DETERMINATION OF SPATIAL DISTRIBUTION OF RADIONUCLIDES

IN ABSORBING MEDIA.

BY

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A Thesis submitted to the Faculty of Science of the University of Surrey for the degree of DOCTOR OF PHILOSOPHY.

AUGUST 1987.
To my parents,

and to my children

Shazia, Nadeem and Sa'adia.
Determination of the spatial distribution and quantification of concentration of pure beta- and photon-emitting radionuclides in absorbing media by external measurements is the subject of this study. Measurements of radiation and the operation of radiation detectors are based on the radiation interactions with matter and the theory governing these interactions has been discussed. Various techniques for localising pure beta- and photon-emitting radionuclides situated inside attenuating media have been suggested on a theoretical basis, and have been experimentally shown to work successfully.

Most of the work is on single photon emission computed tomography (SPECT). The theory of mathematical reconstruction of a two dimensional distribution from its projections is discussed and reconstruction techniques and their relative merits and demerits have been reviewed.

SPECT seeks the determination of absolute regional radionuclide concentrations as a function of time. A SPECT system has been developed by modifying an existing transmission CT scanner and the reconstruction algorithms. The performance of the SPECT system has been tested for a number of point sources and various extended sources in gas, liquid and solid forms. The SPECT scanner in its present design is capable of performing in both the transmission and emission modes. The characteristics of the SPECT scanner, including the detector efficiency, spatial resolution and the effect of collimator size, have been studied experimentally.

The major problems faced by SPECT include the solid angle effect, which influences the collection efficiency, in scattered radiation, and attenuation of photons inside the surrounding medium. These problems together with their various possible solutions have been discussed in
detail. Methods for compensation for solid angle variation, in scattered radiation and photon attenuation have been devised and used successfully to compensate the projection data.
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The use of radiology in medicine and industry started soon after the discovery of X-rays by Roentgen in 1895. Fifty years later following the commercial production of radioactive isotopes the in-depth localisation of radionuclides in absorbing media became equally important in medicine and industry. In medicine the depth of a radioactive source inside an organ has to be determined for the purpose of accurate body burden measurements, or the depth of a particular organ may need to be determined for surgery or for radiotherapy. In nuclear industry the study of spent fuel rods for fission products localisation or the assay of nuclear waste bins to localise the centres of activity are important applications of such measurement techniques. The problem of in-depth localisation of a radioactive source is equivalent to the measurement of the thickness of an attenuating layer between the source and the detector. Analysis of nuclear radiation spectra is widely used to obtain information about the radionuclide composition of a source material. However, a completely different approach is required if the spatial distribution of a given radionuclide is required. Since ionising radiations are attenuated in an energy dependent manner as they pass through matter the localisation of a radionuclide in an absorbing material can be done by analysis of the nuclear radiation spectrum that emerges at the surface of the absorbing medium.

Alpha emitters are difficult to localise in thick absorbers since alpha-particle are heavy charged particle of very short range (~μm), but such radionuclides can be located by their accompanying penetrating gamma-radiation in favourable cases.

The depth of a pure beta-emitting radionuclide can be estimated by
measuring the shift in the maximum energy of the continuous spectrum, caused by the energy loss of beta-particles on their way through matter. Meloni et al [1969] and Van De Geijn [1974, 1976] have used this method to determine the depth of pure beta-emitters in thin absorbers by analysing the beta spectrum at the surface of the absorber. Since beta-particles have fairly short range (-mm) in absorbing materials, this method is applicable only for samples of absorbing media where the thickness of the absorber is less than the maximum range of beta-particles. So, for the cases where the thickness of the absorber between source and detector is greater than the maximum range of beta-particles, one has to look for some other means of localising pure beta-emitters. In this case the depth of a beta-emitter in an absorber can be determined by external measurements of more penetrating radiation, bremsstrahlung, which is produced as the fast electrons are slowed down. Bengtsson [1964], Tubiana et al [1958] and Heidleburg et al [1963] have detected the beta-emitters distributed in large body organs by measuring externally the bremsstrahlung produced. Gaur and Vanketeswaran [1982] have suggested the use of bremsstrahlung spectra to estimate the depth of a point source in absorbing medium. The method used for the work reported in this thesis is based on external measurements of the shift in the bremsstrahlung spectra with increasing thickness of the absorber since the bremsstrahlung photons undergo some degree of attenuation while passing through the absorbing medium. For the purpose of body burden measurements where the beta-emitters have to be estimated quantitatively, this attenuation has to be taken into account. The source strength can be determined from external detection measurements when the depth of the source inside the media is known.

Positron emitters can be localised by the measurement of the annihilation radiation in a coincidence set-up.

Photon emitters in an absorbing medium can be localised by observing the
specific changes in the shape of their spectra caused by differing degree of attenuation between the source and detector. Mohindra and McNeil [1965] were the first to examine both the change in the widths of gamma-peaks and also the ratio of counts in the "valley" just below the full-energy peak to that of full-energy region counts (valley-to-peak ratio) as a function of source depth and photon energy. They found that the valley-to-peak ratio (or scatter-to-peak ratio) was more sensitive to source depth compared to the changes in the full-energy peak width, particularly at low energies. The scatter-to-peak ratio (SPR) was used to estimate the depth of Am-241 (60 keV) deposited in the lungs [Toohey, 1975]. The SPR was also used as a correction for thyroid depth with two uncollimated NaI(Tl) detectors [Wellman and Kereiakes, 1967] and as an organ depth attenuation correction factor using a single collimated NaI(Tl) detector [Meneely et al, 1962] although geometrical considerations required that the source area be smaller than the detector field of view. Its first use with a large-field, parallel-collimated gamma camera which was capable of storing data from a dual energy window (one for scattered radiation, the other for full-energy peak events) was in a quantitative renal uptake study with $^{197}$HgCl$_2$ [Kacperek et al, 1975].

The depth of a photon source can also be determined from the differential attenuation of the radiation from a multi-energy photon source. This technique was proposed by Dolan and Tauxe [1967, 1968] for kidney depth measurements using a solution labelled with I-125 (35keV) and I-131 (364keV), with collimated counters and extended to a rectilinear scanner to image the ratio of counts in the two full-energy peaks in a water phantom. The technique was applied to tomography using labelled I-125/I-131 sodium iodohippurate and depth differences of kidney down to 10mm were observed [Ostrowski and Tothill, 1975]. Kaplan and Ben-Porath [1968] have described a method of depth determination by colour modulation using I-125 and I-131. Oldendorf and Isaka [1969] also used the differential attenuation technique for depth determination. Koral
and Johnston [1977] studied the change in the energy spectrum of an I-131 point source with depth in a water tank. Martin and Rollo [1977] measured the scatter from the 159keV full-energy peak of I-131. They also measured the ratio of the counts in the Compton scatter region (82-112 keV) to those in the full-energy peak region (136-175 keV), and found only a small variation in the SPR with depth. They also employed the differential attenuation of 27keV (X-rays) and 159keV peaks from I-123 to measure the depth, and suggested that this was the more sensitive method. Filipow et al [1979] studied the proportion of scattered radiation in the spectrum with depth from point sources of Xe-133 (28, 81keV), Tc-99m (140keV) and Cr-51 (320keV) in water and perspex with NaI(Tl) and Ge(Li) spectrometers and an Anger camera. They concluded that the counting rate in the Compton scatter region increased more rapidly than that in the "valley".

The advantages of the SPR method for organ depth measurement are apparent. The original scan data is used for the correction, therefore limiting the total patient scan time to one scan, during which the patient position is unlikely to change whereas if a second test or scan is performed, for example for a profile depth scan, the organ of interest may not still lie at the same depth, there is no requirement for any additional radioisotope to be injected into the patient and therefore no extra dose (unlike the doubly labelled I-125/I-131 solution) since the energy spectrum of the radioisotope used in the study is employed. [Kouris et al, 1982]. Modern gamma cameras with more powerful computing facilities should allow fuller exploitation of the method.

Computed tomography (CT) is probably the most significant development in the history of medical imaging since the discovery of X-rays by Roentgen in 1895. The basic principle of CT is that the internal structure of an object can be reconstructed from an infinite set of all possible projections of the object. Tomographic techniques for removing
or minimising the superposition of information with depth has been the goal of a number of systems for producing an image of a desired section of an object at a given depth. This goal has been achieved with the advent of CT. So remarkable is the CT technique that in many cases it generates a dramatic increase in diagnostic information over commercial X-ray shadowgraph techniques. In trying to describe its significance, one may be tempted to use the term "diagnostic breakthrough". Perhaps this can be best appreciated by first considering the earlier imaging techniques.

Conventional focal plane tomography was introduced by Bocage in 1921 in which the X-ray source and film are moved in such that only one plane in the object projects a stationary image and all others become blurred. The image however still contains much unwanted information which reduces contrast of the desired section.

Radon in 1917 developed a mathematical inversion technique relating a set of projections to the original object using Fourier based reconstruction techniques. The first application of mathematical reconstruction was in radioastronomy [Bracewell, 1956] which was later followed in several other fields including optics and electron microscopy [Rowley, 1969; De Rosier and Klug, 1968].

Cormack [1963] developed a tomographic scanner comprising of a Co-60 source and a Geiger detector to perform linear scans at discrete angles around phantoms and produced a cross-sectional image of attenuation coefficients. At about the same time Kuhl and Edwards [1963] developed a scanner that performed linear scans at discrete angles around human objects. Although their algorithm then did not yield an exact reconstruction, it was the first example of a transverse section reconstruction of a radionuclide distribution. The first practical CT machine was commercially realised with the introduction of the EMI brain
scanner in 1972, due largely to the work of Hounsfield [1969]. It was the successful development of EMI X-ray scanner that stimulated a great deal of research and development, and new applications over the past fifteen years. Until now most of the major developments in CT have been in the medical field and it is now an established medical diagnostic tool although its high cost is a barrier to universal use.

The first non-destructive testing (NDT) application of CT was made by Sweeney [1974, 1975]. Several other research groups have since demonstrated the viability of industrial CT [Kruger et al, 1981; Sanderson, 1979; Schlosser et al, 1980; Hopkins et al, 1981; Gilboy et al, 1982; MacCuaig et al, 1985].

Although the mathematical reconstruction of images was first applied in non-medical fields [Bracewell, 1956; De Rosier and Klug, 1968], since then the majority of the research studies and hence the major advancements have been made with respect to the medical imaging applications. However, more recently as CT has become more widely appreciated, the range of applications in non-medical fields, particularly for non-destructive tests in industry, has expanded [Kruger, 1981; Sanderson, 1979; Hopkins et al, 1981; Hunt, 1981; Gilboy et al, 1982; Burstein et al, 1984; Goebbels et al, 1984].

CT can now be divided into two main techniques: Transmission Computed Tomography (TCT) and Emission Computed Tomography (ECT). The major difference between TCT and ECT lies in the fact that ECT seeks to describe the location and intensity of sources of emitted photons in an attenuating medium whereas TCT seeks to determine the distribution of the attenuating medium. ECT was conceived and developed long before X-ray CT, but was not considered a feasible standard imaging technique for a long time. Two distinct modalities of ECT exist: positron emission tomography (PET) and single photon ECT (SPECT).
PET systems depend on the detection of coincident annihilation radiation that accompanies positrons. Positrons in turn are emitted by certain, mostly short-lived, radionuclides. Modern PET systems consist of many detectors surrounding the radioactive object which enables a relatively high detection efficiency to be attained. PET has been very successful, particularly as a research tool in physiology and microbiology and it has the advantage of exploiting physiologically important radionuclides such as C-11, N-13, O-15 and F-18. However most short-lived positron emitters presently in use can only be produced with expensive on-site particle accelerators. On the other hand SPECT has the practical advantage of using commercially available radiopharmaceuticals. SPECT is widely used in nuclear medicine and has great potential for industrial applications. Phillips et al [1981] have used SPECT to measure the distribution of fission products in spent fuel rods from a light water nuclear reactor (LWR) in order to study fuel rod failures caused by pellet-clad interaction.

SPECT is likely to become more important in the near future. This is helped by a concomitant rapid improvement in the computer hardware and software, together with an increasing number of accepted clinical and industrial applications. But despite the renewed interest and progress in SPECT, it still suffers several limitations that pose an obstacle in the determination of absolute regional radionuclide concentrations as a function of time. Collection efficiency, scattered radiation, attenuation and photon utilisation are the fundamental limitations in emission tomography coupled with practical limitations due to non-uniform detector response with depth and poor counting statistics [Keyes, 1981]. Counting statistics limitations are imposed by dose constraints in the case of medical imaging, and by highly attenuating media in case of industrial applications. The use of multi-detector arrays will improve tremendously the collection efficiency and reduce the statistical error.
as well the scanning time both for transmission and emission tomography compared to single detector systems but it will be more costly owing to the large number of detectors required. The high cost of existing medical machines is one important reason why CT has yet to make an impact in industry [Folkard, 1983]. Although the potential and to some extent the applications of each imaging modality that exists today is judged by the parameter measured rather than image resolution or display quality the constraints of total cost will inevitably force decisions to be made between imaging modalities (Brownell, 1982,1984).

The problem of scattered photons being detected in the full-energy peak window is inherent in practical emission (and to a certain extent in transmission) tomography. The scattered photons, if included in the reconstructed image, result in a degraded image, i.e. loss of resolution and contrast. The image degrading effects of scattered photons have been recognised for sometime in nuclear medicine. However, it was not until recently that there has been wide recognition in other imaging areas, e.g. tomography, that the image quality not only depend on the number of the photons recorded but also, and more crucially, on the origin of the photons with respect to the interaction they have undergone before detection [Sanders, 1982]. The number of scattered photons accepted within the full-energy peak window depends on its width and therefore the precise selection of the window. However, at present in commercial scanners the discriminator energy window is normally selected to increase the total recorded counts and no consideration is given to the number of scattered photons contributing to this window. Different methods to eliminating the scatter component has been described with their merits and demerits, theoretically as well as experimentally.

The problem of detector solid angle can be overcome by using the opposed detector arrangement and averaging the values of the two complementary raysums.
The most difficult and fundamental problem in SPECT is the compensation for attenuation of photons inside the radioactive object. In SPECT the projection data results from line integrals of products of source distribution and attenuation along straight lines. Inscattered radiation and geometrical variations response of the detector produce further deviations of the projection data from the correct values. However, these effects are less important than the effect of attenuation. Therefore attenuation compensation is an important consideration in the development of SPECT systems. The effect of photon attenuation is a loss of information content because the intensity of photons is not the true intensity emitted by the source. The degree of attenuation depends on the energy of the emitted photons, the constituents of the surrounding medium and its density and thickness. Ignoring the attenuation and performing the reconstruction using the uncorrected data results in both an attenuated source activity and an erroneous spatial distribution. This may be useful for some qualitative clinical studies [Murphy et al, 1979] but is not adequate for quantitative studies [Lewis et al, 1982] especially bearing in mind that the ultimate goal of SPECT is to determine the absolute regional radionuclide concentration as a function of time.

Different approaches to solving the problem of attenuation have been suggested by different workers but none so far can claim to be universally applicable. The attenuation corrections can be applied prior to the reconstruction (pre-correction techniques), during reconstruction (intrinsic techniques) or after the reconstruction (post-correction techniques). Pre-correction techniques [Budinger and Gulberg, 1977; Kay and Keyes, 1975; Budinger et al, 1979] and post correction techniques [Ruhl et al, 1973; Chang, 1978] are currently used in SPECT systems. Pre-correction techniques are easy to use and are quite adequate for the uniform attenuation case but are not quantitative for variable
attenuation cases. Intrinsic techniques [Budinger and Gulberg, 1974; Hseih and Wee, 1976; Bellini et al, 1979; Gulberg, 1979; Walters et al, 1981, Moore et al, 1982; Tanaka, 1983] properly model the attenuation process and have the potential for accurate quantification of the source distributions, but are not as yet been appreciated in clinical studies.

The rapid technical advancement of transmission tomography especially in the medical field has overshadowed the intrinsic advantages that SPECT has over it. TCT deals mostly with anatomy and structure whereas SPECT has the ability to investigate physiology and kinetics. Some of the reasons for this eclipse are the problems that are still facing SPECT systems which have been mentioned earlier. Both SPECT and TCT suffer from the high cost of the equipment which limits widespread use. Nevertheless the future of SPECT is very encouraging. It has not only been used in the medical field, but has also been used in other fields such as in non-destructive testing to determine the axial and radial distribution of fission products in irradiated fuel rods. This recent development can be of use to nuclear power plant personnel with interests and responsibilities in non-destructive inspection, core physics or fuel performance. Investigations are progressing to use SPECT as a tool for elemental analysis and distribution by utilising the emission of gamma or X-rays emitted from an object as a result of it being irradiated [Balogun, 1986; Rusminarto, 1986].

The scope of this study is to investigate methods to localise and quantify pure beta- and photon-emitters inside absorbing media by external measurements. Most of the work has been concentrated on SPECT. This includes the modification of the transmission scanner to perform in both modes of operation, transmission and emission, and also a review, analysis and investigation of the problems being faced by SPECT and to suggest their possible solution.
Chapter two describes the depth discrimination of pure beta-emitters in an absorbing medium by external measurements of the shift of the maximum in bremsstrahlung spectra. It describes the interactions of beta-particles with matter, range of beta-particles, experimental procedure, results and discussion of the results.

Chapter three describes various depth determination techniques for photon emitting radionuclides in attenuating media. This includes a description of photon interactions with matter, depth determination techniques, experimental procedures, results and discussion of the results.

Chapter four provides more insight into the mathematics of computed tomography paying special attention to SPECT. It includes the description of various mathematical reconstruction techniques, the problem of solid angle of detection, the attenuation of photons and inscattered radiation, and their possible solutions, giving their advantages and disadvantages.

Chapter five describes the characteristics of the SPECT scanner. It describes the different parts of scanner, modification of the transmission scanner to operate in both transmission and emission modes, studies of spatial resolution, effect of collimator length and thickness, and a description of the various radioactive samples used for SPECT work.

Chapter six presents and discusses results from the SPECT scanner. It includes the results of solid angle, inscatter and attenuation correction techniques. It also includes the images from simulated phantom data using iterative reconstruction and attenuation compensation techniques.

Chapter seven concludes this thesis as well as providing some suggestions for future work.
DEPTH DETERMINATION OF A PURE BETA-EMITTER IN A MEDIUM BY BREMSSTRAHLUNG MEASUREMENTS.

2.1 INTRODUCTION

The in-depth localization of a pure beta emitting radionuclide may be determined by measuring the shift in the maximum energy of the continuous spectrum, caused by the energy loss of the beta-rays on their way through the matter [Meloni et al, 1969, Van de Geijn 1974, 1976]. According to Landau [1944] the most probable energy loss depends upon the thickness and density of the material, the primary energy of particles and, to a smaller extent, upon the chemical composition of the material. This method is applicable for samples of absorbing media whose thickness is less than the maximum range of the beta-particles but where the beta emitting radionuclide is sited at a greater depth than the maximum range of beta particles one has to look for other methods.

The depth of a pure beta emitter inside the medium can be estimated by measuring externally the more penetrating bremsstrahlung which is produced [Gaur and Venketswaran, 1982]. The bremsstrahlung photons undergo some degree of attenuation while passing through the medium. For the purpose of body burden measurements where the beta emitters have to be estimated quantitatively, this attenuation has to be taken into account. Tomographic methods are also sometimes used to measure the source strengths for sources distributed inside a medium. The source strength can be deduced from external detector measurements when the depth of the source inside the media is known. It is necessary first to estimate the depth at which the radionuclide is situated. The method used in this experiment is based on the shift of the intensity maximum in the experimentally determined bremsstrahlung spectrum. The studies were
carried out with P-32 (E_{\text{max}} = 1700 \text{ keV}) a pure beta-emitter and Kr-85 a beta-gamma-emitter with maximum beta energy, E_{\text{max}} = 670 \text{ keV}, and gamma-ray energy of 514 keV.

To understand the behaviour of beta particles inside the absorbing medium a knowledge of the interaction processes they undergo is essential, and is discussed in the next section.

2.2 INTERACTION OF BETA-PARTICLES WITH MATTER

Beta-particles are electrons of continuous energy spectrum and are emitted from nuclei in the radioactive decay process. During its passage through material each electron from a beam of beta-rays frequently undergoes interactions with the surrounding atomic electrons and nuclei. Each interaction may cause an angular deflection and the loss of part of the kinetic energy of the penetrating electron. The energy transfer in a single electron-electron interaction can have values up to half the initial energy, but interactions involving small energy exchange are far more probable, [Ruth and Hutchinson, 1962]. Changes take place also in the matter which is penetrated, the constituent atoms are excited or ionised, and dissociation of molecules, changes in the conductivity, and many other processes have been observed [Seigbahn, 1965].

Angular deflections due to elastic scattering which are mainly due to encounters with the nuclei, cause an appreciable spread in the crow-flight path length ("straggling"). As both the straggling and the number of interactions per unit path length increase with decreasing kinetic energy of the electron, the flux loss between the point of emission and detection will be higher for low energy electrons and their probability of detection decreases accordingly. The main interactions of electrons with matter are described briefly in this section. The detailed study of different interactions of electrons with matter have
been given by Bothe [1932], Bethe and Ashkin [1953], Mott and Massey [1948], Seigbahn [1965], and Roy and Reed [1968].

2.2.1 Elastic Scattering by Atomic Nuclei

The incident electrons, interacting with the electrostatic field of the nucleus, will be deflected without emitting any form of radiation, or exciting the nucleus. The probability of this interaction process is high for low energy electrons. The elastic scattering of electrons passing through matter can be divided roughly into four classes:

(i) Single Scattering
(ii) Plural Scattering
(iii) Multiple Scattering
(iv) Back Scattering

2.2.1.1 Single Scattering

If the thickness \( d \) of the absorber is very small, i.e. \( d < l/N\sigma \), where \( N \) is the number of scattering atoms per cm\(^3\) and \( \sigma \) is the interaction cross-section, there will practically be only single scattering, i.e. nearly all the scattered electrons are scattered only by a single event. The probability that an electron with energy \( E \) is scattered during the passage through an absorber of thickness \( d \) and atomic number \( Z \) through an angle \( \Theta \) into the solid angle \( d\Omega \) is given by

\[
W(\Theta) d\Omega = N d \sigma_\Sigma (E, Z, \Theta)
\]  

(2.1)

Where \( d\sigma_\Sigma \) is the differential interaction cross-section. For the pure Coulomb field of a point charge without shielding we get, according to Mott [1929]
\[
\frac{d\sigma}{d\Omega} = q_{\text{Mott}}
\]

where \( q_{\text{Mott}} \) is the Mott cross-section.

### 2.2.1.2 Plural Scattering

When the thickness \( d \) of the absorber is \( d = l/N_0 \), we get plural scattering, i.e., the probability that a scattering through a given angle is due to a number of single scattering processes becomes appreciable. Plural scattering is intermediate between the two extremes of multiple and true single scattering.

### 2.2.1.3 Multiple Scattering

Multiple scattering occurs when the thickness of the absorber is so large that the mean number of scattering processes is greater than about 20. The angular distribution \( W(\theta) \) of the scattered electrons is approximately Gaussian as long as the mean scattering angle is smaller than 20 [Seigbahn, 1965]. The mean square deviation as a result of this scattering in a thickness \( d \) (in cm) is given by

\[
\delta^2 = \frac{4\pi N Z (Z+1) e^2 d}{p^2 v^2} \ln\left[ 4\pi (Z) N d \left( \frac{\hbar}{mv} \right) \right]^{3/4} \]

where
- \( N \) is the number of atoms per \( \text{cm}^3 \)
- \( p \) is the electron momentum
- \( \hbar \) is the angular Planck's constant (\( \hbar = h/2\pi \))
- \( Z \) is the atomic number
- \( m \) is the mass of the electron
- \( e \) is the charge of the electron.
2.2.1.4 Back Scattering

The electrons are back scattered to certain extent by the absorbing material. The degree to which electrons are back scattered depends on several parameters, such as the number of electrons per \( \text{cm}^3 \) and hence on the atomic number of the absorber. For larger values of thickness, \( d \gg 1/N\text{cr} \), the angular distribution becomes of the form \( W(\theta) \propto \cos^2 \theta \). The mean angle of scattering then attains its maximum value \( \theta_{\text{max}} \approx 33^\circ \), and remains constant when the thickness increases still further. Finally, electrons emerge from the absorber also on the side of the incident beam. These electrons are either primary electrons which are deflected in the backward direction, or secondary electrons. The number of back scattered electrons increases up to a certain saturation value as the thickness of the absorber is increased and in theory this value should be reached when the thickness of the absorber is equal to one-half of the maximum path. On average each successive scattering angle increases as each path decreases [Schumacher, 1965]. At an absorber thickness corresponding to \( d_{\text{scat}} \), the back scattered intensity saturates

\[
d_{\text{scat}} = \frac{2}{3} \cdot \frac{116(T)}{T}
\]

where \( T \) is the kinetic energy of the electron.

2.2.2 Elastic Scattering by Atomic Electrons

An incident electron may also be elastically deflected by the Coulomb field of atomic electrons within the absorber. The energy transferred is small compared with the lowest excited state of the atom, therefore it does not go into internal energy of the atom, but rather into its kinetic energy [Fitzgerald et al, 1967]. These collisions are
only important for very low energies.

2.2.3 Inelastic Scattering by Atomic Electrons

Inelastic scattering of electrons by the atomic electrons is the dominant energy loss mechanism for electrons with kinetic energy up to, but not significantly in excess of, the electron's rest mass energy. The interaction of the incident electrons with the atomic electrons in the absorber is characterised by the fact that the average energy transfer to the atoms per collision is very small. Even for very high primary energies the recoiling secondary electron has a mean kinetic energy of only a few eV. The total energy loss after passage through an absorber of thickness \( x \) is therefore the result of very large number of small energy losses. For energies up to nearly 0.5 MeV, the rate of energy loss due to inelastic collision is given by [Price, 1964]

\[
\frac{dE}{dx} \sim \frac{1}{\epsilon v^2}
\]

where \( v \) is the particle energy.

2.2.4 Inelastic Scattering by the Nucleus

The incident electron may experience a deflection by the intense electrostatic field close to the nucleus. As a result, a photon of energy comparable to the energy of the electron is emitted. Energy loss by this process is referred as radiation loss or bremsstrahlung emission. This emission of radiation is discussed in section 2.4. This type of interaction is dominant for electrons with kinetic energy greater than their rest mass energy.
The range of particles through absorbers may be investigated by determination of the absorption curves. A typical absorption curve for mono-energetic electrons and beta-particles is shown in Figure 2.1. The initial part of the absorption curve for electrons is linear while for beta-particles the initial part is much steeper because the low energy beta particles are rapidly absorbed at a small thicknesses, and the resulting curve is a quasi-exponential. If the logarithm of the count rate is plotted against the thickness of the absorber then a linear graph should result, but a sharp cut-off is not reached. The reason is the bremsstrahlung emitted by the electrons and this bremsstrahlung is absorbed more slowly than the electrons. The point where the beta-absorption curve reaches the background level is known as the maximum range of beta particles. This crow-flight penetration range is shorter than the actual path that a beta particle travels before losing all of its energy.

Figure 2.1: Absorption curves for monoenergetic electrons (curve A) and beta-particles (curve B) for same energy.
The total range of beta particles in a given absorber, that is, the length of the flight path before the electron comes to complete stop, can, in principle, be determined from the stopping power of the particles. The stopping power is defined as the energy lost by an incident electron per unit length of path through the absorber, and is given by

\[ \frac{-dT}{dx} = n \sigma \]  \hspace{1cm} (2.6)

where \( n \) is the atomic density of the absorber and \( \sigma \) is the stopping cross-section.

The total range can be achieved by integrating the above equation as

\[ \int_{0}^{R} dx = \int_{E}^{0} \frac{-dT}{n \sigma} \]  \hspace{1cm} (2.7)

but scattering must be allowed for and the resulting computations are quite complex.

Many attempts have been made to formulate the empirical relationship between the range \( R \) and the end-point energy of the beta particles. Some of these are given below:

Feather : \[ R = 0.542E - 0.160 \] ; \( E > 0.8 \text{ MeV} \)

This range-energy relation could be extended to lower energies.

Glendenin : \[ R = 0.542E - 0.133 \] ; \( E > 0.8 \text{ MeV} \)

\[ R = 0.407E^{1.38} \] ; \( 0.15 < E < 0.8 \text{ MeV} \)

Flammersfeld : \[ R = 0.11 [(1+22.4E)^{-1}]^{2.1/2} \] ; \( 0 < E < 3 \text{ MeV} \)
2.4 BREMSSTRAHLUNG EMISSION

The continuous X-ray emission is often called bremsstrahlung, from the German bremsen (=braking, i.e. decelerating) + strahlung (=radiation) [Eisburg and Resnick 1974]. Bremsstrahlung is produced whenever moving charges undergo a series of accelerations or decelerations [Lapp and Andrews 1972] and has a continuous spectrum which rises sharply from zero energy to a maximum intensity, and then decreases slowly towards the higher energy limit. The bremsstrahlung process can be considered as an inverse photoelectric effect: in the photoelectric effect, a photon is absorbed, its energy and momentum going to an electron and a recoiling nucleus; in the bremsstrahlung process: a photon is created, its energy and momentum coming from a colliding electron and nucleus.

The bremsstrahlung process for electrons has been examined thoroughly in a number of works [Bethe 1953, Heitler 1936; Sommerfeld 1939; Schiff 1951]. The classical representation of the bremsstrahlung process for electrons is as follows: an electron at energy $E_0$, moving in the Coulomb field of the nucleus, can change its direction of motion. However, since every change of direction involves an acceleration, then according to classical concepts of charge acceleration, it is probable that in passing a nucleus the electron will emit a quantum of energy $E_r$ and will drop to a new state at energy $E$. But since the nucleus in whose field the photon is emitted is considerably heavier than the electron, it can acquire, in principle, any transfer of momentum. As a consequence there is a probability that after emitting a photon the electron can drop to any

$$R = 0.571E - 0.1611; \quad E > 1 \text{ MeV}$$
observed. In other words, the bremsstrahlung spectrum should be continuous over the range of energies from 0 to \( E_0 - m_0 c^2 \) (where \( m_0 \) and \( c \) are the rest mass of the electron and the speed of light, respectively). At the same time, a most general quantum-mechanical analysis indicates that the differential cross-section for bremsstrahlung production is

\[
d\sigma \sim \frac{2 |H_{if}|^2}{P_0 c/E_0} \quad (2.8)
\]

where \( P_f \) is the density of final states, \( H_{if} \) is the matrix element for the transition from the initial state \( i \) to the final state \( f \) (after photon emission), and \( P_0 \) is the initial electron momentum.

2.5 EXPERIMENTAL STUDIES

The bremsstrahlung emitted by the beta particles while traversing through a medium is of two types: internal bremsstrahlung (IB), which is produced inside the source due to the source material, and external bremsstrahlung (EB), which is produced in the medium surrounding the source. The EB is of greatest practical importance as it is produced in the traversing medium and also depends upon the atomic number (Z) of the medium. The yield of EB is more than that of IB [Goodrich et al, 1953], therefore, the contribution of IB can be neglected in a first approximation. The bremsstrahlung spectrum comprises energies from zero to \( E_{\text{max}} \) and the percentage yield of low energy components is high compared to the higher energies. The lower energy photons undergo a rapid attenuation while passing through the material and therefore the position of the maximum in the bremsstrahlung energy spectrum steadily shifts to the higher energy region. The magnitude of this shift depends essentially on the depth at which the beta source is located and the nature of the attenuating medium.
Water was used as the absorbing material (since water is the convenient tissue-equivalent medium) and the shift in the bremsstrahlung maximum was measured for P-32, a pure beta-emitter of maximum beta-particle energy $E_{\text{max}} = 1700$ keV, and Kr-85, a beta-gamma-emitter with maximum beta-particle energy $E_{\text{max}} = 670$ keV and a photon of 514 keV. The Kr-85 source of beta- and gamma-rays was used to extend the method to isotopes that are of importance in nuclear medicine but have complex decay schemes.

The measurements of bremsstrahlung spectra were carried out with a thallium activated sodium iodide, NaI(Tl), scintillation counter and a lithium drifted germanium, Ge(Li), detector. Some sample spectra are shown in Figure 2.2 for different absorber thicknesses. The experimental arrangements for both detectors are described below.

2.5.1 Experimental set-up with NaI(Tl) detector

The experimental set-up for measuring the bremsstrahlung spectra by using a NaI(Tl) detector consists of a 5cm x 5cm NaI(Tl) scintillation counter with thin beryllium window, a high voltage supply, a pre-amplifier, a linear amplifier, a multichannel analyser (MCA) and a teletypewriter. The output from the detector was fed to the MCA after passing through the pre-amplifier and amplifier. The experiment was done in a broad beam geometry and the detector was calibrated for low energy photons. The resolution was found to be 45% for 60 KeV photons. The pulse height-spectra were measured using an MCA. The data was then printed out by using the teletypewriter. The source and detector arrangement is shown in Figure 2.3(a). A 7.4 MBq Kr-85 source, contained in a thin perspex capsule was placed in a perspex beaker of 9 cm inner diameter at a distance of 15 cm from the detector and bremsstrahlung spectra were obtained by interposing increasing depths of water. The
Figure 2.2: Bremsstrahlung spectra for Kr-85 at 2.5cm (upper) and 10cm (lower) water depths with NaI(Tl) detector.
Figure 2.3(a) Experimental set-up using NaI(Tl) detector.

Figure 2.3(b) Experimental set-up using Ge(Li) detector.
counting time for each measurement was set to be one hour. Source out background counts were subtracted from the spectrum obtained for each water thickness and the 514 keV gamma spectrum was subtracted by replacing the Kr-85 source with a Na-22 source of 511 keV quanta and putting the MCA in subtraction mode. The minimum thickness of water was taken to be 2 cm, more than the range of beta-particles. Therefore, no beta-particles were able to reach the detector.

The same procedure was repeated for P-32 liquid source, 74MBq in 2ml of solution, contained in a small perspex vial of 1mm wall thickness and 10mm inner diameter.

2.5.2 Experimental set-up with Ge(Li) detector

A Ge(Li) detector of 40 cm$^3$ sensitive volume fixed in a lead shield was used for the measurement of bremsstrahlung spectra. The set-up consists of a Ge(Li) detector, a high voltage supply, a spectroscopic amplifier and Nuclear Data 66 (ND66) multichannel analyser with dual port interface and twin ADCs. A block diagram of the detection system is shown in Figure 3.6. The ND66 multichannel analyser is a versatile machine which can be used as an MCA and as a computer terminal, as it is attached to the PRIME 550 computer.

The Kr-85 source was hung in the same perspex beaker, described earlier, but this time the beaker was kept full of distilled water and the source was moved vertically away from the detector as shown in Figure 2.3(b). A photograph of this set-up is shown in Figure 3.8. The measurements of bremsstrahlung spectra were started with 2 cm of distilled water. The amplifier gain was set to be 20x1.1 and ADC gain was set to be 8K and the bremsstrahlung measurements were made for 30 minutes for each increment in the water depth. The same procedure was adopted for the P-32 source and the measurements were taken for every 2 cm of source depth.
The variation in the position of the bremsstrahlung maximum with the water depth is shown in Table 2.1 with NaI(Tl) detector and in Table 2.2 with Ge(Li) detector for Kr-85 and P-32 radionuclides. Table 2.1 indicates that the introduction of 10 cm of water has caused a net energy shift of 17.25 keV for Kr-85 and 27.55 keV net energy shift for P-32 by using the NaI(Tl) detector. The energy corresponding to the bremsstrahlung maximum is plotted against the water thickness in Figure 2.4 for both the sources. Table 2.2 shows a net shift of 16.25 keV in the bremsstrahlung maxima for Kr-85 and 26 keV for P-32 with Ge(Li) detector, with the introduction of 10 cm of water. The bremsstrahlung peak position versus water thickness is plotted in Figure 2.5. (If plotted on the same graph the two sets of data are almost indistinguishable).

From Tables 2.1 and 2.2 it can be seen that the bremsstrahlung peak shift for P-32 beta source is greater than that of Kr-85 for the same thickness of the water. This difference in the peak shift is due to the fact that P-32 emits beta-particles of maximum energy $E_{\text{max}} = 1.70$ MeV, while the maximum energy of the beta particles emitted by Kr-85 is equal to $E_{\text{max}} = 0.76$ MeV. The range of beta-particles in a medium depends on their energy so the low energy beta-particles will be absorbed at lower depths as compared to the high energy beta-particles. Therefore, the bremsstrahlung will be produced near to the point of origin of beta-particles. Also the energy of the bremsstrahlung depends upon the energy of beta-particles, so the maximum energy of the bremsstrahlung produced by the low energy beta particles will be lower than that produced by the high energy beta-particles. As the continuous spectrum bremsstrahlung passes through an absorber, the lower energy photons are attenuated to a greater extent than the higher energy photons, which
FIGURE 2.4:
BREMSSTRAHLUNG PEAK SHIFT WITH WATER THICKNESS FOR Kr-85 AND P-32 WITH NaI(Tl) DETECTOR.
FIGURE 2.5:
BREMSSTRAHLUNG PEAK SHIFT WITH
WATER THICKNESS FOR Kr-85 AND
P-32 WITH Ge(Li) DETECTOR.

PEAK ENERGY (keV)

WATER THICKNESS (CM)
**TABLE 2.1**

Bremsstrahlung peak position versus water thickness with NaI(Tl) detector.

<table>
<thead>
<tr>
<th>Water Thickness (cm)</th>
<th>Peak Energy (keV) with Kr-85</th>
<th>Peak Energy (keV) with P-32</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>31.50 ± 0.8</td>
<td>36.01 ± 0.7</td>
</tr>
<tr>
<td>4</td>
<td>36.75 ± 1.0</td>
<td>41.83 ± 0.9</td>
</tr>
<tr>
<td>6</td>
<td>41.50 ± 1.2</td>
<td>48.88 ± 1.1</td>
</tr>
<tr>
<td>8</td>
<td>45.10 ± 1.4</td>
<td>55.20 ± 1.4</td>
</tr>
<tr>
<td>10</td>
<td>46.20 ± 1.7</td>
<td>60.98 ± 1.7</td>
</tr>
<tr>
<td>12</td>
<td>48.75 ± 1.9</td>
<td>63.56 ± 1.9</td>
</tr>
</tbody>
</table>

**TABLE 2.2**

Bremsstrahlung peak position versus water thickness with Ge(Li) detector.

<table>
<thead>
<tr>
<th>Water Thickness (cm)</th>
<th>Peak Energy (keV) with Kr-85</th>
<th>Peak Energy (keV) with P-32</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>30.20 ± 0.5</td>
<td>34.06 ± 0.5</td>
</tr>
<tr>
<td>4</td>
<td>35.30 ± 0.6</td>
<td>39.78 ± 0.6</td>
</tr>
<tr>
<td>6</td>
<td>39.90 ± 0.8</td>
<td>46.70 ± 0.7</td>
</tr>
<tr>
<td>8</td>
<td>43.54 ± 0.9</td>
<td>52.03 ± 0.9</td>
</tr>
<tr>
<td>10</td>
<td>45.80 ± 1.0</td>
<td>57.66 ± 1.0</td>
</tr>
<tr>
<td>12</td>
<td>46.50 ± 1.2</td>
<td>60.08 ± 1.2</td>
</tr>
</tbody>
</table>
results in progressive hardening of the spectrum and a consequent shift
to higher mean energy, including a shift upward of the maximum in the
spectrum. Both curves in Figures 2.4 and 2.5 shift at a similar rate
initially, but as the attenuation continues, after the softer parts of
the initial spectrum have been removed the remaining spectrum stabilises
around a fairly constant maximum and the energy shift becomes less
pronounced. Since the Kr-85 initially has a lower mean energy than P-32
this flattening off is reached at a lower value of absorber thickness. A
similar behaviour is observed for beta-attenuation and leads to the
quasi-exponential absorption curve shown in Figure 2.1.

The general trend of these results also agrees with those of Gaur and
Venkateswaran [1982] but they observed a larger peak-shift of 30 keV for
8 cm of water using P-32 with a NaI(Tl) detector. In our case additional
measurements were made using the high resolution Ge(Li) detector and was
found that the peak-shift is not more than 25 KeV for 8 cm of water with
both detectors using a P-32 beta source.

The difference in the results could be due to differences in the
geometrical conditions in which the experiments were performed,
differences in the characteristics of the detection system, and possible
difference in the purity of the water used.

Comparing the results of NaI(Tl) and the Ge(Li) detectors, in Tables 2.1
and 2.2, we can see that there is a small difference between the two
results. The discrepancy in the two values may be assigned to the
following reasons.

1: The two different detectors were used in the different set-ups and in
different surroundings.

2: The detectors used have different characteristics. For example
Ge(Li) has much better energy resolution than NaI(Tl) but the latter has
in general a much higher efficiency.
3: The source to detector distance in the case of NaI(Tl) was kept at 15cm throughout the experiment and the water thickness was increased for each measurement but in the case of Ge(Li) the beaker was kept full of water and the source was moved away from the detector through the water for each measurement. So at the time of the very first measurement with the NaI(Tl) detector, there was 2 cm of water and 13 cm of air between source and the detector. So there is a possibility of losing (by out-scattering) some of the bremsstrahlung photons before they reach the detector.

4: Lastly, the error due to the counting statistics is always present.

2.7 CONCLUSION

The comparison of results for Kr-85 and P-32 sources confirms that the position of the maximum in the bremsstrahlung spectrum is dependent on energy of the beta-particles and the degree of attenuation. The shift in the bremsstrahlung maximum is significant even after 10cm of water depth but the rate of shift begins to fall off at larger absorber thicknesses. The experimentally determined bremsstrahlung spectrum is of great importance. The position of the peak intensity can provide a means to estimate the depth of beta emitters and knowing the depth of the source, the source strength can be quantitatively determined which is of great importance in many industrial and medical applications. This method can be efficiently used in low Z media. The depth of the beta-emitter which can be estimated by this method is a function of the beta-particle energy. For a deeper insight into this phenomenon of bremsstrahlung "moderation" it would be interesting to extend such studies to higher Z media which would also be of significance in a wider range of industrial applications.
3.1 INTRODUCTION

The attenuation of photons is the major problem in emission tomography and it is not possible to make reliable quantitative measurements without taking into account the amount by which the emitted photons have been attenuated within the radioactive object. This involves accurate measurement, or at least, excellent approximation of the linear attenuation coefficient of the material and the depth of the source within it. In medical imaging the intervening material is assumed to be homogeneous and the depth of the radioactive source is estimated by the use of geometric or arithmetic means of the data from opposite views [Sanders, 1982]. In other non-medical fields where emission tomography may be applied there is occasionally the additional complication of limited accessibility of the object to be imaged and a generalised shape can not be assumed. In both types of application the need to take opposing views also increases either the number of detectors required in the system or the time taken to complete a scan. Therefore, one has to look for some other feasible methods for depth determination of a radioactive source inside the absorbing medium.

The depth of a radioactive source in an absorbing material can be determined by analysing the spectrum of the radiation that emerges; the deeper the source, the greater is the portion of low energy scattered radiation [Mohindra and McNeil, 1965]. The depth of a gamma-ray source can also be measured using the differential attenuation of the radiation from two isotopes [Dolan and Tauxe, 1968; Oldendorf and Isaka, 1969]. Koral and Johnston [1977] have studied the change in the multi-energy spectrum of an I-131 point source with depth in water. Martin and Rollo
[1977] investigated both the differential attenuation and photon scatter from I-123 to determine a depth correction factor for iodine uptake studies and concluded that the two-energy differential attenuation method was more sensitive. Filipow et al [1979] have studied the variation in the counting rates in different regions of spectra from low energy gamma-ray sources as a function of depth.

The different methods of depth discrimination for photon emitting radionuclides were investigated in the present work. The gamma-ray sources used were Cs-137, Ba-133, Na-22, and Am-241 point sources and a Cs-134 liquid solution as an extended source. The absorbing materials used were water and foam rubber which are both tissue equivalent materials.

Before going into the details of depth determination methods a knowledge of the basic processes by which the photons interact with matter is essential to the understanding of the absorption and scattering of photons, and these are outlined below.

3.2 INTERACTION OF PHOTONS WITH MATTER

Gamma rays and X-rays are two forms of electromagnetic radiation differing only in their origin. Gamma rays are emitted during nuclear transitions of excited nuclei to lower nuclear levels whereas X-rays are produced as characteristic X-rays in transitions of bound electrons between the K,L,M,... shells in atoms or as bremsstrahlung by the deceleration of electrons (or beta-particles).

Several types of photon interactions are possible and the probability that a particular interaction will occur is dependent on the photon energy and the density and the atomic number of the material. Interactions are either with whole atoms or with the constituent
electrons or nuclei and result in either absorption or scattering of the photon.

The basic theory of photon interactions with matter has been established for many years. A detailed study of the interaction processes is given by Davisson and Evans [1952] and Fano et al [1959].

The important interactions in the energy range of a few keV to 5 MeV are the photoelectric absorption, coherent and incoherent scattering and pair production. All of these processes lead to the partial or complete transfer of the photon energy to the electron, resulting either in the complete disappearance of the photon or scattering through an angle. These different interaction processes are discussed briefly in the following sub-sections.

### 3.2.1 PHOTOELECTRIC ABSORPTION

The photoelectric (P.E) absorption is a process in which the incoming photon interacts with the bound electron and is completely absorbed transferring all its energy to the electron which is then ejected from its bound shell. The energy of the photoelectron, \( E_{\text{e}} \), is given by:

\[
E_{\text{e}} = E_{\gamma} - E_{b}
\]  

(3.1)

Where \( E_{\gamma} \) is the incident photon energy and \( E_{b} \) is the binding energy of the ejected electron. The removal of the electron leaves the atom in an ionised state with a vacancy in one of its inner electron shells. This vacancy is filled by rearrangement of electrons in other shells and hence characteristic X-rays may be emitted or, alternatively, the emission of Auger electrons may occur. The P.E absorption in a given shell occurs most favourably if the binding energy is comparable to, but less than, the energy of the incident photon. Therefore for energetic photons it is inner shell electrons that are ejected. This process is the
predominant mode of interaction for low energy photons (upto 100 keV). A number of expressions for photoelectric atomic cross-section, $\sigma^{\text{pe}}$, have been reported [Heitler, 1954; Evans, 1955] but no single analytical expression exists which is valid over all ranges of $E_\gamma$ and atomic number $Z$. Heitler obtained an approximate value for the photoelectric absorption cross-section for photons having energies well above the $K$-absorption edges, by using the Born approximation.

$$\sigma^{\text{BA}} = \frac{1/2 \ 5 \ 4}{4(2) \ 2 \gamma \ 7/2} \frac{Z\alpha \ mc^2}{n \ E_\gamma} \sigma^{\text{Th}}$$

(3.2)

Where $\sigma^{\text{BA}}$ is the Born approximation cross-section for P.E ejection of $s$-state electrons, $E_\gamma$ is the energy of the incident photon, $n$ is the principal quantum number and $\alpha$ is the fine structure constant. $\sigma^{\text{Th}}$ is the Thomson cross-section and is given by:

$$\sigma^{\text{Th}} = \frac{8\pi}{3} \frac{e^2}{m_e c^2}$$

(3.3)

Although this is a poor approximation, it does show the strong $Z$ and $E$ dependence exhibited by the P.E effect. An accurate semi-theoretical treatment of P.E effect is given by Jackson and Hawkes [1981] where corrections for the above approximation have been considered.

Another approximate expression for the P.E absorption cross-section over a wide range of energies is given by:

$$\sigma^{\text{pe}} \propto \frac{Z}{m} \frac{n^\gamma}{E_\gamma}$$

(3.4)

The values of the exponents depend on both $E_\gamma$ and $Z$. At energies less than 100 keV the cross-section is complicated by the absorption edges.
energies greater than 100 keV \( m \) varies between 3 and 3.5 and is greater for low energies. For a given energy range \( m \) is higher for low \( Z \) and decreases with increasing \( Z \). The value of the exponent \( n \) varies between 4 and 5 and depends on \( E_\gamma \). In the energy range \( 0.1 \) to \( 3 \) MeV \( n \) is found to increase from 4 to 4.6 as \( E_\gamma \) increases [Evans 1955]. A plot of P.E absorption cross-section for sodium iodide is shown in Figure 3.1. The discontinuities in the curves or "absorption edges" in the low energy region appear at gamma ray energies which correspond to the binding energies of electrons in the various shells of the absorber atom. The edge lying highest in energy therefore corresponds to the binding energy of the K-shell electrons. For photon energy slightly above the edge, the photon energy is just sufficient to undergo a photoelectric interaction in which a K-electron is ejected from the atom. For photon energies slightly below the edge, this process is no longer energetically possible and therefore the interaction cross-section drops abruptly. Similar, but less prominent, absorption edges occur for low energies for the L,M,..., electron shells of the atom.

The linear attenuation coefficient for P.E absorption is given by

\[
\mu_{\text{pe}} = \sigma_\text{(pe)} \cdot N \quad (\text{mm}^{-1})
\]

Where \( N \) = the number of atoms/\text{mm}^3, and \( \sigma_\text{(pe)} \) is expressed in \text{mm}^2 per atom.

The energy absorption coefficient, \( \mu_{\text{pe}} \), for P.E effect is equivalent to the linear attenuation coefficient since the fraction of the photon energy not converted into the kinetic energy of the photoelectrons represents the excitation energy in the residual atom and is given up as characteristic X-rays or as Auger electrons. These are both absorbed within a distance comparable with the range of the photoelectrons. So the effective energy absorption is represented by \( \mu_{\text{pe}} \), i.e.

\[
\mu_{\text{pe}} = \mu_{\text{pe}} \quad (\text{mm}^{-1})
\]

where the cross-section shows discontinuous jumps [Knoll, 1979]. At
3.2.2 COMPTON SCATTERING

Compton scattering (C.S) is a process in which the incident photon collides with an electron, assumed to be free and at rest. The photon is deflected through an angle w.r.t its original direction. The photon transfers a portion of its energy to the electron which is then described as the recoil electron. Because all angles of scattering are possible, the energy transferred to the electron can vary from zero to a large fraction of the incident photon energy. The relationship between the incident energy $E_i$ and the final energy $E'_i$ of the scattered photon is given by:

$$E'_i = \frac{E_i}{2} \left(1 + \frac{E_i / m_0 c}{1 - \cos \theta}ight)$$  \hspace{1cm} (3.7)

where $m_0 c^2$ is the rest mass energy of electron (0.511 MeV).

The above expression was derived by Compton for photons incoherently scattered by a stationary free electron applying the principles of conservation of energy and momentum. For small scattering angles, very little energy is transferred. Some of the original energy is always retained by the incident photon, even in the extreme case of $\theta = \pi$, which corresponds to the photon being backscattered in a "head-on" collision.

The probability of C.S per atom of the absorber depends on the number of electrons available as scattering targets, and therefore increases linearly with $Z$. The dependence of C.S on gamma ray energy is illustrated in Figure 3.1. Using Dirac's relativistic theory of an electron Klein-Nishina [1929] were able to obtain an equation for the differential cross-section for Compton scattering of a photon by a free electron. For an unpolarized beam of mono-energetic photons the
Klein-Nishina angular distribution function (the differential collision cross-section) per steradian of solid angle is given by:

\[
\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2} \left[ \frac{1}{1+\alpha(1-\cos\theta)} \right]^2 \left[ 1 + \frac{2\alpha}{(1+\cos\theta)(1+\alpha(1-\cos\theta))} \right]^2
\]

where \( r_0 = \frac{e}{m_0c^2} \), and is the classical radius of the electron and \( \alpha = \frac{E_\gamma}{m_0c^2} \).

Equation 3.8 shows that the cross-section falls with the increasing angle, and this occurs more rapidly at higher energies, i.e there is a strong tendency for forward scattering at high values of the incident photon energy. The equation reduces to Thomson's classical formula at low energies when \( \alpha \ll 1 \) i.e.

\[
\frac{d\sigma}{d\Omega} \text{(Th)} = \frac{r_0^2}{2} \left( \frac{1}{1+\cos\theta} \right) \quad (3.9)
\]

Integration of equation 3.8 over all scattering angles gives the total scattering cross-section from a free electron as:

\[
\sigma_c(K.N) = \frac{2}{r_0^2} \left\{ \frac{1}{\alpha^2} \ln(1+2\alpha) - \frac{1}{1+2\alpha} \ln(1+2\alpha) + \frac{1}{2\alpha} \ln(1+2\alpha) - \frac{1+3\alpha}{(1+2\alpha)^2} \right\} \quad (3.10)
\text{mm}^2 / \text{electron}
\]

The values of \( \sigma_c(K.N) \) has been tabulated by Hubble [1969] for a very wide energy range (10keV-100GeV) using the above equation. Hubbell also cited various corrections to the Klein-Nishina values due to binding and radiation effects etc, but these are insignificant over the practical region of energies for most purposes (100keV-10MeV). The incoherent scattering atomic cross-section is given by

\[
\sigma(\text{incoh}) = 2 \sigma_c(K.N) \quad (3.11)
\]
which assumes that all the atomic electrons can be considered to be free. However the Klein-Nishina expression has ignored the effect of the binding of the electron in the atom. For a better evaluation we must consider this to be an inelastic process where one photon is absorbed and another is emitted, resulting in an increase in the external energy of the target atom. We can write the incoherent cross-section in the form:

\[
\frac{d\sigma^{\text{(incoh)}}}{d\Omega} = \frac{d\sigma^{\text{(K.N)}}}{d\Omega} S(q,Z)
\]

where \(S(q,Z)\) is the incoherent scattering function which is the product of the number of electrons per atom and the probability that an atomic electron will actually undergo some transition either to an excited state or to the continuum as a result of receiving a momentum transfer, \(q\). The \(S(q,Z)\) varies between zero to \(Z\) as the momentum transfer varies from zero to infinity. The incoherent function is tabulated by Hubbell [Hubbell, 1969]. The factor \(S(q,Z)\) allows for the Compton cross-section falling off at low energies as binding energy becomes important.

The Compton linear attenuation coefficient can be written as:

\[
\mu_c = \mu_a + \mu_s
\]

where \(\mu_a\) and \(\mu_s\) are absorption and scattering coefficients respectively. The significance of this distinction between scattering and absorption coefficients is particularly important in C.S since only the absorption coefficient represents the detectable effects of the interaction process which are caused by Compton recoil electrons, and the detection of scattered photons depends on further interactions. The energy absorbed per unit volume of an absorber as a result of Compton interaction is equal to \(I\mu_a\), where \(I\) is the incident photon energy.
3.2.3 RAYLEIGH SCATTERING

This is the process in which the photons are scattered by bound atomic electrons and the target atom is neither excited nor ionized. The elastic scattering from different parts of the atomic charge is then 'coherent' i.e there are interference effects. Rayleigh scattering from bound electrons occur when the wave-length of the photons incident on an atom is comparable to or larger than the dimensions of the atom. The bound electrons will then oscillate in fixed phase relationships and the scattered radiation re-emitted by each electron will also be in phase. The differential cross-section is given by

\[
\frac{d\sigma}{d\Omega} = \frac{\sigma_{\text{Th}}}{\Omega} \left[ F(q,Z) \right] \tag{3.14}
\]

where \( \sigma_{\text{Th}} / d\Omega \) is the Thomson scattering cross-section from a single electron and is given by equation 3.9. \( F(q,Z) \) is the atomic form factor which represents the ratio of the amplitude of the coherent scattering by an entire atom to the amplitude of the scattering by a single free electron. It is a function of atomic number of the scattering material and the momentum transferred to the scattering atom, which is itself a function of photon energy and scattering angle. The momentum transfer, \( q \), is given by

\[
q = \frac{1}{\lambda} \sin\left(\frac{1}{2}\theta\right) \tag{3.15}
\]

where \( \lambda \) is the wave-length of the incident radiation and \( \theta \) is the scattering angle.

The value of the form factor decreases with energy since the probability of momentum transfer without energy absorption decreases and, for the
same reason, $F(q, Z)$ decreases as scattering angle increases with the result that 60-70% of Rayleigh scattering interactions cause deflections of less than 30° in most materials.

Rayleigh scattering increases with the atomic number of the scatterer since the binding energy of inner electrons is proportional to $Z$, so that an increasing fraction of the atomic electrons must be considered to be bound. A consequence of the coherent nature of this process is that it is sharply peaked in the forward direction, which taken together with its elastic behaviour means that it generally plays only a minor role in beam attenuation.

### 3.2.4 PAIR PRODUCTION

The pair production (P.P) process occurs in the strong electric field near the nucleus of atoms of the absorbing material, and corresponds to the creation of an electron-positron pair at the point of the complete disappearance of the incident gamma ray photon. Because an energy equal to $2m_0c^2$ is required to create the electron-positron pair, a minimum gamma ray energy of 1.022 MeV is required to make the process energetically possible. In practice the probability of this interaction remains very low until the gamma ray energy approaches twice this value, and therefore pair production is predominantly confined to high gamma ray energies [Knoll 1979]. The excess energy $(E_\gamma - 1.022\text{MeV})$ appears as the kinetic energy shared quasi-randomly by the electron-positron pair and is given by:

$$E_e^- + E_e^+ = E_\gamma - 2m_0c^2 \quad (3.16)$$

Therefore, this process consists of converting the incident gamma ray photon into an energetic electron-positron pair.
There is no simple expression to describe the P.P cross-section, $\sigma_{pp}$, but its magnitude varies approximately with the square of the absorber atomic number, $Z$ [Evans, 1955]. The P.P process is complicated by the fact that the positron is not a stable particle. Once its kinetic energy becomes comparable to the thermal energy of normal electrons, in the absorbing medium, the positron will annihilate with an electron and two annihilation gamma ray photons of energy $0.511\text{MeV}$ each will be emitted in opposite directions to conserve momentum. The time required by the positron to slow down and annihilate is small so that the annihilation radiation appears in virtual coincidence with the original pair production interaction.

The linear attenuation for P.P is simply given by:

$$\mu_{pp} = N\sigma_{pp} \quad (\text{mm }^{-1}) \quad (3.17)$$

In P.P process only a portion of the photon energy appears immediately as K.E of the electron-positron pair so the absorption coefficient can be written as:

$$\mu_{pp} = \mu_{pp}[E_\gamma - 2m_0c^2]/E_\gamma \quad (3.18)$$

The remaining $2m_0c^2$ of the photon energy is given up when the positron annihilates with an electron after being slowed down by collision and radiation losses.

3.3 TOTAL PHOTON ATTENUATION COEFFICIENTS

The probability that a photon will interact when passing through matter is given by the attenuation coefficient. The linear attenuation coefficient $\mu$ is the interaction probability per unit length which is related to the atomic cross-section, by
where $N=N_0\rho/A$, $N_0$ is the Avogadro's number, $\rho$ is the density and $A$ is atomic weight. $\mu_\ell$ is also defined in terms of mean free path as $\mu_\ell=1/\lambda$.

Use of linear attenuation coefficient is hampered by the fact that it varies with the density of the absorber, even though the absorber constituents remain the same [Knoll, 1979]. Therefore the mass attenuation coefficient is much more widely used, and is defined as:

$$\mu_\rho = \mu_\ell/\rho$$ (3.20)

For a given gamma ray energy, the mass attenuation coefficient does not change significantly with the physical state of a given absorber (e.g. $\mu_\rho$ is same for water either liquid or vapour). The mass attenuation coefficient of a compound or mixture of elements is given by:

$$\mu_\rho = \sum_i [W_i (\mu_\ell/\rho)_i]$$ (3.21)

where the $W_i$ factors represent the weight fraction of element $i$ in the compound or mixture.

The interaction of photons is characterised by the fact that each photon interacts individually in single events. Therefore the number of photons removed from a beam either by scattering or absorption, $dI$, is proportional to the thickness of the material $dx$ and the number of incident photons $I$, i.e.

$$dI = -\mu_\ell I dx$$ (3.22)

For a homogeneous material and monochromatic gamma rays, $\mu_\ell$ will be constant and integration of the above equation will give:
where $I_0$ and $I$ are the number of incident and transmitted photon respectively for thickness $x$ of absorber. This means that a collimated photon beam of a particular energy passing through a medium follows an exponential law of attenuation.
The equation 3.23 can be written in terms of mass attenuation coefficient, $\mu_k/\rho$, as

$$I = I_0 \exp(-\mu_k x)$$  \hspace{1cm} (3.23)

$$I = I_0 \exp((\mu_k/\rho) pt)$$  \hspace{1cm} (3.24)

where the product $\rho t$ is known as the mass thickness of the absorber, and is now the significant parameter which determines the degree of attenuation. The thickness of the absorber used in the radiation measurements is therefore often expressed as mass thickness rather than the physical thickness because it is a more fundamental physical quantity. The mass thickness is also a useful concept when discussing the energy loss of charged particles, particularly fast electrons. Even if different absorber materials are involved, a particle will encounter about the same number of electrons passing through absorbers of equal mass thickness. Therefore, the stopping power and range when expressed in units of $\rho t$, are roughly the same for materials that do not differ greatly in $Z$.

As the photon interaction processes are independent of each other, the total linear attenuation coefficient is the sum of attenuation coefficients of all possible interactions. For the energy range 0.01 to 10 MeV the total linear attenuation coefficient can be written as:

$$\mu_k^{(\text{total})} = \mu_{\text{pe}} + \mu_r + \mu_c + \mu_{\text{pp}} \quad (\text{mm}^-1)$$  \hspace{1cm} (3.25)

where $\mu_{\text{pe}}, \mu_r, \mu_c$ and $\mu_{\text{pp}}$ are the total linear coefficients for P.E.
absorption, Rayleigh scattering, Compton scattering and pair production, respectively. Therefore, if a collimated beam of mono-energetic photons of intensity $I_0$ passes through a medium of thickness $x$, the intensity of primary photons that undergo no interaction of any kind is given by:

$$I = I_0 \exp[-\mu_f(tot)x]$$ (3.26)

For a broad beam (or bad) geometry a correction factor, called the "Buildup" factor, is introduced for the additional contribution of the scattered photons within the absorber. The equation 3.23 for this case can be written as

$$I = I_0 B(E_\gamma,t) \exp(-\mu_f x)$$ (3.27)

The magnitude of the buildup factor depends upon the type of the gamma ray detector used, because this will affect the relative weight given to the primary and scattered photons. With a detector that responds only to the primary gamma-rays the buildup factor is unity. As a rough rule of thumb the buildup factor for a thick slab absorber tends to be almost equal to the thickness of the absorber measured in units of mean free path of the incident gamma rays, provided the detector responds to a broad range of energies [Knoll, 1979].

The relative magnitudes of the interaction processes in equation 3.25 for sodium iodide are shown in Figure 3.1. Figure 3.2 illustrates the relative importance of the main interaction processes as a function of atomic number and photon energy. Three areas are defined on the plot within which each of the three processes predominate and the lines defining the regions represent the $(Z,E_\gamma)$ pairs for which the coefficients of neighbouring interactions are equal. It is clear from the graphs that the photoelectric absorption predominates for high $Z$. and
Figure 3.1: Energy dependence of various gamma-ray interaction processes in sodium iodide [Evans, 1955].

Figure 3.2: The relative importance of the three major gamma-ray interactions as a function of atomic number and photon energy.
low photon energy, pair production for high Z and high photon energy and Compton scattering for the domain of intermediate Z and photon energy.

3.4 METHODS OF DEPTH DETERMINATION

The variation of detector response with source depth within an absorber is due to two causes; first the inverse square law effect and second, the attenuation of the radiation in the absorber. The different methods of determining the depth of a gamma ray source are described briefly in this section.

3.4.1 SCATTER-TO-PEAK RATIO (SPR) METHOD

This method exploits the ratio of scattered gamma-rays and the photopeak counts, which varies in a scattering medium. This method was first reported by Mohindra and McNeil [1965], who examined the changes in photopeak resolution and valley-to-peak ratio of counts with source depth and gamma-ray energy, in connection with internal contamination of humans with unknown radioisotopes. They concluded that the variation in valley-to-peak ratio was most sensitive than changes in the effective full-energy peak resolution at low energies, using NaI(Tl) detector. This SPR method is based on the fact that the number of scattered events increases with the depth of the source in the scattering medium. So, the information about the depth of the source can be obtained by measuring the scatter-to-peak ratio of the emerging radiation. Scatter-to-peak ratio is the ratio of the counts in the scatter region to the counts in the full-energy peak. The scatter region is normally defined in terms of the angle of the scattered photons, i.e. single Compton scatter events, which shift the emerging photons to lower energy.

In this method the depth of a point source is determined from the study
of the variations in the SPR with the thickness of the absorber. There are two possible cases, the emission mode (when the source is inside the absorbing medium), and the transmission mode (when the source is outside the absorber). Simple expressions for both the cases can be derived, assuming the small angle scattering events only, as in the following sub-sections.

3.4.1.1 SPR in Emission Mode

In this case the gamma-ray source is inside the absorbing medium and the thickness of the absorber is increased by moving the source away from the detector deeper into the absorber. In Figure 3.3 the total number of scattered photons, $ds$, produced by a thickness 'dx' is given by:

$$ds = I \mu_c dx$$  \hspace{1cm} (3.28)

Where $I$ is effective intensity at distance 'x' from the source and is given by:

$$I = I_0 \exp[-\mu_t(E_y) \cdot x]$$ \hspace{1cm} (3.29)

where $\mu_c$ is the linear Compton scattering coefficient, and $\mu_t$ is the linear total attenuation coefficient of the material for the primary photon energy $E_y$. Therefore the number of forward scattered photons from $dx$ emerging from the surface of the medium and headed for the detector is given by:

$$F ds \cdot \exp[-\mu_t(E_s) \cdot (d-x)] = F I_0 \left[ \exp[-\mu_t(E_s) \cdot (d-x)-\mu_t(E_y) \cdot x] \right] \cdot \mu_c dx$$ \hspace{1cm} (3.30)

where $F$ is the fraction of scattered photons headed for the detector and $\mu_t(E_s)$ is the linear total absorption coefficient for the scattered
Figure 3.3: Small angle scattering from a point source in a scattering medium.

Figure 3.4: Small angle scattering in an absorber from a point source in air behind the absorber.
photon energy $E_s$. If only photons scattered through small angles, $\theta$, are reaching the detector then the difference in energy between the primary and scattered photons is small and it can be assumed that $\mu_T(E_p) - \mu_T(E_s)$. Therefore equation 3.30 becomes

$$F_{ds} = F_0 \exp(-\mu_T d) \mu_c . dx \quad (3.31)$$

Integration of above equation along 'x' gives the total number of single scattered photons, $S_T$, arriving at the detector for a source depth d, as:

$$S_T = \int_0^d I_0 F[\exp(-\mu_T d)] \mu_c . dx$$

or

$$= I_0 F[\exp(-\mu_T d)] \mu_c . d \quad (3.32)$$

Therefore the number of scattered photons recorded by the detector, $S$, is proportional to the number arriving i.e.

$$S = kS_T \quad (3.33)$$

where $k$ is the intrinsic detection efficiency for scattered photons in a defined energy region below the full-energy peak. The number of full-energy peak events recorded, $P$, is given by:

$$P = E_p I_0 \exp(-\mu_T d) \quad (3.34)$$

where $E_p$ is the intrinsic full-energy peak efficiency.

The SPR is therefore, given by

$$S = \frac{F . I_0 \exp(-\mu_T d) \mu_c . k}{P . I_0 \exp(-\mu_T d) . E_p} \quad (3.35)$$

In case of a well collimated beam or small detector, the scattered photons energy is close to the primary energy and self-absorption and
detector response are assumed closely similar for both. Therefore the
SPR for a point source is predicted to vary linearly with depth on this
simple model and

\[ \text{SPR} = \mu_c \cdot d \cdot F \cdot k / E_p \]  (3.36)

where the factor \( F \cdot k / E_p \) is a constant for a given system, \( \mu_c \) is constant
for a given photon energy, and therefore

\[ \text{SPR} \propto d \]  (3.37)

### 3.4.1.2 SPR Method in Transmission Mode

For transmission case a similar expression as in emission case,
can be derived for the relationship between SPR and the thickness of the
absorber placed between a well collimated detector and a point source in
air. In Figure 3.4 a slab of homogeneous material of thickness \( \ell \) is
placed at a distance \( x \) from a point source and the total source to
collimator distance is \( d \). The total number of scattered photons \( ds \)
produced in a thickness \( dt \) of the absorber is

\[ ds = I \mu_c (\text{abs}) dt \]  (3.38)

where \( \mu_c (\text{abs}) \) is the linear Compton scattering coefficient of the
absorber and \( I \) is the source intensity at \( x \) given by:

\[ I = I \exp[-\mu_{T (\text{air})}(E_\gamma) \cdot x - \mu_{T (\text{abs})}(E_\gamma) \cdot t] \]  (3.39)

where \( \mu_{T (\text{air})}(E_\gamma) \) and \( \mu_{T (\text{abs})}(E_\gamma) \) are total linear absorption coefficient
of air and absorber respectively for the primary photon energy \( E_\gamma \).
Therefore the number of scattered photons from \( dx \) reaching the surface
of the absorber is
where \( \mu_T(\text{abs}) (E_s) \) is the linear total absorption coefficient for the scattered photons of energy \( E_s \). If only photons scattered through a small angle, \( \theta_s \), are accepted by the detector then the difference in energy between the primary and scattered photons is small and the approximation that \( \mu_T(\text{abs}) E_g = \mu_T(\text{abs}) (E_s) \) can be applied as before and the right hand side of equation 3.40 becomes as

\[
\int_0^c \mu(\text{abs}) \exp[-\mu(\text{air}) (E_g) \cdot x - \mu(\text{abs}) (E_s) \cdot \ell] \, dt
\]

Integration of this equation with respect to \( t \) gives the total number of scattered photons from an absorber of thickness \( \ell \) recorded by the detector, (after traversing an air path \( 'd-x-\ell' \) in thickness)

\[
S = Fk I \ell \mu(\text{abs}) \exp[-\mu(\text{air}) (E_g) \cdot x - \mu(\text{abs}) (E_s) \cdot \ell - \mu(\text{air}) (E_g) \cdot (d-x-\ell)]
\]

or

\[
S = Fk I \ell \mu(\text{abs}) \exp[-\mu(\text{air}) (E_g) \cdot (d-\ell) - \mu(\text{abs}) (E_g) \cdot \ell]
\]

The number of full-energy peak events recorded by the detector, \( P \), is given by

\[
P = E \int_p \exp[-\mu(\text{air}) (E_g) \cdot (d-\ell) - \mu(\text{abs}) (E_g) \cdot \ell]
\]

where \( E_p \) is the intrinsic full-energy peak detection efficiency.

Then, as before

\[
\text{SPR} = \frac{S}{P} = F k \ell \mu(\text{abs}) / E_p
\]

and as before

\[
\text{SPR} \propto \ell
\]
3.4.1.3 Activity Measurements Of a Distributed Source Using SPR

A distributed source over a large volume can be considered as a collection of point-like sources situated throughout the absorbing medium. A well collimated detector focussed on a distributed source will see the source only along the axis of the collimator and the counts recorded by the detector will be due to the combined effect of all the point sources lying along that line.

Consider a simple case of two point sources lying at distance $x_1$ and $x_2$ from the detector along the axis of the collimator in an absorbing medium. From equation 3.37 we can write

$$\text{SPR} = \frac{V_1}{P_1} = kx \quad (3.46)$$

and

$$\text{SPR} = \frac{V_2}{P_2} = kx \quad (3.47)$$

where $V_1$ and $V_2$ are the counts in the scatter region due to the full-energy peaks from source 1 and 2, and $P_1$ and $P_2$ are the full-energy peak counts due to source 1 and 2.

The number of counts in the full-energy peak due to source 1 recorded by the detector is given by

$$P_1 = A_1 \cdot E_{p_1} \cdot \exp(-\mu x_1)$$

or

$$A_1 = \frac{P_1}{E_{p_1}} \cdot \exp(\mu x_1) \quad (3.48)$$

where $A_1$ is the true activity of source 1 and $E_{p_1}$ is absolute full-energy peak efficiency and $\mu$ is the attenuation coefficient.

Substituting the value of $x$ from equation 3.46
\[ A_1 = \left( \frac{P_1}{E_{p_1}} \right) \exp\left( \frac{\mu V_1}{P_1 k} \right) \] (3.49)

Similarly
\[ A_2 = \left( \frac{P_2}{E_{p_2}} \right) \exp\left( -\frac{\mu V_2}{P_2 k} \right) \] (3.50)

The average activity can be written as
\[ A = \frac{A_1 + A_2}{2} = \frac{1}{2} \left[ \frac{P_1}{E_{p_1}} \exp\left( -\frac{\mu V_1}{P_1 k} \right) + \frac{P_2}{E_{p_2}} \exp\left( -\frac{\mu V_2}{P_2 k} \right) \right] \] (3.51)

For source-to-detector distance greater than the diameter of the absorber absolute efficiency is approximately independent of the source-to-detector distance. Also if the conjugate counts are combined the absolute efficiency is approximately constant along the ray path (sections 4.8.1 and 6.7). Therefore it can be written that \( E_{p_1} \approx E_{p_2} = E_p \).

For \( \mu x \ll 1 \) i.e \( x \ll 1/\mu \), \( \exp(\mu x) \) can be approximated to \( \exp(\mu x) = 1 + \mu x \) (ignoring the higher order terms). Therefore the equation 3.51 can be written as
\[ A = \frac{1}{2} \left[ (P_1 + P_2) + \mu \left( \frac{V_1 + V_2}{k} \right) \right] \] (3.52)

We actually measure \( P = P_1 + P_2 \) and \( V = V_1 + V_2 \), therefore
\[ A = \frac{1}{2E_p} \left[ P + \frac{\mu V}{k} \right] \] (3.53)

In general for \( n \) number of sources lying along a line the above equation can be written as
\[ A = \frac{1}{nE_p} \left[ P + \frac{\mu V}{k} \right] \] (3.54)

where \( A_{av} = (1/n) \{ A_1 + A_2 + \ldots + A_n \} \).

This is a general expression which can be used for any number of point sources and it is true for \( x \ll 1/\mu \), where \( 1/\mu \) is the mean free path.
Therefore for a continuous source distribution the true activity concentration can be computed by measuring the P-values (from conjugate means) and dividing by value of n corresponding to unit volume, and then using expression 3.54 to allow for self-absorption.

For a single source at depth \( x = V/Pk \), the true activity, \( A_o \), can be written as

\[
A_o = \frac{1}{E_p} \left[ P + \frac{\mu P k x}{k} \right] = \frac{P}{E_p} \left[ 1 + \mu x \right]
\]

\[
= \frac{P}{E_p} \exp(\mu x) \quad \text{(as } \exp(\mu x) = 1 + \mu x \text{ for } \mu x < 1) \quad (3.55)
\]

which is the usual equation for exponential attenuation.

For \( \mu = 0 \) the above equation becomes

\[
A = \frac{P}{E_p} \quad (3.56)
\]

which is clearly true.

For two sources it can be written as

\[
A = \frac{A_1 + A_2}{2} = \frac{1}{2E_p} \left[ P + \frac{\mu V}{k} \right]
\]

\[
= \frac{1}{2E_p} \left[ P + \frac{\mu P k x}{k} \right] = \frac{P}{2E_p} \left[ 1 + \frac{\mu x}{av} \right] \quad (3.57)
\]

Again for \( \mu = 0 \) we find the obvious result:

\[
A = \frac{P}{av E_p} \quad (3.58)
\]
This method has been reported by many workers using at least two isotopes of different energy and examining the differential attenuation of the two gamma-ray energies [Dolan and Tauxe 1968, 1969; Oldendorf and Isaka 1969; Martin and Rollo 1977; Kaplan and Ben-Porath 1967, 1968]. If a known source spectrum is present from a mixture of isotopes, each emitting mono-energetic gamma-rays, this method can be used as long as the various isotopes remain mixed together in a constant ratio. If the physical or chemical or biochemical processes alter the relative distribution of sources inside the absorber then the method breaks down. This method is also applicable to single isotope which emit multiple energy gamma rays. The ratio of the counts in the different peaks is determined against the thickness of the absorber. It is based on the fact that, in general, the low energy photons are attenuated relatively more as compared to the high energy photons when passing through an absorbing medium. A simple expression for this method can be derived as follows.

The number of counts \( P_1 \) in the full-energy peak 1 corresponding to the low energy gamma-ray can be written as

\[
P_1 = (I)_{o,1} \cdot \exp(-\mu_1 d)
\]

(3.59)

where \( \mu_1 \) is the attenuation coefficient for that energy, \( d \) is the depth of the source and \( (I)_{o,1} \) is the emitted intensity of full-energy peak 1 gamma-rays.

Similarly the number of counts, \( P_2 \), in the full-energy peak 2 corresponding to the higher energy photons can be written as

\[
P_2 = (I)_{o,2} \cdot \exp(-\mu_2 d)
\]

(3.60)
where \( \mu_2 \) is the attenuation coefficient for the higher energy and \((I)_{o,2}\) is the emitted intensity with no attenuation.
The ratio of the counts in two peaks, peak-to-peak ratio (PPR), is given by

\[
P_{PPR} = \frac{P_2 (I)_{o,2} \exp(-\mu_2 d)}{P_1 (I)_{o,1} \exp(-\mu_1 d)}
\]

If the count rates are normalised, i.e., \((I)_{o,1} = (I)_{o,2}\), then

\[
P_{PPR} = \exp[-(\mu_2 - \mu_1)d]
\]

or

\[
d = \frac{\ln\left(\frac{P_2}{P_1}\right)}{(\mu_2 - \mu_1)}
\]

which shows the depth to be proportional to the logarithm of the counts ratio of the two peaks, and inversely proportional to the difference of the linear attenuation coefficients. The importance of having a large difference in linear attenuation coefficients is emphasised in the above equation since it minimises the uncertainty of depth measurement by ensuring a rapid variation of the high/low energy gamma-ray ratio with depth.

### 3.4.3 INVERSE SQUARE LAW METHOD

According to the inverse square law the response of the detector is inversely proportional to the square of the distance between source and detector. A simple expression can be derived if we assume that count rate in the full-energy peak varies with the distance alone, in cases where self-absorption is negligible (e.g. a radioactive gas). Griffith
[1933] experimentally supported the assumption by observing that the inverse square law was obeyed up to 12 cm from a radium source in a large water phantom and up to 8 cm in a smaller one. But later investigations [Roberts and Honeyburne, 1937; Ter-Pogassian et al, 1952; Van Dillon and Hine, 1952; Wooten et al, 1954] showed the contrary and all agreed that the ratio of exposure in water to exposure in air at a given distance from a source differs from unity, and, in general decreases with increasing distance from the source indicating that absorption in water was not negligible.

An expression can be derived for the depth of the source by including both the effects of the inverse square law and the exponential attenuation together. Let us consider a gamma-ray source situated at a depth 'd' inside an absorber. According to the inverse square and the exponential attenuation law we can write

\[ I(d) = I(0) \exp\{-\mu(\text{abs}) \cdot d\}/d \]  \hspace{1cm} (3.63)

where \(I(d)\) is the measured intensity at the surface of the absorber, \(I(0)\) is the true intensity emitted by the gamma-ray source, and \(\mu(\text{abs})\) is the total attenuation coefficient of the absorber for the gamma-ray emitted by the source.

At a distance 'x' outside the surface of the absorber equation 3.63 can be written as

\[ I(d+x) = I(0) \left[\exp\{-\mu(\text{abs}) \cdot d - \mu(\text{air}) \cdot x\}\right]/(d+x) \]  \hspace{1cm} (3.64)

where \(I(d+x)\) is the measured intensity at the distance 'x' from the surface of the absorber, i.e. the intensity at a distance 'd+x' from the source, and \(\mu(\text{air})\) is the attenuation coefficient of air for the gamma-ray energy emitted by the source.
Dividing equation 3.64 by equation 3.63 we get

\[
\frac{I(d+x)}{I(d)} = \frac{\exp\{-\mu(\text{air}).x\}}{2} \quad [1 + (x/d)]
\]

Substituting the values of the intensities at distance 'd' and 'd+x', \( \mu(\text{air}) \) and the distance 'x' in equation 3.65, the depth 'd' of the gamma-ray source can be determined.

If the gamma-ray emitted by the source is of high energy then the exponential term in equation 3.65 can be replaced by 1 and we get

\[
\frac{I(d+x)}{I(d)} = \frac{1}{2} \quad [1 + (x/d)]
\]

3.4.4 COMPUTED TOMOGRAPHY

Computed tomography (CT) is the most recent development in the medical and industrial field. The depth of a radioisotope inside an absorbing medium can be determined by emission computed tomography (ECT). ECT consists of positron emission tomography (PET) and single photon emission computed tomography (SPECT).

In SPECT the object, in which source is embedded, is scanned by a rectilinear scanner or by a gamma camera from different angles. The data is then processed and an image of the object is reconstructed using different computer programs and source depth is determined from inspection of the image. The SPECT method is discussed in detail in the remaining chapters.
Experimental studies were carried out to study the variation in SPR and PPR with the absorber thickness. Different gamma-ray sources were used with water and tissue equivalent rubber (TER) for different source and detector arrangements. The regions of interest were set over the full-energy peak, at the 'valley' just below the full-energy peak, and around the Compton edge of the spectrum. The width of both the scatter windows were kept the same. Figure 3.5 shows the position of the windows for SPR measurements. The values of SPR and PPR were normalised by taking the first value as a reference value. The detection system, the gamma-ray sources and the absorbers used and the different experimental arrangements are described below.

3.5.1 DETECTION SYSTEM

A block diagram of the equipment used for the measurement of SPR and PPR with depth is shown in Figure 3.6. The detector used was a 40 cm$^3$ coaxial lithium drifted germanium, Ge(Li), detector mounted in a vacuum cryostat. The cryostat is held in an iron frame so that the detector points vertically upwards and the detector is encased in a lead shield to reduce the background radiation. The upper surface of the detector shield is flat and this acts as a base for the apparatus used to mount the source and the absorber over the detector. The detector bias voltage used was +2800 volts and the full-width at half-maximum (FWHM) for 662 keV gammas was found to be 1.79 keV (0.27%). The absolute full-energy peak efficiency curves for different gamma-ray energies at different source-to-detector distances are shown in Figure 3.7.

The signal from the Ge(Li) detector was fed through a Princeton Gamma-Tech (PGT) preamplifier and then into an Ortec 570 shaping amplifier with a 2 microsecond shaping time. The output from the
Figure 3.5: Sketch of the energy spectrum showing the windows chosen for SPR measurements (A). Around the Compton edge (B). Valley region (C). Full-energy peak window.

Figure 3.6: Block diagram of the detection system with Ge(Li) detector
Figure 3.7: Absolute full-energy peak efficiency curves for Ge(Li) detector for different source-to-detector distances.
amplifier was then fed to the Nuclear Data 66 (ND66) multichannel analyser. Different regions of interest were marked on the spectrum and counts were recorded then the spectra were transferred along a serial communication line to the PRIME computer system for analysis.

3.5.2 GAMMA-RAY SOURCES

The sources of gamma-rays used for this experiment were Cs-137, Na-22, Am-241, and Ba-133 standard reference point sources, and a liquid Cs-134 solution in 45 mm diameter perspex bottle as an extended source. The sources used for SPR and PPR measurements are given below with their activity and the gamma-ray energies used for the experiment.

<table>
<thead>
<tr>
<th>ISOTOPE</th>
<th>ACTIVITY (kBq)</th>
<th>GAMMA-RAY ENERGIES USED (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Am-241</td>
<td>37</td>
<td>59.6</td>
</tr>
<tr>
<td>Cs-137</td>
<td>370</td>
<td>662</td>
</tr>
<tr>
<td>Na-22</td>
<td>370</td>
<td>511, 1275</td>
</tr>
<tr>
<td>Ba-133</td>
<td>37</td>
<td>36, 81, 303, 356</td>
</tr>
<tr>
<td>Cs-134 (liq)</td>
<td>370/ml</td>
<td>605, 796</td>
</tr>
</tbody>
</table>

3.5.3 ABSORBING MATERIALS

Water and foam rubber were used as the absorbing materials because they are tissue equivalent materials as their effective atomic number is equivalent to that of soft tissue. A 20cm x 30cm x 20cm perspex tank of 5mm wall thickness was used for water to simulate the chest of a patient. Rubber sheets of dimensions 15cm x 15cm and of 2cm thickness were used. The rubber sheets are made from a fluid rubber compound based on depolymerised natural rubber which can be considered chemically as polyisoprene, i.e. \((C_5H_8)_n\) and is prepared from the raw rubber by subjecting it to mechanical and thermal processes [Asual, 1980].
The absorption of low energy photons by the elements of lower atomic number is mostly due to the Compton and photoelectric absorption processes. Compton absorption depends only on the electron density of the material. The electron density for the rubber is $3.32 \times 10^{23}$ electrons/cm$^3$ very close to $3.34 \times 10^{23}$ electrons/cm$^3$ for water and $3.36 \times 10^{23}$ electrons/cm$^3$ for muscle, and its radiation absorption is closely similar to soft tissue assuming its chemical composition to be $(C_5H_{40}O_{18})_n$ and density to be unity [Stacey 1961].

3.5.4 EXPERIMENTAL SET-UP

3.5.4.1 Using Water

The water tank described in the previous sub-section was used to contain the demineralised water for the experiment. The point source was placed in a perspex vial of 2mm wall thickness, to keep it safe from the water, and was hung in the water tank as is shown in Figure 3.8. Three different data sets were recorded for three different source and detector arrangements. The first data set was obtained by placing the source at the bottom of the tank and recording the spectrum with the MCA. The regions of interest were marked and the counts in the appropriate regions of interest were recorded. Then the source was moved step by step away from the detector and the counts were recorded in each of the regions of interest. This procedure was repeated for all the gamma-ray sources mentioned earlier. The second data set was obtained by collimating the detector with a 20mm diameter and 40mm long lead collimator and repeating the same procedure as above. The third data set was taken by keeping the source-to-detector distance constant at 15cm and increasing the thickness of the water inbetween. Then the SPR and PPR were calculated for each increment in depth as the thickness of water was varied from 0-15cm.
Figure 6.8: Experimental set-up using water
The tissue equivalent rubber (TER) sheets of 15cm x 15cm and 2cm thickness were used as absorbing material instead of water. The water tank in figure 3.8 was removed and the rubber sheets were placed over the detector. Three different data sets were obtained for this case for three different experimental arrangements.

For the first data set the point source was embedded in the centre of a rubber sheet and the rubber thickness was increased by interposing additional sheets inbetween the top sheet (containing the source) and the detector. The second data set was obtained by keeping the source-to-detector distance constant at 30cm and putting the rubber sheets inbetween. The third data set was obtained by collimating the source with a 3mm diameter by 40mm long lead collimator.

The measurements were also made for Cs-134 liquid source by placing it on top of a pile of rubber sheets and increasing the rubber thickness between source and detector. The counts in the respective regions of interest were recorded for every increment in thickness and for every source used, as the thickness of rubber was varied from 0 to 24 cm.

3.6 RESULTS

3.6.1 Cs-137 Point source

The scatter windows were set around the Compton edge and on the 'valley' just below the photopeak and the peak window was set to bracket the 662keV full-energy peak. The SPR values were determined by dividing the counts in the two scatter regions by the photopeak counts. The
Compton scatter-to-peak ratio, $\text{SPR}(C)$, and valley-to-peak ratio, $\text{SPR}(V)$, were determined for water by hanging the source in the water while 15cm deep water tank was kept full. So for the first measurement i.e for the zero thickness of water, 15cm of water was behind the source. The source was then moved away from the detector to increase the thickness of water between source and detector. For the TER absorber the source was placed in the centre of the first sheet and the thickness of TER was increased as the source to detector distance was increased by putting additional TER sheets between the source and the detector. The normalised values of $\text{SPR}(C)$ and $\text{SPR}(V)$ are shown in Figure 3.9 for water and in Figure 3.10 for rubber. It is clear from both the figures that valley-to-peak ratio is more sensitive than the Compton scatter-to-peak ratio for collimated and uncollimated detector arrangements. The variation in $\text{SPR}(C)$ and $\text{SPR}(V)$ for 0-15cm of water is 58% and 110% for collimated and 86% and 158% for the uncollimated detector, respectively. The corresponding values for the TER are 97% and 164% for collimated detector, and 229% and 357% for uncollimated detector for 0-24cm of rubber thickness. This means that the valley-to-peak ratio is almost twice as sensitive as Compton scatter-to-peak ratio.

3.6.2 Am-241 point source

The scatter windows were set around the Compton edge and at the 'valley' just below the 59.6 keV full-energy peak and the peak window was set over the 59.6 keV full-energy peak. The source and detector arrangement was same as described in the previous section. In the case of water the source was hung in the water tank containing 15cm of demineralised water. So for the first measurement i.e for zero thickness of water 15cm of water was behind the source acting as a backscatter medium and the source to detector distance was at its minimum. Then as the source was moved away vertically from the detector the thickness of water between source and detector was increased and so was the
FIGURE 3.9 RESULTS WITH WATER WITH CS-137 SOURCE WITH AND WITHOUT COLLIMATED DETECTOR

FIGURE 3.10 RESULTS FOR RUBBER WITH CS-137 WITH AND WITHOUT COLLIMATED DETECTOR
source-to-detector distance. In the case of TER, to keep the same conditions as for water, the source was put in the centre of the first rubber sheet and for the first measurement the thickness of rubber between source and detector was 2 cm, and 20 cm of rubber was on the back of the source. The thickness of rubber between source and detector was increased by interposing the rubber sheets between the source and the detector and the thickness of the backscatter was decreased as in the case of water. The normalised values of SPR(C) and SPR(V) versus water thickness are shown in Figure 3.11 with the uncollimated detector. The percentage variation from the initial values in SPR(C) is 56% and for SPR(V) is 164% for 15 cm of water thickness. This means that the valley-to-peak ratio is 108% more sensitive than the Compton scatter-to-peak ratio. Figure 3.12 shows the variation in valley-to-peak ratio with and without detector collimator for TER and it can be seen that valley-to-peak ratio for the uncollimated detector is 92% more sensitive than the collimated detector case.

3.6.3 Sodium-22 Point Source

For Na-22 the SPR(C), SPR(V) and PPR (1275 keV/511 keV) were determined. The procedure was the same as described in the previous section. As described previously the SPR's were calculated by taking the ratios of counts in the region around the Compton edge and the counts in the 'valley' just below the 511 keV peak to the counts in the 511 keV photopeak. The normalised values of SPR(C), SPR(V) and PPR against the water thickness are shown in Figure 3.13 for the uncollimated detector arrangement, and for TER the corresponding values are shown in Figures 3.14 and 3.17 for uncollimated and collimated detector arrangements, respectively. The percentage variation in SPR(C), SPR(V) and PPR from the initial values are 97%, 198% and 74% with the uncollimated detector over 15 cm of water thickness, and for the TER 168%, 292% and 102% with the uncollimated detector and 48%, 76% and 85% with the collimated...
FIGURE 3.11 SPR VS WATER THICKNESS WITH AM-241 POINT SOURCE, DETECTOR UNCOLLIMATED.

FIGURE 3.12 VALLEY-TO-PEAK RATIO VS RUBBER THICKNESS WITH AM-241, WITH AND WITHOUT COLLIMATED DETECTOR.
FIGURE 3.13 RESULTS FOR WATER WITH NA-22 UNCOLLIMATED DETECTOR

FIGURE 3.14 RESULTS FOR RUBBER WITH NA-22 SOURCE INSIDE THE RUBBER, DETECTOR UNCOLLIMATED.
detector over 18cm of rubber thickness, respectively. Therefore it can be said that the valley-to-peak ratio is more sensitive for the uncollimated detector case while for the collimated detector case the variation in PPR values is more than the SPR(V) and SPR(C). The results were also obtained for fixed source-to-detector distance, 15cm in the case of water and 30cm in the case of rubber, and are shown in Figures 3.15 and 3.16 for water and rubber for uncollimated source and detector arrangement, respectively.

3.6.4 Barium-133 point source

The Ba-133 source was used to determine the PPR for different pairs of peaks. The source and detector arrangement was same as described in the previous sections. The variation in the ratio of 356keV/36keV peak counts and 356keV/81keV counts are shown in Figure 3.18 for water with the collimated detector. It is obvious from the figure that the PPR for 356keV/36keV is much more sensitive than the PPR for 356keV/81keV peaks. To elaborate the difference between the two ratios the normalised values of PPR are shown in Figures 3.19 and 3.20 for water and TER respectively, with the collimated detector. Results were also obtained for the uncollimated detector and it was found that the PPR was more sensitive for the collimated detector arrangement. The Figures 3.20 and 3.21 show that the percentage variation in the PPR of 356keV/36keV peaks and 356keV/81keV peaks are 1930% and 183% over 15cm of water thickness, and 1600% and 180% over 14cm of rubber thickness, respectively. The Figures 3.21 and 3.22 show the variation in the normalised values of PPR of 356keV/81keV and 356keV/303keV peaks for water and TER respectively, for the collimated detector. It is obvious from the figures that PPR for 356keV/81keV peaks is much more sensitive than PPR for 356keV/303keV peaks. This means that the differential attenuation method is more sensitive for the peaks having large difference in their energies.
FIGURE 3.15
RESULTS FOR WATER WITH NA-22 SOURCE
AT 15 CM, UNEOLLIMATED DETECTOR AND SOURCE

FIGURE 3.16
RESULTS FOR RUBBER WITH NA-22 SOURCE
AT 30CM FROM UNCOLLIMATED DETECTOR.
FIGURE 3.17: RESULTS FOR RUBBER WITH NA-22 WITH COLLIMATED DETECTOR.

FIGURE 3.18 PPR VS WATER THICKNESS WITH BA-133 DETECTOR NOT COLLIMATED.
FIGURE 3.19 PPR VS WATER THICKNESS WITH BA-133 DETECTOR COLLIMATED.

FIGURE 3.20 PPR VS RUBBER THICKNESS WITH BA-133 FOR COLLIMATED DETECTOR.
FIGURE 3.21 PPR US WATER THICKNESS WITH BA-133 COLLIMATED DETECTOR.

FIGURE 3.22 PPR US RUBBER THICKNESS WITH BA-133 COLLIMATED DETECTOR.
FIGURE 3.23 PPR Vs RUBBER THICKNESS WITH BA-133 SOURCE COLLIMATED AND AT 30CM.

FIGURE 3.24 PPR Vs RUBBER THICKNESS WITH BA-133 NCOLLIMATED SOURCE AT 30CM HEIGHT.
Results were also obtained for fixed source-to-detector distance of 30cm and are shown in Figures 3.23 and 3.24 for 356keV/81keV and 356keV/303keV PPRs with collimated and uncollimated source and detector, respectively. The source collimator was 3mm diameter and 40mm long lead collimator. As the figures show the variation in 356keV/81keV normalised PPR is 525% from the initial value for collimated source and 414% for uncollimated source, and for 356keV/303keV PPR the corresponding values are 50% for collimated and 30% for uncollimated source, over 24cm of rubber thickness. This means that the PPR with collimated source and detector is more sensitive than the uncollimated source and detector arrangement.

3.6.5 Cs-134 Liquid source

The Cs-134 solution contained in a 45mm diameter perspex bottle, was used as an extended source to measure the variations in SPR and PPR. The SPR(C), SPR(V) and PPR were determined by setting regions of interest over the Compton edge, the valley next to the 605keV full-energy peak, and on the 605keV and 796keV full-energy peaks. In the case of water the source was hung in a water tank containing 17.5cm of water, and the water thickness was increased by moving the source vertically away from the detector. In the case of TER absorber the source was put over the first rubber sheet and the rubber thickness was increased by putting the additional sheets between the source and detector.

The normalised values of SPR(C), SPR(V) and PPR for water and rubber are shown in Figures 3.25 through 3.28 for uncollimated and collimated detector, respectively. The variations in the SPR(C), SPR(V) and PPR for water are 26%, 35% and 20% for the collimated detector, and 59%, 68% and 23% for the uncollimated detector over 17.5cm variation in water thickness, respectively. For the TER the corresponding values are 67%, 80% and 33%, and 92%, 110% and 42% for the collimated and uncollimated
FIGURE 3.25: RESULTS FOR WATER WITH CS-134 LIQUID SOURCE, DETECTOR UNCOLLIMATED.

FIGURE 3.26: RESULTS FOR WATER WITH CS-134 LIQUID SOURCE, DETECTOR COLLIMATED.
FIGURE 3.27: RESULTS FOR RUBBER WITH CS-134 DETECTOR UNCOLLIMATED.

FIGURE 3.28: RESULTS FOR RUBBER WITH CS-134 DETECTOR COLLIMATED.
detector respectively, for a 24cm variation in rubber thickness. It is clear from the figures that the variations in SPR(C) and SPR(V) are almost double for the uncollimated detector compared to the collimated detector values, but the difference in the PPR values are not significant with the detector collimated or uncollimated.

### 3.7 DISCUSSION

The scatter-to-peak ratio and peak-to-peak ratio versus thickness of the absorber represent two methods by which the depth of a photon source in an absorbing medium can be determined from the measurements of the counts in the different regions of the energy spectrum of the emergent gamma-rays. The measurement of counts in the different regions of the energy spectrum may be performed either by the gamma-camera, a rectilinear scanner, or with a simple set-up shown in Figure 3.8. The two absorbing materials used were the demineralised water and tissue equivalent rubber.

Simple theoretical expressions for the SPR in emission and transmission cases predict a linear relationship between the SPR and the thickness of the absorber but from the results it is obvious that although most of the plots are very close to the straight line but they are not exactly linear. It was thought that this might be due to the fact that the relationship holds provided the assumptions made are true, i.e. that there is no attenuation in the air between the source and detector, that narrow beam geometry holds for a point source of radiation, and that only the low angle single scattered photons are detected. In the case of a fixed source-to-detector distance the attenuation in the air between source and the absorber will be higher for low absorber thicknesses. Although the detector solid angle remains constant for the primary radiation for the fixed source-to-detector distance while for the variable
source-to-detector distance the solid angle varies with the thickness of the absorber, and so the counts recorded in the different regions will be affected by the change in the geometry. In the case of water, for zero thickness of water, the actual absorber thickness was the 5mm thickness of the perspex tank wall and also the source was encased in a perspex vial of 1mm thickness. Kacperek [1977] predicted that for large scattering angles the deviation from linearity is expected and becomes greater as the scattered photon energy decreases.

The results presented in the previous section show that the SPR(V) is more sensitive to depth variations than SPR(C) for the same source and geometry. However the variations in the SPR(C) with depth are significant, and can be used for depth determination purposes. Either method provides a measure of the depth, but SPR(V) varies more rapidly than SPR(C) and, therefore, it can be said that valley-to-peak ratio is the parameter of choice for depth determination of gamma-ray sources inside the absorbing media.

The differential attenuation method is applicable only to gamma-ray sources which emit two or more photons. In Figure 3.13 initially the PPR varies more rapidly than SPR(C) but then as the thickness of absorber increases it varies more slowly than SPR(C). This is due to the fact that as the depth of the source increases, more scattering material between source and detector causes increased multiple scattering. This increases the number of counts at the lower energy regions. This effect becomes more pronounced for large depths particularly due to the fact that more scattering material increases the probability of second scattering collisions. Thus the counts at lower energies, i.e. at higher scattering angles, increases more pronouncedly at greater absorber thicknesses. The degree of collimation of a detection system will affect the shape of the recorded spectra and alter the way they change with depth. With a highly collimated system, the amount of multiply scattered
radiation detected will be decreased because of the limited field of view of the detector, while with low or no collimation the amount of multiply scattered photons detected will be larger. Obviously in the extreme case of perfect collimation only the full-energy peak and 180° backscatter peak would be detected [Filipo et. al, 1979]. So that is why the values of SPR(C) and SPR(V) are higher for the uncollimated detector and relatively lower for the collimated detector, and due to the same reasons the variations in the PPR are relatively greater than the SPR(C) and SPR(V) for the collimated detector and are lower for the uncollimated detector. The multiply scattered radiation from the higher energy can fall in the lower full-energy peak region and so increase the counts in the low energy peak region which therefore decreases the value of the PPR (high/low). With a collimated detector the multiply scattered photons from higher energy gamma-rays are less likely to be detected in the lower energy region and so the PPR value is increased.

The differential attenuation method is more sensitive for pairs of peaks which differ largely in their energy. This is illustrated in Figure 3.19, representing the variation in the PPR [356keV/36keV (barium X-rays)] and PPR (356keV/81keV) full-energy peaks. It can be seen that the former ratio varies more quickly than the latter. This is because the low energy photons are absorbed rapidly as the depth of the absorber increases, so there are less counts in the low energy photopeak region and hence the PPR (high/low) increases as the depth increases. The same is the cases of PPR (356keV/81keV) and PPR (356keV/303keV), shown in Figures 3.21 through 3.24. The PPR (356keV/81keV) varies more rapidly than the PPR (356keV/303keV) for the same reason. This means that the differential attenuation technique for depth determination can, more effectively, be used for gamma-rays having large difference in their energies. For the gamma-ray energies which are not very different the attenuation coefficient differs only by a small amount.
As shown in the previous section the values of SPR and PPR for water and rubber, for the same source and detector geometry are almost equal. This is due to the fact that both the absorbers are the tissue equivalent materials and their effective atomic numbers are very close to each other. The mass absorption coefficient of an element depends on the energy of the gamma-ray and the atomic number, Z, of the element. At most energies the Compton scattering component is a significant part of the total mass absorption coefficient, and is the main component at energies in the range of 0.2 - 3 MeV for most elements. The Compton scatter component depends on the ratio of the atomic number, Z, and atomic mass, A, and hence is approximately independent of Z, since \( \frac{Z}{A} \approx 0.5 \) except for hydrogen. The photoelectric component becomes more important at lower energies and is proportional to between \( \frac{Z^4}{A} \) and \( \frac{Z^5}{A} \). The pair production component becomes important at high gamma-ray energies and proportional to \( \frac{Z^2}{A} \). The Compton absorption which is the main component for the intermediate energies, depends on the electron density of the material. The electron density for water is \( 3.34 \times 10^{23} \) electrons/cm\(^3\) and for TER is \( 3.32 \times 10^{23} \) electrons/cm\(^3\); these values are closely equal to the electron density of soft tissue assuming its density to be unity [Stacey 1961]. So this means that the value of the scatter component will almost be the same for water and rubber and so will be the SPR and PPR.

It has been shown in the previous section that the \( \frac{\text{SPR}(V)}{A} \) is more sensitive to the depth than the \( \text{SPR}(C) \) in all the cases. The differential attenuation method is more sensitive for the cases when the difference in the attenuation coefficients of the respective gamma-ray energies is large. Therefore it can be concluded that for a particular detector and particular geometry, the SPR and PPR techniques can be used as tools to successfully measure the depth of a point gamma-ray source inside an absorbing medium.
The results presented in this chapter can be compared with results of previous workers. Mohindra and McNeil [1965], after a detailed study of the spectra from point sources of a range of isotopes with energies of 0.28 to 1.53 MeV, concluded that the ratio of the minimum counting rate in the 'valley' just below the full-energy peak to that in the photopeak was the parameter that varied most strongly with depth for low energy isotopes and could, for a given geometry and a given detector, be used to measure the depth of a gamma-ray source. But according to Filipow et al [1979] the counting rate in the Compton scatter region increases more rapidly than that in the 'valley'. Koral and Johnston [1977] determined the depth of I-131, and Martin and Rollo [1977] measured the scatter of the 159 keV photons from I-123. The latter authors measured the ratio of counts in the Compton scatter region to those in the photopeak, and found only a small variation with the depth. They also employed the differential absorption of 27keV (X-rays) and 159keV peaks from I-123 to measure the depth, and showed that this was the most sensitive method. Martin and Rollo [1977] also studied the ratio of the counts in the backscatter peak to that of the full-energy peak, and found out that it is insensitive to depth. Our results agree with the results of Mohindra and McNeil [1965], Koral and Johnston [1977], and Martin and Rollo [1977], that the counts in the 'valley' just below the full-energy peak are more sensitive to depth than the counts in the Compton scatter region. The counts in the Compton scatter also varied significantly in our case, but the counts in the 'valley' were more sensitive to depth. The differential attenuation method is equally important and is shown in the previous section that it is the most sensitive method when the gamma-ray energies are substantially different.

The Cs-134 solution contained in a plastic bottle was used to see whether the dimensions of the source affect the SPR and PPR values. It is clear from the Figures 3.25 through 3.28 that there is not much difference in the sensitivity of the SPR and PPR compared with the point source.
measurements. This might be due to the small dimensions of the source (45mm diameter).

The results presented here are specific to the detector and geometries that were employed, but the methods described here does enable depth to be determined by comparing the values of the SPR and PPR with values obtained under the similar conditions. Knowledge of the depth at which a radioisotope is located, will enable the quantity present to be estimated with improved accuracy.

A distributed source throughout a larger volume can be considered as a number of point sources lying in the absorbing medium all over its volume at different depths. The counts recorded by a well collimated detector will represent the total effect of point sources lying along that line. So the counts recorded will be the line integral along that line, which in fact forms the basis of computed tomography and will be discussed in detail in the next chapters.
CHAPTER 4

THEORY OF COMPUTED TOMOGRAPHY

4.1 INTRODUCTION

The mathematical proof that a two or three dimensional object could be reconstructed from an infinite set of all its projections was given by Radon in 1917 who was concerned with gravitational theory. The first example of mathematical image reconstruction from projections was in radio astronomy in 1956 when Bracewell constructed a map of solar micro-wave emission from line integrals across the sun's disc [Bracewell, 1956]. It is worth noting that Bocage in 1921 had produced images by focal plane transmission tomography in medicine [Bocage, 1921], although he used mechanical technique for defining the slice of interest.

The reconstruction problem has arisen and been independently solved in several fields including electron microscopy [De Rosier and Klug, 1968; Gordon et al, 1970], optics [Rowley, 1969], medicine [Kuhl and Edwards, 1963; Cormack, 1963; Hounsfield, 1972]. The solution that all these imaging problems have in common has the same mathematical foundation [Herman, 1979; Ravik, 1979]. The one area of application which has had the greatest impact in general, at least in the public's awareness, is diagnostic medicine.

The reconstruction techniques fall into three broad categories:

(i) Back Projection Techniques
(ii) Analytical Techniques
(iii) Iterative Techniques.

The back projection method, also known as the summation method, was used in the first attempts at reconstruction from projections and Kuhl and Edwards [1963] produced the first radioisotope image by this method.
However, this simple technique suffers from severe image distortion.

Analytical techniques were first applied to reconstruct a gamma-ray image by Cormack [1963] and are now widely used in X-ray tomography commercial scanners, and to a lesser extent in radioisotope imaging. A general characteristic of analytical methods is that they utilise exact formulae in the reconstruction.

Iterative techniques were those used by Bracewell [1956] and Gordon et al [1970] and were utilised in the first model of EMI scanner in 1973 [Hounsfield, 1973] and in other early imaging applications [Schmidlin, 1972; Goiten, 1972; Mayers et al, 1972]. In iterative reconstruction techniques a solution is found by solving a set of algebraic equations.

4.2 FORMATION OF PROJECTIONS

In defining the ray-sum and projection the object's slice of interest is assumed to be thin so that a two-dimensional geometry can be applied. The section of interest is described by two-dimensional density function \( f(x,y) \). In Figure 4.1 we denote the plane under investigation by Cartesian coordinates \((X,Y)\), and the contribution of each point in the object towards the detected signal as the density function \( f(x,y) \). In the transmission computed tomography (TCT) case the density function represents the distribution of linear attenuation coefficient values throughout the object and in emission computed tomography (ECT) it represents the distribution of the radioisotope concentration. The ray paths are more conveniently described in a set of orthogonal coordinates \((r,s)\) rotated at the same angle as the rays to the original \((X,Y)\) axis. Each point along the ray is now specified by coordinates \((r,\theta)\). The projection data are estimates of the line integral of the function \( f(x,y) \), where the line integration is specified by the parameters \( r \) and
Figure 4.1: Representation of the coordinate system. Positions within the object is specified by an \((x,y)\) coordinate while the rays are indicated by a rotated \((r,s)\) system.
The $r$ and $s$ axes are rotated by an angle $\phi$ from the $X$ and $Y$ axes. A line $L(r,\phi)$ can be specified by the pair $(r,\phi)$ where 'r' is the perpendicular distance of the line from the origin $O$ and '$\phi$' is the angle this perpendicular makes with the $X$-axis. The integral of $f(x,y)$ along $s$-axis is termed as the ray-sum or ray-projection.

$$P(r,\phi) = \int_{L(r,\phi)} f(x,y)ds$$  \hfill (4.1)

This equation defines $P(r,\phi)$ as Radon transform of $f(x,y)$.

A complete set of ray-sums at a given angle is called projection or profile.

The equation of any ray-sum could be obtained in the rotating frame of reference from the equation

$$l\cos(\phi-\theta) = r$$  \hfill (4.2)

where 'l' is the distance of a point $(x,y)$ from the origin.

Expanding this equation we obtain

$$r = l\cos\theta\cos\phi + l\sin\theta\sin\phi$$  \hfill (4.3)

From Figure 4.1 $x = l\cos\theta$ and $y = l\sin\theta$. Therefore the coordinate pairs $(x,y)$ that specify the density function along a ray-sum $(r,\phi)$ are given by

$$r = x\cos\phi + y\sin\phi$$  \hfill (4.4)

Ideally $f(x,y)$ is a continuous two-dimensional function and an infinite number of projections are required for reconstruction. In practice, however, $f(x,y)$ is calculated at a finite number of points from a finite number of projections. For data collecting purposes the object is usually limited to a circular domain of diameter $d$, say, and if the image is reconstructed at points arranged rectangularly with spacing $w$, then
there are \( n = \frac{d}{w} \) points along a principal diameter. This is the number of ray-sums contained in a projection. Each square cell of width \( w \) is called a pixel, short for picture element.

### 4.2.1 FORMATION OF PROJECTIONS IN TCT

In Figure 4.1 the line \( \text{L} \) denotes the path of a fine collimated beam of mono-energetic photons from an external collimated source detected by a collimated detector. If the beam enters the scanned object with an intensity \( I(o) \) and the distribution of linear attenuation coefficient relative to \( (r, s) \) is \( \mu(r,s) \), then the intensity of the emergent beam, \( I(r) \), will be given by

\[
I(r) = I(o) \exp[-\int \mu(r,s) \, ds]
\]

or

\[
\int \mu(r,s) \, ds = -\ln[I(r)/I(o)]
\]

and

\[
P(r,\theta) = \int \mu(r,s) \, ds = -\ln[I(r)/I(o)]
\]

Therefore in transmission tomography \( f(x,y) \) represents the distribution of linear attenuation coefficient, \( \mu(r,s) \). By measuring the incident, \( I(o) \), and transmitted, \( I(r) \), beams the value of ray-sum \( P \) can be derived. Measuring a set of ray-sums at a given angle will provide a particular projection for transmission tomography.

### 4.2.2 FORMATION OF PROJECTIONS IN ECT

Let the line in Figure 4.1 be defined by a small well collimated detector whereby the detection of radiation emitted from within the material under study takes place. In single photon emission computed tomography (SPECT), the intensity of photons detected in that direction
is given by

$$A(r,\theta) = \int_{L(r,\theta)} A_0(x,y) \exp[- \int_{L(r,\theta,x,y)} \mu(x',y')ds] ds \quad (4.7)$$

where $A_0(x,y)$ is the activity distribution appropriate to the gamma-ray energies being detected. If the detector is at a distance from the source which is large as compared to the dimension of the object, then the solid angle subtended by the detector at any point along the line $L$ within the object is practically independent of the position of the point in that medium. If there is no attenuation of radiation inside the object, the activity $A(r,\theta)$ would be proportional to the line integral of $A_0$ along $L$. But in practice such conditions are not met. The detector has to be placed as close as possible to the source to achieve high detection efficiency and the attenuation inside the medium is generally not negligible considering the energies of interest in SPECT. Therefore solid angle and attenuation factors have to be considered. The inner integral describes the attenuation of the emitted gamma-ray at a point $(x',y')$ and is evaluated on the segment $L(r,\theta,x,y)$ from $(x,y)$ to the detector [Kouris et al, 1982]. If $(x,y)$ is known, an estimate of $A_0(x,y)$ can be obtained.

In the next sections the different reconstruction techniques will be discussed briefly. The detailed theory of the various reconstruction methods is given by Brooks [1974], Foster [1982], Budinger [1976] and Herman [1974].

4.3 BACK PROJECTION METHODS

The Back projection method, also called the Summation method or Linear Superposition method, was used in the first attempt for reconstruction from projections by Oldendorf [1961]. Kuhl and Edwards
Figure 4.2: Representation of scanning geometry for a circular dense object

Figure 4.3: Back projection for a small dense object showing the star artifact.
[1963] produced the first radioisotope image by this technique. This is the simplest method of reconstruction by which the reconstruction is done by back projecting each profile across the image plane such that the value of each ray-sum is distributed equally between all the points along the ray. In Figure 4.2 a dense circular object is illuminated by an X-ray beam (X) and a shadow (S) is formed on the X-ray film. This process is repeated at several angles around 180° arc so that several radiographs are produced. If a parallel beam of light is passed through each radiograph the result is a series of lines which cross at the position of the original object (Figure 4.3).

Therefore, for a given point the reconstructed density, \( f_b(x, y) \), is the sum of all the ray-sums that pass through it and can be written as:

\[
f_b(x, y) = \sum_{j} P(x', \theta_j) \Delta \theta
\]

where \( \theta_j \) is the jth projection angle, \( \Delta \theta \) is the angular interval between projections and \( m \) is the total number of projections.

The Back Projection method is attractive because it can be implemented easily without the need of a computer or sophisticated mathematics. However the resulting image is only a crude approximation of the original object, and the most striking artifact of Back Projection is the well known star pattern (Figure 4.3) produced by a highly localised object. This occurs because points outside the original object receive some of the back-projected intensity. Even if a great many projections are used, so that the star pattern is blurred and there is still an increase in the background intensity. All points within the object receive these background concentrations from neighbouring points, so that the subtle differences in density can not be distinguished.

There have been attempts to improve the image by partly removing the blurring using a digital computer. Vainshtein [1971] described a
subtracting procedure to increase contrast in the image by making the average density of the reconstruction identical with the estimated average density of the original. Muehllehner and Wetzel [1971] subtracted out the star pattern to a limited degree by looking for high density points, calculating the corresponding star patterns and subtracting them from the image. However, with the advancement of computer technology the emphasis has shifted to more accurate methods and this method is no longer used. It is included here not only for its historical importance but because an understanding of Back-Projection will serve as a basis for understanding the more complex methods to be described in next sections.

4.4 ANALYTICAL METHODS

Analytical reconstruction methods rely on the direct solution of equation 4.6 for the density function \( f(x,y) \). These solutions are designed to remove the star artifact that blurs each image point in the back-projection process. The first analytical reconstruction using \( \gamma \)-rays was performed by Cormack [1963,1964]. But the Fourier reconstruction technique had been introduced by Bracewell [1956] in radioastronomy. Later this technique was used in electron microscopy [De Rosier and Klug, 1968], optical holography [Rowley, 1969], radiography and radioisotopic imaging [Treitiak et al, 1969; Bates and Peters, 1971; Mersereau, 1973; Kay et al, 1974].

Analytical techniques are now widely used in X-ray tomography, e.g in EMI and ACTA scanners, and, to a lesser extent, in radioisotope imaging. The analytical methods can be grouped into two basic categories:

(i). Two-dimensional Fourier Reconstruction

(ii). Filtered Back Projection.
4.4.1 TWO-DIMENSIONAL FOURIER RECONSTRUCTION

The Fourier reconstruction method was first introduced by Bracewell in 1956. This method was independently discovered by De Rosier and Klug [1968] in electron microscopy and by Rowley [1969] in optical holography and has been used or proposed by several authors for radiographic and radioisotopic applications [Brooks, 1974].

The problem posed is to find the best estimate of a set of density function $f(x,y)$ values from a set of projection data $P(r,\theta)$. The image reconstruction is achieved by finding the two-dimensional array of Fourier coefficients. The image will be the inverse Fourier transform of this array. The various interpolation schemes had been discussed in a great deal in the literature [Herman, 1973].

Any function of space or time can be represented as a sum of sine and cosine waves (harmonics) of different frequencies. The amplitude of each harmonic is called the Fourier coefficient. A plot of the projected activity distribution (SPECT) or attenuation coefficients (TCT) as a function of distance constitutes a waveform. This waveform can be approximated by the superposition of a series of sine and cosine waves corresponding to the different Fourier coefficients. Points in Fourier space for which there is no data are estimated by interpolation. By using these Fourier coefficients it is possible to reconstruct the original object pattern [McCullough and Payne, 1977].

The starting point for the derivation of the Fourier method is to represent the two-dimensional density function $f(x,y)$ as a sum of sine and cosine waves, propagating in various directions across the plane, i.e.
\[ f(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(Kx, Ky) \exp[2\pi i (Kxx, Ky)] dKxdKy \quad (4.9) \]

where \( F(Kx, Ky) \) are the Fourier coefficients and the parameters \( Kx \) and \( Ky \) are the wave numbers in the \( X \) and \( Y \) direction. Taking the inverse transform of the above equation gives the Fourier coefficients:

\[ F(Kx, Ky) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) \exp[-2\pi i (Kx X, Ky Y)] dx dy \quad (4.10) \]

If the \((X,Y)\) axes is rotated by an angle \( \theta \) to a new axes \((R,S)\) (Figure 4.4)

![Figure 4.4: Representation of rotation axes.](image)

where

\[ \theta = \tan \left( -\frac{Ky}{Kx} \right) \quad (4.11) \]

and using the rotational transformation

\[ Kx = K \cos \theta \]
\[ Ky = K \sin \theta \]

where

\[ \overline{K} = Kx + Ky \]

and

\[ K = |K| = (Kx + Ky)^{2.1/2} \]
The equation 4.10 becomes

\[ F(K_x, K_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \exp[-2\pi iK(x\cos\theta + y\sin\theta)] \, dx \, dy \quad (4.13) \]

Now both the \((x, y)\) and \((R, S)\) coordinates are Cartesian coordinates therefore \(dx \, dy = dr \, ds\) and \(x\cos\theta + y\sin\theta = r\) (equation 4.4).

Thus the equation 4.13 becomes

\[ F(K_x, K_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \exp(-2\pi iKr) \, ds \, dr \quad (4.14) \]

The \(s\)-integral in the above equation is the ray-sum defined by the equation 4.1. Thus the above equation now becomes

\[ F(K_x, K_y) = \int_{-\infty}^{\infty} P(r, \theta) \exp(-2\pi iKr) \, dr = P(K, \theta) \quad (4.15) \]

where \(P(K, \theta)\) is the Fourier transform of \(P(r, \theta)\) with respect to \(r\). Therefore

\[ F(K_x, K_y) = F(K, \theta) = P(K, \theta) \quad (4.16) \]

This equation states that each Fourier coefficient of \(f(x, y)\) equals the Fourier coefficient of the projection taken at the same angle.

The image reconstruction by this technique is achieved by taking the one-dimensional Fourier transform of the projection, followed by interpolation to provide a two-dimensional array of Fourier coefficients and finally to take the inverse two-dimensional transform to achieve the density function \(f(x, y)\).

The requirement for interpolation arises because the Fourier coefficients obtained from projections do not fall on a rectangular matrix as required.
for the inverse two-dimensional transform [Brooks and Di Chiro, 1974]. Although it is possible to obtain exact interpolation using the sampling theorem [Bracewell, 1965], the large amount of computer time involved has made it prohibitive for most applications. A number of alternative procedures had been developed [Crowther et al, 1970; Mersereau and oppenheim, 1974; Thompson and Bracewell, 1974].

4.4.2 FILTERED BACK-PROJECTION

The equation 4.8 can be written as in the integral form as

\[ f(x,y) = \int_{0}^{\pi} p(r,\theta) \, d\theta \] (4.17)

The Fourier transform of \( p(r,\theta) \) is given by

\[ p(r,\theta) = \int_{-\infty}^{\infty} p(k,\theta) \exp(2\pi ikr) \, dk \] (4.18)

Therefore the equation 4.17 can be written as

\[ f(x,y) = \int_{0}^{\pi} \int_{-\infty}^{\infty} \frac{p(k,\theta)}{\left| k \right|} \exp(2\pi ikr) \, dk \] (4.19)

The above equation is a form of two-dimensional Fourier integral in polar coordinates. Taking the two-dimensional Fourier transform of equation 4.19 and using equation 4.16 we get

\[ F(Kx,Ky) = \frac{p(Kx,Ky)}{|K|} = \frac{F(Kx,Ky)}{|K|} \] (4.20)

where \( F(Kx,Ky) \) are the Fourier coefficients of the back-projected image. This implies that the back-projected image is equal to the true image except that the Fourier coefficients of the true image are divided by the
magnitude of the spatial frequency. Therefore the summation image is closely related to the object. This result suggests that back-projection can be improved if the projections are properly modified in some way before being back-projected. Such modification is called "filtering". The technique of image reconstruction by this method is called 'filtered back-projection' or 'convolution'.

Thus if the profiles shown in Figure 4.5(a) are modified as shown in Figure 4.5(b) by including negative components, spurious contributions to points outside the original object can be removed if filtering is applied.

In Figure 4.5(b), filtered profiles are back-projected to reconstruct the original object. It could be seen that though the point 0 outside the object receives positive contributions for profiles A and B, these are effectively cancelled by the negative effect of profiles C and D. With many projections in use, this should eliminate the star artifacts [Cho et al, 1974; Brooks, 1976].

There exist three forms of filtered back-projection classified according to the type of filtering employed.
4.4.2.1 Fourier Filtering

The Fourier filtering can be obtained by considering the Fourier integral of \( f(x,y) \)

\[
f(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(Kx,Ky) \exp[2\pi i K(x\theta)] \, dK \, d\theta
\]

(4.21)

and in polar coordinates (as shown earlier)

\[
F(Kx,Ky) = P(K,\theta)
\]

(4.22)

then we can write

\[
f(x,y) = \int_{\theta}^{\infty} \tilde{P}(r,\theta) \, d\theta
\]

(4.23)

where

\[
\tilde{P}(r,\theta) = \int_{-\infty}^{\infty} |K| \, P(K,\theta) \exp[2\pi i K r] \, dK
\]

(4.24)

In practice, equation 4.23 is replaced by its discretized form as

\[
f(x,y) = \sum_{j=1}^{m} \tilde{P}(r,\theta_j) \Delta \theta
\]

(4.25)

where \( m \) is the number of projection and \( \Delta \theta \) is the angular interval between each projection. Equation 4.25 can be compared with equation 4.8 of the simple back-projection method except that the back-projected projection \( \tilde{P} \) used in this case has been pre-filtered before back-projection.

The effect of this filtering is to increase the high frequency components of the profile in accordance with their wave number producing a biphasic function with average value of zero [Brooks, 1976].

The procedure for the implementation of this filtering method is to first take the Fourier transform of a projection, multiply each coefficient by the magnitude of the spatial frequencies \( K \), then back-project across the
4.4.2.2 Radon Filtering

Equation 4.24 is in fact the inverse Fourier transform of a product of two functions $P(K, \theta)$ and $|K|$. $P(K, \theta)$ is known to be the Fourier transform of $P(r, \theta)$ and the transform of $|K|$ is $-1/2 \pi r^2$ [Brooks, 1976]. The convolution theorem states that the Fourier transform of a convolution process is equal to the product of the individual Fourier transforms [Bracewell, 1965], we can write using equation 4.24.

$$P^* (r, \theta) = \frac{1}{2} \int_{-\infty}^{\infty} \frac{P(r', \theta)}{2 \pi (r - r')} dr'$$

Integration by parts of this equation will yield

$$P^* (r, \theta) = \frac{1}{2} \int_{-\infty}^{\infty} \frac{P(r', \theta)}{2 \pi (r - r')} dr'$$

which could be evaluated using the Cauchy-Reiman method. The above equation exhibits singularities at $r-r'$ which is source of inaccuracy when it is implemented digitally. Also the squared term in the denominator of equation 4.26 will cause the divergence of that equation. The removal of this source of divergence is the subject of the next filtering method.

4.4.2.3 Convolution Filtering

The divergence of equation 4.26 is caused by the factor $|K|$ in equation 4.24, whose Fourier transform contains a squared term in the
denominator. To avoid the divergence $|K|$ is replaced by $|K| < K_m$, but zero for $|K| > K_m$. $K_m$ is defined as the maximum frequency cut-off value. This band limiting means that equation 4.24 will not be affected by unphysical high frequencies because $P(K,\theta)$ is zero for $|K| > K_m$.

So the Fourier transform of this cut-off version of $|K|$ is given by

$$ \int_{-K_m}^{K_m} |K| \exp(2\pi i K r) \, dK = \int_{-K_m}^{K_m} |K| \exp(2\pi i K r) \, dK + \int_{K_m}^{0} |K| \exp(2\pi i K r) \, dK $$

$$ = 2 \int_{0}^{K_m} |K| \cos(2\pi K r) \, dK \quad (4.28) $$

Integrating the right hand side by parts we get

$$ \int_{-K_m}^{K_m} |K| \exp(2\pi i K r) \, dK = \frac{K_m}{\pi r} \sin(2K_m\pi r) - \frac{2}{\pi r} \sin(\pi K_m r) \quad (4.29) $$

Now for the convolution filtered projection we will have

$$ P^*(r,\theta) = \int_{-\infty}^{\infty} P(r',\theta) \left[ \frac{K_m}{\pi (r - r')} \frac{\sin\{2\pi K_m(r-r')\}}{\sin\{\pi K_m(r-r')\}} - \frac{2}{\pi (r - r')} \right] dr' \quad (4.30) $$

The result obtained for the filter function has been derived in various forms and various approximations have also been used [Ramachandran and Lakshminarayanan, 1971; Bracewell and Riddle, 1967; Shepp and Logan, 1974]. The first term in brackets is the well known sine function which has the effect of removing frequency components greater than $K_m$ in a convolution integral [Bracewell, 1965]. The implication of this is that the $P(r,\theta)$ is left untouched after convolution with this term, and we have

$$ P^*(r,\theta) = K_m P(r,\theta) - \int P(r',\theta) \frac{\sin(\pi K_m(r-r'))}{\pi (r - r')^2} \, dr' \quad (4.31) $$

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Due to band limiting

\[
\sin (\pi K_m (r-r')) = \begin{cases} 
1 & r-r' = \text{odd} \\
0 & r-r' = \text{even} 
\end{cases}
\]  
(4.32)

and the integral in the equation 4.31 can be replaced by a summation, with points spaced at intervals \( w = 1/2 K_m \) [Bracewell 1956]. We get

\[
P(r) = \frac{1}{4w} \sum_{j=1, \text{odd}}^{m} \frac{P(r)}{\pi w (i-j)^2}
\]

with the summation taken over all \( j \) for which \( i-j \) is odd. This method of filtering the projections by convolving them with a function prior to back-projection is most widely used especially in radiography [Ramachandran and Lakhshminarayanan, 1971; Lees et al, 1974; Shepp and Logan, 1974] and most X-ray computer assisted tomography scanners. At Surrey a version of this technique has been used for an X- and gamma-ray transmission scanner [Foster, 1981; Folkard, 1983; MacCuaig, 1986]. Also this technique has been used for the SPECT work described in this thesis.

4.5 ITERATIVE METHODS

The term iterative refers to a method of successive approximations in which an arbitrary starting image is chosen and then successive corrections are made to it, bringing it into better agreement with the measured projections until a satisfactory agreement is obtained.

Iterative techniques have long been used by the mathematicians for solving equations. These methods were first applied to image reconstruction by Bracewell [1956] in radioastronomy, and independently by Gordon et al [1970] in electron microscopy. This technique was used
Iterative techniques are generally used in cases where the number of views is limited and noise is significant or if additional factors such as solid angle and gamma-ray attenuation are present. That is why iterative methods are widely used for radioisotope imaging.

In order to implement this technique the object to be reconstructed is approximated to a square array containing $N = n^2$ cells each of which has a density value $f_i$ ($i = 1, 2, 3, \ldots, N$). The projections are then broken up into strips of width equal to the dimensions of each cell which is normally $1/n$ (Figure 4.6). The ray-sum are then calculated from each pixel intersected by the ray i.e. for the $j$th ray,

$$ P = W_{1j} f_1 + W_{2j} f_2 + \ldots + W_{Nj} f_N $$

where $W_{ij}$ is the weighting factor for the $(i,j)$ cell and defined as the factor which when multiplied by the true value of $(i,j)$ pixel gives its contribution to the expectation value of $P$. This factor incorporates the geometry of the measurement process and in ECT can also incorporate the weighting information to compensate for attenuation.

Since most cells are not intersected by a given ray, most of the $W$s are zero. $W_{ij}$ are calculated in various ways, including the area of the cell intersected by the ray, the length of the ray intersected by the cell and the proximity of the ray to centre of the cell, and represent the contribution of a given cell to the ray-sum.

The equation 4.34 represents a set of matrix equations with the densities $f_i$ as the variables. This equation could, in principle, be solved by
Figure 4.6: Diagramatical illustration of the iterative reconstruction method.

Inverting the matrix $W_{ij}$, i.e.

$$f = \sum_{i}^{m} (W^{-1})_{ij} p_j$$

(4.35)

This method has been used for limited applications [Kashyap and Mittal, 1973; Tewarson, 1973] but in general has been proved impractical [Brooks and Di Chiro, 1974]. This is because of the problems in manipulation of such a large matrix, insufficient number of projections to give a unique solution and above all, data noise and other artifacts will render the projection data to be inconsistent, in which case there is no unique solution.

One method devised to overcome these difficulties is to adjust the density values $f_i$ iteratively until the calculated projections agree closely with the measured projections. In the initial starting image a uniform density distribution is assumed and ray-sums are calculated for these. The density of each pixel is then adjusted to compensate for the difference between calculated and measured ray-sums. When this has been done for all projections the first iteration is completed. This
procedure is then repeated until the desired accuracy in the difference between measured and calculated projection is achieved. Mathematically this can be expressed as

\[
I^l - I^{l-1} = f^l_i + \sum_{j=1}^{m} A_f^l_{ij}
\]

where \( f^l_i \) and \( f_{i}^{l-1} \) are the densities before and after the \( l \)th iteration and \( A_f^l_{ij} \) is the correction applied to the \( i \)th pixel from the \( j \)th ray. At the end of the first iteration one arrives at a solution which is essentially what is obtained for the case of simple back-projection if the starting density was zero throughout the image field. Because this result is not adequate, a correction factor \( A_f^l_{ij} \) is back-projected into the succeeding iteration.

\( A_f^l_{ij} \) are to be calculated in order that, if applied to all cells constituting the \( j \)th ray, the calculated projection will be increased by \( \Delta P_j \).

\[
\Delta P = P^c - P^j
\]

where \( P^c_j \) is the calculated projection.

There are two types of correction procedures, additive and multiplicative. In additive corrections each cell receives a correction in proportion to its weight \( W_{ij} \), i.e.

\[
f^l_{ij} = W^l_{ij} \Delta P_j / \sum_{i=1}^{N} \sum_{j=1}^{2} W_{ij}
\]

The denominator is a normalising factor to ensure that the total change in ray-sum is \( \Delta P_j \) when all the cells are corrected.

In multiplicative corrections each cell receives a correction in proportion to its present assigned density \( f_i^a \), i.e.;
Using equation 4.37 and rearranging it, we get

\[ \frac{a}{f} + \frac{\Delta f}{ij} = \frac{a}{p} \frac{P}{j} \]  \hspace{1cm} (4.40)

which says that the corrected density is equal to the original density multiplied by the ratio of the measured and calculated ray-sums.

There exist three basic iterative reconstruction techniques, classified according to the sequence in which these corrections are made and incorporated during the iteration.

4.5.1 Simultaneous Correction Technique

This technique is also called the iterative least square technique (ILST). This is the simplest method in which all the projections for each iteration are calculated at the beginning and then all corrections are applied simultaneously to all the pixels. However, this results in an over-correction because each cell is reconstructed for every ray passing through it and the iterations therefore oscillate about the correct solution [Bracewell, 1956]. This problem can be solved by applying a damping factor to all the corrections. The choice of damping factor is not critical but in this method lending its name to this technique, it is calculated so as to produce the best least square fit after the iteration [Goiten, 1972]. This method has been applied to both X-ray transmission and gamma-ray emission tomography.
At Surrey, a multipurpose computer program has been developed by the author using the iterative least square technique. The program utilises the RECLBL library routines generated by Herman et al [Herman et al, 1977]. There are two choices of methods, the steepest descent or gradient method and the conjugate gradient method. Both have been used in radioisotope imaging in this study.

4.5.2 Ray-by-Ray Correction or Algebraic Reconstruction Technique (ART)

This technique was discovered by Gordon et al [1970] in the field of electron microscopy who called it the algebraic reconstruction technique (ART). This technique was used in the first version of the EMI scanner [Hounsfield, 1973]. In this method at the beginning of each iteration one ray is calculated and corrections are applied to the ray. This procedure is then repeated for the second ray, and so on, always embodying previous corrections in each new calculation, until all the projections have been treated. This completes one iteration.

It has been found that, in order to ensure that succeeding iterations are independent of each other and avoid error accumulation, the sequence of projections corrected is not chronological so that the angle between the consecutive corrected projections is large [Hounsfield, 1973; Kuhl et al, 1973].

ART is a very efficient method, because it incorporates the corrections during the iteration, without a significant increase in the computation time. Oppenheim [1975] has reported satisfactory results using only two iterations. However, ART is also the most susceptible to noise [Herman and Rowland, 1973; Budinger and Gulberg, 1974]. Noise generally causes inconsistencies in the projection data, so that it may be impossible to
satisfy all projections at the same time. The best approach, then, is to produce density values which are a best least square fit to the projection data.

4.5.3 Point-by-Point Correction or Simultaneous Iteration Technique

The simultaneous iterative reconstruction technique (SIRT) was introduced by Gilbert [1972] in electron microscopy and has also been applied to positron tomography by Schmidlin [1972] and in radioastronomy by Bracewell [1956]. In this method each iteration begins with a particular point and calculations and corrections are applied to all ray-sums that contribute to this point. This procedure is repeated for every other point while taking all the previous corrections into account in the present iteration. Each point (pixel) receives a correction in proportion to its present density.

4.6 DISCUSSION

From the foregoing it is clear that there are a number of algorithms for reconstructing objects from projections and a lot of work has been done by various workers to evaluate individual algorithms, including a few studies of the relative goodness of reconstruction of different algorithms using a variety of objects [Cho and Chan, 1975; Herman and Rowland, 1973; Ramachandran and Lakshminarayanan, 1971; Sweeny, 1973; Smith et al,1973]. However, no quantitative comparison can be made since different workers use different test objects, different measurement criteria and different implementation of the the given algorithms. So generally speaking there exist no generally applicable set of criteria for choosing a reconstruction algorithm.
The suitability of a reconstruction technique is normally expressed in terms of speed, accuracy, image quality and its response to incomplete data. The relative importance of each of these is dependent on the particular application.

It is generally agreed that, for complete data (i.e. data collected at small intervals over all angles) where no interpolation is required, the analytical techniques are the fastest since each projection can be processed immediately after collection and the entire reconstruction is completed shortly after the last projection is measured. But in practice projection data cannot be obtained for all angles between 0 and 180° and therefore, interpolation becomes essential. The back-projection integral has to be replaced by a summation. So band limiting and interpolation are the two factors that affect the accuracy. Band limiting is the loss of high spatial frequencies when the projection is sampled. This is only a significant problem at sharp interfaces between high and low density regions and is normally compensated for by the filter.

Interpolation may be necessary to estimate the values between the data points. If the intervals between data points are sufficiently small then linear interpolation gives adequate results except at sharp edges where the filtered profile changes rapidly.

Iterative techniques are affected by the finite iteration time and a possible lack of convergence of the solution [Brooks and Di Chiro, 1975] especially if noise is present. The exact nature of this problem depends on the selection of the damping factor, sequence of correction etc. If a sufficient number of iterations is done and a proper damping factor is used, the iterative method produces good results at the expense of long computation time.

If the number of projections is not sufficient to fully determine the
image then the data are said to be incomplete. Bracewell and Riddle [1971] suggested that if aliasing is to be avoided and a good reconstruction is to be obtained, the number of projections needed to completely specify a circular-bounded image is

\[ m = n \pi/4 \] (4.41)

where \( n \) is the number of cells spanning a diameter of the object. If more than \( m \) projections are obtained, the resulting image is said to be over-determined and it represents an average of the redundant information. The effect of this is to reduce the noise and minimize interpolation requirements. If fewer than \( m \) projections are available for reconstruction, the image is said to be under-determined. In some cases, especially in medical radioisotope imaging, it may be necessary to reconstruct an image from fewer projections. Therefore the reconstruction program must make assumptions about the missing data. The behaviour of analytic and iterative reconstruction methods are fundamentally different in this respect. The analytic methods assume that the missing projections are similar to the available ones and the missing data is created by interpolation. Iterative methods, however, assume that the image is as smooth as possible, consistent with the available data. Therefore, iterative methods do better for non-symmetrical objects for which analytical methods may introduce errors. Furthermore, the interpolation required to fill the missing data in analytical methods results in increased reconstruction time while for iterative methods the time requirement is actually reduced [Brooks, 1974].

Therefore iterative methods may become the methods of choice in situations where incomplete data is available.
4.7 IMAGE ANALYSIS

4.7.1 POINT SPREAD AND LINE SPREAD FUNCTIONS

The spatial resolution of an imaging system is determined by its ability to resolve two adjacent point objects. One measure of the spatial resolution of a system is the full-width-at-half maximum (FWHM) of an image produced from a small point-like object. The image response against distance is called the point spread function (PSF). If the object is a straight line instead, the image response at right angles to the line is known as a line spread function (LSF). Basically, LSF is equivalent to the PSF integrated along one direction. Thus, mathematically the LSF can be expressed as

\[ L(x) = \int P(x,y) \, dy \]  \hspace{1cm} (4.42)

Where \( P(x,y) \) is the PSF in the \((x,y)\) plane.

The LSF is one dimensional, therefore it is easily treated mathematically and graphically. The LSF can be obtained by imaging a narrow slit but this is impractical since it is impossible to obtain an infinitely thin line source or narrow slit. However, if one side of the slit is removed, then an indirect determination of the LSF can be undertaken by imaging a sharp, opaque, knife-edge object. The image response across the edge is referred to as the unsharpness curve or the edge spread function (ESF). If \( E(x) \) is the ESF of the system, it has been shown that the slope of the ESF at a point \( x \) gives the value of the LSF, \( L(x) \), at that point [Lamberts, 1958].

\[ L(x) = \frac{dE(x)}{dx} \]  \hspace{1cm} (4.43)
The performance of an imaging system can also be described by its spatial frequency response, known as the modulation transfer function (MTF). An ideal imaging system produces images from objects without any loss of informational content. The MTF can be determined experimentally with a test object in which the distribution of the physical quantity of interest varies sinusoidally with known spatial frequency \( f \). MTF is then calculated as the ratio of the output modulation to the input modulation.

\[
MTF(f) = \frac{M_o}{M_i}
\]  

(4.44)

Where \( M_i \) and \( M_o \) are the input and output modulation respectively, and

\[
M_i = \left[ \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} \right]_{\text{input}}
\]

(4.45)

and

\[
M_o = \left[ \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} \right]_{\text{output}}
\]

(4.46)

Where \( I_{\text{max}} \) and \( I_{\text{min}} \) are the maximum and minimum intensity of the response.

The MTF can also be calculated analytically. It has been shown mathematically that the MTF is the Fourier transform of the LSF [Ozek, 1975] i.e.

\[
MTF(f) = \int_{-\infty}^{\infty} LSF(x) \exp(-2\pi i f x) \, dx
\]

(4.47)

By substituting \( LSF(x) \) as given in equation 4.42 the MTF is found to be
The important feature of the MTF is its cascading property; if an imaging system is a linear system and can be separated into several subsystems of individual MTF$_i$ ($i = 1$ to $N$) the total MTF is given by [Kouris, 1982]

\[
MTF = \prod_{i=1}^{N} MTF_i
\]  

\[\text{(4.49)}\]

4.7.3 IMAGE CONTRAST

Brightness is the psychological concept usually associated with the amount of light stimulus. Due to the great adaptive ability of the eye, absolute brightness cannot be accurately judged by the human eye. Two images of equal intensity do not always appear equally bright, but depend on the immediate surrounding which gives different contrast, therefore a quantitative measure of contrast is necessary in image analysis. The term contrast is to emphasize the difference in the intensity of neighbouring regions in images.

Several definitions of contrast have been given by Hall [1979]. In psychology, contrast 'C' refers to the ratio of the difference in intensity of an object $I_0$ and the immediate surroundings $I$.

\[
C = \frac{I_0 - I}{I}
\]  

\[\text{(4.50)}\]

A contrast measurement commonly used in optics describes the contrast 'C' of a spatial frequency grating as

\[
C = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}
\]  

\[\text{(4.51)}\]
where Imax and Imin are the maximum and minimum intensities of the profile. In imaging system this contrast is also known as contrast range or contrast ratio. The contrast of one region with respect to the other can be written as

\[
C = \frac{I_1 - I_2}{I_1 + I_2}
\]  

(4.52)

where I1 and I2 are the intensities of the two regions of interest in the reconstructed image.

The ratio of contrast of the image, Ci, to that of the object, Co, can be used to evaluate the imaging process, i.e

\[
\text{Contrast Ratio} = \frac{C_i}{C_o}
\]  

(4.53)
4.8 SOLID ANGLE AND ATTENUATION COMPENSATION IN SPECT

A beam of photons when passing through matter, may be absorbed or scattered according to the different interaction processes described earlier in chapter 3. Absorption may be thought of as the disappearance of a photon from the beam and scattering as a combination of absorption and emission, where the emission takes place in a new direction [Davison and Evans, 1952]. Both the processes contribute to the attenuation of photons which are initially moving in a particular direction.

The problem of attenuation is one of the major complicating factors in reconstruction with SPECT. Among problems that occur in medical SPECT as a result of attenuation are "hot rim" artifacts and inaccurate data related to perceived asymmetrical organ uptake of injected radionuclides. The ultimate goal of SPECT systems is to determine the absolute regional radionuclide concentration as a function of time [Jaszczak et al, 1980]. The precise estimates of "in-vivo" radioactivity are difficult because the counts recorded by the detector are a function of both the distribution of the source and the self-absorption inside the object. Therefore the position dependent variation has distance-dependent and depth-dependent variation components. The former component is due to the fact that the recorded counts vary as the inverse square of the distance between source and the detector, and the latter is due to the attenuation of the photons within the source. For mono-energetic photons this latter factor has an exponential dependence on depth of the source inside the material.

4.8.1 SOLID ANGLE OR GEOMETRICAL FACTOR CORRECTION

The geometrical factor is caused by the variation in the field of view of the detection system and it poses an obstacle in attaining
quantitative imaging. So a knowledge of solid angle subtended by a
detector at the source is essential in all absolute measurements of the
intensity of the radioactive sources.

In the absence of self-absorption the absolute activity of a radioactive
source can, in principle, be determined by using a detector of known
absolute efficiency. The intrinsic efficiency of a detector can be
defined as

\[ E(\text{int}) = \frac{\text{Number of counts recorded in full-energy peak}}{\text{Number of photons incident on the detector}} \]

i.e

\[ E(\text{int}) = \frac{4\pi Np}{N - \Omega} \] \hspace{1cm} (4.54)

where \( \Omega \) is the solid angle subtended by the detector at the source, \( Np \)
is number of counts recorded in the full-energy peak and \( N \) is number of
photons of a given energy emitted by the source during the time of
measurement.

The solid angle can be defined as

\[ \Omega = \int_{A} \frac{\cos \theta}{r^2} \, dA \] \hspace{1cm} (4.55)

where \( r \) is distance between the source and a surface element \( dA \) on the
detector, \( \theta \) is the angle between the normal to the surface and the source
direction and the integration is carried out over the surface of the
detector. For a point source located along the axis of a right circular
cylindrical detector the solid angle can be written as

\[ \Omega = 2\pi \left[ 1 - \frac{1}{1 + (a/d)^2} \right]^{1/2} \] \hspace{1cm} (4.56)
where 'a' is the radius of the detector and \( d = r + x \), where \( r \) is the source to detector distance and 'x' is the mean depth of interaction within the detector. For \( d \gg a \) the solid angle becomes

\[
\Omega = \frac{2 \pi a^2}{d^2}
\]

For a disc source and a circular detector the solid angle can be written as [Williams, 1966]

\[
\Omega = \int_{0}^{R} \frac{2\pi - \Omega(r)}{R} \, dr
\]

where \( R \) is the radius of the source.

The above equation has been evaluated by Tajuddin [1986] in a Monte Carlo simulation program to calculate the solid angle mathematically. If the value of the solid angle is known then the number of radiation quanta emitted by the source over the measurement period can be determined by the equation 4.54.

This method is not useful for SPECT purposes because the spatial distribution of the radionuclide must be known before running the program which, in fact, is the aim of the SPECT. Also the computer simulation program takes approximately 60 seconds to calculate a single solid angle value which means for a large number of ray-sums it will take unacceptably long computing time. So this method is not feasible for our purposes.

The effect of solid angle on the count rate can be minimised if an optimised geometrical arrangement is used. The opposed detector arrangement has been proven to give more accurate measurements of the activity in human organs "in-vivo" [Arimizu et al, 1969; Williams et al, 1969; Tothill and Galt, 1971]. Any dependence of counting rate upon
source position entirely due to geometrical arrangement is minimised if such a set-up is used.

In Figure 4.9 such an opposed detector set-up is shown. A point source is at a distance $X$ from the centre of a circular object of radius $R$. Two detectors are placed directly opposing each other (same result can be achieved with a single detector by combining results from opposite views); the distance apart is chosen to minimise the solid angle effect. The equations for the outputs of the two detectors can be written as

$$I_a = I_0 \exp(-\mu(R+X))$$

$$I_b = I_0 \exp(-\mu(R-X))$$

where $I_0$ is the detector count in absence of absorber in time $t$, $I_a$ and $I_b$ are the counts recorded by the detectors A and B respectively and $\mu$ is the linear attenuation coefficient of the material comprising the circular object for the particular radiation energy from the source. The
variation in count rate with distance can be reduced if the average of the two detector outputs is taken. The arithmetic mean of the two detectors is given by

\[
\frac{[I_a + I_b]}{2} = I_0 \left[ \exp(-\mu (R-X)) + \exp(-\mu (R+X)) \right] \tag{4.61}
\]

The arithmetic mean still shows some dependence on source position due to attenuation. By contrast the geometric mean of the two counting rates is given by

\[
\sqrt{I_a I_b} = I_0 \exp(-\mu R) \tag{4.62}
\]

It is clear from the above equation that the geometric mean of the opposed detector outputs is independent of the source-to-detector distance. Therefore to minimise both geometrical and attenuation factors the geometric mean gives greater independence of count rate with source depth than the arithmetic mean. Expression 4.62 can be used to derive the count rate corrected for attenuation by dividing the geometric mean by \( \exp(-\mu R) \) where \( \mu \) and \( R \) are known.

The use of the above method improves the point source response of single detecting element e.g a collimator hole. In the case of a large area thin plane source having constant activity per unit area, the response does not depend on the distance from the source if attenuation is ignored and certain approximations are made [Brownell, 1958]. This is because the area on the thin slab of activity within the acceptance solid angle of the detector collimator will increase as the square of the distance while the efficiency of detection per unit area of the slab will decrease with the inverse square of the distance. Therefore these two effects will cancel each other and the overall response will be independent of source depth. This also holds true for the multi-parallel hole collimator where the overlap of fields of view of neighbouring holes, which occur beyond a few centimetres from the collimator face, give
increase in sensitivity proportional to the square of the distance. Therefore the point source response in air is independent of distance over the region of effective overlap. This will not be the case for a single parallel-hole collimator.

SPECT systems, in practice, encounter objects of finite depth which cannot be approximated to thin sources. However they can be considered as a series of thin slabs of large area. The total sensitivity for each slab, neglecting the source attenuation, will be equal. So SPECT systems use the geometric mean as well as arithmetic mean to partially compensate for detection efficiency variations.

4.8.2 ATTENUATION COMPENSATION

The attenuation of photons is a major obstacle to the creation of quantitatively accurate images in SPECT. The problem of photon attenuation always occurs in emission tomography, especially for low energy photons in a high density matrix. The effect of photon attenuation is a loss of the information content. This is due to the fact that the intensity of photons detected is not the true intensity emitted by the source. The attenuation of photons in SPECT may be due to the medium surrounding the source or due to the source itself. The degree of attenuation depends on the energy of the emitted photons, the constituents of the surrounding medium and its density and thickness.

In transmission tomography photon attenuation is the parameter which is measured and each line integral \( P(r,\theta) \) is related to the attenuation coefficients by

\[
P(r,\theta) = \ln\left(\frac{I(r)}{I_0}\right) = \int \mu(r,\theta) \, ds
\]  

(4.63)
where $ds$ is the length of the element with attenuation coefficient $\mu$.

In emission studies the contribution of each element to the projection ray is not a simple additive factor, as in the case of transmission studies. Each element contributes a photon emission concentration (photons/sec-cm$^3$) which is attenuated along the path length between each point and the edge of the object along a projection ray. Thus the intensity of photons measured along one projection view will be significantly different from the intensity measured in the conjugate view ($180^\circ$ apart from the first view) particularly if the distribution of activity is not uniform.

For emission tomography $P(r,\theta)$ is given by

$$P(r,\theta) = \int_{L(r)} A_0(x,y)\, ds \exp\{-\int_{L(r,\theta,x,y)} \mu(x',y')\, ds\}$$

(4.64)

where $\int_{L(r,\theta,x,y)} \mu(x',y')\, ds$ is the line integral of the photon attenuation from the point of its emission to the detector.

Different approaches to solving the problem of attenuation in SPECT have been considered by different workers. Some have derived the images ignoring the attenuation [Kay et al, 1974]. Ignoring the attenuation and performing the reconstruction using the uncorrected data results in both an attenuated source activity and an erroneous spatial distribution. This approach may be used for qualitative clinical studies of regions where attenuation is not significant but it is not adequate for quantitative studies [Murphy et al, 1979; Jaszczak et al, 1980].

There are several ways of attenuation compensation that can be applied in SPECT. The attenuation corrections can be applied prior to the reconstruction (pre-correction or pre-processing techniques), during the reconstruction process (intrinsic techniques) or after the reconstruction (post-correction or post processing techniques). Pre-correction and
post-correction techniques are currently used in SPECT systems. Pre-correction techniques [Budinger and Gullberg, 1977; Kay and Keyes, 1975; Budinger et al, 1979] are easy to use. These techniques reduce the 'hot rim' artifacts and are quite adequate for uniformly distributed source and attenuator configurations but are not quantitative for sources distributed in a large attenuator, especially if the attenuator is non-uniform.

Post-correction techniques [Kuhl et al, 1973; Chang, 1978] do a better job of quantification but are unable to compensate for point response distortions in a non-uniform attenuator [Gullberg et al, 1985]. Intrinsic correction techniques [Budinger and Gullberg, 1974; Hsieh and Wee, 1976; Bellini et al, 1979; Gullberg, 1979; Walters et al, 1981; Moore et al, 1982; Tanaka, 1983] which properly model the attenuation process, have the potential for accurate quantification of the source distributions. This conclusion has been substantiated by a number of comparative studies [Gullberg et al, 1982; Lews et al, 1982; Webb et al, 1984; Pergrale et al, 1984] but not as yet been fully appreciated in clinical studies.

The different correction techniques are described briefly in the next following subsections.

4.8.2.1 PRE-CORRECTION TECHNIQUES

In the pre-correction or pre-processing techniques the attenuation correction is applied to the projection data prior to the conventional convolution or iterative reconstruction methods. There are various ways to compensate for the attenuation by correcting the projection data before reconstructing. The reconstruction algorithm then assumes that the data represents the true unattenuated data and thus does not require any special weighting to compensate for the internal absorption. In
cases where small uniformly distributed source in a uniform low density absorbing material is used, the corrected projection data does represent the true projection data but for large volume source in a high Z material the corrected data are only an approximation to the true projection data. The different pre-correction methods for attenuation compensation are described here.

(a) Simple Conjugate Means

In this method the simple mean of the opposing projection rays 180° apart is taken. There are two approaches used in this method: arithmetic mean [Kay and Keyes, 1975] and geometric mean [Genna, 1966]. These two methods have been discussed in section 4.8.1. These methods compensate only partly for attenuation and are inadequate for the gamma-ray energies usually used in nuclear medicine [Budinger and Gullberg, 1977] but it does reduce the data to a form that is amenable to further modifications.

(b) Hyperbolic Sine Correction

This attenuation correction technique was first applied by Sorenson [1974] to whole body rectilinear scanning and can be used with both the ILST and the convolution reconstruction techniques [Budinger et al, 1979]. In this method the conjugate means are corrected by a factor that assumes a constant attenuation coefficient. This factor can be derived as follows.

The isotope emission counts dAl received at one view are considered as arising from a small thickness increment dx and attenuated by the depth increment in the body; thus dAl can be written as:

\[
dAl = \frac{A_0}{fL} \exp(-\mu x)dx \quad (4.65)
\]
where $L$ is the total thickness through which the ray passes, $f(>0)$ is the linear fraction of the thickness in which the isotope is distributed, and $A_0$ is the true activity of the source. Integration of above equation over the ray length gives:

$$A_1 = \frac{A_0 \exp(-\mu m)}{f \mu (L/2)} \sinh(f \mu L/2) \tag{4.66}$$

where $m$ is the mean source depth.

A similar expression is obtained for the conjugate view

$$A_2 = \frac{A_0 \exp[-(\mu L-m)]}{f \mu (L/2)} \sinh(f \mu L/2) \tag{4.67}$$

The geometric mean, $(A_1A_2)^{1/2}$, is related to the true activity as:

$$A_0 = \frac{(A_1A_2)^{1/2} f \mu (L/2) [\exp(\mu L/2)]}{\sinh(f \mu L/2)} \tag{4.68}$$

Taking the arithmetic mean of the conjugate views gives the true activity as:

$$A_0 = \frac{(A_1+A_2)f(\mu L/2) [\exp(\mu L/2)]}{2\sinh(\mu f L/2) \cosh[\mu(L/2)-m]} \tag{4.69}$$

The above equation contains the source depth dependence while equation 4.68 is independent of the source depth.

A more detailed analysis of this method is given by Sorenson [1974].

The parameter $f \mu L$ can be determined from an assumed attenuation coefficient and a measured thickness through the object for each ray, which can either be measured physically or calculated automatically after a first estimate is made of the image. This method was used by the author with the filtered back-projection method.
This method was suggested by Kay and Keyes [1975]. In this method it is assumed that the attenuation distribution $\mu(x, y)$ is approximated by a constant attenuation coefficient, $\mu$. The summation of the two opposite views can be written as:

$$P(\ell, \theta) + P(-\ell, \theta+\pi) = \int_{0}^{L} A(i,j) [\exp(-\mu \ell) + \exp(-\mu (L-\ell))]$$

(4.70)

where $A(i,j)$ is the true activity and $L$ is the total length through which the projection line passes.

The corrected projection data can be obtained by dividing the sum of the two opposite views by the exponential factor in the above equation. The minimum value of the exponential factor is given by

$$\min \{\exp(-\mu \ell) + \exp(-\mu (L-\ell))\} = 2\exp(-L/2)$$

(4.71)

and the maximum value is given by

$$\max\{\exp(-\mu \ell) + \exp(-\mu (L-\ell))\} = 1 + \exp(-\mu L)$$

(4.72)

The average of these two values is given by:

$$\text{Average} = (1/2) [1 + \exp(-\mu L) + 2\exp(-\mu L/2)]$$

(4.73)

Now the approximations for the corrected projection data are:

$$P(\ell, \theta) = \frac{2[P(\ell, \theta) + P(-\ell, \theta+\pi)]}{1 + \exp(-\mu L) + 2\exp(-\mu L/2)}$$

(4.74)

Then the reconstruction is performed using the corrected projection data. This method was also used by the author with the filtered back-projection
(d) AVERAGE THE EXPONENTIAL FACTOR

This method was suggested by Webb et al [1983]. In this method instead of taking the maximum and minimum of the exponential factor [exp(-μL) + exp(-μ(L-ℓ))], the average of this factor is taken for 0<ℓ<L. This can be written as

\[< \exp(-μL) + \exp(-μ(L-ℓ))> = \frac{2 \exp(-μL/2)}{μL/2} \sinh(μL/2) \]  \hspace{1cm} (4.75)

The corrected projection, therefore, can be written as

\[P(0,θ) = \frac{[P(ℓ,θ) + P(-ℓ,θ+)] (μL/2)}{2 \exp(-μL/2) \sinh(μL/2)} \]  \hspace{1cm} (4.76)

(e) CORRECTION OF DATA USING THE UNCORRECTED TRANSVERSE SECTION

This method involves the correction of each ray \( P(ℓ,θ) \) by a factor that represents what should be the projection data if there were no attenuation. The assumptions are that the uncorrected transverse section represents the isotope concentration diminished by a factor containing the source attenuation coefficient and its mean distance from the edge of the attenuating object. Then the projection data is recalculated by the sum of the products of the uncorrected concentration and \( \exp(μL_{ij}^θ) \) where \( L_{ij}^θ \) is the distance between the picture element \( A(i,j) \) and the edge of the attenuating object in the direction of projection for ray \( k \). Formally, this is expressed as:

\[P(ℓ,θ) = \sum_{i,j} A(i,j) \exp(μL_{ij}^θ) \]  \hspace{1cm} (4.77)
Thus after these corrections a reconstruction is performed by either the ILST or filtered back projection technique.

(f) USE OF A DERIVED CORRECTION MATRIX BASED ON PHANTOM STUDIES

This method, used by Kuhl et al [1973], is based on phantom studies. A correction matrix is derived from the phantom studies and is stored in the computer memory. Then the uncorrected projection data is multiplied with the correction matrix and the reconstruction is performed using the corrected projection values. This method can produce good results for uniform and non-uniform source and attenuator distributions if the appropriate library of corrections is available.

(g) DIFFERENTIAL ATTENUATION METHODS

This method depends upon the differential attenuation of multiple energy gamma-rays. Low energy photons are more attenuated as compared to higher energy photons. The measurement of depth of the gamma-ray source inside the medium using this method is described in chapter 3. The same principle applies to the technique which involves the use of multiple isotopes or a single isotope emitting multiple energy gamma-rays, where advantage is taken of the known attenuation coefficients of different media for different photon energies to determine the attenuation corrections. There are two different approaches possible using the differential attenuation method. The first one was suggested by Budinger and Gullberg [1974] for the medical applications. This method is explained in the following example.

If the photons from Pb-210 (46 keV), Am-241 (60 keV) and Tc-99m (140 keV) were used, the distribution of lung, bone and soft tissue can be
determined by noting

\[-\ln[I/IO] (Pb) = \mu'_{L} + \mu'_{L} + \mu'_{L} = P'(r, \Theta)\]

\[-\ln[I/IO] (Am) = \mu'_{L} + \mu'_{L} + \mu'_{L} = P''(r, \Theta)\]  (4.78)

\[-\ln[I/IO] (Tc) = \mu'_{L} + \mu'_{L} + \mu'_{L} = P'''(r, \Theta)\]

where \(\mu_{L}, \mu_{S}\) and \(\mu_{B}\) are known attenuation coefficients for lung, soft tissue and bone and \(L_{L}, L_{S}\) and \(L_{B}\) are the thickness of lung, soft tissue and bone respectively. The primes denote the coefficients appropriate to the different photon energies. The system of equations 4.78 can be applied to each ray-sum and from this the distribution of lung, soft tissue and bone can be determined using the reconstruction algorithms described earlier.

The second method is related to plutonium assay where it involves measuring ratios of the intensities of two or more gamma-rays emitted by the plutonium and comparing the observed ratios to the ratios of the emitted intensities assuming no absorption [Cline, 1972]. The reason is that since the matrix material preferentially absorbs the lower energy photons, the observed intensity ratio (high/low) of two gamma-rays will be higher in the presence of the matrix.

For two different energy photons we can write

\[I_{1} = (I_{1})_{0} \exp (-\mu_{1}x)\]  (4.79)

\[I_{2} = (I_{2})_{0} \exp (-\mu_{2}x)\]  (4.80)

where \(I_{1}\) and \(I_{2}\) are the observed intensities, \((I_{1})_{0}\) and \((I_{2})_{0}\) are the emitted intensities, and \(\mu_{1}\) and \(\mu_{2}\) are the attenuation coefficients for gamma-ray 1 and 2, respectively and \(x\) is the average distance that both the gamma-rays have to travel. Taking the ratio of the two intensities we get
\[
\frac{I_1}{I_2} = \left(\frac{I_1}{I_2}ight)_0 \exp\left[-(\mu_1 - \mu_2)x\right]
\]

\[
x = \frac{\ln\left[\frac{(I_1/I_2)_0}{(I_1/I_2)}\right]}{(\mu_1 - \mu_2)}
\]

(4.81)

and putting the value of \(x\) in equation 4.80 we get

\[
\left(\frac{I}{I_0}\right)_2 = \frac{1}{I_2} \exp\left\{\frac{\mu_2}{\mu_1 - \mu_2} \ln\left[\frac{I/I_2}{I/I_0}\right]\right\}
\]

(4.82)

This means that the emitted intensity of the higher energy gamma-ray can be determined from the knowledge of the attenuation coefficients for each of the energies and by measuring both the observed bare intensity ratio of gamma-ray 1 and 2 as well as the ratio of their emitted intensities. The attenuation correction by the differential absorption method is useful if we are dealing with inanimate radioactive objects such as radioactive waste packages or nuclear fuel elements; it can also be applied in principle to nuclear medicine.

4.8.2.2 INTRINSIC CORRECTION METHODS

The intrinsic attenuation correction methods compensate for attenuation during the reconstruction process. These methods can be applied to non-uniform source and attenuator distributions as well as for the uniformly distributed source and attenuator cases. Two methods utilising the intrinsic correction technique are described here.
The ILST method [Goiten, 1972; Budinger ang Gullberg, 1977] has been discussed earlier can accurately compensate for attenuation. The problem of attenuation compensation can be overcome in principle if an accurate map of attenuation coefficients is obtained throughout the subject by a transmission scan across the same slice as for the emission scan. This should preferably be done using photons of the same energy as the emission scan, but if CT images at several different energies are measured the attenuation coefficient distribution at intermediate energies can be deduced by interpolation. This provides a priori information for the reconstruction of source activity. In the ILST an iterative method is used and minimisation is carried out to the function

\[ \chi^2 = \sum_k \frac{(P_k - P_k^c)^2}{\sigma_k^2} \]  

(4.83)

where \( P_k^c \) is the estimated projection value given by

\[ P^c = \sum_k^c W f_{ij} \]  

(4.84)

\( P_k \) is the measured projection at the kth ray, \( f_{ij} \) is the intensity (activity distribution) of pixel (i,j) to be reconstructed and \( \sigma_k \) is the uncertainty with which \( P_k \) was measured, which can be estimated as the square root of the observed counts. \( W_{ij} \) is the fraction of \( f_{ij} \) that projects into \( P_k \). In SPECT, as will be seen below, it depends on the model of intensity distribution within each pixel and whether attenuation compensation is involved. Minimisation can be carried out utilising the gradient technique, or method of steepest descent, with parameter scaling to speed convergence [Huesman et al, 1977]. In ILST, the attenuation correction scheme assumes that the emission scan projection data for the transverse section are the summation of pixel concentrations attenuated...
by a factor that is a function of the attenuation between the pixel and the edge of the object. Thus the projection $p_k^c$ is represented by

$$p_k^c = \sum_{ij} f \cdot C_{ij} \cdot f$$

(4.85)

where $F_{ij}$ are the geometrical weighting factors and $C_{ij}$ are the attenuation compensation factors. The attenuation compensation factors $C_{ij}$ can be evaluated after reconstructing the true attenuation coefficients from transmission data using the equation

$$C_{ij} = \exp\left\{ \sum_{ij} L_{i'j'} / \mu_{i'j'} \right\}$$

(4.86)

where the summation is taken over the pixels $(i',j')$ in the projection ray $k$ from the pixel $(i,j)$ in the direction of the measured projection. $L_{i'j'}$ is the length of that portion of a line centred in the projection ray $k$ within the pixel $(i',j')$.

(b) Using Iterative Convolution Method

This method was suggested by Walters et al [1981]. In this technique, the uncorrected projection, $P_k$, is first filtered to give the filtered-projection, $P_k^f$. Back-projection is then performed without any compensation for the attenuation to give the first order reconstruction $f'_{ij}$ such that

$$f'_{ij} = \frac{1}{M} \sum_{k} \sum_{ij} P_k$$

(4.87)

where $M$ is the number of angles. The attenuated projections of the approximated reconstruction $f'_{ij}$ (now call reprojections) are then evaluated by weighting by a known attenuation factor $C_{ij}$ (which can be
evaluated using Equation 4.86)

\[ P' = \sum_{k}^{k} F_{ij} C_{ij} f' \]  

Hence the first-order reprojections, \( P'_k \), contain the effect of attenuation and are expected to match the measured projection data if the reconstructions are exact. This algorithm then subtracts the measured projection data from the reprojection data to give the first-order difference projections, \( \Delta P'_k \)

\[ \Delta P'_k = P_k - P'_k \]  

The difference projections, \( \Delta P'_k \), are then filtered before being back-projected to give the first-order residual reconstruction, \( \Delta f'_{ij} \)

\[ \Delta f'_{ij} = \frac{1}{M} \sum_{k}^{k} F_{ij} \Delta P'_k \]  

The corrections \( \Delta f'_{ij} \) are then subtracted from the previously reconstructed result to give the second order approximation for the reconstruction

\[ f''_{ij} = f'_{ij} - \Delta f'_{ij} \]  

In general the algorithm is described by the following equation

\[ f_{ij}^{n+1} = f_{ij}^n - \Delta f_{ij}^n \]  

The problems with methods that use variable attenuation coefficients are that they require longer time and greater computer memory since two measurements as well as two reconstructions (for each transmission and emission) are needed. In medical applications the radiation dose to the
patients is greater as compared to if only one measurement is used. Hence other ways are sought to compensate for attenuation, for example assuming the attenuation coefficients (which can be derived from either the measurement or theoretical calculation) to be constant throughout the medium.

4.8.2.3 POST CORRECTION TECHNIQUES

The post-correction or post-processing techniques have been used by Kuhl et al [1973] and Chang [1978]. In this method the corrections are applied after reconstruction is performed. Two methods are briefly described here.

The first method is the orthogonal tangent correction (OTC) method which was used by Kuhl et al as an improvement to the processing technique for SPECT. The initial step in this technique is to take an orthogonal pair of tangent data lines and form a matrix distribution that reflects the proportionality that exists between them. The matrix data are then redistributed to conform to the proportionality of each succeeding orthogonal pair of tangent lines without destroying the compatibility achieved in the previous steps. The final section matrix is a high contrast image of the cross-section. The matrix distribution is compatible with all the tangent values and there is a linear relationship between matrix counts and the radioactivity in scanned objects. Hence the true activity can then be determined.

The second method was suggested by Chang (1978). This method involves correcting the transverse section values $A(i,j)$ by a factor that represents the mean attenuation which each element has experienced if the attenuation along rays for all projections is averaged. This can be written as

$$135$$
\[ A'(i,j) = A(i,j) \frac{1}{M} \sum_{\theta=1}^{m} \exp(\mu \rho_{ij}) \] (4.93)

where M is the total number of projections, \( \rho_{ij} \) is the distance from the point \((i,j)\) to the edge of the object along the ray at projection angle \( \theta \).

Using the reconstructed but uncorrected transverse section, the distance along each ray projection is calculated first. These values are then used to calculate the mean \( \exp(\mu \rho_{ij}) \), which is the factor used to change the values for the picture elements of the uncorrected transverse section.

4.9 SCATTER CORRECTION IN SPECT

Over the energy range useful for the tomographic purposes, Compton scattering is generally the dominant contribution to the attenuation of the internally emitted photons. The Compton scattering cross-section decreases with increasing photon energy which suggests that using higher energy sources can reduce the attenuation effect due to scatter. However, although the scattering angular distribution is approximately isotropic at the lower energies used in nuclear medicine, this changes significantly at higher energies, where the scatter becomes predominantly in the forward direction. This is significant since attempts at high energy imaging have been limited by the inability to distinguish scatter from the desired transmitted radiation. These forward scattered photons are comparable to the original energy and are a source of serious degradation to the image although attenuation effects are reduced.

Scattered photons are of lower energy than the primary photons and could, therefore, in theory, be eliminated by setting the baseline of the discriminator window at a photon energy equal to or slightly lower than
the energy of the primary photons. SPECT systems currently use NaI(Tl) detectors which limit their energy resolution to values typically in the range of 10-20% for the gamma-rays useful for nuclear medicine. Therefore some scatter counts do appear in the full-energy peak window in spite of the fact that the scattered photons are of lower energy than the primary photons. The limited energy resolution makes it virtually impossible to eliminate the scattered photons even with the lower level discriminator of the energy window set at energies slightly higher than the primary photons [Axelsson et al, 1984]. This imperfect discrimination against the scattered radiation, which reduces both spatial resolution and signal-to-noise ratio, leads to degradation of the image quality [Beck et al, 1969, 1982]. But unlike the more serious problems of photon attenuation and camera nonuniformity, which produces serious artifacts, the scatter causes only some loss of lesion contrast, slight blurring of the edges of the object and some increase in the apparent radioactivity (due to over-correction for attenuation). In general its effects are subtle and not very disturbing [Oppenheim, 1984]. Nevertheless as one tries to refine techniques and algorithms to achieve more accurate reconstruction of the true activity distribution, one must eventually tackle the problem of Compton inscatter.

There are at least four approaches to scatter corrections

(i) Using effective linear attenuation coefficient

(ii) Computer simulation of scatter component

(iii) Scatter correction by deconvolution

(iv) Scatter window method

4.9.1 Using effective attenuation coefficient

The effective linear attenuation coefficient [Larsson, 1980] is
often used unknowingly, in most cases in which some sort of attenuation correction is carried out. The true linear attenuation coefficient for the 140-keV photons of Tc-99m in water is \( \mu = 0.15 \text{cm}^{-1} \). If this value is used for attenuation correction in a reconstructed cross-section of a large phantom filled with Tc-99m, the radioactivity in the phantom will not appear uniform as it should, but will appear higher round the periphery. The excess radioactivity represents the detected Compton scatter, whose reconstructed image is superimposed on the uniform image produced by the detected primary photons after attenuation correction. Now if one uses a smaller attenuation coefficient, say \( 0.13 \text{cm}^{-1} \) for brain images and \( \mu = 0.12 \text{cm}^{-1} \) or smaller for abdominal images, then one can under-correct for attenuation in such a manner that the central parts of the image model the more intense regions round the edge caused by scattering, resulting in a more uniform radioactivity distribution in a reconstructed image. However, the true activity is over-estimated by this mean, and the overall intensity level must be reduced for accurate quantification.

This method, in effect, replaces some of the primary photons lost through attenuation with scatter photons that are detected in their place. It is a crude method, since it does not solve the problem of loss of contrast and edge sharpness caused by scatter. This method has been investigated by the author and the results are shown in chapter 6.

4.9.2 Computer Simulation of Scatter Component

This method involves the estimation of the scatter component of the image through computer modelling. Simulations of an assumed radioactivity distribution and of the surrounding scattering medium are stored in a computer, and a scatter image is generated from these data either through mathematical techniques involving matrix manipulation
[Egbert and May, 1980] or through Monte Carlo techniques, in which the histories of a large number of scattered photons are simulated and tracked to determine the probability of detection [Beck et al, 1982]. This scatter image is subtracted from the actual detected image. The difference, representing the primary photon component, is then corrected for attenuation using the true tissue value of $\mu(0.15 \text{cm}^{-1})$. In theory this method should result in accurate scatter corrections but in practice it is limited by the "faithfulness" of the simulations, is quite difficult to implement, and makes very heavy demands on the computer memory and time.

4.9.3 Scatter Correction By Deconvolution

In scatter correction of images by deconvolution [Axelsson et al, 1984] the assumption is made that the scatter component blurs the image of the primary photons in a constant and predictable manner. The nature of this blurring is determined by imaging a line source in a scattering medium. Axelsson et al. have showed that the blurring function for superficial and deep radioactivity differ appreciably, so an average blurring function is used. The scatter component is removed from the images by deconvolving them with the function. This approach would be expected to work reasonably well. In situations with a prominent component of superficial activity, as in liver imaging, it would lead to an artifactual "over-correction" for scatter.

4.9.4 Scatter Window Method

In this method the scatter component is measured directly as the object image is acquired. This is done by collecting a separate image in a scatter window at the same time that an image is being collected in the
primary window. The reasonable assumption is made that the events detected in the (lower energy) scatter window are related to the scatter component of the events detected in the full-energy peak window by a constant factor K. The value of K is approximately 0.5 in a typical imaging situation [Jaszczak et al, 1984]. The procedure is to collect the data in separate full-energy peak and scatter windows; subtracting the scatter counts, weighted by a factor of 0.5, from the full-energy peak counts; and applying attenuation corrections to the resulting data using the correct attenuation coefficient value. This method yields an accurate correction for Compton scatter with improved lesion contrast and edge sharpness, and permits quantification of the radioactivity distribution. This approach appears to be a practical solution of the scatter problem, but it requires a dual-energy detection capability, and it will double the computation time; however the data acquisition time will not increase. On the other hand if dual-energy detection facility is not available then the scatter data is collected in a second scan keeping the same scan parameters at the expense of increased data acquisition time. This method for Compton scatter correction in SPECT was used by the author and is described in chapter 6.

In this chapter various reconstruction techniques and the compensation for solid angle, inscatter, and attenuation techniques have been discussed. The merits and demerits of the different compensation techniques will be discussed in chapter 6 of this thesis.
CHAPTER 5

SPECT SCANNER CHARACTERISTICS

5.1 INTRODUCTION

With a medical CT scanner, the projection data for the patient is obtained by rotating the source and detector assembly around the stationary patient. This is mechanically cumbersome to implement and partly accounts for the high cost of the commercial CT scanners. For mobile inanimate objects this is unnecessary. The simplest instrument for computed transverse section tomography of inanimate objects is the rectilinear scanner which moves the sample back and forth, across the radiation beam while keeping the source and detector stationary; this is repeated after rotating the object through a small angle until sufficient angular data are collected for a reconstruction, as pioneered by Kuhl and Edwards [1963]. The work in this thesis under SPECT includes the modification for emission studies of the existing rectilinear gamma-ray scanner, developed and used for transmission studies by previous workers in the Physics Department at Surrey University [Foster, 1981; Folkard, 1983]. The performance and applications of the transmission scanner has been reported in several publications [Gilboy et al, 1982; Gilboy, 1984; Reimers et al, 1984; MacCuaig et al, 1985].

The modifications to the transmission scanner were made to convert it into a viable emission scanning device while retaining its transmission mode capability. Figures 5.1 (a) and (b) show the schematic block diagram of the transmission and emission scanning systems respectively. The scanner in its present form has the capability to perform the two modes of operation by minimal alteration to the set-up. This enables the SPECT system to acquire the attenuation coefficient values by an additional transmission scan for attenuation correction purposes, as
Figure 5.1(a): Block diagram of transmission scanning system.

Figure 5.1(b): Block diagram of emission scanning system.
described in section 4.8.2.

The emission scanner basically consists of the following five components.

(1) Scanner bed
(2) Collimators
(3) Detection and counting electronics
(4) Experimental control and data acquisition
(5) Image reconstruction and display unit.

In SPECT systems only the detector has to be collimated while in transmission tomography the source as well as the detector has to be collimated to define the ray path and to reduce the scattered radiations. It is mechanically quite difficult to move the detector-collimator assembly. Also the counts can not be taken immediately after such an assembly is halted because it may vibrate for some time in between the ray-sums. Movement of both the detector and the test object are minimised or totally eliminated by using a multiple detector array or a position sensitive detector. This will also reduce the scanning time considerably.

5.2 THE SCANNER BED

The scanner bed provides the two degrees of movement, rotation and translation, of the radioactive object. It consists of base, sliding assembly and the detector stand. The linear slide assembly carries a rotating turn-table and linear and rotational motions are controlled by two stepping motors (Fig. 5.1b). The radioactive object to be scanned is placed on the turn-table which has its periphery marked off in degrees. It is moved linearly along an extruded aluminium frame, 80 cm long, controlled by a stepping motor. The stepping motor at one end of the base frame operates an accurate lead screw of 1 mm pitch and the rotation of the lead screw drives the carriage along the bed. Each pulse
delivered to the stepping motor rotates the screw by \(1/400\) of a revolution, which is equivalent to a linear traverse of 2.5 microns, the minimum step length of the system.

The rotating table is a precision aluminium cylinder, of 120mm diameter, with a roller bearing. The minimum step is \(1/100\) of a degree and its maximum speed is 9 degrees per second. The maximum permissible loading is 50kg. A microcomputer (BBC microcomputer model B) controls the movement of the turn-table. TTL pulses from the microprocessor are converted to currents in the stepping motor's coils by two separate drive cards (Digiplan SM Drive Type CD20), one for each stepping motor. The stepping motors and the drive cards use the same 24 Volts DC 7 A Digiplan power supply Module Type PM1200.

A rod with an adjustable collar at either end runs the length of the frame and contact with either of these collars by the carriage operates a microswitch which disables the stepping motor. This is a safety feature to ensure that the sliding assembly does not run onto the end of the frame, which could damage the lead screw. With the collars fitted, the maximum linear traverse, i.e. the largest object that be scanned is 55cm. The complete scanner in emission mode is shown in Figure 5.2.

5.3 THE COLLIMATORS

This part of the apparatus is totally independent of the scanner, and could be replaced by any other collimator depending upon the energy of radiation. Good collimation is required in both transmission and emission tomography. The collimation in SPECT relies only on the detector collimator to define a narrow acceptance cone defining the chosen ray path. One of the factors that enables a three dimensional object to be represented as a series of two dimensional image slices is
Figure 5.2 SPECT scanner
that the radiation that are either emitted from the object (ECT) or traverse through it (TCT) are recorded in such a way that the thickness of the slice is made as thin as possible. The slice of interest is selected by adjusting the position of the collimated detector.

The purpose of the collimator is to limit the field of view of the detector by admitting radiation only from the desired region. A well collimated detector detects only those photons which arrive from a particular region "seen" through the collimator hole. This results in a marked decrease in the amount of detected radiation resulting in reduced sensitivity, or increased counting time. Different types of collimators were tried with low and high energy scans. For low energy photons the collimation is not much of a problem but for energies greater than 60keV the collimation of the detector as well as the shielding of the source for safety of the other workers in the laboratory was a problem. A lead wall was built with lead bricks for source shielding. The detector collimators used are shown in Figure 5.3. The small brass collimators were used for low energies and the big lead collimator was used for higher energy sources. The big lead collimator is a two part collimator, a collimator holder and a collimator insert which fits into the holder and can be replaced by another collimator. The collimator holder is a jacket type with an extension to cover the active length of the detector. Sectional views of the collimator and holder are shown in Figure 5.4.

5.4 DETECTION AND COUNTING ELECTRONICS

The detection and measurement of the radiation emerging from the test sample forms the basis of all the imaging systems where ionising radiation is the probe used to acquire the data for image reconstruction. The detection and counting unit comprises the detector and its high voltage supply, an amplifier to amplify the signals from the detector, a
Figure 5.3: The collimators used
Figure 5.4: Sketch of the collimator and its holder
single channel analyser to select the region of interest in the energy spectrum, and a dual counter/timer to count the pulses occurring in the selected energy region for a fixed time.

5.4.1 The Detector

In gamma ray spectroscopy, above several hundred keV, there are really only three detectors which are widely used: thallium activated Sodium Iodide (NaI(Tl)), Lithium drifted germanium (Ge(Li)) and, more recently, bismuth germanate (BGO), (although Gooda [1987] has recently demonstrated that CsI(Tl) crystals are also very practicable over the range of 60 - 662keV). Spectroscopy below these energies is normally carried out with lithium drifted silicon (Si(Li)), high purity germanium (HPGe) and beryllium windowed NaI(Tl) detectors; at very low energies gas filled proportional counters have been used [Sanders, 1982]. The choice of detector for a particular application also normally depends on the relative importance of energy resolution and efficiency. Scintillation detectors tend to have higher efficiencies due to the detecting medium being of higher atomic number and/or density. Semiconductor detectors however, have a very superior energy resolution and are used where the discrimination between closely spaced gamma ray energies is required.

Gamma rays interact with the detector material through several different interaction processes (as described in chapter 3), transforming their energies into light pulses (scintillators) or electron-hole pairs (solid state detectors). These events are converted into electrical output pulses from a linear amplifier; the amplitude of these pulses is usually directly proportional to the energy of the absorbed quantum and the pulse height spectrum can be measured with a pulse height analyser, or multi channel analyser. The distribution of pulse heights is known as the
gamma ray spectrum.

The detector used with the SPECT scanner consisted of a cylindrical 5cm by 5cm thallium activated sodium iodide, NaI(Tl), crystal hermetically sealed all round in a thin walled aluminium container with a glass window at one end which was optically coupled to the photocathode of an end-window photomultiplier tube (PMT).

The crystal+PMT combination operates in the following manner. The visible light is produced from recoil electron interactions in the crystal. The scintillation photons are transported to the end-window photo-cathode of the PMT, which is coated on the inside with a layer of alkali metals which has the property of releasing an electron when a photon is absorbed. These photo-electrons are accelerated by an electrostatic field to the next electrode called a dynode. Upon striking the dynode, more electrons are released from a secondary electron emitting coating and these in turn are accelerated to further dynodes where even more electrons are produced. This combination of a light sensitive photo-cathode with an electron multiplier results in the name photomultiplier. Finally as many as $10^8$ electrons can be produced from one electron arriving at the first dynode. The pulse of electrons at the anode charges up anode capacitance to form an output pulse. This signal is proportional to the energy of the incoming photon absorbed by the crystal because the number of electrons produced in the PMT is proportional to the amount of light produced in the crystal, which in turn is proportional to the energy deposited in the scintillator by the incoming photon.

The high bias voltage required by the photomultiplier tube is supplied by a Nuclear Enterprise (NE) 4701 high voltage supply in a single width NIM module slotted into a Beni Bin crate. The Beni Bin crate provides the low voltage supplies to all the electronic modules housed in it.
5.4.2 Efficiency and Energy Resolution of NaI(Tl) Detector

It is important in any tomographic system that the detection efficiency be high to allow reasonable statistical accuracy and to minimise the scan time. In addition, in medical use, the radioactivity employed is limited by dose considerations, so it is important to use the emitted radiation as efficiently as possible. The detection efficiency depends on the interaction cross-section of the atoms of the detecting material at the photon energy under investigation. Ideally an output pulse would be produced for every interaction that takes place in the active volume of a detector. However, whilst this is possible with charged particles, the more penetrating radiations such as gamma-rays and neutrons must undergo significant interactions in order to produce a detectable pulse and can travel large distances between interactions, therefore the detectors are rarely 100% efficient.

The detector efficiency can be described in two ways: absolute efficiency and intrinsic efficiency. The absolute efficiency, $E(ab)$, is defined as:

$$E(ab) = \frac{\text{Total number of counts recorded}}{\text{Total number of photons emitted by source}}$$  \hspace{1cm} (5.1)

and is dependent on detector properties and counting geometry.

Intrinsic efficiency, $E(int)$, is defined as:

$$E(int) = \frac{\text{Total number of counts recorded}}{\text{Total number of photons incident on detector}}$$  \hspace{1cm} (5.2)

and is related to the absolute efficiency by:

$$E(int) = E(ab) \times (4\pi/\Omega)$$  \hspace{1cm} (5.3)
where $\Omega$ is the solid angle of the detector seen from the source position.

The more useful and commonly reported efficiency is the intrinsic full-energy peak efficiency, $E(ip)$, and defined as:

$$E(ip) = \frac{\text{Total counts in the full-energy peak of energy } E}{\text{Total number of photons of energy } E \text{ incident on detector}} \quad (5.4)$$

Since the detector is to be used for the detection of a collimated beam of radiation, the intrinsic efficiency is the most useful to know. By using a standard source the intrinsic efficiency can be determined by:

$$E(ip) = 4\pi N/S \cdot \Omega \quad (5.5)$$

where $S$ is the number of photons of particular energy emitted during the measurement period, $N$ is the number of photons recorded in the associated full energy peak and $\Omega$ is the solid angle (in steradians) subtended by the detector at the source and can be found by integrating over the detector face as:

$$\Omega = \int \frac{\cos \theta}{2} \, dA \quad (5.6)$$

where $R$ is the distance between source and a surface element $dA$, and $\theta$ is the angle between its normal and the source direction. This assumes that the detector dimensions are small as compared to the source-to-detector spacing. For the common case of a point source located along the axis of a right circular cylindrical detector (Figure 5.5), $\Omega$ is given by:

$$\Omega = 2 \left(1 - \frac{R}{2 R^{1/2}}\right) \quad (5.7)$$

where $a$ is the radius of the detector and $R$ is the mean distance between
Figure 5.5: Representation of solid angle subtended by the detector at the source.

Figure 5.6: Definition of the detector resolution.
the source and a point inside the detector crystal where the interaction takes place i.e. \( R = d + x \), where \( x \) is the mean depth of interaction in the crystal. For \( R \gg a \), the solid angle reduces to detector plane frontal area \( A_d \), visible at source divided by the square of the distance. Therefore,

\[
\Omega = \frac{\frac{A_d}{2}}{\frac{R}{2}} = \frac{\pi}{R^2}
\]  

(5.8)

Now the effective activity, \( A \), of the source in terms of photon output at the time of measurement is given by:

\[
A = A_0 \cdot P \cdot \exp[-0.693t/(T)]
\]  

(5.9)

where \( A_0 \) is the activity at the time of source calibration, \( T \) is the half-life, \( t \) is the time elapsed since calibration and \( P \) is the number of photons per disintegration. The intrinsic full-energy efficiency can be written as:

\[
E(ip) = \frac{2N}{A_0 \cdot t \cdot P \cdot \exp[-0.693t/(T)] \left[ 1 - R/(R + a) \right]^{1/2}}
\]  

(5.10)

and for \( R \gg a \)

\[
E(ip) = \frac{2^{1/2}}{4N \cdot R}
\]

(5.11)

Another important parameter which controls the choice of detector for particular application is the energy resolution of the detector. It can be defined as the ability to resolve the different peaks of similar energies. It is conventionally expressed in terms of full-width at half-maximum (FWHM) of the nearly Gaussian response to a monoenergetic


Figure 5.7: Energy resolution of NaI(Tl) versus photon energy.

Figure 5.8: Intrinsic efficiency of NaI(Tl) versus photon energy.
input divided by the location of the peak centroid, Ho (Figure 5.6).

Two peaks will be resolved if they are separated by more than the detector FWHM. A good energy resolution allows a smaller energy window to be set which will reduce the contribution of the background continuum to the total count rate.

The energy resolution and the intrinsic full-energy peak efficiency of the NaI(Tl) detector used are shown in Figures 5.7 and 5.8 respectively. It is clear from the figures that for low energies the detector is more efficient but has poorer resolution and for high energies the detector has better resolution but is less efficient. Therefore, when selecting a source and detector for a particular application, the resolution and efficiency of the detector for that energy must be kept in mind.

There are a number of potential sources of fluctuation in the response of a given detector which result in imperfect energy resolution and peak efficiency. These include any drift in the operating characteristics of the detector during the course of measurements, sources of random noise within the detector and the coupled electronics, and the statistical noise arising from the discrete nature of the measured signal itself. In a wide category of detector applications, the statistical noise is the dominant source of fluctuation in the signal and thus sets an important limit on the detector performance.

5.4.3 Spatial Resolution

The spatial resolution is defined qualitatively as the capability of the system to distinguish fine details in the image. A quantitative definition introduces the line spread function, (LSF), described in section 4.7. The spatial resolution obtained by an imaging system
depends on the effective beam size, which is a function of the source and
detector collimator dimensions and also the data sampling frequency.
The spatial resolution of the detector is determined by the type of
collimator used. In our experiments at low energies, for example for
Am-241 and I-125, the replaceable circular and slit brass collimators
were used and for higher energies (Tc-99m, Cs-134 and Cs-137) the slit
shaped and circular lead collimators were used. These collimators were
fitted inside a jacket which shielded the sensitive regions of the
detector.

A few initial tests were carried out using collimators of different
diameter and length. The source used was a Cs-137 source collimated down
to 1mm at a distance of 75mm from the surface of the detector collimator.
For the high positioning accuracy the scan control program (described in
section 5.5) was used just for one projection. The counts in the 662 keV
full-energy peak counted for 60 seconds fixed time. Response curves from
these line scans were obtained and the spatial resolution of the system
was calculated from the FWHM of the curves. Also the signal-to-noise
ratio for each collimator length was calculated and compared. Table 5.1
shows the effect of collimator length in reducing the fraction of photons
transmitted through the surface of the collimator, other than the
collimator hole, to the detector. The response functions of the
collimated detector for collimator of different size and length are shown
in Figures 5.9 to 5.11.

The Figure 5.9 shows the point spread function (PSF) for a 1mm diameter
and 30mm long collimator while Figure 5.10 shows the PSF of a 2mm
diameter and 40mm long collimator for the same detector. The
signal-to-background ratio is improved in the latter case. The
background is due to the detection of scattered and transmitted photons
through the front and sides of the collimator (as the shoulder in Figure
5.9) and increasing the length of the collimator reduces the number of
Figure 5.9: The point spread function for 1mm diameter and 30mm long collimator with NaI(Tl) detector at 75mm away from the collimator surface.
Figure 5.10: the point spread function for 2mm diameter and 45mm long collimator with NaI(Tl) detector at 75mm away from the collimator surface.
Figure 5.11: Point spread function for the 4mm diameter and 45mm long
collimator with NaI(Tl) detector at 75mm away from the
collimator surface.
<table>
<thead>
<tr>
<th>Collimator size (mm)</th>
<th>Collimator length (mm)</th>
<th>Spatial Resolution FWHM (mm)</th>
<th>Signal/Noise ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>30.0 ± 0.5</td>
<td>4.0 ± 0.25</td>
<td>1.76</td>
</tr>
<tr>
<td>2.00</td>
<td>30.0 ± 0.5</td>
<td>4.5 ± 0.25</td>
<td>3.10</td>
</tr>
<tr>
<td>4.00</td>
<td>30.0 ± 0.5</td>
<td>6.0 ± 0.25</td>
<td>5.20</td>
</tr>
<tr>
<td>1.00</td>
<td>45.0 ± 0.5</td>
<td>3.0 ± 0.25</td>
<td>2.66</td>
</tr>
<tr>
<td>2.00</td>
<td>45.0 ± 0.5</td>
<td>4.0 ± 0.25</td>
<td>5.70</td>
</tr>
<tr>
<td>4.00</td>
<td>45.0 ± 0.5</td>
<td>5.5 ± 0.25</td>
<td>7.80</td>
</tr>
</tbody>
</table>

Table 5.1: Spatial resolution and signal-to-noise ratio for different collimators.

transmitted photons and so the signal-to-background ratio improves. Also in the latter case the signal-to-background ratio improves due to the bigger collimator hole size gives an increase in the required signal while the area exposed to the unwanted signals remains constant. Figure 5.11 shows the point spread function for the 4mm diameter and 45mm long collimator. The signal-to-background ratio is even larger than in Figure 5.10.

A spatial response curve for the NaI(Tl) detector is shown in Figure 5.12 which was obtained by scanning the uncollimated detector with a collimated point source of Cs-137. The dip in the central region is probably due to increased dead-layer thickness in the middle of the front face of the NaI(Tl) crystal.

In medical ECT the requirement of good spatial resolution is less demanding than that of TCT. TCT deals with anatomy and structure which
Figure 5.12: Spatial response of sodium iodide detector.
requires good spatial resolution over the whole object while ECT investigates the physiology and kinetics and involves the detection of the presence or absence of the radioactivity in a certain region which is usually well defined in advance. A spatial resolution of 5mm represents the best ECT resolution attained in current medical practice compared with <1mm image resolution in TCT. Although in some industrial applications (fission product distribution in nuclear fuel elements) very high resolution has been achieved (<<1mm) due to very high source strengths that are available in some cases.

5.4.4 The Amplifier

A single width NIM module ORTEC 571 spectroscopic amplifier, with versatile switch selectable pulse shaping and output characteristics, was used. The signals from the PMT anode were fed directly into the amplifier input. The amplifier features extremely low noise (8 microvolts referred to the input using 2 microseconds shaping time and gain > 100), wide gain range (continuously adjustable from 1 to 1500), good overload response and very high count rate capability. It provides symmetrical Gaussian pulse shaping on all ranges. The pulse pile-up effect starts to become significant for count rates in excess of 20000 /second, but it is negligible in our case because the sources used for SPECT purposes were not very active and the count rate was much below this limit.

5.4.5 Single Channel Analyser

An ORTEC 550 single width NIM module single channel analyser (SCA) was used in order to count only those pulses occurring in the full-energy peak or the region of interest. The pulses from the amplifier are fed to
the SCA where they are sorted, according to size of the pulse, into three
groups by two voltage thresholds. Three outputs are available, one from
each group, and only those pulses falling between the two thresholds,
which is called the energy window, were recorded.

The SCA accepts either positive unipolar or bipolar input pulses from
linear amplifier; it examines the input pulses according to their
amplitudes and generates appropriate positive logic output pulses
separately for SCA, lower level discriminator (LLD) and upper level
discriminator (ULD) responses. The Ortec 550 SCA has three modes of
operation namely; an integral mode, normal mode and window mode, and
they can be selected by a toggle switch. There are two ten-turn
potentiometers for setting up the window. The one marked with LLD sets
the lower energy threshold and the other can be used for setting the
upper energy threshold or to define the window width. In this mode the
upper energy threshold is equal to the value set by the LLD plus the
window width. To select a window the two thresholds are set on either
side of the full-energy peak (or the region of interest) and output
pulses within the window are counted. The window can be set up very
conveniently using the SCA output to gate an MCA. The SCA could be
replaced by an MCA, but since the SCA/counter-timer combination was
already interfaced with the scanner system this was used instead.

5.4.6 Dual Counter/Timer

A single width NIM module Canberra Model 2071A dual counter/timer
was used with the system. The positive logic pulses from the SCA were
fed to it. The scaler unit has got two eight decade counters, a crystal
controlled oscillator and presetting logic. It can be used as a preset
time and event counter, which is applicable to both the transmission and
emission scanning, or as a preset event counter and timer, which is
suitable only for transmission scanning. This dual counter/timer device gives the scanner the capability to perform in either emission or transmission mode by simply flicking a switch.

The front panel controls are used to set the system for a transmission scan or emission scan. The two input connectors are marked with "A-IN-B" signifying the input A and input B. The three-position toggle switch marked with "0.01SEC COUNTB 0.01MIN" selects preset function as time or B input events. When time is selected channel A counts external events and when COUNTB is selected, channel A counts time in increments of 0.01 seconds. The "DISPLAY SELECT" switch is a two-position toggle switch which selects the channel A or channel B contents for display on the six digit liquid crystal display (LCD). The LCD includes eight annunciators which describe the display and dynamic state of the unit. A three-digit thumbwheel switch for channel B preset, labelled as "PRESET N M X10 ^P", sets the absolute value of the selected preset function. The "M" sets units, "N" sets tens, and "P" sets the power of 10 by which NM is multiplied.

In our case the preset time and event counter option was selected and the differential SCA was used for emission scanning, and preset count and time recording option was used for transmission scanning.

5.5 EXPERIMENTAL CONTROL AND DATA ACQUISITION

The control of the experiment is achieved by using a BBC model B microcomputer. Five lines are required to fully control the experiment, one for each stepping motor, one to define the linear motor direction and two to the counter/timer for carry and reset pulses. The dual counter/timer has a general purpose interface bus (GPIB) option. The GPIB links the counting module into the IEEE standard communication network, i.e. to the microcomputer via an IEEE controller. The controller
manages bus communications primarily by directing or commanding which device has to send data to other devices (talker), or receives data from other devices (listener) during an operational sequence. The controller may also be interrupted or it may command specific action between devices. As a listener the interface receives start, stop and readout commands from the controller. As a talker it supplies its accumulated data to a peripheral device. The microcomputer has its own keyboard, a compatible disk drive and IEEE controller. For visual display a Microvitec Cub colour visual display unit (VDU) was used.

The scanner has recently been updated with a Hewlett Packard (HP) 9836C microcomputer that has its own colour VDU.

5.5.1 The Experimental Procedure

First the radioactive object to be scanned is placed on the turn-table and a suitable collimator is fitted to the detector. The detector is then aligned with the object on the turn-table so that the collimator hole is in line with the section of the object to be scanned. The PMT high voltage and/or amplifier gain is then adjusted to give suitable output pulses and the desired full-energy peak (or any other region of interest, for example the scatter region) is windowed by using the SCA. The counting time for each ray-sum has to set before starting the scan. The microcomputer is then loaded with the emission scan control program, which has been written and stored on a floppy disk. When this program is loaded it gives the following option list:

1. Average data values no motion
2. Initialise scan parameters
3. Move scanner bed
4. Start scan
Option 1 is applicable to the transmission scanning whereby fixed detector counts and a timer is used, and this calculates the overall time taken by for the whole scan to be completed given the fixed counts.

Option 2 enables the initialisation of the scanning parameters and it asks the values of the following parameters:

- Input step length (mm)
- Input number of steps
- Input angular spacing (degrees)
- 180 or 360 degrees rotation
- Input scan reference number
- Input scan title
- Are details correct?

The dimensions of the radioactive object to be scanned and the detector collimator size controls the choice of these parameters. The 360 degrees rotation is for emission scanning because conjugate projections are needed for solid angle and attenuation compensation purposes.

Option 3 (move scanner bed) is used for the alignment of the radioactive object. Under this option the sliding assembly can be moved linearly either to the left or to the right, and the turn-table can be rotated through any angle.

The option 4 is chosen to start the scan after setting all the parameters. After this option is initiated the whole operation is automated. The turn-table will first rotate by an amount specified by the angular spacing and it will translate by a distance specified by the
step length. The counts for the time selected are then accumulated. At
the end of the fixed time, each value of the ray-sum is read from the
counter A and stored in the microcomputer memory as well as being
displayed graphically on the VDU screen. The microcomputer then
instructs the linear stepping motor controller to move the turn-table to
the next position and counts are again accumulated. This operation is
repeated until the end of the first projection. The data for the
complete projection is then transferred from the microcomputer memory to
the floppy disk and in the meantime the rotary stepping motor controller
receives a command to rotate the turn-table to begin the next projection.
The whole process is repeated until all the projections are completed.

The scan data is held on the floppy disk at the end of the last
projection. Option 5 is then selected to transfer the data to main frame
PRIME computer. The following options are available under this option:

1. Dump data to PRIME
2. Catalog disks
3. Delete files
4. Compact disk data
5. Return to main header

The first option is chosen to transfer the data from the floppy disk to
PRIME at the specified baud rate. Normally the data are stored in the
main frame PRIME computer for easy access and alternately, the data can
be stored on the disk which requires them to be compacted to allow more
memory space for further scan and they are also catalogued for easy
reference according to the scan numbers. After this the option 5 returns
back to the main options list.
This part of the SPECT system consists of a computer suitable for running large programs and its associated software including file handling for the projection data and reconstruction data, and an image analysis package which is accessed via a 16 level grey-scale Sigma 5674 visual display terminal. A Cotron monitor equipped with camera for copying images onto a 35 mm film or Polaroid is connected in parallel with the grey-scale terminal.

5.6.1 PRIMENET and MUM

All the work of image reconstruction and analysis was done on the Surrey University's network of PRIME computers (PRIMENET). The PRIMENET consists of three PRIME 750, one PRIME 950 and one PRIME 550 computers, all operating independently although links exist between them. A PRIME 550 multi-user-mini (MUM) computer has been provided by the SERC. The MUM has 550 kbytes of core and has a one-way link with PRIMENET.

The MUM has been used for most of the processing and storage of the projections and images. The time-sharing facility of the PRIMENET makes the image reconstruction and display process very slow when many users are using the computers. This was one of the reasons for opting for the HP-9836C minicomputer. With this machine it is possible to perform reconstruction and display in addition to controlling the whole scanning procedure. This has made the scanner set-up completely portable. The HP-9836C is a 16-bit system based on the M68000 microprocessor. The complete system consists of two 512k disk drives and a 590 X 312 pixels 16 levels (grey or colour) VDU and a printer.
5.6.2 Image Reconstruction Programmes

The development of the reconstruction and display program for transmission tomography is described at length by Foster [1981] and will be explained here only briefly.

The data transferred from floppy disk to the MUM are processed by a simple FORTRAN program that converts the counts from the dual counter/timer into ray-sums according to equation 4.6, and formats it for analysis by SNARK-75.

SNARK-75 is an interactive program, written in FORTRAN, developed by Herman et al [1975] at the State University of New York and has been implemented on PRIME. It can be considered to consist of three parts namely; data input, reconstruction, and image analysis. Several reconstruction algorithms like ART, SIRT and convolution (described in chapter 4) with a choice of filter functions are available in SNARK-75. The reconstruction program used for SPECT image reconstruction uses a convolution filtered back-projection with the Bracewell/Ramachandran filter. The reason for choosing this algorithm is that it uses less computer time as compared to ART and SIRT. An equivalent ART reconstruction program requires 2 to 5 times as much and SIRT 5 to 15 times longer computing time [Herman and Rowland, 1973].

When data has been processed SNARK creates two files, a projection file and a reconstruction file which contain data about the projection and reconstruction respectively. Programs for decay correction for short lived isotopes such as Tc-99m, scatter correction and attenuation were written separately, and were used to correct the projection data prior to feeding into SNARK. These programs are written in FORTRAN-77 and stored on the PRIME.
Another set of reconstruction algorithms, called 'RECLBL', obtained from the Donner Laboratory University of California [Huesman et al., 1977] has been implemented on the PRIME by the author. RECLBL is a library of computational subroutines for different reconstruction algorithms, suitable for emission and transmission tomography. The subroutines are written in FORTRAN (ANSI standard). The RECLBL library applies to data that represent the projection of density along parallel or divergent sets of straight-line paths (rays) through the object. The algorithm transforms one-dimensional projections from multiple angles around the object to a corresponding transverse section through the object. Three-dimensional information is obtained by stacking successive transverse sections. Since RECLBL is a collection of subroutines, the user must provide a program that performs such functions as: set parameters that define the geometry as well as determine control operations within the library, call display routine of the library, and save results if desired. In addition the user must provide a subroutine for the data input. A block diagram of the structure of the RECLBL library is given in Figure 5.13. The iterative gradient and conjugate gradient reconstruction techniques have been used for simulated and real scan data. Reconstructed images are plotted using the overprinting facility of the line printer with PRIME computers.

5.6.3 Image Display and Analysis

Normally the images are displayed on a Sigma 5674 16-level grey scale terminal provided by the Science and Engineering Research Council (SERC). A Cotron monitor equipped with camera for copying images on a 35mm film or Polaroid, is connected in parallel to the Sigma terminal. A recent addition to it is the colour monitor of HP-9836C microcomputer. For image display using the RECLBL programs the overprinting facility of the line printer with the PRIMENET was used. By overprinting certain
Figure 5.13: Block diagram of the structure of the RECLBL library.
alphanumeric characters a 'pseudo grey-scale' is generated but the contrast of such a method is poor, making fine structures hard to see. The images can be displayed on the Sigma terminal by replacing the alphanumeric characters by the grey levels.

The display and analysis of images, reconstructed using SNARK-75 is performed by the computerised image analysis (CIA) program. CIA is an interactive program, developed by Foster [1981], that provides a wide range of display and analysis facilities. It reads in the projection and reconstruction files created by SNARK and displays the images in few seconds. The package is also capable of windowing the image to display the features of interest, producing histograms of the attenuation values, allowing enlargement of the data, displaying line sections and projections as well as providing quantitative statistical information about the reconstruction. Isometric three-dimensional display and contour graphics options have recently been added to the option list of the CIA.

5.7 GAMMA-RAY SOURCES USED FOR SPECT

Emission imaging is concerned with the representation of an internal three-dimensional distribution of activity by external detection of the internally emitted radiations. In SPECT, as the name suggests, the detection of single photons is carried out and in this study the detection of gamma-rays and characteristic X-rays was carried out.

Gamma-rays are produced as the by-product of a nuclear decay by the de-excitation of excited states of nuclei after nuclear reactions. A radioactive nucleus may decay by one of the several ways namely; spontaneous emission, alpha emission, beta-minus emission, beta-plus emission, electron capture, or isomeric transition with the emission of gamma-rays only. The decay of a nucleus generally results in the
emission of some kind of radiations and the daughter nucleus may be either stable or radioactive.

The selection of a radionuclide for a particular application is governed by various factors. For nuclear medicine purposes the following characteristics of a radionuclide are desirable [Mettler Jr. and Guiberteau, 1983]:

(a) Minimum of particulate emission
(b) Primary photon energy between 50 and 500 keV
(c) physical half-life greater than the time required to prepare material for injection
(d) Effective biological half-life greater than the examination time
(e) Suitable chemical form and reactivity
(f) Chemical stability or near stability of the product.

In nuclear industry, the gamma-rays are by-products of nuclear reactions that usually take place in a nuclear reactor core. Irradiated objects in a neutron flux arising from neutron activation analysis or as components of the reactor core are ideal subjects for investigating the distribution of the induced activity. An obvious case is the distribution of fission products inside fuel rods [Davies et al, 1986].

5.7.1 Technetium-99m

Tc-99m in ammonium pertechnate solution was used for SPECT images. It emits 140.5 keV photons and is the most popular radioisotope in nuclear medicine. Tc-99m has a 6 hours half-life and it decays through isomeric transitions emitting 2.2 keV (98.6%), 140.5 keV (98.6%) and 142.7 keV (1.4%). The isomeric decay is a process in which the nucleus simply changes from a higher to a lower energy level by emitting a photon.
and the daughter nucleus remains the same chemical element as the original nucleus.

The Tc-99m source for emission scanning was obtained from St. Luke's Hospital in Guildford. Only a limited source strength is allowed in the University due to the internal safety regulations. Due to its short half-life the emission scans can not be done properly after a few days. The concentrated source was diluted in the radiation laboratory of the Physics Department and different samples were made for scanning. Because of its short half-life, careful planning is necessary to ensure no time is wasted before scanning starts. A decay correction has to be made to the projection data prior to reconstruction since the time taken by a scan is often longer than the half-life of the source. The decay correction program accounts for the time lost before the scanning starts, and also for counting time for each ray-sum, time between ray-sums measurements, and time between projections using the published half-life for the isotope in question.

5.7.2 Iodine-125

Iodine-125 is a low energy source which decays through electron capture. It emits 35 keV (7%) gamma-ray and 27-32 keV (138%) tellerium K X-rays with a half-life of 60 days. This source was obtained from the Biochemistry Department and the original source strength was 1mCi in 1ml. It was diluted to make 200 ml of the solution by adding demineralised water. The volatilisation of the iodine is the significant problem with this source. Simply opening a vial of sodium iodide at high radioactive concentration can cause minute droplets of up to 74 Bq to become airborne. Three samples of different concentrations and different activity were also made in small vials for spatial resolution tests. The concentration for the successive scans were varied and at the end all
were mixed in a 200 ml solution in a plastic bottle of 50 mm diameter. The decay correction program was used for longer scans. Details of the samples and the results are shown and discussed in the next chapter.

5.7.3 Caesium-137

The Cs-137 has a half-life of 30 years and it decays through beta-minus emission to Ba-137 which is a stable isotope. In 93.5% of beta-transitions the 662keV level in Ba-137m (2.55min) is populated which decays to give the 662keV gamma-ray. This source was obtained from the Radiation Laboratory in the Physics Department. Caesium chloride solution in water was used in a plastic bottle. Due to its high energy the detector collimation and the shielding of the source is the major problem. Different samples in small plastic vials were made for scanning. The details of the samples and the results are given in the next chapter.

5.7.4 Tellerium-123m

Te-123m has a half-life of 119 days and emits 90 keV and 159 keV photons. Due to its 159 keV gamma-ray emission it is an ideal source for nuclear medicine test purposes because it is very close to the 140 keV photons from Tc-99m. Because it has much longer half-life than Tc-99m it was very useful for testing the emission scanner performance for Tc-99m but using 159 keV photons. This source was acquired by a previous worker in the middle of 1982 and by the time the emission scanner was fully operational the source was too weak to obtain a good statistical image. Nevertheless both the gamma-rays were utilised for scanning and the results are shown in the next chapter.
Apart from the above mentioned sources, a mixed energy liquid source containing Cs-134, Ce-144 and Ru-106 radioisotopes, was also used. At the time of use, only the longer lived Cs-134 isotope was present until a useful activity. Attempts were made to do the emission scan of this mixed energy source, contained in a 13 cm diameter plastic bottle, by using Cs-134 peaks at 604 keV and 796 keV. But unfortunately the source was so weak that it could not give good statistical images.

5.7.5 Sealed Sources

Sealed sources were also used to test the performance of the SPECT scanner. These include krypton-85 gas contained in a perspex screw, cobalt-57 embedded in a steel collimator, americium-241 and barium-133 standard gamma-ray point sources. The energies used for emission scans were 514 keV photons from Kr-85, 122 keV from Co-57, 59.6 keV from Am-241, 278 keV and 303 keV photons from Ba-133. The results of these scans are given in the next chapter.

In this chapter the different components of the SPECT scanner has been discussed including the choice of collimators and detector performance (efficiency, energy and spatial resolution). The results from the SPECT scanner along with the corrections for solid angle, inscatter, half-life and attenuation, will be presented in the next chapter.
6.1 INTRODUCTION

Emission tomography maps the two-dimensional radioactivity within the scanned object. The type of information that is provided by a tomographic scan has potentially a wide range of applications both in medicine and in industry, such as the localisation of a radioactive source inside the body, to determine the depth of an organ by measuring the emerging radiation from that organ after administering a radionuclide in the body, or crack and void detection in engineering structures, or burn-up studies of irradiated fuel rods from a nuclear reactor etc etc.

Since the early work in emission tomography [Anger, 1968; Kuhl and Edwards, 1963] nuclear medicine has developed in two complementary directions; viz positron emission tomography (PET) and SPECT. PET works on the detection of the coincident annihilation coincidence radiation from positron emitters. PET systems have the advantage of exploiting physiologically important radionuclides such as C-11, N-13, O-15 and F-18. However most short-lived positron emitting radionuclides presently in use can only be produced with expensive on-site particle accelerators. SPECT has the practical advantage of using commercially available radiopharmaceuticals.

In conventional emission tomography, concentration of given radionuclide in an organ occurs following an intravenous administration of a selected radiopharmaceutical. The spatial and temporal distribution of this concentration function within an organ can provide important information concerning organ function and pathology.
The development of the emission scanner has been described in the last chapter and in this chapter the experimental results from the SPECT scanner will be presented and discussed. A number of scans were carried out both in the transmission and emission mode. Most of the scans were carried out with a view to testing the scanner performance for different situations, i.e. for point sources, a radioactive gas and various extended sources in liquid and solid forms.

A selection of the images obtained are discussed and in some cases the reconstructed data are quantitatively displayed by means of the line scans and analysed using the concept of contrast in the reconstructed vials. Scatter, decay and attenuation corrections, where applicable, are applied to the data and the results are shown for uncorrected and corrected data.

6.2 IMAGE ANALYSIS

Image analysis is fundamental to the field of imaging since the purpose of obtaining an image, in medical or non-medical applications, is to accurately identify features in the original object. The information content of an image is high and hence the process of evaluating and comparing images is complex. Subjective visual inspection of images can give reasonable qualitative results due to the excellent pattern recognition abilities of the human eye. However, the human eye is incapable of detecting small differences quantitatively and hence this is an important aspect of an image, which forms the basis of objective analysis, is absent in the human visual evaluation process.

The principle of describing an information transfer system in terms of its response to a sinusoidal input function was first applied in electronic engineering to analyse communications systems and later in the
field of optics and photography to analyse imaging systems in terms of their spatial frequency response. The introduction of transfer function analysis techniques in radiography in order to quantitatively evaluate imaging systems began in the early 1960's [Rosemann et al, 1964]. Since then both the theoretical and practical aspects of its application in medical imaging have been extensively developed [Rosemann, 1968, 1969; Metz and Doi, 1979]. The ultimate aim of transfer function analysis is to characterise systems in order that the output resulting from a known input can be predicted and also to enable intercomparison of systems. Firstly the generalised system is defined which incorporates every component involved in the production of an image from the original object, for example, in SPECT this includes the effects of scattering of primary photons and attenuation within the object, the detection system response and the reconstruction process.

The quantitative analysis of the input and output functions of systems forms the basis for transfer function analysis techniques. In imaging system various types of input functions are employed, for example, in emission imaging the response of a system to point and line sources, that is, the point and line spread functions, are frequently determined and utilised in the characterisation of systems. However, a more generalised approach is the quantitative analysis of the simple objects and their corresponding images produced by systems using the concept of modulation and contrast.

The definition can be applied to any situation where the distribution pattern is repetitive, such as in sinusoidal distributions. In the generalised form, modulation is defined as the amplitude of a distribution divided by the average value (equation 4.51). The concept of contrast is a generalised form of modulation. Whereas modulation applies to cyclic distributions, contrast can be applied to any type of distribution and the contrast of one region in a distribution with
The ratio of the contrast of the image, $C_i$, to that of the object, $C_o$, can be used to evaluate and compare imaging processes, i.e.

\[
\text{Contrast Ratio} = \frac{C_i}{C_o}
\]  

(6.1)

Therefore quantitative information can be extracted from the image by comparing the contrast in the different regions with the known activity region. This can be done by scanning a known activity source within the object to be scanned, and the activity in different parts of the object can be calculated by comparison. Another way to get quantitative information from the scanned object is to calibrate the imaging system by scanning known activity phantoms and to derive calibration curves for different activities, photon energies and scan parameters. Then the true activity in the scanned object is determined from such calibration curves.

A three-source phantom was prepared by injecting $5\text{ml}$ of $\text{Cs-137}$ solution containing $37\text{ kBq}$, $74\text{ kBq}$ and $148\text{ kBq}$ in three perspex vials of diameter $15\text{mm}$, $20\text{mm}$ and $25\text{mm}$ diameters, respectively. These three sources were placed in water in a container of diameter $76\text{mm}$, and were scanned. The detector collimator used was the big jacket type lead collimator of dimensions $1.5\text{mm} \times 20\text{mm}$. The scan parameters are given below.

- Number of steps = 60
- Number of projections = 90 (over $360\text{ degrees}$)
- Step length = $1.5\text{ mm}$
- Angular spacing = $4\text{ degrees}$
- Counting time per raysum = $30\text{ seconds}$

The reconstructed image of the phantom is shown in Figure 6.1(a) and its
3-dimensional plot is shown in the Figure 6.2(a). The matrix of the phantom was water containing no active solution and, therefore, the contrast of the active vials with respect to the matrix is unity. For this reason one of vials in the phantom was selected to be the reference vial and the contrast for each of the other vials comprising the phantom was calculated with respect to the vial using the equation 4.52. The contrast of the reference vial in the reconstructed image was calculated with respect to the water matrix. The intensity value for water matrix in the reconstructed image is not zero which shows that the reconstruction process is not 100% accurate. Table 6.1 shows the results of the calculations for contrast of the vials in the phantom and in the reconstructed image with respect to vial 1.

The contrast in the image can also be calculated from the 3-dimensional plot of the image as the z-axis of the figure corresponds to the brightness of the image, i.e. to the activity concentration in the image. In Figure 6.2(a) the heights of the peaks corresponds to the activity in the different vials. The height of the peaks were measured and the values of the contrast for the two vials were calculated with respect to the height of the peak corresponding to the reference vial. The reference vial contrast was determined with respect to the base value of the z-axis. The calculated values of contrast from the 3-dimensional plot are also shown in Table 6.1 and they are quite close to the values calculated from the intensity measurements from the VDU screen. This means that the quantitative information can be obtained from the 3-dimensional plots and the use of the VDU grey-scale is not necessary for the measurements.

Another phantom was prepared by using the same three sources but putting them in a line in water in the same container. The scan parameters were kept the same as for the first scan except the counting time for each raysum was increased to 60 seconds. This phantom was scanned twice,
Figure 6.1 Images of contrast phantoms.
Figure 6.2: Three-dimensional representation of the images in Figure 6.1
Table 6.1

Contrast values from the image in Figure 6.1(a).

<table>
<thead>
<tr>
<th>Vial number</th>
<th>Activity in vials kBq/ml</th>
<th>Contrast in phantom</th>
<th>Contrast in image from image</th>
<th>Contrast in image from 3-D plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>148</td>
<td>1</td>
<td>0.817</td>
<td>0.921</td>
</tr>
<tr>
<td>2</td>
<td>110</td>
<td>0.147</td>
<td>0.121</td>
<td>0.130</td>
</tr>
<tr>
<td>3</td>
<td>74</td>
<td>0.333</td>
<td>0.250</td>
<td>0.258</td>
</tr>
</tbody>
</table>

Table 6.2

Contrast values from the images in Figure 6.1(b) and (c).

<table>
<thead>
<tr>
<th>Vial number</th>
<th>Activity in vial kBq/ml</th>
<th>Contrast in phantom</th>
<th>Contrast in image from 662 keV</th>
<th>Contrast in image from 32 keV</th>
<th>Contrast in image from 662 keV</th>
<th>Contrast in image from 32 keV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>148</td>
<td>1.000</td>
<td>0.842</td>
<td>0.765</td>
<td>0.818</td>
<td>0.791</td>
</tr>
<tr>
<td>2</td>
<td>110</td>
<td>0.147</td>
<td>0.138</td>
<td>0.158</td>
<td>0.143</td>
<td>0.176</td>
</tr>
<tr>
<td>3</td>
<td>74</td>
<td>0.333</td>
<td>0.322</td>
<td>0.357</td>
<td>0.304</td>
<td>0.296</td>
</tr>
</tbody>
</table>
first using a 662keV full-energy peak window and then with a 32keV energy window to bracket the barium K X-rays. The reconstructed grey-scale images of the phantom are shown in Figure 6.1 (b) and (c) and their 3-dimensional (isometric) plots in Figure 6.2 (b) and (c), for 662 keV and 32 keV energy windows, respectively. The values of contrast calculated from the image and from the 3-dimensional plots are shown in Table 6.2.

The spatial resolution of the system for this phantom can be determined by measuring the FWHM of the peaks in the corresponding line scans of the vials in the reconstructed image. It can also be determined from the isometric plots of the images by measuring the FWHM of the respective peaks. The spatial resolution obtained from the isometric plots in terms of FWHM of peaks was measured to be 20 mm for the first phantom and 12 mm for the second phantom for both the energy windows. The better spatial resolution for the second phantom can be attributed to the longer counting time than the first. This will be discussed in the next section.

6.3 OPTIMISING THE SCAN PARAMETERS

6.3.1 Number of Projections

The minimum number of projections, M, required to yield a spatial resolution, r, can be determined by the relation

\[ M = \frac{\pi S}{q} \]  

(6.2)

where S is the number of raysums in a projection and q is a constant. Various values of q has been reported depending on the approach used. Cormack [1977] has recommended the value of q=1 while Foster [1982],
Folkard [1983] and Sanders [1982] have used the value $q=2$.

Using filtered back-projection it was found that reasonable results are obtained with the projections greater than or equal to 30 (for $0$ to 180 degrees). However if the experimental time allows then ideally the number of projections would satisfy the condition of

$$M \geq \frac{\pi S}{2}$$

(6.3)

A 3-source phantom was constructed by injecting I-125 solution into vials of diameter 25mm, 20mm and 15mm, and placing them in water in a plastic container of diameter 76mm. The activity in the vials was 10.1 MBq, 7.4 MBq and 3.7 MBq respectively. A steel collimator of 1mm x 7.5mm dimensions was used for the detector and the scans were carried out by windowing the 27keV X-ray peak. The details of this source are given in section 5. The scan parameters are given below:

- Number of Steps = 90
- Number of Projections = 45
- Step length = 1 mm
- Angular spacing = 4 degrees
- Counting Time per raysum = 10 seconds

Another scan was performed by keeping the scan parameters the same except the number of projections was increased to 60. The reconstructed images for 45 and 60 projections are shown in Figures 6.3 (a) and (b) and their three-dimensional plots are shown in Figures 6.4 (a) and (b), respectively. It can be seen that the change in the number of projections has no effect on the quality of the images. Therefore in order to reduce the scan time a small number of projections can be chosen, (although for a given statistical quality the same number of photons must be counted).
Figure 6.3. Images of three source phantom of I-125
Figure 6.4: Isometric plots of the images in Figure 6.3
6.3.2 Number of Raysums

The number of raysums depends upon the collimator size and the size of the object to be scanned. The detector step length is normally equal to the collimator diameter but occasionally when a lower spatial resolution can be tolerated the step length can be set greater than the collimator size. However, if the step length is set less than the collimator diameter no extra information is gained. The same phantom (described in the previous section) was scanned for fewer raysums per projection (45 instead of 90). The scan parameters are given below:

Number of steps = 45
Number of projections = 45
Step length = 2 mm
Angular spacing = 4 degrees
counting time per rasum = 10 seconds

The Figures 6.3(a) and (c) show the reconstructed images of the same phantom with the same parameters except the number of raysums. It is obvious from the figures that the resolution is strongly dependent on the step length and less on the number of projections.

One disadvantage of choosing a smaller step length is that a narrow collimator is required to fully exploit the increase in the resolution, which will lower the count rate and consequently a longer counting time will be needed for given photon statistics. This suggests that the compromise between resolution and scan time could be optimised by using slit collimators of variable widths, as has been used for most of the scans in this work, to allow more photons to be collected by the detector. Such a system allows the collimator to exactly match the step
length. The collimator slit is aligned to the longitudinal axis of the object; if there is little or no variation along this axis the counting efficiency can be improved without loss of image quality in the transverse section.

6.3.3 Counting Time

Counting time is governed by the count rate and maximum available experimental time. A reasonably large number of counts should be collected per raysum to minimise the effect of statistical fluctuations in the final image. Ideally the statistical fluctuations should be very low but when employing isotopic sources and a single detector to scan the object of high attenuation coefficient then lower statistics has to be tolerated.

The counting time can be kept down if the source is active enough to give high count rates, for example in imaging fission products in spent fuel rods, but for the present experimental work we could not use highly active sources due to the shielding problems and the internal safety regulations of the University. This restriction automatically means that longer counting times were required.

The effect of counting time on the image quality was studied. The phantom was made with a 37MBq I-125 source in 110ml of water in a plastic bottle of diameter 76mm. The phantom was scanned for different counting times keeping all the other scan parameters constant. The scan parameters are given below:

- Number of steps = 60
- Number of projections = 45
- Step length = 1.5 mm

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Angular spacing $= 4$ degrees

Counting time per raysum $= 5, 10, 20$ seconds

The detector collimator used was $1.5 \text{mm} \times 20 \text{mm}$ lead slit collimator. Figures 6.5 (a), (b) and (c) show the reconstructed images for $5, 10$ and $20$ seconds counting times for each raysum, respectively. Apparently there is not much difference in the three images. The reason for this is the higher activity of the source so that even in the worst case image is reconstructed from a reasonable number of photons. But it can be noted that the images are getting brighter and well defined as the counting time increases. The comparison of the images for $5$ second and $20$ seconds counting time shows that the image quality is better with the longer counting time. This is also shown in their respective line scans which show the improved statistics for the higher counting time values. Another way to describe the difference in the statistics of the two images is to compare the separation of the two peaks in bottom left quadrant of the figures. These peaks represent pixel histograms due to air (left) and source (right). The two peaks are becoming more and more distinct from each other and more well defined with the increase in the counting time due to the higher total photon count from which the images are reconstructed. The noise in the images was calculated using the expression reported by Foster [1981] and was found to be $2.37 \times 10^{-4}$, $3.14 \times 10^{-4}$ and $4.92 \times 10^{-4}$ in the images for $20, 10$ and $5$ seconds counting times, respectively. This means that the images for longer counting times will be better statistically.

6.4 BACKGROUND CORRECTION

The projection data was corrected for background counts prior to reconstruction. This was done by scanning the radioactive objects with the collimator blocked with sufficient lead to prevent the radiation
Figure 6.5. Images of I-125 phantom with 5, 10 and 20 sec. counting time
passing through the collimator hole. If the object is fairly symmetrical then only one background projection is required. The raysum values of this projection were subtracted from the corresponding raysum values of each projection.

6.5 SCATTER CORRECTION

When a photon is emitted inside an attenuating medium, it suffers a number of collisions within the medium. For photon energies greater than 100 keV, the predominant mode of interaction is Compton scattering. Thus the scattering medium behaves essentially as a secondary source of photons whose intensity and energy varies with the angle [Davies et al, 1985]. Attenuation removes the gamma-ray signal from the ray integral whereas the Compton incoherent results in the addition of sample dependant background signals. The magnitude of this background depends not only on sources within the transverse section of interest, but also on sources in other nearby planes.

Theoretically, one can discriminate against photons using a single channel analyser (as described in section 4.9) since the inelastically scattered photons are of lower energy than the primary photons. However, most ECT systems use NaI(Tl) based detecting systems which typically have an energy resolution in the range of $10 - 20\%$ (Figure 5.6) and require a pulse height window typically twice this value ($-2$FWHM). This inevitably results in the acceptance of scattered photons within the energy window of interest. This situation is even more complicated in the case a multi-energetic spectrum where scattering contributions within the detector from higher energy events are accepted within the selected photopeak window. The effect of this to degrade the resulting images both quantitatively and qualitatively. This generally appears in the form of increased noise which tends to reduce the image resolution
[Jaszczak, 1983,1984; Axelsson, 1984; Pang and Genna, 1979; Ehrhardt et al, 1973; Block and Sanders, 1972] and the appearance in the resulting image of virtual sources. Therefore it is important to find a reliable method to correct for these undesirable components of the detected signal. The growing interest in quantitativity has lead to a wide-spread effort to establish a method to correct for Compton scattering in ECT. The different methods for this scatter correction are outlined in section 4.9. Section 4.9.1 describes a method by which the inscatter correction is achieved during the attenuation correction by using a lower effective value of the attenuation coefficient. This is an easy method and requires no extra work but by using these artificially lower values of the attenuation coefficient the cupping in the middle of the larger size objects will be present even after attenuation correction and it will produce the hot-rim effects on the image. Even if these can be tolerated, this method is only applicable where significant attenuation takes place. For example, in the case of Cs-137 in water contained in a small diameter container, there will not be much attenuation and the projection data may not need compensation for attenuation. In this case the use of the dual energy technique (section 4.9.4) was used. This method rests on the fact that the response of the detecting system to photons is similar irrespective of the source configuration. Thus we can have two energy windows; one for the full-energy peak and the other for the scatter region.

If we denote the scatter counts under the full-energy peak by \( C_p \) and the counts in the scatter window by \( C_s \) then we can write:

\[
C_p = C_s \times K
\]  

(6.4)

where \( K \) is a constant which can be pre-established by using an MCA to integrate the counts in the two windows. The ratio of \( C_s \) to \( C_p \) gives the value of \( K \). For windows chosen \( K \) was 0.5 which did not vary much with photon energy between between 140 keV (Tc-99m) and 662keV (Cs-137).
The value of $K$ was determined by using three phantoms, Cs-137 liquid source, Tc-99m liquid source and Tc-99m liquid mixed with sand and was found to be within $0.5(\pm 7\%)$ for each sample. Thus the dual energy window subtraction method using the mean value of $K$ is capable of estimating the scattered counts in the full-energy peak to approximately 7%. It is, therefore, possible to derive a signal essentially free of scattering components simply by recording the counts within two windows in the spectrum of the matrix under investigation. This method is general, it should work for a single energy, as well as a multi-energy spectrum equally well and moreover it is independent of the geometrical set-up of the scanning system, since $K$ is easily determined for the case in hand. The results have shown a significant improvement qualitatively in the images to which this method has been applied. Further studies are required to establish the quantitative advantage of this method. The simplicity of this technique allows for its use in almost every set-up in ECT.

Another method of scatter correction is reported by Sanders [1982]; this reconstructs the images of the projection data from the two windows separately and then the image for scatter counts is subtracted from the full-energy peak counts image. The resulting image is then supposed to be free of scattering effects. This method is loosely equivalent to our method with $K=1$, but it is slower process since two reconstructions are required; one for scatter counts and the other for full-energy peak events.

The addition of HP-9836C microcomputer to the SPECT scanner allows the collection of data in two channels, one for the full-energy and the other for scatter window. The counts in the scatter window can be subtracted for each raysum after multiplying with the factor $K$, and the net counts recorded by the microcomputer will be the full-energy peak window counts.
free of scatter counts. This will eliminate the need of two separate scans hence reducing the scan time.

6.6 DECAY CORRECTION

The data obtained by scanning objects containing radionuclides of short half-life has to be compensated for their decay. The last projection data will differ considerably from the first projection data if the scan time is longer or comparable to the half-life of the radionuclide. For example Tc-99m has a half-life of 6 hours and if the source is not very active then the scan time is normally greater than the half-life of Tc-99m. It was observed (as is shown in Figure 6.6) that the first projection data differs considerably from the last projection data. The greater the scan time the greater will be the effect of radionuclide decay. Therefore the scan data has to be corrected for decay. The expression for this can be derived as follows:

\[ N(t) = N(0) \exp(-\lambda t) \] \hspace{1cm} (6.5)

where \( N(t) \) = number of radioactive atoms left at time \( t \)
\( N(0) \) = number of radioactive atoms present at \( t=0 \)
and \( \lambda \) = the decay constant.

The half-life, \( T \), is related to the decay constant as

\[ T = \frac{0.693}{\lambda} \] \hspace{1cm} (6.6)

Therefore

\[ N(0) = N(t) \exp[0.693 \times t/T ] \] \hspace{1cm} (6.7)

A computer program was written, in FORTRAN-77, for correction of the projection data for decay. This decay correction software uses the above
equation and it includes the counting time for each raysum plus the time lost between each raysum in reading the data from the counter/timer and storing in the microcomputer memory and moving the object by the step length (2 seconds), the time lost between each projection in transferring the data from the microcomputer memory to the disk and turning the turn-table by the angular step length (2 seconds). The Tc-99m data for all the scans were corrected for decay because of its short half-life. All the other sources used have relatively higher half-life, Cs-137 (30 years), Cs-134 (2.2 years) and I-125 (60 days), so they do not require the decay correction. But to test the software the data from I-125 scan was used to correct for decay. There was not any significant difference in the corrected and uncorrected data because of the total scan time was much less than the half-life of I-125. If the total scan time is, say, 12 hours then the source will decay 1/120 times during this time, and if we have 100 cps in the beginning of the scan then at end of the scan we will have 99.42 cps, which is 0.57% change from the initial value.

Figure 6.7 shows the last projection corrected and uncorrected for decay. The difference in the corrected and uncorrected projection is remarkable and corrected projection is essentially identical to the first projection. This means that the computer software successfully corrects for the decay. The large difference in the corrected and uncorrected data shows that the decay correction is very important for short-lived sources.

6.7 SOLID ANGLE (OR GEOMETRICAL FACTOR) CORRECTION

The effect of solid angle on a point source and an extended source was studied by using the opposed collimated detectors arrangement as described in section 4.8.1. Two NaI(Tl) detectors, of 5cm x 5cm dimensions, were placed 70cm apart facing each other and the sources were
Figure 6.6
First and last projection without any correction.

Figure 6.7: The last projection corrected for decay.
moved inbetween the detectors. The outputs of the two amplifiers were normalised by placing a 370 kBq Cs-137 standard point source in the centre and varying the gains.

Figure 6.8 shows the count rates recorded, from a Cs-137 point source, at detector A, detector B and their arithmetic and geometric means. It can be seen from the figure that the count rates for each detector falls sharply as the source is moved away signifying a large count rate variation if only one detector is used.

30ml of Cs-134 liquid source in a petri dish of 45mm diameter, providing a thin plane circular source, was used as an extended source. The detector's separation was kept at 70cm and the source was moved inbetween the two detectors at 5cm intervals with its cylindrical axis aligned through the detectors. The count rates recorded at detector A, detector B and their arithmetic and geometric means are shown in Figure 6.9.

Comparing the two two figures, for point and extended source, it can be seen that the variations in the count rate for the extended source are much less than the point source. This is due to the fact that the efficiency of detection decreases with the inverse square of the source-to-detector separation. In the case of an infinite thin plane source of uniform activity the counts detected will be the same for different source-to-detector distances since the inverse square law fall-off in detection efficiency will be compensated by an increase in the area of the thin slab of activity seen through the collimator. For the extended source the decrease in count rate is much slower than that of the point source. The residual fall in count rate is due to increased path length for peripheral radiations which will then be attenuated increasingly. For a plane source of finite extent, the edge of the source comes into view at sufficient source-detector separation leading to the usual inverse square fall-off. As it is obvious from both the figures, the count rate variations can be minimised by taking the
Extended source detebtor B and their arithmetic and geometric means for an extended source. For a point source, detector B and their arithmetic and geometric means.

Figure 6.9: Count rate variation detected at detector A.

Figure 6.8: Count rate variation detected at detector A, for a point source.
arithmetic or geometric means of the opposed detector counts. In the case of a point source the geometric mean exhibits minimal variation in the count rate and in the case of an extended source the arithmetic mean shows the smaller variation in the count rate.

In SPECT systems we are mostly dealing with extended sources; for example, in medicine the radionuclide present in an organ, in industry the burn-up studies of fuel rods etc. But occasionally we come across the problem of point source activity measurements where the average of the two opposing detectors can be useful.

With our SPECT scanner the object to be scanned is placed on the turn-table which moves laterally in front of the detector collimator at a fixed distance. Therefore the variation in the count rate is not expected unless the activity is not uniformly distributed when for each projection (as the object is rotated for each projection) the count rate will vary. For that case the scan is carried out over 360 degrees rotation when using a single detector and the conjugate raysums are averaged by taking either arithmetic or geometric means. For a uniform object the arithmetic and geometric means will be the same as the two raysums from opposite views will be the same. A plot of a typical projection from a uniform activity circular source can be seen in Figure 6.6.

The solid angle compensation was carried out by including the choice of arithmetic or geometric means in the attenuation correction computer software.

6.8 ATTENUATION COMPENSATION

The attenuation of the photons inside the radioactive object is the main problem in SPECT. This attenuation can be assumed constant if
the object has uniform density. This assumption makes the attenuation correction process quite easy and attenuation correction can be done by using any of the methods described in section 4.8.2.1. It becomes more complicated for the non-uniform attenuation, i.e. when the density of the object is not uniform. In that case the method of attenuation correction described in section 4.8.2.2 has to be used for accurate results. This requires knowledge of the attenuation coefficient distribution in the object which is obtained by a separate transmission scan of the object with an external photon source of comparable energy to that of the emitted photons by the object itself. This increases the scan time and the computing time because of the dual reconstruction, one for the transmission and one for the emission case.

For trials of attenuation correction methods both the techniques were used. In the case of uniformly distributed sources, like Tc-99m solution in water or I-125 solution in water, the attenuation coefficient was assumed to be constant as the liquid has uniform density. For this case the methods described in sections 4.8.2.1(b) and 4.8.2.1(c) were employed. The scans were performed for 360 degrees rotation. A computer program, in FORTRAN-77, has been developed for attenuation compensation using a constant attenuation coefficient. This attenuation correction software includes the solid angle (or geometrical factor) correction as described in section 4.8.1 together with attenuation correction. The geometrical factor correction includes the choice of arithmetic or geometric mean, and the attenuation correction includes the choice of hyperbolic sine correction (equation 4.69) or the average of the exponential factors correction (equation 4.74) techniques. Since the phantoms used were of symmetrical shape the chord length (L) in equations 4.69 and 4.74 can easily be computed, as the step length is altered, by using Pythagorus' theorem.

Consider a circular object of radius R in Figure 6.10.

We can write that
Figure 6.10: Calculation of the chord length ($L$).

\[ R = \left( \frac{L}{2} \right)^2 + S \]  
(6.8)

Therefore
\[ L = 2(R - S)^{\frac{1}{2}} \]  
(6.9)

and
\[ S = R - n \ell \]  
(6.10)

where $n$ is the number of steps and $\ell$ is the step length which is always incremented by the same value to calculate the next chord length until the whole object is covered. The value of $L$ is calculated from equation 6.9 for each step increment and is substituted in either equation 4.69 or equation 4.74 according to the choice of the attenuation compensation procedure. The decay corrected data (if required) is corrected for attenuation before the reconstruction process.

The edge of the source is defined where the raysum count falls by 90% from one ray to the next. This criterion was found to correspond when the collimator was aligned on the actual edge of the source. The attenuation correction program searches for the edge of the source and from this edge the calculation of the source thickness begins. This threshold was tested by comparing it with the known source radius and was found to work reliably. Providing the source dimensions to the computer software makes the attenuation correction process faster but it is only
applicable when the source dimensions are known which is often not the case. So for the cases where the source is not coincident with the absorber boundary, the edge of the source has to be deduced by actual measurements. Since, for the experimental work in SPECT with phantoms, the dimensions of almost all radioactive objects were known so to make the correction process faster, the actual source radius was given to the correction program.

For the case of variable attenuation coefficient the iterative techniques (described in section 4.8.2.2), using the RECLBL library routines, were employed. Two subroutines of this library used were GRADY, which uses the gradient method, and CONGR, which uses the conjugate gradient method for step calculations. These two methods were used to reconstruct the simulated data for the constant and variable attenuation coefficients. Several scans were simulated for different -scan parameters and the reconstruction was performed. Some scans are discussed here. The simulated scan parameters are given below.

- Number of steps = 100
- Number of projections = 72 (over 360 degrees)
- Step length = 1mm
- Angular spacing = 5 degrees

The results from these methods are shown in Figures 6.11 through 6.16. Figure 6.11 shows the reconstructed image of simulated data for Tc-99m without attenuation correction and a line scan through this image is shown in Figure 6.12. Figure 6.13 shows the image of the same data corrected for constant attenuation coefficient of 0.15 cm$^{-1}$ (for 140 keV photons) with the conjugate gradient method and the line scan through this image is shown in Figure 6.13. The difference between the two images is clearly visible in the plot of the line scans. Figure 6.14 shows the attenuated corrected reconstructed images of a non-uniform
Figure 6.11: Reconstructed image of simulated emission scan data using iterative method.

Figure 6.12: A line scan through the above image.
Figure 6.13: Reconstructed image of the same data corrected for constant attenuation coefficient.

Figure 6.14: Line scan through the above image.
Figure 6.15: Reconstructed image of the same data corrected for attenuation using variable attenuation coefficients.

Figure 6.16: A line scan through the above image.
attenuation object containing Tc-99m with the same method and a line scan through this image is shown in Figure 6.16. Comparing Figures 6.13 and 6.15 we can see that there is not any significant difference between these two. This is due to reason that the simulated phantom was for Tc-99m solution which is supposed to be of uniform density. This means that for a uniform density object the constant attenuation can be assumed.

For the constant attenuation coefficient case only one scan data is given for the emission scan while for the variable attenuation case both transmission and emission scan data has to be provided. The same program can be used for both the cases. The algorithm first calculates the attenuation coefficients for each pixel from the transmission scan and then calculates the attenuation factors for each pixel from the attenuation coefficients and the length of the chord L, and stores it in a file called LUNATIN. This file is kept open for reading and writing of the calculations during the computational process. The emission data is then corrected using the corresponding attenuation factors and corrected projections are then reconstructed using the iterative reconstruction technique.

The reconstructed images of the experimental data from the SPECT scanner for different sources has been shown and discussed under their respective headings.

6.9 EMISSION SCANS OF POINT SOURCES

The scans of point sources were carried out to test the performance of the SPECT scanner as well as the reconstruction technique used, for the point sources. A few scans were done of different point sources. Some are discussed below.
The first scan was done of a 370 kBq Am-241 standard gamma-ray "point" source. It was placed on the turn-table and was scanned by setting the window on the full-energy peak of 59.6 keV photons. The diameter of this source as well as the other standard point sources is approximately 1 mm. The detector collimator was a 1 mm x 10 mm slit and 10.5 cm long. The scan parameters are given below.

- Number of steps = 20
- Number of projections = 30
- Step length = 1 mm
- Angular spacing = 6 degrees
- Counting time per raysum = 120 seconds

The whole scan took 20 hours to complete. The reconstructed image of the Am-241 point source and its three dimensional plot is given in Figure 6.17 (a) and (b) respectively. The image of the point source is clearly visible in the two displays.

The second point source used was a 370 kBq Ba-133. The detector collimator length was reduced to 5.2 cm to reduce the source-to-detector distance. The scan parameters were kept the same as for Am-241 except that the counting time per raysum was reduced to 20 seconds. The scanning was done twice, first for 278 keV energy and second for 303 keV photons. Figure 6.18(a) shows the reconstructed images and the line scans through the images for 303 keV (upper) and 278 keV (lower). The 3-dimensional plots of the images are displayed in Figures 6.18(b) and (c) for 303 keV and 278 keV scans respectively. The pixel size is 1 mm x 1 mm and the total pixels are 324 (18 x 18). The reason for both the images appearing on one side of the image plane is that the source was not properly aligned with the collimator. From inspection of these two figures a few deductions can be made. First, as expected, the magnitude...
Figure 6.17 Am-241 point source image and 3-D plot.

Figure 6.18 Ba-133 point source image and 3-D plots
of the z-axis (scale renormalised automatically to peak value of the
303 keV photons is higher than that of the 278 keV photons. This is due
to the intensity of 303 keV photons (19.7%) being higher than that for
278 keV photons (7.4%). However, the ratio of their intensities do not
match the ratio of their z-axis magnitude. This can be explained by
noting that the Compton scattered events in the detector due to the 303,
356 and 384 keV photons contribute to the counts in the 278 keV
full-energy peak region, increasing the counts in that region, whereas
only that of the weaker 356 keV and 384 keV photons contribute to the
full-energy peak region of 303 keV. Compton tails from higher energy
events are counted in the full-energy peak window due to the relatively
poor resolution of the NaI(T) detector used with the SPECT scanner. The
poor energy resolution of the detector requires a wider energy window.
If the SCA is replaced by an MCA the net peak counts can be extracted to
eliminate these effects.

Another feature is that, even though the source used was a point source
of diameter approximately 1mm both three-dimensional plots show that the
images produced a base of dimensions 8mm x 8mm in size. This is much
bigger than the base of the Am-241 image (Figure 6.17(a) which turns out
to be only 2mm x 2mm in size. This effect can be attributed to the
length of the collimator that differs in the two scans. The expression
for the effect of collimator length on the spatial resolution can be
derived from Figure 6.19 as under.
The spatial resolution is given by
\[ d = \text{FWHM} \]
From the above figure it can be written as
\[ d = GJ = GH + HI + IJ \]
\[ d = 2GH + CE \]
Now since
\[ AC = 2GH \]
so
\[ d = AC + CE \]
Figure 6.19: Trapezium approximation to point source response function.

Now

\[
\frac{AC}{CE} = \frac{b-a}{a}
\]

and

\[CE = 2r\]

so

\[d = \frac{2rb}{a}\]  

(6.11)

where \(r\) is the collimator radius, \(b\) is the source to detector distance and \(a\) is the length of the collimator.

In the above two cases the length of collimator for Am-241 scanning was twice as that for Ba-133 which means that, given a similar collimator radius \((r)\) and source to detector separation \((b)\), the resolution \((d\) or FWHM) is inversely proportional to the length of the collimator \((a)\) as can be seen in equation 6.11. Secondly the energy of the source is increased from 59.6 keV to 278 and 303 keV. The higher energy gamma-rays will penetrate the edges of the collimator, as has seen in section 5.4.3 which will slightly increase its effective diameter. This increase in
energy will also increase the possibility of forward Compton scattering into the detector \textit{even if} the point source is not directly facing the collimator's entrance hole. Scatter causes some loss of image contrast, slight blurring of the edges and some increase in the apparent radioactivity (Oppenheim, 1984). These factors can be seen not only in Figures 6.18 but also in the images of other sources as well in which the edges of the image are not upright (as the physical feature of the plastic containers suggests) but slanting slightly. There is also an apparent increase in radioactivity at the periphery of these images.

6.10 Cobalt-57 in a Steel Collimator

This scan was carried out to test the capability of the SPECT scanner in scanning a source embedded in a container which is highly attenuating except for a small region. A 370 kBq Co-57 source which emits photons of energies 14 keV (9\%), 122 keV (87\%) and 136 keV (11\%) is fixed into one end of a cylindrical steel collimator of dimensions 15mm diameter and 50mm long. The 122 keV photons were used in this trial. The source collimator diameter was 1mm and this source collimator combination is normally used for TCT work. The detector collimator used was a 1mm diameter circular collimator. The scan parameters are given below:

\begin{align*}
\text{Number of steps} & = 60 \\
\text{Number of projections} & = 60 \\
\text{Step length} & = 1\text{mm} \\
\text{Angular spacing} & = 3\text{ degrees} \\
\text{Counting time per raysum} & = 20\text{ seconds}
\end{align*}

The reconstructed images of Co-57 are shown in Figures 6.20(a) and (b) respectively. It can be seen that the source is located at one end of
Figure 6.20. Image of Co-57 in a steel collimator and its 3-D plot.

Figure 6.21. Image of Kr-85 in perspex and its 3-D plot.
the collimator rod in order to produce a fine beam of photons. Comparison can be made with Figure 6.20(c) which shows the sketch of the source assembly. Figure 6.20(c) shows a somewhat T-shape configuration. The tail effect may be due to some photons that are able to penetrate the sides near the source as well as the short end of the collimator through the steel container.

![Collimator Sketch](image)

**Figure 6.20(c):** Sketch of Co-57 source assembly.

This test image proves that the emission scanner and the reconstruction technique used are capable of identifying the position of the source although part of the object in which the source is embedded allows many photons to emerge while its other part highly attenuates them. It suggests that sources embedded in uneven and highly attenuating materials can be scanned and located.

### 6.11 Krypton-85 in Perspex

A 7.4 MBq Kr-85 radioactive gas contained in a perspex screw, 15mm diameter and 25mm long, was scanned. The window was set on the 514 keV full-energy peak. A 1 mm wide and 10cm long slit collimator was made for the detector by putting lead slabs together. The scan parameters are given below.

- Number of steps = 40
- Number of projections = 30
- Step length = 1 mm
- Angular spacing = 6 degrees

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The reconstructed image and its 3-dimensional plot are shown in Figures 6.21(a) and (b) respectively. It can be seen that the radioactive gas which is invisible physically, is visible in the reconstructed image quite clearly. In other words the emission scanner is quite capable of identifying the radioactive gas inside another medium. This example is an unusual example in that the self-absorption within the radioactive medium is negligible, although some attenuation occurs in the surrounding encapsulation.

6.12 Tellerium-123m Liquid Source

This scan was one of the first few scans performed when the emission scanner was still in the development stage. A liquid Te-123m source which emits 90 keV and 159 keV photons was filled into a 50mm diameter plastic bottle. The ND66 multichannel analyser (described in section 2.5.2) was used with the scanner to collect the data instead of the SCA and the counter/timer. The data were collected from the two full-energy peak regions by setting the region of interest on the two full-energy peaks at 90 keV and 159 keV, in order to test the scanner performance for multi-energy scans. The scan parameters are given below.

Number of steps = 80
Number of projections = 30
Step length = 1mm
Angular spacing = 6 degrees
Counting time per raysum = 60 seconds.

The detector collimator used was a 1mm diameter lead collimator. The phantom was placed on the turn-table 150mm away from the detector. The
reconstructed images for 90 keV and 159 keV photons are shown in Figure 6.22(a) upper and lower, respectively. The difference in both the images can be clearly seen from the respective line scans through the image. The cupping in the middle of the 159 keV image is less than the 90 keV photons image. This is due to the higher energy of the former which results in less attenuation. The isometric plots of the images are shown in Figure 6.22(b) and (c) respectively. The cupping in the middle of the images can also be seen in the 3-dimensional plots.

This experiment demonstrates that this scanner set-up is capable of performing multi-energy scans. However the ND66 MCA is a costly machine which is being used for long and low level counting mainly with a Ge(Li) detector. The ND66 MCA can be replaced with two single channel analysers and two counter/timers to collect data in two separate channels from the two different energy regions. The recent version of the emission scanner controlled by the HP-9836C microcomputer has the capability of doing multi-energy scans.

A multiple energy technique to correct for attenuation is described in section 4.8.2 and if a scanner is capable of doing multi-energy scanning then such a correction technique with additional mathematical manipulation can be used for attenuation correction. Alternatively, a simultaneous emission and transmission scan using two energy sources can be carried out with one channel measuring the counts from the transmission source and other channel measuring the counts from the emission source. Then the compensation for attenuation is carried out as described in section 6.8. However, since the energy of the two sources will be different, exact compensation will not be achieved unless results are extrapolated to the energies of interest. The advantage of simultaneous transmission and emission scans is that the total scan time will be reduced compared to the two separate scans, and also the problem of repositioning the sample for successive scans is avoided. Normally
Figure 6.22. Images of Te-123m liquid source and their 3-D plots.
the scans in TCT are performed to higher statistics for adequate accuracy but in this case the transmission scan is performed to provide the attenuation coefficients for correction and lower statistical pression is adequate in this case unless the corrections are very large.

The same dual channel counting system can be used for scatter correction by collecting the scatter counts in the second channel and subtracting them, after normalising by an appropriate factor, from the full-energy window counts.

Multi-energy scanning is equally important in TCT and Compton scattering tomography. A multiple energy photon source can be used for elemental analysis of the scanned object in transmission tomography because the various constituents of the object will pose different attenuation for different photon energies. A problem that always arises is that of the Compton components of the higher energy photons which underly the full-energy peaks of the lower energy photons. Compton scattering tomography will also benefit from multiple energy scanning since several windows can be used to bracket different regions on the Compton spectrum to increase the data rate by using an uncollimated detector. Multi-energy transmission scans can also be used to find the attenuation coefficients of each pixel as a function of energy and the data can be interpolated to find the values for intermediate energies.

6.13 Soil Sample Scans

Some radioactive soil core samples were obtained from the Radiation Laboratories of the Physics Department where they had previously been used in radionuclide protection measurements. These samples now contained mainly Cs-134 since the original short-life nuclides had decayed to a low level. A few samples were tested and a
relatively active sample was chosen for scanning. The plastic sample container diameter was 160mm. Several scans were carried out with different scan parameters and at different positions across the soil column, and three scans will be discussed here. These scans were carried out by windowing the 604 keV, 796 keV full-energy peaks and scattering region (valley) next to the 604 keV full-energy peak. The detector included a 2mm x 20mm lead collimator. These scans were carried out at the same positions and with the same scan parameters which are given below.

<table>
<thead>
<tr>
<th>Number of steps</th>
<th>= 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of projections</td>
<td>= 60 (over 360 degrees)</td>
</tr>
<tr>
<td>Step length</td>
<td>= 2 mm</td>
</tr>
<tr>
<td>Angular spacing</td>
<td>= 6 degrees</td>
</tr>
<tr>
<td>Counting time per raysum</td>
<td>= 60 seconds</td>
</tr>
</tbody>
</table>

The counting time was chosen to be 60 seconds because the sample was not very active and the one scan took about 104 hours to complete. Figure 6.23(a) shows the reconstructed images for 604 keV (upper) and 796 keV (lower) peak windows. Figure 6.23(b) shows the reconstructed images for the scatter window (top left), for 604 keV (top right), for 796 keV (bottom right) and the difference of the 604 keV image and the scattered photon image is shown in the bottom left. From the images it is clear that the Cs-134 is concentrated in one half of the sample and the hot-spot is clearly visible in all the images. The scatter correction does not appear to have improved the image quality since the resulting image shows more spread of activity than the uncorrected image. This implies that the simple image subtraction method of scatter correction is not an effective method. However the electronic selection scheme used did not allow the Compton interference from the 796 keV spectrum to be subtracted from the 604 keV window and since the Compton edge of 796 keV occurs at 603keV it is a particularly important correction in this case.
Figure 6.23. Images of soil sample

Figure 6.24. Images of Cs-137 liquid source.
6.14 Caesium-137 Liquid Source

The main problem with this source was the shielding of the source for safety reasons and the collimation of the detector as described in section 5.7.3. As has been seen in chapter 5 the detector collimator has to be thick enough to prevent the septa penetration. The big jacket type slit collimator of lead (Figure 5.3) was used for this source. A lead wall was constructed for shielding of the source by putting lead bricks together.

A caesium-137 chloride solution in water containing 5.18 MBq in 200ml of solution in a plastic bottle of diameter 76mm was used. Several scans were carried out with different scan parameters and with different energy windows. Some of the scans are discussed below.

The first scan was carried out with the 662 keV gamma photons. The 662 keV full-energy peak was bracketed with the SCA and the counts were collected for a fixed time. The scan parameters for this scan are as follows:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Steps</td>
<td>70</td>
</tr>
<tr>
<td>Number of Projections</td>
<td>90 (over 360 degrees)</td>
</tr>
<tr>
<td>Step length</td>
<td>1.5 mm</td>
</tr>
<tr>
<td>Angular step length</td>
<td>4 degrees</td>
</tr>
<tr>
<td>Counting time per raysum</td>
<td>20 seconds</td>
</tr>
</tbody>
</table>

The scan data was corrected for background radiation as described in section 6.4 and the data was then reconstructed using the filtered back-projection method.

For scatter correction another scan was carried out by windowing the
scatter region (420.8 keV - 594 keV). The scan parameters were kept the same as for the full-energy peak scan. The scatter correction was done by subtracting the scatter region counts, multiplied by the factor K=0.5, from the full-energy counts prior to the reconstruction and the reconstruction was done with the corrected data. The reconstructed images of the uncorrected full-energy peak scan and scatter events scan are shown in Figures 6.24(a) top left and bottom right, respectively. The reconstructed image of the data from the scatter region scan is quite interesting. It is related to the Compton scattering tomography, and work on this branch of tomography is being carried out in this department [Balogun, 1986]. The scatter corrected image is shown in the same figure bottom left. The image quality has certainly improved with the scatter correction.

An interesting thing was noted during the correction processes. Before starting the scan the radioactive object is moved on the turn-table until it is out of the line of sight of the collimator so that for the first few steps the object is not in front of the collimator. The counts collected when the object is not in front of the collimator hole will be due to background or due to transmission through the collimator face or due to scattering from other equipment in the surroundings. We took the average of the first and the last raysum of each projection and subtracted this count from each raysum of the corresponding projection. The data was then reconstructed without any other corrections. The resulting image by this method is shown in the top right of the same figure. The spurious hot-rim effect shown in the reconstructed image of full-energy image has been removed by this method. This technique was then applied to almost all scan data. This is an easy technique and does not require any dual scan for background and scatter correction or any extra electronics for dual channel counting.

Another scan of the same phantom was carried out with the 32 keV
characteristic X-ray peak. The detector collimator and the scan parameters were kept the same except that the counting time was increased to 30 seconds to get more counts. Figure 6.24(b) shows the reconstructed images of scans with 662 keV full-energy peak window (upper) and with the 32 keV energy (lower) along with their respective line scans. The 32 keV scan data was corrected using the constant value of attenuation coefficient for water at this energy (0.027 mm⁻¹).

6.15 Technetium-99m Liquid Source

Technetium-99m is widely used in nuclear medicine and for our experimental work this source was obtained from St. Luke's hospital in Guildford quite a few times. Tc-99m emits several gamma-ray energies but the widely used emissions are the 140.5 keV photons.

A phantom was made of 424 MBq Tc-99m in 220 ml water contained in a plastic container of diameter 116mm. The jacket type 1.5mm x 20mm slit lead collimator was used for detector collimation. The scan parameters are given below.

\[
\begin{align*}
\text{Number of steps} & = 90 \\
\text{Number of projections} & = 120 \text{ (over 360 degrees)} \\
\text{Step length} & = 1.5 \text{ mm} \\
\text{Angular spacing} & = 3 \text{ degrees} \\
\text{Counting time per raysum} & = 4 \text{ second}
\end{align*}
\]

The whole scan took about 15 hours to complete while the actual total time was 12 hours. The additional 3 hours to complete the scan caused by the time required to step the sample between each raysum and between each projection as described in section 6.6. This extra time together with the actual counting time is taken care of in the decay correction.
Figure 6.25. (a) & (b) Tc-99m images corrected for attenuation.

Figure 6.25 (c)&(d) Tc-99m images corrected for different attenuation coefficient values.
Figure 6.26: Three-dimensional plots of Tc-99m
(a). Corrected for decay but not corrected for attenuation.
(b). Corrected for decay and attenuation.
software. The reconstructed image of decay corrected (but uncorrected for attenuation) projection data is shown in Figure 6.25 (a) (lower). The reconstructed image depicts that there is less activity in the central regions of the source as compared to the outer regions. The line scan through the image shows the cupping towards the centre of the source which is due to the greater attenuation of the photons that come from the inner parts of the source.

The attenuation corrections were carried out using the methods described in sections 4.8.2.1(b) and (c). A constant attenuation coefficient was assumed for correction since the source was of uniform density. A linear attenuation coefficient of $0.015\text{mm}^{-1}$ was used for 140keV photons in water [Hubbell, 1969]. The resulting decay and attenuation corrected reconstructed images are shown in Figure 6.25 (a) (upper) using the geometric mean corrector (sine hyperbolic correction method) and in Figure 6.25 (b) (upper) using the average of the exponential factor method. The 3-dimensional plots of the uncorrected and corrected for attenuation images are shown in Figures 6.26 (a) and (b) respectively. There is a remarkable difference between the two images. The line scans through the images, on their respective rights, show the difference quite clearly. It is clear from the two figures that there is not much difference in the two attenuation correction methods. This is due to the fact that the phantom was symmetrical and the activity was uniformly distributed so the arithmetic and geometric means will be the same. Therefore a simplification can be introduced when scanning an object which is known to be symmetrical since the projection $A_1$ and its conjugate $A_2$ will be the same and hence 180 degrees scan is sufficient.

The scatter correction was also carried out using smaller values of the attenuation coefficient as described in section 4.9.1. The resulting images are shown in Figures 6.25 (c) for $0.014\text{mm}^{-1}$ (upper) and $0.013\text{mm}^{-1}$ (lower), and in Figure 5.25 (d) for $0.015\text{mm}^{-1}$ (upper) and
0.0135 mm\(^{-1}\) (lower) attenuation coefficient values. The images are very well defined and apparently there is not much difference between them for these values of attenuation coefficients. But for lower values the hot-rim effects will start appearing, as is slightly evident in the \(\mu = 0.13\) mm\(^{-1}\) image.

Apart from the experimental scan data, computer simulated phantom data for Tc-99m was generated for a uniformly distributed source in tissue equivalent material. This simulated data was then corrected for attenuation for both the constant attenuation and variable attenuation case. In the latter case the simulated data for the transmission scan was also generated. In the constant attenuation case the algorithm first calculates the attenuation factors using the chosen attenuation coefficient and then multiplies the emission data with the corresponding attenuation factors for each pixel. In the variable attenuation case the transmission scan data is first reconstructed to determine the attenuation coefficients for each pixel, then the attenuation factors are calculated for each pixel using the corresponding attenuation values and then the emission data is passed to the algorithm. The reconstruction algorithm then weights every projection according to their respective attenuation factors and the reconstruction is performed using the corrected data. This method is an ideal method for both the uniform and non-uniform objects. But it needs a lot of computing time as well as a large computer memory for attenuation coefficient storage, the calculation and storage of the attenuation factors before weighting each projection with the corresponding attenuation factors followed by reconstruction of the corrected data. The computing time can be considerably decreased by choosing fewer iterations and if a lower accuracy is tolerable then uniform attenuation can be assumed which will decrease the computing time as well as the memory requirements.

If the object to be scanned is symmetrical and the activity in it is
uniformly distributed, then the simple and quick correction methods (as used in this work) along with the filtered back-projection reconstruction techniques give reasonable results.

6.16 Iodine-125 Liquid Source

Iodine-125 is a low energy source and emits 35 keV (7%) and 27–32 keV (138% tellurium K X-rays). This source was obtained from the Biochemistry Department of this University in the amount of 37MBq in 0.5ml of solution. This was diluted by adding de-ionised water to make different phantoms, including the 3-source phantom, constructed for the optimisation of scan parameters as has already been discussed in section 6.3. The remaining 14.8 MBq of this source was diluted in 200ml of water in a plastic bottle of diameter 76mm. This phantom was then scanned by bracketing the 27 keV X-ray peak. Several scans were performed of this phantom with different scan parameters. A selection of them will be discussed here. The scan parameters for the first scan are given below.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of steps</td>
<td>60</td>
</tr>
<tr>
<td>Number of projections</td>
<td>90 (over 360 degrees)</td>
</tr>
<tr>
<td>Step length</td>
<td>1.5 mm</td>
</tr>
<tr>
<td>Angular spacing</td>
<td>4 degrees</td>
</tr>
<tr>
<td>Counting time per raysum</td>
<td>10 seconds</td>
</tr>
</tbody>
</table>

The detector collimator used was the 1.5mm x 20mm jacket type lead collimator. The whole scan took about 17 hours. The data was then corrected for decay, using the reported half-life of 60 days but the corrected and uncorrected data were almost the same as described earlier in section 6.6. The attenuation compensation was done by using equation 4.74 and assuming a constant attenuation coefficient of 0.027 mm⁻¹ throughout the phantom. Figure 6.27 (a) shows the images
Figure 6.27. Images of I-125 liquid source, attenuation corrected and uncorrected.
corrected (upper) and not corrected (lower) together with their line scans (right). The difference between images corrected and uncorrected for attenuation is clearly visible.

Ideally in a uniformly distributed source with no attenuation, the activity per unit volume throughout the source will be constant. This is proved in the line scan through the attenuated corrected image. The line scan shows a lot of spikes which arise due to the poor statistics involved in this scanning. However, if the pixel data is averaged over groups of pixels, it can be deduced that the whole source is almost uniform.

In Figure 6.27 (b) the effect of smoothing on the attenuation corrected image is shown. The smoothed (upper) and not smoothed(lower) images along with their line scans (right) are shown. Although the smoothing is not perfect but the difference in the two images can be seen in the respective line scans (The difference in the shapes of lower and upper quadrants of the picture is due to the pre-defined shapes of different quadrants in the image analysis program).

Another phantom using this source was made by putting all of the I-125 solution in a larger container of diameter 116 mm. The detector collimator was changed to a 5mm diameter circular brass collimator, made for 59.6 keV photons from Am-241 for TCT work. The scan parameters are given below.

Number of steps $= 30$
Number of projections $= 120$ (over 360 degrees)
Step length $= 5$ mm
Angular spacing $= 3$ degrees
Counting time per raysum $= 5$ seconds
Figure 6.28: Three-dimensional plots of I-125 source
(a). Not corrected for attenuation
(b). Corrected for attenuation.
Figure 6.27 (c) shows the reconstructed images of the phantom corrected (lower) and uncorrected (upper) for attenuation together with their line scans. The line scans through the images are less spiky as compared to the line scans through the above images. This can be attributed to the fact that collimator width is changed from 1.5mm to 5mm and so the area of detection has increased which has increased the counts and so the statistics has improved. The 3-dimensional plots of the images are shown in Figure 6.28 (a) and (b). The plot of the uncorrected image shows a dip in the middle at the top while the corrected image plot shows the middle portion is as high as the edges region.

6.17 CONCLUSION

The results presented in this chapter demonstrate the performance of the SPECT scanner and the different correction and reconstruction techniques. The scanner in its present form is capable of distinguishing and localising different types of radioactive sources in different absorbing media. The reconstructed images of point sources as well as that for a radioactive gas tells the success story of the SPECT scanner and the reconstruction algorithms. The effect of different collimators, their thickness and width, as well as the effect of the number of projections, counting time and number of steps was investigated in this chapter.

The results show that the effect of number of projections is small or negligible, as long as the number of projections is greater than or equal to the number of raysums per projection. The number of steps depends on the collimator width. Therefore the collimator dimensions effect the spatial resolution considerably and for better spatial resolution small width collimators are advisable if the count rate penalty is tolerable. But as stated earlier good spatial resolution is generally less important
In ECT the requirement of better spatial resolution is very much demanded because it is concerned with the detailed structure of the object being scanned while in ECT the presence, localisation and concentration of activity is determined over relatively large sub-regions (organs etc). So for ECT work the larger width collimators can be used. The thickness of the collimator should be sufficient to stop the transmission of radiation through the collimator walls to an adequate extent.

The corrections for inscatter, for decay of the short-lived radionuclides and for matrix attenuation of the photons reveal a remarkable difference between the corrected and uncorrected images and show that proper corrections should be made where applicable. For example for Cs-137 in water in a small phantom does not need substantial correction for attenuation when scanning with the 662 keV photons, but there will be more Compton inscattering due to the higher energy. So the scatter correction may be applied to this case, although there is not much difference in this method the one we used for subtraction of counts due to general background and collimator "leakage" radiations. In fact the latter method is more convenient and saves time as well as reducing the need for extra electronics.

A constant attenuation coefficient can be assumed for homogeneous objects and is easily applied where the activity distribution is uniform and this simple and quick attenuation correction method with filtered back-projection reconstruction algorithms is generally adequate. For objects of non-uniform attenuation, the dual scan method should be used to provide an accurate map of attenuation coefficients.

Increasing the number of counts recorded gives better statistical images. This can be achieved by increasing the collimator width, increasing the
counting time or using highly active objects. For industrial applications where longer measurement times can be tolerated, this can be done by increasing the counting time. In the case of fission products inside intensely irradiated nuclear fuel elements, narrow collimators and short counting times can be used to provide high resolution multi-isotopic images of good statistical accuracy in counting times of reasonable duration.
The spatial localisation of radionuclides in absorbing media can be deduced from external measurements of the radiations that reach the surface of the absorbers. The depth of a pure beta-emitter can be determined by studying the changes in the spectrum of beta-rays emerging from thin layers of material. But for the case where the thickness of the absorber between source and detector is greater than the range of beta-particles then the depth can be determined by external measurements of bremsstrahlung which is produced within the absorber. The shift in the intensity maximum of bremsstrahlung with increasing thickness of absorber presents an accurate method for in-vivo determination of depth of a pure beta-emitter by external measurements without disturbing the object. Results from experimental studies highlight that the position of the intensity maximum in the bremsstrahlung spectrum is a function of the energy of beta-particles and the depth of the source inside the absorbing medium.

Initially the shift in the intensity maximum in bremsstrahlung spectra for P-32 and Kr-85 varies at a similar rate, but as the attenuation continues, after the softer parts of the initial spectrum have been removed the remaining spectrum stabilises around a fairly constant maximum and the energy shift becomes less pronounced. This flattening off is reached at a lower value of water thickness for Kr-85 since it has a lower mean energy than P-32.

The experimentally determined bremsstrahlung spectrum is of great practical importance. The position of maximum intensity can provide a means to determine the depth of beta-emitters and knowing the position of the source, its activity can be determined which is of great importance in
many industrial and medical applications. This method can efficiently be used in low Z media and the maximum depth of the beta-emitter which can be estimated by this method is a function of beta-particle energy and the degree of attenuation.

Scatter-to-peak ratio (SPR) and peak-to-peak ratio (PPR) versus thickness of the absorber present two methods by which the depth of a photon source in an absorber can be determined from external measurements of counts in different regions of the energy spectrum of the emergent photons at the surface of the absorbing medium. For the SPR method two scatter windows were chosen, one over the valley just below the full-energy peak and the other around the Compton edge of the spectrum. It was found that the integrated counts in both the scatter regions increase with the source depth, due to the presence of more scattering material inbetween source and detector, while in the full-energy peak they decrease with depth, due to the increase in the attenuation of photons with the increased thickness of absorber. The PPR method (or differential attenuation technique) is only applicable to multi-energy photons sources or in the case of two or more radioisotopes present at the same depth in the absorber.

SPR(V) varies more rapidly with depth than SPR(C) so it can be concluded that SPR(V) versus absorber thickness is the method of choice for mono-energetic photon sources situated inside absorbers.

The values of both SPR and PPR are lower when using a collimated detector. This is due to the fact that the amount of multiply scattered photons detected is reduced with the collimated detector because of the limited field of view, while with low or no collimation the amount of multiply scattered photons detected will be larger. This means that with a highly collimated system the amount of multiply scattered photons detected can be reduced and in case of a perfect collimation only the
full-energy peak and $180^\circ$ backscatter peak would be detected. That is why the values of SPR are higher for the uncollimated detector and relatively lower for the collimated detector, and for the same reasons the variations in the PPR are relatively higher than SPRs for the collimated detector. The change in PPR with the degree of collimation might be due to the Rayleigh scattering which is more important at low energies, but collimation reduces this inscatter component hence reducing the counts in the lower energy peak.

The PPR method is more sensitive for pairs of peaks with large differences in their energy. This is due to the generally more rapid absorption of low energy photons with increasing thickness of absorber, so decreasing the counts in the lower energy peak, and hence the PPR changes with depth. More generally the "softer" of the two photon energies is attenuated more rapidly, so that above the pair production threshold (1.022 MeV) the higher energy peak is attenuated more rapidly.

The values of SPR and PPR for water and tissue equivalent rubber, for the same source and detector geometry and for same absorber thickness, are almost equal. This is because water is also closely tissue equivalent. The results for an extended source, a Cs-134 volume source, are not very different from those with point sources. This is due to the fact that the source dimensions were small compared to the absorber dimensions.

From the results presented in chapter 3 it can be concluded that for a particular detector and particular source-detector geometry, SPR and PPR techniques can be used to successfully measure the depth of a photon source inside attenuating media. The results of SPR and PPR are specific to the detector and geometries that were employed, but these do enable depth to be determined by comparing the values of SPR and PPR with values obtained under similar conditions. Knowledge of the depth at which a
radioisotope is residing will enable the quantity present to be determined with improved accuracy.

With the current development of emission imaging systems, SPECT is now widely used as an important tool in nuclear medicine and is extending its use in nuclear industry. However, its widespread use outside the medical field is limited by the cost and lack of portability. In nuclear industry interest is growing in employing tomographic techniques for non-destructive testing of burnt fuel rods and for the scanning of encapsulated radioactive waste in order to locate, identify and quantify centres of activity. The Surrey transmission gamma-scanner has been modified to perform as a SPECT scanner and in its present form is capable of functioning in both transmission and emission modes, by minimal alteration to the set-up. This SPECT scanner can also perform as a multi-energy scanner by employing dual-channel counting using the HP-9836C microcomputer or by the incorporation of an MCA into the system.

The characteristics of the SPECT scanner have been studied. The energy of the radiations from the radioactive object to be scanned should be taken into account when designing the detector collimator. For photon energies around 60keV the brass and steel collimators used were sufficient but for higher energies a thick lead collimator should be used to avoid the transmission of photons through the face and sides of the collimator.

A number of scans were carried out to test the scanner performance for different situations, i.e. for point sources, a radioactive gas and various extended sources in liquid and solid forms. The results show that the SPECT scanner system is capable of reconstructing and displaying images of the spatial distributions and relative strengths of the sources. The relative source strengths can be converted to absolute activity concentrations by a simple normalisation.
A number of factors have been studied that pose problems in achieving accurate quantitative results in SPECT. The problem of variable detection solid angle or geometrical factor can be overcome by using the opposed detector arrangement. It has been shown that using the geometric mean of the counts from the two directly opposed detectors yields average counts which are almost independent of the source position for point sources, but for extended sources geometric and arithmetic means give almost equal results.

The projection data from a short-lived radionuclide has to be corrected for decay of the radionuclide during the scanning process. The projection data from a Tc-99m extended source showed a large difference in the first and last projection values due to the decay of Tc-99m during the scanning procedure and if the scan data is not corrected for decay it will produce wrong results. The computer software for decay correction has been shown to work perfectly.

The most difficult and fundamental problem in the development of algorithms for ECT is to eliminate the effect of photon attenuation inside the object. Most existing algorithms assume that the detected count rate truly represent a line integral of the source activity. Due to attenuation inside the object, however, the detected count rate is not only an incorrect representation of the line integral of activity but also inconsistent for different viewing angles. Corrections for attenuation of photons, inside the source material and in the medium surrounding the source, is an important aspect of quantitative SPECT.

Ignoring the photon attenuation in objects leads to an artificial depression in the central region of the reconstructed image. For inhomogeneous irregularly shaped objects, the attenuation correction for calculating the distribution of activity throughout the reconstructed
slice will be an unknown factor which can not, in general, be properly
compensated by an assumed correction matrix. In principle, iterative
techniques can provide accurate attenuation compensation, even for
non-uniform attenuation, if an accurate map of attenuation coefficients
in the object is obtained by a transmission scan with object in the same
position as for emission scan. However, this procedure is time consuming
and is not suitable for many clinical applications, since two scans and
two reconstructions have to be performed. Instead, simple and faster
methods based on filtered back-projection can be used on the assumption
that the attenuation is constant within a known region. It has been
shown in chapter 6 that such methods compensate for attenuation
successfully as far as the object is symmetric and attenuation is
constant throughout the object.

The acceptance of in-scattered photons in the projection data for the
full-energy peak window produces image degrading effects in the
reconstructed image. It is possible to discriminate against incoherently
scattered photons by means of pulse height analysis. The extent of
discrimination possible depends on the energy resolution of the detector,
as good energy resolution results in a reduction in the fraction of
Compton scattered photons accepted within the full-energy peak window.
Since our SPECT scanner, like most of SPECT scanners, uses a NaI(Tl)
detector which possesses 10-20% energy resolution over the energy range
of interest, so the full-energy peak window has to be set wider, due to
which more of the scattered photons are also detected in the full-energy
peak window. Therefore the discrimination of scattered photons from the
full-energy peak counts is ineffective by this method and detector
collimation is the more practicable solution.

The method of using a reduced value of the effective attenuation
coefficient to allow for inscatter can produce hot-rim effects in the
final image due to the less activity in the centre (or it results in
under-correction for attenuation). So the provision of a secondary "scatter window" is advisable, although it will result in increased scan time for a double scan or require more electronics for dual-channel scanning. The scatter counts under the full-energy peak window are related to counts in a scatter window of similar width by a factor K whose value was found to be within ±7% of the average value of K=0.5. The counts in the scatter region, after multiplying with the factor K, are subtracted from the full-energy peak window to get the scatter free full-energy peak counts.

Optimisation of scan parameters is essential for optimal production of reconstructed images. The results of the present experiments in this context showed that counting time for each raysum, number of projections and number of raysums effect the quality of the image. The statistical accuracy of the images increases with the increase in the total number of photons collected. This can be achieved by increasing the counting time for each raysum, using higher activity sources or by using a wider collimator. The increase in counting time will increase the overall scan time which is not possible in many applications; for example in nuclear medicine, where long scanning times are not always practicable and may lead to higher doses of radiation to the patient. The use of highly active sources is not always possible, as in our case where higher activity sources could not be used due to the internal safety regulations of the University. But in nuclear industry, where highly active sources are available short counting times and narrow collimators can be used. Conversely, relaxing the spatial resolution by using larger width collimators enables the number of recorded photons to be increased in situations where low activity sources are encountered.

Improvement in the total photon detection efficiency can be achieved by using an array of detectors, however, such an arrangement is costly and an alternative is the position sensitive detector (PSD). High pressure
xenon-filled proportional counter equipped with a carbon fibre anode have been shown to give useful spatial resolution [MacCuaig et al, 1986]. This will decrease the scan time considerably and will improve the detection efficiency.

The use of line scans, isometric plots and calculations of contrast have been shown to be powerful techniques in the quantitative analysis of images. Since human image perception is inadequate in the accurate assessment of images, a form of digitisation is necessary. It is desirable that the quantitative technique employed be easily implemented and that the results can be rapidly obtained. The line scan satisfies both of these requirements and possesses the further advantage that the information, whilst being quantitative, is also visually displayed.

FUTURE DEVELOPMENTS

The method used for depth determination of beta-emitting radionuclides inside a medium can be efficiently used for low Z media. For a deeper insight into the bremsstrahlung "moderation" it would be interesting to extend this method to higher Z media and to higher mean energy sources which would also be of significance in a wider range of industrial applications. Also the application of this method to large volume sources and the effect of collimation has yet to be studied.

For medical purposes phantoms of different body organs can be prepared and curves of bremsstrahlung peak position against the thickness of the "tissue" can be obtained; the depth of a particular source or an organ can be determined by comparing the data with the calibration curve.

Bremsstrahlung from a beta-emitter inside an absorbing medium can also be measured in the SPECT mode and the source distribution can be determined
by inspection of the reconstructed image. This branch of CT, "bremsstrahlung emission tomography", has yet to be explored and is strongly recommended for future work.

Photon-emitters can easily be localised by SPR and PPR measurements. The present study has been confined to the behaviour of three spectral regions, namely; the full-energy peak region, the valley just below the full-energy peak, and the region around the Compton edge in the detector spectrum. The study can be extended for other parts of the energy spectrum, e.g the different parts (lower or upper half) of the full-energy peak, the region between two full-energy peaks and different scatter regions corresponding to different Compton scattering angles. Further work is also required to see the effects due to: forward and back scatter, the dimensions of the source, atomic composition of the scattering medium and the presence of extraneous radioactivity outside the volume under study.

The major drawback of the present SPECT scanner is the long time required to acquire the projection data. There are several experimental variables that influence the scan time including the activity of the source, detection efficiency of the system, number of projections and number of steps per projection. One way to reduce the scan time (or counting more photons for the same time) is to use higher activity sources. This can be applied in industry, e.g scanning of spent fuel rods etc, but for the present experimental work a strong source could not be used due to reasons stated above. Also in medical applications highly active source can not be used in many cases. Therefore the reduction in scan time is only possible by decreasing the total number of data points, that is, by reducing the number of projections and number of raysums per projection. A reduced number of raysums per projection allows the use of larger width collimator at the expense of spatial resolution as the requirements for good spatial resolution are less demanding in ECT than TCT. A
compromise between resolution and scan time can be optimised by using slit collimators of variable width to allow more photons to be collected by the detector. A reasonably large number of photons should be collected to minimise the effect of statistical fluctuations in the final image.

A considerable reduction in scan time can be achieved by using a multi-detector array or a position sensitive detector (PSD). The possibility of transmission scanning with a carbon fibre anode xenon-filled proportional counter (PSPC) has been reported by MacCuaig et al [1986] of our group at Surrey. The inclusion of a PSD in the SPECT system will reduce the scanning time considerably since a 22cm PSPC is equivalent to at least 20 detectors [MacCuaig et al, 1986]. However, this PSPC suffers from low intrinsic efficiency and limited life. This can be overcome by using other position sensitive detectors, e.g. a sodium iodide bar scintillation detector which will increase the intrinsic efficiency by a large factor at higher photon energies [Tajuddin, 1986].

The photon detection efficiency can also be improved by using an array of detectors. At present, a 16-detector scanning system, for transmission studies, is being developed in our group at Surrey. This set-up can easily be adapted to perform in the emission mode with proper electronics and software.

The simple attenuation compensation techniques have been shown to work reasonably well for constant attenuation cases, but for variable attenuation cases, i.e. for irregularly shaped objects having non-uniform attenuation, variable attenuation compensation methods have to be used. The use of RECLBL library routines for constant and variable attenuation compensation have been shown to work for simulated data. They can be used for reconstructing real scan data and the images can be made to display on the grey-level Sigma 5674 terminal by changing the
alphanumeric characters to the appropriate grey-levels or to different colours for colour display on the HP-9836C screen. The iterative methods of attenuation correction and reconstruction can produce accurate results. The only drawback of these methods are the requirements of computation time and large computer memory. The computation time can be reduced considerably by using fewer iterations by relaxing the accuracy requirements, and by optimising the initial parameter values these methods can be made rapidly converging.

It is hoped that this analysis of problems encountered in SPECT together with the various solutions considered in this thesis will soon enable this technique to deliver its full potential in a wide range of medical and industrial applications.
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