Advanced Non-Regenerative Cooperative Relaying Techniques

Mohammad Sadegh Fazel Falavarjani

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UNIVERSITY OF SURREY
Centre for Communication Systems Research Faculty of Engineering and Physical Sciences University of Surrey Guildford, Surrey GU2 7XH, U.K.

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Abstract

In wireless networks, relaying techniques have been proposed to expand the coverage of communication systems and increase the spectral and energy efficiency. In particular, a typical relaying scenario consists of a source, a relay and a destination. The most applicable cooperative relaying techniques are decode and forward (DF), and amplify and forward (AF). DF is a regenerative approach and performs satisfactorily when the source-relay link is strong. Thanks to its regenerative approach, relay can change its transmission format to suit the characteristic of the channel ahead. This will allow the time durations of its receive and transmit phases to be independently adjusted. In contrast, AF has less complexity as well as less cost. It's a non-regenerative scheme that performs a very simple linear processing technique at the relay. Moreover, AF can provide better performance especially when the source-relay link is not reliable enough. However, in the traditional AF, the time durations for the relay-receive and relay-transmit phases are the same. This thesis investigates a flexible cooperative AF scheme that allows separately tuneable phase durations. The achievable rates and the outage probabilities were derived for the proposed AF cooperative protocol. The presented analysis can be used to predict the AF scheme performance and to optimise the duplexing ratio for different network topologies and SNR regions.

In deep static fading channel conditions, it is likely that the transmitted message is not successfully decoded at the destination in the first transmission. In these events, the performance of the system can be improved by retransmitting the message with a suitable combination of channel coding and Automatic Repeat-re-Quest scheme, known as Hybrid ARQ (HARQ). In this thesis, two different HARQ strategies combined with cooperative flexible AF are analysed in order to evaluate the performance of the proposed strategies without any need for time consuming Monte Carlo based evaluation methods.

Furthermore, the flexible AF scheme is extended to multiple antenna scenario. Performance of the flexible AF with multiple antennas is investigated for the case that relay has both receive and transmit channel knowledge. Since the transmit channel knowledge may not always be available at the relay and also in order to reduce the complexity a novel power allocation method based on the maximization of the mutual information of the relay link is proposed when the relay has only the receive channel knowledge.

Key Words:

Amplify and Forward, Throughput, Achievable Rate, Outage Probability, Mutual Information
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"And whoever puts his trust in Allah - then Allah is sufficient for him. Surely, Allah will accomplish his purpose".

Translation: Quran- At-Talaaq - 3
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<th>Description</th>
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<tr>
<td>16-QAM</td>
<td>16-ary Quadrature Amplitude Modulation</td>
</tr>
<tr>
<td>ACK</td>
<td>Acknowledgement</td>
</tr>
<tr>
<td>AF</td>
<td>Amplify-and-Forward</td>
</tr>
<tr>
<td>APP</td>
<td>A Posteriori Probabilities</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
</tr>
<tr>
<td>BAN</td>
<td>Body Area Network</td>
</tr>
<tr>
<td>BER</td>
<td>Bit-Error-Rate</td>
</tr>
<tr>
<td>BICM</td>
<td>Bit Interleaved Coded Modulation</td>
</tr>
<tr>
<td>BLER</td>
<td>Block-Error-Rate</td>
</tr>
<tr>
<td>BSC</td>
<td>Binary Symmetric Channel</td>
</tr>
<tr>
<td>CDF</td>
<td>Cumulative Distribution Function</td>
</tr>
<tr>
<td>CF</td>
<td>Compress-and-Forward</td>
</tr>
<tr>
<td>CNC</td>
<td>Clipping Network Coding</td>
</tr>
<tr>
<td>CRC</td>
<td>Cyclic Redundancy Check</td>
</tr>
<tr>
<td>CSI</td>
<td>Channel State Information</td>
</tr>
<tr>
<td>DFE</td>
<td>Decode-and-Forward</td>
</tr>
<tr>
<td>DTC</td>
<td>Distributed Turbo Codes</td>
</tr>
<tr>
<td>EG</td>
<td>Equal Gain</td>
</tr>
<tr>
<td>EEM</td>
<td>Equal Eigen Mode</td>
</tr>
<tr>
<td>EVD</td>
<td>Eigen Vector Decomposition</td>
</tr>
<tr>
<td>FCSI</td>
<td>Full Channel State Information</td>
</tr>
<tr>
<td>FEC</td>
<td>Forward Error Correction</td>
</tr>
<tr>
<td>FER</td>
<td>Frame-Error-Rate</td>
</tr>
<tr>
<td>HARQ</td>
<td>Hybrid Automatic Repeat re-Quest</td>
</tr>
<tr>
<td>i.i.d.</td>
<td>independent and identically distributed</td>
</tr>
<tr>
<td>IR</td>
<td>Incremental Redundancy</td>
</tr>
<tr>
<td>LDC</td>
<td>Linear Dispersion Code</td>
</tr>
<tr>
<td>LLR</td>
<td>Logarithm of Likelihood Ratio</td>
</tr>
<tr>
<td>MAC</td>
<td>Media Access Control</td>
</tr>
<tr>
<td>MFR</td>
<td>Match Filtering Receiver</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multiple Input Multiple Output</td>
</tr>
<tr>
<td>MISO</td>
<td>Multiple Input Single Output</td>
</tr>
<tr>
<td>MMSE</td>
<td>Minimum Mean Square Error</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
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</tr>
<tr>
<td>MMSEF</td>
<td>Minimum Mean Square Error Filtering</td>
</tr>
<tr>
<td>MSE</td>
<td>Mean Square Error</td>
</tr>
<tr>
<td>MTMR</td>
<td>Multiple-Antenna Transmit Multiple-Antenna Receive</td>
</tr>
<tr>
<td>NACK</td>
<td>Negative Acknowledgement</td>
</tr>
<tr>
<td>NC</td>
<td>Network Coding</td>
</tr>
<tr>
<td>OFDM</td>
<td>Orthogonal Frequency Division Multiplexing</td>
</tr>
<tr>
<td>OWRC</td>
<td>One Way Relay Channel</td>
</tr>
<tr>
<td>p.d.f.</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>PEP</td>
<td>Pair-wise Error Probability</td>
</tr>
<tr>
<td>PER</td>
<td>Packet Error Rate</td>
</tr>
<tr>
<td>PHY</td>
<td>Physical</td>
</tr>
<tr>
<td>QPSK</td>
<td>Quadrature Phase Shift Keying</td>
</tr>
<tr>
<td>RC</td>
<td>Repetition Coding</td>
</tr>
<tr>
<td>RCPC</td>
<td>Rate Compatible Punctured Convolutional Code</td>
</tr>
<tr>
<td>SDF</td>
<td>Selective Decode-and-Forward</td>
</tr>
<tr>
<td>SIC</td>
<td>Serial Interference Cancelation</td>
</tr>
<tr>
<td>SISO</td>
<td>Single Input Single Output</td>
</tr>
<tr>
<td>SISO</td>
<td>Soft-in-Soft-output</td>
</tr>
<tr>
<td>SIMO</td>
<td>Single Input Multiple Output</td>
</tr>
<tr>
<td>SNC</td>
<td>Selective Network Coding</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
</tr>
<tr>
<td>SVD</td>
<td>Singular Value Decomposition</td>
</tr>
<tr>
<td>TDD</td>
<td>Time Division Duplexing</td>
</tr>
<tr>
<td>TDMA</td>
<td>Time-Division-Multiple-Access</td>
</tr>
<tr>
<td>TWRC</td>
<td>Two Way Relay Channel</td>
</tr>
<tr>
<td>UC</td>
<td>Unconstrained Coding</td>
</tr>
<tr>
<td>WiMAX</td>
<td>Worldwide Interoperability for Microwave Access</td>
</tr>
<tr>
<td>ZFR</td>
<td>Zero-Forcing Receiver</td>
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## Notation (List of Symbols)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>Zero-Forcing Receiver</td>
</tr>
<tr>
<td>$N_1$</td>
<td>Time slot size for the first phase (Single input single output scenario)</td>
</tr>
<tr>
<td>$N_2$</td>
<td>Time slot size for the second phase (Single input single output scenario)</td>
</tr>
<tr>
<td>$T_1$</td>
<td>Time slot size for the first phase (Multiple input multiple output scenario)</td>
</tr>
<tr>
<td>$T_2$</td>
<td>Time slot size for the second phase (Multiple input multiple output scenario)</td>
</tr>
<tr>
<td>$S_1$</td>
<td>First node in two way relay channel system</td>
</tr>
<tr>
<td>$S_2$</td>
<td>Second node in two way relay channel system</td>
</tr>
<tr>
<td>$\alpha_j$</td>
<td>SISO channel scalar for the jth link</td>
</tr>
<tr>
<td>$H_j$</td>
<td>MIMO channel Matrix or equivalent channel matrix for the jth link</td>
</tr>
<tr>
<td>$M$</td>
<td>Number of transmit antennas at source</td>
</tr>
<tr>
<td>$R$</td>
<td>Number of receive or transmit antennas at relay</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of receive antennas at the destination</td>
</tr>
<tr>
<td>$R_x$</td>
<td>Covariance matrix of the vector $x$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Power normalization factor at relay (Single input single output scenario)</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Power allocation matrix at relay (Multiple input multiple output scenario)</td>
</tr>
<tr>
<td>$\lambda_{jk}$</td>
<td>K-th eigen mode for the jth link</td>
</tr>
<tr>
<td>$\gamma_j$</td>
<td>Instantaneous SNR for jth link</td>
</tr>
<tr>
<td>$\rho_j$</td>
<td>Average SNR for jth link</td>
</tr>
<tr>
<td>$\text{SNR}_j$</td>
<td>Average SNR of the jth link</td>
</tr>
<tr>
<td>$\rho_{ij}$</td>
<td>Average SNR for link between i-th node and j-th node in two way relay channel system</td>
</tr>
<tr>
<td>$\sigma_j^2$</td>
<td>Variance of the complex additive complex Gaussian noise for the jth link</td>
</tr>
<tr>
<td>$\Delta_j$</td>
<td>SNR offset of jth link compared to $\text{SNR}_2$ in dB</td>
</tr>
<tr>
<td>$\delta_j$</td>
<td>SNR offset of jth link compared to $\text{SNR}_0$</td>
</tr>
<tr>
<td>$I(.)$</td>
<td>Mutual information</td>
</tr>
<tr>
<td>$R$</td>
<td>Associated transmission rate</td>
</tr>
<tr>
<td>$P_{\text{outage}}(.)$</td>
<td>Outage Probability</td>
</tr>
<tr>
<td>$\eta(.)$</td>
<td>Hybrid ARQ Throughput</td>
</tr>
<tr>
<td>$E[T]$</td>
<td>Average air time</td>
</tr>
<tr>
<td>$E[D]$</td>
<td>Average delay time</td>
</tr>
<tr>
<td>$0_M$</td>
<td>$M \times M$ Zero matrix</td>
</tr>
<tr>
<td>$I_M$</td>
<td>$M \times M$ Identity matrix</td>
</tr>
<tr>
<td>$\mathbb{R}$</td>
<td>Real numbers domain</td>
</tr>
<tr>
<td>$\mathbb{C}$</td>
<td>Complex numbers domain</td>
</tr>
<tr>
<td>$E(.)$</td>
<td>Expectation</td>
</tr>
<tr>
<td>$\text{trace}(.)$</td>
<td>Trace function i.e. sum of the diagonal elements of a matrix</td>
</tr>
<tr>
<td>$\text{Pr}(.)$</td>
<td>Probability</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>Characteristic function</td>
</tr>
</tbody>
</table>
Chapter 1 Introduction

1.1 Background

In recent years, demand for fast and high quality flow of information data to homes and business sites has been increased. Some major technologies such as IEEE standard for mobile broadband wireless access, also known as WiMAX, 3GPP long term evolution (LTE) and high-speed downlink packet access (HSDPA) for cellular networks have been proposed in response to the growing demand for high speed internet access by millions of wireless users in every places around the world. In addition to increasing spectral efficiency, improving energy efficiency as well as creating uniform coverage are two main targets of future wireless communication networks. However at the same time, these networks have to combat major problems such as multipath fading, path loss and shadowing effect. Recently, several network structures based on relaying techniques have been proposed in order to provide a solution to above mentioned problems. The relaying technique is a potential solution for enhancing the wireless cellular communications as well as ad-hoc networks in terms of spectral efficiency, energy efficiency, coverage and robustness against channel problems. In this direction the IEEE 802.16 work group develops specifications 802.16j for low cost relays which are suitable for fixed and mobile WiMAX terminals. The mobile multi-hop relay scheme enables mobile stations to communicate with a base station through midway relay stations. In this work group different relay station types are investigated: fixed relay station, nomadic relay station and mobile relay station which is expected to function while it is moving. These specifications support multiple relaying as well as uplink or downlink transmission. Also there are several valuable ongoing research works on the cooperative relaying techniques for body area network (BAN) mesh network.

The major concept in relaying techniques is to get help from some midway nodes, other network nodes or any other devices, to increase spectrum and energy efficiency. A basic relaying scenario consists of a source, a relay and a destination is shown in Figure 1-1. Also two different types of relaying, including mobile relaying and fixed relaying, are presented. As it is clear in both examples, the direct link (source to destination) is highly weakened due to the shadowing created by two high buildings. However, with the introduction of a relay node, a new link (source-relay-
destination link) is created. Therefore the degradation of the source-destination transmission is no longer important.

Historically, the relay concept was first introduced by van der Meulen [2] in 1971. He investigated a system with three nodes known as source, relay and destination. In 1979, Cover and El Gamal significantly developed this relay concept and provided three basic relaying policies [1,3]; “observation where the relay forwards what it has received to the destination, cooperation where if the relay observes a better signal than the destination, it can cooperate with the source by sending a signal that contains a perfect source signal to the destination, facilitation where if the relay receives a corrupted version of what the destination receives, the relay transmits constant signal known at the source and the destination”. Search for the optimal relaying policy in wireless networks has been escalated during past decade. In this regards several relaying schemes have been proposed based on the cooperation and observation policies. In the following, some of these schemes for unidirectional and bidirectional wireless communication systems are introduced.

In one directional communication or one way relay channel (OWRC) scenario, a single source node transmits data to a single destination node with the help of some relay nodes. Various relaying techniques have been proposed for one way relay channel in recent years. These techniques have been divided into mainly three groups including decode and forward (DF), compress and forward (CF), and amplify and forward (AF)[6-7]. Each of these schemes provides efficient performance in certain SNR regions. Finding the suitable SNR region or network topology for each scheme is an important issue.

In the first group named as DF, transmit cooperation or regenerative relaying, the relays are close to source and are able to decode the transmitted data with high reliability and re-encode and
forward the decoded data to the destination in appropriate format and time [22-38],[90]. DF scheme provides improvement in terms of the achievable rate and the outage behaviour when the source-relay link has a good quality [4-5], [22]. In DF, time durations for the relay-receive and relay-transmit phases can be adjusted to improve the overall performance. The quality of the source-relay link limits the performance of DF scheme. If this link undergoes deep fading, the successful decoding of the message at the relay may not be possible. The encoded message at the relay could carry some errors. Forwarding of wrong messages from the relay causes error propagation at the destination. To handle this issue, some researchers have proposed soft DF, in which the relay does soft encoding and forwards the soft values to the destination [39-41]. Moreover a selective DF scheme was proposed [5], where the relay switches to silent mode if it detects errors in the received message. The optimum decoding of the DF scheme has been investigated in [104] while [105] has looked at the BER performance analysis of a cooperative BICM system.

In the second group, CF, also known as receive cooperation, relays are too close to destination, thus reliable transmission from source to the other nodes would be difficult. This scheme is based on the observation policy in Cover et al paper [3] and recently it has received a lot of attention [5], [14], [43-48], [90]. Instead of decoding, the relay helps the destination by providing a distorted version of its observation of the source signal transmitted to the destination. The main concept is that the relay quantizes and compresses the received signal with certain amount of distortion. Then the destination combines the compressed signal with its directly received signal to decode the message. This scheme works well when the link between relay and destination is good, in contrast to DF which performs well when the relay is close to the source.

Third group, named as AF also known as non-regenerative relaying is an efficient cooperation technique when the relays are neither close to the source nor to the destination [18-21],[90]. The relays do not need to fully decode the transmitted data. They can simply amplify what they have received from the source and then forward them to the destination. This scheme has less complexity compared to DF and CF techniques since it only needs very simple linear processing technique at the relays. In addition to having less complexity, AF can perform well in terms of bit error rate (BER) or packet error rate (PER) performance with respect to DF, when the source-relay link is not reliable enough. This scheme also has similar problem to DF scheme. The quality of the source-relay link limits their performance. The achievable diversity-multiplexing trade-off for AF scheme has been discussed in [10,73]. The tradeoffs between network size, rate, and diversity for an AF strategy has been explored in [106]. In [99], the ergodic channel capacities for multiple relay AF half-duplex cooperative systems were explored. Several detection techniques for the non-orthogonal AF protocol based on a multiuser detection approach have been proposed in [101].
In time division duplexing the channel state information (CSI) can become available at a transmitter by reciprocity and in frequency division duplexing CSI can be available through feedback channel. However this feedback causes some signalling overhead. The rate of the appropriate feedback depends on the mobility of the involved nodes or in other words variation of the channel in time domain. Similar to a point-point non-cooperative communication system if the channel knowledge is available at all or some of transmitters, it is possible to apply adaptation techniques in order to improve the performance of cooperative relaying system. Type of adaptation scheme depends on the kind of channel knowledge and which node has the knowledge of which links. In these adaptive techniques by using the transmit channel knowledge, different transmission parameters such as block size, FEC coding, modulation level, MIMO processing, transmit power and so on can be dynamically tuned.

Some of the adaptive schemes need to have transmit CSI available at the transmitters. However some adaptive techniques like ARQ and Hybrid ARQ schemes need only the knowledge of success or failure of the decoding at the receivers [49]-[52], [66]. Destinations send ACK/NAK messages to all or some of the transmitters through the feedback links. This causes small amount of overhead. The main concept behind HARQ is to recover unsuccessful data transmissions by asking for more retransmissions from the transmitters. Then every receiver re-examines the failure or success of its decoding process by properly combining the signals received from all the transmitters and decoding them all together. In this regard, every receiver needs to save all the soft information received from the previous transmissions and combines them with the last received one. Two different type of encoding can be used for HARQ; incremental redundancy (IR) where the data in different retransmissions could be different parts of information or parity data generated by the FEC encoder, or chase combining where the same data is transmitted in all retransmissions [58]-[61].

Recently some researchers have looked at combining cooperative techniques with HARQ. As an example, the two time slots allocated to the source and relay transmission in an orthogonal protocol could be repeated for every transmission. The destination receives the message from the source and the relay and combines them. If the destination fails to decode the message correctly, it asks for another two time slot transmission from the source and the relay. The combination of HARQ and relaying schemes has been investigated in [53]-[57], and [62]-[64] and the diversity-multiplexing trade-off of this HARQ strategy has been analysed in [52]. Combining cooperative relaying techniques with ARQ has been studied in [63]. In this study relay does not retransmit the message after first transmission. Extension of this study to HARQ has been addressed in [54]-[57], [64] and [69]. In these researches, a single relaying technique like DF has been combined with HARQ. The work in [65] proposes combination of HARQ with AF and DF together. In this
study the relay action is varied between AF and DF. This strategy considerably improves PER compared with combination of HARQ with DF only.

Similar to traditional point-point communication, deploying multi antenna transmission and multi antenna reception (MTMR) techniques in all or some of the involved nodes in relaying system improves the performance of these systems and increase their diversity and robustness to fading channel [70-72]. The capacity of MIMO relay channels has been studied in [100]. When the relay node has more than one antenna, it can benefit from it in both directions; in receiving the signal from the source node as well as in forwarding it to the destination. Most of the traditional multi antenna diversity reception as well as transmission schemes could be used in cooperative relaying techniques. In scenarios with only a single destination node, adaptive and non-adaptive schemes could be used for the relay transmission. As an example for adaptive multi antenna transmissions, if the relay-destination link CSI is available at the relay then Eigen beam-forming scheme can be applied. In this scheme several streams of data are sent over the Eigen modes of the relay-destination MIMO channel. In non adaptive transmission scenarios, spatial diversity and multiplexing techniques like space-time block codes can be used over the relay-destination link [11-13]. Performance of the distributed space time codes in AF scheme has been widely explored in [98], [103], [107-109]. In [102], several equalization methods have been proposed for the distributed space-time block codes with AF scheme over frequency-selective fading channels. In contrast to the original AF scheme where the relay acts as a simple equal gain amplifier, in cooperative MIMO scenario, the relay node can be used as a smart pre-coder. In this scenario, by applying efficient power allocation techniques based on the level of CSI available at the relay node, mutual information of the system can be improved. In the case that both the CSI of the source-relay and relay-destination links are known at the relay node, power allocation techniques for the AF scheme have been recently developed [8-9], [82]. The authors have shown great enhancement on the performance of the MIMO AF scheme. Moreover they have provided a linear transceiver design for the AF scheme using transmit CSI at the relay. However, it is clear that the above techniques need extra complexity since they rely not only on the source-relay link CSI similar to the original AF scheme, but as well as on the relay-destination link CSI, which may not always be available. In this case, when only receive CSI is available at the relay, several power allocation schemes has been recently proposed in [9] for traditional MIMO cooperative AF such as the AF matched filter based relaying (MFR) and AF minimum mean square error filtering (MMSEF) techniques.

In the above paragraphs the relaying techniques for OWRC were introduced. However in bidirectional scenarios, where two source nodes want to transmit data to each other through the relay link, the relay forwarding for the first and second nodes are carried out in the separate time or frequency bands if the system takes the same approach as used in OWRC. Notice that at each
time instant, each one of these two nodes acts as a source or a destination. An efficient technique
which allows simultaneous (same time and frequency band) forwarding of the both messages
from the relay, has been proposed in [74]-[75]. The proposed technique employs network coding
for TWRC scheme. The network coding concept is generally based on the combining of the
different received data from the source nodes at the relay in order to improve the reliability as
well as the spectrum efficiency of all the transmissions between the involved nodes in the
network. The relay instead of sending multiple transmissions towards separate destination nodes,
only forwards one single transmission of a network coded signal [74]-[75]. Thus each destination
node is able to extract its desired message out of the network coded signal broadcasted by the
relay. In general a hybrid combination of the channel and network codes performed by all the
involved transmitting nodes creates a distributed turbo network-channel code. As each of the
source nodes has knowledge on its own transmitted data, the source node's receiver is able to
extract the data that it wants to receive. With joint design of channel and network codes, the
redundancy of the coded data is used to improve the data extraction ability and to obtain
necessary error correction level.

1.2 Basic Assumptions

In this section some major assumptions considered throughout this thesis are briefly described.

In the literature, two basic duplexing modes have been defined for a relay scenario: full-duplex
and half-duplex. In the full-duplex mode, the relay can transmit and receive simultaneously in the
same frequency and time. Since there is a great dynamic range between the transmit and receive
signals power for the relay's antenna, it is very difficult to utilize this mode. Therefore it is better
to carry out the transmission in two orthogonal phases: the relay-receive phase (first phase) and
the relay-transmit phase (second phase). This mode is called half-duplex mode where the relay
either receives or transmits at a specific time or frequency slot. During the first phase, the source
broadcasts to the relay and the destination. During the second phase, if the relay is not silent it
transmits a signal to the destination. The half-duplex mode is widely used in several different
systems. Therefore, this thesis focuses on the time-division half-duplex relay channel. The
duplexing ratio, which plays a major role in this thesis, denoted by $a$, represents the percentage of
the first phase in a complete two phased frame.

In this thesis, depending on the status of the source transmission at the second phase, two different
scenarios are considered as shown in Figure 1-2. In the first scenario, known as orthogonal
cooperative relaying protocol, the source keeps silent during the second phase. In contrast, in the
second scenario, known as non-orthogonal cooperative relaying protocol, the source transmits
simultaneously along with the relay during the second phase. The source could transmit either the same signal or a new signal at the second phase.

![Diagram](image)

**Figure 1-2. Relaying protocol** (a) orthogonal cooperative relaying protocol (b) the non-orthogonal cooperative relaying protocol

Figure 1-3 shows the difference between two hop relaying and cooperative relaying. In two hop relaying there is no direct link between the source and the destination. This thesis focuses on the cooperative relaying protocols; orthogonal and non-orthogonal scenarios.

![Diagram](image)

**Figure 1-3. Relaying protocol** (a) two hopping relaying (b) cooperative relaying

This thesis focuses on the single-relay with single antenna scenario in the early chapters and then moves to the single-relay with multiple antenna scenarios. Most of the analysis in this thesis can be straightforwardly extended to multiple-relay scenarios.

Performance of the cooperative relaying techniques depends on the kind of channel knowledge and what node has the channel knowledge of what links. All the links’ CSIs are considered available at the destination. There is no transmit CSI available at the source. Relay does not have
any knowledge about the direct link CSI. In chapter 5, two major cases for availability of CSI at
the relay are considered: at the first case the source to relay link CSI and the relay to destination
link CSI both are available at the relay. However, this case requires extra complexity since it
relies not only on the receive CSI, but as well as transmit CSI, which may not always be available.
Therefore in the second case only the receive CSI is assumed to be available at the relay node.

Notice that although the source, the relay and the destination can be easily synchronized in the
time domain, carrier synchronization cannot be always perfect in realistic systems. In this thesis
the performance of the proposed schemes in synchronized scenarios (time and frequency) are
investigated. Their performance in asynchronous scenarios could be considered as a future work.

It is assumed that for each link the corresponding channel coefficient is a complex Gaussian i.i.d
fading. In multiple antenna scenarios, each member of channel matrix is a complex Gaussian i.i.d
fading. The channel coefficients or channel matrices for separate links are independent. The
channel can remain fixed during a time slot, a block or a frame depending on the assumed
scenario.

The achievable rate, the throughout and the outage probability analysis is done using mutual
information. In this thesis it is assumed that Gaussian code book has been used at the source. For
BER analyses a specific coding and modulation scheme is considered. In this case the achievable
rate may be lower than the case of Gaussian code book.

An effort has been made to deploy consistent notation throughout this thesis. All vectors are
column vectors and denoted with bold non-italic Roman or Greek lower-case letters such as
\( \mathbf{x} \) and \( \mathbf{v} \). Matrices are denoted with bold Italic Roman or Greek upper-case letters such as
\( \mathbf{X} \) and \( \Psi \). Scalars are denoted with italic or non-italic Roman or Greek lower-case letters such as
\( x \) and \( \alpha \). In this thesis only two special row vectors \( \gamma \) and \( \rho \) are denoted with underline Italic
Greek lower-case letters.

### 1.3 Motivation and Objectives of the Research

The overall goal of this thesis is to improve the performance of the cooperative relaying
communication systems and filling the gaps in this research area by presenting new techniques.
The most important issue in cooperative relaying systems is to obtain the knowledge and
necessary guidelines on how and where the different cooperative strategies should be used. It is
important to explore different ideas to propose cooperative communication strategies for different
composition and configuration of the involved nodes such as symmetric and asymmetric topology
of the nodes, etc. During recent years many researchers have investigated cooperative relaying
techniques [18-38]. As a pioneer of research in cooperative relaying, Laneman has worked out the outage behaviour and the diversity gain for AF and DF schemes for an orthogonal cooperative relaying protocol. In contrast with DF scheme, in conventional AF the lengths of the time slots allocated to the relay receive and transmit phases are considered to be equal. This leads to the duplexing ratio of 0.5. In Laneman paper [4] and other works for AF scheme [18-21], the performance results are presented for an equal time slots protocol. The first goal of this thesis is to improve the performance of single relay single antenna AF in terms of the achievable rate and the outage behaviour by proposing a flexible scheme that allows separately tuneable time slots. This adds an extra degree of freedom in optimising the AF scheme along with the other parameters. The second goal is to improve the achievable rate and enhance the performance of traditional cooperative MIMO AF scheme by proposing efficient power allocation techniques. It is necessary to show that proposed techniques provide better average mutual information as well as lower BER in many signal-to-noise-ratio (SNR) configurations.

The BER performance of network coding in traditional two way relay channel in asymmetric channel conditions is highly degraded; this could be because of the asymmetric position of the relay and the source nodes, different channel conditions, and different instantaneous SNR of the source-to-relay links. To overcome this problem, as the third goal of this thesis, two possible candidates are investigated in order to improve the performance of the network coding scheme in asymmetric channel conditions.

1.4 Achievements

The main contributions of this thesis are listed as follows:

- A new flexible AF scheme was proposed which allows a tuneable duplexing ratio for single relay single antenna scenario to increase the achievable rate and the throughput of the traditional AF scheme. The mathematical models for the instantaneous mutual information and average mutual information were derived and employed to find the optimum duplexing ratio in different network topologies and SNR conditions. To more precisely explore the performance, a tight lower bound approximation for the outage probability was derived. The outage analysis enable user to predict the system performance, adjust its parameters and inspect the diversity of the system for any combination of the signal to noise ratio (SNR) of the different links. It was shown that at high SNRs, the outage probability for the optimum duplexing ratio is always lower than the outage probability for half duplexing ratio. Based on the derived outage probability several SNR thresholds were defined to explore the diversity for different SNR regions.
A typical HARQ scheme for a cooperative flexible AF scheme with single relay was proposed in order to improve the throughput of the cooperative scheme. In this direction, the outage and the throughput performance measures were derived for both repetition and unconstrained coding. The presented analysis allows accurate performance evaluation of the flexible AF based cooperative HARQ protocols without need to time consuming Monte Carlo based evaluation methods. This allows identification of the protocol throughput gain over the direct communication and appointing right protocol at right region. Considerable performance gains were observed for some assumed scenarios.

The flexible AF scheme idea was extended to multiple antenna cooperative scenarios. It was shown that with choosing the optimum duplexing ratio, the average mutual information and the BER performance are improved in many SNR configurations. The processing techniques at relay for the flexible AF scheme were presented for the case when relay has both transmit and receive channel knowledge. Also BER upper bound was derived for the presented flexible AF scheme to allow accurate evaluation of flexible AF scheme without need to time consuming Monte Carlo based evaluation methods. In order to consider scenarios where the relay does not have transmit channel knowledge, the power allocation methods were explored when only receive channel knowledge is available at the relay. In this direction a novel power allocation method for non-regenerative cooperative multiple antennas communication was proposed. This method outperforms other common relay receive CSI only power allocation techniques in terms of the mutual information and BER.

Two different possible approaches were proposed to enhance performance of conventional network coding in two way relay channel (TWRC) under asymmetric conditions. The first approach attempts to avoid combining non reliable and reliable data at relay. The second approach is based on the signalling some quality parameters from relay back to source nodes. The proposed approaches are applicable for any combinations of constellations employed in the source and relay nodes and thus allow better spectral efficiency by proper selection of constellations for all three phase transmissions. These schemes perform quite successfully in block fading conditions.

1.5 Organisation of the Thesis (Thesis Outline)

The reminder of the thesis is organised as follows:

In Chapter 2, the conventional AF scheme is described. In section 2.2, basic relay system models for the traditional AF are presented. Two evaluation metrics such as achievable rate and outage
probability are introduced to demonstrate the performance of a relay scheme in section 2.3 and the final section concludes this chapter.

In Chapter 3, a flexible scheme that allows a tuneable duplexing ratio is proposed. Section 3.2 describes the system model for the flexible AF concept. Section 3.3 explores the performance measures including the achievable rate and the outage probability. Section 3.4 looks at derivation of the instantaneous mutual information and the achievable rate. Section 3.5 provides the numerical results for the achievable rate and the throughput performance of the orthogonal AF. Section 3.6 discusses in detail the outage probability analyses and compares analysis result with simulation result. In section 3.7 diversity of the AF scheme is explored in different SNR conditions.

Chapter 4 investigates the performance of a flexible non-regenerative single relay case with HARQ scheme and derives the throughput, the latency and the outage performance for both different encoding schemes and cooperative protocols for any range of SNRs. Section 4.2 describes the system model for the flexible AF concept. Section 4.3 and 4.4 discusses in detail the proposed HARQ techniques at the relay including the HARQ strategies and the probabilities of the outage for HARQ AF model. The throughput performance results of the proposed scheme are compared in section 4.5. Section 4.6 provides the conclusion remarks.

In Chapter 5, a novel scheme that provides variable duplexing ratios for AF MIMO cooperative scheme using linear dispersion Codes (LDC) is proposed. Section 5.2 describes the system model for LDC based AF concept. Section 5.3 discusses in detail the proposed processing technique at the relay and derives upper bound for the presented model. In this section it is considered that the relay is aware of its input and output channel. The BER upper bound for our LDC based cooperative AF model is derived in section 5.3.2 and the throughput and the BER performance results of the proposed scheme are compared in section 5.3.3. It is shown that the flexible AF scheme provides better throughput as well as lower BER in many SNR configurations. At section 5.4, existing power allocation techniques such as MFR and MMSEF are extended to the flexible AF scheme. In this section it is considered that the relay is aware only about its input channel. Then a novel power allocation algorithm is introduced based on the maximization of the relay link mutual information considering equal Eigen mode (EEM). This method is low-complex and still outperforms traditional equal gain (EG) and other existing power allocation techniques such as MFR and MMSEF, both in terms of mutual information and BER, as it is reported in Section 5.4.3. Finally section 5.5 provides conclusion remarks.

In Chapter 6, two new schemes are proposed to improve the performance of the network coding over TWRC system especially in asymmetric conditions, including a selective network coding (SNC) protocol and clipping and network coding (CNC). Section 6.2 describes the system model
for TWRC concept and structure of the joint network and channel decoder. Section 6.3 discusses in detail the proposed selective network coding protocol over TWRC system and the performance of the new selective network coding scheme is compared with the traditional non-selective network coding scheme for different symmetric and asymmetric topology conditions. Then section 6.4 describes in detail the proposed CNC scheme over TWRC system and the performance of the proposed CNC scheme is compared with the traditional non-selective and SNC schemes.

Finally, conclusion and a discussion regarding future work are given in Chapter 7.

It is necessary to mention that, most of the derivations and proofs which are part of this thesis have been moved to the appendix in order to increase readability of the text.

The research carried during the course of this thesis has resulted in following publications:

Chapter 4:


Chapter 5:


Chapter 6:

- Sadegh Fazel, Reza Hoshyar, and Rahim Tafazolli, A Robust Network Coding Scheme over Two Way Relay Channel, ICT-Mobile Summit, Santander, Spain, June 2009.

A full list of publications during my PhD can be found at the end of this thesis.
Chapter 2 Traditional Amplify and Forward

2.1 Introduction

In wireless networks, cooperative and non-cooperative relaying techniques have been employed through some relaying protocols to increase spectral efficiency as well as energy efficiency and to expand the coverage of communication systems of the network. In this direction, three major relaying schemes have been proposed including decode-and-forward (DF), amplify-and-forward (AF), and compress-and-forward (CF). Among these cooperative relaying schemes, the AF scheme has created lots of interest because of its simplicity and low costs. In this scheme, the relay only amplifies the received signal and forwards it to the destination without any encoding or decoding. That is why this scheme is called non-regenerative AF compared to the regenerative scheme, DF. A recent work in [18] has shown that in some SNR regions the Ergodic capacity of the AF scheme can be higher than the capacity of the DF scheme [18]. In general, when the quality of the source-destination and relay-destination links are better than the quality of the source-relay link, the capacity of the AF scheme is higher than the capacity of the DF scheme. Moreover in [19-21], [90] the BER and PER performance of the traditional AF scheme has been evaluated through the analysis of the SNR terms. Also the achievable diversity-multiplexing trade-off for AF scheme has been discussed in [10,73]. One of the AF benefits, which has been shown by Laneman is that the traditional AF scheme can achieve second-order diversity in very high SNR regions [4]. The aim of this chapter is to explore two important aspects of the traditional AF scheme.

The remainder of this chapter is organised as follows. In the next section, basic relay system models for traditional AF are presented. Two evaluation metrics that are used to demonstrate the performance of a relay scheme are given in section 2.3 and the concluding remarks on the traditional AF scheme is provided in section 2.4.

2.2 System Model

In the general case, the three-node relay system in Figure 2-1 is considered. The whole frame is separated into two phases. The solid lines indicate the transmission during the first phase that
Chapter 2 Traditional Amplify and Forward

consists of $M$ symbols and the dashed lines indicate the transmission during the second phase that consists of $M$ symbols. The three links, source-destination, source-relay and relay-destination are indexed by 0, 1 and 2 respectively. A message $w$ belonging to a message set $W = \{1, 2, ..., 2^{|W|}\}$ is transmitted by the source with the associated transmission rate $R$, where $N = 2M$ is the number of symbols in one complete frame. The channel gains for the source-destination, the source-relay and the relay-destination links are defined as $\alpha_0$, $\alpha_1$ and $\alpha_2$ respectively, all modelled by circularly symmetric complex Gaussian distribution with zero mean and unit variance. The channel coefficients are assumed to be i.i.d Rayleigh fading channels. The channel gains remain fixed within one frame and vary independently from one frame to next frame. $\gamma = [\gamma_0, \gamma_1, \gamma_2]$ are defined as the instantaneous SNRs for their corresponding links i.e. $\gamma_j = |\alpha_j|^2 \rho_j$ $j = 0, 1, 2$. Moreover $\rho = [\rho_0, \rho_1, \rho_2]$ represents the average SNRs of these three links.

![Half duplex relay system](image)

Figure 2-1. Half duplex relay system

2.2.1 Conventional Amplify and Forward

In traditional AF scheme, the relay only amplifies the received signal and forwards it to the destination without any encoding or decoding process. Thus the length of two phases of the transmissions is assumed the same i.e. $M = N/2$. The duplexing ratio $\alpha$ is defined as the ratio of the duration of the first phase to the overall duration of the two phases. Therefore $\alpha$ is 0.5. In the following section three different protocols are explored based on the source transmission status; orthogonal or non-orthogonal transmission.

2.2.1.1 Orthogonal Amplify and Forward Transmission

In this protocol, as in Laneman's protocol [4] source is not allowed to transmit during the second phase. During the first phase, the source transmits the signal $x \in \mathbb{C}^M$. It is assumed
that $E_x \left( \text{trace}(xx^H) \right) = M$ where $E_x(.)$ denotes expectation operation with respect to $x$. As a result the received signals at the relay and destination are
\[
\begin{align*}
y_0 &= \alpha_0 \sqrt{\rho_0} x + n_0 \quad \text{Destination} \\
y_1 &= \alpha_1 \sqrt{\rho_1} x + n_1 \quad \text{ Relay}
\end{align*}
\] respectively, where $n_0 \in \mathbb{C}^M$ and $n_1 \in \mathbb{C}^M$ are the additive white Gaussian noise vectors at the relay and the destination respectively. These vectors have i.i.d elements with zero mean and unit variances.

The received signal $y_1$ is amplified with a specific amplifying factor $\beta$ and forwarded during the second phase. In this section the source is not allowed to transmit any signal. Therefore the destination receives
\[
y_2 = \alpha_2 \sqrt{\rho_2} x + n_2
\] where $n_2$ is the noise at the destination during the second phase and follows the same statistic property as $n_0$ and $n_1$ and $x_r = \sqrt{\beta} y_1$. The destination received signal can be rewritten as $y_2 = \alpha_2 \alpha_1 \sqrt{\beta \rho_2} \rho_1 x + \alpha_2 \sqrt{\beta \rho_2} n_1 + n_2$. Then the received signal can be modelled by one equation as follows:
\[
\begin{bmatrix}
y_0 \\
y_2
\end{bmatrix}
= \begin{bmatrix}
\alpha_0 \sqrt{\rho_0} I_M \\
\alpha_1 \alpha_1 \sqrt{\beta \rho_2} \rho_1 I_M
\end{bmatrix} x + \begin{bmatrix}
I_M & 0_M & 0_M \\
0_M & \alpha_2 \sqrt{\beta \rho_2} I_M & 0_M
\end{bmatrix} \begin{bmatrix}
n_0 \\
n_1 \\
n_2
\end{bmatrix}
\] (2-3)

Then the equivalent channel model is defined as $H = \begin{bmatrix}
\alpha_0 \sqrt{\rho_0} I_M \\
\alpha_2 \alpha_1 \sqrt{\beta \rho_2} \rho_1 I_M
\end{bmatrix}$ and the equivalent noise vector as $v = \begin{bmatrix}
I_M & 0_M & 0_M \\
0_M & \alpha_2 \sqrt{\beta \rho_2} I_M & 0_M
\end{bmatrix} \begin{bmatrix}
n_0 \\
n_1 \\
n_2
\end{bmatrix}$.

Finally
\[
R_v = E_{n_0,n_2} (vv^H) = \begin{bmatrix}
I_M & 0_M \\
0_M & (|\alpha_2|^2 \beta \rho_2 + 1) I_M
\end{bmatrix}
\] is the covariance matrix of the equivalent noise vector.
The power factor $\beta$ is used for normalizing power at the relay output. This parameter can be kept fixed or variable per frame depending on the assumed measurement. The following two options can be considered for adjusting this parameter.

In the first option $\beta$ is considered to vary according to the instantaneous SNR of the source-relay link. Since $\mathcal{E}_{x,n}|_{\text{trace}}(x, x^*) = M$, the normalization factor $\beta$ at the relay output is calculated based on the power constraint at relay as follows:

$$\beta \left( 1 + |\alpha|^2 \rho \right) = 1 \Rightarrow \beta = (1 + \gamma)^{-1}$$  \hspace{2cm} (2-4)

This option can only be used for block fading channel conditions. Another option is to fix the normalization factor. In this case the normalization factor remains the same for all the frames according to the average of the instantaneous SNR $\gamma$, i.e. $\rho$. $\beta$ at the relay output is then calculated based on the power constraint at the relay as follows:

$$\mathcal{E}_{0} \left( \beta \left( 1 + |\alpha|^2 \rho \right) \right) \leq 1 \Rightarrow \beta = (1 + \rho)^{-1}$$  \hspace{2cm} (2-5)

This option is useful for fast fading conditions. By knowing the equivalent channel $H$ and $R^{-1}$ at the destination, any detection technique; linear or non linear can be used to decode the transmitted data $x$.

### 2.2.1.2 Non-orthogonal Amplify and Forward Transmission (Repeating the Signal at the Source)

In this protocol, the source can transmit in the second phase the same signal that was transmitted during the first phase. As a result during the second phase the destination receives

$$y_2 = (\alpha_2 \alpha_4 \sqrt{\beta \rho_2 \rho_1} + \alpha_6 \sqrt{\rho_0})x + \alpha_2 \sqrt{\beta \rho_2} n + n_2$$  \hspace{2cm} (2-6)

Consequently, the equivalent signal model is as follows:

$$\begin{bmatrix} y_0 \\ y_2 \end{bmatrix} = \begin{bmatrix} \alpha_0 \sqrt{\rho_0} I_M \\ (\alpha_2 \alpha_4 \sqrt{\beta \rho_2 \rho_1} + \alpha_6 \sqrt{\rho_0}) I_M \end{bmatrix} x + \begin{bmatrix} I_M & 0_M & 0_M \\ 0_M & I_M & 0_M \\ n & n_1 & n_2 \end{bmatrix}$$  \hspace{2cm} (2-7)

Notice that the setting of $\beta$ is the same as in the orthogonal protocol.
2.2.1.3 Non-orthogonal Amplify and Forward Transmission (A New Signal at the Source)

In this case the source transmits the signal \( x_1 \) during the first phase and a new signal \( x_2 \) during the second phase. Then during the second phase the destination receives 

\[
y_2 = \alpha_x x_1 + \alpha_x \sqrt{\beta \rho_2} x_2 + \alpha_x \sqrt{\rho_0} n_1 + n_2
\]

and the equivalent signal model is given by

\[
\begin{bmatrix}
y_0 \\
y_2
\end{bmatrix} =
\begin{bmatrix}
\alpha_x \sqrt{\rho_0} I_M & 0_M \\
\alpha_x \sqrt{\beta \rho_2} I_M & \alpha_x \sqrt{\rho_0} I_M
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} +
\begin{bmatrix}
0_M & 0_M & 0_M \\
\alpha_x \sqrt{\beta \rho_2} I_M & 0_M & 0_M
\end{bmatrix}
\begin{bmatrix}
n_0 \\
n_1 \\
n_2
\end{bmatrix}
\]

(2-8)

Notice that the setting of \( \beta \) is the same as in the orthogonal protocol.

Some detailed information about decode and forward (DF) scheme can be found in appendix B.1. In the following sections two important measures in evaluating the performance of these protocols are introduced.

2.3 Achievable Rate and Outage Behaviour of Amplify and Forward

In the above sections the three different protocols for the traditional AF scheme and their corresponding equivalent signal model were discussed. In this section two important evaluation metrics for evaluating the performance of the AF scheme in the above protocols is introduced. First Ergodic capacity is defined and then achievable rate and outage probability terms.

2.3.1 Ergodic Capacity

According to Shannon capacity theorem [91,94] in 1948, the channel capacity is defined as “The maximum data rate to which information data can be reliably transmitted from a source to a destination over a communication channel with an asymptotically small error probability.” In terms of information theory the channel capacity is the maximum mutual information of input \( x \) and output \( y \) over all possible input distributions \( p(x) \) [16]:

\[
C = \max_{p(c)} I(x; y)
\]

(2-9)

The noisy-channel coding theorem [92-93] expresses that “In a noisy channel for any \( \varepsilon > 0 \) and for any data rate \( R < C \), there is an encoding and decoding scheme that allows the probability of block error at the receiver to be less than \( \varepsilon \) for a sufficiently long code. Also, for any data rate
$R > C$, block error rate at the destination approaches one when the block length increases to infinity.

The Ergodic capacity is defined as the expectation of the instantaneous capacity over a sequence of fading channel realizations. The number of the channel realizations should be sufficiently high such that the statistic properties of the Ergodic fading channel are satisfied. In the appendix A.1 and A.2 two examples for the Ergodic capacity in single antenna and multiple antenna point to point communication scenarios have been explored. In the following section achievable rate is defined.

### 2.3.2 Achievable Rate

For a point to point communication system, the following definition can be given [95]:

**Definition:** "A $(M, N)$ code consists of set of messages $W = \{1, 2, ..., M\}$, an encoding scheme $W \rightarrow \chi^N$ and a decoding scheme $y^N \rightarrow W$ where $\chi$ and $y$ are the channel input and output alphabets. In this scenario, the rate of an $(M, N)$ code is defined as $R = \frac{1}{N} \log_2 M$. The rate $R$ is achievable if for any $\varepsilon > 0$, an $(M, N)$ code exists with $M \geq 2^{NR}$ such that the average probability of error after decoding is smaller than $\varepsilon$ if the length of code $N$ is satisfactorily large".

For example in a block fading channel with instantaneous SNR $\gamma_0$, the rate $R$ is achievable if

$$R \leq C(\gamma_0) = \log_2 (1 + \gamma_0)$$  \hspace{1cm} (2-10)

In Ergodic channel, as it is clear from its name, the process of generating the channel gains is Ergodic and the channel should be varied in time. Thus by averaging the channel gains over time the randomness of the channel gain is removed. By doing expectation over all channel realizations the long term bit rate that can be supported is found.

The achievable rate in an i.i.d Rayleigh fading channel with average SNR $\rho_0$ is:

$$R \leq C(\rho_0) = \int_0^{\infty} \log_2 (1 + \gamma_0) p(\gamma_0) d\gamma_0$$  \hspace{1cm} (2-11)
where $\gamma_0$ has an exponential distribution i.e. $p(\gamma_0) = \frac{1}{\rho_0} e^{-\frac{\gamma_0}{\rho_0}}$ with mean $\rho_0$. Therefore the average achievable rate is a function of the average SNR not the instantaneous SNR.

In contrast in non-ergodic channel, the channel gain is a random variable but it does not change during the transmission of a codebook. Therefore the randomness of the channel gain cannot be removed by averaging over a code time. So this channel cannot support the long-term constant bit rates. As a result in the block fading channel case, the outage capacity should be used instead of the Ergodic capacity or achievable rate. In this case the outage probability for transmission at a specific rate can be measured.

The achievable rate for DF scheme has been provided in appendix B.2. As it is seen for DF scheme the duplexing ratio is not fixed and can be optimized in order to maximize the achievable rate. It has been shown that in DF scheme the duration of the first phase can be decreased if the quality of the source-relay link is enhanced. Similarly when the quality of the relay-destination link is better, the time duration of transmission from relay to destination can be reduced [5,90].

### 2.3.2.1 Achievable Rate for Orthogonal Amplify and Forward

The achievable rate for relaying scheme by using the achievable rate equation for MIMO system (Appendix A.2) is calculated as follows [8]:

$$R(\gamma) \leq I(\gamma) = \frac{1}{2} \log_2 \det(I + H R_x H^H R_v^{-1})$$

(2-12)

where $R_x = \mathcal{E}_x(xx^H)$ is the covariance matrix of the transmit signal $x$, $R_v$ is the covariance matrix of the equivalent noise vector and $H$ is the equivalent channel matrix. After some modifications:

$$R(\gamma) \leq \frac{1}{2} \log_2 \left(1 + \gamma_0 + \gamma_1 \gamma_2 \beta (1 + \gamma_2 \beta)^{-1}\right)$$

(2-13)

For non-Ergodic channel, by replacing $\beta = (1 + \gamma_1)^{-1}$:

$$R(\gamma) \leq I(\gamma) = \frac{1}{2} \log_2 \left(1 + \gamma_0 + \gamma_1 \gamma_2 (1 + \gamma_1 + \gamma_2)^{-1}\right)$$

(2-14)
Therefore if the source has the knowledge about the instantaneous SNRs, it can tune its transmission rate accordingly. For Ergodic channel, by allocating $\beta = (1 + \rho_1)^{-1}$ the average achievable rate is:

$$R(\rho) \leq \mathcal{E} \left( \frac{1}{2} \log_2 \det(I + HH^b R_v^{-1}) \right)$$

(2-15) or

$$R(\rho) \leq \mathcal{E} \left( \frac{1}{2} \log_2 \left( 1 + \gamma_0 + \gamma_1 + \gamma_2 \right) \right)$$

(2-16)

Recall that $\gamma_0, \gamma_1$ and $\gamma_2$ are random variables with exponential distribution. Therefore it is very complicated to find a close form for the achievable rate.

### 2.3.2.2 Numerical Results

In this section some numerical results obtained by extensive Monte Carlo based simulations for the AF orthogonal protocol are provided. Figure 2-2 (a) shows the SNR Contours for the average achievable rate versus average SNRs. In this figure $SNR_0 = 10 \log_{10} (\rho_0)$, $SNR_1 = 10 \log_{10} (\rho_1)$ and $SNR_2 = 10 \log_{10} (\rho_2)$. Several values for $SNR_0$ have been chosen and the SNRs of two other links are allowed to change over a large range of values. Each contour separates two SNR regions i.e direct transmission, $\alpha_{opt} = 1$ and traditional AF, $\alpha_{opt} = 0.5$. These two regions have been emphasized for $SNR_0 = -30$dB (red colour). Recall that in this section optimization is done between traditional AF and direct transmission. Also the average achievable rate has been depicted in Figure 2-2 (b). When $SNR_1$ or $SNR_2$ is very low, the achievable rate is fixed but its value is very low due to very low $SNR_0$. However for high $SNR_1$ or $SNR_2$, the optimum dupplexing ratio $\alpha_{opt} = 0.5$ and thus the achievable rate increases by increasing $SNR_1$ or $SNR_2$. In the next section the outage probability as another measure of the performance of the AF scheme is introduced.
2.3.3 Outage Probability

In a point to point communication system, if both the transmitter and receiver nodes have the instantaneous SNR knowledge of the link between them, the instantaneous transmission rate $R$
can follow the maximum rate \( C(\gamma_0) \). However, the transmitter node usually does not have the transmit CSI. In such a case the transmitter node does not know which rate is optimum at each time instant. In deep fading channel condition, the channel can not support a small selected transmission rate. Therefore the transmission rate \( R \) cannot be appropriately adjusted at the transmitter node. When the transmission rate \( R > C(\gamma_0) \) an outage event happens and consequently the outage probability is defined as \( P_{\text{outage}}(R, \rho_0) = \text{Pr}\{R > C(\gamma_0)\} \). As an example for a block fading channel, the outage probability is

\[
P_{\text{outage}}(R, \rho_0) = \text{Pr}\{R > \log_2(1 + \gamma_0)\} = \text{Pr}\{\left(2^R - 1\right) > \gamma_0\} \quad (2-17)
\]

Notice that the random variable \( \gamma_0 \) has an exponential distribution, thus it is necessary to take all the outage regions into account when the outage probability is calculated. Therefore the outage probability for a single user single antenna point to point communication is:

\[
P_{\text{outage}}(R, \rho_0) = \frac{1}{\rho_0} \int_0^{\rho_0} e^{\rho_0 \gamma_0} d\gamma_0 = 1 - e^{\rho_0 - \rho_0} \quad (2-18)
\]

The outage probability for DF scheme has been provided in appendix B.3. It is seen that in DF scheme by choosing optimum duplexing ratio the outage probability is minimized. In the next section the outage probability for the orthogonal AF protocol is addressed.

### 2.3.3.1 Outage for Orthogonal Amplify and Forward

In the orthogonal AF protocol, an outage event happens when the transmission rate is greater than the instantaneous mutual information i.e. \( R > I(\gamma) \) and therefore the outage probability is \( P_{\text{outage}}(R, \rho) = \text{Pr}\{R > I(\gamma)\} \). Considering block fading channel, by inserting equation (2-14) the outage probability is

\[
P_{\text{outage}}(R, \rho) = \text{Pr}\left\{\left(2^{2R} - 1\right) > \left(\gamma_0 + \frac{\gamma_1 \gamma_2}{1 + \gamma_1 + \gamma_2}\right)\right\} = \text{Pr}\left\{\left(2^{2R} - 1\right) > \left(\gamma_0 + \gamma_{AF}\right)\right\} \quad (2-19)
\]

where \( \gamma_{AF} = \gamma_1 \gamma_2 (1 + \gamma_1 + \gamma_2)^{-1} \). As a result

\[
P_{\text{outage}}(R, \rho) = \int_0^\infty \frac{1}{\rho_0} e^{\rho_0 \gamma_0} \left(\int_0^{\rho_0} f_{\gamma_{AF}}(\gamma_{AF}) d\gamma_{AF}\right) d\gamma_0 = \int_0^\infty \frac{1}{\rho_0} e^{\rho_0 \gamma_0} f_{\gamma_{AF}}(c - \gamma_0) d\gamma_0 \quad (2-20)
\]
where $f_{\gamma_{AF}}(\gamma_{AF})$ and $F_{\gamma_{AF}}(\gamma_{AF})$ are PDF and CDF of $\gamma_{AF}$ respectively and $c = 2^{2R} - 1$. In Rayleigh fading channel, $\gamma_0, \gamma_1$ and $\gamma_2$ are random variables with exponential distribution. Since there are three SNRs and the transmission process is more complicated than a point to point transmission, it is clear that computation of the outage probability for the traditional AF scheme in the block fading channel is very difficult. This issue is discussed with more detail in the next chapter. In this section to show a closed form approximation for the outage probability, an approximation over $\gamma_{AF}$ is considered. If $\gamma_{AF} = \min(\gamma_1, \gamma_2)$ then the CDF of $\gamma_{AF}$ is as follows:

$$F_{\gamma_{AF}}(\gamma_{AF}) = 1 - e^{-\gamma_{AF} \left( \frac{1}{\rho_1} + \frac{1}{\rho_2} \right)}$$

(2-21)

Consequently the outage probability is $P_{\text{outage}}(R, \rho) \approx \int_0^{\rho_0} e^{-\gamma_0 \left( \frac{1}{\rho_1} + \frac{1}{\rho_2} \right)} \left( 1 - e^{-\gamma_0 \left( \frac{1}{\rho_1} + \frac{1}{\rho_2} \right)} \right) d\gamma_0$. After calculating the above integral through some modifications the outage probability is simplified to:

$$P_{\text{outage}}(R, \delta_0, \delta_1, \delta_2, \text{SNR}) \geq 1 - \left( \frac{\delta_0 (\delta_1 + \delta_2)}{\delta_0 (\delta_1 + \delta_2) - \delta_1 \delta_2} \right) e^{-\frac{1}{\text{SNR}} \left( \delta_1 + \delta_2 \right)} + \left( \frac{\delta_1 \delta_2}{\delta_0 (\delta_1 + \delta_2) - \delta_1 \delta_2} \right) e^{-\frac{1}{\text{SNR}} \left( \delta_1 + \delta_2 \right)},$$

where $\delta_1, \delta_2$ and $\delta_0$ are the offsets of $\rho_1, \rho_2$ and $\rho_0$ compared to a reference value SNR. It means: $\rho_k = \delta_k \text{SNR}$. The derivation of the above equation can be found in appendix D.2 when the duplexing ratio is 0.5.

After approximating the exponential function by the first three terms of it's Taylor series i.e. $e^x = 1 + x + \frac{x^2}{2}$, the outage probability $P_{\text{outage}}(R, \delta_0, \delta_1, \delta_2, \text{SNR}) \geq \left( \frac{\delta_0 (\delta_1 + \delta_2)}{2 \delta_1 \delta_2} \right) \left( \frac{1}{\text{SNR}} \right) c$ is obtained which is similar to the probability of the outage calculated in [4]. The outage probability can be considered as a sign for diversity of the every system. Therefore, it is worthwhile to analyse the outage probability. The above equation shows that the exponent of the factor $\left( \frac{1}{\text{SNR}} \right)$ or the slope of the outage curves is 2 so it can be concluded that the AF scheme can provide the diversity of 2.

The throughput of the AF scheme is defined as $\eta(R, \rho) = R \left( 1 - P_{\text{outage}}(\rho, R) \right)$ for a specific transmission rate $R$. The optimum transmission rate can be found as
\[ R^* = \arg \max_{R \in [0, \infty)} R \left( 1 - P_{\text{ outage}} \left( \rho, R \right) \right). \]

Then the optimum throughput for each set of average SNRs is
\[ \eta(\rho) = R^* \left( 1 - P_{\text{ outage}} \left( \rho, R^* \right) \right). \]

### 2.3.3.2 Numerical Results

In this section the throughput results are provided for the scenario where \( \alpha \) is kept fixed and equal to 0.5 or one for all the frames. In order to have a fair comparison, the throughput has to be first optimized with respect to the rate \( R \). The optimisation was carried out over a large range of transmission rates with a fine level of granularity for \( R \). Several values for \( \text{SNR}_q \) have been used and the SNRs of the two other links are allowed to change over a large range of values. Figure 2-3 (a) shows the SNR Contours for the throughput versus average SNRs. Each contour separates two SNR regions i.e the region where the direct transmission is superior and the AF region where \( \alpha_{\text{opt}} = 0.5 \). These two regions are emphasized for \( \text{SNR}_0 = -30 \text{dB} \) (light brown colour). By increasing \( \text{SNR}_0 \) the region with \( \alpha_{\text{opt}} = 0.5 \) becomes smaller. In Figure 2-3(b) the average throughput is depicted for \( \text{SNR}_0 = 0 \text{dB} \) and different SNR offsets \( \Delta_1 = 10 \log_{10} \left( \delta_1 \right) \) and \( \Delta_2 = 10 \log_{10} \left( \delta_2 \right) \). Therefore \( \text{SNR}_1(\text{dB}) = 10 \log_{10} \left( \rho_0 \right), \quad \text{SNR}_2(\text{dB}) = \Delta_1 + \text{SNR}_0(\text{dB}) \) and \( \text{SNR}_2(\text{dB}) = \Delta_2 + \text{SNR}_0(\text{dB}) \). As it is seen when \( \Delta_1 \) or \( \Delta_2 \) are very low, the throughput is fixed. However \( \alpha_{\text{opt}} = 0.5 \) and thus the throughput increases for high \( \Delta_1 \) or \( \Delta_2 \). But how the throughput of AF scheme is affected if \( \alpha \) is set to a value rather than 0.5 or one? In Figure 2-3 (c) the SNR Contours for the throughput versus SNR offsets are shown. By increasing \( \text{SNR}_0 \) the region for which \( \alpha_{\text{opt}} = 0.5 \) becomes larger. In Figure 2-3 (d) an example for the outage probability for \( \Delta_1 = \Delta_2 = 0 \text{dB} \) is provided. This graph shows that the slope of the outage curve for the AF scheme is 2 at high \( \text{SNR}_0 \) and it hence indicates that the AF scheme can provide a diversity of 2 at high SNRs. Notice that at low \( \text{SNR}_0 \) direct transmission performs better.
Chapter 2 Traditional Amplify and Forward

AF: \[ \text{SNR}_C \] onto \( f \) or \( f \) = 1 and \( a = 0.5 \)

Throughput (or \( \text{SNR}_g = 0 \) dB average)

Figure 2-3. The throughput and outage probability (a) the SNR Contours for the orthogonal AF versus SNRs (b) the throughput for \( \text{SNR}_g = 0 \) dB (c) the SNR contours for the orthogonal AF versus SNR offsets (d) the outage probability for \( \Delta_1 = 0 \) dB and \( \Delta_2 = 0 \) dB

2.3.3.3 Further Discussion

As observed, contrarily to decode and forward schemes, the lengths of the time slots allocated to relay reception and forwarding are considered to be equal in the conventional AF. This leads to a duplexing ratio of 0.5 which is mainly due to the assumed amplification function that has no affect on the duration of the processed signal. This obviously is a drawback of the AF as depending on the quality of the source-relay and relay-destination links the duration of one of the two time slots is unnecessarily long. In the one hand direct transmission with the duplexing ratio of one is optimal for some SNR regions and in other hand AF scheme with the duplexing ratio of 0.5 is performing better in other SNR regions. In other words the direct only transmission and the traditional AF are very efficient in some SNR regions and are less efficient in other SNR regions.
Logically, this question could come across a cute mind that if there exist any method that can make a compromise between these two methods and provide better performance? In the next chapter answer to this question is explored by considering a flexible scheme that allows to tune length of time slots and thus to adjust the duplexing ratio. The basic techniques and measures reviewed in this chapter are used as a basis for future investigations.

2.4 Conclusions

In this chapter, the state of the art for the AF relay scheme was reviewed and some initial definitions along with the system models for the AF protocols were introduced. Three different protocols and their corresponding equivalent signal model were discussed. To evaluate the performance of the cooperative relaying AF scheme, two measures: the achievable rate and the outage probability were introduced. Definitely the calculations of these parameters would be very complicated compared to the point to point communication systems. A closed-form approximation for the outage probability of the traditional AF scheme was presented and the behaviour of the traditional AF scheme in terms of the achievable rate and the outage probability in different SNR regions was discussed along with some numerical results. It was shown that the direct only transmission and the traditional AF scheme are very efficient in some SNR regions and are less efficient in other SNR regions. Therefore it is likely to find an AF scheme with the duplexing ratio rather than one and 0.5 which can provide better performance than the direct and traditional AF transmissions. In the next chapter this idea is further investigated.
Chapter 3 Amplify and Forward in Single Antenna Scenario

3.1 Introduction

In conventional AF, the lengths of the time slots allocated to relay reception and forwarding are considered to be equal [5, 18-21]. This leads to the duplexing ratio of 0.5, where duplexing ratio is defined as the ratio of the duration of the first phase to the overall duration of the two phases. The amplification function at the relay does not have effect on the duration of the processed signal. In this chapter a flexible scheme is proposed that allows separately tuneable time slots and thus flexibility in adjustment of the duplexing ratio. This in turn provides user an extra degree of freedom in optimization of the AF scheme.

In order to study this new scheme for AF, analysis in terms of two important measures; achievable rate and outage probability are considered. First the mathematical models for the instantaneous mutual information are provided. The achievable rate and the outage probability performances for different time slot lengths are explored. Later these mathematical models are deployed to find the optimum duplexing ratio in different network topology and SNR conditions. Moreover the analysis results with Monte Carlo simulation are compared in order to verify the proposed mathematical analysis. The provided analysis enables user to predict the system performance and adjust its parameters for any combination of the SNRs of the different links.

The remainder of this chapter is organized as follows. Section 3.2 describes the system model for the flexible AF concept. Section 3.3 describes the performance measurements including the achievable rate and the outage probability. Section 3.4 looks at the derivation of the instantaneous and Ergodic mutual information. In section 3.5, numerical results are provided for the achievable rate and the throughput performance of the orthogonal AF. Section 3.6 discusses in detail the outage probability analysis and compares the analysis result with simulation. Finally in section 3.7 diversity of the AF scheme in different SNR conditions is explored.
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3.2 System Model

Let us assume a three-node cooperative communication system composed of: a source node, a relay node, and a destination node. As shown in Figure 3-1 three existing links; source-destination, source-relay and relay-destination are indexed with 0, 1, and 2, respectively. It is assumed that the relay node is not able to receive and transmit simultaneously. Each transmission frame is divided into two phases: during the first phase the source broadcasts a Gaussian code word \( x_i(w) = (x_1^{(i)}(w), x_2^{(i)}(w), ..., x_{M}^{(i)}(w)) \) to both the relay and the destination where \( x_1^{(i)}(w) \in \mathbb{C}^{N_1} \). During the second phase if the relay cooperates, it transmits a signal sequence \( x_2(w) = (x_2^{(0)}(w), x_2^{(2)}(w), ..., x_2^{(M)}(w)) \) to the destination where \( x_2^{(i)}(w) \in \mathbb{C}^{N_2} \). In this phase depending on the deployed protocol, the source could either transmit to the destination or be silent. In this chapter it is assume that each phase consists of M blocks as shown in Figure 3-1. The durations of the time slots are not necessarily equal. In this chapter the duplexing ratio \( \alpha \) is defined as the ratio of the first phase \( M \times N_1 \) in the whole transmission phase duration \( M \times (N_1 + N_2) \). The duplexing ratio is defined as \( \alpha = N_1 / (N_1 + N_2) \). The duplexing ratio varies between zero and one. When \( N_2 \gg N_1 \), duplexing ratio moves towards zero and when \( N_2 = 0 \) the duplexing ratio is one. It is assumed that the instantaneous SNRs of all the three links \( \gamma = [\gamma_0, \gamma_1, \gamma_2] \) are known to the destination. Also let us assume that both the amplitude and the phase information can be obtained through channel estimation.

![Figure 3-1. System model](image)

For the convenience of the exposition, from now this chapter focuses on one of the blocks and drop the superscripts of the signals and the channel parameters. At the relay, matrix \( A \in \mathbb{C}^{N_2 \times N_1} \) which is a sub-matrix of unitary matrix is multiplied to the received signal. Therefore matrix A should have following feature. \( A^H A = I_{N_1} \) when \( N_1 \leq N_2 \) and \( AA^H = I_{N_2} \) when
Chapter 3 Amplify and Forward in Single Antenna Scenario

In the section 3.4, it will be seen that above assumption for matrix \( A \) highly simplifies the expression for mutual information and later on makes the rate and outage analysis much simpler. Also more detailed information can be found in appendix C.1. It is obvious that by changing the number of rows of this matrix, the duplexing ratio \( \alpha \) is changing.

The received signals at the destination for the first and second time slots are

\[ y_1 = \alpha_0 x_1 + n_0 \quad \text{and} \quad y_2 = \alpha_2 \sqrt{\beta A} x_1 + \alpha_2 \sqrt{\beta A} n_1 + n_2 \]

respectively. \( n_0, n_1 \in \mathbb{C}^{N_1 \times d} \) and \( n_2 \in \mathbb{C}^{N_2 \times d} \) are complex i.i.d noise vectors with zero mean and unit variances. \( \alpha_0, \alpha_1 \) and \( \alpha_2 \) are complex i.i.d fading multipliers with zero mean and variance \( \rho_j; \ j=0,1,2 \) for the source-destination, source-relay and relay-destination links respectively. \( \rho = [\rho_0, \rho_1, \rho_2] \) represents the average SNRs of three links. These channel values are assumed fixed at least during each block. The instantaneous SNRs are \( \gamma_j = [\alpha_j]^2 \ j=0,1,2 \). Therefore \( \mathcal{E}_{\alpha_j}(\gamma_j) = \rho_j \), where \( \mathcal{E}(\cdot) \) stands for expectation over all realizations. Also \( \mathcal{E}_x \left( \text{trace} (x_j x_j^H) \right) = N_1 \). A block diagram for the orthogonal AF has been shown in Figure 3-2. As a result the equivalent model is given by:

\[
\begin{bmatrix}
y_0 \\
y_2
\end{bmatrix} = \begin{bmatrix}
\alpha_0 I \\
\alpha_1 \alpha_2 \sqrt{\beta A}
\end{bmatrix} \begin{bmatrix}
x_1 \\
n
\end{bmatrix} + \begin{bmatrix}
I & 0 & 0 \\
0 & \alpha_2 \sqrt{\beta A} & I
\end{bmatrix} \begin{bmatrix}
n_0 \\
n_1 \\
n_2
\end{bmatrix}
\]

and

\[
R_x = \mathcal{E}_{n_0, n_1, n_2, \alpha_0, \alpha_1} \left( vv^H \right) = \begin{bmatrix}
I & 0 \\
0 & (I + [\alpha_2]^2 \beta AA^H)
\end{bmatrix}
\]

Figure 3-2. Block diagram for the orthogonal AF protocol

Gain factor \( \beta \) is used for normalizing the power at the relay output. This parameter is calculated based on the power constraint at the relay and can be set fixed or variable per frame. In this chapter two options are considered for this parameter. In the first option it is considered that \( \beta \) is
varying according to the instantaneous SNR of the source-relay link. Considering power constraint at the relay output, $\mathcal{E}_{\text{F},\text{in}} \left( \text{trace} \left( x_1 x_2^H \right) \right) = \text{trace} \left( \beta A A^H + |\alpha|^2 \beta A A^H \right) = N_2$, and by assuming $A$ as a sub-matrix of a unitary matrix the normalization factor $\beta$ at the relay output is as follows:

$$\beta = \begin{cases} (1 + \gamma)^{-1} \frac{1 - \alpha}{\alpha} & 0 < \alpha < 0.5 \\ (1 + \gamma)^{-1} & 0.5 \leq \alpha \leq 1 \end{cases}$$  \hspace{1cm} (3-1)$$

More detailed derivation of above equation can be found in appendix C.1. It is assumed that the instantaneous SNRs $\gamma$ are fixed at least during each block (static fading).

In the second option, the normalization factor remains the same for all frames according to the average SNR i.e. $\rho$. Therefore $\beta$ at the relay output is calculated as follows:

$$\mathcal{E}_{\text{F}} \left( \text{trace} \left( \beta A A^H + |\alpha|^2 \beta A A^H \right) \right) = N_2 \Rightarrow \beta = \begin{cases} (1 + \rho)^{-1} \frac{1 - \alpha}{\alpha} & 0 < \alpha < 0.5 \\ (1 + \rho)^{-1} & 0.5 \leq \alpha \leq 1 \end{cases}$$  \hspace{1cm} (3-2)$$

This option is used when the channel is fixed over a block but varies along blocks. The system model and the equivalent signal model for other protocols including the non-orthogonal protocols have been shown in the appendices C-2, C-3 and C-4. In the following section the analysis of the proposed flexible AF scheme is presented.

### 3.3 Performance Measurements

In the previous section the system model and the equivalent signal model for the flexible AF under the orthogonal protocol were introduced. In this section the AF scheme is analysed in terms of two important measures; achievable rate and outage probability. The presented analysis is used for the optimisation of the performance.

#### 3.3.1 Ergodic Achievable Rate

The aim of this section is to find an optimum duplexing ratio to maximize $\mathcal{E}_{\text{F}} \left( I(\gamma, \alpha) \right)$ where $I(\gamma, \alpha)$ is defined as the instantaneous mutual information. The optimum duplexing ratio is
\[ \alpha_{\text{opt}} = \arg\max_{0 \leq \alpha \leq 1} \left( E_{\gamma} \left( I \left( \gamma, \alpha \right) \right) \right) \]. Under Ergodic channel, \( \beta \) remains fixed during all the frames. By Ergodic channel, it means a fixed channel per block but independent i.i.d channel gains over adjacent blocks.

### 3.3.2 Outage Probability Measure

The outage probability and the resulted throughput are the measures to be considered for the performance optimization under static (block) fading channel conditions. The throughput \( \eta \) is defined as \( \eta \left( R, \alpha, \rho \right) = R \left( 1 - P_{\text{out}} \left( \rho, \alpha, R \right) \right) \) where \( R \) is the transmission rate at the source. This throughput is achievable using an ARQ scheme with no limitation on its maximum number of retransmissions. Depending on the CSI status of the involved nodes several scenarios can be considered for maximising the throughput of the system. In this case the channel is assumed fixed during one codeword and the channel values for different code words are i.i.d fading.

At first scenario (global CSI availability) CSIs are available to all the nodes. Since the channel knowledge is available at the source, it is able to tune its transmission rate adaptively. This means the source can change the time slot sizes per frame according to the optimum \( \alpha \). \( \alpha \) is optimised at the frame level as it depends on the instantaneous SNR of the involved links i.e. \( I^* \left( \gamma \right) = \max_{\alpha} I \left( \gamma, \alpha \right) \). In this scenario the transmitted code words are not in the outage and the throughput is the average of transmission rates of the all code words which is equal to the Ergodic achievable rate i.e. \( \eta \left( \rho \right) = E_{\gamma} \left( I^* \left( \gamma \right) \right) \).

At the second scenario only global knowledge about average SNRs is available. Since there is no instantaneous knowledge at the source, it cannot change the time slot sizes per frame and during all the transmissions the duplexing ratio needs to be kept fixed. \( \alpha \) is optimised at an average level with following steps:

1. Calculating the outage probability \( P_{\text{out}} \left( \rho, \alpha, R \right) = P_r \left( I \left( \gamma, \alpha \right) < R \right) \) and
   \[ \eta \left( R, \alpha, \rho \right) = R \left( 1 - P_{\text{out}} \left( \rho, \alpha, R \right) \right) \text{ for } 0 \leq \alpha \leq 1 \text{ and } R \in \left[ 0, +\infty \right) \]
2. \( \eta^* \left( \rho, \alpha \right) = \max_{R \in \left[ 0, +\infty \right)} \eta \left( R, \alpha, \rho \right) \) and \( R^* \left( \rho, \alpha \right) = \arg \max_{R \in \left[ 0, +\infty \right)} \eta \left( R, \alpha, \rho \right) \)
3. \( \rho_{\text{opt}} \left( \rho \right) = \max_{0 \leq \alpha \leq 1} \eta^* \left( \alpha, \rho \right) \text{ and } \alpha^* \left( \rho \right) = \arg \max_{0 \leq \alpha \leq 1} \eta^* \left( \alpha, \rho \right) \).
In the third scenario, relay has full instantaneous SNR knowledge about its input channel i.e. \( \alpha \), therefore following steps can be done to calculate the throughput:

1. \( P_{\text{out}} (\rho_0, \rho_2, \alpha, \gamma_1, R) = P_r \left( I (\gamma_1, \alpha) < R \right) \)

2. \( \eta_{\text{opt}} (\rho) = \max_{\rho \in [0, \infty)} \mathbb{E}_{\alpha} \left( \max_{0 \leq \alpha \leq 1} \left( 1 - P_{\text{out}} (\rho_0, \rho_2, \alpha, \gamma_1, R) \right) \right) \)

Having the knowledge about \( \gamma_1 \), the relay can find the optimum \( \alpha \) and change the size of its output block accordingly. This scenario is applicable only if the orthogonal protocol is deployed.

### 3.4 Mutual Information

In this section the instantaneous mutual information and the achievable rate are derived. These derivations can be extended to non-orthogonal cooperative protocol as well. But for simplicity of the expressions the concentration of this section is on the orthogonal protocol.

#### 3.4.1 Instantaneous Mutual Information

**Lemma 3-1:** The instantaneous accumulative mutual information \( I (\gamma, \alpha) \) normalized over one frame for the orthogonal AF with variable \( \alpha \) is

\[
I (\gamma, \alpha) = \begin{cases} 
I^{(0)} (\gamma, \alpha) & \text{for } 0 < \alpha < 0.5 \\
I^{(2)} (\gamma, \alpha) & \text{for } 0.5 \leq \alpha \leq 1
\end{cases}
\tag{3-3}
\]

where

\[
I^{(0)} (\gamma, \alpha) = \alpha \log \left( 1 + \gamma_0 + \gamma_1 \gamma_{2,\text{eff}} \left( 1 + \gamma_1 + \gamma_{2,\text{eff}} \right)^{-1} \right)
\]

\[
I^{(2)} (\gamma, \alpha) = (2 \alpha - 1) I_0 (\gamma_0) + 2 (1 - \alpha) I^{(0)} (\gamma, 0.5)
\]

\[
\gamma_{2,\text{eff}} = \frac{(1 - \alpha)}{\alpha} \gamma_2
\]

and

\[
I_0 (\gamma_0) = \log_2 \left( 1 + \gamma_0 \right)
\]

It is clear that \( I (\gamma, 0.5) = I^{(0)} (\gamma, 0.5) = I^{(2)} (\gamma, 0.5) \) and \( I (\gamma, 1) = I^{(0)} (\gamma, 1) = I^{(2)} (\gamma, 1) \). As it was seen by choosing matrix \( A \) as a sub-matrix of a unitary matrix, \( A \) is disappeared in the above expression and the mutual information is not affected by changing the contents of the matrix \( A \).
Lemma 3-2: The optimum duplexing ratio to maximize $I(y, \alpha)$ is

$$\alpha_{\text{opt}} = \arg \max \left( I(y, \alpha) \right) = \begin{cases} \alpha_{\text{opt}}^{(1)} & I^{(1)}(y, \alpha_{\text{opt}}^{(1)}) > I^{(2)}(y, \alpha_{\text{opt}}^{(2)}) \\ \alpha_{\text{opt}}^{(2)} & \text{otherwise} \end{cases}$$

(3-4)

where $\alpha_{\text{opt}}^{(1)} = \arg \max_{0<\alpha<0.5} I^{(1)}(y, \alpha)$ and $\alpha_{\text{opt}}^{(2)} = \arg \max_{0.5<\alpha<1} I^{(2)}(y, \alpha) = \begin{cases} 0.5 & 2I_0(y_0) < I^{(1)}(y, 0.5) \\ 1 & 2I_0(y_0) \geq I^{(1)}(y, 0.5) \end{cases}$.

The proof and the derivation of the mutual information for this protocol (lemma 3-1 and lemma 3-2) is described in the appendix C-1. In the following paragraphs some remarks are provided based on the above lemmas.

By rewriting $I^{(1)}(y, \alpha)$ as follows:

$$I^{(1)}(y, \alpha) = \alpha \log \left( \frac{y_1 + y_2}{1 + \frac{y_1}{y_2} \frac{\alpha}{1-\alpha}} \right)$$

(3-5)

It is seen that $y_{12}(y_1, y_2, \alpha) = \frac{y_1}{1 + \frac{y_1}{y_2} \frac{\alpha}{1-\alpha}}$ is a monotonically decreasing function of $\alpha$ and as a result the argument inside of log function is monotonically decreasing function with $y_{12}(y_1, y_2, 0) = y_1$ as a starting point. Thus the $I^{(1)}(y, \alpha)$ is product of one logarithmically decreasing term and one linearly increasing term. So, when $\alpha$ is increasing (for all $0 \leq \alpha$), the function $I^{(1)}(y, \alpha)$ increases from $(0, 0)$ up to an optimum point $(\alpha^*, I^{(1)}(y, \alpha^*))$. Then it starts decreasing. Therefore $I^{(1)}(y, \alpha)$ has a unique optimum at $\alpha^*$. The position of $\alpha^*$ is an important issue. The AF scheme is interested on $\alpha^*$ where $0 \leq \alpha^* < 0.5$ however the position of $\alpha^*$ depends on $y$. By looking at the derivative of $I^{(1)}(y, \alpha)$ at $\alpha = 0.5$ which is

$$I^{(1)}(y, 0.5) = \log \left( 1 + y_0 + y_{12}(0.5) \right) + \frac{-2y_1y_2(1+y_1)/\ln(2)}{(1+y_0+y_{12}(0.5))(1+y_1+y_2)^2},$$

user can find where $\alpha^*$ is. Based on the above argument following remarks could be given.
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3.4.1.1 Remarks

**Remark 1:** If \( I^{(1)}(\gamma, 0.5) \) is positive, it confirms that \( 0.5 \leq \alpha^* \) and \( I(\gamma, \alpha) \) is monotonically increasing until reaches \( \alpha = 0.5 \) and then continue to linearly increase or decrease or remain fixed until \( \alpha = 1 \) since for \( 0.5 \leq \alpha \leq 1 \), relationship between \( I^{(2)}(\gamma, \alpha) \) and \( \alpha \) is linear.

When \( 2I_0(\gamma_0) < I^{(1)}(\gamma, 0.5) \), \( I^{(2)}(\gamma, \alpha) \) is monotonically decreasing with its maximum at \( (\alpha = 0.5, I^{(2)}(\gamma, 0.5)) \). So the optimum duplexing ratio \( \alpha_{opt} = 0.5 \).

When \( 2I_0(\gamma_0) > I^{(1)}(\gamma, 0.5) \), \( I^{(2)}(\gamma, \alpha) \) is monotonically increasing with maximum at \( (\alpha = 1, I^{(2)}(\gamma, 1)) \). So the optimum duplexing ratio is one i.e. \( \alpha_{opt} = 1 \).

When the equality is satisfied \( (2I_0(\gamma_0) = I^{(1)}(\gamma, 0.5)) \) and subsequently \( \gamma_0 = \frac{1}{2}(1 - \sqrt{1 + \frac{4\gamma_1\gamma_2}{1 + \gamma_1 + \gamma_2}}) \) then \( I(\gamma, \alpha) \) remains fixed for \( 0.5 \leq \alpha \leq 1 \). So the optimum duplexing ratio \( \alpha_{opt} \) could be any point between 0.5 and 1.

**Remark 2:** If \( I^{(2)}(\gamma, 0.5) \) is negative it proves that \( 0 \leq \alpha^* < 0.5 \). This means that \( I(\gamma, \alpha) \) starts increasing at first. Then it reaches its maximum at \( \alpha^* \) and finally continues to decrease until it reaches \( \alpha = 0.5 \) (as it is shown in Figure 3-3). Since \( I(\gamma, 1) = I^{(2)}(\gamma, 1) = I^{(1)}(\gamma, 1) < I^{(1)}(\gamma, 0.5) < I^{(1)}(\gamma, \alpha^*) \), \( I^{(2)}(\gamma, \alpha) \) as well as \( I(\gamma, \alpha) \) continue to decrease linearly until they reach \( \alpha = 1 \). In this case \( \alpha_{opt} = \alpha^* \).

**Remark 3:** When \( \gamma_0 \gg \gamma_{12}(\gamma_1, \gamma_2, \alpha) \), it is obvious that \( 1 + \gamma_0 + \gamma_{12} = 1 + \gamma_0 \) and as a result \( I^{(1)}(\gamma, \alpha) = \alpha I_0(\gamma_0) \). So \( I(\gamma, \alpha) \) almost linearly and monotonically increases up to \( (\alpha = 0.5, 0.5I_0) \) and finally it continues to increase linearly to \( (\alpha = 1, I_0) \). Since \( \max_{\alpha \leq 0.5} \gamma_{12}(\gamma_1, \gamma_2, \alpha) = \gamma_1 \), the above condition is satisfied if \( \gamma_0 \gg \gamma_1 \). It also can be concluded that \( 0 \leq \alpha^* < 0.5 \) happens when \( \gamma_0 \ll \gamma_1 \). This behaviour has also been shown in Figure 3-4 (a).
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Flexible AF: Mutual Information Components: $\mathcal{I}^1(\alpha)$, and $\mathcal{I}^2(\alpha)$.

\[ \mathcal{I}^1(\alpha) \]
\[ \mathcal{I}^2(\alpha) \]

derivative $\mathcal{I}^1(\alpha)$

Figure 3.3. The mutual information components versus $\alpha$

\[ \mathcal{I}^1(\alpha) \]
\[ \mathcal{I}^2(\alpha) \]

Figure 3.4. The instantaneous mutual information versus $\alpha$ (a) for different direct SNRs (b) for the different source-relay SNR

Remark 4: By looking at $\gamma_1 / \gamma_2 (\gamma_1, \gamma_2, \alpha) = \gamma_1 / [1 + \{(1 + \gamma_1) / \gamma_2\} (\alpha / (1 - \alpha))]$ it could be found that the ratio $\gamma_1 / \gamma_2$ is an important factor in finding $\alpha^*$. By increasing the ratio $\gamma_1 / \gamma_2$, the $\alpha^*$ is shifted toward half and then less than half. This behaviour has also been shown in Figure 3.4 (b). So in order to have $\alpha_{opt} < 0.5$, $\gamma_0$ must be small and the ratio $\gamma_1 / \gamma_2$ should be large.
Remark 5: When there is no direct link it is just needed to update instantaneous mutual information equation (3-3) by setting $\gamma_0 = 0$. The only difference is $\alpha_{\text{opt}}^{(2)} = \arg \max \limits_{0.5<\alpha<1} I^{(2)}(\gamma,\alpha) = 0.5$. It means that $I^{(2)}(\gamma,\alpha)$ is monotonically decreasing function so it has maximum at $\alpha = 0.5$. By increasing $\gamma_1/\gamma_2$, $\alpha_{\text{opt}}$ is shifted towards less than 0.5.

3.4.2 Ergodic Achievable Rate

When this measure is used, the normalization factor remains fixed for all the transmitted frames. In this case the normalization factor is related to the source to relay average of SNR i.e. $\rho_1$. Ergodic achievable rate is the average of the mutual information over all realizations. By looking at (3-3) the average mutual information for the orthogonal AF with variable $\alpha$ is

$$E_{\gamma}(I(\gamma,\alpha)) = \left\{ \begin{array}{ll} E_{\gamma}(I^{(0)}(\gamma,\alpha)) & 0 < \alpha < 0.5 \\ E_{\gamma}(I^{(2)}(\gamma,\alpha)) & 0.5 < \alpha < 1 \end{array} \right.$$  \hspace{1cm} (3-6)

where $E_{\gamma}(I^{(3)}(\gamma,\alpha)) = (2\alpha - 1)E_{\gamma}(I(\gamma_0)) + 2(1 - \alpha)E_{\gamma}(I(\gamma,0.5))$. The optimum duplexing ratio to maximize $E_{\gamma}(I(\gamma,\alpha))$ is

$$\alpha_{\text{opt}} = \arg \max \limits_{0<\alpha<1} E_{\gamma}(I(\gamma,\alpha)) = \left\{ \begin{array}{ll} \alpha_{\text{opt}}^{(0)} & E_{\gamma}(I^{(0)}(\gamma,\alpha_{\text{opt}}^{(0)})) > E_{\gamma}(I^{(2)}(\gamma,\alpha_{\text{opt}}^{(2)})) \\ \alpha_{\text{opt}}^{(2)} & \text{otherwise} \end{array} \right.$$  

where $\alpha_{\text{opt}}^{(0)} = \arg \max \limits_{0<\alpha<0.5} E_{\gamma}(I^{(0)}(\gamma,\alpha))$ and

$$\alpha_{\text{opt}}^{(2)} = \arg \max \limits_{0.5<\alpha<1} E_{\gamma}(I^{(2)}(\gamma,\alpha)) = \left\{ \begin{array}{ll} 0.5 & 2E_{\gamma}(I(\gamma_0)) < E_{\gamma}(I^{(0)}(\gamma,0.5)) \\ 1 & 2E_{\gamma}(I(\gamma_0)) \geq E_{\gamma}(I^{(0)}(\gamma,0.5)) \end{array} \right.$$  

3.5 Numerical Results

In this section some numerical results obtained are provided by extensive Monte Carlo based simulations for the considered cooperative protocols.
3.5.1 Mutual Information and Achievable Rate

Figure 3-5 investigates the contours of the instantaneous mutual information $I(\gamma, \alpha)$ versus instantaneous SNRs. In this figure $SNR_0 = 10 \log_{10} (\gamma_0)$, $SNR_1 = 10 \log_{10} (\gamma_1)$ and $SNR_2 = 10 \log_{10} (\gamma_2)$. Each contour separates three SNR regions i.e $\alpha_{opt} = 1$, $\alpha_{opt} = 0.5$ and $\alpha_{opt} < 0.5$. These three regions have been emphasized for $SNR_0 = -30dB$ (red colour). This figure shows that the optimum $\alpha$ is not necessarily half or one and in some SNR regions $\alpha < 0.5$ is performing better. This figure depicts that when the instantaneous SNR is available then $\alpha_{opt}$ can easily be decided. But user knows that it is not typical case and in many cases user only knows the average SNR of the links and user needs to decide on $\alpha_{opt}$ on the average level. Also it can be learnt that to get much benefit from $\alpha_{opt} < 0.5$ in average level, $SNR_2$ needs to have narrow distribution (small variance) with average SNR less than zero and around -10 dB but $SNR_1$ can have wider distribution with average SNR around 20 or 30 dB. By increasing $\gamma_0$ from -30dB toward 0dB the region with $\alpha_{opt} < 0.5$ becomes smaller and smaller. For $SNR_0 = 0dB$, no SNR region with $\alpha_{opt} < 0.5$ exists. Interestingly the region for each higher $SNR_0$ is a subset of the region for lower $SNR_0$.

![Flexible AF: SNR Contour with $0 < \alpha \leq 1$ for Instantaneous Mutual Information](image)

Figure 3-5. The SNR contours for the instantaneous mutual information
In the above paragraphs the behaviour of this scheme in the instantaneous mutual information was observed therefore similar behaviour in the average mutual information level is expected. In this respect one of the main issues is the distribution of SNRs which has effect on the performance of the system. Figure 3-6 (a) shows the gain can be achieved for $\alpha_{opt} < 0.5$ in $SNR_0 = -30dB$ for exponential distribution. This gain is defined as the minimum of the difference of the achievable rate for $\alpha_{opt}$ with the achievable rate for $\alpha = 0.5$ or $\alpha = 1$. As it is seen the gain is not high. In this figure $\text{SNR}_0 = 10\log_{10}(\rho_0)$, $\text{SNR}_1 = 10\log_{10}(\rho_1)$ and $\text{SNR}_2 = 10\log_{10}(\rho_2)$. In Figure 3-6 (b) the SNR contours for the achievable rate has been shown which is similar to the SNR contours for the instantaneous mutual information.

![Diagram](image)

**Figure 3-6.** The achievable rate (a) the gain for the exponential distribution with $\text{SNR}_0 = -30dB$ (b) the SNR contours (c) the achievable rate for $\text{SNR}_1 = 20dB$ and $\text{SNR}_2 = -5dB$ (d) the gain for gamma distribution with $\text{SNR}_0 = -30dB$
Also in Figure 3-6 (c) the achievable rate has been shown for a range of $SNR_0$ with $SNR_1 = 20dB$ and $SNR_2 = -5dB$. In this region the optimum duplexing ratio $\alpha_{opt} < 0.5$. Figure 3-6 (d) shows the gain achieved by Gamma distribution $\Gamma (10, \rho_2 / 10)$ with factor 10 and mean equal average SNR. The gain is higher than the exponential distribution. The gamma distribution $\Gamma (k, \theta)$ is based on the two parameters: a scale parameter $\theta$ and a shape parameter $k$. When $k$ is an integer number the Gamma distributed random variable can be considered as the sum of $k$ independent exponentially distributed random variables, each of them has a mean of $\theta$. As a result mean of the Gamma distribution is $k\theta$ [89]. The instantaneous SNR of each link is normalized in order to have mean of Gamma distribution equal to average SNR of corresponding link.

By decreasing $SNR_0$ towards very low values, the optimum $\alpha$ is decreased from $\alpha = 0.5$ toward zero. This behaviour is more clear by looking at 3-D picture (Figure 3-7) of the average mutual information for $SNR_0 = -20dB$. As it is seen in the region around $0dB \leq \Delta_1 \leq 50dB$ and $-20dB \leq \Delta_2 \leq 0dB$, the optimum $\alpha$ is decreasing towards zero. On other regions the optimum $\alpha$ is 0.5.

![Figure 3-7. The optimum $\alpha$ when $0.25 < \alpha \leq 0.5$ for $SNR_0 = -20dB$](image)
Figure 3-8 shows the average mutual information performance of the AF when $0.5 \leq \alpha \leq 1$ for $\Delta_1 = 10dB$ and $\Delta_2 = 10dB$. As it is seen for $SNR_0$ below 6dB, $\alpha = 0.5$ has the highest mutual information. By increasing $\alpha$ toward one, the average mutual information is decreased. However for $SNR_0$ higher than 6dB the direct transmission $\alpha = 1$ has the highest mutual information and by increasing $\alpha$ toward one, the average mutual information is increased. Therefore as it is expected the average mutual information is maximized at $\alpha = 1$ or $\alpha = 0.5$. All the curves meet each other at one crossing point. The position of this crossing point depends on the average SNRs. By changing SNR offsets similar behaviour is seen but the crossing point is different as it is clear in 3-D picture of the optimum $\alpha$ for $SNR_0 = 0dB$. By increasing direct link SNR, more SNR regions is covered with $\alpha = 1$ and less with $\alpha = 0.5$. When there is no direct link the optimum duplexing ratio $\alpha$ is 0.5.

![Figure 3-8. The average mutual information curves when $0.5 \leq \alpha \leq 1$ (a) for $\Delta_1 = 10dB$ and $\Delta_2 = 10dB$ (b) optimum $\alpha$](image)

### 3.5.2 Throughput and Outage Probability

In this section the throughput results are provided for the scenario that $\alpha$ is kept fixed for all frames and then it is optimized in the average level. To have a fair comparison, the throughput has to be first optimized with respect to the rate $R$ and then duplexing ratio $\alpha$. Optimisation was carried out over a large range of transmission rates with a fine level of granularity for $R$ and $\alpha$. Several values for $SNR_0$ have been used and the two other links SNRs are allowed to change over a large range of values. Figure 3-9 (a) shows the contours of the throughput $\eta_{opt}(\rho)$ versus

\[ \text{Figure 3-8. The average mutual information curves when } 0.5 \leq \alpha \leq 1 \text{ (a) for } \Delta_1 = 10dB \text{ and } \Delta_2 = 10dB \text{ (b) optimum } \alpha \]

\[ \text{3.5.2 Throughput and Outage Probability} \]

\[ \text{In this section the throughput results are provided for the scenario that } \alpha \text{ is kept fixed for all frames and then it is optimized in the average level. To have a fair comparison, the throughput has to be first optimized with respect to the rate } R \text{ and then duplexing ratio } \alpha. \text{ Optimisation was carried out over a large range of transmission rates with a fine level of granularity for } R \text{ and } \alpha. \text{ Several values for } SNR_0 \text{ have been used and the two other links SNRs are allowed to change over a large range of values. Figure 3-9 (a) shows the contours of the throughput } \eta_{opt}(\rho) \text{ versus} \]

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average SNRs for $SNR_0 = -30 dB$ and $\alpha_{opt} \leq 0.5$. This contour separates three SNR regions i.e $\alpha_{opt} = 1$, $\alpha_{opt} = 0.5$ and $\alpha_{opt} < 0.5$. This figure shows similar trends as it was seen for the instantaneous mutual information and the achievable rate which the optimum duplexing ratio $\alpha$ is not necessarily half or one and in some SNR regions $\alpha < 0.5$ is performing better. Also in Figure 3-9 (b), the achievable throughput gain has been shown. Throughput gain is defined as $\Delta \eta(\rho) = \min(\eta_{opt}(\alpha', \rho) - \eta(\alpha = 0.5, \rho), \eta_{opt}(\alpha', \rho) - \eta(\alpha = 1, \rho))$. It is interesting to mention that for very low $SNR_0$, there is no region that the system with $1 > \alpha > 0.5$ has better performance.

Figure 3-9. The throughput for the orthogonal AF and the exponential distribution (a) the SNR contour for $SNR_0 = -30 dB$ (b) the throughput gain for $SNR_0 = -30 dB$

Figure 3-10 shows the SNR contours for the throughput $\eta_{opt}(\rho)$ versus average SNRs for $SNR_0 = 0, 10, 20 dB$ and $\alpha_{opt} \geq 0.5$. Here $\Delta_1$ and $\Delta_2$ represent SNR offsets in dB for the source-relay and relay-destination links respectively. The SNR of the link 0 is used as the reference and the SNR of the two other links are adjusted relatively to it: $SNR_1(dB) = \Delta_1 + SNR_0(dB)$ and $SNR_1(dB) = \Delta_2 + SNR_0(dB)$. This contour separates two SNR regions i.e $\alpha_{opt} = 1$ and $1 > \alpha_{opt} > 0.5$. The behaviour of the throughput for high $SNR_0$ is fully opposite to what it was seen for the achievable rate or instantaneous mutual information. In this case, for high $SNR_0$ and high $\Delta_1$ and $\Delta_2$, the optimum duplexing ratio is between one and 0.5 i.e. $1 > \alpha_{opt} > 0.5$. Also
there is no region that $\alpha < 0.5$ has better performance. For example for $\text{SNR}_0 = 10 dB$, shows that for the SNR region $\Delta_1 \geq -5 dB$ and $\Delta_2 \geq -5 dB$, the optimum duplexing ratio $1 > \alpha_{opt} > 0.5$.

![Flexible AF: SNR Contour with $0 < \alpha \leq 1$](image)

Figure 3-10. The throughput for the orthogonal AF and the exponential distribution - the SNR contour for $\text{SNR}_0 = \{0, 10, 20\} dB$

In order to analyse the outage probability of AF scheme mathematically, it is necessary to have an accurate closed form for the outage probability. In the next section this issue is investigated.

### 3.6 Outage Probability Analysis

In order to analyse the outage behaviour of the orthogonal AF with variable $\alpha$ it is needed to find a suitable equation which would be a function of the three average SNRs $\rho$ as well as the duplexing ratio $\alpha$. It means: $P_{\text{outage}}(R, \alpha, \rho) = \text{Pr}(I(\gamma, \alpha) < R) = f(R, \alpha, \rho)$

In order to calculate the outage probability, at first it is necessary to find an accurate closed form for the distribution of $\gamma_{AF}$. However finding exact closed form for the CDF is very difficult. A tight approximation for the CDF could be expressed by using the following lemma.

**Lemma 3-3:** CDF of $\gamma_{AF} = \frac{\gamma \gamma_2}{1 + \gamma_1 + \gamma_2}$ is as follows:
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\[ F_z(z) \geq 1 - \sum_{j=1}^{\alpha} K_j \left( \frac{1}{\text{SNR}}, z \right) e^{-\varphi_j \left( \frac{1}{\text{SNR}} \right)} \]  

(3-7)

where \( K_j \left( \frac{1}{\text{SNR}}, z \right) \) and \( \varphi_j \left( \frac{1}{\text{SNR}}, z \right) \) are the polynomial functions of \( \frac{1}{\text{SNR}} \). It is possible to provide upper bound and lower bound for the CDF:

\[ 1 - \sum_{j=1}^{n} K_j \left( \frac{1}{\text{SNR}}, z \right) e^{-\varphi_j \left( \frac{1}{\text{SNR}} \right)} \geq F_z(z) \geq 1 - \sum_{j=1}^{n} K_j \left( \frac{1}{\text{SNR}}, z \right) e^{-\varphi_j \left( \frac{1}{\text{SNR}} \right)} \]

It is assumed that \( \rho_0 = \delta_{\text{SNR}}, \rho_1 = \delta_{\text{SNR}} \) and \( \rho_2 = \delta_{\text{SNR}} \). The proof and derivation of CDF equation is described in the appendix D.1. Since it is very difficult to find an exact expression for the outage probability, a tight lower bound for the outage probability is proposed in the following lemma.

Lemma 3-4: The outage probability for the orthogonal AF with the duplexing ratio \( \alpha \), the transmission rate \( R \) and the average SNRs \( \rho \) is lower bounded by the following equation:

\[ P_{\text{outage}} (R, \alpha, \rho) \geq 1 - \sum_{j=1}^{n} \chi_j \left( \frac{1}{\text{SNR}} \right) e^{-\zeta_j \left( \frac{1}{\text{SNR}} \right)} \]  

(3-8)

where each \( \chi_j \) and \( \zeta_j \) are both functions of the average SNRs \( \rho_0, \rho_1 \) and \( \rho_2 \), the duplexing ratio and the rate \( R \). \( \chi_j \) and \( \zeta_j \) are the polynomial functions of \( \frac{1}{\text{SNR}} \). In order to reduce complexity of this chapter, the proof and the derivation of the outage probability equations are described in the appendix D.2. The final equations for each \( \chi_j \) and \( \zeta_j \) pair have been shown in the appendix D.2. For \( \alpha = 0.5 \) the outage probability simplifies to:

\[ P_{\text{outage}} \geq 1 - \sum_{j=1}^{n} \chi_j \left( \frac{1}{\text{SNR}} \right) e^{-\zeta_j \left( \frac{1}{\text{SNR}} \right)} \]  

(3-9)

This equation provides much tighter lower bound compared to the outage probability calculated in [4]. In the following section the simulation and analyse result obtained by employing the above expressions are discussed.
3.6.1 Outage Performance Results and Comparison

In this section some simulation and analytical results are provided for the outage probability of the flexible AF scheme. The analytical derivations are further verified by extensive Monte Carlo based simulations. The SNR of the link 0 is used as the reference and the SNRs of the two other links are adjusted relatively to it: \( SNR_1(dB) = \Delta_1 + SNR_0(dB) \) and \( SNR_2(dB) = \Delta_2 + SNR_0(dB) \), where \( \Delta_1 \) and \( \Delta_2 \) represent the SNR offsets of the links 1 and 2, respectively. The transmission rate \( R \) has been set to 2.

3.6.1.1 Duplexing Ratio Half and Greater than Half

Figure 3-11 shows the outage performance of the AF scheme when \( 0.5 < \alpha < 1 \) for \( \Delta_1 = 10dB \) and \( \Delta_2 = 10dB \). As it is seen \( \alpha = 0.588 \) has the lowest outage probability. By increasing \( \alpha \) towards one or decreasing \( \alpha \) towards 0.5, the outage probability is increased. Similar trend of behaviour is observed when the SNR \( \alpha \) is varied. Performance gains of up to 0.1 dB or more are observed compared with \( \alpha = 0.5 \) situation. Notice that by changing SNR offsets similar behaviour is seen.

![Outage for \( \Delta_1 = 10dB \) and \( \Delta_2 = 10dB \)](image)

**Figure 3-11. The outage probability for different \( \alpha : 0.5 \leq \alpha \leq 1 \)**

Figure 3-12 compares analysis and simulation results for the outage performance of the AF scheme when \( 0.5 \leq \alpha \leq 1 \) for \( \Delta_1 = 10dB \) and \( \Delta_2 = 10dB \). As it is seen the curves obtained from analysis are always below their corresponding simulation curves. The simulation curves are much nearer to the exact outage probability and the analysis curves act as a lower bound for the outage
probability. The gap between simulation curve and analysis curve for $\alpha = 0.55$ is around 0.05 dB. These curves confirm the accuracy of the outage probability analysis.

![Figure 3-12. Comparing analysis with simulation result $0.5 \leq \alpha \leq 1$](image)

Figure 3-12 (a) shows a 3-D variation of the optimum duplexing ratio $\alpha$ for $SNR_0 = 30dB$. When the SNR offset of the one of the relaying links are very low, the optimum $\alpha$ is one. But with increasing the SNR offsets the optimum $\alpha$ starts to decrease sharply from one toward 0.5 and finally at medium or high SNR offsets, the optimum $\alpha$ is near 0.57. Figure 3-13 (b) compares the
Figure 3-13. The orthogonal AF: (a) the optimum $\alpha$ for $SNR_0 = 30dB$ (b) the optimum throughput for $SNR_0 = 10dB$
optimum throughput with the throughput of $\alpha = 0.5$ at $\text{SNR}_0 = 10dB$. In this scenario, the transmission rate $R$ was set to one. When the SNR offset of one of the link are low, the optimum throughput remains fixed. In this region the direct transmission $\alpha = 1$ is optimum. But by increasing the SNR offset the optimum throughput starts to increase to higher values than the direct transmission throughput. The maximum throughput is always higher than the throughput for $\alpha = 0.5$. This is similar to the outage probability behaviour; the outage probability for the optimum $\alpha$ is lower than the outage probability when $\alpha = 0.5$.

3.6.1.2 Duplexing Ratio Less than Half

Figure 3-14 compares the analysis and simulation results for the outage performance of the flexible AF when $0 < \alpha \leq 0.5$ for $\Delta_1 = 10dB$ and $\Delta_2 = 10dB$. As it is seen $\alpha = 0.5$ has the lowest outage probability. By decreasing $\alpha$ toward zero, the outage probability is increased. By changing SNR offsets similar behaviour is seen. Also analysis curves are always below their corresponding simulation curves. The gap between simulation curve and analysis curve for the $\alpha = 0.5$ is very small. These curves confirm accuracy of our outage probability analysis.

![Figure 3-14. Comparison between simulation and analysis $0 < \alpha \leq 0.5$](image)

Also Figure 3-15 shows the outage performance of AF scheme when $0 < \alpha \leq 0.5$ for $\Delta_1 = -20dB$ and $\Delta_2 = -20dB$. As it is seen $\alpha = 0.5$ has the lowest outage probability when
SNR₀ is higher than 25dB. But for lower SNR₀, direct transmission is dominant. By decreasing α towards zero, the outage probability is increased.

![Outage probability graph](image)

Figure 3-15. The outage probability for Δ₁ = -20dB and Δ₂ = -20dB

### 3.7 Further Discussion on Diversity

As it was shown in chapter 2, when the average SNR of all the links are increasing together, the traditional AF can provide diversity of two. In this section, the diversity of AF scheme is explored when 0.5 ≤ α ≤ 1. At the previous section, it was shown that the outage probability for the orthogonal AF with the duplexing ratio α and the transmission rate R is lower bounded by the following equation:

\[
P_{\text{outage}}(R, \alpha, \rho) \geq 1 - \sum_{j=1}^{9} \chi_j \left( \frac{1}{\text{SNR}} \right) e^{-\zeta_j \left( \frac{1}{\text{SNR}} \right)}
\]  \hspace{1cm} (3-10)

Let us consider \( \zeta'_j = \text{SNR} \times \zeta_j \), \( j = 1, ..., 9 \) and rewrite \( \chi_j \left( \frac{1}{\text{SNR}} \right) = \chi_{j,1} + \frac{1}{\text{SNR}} \times \chi_{j,2} \), \( j = 1, ..., 9 \).

As a result, \( \zeta'_j, \chi_{j,1}, \) and \( \chi_{j,2} \) become independent of \( \frac{1}{\text{SNR}} \) factor. Then \( e^{-\zeta_j \text{SNR}} \) can be
approximated with a polynomial function of\( \frac{1}{\text{SNR}} \). Using Taylor series, an approximation for the exponential function is\( e^{-\xi_j'/\text{SNR}} \approx 1 - \left( \frac{\xi_j'}{\text{SNR}} \right) + \frac{1}{2} \left( \frac{\xi_j'}{\text{SNR}} \right)^2 \). This approximation is reliable for \( \frac{\xi_j'}{\text{SNR}} \ll 1 \) or \( \xi_j' < \text{SNR} \).

Also for \( \frac{\xi_j'}{\text{SNR}} \gg 1 \) or \( \xi_j' \gg \text{SNR} \), the function \( e^{-\xi_j'/\text{SNR}} \) approaches zero. So by summarising the above expressions, the following expression is obtained:

\[
e^{-\xi_j'/\text{SNR}} = \begin{cases} 
0 & \text{SNR} < \xi_j' \\
1 - \left( \frac{\xi_j'}{\text{SNR}} \right) + \frac{1}{2} \left( \frac{\xi_j'}{\text{SNR}} \right)^2 & \text{SNR} \gg \xi_j'
\end{cases}
\] (3-11)

Therefore \( \text{SNR}_{th,i} = 10 \log 10 \left( \xi_j' \right) \) can be considered as a turning point for \( e^{-\xi_j'/\text{SNR}} \) in (3-10)

where \( th \) stands for the word threshold. By looking at the first factor i.e. \( e^{-\xi_j'/\text{SNR}} \), the first turning point is \( \text{SNR}_{th,i} = 10 \log 10 \left( c \right) \). It is possible to find all the nine turning points for any specific set of the SNR offsets, the transmission rate and the duplexing ratio. Based on the calculated SNR thresholds the following remarks can be provided.

**Remark 1:** It is clear that for SNRs less than \( \min_{j>1} \left( \text{SNR}_{th,j} \right) \) only the first factor i.e. \( e^{-\xi_j'/\text{SNR}} \) is dominant and other \( e^{-\xi_j'/\text{SNR}} \) \( j > 1 \) factors in (3-10) are negligible. So for large range of SNR's between \( \text{SNR}_{th,i} \) and \( \min_{j>1} \left( \text{SNR}_{th,j} \right) \) the outage probability is approximated as

\[
P_{\text{outage}} \approx 1 - \left( 1 - \frac{c}{\text{SNR}} \right) = \frac{c}{\text{SNR}}
\]

where \( c = 2^{2R} - 1 \). Thus the slope of \( P_{\text{outage}} \) curve in logarithmic scale is one.

**Remark 2:** By increasing SNR more than \( \min_{j>1} \left( \text{SNR}_{th,j} \right) \), the first factor i.e. \( e^{-\xi_j'/\text{SNR}} \) approaches one and other factors, \( e^{-\xi_j'/\text{SNR}} \) \( j > 1 \) in (3-10) are taken into account. In this case:
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\[ P_{\text{average}} \geq 1 - \sum_{\{j_{\text{SNR-}i}\}} \left( X_{j,1} + X_{j,2} \left( \frac{1}{\text{SNR}} \right) \right) \left( 1 - \left( \frac{\zeta_j}{\text{SNR}} \right) + \frac{1}{2} \left( \frac{\zeta_j}{\text{SNR}} \right)^2 \right) \]

\[ = \left( 1 - \sum_{\{j_{\text{SNR-}i}\}} X_{j,1} \right) \left( \frac{1}{\text{SNR}} \right) \sum_{\{j_{\text{SNR-}i}\}} \left( X_{j,2} - X_{j,2} \zeta_j \right) \left( \frac{1}{\text{SNR}} \right) \sum_{\{j_{\text{SNR-}i}\}} \left( \frac{1}{2} X_{j,2} (\zeta_j)^2 - X_{j,2} \zeta_j \right) \]

(3-12)

It can be shown that \( \sum_{\{j_{\text{SNR-}i}\}} X_{j,1} \) is negligible and the performance is affected by

\[ \sum_{\{j_{\text{SNR-}i}\}} \left( X_{j,2} - X_{j,2} \zeta_j \right) \] and \[ \sum_{\{j_{\text{SNR-}i}\}} \left( \frac{1}{2} X_{j,2} (\zeta_j)^2 - X_{j,2} \zeta_j \right) \]. Therefore a diversity between one and two can be achieved.

**Remark 3:** By increasing SNR such that \( \text{SNR} > \max_{\text{all} \ j} \left( \text{SNR}_{\text{th-j}} \right) \), all the \( \zeta_j \) factors in (3-10) are taken into account. In this case:

\[ P_{\text{average}} \geq \left( 1 - \sum_{\{j_{\text{SNR-}i}\}} X_{j,1} \right) \left( \frac{1}{\text{SNR}} \right) \sum_{\{j_{\text{SNR-}i}\}} \left( X_{j,2} - X_{j,2} \zeta_j \right) \left( \frac{1}{\text{SNR}} \right) \sum_{\{j_{\text{SNR-}i}\}} \left( \frac{1}{2} X_{j,2} (\zeta_j)^2 - X_{j,2} \zeta_j \right) \]

(3-13)

It is seen that \( \sum_{\{j_{\text{SNR-}i}\}} \left( X_{j,2} - X_{j,2} \zeta_j \right) \) becomes very small compared to \( \sum_{\{j_{\text{SNR-}i}\}} \left( \frac{1}{2} X_{j,2} (\zeta_j)^2 - X_{j,2} \zeta_j \right) \)

and therefore the diversity of two becomes dominant completely.

It is clear that the achievable diversity depends on the geometry (SNR offsets), the transmission rate and the duplexing ratio. Let us discuss the diversity through the following examples.

**Example 1:** Let us assume \( \Delta_i = 10 \log_{10} (\delta) \) \( i = 0,1,2 \). When \( \Delta_i \) and \( \Delta_2 \) are very small for example \( \Delta_1 = -50 \text{dB} \), then all the \( \zeta_j \) which are dependent to \( \delta_i \) and \( \delta_2 \), become very large except the first factor which is \( \zeta_1 = c \). Some typical values for \( \zeta_j \) (for \( \alpha = 0.9 \) and \( R = 1 \)) are as follows:

\[ \zeta_1 = 1.14, \; \zeta_3 = 4.5 \times 10^5, \; \zeta_3 = 1.6 \times 10^5, \; \zeta_5 = 1.6 \times 10^9, \; \zeta_5 = 5.7 \times 10^8, \; \zeta_6 = 1.8 \times 10^7, \; \zeta_7 = 1.3 \times 10^6, \; \zeta_6 = 6.6 \times 10^6, \; \zeta_7 = 4.2 \times 10^5. \]

And SNR turning points are as follows: \( \text{SNR}_{\text{th,1}} = 0.58 \text{ dB}, \; \text{SNR}_{\text{th,2}} = 56 \text{ dB}, \; \text{SNR}_{\text{th,3}} = 50 \text{ dB}, \; \text{SNR}_{\text{th,4}} = 92 \text{ dB}, \; \text{SNR}_{\text{th,5}} = 87 \text{ dB}. \)
SNR_{th,0} = 72 \text{ dB}, \text{SNR}_{th,7} = 61 \text{ dB}, \text{SNR}_{th,8} = 68 \text{ dB}, \text{SNR}_{th,9} = 56 \text{ dB}. \quad \text{Therefore,}
\text{SNR}_{th,1} < \text{SNR}_{th,3} < \text{SNR}_{th,9} < \text{SNR}_{th,2} < \text{SNR}_{th,7} < \text{SNR}_{th,8} < \text{SNR}_{th,6} < \text{SNR}_{th,5} < \text{SNR}_{th,4}.

For SNRs less than SNR_{th,3} = 50 dB only the first factor \( e^{-\frac{S}{\text{SNR}}} \) is dominant and other \( e^{-\frac{S}{\text{SNR}}} \) factors are negligible as a result the slope of \( P_{\text{outage}} \) curve in logarithmic scale is one. By increasing SNR towards more than SNR_{th,3} = 50 dB, gradually other factors come to affect. So the diversity increases gradually from one to two. By increasing SNR to more than SNR_{th,4} = 92 \text{ dB}, the diversity is two completely. Figure 3-16 shows the behaviour of the outage probability curves for this case.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{outage.png}
\caption{The outage probability for \( \Delta_1 = \Delta_2 = -50\text{dB} \)}
\end{figure}

\textbf{Example 2:} By increasing the SNR offset, the SNR thresholds become smaller. For example for \( \Delta_1 = \Delta_2 = 0\text{dB} \) and \( \alpha = 0.9 \), the SNR threshold are:

SNR_{th,1} = 0.58 \text{ dB}, \text{SNR}_{th,2} = 7 \text{ dB}, \text{SNR}_{th,3} = 3 \text{ dB}, \text{SNR}_{th,4} = 42 \text{ dB}, \text{SNR}_{th,5} = 38,
SNR_{th,6} = 23 \text{ dB}, \text{SNR}_{th,7} = 11 \text{ dB}, \text{SNR}_{th,8} = 18 \text{ dB}, \text{SNR}_{th,9} = 7 \text{ dB}

It is observed that a diversity of two can be achieved at lower SNRs as it is shown in Figure 3-17.
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Figure 3-17. The outage probability for $\Delta_1 = \Delta_2 = -10dB$

The above behaviour also becomes more clear in Figure 3-18 where the SNR regions for the diversity one and two is plotted. The transmission rate was set to one and the duplexing ratio is 0.5. As it is seen by increasing the SNR offsets the diversity is shifted from one towards two. This behaviour is also seen in other SNR settings.

Figure 3-18. Diversity contours for $\text{SNR}_0=20\,\text{dB}$
3.8 Conclusion

In this chapter an AF scheme with variable duplexing ratio was proposed and analysed. Two important measures in the analysis; the achievable rate and the outage probability were employed. The instantaneous mutual information and average mutual information were derived for the proposed orthogonal AF cooperative protocol. These derivations could be extended to non orthogonal cooperative protocol. These mathematical models were deployed to find the optimum duplexing ratio in different network topologies and SNR conditions. In term of instantaneous mutual information, it was found that when the instantaneous SNR of the direct link is much lower than the SNR of the first link, the probability to have the optimum duplexing ratio less than 0.5 is higher. Also this probability increases when the source-relay SNR to relay-destination SNR ratio increases. Numerical results for the average achievable rate show that the optimum duplexing ratio less than half happens when average SNR of the direct link is very low. By increasing average SNR of the direct link the SNR region with the optimum duplexing ratio less than 0.5 becomes smaller and smaller.

In the outage probability point of view, numerical results show similar trends as it was seen for the instantaneous mutual information and the achievable rate; the optimum duplexing could be less than half in some SNR regions. However the behaviour of the throughput obtained for the high average SNR of the direct link is completely different. In this SNR region when SNR offsets of the other links are high optimum duplexing ratio is between 0.5 and 1. A tight closed form lower bound approximation was provided for the outage probability. The curves resulted from the outage analysis act as a lower bound for the outage probability. The presented analysis enables us to predict the system performance and adjust its parameters for any combination of the SNR of the different links. It was shown that at high SNRs, the outage probability for the optimum duplexing ratio is always lower than the outage probability for half duplexing ratio.

Also the diversity of AF scheme was explored when the optimum duplexing ratio is equal or greater than 0.5. Based on the outage probability analysis some SNR thresholds were defined in order to explore the diversity for different average SNR regions. It was found that the slope of the outage probability curves in logarithmic scale is one at very low SNR offsets. By increasing SNR gradually diversity increases from one toward two. At very high SNRs diversity is two completely. The position of turning points depends on the network topology (average SNRs).
Chapter 4 Amplify and Forward Combined with HARQ

4.1 Introduction

In previous chapter a flexible scheme was proposed that allows tuneable duplexing ratio. The outage probability and throughput performance of the AF scheme was discussed when every message is transmitted only one time. However in deep fading channel conditions it is more possible that the transmitted message is not successfully decoded at the destination in the first attempt when the source does not have any transmit CSI even though the transmission is supported by cooperative relaying techniques. Retransmitting the message under a proper ARQ process is an efficient solution to this problem. Moreover combining channel coding and ARQ process which is known as Hybrid ARQ (HARQ) can highly improve successful transmission of message [77]. So far two general encoding methods have been used in combination with HARQ including repetition coding (RC) known as chase and unconstrained coding (UC) also known as incremental redundancy (IR). Combination of HARQ strategies over multi-hop and cooperative relaying has been studied in [62] and [64]. The authors in [69] have evaluated the performance of HARQ combined with the cooperative regenerative relaying. This chapter focuses on a flexible non-regenerative single relay case and derives the outage performance for two different HARQ strategies with RC and UC encoding schemes and for any range of SNRs [97]. At first the state transition models of the assumed protocols are presented and employed to analytically calculate the HARQ throughput and latency performance.

The remainder of this chapter is organized as follows. Section 4.2 describes the system model and assumptions for the flexible AF concept. Section 4.3 and 4.4 discusses in detail the proposed HARQ techniques at the relay for two different HARQ strategies. The outage probabilities for the proposed HARQ AF model, as well as their throughput performance results are compared in section 4.5. Section 4.6 provides the conclusion remarks.
4.2 System Model

Similar to previous chapter, a three-node cooperative communication system is assumed that is composed of: a source node, a relay node, and a destination node. As also shown in Figure 4-1 the source-destination, source-relay and relay-destination links are indexed with 0, 1, and 2, respectively. Moreover each transmission frame is divided into two phases. During the second phase depending on the assumed protocol, the source could either transmit to the destination or be silent. Most of the assumptions in the previous chapter also apply to this chapter.

As it was said earlier although the above system utilizes an efficient cooperative relaying technique, the decoding of transmitted message could still fail in the poor instantaneous channel condition. The probability of failure in successful transmission could be much higher when the system faces a block fading condition. In these situations, an error control method based on HARQ can improve performance. In the considered HARQ process, in contrast to regenerative relaying [69], the source node contributes in all the retransmission until the message is correctly decoded at the destination. After each frame the destination sends an ACK or NAK feedback to the source and the relay. The feedback channel is assumed to be ideal. In poor channel conditions number of retransmissions increases rapidly. This issue highly reduces efficiency of the HARQ scheme. So it is necessary to consider a cap over number of retransmissions.

Let us assume two HARQ encoding methods: repetition coding (RC) and unconstrained coding (UC). In the repetition coding, all the code sequences transmitted by the source during first transmission and all retransmissions are the same. In this case after each retransmission, the destination does the maximal ratio combining (MRC) over all the received frames before sending to the decoder. But in the unconstrained coding, the code sequences in different retransmissions
are not necessarily identical. In this chapter the performance analysis of HARQ strategy combined with these two encoding methods is explored including the outage analysis.

4.3 HARQ Transmission- First Strategy

4.3.1 HARQ Strategy

In this strategy it is assumed that the destination decodes the message successfully in following two cases. In the first case, \( n \) blocks (each one with length \( N_1 \)) are transmitted from the source and \( n \) blocks (each one with length \( N_2 \)) are forwarded from the relay. In the second case, \( n \) blocks are transmitted from the source and \( n - 1 \) blocks are forwarded from the relay. It means that in this case the transmission is stopped just after first phase of the last frame transmission. Also for both cases the transmission is stopped after maximum \( N \) frame transmission. Figure 4-2 shows the state transition model for this strategy.

![State Transition Model](image)

**Figure 4-2. The state transition model for the first strategy**

As it is shown in Figure 4-2, the state \( S_{n,k} \) is defined as the state at the \( n \)-th transmission after first phase \( (k = 1) \) or second phase \( (k = 2) \) and the decoding is successful at the destination. The state \( S_{n,k} \) is similar to the state \( S_{n,k} \) but in this case the decoding has failed at the destination. \( p_{n,k}, k = 1, 2 \) are the transition probability for \( S_{n-1,1} \rightarrow S_{n,1} \) and \( S_{n,1} \rightarrow S_{n,2} \) respectively. Similarly \( q_{n,k}, k = 1, 2 \) are the transition probability for \( S_{n-1,1} \rightarrow S_{n,1} \) and \( S_{n,1} \rightarrow S_{n,2} \) respectively. Our goal is to explore the throughput and the outage behaviour of the proposed
system for this strategy combined with some selected protocols. In the next section this issue is investigated.

### 4.3.2 Average Throughput

This section focuses on the deriving the throughput and the outage probability of the first HARQ strategy with the selected cooperative protocols and encoding methods. The average throughput of this strategy is the ratio of the average rate reward \( E[R] \) to the average airtime \( E[T] \) according to [66]. It means \( \eta(R, \alpha, \rho) = \frac{E[R]}{E[T]} \). By appropriate selection of the transmission rate \( R \) and the duplexing ratio \( \alpha \), the throughput of the system can be optimized as 

\[
\eta_{\text{optimum}} = \max_{R, \alpha} \eta(R, \alpha, \rho).
\]

By looking at the state diagram in Figure 4-2, the average airtime is

\[
E[T] = \sum_{k=1}^{2} \sum_{j=1}^{N} Q_{j,k} \tau_{j,k} + \bar{Q}_{N,2} r_{N,2},
\]

where \( \tau_{j,k} = j - (2 - k)(1 - \alpha) \) and average rate rewards is

\[
E[R] = R \sum_{k=1}^{2} \sum_{j=1}^{N} Q_{j,k}.\]

Also the average delay is

\[
E[D] = \sum_{k=1}^{2} \sum_{j=1}^{N} Q_{j,k} d_{j,k} + \bar{Q}_{N,2} d_{N,2},
\]

where \( d_{j,k} \) is the delay of the \( k \)-th transmission and \( d_{N,2} \) is the delay of the N-th transmission. The outage probability after the N-th transmission is \( P_{\text{out}} = \bar{Q}_{N,2} \). In the above equations, \( Q_{n,k} \) is the probability of being at the successful state \( S_{n,k} \), i.e., \( Q_{n,k} = \text{Pr}\{S_{n,k}\} \) and \( \bar{Q}_{n,k} \) is the probability of being at the failed state \( \bar{S}_{n,k} \), i.e., \( \bar{Q}_{n,k} = \text{Pr}\{\bar{S}_{n,k}\} \). In order to calculate the average airtime and the average rate reward it is necessary to calculate the state probabilities.

### 4.3.3 Transition Probabilities

The state probabilities can be recursively calculated using the transition probabilities \( p_{n,k} \) and \( q_{n,k} \). This process is started with the initial value \( \bar{Q}_{0,2} = 1 \) as follows:

\[
\begin{align*}
Q_{a,2} &= q_{a,2} \bar{Q}_{n,1}, & Q_{a,3} &= q_{a,3} \bar{Q}_{n,1} \\
\bar{Q}_{a,2} &= p_{a,2} Q_{n,1}, & \bar{Q}_{a,3} &= p_{a,3} Q_{n,1}.
\end{align*}
\]

Then the transition probabilities are calculated using the outage event probabilities as follows.
\[ p_{n,1} = \Pr \{ I_{n,1} < R \mid I_{n-1,2} < R \} = \frac{\Pr \{ I_{n,1} < R \}}{\Pr \{ I_{n-1,2} < R \}} \]  
\[ p_{n,2} = \Pr \{ I_{n,2} < R \mid I_{n,1} < R \} = \frac{\Pr \{ I_{n,2} < R \}}{\Pr \{ I_{n,1} < R \}} \]  
\[ q_{n,1} = \Pr \{ I_{n,1} \geq R \mid I_{n-1,2} < R \} = 1 - p_{n,1} \]  
\[ q_{n,2} = \Pr \{ I_{n,2} \geq R \mid I_{n,1} < R \} = 1 - p_{n,2} \]  

where \( I_{n,k} \) is the accumulative mutual information normalized over one frame from the first transmission until end of the \( k \)-th time slot of the \( n \)-th transmission. Recall that \( \{ I_{n-1,2} < R \} \subseteq \{ I_{n,1} < R \} \) and therefore \( \Pr \{ I_{n,1} < R, I_{n-1,2} < R \} = \Pr \{ I_{n,1} < R \} \). Now \( F_{n,k}(R) = \Pr \{ I_{n,k} < R \} \) is defined as the probability of the outage after the \( k \)-th time slot of the \( n \)-th transmission. For the starting state \( F_{0,1}(R) = 1 \). In order to calculate the probability of the outage, deriving accumulative mutual information \( I_{n,k} \) is needed. In the next section this issue is investigated.

### 4.3.4 Probability of Outage

#### 4.3.4.1 Accumulative Mutual Information

In order to calculate the outage probability \( F_{n,k}(R) \) it is necessary to derive the accumulative mutual information for the combination of each protocol with each encoding type separately. The accumulative mutual information for the orthogonal AF (Protocol 1 from chapter 3) and the non orthogonal (protocol 2 & 3 from chapter 3) and protocol 4 from chapter 3 have been tabulated in Table 4-1, Table 4-2, Table 4-3, Table 4-4, Table 4-5 and Table 4-6 respectively. In these equations \( \gamma_{i,j} \) is the instantaneous SNR for the \( i \)-th link during \( j \)-th frame. Also \( \alpha_{i,j} \) is the instantaneous channel coefficient for the \( i \)-th link during \( j \)-th frame.
Table 4-1. The accumulative mutual information for the orthogonal AF + UC or RC (protocol 1) for the first frame transmission

<table>
<thead>
<tr>
<th></th>
<th>RC</th>
<th>UC</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>First Time Slot</strong></td>
<td>$I_{1,1} = \alpha \log(1 + \gamma_{0,1})$</td>
<td>$I_{1,1} = \alpha \log(1 + \gamma_{0,1})$</td>
</tr>
<tr>
<td><strong>Second Time Slot</strong></td>
<td>$I_{1,2} = (1 - \alpha) \log \left(1 + \gamma_{0,1} + \gamma_{1,1} \beta_1 \left(1 + \gamma_{2,1} \beta_1 \right)^{-1} \right) + (2\alpha - 1) \log (1 + \gamma_{0,1})$</td>
<td>$I_{1,2} = (1 - \alpha) \log \left(1 + \gamma_{0,1} + \gamma_{1,1} \beta_1 \left(1 + \gamma_{2,1} \beta_1 \right)^{-1} \right) + (2\alpha - 1) \log (1 + \gamma_{0,1})$</td>
</tr>
</tbody>
</table>

The detailed proofs for UC and RC cases have been provided in appendices E-1, E-2, E-3 and E-4.

Table 4-2. The accumulative mutual information for the orthogonal AF + UC or RC (protocol 1) for the n-th frame transmission

<table>
<thead>
<tr>
<th></th>
<th>RC</th>
<th>UC</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>First Time Slot</strong></td>
<td>$I_{n,1} = (1 - \alpha) \log \left(1 + \sum_{j=0}^{n-1} \gamma_{j,1} \beta_1 \left(1 + \gamma_{2,1} \beta_1 \right)^{-1} \right)$</td>
<td>$I_{n,1} = (1 - \alpha) \log \left(1 + \sum_{j=0}^{n-1} \gamma_{j,1} \beta_1 \left(1 + \gamma_{2,1} \beta_1 \right)^{-1} \right)$</td>
</tr>
<tr>
<td></td>
<td>$(2\alpha - 1) \log (1 + \sum_{j=0}^{n-1} \gamma_{j,1})$</td>
<td>$(2\alpha - 1) \log (1 + \sum_{j=0}^{n-1} \gamma_{j,1})$</td>
</tr>
<tr>
<td><strong>Second Time Slot</strong></td>
<td>$I_{n,2} = (1 - \alpha) \log \left(1 + \sum_{j=0}^{n-1} \gamma_{j,1} \beta_1 \left(1 + \gamma_{2,1} \beta_1 \right)^{-1} \right)$</td>
<td>$I_{n,2} = (1 - \alpha) \log \left(1 + \sum_{j=0}^{n-1} \gamma_{j,1} \beta_1 \left(1 + \gamma_{2,1} \beta_1 \right)^{-1} \right)$</td>
</tr>
<tr>
<td></td>
<td>$(2\alpha - 1) \log (1 + \sum_{j=0}^{n-1} \gamma_{j,1})$</td>
<td>$(2\alpha - 1) \log (1 + \sum_{j=0}^{n-1} \gamma_{j,1})$</td>
</tr>
</tbody>
</table>

Table 4-3. The accumulative mutual information for the non-orthogonal AF + UC or RC (protocols 2 & 3) for the first frame transmission

<table>
<thead>
<tr>
<th></th>
<th>RC</th>
<th>UC</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>First Time Slot</strong></td>
<td>$I_{1,1} = \alpha \log (1 + \gamma_{0,1})$</td>
<td>$I_{1,1} = \alpha \log (1 + \gamma_{0,1})$</td>
</tr>
<tr>
<td><strong>Second Time Slot</strong></td>
<td>$I_{1,2} = (2\alpha - 1) \log (1 + \gamma_{0,1}) +$</td>
<td>$I_{1,2} = (1 - \alpha) \log \left(1 + \gamma_{0,1} \beta_1 \right)^{-1}$</td>
</tr>
<tr>
<td></td>
<td>$(1 - \alpha) \log \left(1 + \gamma_{0,1} + \sqrt{\alpha_1} \beta_1 \right)^{-1}$</td>
<td>$I_{1,2} = (1 - \alpha) \log \left(1 + \gamma_{0,1} \beta_1 \right)^{-1}$</td>
</tr>
</tbody>
</table>

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Table 4-4 The accumulative mutual information for the non-orthogonal AF + UC or RC (protocols 2 & 3) for the n-th frame transmission

<table>
<thead>
<tr>
<th></th>
<th>RC</th>
<th>UC</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>First Time</strong></td>
<td>$I_{s1} = (1-\alpha)\log\left(\frac{1+\sum_{\beta}^{\alpha_{s1}}(y_{0,1})}{(1+y_{0,1}\beta_{i})^{-1}}\right)$</td>
<td>$I_{s1} = (1-\alpha)\sum_{\beta}^{\alpha_{s1}}\log\left(\frac{1+y_{0,1}(y_{1,2}\beta_{i}+y_{0,1})(1+y_{0,1})}{(1+y_{0,1}\beta_{i})^{-1}}\right)$</td>
</tr>
<tr>
<td><strong>Slot</strong></td>
<td>$(2\alpha-1)\log\left(1+\sum_{\beta}^{\alpha_{s1}}(y_{0,1})\right)$</td>
<td>$(2\alpha-1)\sum_{\beta}^{\alpha_{s1}}\log(1+y_{0,1})+(1-\alpha)\log(1+y_{0,1})$</td>
</tr>
<tr>
<td><strong>Second Time</strong></td>
<td>$I_{s2} = (1-\alpha)\log\left(\frac{1+\sum_{\beta}^{\alpha_{s2}}(y_{0,2})}{(1+y_{0,2}\beta_{i})^{-1}}\right)$</td>
<td>$I_{s2} = (1-\alpha)\sum_{\beta}^{\alpha_{s2}}\log\left(\frac{1+y_{0,2}(y_{1,2}\beta_{i})y_{0,2}(1+y_{0,2})}{(1+y_{0,2}\beta_{i})^{-1}}\right)$</td>
</tr>
<tr>
<td><strong>Slot</strong></td>
<td>$(2\alpha-1)\log\left(1+\sum_{\beta}^{\alpha_{s2}}(y_{0,2})\right)$</td>
<td>$(2\alpha-1)\sum_{\beta}^{\alpha_{s2}}\log(1+y_{0,2})$</td>
</tr>
</tbody>
</table>

The proof is similar to appendices E-1 and E-3 for orthogonal protocol.

Table 4-5. The accumulative mutual information for the non-orthogonal AF + RC (protocol 4) for the first frame transmission

<table>
<thead>
<tr>
<th></th>
<th>$I_{t1} = \alpha\log(1+y_{0,1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>First Time</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Slot</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Second Time</strong></td>
<td>$I_{t2} = (2\alpha-1)\log(1+y_{0,1}) + (1-\alpha)\log\left(1+y_{0,1} + \frac{y_{1,2}\beta_{i}}{1+y_{0,1}}\right)$</td>
</tr>
<tr>
<td><strong>Slot</strong></td>
<td></td>
</tr>
</tbody>
</table>

Table 4-6 The accumulative mutual information for the non-orthogonal AF + RC (protocol 4) for the n-th frame transmission

|                 | $I_{s1} = (1-\alpha)\log\left(1+\sum_{\beta}^{\alpha_{s1}}(y_{0,1}) + \sum_{\beta}^{\alpha_{s1}}(y_{0,1}y_{1,2}\beta_{i} + y_{0,1})(1+y_{1,2}\beta_{i})^{-1}\right)$ | $(2\alpha-1)\log\left(1+\sum_{\beta}^{\alpha_{s1}}(y_{0,1})\right)$ |
| **First Time**  |                                  |                                                                      |
| **Slot**        |                                  |                                                                      |
| **Second Time** | $I_{s2} = (1-\alpha)\log\left(1+\sum_{\beta}^{\alpha_{s2}}(y_{0,2}) + \sum_{\beta}^{\alpha_{s2}}(y_{0,2}y_{1,2}\beta_{i} + y_{0,2})(1+y_{1,2}\beta_{i})^{-1}\right)$ | $(2\alpha-1)\log\left(1+\sum_{\beta}^{\alpha_{s2}}(y_{0,2})\right)$ |
| **Slot**        |                                  |                                                                      |
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The proof is similar to the appendix E-3 for the orthogonal protocol. In this section the accumulative mutual information were derived. In the next section the outage probability is calculated.

4.3.4.2 Characteristic Function

To be able to calculate the outage probability i.e. \( F_{n,j}(R) = \Pr(I_{n,j} < R) \) at first the characteristic function of \( I_{n,j} \) is obtained. It means: \( \Psi_{n,j} = \mathcal{E}_i \left( e^{-x_{n,j}} \right) \) where \( \mathcal{E} \left( \cdot \right) \) stands for the expectation operation over the random variable \( X \). The characteristic functions of the accumulative mutual information for the orthogonal AF (Protocol 1) during the \( n \)-th frame transmission have been tabulated in Table 4-7 and Table 4-8 for RC and UC encoding respectively.

<table>
<thead>
<tr>
<th>Table 4-7. The characteristic functions for the orthogonal AF + RC during the ( n )-th frame transmission</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RC</strong></td>
</tr>
<tr>
<td>First Time Slot</td>
</tr>
<tr>
<td>( \Psi_{n,1}(x) = \mathcal{E} \left( e^{-x_{n,1}} \right) )</td>
</tr>
<tr>
<td>Second Time Slot</td>
</tr>
<tr>
<td>( \Psi_{n,2}(x) = \mathcal{E} \left( e^{-x_{n,2}} \right) )</td>
</tr>
</tbody>
</table>
Table 4-8 The characteristic functions for the orthogonal AF + UC during the n-th frame transmission

<table>
<thead>
<tr>
<th></th>
<th>( \Psi_{n,1}(s) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Time Slot</td>
<td>[ \mathcal{E} \left( e^{-\alpha \log(1+\gamma_{n,1})} \right) ]</td>
</tr>
<tr>
<td></td>
<td>[ \mathcal{E} \left( e^{-(1-\alpha) \log[1+\gamma_{n,2}+\gamma_{n,3} \cdot (1+\gamma_{n,3})^{i+1}]} \right) ]</td>
</tr>
<tr>
<td></td>
<td>[ \mathcal{E} \left( e^{-(2\alpha-1) \log(1+\gamma_{n,1})} \right) ]</td>
</tr>
</tbody>
</table>

More detail about this derivation can be found in the appendix E-6. Calculation of \( \Psi_{n,1}(s) \) is difficult and an alternative numerical calculation is proposed for a predefined sets of \( s \) and \( \alpha \).

The outage probability \( F_{n,j}(R) = \Pr \{ I_{n,j} < R \} \) can be calculated by using Laplace inversion formula [78]. More detail about this calculation can be found in the appendix E-7. The above outage probability derivation can be repeated in the same way for the other protocols.

### 4.4 HARQ transmission - Second Strategy

#### 4.4.1 HARQ Strategy

The destination decodes the message successfully after the \( m \) time slot transmission from the source. In this strategy the relay transmits if it’s normalized mutual information is not smaller than a threshold i.e. \( \lambda \). In this strategy it is assumed that the source also transmits during the second phase. The transmission is stopped after the \( N \)-th time slot transmission. Figure 4-3 shows the state transition model of the second strategy up to 4 time slot transmission. \( S_{n,a} \) shows the state after the \( m \)-th time slot transmission where decoding has been successful at the destination.
The index $n$ is dependent on the index $m$. For each specific $m$, it can be shown that $n \in \left\{1, \ldots, \left\lfloor \frac{m}{2} \right\rfloor \right\}$. $\overline{S}_{m,n}$ is similar to $S_{m,n}$ but with failed reception at the destination. $p_{m,n}$ is the transition probability for $\overline{S}_{m-1,n} \rightarrow \overline{S}_{m,n}$ or $\overline{S}_{m-\left\lfloor \frac{m}{2} \right\rfloor} \rightarrow \overline{S}_{m,n}$. Similarly, $q_{m,n}$ is the transition probability for $\overline{S}_{m-1,n} \rightarrow S_{m,n}$ or $\overline{S}_{m-\left\lfloor \frac{m}{2} \right\rfloor} \rightarrow S_{m,n}$ respectively. The next section concentrates on the deriving the average throughput for this strategy.

### 4.4.2 Average Throughput

This section focuses on deriving the throughput and the outage probability performance of the second HARQ strategy combined with the selected cooperative protocols and the encoding methods. As it was shown in previous section (4.3) the average throughput of this strategy is the ratio of the average rate reward $E[R]$ to the average airtime $E[T]$ according to [66]. It means that $\eta(R, \alpha, \rho, \lambda) = E[R] / E[T]$. As it is seen the average throughput is a function of the transmission rate $R$, the duplexing ratio $\alpha$ and the threshold $\lambda$. By appropriate selection of $R$, $\alpha$ and $\lambda$, the throughput can be optimized as $\eta_{\text{optimum}} = \max_{R, \alpha, \lambda} \eta(R, \alpha, \rho, \lambda)$. Similar to the previous section (4.3), the average airtime is $E[T] = \sum_{n=1}^{m} \tau_n \sum_{m=1}^{\left\lfloor \frac{m}{2} \right\rfloor} Q_{m,n} + \tau_n \sum_{m=1}^{\left\lfloor \frac{m}{2} \right\rfloor} Q_{\overline{m},n}$, $\tau_n = \left\lfloor \frac{m}{2} \right\rfloor - 2 \alpha \left( \frac{m}{2} - \left\lfloor \frac{m}{2} \right\rfloor \right)$. The average delay is $E[D] = \sum_{m=1}^{N} d_m \sum_{n=1}^{\left\lfloor \frac{m}{2} \right\rfloor} Q_{m,n} + d_n \sum_{n=1}^{\left\lfloor \frac{m}{2} \right\rfloor} \overline{Q}_{m,n}$ where $d_n = \left( \frac{m}{2} \right)$. And the average rate rewards is $E[R] = R \sum_{m=1}^{N} \sum_{n=1}^{\left\lfloor \frac{m}{2} \right\rfloor} Q_{m,n}$. The outage probability after the $N$-th transmission is $p_{\text{out}} = \sum_{n=1}^{\left\lfloor \frac{m}{2} \right\rfloor} Q_{\overline{m},n}$. In the above equations $Q_{m,n}$ is the probability of being at successful state $S_{m,n}$, i.e. $Q_{m,n} = \Pr\{S_{m,n}\}$ and $\overline{Q}_{m,n}$ is the probability of being at failed state $\overline{S}_{m,n}$, i.e. $\overline{Q}_{m,n} = \Pr\{\overline{S}_{m,n}\}$. 
In order to calculate average air time and reward it is necessary to derive the state probabilities.

### 4.4.3 Transition Probabilities

These state probabilities can be recursively calculated using the transition probabilities $p_{m,n}$ and $q_{m,n}$. This process is started with the initial value $Q_{0,0} = 1$ and then $Q_{m,n} = q_{m,n} Q_{m-1,n}$ and $\overline{Q}_{m,n} = p_{m,n} \overline{Q}_{m-1,n}$ when $m$ is odd and $Q_{m,n} = Q_{m,n-1}$ and $\overline{Q}_{m,n} = \overline{Q}_{m,n-1}$ when $m$ is even.

Now it is possible to calculate the transition probabilities using the outage event Probabilities. The transition probabilities for this case have been tabulated in Table 4-9. In these equations $I_{m,n}$ is the accumulative mutual information normalized over one frame from the first slot transmission until end of the $m$-th time slot transmission. $I^{(3)}_{m}$ is the mutual information at the relay normalized over one frame for only $m$-th transmission when $m$ is odd number. Considering the first state $F_{0,0} = 1$.

We define $F_{m,n} = \Pr(I_{m,n} < R)$ when $m$ is odd. $F_{m,n} = \Pr(I_{m,n} < R, I^{(3)}_{m} \geq \lambda)$ and
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\[ J_{m,n} = \Pr \left( I_{m,n} \geq R, I_m^{(i)} \geq \lambda \right) \text{ when } m \text{ is even but } n \text{ is odd}. \]

\[ F_{m,n} = \Pr \left( I_{m,n} < R, I_m^{(i)} < \lambda \right) \]

\[ J_{m,n} = \Pr \left( I_{m,n} \geq R, I_m^{(i)} < \lambda \right) \text{ when } m \text{ and } n \text{ both are even}. \]

### Table 4-9. The transition probabilities

<table>
<thead>
<tr>
<th></th>
<th>( m ) odd</th>
<th>( m ) even</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n ) odd</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p_{m,n} = \Pr \left( I_{m,n} &lt; R \mid I_{m-1,n} &lt; R \right) = \frac{F_{m,n}}{F_{m-1,n}} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( q_{m,n} = \Pr \left( I_{m,n} \geq R \mid I_{m-1,n} &lt; R \right) = 1 - \frac{F_{m,n}}{F_{m-1,n}} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p_{m,n} = \Pr \left( I_{m,n} &lt; R, I_m^{(i)} \geq \lambda \mid I_{m-1,n} \geq \frac{\lambda}{2} \right) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( q_{m,n} = \Pr \left( I_{m,n} \geq R, I_m^{(i)} \geq \lambda \mid I_{m-1,n} \geq \frac{\lambda}{2} \right) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( n ) even</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p_{m,n} = \Pr \left( I_{m,n} &lt; R, I_m^{(i)} &lt; \lambda \mid I_{m-1,n} &lt; \frac{\lambda}{2} \right) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( q_{m,n} = \Pr \left( I_{m,n} \geq R, I_m^{(i)} &lt; \lambda \mid I_{m-1,n} &lt; \frac{\lambda}{2} \right) )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the next section, the accumulative mutual information is derived.

#### 4.4.4 Probability of Outage

##### 4.4.4.1 Accumulative Mutual Information

In order to calculate the outage probability \( F_{m,n} (R, \lambda) \) it is necessary to derive the accumulative mutual information for combination of each protocol and encoding scheme separately. Let us consider combination of the second strategy with RC and UC as well as the non-orthogonal protocols (2 or 3). The accumulative mutual information for these cases has been tabulated in Table 4-10. Let us consider \((a_1 a_2 ... a_n) = \text{dec2bin}(n)\) and \(K = [m/2]\). Recall that \(I_m^{(i)} = \alpha I_{\gamma_{1,m}}\) where \(I(x) = \log_2 (1 + x)\).
### Table 4-10. The accumulative mutual information for the non-orthogonal protocols (2 & 3)

<table>
<thead>
<tr>
<th>RC</th>
<th>m odd</th>
<th>( m = 2K + 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( I_{m,n} = (1 - \alpha) \log_2 \left( 1 + \frac{\sum_j \alpha_j \beta_j \gamma_{a,j}^2}{\sum_j \alpha_j \beta_j} \right) )</td>
<td>( (2\alpha - 1) \log_2 \left( 1 + \sum_j \gamma_{a,j} \right) )</td>
</tr>
<tr>
<td>UC</td>
<td>m odd</td>
<td>( m = 2K + 1 )</td>
</tr>
<tr>
<td></td>
<td>( I_{m,n} = (1 - \alpha) \log_2 \left( 1 + \frac{\sum_j \alpha_j \beta_j \gamma_{a,j}^2}{\sum_j \alpha_j \beta_j} \right) )</td>
<td>( (2\alpha - 1) \log_2 \left( 1 + \sum_j \gamma_{a,j} \right) )</td>
</tr>
</tbody>
</table>

### 4.4.4.2 Characteristic Function

To be able to calculate the outage probability \( F_{m,n}(R, \lambda) \), the characteristic function of \( I_{m,n} \) and \( I_m^{(1)} \) are obtained. Table 4-11 and Table 4-12 show the required characteristic functions. Calculation of \( \Psi_{m,n}(s_1, s_2) \) or \( \Psi_{m,n}(s) \) is complex and an alternative numerical calculation is proposed for a predefined sets of \( (s_1, s_2) \) or \( s \) and \( \alpha \). When \( m \) is odd, the outage probability \( F_{m,n}(R, \lambda) \) can be calculated by using Laplace inversion formula [78]. When \( m \) is even the
outage probability $F_{m,n}(R, \lambda)$ can be calculated by using the Padé rational function [68]. These outage probabilities have been tabulated in Table 4-13.

**Table 4-11. Calculating the outage probabilities using characteristic functions**

<table>
<thead>
<tr>
<th>$m$ odd</th>
<th>$n$ odd</th>
<th>$F_{m,n}(R, \lambda) = \frac{1}{2\pi j} \int_{a+jm}^{a-jm} \int_{a+jm}^{a-jm} \Psi_{m,n}(s_1, s_2) e^{sR + 2s/\lambda} ds_1 ds_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$ even</td>
<td>$m$ even</td>
<td>$F_{m,n}(R, \lambda) = \left(\frac{1}{2\pi j}\right)^{2} \int_{a-jm}^{a+jm} \int_{a-jm}^{a+jm} \Psi_{m,n}(s_1, s_2) e^{sR + 2s/\lambda} ds_1 ds_2$</td>
</tr>
<tr>
<td>$n$ odd</td>
<td>$m$ even</td>
<td>$J_{m,n}(R, \lambda) = \left(\frac{1}{2\pi j}\right)^{2} \int_{a-jm}^{a+jm} \int_{a-jm}^{a+jm} \Psi_{m,n}(s_1, s_2) e^{sR + 2s/\lambda} ds_1 ds_2$</td>
</tr>
<tr>
<td>$n$ even</td>
<td>$m$ even</td>
<td>$J_{m,n}(R, \lambda) = \left(\frac{1}{2\pi j}\right)^{2} \int_{a-jm}^{a+jm} \int_{a-jm}^{a+jm} \Psi_{m,n}(s_1, s_2) e^{sR + 2s/\lambda} ds_1 ds_2$</td>
</tr>
</tbody>
</table>

In the following equations $z_i, i=1,...,M$ are the poles of the Padé rational function $e^z$ and $K_i$ is corresponding residues (refer to appendix E-8). $z_i'$ are the poles of the Padé rational function $e^{-z}$ and $K_i'$ are the corresponding residues. $M$ is the number of the poles. This derivation can be repeated for the other combination of the protocols and encoding schemes.
### Table 4-12: Calculating the characteristic functions using mutual information

<table>
<thead>
<tr>
<th>$m$ odd</th>
<th>$n$ odd</th>
<th>$\Psi_{m,n}(s)$ or $\Psi_{m,n}(s_1, s_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\mathcal{E}(e^{-n_{max}})\bigg</td>
</tr>
<tr>
<td>$n$ even</td>
<td></td>
<td>$\Psi_{m,n}(s_1, s_2) = \mathcal{E}(e^{-n_{max}-s_1 m_{max}})$</td>
</tr>
</tbody>
</table>

### Table 4-13: Calculating the outage probabilities using the Pade rational function - when $m$ is even

<table>
<thead>
<tr>
<th>$n$ odd</th>
<th>$F_{m,n}(R, \lambda) = \sum_{i=1}^{M} \sum_{j=1}^{M} K_i K_j \frac{\Psi_{m,n}(z_i, z_j)}{z_i z_j}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$J_{m,n}(R, \lambda) = \sum_{i=1}^{M} \sum_{j=1}^{M} K_i K_j \frac{\Psi_{m,n}(z_i', z_j')}{z_i' z_j'}$</td>
</tr>
<tr>
<td>$n$ even</td>
<td>$F_{m,n}(R, \lambda) = \sum_{i=1}^{M} \sum_{j=1}^{M} K_i K_j \frac{\Psi_{m,n}(z_i, z_j)}{z_i z_j}$</td>
</tr>
<tr>
<td></td>
<td>$J_{m,n}(R, \lambda) = \sum_{i=1}^{M} \sum_{j=1}^{M} K_i K_j \frac{\Psi_{m,n}(z_i', z_j')}{z_i' z_j'}$</td>
</tr>
</tbody>
</table>
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4.5 Performance Result

In this section some simulation and analytical results are provided for the considered HARQ strategies. The analytical derivations are further verified by extensive Monte Carlo based simulations. The results in this section are provided for the maximum number of 4 frame transmissions. Figure 4-4 shows the throughput performance of the first strategy with UC encoding and the orthogonal protocol for a certain SNR configuration of the three constituent node-to-node links. The SNR of the link 0 $\text{SNR}_o$ is used as the reference and the SNR of the two other links are adjusted relatively to it: $\text{SNR}_1 = \Delta_1 + \text{SNR}_0$ and $\text{SNR}_2 = \Delta_2 + \text{SNR}_0$, where $\Delta_1$ and $\Delta_2$ represent the SNR offsets of the links 1 and 2, respectively.

![Figure 4-4. The throughput for the first strategy and the orthogonal protocol with the UC encoding for $\Delta_1 = 10dB$ and $\Delta_2 = 5dB$](image)

The throughput for the optimum $\alpha$, direct transmission $\alpha = 1$ and the AF scheme $\alpha = 0.5$ with considering the transmission rate $R = 2$ are shown in Figure 4-4. As it is seen at low $\text{SNR}_0$, the optimum throughput curve follows the direct link performance and at very high $\text{SNR}_0$ follows $\alpha = 0.5$ performance but in the middle range SNRs ($5dB \leq \text{SNR}_0 \leq 15dB$) the optimum performance is better than other two cases. It shows that the optimum $\alpha$ is a point between half and one. Similar trend of behaviour is observed when the RC encoding is used. Figure 4-5 shows the optimum $\alpha$ for different $\text{SNR}_0$ values. By increasing $\text{SNR}_0$ the optimum $\alpha$ decreases gradually from 1 to 0.5 for the both RC and UC encoding.
Figure 4-5. The optimum duplexing ratio for the first strategy and the orthogonal protocol and
\[ \Delta_1 = 10\,\text{dB} \text{ and } \Delta_2 = 0\,\text{dB} \]

Figure 4-6 shows the throughput performance of the second strategy with the non-orthogonal protocol and RC encoding. Performance gains of up to 1 and 1.5 dB are observed for the considered simulation scenarios thanks to the flexible AF combined with properly designed HARQ protocols.

Figure 4-6. The throughput for the second strategy and the non-orthogonal protocol with RC and
\[ \Delta_1 = 5\,\text{dB} \text{ and } \Delta_2 = 10\,\text{dB} \]

Also Figure 4-7 compares the RC encoding with the UC encoding. The UC encoding performs better than the RC encoding especially at low and moderate SNRs. This is due to the repetitive structure of the RC code (the source transmits the same codeword over T1 and T2 slots). Also it is
shown that the simulation result is very close to the result obtained by analysis. The optimum transmission rate depends on the SNR configuration of the constituent links. Therefore; the throughput optimization needs the knowledge on the average SNR of all the links. Supplying this knowledge is not practically difficult. The measured average SNRs needs to be reported from the relay and destination nodes to the source node. The availability of this knowledge at source also allows efficient use of the considered protocols in the SNR regions that they provide considerable gain over direct communication.

Figure 4-7. The throughput for the second strategy and the non-orthogonal protocol, analysis versus simulation, RC with UC and $\Delta_1 = 10dB$ and $\Delta_2 = 5dB$.

4.6 Conclusion

In this chapter a typical HARQ scheme was combined with a cooperative flexible AF scheme with single relay. The outage and the throughput performance measures are derived for both repetition and unconstrained coding. The presented analysis allows accurate evaluation of flexible AF based cooperative HARQ protocols without need to time consuming Monte Carlo simulation. This allows identification of the selected protocols and appointing right protocol at right place. The performance gains of up to 1 and 1.5 dB are observed for the considered scenarios thanks to the flexible AF combined with properly designed HARQ protocols. Also the presented analysis lets us to precisely calculate the required transmission rate and frame structure that optimizes the throughput.
Chapter 5 Amplify and Forward in Multiple Antennas Scenario

5.1 Introduction

In previous chapters a flexible AF scheme that allows separately tuneable time slots was proposed. The performance of this scheme was investigated in terms of the outage probability and the achievable rate in a single antenna scenario. In this chapter performance of this scheme is explored in multiple antenna scenarios. In the original AF scheme the relay is used as a simple amplifier. However in cooperative MIMO scenario, it can be utilized as a smart pre-coder, which allows performance improvement through the efficient power allocation techniques based on the level of CSI available at the relay. In the case when both the CSI of the source-relay and relay-destination links are known at the relay, power allocation techniques for the AF scheme have been recently developed [8-9, 82]. They have shown great enhancements on the performance of AF in terms of mutual information by providing a linear transceiver design for amplify and forward scheme using transmit CSI at the relay. In above MIMO cooperative research length of time slots allocated to the relay and the source is considered to be equal. In this chapter a novel flexible MIMO AF scheme is proposed to provide different duplexing ratios using linear dispersion Codes (LDC). LDC scheme disperses sub-streams of data stream in linear combination over space and time. LDC codes are designed to maximize mutual information between the received and transmit signals [79]. By applying LDC codes and distributing data on the time and space, another dimension is opened on the design of linear processing at the relay and the degree of freedom for the AF scheme is increased by considering different time slot lengths for the relay-receive and relay-transmit phases [67]. Considering the case when the relay being aware about its input and output channel, new processing techniques at the relay are introduced for the flexible AF scheme. It is shown that the flexible scheme provides better throughput as well as lower BER in many SNR configurations [67]. However, transmit CSI may not always be available at the relay. Moreover above techniques require extra complexity since they need not only the source to relay link CSI, but also the relay-destination link CSI. This chapter also investigates new techniques in order to improve the performance of MIMO AF but without requiring extra CSI knowledge. In this scenario, several power allocation schemes has been lately proposed in [9] for traditional
Chapter 5 Amplify and Forward in Multiple Antennas Scenario

MIMO cooperative AF such as the AF matched-filter based relaying (MFR) and AF minimum mean square error filtering (MMSEF) techniques. A novel power allocation method is designed for non-regenerative cooperative MIMO scheme [87] then this method is extended to the flexible AF relying scheme based on the system model introduced in Section 5.2.

The remainder of this chapter is organised as follows. Section 5.2 describes the system model for the LDC based AF concept. Section 5.3 discusses in detail the proposed processing technique at the relay in detail. The BER upper bound for the LDC based cooperative AF model is derived in section 5.3.2 and throughput and BER performance results of the proposed scheme are compared in section 5.3.4. At section 5.4, the existing power allocation techniques such as MFR and MMSEF are extended to the flexible AF scheme and a novel power allocation algorithm is introduced based on the maximization of the relay link mutual information considering equal Eigen mode (EEM). This method has low complexity and still outperforms the traditional equal gain and other existing power allocation techniques such as MFR and MMSEF, both in terms of the mutual information and the BER, as it is reported in Section 5.4.3. Finally section 5.5 provides the conclusion remarks.

5.2 System Model

A simple half duplex TDD/TDMA one way relay channel scenario with one source node, one destination node and only one relay is considered here as illustrated in Figure 5-1. The nodes transmission/reception is based on a simple protocol composed of two phases. The source node which equipped with $M$ transmit antenna sends its message, a Gaussian code word $x_i(w) = \{x_i^{(1)}(w), x_i^{(2)}(w), \ldots, x_i^{(N_1)}(w)\}$, where $x_i^{(k)}(w) \in \mathbb{C}^{Q \times 1}$, to the relay which has $R$ receive and transmit antennas and the destination node which equipped with $N$ receive antennas in the first channel phase, $N_1$ (solid line). The first channel phase consists of several time slots each with length $T_1$. The letter $c$ stands for complex numbers. The relay combines the received signal from source through a linear combiner and sends $x_j(w) = \{x_j^{(1)}(w), x_j^{(2)}(w), \ldots, x_j^{(N_2)}(w)\}$, where $x_j^{(k)}(w) \in \mathbb{C}^{Q \times 1}$, in broadcast to destination in the second channel phase, $N_2$ (doted line). The second channel phase also consists of several time slots each with length $T_2$. These channel phases are orthogonal in time and thus, there is no interference between the links. It is assumed that input data are encoded and modulated and then passed through LDC encoder. LDC scheme disperses sub-streams of data stream in linear combination over space and time [79].
In order to reduce complication of the equations hereinafter the block indices are dropped from signals and channels parameters. A block diagram of the system is shown in Figure 5-2. At first phase during the $k^{th}$ LDC time slot with length $T_i$, the transmit signal $x_{i,k}(k), k=1,\ldots,N_i/T_i$, is LDC coded with dispersion matrices $\{A_q,B_q\} \quad q=1,\ldots,Q_0$. The design of the LDC codes depends on the selection of the parameters $T_i,Q_0, M$ and dispersion matrices\[79\]. LDC coded signal with size $T_i \times M$ is transmitted through random complex normal MIMO channels $H_{01} \in \mathbb{C}^{N \times M}$ and $H_{02} \in \mathbb{C}^{N \times M}$ to the relay and the destination respectively. $H_{01}, H_{02}$ are assumed fixed during LDC time slot $T_i$. During the second phase, the relay transmits its desired signal through MIMO channel $H_{12} \in \mathbb{C}^{N \times K}$ to the destination. $H_{12}$ is assumed fixed during LDC time slot $T_2$. $H_{01}, H_{02}$ and $H_{12}$ are defined as random channel matrices having independent and identically distributed (i.i.d) complex Gaussian entries with zero mean with variance $\rho_0, \rho_1$ and $\rho_2$ respectively. In this chapter $\rho_0, \rho_1$ and $\rho_2$ are defined as the average SNR of the source-destination, source-relay and relay-destination links respectively. At the relay and the destination, white complex Gaussian distributed noises with zero mean, unit-variance are added to the received signals. Because of the linear structure of the dispersion matrices $\{A_q,B_q\}$ and the original channels $H_{01}, H_{02}$ and $H_{12}$, the equivalent channel matrices can be calculated\[79\], all known to the destination. It is assumed that the source does not have any information about channel matrices.
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Figure 5-2. Block diagram for the MIMO orthogonal AF

Figure 5-3. Equivalent block diagram for the MIMO Orthogonal AF

For simplicity hereinafter this work is continued with equivalent channel matrices between the source and the relay \( \mathbf{H}_1 \in \mathbb{R}^{2N_T \times 2Q_c} \), the source and the destination \( \mathbf{H}_0 \in \mathbb{R}^{2N_T \times 2Q_b} \) and the relay and the destination \( \mathbf{H}_2 \in \mathbb{R}^{2N_T \times 2Q_d} \) (as it is shown in Figure 5-3). More details about LDC structure can be found in Appendix F-1. Factor 2 is for considering real and imaginary values separately. Now all the complex valued equations can be converted to equivalent real valued equations. So for example for each LDC time slot \( T_1 \):

\[
\begin{align*}
y_0 &= \sqrt{\frac{1}{M}} \mathbf{H}_0 \mathbf{x}_1 + \mathbf{v}_0 \\
y_1 &= \sqrt{\frac{1}{M}} \mathbf{H}_1 \mathbf{x}_1 + \mathbf{v}_1
\end{align*}
\]

(5-1)

where \( \mathbf{x}_1 \) is real valued column-wise transmit signal and \( \mathbf{y}_0 \) and \( \mathbf{y}_1 \) the received signals at the destination and the relay during the LDC time slot \( T_1 \). Furthermore \( \mathbf{v}_0 \in \mathbb{R}^{2NT \times 1} \) and \( \mathbf{v}_1 \in \mathbb{R}^{2MT \times 1} \) are column-wise vectors of independent real valued additive white Gaussian noise entries with zero mean and a variance of \( \frac{1}{2} \sigma^2 \) (recall that \( \sigma^2 = 1 \) is the variance of equivalent complex noise).
Chapter 5 Amplify and Forward in Multiple Antennas Scenario

Generally, \( \mathbf{x} \) represents a real valued column vector consisting of real parts and imaginary parts of any vector \( \mathbf{x}_c \), i.e. \( \mathbf{x} = \begin{bmatrix} \text{real}(\mathbf{x}_c) \\ \text{imag}(\mathbf{x}_c) \end{bmatrix} \). During the second time slot \( T_2 \), the relay applies a linear matrix \( \mathbf{G} \) with size \( 2Q \times 2RT \) to the equivalent channel \( \mathbf{H}_2 \) and transmits the result through the channel to the destination. Then by assuming \( \mathbf{v}_2 \in \mathbb{R}^{2NT_{x}} \) as a real valued additive white Gaussian noise signal:

\[
\mathbf{y}_2 = \mathbf{H}_2 \mathbf{G} \mathbf{x}_1 + \mathbf{v}_2 = \mathbf{H}_2 \mathbf{G} \left( \sqrt{\frac{1}{M}} \mathbf{H}_1 \mathbf{x}_1 + \mathbf{v}_1 \right) + \mathbf{v}_2
\]

(5-2)

Now an equivalent model can be derived for the whole system as \( \mathbf{y} = \mathbf{H} \mathbf{x} + \mathbf{v} \), where \( \mathbf{y} = \begin{bmatrix} \mathbf{y}_0 \\ \mathbf{y}_2 \end{bmatrix} \) is the real valued received vector at the destination during both LDC time slots \( T_1 \) and \( T_2 \) and \( \mathbf{x} = \sqrt{\frac{1}{M}} \mathbf{x}_1 \) is the real value transmitted signal from the source node. Overall equivalent channel matrix is \( \mathbf{H} = \begin{bmatrix} \mathbf{H}_0 \\ \mathbf{H}_2 \mathbf{G} \mathbf{H}_1 \end{bmatrix} \) and equivalent noise vector \( \mathbf{v} \) is

\[
\mathbf{v} = \begin{bmatrix} \mathbf{I}_{2NT_{x} \times 2NT_{x}} \\ \mathbf{0}_{2NT_{x} \times 2NT_{x}} \\ \mathbf{0}_{2NT_{x} \times 2NT_{x}} \\ \mathbf{H}_2 \mathbf{G} \end{bmatrix} \begin{bmatrix} \mathbf{v}_0 \\ \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix}.
\]

Matrix \( \mathbf{R}_v \) is defined as the covariance matrix of signal transmitted from the source \( \mathbf{R}_v = \mathcal{E} \{ \mathbf{x} \mathbf{x}^H \} = (1/2M) \mathbf{I}_{2Q} \), and so \( \text{trace}(\mathbf{R}_v) = T_1 \). The letter \( \mathcal{E} \) stands for expectation and \( \mathcal{H} \) for hermitian function. The factor 2 is applied regarding to the real valued vector \( \mathbf{x} \). It is assumed that total transmit powers of the source and also the relay are equal one. Furthermore \( \mathbf{R}_{v_0}, \mathbf{R}_{v_1}, \mathbf{R}_{v_2} \) are defined as the noise covariance matrices.

\[
\mathbf{R}_{v_0} = \mathcal{E} \{ \mathbf{v}_0 \mathbf{v}_0^H \} = \frac{1}{2} \mathbf{I}_{2NT_{x}}, \\
\mathbf{R}_{v_1} = \mathcal{E} \{ \mathbf{v}_1 \mathbf{v}_1^H \} = \frac{1}{2} \mathbf{I}_{2NT_{x}}, \\
\mathbf{R}_{v_2} = \mathcal{E} \{ \mathbf{v}_2 \mathbf{v}_2^H \} = \frac{1}{2} \mathbf{I}_{2NT_{x}}.
\]

Then overall noise covariance matrix \( \mathbf{R}_v \) is calculated as follows:

\[
\mathbf{R}_v = \frac{1}{2} \begin{bmatrix} \mathbf{I}_{2NT_{x}} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_2 \mathbf{G}^H \mathbf{H}_2^H + \mathbf{I}_{2NT_{x}} \end{bmatrix}.
\]

(5-3)
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The covariance matrix of the signal transmitted by the relay is

\[ \mathbf{R}_{r_n} = E(x_n y_n) (T y_n y_n^H) = \frac{1}{2} \mathbf{I} \left( \frac{1}{M} \mathbf{H}_n \mathbf{H}_n^H + \mathbf{I}_{2 \times 2} \right) \mathbf{I}^{*} \] with constraint \( \text{trace}(\mathbf{R}_{r_n}) = T_2 \).

5.3 First Scenario: Receive/Transmit CSI knowledge Available at Relay

5.3.1 Optimising Power Allocation

Based on the above equivalent channel \( \mathbf{H} \) and covariance matrix of overall noise \( \mathbf{R}_v \), the cooperative mutual information is given by:

\[ I(y; x) = \frac{1}{2} \frac{1}{T_1 + T_2} \log_2 \det \left( \mathbf{R}_v \mathbf{R}_v^{-1} \right) = \frac{1}{2} \frac{1}{T_1 + T_2} \log_2 \det \left( \mathbf{H} \mathbf{R}_v \mathbf{H}_v^H + \mathbf{R}_v \right) \] (5-4)

or

\[ I(y; x) = \frac{1}{2} \frac{1}{T_1 + T_2} \log_2 \det \left( \mathbf{I}_{2 \times 2} + \mathbf{H} \mathbf{R}_v \mathbf{H}_v^H \right) \] (5-5)

where the factor \( 1/2 \) is for real valued effective channel and factor \( T_1 + T_2 \) accounts for the two-phase transmission i.e. each LDC matrix spans over \( T_1 + T_2 \) channel use. The notation \( \det \) represents determinant function. In this chapter duplexing ratio is defined as \( \alpha = T_1 / (T_1 + T_2) \).

Similar to the basic MIMO scenario [8] it can be shown that it is not possible to calculate the upper bound for \( I(y; x) \) directly. The mutual information of the relaying (two-hop) channel when direct link does not exist can be considered as the lower bound for the cooperative scheme. The direct and relay link mutual information, i.e., \( I(y_0; x) \) and \( I(y_2; x) \), can also be worked out such that:

\[ I(y_0; x) = \frac{1}{2} \frac{1}{T_1 + T_2} \log_2 \det \left( \mathbf{I}_{2 \times 2} + \mathbf{H}_0 \mathbf{R}_v \mathbf{H}_v^H \right) \] (5-6)

and

\[ I(y_2; x) = \frac{1}{2} \frac{1}{T_1 + T_2} \log_2 \det \left( \mathbf{I}_{2 \times 2} + \mathbf{H} \mathbf{R}_v \mathbf{H}_v^H \right) \] (5-7)
where \( \tilde{H} = H_2 \Gamma H_1 \) and \( R^{-1}_\psi = R_{\psi} + H_2 \Gamma R_{\psi} \Gamma^\dagger H_2^\dagger = \frac{1}{2} (I_{2M_2} + H_2 \Gamma \Gamma^\dagger H_2^\dagger \). Moreover, \( I(y;x) \) in (5-4) can be re-expressed after some simplifications as

\[
I(y;x) = I(y_0;x) + \frac{1}{2 T_1 + T_2} \log_2 \det \left( I_{2M_2} + \tilde{H} R_{\psi}^{1/2} A^{-1} R_{\psi}^{1/2} \tilde{H}^\dagger R_{\psi}^{-1} \right)
\]

(5-8)

where \( A = I_{2M_0} + R_{\psi}^{1/2} H_2 \Gamma H_1^\dagger R_{\psi}^{1/2} \) is a positive definite matrix. Therefore, according to (5-6) and (5-7), \( I(y;x) \leq I(y_0;x) + I(y_2;x) \). Furthermore, it can easily be proved that \( I(y;x) \geq \min \{ I(y_0;x), I(y_2;x) \} \). As a result, \( I(y;x) \) can be increased by maximizing \( I(y_2;x) \) or equivalently by optimizing \( \Gamma \) at the relay. Now the optimizing of \( \Gamma \) at the relay is explored for the relaying scenario. The mutual information of Multihop channel \( I(y_2;x) \) can be expressed as:

\[
I(y_2;x) = \frac{1}{2 T_1 + T_2} \log_2 \det \left( \frac{I_{(2MT_1)} + H_2 \Gamma \Gamma^\dagger H_2^\dagger + \frac{1}{M} H_1 \Gamma H_1^\dagger \Gamma^\dagger H_2^\dagger}{I_{(2MT_2)} + H_2 \Gamma \Gamma^\dagger H_2^\dagger} \right)
\]

subject to \( \text{trace}(\frac{1}{2} \Gamma (\frac{1}{M} H_1 H_1^\dagger + I_{(MT_1)}) \Gamma^\dagger) \leq T_2 \).

Figure 5-4. Diagonalisation and per mode power allocation

Here it is assumed that the relay is aware of its input equivalent channel \( H_1 \) and forward equivalent channel \( H_2 \). Furthermore in order to reduce complexity of the equations it is assumed that \( Q_1 = RT_2 \). It can be proved that matrix \( \Gamma \) that maximizes multihop mutual information (5-9)
Chapter 5 Amplify and Forward in Multiple Antennas Scenario

is given by: \( \Gamma = \tilde{V}_2 \tilde{U}_1^T \) where \( U_1 \) is left singular matrix of the equivalent channel matrix \( H_1 = U_1 \Lambda^{1/2}_1 V^*_1 \) and \( \tilde{V}_2 \) is permuted right singular matrix of the equivalent channel matrix \( H_2 = U_2 \Lambda^{1/2}_2 V^*_2 \) i.e. \( \tilde{V}_2 = PV_2 \) (Figure 5-4). \( U_1 \in \mathbb{R}^{2RT_1 \times 2RT_1} \), \( V_1 \in \mathbb{R}^{2Q_1 \times Q_1} \), \( U_2 \in \mathbb{R}^{2NT_2 \times 2NT_2} \) and \( V_2 \in \mathbb{R}^{2RT_1 \times 2RT_1} \) are unitary matrices. \( P \in \mathbb{R}^{2RT_1 \times 2RT_1} \) is a mode permutation matrix. \( \Lambda_1 \in \mathbb{R}^{2RT_1 \times 2Q_1} \) and \( \Lambda_2 \in \mathbb{R}^{2NT_2 \times 2RT_1} \) are two rectangular diagonal matrices. \( \hat{\Lambda}_1 = \Lambda_1^{1/2} \Lambda_1^{1/2} \)

where \( \hat{\Lambda}_1 \in \mathbb{R}^{2RT_1 \times 2RT_1} \) is a diagonal matrix with diagonal elements \( \lambda_{i,i} \in \mathbb{R}^+ \), which are sorted in descending order. Notice that \( \lambda_{i,j} \neq 0 \) for \( k \in [1,K_1] \) and that \( \lambda_{i,k} = 0 \) for \( k \in [K_1+1,2RT_1] \)

where \( K_1 = \min(2RT_1,2Q_0) \). \( \hat{\Lambda}_2 = \Lambda_2^{1/2} \Lambda_2^{1/2} \) where \( \hat{\Lambda}_2 \in \mathbb{R}^{2NT_2 \times 2RT_1} \) is a diagonal matrix with diagonal elements \( \lambda_{2,i} \in \mathbb{R}^{+} \), which are sorted in descending order. Notice that \( \lambda_{2,i} \neq 0 \) for \( k \in [1,K_2] \) and that \( \lambda_{2,k} = 0 \) for \( k \in [K_2+1,2NT_2] \) where \( K_2 = \min(2NT_2,2RT_1) \). \( \hat{\Gamma} \in \mathbb{R}^{2RT_1 \times 2RT_1} \)

Here it is seen that \( \hat{\Gamma} \) can have different column and row size depending to value of \( K_1 \) and \( K_2 \) as follows:

\[
\hat{\Gamma} = \begin{cases} 
[\hat{\Gamma} \\ 0] & K_2 < K_1 \\
\hat{\Gamma} & K_2 = K_1 \\
[0] & K_2 > K_1 
\end{cases}
\]

where \( \hat{\Gamma} \) is a diagonal matrix which performs power allocation at the relay. Considering \( \tilde{\Lambda}_1^{1/2} = P \Lambda_1^{1/2} \) where \( P \) is a mode permutation matrix and by some modification over equation (5-5) the following equation is obtained:

\[
I(y_2;x) = \frac{1}{2} \frac{1}{T_1 + T_2} \sum_{k=1}^{K} \log_2 \left( \frac{1}{2} \left( 1 + \hat{\lambda}_{2k} \hat{\lambda}_{2k} \right) + \gamma_{k1} \hat{\lambda}_{1k} \hat{\lambda}_{2k} \right) \left( \frac{1}{2} \left( 1 + \hat{\lambda}_{2k} \hat{\lambda}_{2k} \right) \right) \tag{5-10}
\]

Then:

\[
I(y_2;x) = \frac{1}{2} \frac{1}{T_1 + T_2} \sum_{k=1}^{K} \left( \log_2 \left( 1 + \hat{\lambda}_{2k} \hat{\lambda}_{2k} \right) + 2 \gamma_{k1} \hat{\lambda}_{1k} \hat{\lambda}_{2k} \right) - \log_2 \left( 1 + \hat{\lambda}_{2k} \hat{\lambda}_{2k} \right) \tag{5-11}
\]
where $K$ is minimum of number of eigenvalues of channels $H_1$ and $H_2$. Also the constraint $\sum_{i=1}^{K} \left( \frac{1}{2} + \gamma_{ik} \tilde{\lambda}_{ik} \right) \leq T_2$ needs to be satisfied. $\tilde{\gamma}_{2k}$ is assumed as the $k^{th}$ diagonal element of matrix $\tilde{\Gamma}^{\text{th}}$ and $\tilde{\lambda}_{1k}$ and $\tilde{\lambda}_{2k}$ as the squared of $k^{th}$ eigenvalues of $\Lambda_1^{1/2}$ and $\Lambda_2^{1/2}$ respectively. The optimization problem will be:

maximizing rate:

$$\sum_{i=1}^{K} \log_2 \left( \frac{1}{2} + \gamma_{ik} \tilde{\lambda}_{ik} \right) - \log_2 \left( \frac{1}{2} + \gamma_{ik} \tilde{\lambda}_{ik} \right),$$

subject to the relay power constraint:

$$\sum_{i=1}^{K} \left( \frac{1}{2} + \gamma_{ik} \tilde{\lambda}_{ik} \right) \leq T_2.$$  

A Lagrange method for solving this problem has been provided in Appendix F.3. The optimum solution for $k = 1, \ldots, K$ using Lagrange method is as follows:

$$\left( \frac{1}{2} + \gamma_{ik} \tilde{\lambda}_{ik} \right) \bar{P}_{2k} = \max \left( 0, \left[ \mu \left( \gamma_{ik} \tilde{\lambda}_{ik} + \left( \frac{\gamma_{ik} \tilde{\lambda}_{ik}}{2 \tilde{\lambda}_{2k}} \right)^2 - \frac{1}{2 \tilde{\lambda}_{2k}} \right) \right] \right).$$

(5-13)

Where $\mu$ is the Lagrange multiplier and its starting point is $\mu_{\text{ini}} = \max \left( \frac{1}{2} + \gamma_{ik} \tilde{\lambda}_{ik} \right)$ for $k \in [1, K]$ and in order to update $\mu$ the Newton-Raphson algorithm can be used. This method is lowly complex since it is implemented by employing the Newton-Raphson method which requires on average less than 10 iterations to obtain $\mu^*$ for $\varepsilon = 10^{-5}$. Ordering of squared eigenvalues $\tilde{\lambda}_{1k}$ and $\tilde{\lambda}_{2k}$ (permutation matrix $P$) also needs to be optimized. From the simulation results it is found that sorting eigenvalues in decreasing order maximize the mutual information.

5.3.1.1 Another Method for Deriving Optimised Power Gains

Back to equation (5-7) by using $R_{nh} = \frac{1}{2} I_{K \times K} + H_n H_n^H$: 

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The equation (5-14) can be expressed by applying the matrix determinant lemma
\[(\det(A + UV^T)^{\gamma}) = \det(I + V^T A^{-1} U) \det(A) \] (83) as follows:

\[
I(y_2; x) = \frac{1}{2 T_1 + T_2} \log_2 \det\left( \frac{2\Gamma R_y \Gamma^T}{\Gamma^T \Gamma} \right) + \frac{1}{2 T_1 + T_2} \log_2 \det\left( \frac{(2\Gamma R_y \Gamma^T)^{-1} + H_2^T H_2}{\Gamma^T \Gamma} \right)
\] (5-15)

The above equation simplifies as

\[
I(y_2; x) = \frac{1}{2 T_1 + T_2} \log_2 \det(S) \quad \text{when} \quad \|2\Gamma R_y \Gamma^T\|_F \gg 1,
\]

\[
\|\Gamma^T \Gamma\|_F \gg 1 \quad \text{and} \quad N \geq R,
\]

where \( S = \frac{2\Gamma R_y \Gamma^T}{\Gamma^T \Gamma} \). As described in [8], according to the Hadamard determinant theorem [84], \( I(y_2; x) \) will be maximized under a total power constraint if the matrix \( S \) within the determinant in (5-15) is diagonal. The matrix \( R_y \) can be decomposed via eigenvalue decomposition as

\[
R_y = U \Lambda U^H,
\]

where \( U \in \mathbb{C}^{2RT_1 \times 2RT_1} \) is a unitary matrix and \( \Lambda \in \mathbb{R}^{2RT_1 \times 2RT_1} \) is a diagonal matrix with diagonal elements \( \lambda_k \in \mathbb{R}^+ \) for \( k \in [1, 2RT_1] \), i.e., \( \mathbb{R}^+ = \{ x \in \mathbb{R} | x \geq 0 \} \), which are sorted in descending order. Therefore, by setting \( \Gamma = \tilde{V}_2 \tilde{\Gamma} U^H \) in \( S \), \( S \) is diagonal and thus (5-14) is maximized.

Since both the source to relay and the relay to destination CSI are known at the relay, the optimum power allocation is \( \Gamma = \tilde{V}_2 \tilde{\Gamma} U^H \), and therefore, (5-13) can be re-expressed after some simplifications as follows
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\[ J(y_2;x) = \frac{1}{2T_1+T_2} \log_2 \det \left( (I_{2Mx_1} + 2H_2\Gamma R_y \Gamma^H H_2^H) (I_{2Mx_1} + H_2\Gamma^H H_2^H)^{-1} \right) \]  

subject to \( \text{trace}(\Gamma R_y \Gamma^H) \leq T_2 \). Furthermore with substituting \( R_y = U\Lambda U^H \) and \( H_2 = U_2\Lambda_2^{1/2}V_2^H \)

\[ J(y_2;x) = \frac{1}{2T_1+T_2} \sum_{k=1}^{K} \left( \log_2 \left( 1 + 2\tilde{\gamma}_{2,k}^2\lambda_k \right) - \log_2 \left( 1 + \lambda_k \right) \right) \]  

where \( K \) is the minimum of number of eigenvalues of matrices \( R_y \) and \( H_2 \). Then, the values of \( \tilde{\gamma}_{2,k} \) that maximize (5-14) are obtained by solving the following problem

\[ \max_{\{\tilde{\gamma}_{2,1}, \tilde{\gamma}_{2,2}, \ldots, \tilde{\gamma}_{2,K}\}} J(y_2;x) \]

Subject to

\[ \tilde{\gamma}_{2,k} \geq 0 \quad \text{and} \quad \sum_{k=1}^{K} \tilde{\gamma}_{2,k} \lambda_k \leq T_2 \]  

(5-18)

The optimum solution for this modified problem is obtained by solving its Karush-Kuhn-Tucker (KKT) conditions [85], such that

\[ \gamma_{\mu} = \max \left( 0, \frac{1}{4\lambda_{2,k}^2 - 1} \left\{ \frac{\lambda_k}{\lambda_{2,k}^2 (2\lambda_k - 1)} \right\} \right) \]  

(5-19)

where \( \mu \geq 0 \) is the Lagrange multiplier that needs to be adjusted according to the power constraint in (5-18). The Lagrange method for solving this problem is similar to Appendix F.3.

The starting value for \( \mu \) is \( \mu_{\text{in}} = \max_{k \in [1,K]} \left\{ \frac{\lambda_k}{\lambda_{2,k}^2 (2\lambda_k - 1)} \right\} \) then \( \mu \) is be updated by using the Newton-Raphson method [86] until \( \mu^* \) is obtained. The value of \( \mu^* \) must fulfill the following inequality

\[ f(\mu^*) = \sum_{k=1}^{K} \tilde{\gamma}_{2,k} \lambda_k - T_2 < \varepsilon \quad \text{and} \quad \varepsilon \ll 1. \]

Ordering of squared eigenvalues \( \lambda_k \) and \( \lambda_{2,k} \) (permutation matrix \( P \)) also needs to be optimized. Block diagram of the proposed algorithm in order to find the optimum gains has been shown in Figure 5-5.
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5.3.2 BER Analysis for Cooperative Bit interleaved Coded Modulation Amplify and Forward

An encoder at the source uses binary code $C$ and generates code sequence $c \in C$. A bit interleaver $\pi$ in the bit interleaved coded modulation (BICM) scheme establishes a one-to-one correspondence $\pi: k \rightarrow (k', i)$, where $k$ denotes the time ordering of the bit sequence before interleaving, $k'$ denotes the time ordering of the modulated signals, and $i$ indicates the position of bit in the label of the signal. Modulator $(\mu, \mathcal{X})$ is done at the source. The modulator is memory less and is over the signal set $\mathcal{X} \in \mathbb{C}^L$ with size $|\mathcal{X}| = 2^m$ and uses one-to-one binary labelling map $\mu: \{0,1\}^m \rightarrow \mathcal{X}$. $\mathbb{C}^L$ is the complex $L$-dimensional Euclidean space. The ML criteria for decoding the observations $\mathbf{y}$ given CSIs $\mathbf{H}_0$, $\mathbf{H}_1$, and $\mathbf{H}_2$ at the destination is written as

$$
(c) = \arg \max_{\hat{c}} p(y | c)
$$

where $\mathbf{H}$ is the equivalent channel of whole cooperative system. Let $P(c \rightarrow \hat{c})$ denote the pair-wise error probability (PEP) as the probability that the decoder at the destination prefers $\hat{c}$ to $c$. $d$ denotes the Hamming distance between the corresponding component code-words of $c$ and $\hat{c}$. The PEP will be a function of $d$, $\mu$ and $\mathcal{X}$,
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$P(\xi \rightarrow \hat{\xi}) = f(d, \mu, \chi)$. The PEP can be used in conjunction with usual union bound to upper bound the BER. It means that for the convolutional codes rate $k_c / n_c$, the bit probability is:

$$P_b \leq \frac{1}{k_c} \sum_{d=1}^{\infty} W_f(d) f(d, \mu, \chi)$$

where $W_f(d)$ is the total input weight of the error events at hamming distance $d$ [80]. Now, this section focuses on the calculation of the PEP i.e. $f(d, \mu, \chi)$.

For the derivation of the BICM union bound:

$$f_u(d, \mu, \chi) = \frac{1}{2\pi j} \int_{-j\infty}^{+j\infty} [\psi(s)]^d ds$$  \hspace{1cm} (5.20)

where $\psi(s) = \frac{1}{mQ_0 2^{n_0}} \sum_{l=1}^{2^{n_0}} \sum_{b=0}^{n_0} \sum_{\chi(l,b)} \Phi_{\Delta(x_l,z)}(s)$.

$x_1 = \sqrt{M} x$ and $x_l = \sqrt{M} z$ are the signal vectors corresponding to the paths $\xi$ and $\hat{\xi}$. Equation (5.20) can be calculated using the scheme mentioned in [78]; similar to [81] removing irrelevant error events in (5.20) will result in the cooperative BICM Expurgated Bound. $\Phi_{\Delta(x_l,z)}(s)$ is the Laplace transforms of the probability density function of the metric difference $\Delta(x_l,z)$. It means:

$$\Phi_{\Delta(x_l,z)}(s) = E_{s,0} \left[ e^{-s\Delta(x_l,z)} \right]$$

where
\[ A(x, z) = \log(p_{h_0, h_1, h_2}(y|x)) - \log(p_{h_0, h_1, h_2}(y|z)) \] (5-21)

Here it is possible to separate the direct transmission and the relay transmission:
\[ A(x, z) = \Delta_{\text{direct}}(x, z) + \Delta_{\text{relay}}(x, z) \]

Considering \((x_i - z_i)^\top H_0^2 H_0(x_i - z_i) = h_0^\top Q(x_i, z_i) h_0\) [81]
where \(Q(x_i, z) = (x_i - z)(x_i - z)^\top \otimes I_{2N_0}\) \((\otimes\) is used for Kronecker tensor product) and \(h_0 = \text{vec}(H_0)\) is a column vector consisting of all the elements of \(H_0\) and also
\[ C_{h_0, \text{norm}} = \frac{1}{2} E(h_{0, \text{norm}} h_{0, \text{norm}}^\top) (C_{h_0, \text{norm}}^\top) \text{ depends on the specification of the used LDC code where} \]
\[ h_{0, \text{norm}} = \sqrt{1/\rho_0} h_0 \):

\[
\begin{align*}
\Phi_{\Delta_{\text{direct}}(x, z)}(s) &= E_{H_2} \left[ e^{(-s + s^2)h_0^\top Q(x_i, z_i) h_0} \right] \\
&= \left[ \det(I + \frac{\rho_0^2}{M} (s - s^2)Q(x_i, z_i) C_{h_0, \text{norm}}) \right]^{-1}
\end{align*}
\] (5-22)

For the relay link, by considering \(\tilde{H}_2, \text{norm} = H_2, \text{norm} \Gamma H_1, \text{norm}\), \(\tilde{v} = H_2 \Gamma v_1 + v_2\) and assuming \(R' = (\rho_2 H_2, \text{norm} \Gamma^\top H_2, \text{norm} + I_{2N_0})\) then after getting expectation over noise vectors:

\[
\begin{align*}
\Phi_{\Delta_{\text{relay}}(x, z)}(s) &= E_{H_1, H_2} \left[ e^{(-s + s^2)(x_i - z_i)^\top \tilde{H}_2, \text{norm} \Gamma^\top \tilde{H}_2, \text{norm}(x_i - z_i)} \right] \\
&= \Phi_{\Delta_0(x_i, z_i)}(s) \Phi_{\Delta_{\text{norm}}(x_i, z_i)}(s) \Phi_{\Delta_{\text{norm}}(y_i, x_i)}(s)
\end{align*}
\] (5-23)

Recall that \(\Gamma\) is the function of \(\rho_1, \rho_2, H_1, \text{norm}\) and \(H_2, \text{norm}\). By considering \(H_1 = \sqrt{\rho_1} H_1, \text{norm}\) and \(H_2 = \sqrt{\rho_2} H_2, \text{norm}\), for the relay link, it is necessary to do expectation over the channel matrices numerically. Then finally \(\Phi_{\Delta_0(x_i, z_i)}(s) = \Phi_{\Delta_{\text{norm}}(x_i, z_i)}(s) \Phi_{\Delta_{\text{norm}}(y_i, x_i)}(s)\) can be calculated numerically for the different values of \(s\).

### 5.3.3 Performance Result

The performance of our proposed flexible LDC-based AF scheme has been examined for different number of transmit/receive antennas as well as different SNR configurations. Due to the lack of space in this section the performance results are provided for a system with two transmit antennas at the source, two receive/transmit antennas at the relay and two receive antenna at the destination.
for different SNR configurations. Here an LDC code from [79] for 2x2 MIMO system is used. One tap complex independent and identically distributed (i.i.d) Rayleigh fading channel coefficients are multiplied with transmitted signals. Coherency time of the channel has been set to max\(T_1, T_2\). Figure 5-7 shows the average mutual information for different duplexing ratios for \(\rho_{\text{a,d}} = 4\text{dB}\) and \(\rho_{\text{a,f}} = 15\text{dB}\). As it is seen during \(\rho_{\text{a,d}}\) from 6 dB to 9 dB, the curve with \(\alpha = 8/10\) has the better average mutual information than the direct transmission (\(\alpha = 1\)) and \(\alpha = 5/10\). During \(\rho_{\text{a,f}}\) from 9 dB to 16 dB, the curve with \(\alpha = 6/10\) has better average mutual information. But for \(\rho_1 \geq 16\text{dB}\) the curve with \(\alpha = 5/10\) has the best average mutual information. Also the region that the average mutual information gain can be achieved compared with \(\alpha = 5/10\) and \(\alpha = 1\) has been highlighted. Here it is seen clearly that by choosing the optimum duplex ratio the average mutual information can be increased. Figure 5-8 shows the amount of the achieved SNR gain in \(\rho_{\text{a,d}}\) compared to \(\alpha = 1\) and \(\alpha = 5/10\) for \(\rho_{\text{a,d}} = 4\text{dB}\) and \(\rho_{\text{a,f}} = 15\text{dB}\) if a fixed average mutual information is considered. Near 3dB gain can be achieved at the transmission rate near 3bps/Hz. Figure 5-9 compares the average mutual information for \(\alpha = 1/2\) (\(T_1 = 2\) and \(T_2 = 2\)) and \(\alpha = 2/3\) (\(T_1 = 2\) and \(T_2 = 1\)) for \(\rho_{\text{a,d}} = 5\text{dB}\) and \(\rho_{\text{a,f}} = 15\text{dB}\). As it is seen during \(\rho_{\text{a,d}}\) from 8 dB to 16 dB, the curve with \(\alpha = 2/3\) has better average mutual information than \(\alpha = 1\) and \(\alpha = 1/2\). Let’s look at the BER performance results. For \(\alpha = 2/3\) at the source node, the information bit sequences are encoded with an optimum 1/2 rate systematic recursive convolutional code with generator polynomial (13, 15) in octal representation. Then the interleaved coded bits are QPSK modulated and LDC encoded and transmitted to other nodes. Coherency time of the channel has been set to max\(T_1, T_2\) = 2. For \(\alpha = 1/2\) in order to provide the BER result for the same total rate, appropriate puncturing is applied after the 1/2 rate convolutional code to convert the total coding rate to 2/3 which in turn converts the total rate of the AF scheme with \(\alpha = 1/2\) to a value equals to total rate of the AF scheme with \(\alpha = 2/3\). As it is seen in Figure 5-10 the scheme with \(\alpha = 2/3\) performs better than \(\alpha = 1/2\) in all considered SNRs for \(\rho_{\text{a,d}} = 5\text{dB}\). In Figure 5-11 the BER performance for \(\alpha = 1/2\) (\(T_1 = 2, T_2 = 2\)) is compared with \(\alpha = 2/3\) (\(T_1 = 2, T_2 = 1\)) in a SISO system. The coding and modulation schemes are same as above. As it is seen scheme with \(\alpha = 2/3\) is performing better than \(\alpha = 1/2\) in considered SNRs for \(\rho_{\text{a,f}} = 5\text{dB}\). Expurgated BER upper bounds for \(\alpha = 2/3\) have been provided using BER analysis done in section 5.3.2.
Figure 5-7. Average throughput comparison for different duplexing ratios for $\rho_0 = 4dB$ and $\rho_1 = 15dB$. The region that the average mutual information gain can be achieved compared with $\alpha = 5/10$ and $\alpha = 1$ has been highlighted by red colour.

Figure 5-8. Achieved SNR gain for flexible AF scheme compared to direct transmission $\alpha = 1$ and traditional AF $\alpha = 5/10$ at different rates when $\rho_0 = 4dB$ and $\rho_1 = 15dB$. This figure shows the region that the flexible AF scheme provides gain compared with $\alpha = 5/10$ and $\alpha = 1$. 
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Figure 5-9. Average throughput comparison for different duplexing ratios for $\rho_1 = 5dB$ and $\rho_2 = 15dB$. The region that the average mutual information gain can be achieved compared with $\alpha = 5/10$ and $\alpha = 1$ has been highlighted by red colour.

Figure 5-10. BER performance comparison for duplexing ratios $1/2$ and $2/3$ for $\rho_1 = 5dB$.

Figure 5-11. BER performance comparison for duplexing ratios $1/2$ and $2/3$ for $\rho_2 = 5dB$.
Figure 5-12 shows 3-D view of average mutual information for fixed \( \text{SNR}_2 = \rho_{2_{\text{dB}}} = 15 \text{dB} \) when the \( \text{SNR}_1 = \rho_{1_{\text{dB}}} \) and \( \text{SNR}_q = \rho_{q_{\text{dB}}} \) are varying. These curves are very useful in selecting the duplexing ratio in order to provide maximum average mutual information performance for each specific SNR region for proposed AF scheme.

![Figure 5-12. 3-D Average mutual information comparison for different duplexing ratios for \( \text{SNR}_2 = 15 \text{dB} \)](image)

In this section performance of the flexible AF was investigated when the transmit/receive CSI are both available at the relay but having the transmit CSI at the relay requires extra complexity. So it is valuable to look at performance of this system when only receive CSI is available at the relay.

## 5.4 Second Scenario: Amplify and Forward Relay Receive Knowledge Only Power Allocation Methods

In previous section the performance of the AF scheme was investigated when the transmit/receive CSI are available at the relay. However, above techniques required extra complexity. In this section, power allocation techniques which rely only on the use of receive CSI at the relay node are investigated. At first the existing AF schemes are extended to the flexible AF scheme. Then, a novel power allocation method is derived. In this power allocation method, it is assumed that all eigenmodes of the relay to destination link have the same quality, i.e., equal Eigen mode (EEM), as it is assumed in the AF EG scheme.
5.4.1 Traditional Amplify and Forward Power Allocation Methods

In the traditional equal gain AF scheme (AF EG), at the relay the received signal is multiplied by a scalar power factor $\beta$ and the result is transmitted to destination. This power factor is calculated based on the power constraint at the output of the relay. Recall that the total transmit power of the relay node is constrained as $\text{trace}(R_{y_j}) = T_2$ where $R_{y_j} = \frac{1}{2} \Gamma \left( \frac{1}{M} H_j R_j + I_{2R_j} \right) \Gamma^H = \Gamma (R_j) \Gamma^H$. According to the flexible MIMO AF system model, for the EG AF scheme $\Gamma = \beta I_{2R_j \times 2R_j}$ where $I_{2R_j \times 2R_j}$ is a rectangular identity matrix. Consequently, $\beta$ can be expressed as $\beta = \sqrt{\frac{T_2}{\text{trace}(JR_j)}}$ where for EG AF scheme $J = I_{2R_j \times 2R_j}$. Therefore, this approach does not need the explicit knowledge of $H_j$. It only needs knowledge about $R_j$. Let’s look at the AF EG power allocation policy in terms of eigenmodes of the source-relay channel. It is seen that this policy does not consider any difference between different eigenmodes of the source-relay channel. Also, the power allocation is performed by assuming that the eigenmodes of the relay-destination channel have the same quality. Therefore $\Gamma$ is designed such that the power at the relay is evenly distributed amongst the available eigen modes of the relay to destination link. That is why this approach is named as AF equal gain scheme.

In addition to equal gain method, recently in [9], several AF based power allocation methods for a basic MIMO scenario have been proposed. In their scenario the time slot lengths are equal i.e. $T_1 = T_2$, proposed. These methods include match filtering (MFR), zero forcing (ZFR) and minimum mean square error (MMSEF). In this section these methods are extended to the flexible AF scheme. In the extended methods the power allocation matrix at the relay is considered as $\Gamma = \beta J$, where $\beta$ is expressed as $\beta = \sqrt{\frac{T_2}{\text{trace}(JR_j)}}$, but for these methods, $J = VH_j^H$, $J = VH_j^H$ and $J = VH_j^H R_j^{-1}$, for MFR, ZFR and MMSEF respectively. Note that $H_j^H = \left( H_j^H H_j \right)^{-1} H_j^H$. The matrix $V$ is defined as $V = I_{R_j}$, $V = \left( V_{4R_j} \right)$ and $V = \left[ V_1, \ 0_{2R_j \times (2R_j-2Q_j)} \right]^H$, if $Q_j = RT_j$, $Q_j > RT_j$ and $Q_j < RT_j$ respectively, where $V_1$ is a
matrix that contains the $2Q_0$ right-singular vectors of $H$, $V_{i \in [2Q_0]}$ is the matrix that contains the first $2RT_z$ columns of $V_i$ and $0_{2Q_0 \times (2RT_z - 2Q_0)}$ a $2Q_0 \times (2RT_z - 2Q_0)$ matrix of zeros.

Let us look at these methods in terms of Eigen modes of the source-relay channel. It is observed that the MFR power policy at the relay tries to give more power to the stronger eigenmodes and less power to the weaker eigenmodes. In other word the MFR maximizes the output SNR of the relay. This technique promises to perform well when the relay and the destination are close to each other, i.e., $\rho_2 \gg \rho_1$. However, ZFR and MMSEF power policy at the relay tries to invert the power of each eigenmode in order to provide equal power for all the eigenmodes at the output of the relay. In other word both of these methods try to remove the inter symbol interference. When the source and the relay are very close i.e $\rho_1 \gg \rho_2$, ZFR and MMSEF have an equivalent power allocation matrix $\Gamma$ and therefore in both methods the transmit signal from the relay to the destination can be approximated as $\sqrt{\rho_2 / \rho_1}x$. Thus an early decision by decoding is taken at the relay on the transmit signal $x$ and then amplifies the decoded signal with $\sqrt{\rho_2 / \rho_1}$. As a result, it is expected that the ZFR and MMSEF perform better than equal gain and MFR when $\rho_1 \gg \rho_2$.

Obviously their performance is worst when $\rho_2 \gg \rho_1$. More detailed derivation of MMSE factor can be found in Appendix F-2.

### 5.4.2 Equal Eigen Mode Power Allocation Method

In the EEM power allocation method, the $\Gamma$ matrix is designed to maximize $I(y_2;x)$ by assuming that all Eigen modes of $H_2$ have the same quality, and by taking into account the quality difference of the source to relay link Eigen modes. This method is a modified version of the methods that were proposed in section 5-3 for the case where both the source-relay and relay-destination CSIs are known at the relay. As it was shown in section 5-3, the optimal power allocation is expressed as $\Gamma = V_2 \tilde{U}^H_2$. And also in section 5.3.4 as $\Gamma = V_2 \tilde{U}^H_2$. Then, the values of $\gamma_{2,k}$ that maximize (5-18) are obtained by solving an optimization problem. In this section, it is considered that $H_2$ is not available at the relay, hence, the problem in (5-18) cannot be directly solved. This optimization problem can be reformulated by assuming that all the eigenmodes of $H_2$ are equivalent. In other words, $\lambda_{2,k} = \rho_2$ in (5-18) for all $k = 1, ..., K$. By substituting $\lambda_{2,k} = \rho_2$ the optimization problem will be Maximizing:
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\[
\sum_{k=1}^{K} \log_2 \left( 1 + 2 \lambda_k \tilde{\gamma}_k \rho_k \right) - \log_2 \left( 1 + \tilde{\gamma}_k \rho_k \right)
\]  

(5-25)

subject to \( \sum_{k=1}^{K} (\tilde{\gamma}_k \lambda_k) \leq T_2 \).

The optimum solution for this modified problem is again obtained by solving its Karush-Kuhn-Tucker (KKT) conditions [85], such that

\[
\tilde{\gamma}_k = \max \left( 0, \frac{1}{4 \rho_k \lambda_k} \left[ \sqrt{(2 \lambda_k - 1)(8 \rho_k \mu + (2 \lambda_k - 1) - (2 \lambda_k + 1))} \right] \right)
\]  

(5-26)

where \( \mu \geq 0 \) is the Lagrange multiplier that needs to be adjusted in order to satisfy the power constraint in (5-25). The Lagrange method for solving this problem is similar to Appendix F.3. The starting value for \( \mu \) is \( \mu_{\text{init}} = \max_{k=1,K} \left\{ \frac{\lambda_k}{\rho_k (2 \lambda_k - 1)} \right\} \) then \( \mu \) is be updated by using the Newton-Raphson method [86] until \( \mu \) is obtained. The value of \( \mu \) must fulfill the inequality \( f(\mu^*) = \sum_{k=1}^{K} \gamma_k \lambda_k - T_2 < \varepsilon \), for \( \varepsilon \ll 1 \). Following flow diagram (Figure 5-13) shows the steps of the EEM algorithm.

Figure 5-13. Block diagram for the EEM power allocation algorithm

Since the EEM method only requires the knowledge of \( R_K \) at the relay, similar to equal gain AF method and it is implemented by using the Newton-Raphson method this approach has low
complexity. Thus, the EEM algorithm allows us to obtain quickly the power allocation matrix $\Gamma$ that maximizes (5-18) when $\lambda_{2,k} = \rho_2$ for all $k = 1, \ldots, K$. However, the eigenvalues $\lambda_{2,k} = \rho_2$ of $H_2$ are in general most unlikely to be all equal to one. Considering statistical knowledge about $H_2$, further improvement could be achieved. More detailed about this issue can be found at [87].

5.4.3 Numerical Result

The new power allocation method for our proposed LD C based AF scheme introduced in Section 5.4, has been compared against the EG, MFR and MMSEF methods, in terms of average mutual information and BER performance for various number of transmit/receive antennas as well as different SNR settings. In this section the performance results are provided for a system with two transmit antennas at source, two receive/transmit antennas at the relay and two receive antennas at destination for different SNR configurations. In the simulation result, $\text{SNR}_0$ is denoted as the SNR of the source to destination link, $\text{SNR}_1$ as the SNR of the source to relay link and $\text{SNR}_2$ as the SNR of the Relay to destination link. Furthermore, for the EEM method, $\epsilon = 10^{-5}$. In Figure 5-14, the cooperative mutual information performance of the various power allocation methods which described in Section 5.4, are compared for $M = R = N = 2$, $\alpha = 0.5$, $\text{SNR}_0=4\text{dB}$ and $\text{SNR}_2=15\text{dB}$. The EEM method performs as good as EG for low and medium values of $\text{SNR}_1$ and much better than EG for high $\text{SNR}_1$ values. The EEM method also outperforms MMSEF and MFR for any $\text{SNR}_1$ values.
Ergodic mutual Information for SNR\(_g\)-4dB and SNR\(_j\)-15dB

Figure 5-14. Cooperative mutual information performance of various power allocation methods for 
\( M = R = N = 2, \alpha = 0.5, \text{SNR}_g = 4\text{dB and SNR}_j = 15\text{dB} \)

In the BER performance evaluation, at the source, a bit interleaved coded modulation (BICM) model is used similar to section 5.3. In Figure 5-15 and Figure 5-16, the cooperative BER performance of the various power allocation methods described in Section 5.4 are presented as well as the FCSI method (section 5.3), for \( M = R = N = 2 \) and \( M = R = N = 4 \), respectively, and considering \( \alpha = 0.5, T_1 = T_2 = 1, \text{SNR}_1 = 10\text{dB and SNR}_2 = 15\text{dB} \). The graphs indicate that the EEM method outperforms all the other traditional techniques including EG, MFR and MMSEF, when only receive CSI is available at the relay. The MFR method shows the worst performance compared to the others. As a benchmark, the BER performance of the FCSI method where the Transmit/receive CSI are both available at the relay is also displayed. This result shows the performance improvement that the knowledge of the transmit CSI at the relay can bring to the system, but at the cost of higher complexity compared to the EEM method. Similar behaviour can be observed for \( M = R = N = 4 \), but with larger performance difference between the various power allocation techniques, especially, between the FCSI method and the receive CSI only techniques. In this SNR setting, since SNR\(_1\) and SNR\(_2\) are fairly high in comparison with SNR\(_g\), the performance of the direct communication is worse than the others, and consequently, the direct communication performance is not shown here.
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Perfor mance of cooperative AF for SNR$^1=10$dB and SNR$^2=15$dB

Figure 5-15. Cooperative BER performance of various power allocation methods for $M = R = N = 2$, $\alpha = 0.5$, $T_1 = T_2 = 1$, $SNR^1=10$dB and $SNR^2=15$dB

Figure 5-16. Cooperative BER performance of various power allocation methods for $M = R = N = 4$, $\alpha = 0.5$, $T_1 = T_2 = 1$, $SNR^1=10$dB and $SNR^2=15$dB

Figure 5-17 shows average mutual information for different duplexing ratios for $SNR^1=4$dB and $SNR^2=15$dB. As it is seen during $SNR^1$ from $6$dB to $11$dB, the curve with $\alpha = 8/10$ has better average throughput than direct transmission ($\alpha = 1$) and $\alpha = 5/10$. During $SNR^1$ from $11$dB to $22$dB, the curve with $\alpha = 6/10$ has better average throughput. But for $SNR^1 \geq 22$dB the curve with
\( \alpha = 5/10 \) has the best throughput. Here it is seen clearly that by choosing optimum duplex ratio the average throughput can be increased.

Figure 5-17. Average mutual information comparison for different duplexing ratios for \( \text{SNR}_1 = 4\, \text{dB} \) and \( \text{SNR}_2 = 15\, \text{dB} \)

Figure 5-18 compares the average mutual information for cases that transmit CSI available at the relay with cases that transmit CSI is not available for \( \text{SNR}_1 = 0\, \text{dB} \) and \( \text{SNR}_2 = 10\, \text{dB} \). The cases with transmit CSI always provide better performance than cases with only receive CSI. This figure shows that by increasing \( \text{SNR}_1 \), in order to get better average mutual information, the duplexing ratio \( \alpha \) needs to decrease toward half gradually. These curves are very helpful in selection of the duplexing ratio which provides maximum average mutual information for each specific SNR region for the proposed AF system.
5.5 Conclusion

In this chapter a novel method was proposed to increase average mutual information of cooperative MIMO AF scheme with choosing the optimum duplexing ratio. For each SNR region the optimum duplexing ratio is different. The processing techniques at the relay for this new AF scheme was presented for the case when the relay has knowledge about its input and output channel. It was shown that with applying this new flexible scheme and choosing the optimum duplexing ratio, much better average mutual information can be achieved as well as improvement in the BER performance in many SNR configurations. In order to reduce complexity of the relay process, the power allocation methods were explored for the case that only receive-CSI is available at the relay. Traditional power allocation methods were extended to flexible AF and one novel power allocation method (EEM method) for cooperative MIMO AF scheme was proposed. The EEM method which has a low complexity is based on the maximization of the mutual information of the relay link when only receive CSI is available at the relay. Numerical results presented in this chapter showed that this method outperform other common relay receive CSI only power allocation techniques in terms of mutual information and BER.
Chapter 6  Two Way Relay Channel

6.1 Introduction

Most of the cooperative relaying schemes have been proposed for one way relay channel (OWRC) scenario, where a single source node transmits data to a single destination node with the help of some relay nodes. As an example in the DF scheme the relay decodes the received signal from the source and encodes it appropriately before forwarding it to the destination.

In bidirectional scenarios, when two nodes want to transmit data to each other by receiving help from a relay, the relay forwarding for the first and second nodes should be carried out in the separate time or frequency bands if the one way relay channel (OWRC) approach is being used. A much more efficient approach which allows the relay to forward both messages simultaneously (same time and frequency band) has been proposed in [74]-[75]. The proposed approach employs network coding technique to enable each node to extract its desired message from the signal that the relay broadcasts. The network coding concept is mainly based on combining different traffics data at the intermediate nodes in an attempt to improve the overall reliability and achievable throughput of all the possible communications across the network. Thanks to the broadcast nature of the wireless medium and changing transmission and reception role of each node, a receiving node will have enough prior knowledge on some parts of the other traffics data and hence will be able to extract its own desired data out of the mixed received signal. This will improve the spectrum efficiency of the radio system as multiple transmissions towards separate receiving nodes will be replaced by one single transmission of a network coded signal [74]-[75]. In practical networks several important aspects have considerable effect on the performance of TWRC including the network topology in terms of the position of the nodes and the channel conditions of the constituting point to point links in form of the instantaneous signal to noise ratios (SNR) between the involved nodes and the relay. In order to improve performance of the network coding over TWRC system especially in asymmetric conditions two schemes are proposed in this chapter including a selective network coding (SNC) protocol and clipping and network coding (CNC) [96].

This chapter is organised as follows. Section 6.2 describes the system model for TWRC concept, exploiting network coding in wireless cooperation and structure of the joint network and channel
decoder. Section 6.3 discusses in detail the proposed selective network coding protocol over TWRC system and the performance of the new selective network coding scheme is compared with the traditional non-selective network coding scheme for different symmetric and asymmetric topology conditions. Then section 6.4 describes in detail the proposed CNC scheme over TWRC system and the performance of the proposed CNC scheme is compared with the traditional non-selective and SNC scheme.

6.2 System Model

6.2.1 Two Way Relay Channel

A basic TDD/TDMA TWRC scenario with two source nodes and only one relay has been shown in Figure 6-1. Different orthogonal channels such as different frequency bands or spreading codes also can be used for TWRC system. But in the following sections, it is supposed that each node transmits at separate channel time slot and no simultaneous transmission is allowed during one time slot. The first node (S1) transmits its message to the relay (R) and the second node in the channel time slot 1 (solid line), then the second node (S2) sends its message to the relay and the first node during the second channel time slot (dashed line). The relay combines the received information of the first node and the second node through a network encoder and sends back to the first and second nodes in the third channel time slot (dotted line). The channel time slots 1, 2 and 3 are orthogonal in time. The transmit (TX) and receive (RX) status of each of the involved nodes in different time slots has been tabularised in Figure 6-1.

![Figure 6-1. Basic two way relay channel model](image)

As it is seen, instead of allocating two separate time slot for communications from the relay to the S1 and S2, only one time slot is used and so one time slot will be saved. This is done by utilizing
the network coding concept over TWRC system. In all following sections the average SNRs for the S1-S2 link (direct link), S1-R link and S2-R link have been shown by $\rho_{12} - \rho_{1R}$ and $\rho_{2R}$ respectively. The SNR offset of the S1-R link compared to the direct link has been shown by $\Delta_{1R}$ and the SNR offset of the S2-R link compared to the direct link by $\Delta_{2R}$. It means $\rho_{1R} = \rho_{12} + \Delta_{1R}$ and $\rho_{2R} = \rho_{12} + \Delta_{2R}$. Here it is assumed that $\rho_{21} = \rho_{12}$, $\rho_{11} = \rho_{1R}$ and $\rho_{22} = \rho_{2R}$.

### 6.2.2 Iterative Joint Network and Channel Decoder Structure

Let assume that S1 wants to send a bit sequence $u^{(1)} = (u_1^{(1)}, u_2^{(1)}, \ldots, u_m^{(1)})$ to S2 and S2 also wants to send a bit sequence $u^{(2)} = (u_1^{(2)}, u_2^{(2)}, \ldots, u_m^{(2)})$ to S1. The information bits $u^{(1)}$ and $u^{(2)}$ are encoded with channel codes $C_1$ and $C_2$ resulting in the coded sequences $c^{(1)} = (c_1^{(1)}, c_2^{(1)}, \ldots, c_m^{(1)}) = C_1(u^{(1)})$ and $c^{(2)} = (c_1^{(2)}, c_2^{(2)}, \ldots, c_m^{(2)}) = C_2(u^{(2)})$, respectively. These coded sequences then appropriately interleaved, modulated with the suitable constellation and transmitted to the other nodes as it is shown in Figure 6-2.a and Figure 6-2.b. Each modulated component belongs to finite signal alphabet $\Psi_1 \subset \mathbb{C}$ and $\Psi_2 \subset \mathbb{C}$ for S1 and S2 respectively ($\mathbb{C}$ represents the complex plane). The relay at the first and second time slots receives signal sequences $y^{(1,R)} = (y_1^{(1,R)}, y_2^{(1,R)}, \ldots, y_m^{(1,R)})$ and $y^{(2,R)} = (y_1^{(2,R)}, y_2^{(2,R)}, \ldots, y_m^{(2,R)})$ from S1 and S2 through vector channels $p(y^{(1,R)} | C_1(u^{(1)}))$ and $p(y^{(2,R)} | C_2(u^{(2)}))$, respectively. Then the relay decodes the received signals that results in decoded bit sequences $\hat{u}^{(1,R)} = (\hat{u}_1^{(1,R)}, \hat{u}_2^{(1,R)}, \ldots, \hat{u}_m^{(1,R)})$ and $\hat{u}^{(2,R)} = (\hat{u}_1^{(2,R)}, \hat{u}_2^{(2,R)}, \ldots, \hat{u}_m^{(2,R)})$. Here it is assumed that $\hat{u}^{(1,R)}$ and $\hat{u}^{(2,R)}$ contain only hard bit values 0 and 1. The relay employs a network coding method to generate the coded data $u^R = NC(\hat{u}^{(1,R)}, \hat{u}^{(2,R)})$ from the decoded data ($\hat{u}^{(1,R)}$ and $\hat{u}^{(2,R)}$) received from S1 and S2 and then after appropriate interleaving and modulation transmits the network coded signal towards the both S1 and S2. The general structure of the relay has been shown in Figure 6-2.c. Each modulated component belongs to finite signal alphabet $\Psi_x \subset \mathbb{C}$. $\Psi_1$, $\Psi_2$ and $\Psi_x$ are not necessarily identical. Signal alphabets $\Psi_j, j=1,2,R$ have associated one-to-one binary labelling maps $\mu_j: [0,1]^n \rightarrow \Psi_j$ where $n_j = \log_2 |\Psi_j|$ and modulated symbols are subject to power constraints related to S1, S2 and R. Frequency-flat fading channels are assumed between any pair of the transmitting-receiving nodes. All receivers at the relay, S1 and S2 are assumed to have
perfect channel knowledge on their corresponding connected channels. No channel knowledge is assumed to be available at corresponding transmitters.

Figure 6-2. General structure of the baseband transmitter and receiver of different nodes in TWRC system (a) SI Transmitter (b) S2 Transmitter (c) Relay Receiver and Transmitter (d) S2 Receiver (e) S1 Receiver

In general a hybrid composition of the channel and network codes will form a distributed turbo network-channel code. As each of the S1 and S2 has knowledge on its own transmitted data, the node’s receiver will be able to extract it’s desire data. Each receiving node uses an appropriately modified turbo receiver to iteratively decode and extract its own desired data. General structure of an iterative joint network and channel decoding has been shown in Figure 6-2. (d) and Figure 6-2. (e). The S2 performs the joint iterative network and channel decoding in order to extract $\hat{u}^{(1)}$ from the received signal.
the direct signal \( y^{(1,2)} = (y_1^{(1,2)}, y_2^{(1,2)}, \ldots, y_6^{(1,2)}) \) and relayed signal \( y^{(R,2)} = (y_1^{(R,2)}, y_2^{(R,2)}, \ldots, y_6^{(R,2)}) \) received through the vector channels \( p(y^{(1,2)} | C_1(u^{(1)})) \) and \( p(y^{(R,2)} | NC(u^{(1,R)}), u^{(2,R)}) \), respectively. Recall that the S2 has already full information about its own data \( u^{(2)} \). It means that \( \hat{u}^{(1)} = f(y^{(1,2)}, y^{(R,2)}, u^{(2)}) \), where iterative decoding process has been represented by \( f(\cdot) \). Similar to the S2, the joint network and channel decoding can be done at the S1 to extract \( \hat{u}^{(2)} \) from the directly received signal \( y^{(2,1)} \) and the indirect signal \( y^{(R,2)} \). It means that \( \hat{u}^{(2)} = f(y^{(2,1)}, y^{(R,1)}, u^{(1)}) \).

6.3 Selective Network Coding

In symmetric cases, the relay receives data from all the source nodes with similar reliability. Under reliable source-to-relay link conditions, combing these data through the network coding and forwarding to the other nodes will improve all the communications and the respective throughputs. But in realistic scenarios, generally the system faces asymmetric conditions, because of the asymmetric position of the relay and the source nodes, different channel conditions, and different instantaneous SNR of the source-to-relay links. The relay combines data received with different reliability through the network encoder and broadcasts it to the other nodes. Since this network coded sequence does not have good reliability any more, it could not help the iterative joint network and channel decoder, and even it may corrupt final decoding performance. Let us consider again a TWRC system with only two source nodes and one relay node. In symmetric cases, the distance between the relay and both source nodes is considered to be equal and it is assumed that the S1-R link (between the S1 and the relay) and the S2-R link (between the S2 and the relay) have similar instantaneous SNR and hence similar instantaneous BER performances at each time slot. Since the received data from both links provides similar reliability, combing these data through the network code at the relay will help both source nodes to highly improve their final performances as well as their throughputs.

In asymmetric cases, including asymmetric position of the relay where distances between the relay and the nodes S1 and S2 are not equal or having different static and dynamic fading channel conditions or different instantaneous SNR for the links S1-R and S2-R. For example, let's consider the relay is much closer to the S2 and therefore the link between the relay and far node (S1) experiences much lower instantaneous SNR than the closer node (S2). In this case, reliability of the data received by the relay from the S1 will be much lower than reliability of the data received from the S2. The relay combines these low and high reliable data by the network coding.
and transmits the resulted sequence to the both S1 and S2. Since this network coded sequence does not have good reliability, not only it could not help the iterative joint network and channel decoder, but also it may corrupt its final performance. To combat with this problem, it is necessary to avoid combing non reliable and reliable data at the relay. The relay can combine data from different nodes if they, all, have been received through the reliable channel. It means that the relay should cooperate only if it successfully decodes the message. Based on this scheme a selective network coding protocol over TWRC system is introduced. In this protocol, the relay decides whether or not to do cooperation for one or both pairs of nodes based upon its received signal condition from the both links (S1-R and S2-R) separately. The relay only does network coding when it receives the data from both nodes successfully. If one of the transmissions from the nodes to the relay fails, then the relay will only encode and forward the data that has been received successfully.

### 6.3.1 Selective Protocol

Reliability of data at the relay can be checked by using CRC check over the received data from S1 and S2 separately. In this selective protocol, the relay takes four different actions ($\zeta$) based on the quality of S1-R and S2-R links separately as follows.

**Action 1: Silence ($\zeta = 1$)**

If CRC of the both decoded data $\hat{u}^{(1, R)}$ and $\hat{u}^{(2, R)}$ fails then the relay node will transmit nothing. The final decoding of the data at the receiver of the both nodes will be based only on the directly received sequences and the channel codes $C_1$ and $C_2$, respectively. It means that: $\hat{u}^{(1)} = f(y^{(1, 2)})$ and $\hat{u}^{(2)} = f(y^{(2, 2)})$.

**Action 2: Decode and Forward (DF) ($\zeta = 2$)**

If only CRC of the decoded data from S2 succeeds, then the relay node will encode only the decoded data from the S2 ($u^R = \hat{u}^{(2, R)}$) through the code $C_u$ and forward it toward S1. The final decoding of data at the receiver of S2 will be based only on the directly received sequence and channel code $C_1$, but at the receiver of the S1, the distributed turbo decoding is made on the directly received and the relayed sequences by taking into account the channel code $C_2$ and $C_u$. It means that: $\hat{u}^{(1)} = f(y^{(1, 2)})$ and $\hat{u}^{(2)} = f(y^{(R, I)}, y^{(R, I)}))$.

**Action 3: DF ($\zeta = 3$)**
If only CRC of the decoded data from S2 fails, then the relay node will encode only the decoded data from the S1 (\( u^R = \tilde{u}^{(1,R)} \)) through the code \( C_R \) and forward it toward the S2. The final decoding of the data at the receiver of the S1 will be based only on the directly received sequence and the channel code \( C_2 \) \( (\tilde{u}^{(2)} = f(y^{(2,2)})) \) but at the receiver of the S2, the distributed turbo decoding is made on the directly received and relayed sequences by considering channel code \( C_1 \) and \( C_R \). It means that: \( \tilde{u}^{(1)} = f(y^{(1,2),y^{(0,2)}}) \) and \( \tilde{u}^{(2)} = f(y^{(2,1)}) \).

Action 4: NC (\( \xi = 4 \))

If CRC of the both decoded data \( \tilde{u}^{(1,R)} \) and \( \tilde{u}^{(2,R)} \) succeeds then the relay node will encode the decoded data from the both nodes through the network code NC, \( u^R = NC(\tilde{u}^{(1,R)}, \tilde{u}^{(2,R)}) \) and forward it toward the both nodes. At the receiver of the both nodes, the iterative joint network and channel decoding is made on the directly received and the indirect sequences by considering the channel code \( C_1 \) or \( C_2 \) and the network code NC. It means that: \( \tilde{u}^{(1)} = f(y^{(1,2),y^{(0,2)}, u^{(2)}}) \) and \( \tilde{u}^{(2)} = f(y^{(2,1),y^{(1,1)}, u^{(1)}}) \).

### 6.3.2 Performance Result

In this section the performance of the selective and non-selective network coding over TWRC system is compared. The performance improvement of this new scheme has been studied in many different scenarios. At both S1 and S2, the information bit sequences are encoded with a 1/2 rate systematic recursive convolutional code with generator polynomial \( (13, 15) \) in octal representation. Then the interleaved coded bits are QPSK modulated and transmitted to other nodes. At the relay the hard decoding of the data from both nodes is done separately. It employs same convolutional code as the network code to generate the network coded data from the hard decoded sequence received from other two nodes following appropriate interleaving. Then only the parity bits part of the network coded sequence are interleaved and modulated with QPSK constellation and are sent to the both nodes S1 and S2. In the DF case, the decoded bits from the desire node are encoded with the same convolutional code and then the interleaved coded bits are QPSK modulated and transmitted to another node. The number of iterations needed for the iterative joint network and channel decoding has been set to ten. One tap complex Gaussian Rayleigh fading channel coefficients are multiplied with transmitted symbols during simulation. It is assumed that channel coefficients of the each individual links are kept constant during each time slot (block fading channel or static channel) and the channel coefficients for all the links are independently generated.
Figure 6-3.a compares the final BER performance of the S1 to S2 transmission for the selective and non selective schemes, different asymmetric configurations, and block fading condition. Here \( \Delta_{2R} \) has been considered equal to 5dB in all cases and \( \Delta_{1R} \) is changing from -5 dB to 5 dB. As a result the quality of the S2-R link is much better than other links. The BER performance curves of the selective and traditional non-selective schemes have been distinguished by SNC and NC at the legend of the BER figure, respectively. As it is seen, when \( \Delta_{1R} \) is equal to -5 dB, the BER performance of the non selective (network coding only) is even worse than the performance of the direct link. In this case, the quality of the S1-R link is very low. As a result the network coding produces non reliable data by combining the high reliable data received from the S2 with the non reliable data from S1. This new non reliable network coded data is sent to S2 through a very good quality link (recall that \( \Delta_{2R} =5 \) dB). The S2 wrongly regards the relayed data as more reliable than the data received from direct transmission and employs it in the iterative joint network and channel decoder. By this fault, the iterative network and channel decoding instead of improving the link performance, corrupts direct received data and demonstrates a performance much worse than the direct link.

![Figure 6-3. Performance of the selective TWRC (S1 to S2 cooperative transmissions) in block fading channel (a) BER performance (b) Time percentage for different relay actions](image)

Now by looking at the BER performance of the selective scheme (SNC), it is observed that with changing the relay action appropriately, above corruption in the performance of the network coding scheme is avoided. For \( \Delta_{1R} =-5 \) dB, the BER performance of the selective scheme is much better than the direct link BER performance and much better than the non selective Network coding. For example, at \( BER =10^{-3} \) the selective network coding shows around 10 dB gain
compared to the direct link BER performance. By increasing $\Delta_{1R}$, the BER performance of the non selective scheme is getting better than the direct link. But by applying the selective network coding scheme and choosing appropriate action for the relay this improvement will be much higher. As it is seen by increasing $\Delta_{1R}$, the BER performance of the selective scheme is enhanced. However at the high direct link SNR, $\rho_{12}$, the BER performance of the selective scheme with different $\Delta_{1R}$ are merging together. The final BER performance of the cooperative transmission from the S2 to the S1 shows similar trend even with better BER performance. Figure 6-3.b compares the percentage of time allocated to each action of the relay ($\zeta = 1, ..., 4$) during simulation period considering transmission from the relay node to the second node. When $\zeta = 1$ or $\zeta = 2$, the relay sends nothing to the S2. When $\zeta = 3$ the relay node only sends data received from the S1 to the S2 and when $\zeta = 4$ the relay transmits the network coded data to the both nodes.

![Figure 6-4. performance of the selective TWRC (SI to S2 cooperative transmissions) in AWGN channel (a) BER performance (b) Time percentage for different relay actions](image)

As it is seen with increasing $\rho_{12}$ number of channel time slots which the relay does not transmit to S2 is decreasing and more network coding action is expected at the relay. Increasing $\Delta_{1R}$ from -5dB to 0 dB has similar outcome. Since the quality of the S1-R and S2-R links are very good at the higher SNR, all the data will be network coded and hence the performance of the selective scheme converges toward the non selective (network coding only) performance. For comparison, final BER performance of the S1 to S2 communication for the selective and non selective schemes in AWGN channel has been shown in Figure 6-4.a. Also the percentage of time allocated
to each action of the relay \((\zeta = 1,\ldots, 4)\) during simulation period has been plotted in Figure 6-4.b. In this figure \(\Delta_{2R}\) has been considered equal to 2dB in all cases and \(\Delta_{1R}\) is changing from -2 dB to 2 dB.

![Performance of Selective Network Coding over TWRC](image)

**Figure 6-5.** Performance of the selective network coding over TWRC (S1 to S2 cooperative transmissions) in i.i.d Rayleigh Fading (a) Percentage for different relay actions (b) BER performance versus direct link SNR \((\rho_{12})\) (c) BER performance versus offset between S1 and relay \((\Delta_{1R})\)

Now, performance of the selective and non-selective network coding over TWRC system is compared in independent and identically distributed (i.i.d) Rayleigh fading channel. The number of iterations needed for the iterative joint network and channel decoding has been set to five. Before adding White Gaussian noise (AWGN), one tap complex i.i.d Rayleigh fading channel
coefficients are multiplied with transmitted signals during simulation. It is assumed that channel coefficients of the each individual links are independently generated. Here $\Delta_{2R}$ has been considered equal to 5dB in all cases and $\Delta_{1R}$ is changing from -5 dB to 5 dB. As a result the quality of the S2-R link is much better than other links.

Figure 6-5.a compares a percentage of time allocated to each action of the relay ($\zeta = 1, ..., 4$) during simulation period considering transmission from the relay node to the second node. As it is seen by increasing $\rho_{12}$, number of channel time slots which the relay does not transmit to the S2 is decreasing and more network coding action is expected at the relay. Increasing $\Delta_{1R}$ from -5dB to 0 dB has similar outcome. The percentage of action $\zeta = 3$ is very low compare to the other relay actions. Since the quality of the S1-R and S2-R links are very good at the higher SNR, all the data will be network coded.

In Figure 6-5.b when $\Delta_{1R}$ is -5 dB, the quality of the S1-R link is very low. As a result the network coding produces non reliable data by combining the high reliable data received from the S2 with the non reliable data from the S1. This new non reliable network coded data is sent to the S2 through a very good quality link (recall that $\Delta_{2R} = 5$ dB). The S2 wrongly regards the relayed data as more reliable than the data received from direct transmission and employs it in the iterative joint network and channel decoder. As it is seen the BER performance of the non selective scheme is worse than the performance of the direct link. Above corruption in the performance of the network coding scheme is avoided by changing the relay action appropriately through the selective scheme (SNC). It is observed that for $\Delta_{1R} = -5$ dB, the BER performance of the selective scheme is slightly better than the direct link BER performance and much better than the traditional non selective Network coding.

As it is seen in Figure 6-5.c, the BER performance of the non selective scheme is getting better than the direct link by increasing $\Delta_{1R}$. However by choosing appropriate action for the relay this enhancement will be much higher. By increasing $\Delta_{1R}$, more improvement is seen on the performance of the selective scheme. At the high direct link SNR $\rho_{12}$, since the performance of all individual links will be good, the BER performance of the selective scheme with different $\Delta_{1R}$ are merging together. The final BER performance of the cooperative transmission from the S2 to the S1 shows similar trend even with better BER performance.
6.4 Enhanced and Robust Network Coding Scheme for Asymmetric Conditions

As you may have seen in previous section in asymmetric conditions, combing data thorough the network coding could not help the iterative joint network and channel decoder, and even it may corrupt final decoding performance with forwarded erroneous data from the relay. In this section a new approach is proposed in order to improve performance of network coding scheme. The enhanced network coded scheme is called CNC where the letters ‘C’ stands for the clipping of Reliability of the relay. The clipping of bit reliabilities at the relay for decode and forward in one way relay channel has been proposed in [76]. Here this method is extended to network coding scheme in two way relay channel and then necessary modifications and also differences of the clipping in TWRC with clipping in OWRC (DF) are explained. The performance of this scheme will be later on compared with selective network coding (SNC) protocol over TWRC system.

6.4.1 New Structure for Iterative Joint Network and Channel Decoder with Clipping Relay Reliability

In order to explain our new proposed scheme an equivalent BSC model of a basic TWRC system consist of two sources $S_1$ and $S_2$ and the relay is adopted as it is shown in Figure 6-6. Here the link from the encoder of $S_1$ and the decoder of the relay is considered as a binary symmetric
channel $BSC(q_1)$ with bit error probability of $q_1$. Similarly the link between S2 and the relay has been shown with $BSC(q_2)$ and bit error probability of $q_2$. The reliability of each BSC channel is as follows:

$$l_{q,j} = \log((1-q_j)/q_j), j=1,2$$  \hspace{1cm} (6-1)

As it is seen from equivalent BSC model, the decoded and then re-encoded bits at the relay are not perfectly reliable. Obviously their network coding and forwarding from the relay to sources S1 and S2 cannot increase their reliability, thus it can be concluded that the soft values calculated at S1 based on error free data of the relay should be clipped with $\pm l_{q,2}$ levels and at S2 with $\pm l_{q,1}$ levels. Clipping function over input argument $\lambda$ with levels $\pm l_{q,j}$ is made as follows:

$$clip(\lambda, l_{q,j}) = \begin{cases} l_{q,j}, & l_{q,j} \leq \lambda \\ \lambda, & -l_{q,j} \leq \lambda \leq l_{q,j} \\ -l_{q,j}, & \lambda \leq -l_{q,j} \end{cases}$$  \hspace{1cm} (6-2)

To be able to use the CNC scheme, the relay has to estimate $l_{q,1}$ and $l_{q,2}$ and send it to S2 and S1 respectively. The signaling rate depends on the channel variation and at most it needs to be signaled every transmitted frame. In traditional detection scheme for TWRC system when network coding is done at the relay, S2 detector has knowledge on its own transmitted data and uses this knowledge with indefinite reliability to extract its desired data through iterative joint channel and network decoding. In new scheme, considering equivalent BSC channel model, this obvious data also needs to be limited if they are utilized in iterative scheme. Similar method needs to be done at S1 detector. Figure 6-7 shows new detector structure for S2. Extrinsic information LLRs after SISO channel decoder and also at output of network decoder should be clipped with $\pm l_{q,1}$ levels. S2 own bipolar data $u^{(2)}$ should also be multiplied with $l_{q,2}$ levels.
6.4.2 Relay Reliability Estimation

To calculate reliability levels it is required to estimate the error rate of the bits decoded at the relay. Here using the estimation techniques proposed in [76] is recommended. In proposed scheme, BER of decoded bits has been estimated from LLR values by using Gaussian approximation (GA) model assuming that the relay decoder is able to provide the soft-decoded values.

6.4.3 Performance Comparison

In this sub-section performance of the new CNC is compared with traditional network coding scheme as well as selective network coding (SNC) over TWRC system. The setting of transmitter at S1 and S2 and relay are similar to section 6.3.2. The number of iterations needed for the iterative joint network and channel decoding has been set to ten. Before adding White Gaussian noise (AWGN), one tap complex static (block) Rayleigh fading channel coefficients are multiplied with transmitted signals during simulation. It is assumed that channel coefficients of the each individual links are independently generated and are constant during each time slot. Here $\Delta_{2R}$ has been considered equal to 5dB in all cases and $\Delta_{1R}$ is changing from -5 dB to 5 dB. In Figure 6-8 the BER performance curves of CNC and traditional schemes have been distinguished by CNC and NC at the legend of all related figures, respectively. When $\Delta_{1R}$ is -5 dB, the quality of the S1-R link is very low. As a result in traditional scheme, the network coding produces non reliable data by combining the high reliable data received from S2 with the non reliable data from S1. This non reliable network coded data is sent to S2 through a very good quality link (recall that $\Delta_{2R} = 5$ dB). S2 wrongly regards the relayed data as more reliable than the data received from
direct transmission and employs it in the iterative joint network and channel decoder. As it is seen the BER performance of the traditional scheme is worse than the performance of the direct link. Above corruption in the performance of the network coding scheme is avoided appropriately through the clipping scheme (CNC). It is observed that for $\Delta_{1R} = -5$ dB, the BER performance of the CNC scheme is much better than the direct link BER performance and the traditional Network coding. As it is seen, the BER performance of the traditional scheme is getting better than the direct link by increasing $\Delta_{1R}$. However by choosing new clipping scheme this enhancement will be much higher and with higher diversity. By increasing $\Delta_{1R}$, more improvement is seen on the performance of the CNC scheme. The final BER performance of the cooperative transmission from S2 to S1 shows similar trend even with better BER performance. Figure 6-9 shows the performance of CNC scheme for different direct SNR values $\rho_{12}$ versus iteration number and $\Delta_{1R}$. As it is seen by increasing iteration number or $\Delta_{1R}$ performance of CNC scheme improves. Most of the improvement in BER performance is achieved after first iteration.

![Figure 6-8. BER performance of the CNC over TWRC in Block Fading channel (S1 to S2 cooperative transmissions) versus direct link SNR ($\rho_{12}$)](image)

performance of the CNC scheme. The final BER performance of the cooperative transmission from S2 to S1 shows similar trend even with better BER performance. Figure 6-9 shows the performance of CNC scheme for different direct SNR values $\rho_{12}$ versus iteration number and $\Delta_{1R}$. As it is seen by increasing iteration number or $\Delta_{1R}$ performance of CNC scheme improves. Most of the improvement in BER performance is achieved after first iteration.
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Figure 6-9. BER performance of the CNC over TWRC (SI to S2 cooperative transmissions) versus offset $\Delta_{1R}$ and iteration

Figure 6-10. compares performance of CNC scheme with SNC scheme. It is observed that performance of both scheme are much better than traditional scheme. However in the most of the cases CNC scheme shows better performance compared with selective scheme.

Figure 6-10. BER performance of the CNC and SNC over TWRC system in block fading (SI to S2 cooperative transmissions) versus direct link SNR ($\rho_{12}$)

6.5 Conclusion

In this chapter two new enhancement schemes to improve performance of two way relay channel were proposed. It was shown that although the network coding concept at the relay over TWRC
provides good performance in symmetric conditions where the reliability of the received data at the relay from all the source nodes are similar, but the performance of this scheme in asymmetric conditions is not perfect due to the combining data with different reliability at the relay. In order to avoid combining non reliable and reliable data at the relay, a selective network coding protocol over TWRC system was used in section 6.3. Based on this protocol the relay can cooperate only if successfully decodes the message from one of the nodes and it only does network coding when it receives data from all the source nodes successfully. The performance of this selective scheme was compared with the traditional network coding over TWRC in many different asymmetric cases and it was shown that by taking appropriate action at the relay, significant improvement can be achieved compared to the traditional non selective network coding over TWRC system. In section 6.4 another possible approach, i.e. CNC, was introduced to enhance the original network coding scheme over TWRC. This approach is based on signalling some quality parameters from the relay back to the source nodes. The signalling overhead will be at most one scalar value per transmitted frame of data per source. Under low mobility conditions the overhead can be further reduced. The quality parameter is used at the source to clip the bit reliabilities obtained from indirect route and hence will avoid fictitious domination of the indirect link. The proposed approach is applicable for any combinations of the constellations deployed at the source and the relay. As a result better spectrum efficiency can be achieved by proper selection of the constellations for all three phases (time slot) transmissions. This scheme performs quite successfully in block fading channels.
Chapter 7  Conclusion and Future Directions

7.1 Conclusive Summary

In a half duplex relaying system a time/frequency domain based duplexing is essential to avoid simultaneous reception/transmission by the relay node. For a time domain based duplexing, in a traditional AF scheme, in contrast with DF scheme, the time slot duration of the relay receive and relay transmit phases are the same and the duplexing ratio forces to be fixed at 0.5. This is mainly due to the assumed amplification function that has no affect on the duration of the forwarded signal. This clearly is a disadvantage for the traditional AF scheme as depending on the quality of the source-relay and relay-destination links the duration of one of the two time slots will be unnecessarily long. In this direction, this thesis investigated a flexible AF scheme that allows separately tuneable time slots and thus flexibility in adjustment of the duplexing ratio. This as a result will give us an extra degree of freedom in optimization of the AF scheme.

In order to lay the basis for the analysis of this flexible scheme, chapter 2, reviewed the state of the art techniques for the AF relay scheme in brief and introduced some preliminary definitions, and the system model for the cooperative relaying protocols. A closed form approximate for the outage probability for the traditional AF was shown. It was shown that the direct only transmission and the traditional AF scheme are very efficient in some SNR regions and are less efficient in other SNR regions.

In chapter 3, a flexible scheme with variable duplexing ratio was proposed to improve the mutual information and the throughput of the SISO AF scheme. In this direction, the achievable rate and the outage probability was analysed to optimise the performance of the proposed scheme. The mathematical models for the instantaneous mutual information and average mutual information were derived and employed to find the optimum duplexing ratio in different network topologies and SNR conditions. By looking at the instantaneous mutual information, it was found that when instantaneous SNR of direct link is much lower than SNR of first link, the optimum duplexing ratio moves towards the values less than 0.5. Also this behaviour could happen when the ratio of the source-relay SNR to relay-destination SNR increases. Numerical results for the achievable rate showed that the optimum duplexing ratio less than half happens when the average SNR of the direct link is very low, the average SNR of the second link is less than zero and the average SNR
range of the first link is between 20 and 30dB. Notice that this observation is valid for the single antenna single relay scenario. By increasing the average SNR of the direct link this SNR region becomes smaller and smaller. By looking at the outage probability, numerical results showed similar trends as it was seen for the instantaneous mutual information and the achievable rate; the optimum duplexing ratio can be less than half in some SNR regions. However the behaviour of the throughput for high average SNR of the direct link was completely different. In this case when the SNR offsets of the first and second links are high the optimum duplexing ratio is between 0.5 and 1. To further explore this issue, a tight lower bound approximation for the outage probability was derived. It was shown that at high SNRs, the outage probability for the optimum duplexing ratio is always lower than the outage probability for half duplexing ratio. The outage analysis enable user to predict the system performance and adjust its parameters for any combination of the SNR of the different links. Also it was found that the slope of the outage probability curves in logarithmic scale is one at very low SNR offsets. By increasing the SNR gradually the diversity increases from one towards two. At very high SNRs diversity will be two completely. The position of turning points depends on the network topology (average SNR).

Chapter 4 considered a typical HARQ scheme for a cooperative flexible AF scheme with single relay in order to improve the throughput of the cooperative scheme. In this regards, the outage and throughput performance measures were analysed for both repetition and unconstrained coding. The presented analysis allows accurate evaluation of the flexible AF based cooperative HARQ protocols without need to time consuming Monte Carlo based evaluation approaches. This will allow identification of the protocol throughput gain over the direct communication and appoint right protocol at right condition. Considerable performance gains were observed for the considered simulation scenarios thanks to the flexible AF combined with properly designed HARQ protocols.

In chapter 5 a flexible MIMO AF method was proposed to increase average mutual information of the cooperative AF scheme with choosing the optimum duplexing ratio. The processing techniques at the relay for this new AF scheme was presented for the case when the relay has knowledge about its input and output channel. It was shown that with applying this new flexible scheme and choosing the optimum duplexing ratio, much better average mutual information can be achieved as well as improvement in BER performance in many SNR configurations. In order to reduce extra complexity, cost and bandwidth of the above technique the power allocation problem was explored when only receive CSI is available at the relay. Traditional power allocation methods were extended to the flexible AF scheme and a novel power allocation method for non-regenerative cooperative MIMO communication was proposed. This method which has low complexity is based on the maximization of the mutual information of the relay link.
Numerical results were shown that this method outperforms the other common relay receive CSI only power allocation techniques in terms of the mutual information and the BER.

Chapter 6 proposed two enhancement schemes to improve the performance of two way relay channel (TWRC). It was shown that although the network coding concept at the relay over TWRC provides a good performance in symmetric conditions where the reliability of the received data at the relay from all the source nodes are similar, but the performance of this scheme in asymmetric conditions is degraded due to the combining data with different reliabilities at the relay. In order to avoid combining non reliable and reliable data at the relay, a selective network coding protocol over TWRC system was proposed. Based on this protocol the relay can cooperate only if successfully decodes the message from one of the nodes and it only does network coding when it receives data from all the source nodes successfully. It was shown that by applying suitable action at the relay, significant improvement can be achieved compared to the traditional non selective network coding over TWRC. Also another possible approach, i.e. CNC, was introduced to enhance the original network coding scheme over TWRC. This approach is based on signalling some quality parameters from the relay back to the source nodes. The signalling overhead will be at most one scalar value per transmitted data frame per source. The proposed approach is applicable for any combinations of the constellations employed in the source and relay nodes and thus allows better spectrum utilization by proper selection of the constellations for all the three phases of the two way transmissions. These schemes performs quite successfully in block fading channels.

It is worth noting that a number of issues still need further investigation. These are left for future work, which are discussed in the next section.

### 7.2 Future Work

In this thesis, most of the performance analysis was based on the orthogonal protocol where the source does not transmit during the second phase. Although the complexity of this protocol is much lower than non-orthogonal protocol, some performance loss is expected. All the techniques could be extended to the non orthogonal protocols where the source is allowed to transmit in the second phase. Several cases can be considered including the source transmits the same or a new signal at the second phase. Similar procedure but more complex needs to be taken in order to optimize mutual information and the throughput of the flexible AF scheme in these protocols.

In chapter 4, two HARQ cooperative relaying strategies were developed both based on the flexible AF. There are many other scenarios that are worth investigating including combination of DF and flexible AF schemes. The same practise from chapter 4 can be applied in order to
Chapter 7 Conclusion

calculate the outage probability and the throughput of the proposed scheme. Also it is valuable to look at more complex strategy, combination of HARQ with all the known cooperative relaying schemes including flexible AF, DF, CF in order to find the optimum throughput and then define the optimum strategy in every SNR region for cooperative relaying combined with HARQ.

This thesis investigated only a single-user relay scenario. Cooperative relaying concept can be integrated into a multiple access channel (MAC) where several users transmit to a base station (uplink) or broadcast channel where a base station transmit to several users (downlink). In a single relay uplink scenario, the mutual information of the relaying link for each user can be calculated separately. Obviously, each user signal causes interference over the other users' signals. In this case a serial interference cancelation (SIC) technique can be applied to remove the interference created by other users. Then the total mutual information of the relay link for multiuser case can be calculated by the sum of the mutual information of all the users. It is expected that the equation of the total relay link mutual information in the multiuser case will be similar to the single user case and as a result the power allocation algorithms for the single user can be applied in multiuser case with some minor changes. But allocation different duplexing ratio to users looks very challenging.

It is also interesting to look at the extension of the cooperative relaying techniques to the multicarrier (OFDM) scenario. In multicarrier scenario the frequency-selective channel is divided into a number of flat sub-channels. Then the power allocation methods which were investigated in chapter 5 can be applied to each sub-channel. Also finding how to pair the sub-channels allocated for the relay-receive and relay-transmit phases are another important issue. Another important issue is to know how to distribute the total power across all sub-channels within OFDM symbol. If the power allocation optimization only considered per sub-channel, then the whole power allocation scheme for OFDM symbol will be suboptimum. An optimum power allocation method considers the power allocation among different sub-channels as well as within each sub-channel.

In a multi-hop scenario the transmitted message from a source pass through several relays before receiving at a destination. It is interesting to know how the flexible AF scheme can be applied to this scenario. At first a closed form mathematical model for the mutual information of the multi-hop relaying should be found and then the outage probability analysis can be done using this mathematical model. In this scenario finding a proper method for allocating unitary matrix and selecting duplexing ratio for each relay is an important issue.

The focus of this thesis was on the single-relay with single antenna case in early chapters and then on the single-relay with multiple antenna case. Most of the analysis in this thesis can be extended to multiple-relay with single antenna and multiple antenna cases. Similar procedure can be done to find the mathematical model for mutual information and then outage probability.
Performance of the cooperative relaying techniques depends on the kind of channel knowledge and what node has the knowledge of what links. In this thesis it was considered that all the links CSIs are available at the destination. There is no transmit CSI available at the source. Also CSI of the direct transmission link is not available at the relay. In chapter 5, at the relay with MIMO capability two CSI cases were considered: for the first case the transmit-and-receive-CSIs both are available. For the second case only the receive CSI is available. The performance of investigated schemes can be also examined in the following scenarios individually:

- CSI about the source to destination and source-relay links are available at source
- CSI about the source to destination, the first link (source to relay) and the second link (relay to destination) are available at the source
- CSI about the source to destination link is available at the relay

Following the practise in chapter 5 for MIMO flexible AF, optimum power allocation techniques at the relay as well as the source needs to be investigated for above CSI assumptions. In these cases the optimization of the power allocation method at the source looks very interesting if it has transmit-CSI about the source-relay link and the source-destination link. Since the source needs to transmit to the relay and the destination at the same time, the source may need to choose between the relay link and the destination link to do beam-forming in suitable direction. This issue needs more investigation.

In this thesis it was assumed that CSI is ideal wherever it is available. However in realistic systems ideal CSI may not be available and there may be some delay or distortion in CSI. Thus it is good to further study the effect of the channel estimation error or the feedback channel error at different nodes on the mutual information by assuming imperfect knowledge of the channel at the involved nodes.

This thesis investigated the performance of proposed schemes in synchronized scenarios (time and frequency). In practical systems it is possible to come across the asynchronous case where a phase mismatch which is a random variable and has a uniform distribution can be considered as part of the channel gain at the relay or destination. It is valuable to look at the performance of proposed flexible scheme in asynchronous scenarios. In this case, the metric such as the mutual information must be averaged over this phase mismatch.

The main motivation of this thesis was to propose novel cooperative techniques to increase spectrum-efficiency of the wireless communication systems. However energy-efficiency and complexity are also important players in practical systems. Due to the environmental economic facts energy consumption has emerged as an important factor in wireless communication. In this
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regards the main focus should be on the efficient cooperative techniques which minimize the energy consumption, reduce interference to other users and maximize the battery life. The proposed cooperative relaying schemes should be examined in term of the energy efficiency and complexity.

In theoretical analysis in chapter 3, the exponential distribution was used in order to analyse the outage probability and the achievable rate for the flexible AF scheme. However it is interesting to look at the effect of other distributions such as gamma distribution in the performance of the proposed scheme. Also in that chapter, some tight lower bounds were derived for the outage probability and some interesting remarks about the diversity of the proposed scheme were made. But this issue needs more attention since it is needed to provide more sophisticated equation for the outage probability and then the diversity of the proposed scheme. The outage probability equation should be such that the effect of the duplexing ratio and the transmission rate can clearly be seen. Then based on the derived equations, major design rules for the proposed scheme should be defined.

For achievable rate, throughout and outage probability analysis, this thesis dealt with the mutual information and therefore it was assumed that Gaussian code book has been used at the source. It is known that Gaussian code book should be used in order to achieve the optimal capacity. However in practical systems finite alphabet modulation constellations are used. It is obvious that the achievable rate and the outage probability are affected by this limitation. For the outage and achievable rate analysis similar to the BER analyses a finite alphabet modulation scheme also can be considered. In this case the achievable rate may be lower than the one with Gaussian code book. This issue needs further investigation.
Appendix A

In the following sections two examples for the Ergodic capacity are explored.

A.1 Single Antenna (Shannon Law)

In a single user point to point SISO fading channel, the received signal can be expressed as
\[ y_0 = x_0 + n_0 \]
where \( x \in \mathbb{C} \) is the transmitted symbol, \( y_0 \in \mathbb{C} \) is received symbol, \( n_0 \in \mathbb{C} \) is additive white Gaussian noise with zero mean and variance \( N_0 \) and \( x_0 \in \mathbb{C} \) is a complex channel gain of the i.i.d fading process with zero mean and unit variance. Moreover the transmit symbol sequence is subject to an average power constraint \( E_s(x^Hx) \leq P_s \). Then the Ergodic capacity of this fading channel is given by:

\[
C = E_{n_0} \left( \log_2 \left( 1 + \rho_0 |x|^2 \right) \right)
\]

where \( \rho_0 = P_s / N_0 \).

A.2 Multiple Antenna

In a single user point to point MIMO fading channel with \( M_t \) transmit and \( M_r \) receive antennas, the received signal can be expressed as
\[ y_0 = H_0 x + n_0 \]
where \( x \in \mathbb{C}^{M_r \times 1} \) denotes the transmitted symbol vector and \( y_0 \in \mathbb{C}^{M_r \times 1} \) is the received symbol vector and \( n_0 \in \mathbb{C}^{M_r \times 1} \) is the additive white Gaussian noise vector with zero mean and variance \( N_0 \). Furthermore the transmitted signal vector is subject to an average power constraint \( E_s(x^Hx) \leq P_s \), where \( E_{n_0}(n_0^Hn_0^H) = N_0 I_{M_r} \) and \( H_0 \in \mathbb{C}^{M_r \times M_t} \) is the complex Gaussian random channel matrix.

The channel capacity of the single user MIMO channel is given by
\[
C = \log_2 \det \left( I + \frac{1}{N_0} H_0 R_x H_0^H \right)
\]
where \( R_x = E_x(x^Hx) = \frac{P_s}{M_t} I_{M_t} \). Therefore the Ergodic capacity [15,17,95] is expressed by

\[
C = E_{n_0} \left( \log_2 \det \left( I + \frac{\rho_0}{M_t} H_0 H_0^H \right) \right)
\]

where \( \rho_0 = P_s / N_0 \). In the following section achievable rate is defined.
Appendix B

B.1 Decode and Forward Transmission

In this thesis our main focus is on the AF scheme. However it is useful to provide a brief description about the DF scheme. In the DF scheme the source transmits the signal $x_1$ during the first phase. The received signal is decoded and re-encoded at the relay. Depending on the assumed coding scheme at the relay several cases can be considered. In one case the relay can encode the message with the same encoder as the source encoder and transmit the signal $x_1$. In another case the relay can employ a different encoder and transmit a new signal $x_2$. In all the cases the destination received signal at the first phase is $y_0 = \alpha \sqrt{\rho_1} x_1 + n_0$. However the received signal for different cases at the second phase is as follows:

$$y_2 = \begin{cases} 
\alpha \sqrt{\rho_2} x_1 + n_2 & \text{orthogonal repeating at the relay} \\
(\alpha_2 \sqrt{\rho_2} + \alpha_0 \sqrt{\rho_0}) x_1 + n_2 & \text{non-orthogonal repeating at the relay} \\
\alpha_2 \sqrt{\rho_2} x_1 + \alpha_0 \sqrt{\rho_0} x_2 + n_2 & \text{non-orthogonal new code at the relay}
\end{cases}$$

(B-1)

B.2 Achievable Rate for Decode and Forward

The mutual information between the source and the relay which is normalized over a frame is $I(x_1; y_0) = \log_2 (1 + \gamma_1)$. It shows that the message is successfully decoded if the transmission rate $R \leq \frac{1}{2} \log_2 (1 + \gamma_1)$. The average mutual information at the destination when the relay repeats the signal and the source also transmits at the second phase is as follows [90]:

$$I(x_1; y_0, y_2) = \log_2 \left(1 + \gamma_0 + \sqrt{\gamma_0 + \gamma_2} \right).$$

As a result the total achievable rate is

$$R \leq \frac{1}{2} \min \varepsilon_2 \left( \log_2 \left(1 + \gamma_0 + \sqrt{\gamma_0 + \gamma_2} \right), \log_2 (1 + \gamma_1) \right)$$

(B-2)

When the relay transmits a new signal at the second phase and the source also transmits, joint decoding can be done at the destination. According to [5], the achievable rate for this case is expressed by
Appendix B

\[ R \leq \max_{\alpha} \min_{\gamma_0} \mathcal{E}(\alpha \log_2 (1+\gamma_1), \alpha \log_2 (1+\gamma_0) + (1-\alpha) \log_2 \left(1+\gamma_0 + \sqrt{\gamma_0^2 + \gamma_1^2}\right)) \]  \hspace{1cm} (B-3)

As it is seen the duplexing ratio is not fixed and can be optimized in order to maximize the achievable rate. It has been shown that the duration of the first phase can be decreased if the quality of the source-relay link is enhanced. Similarly when the quality of the relay-destination link is better, the time duration of transmission from relay to destination can be reduced [5,90].

B.3 Outage for Decode and Forward

When the relay repeats the signal and the source also transmits at the second phase, according to [41,90], the overall outage probability can be expressed as

\[ P_{\text{outage}}(R, \beta) = \Pr \left( \log_2 \left(1+\gamma_0 + \sqrt{\gamma_0^2 + \gamma_2^2}\right) < 2R \right) \Pr (\log_2 (1+\gamma_1) > 2R) + \Pr (\log_2 (1+\gamma_1) < 2R) \]  \hspace{1cm} (B-4)

When the relay transmits a new signal at the second phase and the source also transmits in this phase the outage probability is a function of the duplexing ratio as follows

\[ P_{\text{outage}}(R, \beta, \alpha) = \Pr \left( \alpha \log_2 (1+\gamma_0) + (1-\alpha) \log_2 \left(1+\gamma_0 + \sqrt{\gamma_0^2 + \gamma_2^2}\right) < R \right) \times \]  \hspace{1cm} (B-5)

\[ \Pr (\alpha \log_2 (1+\gamma_1) > R) + \Pr (\alpha \log_2 (1+\gamma_1) < R) \]

By choosing optimum duplexing ratio the outage probability is minimized.
Appendix C

C.1 Driving Mutual Information for Orthogonal AF (Lemma 3-1 in chapter 3)

As it is seen in Figure C-1 during the first phase the source transmits the signal code sequence \( x \in \mathbb{C}^{N_d \times 1} \) to the relay and the destination. During the second phase only the relay transmits to the destination. At the relay, matrix \( A \in \mathbb{C}^{N_r \times N_t} \) is multiplied to the received signal. The received signals at the destination are \( y_0 = \alpha_0 x + n_0 \) and \( y_2 = \alpha_2 \sqrt{\beta} A \alpha_3 x + \alpha_4 \sqrt{\beta} A n_1 + n_2 \) where \( n_0 \in \mathbb{C}^{N_d \times 1} \), \( n_1 \in \mathbb{C}^{N_r \times 1} \) and \( n_2 \in \mathbb{C}^{N_d \times 1} \) are i.i.d noise vectors with zero mean and unit variances. \( \alpha_0, \alpha_1 \) and \( \alpha_2 \) are complex i.i.d fading multipliers with zero mean and variance \( \rho_j; \ j=0,1,2 \) for source-destination, source-relay and relay-destination respectively. \( \beta \) is used for normalizing power at the relay output. As a result we have:

\[
\begin{bmatrix}
    y_0 \\
    y_2
\end{bmatrix} =
\begin{bmatrix}
    \alpha_0 I \\
    \alpha_2 \sqrt{\beta} A
\end{bmatrix}
\begin{bmatrix}
    x \\
    n_0
\end{bmatrix}
+ \begin{bmatrix}
    I & 0 & 0 \\
    0 & \alpha_3 \sqrt{\beta} A & I
\end{bmatrix}
\begin{bmatrix}
    n_1 \\
    n_2
\end{bmatrix}
\text{ and } R_v = \begin{bmatrix}
    I & 0 \\
    0 & (I+|\alpha_2|^2 \beta A A^H)
\end{bmatrix}
\]

Lemma 3-1: The instantaneous accumulative mutual information \( I(y, \alpha) \) normalized over one frame for the orthogonal AF with variable \( \alpha \) is

\[
I(y, \alpha) =
\begin{cases}
  I^{(1)}(y, \alpha) & 0 < \alpha < 0.5 \\
  I^{(2)}(y, \alpha) & 0.5 \leq \alpha \leq 1
\end{cases}
\]
where
\[ I^{(0)}(\chi, \alpha) = \alpha \log \left( 1 + \gamma_0 + \gamma_1 \gamma_{2,\text{eff}} \left( 1 + \gamma_1 + \gamma_{2,\text{eff}} \right)^{-1} \right) \]

\[ I^{(2)}(\chi, \alpha) = (2\alpha - 1) I_0(\gamma_0) + 2(1 - \alpha) I^{(0)}(\chi, 0.5) \]

\[ \gamma_{2,\text{eff}} = \frac{(1 - \alpha)}{\alpha} \gamma_2 \]

\[ I_0(\gamma_0) = \log_2 (1 + \gamma_0). \]

In following sections two proofs for the above lemma are provided. In the first proof, it is considered that matrix A is a sub-matrix of a unitary matrix and in the second proof matrix A is assumed to be a hermitian matrix.

**C.1.1 Lemma:**

If A is \( M \times N \) and B is \( N \times M \) matrix for any \( \alpha \) and \( \beta \) we have:

\[ \det (\alpha I_M + \beta AB) = \alpha^{M-N} \det (\alpha I_N + \beta BA) \]

**Proof:**

\[ \det (\alpha I_M + \beta AB) = \det(\alpha I_M) \det(\frac{1}{\alpha} I_M + \beta BA) = \alpha^N \det(I_M + \frac{\beta}{\alpha} BA) \]

\[ = \alpha^N \det \left( \frac{1}{\alpha} I_N \right) \det(\alpha I_N + \beta BA) = \alpha^M \left( \frac{1}{\alpha} \right)^N \det(\alpha I_N + \beta BA) = \alpha^{M-N} \det(\alpha I_N + \beta BA) \]

**C.1.2 First proof for Lemma 3-1 (Unitary matrix):**

**First case:** \( N_1 \geq N_2 \) i.e. \( 0.5 \leq \alpha \leq 1 \)

We calculate the accumulative mutual information normalized over one frame:

\[ I^{(2)}(\chi, \alpha) = \frac{1}{N_1 + N_2} \log \det \left( I_{(N_1+N_2)} + HH^H + \beta AA^H \right) \]

\[ = \frac{1}{(N_1 + N_2)} \log \det \left( \begin{bmatrix} \left( 1 + |\alpha_0|^2 \right) I_{N_1} & \alpha_0 \alpha_2^* \sqrt{\beta} A^H \left( I_{N_2} + |\alpha_2|^2 \beta A A^H \right)^{-1} \\ \alpha_0^* \alpha_2 \sqrt{\beta} A I_{N_1} + |\alpha_2|^2 \beta A A^H \left( I_{N_2} + |\alpha_2|^2 \beta A A^H \right)^{-1} \end{bmatrix} \right) \]

\[ (C-1-1) \]

According to the matrix determinant lemma i.e: \( \det \left( \begin{bmatrix} E & F \\ G & H \end{bmatrix} \right) = \det(E) \det(H - GE^{-1}F) \) we have:
According to the lemma in C.1.1:

\[ I^{(2)}(\gamma, \alpha) = \frac{1}{(N_1 + N_2)} \left( \log \left( 1 + |\alpha_0|^2 \right) I_{N_2} + \log \left( 1 + |\alpha_1|^2 \right) I_{N_2} + \log \left( 1 + |\alpha_2|^2 \beta AA^h \left( I_{N_2} + |\alpha_2|^2 \beta AA^h \right)^{-1} \right) \right) \]

(C-1-2)

\[ I^{(3)}(\gamma, \alpha) = \frac{1}{(N_1 + N_2)} \left( (N_1 - N_2) \log \left( 1 + |\alpha_0|^2 \right) + \log \left( 1 + |\alpha_0|^2 \right) I_{N_2} \right) \]

\[ + \log \left( 1 + |\alpha_1|^2 \beta AA^h \left( I_{N_2} + |\alpha_2|^2 \beta AA^h \right)^{-1} \right) \]

(C-1-3)

where, \( \gamma_j \) j=0,1,2 are the instantaneous SNRs for their corresponding links and \( \gamma_j = |\alpha_j|^2 \) j = 0, 1, 2. Since A is sub-matrix of an unitary matrix then \( AA^h = I_{N_2} \). It is clear that \( AA^h A = I_{N_1} \) for \( N_2 < N_1 \)

\[ I^{(3)}(\gamma, \alpha) = \frac{1}{(N_1 + N_2)} \left( (2\alpha - 1) \log \left( 1 + |\alpha_0|^2 \right) + \log \left( 1 + |\alpha_0|^2 \right) I_{N_2} \right) \]

\[ + \log \left( 1 + \gamma_0 \gamma_2 \beta AA^h \left( I_{N_2} + \gamma_2 \beta AA^h \right)^{-1} \right) \]

(C-1-4)

Power Constraint at relay:

\[ E_{x_n, \alpha} \left( \text{trace} \left( x_n x_n^h \right) \right) = \text{trace} \left( \beta AA^h + |\alpha_0|^2 \beta AA^h \right) = N_2 \]

Since \( AA^h = I_{N_2} \) then \( \beta = \left( 1 + |\alpha_0|^2 \right)^{-1} = \left( 1 + \gamma_1 \right)^{-1} \)

By substituting \( \beta = \left( 1 + \gamma_1 \right)^{-1} \) in equation (C-1-4) finally we have:
Appendix C

\( I^{(o)}(\gamma, \alpha) = (2\alpha - 1) \log (1 + \gamma_0) + (1 - \alpha) \log \left(1 + \gamma_0 + \gamma_2 (1 + \gamma_1 + \gamma_2)^{-1}\right) \)  \hspace{1cm} (C-1-5)

Second case; \( N_1 \leq N_2 \) and \( 0 < \alpha \leq 0.5 \)

We calculate the accumulative mutual information normalized over one frame:

\[
I^{(o)}(\gamma, \alpha) = \frac{1}{(N_1 + N_2)} \log \det \left( I_{N_1+N_2} + HH^H R^A_2 \right) = \frac{1}{(N_1 + N_2)} \log \det \left( I_{N_1} + HH^H R^A_2 \right) \tag{C-1-6}
\]

\[
= \frac{1}{(N_1 + N_2)} \log \det \left( I_{N_1} + |\alpha_0|^2 I_{N_1} + |\alpha_1|^2 |\alpha_2|^2 \beta A^H \left( I_{N_2} + |\alpha_2|^2 \beta AA^H \right)^{-1} A \right)
\]

Since \( A \) is a sub-matrix of an unitary matrix \( A^H A = I_{N_1} \) and it is clear that \( AA^H \neq I_{N_2} \) for \( N_1 < N_2 \). According to matrix inversion lemma

\[
\left( I_{N_1} + |\alpha_2|^2 \beta AA^H \right)^{-1} = I_{N_2} - |\alpha_2|^2 \beta A \left( I_{N_1} + |\alpha_2|^2 \beta AA^H \right)^{-1} A^H.
\]

Since \( A \) is a sub-matrix of a unitary matrix \( \left( I_{N_1} + |\alpha_2|^2 \beta AA^H \right)^{-1} = \left( 1 + |\alpha_2|^2 \beta \right)^{-1} I_{N_1} \). By replacing in equation (C-1-6) we have:

\[
I^{(o)}(\gamma, \alpha) = \frac{1}{(N_1 + N_2)} \log \det \left( I_{N_1} + |\alpha_0|^2 I_{N_1} + |\alpha_1|^2 |\alpha_2|^2 \beta A^H \left( I_{N_2} - |\alpha_2|^2 \beta \left( 1 + |\alpha_2|^2 \beta \right)^{-1} AA^H \right) A \right) \tag{C-1-7}
\]

Then;

\[
I^{(o)}(\gamma, \alpha) = \frac{1}{(N_1 + N_2)} \log \det \left( I_{N_1} + |\alpha_0|^2 I_{N_1} + |\alpha_1|^2 |\alpha_2|^2 \beta \left( 1 - |\alpha_2|^2 \beta \left( 1 + |\alpha_2|^2 \beta \right)^{-1} \right) I_{N_1} \right)
\]

\[
= \frac{1}{(N_1 + N_2)} \log \det \left( I_{N_1} + |\alpha_0|^2 I_{N_1} + |\alpha_2|^2 |\alpha_2|^2 \beta \left( 1 + |\alpha_2|^2 \beta \right)^{-1} I_{N_1} \right)
\]

\[
I^{(o)}(\gamma, \alpha) = \alpha \log \left( 1 + |\alpha_0|^2 + |\alpha_1|^2 |\alpha_2|^2 \beta \left( 1 + |\alpha_2|^2 \beta \right)^{-1} \right) \tag{C-1-8}
\]

Finally:

\[
I^{(o)}(\gamma, \alpha) = \alpha \log \left( 1 + \gamma_0 + \gamma_2 \beta (1 + \gamma_1 \beta)^{-1} \right) \tag{C-1-9}
\]

Power constraint at relay:

\[
\mathcal{E}_{x_{n_0},x_{n_1}}(trace(x_{n_0}x_{n_1}^H)) = trace \left( \beta AA^H + |\alpha_1|^2 \beta AA^H \right) = N_2 \text{ Since } trace(\AA^H) = N_1 \text{ then}
\]
Appendix C

\[ \beta = \left( 1 + |\alpha|^2 \right)^{-1} = \left( 1 + \gamma_i \right)^{-1} \left( \frac{N_2}{N_1} \right) = \left( 1 + \gamma_i \right)^{-1} \left( \frac{1 - \alpha}{\alpha} \right) \]

By replacing in the equation (C-1-9) we have:

\[ I^0(\chi, \alpha) = \alpha \log \left( 1 + \gamma_0 + \frac{\gamma_1 (1-\alpha) \gamma_2}{\alpha (1+\gamma_i + \frac{1-\alpha}{\alpha} \gamma_2)} \right) \]

if we consider the effective SNR as:

\[ \gamma_{2,\text{eff}} = \frac{(1-\alpha)}{\alpha} \gamma_i \], then we have:

\[ I^0(\chi, \alpha) = \alpha \log \left( 1 + \gamma_0 + \frac{\gamma_1 \gamma_{2,\text{eff}}}{1 + \gamma_i + \gamma_{2,\text{eff}}} \right) \]  

(C-1-10)

C.1.3 Second proof for Lemma 3-1 (Hermition matrix):

Here we consider that matrix \( A \) is hermition matrix. If \( AA^H = UDU^H \) where \( U \in \mathbb{C}^{N_2 \times N_2} \) is a unitary matrix and \( D \) is a \( N_2 \times N_2 \) diagonal matrix with elements \( d_k \) for \( k = 1, \ldots, N_2 \). It is clear that when \( N_2 \geq N_1 \) the last \( N_2 - N_1 \) diagonal elements of \( D \) will be zeros, i.e. \( d_k = 0 \) for \( k = N_1 + 1, \ldots, N_2 \). Then according to the equation (C-1-3) we have:

\[ I(\chi, \alpha) = (2\alpha - 1) \log (1 + \gamma_0) + \frac{1}{(N_1 + N_2)} \log \det \left( (1 + \gamma_0) I_{N_1} + \gamma_i \gamma_2 \beta AA^H (I_{N_1} + \gamma_i \beta AA^H)^{-1} \right) \]  

(C-1-11)

We define:

\[ I_2(\chi, \alpha) = \frac{1}{(N_1 + N_2)} \log \det \left( (1 + \gamma_0) I_{N_1} + \gamma_i \gamma_2 \beta AA^H (I_{N_1} + \gamma_i \beta AA^H)^{-1} \right) \]  

(C-1-12)

Since:

\[ \det \left( (1 + \gamma_0) I_{N_1} + \gamma_i \gamma_2 \beta AA^H (I_{N_1} + \gamma_i \beta AA^H)^{-1} \right) = \det \left( U \left( (1 + \gamma_0) I_{N_1} + \gamma_i \gamma_2 \beta D (I_{N_1} + \gamma_i \beta D)^{-1} \right) U^H \right) \]

\[ = \det \left( (1 + \gamma_0) I_{N_1} + \gamma_i \gamma_2 \beta D (I_{N_1} + \gamma_i \beta D)^{-1} \right) = \prod_{k=1}^{N_2} \det \left( \left( 1 + \gamma_0 \right) + \gamma_i \gamma_2 \beta d_k \left( 1 + \gamma_i \beta d_k \right)^{-1} \right) \]

Therefore:
$I_2(\gamma, \alpha) = \frac{1}{(N_1 + N_2)} \sum_{k=1}^{N_1} \log \left( (1 + \gamma_0) + \gamma_1 d_k (1 + \gamma_2 d_k)^{-1} \right)$ \hfill (C-1-13)

Now we consider the power constraint at the relay:

$E_{x_i, a, o_i} \left( \text{trace} (x_i x_i^H) \right) = \text{trace} (\beta AA^H + \gamma_1 \beta AA^H) = \beta (1 + \gamma_1) \text{trace} \left( AA^H \right) = \beta (1 + \gamma_1) \left( \sum_{k=1}^{N_2} d_k \right) = N_2$

Since the scaling for $d_k$'s will not affect on $I_2$ so we fix $\sum_{k=1}^{N_2} d_k = N_2$ then $\beta = (1 + \gamma_1)^{-1}$

Then:  

$I_2(\gamma, \alpha) = \frac{1}{(N_1 + N_2)} \sum_{k=1}^{N_2} \log \left( (1 + \gamma_0) + \gamma_1 d_k (1 + \gamma_1 + \gamma_2 d_k)^{-1} \right)$. We need to find $d_k$'s which maximize $I_2(\gamma, \alpha)$. We suggest to use Lagrange technique as follows:

$\frac{\partial}{\partial d_k} \log \left( (1 + \gamma_0) + \gamma_1 d_k (1 + \gamma_1 + \gamma_2 d_k)^{-1} \right) = \lambda \quad \forall d_k > 0$ then

$\frac{1}{(1 + \gamma_0) + \gamma_1 d_k (1 + \gamma_1 + \gamma_2 d_k)^{-1}} \times \frac{\gamma_2 d_k (1 + \gamma_1)^{-1}}{(1 + \gamma_1 + \gamma_2 d_k)^{-2}} = \lambda \quad \forall d_k > 0$. As a result all the $d_k$'s should be equal in order to have the above equality.

For $N_2 \leq N_1$ or $0.5 \leq \alpha \leq 1$: $d_k = 1$ for $k = 1, \ldots, N_2$ then

$I_2(\gamma, \alpha) = (1 - \alpha) \log \left( (1 + \gamma_0) + \gamma_1 (1 + \gamma_2)^{-1} \right)$ \hfill (C-1-14)

For $N_2 > N_1$: $d_k = \frac{N_2}{N_1}$ for $k = 1, \ldots, N_1$ and $d_k = 0$ for $k = N_1 + 1, \ldots, N_2$ then

$I_2(\gamma, \alpha) = \log \left( (1 + \gamma_0) + \gamma_1 \left( \frac{1 - \alpha}{\alpha} \right) \left( (1 + \gamma_1) + \left( \frac{1 - \alpha}{\alpha} \gamma_2 \right)^{-1} \right) \right) + (1 - 2\alpha) \log \left( (1 + \gamma_0) \right)$ \hfill (C-1-15)

Now we can substitute $I_2(\gamma, \alpha)$ in the equation (C-1-11).

Based on the above proofs we can conclude that when the unitary matrices or their sub-matrices are employed at the relay following relationship exists between the instantaneous mutual information and the duplexing ratio:
Appendix C

\[ I(y, \alpha) = \begin{cases} 
\alpha \log \left( 1 + y_0 + \gamma_1 \left( \frac{1 - \alpha}{\alpha} \gamma_2 \right) \left( 1 + \gamma_1 + \left( \frac{1 - \alpha}{\alpha} \gamma_2 \right)^{-1} \right) \right) & 0 \leq \alpha \leq 0.5 \\
(2\alpha - 1) \log (1 + y_0) + (1 - \alpha) \log \left( 1 + y_0 + \gamma_2 \left( 1 + \gamma_1 + \gamma_2 \right)^{-1} \right) & 0.5 \leq \alpha \leq 1 
\end{cases} \]  
(C-1-16)

By replacing \( I^{(1)}(\gamma, \alpha) = \alpha \log \left( 1 + y_0 + \gamma_1 \gamma_2 (1 + \gamma_1 + \gamma_2)^{-1} \right) \) and
\[ I^{(2)}(\gamma, \alpha) = (2\alpha - 1) I_0(\gamma_0) + 2(1 - \alpha) I^{(1)}(0.5) \]
where \( I_0(\gamma_0) = \log (1 + \gamma_0) \) and
\[ \gamma_{2,\text{eff}} = \frac{(1 - \alpha)}{\alpha} \gamma_2 \]
we have:
\[ I(y, \alpha) = \begin{cases} 
I^{(1)}(\gamma, \alpha) & 0 < \alpha < 0.5 \\
I^{(2)}(\gamma, \alpha) & 0.5 \leq \alpha \leq 1 
\end{cases} \]  
(C-1-17)

Since \( I_0(\gamma_0) \) and \( I^{(1)}(0.5) \) are independent from \( \alpha \) when \( 0.5 \leq \alpha \leq 1 \), it is clear that the optimum duplexing ratio which maximize the instantaneous mutual information when \( 0.5 \leq \alpha \leq 1 \) is:
\[ \alpha_{\text{opt}}^{(2)} = \arg \max_{0.5 \leq \alpha \leq 1} I^{(2)}(\gamma, \alpha) = \begin{cases} 
0.5 & 2I_0(\gamma_0) < I^{(1)}(\gamma, 0.5) \\
2I_0(\gamma_0) & I^{(1)}(\gamma, 0.5) 
\end{cases} \]  
(C-1-18)

The optimum \( \alpha \) can be found numerically by using Newton-Raphson algorithm.
\[ \alpha_{n+1} = \alpha_n - \frac{f(\alpha_n)}{f'(\alpha_n)} \]  
(C-1-19)

Where:
\[ f(\alpha) = \log_2 \left( 1 + y_0 + \beta \gamma_1 \gamma_2 (1 + \gamma_1 + \beta \gamma_2) \right) - \frac{\gamma_2 (1 + \gamma_1)(1 + \beta)}{\left( (1 + y_0)(1 + \gamma_1 + \beta \gamma_2) + \beta \gamma_1 \gamma_2 (1 + \gamma_1 + \beta \gamma_2) \right) \ln(2)} \]

and
C.2 Mutual Information for Non Orthogonal AF when source transmits new signal – Protocol 2 Non orthogonal AF (UC)

As it is shown in Figure C-2 in this protocol we consider that the source encodes a new codeword from the message \( w \) i.e. \( x_2 = x_2(w) \) and simultaneously transmits with the relay during the second phase.

The received signal at the second phase will be \( y_2 = \alpha_2 \sqrt{\beta A} \alpha_1 x_1 + \alpha_0 x_2 + \alpha_2 \sqrt{\beta A} n_1 + n_2 \) and the equivalent signal model as follows:

\[
\begin{bmatrix}
  y_0 \\
  y_2
\end{bmatrix} =
\begin{bmatrix}
  \alpha_0 I & 0 \\
  \alpha_0 \alpha_2 \sqrt{\beta A} & \alpha_0 I
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2
\end{bmatrix} +
\begin{bmatrix}
  I & 0 & 0 \\
  0 & \alpha_2 \sqrt{\beta A} & I
\end{bmatrix}
\begin{bmatrix}
  n_0 \\
  n_1 \\
  n_2
\end{bmatrix} \quad (C-2-1)
\]
Appendix C

C.2.1 Mutual information

\[
I^{(2)}(\gamma, \alpha) = \frac{1}{(N_1 + N_2)} \log \det \left( I_{(N_1 + N_2)} + HH^H \right)
\]

\[
= \frac{1}{(N_1 + N_2)} \log \det \left[ \left( 1 + |\alpha|^2 \right) I_{N_2} + |\alpha|^2 |\alpha|^2 I_{N_2} + |\alpha|^2 |\alpha|^2 I_{N_2} - \left( 1 + |\alpha|^2 \right) I_{N_2} \right]
\]

The accumulative mutual information normalized over one frame is:

\[
I^{(2)}(\gamma, \alpha) = \frac{1}{(N_1 + N_2)} \log \det \left( I_{(N_1 + N_2)} + HH^H \right)
\]

\[
(2\alpha - 1) \log (1 + \gamma) + \frac{1}{(N_1 + N_2)} \log \det \left( I_{N_2} + \gamma \left( 1 + \gamma \right) I_{N_2} \right)
\]

\[
(C-2-2)
\]

By considering \( A \) as a sub-matrix of a unitary matrix and \( N_1 \geq N_2 \) we have:

\[
I^{(2)}(\gamma, \alpha) = (2\alpha - 1) \log (1 + \gamma_0) + (1 - \alpha) \log \left( 1 + \gamma + \gamma_0 \left( 1 + \gamma_0 \right) \left( 1 + \gamma \right) \right)
\]

\[
(C-2-3)
\]

C.3 Mutual Information for Non Orthogonal AF when source repeats transmitted signal-Protocol 3 Non orthogonal AF (RC)

In this protocol as it is shown in Figure C-3 we consider that the source transmits a transformed version of the first phase signal code sequence during the second phase. The signal received at the destination during the second phase is \( y_2 = \alpha_2 \sqrt{b} A x_1 + \alpha_0 B x_1 + x_2 \). During the second phase matrix \( B \) with size \( N_2 \times N_1 \) is applied to the transmitted sequence at the source. In order to normalize the source transmit power \( \text{trace}(BB^H) \leq N_2 \).
The equivalent signal model is as follows:

\[
\begin{bmatrix}
    y_0 \\
    y_2
\end{bmatrix} = \begin{bmatrix}
    \frac{\alpha_0}{\alpha_1 + \sqrt{\beta}A + \alpha_0 B} \\
    \alpha_0 + \frac{1}{\alpha_1}
\end{bmatrix} \begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix} + \begin{bmatrix}
    \mathbb{I} & 0 & 0 \\
    0 & \alpha_1 + \sqrt{\beta}A & I
\end{bmatrix} \begin{bmatrix}
    n_0 \\
    n_1 \\
    n_2
\end{bmatrix}
\]

\(\text{dim}(B) = N_2 \times N_1\)

**C.3.1 Mutual information**

\[
I_{\text{ch}}(y; \alpha) = \frac{1}{N_1 + N_2} \log \det \left( I_{n_1, n_2} + \mathbf{H}^\dagger \mathbf{R} \mathbf{H} \right)
\]

\[
= \frac{1}{N_1 + N_2} \log \det \left( \left( 1 + |\alpha|_1^2 \right) I_{n_1} + \alpha \left( \alpha_1 + \sqrt{\beta} A + \alpha_0 B \right) \left( I_{n_2} + |\alpha|_1^2 \beta A A^\dagger \right)^{-1} \left( \alpha_1 + \sqrt{\beta} A + \alpha_0 B \right)^\dagger \left( I_{n_2} + |\alpha|_1^2 \beta A A^\dagger \right)^{-1} \right)
\]

Then the accumulative mutual information normalized over one frame will be:

\[
I_{\text{ch}}(y; \alpha) = \frac{1}{N_1 + N_2} \log \det \left( I_{n_1, n_2} + \mathbf{H}^\dagger \mathbf{R} \mathbf{H} \right)
\]

\[
= \left( 2 \alpha - 1 \right) \log \left( 1 + \gamma_1 \right) + \frac{1}{N_1 + N_2} \log \det \left( \left( 1 + \gamma_1 \right) I_{n_1} + \left( \alpha_1 + \sqrt{\beta} A + \alpha_0 B \right) \left( \alpha_1 + \sqrt{\beta} A + \alpha_0 B \right)^\dagger \left( I_{n_2} + \gamma_1 \beta A A^\dagger \right)^{-1} \right)
\]

\(\text{(C-3-2)}\)
By considering $A$ as sub-matrix of a unitary matrix, $N_1 \geq N_2$ and $B = A$ we have:

\[ I^{(2)}(\gamma, \alpha) = (2\alpha - 1) \log(1 + \gamma_0) + (1 - \alpha) \log \left(1 + \gamma_0 + \left| \alpha_1 \alpha_2 \sqrt{\beta} + \alpha_0 \right|^2 \left(1 + \beta \gamma_2 \right)^{-1} \right) \]  

\[ (C-3-3) \]

C.4 Mutual Information for Non Orthogonal AF using Alamouti scheme – Protocol 4 Non orthogonal AF (RC) with Alamouti at relay

This protocol is similar to protocol 3 except that at the relay an Alamouti space time code [42] is applied over the received signal before it is passed through a linear matrix $A$. If we consider $x = (x_1, x_2, \ldots, x_{N_t})$ as the transmitted sequence from the source during the first phase then $y_1 = (y_{1,1}, y_{1,2}, \ldots, y_{1,N_t})$ is the received signal at the relay. As a result the Alamouti coded signal is $(y_{1,1}, y_{1,3}, \ldots, y_{1,N_t - 1}, -y_{1,2}, -y_{1,4}, \ldots, -y_{1,N_t})$.

Let's consider: $A_{N_t \times N_t} = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}$, $B_{N_t \times N_t} = \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix}$ in order to separate two half of the input signal of the relay. The equivalent signal model of this protocol is as follows:

\[ \begin{bmatrix} y_{0,1}^* \\ y_{0,2}^* \\ y_{1,1}^* \\ y_{1,2}^* \end{bmatrix} = \begin{bmatrix} \alpha_0 \mathbb{I} & 0 \\ 0 & \alpha_0 \mathbb{I} \\ \alpha_2 B_1 & -\alpha_2 \sqrt{\beta} A_2 \\ \alpha_2 \sqrt{\beta} A_1^* & -\alpha_2 B_2^* \end{bmatrix} \begin{bmatrix} x_{1,1}^* \\ x_{1,2}^* \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\alpha_2 \sqrt{\beta} A_2 & \mathbb{I} & 0 & 0 \\ 0 & 0 & 0 & -\alpha_2 \sqrt{\beta} A_1^* & \mathbb{I} & 0 \\ n_{0,1} \\ n_{0,2} \\ n_{1,1} \\ n_{1,2} \end{bmatrix} \]

\[ (C-4-1) \]

C.4.1 Mutual Information

If $A_2 = A_1^*$ and $B_2 = B_1^*$ and $A_2 = B_2$ then the accumulated mutual information normalized over one frame is:
Appendix C

\[
I^{(3)}(\gamma, \alpha) = \frac{1}{N_1 + N_2} \log \det \left( I_{(N_1+N_2)} + H^* R_v^* H \right) = \\
\frac{1}{N_1 + N_2} \left[ \log \left( 1 + |\alpha_0|^2 \right) I_{N_1/2} + \right. \\
\left. \log \left( |\alpha_1|^2 |\alpha_2|^2 \beta + |\alpha_0|^2 \right) A_2 A_2^* \left( I_{N_2/2} + |\alpha_2|^2 \beta A_2 A_2^* \right)^{-1} \right] \\
+ (N_1 - N_2) \log \left( 1 + |\alpha_0|^2 \right)
\]

or

\[
I^{(3)}(\gamma, \alpha) = \frac{1}{N_1 + N_2} \log \det \left( I_{(N_1+N_2)} + H^* R_v^* H \right) = \\
\frac{2}{N_1 + N_2} \log \det \left( 1 + \gamma_0 I_{N_1/2} + (\gamma_2 \beta + \gamma_0) A_2 A_2^* \right) \\
\left( I_{N_2/2} + \gamma_2 \beta A_2 A_2^* \right)^{-1} \\
+ (2\alpha - 1) \log \left( 1 + |\alpha_0|^2 \right)
\]

(C-4-2)

By considering \( A_2 \) as a sub-matrix of an unitary matrix i.e. \( A_2 A_2^* = I_{N_2/2} \) and \( N_1 \geq N_2 \), then

\[
I^{(3)}(\gamma, \alpha) = \frac{1}{(N_1 + N_2)} \log \det \left( I_{(N_1+N_2)} + H^* R_v^* H \right) = (1 - \alpha) \log \left( 1 + \gamma_0 + (\gamma_2 \beta + \gamma_0) (1 + \gamma_2 \beta)^{-1} \right) \\
+ (2\alpha - 1) \log \left( 1 + \gamma_0 \right)
\]

(C-4-3)
Appendix D

D.1 Lemma 3-3 in chapter 3

In this section, we want to find the probability distribution of \( z = xy(1 + x + y)^{-1} \). We can express

\[
y = \frac{z(x+1)}{x-z},
\]

where \( x \) and \( y \) are independent and both have the exponential distribution with the average \( \lambda_x \) and \( \lambda_y \), respectively. It means: \( f_x(x) = \frac{1}{\lambda_x} e^{-\frac{x}{\lambda_x}} \) and \( f_y(y) = \frac{1}{\lambda_y} e^{-\frac{y}{\lambda_y}} \).

Let's look at the CDF of \( z \):

\[
F_z(z) = \Pr \left( \frac{xy}{(1 + x + y)} < z \right) = \Pr \left( \frac{xy}{(1 + x + y)} < z \right) = \\
\Pr \left( \min(x, y) < z \right) + \Pr \left( \frac{xy}{(1 + x + y)} < z | x \geq z, y \geq z \right)
\]

(D-1-1)

Let us consider the first part of the above equation:

\[
\Pr \left( \min(x, y) < z \right) = 1 - \Pr \left( \min(x, y) \geq z \right) = \\
1 - \Pr(x \geq z \& y \geq z) = 1 - \Pr(x \geq z) \Pr(y \geq z) = \\
1 - (1 - \Pr(x < z))(1 - \Pr(y < z)) = 1 - e^{-\frac{1}{\lambda_x}} e^{-\frac{1}{\lambda_y}} = 1 - e^{-\frac{1}{\lambda_x} + \frac{1}{\lambda_y}}
\]

(D-1-2)

The second part of \( F_z(z) \) i.e.

\[
P(S) = \Pr \left( \frac{xy}{(1 + x + y)} < z | x \geq z, y \geq z \right) = \int_{x=z}^\infty f_x(x) \int_{y=z}^{\frac{x(x+1)}{x-z}} f_y(y) dy dx
\]

is the probability of the area between the curve \( y = \frac{z(x+1)}{x-z} \) and the lines \( y = z \) and \( x = z \). As it is seen the boundaries of this integral are mathematically complex and we can not expect to find a polynomial exponential form for this part as the integral is very difficult to be solved.
We are looking for a polynomial exponential form for the CDF of $z$. The best way to achieve this is to approximate the $y = \frac{z(x+1)}{x-z}$ curve with several sequential piecewise lines tangent to the original curve. It means: $y = \begin{cases} a_1 - \eta_1 x & z \leq x \leq b_1 \\ a_2 - \eta_2 x & b_1 \leq x \leq b_2 \\ \vdots & \vdots \\ a_n - \eta_n x & b_{n-1} \leq x \leq b_n \end{cases}$ will provide a good approximation to the original curve. In this equation $\eta_j$ is the slope of the j-th line, $a_j$ is the crossing point of this line with $x = 0$ line, $b_j = z$ and $b_j$, $1 \leq j \leq n-1$ are the crossing points of the j-th line with (j+1)-th line. As a result $b_j > b_{j-1}$. $b_n$ is the crossing point of the last line with $y = z$ line.

Therefore $P(A)$ as the probability of the area between the tangent lines and the lines $y = z$ and $x = z$ as it is shown in Figure D-1, will be a lower bound to the $P(S)$. It means: $P(S) \geq P(A)$.

Where $P(A) = \sum_{j=1}^{n} \int_{a_j}^{b_j} f_X(x) \int_{y=x}^{\eta_j x} f_Y(y) dy dx$

We can show that: $b_j = z + 2\sqrt{z(z+1)} \left( \frac{\sqrt{\eta_j} - \sqrt{\eta_{j+1}}}{\eta_j - \eta_{j+1}} \right)$ and each line is tangent to the original curve at $x = d_j = z + \sqrt{\frac{z(z+1)}{\eta_j}}$. Figure D-1 shows the three line approximation. These three lines are tangent to the original curve at $x = d_1$, $x = d_2$ and $x = d_3$ points respectively. We have learned that three line approximation provides us a tight approximation for the CDF of $z$. 
Also we found that a good choice for the slope of the lines are: \( \eta_1 = \frac{\lambda_3}{\lambda_1}, \eta_2 = \mu \eta_1 \) and \( \eta_3 = \frac{1}{\mu} \eta_2 \) where \( \mu \) is the scaling factor and \( \mu > 1 \). With the above assumption we carry on to calculate \( P(A) \). Later on we consider that \( \sqrt{z(z+1)} = z + \frac{1}{2} \). After tedious calculation we can simplify the CDF of \( z \) as a function \( \lambda_x \) and \( \lambda_y \) as follows:

\[
F_z(z) \geq 1 - \sum_{j=1}^{3} K_j \left( \frac{1}{SNR}, z \right) e^{-\frac{1}{SNR z}}
\]

where \( K_j \left( \frac{1}{SNR}, z \right) = \frac{2}{1-\mu}, K_2 \left( \frac{1}{SNR}, z \right) = \mu + 1 \frac{1}{\mu - 1} + \frac{1}{\mu - 1} + \frac{1}{\mu - 1} (1 + 2z) \) and

\[
\varphi_1 \left( \frac{1}{SNR}, z \right) = \frac{1}{SNR} \left( \sqrt{\frac{\mu}{\delta_x \delta_y}} + \frac{2\sqrt{\mu}}{\delta_x \delta_y} + \frac{1}{\delta_x} + \frac{1}{\delta_y} \right) z
\]

and

\[
\varphi_2 \left( \frac{1}{SNR}, z \right) = \frac{1}{SNR} \left( \frac{1}{\sqrt{\delta_x \delta_y}} + \frac{2}{\sqrt{\delta_x \delta_y}} + \frac{1}{\delta_x} + \frac{1}{\delta_y} \right) z
\]

where we consider \( \lambda_x = \delta_x SNR \) and \( \lambda_y = \delta_y SNR \). We can rewrite the above equation as follows:
Appendix D

D.2 Lemma 3-4 in chapter 3

D.1.1 When Duplexing Factor is greater than half

We showed that the instantaneous mutual information for the flexible AF scheme is as follows:

\[ I(\gamma, \alpha) = (2\alpha - 1) \log(1 + \gamma_0) + (1 - \alpha) \log(1 + \gamma_0 + \gamma_1(1 + \gamma_1 + \gamma_2)^{-1}) \]

Where \( \gamma = (\gamma_0, \gamma_1, \gamma_2) \) is a vector containing all three instantaneous SNRs. By replacing \( \gamma_{AF} = \gamma_1(1 + \gamma_1 + \gamma_2)^{-1} \) and \( \lambda = (2\alpha - 1) \) we have:

\[ I(\gamma, \alpha) = \lambda \log(1 + \gamma_0) + \frac{(1 - \lambda)}{2} \log(1 + \gamma_0 + \gamma_{AF}) \]

(D-2-1)

Considering the transmission rate \( R \), the outage happens in the region where \( I(\gamma, \alpha) < R \).

\[ \lambda \log(1 + \gamma_0) + \frac{(1 - \lambda)}{2} \log(1 + \gamma_0 + \gamma_{AF}) = R \]

shows the boundary of this region. In this boundary

\[ \gamma_{AF}^* = 2^{\frac{2R}{1 - \lambda} - \frac{2\lambda \log(1 + \gamma_0)}{1 - \lambda}} - \gamma_0 - 1 = 2^{\frac{2R}{1 - \lambda} - \frac{2\lambda}{1 - \lambda}} (1 + \gamma_0)^{\frac{1 - \lambda}{1 - \lambda}} - \gamma_0 - 1 \]

(D-2-2)

The outage probability will be the probability of the area between this boundary and \( \gamma_{AF} = 0 \), and \( \gamma_0 = 0 \) axis. Let us look at the outage probability equation.

\[ P_{outage} = \Pr(I(\gamma, \alpha) < R) = \int_{0}^{\gamma_{AF}^*} \int_{0}^{\gamma_0} f_{\gamma_0} (\gamma_0) f_{\gamma_{AF}} (\gamma_{AF}) d\gamma_{AF} d\gamma_0 \]

(D-2-3)

Let's assume \( f_{\gamma_0} (\gamma_0) = \frac{1}{\lambda_0} e^{-\gamma_0/\lambda_0} \) as the PDF of \( \gamma_0 \) and \( f_{\gamma_{AF}} (\gamma_{AF}) \) as the PDF of \( \gamma_{AF} \). By replacing the CDF of \( \gamma_{AF} \) \( (F_{\gamma_{AF}} (\gamma_{AF})) \) which was calculated in previous lemma (D-1) we have:
D.1.2 Outage Probability Equation When Duplexing Factor is equal half

For $\alpha = \frac{1}{2}$ the $\gamma^\prime_{AF}(\gamma_0, \alpha)$ curve is converted to a line: $\gamma^\prime_{AF}(\gamma_0) = 2^{2R} - \gamma_0 - 1$. Then by using CDF of $\gamma^\prime_{AF}$, the outage probability integral (D-2-4) can easily calculated as follows:

$$P_{\text{outage}} = 1 - \sum_{j=1}^{\infty} x_j \left( \frac{1}{\text{SNR}} \right) e^{-\frac{1}{x_j(\text{SNR})}}$$

(D-2-5)

where $x_1 = 1$, $x_2 = \left( \frac{2}{(1-\mu)(\delta_0^2-1)} \right)$, $\zeta_1 = \frac{1}{\text{SNR}} \frac{c}{\delta_0}$, $\zeta_2 = \frac{1}{\text{SNR}} \left( \frac{c + \sqrt{\mu}}{\delta_0} \right)$

$$x_3 = \left( \frac{\sigma_0 + \frac{1}{\text{SNR}} \sigma_3 (\sigma_0 \delta_0 - 1) + 2 \sigma_3 \delta_0}{(\sigma_0 \delta_0 - 1)^2} \right)$$

$\zeta_3 = \frac{1}{\text{SNR}} \left( \frac{c + \frac{1}{\sqrt{\delta_1 \delta_2}}}{\delta_0} \right)$

$$x_4 = \left( -\frac{2}{(1-\mu)(\delta_0^2-1)} \right)$$

$\zeta_4 = \frac{1}{\text{SNR}} \left( c \sigma_1 + \frac{\sqrt{\mu}}{\sqrt{\delta_1 \delta_2}} \right)$

$$x_5 = \left( -\frac{2 \sigma_0 + \frac{1}{\text{SNR}} (2c+1) \sigma_3 (\sigma_0 \delta_0 - 1) + 2 \sigma_3 \delta_0}{(\sigma_0 \delta_0 - 1)^2} \right)$$

$\zeta_5 = \frac{1}{\text{SNR}} \left( c \sigma_3 + \frac{1}{\sqrt{\delta_1 \delta_2}} \right)$

Here we consider $\delta_1$, $\delta_2$ and $\delta_0$ as the offset of $\lambda_1$, $\lambda_2$ and $\lambda_0$ compared to the reference value SNR. It means: $\lambda_s = \delta_1 \text{SNR}$, $\lambda_2 = \delta_2 \text{SNR}$, $\lambda_0 = \delta_0 \text{SNR}$.

Also $\sigma_1 = \frac{2(\sqrt{\mu} + \frac{1}{\delta_1 \delta_2} + \frac{1}{\delta_1 \delta_2})}{\sqrt{\delta_1 \delta_2}}$, $\sigma_2 = \left( \frac{2}{\sqrt{\delta_1 \delta_2}} + \frac{1}{\delta_1 \delta_2} \right)$, $\sigma_3 = \frac{\sqrt{\mu} - 1}{\sqrt{\delta_1 \delta_2} (\sqrt{\mu} + 1)}$ and

$\sigma_4 = \frac{\mu + 1}{\mu - 1}$ where $c = 2^{2R} - 1$. $K_1, K_2, K_3, \varphi_1, \varphi_2, \delta_1, \delta_2, \delta_3$ are obtained from the lemma for the CDF of $\gamma^\prime_{AF}$. 

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### D.1.3 Outage Probability Equation when duplexing ratio is greater than half

But with increasing $\alpha$ towards one, $0.5 < \alpha < 1$, the boundary curve $\gamma_{AF}^* (\gamma_0, \alpha)$ is getting to be more complex. Therefore calculating the outage probability integral in the equation (D-2-4) will be difficult. Here we propose to use a piecewise linear equation to approximate $\gamma_{AF}^* (\gamma_0, \alpha)$ as it is shown in figure D-2. In this case the lower bound for the outage probability is

$$P_{\text{outage}} = \sum_{j=1}^{n} \int_{d_{j-1}}^{d_j} \frac{1}{\lambda_0} e^{-\tau_0 \lambda_0} F_{\gamma_{AF}} (a_j - \eta_j, \gamma_0) d\gamma_0$$

where $\eta_j$ is the slope of the j-th line and $a_j$ is the crossing point of this line with $\gamma_0 = 0$ line. $d_0 = 0$. $d_j$, $1 \leq j \leq n-1$ are the cross points of the line j with j+1-th line. $d_n$ is the cross point of the last line with $\gamma_{AF} = 0$. All the lines are assumed tangent to the original curve.

![Figure D-2. Three line approximation for calculating the outage probability](image)

We found that this three lines approximation is enough for calculating the outage probability. Note that second line is tangent to the curve at the point with $\gamma_0 = 2^R - 1$ and third line at ($\gamma_0 = 2^{1/R} - 1, \gamma_{AF} = 0$). We select the slope of first line such that the area between these lines and both axis is maximized. It is clear that all the slopes are dependent to the variable $\alpha$. After tedious calculation the outage probability is as follows:
Appendix D

\begin{equation}
P_{\text{total}}(\text{SNR}, \alpha) = 1 - \sum_{j=1}^{N} \zeta_{j} \left( \frac{1}{\text{SNR}}, \alpha \right) e^{-\zeta_{j}(\frac{1}{\text{SNR}}, \alpha)}
\end{equation}

where \( \zeta_{j} \left( \frac{1}{\text{SNR}}, \alpha \right) = \sum_{i} \zeta_{i,j} \left( \alpha \right) \frac{1}{\text{SNR}} \) and \( \zeta_{i,j} \left( \alpha \right) = \sum_{\delta} \chi \left( \frac{1}{\text{SNR}}, \alpha \right) \left( \frac{c}{\delta_{0} + \sqrt{\delta_{0} \delta_{2}}} \right) \)

\( \chi = 1 \)

\[ \zeta_{1} = \left. \frac{1}{\text{SNR}} \right|_{\delta_{0}, \delta_{2}} \chi_{2} = \left. \frac{2}{(1-\mu)(\eta_{\delta_{0}} - 1)} \right|_{\delta_{0}, \delta_{2}} \]

\[ \zeta_{2} = \left. \frac{1}{\text{SNR}} \right|_{\delta_{0}, \delta_{2}} \chi \left( \frac{c}{\delta_{0} + \sqrt{\delta_{0} \delta_{2}}} \right) \]

\[ \chi_{3} = \left. \frac{\left( \omega_{4} + \frac{1}{\text{SNR}}(\eta_{\delta_{0}} - 1) + 2\eta_{\delta_{0}} \right)}{\left( \eta_{\delta_{0}} - 1 \right)^{2}} \right|_{\delta_{0}, \delta_{2}} \]

\[ \zeta_{3} = \left. \frac{1}{\text{SNR}} \right|_{\delta_{0}, \delta_{2}} \chi \left( \frac{c}{\delta_{0} + \sqrt{\delta_{0} \delta_{2}}} \right) \]

\[ \chi_{4} = \left. \frac{2}{(1-\mu)(\eta_{\delta_{0}} - 1)} \right|_{\delta_{0}, \delta_{2}} \]

\[ \zeta_{4} = \left. \frac{1}{\text{SNR}} \right|_{\delta_{0}, \delta_{2}} \chi \left( \frac{c}{\delta_{0} + \sqrt{\delta_{0} \delta_{2}}} \right) \]

\[ \chi_{5} = \left. \frac{\left( \omega_{4} + \frac{1}{\text{SNR}}(1+2\eta_{\delta_{0}})(\eta_{\delta_{0}} - 1) + 2\eta_{\delta_{0}} \right)}{\left( \eta_{\delta_{0}} - 1 \right)^{2}} \right|_{\delta_{0}, \delta_{2}} \]

\[ \zeta_{5} = \left. \frac{1}{\text{SNR}} \right|_{\delta_{0}, \delta_{2}} \chi \left( \frac{c}{\delta_{0} + \sqrt{\delta_{0} \delta_{2}}} \right) \]

\[ \chi_{6} = \left. \frac{2(\eta_{\delta_{0}} - 1)}{(1-\mu)(\eta_{\delta_{0}} - 1)(\eta_{\delta_{0}} - 1)} \right|_{\delta_{0}, \delta_{2}} \]

\[ \zeta_{6} = \left. \frac{1}{\text{SNR}} \right|_{\delta_{0}, \delta_{2}} \chi \left( \frac{c}{\delta_{0} + \sqrt{\delta_{0} \delta_{2}}} \right) \]

\[ \chi_{7} = \left. \frac{2(\eta_{\delta_{0}} - 1)}{(1-\mu)(\eta_{\delta_{0}} - 1)(\eta_{\delta_{0}} - 1)} \right|_{\delta_{0}, \delta_{2}} \]

\[ \zeta_{7} = \left. \frac{1}{\text{SNR}} \right|_{\delta_{0}, \delta_{2}} \chi \left( \frac{c}{\delta_{0} + \sqrt{\delta_{0} \delta_{2}}} \right) \]

\[ \chi_{8} = \left. \frac{\left( \omega_{4} + \frac{1}{\text{SNR}}(1-2\beta_{0})(\eta_{\delta_{0}} - 1) + 2\eta_{\delta_{0}} \right)}{\left( \eta_{\delta_{0}} - 1 \right)^{2}} \right|_{\delta_{0}, \delta_{2}} \]

\[ \zeta_{8} = \left. \frac{1}{\text{SNR}} \right|_{\delta_{0}, \delta_{2}} \chi \left( \frac{c}{\delta_{0} + \sqrt{\delta_{0} \delta_{2}}} \right) \]

\[ \chi_{9} = \left. \frac{\left( \omega_{4} + \frac{1}{\text{SNR}}(1-2\beta_{0})(\eta_{\delta_{0}} - 1) + 2\eta_{\delta_{0}} \right)}{\left( \eta_{\delta_{0}} - 1 \right)^{2}} \right|_{\delta_{0}, \delta_{2}} \]

\[ \zeta_{9} = \left. \frac{1}{\text{SNR}} \right|_{\delta_{0}, \delta_{2}} \chi \left( \frac{c}{\delta_{0} + \sqrt{\delta_{0} \delta_{2}}} \right) \]
In above equations: 
\[ \beta = \eta_d - a \cdot \eta_2 = \left( \frac{2\lambda}{1-\lambda} \right)^{2^\alpha - 1} \cdot \eta_3 = \frac{1+\lambda}{1-\lambda}, \]
\[ a_2 = 2^{2^\alpha} \left( \frac{1+\lambda}{1-\lambda} \right)^{-1}, \quad a_3 = \left( \frac{1+\lambda}{1-\lambda} \right)^{2^{2^\alpha - 1}}, \quad d_1 = \left( \frac{a_1-a_2}{\eta_1-\eta_2} \right) \quad \text{and} \quad d_2 = \left( \frac{a_2-a_3}{\eta_2-\eta_3} \right) \]
where \( \lambda = 2\alpha - 1 \).

Also: 
\[ \sigma_1 = \left( \frac{2\sqrt{\mu}}{\sqrt{\sigma_1 \sigma_2} + 1 + \frac{1}{\delta_2}} \right), \quad \sigma_2 = \left( \frac{2}{\sqrt{\sigma_1 \sigma_2} + 1 + \frac{1}{\delta_2}} \right) \]
\[ \sigma_3 = \frac{\sqrt{\mu} - 1}{\sqrt{\delta_1 \delta_2} (\sqrt{\mu} + 1)} \quad \text{and} \quad \sigma_4 = \frac{\mu + 1}{\mu - 1}. \]

### D.1.4 Outage probability equation for when duplexing ratio is less than half

We have showed that the instantaneous mutual information for \( 0 < \alpha \leq 0.5 \) is as follows:

\[ I(\gamma, \alpha) = \alpha \log \left( 1 + \gamma_0 + \gamma_1 \gamma_2, \left( 1 + \gamma_1 + \gamma_2 \right) \right) \]  
(D-2-7)

By replacing \( \gamma_{AF} = \gamma_1 \gamma_2, \left( 1 + \gamma_1 + \gamma_2 \right) \) where \( \lambda = (2\alpha - 1) \) we have:

\[ I(\gamma, \alpha) = \frac{(1+\lambda)}{2} \log \left( 1 + \gamma_0 + \gamma_{AF} \right) \]  
(D-2-8)

The boundary for the outage will be \( \gamma_{AF} = \gamma_0, \alpha = 2^{2^\alpha} - 1 \). Therefore the outage probability is calculated by using the same equation used for \( \alpha = 0.5 \) except that \( c = 2^{2^\alpha} - 1 \) and \( \delta_2, \delta_2 = \frac{1-\alpha}{\alpha} \delta_1, \delta_2 = \frac{1-\lambda}{1-\lambda} \delta_2 \) is employed instead of \( \delta_2 \).

### D.1.4.1 When Duplexing Factor is less than half without direct link

The outage probability equation for the case that there is no direct link is calculated by replacing \( \delta_0 = 0 \). The outage probability equation will be very simple as follows:

\[ P_{\text{outage}} = 1 - \sum_{j=1}^{2^{2^\alpha}} \hat{c}_j \left( \frac{1}{\text{SNR}} \right) e^{-c_j \left( \frac{1}{\text{SNR}} \right)} \]

(D-2-9)
where

\[ \chi_1 = \frac{2}{(1 - \mu)} \quad \text{and} \quad \zeta_1 = \frac{1}{\text{SNR}} \left( \alpha_c + \frac{\sqrt{\mu}}{\sqrt{\delta_1 \delta_{\text{eff}}}} \right) \quad \text{and} \quad \chi_2 = \frac{1}{\text{SNR}} \alpha \left( 1 + 2 \alpha \right) \]

\[ \zeta_2 = \frac{1}{\text{SNR}} \left( \alpha_c + \frac{1}{\sqrt{\delta_1 \delta_{\text{eff}}}} \right) \quad \text{and} \quad c = 2^{\frac{R(1 + \lambda)}{\alpha}} - 1 \quad \text{and} \quad \delta_{\text{eff}} = \frac{1 - \alpha}{\alpha} \delta_1, \delta_2 = \frac{1 - \lambda}{1 + \lambda} \delta_1, \delta_2. \]

**D.1.4.2 When Duplexing Factor is greater than half without direct link**

In this case we can use the equation (D-2-9) except that \( c = 2^{\frac{R(1 + \lambda)}{\alpha}} - 1 \) and \( \delta_{\text{eff}} = \delta_2. \)
Appendix E

E-1  Mutual Information for n Transmission for Orthogonal AF with UC System

By considering the signal model for the orthogonal AF i.e. $H_j = \begin{bmatrix} \alpha_{o,j} I_{N_1 \times N_1} \\ \alpha_{o,j} \alpha_{x,j} \sqrt{\beta_j} A \end{bmatrix}_{(N_1+N_2) \times N_1}$ and employing the proof from appendix E-2, for any $n$, we can show that:

$$\log \det \left( I + \left( \alpha_{x,j} \right)^2 \beta_j A A^H \right) = \sum_{j=1}^{N} \log \left( 1 + \left| \alpha_{o,j} \right|^2 \right) \quad (E-1-1)$$

Since $\det(\alpha I + \beta A A^H) = \alpha^{N-H} \det(\alpha I + \beta B A)$ (for proof refer to appendix E-5). If $A A^H = I_{N_2}$ and $A^H A = \begin{bmatrix} I_{N_2} & 0_{N_2 \times (N_1-N_2)} \\ 0_{(N_1-N_2) \times N_2} & 0_{(N_1-N_2) \times (N_1-N_2)} \end{bmatrix}$ then

$$\frac{1}{(N_1+N_2)} \log \det \left( I + \left( \alpha_{o,j} \right)^2 \beta_j \right) = \begin{bmatrix} \sum_{j=1}^{N} \log \left( 1 + \left| \alpha_{o,j} \right|^2 \right) \\ \sum_{j=1}^{N} \log \left( 1 + \left| \alpha_{x,j} \right|^2 \beta_j \right) \end{bmatrix} \quad (E-1-2)$$

By substituting $\gamma_{i,j} = \left| \alpha_{i,j} \right|^2$
\[
\frac{1}{(N_1 + N_2)} \log \det \left( I_{n(N_1+N_2)} + HH^* \mathbf{R}_v \right) = (1-\alpha) \sum_{j=1}^{\beta} \log \left( 1 + \gamma_{0,j} + \gamma_{1,j} \gamma_{2,j} \beta_j \left( 1 + \gamma_{2,j} \beta_j \right)^{-1} \right) + (2\alpha-1) \sum_{j=1}^{\beta} \log \left( 1 + \gamma_{0,j} \right)
\]

(E-1-3)

If \( H_n = \begin{bmatrix} \alpha_{n,n} \end{bmatrix}_{N_1 \times N_1} \) and \( \mathbf{R}_{v_j}^{-1} = I_{M \times M} \), then

\[
\log \det \left( I_{n(N_1+N_2)} + HH^* \mathbf{R}_v \right) = \sum_{j=1}^{\beta} \log \det \left( I_{N_1} + |\alpha_{n,j}|^2 \mathbf{I}_n + AA^* |\alpha_{n,j}|^2 |\alpha_{2,j}|^2 \beta_j \left( 1 + |\alpha_{2,j}|^2 \beta_j \right)^{-1} \right) + N_2 \log \left( 1 + |\alpha_{n,j}|^2 \right) + (N_1 - N_2) \sum_{j=1}^{\beta} \log \left( 1 + |\alpha_{n,j}|^2 \right)
\]

(E-1-4)

If \( AA^* = I_{N_1} \) and \( A^* A = \begin{bmatrix} \mathbf{I}_{N_2} & 0_{N_2 \times (N_1-N_2)} \\ 0_{(N_1-N_2) \times N_2} & 0_{(N_1-N_2) \times (N_1-N_2)} \end{bmatrix} \), then

\[
\frac{1}{(N_1 + N_2)} \log \det \left( I_{n(N_1+N_2)} + HH^* \mathbf{R}_v \right) = \sum_{j=1}^{\beta} \log \det \left( 1 + |\alpha_{n,j}|^2 + |\alpha_{2,j}|^2 \beta_j \left( 1 + |\alpha_{2,j}|^2 \beta_j \right)^{-1} \right) + (2\alpha-1) \sum_{j=1}^{\beta} \log \left( 1 + |\alpha_{n,j}|^2 \right) + (1-\alpha) \log \left( 1 + |\alpha_{n,j}|^2 \right)
\]

(E-1-5)

By substituting \( \gamma_{i,j} = |\alpha_{i,j}|^2 \) we have:

\[
\frac{1}{(N_1 + N_2)} \log \det \left( I_{n(N_1+N_1)} + HH^* \mathbf{R}_v \right) = (1-\alpha) \sum_{j=1}^{\beta} \log \left( 1 + \gamma_{0,j} + \gamma_{1,j} \gamma_{2,j} \beta_j \left( 1 + \gamma_{2,j} \beta_j \right)^{-1} \right) + (2\alpha-1) \sum_{j=1}^{\beta} \log \left( 1 + \gamma_{0,j} \right) + (1-\alpha) \log \left( 1 + \gamma_{0,x} \right)
\]

(E-1-6)

E-2 Mutual Information for n Transmission for UC – General Case

Lemma:
During j-th transmission, a sequence $x_j \in \mathbb{C}^{M \times 1}$ is transmitted through the jth complex normal channel $H_j \in \mathbb{C}^{N \times M}$ and then zero mean white Gaussian noise $v_j \in \mathbb{C}^N$ with covariance matrix $R_{v_j}$ is added to the received signal. As a result the signal model for all the transmission will be as follows:

$$
\begin{bmatrix}
    y_1 \\
    y_2 \\
    \vdots \\
    y_n
\end{bmatrix} =
\begin{bmatrix}
    H_1 & 0 & \cdots & 0 \\
    0 & H_2 & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & \cdots & H_n
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2 \\
    \vdots \\
    x_n
\end{bmatrix} +
\begin{bmatrix}
    v_1 \\
    v_2 \\
    \vdots \\
    v_n
\end{bmatrix}
$$

(E-2-1)

Here we want to prove that

$$
\log \det \left( I_{nN} + HH^t R_{v_j}^{-1} \right) = \sum_{j=1}^N \log \det \left( I_N + H_j H_j^t R_{v_j}^{-1} \right)
$$

where $R_{v_j}^{-1} = \begin{bmatrix} R_{v_1} & 0 & \cdots & 0 \\
0 & R_{v_1} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & R_{v_1} \end{bmatrix}$.

Proof:

$$
I_{nN} + HH^t R_{v_j}^{-1} =
\begin{bmatrix}
    I_N + H_1 H_1^t R_{v_1}^{-1} & 0 & \cdots & 0 \\
    0 & I_N + H_2 H_2^t R_{v_2}^{-1} & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & \cdots & I_N + H_n H_n^t R_{v_n}^{-1}
\end{bmatrix}
$$

$$
\log \det \left( I_{nN} + HH^t R_{v_j}^{-1} \right) = \sum_{j=1}^N \log \det \left( I_N + H_j H_j^t R_{v_j}^{-1} \right)
$$

(E-2-2)
E-3 Mutual Information for n Transmission for Orthogonal AF with RC System

By considering the signal model for the orthogonal AF i.e. \( \mathbf{H}_j = \begin{bmatrix} \alpha_{0,j} & \mathbf{I}_{N_1} \\ \alpha_{1,j} & \alpha_{2,j} \sqrt{\beta_j} \mathbf{A} \end{bmatrix} \),

\[
\mathbf{R}_n^{-1} = \begin{bmatrix} \mathbf{I}_{N_1 \times N_1} & \mathbf{0}_{N_1 \times N_2} \\ \mathbf{0}_{N_2 \times N_1} & \left( \mathbf{I} + \left| \alpha_{2,j} \right|^2 \beta_j \mathbf{A} \mathbf{A}^H \right)^{-1} \end{bmatrix}_{(N_1+N_2) \times (N_1+N_2)}
\]

and employing the proof from appendix E-4

for any \( n \), we can show that:

\[
\log \det \left( \mathbf{I}_{N_1+N_2} + \mathbf{H}^H \mathbf{H} \right) = \log \det \left( \mathbf{I}_{N_1} + \sum_{j=1}^{n} \left( \left| \alpha_{0,j} \right|^2 \mathbf{I}_{N_1} + \mathbf{A} \mathbf{A}^H \left| \alpha_{1,j} \right|^2 \left| \alpha_{2,j} \right|^2 \beta_j \left( \mathbf{I}_{N_1} + \left| \alpha_{2,j} \right|^2 \beta_j \mathbf{A} \mathbf{A}^H \right)^{-1} \right) \right) +
\]

\[
(N_1-N_2) \log \left( 1 + \sum_{j=1}^{n} \left| \alpha_{0,j} \right|^2 \right)
\]

(E-3-1)

Since \( \det \left( \mathbf{A} \mathbf{A}^H + \mathbf{B} \mathbf{B}^H \right) = \det \left( \mathbf{A} \mathbf{A}^H \right)^{n} \det \left( \mathbf{B} \mathbf{B}^H \right) \) (for proof refer to appendix E-5). If \( \mathbf{A} \mathbf{A}^H = \mathbf{I}_{N_1} \) and \( \mathbf{A}^H \mathbf{A} = \begin{bmatrix} \mathbf{I}_{N_2} & \mathbf{0}_{N_2 \times (N_1-N_2)} \\ \mathbf{0}_{(N_1-N_2) \times N_2} & \mathbf{0}_{(N_1-N_2) \times (N_1-N_2)} \end{bmatrix} \) then

\[
\frac{1}{(N_1+N_2)} \log \det \left( \mathbf{I}_{N_1+N_2} + \mathbf{H}^H \mathbf{H} \right) = \frac{1}{(N_1+N_2)} \left[ N_1 \log \left( 1 + \sum_{j=1}^{n} \left| \alpha_{0,j} \right|^2 + \left| \alpha_{1,j} \right|^2 \left| \alpha_{2,j} \right|^2 \beta_j \left( 1 + \left| \alpha_{2,j} \right|^2 \beta_j \right)^{-1} \right) \right] +
\]

\[
(N_1-N_2) \log \left( 1 + \sum_{j=1}^{n} \left| \alpha_{0,j} \right|^2 \right)
\]

(E-3-2)

Then:

\[
\frac{1}{(N_1+N_2)} \log \det \left( \mathbf{I}_{N_1+N_2} + \mathbf{H}^H \mathbf{H} \right) = \alpha \log \left( 1 + \sum_{j=1}^{n} \left| \alpha_{0,j} \right|^2 + \left| \alpha_{1,j} \right|^2 \left| \alpha_{2,j} \right|^2 \beta_j \left( 1 + \left| \alpha_{2,j} \right|^2 \beta_j \right)^{-1} \right) +
\]

\[
(2\alpha-1) \log \left( 1 + \sum_{j=1}^{n} \left| \alpha_{0,j} \right|^2 \right)
\]

(E-3-3)

By substituting \( \gamma_{i,j} = \left| \alpha_{i,j} \right|^2 \) we have
Appendix E

\[
\frac{1}{(N_1 + N_2)} \log \det \left( I_{(N_1 + N_2)} + HH^T R_{V}^{-1} \right) = \\
(1 - \alpha) \log \left( 1 + \sum_{j=1}^{n} \left( \gamma_{0,j} + \gamma_{1,j} \beta_j \left( 1 + \gamma_{2,j} \beta_j \right)^{-1} \right) \right) + \\
(2\alpha - 1) \log \left( 1 + \sum_{j=1}^{n} (\gamma_{0,j}) \right)
\]

(E-3-4)

If \( H_n = \left[ \alpha_{0,n} I_{N_1+N_2} \right] \) and \( R_{V}^{-1} = I_{N_1+N_2} \) then:

\[
\frac{1}{(N_1 + N_2)} \log \det \left( I_{(N_1 + N_2)} + HH^T R_{V}^{-1} \right) = \\
= \frac{1}{(N_1 + N_2)} \log \det \left( I_{N_1} + \sum_{j=1}^{n} \left( |\alpha_{0,j}|^2 I_{N_1} \right) + \sum_{j=1}^{n} \left( \alpha A^T |\alpha_{j}|^2 |\alpha_{2,j}|^2 \beta_j \left( I_{N_1} + |\alpha_{2,j}|^2 \beta_j A A^T \right)^{-1} \right) \right) + \\
(N_1 - N_2) \log \left( 1 + \sum_{j=1}^{n} (|\alpha_{0,j}|^2) \right)
\]

(E-3-5)

If \( AA^T = I_{N_1} \) and \( A^T A = \left[ \begin{array}{cc} I_{N_2} & 0_{N_2 \times (N_1 - N_2)} \\ 0_{(N_1 - N_2) 	imes N_1} & 0_{(N_1 - N_2) \times (N_1 - N_2)} \end{array} \right] \) then

\[
\frac{1}{(N_1 + N_2)} \log \det \left( I_{(N_1 + N_2)} + HH^T R_{V}^{-1} \right) = \alpha \log \left( 1 + \sum_{j=1}^{n} (|\alpha_{j}|^2) + \sum_{j=1}^{n} \left( |\alpha_{j}|^2 |\alpha_{2,j}|^2 \beta_j \left( 1 + |\alpha_{2,j}|^2 \beta_j \right)^{-1} \right) \right) + \\
(2\alpha - 1) \log \left( 1 + \sum_{j=1}^{n} (|\alpha_{0,j}|^2) \right)
\]

(E-3-6)

By substituting \( \gamma_{0,j} = |\alpha_{j}|^2 \) we have

\[
\frac{1}{(N_1 + N_2)} \log \det \left( I_{(N_1 + N_2)} + HH^T R_{V}^{-1} \right) = (1 - \alpha) \log \left( 1 + \sum_{j=1}^{n} (\gamma_{0,j}) + \sum_{j=1}^{n} \left( \gamma_{1,j} \gamma_{2,j} \beta_j \left( 1 + \gamma_{2,j} \beta_j \right)^{-1} \right) \right) + \\
(2\alpha - 1) \log \left( 1 + \sum_{j=1}^{n} (\gamma_{0,j}) \right)
\]

(E-3-7)

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Lemma:

During j-th transmission, a sequence \( x_j \in \mathbb{C}^{Mx} \) is transmitted through the jth complex normal channel \( H_j \in \mathbb{C}^{N \times M} \) and then white Gaussian noise \( v_j \in \mathbb{C}^{N} \) with mean zero and covariance matrix \( R_{v_j} \) is added. Here we assume that \( x_j = x \) for all the transmissions. As a result the signal model for all the transmission will be as follows:

\[
\begin{bmatrix}
y_1 \\
y_2 \\
\vdots \\
y_n
\end{bmatrix} =
\begin{bmatrix}
H_1 \\
H_2 \\
\vdots \\
H_n
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_n
\end{bmatrix} +
\begin{bmatrix}
v_1 \\
v_2 \\
\vdots \\
v_n
\end{bmatrix}
\tag{E-4-1}
\]

Here we want to prove that

\[
\log \det \left( I_{nN} + HH^* R_{v_j}^{-1} \right) = \log \det \left( I_N + H_j^* R_{v_j}^{-1} H_j + H_j^* R_{v_j}^{-1} H_j + \ldots + H_j^* R_{v_j}^{-1} H_j \right)
\tag{E-4-2}
\]

First Proof:

\[
det \left( I_{nN} + HH^* R_{v_j}^{-1} \right) = det \left( \begin{bmatrix}
I_N + H_1^* R_{v_1}^{-1} & H_2^* R_{v_1}^{-1} & \ldots & H_N^* R_{v_1}^{-1} \\
H_1^* R_{v_1}^{-1} & I_N + H_2^* R_{v_1}^{-1} & \ldots & H_N^* R_{v_1}^{-1} \\
\vdots & \vdots & \ddots & \vdots \\
H_1^* R_{v_1}^{-1} & H_2^* R_{v_1}^{-1} & \ldots & I_N + H_N^* R_{v_1}^{-1}
\end{bmatrix} \right)
\tag{E-4-3}
\]

\[
= det \left( \begin{bmatrix}
I_{nN} + \Sigma G^* R_{v_1}^{-1} & GH^* R_{v_1}^{-1} \\
H_N^* G^* R_{v_1}^{-1} & I_N + H_N^* R_{v_1}^{-1}
\end{bmatrix} \right)
\]

(E-4-3)
Appendix E

Where: \( \mathbf{R}_{\nu_0}^{-1} = \begin{bmatrix} R_{\nu_1}^2 & 0 & \ldots & 0 \\ 0 & R_{\nu_2}^2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & R_{\nu_r}^2 \end{bmatrix} \) and \( \mathbf{G} = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_{n^2} \end{bmatrix} \)

\[
\begin{align*}
\det\left( \begin{bmatrix} I_{n^2} + G^\dagger R_{\nu_0}^{-1} + G \mathbf{H} \mathbf{R}_{\nu_0}^{-1} & \mathbf{G} \mathbf{H} \mathbf{R}_{\nu_0}^{-1} \\ \mathbf{H} \mathbf{G} \mathbf{H} \mathbf{R}_{\nu_0}^{-1} & I_{n^2} + \mathbf{H} \mathbf{G} \mathbf{H} \mathbf{R}_{\nu_0}^{-1} \end{bmatrix} \right) \\
= \det\left( I_{n^2} + G^\dagger R_{\nu_0}^{-1} \right) \det\left( I_{n^2} + \mathbf{H} \mathbf{G} \mathbf{H} \mathbf{R}_{\nu_0}^{-1} \right) \\
= \det\left( I_{n^2} + G^\dagger R_{\nu_0}^{-1} \right) \det\left( I_{n^2} + G^\dagger R_{\nu_0}^{-1} \left( I_{n^2} + G^\dagger R_{\nu_0}^{-1} \right)^\dagger \mathbf{H} \mathbf{G} \mathbf{H} \mathbf{R}_{\nu_0}^{-1} \right) \\
= \det\left( I_{n^2} + G^\dagger R_{\nu_0}^{-1} \right) \det\left( I_{n^2} + \left( I_{n^2} + G^\dagger R_{\nu_0}^{-1} \right)^\dagger \mathbf{G} \mathbf{H} \mathbf{G} \mathbf{H} \mathbf{R}_{\nu_0}^{-1} \right) \\
= \det\left( I_{n^2} + \mathbf{H} \mathbf{G} \mathbf{H} \mathbf{R}_{\nu_0}^{-1} \right) \det\left( I_{n^2} + \left( I_{n^2} + G^\dagger R_{\nu_0}^{-1} \right)^\dagger \mathbf{G} \mathbf{H} \mathbf{G} \mathbf{H} \mathbf{R}_{\nu_0}^{-1} \right)
\end{align*}
\]

Using matrix inversion lemma:

\[
\begin{align*}
\det\left( I_{n^2} + G^\dagger R_{\nu_0}^{-1} \right) \det\left( I_{n^2} + \left( I_{n^2} + G^\dagger R_{\nu_0}^{-1} \right)^\dagger \mathbf{G} \mathbf{H} \mathbf{G} \mathbf{H} \mathbf{R}_{\nu_0}^{-1} \right) \\
= \det\left( I_{n^2} + \mathbf{H} \mathbf{G} \mathbf{H} \mathbf{R}_{\nu_0}^{-1} \right) \det\left( I_{n^2} + \left( I_{n^2} + G^\dagger R_{\nu_0}^{-1} \right)^\dagger \mathbf{G} \mathbf{H} \mathbf{G} \mathbf{H} \mathbf{R}_{\nu_0}^{-1} \right) \\
= \det\left( I_{n^2} + \mathbf{H} \mathbf{G} \mathbf{H} \mathbf{R}_{\nu_0}^{-1} \right) \det\left( I_{n^2} + \left( I_{n^2} + G^\dagger R_{\nu_0}^{-1} \right)^\dagger \mathbf{G} \mathbf{H} \mathbf{G} \mathbf{H} \mathbf{R}_{\nu_0}^{-1} \right) \\
= \det\left( I_{n^2} + \mathbf{H} \mathbf{G} \mathbf{H} \mathbf{R}_{\nu_0}^{-1} \right) \det\left( I_{n^2} + \left( I_{n^2} + G^\dagger R_{\nu_0}^{-1} \right)^\dagger \mathbf{G} \mathbf{H} \mathbf{G} \mathbf{H} \mathbf{R}_{\nu_0}^{-1} \right)
\end{align*}
\]

Second proof:

\[
\log \det \left( I_{n^2} + \mathbf{H} \mathbf{G} \mathbf{H} \mathbf{R}_{\nu_0}^{-1} \right) = \log \det \left( I_{n^2} + \left( I_{n^2} + G^\dagger R_{\nu_0}^{-1} \right)^\dagger \mathbf{G} \mathbf{H} \mathbf{G} \mathbf{H} \mathbf{R}_{\nu_0}^{-1} \right)
\]

since \( \det \left( I_{n^2} + A_{n^2,n^2} B_{n^2,n^2} \right) = \det \left( I_{n^2} + B_{n^2,n^2} A_{n^2,n^2} \right) \)

\[
I_{n^2} + \mathbf{H} \mathbf{G} \mathbf{H} \mathbf{R}_{\nu_0}^{-1} H = I_{n^2} + \begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_2 & \ldots & \mathbf{H}_{n^2} \\ \mathbf{H}_1 & \ldots & \mathbf{H}_{n^2} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_1 & \mathbf{H}_2 & \ldots & \mathbf{H}_{n^2} \end{bmatrix} \begin{bmatrix} R_{\nu_1}^{-1} & 0 & \ldots & 0 \\ 0 & R_{\nu_2}^{-1} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & R_{\nu_r}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_{n^2} \end{bmatrix}
\]

\[
= I_{n^2} + \mathbf{H}_1 \mathbf{R}_{\nu_1}^{-1} \mathbf{H}_1 + \mathbf{H}_2 \mathbf{R}_{\nu_2}^{-1} \mathbf{H}_2 + \ldots + \mathbf{H}_{n^2} \mathbf{R}_{\nu_{n^2}}^{-1} \mathbf{H}_{n^2}
\]

E-5 Lemma

Lemma: for any \( \alpha \) and \( \beta \) greater than zero we have: \( \det(\alpha I_{n^2} + \beta \mathbf{A} \mathbf{B}) = \alpha^{n^2} \det(\alpha I_{n^2} + \beta \mathbf{A}) \)

Proof:
\[ \det(\alpha I_n + \beta AB) = \det(\alpha I_n) \det(I_n + \frac{\beta}{\alpha} AB) = \alpha^n \det(I_n) \det(\alpha I_n + \beta BA) = \alpha^n \left( \frac{1}{\alpha} \right) \det(\alpha I_n + \beta BA) = \alpha^{n-N} \det(\alpha I_n + \beta BA) \]

**E-6 Characteristic Function for Orthogonal AF + UC**

The characteristic function of \( I_{n,j} \) is \( \Psi_{n,j}(s) = \mathcal{E}\left(e^{-i\omega_j s}\right) \)

\[
\Psi_{n,1}(s) = \mathcal{E}\left(e^{-\left( -\frac{(1-\alpha)\sum_{j=1}^{n-1} \log(1+\gamma_j + \gamma_{j,j}\beta_j(1+\gamma_{j,j}\beta_j)^{-1})}{(2\alpha-1)\sum_{j=1}^{n} \log(1+\gamma_{j,j})} - \log(1+\gamma_{j,j}) \right)} \right)
\]

\[
\Psi_{n,2}(s) = \mathcal{E}\left(e^{-\left( -\frac{(1-\alpha)\sum_{j=1}^{n-1} \log(1+\gamma_j + \gamma_{j,j}\beta_j(1+\gamma_{j,j}\beta_j)^{-1})}{(2\alpha-1)\sum_{j=1}^{n} \log(1+\gamma_{j,j})} - \log(1+\gamma_{j,j}) \right)} \right)
\]

(E-6-1)
Appendix E

E-7 Laplace Inversion Formula

Laplace inversion formula of the characteristic function $\Psi_{\eta_2}(s)$ is as follows:

$$ F_{\eta_2}(R) = \frac{1}{2\pi j} \int_{a-j\infty}^{a+j\infty} \frac{\Psi_{\eta_2}(s)e^{st}}{s} ds \quad (E-7-1) $$

then

$$ \frac{1}{2\pi j} \int_{a-j\infty}^{a+j\infty} \left[ \Psi_{\eta_2}(s) \right] \frac{ds}{s} = \frac{1}{2N} \sum_{n=1}^{N} \left[ Re\left[ \Psi_{\eta_2}(c+je\tau_k) \right] + \tau_k \operatorname{Im}\left[ \Psi_{\eta_2}(c+je\tau_k) \right] \right] $$

More details about the derivation can be found in [78]. We can choose the number of points, N for example 50 and $c = \frac{1}{2}$ and $\tau_k = \tan\left( \frac{(2k-1)\pi}{2N} \right)$ $k = 1, ..., N$.

E-8 Pade Rational Function

$$ \left( \frac{1}{2\pi j} \right)^2 \int_{a-j\infty}^{a+j\infty} \int_{b-j\infty}^{b+j\infty} \frac{\Psi_{n,m}(s_1, s_2)e^{s_1 + s_2}}{s_1 s_2} ds_1 ds_2 = \sum_{i=1}^{M} \sum_{j=1}^{M} \frac{\Psi_{n,m}(\frac{s_i}{R}, \frac{s_j}{\lambda})}{s_i s_j} \quad (E-8-1) $$

where $z_i$ are the poles of the Pade rational function $e^z$ and $K_i$ is corresponding residues as follows:

$$ e^z = \frac{\sum_{i=1}^{N} (M+N-i) \binom{N}{i} z_i^i}{\sum_{i=1}^{M-N} (-1)^i (M+N-i) \binom{M}{i} z_i^i} = \sum_{i=1}^{N} \frac{K_i}{z - z_i} $$

(E-8-2)

for $M > N$
Appendix F

F-1 LDC Structure

Let us consider a point to point MIMO scenario with $M_t$ antenna at the transmitter and $M_r$ antenna at the receiver. This section shows how the equivalent channel matrix $H_{equ}$ is calculated based on the LDC dispersion code and channel matrix $H_{M_r \times M_t}$. The matrix $S$ is transmitted over $M_t$ antenna during $T$ time samples. Let us assume that channel is fixed during these $T$ time samples.

$$S_{TM_t} = \sum_{q=1}^{Q} (s_{R,q} A_q + js_{I,q} B_q) \quad (F-1-1)$$

$A_q$ and $B_q$, $q=1:Q$ are dispersion matrices, $s_{R,q}$ and $s_{I,q}$, $q=1:Q$ are real parts and imaginary parts of original symbols respectively. During $T$ time samples, $Q$ symbols are transmitted.

We define: $A_{q,R} = [A_{R,q}, -A_{I,q}]$, $A_{q,I} = [A_{I,q}, A_{R,q}]$, $B_{q,R} = [-B_{R,q}, -B_{I,q}]$ and $B_{q,I} = [B_{I,q}, B_{R,q}]$, where column-wise vectors $h_{R,n}$ and $h_{I,n}$ are real part and imaginary part of nth row of channel matrix $H_{M_r \times M_t}$, respectively then

$$
\begin{bmatrix}
    y_{R,1} \\
    \vdots \\
    y_{R,M_r} \\
    y_{I,1} \\
    \vdots \\
    y_{I,M_r}
\end{bmatrix} = \sqrt{\frac{\rho}{M_t}} H_{equ}
\begin{bmatrix}
    s_{R,1} \\
    \vdots \\
    s_{R,Q} \\
    s_{I,1} \\
    \vdots \\
    s_{I,Q}
\end{bmatrix} +
\begin{bmatrix}
    v_{R,1} \\
    \vdots \\
    v_{R,M_r} \\
    v_{I,1} \\
    \vdots \\
    v_{I,M_r}
\end{bmatrix} \quad (F-1-2)
$$

where column-wise vectors $y_{R,n}$ and $y_{I,n}$ are real and imaginary part of the received signal at nth receive antenna respectively and column-wise vectors $v_{R,n}$ and $v_{I,n}$ are real and imaginary parts of the noise at nth receive antenna respectively. $\rho$ is the average SNR of the link. As the result the equivalent real channel matrix is given by:
Let us suppose a basic linear MIMO model i.e. \( y = Hx + v \) where \( x \) is the transmitted signal, \( y \) is the received signal and \( H \) and \( v \) are the MIMO complex normal channel and additive white Gaussian noise vector respectively. Let's assume \( \hat{x} = \Gamma y \) as the estimated signal. Matrix \( \Gamma \) should minimise error between the transmitted signal and the estimated signal i.e. \( E\{(x - \hat{x})(x - \hat{x})^H\} \)

As result following orthogonality condition should be satisfied:

\[
E\{(x - \hat{x})y^H\} = 0 \Rightarrow \Gamma = \frac{E\{xy^H\}}{E\{yy^H\}}
\]

(F-2-1)

By replacing \( y = Hx + v \) and considering that the transmitted signal and the noise are independent we have:

\[
E\{yy^H\} = HR_xH^* + R_v
\]

(F-2-2)

\[
E\{xy^H\} = R_xH^*
\]

(F-2-3)

where \( R_x = E\{xx^H\} \) and \( R_v = E\{vv^H\} \) are covariance matrices of the transmitted signal and the noise vector respectively.

As a result:

\[
\Gamma = R_xH^*\left(\frac{R_xH^*}{R_xH^* + R_v}\right)^{-1}
\]

(F-2-4)

Employing the MMSE power allocation method in the flexible AF scheme is straight forward.
F.3 Lagrange method for Section 5.3.1

In this section we like to find optimum solution for following optimization problem using Lagrange method:

Maximizing rate:

\[
\sum_{k=1}^{K} \log_2 \left( \frac{1}{2} (1 + \frac{1}{2} \lambda_{2,k}) + \gamma_k \lambda_k \right) - \log_2 \left( \frac{1}{2} (1 + \frac{1}{2} \gamma_k \lambda_k) \right),
\]

subject to the relay power constraint:

\[
\sum_{k=1}^{K} \left( \gamma_k \left( \frac{1}{2} + \lambda_k \right) \right) \leq T_2.
\]

\(\gamma_k\), \(\lambda_k\) and \(\lambda_{2,k}\) are known and have positive values. Also \(T_2\) and \(K\) are known. We like to find the optimum values for \(\gamma_{2,k}\) \(k = 1, \ldots, K\). It is clear that:

\[
\sum_{k=1}^{K} \log_2 \left( \frac{1}{2} (1 + \frac{1}{2} \lambda_{2,k}) + \gamma_k \lambda_k \right) - \log_2 \left( \frac{1}{2} (1 + \frac{1}{2} \gamma_k \lambda_k) \right) = \sum_{k=1}^{K} \log_2 \left( \frac{1}{2} + \frac{2 \gamma_k \lambda_k \lambda_{2,k}}{1 + \frac{1}{2} \gamma_k \lambda_k} \right)
\]

We define the Lagrange function as:

\[
f (\gamma_{21}, \gamma_{22}, \ldots, \gamma_{2K}) = \sum_{k=1}^{K} \log_2 \left( \frac{1}{2} + \frac{2 \gamma_k \lambda_k \lambda_{2,k}}{1 + \frac{1}{2} \gamma_k \lambda_k} \right) + \omega \left( \sum_{k=1}^{K} \gamma_{2,k} \left( \frac{1}{2} + \lambda_k \right) \right) - T_2
\]

Then we need to find \(\gamma_{2,k}\) for \(k = 1, \ldots, K\) where the gradients \(\frac{\partial f}{\partial \gamma_{2,k}} = 0\) \(k = 1, \ldots, K\). In order to reduce complexity of the following equations the index \(k\) is removed from the next equations.

Therefore \(\frac{\partial f}{\partial \gamma_2} = \frac{2 \gamma_1 \lambda_2}{(1 + \gamma_2 \lambda_2)(1 + \gamma_2 \lambda_2 + \gamma_2 \lambda_2 \lambda_2)} + \omega \left( \frac{1}{2} + \gamma_1 \lambda_1 \right) = 0\) and then

\[2 \gamma_2 \lambda_2 + \omega \left( \frac{1}{2} + \gamma_1 \lambda_1 \right) (1 + \gamma_2 \lambda_2)(1 + \gamma_2 \lambda_2 + 2 \gamma_1 \lambda_2 \lambda_2) = 0.\]

By further simplification of the above equation we have:

\[2 \mu \gamma_2 \lambda_2 \left( \frac{1}{2} + \gamma_1 \lambda_1 \right)^2 + \omega \gamma_2 \lambda_2 \left( 2 \left( \frac{1}{2} + \gamma_1 \lambda_1 \right) + \left( \frac{1}{2} + \gamma_1 \lambda_1 \right) \right) + \mu \left( \frac{1}{2} + \gamma_1 \lambda_1 \right) + 2 \gamma_1 \lambda_2 \lambda_2 = 0.\]
Appendix F

By dividing over \(\frac{1}{2} + \gamma_i \lambda_i\) and \(\mu\) then by substituting \(\mu = -\frac{1}{\omega}\) we have:

\[
2\gamma_2^2 \lambda_2^2 \left(\frac{1}{2} + \gamma_i \lambda_i\right) + 2\gamma_2 \lambda_2 (1 + \gamma_i \lambda_i) + 1 - \frac{2\gamma_2 \lambda_2 \lambda_2}{\left(\frac{1}{2} + \gamma_i \lambda_i\right)} \mu = 0
\]

By solving above equation

\[
\gamma_2 \lambda_2 = \frac{-2(1+\gamma_i \lambda_i) + \sqrt{4(1+\gamma_i \lambda_i)^2 - 8\left(\frac{1}{2} + \gamma_i \lambda_i\right)\left(1 - \frac{2\gamma_2 \lambda_2 \lambda_2}{\left(\frac{1}{2} + \gamma_i \lambda_i\right)} \mu\right)}}{4\left(\frac{1}{2} + \gamma_i \lambda_i\right)}
\]

and then

\[
\gamma_2 \left(\frac{1}{2} + \gamma_i \lambda_i\right) = \frac{\sqrt{4(1+\gamma_i \lambda_i)^2 - 8\left(\frac{1}{2} + \gamma_i \lambda_i\right)\left(1 - \frac{2\gamma_2 \lambda_2 \lambda_2}{\left(\frac{1}{2} + \gamma_i \lambda_i\right)} \mu\right)}}{4\lambda_2} - 2(1+\gamma_i \lambda_i)
\]

Finally the optimum solution is as follows:

\[
\gamma_2 \left(\frac{1}{2} + \gamma_i \lambda_i\right) = \sqrt{\mu \left(\frac{\gamma_2 \lambda_2 \lambda_2}{\lambda_2} \right) + \left(\frac{\gamma_2 \lambda_2 \lambda_2}{2\lambda_2}\right)^2 - \frac{\gamma_2 \lambda_2 \lambda_2}{2\lambda_2} - \frac{1}{2\lambda_2}}
\]  \hspace{1cm} (F-3-3)

or by inserting \(k\) index in the equations and by considering that \(\left(\frac{1}{2} + \gamma_k \lambda_k\right)\gamma_{2k}\) must be zero or a positive value:

\[
\gamma_{2k} \left(\frac{1}{2} + \gamma_k \lambda_k\right) = \max \left(0, \left[\sqrt{\mu \frac{\gamma_k \lambda_k \lambda_k}{\lambda_{2k}} + \left(\frac{\gamma_k \lambda_k \lambda_k}{2\lambda_{2k}}\right)^2 - \frac{\gamma_k \lambda_k \lambda_k}{2\lambda_{2k}} - \frac{1}{\lambda_{2k}}}\right]\right)
\]  \hspace{1cm} (F-3-4)

In order to update \(\mu\) the Newton-Raphson algorithm can be used.

\[
\mu_{\text{new}} = \max \left(\frac{1}{2} \left(\frac{1}{2} + \gamma_k \lambda_k\right) / (\gamma_k \lambda_k \lambda_k) \right) \text{ for } k \in [1,K]
\]  \hspace{1cm} (F-3-5)
\[ \mu_{\text{new}} = \mu_{\text{old}} - \frac{g(\mu_{\text{old}})}{g'(\mu_{\text{old}})} \]

where

\[ g(\mu) = \sum_{k=1}^{K} \left[ \frac{\gamma_{2k} (\frac{1}{2} + \gamma_{1k} \lambda_{1k})}{\lambda_{2k}} - T_2 \right] \]

and therefore

\[ g(\mu) = \sum_{k=1}^{K} \left[ \sqrt{\frac{\mu \gamma_{1k} \lambda_{1k}}{\lambda_{2k}} + \left( \frac{\gamma_{1k} \lambda_{1k}}{2 \lambda_{2k}} \right)^2} - \left( \frac{\gamma_{1k} \lambda_{1k}}{2 \lambda_{2k}} \right) - \frac{1}{2 \lambda_{2k}} \right] - T_2 \]
List of Publications


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