ENTRAINMENT, MIXING AND MASS TRANSFER
STUDIES ON SIEVE TRAYS WITH 1-INCH
DIAMETER PERFORATIONS.

A thesis submitted to the University of Surrey for
the Degree of Doctor of Philosophy in the Faculty
of Engineering

by

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SUMMARY

Performance studies on sieve trays with 1 inch diameter perforations are reported. Few published studies exist on such large hole sizes as compared with other sizes such as 3/8 and 3/16 inch diameter holes. Two columns, one rectangular (3 ft x 1 ft) and the other round (32 inch diameter) were examined in a large pilot rig.

The hydrodynamics of the columns have been investigated with respect to tray pressure drops, aeration and froth properties, and entrainment of liquid by vapour on the trays. Dry tray pressure drop studies have been used extensively to examine the validity of some of the commonly employed published correlations and to determine their applicability to large hole sizes.

An extensive study of entrainment has been carried out to evaluate the effects of tray spacing and operating variables. The results have been successfully correlated by equations previously established by Kister et al. This is important as no results appear to be available for such large hole sizes.

The mass transfer efficiency of the absorption system, air-CO₂-water has been reported with a critical examination of sampling technique and method of analysis. These are shown to critically affect the evaluation of solute concentration in solution.

The degree of liquid mixing has been investigated by a tracer injection technique. Liquid residence time and mixing
Parameters as summarized by the Peclet number and eddy diffusivity have been obtained and their significance and limitations discussed.

Values of the mass transfer coefficient, $k_{La}$ obtained from the efficiency studies varied between 0.091 and 0.147 sec$^{-1}$ for the round column and 0.139 and 0.25 sec$^{-1}$ for the rectangular column, over the range of operating variables investigated.

The studies establish more clearly the significance of large diameter perforations in the context of tray performance and fill a gap in the knowledge of the performance of such hole sizes.

Results have been correlated wherever possible in a form suitable for design purposes.
ACKNOWLEDGEMENTS

I wish to thank my supervisor Dr. W. J. Thomas, Reader in Chemical Engineering, University of Surrey, for his guidance, inspiration and invaluable encouragement through the duration of this research.

The departmental technical staff were most helpful in the building of the extensive research apparatus, and instrumentation.

Special thanks to my wife, Bunmi and little Lola for holding the fort and believing.

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CHAPTER ONE

GENERAL CONSIDERATIONS

1.1 Basic Design Calculations for Bubble Tray Columns

The performance of gas-liquid contacting devices for absorption and distillation processes has been and still is an important aspect of research and development in the chemical industry.

Many types of gas-liquid contacting devices are employed in chemical and physical processes; of these the most popular are various modifications of the packed tower and the bubble tray column. The study of these columns reduces primarily to the understanding of the basic principles of mass transfer between phases by diffusion, the rate or the capacity of the transfer process and the performance of the different types of contacting equipment. Impetus for such studies stems largely from the design engineer's need to know the performance profiles of these devices.

The most frequently used gas-liquid contacting device for distillation and absorption processes, is the multiple tray or plate column. Its preference over the packed tower is its ability to provide positive and intimate contact between the two streams, avoiding some of the problems associated with bypassing and back-mixing. All the gas is thoroughly dispersed through all the liquid which is then discharged, collected, mixed and redistributed on the next tray below. This more positive control of countercurrent
action plus the more certain performance give the plate column an advantage over the packed tower.

Bubble cap trays were almost exclusively used up till 1950 in preference to sieve trays, the first documented use of which has been credited to Cellier in 1818 (Forbes, 1948) and Coffey in 1832 (Perry, 1950). The earlier hesitation in developing and expanding the use of sieve tray columns has been attributed to their lack of stability over a wide range of operating conditions, but in recent years as a result of improved design methods and materials of construction leading to good fluid-handling capacity because of the absence of obstructions to the flow of fluids, together with design for particular capacity and design duties, it is generally recognised that the advantages offered by sieve tray columns far outweigh their disadvantages in use. Economic considerations, in particular the lower cost of tray fabrication and installation, also favour the preference of sieve trays over conventional bubble-cap tray assemblies.

The successful design and operation of sieve-tray columns depend to a large extent on the related parametric factors of fluid-handling capacity, tray geometric factors, hydrodynamics, e.g. frictional pressure drop, liquid hold-up, froth height, flooding and entrainment, type and degree of mixing, and most important of all, the contacting efficiency. The design engineer thus has a problem of predicting performance profile data for these trays taking into consideration the system, flow and device parameters listed above.
Considerable progress has been made in recent years into the understanding and solving of problems associated with the performance of sieve-tray columns resulting in the proposal of a variety of empirical and quasi-empirical correlations. Perhaps the most extensive single effort in this enterprise has been the A.I.Ch.E. research programmes that were carried out at the Universities of Michigan, Delaware and North Carolina, culminating in the A.I.Ch.E. Bubble Tray Design Manual. The research was highly commendable but paradoxically unknown to the original researchers; the work was found to be deficient in more ways than one by many investigators. Ironically, the masses of correlations proposed in subsequent studies by various other workers have shown a general inconsistency when comprehensively tested. The state of the art was aptly summarised by H.E. Eduljee (1966) who said, "The various authors appear to have based their equations on their own works, and not to have taken into account the works of others. The claims made about their accuracy of prediction seldom meets with a wider test". Also, most of these studies have concentrated efforts on performance characteristics of small-diameter perforation columns, largely ignoring large-diameter perforation columns for which there is now extensive use in the process industry. While the correlations proposed in the A.I.Ch.E. Bubble Tray Manual have been generally accepted to work more satisfactorily when applied to bubble-cap trays than sieve trays, so also it seems, from the small number of studies so far carried out, that correlations obtained in studies on small-diameter columns are not necessarily valid when applied to
There thus appears a yawning gap in the understanding of the performance characteristics of large-perforation columns which calls for a need for more investigations into the operating and design variables which affect these columns.

1.2 General Concepts of Hydrodynamics of Sieve-Tray Columns

A satisfactory tray design should satisfy both economic and design requirements. To be most economical, the tray design should result in the smallest diameter tower to meet the maximum design rates and the highest efficiency at the maximum and the minimum design rates. The pressure drop should be as low as practicable to meet design requirements and provide a suitable efficiency.

Thus the design engineer takes the following factors into consideration in designing a bubble tray column:

1. the provision of a high degree of contact between the vapour and the liquid phases allowing equilibrium between the phases to be approximated with a high degree of accuracy;

2. provision of an adequate liquid flowpath on the plates and in the downcomers such that the liquid-handling capacity of the column is within required limits;

3. minimum of pressure drop throughout the column while providing the necessary contact between the phases.
Generally, tray columns are classified according to the mode of liquid flow in relation to the vapour flow, either as (a) crossflow or (b) countercurrent flow. For the majority of plate columns, the flow of vapour and liquid is countercurrent but the nature of fluid contact on each plate is best described as "crossflow". Fig. 1.1 is a schematic diagram of such a plate.

The sieve-tray column is usually a round crossflow contactor with a large number of small round perforations on each tray. The liquid is introduced at one side through a downflow pipe or a downcomer, flows over the inlet weir, IW, and over an unperforated inlet calming section, A, onto the perforated zone or the active tray length, BC, of the tray. Over this active length, interaction of the vapour and liquid occurs, with the net liquid movement being in crossflow to the ascending vapour stream. This interaction is non-uniform; the vapour erupts through the liquid intermittently and randomly forming a transient surging froth. The froth provides a high interfacial area and the turbulence essential for efficient vapour-liquid interaction. For design purposes, however, it is assumed for simplification purposes that there is a steady uniform flow of vapour through the perforations.

The froth zone can be observed through windows in the columns, it occupies nearly all of the tray, and in it most of the mass transfer takes place. Its observable height above the tray floor is designated as $Z_f$, although its upper limit is not easily defined as it varies across the tray as is shown in Fig. 1.1. The frothing mass has been observed
Fig. 1.1 Sieve Tray Dynamics Schematic Diagram
and shown by many workers to change its character from liquid continuous to vapour continuous ("spray") depending on the flow regime in which the tray is being operated.

Liquid descends by gravity from the tray above, over an exit calming section over which some of the aerated mass collapses, and the exit weir, through a downcomer onto the lower tray. The liquid level on the tray or the effective hydrostatic head, $Z_d$, is determined by the height of this exit weir. The height of the liquid crest over the weir is designated as $h_{ow}$.

Downcomers are provided to conduct the flow of liquid between trays, and are designed to provide adequate liquid-handling capacity for the column, while at the same time occupying a minimum of tray cross-sectional area, so that the active area of the tray will be a maximum. Downcomers may be circular or segmental in shape, single or multiple, and offer cross, reverse, doublepass cascade or four pass flow patterns. Generally, segmental crossflow downcomers which provide maximum utilisation of column area and therefore greatest downflow capacity are most commonly used. The type of flow in downcomers has long been a moot point. While some investigators believe that the pattern is mainly by plug flow, others believe in the liquid circulation theory. The former believe that vapour disengagement occurs and that liquid circulation results in vapour being carried onto the tray below with a subsequent lowering of efficiency and an increase in pressure drop; the latter school of thought believes that practically all the entrained air is carried
to the tray below with only a minimal effect on column efficiency or pressure drop.

Hole size and arrangement of holes depend on the service for which the column is intended and the designer's preference, but can range from 1/16 to over 1 inch in diameter on a triangular or square pitch arrangement.

Because of the diversity of definitions encountered in published literature, it is necessary to define certain terms that would be used in subsequent sections:

1. Active Area: the area containing the perforations and enclosed by the tower walls, the outer weir and the inlet edge of the tray. It is usually equal to the tower area less the sum of the area of the downcomer and the downcomer apron.

2. Tower Area: the cross-sectional area of the column.

3. Hole Area: the total perforated area open to vapour flow.

4. Net Area for Vapour Flow: this is equal to the tower cross-sectional area less the area of one downcomer.

5. Free area: same as above in (4).

6. Area of Active Holes: total perforated area less the area occupied by the tray calming sections.

7. Fraction Hole Area or Free Area: the ratio of the area of active holes to the plate active area. The percent hole or free area is the fraction hole area times 100.
Certain flow conditions result in tray inoperability. The limits of operability for plate column are imposed by flow and design characteristics. Chief of these conditions are flooding and entrainment.

1.2.1 Flooding and Entrainment

Column flooding begins when the vapour and liquid flows become greater than the equipment capacity.

Hausch (1964) states that column flooding can be caused by any one of the following:-

(a) excessive entrainment of liquid
(b) froth build-up and overloading of downcomers
(c) obstruction to vapour flow in the perforations
(d) obstruction in the downcomers
(e) poor design of the feed inlet system
(f) poor design of the bottom tray

It is generally believed that there are three basic mechanisms which may cause a column to flood:-

(a) systems limitations
(b) liquid back-up in downcomer
(c) jetting or excessive entrainment

Each system has a limiting capacity which cannot be exceeded by changing the tray design or by increasing the tray spacing. This limiting capacity is mainly associated with
the vapour/liquid spray interaction within the intertray spacing, but to some extent depends on the tray design. The phenomenon occurs when there is a substantial net upward flow of liquid relative to the total liquid flow and is a function of the terminal velocity of the liquid drops in the intertray space.

Fair (1958, 1961) believes that flooding may be brought on either by excessive entrainment or by liquid backup in the downcomer. Liquid back-up in the downcomer occurs when the downcomer fills with aerated liquid which cannot be accommodated by the downcomer and the liquid then accumulates in the plate active tray region. Entrainment or jetting is the projection of massive quantities of liquid to the tray above by the eruption of vapour through the frothing mass on the tray below.

Souders and Brown (1934) have defined entrainment as the quantity of liquid which is carried with the gas from a plate up to and on to the plate above per unit time. The mechanism of entrainment generation is a consequence of the complex mechanics of bubble formation and disintegration caused by the dynamic action of the vapour on the tray. Extensive studies have been conducted with regards to the mechanism of entrainment on sieve trays.

Newitt et al. (1954) utilising high-speed photographs of bubbles bursting at an air-liquid interface on a single orifice found that entrainment from a single bubble formation was the result of the disintegration of the bubble at the
surface, followed by the development of a vertical jet flume caused by the rush of liquid into the cavity produced by the disintegration.

Spells and Bakowski (1950, 1952) had earlier used the same technique to evaluate the mechanism of bubble formation in single and multiple perforations. They observed two mechanisms:

1. a "deep mechanism" in which discrete bubbles are formed;
2. a "shallow mechanism" in which channelling results in a continual passage of vapour without the formation of bubbles.

The effect of entrainment is detrimental to the output and efficiency of absorbers and fractionating columns and has long been recognised as one of the chief limiting factors governing the capacity of such columns. Entrainment counteracts the mass transfer process, lowering the plate efficiency and contaminating the product.

1.3 Principles of Mass Transfer in Sieve Tray Columns

In gas-liquid contacting devices, mass transfer problems are centred around the important steps of reaction and separation.

Any study of the absorption process requires an accurate knowledge of the gas-liquid equilibrium relationships of the phases involved. Such data are available for many of the important gas-liquid systems and may be found in the literature and the International Critical Tables. Nomographs rela-
ting equilibrium concentrations in the gas and liquid phases with temperature exist for a number of the more important cases.

The equilibrium relationship provides a means of evaluating the real driving force and hence the overall mass transfer rate.

Gas absorption may be defined as the removal of one or more selected components by absorption into a suitable liquid. Gas absorption may be either physical, i.e. no chemical reaction between the phases or, chemical, in which the gases often enter into a chemical reaction with the absorbing liquids.

1.3.1 Mass Transfer Theories

Several theories have been proposed for mass transfer rates in gas absorption. Classically, the Lewis-Whitman (1924) stagnant film model has been used to develop correlations of phase transfer rates. This model, which has been a great aid in visualising the process of interphase mass transfer between two fluids, assumes the existence of a stagnant interfacial boundary layer (film) between the liquid phase and the gas phase, through which the dissolved molecules, in turbulent motion, can only pass by means of molecular diffusion. The total resistance to mass transfer is assumed to reside in the films. This theory is unrealistic and is by no means truly representative of the transfer mechanism occurring in absorption as it could not be serious-
ly contemplated that such a discontinuity really exists.

Other theories have been advanced. These include the different variations of the penetration or surface-renewal model. These models suggest that the interface is continuously being impinged on by eddies and that in these eddies mass transfer is controlled by unsteady molecular diffusion. The model originally proposed by Higbie (1935) assumes that every element is exposed to the gas for the same length of time, during which absorption takes place by unsteady state molecular diffusion into the liquid before the surface element is mixed with the bulk again. This model is also unrealistic because of the assumption of equal contact time. A variation of this model later proposed by Danckwerths (1951) supposes that the probability of exchange of an element of surface with fresh liquid is independent of the length of time the element has been in contact with the gas.

A summary of the three models is shown below:

Two film: \[ K_v = \frac{D_v}{\delta_v} \] (1.1)

Penetration: \[ K_L = 2 \left( \frac{D_L}{\pi \theta'_L} \right)^{0.5}, \quad K_v = 2 \left( \frac{D_v}{\pi \theta'_v} \right)^{0.5} \] (1.2)

Surface renewal: \[ K_L = (S_L D_L)^{0.5}, \quad K_v = (S_v D_v)^{0.5} \] (1.3)

where

\[ K_L, K_v = \text{liquid and vapour mass transfer coefficient respectively} \]

\[ D_L, D_v = \text{molecular liquid/vapour phase diffusivities (ft}^2/\text{sec}) \]
\[ \delta = \text{stagnant film thickness (ft)} \]
\[ \theta' = \text{renewal time (sec)} \]
\[ S = \text{fractional renewal} \]

1.3.2 Mass Transfer Equations

These equations were developed on the assumption that true equilibrium exists at the interface between the gas phase and the liquid phase. Thus, at any point in an absorption process operating continuously at "steady state" conditions, the rate of mass transfer can be represented by:

\[ N_A = k_L (C_i - C_L) \quad (1.4) \]

and

\[ N_A = k_G (P_g - P_i) \quad (1.5) \]

where

- \( N_A \) = rate of mass transfer (lb moles/hr ft²)
- \( k_L \) = liquid film coefficient (ft/hr)
- \( k_G \) = gas film coefficient (lb moles/hr ft² atm)
- \( C_i, C_L \) = concentration of solute at interface and liquid bulk respectively (lb moles/ft³)
- \( P_g, P_i \) = partial pressure of component in gas bulk and at interface respectively (atm)

To facilitate the practical use of equations (1.4) and (1.5) when interfacial conditions are unknown, two overall coefficients are defined and employed in terms of overall driving forces:

\[ N_A = K_L (C^* - C_L) \quad (1.6) \]
and

\[ N_A = K_g (P_g - p^*) \]  \hspace{1cm} (1.7)

where

\[ C^* = \text{concentration that would exist in the liquid phase if the bulk of the liquid were in equilibrium with the bulk vapour phase} \]

\[ p^* = \text{partial pressure of the diffusing component which would exist in the bulk if the vapour were in equilibrium with the bulk liquid phase} \]

1.3.3 Mass Transfer Coefficients

As stated above, overall mass transfer coefficients have been substituted for individual coefficients because of the inherent difficulties of measuring the interfacial parameters. The overall mass transfer coefficients are only used when Henry's law applies, i.e. the equilibrium relationship between the partial pressure and the concentration in the liquid is linear over the concentration range involved.

When Henry's law, \((p = HC)\) holds, \(K_G\) and \(K_L\) can be shown to be related to the film coefficients by the following equations:

\[
\frac{1}{K_G} = \frac{1}{k_G} + \frac{n}{k_L} \hspace{1cm} (1.8)
\]

\[
\frac{1}{K_L} = \frac{1}{k_L} + \frac{1}{mK_G} \hspace{1cm} (1.9)
\]

and

\[ m K_G = K_L \hspace{1cm} (1.10) \]

where
Thus, for very soluble gases, e.g. ammonia in water, where \( m \) is very small, \( K_G = k_G \). For very low solubility gases, e.g. carbon dioxide in water, \( m \) is very large and \( K_L = k_L \). These two situations give rise to expressions gas film and liquid film control respectively.

The independent determination of \( K_L(K_G) \) and \( a \), the interfacial area, is notoriously difficult in as much as the liquid condition existing on the tray cannot be reproduced independently. A device with a fixed interfacial area to enable us to measure \( K_L \) would not produce the same \( K_L \) as in the frothing mass. The measurement of \( a \), is very difficult, although it has been done. Therefore the technique of recording values for the product \( K_La \) or \( K_Ga \) has been developed.

1.4 Tray Efficiency

The resistance to mass transfer between gas and liquid on a tray is expressed in terms of a ratio of the change in gas composition to the maximum possible change. This ratio is defined as the plate efficiency.

Many different approaches have been used to define the efficiency of sieve trays. The simplest definition is that of the overall plate efficiency which suffers in that it tries to describe the behaviour of a whole column with one number, even though conditions are changing along the tray.
Efficiency is better described in terms of the actual conditions on individual trays. This may be in terms of the rates of mass and heat transfer, the interfacial area, the contact times and the extent of mixing in the phases. Each tray of course has a dependence of the efficiency on the mechanism of transfer.

The methods of predicting tray efficiency vary from the empirical to the semi-theoretical approach of the A.I.Ch.E. Bubble Tray Manual, which is based on a simplified tray model and the concept, due to Gerster (1949).

Murphree defined efficiency in terms of the vapour composition change by:

\[ E_{MV} = \frac{y_n - y_{n+1}}{y^* - y_{n+1}} \quad (1.11) \]

where \((y_n - y_{n+1})\) is the change in average composition of the vapour in passing through the tray, and \(y^*_n\) is the composition of the vapour in equilibrium with the liquid leaving the tray.

If the major part of the resistance to mass transfer is in the liquid phase, it is more convenient to express efficiency in terms of liquid compositions:

\[ E_{ML} = \frac{x_{n-1} - x_n}{x_{n-1} - x^*_n} \quad (1.12) \]

While the Murphree efficiency is the tray efficiency of interest to the designer, the efficiency that can be most
easily correlated or predicted is the overall point efficiency. This is more closely related to the resistance to interphase diffusion than any other plate efficiency. It is defined in terms of point composition as:

\[ E_{OG} = \frac{y - y_n + 1}{y^* - y_n + 1} \]  

(1.13)

where the y's are taken above and below a single position on the tray. The liquid composition is assumed constant along any vertical length. The maximum value for the point efficiency is 100%. Similarly, for the liquid phase:

\[ E_{OL} = \frac{x_n - x}{x_n - x^*} \]  

(1.14)

The relationship between the point efficiency and the Murphree plate efficiency depends on the degree of liquid mixing on the tray.

For complete mixing:

\[ E_{OG} = E_{MV} \]  

(1.15)

For no mixing:

\[ E_{MV} = \frac{L_m}{G_m} \left[ e^{m \frac{G_m}{L_m} \cdot E_{OG} - 1} \right] \]  

(1.16)

The relationship between the efficiencies based on liquid and gas compositions is:

\[ E_{MV} = \frac{E_{ML}}{E_{ML} + \lambda (1 - E_{ML})} \]  

(1.17)

where
1.4.1 Transfer Units

This concept was developed by Colburn and Chilton (1935) as an alternative method of calculating transfer operations where the concept of equilibrium stages is inconvenient.

Consider conditions existing in a differential slice of a column operating under steady state, a material balance around this differential slice of height, dZ, is given by:

\[ dN = G_m \, dy = L_m \, dx \quad (1.18) \]

where

\[ G_m, L_m = \text{gas and liquid flowrates (lb moles/hr)} \]

Noting that

\[ dN = N_A \cdot A \cdot a \, dZ \]

where

\[ A = \text{column cross-sectional area} \]
\[ a = \text{interfacial area between the two phases per unit volume of column} \]

Then

\[ N_A \cdot A \cdot a \, dZ = G_m \, dy = L_m \, dx \quad (1.19) \]

but

\[ N_A = K_G \cdot P (y - y_i) = K_L \cdot C_T (x_i - x) \quad (1.20) \]
Combining equations (1.19) and (1.20) for the gas and liquid phases respectively, and noting that $y$ and $x$ decrease as $Z$ increases,

\[
\frac{-K_G a P A}{G_M} = \frac{dy/dZ}{(y-y_i)}
\]

(1.21)

and

\[
\frac{-K_L a C_T A}{L_M} = \frac{dx/dZ}{(x_i-x)}
\]

(1.22)

where

\(G_M', L_M'\) = gas and liquid flowrates respectively

(1 lb moles/hr ft²)

Integrating over the entire length of gas travel between $y_n$ and $y_{n-1}$ and $x_n$ and $x_{n-1}$ respectively and rearranging,

\[
\frac{K_G a P Z}{G_M'} = - \ln \left[ 1 - \frac{y_n - y_{n-1}}{y_n^* - y_{n-1}} \right]
\]

(1.23)

and

\[
\frac{K_L a C_T Z}{L_M'} = - \ln \left[ 1 - \frac{x_{n-1} - x_n}{x_{n-1}^* - x_n^*} \right]
\]

(1.24)

The expression on the right-hand side of both equations (1.23) and (1.24) is simply a transfer requirement the column must achieve and therefore a measure of its efficiency. The quantity on the left-hand side defines the difficulty of the transfer operation.

The reciprocals of the quantities $K_G a P/G_M'$ and $K_L a C_T/L_M'$ are known as the height of transfer unit, $H$.

\[
H_G = \frac{G_M'}{K_G a P}
\]

(1.25)
and

$$H_L = \frac{L_M'}{K_L a C_T}$$  \hspace{1cm} (1.26)

Rearranging the expressions on the left-hand side of equations (1.23) and (1.24) respectively gives:

$$\frac{K_G a P Z}{G_M'} = \frac{Z}{G_M'/K_G a P} = \ln (1 - E_M)$$  \hspace{1cm} (1.27)

and

$$\frac{K_L a C_T Z}{L_M'} = \frac{Z}{L_M'/K_L a C_T} = - \ln (1 - E_M)$$  \hspace{1cm} (1.28)

The left-hand side expressions in equations (1.27) and (1.28), \(Z/G_M'/K_G a P\) and \(Z/L_M'/K_L a C_T\), represent the height of the column divided by the height of transfer unit and are known as the "number of transfer units", \(N\).

Thus,

$$N_G = \frac{Z}{G_M'/K_G a P}$$  \hspace{1cm} (1.29)

$$N_L = \frac{Z}{L_M'/K_L a C_T}$$  \hspace{1cm} (1.30)

Similarly,

$$H_{OG} = \frac{G_M}{K_{OG} a P}$$  \hspace{1cm} (1.31)

$$N_{OG} = \frac{Z}{G_M'/K_{OG} a P}$$  \hspace{1cm} (1.32)

and

$$H_{OL} = \frac{L_M}{K_{OL} a C_T}$$  \hspace{1cm} (1.33)

$$N_{OL} = \frac{Z}{L_M'/K_{OL} a C_T}$$  \hspace{1cm} (1.34)
where

\[ \text{where} \]
\[ H_{\text{OG}}, H_{\text{OL}} = \text{overall height of transfer unit} \]
\[ N_{\text{OG}}, N_{\text{OL}} = \text{overall number of transfer unit} \]

1.5 Residence Time and Liquid Mixing

The structure of a model for any chemical engineering process involving fluid flows is affected chiefly by the flow patterns prevailing in the system and this is reflected in the distribution of residence times of fluid elements in their passage through that system.

Fluid flow through a system can either be ideal or non-ideal. Two types of ideal flow are regarded as the limiting cases: plug or "piston" flow, and perfectly mixed flow. Intermediate between the two ideal cases is backmixing.

In the plug flow model, the vessel contents are assumed to proceed in a plug progress along the vessel, with no interchange of material in the plug, with material in either leading or following plugs taking place, and the vessel contents are uniformly distributed in a direction normal to that of flow, i.e. the fluid velocity gradient is uniform over the entire cross-section of the vessel.

The perfectly mixed flow model assumes that the vessel contents are completely homogenous down to a molecular scale. No differences exist between the various portions of fluid elements in the vessel, and the outlet stream properties are identical to the bulk vessel-fluid properties.
The flow patterns in plate columns never fully follow these two idealised flow patterns. The interaction between the phases results in mixing of the fluid in both longitudinal and lateral directions and this is termed "non-ideal" flow.

The concentration gradient that exists depends on the degree of mixing on the tray. A concentration gradient is established from inlet to exit weir by virtue of mass transfer. This is offset by the mixing process and the final gradient is some complex function of mass transfer and liquid mixing. This concentration gradient is a function of the magnitude of backmixing of the liquid that occurs on the plate. If the mixing were perfect then the composition of the liquid on the plate would be uniform at all points, i.e. the point efficiency is the same as the plate efficiency. In practice, however, this is far from true as only partial mixing of the liquid takes place, with the result that the plate efficiency may be greater than 100%. Methods of evaluating the type of mixing and therefore calculating the degree of mixing in plate columns are desirable if plate efficiencies are to be calculated with any degree of confidence.

The evaluation of type and degree of mixing requires a knowledge of the complete flow pattern of the fluid on the plate. The most commonly employed method is a stimulus-response technique using a tracer material injected into the flowing liquid. The stimulus or input signal is the tracer introduced in a known manner into the liquid inlet stream and the response or output signal is then recorded
at certain locations, such as the exit of the tray as a function of time. The extent of liquid mixing can then be evaluated from the distribution of residence times in the liquid phase.

The use of the residence time distribution concept stems largely from the comprehensive treatment by Danckwerts (1953) of a class of continuous flow systems for which the residence time distribution can be obtained by the stimulus-response technique.
CHAPTER TWO

LITERATURE SURVEY

2.1 Hydrodynamics

An accurate prediction of the hydrodynamic characteristics of a perforated plate is necessary since they affect both tray capacity and stability.

A great number of publications have appeared in the literature in recent years concerned with sieve tray hydraulics; too many to comment on in detail. Only a few will be referred to as being relevant to the present studies.

2.1.1 Dry-tray Pressure Drop

The dry-tray pressure drop is the vapour pressure drop due to vapour flow through a dry perforated plate with no liquid flow on the plate.

Many of the published investigations have correlated the dry-plate pressure drop in terms of an orifice-flow model while others have used an empirical correlation of dry pressure drop as a function of vapour velocity. The perforated tray in reality is a system of orifices; trays differ in the number of perforations and their size, which may be arranged in specific patterns. Other differences are in the tray thickness, roughness of surfaces, and sharpness of the inlet edge of the perforations.
Mayfield et al. (1952) used the orifice model for correlating dry tray pressure drop as a function of vapour volumetric flowrate. They studied dry pressure drop in two round experimental columns of 6.0 and 6⅛ inch diameter respectively, as a function of hole size (1/8, 3/16, and 1/4 inch), plate thickness (1/4 and 3/8 inch), hole pitch (1/2, 3/4 and 1 inch triangular) and number of holes. For the 6 inch diameter column, they obtained an average orifice coefficient of 0.85 and for the 6⅛ inch diameter column, they reported an orifice coefficient ranging from 0.68 to 0.94.

Arnold et al. (1952) developed a correlation for dry-plate pressure drop using a modified orifice equation. They used a 15 inch diameter column with plates varying in hole sizes from 0.038 to 0.312 inch, total perforation area from 6.94% to 28.8%, and holes arranged either in a triangular or square pitch. They correlated the measured data by a simplified orifice equation given by:

\[ U_Q = C_D \left( 2g \frac{h_{DP}}{h} \right)^{1/2} \]  

(2.1)

where

- \( U_Q \) = average linear air velocity through the holes (ft/sec)
- \( h_{DP} \) = pressure drop across plate (ft of fluid flowing)
- \( C_D \) = orifice coefficient

The coefficients calculated in this way were plotted as a function of the air velocity through the perforations, with their diameters as parameters. They reported values of the orifice coefficients varying from 0.55 to 0.85, and observed an increase of the coefficient with an increase in free area,
and with a decrease in hole size. They also observed a higher value of the coefficient with a triangular pitch arrangement than with a square pitch arrangement. This difference was assumed to be due to the closer distribution of the perforations in the former arrangement. The authors further proposed a more useful form of equation (2.1) based on experimentally obtained data:

\[ \Delta P_D = k u_0^1.8 \]  

(2.2)

where \( k \) is an empirical dimensional coefficient which is independent of flowrates over the range studied. \( C_D \) in equation (2.1) is dependent on air flowrates. However, the authors were only able to apply their results to plate thickness-hole diameter ratios greater than unity.

Kamei et al. (1954) extended the orifice model by correlating the coefficient of exit flow as a function of the tray thickness and hole diameters. They suggested that the pressure drop was not influenced by neighbouring perforations and correlated their data by:

\[ h_{DP} = \frac{U_o^2 \rho g}{2C_D^2} \]  

(2.3)

Eduljee (1958) reported discharge coefficients of 0.72, 0.775 and 0.83 for holes of diameter 1/8 and 1/4 inch respectively. They used a \( t/d_o \) ratio of unity.

Hunt et al. (1955) studied dry-plate pressure drop in a 6 inch diameter column with holes of 1/8 to 1/2 inch diameter. They correlated their data graphically with an
equation representing the sum of pressure losses due to sudden contraction and expansion of the vapour in passing through the perforations.

\[ h_{DP} = 1.14 \frac{U_o}{2g_c} \left[ 0.4 \left( 1.25 - \frac{A_o}{A_c} \right) + \left( 1 - \frac{A_o}{A_c} \right)^2 \right] \]  

(2.4)

where

- \( A_o \) = total perforation area (ft\(^2\))
- \( A_c \) = area of column available for vapour flow (ft\(^2\))

They found their correlation to be satisfactory when applied to a variety of gases other than air, provided that \( t/d_o \) was equal to or greater than 0.9. They listed certain differences when their correlation was applied to the data of Mayfield et al. (1952) and Arnold et al. (1952).

Hughmark and O'Connell (1957) used a generalised orifice coefficient equation:

\[ h_{DP} = \frac{0.003 F_A^2 (1 - \beta^2)}{C_D^2} \]  

(2.5)

where

- \( F_A = U_o \sqrt{\rho_g} \)
- \( \beta = \text{fraction free area of active tray} \)

They presented a graphical correlation of the orifice coefficient as a function of hole diameter to tray thickness ratio and compared their data with those of Arnold et al., Mayfield et al., Kamei et al., Hunt et al., Jones and Pyle (1955), and others, claiming an average deviation of 10% for all the points, 90% of which had a deviation of less than 20%.
Kolodzie and Van Winkle (1957) used a semi-empirical approach to correlate orifice coefficient with Reynold's number, \( t/d_o \) ratio, and pitch-to-diameter ratio. The equipment employed was a specifically designed 3 inch internal diameter pyrex pipe with perforated plates of hole sizes varying from \( \frac{1}{16} \) to \( \frac{1}{4} \) inch diameter flanged between two sections of the pipe. Pitch-to-hole diameter ratios from 2.0 to 5.0 and \( t/d_o \) ratios from 0.33 to 4.0 were used. By dimensional analysis, they obtained dimensionless numbers which formed the basis for correlating their experimental data thus:

\[
C_D = f \left( \frac{d_o U_o \rho g}{\mu g}, \frac{d_o}{p}, \frac{t}{d_o}, \frac{A_f}{A_t} \right)
\]  

(2.6)

where

\( P = \) pitch of perforations (in)

\( A_f = \) actual plate free area (ft\(^2\))

\( A_t = \) maximum free area possible if the total area of the plate were perforated at the same \( P/d_o \) ratio and pattern

The orifice coefficient was calculated from:

\[
U_o = C/Y \sqrt{\frac{2 g h_{DP}}{\rho g \left[ 1 - \left( \frac{A_o}{A_c} \right)^2 \right]}}
\]  

(2.7)

where

\( Y = \) expansion factor = 1

They correlated the curves of orifice coefficient with plate geometry by:
where

\[ C_D = K \left( \frac{d_o}{P} \right)^{0.10} \]  \hspace{1cm} (2.8)

where

\[ K = \text{function of the ratio } t/d_o \text{ for varying Reynolds number } (2,000-20,000). \]

McAllister et al. (1958) extended the range of equations proposed by Hunt et al. (1955), and others, to include frictional losses through the perforations. The proposed correlation included the effect of variations in thickness-to-hole diameter ratios and was represented by:

\[ h_{DP} = K \left[ 0.4 \left( 1.25 - \frac{A_o}{A_c} \right) + \left( 1 - \frac{A_o}{A_c} \right)^2 + 4f \left( \frac{t}{d_o} \right) \right] \frac{U_o^2}{2g_c} \]  \hspace{1cm} (2.9)

where \( K \) is the dry-plate factor summarising the influence of a great number of parallel openings, the irregularity of the velocity gradient inside each opening and the so-called "Couette correction", because the members of the different losses in equation (2.9) were derived for single orifices. They plotted the slope of \( K \) as a function of \( t/d_o \) using their data and the data of Hunt et al., Mayfield et al., and others. They also correlated the data of these workers with those calculated by equation (2.9) and claimed a small average deviation from the line of equality.

Smith and Van Winkle (1958) carried out an investigation similar to one previously carried out by Kolodzie and Van Winkle (1957), but covering Reynolds number from 400-3,000. They claimed an accuracy of within 5% when their correlation was tested with the data of Arnold et al. (1952), Hunt et al.
Huang and Hodson (1958) developed a nomograph for the solution of equation (2.4), claiming a deviation of within 30% when applied to the data of Arnold et al. (1952), and Mayfield et al. (1952).

Harris and Roper (1962), Teller and Cheng (1963), and Thomas and Ogboja (1978), have also reported data based on the orifice model.

2.1.2 Total Pressure Drop

Thomas and Ogboja (1978) in their publication on the hydraulic studies of sieve tray columns have presented a state-of-the-art review based on previously reported correlations in the literature.

The total pressure drop across a perforated tray has been established to be a summation in part of:

(i) the dry tray pressure drop, $h_{DP}$
(ii) the dynamic head at the tray floor, $Z_D$
(iii) the residual pressure drop, $h_R$
(iv) the momentum head loss over the tray, $h_M$

i.e.

$$h_T = h_{DP} + Z_D + h_R + h_M$$  \hspace{1cm} (2.10)

Mayfield et al. (1952) presented an additive model for sieve tray pressure drop, and in general established that the presence of liquid on the tray had no marked effect on orifice
coefficients obtained on the dry tray, with pressure drop through the wet tray agreeing to within 0.2 inch of water with pressure drops through the dry tray. They presented an equation for total pressure drop:

\[ h_T = h_{DP} + h_L = h_{DP} + \beta (h_w + h_{ow}) \]  \hspace{1cm} (2.11)

where

- \( h_L \) = effective liquid seal on tray floor (in)
- \( h_w \) = outlet weir height (in)
- \( h_{ow} \) = head of clear liquid over weir (in)
- \( \beta \) = aeration factor of liquid on tray

They presented graphical correlations of \( \beta \) as a function of the clear liquid head and the volumetric air flowrate.

Arnold et al. (1952) used an additive model to present graphical correlations of total tray pressure drop as a function of air mass velocity with water mass velocity as parameter. They advanced an expression for approximate prediction of the total pressure drop:

\[ h_T = R (h_{DP} + S) \]  \hspace{1cm} (2.12)

where

- \( R \) = empirical correction factor dependent on \( S \)
- \( S \) = static liquid seal (\( h_w \)) (in)

They claimed that the use of the correction factor permitted estimation of pressure drop to within 5% for values of liquid flowrate around 1000 lb/hr ft².

Hunt et al. (1955) further developed the additive model
They investigated the residual pressure drop as a function of hole air velocity and found it to be small in all instances and to be correlated better as a function of gas kinetic energy ($\rho_v u_o^2/2g_c$) than as a function of gas velocity alone. They also concluded that the effect of surface tension was negligible.

Hughmark and O'Connell (1957) estimated total pressure drop from the dry pressure drop and an effective head which combined the hydrostatic head on the tray and the head required to form the bubbles and force them through the liquid. They correlated this effective head as a function of total submergence which is given as a sum of weir height, height of liquid crest over weir given by the Francis formulae and one half of the hydraulic gradient which they calculated using a modified form of the correlation for bubble-cap trays developed by Klein (1950).

McAllister et al. (1958) investigated the effect of $t/d_o$ and liquid and vapour flowrates on the residual pressure drop.

Bernard and Sargent (1966) working under actual distillation conditions to study pressure drop using the additive model, obtained residual pressure drops which disagreed with the findings of other workers and showed that the total pressure drop levelled out at hole velocities greater than
Various correlations have been established for the total pressure drop using regression analysis. Thomas and Campbell (1967) correlated their data on the air-water system by:

\[ h_T = 1.34 F_A^2 + 0.018 L + 0.62 W + 1.22 \]  \hspace{1cm} (2.14)

where

\[ L = \text{liquid flowrate (gall/min ft of weir)} \]
\[ W = \text{weir height (in)} \]

Thomas and Haq (1976), working on the air-water system, correlated their data by:

\[ h_T = 0.34 F_A^2 + 0.03 L + 0.7 W + 1.22 \]  \hspace{1cm} (2.15)

Thomas and Ogboja (1978) obtained for studies in a rectangular column:

\[ h_T = 0.024 L + 0.2 F_A^2 + 3.66 \]  \hspace{1cm} (2.16)

and in a round column:

\[ h_T = 0.038 L + 0.533 F_A^2 + 2.0 \]  \hspace{1cm} (2.17)

Similar correlations derived by regression analysis have been presented by Harris and Roper (1962).

2.1.3 Froth Height

Gerster et al. (1949), Foss and Gerster (1956) and Gilbert (1959) have reported froth height data measured visually for the air-water system. Essentially, they all
found that the froth height increased with an increase in liquid and gas flowrates. None of them offered a correlation.

Zenz (1954) reported froth height data obtained on plates with holes of 1/16, 1/8 and 1/4 inch diameter. He correlated vapour velocity and liquid submergence with froth height as parameter.

The A.I.Ch.E. Bubble Tray Design Manual (1958) reported froth height data obtained on a cyclohexane-n-heptane system, as a function of F-factor and weir height. The correlation obtained by regression analysis is given by:

\[ Z_f = 2.53 F_A^2 + 1.86 W - 1.6 \]  \hspace{1cm} (2.18)

The University of Delaware Final Report also reported froth height data correlated by regression analysis:

\[ Z_f = 0.73 W + 3.24 F + 0.084 L \]  \hspace{1cm} (2.19)

Friend et al. (1960), in their studies of entrainment on a sieve plate presented a graphical correlation of froth height measured visually as a function of vapour density, liquid flowrate and weir height. They estimated that the error of observation was ± 1 inch and observed the froth height to increase with increasing vapour and liquid rates.

Bain and Van Winkle (1961) found the froth height to be a linear function of liquid rate and independent of hole size. They proposed that the froth height may be represented by:
\[ Z_f = \frac{(0.094 + 0.014 h_w^2) L}{1000} + I \]  (2.20)

It was correlated as a function of gas flowrate with weir height as parameter. They, however, observed that the froth height measured visually has limited accuracy.

Harris and Roper (1962) correlated their froth height data using regression analysis, with the F-factor, weir height and liquid rate:

\[ Z_f = 1.5 + F + 0.75 W + 0.1 L_w \]  (2.21)

where

\[ L_w = \text{liquid rate (U.S. gall/min ft of mean plate width)} \]

Thomas and Campbell (1967), Thomas and Haq (1976), and Thomas and Ogboja (1978) have similarly correlated froth height by regression analysis as a function of F-factor, liquid flowrate and weir height.

Lockett et al. (1979) measured the froth density as a function of froth height by an innovative method. They used gamma ray absorption and from their studies concluded that the froth had no distinct upper surface. They also found that the froth height as measured visually corresponded to a height where the froth density was approximately 10% by volume of liquid.

Lockett et al. (1980) used high-speed photographic technique to measure froth heights in an air-CO$_2$ water system.

Baker and Self (1962), while studying liquid mixing
effects on a sieve tray presented a correlation for froth height as follows:

\[ Z_f = 0.116 + 0.00395 L + 0.088 W + 0.0384 U_A \]  
\[ (2.22) \]

They also gave the following operating limits for their correlation:

- \( L = 10 - 50 \) gall/min ft weir height
- \( U_A = 2 - 6.5 \) ft³/sec ft² of bubbling area
- \( W = 2 - 4 \) inch

2.1.4 Liquid Hold-up on Tray

Prior to 1957, the liquid hold-up on the tray was assumed to be the sum of the weir height plus the liquid crest over the weir, but in 1957, however, the concept of liquid hold-up was redefined.

Earlier clear liquid head prediction methods available in the literature for sieve trays consisted of variations of the classical Francis weir equation which is given by Smith (1963) for circular cross-sectional columns as:

\[ h_{ow} = 0.48 F_w (L/W)^{0.67} \]  
\[ (2.23) \]

where

- \( L = \) liquid flowrate (gall/min)
- \( W = \) weir length (in)

The correction factor, \( F_w \), was introduced by Bolles (1946) to account for the constricting effect of the tower on the flow of liquid over the weir when segmental downcomers are
used and a correlation graph of \( F_W \) against liquid flow-rate with ratio of weir length to tower diameter or parameter was given.

The liquid crest over the weir as calculated by equation (2.23) was then used to determine the clear liquid head, \( Z_C \), by:

\[
Z_C = W + h_{OW}
\]  

Criticism of this method of predicting the clear liquid head centred on the fact that the Francis formula was derived for the flow of unaerated liquid over the weir, while in reality, on a sieve plate, the gas-liquid interaction is an often violet, non-uniform, two-phase mixture with non-uniform flow patterns. Furthermore, the equation lacks a vapour rate dependence which is prevalent on sieve trays.

Thomas and Campbell (1967) have shown that although equation (2.23) may hold, the use of equation (2.24) did not give values of \( Z_C \) comparable with values obtained by other methods.

More empirical correlations involving the measurement of the clear liquid head with manometers flush with the tray floor have been reported. The readings obtained by this method are commonly supposed to represent the height to which the foam would collapse in the absence of vapour flow, and for this reason, they are often referred to as the equivalent clear liquid height.
The manometer method was used by Foss and Gerster (1956). They observed a decrease in the averaged value of the clear liquid head with increase in gas flowrate and an increase when the liquid flowrate and weir heights were increased.

The effective head concept was introduced by Hughmark and O'Connell (1957). They correlated effective head with total submergence on tray. They defined the effective head, $h_e$, as the sum of the hydrostatic head plus the head to form the bubbles and to force them through the aerated mixture. The effective head in actuality is the static liquid head on the tray.

Harris and Roper (1962) presented data for clear liquid head obtained on a plate of 3/16 inch diameter holes over a range of gas and liquid rates. They correlated their data as a function of F-factor, weir height and liquid flowrate using regression analysis:

$$Z_c = 0.25 + 0.58 W + 0.03 L_w - 0.28 WF$$

where

$L_w$ = liquid flowrate (U.S. gall/(min) ft of mean plate width)

A similar regression equation was earlier presented in the A.I.Ch.E. Bubble Tray Manual (1958):

$$Z_c = 1.65 + 0.19 W + 0.02 L - 0.65 F$$

Barker and Self (1962) correlated liquid hold-up for a sieve plate with 2, 3 and 4 inch outlet weirs. The data were well represented by:
\[ h_L = 0.078 + 0.00145 L + 0.031 W - 0.013 U \]  (2.26)

where

\[ h_L = \text{clear liquid hold-up (ft)} \]

Bernard et al. (1964) calculated liquid hold-up on a tray by an indirect measurement of foam density using a gamma-radiation technique. They also used flow manometers, and found a difference between the results of the two methods. The manometric readings corrected for surface tension effects give the dynamic head to which it is necessary to add the momentum head of vapour to give the equivalent clear liquid head:

\[ Z_c = Z_D + Z_M \]

\[ = Z_D + \frac{\rho}{\rho_1} \frac{U_s}{g_c} (U_o - U_s) \]  (2.27)

Lemieux and Scotti (1969) measured a tray seal pot level as the total depth of clear liquid at the upstream end of the flow section, and showed it to increase with increasing liquid rate and increasing vapour rate, but decreased with increasing hole size at constant vapour rates. They used a manometric method.

Thomas and Campbell (1967) compared three methods for calculating the liquid hold-up. While results obtained by the use of equation (2.27) and equation (2.13) show good agreement, except at high \( F_A \) values, the results obtained from their residence time studies gave values higher than those of the other two methods. They concluded that calcu-
lation of liquid hold-up from residence time data was not recommended. They also advanced a correlation for the dynamic liquid head:

\[
Z_D = 0.19 W - 0.40 F_A - 0.013 L + 1.56 \quad (2.28)
\]

Similar regression equations were proposed by Thomas and Haq (1976) and Thomas and Ogboja (1978).

More recent studies have attempted to correlate the liquid hold-up at the froth-to-spray transition. Jeronimo and Sawistowski (1973) have correlated the transition hold-up data of Pinczewski and Fell (1972) with the equation (air-water system):

\[
h_{Lt} = \frac{1.059 A_f^{-0.791} d_o^{-0.833}}{1 + 0.013 L^{-0.590} A_f^{-1.790}} \quad (2.29)
\]

where

\[ h_{Lt} = \text{liquid hold-up of the froth to spray transition (mm)} \]
\[ A_f = \text{fractional tray free area} \]
\[ d_o = \text{hole diameter (mm)} \]
\[ L = \text{liquid rate per unit weir length (m}^3(\text{hm})^{-1}) \]

2.1.5 Aeration Factor and Foam Density

Pressure losses above the holes on a tray can be grouped into two major classes:

1. surface tension loss

2. loss due to flow through the aerated mass plus those for static effects in the vapour space.
The second group of losses are usually correlated by means of an aeration factor, $\beta$, defined in Smith (1963) as:

$$\beta = \frac{h_T - h_H}{W + h_{ow}}$$

where

$h_H$ = head loss due to vapour flow through perforation (in. liquid)

Mayfield et al. (1952) were the first to correlate aeration factor with clear liquid head. A calculated aeration factor was presented as a function of calculated clear liquid head on the tray for hole sizes of 1/8 and 3/16 inch diameter, and a range of weir heights. They defined the aeration factor as:

$$\beta = \frac{h_T - h_{DP}}{W + h_{ow}}$$

$h_{ow}$ was calculated from the Francis Weir formula. They also correlated the aeration factor as a function of volumetric air rate with liquid rate as a parameter. They found the aeration factor to increase slightly with increase in gas rate with no discernible trend with variations in liquid rate.

Due to the vagaries of the dry-tray pressure drop correlations and the unknown interactions of dry and aerated effects, a more direct and satisfactory approach to evaluating $\beta$ is based on direct measurement of the relative froth density, $\phi$. The relative froth density is defined in Smith (1963) as:

$$\phi = \frac{Z_C}{Z_f}$$
Hutchinson et al. (1949) have shown that the relative froth density and the aeration factor are related by:

\[ \beta = \frac{\phi + 1}{2} \]  \hspace{1cm} (2.33)

if the phases are separated in the calming sections. Although, in practice, this is unlikely, Thomas and Shah (1964), Thomas and Campbell (1967), and others, have shown the equation to be approximately true. They found that aeration factors determined from dynamic and pressure head measurements appeared to be relatively insensitive to variations in gas flowrate, whereas values calculated from the froth density decreased with increase in gas rate, while both showed an increase with increase in liquid rate.

Prince (1960) related aeration factor for a number of gas/liquid systems, to liquid-to-gas flow ratio and tray submergence, on a 6 inch diameter sieve tray using foam baffles.

Bernard and Sargent (1966) measured froth density directly using gamma radiation absorption. They presented foam density profiles with very peculiar shape which they, however, claimed were reproducible. They also concluded that the foam density was affected by weir height, vapour rate and pitch-to-diameter ratio.

The same gamma ray absorption technique was used by Lockett et al. (1979) to measure and produce foam density profiles.

Thomas and co-workers have all reported froth density factor values of between 0.2 to 0.4.
2.1.6 Liquid Entrainment

Extensive studies have been reported in the literature to correlate absolute entrainment from plate columns with flow and tray geometry variables. Much of the earlier studies conducted by Souders and Brown (1934), Pyott (1935), Spells et al. (1950, 1952), Simpkin et al. (1954), Atteridge et al. (1956), Eduljee (1958), and others, were derived from bubble-cap trays for which empirical and quasi-empirical correlations applicable to specific tray geometries or operating conditions were presented.

With the increased use of perforated trays, a number of studies have been conducted into the performance characteristics of such trays. The inconsistencies reported in the use of correlations derived from these studies may be due in part to the diversity of column geometries and methods of measurements employed.

While most of the investigations have concentrated efforts on the prediction of entrainment, very few have been conducted into comparing the methods by which the predicted entrainment was measured.

Calcaterra et al. (1968) studied entrainment in an air-water system and compared values of entrainment measured by two principal methods. They found that entrainment values were higher when measured by the "position-capture" method than by the measurement of "free" entrainment. The "position-capture" method involves measuring the amount of entrainment actually carried over to the tray above the
operating tray, while the second method measures "free" entrainment which is the amount of entrainment that reaches a given height above the operating tray.

They explained this anomaly by suggesting the formation of a vertical jet flume by contraction loss as the vapour passes through the perforations. According to them, entrainment results when liquid droplets are picked up in these flumes. When the larger drops have lost their initial momentum, they fall back onto the operating tray, but if the plate spacing and the initial momentum are such that the drops are carried close to the collector tray, then they are picked up by the increased vapour velocity of the flumes and are carried into the collector tray. However, without a collector tray above the operating tray, 100% free area, there is no formation of flumes, and the drops eventually fall back onto the tray. Calcaterra et al. illustrated this relationship by Fig. 2.1.

Fig. 2.1 Diagrammatic Explanation of Relationship between "Free" and "Captured" Entrainment
Molokanov et al. (1969) used a photocolorimetric technique to measure the amount of "captured" entrainment by the tracer injection method. They compared their data with data obtained in the same studies using a specially designed trap to measure "free" entrainment and proposed that a correction factor should be applied to data obtained by the latter method. They concluded that the measurement of entrainment should be made using methods which would not involve the introduction of extraneous devices that might modify the nature of the vapour flow and as a consequence influence the entrainment actually occurring.

Many of the published studies have correlated entrainment as a function of tray design parameters, gas and liquid loading and system physical properties. As previously noted, there remains a general inconsistency in the published studies and no generally accepted correlation is available.

Hunt et al. (1955) related entrainment to superficial velocity, surface tension and effective plate spacing. They used a "dry" collector tray of similar configuration as the test tray and established an empirical relationship for weight entrainment given by:

$$ e_w = 0.22 \left( \frac{U_c}{S'} \right)^{32} $$

\[ (2.34) \]

where

- $e_w$ = weight entrainment ratio lb liquid/lb gas
- $\sigma$ = surface tension of liquid on tray (dynes/cm)
- $U_c$ = column velocity (ft/sec)
- $S'$ = effective plate spacing, i.e. actual plate spacing minus froth height
In order to use this correlation effectively, it is necessary to know the froth height, which has been pointed out by Bain and Van Winkle (1961) as having a limited accuracy of measurement due to the sloshing and frothing on the tray. Although the correlation was obtained primarily for 1/4 inch diameter holes using the air-water system, Hunt et al (1955) found it to be satisfactory when applied to both 1/8 and 3/8 inch diameter holes and with other gases, although it is limited to superficial column velocities less than 10 ft/sec. Their correlation gave entrainment values three times higher for bubble cap trays when applied to the data of Holbrook and Baker (1934), and Sherwood and Jenny (1935).

Jones and Pyle (1955) employed a tracer-injection method in an acetic-water system; they found entrainment values exhibited by sieve trays to be 20% of values exhibited by bubble-cap trays.

Atteridge et al. (1956) studied bubble-cap tray entrainment with variations in liquid-path length, number of slots and caps, cap spacing and liquid flowrate. They concluded that different mechanisms govern entrainment at different liquid rates.

Friend et al. (1960) measured entrainment in a sieve tray column with the air-water system, as a function of tray spacing, hole diameter and tray thickness. They employed a "dry" collector tray of similar configuration as the test-tray, and found that entrainment generally increased with increasing weir length, vapour rate and hole size,
decreased with tray spacing and hole area, and is unaffected by plate thickness and the angle of inclination of the tray. They indicated that the Hunt correlation gave higher values than the measured values, and recommended correlating data by the form of equation developed by Simpkin (1954).

Bain and Van Winkle (1961) related entrainment to hole size, plate spacing and liquid-to-gas flow ratio. Their data were obtained on a 30 inch diameter sieve tray column, employing the air-water system, and was satisfactorily correlated by the following equation:

\[ \ln E = k \ln \left( \frac{d_o}{S} \right)^{\frac{1}{L}} \left( \frac{1}{G} \right)^f + B \]  

(2.35)

where

- \( E \) = entrainment (lb water/100 lb air)
- \( K \) and \( B \) = constants for each hole diameter
- \( S \) = plate spacing
- \( L \) = liquid flowrate (lb/hr ft of weir length)
- \( G \) = gas flowrate (lb/hr ft\(^2\) of tower cross-section)
- \( f \) = function of weir height and hole diameter
- \( g \) = function of hole size and gas flowrate

They claimed that approximately 90% of the over 900 points of data used fell within \( \pm 25\% \) of the proposed correlation. The functions \( f \) and \( g \) were correlated graphically. The entrainment collection device consisted of V-shaped troughs oriented parallel to the direction of air flow. Such a configuration was considered by Teller et al. (1963) to give a significant difference in magnitude of captured entrainment compared with a conventional sieve tray. The appli-
cability of the proposed correlation to weir heights greater than 2 inch and hole sizes greater than 1/4 inch diameter is doubtful, as it was suggested by Arnold et al. (1952) that higher hole sizes may give higher entrainment values and lead to optimistic design.

One method of calculating entrainment, designs for a given fractional approach to an assumed 100% entrainment top-operating limit. Souders and Brown (1934) from a consideration of force balance on an "average" suspended liquid droplet, defined this top-operating limit in terms of an allowable vapour velocity or capacity parameter, $C_{sb}$:

$$C_{sb} = U_{VN} \left( \frac{\rho_v}{\rho_l - \rho_v} \right)^{0.5}$$

(2.36)

where

$$U_{VN} = \text{velocity of vapour based on net area (ft/sec)}$$

$\rho_l, \rho_v = \text{liquid and vapour densities respectively (lb/ft}^3)$$

The capacity parameter, $C_{sb}$, was related to the flooding capacity, of bubble-cap trays by Fair and Matthews (1958), and later, of sieve trays, by Fair (1961), through plate spacing and a flow parameter, $F_{LV}$. This parameter accounts for liquid flow effects on the trays and in reality is a ratio of liquid-vapour kinetic energy effects. It is defined as:

$$F_{LV} = \frac{L}{G} \left( \frac{\rho_v}{\rho_l} \right)^{0.5}$$

(2.37)

where

$L/G = \text{liquid/gas mass ratio}$

The correlation developed was presented graphically and is reproduced in Fig. 2.2. Subject to limitations imposed by
Fig. 2.2 Flooding Capacity for Bubble-cap and Sieve Trays
[Fair (1951), Fair and Matthews (1958)] (Reproduced from Von Winklo (1967) p.525)
the authors, the chart may be used to calculate the flooding vapour velocity:

$$U_{N,\text{flooding}} = C_{sb} \left( \frac{\rho_L - \rho_V}{\rho_V} \right)^{0.5}$$

(2.38)

The authors also presented a design chart showing a correlation relating fractional entrainment and the flow parameter, $F_{LV}$ with percent of flood as a parameter. The chart is reproduced in Fig. 2.3 and is claimed to be accurate within ±15%. The fractional entrainment, $\psi$, defined in terms of gross liquid downflow is given by:

$$\psi = \frac{e}{L + c}$$

(2.39)

where

$$e = \text{entrainment (lb or moles/time)}$$

and the approach to flooding given in terms of percentage flood can be determined from:

$$\% \text{ flood} = \frac{U_{N,\text{design}}}{U_{N,\text{flooding}}} \times 100$$

(2.40)

where

$$U_{N,\text{design}} = \text{vapour velocity based on net area for vapour flow (ft/sec)}$$

Fair also points out that the use of Figs. 2.2 and 2.3 is valid only when the following restrictions are observed:

(1) low or non-foaming systems

(2) weir height less than 15% of plate spacing

(3) hole diameters less than or equal to 1/4 inch

(4) ratio of hole area to plate active area $A_0/A_a$, equal to or greater than 0.1.
Fig. 2.3  Entrainment Correlation [Fair (1961); Fair and Matthews, (1958).] (Reproduced from Van Winkle, (1967) p.538)
These restrictions make the applicability of these correcting charts to large hole sizes, i.e. greater than 1/4 inch, very doubtful. Although it is generally known that larger hole sizes increase entrainment, the specific effect on the flood point is not known.

Attempts have been made to present correlations for hole sizes greater than 1/4 inch as such sieve trays are finding increasing use in industry.

Lemieux and Scotti (1969) studied entrainment from 1/2 and 1 inch diameter hole trays using the air-water system. Entrained droplets were collected on an upper tray of the same configuration as the test tray. They presented graphical correlations of entrainment (lb/lb air), with liquid flowrate (gall/hr in), and a vapour velocity ratio, $Z_p$, defined as:

$$Z_p = \frac{\text{vapour velocity in holes}}{\text{allowable vapour velocity}} \quad (2.41)$$

The allowable vapour velocity derived from Souders and Brown is given by:

$$0.227 \left( \frac{\rho_L - \rho_V}{\rho_V} \right)^{0.5} \quad (2.42)$$

They concluded that at low liquid rates, larger holes produce entrainment values 4-5 times the values shown by smaller holes, and twice at high liquid rates.

Thomas and Ogboja (1978) measured entrainment on a 3 ft$^2$ rectangular column with 1 inch diameter perforations using the air-water system. They employed the "position-capture"
method and used their entrainment data to test the validity for large hole sizes of the correlations proposed by Bain and Van Winkle (1961) and Fair (1961). They indicated that the established correlations fall short of predicting satisfactorily the entrainment values either for large holes or small holes contained in large columns. They established an empirical correlation for entrainment as a function of hole velocity, liquid loading and tray spacing:

\[ S'' \psi_V = k \left( \frac{U_o}{L'_{W''} S''} \right)^x \]  

(2.43)

where

\[ \psi_V = \text{entrainment factor} = \frac{e_v}{V} + e_v \]
\[ e_v = \text{liquid entrainment (lb/lb air)} \]
\[ V = \text{air flow rate (lb/hr)} \]
\[ S'' = \text{tray spacing (ft)} \]
\[ k = \text{constant, function of liquid rate} \]
\[ L'_{W''} = \text{liquid rate (lb/sec ft of weir)} \]
\[ U_o = \text{hole air velocity (ft/sec)} \]

Their correlation does not require a knowledge of the froth height, \( Z_f \).

Kharbanda and Ju Chin Chu (1970) correlated entrainment data for 3.5 to 18 inch plate spacing with the water-water and ethanol-water systems, as a function of \( F_A \)-factor. They proposed two correlations:

**Empirical:**

\[ \log (E \times 10^3) = 11.1/S + 14.9 U_c \sqrt{\rho_v} \]  

(2.44)

where

\[ E = \text{entrainment (lb/lb vapour)} \]
S = plate spacing (in)

\( U_c = \) column vapour velocity (ft/sec)

Semi-empirical:

\[
E = \frac{m (\rho_L - \rho_v)^{0.5} n}{U_c \cdot A_c \cdot \rho_v^{1.5}}
\]

(2.45)

where

\( m = \) factor of plate spacing

\( A_c = \) superficial column area (ft²)

\( m \) is given as a correlation of vapour velocity through the perforations. They do not state what the constant \( n \) represents and they concluded that their correlation works satisfactorily with the water-water system but not for the ethanol-water system due to the system's physical properties.

More recent studies on entrainment have concentrated on analysing the effect of flow regimes.

Lockett et al. (1976) have demonstrated that the sudden decrease observed in entrainment in sieve plates when the liquid flowrate is increased is caused by a change from froth to spray in the operating regime. They measured entrainment by the tracer-injection method using a collector tray similar to the test tray in an 18 inch diameter column with a plate spacing of 12 inches. They presented graphical correlations of their data as a function of hole diameter, liquid rate and hole velocity, and used their observations to explain results obtained by Bain and Van Winkle (1961).

Kister et al. (1981) have applied these changes in ope-
rating regime criteria to the entrainment data of Pinczewski et al. (1974), Raper (1980), Bain and Van Winkle (1961), Friend et al. (1960), Lemieux and Scotti (1969), Lockett et al. (1976), and Thomas and Ogboja (1978), to develop entrainment data applying specifically to the spray regime and utilised these data to establish both the effect of tray design parameters on entrainment, and a spray regime correlation. Kister et al. studied factors such as tray free area, hole diameter, weir height, tray spacing, tray geometry and liquid and gas hold-up as having an important influence on entrainment. Their correlation was conveniently expressed in dimensionless ratios given as:

\[
E = a \left( \frac{\rho_G U_s^3}{\rho_L L g_c} \right)^a \left( \frac{U_s \rho_L \mu_G}{\rho_G \sigma_L} \right)^b \left( \frac{d_H}{h_L} \right)^c \left( \frac{S}{h_L} \right)^e \tag{2.46}
\]

where

- \( E \) = entrainment (kg liquid/kg vapour)
- \( \rho_G \) = gas density (kg m\(^{-3}\))
- \( \rho_L \) = liquid density (kg m\(^{-3}\))
- \( U_s \) = superficial velocity (m s\(^{-1}\))
- \( L \) = liquid rate per unit weir length (m\(^3\)(hm)\(^{-1}\))
- \( \mu_G \) = gas viscosity (kg (mh)\(^{-1}\))
- \( \sigma_L \) = liquid surface tension (m N m\(^{-1}\))
- \( d_H \) = hole diameter (mm)
- \( h_L \) = liquid hold-up (mm)
- \( S \) = tray spacing (mm)

It might be worth noting that the last three groups, previously developed by Banerjee et al. (1969) for entrainment from a single orifice with no cross-flow, represent the ratio
of hole Weber number to hole Reynolds number, and the principal dimensions of tray geometry. The first group introduced by Kister et al. to account for the effect of liquid cross-flow is actually the product of the hole Froude's number and the hole Reynolds number.

\[
\text{Hole Froude's number} = \frac{U_s^2}{d_H g_c} \quad (2.47)
\]

\[
\text{Hole Reynolds number} = \frac{d_H U_s \rho_G}{\mu_G} \quad (2.48)
\]

\[
\text{Fr} \times \text{Re} = \frac{U_s^3 \rho_G}{\mu_G g_c} \quad (2.49)
\]

\[
\mu_G, \text{ the gas viscosity (kg (mh)}^{-1} \text{) is}
\]

\[
\mu_G = L \rho_L \quad (2.50)
\]

where

\[
L = \text{liquid flowrate per unit weir length (m}^3\text{(hm)}^{-1})
\]

\[
\rho_L = \text{liquid density (kg m}^{-3})
\]

Thus,

\[
\text{Fr} \times \text{Re} = \frac{U_s^3 \rho_G}{\mu_G g} = \frac{U_s^3 \rho_G}{\rho_L L g_c} \quad (2.51)
\]

The final correlation, for the air/water system is:

\[
E a \frac{U_s^{(3a+b)} d H^c S^e}{L^a h_L^{(c+e)}} \quad (2.52)
\]

Using the experimentally measured entrainment values, they obtained the following exponents in equations (3.13) and (3.16):

\[
a = b = 1.17
\]

\[
c = e = -2.34
\]

The final correlation then reduces to:
They claim an average deviation of 12% and express doubts as to the validity of their correlation for low tray free area (5% or less) operated at low liquid rates.

Some recent studies have preferred to investigate entrainment from the standpoint of mechanism of drop formation, projection velocities, drag and settling forces, and projection height.

Cheng and Teller (1961) analysed the behaviour of entrained drops in the vapour space of a sieve tray containing 5/32 inch diameter perforations using the air-water system. A drop interceptor was used to measure "free" entrainment and drop-size distribution. They found the drop-size distribution to be logarithmic and presented an equation for entrainment which they claimed correlates the data with an average deviation of 28%.

Teller and Rood (1962) employed high-speed photographic technique to study the entrainment, especially the phenomena of bubble generation, disintegration and coalescence on a sieve tray containing three 3/16 inch diameter holes contained in a narrow (3 x 8 inch) plexiglass column. They employed the air-water system and the method of Cheng and Teller (1961) for capture and measurement of entrainment. They studied entrainment as a function of hole pitch, tray spacing and particle size distribution. They found that for multiple perforations, shear of the liquid walls by the vapour flumes
rather than bubble burst is the major mechanism of surface
development for creation of entrainment. They also observed
an increased tendency for staggered or non-simultaneous bubble
appearance when hole pitch is increased resulting in turn in
a decrease in the degree of coalescence between bubbles.
Coalescence releases kinetic energy which manifests itself
as an increase in vapour velocity entering the froth zone,
and a decrease in the diameter of the flume. This was adduced
as the reason for the decrease in entrainment resulting from
an increase in hole pitch. Teller and Rood also observed that
the variation of entrainment as a function of tray spacing
is due to the variation in size of particles projected into
the vapour zone as a function of the flume velocity. Large
particle sizes are projected as the hole pitch and tray
spacing are increased.

Bejnnerjee et al. (1969) studied drop size of entrainment
as a function of tray geometry, vapour velocity, sampling
height and liquid transport properties using the air-water
system with single and multiple orifices. They used the
noise pattern of air flow through an orifice, amplified on
an oscilloscope, to observe the change in operating regime.
They observed that the mean drop size, increases to a maximum
and then decreases as the gas velocity is increased with the
change from froth to spray regime being indicated by the
peak, increases with increasing hole diameter, decreases with
increasing sampling height, increases with increasing liquid
surface tension, is unaffected by changes in liquid viscosity
and increases with the number of perforations. Bernerjee et al.
proposed that in the froth regime, drop formations by bubble bursting is dependent on bubble dynamics, i.e. drop frequency, size distribution, velocity, path of travel under inertial, gravitational and liquid transport properties, while in the spray regime, where bubble dynamics are insignificant, hole gas dynamics predominate in addition to liquid transport properties. Using dimensional analysis, Be\textquotesingle merjee et al. presented a correlation for drop size of entrainment from single orifices as a function of the orifice Reynolds number, Froude's number, Weber number, drag and settling forces, and apparatus geometry. In later studies (1969), they calculated entrainment values from the Sauter mean drop diameter and the total number of drop entrained by the method developed by May (1945), and presented a correlation for entrainment as a function of the dimensional groups they developed earlier (1961).

\[
\text{Froth Zone} = \frac{U_o^3 \rho_g}{\mu_g \rho_c} < 4 \times 10^7
\]

\[
E' = 3.1 \times 10^{-23} \left[ \left( \frac{U_o^3 \rho_g}{\mu_g \rho_c} \right)^{0.8} \left( \frac{d_H U_o^2 \rho_L}{\sigma_L} \right)^{0.8} \left( \frac{d_h}{h_s} \right)^{1.6} \left( \frac{h_s}{H} \right)^{2.8} \right] \tag{2.54}
\]

\[
\text{Spray Zone} = \frac{U_o^3 \rho_g}{\rho_g \rho_c} > 5 \times 10^7
\]

\[
E' = 9.44 \times 10^{-9} \left[ \left( \frac{U_o^3 \rho_g}{\mu_g \rho_c} \right)^{0.8} \left( \frac{d_H U_o^2 \rho_L}{\sigma_L} \right)^{0.8} \left( \frac{d_h}{h_s} \right)^{1.6} \left( \frac{h_s}{H} \right)^{2.8} \right] \tag{2.55}
\]

where

- \( E' \) = entrainment (g liquid/cm\(^2\)sec/g air/cm\(^2\)sec)
- \( U_o \) = orifice air velocity (cm sec\(^{-1}\))
- \( \mu_g \) = viscosity (g (cm sec\(^{-1}\)))
\( \sigma_L \) = surface tension (dynes cm\(^{-1}\))

\( d_H \) = orifice diameter (cm)

\( h_s \) = orifice submergence (cm)

\( H \) = sampling height (cm)

Bennerjee et al observed that the entrainment rate was satisfactorily correlated to within \( \pm 20\% \) maximum deviation. However, they offer no correlation for entrainment in multiple orifices.

2.2 Mass Transfer Efficiency in Sieve Plate Columns

The reliable prediction of mass transfer efficiency in sieve plate columns stands as an unsolved and challenging problem. Much pertinent work has appeared in the literature but such efforts apparently have not led to an understanding of the mass transfer processes on sieve plate columns. The mechanics of vapour-liquid interaction are exceedingly complex and macroscopic measurements do not delineate the more microscopic mechanisms that participate in the transfer process.

Predictions of tray efficiencies has in the past depended largely on the designers' experience with similar columns operating under the anticipated operating conditions.

2.2.1 Tray Efficiency

The beginning of the concept of tray efficiency was due to Murphree (1925) who defined two plate efficiencies in terms of either an overall change in concentration or a point change.
He assumed that the liquid on the trays was completely mixed and defined the overall plate efficiency as:

\[ E_{MV} = \frac{y_n - y_{n+1}}{y^*_{n} - y_{n+1}} \quad (2.56) \]

and the point efficiency as:

\[ E_{OG} = \frac{y - y_{n+1}}{y^* - y_{n+1}} \quad (2.57) \]

where

- \( y \) = mole fraction of vapour
- subscripts \( n \) and \( n+1 \) refer to the outlet and inlet vapour streams respectively, and \( y^* \) is the mole fraction of solute in gas corresponding to composition of liquids on (and leaving) the plate. The Murphree vapour efficiency is based on a constant liquid composition and a changing vapour composition. However, in some processes, for example, absorption of relatively insoluble gases, it is necessary to define a liquid efficiency since the gas composition changes only slightly while there is a progressive change in the liquid composition across the tray. Thus, in a similar manner:

\[ E_{ML} = \frac{x_{n-1} - x_n}{x_{n-1} - x^*_{n}} \quad (2.58) \]

where

- \( x^*_{n} \) is the liquid composition in equilibrium with the constant vapour composition.

The overall efficiency can by observation exceed 1.0, while the point efficiency by definition cannot exceed 1.0. The applicability of these efficiencies is obviously limited
Lewis (1936) assumed no mixing whatsoever in the direction of liquid flow, although significant but incomplete mixing was actually found in this study. He considered three possibilities:

1. perfect vertical liquid mixing
2. no horizontal liquid mixing (in the direction of flow)
3. perfect mixing of entering vapour

He derived a relationship between the Murphree overall tray and point efficiencies, for a straight equilibrium time:

\[ E_{MV} = \exp \left( \frac{\lambda E_{OG} - 1}{\lambda} \right) \]  

where

\[ \lambda = \frac{m}{G_m/L_m} \]  

and \( m \) = slope of equilibrium line

\( G_m \) = gas rate (lb moles/hr)

\( L_m \) = liquid rate (lb moles/hr)

Lewis showed that a point efficiency of, say 70\%, may give plate efficiencies of 110\% or 120\% on an actual plate.

In a pioneering paper, Geddes (1946) suggested a more fundamental approach to the problem. He developed a predictive model for \( E_{OV} \), using a discrete bubbling concept and the Higbie (1935) semi-infinite stagnant two-film theory. He advocated the separate study of the factors which determine the bubble size, the time of contact, and the individual mass transfer coefficients.
Shortly afterwards, Gerster et al. (1949) developed the transfer unit approach to tray efficiency. They showed that plate efficiencies may be predicted by the addition of separate liquid and gas phase resistances. Later, while studying the effect of tray geometric factors and operating variables on gas and liquid film efficiencies, Gerster et al. (1951) presented tentative predictive empirical correlations which were successfully used for determining the plate efficiencies of the water-methanol system.

The work of Gerster led to an industry-funded research programme, carried out at the Universities of North Carolina (1959), Michigan (1960), and Delaware (1958), and ultimately to the A.I.Ch.E. Bubble Tray Manual (1958). These studies developed procedures for the prediction and correlation of tray efficiencies in many industrial applications, and although their recommendations have been widely criticised, (Eduljee, 1965, Standart, 1974, etc.), practical design methods are still generally based on the framework given in the Manual.

The North Carolina College Final Report (1959) extended the two-film model and expressed the number of transfer units in terms of the contact times:

\[ N_L = K_L \cdot t_L \]
\[ N_G = K_G \cdot t_g \]  

where \( t_L, t_g \) = liquid and gas contact time respectively.
Other studies have been conducted based on the theoretical approach of Gerster et al. (1949, 1951).

West, Gilbert and Shimizu (1952) reported data for gas and liquid film resistances while studying the factors which determine bubble size, time of contact between the gas and liquid and individual mass transfer coefficients.

Chu et al. (1951) have derived a very complex correlating equation for the Murphree plate efficiency in terms of a number of parameters, amongst which are: diffusivities of both vapour and liquid, total vapour pressure, molecular weight of liquid, equivalent bubble diameter, weir dimensions, etc. Their equation is too complicated to use.

Calderbank and co-workers (1950, 1960, 1962) have reported studies on the fundamental factors affecting mass transfer using various techniques, and then related them to the number of transfer units and tray efficiency.

Hay and Johnson (1960) reported results for a gas-film controlled distillation system and substantiated equation (2.61) from the North Carolina College Final Report (1959). They obtained a linear dependence of the number of gas phase transfer units on the ratio of vapour hold-up to linear velocity.

Garner and Potter (1960) have also shown the same relationship:

\[ N_G = 0.502 + \frac{D_G}{\pi^2} \frac{t_G}{r^2} \]  

(2.62)
where
\[ t_G = \beta / U_G \]
and \( \beta \) = gas hold-up on the tray \((\text{ft}^3/\text{ft}^2 \text{ of bubbling area})\)

\[ U_G = \text{gas phase velocity (ft/sec)} \]

\[ r = \text{bubble radius} \]

\[ D_G = \text{gas phase diffusivity (ft}^2/\text{hr}) \]

A number of investigators have studied the effect of column variables on tray efficiency.

Ellis and Moyade (1959) studied sieve tray efficiencies in a 4½ inch diameter column with hole sizes of 1/4, 1/8 and 3/16 inch diameter. Liquid- and gas-film efficiencies were obtained from desorption of oxygen with water and humidification of air experiments. They showed that the liquid film efficiency decreased with liquid rate at low air rates, but was independent of liquid rate at high air rates, while vapour-film efficiency increased with liquid rates and decreased with air rate. Ellis and Moyade also showed that liquid film efficiency was independent of hole diameter while gas-film efficiency was a maximum for the small hole diameters, changing little with increased hole size. They found the efficiencies to increase with free area.

Finch and Van Winkle (1964), reviewing the studies of other investigators, presented a statistical correlation of effect of tray variables on efficiency and confirmed the results found earlier. They showed that point efficiency increased with liquid path length and was independent of hole diameter.
Several workers, Teller et al. (1963), Kharbanda and Ju Chin Chu (1970), etc., have found the plate efficiency to increase to a maximum and then decreases with increasing vapour velocity. Others still, Ashley and Haselden (1973) and Lockett et al. (1979), have found a decrease in plate efficiency with an increase in superficial gas velocity. Lockett and Uddin (1980) found an increase of $E_{ML}$ and a decrease in $E_{MV}$ with increased superficial gas velocity.

Thomas and Haq (1976) and Thomas and Ogboja (1982) have reported that in general, tray efficiency decreased as the liquid rate increased.

Other investigators have studied the effect of system physical properties, chiefly surface tension, on tray efficiency. Systems studied have been classified into positive and negative groups, depending on the direction of the surface tension change. The most general conclusion drawn by these investigators, Zuiderweg and Hermens (1958), Barker and Choudhury (1959), Bainbridge and Sawistowski (1964), Ellis and Biddulph (1967), amongst others, was that there is a reduction in plate efficiency caused by changes in liquid viscosity. Explanations offered for this phenomenon have varied from variations in interfacial area (Zuiderweg and Hermens), interfacial turbulence effects, thermal effects to variations in individual phase resistances. Balcowski (1963) believed that viscosity itself had no significant effect on the rate of mass transfer and that the observed decrease in tray efficiency was due to a decrease of solubility coefficient of the solute in the solvent.
2.2.2 Other Definitions and Correlations of Tray Efficiency

Bakowski (1969) has defined a liquid-phase efficiency in terms of solute concentrations:

\[ E_L = \frac{C_i - C_o}{C_i - C^*} \]  

(2.63)

where \( C_i \) and \( C_o \) are the concentrations of solute in liquid entering and leaving the tray respectively.

Nord (1946) gave a correlation of the Murphree vapour efficiency in terms of a local diffusion efficiency, \( E_D \).

\[ \frac{Y_2 - Y_1}{Y_i - Y_1} = E_D = 1 - \exp \left( -\frac{DPS}{RTLV'} \right) \]  

(2.64)

where

- \( D \) = diffusivity
- \( P \) = total pressure
- \( S \) = interfacial area of bubble
- \( \tau \) = total time of contact on plate
- \( L \) = diffusion film thickness
- \( V' \) = moles of vapour in bubble

Carey (1930) defined a temperature or thermal efficiency:

Vapour phase:

\[ E_{T_v} = \frac{T_n - T_n + 1}{T_n^* - T_n + 1} \]  

(2.65)

where

- \( T \) = temperature of vapour
- \( T_n^* \) = vapour temperature in equilibrium with that of the liquid leaving the \( n^{th} \) tray
Liquid phase:

\[ E_{TL} = \frac{T_n - T_{n-1}}{T_n^* - T_{n-1}} \]  

(2.66)

where

\[ T_n^* = \text{temperature of liquid in equilibrium with that of the vapour leaving the } n^{\text{th}} \text{ tray.} \]

Nord (1946) found an agreement within 0.5% between \( E_D \) and \( E_T \).

2.3 Mixing Effects on Tray Efficiencies

The degree of liquid mixing on a tray has an important bearing on the Murphree efficiency as a result of a concentration gradient in the liquid on the tray. The concentration gradient is the result of the degree of backmixing on the plate.

This was first studied by Lewis (1936) who derived a relationship for the Murphree overall tray and point efficiencies and showed that if the liquid on the tray was completely mixed, then

\[ E_{MV} = E_{OG} \]  

(2.67)

and if there was incomplete mixing,

\[ E_{MV} = \frac{\exp (\lambda E_{OG}) - 1}{\lambda} \]  

(2.68)

He also showed that in the absence of total mixing, \( E_{MV} \) may be greater than 100% though \( E_{OG} \) is always less than 100%.

This is even more so in commercial-size trays where only
partial mixing of the liquid occurs. It is therefore desirable for design purposes, to investigate quantitatively, the mixing parameters if plate efficiencies are to be predicted with any confidence.

Several concepts have emerged to explain the phenomenon of liquid mixing and its effect on the tray efficiency.

2.3.1 Mixed Pool Model

Kirschbaum (1948) described liquid mixing on plates in terms of a number of perfectly mixed pools. He assumed the plate to be divided in the direction of flow into a number of discrete, equal-sized, perfectly mixed pools through which the liquid flowed until it reached the exit weir, the reasoning being that a plate with a single pool corresponded to a perfectly-mixed plate, while one with an infinite number of pools corresponded to an unmixed plate. When applied to sieve plates, he made no attempt to relate the model to any actual flow process on the plates, rather it was offered as a convenient numerical parameter.

Gautreaux and O'Connell (1955) revived this concept and presented an equation relating point and plate efficiencies:

$$E_{MV} = \lambda^{-1} \left[ \left( 1 + \frac{\lambda E_{OG}}{n} \right)^n - 1 \right]$$

where

$\lambda$ = ratio of the slopes of operating and equilibrium lines

$n$ = number of pools
Goutreaux and O'Connell also showed that the liquid-mixing effect tends to become more important with an increase in liquid path length. It is interesting to note from equation (2.69) that when the concept of a perfectly-mixed pool was applied, that is, \( n = 1 \),

\[
E_{MV} = E_{OG} \quad (2.70)
\]

and when \( n = \infty \). An unmixed plate,

\[
E_{MV} = \lambda^{-1} \left( \exp(\lambda E_{OG}) - 1 \right) \quad (2.71)
\]

thus confirming the earlier work of Lewis (1936).

Molokanov (1968) also used this pool concept, assuming that the liquid is divided into \( n \) layers within which the vapour is completely mixed. The degree of mixing is characterised by the number of completely mixed sections. For binary mixtures, he derived the relationship:

\[
E_{OG} = 1 - \left( 1 - \frac{N_{OG}}{n} \right)^{-n} \quad (2.72)
\]

He states that for complete mixing, \( n = 1 \), and

\[
E_{OG} = \frac{N_{OG}}{1 + N_{OG}} \quad (2.73)
\]

while for no mixing or complete displacement of the vapour,

\[
E_{OG} = 1 - \exp(-N_{OG}) \quad (2.74)
\]

2.3.2 Recycle Stream Concept

This was proposed by Oliver and Watson (1956) to define the mixing parameter in terms of a concentration "jump" at the outlet weir. They assumed that liquid mixing is affected
by a certain fraction of the liquid at the outlet weir being recirculated to the inlet weir where it is mixed with the incoming liquid. Their experimental determination of the parameter which characterises the mixing unfortunately depended on concentration samples being obtained in the area immediately preceding and following the inlet weir; an area noted for its instability. The derived relationship between the point and overall efficiencies was given as:

$$E_{OCT} = \frac{\ln [1 + \lambda (1 - F) E_{MV}]}{\lambda (1 - F)}$$

(2.75)

where

- $E_{OCT}$ = true local efficiency
- $F$ = fractional mixing concentration change across inlet weir directed by total change across plate

The fractional mixing parameter, $F$, was considered to be a function of the power input per unit volume for a given geometrical shape and a given system. Oliver and Watson's results showed a large amount of scatter and the authors admitted that the accuracy of their mixing correlation was questionable.

2.3.3 The "Splashing" Concept

This was introduced by Johnson and Marangozis (1958) while carrying out mixing studies on 0.75 ft$^2$ perforated trays with 1/8 inch hole diameter and varying number of perforations. They considered splashing of the liquid to be the mechanism of mixing on the tray. A visual observation revealed a thin liquid layer at the bottom of the tray.
while the remainder of the liquid hold-up was in the form of eddies and spray or foam which extended from the liquid layer to the top of the froth, swirling and falling back again on to the plate. This turbulent motion was supposed to cause the liquid to "splash" from place to place, mixing with adjacent liquid, upstream or downstream. They established a relationship between point and plate efficiencies in terms of a mixing factor, $\beta$, due to liquid splashing:

$$E_{MV} = \frac{EOG (1 - \exp (-\theta_1 Z_0))}{\theta_1 Z_0}$$  \hspace{1cm} (2.76)

$$\theta_1 = \frac{1}{2\beta} - \sqrt{\frac{1}{4\beta^2} + \frac{\lambda EOG}{\beta Z_0}}$$  \hspace{1cm} (2.77)

where

- $Z_0 =$ length of tray (in)
- $\beta = Q_F Z_F - Q_B Z_B$
- $Q_F, Q_B =$ fraction of liquid rate splashing, downstream and upstream respectively
- $Z =$ distance measured on the plate (in)

Johnson and Marangozis correlated the mixing factor with an average deviation of $\pm 14\%$ by an empirical equation.

2.3.4 Eddy Diffusion Model

This model supposes that liquid mixing on a plate can be interpreted by the simple laws of diffusion. When turbulent conditions exist in a fluid, mass transfer by eddies is considered to be analogous to molecular diffusion. This eddy diffusion mechanism could then be used on sieve trays if it is
assumed that the vapour-liquid interaction produces random eddies. The rate of transfer of a component, downstream through a unit area, by eddy diffusion is given by:

\[-D_e \frac{dc}{dw} \quad \text{or} \quad -D_e \rho \frac{dx}{dw}\]  

(2.78)

where

\[D_e = \text{eddy diffusivity (ft}^2/\text{sec)}\]

The eddy diffusivity represents quantitatively the degree of turbulence in the dispersion and a knowledge of this coefficient is necessary for the application of any method based on this model.

For the case of a completely mixed inlet vapour, workers at the University of Delaware (1958) have derived an expression embracing the eddy diffusion coefficient in a Peclet number, Pe, based on foam velocity and length of plate.

\[E_{MV} = \frac{1 - \exp - (n + Pe)}{(n + Pe) \left[1 + (n + Pe)/\eta \right]} + \frac{\exp \eta - 1}{\eta \left[1 + \eta/(n + Pe) \right]}\]

(2.79)

where

\[\eta = \frac{Pe}{\xi} \left\{ \left[ 1 + \frac{4\lambda EOG}{Pe} \right]^{\frac{1}{2}} - 1 \right\}\]

(2.80)

and

\[Pe = \frac{u Z_L}{D_e} \quad \text{or} \quad \frac{Z_L^2}{D_e t_L}\]

(2.81)

where

\[u = \text{average velocity of liquid flowing across tray (ft/sec)}\]

\[Z_L = \text{length of plate (ft)}\]

\[t_L = \text{average liquid residence time}\]
For sieve trays, the Bubble Tray Manual gives the eddy diffusion coefficient, $D_e$ in a correlated form as:

$$ (D_e)^{0.5} = 0.0124 + 0.0171 U_c + 0.0025 L + 0.015 h_w \quad (2.82) $$

where

$U_c$ = superficial gas velocity (ft/sec)

$L$ = liquid flow rate of path (U.S. gall/min ft)

$h_w$ = weir height (in)

As equation (2.79) is somewhat cumbersome to use, numerical solutions at selected values of $P_e$ and $E_{OG}$ were worked out on a digital computer and the results presented in graphical and tabulated form.

The eddy diffusion model was extended by Gilbert (1959) using the frequency response method earlier adopted by Kramers and Alberda (1953) who showed that a characteristic mixing parameter could be determined for a continuous flow system by comparison of the experimental and theoretical frequency responses, if it was assumed that the mixing process can be represented mathematically as a combination of plug flow and longitudinal mixing. Gilbert derived a cumbersome relationship relating the point and tray efficiencies. He introduced a "modified" Peclet number which was correlated as a function of froth density, and for sieve plates was given by:

$$ \frac{D}{uh} = 0.25 \left( \frac{h_f}{h} \right)^3 \frac{1}{u} \quad (2.83) $$

where

$D$ = longitudinal eddy diffusivity in the liquid (cm$^2$/sec)

$h_f$ = froth height (cm)
u = mean liquid velocity (cm/sec) = $Z_L/t_L$

h = clear-liquid height (cm) after the collapse of froth = $V/w Z_L$

w = plate width (cm)

Barker and Self (1962) have measured the degree of mixing in terms of a longitudinal eddy coefficient by an unsteady- and a steady-state tracer technique, on a 5 ft 8 in. by 13½ in. sieve plate with 3/4 inch diameter holes. They correlated their data by the following equation:

$$D_E = 0.01298 U c^1 n^1 + 0.3024 h_L - 0.0605 \quad (2.84)$$

where

$$h_L = \text{liquid hold-up on the plate (ft)}$$

Barker and Self compared the results obtained with those of Gilbert (1959), Foss et al. (1958), and the A.I.Ch.E. University of Delaware Report (1958). They found that all the results could be correlated with this relationship.

$$\frac{2 D_E}{V^3 h_L} = C_1 \left( \frac{h_L V}{h_f} \right) - C_2 \quad (2.85)$$

where

$$C_1 \text{ and } C_2 \text{ are constants, functions of weir height}$$

$$h_f = \text{froth height (ft)}$$

$$V = \text{froth velocity} = L_w/h_L \text{ (ft/sec)}$$

$$L_w = \text{liquid rate (ft}^3/\text{sec per ft weir length)}$$

Sterbaceck (1968) reviewed some published prediction methods for the dispersion coefficient:

\[
(D_L)^{0.65} = 0.0038 + 0.0171 \frac{U_c}{U_{L_v}} + \frac{0.066L_v}{\frac{1}{2}(d_k + W)} + 0.1799 \ h_w
\]

(2.86)

where

- \(D_L\): liquid dispersion coefficient (m\(^2\)/s)
- \(L_v\): volumetric flowrate of liquid (m\(^3\)/s)
- \(d_k\): column diameter (m)
- \(W\): weir length (m)
- \(h_w\): weir height (m)

2. Barker and Self (1962):

(a) \(h_w > 1\) inch

\[
D_L = 1.66 \times 10^{-3} (1 - Y)^{-3} U_L^{0.69} \ h_o
\]

(2.87)

(b) \(h_w = 1\) inch

\[
D_L = 3.58 \times 10^{-4} (1 - Y)^{-3} U_L^{-0.02} \ h_o
\]

(2.88)

where

- \(Y\): fractional gas hold-up
- \(U_L\): liquid velocity (m/s)
- \(h_o\): true clear liquid height (m)

3. Harada (1962):

\[
D_L = 0.011 \ h_f \ U_G (U_{OG} \ d_o)^{-0.37} Y^{-1}
\]

(2.89)

where

- \(h_f\): froth height (m)
- \(U_G\): gas velocity (m/s)
- \(U_{OG}\): gas velocity through perforations (m/s)
- \(d_o\): diameter of perforations (m)
4. Danilchev, Planovskii and Chekhov (1964):

\[ \text{Pe}_L = 0.22 \text{Re}_L^{0.5} \text{Re}_G^{0.5} \] (2.90)

where

\[ \text{Pe}_L = \text{liquid Peclet number} = \frac{U_L L_t}{D_L} \]
\[ L_t = \text{plate length (m)} \]
\[ \text{Re}_j = \text{Reynolds number (j = gas or liquid)} = \frac{U_j \rho h_0}{\mu} \]
\[ U_j = \text{velocity of gas or liquid (m/s)} \]
\[ \mu = \text{viscosity (kg/ms)} \]

Sterbaceck (1968) found that all the relationships showed a general inconsistency with each other and proposed a correlating equation for liquid Peclet number given by:

\[ \text{Pe}_L = \frac{K_1 [(1 - Y) \rho_L + Y \rho_G]}{K_2 (1 - R_0)^2 U_G^2 \rho_G + \rho_L h_0 g_c} \] (2.91)

where

\[ K_1, K_2 = \text{constants, 42.1 and 1.14 for sieve trays resp.} \]
\[ \rho_L, \rho_G = \text{liquid and gas densities respectively (kg/m}^3) \]
\[ R_0 = \text{plate free area} \]

Sterbaceck claimed a mean deviation of \( \pm 15.5\% \) for sieve trays.

Kafarov et al. (1968) claimed that the tray efficiencies found experimentally differed from efficiencies calculated by the diffusion model and explained that this was due to the fact that the diffusion model does not suitably account for the mixing process. They proposed a combined model which consists of successive connecting cells in a zone of complete mixing with a length of 100-150 mm, and a zone in which the mixing is described by the diffusion model. They related the
point and tray efficiencies by the following equation:

\[ \frac{E_{MV}}{E_{OG}} = K \cdot A + (1 - K) B \quad (2.92) \]

where

\[ K = \text{fraction of the tray active area where there is complete mixing of the liquid} \]

\[ A = \frac{\exp(-\beta - \text{Pe})}{1 + \frac{\beta + \text{Pe}}{\beta}} + \frac{\exp(\beta)}{1 + \frac{\beta}{\beta + \text{Pe}}} \quad (2.93) \]

\[ B = \frac{1 - \exp(-\beta - \text{Pe})}{(\beta + \text{Pe}) \left(1 + \frac{\beta + \text{Pe}}{\beta}\right)} + \frac{\exp(\beta - 1)}{1 + \frac{\beta}{\beta + \text{Pe}}} \quad (2.94) \]

and

\[ \beta = \frac{\text{Pe}}{2 \left(1 + 4 \lambda \frac{\text{EOG}}{\text{Pe} - 1}\right)^{0.5}} \quad (2.95) \]

These equations reduce to equations (2.79) and (2.80) obtained by the University of Delaware if the mixing process is simply by eddy diffusion.

2.3.5 The Residence Time Concept

The degree of mixing on a sieve plate can also be characterised by measurement of the residence times of the liquid elements flowing across the plate. This developed from the publication of Danckwerts (1953) on the residence time concept of continuous flow systems.

Foss (1957) used this approach in his research into liquid mixing on sieve plates by applying a step change to the inlet salt concentration and obtaining diagrams of salt con-
centration, expressed as a fraction of the step change, against time at various distances from the inlet (C-diagram).

Foss, Gerster and Pigford (1958) used this distribution of residence time concept to derive expressions for the tray efficiencies based on the assumptions that the gas passes uniformly through the liquid in plug flow and that the operating and equilibrium lines are straight. For the vapour phase they obtained:

\[ E_{MV} = \frac{1 - \int_0^\infty \exp (-\lambda E_{OG} t/\tau) f(t) \, dt}{\lambda \int_0^\infty \exp (-\lambda E_{OG} t/\tau) f(t) \, dt} \]  \hspace{1cm} (2.96)

and for the liquid phase:

\[ E_{ML} = \frac{1 - \int_0^\infty \exp (-\lambda E_{OG} t/\tau) f(t) \, dt}{1 - \lambda^{-1} \left[ 1 - \int_0^\infty \exp (-\lambda E_{OG} t/\tau) f(t) \, dt \right]} \]  \hspace{1cm} (2.97)

where

\[ \tau = \text{mean age of the fluid elements leaving tray (sec)} \]

Calculation of the plate efficiency by equations (2.96) and (2.97) requires detailed knowledge of the distribution function and this becomes tedious when the information is in tabular or graphical form. The authors offered a simplified method by employing an approximate analytical representation of the distribution function. They correlated the measured residence time in the form:

\[ f(\theta) = a \theta^\beta \exp (\gamma \theta) \]  \hspace{1cm} (2.98)

where

\[ \theta = \text{dimensionless time} = t/\tau \]

\[ a, \beta, \gamma = \text{constants} \]
and proposed for the case of liquid phase controlling mass transfer resistance,

\[ F_{ML} = 1 - (1 + N_L \sigma^2)^{-1/\sigma^2} \quad (2.99) \]

where

\[ \sigma^2 = \text{dimensionless variance} \]

\[ N_L = \text{number of liquid-phase transfer units} \]

\[ N_L = \frac{K_L a Z_f}{L} \]

\[ K_L a, \text{ the liquid-phase mass transfer coefficient was measured by a steady state method with a large pool of completely mixed froth so as to be representative of mass transfer on large-scale large-scale trays. They derived an equation of the form:} \]

\[ K_L a = 1197 - 3478 Z_f + 18900 \phi + 1654 \phi Z_f - 43160 \phi^2 \quad (2.100) \]

where

\[ \phi = \text{froth density} = Z_c/Z_f \]

\[ Z_c = \text{height of clear liquid on tray (ft)} \]

\[ Z_f = \text{froth height (ft)} \]

Campbell (1965) and Thomas and Campbell (1967) have extended the Foss et al. approach to derive an equation for \( F(\theta) \) using equation (2.98) as a suitable function for correlation. The derived equation is:

\[ f(\theta) = \frac{\theta(1-\sigma^2)/\sigma^2}{\sigma^2/\sigma^2} \cdot e^{-\theta/\sigma^2} \cdot \Gamma(1/\sigma^2) \quad (2.101) \]

where

\[ \Gamma = \text{gamma function} \]
Substitution of equation (2.101) into equation (2.97) yielded the relationship between \( E_{ML} \) and \( N_L \), and when \( \lambda \) is much greater than one, they showed that equation (2.97) is reduced to the form given in equation (2.99).

Shore and Haselden (1969) recalculated the results of Foss (1957) to yield values of the eddy diffusion coefficient which was correlated by:

\[
D_e = 0.48 \, H \cdot K_s^{0.63}
\]  

(2.102)

where

\[
H = \text{foam height (ft)}
\]

\[
K_s = \frac{U_c}{(\rho_y/\rho_L - \rho_y)^{0.5}}
\]

Levenspiel and Smith (1957) showed that the variance can be conveniently related to the longitudinal dispersion coefficient. Van der Laar (1958) used a similar eddy diffusion approach to solve equations for a variety of systems by Laplace transforms. These authors' works depended on being able to represent the tracer injection by a delta function, a mathematical idealisation which physically can only be approximated since it requires a finite amount of tracer to be injected in zero time.

To circumvent the inherent difficulty of using a delta function, Aris (1959), Bischoff (1960) and Bischoff and Levenspiel (1962) have described a method that does not require a perfect delta function input. The method involves taking concentration measurements at two points, both within the test section, rather than at only one point. The variances of the experimental concentration curves were cal-
culated as before and their differences found. The difference was related to the dispersion coefficient. Bischoff and Levenspiel (1962) also presented design charts which may be used to estimate the position of the measuring point sufficiently far from the end of the system to obviate end effects.
CHAPTER THREE

THEORETICAL CONSIDERATIONS

3.1 Gas Absorption Theories

Most of the theories developed are based on the following assumptions:

1. The total resistance to mass transfer is the sum of the resistance of each phase and the resistance of the interface. However, in many cases the interfacial resistance may be taken as negligible. In this case, the total resistance to mass transfer is the sum of the resistance of the individual phases.

2. Equilibrium is established at the interface and this is more rapid than the change of average concentration in either bulk fluid.

The two major models are the film theory and the penetration theory.

3.1.1 Film Theory

This assumes that a region exists between the two phases where steady state molecular diffusion is the transfer mechanism. Although this gross oversimplification of conditions at the interface is the weakest element of the theory, that is, the posting of a stagnant film of a definite thickness, the theory does give an insight into the process of interphase mass transfer. Fig. 3.1 illustrates this phenomenon.
Fig. 3.1 Concentration Distribution in Phases during Mass Transfer
According to the theory, steady-state mass transfer occurs by molecular diffusion across a stagnant or laminar-flow film at the interphase between the phases. It is assumed that the total resistance to mass transfer resides in the film with a mass concentration gradient arising only inside this film in bulk fluids.

The mass transfer across a unit area of interface per unit time is given by:

\[ N_A = -\frac{D}{\delta_{eff}} (C_o - C_i) = -K (C_o - C_i) \]  \hspace{1cm} (3.1)

where

\( C_o, C_i \) = average concentrations in bulk fluid and interface respectively

\( D \) = molecular diffusion coefficient

\( \delta_{eff} \) = effective film thickness

\( K \) is a mass transfer rate coefficient given by:

\[ K = \frac{D}{\delta_{eff}} \] \hspace{1cm} (3.2)

Equation (3.1) reveals a major flaw of the model by indicating direct proportionality between the mass transfer rate and the molecular diffusion coefficient. In practice, turbulence in the bulk only gradually diminishes as the film surface is approached, and consequently the transition from eddy diffusion (where \( N_A \propto D^0 \) or mass transfer rate is dependent on molecular diffusion) to molecular diffusion (where \( N_A \propto D^{10} \)) is gradual. A more reasonable value for the exponent of \( D \) in equation (3.1) would be some value between zero and unity. This is in fact found to be the
the case and this discrepancy is a direct result of assuming a definite film thickness $\delta_{\text{eff}}$.

3.1.2 The Penetration Theory

This model assumes that the interface is continually being impinged upon by eddies, and that in these eddies, mass transfer is controlled by an unsteady time-dependent process.

The Higbie penetration or surface renewal model (1935) assumes mass transfer to occur during brief, repeated contacts of the phases with the interface, the motion being generated by turbulent fluctuations in the bulk fluid. Fresh liquid elements continually replace those interacting with the interface, and consequently mass transfer is effected by the interface being systematically renewed. The exposure time of the fluid elements to mass transfer effects at the interface is so short that steady-state is never attained. Any transfer is due to unsteady molecular diffusion. The eddies are all assumed to remain at the interface for the same time duration, $\tau$. For this time duration, gas diffuses into the liquid eddy in a manner described by the diffusion equation:

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial y^2}$$  

(3.3)

where

- $c =$ local gas concentration
- $y =$ distance from the interface

For a small diffusion rate, and assuming that the time duration $\tau$ is small, if boundary conditions are applied, the average
rate of mass transfer is then:

\[ N_A = \int_0^{\tau} \frac{(C_i - C_0)(D/\pi t)^{1/2}}{\tau} \, dt = 2 \left( C_i - C_0 \right) \left( \frac{D}{\pi \tau} \right)^{1/2} \quad (3.4) \]

This gives a mass transfer coefficient for the model,

\[ K = 2 \left( \frac{D}{\pi \tau} \right)^{1/2} \quad (3.5) \]

Compared to equation (3.2) for the film theory, this theory gives the desired \( N_A \propto D^{1/2} \).

3.1.3 Random Surface Renewal Theory

This theory, proposed by Danckwerts (1951), is a modification of the penetration theory. The theory eliminates assumption of each fluid element being exposed to mass transfer at the interface for a certain time duration, \( \tau \). It proposes that the fluid elements can have a surface residence time from zero to infinity. This results from the assumption that the probability of any given surface being replaced by another is independent of how long it has been on the surface. If \( s \) is the fractional rate of renewal of elements of any age group, then

\[ f = s \exp(-st) \quad (3.6) \]

where

\[ s = \text{probability density for any given element of area to be exposed to the surface for a time } t, \text{ before being replaced.} \]

By combining equation (3.6) with the mass transfer rate due to one fluid element's residence time \( t \),
Thus the steady state mass transfer rate per unit interfacial area is then obtained from:

\[ N_A = -D \left( \frac{\partial c}{\partial y} \right)_{y=0} = (C_i - C_o) \left( \frac{D}{\pi \tau} \right)^{\frac{1}{2}} \]  

(3.7)

Thus, the mass transfer coefficient is given by:

\[ K = (Ds)^{\frac{1}{2}} \]  

(3.8)

This equation still has an unknown quantity, \( s \), like the earlier model for which the time duration, \( \tau \), was also unknown. Unlike \( \tau \), which cannot be easily obtained due to the residence times not being constant in reality, \( s \) has been obtained under controlled conditions (Lamb et al., 1969).

3.1.4 Further Theories

A modification of the Random Surface Renewal theory, proposed by Hanratty (1956) leaves the probability function as the unknown factor (instead of its functional argument, \( s \)), and equation (3.8) then becomes:

\[ K = \frac{1}{(\pi)^{\frac{1}{4}}} \int_{0}^{\infty} \phi(t) (D/t)^{\frac{1}{2}} \, dt \]  

(3.10)

Averaged over all time durations, \( \tau \), and assuming this probability function to take the form

\[ \phi(\tau) = A e^{-({\tau}/T)^n} \]  

(3.11)

where \( n, A, T \) are constants, the obvious condition on \( \phi \) is given by:
\[ \int_0^\infty \phi(t) \, dt = 1 \] 

(3.12)

The solutions of equations (3.10), (3.11) and (3.12) generate the following relations between the constants \( A \) and \( T \), and hence between the mass transfer coefficients \( K \) and \( T \).

\[
\begin{align*}
n = 1: & \quad A = \frac{1}{T}, \quad K = (D/T)^{\frac{1}{2}} \\
n = 2: & \quad A = \frac{2}{\pi^{\frac{1}{2}} T}, \quad K = \frac{\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{1}{4}\right)} \left(\frac{D}{T}\right)^{\frac{1}{2}} \\
n = 3: & \quad A = \frac{3}{T \Gamma\left(\frac{1}{3}\right)}, \quad K = \frac{\Gamma\left(\frac{2}{3}\right)}{\Gamma\left(\frac{1}{3}\right)} \left(\frac{D}{T}\right)^{\frac{1}{2}} \\
n = 4: & \quad A = \frac{4}{T \Gamma\left(\frac{1}{4}\right)}, \quad K = \frac{\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{1}{4}\right)} \left(\frac{D}{T}\right)^{\frac{1}{2}}
\end{align*}
\]

(3.13)

and equation (3.5) for \( n = 0 \). For known values of the mass transfer coefficient, the case \( n = 0 \) and \( n = 1 \) provide the best correlation with measured concentration distributions.

Another approach developed by Toor and Marchello (1963) tries to unify the film theory and the penetration model into one theory by noting that in the film theory \( K \propto D \), whereas in the penetration model, \( K \propto (D)^{\frac{1}{2}} \), with the assumption that the fluid elements or eddies have effectively infinite depth. Hence \( K \propto D^n \), where \( n \) is dependent on the physical conditions, and the two models might be linked by either allowing for a finite liquid element thickness or for eddies of a limited time duration. The authors found that for large values of \( D \) and \( \tau \) the Higbie model decomposes to the film model (rapid penetration), and conversely, for small \( D \) and \( \tau \), this film penetration model approaches the penetration model. With this approach, instead of equation (3.8) the mass transfer
rate becomes:

\[ K = (D)^{\frac{1}{4}} \coth \left( \frac{S \gamma_e^2}{D} \right)^{\frac{1}{4}} \] (3.14)

where \( \gamma_e \) is the finite element thickness.

Rosen and Krylov (1966) have modified the film theory by taking account of the actual conditions near the interface. Their theory, known as the Border Diffusion Layer Model, considers the absence of a clearly defined layer thickness, effect of liquid motion on mass transfer, and molecular and convective diffusion both radially and tangentially. They proposed a four-layered structure: (1) a main turbulent stream; (2) a turbulent boundary layer; (3) a viscous sublayer; and (4) a diffusion sublayer. Transfer of matter is mainly by turbulent eddies from (1) to (2), momentum transfer by molecular viscosity from (2) to (3) and molecular diffusion from (3) to (4). Although transfer by turbulent eddies is present in all four layers, its effect decreases rapidly from layer (1) to (4).

3.2 Choice of Model

All the more important models make an interpretative analysis of absorption apparatus performance possible, even though they might not be strictly true. It was found from the proposed theories that for the steady-state film theory

\[ K \propto D \] (3.15)

and for the unsteady-state model (penetration, surface renewal)
the unsteady-state models would generally be more favoured, because (a) they are more representative of the real physical situation, and (b) the dependence of mass transfer rate on the square root of diffusivity is borne out by observations. Also, the film model takes no account of fluid-mechanical aspects. Nevertheless, the principle of additivity of resistances is still quite a useful concept for obtaining at least approximate results, and was satisfactorily used by the A.I.Ch.E. research workers at the University of Michigan (1960), and the principle underlies modern methods of designing plate columns for absorption processes.

3.3 Additivity of Resistances

Mass transfer equations are usually expressed by a general form given by:

\[
\text{Rate} = \text{Conductance} \times \text{Driving force}
\]

Rearranging equations (1.21) and (1.22), with corresponding equations for individual mass transfer coefficients, and integrating over the length of gas travel, the rate of mass transfer, \(N_A\), is given by:

\[
N_A = K_{OG} a (y^* - y) PA \, dZ = K_G a (y_1 - y) \, PA \, dZ \quad (3.17)
\]

and

\[
N_A = K_{OL} a (x - x^*) C_T A \, dZ = K_L a (x - x_i) C_T A \, dZ \quad (3.18)
\]

\(K_{OG}, K_G, K_{OL}\) and \(K_L\), the mass transfer coefficients are measures of conductance, the reciprocal of which is resistance. Each system is represented by four resistances, one overall.
and one individual, each for the gas and liquid phases. Equations (3.17) and (3.18), based on the two-film model, give rise to the so-called "addition of resistances".

For a system for which the equilibrium relationship is linear over the range of concentrations being considered

\[ y^* = m x + b \]  

(3.19)

For no resistance to mass transfer at the interface, \( y_i \) is in equilibrium with \( x_i \), and

\[ y_i = m x_i + b \]  

(3.20)

Subtracting equation (3.20) from equation (3.19):

\[ y^* - y_i = m(x - x_i) \]  

(3.21)

or

\[ y^* - y_i + y_i - y = y^* - y = m(x - x_i) + y_i - y \]  

(3.22)

Thus,

\[ \frac{N_A}{K_{OG} \, a \, P \, A \, dZ} = \frac{N_A}{K_L \, a \, C_T \, A \, dZ} + \frac{N_A}{K_G \, a \, P \, A \, dZ} \]  

(3.23)

Rearranging equation (3.23),

\[ \frac{1}{K_{OG} \, a} = m \frac{P/C_T}{K_L \, a} + \frac{1}{K_G \, a} \]  

(3.24)

Similarly,

\[ (x - x_i) + (x_i - x^*) = (x - x^*) = (x - x_i) + 1/m \, (y_i - y) \]  

(3.25)

and

\[ \frac{N_A}{K_{OL} \, a \, C_T \, A \, dZ} = \frac{N_A}{K_L \, a \, C_T \, A \, dZ} + \frac{1}{m} \cdot \frac{N_A}{K_G \, a \, P \, A \, dZ} \]  

(3.26)

Rearranging,
\[
\frac{m \text{ P/C}_T}{K_{OL} a} = \frac{m \text{ P/C}_T}{K_L a} + \frac{1}{K_G a}
\]  
(3.27)

Combining equations (3.24) and (3.27),

\[
\frac{1}{K_{OG} a} = \frac{m \text{ P/C}_T}{K_L a} + \frac{1}{K_G a} = \frac{m \text{ P/C}_T}{K_{OL} a}
\]  
(3.28)

In terms of the number of transfer units, and from equations (1.29), (1.30), (1.33) and (1.34), it can be shown that:

\[
\frac{1}{N_{OG}} = \frac{1}{N_G} + \left( \frac{m \text{ G}_m}{L_m} \right) \frac{1}{N_L} = \left( \frac{m \text{ G}_m}{L_m} \right) \frac{1}{N_{OL}}
\]  
(3.29)

3.4 Residence Time and Liquid Mixing

3.4.1 Residence Time Distribution Function

The structure of a mathematical model for any chemical engineering process involving fluid flows is affected chiefly by the flow patterns prevailing in the system.

Two types of abstract or ideal flows are commonly used as limiting cases of flow patterns: these are the plug or piston flow and the perfectly mixed or backmix flow. The relevant definitions have been given in an earlier section (Section 1.5). Although most elementary equipment design methods are based on these two ideal, abstract concepts of flow, between these two extremes lie flow patterns found in actual processes. The effect of this deviation from ideality is reflected in the distribution of residence times of fluid elements in their passage through that system. This manifests itself in affect-
ing the mass transfer efficiency of the system and the extent of this deviation can be obtained from the residence time distribution functions.

Quantitative definitions of the residence time distribution functions developed by Danckwerts have been used for obtaining the age distribution functions. The residence time of a fluid element is defined as the time that elapses from the time the element enters the vessel to the time it leaves it. The age of a fluid element at a given instant of time is the time that elapses between the element's entrance into the vessel and the given instant and is of course less or equal to the residence time.

If a fluid flows at a steady rate $Q$ at all time through a vessel of volume $V$, the mean residence time of the fluid is defined as:

$$\bar{\tau} = \tau = \frac{V}{Q} \quad (3.30).$$

3.4.2 The Internal Age Distribution Function: $I(t)$

From the definition of the age of a fluid element in a vessel it is evident that the vessel contains fluid of varying ages. The fraction of fluid elements between ages $t$ and $t + \Delta t$ in the vessel is given by $I(t)\Delta t$, where the functional notation $I(t)$ is the internal age-distribution frequency of the fluid elements. Since $I(t)$ is a continuous function, it is normalised by letting the sum of all fractions be unity:

$$\int_{0}^{\infty} I(t) \, dt = 1 \quad (3.31)$$
This equation implies that all fluid has an age somewhere between 0 and $\infty$.

The fraction of vessel contents younger than a specified age $t$ is

$$\int_0^t I(t') \, dt'$$

while the fraction older than $t$ is

$$\int_t^\infty I(t') \, dt' = 1 - \int_0^t I(t') \, dt'$$

This is illustrated in Fig. 3.2.

![Fig. 3.2 Characteristic Distribution of Ages](image)

3.4.3 The Age Distribution of the Exit Stream: $E(t)$

The residence-time distribution or the age-distribution frequency of the exit stream from a vessel are two different names for the same function, $E(t)$. It is also known as the external age-distribution function.
The fraction of exit age itself is \( E(t) \Delta t \) and the function is normalised so that:

\[
\int_0^\infty E(t) \, dt = 1 \quad (3.34)
\]

The fraction of fluid in the exit stream younger than age \( t \) is given by:

\[
\int_0^t E(t') \, dt' \quad (3.35)
\]

while the fraction older than \( t \) is

\[
\int_t^\infty E(t') \, dt' = 1 - \int_0^t E(t') \, dt' \quad (3.36)
\]

This is illustrated in Fig. 3.3.

![Characteristic Distribution of Residence Times](image)

**Fig. 3.3 Characteristic Distribution of Residence Times Equivalent to the Age-Distribution of the Exit Stream.**

The mean of the residence-time distribution is found from the first moment

\[
\tau = \int_0^\infty t \, E(t) \, dt = V/Q \quad (3.37)
\]

Similarly, the mean age of fluid elements inside the vessel is

\[
\bar{t}_1 = \int_0^\infty t \, I(t) \, dt \quad (3.38)
\]
3.4.4 Relations between the Age Distribution Functions

The age-distribution functions are related through the unsteady-state macroscopic age population balance which expresses the continuity of ages of fluid elements.

Calling all fluid entering the vessel at \( t > 0 \) "new" fluid and the existing contents at \( t = 0 \) "old" fluid, then at some time \( t \), the fraction of "new" fluid is

\[
\left\{ \text{Amount of "new" fluid in vessel} \right\} = V \int_0^t I(t') \, dt' \quad (3.39)
\]

and the amount of "old" fluid that has left the vessel at all times from \( 0 \) to \( t \) is

\[
\left\{ \text{Amount of "old" fluid gone from vessel} \right\} = \int_0^t Q \, dt' \int_{t'}^\infty E(t'') \, dt'' \quad (3.40)
\]

then by a mass balance,

\[
V \int_0^t I(t') \, dt' = Q \int_0^t dt' \int_{t'}^\infty E(t'') \, dt'' \quad (3.41)
\]

Differentiating both sides with respect to time, and introducing equation (3.37),

\[
\bar{\tau} I(t) = \int_{t'}^\infty E(t') \, dt' = 1 - \int_0^t E(t') \, dt' \quad (3.42)
\]

Differentiating once more,

\[
E(t) = -\bar{\tau} \frac{d}{dt} I(t) \quad (3.43)
\]

The age-distribution functions can also be expressed in dimensionless form. Dimensionless time is given by:

\[
\theta = \frac{t}{\bar{\tau}} \quad (3.44)
\]
Thus, \( E(\theta) \, d\theta = E(t) \, dt \), and \( I(\theta) \, d\theta = I(t) \, dt \), so that

\[
E(\theta) = \bar{E} E(t) \tag{3.45}
\]

\[
I(\theta) = \bar{I} I(t) \tag{3.46}
\]

and

\[
E(\theta) = -\frac{d}{d\theta} I(\theta) \tag{3.47}
\]

### 3.4.5 Experimental Measurements of Age Distribution Functions

The experimental determination of the age-distribution functions is accomplished by a stimulus-response technique which involves the injection of a tracer material, the stimulus, in the inlet-fluid stream, and the measurement of the concentration in the exit stream, the response. The tracer is injected in a known fashion which may be a step or sudden jump, a pulse, a sinewave or other cyclic signal and even a random signal with known properties. These signals and their responses are shown in Fig. 3.4.

![Fig. 3.4 Stimulus-response techniques](Levenspiel, 1972 p.256)
3.4.6 The F-curve

The dimensionless response to an up-step injection of tracer is called the F-curve. With no tracer initially present, and a step function (in time) of tracer is introduced into the inlet fluid stream, then the dimensionless concentration-time curve for tracer in the exit stream, that is, the F-curve and $\theta$ given by equation (3.44) is shown in Fig. 3.5. The F-curve rises from 0 to 1.

3.4.7 The C-curve

The dimensionless response to an impulse injection of tracer is called the C-curve. With no tracer initially present, if an instantaneous pulse or shot of tracer is injected into the inlet-fluid stream, the dimensionless response-time curve C is shown in Fig. 3.6 The pulse of tracer is mathematically termed an impulse or Dirac delta function.
The area under the C-curve is always unity, or

\[ \int_{0}^{\infty} C \theta \, d\theta = \int_{0}^{\infty} C(\theta) \, d\theta = 1 \]  

(3.48)

so that

\[ C_0 = \int_{0}^{\infty} C \, d\theta = 1/t \int_{0}^{\infty} C(t) \, dt \]  

(3.49)

where \( C_0 \) is the reference concentration of tracer, which can be found from the area under the measured output curve or from a knowledge of the quantity of tracer injected into the pulse and the volume of the vessel.

3.4.8 Relation between the F, C, I and E-curves in Closed Vessels

For a closed vessel, there is a simple relationship between the E and C curves, and the I and F curves. A closed vessel is defined as one in which there is no back diffusion of any sort at the entrance and exit, that is, materials enter and leave solely by plug flow. Most real systems
approximately satisfy this requirement since the inlet and outlet pipes are frequently smaller than the vessel.

Suppose \( t = 0 \) is taken as the instant of a pulse injection, then all tracer elements of fluid have the same starting time for their ages. Thus the outlet concentration time or C-curve is also a record of the age fraction of fluid elements (the tracer) that entered at \( t = 0 \), and left at \( t = t \), that is, the \( E \)-curve. Thus,

\[
C (t) = E (t) = \frac{1}{\tau} E (t)
\]

(3.50)

By an age balance,

\[
\begin{bmatrix}
\text{Amount of tracer} \\
\text{remaining in vessel}
\end{bmatrix}
= \begin{bmatrix}
\text{Amount of tracer} \\
\text{not leaving vessel}
\end{bmatrix}
\]

or

\[
VI (t) = Q [1 - F (\theta)]
\]

(3.51)

From equation (3.46)

\[
I (\theta) = \frac{1}{\tau} I (t)
\]

Substituting for \( I (t) \) in equation (3.51) and noting that \( \tau = V/Q \),

\[
I (\theta) = 1 - F (\theta)
\]

(3.52)

Similarly, from equation (3.42),

\[
\frac{1}{\tau} I (t) = 1 - \int_0^t E (t') dt'
\]

Substituting for \( \frac{1}{\tau} I (t) \) from equation (3.46),

\[
I (\theta) = 1 - \int_0^t E (t') dt'
\]

(3.53)

and substituting for \( I (\theta) \) from equation (3.52), it can be
shown that

$$F (\theta) = \int_{0}^{\theta} C (\theta') d\theta'$$

(3.54)

= fraction of material in exit stream younger than age $t$ (from eq. 3.35)

But from equation (3.50),

$$C (\theta) = \bar{T} E (t)$$

Substituting for $E (t')$ in equation (3.54),

$$F (\theta) = \int_{0}^{\theta} C (\theta') d\theta'$$

(3.55)

also

$$C (\theta) = \frac{dF(\theta)}{d\theta}$$

In summary, the theoretical age-distribution functions, $E (\theta)$ and $I (\theta)$ can be experimentally measured with ease for a closed vessel through the use of the impulse and step responses, $C (\theta)$ and $F (\theta)$, respectively. Table 3.1 summarises the various results:

**Table 3.1** Relations between the age distribution functions

<table>
<thead>
<tr>
<th>Experimental measures</th>
<th>Abstract definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C (\theta) = \frac{dF(\theta)}{d\theta}$</td>
<td>$E (\theta) = \bar{T} E (t) = \frac{-dI(\theta)}{d\theta} = -\bar{T} \frac{dI}{dt}$</td>
</tr>
<tr>
<td>$F (\theta) = \int_{0}^{\theta} C (\theta') d\theta'$</td>
<td>$1 - I (\theta) = 1 - \bar{T} I (dt) = \int_{0}^{\theta} E (\theta') d\theta' = \int_{0}^{\theta} E (t) dt'$</td>
</tr>
</tbody>
</table>
3.4.9 Age-Distribution Functions applied to Perfect Mixing and Plug Flow

Perfect mixing assumes that the contents are perfectly homogeneous and have the same composition as the exit stream. By a material balance

\[ Q C_o = Q C + V \frac{dc}{dt} \]  

or

\[ C_o = C + \bar{T} \frac{dc}{dt} \]

which for \( C \) at \( t = 0 \) is given by:

\[ C/C_o = 1 - (\exp - t/\bar{T}) \]

\[ = 1 - (\exp - \theta) \]  

which by definition is

\[ C/C_o = F (\theta) \]

Consequently,

\[ I (t) = \frac{1}{\bar{T}} [1 - F (\theta)] = \frac{1}{\bar{T}} e^{-t/\bar{T}} \]  

\[ E (t) = \frac{dF (\theta)}{dt} = \frac{1}{\bar{T}} e^{-t/\bar{T}} \]  

\[ I (\theta) = e^{-\theta} = E (\theta) \]

From equation (3.62) the internal age distribution, \( I \), and the exit age distribution, \( E \), are equal and this is true for perfect mixing, since the fluid within the vessel has the same composition as the exiting fluid.

In plug flow, all materials pass through the vessel with no mixing whatsoever. Thus, for a step input, the interface between the tracer and non-tracer fluids "marches"
down the vessel, exiting in a time equal to the mean residence time. Thus the $F(0)$ curve is a step function. A unit step function is defined by:

$$U(t - \bar{t}) = \begin{cases} 0, & t < \bar{t} \\ 1, & t > \bar{t} \end{cases}$$  \hspace{1cm} (3.63)

Thus,

$$F(0) = U(t - \bar{t})$$  \hspace{1cm} (3.64)

and

$$I(t) = 1/\bar{t} \left[ 1 - F(0) \right] = 1/\bar{t} \left[ 1 - U(t - \bar{t}) \right]$$  \hspace{1cm} (3.65)

or

$$E_C = \int_{-\infty}^{\infty} U(t - \bar{t}) dt = \delta(t - \bar{t})$$  \hspace{1cm} (3.66)

$$I(\theta) = 1 - U(\theta - 1)$$  \hspace{1cm} (3.67)

$$E(\theta) = \delta(\theta - 1)$$

where $\delta(x)$ = Dirac delta function or impulse function.

Figs. 3.7(a) and 3.7(b) show the shapes of these curves for perfect mixing and plug flow. For real processes, where the mixing is intermediate between these two ideal conditions, the curves have shapes indicated in Fig. 3.7(c).

3.4.10 Mean and Variance of the Distribution

The mean residence time, $\bar{t}$, is defined by:

$$\bar{t} = \mu = \frac{\int_{0}^{\infty} t C(t) dt}{\int_{0}^{\infty} C(t) dt}$$  \hspace{1cm} (3.68)

$\bar{t}$ is also known as the first moment about the origin, or the mean or centroid of the distribution. The symbol $\mu$ is also commonly used.
Fig. 3.7 Output and Internal Age-Distributions for the Two Ideal Limiting Cases of Mixing and an Intermediate Case
The integrals can be approximately evaluated by summation instead of integration. Thus,

\[ \mu \approx \frac{\sum t \cdot c \cdot \Delta t}{\sum c \cdot \Delta t} \quad (3.69) \]

where \( \Delta t \) is the sampling interval.

For closed vessels, the mean residence time, \( \bar{\tau} \), given by \( V/Q \) is equal to \( \mu \). For real systems or open vessels, the values of \( \bar{\tau} \) and \( \mu \) will be different and this difference may be used as a measure of the deviation from the ideal case of plug flow.

The variance represents the square of the spread of the distribution and it is a measure of the degree of the spread of the residence time about the mean. It is defined by:

\[ \sigma^2 = \frac{\int_0^\infty (t - \mu)^2 C(t) \, dt}{\int_0^\infty C(t) \, dt} \quad (3.70) \]

\[ = \frac{\int_0^\infty t^2 C(t) \, dt}{\int_0^\infty C(t) \, dt} - \mu^2 \quad (3.71) \]

By summation,

\[ \sigma^2 \approx \frac{\sum t^2 \cdot C \cdot \Delta t}{\sum C \cdot \Delta t} - \mu^2 \quad (3.72) \]

When applied to the C, E and I distributions, the denominators of equations (3.68) and (3.71) are unity. Thus, for the C distribution in dimensionless time,

\[ \sigma^2 = \sum \theta^2 \cdot C \cdot \Delta \theta - 1 \quad (3.73) \]
3.4.11 The Dispersion Model

The quantitative shapes of the age-distribution curves make it possible to estimate the degree of mixing in a process, and the variance can usually be used to obtain a quantitative value for comparative purposes.

Many models have been proposed for the characterisation of the degree of mixing and flow patterns in a process.

The basis of the dispersion model is the superimposition of some degree of backmixing on plug flow through a vessel, and obeying the formal law of diffusion. The parameter of this model is the turbulent diffusion coefficient or the longitudinal dispersion coefficient, $D_E$. In developing this one-parameter model, the following assumptions are made.

1. the change in concentration is a continuous function of the distance along the tube;

2. the concentration is constant in a given cross-section;

3. the space velocity of the stream and the longitudinal dispersion coefficient remain unchanged along the length and over the cross-section of the stream.

With these assumptions, the model can be represented mathematically by:

$$ \frac{\partial C}{\partial t} = -U \frac{\partial C}{\partial x} + D_E \frac{\partial^2 C}{\partial x^2} $$

(3.74)

The term $D_E \frac{\partial^2 C}{\partial x^2}$ allows for the turbulent diffusion or dispersion and $D_E$, the longitudinal dispersion coefficient,
can be determined experimentally. In experimental determination, the coefficient is usually presented as a dimensionless group known as the Peclet number, Pe, thus:

\[ \text{Pe} = \frac{UL}{D_E} \] (3.75)

The features of the dispersion number or the reciprocal of the Peclet number, that is, \( D/UL \) has been given by Levenspiel (1972) as follows:

\[ \frac{D}{UL} \rightarrow 0 \quad \text{negligible dispersion, hence plug flow} \]
\[ \frac{D}{UL} \rightarrow \infty \quad \text{large dispersion, hence mixed flow} \]

Equation (3.74) for convenience in analysis, is usually placed in dimensionless form by making the following definitions:

Dimensionless Time:
\[ t/\bar{t} = Ut/L = Qt/V \]
where
\[ Q = \text{volumetric flowrate} \]
\[ V = \text{volume of vessel} \]

Dimensionless Length:
\[ Z = x/L \]
where \( L = \text{length between the injection point and the measurement point} \)

Recalling the assumption of a constant space velocity, equation (3.74) reduces to:

\[ \frac{3c}{\theta} + \frac{3c}{z} = \frac{1}{\text{Pe}} \frac{3^2c}{z^2} \] (3.76)
The analytical solution of equation (3.76) is an aduous task and since Pe must be obtained by experiment for any simulation, an alternative approach of utilising the equation was developed by Levenspiel and Smith (1957). They showed that the variance can be conveniently related to the dispersion coefficient. Bearing in mind that the dispersion coefficient is contained in the Peclet number, the mean and variance could be related to Pe for various boundary conditions. Levenspiel and Bischoff (1963) have given a number of such relations with appropriate references. A summary of these relationships is given here for two of the many boundary conditions encountered with large extents of dispersion: (1) the doubly infinite pipe (or open vessel), and (2) the closed pipe, in Table 3.2.

For small extents of dispersion, with no significant spreading of the tracer, the solution of equation (3.76) is given by:

\[ C_\theta = \frac{1}{2} \left( \frac{Pe}{\pi} \right)^{\frac{1}{2}} \exp \left[ -\frac{Pe (1 - \theta)^2}{4} \right] \]  

(3.77)

and

\[ \mu = 1 \]  

(3.78)

or

\[ \sigma^2 = \frac{2}{Pe} \]  

(3.79)

The solutions of equation (3.76) have also been presented as a series of curves of the dimensionless impulse response concentration, \( C_\theta \), versus time \( \theta \) by Levenspiel and Smith (1957). One such a plot is shown in Fig. 3.8 for the various extents of mixing.
Table 3.2 Relation between moments and Peclet number for models with large extents of axial dispersion

<table>
<thead>
<tr>
<th>Boundary conditions</th>
<th>Solution of equation (3.76)</th>
<th>Moments</th>
</tr>
</thead>
</table>
| (1) Doubly infinite pipe 
Undisturbed flow 
at boundaries 
same $D_E$ everywhere | $C_\theta = \frac{1}{2} \left( \frac{\text{Pe}}{\pi \theta} \right)^{\frac{1}{2}} \exp \left[ - \frac{\text{Pe} \left( 1 - \theta \right)^2}{4 \theta} \right]$ | $\mu = 1 + \frac{2}{\text{Pe}}$ |
|                     | $\sigma_\theta^2 = \frac{2}{\text{Pe}} + \frac{8}{\text{Pe}^2}$ |         |
| (2) Closed pipe 
Plug Flow $D = 0$ 
Plug Flow $D = 0$ | $C_\theta = e \left( \frac{\text{Pe}}{2} \right)^\infty \sum_{n=1}^{\infty} \frac{\left( -1 \right)^{n+1}}{4 \text{a}_n^2 + 4 \text{Pe} + \text{Pe}^2} \cdot e^{-\text{a}_n \theta}$ | $\mu = 1$ |
|                     | $\sigma_\theta^2 = \frac{2}{\text{Pe}} - \frac{2}{\text{Pe}^2} \left[ 1 - \exp(-\text{Pe}) \right]$ |         |
|                     | $\alpha_n = \frac{\text{Pe}^2 + 4 \text{a}_n^2}{4 \text{Pe}}$ |         |
|                     | $\tan \alpha_n = \frac{4 \text{Pe}}{4 \text{a}_n^2 - \text{Pe}}$ |         |
Fig. 3.8 C curves in Closed Vessels as predicted by Dispersion Model (from Levenspiel, 1972 p. 277)
Van der Laan (1958) has also studied the equation for the open-closed vessel configuration. The moments are given by:

\[ u = 1 + \frac{1}{Pe} \tag{3.81} \]

and

\[ \sigma_\theta^2 = \frac{1}{Pe^2} \left( 2Pe + 3 \right) \tag{3.82} \]

Thomas and Campbell (1967) have developed Danckwert's solution of equation (3.76) and established independently that:

\[ \sigma^2 = \frac{2}{U^3} \frac{Z_L D}{U} \left( 1 + \frac{5}{2Pe} \right) \tag{3.83} \]

where

- \( Z_L = \) length of plate (ft)
- \( Pe = \frac{Z_L U}{D} \)
- \( U = \) axial velocity of fluid (ft/sec)

They further established that when

\[ 1 \gg \frac{5}{2Pe}, \text{ then} \]

\[ \sigma^2 = \frac{2 Z_L D}{U^3} \tag{3.84} \]

and

\[ \sigma_\theta^2 = \sigma^2 / \bar{\tau} = \frac{2 \bar{\tau} D}{Z_L^2} \tag{3.85} \]

Since, for small extents of dispersion, the shape of the C-curves is insensitive to boundary conditions, whether open or closed, the variances have been established by Levenspiel (1972) to be additive, i.e.

\[ \sigma_{\text{overall}}^2 = \sigma_a^2 + \sigma_b^2 + \ldots + \sigma_n^2 \tag{3.86} \]

This additive property allows for the treatment of any one-
shot tracer input, no matter what its shape, and also for the extraction from it, the variance of the C-curve of the vessel.

3.5 Combining the Residence Time Distribution Function with Plate Efficiency

Foss et al. (1958) and Campbell (1965) have established that for partial liquid mixing on a tray, it can be shown that:

\[
E_{ML} = \frac{1 - \int_0^\infty \exp (-\lambda E_{OG} \theta) f(t) \, dt}{1 - \lambda^{-1} \left[1 - \int_0^\infty \exp (-\lambda E_{OG} \theta) f(t) \, dt\right]} \tag{3.87}
\]

For a liquid phase controlled system such as carbon dioxide-air-water, the Henry's Law constant, \(m\), will be large and \(\lambda >> 1\). Equation (3.87) therefore approximates to:

\[
E_{ML} = 1 - \int_0^\infty \exp (-\lambda E_{OG} \theta) f(t) \, dt \tag{3.88}
\]

Also,

\[
N_L = \frac{K_L a z_f}{L} = \lambda E_{OG} \tag{3.89}
\]

Thus,

\[
E_{ML} = 1 - \int_0^\infty \exp (-N_L \theta) f(t) \, dt \tag{3.90}
\]

Campbell (1965) has developed an equation for the residence time distribution function, \(f(\theta)\), employing a gamma function, \(r\),

\[
f(\theta) = \frac{\left(\frac{1-\sigma^2}{\sigma^2}\right) \exp (-\theta/\sigma^2)}{\sigma^2 \Gamma (1/\sigma^2)} \tag{3.91}
\]
By substitution into equation (3.90) Campbell obtained for $\lambda >> 1$,

$$E_{ML} = 1 - (1 + N_L \sigma^2)^{-1/\sigma^2}$$

(3.92)

3.6 Sater Tailing Technique for Response Curve Analysis

Levenspiel and Smith (1957) have shown that the tail of the response to a perfect delta function input would decay exponentially, with a long "tail". Skewness of this sort was also found by Sater and Levenspiel (1966) who pointed out the attendant large error in evaluation of the mean and variance when there are small errors in the evaluation of the "tail". They represented the exponential decay as:

$$C(t) = A e^{-qt}$$

(3.93)

where

- $t =$ time after injection of tracer
- $A$ and $q =$ constants

They plotted data from the right-hand side of the peak of the response curve to a "cut-off" point on semi-log coordinates and found the relationship in equation (3.93) to be linear. The line was extrapolated beyond this "cut-off" point and the extrapolated data were used to calculate the contributions of that portion of the curve to the total moments. The following derivations were established for the determination of the mean and variance where the response curve has a "long tail".
The mean of the response curve, \( \mu \), is given by equation (3.68) as:
\[
\mu = \frac{\int_0^\infty t C(t) \, dt}{\int_0^\infty C(t) \, dt}
\]
and the variance by equation (3.71) as
\[
\sigma^2 = \frac{\int_0^\infty t^2 C(t) \, dt}{\int_0^\infty C(t) \, dt} - \mu^2
\]

If the cut-off point commences at a time, \( t_c \), then for \( t > t_c \).
\[
\int_0^\infty C(t) \, dt = \int_{t_c}^\infty A e^{-qt} \, dt = \left[ -A e^{-qt/q} \right]_{t_c}^\infty = \frac{A e^{-qt_c}}{q} \tag{3.94}
\]
\[
\int_{t_c}^\infty C(t) \, dt = \int_{t_c}^\infty A e^{-qt} \, dt = \left[ -\frac{A e^{-qt}}{q^2} (qt + 1) \right]_{t_c}^\infty = \frac{A}{q^2} e^{-qt_c} (qt_c + 1) \tag{3.95}
\]
\[
\int_{t_c}^\infty t^2 C(t) \, dt = \int_{t_c}^\infty t^2 A e^{-qt} \, dt = \left[ -\frac{A e^{-qt}}{q^3} (q^2t^2 + 2qt + 2) \right]_{t_c}^\infty = \frac{A}{q^3} e^{-qt_c} (q^2t_c^2 + 2qt_c + 2) \tag{3.96}
\]

Thus, with Sater's correction applied,
\[
\mu = \frac{\Delta t \sum t_i C_i}{\Delta t \sum C_i} + \frac{A e^{-qt_c}}{q^2} e^{-qt_c} (qt_c + 1) \tag{3.97}
\]
and
\[
\sigma^2 = \frac{\Delta t \sum t_i^2 C_i}{\Delta t \sum C_i} + \frac{A}{q^3} e^{-qt_c} (q^2t_c^2 + 2qt_c + 2) \tag{3.98}
\]
Thus, the first terms of each equation represent the moments when \( t < t_c \) and the second term is the contribution for \( t > t_c \).

Because of the unwieldiness of equations (3.97) and (3.98), and for speedy and accurate determinations, Millington (1971) has written a computer program to calculate the moments, corrected by the Sater Method. For the present work, the program has been modified to average the moments from five separate response curves for each column condition so as to negate the fluctuations that may arise from flow instabilities.
CHAPTER FOUR

EXPERIMENTAL PROGRAMME

It is generally recognised in industry that the use of small diameter perforations in processes where rust and sediment deposition are prevalent either during construction or initial start-up, could lead to maloperation of the column. Small holes are also susceptible to high pressure drops and limited capacity. Normally, in processes where these problems occur, the practice is to use bubble-cap trays.

However, with the accumulation of data and information on the performance of mass transfer apparatus in recent years, design engineers have become more adventurous and have been using sieve trays where previously they would have used, say bubble cap or valve trays. They have also been encouraged to circumvent some of the problems associated with small perforations (blockage, etc.) by using large hole perforated trays. Owing to the limited information available on the performance of these trays, it therefore becomes pertinent that investigations are carried out into the characteristics of these trays.

This research was initiated to investigate the factors affecting the performance of distillation and absorption columns employing an air-water simulator in two sieve tray columns - one rectangular and the other round, and both containing trays with 1 inch diameter perforations.

Preliminary investigations and calibrations were carried
out prior to the start of the main experimental programme, which are conveniently divided into three sections.

4.1 Hydrodynamic Study

This study investigates the factors affecting the vapour handling capacity of the columns. These factors were investigated as functions of the operational and geometric column parameters on the hydrodynamic behaviour of the aerated liquid mass flowing across the sieve tray.

The variables studied were air and liquid flow rates. The effects of the parametric changes on the total plate pressure drop, dynamic liquid head profile across the tray, froth height and liquid entrainment upward from plate to plate were measured. Measurements were also made of the dry plate pressure drop across the sieve tray and a specially designed experimental apparatus.

To simulate industrial conditions, liquid rates up to 3000 gall/hr/ft of weir and air-flowrates up to 40,000 ft³/hr were used.

4.2 Liquid Phase Residence Time Distribution Study

The residence time of a fluid element is the time that elapses from the time the element enters the vessel to the time it leaves it. Liquid flowing across a tray seldom if ever travels from inlet to outlet without mixing in the longitudinal direction. Usually, the mixing is incomplete and a concentration gradient in the liquid phase thus ensues.
The approach to the problem is therefore based on the supposition that mixing of the flowing liquid will cause some of the liquid to reside on the tray for periods longer and/or shorter than the period of residence of some other portions of liquid. In other words, mixing of the liquid should produce an entire spectrum or distribution of liquid residence times. If the actual distribution of residence times could be determined, this would give a good general indication of how much and what type of mixing occurred and a tool to quantitatively measure the degree of liquid mixing. This distribution of residence times could be used further with a good degree of accuracy to predict the performance of the column.

The liquid-phase residence time distribution and therefore the extent of liquid mixing on the tray were studied as a function of liquid and vapour loading, and column geometry. Investigations were conducted on the tray only for the round column, and additionally, tray plus downcomer and downcomer only for the rectangular column.

4.3 Mass Transfer Study

The system air-carbon dioxide-water was used in this study. Although other gases, e.g. nitrogen, ethylene and oxygen could have been used, carbon dioxide was by far the most convenient for the apparatus and method of analysis used. It has a low solubility in water and may be regarded as entirely liquid-phase controlled.
The absorption of carbon dioxide into water was studied as a function of liquid and air flowrates, and column geometry.

Fairly high carbon dioxide concentration in the gas phase (5% by vol.) was employed to obtain efficiency values as accurately as is possible.
CHAPTER FIVE

DESCRIPTION OF APPARATUS

5.1 Primary Equipment

The experimental studies described in the previous and subsequent chapters were all carried out using two columns each with two plates. Many features of the apparatus remained the same throughout each of the major parts of the work, although for the pressure drop and entrainment studies, additional equipment was designed and/or modifications were made, both to the column and trays. The general physical data for the columns are described first while details concerning design of additional equipment, modifications to the perforated plates and column, tracer injection system and the apparatus used in measurements follow later.

5.1.1 Physical Data for Sieve Tray and Downcomer

5.1.1.1 Rectangular Column

A. Sieve Tray

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of tray</td>
<td>3 ft</td>
</tr>
<tr>
<td>Width of tray</td>
<td>1 ft</td>
</tr>
<tr>
<td>Thickness of tray</td>
<td>1/8 in.</td>
</tr>
<tr>
<td>Length of tray weir-weir</td>
<td>24.1 in.</td>
</tr>
<tr>
<td>Perforation diameter</td>
<td>1 in.</td>
</tr>
<tr>
<td>Perforation pitch</td>
<td>3 in. (A)</td>
</tr>
<tr>
<td>Total number of holes</td>
<td>32</td>
</tr>
<tr>
<td>Number of active holes</td>
<td>18</td>
</tr>
<tr>
<td>Length of inlet calming section</td>
<td>5 1/8 in.</td>
</tr>
<tr>
<td>Length of outlet calming section</td>
<td>7 5/8 in.</td>
</tr>
</tbody>
</table>
Distance between last row of holes and column wall
Area of active holes
Area of perforated zone
% free area = Area of active holes/plate active area
Length of weir
Height of weir
Total superficial area
Area available for vapour flow, Net Area, \( A_N = A_c - A_d \)
Tray spacing

B. Downcomer

Height of downcomer
Breadth of downcomer
Inside width of downcomer
Height of downcomer exit
Width of downcomer exit

5.1.1.2 Round Column

A. Sieve Tray

Diameter of tray
Thickness of tray
Perforation diameter
Perforation pitch
Total number of holes
Number of active holes
Length of inlet calming section
Length of outlet calming section
Distance between last row of holes and column wall
Area of active holes
Area of perforated zone
% Free area = Area of active holes/plate active area
Area between weirs
Total superficial area
Length of weir
Height of weir
Tray spacing

5.58 ft²
24 in.
3 in.
40 in.

B. Downcomer

Height of downcomer
Maximum width of segmental downcomer
Height of downcomer exit
Width of downcomer

40 in.
24 in.
3 in.
24 in.

5.1.2 General Features of Columns

The main apparatus consisted of two columns placed side by side mounted in a steel girder framework with a substantial spacious wooden platform and access stair (see Fig. 5.1a,b). In general, it was desired that the columns be made as flexible as possible to permit easy and rapid dismantling and assembling of trays, downcomers and weirs, although this was more difficult to achieve with the round column due to the weight and fragility of its material of construction, and the attendant cost of replacement as a result of damage. On the other hand, the rectangular column, because of its robust and rigid construction was more accessible for dismantling and reassembling.

Both columns had common air and water circuits, with valves being provided in the circuits to enable the ease of switching flow from one column to the other. Each had a separate exit downpipe emptying into a tank placed below it.
Fig. 5.1a The main pilot rig
Fig. 5.1b The absorption columns
The sieve plates had features common to all runs, and were made of 1/8th inch thick stainless steel to which was bolted or soldered the segmental downcomers, weirs, downflow pipes and pressure taps. The perforations in the plates were carefully machined to remove burrs and other imperfections. Stainless steel was chosen to preclude changes in hole diameter as a result of corrosion during prolonged periods of operation.

Segmental downcomers and overflow weirs were chosen to provide single crossflow of liquid on the plate since it was believed this could be representative of industrial practice. Weirs were installed at the entry to each tray to reduce flow irregularities from the upper plate downcomer, as well as to provide a more or less uniform velocity profile across the tray. Headroom was adequate for subsequent studies upon addition of necessary auxiliary equipment. Except for the residence time and entrainment studies, the liquid stream was recycled while the gas stream was recycled for all runs.

5.1.2.1 The Rectangular Column

The sieve tray simulator is shown diagrammatically in Fig. 5.2. It consisted of three stainless steel rectangular sections, each provided with a large perspex window to permit unobstructed visual observation of the air-water dispersion formed at all times. The column, for the same reason, was fitted with external downcomers constructed in perspex.
Fig. 5.2 Rectangular Column in section.
The test tray was mounted between the top and middle sections, each of which was 3 ft long, 1 ft wide and 2.5 and 2 ft high respectively. A second tray was mounted between the middle section and a 2 ft 4 in. long x 1 ft wide windbox to promote effective distribution of the air prior to entering the perforations in the test tray.

The air entered the windbox, via a 6 inch internal diameter plastic pipe perpendicular to the lower tray and left from the top section via a 6 inch internal diameter pipe which was covered with a mist eliminator stainless steel mesh to prevent excessive carry-over of liquid into the fan.

Pressure taps were soldered into the sides of the column.

5.1.2.2 The Round Column

This column was made up of two 32 inch internal diameter pyrex-glass cylinders, a 40 inch-high top and middle sections, and a 2 ft 2 inch x 14 inch. x 1 ft. windbox constructed from stainless steel. A glass-walled column was chosen so that plate performance might be easily and visually monitored at all times. The two stainless steel perforated plates were held, bolted to flanges which were sealed with PTFE gaskets. Chord type inlet and exit weirs 3 inch high were bolted to each tray. Stainless steel downpipes and internal segmental downcomers were used.

The air was metered through a 6 inch internal diameter plastic pipe into the windbox and exited through another
6 inch internal diameter stainless steel pipe welded to a stainless steel cover bolted to a flange and sealed with a PTFE gasket round the top of the upper glass section.

5.1.3 The Air Flow Circuit

The air and air-carbon dioxide mixture was circulated by means of a single inlet, 1.5 inch width narrow pattern high pressure "Sturtevant" (No. 30 GV 7/30) fan driven by belts from a 20 H.P. totally enclosed squirrel cage fire-proof motor.

A schematic diagram of the air circuit is shown in Fig 5.3. The air from the fan entered the windbox of the columns through a long trombone-shaped length of horizontal plastic pipe, 6 inch internal diameter, and was metered with a British Standard orifice plate placed along the length using D and D/2 pressure tappings, controlled by means of a gate valve provided with threaded screws for fine adjustments. The valve was fitted to the suction side of the fan.

The air inlet at the base of the windbox was opened to the atmosphere through a T-piece to impose a neutral point (where there was no positive or suction pressure) which would otherwise move through the system creating instability.

For all the studies, a closed circuit air loop was employed.
Fig. 5.3. The Air Circuit
5.1.4 The Liquid Flow Circuit

The liquid circulation system is diagrammatically shown in Fig. 5.4. Essentially, it consists of two 30 gallon capacity holding-tanks from which the water was pumped by means of a double gear stainless steel pump, manufactured by Automatic Pumps Ltd., driven by a flame and explosion proof 3 H.P. motor, and capable of delivering an output of 3600 gallons per hour against a liquid head of 30 ft. The liquid is pumped through a 60 ft length of 2 inch internal diameter copper pipe, via a preheating/cooling-arrangement into an open header tank placed on top of the steel girder framework. A calibrated rotameter placed in the circuit between the preheater/cooler and the header tank was used to measure the rate of flow. The liquid from the header tank then flowed under gravity through a 3 inch plastic pipe into the stainless steel inlet downpipe, whence it was discharged behind the inlet weir of the upper tray.

After flowing over the calming section, the perforated region and the exit weir of the upper tray, and down the segmental downcomer onto the lower tray, over the exit weir, the liquid is discharged through the lower tray downflow pipe into the holding tank. The lower tray downflow pipe is extended to within 6 inch of the bottom of the holding tank to form a liquid seal inside the pipe. Wire mesh is provided at the entrance to the pump to prevent entrained particles and air bubbles in the liquid from entering the pump.
5.2 Miscellaneous Equipment

This section describes details of all other equipment used in addition to modifications made either to the column or sieve trays during some of the experimental work.

5.2.1 Pressure Drop, Dynamic Head and Froth Height Measurements

The dry plate pressure drop was measured using a water manometer both for the two columns and an experimental rig designed specifically to measure pressure drop across perforated plates at known air flow rates.

The details of the auxiliary trays used in the column and the experimental rig are listed in Table 5.1. The design of this rig was based on a similar one previously used by Kolodzie and Van Winkle (1957) and it is illustrated in Fig. 5.5. It consists of two sections, a downstream section 12 inch internal diameter and 35\(\frac{1}{2}\) inch long, and an upstream section also 12 inch internal diameter and 24 inch long, between which perforated plates of different dimensions could be flanged. Both sections were constructed from stainless steel. The downstream section was connected to the air circuit on the suction side of the fan with a 48 inch length of straight 6 inch internal diameter pipe. Pressure taps were provided 6 inch downstream and 6 inch upstream, and the air was exhausted to the atmosphere. The dry plate pressure drop was measured with a water manometer at the low flowrates and with a mercury manometer at the high air flowrates.
### Table 5.1 Auxiliary Tray Details

<table>
<thead>
<tr>
<th>Column</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>length of trays</td>
<td>3 ft</td>
<td>3 ft</td>
</tr>
<tr>
<td>width of trays</td>
<td>1 ft</td>
<td>1 ft</td>
</tr>
<tr>
<td>thickness of trays</td>
<td>0.125 in</td>
<td>0.125 in</td>
</tr>
<tr>
<td>hole diameter</td>
<td>1 in</td>
<td>1 in</td>
</tr>
<tr>
<td>pitch of perforations</td>
<td>3 in (A)</td>
<td>3 in (A)</td>
</tr>
<tr>
<td>total number of holes</td>
<td>32</td>
<td>32</td>
</tr>
<tr>
<td>number of active holes</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>length of inlet calming section</td>
<td>7.5 in</td>
<td>7.5 in</td>
</tr>
<tr>
<td>length of outlet calming section</td>
<td>7.625 in</td>
<td>7.625 in</td>
</tr>
<tr>
<td>distance between last row of holes and column wall (top)</td>
<td>1.75 in</td>
<td>3.375 in</td>
</tr>
<tr>
<td>distance between last row of holes and column wall (bottom)</td>
<td>3.25 in</td>
<td>3.25 in</td>
</tr>
<tr>
<td>area of active holes</td>
<td>0.0545 ft²</td>
<td>0.0436 ft²</td>
</tr>
<tr>
<td>area of perforated zone</td>
<td>0.4362 ft²</td>
<td>0.335 ft²</td>
</tr>
<tr>
<td>% free area</td>
<td>12.5</td>
<td>13.0</td>
</tr>
</tbody>
</table>

### Experimental Rig for Pressure drop measurements

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>diameter of trays</td>
<td>12 in</td>
<td>12 in</td>
</tr>
<tr>
<td>thickness of trays</td>
<td>0.125 in</td>
<td>0.125 in</td>
</tr>
<tr>
<td>hole diameter (in)</td>
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<td>1</td>
</tr>
<tr>
<td>hole pitch (triangular)(in)</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>number of active holes</td>
<td>10</td>
<td>22</td>
</tr>
<tr>
<td>% free area</td>
<td>6.94</td>
<td>15.28</td>
</tr>
</tbody>
</table>
Fig. 5.5 The "Experimental Rig"
The wet plate pressure drop in the column was measured using water manometers.

The dynamic head profiles along the length of the tray were obtained using a series of single leg manometers, each of which had one leg flush with the floor of the tray and the other end connected through a manifold to the vapour space above the test tray. A manifold provided with nine single leg manometers was used and the locations of the manometers are shown diagrammatically in Fig. 5.6.

The froth height was obtained by visual observation through the perspex windows, both head on and sideways, of the rectangular column at four points, and through a narrow slit in a piece of thin metal sheet bent to take the curvature of the round column. The sheet was mounted on a swivel arrangement and a bracket, such that it could be moved about. The measurement of froth height at these four points, inlet weir, tray centre, exit weir and sideways-on, was necessary to minimise the errors that usually arise due to an inexact definition of the boundary between the top of the froth and the gas phase. Due to the geometry of the round column, it was possible and necessary to obtain froth height measurements at only one point, the tray centre.

5.2.2 Mass Transfer Equipment

The mass transfer system investigated was the absorption of carbon dioxide from an air-carbon dioxide mixture into water flowing on the tray. Due to the nature of the absorption process, the quality of information that could be obtained
Pressure drop and Dynamic head Measurement

Z_d = Dynamic head profile
h_T = Total pressure drop

Fig. 5.6
with this system was limited by the frequency, speed and accuracy with which the analyses of the gas and liquid samples could be made.

The equipment could be divided into three sections:

(i) carbon dioxide injection
(ii) gas sampling
(iii) liquid sampling and analysis.

5.2.2.1 Carbon Dioxide Injection

Carbon dioxide was injected into the air stream at a point below the gate valve on the suction side of the fan from two pressurised cylinders operated cocurrently. The point of injection was chosen to enable complete mixing of the carbon dioxide-air mixture along the entire length of pipeline before the gas was sampled for analysis or fed into the column.

The carbon dioxide flowrate from the cylinders was controlled by fine-thread valves which were electrically heated to overcome the problems of freezing and subsequent blockage of the nozzles associated with high flowrates of carbon dioxide from cylinders.

A 5-litre capacity cylinder was provided between the gas cylinders and a rotameter used for metering the carbon dioxide, to act as a pressure stabiliser damping out oscillations in the CO₂ flow.
5.2.2.2a. Gas Sampling

The concentration of carbon dioxide in the air, entering the tray, was determined using an infra-red gas analyser. Infra-red gas analysers are sensitive to humidity and as of necessity it was required that the gas mixture being sampled should be as dry as is possible. By the nature of the physical process taking place in the tray during these mass transfer experiments, it was expected that entrainment of liquid by the air mixture would ensue, and as a result, considerable thought was directed at designing a gas sampling circuit that would remove most if not all of the entrained liquid before analysis. This gas sampling circuit is shown diagrammatically in Fig. 5.7.

The gas sample was obtained inside the column at a point above the lower tray. The gas mixture was withdrawn through a 3/8 inch copper tubing, the end of which carried a cone projecting into the vapour space above the lower tray and aligned perpendicularly to the centre of the perforated region of the lower tray. The sampling tube was designed to avoid entrainment of liquid drops. The withdrawn gas-mixture sample was pumped into another drop separator and a cooler, from where it was then led through three U-tubes containing silica gel, to remove the last traces of liquid, into the infra-red gas analyser. The gas-mixture flowrate was metered by means of a rotameter attached to the analyser and the percentage of carbon dioxide in the mixture was continuously read off the meter on the infra-red analyser.
Fig. 5.7 The Gas Sampling Circuit
The sampling line was continuously purged with nitrogen for 10 minutes before and after each run. No attempt was made to analyze the gas leaving the test tray because the change in composition of the gas across the tray was too small to be detected by the infra-red analyser.

5.2.2.2b Measuring Principles of the Infra-Red Analyser

There are three principal methods of analysing a carbon dioxide-air mixture:

(i) Chemically, e.g. the Winkler method or gravimetric absorption in concentrated caustic potash solution.

(ii) Volumetrically, e.g. the Orsat or Haldane method.

(iii) Physically, e.g. gas chromatography, polarography, thermal conductivity, interferometry, mass spectroscopy, pH method and infra-red analysis.

The nature of the mass-transfer process required that an analytical method be chosen which would be rapid, easy and afford a means of direct and continuous monitoring of the carbon dioxide concentration level in the inlet gas mixture, since this level had to be kept fairly constant during the experiments. For these reasons, the first two methods were rejected and of the third method, the infra-red technique offered the best prospects. In addition to the reasons previously stated, cost also had to be considered. The infra-red analyser used was the cheapest but most efficient technique and was also readily available.
The infra-red gas analyser used in this work was a carbon dioxide gas analyser manufactured by The Analytical Development Company (A.D.C.) Ltd. It is a non-dispersive double-beam analyser in which the gas is passed continuously through an optical cell. The analyser output meter is calibrated in terms of gas concentration and although the calibration is not absolute, it could be established by standardising with a known gas mixture, 10% CO₂ + 90% N₂ in this case. The accuracy of calibration is specified to be better than ±1% of full scale deflection over the full scale. It has a measuring range of 0-30% CO₂.

5.2.2.2c Principle of Operation of Infra-Red Analyser

The essential parts of the instrument are shown in the schematic drawing of Figure 5.8 and the set-up in Figure 5.9.

The heart of the instrument is an A.D.C. highly selective "Luft-type" infra-red detector mounted at one end of an optical bench and consisting of an aluminium alloy body containing a sealed sample of carbon dioxide gas. It is divided by a pressure-sensitive diaphragm into two chambers, the energy-receiving cells, fitted with windows, being in communication with each chamber. Infra-red energy reaching the gas in the cells is absorbed by molecular vibration at the frequencies characteristic of the gas, and then converted into translational energy which heats the gas. Since the chambers are sealed, the heat causes the gas pressure to rise and act upon the diaphragm. Insulated electrodes forming capacitors on either side of the diaphragm are connected to an electronic bridge circuit to detect diaphragm movement and hence the difference in energy
Fig. 5.8 Principle of the Infra-red Gas Analyser
Fig. 5.9 The infra-red analyser set-up for mass transfer studies
received between the detector cells.

The infra-red radiation source consists of a heated wire in a quartz tube mounted in the source block, which is located at the opposite end of the optical bench to the detector. Radiation from the source is reflected within the mounting block and passes out of a symmetrical pair of rectangular apertures into the analysis tube set. The set consists of a pair of tubes which are internally highly polished and gold-plated to ensure high transmission of radiation into the detector cells. One tube is used for the gas to be measured whilst the opposite tube is for reference, and is filled with dry air. The sample cell is divided by windows into various lengths to give different ranges of sensitivity, the unused sample cells generally being flushed with a non-infra-red absorbing gas such as nitrogen. The radiation from the source is interrupted by a rotating 2-bladed shutter driven by a synchronous motor. This "chopping" is necessary to eliminate spurious signals which would otherwise be caused by slow thermal changes and also provides an alternative signal more convenient for amplification.

To enable the analyser to be set to read zero on the meter to correspond to zero gas concentration in the sample cells, a zero control, operated from the front panel, which moves a shutter blade across either of the source block windows to alternate the sample or reference beam as appropriate and a phase control which moves the source block across the plane of the rotating shutter providing a precise adjustment to ensure that as the shutter rotates it exposes and obscures the refe-
rence and sample beams simultaneously, thus giving no net output from the detector due to asymmetrical exposure, are provided.

An infra-red sensor mounted in the reference tube detects the position of the rotating shutter and provides signal to the electronic circuit which then converts and amplifies the signal. This gives the analyser output signal.

5.2.2.3a Liquid Sampling

The determination of efficiency required a knowledge of the carbon dioxide concentration in the liquid at the tray inlet, tray outlet and in equilibrium with the inlet gas mixture.

The carbon dioxide concentration at the tray inlet and outlet were determined from liquid samples collected at these points respectively, in specially designed 500 cc conical flasks, diagrammatically shown in Figure 5.10. These have been described earlier by Varshney (1964). The conical flasks were provided with a bottom glass-tube connection, A, which was used for introducing the liquid sample and an upper connection, B, for venting air during this operation. Reagents were added to the flask through rubber stoppers.

The equilibrium carbon dioxide concentration was obtained from analysis of a liquid sample contained in a gas-washing bottle through which the carbon dioxide-air mixture bled from the column was bubbled. Details of the wash-bottle arrangement are shown in Figure 5.11. The column was on the suction side of the neutral point in the air circuit and so the absolute
Fig. 5.10 Liquid Sampling Flask

Fig. 5.11 Equilibrium Sample Bottle
pressure inside was below atmospheric. As a result, the connections to the bottle were designed to withdraw the liquid sample inside it without bringing about changes in composition. A three-way tap $T_1$ was used for connecting and disconnecting the wash-bottle from the gas-mixture supply. When the bottle was connected, the gas mixture flowed through the three-way tap into the bottle and out through limb A into the column. By adjusting the tap, the limb B could be connected directly to the column such that the wash-bottle was bypassed by the gas flow. The pressure inside the bottle was monitored by means of a water manometer connected to P while the temperature was read off a thermometer fitted to the bottle at T. Withdrawal of samples was made by connecting the bottom limb of the sampling bottle (Fig. 5.10) to drain D, and the upper limb to point C of the wash bottle.

5.2.2.3b Analysis of Liquid Samples

The liquid samples obtained were analysed by means of an automatic titrator manufactured by Radiometer. The automatic titrator is shown in Fig. 5.12. It is made up basically of two parts.

Auto_Burette_Unit

The auto burette is a fully automatic, motor-driven piston burette with digital read-out. All functions can be operated by push buttons or by remote control, e.g. from an automatic titrator. Its simplicity and great precision make it suited for routine analysis.
Fig. 5.12 The automatic titration equipment
The autoburette is equipped with a piston in an airtight, precision glass cylinder of 50 ml capacity which ensures a highly accurate delivery of titrant. The piston is moved by a dc motor via an 8-speed gear-train. This motor can be activated by push buttons or by signals from an external source, e.g. the titrator. When the titration has been completed, the glass cylinder can be refilled automatically by activating a push button. The rate of titrant delivery can be varied from 1.25-2.5-5-10-20-40-80-160% of total burette volume per minute. The autoburette is claimed to be accurate to within ±50 μl ± 0.2%. All parts directly in contact with the titrant are made of either glass, polyethylene or rubber.

The titrant is delivered into a titration assembly which is fitted with a stirrer, glass and calomel reference electrodes and an inlet through which an inert gas, in this application, Nitrogen, could be passed to maintain a blanket over the solution in the titration vessel thus preventing contamination by atmospheric carbon dioxide.

The signals from the electrodes are fed to a titrator unit.

Titrator

The Radiometer TTT 60 titrator is an automatic titration control unit designed for use in conjunction with a pH meter. It monitors the autoburette. It is equipped with push buttons for increasing or decreasing pH values during titrations, and a proportional band selector which can be preset to prevent overshooting of the end-point. The proportional band selector
reduces titrant delivery, automatically starting and stopping the autoburette so that the titrant is delivered in small increments as the end-point is approached. The TTT 60 is also equipped with a time delay selector such that when the time interval between two titrant increments as selected using the proportional band selector exceeds the time interval preselected by means of this selector, the titrant delivery is definitively blocked, this being particularly useful in an application of this sort where the end-point is not so sharply defined; and an end-point selector which is used to set the required end-point. It is a friction-drive potentiometer with coarse- and fine-adjustment.

The TTT 60 was used in conjunction with a Radiometer PHM 61 laboratory pH meter.

Standard solutions of sodium hydroxide and barium chloride were used in the analysis.

5.2.3 Liquid Phase Residence Time Equipment

The concept of residence time distribution stems from the publication of Danckwert (1953) which analysed generally a class of continuous flow systems for which residence time distribution is readily identifiable by tracer injection techniques. Since these systems contain one inlet and one outlet through which a single incompressible fluid flows at a steady state, an arrangement can be obtained for determining the residence time distribution by tagging material that enters by means of an impulse injection of a suitable
tracer whose outflow concentration is then recorded. It is implicitly assumed that the tracer particles have the same flow behaviour as the elements of the fluid. However, it has been found by Danckwerts that the assumptions may not hold if molecular diffusion of the tracer material is appreciable. For the present application, this effect would be undetectable experimentally.

5.2.3.1 Choice of Tracer

The selection of a tracer was governed by the need for a rapid and continuous method of measurement of the tracer concentration. It was observed that in some instances, the mean liquid residence time could be as low as four or five seconds and this necessitated the use of a measuring technique which was continuous, rapidly responding and sensitive. Measurement of the colour intensity of the dye, Nigrosine with a photocell detector, which has been successfully used by Campbell (1965), and others was considered to satisfy these conditions.

The dye is very soluble in water (17% at 20°C), gives an intense dark blue coloration in solution and is detectable down to a concentration of 150 p.p.m.

A 5% solution of the dye was used for all the runs.

5.2.3.2. Dye Injection and Detection

The successful application of the tracer technique demands that the inlet signal should be an instantaneous pulse injection of the tracer material into the liquid just as it enters
the tray. This calls for an instantaneous pulse source of tracer at the liquid inlet. This requirement was approximated in this investigation by injecting the tracer at three evenly spaced points in the traverse direction of the liquid cross-section. This approximation is valid if the tracer from an individual point merges with tracer from surrounding points in a time shorter than the time it would take the tracer to reach the photocell and the tracer injection technique employed was considered to assure the validity of this approximation. Each injection port had a Schrader valve fitted to its end to prevent leakage of tracer on to the tray after each injection.

The measurement of tracer concentration may be made on a sample portion of the fluid stream, as a representative of the total, or on the total fluid stream itself, internally or externally, continuously or as discrete samples. In this application, the light absorbent property rather than the actual concentration of the tracer was continuously monitored using an internal photocell detector unit, after it had been ensured that a linear relationship exists between the light absorption property and concentration of the tracer.

The photocell detector unit is pictured in Figs. 5.13a and 5.13b, and was constructed by the University of Surrey, Chemical Engineering Technical Department.

The detection unit consisted of two water-tight rectangular brass boxes both fitted with a ground glass window positioned facing each other at each end of a trough through which the liquid could flow. One of the boxes housed a 36W
Fig. 5.13a Components of dye detection unit

Fig. 5.13b The dye detection unit
tungsten halogen lamp and a small concave lens fitted into the window to produce an intense parallel beam of light. The other box housed a selenium photocell, type 001-55.008, so positioned that when the two boxes faced each other, the light beam illuminated the active film of the photocell. The trough was divided into two compartments by an underflow weir, the first compartment, a rectangular channel, representing the liquid inlet into the detector, and the second compartment housing the intense beam of light. Liquid entered the first compartment through a hood covered with a strip of wire mesh to disengage air bubbles, flowed down the channel and under an underflow weir into the second compartment where it was discharged into the column over an upright weir. The weir ensured that the second compartment was filled with liquid at all times.

The output from the photocell was monitored by passing the signal through an electronic amplifier provided with zero and full scale adjustments. This unit controlled the detection and initiated logging. Dye injection was controlled by a timer unit which was equipped with a push button to drive a solenoid valve and at the same time initiate logging via a relay system linking the amplifier, data logger and the punch drive. The amplifier became saturated at 12.50 volts and so the injection time could be adjusted to ensure that an output of 10.50 volts was not exceeded. The amplifier was also provided with a manual overload input to mark the tape with an end of data character. The timer had a range of 0.1-10 seconds.
The tracer was held in a tank which was constructed from a five-foot length of six inch diameter steel pipe and provided with an opening to a 60 psi pressure source. Dye was fed from the bottom of the tank through a 1/4 inch copper tubing to the injection ports in the column via a solenoid valve operated automatically by the timer unit.

**The Data logger**

The dye concentration expressed in terms of a voltage was fed to a compact Data logger where it was displayed on a Solatron digital voltmeter (DVU) LM 1420 which was coupled to a scanner (LU 1461) and an encoder or punch drive (LU 1718) to drive an Addo tape punch. The scanner was equipped with a pulse repeat frequency (P.R.F.) control to adjust the speed of scanning.

A P.R.F. setting of

- 0.5 gave a reading every 2 seconds
- 1 gave a reading every 1.0 seconds
- 2 gave a reading every 0.5 seconds
- 3 gave a reading every 0.333 seconds

The P.R.F. could be set to a convenient value depending on the velocity of fluid flow on the tray.

A schematic diagram of the residence time system block is shown in Fig. 5.14a and the data logger in Fig. 5.14b.

5.2.4 **Entrainment Measurement Equipment**

Entrainment of liquid by vapour in a column is usually
Fig. 5.14c The Residence Time Study: Equipment
found by one of two methods:

(a) "Free" entrainment - amount of liquid which reaches a given height above the test tray;

(b) "Captured" entrainment - amount of liquid collected on the tray above the test tray.

Of the two methods, "captured" entrainment which is the more industrially important, has been shown to provide the more reliable entrainment data (section 2.1.6). The "free" entrainment method involves the introduction of an extraneous measuring device into the vapour space above the test tray to collect the entrained liquid. Although this method is simple, it not only modifies the pattern of the air or vapour flow occurring in the column as a whole, but also influences the entrainment actually occurring. From the foregoing, it was therefore expedient to obtain the entrainment data by the method of "captured entrainment".

The entrainment from the lower to upper plate was measured by a technique which has been employed by several investigators - namely, analyzing quantitatively the amount of a tracer material in the upper plate as a result of entrainment of the tracer placed on the lower plate. In this application, the tracer used was Nigrosine dye and the top plate, of identical design to the lower test plate, was used as the entrainment collection tray. Entrainment data were obtained for the basic 1 inch diameter hole tray. For this investigation, modifications were made to the basic equipment as follows:-
(a) Three rectangular sections were fabricated out of stainless steel to replace the original 24 inch-high middle section of the column. These were of the same basic dimensions as the original section except for variations in height, to give a 12, 15 and 18 inch plate spacing respectively. The sections were provided with large perspex windows for visual observation of the frothing liquid in them.

(b) As for other experiments, a closed circuit air loop was employed but the liquid flow circuit was modified for a once-through flow through the column and separate liquid flow circuits were provided for the test and collector trays.

Fig. 5.15 is a schematic diagram of the entrainment studies set-up and Fig. 5.1 shows the rectangular column set up for the studies.

Since the objective of this investigation was to simulate and measure entrainment carried from the bottom tray to the top tray, considerable effort was directed at eliminating non-uniform dye/water mixing conditions on the tray as would be expected if the dye was injected at a point source. In consideration of this, a device was constructed that would inject dye solution into the liquid on the tray at several points in the horizontal cross-section of the frothing mass. This consisted of a horizontal copper tube, 1/8 inch in diameter and 16 inch long in which holes 0.0135 inch in diameter were drilled on both sides of a horizontal axis - at 2 inch spacing. The tube was laid horizontally along the central axis of the bottom tray parallel to the direction of liquid flow. In operation, this device produced a mass of horizontal
Fig. 5.15 Schematic Diagram of Entrainment Measuring Apparatus
high velocity dye stream ensuring that each element of liquid was completely mixed with the dye and no dead spaces were formed.

Dye solution was supplied to the distributor from a closed tank which was held under 60 lbs/sq. inch air pressure during a run. The holding tank has been described in an earlier section. A 10% solution of the dye was continuously introduced into the column through the injection ports, points X, of the copper tube as shown in Fig. 5.15, via a solenoid valve. Liquid samples were obtained at points S₁ and S₂ using solenoid valves activated by a venner timer to withdraw the samples. The sample lines were purged with air and washed out with fresh water after each sampling to prevent contamination from previous samples. A Pye Unicam Ultraviolet Spectrophotometer SP 1800 was used to quantitatively analyse the samples.

The spectrophotometer was a manually operated double-beam grating instrument with a solid state circuit measuring the logarithm ratio of reference and sample beam light intensities. The absorbance output was displayed on a meter with a linear absorbance scale. A linear wavelength scale with a range of 190-850 nm was provided. The output of the spectrophotometer was sent to an external Unicam AR 25 Series recorder, thus providing a permanent record of the sample absorbance with wavelength. The spectrophotometer used an air cooled deuterium arc and a tungsten filament lamp as its sources with a static beam splitter. The beams were chopper modulated. A venetian blind photomultiplier was used as detector. The SP 1800 had
an accuracy of ±1% of full scale on all ranges with a 1% of full scale photometric reproducibility. The spectrophotometer and chart recorder are shown in Fig. 5.16.

The spectrophotometer was provided with a zero control which is a ten turn control used to set the absorbance zero, a zero shift which offsets the absorbance zero by approximately -1 absorbance units and used in conjunction with the zero control, enabled the maximum sensitivity to be used when small absorbance changes were studied, a meter range control used to select the meter range from 0-0.2, 0-0.5, 0-10 or 0-2.0 absorbance units, and a wavelength control and indicator used to set the wavelength range in which the sample showed the strongest absorbance. The wavelength scale is calibrated linearly in nanometers.

Samples were placed in two cells which in turn were inserted in cell compartments placed opposite and in the sample beam. The meter range was set as required, depending on the concentration of dye samples collected, after the wavelength control had been adjusted to the range over which the absorbance was strongest. As no published data could be found on the dye, Nigrosine, this wavelength range was obtained by a method of trial and error, and the dye solution was found to exhibit the strongest absorbance at 330 nm. The two cells contained the dye solution in the sample compartment and tap water in the reference compartments respectively. For setting the absorbance zero, both cells were filled with tap water.

To obtain direct readout of concentration from the chart
Fig. 5.16 The spectrophotometer assembly
recorder, it was necessary to establish firstly that the solution obeyed Beer's Law. A calibration curve of concentration against absorbance was therefore obtained, and is shown in Fig. 5.17.
Fig 5.17 Spectrophotometer Calibration Curve
CHAPTER SIX

DETAILS OF EXPERIMENTAL PROCEDURE

The experimental procedures outlined in the following sections were designed with the abstraction of the maximum number of accurate and precise data in mind.

6.1 Hydrodynamic Study

As for all runs, the first step was the attainment of steady state in the simulators. The fan was first switched on and air circulated through the column. After 10 minutes the pump was turned on to continuously recirculate the water from the holding tanks. The circulation of fluids went on for a further 20 minutes, allowing the simulators to attain steady state, after which the air and water were then set to the desired values.

6.1.1 Pressure Drops and Dynamic Heads

When steady state had been attained as noted from the constant temperature readings, the temperatures of the gas and liquid streams were noted. All air bubbles inside the rubber tubings connected to the pressure drop and dynamic head manometers were carefully removed and measurements were taken of the total pressure drop, froth height and the dynamic heads. Many readings were obtained for each pressure measurement and later averaged. The dynamic head measurements were corrected for surface tension effects by subtracting
capillary rise from the averaged measurements.

Dry plate pressure drop measurements were obtained, with no liquid flow on the tray, after the preliminary adjustments as previously described had been carried out. For these runs, a liquid seal was maintained around the downflow pipe into the water tank, and the downcomers, and the water downflow pipe into the upper tray were effectively sealed.

The experimental procedure for the measurement of dry plate pressure drop in the specially designed experimental pipe based on Kolodzie and Van Winkle (1957), was essentially the same as that adopted for the simulators. The runs were commenced at high flowrates and the readings were taken at decreasingly lower rates until the pressure drop was too small to be measured.

6.1.2 Measurement of Entrainment

In these runs, modifications were made to the water flow circuit as previously described (Section 5.2.4). This was necessary as a dye build-up would have occurred in the holding tanks, contaminating the water metered into the column and so making an accurate quantitative measurement of the entrained dye on the top collector tray difficult or impossible. Thus water from the mains, set at 5 gall/min was continuously metered on to the top collector tray, while a flow of water, which when added to the 5 gall/min coming into the bottom test tray from the tray above corresponded to the desired water flowrate, was pumped from the holding tank on to the bottom
test tray. The contaminated water from the test tray was continually run into another holding tank and run to waste. Thus, it was ensured that only fresh and uncontaminated water was metered on to the two trays before the injection of the tracer.

Start-up procedure here was identical to that used in other runs. The fan was first switched on to circulate the air for 10 minutes, after which water was metered on to the top collector tray from the mains and water from the holding tank pumped into the test tray. Thus, for a desired flowrate of 30 gall/min on the test tray, 5 gall/min was metered from the mains on to the top tray and 25 gall/min was pumped on to the test tray. The fluids were allowed to circulate for 30 minutes before each run to ensure a constant flow of water of the desired rate on the test tray.

When thermal equilibrium had been attained, the tracer was continuously injected on to the test tray for 3 minutes, from the holding tank before the commencement of sampling to ensure that the dye and the water were thoroughly mixed and a uniform concentration of dye solution was obtained at all points on the tray. The sampling lines were flushed with clean water and purged with air. The Venner timer was then set to operate for 2 minutes, during which time liquid samples were withdrawn via the solenoid valves attached to the top tray and the bottom tray exits respectively.

A preliminary experiment was conducted to determine the flowrate of dye to be injected onto the test tray. This was
found not to be critical as long as the flowrate was such as to give a visible coloration of the water on the top collector tray and therefore a measurable amount of dye.

Care was taken that the flowrate of dye on to the test tray remained constant from the time of injection to the end of the run.

The samples collected were analysed quantitatively with a Pye Unicam Ultraviolet Spectrophotometer to determine the amount of dye contained in the liquid sample on the test tray, and the amount of dye in the liquid sample from the collector tray representing what was actually carried over by entrainment.

Entrainment was investigated as a function of liquid flowrate and air flowrate.

6.2 Liquid Phase Residence Time Study

In these runs, considerable care was excercised to ensure that meaningful and accurate results were obtained.

Preliminary experiments were conducted to determine the optimum size of tracer that would not only conform with conditions for a pulse injection of tracer but would also give a measurable residence time ensuring the accuracy of the determination. The initial concentration of dye in the system is of considerable importance in determining the accuracy of the ultimate measured mean residence time. If the residence time is known approximately, e.g. 50 seconds, then
sufficient dye must be added initially to ensure that at this time and afterwards, there is sufficient dye left in the exit liquor not only to be measurable but with accuracy.

The above is related to the initial concentration of dye in the storage vessel and the time taken to inject this dye into the apparatus. From a practical point of view, it can be clearly demonstrated, in this application, that an insufficient volume of dye is injected in, for example, 0.1 seconds relative to the large volumetric flow of liquid in the apparatus to give a measurable dye strength in the exit liquor after even a small interval of time. By trial and error, it was found that in his application, an injection time of over 1.5 seconds and 0.15 seconds for runs in the round and rectangular columns respectively gave a sensible and measurable concentration of dye in the exit liquid, even beyond the cut-off point. With insufficient dye in the system, there is no doubt that the measured residence time would be smaller than it actually is.

Care was also taken to ensure that the amount of dye injected did not cause the detector to operate outside the range of linear operation.

During these runs, water was supplied by a large capacity constant-head system on a once-through basis. Since the water leaving the tray contained dye, it was discharged into a separate tank from the one from which water was being pumped, and wasted to a drain. Thus only fresh water was circulated in the columns before the injection of dye.
Air and water were set to the desired values and allowed to circulate for a period long enough for the system to reach equilibrium. The optimum size of tracer was injected via a solenoid valve activated by the Venner timer which also initiated detection and logging simultaneously. This volume of dye injected was obtained by adjusting the air pressure within the dye holding tank.

For each set of operating conditions, the response to five separate injections was determined, and a mean average value was used in the calculations. The mean residence time distribution function was studied as a function of liquid and air flow rates on the tray alone for the round column and in addition, on the tray plus downcomer and the downcomer alone for the rectangular column.

6.3 Mass Transfer Study

The preliminary adjustments of the air and liquid flow systems were similar to those previously described. In this study, however, the air and water flows were recirculated, the water circuit being set up such that fresh water was continuously fed into the holding tanks and an equal volume of water continuously bled off to waste.

Carbon dioxide was injected into the air circuit on the suction side of the fan to promote adequate mixing with the air before the gas mixture entered the columns. The flow-rate of carbon dioxide was kept high initially, so that a rapid build-up of CO\textsubscript{2} to the required concentration could be
achieved as quickly as possible. The ADC CO₂ infra-red gas analyser was used to monitor this build-up and subsequently to keep the carbon dioxide concentration in the air constant at 5%. Slight variations of this concentration were corrected by adjustment of the heated valves on the carbon dioxide cylinders.

With the CO₂/air mixture held constant at 5%, a gas mixture sample, bled from the column, was bubbled into 150 ml of tap water contained in the gas-washing bottle previously described (Section 5.2.2.3) to determine the equilibrium solubility of the carbon dioxide in the tap water being used on the tray.

The circulation of carbon dioxide-air mixture was maintained for 60 minutes to ensure steady-state conditions. After this period, the inlet and outlet air, and water temperatures, the absolute pressure above the test tray, and the barometric pressure were recorded. The temperatures were measured by thermocouples placed at the inlet and outlet weirs of the test tray for water, and by thermocouples placed at right angles to the direction of flow in the inlet and outlet air ducts to and from the column, for air. This arrangement for air ensured that the temperatures recorded were the true dry bulb temperatures of the inlet and exit air.

6.3.1 Collection of Liquid Samples

Liquid samples were collected, before the inlet and after the outlet weirs, into the specially designed conical
flasks which were described previously in Section 5.2.2.3 and shown in Fig. 510.

The sampling lines and the conical flasks were continuously flushed with nitrogen gas for 15 minutes prior to sampling. The sampling flasks were connected to their appropriate sampling positions by their lower limb while the upper limb was connected to the suction side of the column. In order to arrest the dissolved gas in the liquid sample, 25 ml of 0.05 N NaOH was carefully pipetted into each sampling flask and made up to 125 ml with the liquid sample. This obviated the risk of gas either flashing or desorbing due to changing temperature and pressure. The sampling arrangement also ensured that the same atmosphere was kept in the column and over the flasks during sampling.

After the collection of samples, the conical flasks were quickly disconnected, plugged and vigorously stirred. They were kept in a constant temperature bath until they were analysed.

Three samples were collected at each sampling position over an interval of 15 minutes, during each run.

A similar procedure was employed for the withdrawal of the liquid samples from the gas-washing bottle. The lower limb of the conical flask was attached to drain D of the wash bottle, while the upper limb was attached to position C of the bottle, thus ensuring that when samples were being withdrawn, both the conical flask and gas-washing bottle were in pressure equilibrium. As in other runs, 25 ml 0.05 N NaOH
pipetted into the conical flask was made up to 125 ml with the sample.

Care was also taken to ensure that the pressure inside the wash bottle remained more or less constant during sampling as a sudden change could result in the carbon dioxide flashing out of solution.

All the samples collected were subsequently analysed for carbon dioxide content using the automatic titrator.

A description of the analysis is given in Appendix A1.3.
CHAPTER SEVEN

EXPERIMENTAL RESULTS AND DISCUSSIONS

7.1 Hydrodynamic Studies

7.1.1 Dry Plate Pressure Drop

By measuring and correlating pressure drops for dry trays, many investigators have sought to find a basis for the calculation of the total pressure drop across perforated trays at definite liquid loadings. Thus, the proposed correlations for such calculations usually include the value of the dry tray pressure drop.

The dry tray pressure drop has been investigated as a function of vapour velocity, the number of perforations, and the plate thickness-to-hole diameter ratio, t/d₀. Attempts have also been made to correlate the data obtained on the basis of the work of McAllister et al. (1958).

The experimental results of this work are shown in Tables A2.1.1 and A2.1.2 for the Rectangular Column, Table A2.1.3 for the Round Column, and Tables A2.1.4-A2.1.6 for the "experimental rig".

Data for the Round Column (39 holes) and the Rectangular Column (18 holes) have been correlated graphically as shown in Fig. 7.1 on the basis of the simplified form of the orifice equation:

\[ Uₒ =  \frac{2g_c \rho_L h_{DP}}{12 \rho_v} \left( \frac{2g_c \rho_L h_{DP}}{12 \rho_v} \right)^{0.5} \]  

(7.1)
FIG 7.1 Correlation of hole velocity with dry plate pressure drop
FIG 7.2 Correlation of dry plate pressure drop with $F_A$-FACTOR
Fig 7.3 Graph of Rect. Column dry plate pressure drop versus $F_A$-FACTOR
where

\[ U_0 = \text{linear velocity of vapour through perforations (ft/sec)} \]

\[ h_{DP} = \text{dry plate pressure drop (in. water)} \]

\[ C_D = \text{orifice coefficient} \]

The use of equation (7.1) to correlate the data by plotting the square of the velocity through the perforations against the dry plate pressure drop should result in a straight line of gradient \( \frac{C_D^2 \cdot \frac{2g_c \cdot p_L}{12 \rho_v}}{1} \).

In the present studies, in the context of the above, the following gradients were obtained:

**Round Column:** 2144.14
from which \( C_D = 0.673 \)

**Rectangular Column:** 2608.7
from which \( C_D = 0.743 \).

A summary of the orifice coefficients obtained for the various modifications of the 1 inch hole diameter trays used in the studies on the Rectangular Column is shown below in Table 7.1. Upon the inspection of Table 7.1, a striking trend is the increase of the orifice coefficient with increasing free area and increasing \( t/d_o \). This is consistent with the fact that the higher the ratio \( t/d_o \) and thus the thickness of the plate, the greater the value of \( C_D \), since, generally, a nozzle has a large coefficient of discharge, while a thin-edge orifice has a small value of \( C_D \). Also, for any gas velocity and hole diameter, the pressure drop would be expected to decrease with increasing free area.
Table 7.1 Rectangular Column orifice coefficients

<table>
<thead>
<tr>
<th>Number of perforations</th>
<th>Plate thickness (in)</th>
<th>% Free area</th>
<th>C_D</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.125</td>
<td>13.0</td>
<td>0.804</td>
</tr>
<tr>
<td>10</td>
<td>0.125</td>
<td>12.5</td>
<td>0.768</td>
</tr>
<tr>
<td>18</td>
<td>0.125</td>
<td>11.8</td>
<td>0.743</td>
</tr>
<tr>
<td>18</td>
<td>0.250</td>
<td>11.8</td>
<td>0.722</td>
</tr>
<tr>
<td>18</td>
<td>0.375</td>
<td>11.8</td>
<td>0.729</td>
</tr>
<tr>
<td>18</td>
<td>0.500</td>
<td>11.8</td>
<td>0.755</td>
</tr>
<tr>
<td>18</td>
<td>0.625</td>
<td>11.8</td>
<td>0.764</td>
</tr>
<tr>
<td>18</td>
<td>0.750</td>
<td>11.8</td>
<td>0.790</td>
</tr>
</tbody>
</table>

These results confirm in general those of earlier studies, notably Arnold et al. (1952), Hughmark and O'Connell (1957), and McAllister et al. (1958). These authors have independently established that the dry tray pressure drop is a function chiefly of the % free area and the plate thickness-to-hole diameter ratio amongst other factors. McAllister et al. considered the dry tray pressure drop to be the sum of the contraction, expansion and frictional losses which are related by the following equation:

$$h_{dp} = k \left[ 0.4 \left( 1.25 - \frac{A_o}{A_t} \right) + 4f \left( \frac{t}{d_o} \right) + \left( 1 - \frac{A_o}{A_t} \right)^2 \right] \frac{U_o^2}{2g_c}$$

(7.2)

In the present study, the frictional losses were minimal for the Rectangular Column; thus, the dry plate pressure drop
is a function mainly of the contraction and expansion losses. For low values of plate thickness, the vena contracta would be located downstream of the perforations and would be smaller in diameter than the perforation. Consequently, the sum of the concentration and expansion losses should result in a smaller value of the orifice coefficient as shown in Table 7.1. As the plate thickness increases, the minimum cross-section of the vena contracta approaches the diameter of the perforations, and hence a constant orifice coefficient close to unity. The orifice coefficients obtained in these studies are well within the range of 0.6 to 0.85 reported in the literature by these authors for varying hole sizes, although it cannot be conclusively proved from the result of the present studies that the use of large diameter perforations would be advantageous in reducing pressure drop. However, one is inclined to agree with Friend and Lemieux (1956) that the hole size generally does not limit stability and the flexibility of operation in properly designed trays, although a small error in drilling or punching in trays with small holes would introduce a greater effect on the pressure drop.

The difference in the orifice coefficients obtained for the round and rectangular columns could be the result of a more stable operation in the former as a consequence of the higher tray spacing (40 inch) used compared with 24 inch for the rectangular column. A high tray spacing would be expected to give a more uniform distribution of vapour between trays thus reducing losses associated with sudden contraction at the entrance to the perforations.
Figs 7.4 and 7.5 compare the experimental dry plate pressure drops obtained in the present studies with those calculated by the use of equation (7.2) proposed by McAllister et al. (1958). They introduced the factor 'k' to account for the influence of the great number of parallel openings, the irregularity of the velocity gradient inside the openings and the so-called "Couette correction", because the individual losses in equation (7.2) were derived for a single orifice. For the calculations, the factor 'k' was obtained from the correlation graph presented by McAllister et al. (1958) and reproduced in Fig. 7.6. From Fig. 7.4, it is observed that while the data for the round column were well correlated and those for the 18 perforations of the Rectangular Column show a reasonable agreement, the correlation breaks down when applied to trays with a low % free area. Comparison of the values calculated from equation (7.2) and the experimental data for variations in plate thickness in the Rectangular Column are in good agreement as shown in Fig. 7.5. In conclusion, the correlation proposed by McAllister et al. holds well when the plate thickness is varied at a constant hole diameter. This is further borne out as shown in Fig. 7.6 where the correction factors, k, obtained by plotting the right-hand side of equation (7.2) against the dry plate pressure drops from the present studies have been superimposed on the original graph of McAllister et al. (1958). The factors have also been obtained for the data from the "experimental rig" and are also shown on the same graph.
FIG 7.4 Comparison of calculated and expt. dry plate pressure drop
Experimental Dry-Plate Pressure Drop (Ft. water) vs. Log10

Calculated Dry-Plate Pressure Drop (Ft. water) Eqn. 7.2
- PLATE THICKNESS = 0.125-in
- PLATE THICKNESS = 0.25-in
- PLATE THICKNESS = 0.375-in
- PLATE THICKNESS = 0.5-in
- PLATE THICKNESS = 0.750-in

FIG 7.5 Comparison of calculated and expt. dry plate pressure drops
McAllister et al (1958) Correlation Line

Pipe: $N = \text{holes}$ $t = 0.125\text{in}$ $do = 1.0\text{in}$
- 22 holes
- 61 holes $= 0.375\text{in}$
- 211 holes
- 547 holes $= 0.125\text{in}$
- 881 holes

Rectangular Col: $N = 18\text{holes}$ $t = 0.125\text{in}$ $do = 1$
- 10 holes
- 8 holes

Rectangular Col: $N = 18\text{holes}$ $t = 0.250\text{in}$ $do = 1$
- $0.375\text{in}$
- $0.50\text{in}$
- $0.625\text{in}$
- $0.75\text{in}$

Fig 7.6 Dry Plate Coefficient, $k$ as a function of $t/do$
In the work on the "experimental rig", a uniform pipe was used and therefore the results tend to conform with the correction factor, k, in the McAllister curve despite the different arrays of holes and the number of holes on the tray.

In the rectangular column itself, the number of holes on the tray have an effect on the agreement with the McAllister curve and clearly suggests that with each array of holes, that is, for each different number of holes on the tray, it would appear based on 'k' that for the low t/d values (0.125) there would be different curves parallel to the McAllister curve shown in Fig. 7.6. Further work would have to be done with different percentage free area at higher t/d values to confirm this. However, there is nothing to suggest that this would be so from the results obtained on the "experimental rig" where different number of holes and arrays were used.

Therefore, the asymmetric geometry of the actual sieve tray holes in the column have an interactive effect different from the straight pipe. The factor 'k' will depend on all the geometric factors including the diameter of the column itself. The constraining factor is that there is no way of predicting 'k' from first principles as the hydrodynamic conditions cannot be defined. However, it would be apparent from equation (7.2) and Fig. 7.6 that as 'k' falls, the pressure drop decreases, provided a constant percentage free area is maintained.
7.1.2 Total Pressure Drop

The total tray pressure drop has been investigated as a function of the liquid and the air flowrates. The results obtained are shown in Tables A2.1.7-A2.1.10 for the rectangular column, and Tables A2.1.11-A2.1.14 for the round column.

It is apparent from previous investigations that the total pressure drop is a function of vapour and liquid flowrates and weir height. The effect of weir height has been extensively examined by previous workers, Campbell (1965), Millington (1971), and was not covered in the present studies. For correlation purposes, the vapour flowrate is expressed in terms of an $F_A$-factor based on the air velocity over the plate active area.

The total tray pressure drop data have been correlated against the liquid flowrate and the square of the $F_A$-factor as represented graphically in Figs. 7.7 and 7.8 respectively for the rectangular column, and Figs. 7.9 and 7.10 respectively for the round column.

Fig. 7.7 shows a family of parallel curves when the rectangular column total pressure drop was plotted against the liquid flowrate with the $F_A$-factor as parameter. It is interesting to note that Campbell (1965), Millington (1971) and other workers obtained a linear relationship between the total pressure drop and the liquid flowrate. An examination of Fig. 7.7 would reveal that this relationship holds true also for the present studies, although at liquid rates above 15 gpm/ft weir. The deviation obtained at low liquid rates,
**FIG 7.7 Total pressure drop versus Liquid flowrate (Rectangular Column)**

- \( h_T \) (in, water)
- \( L_w \) (gpm/Ft Weir)

- \( F_A \)-Factor 1.72
- \( F_A \)-Factor 2.20
- \( F_A \)-Factor 2.60
- \( F_A \)-Factor 2.80
Total Pressure Drop, $h_T$ (in. water)

FIG 7.8 Graph of Total pressure drop versus Square of $F_A^2$ FACTOR (Rect. Col.)
that is, below 15 gpm/ft weir can be attributed to oscillations on the tray at these liquid rates.

Fig. 7.8 shows the correlation of total pressure drop with the square of the $F_A$-factor at constant liquid flowrates. The results indicate a linear relation between $h_T$ and $F_A^2$, thus confirming the results of Arnold et al. (1952), Hunt et al. (1955), Thomas and Campbell (1967) and Thomas and Ogboja (1978).

The data for the round column are well correlated as represented graphically in Figs. 7.9 and 7.10.

The results of the rectangular and round columns are shown compared in Fig. 7.11 at an $F_A$-factor of 1.72. As was explained earlier, a linear relation between $h_T$ and $L_w$ holds for flowrates greater than 15 gpm for the rectangular column, hence the disparity between the data obtained may be due to the difference in column geometries.

A multiple regression analysis of the data shows the results to be satisfactorily correlated as follows:

**Rectangular Column:**

- $L_w = 10-50$ gpm/ft weir;
  
  \[ h_T = 2.32 L_w^{0.114} \exp (0.0207 F_A^2) \]  
  
  \[ 0.18\% \text{ Standard Deviation} \]

- $L_w > 15$ gpm/ft weir:
  
  \[ h_T = 0.015 L_w + 0.213 F_A^2 + 2.83 \]  
  
  \[ 2.7\% \text{ Standard Deviation} \]
Liquid Flowrate, $L_w$ (gpm/Ft Weir length)

- $F_a$-FACTOR 1.40
- $F_a$-FACTOR 1.50
- $F_a$-FACTOR 1.60
- $F_a$-FACTOR 1.72

FIG 7.9 Total pressure drop versus Liquid Flowrate (ROUND COLUMN)
FIG 7.18 Graph of Total pressure drop versus Square of $F_A\cdot$FACTOR(ROUND)

$F_A\cdot$FACTOR (F_A^2)

- LW = 5 gpm/ft weir
- LW = 10 gpm/ft weir
- LW = 15 gpm/ft weir
- LW = 20 gpm/ft weir
- LW = 25 gpm/ft weir
Fig 7.11 Comparison of Total pressure drops.
Round Column:

\[ h_T = 0.0239 L_w + 0.259 F_A^2 + 2.425 \]  \hspace{1cm} (7.5)

0.83% Standard Deviation

The constants account for the effect of weir height and other geometric factors, for example, percentage free area.

7.1.3 Liquid Holdup on the Tray

Liquid holdup is considered to be one of the basic liquid-side dependent variables in sieve-tray column operation, because it has a direct influence on liquid-phase mass transfer and on loading behaviour. The degree of aeration on a tray is the result of vapour bubbling through the liquid depth on the tray and in turn influences the tray performance such as pressure drop, entrainment and plate efficiency. The liquid holdup is one of the major variables which affect this degree of aeration.

The liquid holdup in this investigation has been measured in terms of the dynamic head of liquid on the tray by the use of a series of manometers installed in the tray floor. Since the dynamic head varies in height from the inlet weir to the outlet weir, the reported values represent the averaged sum of the measurements over the tray length. These values have also been corrected for surface tension effects as a result of the manometers having one leg flush with the tray floor and the other projecting into the vapour space above the tray.
7.1.3.1 Dynamic Liquid Head, $Z_D$

The dynamic liquid head was investigated as a function of liquid and vapour flowrates. The results are represented graphically in Figs. 7.12 and 7.13 for the rectangular column respectively, and Fig. 7.14 for the round column.

Fig. 7.12 indicates a linear increase of the dynamic head with increasing liquid flowrate at constant values of the vapour flowrate, while Fig. 7.13 shows a linear decrease of the dynamic head with increasing $F_A$-factor at fixed liquid rates. The same trend has been noted for the data obtained on the round column as represented in Fig. 7.14.

The experimental data have been satisfactorily correlated by the following regression equations:

**Rectangular Column:**

$$Z_D = 0.0169 L_w - 0.329 F_A + 2.514 \quad (7.6)$$

**Round Column:**

$$Z_D = 0.0266 L_w - 0.980 F_A + 3.08 \quad (7.7)$$

7.1.3.2 Static Liquid Head, $Z_C$

Hughmark and O'Connell (1957) introduced the effective head concept which they correlated against the total submergence. This effective head in reality is the static liquid head or liquid holdup on the tray.

Bernard and Sargent (1966) working under actual distillation conditions, later observed that the liquid holdup
Mean Dynamic Head, $Z_D$ (in. water) -198-

Liquid Flowrate, $L_w$ (gpm/ft Weir)

- $F_A$ - FACTOR 1.72
- $F_A$ - FACTOR 2.20
- $F_A$ - FACTOR 2.60
- $F_A$ - FACTOR 2.80

Fig 7.12 Mean Dynamic Head versus Liquid Flowrate (Rectangular Column)
Graph of Mean Dynamic Head versus $F_A$-Factor (Rectangular Column)

- Liquid Flowrate=10 gpm/ft Weir
- Liquid Flowrate=20 gpm/ft Weir
- Liquid Flowrate=30 gpm/ft Weir
- Liquid Flowrate=40 gpm/ft Weir
- Liquid Flowrate=50 gpm/ft Weir
Mean Dynamic Head, $Z_D$ (in. water)

Liquid Flowrate, $L_w$ (gpm/ft Weir)

- $F_A$-FACTOR 1.40
- $F_A$-FACTOR 1.50
- $F_A$-FACTOR 1.60
- $F_A$-FACTOR 1.72

Fig 7.14 Mean Dynamic Head versus Liquid Flowrate (Round Column)
was underestimated by the exclusion of a term due to the head lost by the vapour leaving the tray or the momentum head, \( Z_m \).

The momentum head is defined by:

\[
Z_M = \frac{\rho_v}{\rho_L} \cdot \frac{U_C}{g_c} \cdot (U_O - U_C)
\]  

(7.8)

where

- \( U_C \) = superficial vapour velocity (ft/sec)
- \( U_O \) = vapour velocity through the perforations (ft/sec)

They suggested that the liquid holdup be calculated from the sum of the measured dynamic head and the momentum head:

\[
Z_C = Z_D + Z_M
\]  

(7.9)

Values of the momentum head calculated from equation (7.8) have been correlated as a function of \( F_A \)-factor as shown in Fig. 7.15.

Using these values of \( Z_m \), the liquid holdup was computed from equation (7.9) and is shown plotted against liquid flowrate in Fig. 7.16 for the rectangular column and Fig. 7.17 for the round column. In both cases, the static head increases linearly with liquid flowrate and decreases linearly with \( F_A \)-factor. These results are in good agreement with those reported by Thomas and co-workers (1967, 1978).

7.1.3.3 Liquid Holdup Calculated from Total Pressure Drop

Mayfield et al. (1952), Arnold et al. (1952), Hunt et al. (1955) and Thomas and co-workers (1967, 1978) have all used and recommended the additive model of sieve tray pressure drop. Generally, the expression used is given by:
Fig 7.15  Graph of Momentum Head versus $F_A$-Factor
Mean Static Head, \( Z_c \) (in. water)

Liquid Flowrate, \( L_w \) (gpm/ft Weir)

\[ \text{Liquid Flowrate, } L_w \text{ (gpm/ft Weir)} \]

- \( F_A \)-FACTOR 1.72
- \( F_A \)-FACTOR 2.20
- \( F_A \)-FACTOR 2.60
- \( F_A \)-FACTOR 2.80

Fig 7.16 Mean Static Head versus Liquid Flowrate (Rectangular Column)
Fig 7.17 Mean Static Head versus Liquid Flowrate (Round Column)
where

\[ h_T = h_{DP} + h_F + h_R \]  \hspace{1cm} (7.10)

Mayfield et al. (1952) found that for their air-water system, the wet tray pressure drop was consistently 0.1-0.2 inch higher than the sum of the measured manometric head and the dry tray pressure drop. This work was confirmed by Arnold et al. (1952) who found their results to be satisfactorily correlated with a standard deviation of 0.12 inch water in all cases.

In the present studies, the liquid holdup calculated from equation (7.10) is designated as \( Z_c(h_T) \). Using a value of 0.2 inch water for the residual pressure drop \( Z_c(h_T) \) can be calculated from:

\[ Z_c(h_T) = h_T - h_{DP} - 0.2 \]  \hspace{1cm} (7.11)

The values of the computed liquid holdup are shown in Tables A2.1.7-A2.1.14 and although they are in good agreement with those calculated from the dynamic head data at values of the \( F_A \)-factor less than or equal to 1.72, slight differences are obtained in the magnitude of the two values. These differences are magnified at high \( F_A \) values.

Thomas and co-workers (1967, 1978) have pointed out that equation (7.10) used for calculating \( Z_c(h_T) \) is not strictly valid if it is considered that \( h_F \) by definition is the manometric head measured at the tray floor or \( Z_D \), that is,

\[ Z_c(h_T) = h_F = Z_D \]
However, this definition has been found to be deficient as the values of the computed $Z_c(h_T)$ are greater than $Z_D$. Also, if $Z_D$ is used in equation (7.10) to compute the residual pressure drop, $h_R$, the values obtained are usually greater than the 0.2 inch water gauge proposed. They therefore recommended that $h_R$ be subdivided such that:

$$h_R = h_R' + Z_M$$

where $h_R' = 0.2$ in. water gauge

The total pressure drop across the wet tray will then be:

$$h_T = h_{DP} + h_F + Z_M + h_R' \quad (7.12)$$

and if $h_F = Z_D$, 

$$h_T = h_{DP} + h_F' + h_R' \quad (7.13)$$

By definition,

$$h_F' = h_F + Z_M = Z_D + Z_M = Z_C$$

but from the computed results, $h_F'$ is not equal to $Z_C$. It therefore seems logical that a correction factor needs be applied. In the present studies,

$$h_F' = 1.12 \ Z_C \quad (7.14)$$

This compares very well with $1.14 \ Z_C$ obtained by Thomas and Campbell (1967) working with $1/8$ inch hole diameter. A comparison of the clear liquid head data for the rectangular and round columns shows that a higher clear liquid head is obtained in the rectangular column than in the round column. This may be explained on the basis of the results of Lockett and Gharani (1979) who have shown that on rectangular columns,
the small downcomer width restricts flow and therefore increases the liquid hold-up. This restriction, even though present in round columns, would have less impact as the column diameter is increased.

7.1.4 Froth Height on the Tray, \( Z_P \)

Prediction of the froth height is important for two reasons:

(a) The froth height is a measure of the interfacial area available for mass transfer and hence to some extent determines the efficiency of the transfer process.

(b) The distance between the top of the froth and the plate above affects the degree of entrainment. Hence, the greater the froth height, the higher the tray spacing must be in order to get an adequate froth-free head room.

Since the exact boundary between the top of the frothing mass and the vapour phase is inexact, the froth height in the present studies has been obtained at various locations along the length of the tray. Also, because of the instability of the frothing mass, a number of readings have been obtained and averaged to minimise errors due to human judgement. In this manner, it is estimated that the error of observation is ±0.25 in.

The froth height was investigated as a function of liquid and vapour flowrates and the data are shown correlated graphically in Fig. 7.18 for the rectangular column, and Fig. 7.19 for the round column. Inspection of the graphs shows that the froth height increases linearly with the liquid flowrate and air flowrates.
Fig 7.18 Froth Height versus Liquid Flowrate (Rectangular Column)
Fig 7.19 Froth Height versus Liquid Flowrate (Round Column)
The data have been adequately correlated by the following equations:

**Rectangular Column:**

\[ Z_f = 0.0858 L_w + 1.39 F_A + 2.61 \]  \hspace{1cm} (7.15)

**Round Column:**

\[ Z_f = 0.128 L_w + 3.64 F_A + 1.22 \]  \hspace{1cm} (7.16)

The results obtained are in agreement with trends reported by the AIChE workers at Delaware (1958), and Thomas and co-workers (1967, 1978).

Data obtained from studies on the rectangular and round columns are compared in Fig. 7.20, inspection of which indicates that at constant liquid and vapour flowrates, values obtained in the latter column are very much higher than those obtained in the former. This may be attributed to the difference in column geometric factors, design and materials of construction. The walls of the stainless-steel rectangular column would be more likely to break up the froth than the glass-walled round column due to imperfections on these surfaces. Also, a result of the weir constriction effects noted earlier would be to increase liquid holdup and hence decrease the froth height.

7.1.5 **Aeration Factor and Froth Density Factor**

As previously discussed in Section 7.1.4, the prediction of the degree of frothing to be expected in a distillation column is important both from a hydrodynamic and mass transfer point of view. The prediction of degree of frothing has evolved
Fig 7.28 Comparison of Froth Height Data

\textbf{Liquid Flowrate, }L_w \textbf{ (gpm/Ft Weir)}

- \text{ROUND COLUMN } F_A = 1.72
- \text{RECTANGULAR COLUMN } F_A = 1.72

\textit{Fig 7.20 Comparison of Froth Height Data}
around two concepts: the aeration factor and the froth density factor.

7.1.5.1 Aeration Factor

The aeration factor is defined here as the ratio of the observed pressure drop through the frothing mass on the tray to the calculated clear liquid depth on the tray; the clear liquid depth is defined as the sum of the outlet weir height and the liquid crest over the weir calculated by the Francis weir formula.

For design purposes, Mayfield et al. (1952), Thomas and Campbell (1967), amongst others, have computed values of the aeration factor from measurements of the total pressure drop:

\[ \beta = \frac{h_T - h_{DP}}{h_w + h_{ow}} \quad (7.17) \]

Such correlations are only approximate and are accordingly not definitive as they are subject to the vagaries of the dry tray pressure drop correlations. This is due to the fact that hole irregularities (punch direction, smoothness of holes, tolerances, etc.) may significantly affect the dry tray pressure drop. Also, the interactions of dry and aerated effects are relatively unknown.

In the present studies, only the directly measured static head, \( Z_c \) was accepted in the aeration factor data. Thus,

\[ \beta_d = \frac{Z_c}{h_w + h_{ow}} \quad (7.18) \]
The liquid crest over the weir, \( h_{OW} \) is calculated from the equation given in Coulson and Richardson (2nd Ed., 1976) as

\[
h_{OW} = \left(\frac{L}{2.6 L_w}\right)^{2/3}
\]

(7.19)

where

- \( L \) = liquid flowrate (gall/min)
- \( L_w \) = weir length (in)
- \( h_w \) = height of weir (in.)

Values of \( \theta_d \) computed from equation (7.18) are shown plotted in Fig. 7.21 as a function of liquid flowrate and in Fig. 7.22 as a function of \( F_A \)-factor.

From Fig. 7.21, it is observed that the aeration factor is not much affected by the liquid flowrate, increasing linearly but slowly as the liquid rate is increased. For example, the aeration factor for the rectangular column at \( F_A = 1.72 \) increases less than 5% from 0.651-0.680 for an increase in liquid flowrate from 10-50 gall/min/ft weir. The same trend is observed for the round column, although the increase in aeration factor here is more than was obtained on the rectangular column; an increase of about 11% from 0.506-0.562 for an increase in liquid flowrate from 5.0-25 gall/min/ft weir.

From Fig. 7.22, the aeration factor is found to decrease with increasing \( F_A \)-factor.

In the absence of aeration on the tray, the liquid holdup is equal to the sum of the weir height and the liquid crest over the sum, that is,
Aeration Factor $\Phi_A$

Liquid Flowrate, $L_v$ (gpm/Ft Weir)
- Rectangular Column $\Phi_A=1.72$
- Rectangular Column $\Phi_A=2.00$
- Rectangular Column $\Phi_A=2.60$
- Rectangular Column $\Phi_A=2.80$
- Round Column $\Phi_A=1.40$
- Round Column $\Phi_A=1.50$
- Round Column $\Phi_A=1.60$
- Round Column $\Phi_A=1.72$

Fig 7.21 Tray Aeration Factor ($\Phi_d$) versus Liquid Flowrate
Aeration Factor (\( F_a \))

- Rectangular Column \( L_w=10 \) gpm/ft Weir
- \( L_w=30 \) gpm/ft Weir
- \( L_w=50 \) gpm/ft Weir
- Round Column \( L_w=5 \) gpm/ft Weir
- \( L_w=15 \) gpm/ft Weir
- \( L_w=25 \) gpm/ft Weir

Fig 7.22 Aeration Factor as a Function of \( F_a \)-Factor
An examination of Fig. 7.22 shows that from the trend of the curves obtained, it is quite possible to extrapolate them to an aeration factor value of unity. This agrees with the results of Foss and Gerster (1956). However, Mayfield et al. (1952) have observed aeration factors above unity, particularly with zero weir setting.

It might be expected that the higher value of the aeration factor obtained for the rectangular column when compared with the round column would result in the rectangular column having a higher froth height, but as has been shown in Section 7.1.4, this is not so. The reasons for this are not immediately apparent, but may be explained on the basis of weir constriction and wall effects which would be more marked in the rectangular column and prevent the formation of a stable froth. Also, the Francis weir concept from which \( h_{ow} \) is calculated applies to the flow of unaerated liquid, but in sieve trays it is more correct to consider liquid flowing over the weir as a froth rather than a clear liquid. Furthermore, the equation lacks a vapour rate dependency which might be significant when it is considered that, for example, at an \( F_A \) of 1.72, the vapour flowrate in the round column is 700 ft\(^3\)/min, while in the rectangular column, it is 321 ft\(^3\)/min. Thus, the use of equation (7.18) for computing \( \beta_d \) is questionable and should be used with caution in comparing columns of different geometric factors, in particular when \( h_{ow} \) is obtained from the Francis Weir formula.
Aeration factors based on $Z_c(hT)$ have also been calculated, that is, from

$$\beta_h = \frac{h_T - h_{DP} - h_R}{h_W + h_{OW}}$$  \hspace{1cm} (7.22)

Values of $\beta_d$ have been found to be consistently higher than the corresponding values of $\beta_h$, especially at higher $F_A$-values. This is to be expected since values of $Z_c$ are higher than the corresponding $Z_c(hT)$.

Over the range of air and liquid flowrates employed in these studies, the values of aeration factor obtained are in the range:

- Rectangular Column : 0.609 - 0.680
- Round Column : 0.506 - 0.634.

These are in good agreement with values reported by Mayfield (1952) and Thomas and co-workers (1967, 1976, 1978).

7.1.5.2 Froth Density Factor, $\Phi$

The froth density is defined as the fraction of the volume of unaerated liquid to the volume of froth. At any point on the tray, this may be redefined as the ratio of the liquid holdup on the tray to the froth height, thus

$$\Phi_d = \frac{Z_D + Z_M}{Z_f} = \frac{Z_c}{Z_f}$$  \hspace{1cm} (7.23)

where

- $Z_c = \text{static liquid head (in water)}$
- $Z_f = \text{froth height (in)}$
A second definition of the froth density factor is given on the basis of the total pressure drop and is defined by:

\[ \phi_h = \frac{h_T - h_{DP} - h_R}{Z_f} = \frac{Z_c (h_T)}{Z_f} \]  

(7.24)

As might be expected from discussions in the earlier sections, \( \phi_h \) would be subject to the uncertainties inherent in its definition. For comparison purposes and completeness, the foam density factor has been evaluated for each method.

The froth density factor based on the dynamic head on the tray, \( \phi_d \) has been plotted as a function of liquid rate and \( F_A \)-factor as shown in Figs. 7.23 and 7.24 respectively. \( \phi_d \) is seen to decrease only slightly but linearly with increasing liquid rate as shown in Fig. 7.23, although the decrease is so small that to all intents and purposes \( \phi_d \) may be assumed constant over the operating range employed. This is reflected more on the round column than on the rectangular column where the rate of decrease, though slight, cannot be dismissed as insignificant. However, the values of \( \phi_d \) may be considered to be relatively insensitive to liquid flowrate.

The variation of \( \phi_d \) with \( F_A \) as shown in Fig. 7.24 indicates a decrease in the value of \( \phi_d \) with increasing air flowrate as has been found by Gerster et al. (1951), Foss and Gerster (1958), Bernard et al. (1964) and Thomas and Campbell (1967). This decrease in \( \phi_d \) is to be expected as there is a visible increase in the volume of the froth as the gas flowrate is increased as a result of a corresponding decrease in the clear liquid holdup.
Fig 7.23 Tray Froth Density Factor ($\phi_d$) versus Liquid Flowrate
Fig 7.24 Froth Density Factor as a Function of $F_A$-Factor
The comparison of $d$ and the aeration factor based on total pressure drop, $h$ shows only a slight difference in the values obtained, especially on the rectangular column. Values obtained on the round column can more or less be deemed to be in agreement. Both trends are to be expected since for the rectangular column, $Z_c$ is greater than $Z_c(hT)$ at high air flowrates, while for the round column, there is a good agreement between the two values.

The froth density factor offers a more direct and satisfactory approach to evaluating the aeration factor $\beta$ as has been shown by Hutchinson who related the froth density and the aeration factor by:

$$\beta = \frac{\phi + 1}{2}$$

(7.25)

Over the range of air and liquid flowrates employed, the froth density factor, $\phi_d$ varies from:

Rectangular Column : 0.255 - 0.386  
Round Column : 0.200 - 0.280

for which the aeration factors calculated from equation (7.25) are as follows:

Rectangular Column : 0.628 - 0.693  
Round Column : 0.600 - 0.640

whereas the actual range encountered is 0.609-0.680 for the rectangular column, and 0.506-0.534 for the round column. Both results show a very good agreement and the experimental data from the present work are well correlated by the equation.
These values are also in good agreement with those obtained by Thomas and Campbell (1967).

7.2 Liquid-to-Vapour Entrainment

A reliable means of predicting and correlating liquid entrainment on operating sieve trays is necessary to avoid the adverse effects of entrainment on column capacity and tray efficiency. The effect of entrainment is to reduce tray efficiency and in extreme cases limit column capacity by causing flooding.

The phenomenon of entrainment has been extensively studied but still remains an area in which there is very little agreement in the diverse correlations proposed. Most of the work has been conducted mainly on small diameter perforations, and very little has been reported on larger perforations, these being notably by Lemieux and Scotti (1969) and Thomas and Ogboja (1978).

Lemieux and Scotti (1969) studied entrainment in sieve trays with hole diameter ranging from 1/2 inch to 1 inch. They correlated their data graphically. Thomas and Ogboja (1978) studied entrainment on a sieve tray containing 1 inch diameter perforations and compared their data with those of Hunt et al. (1955) and Bain and Van Winkle (1961). However, more work remains still to be done, especially on these large diameter perforations as most of the published correlations have been obtained for smaller hole sizes and have been found deficient when applied to hole sizes outside the range for
In the present study, "free" entrainment has been measured, rather than by the "position-capture" method which would have involved the introduction of an extraneous device into the vapour space of the operating tray to measure the entrainment that reaches a certain height above this tray. The extent to which such a device interferes with the operation of the column still remains unknown. Thus, the method used involved measuring "free" entrainment, that is, the entrained liquid that was actually carried over and onto the tray above the operating tray, thereby eliminating the errors inherent in the former method.

Entrainment was measured as a function of liquid and air flowrates at 18 inch, 15 inch and 12 inch plate spacing. The experimental results are shown in Tables A2.2.1-A2.2.15. The entrainment data are shown in Fig. 7.25, plotted as a function of liquid flowrate with air flowrate, $F_A$, as a parameter at the three tray spacings used. For reasons of clarity, the results have been presented at only two $F_A$-factors; however, the same trend is observed at the other $F_A$-factors. Fig. 7.25 shows that entrainment varies with liquid flowrate in a strange fashion entirely empirical. This suggests that different mechanisms govern the generation of entrainment under different flow conditions. For the 18 inch tray spacing, it is noted that entrainment decreases with increasing liquid flowrate and this is in agreement with results reported previously by Lemieux and Scotti (1969) at the same tray spacing.
Fig 7.25 Entrainment as a function of Liquid Flowrate

Liquid Flowrate, $L_w$ (gpm/Ft Weir)

- Tray Spacing=18in, $F_A=2.60$
  - 15in, ,,  
  - 12in, ,,  

- Tray Spacing=18in, $F_A=3.00$
  - 15in, ,,  
  - 12in, ,,  

Entrainment: $E$ (lb water/100lb air)
At both 15 inch and 12 inch spacing, the value of the measured entrainment is seen to decrease from 10-20 gall/min/ft weir, rising sharply as the liquid rate is increased. As much as great care was taken to ensure that the study was conducted in the froth regime only, the high value of entrainment obtained at 10 gall/min tends to indicate that this was not so. Visual observation also showed that at this liquid rate a considerable amount of fine mist was being propelled into the vapour space above the operating tray which became reduced when the flowrate was increased. It is therefore not inconceivable that the tray was being operated in the spray regime at this liquid flowrate. A similar trend has been recorded by Lockett et al (1976) who found a sudden change in entrainment at the transition from spray to froth. The difference in the trend observed from 20-50 gall/min for the 18 inch spacing on the one hand, and the 12 and 15 inch spacings on the other hand, may be explained from the point of view of the momentum of the entrained drops. At the 18 inch tray spacing, as the liquid rate is increased, the frothing on the tray becomes more stable, the projection momentum of the entrained drops therefore decreases and the entrainment decreases. As the tray spacing is reduced from 18-15 inch, even though the same effect is obtained, now the top of the froth is close enough to the tray above for the projected drops to actually get carried over and on to the tray, thereby increasing the amount of entrainment. Fig. 7.25 also shows that entrainment is strongly influenced by the tray spacing. This is as might be expected since the
smaller the inter-tray spacing, the more the liquid that would be carried over.

Fig. 7.26 shows the entrainment data as a function of vapour velocity through the active area of the column with plate spacing and liquid flowrate as parameters. Generally, entrainment increases as the air flowrate is increased. The higher the vapour velocity, the higher would be the capacity of the vapour to entrain liquid at the froth surface. The results are in agreement with those of Friend et al. (1960), Kharbanda (1970) and Lemieux and Scotti (1969).

The experimental data from the present work have been very well correlated by a series of parallel lines at different liquid flowrates as shown in Fig. 7.27. The correlation equation obtained by multiple regression analysis is given by:

\[
\log (S''\psi_v) = 3.14 \log \left( \frac{U_o}{L_w' S''} \right) + 0.352 L_w' - 6.78 \quad (7.26)
\]

where

- \( S'' \) = tray spacing (ft)
- \( \psi_v \) = fractional entrainment, lb liquid entrained per lb vapour flow
- \( U_o \) = air velocity through the perforations (ft/sec)
- \( L_w' \) = liquid rate (lb/sec (ft of weir))

The average standard deviation is 16.6%. It is observed from Fig. 7.27 that the data for the 12 inch tray spacing show a lot of scatter. Such tray spacings, however, are rarely encountered in industry. Equation (7.26) does not require the froth height. This is to some extent an advantage as some other equations (see later) require a knowledge of the froth
Fig 7.26 Variation of Entrainment with Vapour Velocity
Fig 7.27 Correlation of Entrainment Data
height, for example \((S - Z_f)\) which is difficult to obtain with any degree of accuracy because of the turbulent interaction of the fluid systems. The correlation equation also takes into account the effect of the hole size.

The experimental data have also been correlated on the basis of the equation of Hunt et al. (1955) and the results are shown in Fig. 7.28. The best line through the experimental data is given by:

\[
E = 0.76 \left( \frac{U_A}{S - Z_f} \right)^{1.63}
\]  \hspace{1cm} (7.27)

where

- \(E\) = weight entrainment (lb liquid/100 lb air)
- \(U_A\) = vapour velocity based on plate active area (ft/sec)
- \(S\) = tray spacing (in)
- \(Z_f\) = froth height (in)

The standard deviation about the regression line is 14%. The Hunt-type equation, although theoretically correct in that it considers the difference between the top of the froth and the tray above as the "effective" tray separation rather than the physical tray spacing, has been criticized by Thomas and Ogboja (1978), Bain and Van Winkle (1961) and Friend et al. (1960), amongst others. These authors have pointed out that the froth height has only a limited accuracy of measurement, thus its inclusion in the Hunt et al. (1955) correlation makes its applicability to column design very doubtful. Friend et al. (1960) have estimated that the froth height measurement obtained visually is only accurate to \(\pm 1\) inch.
Fig 7.20 Correlation of Entrainment based on Hunt et al (1955)
They claimed that the value of the froth height has a large effect on the predicted value of entrainment especially at low tray spacings. Hunt et al.'s correlation equation given by:

\[
E = 0.22 \left[ \frac{U_C}{S - Z_f} \right]^{32}
\]

was obtained from studies on hole sizes less than 1/4 inch. Since the present study relates to 1 inch diameter holes, it is not surprising that the indices of equations (7.27) and (7.28) show such a discrepancy. This is in agreement with the findings of Thomas and Ogboja (1978), and Friend et al. (1960) who also found that a line drawn through the data would have a different slope from the Hunt correlation.

7.2.1 Comparison with Other Published Correlations

The experimental data have been correlated in the form suggested by Simpkin et al. (1954) for bubble caps. Their correlation equation was given as:

\[
\log \left( \frac{E_W}{A_f(h_w + h_{ow} - h_e + h_s)} \right) = -3.95 + 27.3/S + 10.75 U_0 \left( \frac{\rho_v}{\Delta \rho} \right)^{1/2}
\]

where

- \( E_W \) = entrainment rate (lb/min)
- \( A_f \) = column cross-sectional area available for vapour flow (ft\(^2\))
- \( h_w \) = weir height (in)
- \( h_{ow} \) = weir crest (in)
- \( h_e \) = slot elevation (in)
- \( h_s \) = slot opening (in)
- \( \Delta \rho = \rho_L - \rho_v \)
For this use, the elevation of the top of slot ($h_e$) and the slot opening ($h_s$) have been assumed to be zero. The results obtained are shown plotted in Fig. 7.29. The best line through the points is given by:

$$\log \left( \frac{F_w}{h_w + h_{ow}} \right) = -4.85 + 33.81/S + 13.3 \ U_o \ \left( \frac{\rho_v}{\Delta \rho} \right)^{1/3} \ (7.30)$$

with a standard deviation of 9.6%. The line obtained by Simpkin et al. is also given in Fig. 7.29 for comparison. It is observed that most of the experimental points fall below that for bubble caps. Although all the factors used in the correlation are readily available, the main criticism of this correlation lies in the use of the liquid crest over the weir, $h_{ow}$ as determined by the Francis weir concept, the failings of which were discussed in section (7.1.5.1). Hence its validity for use in predicting entrainment remains doubtful.

Kister et al (1981) have correlated their entrainment data in terms of four dimensionless groups as:

$$E \propto \left( \frac{\rho_G U_s}{\rho_L L g_c} \right)^a \left( \frac{U_s \rho_L \mu_G}{\rho_G \sigma_L} \right)^b \left( \frac{d_H}{h_L} \right)^c \left( \frac{S}{h_L} \right)^d \ (7.31)$$

where

- $U_s = $ superficial velocity (ft/sec)
- $L = $ liquid rate (ft$^3$/hr ft of weir)
- $h_L = $ liquid holdup (in)
- $d_H = $ hole diameter (in)

For the air-water system, Kister et al found that equation (7.31) reduces to:
Fig 7.29 Correlation of Entrainment based on Simpkin et al. (1959)
As has been shown in Section (2.1.6), the first dimensionless group is the product of the Reynolds number and the Froude number, that is:

\[ Fr \times Re = \frac{U_s^2}{d_H \rho_L} \times \frac{d_H U_s \rho_G}{\mu_G} \]

\[ = \frac{U_s^3 \rho_G}{\mu_G \cdot \rho_L} \quad \text{(7.34)} \]

and since \( \mu_G \), the gas viscosity \((\text{lb/(ft h)}^{-1})\) is

\[ \mu_G = L \rho_L \]

where

\[ L = \text{liquid flowrate per unit weir length (ft}^3/\text{hr ft)} \]

\[ \rho_L = \text{lb/ft}^3 \]

Equation (7.33) implies that the liquid entrainment is strongly influenced by the liquid holdup and indirectly by the Reynolds and Froude numbers. The use of the Froude number emphasises the importance of the effect of fluid physical properties which in the case of the air/water system is minimal as is evident from the disappearance of these dimensionless groups in the final equation. The Froude number is typically the ratio of inertial force to the gravitational force, and its use implies that the forces required to overcome the movement upward of
the entrained drops and stop them reaching the tray above might be an important though overlooked parameter in predicting entrainment in systems other than water. Obviously, the projection velocity of the entrained drops would be an influential factor in the amount of entrainment generated. The height an entrained drop reaches before either falling back on the tray or getting caught in the flume formed at the entrance to the perforations would depend on its initial projection velocity, which in turn would depend partly on the fluid physical properties and the diameter of the perforation. Thus, the Reynolds and Froude number represent the balance between the drag and settling forces of the drops of entrainment.

On the basis of the Kister et al. (1981)-type correlation equation, the entrainment data have been correlated as shown in Fig. 7.30 and by regression analysis, the following correlation equation has been obtained:

\[
E \propto \left( \frac{U_s}{L_\infty} \right) \left( \frac{Z_C}{(d_o \cdot S) \alpha} \right)^{0.35}
\]

(7.35)

The standard deviation about the regression line is 28%. From Fig. 7.30 it is noted that the worst deviations from the regression line occur at the tray spacing of 12 inch, while data at 18 inch and 15 inch are well correlated. In general, the agreement of the index of 4.85 obtained in equation (7.35) with (4.68) obtained by Kister et al. is encouraging. Also, the liquid holdup is seen to be an important parameter in correlating entrainment.
Fig 7.30 Correlation of Entrainment Data based on Kister et al (1981)
The experimental data have also been interpreted on the basis of the Bain and Van Winkle (1961) correlation. The authors correlated their entrainment data by plotting entrainment against a term 'M' (defined later in the text). Correlation was by the relation:

$$\log E = K \left( \frac{D}{S} \frac{1000}{L} \right)^g \left( \frac{G}{1000} \right)^f \quad (7.36)$$

where

- $E =$ entrainment, lb water/100 lb air
- $D =$ hole diameter (in)
- $S =$ plate spacing (in)
- $L =$ liquid rate (lb/hr ft weir)
- $G =$ gas rate (lb/hr ft$^2$ tower cross-section)
- $K =$ constant
- $g,f =$ coefficients

For the sake of simplicity, the right-hand side of equation (7.36) would be referred to as 'M'. The authors drew curves for three hole sizes, 1/16 inch, 1/8 inch and 1/4 inch, the position of these curves relating to $D/S$, $L$, $G$ and the correlating power indices of '$g$' and '$f$'. The exponent '$g$' is a function of hole size and gas rate, and was graphically correlated as a function of gas flowrate, while '$f$', a function of weir height and hole diameter, was graphically correlated as a function of weir height with hole diameter as parameter. The specific graphs are shown both in Bain and Van Winkle's paper (1961), and summarized by Van Winkle in his book (p.540).

In this work, '$g$' and '$f$' have been obtained by interpolation to the 1 inch diameter holes used in the present
studies and a review of the method is given in the paper by Thomas and Ogboja (1978). Fig. 7.31 shows the curves, reproduced from Bain and Van Winkle (1961) and the experimental data from this study. Close observation shows that there is an upper limit of G and L used in the experiment conducted by the authors. They used gas rates up to 1,680 lb/hr ft\(^2\) and liquid rates up to 11,900 lb/hr ft of weir. For the present study, liquid rates up to 30,000 lb/hr ft of weir and gas rates up to 1025 lb/hr ft\(^2\) were used. It was tacitly assumed by the authors that extrapolation of the three curves to higher values of 'M' could be made by extension of the curves and it is fair to comment that the absence of experimental data for larger hole sizes such as 1 inch diameter led them to believe that such an extrapolation was valid. The highest value of entrainment encountered by the authors at the most extreme flow conditions used was 2.73 lb/100 lb air.

The results of the present study reported in this thesis for 1 inch holes show that entrainment values at higher gas and liquid rates than those used by Bain and Van Winkle (1961) are quite consistent in themselves but are not correlated very well by the upper extrapolation curves suggested by Bain and Van Winkle. The trend of the experimental data points obtained in this study and shown in Fig. 7.31 suggests that a curve could be drawn through them to give a reasonable correlation with liquid rates up to 30,000 lb/hr ft well in excess of that used by Bain and Van Winkle. One can only conclude that the interactions of the phases on the tray for 1 inch holes, in some ways, alter the entrainment values causing them
Fig 7.31 Entrainment Correlation based on Bain and Van Vinkle(1961)
to differ for these at low liquid rates. Results from the present study confirm the magnitude of $E$ and as described in the experimental section, each result was painstakingly repeated several times to establish consistency.

It is satisfying to observe that there is a correlation between the entrainment values and flow conditions for 1 inch diameter holes, but it leaves some reasonable doubt as to the effectiveness of the Bain and Van Winkle method (1961) when applied to large hole sizes. Some indication of this disagreement was found by Thomas and Ogboja (1978) but no comment was made at that time on the probability that the reference datum for the 1 inch holes as given by the Bain and Van Winkle — extrapolated curves shown in Fig. 7.31 was wrong. Further discussion of this point would involve very considerable study of any interactions and how they differ from small holes. However, one should proceed with caution in attempting to predict entrainment for large holes on the basis of Bain and Van Winkle's work. A more reliable prediction could be obtained from the work of Kister et al (1981).

A comparison has also been made with the flooding curves of Fair and Matthews (1958) which are commonly used for estimating fractional entrainment on the basis of a flow parameter, $F_{LV}$ which accounts for the liquid-vapour kinetic energy effects. The flow parameter is defined by:

$$F_{LV} = \frac{L}{G} \left( \frac{\rho_v}{\rho_L} \right)^{0.5}$$

(7.37)

where
\( G = \text{mass flowrate of vapour (lb/hr)} \)

The fractional entrainment is computed from the following equation:

\[
\psi_L = \frac{e}{1 - e}
\]  

(7.38)

where

\( \psi_L = \text{fractional entrainment, moles/mole gross downflow} \)

\( e = \text{entrainment weight ratio, lb liquid entrained/lb of liquid (= E (L/V))} \)

\( L = \text{liquid flowrate (lb/hr ft}^2\text{)} \)

\( V = \text{vapour flowrate (lb/hr ft}^2\text{)} \)

\( E = \text{entrainment (lb/lb air)} \)

The entrainment data from the present work have been correlated as fractional entrainment \( \psi_L \) as a function of the flow parameter, \( F_{LV} \) in Fig. 7.32. As is observed, all the entrainment values lie on parallel lines which are not in agreement with the flooding curves of Fair, but cut across them. The reasons for this cannot be easily deduced, but a significant point lies in the restrictions on the use of Fair's curve:

(a) the curves are only valid for use with hole diameters less than or equal to 1/4 inch. (b) Weir height less than 15% of plate spacing. (c) Ratio of hole area to plate active area greater than or equal to 0.1. In the present work, it seems that only the third condition has been satisfied. It can be concluded without any doubt that the Fair correlation in its present form is certainly not applicable to hole sizes outside the range for which it was formulated.
Fig 7.32 Correlation of Entrainment Data with % Flood
7.3 Mass Transfer Efficiency

The effect of gas rate, liquid rate and column geometry upon the efficiency of absorption of small amounts of carbon dioxide, CO₂ from an air-CO₂ mixture into water has been studied. The mass transfer resistance of this system is all within the liquid phase because of the low solubility of CO₂ in water. Hence, the system is liquid-phase controlled and the efficiency values so obtained may be regarded as representing the pure liquid phase efficiencies.

After liquid compositions were determined by titration, the Murphree liquid tray efficiency, E_{ML} was calculated by:

\[ E_{ML} = \frac{x_n - x_{n-1}}{x^*_n - x_{n-1}} \]  

(7.39)

where \((x_n-x_{n-1})\) represents the change in the average composition of the liquid across the tray and \(x^*_n\) is the composition of the liquid in equilibrium with the vapour leaving the tray. Results are shown in Tables A.2.5.1-2.5.4.

The Murphree liquid tray efficiency data are shown as a function of liquid rate, air flowrate and column geometry in Fig. 7.33. This plot shows that the efficiencies decreased with increasing liquid rate sharply at low liquid rates and then gradually at high liquid rates, increased slightly with increasing air flowrate and at the same liquid rate, were higher for the rectangular column than the round column. These results, in general, confirm those of the A.I.Ch.E. Delaware workers (1958), Ellis and Moyade (1959), Thomas and
Murphree Liquid Efficiency, $E_{\text{M,L}}(\%)$

- Round Column $F_A=1.50$
- $F_A=1.72$
- Rectangular Column $F_A=1.72$
- $F_A=2.20$

Fig 7.33 Plate Efficiency as a Function of Liquid Flowrate
Campbell (1967), Lockett and Uddin (1980) amongst numerous others.

The decrease of efficiency with increasing liquid flow-rate, although logically incorrect since there is more liquid on the tray to absorb more CO$_2$ from the mixture, may in fact be due to the lower contact time of the vapour with the liquid at the high rates, thus resulting in less CO$_2$ being absorbed at these high rates. This relationship will be discussed further in later sections.

Although there is a slight increase of efficiency with increasing vapour flowrate, the increase in efficiency for both columns is so small as to be insignificant. For example, an increase of 100.5 ft$^3$/min or 31% in the air flowrate for the rectangular column only achieves a 1% increase in efficiency.

An analysis of equation (7.39) can be very revealing inasmuch as it affects the magnitude of the efficiency values. For example, $(x_n - x_{n-1})$ is the absorption of CO$_2$ in the liquid leaving the $n^{th}$ tray. The question arises of the significance of the values of $x_n$, $x_{n-1}$ and $(x_n - x_{n-1})$ with a fixed CO$_2$ concentration in the gas and how important the inlet concentration $x_{n-1}$ is in affecting the amount of transfer of CO$_2$ occurring. Clearly, the closer to zero the value of $x_{n-1}$, then the higher the driving force of the gas to the liquid at a fixed liquid rate, and therefore the higher would be the take-up of CO$_2$ by the liquid. In the extreme, as the inlet liquid composition, $x_{n-1}$ approaches $x_n^*$, then the driving force is greatly reduced (the driving force being $x_n^* - x_{n-1}$, where $x_n^* = \frac{y_n}{H}$) and
when \( x_{n-1} = x^*_{n} \), then clearly no more transfer takes place. Therefore, the concentration of the inlet liquid to the \( n^{th} \) tray is important in determining the take-up of CO\(_2\) at a fixed CO\(_2\)-air composition and liquid flowrate. Also, if \( x^*_n >> x_{n-1} \), then the denominator tends to be too large but the absolute value clearly depends on \( x^*_n \) which is \( y_n/H \). Therefore, two effects are obtained:

(a) for a low-solubility gas like CO\(_2\) in water, there is no great change in the CO\(_2\) content of the gas as it passes through a pilot-plant column with a limited number of trays. If there are a large number of trays, then more CO\(_2\) will be absorbed, but no marked absorption occurs unless the operation is under pressure. This is so in industry where absorption occurs at 25 atm. This means that in the present kind of study, \( x^*_n \) varies very little.

(b) \( x_{n-1} \) or the inlet liquid to each tray will change progressively down the column - increasing. The question then arises as to the significance of the change of \( x_{n-1} - x_n \) or \( x_{in} - x_{out} \) for each tray relative to \( x^*_n \) (i.e. \( x_{out}^* \)) and \( x^*_n - x_{n-1} \).

These factors appear to be glossed over or neglected in the general research studies reported in the literature. The interaction between the elements in the \( E_{aq} \) equation are dependent on the type of tray, air rate, hole size, liquid back-mixing on the tray, the temperature, the pressure, the question of solubility, and to some extent, the physico-chemical properties of the system.
It is just possible therefore that many low efficiencies reported for gas absorption are due to changes arising from the insoluble nature of gases, the small mass transfer changes along the column (compared with distillation) etc. If the uptake of CO₂ is small, or the change in take-up is small with changing conditions, compared with the arbitrary value of \( x^*_n \), then it is possible that if:

\[
\frac{d(x_n - x_{n-1})}{dt} < \frac{d(x^*_n - x_{n-1})}{dt}
\]  

(7.40)

then \( E_{ML} \) will decrease sometimes alarmingly to 10, 20 or 30%, and this level of efficiency is more a function of the inadequacy of the Murphree concept than of the true efficiency of the tray.

7.4 Residence Time Distribution and Liquid Mixing

7.4.1 Mean Residence Time

The results obtained from the impulse dye injection studies described in Section 5.2.3.2 and 6.2 were processed on the University of Surrey Prime Computer using a program written in Surrey Algol (Appendix A.3) which incorporated the Sater Tailing technique described in Section 3.6.

The mean residence time and the variance were calculated from equations (3.69) and (3.72) respectively. The experimental results are shown in Tables A2.3.1-A2.3.4.

The mean residence time was studied as a function of liquid flowrate, air flowrate and column geometry, on the
tray only for the Round Column and the tray, downcomer only and tray plus downcomer for the Rectangular Column.

The variation of mean residence time with liquid flowrate for the round column is shown in Fig. 7.34 from which it is noted that the mean residence time falls steeply initially and then gradually as the liquid flowrate is increased, and finally tailing off with further increases. It is also apparent from Fig. 7.34 that the mean residence time is little affected by an increase in air flowrate. The results of the studies on the Rectangular Column are shown in Figs. 7.35-7.37. In general, it is seen that the variation of the mean residence time with liquid flowrate follows much the same trend as in the round column, irrespective of which aspect of the column was being investigated. Figs. 7.38 and 7.39 indicate the variation of mean residence time with air flowrate for the Round and Rectangular Columns respectively, and show that, in effect, the mean residence time is virtually independent of the air flowrate although a slight decrease of the mean residence time is observed at low liquid rates. This effect of the air flowrate on the mean residence time can be explained on the basis of the liquid holdup on the tray as discussed in Section 7.1.3.2. An increase in the air rate leads to a decrease in the liquid hold-up on the tray and therefore it would be expected that with less liquid being held up, there would be a corresponding decrease in the mean residence time.

Fig. 7.37 compares the mean residence time data observed for the tray, downcomer and the combined tray plus downcomer. As would be expected from the longer pathlength, the mean
Liquid Flowrate, \( \text{Lw (gpm/ft Weir)} \)

- \( F_A = 1.40 \)
- \( F_A = 1.50 \)
- \( F_A = 1.60 \)
- \( F_A = 1.72 \)

Fig 7.34  Mean Residence Time versus Liquid Flowrate (Round Column)
Mean Residence Time, $\mu$ (secs)

Liquid Flowrate, $L_w$ (gpm/Ft Weir)

Rectangular Column: $F_A$-Factor = 1.72

$\therefore$ : $F_A$-Factor = 2.20

$\therefore$ : $F_A$-Factor = 2.60

Fig 7.35 Variation of Mean Residence Time with Liquid Flowrate on Tray
Mean Residence Time, $\mu$ (secs)

Liquid Flowrate, $L_w$ (gpm/Ft Weir)

Rectangular Column: $F_A$-Factor = 1.72

,  : $F_A$-Factor = 2.20

,  : $F_A$-Factor = 2.60

Fig 7.36 Mean Residence Time versus Liquid Flowrate (Tray+DC)
Fig 7.37 Comparison of Mean Residence Time on Tray, Tray+DC, and DC only
Fig 7.38 Mean Residence Time on Tray against F_A-Factor (Round Column)

- Liquid Flowrate = 5gpm/Ft Weir
- = 10gpm/Ft Weir
- = 15 gpm/Ft Weir
- = 22.5 gpm/Ft Weir
- = 25gpm/Ft Weir
Fig 7.39 Mean Residence Time against $F_A$-Factor (Rectangular Column)
residence time on the combined system is higher than for the separate tray and downcomer systems. It is also observed that the mean residence time in the downcomer is slightly higher than on the tray. This contradicts the assumption usually held that the flow in the downcomer generally approaches plug flow. In the opinion of this author, this is a misconception. Firstly, liquid flow over the weir into the downcomer is not by "free fall" but rather by the incoming liquid impinging against the downcomer walls causing considerable sloshing and frothing. Secondly, it was observed that there is an upward flow of vapour issuing through the downcomer skirt from the tray below. This in effect causes "choking" at the exit from the downcomer, leading to liquid backup in the downcomer. These effects serve as an obstruction to the flow of liquid outwards from the downcomer, thereby causing the liquid to reside in the downcomer for longer than is necessary. On the other hand, the tray suffers no such physical obstructions other than those due to backmixing. It would therefore be suggested that the use of the plug flow concept for design should be made with caution when considering downcomers.

The difference between the mean residence time obtained for the combined tray plus downcomer and the tray only, is shown compared with the mean residence time measured experimentally for the downcomer only.
Table 7.2 Comparison of measured and "calculated" mean residence time for the downcomer

<table>
<thead>
<tr>
<th>$L_w$ (gpm/ft weir)</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{T+DC} - \mu_{tray}$ (sec)</td>
<td>14.0</td>
<td>11.6</td>
<td>9.9</td>
<td>7.5</td>
<td>6.3</td>
<td>5.0</td>
<td>4.6</td>
<td>4.8</td>
<td>4.3</td>
</tr>
<tr>
<td>$\mu_{downcomer}$ (sec)</td>
<td>34.7</td>
<td>23.6</td>
<td>20.1</td>
<td>16.2</td>
<td>11.6</td>
<td>9.6</td>
<td>8.3</td>
<td>6.8</td>
<td>6.9</td>
</tr>
</tbody>
</table>

The magnitude of the difference, $\mu_{T+DC} - \mu_{TRAY}$, should in theory reflect the magnitude of the mean residence time in the downcomer, but as is shown in Table 7.2, this actually does not reflect the absolute value of $\mu_{downcomer}$. Therefore, it can be concluded that the combined system and the tray do not exert a resistance in series to the liquid flow and the mean residence times are certainly not additive.

The A.I.Ch.E. Bubble Tray Manual recommends that the average residence time of liquid on the tray be computed from the equation:

$$\mu = \bar{t}_L = 37.4 \frac{Z_c Z_L}{L_w}$$

(7.41)

where $Z_c$ = liquid holdup on tray (in$^3$/in$^2$)

$Z_L$ = liquid pathlength between inlet and outlet weirs (ft)

$L_w$ = liquid flowrate (gall/min ft weir)

$Z_c$ was correlated by

$$Z_c = 1.65 + 0.19 w - 0.65 F_A + 0.02 L_w$$

(7.42)

Inspection of equation (7.41) shows that 37.4 is a coeffi-
cient entirely related to the units used in the equation and the actual definition of $\bar{t}_L$ is given by:

$$\mu = \bar{t}_L = \frac{Z_c \cdot Z_L \cdot W}{Q}$$

where $L_w = Q/W$ (gall/min ft of weir)

that is,

$$\mu = \bar{t}_L = \frac{\text{active volume of fluid}}{\text{volumetric flowrate}}$$

The above definition refers to plug flow. The liquid hold-up, $Z_c$ is susceptible to the tray condition changing constantly i.e. oscillations and since the equation used for calculating it, Equation (7.42) is for specific conditions only, the question arises as to its universality. The results of this study and those calculated by equation (7.41) are shown compared in Fig. 7.40. This has also been done for the case of the 3/8 inch hole diameter used by Haq (1972) for comparison purposes. Fig. 7.40 shows a considerable variation in the values of the experimental and the calculated mean residence times, equation (7.41). The low values of the calculated mean residence time would suggest little or no mixing on the trays and therefore plug flow as has previously been discussed while the high values obtained experimentally suggest that the liquid on the trays is not in plug flow and that partial mixing does occur.

7.4.2 The Variance of the Residence Time Distribution

Another parameter which can be measured directly from the experimental results is the variance of the mean residence time
Liquid Flowrate, $L_w$(gpm/Ft Weir)

Round Column: $F_A$-Factor = 1.72
Rectangular Column: $F_A$-Factor = 1.72

Round Column: $F_A$=1.72 Calculated
Rectangular Column: $F_A$=1.72 Calculated
HAG: Rect. Column do=0.375" $F_A$=2.1

FIG 7.40 Comparison of Mean Residence Time on Tray Only
distribution or the second moment. The variance characterizes the amount of dispersion of the data from the mean and is thus a measure of the extent of liquid mixing on the tray.

The time-based variance has been plotted as a function of liquid flowrate on the tray for both columns as shown in Fig. 7.41 and is seen to decrease rapidly with increasing liquid rate, but flattens out to an almost constant value as the liquid rate is further increased. This is in agreement with the results of Thomas and Campbell (1967), Gilbert (1959) and Thomas and Haq (1976). Fig. 7.41 also shows that the time-based variance is a function of column design. A higher value of the time-based variance is obtained on the rectangular column than on the round column, especially at low liquid rates. This implies that the degree of mixing in each column differs. This can be explained in terms of the effect of weir constriction. As a result of the size of the round column, this effect is minimal at comparatively low flowrates, but as the liquid flowrate is increased, a more significant restriction is likely to exist on the round column, especially in the vicinity of the two ends of the chordal downcomer segment. The effect of this is to increase liquid holdup in the round column at these high flowrates and thereby the time based variance. At low liquid rates, where the effect of the weir constriction is not so significant for the round column, a lower liquid holdup is obtained and therefore a lower value of the time based variance.

The variances obtained from studies on the tray, downcomer
Fig 7.41 Time-based Variance as a Function of Liquid Flowrate (Tray)
Fig 7.42 Variation of Time-based Variance with $F_A$-Factor (Tray only)
and the combined tray plus downcomer system of the rectangular column have been compared in Fig. 7.43. The trend remains the same and additionally, it is noted that the variances are additive. The variance obtained for the combined system is about equal to the sum of the variance of the separate tray and downcomer systems. This is in agreement with the additivity of variances pointed out by Levenspiel (1972). Thus,

$$\sigma^2 (\text{tray + DC}) = \sigma^2 (\text{tray}) + \sigma^2 (\text{DC})$$  \hspace{1cm} (7.43)

### 7.4.3 Eddy Diffusion Model for Liquid Mixing

Several authors have previously applied the eddy diffusion concept to mixing problems in continuous flow systems. If mixing can be shown to obey the diffusion equation the assumption of an eddy-diffusion mechanism will yield the maximum useful information than the considerably more complicated residence time distribution approach. The eddy-diffusion model has been used by Foss et al. (1958), A.I.Ch.E. researchers (1958), Gilbert (1959), Barker and Self (1962), Thomas and Campbell (1967) amongst others, to quantify the degree of liquid mixing on sieve trays. Since the concentration gradient in the liquid on bubble plates is a function of the degree of back-mixing, a precise knowledge of the degree of mixing would enable the calculation to be made with more confidence, of the plate efficiencies.

It has been shown in Section 3.4.11 that the approach of Van der Laan (1958) can be used to solve the eddy-diffusion equation (3.76) for a doubly infinite pipe. The doubly infinite pipe is assumed to be more closely related to the mixing con-
Liquid Flowrate, $L_v$ (gpm/ft Weir)

Rectangular Column:  Tray $F_A = 1.72$

; ; : Downcomer $F_A = 1.72$

; ; : Tray + Downcomer $F_A = 1.72$

Fig 7.43 Comparison of Time-based Variance for Tray, DC, and Tray+DC
ditions on the tray in the present studies than the closed pipe concept which assumes that the mixing pattern at the inlet and outlet to the tray is related to plug flow.

Based on the doubly infinite pipe concept, Van der Laan (1958) has shown that the dimensionless variance can be defined by:

$$\sigma_0^2 = \frac{\sigma^2}{\mu} = \frac{2}{Pe} + \frac{8}{Pe^2} \quad (7.44)$$

where \( Pe \) is the liquid phase Peclet number.

Equation (7.44) can be solved quadratically for \( Pe \) and is given by:

$$Pe = \frac{1 + \sqrt{1 + 8 \sigma_0^2}}{\sigma_0^2} \quad (7.45)$$

where \( \sigma_0^2 \) is the dimensionless variance calculated from the equation:

$$\sigma_0^2 = \frac{\sigma^2}{\mu^2} \quad (7.46)$$

where

\( \sigma^2 = \) time-based variance (sec$^2$)
\( \mu = \) mean residence time (sec)

The dimensionless variance has been plotted as a function of liquid flowrate as shown in Figs. 7.44 and 7.45 for the round and rectangular columns respectively. Fig. 7.46 shows a comparison of the data obtained for the two columns. The most significant observation from Figs. 7.44 and 7.45 is the considerable amount of scatter of the experimental points about the correlation lines. The same variation has been noted in
Fig 7.44 Dimensionless Variance as a Function of Liquid Flowrate on Tray
Fig 7.45 Dimensionless Variance as a function of Liquid Flowrate on Tray
Fig 7.46 Comparison of Dimensionless Variance of Liquid on Tray only
the work of Thomas and Ogboja (1982), although Thomas and Haq (1976) and Campbell (1965) showed a good correlation of the two parameters. The reasons for this are not clear, but it must be noted that Thomas and Haq employed 3/8 inch hole diameter and Campbell employed 1/8 inch hole diameter compared to 1 inch diameter employed in this and Ogboja's studies. It would therefore appear that there is either a considerable difference in the mixing patterns between small and large diameter perforations or that the parameters chosen are not the best available. This would in effect call into question the universality of the use of the dimensionless variance to characterize liquid mixing on sieve plates, but may be of use as a first approximation. A comparison of the values of the dimensionless variance obtained in the round and rectangular columns is shown in Fig. 7.46. The larger round column gives a lower value for $\sigma_0^2$ and if this is any indication of the extent of mixing, it can be concluded that more liquid mixing is obtained on the rectangular column. This could be due to the presence of stagnant liquid layers around the perimeter of the tray in the round column due to wall effect as a result of the curvature of the walls.

In addition, the discrepancy can be considered to be related to the differences in the nature of the froth movement on the plate in the two columns. The froth movement in the smaller rectangular column was observed to be of a pulsating sinusoidal type, oscillating from side to side along the length of the tray while on the bigger round column, a uniform vertical oscillation was obtained between the plate
sides perpendicular to the liquid flow. The first type of motion would tend to promote backmixing while the second type would suppress it. Logically, the effect of more backmixing would be to increase the residence time of the liquid on the tray, and therefore the time of contact between the vapour and the liquid phase. This, in effect, in absorption, should increase the uptake of gas by the liquid and therefore the efficiency of the tray. This would therefore confirm results obtained for the two columns as already discussed in Sections 7.3 and 7.4.1.

An attempt to correlate the dimensionless variance with air flowrate did not yield any satisfactory relationship.

Another means of quantifying the degree of mixing is by the use of the Peclet number which has variously been used to measure the departure of a system from plug or completely mixed flow. In this study, the Peclet number has been computed from equation (7.45) and it may be considered as a ratio of the bulk mass transport to the diffusive mass transport, that is, the ratio of the velocity of the fluid through the system to the hypothetical velocity of the fluid moved by the eddy diffusion mechanism. Thus, as shown in Section 3.4.11, when the Peclet number tends to zero, a large amount of dispersion is obtained and the system tends to mixed flow. Conversely, when the Peclet number tends to infinity, a negligible amount of dispersion results and the system tends to plug flow.

The Peclet number computed from the results of the present studies is tabulated in Tables A2.4.1-A2.4.3.
Fig 7.47 Variation of Peclet Number with Liquid Flowrate (Tray only)

- Liquid Flowrate, $L_w$ (gpm/ft Weir)
  - Round Column $F_A = 1.72$
  - Rectangular Column $F_A = 1.72$

Peclet Number, $P_e$
Its variation with liquid flowrate has been plotted as shown in Fig. 7.47 from which it is observed that although there is a lot of scatter of the experimental points about the correlation line, the general trend indicates an increase of the Peclet number with increasing liquid flowrate. An inspection of the tabulated results reveals that it is more or less independent of the air flowrate. The findings are in agreement with those of Barker and Self (1962).

Below is a table of Peclet numbers for various mixing conditions abstracted from Levenspiel (1972).

Table 7.3 Relationship between mixing conditions, dispersion number and Peclet number (from Levenspiel (1962))

<table>
<thead>
<tr>
<th>Mixing pattern</th>
<th>Dispersion No. D/uL</th>
<th>Peclet No. Pe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plug flow</td>
<td>0</td>
<td>∞</td>
</tr>
<tr>
<td>Small amount of dispersion</td>
<td>0.002</td>
<td>500</td>
</tr>
<tr>
<td>Intermediate amount of dispersion</td>
<td>0.025</td>
<td>40</td>
</tr>
<tr>
<td>Large amount of dispersion</td>
<td>0.2</td>
<td>5</td>
</tr>
<tr>
<td>Mixed flow</td>
<td>∞</td>
<td>0</td>
</tr>
</tbody>
</table>

The range of dispersion number and Peclet number obtained from the two columns in the present studies are 0.133-0.092 and 7.5-10.8 respectively. The values obtained when viewed with respect to those in Table 7.3 would suggest a mixing
pattern in the two columns that may be described as having either an intermediate or a large amount of dispersion. Millington (1971), while working on the rectangular column used in the present studies but on 1/8 inch diameter perforations obtained values of Pe ranging from 19.0-26.0. Barker and Self (1962) have reported values ranging from 40-120, while Sterbacek (1968) has reported values ranging from 0.25-14. All these results would indicate that the mixing on sieve plates is far removed from plug flow, lying more between intermediate mixing and perfect mixing. However, the interpretation of and the significance that may be attached to the value of the Peclet number, should be viewed with some caution. The range of figures listed above would suggest that there may be circumstances when the effects associated with these values are not far removed from those which would occur when a Pe of zero or infinity was reached. This leads to some confusion and uncertainty and in effect throws wide open the whole concept of the use of the Peclet number for qualitatively and quantitatively assessing the extent of mixing in a system. The concept has its basis in flow through pipes which when applied to flow in a system as has been used in this and other studies would be a gross oversimplification of the conditions really existing in the systems. There is no doubt about the usefulness of the concept as a measure of mixing in the intermediate ranges and the performance of the apparatus in which the mixing occurs but the interpretation of the absolute value of the Peclet number for defining the two extreme cases of plug flow and perfect mixing leaves much to be desired.
Figs. 7.48 and 7.49 are typical response curves obtained directly from the results of the mixing studies and both show that the columns are being operated with mixing patterns intermediate between plug flow and perfect mixing.

7.4.3.1 Correlation of Eddy Diffusion Coefficient

In attempting to characterize the degree of mixing on perforated plates, an analysis is usually drawn between the mixing and molecular diffusion. The dispersion coefficient or the eddy diffusion coefficient therefore becomes the equivalent of the molecular diffusion coefficient and a useful tool for measuring the degree of mixing during flow.

The eddy diffusion coefficient in the present work has been calculated from:

\[ D_E = \frac{V_f Z}{Pe} \]  

(7.47)

where \( V_f \), the froth velocity, is defined by:

\[ V_f = \frac{Q_L}{Z_C} \]  

(7.48)

\( Q_L = \) liquid rate (ft\(^3\)/sec ft of weir)
\( Z_C = \) static head or liquid holdup on tray (ft)
\( Z = \) distance from inlet weir to sampling point (ft)
\( Pe = \) Peclet number (equation (7.45))

The calculation of the value of the eddy diffusion coefficient, \( D_E \), has been made by Foss (1957), Foss et al (1958) and by Barker and Self (1962).
RUN 2

Fig. 7.48 A typical response curve from the residence time study on the Round Column

$F_A = 1.50 \quad L_w = 22.5 \text{ gal/min ft of weir}$
Fig. 7.49 A typical response curve from the residence time study on the Rectangular Column
\[ F_A = 2.60 \quad L_w = 30 \text{ gall/min ft. of weir} \]
Foss et al. (1958) have shown that the froth momentum is a fundamental variable in the correlation of $D_E$ because it affords some measure of the froth's tendency to remain in uniform motion and that the higher the froth momentum, the more difficult it would be for any external force to change the velocity vector of a given froth element. Sterbacek (1968) has successfully related the dispersion coefficient to the energy dissipated per unit volume of froth.

A problem that arises in reading and assessing the above work is the meaning implied by the use of the word "froth", for example in froth velocity, $V_f$. The only meaning that suggests itself is that it refers to the "aerated mass of fluid" (gas/liquid) on the tray. In this context, an assumption is made that the velocity of the aerated mass or froth on the tray corresponds to the liquid velocity as defined by:

\[ \text{Volumetric liquid flowrate across the tray} \]
\[ \text{Liquid holdup on the tray} \]

Further, this can be defined by equation (7.48). The measurement of the froth velocity would be very difficult as the froth comprises two phases. The proven partial mixing taking place on the tray (as shown by the Peclet number) seems to be contradictory to a uniform froth velocity and further, the definition given in equation (7.47) as being applicable is open to some comment. The use of the active tray length or $Z$, usually referred to plug flow, may not be strictly true. Hence, the value of $D_E$ which depends on $V_f$ and $Z$ may not be therefore an absolute but approximate value. This being so,
one still has to use the definition of equation (7.47) in the absence of any other reliable methods.

The eddy diffusion coefficients calculated from equation (7.47) are shown in Tables A2.4.1-A2.4.2. It has been found that these results are well correlated by the following equations:

**Rectangular Column:**

\[ D_E = 0.01663 \ U_A^{0.46} + 1.620 \ Z_C - 0.3216 \] (7.49)

**Round Column:**

\[ D_E = 0.03435 \ U_A^{0.44} + 1.12 \ Z_C - 0.2584 \] (7.50)

where \( D_E \) = eddy diffusion coefficient \((\text{ft}^2/\text{sec})\)

\( U_A \) = superficial air velocity based on the plate active area \((\text{ft/sec})\)

\( Z_C \) = liquid holdup on plate \((\text{ft})\)

A comparison of the experimental \( D_E \) values and those predicted by equations (7.49) and (7.50) are shown plotted in Figs. 7.50 and 7.51 for the rectangular and round columns respectively. For the rectangular column, the average standard deviation of the experimental points from the correlation line is 1.1% and for the round column 0.5%.

The eddy diffusion is seen to increase with the liquid holdup on the tray and in effect with liquid flowrate, since an increase in the latter will produce a corresponding increase in the liquid holdup as has been shown in Section 7.1.3.2. The eddy diffusion coefficient is also found to be a function of the air flowrate. All these results are in agreement with those of Foss et al. (1958), Gilbert (1959) and in particular
Eddy-Diffusivity, $D_E (\text{ft}^2/\text{sec})$

- $D_E$ calculated from Equation (7.49)

$F_A$-Factor = 1.72
$F_A$-Factor = 2.20
$F_A$-Factor = 2.60

Fig 7.50 Correlation of Eddy-Diffusivity Results (Rectangular Column)
Eddy-Diffusivity, $D_E (ft^2/sec)$

$D_E$ calculated from Equation 7.50.

- $F_A$-Factor = 1.40
- $F_A$-Factor = 1.50
- $F_A$-Factor = 1.60
- $F_A$-Factor = 1.72

Fig 7.51 Correlation of Eddy-Diffusivity Results (Round Column)
Barker and Self (1962). The satisfactory correlation of the eddy diffusion coefficient data is seen as an indication of the considerable care exercised in conducting the dye studies experiments.

The trends reported for the variation of $D_E$ with air and liquid rates have also been reported in the A.I.Ch.E. Bubble Tray Manual. Here, $D_E$ was correlated by:

$$(D_E)^{0.5} = 0.0124 + 0.0171 U_A + 0.0025 L_w' + 0.015 W \quad (7.51)$$

for sieve plates and 3 inch diameter bubble caps.

where $L_w'$ = liquid flowrate (US gal/min ft weir length)

A comparison of the results obtained in the present studies and those calculated by equation (7.51) is presented in Fig.7.52. The results of Gilbert (1959) and Barker and Self (1962) are abstracted from an equivalent plot in the paper by the latter authors (1962) and are also shown in Fig. 7.52. None of the plotted experimental data seem to be well correlated by equation (7.51) and from the scatter of the results, it would appear that the parameters chosen in the equation are not the best available.

Barker and Self (1962) have suggested the use of a modified form of an equation proposed by Foss et al. (1958) to correlate liquid mixing results in terms of the liquid hold-up and the froth momentum on the plate. The former workers modified the equation proposed by Foss et al. (1958), transcribing the "rate of exchange of variance per unit of liquid hold-up ($2 D_E/V_f^3$) versus froth momentum ($Z_{c}/Z_f$, froth
Fig 7.52 Comparison of Experimental and Predicted Eddy-Diffusivity Values
density \times V_f$, froth velocity) into a relationship between 
\( \frac{D_E}{V_f^3 h_L} \) and \( h_L V_f / h_f \). Their data was found to be well 
correlated by an equation of the form:

\[
\frac{2 D_E}{V_f^3 Z_c} = k_1 \left[ \frac{Z_c V_f}{Z_f} \right]^{-k_2}
\] (7.52)

where

- \( k_1 \) and \( k_2 \) are constants
- \( h_f, Z_f \) = froth height (ft)
- \( h_L, Z_c \) = liquid holdup (ft)
- \( V_f \) = froth velocity (ft/sec)

In the present work, the experimental data are well 
correlated by the following equations:

**Rectangular Column:**

\[
\frac{2 D_E}{V_f^3 Z_c} = 0.0676 \left[ \frac{Z_c V_f}{Z_f} \right]^{-2.7}
\] (7.53)

with a 10% standard deviation.

**Round Column:**

\[
\frac{2 D_E}{V_f^3 Z_c} = 0.0537 \left[ \frac{2 c V_f}{Z_f} \right]^{-2.27}
\] (7.54)

with a 10% standard deviation.

The best line through the data of Foss et al. (1958) 
and Barker and Self (1962) have been reproduced from the 
latter's paper and are shown alongside the results obtained 
from the present studies on the rectangular column in Fig. 7.53.
Fig 7.53 Correlation of Eddy-diffusivity Data (Rectangular Column)
For the sake of clarity, the results of the work on the round column are shown separately in Fig. 7.54. From Figs. 7.53 and 7.54, it is apparent that values of the eddy diffusion coefficient are well correlated in terms of the liquid hold-up, the froth height and the froth velocity.

The apparent discrepancies between the results of the three independent studies reported in Fig. 7.53 have no obvious explanation but might be due partly to the different hole sizes and tray geometries employed. Thomas and Haq (1976) have shown that the hole size has a marked effect on the eddy diffusivity and therefore the degree of mixing. They found that the eddy diffusion coefficient increases with hole diameter. Foss et al. (1958) carried out their studies on sieve plates of dimensions 9\(\frac{1}{2}\) inch x 45 inch with 3/16 inch diameter holes giving 4\% and 11\% free area, while Barker and Self employed a sieve plate 68\(\frac{1}{2}\) inch x 13 inch with 3/16 inch diameter holes and a 5\% free area. The present work has been carried out on a sieve plate 36 inch x 12 inch with 1 inch diameter holes and a 12\% free area. The afore-mentioned might in fact be a confirmation of the findings of Thomas and Haq (1976). Also, Barker and Self have attempted to justify the difference between their and Foss et al.'s results on the basis of the methods employed for calculating \(D_E\). Barker and Self computed \(D_E\) from values of the Peclet number which they obtained as the gradient of a linear plot of the ratio of tracer concentration at the sampling point and tracer concentration at the injector grid against (1-w) where w is the ratio of the total tray length and the distance from the inlet.
Fig 7.54 Correlation of Eddy-Diffusivity Data for Round Column
weir to the sampling point. Foss et al., on the other hand, correlated their liquid mixing data by plotting the rate of increase of variance per unit tray length, \( \sigma_t^2 \sqrt{w_T Z} \) against the product of froth density and froth velocity or froth momentum. Barker and Self have converted this for use with \( D_E \) measurements to give the plot shown in Fig. 7.53.

7.4.4 Liquid Residence Time and Tray Efficiency

The tray efficiency is a measure of the mass transfer process occurring as a result of the contact between the liquid and vapour phases on a bubble tray and the value of the efficiency obtained would in effect depend on the length of contact of these two phases. The mean residence time of the liquid phase is a measure of the length of time taken by the liquid elements in traversing this tray. Hence, any variable which affects the mean residence time of the liquid on the tray would in effect influence the mass transfer efficiency.

Fig. 7.55 has been presented to show the variation of the mass transfer efficiency with variations in the mean liquid residence time. It is clear that increasing the mean residence time produces a corresponding increase in the mass transfer efficiency which approaches 100% asymptotically as the mean residence time tends to infinity. This is in agreement with the results obtained by Thomas and Haq (1976) and can be attributed to the fact that the effect of increasing the mean residence time is to increase the contact time between the vapour and liquid phases, thereby increasing the up-take of the solute and therefore increasing the mass transfer efficiency.
Fig 7.55 Effect of Mean Residence Time on Tray Efficiency
The deviations between the results obtained for the rectangular column and those obtained for the round column are due to the differences in the mean residence time recorded for the two columns as has been shown in Fig. 7.40. At low liquid flowrates, the rectangular column has a higher mean residence time than the round column, and therefore would be expected to show a higher efficiency at these low flowrates. However, with further increases in the liquid flowrate, as a consequence of the reasons discussed in Section 7.4.3, the residence time of the liquid in the round column becomes higher than that of the rectangular column and thus as shown in Fig. 7.55, the efficiency obtained is correspondingly higher than for the rectangular column.

Gerster et al. (1949) have attempted to explain the slight variations in efficiency with increasing gas flowrate in terms of the contact time between the phases. They pointed out that this could be due to the interaction of counteracting effects of variables involved. They pointed out that increasing the gas rate would have three effects:

a) decrease the contact time between the phases and therefore decrease the amount of mass transfer;

b) increase the froth height and thereby increase the contact time;

c) increase the degree of aeration of the liquid, which increases the interfacial area between phases and thus increase the amount of mass transfer.
The net effect therefore is the combination of an increase in mass transfer on the one hand and a decrease in mass transfer on the other hand, and if these effects are equal in magnitude, the consequence would be for them to cancel out, therefore leaving the tray efficiency unchanged with increasing air flowrate.

Foss et al. (1956) have also shown that the lack of variation of the tray efficiency with air flowrate is due to the opposing effects of an increase in interfacial area on the one hand and a decrease in residence time on the other hand.

The results of this study reveal that the mean residence time is not significantly affected by variations in the air flowrate.

7.4.5 Liquid Mixing and Tray Efficiency

For the interpretation of mass transfer data on a sieve plate, use of the number of transfer units, \( N_L \) is preferred over the more obvious plate efficiency, \( E_{ML} \), because it is more related to the fundamental mass transfer equations.

The relationship between \( E_{ML} \) and \( N_L \) depends on the degree of liquid mixing on the bubble tray and correlations have been proposed for the two extreme cases of plug flow and complete mixing, thus:

\[
\text{Plug Flow: } N_L = -2.3 \log (1 - E_{ML})
\]
Complete Mixing: 

\[ N_L = \frac{E_L}{1 - E_L} \quad (7.56) \]

where \( E_L \) is the liquid-phase point efficiency.

Both of these equations are known to give results which can be seriously in error when applied to cases of partial mixing. Thus, for the intermediate case of liquid mixing on sieve trays, the mixing model of Foss et al. (1958) has been used to relate \( E_{ML} \) and \( N_L \) by the equation:

\[ E_{ML} = 1 - (1 + N_L \sigma^2)^{-1/2} \quad (7.57) \]

where \( \sigma^2 \) = dimensionless variance.

Equation (7.57) has been solved for \( N_L \) and is given by:

\[ N_L = \frac{(1 - E_{ML})^{-\sigma^2} - 1}{\sigma^2} \quad (7.58) \]

The Bubble Tray Manual has proposed a correlation for computing \( N_L \) given by:

\[ N_L = [ (1.065 \times 10^8) D_L ]^{0.06} (0.26 F_A + 0.15) t_L \quad (7.59) \]

where

\[ D_L = \text{liquid diffusivity (ft}^2/\text{hr}) \]
\[ t_L = \text{time liquid residence time as given by equation (7.41).} \]

Values of the number of liquid mass transfer units have been calculated by equations (7.58) and (7.59) for comparison. Values obtained from equation (7.58) have been tabulated in Tables A2.5.1-A2.5.4.
It would appear from Figs. 7.44 and 7.45 that the dimensionless variance varies little with liquid flowrate. It therefore follows from equation (7.58) and of course (7.59) that $E_{ML}$ and $N_L$ would be expected to show the same trends with respect to variations in liquid flowrate and air flowrate. If this is true, then the behaviour of $E_{ML}$ with an independent variable can be predicted directly from the behaviour of $N_L$ with that variable.

Fig. 7.56 shows the effect of the variation of liquid flowrate on the number of liquid phase transfer units, $N_L$ calculated from equation (7.58). It is apparent from Fig. 7.56 that $N_L$ shows a similar variation with variations in liquid flowrate as $E_{ML}$. They both decrease with increasing liquid rate and are insensitive to increasing air flowrate. This decrease of $N_L$ with increasing liquid flowrate is a direct function of the decrease of mean residence time with increasing liquid flowrate. This dependence of $N_L$ on the residence time has been proposed by A.I.Ch.E. workers at both the University of Delaware (1958) and the North Carolina State College (1959). They showed that:

$$N_L = k_L a t_L$$  \hspace{1cm} (7.60)

where

$k_L$ = liquid phase mass transfer coefficient (ft/hr)

$a$ = interfacial area per unit volume of liquid holdup (1/ft)

$t_L$ = true liquid residence time (sec)

Thus, a plot of $N_L$ against the liquid residence time
Number of Transfer Units, \( N \)

Liquid Flowrate, \( L_w \) (gpm/Ft Weir)

- Round Column \( F_A = 1.72 \)
- Rectangular Column \( F_A = 1.72 \)
- \( F_A = 1.50 \)
- \( F_A = 2.20 \)

Fig 7.56 Variation of Number of Transfer Units with Liquid Flowrate
should result in a straight line of slope $k_L a$. Values of $N_g$

...
Fig 7.57 Variation of Number of Transfer Units with Mean Residence Time
Fig 7.58 Comparison of Measured and Calculated Number of Transfer Units
Fig 7.59 Variation of Mass Transfer Coefficient with Liquid Flowrate
Table 7.4 Variation of $k_{La}$ with air flowrate

<table>
<thead>
<tr>
<th>Column</th>
<th>$L_w$ (gpm/ft weir)</th>
<th>$F_A$</th>
<th>$k_{La}$ (sec$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Round</td>
<td>10.0</td>
<td>1.50</td>
<td>0.115</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.72</td>
<td>0.147</td>
</tr>
<tr>
<td>Rectangular</td>
<td>15.0</td>
<td>1.72</td>
<td>0.139</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.20</td>
<td>0.158</td>
</tr>
</tbody>
</table>

From Table 7.4 it is seen that for the rectangular column, a 28% increase in $F_A$ yields a 14% increase in $k_{La}$. The significance of this change would depend on the subsequent effect it has on tray efficiency, which for the constant liquid rate is found to increase by 2.3%. This might be of significance from an industrial point of view. Generally, an increase in $k_{La}$ would serve to increase the dispersion of the gas within the liquid and therefore increase the tray efficiency. This deduction would partly account for the higher efficiency values obtained in the rectangular column compared with the round column. A higher $k_{La}$ means a higher dispersion on the tray, and therefore a higher value of the efficiency.

Figs. 7.57 and 7.58 also show the comparison of $N_L$ values calculated from equations (7.58) and (7.59). From the graphs, although the same variation of $N_L$ with mean residence time and liquid flowrate have been obtained from equation (7.59),
This equation is also seen to overestimate values of $N_L$. It has previously been shown in section 7.4.1 that the use of equation (7.41) proposed in the Bubble Tray Manual for calculating the mean residence time seriously underestimates the mean residence time. The effect of the interaction of $N_L$ and mean residence time computed from the equations in the Bubble Tray Manual would be to grossly overestimate $k_L a$. Therefore, the validity of these equations for use in conditions of intermediate mixing is questionable.
CHAPTER EIGHT

CONCLUSIONS

The conclusions drawn from the present investigations will be considered under three sections:

1. Hydrodynamic Studies
2. Mass Transfer Study
3. Liquid phase residence time distribution and mixing

8.1 Hydrodynamic Studies

The experimental results have been reported in the form of correlation equations as the case may be.

8.1.1 Dry Plate Pressure Drop

The dry plate pressure drop was investigated as a function of vapour velocity, number of perforations and plate thickness-to-hole diameter ratio.

The experimental data have been correlated as follows:

\[
\text{Round Column: } U_0 = 0.673 \left( \frac{2g \rho L h_{DP}}{12\rho_v} \right)^{0.5}
\]

\[
\text{Rectangular Column: } U_0 = 0.743 \left( \frac{2g \rho L h_{DP}}{12\rho_v} \right)^{0.5}
\]

The results of the present work confirm the dependence of the dry tray pressure drop on the tray free area and plate thickness-to-diameter ratio. The McAllister equation (7.2)
in its present form is adequate for the prediction of dry tray pressure drop. However, the indication is that for low percentage free area, the equation in its present form needs to be modified.

The dry plate pressure drop decreases with decreasing values of the correlation factor 'k'. It might also be possible to obtain pressure drop data on simulators, which could then be scaled up for application to larger columns.

8.1.2 Total Pressure Drop

The total pressure drop was investigated as a function of liquid and air flowrates. The experimental data, obtained by use of a manometer across the operating plate have been satisfactorily correlated by these equations:

**Rectangular Column:**

\[ h_T = 2.32 \ L_w^{0.114} \ \exp(0.0207 \ F_A^2) \quad (L_w = 10-50 \ gpm/ft \ weir) \]

and

\[ h_T = 0.015 \ L_w^5 + 0.213 \ F_A^2 + 2.83 \quad (L_w > 15 \ gpm/ft \ weir) \]

**Round Column:**

\[ h_T = 0.0239 \ L_w^5 + 0.259 \ F_A^2 + 2.425 \]

These equations predict an increase of the total pressure drop with increasing liquid and air rates.

8.1.3 Froth Height on Tray

The froth height was measured visually and is only accurate
to ±0.25 inch, but the experimental data have been well correlated by:

**Rectangular Column:**

\[ Z_f = 0.0850 \ L_w + 1.39 \ F_A + 2.61 \]

**Round Column:**

\[ Z_f = 0.128 \ L_w + 3.64 \ F_A + 1.22 \]

The froth height is predicted to increase with increasing liquid and air rates.

### 8.1.4 Liquid Hold-up on the Tray

The liquid hold-up on the tray has been derived from measurements of the dynamic head which were later correlated for the effect of the momentum head of the gas.

\[ Z_c = Z_D + Z_M \]

The dynamic head, \( Z_D \) was measured using monometers installed in the tray floor and after corrections for surface tension effects, the experimental data have been correlated by:

**Rectangular Column:**

\[ Z_D = 0.0169 \ L_w - 0.329 \ F_A + 2.514 \]

**Round Column:**

\[ Z_D = 0.0266 \ L_w - 0.980 \ F_A + 3.08 \]

These equations predict an increase of \( Z_D \) with increasing liquid rate but with decreasing air rate.
The liquid hold-up was also calculated on the basis of dry tray pressure drop, total pressure drop and residual pressure drop data. The hold-up data were obtained from the equation:

$$h_F = 1.12 \frac{Z_c(h_T)}{h_T} = h_T - h_{DP} - h_R$$

Values of $Z_c(h_T)$ calculated thus have been found to be generally smaller from those calculated from $Z_D$ correlations.

### 8.1.5 Aeration Factor

The aeration factor was computed as the ratio of the liquid hold-up to the sum of the weir height and the weir crest. In the present work, two values of the aeration factor were obtained, one for the hold-up based on dynamic head and the second for holdup based on total pressure drop data. Within the range of flow variables studied, the values of aeration factor obtained are within the range:

- Rectangular Column : $\beta = 0.609 - 0.680$
- Round Column : $\beta = 0.506 - 0.634$

### 8.1.6 Froth Density Factor

The froth density factor was calculated on the basis of the ratio of liquid hold-up to the froth height. The froth density was computed separately as a function of hold-up based on dynamic head data and hold-up based on total pressure drop data respectively. Over the range of fluid flowrates studied, values of froth density factor obtained fell within
the following ranges:

Rectangular Column: \( \phi = 0.255 - 0.386 \)
Round Column : \( \phi = 0.200 - 0.280 \)

8.1.7 Liquid Entrainment

The entrainment of liquid by vapour has been studied as a function of liquid and vapour rates, and as a function of tray spacing.

The results of the investigations are as follows:

a) Quantitative relationships have been obtained relating entrainment to the tray spacing, vapour and liquid rates, froth height, clear liquid head and hole diameter.

b) Variation of entrainment with liquid rate is entirely empirical, depending on different mechanisms under different vapour flowrate conditions.

c) Entrainment increases with vapour rate in general.

d) Entrainment increases with decreasing tray spacing.

e) The use of some established correlations is doubtful when applied to hole diameters larger than the range for which they were derived.

f) The Fair and Matthews (1959) flooding chart cannot be successfully applied to hole sizes outside the range for which the chart was derived.

g) The experimental data from the present work have been successfully correlated by two equations:
Log ($S'' \psi_v$) = 3.14 log $\left( \frac{U_0}{L_w S''} \right)$ + 0.352 $L_w' - 6.78$

and based on Kister et al. (1981)

$E \alpha \left( \frac{U_s}{L^{0.25}} \right)^{4.85} \left( \frac{Z_C}{(d_o S)^{1/4}} \right)$

8.2 Mass Transfer Study

The mass transfer efficiency was studied as a function of air and liquid rates. The experimental data were successfully correlated against liquid flowrate.

For the air-$CO_2$-water system employed in this study, the Murphree liquid efficiency is shown to decrease with increasing liquid rate and to increase slightly with increasing air rate.

The concentration of the solute at the tray inlet is significant in determining the up-take of the solute and therefore the resulting mass transfer efficiency.

The accuracy of the experimental data depends to a large extent on the method of sample withdrawal and care taken in the absorption analysis.

8.3 Liquid-phase Residence Time Distribution and Liquid Mixing

The liquid-phase mean residence time was investigated as a function of liquid and air flowrates, separately for the tray, downcomer and the combined tray plus downcomer system as a function of liquid and air flowrate and column geometry.
The following are the deductions from these investigations:

a) The mean residence time for all the systems studied was found to decrease sharply with increasing liquid flowrate, but subsequently tailed off to an almost constant value with further increases in the liquid rate. This has been related to the increase of liquid hold-up with liquid flowrate.

b) The mean residence time, to a good approximation, is insensitive to increased air flowrate. This also ties in with the insensitivity of the liquid hold-up to air rates.

c) While the mean residence times for the separate systems are not additive, the addition of the separate time based variances for the tray and downcomer approximate to the time-based variance obtained for the combined tray plus downcomer.

d) The dimensionless variance may only be used as an approximation and with caution is characterising the degree of liquid mixing on large diameter trays. The characteristics that have been previously reported in studies in small diameter perforations were not observable in this investigation.

e) The degree of liquid mixing is best characterised in terms of the eddy diffusion coefficient. The eddy diffusion coefficient data have been successfully correlated by the following equations:
De = 0.01663 U_A^{1.620} + 1.620 Z_C - 0.3216

Round Column:

De = 0.03435 U_A^{1.12} + 1.12 Z_C - 0.2584

The equation predicts an increase in De with increase in the liquid and air flowrates.

f) The correlation suggested by Barker and Self (1962), has been successfully used to correlate the data of the present studies. This correlation relates the eddy diffusion coefficient to the liquid hold-up, the froth height and the froth velocity. For the present studies, the following correlations were obtained:

Rectangular Column:

\[
\frac{2D_e}{V_f^3 Z_C} = 0.0676 \left( \frac{Z_C V_f}{Z_f} \right)^{-2.47}
\]

Round Column:

\[
\frac{2D_e}{V_f^3 Z_C} = 0.0537 \left( \frac{Z_C V_f}{Z_f} \right)^{-2.27}
\]

g) The absolute value of the Peclet numbers obtained in these studies suggest that the mixing patterns in the two columns are close to intermediate amount of dispersion. The range of Peclet number obtained in these studies is 7.5-10.8.
8.3.1 Liquid Mixing and Mass Transfer Efficiency

The tray efficiency has been shown to be a function of the mean residence time. The Murphree Liquid Efficiency increases with increasing mean residence time as a result of the increase in contact time between the phases.

The number of liquid-phase transfer units increases linearly with the mean residence time, thus confirming the theoretically derived equation:

\[ N_L = k_L a \bar{r}_L \]

The mass transfer coefficient is independent of liquid flowrate but increases with increasing air flowrate.

Finally, a summary of the above conclusions can be made as follows:

a) Data have been obtained for 1 inch diameter perforations in large pilot rigs.

b) An extensive critique on the dry tray pressure drop has been presented as very few studies have been published with respect to large hole sizes. Special reference has been made to the extensively used McAllister equation and its application to large hole sizes.

c) Great trouble was taken over obtaining the efficiency data. Although many values are quoted in the literature, there exists a great deal of doubt as to their validity. A careful examination shows that considerable errors can
be involved due to the methods of sampling and analysis of the solutions. Varshney (1964) devoted a major part of his thesis to comparing the methods of sampling and analyses.

The method employed in the present study devoted great care in the analysis using the radiometer autoanalyser which measured to 0.04% accuracy.

d) Particular attention and care were taken in the measurement of entrainment using a two-tray independent circulation system. Little work is available on entrainment in large hole sizes.

e) Data have been obtained for the mean residence time of the liquid on the tray only, downcomer only, and the combined tray plus downcomer system as applicable to large hole sizes. Very little work has been published with respect to the mean residence time of liquid on sieve trays with large perforations.
NOMENCLATURE

A constant in eq. (3.11)

A\textsubscript{C} cross-sectional area of column, ft\textsuperscript{2}

A\textsubscript{f} plate free area, ft\textsuperscript{2}

A\textsubscript{o} total area of holes per tray, ft\textsuperscript{2}

A\textsubscript{t} maximum free area possible if the total area of the plate were perforated at the same pitch-to-hole diameter ratio and pattern, ft\textsuperscript{2}

a interfacial area per unit volume of column, ft\textsuperscript{2}/ft\textsuperscript{3}

B constant in eq. (2.35)

C molar concentration, lb moles/ft\textsuperscript{3}

C\textsubscript{D} orifice coefficient

C\textsubscript{i} molar concentration at interface, lb mole/ft\textsuperscript{3}

C\textsubscript{L},C\textsubscript{O} molar concentration in bulk of fluid, lb mole/ft\textsuperscript{3}

C\textsuperscript{*} molar concentration in liquid phase in equilibrium with partial pressure, P\textsubscript{G} in gas phase, lb mole/ft\textsuperscript{3}

C\textsubscript{sb} vapour capacity parameter

C\textsubscript{T} total molar concentration, lb mole/ft\textsuperscript{3}

D\textsubscript{E} eddy diffusion coefficient, ft\textsuperscript{2}/sec

D\textsubscript{G},D\textsubscript{V} gas phase diffusivity, ft\textsuperscript{2}/sec

D\textsubscript{L} liquid phase diffusivity, ft\textsuperscript{2}/sec

d\textsubscript{k} column diameter, as defined in eqn. (2.86), m

d\textsubscript{o} perforation diameter, in

E entrainment ratio, lb liquid/100 lb air

E' rate of entrainment ratio, as defined in eqns. (2.54,255), (g liquid/cm\textsuperscript{2}sec)/(g air/cm\textsuperscript{2}sec)

E\textsubscript{ML} Murphree liquid efficiency

E\textsubscript{MV} Murphree vapour efficiency

E\textsubscript{OG} overall gas phase efficiency

E\textsubscript{OGT} true local efficiency
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{OL}$</td>
<td>overall liquid phase efficiency</td>
</tr>
<tr>
<td>$E_{TV}$</td>
<td>temperature or thermal efficiency</td>
</tr>
<tr>
<td>$E_w$</td>
<td>rate of entrainment, lb/min</td>
</tr>
<tr>
<td>e</td>
<td>entrainment (lb or moles/time)</td>
</tr>
<tr>
<td>$e_w$</td>
<td>weight entrainment ratio, lb liquid/lb air</td>
</tr>
<tr>
<td>$F_A$</td>
<td>vapour-liquid flow parameter</td>
</tr>
<tr>
<td>$F_{LV}$</td>
<td>Froude number, defined in eqn. (2.47)</td>
</tr>
<tr>
<td>Fr</td>
<td>correction factor, defined in eqn. (2.33)</td>
</tr>
<tr>
<td>f</td>
<td>coefficient (in eqn. 2.35)</td>
</tr>
<tr>
<td>$G_{M'}$</td>
<td>molar flowrate of gas per unit bubbling area, lb mole/hr ft$^2$</td>
</tr>
<tr>
<td>$G_M$</td>
<td>molar flowrate of gas, lb mole/hr</td>
</tr>
<tr>
<td>g</td>
<td>coefficient (in eqn. 2.35)</td>
</tr>
<tr>
<td>$g_C$</td>
<td>acceleration due to gravity, ft/sec$^2$</td>
</tr>
<tr>
<td>$H_{OL}$</td>
<td>overall height of transfer unit, ft</td>
</tr>
<tr>
<td>$h_{DP}$</td>
<td>dry plate pressure drop, in. water</td>
</tr>
<tr>
<td>$h_L$</td>
<td>effective liquid seal or tray floor, in. water</td>
</tr>
<tr>
<td>$h_M$</td>
<td>momentum head, in. water</td>
</tr>
<tr>
<td>$h_{OW}$</td>
<td>head of clear liquid over weir, in.</td>
</tr>
<tr>
<td>$h_R$</td>
<td>residual pressure drop, in. water</td>
</tr>
<tr>
<td>$h_T$</td>
<td>total pressure drop, in. water</td>
</tr>
<tr>
<td>$h_w$</td>
<td>weir height, in.</td>
</tr>
<tr>
<td>K</td>
<td>function of $t/d_o$, defined in eqns. (2.8,2.9,7.2)</td>
</tr>
<tr>
<td>K</td>
<td>constant in equation (2.35)</td>
</tr>
<tr>
<td>$K_{OG}$</td>
<td>overall gas phase mass transfer coefficient, lb mole/hr ft$^2$</td>
</tr>
<tr>
<td>$K_{OL}$</td>
<td>overall liquid phase mass transfer coefficient, lb mole/hr ft$^2$</td>
</tr>
<tr>
<td>$k_G$</td>
<td>gas film mass transfer coefficient, lb mole/hr ft$^2$</td>
</tr>
<tr>
<td>$k_L$</td>
<td>liquid film mass transfer coefficient, lb mole/hr ft$^2$</td>
</tr>
</tbody>
</table>
L, L_w  liquid flowrate, gall/min ft weir length
L_M    molar liquid flowrate, lb mole/hr ft^2
L_w'   liquid rate, lb/sec ft weir
m      slope of equilibrium curve y/x
m      factor of plate spacing, defined in eqn. (2.45)
N_A    molar rate of absorption of A per unit area, lb mole/hr ft^2
N_G    number of vapour-phase transfer units
N_L    number of liquid-phase transfer units
N_OG   number of overall transfer units (vapour)
N_OL   number of overall transfer units (liquid)
n      number of pools, defined in eqn. (2.69)
P      total pressure, atm
p      pitch of perforation, in
p      partial pressure, atm
Pe     Peclet number, defined in eqns. (7.45, 7.47)
Q      air flowrate, ft^3/min
R      universal gas constant
R      correction factor (in eqn. 2.12)
Re     Reynolds number (in eqn. (2.48)
r      bubble radius
S      tray spacing, in
S'     effective tray spacing (in) S-Z_f
S''    tray spacing, ft
S      interfacial area of bubble (in eqn. 2.64)
s      probability density
T      temperature
T      constant in eqn. (3.11)
t      plate thickness, in
T_L    liquid contact time, sec
\( t_g \)  
\( U \)  
\( U_A \)  
\( U_c \)  
\( U_o \)  
\( U_s \)  
\( U_{VN} \)  
\( V \)  
\( W \)  
\( x \)  
\( Y \)  
\( Y \)  
\( y \)  
\( y_e \)  
\( y^* \)  
\( Z \)  
\( Z_c \)  
\( Z_{c(hT)} \)  
\( Z_D \)  
\( Z_f \)  
\( Z_M \)  
\( Z_o \)  
\( Z_P \)  

Greek letters

\( \alpha \)  
\( \beta \)  
\( \beta_d, \beta_h \)
-313-

\( \beta \) constant in equation (2.98)

\( \delta \) stagnant film thickness, ft

\( \gamma \) constant in equation (2.98)

\( \Gamma \) gamma function

\( \lambda \) mass transfer rate, \( \frac{G_m}{L_m} \), absorption factor

\( \mu \) mean residence time, sec

\( \mu_g \) viscosity of vapour, \( \text{lb/(hr)(ft)} \)

\( \phi_d, \phi_h \) relative froth density based on dynamic head and total pressure head data respectively

\( \psi \) fractional entrainment

\( \Delta_{\rho} \) density difference, \( \rho_L - \rho_V \)

\( \rho_L \) density of liquid, \( \text{lb/ft}^3 \)

\( \rho_g, \rho_v \) density of vapour, \( \text{lb/ft}^3 \)

\( \sigma \) surface tension of liquid, dynes/cm

\( \sigma^2 \) time based variance, sec\(^2\)

\( \sigma^2_{\theta} \) dimensionless variance

\( \tau \) mean age of liquid leaving tray, sec

\( \theta \) dimensionless time

\( \theta' \) renewal time, sec

**Subscripts**

i interface

g gas

l liquid

v vapour
APPENDIX

A.1 CALIBRATION OF INSTRUMENTS

The preliminary calibrations of the various instruments used in this study were carried out according to standard procedures. The procedures are described below for the air orifice meter, the infra-red gas analyser and the automatic titration equipment.

A.1.1 Calibration of the Air Orifice Meter

A standard orifice meter, 3 inch internal diameter corresponding to B.S.S. 1042 (1943), with tappings located at D and D/2 of the plate was used for metering the air flow. This was installed in the 6 inch internal diameter pipe, upstream of the column.

The method outlined in B.S.S. 1042 (1943) was used to obtain the relationship:

\[ Q = 132.5 \sqrt{h_o} \]  \hspace{1cm} (A.1)

where

- \( Q \) = flowrate of saturated air (ft\(^3\)/min)
- \( h_o \) = pressure head drop across the orifice (in H\(_2\)O)

The gas flowrate has been conveniently expressed in terms of a factor based on the plate active area, that is, the area of the plate containing the perforations and defined as:

\[ F_A = U_a \sqrt{\rho_v} \]  \hspace{1cm} (A.2)

where
U_a = vapour velocity based on plate active area (ft/sec)
\( \rho_v = \) vapour density (lb/ft\(^3\))

Thus, for rectangular column operation at 20°C, \( F_A \) is related to \( h_0 \) by:

\[
F_A = \frac{0.710}{\sqrt{h_0}}
\]  \hspace{1cm} (A.3)

and for the round column,

\[
F_A = \frac{0.325}{\sqrt{h_0}}
\]  \hspace{1cm} (A.4)

where \( \rho_v = 0.0708 \) lb/ft\(^3\).

A.1.2 Calibration of Infra-red Gas Analyser

Calibration of the gas analyser was carried out every four hours during use. A supply of clean dry CO\(_2\)-free nitrogen was first pumped through the analyser for 15 minutes to purge it of any traces of CO\(_2\) that might have been trapped in it during operation. With the air flowing through the sample cell, the zero control knob was adjusted until the meter read zero. A cylinder of compressed gas mixture containing 10% CO\(_2\) + 90% Nitrogen was then attached to the analyser and the gas mixture pumped through it. The span control was adjusted until the correct concentration reading was obtained on the analyser. The reading was observed for a further 10 minutes to ensure that the analyser was stable. The analyser was once more purged with the CO\(_2\)-free Nitrogen for zero reading.
A.1.3 Mass Transfer Studies. Liquid Sample Analysis

The procedure for withdrawal of liquid sample for the mass transfer studies has been described in detail in Section 5.2.2.3a. The liquid samples collected were analysed using a Radiometer Auto-titrator, the component parts of which were described in Section 5.2.2.3b.

Liquid samples from the absorption process in the columns were collected in specially designed wash-bottles holding 25ml 0.05 N Sodium Hydroxide solution. Three samples each of 100ml were collected at each run.

A1.3.1 Analytical Technique

The analytical methods for the determination of dissolved carbon dioxide in water samples are very well covered in most textbooks on quantitative inorganic analysis.

When carbon dioxide is dissolved in water, carbonic acid is formed:

\[ \text{CO}_2 + \text{H}_2\text{O} \rightleftharpoons \text{H}_2\text{CO}_3 \]  \hspace{1cm}(A.5)

However, most of the carbon dioxide is present in the free state and only a small part (less than 1%) is present as carbonic acid.

Free carbon dioxide can be titrated as a monobasic acid when reacted with an alkali. The reaction is:

\[ \text{H}_2\text{CO}_3 + \text{NaOH} \rightarrow \text{NaHCO}_3 + \text{H}_2\text{O} \]  \hspace{1cm}(A.6)
\[ \text{NaHCO}_3 + \text{NaOH} \rightarrow \text{Na}_2\text{CO}_3 + \text{H}_2\text{O} \]  \hspace{1cm}(A.7)
The carbonic acid reacts instantaneously with the base and the 
equilibrium of reaction (A.5) having been disturbed, more 
carbon dioxide goes into solution to form carbonic acid before 
reacting with alkali. Thus, the net reaction is:

\[ \text{CO}_2 + 2\text{NaOH} \rightarrow \text{Na}_2\text{CO}_3 + \text{H}_2\text{O} \]  \hspace{1cm} (A.8) 
i.e. 1 mole \text{CO}_2 requires 2 moles \text{NaOH}.

However, because of the low concentration of \text{CO}_2 originally 
dissolved in the water, there would now be an excess of \text{NaOH} 
in solution and by a suitable method of analysis, the amount 
of excess \text{NaOH} can be determined. Since the original value 
of \text{NaOH} used is known, the amount of \text{CO}_2 that reacted may now be obtained.

Thus, liquid samples from the columns were run into \text{NaOH} 
to arrest the reaction of \text{CO}_2 with the water on the tray. The 
resultant solution, containing \text{Na}_2\text{CO}_3 and an excess of \text{NaOH} 
was analysed for excess \text{NaOH}.

The procedure used involved precipitating out the carbonate 
with barium chloride solution to form insoluble barium carbonate. 
A measured excess of barium chloride is added to a known volume 
of the sample mixture:

\[ \text{Na}_2\text{CO}_3 + \text{BaCl}_2 \rightarrow \text{BaCO}_3 + 2\text{NaCl} \]  \hspace{1cm} (A.9) 
\[ 2\text{NaOH} + \text{BaCl}_2 \rightarrow \text{Ba(OH)}_2 + 2\text{NaCl} \]  \hspace{1cm} (A 10)

The excess barium chloride, now reacted with \text{NaOH} to form 
soluble barium hydroxide, is back titrated with standard acid, 
using a suitable indicator, without filtering off the insoluble 
barium carbonate:
The next reaction therefore is:

\[ \text{NaOH} + \text{HCl} \rightarrow \text{NaCl} + \text{H}_2\text{O} \quad \text{(A.12)} \]

i.e. 1 mole NaOH requires 1 mole HCl for neutralization.

Since barium carbonate is the salt of a strong base and a very weak acid, it is strongly hydrolyzed in solution and has a pH of 8.6. Its solubility, however, is very small and is, moreover, diminished by the excess of barium salt present. Since a suspension of barium carbonate in a solution containing barium chloride is neutral toward phenolphthalein, a colour change from purple to colourless in the white suspension can be observed very sharply. Hence, phenolphthalein (colour change pH range 8.3-10.0) was employed as an indicator.

Standard solutions of HCl and carbonate-free NaOH purchased from B.D.H. Chemicals Ltd. were used for all analyses. All titrations were conducted in a closed flask under an atmosphere of inert Nitrogen gas to prevent escape of carbon dioxide from solution and contamination by atmospheric CO\(_2\). Extreme care had to be ensured, especially at the end point to make sure that the change of colour was permanent because the end-point of carbon dioxide solutions has a tendency for being indistinct.

The difference between the volume of HCl solution required in this titration and that required to neutralize another aliquot portion containing distilled and CO\(_2\)-free water, NaOH and BaCl\(_2\), therefore gives the volume of HCl equivalent to CO\(_2\) and bicarbonate content in the water. The bicarbonate content of the tap water used on the tray was determined by

\[ \text{Ba(OH)}_2 + 2\text{HCl} \rightarrow \text{BaCl}_2 + 2\text{H}_2\text{O} \quad \text{(A.11)} \]
titrating the known amount of water with standard HCl solution to a pH of 5.7 using methyl red indicator (colour change range: pH 4.4-6.2).

Each sample was made up from 100 ml CO2/H2O sample from the tray run into 25 ml 0.05 N NaOH contained in a stoppered sampling flask placed in a tank of crushed ice supercooled with salt. The sampling procedure has been described in Section 5.2.2.3a. Three samples each were taken for the inlet and outlet simultaneously and at intervals of 5 minutes.

A.1.3.2 Titration Procedure

The stoppered sampling flask was stirred vigorously on a magnetic stirrer for 10 minutes and then heated to 70°C. 10% BaCl2 solution was then added until no further precipitate was produced. This was found to be about the 25 ml mark for all analyses. The mixture was then vigorously stirred and rapidly cooled to room temperature in a water bath. 5 drops of phenolphthalein was added through the stopper and the mixture was titrated with 0.05 N HCl to the phenolphthalein end point using the automatic titrator. All titrations took place with a nitrogen gas blanket over the sample. The final volume of HCl required for neutralization corresponded to the amount of excess NaOH in the sample.

A blank solution, made up from 100ml tap water run through a stopper into 25 ml 0.05 N NaOH, was also analysed employing the same procedure outlined above.
The bicarbonate content of the tap water used was separately determined by titration with standardised 0.05 N HCl using methyl orange as indicator.

Owing to the large consumption of water during the efficiency runs, ordinary tap water had to be used for absorbing the CO$_2$ on the tray. Tap water contains dissolved salts and it becomes necessary to determine the solubility of CO$_2$ in the tap water used since solubility data quoted in the literature are for CO$_2$ in pure water. The procedure used has been outlined in Section 5.2.2.3. The liquid samples thus collected were analysed as discussed above. In this manner, errors arising from the analytical method tended to cancel out in the efficiency expression (equation 7.39), since the values of $x_n$, $x_{n-1}$ and $x^*_n$ have all been obtained by the same method.

A.1.3.3 Analysis of Titration Results

The average from 3 titrations was used in each run to calculate the CO$_2$ content. The method used for calculating the efficiency is outlined below:

Rectangular Column: $F_A = 1.72$

Liquid Flowrate: 10 gpm = 45.46 L/min = 2520 gmole/min.
Volume of HCl required for blank solution = 24.8 ml
Volume of HCl required for bicarbonate in tap water = 3.2 ml.

Tray Inlet

Volume of HCl required for excess NaOH in sample = 20.7 ml

$\therefore$ Volume of HCl equivalent to CO$_2$ in sample = 24.8 - 20.7 - 3.2

= 0.9 ml.
From equation (A.12):

1 mole NaOH requires 1 mole HCl for neutralisation

\[ \therefore \text{0.9 ml 0.05 N HCl } \equiv \text{0.9 ml 0.05 N NaOH.} \]

1 ml 0.05 N NaOH contains \( 2 \times 10^{-3} \) g NaOH

\[ \therefore \text{0.9 ml 0.05 N NaOH contains } 1.8 \times 10^{-3} \text{g NaOH} \]

\[ = 4.5 \times 10^{-5} \text{ g mole NaOH.} \]

From equation (A.8):

1 mole \( \text{CO}_2 \) requires 2 moles NaOH

\[ \therefore \text{CO}_2 \text{ in inlet sample } = 2.25 \times 10^{-5} \text{ g mole/100 ml} \]

\[ = 2.25 \times 10^{-7} \text{ g mole/ml CO}_2 \]

Liquid flowrate = 45.46 l/min

\[ \therefore \text{CO}_2 \text{ uptake } = 2.25 \times 10^{-7} \times 45.46 \times 1000 \]

\[ = 0.01023 \text{ g mole/min} \]

\[ \therefore \text{CO}_2 \text{ mole fraction in inlet sample} \]

\[ = \frac{0.01023}{2520 + 0.01023} \]

\[ = 4.059 \times 10^{-6} \]

Tray Outlet

Volume of HCl required for excess NaOH = 13.94 ml

Volume of HCl equivalent to \( \text{CO}_2 \) in sample = 24.8 - 13.94 - 3.2

\[ = 7.66 \text{ ml} \]

Repeating the calculations outlined above,

\[ \text{CO}_2 \text{ mole fraction in outlet sample } = 3.455 \times 10^{-5} \]
**Equilibrium Sample**

Volume of HCl required for excess NaOH = 13.95 ml

\[ \text{CO}_2 \text{ mole fraction in equilibrium sample} = 3.4425 \times 10^{-5} \]

Temperature in bottle = 20.8°C

Henry's constant from Int. Critical Tables at this temp. = 1.1046 \times 10^6

Temperature of liquid on tray = 18.85°C

Henry's constant from Int. Critical Tables at this temp. = 1.044 \times 10^6

\[ \text{Mole fraction of CO}_2 \text{ in equilibrium sample corrected to tray temperature} = \frac{3.4425 \times 10^{-5} \times 1.1046}{1.044} \]

\[ = 3.6423 \times 10^{-5} \]

Pressure in equilibrium sample flask = 4.85 mm Hg

Atmospheric pressure = 769.5 mm Hg

Vapour pressure of water at 20.8°C = 18.422 mm Hg

\[ \text{Corrected pressure in sample flask} = 746.22 \text{ mm Hg} \]

Absolute pressure inside column = 8.60 mm Hg

Outlet air temperature = 19.1°C

Vapour pressure of water at 19.1°C = 16.58 mm Hg

\[ \text{Corrected pressure inside column} = 744.32 \text{ mm Hg} \]

\[ \text{Mole fraction of CO}_2 \text{ in equilibrium sample corrected to column pressure} = \frac{3.6423 \times 10^{-5} \times 744.32}{746.22} \]

\[ = 3.633 \times 10^{-5} \]
Hence,

\[
E_{mL} = \frac{3.455 \times 10^{-5} - 4.059 \times 10^{-6}}{3.633 \times 10^{-5} - 4.059 \times 10^{-6}}
\]

\[
= 0.945
\]

\[
= 94.5\%
\]

CO₂ in inlet air = 5%

\[\therefore \text{Henry's constant from experiment} = \frac{0.05 \times 744.32}{3.633 \times 10^{-5}}\]

\[= 1.0244 \times 10^6\]

Henry's constant obtained from I.C.T. \[= 1.1044 \times 10^6\]

Henry's constant obtained experimentally \[= 1.0244 \times 10^6\]

\[= 1.02.\]
APPENDIX A2: EXPERIMENTAL RESULTS
## A2.1 PRESSURE DROP MEASUREMENTS

Table A2.1.1 Rectangular Column dry plate pressure drop

Hole diameter = 1 inch  Plate thickness = 1/8 inch

<table>
<thead>
<tr>
<th>Number of Perforations</th>
<th>18</th>
<th>10</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Q</strong> (ft³/min)</td>
<td><strong>U₀</strong> (ft/sec)</td>
<td><strong>U₀²</strong> (ft/sec)²</td>
<td><strong>Fₐ</strong></td>
</tr>
<tr>
<td>94.79</td>
<td>16.11</td>
<td>259.38</td>
<td>0.51</td>
</tr>
<tr>
<td>134.06</td>
<td>22.78</td>
<td>518.75</td>
<td>0.72</td>
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<tr>
<td>216.80</td>
<td>36.83</td>
<td>1356.74</td>
<td>1.16</td>
</tr>
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<td>273.22</td>
<td>46.42</td>
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<td>1.46</td>
</tr>
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<td>320.92</td>
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<td>1.72</td>
</tr>
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<td>360.48</td>
<td>61.25</td>
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<td>1.93</td>
</tr>
<tr>
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<td>67.45</td>
<td>4549.06</td>
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</tr>
<tr>
<td>432.00</td>
<td>73.40</td>
<td>5387.05</td>
<td>2.31</td>
</tr>
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<td>462.14</td>
<td>78.52</td>
<td>6165.17</td>
<td>2.48</td>
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<td>489.74</td>
<td>83.21</td>
<td>6923.35</td>
<td>2.62</td>
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<td>517.19</td>
<td>87.87</td>
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<td>2.77</td>
</tr>
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<td>531.69</td>
<td>90.33</td>
<td>8160.38</td>
<td>2.85</td>
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<tr>
<td>552.73</td>
<td>93.91</td>
<td>8818.79</td>
<td>2.96</td>
</tr>
</tbody>
</table>
Table A2.1.2 Rectangular Column dry plate pressure drop

Hole diameter = 1 inch  Number of perforations = 18

<table>
<thead>
<tr>
<th>Plate thickness (inch)</th>
<th>1/8</th>
<th>1/4</th>
<th>3/8</th>
<th>1/2</th>
<th>5/8</th>
<th>3/4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Q (ft³/min)</td>
<td>U₀ (ft/sec)</td>
<td>h₀</td>
<td></td>
<td>h₀</td>
<td></td>
</tr>
<tr>
<td></td>
<td>94.79</td>
<td>16.11</td>
<td>0.51</td>
<td>0.010</td>
<td>0.010</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>134.06</td>
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<td>2.96</td>
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Table A2.1.3 Round Column dry plate pressure drop
No. of perforations = 39  t = 1/8 in.  do = 1 in.

<table>
<thead>
<tr>
<th>Q (ft³/min)</th>
<th>Uo (ft/sec)</th>
<th>Uo² (ft/sec)²</th>
<th>FA</th>
<th>hDP (in. H₂O)</th>
</tr>
</thead>
<tbody>
<tr>
<td>337.81</td>
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<td>701.52</td>
<td>0.38</td>
<td>0.35</td>
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<tr>
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<td>31.34</td>
<td>982.13</td>
<td>0.98</td>
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<td>0.59</td>
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<td>519.97</td>
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<td>0.94</td>
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<td>621.48</td>
<td>48.73</td>
<td>2374.38</td>
<td>1.53</td>
<td>1.10</td>
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<td>52.46</td>
<td>2752.12</td>
<td>1.64</td>
<td>1.26</td>
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<td>708.60</td>
<td>55.56</td>
<td>3086.69</td>
<td>1.74</td>
<td>1.46</td>
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<td>723.31</td>
<td>56.71</td>
<td>3216.21</td>
<td>1.78</td>
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Table A2.1.4 Experimental rig dry plate pressure drop
do = 1/4 in.  t = 1/8 in.

<table>
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<tr>
<th>Number of perforations</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Q (ft³/min)</td>
<td>Uo (ft/sec)</td>
<td>Uo² (ft/sec)²</td>
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<tr>
<td>94.79</td>
<td>33.87</td>
<td>1147.2</td>
</tr>
<tr>
<td>134.06</td>
<td>47.90</td>
<td>2294.4</td>
</tr>
<tr>
<td>216.80</td>
<td>77.46</td>
<td>6000.0</td>
</tr>
<tr>
<td>273.22</td>
<td>97.62</td>
<td>9529.7</td>
</tr>
<tr>
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<td>114.66</td>
<td>13146.9</td>
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<td>20118.6</td>
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Table A2.1.5 Experimental rig dry rate pressure drop
\(d_0 = 3/8 \text{ in.} \quad t = 1/8 \text{ in.}\)

<table>
<thead>
<tr>
<th>Number of perforations</th>
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<th>211</th>
</tr>
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<tbody>
<tr>
<td>(Q) (ft(^3)/min)</td>
<td>(U_0) (ft/sec)</td>
<td>(U_0^2) (ft/sec(^2))</td>
</tr>
<tr>
<td>94.79</td>
<td>33.77</td>
<td>1140.4</td>
</tr>
<tr>
<td>134.06</td>
<td>47.75</td>
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</tr>
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<td>77.23</td>
<td>5964.5</td>
</tr>
<tr>
<td>273.22</td>
<td>97.33</td>
<td>9473.1</td>
</tr>
<tr>
<td>320.92</td>
<td>114.32</td>
<td>13069.1</td>
</tr>
<tr>
<td>360.48</td>
<td>128.41</td>
<td>16489.1</td>
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<tr>
<td>396.98</td>
<td>141.41</td>
<td>19996.8</td>
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<td>174.46</td>
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<td>196.90</td>
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Table A2.1.6 \(d_0 = 1 \text{ in.} \quad t = 1/8 \text{ in.}\)

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<th>10</th>
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</thead>
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<tr>
<td>(Q) (ft(^3)/min)</td>
<td>(U_0) (ft/sec)</td>
<td>(U_0^2) (ft/sec(^2))</td>
</tr>
<tr>
<td>94.79</td>
<td>28.99</td>
<td>840.4</td>
</tr>
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<td>6982.3</td>
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<td>320.92</td>
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<tr>
<td>360.48</td>
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<td>10</td>
<td>15</td>
</tr>
<tr>
<td>---------------</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>$h_T$ (in. water)</td>
<td>3.48</td>
<td>3.62</td>
</tr>
<tr>
<td>$Z_D$ (in. water)</td>
<td>2.13</td>
<td>2.20</td>
</tr>
<tr>
<td>$Z_m$ (in. water)</td>
<td>0.13</td>
<td>0.13</td>
</tr>
<tr>
<td>$Z_C$ (in. water)</td>
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<td>2.33</td>
</tr>
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<td>$Z_C(h_T)$</td>
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<td>2.28</td>
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<td>$h_{ow}$ (in. water)</td>
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<td>0.61</td>
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<tr>
<td>$Z_f$ (in. water)</td>
<td>5.86</td>
<td>6.26</td>
</tr>
<tr>
<td>$\beta_d$</td>
<td>0.651</td>
<td>0.645</td>
</tr>
<tr>
<td>$\beta_h$</td>
<td>0.617</td>
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<td>$\phi_d$</td>
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<td>$\phi_h$</td>
<td>0.365</td>
<td>0.364</td>
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Table A2.1.7 Rectangular Column
Air flowrate, $Q = 321.02 \text{ ft}^3/\text{min}$
$F_A = 1.72$
Table A2.1.8  Air flowrate, $Q = 410.54 \text{ ft}^3/\text{min}$
$F_A = 2.20$

<table>
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<th>$L_w$ (gpm/ft)</th>
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<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_T$ (in. water)</td>
<td>3.79</td>
<td>3.99</td>
<td>4.13</td>
<td>4.21</td>
<td>4.32</td>
<td>4.32</td>
<td>4.47</td>
<td>4.49</td>
<td>4.58</td>
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<tr>
<td>$Z_D$ (in. water)</td>
<td>1.98</td>
<td>2.03</td>
<td>2.15</td>
<td>2.24</td>
<td>2.29</td>
<td>2.42</td>
<td>2.47</td>
<td>2.55</td>
<td>2.64</td>
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<tr>
<td>$Z_m$ (in. water)</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
</tr>
<tr>
<td>$Z_c$ (in. water)</td>
<td>2.19</td>
<td>2.24</td>
<td>2.36</td>
<td>2.45</td>
<td>2.50</td>
<td>2.63</td>
<td>2.68</td>
<td>2.76</td>
<td>2.85</td>
</tr>
<tr>
<td>$Z_c$ (in. water)</td>
<td>1.73</td>
<td>1.93</td>
<td>2.07</td>
<td>2.15</td>
<td>2.26</td>
<td>2.31</td>
<td>2.41</td>
<td>2.43</td>
<td>2.52</td>
</tr>
<tr>
<td>$h_{ow}$ (in. water)</td>
<td>0.47</td>
<td>0.61</td>
<td>0.74</td>
<td>0.86</td>
<td>0.97</td>
<td>1.08</td>
<td>1.18</td>
<td>1.28</td>
<td>1.37</td>
</tr>
<tr>
<td>$\beta_d$</td>
<td>0.631</td>
<td>0.620</td>
<td>0.631</td>
<td>0.635</td>
<td>0.630</td>
<td>0.645</td>
<td>0.641</td>
<td>0.645</td>
<td>0.652</td>
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<td>$\beta_h$</td>
<td>0.498</td>
<td>0.538</td>
<td>0.553</td>
<td>0.557</td>
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<td>0.566</td>
<td>0.576</td>
<td>0.568</td>
<td>0.577</td>
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<tr>
<td>$\phi_d$</td>
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<td>0.324</td>
<td>0.318</td>
<td>0.314</td>
<td>0.305</td>
<td>0.303</td>
<td>0.294</td>
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<td>0.279</td>
<td>0.275</td>
<td>0.276</td>
<td>0.266</td>
<td>0.264</td>
<td>0.255</td>
<td>0.254</td>
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Table A2.1.9  Air flowrate, $Q = 485.21 \text{ ft}^3/\text{min}$
$F_A = 2.60$

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<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_T$ (in. water)</td>
<td>4.17</td>
<td>4.39</td>
<td>4.52</td>
<td>4.65</td>
<td>4.76</td>
<td>4.80</td>
<td>4.86</td>
<td>4.92</td>
<td>5.04</td>
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<tr>
<td>$Z_D$ (in. water)</td>
<td>1.86</td>
<td>1.92</td>
<td>2.01</td>
<td>2.08</td>
<td>2.15</td>
<td>2.26</td>
<td>2.36</td>
<td>2.40</td>
<td>2.47</td>
</tr>
<tr>
<td>$Z_m$ (in. water)</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>$Z_c$ (in. water)</td>
<td>2.16</td>
<td>2.22</td>
<td>2.31</td>
<td>2.38</td>
<td>2.45</td>
<td>2.56</td>
<td>2.66</td>
<td>2.70</td>
<td>2.77</td>
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<tr>
<td>$Z_c$ (in. water)</td>
<td>1.37</td>
<td>1.59</td>
<td>1.72</td>
<td>1.85</td>
<td>1.96</td>
<td>2.00</td>
<td>2.06</td>
<td>2.12</td>
<td>2.24</td>
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<tr>
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<td>0.74</td>
<td>0.86</td>
<td>0.97</td>
<td>1.08</td>
<td>1.18</td>
<td>1.28</td>
<td>1.37</td>
</tr>
<tr>
<td>$Z_f$ (in. water)</td>
<td>7.10</td>
<td>7.51</td>
<td>7.94</td>
<td>8.42</td>
<td>8.82</td>
<td>9.24</td>
<td>9.66</td>
<td>10.06</td>
<td>10.51</td>
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<td>0.618</td>
<td>0.616</td>
<td>0.612</td>
<td>0.627</td>
<td>0.636</td>
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<td>0.634</td>
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<td>0.490</td>
<td>0.493</td>
<td>0.495</td>
<td>0.513</td>
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<tr>
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<td>0.291</td>
<td>0.283</td>
<td>0.278</td>
<td>0.277</td>
<td>0.275</td>
<td>0.268</td>
<td>0.264</td>
</tr>
<tr>
<td>$\phi_h$</td>
<td>0.193</td>
<td>0.212</td>
<td>0.217</td>
<td>0.220</td>
<td>0.222</td>
<td>0.216</td>
<td>0.213</td>
<td>0.211</td>
<td>0.213</td>
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Table A.2.1.10  Air flowrate, $Q = 552.49 \text{ ft}^3/\text{min}$  
$F_A = 2.80$

<table>
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<th>$L_w$ (gpm/ft)</th>
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<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_T$ (in. water)</td>
<td>4.37</td>
<td>4.60</td>
<td>4.80</td>
<td>4.88</td>
<td>4.96</td>
<td>5.04</td>
<td>5.15</td>
<td>5.20</td>
<td>5.26</td>
</tr>
<tr>
<td>$Z_D$ (in. water)</td>
<td>1.79</td>
<td>1.85</td>
<td>1.93</td>
<td>2.01</td>
<td>2.08</td>
<td>2.16</td>
<td>2.24</td>
<td>2.31</td>
<td>2.40</td>
</tr>
<tr>
<td>$Z_m$ (in. water)</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td>$Z_c$ (in. water)</td>
<td>2.14</td>
<td>2.20</td>
<td>2.28</td>
<td>2.36</td>
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<td>2.59</td>
<td>2.66</td>
<td>2.75</td>
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<tr>
<td>$Z_c$ (in. water)</td>
<td>1.15</td>
<td>1.38</td>
<td>1.58</td>
<td>1.66</td>
<td>1.74</td>
<td>1.82</td>
<td>1.93</td>
<td>1.98</td>
<td>2.04</td>
</tr>
<tr>
<td>$Z_f$ (in. water)</td>
<td>7.36</td>
<td>7.79</td>
<td>8.18</td>
<td>8.63</td>
<td>9.08</td>
<td>9.51</td>
<td>9.90</td>
<td>10.40</td>
<td>10.80</td>
</tr>
<tr>
<td>$\beta_d$</td>
<td>0.617</td>
<td>0.609</td>
<td>0.610</td>
<td>0.611</td>
<td>0.612</td>
<td>0.615</td>
<td>0.620</td>
<td>0.621</td>
<td>0.629</td>
</tr>
<tr>
<td>$\beta_h$</td>
<td>0.331</td>
<td>0.382</td>
<td>0.422</td>
<td>0.430</td>
<td>0.438</td>
<td>0.446</td>
<td>0.462</td>
<td>0.463</td>
<td>0.467</td>
</tr>
<tr>
<td>$\phi_d$</td>
<td>0.291</td>
<td>0.282</td>
<td>0.279</td>
<td>0.273</td>
<td>0.268</td>
<td>0.264</td>
<td>0.262</td>
<td>0.256</td>
<td>0.255</td>
</tr>
<tr>
<td>$\phi_h$</td>
<td>0.156</td>
<td>0.177</td>
<td>0.193</td>
<td>0.192</td>
<td>0.092</td>
<td>0.191</td>
<td>0.195</td>
<td>0.190</td>
<td>0.189</td>
</tr>
</tbody>
</table>
### Table A2.1.11 Round Column

Air flowrate, $Q = 570.14 \text{ ft}^3/\text{min}$

$F_A = 1.40$

<table>
<thead>
<tr>
<th>$L_w$ (gpm/ft)</th>
<th>5.0</th>
<th>7.5</th>
<th>10.0</th>
<th>12.5</th>
<th>15.0</th>
<th>17.5</th>
<th>20.0</th>
<th>22.5</th>
<th>25.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_T$ (in. water)</td>
<td>3.05</td>
<td>3.11</td>
<td>3.18</td>
<td>3.24</td>
<td>3.28</td>
<td>3.37</td>
<td>3.43</td>
<td>3.46</td>
<td>3.53</td>
</tr>
<tr>
<td>$Z_D$ (in. water)</td>
<td>1.84</td>
<td>1.92</td>
<td>1.98</td>
<td>2.02</td>
<td>2.08</td>
<td>2.17</td>
<td>2.28</td>
<td>2.31</td>
<td>2.35</td>
</tr>
<tr>
<td>$Z_m$ (in. water)</td>
<td>0.09</td>
<td>0.09</td>
<td>0.09</td>
<td>0.09</td>
<td>0.09</td>
<td>0.09</td>
<td>0.09</td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td>$Z_c$ (in. water)</td>
<td>1.93</td>
<td>2.01</td>
<td>2.07</td>
<td>2.11</td>
<td>2.17</td>
<td>2.26</td>
<td>2.37</td>
<td>2.40</td>
<td>2.44</td>
</tr>
<tr>
<td>$Z_c$ (in. water)</td>
<td>1.92</td>
<td>1.98</td>
<td>2.05</td>
<td>2.11</td>
<td>2.15</td>
<td>2.24</td>
<td>2.30</td>
<td>2.33</td>
<td>2.40</td>
</tr>
<tr>
<td>$h_{ow}$ (in. water)</td>
<td>0.30</td>
<td>0.39</td>
<td>0.47</td>
<td>0.54</td>
<td>0.61</td>
<td>0.68</td>
<td>0.74</td>
<td>0.80</td>
<td>0.86</td>
</tr>
<tr>
<td>$Z_f$ (in. water)</td>
<td>6.89</td>
<td>7.22</td>
<td>7.55</td>
<td>7.88</td>
<td>8.22</td>
<td>8.54</td>
<td>8.88</td>
<td>9.21</td>
<td>9.54</td>
</tr>
<tr>
<td>$\beta_d$</td>
<td>0.595</td>
<td>0.593</td>
<td>0.596</td>
<td>0.596</td>
<td>0.601</td>
<td>0.614</td>
<td>0.634</td>
<td>0.632</td>
<td>0.632</td>
</tr>
<tr>
<td>$\beta_h$</td>
<td>0.582</td>
<td>0.584</td>
<td>0.591</td>
<td>0.596</td>
<td>0.596</td>
<td>0.609</td>
<td>0.615</td>
<td>0.613</td>
<td>0.622</td>
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<tr>
<td>$\phi_d$</td>
<td>0.280</td>
<td>0.278</td>
<td>0.274</td>
<td>0.268</td>
<td>0.264</td>
<td>0.265</td>
<td>0.267</td>
<td>0.260</td>
<td>0.256</td>
</tr>
<tr>
<td>$\phi_h$</td>
<td>0.279</td>
<td>0.274</td>
<td>0.272</td>
<td>0.268</td>
<td>0.262</td>
<td>0.262</td>
<td>0.259</td>
<td>0.253</td>
<td>0.252</td>
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</table>
Table A2.1.12 Air flowrate, Q = 610.86 ft$^3$/min
$F_A = 1.50$

<table>
<thead>
<tr>
<th>$L_w$ (gpm/ft)</th>
<th>5.0</th>
<th>7.5</th>
<th>10.0</th>
<th>12.5</th>
<th>15.0</th>
<th>17.5</th>
<th>20.0</th>
<th>22.5</th>
<th>25.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_T$ (in. water)</td>
<td>3.13</td>
<td>3.19</td>
<td>3.25</td>
<td>3.31</td>
<td>3.36</td>
<td>3.43</td>
<td>3.49</td>
<td>3.54</td>
<td>3.61</td>
</tr>
<tr>
<td>$Z_D$ (in. water)</td>
<td>1.72</td>
<td>1.81</td>
<td>1.90</td>
<td>1.95</td>
<td>2.00</td>
<td>2.09</td>
<td>2.14</td>
<td>2.21</td>
<td>2.30</td>
</tr>
<tr>
<td>$Z_m$ (in. water)</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>$Z_C$ (in. water)</td>
<td>1.82</td>
<td>1.91</td>
<td>2.00</td>
<td>2.05</td>
<td>2.10</td>
<td>2.19</td>
<td>2.24</td>
<td>2.31</td>
<td>2.40</td>
</tr>
<tr>
<td>$Z_C$ (in. water) (hT)</td>
<td>1.86</td>
<td>1.92</td>
<td>1.98</td>
<td>2.04</td>
<td>2.09</td>
<td>2.16</td>
<td>2.22</td>
<td>2.27</td>
<td>2.34</td>
</tr>
<tr>
<td>$h_{ow}$ (in. water)</td>
<td>0.30</td>
<td>0.39</td>
<td>0.47</td>
<td>0.54</td>
<td>0.61</td>
<td>0.68</td>
<td>0.74</td>
<td>0.80</td>
<td>0.86</td>
</tr>
<tr>
<td>$Z_f$ (in. water)</td>
<td>7.20</td>
<td>7.55</td>
<td>7.90</td>
<td>8.25</td>
<td>8.60</td>
<td>8.96</td>
<td>9.31</td>
<td>9.66</td>
<td>10.01</td>
</tr>
<tr>
<td>$\theta_d$</td>
<td>0.552</td>
<td>0.563</td>
<td>0.576</td>
<td>0.579</td>
<td>0.582</td>
<td>0.595</td>
<td>0.599</td>
<td>0.608</td>
<td>0.622</td>
</tr>
<tr>
<td>$\theta_h$</td>
<td>0.564</td>
<td>0.567</td>
<td>0.571</td>
<td>0.577</td>
<td>0.580</td>
<td>0.588</td>
<td>0.594</td>
<td>0.598</td>
<td>0.607</td>
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<tr>
<td>$\psi_d$</td>
<td>0.253</td>
<td>0.253</td>
<td>0.253</td>
<td>0.248</td>
<td>0.244</td>
<td>0.244</td>
<td>0.240</td>
<td>0.239</td>
<td>0.240</td>
</tr>
<tr>
<td>$\psi_h$</td>
<td>0.259</td>
<td>0.255</td>
<td>0.251</td>
<td>0.248</td>
<td>0.243</td>
<td>0.241</td>
<td>0.239</td>
<td>0.235</td>
<td>0.234</td>
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Table A2.1.13  Air flowrate, \( Q = 651.59 \text{ ft}^3/\text{min} \)
\( F_A = 1.60 \)

<table>
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<tr>
<th>( L_w ) (gpm/ft)</th>
<th>5.0</th>
<th>7.5</th>
<th>10.0</th>
<th>12.5</th>
<th>15.0</th>
<th>17.5</th>
<th>20.0</th>
<th>22.5</th>
<th>25.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_T ) (in. water)</td>
<td>3.21</td>
<td>3.26</td>
<td>3.30</td>
<td>3.38</td>
<td>3.46</td>
<td>3.49</td>
<td>3.57</td>
<td>3.61</td>
<td>3.69</td>
</tr>
<tr>
<td>( Z_D ) (in. water)</td>
<td>1.67</td>
<td>1.69</td>
<td>1.76</td>
<td>1.86</td>
<td>1.91</td>
<td>1.98</td>
<td>2.03</td>
<td>2.10</td>
<td>2.19</td>
</tr>
<tr>
<td>( Z_m ) (in. water)</td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
</tr>
<tr>
<td>( Z_c ) (in. water)</td>
<td>1.78</td>
<td>1.80</td>
<td>1.87</td>
<td>1.97</td>
<td>2.02</td>
<td>2.09</td>
<td>2.14</td>
<td>2.21</td>
<td>2.30</td>
</tr>
<tr>
<td>( Z_c ) (in. water) ((hT))</td>
<td>1.80</td>
<td>1.85</td>
<td>1.89</td>
<td>1.97</td>
<td>2.05</td>
<td>2.08</td>
<td>2.16</td>
<td>2.20</td>
<td>2.28</td>
</tr>
<tr>
<td>( h_{ow} ) (in. water)</td>
<td>0.30</td>
<td>0.39</td>
<td>0.47</td>
<td>0.54</td>
<td>0.61</td>
<td>0.68</td>
<td>0.74</td>
<td>0.80</td>
<td>0.86</td>
</tr>
<tr>
<td>( \beta_d )</td>
<td>0.539</td>
<td>0.531</td>
<td>0.539</td>
<td>0.556</td>
<td>0.560</td>
<td>0.568</td>
<td>0.572</td>
<td>0.582</td>
<td>0.596</td>
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<tr>
<td>( \beta_h )</td>
<td>0.544</td>
<td>0.544</td>
<td>0.543</td>
<td>0.555</td>
<td>0.567</td>
<td>0.564</td>
<td>0.576</td>
<td>0.578</td>
<td>0.590</td>
</tr>
<tr>
<td>( \phi_d )</td>
<td>0.234</td>
<td>0.226</td>
<td>0.226</td>
<td>0.228</td>
<td>0.225</td>
<td>0.225</td>
<td>0.222</td>
<td>0.222</td>
<td>0.224</td>
</tr>
<tr>
<td>( \phi_h )</td>
<td>0.236</td>
<td>0.232</td>
<td>0.227</td>
<td>0.228</td>
<td>0.228</td>
<td>0.223</td>
<td>0.224</td>
<td>0.220</td>
<td>0.221</td>
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</table>
Table A2.1.14  Air flowrate, \( Q = 700.46 \text{ ft}^3/\text{min} \)
\( F_A = 1.72 \)

<table>
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<tr>
<th>( L_w (\text{gpm/ft}) )</th>
<th>5.0</th>
<th>7.5</th>
<th>10.0</th>
<th>12.5</th>
<th>15.0</th>
<th>17.5</th>
<th>20.0</th>
<th>22.5</th>
<th>25.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_T ) (in. water)</td>
<td>3.31</td>
<td>3.38</td>
<td>3.44</td>
<td>3.49</td>
<td>3.55</td>
<td>3.61</td>
<td>3.66</td>
<td>3.75</td>
<td>3.80</td>
</tr>
<tr>
<td>( Z_D ) (in. water)</td>
<td>1.54</td>
<td>1.63</td>
<td>1.69</td>
<td>1.93</td>
<td>1.81</td>
<td>1.83</td>
<td>1.92</td>
<td>2.01</td>
<td>2.04</td>
</tr>
<tr>
<td>( Z_m ) (in. water)</td>
<td>0.13</td>
<td>0.13</td>
<td>0.13</td>
<td>0.13</td>
<td>0.13</td>
<td>0.13</td>
<td>0.13</td>
<td>0.13</td>
<td>0.13</td>
</tr>
<tr>
<td>( Z_c ) (in. water)</td>
<td>1.67</td>
<td>1.76</td>
<td>1.80</td>
<td>1.86</td>
<td>1.94</td>
<td>1.96</td>
<td>2.05</td>
<td>2.14</td>
<td>2.17</td>
</tr>
<tr>
<td>( Z_c ) (in. water)</td>
<td>1.71</td>
<td>1.78</td>
<td>1.84</td>
<td>1.89</td>
<td>1.95</td>
<td>2.01</td>
<td>2.06</td>
<td>2.15</td>
<td>2.20</td>
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<tr>
<td>( Z_f ) (in. water)</td>
<td>8.33</td>
<td>8.60</td>
<td>8.86</td>
<td>9.12</td>
<td>9.38</td>
<td>9.64</td>
<td>9.91</td>
<td>10.17</td>
<td>10.43</td>
</tr>
<tr>
<td>( \beta_d )</td>
<td>0.506</td>
<td>0.519</td>
<td>0.519</td>
<td>0.525</td>
<td>0.537</td>
<td>0.533</td>
<td>0.548</td>
<td>0.563</td>
<td>0.562</td>
</tr>
<tr>
<td>( \beta_h )</td>
<td>0.517</td>
<td>0.524</td>
<td>0.529</td>
<td>0.533</td>
<td>0.539</td>
<td>0.545</td>
<td>0.550</td>
<td>0.565</td>
<td>0.569</td>
</tr>
<tr>
<td>( \phi_d )</td>
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<td>0.205</td>
<td>0.203</td>
<td>0.204</td>
<td>0.207</td>
<td>0.203</td>
<td>0.207</td>
<td>0.210</td>
<td>0.208</td>
</tr>
<tr>
<td>( \phi_h )</td>
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<td>0.207</td>
<td>0.207</td>
<td>0.207</td>
<td>0.208</td>
<td>0.208</td>
<td>0.208</td>
<td>0.211</td>
<td>0.211</td>
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</table>
### A2.2 RECTANGULAR COLUMN ENTRAINMENT

#### Table A2.2.1 \( L_w = 10 \text{ gpm/ft weir} \)
Tray spacing = 12 in.

<table>
<thead>
<tr>
<th>( F_A )</th>
<th>( F_{LV} )</th>
<th>( E_w ) (lb/min)</th>
<th>( E ) (lb/100 lb air)</th>
<th>( \psi_L )</th>
<th>( \psi_V )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2</td>
<td>0.088</td>
<td>1.595</td>
<td>4.81</td>
<td>0.0162</td>
<td>0.0839</td>
</tr>
<tr>
<td>2.6</td>
<td>0.074</td>
<td>2.587</td>
<td>6.60</td>
<td>0.0266</td>
<td>0.119</td>
</tr>
<tr>
<td>2.8</td>
<td>0.069</td>
<td>3.250</td>
<td>7.70</td>
<td>0.0336</td>
<td>0.141</td>
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<td>3.0</td>
<td>0.064</td>
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<td>0.0419</td>
<td>0.167</td>
</tr>
<tr>
<td>3.4</td>
<td>0.057</td>
<td>6.150</td>
<td>12.00</td>
<td>0.0655</td>
<td>0.239</td>
</tr>
</tbody>
</table>

#### Table A2.2.2 \( L_w = 20 \text{ gpm/ft weir} \)
\( S = 12 \text{ in.} \)

<table>
<thead>
<tr>
<th>( F_A )</th>
<th>( F_{LV} )</th>
<th>( E_w ) (lb/min)</th>
<th>( E ) (lb/100 lb air)</th>
<th>( \psi_L )</th>
<th>( \psi_V )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2</td>
<td>0.175</td>
<td>1.200</td>
<td>3.62</td>
<td>0.00604</td>
<td>0.0618</td>
</tr>
<tr>
<td>2.6</td>
<td>0.148</td>
<td>1.960</td>
<td>5.00</td>
<td>0.00990</td>
<td>0.0875</td>
</tr>
<tr>
<td>2.8</td>
<td>0.137</td>
<td>2.469</td>
<td>5.85</td>
<td>0.0125</td>
<td>0.104</td>
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<tr>
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<td>0.128</td>
<td>2.939</td>
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<td>0.0149</td>
<td>0.117</td>
</tr>
<tr>
<td>3.4</td>
<td>0.113</td>
<td>4.510</td>
<td>8.80</td>
<td>0.0231</td>
<td>0.165</td>
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</table>

#### Table A2.2.3 \( L_w = 30 \text{ gpm/ft weir} \)
\( S = 12 \text{ in.} \)

<table>
<thead>
<tr>
<th>( F_A )</th>
<th>( F_{LV} )</th>
<th>( E_w ) (lb/min)</th>
<th>( E ) (lb/100 lb air)</th>
<th>( \psi_L )</th>
<th>( \psi_V )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2</td>
<td>0.262</td>
<td>1.718</td>
<td>5.18</td>
<td>0.00576</td>
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</tr>
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<td>0.131</td>
</tr>
<tr>
<td>2.8</td>
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<td>7.85</td>
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<td>0.144</td>
</tr>
<tr>
<td>3.0</td>
<td>0.192</td>
<td>4.120</td>
<td>9.11</td>
<td>0.0139</td>
<td>0.172</td>
</tr>
<tr>
<td>3.4</td>
<td>0.170</td>
<td>4.510</td>
<td>13.50</td>
<td>0.0236</td>
<td>0.165</td>
</tr>
</tbody>
</table>
Table A2.2.4  \( L_w = 40 \text{ gpm/ft weir} \)
\( S = 12 \text{ in.} \)

<table>
<thead>
<tr>
<th>( F_A )</th>
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<th>( E_w ) (lb/min)</th>
<th>( E ) (lb/100 lb air)</th>
<th>( \psi_L )</th>
<th>( \psi_V )</th>
</tr>
</thead>
<tbody>
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<td>2.2</td>
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<td>0.00525</td>
<td>0.113</td>
</tr>
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<td>0.00860</td>
<td>0.163</td>
</tr>
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<td>10.50</td>
<td>0.0112</td>
<td>0.203</td>
</tr>
<tr>
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<td>0.256</td>
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<td>12.00</td>
<td>0.0138</td>
<td>0.239</td>
</tr>
<tr>
<td>3.4</td>
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<td>0.0199</td>
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Table A2.2.5  \( L_w = 50 \text{ gpm/ft weir} \)
\( S = 12 \text{ in.} \)

<table>
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<th>( E_w ) (lb/min)</th>
<th>( E ) (lb/100 lb air)</th>
<th>( \psi_L )</th>
<th>( \psi_V )</th>
</tr>
</thead>
<tbody>
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<td>2.2</td>
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<td>7.70</td>
<td>0.00513</td>
<td>0.141</td>
</tr>
<tr>
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<td>4.234</td>
<td>10.80</td>
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<td>0.210</td>
</tr>
<tr>
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<td>0.344</td>
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<td>12.10</td>
<td>0.0103</td>
<td>0.242</td>
</tr>
<tr>
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<td>5.969</td>
<td>13.20</td>
<td>0.0121</td>
<td>0.270</td>
</tr>
<tr>
<td>3.4</td>
<td>0.283</td>
<td>7.790</td>
<td>19.50</td>
<td>0.0204</td>
<td>0.324</td>
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Table A2.2.6  \( L_w = 10 \text{ gpm/ft weir} \)
\( S = 15 \text{ in.} \)

<table>
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<th>( E_w ) (lb/min)</th>
<th>( E ) (lb/100 lb air)</th>
<th>( \psi_L )</th>
<th>( \psi_V )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2</td>
<td>0.088</td>
<td>0.468</td>
<td>1.41</td>
<td>0.00470</td>
<td>0.0290</td>
</tr>
<tr>
<td>2.6</td>
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<td>0.772</td>
<td>1.97</td>
<td>0.00778</td>
<td>0.041</td>
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<tr>
<td>2.8</td>
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<td>2.66</td>
<td>0.0114</td>
<td>0.056</td>
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### Table A2.2.7  \( L_w = 20 \text{ gpm/ft weir} \)

\( S = 15 \text{ in.} \)

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<th>( F_A )</th>
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<th>( E_w )</th>
<th>( E )</th>
<th>( \psi_L )</th>
<th>( \psi_V )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2</td>
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<td>0.85</td>
<td>0.00141</td>
<td>0.0173</td>
</tr>
<tr>
<td>2.6</td>
<td>0.148</td>
<td>0.470</td>
<td>1.20</td>
<td>0.00236</td>
<td>0.0246</td>
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<tr>
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<td>1.65</td>
<td>0.00349</td>
<td>0.0341</td>
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<td>0.00498</td>
<td>0.0456</td>
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### Table A2.2.8  \( L_w = 30 \text{ gpm/ft weir} \)

\( S = 15 \text{ in.} \)

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<th>( E_w )</th>
<th>( E )</th>
<th>( \psi_L )</th>
<th>( \psi_V )</th>
</tr>
</thead>
<tbody>
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<td>0.93</td>
<td>0.00103</td>
<td>0.0190</td>
</tr>
<tr>
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<td>0.222</td>
<td>0.510</td>
<td>1.30</td>
<td>0.00170</td>
<td>0.0267</td>
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<td>1.75</td>
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<td>0.0484</td>
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<tr>
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### Table A2.2.9  \( L_w = 40 \text{ gpm/ft weir} \)

\( S = 15 \text{ in.} \)

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<th>( F_A )</th>
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<th>( E_w )</th>
<th>( E )</th>
<th>( \psi_L )</th>
<th>( \psi_V )</th>
</tr>
</thead>
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<td>1.00</td>
<td>0.00083</td>
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</tr>
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<td>0.00139</td>
<td>0.0292</td>
</tr>
<tr>
<td>2.8</td>
<td>0.275</td>
<td>0.815</td>
<td>1.93</td>
<td>0.00204</td>
<td>0.0401</td>
</tr>
<tr>
<td>3.0</td>
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<td>0.00283</td>
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Table A2.2.10  \( L_w = 50 \text{ gpm/ft weir} \)
\( S = 15 \text{ in.} \)

<table>
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<th>( E_w ) (lb/min)</th>
<th>( E ) (lb/100 lb air)</th>
<th>( \psi_L )</th>
<th>( \psi_V )</th>
</tr>
</thead>
<tbody>
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<td>1.09</td>
<td>0.00723</td>
<td>0.0223</td>
</tr>
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<td>1.57</td>
<td>0.00123</td>
<td>0.0324</td>
</tr>
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<td>0.984</td>
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<td>0.0020</td>
<td>0.0487</td>
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<td>0.00256</td>
<td>0.0594</td>
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<td>2.101</td>
<td>4.10</td>
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Table A2.2.11  \( L_w = 10 \text{ gpm/ft weir} \)
\( S = 18 \text{ in.} \)

<table>
<thead>
<tr>
<th>( F_A )</th>
<th>( F_{LV} )</th>
<th>( E_w ) (lb/min)</th>
<th>( E ) (lb/100 lb air)</th>
<th>( \psi_L )</th>
<th>( \psi_V )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.294</td>
<td>0.89</td>
<td>0.00295</td>
<td>0.0217</td>
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<td>0.478</td>
<td>1.22</td>
<td>0.00480</td>
<td>0.030</td>
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<td>1.59</td>
<td>0.00676</td>
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<td>0.064</td>
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<td>0.0520</td>
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<tr>
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<td>0.00175</td>
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Table A2.2.12  \( L_w = 20 \text{ gpm/ft weir} \)
\( S = 18 \text{ in.} \)

<table>
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<th>( F_A )</th>
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<th>( E_w ) (lb/min)</th>
<th>( E ) (lb/100 lb air)</th>
<th>( \psi_L )</th>
<th>( \psi_V )</th>
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<tbody>
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<td>0.64</td>
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<td>0.0156</td>
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<tr>
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<td>0.0366</td>
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<tr>
<td>3.4</td>
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<td>0.00515</td>
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Table A2.2.13 \( L_w = 30 \text{ gpm/ft weir} \)
\( S = 18 \text{ in.} \)

<table>
<thead>
<tr>
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<th>( E_w ) (lb/min)</th>
<th>( E ) (lb/100 lb air)</th>
<th>( \psi_L )</th>
<th>( \psi_V )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2</td>
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<td>0.50</td>
<td>0.00055</td>
<td>0.0122</td>
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Table A2.2.14 \( L_w = 40 \text{ gpm/ft weir} \)
\( S = 18 \text{ in.} \)

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<tr>
<th>( F_A )</th>
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<th>( E_w ) (lb/min)</th>
<th>( E ) (lb/100 lb air)</th>
<th>( \psi_L )</th>
<th>( \psi_V )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.00035</td>
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Table A2.2.15 \( L_w = 50 \text{ gpm/ft weir} \)
\( S = 18 \text{ in.} \)

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<th>( F_{VL} )</th>
<th>( E_w ) (lb/min)</th>
<th>( E ) (lb/100 lb air)</th>
<th>( \psi_L )</th>
<th>( \psi_V )</th>
</tr>
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<tbody>
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<td>0.00041</td>
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### A2.3 RESIDENCE TIME STUDIES

Table A2.3.1 Round Column

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<th>$L_w$ (gpm/ft weir)</th>
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<th>$\sigma^2$ (sec$^2$)</th>
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### A2.4 EXPERIMENTAL AND CALCULATED MIXING PARAMETERS

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Cont'd.
<table>
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<tr>
<th>FA</th>
<th>( L_W ) (gpm/ft weir)</th>
<th>Pe exp. Eqn. 7.45</th>
<th>Dp exp. (ft(^2)/sec) Eqn. 7.47</th>
<th>Dp calc. (ft(^2)/sec) Eqn. 7.51</th>
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<tbody>
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Table A2.4.2 Rectangular Column (tray only)

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<th>Pe exp. (Eqn.7.45)</th>
<th>$D_{e} \text{ exp. (Eqn.7.47)}$</th>
<th>$D_{e} \text{ calcld. (Eqn. 7.51)}$</th>
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Table A2.4.3 Rectangular Column mixing

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<th>L_w (gpm/ft weir)</th>
<th>Pe exp. (downcomer only) Eqn. 7.45</th>
<th>Pe exp. (tray and downcomer) Eqn. 7.45</th>
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### A2.5 GAS ABSORPTION

#### Table A2.5.1 Round Column, \( F_A \) 1.50

<table>
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<tr>
<th>( L_w ) (gpm/ft weir)</th>
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<th>12.5</th>
<th>15.0</th>
<th>17.5</th>
<th>20.0</th>
<th>22.5</th>
<th>25.0</th>
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<tbody>
<tr>
<td>( \mu ) (sec)</td>
<td>33.06</td>
<td>27.72</td>
<td>22.45</td>
<td>20.63</td>
<td>18.64</td>
<td>14.46</td>
<td>15.13</td>
<td>13.65</td>
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<tr>
<td>( \sigma_g^2 )</td>
<td>0.3421</td>
<td>0.3758</td>
<td>0.3346</td>
<td>0.3126</td>
<td>0.3174</td>
<td>0.3676</td>
<td>0.3223</td>
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<td>( E_{mL} )</td>
<td>92.5</td>
<td>87.8</td>
<td>81.0</td>
<td>78.0</td>
<td>74.2</td>
<td>72.0</td>
<td>69.9</td>
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<tr>
<td>( N_L )</td>
<td>4.168</td>
<td>3.206</td>
<td>2.221</td>
<td>1.936</td>
<td>1.693</td>
<td>1.623</td>
<td>1.466</td>
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<td>( K_{La} )</td>
<td>0.126</td>
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<td>0.094</td>
<td>0.091</td>
<td>0.112</td>
<td>0.097</td>
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Table A2.5.2  Round Column Absorption  $F_A = 1.72$

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<th>12.5</th>
<th>15.0</th>
<th>17.5</th>
<th>20.0</th>
<th>22.5</th>
<th>25.0</th>
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<td>$\mu$ (sec)</td>
<td>28.70</td>
<td>23.02</td>
<td>21.29</td>
<td>17.76</td>
<td>17.39</td>
<td>15.13</td>
<td>14.92</td>
<td>13.03</td>
</tr>
<tr>
<td>$\sigma_g^2$</td>
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<td>0.3642</td>
<td>0.3413</td>
<td>0.3710</td>
<td>0.3182</td>
<td>0.2974</td>
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<tr>
<td>$E_{mL}$</td>
<td>94.5</td>
<td>89.0</td>
<td>82.9</td>
<td>78.5</td>
<td>74.9</td>
<td>72.5</td>
<td>71.2</td>
<td>68.0</td>
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<td>5.186</td>
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<td>2.424</td>
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<td>1.574</td>
<td>1.502</td>
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<td>0.104</td>
<td>0.100</td>
<td>0.105</td>
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<td>$L_w$ (gpm/ft weir)</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td>30</td>
<td>35</td>
<td>40</td>
<td>50</td>
</tr>
<tr>
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<td>----</td>
<td>----</td>
<td>----</td>
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<td>----</td>
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<tr>
<td>(sec)</td>
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<td>8.63</td>
<td>7.97</td>
<td>5.35</td>
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<td>0.3617</td>
<td>0.3820</td>
<td>0.3238</td>
<td>0.3255</td>
<td>0.3414</td>
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<td>$E_mL$ (%)</td>
<td>96.9</td>
<td>86.9</td>
<td>80.2</td>
<td>76.0</td>
<td>72.0</td>
<td>70.0</td>
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<td>3.002</td>
<td>2.242</td>
<td>1.814</td>
<td>1.577</td>
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<td>1.134</td>
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<td>0.155</td>
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Table A2.5.3 Rectangular Column Absorption (tray only) F.1.72
Table A2.5.4 Rectangular Column Absorption  
(tray only)  \( F_A \ 2.20 \)

| \( L_w \)  
<table>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(gpm/ft weir)</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td>30</td>
<td>35</td>
<td>40</td>
<td>50</td>
</tr>
</tbody>
</table>
| \( \mu \)  
| (sec) | 31.85 | 20.51 | 14.16 | 12.09 | 9.57 | 8.17 | 7.32 | 6.42 |
| \( \sigma_\theta^2 \) | 0.3484 | 0.3353 | 0.3722 | 0.3331 | 0.2861 | 0.2877 | 0.2518 | 0.3508 |
| \( E_{mL} \) (%) | 97.8 | 88.9 | 81.5 | 77.0 | 73.5 | 70.5 | 68.9 | 63.5 |
| \( N_L \) | 7.98 | 3.25 | 2.348 | 1.896 | 1.616 | 1.463 | 1.358 | 1.209 |
| \( K_{la} \) | 0.250 | 0.158 | 0.166 | 0.157 | 0.169 | 0.179 | 0.186 | 0.188 |
APPENDIX A3: COMPUTER PROGRAM FOR RESIDENCE TIME AND MIXING STUDY
BEGIN COMMENT ********** MIXING STUDY**********
**MEAN AND VARIANCE ANALYSIS USING THE SATER**
*******************CORRECTION*******************
WRITTEN BY C.A.MILLINGTON SEPT 72
MOD 18 APR 82 (PRIME)
ICL I/O REMOVED
SATER ABORTION INCLUDED
PLOTTING COMPATIBLE
FROZEN VERSION 26:04:82
**************************************************.

COMMENT COMPILE USING ALGOL MIX ;
PROCEDURE SPACE(N);
VALUE N; INTEGER N;
FOR N:=N STEP -1 UNTIL 1 DO
PRINT PREFIX(''),";
REAL INT,A,SUMX,SUMY,SUMXY,SUMXSQ,SUMC,SUMCT,SUMCTSQ,K,
CUTOFF,INTC,INTCT,INTCTSQ,P,Q,RATIO,SIGMU,PEC,UL,LL,FL,PU,
TBAR,B,CC,MAX;
INTEGER M,N,SYST,I,J,D,R,ST;
REAL ARRAY C[0:300,1:6],F,G,MEAN,VAR[1:6];
INTEGER ARRAY TITLE[1:60],LENGTH,TRACK,FAIL,DROP,TOP[1:6];
R:=0;
OPEN('TAPE',11,1);
OPEN('SYSTEM',12,1);
OPEN('TITLE',13,1);
OPEN('MIXOUT',14,2);
CLOSE(14);
OPEN('MIXOUT',14,5);
OPEN('MIXOUT',14,2);
PUNCH(1);
NEWLINE(1);
PRINT PREFIX(''),*********** PROGRAM NOW RUNNING ***********';
NEWLINE(1);
PRINT PREFIX(''),********* MIX MOD 18 FROZEN 26:04:82 ********;
NEWLINE(1);

START: R:=R+1;
FOR J:=1 STEP 1 UNTIL 6 DO
FOR I:=1 STEP 1 UNTIL 300 DO
BEGIN C[I,J]:=0;
END;
COMMENT DATA FROM FILE SYSTEM WILL BE READ;
READER(12);
READ N;
IF N LE 0 THEN GOTO FINISH;
COMMENT DATA FROM FILE TITLE WILL BE READ;
READER(13);
ST:=1;
INSTRING(TITLE,ST);
PUNCH(14);
IF R#0 THEN PRINT '"B:214''';
IF R=1 THEN
BEGIN NEWLINE(1);
PRINT PREFIX(''),"*** RUN BY MIX MOD 18 FROZEN 26:04:82 ***";
NEWLINE(1);
PRINT PREFIX(''),"*********** MIX MOD 18 FROZEN 26:04:82 ***********";
NEWLINE(1);
END
ELSE NEWLINE(2);
PRINT PREFIX(''),"THE DATA BEING PROCESSED IS TITLED";
ST:=1;
OUTSTRING(TITLE,ST);
NEWLINE(1);
PUNCH(1);
ST:=1;
OUTSTRING(TITLE,ST);
NEWLINE(1);
IF N>6 THEN GOTO FOOL;
READER(12);
READ INT;
READ SYST;
IF SYST>5 THEN GOTO FOOL;
J:=1;
COMMENT DATA IN FILE TAPE WILL BE READ AND TESTED FOR MAXIMUM LENGTH;
READER(11);
ALPHA: READ C[0,J];
MAX:=0;
FOR I:=1 STEP 1 UNTIL 301 DO
BEGIN READ A;
A:=ABS(A);
IF A>2200 THEN GOTO BETA;
IF I=300 THEN GOTO GAMMA;
IF I=1 THEN B:=A ELSE C[I,J]:=A-B;
IF C[I,J]LE 0 THEN C[I,J]:=0;
IF C[I,J]GE MAX THEN BEGIN MAX:=C[I,J];
TOP[J]:=I;
END;
END;
BETA: LENGTH[J]:=I-1;
IF J=N THEN GOTO DELTA;
J:=J+1;
GOTO ALPHA;
GAMMA: PUNCH(14);
SPACE(5);
PRINT PREFIX(' '), 'DATA TRACK';
SPACE(1);
PRINT PREFIX(' '), DIGITS(1), J;
SPACE(1);
PRINT PREFIX(' '), 'OF THIS RUN HAS MORE THAN 299 VALUES';
NEWLINE(1);
SPACE(5);
PRINT PREFIX(' '), 'CONSULT DR. MILLEIGHTON BEFORE RESUBMITTING DATA';
NEWLINE(1);
GOTO QUIT;
DELTA: J:=1;
COMMENT THIS LOOP TESTS DATA FOR CONVERGENCE TO INITIAL ZERO LEVEL
MINIMUM PERMISSIBLE LENGTH AND MINIMUM DEFLATION;
PHI: IF TOP[J]GE LENGTH[J] THEN GOTO NU;
IF C[TOP[J],J]LE 300 THEN GOTO PEAK;
D:=0;
FOR I:=TOP[J] STEP 1 UNTIL LENGTH[J] DO
BEGIN A:=C[I,J];
D:=D+1;
IF A<40 THEN
BEGIN IF A=0 THEN C[I,J]:=C[I-1,J];
TRACK[J]:=I;
IF D LE 8 THEN GOTO NU;
FAIL[J]:=1;
GOTO TEST;
END;
END;
IF I=LENGTH[J] THEN
BEGIN PUNCH(14);
SPACE(5);
PRINT PREFIX(' '), 'DATA TRACK';
SPACE(1);
PRINT PREFIX(' '), DIGITS(1), J;
SPACE(1);
PRINT PREFIX(' '), 'OF THIS RUN HAS NOT CONVERGED';
NEWLINE(1);
SPACE(5);
PRINT PREFIX(''), 'THE DATA TRACK HAS BEEN IGNORED';
NEWLINE(1);
FAIL[J] := 0;
GOTO TEST;

END;

PEAK: PUNCH(14);
SPACE(5);
PRINT PREFIX(''), 'DATA TRACK ';
PRINT PREFIX(''), DIGITS(1), J;
SPACE(1);
PRINT PREFIX(''), 'OF THIS RUN DID NOT HAVE ENOUGH DYE ';
PRINT PREFIX(''), 'INJECTED';
NEWLINE(1);
SPACE(5);
PRINT PREFIX(''), 'THE DATA TRACK HAS BEEN IGNORED';
NEWLINE(1);
FAIL[J] := 0;

TEST: IF J = N THEN GOTO THETA;
J := J + 1;
GOTO PHI;

NU: PUNCH(14);
SPACE(5);
PRINT PREFIX(''), 'DATA TRACK';
SPACE(1);
PRINT PREFIX(''), DIGITS(1), J;
SPACE(1);
PRINT PREFIX(''), 'OF THIS RUN HAS TOO LITTLE DATA FOR ACCURATE ';
PRINT PREFIX(''), 'ANALYSIS';
NEWLINE(1);
SPACE(5);
PRINT PREFIX(''), 'THEIR WAS AN EQUIPMENT FAULT OR THE PRF WAS ';
PRINT PREFIX(''), 'TOO SMALL';
NEWLINE(1);
SPACE(5);
PRINT PREFIX(''), 'THE DATA TRACK HAS BEEN IGNORED';
NEWLINE(1);
FAIL[J] := 0;
GOTO TEST;

COMMENT A NUMBER OF VALID DATA TRACKS ARE NOW IN STORE IN MATRIX C[I,J]
A CHECK FOR A SUFFICIENT NUMBER OF VALID TRACKS WILL BE MADE;

THETA: A := 0;
FOR J := 1 STEP 1 UNTIL N DO
BEGIN A := A + FAIL[J];
END;
A := A / N;
IF A < 0.5 THEN GOTO LAMBDA;

COMMENT THE TAIL OF VALID DATA TRACKS WILL BE FITTED TO AN EXPONENTIAL
DECAY AND THE POINTS SMOOTHED TO THE BEST LINE;
J := 1;

MU: SUMX := SUMXY := SUMXSQ := SUMY := 0;
IF FAIL[J] = 1 THEN GOTO RHO;
IF J = N THEN GOTO SIGMA;
J := J + 1;
GOTO MU;

RHO: D := 0;
PUNCH(14);
M := TOP[J] + ((TRACK[J] - TOP[J]) DIV 2);
FOR I := M STEP 1 UNTIL TRACK[J] DO
BEGIN CC := LN(C[I, J]);
SUMX := SUMX + ((I - M) * INT);
SUMY := SUMY + CC;
SUMXY := SUMXY + (CC * (I - M) * INT);
\[
\text{SUMXSQ} = \text{SUMXSQ} + ((I - M) \times (I - M) \times \text{INT} \times \text{INT})
\]
\[
D = D + 1;
\]
\[
\text{NEWLINE(1)};
\]
\[
\text{PRINT PREFIX('} , '\text{CURVE FIT DATA RUN}', \text{DIGITS(1)}, J);
\]
\[
\text{NEWLINE(1)};
\]
\[
\text{PRINT PREFIX('} , 'SUMX=', \text{SCALED(8)}, \text{SUMX});
\]
\[
\text{SPACE(3)};
\]
\[
\text{PRINT PREFIX('} , 'SUMY=', \text{SCALED(8)}, \text{SUMY});
\]
\[
\text{SPACE(3)};
\]
\[
\text{PRINT PREFIX('} , 'SUMXY=', \text{SCALED(8)}, \text{SUMXY});
\]
\[
\text{SPACE(3)};
\]
\[
\text{PRINT PREFIX('} , 'D=', \text{DIGITS(3)}, D);
\]
\[
\text{NEWLINE(1)};
\]
\[
\text{F}[J] = \frac{((\text{SUMY} \times \text{SUMXSQ}) - (\text{SUMX} \times \text{SUMXY}))}{(D \times \text{SUMXSQ}) - (\text{SUMX} \times \text{SUMX})};
\]
\[
\text{PRINT PREFIX('') , 'F=' , SCALED(8), F[J]};
\]
\[
\text{SPACE(4)};
\]
\[
\text{G}[J] = \frac{(D \times \text{SUMXY}) - (\text{SUMX} \times \text{SUMY})}{(D \times \text{SUMXSQ}) - (\text{SUMX} \times \text{SUMX})};
\]
\[
\text{PRINT PREFIX('') , 'G=' , SCALED(8), G[J]};
\]
\[
\text{NEWLINE(1)};
\]
\[
\text{IF } G[J] \geq 0.0 \text{ THEN}
\]
\[
\text{BEGIN DROP}[J]: = 0;
\]
\[
\text{TRACK}[J]: = \text{TRACK}[J] + ((\text{LENGTH}[J] - \text{TRACK}[J]) \text{ DIV 2});
\]
\[
\text{NEWLINE(1)};
\]
\[
\text{GOTO REP};
\]
\[
\text{END}
\]
\[
\text{ELSEIF } \text{DROP}[J]: = 1;
\]
\[
\text{FOR } I: = M \text{ STEP 1 UNTIL } \text{TRACK}[J] \text{ DO}
\]
\[
\text{BEGIN } C[I, J]: = \exp(F[J] + (G[J] \times (I - M) \times \text{INT}));
\]
\[
\text{END};
\]
\[
\text{F}[J] = F[J] - (G[J] \times (M - 1) \times \text{INT});
\]
\[
\text{PRINT PREFIX('') , 'MODIFIED F=' , SCALED(6), F[J]};
\]
\[
\text{NEWLINE(1)};
\]
\[
\text{REP: IF } J = N \text{ THEN GOTO SIGMA};
\]
\[
J: = J + 1;
\]
\[
\text{GOTO MU};
\]
\[
\text{LAMBDA: PUNCH(14)};
\]
\[
\text{SPACE(5)};
\]
\[
\text{PRINT PREFIX('') , 'MORE THAN HALF THE DATA TRACKS OF THIS RUN';}
\]
\[
\text{SPACE(1)};
\]
\[
\text{PRINT PREFIX('') , 'ARE INVALID';}
\]
\[
\text{NEWLINE(1)};
\]
\[
\text{SPACE(5)};
\]
\[
\text{PRINT PREFIX('') , 'THE RUN HAS BEEN ABANDONED';}
\]
\[
\text{NEWLINE(1)};
\]
\[
\text{SPACE(5)};
\]
\[
\text{PRINT PREFIX('') , 'CONSULT DR MILLINGTON';}
\]
\[
\text{NEWLINE(1)};
\]
\[
\text{GOTO START};
\]
\[
\text{SIGMA: J: = 1;}
\]
\[
\text{COMMENT THE MEAN AND VARIANCE OF THE VALID RESPONSE CURVES}
\]
\[
\text{WILL BE CALCULATED USING THE SATER CORRECTION;}
\]
\[
\text{CHI: IF FAIL[J] = 1 THEN GOTO TAU;}
\]
\[
\text{MEAN[J]: = 0;}
\]
\[
\text{VAR[J]: = 0;}
\]
\[
J: = J + 1;
\]
\[
\text{IF } J = N + 1 \text{ THEN GOTO PI ELSE GOTO CHI;}
\]
\[
\text{TAU: SUMC: = SUMCT: = SUMCTSQ: = 0;}
\]
\[
\text{FOR } I: = 1 \text{ STEP 1 UNTIL (TRACK[J] - 1) DO}
\]
\[
\text{BEGIN SUMC: = SUMC + (C[I, J] \times \text{INT});}
\]
\[
\text{SUMCT: = SUMCT + (C[I, J] \times \text{INT} \times (I - 1) \times \text{INT});}
\]
\[
\text{SUMCTSQ: = SUMCTSQ + (C[I, J] \times \text{INT} \times (I - 1) \times (I - 1) \times \text{INT} \times \text{INT});}
\]
\[
\text{END;}
\]
\[
\text{IF } \text{DROP}[J] = 0 \text{ THEN}
BEGIN

INTC:=0.0;
INTCT:=0.0;
INTCTSQ:=0.0;
END
ELSE BEGIN
K:=EXP(F[J]);
G[J]:=-G[J];
CUTOFF:=(TRACK[J]-2)*INT+(INT/2);
INTC:=(K/G[J])*EXP(-G[J]*CUTOFF);
INTCT:=(K/(G[J]*G[J]))*EXP(-G[J]*G[J]*CUTOFF)*
(Q[J]*CUTOFF+1);
INTCTSQ:=(K/(G[J]*G[J]*G[J]))*EXP(-G[J]*CUTOFF)*
((G[J]*CUTOFF)+2);
END;
PUNCH(14);
NEWLINE(1);
PRINT PREFIX(''), 'CALCULATION DATA FROM TRACK ',DIGITS(1),J;
NEWLINE(1);
PRINT PREFIX(''), 'SUMC=', SCALED(8),SUMC;
SPACE(4);
PRINT PREFIX(''), 'SUMCT=', SCALED(8),SUMCT;
SPACE(4);
PRINT PREFIX(''), 'SUMCTSQ=', SCALED(8),SUMCTSQ;
SPACE(4);
IF DROP[J]=0 THEN
BEGIN
PRINT '***SATER CORRECTION ABORTED***';
NEWLINE(2);
GOTO EVAL;
END;
NEWLINE(1);
PRINT PREFIX(''), 'INTC=', SCALED(8),INTC;
SPACE(4);
PRINT PREFIX(''), 'INTCT=', SCALED(8),INTCT;
SPACE(4);
PRINT PREFIX(''), 'INTCTSQ=', SCALED(8),INTCTSQ;
SPACE(4);
PUNCH(14);
NEWLINE(1);
EVAL: MEAN[J]:=(SUMCT+INTCT)/(SUMC+INTC);
VAR[J]:=((SUMCTSQ+INTCTSQ)/(SUMC+INTC))-(MEAN[J])^2;
IF MEAN[J]>1.0E6 THEN MEAN[J]:=1.0E6;
IF VAR[J]>1.0E6 THEN VAR[J]:=1.0E6;
IF J=N THEN GOTO PI;
J:=J+1;
GOTO CHI;
COMMENT RESULTS TO FILE OUTPUT;
PI: PUNCH(14);
PRINT 'B:214';
NEWLINE(6);
SPACE(36);
PRINT PREFIX(''), '**********RESULTS OF THIS RUN**********';
NEWLINE(2);
J:=1;
P:=0;
Q:=0;
KAPPA: IF FAIL[J]=1 THEN GOTO OMIGA;
PUNCH(14);
SPACE(10);
PRINT PREFIX(''), 'TRACK';
SPACE(1);
PRINT PREFIX(''), DIGITS(1),J;
SPACE(1);
PRINT PREFIX(''), 'GIVES NO VALID RESULTS';
NEWLINE(1);
J:=J+1;
IF J=N+1 THEN GOTO EPSILON ELSE GOTO KAPPA;

OMIGA: PUNCH(14);
SPACE(13);
PRINT PREFIX('', 'MEAN OF TRACK');
SPACE(1);
PRINT PREFIX('', DIGITS(1), J);
SPACE(1);
PRINT PREFIX('', '=');
PRINT PREFIX('', ALIGNED(7, 3), MEAN[J], ');'';
PRINT PREFIX('', 'secs');
P:=P+MEAN[J];
SPACE(6);
PRINT PREFIX('', 'VARIANCE OF TRACK');
SPACE(1);
PRINT PREFIX('', DIGITS(1), J);
SPACE(1);
PRINT PREFIX('', '=');
PRINT PREFIX('', ALIGNED(7, 3), VAR[J], ');'';
PRINT PREFIX('', 'secs'2');
Q:=Q+VAR[J];
NEWLINE(1);
IF J=N THEN GOTO EPSILON;
J:=J+1;
GOTO KAPPA;
EPSILON: PUNCH(14);
NEWLINE(2);
PRINT PREFIX(' ', 'AVERAGE MEAN =');
PRINT PREFIX('', ALIGNED(7, 3), P/(A*N), '');
PRINT PREFIX('', 'secs');
SPACE(12);
PRINT PREFIX(' ', 'AVERAGE VARIANCE =');
PRINT PREFIX('', ALIGNED(7, 3), Q/(A*N), '');
PRINT PREFIX('', 'secs'2');
NEWLINE(3);
TBAR:=P/(A*N);
IF SYST GE 4 THEN GOTO PLOT;
COMMENT IF THE SYSTEM CONTAINS A PLUG FLOW SECTION THEN THE PECLET
NUMBER FOR BOTH THE VAN DER LAAN OPEN AND CLOSED PIPE MODELS
WILL BE DETERMINED;
SIGMU:=(Q*N*A)/(P*P);
IF SIGMU<-0.2500 THEN GOTO HELL;
M:=1;
PUNCH(14);
NEWLINE(2);
SPACE(27);
PRINT PREFIX('', 'PECLET NUMBER FOR DOUBLY INFINITE SYSTEM =');
PEC:=((2-(4*SIGMU)-(((4*SIGMU)-2)^2)-
((16*SIGMU)*(SIGMU-2))^.5))/(2*SIGMU);
IF PEC>0 THEN GOTO PECPRINT;
PLUS: PEC:=((2-(4*SIGMU)+(((4*SIGMU)-2)^2)-
((16*SIGMU)*(SIGMU-2))^.5))/(2*SIGMU);
M:=2;
IF PEC>0 THEN GOTO PECPRINT ELSE GOTO LOOK;
PCEPRINT: PUNCH(14);
SPACE(1);
PRINT PREFIX('', 'PECLET NUMBER FOR DOUBLY INFINITE SYSTEM =');
PEC:=((2-(4*SIGMU)-(((4*SIGMU)-2)^2)-
((16*SIGMU)*(SIGMU-2))^.5))/(2*SIGMU);
IF PEC>0 THEN GOTO PECPRINT;
PLUS: PEC:=((2-(4*SIGMU)+(((4*SIGMU)-2)^2)-
((16*SIGMU)*(SIGMU-2))^.5))/(2*SIGMU);
M:=2;
IF PEC>0 THEN GOTO PECPRINT ELSE GOTO LOOK;
HELL: PUNCH(14);
NEWLINK(2);
SPACE(10);
PRINT PREFIX('', 'SOLUTION OF EDDY DIFFUSION EQUATION IMPOSSIBLE');
NEWLINE(1);
SPACE(22);
PRINT PREFIX('', 'CONSULT DR MILLINGTON');
NEWLINK(1)
GOTO PLOT;
LOOK:  RATIO:=-0.5+((5^0.5)/2);
UL:=2/SIGMU;
LL:=0;
D:=0;
MAXSEEK:  PEC:=UL-(RATIO*(UL-LL));
FL:=(2/PEC)-((2/(PEC*PEC))*(1-EXP(-PEC)));
PEC:=LL+(RATIO*(UL-LL));
FU:=FL+(RATIO*(UL-LL));
IF FL>FU THEN UL:=LL+(RATIO*(UL-LL)) ELSE
    LL:=UL-(RATIO*(UL-LL));
IF UL-LL<0.01 THEN GOTO ZEROSEEK ELSE
    IF D>40 THEN GOTO LAM1;
    D:=D+1;
GOTO MAXSEEK;
ZEROSEEK:  LL:=UL;
UL:=UL-(RATIO*(UL-LL));
D:=0;
M:=3;
AGAIN:  PEC:=(UL+LL)/2;
PUNCH(14);
NEWLINE(2);
SPACE(30);
PRINT PREFIX(''),'PECLET NUMBER OF CLOSED PIPE SYSTEM =';
GOTO PECPRINT;
LAM1:  PUNCH(14);
NEWLINE(1);
SPACE(10);
PRINT PREFIX(''),'MAXSEEK HAS NOT CONVERGED - CONSULT DR LAMB';
GOTO PLOT;
LAM2:  PUNCH(14);
NEWLINE(1);
SPACE(10);
PRINT PREFIX(''),'ZEROSEEK HAS NOT CONVERGED - CONSULT DR LAMB';
COMMENT THE FOLLOWING PACKAGE TAKES THE VALID DATA TRACKS AND 
EVALUATES A MEAN DIMENSIONLESS DATA SET WHICH IS 
USED TO EVALUATE THE P-CURVE;
PLOT: BEGIN REAL CZERO,HB;
    REAL ARRAY TOT,RATIO[1:6],CDIM,TDIM[1:300],
    ACC[0:300];
    FOR I:=1 STEP 1 UNTIL 300 DO
        BEGIN CDIM[I]:=O;
            TDIM[I]:=O;
            ACC[I]:=O;
        END;
        ACC[0]:=O;
        TOT[1]:=TOT[2]:=TOT[3]:=TOT[4]:=TOT[5]:=TOT[6]:=O;
        M:=O;
        J:=1;
        TOTAL: IF FAIL[J]#1 THEN GOTO CHECK;
            M:=M+1;
            FOR I:=1 STEP 1 UNTIL TRACK[J] DO
                BEGIN TOT[M]:=TOT[M]+C[I,J];
                    C[I,M]:=C[I,J];
                END;
                TRACK[M]:=TRACK[J];
CHECK:  
J:=J+1;
IF J LE N THEN GOTO TOTAL;
N:=N;
FOR M:=1 STEP 1 UNTIL N DO
BEGIN 
BEGIN RATIOM[M]:=TOT[J]/TOT[M];
FOR I:=1 STEP 1 UNTIL TRACK[M] DO
BEGIN 
C[I,M]:=(C[I,M]*RATIOM[M];
ACC[I]:=(ACC[I]+C[I,M]/N);
END;
END;
CZERO:=0;
FOR I:=1 STEP 1 UNTIL 300 DO
BEGIN 
CZERO:=CZERO+(ACC[I]*INT/TBAR);
END;
J:=O;
M:=O;
HB:=O;
PUNCH(15);
PRINT DIGITS(2),R;
FOR I:=1 STEP 1 UNTIL 300 DO
BEGIN
BEGIN CDIML[I]:=ACC[I]/CZERO;
ACC[I]:=(ACC[I]+(CDIML[I]*INT/TBAR));
TDIML[I]:=(I-1)*INT/TBAR;
IF ACC[I]<1 THEN ACC[I]:=1.0;
END;
COMMENT F AND TDIM DATA OUTPUT TO THE PLOT FILE;
NEWLINE(1);
PRINT PREFIX(' '),ALIGNED(2,6),TDIML[I];
SPACE(5);
PRINT PREFIX(' '),ALIGNED(1,6),ACC[I];
IF M=1 THEN HB:=HB+((ACC[I]+ACC[I-1])*INT/(2*TBAR));
M:=O;
IF TDIML[I]<1.0 THEN
BEGIN HB:=HB+((ACC[I]+ACC[I-1])*(1.0-TDIML[I-1])/2);
M:=O;
IF TDIML[I]<1.0 THEN
BEGIN HB:=HB+((ACC[I]+ACC[I-1])*INT/(2*TBAR));
END;
END;
IF ACC[I]<1.0 THEN
BEGIN HB:=HB+((ACC[I]+ACC[I-1])*(1.0-TDIML[I-1])/2);
END;
LOOP: END;
PUNCH(14);
NEWLINE(2);
SPACE(40);
PRINT PREFIX(' '),'HOLDBACK=',ALIGNED(2,6),HB;
GOTO START;
END;
FOOL: PUNCH(14);
SPACE(5);
PRINT PREFIX(' '),'ERROR IN SYSTEM DATA - EITHER N>6 OR SYST>5';
NEWLINE(1);
SPACE(5);
PRINT PREFIX(' '),'BE MORE CAREFUL IN FUTURE';
NEWLINE(1);
SPACE(5);
PRINT PREFIX(' '),'THE PROGRAM HAS BEEN HALTED';
NEWLINE(1);
GOTO STOP;
QUIT: PUNCH(14);
SPACE(5);
PRINT PREFIX(' '),'ERROR IN DATA ON TAPE';
SPACE(1);
PRINT PREFIX(' '),'EITHER 2300 OMITTED OR TOO MANY READINGS';
NEWLINE(1);
SPACE(5);
PRINT PREFIX(' '),'BE MORE CAREFUL IN FUTURE';
NEWLINE(1);
GOTO STOP;
FINISH: PUNCH(14);
NEWLINE(4);
PRINT PREFIX(' '),'THE PROCESSING OF DATA IS NOW COMPLETE';
SPACE(1);
PRINT PREFIX('', 'PLEASE STUDY THE OUTPUT CAREFULLY');
NEWLINE(1);
SPACE(5);
PRINT PREFIX('', 'DR MILLINGTON WILL ASSIST IN THE INTERPRETATION');
SPACE(1);
PRINT PREFIX('', 'OF THE RESULTS');
NEWLINE(2);

STOP: SPACE(35);
PRINT PREFIX('', '**********END OF PRINTOUT**********');
CLOSE(11);
CLOSE(12);
CLOSE(13);
PUNCH(1);
NEWLINE(1);
PRINT PREFIX('', '***************RUN COMPLETE***************');
NEWLINE(2);
PRINT PREFIX('', 'PLEASE EXAMINE FILE <MIXOUT> FOR YOUR RESULTS');
NEWLINE(1);
CLOSE(14);
R:=9999;
PUNCH(15);
PRINT DIGITS(4), R;
NEWLINE(1);
CLOSE(15);
END;
REFERENCES

46. Friend, L. and E.J. Lemieux: Oil Gas J., 54 (64) 88 (1956).


