The Distortion of Ultra-Wideband Signals in the Environment

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Summary

The need for higher data rates in wireless communication systems and the lack of available frequency space has led the researchers in developing new technologies, such as UWB. Ultra-wideband systems utilise signals with very short duration and very large bandwidth, from which the benefits are twofold; more information can be exchanged and coexistence with narrowband technologies is possible, as such systems perceive UWB as noise and interference is thus avoided. The increased bandwidth offers multipath resolution, which when properly treated can enhance system's performance and reliability.

Signal distortion however, may prohibit system's optimum performance. It is more efficiently treated directly into the time-domain. The lack of information on the propagation of UWB signals in a complex environment, other than single reflected and diffracted waves, was tackled in this work. Easy-to-use and accurate reflection and transmission coefficients for a wave impinging on a dielectric slab were deduced. An algorithm for the prediction of a signal, which is multiply diffracted waves on an number of absorbing knife-edges and/or imperfect conducting wedges was also implemented. The algorithm accurately took into account the arisen higher-order fields, that are created in such cases, according to the Uniform Theory of Diffraction.

A tool for signal prediction in a complex environment was constructed, based on the environment discretisation into tiles and segments. Its validity was initially tested for the narrowband case, by comparing its predictions with measurement, with a quite good agreement. Then, it was modified, so as to incorporate the dispersive nature of the channel. This was accomplished in the time-domain, where the TD ray-trace model not only predicts the arrival times of the multipaths but also their shape. The results were compared with measurement data collected with a VNA.

Key words: UWB, signal distortion, diffraction, ray-trace
Acknowledgements

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My time in the office would not have been pleasant, if not for the people there. Everyone had something to give, for which I will be forever grateful. I would like to name them all, but the space here is very limited. However, they will always remain in my heart.

Finally, this work is dedicated to my family, without whose support this quest would have been just a dream.
# Contents

## 1 Introduction

1.1 Wireless Communications ................................................................. 1  
1.2 Motivation ....................................................................................... 2  
1.3 Thesis Outline .................................................................................. 3  
1.4 Achievements .................................................................................. 5  
  1.4.1 Reflection and Transmission through a Dielectric Slab .............. 5  
  1.4.2 Multiple Time-Domain Diffraction .......................................... 6  
  1.4.3 Narrowband Ray-Trace Model for Macrocells ....................... 6  
  1.4.4 Time-domain Ray-Trace Simulator ............................................ 6  
1.5 List of publications ........................................................................ 7  

## 2 Ultra-Wideband Communications

2.1 Impulse Radio .................................................................................. 8  
2.2 Regulations on UWB Radio ............................................................. 10  
2.3 Ultra-Wideband Waveforms ............................................................ 13  
2.4 Modulation of UWB Radio ............................................................... 16  
  2.4.1 Single-Band Modulation Techniques ....................................... 18  
  2.4.2 Multiband Modulation Techniques  .......................................... 20  
2.5 UWB Receivers ............................................................................... 26  
2.6 Conclusions ................................................................................... 29  

## 3 The Wireless Channel

3.1 Radio Wave Propagation ................................................................. 30  
3.2 Inverse Techniques ......................................................................... 32  
3.3 Reflected Waves ............................................................................ 33
Contents

3.3.1 Half-plane Reflection ......................................................... 34
3.3.2 Half-plane in the Time-Domain ........................................ 35
3.4 Diffraction theory .............................................................. 39
  3.4.1 Geometrical/Uniform Theory of Diffraction .................. 40
  3.4.2 Time-domain UTD ......................................................... 44
3.5 Scattering ........................................................................... 46
3.6 Conclusions ...................................................................... 47

4 Multiple Time-Domain Interactions ........................................... 48
  4.1 Multiple-Interactions ......................................................... 48
  4.2 Layer Model ...................................................................... 49
    4.2.1 Frequency-Domain Approach ................................. 50
    4.2.2 Time-Domain Approach ......................................... 52
    4.2.3 Numerical Evaluation - Comparison ...................... 54
  4.3 Multiple-Diffraction Phenomena ........................................ 58
    4.3.1 Frequency-Domain Approach ................................. 58
    4.3.2 Time-Domain Approach ......................................... 62
    4.3.3 Numerical Evaluation - Comparison ...................... 65
  4.4 Conclusions ...................................................................... 71

5 Ray-Trace Algorithm ............................................................. 73
  5.1 Field Prediction ................................................................. 73
  5.2 Ray-Trace Models ............................................................ 75
    5.2.1 Image Method ............................................................ 75
    5.2.2 Shooting-and-Bouncing Ray-Launching .................. 76
    5.2.3 Acceleration Algorithms ......................................... 77
  5.3 Parameters in Ray-Trace Algorithms ................................. 78
  5.4 Database Preprocessing Method ......................................... 79
    5.4.1 Database Preprocessing ........................................... 81
    5.4.2 Ray-trace Algorithm ................................................ 83
  5.5 Measurement Campaign .................................................... 86
  5.6 Comparison Analysis ........................................................ 86
  5.7 Conclusions ...................................................................... 96
6 Indoor Narrowband and Time-Domain Ray-Trace Model 97
  6.1 Introduction ................................................. 97
  6.2 Indoor Measurements .................................... 99
  6.3 Indoor Narrowband Ray-Trace Model .............. 102
    6.3.1 Comparison - Results ......................... 102
  6.4 Time-domain Ray-Trace .................................. 106
    6.4.1 Current algorithm .............................. 107
    6.4.2 Hermitian Processing ......................... 108
    6.4.3 Antenna Effects ............................... 109
    6.4.4 Comparison - Results ....................... 110
  6.5 Conclusions ............................................. 116

7 Conclusions 118
  7.1 Overview and Discussion ............................ 118
  7.2 Limitation Factors for Signal Prediction .......... 120
    7.2.1 Environmental Modeling ....................... 121
    7.2.2 Ray-trace Limitations - Memory Requirements 122
    7.2.3 Multiple TD Diffracted Signals ............... 122
  7.3 Conclusion - Future Work .......................... 123

A Derivation of the time-domain reflection coefficient for the half-plane 125

B The relationship between the transition function and the complementary error function 129

Bibliography 131
List of Figures

2.1 UWB unlicensed spectrum limits .................................................. 13
2.2 Gaussian pulses in time-domain ..................................................... 15
2.3 Gaussian pulses in frequency-domain ............................................. 17
2.4 The waveforms of different modulation types ................................. 21
2.5 The spectrums of the different modulation options ....................... 22
2.6 MB-UWB in time- and frequency-domain .................................... 23
2.7 MB-OFDM modulation technique ................................................ 25
2.8 DS-UWB modulation technique .................................................. 26
2.9 Multiband DS-UWB modulated signals ........................................ 27
3.1 Half-plane reflection and transmission .......................................... 34
3.2 The frequency response of the Fresnel reflection coefficients for soft and hard polarised waves ...................................................... 36
3.3 The approximation of the sum for the TD reflection coefficient ....... 37
3.4 TD and inverse Fourier transformed FD reflected waves ............... 38
3.5 Knife-edge diffraction geometry .................................................. 40
3.6 Diffraction according to the Geometrical Theory of Diffraction ...... 41
3.7 UTD diffraction geometry ............................................................ 43
3.8 The frequency response of the UTD diffraction coefficient for a knife-edge and a wedge. .......................................................... 44
3.9 Diffracted pulses in the TD ......................................................... 46
3.10 Scattering effects in reflection ..................................................... 47
4.1 Internal reflections in a layer model .............................................. 49
4.2 The reflection and transmission frequency response for a layer model 51
4.3 Reflected pulses on a lossless dielectric slab .................................. 55
List of Figures

4.4 Transmitted pulses through a lossless dielectric slab. ........................................ 55
4.5 Reflected pulses on a lossy dielectric slab. .......................................................... 56
4.6 Transmitted pulses through a lossy dielectric slab. ............................................ 57
4.7 Multiple-diffracted rays ....................................................................................... 60
4.8 Received signal in the grazing incidence of five knife-edges. ......................... 66
4.9 Received signal in the grazing incidence of five wedges. ............................... 66
4.10 The propagation path of a quadruple-diffracted signal. ................................. 68
4.11 The effect of a reduced time-step in the convolution accuracy. ................... 69
4.12 The received signal for a cascade of different objects. ................................... 70
4.13 The received signal for a cascade of different objects and unequal heights. .... 71

5.1 Ray-trace methods ............................................................................................ 76
5.2 The environmental discretisation into tiles and segments. ............................... 79
5.3 Reflection conditions. ....................................................................................... 80
5.4 The architecture for the Database Preprocessing. ........................................... 82
5.5 The tree structure of the ray-trace algorithm. .................................................. 84
5.6 The architecture for the ray-trace algorithm. .................................................... 85
5.7 Measurement results for Portland Place, London. ........................................... 87
5.8 Measurement results for Holborn, London. ..................................................... 88
5.9 Measurement results for Kingsland, London. .................................................... 89
5.10 Comparison for the first scenario in Portland Place area. ............................... 91
5.11 Comparison for the second scenario in Portland Place area. ......................... 92
5.12 Comparison for the Holborn area. ..................................................................... 93
5.13 Comparison for the Kingsland area. ............................................................... 95

6.1 The measurement apparatus .............................................................................. 99
6.2 The radiation pattern of the antennas ............................................................... 100
6.3 The elevation pattern of the antennas ............................................................... 101
6.4 Typical channel frequency response ................................................................. 101
6.5 The second floor plan of the CCSR building .................................................... 102
6.6 Narrowband propagation for a corridor ............................................................ 104
6.7 An example of predicted rays in the indoor scenario. ...................................... 104
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.8</td>
<td>Deeply obstructed propagation path</td>
<td>106</td>
</tr>
<tr>
<td>6.9</td>
<td>Hermitian Processing</td>
<td>109</td>
</tr>
<tr>
<td>6.10</td>
<td>UWB vs. NB path loss</td>
<td>111</td>
</tr>
<tr>
<td>6.11</td>
<td>UWB channel response</td>
<td>113</td>
</tr>
<tr>
<td>6.12</td>
<td>The predicted TD path loss</td>
<td>114</td>
</tr>
<tr>
<td>6.13</td>
<td>TD Gaussian doublets</td>
<td>115</td>
</tr>
<tr>
<td>6.14</td>
<td>Predicted received waveforms in the TD for two different signals</td>
<td>116</td>
</tr>
</tbody>
</table>
List of Tables

2.1 FCC Spectral Mask ................................................................. 11
2.2 EC Spectral Mask ................................................................. 12
4.1 Comparison of time-domain prediction with IFFT ................. 71
5.1 Comparison results for the ray-trace algorithm .................... 96
6.1 Wall parameters ................................................................. 103
6.2 Comparison results for the indoor ray-trace algorithm ............ 105
6.3 Comparison results for the TD-UWB ray-trace algorithm ......... 115
Chapter 1

Introduction

1.1 Wireless Communications

Today's epoch is characterised by a revolution of wireless communications. The domination of wired systems in the previous decades has started to decline due to advances in satellite communications, mobile telephony and wireless personal area networks. Expensive and tedious infrastructure may be avoided and the wireless connectivity offers simplicity and mobility for the user.

In a wireless communication system, the signal to be transmitted is sampled and converted into a digital format, as in a stream of binary digits (bits). The bit stream is then encoded/interleaved, in order to make transmission more reliable by applying error-correction techniques or data compression. In the next step, modulation occurs, where the stream is mapped onto a pattern, which either governs the shape, the amplitude, the phase or the frequency of a carrier and the resulting modulated signal is sent to the transmitting antenna. The excited waveform traverses the radiowave channel before it is received, where the inverse conversion occurs; the incoming wave is demodulated, decoded/de-interleaved, so as to be transformed into the original signal.

The accurate conversion of the received signal to its original state however is not a straightforward task, as the received waveform has been corrupted by the propagation channel. The radiowave medium is actually the most variable system component that
1.2. Motivation

places fundamental limitations on the performance of wireless communication systems. The transmission path often takes place over, through and around obstacles in the environment, including buildings, trees, offices, partitions and other objects, either small or large. A correct prediction of the channel's effects would lead to an enhancement of the system's performance and its reliability.

There are many methods for estimating the received signal, where methods vary widely in their approach, complexity and accuracy. Empirical models rely upon a combination of systematic interpretation of collected measurements together with some elementary physical laws. However, in their simplicity, they suffer from inaccurate predictions, when the environment changes, and they offer no physical insight into the mechanisms by which propagation occurs. On the other hand, physical models rely on the description of the environment, in terms of the location and dimensions of the surrounding objects, where the electromagnetic laws are applied. The physical phenomena, like reflection, diffraction or scattering, are taken into account, offering thus more accurate predictions. However, the performance of such a tool does not only depend on the accuracy of the electromagnetic analysis, but also on the accuracy of the environment representation and simplification, as the path profile is sometimes needed to be replaced with a series of canonical forms. Therefore, the accumulation of possible errors may result in large deviations from the correct value. Finally, the antenna itself affects the field prediction; the intensity of the excited rays depends on the frequency and the radiating angle. Thus, knowledge of the antenna transfer function would be desired [1,2] and careful design should be conducted in order to ensure optimum performance.

1.2 Motivation

One of the greatest benefits of ultra wideband radio is its inherent capability of resolving the multipath components (MPCs). This multipath resolution is an immediate derivation from the large bandwidth of the transmitted signals. Therefore, the MPCs can be distinguished and received and, if they are properly treated, they can enhance considerably the quality of the received signal and the data rate of the system. However, the large bandwidth also creates increased challenges in the receiver design; imperfect
synchronisation and pulse distortion may deviate the system from its optimum performance.

The most common UWB receiver is the RAKE receiver [3-7], where the arriving multipath on each finger is matched to a template. If the template matches correctly the shape of the received signal, maximum performance is then achieved. For the UWB channel modeling, it is either assumed the channels have no distortion, where the received signal is a replica of the transmitted one, or a priori knowledge of the channel, where the propagation effects on the received signal are assumed to be known. Such models can adequately describe the narrowband propagation channel, but they are unrealistic or very optimistic for a UWB channel. The actual environment is by nature diverse, either outdoor or indoor; the building sizes and shapes, the surrounding environment like trees, hedges, cars and sign posts, interior decoration, as well as their electrical properties are not easily defined. Therefore, distortion would be different on every path and a statistical analysis may be insufficient, if it is not complemented by a site-specific prediction.

The lack of information on the propagation of UWB signals in a complex environment was the driving force for this thesis and an attempt to construct a site-specific tool that would incorporate the dispersion nature of the channel in an efficient way. The ray-trace model can provide with information not only for indoor channels, but for outdoor environments as well, whether considering narrowband communication systems or ultrawideband ones. The information would give an insight on the distortion mechanisms that are necessary for the correct prediction of the signal shape and truly enhance the receiver’s performance.

1.3 Thesis Outline

UWB is an exciting new technology, with a long history. This antithesis is expressed by the fact that even though the first radio experiments, in the beginning of the previous century, were pulse based, only recently the research community and industry are trying to find ways of exploiting such systems. The regulation governing UWB systems and the benefits of impulse radio are presented in Chapter 2, which acts as an introduction
into ultra wideband technology. An UWB system involves the transmission of very short pulses, like Gaussian monocycles or doublets, with sub-nanosecond duration and consequently a very large bandwidth. Possible modulation schemes may include single-band or multiband signals, depending on the available resources and the propagation channel. Receiver designs and challenges which affect the system performance are also briefly discussed.

The propagating signal is reflected, refracted or diffracted by the various objects in the environment and therefore distortion and temporal dispersion inevitably occurs. It is more advantageous to work directly into the time-domain, if closed-form solutions exist, than inverting the frequency-domain solutions into the time-domain by using inverse Fourier transforms. Previous work and examples of time-domain coefficients are presented in Chapter 3. Such are the TD solution for the half plane reflection [8] and solutions for single diffracted waves [9–11]. The occurring signal distortion is exhibited and correctly predicted.

However, in a practical propagation channel, the signal is expected to undergo more than one interaction. Such may be the multiple internal reflections that occur when a signal propagates through a dielectric slab and the case of multiple diffracted pulses. Therefore the existing solutions need be extended so as to provide correct representation of these phenomena in the TD, which is the content of Chapter 4. New easy-to-use and accurate TD coefficients for the reflection and transmission of waves through a dielectric slab are derived. These solutions take into account the multiple internal reflections of rays trapped in the dielectric medium. The response of a path that is diffracted on many obstacles is also presented. The TD representation of the multiple diffracted field is based on the high frequency approximation of the Uniform Theory of Diffraction (UTD). The proposed solution accommodates the second order diffracted field in an efficient algorithm and provides with correct predictions of the received waveform.

In Chapter 5, a novel model for ray-tracing is introduced. It is a combination of image theory and ray-launching, which benefits more by their strengths and loses less by their shortcomings. The model is divided into two stages and each of the stages is
1.4. Achievements

presented. The key element of the model is the discretisation of the environment into tiles and segments. By noting and storing their respective relationships, in terms of the angles they form with each other, a great simplification on the actual path-finding can be made. The predictions of the model are compared against measurement data, with satisfying results.

The ray-trace model is used for indoor field predictions in Chapter 6, so as to strengthen its value and prove its versatility. The received field in an indoor environment was simulated and compared with measurements. Again the comparison of the field predictions with the measurements are in a quite good agreement. The ray-trace model was transformed into the time-domain, so as to accommodate the dispersion nature of the channel. The TD path-finding model accurately tracks the per path distortion, to form the channel response for every receiver location. The received waveform would then be the convolution of the transmitted signal with the predicted response. The comparison with measured channel responses in an indoor environment encouragingly supports the TD model's merit.

Finally, Chapter 7 gives an overview of the current thesis and a discussion on its achievements, suggesting some possible extensions.

1.4 Achievements

1.4.1 Reflection and Transmission through a Dielectric Slab

New easy-to-use and accurate reflection and transmission coefficients were derived, for a wave impinging on a dielectric slab. The coefficients include in a straightforward manner a great number of internal reflected waves that are trapped inside the slab. In many cases, these multiple reflected signals may carry significant energy, but they also need to be considered if symbol interference is to be avoided.
1.4. Achievements

1.4.2 Multiple Time-Domain Diffraction

A novel algorithm for predicting the received signal after being diffracted on a number of obstacles is developed. The algorithm incorporates the arisen second-order diffracted field, as this is expressed in the UTD formulation [12–14], directly into the time-domain. The solution thus includes the amplitude and slope terms of the diffracted waves, providing accurate results and predicting correctly the received waveform.

1.4.3 Narrowband Ray-Trace Model for Macrocells

A new ray-trace model is constructed, based on the idea of an intelligent preprocess of the database. The model, in the first step, discretises the environment into tiles and segments, recording the angles they form with each other. This information is stored into memory and it is irrelevant to the position of the transmitter. In the second stage, the transmitter is placed and the actual path-finding occurs. It simplifies the path-finding into a search in a look-up table. The ray-trace model can also accurately predict the field in an indoor environment. The main propagation mechanisms however are reflection and transmission through walls that the indoor ray-trace model can easily identify. The proposed ray-trace model combines the strengths of image theory with ray-launching and the comparison of the predictions with measurements prove its validity and versatility.

1.4.4 Time-domain Ray-Trace Simulator

Having established the correctness of the ray-trace model for narrowband communications, it is used as a base for the incorporation of the frequency selective nature of the channel. This is accomplished in an efficient way by working directly in the time-domain. The TD ray-trace model can track the different alterations on the signal’s shape, as it traverses the many possible routes, before it is received. The time-domain simulator can predict a great number of multipath components, with their respective delays, and construct the channel response for every specific location. The convolution of that response with any signal would give the received waveform, which can be used for channel characterisation and performance estimation.
1.5 List of publications

Journals


Conferences


Chapter 2

Ultra-Wideband Communications

2.1 Impulse Radio

Wireless communications until now have been dominated by the use of narrowband signals, which are characterised with small bandwidth. However, the need for higher data rates and the crowding of the available frequency spectrum are the driving forces for seeking more appropriate technologies, such as ultra-wideband (UWB). Ultra-wideband is a new wireless technology with very promising characteristics. It employs the transmission and reception of very short pulses that resemble impulses and this is why it is also referred as impulse radio (IR). Even though researchers have experimented with impulses for more than a century — the first radio transmission experiments by Hertz in 1887 employed the radiation of electromagnetic waves created by a spark gap apparatus — due to the difficulties in the receiver design, research was focused into narrowband communications. However, with the advance of technology, today UWB is a feasible option.

The UWB pulses have very short duration, a fraction of a nanosecond, and therefore their bandwidth is extremely large. Shannon proved that the more the signals are spread in bandwidth, in a similar manner to the background noise, the more information can carry [15]. Therefore, it would be more efficient to transmit signals with low power density and wider bandwidth, than high power signals with narrow bandwidth. This primarily led to the deployment of spread-spectrum communications, where the signal
is deliberately spread to a larger bandwidth. Spread-spectrum has been applied for third generation mobile communication systems (3G). By reducing the duration of the pulses even more, and hence increasing their bandwidth, the system resembles impulse radio, which is the key attribute of UWB communications.

Some of the benefits of UWB communications are quite evident, as an immediate result of the increased bandwidth, and some are not so easily distinguished. The reader can refer to [16–18] for a more detailed account. The most compelling aspect is the possibility of achieving very high data-rates, or when this is not required, to vary the rate of the transmitted data by reducing the number of pulses per symbol so as to have more reliable communication links. UWB signals are very resistant to severe multipath, in the sense that scatterers which are not very close together can be distinguished and resolved. This inherent good time-domain resolution property of UWB allows for location and tracking applications. The low power spectral density and the noise-like characteristics of UWB signals provide them with the ability of coexisting with other wireless devices and especially narrowband ones. They are also less susceptible to jamming and they are good candidates for covert and secure communications, due to their low-probability-of-detection (LPD). The elimination of many components, such as up- and down-converters, and the ability of directly modulating a pulse onto an antenna intrigues the industry and the researchers so as to manufacture transceivers with low-cost and complexity. Another interesting feature of UWB systems, especially when operating in multiband mode, is the flexibility in the spectrum usage; since the regulations concerning UWB vary from country to country, careful tailoring of the radiated spectrum may be necessary, so as to obey the local legislation and/or avoid strong narrowband interferers, such as U-NII radio band.

The applications of UWB are numerous and span on many different areas. The most straightforward application is in multimedia communications and wireless personal (WPAN) or body area networks (BAN). A huge amount of data can be exchanged between different devices, like wearable electronic equipments (such as MP3 players and palmtops) or in-home entertainment devices (like HDTV and DVD players), leading to an all-wireless revolution [19,20]. The large frequency spectrum and especially the low frequency components can be used in the implementation of ground penetrating
radars. This is very useful for geophysical prospecting, archaeology, civil engineering, environmental engineering, and defence technologies as a non-invasive sensing tool. These radars have the ability in operating in ambient temperature, which is not feasible for infra-red radars. Another extension of their use is in medicine, for biomedical purposes, in imaging the internal organs of a body without the harmful effects and the high cost of X-rays and magnetic resonance imaging (MRI) scans.

The rest of this chapter will cover in a little more detailed manner the basic principles of UWB communications. Starting from the current official regulations for unlicensed radio in the USA and the EU, which also includes UWB transmission, topics like UWB waveforms and modulation techniques will be covered. Possible receiver designs will also be briefly presented.

2.2 Regulations on UWB Radio

The first regulations for UWB radio were set out by the Federal Communication Committee (FCC) in the United States [21]. FCC defined spectral masks, specifying the power limits for the transmission of unlicensed radio, which also includes UWB transmission.

According to FCC, a UWB transmitter is

"an intentional radiator that, at any point in time, has a fractional bandwidth equal or greater than 0.20 or has a UWB bandwidth equal to or greater than 500 MHz, regardless of the fractional bandwidth".

The bandwidth is considered to be the frequency band bounded by the points that are 10 dB below the highest radiated emission. If the upper bound is designated as \( f_H \) and the lower one as \( f_L \), then the fractional bandwidth equals to \( B_{FC} = 2 \cdot (f_H - f_L)/(f_H + f_L) \).

Under these definitions, the radiated power spectral density for indoor UWB systems shall not exceed the average limits when measured using a resolution bandwidth of 1 MHz, of those stated in Table 2.1. The spectral masks for other cases, such as ground
2.2. Regulations on UWB Radio

Table 2.1: FCC Spectral Mask

<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>EIRP (dBm/MHz)</th>
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<tbody>
<tr>
<td>960-1610</td>
<td>-75.3</td>
</tr>
<tr>
<td>1610-1990</td>
<td>-53.3</td>
</tr>
<tr>
<td>1990-3100</td>
<td>-51.3</td>
</tr>
<tr>
<td>3100-10600</td>
<td>-41.3</td>
</tr>
<tr>
<td>Above 10600</td>
<td>-51.3</td>
</tr>
</tbody>
</table>

penetrating radars, wall imaging systems, surveillance systems, hand-held devices and outdoor UWB communications can also be found in [21].

In a similar manner, the European Commission (EC) issued a ruling on the operation of unlicensed UWB radio systems [22]. According to the EC, equipment using ultra-wideband technology means

"equipment incorporating, as an integral part or as an accessory, technology for short-range radio communication, involving the intentional generation and transmission of radio-frequency energy that spreads over a frequency range wider than 50 MHz, which may overlap several frequency bands allocated to radiocommunication services".

These devices may not cause harmful interference and no claims for protection of these devices from other interfering radio communication services may be made. It should be noted that, in contrast to the FCC regulation, UWB is defined as signals with bandwidth greater than 50 MHz. The suggested power spectral densities, in the absence of mitigation techniques, are presented in Table 2.2. However, it is very unlikely systems that employ signals with the lowest permitted bandwidth will be implemented, as the total transmission power needs to be quite low and hence the performance will be very poor.

Ultra-wideband equipment may be allowed to transmit with power spectral densities higher than the ones stated above, if appropriate mitigation techniques, like detect-and-
avoid or low-duty-cycle, are applied. It is evident that the major spectrum allocation for UWB use is confined, temporarily, in the region between 4.2 to 4.8 GHz and above 6 GHz.

In UK, it is recommended by Ofcom to relax the limits of EIRP spectral densities for frequencies higher than 10.6 GHz, with the lower bound to be equal to \[23\]

1. \[-85 + 20 \cdot \log(f_{GHz}/10.6) \text{ dBm/MHz}\] (mean EIRP density)

for frequencies above 10.6 GHz which support sensitive services such as radio astronomy and exploration satellites; and

2. \[-65 + 20 \cdot \log(f_{GHz}/10.6) \text{ dBm/MHz}\] (mean EIRP density)

for all other frequencies above 10.6 GHz.

where \(f_{GHz}\) represents the frequency in units of GHz. The proposed mask though, shall serve as guidance only.
2.3. Ultra-Wideband Waveforms

The mean EIRP spectral density limits for UWB devices, as these were set by the regulatory bodies in the United States and the European Union are summarized in Fig. 2.1, as well as Ofcom's recommendations for the transmission of UWB radio waves for frequencies above 10.6 GHz.

![Figure 2.1: The spectral limits for the operation of unlicensed UWB radio transmission. For the EC mask, the permanent values were used.](image)

The work in this thesis is not affected by the decision of which definition for UWB is followed, as it will be shown in the following chapters. However, since the benefits of using greater band widths are quite evident, and with the notion in mind of Shannon rule, it is expected that a viable UWB system will incorporate signals with hundreds of MHz bandwidth.

2.3 Ultra-Wideband Waveforms

The ultra-wideband waveforms play a very essential role in the deployment of UWB systems, because not only they need to spread the power effectively in the frequency domain, but also they need to conform with the regulations imposed by the official bodies. People are more keen in utilising harmonic waves, and therefore it would be more natural similar kinds of waveforms to be excited. These might include sinusoidal
2.3. Ultra-Wideband Waveforms

electromagnetic waves whose amplitude of oscillation decreases exponentially with time, so as to reduce the duration of the pulse. However, such kinds of emissions are classified as damped sine (class B) emissions and they are prohibited, since they increase the noise to frequency adjacent bands. Damped sine waves can be represented mathematically as [17]

\[ d(t) = Ae^{-\lambda t} \sin(2\pi f_0 t) \]  

(2.1)

where \( A \) is the amplitude of the wave, \( \lambda \) is the exponential decay coefficient, \( f_0 \) is the frequency of oscillation of the sine wave and \( t \) is the time.

Therefore, non-damped waves need to be excited, which must be characterised by fast rise and fall times, so as to broaden the spectrum and have small energy content. For effective radiation, it is desired these pulses to contain zero dc content, and also to be orthogonal, in order to be uniquely demodulated. There are many candidates that can serve as UWB signals; Gaussian, Rayleigh, Laplacian, and cubic pulses [24], whose mathematical representations are similar to the corresponding mathematical functions, modified Hermitian pulses [25,26], as well as orthogonal prolate spheroidal wave functions [27,28]. The most commonly used are the Gaussian pulses and their derivatives, due to their simplicity and low out-of-band energy [29]. The Gaussian pulse is given by

\[ g_0(t) = \sqrt{\frac{E}{\tau \cdot \sqrt{\pi/2}}} \cdot e^{-\left(t/\tau\right)^2} \]  

(2.2)

where \( E \) is the desired energy of the pulse, \( \tau \) is a the full width half maximum pulse duration, a parameter that actually controls the duration of the pulse and consequently its bandwidth, and \( t \) is the time [17]. The first derivative of (2.2) is called the Gaussian monocycle and it can be written as

\[ g_1(t) = -2 \cdot \sqrt{\frac{\tau E}{\sqrt{\pi/2}}} \cdot \frac{t}{\tau^2} \cdot e^{-\left(t/\tau\right)^2}. \]  

(2.3)

The second derivative, also known as Gaussian doublet, is defined as

\[ g_2(t) = \frac{-2}{\tau} \cdot \sqrt{\frac{\tau E}{3 \cdot \sqrt{\pi/2}}} \left(1 - \frac{2t^2}{\tau^2}\right) \cdot e^{-\left(t/\tau\right)^2}. \]  

(2.4)

The above pulses are depicted in the time-domain in Fig. 2.2. The durations of the pulses are in the subnanosecond region. A damped sine wave is presented as well for
2.3. Ultra-Wideband Waveforms

Comparison. It can be noticed that the Gaussian pulses confine the energy more tightly than the damped sine wave, where energy is leaked to adjacent bins (where bin is an increment in the time axis). Another point to notice is that the number of the zero crossings of the differentiated pulses increases with respect to the order of differentiation of the initial Gaussian pulse in (2.2), and also that the energy of the higher-order pulses is more spread over time.

Figure 2.2: The pulses of (2.2), (2.3), (2.4) are displayed here, along with a damped sine wave, as this is given in (2.1). The pulses have normalised amplitudes. For the Gaussian pulses, \( \tau = 70 \) ps and for the damped sine wave, \( f_0 = 4.5 \text{ GHz} \), \( \lambda = 6 \cdot 10^9 \) and \( A = 112 \text{ kV/m} \).

In the above waveforms it is assumed that the antenna effects on the transmitted pulses are minimal. However this is not quite true; the antenna radiation pattern depends on the frequency and the radiation angle. Therefore, its behaviour will distort the transmitted pulse shape [1,2,30] and is actually approximated as a differentiator both at the transmitter and receiver, i.e. the signal immediately after the receiver antenna will be the second derivative of the signal just before the transmitting antenna [18].

The frequency contents of the aforementioned signals can be found with the help of the Fourier transformation. Hence, the frequency spectrum of the Gaussian pulse is

\[
G_0(f) = \sqrt{\frac{\pi E}{\tau \sqrt{\pi/2}}} \cdot \tau \cdot e^{-(\pi \tau f)^2}.
\]  

(2.5)
Similarly, the representations of the Gaussian monocycle and doublet in the frequency domain are given by

\[ G_1(f) = \sqrt{\frac{\pi \tau}{\sqrt{\pi} \tau}} \cdot (j2\pi f)^2 \cdot e^{-\left(\pi \tau f^2\right)^2} \]  \hspace{1cm} (2.6)

\[ G_2(f) = \sqrt{\frac{\pi \tau}{3 \sqrt{\pi} \tau}} \cdot (j2\pi f)^2 \cdot e^{-\left(\pi \tau f^2\right)^2} \]  \hspace{1cm} (2.7)

and, finally, the spectrum of a damped sine wave is written as

\[ D(f) = \frac{1}{\sqrt{\lambda^2 - 4\pi^2 (f^2 - f_0^2)}} + j4\pi \lambda f \]  \hspace{1cm} (2.8)

The frequency spectra of these waves can be seen in Fig. 2.3. As higher derivatives of the Gaussian pulse are used, the effective bandwidth is increased and the centre frequency is shifted to higher values. In the case of UWB technology, the centre frequency signifies the frequency where the highest power emission occurs, in contrast to narrowband communications where it signifies the carrier modulation frequency. In UWB radio, there is no need for carrier frequency, and this is one of the reasons why UWB systems are considered to be of low-complexity. However, it should be mentioned that this is true when single-band modulation is applied, i.e. the whole available bandwidth is excited simultaneously for every signal. It is possible to divide the frequency spectrum into smaller bands, as it will be shown in the next section, realizing thus a multiband system, where the use of a carrier frequency will be necessary. Finally, it should be also remarked that the frequency spectrum of a Gaussian doublet fits best the FCC mask, and this is the main reason why it is excessively used in simulations.

2.4 Modulation of UWB Radio

The extremely high bandwidth and the exceptionally narrow pulses of UWB signals make the use of conventional narrowband modulation techniques very difficult. For this reason, the concept of modulation in UWB differs slightly with that of narrowband communications; in UWB, modulation is the mapping of a binary data stream into corresponding waveforms, rather than the translation of the signal to a carrier frequency. There are various UWB modulation schemes, but the right choice depends on the application, the design specifications and constraints, range, transmission and reception.
2.4. Modulation of UWB Radio

Figure 2.3: The pulses of Fig. 2.2 are displayed here in the frequency domain. The parameters used are the same as in Fig. 2.2. The spectrums are presented in the dB scale and have normalised magnitudes.

power, quality of service, regulatory requirements, hardware complexity, data rate, reliability of the channel and capacity [16]. Some of the possible modulation schemes are binary phase shift keying (BPSK), pulse amplitude modulation (PAM), on-off keying (OOK), pulse position modulation (PPM), pulse interval modulation (PIM) and pulse shape modulation (PSM). Predominantly, the above modulation schemes are used in a binary communication system, that is there are two distinct and possible states that can be described by the symbol which correspond either to bit 0 or 1. However, higher-order modulation can be achieved (like $M$-ary PAM and $M$-ary PPM), but this implies a compromise between data rate and bit error rate (BER) performance; high-order modulation leads to higher data rate but also to poor BER in noisy channels, whereas with low-order modulation, lower data rates and reliable performance in poor channel conditions can be achieved.

The aforementioned modulation techniques are described as single-band techniques (as well as carrierless or IR signalling), since all the available frequency band is excited simultaneously and the resulted signals resemble a train of impulses. However, the transmission of UWB signals over multiple frequency bands or over multiple carriers
can also be considered. In this case, we are leaning towards carrier based modulation, which is also referred as multiband UWB. Such techniques are (multiband) MB-UWB, MB-OFDM (orthogonal frequency division multiplexing) and (direct-sequence) DS-UWB.

The available modulation options will be discussed in the following sections, starting with the single-band modulation techniques, which is subsequently followed by the proposed multiband UWB modulation options.

2.4.1 Single-Band Modulation Techniques

In single-band modulation, pulses that are spread in the whole available frequency spectrum are used. Each transmitted symbol can be represented by a series of such pulses, so as to achieve low duty cycle and high processing gain. Mathematically, such modulation technique is expressed as

$$s^{(u)}(t) = \sum_{j=-\infty}^{\infty} \sqrt{\frac{E_s}{N_s}} d_j^{(u)} \beta_{[j/N_s]}^{(u)} \beta_{tr}^{(u)} \left( t - jT_f - \delta^{(u)}_{[j/N_s]} \right)$$

(2.9)

where $s^{(u)}(t)$ is the transmitted symbol for the $u$-th user, $j$ is the frame index, $E_s$ is the energy of each symbol and $N_s$ is the number of pulses per symbol. $T_f$ is the nominal time duration between two symbols, i.e. the frame interval, $\delta$ is the time modulation index, if the modulation scheme that is used is PPM, and $T_c$ is the chip duration which is larger than the pulse duration. $\beta_{[j/N_s]}^{(u)}$ and $\delta^{(u)}_{[j/N_s]}$ change the amplitude of the pulses and their time position in the frame, respectively. $\beta_{tr}^{(u)}$ modifies the shape of the pulses and $\lfloor \cdot \rceil$ signifies a floor integer. $d_j^{(u)}$ and $d_j^{(u)}$ are pseudo-random codes unique to user $u$ and are used to employ TH- (time-hopping) and DS-UWB (direct-sequence). Time-hopping randomizes the position of the pulse inside the frame interval, whereas direct-sequence randomizes the polarity of the transmitted pulses. TH and DS techniques are used not only for incorporating more users in the system, i.e. implementing multiple-access (MA), but also for smoothing the strong spectral lines that are the result of the repetition of the pulses in every frame. All the possible single-band modulation techniques can be described by (2.9), by the appropriate control of the relevant parameters. For example, with parameter $\beta_{[j/N_s]}^{(u)}$ OOK, BPSK, PAM, or
2.4. Modulation of UWB Radio

$M$-ary PAM can be employed [24,31–33], whereas by controlling parameter $\delta a^{(u)}_{(j/N_s)}$, PPM, $M$-ary PPM, or PIM can be realised [34–38], and finally by varying the shapes of the pulses through $p^{(s)}_M$, PSM can be achieved [25,27,39].

The most commonly used modulation schemes are BPSK and PPM, which are presented in Fig. 2.4. BPSK modulated signals have their polarity dictated by the transmitted bit, whereas in PPM, one of the two bits is transmitted with a delay $\delta$. This is more evident in Fig. 2.4a, where the transmitted message ([0 1 1 0 1]) is the same for both modulation schemes, and there is one pulse per symbol, i.e. $N_s = 1$. The pulse repetition time $T_f$ is 1.4 ns and the time delay $\delta$ for the PPM case is 0.5 ns. It should be noted that this value of $T_f$ is not a realistic one, as it would lead to a system with a very poor performance, due to the collisions with the received waveforms coming from the different multiple paths of the environment. It was chosen here only for better representing the two modulation options. In a proper UWB system, $T_f$ must be greater than the channel excess delay, so as intersymbol interference (ISI) to be avoided, leading thus to symbols with very low duty cycle. The transmitted pulse which is used for the simulations is a Gaussian doublet, as given in (2.4) with $\tau = 70$ ps. As seen in the figure, bit 1 is transmitted with a delay $\delta$ for PPM, whereas it is transmitted in the beginning of the frame for BPSK modulation. On the contrary, bit 0 forces the symbol to change its polarity for BPSK, whereas the respective PPM symbol remains unchanged. BPSK and PPM, as well as the other modulation techniques, like OOK and PAM, experience strong frequency peaks, due to the repetition of the pulses, as observed in Fig. 2.5a. The frequency spectrums presented in the figure were calculated using numerical fast-Fourier transform algorithm (FFT) on a message which contained 100 bits, with frame interval $T_f = 1.4$ ns and $N_s = 1$. These peaks comprise a great problem, as the mean transmitted power must be decreased, so as not to violate the power spectral masks which are imposed by the different regulatory bodies. Randomisation techniques can be adopted so as to overcome this impediment, with time-hopping (TH) and direct-sequence (DS) being two possible solutions. An example of such a modulation scheme is demonstrated in Fig. 2.4b. Each transmitted bit is consisted of three pulses ($N_s = 3$) and for simplicity only TH-PPM, DS-PPM and TH-DS-PPM are considered. The signature for the TH is $c = [4 \ 4 \ 3]$ and the
2.4. Modulation of UWB Radio

direct-sequence is \( d = [-1 \ -1 \ 1] \). The chip interval is \( T_c = 0.7 \) ns, which is slightly larger than the pulse duration, and the frame time is \( T_f = 3.5 \) ns. TH and DS can also be used for introducing more users \( u \) in the system, by assigning to each user different unique codes \( d^{(u)} \) and \( c^{(u)} \). The frequency contents of these randomised signals exhibit the desired smoothness, especially if DS-PPM or TH-DS-PPM are used (Fig. 2.5b). Again, a message of 100 bits was used for the frequency domain representation.

2.4.2 Multiband Modulation Techniques

Another approach to modulating UWB signals is the partitioning of the available bandwidth into smaller bands of frequencies. Each band needs to be equal or greater than 500 MHz, so as not to violate the FCC regulation. Such a system offers flexibility and a more controllable operation, which is much desired since the UWB ruling is not globally in agreement. Therefore it may be necessary to adjust the excited frequency spectrum by turning some bands on or off, so as to conform with the local laws and the issued power spectral masks. The ability of avoiding strong narrowband interferers is also another positive attribute of multiband modulation, as well as the advanced spectral efficiency, when compared with the single-band modulation options, due to the minimally densely subcarrier spacing.

Multiband modulation techniques involve pulsed-based or multi-carrier-based solutions. Representatives of the former are MB-UWB and DS-UWB, whereas the most known multi-carrier multiband solution is MB-OFDM.

MB-UWB was initially proposed by Intel Corp., before joining the Multiband OFDM Alliance (MBOA) and WiMedia Alliance [40], and it is TD/FDMA approach. The spectrum is divided into bands with bandwidth more than 500 MHz, which is utilised through tailored pulses. The number of the bands can be adjustable, nevertheless the maximum number of bands cannot exceed 15 (7500 MHz/500 MHz). The modulation is achieved by an appropriate concatenation of these pulses, but the decision on the transmission band can also be based on throughput requirements, interference environment etc. The pulses do not interfere with each other, since they occupy different portion of the frequency spectrum, as it can be seen in Fig. 2.6, and hence they are
2.4. Modulation of UWB Radio

Figure 2.4: (a) BPSK and PPM modulated signals. PPM exhibits a time delay $\delta$, whereas BPSK exhibits polarity changes, with respect to the transmitted message. $T_f = 1.4$ ns and $N_s = 1$. (b) Time-hopping and direct-sequence techniques can be applied for randomisation purposes. Here, $N_s = 3$ and $T_f = 3.5$ ns. The codes for the TH is $c = [4 4 3]$ and for the DS is $d = [-1 -1 1]$. Also, TH and DS codes can be used simultaneously for better performance.
2.4. Modulation of UWB Radio

Figure 2.5: The frequency spectrums for the different modulation options. (a) The spectrums exhibit strong spikes which would lead to a decrease to the mean transmitted power. These spikes are closely related with the frame time $T_f = 1.4$ ns, or more correctly with the inverse of that, which is the frame repetition frequency. (b) Randomisation techniques smooth the spectrum, especially if DS-PPM or TH-DS-PPM is implemented. Here, the values $T_f = 3.5$ ns and $N_s = 3$ pulses were used.
orthogonal.

Figure 2.6: A possible realisation of MB-UWB may employ pulses with approximately 700 MHz bandwidth at 10 dB. (a) The message can be formed as a train of such pulses. (b) These pulses are almost orthogonal, contributing very little to the inter-pulse interference.
2.4. Modulation of UWB Radio

MB-OFDM is a multi-carrier modulation technique which is based on the well known OFDM technology, adapted for UWB systems [41–43]. The number of subcarriers that will be used, as well as other parameters like guard intervals and the duration of the cyclic or the zero-padded prefix, is under investigation. In [42], it was proposed the whole available spectrum to be divided into 14 bands, 528 MHz each (Fig. 2.7a). The bands would belong to 5 band groups, where the first band group is mandatory, with the rest being optional. Each band consists of 128 sub-carriers (tones) with 4.125 MHz spacing, of which 100 tones are used for data information, 12 tones for carrier and phase tracking, 10 tones are used for user-definition and the last 6 tones remain idle. The 122 sub-carriers are modulated using quadrature phase-shift keying (QPSK). The main difference of MB-OFDM with a conventional OFDM system is the use of a time-frequency code, from the MB-OFDM behalf, which appoints the band in the band group and the time when the OFDM symbol will be transmitted. This allows for frequency diversity, as well as for multiple access. An example of such a scheme can be seen in Fig. 2.7b. The symbol’s duration is 312.5 ns, which includes 60.6 ns for zero-padding or cyclic prefix and 9.5 ns for guard interval. The duration of the prefix determines the amount of multipath energy that is tolerated, minimizing hence the intersymbol interference (ISI). The guard interval is necessary in order for the transmitter and the receiver to switch between the different frequency bands. MB-OFDM is not an UWB technology with the strict meaning, as it resembles the transmission of 128 narrowband radios. The maximum data rate that can be achieved, according to the proposal in [42], is 480 Mbps.

In contrast to MB-OFDM, DS-UWB is closer to the traditional impulse radio approach [44–46]. It supports two independent bands of operation; the lower band occupies the spectrum between 3.1 and 4.85 GHz, and the higher band occupies the spectrum between 6.2 and 9.7 GHz, with 1.75 and 3.5 GHz bandwidth respectively. The application of the multiband DS-UWB is similar to the single band direct-sequence technique, only that the chip symbols have longer duration, due to the decrease in the signal bandwidth, as well as the use of a carrier wave which may be necessary. The data can be BPSK modulated symbols, in the simple case (mandatory for all devices), or 4BOK modulated ones. Each transmitted symbol is composed by a sequence of UWB pulses.
2.4. Modulation of UWB Radio

Figure 2.7: (a) The frequency spectrum in MB-OFDM has been divided into 14 bands, which belong to 5 band groups. Each band has 528 MHz bandwidth and it is divided into 122 subcarriers. (b) The OFDM symbols are transmitted in different bands of the band group, according to the time-frequency code. The OFDM symbol is 242.42 ns long and 60.6 ns CP and 9.5 ns guard interval have been added. The time-frequency code in this example is [2 3 1].

(1-24 pulses or chips per symbol), depending on the length of the spreading code. In BPSK modulation, the data bit determines the polarity of the transmitted spreading code (Fig. 2.8), whereas in the 4-BOK modulation scheme, the data bit stream is divided into blocks of two bits and each block is mapped to one of the two possible spreading codes with different polarity (positive or negative).

The pulse shape must be tailored in such manner that it fits the regulatory power limits. Such possible pulses can be generated by the modulation of a Gaussian pulse with the appropriate bandwidth in the desired frequency. Therefore, such a signal, with energy $E$, can be written as

$$g(t) = 2\sqrt{\frac{E \cdot B}{\sqrt{\pi}}} e^{-\frac{(t-B)^2}{2}} \cdot \cos(2\pi fct)$$

(2.10)
Figure 2.8: An example of a DS-UWB transmitted symbol. Three BPSK bits are spread by the code $c=[1,1,-1,1]$.

where $B$ is the bandwidth of the signal and $f_c$ is the centre frequency. The different data rates that are supported by this scheme are realised by the use of error correction codes of variable lengths; the proposed highest data rate is 1320 Mbps.

MB-OFDM and DS-UWB were the main technology proposals for standardising UWB systems in the IEEE 802.16 Task Group 3a (TG3a), each of them backed up from different industry alliances. However, TG3a had to be withdrawn, as it was unable to reach into an agreeable from all parts solution and it was left for the market to decide which technology will prevail. This does not imply that every other possible solution is excluded. Every technology regarding UWB is allowed to be deployed, as long as it is compliant with the FCC rules and the issued power spectral mask. Clearly, each solution has its own design challenges and trade-offs, that will be briefly discussed in the following section.

2.5 UWB Receivers

The wireless channel is a collection of multiple copies of the transmitted signal, with each copy having a different time-of-arrival, attenuation and phase. These copies are the result of the flow of the transmitted information via the natural diversity of multiple paths. UWB systems have inherently the ability to finely resolve these multipath components (MPCs) of the channel. This is a consequence of the extremely large band-
Figure 2.9: (a) The frequency spectrum in DS-UWB has been divided into 2 bands; the lower-band from 3.1 to 4.85 GHz and the higher-band from 6.2 to 9.7 GHz, with 1.75 and 3.5 GHz bandwidth at -10 dB respectively. The two pulses occupy different areas of the spectrum and therefore they are orthogonal. (b) The two pulses for DS-UWB. The low-band pulse has longer duration, whereas the energy of the high-band pulse is more confined.
2.5. UWB Receivers

The width of the UWB signals; the larger the bandwidth is, the more multipath components can be resolved. Multipath resolution leads to an elimination of significant fading, as the fading margin can be reduced [47, 48]. However, the design of a receiver that can take full advantage of the multipath diversity is very arduous.

Rake receivers have been initially proposed for spread-spectrum communications [3–5], but they can be also used for UWB reception [6, 7, 49]. They have the ability of tuning themselves to the multiple paths, process and combine them, so as to achieve a correct symbol detection. The number of the MPCs that can be processed is equal to the number of fingers of the Rake receiver. However, since the transmitted power is distributed into many multipaths and given the fine resolution of UWB, it is expected that the power in each multipath will be very low and therefore a large number of fingers will be needed so as to enhance the performance of the receiver. This demands high computational and hardware complexity, which is in contrast to the promise of practical, simple and low-power UWB transceiver design. A viable receiver design would be a trade-off between complexity and performance [7, 49].

Proper Rake operation requires information about the channel, the delays and the attenuation factors of the multiple paths. These parameters are collectively called channel estimation. If these are known a priori, the receiver benefits from the multipath diversity. However, in a real scenario, such knowledge of the channel is not available, and timing offsets, even of the fraction of a nanosecond and pulse distortion can cause performance losses. Timing synchronisation can be attained using the maximum-likelihood (ML) criterion [50, 51], nevertheless this might require sampling the received signal with very fast analogue-to-digital converters (ADC) and thus high power consumption.

An alternative to Rake reception is the data-aided receiver, where pilot waveforms are transmitted periodically so as to optimise the performance of the receiver without the knowledge of the channel [52–54]. In such reception, a segment of the received waveform serves as a template for the demodulation process. However, pilot-aided schemes need perfect timing synchronisation and suffer from reduced data-rates, as some waveforms are used for channel estimation. Dirty templates have also been proposed for timing and channel estimation [55, 56]; they are called dirty because they are noisy, distorted
and they are subjected to an unknown time offset. In such a scheme, the channel estimation is not attained by pilot symbols, but by the previous information symbol and therefore there is no loss in the data-rate, and also it benefits from the symbol-rate sampling, reducing thus the complexity and the power requirements.

2.6 Conclusions

Ultra-wideband radio employs the transmission of signals with very short duration and large bandwidth. However, low power spectral masks have been issued by different regulatory bodies, like FCC and EC, so as to avoid interference with the existing technologies operating in the same frequencies. The advantages of UWB systems can be also considered as disadvantages; the extremely large bandwidth offers multipath diversity and high data-rates, but on the other hand, it leads to complex receiver designs and pulse distortion. The latter is mainly addressed by this thesis, where the effects of the environment on the pulse shape will be studied.
Chapter 3

The Wireless Channel

3.1 Radio Wave Propagation

The radio channel plays a very important role in wireless communications, as it imposes fundamental limitations on their performance. The propagating wave travels multiple different paths before it is received, different not only in length but also in nature; the transmission path can vary from simple line-of-sight (LOS) to one that is severely obstructed. This environmental clutter affects the electromagnetic wave by amending its direction, amplitude, phase and time-of-arrival. However, the radio channel is extremely random and unpredictable, due to its non stationary nature. If such a behaviour is ignored though, then the strength of the electromagnetic field can be fairly predicted, if the propagation mechanisms for the transmitted wave are considered.

The propagation mechanisms reveal the way the electromagnetic wave interacts with the environmental objects and more precisely with the charged particles of the medium. These mechanisms are quite diverse but can generally be attributed to reflection, refraction, scattering and diffraction. These effects are defined by the electrical properties of the material, as they are expressed by its constitutive parameters, which are the electrical permittivity $\varepsilon$ (F/m), the magnetic permeability $\mu$ (H/m) and the conductivity $\sigma$ (S/m). The constitutive parameters control the portion of the electromagnetic wave that is reflected, refracted or diffracted, through the corresponding coefficients. These coefficients also depend on the geometry of the environment, the incident angle and
the operating frequency. For narrowband communications, the frequency dependency is expressed by a complex scalar, which alters the amplitude and the phase of the impinging wave, but not its shape. However, the frequency selectivity of these coefficients has a severe effect in UWB radio, as due to its extremely large bandwidth, portions of the frequency spectrum are affected differently, hence distorting the signal. Not proper consideration of the signal distortion leads to a degradation into the system’s performance.

It is natural to treat such impulse-like phenomena in the time-domain (TD), where all the frequencies are treated simultaneously. All the necessary parameters that need to be calculated in an UWB system, such as the number of multipaths, the delay, the power and the distortion of every path are easily obtained from the TD profile. It also appears to be more efficient to work directly in the TD than applying numerical inverse fast Fourier transform algorithms to convert frequency-domain (FD) solutions into the TD. The latter would involve the calculation of the radio propagation effects for each frequency separately, which is very inefficient for very large bandwidths. Hence, TD is the preferred domain, as long as we can employ simple closed forms for the prediction of the radio wave propagation for every path. The resulted path impulse response is then considered to be a general solution, irrespective of bandwidth, that can be convolved with any transmitted signal. The TD coefficients can then be inserted into channel models, which will thus include the signal distortion, and will present better accuracy, as they will predict more precisely the received field.

In this chapter, the theory behind the inversion techniques is presented. The coefficients for the propagation mechanisms, reflection and diffraction, for single interaction occurrences are introduced in the frequency- and the time-domain. The accuracy of the TD solutions is compared against the numerical inverse Fourier transform of the corresponding coefficients, as these are expressed in the frequency domain. Finally, refraction and scattering are also discussed, for completeness on the wireless propagation mechanisms.
3.2 Inverse Techniques

In order to find the TD solution when the FD solution is known, the inverse Fourier transform integral can be used [57], i.e.

\[ f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)e^{i\omega t} d\omega \]  

(3.1)

where \( f(t) \) is the TD solution, \( F(\omega) \) is the FD representation and \( \omega \) is the angular frequency. It is possible to avoid integrating in the negative frequencies by taking the one-sided inverse Fourier transform, which is defined as

\[ \tilde{f}(t) = \frac{1}{\pi} \int_{0}^{\infty} F(\omega)e^{i\omega t} d\omega \]  

(3.2)

where \( \tilde{f}(t) \) is the analytic (complex) TD solution. Since the interaction mechanisms are causal functions, i.e. they are zero for negative times, the real part of \( F(\omega) \) must be an even function and its imaginary part must be an odd one. Then, the analytic TD solution, as it is calculated in (3.2), can be written as [57]

\[ f(t) = \Re\{\tilde{f}(t)\} \]  

(3.3)

where \( \Re \) represents the real part of the analytic TD solution. Therefore, the real part of the analytic TD solution in (3.2) corresponds to the desired TD function.

In most cases, it might be easier to find the TD solution by using inverse Laplace transform, as the latter has been extensively studied by researchers and there is a plethora of Laplace-pair tables in the literature [58]. The translation of the integral in (3.1) into an inverse Laplace one can be achieved by substituting \( j\omega \) with \( s \). However, for a correct translation, the poles of the new Laplace function \( F(s) \) should lay on the left-hand side of the imaginary axis of the \( s \)-plane, otherwise the solutions will not converge as \( t \) tends to infinity.
In order to determine the value of $f(0)$, the integrals in (3.1) or (3.2) must be evaluated for $t = 0$. However, in many cases the evaluation of the above integral might not be feasible for the whole interval $[0, \infty)$. Instead, without loss of generality, a maximum angular frequency $\omega_{\text{max}}$ may be introduced, which needs to be greater than the maximum angular frequency of the transmitted signal. In this case, the integral can be evaluated either in closed form or numerically.

### 3.3 Reflected Waves

When an electromagnetic wave propagating in a medium impinges upon another medium having different electrical properties, the wave is partially reflected and partially transmitted. If the boundary surface has dimensions much greater than the wavelength of the propagating wave, is smooth and the point of incidence is not close to the edges of the surface, then the electric field intensity of the reflected and transmitted wave may be related to the incident wave in the medium of origin through the Fresnel amplitude reflection coefficient $R$. The amplitude reflection coefficient depends on the electrical properties of the two media involved, the angle-of-incidence, the frequency and the polarisation of the wave. The polarisation describes the orientation of the oscillation of the electric and magnetic field, which for an electromagnetic wave belongs to a plane normal to the direction of the wave, i.e. electromagnetic waves are transversal waves. Polarisation plays a very important role in phenomena related with the wave propagation.

The Fresnel reflection coefficient $R$ applies for waves impinging on the boundary surface that separates two linear, isotropic, homogeneous media with different electrical properties (Fig. 3.1). The incident, reflected, refracted wave vector, along with the normal to the boundary vector, lie on a plane, that is termed as the plane-of-incidence. In that aspect, the waves can be either hard polarised (i.e the electric fields lie parallel to the plane-of-incidence) or soft polarised (i.e. the electric fields are normal to the plane-of-incidence). Any other possible polarisation can be analysed to the previous two vertical components.
3.3. Reflected Waves

![Figure 3.1: The reflected and transmitted fields for an incident electric field, (a) with soft polarisation, and (b) with hard polarisation. The dotted circle signifies a vector with direction outwards the page plane. E is the electric field, B is the magnetic field, and \( s \) is the direction vector.]

3.3.1 Half-plane Reflection

By applying the boundary conditions on the incident, reflected and refracted waves, the laws for reflection and refraction can be derived. Therefore, if \( \theta_i \), \( \theta_r \), and \( \theta_t \) are the incident, reflected and refracted angles with respect to the normal vector to the boundary plane, then it is [59]

\[
\theta_i = \theta_r \quad (3.5)
\]

\[
n_1 \sin(\theta_i) = n_2 \sin(\theta_t) \quad (3.6)
\]

where \( n_i \) is the refractive index of the medium, with \( n_i = \sqrt{\varepsilon_r i \cdot \mu_r i} \), and \( \varepsilon_r i \), \( \mu_r i \) are the relative permittivity and permeability of the matter respectively. The Fresnel reflection coefficients \( R_{s,h} \), for soft and hard polarised waves are [60]

\[
R_a = \frac{n_1 \cos \theta_i - n_2 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t} \quad (3.7)
\]

\[
R_h = \frac{-n_1 \cos \theta_i + n_2 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t} \quad (3.8)
\]

If a dielectric material is lossy, it will absorb energy and its electrical properties may
be described by a complex dielectric constant given by

\[ \epsilon_{r,\text{com}} = \epsilon_r - j \frac{\sigma}{\omega \epsilon_0} \]  

(3.9)

where \( \omega \) is the angular frequency and \( \epsilon_0 \) is the permittivity of the free space, with \( \epsilon_0 = 8.85 \cdot 10^{-12} \text{ F/m} \). In general, \( \epsilon_r \) and \( \sigma \) can be considered independent of frequency, a fair approximation for common materials that are used in buildings, like bricks, wood, glass etc., and the frequency range of UWB, but not true when considering on-body propagation [61]. Therefore, assuming that the first medium is free space (\( \epsilon_r = 1 \), \( \sigma = 0 \)) and also \( \mu_r,1 = \mu_r,2 = 1 \), the amplitude reflection coefficients for soft and hard polarisation are given by

\[ R_s = \frac{\cos \theta - \sqrt{\epsilon_{r,\text{com}} \sin^2 \theta}}{\cos \theta + \sqrt{\epsilon_{r,\text{com}} \sin^2 \theta}} \]  

(3.10)

\[ R_h = \frac{\epsilon_{r,\text{com}} \cos \theta - \sqrt{\epsilon_{r,\text{com}} \sin^2 \theta}}{\epsilon_{r,\text{com}} \cos \theta + \sqrt{\epsilon_{r,\text{com}} \sin^2 \theta}} \]  

(3.11)

If the second medium is a perfect dielectric (lossless), then its conductivity is zero, and therefore the reflection coefficients are independent of frequency, as it can be deduced by (3.9), (3.10) and (3.11). On the other hand, perfect conductors have infinite value of \( \sigma \), and hence \( R_{s,h} = \mp 1 \) [60], which is again independent of frequency. However, the materials that are used in a normal environment are neither perfect dielectrics nor perfect conductors, and thus the coefficients in (3.10) and (3.11) depend, among other parameters, on frequency. In Fig. 3.2, the magnitudes of the reflection coefficients for soft and hard polarisation with respect to \( \sigma \) and \( f \) are plotted. As it is witnessed, an increase in the conductivity \( \sigma \) corresponds to a dependency upon the operating frequency. In UWB radio, this dependency is translated into signal distortion. Also, it should be noted that soft polarised waves are more resistant to reflection than hard polarised ones.

### 3.3.2 Half-plane in the Time-Domain

The reflection coefficient for the half-plane (ground reflection) in the time-domain was initially treated in [8] and in [62–64]. The derivation is the TD coefficient is based on
3.3. Reflected Waves

Figure 3.2: The frequency response of the Fresnel reflection coefficients (a) for soft and (b) hard polarised waves with respect to the conductivity and the frequency. The dielectric constant is $\varepsilon_r = 5$ and the incident angle $\theta = \pi / 3$.

the inverse Laplace transform of the Fresnel amplitude reflection coefficients, as these are given in (3.10) and (3.11). The $j\omega$ is substituted with $s$ and the inverse operation is performed. Thus, the time-domain representation of the reflection coefficient is given in (Appendix A)

$$r_{s,h}^N(t) = \pm \left[ K_{s,h} \delta(t) + \frac{4\kappa_{s,h}}{1 - \kappa_{s,h}^2} e^{-\alpha t} \sum_{k=0}^{N-1} f_k \right]$$  \hspace{1cm} (3.12)

where $s,h$ represent soft or hard polarisation, $K_{s,h} = \frac{1 - \kappa_{s,h}}{1 + \kappa_{s,h}}$, $\kappa_s = \frac{\cos \theta}{\sqrt{\varepsilon_r - \sin^2 \theta}}$ and $\kappa_h = \sqrt{\varepsilon_r - \sin^2 \theta}$, $\alpha = \frac{\sigma}{2\pi\tau_0}$ and the plus sign corresponds to hard polarisation whereas the minus sign to soft polarisation. Finally, $N$ represents the number of terms of $f_k$ that are needed to be taken account, with

$$f_k = \frac{(-\alpha t/2)^k}{tk!K^k} \left[ (A + k)X + \sum_{\ell=0}^{k-1} \frac{(\ell-k)(-A)^\ell}{\ell!} \right]$$  \hspace{1cm} (3.13)

where $A = K\alpha t/2$ and $X = e^{-A}$. However, for typical values of $\sigma$ (e.g. $\sigma = 0.05$ S/m) and $\varepsilon_r$ (e.g. $\varepsilon_r = 5$), the above sum can be approximated with only the first term of $f_k$ (Fig. 3.3) [63]. This approximation though is not valid for late times, as it will introduce great errors in the prediction [8]. Nevertheless, this is not the case for UWB signals, due to their short temporal duration. Therefore, the reflection coefficient for
the half-plane, for soft or hard polarised waves is given by

\[ r_{s,h}(t) = \mp \left( K_{s,h} \delta(t) + K_{s,h} \frac{2K_{s,h} \alpha}{1 - K_{s,h}^2} e^{-(1+K_{s,h}/2)\alpha t} \right). \]  

(3.14)

**Figure 3.3:** The approximation of the sum for the TD reflection coefficient. The lines represent the approximation of the sum with the first, the second or the third term, as well as the actual sum of \( f_k \). Here, \( f_{N-1}^{\text{norm}} = \frac{\sum_{n=0}^{N-1} f_k}{\max(\sum_{n=0}^{N-1} f_k)} \) is the normalised sum and \( e^{-\alpha t} f_{N-1}^{\text{norm}} \) is plotted.

Equation (3.14) is used for the prediction of a reflected signal. The initial pulse is a Gaussian doublet with \( \tau = 70 \) psec. The total path distance that the signal has traversed is 1.2 m. The shapes of the received waveforms are plotted in Fig. 3.4 for soft and hard polarised waves. The signals are normalised with the maximum absolute value of the two received waveforms, the \( r_{\text{FFT}}(t) \) and the \( r_{\text{TD}}(t) \), in each case. In the upper set of graphs, \( \sigma \) is set to zero. In that case it is expected, following the discussion in the previous section, that the signal will be undistorted. Indeed, such a behaviour is observed and also the TD predicted shape agrees extremely well with the inverse Fourier transform of the received signal as this is given in the frequency domain. The dielectric constant of the reflective surface is \( \varepsilon_r = 5 \) and the angle-of-incidence is \( \theta = 45^\circ \). If the conductivity is increased, as in the middle pair of graphs where \( \sigma = 0.1 \) S/m, pulse distortion is observed, as it was though expected, since the reflection coefficient is then frequency selective. Finally, for a large angle \( \theta \), as in the last pair of graphs in Fig. 3.4
3.3. Reflected Waves

Figure 3.4: The comparison of normalised reflected signals, when computed directly in the time-domain and numerically inverse Fourier transform the frequency-domain solutions. Soft polarised signals are presented on the left, whereas hard polarised signals are presented on the right. In (a) and (b) the dielectric constant is $\epsilon_r = 5$ and the conductivity is $\sigma = 0$, while the angle-of-incidence is $\theta = 45^\circ$. In (c) and (d), $\sigma$ is increased, so as $\sigma = 0.1$ S/m. In (e) and (f), $\epsilon_r = 5$, $\sigma = 0.1$ S/m and $\theta = 85^\circ$. 
3.4. Diffraction theory

where $\theta = 85^\circ$, the prediction of the received waveform in the TD agrees extremely well with the inverse Fourier transform of the FD solution. Therefore, the approximation on the sum of (3.13), where only the first term is considered, is a valid one, mainly due to the short duration of the UWB pulse. In the latter example, a noteworthy remark as well is the reverse polarity of the hard polarised waveform with respect to the polarities of the previous examples, which however was anticipated by the propagation theory. In all the cases, an error of less than 0.05% is observed, which proves the validity of the TD solutions.

3.4 Diffraction theory

Diffraction is the most important propagation mechanism for radio propagation as it is the main reason for the existence of electromagnetic energy in shadowed regions, that are regions where the direct ray is obstructed, and the most difficult to model and predict. Diffraction characterises the deviation of wave rays from rectilinear propagation, and this effect is a general attribute of wave phenomena occurring whenever a portion of a wavefront is obstructed in some way, i.e the waves 'bend' around the object. Closed form solutions describing such complex events are seldom, however some rigorous solutions of a small number of diffraction problems have been found, like the diffraction of a plane wave by a perfectly conducting semi-infinite plane screen which was introduced by Sommerfeld in 1896. Nonetheless, due to mathematical difficulties, most practical cases are approached by approximate methods.

Diffraction theory in physical or wave optics is based on Huygens-Fresnel principle, which states that every unobstructed point of a wavefront, at a given instant, serves as a source of spherical secondary wavelets (with the same frequency as that of the primary wave). The amplitude of the optical field at any point beyond is the superposition of all these wavelets (considering their amplitudes and relative phases) [65]. These secondary wavelets cause the presence of electromagnetic energy in shadowed regions. Under Huygens principle and the solution of Fresnel-Kirchhoff integrals, when applied on a
3.4. Diffraction theory

semi-infinite absorbing plane, its path-loss due to diffraction effects is given by [60]

\[ L = \left( \frac{1 + j}{2} \right) \int_{\nu}^{\infty} e^{-j \frac{\pi}{2} \nu \frac{d}{h}} d\nu \]  

(3.15)

where \( \nu \) is the Fresnel diffraction coefficient, which for \( d_1, d_2 \gg h \) (Fig. 3.5) is given by

\[ \nu = \theta \sqrt{\frac{2d_1d_2}{\lambda(d_1 + d_2)}}. \]  

(3.16)

The diffraction phenomena are independent of the environment involved, i.e. indoor or outdoor. The same principles apply for both situations. Therefore, the knife-edge in Fig. 3.5 can be envisioned as a partition diffraction incidence, where the formula of (3.15) can be equally applied.

![Figure 3.5: The direct path between transmitter and receiver is obstructed by a knife-edge. The attenuation of the received field is given by (3.15), assuming that the ground reflections are ignored.](image)

3.4.1 Geometrical/Uniform Theory of Diffraction

However, diffraction theory based on the wave approach is very difficult to apply on complex geometrical obstacles, as it may be too computationally demanding. A solution for simplifying this aspect is to consider the waves as rays, similar to the reflection or refraction cases. Geometrical Optics (GO) though, which apply in the limiting case when \( \lambda \) tends to zero, fail to describe such phenomena. An attempt to compensate for the shortcomings of Geometrical Optics was made through the Geometrical Theory of Diffraction (GTD), which is a high frequency analysis technique developed by Keller [66]. In contrast with geometrical optics, in GTD diffracted rays do exist and they are produced whenever a structure causes a discontinuity in a GO field by creating
shadow regions. Based on a generalised Fermat’s principle, Keller formulated a law of diffraction that determines the location of a diffraction point and the direction of propagation of a diffracted ray. Following Sommerfeld's solution which indicates that the diffracted rays propagate along parallel cones with the edge as their common axis, this law states that a diffracted ray and the corresponding incident ray make equal angles with the edge at the point of diffraction, provided they are at the same medium. They lie on opposite sides of the plane normal to the edge at the point of diffraction. Hence, one incident ray results into infinite number of diffracted rays, all lying on the diffracted cone and propagating to all possible directions (Fig. 3.6a). The total field at a specific position is the summation of all the possible rays.

\[
E_d(s) = E_i(Q) \cdot D \cdot A(s) \cdot e^{-jks}
\]  

where \(E_i(Q)\) is the incident field at the diffraction point \(Q\), \(D\) is the diffraction coefficient, which incorporates the diffraction effects, \(A(s)\) is the spreading factor and \(e^{-jks}\) is a phase term. The spreading factor \(A(s)\) describes the variance of the amplitude of the field along the diffracted ray and it is given by \([12]\) 

\[
A(s) = \begin{cases} 
\frac{1}{\sqrt{s_1}} & \text{for plane wave incidence,} \\
\sqrt{\frac{s_0}{s_1 (s_0 + s_1)}} & \text{for spherical wave incidence.}
\end{cases}
\]
3.4. Diffraction theory

Geometrical Theory of Diffraction suffers from a number of problems itself, most notably the failure in predicting the field near and on the shadow boundaries (Fig. 3.6b), where the predicted diffracted field becomes infinite. This inconsistency is corrected by the Uniform Theory of Diffraction (UTD) which was proposed in [12], and it is accomplished by multiplying the diffraction coefficient with a transition function which approaches zero at the same rate as the diffraction coefficient becomes singular at the shadow boundaries. The suggested solution then offers a smooth (uniform) field prediction.

The application of UTD in diffraction problems is translated into finding the appropriate diffraction coefficients $D$. For an absorbing knife-edge problem, this coefficient is given by

$$D_{\text{knife-edge}}(\alpha) = \frac{-e^{-j\pi/4}}{2\sqrt{2\pi k \cos(\alpha/2)}} F \left[ 2kL \cos^2(\alpha/2) \right]$$  \hspace{1cm} (3.19)

where $\alpha \equiv \phi - \phi'$ (Fig. 3.7a), $k$ is the wavenumber and $L$ is the distance parameter given by

$$L = \frac{s_0 \cdot s_1}{s_0 + s_1}$$  \hspace{1cm} (3.20)

$s_0$ is the distance between the source point and the diffraction point and $s_1$ is the distance between the diffraction point and the field point. Finally, $F$ is the transition function which is written as [67]

$$F[X] = 2j\sqrt{X}e^{j\pi X} \int_{\sqrt{X}}^\infty e^{-ju^2}du$$  \hspace{1cm} (3.21)

The diffraction coefficient for a nonperfectly conducting wedge is given by [12, 68, 69]

$$D_{\text{wedge}}^{n,h} = R_{o,n,h}D_1 + D_2 + R_{o,n,h}D_3 + R_{n,h}D_4$$  \hspace{1cm} (3.22)

where $R_{o,n,s,h}(\psi)$ is the Fresnel reflection coefficient for the $o$- and $n$-face for soft and hard polarised waves respectively, as it is given in (3.10) and (3.11). The soft and hard polarisation cases are defined with respect to the plane-of-incidence, which is the plane that contains the vectors $\vec{s}_0$ and $\vec{e}$ in Fig. 3.6a, and the plane-of-diffraction, the plane which contains the vectors $\vec{s}$ of every diffracted ray and $\vec{e}$. As it can be seen, there is an infinite number of diffraction planes. The incident and diffracted field can then be depolarised into two components, one that lies parallel to the corresponding plane,
3.4. Diffraction theory

Figure 3.7: Diffracted rays according to UTD, for (a) knife-edge and (b) wedge. The angles $\phi$ and $\phi'$ are measured from the $o$-face.

the soft component, and one that is perpendicular to that, the hard component. The argument $\psi$ for the $o$-case is $\pi/2 - \phi'$, whereas for the $n$-case is $\psi = \pi/2 - n\pi + \phi$, with $n = 2 - \alpha_{int}/\pi$ and $\alpha_{int}$ is the internal angle of the wedge (Fig. 3.7b). If the wedge is a perfect conductor, then $R_{s,h} = \mp 1$. The coefficients $D_i$ are given by

$$D_i = \frac{-e^{-i\pi/4}}{2n\sqrt{2\pi k}} \cot(\alpha_i)F[x_i]$$

(3.23)

where $\alpha_1 = \frac{1}{2n}[\pi + (\phi - \phi')]$, $\alpha_2 = \frac{1}{2n}[\pi - (\phi - \phi')]$, $\alpha_3 = \frac{1}{2n}[(\pi - (\phi + \phi')]$, $\alpha_4 = \frac{1}{2n}[\pi + (\phi + \phi')]$ and $x_i = 2kn^2\sin^2(\alpha_i)$ for $i = 1, 2, 3, 4$.

The diffraction coefficients are characterised from a frequency selectivity and their response reduces as the frequency increases, which is in agreement with the geometrical/uniform theory of diffraction (Fig. 3.8). Diffraction coefficients for convex surfaces can also be found in [70], but such obstacles are not considered in the thesis.
3.4. Diffraction theory

3.4.2 Time-domain UTD

The representation of diffraction phenomena in the time-domain can be achieved by the inverse transform of the frequency-domain diffraction coefficients, as these are given by the uniform theory of diffraction. UTD is the preferred diffraction theory due to its simplicity and ray interpretation. Thus, TD-UTD solutions are produced, that are valid for early-to-intermediate times, since UTD is an asymptotic solution in the frequency domain that remains accurate for medium-to-high frequencies. Similar results can be obtained if the spectral theory of transients (STT) is utilised, which is valid for all times [71]. However, STT is applicable only to some special canonical geometries, whereas TD-UTD can be applied to more general and complex geometries, with less accuracy for late times though, as suggested in [10]. Nonetheless, since the TD-UTD solutions are implemented for UWB signals, which are characterised by short temporal durations, or equally stated, the UWB band is in the frequency band where UTD approximations apply, hence, it is expected that the inaccuracy in the late times will not affect the received pulse’s shape prediction.

In the UTD formulation, the received waveform in the TD, for a pulse that has been
3.4. Diffraction theory

diffracted on an edge, is given by

\[ E_d^Q(t) = A(s)E_i^Q(t) \ast d(t) \ast \delta(t - s/c) \]  \hspace{1cm} (3.24)

where \( E_i^Q(t) \) is the incident waveform in the time-domain at the diffraction point \( Q \) and \( E_d^Q(t) \) is the diffracted waveform that is \( s \) meters away from \( Q \). \( d(t) \) is the diffraction coefficient in the time-domain and \( A(s) \) is the spreading factor, which is given in (3.18). \( \delta(t) \) is Dirac’s delta-function, \( c \) is the speed of light which equals to \( c = 3 \cdot 10^8 \) m/s and the asterisk * denotes convolution.

For an absorbing knife-edge, the diffraction coefficient in the frequency-domain is given in (3.19). The transition function is related with the complementary error function as (Appendix B)

\[ F[s] = \sqrt{j\pi se^{j\pi}s} \text{erfc}(\sqrt{j\pi}s). \]  \hspace{1cm} (3.25)

Hence, by using the Laplace pair [58]

\[ L^{-1}\{e^{\pi\text{erfc}(\sqrt{s})}\} = \frac{\sqrt{\pi}}{\pi\sqrt{t}(t + \alpha)} \]  \hspace{1cm} (3.26)

where \( L^{-1}\{\cdot\} \) represents the inverse Laplace transform, the TD diffraction coefficient for the knife-edge case is given by

\[ d_{\text{Knife-edge}}(t) = -\frac{L}{\pi\sqrt{2c}} \frac{\cos(\alpha/2)}{\sqrt{t + \gamma}} \cdot u(t) \]  \hspace{1cm} (3.27)

where \( L \) is the distance parameter, \( \gamma = 2L\cos^2(\alpha/2)/c, \alpha \) is given in (3.19) and \( u(t) \) is the Heaviside step function.

Similar to the knife-edge diffraction incidence and following the UTD diffraction coefficient formulation for a wedge obstacle, as this is given in (3.22), the general TD wedge diffraction coefficient \( d_{\text{Wedge}}^{s,h}(t) \), for soft or hard polarisation, is given by

\[ d_{\text{Wedge}}^{s,h}(t) = r_{o,s,h}(t) \ast r_{ns,h} \ast d_1(t) + d_2(t) + r_{o,s,h}(t) \ast d_3(t) + r_{ns,h} \ast d_4(t) \]  \hspace{1cm} (3.28)

where \( r_{o,n,s,h}(t) \) is given in (3.14), and \( d_i(t) \), for \( i = 1, \ldots, 4 \), is given in [10], which can be reformulated in a simpler form as

\[ d_i(t) = -\frac{nL}{2\pi\sqrt{2c}} \frac{\sin(2\alpha_i)}{\sqrt{t + \gamma_i}} \]  \hspace{1cm} (3.29)

where \( \alpha_i \) can be found in (3.23) and \( \gamma_i = 2Ln^2\sin^2(\alpha_i)/c. \)
3.5 Scattering

The Fresnel reflection coefficient applies for smooth, flat surfaces, where specular (mirror-like) reflection occurs. However, the non-uniformity of the reflective surface may diffuse the reflected energy into all directions, reducing thus its field at the expected path (Fig. 3.10). This behaviour is termed scattering and it is related with the surface protuberances. Similar behaviour can be experienced in the reflection of an object on a water surface; when the water is still, the reflection is specular, whereas the roughness of the water creates a blurred reflected image due to the diffuse reflection.

In an indoor environment, the surfaces involved may be considered smooth when compared with the signal’s wavelength, therefore TD solutions for the scattering case are not available. However, it is presented here in the frequency-domain for the sake of completeness.

The surface roughness is tested using the Rayleigh criterion, which defines a critical height \( h_c \), for a given angle-of-incidence \( \theta_i \). This critical height is given by [60]

\[
h_c = \frac{\lambda}{8 \sin \theta_i}
\]

where \( \lambda \) is the wavelength of the impinging wave. A surface with maximum to minimum height difference \( \Delta h \) smaller that \( h_c \) is considered smooth, whereas the opposite occurs
3.6 Conclusions

The basic propagation mechanisms were presented in this chapter. These mechanisms involve the reflection, diffraction and scattering of the traversing ray. The impact of these processes on the impinging wave is governed not only from the geometry and the material of the obstructing body, but from the frequency as well. This frequency dependency affects greatly an UWB signal, as it leads to signal distortion. However, this signal distortion can be directly predicted in the TD, if appropriate TD coefficients are used. In the next chapter, the impact of multiple interactions on the transmitting pulse is presented and new time-domain solutions to encompass these effects are introduced. These multiple interactions would involve the reflection and refraction of waves on walls with finite width and multiple diffraction phenomena.
Chapter 4

Multiple Time-Domain Interactions

4.1 Multiple-Interactions

In the previous chapter, the basic propagation mechanisms in frequency- and time-domain were presented. However, these mechanisms are applied for single interaction incidences, where the traversing wave is reflected, diffracted or refracted once before it is received. In a real environment though, the propagating ray may undergo more than one interaction. Such phenomena are characterised in the frequency-domain as the multiplication of the corresponding coefficients, which are translated in the TD as convolutions. Nevertheless, it may be more efficient if general solutions, which describe such occurrences directly in the TD, are found.

In the first part of this chapter, the reflection and transmission coefficients for describing the multiple internal reflections in a layer model (slab) in the frequency-domain are considered. These coefficients are translated into the time-domain into simple and easy-to-use solutions, being able to accurately predict a large number of the internally reflected pulses. They are also characterised by a very good agreement when compared with the numerical inverse Fourier transform of the corresponding FD solutions. In the second part of the chapter, an algorithm for predicting multiple diffraction phenomena
4.2. Layer Model

The analysis of the boundary model in Chapter 3 is a general approach concerning the reflection of a propagating wave on the interface of two materials with different constitutive parameters. However, the boundary model can not accurately predict the reflected field on a real environment, as it oversees the natural dimensions of the media involved. This discrepancy can be upheaved if the layer model is used. In such a model, the space separating two media is occupied by a layer material, in where the refracted wave is again partially reflected and transmitted upon meeting the end interface of the layer medium, resulting in successive multiple internal reflections (Fig. 4.1). The layer model is a good approximation for describing slabs, walls and in general alike obstacles in the environment.

Figure 4.1: The incident wave is partially reflected and partially transmitted. Due to the finite width of the wall though, internal reflections occur, resulting in multiple reflected and transmitted rays. The total reflected and transmitted field is the summation of all the individual waves and can be assumed to follow the designated paths. The wall extends infinitely on the z-direction.

In the TD is also presented. The algorithm is based on a novel slope-UTD solution, that has been proven to provide with accurate field predictions with less computation time, when compared against other multiple diffraction solutions. Again, the comparison against the numerical inverse Fourier transform of the corresponding FD solutions verifies the validity of the method.
4.2. Layer Model

4.2.1 Frequency-Domain Approach

The total reflected and transmitted fields are the summation of all these multiple interacting waves. If we assume that all the reflected and transmitted energy follows a single path (Fig. 4.1), an approximation that is quite valid for small angles-of-incidence and a small wall width \( d \), and the outer media are air (air-wall-air model), then the reflection and transmission coefficients, \( R^d_{s,h} \) and \( T^d_{s,h} \), for the boundary model, assuming that the medium is made of a linear, homogeneous material with smooth surfaces, are given by \([72,73]\)

\[
R^d_{s,h} = \frac{1 - e^{-j2\delta}}{1 - R^2_{s,h} e^{-j2\delta}} R_{s,h}
\]

\[
T^d_{s,h} = \frac{(1 - R^2_{s,h}) e^{-j(\delta-kd)}}{1 - R^2_{s,h} e^{-j2\delta}}.
\]

where \( \delta = kd\sqrt{\epsilon_{r,com} - \sin^2 \theta_i} \), \( k = 2\pi/\lambda \), \( \lambda \) is the wavelength of the excited wave and \( \epsilon_{r,com} \) is the complex dielectric constant given by (3.9). \( R_{s,h} \) is the Fresnel reflection coefficient for the half-plane and \( s,h \) signify soft or hard polarisation. Finally, \( \delta \) is the width of the wall and \( \theta_i \) is the angle-of-incidence with respect to the normal to the boundary surface vector.

The superposition of the multiple waves causes constructive and destructive interference. The internally reflected waves can be in-phase or out-of-phase, depending on the width of the wall, the angle-of-incidence and the constitutive parameters. This behaviour is translated into the total reflection and transmission coefficients frequency response as deep fades, which are more evident in Fig. 4.2a and b. Here, the reflection and transmission coefficients for soft polarisation are presented with respect to the frequency and the dielectric constant. The width of the wall \( d \) is 15 cm and the incident angle \( \theta \) is \( \pi/3 \), while the conductivity \( \sigma \) is set to zero. However, as \( \sigma \) increases, the deep fades diminish (Fig. 4.2c,d), since the wall absorbs some of the wave's energy in every internal reflection and therefore the subsequent waves contribute less to the total received field. In these graphs, the dielectric constant \( \epsilon_r \) is set equal to 5, while the width of the wall and the incident angle are kept the same as before \((d = 15 \text{ cm} \text{ and } \theta = \pi/3)\). For higher values of \( \sigma \), a smooth response is witnessed (Fig. 4.2e,f), which though is anticipated, as the behaviour of the wall starts resembling the behaviour of a
Figure 4.2: The reflection and transmission coefficients for soft polarised waves with angle-of-incidence $\theta = \pi/6$ and for a wall with width $d = 15$ cm. In (a) and (b) the coefficients are plotted with respect to the dielectric constant and frequency, for $\sigma = 0$. In (c) and (d), the dielectric constant is kept equal to 5, varying the conductivity and the frequency. In (e) and (f) the responses of the coefficients are presented against the frequency, for certain values of $\sigma$. 
nonperfect conductor. It should also be noted, that in the latter case, the transmission
coefficient tends to zero, as the refracted wave is very weak and most of its energy is
absorbed by the intermediate medium. Similar results can be obtained if hard polarised
waves are considered.

4.2.2 Time-Domain Approach

Time-domain solutions for the reflection and transmission through a dielectric slab were
presented in [74] and [63]. However, in the former solution it was assumed that only a
single ray was transmitted through the slab, whereas in the latter, a limited number of
internal reflections are considered, as the complexity of the solution is greatly increased
with the number of the internal reflections. New and easy-to-use TD solutions for the
reflection and transmission through a slab are introduced in [75]. The number of the
internal reflections that need to be taken into account can be easily calculated by the
slab's constitutive parameters, its width and the angle of incidence. It is shown that, in
some cases, even the third or fourth internally reflected signal carries significant energy.
The above TD solutions are convolved with the excited waveform and the resulted
signals are compared with the numerical inverse Fourier transform of the solutions as
they were calculated in the FD. The comparison shows an extremely good agreement.

The reflection coefficient for a slab with thickness $d$ is given in (4.1). It can be written
as a three-term product, and thus the inverse Laplace transform will be the convolution
of these, i.e.

$$r_{s,h}^d(t) = \mathcal{L}^{-1} \left\{ \frac{1}{1 - R_{s,h}^2 e^{-j2\delta}} \right\} * \mathcal{L}^{-1} \left\{ 1 - e^{-j2\delta} \right\} * \mathcal{L}^{-1} \{ R_{s,h} \} \quad (4.3)$$

where the asterisk $*$ denotes convolution, and $\mathcal{L}^{-1}\{\cdot\}$ represents an inverse Laplace
transformation. The last term of (4.3) equals to the TD reflection coefficient for the
half-plane $r_{s,h}(t)$, as this was calculated in (3.14). Using the same approximation as
in [8], i.e. $\alpha/s \gg 1$, where $\alpha = \sigma/(2\varepsilon_0 \varepsilon_r)$ and $s = j\omega$ (which approximation is true for
dielectric materials and high frequencies), it can be written that $j2\delta \approx \gamma(s + \alpha)$, where
$$\gamma = (2d/c)\sqrt{\varepsilon_r - \sin^2 \theta},$$
and $c$ is the speed of light in free space. Thus,

$$\mathcal{L}^{-1} \left\{ 1 - e^{-j2\delta} \right\} = \delta(t) - e^{-\gamma \delta}(t - \gamma). \quad (4.4)$$
4.2. Layer Model

Since $s \gg \alpha$, then $\sqrt{s^2 - \alpha^2} \approx s$, and therefore, if we use (A.11) in Appendix A, keeping only the first term of the sum and using binomial expansion, the reflection coefficient at the denominator can be written as

$$R_{s,h}^2(s - \alpha) \approx K_{s,h}^2 \left( 1 + \frac{4\kappa_{s,h}}{1 - \kappa_{s,h}^2} \frac{\alpha}{s} \right)$$  \hspace{1cm} (4.5)

where $K_{s,h} = (1 - \kappa_{s,h})/(1 + \kappa_{s,h}), \kappa_a = \cos \theta / \sqrt{\epsilon_r - \sin^2 \theta}$, and $\kappa_h = \sqrt{\epsilon_r - \sin^2 \theta}/(\epsilon_r \cdot \cos \theta)$. The first term in the right hand side of (4.3) can then be expanded into series:

$$K_{s,h}^2 \left( 1 + \frac{4\kappa_{s,h}}{1 - \kappa_{s,h}^2} \frac{\alpha}{s} \right) e^{-\gamma s}$$

$$\approx \sum_{n=0}^{\infty} \left( K_{s,h}^{2n} e^{-n\gamma s} + K_{s,h}^{2n} \frac{4\kappa_{s,h} n \alpha}{1 - \kappa_{s,h}^2} e^{-n\gamma s} \right).$$  \hspace{1cm} (4.6)

The identities $f(t) = e^{-at} \mathcal{L}^{-1}\{F(s - \alpha)\}$ and $f(t) \cdot \delta(t - t_0) = f(t_0) \cdot \delta(t - t_0)$ may be used and thus (4.6) can be represented in the TD by

$$h_R(t) = \sum_{n=0}^{\infty} \left( K_{s,h}^{2n} e^{-n\gamma s} \delta(t - n\gamma) + K_{s,h}^{2n} \frac{4\kappa_{s,h} n \alpha}{1 - \kappa_{s,h}^2} e^{-n\gamma s} \delta(t - n\gamma) \right).$$  \hspace{1cm} (4.7)

Finally, combining (4.3), (4.4), and (4.7), the TD reflection coefficient for the slab is

$$\tau_{s,h}^d(t) = (h_R(t) - e^{-\alpha t} h_R(t - \gamma)) * \tau_{s,h}(t).$$  \hspace{1cm} (4.8)

The transmission coefficient for a slab with thickness $d$ is found in (4.2). Therefore, in a similar derivation to the TD slab reflection coefficient, the TD transmission coefficient will be

$$\tau_{s,h}^d(t) = \mathcal{L}^{-1} \left\{ \frac{1 - R_{s,h}^2}{1 - R_{s,h}^2 e^{-\gamma d}} \right\} \mathcal{L}^{-1} \left\{ e^{-j(\delta - kd)} \right\}$$  \hspace{1cm} (4.9)

The last term of (4.9) can be written as $-j(\delta - kd) \approx -\alpha \gamma / 2 - (\gamma / 2 - d/c) \cdot s$, where $s$, $\alpha$, and $\gamma$ were previously defined. Hence

$$\mathcal{L}^{-1} \left\{ e^{-j(\delta - kd)} \right\} = e^{-\alpha \gamma / 2} \cdot \delta(t - (\gamma / 2 - d/c)).$$  \hspace{1cm} (4.10)

The reflection coefficient is given by (4.5), thus the first term of the right hand side of (4.9) can be expanded into series, keeping only the first order terms of $\alpha/s$, i.e.

$$H_T = \frac{1 - R_{s,h}^2}{1 - R_{s,h}^2 e^{-j2\delta}} \approx H_T(s - \alpha) \approx$$

$$\frac{4\kappa_{s,h}}{(1 + \kappa_{s,h})^2} \cdot \sum_{n=0}^{\infty} \left( K_{s,h}^{2n} e^{-n\gamma s} + \frac{4n\kappa_{s,h}}{1 - \kappa_{s,h}^2} - K_{s,h} \right) K_{s,h}^{2n} \frac{\alpha}{s} e^{-n\gamma s}.$$  \hspace{1cm} (4.11)
Therefore, the time domain representation of (4.11) is given by

\[
\hat{h}_T(t) = \frac{4\kappa_{s,h}}{(1 + \kappa_{s,h})^2} \sum_{n=0}^{\infty} \left( K^{2n}_{s,h} e^{-n\alpha\gamma} \delta(t - n\gamma) + \left( \frac{4n\kappa_{s,h}}{1 - \kappa_{s,h}^2} - K_{s,h} \right) \alpha K^{2n}_{s,h} e^{-n\alpha\gamma} \delta(t - n\gamma) \right) \tag{4.12}
\]

and the TD transmission coefficient for the slab equals to

\[
\tau^d_{s,h}(t) = e^{-\alpha\gamma/2} \cdot h_T(t - (\gamma/2 - d/c)) \tag{4.13}
\]

### 4.2.3 Numerical Evaluation - Comparison

The formulas in (4.8) and (4.13) are used to predict the shape and the amplitude of the received signal and to be compared against the numerical inverse Fourier transform of the resulted field as calculated in the frequency domain. It is assumed that there are no interactions from the sides of the slab, or equally that the slab is infinitely extended.

The excited spherical wave is assumed to be a Gaussian doublet, which is given by [17]

\[
s = \frac{1}{\tau} \sqrt{\frac{2}{3\sqrt{\pi/2}}} \left( 1 - 2 \frac{t^2}{\tau^2} \right) e^{-t^2/\tau^2} \tag{4.14}
\]

where \( \tau \) is the full width half maximum pulse duration, with the value \( \tau = 0.1 \) ns being used for the simulations. The received signal is the convolution of the TD coefficients (4.8) and (4.13) with (4.14).

The maximum number of the internal reflections that needs to be taken into account can be calculated by the reflection and transmission coefficients. If a limit \( l \) is set for the ratio of the \( N \)-th reflected signal with respect to the first arriving one, the maximum number \( N \) of internal reflections that must be taken into account will be given by

\[
N_R = \left\lfloor \frac{\ln l}{2 \ln K_{s,h} - \alpha\gamma} \right\rfloor \quad \text{and} \quad N_T = \left\lfloor \frac{\ln l}{2 \ln K_{s,h} - \alpha\gamma} \right\rfloor \tag{4.15}
\]

for the reflection and transmission case respectively, where the brackets \( [\cdot] \) denote rounding towards the greater integer.

In Fig. 4.3 and Fig. 4.4, the reflection and transmission through a lossless dielectric slab for a hard polarized wave is considered. The thickness of the slab is \( d = 20 \) cm,
4.2. Layer Model

Figure 4.3: Reflected signals for an impinging hard polarized pulse on a lossless slab. The thickness of the slab is $d = 20$ cm, $\varepsilon_r = 4.4$, and the incident angle is $\theta = 0$. The proposed TD reflection solution and the numerical inverse Fourier transform of the corresponding FD solution are in an extremely good agreement.

Figure 4.4: The transmitted through the slab received signal, where the parameters are the same as in Fig. 4.3. The proposed TD solution can predict accurately even the very weak pulses.
4.2. Layer Model

Figure 4.5: The result of a soft polarized signal when reflected on a lossy slab. The thickness of the slab is \( d = 20 \) cm, \( \varepsilon_r = 4.4 \), and \( \sigma = 0.018 \) S/m. The angle of incident is \( \theta = \pi/8 \). The received signals are distorted by the slab; however, the proposed TD solution produces quite correct results.

and its relative dielectric constant is \( \varepsilon_r = 4.4 \). The angle of incidence is \( \theta = 0 \). The limit \( l \) is set to -40 dB assuming that signals that are below this limit, relative to the first pulse peak, will not be taken into account. Since the conductivity is zero, it is expected that the signal will be attenuated but undistorted. Indeed, as it can be observed in the figures, the latter is true. Also, the traversing signal is reflected internally in the sides of the slab, and in each internal reflection some of the energy of the wave is refracted to the neighbouring medium of the boundary surface, a behaviour that affects equally all the frequency spectrum of the excited pulse, and therefore the energy of each subsequent pulse is decreased. The time between two consequent pulses is equal to \( \gamma \), and it is affected not only from the thickness \( d \) of the slab, but also from \( \varepsilon_r \) and \( \theta \). It can be also noticed, through the insets of Fig. 4.3 and Fig. 4.4, that the proposed TD solutions can predict the received signal even for very weak reflected signals. The number of reflections to be considered is governed by the choice of the limit \( l \) through (4.15). If this threshold is reduced, it would result in predicting even the late-time low-powered arriving pulses in Fig. 4.3 and Fig. 4.4.

The case of the reflection and transmission through a lossy slab is considered in Fig.
4.2. Layer Model

Figure 4.6: The received signal for a wave transmitted through a lossy slab. The parameters are the same as in Fig. 4.5.

4.5 and Fig. 4.6 respectively. The thickness \( d \) of the slab is 30 cm, and the incident angle is \( \theta = \pi/8 \). The slab has \( \epsilon_r = 4.4 \) and \( \sigma = 0.018 \) S/m, which correspond to brick’s constitutive parameters. The excited signal has soft polarization and the limit \( \ell \) was set to -30 dB. The conductivity of the slab has caused the received signals to be distorted; the more times the ray travels inside the slab, the more distorted it gets, as this can be seen in the insets of Fig. 4.5 and Fig. 4.6. This is a result of the dissipation of energy inside the slab, which is analogous to the frequency. Therefore, portions of the excited frequency spectrum are treated differently, resulting thus to signal distortion. Also, it can be noticed that in the case of reflection, from either a lossless (Fig. 4.3) or a lossy (Fig. 4.5) slab, the reflected pulses have inverse polarization with respect to the first arriving pulse, whereas this is not true for the transmission case (Fig. 4.4 and Fig. 4.6). Again, the good agreement of the proposed TD coefficients with the numerical inverse Fourier transform of the FD ones validates our solutions. However this agreement tends to deteriorate as \( \sigma \) increases. This is a result of the approximations made during the time-domain derivation in (4.6), which results in a small disagreement of the predicted and theoretical pulse shapes for late times.
4.3 Multiple-Diffraction Phenomena

For multiple diffraction phenomena, there are methods in which the forecasted field is calculated by the use of single diffraction processes based on the Fresnel-Kirchhoff theory [76–78], and thus their result is inaccurate, or methods that use multiple diffraction integrals [79, 80], rendering their application cumbersome. Accurate predictions can also be achieved if higher order-terms of the UTD diffraction coefficients are used [81]. However, even these solutions suffer from complexity, which though can be alleviated if an appropriate recursive algorithm is applied [13, 82]. The new slope-UTD method only involves a second-order diffraction process, decreasing the computational time and increasing the accuracy of the prediction when compared to the existing multiple diffraction methods [83]. The new algorithm is applied to multishaped canonical objects as well [14, 84].

4.3.1 Frequency-Domain Approach

In the general case of a multi-diffracted ray, using the UTD formulation, the field after \( N \) obstacles, which contains an amplitude and a slope term, is given by

\[
E_N = \left[ E_{N-1}D_{N-1} + \frac{\partial E_{N-1}}{\partial n} d_{SN-1} \right] \cdot A_{N-1}(s_{N-1}) \cdot e^{-jk\phi_{N-1}}
\]  

(4.16)

where \( E_{N-1} \) is the field at the \( N-1 \)-th object, \( D_{N-1} \) is the diffraction coefficient of the \( N-1 \)-th obstacle and \( d_{SN-1} \) is the slope diffraction component which is written as

\[
d_{SN-1} = \frac{1}{jk} \frac{\partial D_{N-1}}{\partial \phi_{N-1}}
\]  

(4.17)

The spreading factor for a spherical wave, after the occurrence of \( N-1 \) diffraction incidences, is written as

\[
A_{N-1}(s_{N-1}) = \sqrt{\frac{s_0 + s_1 + \cdots + s_{N-2}}{s_{N-1}(s_0 + s_1 + \cdots + s_{N-1})}}
\]  

(4.18)

and \( \frac{\partial E_{N-1}}{\partial n} \) is the directional derivative of the \( E_{N-1} \) field from the \( N-2 \)-th to the \( N-1 \)-th obstacle, which can be rewritten as

\[
\frac{\partial E_{N-1}}{\partial n} = -\frac{1}{s_{N-2}} \frac{\partial E_{N-1}}{\partial \phi_{N-2}}
\]
As it can be seen, the multiple-diffraction solution involves first and second partial derivatives of the diffraction coefficients with respect to \( \phi \) and \( \phi' \). Using the derivative of the transition function, which is given by \[67\]

\[
d F[x] \over dx = j(F[x] - 1) + F[x] \over 2x, \tag{4.20}
\]

the partial derivatives of the diffraction coefficients for the knife-edge and wedge case can be deduced. Therefore, for the knife-edge case, it will be

\[
\frac{\partial D}{\partial \phi \partial \phi'} = \frac{L \cos(\alpha/2)}{\sqrt{2\pi}} \sqrt{j k} \sin(\alpha/2) \left(1 - F[x]\right) \tag{4.21}
\]

where the minus sign corresponds to differentiation with respect to \( \phi \) and the plus one with respect to \( \phi' \). The notation \( \partial / \partial \phi \partial \phi' \) represents partial differentiation with respect either to \( \phi \) or \( \phi' \) and \( x = 2kL \cos^2(\alpha/2) \). The second partial derivative of the diffraction coefficient is applied with respect to \( \phi \) and \( \phi' \). Thus

\[
\frac{\partial^2 D}{\partial \phi \partial \phi'} = \frac{L \cos(\alpha/2)}{2\sqrt{2\pi}} \sqrt{j k} \cdot
\left(1 - F[x] - j kL \sin^2(\alpha/2) \left(1 - F[x]\right) + \tan^2(\alpha/2)F[x]\right). \tag{4.22}
\]

The \( L \) parameters are necessary so as to enforce continuity in the field prediction. The calculation of these parameters for the amplitude and slope field terms is achieved through \[14\]

\[
L_{mnk} = \frac{E_{mn}(s_{mn} + s_{nk})}{[E_{mn}(s_{mn})A_{r}(s_{nk})e^{-jk_{s_{nk}}}]}^2 \tag{4.23}
\]

\[
L_{s_{mnk}} = \frac{s_{nk}\partial E_{mn}(s_{mn} + s_{nk})/\partial n}{[\partial E_{mn}(s_{mn})/\partial nA_{r}(s_{nk})e^{-jk_{s_{nk}}}]}^{2/3} \tag{4.24}
\]

with \( E_{mn}(s) \) representing the field at obstacle \( n \) caused by obstacle \( m \) which is \( s \) meters away (if the subscript is 0 then it refers to the transmitter position), and \( r \) is the order of diffraction. The \( L \) parameter in (4.23) is used in the diffraction coefficient for the amplitude term, whereas the \( L_s \) parameter in (4.24) is used in the slope term (eq. (4.17)).
4.3. Multiple-Diffraction Phenomena

Figure 4.7: A multiple-diffracted ray on a knife-edge and a wedge. (a) The distance between the obstacles and the transmitter and the receiver is \( d = 2 \) m, and the height \( h = 2 \) m. The wedge has \( \epsilon_r = 5.5 \) and \( \sigma = 0.018 \) S/m, with internal angle \( \alpha_{\text{int}} = \pi/5 \). The incoming wave is soft polarised. (b) The frequency response of a multiple-diffracted ray exhibits a frequency dependency. \( D_{\text{TOT}} \) is the total diffraction loss of the received waveform with respect to the transmitted one.
Similarly to the knife-edge solution, the derivative of the wedge diffraction coefficient can be calculated by differentiating (3.22) with respect to \( \phi \) or \( \phi' \). Hence,

\[
\frac{\partial D_{s,h}}{\partial \phi \partial \phi'} = \frac{\partial R_{os,h}}{\partial \phi'} R_{ns,h} D_1 + R_{os,h} \frac{\partial R_{ns,h}}{\partial \phi} D_1 + R_{os,h} R_{ns,h} \frac{\partial D_1}{\partial \phi} + \frac{\partial D_2}{\partial \phi \partial \phi'} + \frac{\partial D_3}{\partial \phi} + R_{os,h} \frac{\partial D_3}{\partial \phi'} + \frac{\partial R_{ns,h}}{\partial \phi} \frac{\partial D_4}{\partial \phi'} + R_{os,h} \frac{\partial R_{ns,h}}{\partial \phi} \frac{\partial D_4}{\partial \phi'}
\]

(4.25)

where if the derivative is calculated with respect to \( \phi' \), then the derivative of the \( R_{ns,h} \) is set to zero and vice versa. The derivatives of the reflection coefficients are given by [85]

\[
\frac{\partial R_s(\psi)}{\partial \psi} = \frac{-2 \sin \psi (\varepsilon_{r,com} - 1)}{\sqrt{\varepsilon_{r,com} - \sin^2 \psi} \left( \cos \psi + \sqrt{\varepsilon_{r,com} - \sin^2 \psi} \right)}
\]

(4.26)

\[
\frac{\partial R_h(\psi)}{\partial \psi} = \frac{-2 \varepsilon_{r,com} \sin \psi (\varepsilon_{r,com} - 1)}{\sqrt{\varepsilon_{r,com} - \sin^2 \psi} \left( \varepsilon_{r,com} \cos \psi + \sqrt{\varepsilon_{r,com} - \sin^2 \psi} \right)}
\]

(4.27)

with \( \partial \psi / \partial \psi' = -1 \) and \( \partial \psi / \partial \phi = 1 \). The derivative of \( D_i \) is found to be

\[
\frac{\partial D_i}{\partial \alpha_i} = \frac{\alpha_j \pi / 4}{2n \sqrt{2 \pi k}} \left[ F[x_i] + i 2 x_i \cot^2 (\alpha_i) (1 - F[x_i]) \right]
\]

(4.28)

where the \( \alpha_i \) and \( x_i \) are stated in (3.23) and the partial derivatives of \( \alpha_i \) with respect to \( \phi \) or \( \phi' \) are easily deduced.

Finally, the second derivative of the diffraction coefficient is

\[
\frac{\partial^2 D_{s,h}}{\partial \phi \partial \phi'} = \frac{\partial R_{os,h}}{\partial \phi'} \frac{\partial R_{ns,h}}{\partial \phi} D_1 + \frac{\partial R_{os,h}}{\partial \phi} R_{ns,h} \frac{\partial D_1}{\partial \phi} + \frac{\partial R_{ns,h}}{\partial \phi} \frac{\partial D_1}{\partial \phi'} + R_{os,h} \frac{\partial ^2 D_1}{\partial \phi \partial \phi'} + R_{os,h} \frac{\partial ^2 D_2}{\partial \phi \partial \phi'} + \frac{\partial R_{ns,h}}{\partial \phi} \frac{\partial ^2 D_3}{\partial \phi \partial \phi'} + R_{os,h} \frac{\partial R_{ns,h}}{\partial \phi} \frac{\partial ^2 D_4}{\partial \phi \partial \phi'}
\]

(4.29)

with

\[
\frac{\partial^2 D_i}{\partial \phi \partial \phi'} = \frac{\pi e^{-j \pi / 4}}{8 \pi n^3 \sqrt{2 \pi k}} \cot (\alpha_i) \left[ F[x_i] - j 2 x_i (1 - F[x_i]) \right] - j 2 x_i \cot^2 (\alpha_i) F[x_i] - 4 x_i^2 \cot^2 (\alpha_i) (1 - F[x_i])
\]

(4.30)

where for \( i = 1, 2 \) the minus sign shall be taken into account, whereas for \( i = 3, 4 \) the plus one.
4.3. Multiple-Diffraction Phenomena

4.3.2 Time-Domain Approach

A TD double wedge solution, based on slope-UTD, was presented in [86]. The two-dimensional single- and double-diffraction solutions are extended to a multiple-diffraction solution, with an arbitrary number of objects. In the proposed algorithm [87], it is possible to have obstacles with different shape across the path, such as knife-edges and wedges. The impulse response of every path is calculated and it is convolved with the transmitted pulse to produce the received path signal. The total received signal is the summation of every possible ray, as in the FD solutions. This signal is compared with the numerical inverse fast Fourier transform of the corresponding solution in the FD, so as to validate our method. The results show a very good agreement between the two solutions with the TD solution offering much shorter computation times.

Equation (4.16) is translated into the TD as

\[ e_N(t) = \left[ e_{N-1}(t) * d_{N-1}(t) + e_{N-1}^d(t) * d_{N-1}(t; \phi') \right] \cdot A_{N-1}(s_{N-1}) * \delta(t - s_{N-1}/c) \]  

(4.31)

where \( e_{N-1}(t) \) is the TD signal in the \( N - 1 \)-th object, \( d_{N-1}(t) \) is the diffraction coefficient of the \( N - 1 \)-th object, \( e_{N-1}^d(t) \) is the TD directional derivative of the field and \( d_{N-1}^d(t; \phi') \) is the derivative of the diffraction coefficient with respect to \( \phi' \). The two last coefficients are connected with (4.17) and (4.19), and more specifically, they are the inverse Fourier transform of \( \frac{1}{\sqrt{2\pi}} \frac{\partial E_{N-1}}{\partial t} \) and \( \frac{1}{\sqrt{2\pi}} \frac{\partial D_{N-1}}{\partial \phi} \). The latter method is necessary so as the functions to be inversed to be bound; that means there is a value \( M \in \mathbb{R} \), \( \mathbb{R} \) is the field of real numbers, for which \( |F(\omega)| < M \) as \( \omega \) tends to infinity. Thus, it will be

\[ e_{N-1}^{\text{der}}(t) = \left[ e_{N-2}(t) * d_{N-2}^{\text{der}}(t; \phi) + e_{N-2}^{\text{der}}(t) * d_{N-2}^{\text{der, slope}}(t) \right] \cdot \frac{A_{N-2}(s_{N-2})}{s_{N-2}} * \delta(t - s_{N-2}/c) \]  

(4.32)

where \( d_{N-2}^{\text{der, slope}}(t) \) is the inverse Fourier transform of \( \frac{1}{\sqrt{2\pi}} \frac{\partial D_{N-2}}{\partial \phi} \).

In order to complete the general multiple-diffraction solution in time-domain following (4.31) and (4.32), the derivative and the derivative of the slope diffraction coefficient should be specified. For the rest of the section, the \( L \) parameters will be assumed to
be constant with respect to frequency so as to reduce the complexity of the solutions. The latter is true when the receiver lies away from the shadow boundaries. Even if the receiver is close to the shadow boundary, the above assumption does not increase greatly the error in the prediction.

The derivative of the knife-edge diffraction coefficient was presented in (4.21). Hence, if we use the Laplace pair [58]

$$\mathcal{L}^{-1}\left\{1 - \sqrt{\pi \alpha e^{-\alpha^2}} \text{erfc} \left(\sqrt{\alpha s}\right)\right\} = \frac{\sqrt{\alpha}}{2 \cdot (t + \alpha)^{3/2}} \cdot u(t)$$  \hspace{1cm} (4.33)

where \(\mathcal{L}^{-1}\cdot\) represents the inverse Laplace transformation, the time domain representation of (4.21) will be

$$d^{\text{der}}(t; \phi, \phi') = \frac{L \sqrt{\gamma}}{2 \sqrt{2\pi}} \cdot \sin(2s\alpha) \cdot \frac{\sqrt{\gamma}}{(t + \gamma)^{3/2}} \cdot u(t)$$  \hspace{1cm} (4.34)

where the minus sign corresponds to differentiation with respect to \(\phi\) and the plus sign refers to differentiation with respect to \(\phi'\). The derivative of the slope diffraction coefficient, as this is given in FD in (4.22), in the TD can be found by taking the TD of each individual component and adding them together. Therefore, the final result will be

$$d^{\text{der, slope}}(t) = \frac{L \sqrt{\gamma}}{2 \pi \sqrt{2}} \cdot \cos(\alpha/2) \cdot \frac{\sqrt{t} \left(t + \frac{2L}{c} \left(1 + \sin^2(\alpha/2)\right)\right)}{(t + \gamma)^{3/2}} \cdot u(t).$$  \hspace{1cm} (4.35)

When the obstacle is a nonperfectly conducting wedge, the derivative of the diffraction coefficient is

$$d^{\text{der}}_{\gamma, h}(t; \phi, \phi') = r^{\text{der}}_{\gamma, h}(t) \ast r_{n\gamma, h}(t) \ast d_1'(t) + r_{o\gamma, h}(t) \ast r^{\text{der}}_{n\gamma, h}(t) \ast d_4'(t) +$$

$$r_{o\gamma, h}(t) \ast r_{n\gamma, h}(t) \ast d_1^{\text{der}}(t; \phi, \phi') + d_2^{\text{der}}(t; \phi, \phi') +$$

$$r^{\text{der}}_{n\gamma, h}(t) \ast d_3'(t) + r_{o\gamma, h}(t) \ast d_3^{\text{der}}(t; \phi, \phi') +$$

$$r^{\text{der}}_{n\gamma, h}(t) \ast d_4'(t) + r_{n\gamma, h}(t) \ast d_4^{\text{der}}(t; \phi, \phi').$$  \hspace{1cm} (4.36)

When the differentiation is calculated with respect to \(\phi\), the derivative of \(r_{o\gamma, h}(t)\) is set to zero, whereas when the differentiation is calculated with respect to \(\phi'\), the derivative of \(r_{n\gamma, h}(t)\) is set to zero. Also, since

$$\mathcal{L}^{-1}\left\{\frac{1}{\sqrt{s}} e^{\alpha s} \text{erfc} \left(\sqrt{\alpha s}\right)\right\} = \frac{1}{\sqrt{\pi (t + \alpha)}} \cdot u(t),$$  \hspace{1cm} (4.37)
it will be

\[ d_d^l(t) = \frac{c}{2n\sqrt{2\pi}} \cdot \cot(\alpha_l) \cdot \frac{\sqrt{\gamma_l}}{(t + \gamma_l)^{1/2}} \cdot u(t) \]

(4.38)

\[ d_t^{der}(t; \phi, \phi') = \frac{\partial \alpha_l}{\partial \phi \partial \phi'} \cdot \frac{c\sqrt{\gamma_l}}{2n\sqrt{2\pi}} \cdot \frac{t + 2Ln^2/c}{(t + \gamma_l)^{3/2}} \cdot u(t) \]

(4.39)

where \( \alpha_l \) and \( \gamma_l \) were defined in (3.23) and (3.29) respectively and \( \frac{\partial \alpha_l}{\partial \phi \partial \phi'} \) can be easily calculated. The derivative of the reflection coefficient in TD is given by

\[ r_a^{d>c}(t; \psi) = \pm \left[ \frac{2}{(1 + \kappa_{s,h})^2} \delta(t) - 2 \frac{1 - \kappa_{s,h}^2 + \kappa_{s,h} \alpha_l}{(1 + \kappa_{s,h})^4} - e^{-(1+\kappa_{s,h}/2)\alpha_l} \right] \cdot \frac{\partial \kappa_{s,h}}{\partial \psi} \cdot u(t) \]

(4.40)

where \( \alpha = \sigma/(2\epsilon_r\epsilon_0) \), \( \kappa_{s,h} = (1 - \kappa_{s,h})/(1 + \kappa_{s,h}) \), \( \kappa_s = \cos \psi/\sqrt{\epsilon_r - \sin^2 \psi} \), and \( \kappa_h = \sqrt{\epsilon_r - \sin^2 \psi}/(\epsilon_r \cdot \cos \psi) \). For the \( o \)-case, \( \psi = \pi/2 - \phi' \) and the minus sign corresponds to soft polarisation whereas the plus sign refers to hard polarisation. For the \( n \)-case, \( \psi = \pi/2 - \pi + \phi \) and the minus sign corresponds to hard polarisation whereas the plus sign refers to soft polarisation. Finally,

\[ \frac{\partial \kappa_s}{\partial \psi} = \frac{\sin \psi(1 - \epsilon_r)}{(\epsilon_r - \sin^2 \psi)^{3/2}} \]

(4.41)

\[ \frac{\partial \kappa_h}{\partial \psi} = \frac{\sin \psi(\epsilon_r - 1)}{\epsilon_r \cos^2 \psi \sqrt{\epsilon_r - \sin^2 \psi}} \]

(4.42)

The derivative of the slope diffraction coefficient in TD for the wedge case can be written as

\[ d_{s,h}^{\text{slope}}(t) = r_{\text{cs},h}(t) \cdot d_{1}^{\text{slope}}(t) + r_{\text{os},h}(t) \cdot d_{1}^{\text{slope}}(t; \phi') + r_{\text{os},h}(t) \cdot d_{2}^{\text{slope}}(t; \phi') + r_{\text{os},h}(t) \cdot d_{3}^{\text{slope}}(t; \phi') + r_{\text{ns},h}(t) \cdot d_{4}^{\text{slope}}(t; \phi') \]

(4.43)

The coefficient \( d_{1}^{\text{int}}(t) \) can be calculated by using the identity

\[ \mathcal{L}^{-1} \left\{ \frac{F(s)}{s} \right\} = \int_0^t f(u)du \]

(4.44)

where \( \mathcal{L}^{-1} \{F(s)\} = f(t) \) and thus

\[ d_{1}^{\text{int}}(t) = \int_0^t d_1(u)du \]

\[ = \frac{Ln\sqrt{\alpha}}{\pi\sqrt{2}} \cdot \frac{\sin(2\alpha_l)}{\sqrt{\gamma_l}} \cdot \tan^{-1} \left( \frac{t}{\gamma_l} \right) \cdot u(t). \]

(4.45)
Consequently, $d_i^{slopes}$ and $d_i^{sec.slopes}$ can be calculated by differentiating $d_i^{int}$ and $d_i^{slopes}$ with respect to $\phi$ or $\phi'$. In this case,

$$
d_{i}^{slopes}(t, \phi, \phi') = \frac{\partial d_{i}^{int}}{\partial \phi \partial \phi'} = \frac{\partial \alpha_i}{\partial \phi \partial \phi'} \cdot \frac{L \sqrt{2c}}{\pi} \cdot \sin^{2}(\alpha_i) \cdot \tan^{-1}\left(\frac{t}{\gamma_i}\right) + \frac{\cos^{2}(\alpha_i) \cdot \sqrt{t}}{t + \gamma_i} \cdot u(t) \quad (4.46)
$$

$$
d_{i}^{sec.slopes}(t) = \frac{\partial^{2} d_{i}^{slopes}}{\partial \phi \partial \phi'} = \frac{\partial \alpha_i}{\partial \phi} \cdot \frac{\partial \alpha_i}{\partial \phi'} \cdot \frac{L \sqrt{c}}{\pi \sqrt{2}} \cdot \sin(2\alpha_i) \cdot \frac{1}{\sqrt{\gamma_i}} \cdot \tan^{-1}\left(\frac{t}{\gamma_i}\right) - \frac{\sqrt{t} \cdot \left(3t + \frac{2L \ln(2 + \sin^{2}(\alpha_i))}{e}\right)}{(t + \gamma_i)^{2}} \cdot u(t) \quad (4.47)
$$

and the partial derivative of $\alpha_i$ with respect to $\phi$ or $\phi'$ can be easily calculated by following their definition in (3.23).

### 4.3.3 Numerical Evaluation - Comparison

The above formulas are used to predict the shape of the received signal and to be compared against the numerical inverse fast Fourier transform of the resulted field as calculated in the frequency domain.

The excited wave which is used in the simulations is a Gaussian doublet in TD, as this is given in (4.14), with a value of $\tau = 0.1$ ns. The impulse response of every possible ray is calculated and then the received signal is regarded as the sum of the convolutions of the transmitted signal in the TD with every path response. The values of the $L$ parameters that are used in the TD solutions are the values that correspond to a frequency which is in the centre of the excited spectrum.

The case of five knife edges in grazing incidence is presented in Fig. 4.8. The knife-edges are 2 m apart and the total path is 12 m. According to [13], in such configuration there are 1 LOS ray, 5 single-diffracted rays, 10 double-diffracted ones, 10 triple-diffracted rays, 5 quadruple-diffracted ones and 1 quintuple-ray, with the assumption that there are no reflections from the ground, as we are interested only in the diffraction losses. It is known from theory that for equally spaced knife-edges the diffraction loss is independent of the frequency. Therefore it is expected that the received waveform will be attenuated but undistorted. Indeed, as it can be seen in Fig. 4.8, the predicted
4.3. Multiple-Diffraction Phenomena

Figure 4.8: The received signal in the grazing incidence of five knife-edges. The distances between the knife-edges, the transmitter and the receiver are 2 m and their heights are 2 m.

Figure 4.9: The received signal in the grazing incidence of five wedges. The wedges are metallic with internal angle of $\pi/5$ rads. The distances between the wedges, the transmitter and the receiver are 2 m and their heights are 2 m.
waveform agrees with the latter statement and that also the proposed time-domain solution agrees exceptionally well with the numerical inverse fast Fourier transform of the FD solution. Similar results can be achieved if the knife-edges are substituted with wedges, as in Fig. 4.9. In this scenario, all the wedges are metallic and they have the same internal angle, \( \alpha_{int} = \pi/5 \) rads and the transmitted signal has soft polarization. In this case though, the diffraction loss is less than in the knife-edge scenario, and also the received signal is slightly distorted, which nevertheless was anticipated. Still, the predicted received signal agrees very well with the numerical inverse fast Fourier transform of the FD solution. Similar results can be achieved if a hard polarized signal is excited.

As mentioned above, the \( L \) parameters were considered to be independent of frequency so as to reduce the complexity of the TD solutions. However, by inspecting (4.23) and (4.24) it can be observed that the previous statement is not true and therefore the aforementioned assumption will introduce an error in the predicted TD solution. This is more evident if a configuration such as in Fig. 4.10 is used. The wedges are set in such way, so that there can be only one ray, which is quadruple-diffracted, and so that also all the obstacles would lie in the shadow region of the previous obstacle and very near to the shadow boundary. It shall be mentioned that it is not possible to define a transition region when we are dealing with UWB pulses, like in narrowband signals, because such region depends, among other parameters, on the operating frequency \[67\]. Since in an UWB signal many frequencies coexist, the transition region will not be constant for a certain configuration. The heights of the transmitter and the receiver are 2 m, and all the distances between the obstacles are 3 m. The wedges are metallic with \( \alpha_{int} = \pi/4 \) rads, and their heights are \( h_1 = h_4 = 2.1 \) m, and \( h_2 = h_3 = 2.18 \) m. In this case, the TD solution differs from the numerical inverse Fourier transform of the FD solution, as was expected based on the \( L \) parameter assumption (Fig. 4.11). However, this is not the only reason that introduced an error in the predicted signal. The way of evaluating the convolution also affects the result. In this thesis, the convolution was assumed to be the product of the convolution sum of the two TD functions and the time step \( dt \), where \( dt \) is the time difference between two consecutive points that is needed to represent the function. This procedure corresponds in evaluating the convolution integral as a
4.3. Multiple-Diffraction Phenomena

Figure 4.10: The propagation path of a quadruple-diffracted signal. The four wedges are metallic with $\alpha_{int} = \pi/4$ rads, $h_1 = h_4 = 2.1$ m, and $h_2 = h_3 = 2.18$ m. The height of the transmitter and the receiver is 2 m. The distance between each consequent object is 3 m. Shadow Boundary $mn$ corresponds to the shadow boundary of object $n$ with respect to object $m$.

If one or both of the functions are very sensitive to time, which is the case for the path impulse response for objects close to the shadow boundaries, the latter method will introduce an error. A remedy for that is to decrease $dt$, as it was done in the lower inset of Fig. 4.11, but doing so the number of points that are needed to represent the solution in TD will increase. Therefore, there is a trade-off between the error and the number of points that are needed so as to present the received signal in the time domain properly. This effect becomes less important when we are away from the shadow boundaries.

The proposed solution can also be applied when there are different types of objects along the propagation path. Such a scenario is presented in Fig. 4.12. The outer objects are absorbing knife-edges with height $h_{KE} = 2$ m, whereas the middle object is a nonperfectly conducting wedge with internal angle $\alpha_{int} = \pi/5$ rads, relative dielectric constant $\varepsilon_r = 10$, conductivity $\sigma = 0.1$ S/m and with a variable height. The distances between each object are equal to 2 m. The transmitter and the receiver are in the same level ($y = 0$), and it is assumed that the transmitter sends pulses every 5 ns. In each transmission time, the height of the wedge $h_W$ is increased by 1 m, with initial height $h_W = 0$. It is possible to separate two rays, if the time that is needed for the signal to traverse the path difference between the rays is greater than the signal duration, as it is shown in the second received signal in Fig. 4.12. Another remark is that
Figure 4.11: The predicted TD signal and the numerical inverse Fourier transform of the FD signal for the propagation path of Fig. 4.10. For the lower inset, the time difference $dt$ was smaller that the one used in the upper inset.

Distortion is not only caused because of the frequency selectivity of the interactions but also because of the inter-symbol interference, when the pulses arriving from two different paths collide, as it can be seen in the last arriving signal. For heights of the wedge smaller than $2\ m$, the two main contributions in the received signal come from a double-diffracted ray on the first and last obstacle and a triple-diffracted one. As $h_w$ becomes greater than $2\ m$, the double-diffracted path is obstructed and this is why the received energy is reduced. However, when $h_w$ is $4\ m$, there is a single-diffracted path, which explains the peak on the received energy as seen in the fifth arriving signal in Fig. 4.12, as well as the fact that the received signal is less distorted. Finally, when $h_w$ increases further, so does the length of the propagation path. The transmitter and the receiver are getting deeper into the shadow region of the wedge as well, and therefore the received energy will decrease until it vanishes. Again, the proposed TD solution agrees exceptionally well with the inverse Fourier transform of the theoretical solution.

In the last example (Fig. 4.13), there is a double obstruction between the transmitter and the receiver. The versatility and the validity of the current algorithm is proved by this example as well, as the position of the objects is quite random. The transmitter is at $0.5\ m$ above the ground and $3\ m$ away from it, there is a nonperfectly conducting
4.3. Multiple-Diffraction Phenomena

Figure 4.12: The received signal for a cascade of different objects. The transmitter and the receiver are on the same level \( h_{T_x} = h_{R_x} = 0 \text{ m} \). The two knife-edges have heights equal to 2 m and the height of the wedge is varied. The wedge has internal angle \( \alpha_{int} = \pi/5 \text{ rads} \), dielectric constant \( e_r = 10 \), and conductivity \( \sigma = 0.1 \text{ S/m} \). The transmitter sends pulses every 5 ns and in every transmission time, the height of the wedge is increased by 1 m, with initial height \( h_w = 0 \text{ m} \) and maximum height 6 m.

Another benefit of the proposed algorithm is the saving on the computation time when the prediction of the received waveform is carried out directly in the time-domain rather than calculating the numerical inverse Fourier transform of the corresponding FD received signal. This can be seen in Table 4.1, where the average ratios of the computation times of the two methods, \( T_{IFFT}/T_{TD} \), are presented, for scenarios similar to Fig. 2 with 4, 5 and 6 edges. The time-domain solutions outperform the IFFT frequency-domain ones in terms of computation time and as the number of edges increases, the former solutions can be calculated twice as fast as the latter ones.
4.4 Conclusions

Figure 4.13: The received signal for a cascade of different objects and unequal heights between the transmitter and the receiver. The first obstacle is a nonperfectly conducting wedge, with internal angle $\alpha_{\text{int}} = \pi/6$ rads, dielectric constant $\varepsilon_r = 10$, conductivity $\sigma = 0.05$ S/m and height equal to 2 m. The second object is an absorbing knife-edge, with height 3 m. The transmitter is at 0.5 m and the receiver at 2 m.

Table 4.1: Average Ratios of the computation times of the two methods

<table>
<thead>
<tr>
<th>Number of edges</th>
<th>$T_{\text{IFFT}}/T_{\text{TD}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1.56</td>
</tr>
<tr>
<td>5</td>
<td>1.79</td>
</tr>
<tr>
<td>6</td>
<td>2.20</td>
</tr>
</tbody>
</table>

4.4 Conclusions

In this chapter, time-domain solutions for multiple interactions were presented. In the first part, TD reflection and transmission coefficients for the slab case were presented, whereas in the second part, multiple diffraction phenomena were considered.

The proposed solutions for the slab case can predict accurately a great number of internally reflected rays, in simple and easy-to-use closed forms. The very good agreement of the presented coefficients with the numerical inverse Fourier transform of the corresponding FD ones proves the validity of our method.
The analytical TD formulas for the multiple-diffraction case represents a more realistic environment, and the agreement of the TD formulas with the numerical inverse Fourier transform of the FD solution validates the proposed algorithm. The results show that the signal is distorted greatly as the number of the diffraction obstacles increases. It is possible to distinguish two signals which follow different path routes, when their path difference is larger than the pulse duration of the transmitted pulse. The latter is inherited in the good resolution properties of UWB signals. Different types of canonical forms can be considered by the proposed algorithm, if similar expressions to (4.34)-(4.47) are found. It is also more efficient to work directly in time-domain than numerically inverse Fourier transform the frequency domain solution, in terms of computation time.

In contrast to 2D diffraction phenomena, in the three-dimensional case, the edge needs not be straight or infinitely long. In addition, the incident and diffracted rays in general are not perpendicular to the edge. Therefore, it would be necessary to resolve the incident and diffracted fields into their hard and soft components [67], and to use the dyadic diffraction coefficients, similar to [12], transformed in the TD, and subsequently to calculate the diffracted field, following the proposed in this paper algorithm. Finally, for an edge with finite length, it would also be necessary to add additional components, which would represent the diffracted fields arisen from the corners of the edge [67].
Chapter 5

Ray-Trace Algorithm

5.1 Field Prediction

The constant advances in cellular communication systems and the rapid demands in capacity have led engineers into developing intelligent methods for accommodating more users in a more efficient manner. Such a goal can be achieved if appropriate tools for the accurate prediction of the received field strength are utilised, which will improve the system performance and define the actual boundaries of a cell, providing a better exploitation of the existing resources. These tools are based on propagation prediction models and can be categorised as empirical, theoretical and deterministic models [60,88].

Empirical models are derived from the statistical analysis of a large number of measurements, where an appropriate function is fitted into the measurement data. Therefore, they are based on extensive and expensive collection of data and they are only accurate for that specific environment, where the measurements were conducted, or for an environment with similar characteristics. Empirical models are a very attractive choice due to their simplicity and negligible computational requirements, but they do suffer from great inaccuracies, especially when distinctive data features, such as high buildings, exist in the environment, failing as well in providing a physical insight into the mechanisms by which the propagation occurs. The subjective classification of the
environment into rural, suburban or urban may also contribute to an incorrect prediction. Empirical models are usually applied for macrocells. One of the most widely used empirical model is Okumura model, which is applied for signal prediction in urban areas [60]. This model is valid for frequencies in the range from 150 MHz to 1920 MHz and distances of 1 km to 100 km. It can be used for base station antenna height ranging from 30 m to 1000 m and it is based on a set of curves that give the median attenuation relative to free space. Based on Okumura model, Hata provided an empirical formulation of the graphical path-loss data provided by Okumura and supplied correction equations for application to other situations [89]. Hata model is valid for frequencies ranging from 150 MHz to 1500 MHz.

Different from the empirical models, theoretical models assume simplified geometry and ideal conditions, such as uniform building height or space between buildings. They provide a physical basis for the prediction models, however they are inaccurate for practical environments. Such a model is the Walfisch-Bertoni model [90], where the buildings are supposed to be equidistant with equal heights and the main propagation mechanism is the over-the-rooftop diffraction. Saunders and Bonar generalised that model in the Flat Edge model, allowing the transmitting base station to be at any height, even beneath the rooftop level, as well as allowing the number of buildings to be small [91]. The Ikegami model is based on a simplified deterministic approach, where with a detailed map of buildings positions and shapes single diffracted and single reflected rays are traced [92]. Diffraction is calculated using a single edge approximation at the building nearest the mobile and the wall reflection loss is fixed at a constant value. The Walfisch-Bertoni model has been combined with the Ikegami model for diffraction down to street level plus some empirical correction factors to improve agreement with measurements in a single integrated model, the COST 231/Walfisch-Ikegami model.

Site-specific models use detailed building and terrain databases for the prediction of the field strength at the receiver position. They apply well-known electromagnetic techniques to a site-specific description of the environment and therefore they need a large computational overhead, especially if the area under test is large. Nonetheless, they provide relatively accurate predictions, as well as a number of other necessary parameters, such as the rms delay spread and the angle-of-arrival, avoiding thus the
need for expensive and time consuming field measurements. The increase in CPU power and the existence of Graphical Information System (GIS) databases made site-specific models a feasible solution. However, there is a trade-off between prediction accuracy and computation time, as more levels of interactions need be considered, like in the case for receivers that are far away from the transmitting point and well beneath the rooftop level.

In this chapter an account of the recent advances in ray-tracing is presented and a novel deterministic algorithm for the field prediction is introduced. The algorithm uses a preprocessing step, where information about the environment is found and saved into memory, reducing thus the need for the calculation of similar operations, as well as the actual time of the ray-trace implementation. The algorithm is then compared with measurements and the final section concludes the chapter.

5.2 Ray-Trace Models

Site-specific methods may include ray-trace algorithms and finite-difference time-domain methods (FDTD). FDTD is an accurate method which can be applied to small-to-medium sized areas, but the error would increase for larger areas [93]. Ray theory emerged as a highly promising procedure for providing an accurate site-specific means to obtain useful simulations. In such an algorithm, rays from the transmitter to the receiver will be traced and their field will be calculated according to the geometrical optics and the uniform theory of diffraction. The most commonly used ray-trace methods are the image method and the shooting-and-bouncing ray launching algorithm [94].

5.2.1 Image Method

The image method is a simple and accurate method for determining the possible ray trajectories between the transmitter and the receiver. Based on optics theory, in order to determine the existence of a wall reflection, the image of the transmitter with respect to the wall plane is specified, \( T_{W_1} \), and the intersection point \( r_1 \) of the line \( T_{W_1}R \) with wall \( W_1 \) plane is computed (Fig. 5.1a). A reflection occurs if \( r_1 \) lies inside the wall's
boundaries. For a second level reflection occurrence, the image of $T_{W_1}$ with respect to the second wall plane $W_2$, $T_{W_1}W_2$, is determined and the second intersection point $r_{12}$ is computed. If this point belongs on the wall, in order the second level reflection to occur, the intersection point of the line $\overline{T_{W_1}r_{12}}$ with the first wall plane must lie inside the wall. This procedure is repeated for every wall and image.

The image method is an accurate method, but suffers from inefficiency when the number of walls involved is large and the reflection occurrences are high. The representation of diffraction phenomena is a very tedious task in image theory, as the diffraction points act as virtual transmitters and therefore their images with respect to the walls need be taken into account as well, slowing down considerably the prediction algorithm. This limits the number of total diffraction occurrences and, in general, a practical ray-trace algorithm realisation based on image theory would involve a trade-off between accuracy and the desired level of interactions [95].

![Figure 5.1](image)

**Figure 5.1:** (a) Determination of rays according to image theory. (b) Ray launching technique. Rays are launched from the transmitter and their paths are traced. The radius of the reception sphere for each ray depends on its total traversed path.

### 5.2.2 Shooting-and-Bouncing Ray-Launching

In ray-launching, rays are launched from the transmitting antenna and then they are traced to see whether they hit any object or are received by the receiving antenna (Fig. 5.1b) [96]. When an object is hit, reflection, transmission, diffraction or scattering will
occur, depending on the geometry and the electric properties of the object. Due to the 
discrete nature of the ray-launching algorithm, the ray is considered to have reached 
the receiver, when it passes through a sphere with radius \( r = \frac{\alpha d}{\sqrt{3}} \), centered on the 
receiver point, where \( \alpha \) is the angular separation of two adjacent rays and \( d \) is the total 
propagation path.

However, the reception spheres between adjacent receivers may overlap, leading to dou­
ble counting rays. The angular separation of the transmitting rays affect the perfor­
mance of the algorithm as well; if it is very small, the number of rays that are launched 
are very large and the computation time increases considerably, whereas if \( \alpha \) is very 
large, the possibility of omitting objects increases. Additionally, diffraction edges must 
be treated as ray-launching sources, leading to a very time consuming algorithm.

In ray-launching, the rays are effectively represented as ray cones. However, a variation 
of ray-launching is ray-tubes where instead of ray cones, three dimensional tubes are 
launched [97]. A hybrid method, combining image theory and ray-launching, is also 
possible [98]. Ray-launching is used to quickly identify a possible ray trajectory and 
the exact reflection positions are calculated using the image theory.

5.2.3 Acceleration Algorithms

Ray-tracing methods can become very inefficient when the simulated area is very large 
and higher levels of interactions need to be considered. Nonetheless, their performance 
can be accelerated by reducing the number of objects on which actual ray-object inter­
section will be performed, reducing thus the total number of the calculations. All ac­
celeration methods concern the preprocessing of the propagation environments and/or 
the positions of the transmitter and/or the receiver. Such methods are the Angular 
Z-Buffer (AZB) and the ray-path search algorithm.

The AZB consists in dividing the space seen from the source in angular regions and 
storing the facets in the regions where they belong [99]. In this way, for each ray only 
the facets stored in the same with the ray region need be analysed. When the ray is 
reflected, the algorithm is applied in a similar manner for the image of the transmitter, 
but the angular window region corresponds to the actual reflection space, while for
diffracted rays, the minimum and maximum angles with respect to the edge facet and edge vector are used.

The ray-trace algorithm can also be accelerated if the environmental elements on which actual ray-tracing is performed are reduced [100]. This will decrease the computation time greatly as less calculations will be needed. The acceleration is realised through a preprocessing of the data, where a visibility graph, which resembles a tree-like list, is constructed. The possible interactions between the points form the branches of the tree and they are stored in the memory. The visibility graph can be constructed through a polar-sweep algorithm, in which the objects that are visible or partially visible by the transmitter are recognised through sweep lines, storing their respective angular positions, similarly to the AZB algorithm. For diffraction incidents, the polar sweep algorithm is performed in the area, which is contained by the walls that make up the edges. Another approach in building the visibility graph is the bounding boxes [100]. The area is split into smaller areas, defined by flat-top boxes each with height equal to the highest building inside that box. The visibility algorithm is performed initially in the box where the transmitter is located. Only if the outer bounds of the box are visible the visibility algorithm is executed for the neighbouring box as well, reducing thus the total number of elements in which the intersection test is performed.

5.3 Parameters in Ray-Trace Algorithms

Ray-trace algorithms are very powerful tools for relatively accurate field predictions in areas where digital information about the environment exists. However, even such algorithms suffer from great errors if certain parameters are not taken properly into account.

The data about the buildings may suffer from inaccuracies, which will affect the predicted result. Also, complexities concerning their structure may lead to large memory requirement and computation time and a simplification on their shape may be desired. The electrical properties of the walls that are involved in the site-specific prediction are not known \textit{a-priori} and an approximation on their effective values need be made [96]. Also, due to the irregularities on the walls and small objects in the propagation path,
like sign posts, pedestrians and cars, the propagating energy is scattered in many di­rections and in many cases it needs to be considered in the model [101]. Finally, the maximum level of interactions and number of rays affect the performance and accuracy of the prediction and they need careful consideration.

**Figure 5.2:** The walls of the environment are divided into tiles and the edges are divided into segments and their respective angles are stored in a file.

### 5.4 Database Preprocessing Method

A novel ray-trace algorithm is constructed based on a database preprocessing method [102–106]. Since the visibility relations between walls and edges are independent of the position of the transmitter, it is possible to accelerate the time consuming process of path finding by a single intelligent preprocessing of the database. In this preprocessing, all walls of the database are subdivided into tiles and all edges are subdivided into segments (Fig. 5.2). The discretisation of the database leads to an automatisation in the ray-path finding.

For a reflection to occur, both the transmitter and the receiving point need to be visible from the centre of the tile (Fig. 5.3). Therefore, if the conditions $[\phi_{1,\text{min}}, \phi_{1,\text{max}}] \cap [180-\phi_{2,\text{max}}, 180-\phi_{2,\text{min}}] \neq \emptyset$ and $[\theta_{1,\text{min}}, \theta_{1,\text{max}}] \cap [180-\theta_{2,\text{max}}, 180-\theta_{2,\text{min}}] \neq \emptyset$ are satisfied, a single reflected ray between the transmitter and the receiving point exists and the
5.4. Database Preprocessing Method

Figure 5.3: Reflection conditions.

reflection point is assumed to be the centre of that tile. In reality, the above conditions indicate that the actual reflection point lies inside the tile, but it is approximated as the tile centre.

Similar operations are performed for the diffracted rays when segments are illuminated. In that case, only points inside the two diffraction cones that are formed by the angles $180 - \theta_{\text{max}}$ and $180 - \theta_{\text{min}}$ (Fig. 5.2b) are considered. The angles $\phi_{\text{min}}$ and $\phi_{\text{max}}$ are essential for the field calculation and also if further simplification in the algorithm is desired (e.g. only non-line-of-sight points are taken into account).

However, in [102–106] the prediction is restricted to a small number of interactions or to the insertion of an empirical model (COST 231/Walfisch-Ikegami model) to complement the prediction for receiver points far away from the transmitter. This is a result of the increased error, due to the double counting rays that the algorithm experiences, similar to the ray-launching algorithm.

The double-counting rays increase not only the prediction error but also the memory requirements of the algorithm, as the number of the predicted rays grows exponentially, even though most of them are spurious. One way of combating such a problem is by checking the history of each ray; if the ray has the same wall history as a previous ray, then one of them is discarded. However, it is not possible to have an accurate criterion for which ray is the correct one. Another solution is to perform the image test. Since the knowledge of the walls involved in the propagating ray is available, the image test is
quite fast, and actually the computation time that is spent for the test is compensated by the time that is saved when discarding the spurious nodes.

5.4.1 Database Preprocessing

After discretising the database, the visibility relationships between all tiles, segments and receiver points are determined in the preprocessing stage. The visibility relations are given by the line-of-sight criterion between the centers of the tiles (or segments). This leads to a simplification of the path finding problem, i.e. possible interaction points are only the centers of the tiles and segments.

If there is a line-of-sight condition between a tile and a receiving point, the four connecting straight lines from the receiving point to the corners of the tile are considered (Fig. 5.2). By projecting these four lines into two perpendicular planes, four angles are determined which give an adequate description of the visibility relation. Similar computations for the visibility relations between tiles and tiles, tiles and segments, segments and segments, segments and receiving points are performed in the following steps and are also stored in a file. Essentially, this technique integrates the image method with the ray-launching.

The projection of the connecting straight lines in the two planes is very important, as it provides with adequate information for the determination of the possible reflection and diffraction points in the ray-trace stage. Also since the angles continue to the neighboring tiles or segments, a very accurate computation of the rays is possible even if the sizes of the tiles or segments are large (up to 5 or 10 meters, depending on the database).

The architecture for the preprocessing step is given in the diagram in Fig. 5.4. After the simulation area is determined, as well as the dimensions of the tiles, the algorithm performs the following steps:

Wall..File: The buildings in the specified area around the central point are read from the database and the rest of the environment (vegetation, canals) is ignored. Normally, such a database includes the three-dimensional coordinates of the buildings'
5.4. Database Preprocessing Method

Figure 5.4: The architecture for the Database Preprocessing.

roofs. The algorithm uses this information to define all the necessary walls. However, due to the nature of the method which needs the tiles to be rectangular, the buildings are simplified as flat top boxes with heights equal to the median height of their respective roofs.

Information about the ground level is also read from the available digital database. The average ground level for each building is computed and subtracted from its height, so as to have a more realistic representation of the environment. The ground though, is considered flat.

**Building Visibility:** A version of a 3D polar-sweep algorithm is applied to determine the visibility relations between buildings and accelerate the shadowing test between the points. Instead of constructing a visibility tree for every point, a process that would be very time consuming, the visibility relations between buildings are established. Then, in the shadowing test process only the walls from these buildings need to be considered.

**Tiles Coordinates:** The walls are divided into tiles and the coordinates of their centers and their tile vertices are calculated. The size of the tiles on each wall is altered according to the wall’s dimensions, so as to have equal sized tiles on that specific wall. The roofs are not divided into tiles, due to their irregular shapes. Tiles that belong to two walls are discarded. The ground is also divided into tiles and the ground points that are inside the buildings are discarded as well.
Segments Coordinates: The segments for every edge are calculated. If the edge belongs to only one wall, then it is categorised as a knife-edge. If it belongs to two walls, then it is categorised as a wedge and if it belongs to more than two wall, it is checked if either the current wall lies between other walls, in which case the current segment is discarded, or else, the other wall with which it forms the greatest angle is determined and the wall is designated as a wedge. The maximum internal angle of the wedge is determined.

Receiver Points: The coordinates of the receiving points are added in the database. These may form a grid or be actual measurement points.

Tile Angles: This submodule is the actual core of the preprocessing algorithm. Using the information during the visibility algorithm, the points that are visible from every tile are calculated by applying the intersection test and their respective angles, as shown in Fig. 5.2a, are saved into memory.

Segment Angles: Similar to the Tile Angles submodule, the visible points for every segment are noted down and their forming angles (Fig. 5.2b) are stored in a file.

Save Information: The information about the tiles, segments, receiver points, their respective angles and the walls of the environment is saved in the memory.

5.4.2 Ray-trace Algorithm

The result of the preprocessing of the building database is a tree structure, containing tiles, segments and receiving points of the predicting area (Fig. 5.5).

Every branch of the tree symbolizes a visibility relationship between the two corresponding elements. In the first step of the prediction process, only the tiles, segments and receiving points, which are visible from the transmitter have to be determined, as well as their respective angles. Subsequently, path finding can be achieved in a similar manner to the ray-launching algorithm by recursively processing all visible elements and checking if the specific conditions for reflection or diffraction are fulfilled. The ray search is stopped, if a receiving point or a given maximum number of interaction is
5.4. Database Preprocessing Method

reached. Finally, the field strength is summed up at every receiving point. The preprocessing of the database reduces the actual path finding into the corresponding search in the tree structure.

The visibility relations stored in the tree (all layers except the first layer) are independent of the transmitter location and can be used for all predictions within the same database. Only the relations in the first layer of the tree depend on the location of the transmitter and must be computed in the beginning of the prediction process. Therefore, if the transmitter location is altered, only the first layer of the tree is recalculated, leaving the rest of the tree trunk unaffected, enhancing thus the speed for the prediction result.

The architecture of the ray-trace algorithm is illustrated in Fig. 5.6. After the determination of the necessary parameters (transmitter position, levels of interactions, effective electrical properties of the walls, their thickness and the rough factor) the algorithm is divided into five steps and within each step the following actions are performed:

**Load_Data:** The information about the current environment, as it is saved in the preprocess step, is read from a file.

**Insert.Tx:** The visible points from the transmitter and their respective angles are calculated. It corresponds to the root of the tree in Fig. 5.5.
5.4. Database Preprocessing Method

Ray Tracing Algorithm

- Load_Data
- Insert_Tx
- Find_Nodes
- Find_Total_Field
- Save Results
- End

- The transmitter coordinates, the number of reflections, diffractions and the total number of interactions are determined, as well as the roughness factor, walls' thickness and effective walls' electrical properties.

Figure 5.6: The architecture for the ray-trace algorithm.

**Find_Nodes:** The basic core of the ray-trace algorithm. Every point, either it is a tile, a segment or a receiving point is considered as a node, and its field is calculated. Only the non-receiver points can give children nodes (Fig. 5.5). The image test is performed for the children of each node, discarding the spurious ones, and the decomposition of the incident field into its soft and hard components occurs. The reflected field is computed using the reflection coefficients for the air-layer-air model. For the diffraction case, the UTD diffraction coefficients are implemented. In the case of consecutively diffracted rays, a second-order diffracted field is also considered. Finally, since diffraction phenomena are very sensitive to the impinging angle, the actual angles between the parent and the child nodes of the current node are found, so as to minimise the error in the prediction.

**Find_Total_Field:** The total field for a specific receiver \( E_i \) is computed by coherently adding the fields of the nodes that correspond to that receiver, i.e.

\[
E_i = \sqrt{E_1^2 + E_2^2 + E_3^2 + \ldots}
\]  

(5.1)

**Save Results:** The results from the ray-tracing and the environment are stored in a file.
5.5 Measurement Campaign

A measurement campaign was conducted in different places in London and for different configurations [107]. A continuous wave mode transmitter was used, with frequency 2.1 GHz. The transmitter was set on a stationary place, whereas the receiver was mounted on a moving vehicle. The receiver was recording 1,000 samples per meter of traveling distance and the large-scale average value was stored, repressing thus the fast fading effects.

Three different locations were measured in London; Portland Place area (Fig. 5.7), a dense urban area with high buildings, low vegetation and wide streets, Holborn (Fig. 5.8), a dense urban area with high buildings, high vegetation and narrow streets, and Kingsland (Fig. 5.9), an urban area with low buildings, medium height vegetation and wider streets. The transmitter and the receiver were set to various heights and the maximum path loss the receiver could measure was 147 dB.

5.6 Comparison Analysis

Path loss predictions for the aforementioned areas were performed by the ray-trace algorithm. In order to include border effects, the total simulated area for each case is set to 1 km x 1 km. After the determination of the buildings, the walls are divided into tiles and segments, as described in the previous sections. The tile size plays a significant part in the algorithm implementation; if it is very large, then a larger prediction error is expected, whereas small tiles increase considerably the memory requirements, which must be compensated by a reduction in the total simulating area. Therefore, there is a trade-off between simulation area, tile size, accuracy and efficiency. The tile size is chosen to be 10 m, which gives fairly accurate predictions [102,103].

The walls are assumed to be flat, but in order to include the surface protuberances, a constant Rayleigh roughness factor is set equal to 0.6. The noise floor in every measurement place is considered to be the maximum measured path loss. In order to have a fairer comparison, this limit was also incorporated into the prediction result, i.e. if the predicted path loss at a receiver position is greater than the noise floor,
Figure 5.7: Measured path loss for Portland Place: Tx is at 3 m height and Rx is (a) at 1.5 m and (b) at 0.5 m.
Figure 5.8: Measured path loss for Holborn area: (a) Tx is at 3 m and Rx is at 0.5 m, (b) Tx and Rx are at 1.5 m height.
Figure 5.9: Measured path loss for Kingsland: (a) Tx and Rx are at 1.5 m and (b) Tx and Rx are at 0.5 m.
5.6. Comparison Analysis

then it is replaced by that limit. The maximum level of interactions is eight, with no restrictions on the maximum number of reflections or diffractions, meaning that a ray can be reflected or diffracted eight times, or with any combination between these two. Finally, the transmission through buildings is considered negligible.

Since information about the electrical properties of the buildings in the environment is not available, the effective values of the constitutive parameters need to be approximated. Through simulations, it is found that the values $\varepsilon_r = 10$ and $\sigma = 8 \text{ S/m}$ give satisfying results. The high value of $\sigma$ is anticipated, as the wall surfaces are a complex pattern of brick, metal and glass [98]. The thickness of the wall does not affect the predicted path loss and therefore it is assumed to be 30 cm for all the walls.

Comparing the simulation results with the measurement data (Fig. 5.10a), a quite good agreement is observed. The prediction of the path loss for LOS receivers that are very close to the transmitter (which correspond to the lower points in Fig. 5.10c) is in extremely good accordance with the measured one. However, as the distance increases, the predicted path loss is underestimated, due to the effects of obstacles in the environment that are not considered in the modeling and may obstruct the direct path, like trees and cars. Also, the inclusion of more objects in the first Fresnel zone (as a result of the small height of the receiving antenna), which increase the actual propagation loss, contributes to the incorrect prediction.

For the NLOS receivers, diffraction is the main propagation mechanism. Even though the algorithm can predict a large number of rays, it is possible that some diffracted paths may be omitted, due to the segmentation of the edges. The simplification on the roofs' shapes and heights may have also affected the field prediction, especially for receiver points that are separated from the transmitter by a single building (Fig. 5.10d), as well as that the penetrated field through the buildings is not negligible for that case.

If the height of either the transmitter or the receiver is decreased, it is expected that the path loss would increase, due to the possible blockage of the rays. This is evident when the path loss exponents for the LOS receivers are examined (Fig. 5.10d and 5.11d), where the increase in the path loss exponent verifies the latter statement. However,
5.6. Comparison Analysis

Figure 5.10: The comparison of the ray-trace algorithm results with the measurement data for the Portland Place area, where the transmitter is at 3 m height and the receivers are at 1.5 m. (a) The prediction error for every measurement point. (b) The predicted results agree well with the measurements. (c) The predicted path loss is plotted against the measured path loss. The line corresponds when they are in perfect agreement. (d) The path loss exponents for LOS and NLOS receiver points.
5.6. Comparison Analysis

Figure 5.11: The comparison of the ray-trace algorithm results with the measurement data for the Portland Place area, where the transmitter is at 3 m height and the receivers are at 0.5 m. (a) The prediction error for every measurement point. (b) The predicted results do not agree well with the measurements. (c) The predicted path loss is plotted against the measured path loss. The line corresponds when they are in perfect agreement. (d) The path loss exponents for LOS and NLOS receiver points.
5.6. Comparison Analysis

Figure 5.12: The comparison of the ray-trace algorithm results with the measurement data for the Holborn area. In the left side, the Tx height is 3 m and the Rx is at 0.5 m, whereas in the right side, the Tx and the Rx are at 1.5 m. In (a) and (b), the measured and predicted path loss are plotted for every receiving point. In (c) and (d), the predicted path loss is drawn with respect to the measured one. In (e) and (f), the path loss with respect to distance is plotted.
such a behaviour is not demonstrated for NLOS positions. This is partly explained by the fact that the main propagation mechanism for receiver points away from the transmitter is the over-the-rooftop diffraction, which is unaffected by small changes in the transmitter’s or receiver’s height. Another reason is also the fact that the noise floor is not constant for every measurement scenario, which is apparent in Fig. 5.12a and b. The noise floor may also vary during the same measurement scenario, possibly due to weather conditions or overheating of the receiver (Fig. 5.11b), which contributes to the increase of the prediction error as well.

Another important parameter, affecting the path loss prediction, is the local surrounding of the transmitter and receiver during the measurement run. In contrast to the ray-trace algorithm, which produces smooth results for receiver points that are close to each other, the local environment of the transmitter and the receiver is not static (large vehicles like buses and lorries may block the antennas for a certain period) and therefore may lead to a high variation of the received field in a local area and an increase in the prediction error.

Finally, Kingsland is a typical residential area with low buildings and high trees along the roads. Even though, the over-the-rooftop diffraction is expected to be quite dominant, the high trees block many of the possible paths and this is why the path loss is underestimated (Table 5.1). Nonetheless, it can provide with more accurate results than in a similar scenario in a more complex environment (Fig. 5.12d and 5.13c).

The overall comparison results of the ray-trace algorithm with the measurements are presented in Table 5.1. It shall be noted, that only the NLOS receivers were used for the calculation of these statistics, as these points are more resilient to the unmodeled objects in the environment. In general, it is observed that as the receiver height is decreased, the more difficult it is to accurately predict the path loss, as the probability of obstacles blocking the rays is increasing.
5.6. Comparison Analysis

Figure 5.13: The comparison of the ray-trace algorithm results with the measurement data for the Kingsland area. In the left side, the Tx and the Rx are at 1.5 m above the ground, whereas in the right side, the Tx and the Rx height is 0.5 m. In (a) and (b), the measured and predicted path loss are plotted for every receiving point. In (c) and (d), the predicted path loss is drawn with respect to the measured one. In (e) and (f), the path loss with respect to distance is plotted.
5.7. Conclusions

A fully 3D ray-trace model was presented in this chapter. The model is based in an intelligent preprocess of the environment data, which enables us to avoid repeating steps during the ray-tracing procedure and augmenting hence its efficiency. The ray-trace simulator provides encouraging results when compared with measurements, even though the transmitter, in contrast to other ray-tracing tools in the literature, is placed well beneath the rooftop level. No empirical formulations have been applied to enhance its performance and a flat terrain was also assumed, taking into consideration though the local ground level for each building.

Nonetheless, inaccuracies on the actual locations of the transmitter and receivers, as well as on the actual positions of the buildings, may have deteriorated the quality of the predictions, especially when these are performed at street corners or, in general, near the shadow boundaries of the obstacles. Another limiting factor of the deterministic models is the simplification of the walls as flat surfaces and the approximation on their effective electrical parameters.

The current algorithm can be applied for indoor wave propagation phenomena, as well. This is demonstrated in the next chapter, as well as the modifications needed, to implement a UWB ray-trace model in the TD, so as to accommodate the dispersive nature of the channel.

### Table 5.1: Comparison results for the ray-trace algorithm

<table>
<thead>
<tr>
<th>Place</th>
<th>Tx Height (m)</th>
<th>Rx Height (m)</th>
<th>Mean Error (dB)</th>
<th>Std (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portland</td>
<td>3</td>
<td>1.5</td>
<td>-3.81</td>
<td>8.46</td>
</tr>
<tr>
<td>Portland</td>
<td>3</td>
<td>0.5</td>
<td>-4.46</td>
<td>9.25</td>
</tr>
<tr>
<td>Holborn</td>
<td>3</td>
<td>0.5</td>
<td>-2.37</td>
<td>11.46</td>
</tr>
<tr>
<td>Holborn</td>
<td>1.5</td>
<td>1.5</td>
<td>-3.03</td>
<td>8.38</td>
</tr>
<tr>
<td>Kingsland</td>
<td>1.5</td>
<td>1.5</td>
<td>1.07</td>
<td>8.25</td>
</tr>
<tr>
<td>Kingsland</td>
<td>0.5</td>
<td>0.5</td>
<td>4.72</td>
<td>9.71</td>
</tr>
</tbody>
</table>
Chapter 6

Indoor Narrowband and
Time-Domain Ray-Trace Model

6.1 Introduction

The increase in the market for indoor wireless facilities is very evident, both for commercial and domestic purposes. The need for high-speed data-rate services and the reduced cell size of the mobile networks made the planning of such systems of high importance. A proper exploitation of the resources will not only have a beneficiary effect financially, but also the wireless provider will be able to comply with health regulations, an area that has attracted increased interest nowadays.

Statistical models can be applied adequately in macrocellular channels. However, the variations on building shapes, size and type of construction materials demand a more accurate representation of the propagation environment, as their characteristics will differ greatly from building to building. Measurements are a very tedious and expensive process, that provide accurate results as long as the channel has not changed greatly, as well as offering a limited database for statistical analysis. On the other hand, simulations of the propagation effects in a site-specific environment offer a flexibility in the channel changes and a potential increase in the statistical input dataset.

The purpose of the simulation is to determine the propagation loss and the time dis-
6.1. Introduction

persion of the channel. Candidates for indoor field predictions are algorithms based on the method of moments (MoM), finite-difference time-domain (FDTD) or ray-tracing (RT). In MoM, the differential equations that describe the physical phenomena, formulated as integral equations, are solved by applying specific boundary values into the integral equation [108, 109]. As a result, the integral equations can then be used to calculate numerically the field at any point inside the surface boundary. The boundary conditions can be calculated by discretising the boundary surface into small areas (less than the signal wavelength) and apply a suitable fundamental solution (Green’s functions). MoM provides with very accurate results but the treatment of inhomogeneous and non-linear problems is quite cumbersome, as well as it can be very computation intensive if the modeling area is large and the signal wavelength is small. In FDTD, the entire space is divided into cells, in contrast to MoM where the discretisation occurs on a surface, and the electromagnetic fields are calculated in each cell for every instant of time, until a steady-state behaviour is observed [110, 111]. It is a time-domain solution and therefore it can predict the response of the channel over a wide range of frequencies, it is simple to implement, as the dielectric properties of each cell are easily assigned, and it can achieve comparable accuracy with the MoM. However, the cell size is related to the maximum excited frequency and therefore for large areas and high frequencies the discretisation step must be small and as a result it will increase the memory requirements [93, 97, 112]. On the other hand, ray-tracing models can upheave such complexities, providing faster and accurate results, comparable to those achieved with the MoM and FDTD methods [97, 113, 114].

In this chapter, the ray-trace algorithm that was introduced in the previous chapter is applied to indoor propagation environments and it is compared against measurements. The comparison exhibits a quite good agreement, which strengthens the versatility of the current algorithm. The model is then modified, so as to accommodate the frequency selective nature of the propagation channel. Again, the predicted waveforms are compared against measurements, with encouraging results.
6.2 Indoor Measurements

The frequency response of the indoor channel was measured with a 2-port vector network analyzer (VNA) [115]. The VNA (ZVCE by Rohde & Schwarz) can measure the frequency response from 20 KHz to 8 GHz. The VNA sweeps the band of interest with sinusoidal signals and determines the frequency and time response of the channel from the returned output.

The band of interest is the 3 GHz to 8 GHz, which corresponds to a large part of the available UWB band (3.1 GHz to 10.6 GHz). The VNA limitations did not allow measurement of the whole UWB band. The 5 GHz-bandwidth was divided into 1601 points, which equals to a frequency separation of $\Delta f = 3.125$ MHz. The transmitted signal in port 1 was inserted into a 30-dB amplifier (ZVE-8G by Mini-Circuits) and then fed to an approximate 90 m low-loss coaxial cable (Aircom Plus). The receiver antenna was connected to the second port of the VNA via an approximate 10 m cable (Fig. 6.1). In order to exclude the effects of the amplifier and the cables in the path loss measurements, the apparatus was calibrated by connecting the cables of the two ports at the antennas’ points.

![Figure 6.1](image)

Figure 6.1: In (a) the block diagram of the measurement apparatus, and in (b) a picture of the UWB antenna that was used.

The transmitter and receiver antennas were omni-directional wideband antennas (EM-6865 by Electro-Metrics). Their radiation patterns were measured in the anechoic chamber (Fig. 6.2 and 6.3). Indeed, the azimuth pattern exhibits an omni-directional characteristic (Fig. 6.2a). The mean value of the gain for each frequency is calculated
and the frequency vs. gain is plotted in Fig. 6.2b. A polynomial fit is applied in the data, in order to calculate the gain for the intermediate frequencies. The shape of the elevation pattern is approximately the same for all the frequencies and therefore the mean measured elevation pattern is calculated (Fig. 6.3). However, it demonstrates an unexpected behaviour; the nulls do not align with the vertical axis, but there is an offset of about 30°. Apart from errors in the measurement setup, the offset may be a result of the antenna construction; it consists of two brass biconical elements connected point-to-point to form a hourglass shaped antenna, whose radiation center may not be the actual physical center of the antenna. Nonetheless, it is not possible to know the exact direction of the null axis during the measurement campaign and hence the averaged elevation pattern will be used for simulation purposes (Fig. 6.3).

Since the distances involved are quite large, it is expected that the majority of the rays would arrive in an angle range, which is close to the equatorial plane, and thus the dependency of the radiation pattern on the incoming angle can be ignored, following a simplistic approximation of the antenna effects.

A typical frequency response of the channel, the $s_{21}$ parameter, as it was measured with the VNA can be seen in Fig. 6.4. The magnitude $|H(f)|$ of the response is in dB and the phase in radians. The frequency selective nature of the channel results in deep nulls at certain frequencies in the magnitude response and phase jumps in its phase.

---

**Figure 6.2:** The radiation pattern of the antenna. (a) The antenna is quite omni-directional. (b) A linear fit of the mean gain at every frequency.
6.2. Indoor Measurements

The measurements were conducted on the second floor of the Centre for Communication Systems Research (CCSR) building of the University of Surrey (Fig. 6.5). It is a typical brick wall building with offices, meeting rooms and an anechoic chamber. In order to keep the channel stationary, the measurements took place during the night, when there were very few people in the vicinity. The transmitter was positioned in various places and for each transmitter place the receiver antenna was moved to different locations so as to cover LOS and NLOS situations. However, it is evident from the measurement frequency responses, that the high frequency components for some receiver positions were erroneous, possibly due to a failure of a system's component (amplifier) or to the

Figure 6.3: The mean elevation pattern of the antenna.

Figure 6.4: Typical channel frequency response. The magnitude in (a) is in dB and the phase in (b) in radians.
large value of the IF bandwidth (and hence increased noise). Therefore, only the band between 3 GHz to 6 GHz is used for comparison. These results are discussed in the following sections.

![Figure 6.5: The second floor plan of the CCSR building](image)

6.3 Indoor Narrowband Ray-Trace Model

The algorithm, as in the outdoor case, is divided into two stages: the preprocessing of the data and the ray-tracing. The building plan is read from a file, if such a file exists, or it is manually imported into the PC. The walls and edges are then specified and the environment is discretised into tiles and segments, calculating the angles between them, in the same way as shown in the previous chapter. Penetration through the walls is permitted, except in the case of the elevator and the anechoic chamber. The reflection and transmission coefficients that are used are the ones for the air-wall-air model. Diffraction in indoor environment is not the predominant propagation phenomenon, but still it needs to be considered. The characteristics for every wall are specified and stored in memory and can be found in Table 6.1. The tile and segment sizes are 0.5 m.

6.3.1 Comparison - Results

In order to compare the indoor ray-trace algorithm with measurements, the channel response for $f = 4.5$ GHz was chosen at each location. The antennas' gain for that
6.3. Indoor Narrowband Ray-Trace Model

Table 6.1: Wall parameters

<table>
<thead>
<tr>
<th>Type</th>
<th>$\varepsilon_r$</th>
<th>$\sigma$ (S/m)</th>
<th>d (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brick (external)</td>
<td>5.5</td>
<td>0.018</td>
<td>30</td>
</tr>
<tr>
<td>Brick (internal)</td>
<td>4.5</td>
<td>0.018</td>
<td>15</td>
</tr>
<tr>
<td>Glass</td>
<td>4</td>
<td>0.0005</td>
<td>2</td>
</tr>
<tr>
<td>Wood</td>
<td>2</td>
<td>0.008</td>
<td>10</td>
</tr>
<tr>
<td>Plaster</td>
<td>3.5</td>
<td>0.015</td>
<td>10</td>
</tr>
<tr>
<td>Floor</td>
<td>15</td>
<td>0.018</td>
<td>40</td>
</tr>
<tr>
<td>Ceiling</td>
<td>15</td>
<td>5</td>
<td>40</td>
</tr>
</tbody>
</table>

frequency is approximately 0.65 dBi (Fig. 6.2b). In the first scenario, the transmitter was placed in a corridor and the receiver followed a L-shaped path, both in a direct and an obstructed route (Fig. 6.6a).

The first receiver points in Fig. 6.6, that run along Line 1 (points 1 to 84), are directly visible from the transmitter and, due to the corridor environment, waveguide effects are expected, resulting in the path loss not following the square law distance dependence [116]. Indeed, such a behaviour is supported by the measurements, as well as by the simulations (Fig. 6.6c). An example of the traced paths can be seen in Fig. 6.7a. The receivers in Line 2 (points 85 to 111), starting from a NLOS location in the top in Fig. 6.6a and b, and while running to the bottom, pass through a LOS situation, before ending in a deep NLOS position. The ray-trace model have correctly predicted such behaviour in the path loss. Receivers along Line 3 (points 112 to 141) are NLOS points that are getting deeply obstructed and therefore their path-loss is expected to constantly increase, as it is correctly predicted. The ray-trace results agree extremely well with the measurements and major changes in the propagation characteristics are accurately tracked. It should be noted though that the simulation model offers a smooth path loss prediction, in contrast to the actual measurements, where the statistical nature of the channel is responsible for the local fluctuations.

As the path gets deeply obstructed, the propagation error for every interaction adds
Figure 6.6: Narrowband propagation for a corridor. (a) The measured path loss and (b) the prediction error. (c) The ray-trace prediction follows the path loss changes correctly. (d) The simulated path loss prediction against the measured loss. The transmitter and the receiver heights are 1.37 m.

Figure 6.7: An example of predicted rays in the indoor case. In (a), the receiver is in the corridor, in LOS position, whereas in (b), the receiver is placed inside a room, with no direct sight of the transmitter.
up and hence the overall prediction error of the path loss increases. This is more evident in Fig. 6.8, where all the receiver points are in a non-line-of-sight location. The fluctuations of the measured field in adjacent points are a result of the arriving multipath components, which are summed up at the receiver antenna destructively or constructively. Also, the objects inside the rooms were not taken into account in the simulations, affecting thus the prediction results, especially in the case of the cluttered meeting rooms. However, it should be noted that for the receiver points running along Line 3 in Fig. 6.8a (points 37 to 70), where the route is getting into a more shadowed area (to the right of Line 3), it is expected the path loss to increase, which though it is not evident from the measurements. A reason for that might be the presence of a strong interferer. Nonetheless, even for such an adverse scenario, the comparison of the predicted path loss against the measured one lies within an acceptable limit (Table 6.2). Again, the number of rays traced by the model is quite sufficient (Fig. 6.7b).

The overall performance of the ray-trace algorithm is depicted in Table 6.2. Ray-tracing algorithms calculate the path loss with geometrical accuracy, where the dimensions of the walls affect the prediction [117]. Even though the walls are considered to have a certain width for the reflection and transmission coefficients' calculations, in the actual ray-trace they are treated as infinitely thin. It is very unlikely the walls to be homogeneous as well, introducing thus another error in the prediction result. Uncertainties in the building construction or the actual receiver positions, or even in the information of the surrounding objects of the environment have an impact in the simulated field, especially for those points that lie near the shadow boundaries between LOS and NLOS areas.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Mean Error (dB)</th>
<th>Std (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corridor</td>
<td>0.04</td>
<td>5.75</td>
</tr>
<tr>
<td>NLOS</td>
<td>-0.50</td>
<td>8.37</td>
</tr>
</tbody>
</table>
6.4 Time-domain Ray-Trace

In narrowband channel modeling, the path loss prediction occurs in the frequency-domain. However, when wideband phenomena are treated, parameters like maximum excess delay, rms delay spread and number of multipaths are calculated from the channel’s time-domain response. It appears thus to be more efficient, as well as more natural, to work directly in the time-domain, if closed-form TD solutions are available, than to apply numerical inverse Fourier transforms on the frequency-domain results [87]. Another benefit of working in the time-domain is the fact that TD solutions are always valid, irrespective of the transmitted signal’s bandwidth, whereas in FD, an increase in the bandwidth leads to an increase in the number of points, rendering hence the FD...
prediction to be very inefficient in terms of memory requirements.

Narrowband models are usually applied for wideband channels, where the channel response is assumed to be a train of properly weighted delayed impulses. However, such models do not consider the effects of the channel on the shape of the transmitted signal, which is a result of its frequency dispersive nature and leads to pulse distortion. Signal distortion is unavoidable and affects the system's performance [118]. Proper modeling of the channel response is desired, especially when equipment limitations cannot provide adequate information on the channel characteristics.

6.4.1 Current algorithm

A time-domain ray-trace model is constructed, using the narrowband one as its basis. Nevertheless, since the model is in time-domain, it has the capability of including the frequency selective characteristics of the environment in the prediction and thus the distortion that it causes in the propagating signals. The per path response is calculated directly in the TD by incorporating the TD coefficients that were introduced in the previous chapters. The response of each path is the convolution of every interaction the wave undergoes in the TD (reflection, refraction or diffraction). Thus, the response of the \( m \)-th multipath, for a receiver at an arbitrary location \( P \), is given by

\[
\begin{align*}
\hat{h}_m(t - \hat{t}^P_m) &= \prod_i r_m^i(t) \star \prod_j t_m^j(t) \star \prod_k d_m^k(t) \star \delta(t - \hat{t}^P_m)
\end{align*}
\]

where \( \prod \ast \) denotes convolution product, \( r_m^i(t) \), \( t_m^j(t) \) and \( d_m^k(t) \) are the reflection, transmission and diffraction coefficients in the TD respectively for the \( m \)-th multipath and \( \delta(t - \hat{t}^P_m) \) represents the elapsed time \( \hat{t}^P_m \) that is required for the signal to traverse the total path. The total received signal in that location would be then the convolution of the transmitted signal with the summation of the responses of all the arriving multipaths, i.e.

\[
wp(t) = s(t) \ast \sum_{m=1}^{L} \hat{h}_m(t - \hat{t}^P_m)
\]
where \( w_P(t) \) and \( s(t) \) are the received and transmitted pulses and \( L \) is the number of multipaths.

Previous work on TD ray-trace models either incorporated incomplete formulation or simplified structures or there was not comparison with measurements [62, 64, 119, 120]. In the current algorithm, an actual environment is considered (the second floor of the CCSR building). The propagating pulse is allowed to be reflected, refracted or diffracted numerous times, before it is received and the predicted channel response is compared against measurements. The signal, upon impinging a surface, is depolarised into its hard and soft components. However, it should be noted that in contrast to the narrowband case, due to the fact that the ratio of the two components, soft and hard, is not constant in time, it will lead to a propagating wave whose polarisation rotates in time. Nonetheless, the representation of such a phenomenon would require a large memory, in the sense that for every instance of time a polarisation vector should be issued. Therefore, in order to simplify the ray-trace algorithm, the polarisation vector of the impinged wave is calculated with respect to the maximum values of each polarisation component.

### 6.4.2 Hermitian Processing

The VNA measures the response of the channel for the defined frequencies. In order to compare it with the simulations, the frequency-domain data needs to be converted into the time-domain. This is achieved using the Hermitian Processing [18, 121]. In doing so, the response of the passband signal is zero-padded from the lowest frequency down to DC and from the maximum frequency up to an arbitrary high frequency, and then it is conjugate reflected to the negative frequencies (Fig. 6.9). The expanded response is transformed into the time-domain using IFFT. The benefit of the Hermitian approach is that the corresponding TD signal is a real one, with greater resolution when compared with the baseband method, where the measured response is transformed directly into the time-domain.
6.4. Time-domain Ray-Trace

Figure 6.9: The Hermitian Processing. The signal in (a) is zero-padded in (b) and it is conjugate reflected to the negative frequencies in (c).

6.4.3 Antenna Effects

The VNA data includes the antenna effects as well, i.e.

\[ H_{Meas}(f) = G_{Antennas}(f) \cdot H_{FSL}(f) \cdot H_{Channel}(f) \]  \hspace{1cm} (6.3)

where \( G_{Antennas}(f) \) is the gain of the two antennas, \( H_{FSL} \) is the free space loss (FSL) and \( H_{Channel}(f) \) is the actual response of the channel. The antenna effects were calibrated by dividing the measured radio channel with the averaged antenna gain (Figure 6.2b). The direction dependence of the antenna impulse response is thus ignored, so as to simplify the calibration process. However, since the UWB antennas have complex patterns that vary with frequency and radiation angle, the actual deconvolution process, in a more thorough analysis, would require a separate computation for the different directions of the incoming multipath components.

The ray-trace model provides the response of the channel, without the free space loss. The free space loss is a result of the antenna aperture, i.e. the ability of the antenna to gather the surrounding radiating energy. It affects the signal's shape, as it acts like a
low-pass filter [17]. The FSL, excluding the distance dependence which is incorporated in the channel response, is given by

$$H_{\text{FSL}} = \frac{\lambda}{4\pi}$$

(6.4)

where $\lambda$ is the signal’s wavelength and equals $\lambda = c/f$, where $c$ is the speed of light in free space and $f$ is the frequency of the signal. Therefore, since $\mathcal{F}^{-1}\{\frac{a}{\omega}\} = \text{sgn}(t)$, the time-domain representation of the FSL will be

$$h_{\text{FSL}}(t) = \frac{jc}{4\text{sgn}(t)}$$

(6.5)

where sgn is the signum function and $\mathcal{F}^{-1}$ signifies inverse Fourier transformation.

The received waveform thus, in the TD, is written as

$$r(t) = s(t) * h_{\text{FSL}}(t) * h_{\text{Channel}}(t)$$

(6.6)

where $s(t)$ is the transmitted signal. Since

$$f_1(t) * (f_2(t) * f_3(t)) = (f_1(t) * f_2(t)) * f_3(t)$$

(6.7)

and if $f(t)$ and $g(t)$ are integrable functions, with $\frac{df}{dt}g(t) = f(t)$ and $\lim_{t \to \pm \infty} f(t) = \lim_{t \to \pm \infty} g(t) = 0$, then

$$\text{sgn}(t) * f(t) = 2 \int f(t) \, dt = 2g(t)$$

(6.8)

and therefore

$$r(t) = w(t) * h_{\text{Channel}}(t)$$

(6.9)

where $w(t)$ is the indefinite integral of $s(t)$.

6.4.4 Comparison - Results

The path loss for wideband signals is given by [122]

$$PL = \frac{1}{BW} \int_{f_{\text{min}}}^{f_{\text{max}}} |H(f)|^2 \, df$$

(6.10)

Comparing the measured path loss for the UWB case, where the whole bandwidth of 3 to 6 GHz is used, with the narrowband one at 4.5 GHz, an increase in performance...
6.4. Time-domain Ray-Trace

is anticipated (Fig. 6.10). This is expressed as a decrease in the total path loss, due to the fact that the multipath components can now be distinguished and exploited, enhancing thus the total received signal’s energy. The points in the graph correspond to the actual measurement points along the lines in Fig. 6.6a and Fig. 6.8a. It is also observed that there is a decrease in the multipath fading, which is represented by the smoothness of the path loss curve, when compared to the narrowband case, which makes UWB signals more robust in multipath environments [48].

![Figure 6.10: The comparison of the UWB measured path loss against the narrowband one, at frequency of 4.5 GHz, for the same scenarios as in Sec. 6.3.1](image)

The ray-trace model provides channel responses assuming almost an infinite bandwidth. In order to compare it with the measurement data, the same conditions should be simulated. Therefore, a sinc wave is assumed to be excited in the TD, which represents the transmission of a square pulse in the FD from 3 to 6 GHz band, similarly to the VNA. The sinc wave is thus written, appropriately shifted in the band of interest [57],

\[
s(t) = 2 \cdot \cos(2\pi St) \cdot \frac{\sin(2\pi Lt)}{\pi t} \tag{6.11}
\]

where \( S = 4.5 \) GHz is the middle of the exciting spectrum, and \( L = 1.5 \) GHz is a half of the total bandwidth. In order to include the free space loss component, it will be

\[
w(t) = \frac{jc}{\pi} \cdot \int \cos(2\pi St) \cdot \frac{\sin(2\pi Lt)}{t} \, dt = \frac{jc}{\pi} \left( \text{si}(2\pi L - St) + \text{si}(2\pi L + St) \right) \tag{6.12}
\]

where \( \text{si}(t) \) is the sine integral. Therefore, the predicted impulse response is

\[
h_{predicted}(t) = w(t) \ast h_{channel}(t). \tag{6.13}
\]
The predicted TD channel response is compared against the measured one (Fig. 6.11). For LOS receivers that are close to the transmitter, the predicted and measured received waveforms are in an extremely good agreement (Fig. 6.11a). The main contributions come from the direct path and first order reflections, which are correctly tracked by the ray-trace algorithm. An offset of about a fraction of nanosecond is also observed for some predicted multipath signals, which is though explained by the actual construction of the ray-trace algorithm; the environment has been discretised and the actual reflection points are assumed to be the centre of the tiles. Therefore, there is a path difference, of the order of the tile size, on the total predicted traversed distance, which the ray has traveled, with the actual one. Also, inaccuracies in the actual receiver positions with the simulated ones explain the small timing mismatch for some LOS points.

As the receiver travels further away from the transmitter, the multipath components contribute more to the total received energy and therefore they play a more important role in the system's performance (Fig. 6.11b). However, the ray-trace algorithm can only predict a number of them, due to the limitations on the available memory; there is a trade-off between the level of interactions, tile size and time resolution, which affects the accuracy of the ray-trace model.

For NLOS receivers (Fig. 6.11c and d), the energy is spread over many multipaths. Inaccuracies in the modeling of the environment, like the existence of unmodeled objects and the actual electrical properties and dimensions of the modeled walls, are more evident and affect more severely the predicted waveform. Nonetheless, a quite good agreement with the measurements is observed.

The path loss prediction is compared against the measured one (Fig. 6.12). The predicted path loss follows correctly the measured path loss, in almost all of the cases. However, as the energy is scattered more evenly in the multipath components, especially in deep NLOS situations, as in the last points of the first and second scenarios (Fig. 6.12a and c), more levels of interactions are needed to predict an adequate number of rays. This is analogous to a RAKE receiver; the fingers of the receiver correspond to the levels of interaction. As more fingers are considered (more levels in the algorithm), more
6.4. Time-domain Ray-Trace

Figure 6.11: The measured and predicted channel response for receiver points of the first scenario. (a) A close to the transmitter LOS receiver point, (b) a distant LOS point, (c) a NLOS point and (d) a deep NLOS received waveform.

multipath energy is received (predicted) [47]. Nonetheless, due to memory restrictions, there is a limitation on the permitted level of interactions and thus the prediction error increases for deep NLOS receivers. On the other hand, in a practical UWB system, the receiver will be within $10\text{m}$ from the transmitter, for which the current algorithm gives good results.

Another important observation is the large disagreement of the path loss prediction between points 40 to 70 in Fig. 6.12c, which travel along Line 3 in Fig. 6.8a. Intuitively, it is expected that the path loss would increase, as the receiver points are getting deeper in an NLOS situation, similar to the ray-trace model prediction. However, this is not the
6.4. Time-domain Ray-Trace

Figure 6.12: The predicted TD path loss is compared against the measured one. In the measured path loss, it is assumed that all the multipath components have been detected and received.

case with the measurements. This might be due to the presence of an unidentified strong interferer or to a possible momentary failure of the equipment (due to overheating). This large disagreement also partly explains the large standard deviation of the results in Table 6.3.

Measurements offer only a limited set of data and they are very time consuming. Additionally, the measurement of a very large bandwidth would require expensive equipment. In practical terms also, an increase in the bandwidth would mean an increase in the number of points, which by itself imposes restrictions on the actual measurement capabilities. Finally, the channel information, which is acquired by the measurement
6.4. Time-domain Ray-Trace

Table 6.3: Comparison results for the TD-UWB ray-trace algorithm

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Mean Error (dB)</th>
<th>Std (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corridor</td>
<td>3.66</td>
<td>4.80</td>
</tr>
<tr>
<td>NLOS</td>
<td>0.91</td>
<td>7.23</td>
</tr>
</tbody>
</table>

campaign, is only valid for that frequency band.

On the other hand, the TD ray-trace model can be applied for almost an infinite bandwidth (practically, the maximum bandwidth is $2/\Delta t$, where $\Delta t$ is the time step discretisation). The predicted channel responses can be convolved with any signal to produce the received waveform at a specific location. To demonstrate that, it is assumed that two Gaussian doublet signals with $\tau_1 = 50$ ps and $\tau_2 = 200$ ps are excited (Fig. 6.13), with equal energies, which are given by

$$s(t) = -\frac{1}{\tau} \sqrt{\frac{\tau}{3 \cdot \sqrt{\pi/2}}} \left(1 - \frac{2 \cdot t^2}{\tau^2}\right) \cdot \exp\left(-\frac{t^2}{\tau^2}\right).$$

(6.14)

The free space loss component would include the indefinite integral of $s(t)$ and it would
6.5. **Conclusions**

Thus, the received waveforms of the two signals are

\[ r(t) = w(t) \ast h_{\text{channel}}(t). \]  

(6.16)

These are depicted for a receiver location in Fig. 6.14. The received waveform in Fig.

![Graphs](image)

**Figure 6.14**: The received waveforms for the same location for two Gaussian doublets with different \( \tau \). (a) \( \tau = 50 \) ps and (b) \( \tau = 200 \) ps.

6.14a offers a finer resolution than the one in Fig. 6.14b. This is very well expected since the bandwidth of the first signal is much larger than the bandwidth of the second signal. Due to multipath collisions, the received waveform in the second case is more distorted than the first one, which will degrade the performance of the system. It is also evident that the received signals do not resemble the transmitted ones, which is a very important information for the receiver design.

### 6.5 Conclusions

An indoor ray-trace model model was presented in this chapter. The model was based on an intelligent database preprocessing algorithm with good results for narrowband signals. The mean path loss was accurately predicted, with a standard deviation within acceptable limits.
6.5. Conclusions

The same model was modified to accommodate the frequency selective nature of the environment. Doing so, the signal distortion can be predicted, offering a valuable tool for system planning. The ray-trace model operates in an efficient manner directly in the TD, providing with a good insight of the propagation phenomena.

The performance of the TD ray-trace algorithm is however affected by the available memory, as more interactions may be needed to achieve a good prediction. This is the case especially for deep NLOS receiver points. Nonetheless, since it is expected that UWB systems would operate in office environments, within a 10 m distance from the transmitter, the performance of the current ray-trace model is quite encouraging.

The ray-trace model also showed that inaccuracies in the modeling affect the prediction error. This error is quite small in the narrowband case but it has a greater impact when transmitting UWB signals, due to the increased resolution they can offer, which can provide with a more detailed information of the environment.
Chapter 7

Conclusions

7.1 Overview and Discussion

The work in this thesis focused on the prediction of the distortion of UWB signals as these propagate in the environment. The change on their waveform shape affects the receiver design and consequently its performance.

In the first part of the thesis, Chapter 2, an introduction into the ultra wideband technology was presented. UWB employs signals with extremely large bandwidth and therefore, it is inevitable to coexist with other technologies operating in the same frequency band. In order to minimise the interference with these systems, very low power masks are issued by the regulatory bodies, in which UWB systems are confined to operate. A key benefit of the large bandwidth is the increase in the data rate and the multipath resolution that can be achieved, as the multipath components can now be distinguished and resolved. Also, a decrease in the fading margin, as well as an enhancement in the system’s performance is anticipated, when compared to narrowband systems. On the other hand, in order to exploit the multipath resolution, the receiver needs to be designed more elaborately, increasing its complexity, which also means to be able to compensate the signal distortion that unavoidably occurs.

The waveform distortion was initially treated in Chapter 3. A signal, when propagating in the environment, undergoes a number of interactions, like reflection, refraction
and diffraction. These phenomena are dependent on frequency. Therefore, a UWB signal, due to its large bandwidth, will have a different response in the lower and higher frequencies and thus distortion occurs. It is more natural to treat such transient phenomena directly into the time-domain, but it also appears to be more efficient, than to apply inverse Fourier techniques on the respective frequency domain solutions. Previous work included the TD reflection coefficients for hard and soft polarised waves for the half-plane [8] and TD solutions for diffracted signals on a variety of canonical objects [9–11]. For diffraction phenomena, the uniform theory of diffraction is proved to provide with accurate results for high frequencies [12] and therefore it is the base for the TD solutions. However, the propagating wave is hardly expected to be obstructed by a single object. The extension of the effects of multiple interactions on a propagating signal was treated in Chapter 4. More precisely, accurate and easy-to-use formulations for the reflection and transmission coefficients for the slab case were deduced, which accommodate the multiple internal reflections [75]. In contrast to other existing solutions [63], the proposed ones accurately predict a great number of internal reflections without increasing the solutions’ complexity. An algorithm to predict the multi-diffracted waveform is also presented, which correctly track the changes in its shape, when a number of absorbing or imperfect conducting obstacles block the traversed path [87]. This was accomplished by representing in closed-forms the higher order diffracted fields in the time-domain. It was shown that multi-diffraction phenomena distort greatly the transmitted signal, but also that it is more effective to work directly in the time-domain than in the frequency-domain and applying an inverse Fourier transformation.

As a next step, a path-finding model was constructed. The model was presented in Chapter 5 and it was based on an intelligent preprocessing of the database. It is divided into two parts; the database preprocess and the ray-trace. In the first step, the environment is discretised into tiles and segments and their respective angles are calculated and stored into memory. In the ray-trace step, the transmitter is inserted and the actual ray tracing occurs. This is achieved by checking if the angles fulfill certain conditions, which practically translates the path finding into an appropriate search in a look-up table. The current model combines the strengths of the image theory and ray-
limitation Factors for Signal Prediction

launching, overcoming some of their weaknesses; diffraction can be treated more easily and all the possible impinging objects are recognised. However, it does suffer from some of their shortcomings; double-counting rays need to be recognised and excluded, a process that slows down the algorithm, and also the memory requirements may be high, if the tile size is small and the simulation area is large. Nonetheless, it was shown that even for tile size equal to 10 m, fairly accurate results can be achieved for areas of 1 km x 1 km, without increasing greatly the processing power. The comparison of the predicted path loss with narrowband measurements in various places in London proved the validity of the algorithm. A great number of paths are predicted and there is no restriction on the maximum number of diffractions occurring in each ray, strengthening its value.

The versatility of the ray-trace algorithm was presented in Chapter 6. The model can be equally applied in an indoor environment. Similarly to the outdoor case, the building is modeled in a PC and saved in a file. Again every wall and edge are discretised into tiles and segments and their relations are stored into memory. The ray-trace step is performed and the field for every location is calculated. The comparison with the measurements showed a quite good agreement. The algorithm was also modified, so as to incorporate the frequency selective nature of the propagation environment. The TD ray-trace model predicts the per path signal distortion in an efficient manner. In contrast to frequency domain solutions, the current model is equally applied for signals with variable bandwidth, without any change in its operation, whereas an increase in the number of frequency point would be necessary in the former case, increasing thus the memory requirements. The tool can then be used for capacity coverage of UWB systems and cell planning.

7.2 Limitation Factors for Signal Prediction

The deterministic prediction of the received field offers a great capability in system planning. The mean path loss can be estimated and a better exploitation of the available resources can be achieved. Nonetheless, site-specific prediction is not a panacea. Limiting factors can degrade the efficiency of the model and the actual random nature
7.2. Limitation Factors for Signal Prediction

of the real environment prohibits the construction of the perfect model. Some of these limitations are presented here.

7.2.1 Environmental Modeling

The actual representation of the surrounding environment relies on the existing information about the locations and heights of the buildings in the area, as well as the ground level. This information is acquired through topographical surveys and it is stored in a file with a specific format. However, it is difficult to have position accuracy better than 0.5 m, which is in most cases greater than the wavelength, and therefore the accuracy of the ray-trace prediction is in question. Also, the walls are assumed to have a certain width, which is used in the coefficient calculations. However, when the actual ray-trace occurs, the walls are represented as infinitely thin and therefore the site-specific path searching, being an absolute geometrical process, may allow spurious rays. The imperfections of the surrounding modeling are more evident in the TD ray-trace algorithm, since the distortion and delay occurring on each path can be viewed. Therefore, the shape of the predicted channel response will be different from the measured one.

Information about the electrical properties of the buildings is not given and effective values need to be approximated. The variations on the walls' surfaces, due to windows, doors or other decorative features, may have an impact on the impinging waves, as they will disperse the reflecting energy. Energy can also be scattered due to the presence of movable objects, like people or cars, especially if the height of the transmitter and/or the receiver is low, or small sized obstacles that are usually present in a typical office environment. This energy is sometimes needed to be considered, as it may have a remarkable contribution to the received field.

Furthermore, the shape of the objects is limited to certain canonical objects. The edges are considered sharp and convex obstacles can only be described as multi-faceted surfaces. This may affect the radiowave propagation and increase the prediction error.
7.2. Limitation Factors for Signal Prediction

7.2.2 Ray-trace Limitations - Memory Requirements

Except from the imperfections on the environmental description, the limitations of the ray-trace algorithm itself have an impact on the predicted field. The buildings are simplified as flat top boxes, where information about the shape of the roof is lost. Such information affects mainly the over-the-rooftop propagation, which is important especially for receivers far away from the transmitter.

A large simulated area leads to a large memory requirement. Therefore, a simplification on the available data is sometimes needed, through a sophisticated process; very small or hidden walls can be discarded and the multi-faceted corners may be treated as wedges. Such an action may affect the total result but it would be necessary for a more effective algorithm. The inclusion of the roof shapes, as well as rounded objects, will lead to a complex algorithm, which however may be unrealistic for urban areas. The memory requirements of the proposed ray-trace algorithm in this work also depend on the level of the wall discretisation. Small tile and segment size may provide with accurate predictions, but on the other hand would increase the necessary processing power. A trade-off between the accuracy and the efficiency needs to be established.

The accuracy of the prediction is also in question for the TD path-finding algorithm. Due to the increased bandwidth, many multipaths can be resolved. The comparison between the measurements and the predictions showed that the performance of the latter is analogous to the number of multipaths that are predicted. However, in order to predict an adequate number of multipaths, a higher level of interactions may be necessary, especially for NLOS receivers, increasing thus considerably the required memory.

7.2.3 Multiple TD Diffracted Signals

In UTD formulation, the diffracted field is continuous over the whole space. However, in multiple diffracted fields, this continuity is enforced if second order diffracted fields are considered, with the appropriate distance or L-parameters [13,14]. The prediction of multiple diffracted waveforms in the TD is very accurate when the objects do not lie
in the transition regions, where the distance parameters can be thought as constant. However, these parameters are actually dependent on frequency, and for objects in the transition regions, would lead to a sharper response of the diffracted path. Therefore, the assumption of constant $L$-parameters inserts an inherent error of the predicted waveform. In practical terms though, this error can be alleviated if a finer discrete time representation of the response is used [87]. However, this would increase the memory requirements, slowing down the prediction, through the increase of the needed time for performing the convolution step, rendering an actual algorithm ineffective.

7.3 Conclusion - Future Work

In this thesis, it was shown that it is possible to predict the received shape of a propagating signal directly into the time-domain, if appropriate closed-forms are available. In contrast to many proposed channel models, the actual received signal is severely distorted through its interaction with the environment, which distortion needs to be addressed in the receiver design for enhanced performance. Empirical models may be a solution for narrowband propagation channels, but clearly the increased bandwidth of UWB technology seeks for more elaborate and deterministic solutions. The advances in computer resources will overcome some of the shortcomings of the site-specific approaches, making them an indispensable tool for radio planning.

The performance of such algorithms may be elevated if more TD coefficients describing the environment are inserted, as well as its combination with statistical approaches, to compensate for the random nature of the propagating channel. Appropriate forms to describe reflected and diffracted fields from convex surfaces (ellipsoids, cylinders) would increase the available options for a correct environment description. Such coefficients, however, would also need to describe the higher order diffracted fields, in a similar way as described in Chapter 5, so as to produce a correct prediction in multiple diffracted phenomena. The antenna also plays a very important part in a correct signal prediction. Since realistic antennas do have a flat response with respect to frequency, it is anticipated that they will distort the UWB waveforms greatly and proper knowledge of their response would need be taken into great consideration.
From a programming point of view, it is always useful to make the ray-trace tool, narrowband and TD, even more efficient, in terms of computational time and memory requirements. Experience declares that there is always room for improvement. Finally, it would also be interesting to examine how distortion affects the performance of multishaped modulation schemes. Since the complex environment greatly distorts the propagating signals, which distortion however can now be predicted, it would have an adverse effect on their orthogonality as well.
Appendix A

Derivation of the time-domain reflection coefficient for the half-plane

For the hard polarisation, the Fresnel reflection coefficient is written as

$$R_h(\omega) = \frac{\varepsilon_{r,\text{com}} \cos \theta - \sqrt{\varepsilon_{r,\text{com}} - \sin^2 \theta}}{\varepsilon_{r,\text{com}} \cos \theta + \sqrt{\varepsilon_{r,\text{com}} - \sin^2 \theta}}$$

$$= \frac{\left(\varepsilon_r - j \frac{\sigma}{\omega \varepsilon_0}\right) \cos \theta - \sqrt{\varepsilon_r - j \frac{\sigma}{\omega \varepsilon_0} - \sin^2 \theta}}{\left(\varepsilon_r - j \frac{\sigma}{\omega \varepsilon_0}\right) \cos \theta + \sqrt{\varepsilon_r - j \frac{\sigma}{\omega \varepsilon_0} - \sin^2 \theta}}, \quad (A.1)$$

since $\varepsilon_{r,\text{com}} = \varepsilon_r - j \frac{\sigma}{\omega \varepsilon_0}$. If $j \omega$ is substituted with $s$ and set $\tau = \sigma / \varepsilon$, where $\varepsilon = \varepsilon_r \cdot \varepsilon_0$, it will be

$$R_h(s) = \frac{\varepsilon_r (1 + \frac{\tau}{s}) \cos \theta - \sqrt{\varepsilon_r (1 + \frac{\tau}{s}) - \sin^2 \theta}}{\varepsilon_r (1 + \frac{\tau}{s}) \cos \theta + \sqrt{\varepsilon_r (1 + \frac{\tau}{s}) - \sin^2 \theta}} \quad (A.2)$$

After some manipulation, the above formula can be expressed as

$$R_h(s) = \frac{\varepsilon_r (s + \tau) \cos \theta - \sqrt{\varepsilon_r s^2 + \varepsilon_r s \tau - s^2 \sin^2 \theta}}{\varepsilon_r (s + \tau) \cos \theta + \sqrt{\varepsilon_r s^2 + \varepsilon_r s \tau - s^2 \sin^2 \theta}}$$

$$= \frac{\varepsilon_r (s + \tau) \cos \theta - \sqrt{\left(\varepsilon_r - \sin^2 \theta\right)s^2 + \varepsilon_r s \tau}}{\varepsilon_r (s + \tau) \cos \theta - \sqrt{\left(\varepsilon_r - \sin^2 \theta\right)s^2 + \varepsilon_r s \tau}}$$

$$= \frac{(s + \tau) \varepsilon_r \cos \theta - \sqrt{\varepsilon_r - \sin^2 \theta} \sqrt{s^2 + \varepsilon_r s \tau (\varepsilon_r - \sin^2 \theta)^{-1}}}{(s + \tau) \varepsilon_r \cos \theta + \sqrt{\varepsilon_r - \sin^2 \theta} \sqrt{s^2 + \varepsilon_r s \tau (\varepsilon_r - \sin^2 \theta)^{-1}}}$$

125
By setting $\beta = \frac{\sqrt{\epsilon_r - \sin^2 \theta}}{\epsilon_r \cos \theta}$ and $\gamma = \frac{\tau}{1 - \sin^2 \theta / \epsilon_r}$, the result for the reflection coefficient for the hard polarisation is

$$R_h(s) = \frac{s + \tau - \beta \sqrt{s(s + \gamma)}}{s + \tau + \beta \sqrt{s(s + \gamma)}}$$

(A.4)

Using similar derivation for the soft polarisation, it is

$$R_s(s) = \frac{\sqrt{s - \epsilon_r \beta \sqrt{s + \gamma}}}{\sqrt{s + \epsilon_r \beta \sqrt{s + \gamma}}}$$

(A.5)

For large values of $\epsilon_r$ and small values of $\theta$ so that $\sin^2 \theta / \epsilon_r \ll 1$, $\gamma$ is approximately equal to $\tau$. Thus, the coefficients for the two possible polarisations are written as

$$R_h(s) \approx \frac{\sqrt{s + \tau - \beta \sqrt{s}}}{\sqrt{s + \tau + \beta \sqrt{s}}}$$

(A.6)

$$R_s(s) \approx \frac{\sqrt{s + \tau - (\epsilon_r \beta)^{-1} \sqrt{s}}}{\sqrt{s + \tau + (\epsilon_r \beta)^{-1} \sqrt{s}}}$$

(A.7)

or more generally

$$R(s) \approx \pm \frac{\sqrt{s + 2\alpha - \kappa \sqrt{s}}}{\sqrt{s + 2\alpha + \kappa \sqrt{s}}}$$

(A.8)

where $\alpha = \tau/2$, and $\kappa = \beta$ for hard polarisation, while $\kappa = (\epsilon_r \beta)^{-1}$ for soft polarisation, and the plus sign corresponds to hard polarised waves, whereas the minus one for soft polarised waves. Then

$$R(s - a) = \pm \frac{\sqrt{s + a - \kappa \sqrt{s - a}}}{\sqrt{s + a + \kappa \sqrt{s - a}}}$$

$$= \pm \frac{(1 - \kappa)s + (1 - \kappa)\sqrt{s^2 - \alpha^2} + (1 + \kappa)\alpha}{(1 + \kappa)s + (1 + \kappa)\sqrt{s^2 - \alpha^2} + (1 - \kappa)\alpha}$$

$$= \pm \frac{(1 - \kappa)\left(s + \sqrt{s^2 - \alpha^2} + \frac{1 - \kappa}{1 + \kappa} \alpha\right)}{(1 + \kappa)\left(s + \sqrt{s^2 - \alpha^2} + \frac{1 + \kappa}{1 + \kappa} \alpha\right)}$$

$$= \pm \frac{(1 - \kappa)\left(s + \sqrt{s^2 - \alpha^2} + \frac{1 - \kappa}{1 + \kappa} \alpha + \frac{2\kappa - 1}{1 + \kappa} \alpha\right)}{(1 + \kappa)\left(s + \sqrt{s^2 - \alpha^2} + \frac{1 - \kappa}{1 + \kappa} \alpha\right)}$$

$$= \pm \left(K + \frac{4\kappa}{1 + \kappa} \frac{\alpha}{S + aK}\right)$$

(A.9)

where $K = \frac{1 - \kappa}{1 + \kappa}$ and $S = s + \sqrt{s^2 - \alpha^2}$. The reflection coefficient can be expanded into series, if it is assumed that $|\frac{aK}{S}| < 1$. Then it will be

$$\frac{\alpha}{S + aK} = \frac{a}{S} \left(1 + \frac{\alpha K}{S}\right)^{-1}$$
\[
\sum_{n=0}^{\infty} (-1)^n (\alpha K)^n S^{-n} = \frac{\alpha}{S} \sum_{n=0}^{\infty} (-1)^n (\alpha K)^n S^{-n} = \frac{1}{K} \sum_{n=1}^{\infty} (-1)^{n+1} (\alpha K)^n S^{-n}. \tag{A.10}
\]

Therefore, if the above series is imported into the reflection coefficient, the result will be

\[
R(s-\alpha) = \pm \left( K + \frac{4\kappa}{(1+\kappa)^2} \frac{1}{K} \sum_{n=1}^{\infty} (-1)^{n+1} (\alpha K)^n S^{-n} \right) = \pm \left( K + \frac{4\kappa}{1 - \kappa^2} \sum_{n=1}^{\infty} (-1)^{n+1} (\alpha K)^n S^{-n} \right). \tag{A.11}
\]

By the use of the inverse Laplace transform identity \( L^{-1} \{ F(s-\alpha) \} = e^{\alpha t} f(t) \), where \( f(t) \) is the inverse Laplace transform of \( F(s) \), and \( L^{-1} \{ \left( \frac{n}{(s+\sqrt{\beta^2 - \alpha^2})^n} \right) \} = \frac{n}{i} I_n(\alpha t) \), where \( I_n(\alpha t) \) is the modified Bessel function of the first kind and \( n \)-th order, the reflection coefficient is written as

\[
r_{s,h} = \pm \left[ K_{s,h} \delta(t) + \frac{4\kappa_s h}{1 - \kappa_s^2} \frac{e^{-\alpha t}}{t} \sum_{n=1}^{\infty} (-1)^{n+1} n K_{s,h}^n I_n(\alpha t) \right]. \tag{A.12}
\]

The formula above represents the time-domain coefficient of the reflection coefficient. The convolution of this coefficient with an incident signal gives the reflected effect directly in the time-domain. However, (A.12) is difficult to be utilized due to the infinite sum. The modified Bessel function can be expanded into series, and thus

\[
I_n(\alpha t) = \sum_{k=0}^{\infty} \frac{(\alpha t/2)^{2k+n}}{k!(n+k)!}. \tag{A.13}
\]

Therefore, the infinite sum in (A.12) is written as

\[
\sum_{n=1}^{\infty} (-1)^{n+1} n K_{s,h}^n I_n(\alpha t) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{nK^n}{t} \sum_{k=0}^{\infty} \frac{(\alpha t/2)^{2k+n}}{k!(n+k)!} = \sum_{k=0}^{\infty} f_k \tag{A.14}
\]

with

\[
f_k = \sum_{n=1}^{\infty} (-1)^{n+1} n K^n \frac{(\alpha t/2)^{2k+n}}{t k!(n+k)!}. \tag{A.15}
\]
Equation (A.15) can be written, if $A = K\alpha t/2$ and $X = e^{-A}$ are set, as

$$f_k = \frac{(-\alpha t/2)^k}{tk!K^k} \left( (A + k)X + \sum_{\ell=0}^{k-1} \frac{(\ell - k)(-A)^\ell}{\ell!} \right). \quad (A.16)$$

Finally, the number of terms in (A.16) depend on the convergence of the series, which subsequently depends on the electrical properties of the material on which the reflection occurs and the incident angle. If a number $N$ of terms is assumed to be sufficient, then the TD reflection coefficient for the half-plane, for soft or hard polarisation is written as

$$r_{s,h}^N(t) = \pm \left[ K_{s,h} \delta(t) + \frac{4K_{s,h}}{{K_{s,h}}^2 - 1} e^{-\alpha t} \sum_{k=0}^{N-1} f_k \right]. \quad (A.17)$$
Appendix B

The relationship between the transition function and the complementary error function

The transition function is written as

\[ F[X] = 2j\sqrt{X}e^{jkx} \cdot \int_{\sqrt{X}}^{\infty} e^{-jtu^2} du \]  \hspace{1cm} (B.1)

and the complementary error function as

\[ \text{erfc}(z) = \frac{2}{\sqrt{\pi}} \cdot \int_{z}^{\infty} e^{-u^2} du. \]  \hspace{1cm} (B.2)

If \( u \) is substituted with \( j t \) and \( z \) with \( \sqrt{x} \) in (B.2), it will be

\[ \text{erfc}(\sqrt{x}) = \frac{2\sqrt{j}}{\sqrt{\pi}} \cdot \int_{\sqrt{x}/j}^{\infty} e^{-jt^2} dt. \]  \hspace{1cm} (B.3)

Then, if \( \sqrt{x}/j = \sqrt{s} \Rightarrow \sqrt{x} = \sqrt{j}s \), (B.3) becomes

\[ \text{erfc}(\sqrt{j}s) = \frac{2\sqrt{j}}{\sqrt{\pi}} \cdot \int_{\sqrt{s}}^{\infty} e^{-jt^2} dt \]

\[ \int_{\sqrt{s}}^{\infty} e^{-jt^2} dt = \frac{\sqrt{\pi}}{\sqrt{2\sqrt{j}}} \cdot \text{erfc}(\sqrt{j}s) \]

\[ 2j\sqrt{s}e^{js} \cdot \int_{\sqrt{s}}^{\infty} e^{-jt^2} dt = 2j\sqrt{s}e^{js} \frac{\sqrt{\pi}}{\sqrt{2\sqrt{j}}} \cdot \text{erfc}(\sqrt{j}s). \]  \hspace{1cm} (B.4)
Therefore, the transition function can be written with respect to the complementary error function as

\[ F[s] = \sqrt{\pi s} e^{js} \text{erfc}(\sqrt{js}). \]  \hspace{1cm} (B.5)
Bibliography


[81] P. D. Holm, “UTD-diffraction coefficients for higher order wedge diffracted fields,”  


digital broadcasting coverage prediction,” IEEE Trans. Broadcast., vol. 46, no. 3,  


function for mobile radio wave propagation,” IEE Electron. Lett., vol. 27,  


