Single Relay Cooperative
Transmission/Reception Techniques

Yinan Qi

Submitted for the Degree of
Doctor of Philosophy
from the
University of Surrey

Centre for Communication Systems research
Faculty of Electronics and Physical Sciences
University of Surrey
Guildford, Surrey GU2 7XH, UK

July 2009

© Yinan Qi 2009
Abstract

One of primary objectives of future wireless systems is to provide uniform high rate coverage. Essentially, a wireless system must take multipath fading, path loss and shadowing effect into consideration. In the pursuit of schemes that will provide a solution to these problems, various granular and distributed network architectures based on relaying techniques are emerging. In particular, three basic relaying schemes have been proposed including amplify-and-forward (AF), decode-and-forward (DF) and compress-and-forward (CF).

This thesis has investigated the performance of various single-relaying schemes from the theoretical point of view and established a fundamental understanding of relay-assisted communication systems. An improved version of DF — Soft DF — has been evaluated through the derived upper bounds of Bit-Error-Rate (BER) and Block-Error-Rate (BLER). For the most complicated relaying scheme — CF, we proposed some novel techniques to efficiently improve its performance.

Moreover, both theoretic and simulation results have shown that one single-relaying scheme cannot always outperform the others over all channel conditions. In this regard, the relaying schemes need to be deployed in a hybrid fashion to adapt to the current channel realization. We have provided two hybrid relaying schemes in this thesis and investigated the integration of the hybrid relaying schemes with hybrid retransmission protocols. The proposed integrated technique is named as H²-ARQ-Relaying schemes. Moreover, we have proposed two H²-ARQ-Relaying strategies that efficiently exploit a hybrid CF/DF forwarding scheme, where the relay, based on its decoding status, could dynamically switch between CF and DF and analyzed the performance analysis has been performed for both strategies mainly in terms of the obtained outage probabilities and throughputs. Considering the availability of Channel State Information (CSI) at the relay in practical implementations, we have proposed modified CF/DF-based H²-ARQ-Relaying strategies with limited CSI feedback accordingly.

Key words: cooperative communication, Wyner-Ziv coding, Channel State Information (CSI), Hybrid Automatic Retransmission re-Quest (HARQ), H²-ARQ-Relaying
Acknowledgements

It is with a great pleasure that I take this opportunity to thank numerous people who made my Ph.D. years at the University of Surrey such a lovely and rewarding experience.

My deepest gratitude goes first and foremost to my supervisors, Dr. Reza Hoshyar and Prof. Rahim Tafazolli, for their constant encouragement and guidance. Dr. Reza has walked me through all the stages of the writing of this thesis. Without his consistent and illuminating instructions, this thesis could not have reached its present form. I have learned greatly from their strong knowledge, out-of-the-box thinking, and exceptional insight on my work.

I would like to express my heartfelt gratitude to Dr. Yi Ma and Dr. John Thompson. They have been taking a time out of their busy schedule for reading this thesis and I had a great pleasure of having a great discussion in my Ph.D. examination. I would also like to acknowledge Dr. Haitham Cruickshank for being the Chair of my Ph.D. examination committee.

I also owe my sincere gratitude to my friends and my colleagues in CCSR who gave me their help and time in listening to me and helping me work out my problems during the difficult course of the thesis. Not to forget, I would like to thank Alison and Elizabeth for always being helpful to me throughout my stay in CCSR.

At last, my thanks would go to my beloved parents, Quanqing Qi and Yuqing Wang, for their loving considerations and great confidence in me all through these years. Their careful attention and love will forever grow in my heart.
CONTENTS

List of Acronyms ............................................................................................................................................. iv
List of Symbols ................................................................................................................................... vi
List of Figures ................................................................................................................................................viii
List of Tables .................................................................................................................................................... xi

Chapter 1 Introduction ....................................................................................................................................1
  1.1 Background ............................................................................................................................................ 1
  1.2 Motivations and Objectives ......................................................................................................................3
  1.3 Main Contributions .....................................................................................................................................5
  1.4 Publications ........................................................................................................................................... 6
  1.5 Thesis Outline ....................................................................................................................................... 7

Chapter 2 System Model ........................................................................................................................ 9
  2.1 Introduction .....................................................................................................................................................9
  2.2 Relay System Model .....................................................................................................................................10
    2.2.1 Notation Rules .......................................................................................................................................10
    2.2.2 Conventional Two-Node Communication System ..............................................................................11
    2.2.3 Relay System .........................................................................................................................................11
      2.2.3.1 RC-based Relay System ................................................................................................................13
      2.2.3.2 UC-based Relay System .................................................................................................................14
  2.3 Evaluation Metrics .......................................................................................................................................15
  2.4 Conclusions ....................................................................................................................................................17

Chapter 3 Amplify-and-forward, Decode-and-forward and Soft Decode-and-forward ..................18
  3.1 Introduction ..................................................................................................................................................18
  3.2 Achievable Rate and Outage Behavior of AF ..............................................................................................19
    3.2.1 Achievable Rate of AF ..........................................................................................................................19
    3.2.2 Outage Behavior of AF .........................................................................................................................22
      3.2.2.1 Outage Behavior of AF with Feedback of CSI .............................................................................22
      3.2.2.2 Outage Behavior of AF without Feedback of CSI ........................................................................24
    3.2.3 Simulation Results and Discussions for AF .........................................................................................24
  3.3 Achievable Rate and Outage Behavior of DF ...............................................................................................27
    3.3.1 Repetition Coding (RC) .......................................................................................................................27
    3.3.2 Unconstrained Coding (UC) ................................................................................................................29
    3.3.3 Simulation Results and Discussions of DF .........................................................................................33
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.3</td>
<td>Hybrid Relaying Scheme II</td>
<td>90</td>
</tr>
<tr>
<td>5.3.1</td>
<td>Outage Behavior for Two Extreme Cases</td>
<td>90</td>
</tr>
<tr>
<td>5.3.2</td>
<td>Limited CSI with a Limited Feedback Link</td>
<td>93</td>
</tr>
<tr>
<td>5.3.3</td>
<td>Simulation Results, Comparisons and Discussions</td>
<td>97</td>
</tr>
<tr>
<td>5.4</td>
<td>Conclusions</td>
<td>99</td>
</tr>
<tr>
<td>Chapter 6</td>
<td>Hybrid Automatic Repeat re-Quest (HARQ) for the Relay System</td>
<td>101</td>
</tr>
<tr>
<td>6.1</td>
<td>Introduction</td>
<td>101</td>
</tr>
<tr>
<td>6.2</td>
<td>H²-ARQ-Relaying Strategies</td>
<td>102</td>
</tr>
<tr>
<td>6.2.1</td>
<td>H²-ARQ-Relaying Strategy 1 (HS_S1)</td>
<td>103</td>
</tr>
<tr>
<td>6.2.2</td>
<td>H²-ARQ-Relaying Strategy 2 (HS_S2)</td>
<td>106</td>
</tr>
<tr>
<td>6.3</td>
<td>Throughput Analysis</td>
<td>108</td>
</tr>
<tr>
<td>6.3.1</td>
<td>Outage Analysis</td>
<td>108</td>
</tr>
<tr>
<td>6.3.2</td>
<td>State and Transition Probabilities</td>
<td>111</td>
</tr>
<tr>
<td>6.3.3</td>
<td>Throughput Analysis</td>
<td>112</td>
</tr>
<tr>
<td>6.4</td>
<td>Modified H²-ARQ-Relaying Strategy with One Bit Feedback</td>
<td>114</td>
</tr>
<tr>
<td>6.5</td>
<td>Modified H²-ARQ-Relaying Strategy with Two Bits Feedback</td>
<td>118</td>
</tr>
<tr>
<td>6.6</td>
<td>Performance Analysis of Modified Strategies</td>
<td>119</td>
</tr>
<tr>
<td>6.6.1</td>
<td>Outage Analysis</td>
<td>119</td>
</tr>
<tr>
<td>6.6.2</td>
<td>State and Transition Probabilities</td>
<td>121</td>
</tr>
<tr>
<td>6.6.3</td>
<td>Throughput Analysis</td>
<td>122</td>
</tr>
<tr>
<td>6.7</td>
<td>Simulation Results, Comparisons and Discussions</td>
<td>123</td>
</tr>
<tr>
<td>6.7.1</td>
<td>H²-ARQ-Relaying Strategies</td>
<td>125</td>
</tr>
<tr>
<td>6.7.2</td>
<td>Modified H²-ARQ-Relaying Strategies</td>
<td>128</td>
</tr>
<tr>
<td>6.8</td>
<td>Conclusions</td>
<td>132</td>
</tr>
<tr>
<td>Chapter 7</td>
<td>Conclusions and Future Work</td>
<td>134</td>
</tr>
<tr>
<td>7.1</td>
<td>Conclusions</td>
<td>134</td>
</tr>
<tr>
<td>7.2</td>
<td>Future Work</td>
<td>137</td>
</tr>
<tr>
<td>Appendix A</td>
<td></td>
<td>139</td>
</tr>
<tr>
<td>Appendix B</td>
<td></td>
<td>141</td>
</tr>
<tr>
<td>Appendix C</td>
<td></td>
<td>142</td>
</tr>
<tr>
<td>Appendix D</td>
<td></td>
<td>144</td>
</tr>
<tr>
<td>Appendix E</td>
<td></td>
<td>148</td>
</tr>
<tr>
<td>Appendix F</td>
<td></td>
<td>151</td>
</tr>
<tr>
<td>References</td>
<td></td>
<td>155</td>
</tr>
</tbody>
</table>
# List of Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>3GPP</td>
<td>3rd Generation Partnership Project</td>
</tr>
<tr>
<td>ACK</td>
<td>Acknowledgement</td>
</tr>
<tr>
<td>AF</td>
<td>Amplify-and-Forward</td>
</tr>
<tr>
<td>APP</td>
<td>A Posteriori Probabilities</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
</tr>
<tr>
<td>asy</td>
<td>asynchronous</td>
</tr>
<tr>
<td>BER</td>
<td>Bit-Error-Rate</td>
</tr>
<tr>
<td>BLER</td>
<td>Block-Error-Rate</td>
</tr>
<tr>
<td>BrF</td>
<td>Broadcasting Frame</td>
</tr>
<tr>
<td>CSCGD</td>
<td>Circularly Symmetric Complex Gaussian Distribution</td>
</tr>
<tr>
<td>CDMA</td>
<td>Code-Division-Multiple-Access</td>
</tr>
<tr>
<td>CF</td>
<td>Compress-and-Forward</td>
</tr>
<tr>
<td>CSI</td>
<td>Channel State Information</td>
</tr>
<tr>
<td>CoF</td>
<td>Complete Frame</td>
</tr>
<tr>
<td>D</td>
<td>Destination</td>
</tr>
<tr>
<td>DTC</td>
<td>Distributed Turbo Codes</td>
</tr>
<tr>
<td>DF</td>
<td>Decode-and-Forward</td>
</tr>
<tr>
<td>FDF</td>
<td>Fixed Secode-and-Forward</td>
</tr>
<tr>
<td>FER</td>
<td>Frame-Error-Rate</td>
</tr>
<tr>
<td>HSDPA</td>
<td>High-Speed Downlink Packet Access</td>
</tr>
<tr>
<td>HARQ</td>
<td>Hybrid Automatic Repeat re-Quest</td>
</tr>
<tr>
<td>i.i.d.</td>
<td>independent and identically distributed</td>
</tr>
<tr>
<td>INR</td>
<td>Incremental Redundancy</td>
</tr>
<tr>
<td>LDPC</td>
<td>Low-Density-Parity-Check</td>
</tr>
<tr>
<td>LLR</td>
<td>Logarithm of Likelihood Ratio</td>
</tr>
<tr>
<td>LTE</td>
<td>Long Term Evolution</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-------------</td>
</tr>
<tr>
<td>MAC</td>
<td>Media Access Control</td>
</tr>
<tr>
<td>MSE</td>
<td>Mean Square Error</td>
</tr>
<tr>
<td>NACK</td>
<td>Negative Acknowledgement</td>
</tr>
<tr>
<td>p.d.f.</td>
<td>probability density function</td>
</tr>
<tr>
<td>PEP</td>
<td>Pairwise Error Probability</td>
</tr>
<tr>
<td>PHY</td>
<td>Physical</td>
</tr>
<tr>
<td>QF</td>
<td>Quantize-and-Forward</td>
</tr>
<tr>
<td>R</td>
<td>Relay</td>
</tr>
<tr>
<td>RC</td>
<td>Repetition Coding</td>
</tr>
<tr>
<td>RCPC</td>
<td>Rate Compatible Punctured Convolutional</td>
</tr>
<tr>
<td>RD</td>
<td>Rate-Distortion</td>
</tr>
<tr>
<td>RSAC</td>
<td>Recursive Systematic Convolutional Code</td>
</tr>
<tr>
<td>RTF</td>
<td>Relay transmission Frame</td>
</tr>
<tr>
<td>S</td>
<td>Source</td>
</tr>
<tr>
<td>syn</td>
<td>synchronized</td>
</tr>
<tr>
<td>SDF</td>
<td>Selective Decode-and-Forward</td>
</tr>
<tr>
<td>SDMA</td>
<td>Space-Division-Multiple-Access</td>
</tr>
<tr>
<td>SISO</td>
<td>Soft-in-Soft-output</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal Noise Ratio</td>
</tr>
<tr>
<td>SoDF</td>
<td>Soft Decode-and-Forward</td>
</tr>
<tr>
<td>SW</td>
<td>Slepian-Wolf</td>
</tr>
<tr>
<td>TCQ</td>
<td>Trellis-Coded Quantization</td>
</tr>
<tr>
<td>TDMA</td>
<td>Time-Division-Multiple-Access</td>
</tr>
<tr>
<td>UC</td>
<td>Unconstrained Coding</td>
</tr>
<tr>
<td>WEF</td>
<td>Weight Enumerating Function</td>
</tr>
<tr>
<td>WiMAX</td>
<td>Worldwide Interoperability for Microwave Access</td>
</tr>
<tr>
<td>WZ</td>
<td>Wyner-Ziv</td>
</tr>
</tbody>
</table>
LIST OF SYMBOLS

\( \alpha \) \quad \text{Half-duplexing ratio}

\( \beta_r \) \quad \text{Amplifying gain at the relay in AF}

\( c_i \) \quad \text{Channel gain of link } i

\( \theta_i \) \quad \text{Phase of the } c_i

\( \mu_i \) \quad \text{Average SNR of link } i

\( \gamma_i \) \quad \text{Instantaneous SNR of link } i

\( \gamma_w \) \quad \frac{\sigma_i^2}{\sigma_r^2}

\( \rho \) \quad \frac{\gamma_0}{\gamma_1}, \text{ denoted as destination and relay SNR ratio}

\( \sigma_i^2 \) \quad \text{Noise variance at the relay}

\( \sigma_r^2 \) \quad \text{Noise variance at the destination}

\( \sigma_w^2 \) \quad \text{Compression Noise variance}

\( \Phi \) \quad \text{Random Reward}

\([\text{con}]\) \quad \text{Indication function whose value is 1 when } \text{con} \text{ is satisfied and 0 otherwise.}

\( \mathcal{N}(\mu, \sigma^2, \sigma^2) \) \quad \text{Complex Circularly Symmetric Gaussian Distribution with mean } \mu, \text{ variance of real part } \sigma^2, \text{ and variance of imaginary part } \sigma^2.

\text{inf} \quad \text{Infimum}

\( D \) \quad \text{Distortion}

\( d_N \) \quad \text{Hamming distance}

\( E_s \) \quad \text{Transmitted energy per symbol at the source}

\( E_r \) \quad \text{Transmitted energy per symbol at the relay}

\( E \{ \gamma \} \) \quad \text{Expectation with respect to } \gamma

\( f_i(\cdot) \) \quad \text{Relay encoding function}

\( H(X) \) \quad \text{Entropy of argument } X

\( I(X;Y) \) \quad \text{Mutual Information of } X \text{ and } Y

\log(x) \quad \text{Logarithm of argument } X
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Pr(e)$</td>
<td>The probability of event $e$</td>
</tr>
<tr>
<td>$Q(x)$</td>
<td>$Q$-function</td>
</tr>
<tr>
<td>$Q(x)$</td>
<td>Quantization Function</td>
</tr>
<tr>
<td>$Q_{k}(x)$</td>
<td>$K$-bit quantization</td>
</tr>
<tr>
<td>QPSK</td>
<td>Quadrature Phase Shift Keying</td>
</tr>
<tr>
<td>$R(D)$</td>
<td>Rate-Distortion Function</td>
</tr>
<tr>
<td>$R_{RD}(D)$</td>
<td>Classic RD function of coding $X$ with side information $Y$ available at the encoder and decoder</td>
</tr>
<tr>
<td>$R_{WZ}(D)$</td>
<td>RD function of WZ coding</td>
</tr>
<tr>
<td>$s$</td>
<td>Compressed index</td>
</tr>
<tr>
<td>$SoM(.)$</td>
<td>Soft mapping function</td>
</tr>
<tr>
<td>$T_{F}$</td>
<td>Duration of a whole frame including two phases</td>
</tr>
<tr>
<td>$X'(w)$</td>
<td>Codebook containing all the codewords transmitted by the source in phase 1</td>
</tr>
<tr>
<td>$X_{c}(w)$</td>
<td>Codebook containing all the codewords of compressed index $s$ in CF scheme</td>
</tr>
<tr>
<td>$X^{2}(w)$</td>
<td>Codebook containing all the codewords to be transmitted by the source during phase 2</td>
</tr>
<tr>
<td>$X_{o}^{2}(w)$</td>
<td>Codebook containing all the codewords to be transmitted by the relay during phase 2 in DF scheme</td>
</tr>
<tr>
<td>$Z_{r}$</td>
<td>Complex Gaussian noise at the relay</td>
</tr>
<tr>
<td>$Z_{d}$</td>
<td>Complex Gaussian noise at the destination</td>
</tr>
</tbody>
</table>
# List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1.1</td>
<td>A relay system in the urban environment</td>
<td>1</td>
</tr>
<tr>
<td>Figure 1.2</td>
<td>Half-duplex relaying protocol</td>
<td>3</td>
</tr>
<tr>
<td>Figure 2.1</td>
<td>Two-node communication system</td>
<td>11</td>
</tr>
<tr>
<td>Figure 2.2</td>
<td>Half-duplex relay system</td>
<td>12</td>
</tr>
<tr>
<td>Figure 3.1</td>
<td>$f(\beta_c)$</td>
<td>20</td>
</tr>
<tr>
<td>Figure 3.2</td>
<td>Achievable Rate</td>
<td>25</td>
</tr>
<tr>
<td>Figure 3.3</td>
<td>Achievable Rate, $\gamma_0=0$ dB, (a) $\gamma_2=-10$ dB</td>
<td>26</td>
</tr>
<tr>
<td>Figure 3.4</td>
<td>Outage Probabilities</td>
<td>26</td>
</tr>
<tr>
<td>Figure 3.5</td>
<td>$\gamma_a$</td>
<td>26</td>
</tr>
<tr>
<td>Figure 3.6</td>
<td>Decode-and-forward with unconstrained coding</td>
<td>30</td>
</tr>
<tr>
<td>Figure 3.7</td>
<td>Optimal half-duplex ratio $\alpha$ for DF</td>
<td>31</td>
</tr>
<tr>
<td>Figure 3.8</td>
<td>Achievable Rate and optimal half-duplex ratio $\alpha$</td>
<td>34</td>
</tr>
<tr>
<td>Figure 3.9</td>
<td>Achievable Rate, $\gamma_0=0$ dB, (a) $\gamma_2=-10$dB</td>
<td>35</td>
</tr>
<tr>
<td>Figure 3.10</td>
<td>Outage probability</td>
<td>36</td>
</tr>
<tr>
<td>Figure 3.11</td>
<td>Soft decode-and-forward</td>
<td>37</td>
</tr>
<tr>
<td>Figure 3.12</td>
<td>$\gamma_{in}$ and $\gamma_{out}$ relationship, RSCC with generator polynomials (5/7)</td>
<td>39</td>
</tr>
<tr>
<td>Figure 3.13</td>
<td>BER, Sim: simulation, UB: upper bound</td>
<td>43</td>
</tr>
<tr>
<td>Figure 3.14</td>
<td>BLER, Sim: simulation, UB: upper bound</td>
<td>43</td>
</tr>
<tr>
<td>Figure 3.15</td>
<td>Upper bounds for space-time coding</td>
<td>44</td>
</tr>
<tr>
<td>Figure 4.1</td>
<td>Compress-and-forward System</td>
<td>47</td>
</tr>
<tr>
<td>Figure 4.2</td>
<td>Achievable Rate for CF and optimal $\alpha$</td>
<td>50</td>
</tr>
<tr>
<td>Figure 4.3</td>
<td>Achievable rate</td>
<td>51</td>
</tr>
</tbody>
</table>
Figure 4.4. Outage probability................................................................................................................ 52
Figure 4.5. Source encoder/decoder with/without side information......................................................... 53
Figure 4.6. Practical CF model.................................................................................................................. 54
Figure 4.7 (a) 4-state Trellis (b) Codebook............................................................................................ 61
Figure 4.8. BER & Distortion, AWGN. (a) BER (b) Distortion................................................................. 63
Figure 4.9. Coding Rate ........................................................................................................................ 63
Figure 4.10. Achievable Rate.................................................................................................................... 63
Figure 4.11. BER in block fading channel. $\mu_1=\mu_0+2\text{dB}$ ............................................................. 64
Figure 4.12. Distortion ............................................................................................................................ 64
Figure 4.13. Multilevel SW coding at the relay....................................................................................... 67
Figure 4.14. Multilevel SW decoding at the destination.......................................................................... 69
Figure 4.15. Separate decoding and decompressing .............................................................................. 71
Figure 4.16 Joint Turbo Decoding & Decompressing .......................................................................... 72
Figure 4.17. BER of the first level. AWGN, $\mu_1=\mu_0+2\text{dB}$, $\mu_2=2\text{dB}$ .................................................. 73
Figure 4.18. BER of the second level with different information passing. (a) $S_1$; (b) $S_2$; (c) $S_3$; (d) $S_1$, $S_2$ and $S_3$ .......................................................................................................................... 75
Figure 5.1. BER of different levels......................................................................................................... 81
Figure 5.2. Source with a multilevel coding structure ......................................................................... 83
Figure 5.3. 64-QAM ............................................................................................................................... 84
Figure 5.4. Hybrid DF/CF....................................................................................................................... 85
Figure 5.5. Destination ........................................................................................................................... 86
Figure 5.6. $\mu_1=16\text{dB}$ without compression. Phase 1: 64QAM; Phase 2: QPSK .................................. 88
Figure 5.7. $\mu_1=20\text{dB}$ without compression.......................................................................................... 88
Figure 5.8. $\mu_1=20\text{dB}$ with compression, 64-QAM............................................................................ 89
LIST OF TABLES

TABLE 4.1 ........................................................................................................................................................74
TABLE 5.1 .......................................................................................................................................................... 93
TABLE 6.1 ....................................................................................................................................................... 105
TABLE 6.2 ....................................................................................................................................................... 119
TABLE 6.3 ....................................................................................................................................................... 124
TABLE 6.4 ....................................................................................................................................................... 128
TABLE D.1 .....................................................................................................................................................147
TABLE D.2 .....................................................................................................................................................147
Chapter 1 Introduction

1.1 Background

Recently, there has been escalated demand for rapid, low latency and high volume communication of information to homes and business premises. As the need for high speed access by end-users in millions of locations evolved, particularly fueled by the widespread adoption of the internet, some technologies such as High-Speed Downlink Packet Access (HSDPA), 3rd Generation Partnership Project Long Term Evolution (3GPP LTE) for mobile phone networks, and IEEE standard for mobile broadband wireless access, also known as 'mobile Worldwide Interoperability for Microwave Access (WiMAX)', have been proposed to provide high speed data transmission. One of main objectives of future wireless systems is to provide uniform high rate coverage. Essentially, the system must take multipath fading, path loss and shadowing effect into consideration. In the pursuit of schemes that will provide a solution to these problems, various granular and distributed network architectures based on relaying techniques are emerging. For instance, two projects that focused on the research of cooperative communications have been launched by the 7th Framework Programme of the European Commission within the Centre for Communication Systems Research in the University of Surrey. One is FIREWORKS (Flexible Relay Wireless OFDM-based networks) and another is ROCKET (Reconfigurable OFDMA-based Cooperative Networks Enabled by Agile Spectrum Use).

As depicted in Figure 1.1, in both mobile relay and fixed relay cases, the direct source-destination link is impaired because the link has been blocked by a group of buildings. However, with the help of the relay node, another independent link, i.e. source-relay-destination, can be established. This link does not suffer from shadowing effect because no obstacles appear in the link.

![Figure 1.1. A relay system in the urban environment. (a) mobile relay (b) fixed relay](image)

The basic idea behind relaying is to receive help from some radio nodes, called relays, to perform more spectrum and power efficient communications. Relay nodes can be specifically devoted network nodes or other
user devices in the vicinity. The relay model was first introduced by van der Meulen [1], where a communication system with three nodes denoted as source (S), relay (R) and destination (D), respectively, was investigated. There are three links between these nodes and each link has an input/output pair. Cover and El Gamal have substantially developed the relay model and provided three basic relaying principles [2]:

- **Cooperation**: If the relay receives a better signal than the destination, it is able to cooperate with the source by sending a signal that compromises a perfect source signal to the destination.

- **Facilitation**: If the relay sees a corrupted version of what the destination sees, the relay transmits a constant signal which is known to the source and the destination to open the channel between the source and the destination.

- **Observation**: Alternative information can be sent by the relay. This information does not comprise a perfect source signal, thus precluding pure cooperation, and is not constant, thus precluding simple facilitation. Instead, the relay forwards what it has observed to the destination.

While the optimal relaying strategy in wireless networks has not yet been fully understood, several relaying schemes developed based on the principles (especially for cooperation and observation) have been suggested in the literature. Among them, the simplest scheme is amplify-and-forward (AF) [12]-[15], in which the relay just forwards what it has received with a proper amplification gain. Another well developed scheme is decoded-and-forward (DF) [16]-[32], where the relay decodes, re-encodes and forwards the received signal to the destination. DF has shown improvement in terms of the achievable rate and outage behavior when the relay is close to the source [4]-[5], [16]. Both schemes share a common weakness: their performance is constrained by the quality of the source-relay link. If this link endures deep fading, for AF the relay will forward nothing but mostly its own noise, and for DF the relay will not be able to successfully decode and forwarding of erroneous messages will lead to error propagation at the destination. To cope with this problem, the selective DF scheme was proposed [5], where the relay is turned off when errors are detected. In addition, some researchers have proposed an improved DF scheme, namely soft DF, in which the relay conducts soft encoder and forwards the soft values of the re-encoded message to the destination [33]-[35]. This scheme exhibits the capability of mitigating the error propagation. Recently, a scheme called compress-and-forward (CF) based on the observation principle in Cover’s paper has drawn considerable attention [4], [8], [52]-[53], [62]-[63]. Instead of decoding, the relay quantizes the received signal, compresses the quantized version and forwards the compressed version to the destination. At the destination, the signal received from the source directly will serve as side information to reconstruct the relay’s observation. The whole process including quantization, compression, decompression and reconstruction can be carried out through Wyner-Ziv coding [49]-[50]. The destination then tries to decode the message by joint processing of the received signals and the reconstructed observation. Studies on CF revealed that it outperforms DF when the relay is close to the destination, or generally speaking, the relay-destination link is very strong.
Besides the single-relay with single antenna scenario, the case where the relay is equipped with multiple antennas and the case where multiple relay nodes are deployed at the relay have been investigated in [110]-[112]. However, this thesis will focus on the single-relay with single antenna case. Most of the analysis in this thesis can be easily extended to multiple-antenna and multiple-relay cases. There are two basic working modes for a relay system: full-duplex and half-duplex. In the full-duplex mode, the relay is assumed to be able to transmit and receive simultaneously in the same band. Due to the large dynamic range between the incoming and outgoing signals through the same antenna elements, the full-duplex mode is regarded difficult to implement. In contrast, in the half-duplex mode, the relay is normally assumed to work in a time-division or frequency-division manner, where it either receives or transmits at a given time and band instance. Compared with the full-duplex mode, the half-duplex mode, especially the time-division manner, is more feasible and widely accepted. Therefore, we will focus on the time-division half-duplex relay channel in this thesis. The entire frame is separated into two phases: the relay-receive phase (phase 1) and the relay-transmit phase (phase 2). During the first phase, the source broadcasts to the relay and the destination. In phase 2, the relay is active and transmits to the destination. The duplexing ratio, denoted by $\alpha$, represents the percentage of the first phase in a complete frame.

### 1.2 Motivations and Objectives

The overall aim of the research is to establish a fundamental understanding about the cooperative communication systems and provide novel techniques which are capable of achieving improved performance. Over the past decade, many researchers have investigated the AF and DF schemes [12]-[32]. Laneman has derived the outage behavior and the diversity gain for both schemes in his landmark paper [5]. However, the derivation is based on a relatively simple relaying protocol as shown in Figure 1.2, denoted by relaying protocol I in which the source keeps silent during the second phase. Other researchers, especially for AF [12]-[15], also presented their results in this simple protocol. As a result, the amplification mechanism at the relay is quite simple. In contrast, our work will provide the outage and diversity analysis in a general relaying protocol, denoted by relaying protocol II, where the source transmits with the relay simultaneously in a cooperative manner during phase 2. In such a case, the optimization of the amplification function for the AF scheme requires further insight.
For the DF scheme, different coding strategies including repetition coding (RC) and unconstrained coding (UC) have been investigated [4], [5]. The outage performance of UC-based DF has been derived in [4]. We will examine the achievable rate and the outage behavior of RC-based DF for theoretical completeness and a thorough comparison of different DF schemes will be addressed in this thesis. We noticed that although the source and the relay can be easily synchronized in the time domain, carrier synchronization is challenging due to the usage of phase-locking-separated microwave oscillators, which is formidable in practical systems. Without carrier synchronization, random phase difference will be introduced. In light of this observation, we will investigate the performance of DF in synchronized and asynchronous scenarios respectively.

Some practical unconstrained cooperative coding schemes, in particular, distributed turbo coding, have been investigated in DF, where the source signal and the relay signal form the two components of a turbo code [16]-[17], [26]-[28]. As the two components are encoded separately, this coding scheme is named as distributed turbo coding. In most of the previous work, it is always assumed that the relay is able to successfully decode the message from the source, i.e. the source-relay link is assumed to be error-free. However, due to the fact that the wireless channel suffers from fading from time to time, there is a possibility that the instantaneous signal-to-noise ratio (SNR) of the source-relay link is below the required SNR of successful decoding. In some recent work, a soft decode-and-forward (SoDF) scheme has been proposed, where the relay’s decoding and re-encoding process is conducted by a soft-in-soft-out (SISO) decoder followed by a soft encoder. A weakness of current research on SoDF is that the performance evaluation is simulation-oriented and lack of theoretical analysis. We will provide an analytical approach to derive the theoretical upper bounds of bit-error-rate (BER) and block-error-rate (BLER) for SoDF. By doing this, the performance of a SoDF system can be easily predicted by the upper bounds.

We will then provide the theoretical derivations of the achievable rate, outage behavior and diversity gains for RC- and UC-based CF. As a key technique, CF, especially its practical implementations draws much less attention than AF and DF. The reason is that a crucial part in CF — Wyner-Ziv (WZ) coding — has not been well understood. However, in the research of sensor networks, as one of the enabling techniques, WZ coding has been undergoing a quiet revolution [49]-[61]. We will extend the state of the art WZ coding techniques in sensor networks to the relay-assisted communication systems. Realizing the promise of the CF scheme requires more than a mere extension; it relies on novel techniques which are specially designed for relay channels. We will propose some novel techniques for WZ coding which can improve the performance of CF efficiently.

Moreover, based on the analysis and simulation results obtained, each single relaying scheme cannot always outperform other schemes over all channel conditions. In a fading environment, the instantaneous SNRs always change in a wide range. In this regard, the relaying schemes should be deployed in a hybrid fashion to adapt itself to the current channel realizations. We will provide two hybrid relaying schemes in this thesis. One scheme is based on the concept of multilevel coding and DF/CF is deployed simultaneously in different coding levels. Another scheme is based on the relay’s decoding status and different relaying schemes are carried out in a time-division manner. As pointed out in [8] and [49], for the CF scheme, perfect global channel state
information (CSI) is required at the relay node for adjusting its compression rate and performing ideal WZ coding. Part of this CSI related to the relay-destination link can be obtained either through feedback or exploitation of reciprocity of the channel [101], and the other part related to the source-destination link can be provided through feedback from the destination. Anyhow provision of global CSI at the relay is not affordable in most of the practical conditions. To tackle this problem, we will propose a practical hybrid relaying scheme with very relaxed requirements on the CSI knowledge, where very few extra bits to convey, from the destination to the relay, partial but useful CSI of the source- and relay-destination links are fed back. This strategy greatly reduces the amount of feedback information, thus making the whole system more practical to implement.

Despite the exploitation of an efficient cooperative relaying strategy the transmitted packet could still be lost due to the instantaneous channel condition and noise realization. The packet loss could be even more severe when the system is operating under a static (block) fading condition and the transmitter is not able to properly tune its transmission rate due to lack of sufficient level of channel knowledge. Packet-oriented data transmission calls for the usage of error control methods based on Hybrid Automatic Repeat re-Quest (HARQ). The application of HARQ protocols to a cooperative relay channel will lead to two classes of strategies: H-ARQ-Relaying, where HARQ is used in conjunction with a conventional forwarding scheme, usually DF, and H^2-ARQ-Relaying, where both retransmission protocol and relaying scheme are hybrid. In this thesis, we will propose two H^2-ARQ-Relaying strategies that efficiently exploit a hybrid CF/DF forwarding scheme, where the relay, based on its decoding status, could dynamically switch between CF and DF and analyzed the performance of both strategies mainly in terms of outage probabilities and obtained throughputs. We also considered the availability of CSI at the relay in practical implementations and will propose modified CF/DF-based H^2-ARQ-Relaying strategies with limited CSI feedback accordingly.

1.3 Main Contributions

My main contributions are listed as follows:

1. The upper bounds of the Bit-error-rate (BER) and Block-error-rate (BLER) of soft decode-and-forward were evaluated [con.7]. We noticed that the quality of the source-relay link limits the overall performance because of the error propagation problem and used a new technique, namely soft DF (SoDF), to deal with this problem. The main contribution of our work is to evaluate the performance of the SoDF technique for distributed turbo code by deriving very tight upper bounds for BER and BLER.

2. Efficient quantization techniques for a CF system were provided [con.5] and [con.8]. A practical WZ coding structure consisting of a quantizer and a soft multilevel Slepian Wolf (SW) compressor. We extended the rate-distortion quantization [68] into the relay scenario and proposed a new quantization measure which was based on the maximization of the achievable rate.
3. A new soft multilevel SW decoding structure was proposed [con.6]. Moreover, a turbo-like joint decoding and decompressing framework was also suggested, where the decoder and the decompressor cooperate through an iterative structure.

4. Multilevel coding was incorporated with the cooperative relaying concept to produce a hybrid relaying scheme, where the source has a layered encoding structure and DF/CF is deployed in different coding levels simultaneously. This scheme is denoted as hybrid relaying scheme I.

5. Another hybrid relaying scheme (hybrid relaying scheme II) capable of adapting itself to the current channel state was introduced, where different relaying schemes are deployed in a time-division manner based on the relay's decoding status [con.1]. We then analyzed the performance of hybrid relaying scheme II in terms of outage probabilities and expected throughputs. We noticed that the availability of CSI at the relay makes great impact to the system's performance and developed a more practical hybrid relaying scheme with partial CSI feedback.

6. We proposed two H2-ARQ-Relaying strategies for the relay system and analyzed their performance in terms of the obtained average airtime and throughputs [con.2]-[con.4], [jou.1]-[jou.2]. Then, we considered practical implementation issues by utilising the feedback channel between the destination and the relay to convey, in addition to the acknowledgement bit, a few extra bits carrying partial CSI and proposed modified H2-ARQ-Relaying strategies accordingly.

Some preliminary research on multiuser relay systems has also been addressed but not included in this thesis [con.9].

1.4 Publications

The study of this research has lead to a number of publications:


- [con.3] Yinan Qi, Reza Hoshyar, and Rahim Tafazolli, “A new ARQ protocol for Hybrid DF/CF relay scheme,” accepted by IEEE Vehicular Technology Conf. (VTC’09), Barcelona, Spain, April, 2009.

- [con.4] Yinan Qi, Reza Hoshyar, and Rahim Tafazolli, “Efficient ARQ protocol for Hybrid relay schemes with limited Feedback,” same as above.
CHAPTER 1 INTRODUCTION

- [con.5] Yinan Qi, Reza Hoshyar, and Rahim Tafazolli, “A Novel Quantization Scheme in Compress-and-Forward Relay System,” *same as above*.


- [con.9] Yinan Qi, Reza Hoshyar, and Rahim Tafazolli, “CDMA Based Multiuser Relay System Using Modified Turbo MUD through Conference Cooperation” *same as above*.

The following publications were also submitted and currently under review:


1.5 Thesis Outline

The thesis is divided into seven chapters, the first being this introduction in which the overview, objectives of the research, novel contributions and achievements are presented.

In chapter 2, we will briefly review the state of the art relaying techniques and establish the system model for the theoretical analysis throughout the paper. In addition, some important evaluation measurements which are useful indicators for the performance of a communication system will be presented.

Chapter 3 will begin with the derivation of the achievable rate of AF under relay protocol II. The amplification function will be optimized and the outage behavior will also be investigated. Furthermore, we will analyze the performance of RC- and UC-based DF and examine the optimal selection of the duplexing ratio. Both schemes will be analyzed in the context of synchronized and asynchronous scenarios respectively. Moreover, we will introduce the SoDF relaying scheme and provide the theoretical upper bounds to evaluate its BER and BLER performance.
In chapter 4, we will focus on the CF scheme. Starting from the theoretical analysis for RC-based CF, we will then introduce QF and compare their performance with UC-based CF. Afterwards, we will move to the practical implementation of WZ coding which consists of a quantizer and a SW encoder. A new quantization scheme will be proposed and compared with other schemes in the literature. A novel soft multilevel SW decoding algorithm and a turbo-like joint decoding and decompressing structure will be presented.

Chapter 5 will provide two hybrid relaying schemes. We will first investigate the hybrid relaying scheme using multilevel coding, denoted as hybrid relaying scheme I. Its achievable rate will be derived and its performance will be evaluated through simulation. Another hybrid relaying scheme where different relaying schemes are carried out in different frames according to the current channel realizations will be introduced in this chapter as well. Based on different levels of CSI availability at the relay, we will first provide two hybrid relaying schemes with full CSI and then develop a scheme under the assumption of limited CSI.

Two H2-ARQ-Relaying strategies with full channel knowledge at the relay will be proposed and the performance, in terms of the outage probabilities and obtained throughputs, will also be studied in chapter 6. Then we will consider the limited CSI case and propose two modified H2-ARQ-Relaying strategies with different levels of partial CSI feedback. The proposed modified strategies have very relaxed feedback requirements and with a few feedback bits for provisioning of partial CSI are able to operate close to their equivalent version with unlimited CSI feedback. The modified strategies allow for more flexible operations at the relay node according to its decoding status and the CSI feedback.

Finally, chapter 7 will conclude the thesis, provide suggestions, and propose ideas for future work for our research.
Chapter 2 System Model

2.1 Introduction

In wireless networks, relaying techniques have been traditionally used to extend the coverage of communication systems. However, in recent years, other relaying schemes to assist in the communications between the source and destination via some cooperation protocols have emerged. By controlling medium access between the source and the relay coupled with appropriate modulation or coding strategies in such cooperative schemes, it has been found that the diversity gain of the system can be improved. In particular, three basic relaying schemes have been proposed including amplify-and-forward (AF), decode-and-forward (DF) and compress-and-forward (CF).

Among various cooperative protocols, AF is one of the basic modes and has attracted lots of attention in recent years due to its simplicity and low expenses. Being a non-regenerative scheme, AF only allows the relay to amplify and forward the received signal to the destination without any coding or decoding process. Laneman has, in his landmark paper [5], pointed out that AF is able to achieve second-order diversity in very high SNR regions. A recent work indicated that the ergodic capacity of AF can be higher than that of DF with certain SNR settings [12]. In particular, when the source-relay link is statistically worse than the other two links, AF is able to outperform DF in capacity. In addition, the analytical SNR expression of AF is derived for the purpose of evaluation in terms of BER or FER [13]-[15], and the problem of optimal power allocation was also addressed [13].

Two basic relaying schemes have been introduced in the classic work of Cover and El Gamal [2]. One is decode-and-forward (DF), where the relay decodes the received message completely, re-encodes the message and fully or only partially forwards the decoded message to the destination. It has been proved that DF is able to achieve the capacity of the degraded relay channel. The information-theoretical analysis of the outage behavior in Laneman's work [5] indicated that fixed decode-and-forward (FDF), where the relay always forwards the decoded message even when the decoding is unsuccessful, does not achieve diversity. In contrast, selective DF in which the relay keeps silent when it is not able to decode the message achieves full cooperative diversity in high SNR regions. The diversity analysis was extended to multi-relay cases in [22]-[25], where a group of relays is deployed and various combining techniques including selection combining and maximum ratio combining (MRC) are used. Moreover, some works with focus on practical implementations of DF could be found in [16]-[17], [26], [29]. A conventional simple coding scheme is repetition coding (RC) [26], where the relay re-encodes the decoded message with an identical encoder as the source. The destination will receive two versions of the same signal, where one version is from the source directly and another is the re-encoded signal at the relay, and these two versions can be combined through some combination techniques. Authors of [26] to [28] have developed cooperative and distributed coding schemes, namely distributed turbo code (DTC), to further improve the performance. DTC is able to perform close to the theoretic outage probability bound of a
relay channel. In addition, some researchers provided efficient implementation of binning using bilayer low-density parity-check (LDPC) codes [29]-[32]. Generally speaking, both DTC and bilayer LDPC can be classified as UC.

Another basic relaying scheme is compress-and-forward (CF) which has drawn more and more attention recently [1]-[4], [8], [62]-[63]. In CF, the operation at the relay is quite different from DF. Instead of decoding, the relay compresses the received signal and transmits the compressed version to the destination. As a result, at the destination, the signal received from the source during phase 1 will serve as side information to reconstruct the relay's observation. Then the destination tries to decode the message by joint processing of the received signal and the reconstructed observation of the relay. To clarify the point, we assume an extreme case in which the relay and the destination can fully share their observations without any distortion. In such a case, the relay and destination can enjoy full receive diversity. In contrast to DF, which normally exhibits superior performance when the relay is close to the source, CF is desired under the condition of a weak source-relay link and a strong relay-destination link.

The rest of the chapters are organized as follows. In the next section, the system model, common definitions and some notation rules are presented. Some evaluation metrics used to demonstrate the performance of a relay scheme in a quantified manner are given in section 2.3 and the final section concludes.

2.2 Relay System Model

2.2.1 Notation Rules

Before starting to describe the system mode, we give a few words about the adopted notations and definitions in this thesis:

- Random variables are represented by upper-case letters, e.g. X. We use bold letters to represent vectors and matrices, e.g. X. Unless specified, the subscripts and superscripts of the considered signals denote nodes and relay-receive or -transmit phases respectively.

- An m-vector is defined as a vector with m elements. A random m-vector is an m-vector with m independent and identical distributed (i.i.d.) random elements.

- Only memoryless channel is considered. The mutual information between two random m-vectors X and Y is given by

\[
I(X;Y) = \sum_{k=1}^{m} I(X_k; Y_k)
\]
where \( k \) is the index, for \( 1 \leq k \leq m \). Now assuming two representative random variables \( X \) and \( Y \) that follow the same distribution as \( X_k \) and \( Y_k \), we have mutual information per element as \( I(X_k; Y_k) = I(X; Y) \). Then the mutual information can be expressed as

\[
I(X; Y) = mI(X; Y)
\]

This property can be easily checked to see its validity for the entropy and conditional entropy expressions. Since most of the analysis in this paper is based on these information measures, we use per element expressions and ignore the index \( k \) for simplicity.

- Sequences of a scalar \( x \) and a vector \( X \) are defined as \( x = (x_1, \ldots, x_n) \) and \( X = (X_1, \ldots, X_n) \) respectively. We also define truncated sequences as \( x''_n = (x_1, \ldots, x_n) \).
- In all the logarithm expressions, natural logarithm is used and thus nat but not bit is used as the unit of information.

### 2.2.2 Conventional Two-Node Communication System

We first consider a two-node communication system consisting of a transmitter and a receiver as shown in Figure 2.1.

![Figure 2.1. Two-node communication system](image)

We can borrow some definitions from [3]. The discrete channel can be denoted as \( (X, p(Y|X), Y) \), where \( X \) and \( Y \) represent two finite sets, \( p(Y|X) \) is the probability distribution on \( Y \), and \( X \in X \) and \( Y \in Y \). An \((M, N)\) code can be defined on this channel.

**Definition 1**: An \((M, N)\) code can be defined to consist of an integer set \( \mathcal{J} = \{1, 2, \ldots, M\} \), an encoding function \( f: \mathcal{J} \to \mathcal{X}^N \) and a decoding function \( g: \mathcal{Y}^N \to \mathcal{J} \).

**Definition 2**: If a message \( w \) belongs to \( \mathcal{J} \), the probability of decoding error is \( \lambda(w) = \Pr\{g(Y) \neq w\} \). The maximal probability of error for the \((M, N)\) code is defined as

\[
\lambda_* = \max_{w \in \mathcal{J}} \lambda(w)
\]

### 2.2.3 Relay System
The definitions can be extended to a relay system. The discrete relay channel is defined in [2] as $(X_s \times X_r, p(Y_n|Y_d, X_s, X_r), \mathbb{Y}_r \times \mathbb{Y}_d)$, where $X_s$, $X_r$, $\mathbb{Y}_r$ and $\mathbb{Y}_d$ represent four finite sets, $p(Y_n|Y_d, X_s, X_r)$ is the probability distribution on $\mathbb{Y}_r \times \mathbb{Y}_d$, $(X_s, X_r)$ is the input to the channel belonging to $X_s \times X_r$, and $(Y_n, Y_d)$ is the output of the channel belonging to $\mathbb{Y}_r \times \mathbb{Y}_d$. The relay channel not only has more inputs and outputs, but its error probability is also more complicated. We give the general definition of a code on a relay channel as follows.

**Definition 3**: An $(M, N)$ code on a relay channel can be defined as a code that consists of an integer set $\mathbb{J} = \{1, 2, \ldots, M\}$, an encoding function

$$f : \mathbb{J} \rightarrow \mathcal{X}^N$$

a set of relay functions

$$X_{r,n} = f_n(Y_1, Y_2, \ldots, Y_{n-1}) \quad n \leq N$$

and a decoding function $g : \mathbb{Y}^N \rightarrow \mathbb{J}$.

In this thesis, we consider the three-node relay system in Figure 2.2, where $c_0$, $c_1$ and $c_2$ represent the channel gains between the source and the destination, the source and the relay and the relay and the destination, all modeled by circularly symmetric complex Gaussian distribution (CSCGD) with zero mean and unit variance. In particular, we assume that the channels are quasi-static Rayleigh fading channels and the channel gains are constant within one frame and vary independently from one frame to another. In other words, this thesis is focused on the fixed relay scenario or low mobility relay scenario. However, Additive White Gaussian Noise (AWGN) channel is also considered for some particular cases. The whole frame is separated into two phases. The solid lines indicate the transmission in phase 1 and the dashed lines indicate phase 2. A message $w$...
belonging to a message set \( W = \{1, 2, \ldots, e^N\} \) is transmitted by the source with rate \( R \), where \( N \) is the number of symbols in one complete frame and \( R \) is the associated transmission rate in nats/use.

### 2.2.3.1 RC-based Relay System

We first investigate the case where RC is deployed. When RC is applied, the duplexing ratio \( \alpha \) is fixed at 0.5. During the first phase, the source broadcasts \( X_i \), and the received signals at the relay and destination are

\[
\begin{align*}
Y_i &= c_1 \sqrt{\mu_1} X_i + Z_i,
Y_i' &= c_2 \sqrt{\mu_2} X_i + Z_i',
\end{align*}
\]

(2.1)

respectively, where \( \mu_0, \mu_1 \) and \( \mu_2 \) represent the average SNRs of three links and \( Z_i \) and \( Z_i' \) are the noises at the relay and the destination respectively during phase 1 with i.i.d elements modeled by CSCGD with zero mean and unit variances \( \sigma_i^2 = 1 \) and \( \sigma_i'^2 = 1 \) respectively. Note that this is a normalized expression because the variance of the noise is set as 1.

For AF, the received signal \( Y_i \) is amplified with certain amplifying gain \( \beta \) and forwarded during phase 2. For DF, the relay tries to decode message \( w \) with \( Y_i \) and re-encodes \( w \) using the same encoder as the source. For CF, the relay quantizes and compresses the received signal into bin index \( s \), which will be encoded by \( X_{s,c} \). Here we allow the source to transmit the same signal as in phase 1 and the destination receives

\[
\begin{align*}
Y_d &= c_1 \sqrt{\mu_1} X_i + c_0 \sqrt{\mu_0} X_i + Z_d,
Y_d' &= c_2 \sqrt{\mu_2} X_i + Z_d',
\end{align*}
\]

(2.2)

where \( Z_d \) is the noise at the relay during phase 2 and follows the same statistic property as \( Z_d' \) and

\[
X_i^2 = X_i^1, \quad X_i^2 = \begin{cases} 
\beta Y_i, & \text{AF} \\
X_i^1, & \text{DF} \\
X_{s,c}^1, & \text{CF}
\end{cases}
\]

It follows that the destination received signal can be rewritten, respectively, as

\[
\begin{align*}
Y_d^2 &= \begin{cases} 
(\beta, c_1, c_2) \sqrt{\mu_1, \mu_2} X_i + \beta, c_2\sqrt{\mu_2} Z_i', & \text{AF} \\
(c_2, c_1) \sqrt{\mu_2} X_i + Z_d', & \text{DF} \\
c_0 \sqrt{\mu_0} X_i + c_2 \sqrt{\mu_2} X_{s,c}^2 + Z_d', & \text{CF}
\end{cases}
\]

(2.3)
For a synchronized system it is assumed that each node has been perfectly synchronized with respect to block, symbol and carrier. In such a case, for AF and DF schemes, the relay and the destination is able to perform beamformed transmission in the second phase. Therefore, the phase difference between different channel gains can be cancelled. However, carrier synchronization requires phase-locking-separated microwave oscillators which are very challenging in practical implementation. In light of this observation, we should take the asynchronous case into consideration as well. At the destination, the received signals during two phases are combined through some combination techniques such as MRC [10] and decoded for AF and DF. In CF, the destination first decodes $X_{c}^{2}$ to get the index $s$. Based on $s$, the observation of the relay is estimated. Then it cancels $c_2 \sqrt{\mu_2}X_{c}^{2}$ from $Y_{j}^{1}$ to get $\hat{Y}_{j}^{1} = Y_{j}^{1} - c_2 \sqrt{\mu_2}X_{c}^{2}$ and combines $Y_{d}^{1}, \hat{Y}_{d}^{1}$ and the estimation of the relay received signal to decode message $w$.

For the RC-based relay system, we define two codebooks:

1) $\mathcal{X}_c'(w)$: Each codeword $X_{c}^{1}(w) \in \mathcal{X}_c'(w)$ is a random $\alpha N$-vector following power constraint $\frac{1}{\alpha N} \sum_{n=1}^{N} |X_{c}^{1}(n)|^2 \leq 1$.

2) $\mathcal{X}_c^2(s)$: Each codeword $X_{c}^{2}(s) \in \mathcal{X}_c^2(s)$ is a random $(1-\alpha)N$-vector following the same power constraint, where $s$ is the index obtained after compression and will be detained later.

2.2.3.2 UC-based Relay System

Now we turn to the more general scenario, where UC is deployed and the duplexing ratio $\alpha$ can be flexible in the region $(0, 1]$. Note that UC is only proposed for DF and CF. For the DF scheme, the relay still attempts to decode the message. Upon successful decoding, in the synchronized scenario, the message will be re-encoded by $X_{d}^{2} = X_{d}^{1}$ which is different with $X_{c}^{1}$. If the system is asynchronous, the relay re-encodes the message with $X_{d}^{2} = X_{d}^{1}$, $X_{c}^{2}$. For CF, the process is same as the RC-based case and $X_{d}^{2} = X_{d}^{1}$ is transmitted during the second phase. At the same time, the source encodes message $w$ with a new codeword $X_{c}^{2}$ and transmits with the relay simultaneously. The signal received by the destination is given as

$$Y_{d}^{2} = c_0 \sqrt{\mu_0}X_{c}^{2} + c_2 \sqrt{\mu_2}X_{c}^{2} + Z_{d}^{2}$$

(2.4)

where

$$X_{d}^{2} = \begin{cases} X_{d}^{1}, & \text{syn DF} \\ X_{d}^{1}, & \text{asy DF} \\ X_{c}^{2}, & \text{CF} \end{cases}$$
In DF, the destination first combines $X_2$ and $X'_2$. The resultant signal will be used with $Y'_j$ for joint decoding of the main information. For CF, the destination starts from reconstructing the estimation of the relay’s observation and the resultant estimation, $Y'_j$ and $Y'_j$ are jointly processed to decode the message $w$.

Two codebooks are defined for the UC-based relaying system as follows.

1) $X_2^2(w)$: Each $X_2^2(w) \in X_2^2(w)$ is a random $(1-\alpha)N$-vector following the same power constraint.

2) $X_2^0(w)$: Each $X_2^0(w) \in X_2^0(w)$ is a random $(1-\alpha)N$-vector with the same power constraint and forms a MISO code with $X_2^j$.

It must be emphasized that this system model is only used for the purpose of theoretical analysis. That means no specific coding or modulation schemes are considered here. If practical design is addressed such as the practical CF system design in chapter 4, we will resort to a more detailed signal model.

2.3 Evaluation Metrics

We will introduce some important evaluation metrics used in this thesis. With the definitions for the two-node communication system, we have the following definition:

**Definition 4**: The rate of an $(M, N)$ code is defined as $R = \frac{1}{N} \log M$ nats/transmission. $R$ is said to be **achievable** if for any $\epsilon > 0$, there exists an $(M, N)$ code with $M \geq e^{NR}$ such that $\lambda_m < \epsilon$ if $N$ is sufficiently large.

For a block fading channel with instantaneous SNR $\gamma$, rate $R$ is achievable if

$$ R \leq C(\gamma) = \log(1 + \gamma) $$

If both nodes have the instantaneous SNR knowledge, it is possible to transmit with rate $R = C(\gamma)$. However, the channel state information (CSI) is usually not assumed to be available at the transmitter such that the transmission rate $R$ cannot be properly tuned. In such a case, since the channel suffers from fading from time to time, even with small $R$, there is some possibility that the selected rate is not supported by the current channel realizations. An outage event happens when $R > C(\gamma)$. The **outage probability** is therefore defined as

$$ P^o = \Pr\{R > C(\gamma)\} $$

For a block fading channel, the outage probability is

$$ P^o = \Pr\{R > \log(1 + \gamma)\} $$
Since $\gamma$ is a random variable following exponential distribution, the outage probability has to be averaged over all outage regions as

$$P_{\text{out}} = \int_0^{\infty} p(\gamma) d\gamma$$

where $p(\gamma)$ is the pdf of $\gamma$.

The achievable rate and outage probability are good indicators for the performance of a communication system from the theoretical point of view. In practical implementation, a more straightforward indicator is the error rate. In telecommunications, an error rate is the ratio of the number of bits or blocks incorrectly received to the total number of bits or blocks transmitted, denoted as bit error rate (BER) and block error rate (BLER) respectively. These two rates can be obtained through simulation. But sometimes, we can derive the theoretical expressions for BER or BLER.

When analyzing the error performance, it will be assumed that, without loss of generality, the all-zero codeword $C_0$ is transmitted. An error event will occur when a codeword $C_c$ with non-zero weight is decoded. For the block fading channel, the error probability is

$$\Pr(C_0 \rightarrow C_c | \gamma) = Q(\sqrt{2n_H} \gamma)$$

where $n_H$ is the Hamming weight of $C_c$ and $Q(x)$ is the Gaussian Q-function. The Pairwise Error Probability (PEP) is the probability that $C_c$ with specific weight is decoded when a zero codeword is transmitted. We should consider all possible $C_c$ to get the overall error probability by using the Weight Enumerating Function (WEF). The detailed derivation can be referred to chapter 3.

For a relay system, we have the relay functions appearing in the definition of the $(M, N)$ code. With the same definition for the achievable rate, it is reasonable to expect that the achievable rate has a more complicated expression. For the block fading channel, it is apparent that computation of the outage probability is complicated due to the fact that there are three SNRs and the transmission mechanism is more sophisticated. It is an interesting issue requiring further insight. Moreover, according to Laneman's work, outage probability can serve as an indicator for diversity. Therefore, it is of great worth to analyzing.

The BER and BLER analysis could be difficult for relaying schemes. The DF-based relay system with DTC can be evaluated through analytically derived upper bounds of BER and BLER but for other schemes, we have to resort to simulation to evaluate the performance.

Moreover, we also should consider spectrum-efficiency, energy-efficiency and complexity of the relay system. Basically, the novel techniques we proposed focused on the spectrum-efficiency aspect. However, complexity is also an important issue.
2.4 Conclusions

In this chapter, we briefly reviewed the state of the art techniques for the relay system and introduced some preliminary definitions, notation rules and the system model for the theoretical analysis throughout the thesis. In addition, some important evaluation metrics which are useful indicators for the performance of the communication system from the information-theory or implementation points of view were presented.

Three relaying schemes including AF, DF and CF were introduced in this chapter. AF is a non-regenerative scheme because the relay does not attempt to decode or compress the message. DF and CF can both be classified as regenerative schemes as the relay tries to decode or compress the received message. The system model with two coding schemes (RC and UC) was presented. Our work focused on relay protocol II where the source is allowed to transmit during the second phase. Synchronization problem was also addressed and in the following chapters, the effect of synchronization on the system’s performance will be demonstrated.

To evaluate the performance of the relay system from the information-theory point of view, we introduced two metrics: the achievable rate and outage probability. It has been shown that for a relay channel, these two metrics require more complicated calculations because, as opposed to a two-node communication system where only one link is considered, we need to take all the three links into consideration. We also introduced BER and BLER to evaluate the performance of the relay system and pointed out that although it is possible to derive the expressions for BER and BLER for some relaying schemes, we have to resort to simulation to evaluate these two metrics because some of the relaying schemes are very difficult to be analyzed theoretically.
Chapter 3 Amplify-and-forward, Decode-and-forward and Soft Decode-and-forward

3.1 Introduction

Before the recent emergence of research interests shown in CF, AF and DF have been the focus of the study in relay-assisted communication systems. In this chapter, the performance of AF is analyzed in the context of relay protocol II, where the source is involved in the transmission during phase 2. The optimization problem of the amplification function is highlighted and the effect of phase synchronization is examined. It comes to our attention that the optimization of the amplification function will depend on the availability of CSI at the relay. Therefore, with different levels of CSI availability, the optimal amplification function will have different forms. Regarding the DF scheme, we start from the derivation of the achievable rate and outage behavior for the RC-based relay system. The UC-based DF scheme has been investigated in [4]. The emphasis of the theoretic analysis is on the optimization of the duplexing ratio and the phase synchronization problem.

The implementation issues of the DF scheme are also discussed. Distributed turbo codes (DTC) have been developed and proved to perform close to the theoretic outage probability bound of a DF-based cooperative communication system [26]-[28]. However, in some of the existing analysis of the distributed turbo coding schemes, it is normally assumed that the source-relay link is error-free. But in a practical environment, this link usually experiences block fading from time to time and sometimes its instantaneous SNR is not high enough to ensure the error-free decoding at the relay. In cases of decoding failure, the relay obtains erroneous messages and the forwarding of erroneous messages will lead to error propagation at the destination. An alternative distributed turbo coding structure, namely soft decode-and-forward (SoDF), has been proposed to deal with this problem [33]-[35]. Instead of making hard decisions, the relay decodes the received signal with a soft-input-soft-output (SISO) decoder and calculates a posteriori probabilities (APP) of the information symbols. Then the output soft values are interleaved and the relay performs soft encoding by calculating the soft values of the parity check symbols based on the APP of the information symbols and the soft values are forwarded to the destination. The simulation results in [33] and [34] indicated that SoDF is able to mitigate the error propagation in practical relay systems using distributed turbo codes. The main objective of our work is to derive the upper bounds of the SoDF scheme's error rates. With these bounds, there is no need to resort to a large amount of simulation to evaluate the performance of a particular DTC as in [33] and [34]. Using the derived upper bounds, we are able to reliably predict the error performance of a SoDF system.

The rest of the chapters are organized as follows. The achievable rate of AF under protocol II is derived in the next section, where the amplifier gain function is optimized. The outage behavior and the influence of the channel state information (CSI) availability on the outage probabilities are analyzed in the next section as well. In section 3.3, we begin with the derivation of the achievable rate and the analysis of the outage behavior for
RC-based DF and extend the analysis to the more general UC case. In section 3.4, SoDF scheme is introduced and its upper bounds of both BER and BLER are derived. Finally, concluding remarks are drawn in section 3.5.

3.2 Achievable Rate and Outage Behavior of AF

Suppose a frame consists of \( N \) symbols (\( N \) is assumed to be an even integer), the duplexing ratio \( \alpha \) is fixed at 0.5 and the duration of the first phase is exactly half of the whole block. Recall (2.3) in the previous chapter, the destination received signal can be rewritten as

\[
Y_d = (\beta, c, c_1 \sqrt{\mu_1} + c_2 \sqrt{\mu_2}, X_1 + \beta, c_1 \sqrt{\mu_1}, Z_1 + Z_2)
\]

The amplification gain \( \beta \) is subject to the power constraint

\[
\beta \leq \sqrt{\frac{1}{\gamma_i + 1}}
\]

where \( \gamma_i = |c_i|^2 / \mu_i \) are the instantaneous SNRs in the source-destination, source-relay and relay-destination links for \( i = 0, 1 \) and 2 respectively.

3.2.1 Achievable Rate of AF

In Laneman's protocol [5], the source is not involved in the transmission during the second phase and the achievable rate is maximized when the relay transmits with full power. In this section, we will investigate the more general model for protocol II and allow the source to transmit in phase 2.

The mutual information of AF with relay protocol II is derived in Appendix A and given as

\[
l(X_i; Y_d) = \log \left(1 + \gamma_0 \frac{\beta, c, c_1 \sqrt{\mu_1} + c_2 \sqrt{\mu_2}}{1 + \beta, \gamma_2} \right) = \log \left(1 + \gamma_0 \frac{\beta, \sqrt{\gamma_2} Y_2 e^{j \theta_2} + \sqrt{\gamma_2} Y_2}{1 + \beta, \gamma_2} \right)
\]

(3.1)

where \( \theta_i \) is the phase of the channel gain \( c_i \) which follows uniform distribution in the region \([-\pi, \pi]\), and

\[
Y_d = [Y_d, Y_d^2]^T.
\]

Let \( \theta = \theta_0 \theta_1 \theta_2 \), if each node in the system has been synchronized in phase, \( \theta \) is zero and the mutual information is given as

\[
l(X_i; Y_d) = \log \left(1 + \gamma_0 + \frac{\beta, \sqrt{\gamma_2} Y_2 + \sqrt{\gamma_2} Y_2}{1 + \beta, \gamma_2} \right) = \log(1 + \gamma_0 + l(\beta,))
\]

(3.2)
We need to maximize the above equation by optimizing the selection of $\beta_r$. Since it is not straightforward to ascertain if $f(\beta_r)$ is an increasing or decreasing function of $\beta_r$, we resort to its first order derivative, which can be expressed as

$$
\frac{\partial f(\beta_r)}{\partial \beta_r} = \frac{2\beta_r \sqrt{\gamma_1 \gamma_2}}{1 + \beta_r^2 \gamma_1^2}
$$

(3.3)

The second term in the nominator will decide whether the first order derivative is positive or negative. If $\beta_r < \sqrt{\gamma_1 / \gamma_2 \gamma_3}$, $f(\beta_r)$ increases with $\beta_r$ and is maximized when $\beta_r = \sqrt{\gamma_1 / \gamma_2 \gamma_3}$. Otherwise, $f(\beta_r)$ decreases with $\beta_r$ and the maximal value of $f(\beta_r)$ is achieved when $\beta_r = 0$. However, since $\beta_r$ is constrained in the region $[0, \sqrt{1/(1 + \gamma_1)]}$, this maximal value is not always achievable in this region. If $\sqrt{\gamma_1 / \gamma_2 \gamma_3}$ is in this region as shown in Figure 3.1(a), $f(\beta_r)$ can be maximized at the point $\beta_r = \sqrt{\gamma_1 / \gamma_2 \gamma_3}$. But if $\sqrt{\gamma_1 / \gamma_2 \gamma_3}$ is outside the region as shown in Figure 3.1(b), $f(\beta_r)$ is a monotonically increasing function of $\beta_r$ and can be maximized by letting $\beta_r = \sqrt{1/(1 + \gamma_1)}$. Hence, the mutual information is given as

$$
I_{AF,syn} = \begin{cases} 
\frac{1}{2} \log(1 + \gamma_0 + f(\sqrt{\gamma_1 / \gamma_2 \gamma_3})), & \gamma_1 \leq \sqrt{\gamma_0 \gamma_2 + \frac{1}{4} - \frac{1}{2}} \\
\frac{1}{2} \log(1 + \gamma_0 + f(\sqrt{1/(1 + \gamma_1)})), & \gamma_1 > \sqrt{\gamma_0 \gamma_2 + \frac{1}{4} - \frac{1}{2}} 
\end{cases}
$$

(3.4)
The rationale behind (3.4) can be explained as follows. If the link between the source and the relay is above a certain threshold, the relay’s observation is of better quality and it is desirable to transmit the observation with greater power. Consider an extreme case where the relay is absolutely confident about what it has received from the source, i.e. $y_1$ approaches infinity. The relay should transmit with maximal power as indicated by the second case of (3.4). On the contrary, if the source-relay link is not that good, the relay should control its transmitting power. Consider another extreme case where the relay receives nothing useful from the source, i.e. $y_1$ approaches 0. There is no need to transmit the amplified noise to the destination and the relay should keep silent.

We have obtained the achievable rate of AF in the phase synchronization scenario. But the phase synchronization through phase-locking is very challenging in practical systems. In light of this implementation problem, we should also consider the asynchronous case where $\theta$ is a random variable and follows uniform distribution in the region $[-\pi, \pi)$. In such a case, the mutual information should be averaged with respect to $\theta$ and given as (see Appendix A for proof)

$$I_{af}(\beta_1, \gamma) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \log \left( 1 + \gamma_0 + \frac{\beta_1 \sqrt{y_1^2 + y_0^2}}{1 + \beta_1^2 y_2} \right) d\theta$$

where $\gamma = [y_0, y_1, \gamma_1]^T$. Since the second term in the above equation is in the region $[-\log 2, 0]$, we can define a trivial upper bound of (3.5) by ignoring the second term as

$$I_{af}^{\text{ub}} = \log \left( 1 + \gamma_0 + \frac{\beta_1 \sqrt{y_1^2 + y_0^2}}{1 + \beta_1^2 y_2} \right)$$

Although $\beta_1$ should be optimized to maximize (3.5), this equation is too complicated to be optimized. Hence with a bounded second term, our optimization is based on the trivial upper bound $I_{af}^{\text{ub}}$ only. For $\beta_1$, we rewrite $I_{af}^{\text{ub}}$ as

$$I_{af}^{\text{ub}} = \log \left( 1 + \gamma_0 + \frac{\gamma_1 + y_1}{1 + \beta_1^2 y_2} \right) \tag{3.6}$$

It is easy to see that the optimal selection of $\beta_1$ is actually related with instantaneous SNRs $y_0$ and $y_1$. Define destination and relay SNR ratio $\rho$ as $\gamma_0/\gamma_1$. If $\rho$ is larger than 1, $I_{af}^{\text{ub}}$ is a decreasing function of $\beta_1$ and $\beta_1$ should
be minimized as zero; otherwise, $\beta_0$ should be maximized. In other words, if the link between the source and the
destination is better than the source-relay link, the optimal amplification strategy of the relay is to keep silent;
otherwise, the relay should forward the received signal with full power and the achievable rate is

$$I_{AF_{\text{say}}} = \begin{cases} \frac{1}{2} \log(1 + 2\gamma_0), & \rho < 1 \\ \frac{1}{2} \log \left( \frac{1}{\sqrt{\gamma_1 + 1}} \right), & \rho \geq 1 \end{cases}$$

In both the synchronized and the asynchronous scenarios, the relay does not always amplify the received signal
with maximal power as in Laneman's protocol. Instead, the relay carefully selects its transmitting power
according to the current channel realizations. The intuition behind its selection is based on the quality of its
observation: the better the quality, the greater the power.

### 3.2.2 Outage Behavior of AF

With the derived achievable rate, the outage probability can be expressed as,

$$P_{AF}^{\text{out}}(R) = \Pr \{ I_{AF} < R \}$$

It must be emphasized that, as opposed to the achievable rate analysis, where the CSI (including $\gamma_1$ and $\theta$) is
known globally, in outage analysis it is assumed that CSI is available to the receiver's side only. In Laneman's
protocol, where the amplification function is a fixed function of $\gamma_1$, it is possible to get the optimal $\beta_0$, without
knowing the global CSI. However, in protocol II, to optimize $\beta_0$, $\gamma_0$ and $\gamma_2$ are needed at the relay. If a feedback
link is established between the destination and the relay, $\gamma_0$ and $\gamma_2$ can be available at the relay and the optimal
$\beta_0$ can be obtained. The outage probability under this assumption will be investigated first.

#### 3.2.2.1 Outage Behavior of AF with Feedback of CSI

For the synchronized case, according to the current channel realization, the outage events can be separated into
two exclusive classes. One class is

$$\{ I_{AF_{\text{say}}} < R \} \cap \{ \gamma_1 + \gamma_1^2 \leq \gamma_0 \gamma_2 \}$$

The probability of this class is given as

$$\Pr \{ \{ I_{AF_{\text{say}}} < R \} \cap \{ \gamma_1 + \gamma_1^2 \leq \gamma_0 \gamma_2 \} \}$$

Another class of outage events is

$$\{ I_{AF_{\text{say}}} < R \} \cap \{ \gamma_1 + \gamma_1^2 > \gamma_0 \gamma_2 \}$$

(3.7)
The corresponding outage probability is given as

$$\Pr\{\{I_{AF,\text{syn}} < R\} \cap \{\gamma_1 + \gamma_2^2 \leq \gamma_0 Y_2\}\}$$

(3.8)

Note that both \(\gamma_0\) and \(\gamma_2\) should be fed back to the relay because \(\beta_r\) is a function of them. Here we assume that \(R\) is not known to the relay, otherwise, the outage probability will be degraded to the direct transmission case.

For the asynchronous case, we first consider the situation in which the source-destination link is of better quality than the link between the source and the relay \((\rho>1)\). In such a case, \(\beta_r\) should be minimized as 0. The outage event is

$$\{I_{AF,\text{asy}} < R\} \cap \{\rho > 1\}$$

The corresponding probability is,

$$\Pr\{\{I_{AF,\text{asy}} < R\} \cap \{\rho > 1\}\} = \frac{\mu_0}{\mu_0 + \mu_1} \left(1 - \exp\left(-\frac{\mu_0 + \mu_1}{\mu_0 \mu_1} \frac{e^{2\rho}-1}{2}\right) \right) - \exp\left(-\frac{e^{2\rho}-1}{2\mu_0}\right) \left(1 - \exp\left(-\frac{e^{2\rho}-1}{2\mu_1}\right)\right)$$

(3.9)

If \(\rho\) is smaller than 1, \(\beta_r\) should be chosen as \(\sqrt{1/(1+\gamma_1)}\) to maximize \(I_{AF,\text{asy}}\) and the corresponding outage event is

$$\{I_{AF,\text{asy}} < R\} \cap \{\rho < 1\}$$

The outage probability can be calculated as

$$\Pr\{\{I_{AF,\text{asy}} < R\} \cap \{\rho \geq 1\}\}$$

(3.10)

Here optimal \(\beta_r\) is a function of \(\gamma_1\) conditioned to the fact that \(\rho\) is smaller than 1. The feedback mechanism can be very simple. At the destination, 1 bit is used to indicate if \(\rho\) is smaller than 1 and this bit is fed back to the relay. This feedback bit provides the information about how to decide optimal \(\beta_r\).

Since the two classes of events are mutually exclusive, the overall outage probability is the summation of two probabilities, given as

$$p_{\text{out}}^{AF} = \begin{cases} \Pr\{\{I_{AF,\text{syn}} < R\} \cap \{\gamma_1 + \gamma_2^2 > \gamma_0 Y_2\}\} + \Pr\{\{I_{AF,\text{asy}} < R\} \cap \{\gamma_1 + \gamma_2^2 \leq \gamma_0 Y_2\}\}, & \text{syn} \\ \Pr\{\{I_{AF,\text{asy}} < R\} \cap \{\rho < 1\}\} + \Pr\{\{I_{AF,\text{asy}} < R\} \cap \{\rho \geq 1\}\}, & \text{asy} \end{cases}$$

(3.11)
3.2.2.2 Outage Behavior of AF without Feedback of CSI

Without a feedback link, $y_0$ and $y_2$ are not available at the relay and therefore the optimal $\beta_r$ cannot be obtained. However, the optimization principle of $\beta_r$ is still valid: the better the quality of the received signal at the relay, the greater the transmitting power. In this regard, we can use a simple selection mechanism by incorporating a pre-set threshold $y_u$ and define the amplifier function as

$$\beta_r = \begin{cases} \sqrt{1/(1+y_1)}, & y_1 \geq y_u \\ 0, & y_1 < y_u \end{cases}$$

(3.12)

The threshold can be obtained numerically. After gaining some insight from the numerical results, we can choose $y_u = (e^{2\theta} - 1)/2$. With (3.12), we rewrite (3.7) to (3.10) as

$$\Pr\{\{I_{AF,om} < R\} \cap \{y_1 < y_u\}\} = \Pr\{\{I_{AF,om} < R\} \cap \{y_1 < y_u\}\} = \left[1 - \exp\left(-\frac{e^{2\theta} - 1}{2\mu_0}\right)\right] \left[1 - \exp\left(-\frac{y_u}{\mu_1}\right)\right]$$

(3.13)

The outage probabilities are given in a similar way as (3.11).

3.2.3 Simulation Results and Discussions for AF

Figure 3.2 suggests that the achievable rate of AF is closely related with the SNRs of the source-relay and relay-destination links. If either link is of poor quality, AF performs worse than direct transmission. This observation can be explained as follows. On the one hand, the gain of AF stems from the fact that the destination receives more than one versions of the same signal in two phases and is able to enjoy the diversity gain. The quality of the version forwarded by the relay is determined by $y_1$ and $y_2$. On the other hand, AF suffers from the bandwidth efficiency loss due to the usage of RC. If either of $y_1$ and $y_2$ is small, the quality of the relay transmitted version becomes poor. Therefore, the gain from diversity is relatively small and is offset by the bandwidth efficiency loss, thus leading to a lower achievable rate than direct transmission. But if both $y_1$ and $y_2$ are large, the diversity gain dominates the system’s performance and AF demonstrates a higher achievable rate. Figure 3.2 also indicates that, compared with Laneman’s protocol, allowing the source to transmit during the second phase improves the achievable rate.
Chapter 3 Amplify-and-forward, Decode-and-forward and Soft Decode-and-forward

Figure 3.2. Achievable Rate ($y_2=0$ dB, the left and right ones are from the front and reverse angles respectively)

We also depict the achievable rates with different $y_2$ in Figure 3.3, which are transverse sections of the two dimensional Figure 3.2. In any circumstance, AF with synchronized phase outperforms other AF schemes. Without phase synchronization, protocol II AF is still better than protocol I AF and is close to the trivial upper bound. When $y_1$ is smaller than 0 dB, the achievable rate of asynchronous protocol II AF does not change with $y_1$ because the optimal $\beta_s$ is 0 and the relay is silent. When $y_2$ is very small (-10 dB), Figure 3.3(a) suggests that direct transmission is the best choice. When $y_2$ is increased to 0 dB, AF with synchronized phase is able to outperform direct transmission. If we increase $y_2$ further to 20 dB, all of the AF schemes show considerable gains over direct transmission when the source-relay link is in the high SNR region.

We present the outage performance of different AF schemes with or without CSI feedback in Figure 3.4. All of the AF schemes have the same slope which indicates the diversity. The fact that a factor of 10 decrease in the outage probability requires additional 5 dB of SNR for all AF schemes means that the protocol II AF schemes enjoy the same second-order diversity as protocol I in high SNR regions. However, when we allow the source to transmit during phase 2, the outage probability can be further reduced. The availability of CSI at the relay actually has little influence on the outage behavior as the outage probability with CSI is very close to that of no CSI case, where the optimal $\gamma_{th}$ is chosen as $(e^{2R} - 1)/2$. $\gamma_{th}$ can be achieved by numerical methods and the result is compared with our setting as shown in Figure 3.5. The numerical results suggest that $\gamma_{th}$ should be exact equal to $(e^{2R} - 1)/2$ for a wide range of rate $R$. 

25
Figure 3.3. Achievable Rate. $\gamma_0=0$ dB (syn: synchronized; asy: asynchronous; UB: trivial upper bound (3.6)). (a) $\gamma_2=-10$ dB, (b) $\gamma_2=0$ dB, (c) $\gamma_2=20$ dB.

Figure 3.4. Outage Probabilities. $\mu_2=\mu_1=\mu_2$ dB, $R=0.1$. Figure 3.5. $\gamma_L$. Numerical Results VS $(e^{2R}-1)/2$. 26
3.3 Achievable Rate and Outage behavior of DF

3.3.1 Repetition Coding (RC)

For the RC-based DF scheme, the same encoder is deployed at both the source and the relay and the duplexing ratio \( \alpha \) is fixed at 0.5 due to the usage of RC. Suppose the length of a frame is \( N \) (still, \( N \) is assume to be an even integer here), in the first phase, the average mutual information between the source and the relay is,

\[
I(X'_1; Y'_1) = \log(1 + r_1)
\]

The relay can successfully decode the message if

\[
R \leq \frac{1}{2} I(X'_1; Y'_1)
\]

Upon successful decoding, the relay then re-encodes the message with the same encoder as the source. During phase 2, the relay transmits the encoded message which is exactly same as the signal transmitted by the source in the first phase. Given that relay protocol II comes into consideration here, we allow the source to transmit the same codeword simultaneously. Similar to the derivation in AF schemes, the mutual information is derived in Appendix A and given as

\[
I(X'_1; Y_d) = \log \left(1 + r_0 + \sqrt{y_1^2 + y_0^2} \right)
\]

(3.14)

where \( Y_d = [y'_1, y'_0]^T \), \( \theta_a \) is the difference between the phase of the relay-destination link and the source-destination link. If the relay and the source have been synchronized in phase before transmission, i.e. the phase difference has been compensated prior to transmission, \( \theta_a \) is zero and \( R \) can be achieved if

\[
R \leq \frac{1}{2} I(X'_1; Y_d) = \frac{1}{2} \log \left(1 + r_0 + \sqrt{y_1^2 + y_0^2} \right)
\]

Combine two equations and the achievable rate is the minimum,

\[
C_{RC,am} = \min \{C'_1, C'^2_{RC,am}\}
\]

(3.15)

where

\[
C'_1 = \frac{1}{2} \log(1 + r_1), \quad C'^2_{RC,am} = \frac{1}{2} \log \left(1 + r_0 + \sqrt{y_1^2 + y_0^2} \right)
\]
If the transmission of the relay and the source is not synchronized in phase, $\theta_s$ is modeled by uniform distribution in the region $[-\pi, \pi)$ and the mutual information should be averaged with respect to $\theta_s$ as

$$I(X'_s; Y_s) = \frac{1}{2\pi} \int \log \left(1 + \gamma_s + \sqrt{\gamma_s e^{-j\theta_s} + \sqrt{\gamma_s^*}} \right) d\theta_s = \log (1 + 2\gamma_s + \gamma_s^*) + \log \left(1 + \frac{1}{2} \sqrt{1 - \frac{4\gamma_s \gamma_s^*}{(1 + 2\gamma_s + \gamma_s^*)^2}} \right)$$

where the second term is in the region $[-\log 2, 0]$ and therefore, the average mutual information is upper bounded by the first term. Comparing the mutual information of the synchronized and asynchronous cases, we can easily conclude that the synchronized transmission has larger average mutual information. The achievable rate in the asynchronous scenario is given as

$$C^i_{sc, asy} = \min \{C^i_s, C^i_{sc, asy} \}$$

where

$$C^i_{sc, asy} = \frac{1}{2} \log \left(\frac{1 + 2\gamma_s + \gamma_s^* + \sqrt{(1 + 2\gamma_s + \gamma_s^*)^2 - 4\gamma_s \gamma_s^*}}{2} \right)$$

In order to gain a thorough understanding of the DF scheme's outage behavior, we need to examine two decoding structures at the relay. Firstly, the relay is required to re-encode and forward the received message without considering whether the message can be successfully decoded or not, namely fixed decode-and-forward (FDF). Secondly, we give the relay more flexibility by allowing it to keep silent if it is not able to decode the message, termed as selective decode-and-forward (SDF).

Since the link between the source and the relay is not error-free, there is some possibility that the number of errors is beyond the error correction capability of the decoder at the relay. In such a case, for FDF, the relay will forward erroneous messages to the destination and the error will propagate in the decoding of the destination. The failure of the destination's decoding is a direct consequence of the unsuccessful decoding at the relay. Hence, the outage probability of such events is the probability of erroneous decoding at the relay and can be given by

$$\Pr \{C^i_s < R \} = \Pr \left\{ \frac{1}{2} \log (1 + \gamma_s) < R \right\} = 1 - \exp \left( -\frac{e^{2\gamma_s} - 1}{\mu} \right)$$

If the relay successfully decodes the message, there is still a possibility that the destination detect errors. We consider the synchronized and asynchronous cases together and define

$$C^i_{sc, asy} = \min \{C^i_s, C^i_{sc, asy} \}$$

(3.16)
Chapter 3 Amplify-and-forward, Decode-and-forward and Soft Decode-and-forward

The outage probability at the destination is given as

\[ \Pr\{C_{sc}^2 < R | C_i^1 \geq R\} = \frac{\Pr\{C_{sc}^2 < R, C_i^1 \geq R\}}{\Pr\{C_i^1 \geq R\}} = \Pr\{C_{sc}^2 < R\} \]

Here \( C_{sc}^2 \) is a function of \( \gamma_0 \) and \( \gamma_2 \) and \( C_i^1 \) is a function of \( \gamma_1 \) only. Since the two parts are statically independent to each other, the conditioned probability can be simplified. It follows that the overall outage probability can be expressed as in [5]

\[
P_{out} = \Pr\{C_i^1 < R\} + \Pr\{C_i^1 \geq R\} \Pr\{C_{sc}^2 < R | C_i^1 \geq R\} = \Pr\{C_i^1 < R\} + \Pr\{C_i^1 \geq R\} \Pr\{C_{sc}^2 < R\}
\]

(3.17)

For SDF, during the second phase the relay does not transmit if detecting errors. Therefore, the source is the only active node and it sends the same codeword for the second time. Letting \( \gamma_2=0 \) in \( C_{sc}^2 \), the average mutual information is given as

\[ I = \frac{1}{2} \log(1 + 2\gamma_0) \]

In such a case, the outage event is equivalent to the event \( \{I < R\} \cap \{C_i^1 < R\} \). If the relay has been able to decode the message, the destination is able to enjoy mutual information \( C_{sc}^2 \) and the outage event is \( \{C_{sc}^2 < R\} \cap \{C_i^1 \geq R\} \). Since the two event classes are mutually exclusive, the overall outage event probability can be expressed as the summation of probabilities of two outage classes, namely

\[
P_{out}^{SDF} = \Pr\{I < R, C_i^1 < R\} + \Pr\{C_{sc}^2 < R, C_i^1 \geq R\} = \Pr\{I < R\} \Pr\{I < R\} + \Pr\{C_{sc}^2 < R\} \Pr\{C_i^1 \geq R\}
\]

(3.18)

The closed-form distribution of \( C_{sc}^2 \) is difficult to be calculated directly. However multiple integrations can help us to approach the answer. Comparing the outage probabilities of FDF and SDF, SDF has a smaller first term and the same second term, which means that SDF has better outage performance because the strategy guarantees that its performance will never be worse than direct transmission. However, it is important to point out that RC suffers from potential loss in bandwidth efficiency because the same codeword is transmitted in two phases repeatedly.

3.3.2 Unconstrained Coding (UC)
Chapter 3 Amplify-and-forward, Decode-and-forward and Soft Decode-and-forward

Compared with RC, unconstrained coding (UC) is more efficient in the sense that rather than MRC, it performs joint decoding at the destination and the duplexing ratio $\alpha$ can be optimized according to the current channel realizations. As depicted in Figure 3.6, a different encoder $C_r$ is deployed at the relay and the destination is able to perform joint decoding.

![Figure 3.6. Decode-and-forward with unconstrained coding.](image)

According to [4], the achievable rate has the similar form as RC and is given by the minimum,

$$C_{UC,\text{syn}} = \max_{\alpha} \min\{C_1, C_{UC,\text{syn}}^2\}$$

(3.19)

where

$$C_1 = \alpha \log(1 + \gamma_1), \quad C_{UC,\text{syn}}^2 = \alpha \log(1 + \gamma_2) + (1 - \alpha) \log\left(1 + \sqrt{\gamma_2^2 + \gamma_1^2}\right)$$

Note that the duplexing ratio $\alpha$ is not fixed as in RC and should be optimized to maximize the achievable rate. In order to get the optimal $\alpha$, we will separate the whole space into two parts: $\gamma_1 < \gamma_0$ and $\gamma_1 > \gamma_0$. If $\gamma_1 < \gamma_0$, we have

$$C_1' < C_{UC,\text{syn}}^2 = \alpha \log(1 + \gamma_2) + (1 - \alpha) \log\left(1 + \sqrt{\gamma_2^2 + \gamma_1^2}\right)$$

The achievable rate is

$$C_{UC,\text{syn}} = \max_{\alpha} \min\{C_1', C_{UC,\text{syn}}^2\} = \max_{\alpha} \alpha \log(1 + \gamma_1) = C_{UC} = \log(1 + \gamma_0)$$

30
where $C_{sd}$ is the mutual information for direct transmission. To maximize $C_{sc}$, $a$ should be 1. If $\gamma_1 > \gamma_0$, $C^i_{s}$ is an increasing function of $a$ and $C^2_{sc}$ is a decreasing function of $a$. As depicted in Figure 3.7, the maximum of (3.19) is achieved when $C^i_{s} = C^2_{sc}$. 

The optimal $a$ is given by 

$$a_{opt} = \log\left(1 + \left(\sqrt{\gamma_0} + \sqrt{\gamma_2}\right)^2\right) / \log\left(1 + \gamma_1\right) \left(1 + \left(\sqrt{\gamma_0} + \sqrt{\gamma_2}\right)^2\right) / (1 + \gamma_0)$$ 

(3.20)

The optimal $a$ is a decreasing function of $\gamma_1$ with fixed $\gamma_0$ and $\gamma_2$ and an increasing function of $\gamma_2$ with fixed $\gamma_0$ and $\gamma_1$. This implies that the length of phase 1 should be shortened if the source-relay link is improved and prolonged while the relay-destination link is getting stronger. The rationale behind is that when the source-relay link becomes better, the same amount of information can be transmitted within less time through this link. Thus the duration of phase 1 is reduced. When the relay-destination link gets better, less time is required when transmitting the same amount of information from the relay to the destination. Hence the length of phase 2 can be shortened, i.e. phase 1 is prolonged.

If the source and the relay are not synchronized, the relay transmits $X^r_{\alpha}$ instead to form a MISO code with the source transmitted codeword. In such a case, the achievable rate is given by 

$$C_{sc,as} = \max_{a} \min\{C^i_{s}, C^2_{sc}\}$$

(3.21)

where
\[ C_{uc,asy}^1 = \alpha \log(1 + \gamma_s) + (1 - \alpha) \log(1 + \gamma_s + \gamma_i) \]

The optimal \( \alpha \) is given as

\[ \alpha_{opt} = \frac{\log(1 + \gamma_s)}{\log\left( \frac{1 + \gamma_s}{1 + \gamma_s + \gamma_i} \right)} \]

(3.22)

The derivation of the outage behavior of UC is similar to RC. Due to the error propagation problem, FDF requires successful decoding at both the relay and the destination. Define

\[ C_{uc}^2 = \begin{cases} C_{uc,asy}^1 & \text{sym} \\ C_{uc,asy}^1 & \text{asy} \end{cases} \]

The outage events are equivalent with the set

\[ \{C^i < R\} \cup \{C^i \geq R\} \cap \{C_{uc}^2 \geq R\} \]

where the first part is the outage events at the relay and the second part represents the situations where the relay has been able to decode but the final decoding at the destination fails. Since the events considered are mutually exclusive, the outage probability can be expressed as a summation of two parts

\[ P_{uc,FDF}(\alpha) = \Pr\{C^i < R\} + \Pr\{C^i \geq R\} \cap \{C_{uc}^2 \geq R\} = \Pr\{C^i < R\} + \Pr\{C^i \geq R\} \cap \{C_{uc}^2 \geq R\} \]

(3.23)

Regarding SDF, if the relay cannot decode, it keeps silent and the source sends \( X_2 = X_1 \). At the destination, these two codes will not be combined through MRC but jointly processed. The average mutual information in such a scenario is given as

\[ I_{sd} = \alpha \log(1 + \gamma_s) + (1 - \alpha) \log(1 + \gamma_s) = \log(1 + \gamma_s) \]

This equation indicates that the relay system behaves just like a two-node system with direct transmission. It will not suffer from the loss in bandwidth efficiency as RC, where the bandwidth efficiency is halved because the same codeword is transmitted twice. If the relay correctly decodes the message, the system is able to enjoy transmit diversity during phase 2. The outage event is given as
3.3.3 Simulation Results and Discussions of DF

The previous results of AF have suggested that relay-assisted communication systems cannot always outperform direct transmission. The same question applies for DF. We need to investigate the SNR regions where DF is superior to direct transmission.

Figure 3.8 depicts the achievable rates of synchronized RC- and UC-based DF schemes and the optimal duplexing ratio $\alpha$ for the UC case. Basically, UC outperforms RC in any SNR region. However, just like AF, the superior area of DF is bounded and shrinking when $\gamma_0$ increases. For the RC-based DF scheme, the superior area is smaller than that of the UC-based scheme and its achievable rate is even lower than direct transmission in some SNR regions. The loss can be explained by the fixed duplexing ratio and reduced bandwidth efficiency.

For the UC-based scheme, DF outperforms direct transmission when $\gamma_1 >\gamma_0$. In contrast, if $\gamma_1 \leq \gamma_0$, optimal $\alpha$ is 1 and DF is degraded to direct transmission. When $\gamma_1 >\gamma_0$, with fixed $\gamma_2$, optimal $\alpha$ decreases with increased $\gamma_1$ because with a better source-relay link, the relay is able to decode in a more timely fashion, thus reducing the time spent in phase 1. If $\gamma_1$ is fixed, the optimal $\alpha$ increases with $\gamma_2$. This observation complies with our derivations in previous sub-sections.

We also illustrate the achievable rates with different $\gamma_2$ when $\gamma_0 = 0$ dB and compare them with synchronized AF. All the figures are transverse sections of the two dimensional Figure 3.9. As demonstrated, the UC-based DF scheme outperforms other schemes including RC-based DF and AF, and performs no worse than direct transmission. With phase synchronization, the achievable rate can be improved. However, the improvement becomes smaller when the relay-destination link is of higher quality. In contrast, the achievable rate of the RC-based DF and AF schemes could be lower than direct transmission when $\gamma_1$ is low. There is a flat part for the RC-based DF curve because with fixed $\gamma_2$ and $\gamma_0$, when $\gamma_1$ is increased such that

$$\frac{1}{2} \log (1 + \gamma_1) \geq \frac{1}{2} \log \left( 1 + \gamma_2 + \sqrt{\gamma_2 + \sqrt{\gamma_2}} \right)$$

the achievable rate is fixed as $C_{RC,om}^{-} = \min \left( C_i^1, C_{RC,om}^2 \right) = C_{RC,om}^1$. We then focus on the best scheme: UC-based DF whose performance is closely related with the quality of the source-relay link. If this link is of low quality, its achievable rate performs exactly the same as direct transmission. Only when this link is improved, does the achievable rate become higher than direct transmission and increases with $\gamma_1$. It implies that the UC-based DF
scheme can only demonstrate cooperative gain over direct transmission when the source-relay link is relatively strong.

Figure 3.8 Achievable Rate and optimal half-duplex ratio $\alpha$
Figure 3.9 Achievable Rate. $\gamma_0=0$ dB. (a) $\gamma_2=-10$ dB, (b) $\gamma_2=0$ dB, (c) $\gamma_2=20$ dB.

The outage behavior is presented in Figure 3.10. The slopes indicate the diversity gains of the corresponding protocols. For FDF, a factor of 10 decrease in outage probability requires additional 10 dB of SNR, which is the same as direct transmission. But for SDF, a factor of 10 decrease requires only 5 dB of SNR. The observation conforms to the conclusions of Laneman's paper — a fixed system cannot enjoy the second-order diversity gain as the selected DF scheme. By allowing the source to be involved in the transmission during the second phase, the UC-based DF scheme achieves about 4 dB and 2 dB gains over protocol 1 in synchronized and asynchronous cases respectively.

The advantage of the UC-based DF scheme over the RC-based DF scheme is also demonstrated in the outage behavior. For SDF, the UC-based scheme features a 3 dB gain in the synchronized case and a 1.5 dB gain in the asynchronous case. Even for FDF, the UC-based scheme still outperforms the RC-based scheme for about 2 dB. The AF scheme is also presented in Figure 3.10. Its outage performance is close to the asynchronous RC-based scheme but is about 1 dB worse than the asynchronous UC-based scheme.
3.4 Soft Decode-and-forward (SoDF)

In previous sub-sections, the UC-based DF scheme has been proved to have the best performance in terms of the achievable rate and outage probabilities. The distributed turbo code (DTC) which is a cooperative and distributed coding scheme have been developed for the UC-based DF relay system. In the first phase, the source encodes the information and broadcasts to both the relay and the destination. The relay decodes the message and interleaves it before re-encoding. After that, the relay partially forwards the encoded message (usually only the parity-check part) to the destination. However, the weakness of DTC is the error propagation problem caused by the erroneously decoded and forwarded messages by the relay. We will introduce an advanced DF scheme which is able to mitigate the error propagation, namely soft DF (SoDF), and derive the upper bounds for its BER and BLER.

3.4.1 Signal Model and the Soft Coding Algorithm

In this section, a detailed signal model for SoDF is introduced by taking coding and modulation schemes into consideration. Usually, a DTC is formed by two recursive systematic convolutional code (RSCC) components. The two RSCCs can differ in generator polynomials and coding rates. For simplicity, we assume that the two RSCCs are identical with coding rate 1/2 and binary phase-shift keying (BPSK) modulation is applied. The system is shown in Figure 3.11.
Chapter 3 Amplify-and-forward, Decode-and-forward and Soft Decode-and-forward

Figure 3.11. Soft decode-and-forward (SoDF)

The source is assumed to be silent during phase 2 in the first place and it will be extended to the space-time coding scenario later, where we allow the source to transmit. The source transmits a binary information vector \( \mathbf{D} \) with length \( K \), denoted as \( \mathbf{D} = (d_1, \ldots, d_k, \ldots, d_K) \). At the source, \( \mathbf{D} \) is first encoded by a RSCC encoder at rate 1/2 and the coded information is represented as

\[
\mathbf{C} = (d_1, p_1, \ldots, d_k, p_k, \ldots, d_K, p_K)
\]

where \( p_k \) is the parity check bit of \( d_k \). After encoding, \( \mathbf{C} \) is BPSK modulated and the resultant signal broadcasted by the source is expressed as

\[
\mathbf{X}_s = (x'_1, x'_2, \ldots, x'_k, \ldots, x'_K, x'_K)
\]

where \( x'_k \) and \( x'_k \) are the modulated information symbol and parity check symbol respectively and \( x_s \in \{0,1\} \).

The received signals at the relay and the destination are

\[
\begin{align*}
\mathbf{Y}_1 &= c_1 \sqrt{E_s} \mathbf{X}_s + \mathbf{Z}_1, \\
\mathbf{Y}_2 &= c_2 \sqrt{E_s} \mathbf{X}_s + \mathbf{Z}_2
\end{align*}
\]

respectively, where \( E_s \) is the transmitted energy per symbol at the source. The received signal can also be expressed in a similar way as

\[
\begin{align*}
\mathbf{Y}_1' &= \left( y'_{1,1}, y'_{1,2}, \ldots, y'_{1,k}, y'_{1,k}, \ldots, y'_{1,K}, y'_{1,K} \right) \\
\mathbf{Y}_2' &= \left( y'_{2,1}, y'_{2,2}, \ldots, y'_{2,k}, y'_{2,k}, \ldots, y'_{2,K}, y'_{2,K} \right)
\end{align*}
\]

37
Amplify-and-forward, Decode-and-forward and Soft Decode-and-forward

Chapter 3 Amplify-and-forward, Decode-and-forward and Soft Decode-and-forward

where $y'_k$ and $y''_k$ are the received information symbol and parity check symbol respectively at the relay and $y'_x$ and $y''_x$ are received signals at the destination.

For a standard DTC system, the relay decodes the received message based on $Y'_r$. The decoded binary information stream is then interleaved and re-encoded with another RSCC encoder. Only the parity check part of the re-encoded message is modulated and forwarded to the destination during phase 2. The received signal is

$$Y_d^2 = c_2 \sqrt{E_r} X^2 + Z_d^2$$

where $E_r$ is the transmitted energy per symbol at the relay. $Y'_r$ and $Y'_d$ form two components of a standard DTC.

In SoDF, the relay starts to decode the message with a SISO decoder which uses the modified BCJR algorithm [36]-[39] to calculate the logarithm of likelihood ratio (LLR) values of the information symbols, denoted as $A(d_i)$. With LLR values, the APP, denoted as $Pr\{d_i = i | Y'_r\}$, can be calculated. Based on APPs of the information symbols, the relay is able to calculate the soft values of the interleaved information symbols. Suppose the interleaved information symbol vector is $\mathbf{D} = [d_1, \ldots, d_i, \ldots, d_K]$, the APPs of $\mathbf{D}$ can be easily obtained from $Pr\{d_i = i | Y'_r\}$. According to [33], the APPs of the parity check symbols $Pr\{p_k = i | Y'_r\}$ can be expressed in a recursive way. Based on [33] and [41], the according LLR values can be approximated by a Gaussian noise model as

$$A(p_i) = L_m(1 - 2\tilde{p}_i) + Z_s$$

where $L_m$ reflects the signal reliability and $Z_s$ is a zero mean Gaussian noise with variance $\sigma^2$. The achieved soft values will be transmitted under certain power constraints. Assuming the relay is subject to the same power constraint as the source, the soft values should be normalized before transmission. The transmitted signals are expressed as

$$\tilde{x}_{j} = \sqrt{E_r} \left(L_m g_k + Z_s\right)$$

where

$$E_r = \frac{E_r}{L_m^2 + \sigma^2}, \quad g_k = 1 - 2\tilde{p}_k$$

The signal received at the destination is

$$Y_d^2 = c_2 X^2 + Z_d^2 = c_2 \sqrt{E_r} L_m g + \left(c_2 \sqrt{E_r} Z + Z_d^2\right)$$
Y'_1 and Y'_2 form a turbo code at the destination, which can be decoded through information exchange between two components.

3.4.2 Upper Bounds of Bit-error-rate (BER) and Block-error-rate (BLER)

To evaluate the performance of SoDF, we first investigate the relationship between the input SNR \( \gamma_{in} \), defined as the instantaneous SNR of the source-relay link and the output SNR \( \gamma_{out} \), defined as the instantaneous output SNR after the SISO decoding and soft re-encoding process. With this relationship, the pairwise error probability (PEP) can be obtained according to the results in [16], [40] and [42].

3.4.2.1. Relationship between \( \gamma_{in} \) and \( \gamma_{out} \) at the relay

Define \( \gamma_{out} \) as a function of \( \gamma_{in} \):

\[
\gamma_{out} = \frac{E \sqrt{2}}{\sigma_{in}^2} = f(\gamma_{in}), \quad \gamma_{in} = \frac{E |c|^2}{\sigma_{in}^2}
\]

Since the closed-form expression for the function \( f(\gamma_{in}) \) has not yet been derived, we resort to the Monte Carlo method to exhibit the in-out relationship curve. As seen in Figure 3.12, this curve can be separated into 2 parts. The first part is when \( \gamma_{in} \) is larger than 0dB and can be approximated by the dash and dot curve. When \( \gamma_{in} \) is smaller than 0dB, it is approximated by the dash curve.

The approximation is expressed as

\[
\gamma_{out}(dB) = \begin{cases} 5 \cdot \gamma_{in} - 8, & \gamma_{in} \geq 0dB \\ 0.9 \cdot \gamma_{in} - 8, & \gamma_{in} < 0dB \end{cases}
\]
Although the curve can be approximated more precisely by polynomials with multiple terms, the simulation results show that the current level of approximation is adequate at establishing a tight upper bound.

3.4.2.2 Pairwise Error Probability (PEP)

When analyzing the error performance, it will be assumed that, without loss of generality, an all-zero codeword \( C_0 \) is transmitted. An error event occurs when a codeword \( C_e \) with nonzero weight is decoded. The PEP is the probability of the occurrence of these error events. According to [16], [40] and [42], when the length of the codeword is \( 2K \), the PEP conditioned on instantaneous SNR can be expressed as

\[
P(C_0 \rightarrow C_e | \gamma) = Q\left(\sqrt{\sum_{i \in U} \gamma_i}\right)
\]

where \( \gamma = (\gamma_1, \gamma_2, \ldots, \gamma_{2K}) \) and \( U \) is a position set where the \( k \)-th bit of \( C_e \) is not zero. The cardinality of \( U \) is \( d \).

For a standard DTC based relay system, \( C_e \) consists of three parts: the information part, the first parity check part from the source and the second parity check part from the relay. In the quasi-static fading scenario, the channel gains keep constant within one block. Since the first two parts both originate from the source, the instantaneous SNRs are same for each bit of the two parts. The claim also applies to the parity check part from the relay. The PEP is given as

\[
P(C_0 \rightarrow C_e | \gamma) = Q\left(\sqrt{2d_1 \gamma_0 + 2d_2 \gamma_2}\right)
\]

where \( d_1 \) is the number of non-zeros weights in the information and first parity check part, \( d_2 \) is the number of non-zeros weights in the second parity check, and \( d = d_1 + d_2 \). It must be pointed out that this expression implies that the source-relay link is error-free.

In SoDF, the relay conducts SISO decoding followed by soft encoding. The SNR of the second parity check part is different to the standard DTC. Define an equivalent SNR \( \gamma_{eq} \) as

\[
\gamma_{eq} = \frac{E_{c_2}^2 |c_2|^2 \gamma_{out}}{E_{c_2}^2 |c_2|^2 + \sigma_s^2 (1+\gamma_{out})}
\]

The PEP can be expressed as

\[
P(C \rightarrow C_{eq} | \gamma) = Q\left(\sqrt{2d_1 \gamma_0 + 2d_2 \gamma_{eq}}\right)
\]

The equivalent SNR \( \gamma_{eq} \) is a function of \( \gamma_{out} \) and can be approximated as a function of \( \gamma_{in} \).
\[ \gamma_w = \begin{cases} \frac{E_{\gamma_0}\gamma_0^{\gamma_0} / 10^{-0.8}}{E_{\gamma_2} + \sigma_2^2 (1 + \gamma_m^{0.5} / 10^{-0.8})}, & \gamma_m > 1 \\ \frac{E_{\gamma_0}\gamma_0^{0.5} / 10^{-0.8}}{E_{\gamma_2} + \sigma_2^2 (1 + \gamma_m^{0.0} / 10^{-0.8})}, & \gamma_m < 1 \end{cases} \] (3.26)

where \( \gamma_i = |c_i|^2 \) for \( i = 0, 1, 2 \). It follows that the PEP is a function of \( \gamma_0 \) and \( \gamma_m \) as well. Because \( \gamma_i \) are random variables, the unconditional PEP should be averaged with respect to \( \gamma_i \):

\[ P(C_0 \rightarrow C_i) = E_{\gamma_0, \gamma_m} \left( Q\left(\sqrt{2d_i\gamma_0 + 2d_i\gamma_m}\right) \right) \] (3.27)

### 3.4.2.3 Weight Enumerating Function (WEF) of RSCC

The PEP is the probability that \( C_s \) with specific weight is decoded when an all zero codeword is transmitted. All possible \( C_s \) should be considered to obtain the overall error probability. There are two ways to evaluate the analytical upper bound for the BER performance of turbo codes. Divsalar used the transfer function [43] and Benedetto introduced the concept of a uniform interleaver [44]. We use the second method. Based on the results in [44], the conditional WEF of terminated turbo codes is proposed in [45] as

\[ A^{\text{new}}_w(X, Z, K) = A^{(c)}_w(X, K) \times A^{(c)}_w(Z, K) \]

where \( A^{(c)}_w(X, K) \) and \( A^{(c)}_w(Z, K) \) are the conditional WEF of component terminated convolutional codes in turbo codes, \( X \) and \( Z \) are dummy variables and \( w \) is the Hamming weight of the input information sequence. \( A^{(c)}_w(X, K) \) and \( A^{(c)}_w(Z, K) \) can be calculated as in [46]. \( A^{\text{new}}_w(X, Z, K) \) is given in Appendix B for the RSCC we used. BLER and BER are

\[ P_{\text{BLER}} \leq E_{\gamma_0, \gamma_m} \left\{ \sum_{i,x,w} a_{n,x,w} P(C_0 \rightarrow C_s | \gamma_0, \gamma_m, \gamma_i) \right\}, \]

\[ P_{\text{BER}} \leq E_{\gamma_0, \gamma_m} \left\{ \sum_{i,x,w} K a_{n,x,w} P(C_0 \rightarrow C_s | \gamma_0, \gamma_m, \gamma_i) \right\} \]

where \( a_{n,x,w} \) denotes the corresponding coefficients of \( A^{\text{new}}_w(X, Z, K) \). However, the upper bounds are tight only for fast fading channels. For slow fading channels, in order to get tight upper bounds, the limit-before-average technique [47] is utilized.
The upper bounds can be achieved through numerical integrations.

### 3.4.3 Space-Time Cooperation in Soft Decode-and-forward

We have proved in previous chapters that the relay system's performance can be improved if the source is permitted to transmit with the relay cooperatively during phase 2. Therefore, we extend the analysis of the previous section to the more general scenario.

To further improve the SNR of the received signal in phase 2, a simple space-time coding scheme called Alamouti scheme [48] is deployed during the second phase. Under the current settings, the source interleaves, encodes the information symbols and forwards its parity check part in a cooperative manner. There are $K$ symbols that should be transmitted during the second phase. With Alamouti scheme, the SNR at the destination after combination can be easily obtained as

$$\tilde{\gamma}_{eq} = \frac{|c_2|^2}{\sigma^2 + \sigma_d^2}$$

with $\sigma^2 = |c_1|^2 E_r \sigma^2 = |c_1|^2 E_r \sigma^2 = |c_1|^2 E_r \frac{1}{\gamma_{in} + 1}$. $\tilde{\gamma}_{eq}$ can be approximated as a function of $\gamma_{in}$ and rewritten as

$$\tilde{\gamma}_{eq} = \frac{E_r |c_2|^2 (1 + \gamma_{out})}{E_r \gamma_{out} + \sigma_d^2 (1 + \gamma_{out})}$$

Comparing $\tilde{\gamma}_{eq}$ with $\gamma_{eq}$ in (3.26), the nominator is increased by the term $E_r |c_2|^2 (1 + \gamma_{out})$ and the denominator is decreased because the first term is multiplied by $\gamma_{out} < 1$. Apparently, the equivalent SNR is improved by the space-time coding.

### 3.4.4 Simulation Results and Discussions for Soft Decode-and-forward

For the purposes of comparison, we provide simulation results and derive the upper bounds for below cases:
• Direct Transmission, where the source encodes the information symbols with turbo code and transmits to the destination without assistance from the relay. Its upper bound can be easily obtained by letting $y_Q = y_2$ through (3.25) and (3.28).

• Perfect decode-and-forward, where the source-relay link error-free.

Figure 3.13 and 3.14 display the simulation results and the derived upper bounds of BER and BLER for SoDF and other schemes, where we keep the average SNR between the source and the relay constant and increase the SNRs of the other two links simultaneously. The performance of DF reaches an error floor when $\mu_0$ and $\mu_2$ are increased to a high region where its BER and BLER are mainly determined by the quality of the source-relay link. If this link's quality cannot be improved, the error propagation caused by the erroneous decoding at the relay will compromise the overall performance. However, the BER of SoDF keeps decreasing and reaches no floors in the range of evaluated SNR. However, its BER performance cannot outperform perfect DF because the noise introduced in the transmission from the source to the relay should also be considered at the final decoding.

The upper bounds we derived using the limit-before-average technique are very tight. With such upper bounds, there is no need to evaluate the performance of a relay system through simulation.

![Figure 3.13 BER.](image1)

![Figure 3.14 BLER.](image2)

Under the assumption that the source-relay link keeps unchanged during two phases, Figure 3.15 compares the analytical bounds of three different scenarios:

- The source remains silent in phase 2.
- The relay remains silent in phase 2.
- The source and the relay implement Alamouti scheme to cooperate during phase 2.
The results demonstrate that with space-time coding, the performance of SoDF can be further improved. This observation conforms to our previous discussions.

![Figure 3.15 Upper bounds for space-time coding.](image)

### 3.5 Conclusions

Several AF schemes for different system settings have been presented in this chapter. It has been concluded that Protocol II AF with phase synchronization is the best scheme in the sense that it outperforms other AF schemes in the achievable rate and has the smallest outage probability in high SNR regions. However, phase synchronization is challenging in practical implementation. Therefore, we also considered the asynchronous case where the mutual information should be averaged with respect to a random phase $\theta$. Since the mutual information expression in the asynchronous case is very complex, the trivial upper bound was investigated instead. The optimization of the amplification function was studied for both cases. The results demonstrate the superior performance of protocol II AF over Laneman's AF scheme. Another important observation we have made is that none of the AF schemes is always able to be superior to direct transmission. As long as either of $\gamma_1$ and $\gamma_2$ is very small, the system can barely benefit from diversity and therefore, the factor that dominates the system's performance will be the bandwidth efficiency loss. When both $\gamma_1$ and $\gamma_2$ are improved to a high level, AF schemes are able to enjoy a higher achievable rate than direct transmission. In addition, we have pointed out that although protocol II AF has a smaller outage probability, its diversity order, which can be roughly illustrated by the slope of the outage probability, is the same as protocol I. According to [5], in high SNR regions, the diversity order is 2. In spite of the fact that the optimization of the amplification function requires CSI at the relay, we can choose $\beta$ based on a pre-set threshold according to (3.12). The simulation results suggest that this sub-optimal amplification function without CSI performs very close to the optimal case where the relay has sufficient CSI.
Moreover, a comprehensive study of the DF scheme has been presented. The achievable rates and outage probabilities have been derived for RC- and UC-based DF schemes. We also investigated the effects of phase synchronization and the optimization problem for the duplexing ratio $\alpha$. Out of all the issues under examination, the effect of phase synchronization is an important one. Normally, a synchronized system is performing better than an asynchronous system. Analysis of the achievable rates indicates that there is a superior area for the DF schemes, in which DF enjoys a higher achievable rate than direct transmission. For the UC-based DF scheme, this region is bounded by the straight line $y_1 = y_0$. If $y_1 \geq y_0$, its achievable rate is higher than direct transmission; otherwise, its performance is the same as direct transmission due to the fact that $\alpha$ is optimized at 1 when $y_1 < y_0$. It means that the source will take full responsibility of transmission during the whole frame (phase 1 and phase 2) and ensures that the UC-based scheme's achievable rate is no worse than direct transmission. The results imply that for the UC-based DF scheme, the quality of the source-relay link is essential and its cooperative gain over direct transmission can only be achieved when this link is relatively strong. However, for the RC-based DF scheme, $\alpha$ is fixed and the system suffers from the bandwidth efficiency loss due to repeated transmission of the same codeword. These effects cause its achievable rate to be lower than direct transmission in certain SNR regions. The diversity gain of different DF schemes can be exhibited by their outage probabilities. With fixed DF, where the relay forwards whenever its received message is successfully decoded or not, the diversity is just the same as direct transmission. Only when selective DF is applied, where the relay keeps silent if unable to decode, is the relay system able to enjoy second-order diversity. Interestingly, the AF scheme enjoys second-order diversity and performs close to the RC-based DF scheme. Due to its simple structure compared with the DF scheme, AF could be a better choice if the main objective is to balance the complexity and performance. However, in half-duplex channel, the received analog signal during phase 1 should be stored in the relay for AF. Sometimes, it is not efficient and DF is a better choice.

We then focused on the practical implementation of DF schemes. A distributed turbo codes based system is introduced. It has been pointed out that the quality of the source-relay link limits the overall performance because of the error propagation problem. A new technique, namely soft decode-and-forward, was introduced to cope with this problem, where the SISO decode is followed by a soft encoder at the relay. The main contribution of our works is to evaluate the performance of the SoDF technique by deriving very tight upper bounds for BER and BLER. We used log likelihood ratio (LLR) values and the Gaussian approximation model to mimic the input and output SNR relationship at the relay. The pairwise error probability (PEP) was derived and the weight enumerating function (WEF) of the RSCC was investigated. In order to obtain tight bounds, the limit-before-average technique was considered. The analysis was extended to the space-time cooperation framework finally. Based on the simulation and analytical results, SoDF outperforms DF which reaches an error floor if the source-relay link cannot be guaranteed to be error-free. The upper bounds we derived are very tight. When space-time coding scheme is considered, the performance of SoDF can be further improved. Here we should note that different codes may have different in-out SNR relationships and we need to calculate the in-out curve through simulations for given codes. But once we obtain the curves, they can be stored for future usage.
Chapter 4 Compress-and-forward (CF)

4.1 Introduction

In this chapter, we will present another fundamental relaying scheme introduced by Cover [2], namely compress-and-forward (CF), where rather than decoding, the relay forwards its observation to the destination. Some recent work has been done for CF in [114]-[117]. In CF, during the first phase, the signals received by the relay and the destination respectively both originate from the same source and contain a common term \( X \). Therefore the two signals are correlated and this fact provides the possibility to transmit the observation of the relay at a reduced rate, i.e. the relay received signal can be compressed. Wyner-Ziv coding is an efficient method for compressing correlated sources which are separately located [49], [50]. This issue was first investigated in the area of distributed sensor array communication systems [51]-[61], where although each sensor images a common scene independently, their transmitting information is highly correlated. Then the question is raised: what is the least number of bits required for the transmission of correlated but separately located sources? Slepian-Wolf (SW) coding provides the answer for lossless coding [70] for the discrete alphabet.

SW coding was extended to lossy coding of continuous-valued sources by Wyner and Ziv [49], [50], where the coding is lossy with respect to a fidelity criterion rather than lossless. Zamir presented a structured algebraic binning scheme based on nested lattice codes and proved that the Wyner-Ziv rate-distortion function in the quadratic Gaussian case is asymptotically achievable [59], [60]. Servetto also explored similar nested lattice construction in [58] and proved its performance is asymptotically approaching the theoretic bound. But lattice codes, especially for the high dimension types, are practically difficult to implement. Recently, a more feasible structure consisting of a quantizer and a SW encoder (or compressor) was proposed in [51]-[53] for distributed source coding. In such a system, the signal is first quantized and the achieved quantization bin index can be SW coded because it belongs to a discrete alphabet. Based on this prototype, the Wyner-Ziv problem is actually decoupled into two separate parts: quantization and SW coding. The optimization of the quantizer has been addressed in [68] for distributed source coding and the SW coding has been explored in [74]-[76]. In this chapter, we will extend this structure to the relay scenario and propose a novel quantization technique.

We will start from the analysis of the achievable rate and outage behavior in the next section. Since WZ coding structure requires a high level of complexity compared with AF and DF, CF can be replaced by an alternative technique called quantize-and-forward (QF), where the relay sends its observation without any compression, thus reducing the complexity of the compression/decompression process. We will also analyze QF. A practical WZ coding scheme, which is the most important part of CF, is provided in section 4.3. The conclusions are drawn in the final section.

4.2 Achievable Rate and Outage Behavior of CF

46
4.2.1 Achievable Rate and Outage Probability for RC-based CF

The system model is introduced in chapter 2 and can be depicted in Figure 4.1. The duplexing ratio $\alpha$ is fixed at 0.5 for the RC-based scheme.

In the first phase, the source broadcasts $X'$ and the relay quantizes the received signal into some intermediate bin index which will be SW coded into bin index $s$. $s$ is encoded by $X_c = X_s$ and forwarded by the relay during the second phase. Meanwhile, the source transmits $X_c = X'$ for the second time. $X_c$ and $X_c'$ form a multiple-access-channel (MAC). The destination starts from decoding the SW coded bin index $s$ by treating $X_c'$ as a noise. Once $s$ is obtained, with the help of the side information $Y'$, the compressed bin index is decompressed and utilized to reconstruct the observation of the relay, which is actually the estimation of $Y'$, denoted as $\hat{Y}'$. The achievable rate is given as (see Appendix C for derivation)

$$C_{ac} = \frac{1}{2} I(X'_s; Y'_d) = \log \det \left( I_{s} + \left( C C^H \right) \left( E(ZZ^H) \right)^{-1} \right)$$

(4.1)

where $C^H$ represents Hermitian transpose of matrix $C$, $Z_{d'}$ is a Gaussian distributed variable independent of $Y'_d$ and $Y'_s$ with variance $\sigma_d^2$ and

$$Y_d = [Y'_d, Y'_s, c_s]$$

$$C = [c_0 \sqrt{\mu_0}, c_0 \sqrt{\mu_0}, c_1 \sqrt{\mu_1}]$$

$$Z = [Z'_d, Z'_s, Z'_r + Z_{d'}]$$

$$Y'_d = Y'_d - c_1 \sqrt{\mu_1} X'^2$$

Figure 4.1. Compress-and-forward System
\[ \sigma^2_n = \left(1 + \frac{\gamma_1}{1 + \gamma_0}\right) \left(e^{\nu/\mu} - 1\right) \]

With this achievable rate, the outage probability can be easily obtained as

\[ P_{ou}^c = \Pr\{C^c < R\} = \frac{1}{\gamma} \left[ C^c < R \right] \]

(4.2)

4.2.2 The Achievable Rate and Outage Probability for UC-based CF

In contrast to the RC-based scheme, where the source transmits exactly the same codeword during phase 2, the source transmits \( X^2_i \neq X^1_i \) and the duplexing ratio \( \alpha \) can be flexible in the UC-based CF scheme. The compression/decompression process at the relay and destination respectively are the same as the RC-based scheme. With some modifications of the results in [4], the overall accumulated mutual information can be expressed as

\[ C^c_{uc}(\alpha) = \alpha I\left(X^1_i; Y^1_i, Y^2_i\right) + (1 - \alpha) I\left(X^2_i; Y^2_i | X^1_i\right) = \alpha \log\left(1 + \gamma_0 + \frac{\gamma_1}{1 + \sigma^2_n}\right) + (1 - \alpha) \log(1 + \gamma_0) \]

(4.3)

The first term is the mutual information transmitted with the help of the relay and the second term is conveyed directly from the source during phase 2. \( C^c_{uc}(\alpha) \) should be optimized with respect to \( \alpha \). However, since \( \sigma^2_n \) is a function of \( \alpha \), it turns out that \( C^c_{uc}(\alpha) \) is a very complex function of \( \alpha \) and the closed-form expression for \( \alpha \) cannot be derived. Therefore, we resort to numerical methods. The outage probability given \( \alpha \) is

\[ P_{ou}^c(\alpha) = \Pr\{C^c_{uc} < R\} = \frac{1}{\gamma} \left[ C^c_{uc} < R \right] \]

The duplexing ratio \( \alpha \) should be optimized to minimize the outage probability, hence we have

\[ P_{ou}^c = \min_{\alpha} P_{ou}^c(\alpha) \]

(4.4)

The achievable rate of the CF scheme is complicated and an analytical expression is difficult to derive. We have to use numerical integrations to calculate it and optimize the duplexing ratio \( \alpha \).

4.2.3 Quantize-and-forward (QF)
In previous sections, the quantized signals can be compressed at the relay because of the existence of side
information. However, the compression/decompression process requires a large amount of calculations.
Therefore, we introduce another scheme, where the relay quantizes its observation and forwards the quantized
signal without compression. In such a case, we assume
\[ \hat{Y}_r = Y_r + \hat{W} \]
where \( \hat{W} \) is independent to \( Y_r \) and follows zero mean CSCGD with variance \( \sigma_w^2 \). The mutual information
between \( \hat{Y}_r \) and \( Y_r \) is
\[ I(\hat{Y}_r; Y_r) = H(\hat{Y}_r) - H(\hat{Y}_r | Y_r) = H(\hat{Y}_r) - H(\hat{W}) = \log \frac{\gamma_1 + 1 + \sigma_w^2}{\sigma_w^2} \]
The transmission rate of the quantized signal is constrained by the rate \( R_0 \) as
\[ R_0 \geq \alpha \log \frac{\gamma_1 + 1 + \sigma_w^2}{\sigma_w^2} \]
such that
\[ \sigma_w^2 \geq \frac{(\gamma_1 + 1)}{e^{R_0/\alpha} - 1} \]
The achievable rate and outage probabilities of QF have the same expression with CF only with substitution of
\( \sigma_w^2 \) by \( \sigma_w^2 \).

Here we must emphasis that in the outage analysis, calculations of the available rate at the relay and the ideal
transmission rate of the compressed signal requires global CSI (specifically, the amplitudes information because
it decides the statistical properties of the signals). It implies that although the source does not require any CSI
knowledge, the relay needs to be made aware of the instantaneous SNRs of all the three links, which could be
achieved through the feedback from the destination. However, a large amount of information feedback is not
always feasible in practical communication systems. Normally, we assume that the feedback link is extremely
limited. The discussions involving the limited feedback link will be left to Chapter 5. At the current state, global
CSI is assumed to be available at the relay.

4.2.4 Numerical Results and Discussions for CF
The achievable rates of RC-based CF, UC-based CF and QF are depicted in Figure 4.2, where the duplexing ratio $\alpha$ is optimized numerically. Given the same instantaneous SNRs, the variance of the compression noise produced in the QF scheme is larger than that in the CF scheme. Hence in the achievable rate expression (4.3),
the first term \( I(X'_1;Y'_f,Y'_{nd}) \) is relatively smaller in QF and as a result, the associated optimal duplexing ratio \( \alpha \) could be smaller compared with CF. When the relay-destination link is improved, the same amount of information can be conveyed in a shorter phase 2 whose duration is associated with \( 1-\alpha \). It explains the fact that when we fix \( y_0, y_1 \), \( \alpha \) is increasing with \( y_2 \).

Interestingly, if \( y_0 \) is low, there is no much difference between the UC-based CF and QF despite their difference in complexity. The achievable rate of RC-based CF is even smaller than UC-based QF due to the usage of repetition codes. When \( y_0 \) increases, the gap between the UC- and RC-based CF schemes becomes wider. A crucial feature of UC-based CF and QF is that they always outperform direct transmission in the achievable rate regardless of the SNR settings. In contrast, the UC-based DF has a superior area. Inside the area, its achievable rate is higher than direct transmission; otherwise, the achievable rate is exactly the same as direct transmission. For RC-based CF, its superior area is also limited due to the fixed duplexing ratio and the bandwidth efficiency loss. To compare CF with other schemes, we also illustrate the achievable rates of different schemes in a generic path loss model according to the practical transmission setting with frequency 2.4GHz, path loss coefficients 3, and free-space reference \( d_0 \). The path loss is \( L_p(d) = 10 \log_{10}(d/d_0) \), where we assume that the system loss is 0 at \( d_0 \). The distance between the source and the destination is \( r = 10d_0 \), and the relay is in the middle of the source-destination link. The distance from the source to the relay is \( d \) and the distance between the relay and the destination is \( r-d \).

Figure 4.3. Achievable rate.
As seen from Figure 4.3, the superior performance areas of UC-based CF and DF have been identified. When the relay is close to the source, DF outperforms CF. In contrast, when the relay is moving towards the destination, the achievable rate of UC-based CF outperforms DF eventually. Both RC-based CF and AF schemes are able to outperform direct transmission in certain regions. But there achievable rates are lower than UC-based relaying schemes. Interestingly, although the achievable rate of CF is degraded when the source is kept silent during phase 2, in the CF superior, the performance loss is quite small. However, the decoding complexity can be greatly reduced when the source is silent during phase 2 because we can combine the destination received signal and the estimation of the relay received signal and the combined signal can be used for decoding directly without resorting to joint processing. Considering the fact that complexity is one of the main concerns in practical implementation, in the practical CF techniques which will be introduced in the later sub-sections in this chapter, we assume the relay is silent during phase 2. As a result, the decoding structure at the destination is greatly simplified at the expense of small performance loss.

The outage behavior is presented in Figure 4.4. The outage behavior for UC-based CF, QF and RC-based CF is quite similar and outperforms UC-based SDF for about 2 dB. The fact that a factor of 10 decrease in slope requires 5 dB of SNR indicates that QF and CF schemes enjoy the same second order diversity as UC-based SDF. However, we should keep in mind that these results are based on the full availability of CSI at the relay.

![Figure 4.4. Outage probability. R=0.1.](image)

4.3 Wyner-Ziv Coding Design

Consider the communication systems in Figure 4.5, where $X$ and $Y$ are correlated discrete-alphabet random variables. If $X$ is encoded independently as in Figure 4.5(a), according to Shannon’s source coding theory [3], a rate $H(X)$ is sufficient to ensure $X$ can be recovered without any distortion, i.e. $\hat{X} = X$, where $H(X)$ represents the entropy of $X$. If $Y$ (referred to as side information) is available at both the encoder and the decoding as
shown in Figure 4.5(b), $X$ can be encoded at the theoretical rate of its conditional entropy given $Y$, $H(X|Y)$. But what will happen if $Y$ cannot be accessed at the encoder, i.e. $X$ and $Y$ are encoded separately?

One simple solution is to encode $X$ and $Y$ separately with rates $H(X)$ and $H(Y)$ respectively and the total rate will be $H(X) + H(Y)$. However, in a landmark paper [70], Slepian and Wolf stated that if the decoder has the access to $Y$, whether or not the encoder knows about $Y$, the number of bits used for encoding $X$ is the same. In other words, a total rate $H(X,Y)$, which is smaller than $H(X) + H(Y)$ when $X$ and $Y$ are correlated, is sufficient to ensure $\hat{X} = X$ and $\hat{Y} = Y$. Hence we can encode $Y$ at rate $H(Y)$ and $X$ at rate $H(X|Y)$. Wyner and Ziv extended the lossless coding to the lossy coding scenario. Before we introduce Wyner-Ziv (WZ) coding, we need to define the rate-distortion (RD) function as in [3].

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure4.5.png}
\caption{Source encoder/decoder with/without side information}
\end{figure}

**Definition 5:** The rate distortion function for an i.i.d. source $X$ with distribution $p(x)$ and bounded distortion function $d(x, \hat{x})$ is equal to the associated information rate distortion function. Thus,

$$R(D) = \min_{p(\hat{x}|x)} \sum_{x,d(x,\hat{x})} \mathbb{E}[d(x,\hat{x})] I(X;\hat{x})$$

is the minimum achievable rate at distortion $D$, where the minimization is over all conditional distributions $p(\hat{x}|x)$ for which the joint distribution $p(\hat{x}, x) = p(\hat{x}|x)p(x)$ satisfies the expected distortion constraint $\mathbb{E}[d(x,\hat{x})] \leq D$.
Wyner and Ziv investigated the rate distortion problem with side information $Y$ at the decoder and concluded that for both discrete and continuous alphabets, with distortion metrics $d(x, \hat{x})$, there exists a function $\hat{x}(A, Y)$ satisfying $E[d(x, \hat{x})] \leq D$ and the rate-distortion function is

$$R_{wz}(D) = \inf I(X, A | Y)$$

where $A$ is an auxiliary variable and $Y \rightarrow X \rightarrow A$ forms a Markov chain. The infimum is over all possible distributions of $A$. The bound of $R_{wz}(D)$ is given as

$$R_{wz}(D) \geq R_{xp}(D) = \min_{p(x|y)} \sum_{x,y} d(x,y) p(x,y) \log \left( \frac{1}{p(x|y)} \right)$$

where $R_{xp}(D)$ is the classic RD function of encoding $X$ with side information $Y$ available at both the encoder and the decoder. It means that, when the side information is not known to the encoder, the number of bits required to encode $X$ under certain distortion constraint is no lower than the case where the side information $Y$ is also available to the encoder, i.e. WZ coding suffers from rate loss.

When $X$ is discrete and $D$ approaches zero, the WZ problem is degenerated to the Slepian-Wolf (SW) problem with

$$R_{wz}(D) = R_{xp}(D) = H(X | Y)$$

As suggested in [53], we will use a WZ coding structure consisting of a quantizer followed by a SW encoder to solve the compression/decompression problem in the CF scheme. The analysis is based on a practical CF model depicted in Figure 4.6. Note that the decoding at the destination is simplified by prohibiting the source from
transmitting in phase 2. The reason has been explained in the previous sub-section. The WZ problem is decoupled and two parts will be investigated independently in the following sub-sections.

4.3.1 Quantizer Design

The typical design method of a quantizer in the WZ problem involves certain partitions that result in minimum distortion between the original signal and the reproduced version. The well known Lloyd-Max algorithm [64], [67] is able to achieve this purpose. However, in CF, it has been shown that the quantization scheme not only has impact on the distortion but also on the transmission rate of the compressed signal [52], [53]. While small distortion is desirable, the transmission rate of the compressed signal should also be as low as possible to ensure successful decoding of the compressed bin index. A quantization scheme called rate-distortion scheme (RDS) which takes both the compression rate and distortion into consideration has been proposed for distributed source coding [68], where the optimization objective of the design is neither pure distortion nor compression rate but is the weighted summation of two parts. At the receiver, the quantized signal is reconstructed with the help of the side information through WZ estimation [49]. However, in a relay system, considering that the achievable rate is the main objective, we propose a quantization algorithm directly based on the maximization of the achievable rate. The corresponding signal combination structure at the destination is provided as well. Finally, Trellis-Coded Quantization (TCQ) will be incorporated and the scalar quantization schemes will be extended to the vector quantization scenario [69].

The source transmits binary information vector \( D \) with length \( K \), denoted as

\[
D = (d_1, \ldots, d_1, \ldots, d_K)^	op
\]

where \( d_i \in \{0,1\} \). \( D \) is encoded and the resultant coded information is represented by

\[
C = (c_1, \ldots, c_n, \ldots, c_{N_t})^	op
\]

where \( N_t = aN \) is the length of the code and the coding rate is \( K/N_t \). After encoding, \( C \) is modulated. Here for simplicity, we assume BPSK modulation. However, it can be extended to high order modulation with same principles applied. The modulated signal to be broadcasted by the source is expressed as

\[
X_i(n) = (X_i(1), \ldots, X_i(n), \ldots, X_i(N_t))^	op
\]

where \( X_i(n) \in \{-1,1\} \). We assume that the phase of channel gains of the source- and source-destination links has been compensated at the receiver side, therefore \( c_0 \) and \( c_1 \) are real random variables modeled by Rayleigh distribution. At time \( n \), \( Y_i(n) \) is quantized and the quantization function \( Q \{ Y_i(n) \} \) is determined by two parameters as below

- Partition \( U = (u_1, u_2, \ldots, u_{L+1}) \), where \( u_1 = -\infty \) and \( u_{L+1} = \infty \).
Chapter 4 Compress-and-forward (CF)

- Representation Code set \( R_C = \{ r_{ci}, r_{c2}, ..., r_{cL} \} \), where \( r_{ci} \) lies in the region \( \Gamma_C = [u_i, u_{i+1}] \).

If \( Y'_i(n) \) lies in the region \( \Gamma_i \), it is quantized as

\[
Q \{ Y'_i(n) \} = \hat{Y}'_i(n) = r_{cj}
\]

Suppose there is a set \( \mathcal{V} \) which consists of all the binary sequences with length \( \log_2 L \), we can establish a one-to-one mapping between \( R_C \) and \( \mathcal{V} \). It follows that

\[
M \{ Q \{ Y'_i(n) \} \} = v_n = (v_{n,1}, ..., v_{n,M})^T, \quad v_{n,m} \in \{0,1\}
\]

where \( v_{1}(n) \) and \( v_{M}(n) \) are the most and least significant bits respectively, \( M \{ \} \) denotes the mapping from set \( R_C \) to \( \mathcal{V} \) and \( M = \log_2 L \). By the end of phase 1, we have \( \mathbf{V} = (v_1, ..., v_{M}) \) which is then compressed by the SW encoder. The obtained index is encoded by the error protection code \( C_p \) and forwarded to the destination during phase 2. At the destination, after demodulation, decoding and decompression, \( \mathbf{V} \) is obtained. Since our focus in this section is on the quantization part only, we assume that the SNR of the relay-destination link is high enough to ensure that the decoding and decompression process will introduce no error, i.e. \( \mathbf{V} = \mathbf{V} \). With \( \mathbf{V} \), the estimation of the relay's observation can be reconstructed and utilized for the final decoding of the main information.

4.3.1.1 Lloyd-Max Quantizer Design

The mean-square distortion \( D \) for a quantizer with partition \( U \) can be defined as

\[
D = E \left\{ \left( X - Q \{ X \} \right)^2 \right\} = \sum_{i=1}^{\infty} \sum_{x \in \Gamma_i} \frac{E \left\{ (X - r_{cj})^2 \right\}}{p(x)} = \sum_{i=1}^{\infty} \int_{\Gamma_i} p(x) (X - r_{cj})^2 dx
\]

where \( p(X) \) is the probability density function (pdf) of \( X \). Letting the partial derivatives of \( D \) with respect to \( R_C \) equal to zero, we have

\[
r_{cj} = E \{ X \mid X \in \Gamma_j \} = \frac{\int_{\Gamma_j} x p(x) dx}{\int p(x) dx} \quad \text{and} \quad u_j = \frac{1}{2} (r_{cj-1} + r_{cj})
\]

If \( r_{cj} \) can be expressed in a closed-form function, \( u_j \) can be explicitly solved. However, it is rarely this luck and therefore, the problem has to be solved numerically. First, pick an initial guess \( U^{(0)} \) for the partition and update \( r_{cj} \) as
Then update the partition \( U \) as

\[
U_i^{(n+1)} = \frac{1}{2} \left( U_i^{(n)} + U_{i+1}^{(n)} \right)
\]

It has been proved that \( U^{(i)} \) is able to converge to optimal \( U \) when \( i \) approaches infinity [67]. The quantization algorithm can be applied to \( Y_i(n) \) directly.

### 4.3.1.2 Rate-Distortion Quantization

In the framework of distributed source coding, the performance of the Lloyd-Max quantizer can be improved by exploiting the statistical dependence between the source and the side information. Some recent approaches for the design of optimal quantizers have been proposed in [65]-[67]. Some heuristic algorithms have been proposed and the Lloyd algorithm is extended to distribute source coding with the objective of minimizing the distortion only. A recent work proposed a new design of quantizers by taking both the distortion and the source coding rate into consideration [68]. In this model, the statistical dependence between the source and the side information is exploited as well. We will extend this algorithm into a relay system and design the optimal quantizer in the sense that both the distortion and the source coding rate are optimized.

Suppose the relay’s observation is reconstructed as \( \hat{Y}_i(n) \), with mean square error (MSE) distortion

\[
d(Y_i(n), \hat{Y}_i(n)) = (Y_i(n) - \hat{Y}_i(n))^2
\]

the expected distortion can be expressed as

\[
D = E \left( Y_i(n) - \hat{Y}_i(n) \right)^2
\]

Hereafter, for the explicit expression purpose, we ignore the time index \( n \). In the meanwhile, the ideal SW encoding rate \( R_{sw} \) is, with the existence of the side information, given as \( H(v | Y'_i) \). Here the source is entropy encoded by exploiting the statistical correlation between \( v \) and \( Y'_i \). This rate is also a crucial issue we need to consider and is desired to be as small as possible. The distortion and the rate are derived in Appendix D. By taking both the distortion and the source encoding rate into consideration, we form the Lagrangian cost as

\[
J = \lambda D + (1 - \lambda) R_{sw}
\]

\[(4.5)\]
where $\lambda$ is a number in the region $[0, 1]$. The previous Lloyd-max algorithm can be regarded as a special case of the rate-distortion algorithm by letting $\lambda=1$. Our objective is to find the optimal $U^{opt}$ such that

$$U^{opt} = \arg\min_u J$$

(4.6)

The searching for $U^{opt}$ is based on a modified Lloyd algorithm, which will be introduced later because the new quantization scheme we proposed is also based on this algorithm.

4.3.1.3 Achievable Rate Based Quantization

In the rate distortion quantizer design, the objective function is the Lagrangian cost $J$ which is a weighted summation of the rate and the distortion. The statistical dependence between the source and the side information is exploited during the optimization process. However, since the rate-distortion quantizer is originally designed for distributed source coding systems in sensor networks, it may not be the optimal choice for a relay system. Compared with $J$, the achievable rate is a more important measurement in a relay system. Hence, instead of $J$, we use the achievable rate as our optimization objective. We propose a quantizer design algorithm dedicated to the relay system and take complexity into consideration.

As depicted in Figure 4.1, the reconstructed relay’s observation $\hat{Y}_i$ will be used in company with $Y_i$ to decode the information. Here we assume that a SISO decoder is deployed because of its better performance compared with the hard decision decoder. The input LLR values which serve as the initial values for the SISO decoder is given as

$$LLR_v = \log \frac{p(\hat{Y}_i, Y_i | X_i = +1)}{p(\hat{Y}_i, Y_i | X_i = -1)} = \log \frac{p(\hat{Y}_i | X_i = +1)}{p(\hat{Y}_i | X_i = -1)} + \log \frac{p(Y_i | X_i = +1)}{p(Y_i | X_i = -1)}$$

We notice that the reconstruction of $\hat{Y}_i$ requires a large amount of calculations produced by WZ estimation. Given the fact that the bin index $v$ could be quite informative, to reduce the calculation complexity, we define the new input LLR values based on $v$ directly as

$$LLR_v = \log \frac{p(v, Y_i | X_i = +1)}{p(v, Y_i | X_i = -1)} = \log \frac{p(v | X_i = +1)}{p(v | X_i = -1)} + \log \frac{p(Y_i | X_i = +1)}{p(Y_i | X_i = -1)}$$

where the first term can be obtained with fewer calculations.

Based on the new definition of the initial LLR values, we present our achievable rate based quantizer design method. Recall the results in the previous chapters, given a partition $U$, the achievable rate is

$$R = \max_{0 \leq \alpha \leq 1} \alpha I(X_i; Y_i | X_i) + (1-\alpha) I(Y_i; Y_i | X_i)$$

58
with constraints

\[
R_0 \leq (1 - \alpha) I(X'_1; Y'_2^2), \quad R_0 \geq \alpha [I(Y'_1; W) - I(Y'_2; W)]
\]

The fact that \( v \) is used instead of \( \hat{Y}_1 \) to produce the initial LLR values implies that we do not need to reconstruct \( \hat{Y}_1 \) and the achievable rate can be changed accordingly as

\[
R = \max_{\theta \in S, \mu} \alpha I(X'_1; v, Y'_1) + (1 - \alpha) I(X'_1; Y'_2 | X'_1)
\]

where the second constraint is

\[
R_0 \geq \alpha [I(Y'_1; v) - I(Y'_2; v)] = \alpha [H(v | Y'_1) - H(v | Y'_2)] = \alpha I(Y'_1; v | Y'_2)
\]

Here \( \alpha \) is due to the fact that for the given partition \( U \), \( v \) is related to \( Y'_1 \) only. It is reasonable to assume the existence of an optimal partition \( U \) which is able to maximize the achievable rate. Since we assume the source does not transmit during phase 2, the achievable rate can be maximized as

\[
R = \max_{\theta \in S, \mu} \alpha I(X'_1; v, Y'_1)
\]

where the second constraint is changed accordingly as

\[
R_0 \geq \alpha I(Y'_1; v | Y'_2) = \alpha H(v | Y'_2)
\]

Here \( \alpha \) is due to the fact that for a given quantizer, \( v \) is a definite consequence of \( Y'_1 \). It follows that

\[
\alpha \leq \frac{I(X'_1; Y'_2)}{I(X'_1; Y'_2) + H(v | Y'_2)}
\]

\( R \) is an increasing function of \( \alpha \), it is maximized when \( \alpha \) is maximized as,

\[
R = \max_v \frac{I(X'_1; Y'_2)}{I(X'_1; Y'_2) + H(v | Y'_2)} I(X'_1; v, Y'_2)
\]

where

\[
I(X'_1; v, Y'_2) = H(v, Y'_2) - H(v, Y'_2 | X'_1)
\]

The joint entropy and conditional joint entropy are given by
Chapter 4 Compress-and-Forward (CF)

\[ H(v, Y_j') = -\sum_{i=1}^{\infty} \int p(v, Y_j') \log p(v, Y_j') dY_j' \]

\[ H(v, Y_j' | X_i^j) = -\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \int p(v, Y_j' | X_i^j) \log p(v, Y_j' | X_i^j) dY_j' \]

If the relay is very close to the destination, which is a reasonable assumption because only in such circumstances CF outperforms DF, \( I(X_i^j; Y_j') \) could be very large compared with \( H(v | Y_j') \). Hence \( \alpha \) is approaching 1 and the achievable rate can be approximated as

\[ R \approx \max_u I(X_i^j; v, Y_j') \]

(4.7)

In order to achieve the optimization objective, the Lloyd algorithm is extended for distributed source coding as follows:

1. Set the maximum iteration limit as \( IT_{\text{max}} \), make the initial iteration counter \( t=0 \) and choose an initial guess for partition \( U \), which usually can be uniform quantizer.
2. Set counter \( t=1 \).
3. Keep \( u_1 \) and \( u_{L+1} \) fixed, update \( u_{i+1} \) to optimize the objective function. For the rate distortion algorithm, the objective function is the weighted summation of the distortion and the rate and should be minimized according to (4.5). For the achievable rate based algorithm, our objective is to maximize the achievable rate according to (4.7).
4. If \( t \leq L-1 \), update \( t \) as \( t+1 \) and go to step 3; otherwise, go to the next step.
5. If \( t = IT_{\text{max}} \), go to the next step; otherwise, set \( t=1 \) and go back to step 2.
6. Finish.

In the modified Lloyd algorithm described above, the elements of partition \( U \) are updated except \( u_1 \) and \( u_{L+1} \) because they are equal to \(-\infty\) and \(+\infty\) respectively. The quantization algorithm will be applied to \( Y_j' \). We notice that the distribution of \( Y_j' \) is symmetric. Hence, the partition \( U \) should be symmetric as well. In this regard, the optimization can be applied to the part \( 1 \leq L/2 \) only to reduce the overall calculation complexity. We must emphasize that although the modified Lloyd algorithm is able to make \( J \) decreasing and \( R \) increasing for the rate-distortion and the achievable rate based algorithms respectively, it does not ensure that with the obtained optimal partition \( U \), the objective function \( J \) and \( R \) are global extreme values. However, the simulation results indicate that even under the current level of optimization, the performance is satisfactory.

4.3.1.4 Vector Quantization: Trellis Coded Quantizer

The previous schemes have been of scalar quantization types. Motivated by the fact vector quantization is able to achieve smaller distortion, we extend our algorithm to the vector quantizer design. Since Trellis coded
quantization (TCQ) has been proved to be an efficient quantization scheme [69], we will build such quantizers based on the achievable rate based scalar quantizer. Generally, M-bits TCQ is based on (M+1)-bit scalar quantization. Take M=2 as an example, the 3 bits scalar quantization is shown in Figure 4.7.

![Figure 4.7 (a) 4-state Trellis (b) Codebook](image)

The representation codebook consists of 8 points which are separated into 4 groups \{D1, D2, D3, D4\}. Each group has 2 elements. In a 4-state trellis, we use one bit (path bit) to denote the path and one of the 4 groups is selected accordingly. Another bit denotes which element in the selected groups is chosen. By doing this, we can use 2 bits to denote an element in an 8-element codebook.

The extension is quite straightforward. After we obtain the partition U with size (L+1) from any of the two algorithms, we build the representation codebook \(C_{TCQ}\) with size L as follows,

\[
c_{L/2} = u_{L/2}/2; \\
cc_{u-1} + c_{u+1} = 2u_{u/2}; \\
cc_1 + c_3 = 2u_2
\]

As mentioned previously, the other half part of \(C_{TCQ}\) is a symmetric mirror image. Then the TCQ algorithm can be applied based on codebook \(C_{TCQ}\).

4.3.1.5 Simulation Results, Comparisons and Discussions

The performance of the achievable rate based algorithm, rate-distortion algorithm and TCQ algorithm are compared. In order to exhibit the performance of different quantization schemes solely, we assume that the compression/decompression process introduces no error. Same duplexing ratio \(\alpha=0.9\) is applied to different schemes. The relay-destination link is strong enough such that \(R_d \leq (1-\alpha)\log(1+\mu_d)\). It ensures successful reception of compressed signals. Taking this constraint into consideration, we choose \(\mu_d\) as 35dB. These quantizers are deployed in the same relay system where the source information is encoded by convolutional
codes and the modulation scheme is BPSK. We investigated the BER performance of such a relay system in the AWGN channel first. Supposing 2-bits quantization, \( L \) is equal to 4 for scalar quantizers. For the 2-bit TCQ algorithm, a representation code \( C_{TCQ} \) with size \( L = 8 \) is required. The corresponding partitions are listed in the Appendix D.

The simulation results for \( L=4 \) are shown in Figure 4.8 (a) and (b). For the comparison purpose, the no-distortion case where \( Y_r \) is perfectly received by the destination is also depicted (ID). The BER performance of the relay system with the rate-distortion quantizer is improved with increased \( \lambda \). It suggests that if we put more emphasis on the distortion, the BER curve is able to approach no-distortion one. However, an interesting thing is that the relay system with the achievable rate based quantizer outperforms that with the rate-distortion quantizer, whereas the distortion of the new quantizer is larger.

When TCQ is considered, as seen from Figure 4.8(b), with the same number of quantization bits, the distortion of TCQ is smaller than its corresponding scalar version. Interestingly, although TCQ offers slight improvement in distortion, the BER performance of the TCQ-based relay systems are not as good as the scalar quantizer based one. These results imply that the quantization scheme plays an important role in the system’s performance but distortion is not the most crucial factor that dominates BER performance. In contrast, lower BER is actually achieved through maximization of the achievable rate.

The source coding rate for the compressed index is shown as an indicator for the efficiency of compression in Figure 4.9 (the lower the conditional entropy, the higher the compression rate). With decreased \( \lambda \), the coding rate of the rate-distortion quantizer is reduced because we put more emphasis on it. The rate of the new algorithm is similar to the rate-distortion quantizer when \( \lambda = 1 \). It means that the new algorithm is able to outperform the rate-distortion quantizer in BER performance while achieving similar compression efficiency at the same time. As seen from Figure 4.10, the new algorithm obtains the highest achievable rate.
Figure 4.8. BER & Distortion, AWGN. (a) BER. (b) Distortion. ($\mu_1 = \mu_0 + 2\text{dB}$, ID: ideal case where perfect relay’s observation is available to the destination; RDS: rate-distortion quantization scheme, defined in section 4.3.1.2; ARBQS: achievable rate based quantization scheme, defined in section 4.3.1.3; TCQ: Trellis coded quantization scheme.)

Figure 4.9. Coding Rate

Figure 4.10. Achievable Rate

Simulation is also conducted in a block fading scenario in Figure 4.11. Although the new algorithm still outperforms rate-distortion schemes, the improvement is not as large as that in the AWGN channel because the BER performance is mainly dominated by fading rather than the selected quantization scheme.
Figure 4.11. BER in block fading channel. $\mu_1 = \mu_0 + 2$ dB.

Figure 4.12 plots the distortion of different schemes. Here we search for an optimal partition for each block. The rate-distortion scheme with $\lambda = 1$ still has the smallest distortion; while at the same time, the new algorithm has larger distortion but better BER performance.

Figure 4.12. Distortion in fading Channels. $\mu_1 = \mu_0 + 2$ dB.

In this sub-section, we extended the rate-distortion quantizer design algorithm into a relay system and developed a new achievable rate based quantizer. The new scheme outperforms the other schemes in both AWGN and block fading cases. It also displays the advantages in higher compression efficiency and reduced complexity.

4.3.2 Slepian Wolf (SW) Coding
After quantization, the bin index belongs to a discrete alphabet which, as Slepian and Wolf have pointed out, features lossless source coding based on random binning when the side information is available at the decoder. The random binning technique is deployed by partitioning the space of all possible outcomes of a random source into disjoint subsets, referred to as bins [70]. However, the weakness of the random binning theory is mentioned in Shannon’s channel coding theorem [3]: it is asymptotic and non-constructive. The close connection of source coding and channel coding has been first noticed in [57], where the authors suggested that linear channel codes be used as a constructive approach to SW coding. The basic idea was to partition the space of all possible source outcomes into disjoint bins that are the cosets of some “good” linear channel code for the specific correlation model. Consider the case of binary symmetric sources and Hamming distance measure, with a linear \((n, k)\) binary block code, there are \(2^{n-k}\) distinct syndromes, each indexing a bin of \(2^n\) binary words of length \(n\). Each bin is a coset code of the linear binary block code, which means that the Hamming distance properties of the original linear code are preserved in each bin. In compressing, a vector of \(n\) input bits is mapped into its corresponding \((n-k)\)-bit syndrome, achieving a compression ratio of \(n : (n - k)\).

To explain the SW problem more clearly, let us consider a simple example in [54]. Assume \(X\) and \(Y\) are equiprobable binary triplets with \(X, Y \in \{0, 1\}^3\) that differ at most in one position, i.e. the Hamming distance \(d_H(X, Y)\) between \(X\) and \(Y\) is no larger than 1. The entropies of \(X\) and \(Y\) are equal to 3 bits. When \(X\) and \(Y\) are encoded separately, 6 bits are required. If we know \(Y\), only four choices of \(X\) with equal probability are available. For example, when \(Y = 000\), \(X \in \{000, 110, 101, 011\}\). Hence we need 2 extra bits to indicate \(X\). The conditional entropy \(H(X|Y)\) is equal to 2 bits. For joint encoding of \(X\) and \(Y\), three bits are needed to convey \(Y\) and two additional bits to index the four possible choices of \(X\) associated with \(Y\), thus instead of \(H(Y) + H(X) = 6\) bits, a total of \(H(X, Y) = H(Y) + H(X|Y) = 5\) bits are sufficient.

For source coding as depicted in Figure 4.5(c), the side information \(Y\) is known accurately at the decoder but not at the encoder. According to the SW theorem, it is still possible to send \(H(X|Y) = 2\) bits instead of \(H(X) = 3\) bits for \(X\) and decode it without loss at the joint decoder. This can be done by first partitioning the set of all possible outcomes of \(X\) into four bins \(B_{00}, B_{01}, B_{10}, B_{11}\) with \(B_{00} = \{000, 011\}, B_{01} = \{001, 110\}, B_{10} = \{010, 101\}\) and \(B_{11} = \{111, 100\}\) and then sending two bits for the index \(v\) of the bin \(B_v\) which \(X\) belongs to. The bins are formed in a way that each of them has two elements with Hamming distance \(d_H = 3\). For joint decoding with \(v\) and the side information \(Y\), we choose in bin \(B_v\) the \(X\) with \(d_H(X, Y) \leq 1\). Unique decoding is guaranteed because the two elements in each bin \(B_v\) have Hamming distance \(d_H = 3\). Thus we achieve the SW limit of \(H(X, Y) = H(Y) + H(X|Y) = 3 + 2 = 5\) bits in this example. This example can be analyzed within the framework of coset codes and syndromes. We form the parity-check matrix \(H\) of rate 1/3 repetition channel code as

\[
H = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}
\]

For all the \(X \in B_v\), the index \(v\) of the bin \(B_v\) is the syndrome \(v = XH^T\). Sending the 2 bits syndrome \(v\) instead of the original 3 bits \(X\) achieves a compression ratio of 3: 2. In partitioning the eight \(X\) according to their
syndromes into four disjoint bins $B_v$, we preserve the Hamming distance properties of the repetition code in each bin. This partitioning is critical because it ensures the same decoding performance for different syndromes. In channel coding, the set of vectors $X$ satisfying $v = XH^T$ is called a coset code $C_v$ of the linear channel code $C_0$ with all zero syndrome. It is easy to see that each coset code corresponds to a bin $B_v$. The Hamming distance between the codewords of the linear rate 1/3 repetition channel code $C_0$ is the same as the codewords of each coset code $C_v$. Given the index $v$, the side information $Y$ can indicate its closest code word in $C_v$, and $X$ can be recovered.

The concept can be extended to more general cases where a linear channel code with its coset codes can be used to do the binning and hence to construct a SW code. In [55], the correlation between the source $X$ and the side information $Y$ can be modeled with a virtual "correlation channel" with input $X$ and output $Y$. For a received syndrome $v$, the decoder uses $Y$ together with the correlation statistics to determine which codeword of the coset code $C_v$ was the input to the "correlation channel". So if the linear channel code $C_0$ is an efficient channel coding for the "correlation channel", the SW source code defined by the coset code $C_v$ is also a good source code for this type of correlation.

Some state of the art channel codes such as turbo codes and low-density-parity-check (LDPC) codes have been proposed for SW coding [74]-[76], focusing on the case where the source and the side information are correlated binary sequences. In [52] and [53], the source and side information are assumed to be continuous random variables that follow joint Gaussian distribution and a practical Wyner-Ziv coding structure consisting of a quantizer and a multi-level SW encoder was proposed. However, the correlation models that are reasonable in sensor networks is suitable in a CF relay system, where the signals received at the relay and the destination share a common part and are correlated in a different but more complicated manner. Furthermore, in a relay-assisted communication system, the compressed data is transmitted in the relay-destination link which, as opposed to previous works where the compressed data are assumed to be perfectly received by the receiver, is not error-free. Hence in order to maintain an error-free transmission of the compressed data, it should be protected by means of protection coding. We will extend the multi-level SW coding structure into a half-duplex relay scenario and incorporate the soft processing concept to develop a novel soft multi-level SW coding scheme. Moreover, we noticed the relation between the channel code used in the relay-destination link and the SW code used for compression. This relation inspires the idea of joint processing of two codes to improve the performance. For this purpose, a joint turbo-like processing structure will be proposed. Since it has been pointed out that LDPC codes are suitable for SW coding for their near-capacity property [72] and [73], SW codes are deployed through systematic LDPC codes.

4.3.2.1 Signal Model and Multilevel Slepian Wolf Coding

The encoding model is shown in Figure 4.13. $Y'_r(n)$ is splitted into multiple bit planes by the quantizer and we get a binary bin index matrix $V$ is obtained as
\[ V = (v_1, \ldots, v_m) = \begin{pmatrix} v_1(1) & \ldots & v_1(aN) \\ \vdots & \ddots & \vdots \\ v_m(1) & \ldots & v_m(aN) \end{pmatrix} = \begin{pmatrix} V_1 \\ \vdots \\ V_M \end{pmatrix} \]

The multi-level SW coding is carried out in the following way. All of the most important bits which belong to the first bit plane form the first line vector \( V_1 \). Since \( Y_f(n) \) is an \( i.i.d. \) random variable, for a given quantizer, we have

\[ H(v(n) | Y_f(n)) = H(v(m) | Y_f(m)) \forall n \neq m \]

---

**Phase 1**

---

**Phase 2**

---

**Relay**

![Figure 4.13. Multilevel SW coding at the relay](image)

This equation means that the bits belonging to the same bit plane can be compressed at the same rate and the time index \( n \) can be omitted, which forms the basis of the multi-level SW coding. \( V_1 \) is compressed by the first level SW code at a rate

\[ R_i = H(v_i | Y_f) = H(v_i(n) | Y_f(n)) \]

In particular, the line vector \( V_1 \) is compressed as

\[ G_i = V_i H_f = (G_i(1), \ldots, G_i(R; aN)) \]
where $H_1$ is the $R_1\alpha N$-by-$\alpha N$ parity check matrix of a code $C_{SW,1}$ for the first level, $R_1$ is the compression rate for the first level and $G_1$ is actually the syndrome of $V_1$. The process can be repeated for the rest of the levels. For instance, the line vector $V_m$ is compressed as

$$G_m = V_m H_m^T = [G_m(1),...,G_m(R_m\alpha N)]$$

where $H_m$ is the $R_m\alpha N$-by-$\alpha N$ parity check matrix of a code $C_{SW,m}$ for the $m$th level and $G_m$ is the syndrome of $V_m$. Finally, the compressed $G_m$ are concatenated before transmission as

$$G = (G_1,...,G_M)$$

where the number of elements in $G$ is $\alpha N \sum_{m=1}^{M} R_m$. $G$ is then forwarded to the destination. However, in practical wireless channels, the relay-destination link suffers from occasional block fading and the instantaneous SNR may not always be high enough to ensure a successful reception of uncoded $G$. Therefore, as opposed to [74], where $G$ is assumed to be perfectly received, we encode $G$ by another systematic LDPC code $C_p$ to protect it the relay-destination link. With this error protection, we can guarantee that $G$ can be received with few errors, i.e. the relay-destination link is almost error-free. The coded binary symbol vector is $C_p = [G,P]$, where $P$ is the parity check part. Suppose the coding rate of $C_p$ is $R_{Cp}$, the number of elements in $P$ is $(1- R_{Cp})(\alpha/N)N$. The destination structure is demonstrated in Figure 4.14. The received $Y_j$ during phase 2 is first fed into the soft-in-soft-output (SISO) LDPC decoder. The input likelihood ratio (LLR) values are given as

$$L_n = [L_n(1),...,L_n(n),...,L_n((\alpha-1)N)]$$

where

$$L_n(n) = \frac{2e_3 \sqrt{\mu_n}}{\sigma} X^+(n)$$

After SISO decoding, the output LLR values for the compressed data $G$ is $L_c = \{L_{c_1},...,L_{c_M}\}$, where $L_{c_m} = (L(G_{m,1}),...,L(G_{m,\alpha N_m}))$ and $L(G_{m,j})$ represents the LLR value of the $j$th bit of the $m$th level compressed data. From previous definitions, we can see that $G_{m,j}$ is also the $s$th bit of $G$, where $s$ is equal to $\left(\alpha N \sum_{m=1}^{M} R_m + j\right)$. 
According to [73], \( L(G_{m,j}) \) is given by
\[
L(G_{m,j}) = \log \frac{\Pr(G_{m,j} = 0 | Y_j = y_d, \text{con1, con2})}{\Pr(G_{m,j} = 1 | Y_j = y_d, \text{con1, con2})}
\]
where \( \text{con1} \) and \( \text{con2} \) are two conditions related with the Belief Propagation (BP) algorithm and can be defined as
- \( \text{con1} \): the check equations involving \( G_{m,j} \) are satisfied.
- \( \text{con2} \): messages sent to \( G_{m,j} \) from all check nodes of the LDPC code.

If hard decision is made regarding \( L_G \), the SW decoding algorithm in [74] can be carried out directly. However, since soft processing always improves performance, we can deliver the soft values to the SW decoder and develop a soft algorithm which will be detailed later.

The SW decoder has the similar structure to a SISO decoder. When the first level of the compressed data \( G_1 \) is decompressed, with hard decision, the result is \( V_1 \) which consists of all the most significant bits for the quantized \( Y_1' \). \( V_1 \) is required by the SW decoders in the following levels and will then be passed to lower levels. This process will be repeated such that \( V_1 \) to \( V_{(m-1)} \) are available in the \( m \)th level. On the contrary, in the soft scheme, instead of \( V_m \), the output of the SW decoder, i.e. the LLR value of \( V_m \) is delivered to the SW decoders in the following levels directly. Similarly, we define \( L_{V_m} = \{ L(V_m), \ldots, L(V_{m,N}) \} \), which, according to [73], can be expressed as
\[
L(V_{m,n}) = \log \frac{\Pr(V_{m,n} = 0 | V_d = y_d, L_{G_m}, L_{r_1}, \ldots, L_{r_N}, \text{con3, con4})}{\Pr(V_{m,n} = 1 | V_d = y_d, L_{G_m}, L_{r_1}, \ldots, L_{r_N}, \text{con3, con4})}
\]

69
where \( L_j \) to \( L_{m-1} \) are obtained from upper levels 1 to \((m-1)\), and \(\text{con3} \) and \(\text{con4} \) are conditions related to the BP process in the SW decoder, denoted as

- \(\text{con3} \): the check equations involving \( v_{mj} \) are satisfied.
- \(\text{con4} \): messages passed to \( v_{mj} \) from all check nodes.

Moreover, according to the expressions of the LLR values, we noticed that the SISO decoder of \( C_p \) and the SW decoders are not independent, which arose the idea of introducing a joint processing structure between them. As a result, a turbo-like joint processing structure is proposed, where the extrinsic information is allowed to be exchanged between two parts to achieve better performance.

### 4.3.2.2 Soft Slepian Wolf Decoding Using LDPC Codes

In this section, we introduce the novel soft decoding algorithm. The BP algorithm can be implemented here as usual, where the difference is that the syndrome is not an all-zero vector but a vector whose elements are LLR values. By doing this, we take the soft values of the syndrome into consideration. The detailed BP algorithm is provided in the Appendix E. From the results in Appendix B, it is easy to see that the derived equation is identical with the results in [74] if the LLR values of the syndrome are approaching infinity. In our algorithm, the initial LLR values of the syndrome \( L_j \) in the \( m \)th level is given by \( L(J) \).

### 4.3.2.3 Soft Information Passing in the Multilevel Slepian Wolf Decoding

In [53], the decompressed information \( V_{m-1} \) is passed to the following levels such that \( V_1 \) to \( V_{m-1} \) are available at the SW decoder in the \( m \)th level. A hypothetical correlation channel can be established between the input \( v_{mj} \) and the output \( (y_j(n), y_{n,1}, \ldots, y_{n,m-1}) \) and the SW decoder can be regarded as a channel decoder for this hypothetical channel. Therefore, the BP algorithm can be carried out as well with initial LLR values

\[
L(v_{mj}) = \log \frac{\Pr(y_j(n) = y_j^*(n), v_{mj} = b_{m,1}, \ldots, v_{m-1,m} = b_{m-1,m} | v_{mj} = 0)}{\Pr(y_j(n) = y_j^*(n), v_{mj} = b_{m,1}, \ldots, v_{m-1,m} = b_{m-1,m} | v_{mj} = 1)}
\]

Given the fact that \( X_j^*(n) \), \( y_j^*(n) \) and \( y_j^*(n) \) are independent to each other, if \( L_{mj} \) is delivered to the \( m \)th decompressor, the output of the hypothetical channel is replaced by \( y_j^*(n) \), and the corresponding initial LLR values are given in Appendix E as
\[
L(v_{n,n}) = \log \frac{\sum_{s_{n-1} \in \{0,1\}} \sum_{s_{n-2} \in \{0,1\}} \left( \sum_{s_{n-3} \in \{0,1\}} \cdots \left( \sum_{s_{n-k} \in \{0,1\}} \prod_{i=1}^{k} \Pr^{\text{CF}}_{s_i} P_r^{\text{CF}(\text{sym}-1)} \prod_{m=0}^{k-1} \Pr^{\text{CF}(\text{sym}-1)} \right) \right) \right)}{\sum_{s_{n-1} \in \{0,1\}} \sum_{s_{n-2} \in \{0,1\}} \left( \sum_{s_{n-3} \in \{0,1\}} \cdots \left( \sum_{s_{n-k} \in \{0,1\}} \prod_{i=1}^{k} \Pr^{\text{CF}}_{s_i} P_r^{\text{CF}(\text{sym}-1)} \prod_{m=0}^{k-1} \Pr^{\text{CF}(\text{sym}-1)} \right) \right) \right)} - \log \frac{\Pr(v_{n,n} = 0)}{\Pr(v_{n,n} = 1)}
\]

(4.8)

with definitions

\[
\Pr^{\text{CF}}_n = \Pr(\gamma^1_n(n) = \gamma^0_n(n) | X^1_n(n)), \quad \Pr^{\text{CF}(\text{sym}-1)}_n = \Pr(v_{\text{sym}} = 0 | v_{\text{sym}}, \ldots, v_{\text{sym}-1,n} X^1_n(n))
\]

\[
\Pr^{\text{CF}(\text{sym}-1)}_n = \Pr(v_{\text{sym}} = 1 | v_{\text{sym}}, \ldots, v_{\text{sym}-1,n} X^1_n(n)), \quad \Pr^{v_{\text{sym}}} = \Pr(v_{\text{sym}} | v_{\text{sym}}, \ldots, v_{\text{sym}-1,n} X^1_n(n))
\]

\[
\Pr^{v_{\text{sym}}} = \Pr(v_{\text{sym}} | X^1_n(n))
\]

\(\Pr^{v_{\text{sym}}}\) can be obtained from the soft values passed from upper levels. In the previous SW decoding algorithm, when decoding in the \(m\)th level, \((v_1, \ldots, v_{m-1})\) is deemed as a deterministic vector. In contrast, we treat \((v_1, \ldots, v_{m-1})\) as a random vector in the new algorithm. In this regard, it is possible to exploit the soft value of \((v_1, \ldots, v_{m-1})\) and benefit from the soft information passing.

4.3.2.4 Joint Turbo-like Decoding and Decompressing

For simplicity, we only address the joint processing between the first SW decoder and the decoder of \(C_p\), i.e. we assume that \(G = G_1\). The extension to the multilevel case is quite straightforward. The decoding process of error protection code \(C_p\) and the compression code \(C_{\text{CF}}\) is shown in Figure 4.15.

![Figure 4.15. Separate decoding and decompressing.](image-url)

\(s\): observed syndrome, \(f\): parity check nodes, \(v\): observed variable nodes (through side information)
In the separate mode, $C_p$ is decoded first and the output soft values of the compressed data are delivered to the SW decoder. We notice the similarity of the whole structure to the turbo codes with two LDPC component codes and propose a joint turbo-like algorithm as follows:

Step1: Initialize the iteration counter $t=1$ and the initial LLR values $L_{C_{p,in}}$ are given as $(L_{i,m}, L_{P,m})$. $L_{P,m}=(L(P_1),\ldots, L(P_1))$ is the initial LLR value of the parity check part $P$, given as

$$L_a(P_i) = \log \frac{\Pr(P_a = 0 | Y_a^i (n + R_{c_p}(1 - \alpha)N) = Y_a^i (n + R_{c_p}(1 - \alpha)N))}{\Pr(P_a = 1 | Y_a^i (n + R_{c_p}(1 - \alpha)N) = Y_a^i (n + R_{c_p}(1 - \alpha)N))}$$

where $P=(1- R_{c_p})(1-\alpha)N$. $L_{C_{p,in}}$ is fed into the SISO decoder and the output LLR values are $L_G$. The initial extrinsic information from the SISO decoder to the SW decoder is $\Omega_a = L_a$.

Step2: At the SW decoder

i. $\Omega_a$ is treated as extrinsic information, decompress $C_{sw}$ to obtain $L_{iV}$,

ii. Based on $L_{iV}$, update $L_a$ as

$$L(G_j) = L(F_j) = \gamma / \prod_{i} \alpha_j \cdot \phi(\nu_i) + \sum \phi(\beta_i)$$

iii. Update $\Lambda_a = L_a - \Omega_a$

Step3: $\Lambda_a$ is passed to the SISO decoder of $C_p$ and treated as extrinsic information. Update the input LLR values for the information part of $C_p$ as $L_{i_c,m} = L_{i_c,m} + \Lambda_a$ and decode $C_p$.

Step4: After decoding, we will obtain new output LLR values for the compression data $G$ and update $\Omega_a = L_a - \Lambda_a$. If $t=$Maximal iteration numbers, go to step 5; otherwise, $t=t+1$ and pass $\Omega_a$ to the SW decoder. Go to step 2;

Step5: Finish.

The whole process is shown in Figure 4.16.

![Figure 4.16 Joint Turbo Decoding & Decompressing](image)
4.3.2.5 Simulation Results

The novel joint processing structure is evaluated in the AWGN channel first. The simulation scenario is set as follows:

- \( M = 2 \). This means that two-level or two-bit plane SW coding is deployed.
- A Systematic LDPC code \( C_{SW} \) with length 204, coding rage 1/2 is used for the SW coding in two levels.
- For simplicity, the compressed data \( G_1 \) and \( G_2 \) achieved by the SW encoders in two levels respectively will not be concatenated before error protection coding. Instead, they are separately encoded with error protection codes \( C_{p,1} \) and \( C_{p,2} \) respectively. By doing this, the joint turbo-like decoding and decompressing process is constrained within a single level (bit plane). For example, \( C_{p,1} \) only interacts with \( C_{SW,1} \). After encoding, \( C_{p,1} \) and \( C_{p,2} \) are concatenated. The extension to the case where \( G_1 \) and \( G_2 \) are concatenated before error protection coding is not difficult.
- A systematic LDPC code with length 204 and coding rate 1/2 is used for \( C_{p,1} \) and \( C_{p,2} \).
- The achievable rate based quantization scheme introduced in previous sections is used.

The actual coding rate for the level 1 and level 2 SW coding are the same and fixed at 0.5 for simplicity but the ideal coding rates, which can be obtained via the conditional entropies according to [54], are different. In future work, we can extend this scenario to use flexible compression rate. The ideal rates are listed in TABLE 4.1 and the gap between the ideal and actual rates is also listed.

![Figure 4.17. BER of the first level. AWGN, \( \mu_1 = \mu_0 + 2 \) dB, \( \mu_2 = 2 \) dB](image1)

Figure 4.17 depicts the BER in the first level. When the SNR is lower than 4 dB, three BER curves are almost flat and maintains in a high level. The work in [54] presented an explanation for this phenomenon. The error stems from two sources. On the one hand, the relay-destination link might not be strong enough to support error-free transmission and the error in this link contributes to the overall BER. On the other hand, the SW decoding also produces errors which account for the overall BER as well. The SW decoding error rate is closely

73
related to the gap $\Delta$ between the actual and ideal coding rates. When $\Delta$ is below a certain threshold, the decompression error rate remains almost flat at a high level; otherwise, the error rate drops sharply to a very low level. When the SNR is below 4 dB, the gap $\Delta$ is still lower than that potential threshold. In such a case, the overall error performance is dominated by the SW decoding error and the BER remains almost unchanged. If the SNR is above 4 dB, $\Delta$ exceeds the potential threshold and the SW decoding error rate decreases sharply. Now the Errors introduced in the relay-destination link is dominating the overall BER instead. With a hard decoder, the errors will propagate when the erroneous syndromes are further utilized in the SW decoding, thus leading to a high error rate in the first level. The error propagation does not only occur within one single level, but it also happens when the decompressed information is passed from upper levels to lower levels. This error propagation process greatly degrades the system's performance. But if we treat syndromes as soft values, the error propagation can be avoided and a far lower BER is achievable. When the joint turbo-like decoding and decompressing concept is incorporated, the performance can be further improved.

### Table 4.1

<table>
<thead>
<tr>
<th>$\gamma$ (dB)</th>
<th>Ideal coding rate</th>
<th>Actual coding rate</th>
<th>Gap $\Delta_1$ and $\Delta_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.502(SW1)</td>
<td>0.5(SW1)</td>
<td>-0.002(SW1)</td>
</tr>
<tr>
<td></td>
<td>0.098(SW2)</td>
<td>0.5(SW2)</td>
<td>0.402(SW2)</td>
</tr>
<tr>
<td>1</td>
<td>0.442(SW1)</td>
<td>0.5(SW1)</td>
<td>0.058(SW1)</td>
</tr>
<tr>
<td></td>
<td>0.096(SW2)</td>
<td>0.5(SW2)</td>
<td>0.404(SW2)</td>
</tr>
<tr>
<td>2</td>
<td>0.372(SW1)</td>
<td>0.5(SW1)</td>
<td>0.128(SW1)</td>
</tr>
<tr>
<td></td>
<td>0.094(SW2)</td>
<td>0.5(SW2)</td>
<td>0.406(SW2)</td>
</tr>
<tr>
<td>3</td>
<td>0.297(SW1)</td>
<td>0.5(SW1)</td>
<td>0.203(SW1)</td>
</tr>
<tr>
<td></td>
<td>0.091(SW2)</td>
<td>0.5(SW2)</td>
<td>0.409(SW2)</td>
</tr>
<tr>
<td>4</td>
<td>0.220(SW1)</td>
<td>0.5(SW1)</td>
<td>0.280(SW1)</td>
</tr>
<tr>
<td></td>
<td>0.090(SW2)</td>
<td>0.5(SW2)</td>
<td>0.410(SW2)</td>
</tr>
<tr>
<td>5</td>
<td>0.150(SW1)</td>
<td>0.5(SW1)</td>
<td>0.350(SW1)</td>
</tr>
<tr>
<td></td>
<td>0.089(SW2)</td>
<td>0.5(SW2)</td>
<td>0.411(SW2)</td>
</tr>
<tr>
<td>6</td>
<td>0.092(SW1)</td>
<td>0.5(SW1)</td>
<td>0.408(SW1)</td>
</tr>
<tr>
<td></td>
<td>0.088(SW2)</td>
<td>0.5(SW2)</td>
<td>0.412(SW2)</td>
</tr>
</tbody>
</table>

SW1: level 1 SW2: level 2 $\Delta =$ real coding rate - ideal transmission rate.
For the second level SW decoding, three scenarios are considered:

1. The SW decoding in the first level is error-free and perfect $V_1$ is passed to the second level ($S1$).
2. $V_1$ is obtained from joint processing and passed to level 2 ($S2$)
3. $V_1$ is obtained from joint processing but $L_{v_l}$ is passed to the second level instead ($S3$)

The BER performance for different scenarios is shown in Figure 4.18. In Figure 4.18 (a), we assume scenario $S1$. As shown in the TABLE 4.1, the gap $A_2$ is quite large in the second level. Hence the dominating factor for the overall BER performance is the errors introduced in the relay-destination link.

![BER performance graphs](image)

Figure 4.18 BER of the second level with different information passing. (a) $S1$; (b) $S2$; (c) $S3$; (d) $S1$, $S2$ and $S3$. (AWGN, $\mu_1 = \mu_2 + 2$ dB, $\mu_3 = 2$ dB)

Apparently, by treating the syndrome as soft values, soft SW decoding is able to achieve a far lower BER. With joint processing of the SISO decoder and the SW decoder, the performance can be further improved about 3 dB. When scenarios $S2$ or $S3$ are assumed, the BER keeps at a high level when the SNR is below 4 dB because in
this SNR region, the error rate of $V_1$ is retained at a high level and these errors will propagate to the SW decoder in the second level. When the SNR is higher than 4 dB, the same trend as level 1 is observed.

We also compare the results for all the three scenarios in Figure 4.18 (d) to indicate the effect of soft information passing between levels. As depicted, soft passing achieves lower BER and is able to approach the lower bound with increased SNR.

To reveal the advantage of CF, the performance of DF and CF schemes are compared under below assumptions:

- A distributed turbo code (DTC) is deployed in the DF-based relay system. Two identical RSCCs with generator matrix (1, 5/7) are used as two components of the DTC.
- To achieve a fair comparison, we assume for the CF scheme that the same RSCC is used at the source.
- The new joint processing structure is incorporated in the CF scheme.

We investigate the case where the average SNRs of the source-relay and source-destination links are kept constant at a low level and the average SNR in the relay-destination link increases in a wide range. Two different SNR settings are examined. The compression rate is chosen adaptively based on the SNR settings. For the first case where $\mu_0=\mu_1=0$, efficient compression cannot be applied to the first level in the multilevel SW decoder. As a result, the compression rate is set at 1, i.e. no compression for the first level. In the second level, the compression rate is chosen as $1/2$. For the second case where $\mu_0=\mu_1=4$ case, the compression rate for both levels are set at $1/2$. For comparison purposes, two scenarios are considered as below:

- Ideal decompression (IC) where we assume that the compressed index can be recovered without any error.
- Co-located (CL) where the relay is assume to be co-located with the destination such that they can share all the information. However, the source- and source-destination links are assumed to be independent with each other, thus achieving full receive diversity.

The simulation results are displayed in Figure 4.19 and Figure 4.20. The BER of the DF scheme remains almost unchanged in Figure 4.19 because its performance is mainly constrained by the low quality of the source-relay link. When this link is improved in Figure 4.20, this constraint is relaxed and the DF's performance can be improved with increased $\mu_2$ until it reaches an error floor in the high $\mu_2$ region where the DF's performance is constrained by the source-relay link again. The performance of CL and IC is constant because we assume no compression errors are introduced, i.e. the destination knows exactly the quantized index of the signal received by the relay. The gap between them is due to the quantization noise.
The BER of CF is able to approach the IC case when the relay-destination link is being improved and the number compression error is getting smaller. Finally, when the relay-destination link is sufficiently good, it can be regarded as error-free, i.e., no errors are produced in the compression/decompression process. As a result, the BER of CF is able to approach the IC case and outperform DF eventually.

The simulation is extended to block fading scenarios as shown in Figure 4. 21. Here we only compress the bits in the second bit plane. The same trend has been observed for CL and IC cases. The BER of DF decreases slowly until reaching an error floor due to the limitation of the source-relay link. In contrast, the BER of CF is able to approach the BER of IC when the relay-destination link is getting stronger and the number of decompression errors becomes smaller.

4.4 Conclusions
We presented the compress-and-forward (CF) scheme in this chapter and provided the analysis for the achievable rate and outage behavior for RC- and UC-based schemes. We further considered the complexity problem and proposed a quantize-and-forward (QF) scheme, where the relay’s observation is quantized and forwarded to the destination without compression. Moreover, we addressed an important aspect in the CF-based relay system: Wyner-Ziv (WZ) coding. A practical WZ coding structure consisting of a quantizer and a multi-level SW compressor was presented. A novel quantizer design scheme dedicated to the CF scheme and a new soft multi-level SW coding structure were proposed.

The achievable rate analysis revealed that the UC-based CF scheme has the advantage of always being able to outperform direct transmission. In contrast, when RC is implemented, due to the fixed duplexing ratio and bandwidth efficiency loss, the achievable rate could be lower than direct transmission in some SNR regions. The comparisons between the UC-based CF and DF schemes suggest that DF is more suitable when the relay is close to the source and the link between the two nodes is relatively strong; in contrast, CF is a better candidate when the relay is close to the destination and corresponding link is of high quality. We reached an important conclusion that will guide the implementation of novel techniques for CF. If the source is silent during phase 2, the performance loss in terms of the achievable rate is quite small. However, the decoding structure at the destination can be greatly simplified because simple combination schemes can be applied for decoding. In contrast, if the source is allowed to transmit in phase 2, we have to resort to a joint processing of the destination received signals in two phases and the estimated relay’s observation. Therefore, we decided to use the simpler relay protocol I in practical implementation at the expense of small performance loss.

The outage behavior was also presented. The interesting thing is that despite the lower achievable rates of RC-based CF and UC-based QF, they enjoy the same second-order diversity as UC-based CF and all the CF and QF schemes have smaller outage probabilities than the UC-based SDF scheme.

Besides the theoretical analysis, we also addressed the most important part of the CF scheme: WZ coding. The WZ coding problem is decoupled into two parts: quantization and SW coding and studied separately. The definition of quantization is the division of a quantity into a discrete number of small parts, often assumed to be integral multiples of a common quantity. Numerous quantization algorithms have been proposed for an independent source $X$ belonging to a continuous alphabet with the optimization objective to minimize the distortion. A representative design algorithm, namely Lloyd-Max quantization was examined in this chapter. However, in WZ problem, an important difference is the existence of side information which is correlated with source $X$. Some recent approaches took side information into consideration but the optimization objective is still to minimize the distortion only. Some researchers noticed that in the WZ problem, besides the distortion, the source coding rate is also an important parameter and proposed the design algorithm with Lagrangian optimization objective which is the weighted summation of the distortion and the source coding rate. We extended this concept into the relay scenario and examined its performance. Our main contribution was to develop a new quantization scheme based on the achievable rate of the relay system and provided a new decoding scheme accordingly with reduced complexity. The new quantization scheme achieved the highest rate
and displayed a better BER performance than the rate distortion algorithm. However, it produces larger distortion. This fact gave us a hint that in a practical relay system, the most crucial factor which could influence the BER performance is not the distortion.

In addition, a soft multi-level SW coding structure was provided. The relay received signal is split into multiple bit planes by a quantizer. All the bits belong to the same bit plane are compressed together at the same rate. One bit plane corresponds to one level in the multi-level SW coding structure. In light of the observation that the relay-destination link may not always be strong enough for the error-free reception of the compressed data, the compressed data should be protected by some error protection code $C_p$. In the new structure, the SISO decoder of $C_p$ produces soft values of the compressed data and the achieved soft values are delivered to the soft SW decoders to be decompressed. The soft process within one single bit plane (level) efficiently avoids the error propagation from the decoder of $C_p$ to the soft SW decoder. Moreover, in the multilevel SW decoder, the decompressed data should be passed from upper levels to lower levels. This process is also “softened” by delivering the soft values of the decompressed data to the lower levels instead. By doing this, we efficiently reduced the risk of error propagation during the information passing process between different levels. Finally, we noticed the statistical relation between the SISO decoder and the SW decoder and developed a joint turbo-like structure to further improve the performance through extrinsic information exchange. A practical CF system incorporating the novel techniques was compared with DF in both AWGN and fading channels. The simulation results indicated that when the relay-destination link is sufficiently strong, the CF scheme is able to outperform DF in certain SNR regions and approach the performance of the case where the relay and the destination are co-located.
Chapter 5 Hybrid Relaying Schemes

5.1 Introduction

In previous chapters, various relaying schemes have been investigated. Each scheme has its own favorable area. For the AF scheme, the destination and relay SNR ratio $\rho$ is a crucial measurement, which determines the amplification function. The performance of DF is constrained by the source-relay link. If this link is weak, erroneous information will be decoded at the relay and the transmission of incorrect messages in turn will result in error propagation at the destination and degrade the overall performance. CF is suitable for the scenario where the relay-destination link is very strong and the compressed data can be received by the destination without or with a very small number of errors.

However, the wireless channels suffer from fading occasionally. Even when the relay is close to the source or destination, the instantaneous SNRs of the links are not always apposite for one single relaying scheme. For instance, the relay cannot successfully decode every block no matter how close it is to the source. Thus DF cannot be ensured to be successful all the time. We need to develop a flexible relay system which is able to adapt itself to the dynamics of the channel.

The first hybrid relaying scheme we proposed incorporates the multilevel coding concept in a relay system, where the source has a layered encoding structure. According to [77], the BERs of different coding levels exhibit considerable difference when appropriate labelling schemes are chosen. Figure 5.1 plots the BER performance of different levels for 64-QAM signals with 6-level coding. The fact that the BER gap between different levels is quite large motivates us to deploy DF in upper coding levels, where decoding is more likely to be successful, and in the mean time CF in the lower coding levels, where the error probability of decoding increases significantly. In such a scheme, the system's complexity is an important issue that deserves considerable insight. As discussed in the last chapter, to approach the ideal compression rate, multilevel SW compressor is required [53], where each level requires one compressor/decompressor pair in the relay and destination respectively. From an implementation point of view, CF requires far more complexity than the DF technique. By deploying CF in the lower levels only, the error propagation is precluded and with less compressor/decompressor pairs required, the overall complexity is reduced.

In the first hybrid scheme, DF and CF are carried out simultaneously. We will propose another hybrid relaying scheme where DF and CF are performed in different time according to the current channel realizations. We noticed that in a block fading scenario, if the CSI is known globally, an ideal hybrid relaying scheme can be utilized in which each node calculates the achievable rates of different relaying schemes under current channel realizations before the transmission of each block, and wisely chooses the scheme with the highest achievable rate. The source is able to tune its transmission rate to the achievable rate $R$. 

where $R_{DF}$ and $R_{CF}$ represent the achievable rates of DF and CF respectively. In a two-node communication system, at the transmitter side CSI can be obtained through feedback from the receiver or exploitation of the reciprocity property of the channel [101]. Apparently, a relay system with three nodes requires a more sophisticated strategy to make CSI of three links known globally. Anyhow provision of global CSI at each node is not affordable in most of the practical conditions. A practical assumption is that the channel knowledge is only available at the receiver, i.e. the source has not access to CSI of any link, the knowledge of the source-relay link is available at the relay and the destination is able to estimate the source- and relay-destination links. In such a case, the absence of CSI at the source and relay prevents us from applying the ideal hybrid scheme, but the adaptive concept could still be incorporated, where the relay re-encodes the received signal if it is able to decode successfully and compresses the received signal otherwise. In other words, the relay works in DF or CF mode adaptively according to its decoding status. When operating in DF mode, the relay only requires the channel knowledge of the source-relay link. However, as pointed out in [4], [49], the original CF technique requires global CSI (especially the instantaneous SNRs which determines the statistics of the signal) at the relay to adjust its compression rate and carry out ideal Wyner-Ziv coding. To address this problem, a limited feedback link from the destination to relay was established to convey partial CSI of the source-destination link in [8] and the results showed some improvement in terms of the expected throughput. However, the weakness of their work is that it is assumed that the relay is extremely close to the destination such that the channel between them keeps constant and known to the relay. This assumption brought two problems: one is that the closely located relay and destination could lead to high correlation that could violate the independence assumption for the source-relay and source-destination links; another problem is that the hybrid relaying scheme, which enjoys the benefits provided by both DF and CF, should perform well when the relay is neither extremely close to the source nor to the destination. Therefore, in this thesis we accepted a more general
assumption that the relay could be located at any position and the relay-destination link also suffers from fading. The feedback should carry partial CSI of both the source- and relay-destination links. Our results revealed that the relay-destination link is crucial to the outage behavior of the relay system and presented some new insight into the hybrid relay system.

The rest of the chapter is organized as follows. In the next section, we will introduce the hybrid relaying scheme based on multilevel coding. The research of this scheme is simulation-oriented and its BER performance is evaluated through simulation. Another hybrid relay model is introduced in section 5.3 and this hybrid scheme is further developed under the assumption of partial CSI feedback through a limited feedback link. Conclusions are drawn in the final section.

5.2 Hybrid Relaying Scheme I

5.2.1 Multilevel Coding Strategy

The multilevel coding structure is depicted in Figure 5.2. Messages $w_1$ to $w_N$ are mapped to $N$ binary vectors $b_1$ to $b_N$,

$$ b_n = [b_n^1, \ldots, b_n^{K_C}] $$

where $N$ is the total number of coding levels, $K_C$ is the length of the information part in the $n$th level and $1 \leq n \leq N$. $b_1$ to $b_N$ are parallel encoded into $s_1$ to $s_N$ independently at rates $R_1$ to $R_N$ respectively,

$$ s_n = [s_n^1, \ldots, s_n^{K_C}] $$

where $N_{c,n}$ is the length of coded $b_n$ and the coding rate is $K_{c,n}/N_{c,n}$. Here we assume that the length of $b_n$ and the coding rate $R_n$ for different levels are the same, i.e. $K_{c,n} = K_c$ and $N_{c,n} = N_c \forall n$. The coded information $s_1$ to $s_N$ from a matrix $S$, given by

$$ S = \begin{bmatrix} s_1 \\ \vdots \\ s_N \end{bmatrix} = \begin{bmatrix} s_1^1 & \cdots & s_1^{K_C} \\ \vdots & \ddots & \vdots \\ s_N^1 & \cdots & s_N^{K_C} \end{bmatrix} = (\bar{s}_1, \ldots, \bar{s}_{N_c}) $$

where

$$ \bar{s}_{n_c} = [s_{n_c}^1, s_{n_c}^2, \ldots, s_{n_c}^{K_c}]^T $$

$S$ is modulated in a cross level manner as shown in Figure 5.2. At time $n_c$ ($1 \leq n_c \leq N_c$), the column vector $\bar{s}_{n_c}$ is mapped into a point in the constellation, denoted as $X_n(\bar{s}_{n_c})$. Suppose Gray labeling is applied, $s_{n_c}^1$ is the most significant bit and $s_{n_c}^{K_c}$ is the least significant bit.
Each $y^s$ is mapped into a point in the constellation.

Figure 5.2. Source with a multilevel coding structure.

With the relay model introduced previously, the received signal at the relay is $y^1_r$. The relay starts by processing $y^1_r$ with a soft demapper and a $N \times N_c$ matrix $U$ representing the soft values is obtained as

$$U = \text{SoM}(S) = \begin{pmatrix} u^1_1 & \cdots & u^N_1 \\ \vdots & \ddots & \vdots \\ u^1_N & \cdots & u^N_N \end{pmatrix} = \begin{pmatrix} u^1_1 \\ \vdots \\ u^N_N \end{pmatrix}$$

where $\text{SoM}(.)$ represents the soft mapping and

$$u^s_j = \log \frac{P(s_j^+ = 0 \mid y^1_r(n_r))}{P(s_j^+ = 1 \mid y^1_r(n_r))}$$

In the conventional DF scheme, $u_n$ is fed into the SISO decoder followed by a hard decision function in the $n$th level and $s_n$ is obtained. From $s_n$ and $\hat{b}_n$ can be extracted (if decoding is successful, $\hat{b}_n$ is equal to $b_n$) and re-encoded. This process is carried out in $N$ levels parallel. Finally, the relay could either modulate the coded information in the same cross level manner as the source, or forward the coded information in a time-division manner for different levels. However, it has been pointed out that the BERs in coding levels are significantly different. For Gray labeling, the higher the level, the larger the possibility of successful decoding is [77]. The fact motivates us to decode upper $N_u$ levels only and leave lower $N_l$ levels for CF $(N=N_u+N_l)$. For instance, there are 6 levels in 64-QAM $(N=6)$ as shown in Figure 5.3. Suppose distributed turbo codes are utilized in the two highest levels $(N=2)$ only, which together indicate the quadrant where the signal lies. The received signals are decoded, denoted as $\hat{s}_1$ and $\hat{s}_2$. Information $\hat{b}_1$ and $\hat{b}_2$ are extracted, re-encoded and partially forwarded, i.e. only the parity check part is forwarded. With $\hat{s}_1$ and $\hat{s}_2$, we can resolve the quadrant where $X^1_r(n_r)$ is. In the following steps $Y^1_r(n_r)$ (the black square point) in the large coordinates can be represented by $y^1_r(n_r) = y^1_r(n_r) - 4p \left( + \sqrt{-1} \right)$ in the smaller coordinates, where $p$ is shown in Figure 5.3.

83
Instead of 64-QAM in the large coordinates, we are now facing a 16-QAM signal in the small coordinates. As opposed to conventional CF, where $Y'(n_i)$ is quantized into a binary bin index with $K$ bits, it is reasonable to use a smaller number of bits to quantize $Y'(n_i)$. Supposing $K'$ bits quantization ($K' < K$), we have

$$\mathbf{V} = (\mathbf{v}_1, \ldots, \mathbf{v}_{n_c}) = \begin{pmatrix} v'_1 & \cdots & v'_{K'} \\ v_{K'} & \cdots & v'_{2K'} \\ \vdots & \ddots & \vdots \\ v_{n_cK'} & \cdots & v_{n_c2K'} \end{pmatrix} = \begin{pmatrix} \mathbf{V}_1 \\ \vdots \\ \mathbf{V}_{n_c} \end{pmatrix}$$

where $\mathbf{V}_n = \mathbf{Q}_{x'}(y'_{(n_i)}) = \begin{pmatrix} v^x_{n_1} \\ \vdots \\ v^x_{n_{K'}} \end{pmatrix}$ and $\mathbf{Q}_{x'}(\cdot)$ represents a $K'$-bit quantization function. For the selected quantization scheme, $v^x_{n_1}$ is the most significant bit and $v^x_{n_{K'}}$ is the least significant bit. Note that the meaning of significant bit here is different from the definitions in the modulation scheme. In this regard, we denote them as the most or least significant quantization bit. The matrix $\mathbf{Q}$ is compressed by a $K'$-level SW encoder, where each line vector $\mathbf{V}_n$ is compressed by an independent SW encoder at a particular compression rate and $K'$ SW encoders are required in total. After compression, we have

$$\mathbf{G} = [\mathbf{G}_1, \ldots, \mathbf{G}_{K'}, \ldots, \mathbf{G}_{n_c}]$$

where $\mathbf{G}_n = [G'_1, \ldots, G'_{K'}, \ldots, G'_{n_cK'}]$ is the compressed $\mathbf{V}_n$ and $R_c(k)$ is the compression rate of the $k$th level SW encoder. The compressed data $\mathbf{G}$ should be protected by an error-protection code $\mathbf{C}_p$ with coding rate $R_{cp}$ to combat the errors introduced in the relay-destination link. The entire process is depicted in Figure 5.4.
Upper levels decoded information is used to reduce the size of the constellation.

Figure 5.4. Hybrid DF/CF at the relay.

Compared with conventional CF, the number of quantization bits becomes smaller \((K' < K)\) and the total number of required compressor/decompressor pairs is reduced accordingly, thus leading to reduced complexity.

The total number of bits required to be transmitted during the relay-transmit phase can be calculated. Suppose the coding rate of the \(n\)th level \((n \leq N_u)\), where DF is performed, is \(R_{DF,n}\), the number of parity check bits in \(N_u\) levels is

\[
B_{DF} = \sum_{n=1}^{N_u} K_{x,n} \left( \frac{1 - R_{DF,n}}{R_{DF,n}} \right)
\]

When \(N_u \rightarrow N_l\), we have the total number of transmitted bits for conventional DF. For the lower levels where CF is carried out, we have

\[
B_{CF} = \sum_{k=1}^{K} N_l R_{c}(k)
\]

It follows that the total number of bits transmitted during the second phase is
\[ B_H = B_{cp} + B_{cr} \]

Similarly, we can derive the number of bits used for conventional CF as

\[ B_{cr} = \frac{\sum_{k=1}^{N_c} R_c(k)}{R} \]

With proper selection of the coding and compression rates, \( B_H \) could be smaller than \( B_{cr} \), i.e. the hybrid scheme is able to transmit fewer bits, thus under the condition that the same modulation scheme is deployed, more efficient transmission can be achieved due to the reduction of the second phase's duration.

At the end of phase 2, the destination carries out 2-stage decoding. In the first stage, the upper level information is decoded with turbo decoders and based on the decoded bits, \( y'(n_1) \) can be transplanted to the smaller coordinates as well, where it is represented by \( y'(n_r) \). \( y'(n_r) \) serves as side information to reconstruct the estimation of \( y'(n_r) \): \( \hat{y}'(n_r) \). The reconstructed estimation \( \hat{y}'(n_r) \) is then combined with \( y'(n_r) \) to decode in the lower levels. The block diagram is presented in Figure 5.5.

![Diagram](attachment:DestinationStructure.png)

**Figure 5.5. Destination Structure.**

### 5.2.2 Simulation Results and Discussions

The settings for our simulations are given as follows:

- Gray labeled six-level 64-QAM modulation scheme is used in the first phase.
- The same distributed turbo code with two RSCC components with rate 1/2 is applied in each level.
- DF is performed for level 1 and 2, and for the remaining four lower levels, we use CF.
• LDPC codes with rate 2/3 are used for compression.

• 3-bit quantizer coupled with a 3-level SW encoder is deployed per dimension (two dimensions). Only the lowest level is compressed. We leave the information in the first and second levels uncompressed because their ideal compression rates are close to 1, which means the information cannot be efficiently compressed in practical implementations.

• The compressed bits are protected by a LDPC code with rate 2/3 in the relay-destination link during phase 2, and the modulation scheme is Quadrature Phase Shift Keying (QPSK).

For the purposes of comparison, we also need to explore the performance of conventional DF and CF. For the conventional DF scheme, the same distributed turbo codes are used. However, no multilevel coding is incorporated. At the source, as opposed to the multilevel coding structure where the column vector of the coded information matrix $S$ is mapped into a 64-QAM signal, $S$ is modulated line by line. In other words, the source structure is simplified to a single encoder followed with a 64-QAM modulator. The same setting applies to the conventional CF scheme. In the conventional CF scheme, the relay uses 4-bit quantizer coupled with a 4-level SW encoder per dimension (two dimensions). Only the lowest two levels are compressed for the same reason and the same LDPC code is used for the purpose of error-protection.

Compared with conventional CF, the total number of compressor/decompressor pairs is reduced by half. Suppose the length of the uncoded information sequence at the source is $K_c$ in each level. At the relay, the total number of the bits required to be transmitted during phase 2 for conventional CF is $2K_c \cdot \frac{2}{3} \cdot 2 + 2K_c \cdot 2$ per dimension. The first term is from the compressed two lower levels and the other term is from the uncompressed two upper levels in the 4-level SW encoder. These bits should be protected by the LDPC code with rate 2/3 in the relay-destination link and the total number of bits is $10K_c$ per dimension. With regard to the hybrid scheme, the total number of bits before error protection is $K_c + \left(2K_c \cdot \frac{2}{3} + 2K_c \cdot 2\right)$. The first term originates from the parity check part of the upper level and the second term in the parentheses is related with the compression process. The total number of bits with error protection is $9K_c$ per dimension. The duration of phase 2 is reduced by 10%.
Figure 5.6. $\mu_1=16$dB without compression. Phase 1: 64QAM; Phase 2: QPSK.

Figure 5.7. $\mu_1=20$dB without compression.

Figure 5.6 and Figure 5.7 depict the BER performance without compression. In other words, it can be regarded as hybrid DF/QF, instead of hybrid DF/CF. Here the duration of the first phase is same for all the schemes and the duration of the second phase varies. It is easy to see that conventional DF reaches an error floor when $\mu_0$ and $\mu_2$ increases to 8 dB. However, the BERs of both the hybrid and conventional CF schemes reach a lower level than the error floor of DF if $\mu_0$ and $\mu_2$ are sufficiently high. The hybrid scheme outperforms the conventional CF in the low SNR region and has a similar performance in the high SNR region. In the low SNR region, the FEC code $C_p$ will be decode with a high error rate. For the conventional CF, the most significant quantization bits suffer from this high error rate, which in turn will propagate and cause more errors. However, in the hybrid
scheme, DF is performed in the upper levels and the scheme can benefit from transmit diversity. In the meantime, CF is deployed in the lower levels to enjoy receive diversity. The possibilities of enjoying transmit/receive diversity at the same time explain the better performance of the hybrid scheme. When SNR is increased, the receive diversity will dominate the performance and the BER curves of conventional CF and the hybrid scheme becomes closer. It can be expected that the hybrid scheme should reach some error floor as well when \( \mu_1 \) and \( \mu_0 \) increases further. However, we can safely claim that this error floor is much lower than that in the conventional DF scheme because it is mainly determined by the BERs of upper levels only.

Figure 5.8 shows BER performance when compression is taken into consideration. The BER curves of the hybrid and conventional CF schemes decrease slowly when \( \mu_0 \) and \( \mu_2 \) are below 16 dB and drop sharply when they are above 16 dB. The reason has been explained in the previous chapter. When the SNRs are not sufficiently high, the multilevel SW decoder suffers decompression errors seriously [54], [74]. Only when SNRs are in a high region, will the number of decompression errors drop sharply. The hybrid scheme outperforms the conventional CF scheme in a wide range of SNR settings. It can be interpreted as follows. In the conventional CF scheme, the fact that more bits are required to quantize the signal results in more levels in the multilevel SW encoder/decoder structure accordingly. Each level suffers from decompression errors. On the contrary, for the hybrid scheme, the reduced number of quantization bits and the corresponding SW compressor/decompress pairs (\( K' < K \)) result in a decrease in decompression errors. Since the overall BER performance is dominated by decompression errors, the hybrid scheme with fewer decompression errors outperforms the conventional CF scheme. However, we must clarify that when \( \mu_0 \) and \( \mu_2 \) increases to a very high region, CF outperforms the hybrid technique eventually.

![Figure 5.8. \( \mu_1=20\text{dB} \) with compression, 64-QAM](image-url)
5.3 Hybrid Relaying Scheme II

In the multilevel coding based hybrid relaying scheme, DF and CF are deployed at simultaneously in different coding levels. In this section, we will explore another hybrid scheme where DF and CF are carried out adaptively according to the current channel realization.

5.3.1 Outage Behavior for Two Extreme Cases

We first examine two extreme cases whose performance serves as upper and lower bounds. More specifically, we begin with the scenario where full CSI is known to the relay, which is a lower bound for outage performance.

**Case 1: Ideal Hybrid Relaying Scheme (DC)**

In this scenario, we assume that global CSI (both instantaneous SNRs and phase information included) is known to the relay by means of feedback or exploitation of reciprocity. The relay is able to adjust its compression rate and perform ideal Wyner-Ziv coding when working in CF mode. We denote this scheme as ideal hybrid DF/CF (DC).

During the first phase, a message \( w \in \mathcal{W} = \{1, 2, \ldots, 2^N\} \) is encoded with \( X_1 \) and broadcasted by the source. Using the relay model in chapter 2, the received signals at the relay and the destination are \( Y_i \) and \( Y_j \) respectively. The mutual information conveyed to the destination and the relay during phase 1 are denoted as \( I^c(y) \) and \( I^r(y) \) respectively, where \( y = \{y_1, y_2\} \) and \( y = \{\gamma_i^2, \mu_i\} \) are the instantaneous SNRs that follow exponential distribution with mean \( \mu_i \) for \( i = 0, 1, 2 \). At the end of the first phase, the relay tries to decode \( w \).

Define the outage event set and its complementary set in the source-relay link as

\[
E^m := \{ I(y) < R \}, \quad \overline{E}^m := \{ I(y) \geq R \}
\]

Let \( dec \) indicates if the relay has been able to decode the message with value 1 for decoding success and 0 for decoding failure, we can define the relay encoding function as

\[
X_2 = f_r(Y_1) = \begin{cases} X_{2,0} (w), & dec = 1 \\ X_{2,c} (s), & dec = 0 \end{cases}
\]

where the subscript \( D \) and \( C \) denote DF and CF respectively. \( f_r(\cdot) \) is a function of \( Y_1 \) with implicit functionality to the assumed channel knowledge availability at the relay node. Depending on the success of the decoding at the relay node, two following actions are envisaged to be taken:

- **Relay Decoding Success (\( dec = 1 \))**: the relay node re-encodes the message \( w \) to \( X_{2,0} = X_1 \). With perfect CSI at the relay, the phase synchronization can be easily achieved such that \( X_{2,0} \) and \( X_1 \) can be transmitted in a beamformed manner during phase 2. The received signal at the destination is given as
The destination decodes the message by joint processing of the received $Y_d^1$ and $Y_d^2$. The relay system enjoys the capacity of the synchronized MISO channel in phase 2, denoted as $I^{R2}(y)$. The overall mutual information transmitted to the destination in a complete frame is the summation $I^{RC}$ and $I^{R3,1}$, denoted as $I^{DF}(y)$. The outage probability is given as

$$p_a(R;\mu,\alpha) = \Pr\{I^{DF}(y) < R | E_{\text{out}}\}$$

(5.3)

where $\mu$ represents $[\mu_1,\mu_2,\mu_3]^T$.

*Relay Decoding Failure (dec=0):* The relay performs a CF scheme using Wyner Ziv coding. The details of the operation at the destination in CF have been described in chapter 4. The mutual information accumulated at the destination in a complete frame is denoted as $I^{CF}(y)$. According to previous results, the outage probability can be calculated as

$$p_c(R;\mu,\alpha) = \Pr\{I^{CF}(y) < R | E_{\text{out}}\}$$

(5.4)

Calculations of $I^{RC}(y)$, $I^{R}(y)$, $I^{R3}(y)$, $I^{DF}(y)$ and $I^{CF}(y)$ are straightforward and can be found in previous chapters. For convenience of readers, they are listed in TABLE 5.1.

Since the events in two sets where $\text{dec}$ is equal to 1 and 0 respectively are mutually exclusive, the overall outage probability can be expressed as

$$p(R;\mu,\alpha) = p_a(R;\mu,\alpha)\Pr(E_{\text{out}}) + p_c(R;\mu,\alpha)\Pr(E_{\text{out}})$$

(5.5)

The resulted outage probability is a function of duplexing ratio $\alpha$. Therefore the probability can be minimized with respect to $\alpha$, given as

$$p_{\text{out}}(R;\mu) = \min_{\alpha} p(R;\mu,\alpha)$$

*Case 2: No CSI at the Relay (DQ)*

If the knowledge of $c_0$ and $c_2$ is not available at the relay, phase synchronization is difficult to be achieved when the system carries out the DF scheme. As a result, during the second phase the relay transmits $X^2_d = X^2; \beta$ instead and these two codes form a MISO code. The outage probability in DF mode can be derived in a similar way as *Case 1*. The only difference is that $I^{DF}(y)$ is replaced by $I^{DF}(y)$.
If the relay cannot decode, without the information of $y_0$ and $y_2$ (note that in the expressions of the outage probabilities, the phase information is not important, we are hereby dealing with $y_0$ and $y_2$ only), ideal Wyner-Ziv coding cannot be conducted because the relay has no sufficient information to adjust its compression rate to the ideal level. In such a case, we can use the quantize-and-forward (QF) scheme proposed in chapter 4 instead, where no compression is deployed. Meanwhile, without the instantaneous SNRs information, the available rate at the relay is unknown. The transmission rate of the quantized signals is chosen as $R_r$ discretionarily. The signal received by the destination in phase 1 no longer serves as side information and the estimation of $Y_r^j$ is reconstructed independently. The mutual information after a complete frame in QF mode is denoted as $I^{QF}(y)$ and listed in TABLE 5.1.

Since the selection of $R_r$ is arbitrary, there is a possibility that the transmission in the relay-destination link fails. The outage event set and its complementary set are defined in this link as

$$E^{\text{out}}_{\text{rel}} := \{R_r < R\}, \quad \bar{E}^{\text{out}}_{\text{rel}} := \{R_r \geq R\}$$

If outage events occur in this link, the destination is not able to successfully cancel the relay transmitted signal from $Y_j^j$ and the transmission in phase 2 will not bring any useful information at the destination. The mutual information accumulated is from the source's transmission during the first phase only and the outage probability is

$$p_o(R;\mu,\alpha, R_r) = \Pr(I^R(y) < R, E^{\text{out}}_{\text{rel}} | E^{\text{out}}_{\text{rel}})$$

If the transmission in the relay-destination link is successful, the outage probability is given as

$$p_o(R;\mu,\alpha, R_r) = \Pr(I^R(y) \geq R, E^{\text{out}}_{\text{rel}} | E^{\text{out}}_{\text{rel}})$$

It follows that the outage probability in such a case can be expressed as

$$p(R;\mu,\alpha, R_r) = p_o(R;\mu,\alpha) \Pr\{E^{\text{out}}_{\text{rel}}\} + \left(p_o(R;\mu,\alpha, R_r) + p_o(R;\mu,\alpha, R_r)\right) \Pr\{E^{\text{out}}_{\text{rel}}\}$$

The outage probability can be minimized with respect to $\alpha$ and $R_r$,

$$p^\text{out}_{\text{min}}(R;\mu) = \min_{\alpha, R_r} p(R;\mu,\alpha, R_r)$$

(5.6)
### TABLE 5.1
Accumulated mutual information at the relay and the destination

<table>
<thead>
<tr>
<th>Node</th>
<th>Scenario</th>
<th>Mutual Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>S Broadcasts</td>
<td>$I^S(\gamma) = \alpha I(X'_1; Y'_1) = \alpha \log(1 + \gamma)$</td>
</tr>
<tr>
<td>D</td>
<td>S Broadcasts</td>
<td>$I^{DC}(\gamma) = \alpha I(X'_1; Y'_1) = \alpha \log(1 + \gamma_o)$</td>
</tr>
<tr>
<td></td>
<td>Direct transmission</td>
<td>$I^{DT}(\gamma) = I(X'_1; Y'_1) = \log(1 + \gamma)$</td>
</tr>
<tr>
<td></td>
<td>R decodes successfully, phase 2 only in the asynchronous DF case</td>
<td>$I^{DS1}(\gamma) = (1 - \alpha)I(X'_1, X'_2; Y'_2) = (1 - \alpha)\log(1 + \gamma + \gamma_s)$</td>
</tr>
<tr>
<td></td>
<td>R decodes successfully, phase 2 only in the synchronized DF case</td>
<td>$I^{DS2}(\gamma) = (1 - \alpha)I(X'_1, Y'_2) = (1 - \alpha)\log\left(1 + \sqrt{\gamma} + \sqrt{\gamma_s}\right)$</td>
</tr>
<tr>
<td></td>
<td>R decodes successfully, complete frame in the asynchronous DF case</td>
<td>$I^{DST}(\gamma) = I^{DC}(\gamma) + I^{DS2}(\gamma)$</td>
</tr>
<tr>
<td></td>
<td>R decodes successfully, complete frame in the synchronized DF case</td>
<td>$I^{DST}(\gamma) = I^{DC}(\gamma) + I^{DS2}(\gamma)$</td>
</tr>
<tr>
<td></td>
<td>R detects errors, complete frame in CF mode</td>
<td>$I^{DT}(\gamma) = \alpha I(X'_1; \tilde{Y}'_1, Y'_2) + (1 - \alpha)I(X'_2; Y'_2</td>
</tr>
<tr>
<td></td>
<td>R detects errors, complete frame in QF mode</td>
<td>$I^{DT}(\gamma) = \alpha I(X'_1; \tilde{Y}'_1, Y'_2) + (1 - \alpha)I(X'_2; Y'_2</td>
</tr>
</tbody>
</table>

$\tilde{Y}'_1$: the estimation of $Y'_1$; $\gamma = \sigma^2 / \sigma^2_s$ (here $\gamma_s = 1 / \sigma^2_s$ because $\sigma^2_s = 1$); $\sigma^2$ is the compression noise and $\gamma = g(\alpha, \gamma_o, R_o) = (\exp(R/\alpha) - 1)(1 + y_0/(1 + y_0)); y_0 = \sigma^2 / \sigma^2_s$ (here $\gamma_o = 1 / \sigma^2_o$); $\sigma^2$ is the quantization noise and $\gamma = g(\alpha, \gamma, R) = (\exp(R/\alpha) - 1)(1 + \gamma) = \gamma_o$; Available rate on the relay-to-destination link and $R_o = g(\alpha, \gamma_o, \gamma) = (1 - \alpha)\log(1 + \gamma_y/(1 + y_0)); R_o$: the transmission rate of the quantized signals at the relay.

#### 5.3.2 Limited CSI with a Limited Feedback Link

In the previous section, QF was applied with discretionary $R$, when CSI is not available at the relay. In chapter 5, we have already shown the superior achievable rate performance of CF compared with QF. If a limited feedback link can be established to convey partial CSI, we may still be able to apply CF but not QF to achieve better performance. Since we have relaxed the constraint in [8] that the relay is co-located with destination, the instantaneous SNR of the relay-destination link $\gamma_f$ is no longer a constant and not known to the relay.
CSI of this link has to be conveyed through the feedback link as well. We decouple the problem by first investigating the influence of the limited feedback of $y_2$, then $y_0$ and combine the results finally.

**Case 3:** Perfect $y_0$, partial $y_2$ through a limited feedback link (DCT)

A straightforward approach for limited feedback of $y_2$ is to quantize it. Suppose a $L$-bit quantizer with quantization levels $U=(u_0,...,u_M)$ is used, where $u_0=0$, $u_M=+\infty$ and $M=2^L$. When $y_2$ is in the region of $[\mu_2u_i, \mu_2u_{i+1})$, to avoid the outage events in the relay-destination link, we can choose the transmission rate of the compressed signal at the relay as $R_0' = f(\alpha, y_0, \mu_2u_i)$ such that

$$R_0' = f(\alpha, y_0, \mu_2u_i) \leq f(\alpha, y_0, y_1) = R_0$$

This selection will ensure the transmission rate of the compressed signal is lower than the relay supported rate $R_0$. The corresponding $y_{w_j}$ is equal to $g(\alpha, y_0, y_1, R_0)$ and the accumulated mutual information can be expressed as $I_{\alpha y_1}^f(y, u)$. Define event sets

$$E'_i = \{u_i \leq y_1 < u_{i+1}\}$$

The outage probability in the region $[u_i, u_{i+1})$ is given by

$$p_{c_i} (R; \mu, \alpha, U) = \Pr \{I_{\alpha y_1}^f(y, u) < R, E'_i | E''_w\}$$

If $y_2$ is smaller than $\mu_2u_i$, the lower quality of the relay-destination link will lead to inefficient transmission of the compressed signals. In this regard, we keep the relay silent and let the source transmit alone during the second phase. The information accumulated at the destination is $I^D(y)$ as shown in TABLE 5.1. The outage probability is

$$p_{c_i} (R; \mu, \alpha, U) = \Pr \{I^D(y) < R, E''_w | E''_w\}$$

This scheme is denoted as the hybrid DF/CF/Direct Transmission (DCT) scheme. The overall outage probability in CF mode is the summation of outage probabilities when $y_2$ is in different regions, denoted as

$$p(R; \mu, U, \alpha) = p_o(R; \mu, \alpha) \Pr \{E''_w\} + \sum_{i=0}^{M-1} p_{c_i} (R; \mu, U, \alpha) \Pr \{E''_w\}$$

The outage probability should be minimized by with respect to $\alpha$ and $U$,

$$p_{\text{min}}(R; \mu) = \min_{\alpha, U} p(R; \mu, U, \alpha)$$

(5.7)
Case 4: Perfect $\gamma_0$, partial $\gamma_0$ through the limited feedback link (DQCT)

In this case, the feedback scheme in [8] can be directly deployed. Two bits are used to indicate if $\gamma_0$ is in the region $[a\mu_0, b\mu_0]$. If $\gamma_0$ is smaller than $a\mu_0$, the amount of side information at the destination is too small to be used for efficient Wyner-Ziv coding. In such a scenario, QF is applied by choosing $R_{o,a}^{\text{QF}}(a,a_{0},y_2) \leq R_0$ to guarantee successful transmission in the relay-destination link. The accumulated information $I_{n}^{\text{OF}}(\gamma,a)$ can be easily found according to TABLE 5.1 and the corresponding outage probability is given as

$$P_a(R;\mu,a) = \Pr\{I_{n}^{\text{OF}}(\gamma,a) < R, \gamma_0 < a\mu_0 | E_n\}$$

where $a_{0b}=[a, a, b]^T$.

If $\gamma_0$ is larger than $b\mu_0$, the power the source transmitted signal, which actually acts as an additional noise when decoding the compressed bin index, is relatively large, thus leading to degraded performance. In this circumstance, CF is not a good choice due to the enormous complexity of Wyner-Ziv coding and the poor gain in terms of the achievable rate. So we keep the relay silent and allow the source to transmit alone in the second phase. In particular, here we choose [8]

$$b = \frac{\exp(-e^x - 1)}{\mu_0}$$

(5.8)

$b$ is chosen as (5.8) for two reasons: one is that this selection simplifies the outage probability calculation due to the fact that the outage probability will be zero if $\gamma_0$ is larger than $b\mu_0$; another reason is that the numerical results have indicated the appropriateness of this choice.

If $\gamma_0$ lies between $a\mu_0$ and $b\mu_0$, the system is able to work in CF mode. In order to ensure successful Wyner-Ziv decoding, some constraints have to be satisfied. It follows that we can choose the relay transmission rate as

$$R_{o,ab} = f(\alpha, b\mu_0, \gamma_2) \leq R_0$$

Resorting to TABLE 5.1, the mutual information $I_{n}^{\text{CF}}(\gamma,a,b)$ is obtained by choosing $I_{n}^{\text{OF}}(a,\mu_{0},y_2)$ and the corresponding outage probability is

$$P_a(R;\mu,a) = \Pr\{I_{n}^{\text{CF}}(\gamma,a,b) < R, a\mu_0 \leq \gamma_0 \leq b\mu_0 | E_n\}$$

This scheme is denoted as the hybrid DF/QF/CF/Direct Transmission (DQCT) scheme. The overall outage probability is given by the summation of the outage probabilities in different scenarios and can be minimized with respect to $\alpha$, $\omega$ and $b$, 

95
\[ p(R; \mu, \alpha) = p_b(R; \mu, \alpha) \Pr\{ \mathcal{E}_w^{\mu} \} + (p_a(R; \mu, \alpha) + p_{aw}(R; \mu, \alpha)) \Pr\{ \mathcal{E}_w^{\mu} \} \]
\[ p_{aw}^{\mu}(R; \mu, \alpha) = \min_{b, a} p(R; \mu, \alpha) \]

(5.9)

Case 5: Partial \( \gamma_0 \) and \( \gamma_2 \) through a limited feedback link (HR _{CSI})

If both \( \gamma_0 \) and \( \gamma_2 \) are restricted, the two above schemes can be combined to develop a more flexible one that is able to switch among DF, CF, QF and direct transmission. The whole instantaneous SNRs space is separated into several operational regions, in each of which one single relaying scheme is conducted.

1) If \( \gamma_0 > b \mu_0 \) the relay keeps silent and the source transmits alone during phase 2. According to (5.8), the outage probability is zero.

2) If \( \gamma_0 < b \mu_0 \) and \( \gamma_2 < \mu_{2+1} \), CF cannot be efficiently conducted and the relay system works in the operational region of direct transmission. Based on the selection of \( b \) according to (5.8), the outage events will definitely happen and the outage probability is given by,

\[ p_{aw}^{\mu}(R; \mu, \alpha) = \Pr\{ \gamma_0 < b \mu_0, \mathcal{E}_w^{\mu} \} \]

3) If \( a \mu_0 \leq \gamma_0 \leq b \mu_0 \) and \( u \mu_2 \leq \gamma_2 \leq u_{2+1} \mu_2 \), the system works in CF mode and the achievable rate \( I_{aw}^{\mu}(\gamma, u, \alpha, b) \) can be easily obtained with \( \gamma_{aw} = g(\alpha, a \mu_0, \gamma_1, f(\alpha, b \mu_0, u, \mu_2)) \). The outage probability is

\[ p_{aw}^{\mu}(R; \mu, \alpha, U) = \Pr\{ I_{aw}^{\mu}(\gamma, u, \alpha, b) < R, a \mu_0 \leq \gamma_0 \leq b \mu_0, \mathcal{E}_w^{\mu} \} \]

4) If \( \gamma_0 < a \mu_0 \) and \( \mu_{2+1} \leq \gamma_2 < \mu_{2+1} \), the system works in QF mode and the achievable rate \( I_{aw}^{\mu}(\gamma, u, \alpha) \) is obtained when \( \gamma_{aw}^{\mu} = q(\alpha, \gamma_1, f(\alpha, a \mu_0, u, \mu_2)) \). The outage probability is given as

\[ p_{aw}^{\mu}(R; \mu, \alpha, U) = \Pr\{ I_{aw}^{\mu}(\gamma, u, \alpha) < R, \gamma_0 < a \mu_0, \mathcal{E}_w^{\mu} \} \]

This scheme is denoted as hybrid relay with partial CSI (HR _{CSI}). Define \( U_{aw}^{\mu} = [a, b, \alpha, u_0, ..., u_{2+1}] \). Taking the outage probabilities in all of the operational regions into consideration, the outage probability can be minimized as

\[ p(R; \mu, U_{aw}^{\mu}) = p_b(R; \mu, \alpha) \Pr\{ \mathcal{E}_w^{\mu} \} + (p_a(R; \mu, U_{aw}^{\mu}) + \sum_{b, a} (p_{aw}(R; \mu, U_{aw}^{\mu}) + p_{aw}(R; \mu, U_{aw}^{\mu}))) \Pr\{ \mathcal{E}_w^{\mu} \} \]
\[ p_{aw}^{\mu}(R; \mu) = \min_{U_{aw}^{\mu}} p(R; \mu, U_{aw}^{\mu}) \]

(5.10)
The basic idea is shown in Figure 5.9, where the SNR space is divided into several operational regions.

Figure 5.9

5.3.3 Simulation Results, Comparisons and Discussions

The outage probabilities are optimized numerically by resorting to multilevel integrations obtained by the Monte Carlo simulations. The expected throughput, which is defined as the multiplication of the rate $R$ and the probability of successful decoding at this $R$, is given in [8] as

$$T = R \cdot (1 - \Pr(\text{outage events}))$$

Figure 5.10 depicts the expected throughput, where selective DF means that the relay is turned off when not able to successfully decode and conventional CF stands for the case that the relay carries out the CF scheme without trying to decode. These two schemes can be regarded as special cases of the hybrid schemes and their performance is evaluated for the purposes of comparison.

The results show that if full CSI is available at the relay, the hybrid scheme achieves the highest expected throughput. In contrast, when CSI is not known to the relay, selective DF outperforms the hybrid relaying scheme $DQ$. This phenomenon can be explained as follows. When the selected transmission rate of the compressed signals $R_r$ increases, in one aspect, if the transmission of the quantized signals in the relay-destination link is successful, the achievable rate of the QF operation can be improved; in another aspect, the outage probability in the relay-destination link increases as well, thus leading to a smaller probability for the relay system to enjoy the achievable rate of QF. These two effects work in reverse directions. The simulation results indicate that the latter effect dominates the final expected throughput. As a result, the system's expected throughput increases when $R_r$ decreases and reaches the optimal curve when $R_r$ approaches 0, which is actually the expected throughput of selective DF.
Figure 5.10. Expected throughput. $\mu_0-\mu_1=5\text{dB}, \mu_2=15\text{dB}$

Figure 5.11 displays the outage performance of the hybrid relaying scheme with partial CSI. Here we assumed that only 1 bit is used to convey the information of $\gamma_2$ because the error-free feedback link should be strictly limited. In other words, $U$ is defined as $\{0, u, +\infty\}$. As seen in Figure 5.11, if only $\gamma_0$ is limited, the relay is able to switch between DF, QF, CF and direct transmission modes adaptively. This flexibility keeps the overall performance in terms of both the outage probability and expected throughput close to the ideal hybrid relaying scheme when full CSI is available at the relay. However, the limitation on $\gamma_2$ allows the system to switch between DF, CF and direct transmission modes only. No QF operations will be conducted. The difference in flexibility explains the differences in outage performance. If both $\gamma_0$ and $\gamma_2$ are limited, the hybrid relaying scheme ($HR\_PCSI$) outperforms selective DF in outage behavior. This fact indicates that with extremely limited feedback of the knowledge of $\gamma_0$ and $\gamma_2$ (only 3 bits here), despite the fact that CF or QF operations are not ideal, the relay system is able to enjoy received diversity provided by CF or QF and the outage behavior of the relay system can be improved through application of a more flexible hybrid relaying scheme.

In order to present the effect of receive diversity, the expected throughput is depicted Figure 5.12, where the relay-destination link approaches a high level and the qualities of other two links are kept constant in a low level. As can be seen in Figure 5.12, the expected throughput of selective DF is saturated in a low level when $\gamma_2$ increases. In contrast, the hybrid relaying scheme $DC$ increases with $\gamma_2$ and approaches a high level. $HR\_PCSI$ with partial CSI feedback shows the same trend as $DC$ because it is able to benefit from receive diversity as well when the system is in the operational region of CF. Apparently, its expected throughput is lower than $DC$ for two reasons. One is that as opposed to the ideal hybrid relaying scheme $DC$, the system is not always in the CF operational region when the decoding at the relay fails; another is that with partial CSI, the relay cannot adjust its transmission rate of the compressed signals ideally, thus leading to a degraded achievable rate.
5.4 Conclusions

In this chapter, we proposed two hybrid relaying schemes. In the first scheme, multilevel coding is incorporated into the cooperative relaying concept, where the source possesses a layered coding structure. The BER differences in different coding levels bring a possibility that the relay is able to decode successful in the upper levels but fails to decode in the lower levels. The proposed hybrid relaying scheme is able to cope with this problem. In the new scheme, the relay deploys DF in the upper levels and CF in the lower levels. The simulation results showed that the hybrid relaying scheme is able to lower the error floor faced by conventional DF. Compared with conventional CF, the hybrid scheme reduces the complexity and improves the bandwidth efficiency when the coding and compression rates are selected carefully. It outperforms the conventional CF.
scheme in terms of BER because fewer decompression errors are introduced. These results demonstrated that
the hybrid scheme finely balances the BER performance, complexity and bandwidth efficiency.

As opposed to the first hybrid relaying scheme, where DF and CF are carried out simultaneously in different
coding levels, another hybrid relaying scheme works in a time-division manner. The relay switches between
different relaying schemes including DF, CF, QF and direct transmission adaptively according to the current
channel realizations. We noticed that the original CF scheme requires global CSI at the relay. The global CSI
can be obtained through feedback or exploitation of reciprocity of the channel. Anyhow provision of global CIS
is unrealistic in most of practical implementations. To cope with this problem, we established a limited
feedback link between the destination and the relay to convey partial CSI (only 3 bits in our results) and
developed a practical hybrid relaying scheme accordingly.

The results showed that the availability of CSI at the relay plays an important role. When full CSI is known to
the relay, as opposed to DF, which targets transmit diversity only, the hybrid scheme benefits from either of
transmit/receive diversity based on different channel and noise realizations. In contrast, when no CSI is
available to the relay, selective DF outperforms the hybrid relaying scheme in terms of the outage probability
and expected throughput.

In the practical hybrid relaying scheme, several bits are used to convey partial information of the relay- and
source-destination links in a limited feedback link between the destination and the relay. These bits determine
the operational regions of the relay system. Based on the decided operational regions, various relaying schemes
including DF, CF, QF and direct transmission can be conducted at the relay adaptively. In the practical scheme,
when working in CF or QF mode, the relay cannot adjust its transmission rate of the compressed or quantized
signals to the ideal one but to a sub-optimal one instead. The system suffers a loss for this suboptimal selection.
However, the practical hybrid relaying scheme is still able to enjoy receive diversity when decided operational
region is within the region of CF or QF.
Chapter 6 Hybrid Automatic Repeat re-Quest (HARQ) for the Relay System

6.1 Introduction

One of the main objectives of the future wireless systems is provision of uniform high rate coverage. Being potential solutions for improving wireless systems' performance, it is quite natural that both Hybrid Automatic Repeat re-Quest (HARQ) and relaying schemes can be combined in a complementary way to further exploit the degrees of freedom of a wireless media and attain better performance.

Despite the exploitation of an efficient cooperative relaying strategy the transmitted packet could still be lost due to the instantaneous channel condition and noise realization. The packet loss could be even more severe when the system is operating under static (block) fading condition and the transmitter is not able to properly tune its transmission rate due to the lack of sufficient level of channel knowledge. Retransmission techniques based on automatic repeat request, i.e. ARQ [78]-[80] and its advanced hybrid types that combine forward error correction (FEC) with ARQ, commonly known as Hybrid ARQ (HARQ), will be the natural choices to circumvent this problem and guarantee correct data packet delivery. Some studies of HARQ protocols regarding throughput analysis, error rate and average delay in two-node communications can be found in [81]-[84], [102].

Common encoding techniques for HARQ are repetition coding (RC) with chase combing and unconstrained coding (UC) with incremental redundancy (INR) respectively [90]-[93]. The emphasis of this paper is on HARQ, especially INR as it is capable of offering higher throughput [81]. Considering that the RC based HARQ performs weaker than INR and also the fact that the extension of the presented analysis to repetition coding is a straightforward practice, RC based HARQ will not be considered in this paper.

The combination of HARQ and relaying schemes has been addressed in [85]-[89], and [97]-[99]. In [97], the diversity-multiplexing-delay tradeoff was analyzed. Lin Dai studied the application of adaptive cooperation with ARQ in [98], where the relay will not be involved in the cooperation if errors are detected. The work has been further extended to HARQ in [86]-[89], [99] and [105]. In all of these researches, HARQ is deployed with a single non-hybrid forwarding scheme, say, DF. Therefore, we name it as H-ARQ-Relaying strategy, which means HARQ is combined with a non-hybrid relay system which does not attempt to dynamically switch between different forwarding schemes.

However, since we have highlighted in the previous chapter that if the relay can adaptively switch between different forwarding modes, the outage behavior can be greatly improved, it is natural to expect further improvement through the combination of a hybrid relay system and an HARQ mechanism. We name this kind of combined strategy as H^2-ARQ-Relaying because not only the retransmission scheme but the relaying scheme is also hybrid. Some preliminary work on H^2-ARQ-Relaying has been done in [100], where the relay is allowed
to switch between AF and DF. Compared with conventional HARQ strategies based on DF only, the new relay system is able to enjoy certain level of flexibility and exhibits significant improvement in FER.

Based on the existing analysis in the literature on application of HARQ in a relay system, several limitations and drawbacks are evident: 1) as we mentioned the performance of AF/DF scheme is fundamentally limited by the source-to-relay link. 2) If AF is applied, the relay also suffers from bandwidth loss due to the necessity of RC implementation, as will be shown by some simulation results presented later in this paper. 3) Moreover, in previous approaches, the source is assumed to be silent when the relay is transmitting. In fact, we have argued in previous sections that the system’s performance would be improved by allowing the source and the relay transmit simultaneously in a cooperative manner. 4) Finally, no throughput analysis for a hybrid AF/DF relay system has been provided. In this section, a generalization of the previous models is presented which will allow for simultaneous transmission of both the source and the relay. To overcome the weakness of both AF and DF forwarding, we assume hybrid CF/DF forwarding and propose two novel H²-ARQ-Relaying strategies. These two strategies are different in their feedback mechanisms. The throughput performances of both strategies will be investigated through an analytical framework. The proposed strategies and their performance analysis are among the main contributions of this work and, to the best of the authors’ knowledge, are not yet in the literature.

However the original CF technique requires global CSI knowledge at the relay node (especially the amplitude information for the statistics of received signals) at the relay node to be able to adjust its compression rate and perform an ideal Wyner-Ziv coding. Part of this CSI related to relay-to-destination link can be obtained either through feedback or through reciprocity of the channel [101], the other part related to source-to-destination can be provided through feedback from destination. Anyhow provision of global CSI at the relay is not affordable in most of the practical conditions. We propose modified H²-ARQ-Relaying strategies with a very relaxed requirement on CSI knowledge. The modified schemes require feedback of very few extra bits in addition to the acknowledge bit to convey, from the destination to the relay, partial but useful CSI of the source- and relay-destination links. The strategies greatly reduce the amount of feedback information while performing close to its equivalent strategy with unlimited CSI, thus making the whole system more practical to implement.

The rest of the chapter is organized as follows. Two H²-ARQ-Relaying strategies with full channel knowledge at the relay are proposed and the performance, in terms of the outage probabilities, average reward, average airtime and throughput, is also studied in section 6.2 and 6.3 respectively. We investigate these two strategies because they not only form the basis for the more practical scheme introduced later but also could serve as the theoretic upper bounds for the throughput. Two modified H²-ARQ-Relaying strategies with partial CSI feedback bits are investigated in section 6.4 and 6.5 respectively and their performance is also examined in section 6.6. Section 6.7 gives simulation results and the final section concludes.

6.2 H²-ARQ-Relaying Strategies
We introduce the basic concept of the hybrid relaying scheme with feedback in Figure 6.1. We assume that the instantaneous SNRs of all three links are known to the relay through feedback or exploitation of the reciprocity property. However, this knowledge is not available at the source, thus the source is not able to tune its transmission rate adaptively. For the receiver side, we assume that both the amplitude and the phase information can be obtained through channel estimation. Maximum of $N$ frames are assumed to be allocated to achieve reliable transmission of one single message $w$. We expand the codebooks defined in chapter 2 as $X_{s,n}^1(w), X_{s,n}^2(w), X_{r,n}^2(w)$ and $X_{r,c,n}^2(s_n)$, for $n=1,...,N$, and define the relay encoding function as

$$X_{r,n}^2 = f_{r,n}(i_n, ..., Y_{r,s}) = \begin{cases} X_{r,n}^2(w), & dec(n) = 1 \\ X_{r,c,n}^2(s_n), & dec(n) = 0 \end{cases}$$

Now $f_{r,n}(.)$ is a function of all the previous received signals. Based on the codebooks defined, we propose two $H^2$-ARQ-Relaying strategies for a hybrid CF/DF relay system.

6.2.1 $H^2$-ARQ-Relaying Strategy 1 (HS_SI)

As depicted in the right part of Figure 6.1, the feedback channels are utilized to convey information about the decoding status at the relay and destination. During the first phase of the $n$th frame, the source broadcasts $X_{s,n}^1$. If, based on $Y_{d,n}$ and all the previously buffered signals, the message is successfully decoded by the destination, an ACK is sent back to the source. The transmission of current message stops. On the contrary, if the
destination detects errors, it will wait for the second phase to further receive signals from both the source and relay nodes. At the same time, if the destination has not been able to decode the message, the relay will try to decode through joint processing of $Y^s_{t,n}$ and previously received signals. Depending on its success in decoding it will perform CF or DF. Different actions will be applied based on the decoding status of the relay:

**Relay Decoding Success:** An ACK is sent back from the relay to the source and the decoded message is re-encoded with $X^s_{D,n}$ to form a MISO code with $X^s_{s,n}$. The relay and the source transmit $X^s_{D,n}$ and $X^s_{s,n}$ respectively in a cooperative manner during phase 2. Afterwards, the source and the relay will keep on working in DF mode to benefit from transmit diversity produced by the MISO channel. The phase 2 will then be repeated until an ACK is received from the destination or the retransmission limit is reached.

**Relay Decoding Failure:** The relay switches to CF mode and sends a NAK back to the source. The CF operation has been described in the previous sections. Finally, the destination attempts to decode by joint processing $Y^s_{D,n}$, $Y^r_{t,n}$ and $Y^d_{t,n}$. Upon successfully decoding, an ACK is sent back. Otherwise, the destination stores $Y^s_{D,n}$, $Y^r_{t,n}$ and $Y^d_{t,n}$ or the weighted combination of them with their previous versions and feeds back a NAK message. As long as $n<N$, in the following $(n+1)$th frame, the source starts to broadcast again; otherwise, retransmission will be stopped and HARQ failure will be announced.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Message</th>
<th>Frame</th>
<th>Destination</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>1</td>
<td>1</td>
<td>NAK</td>
</tr>
<tr>
<td>T1</td>
<td>1</td>
<td>2</td>
<td>NAK</td>
</tr>
<tr>
<td>T1</td>
<td>1</td>
<td>3</td>
<td>ACK</td>
</tr>
<tr>
<td>Case 1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 6.2. $H^2$-ARQ-Relaying Strategy 1 (HS_SI) $T_1$: phase 1 with duration $aT_F$, $T_2$: phase 2 with duration $(1-a)T_F$. The upper part indicates three cases of successful decoding, the middle part indicates two cases of outage events in CF and DF modes respectively and the lower part depicts frame types.

Figure 6.2 sketches different scenarios. In case 1, $w$ is successfully received at the end of phase 1 in the $K$th frame, while in case 2 and 3, $w$ is correctly decoded when the relay works in CF and DF mode respectively. Figure 6.2 also depicts two cases where the message is discarded because the retransmission limit $N$ is reached.
Chapter 6 Hybrid Automatic RepeAt re-Quest (HARQ) for the Relay System

in CF and DF mode respectively. Based on the illustration of the lower part in Figure 6.2, we define six types of frames in TABLE 6.1. Four of them appear in this strategy and the fifth one will appear in the AF/DF H2-ARQ-Relaying strategy.

<table>
<thead>
<tr>
<th>Type</th>
<th>Name</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CoF(C)</td>
<td>Complete Frames consist of two phases, operating in CF mode in phase 2</td>
</tr>
<tr>
<td>2</td>
<td>BrF</td>
<td>Broadcasting Frames consist of phase 1 only</td>
</tr>
<tr>
<td>3</td>
<td>CoF(D)</td>
<td>Complete Frames consist of two phases, operating in DF mode in phase 2</td>
</tr>
<tr>
<td>4</td>
<td>RtF(D)</td>
<td>Relay-transmit Frames consist of phase 2 only, operating in DF mode</td>
</tr>
<tr>
<td>5</td>
<td>CoF(A)</td>
<td>Complete Frames consist of two phases, operating in AF mode in phase 2</td>
</tr>
<tr>
<td>6</td>
<td>CoF(Di)</td>
<td>Complete Frames consist of two phases, operating in direct transmission mode in phase 2</td>
</tr>
</tbody>
</table>

Recall our assumption of block fading channels, where the channel gains are constant during one frame. Here we assume that this assumption is valid for each category of frames and provide some explanations to our settings. In a practical communication system, a real physical frame (PF) is typically longer than the frame we have defined. For instance, in a time division multiple access system, the information of multiple users can be transmitted within one PF consisting of several sub-frames. Each sub-frame carries information of one user. The frame we define is actually corresponding to one sub-frame. If the relay but not the destination has decoded successfully in the sub-frame of one PF, in the next PF, only the relay-transmit phase is preserved and, therefore, the duration of the sub-frame for that particular user can be reduced from $T_r$ to $(1-\alpha)T_r$. The spare time equal to the duration of phase 1 can be allocated to other users. We assume that the transmission of the next sub-frame of a user is spaced enough along the time domain (or frequency domain if a multi-carrier modulation is used) to ensure independency of channel gains. This is a reasonable assumption as the retransmission schemes insisting on several transmissions within their channel coherence time will lose performance and will become totally inefficient. Therefore, we can safely use the uncorrelated block faded frames assumption for any considered type of frames.
State diagram of strategy 1 is presented in Figure 6.3. State $B_n$ stands for the state where both the relay and the destination detect errors in the previous $(n-1)$ frames and the source is ready to broadcast message $w$ in the $n$th frame. State $D_n$ defines the state where the destination detects errors at the end of phase 1 but the relay decodes successfully and the system is ready to work in DF mode. State $DF_{m,n}$ is the state where the system has entered to DF mode in the $n$th frame, and the relay-transmit phase has been repeated $m$ times but the destination still detects errors. When $m=N-n+1$, the transmission for the current message ends up with unsuccessful decoding. State $C_n$ defines the stage where the relay still has not been able to decode $w$ in phase 1 of $n$th frame and is ready to do CF during phase 2. State $S$ denotes the state where the destination has successfully decoded the message $w$. State $F_n$ indicates the state where the transmission of current message fails after $N$ CF operations and $w$ is discarded.

6.2.2 H$^2$-ARQ-Relaying Strategy 2 ($HS_S2$)

In strategy 1, the system needs a sophisticated feedback mechanism, where ACK/NAK is sent from the relay and destination. On the contrary, we propose strategy 2 that only requires feedback from the destination. Unlike $HS_S1$, where the relay attempts to decode during phase 1 of each frame, here we allow the relay to decode only at the end of phase 1 of the first frame. Upon successful decoding at the relay, the system will keep on working in DF mode and phase 2 will then be repeated. If the relay's first decoding attempt fails, it performs CF. Till this step, it is similar with $HS_S1$. However, when the destination cannot decode at the end of the first phase 2, as opposed to the strategy 1, where a new frame starts and the source begins to broadcast, here we let the relay keep on working in CF mode and phase 2 will then be repeated until successful decoding or reaching the limit $N$. In other words, the relay will observe once during the first phase 1 and store its observation in the buffer. After that, the relay stops observation but compresses and forwards the same observation repeatedly in the following phase 2 transmissions. At the end of each phase 2, the destination will reconstruct an estimation of
Chapter 6 Hybrid Automatic Repeat re-Quest (HARQ) for the Relay System

relay’s observation, only with different compression noise and combine all the estimations and \( Y_{d,n} \) first. To investigate the realized mutual information in the combination process, we provide *proposition 6.1* as below, which is a direct generalization of [8, Proposition 3].

**Proposition 6.1:** In the HARQ strategy in which the relay observes only once and compresses and forwards its observation \( Y_{r,j} \) repeatedly for \( n \) times, and the destination reconstructs a series of estimations for the same \( Y_{r,j} \), denoted as \( \hat{Y}_{r,j} = (\hat{Y}_{r,j}[1], \hat{Y}_{r,j}[2], \ldots, \hat{Y}_{r,j}[n]) \), the mutual information conveyed with the help of the relay is given by,

\[
I_r^c (\hat{Y}) = \log \det \left( I_n + \left( CC^* \right) \left( BR_n B^* \right)^{-1} \right)
\]

\((6.1)\)

where

\[
C = \begin{bmatrix}
\mu_1 \sqrt{\mu_2} \sqrt{\mu_3}, \ldots, \sqrt{c_1} c_2, \ldots, c_n \sqrt{\mu_1}
\end{bmatrix},
B = \begin{bmatrix}
1 & 0 & 0 & \cdots & 0 \\
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
0 & 0 & 0 & \cdots & 1
\end{bmatrix},
Z = [Z_{d,1}, Z_{r,1}, Z_{r,2}, \ldots, Z_{r,n}]^{[n]}.
\]

Here \( I_n \) is \( n \times n \) identity matrix, \( Z_t \) is zeros mean circular symmetric Gaussian distributed random variable with variance \( \sigma^2 \) (compression noise), the noise vector has covariance matrix \( R_n = E\{ZZ^*\} = \text{diag}(1, 1, 1, \ldots, 1) \) and \( \rho_r = g(\alpha, \gamma_{0,1}, \gamma_{1,1}, R_0) \), where function \( g(.) \) can be found in TABLE 5.1 and \( R_{0,n} \) is the available rate of the relay. The proof of this proposition is presented in the Appendix F.

---

![Table 6.4](image)

Figure 6.4. H^2-ARQ-Relaying Strategy 2 (HS_S2). The upper part indicates three cases of successful decoding and the lower part depicts two cases of outage events in CF and DF modes respectively.
Then we use the combined signal and $\tilde{Y}_j^2$ for later joint decoding. Figure 6.4 sketches the strategy, where case 1 represents the scenario where the message is decoded by the destination in phase 1 of the first frame, case 2 and 3 indicate the scenarios where the message is successfully decoded in CF or DF modes respectively, and case 4 and 5 demonstrate strategy failures in CF and DF modes respectively. Figure 6.5 presents the state diagram. The state definition is almost the same as strategy 1. The difference lies in the CF part, where new state $CF_n$ is defined to indicate that phase 2 has been repeated $n$ times in CF mode. Note that in this strategy, there is no need to feed back from the relay to the source because the source will transmit $X^2_{k,n}$ regardless of the decoding status of the relay in the followed phase 2.

![State Diagram of the CF/DF-based H^2-ARQ-Relaying strategy 2 (HS_S2).](image)

### 6.3 Throughput Analysis

In this section the main performance parameters including outage probability and throughput are derived. Our analysis is based on the assumptions in the previous section and two new assumptions: one is the usage of capacity achieving error correction codes, which with circularly symmetric complex Gaussian distributed input assumption allow us to take an analytical approach and drive performance bounds for any kind of practical error correction coding and modulation; another is that the acknowledged (ACK) or not acknowledged (NCK) message is sent in delay- and error-free feedback links.

#### 6.3.1 Outage Analysis

In order to analyze the outage probability, we need to derive the accumulated mutual information after $n$ transmissions for two strategies respectively.

**$H^2$-ARQ-Relaying Strategy 1:** Only the first four types of frames appear in this strategy. Define the valid frame type set $J = \{1, 2, 3, 4\}$ for strategy 1. Let $I^0(p)$ denote the conveyed mutual information in a type $i$ frame, where
The accumulated mutual information at the destination after \( n \) frames can be expressed as

\[
I_d(y;\ell) = \sum_{i=1}^{M} f^{(i)}(y_i)
\]

(6.2)

where \( t_i \) is defined as the type attribute of the \( i \)th frame, \( \ell=(t_1,\ldots,t_n) \) is the sequence of the frame types, \( \gamma=(\gamma_1,\ldots,\gamma_n) \) is the sequence of the instantaneous SNRs in three links in the \( n \) frames, and \(|\ell|\) is the length of the sequence \( \ell \). Frame type sequence \( \ell \) is not valid for any combination of different types but defined by the state transition diagram and the 5 possible cases are displayed in Figure 6.2. We also define the following mapping function to identify the total number of frames using a specific frame type \( j \) as

\[
m_j: m_j(\ell) \rightarrow \sum_{i=1}^{M} \left[ t_i = j \right]
\]

(6.3)

where \([\cdot]\) with a condition inside is an indicator function whose values is 1 when the condition is satisfied and 0 otherwise. The accumulated mutual information at the relay after \( k \) transmissions is given as

\[
I_r(y;\ell) = \sum_{i=1}^{M} f^{(i)}(y_i)
\]

(6.4)

where \( k \leq n \). If \( k \) is equal to \( n \), \( I_r(y;\ell) \) and \( I_s(y;\ell) \) are the accumulated mutual information after \( n \) frames respectively. However, sometimes we are interested in the cases where \( k \) is not equal to \( n \) to derive the state probabilities. This is the reason why we introduce the relay-receive index \( k \).

To be able to obtain all the state probabilities of the considered state transition diagrams, we need to calculate outage probabilities of the form \( \Pr(I_s(y;\ell) < R, I_r(y;\ell') < R) \). Generally, this expression will depend on the frame type sequence and \( k \). However it is straightforward to show that the considered probability is the same for all frame type sequences that have the same number of frame types:

\[
\Pr(I_s(y;\ell) < R, I_r(y;\ell') < R) = \frac{1}{\lambda} F(R; m(\ell'), m(\ell', \alpha))
\]

(6.5)
where \( m(t) = (m_1(t), \ldots, m_J(t)) \) and \( J = \begin{cases} 4 \end{cases} \). Define \( n = m(t^*_1) \) and \( \bar{n} = m(t^*_{1+1}) \), where \( n_j = m_j(t^*_1) \) and \( \bar{n}_j = m_j(t^*_{1+4}) \). We only need to calculate outage probabilities \( F(R; n, \bar{n}, \alpha) \) for all possible values of \( n \) and \( \bar{n} \) that we may come across for different frame type sequences and relay-receive time index \( k \). In order to calculate the outage probability, we resort to 2-Dimensional characteristic function of \( 1_s((y; t^*_1) \) and \( 1_s((y^*_1; t^*_1) \)

\[
E\{\exp(-s_1 I_y(s; t^*_1) - s_2 I_y(s^*_1; t^*_1))\} = E\{\prod_{j \in J_1} \exp(-s_j I_y(s; t^*_1) - s_j I_y(s^*_1; t^*_1))\} \prod_{j \in J_2} \exp(-s_j I_y(s; t^*_1))
\]

\[
= \prod_{j \in J_1} \phi_f(s; t^*_1, t^*_1) \prod_{j \in J_2} \phi_f(s; t^*_1, t^*_1) \Psi(s; m(t^*_1), m(t^*_{1+1}), \alpha)
\]

(6.6)

where \( s = (s_1, s_2) \) and \( \phi(s; t^*_1) = E\{\exp(-s_1 I_y(s; t^*_1) - s_2 I_y(s^*_1; t^*_1))\} \), \( \phi(s; t^*_1) = E\{\exp(-s_2 I_y(s; t^*_1))\} \), for \( j \in J_2 \). It follows that

\[
\Psi(s; n, \bar{n}, \alpha) = \prod_{j \in J_1} \phi_f(s; t^*_1, t^*_1) \prod_{j \in J_2} \phi_f(s; t^*_1, t^*_1)
\]

(6.7)

Depending on the adopted strategy a specific subset of possible sequence types will be valid and consequently only a subset of the above defined functions will appear in the function \( \Psi(s; n, \bar{n}, \alpha) \). In particular, only \( \phi_1(s; t^*_1) \) for \( 1 \leq j \leq 3 \), and \( \phi_1(s; t^*_1) \) for \( 2 \leq j \leq 4 \) should be calculated. The outage probabilities can be expressed by the Laplace inversion formula and approximated as [104],

\[
F(R; n, \bar{n}, \alpha) = \frac{1}{(2\pi i)^2} \int_{d_1}^{d_1+\pi i} \int_{d_2}^{d_2+\pi i} \Psi(s; n, \bar{n}, \alpha) e^{s_1 s_2} ds_1 ds_2
\]

\[
\approx \sum_{i=1}^{d_1} \sum_{j=1}^{d_2} K_i K_j \Psi(z_i, R, z_j, \alpha)
\]

(6.8)

where \( d_1 \) and \( d_2 \) are proper constants, \( z_i \) are the poles of the Padé rational function and \( K_i \) is the corresponding residues. The close-form expressions for function \( \phi_f \) and \( \phi_f \) are difficult to calculate, but with the approximation of \( F(R; n, \bar{n}, \alpha) \), we only need to calculate their values at the \( M \) poles \( z_n \) which we can resort to numerical methods.
AF/DF-based $H^2$-ARQ-Relaying: The above approximation of outage probabilities can be easily extended to the AF/DF $H^2$-ARQ-Relaying strategy, where the valid frame type set is $J_{AF} = \{2, 3, 4, 5\}$. The mutual information through AF is

$$I^{AF}(\gamma) = I^{AF}(\gamma) = \alpha \log\left(1 + \gamma_0 + \left(\beta_e^2 \gamma_2 + \gamma_2\right) \left(1 + \beta_e^2 \gamma_2\right)\right)$$

where $\alpha$ is fixed at 0.5, $\beta_e$ is the amplifying gain at the relay and $I^{AF}$ is the trivial upper bound of the maximum mutual information conveyed with one AF operation according to (3.6).

$H^2$-ARQ-Relaying Strategy 2: For the DF branch of the this strategy, the outage probabilities can be easily obtained by letting $n_t$ equal to 0 and approximated by (6. 8). We will investigate the outage probabilities in the CF branch, where the relay detects errors in phase 1 of the first frame. In this circumstance, according to (6. 1), the accumulated mutual information after $n$ transmissions is

$$I_s(\gamma; t) = I^{CF}_{s} (\gamma) + (1 - \alpha) \sum_{i=1}^{n} \log\left(1 + \gamma_{s,i}\right)$$

(6. 9)

where the second term comes from the transmissions by the source during phase 2 of each frame. The outage probability in the CF branch is given by,

$$Pr\left(I_s(\gamma; t) < R, I_s(\gamma; t) = 1 < R\right) = G(R; \alpha)$$

(6. 10)

Note that the relay receives only in the first frame and therefore the outage probability will be determined by $I_s(\gamma; t) = I^s(\gamma_1)$. Then we need to calculate $G(R; \alpha)$ for the CF branch, which unfortunately cannot be approximated as in strategy 1 but we can use the Monte-Carlo method to calculate.

6.3.2 State and Transition Probabilities

In general, for two events sets $E_1$ and $E_2$, $Pr(E_1|E_2)$ is equal to $Pr(E_1, E_2)/Pr(E_2)$ and if $(E_1 \subseteq E_2)$, then $Pr(E_1|E_2)$ is equal to $Pr(E_1)$. In the state diagrams, the transition probability to state $j$ from any of its adjacent incoming state $i$ ($i \neq j$) can be expressed as

$$Pr(\text{state } i \rightarrow \text{state } j) = Pr(\text{state } j | \text{state } i)$$

$$= \frac{Pr(\text{state } j)}{Pr(\text{state } i)}$$

(6. 11)

111
It is straightforward to check that the state diagrams presented so far have the property that except state \( S \), for any state transition the event associated to a new state is a subset of the corresponding event of the previous state. Therefore we can first calculate the state probabilities and then obtain the transition probabilities through (6.11).

**H2-ARQ-Relaying Strategy 1:** We start from state \( B_n \) whose probability is the probability that neither the relay nor the destination is able to decode the message in previous \((n-1)\) frames. With assumed \( F(R;\emptyset,0,0,\alpha)=1 \), where \( \emptyset \) is an all zero vector, the state probability is \( \Pr(B_n)=F(R;(n-1,0,0,0),\emptyset,\alpha) \) with \( k=(n-1) \). It follows that the probability of state \( F_n \) is given as \( \Pr(F_n)=F(R;(N,0,0,0),\emptyset,\alpha) \) with \( k=N \). When the system operates in CF mode and the current state is \( C_n \), it implies that the decoding at the relay and the destination fails at the end of phase 1 in the \( n \)th frame. The probability is \( \Pr(C_n)=F(R;(n-1,1,0,0),\emptyset,\alpha) \) with \( k=n \). As it is seen, the calculation of the above state probabilities requires one outage calculation per state. On the contrary, if the relay has been able to decode the message, the system moves to state \( D_n \) and the corresponding probability is given by,

\[
\Pr(D_n) = \Pr\left( \left( y_i < R, I, \left( z_i^{-1}; t_i^{-1} \right) < R, I, \left( y_i < R \right) \right) \right) - \Pr\left( \left( y_i < R, I, \left( z_i^{-1}; t_i^{-1} \right) < R \right) \right)
\]

where \( k=(n-1) \) and \( n \) for the first and the second term respectively, and the length of \( \tau \) is \( n \). When the system is in state \( DF_{m,n} \), it means that \((n+m-1)\) frames \((n-1)\) CoF(C), \( 1 \) CoF(D) and \((m-1)\) Rtf(D)) have been transmitted and phase 2 has been repeated \( m \) times. Therefore, the probability of state \( DF_{m,n} \) is

\[
\Pr(DF_{m,n}) = F(R;(n-1,0,0,0),(0,0,1,0),(0,0,0,0),\alpha) - F(R;(n-1,1,0,0),(0,1,0,0),\alpha)
\]

The calculation of the probabilities of \( D_n \) and \( DF_{m,n} \) requires two outage calculations per state.

The transition probabilities can be easily obtained through (6.11). There are three routes stemming from state \( B_n \). As the probabilities of all the state transitions emanating from a single state must add up to 1, we have \( \Pr(B_n \rightarrow S)=1-\Pr(B_n \rightarrow D_n)-\Pr(B_n \rightarrow C_n) \). Similarly, we can obtain \( \Pr(C_n \rightarrow S) \) and \( \Pr(DF_{m,n} \rightarrow S) \).

**H2-ARQ-Relaying Strategy 2:** The state probabilities of \( D \) and \( DF \) can be easily obtained through similar process in the first strategy by letting \( n=1 \) in the expression of \( D_n \) and \( DF_{m,n} \). State \( CF_n \) indicates that the relay and destination detect errors in the first phase 1 and work in CF mode afterwards, and the destination fails to decode after \( n \) successive phase 2. The corresponding state probability is \( \Pr(CF_n) = G(R;n,\alpha) \). The rest of the transition probabilities can be obtained according to (6.11).

**6.3.3 Throughput Analysis**
Chapter 6 Hybrid Automatic Repeat re-Quest (HARQ) for the Relay System

With transition probabilities, we can apply the renewal-reward theorem [81], [102] to evaluate the throughput by investigating the random reward $\Phi$ and average airtime $T$ in two strategies. Here we assume the duration of one frame $T_f=1$.

HARQ-Relaying Strategy 1: In order to evaluate the average airtime, we need to use the state and the state transition probabilities. We first investigate the successful decoding events depicted in Figure 6.2. $T_{ss}(K)$ stands for the average time associated with case 1 where the destination decodes in phase 1 of the $K$th frame and is given by

$$T_{ss} = \sum_{k=1}^{K} P_{ss}(K)[(K-1)+\alpha], \quad P_{ss}(K) = Pr(B_k)Pr(B_k \rightarrow S)$$

$P_{DF}(L,K)$ defines the situations shown by case 2 of Figure 6.2, where the relay decodes in the $L$th frame but the destination cannot decodes until the $K$th frame ($K>L$), and the associated average time is

$$T_{DF} = \sum_{L=1}^{K} \sum_{K=L}^{N} P_{DF}(L,K)[L+(K-L)(1-\alpha)], \quad P_{DF}(L,K) = \rho_{DF}(L,K)$$

where $\rho_{DF}(L,K)$ is initialized as $Pr(D_2)Pr(D_2 \rightarrow S)$ when $K=L$ and equal to $Pr(DF_{(K<L)})Pr(DF_{(K>L)} \rightarrow S)$ for $K>L$. For those situations where the system is able to decode in CF mode in the $M$th frame, the average time associated with this event is defined by,

$$T_{CF} = \sum_{M=1}^{K} P_{CF}(M)M, \quad P_{CF}(M) = Pr(C_M)Pr(C_M \rightarrow S)$$

In above cases, the transmission is successful and $\Phi=S$. When outage events depicted in Figure 6.2 happen, $\Phi$ equals to zero and the related time should also be considered. The outage events happen in CF mode as depicted by case 4 and the average time is $T_{CF}^\text{out} = Pr(F_2)N$. On the contrary, the outage events may happen in DF mode as shown in case 5 of Figure 6.2 when the relay decodes the message in frame $L$, and the average time is given by,

$$T_{DF}^\text{out} = \sum_{L=1}^{K} P_{DF}^\text{out}(L)[L+(N-L)(1-\alpha)], \quad P_{DF}^\text{out}(L) = Pr(DF_{N-L+1,L})$$

Taking all possible outage events into consideration, since the outage events in DF and CF mode are mutually exclusive, the overall outage probability is

$$P^\text{out} = Pr(F_2) + \sum_{L=1}^{N} Pr(DF_{N-L+1,L})$$

The average random reward and the average airtime can be written as
\[ E\{\Phi\} = R\left(1 - P^\text{out}\right) \]
\[ E\{T\} = T_{\text{BS}} + T_{\text{CF}} + T_{\text{DF}} + T_{\text{out}} \]

The average throughput is \([81],[102]\]

\[ \eta(R,\alpha) = \frac{E\{\Phi\}}{E\{T\}} = \frac{R\left(1 - P^\text{out}\right)}{E\{T\}} \]

(6.12)

The resulted throughput is a function of the duplexing ratio \(\alpha\) and the selected rate \(R\). Therefore the average throughput can be maximized with respect to \(\alpha\) and \(R\) as

\[ \eta_{\text{max}} = \max_{R,\alpha} \eta(R,\alpha) \]

(6.13)

**\(H^2\)-ARQ-Relaying Strategy 2:** For HS\_S2, the first frame is the only complete frame. If the destination decodes the packet at the end of the first phase 1 (Case 1), the average time used is \(T_{\text{BS}} = \Pr(B\rightarrow S)\alpha\). If the destination decodes the packet in the \(K\)th frame in CF mode (Case 2), the average time is

\[ T_{\text{CF}} = \sum_{K=1}^{\infty} P_{\text{CF}}(K)\left[1 + (K-1)(1-\alpha)\right], \]

where \(P_{\text{CF}}(K)\) is initialized as \(\Pr(C)\Pr(C\rightarrow S)\) when \(K=1\) and equal to \(\Pr(\text{CF}_{K-1})\Pr(\text{CF}_{K-1}\rightarrow S)\) for \(K>1\). If outage events happen in CF mode, the average time is

\[ T_{\text{out}} = \Pr(\text{CF}_K)\left[1 + (N-1)(1-\alpha)\right]. \]

Because the DF part of strategy 2 is symmetric with CF part in the stage diagram, the time associated with successful decoding and outage events in DF mode can be obtained in a similar way as

\[ T_{\text{DF}} = \sum_{K=1}^{\infty} P_{\text{DF}}(K)\left[1 + (K-1)(1-\alpha)\right] \]

and

\[ T_{\text{out}} = \Pr(\text{DF}_K)\left[1 + (N-1)(1-\alpha)\right], \]

respectively, where \(P_{\text{DF}}(K)\) is initialized as \(\Pr(D)\Pr(D\rightarrow S)\) when \(K=1\) and equal to \(\Pr(\text{DF}_{K-1})\Pr(\text{DF}_{K-1}\rightarrow S)\) for \(K>1\). The throughput expression of the second strategy can be obtained according to (6.12) and (6.13) as well, where \(P^\text{out} = \Pr(\text{CF}_0) + \Pr(\text{DF}_0)\).

**6.4 Modified \(H^2\)-ARQ-Relaying Strategy with One Bit Feedback**

A relay system as shown in Figure 6.1 is considered. We assume that the phase and amplitude of the channel is only known to the receiver side through channel estimation, but the relay has limited knowledge of the amplitudes of the source-to-destination and relay-to-destination links through the error-free feedback link from the destination.
We first provide a modified strategy with one bit CSI feedback, denoted as MHS(1). In phase 1 of the nth frame, a message wεW is encoded into aNc symbols, denoted by X_{r,n} (without loss of generality, we assume that a is carefully selected such that aNc and (1-a)Nc are integers), and broadcasted by the source. The destination tries to decode the message. Note that the decoding process is not only based on the received signal in the current frame n but the buffered signals in the previous (n-1) frames will also be used. If the decoding is successful, an ACK is sent back to the source and the transmission of the current message ends. Otherwise, the destination will wait for further received signals in the second phase. With CSCGD input, the mutual information conveyed to the destination and the relay during phase 1 of the nth frame can be found in TABLE 5.1.

As we assumed, the relay has limited knowledge of the two links from the source and relay to the destination respectively. One indication bit b_n is used to carry partial CSI in the nth frame. If \gamma_2,n is in the region [\mu_2A_1, +\infty) and in the mean time, \gamma_0,n lies in the region [\mu_0A_1, \mu_0A_2], b_n is equal to one; otherwise, 0. This indication bit can be sent back through the error- and delay-free feedback link from the destination to the relay and the source. Depending on the success of the decoding at the relay node, two following actions are envisaged to be taken:

**Relay Decoding Success:** if the relay has been successfully decoded by joint processing Y_{r,n} and previously received signals, an ACK is sent back to the source. In the mean time, w is re-encoded to X_{r,n}^2 = X_{r,n}^1 and X_{r,n}^1 and X_{r,n}^2 are transmitted simultaneously by the relay and the source nodes in phase 2 of the nth frame. The relay system enjoys the capacity of the MISO channel, denoted as f^{BC}(\gamma) in TABLE 5.1.

The destination decodes the message by joint processing of the received Y_{d,n} and Y_{d,n}^2. The overall mutual information transmitted to the destination in a complete frame is the summation f^{BC} and f^{RS}, denoted as f^{DF} in TABLE 5.1.

**Relay Decoding Failure (dec=0):** without successful decoding, the relay will check the indication bit b_n. If b_n=1, it will perform the CF scheme using Wyner Ziv coding. However, it has been pointed out that in ideal Wyner-Ziv coding, the relay, while ignorant of actual realization of side information, requires CSI of the source-to-destination and relay-to-destination links to calculate the ideal transmission rate of the compressed signal and the available rate at the relay [8], [49]-[50]. The only available information about the two links at the relay is the indication bit. As a direct consequence of [Corollary 1, 8], proposition 6.2 is provided to find the transmission rate of the compressed signal under this non-ideal scenario.

**Proposition 6.2:** In a half-duplex relay system in which only the destination has global CSI knowledge, if the relay has the source-to-relay link CSI and one indication bit that indicate whether \gamma_0 and \gamma_2 lie in the region [A, B] and [C, +\infty) simultaneously, the relay node can deploy CF and the transmission rate of the compressed signal is

\[ R_c = \alpha \log \left( \frac{1 + \gamma_0 + \gamma_2}{(1 + A)} \right) \]
where

\[ \gamma_z = h(\alpha, A, B, C, \gamma_l) = \frac{(1 + C(1 + B))^{(1-\alpha)A} - 1}{1 + \gamma_l / A} \]

The average mutual information conveyed to the destination in one frame is

\[ I_{CY}(\gamma) = \alpha \log(1 + \gamma_z + \gamma_l / (1 + \gamma_l)) + (1 - \alpha) \log(1 + \gamma_z) \]

(6.14)

Proof: The proof of this proposition is presented in Appendix F.

In the mean time, the source still sends \( X_{2,n}^2 \) during phase 2. The following operation at the destination will be the same as \( HS_{-S1} \).

If \( b_n = 0 \), the current frame \( n \) is stopped without phase 2 and the system starts a new frame and goes back to the broadcasting phase. The reason for our setting can be justified as follows. If \( \gamma_{2,n} \) is smaller than \( \mu_1a_1 \) or \( \gamma_{1,n} \) is in \([\mu_2a_1, +\infty)\) but \( \gamma_{0,n} \) is larger than \( \mu_2a_2 \) (\( b_n = 0 \)), the power of the relay transmitted signal is relatively small when compared with source transmitted signal that actually acts as an additional noise when decoding the bin index \( s_n \). In such a case, the destination is not able to decode the signal from the relay and this signal will act as additional noise and degrade the performance. Therefore it would be better to prohibit the relay node from forwarding any signal and let the source broadcast again. If \( \gamma_{2,n} \) is in \([\mu_2a_1, +\infty)\) but \( \gamma_{0,n} \) lies in the region \([0, \mu_2a_2)\) (\( b_n = 0 \)), the 'amount' of side information is insufficient for efficient CF [8] and the system should go back to broadcasting phase as well.

The operation process can be summarized by the flow chart in the Figure 6.6(a). Note that the complete frame structure that consists of two phases will not be kept in some circumstances. Even with two-phase structure, the relay node could be working in different modes. The types of frames have been defined in TABLE 6.1, out of which the first four types appear in this strategy.
Figure 6.6. Flow charts of two modified H2-ARQ-Relaying strategies. (a) one bit CSI feedback (b) two bits CSI feedback.

The state diagram is shown in Figure 6.7. $B_{n1}$ is defined as the state where in previous $(n-1)$ frames, there are $n_1$ type 1 frame where CF operation occurs and $(n-1-n_1)$ type 2 frames with broadcasting operations and the destination has not been able to decode. When the system is in state $B_{n1}$, upon successful decoding at the relay, the system is ready to work in DF mode and the state transits to $D_{n1}$. When the phase 2 is repeated for $m$ times and the destination is still not able to decode the message, the system is in state $DF_{m,n1}$. If the relay detects errors when the current state is $B_{n1}$, with $b_h=1$, the state transits to $C_{n1}$, where the relay is ready to perform CF, otherwise, the system transits to state $B_{n+1,n1}$ directly. State $S$ denotes the state where the destination successfully decodes the message. To keep the Figure 6.7 concise, we use a combination state $G_{n1}$ to indicate the DF operations as shown in the lower part.
6.5 Modified $H^2$-ARQ-Relaying Strategy with Two Bits Feedback

In the previous strategy, when the instantaneous SNR between the source and the destination is in the region $[\mu(n-1), +\infty)$ in the $n$th frame, the CF operation is not efficient because the compressed signal forwarded from the relay will not be decoded successfully by the destination, thus leading to degraded performance. Therefore, we prohibit the relay from transmitting. However, we notice that if we keep the relay silent and let the source transmit alone in the second phase, the amount of the accumulated mutual information through this direct transmission is sufficiently large because at least we have

$$I^0(\gamma_n) = I(X_n^1; Y_n^1) = \log(1 + \gamma_n) \geq \log(1 + \mu(n-1))$$

If we choose $\alpha$ such that
\[
\log(1 + \mu_0 \alpha_1) = R
\]  

(6.15)

we can promise successful decoding by the end of frame \( n \) through direct transmission even without resorting to the buffered signals in previous \((n-1)\) frames. Now we propose a modified strategy with two indication bits \((b_n, c_n)\), denoted as \(MHS(2)\). There are four possible values for these two bits from 00 to 11 and their corresponding actions are summarized in TABLE 6.2. The operation process flow chart is depicted in Figure 6.6(b). Note that we have the fifth type of frame in which the relay keeps silent during the whole frame in this strategy.

The state diagram of this strategy can be obtained through simple extension from the previous strategy by inserting new state \(D_{in,ni}\) between \(B_{in,ni}\) and \(S_{in,ni}\) is defined as the state where the source is the only active node to transmit in the second phase of the \(n\)th frame. With selection of \(\alpha_3\) according to (6.15), we can be sure that this direct transmission will lead to successful decoding at the destination.

<table>
<thead>
<tr>
<th>(b_n, c_n)</th>
<th>(\gamma) regions</th>
<th>Actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>(\gamma_0, \in(0, \mu_0 \alpha_2)), (\gamma_2, \in[\mu_0 \alpha_1, +\infty))</td>
<td>The current frame ends and a new frame starts by broadcasting from the source.</td>
</tr>
<tr>
<td>01</td>
<td>(\gamma_0, \in(0, \mu_0 \alpha_2)), (\gamma_2, \in(0, \mu_0 \alpha_1))</td>
<td>The current frame ends and a new frame starts by broadcasting from the source.</td>
</tr>
<tr>
<td>10</td>
<td>(\gamma_0, \in[\mu_0 \alpha_2, \mu_0 \alpha_3]), (\gamma_2, \in[\mu_0 \alpha_1, +\infty))</td>
<td>The system operates in CF mode in phase 2</td>
</tr>
<tr>
<td>11</td>
<td>(\gamma_0, \in[\mu_0 \alpha_3, +\infty))</td>
<td>The relay keeps silent and the system operates in direct transmission mode in phase 2</td>
</tr>
</tbody>
</table>

6.6 Performance Analysis of Modified Strategies

In this section the main performance parameters including outage probability and throughput are derived. Our analysis is based on the same assumptions in the previous sections with the only difference that full CSI is not available at the relay.

6.6.1 Outage Analysis

Only the first four types of frames appear in this strategy. Define the valid frame type set \(J = \{1,\ldots,J\}\), where for strategy 1 \(J=4\) and for strategy 2 \(J=5\). Let \(I_0(\gamma)\) denote the conveyed mutual information in a type \(t\) frame,
where $t \in J_n$, then we have $f^{(1)}(y) = f^{CF}(y)$, $f^{(2)}(y) = f^{BC}(y)$, $f^{(3)}(y) = f^{DF}(y)$, $f^{(4)}(y) = f^{PS}(y)$ and $f^{(5)}(y) = f^{AR}(y)$. The accumulated mutual information at the destination after $n$ frames and at the relay after $k$ transmissions can be obtained according to (6.2) and (6.3).

In the modified strategies once the relay has been able to successfully decode or $(b_n, c_n) = 11$, the system will keep on operating in DF or direct transmission mode. It implies that the frame type sequence $\ell$ has the form of two concatenated sub-sequences $\ell = (\ell^s_n, \ell^e_n)$, where $\kappa = K(\ell) = \max \{i | \ell_i \in \{1, 2\}\}$, i.e. $\ell^s_n$ is a sequence consisting 1 and 2 only and $\ell^e_n$ is formed with 3, 4 and 5 only. The number of type 1 frames in $n$ frames is $\kappa_1 = m_1(\ell) = m_1(\ell^s_n)$. Clearly, there are $\binom{\kappa}{\kappa_1}$ valid $\ell^s_n$ in total. For a particular $\kappa$ and $\kappa_1$, we define the set of frame type sequences as

$$T(\kappa_1, \kappa) = \{ \ell | \ell \in \{1, 2\} \}$$

and for any $\ell \in T(\kappa_1, \kappa)$, we define the position set as $\chi(\ell; j) = \{i | \ell_i = j\}$ for $j \in \{1, 2\}$. Obviously, $|\chi(\ell; 1)| = \kappa_1$ and $|\chi(\ell; 2)| = \kappa - \kappa_1$. Under the block fading assumption, it is straightforward to show that for any two frame type sequences $a, b$ with length $n$, $a^s_n = b^s_n$, and $a^e_n, b^e_n \in T(\kappa_1, \kappa)$, their corresponding mutual information will be the same: $I_a(y; \ell) = I_b(y; \ell)$. This is a direct consequence of the claim 1 in Appendix F. In light of this observation, we can choose a representative frame type sequence $\ell$ whose sub-sequence $\ell^s_n$ has position sets $\chi(\ell; 1) = \{\kappa_1 + 1, \ldots, \kappa\}$ and $\overline{\chi}(\ell^e_n; 2) = \{1, \ldots, \kappa_1\}$. The outage probabilities for $\ell$ can be expressed as

$$Pr(I_a(y; \ell) < R, I_b(y; \overline{\ell}) < R, (\ell^s_n, \overline{\ell}^e_n) = f_a(R; m(\ell), m(\overline{\ell}), \kappa_1, \alpha)$$

(6.17)

where $m(\ell) = (m_1(\ell), \ldots, m_j(\ell))$. For any frame type sequence whose first sub-sequence belongs to the same set $T(\kappa_1, \kappa)$, the outage probability is the same according to claim 1. Since there are $\binom{\kappa}{\kappa_1}$ possible sequences with the same outage probability, the overall outage probability can be expressed as

$$Pr(I_a(y; \ell) < R, I_b(y; \overline{\ell}) < R, K(\ell) = \kappa, \ell^e_n \in T(\kappa_1, \kappa)) = \frac{\alpha}{\binom{\kappa}{\kappa_1}} f_a(R; n, \kappa_1, \alpha)$$

(6.18)
where we define $n = m(\xi_i^j)$ and $\bar{n} = m(\xi_{\bar{i}}^j)$ and $n_j = m_j(\xi_i^j)$ and $\bar{n}_j = m_j(\xi_{\bar{i}}^j)$.

6.6.2 State and Transition Probabilities

We derive the state probabilities of $MHS(2)$ only because for $MHS(1)$, the probabilities can be easily obtained through some simplifications. $F_a(R; \theta, \theta, 0, \alpha)$ is initialized as 1. Based on (6.18), the state probability of $B_n$ can be expressed as

$$
\Pr(B_n) = \Pr(1_x | \xi_i^j < R, I, \xi_{\bar{i}}^j < R, K(t) = n-1, \xi_{\bar{i}}^j \in T(n, n-1))
= F_a(R; (n, n-1-n_i, 0, 0), \theta, n-1, \alpha)
$$

where $|\eta| = k = (n-1)$. In state $B_n$, the source broadcasts. If only the relay successfully decodes the message, the system moves to state $D_n$ and the state probability is

$$
\Pr(D_n) = \Pr(1_x | \xi_i^j < R, I, \xi_{\bar{i}}^j < R, K(t) = n-1, \xi_{\bar{i}}^j \in T(n, n-1))
= F_a(R; (n, n-1-n_i, 0, 0), (0, 1, 0, 0), n-1, \alpha) - F_a(R; (n, n-1-n_i, 0, 0), \theta, n-1, \alpha)
$$

where $|\eta| = m$. Afterwards, the system works in DF mode. When phase 2 has been repeated for $m$ times, the system is in state $DF_m$ and the state probability is

$$
\Pr(DF_m) = F_a(R; (n, n-1-n_i, 0, 0), (0, 0, 1, m-1), n-1, \alpha)
= F_a(R; (n, n-1-n_i, 1, 0), (0, 0, 0, m-1), n-1, \alpha)
$$

If the relay detects errors by the end of phase 1 of the $n$th frame, the latest added mutual information at the destination is $I^{bc}(\gamma_i)$. With $(b_n, c_n) = 10$, we have $t = 1$ and the system transits to state $C_n$ where the relay is ready to perform the CF scheme. The state probability of $C_n$ can be expressed as

$$
\Pr(C_n) = \Pr(1_x | \xi_i^j < R, I, \xi_{\bar{i}}^j < R, K(t) = n, \xi_{\bar{i}}^j \in T(n, n))
= \binom{n-1}{n_i-1} \Pr(1_x | \xi_i^j < R, I, \xi_{\bar{i}}^j < R, K(t) = n, \xi_{\bar{i}}^j < R, K(t) = n, \xi_{\bar{i}}^j, \xi_{\bar{a}})
$$

If $(b_n, c_n) = 11$, the system transits to state $D_{i_m}$ and the state probability is

$$
\Pr(D_{i_m}) = \Pr(1_x | \xi_i^j < R, I, \xi_{\bar{i}}^j < R, K(t) = n-1, \xi_{\bar{i}}^j \in T(n, n-1), (b_n, c_n) = 11)
= \binom{n-1}{n_i} \Pr(1_x | \xi_i^j < R, I, \xi_{\bar{i}}^j < R, K(t) = n-1, \xi_{\bar{i}}^j < R, K(t) = n-1, \xi_{\bar{i}}^j, \xi_{\bar{a}}, (b_n, c_n) = 11)
$$

where $|\eta| = m$. Note that $\Pr(C_n)$ and $\Pr(D_{i_m})$ cannot be expressed by $F_a$ function.
We have two general principles for the transition probabilities. First, for two events \( E_1 \) and \( E_2 \), \( \Pr(E_1|E_2) \) is equal to \( \Pr(E_1,E_2)/\Pr(E_2) \) and if \( E_1 \cap E_2 \), then \( \Pr(E_1,E_2) \) is equal to \( \Pr(E_1) \).

Secondly, the probabilities of all the state transitions emanating from a single state must add up to 1. Based on these two principles, we can get the transition probabilities.

### 6.6.3 Throughput Analysis

With transition probabilities, we can apply the renewal-reward theorem [81] to evaluate the throughput by investigating the random reward \( \Phi \) and average airtime \( T \) in two strategies. Here we assume the duration of one frame \( T_f=1 \). To evaluate the average airtime, we investigate the time devoted to a certain state. Let \( q(l,m) = m + (l-m)\alpha \). \( T_{\text{BS}}(K,n_i) \) stands for the average time associated with case where \( t_f^{K-l} \in T(n_i, K-1) \) and the destination decodes in phase 1 of the \( K^{th} \) frame, given by,

\[
T_{\text{BS}}(K,n_i) = P(B_{K,n_i})P(B_{K,n_i} \rightarrow S)[q(K-1,n_i) + \alpha]
\]

We should consider all possible \( K \) and \( n_i \) and it follows that,

\[
T_{\text{BS}} = \sum_{K=1}^{\infty} \sum_{n_i=0}^{\infty} T_{\text{BS}}(K,n_i)
\]

\( T_{\text{DFS}}(L,K,n_i) \) defines the situations where \( t_f^{L-1} \in T(n_i, L-1) \), the relay decodes in the \( L^{th} \) frame, but the destination cannot decodes until the \( K^{th} \) frame \((K>L)\), and the associated average time is

\[
T_{\text{DFS}}(L,K,n_i) = P^{\Phi}(L,K,n_i)\left[q(L-1,n_i) + 1 + (K-L)(1-\alpha)\right]
\]

where \( P^{\Phi}(L,K,n_i) \) is initialized as \( \Pr(D_{L,n_i})\Pr(D_{L,n_i} \rightarrow S) \) when \( K=L \) and equal to \( \Pr(DF_{K,L,n_i})\Pr(DF_{K,L,n_i} \rightarrow S) \) for \( K>L \). It follows that

\[
T_{\text{DFS}} = \sum_{L=1}^{\infty} \sum_{K=1}^{\infty} \sum_{n_i=0}^{\infty} T_{\text{DFS}}(L,K,n_i)
\]

For those situations where \( t_f^{M} \in T(n_i, M) \), and the system is able to decode in CF mode in the \( M^{th} \) frame, the average time associated with this event is defined by,

\[
T_{\text{CF}} = \sum_{M=1}^{\infty} \sum_{n_i=1}^{\infty} T_{\text{CF}}(M,n_i), \quad T_{\text{CF}}(M,n_i) = \Pr(C_{M,n_i})\Pr(C_{M,n_i} \rightarrow S)q(M,n_i)
\]

If \( t_f^{M-1} \in T(n_i, M-1) \), and the system is able to decode in direct transmission mode in the \( M^{th} \) frame, we have
where \( \Pr(D_{iM_\text{out}} \rightarrow S) = 1 \) according to (6.15).

In above cases, the transmission is successful and \( \Phi = R \). When the retransmission limit is reached and the failure of HARQ is announced, \( \Phi \) equals to zero and the related time should also be considered. The outage events could happen in CF or broadcasting mode and the average time is

\[
T_{\text{out}} = \sum_{n=1}^{L-1} \sum_{j=0}^{N-1} \Pr(DF_{K+n,L-1,n}) \cdot q(L-1,n) + 1 + (N-L)(1-\alpha)
\]

The average random reward and the average airtime can be written as

\[
E\{\Phi\} = R(1 - P_{\text{out}}), \quad E\{T\} = T_{BS} + T_{CPS} + T_{DF} + T_{DS} + T_{CS} + T_{DF}^{\text{out}}
\]

The average throughput is \([81]\)

\[
\eta(R,\alpha) = \frac{E\{\Phi\}}{E\{T\}} = \frac{R(1 - P_{\text{out}})}{E\{T\}}
\]

The resulted throughput is a function of the duplexing ratio \( \alpha \), the selected rate \( R \) and parameters \( \alpha_1 \), \( \alpha_2 \) and \( \alpha_3 \). Therefore the average throughput can be maximized as

\[
\eta_{\text{max}} = \max_{R,\alpha,\alpha_1,\alpha_2,\alpha_3} \frac{E\{\Phi\}}{E\{T\}}
\]

### 6.7 Simulation Results, Comparisons and Discussions

In this section, we provide some results on the maximum achievable throughput (\( \eta_{\text{max}} \)) for the proposed H2-ARQ-Relaying and modified strategies and compare their performance with the other state of the art H-ARQ-Relaying and AF/DF based H2-ARQ-Relaying ones. The maximum retransmission limit \( N \) is 4. The benchmark strategies are listed in TABLE 6.3.
## TABLE 6.3

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>H-ARQ with direct transmission ((DT))</td>
<td>The relay keeps silent throughout the transmission.</td>
</tr>
<tr>
<td>H-ARQ-Relaying with co-located relay and destination ((CRD))</td>
<td>We assume that the relay and destination are so close such that they share all the information and can be regarded as a receiver with two antennas. However, (y_0) and (y_1) are independent. Therefore, full receive diversity is achieved.</td>
</tr>
<tr>
<td>H-ARQ-Relaying with conventional DF using unconstrained coding ((DF_U))</td>
<td>This scheme is based on selective DF, where when the relay detects errors, it keeps silent during phase 2 and the source is allowed to transmit alone, otherwise, the system works in the DF mode and phase 2 will be repeated. Unconstrained coding is used and the duplexing ratio can be optimized flexibly.</td>
</tr>
<tr>
<td>H(^2)-ARQ-Relaying with hybrid AF/DF ((HAD))</td>
<td>The relay performs AF when detecting errors and DF when successfully decoding. Note that in this case, (\alpha) is fixed at 0.5 and repetition coding is used within one frame when AF is performed.</td>
</tr>
</tbody>
</table>

All of the benchmark strategies can be regarded as some special realizations of \(HS\_SI\) and evaluated in the similar way as \(HS\_SI\). For instance, the state transition diagram of \(HAD\) is same as \(HS\_SI\) except that the state \(C_n\) is replaced by \(A_n\) where the system is ready to perform AF. The analytical results have been verified by the Monte-Carlo simulations. Note that although repetition coding is used in \(HAD\) within one single frame when AF is applied, to combine all the previously buffered signals in different frames, we still use INR. It means that the mutual information but not the instantaneous SNR in different frames is added at the destination. The throughput performance is evaluated in a large scale fading model according to the practical transmission setting with frequency 2.4GHz, path loss coefficients 3, and free-space reference \(d_0\). The path loss is \(L_p(d)=10\log_{10}(d/d_0)\), where we assume that the system loss is 0 at \(d_0\). The distance between the source and the destination is \(r=10d_0\). The source and the destination are assumed to be placed in the foci of the ellipse \((d/r)^2/(e/2)^2+(y/r)^2/(b/r)^2=1\), for \(1<e<+\infty\). The relay is moving along the ellipse as shown in Figure 6.8. With this assumption, we are able to investigate the relay system’s performance when the relay is not on the line segment between the source and the destination.
6.7.1 $H^2$-ARQ-Relaying Strategies

Figure 6.9 depicts the throughputs for various protocols, where the curves are obtained through the approximation (6.6)-(6.8) except the CF branch of $HS_S2$, for which we resorted to the Monte-Carlo based numerical multiple integrations. Basically, for any strategy, the throughputs are upper bounded by $CRD$ whose throughput is decreasing when the average SNR in the source-to-relay link decreases. From Figure 6.9, we can see that except $CRD$, $HS_S1$ is always performing best and able to approach the performance of $CRD$. It can be interpreted as follows. When the relay is moving towards the destination, the successful decoding probability at the relay decreases and the source-to-relay link is getting weaker. Therefore, in such a condition if the system insists on using either AF or DF forwarding schemes, none of transmit/receive diversity will be obtainable. In contrast, when the hybrid CF/DF relaying scheme is used, the system will be able the receive diversity through CF operation. Note that the throughput of $HS_S2$ is smaller than that of $DF_U$ when the relay is close to the source but eventually outperform $DF_U$ as the relay moves to the destination because it is able to enjoy receive diversity as well. However, in $HS_S2$, the relay observes only once in the first phase and this observation will be compressed and forwarded repeatedly. Therefore, the quality of the first observation is crucial for the overall performance. On the contrary, for $HS_S1$, different received signals are observed during a series of phase 1 and certain level of time diversity is achieved through the retransmission process. Therefore, we can claim that the performance of $HS_S2$ is mainly based on the channel condition during the first phase and is not able to benefit from the time diversity as $HS_S1$ does. This explains the performance gap between the two strategies. Compared with $DF_U$ with fixed $a$, $HAD$ is able to enjoy certain level of flexibility and its throughput is improved. As we mentioned in the introduction section, $HAD$ suffers from the fixed duplexing ratio due to the usage of repetition coding and its throughput is therefore smaller than $DF_U$ with optimal $a$. However, we must note that the AF/DF scheme we presented is not fully optimized. More sophisticated AF schemes with variable $a$ can be combined with DF [103] and the performance can be further improved, but this is out of the scope of this thesis.
Another advantage of $HS_S1$ is that the influence of fixed $\alpha$ is smaller than $DF_U$. This property is essential as the optimization of $\alpha$ for a real system will require exchange of some control messages thus an increase in signaling overhead. In such a case, we have to calculate the optimal $\alpha$ for every position of the relay from $d=0.5$ to $0.5$.

Figure 6.9. Throughput $\eta_{\text{max}}$ as a function of $\mu_2$ with ellipse model ($e=1.2, b/r=e/6$), $N=4$. DT: H-ARQ with direct transmission, CDF: H-ARQ-Relaying with co-located relay and destination, HAD: AF/DF-based $H^2$-ARQ-Relaying with repetition coding (when AF is applied), $DF_U$: DF-based H-ARQ-Relaying with unconstrained coding, $HS_S1$: CF/DF-based $H^2$-ARQ-Relaying strategy 1 using unconstrained coding, $HS_S2$: CF/DF-based $H^2$-ARQ-Relaying strategy 2 using unconstrained coding.

Figure 6.10 shows the optimal $\alpha$ which is optimized over grid, and Figure 6.11 depicts the corresponding ratios of the average time in DF and CF modes over the total average time, denoted by $\rho_D$ and $\rho_C$ respectively, and can be expressed as

$$\rho_C = (1-\alpha) \left[ \frac{\sum_{k=1}^{K} \left[ P_{cl} (K) + Pr(DF_{n-x,k},k) \right] (K-1) + P_{cr} (K) K}{\sum_{k=1}^{K} \sum_{l=1}^{L} P_{DF} (L,K) (L-1) + Pr(F_1) N} \right] / E\{T\}$$

$$\rho_D = (1-\alpha) \left[ \sum_{k=1}^{K} Pr(DF_{n-x,k},k) (N-K+1) + \sum_{k=1}^{K} \sum_{l=1}^{L} P_{DF} (L,K) (K-L+1) \right] / E\{T\}$$

126
Chapter 6 Hybrid Automatic Repeat re-Quest (HARQ) for the Relay System

There are two factors that could influence $p_D$ and $p_C$: one is the average number of DF or CF operations and another is the duration of the phase 2, i.e. $(1-\alpha)T_r$. For the DF part, when the relay is moving towards the source ($d$ decreases), the system is more likely to operate in the DF mode and the duration of phase 2 should become shorter as shown in Figure 6.10. Both factors reduce $p_D$. If SNR setting is getting more and more in favor of the CF mode ($d$ increases), although the fact that the average number of CF operations increases tends to increase $p_C$, the shortened second phase works in the reverse direction. Therefore $p_C$ is not monotonically changing with $d$. Only with fixed $\alpha$, $p_C$ is a non-decreasing function of $d$. Flexibility of switching between DF and CF mode based on the decoding status of the relay offers a possibility to enjoy the benefits of both relaying schemes. For the AF/DF-based H²-ARQ-Relaying strategy, the normalized average time of AF/DF operations, will be the same as above expressions with $p_C$ replaced with $p_A$ this time, where $p_A$ denotes AF mode portion. As it is seen from Figure 6.11, this strategy follows the same trend as $HS_S1$ with fixed $\alpha$ and can also enjoy some level of flexibility by adaptively switching between AF and DF.

![Figure 6.10. Optimal $\alpha$ (ellipse model with $e=1.2$, $b/r=e/6$).](image-url)
6.7.2 Modified $H^2$-ARQ-Relaying Strategies

In this section, we provide some results on the maximum achievable throughput ($\eta_{\text{max}}$) for the proposed modified $H^2$-ARQ-Relaying strategies and compare their performance with the other state of the art $H$-

<table>
<thead>
<tr>
<th>Strategy</th>
<th>CSI feedback Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DT$</td>
<td>AB ($f_0$)</td>
</tr>
<tr>
<td>$CRD$</td>
<td>AB ($f_0$)</td>
</tr>
<tr>
<td>$DF_U$</td>
<td>AB ($f_0$, $f_1$)</td>
</tr>
<tr>
<td>$HAD$</td>
<td>AB ($f_0$, $f_1$)</td>
</tr>
<tr>
<td>$HCD$</td>
<td>AB ($f_0$, $f_1$)</td>
</tr>
<tr>
<td>$MHS(1)$</td>
<td>AB ($f_0$, $f_1$), $b_h$($f_0$)</td>
</tr>
<tr>
<td>$MHS(2)$</td>
<td>AB ($f_0$, $f_1$), $b_h$($f_0$)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Strategy</th>
<th>CSI feedback Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CRD$</td>
<td>AB ($f_0$, $f_1$)</td>
</tr>
<tr>
<td>$DF_U$</td>
<td>AB ($f_0$, $f_1$)</td>
</tr>
<tr>
<td>$HAD$</td>
<td>AB ($f_0$, $f_1$)</td>
</tr>
<tr>
<td>$HCD$</td>
<td>AB ($f_0$, $f_1$)</td>
</tr>
<tr>
<td>$MHS(1)$</td>
<td>AB ($f_0$, $f_1$), $b_h$($f_0$)</td>
</tr>
<tr>
<td>$MHS(2)$</td>
<td>AB ($f_0$, $f_1$), $b_h$($f_0$)</td>
</tr>
</tbody>
</table>

N/A: not applicable; AB: Acknowledgement Bit; $b_h$: indication bit; $f$: the AB is obtained through feedback link $i$, CSI or $b_h$ is obtained through the feedback link $i$; $r$: the CSI is obtained by exploiting the reciprocity property of link $i$
ARQ-Relaying and $H^2$-ARQ-Relaying ones. Their corresponding channel knowledge requirements are summarized in TABLE 6.4.

Fig. 6.12 depicts the throughputs for various protocols, where the curves of benchmark strategies are obtained through the approximation and for the curves of the modified strategies, we resorted to the Monte-Carlo based numerical multiple integrations. Basically, $H^2$-ARQ-Relaying strategies outperform their corresponding H-ARQ-Relaying versions and their throughputs are bounded by CRD case where full receive diversity is achieved. However, HAD suffers from the fixed duplexing ratio due to the usage of repetition coding and its throughput is therefore smaller than $DF_U$ with optimal $\alpha$. When full CSI is not available at the relay, partial CSI is conveyed through the limited feedback link. The limited CSI will lead to performance loss for the CF/DF-based modified $H^2$-ARQ-Relaying strategies. With only one bit CSI feedback, $MHS(1)$ is not able to outperform $DF_U$ when the relay is relatively close to the source, i.e. the average SNR settings are in favor of DF. When we have two bits feedback, $MHS(2)$ performs close to HCD with unlimited CSI at the relay and shows significant improvement with $DF_U$ and HAD because it is able to enjoy certain level of receive diversity when CF is deployed. Clearly, both modified strategies are not able to be approaching full diversity as HCD because when the decoding at the relay fails, the system is not always in the operational region of CF.

![Figure 6.12. Throughput $\eta_{\text{max}}$ as a function of $\mu_2$ with ellipse model ($e=1.2$, $b/r=e/6$). $N=4$. $MHS(1)$: modified CF/DF-based $H^2$-ARQ-Relaying strategy with one bit CSI feedback; $MHS(2)$: modified CF/DF-based $H^2$-ARQ-Relaying strategy with two bits CSI feedback](image)
It is interesting to look at the optimal selection of $a_1$, $a_2$ and $a_3$ that are obtained numerically. Fig. 6.13 displays the optimal $\mu_2a_1$(dB) for two modified strategies. As the SNR of the relay-to-destination link increases, the optimal $\mu_2a_1$ increases as well. The intuition behind can be explained as follows. The actual transmission rate of the compressed signal at the relay is an increasing function of $\mu_2a_1$. With a better relay-destination link, it is reasonable to transmit the compressed signal with higher rate. Therefore, optimal $\mu_2a_1$ should be increased to provide higher transmission rate in the relay-to-destination link.

Figure 6.13. Optimal $\mu_2a_1$ (dB)

The optimal $\mu_0a_2$(dB) and $\mu_0a_3$(dB) are depicted in Fig. 6.14. Note that in $MHS(2)$, $\mu_0a_3$ is chosen as $(e^R-1)$, where $R$ is changing when the relay is moving towards the destination. Rests of the parameters are obtained numerically. For both strategies, $\mu_0a_2$ decreases with $r/d$ at a small slope. In $MHS(1)$, when $y_0a_2$ is larger than $\mu_0a_2$ in the $n$th frame, the current frame is ended after phase 1 because CF cannot be deployed efficiently. However, we ignore the fact that since the quality of the source-to-destination link is lower bounded ($y_0a_2$), by allowing the source to transmit solely during phase 2 of the $n$th frame, the destination stands a chance of successful decoding. Since the numerically obtained optimal $\mu_0a_2$ is larger than $(e^R-1)$ as shown in the Fig. 6.14, it means that the direct transmission will definitely lead to successful decoding at the destination even without the help of the buffered signals in previous $(n-1)$ frames. When the current frame is stopped without phase 2, we actually give up this opportunity of successful decoding and will waste time to start a new frame. However, in $MHS(2)$, we set $\mu_0a_2=(e^R-1)$ and allow the source to transmit solely during phase 2 when $y_0a_2$, where successful decoding is guaranteed at the end of frame $n$. It is more time efficient. This difference contributes to the performance gap between two strategies.
The region between $\mu_0a_2$ and $\mu_0a_3$ is the operational region for CF. When the relay is not close to the destination, the CF operational region is very small because the system is more likely to perform DF. When the relay is moving towards the destination, $\mu_0a_2$ decreases more rapidly than $\mu_0a_3$ and the CF operational region becomes wider. The intuition behind is that since the SNR settings are becoming in favor of CF, we should enlarge the CF operational region to give the system more opportunities to deploy CF.
Since the optimal parameters are obtained numerically, it is required to examine their sensitivities, i.e. to explore the throughput performance when the selection of the parameters strays away from their optimal values. In $MHS(2)$, when the relay is very close to the source ($r/d=-0.5$), the probability that the system is working in CF mode is very small. In such a case, $a_1$ is not sensitive as seen from Fig. 6.15. When the relay is moving towards the destination, the system is more likely to work in CF mode. Thus the throughput is becoming more sensitive to $a_1$, which is highly related with the CF operational region. If $a_1$ is selected smaller than the optimal one, we actually allow inefficient CF operations when the relay-to-destination link is relatively weak; otherwise, when $a_1$ is too large, we prohibit efficient CF operations when the relay-to-destination link is relatively strong. Both settings lead to performance degradation. The same trend was observed in $MHS(1)$.

Parameter $a_2$ shows the same sensitivity property as $a_1$. For $a_2$, it is chosen as $(e^k-1)$ in $MHS(2)$. So we only discuss its sensitivity in $MHS(1)$. When the relay is close to the source, same as $a_1$ and $a_2$, change of $a_3$ causes very small variation in the obtained throughput. Its shows more sensitivity when the system is more likely to operate in CF mode, i.e. the relay is close to the destination as shown in Fig. 6.16. When $a_3$ decreases, the CF operational region is shrinking and the probability of benefiting from receive diversity becomes smaller, thus degrading the throughput performance.

![Figure 6.16. Sensitivity of $a_3$ ($r/d=0.5$).](image)

6.8 Conclusions

In this chapter we consider integration of retransmission protocols with advanced hybrid relaying schemes. The most efficient form of retransmission protocols is obtained through hybrid use of forward error correction (FEC) and Automatic Repeat re-Quest (ARQ) and is commonly known as Hybrid ARQ (HARQ). On the other hand a hybrid approach is also proved to be useful for cooperative relaying where the common forwarding schemes, i.e. decode and forward (DF), amplify and forward (AF), and compress and forward (CF), are properly combined to better exploit the instantaneous channel conditions. The application of HARQ protocols to a
cooperative relay channel will lead to two classes of strategies: H-ARQ-Relaying, where HARQ is used in conjunction with a conventional forwarding scheme, usually DF, and H²-ARQ-Relaying, where both the retransmission protocol and the relaying scheme are hybrid. We propose two H²-ARQ-Relaying strategies that efficiently exploit a hybrid CF/DF forwarding scheme, where the relay, based on its decoding status, could dynamically switch between CF and DF. We analyze the performance of both strategies mainly in terms of outage probabilities and throughput. The proposed H²-ARQ-Relaying strategies exhibit significant improvement compared with DF-based H-ARQ-Relaying as well as AF/DF-based H²-ARQ-Relaying. The hybrid CF/DF forwarding attains either of transmit and receive diversities depending on the current channel condition, while the non-hybrid forwarding schemes only target one of these diversities. We also consider practical implementation issues and propose a modified CF/DF-based H²-ARQ-Relaying strategy with partial channel state information (CSI). The proposed modified strategy has a very relaxed feedback requirement and with only one extra feedback bit for provisioning of partial CSI is able to operate close to its equivalent version with unlimited CSI feedback. The modified strategy allows for more flexible operation of the relay node according to its decoding status and the partial CSI feedback.

In the first part, we developed H²-ARQ-Relaying strategies in a hybrid relay system that enables a pair of terminals (relay and destination) with a single antenna to exploit spatial diversity. A flexible approach was taken to combine both CF and DF strategy enabling system to efficiently adapt itself to realized dynamic channel conditions. Two strategies are proposed and their performance is analyzed under the assumption of full CSI at the relay. These two strategies show significant improvement over those strategies based on conventional DF and hybrid AF/DF. The main advantage of the proposed hybrid scheme of CF/DF is that it can achieve either of the transmit and receive diversities depending on the realized channel condition, while the other schemes only target one of these diversities and as a result exhibit a saturated performance in a low level under certain conditions that are detailed in the previous section.

Moreover, we noticed that the global CSI is required when CF is deployed at the relay. This assumption is reasonable when reciprocity holds for the relay-destination link and/or this link is strong enough to support the feedback of full CSI knowledge. When the provision of full channel knowledge is not affordable, we proposed modified H²-ARQ-Relaying strategies. The proposed strategies require a very small number of extra feedback bits from the destination to inform relay of the decided operational region. With modified strategies, in certain practical conditions the throughput is significantly improved compared with other cases. The main advantage of the proposed strategies is that it can achieve transmit diversity when DF is deployed and sometimes receive diversities depending on the realized channel condition, while the other schemes only target one of them.
Chapter 7 Conclusions and Future Work

7.1 Conclusions

In this thesis, a comprehensive investigation for the cooperative communication systems was presented and fundamental understanding from both the theoretical and practical points of view was achieved. We considered, as opposed to the landmark work in [5], where the source is silent when the relay node is active, a more general model by allowing the source to transmit with the relay simultaneously in a cooperative manner. Typical relaying schemes including amplify-and-forward (AF), decode-and-forward (DF) and compress-and-forward (CF) were studied and their achievable rates and outage probabilities were derived in a half-duplex relay channel. Both repetition coding (RC) and unconstrained coding (UC) were investigated. We also noticed that during the second phase, although the source and the relay can be easily synchronized in the time domain, carrier synchronization is very challenging in practical systems. In this regard, both synchronized and asynchronous scenarios were examined.

For the AF scheme, the optimization of the amplification function at the relay was highlighted. As opposed to previous work where the relay always transmits with full power, we found that for the synchronized relay channel, the amplification function cannot be optimized without the knowledge of the instantaneous SNRs of all the three links. For the asynchronous case, to optimize the amplification function, the relay requires to know if the instantaneous SNR of the source-relay link is higher than that of the source-destination link. Despite the fact that the amplification functions for synchronized and asynchronous cases are in different formations, the principle behind is the same: the better the received signal, the greater the transmitting power. Based on this principle, a sub-optimal amplification function without any CSI requirement was proposed for implementation convenience. If the quality of the source-relay link is above some pre-set threshold, the relay transmits with full power; otherwise, the relay is silent. Apparently, the same rule is applied here and the quality of the received signal decides the behavior of the relay. The results have shown that the AF scheme is not always able to outperform direct transmission because it suffers from bandwidth loss due to the usage of RC. The usage of sub-optimal amplification function only causes negligible performance loss. The results of the outage probabilities suggest that whenever the source is active or not during phase 2, the AF scheme enjoys the same second-order diversity in the high SNR regions. However, if the source is active during phase 2, the AF scheme could achieve a smaller outage probability.

In the DF scheme, unconstrained coding (UC) can be applied and the relay system will not suffer from the bandwidth efficiency loss. The phase synchronization problem is an important issue. With phase synchronization, beamforming can be applied and the relay system is able to achieve better performance than an asynchronous system. The achievable rate analysis indicates that the UC-based DF scheme achieves the best performance but it shares the same weakness as the AF scheme: its performance is constrained by the source-relay link. If this link is weaker than the source-destination link, the UC-based DF scheme has the same
achievable rate as direct transmission. The diversity gains of different DF schemes have been shown through their outage probabilities. With fixed DF, where the relay forwards whenever its received message is successfully decoded or not, the diversity is just the same as direct transmission. Only when selective DF is applied, where the relay keeps silent if not able to decode, the relay system is able to enjoy second-order diversity. By allowing the source to transmit during the second phase, both the RC- and UC-based DF schemes demonstrates considerable gains.

An advanced DF scheme, namely soft DF (SoDF), was investigated and the upper bounds of BER and BLER were derived. In SoDF, the decoder at the relay is a SISO decoder and the obtained soft values of the received message are soft encoded. By doing this, the error propagation can be mitigated. In order to approximate the distribution of the output values of the soft encoder, a Gaussian approximation model was used. We derived the pairwise error probability (PEP) and investigated the weight enumerating function (WEF) of the distributed turbo codes (DTC). In order to make the upper bounds tight, the limit-before-average technique was applied. The analysis was extended to the space-time cooperation framework. Based on the simulation and analytical results, as opposed to the conventional DF scheme which reaches an error floor because of the error propagation, the SoDF scheme shows significant improvement in BER and BLER and the upper bounds derived are very tight.

We also studied the RC- and UC-based CF scheme and introduced a complexity reduced scheme: UC-based quantize-and-forward (QF). As opposed to the DF scheme, the UC-based CF and QF schemes have the advantage of always being able to outperform direct transmission. The results revealed that CF is a more competitive candidate when the relay is close to the destination, i.e. the relay-destination is of high quality. The interesting thing is that regardless of coding strategies applied, all the schemes enjoy the same second-order diversity as the AF and DF schemes.

The practical implementation of CF involves quantization and SW coding. Our main contribution was to develop a new quantization scheme based on the achievable rate of the relay system and provide a decoding scheme accordingly with reduced complexity. The design objective of the new quantization scheme is to maximize the achievable rate instead of the minimization of the distortion. This scheme helps the relay system to achieve lower BER than other quantization algorithms.

After quantization, the achieved index should be compressed efficiently. The compression/decompression design was addressed in this thesis. A soft multilevel SW coding/decoding structure was proposed and its performance was evaluated through simulation. This new structure has the ability to combat the errors introduced in the relay-destination link during the transmission of the compressed data. In the multilevel structure, the received signal is quantized and split into multiple bit planes and binary channels codes are used to implement SW coding of each of them. Each bit plane is corresponding to a SW coding level and the new design allows for the soft information passing within one single level. The compressed data is protected by a code $C_p$ and at the destination a SISO decoder is implemented to decode $C_p$. Within one single level, the
output of the SISO decoder, i.e. the soft values of the compressed data are delivered to a soft SW decoder which is able to process soft values. The decompression in lower levels requires the decompressed information of upper levels. This process can also be "soften" by passing the soft values in upper levels instead. Finally, we noticed the statistical relation between the SISO decoder of \( C_p \) and the SW decoders and developed a joint turbo-like structure to further improve the performance through extrinsic information exchange.

Furthermore, the research is extended to more flexible relaying schemes. Multilevel coding is incorporated with cooperative communications. We exploited the property that BERs differ in different coding levels when an appropriate labeling scheme is implemented. This property triggers the usage of a hybrid relaying scheme in which the upper levels of the multilevel codes are decoded but the lower levels are compressed. This hybrid relay was proved to be able to combat the error floor problem faced by the conventional DF scheme. At the same time, compared with the conventional CF scheme, the hybrid scheme is capable of reducing the system's complexity and improving the transmission efficiency by shortening the duration of phase 2. A good balance among the BER performance, complexity and transmission efficiency could be achieved. Hybrid relaying schemes can also be implemented in a time-division fashion, where the relay is able to switch to DF, QF and CF according to the current channel realizations. We noticed that the usage of the CF scheme requires global CSI at the relay. However, full CSI knowledge assumption is unrealistic in a practical relay system. To cope with this problem, we established a limited feedback link between the destination and the relay to transmit partial but useful CSI. Compared with conventional relaying schemes, the hybrid relaying scheme shows significant improvement in its outage behavior.

Finally, we considered integration of retransmission protocols with advanced hybrid relaying schemes. The most efficient form of retransmission protocols is obtained through hybrid use of forward error correction (FEC) and Automatic Repeat re-Quest (ARQ) and is commonly known as Hybrid ARQ (HARQ). On the other hand a hybrid approach is also proved to be useful for cooperative relaying where the common forwarding schemes, i.e. decode-and-forward (DF), amplify-and-forward (AF), and compress-and-forward (CF), are properly combined to better exploit the instantaneous channel conditions. The application of HARQ protocols to a cooperative relay channel will lead to two classes of strategies: H-ARQ-Relaying, where HARQ is used in conjunction with a conventional forwarding scheme, usually DF, and H2-ARQ-Relaying, where both the retransmission protocol and the relaying scheme are hybrid. We proposed two H2-ARQ-Relaying strategies that efficiently exploit a hybrid CF/DF forwarding scheme, where the relay, based on its decoding status, could dynamically switch between CF and DF. We analyzed the performance of both strategies mainly in terms of outage probabilities and throughput. The proposed strategies exhibit significant improvement compared with DF-based H-ARQ-Relaying as well as AF/DF-based H2-ARQ-Relaying. The hybrid CF/DF forwarding attains either of transmit and receive diversities depending on the current channel condition, while the non-hybrid forwarding schemes only target one of these diversities. We also considered practical implementation issues and proposed a modified CF/DF-based H2-ARQ-Relaying strategy with partial CSI. The proposed modified strategy has a very relaxed feedback requirement and with only one extra feedback bit for provisioning of partial CSI.
able to operate close to its equivalent version with unlimited CSI feedback. The modified strategy allows for more flexible operation of the relay node according to its decoding status and the partial CSI feedback.

7.2 Future Work

Although the results of this research have significantly contributed towards the cooperative communication systems, there are still numerous opportunities of research and improvements.

In this thesis, only RC was implemented in the AF scheme. Due to the usage of RC, the AF scheme suffers from the bandwidth loss and a fixed duplexing ratio. However, we must note that the AF scheme presented is not fully optimized. A more sophisticated flexible AF scheme with variable duplexing ratio has been studied in [103] and the performance can be further improved.

Distributed turbo codes have been proved to approach the theoretical bound and are widely implemented in the DF scheme. However, the convergence property of the distributed turbo codes is still an open problem. The convergence property can be evaluated through the extrinsic information transfer (EXIT) chart. The EXIT chart analysis for the distributed turbo codes will provide another method to evaluate the error rate performance and this research will lead to the optimization of the iteration numbers. The EXIT chart can be extended to the soft DF scheme. It will provide insight into the soft encoding process at the relay.

Till this date, the most important part of the CF scheme — the Wyner-Ziv problem — has not been well understood. The closed-form rate-distortion functions for Wyner-Ziv problem have only been derived for correlated discrete binary sources and Gaussian distributed continuous sources. As mentioned, the correlation model in a relay system is more complicated and the corresponding rate-distortion function is an important issue to be investigated in the future.

In this thesis, although the theoretical analysis is based on a general relaying model where the source is allowed to transmit during phase 2 in a cooperative manner, the novel CF techniques were implemented in a simple relaying protocol where the source is silent during phase 2. This simple protocol greatly reduced the decoding complexity at the destination at an expense of small performance loss. However, our techniques should be extended to the general relay model in the future.

In the design of Slepian Wolf coding, the decoder for the error-protection code of the compressed signal is allowed to interact with the SW decoder through a turbo-like structure. It has been shown that this structure is able to improve the decompression error rate. This joint processing concept can be further extended to the whole system, where the decoder for the main information from the source can be involved. The extrinsic information can be delivered among three components: the decoder for the compressed data, the SW decoder and the decoder for the main information.
The multilevel coding concept was incorporated and hybrid DF/CF relaying scheme was investigated, where the boundary of DF and CF operations is fixed and selected in a heuristic way. This boundary can be more flexible and adaptive to the current channel realizations. The decoding starts from the highest level and stops at the first level when encountering errors. The rest of levels will be left to CF. The adaptive concept will give more flexibility to the relay node.

In this thesis, we developed H2-ARQ-Relaying strategies in a hybrid relay system that enables a pair of terminals (relay and destination) with a single antenna to exploit spatial diversity. Generally speaking, in addition to the channel circumstances described in this thesis, there are many other scenarios that are worth investigating. The relaying scheme can be more flexible by switching between more operational modes such as silent mode, amplify-and-forward, decode-and-forward, compress-and-forward and even quantize-and-forward to achieve the perfect balance between throughput and complexity. Moreover, the resource allocation can be incorporated and yield further enhancements. It would be of great interest to examine this problem in the future.

As examined in the thesis, only single relay case was considered. In wireless networks, there could be more than one relay nodes working at the same time [110]-[112]. These relays can be divided into different groups according to their working modes. For instance, for those close to the source, DF can be carried out and other relay nodes close to the destination can deploy CF. The performance for this kind of system is an interesting problem and is worth further investigating. Moreover, only two hop relaying was addressed in this thesis. It is possible that the signal transmitted by the source should be relayed more than one time to reach the destination. The theoretical analysis for multi-hop relay is still an open problem.

Finally, we only dealt with a single-user relay system. Cooperative relaying concept can incorporated into a multiple access channel (MAC) where multiple user terminals transmit through a code division multiple access (CDMA) mechanism to a single base station (BS). A relay station (RS) could help the BS to detect the users’ signals. Turbo multi-user detection (MUD) [107]-[109] can used at both the BS and the RS to exploit the error control coding structure of the signals. For this multiuse case, CDMA can be extended to SDMA where MIMO systems will be taken into consideration. Study on overloaded conditions is also an important issue especially for SDMA schemes, where the number of receive antennas at either of the relay and the destination nodes leads to an overloaded condition. It is believed that turbo MUD will be powerful to satisfactorily operate even under heavily loaded conditions. The extension to the multi-carrier scenario as shown in [114] is also an interesting problem.
Appendix A

In this appendix, we derive the achievable rate of AF and DF with relay protocol II. For AF, rewrite in an equivalent channel format as

\[ Y_d = CX_d' + BZ \]

where

\[
Y_d = \begin{bmatrix} Y_d^1 \\ Y_d^2 \end{bmatrix}, \quad C = \begin{bmatrix} \epsilon_1 \sqrt{\mu_0} & \beta_1 \epsilon_1 \sqrt{\mu_0} \\ \beta_2 \epsilon_2 \sqrt{\mu_2} & \beta_1 \epsilon_1 \sqrt{\mu_0} + \beta_2 \epsilon_2 \sqrt{\mu_2} \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ \beta_2 \epsilon_2 \sqrt{\mu_2} & 0 \end{bmatrix}, \quad Z(n) = \begin{bmatrix} Z^1_d \\ Z^2_d \end{bmatrix}
\]

where superscript \( T \) represents transpose. Using vector results in [9], the mutual information is maximized with zero-mean, circularly symmetric complex Gaussian distributed (CSCGD) input \( X_d' \) as

\[
l(X_d'; Y_d) = \log \det \left( \mathbf{I}_2 + (CC^T)(BE(ZZ^T)B^*)^{-1} \right)
\]

where \( \mathbf{I}_2 \) is a 2-by-2 identity matrix and the covariance matrix \( E(ZZ^T) = \text{diag}\{\sigma_1^2, \sigma_2^2, \sigma_3^2\} \). Since the noise power is set as 1, the covariance matrix is a 3-by-3 identity matrix. The rest terms can be calculated as

\[
CC^T = \begin{bmatrix} \gamma_0 \\ \gamma_0 \end{bmatrix}, \quad (BE(ZZ^T)B^*)^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 + \beta_2^2 \gamma_2 \end{bmatrix}
\]

Hence, the mutual information is given as

\[
l(X_d'; Y_d) = \log \left( 1 + \gamma_0 \right) + \frac{\beta_1 \epsilon_1 \sqrt{\mu_0} + \beta_2 \epsilon_2 \sqrt{\mu_2}}{1 + \beta_2^2 \gamma_2}
\]

\[
= \log \left( 1 + \gamma_0 \right) + \frac{\beta \sqrt{\gamma_0 \gamma_2^2 (\sigma_1^2 + \sigma_2^2)} + \beta \sqrt{\gamma_0}}{1 + \beta_2^2 \gamma_2}
\]

If in the asynchronous case, the achievable rate is averaged with respect to \( \theta \) as
\[ I_{sd} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \log \left( 1 + \gamma_0 + \frac{\beta_\infty \sqrt{\gamma_y \gamma_2} e^{-j\theta} + \beta_\infty \sqrt{\gamma_y}}{1 + \beta_\infty \gamma_2} \right) d\theta \]

\[ = \frac{1}{2\pi} \int_{-\pi}^{\pi} \log \left( 1 + \gamma_0 + \frac{\beta_\infty \gamma_y \gamma_2 + \beta_\infty \gamma_2 + 2\beta_\infty \beta_\infty \sqrt{\gamma_y \gamma_2} \cos \theta}{1 + \beta_\infty \gamma_2} \right) d\theta \]

\[ = \frac{1}{\pi} \int_{0}^{\pi} \log \left( 1 + \gamma_0 + \frac{\beta_\infty \gamma_y \gamma_2 + \beta_\infty \gamma_2 + 2\beta_\infty \beta_\infty \sqrt{\gamma_y \gamma_2} \cos \theta}{1 + \beta_\infty \gamma_2} \right) d\theta \]

\[ \overset{(1)}{=} \log \left( \frac{1 + \gamma_0 + \beta_\infty \gamma_y \gamma_2 + \beta_\infty \gamma_2 + \beta_\infty \gamma_y}{1 + \beta_\infty \gamma_2} + \frac{1 + \gamma_0 + \beta_\infty \gamma_y \gamma_2 + \beta_\infty \gamma_2}{1 + \beta_\infty \gamma_2} \right) - \frac{4\beta_\infty \beta_\infty \gamma_y \gamma_2}{(1 + \beta_\infty \gamma_2)^2} \]

where step (1) is derived according to [8].

For DF, similar to the derivation in AF schemes, we reorganize the equations in its equivalent form as

\[ Y_d = CX_d' + Z \]

where

\[ Y_d = \begin{bmatrix} Y_1' \\ Y_2' \end{bmatrix}, \quad C = \begin{bmatrix} c_0 \sqrt{\mu_0} & c_1 \sqrt{\mu_1} + c_0 \sqrt{\mu_2} \\ c_0 \sqrt{\mu_0} & c_1 \sqrt{\mu_1} + c_0 \sqrt{\mu_2} \end{bmatrix}, \quad Z = \begin{bmatrix} Z_1' \\ Z_2' \end{bmatrix} \]

According to [9], the mutual information is given by

\[ I(X_d', Y_d) = \log \det \left( I_2 + (CC^H)(E(ZZ^H))^{-1} \right) \]

where

\[ CC^H = \begin{bmatrix} \gamma_0 & c_0 c_2 \sqrt{\mu_0 \mu_1 + \gamma_0} \\ c_0 \sqrt{\mu_0 \mu_1 + \gamma_0} & c_1 \sqrt{\mu_1 + \gamma_0} \end{bmatrix} \]

140
Appendix B

We give the WEF of the RSCC we used in SoDF. Here we only get accurate expression of the terms with \( w+z+x < 2^t \) free distance for the RSCC we used, where \( z \) and \( x \) are parity check weights.

\[
A^\text{turbo}_r (X,Z,K) = \left( X^4 Z^4 + X^4 Z^4 + X^4 Z^4 + X^4 Z^4 \right) K^1 \binom{K}{2}
\]

\[
A^\text{turbo}_w (X,Z,K) = \left( X^2 Z^2 + X^2 Z^2 + X^2 Z^2 + X^2 Z^2 \right) K^3 \binom{K}{3}
\]

\[
A^\text{turbo}_j (X,Z,K) = \left( X^2 Z^2 + X^2 Z^2 + X^2 Z^2 + X^2 Z^2 \right) K^1 \binom{K}{4}
\]

\[
A^\text{turbo}_i (X,Z,K) = \left( X^2 Z^2 + X^2 Z^2 + X^2 Z^2 + X^2 Z^2 \right) K^1 \binom{K}{5}
\]

\[
A^\text{turbo}_a (X,Z,K) = X^2 Z^2 K^1 \binom{K}{6}
\]

\[
A^\text{turbo}_f (X,Z,K) = X^2 Z^2 K^1 \binom{K}{7}
\]

Where \( A(X,Z,K) \) is the WEF function of the turbo code. The contribution of the rest terms can be overbounded as [45],

\[
A^\text{turbo}_{1/2} (X,Z,K) = 2^e \ X^{K/2} Z^{K/2}
\]
Appendix C

This appendix gives the achievable rate of the RC-based compress-and-forward scheme. For RC, the destination starts from decoding the Slepian-Wolf coded bin index $i$ by treating $X_i^2$ as noise. With CSCGD input, it can do so if

$$R_0 \leq (1-\alpha)I(X_i^1;Y_i^2) = (1-\alpha)\log\left(1 + \frac{Y_i}{1+\gamma_s}\right)$$

(C.1)

where $R_0$ is the available rate at the relay. According to [49], $\hat{Y}_i^2$ can be expressed as

$$\hat{Y}_i^2 = Y_i^1 + Z_w + kY_o^1 = (c_i\sqrt{\mu_i} + k\sqrt{\mu_o}X_i^1 + Z_w + kZ_o^1$$

where $k$ is a constant that is given in [50] but does not influence the final results and therefore can be set as 0, and $Z_w$ is a Gaussian variable independent of $Y_i^1$ and $Y_o^1$ with variance $\sigma_w^2$. The transmission rate of the compressed signal should also be lower than the available rate at the relay

$$R_0 \geq \alpha\left[ I(Y_o^1;W) - I(Y_o^1;W) \right] = \alpha\left[ \log\left(1 + \frac{\sigma_w^2}{1+\gamma_s}\right) - \log\left(\sigma_w^2\right) \right]$$

(C.2)

where $W$ is an auxiliary random variable of the form $Y_o^1 + Z_w$. Therefore, insert (C.2) to (C.1), the compression noise can be minimized as

$$\sigma_o^2 = \frac{1 + \frac{Y_o}{1+\gamma_s}}{e^{\delta/\gamma_s} - 1}$$

(C.3)

Define $\hat{Y}_i^3 = Y_o^3 - c_i\sqrt{\mu_i}X_i^1$, the received signal can also be rewritten in an equivalent channel model

$$Y_o = CX_o^1 + Z$$

where
According to [9], the mutual information between $X_i$ and $Y_s$ is given by

$$I(X_i; Y_s) = \log \det \left( I_s + (CC^H)(E(ZZ^H))^{-1} \right)$$

Then we obtain the achievable rate. 
Appendix D

In this appendix, we derived the distortion and the transmission rate of the compressed signals.

If we have already decoded the index \( v \), with side information \( Y_j \), the optimal reconstruction function is achieved by minimizing \( D \) over all choices of the reconstruction function [68], namely

\[
\hat{y}_i = \arg\min_{y_i} E\{D(y_i, \hat{y}_i) | v, Y_j \}
\]

With the MSE distortion measurement, we have

\[
\hat{y}_i = E\{y_i | v, Y_j \}
\]

It follows that the expected distortion for the given \( v \) and \( Y_j \) can be expressed as

\[
D_i = E\{(y_i - \hat{y}_i)^2 | v, Y_j\} = E\{y_i^2 | v, Y_j\} - \hat{y}_i^2
\]

The distortion should be averaged over all possible \( v \) and \( Y_j \). We need to start from \( \hat{y}_i \), which can be expressed as

\[
\hat{y}_i = E\{y_i | v, Y_j\} = \int y_i p(y_i | v, Y_j) dy_i = \int y_i \frac{p(y_i, v, Y_j)}{p(Y_j, v)} dy_i
\]

The source signal follows discrete distribution with probability \( p \) and \( 1-p \) at -1 and 1 respectively. Here we assume that the signal is equally distributed, i.e. \( p=0.5 \). Given \( X_i, Y_j \) and \( Y_j \) are independent to each other and their joint distribution is

\[
p(x_i, y_j) = \sum_{x_i=-1}^{1} p(x_i | X_i)p(x_i)
\]

Similarly, the joint distribution of \( v \) and \( Y_j \) is

\[
p(v, y_j) = \sum_{x_i=-1}^{1} p(v | x_i)p(x_i)
\]

Given \( X_i \), the distribution of \( Y_j \) and \( Y_j \) are given as

\[
p(x_i, y_j) = \sum_{x_i=-1}^{1} p(x_i | X_i)p(x_i)
\]
\[
p(Y'_i | X'_i) = \frac{1}{\sqrt{2\pi\sigma^2_i}} \exp \left( -\frac{(Y'_i - c_i \sqrt{E_i X'_i})^2}{2\sigma^2_i} \right)
\]
\[
p(Y''_i | X''_i) = \frac{1}{\sqrt{2\pi\sigma^2_i}} \exp \left( -\frac{(Y''_i - c_i \sqrt{E_i X''_i})^2}{2\sigma^2_i} \right)
\]
respectively, and we have
\[
P(v | X'_i = \pm 1) = \frac{1}{2} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2_i}} \exp \left( -\frac{(Y'_i \pm \sqrt{E_i c_i})^2}{2\sigma^2_i} \right) dY'_i
\]
\[
= \frac{1}{4} \left[ \text{erf} \left( \frac{u_i}{\sqrt{2\sigma^2_i}} \right) - \text{erf} \left( \frac{u_i \pm \sqrt{E_i c_i}}{\sqrt{2\sigma^2_i}} \right) \right]
\]
It follows that
\[
\bar{E}' = \frac{1}{2} \int_{-\infty}^{\infty} Y'_i p(Y'_i, X'_i) dY'_i = \frac{1}{2} \frac{p(Y'_i | X'_i = -1)}{p(v, Y'_i)} E'(u_i, u_{i,n}) + \frac{1}{2} \frac{p(Y'_i | X'_i = 1)}{p(v, Y'_i)} E'(u_i, u_{i,n})
\]
\[
\text{where } E' \text{ and } E'' \text{ are expectations of } Y'_i \text{ given } X'_i \text{ is } +1 \text{ and } -1 \text{ respectively, denoted as}
\]
\[
E'(u_i, u_{i,n}) = \frac{1}{2} \frac{p(Y'_i | X'_i = -1)}{p(v, Y'_i)} E'(u_i, u_{i,n}) + \frac{1}{2} \frac{p(Y'_i | X'_i = 1)}{p(v, Y'_i)} E''(u_i, u_{i,n})
\]
\[
= \frac{1}{\sqrt{2\pi\sigma^2_i}} \left[ \text{erf} \left( \frac{u_i}{\sqrt{2\sigma^2_i}} \right) - \text{erf} \left( \frac{u_i \pm \sqrt{E_i c_i}}{\sqrt{2\sigma^2_i}} \right) \right]
\]
Similarly, the first term of the distortion \(D_1\) can be expressed as
\[
E(Y''_i | v, Y'_i) = \frac{1}{2} \frac{p(Y'_i | X'_i = -1)}{p(v, Y'_i)} E^{2\prime}(u_i, u_{i,n}) + \frac{1}{2} \frac{p(Y'_i | X'_i = 1)}{p(v, Y'_i)} E^{2\prime}(u_i, u_{i,n})
\]
where \(E^{2\prime}_1\) and \(E^{2\prime}_2\) are expectation of \(\{Y'_i\}_y\) given \(X'_i\) is +1 and -1 respectively,
The distortion $D_i$ is expressed as a function of $u_i$, $u_{i-1}$, and $Y_j$ and should be averaged over all possible $Y_j$ as

$$D_i(u_i, u_{i-1}) = \int \left[ \int_E [E[y_i | Y_j = v, Y_j = v'] - E[y_i | Y_j = v] dY_j] p(Y_j | v) dY_j \right] \frac{P(Y_j | v)}{p(v)} dY_j$$

where $P(v)$ is the probability that the index after quantization is equal to $v$, given as

$$P(v) = \sum_{x_i} p(v | x_i) p(x_i)$$

and

$$= \frac{1}{2} \int \frac{1}{\sqrt{2\pi \sigma_i^2}} \exp \left( -\frac{Y_j^2 + \sqrt{E_i} c_i}{2\sigma_i^2} \right) dY_j + \frac{1}{2} \int \frac{1}{\sqrt{2\pi \sigma_i^2}} \exp \left( -\frac{Y_j^2 - \sqrt{E_i} c_i}{2\sigma_i^2} \right) dY_j$$

$$= \frac{1}{4} \left[ \text{erf} \left( \frac{u_i + \sqrt{E_i} c_i}{\sqrt{2\sigma_i^2}} \right) - \text{erf} \left( \frac{u_i - \sqrt{E_i} c_i}{\sqrt{2\sigma_i^2}} \right) \right]$$

The distortion is then averaged over all possible $v$

$$D = \sum_{i=1}^{L} P(v) \cdot D_i(u_i, u_{i-1})$$

$H(v | Y_j)$ can be expressed as

$$R_{\text{aw}} = H(v | Y_j) = -\sum_{i=1}^{L} \int \mathcal{P}(v, Y_j) \log \mathcal{P}(v | Y_j) dY_j$$

2-bits partition and 3-bits partition for achievable rate based quantizer.
### TABLE D.1

$L=4$ quantizer

<table>
<thead>
<tr>
<th>$\gamma_0$ (dB)</th>
<th>RDS $\lambda=0.5$</th>
<th>RDS $\lambda=1$</th>
<th>ARBQS</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>$u_2=-4.66$</td>
<td>$u_2=-1.46$</td>
<td>$u_2=-0.91$</td>
</tr>
<tr>
<td>-1.5</td>
<td>$u_2=-4.45$</td>
<td>$u_2=-1.41$</td>
<td>$u_2=-0.87$</td>
</tr>
<tr>
<td>-1</td>
<td>$u_2=-4.26$</td>
<td>$u_2=-1.35$</td>
<td>$u_2=-0.78$</td>
</tr>
<tr>
<td>-0.5</td>
<td>$u_2=-4.08$</td>
<td>$u_2=-1.29$</td>
<td>$u_2=-0.75$</td>
</tr>
<tr>
<td>0</td>
<td>$u_2=-3.90$</td>
<td>$u_2=-1.23$</td>
<td>$u_2=-0.72$</td>
</tr>
<tr>
<td>0.5</td>
<td>$u_2=-3.74$</td>
<td>$u_2=-1.18$</td>
<td>$u_2=-0.69$</td>
</tr>
</tbody>
</table>

### TABLE D.2

$L=8$ quantizer

<table>
<thead>
<tr>
<th>$\gamma_0$ (dB)</th>
<th>ARBQS</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>$[u_2, u_3, u_4] = [-1.53, -0.90, -0.42]$</td>
</tr>
<tr>
<td>-1.5</td>
<td>$[u_2, u_3, u_4] = [-1.46, -0.86, -0.40]$</td>
</tr>
<tr>
<td>-1</td>
<td>$[u_2, u_3, u_4] = [-1.39, -0.83, -0.38]$</td>
</tr>
<tr>
<td>-0.5</td>
<td>$[u_2, u_3, u_4] = [-1.33, -0.79, -0.36]$</td>
</tr>
<tr>
<td>0</td>
<td>$[u_2, u_3, u_4] = [-1.25, -0.73, -0.34]$</td>
</tr>
</tbody>
</table>
Appendix E

The detailed BP algorithm for the soft SW decoding is presented in this Appendix. Before we describe the algorithm, some general definitions are given below:

- \( c_i \): variable nodes of the LDPC codes
- \( f_j \): check nodes of the LDPC codes
- \( M_v \) = \{messages from all variable nodes\}
- \( M_c \) = \{messages from all check nodes\}
- \( M_v(\sim i) \) = \{messages from all variable nodes except node \( c_i \)\}
- \( M_c(\sim j) \) = \{messages from all check nodes except node \( f_j \)\}
- \( P_i = \Pr(c_i = 1 | \text{channel sample } i) \)
- \( S_i \) indicates the events that all the check equations involving \( c_i \) are satisfied
- \( q_i(b) = \Pr(c_i = b | S_i, \text{channel sample } i, M_v(\sim i)) \)
- \( r_j(b) = \Pr(\text{check equation } f_j \text{ is satisfied} | c_i = b, M_v(\sim i)) \)
- \( P_j(b) = \Pr(\text{check node } f_j = b) \)
- \( Q_i(b) = \Pr(c_i = b | S_i, \text{channel sample } i, M_c) \)
- \( F_j(b) = \Pr(\text{check equation } f_j \text{ is satisfied} | c_i = b, P_j(b), M_c) \)
- \( L \), \( L(r) = \log \left( \frac{P_r(0)}{P_r(1)} \right) \), \( L(q) = \log \left( \frac{q(0)}{q(1)} \right) \)
- \( L(F) = \log \left( \frac{F(0)}{F(1)} \right) \), \( L(Q) = \log \left( \frac{Q(0)}{Q(1)} \right) \)
- \( L(q_i) \) is the initial values of \( L(q_i) \).

**Gallager theorem:** Consider a sequence of \( M \) independent binary digits \( c_i \) for which \( \Pr(c_i = 1) = p_c \). Then the probability that a binary sequence with \( M \) elements contains an even number of 1’s is

\[
\frac{1}{2} + \frac{1}{2} \prod_{i=1}^{M} (1 - 2p_c)
\]

In view of this theorem, if check node \( j \) is equal to 0, the probability of the check equations \( f_j \) being satisfied is \( 0.5 + 0.5 \prod_{i \neq j} (1 - 2q_{r_i}(1)) \); otherwise, if the check node \( j \) is equal to 1, the probability is \( 0.5 - 0.5 \prod_{i \neq j} (1 - 2q_{r_i}(1)) \). Hence, we have
It follows that

\[ 1 - 2r_p(t) = (P_g(0) - P_g(1)) \prod_{t \in \Omega}(1 - 2q_{r,j}(t)) \]

The above equation can be expressed in LLR formation as

\[
\tanh\left(\frac{1}{2}L(r_p)\right) = \tanh\left(\frac{1}{2}L_q\right) \prod_{t \in \Omega} \tanh\left(\frac{1}{2}L(q_{r,j})\right)
\]

Let

\[ L(q_q) = \alpha_{ij} \beta_{ij}, \quad L_j = \gamma_j \nu_j, \quad \alpha_{ij} = \text{sign}[L(q_q)], \quad \beta_{ij} = |L(q_q)|, \quad \gamma_j = \text{sign}[L_j], \quad \nu_j = |L_j| \]

\[ L(r_p) \] can be updated as

\[
L(r_p) = \gamma_j \prod_{t \in \Omega} \alpha_{ij} \cdot 2 \tanh^{-1}\left(\tanh\left(\frac{1}{2} \nu_j\right) \prod_{t \in \Omega} \tanh\left(\frac{1}{2} \beta_{ij}\right)\right)
\]

\[ = \gamma_j \prod_{t \in \Omega} \alpha_{ij} \cdot 2 \tanh^{-1}\left[\log\left(\tanh\left(\frac{1}{2} \nu_j\right)\right) + \sum_{t \in \Omega} \log\left(\tanh\left(\frac{1}{2} \beta_{ij}\right)\right)\right] \]

\[ = \gamma_j \prod_{t \in \Omega} \alpha_{ij} \cdot \varphi(\nu_j) + \sum_{t \in \Omega} \varphi(\beta_{ij}) \]

where we define

\[ \varphi(x) = -\log\left(\tanh\left(x/2\right)\right) = \log\left(\frac{e^x + 1}{e^x - 1}\right) \]

End of the algorithm.

The initial LLR values of \( L(q_{\text{init}}) \) can be expressed as
$L(v_{n,a}) = \log \frac{Pr(Y_2^1(n) = y_2^1(n) | v_{n,a} = 0)}{Pr(Y_2^1(n) = y_2^1(n) | v_{n,a} = 1)} = \log \frac{Pr(Y_2^1(n) = y_2^1(n), v_{n,a} = 0)}{Pr(Y_2^1(n) = y_2^1(n), v_{n,a} = 1)} - \log \frac{Pr(v_{n,a} = 0)}{Pr(v_{n,a} = 1)}$

$= \log \frac{\sum_{Y_2^1(n) \in \{0,1\}} Pr(Y_2^1(n) = y_2^1(n) | v_{n,a} = 0) Pr(v_{n,a} = 0 | Y_2^1(n))}{\sum_{Y_2^1(n) \in \{0,1\}} Pr(Y_2^1(n) = y_2^1(n) | v_{n,a} = 1) Pr(v_{n,a} = 1 | Y_2^1(n))}

= \log \frac{\sum_{v_{n,a}, x_{1} \in \{0,1\}} \prod_{X_{1}^1(n) \in \{0,1\}} \sum_{x_{1}^1(n) \in \{0,1\}} Pr_{\text{in}}^{|X_{1}^1(n)|} \prod_{i=1}^{n-1} Pr_{\text{in}}^{-i}}{\sum_{v_{n,a}, x_{1} \in \{0,1\}} \prod_{X_{1}^1(n) \in \{0,1\}} \sum_{x_{1}^1(n) \in \{0,1\}} Pr_{\text{in}}^{|X_{1}^1(n)|} \prod_{i=1}^{n-1} Pr_{\text{in}}^{-i}}

= \log \frac{\prod_{i=1}^{n-1} Pr_{\text{in}}^{-i}}{\prod_{i=1}^{n-1} Pr_{\text{in}}^{-i}}

(E. 4)

with definitions

$Pr_{\text{in}}^{|X_{1}^1(n)|} = Pr(Y_2^1(n) = y_2^1(n) | X_{1}^1(n))$

$Pr_{\text{in}}^{|X_{1}^1(n)|} = Pr(v_{n,a} = 0 | v_{n-a}, ..., v_{n-1-a}, X_{1}^1(n))$

$Pr_{\text{in}}^{|X_{1}^1(n)|} = Pr(v_{n,a} = 1 | v_{n-a}, ..., v_{n-1-a}, X_{1}^1(n))$

$Pr_{\text{in}}^{|X_{1}^1(n)|} = Pr(v_{n,a} = 0 | v_{n-a}, ..., v_{n-1-a}, X_{1}^1(n))$

where $= \ $ is due to the fact that

$Pr(v_{n,a} = 0, v_{n-1,a}, ..., v_{n,a} | X_{1}^1(n)) = Pr(v_{n,a} = 0 | v_{n-a}, ..., v_{n-1-a}, X_{1}^1(n)) Pr(v_{n-a}, ..., v_{n-1-a} | X_{1}^1(n))$

$= Pr(v_{n,a} = 0 | v_{n-a}, ..., v_{n-1-a}, X_{1}^1(n)) Pr(v_{n-a}) Pr(v_{n-a}, ..., v_{n-1-a} | X_{1}^1(n)) Pr(v_{n-a}, ..., v_{n-1-a} | X_{1}^1(n))$

$= Pr_{\text{in}}^{|X_{1}^1(n)|} \prod_{i=1}^{n-1} Pr_{\text{in}}^{-i}$
APPENDIX F

Proof of Proposition 6.1:

If \( n=1 \), this problem is simplified as a normal CF case. According to [49], we define an auxiliary random variable as 
\[ W_i = Y_i + Z_i \], where \( Z_i \) is a zeros mean circularly symmetric complex Gaussian distributed random variable with variance \( \bar{\sigma}_i^2 \) and independent with \( Y_i \) and \( Y_{j,i} \). The estimation of \( Y_{j,i} \) can be reconstructed as \( \bar{Y}_{j,i} = W_i + \delta Y_{j,i} \), where \( \delta \) is a constant. The mutual information conveyed with help of the relay can be expressed as

\[
I(X_{i,i};\bar{Y}_{j,i},Y_{j,i}) = \log \left( 1 + \gamma_{i,i} + \gamma_{j,i} / \left( 1 + \bar{\sigma}_i^2 \right) \right)
\]

(F. 1)

Notice that in above equations, \( \delta \) does not appear. We are free to set \( \delta \) as zero in the following derivations. If \( n > 1 \), we can define a sequence of auxiliary random variables in a similar way as,

\[
W = Y + Z
\]

where \( W = [W_1, \ldots, W_{n-1}]^T \), \( Y = [Y_{1,1}, \ldots, Y_{i,1}]^T \), and \( Z = [Z_1, \ldots, Z_{n-1}]^T \). With side information \( Y_{j,i} \), the destination will generate \( n \) versions of estimation for \( Y_{j,i} \). Define \( Y_d = [\hat{Y}_{d,1}, \hat{Y}_{d,1}[1], \ldots, \hat{Y}_{d,1}[n]]^T \), where \( \hat{Y}_{d,1}[i] \) is the \( i \)th estimation of \( Y_{j,i} \). \( Y_d \) can be expressed in the form of

\[
\begin{bmatrix}
Y_{d,1} \\
\cdot \\
Y_{d,1}[n]
\end{bmatrix} = \begin{bmatrix}
\begin{bmatrix}
\sqrt{\mu_0} & 1 & 0 & 0 & \cdots & 0 & 0
\end{bmatrix}^T \\
\cdot \\
\begin{bmatrix}
\sqrt{\mu_i} & 0 & 1 & 1 & 0 & \cdots & 0
\end{bmatrix}^T
\end{bmatrix} \begin{bmatrix}
Y_{1,i} \\
X_{2,i} \\
\vdots \\
X_{n,i}
\end{bmatrix} + \begin{bmatrix}
\begin{bmatrix}
\sqrt{\mu_0} & 0 & 1 & 0 & \cdots & 0 & 0
\end{bmatrix}^T \\
\cdot \\
\begin{bmatrix}
\sqrt{\mu_i} & 0 & 1 & 0 & \cdots & 0 & 0
\end{bmatrix}^T
\end{bmatrix} X_{2,i} + \begin{bmatrix}
Z_{d,1} \\
\cdot \\
Z_{d,1}[n]
\end{bmatrix}
\]

(F. 2)

Borrowing from results in [5] and [11], the mutual information \( I(X_{i,i},Y_d) \) is

\[
I(X_{i,i},Y_d) = \log \det \left( I_{n,n} + (CC^\ast)(BR_nB_n^\ast)^{-1} \right)
\]

(F. 3)

The variance \( \bar{\sigma}_i^2 \) can be determined according to [4] as

151
Appendix F

\[
R_{a,i}/\alpha \geq I(Y_{a,i}';W_i) - I(Y_{a,i};W_i) \\
= \log \left( 1 + 1/\delta_{a,i}^2 + \gamma_{a,i} / ((1 + \gamma_{a,i}) \delta_{a,i}^2) \right) \\
= \log \left( 1 + \bar{r}_{a,i} + \bar{r}_{a,i} \gamma_{a,i} / (1 + \gamma_{a,i}) \right)
\]

It follows that \( \check{\gamma}_{a,i} = g(\alpha, \gamma_{a,i}, \gamma_{a,i}, R_{a,i}) \). The proof is completed.

Proof of Proposition 6.2

The relay supported transmission rate is

\[
R_a = (1 - \alpha) \log \left( 1 + \gamma_a / (1 + \gamma_a) \right) 
\]

(F.4)

If \( \gamma_0 \) and \( \gamma_2 \) are in the region \([A, B]\) and \([C, +\infty)\) respectively, we can carefully choose the transmission rate of the relay as

\[
R_r = (1 - \alpha) \log \left( 1 + C / (1 + B) \right) \leq R_a
\]

(F.5)

This selection will keep the link between the relay and the destination out of the outage area. The ideal transmission rate of the compressed signal is given by \([49]\),

\[
R_c = \alpha \left( I(Y'_r;W) - I(Y'_r;X_r) \right) \\
= \alpha \log \left( 1 + \frac{\gamma_{r,c} + \gamma_{r,c} \gamma_r}{1 + \gamma_r} \right)
\]

(F.6)

where \( W \) is an auxiliary random variable. The real transmission rate of the compressed signal is selected as

\[
R_{r,c} = \alpha \log \left( 1 + \gamma_{r,c} + \gamma_{r,c} \gamma_r / (1 + \gamma_r) \right) \geq R_c
\]

(F.7)

This selection will ensure the observation of the relay can be recovered with small distortion. Let \( R_r \geq R_{r,c} \), we can keep \( R_d \geq R_c \). According to \([4]\), the mutual information at the destination can be expressed as

\[
I^{cr}(Y) = \alpha I(X';Y'_r, Y'_r) + (1 - \alpha) I(X';Y'_r | X'_r) \\
= \alpha \log \left( 1 + \gamma_{r,c} + \gamma_{r,c} \gamma_r / (1 + \gamma_{r,c}) \right) + (1 - \alpha) \log (1 + \gamma_{r,c})
\]

152
which is a monotonically increasing function of \( \gamma_t \). Let \( R_t = R, \gamma_t \) can be maximized as

\[
\gamma_t = h(\alpha, A, B, C, \gamma_t) = \frac{(1 + C(1 + B))^{(n-a)/A} - 1}{1 + \gamma_t/A}
\]

We can deploy CF and recover the observation with small distortion.

End of proof.

**Claim 1**: Supposing that after \( n \) frames, there are two routes \( r_a \) and \( r_b \) with different frame type sequence \( a \) and \( b \) that belong to the same set \( T(n, n) \), where \( n_t = m_t(a) = m_t(b) \), the accumulated mutual information through two routes and the probabilities of the two routes are the same.

**Proof**: We can assume that position sets for \( r_a \) are

\[
\begin{align*}
\chi_a = \chi(a; 1) &= \{x_a(1), \ldots, x_a(n_t)\}, \\
\chi_a = \chi(a; 2) &= \{x_a(1), \ldots, x_a(n - n_t)\}
\end{align*}
\]

and for \( r_b \),

\[
\begin{align*}
\chi_b = \chi(b; 1) &= \{x_b(1), \ldots, x_b(n_t)\}, \\
\chi_b = \chi(b; 2) &= \{x_b(1), \ldots, x_b(n - n_t)\}
\end{align*}
\]

The probability of the routes \( r_a \) is given as

\[
\begin{align*}
Pr(r_a) &= Pr \left( I_a(y; a) < R | I_a(y; a) < R, \chi_a, \bar{\chi}_a \right) \\
&= E \left[ I_a(y; a) < R | I_a(y; a) < R \right] | \chi_a, \bar{\chi}_a \right)
\end{align*}
\]

(F. 8)

where \([\cdot]\) with equalities or inequalities inside represents the indication function whose values is 1 when the conditions inside are satisfied and 0 otherwise, \( E \{ f(x) | e \} \) is the conditional expectation with respect to \( x \) given event \( e \) and

\[
I_a(y; a) = \sum_{i=1}^{n} I_{a,x_i}(y) + \sum_{i=n+1}^{n-k} I_{a,x_i}(y), \quad I_b(y; a) = \sum_{i=1}^{n} I_{b,x_i}(y)
\]

Let \( v_{a,x_i(k)} = y_{a,x_i(k)}, k = 0, 1, 2, \ 1 \leq i \leq n_t \) and \( v_{b,x_i(k)} = y_{b,x_i(k)}, k = 0, 1, 2, \ 1 \leq j \leq n - n_t \), with proper variables substitution, we have.

153
\[ I_d (y; z) = \sum_{n=1}^{N} I_{mn}^{cr} \left( \left( y_{n,z}(t), y_{n,z}(t), y_{n,z}(t) \right) \right) + \sum_{n=1}^{N} I_{nc} \left( y_{n,z}(t) \right) \]
\[ = \sum_{n=1}^{N} I_{mn}^{cr} \left( \left( y_{n,z}(t), y_{n,z}(t), y_{n,z}(t) \right) \right) + \sum_{n=1}^{N} I_{nc} \left( y_{n,z}(t) \right) = I_d (y; z) \]  

(F. 9)

It follows that

\[ \Pr (r) = E \left\{ I_d (y; z) < R, I, (y; z) < R \right\} | x, \bar{x} \}
\[ = E \left\{ I_d (y; z) < R, I, (y; z) < R \right\} | x, \bar{x} \} = \Pr (r) \]  

(F. 10)

End of proof.
References


156


158


159


161


