 6.4 that relaxation leads to a significant reduction in the mean stress hardening rate at room temperature, but at 77 K relaxation is suppressed considerably. This indicates that the underlying mechanism is (at least partly) assisted by thermal activation. It has, therefore, been attempted here to describe the relaxation mechanism in the aluminium\Saffil composites by adapting the model of Atkinson et al. (1974), developed for copper-silica (see section 2.6). This is described in section 7.3.

7.2 Matrix hardening

An attempt will be made in this section to quantify the hardening of the matrix in the aluminium\Saffil composites by application of Pedersen's (1990) modified Orowan-Wilson model. In this model the peak stresses of the hysteresis loops in the Bauschinger experiment are used to obtain the peak stress hardening rate, which is used in addition to the mean stress hardening rate. For the copper-tungsten composites the plots of peak stress versus plastic strain are linear, in accord with the observed linearity of the overall hardening in tensile testing (Kelly and Lilholt, 1969). The feature about using the peak stress is that it contains both mean stress and matrix hardening. This enabled Pedersen
(1990) to separate the plastic friction component of the source shortening stress from the overall hardening.

7.2.1 Evaluation of the plastic friction coefficient

The requirement of Pedersen's model is that both the mean stress and peak stress curves are linear in plastic strain. The mean stress hardening in the aluminium\Saffil composites is, in general, not linear over the whole plastic strain range tested (0.05% < \epsilon_{pc} < 0.4%), but for strains less than 0.2%, the curves are approximated linear (see chapter 6). The peak stress of the hysteresis loops in tension and compression has been evaluated and the resulting curves are shown in figs. 7.1a and b (room temperature) and 7.2a and b (77 K). At room temperature the curves for pure aluminium are linear, but for the composites they are all non-linear and concave. The results at 77 K have the interesting feature that the curves for pure aluminium are slightly convex, the curves for \( f = 7\% \) are linear, and the curves for the higher volume fraction composites are non-linear and concave. The real origin of the linearity of the composite with \( f = 7\% \) is, therefore, questionable. It might be that for \( f = 7\% \) the convex character of the curves for pure aluminium is exactly opposed (fortuitously) by the fibres, in which case the linearity cannot be taken to show that the modified Orowan-Wilson model is really applicable here. With this reservation in mind it will, however, be attempted to use the linearity to estimate the matrix friction coefficient \((a/A, \text{ see section 2.6)}\).

The Pedersen model gives an expression for the ratio of the plastic component of the source shortening stress \((\alpha)\) and the plastic component of the mean stress \((A)\)

\[
\frac{\alpha}{A} = \frac{\theta_P}{\theta_A} - 1
\]  

(2.30)

\( \theta_P \) is the peak stress hardening rate and \( \theta_A \) the mean stress hardening rate. These quantities have been evaluated for the \( f = 7\% \) specimens at 77 K for which the peak stress curves are linear and the results are shown in table 7.1. For the theoretical predictions, \( A \) is obtained by using eqs. 2.26 and 2.29 and \( \alpha \) by using eqs. 2.32 and 2.34. Because the 1 and 2 directions in the aluminium\Saffil composites are not identical, eq. 2.34 gives rise to two values of \( \alpha \); one by using \( a_2 \) and \( a_3 \) and the other by using \( a_1 \) and \( a_3 \) (the \( a_i \) components are defined by eq. 2.32). \( \alpha \) has been calculated for both cases and the results are given in table 7.1.
The source shortening model combined with the mean field theory predicts the ratio of $\alpha/A$ to be between 0.6 and 0.7. This is similar in magnitude to the value calculated by Pedersen (1990) for continuous tungsten fibre reinforced copper ($\alpha/A \sim 0.45$). The experimental value of $3.8 \pm 0.1$ is about 6 times higher than the predictions. Hence, the conclusion is that the modified Orowan-Wilson model cannot be used to analyse Bauschinger experiments in these aluminium\Saffil composites. Pedersen (1990) discusses the limitations of the model in terms of fibre diameter and says the model is limited to composites whose fibres are not too thin; break-down of the source-shortening model may occur when the effective interfibre spacing is similar to the actual interfibre spacing. For the Cu-W composites this occurs for fibre diameters of the order of $a \mu m$. This limit has not been calculated for the composite studied here, but since the fibre diameter is small (about $3 \mu m$), it may be close to the limits of applicability of the model. Pedersen (1985a) mentions that eq. 2.30 is expected to be valid for relaxed composites if the relaxation reduces $\theta_F$ and $\theta_A$ in the same proportion.

In view of the high measured value of $\alpha/A$ (about 4) it is interesting to compare the peak stress and mean stress curves in this composites. This is done in figs. 7.3a-d for specimens with $f=7\%$ and $15\%$ at (a and b) room temperature and (c and d) 77 K. The figures show that the mean stress contributes only about 20% to the overall (tension) peak stress and matrix hardening is therefore responsible for the majority of the overall flow stress of the composite. This is in contrast to the results of Pinnel and Lawley (1970) and Isaacs and Mortensen (1992), which found no constraint effect in continuous fibre reinforced pure aluminium. Their results are, however, obtained in monotonic testing and may not be directly comparable with the present results, which are obtained in Bauschinger experiments.

7.2.2 Summary

The conclusions of this section are as follows. Pedersen's modified Orowan-Wilson model is not applicable to the aluminium\Saffil composite for two reasons. Firstly, because the peak stress curves are generally not linear, and secondly, because the measured value of $\alpha/A$ for the two specimens which have linear peak stress curves, is six times higher than the predicted value. The observed discrepancy could be caused by a different relaxation behaviour of the mean stress and the source shortening stress. It is also possible that the diameter of the Saffil fibres is close to the limits of applicability of the model.
7.3 A model for relaxation of the mean stress

The model which will be developed here is based on the assumption that the rate controlling mechanism of relaxation is cross slip of screw dislocations. In copper-silica, where the reinforcement is in the form of spherical particles, the conversion of Orowan loops into dipoles of prismatic loops with the aid of cross slip has been observed by Brown and Stobbs (1971b) who showed that this mechanism is capable of reducing the mean stress in the matrix. A similar relaxation process can take place even when an Orowan loop encircles a continuous fibre (Kelly, 1972; Lilholt, 1977). Relaxation in continuous fibre copper-tungsten composites has been observed by Lilholt (1977), and Pedersen (1985a). It has also been shown by Schoeck and Seeger (1955) that the activation energy for cross slip in aluminium is smaller than in copper, because of the much higher stacking fault energy in aluminium (see section 2.6). Hence, for a given temperature and applied stress, the rate of cross slip is expected to be higher in aluminium than in copper.

7.3.1 Relaxation of the mean stress in aluminium\Saffil composites

The model proposed here is based on the work of Atkinson et al. (1974), and it is assumed that cross slip of screw segments of Orowan loops is responsible for the observed relaxation of the mean stress. The first task is to derive the equation relating the number of Orowan loops around each inclusion, \( n_f \), to the plastic strain imposed on the composite. If the inclusions are spherical with a diameter \( d \), the expression is

\[
 n_f = \frac{2d}{b} e_p
\]  

(7.1)

where \( b \) is the Burgers vector and \( e_p \) the plastic strain (Atkinson et al., 1974; Ashby, 1971). To derive a similar expression for long fibre composites, some simplifications have to be made. It is assumed that fibres of length \( l_f \) are aligned parallel to the loading axis, and that the glide plane normal and the glide direction are at 45° to the tensile axis. The number of Orowan loops per fibre, \( n_0 \), can then be expressed in terms of the plastic strain (see Appendix 4) and the result is

\[
 n_f = \frac{\sqrt{2} l_f}{b} e_p
\]  

(7.2)
The problem now is to find the rate of loss (or conversion) of Orowan loops, \( \frac{dn_l}{dt} \) (\( n_l \) is the number of Orowan loops which are lost or converted). In the absence of relaxation, the imposed (experimental) plastic strain, \( e_p \), would be equal to the residual elastic strain deduced from measurements of the mean stress. However, when relaxation occurs, the residual elastic strain is reduced to \( e_p^* \) (see fig. 7.4a). Fig 7.4b shows three types of behaviour relating to relaxation. Curve 1 represents the unrelaxed composite \( (e_p = e_p^*) \) and curve 2 a linear relaxation behaviour \( (\frac{de_p^*}{de_p} constant) \) for \( e_p < e_p^* \) (part A) and non-linear relaxation for \( e_p > e_p^* \) (part B). Curve 3 is unrelaxed up to \( e_p^* \), but completely relaxed for \( e_p > e_p^* \) \( (\frac{de_p^*}{de_p} = 0) \).

Atkinson et al. (1974) assume, on the basis of their experimental results, that relaxation in copper-silica is represented by curve 3 in fig. 7.4b. Once relaxation begins, it is complete and the Orowan loops are relaxed at the same rate as they are generated. This approach allows eq. 7.1 to be used directly (with \( n_l = n_d \))

\[
\frac{dn_l}{dt} = 2d \frac{de_p}{b \frac{de_p}{dt}}
\]  

(7.3)

This approach cannot be used for the aluminium/Saffil composites because relaxation is better described by curve 2 in fig. 7.4b and hence some modification is required.

Because the number of Orowan loops around an inclusion in an unrelaxed composite is proportional to the plastic strain, \( e_p \), it seems reasonable to assume that the number of Orowan loops which are relaxed, \( n_l \), is proportional to the difference between the unrelaxed and relaxed strains, \( (e_p - e_p^*) \), which gives

\[
\frac{dn_l}{dt} \propto \frac{d}{dt} (e_p - e_p^*)
\]  

(7.4)

The relation between \( e_p^* \) and \( e_p \) for linear relaxation can be found as follows. \( e_p^* \) is directly proportional to \( e_p \) with \( e_p^* = ke_p \) where \( k \) is a constant (see fig. 7.4b, curve 2, part A). It then follows that \( \frac{de_p^*}{de_p} = k \) and hence

\[
e_p^* = \left( \frac{de_p^*}{de_p} \right) e_p
\]  

(7.5)

Eq. 7.4 can then be written as

\[
\frac{dn_l}{dt} \propto \left[ 1 - \frac{de_p^*}{de_p} \right] \frac{de_p}{dt}
\]  

(7.6)
where \( \frac{de_p}{dt} \) is the imposed (experimental) plastic strain rate. The constant of proportionality can be found by considering the limiting cases. For an unrelaxed composite, \( \frac{de^*}{de} = 1 \) and \( \frac{dn}{dt} = 0 \). In a composite which is completely relaxed, that is, when the Orowan loops are relaxed at the same rate as they are generated, then \( \frac{de^*}{de} = 0 \). In the latter case it is then clear that the constant of proportionality is given by eq. 7.2

\[
\frac{dn_i}{dt} = \sqrt{2l_f} \left[ 1 - \frac{de^*}{de} \right] \frac{de}{dt} \tag{7.7}
\]

Eq. 7.7 is an extension of the work of Atkinson et al. (1974) and contains their model as a special case when \( \frac{de^*}{de} = 0 \). The relaxation rate, \( \frac{de^*}{de} \), can be obtained for each fibre volume fraction by comparing the measured (relaxed) and calculated (unrelaxed) mean stress hardening rates at small plastic strains.

The next step is to describe the thermal activation of cross slip. The rate of activation of a dislocation segment over an obstacle is

\[
\Gamma = v \exp \left( -\frac{\Delta G}{kT} \right) \tag{7.8}
\]

where \( v \) is the dislocation attempt frequency and \( \Delta G \) the appropriate Gibbs free energy barrier (Gibbs, 1969). Atkinson et al. (1974) use the Debye frequency for \( v_D \) and equate \( \Gamma \) to the rate of loss of Orowan loops (i.e., the rate at which cross slip occurs). Eqs. 7.7 and 7.8 then give the relaxation rate in the aluminium\Saffil composites as

\[
\frac{dn_i}{dt} = \sqrt{2l_f} \left[ 1 - \frac{de^*}{de} \right] \frac{de}{dt} = v_D \exp \left( -\frac{U - Vr}{kT} \right) \tag{7.9}
\]

where \( U \) is the activation energy and \( V \) the activation volume (see section 2.6).

A decision must now be made about what value of the stress \( \tau \) to use in this equation. Atkinson et al. (1974) use the mean shear stress in the matrix at which relaxation begins (see curve 3 in fig. 7.4b). When the relaxation behaviour is as shown by curve 2 in fig. 7.4b (part A), it is not clear which value of \( \tau \) to use. The approach used here is to use the value of the mean stress at \( e_p = 0.2\% \), which is about half way between the lower and upper limits of the plastic strain range used. The average of all the results for the mean stress at \( e_p = 0.2\% \) was then calculated at each fibre volume fraction and half of that value is used for \( \tau \) (it has been assumed here that the normal of the glide planes and the glide directions are both at a 45° angle to the loading axis and \( \tau \) can then be taken to be half the
measured mean stress). Eq. 7.9 was then solved to give the activation energy and volume for each fibre volume fraction. The parameters used are shown in table 7.2a and the results for U and V are given in table 7.2b.

The results for the activation energy as a function of fibre volume fraction are shown in fig. 7.5. The activation energy is independent of fibre volume fraction with an average value of $1.3 \pm 0.1 \text{ eV}$ (error estimated from the scatter). This can be interpreted to mean that the mechanism of relaxation is the same for all fibre volume fractions. In an attempt to identify the relaxation process, some literature values of activation energy for cross slip and diffusion in aluminium and copper are shown in table 7.3. Looking first at copper, the activation energy for cross slip in copper has been estimated theoretically by Schoeck and Seeger (1955) to be about 10 eV. Detailed numerical calculations have recently been published by Duesbery et al. (1992) for copper, giving an activation energy between 1.5 and 2.0 eV (dependent on crystal orientation). This is about an order of magnitude less than the prediction of Schoeck and Seeger, and close to the value of 0.9 eV obtained experimentally by Atkinson et al. (1974).

Turning now to aluminium, which has a higher stacking fault energy, it is to be expected that the activation energy for cross slip is lower than in copper. Hence the value of 1.5-2.0 eV calculated for copper (Duesbery et al. 1992) can be taken to be an upper limit for aluminium. Schoeck and Seeger (1955) estimated the activation energy for cross slip in aluminium to be about 1.0 to 1.1 eV although the basis of their calculation is questionable. The high stacking fault energy results in a small separation ($d_0$) of the partials ($d_0\sim3b$, see section 2.6), which means that the dimensions of a constriction necessary for cross slip to occur is only of the order of a few Burgers vectors. However, the meaning of a "constriction" at an atomistic level is not clear, and hence, it is very difficult to estimate theoretically the activation energy for aluminium (Duesbery, 1992). Fortunately, experimental measurements of the activation energy are available. Sherby et al. (1957) and Dorn and Jaffe (1961) rapidly changed the temperature during creep tests at constant stress and measured the resulting strain rates. These measurements are in agreement and give values of about 1.2 eV, which is in remarkable accord with the present estimate of $1.3 \pm 0.1 \text{ eV}$.

Relaxation based on diffusive processes is an alternative to cross slip. The activation energy for lattice diffusion in aluminium is 1.47 eV (see table 7.3), which is not incompatible with the results obtained here, although it has been shown that dislocation climb is important in aluminium only for temperatures greater than about 370 K (Sherby
et. al., 1957; Dorn and Jaffe, 1961). The value for boundary diffusion is 0.87 eV, which is somewhat less than the measured activation energy.

Hence, considerations of the magnitude of activation energy show that it is not possible to exclude diffusion as a potential relaxation mechanism, although there is excellent agreement between the present estimate and values obtained by others for cross slip in pure aluminium. Before discussing the activation volume it is worth pointing out that the activation energy does not change significantly if \( \tau \) is evaluated at a different plastic strain value. For \( f=7\% \), using the mean stress at \( \varepsilon_p=0.1\% \) gives \( U = 1.34 \text{ eV} \) and at \( \varepsilon_p=0.3\% \), \( U = 1.41 \text{ eV} \); for \( f=15\% \), using the mean stress at \( \varepsilon_p=0.1\% \) gives \( U = 1.35 \text{ eV} \) and at \( \varepsilon_p=0.3\% \), \( U = 1.19 \text{ eV} \). However, the estimate of the activation volume is affected if a different plastic strain is used.

The activation volumes derived here are shown in fig. 7.6 as a function of fibre volume fraction. Using a different estimate of \( \tau \), the values of the activation volume change, but the change is less than an order of magnitude. Experimental and theoretical estimates of the activation volume for cross slip in aluminium are not available in the literature. Theoretical calculations are problematic, as discussed above, because of the high stacking fault energy in aluminium (Duesbery, 1992). The values for copper (experimental and theoretical), are between 335 b\(^3\) and 475 b\(^3\) (Bonneville and Escaig, 1979; Duesbery et al., 1992). The literature value for the activation volume for lattice diffusion in aluminium is 0.62 b\(^3\), which is about three orders of magnitude smaller than measured here. This can be taken as an indication that the rate controlling mechanism responsible for relaxation of the mean stress is unlikely to be diffusion. In view of these comparisons for the activation volume, the indications are that cross slip is more likely to be the rate controlling mechanism than diffusion.

In considering the significance of the change in the activation volume, it is interesting that the activation volume decreases with increasing fibre volume fraction in the same way as the interfibre spacing (see fig. 7.6), assuming square packing of continuous aligned fibres with a diameter of 3\( \mu \)m. This leads to the possibility that the activation volume is related to the dislocation/obstacle configuration. If the width of extended dislocations in aluminium, in units of the Burgers vector, is 3 (see section 2.6), the spacing of obstacles on the dislocation lines can be interpreted to be (Gibbs, 1969; Conrad et al., 1960)

\[
I = \frac{V}{3b^2}
\]  

(7.10)  

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Using the data from table 7.2b, l can be calculated and is found to decrease from about 110 nm for f=7% to about 40 nm for f=23%, which is about 70 - 80 smaller than the interfibre spacing. It appears then that the obstacles on the dislocation lines are unlikely to be the fibres themselves.

Nategh et al. (1992) measured dislocation densities in as-cast aluminium\Saffil composites with f = 10% (their material is the same as used in this project) and observed a dislocation density of \( \rho = 4.7 \times 10^{13} \, \text{m}^{-2} \). This gives an approximate spacing of dislocations (1/\( \sqrt{\rho} \)) between 80 and 145 nm. This is a similar result as obtained by using the activation volume to estimate the spacing of obstacles on a dislocation line (about 70 nm for the composites with f=10%) and suggests that there might be a relationship between the activation volume and the geometry of the dislocation configuration. However, as discussed above, using a different estimate of \( \tau \) will change the activation volume and hence the estimate of l in eq. 7.10. It must also be said that any correlation of activation volume and dislocation geometry is necessarily tentative, because in general the correlation is far from being simple (Hirth and Lothe, 1982; Duesbery et al., 1992).

### 7.3.2 Summary

The model of Atkinson et al. (1974) for cross slip in copper silica has been modified and applied to the aluminium\Saffil composites studied here. The estimated activation energy is independent of fibre volume fraction with an average of 1.3 ±0.1 eV. This has been interpreted to mean that the relaxation mechanism is the same for all fibre volume fractions. The magnitude of the activation energy is smaller than for cross slip in copper, as was expected, and in good agreement with experimental estimates for aluminium, which show it to be 1.2 eV (Sherby et al., 1957; Dorn and Jaffe, 1961; Jaffe and Dorn, 1962). Additional evidence is provided by estimates of the activation volume, which was observed to decrease with increasing fibre volume fraction. Its magnitude is of the same order as that for cross slip in copper, but estimates for aluminium are not known. The activation volume was found to be about three orders of magnitude higher than for lattice diffusion in aluminium, which indicates that the relaxation mechanism is unlikely to be diffusion. Thus, the combined evidence from considerations of activation energy and activation volume indicates that the rate controlling mechanism for relaxation of the mean stress is cross slip of screw dislocations.
Table 7.1  Measured values of $\theta_F$, $\theta_A$, and $\alpha/A$ for two specimens with $f=7\%$ (LT071 and LT072), tested at 77 K. The theoretical prediction for $\alpha/A$ are shown for comparison.

<table>
<thead>
<tr>
<th>$f$</th>
<th>Temp. K</th>
<th>$\theta_F$ GPa</th>
<th>$\theta_A$ GPa</th>
<th>$\alpha/A$ Measured</th>
<th>$\alpha/A$ Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>77</td>
<td>23.6</td>
<td>4.8</td>
<td>3.9</td>
<td>0.62 (using $a_2$ and $a_3$)</td>
</tr>
<tr>
<td>7</td>
<td>77</td>
<td>23.3</td>
<td>4.9</td>
<td>3.7</td>
<td>0.70 (using $a_1$ and $a_3$)</td>
</tr>
</tbody>
</table>

Table 7.2a  The parameters needed for the evaluation of $U$ and $V$ in eq. 7.9. The Debye frequency for aluminium is $9 \cdot 10^{12}$ sec$^{-1}$ and the Burgers vector is $2.86 \cdot 10^{-10}$ m (Ashby and Brown, 1980). The length of the fibres, $l_f$, was taken to be $400 \mu$m, and the strain rate at $e_p=0.2\%$ was $10^{-3}$ sec$^{-1}$.

<table>
<thead>
<tr>
<th>$f$</th>
<th>No. of specimens</th>
<th>$&lt;\sigma&gt;_M$ at $e_p=0.2%$ MPa</th>
<th>$\tau$ MPa</th>
<th>$1 - \frac{de_p^*}{de_p}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Room Temperature</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>8.6 $\pm 0.2$</td>
<td>4.3</td>
<td>0.627</td>
</tr>
<tr>
<td>10</td>
<td>7</td>
<td>12.2 $\pm 0.4$</td>
<td>6.1</td>
<td>0.596</td>
</tr>
<tr>
<td>15</td>
<td>2</td>
<td>16.3 $\pm 0.7$</td>
<td>8.1</td>
<td>0.680</td>
</tr>
<tr>
<td>20</td>
<td>3</td>
<td>22.9 $\pm 1.1$</td>
<td>11.4</td>
<td>0.721</td>
</tr>
<tr>
<td>23</td>
<td>2</td>
<td>30.2 $\pm 0.1$</td>
<td>15.1</td>
<td>0.671</td>
</tr>
<tr>
<td></td>
<td>77 K</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>13.7 $\pm 0$</td>
<td>6.8</td>
<td>0.304</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>20.4 $\pm 2.3$</td>
<td>10.2</td>
<td>0.253</td>
</tr>
<tr>
<td>15</td>
<td>2</td>
<td>27.1 $\pm 1.8$</td>
<td>13.5</td>
<td>0.397</td>
</tr>
<tr>
<td>20</td>
<td>2</td>
<td>35.6 $\pm 0.4$</td>
<td>17.8</td>
<td>0.464</td>
</tr>
<tr>
<td>23</td>
<td>2</td>
<td>44.9 $\pm 0.1$</td>
<td>22.4</td>
<td>0.467</td>
</tr>
</tbody>
</table>
Table 7.2b  The activation energy and activation volume derived from applying eq. 7.9 to the data of table 7.1a.

<table>
<thead>
<tr>
<th>f %</th>
<th>Activation energy ( eV )</th>
<th>Activation volume in units of ( b^3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>1.28</td>
<td>1120</td>
</tr>
<tr>
<td>10</td>
<td>1.20</td>
<td>696</td>
</tr>
<tr>
<td>15</td>
<td>1.19</td>
<td>518</td>
</tr>
<tr>
<td>20</td>
<td>1.32</td>
<td>449</td>
</tr>
<tr>
<td>23</td>
<td>1.43</td>
<td>389</td>
</tr>
</tbody>
</table>

Table 7.3  Comparison of data in the literature for the activation energy and the activation volume for cross slip and diffusion in pure aluminium and pure copper.

<table>
<thead>
<tr>
<th></th>
<th>aluminium</th>
<th>copper</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>activation energy ( eV )</td>
<td>activation volume/b^3</td>
</tr>
<tr>
<td>Cross slip</td>
<td>( 1.2 \ eV^{(1)} )</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>( 1.0 - 1.1 \ eV^{(2)} )</td>
<td>0.9 \pm 0.1 eV^{(7)}</td>
</tr>
<tr>
<td>Lattice diffusion</td>
<td>( 1.47 \ eV^{(3)} )</td>
<td>0.62^{(4)}</td>
</tr>
<tr>
<td>Boundary diffusion</td>
<td>( 0.87 \ eV^{(3)} )</td>
<td>--</td>
</tr>
</tbody>
</table>

1. Sherby et al., (1957); Dorn and Jaffe, (1961); Jaffe and Dorn, (1962).
Figure 7.1

Peak stress for pure aluminium and the composites at room temperature. a) and b) show typical values of the peak stress in tension and compression. The fibre volume fraction (in %) is shown for each curve.
Figure 7.2

Peak stress for pure aluminium and the composites at 77 K. a) and b) show typical values of the peak stress in tension and compression. The fibre volume fraction (in %) is shown for each curve.
Figure 7.3
A comparison of peak stress and mean stress curves for f=7% and 15% at (a and b) room temperature and (c and d) 77 K.
a) A schematic explanation of the residual elastic strain. In the unrelaxed composite the residual elastic strain is equal to the imposed plastic strain, \( e_p^* \). When relaxation occurs, the residual elastic strain decreases and \( e_p^* < e_p \).

b) Schematic variation of the residual elastic strain (or the mean stress) with imposed plastic strain (adapted from Brown, 1979).

1. Unrelaxed composite, \( e_p^* = e_p \).
2. Linear relaxation for \( e_p < e_p' \), non-linear relaxation for \( e_p > e_p' \).
3. No relaxation for \( e_p < e_p' \), complete relaxation for \( e_p > e_p' \).
Figure 7.5

The activation energy evaluated for the aluminium\Saffil composites by applying eq. 7.9 to the data in table 7.1a.

Figure 7.6

The activation volume evaluated for the aluminium\Saffil composites (scale on left side). The solid curve is the interfibre spacing, $s$, calculated by assuming a square packing of continuous aligned fibres with a diameter of $d = 3 \, \mu m$ (scale on right). The formula for $s$ is

$$s = \left[ \left( \frac{\pi}{4f} \right)^{\frac{1}{2}} - 1 \right] d$$
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8.1 Conclusions

The main conclusions of this thesis are as follows.

1. The Eshelby S tensor for a planar random array of continuous and discontinuous fibres has been calculated. The results complement previous calculations by other workers for different geometries of reinforcement.

2. The aluminium\Saffil composites have been found to be anisotropic in plastic strain because the fibres lie in the 2-3 plane and load is applied in the 3 direction. A method is proposed for quantifying the transverse plastic strains which consists of measuring the plastic strain of the composite parallel to orthogonal axes. The plastic strains thus obtained are used in the calculations of the mean stress hardening rate.

3. An analysis of cyclic Bauschinger experiments has been proposed which allows the mean and thermal stress to be separated. Such a separation is essential when the permanent softening is history dependent. The proposed analysis of the mean stress has been used to analyse the results of Bauschinger experiments. The prediction of the mean stress hardening rate, which is based on the mean field theory and the Eshelby S tensor for the composite, is in good agreement with experimental results for low fibre volume fractions ($f=7\%$ and 10\%) at 77 K. Less good agreement was found for higher volume fractions at 77 K and all fibre volume fractions at room temperature. The discrepancy between experiment and theory suggests that extensive relaxation of the mean stress has occurred, especially at room temperature. The relaxation is suppressed considerably at 77 K. The results also show that the degree of relaxation increases with fibre content.
4. The proposed separation of the thermal stress has been applied to the results from the cyclic Bauschinger experiments. The estimated magnitude of the thermal stress in virgin specimens is in quantitative agreement with measurements of the thermal stress in monotonic tests. The thermal stress is independent of fibre volume fraction for a given temperature and its value at 77 K is about a factor of two larger than at room temperature. The constancy of the thermal stress at a given temperature indicates that relaxation of the thermal stress increases with fibre content.

5. A model has been proposed for the thermally activated relaxation of the mean stress. An equation is derived for the number of Orowan loops per fibre and it is assumed in the model that the rate controlling mechanism of relaxation is cross slip of screw segments of Orowan loops. The estimated magnitude of the activation energy is found to be independent of fibre volume fraction and equal to 1.3 ±0.1 eV. The estimated activation volume decreases with increasing fibre content and is about three orders of magnitude larger than for diffusion in aluminium. The combined evidence for activation energy and volume suggests that the rate controlling mechanism of relaxation of the mean stress is cross slip of screw dislocations.

6. Pedersen's modified Orowan-Wilson model for matrix hardening in metal matrix composites has been applied to the experimental results and it is found to be inapplicable for two reasons. Firstly, the model requires that both the mean stress and the peak stress of the hysteresis loops are linear in plastic strain, but the peak stress curves for the aluminium/Saffil composites are generally non-linear. Secondly, the measured value of $\alpha/A$, which is the ratio of the plastic friction coefficient and the plastic mean stress coefficient, for those specimens which have linear peak stress curves, is about six times higher than the predicted value. This is probably due to either relaxation which reduces the rate of increase of the mean stress and peak stress in different proportions, or to the fact that the fibre diameter is close to the lower limit for applicability of the model.
8.2 Suggestions for further work

Some suggestions for further work will now be discussed.

1. The Eshelby S tensor for a planar random array of fibres has been calculated in this work. A direct extension is to calculate the S tensor for reinforcements which are random in three dimensions. This can be done by solving the general expression for the average of a tensor over all directions (eq. 3.6).

2. The analysis to separate the mean and thermal stresses in Bauschinger experiments has been shown to give unambiguous results for the mean stress hardening rate and the magnitude of the thermal stress in the aluminum\Saffil composites. A problem with the analysis is the interpretation of the non-zero intercept of the mean stress term when plotted against the plastic strain amplitude. To clarify the situation, the mean stress in a cyclic Bauschinger experiment should be measured by a direct method (e.g., X-ray or neutron diffraction) for comparison with the analysis used here which is based on the permanent softening.

3. The combined experimental evidence of the measured activation energy and activation volume supports the view that the rate controlling mechanism for relaxation of the mean stress is cross slip. One way of identifying the relaxation mechanism is to carry out TEM studies on a cycled material. If cross slip is the rate controlling mechanism of relaxation, it is expected that debris, in the form of prismatic loops, will be found in the matrix.
Appendices

Appendix 1: Calculation of the Eshelby $S$ tensor for a planar random array of fibres

Table A1.1 - A1.2
Figure A1.1

Appendix 2: Persistent slip bands in aluminium\Saffil composites

Figures A2.1 - A2.2

Appendix 3: Elastic constants

Table A3.1 - A3.2

Appendix 4: Derivation of the number of Orowan loops per fibre

Figures A4.1 - A4.3

Appendix 5: Published paper
Appendix 1

Calculations of the Eshelby S tensor for a planar random array of fibres

The problem is to find the solution of equation 3.8 for the 6561 terms involved. A personal computer was used for the calculations and the programming language was QuickBASIC. The method is suitable only for finding the average value of a fourth rank tensor in one plane, or in other words, computing the mean value of the tensor over all orientations in that plane.

The matrix for an anti-clockwise rotation of \( \theta \) in the 2-3 plane (fig. A1.1) is

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & \cos\theta & -\sin\theta \\
0 & \sin\theta & \cos\theta
\end{pmatrix}
\]  

(A1.1)

There are only five \( \lambda_{ij} \) terms which are generally not zero, two of which are equal \( \lambda_{22} = \lambda_{33} \). An overview of the steps in the calculation will now be given, and a detailed description follows.

1. First, all possible combinations of \( \lambda_{ij} \) (which are generally not zero) were generated and the integral calculated by hand. The value of the integral was then stored in a random access computer file.

2. A computer program was then written which checks all 6561 combinations and does the following for each combination.
   i) The program identifies the four lambda terms.
   ii) It then checks if any of them is zero. If so, the program skips this combination and goes to the next since zero terms are of no interest.
   iii) If none of the lambda terms is zero, the program finds the value of the integral using the file created in step 1.
   iv) If the value of the integral is not zero, the program prints out the value along with the 8 indexes.

These steps will now be described in more detail.

1. Before the computer program was written, the integration had to be solved by hand and the results stored in a random access file. Since the number of different non-zero lambda terms is only 4 (there are 5 non-zero lambda terms but two of them are equal) it was possible to generate by hand all (35) combinations of these 4 terms and calculate the
integral for each. A way had then to be found to make the results of the integration intelligible to the computer program. The solution to that problem is presented in step iii below. To facilitate the mapping of all combinations of the 4 lambda terms which are generally not zero, the letters a, b, c and d were assigned to each of them as shown here:

$$\begin{align*}
\lambda_{11} &= 1 : a \\
\lambda_{22} &= \lambda_{33} = \cos\theta : b \\
\lambda_{32} &= \sin\theta : c \\
\lambda_{23} &= -\sin\theta : d \\
\lambda_{12} &= \lambda_{13} = \lambda_{21} = \lambda_{31} = 0
\end{align*}$$

The number of different combinations of a, b, c and d, where order is not important, is 35. They are all given below.

$$S_{ijkl} = \frac{S_{mnop}}{2\pi} \int_0^{2\pi} \lambda_{im}\lambda_{jn}\lambda_{ko}\lambda_{lp} d\theta = \int_{mnop} \cdot \text{Integral} \quad (A1.2)$$

<table>
<thead>
<tr>
<th>$\lambda_{im}\lambda_{jn}\lambda_{ko}\lambda_{lp}$</th>
<th>Integral</th>
<th>$\lambda_{im}\lambda_{jn}\lambda_{ko}\lambda_{lp}$</th>
<th>Integral</th>
</tr>
</thead>
<tbody>
<tr>
<td>a$^4$ = 1</td>
<td>1</td>
<td>b$^4$ = $\cos^4\theta$</td>
<td>%</td>
</tr>
<tr>
<td>a$^3$b = $\cos\theta$</td>
<td>0</td>
<td>b$^3$a = $\cos^3\theta$</td>
<td>0</td>
</tr>
<tr>
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119
Of the 35 integrals, 21 are zero and 14 non zero. The next task was to write a program which can use the results of the integration calculated here.

2. The computer program which was written to tackle the computation, consists essentially of four nested loops for i, j, k and l, containing within them four nested loops for m, n, o and p. For each of the 81 sets of ijkl the program checks through all 81 combinations of mnop.

i) The next step was to devise a way of identifying the lambda terms by virtue of the 8 indices which appear in the integral (see step 1 above). Ideally, the two indices specifying each lambda term would be used directly by the program to identify each term. However, because the file containing the nine values of \lambda_{ij} has to be a random access file, each lambda has to be labelled with a single number (this is a feature of the programming language). The two indices i and j can therefore not be used directly. Instead a new label system was devised, as shown here:

\begin{align*}
\lambda_{ij} &= \begin{pmatrix}
11 & 12 & 13 \\
21 & 22 & 23 \\
31 & 32 & 33
\end{pmatrix} \\
&\rightarrow \\
&\begin{pmatrix}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9
\end{pmatrix}
\end{align*} 

(A1.3)

Hence, to calculate the new labels from the indices i and j:

- if i = 1 then new label = j
- if i = 2 then new label = j + 3
- if i = 3 then new label = j + 6

ii) Now that the lambda terms are known, the program checks if any of them is zero. As eq. A1.1 shows, 4 of the lambda terms are zero and therefore, if they appear in the integral (eq. A1.2 and 3.8), the whole term becomes zero and no further consideration is needed. To check this, a special matrix was designed, whose purpose was solely to find the zero lambda terms:

\begin{equation}
\lambda_{\text{test}} = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 1 \\
0 & 1 & 1
\end{pmatrix}
\end{equation}

The new labels calculated in this last step are used to obtain the values of the 4 corresponding components from this test matrix and their product calculated

\text{Product} = (\lambda_{im}\lambda_{jn}\lambda_{ko}\lambda_{lp})_{\text{test}}
If this product is zero, there is no need to consider this combination any further and the
program proceeds to the next combination. This step reduces dramatically both the files
needed to store the integrals and the computing time.

iii) The problem in this step was how to use a knowledge of the lambda terms to find
the value of the integral. The crux of the problem is that the order of the terms in the
integrand (eq. A1.2) does not matter and, in fact, the realisation of this leads to the
solution of the problem. The value of the integral can be found by taking the four new
indices specifying each combination and multiplying them together. The product is then
taken as the label of the file which stores the value of the integral. The order of the
lambda terms in a combination does not matter for the value of the product and hence, the
problem is solved.

One example of how this works is as follows. The combination $\lambda_{32}\lambda_{32}\lambda_{23}\lambda_{23}$ ($c^2d^2$)
stands for $\sin^4\theta$ and $\lambda_{32}$ and $\lambda_{23}$ have the labels 8 and 6 respectively. The product of the
new labels gives $\lambda_{32}\lambda_{32}\lambda_{23}\lambda_{23} \rightarrow 8 \times 6 \times 6 = 2034$. This is the number of the record in the
file which stores the value of the integrals (in this case %). As there are two lambda terms
which are $\cos\theta$ ($\lambda_{22}$ and $\lambda_{33}$), the number of possible combinations is 70, or twice that in
the first step.

iv) Finally, the program prints out the value of the integral together with the 8 indices.
For a given set of $ijkl$ there are 81 combinations of $mnop$. Most of these combinations are
zero, either because they contain one or more of the zero lambda terms, or because the
integral is zero. Hence, only 177 terms of the 6561 survive. The maximum number of
components for a given set of $ijkl$ is 8 (e.g. $S^{2222}$). The 177 components are given at the
end of this appendix in table A1.1 and the computer program is given in table A1.2.

It should be noted that the results derived here can be used to find the average of
any fourth rank tensor in the 2-3 plane. The raw results of the calculations are given in
table A1.1, which contains all 177 terms in eq. 3.8 which have a non-zero integral. Brown
and Clarke (1975 and 1977) calculated the $S$ tensor for continuous and short fibres aligned
parallel to the loading axis (the 3 axis). The information in table A1.1 is then used on the
continuous and short fibre $S$ tensor to find their planar average. A few examples of how
the transformed $S$ tensor is constructed are given below.

The lines in table A1.1 are numbered from 1 to 177 for convenience. The first
component is $S_{1111}$ and the table contains only one line (Nr. 1) with the combination 1111
for $ijkl$. In that line the value of the integral, which is the coefficient of the "old" $S_{1111}$ term, is 1 and hence,

$$S'_{1111} = 1 \cdot S_{1111}$$

Here only one term survives the transformation, the other 80 are zero. The primed components are those of the (new) transformed planar tensor and the components without prime are those of the (old) aligned fibre tensor, as calculated by Brown and Clarke (1975, 1977) and given in table 2.1. The transformation does then not change this component.

The second component is $S_{1122}$ and for $ijkl$ there are two lines with that combination of $ijkl$ (lines 2 and 3). The integral in both lines is 0.5, so

$$S'_{1122} = 0.5 \cdot S_{1122} + 0.5 \cdot S_{1133}$$

Inserting the values for $S_{1122}$ and $S_{1133}$ from table 2.1 into this expression yields

$$S_{1122} = \frac{8v-1}{16(1-v)}$$

The third example is $S'_{2222}$ which is more complicated to calculate because there are 8 lines in the table with $ijkl$ as $2222$ (lines 36 to 43). Using the table to find the value of the integrals for each term gives

$$S'_{2222} = \frac{1}{6} S_{2222} + \frac{1}{6} S_{2233} + \frac{1}{6} S_{2323} + \frac{1}{6} S_{3232} + \frac{1}{6} S_{3322} + \frac{1}{6} S_{3333}$$

$$S_{2222} = \frac{23-16v}{64(1-v)}$$

The resulting Eshelby $S$ tensor for a planar array of fibres, is given in table 3.1 for continuous and short fibres.
The values of the integral in eq. A1.2 (see also eq. 3.8) for all the (177) non-zero integrals. The program which was written to do the calculations is given in table A1.2. See text for an explanation of how the table is used.

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Number of non-zero terms is: 177
Total number of terms is: 6561

125
Table A1.2  The program which was written to solve eq. A1.2 (which is the same as eq. 3.8). Running the program results in a print-out which is given in table A1.1.

REM This program calculates the planar average of a fourth order REM tensor and takes the integral from 0 to 2pi. The file which REM stores the value of the integrals is called "INTEGRAL.DAT". REM The file storing the matrix used to check if any of the lambda REM terms are zero, is "LAMBDA.DAT". This program is called REM "FINALT.BAS" and was made by Birgir Johannesson in 1991.

OPEN "LAMBDA.DAT" FOR RANDOM AS #1 LEN = 2  ' Length for integer OPEN "INTEGRAL.DAT" FOR RANDOM AS #2 LEN = 4  ' Length for single precison

NUMI% = 0  
NUMJ% = 0  
NUMK% = 0  
NUML% = 0  
dummy% = 0  
NUMB% = 0

REM The program first prints the header of the printout, REM showing which column is what. The first column is REM the line number, the next 8 contain the indices and REM the last contains the value of the integral.

LPRINT TAB(15); "Nr."; SPC(7); "i"; SPC(3); "j"; SPC(3); "k"; SPC(3); "l"; SPC(5); "m"; SPC(3); "n"; SPC(3); "o"; SPC(3); "p"; SPC(5); "Integral"

REM Here the loops start.

FOR I = 1 TO 3  
FOR J = 1 TO 3  
FOR K = 1 TO 3  
FOR L = 1 TO 3  
FOR M = 1 TO 3  
FOR N = 1 TO 3  
FOR O = 1 TO 3  
FOR P = 1 TO 3  
NUMB% = NUMB% + 1  ' NUMB% will increase to 6561

IF I = 1 THEN  
    NUMI% = M  ' The new indices for
ELSEIF I = 2 THEN  
    NUMI% = M + 3  ' the lambda terms are
ELSEIF I = 3 THEN  
    NUMI% = M + 6
END IF

IF J = 1 THEN  
    NUMJ% = N
ELSEIF J = 2 THEN  
    NUMJ% = N + 3
ELSEIF J = 3 THEN  
    NUMJ% = N + 6
END IF

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<th>Table A1.2</th>
<th>The program which was written to solve eq. A1.2 (which is the same as eq. 3.8). Running the program results in a print-out which is given in table A1.1.</th>
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<td>REM</td>
<td>This program calculates the planar average of a fourth order tensor and takes the integral from 0 to 2π. The file which stores the value of the integrals is called &quot;INTEGRAL.DAT&quot;. The file storing the matrix used to check if any of the lambda terms are zero, is &quot;LAMBDA.DAT&quot;. This program is called &quot;FINALT.BAS&quot; and was made by Birgir Johannesson in 1991.</td>
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<td>REM</td>
<td>The program first prints the header of the printout, showing which column is what. The first column is the line number, the next 8 contain the indices and the last contains the value of the integral.</td>
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<td>TAB(15); &quot;Nr.&quot;; SPC(7); &quot;i&quot;; SPC(3); &quot;j&quot;; SPC(3); &quot;k&quot;; SPC(3); &quot;l&quot;; SPC(5); &quot;m&quot;; SPC(3); &quot;n&quot;; SPC(3); &quot;o&quot;; SPC(3); &quot;p&quot;; SPC(5); &quot;Integral&quot;</td>
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<td>REM</td>
<td>Here the loops start.</td>
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<td>= NUMB% + 1  ' NUMB% will increase to 6561</td>
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<td>NUMI% = M  ' The new indices for</td>
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<td>ELSEIF</td>
<td>I = 2 THEN</td>
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<td>NUMI% = M + 3  ' the lambda terms are</td>
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<td>NUMI% = M + 6</td>
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<td>J = 3 THEN</td>
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<td>NUMJ% = N + 6</td>
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<td>END IF</td>
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IF $K = 1$ THEN
    NUMK% = 0
ELSEIF $K = 2$ THEN
    NUMK% = $O + 3$
ELSEIF $K = 3$ THEN
    NUMK% = $O + 6$
END IF

IF $I = 1$ THEN
    NUML% = $P$
ELSEIF $I = 2$ THEN
    NUML% = $P + 3$
ELSEIF $I = 3$ THEN
    NUML% = $P + 6$
END IF

REM File #1 is "LAMBDA.DAT" which is used to check if
REM any of the lambda terms are zero
GET #1, NUMI%, LAMBDAIM%
GET #1, NUMJ%, LAMBDAJN%
GET #1, NUMK%, LAMBDAKO%
GET #1, NUML%, LAMBDAHP%

VALUE% = LAMBDAIM% * LAMBDAJN% * LAMBDAKO% * LAMBDAHP%
REM If VALUE% = 0, there is no need to proceed further
REM and the program can go to the next combination. The
REM statement in the next line checks if VALUE% is zero.
IF VALUE% <> 0 THEN ' If it is not zero,
    ' then continue here

PRODUCT% = NUMI% * NUMJ% * NUMK% * NUML%
REM "PRODUCT%" is the number of the record in "INTEGRAL.DAT"
REM which stores the value of the integral
GET #2, PRODUCT%, integral!
REM The program now has the value of the integral and
REM provided it is not zero it prints it out along with
REM the indices. The first column contains the line
REM number in the table and the last contains the integral.
IF integral! <> 0 THEN ' carry on
    dummy% = dummy% + 1
    'To keep track of
    'non-zero elements
REM The purpose of the next two IF statements is to print
REM the header again at the beginning of a new page in
REM the printout
IF dummy% = 61 THEN
    LPRINT TAB(15); "Nr."; SPC(7); "i"; SPC(3); "j";
    LPRINT SPC(3); "k"; SPC(3); "l"; SPC(5); "m";
    LPRINT SPC(3); "n"; SPC(3); "o"; SPC(3); "p";
    LPRINT SPC(5); "Integral"
    LPRINT
END IF
IF dummy% = 121 THEN
LPRINT TAB(15); "Nr."; SPC(7); "i"; SPC(3); "j"; :
LPRINT SPC(3); "k"; SPC(3); "l"; SPC(5); "m"; :
LPRINT SPC(3); "n"; SPC(3); "o"; SPC(3); "p"; :
LPRINT SPC(3); "Integral"
LPRINT
END IF

REM And finally, here comes the result
LPRINT TAB(15); dummy%; TAB(24); I; SPC(1); J; :
LPRINT SPC(1); K; SPC(1); L; SPC(3); M; SPC(1); :
LPRINT N; SPC(1); O; SPC(1); P; SPC(4); integral!
END IF

END IF

NEXT P
NEXT O
NEXT N
NEXT M
NEXT L
NEXT K
NEXT J
NEXT I

LPRINT TAB(15); "Number of non-zero terms is:"; SPC(3); dummy%
LPRINT TAB(15); "Total number of terms is";
CLOSE #1
CLOSE #2
END
Figure A1.1

Anti-clockwise rotation of $\theta$ in the 2-3 plane.
Appendix 2

Persistent slip bands in aluminium\Saffil composites

During preliminary experiments on composites cycled to high cumulative plastic strains, it was noted that slip bands appeared on the surface of the aluminium\Saffil composites. To establish whether these slip bands are persistent or not, a controlled experiment was carried out at room temperature on a composite specimen with fibre volume fraction \( f = 10\% \).

A specimen (no. RT108) was polished by the method described in section 4.4.2 and after polishing, the specimen was annealed at 350°C in a vacuum for one hour. It was then cycled at a plastic strain amplitude of 0.3% for 550 cycles and the surface photographed in an SEM microscope. The specimen was repolished and the surface photographed again in the SEM. Finally, the specimen was recycled at the plastic strain amplitude of 0.3% for 221 cycles and the surface photographed in the SEM for the third time.

Fig. A2.1 shows SEM images of (a) a polished surface of a specimen which had not been fatigued and (b) the surface of specimen RT108 after the initial cycling at 0.3% plastic strain amplitude. Slip bands in the matrix are clearly visible as bright lines in fig. A2.1b and they can easily be distinguished from polishing scratches which run roughly parallel to the axis of loading (which is horizontal in the figures).

The photographs taken of the surface of the specimen after each stage of the experiment are shown in figs. A2.2a-f. The slip lines (a and b) are clearly seen to consist of extrusions and intrusions, indicating repeated slip. The specimen was then polished to remove the slip lines but the same areas can be identified readily (c and d). The specimen was recycled at the same plastic strain amplitude and some of the slip lines which appeared after the first set of cycles reappear (e and f). The small number of cycles here could be the reason why not all the slip extrusions have reappeared and the mechanical polishing is also bound to introduce some damage into the surface which might disrupt the repeat formation of all of the extrusions.

The conclusion of this experiment is that persistent slip bands can form in aluminium\Saffil composites in cyclic deformation.
SEM images of composite specimens with f=10%. a) Surface after polishing. b) Surface after polishing, annealing and 550 cycles at a plastic strain amplitude of 0.3%. The nearly horizontal polishing scratches in both a) and b) are approximately parallel to the loading axis. The slip bands in b) can easily be distinguished from polishing scratches.
Figure A2.2

SEM images from surface of a composite specimen (f=10%) cycled at room temperature at a plastic strain amplitude of 0.3%. a) and b): After polishing, annealing and 550 cycles. c) and d): The same areas as in a) and b) after polishing to remove the slip extrusions. e) and f): Same areas again after further 221 cycles. Some of the extrusions reappear.
Appendix 3
Elastic constants

In this appendix it is shown how the stiffness matrices of the fibres, \( C_p \), and matrix, \( C_M \), are obtained and some relations used in this thesis between the elastic moduli are given. It is assumed that the fibres and matrix are elastically isotropic. The treatment here is based on work by Nye (1990) and Kelly and Macmillan (1986).

In elastic deformation the general relation between stress \((\sigma_{ij})\) and strain \((e_{ij})\) is

\[
\sigma_{ij} = \mathbf{C}_{ijkl} e_{kl}
\]

(A3.1)

where all indices assume the values 1, 2 or 3 and the summation convention applies. \( C_{ijkl} \) is the stiffness tensor and summation is taken over repeated indices. Because of the symmetry of the stress and strain tensors it is possible to write eq. A3.1 in a contracted matrix notation (Nye, 1990). The stress and strain tensors are then written as

\[
\begin{pmatrix}
\sigma_{11} & \sigma_{12} & \sigma_{13} \\
\sigma_{21} & \sigma_{22} & \sigma_{23} \\
\sigma_{31} & \sigma_{32} & \sigma_{33}
\end{pmatrix}
\quad
\begin{pmatrix}
e_{11} & e_{12} & e_{13} \\
e_{21} & e_{22} & e_{23} \\
e_{31} & e_{32} & e_{33}
\end{pmatrix}
\]

(A3.2)

and eq. A3.1 for elastically isotropic materials becomes

\[
\begin{pmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\sigma_4 \\
\sigma_5 \\
\sigma_6
\end{pmatrix}
= \begin{pmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{pmatrix}
\begin{pmatrix}
e_1 \\
e_2 \\
e_3 \\
e_4 \\
e_5 \\
e_6
\end{pmatrix}
\]

(A3.3)

with

\[
C_{ii} = 2G + \lambda \\
\text{if } i = 1, 2 \text{ or } 3,
\]

(A3.4)

\[
C_{ii} = \frac{1}{2}(C_{11} - C_{12}) = G \\
\text{if } i = 4, 5 \text{ or } 6,
\]

(A3.5)

\[
C_{12} = C_{13} = C_{23} = \lambda
\]

(A3.6)

and

\[
C_{ij} = C_{ji}
\]

(A3.7)
\( \lambda \) is the Lame constant and \( G \) the shear modulus. There are only two independent \( C_{ij} \) constants (\( C_{11} \) and \( C_{12} \)) and they can be expressed in terms of any two of \( G, E \) and \( v \), where \( E \) is the Young's modulus and \( v \) the Poisson's ratio.

The Young's modulus and Poisson's ratio used here are 70 GPa and 0.346 for aluminium (Ashby and Brown) and 285 GPa (Birchall, 1983) and 0.233 (Ashby and Brown, 1981) for alumina, respectively. Some relations between the elastic moduli of isotropic materials are given in Table A3.1 and the elastic moduli and stiffness matrices of aluminium and alumina are given in Table A3.2.

Table A3.1. Relations between elastic moduli of an isotropic solid (from Kelly and Macmillan, 1986).

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Expressed in terms of</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( E, v )</td>
</tr>
<tr>
<td>( E )</td>
<td>Young's modulus</td>
</tr>
<tr>
<td></td>
<td>( E )</td>
</tr>
<tr>
<td>( v )</td>
<td>Poisson's ratio</td>
</tr>
<tr>
<td></td>
<td>( v )</td>
</tr>
<tr>
<td>( G )</td>
<td>Shear modulus</td>
</tr>
<tr>
<td></td>
<td>( G )</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Lame's constant</td>
</tr>
<tr>
<td></td>
<td>( \lambda )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Aluminium</th>
<th>Alumina</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E )</td>
<td>70.0 GPa</td>
<td>285 GPa</td>
</tr>
<tr>
<td>( G )</td>
<td>26.0 GPa</td>
<td>115.6 GPa</td>
</tr>
<tr>
<td>( v )</td>
<td>0.346</td>
<td>0.233</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>58.4 GPa</td>
<td>100.9 GPa</td>
</tr>
<tr>
<td>( C_{11} )</td>
<td>110.5 GPa</td>
<td>332.0 GPa</td>
</tr>
<tr>
<td>( C_{12} )</td>
<td>58.4 GPa</td>
<td>100.9 GPa</td>
</tr>
<tr>
<td>( C_{44} )</td>
<td>26.0 GPa</td>
<td>115.6 GPa</td>
</tr>
</tbody>
</table>
Appendix 4

Derivation of the number of Orowan loops per fibre

In this appendix the equation which relates the number of Orowan loops per fibre, \( n_f \), to the tensile plastic strain, \( \varepsilon_p \), is derived (eq. 7.2).

In the composites, the fibres are arranged in a planar random array. However, to simplify the problem, it is assumed that the fibres are unidirectional, and aligned parallel to the loading axis. A consequence of this is that the expression for the number of loops derived here will be an upper limit for the aluminium\(\backslash\)Saffil composite.

The geometry used in the calculation is shown in figs. A4.1 and A4.2. Fig. A4.1 shows the cross section (area \( d \times d \)) and fig. A4.2 shows two slip planes spaced a distance \( h \) apart. Fig. A4.3 shows a composite with a single fibre of length \( l_f \). It is assumed that the fibres have a square packing, that the fibre diameter is \( 2r \) and the spacing between the fibres is \( s \). Dislocations with Burgers vector \( b \) glide on planes which are at \( 45^\circ \) to the loading axis, and \( n_f \) is derived in terms of the shear strain, \( \gamma_p \). The results can then be expressed in terms of the tensile plastic strain \( \varepsilon_p \).

The first step is to calculate the tensile strain which results if \( N \) dislocations glide on average a distance \( <x> \), each dislocation gliding a distance \( x_i \). The average distance \( <x> \) can be written as

\[
<x> = \frac{1}{N} \sum_{i=1}^{N} x_i
\]  

(A4.1)

The shear displacement which results if a dislocation moves by \( x_i \) is (note that \( 0 \leq x_i \leq \sqrt{2}d \))

\[
D_i = \frac{b}{\sqrt{2}d} x_i
\]  

(A4.2)

The shear displacement resulting from \( N \) dislocations moving on average a distance \( <x> \) is therefore

\[
D_N = \frac{b}{\sqrt{2}d} \sum_{i=1}^{N} <x_i>
\]  

(A4.3)

and using eq. A4.1

\[
D_N = \frac{Nb}{d} <x>
\]  

(A4.4)
Hence, the shear strain is (see fig. A4.2)

\[ \gamma_p = \frac{D_N}{h \sqrt{\frac{1}{2}}} = \frac{Nb}{hd} <x> \quad (A4.5) \]

The next step is to calculate the number of fibres, \( n_f \), which a dislocation passes when it moves a distance \( x_i \). In an unrelaxed composite, this number is equal to the number of Orowan loops left behind by the dislocation. The number of fibres in a length \( d \) parallel to the sides of the specimen is

\[ N_d = \frac{d}{(2r + s)} \quad (A4.6) \]

When a dislocation moves a distance \( x_i \) on a glide plane at 45° to the tensile axis, it passes \( n_i \) fibres, where

\[ n_i = N_d \left( N_d \frac{x_i}{\sqrt{2}d} \right) \quad (A4.7) \]

Using eq. A4.6 for \( N_d \) gives \( n_i \) as

\[ n_i = \frac{dx_i}{\sqrt{2}(2r + s)^2} \quad (A4.8) \]

If \( N \) dislocations move on the average by \( <x> \), the number of Orowan loops, \( n_N \), can be found by using eqs. A4.8 and A4.1, so that

\[ n_N = \sum_{i=1}^{N} n_i = \frac{d \sum_{i=1}^{N} x_i}{\sqrt{2}(2r + s)^2} = \frac{dN}{\sqrt{2}(2r + s)^2} <x> \quad (A4.9) \]

Combining eqs. A4.5 and A4.9 to eliminate \( <x> \) gives

\[ \gamma_p = \frac{Nb}{hd} <x> = \frac{N b \sqrt{2} n_N (2r + s)^2}{hd} \frac{dN}{h} \frac{(2r + s)^2}{d^2} \quad (A4.10) \]

But the total number of fibres in a cross section of the composite is

\[ N_f = \frac{d^2}{(2r + s)^2} \quad (A4.11) \]

so eq. A4.10 becomes

\[ \gamma_p = \frac{\sqrt{2}b}{h} \left( \frac{n_N}{N_f} \right) \quad (A4.12) \]
$n_N$ is the total number of Orowan loops, associated with a length $h$. The total number of Orowan loops per fibre is $n_N(l_f/h)$ and if $N_f$ is the total number of fibres in a cross section of the composite, then the number of Orowan loops per fibre, $n_f$ is

$$n_f = \frac{n_N}{N_f} \frac{l_f}{h} \quad (A4.13)$$

Hence, the number of Orowan loops per fibre, expressed in terms of the plastic shear strain $\gamma_p$, is

$$n_f = \frac{l_f}{\sqrt{2} b} \gamma_p \quad (A4.14)$$

What now remains is to express $n_f$ in terms of the tensile plastic strain, $e_p$. Since the normal to the glide plane, and the glide direction are a $45^\circ$ to the tensile axis (see fig. A4.2),

$$\gamma_p = 2 e_p \quad (A4.15)$$

and hence

$$n_f = \frac{\sqrt{2} l_f}{b} e_p \quad (A4.16)$$
Figure A4.1  A schematic view of the composite cross section as it is assumed to be in the calculations. The aligned fibres have a diameter 2r and length l_f and are arranged in a square array with a separation s. The sides of the specimen are each of length d.

\[
e_p = \text{tensile strain} \\
\gamma_p = \text{shear strain} \\
e_p = \frac{\delta l}{h} \\
\gamma_p = \frac{\delta x}{h \sin \theta} \\
\cos \theta = \frac{\delta l}{\delta x} \\
\rightarrow e_p = \gamma_p \cos \theta \sin \theta
\]

Figure A4.2  Two slip planes spaced h apart, at an angel of \(\theta\) to the tensile axis. This figure shows how the relation between tensile and shear strains is derived.
Figure A4.3  A schematic diagram of a composite containing a single fibre of length $l_f = 2h$, with dislocations moving on planes whose normals are at $45^\circ$ to the loading axis. $h$ is the spacing between the glide planes. If one dislocation on each slip plane glides across the full width of the specimen, the number of Orowan loops per fibre is 2.
Appendix 5
Published paper

One paper has been published from the work presented in this thesis:

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