LIQUID - LIQUID EXTRACTION IN AN
OSCILLATING BAFFLE COLUMN.

Thesis submitted for the degree of Doctor of Philosophy
in the Faculty of Engineering of the University of London.

Yao-Tyng Chiu.
Chemical Engineering Department,
Battersea College of Technology,
London.
November, 1966.
ACKNOWLEDGMENTS.

This work was carried out under the supervision of Mr. W. J. Thomas Reader in the Chemical Engineering department of Battersea College of Technology. The author wishes to express his sincere gratitude to Dr. Thomas for his constant help, advice and encouragement. Thanks are due to Battersea College of Technology for their generous grant towards laboratory facilities, computer equipment and the scholarship awarded to the writer.

Helpful discussions with many people took place and in particular grateful acknowledgement is due to Professor W. E. Williams, and Mr. H. Ream.

The assistance of the laboratory technicians in the Chemical Engineering Department in the construction and maintenance of the apparatus is gratefully acknowledged. The author also wishes to thank Mr. W. Sledsieski for his help in photographic matters.
ABSTRACT.

The oscillating baffle column presently described was invented by W. J. Thomas (1). It is a differentially operating continuous counter-current extractor. The experimental work is concerned with establishing the efficiency of the extractor under different conditions of operation up to the flooding point. Oscillating speeds and liquid flow rates were varied while other parameters were maintained constant. A direct comparison with the efficiency of operation of a Rotating Disc Contactor was made by carrying out the identical experiments in each case.

The general differential equations for the extract and raffinate phases in a differential contactor model have been derived by several methods. The relationship between the sign of each term in the differential equation and the direction of phase flow has been considered. The overall mass transfer coefficient in the extract phase, $k_a$, has been measured along the column. It was found that a reversal in the trend in $k_a$ occurs depending on the speed of oscillation. At low oscillating speed $k_a$ is larger at the bottom of the column than at the top and at high oscillating speeds the reverse is true.

Three methods of calculating the longitudinal mixing coefficient from experimental data have been employed. The method using
integration by parts is considered to be the most accurate and reliable.

A dye injection technique was used to experimentally determine the C curve for different operating conditions for the Oscillating Baffle and the Rotating Disc Columns. A gamma function was used to relate the C curve to the variance and the longitudinal dispersion coefficient.
Chapter 1 - Introduction                                                                                           Page.

Chapter 2 - Brief description of counter-current extractors.                                                        3

I. Gravity type - no mechanical aid                                                                                3
   a. Wetted wall column                                                                                            3
   b. Spray towers                                                                                                3
   c. Baffle towers                                                                                               4
   d. Packed towers                                                                                                5
   e. Perforated plate towers                                                                                      6

II. Gravity type - mechanical aid                                                                                   6
   a. Rotary annular extractor                                                                                        6
   b. Rotating Disc Column                                                                                        7
   c. Mixco (Oldshue-Rushton) contactor                                                                            7
   d. Sheibel extractors                                                                                        8
   e. Pulse column                                                                                                 8

III. Centrifugal type extractors                                                                                     8
Chapter 3 - Theory

I. Equipment performance
   a. operating condition - steady state
   b. number of transfer units
   c. "True" number of transfer units
   d. "Apparent" number of transfer units

II. Differential equation of a true differential contactor model
   a. Case I ($\rho_{\text{organic}} > \rho_{\text{water}}$)
   b. Case II($\rho_{\text{organic}} < \rho_{\text{water}}$)

III. Differential equation based on Fick's law
   a. Case I ($\rho_{\text{organic}} > \rho_{\text{water}}$)
   b. Case II($\rho_{\text{organic}} < \rho_{\text{water}}$)

IV. Differential equation based on Miyauchi et. al.

V. Examination of the validity of the differential equation

VI. Longitudinal mixing
VII. Mean residence time study,

a. Distribution of residence time.

b. C-curve and F-curve.

c. Relationship of C, F, I and E curves

d. Measurement of the deviation of fluid in a vessel from ideality

(1) Hold-back
(2) Segregation.

e. Mean and variance of C-curve.

f. The relationship between exit age distribution function and its variance.

g. Relationship between the longitudinal dispersion coefficient and variance of the mean residence time distribution function.

Chapter 4 - Scope of experimental study.

I. Choice of liquid system.

II. Pilot plant experiments.

III. Equilibrium.
IV. Experimental determination of equilibrium values.

1. Automatic titrator.

2. Technique.

3. Results.

V. Correlation of equilibrium results.

VI. Experimental programme.

1. Mass transfer study,

2. Investigation of the longitudinal mixing.


4. Investigation of the continuous phase residence time distribution in the extracting column.

Chapter 5 - Description of pilot plant.

I. General apparatus.

II. Oscillating Baffle Column

1. Column.

2. The internal baffle.

3. Driving force mechanism for the oscillating system.
III. Rotating Disc Contactor.

1. Internal baffle.

2. Driving force for shaft in Rotating Disc Contactor.

IV. Discussion of model of O.B.C. and R.D.C.

Chapter 6 - Experimental technique.

I. General technique.

II. Sampling.

III. Mass transfer runs.

IV. Resident time experiments.

(i) Apparatus and technique.

1. Tracer.

2. Inlet signal.

3. Outlet signal.

4. Absorptiometer.

(ii) Resident time runs.

Chapter 7 - Experiment results of mass transfer in Oscillating Baffle Column.

I. Method of calculation of $\left( H_{oy} \right)_{M}$, $(K_a)_{av}$, and $\left( H_{oy} \right)_{M}$ and its results.

II. Variation of $K_a$ along the column and its results.
III. Experimental values for eddy diffusion coefficient based on the mass transfer runs

1. Method one. 132
2. Method two. 133
3. Method three. 137

IV. Experimental results for mean residence time runs. 145

V. Discussion 154

Chapter 8 Experiment results of mass transfer in Rotating Disc Contactor.

I. Results of \((N_{Oy})_M\), \((K_a)_{av}\), and \((H_{Oy})_M\). 154

II. Results of variation of \(K_a\) along the column 176

III. Results of eddy diffusion coefficient based on mass transfer runs. 176

IV. Experimental results for mean residence time runs. 176

Chapter 9 Comparison of experimental results of ODC and RDC 198

Chapter 10 Summary and conclusion 203

Nomenclature.

References.
Appendix 1. The relationship between C curve and its variance.

Appendix 2. The solution of Fick's second law.

Appendix 3. The relationship between variance (σ²) and D/UL from a Gamma function.

Appendix 4. The best curve fitting for equilibrium curve.

Appendix 5. Examples of calculation and its computer programme.

Appendix 6. An example of examining the validity of differential equation by using Solatron "274" Analogy system Computer.
<table>
<thead>
<tr>
<th>Figure number</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>11</td>
</tr>
<tr>
<td>3.2</td>
<td>17</td>
</tr>
<tr>
<td>3.3</td>
<td>17</td>
</tr>
<tr>
<td>3.4</td>
<td>22</td>
</tr>
<tr>
<td>3.5</td>
<td>22</td>
</tr>
<tr>
<td>3.6</td>
<td>26</td>
</tr>
<tr>
<td>3.7</td>
<td>32</td>
</tr>
<tr>
<td>3.8</td>
<td>32</td>
</tr>
<tr>
<td>3.9</td>
<td>35</td>
</tr>
<tr>
<td>3.10</td>
<td>36</td>
</tr>
<tr>
<td>3.11</td>
<td>40</td>
</tr>
<tr>
<td>3.12</td>
<td>42</td>
</tr>
<tr>
<td>4.1</td>
<td>54</td>
</tr>
<tr>
<td>4.2</td>
<td>57</td>
</tr>
<tr>
<td>5.1</td>
<td>64</td>
</tr>
<tr>
<td>5.2</td>
<td>66</td>
</tr>
</tbody>
</table>
5.3 The column

5.4 The OBG internal baffle

5.5 The RDC internal baffle

5.6 Flow pattern in RDC predicted by Reman

5.7 Flow pattern observed in RDC by Kung
( rotor speed $< 300 \text{ ft/m}$ )

5.8 Flow pattern observed in RDC by Kung
( rotor speed $> 300 \text{ ft/m}$ )

5.9 Flow pattern of Oscillating Baffle Column

6.1 The rotameter calibrating curve of organic phase

6.2 The rotameter calibrating curve of aqueous phase

6.3 The calibrating curve of oscillating baffle speed

6.4 The calibrating curve of rotating disc speed

6.5 The calibrating end point of titration

6.6 The sampling arrangement

6.7 The arrangement of the sample cock

6.8 Residence time measuring equipment

6.9 The absorptiometer

6.10 The optical system of the absorptiometer

6.11 The photocell electric circuit
<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1 to</td>
<td>The concentration profile of OBC mass transfer run</td>
</tr>
<tr>
<td>7.11</td>
<td>110 to</td>
</tr>
<tr>
<td>7.12 to</td>
<td>The concentration profile of RDC mass transfer run</td>
</tr>
<tr>
<td>7.23</td>
<td>161 to 1.72</td>
</tr>
<tr>
<td>7.24 to</td>
<td>The variation of $K_a$ value along the column in OBC mass transfer run</td>
</tr>
<tr>
<td>7.32</td>
<td>123 to 1.31</td>
</tr>
<tr>
<td>7.33 to</td>
<td>The variation of $K_a$ value along the column in RDC mass transfer run</td>
</tr>
<tr>
<td>7.42</td>
<td>130 to 1.38</td>
</tr>
<tr>
<td>7.43</td>
<td>The actual recorder traces of dye concentration in the aqueous phase at OBC single phase run</td>
</tr>
<tr>
<td>7.44</td>
<td>146</td>
</tr>
<tr>
<td>7.45</td>
<td>The actual recorder traces of dye concentration in the aqueous phase at OBC two phases run</td>
</tr>
<tr>
<td>7.46</td>
<td>147</td>
</tr>
<tr>
<td>7.47</td>
<td>$D$ versus oscillating speed in ODC single phase run</td>
</tr>
<tr>
<td>7.48</td>
<td>152</td>
</tr>
<tr>
<td>7.49</td>
<td>$D$ versus oscillating speed in OBC two phases run</td>
</tr>
<tr>
<td>7.50</td>
<td>153</td>
</tr>
<tr>
<td>7.51</td>
<td>$(H_{OY})_H$ versus oscillating speed in ODC low oscillating speed run</td>
</tr>
<tr>
<td>7.52</td>
<td>157</td>
</tr>
<tr>
<td>7.53</td>
<td>The actual recorder traces of dye concentration in the RDC single phase run</td>
</tr>
<tr>
<td>7.54</td>
<td>194</td>
</tr>
<tr>
<td>7.55</td>
<td>The actual recorder traces of dye concentration in the RDC two phases run</td>
</tr>
<tr>
<td>7.56</td>
<td>195</td>
</tr>
<tr>
<td>7.57</td>
<td>$(H_{OY})_H$ versus oscillating speed in OBC high oscillating speed run</td>
</tr>
<tr>
<td>7.58</td>
<td>196</td>
</tr>
</tbody>
</table>
7.51 \((H_{OY})_M\) versus oscillating speed in RDC rotating speed run.

8.1 Mean residence time versus oscillating speed in OBC constant flow rate ratio runs.

8.2 Mean residence time versus rotating speed in RDC 30 l/hr constant flow rate ratio run.

8.3 \(H_{OY}\) versus mean residence time in OBC 30 l/hr constant flow rate ratio run

8.4 \(H_{OY}\) versus mean residence time in RDC 30 l/hr constant flow rate ratio run
LIST OF TABLES.

<table>
<thead>
<tr>
<th>Table No.</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>The important physical properties of the carbon tetrachloride-acetic acid-water system</td>
<td>51</td>
</tr>
<tr>
<td>4.2</td>
<td>The equilibrium experimental result.</td>
<td>56</td>
</tr>
<tr>
<td>7.1</td>
<td>The experimental results of O.B.C. mass transfer run.</td>
<td>97-99</td>
</tr>
<tr>
<td>7.2</td>
<td>The results of the variation of $K_a$ along the column in O.B.C. mass transfer run.</td>
<td>113-115</td>
</tr>
<tr>
<td>7.3</td>
<td>The experimental results of R.D.C. mass transfer run.</td>
<td>159-160</td>
</tr>
<tr>
<td>7.4</td>
<td>The results of the variation of $K_a$ along the column in R.D.C. mass transfer run</td>
<td>174</td>
</tr>
<tr>
<td>7.5</td>
<td>The results of $(N_{oy})<em>M$, $(H</em>{oy})_M$, and $K_a$ in O.B.C. mass transfer run.</td>
<td>117-122</td>
</tr>
<tr>
<td>7.6</td>
<td>The results of $(N_{oy})<em>M$, $(H</em>{oy})_M$, and $K_a$ in R.D.C. mass transfer run.</td>
<td>177-178</td>
</tr>
<tr>
<td>Chapter</td>
<td>Description</td>
<td>Pages</td>
</tr>
<tr>
<td>---------</td>
<td>------------------------------------------------------------------------------</td>
<td>---------</td>
</tr>
<tr>
<td>7.7</td>
<td>The results of $E_y$ in O.B.C. mass transfer run.</td>
<td>138-144</td>
</tr>
<tr>
<td>7.8</td>
<td>The results of $E_y$ in R.D.C. mass transfer run.</td>
<td>189-192</td>
</tr>
<tr>
<td>7.9</td>
<td>The results of mean residence time study in O.B.C. run.</td>
<td>149-151</td>
</tr>
<tr>
<td>7.10</td>
<td>The results of mean residence time study in R.D.C. run.</td>
<td>196-197</td>
</tr>
</tbody>
</table>
CHAPTER I

Introduction

Extraction is a process for separating a homogeneous mixture of two or more substances into its components or groups of components by using a liquid which is partially or wholly immiscible with the original solution.

There is now a considerable number of extractors available. Some of which are described later. Of particular interest are those in which extraction is aided by some mechanical means of agitation. The two types which have received most attention in recent times are the Rotating Disc Contactor and the Pulsed Column. The present study is concerned with the investigation of the performance of a new extractor (1) and a comparison with a Rotating Disc Contactor working under identical conditions.

The new contactor is a mechanical device which induces an oscillation in the liquid system in a vertical column. The constructional details are given later in the thesis but it is of value to mention here the mode of operation.

The oscillation of a perforated baffle in the flowing liquids produces a dispersion of one liquid phase into the other. The clockwise action only partially moves the liquid
buck as there is a slip of liquid through the perforations. This is also true in the anti-clockwise direction. Turbulence is induced in the liquid by virtue of the bulk movement, by the movement of liquid through the perforations and by the change of direction of the oscillation. This induced turbulence provides energy required for the break-up of the dispersed phase into droplets. The extent of oscillation is a significant factor and the equipment presents opportunities for better control. The amplitude and speed of oscillation, the hole sizes and number, and various other design features can be adjusted to meet the needs of the physical properties of the system.

Before proceeding to describe the experiments carried out it is worth while considering briefly the types of extraction equipment generally available.
CHAPTER 2

Brief description of counter current extractors.

A general discussion of extractors is given by Treybal (2) but for convenience a short summary is given below. They can be broadly divided into two classes — gravity and centrifugal.

The gravity types can be further subdivided into those which are mechanically aided and those which are not. It is obvious that in gravity types the heavy phase is introduced at the top of the equipment and the light phase at the bottom.

I. Gravity types — no mechanical aid.

a. Wetted-wall column.

In such cases a thin film of heavy phase liquid flows along the inside of a narrow vertical circular tube countercurrent to a light phase central core. In common with other wetted wall apparatus, the main advantage is the known interfacial area presented for mass transfer. A disadvantage of this type is that they are difficult to operate due to instability of the interface and because of wall effect factors. Some example of the use of wetted-wall column are given by Murpy et al (3), Murdoch et al (4) and Treybal et al (5).

b. Spray towers.

These are the simplest kind of extractor being
empty shells. Means are provided for introducing and removing the phases at the top and bottom. The position of the interface is regulated by a loop arranged at the heavy phase outlet and pressure drop is regulated at the heavy phase outlet. Flemming and Johnson (6, 7) give results of experiments in spray towers. Other workers (8, 9, 10, 11) discuss the various problems of dispersing and mixing.

Longitudinal mixing is due to a combination of turbulent eddy mixing in the axial direction and to a back-mixing of the continuous phase induced by the dispersed phase droplets. This longitudinal mixing of the continuous phase can be appreciable and leads to reduced extraction rates. As shown by Flemming and Johnson (6, 7) the recirculating effect can be limited by operating at high flow rates when the hold-up and interfacial area are large.

c. Baffle towers.

These are cylindrical columns containing horizontal baffles to direct the flow of the liquids. There are three principal types; disc-and-doughnut, side to side and centre to side.

The baffles increase the residence time of the phase, and increases the mass transfer by virtue of the coalescence and redispersion of the discontinuous phase. The dispersed
liquid flows along the baffle in a thin film and then leaves the edge of the baffle in a broken sheet.

Morello and Proffenberger (12) have listed a few such installations but little information of a basic character are available. Generally it can be said that they have low capacity and a limited interfacial area.

d. Packed towers.

The packing serves to disperse the phase so providing the necessary larger interfacial area for a reasonable transfer rate, and also prevents vertical recirculation of the continuous phase which would give rise to poor extraction efficiencies.

A nozzle or distributor initially breaks up the dispersed phase into droplets. The packing material is chosen such that it is wetted preferentially by the continuous phase to limit the coalescence of the dispersed phase droplets.

The extent of axial mixing is much less than that for spray tower as shown by Vermeulen and Miyauchi (13,14,15) using a kerosene-mineral oil-water system in sphere and ring type packing. According to Vermeulen et al the corrected height of a transfer unit in the continuous phase may be as low as 15% of the uncorrected value due to longitudinal mixing. Pratt
et al (16,17,18,19) have reported work on hold-up and phenomena in packings.

e. **Perforated plate towers.**

Dispersion is achieved by a phase passing through the perforations of a series of plates. Coalescence occurs partially between each plate and the phase is redispersed. The repeated dispersion enhances the extraction rate. The continuous phase flows across the plate and passes from plate to plate by means of downcomers. It would appear that the hole diameter and perforated area have little effect on efficiency but affect the capacity (20). The column construction is similar to a distillation column. Examples are given by Treybal (2), Grunewald et al (21) and others (22,23,24,25,26,27). The columns are fairly expensive to fabricate and other problems of hole blockage under industrial conditions have to be considered.

II. **Gravity type - mechanical aid.**

a. **Rotary annular extractor.**

This is the simplest type consisting of concentric cylinders. The inner cylinder rotates and sets up eddy forces which disperse one phase in the other. Tayler et al (28), Davis et al (29) and Thornton et al (30) have carried out experiments with small scale equipment. They considered such
aspects as rotor speed on extraction on hold-up. No references seem to be available on industrial scale apparatus.

b. Rotating disc contactor.

This type of contactor has found increasing use in the last few years. It was originally developed by the Shell Company (31). The contactor comprised a shell containing a central rotor. The rotor is made up of a shaft on which are mounted a series of horizontal circular discs. The column is divided into a series of compartments by lateral baffles a rotor blade or disc to each compartment.

A number of workers (31,32,33,34,35) carried out the work of the contactor. Most work has been carried out on laboratory scale column (approx. 3" diameter) to investigate the effect of flow conditions and phase ratio at different rotor speeds on separating efficiency, hold-up and capacity.

The contactor will be discussed in greater detail later in the present work and therefore will not be considered further at this point.

c. Mixco Lightnin Contactor (Oldshue-Rushton).

This type is similar to Rotating Disc contactor but instead of the flat rotating discs it consists of turbine impellers. Although commercial application has been made, most data is available for small scale apparatus (37).
d. **Scheibel extractors.**

These are well described by Treybal (2). The early type still recommend for column with diameter less than 1 ft. consists of paddle type impellers alternating with short packed section. The packing is of woven wire mesh. The newer type consists of impellers and stator rings but not packing. The new design has been described by Scheibel (38,39). Most work refers to laboratory scale and although there have been some large scale industrial use, few references are available.

e. **Pulse column.**

In this column a rapid reciprocating vertical action is given to a series of perforated plates mounted on a central shaft or the same effect is attained by fixing the plates and pulsing the liquid contents.

Extensive work has been carried out to investigate the performance (40,41,42,43,44,45,46,47). The column has been used fairly extensively in the field of extraction of radioactive material. Especially reference should be made to Thornton (43) and Marr and Babb (45).

III. **Centrifugal type extractors.**

Of these extractors the most popular are those of Podbielniak (48) and Luwesta (49, 50). As the title suggests the dispersion is achieved by centrifugal action in pump-like
machine. Some advantages are a minimum use of expensive solvent with a small quantity of raffinate for subsequent reworking where small residence times are vital to avoid chemical destruction of the product no other type of contactor can compete.

There is the obvious advantage of space saving. Disadvantages are the high initial and operating costs and the limited capacity under certain circumstances.
CHAPTER 3

Theory

I. Equipment performance.

Reference should be made to Treybal (2), Bird et al (51), Sherwood and Pigford (52) and Miyauchi and Vermeulen (53,14) for elementary theoretical treatment of mass transfer.

a. Operating condition — steady state.

Consider two immiscible phases which are contacted countercurrently with the transfer of a solute. If we restrict the development for convenience to an organic (heavy) phase and a water (light) phase, then according to our definition of $x$ and $y$ the appropriate concentrations at the ends of the column and over a differential section are given in Fig. 3.1.

Taking a material balance between $z=0$ and $z=z$ then,

$$V_y y_B + V_x x = V_x x_B + V_y y \quad \ldots \ldots \ldots (1)$$

Rearranging,

$$y = \frac{V_x}{V_y} \cdot (x - x_B) - y_B \quad \ldots \ldots \ldots (2)$$

If $R = \text{solvent ratio}$, $y_B = 0$ (pure solvent) then from Eqn. (2), we have;

$$y = \frac{1}{R} \cdot (x - x_B)$$
Fig. 3.1 Differential contact model
At the top of the column
\[ y_T = \frac{1}{R} \cdot (x_T - x_B) \] ........ (3)

A further simplification can be made when \( R=1 \), then
\[ y = x - x_B \] ........ (4)

Equations (3) and (4) define the operating condition in the column and give a line when \( x_B \) is a constant under steady state.

b. Number of transfer units.

From a consideration of the mass transfer of solute occurring over the differential slice \( dz \) we have;

Organic phase:
\[ dN = K_x a \cdot A (x - x_e) dz \]
\[ = (K_a)_x \cdot A (x - x_e) dz \] ........ (5)

Water phase:
\[ dN = K_y a \cdot A (y_e - y) dz \]
\[ = (K_a)_y \cdot A (y_e - y) dz \] ........ (6)

A material balance on the solute over the differential section gives:
\[ dN = v_x \cdot dx \] ........ (7)
\[ = v_y \cdot dy \] ........ (8)

By substitution of \( dN \) from Eqn.(5) into Eqn.(7) and Eqn.(6) into Eqn.(8) and integrating between appropriate
limits, we have:

\[
\frac{( Ka )}{V_x} \int_{z=0}^{z=h} \frac{dz}{dx} = \int_{x_B}^{x_T} \frac{dx}{x-x_e} = N_{ox} \tag{9}
\]

\[
\frac{( Ka )}{V_y} \int_{z=0}^{z=h} \frac{dz}{dy} = \int_{y_B}^{y_T} \frac{dy}{y_e-y} = N_{oy} \tag{10}
\]

Where \( N_{ox} \) and \( N_{oy} \) are the number of overall transfer units. The heights of a transfer unit are given by

\[
H_{ox} = \frac{h}{N_{ox}} = \frac{( Ka )}{V_x} \tag{11}
\]

\[
H_{oy} = \frac{h}{N_{oy}} = \frac{V_y}{( Ka )} \tag{12}
\]

Miyauchi (53) referred to the transfer units as given in Eqns. (9) and (10) as the "Measured" number of transfer units. So that strictly speaking from Eqns. (9), (10), (11) and (12), we have

\[
(N_{oy})_M = \int_{y_B}^{y_T} \frac{dy}{y_e-y} = \frac{( Ka )}{V_y} h \tag{13}
\]

and

\[
(N_{ox})_M = \int_{x_B}^{x_T} \frac{dx}{x_e-x} = \frac{( Ka )}{V_x} h \tag{14}
\]

Two other definitions of numbers of transfer units were introduced by Miyauchi for purpose of comparison.

c. "True" number of mass transfer units \(( N_{ox})_T, (N_{oy})_T\)

For any defined equilibrium condition and piston
flow i.e. no back mixing, Miyauchi proposes the following relationship:

\[
\begin{align*}
(N_{ox})_T &= (Ka)_x \frac{h}{V_x} \\
(N_{oy})_T &= (Ka)_y \frac{h}{V_y}
\end{align*}
\] ........ (15)

Where \((Ka)_x\), \((Ka)_y\) are the so defined "true" overall mass transfer coefficients based on the ideal condition.

d.

"Apparent" number of transfer units \((N_{ox})_p\), \((N_{oy})_p\)

If the operating and equilibrium conditions are linear that is

\[y_e = mx\]

and Eqn. (4) applies, then Eqns. (9) and (10) can be integrated to give a log mean relationship

\[
(N_{ox})_p = \frac{x_T - x_B}{(\Delta x)_{lm}} \\
(N_{oy})_p = \frac{y_T - y_B}{(\Delta y)_{lm}}
\] ........ (17)

where

\[
(\Delta x)_{lm} = \frac{(x_T - x_{eT}) - (x_B - x_{eB})}{\ln \left\{ (x_T - x_{eT})/(x_B - x_{eB}) \right\}}
\]

\[
(\Delta y)_{lm} = \frac{(y_{eT} - y_T) - (y_{eB} - y_B)}{\ln \left\{ (y_{eT} - y_T)/(y_{eB} - y_B) \right\}}
\]
When conditions are such as to give "piston flow" then it is clear that \( N_{oy}'_M = N_{oy}'_T \). The deviation from piston flow due to back mixing is represented by the difference in these quantities as shown in the following equation where the correction term is introduced to allow for the deviation due to back mixing.

\[
N_{ox}'_T = N_{ox}'_M + \text{(correction term)}_x \quad \cdots \quad (19) \\
N_{oy}'_T = N_{oy}'_M + \text{(correction term)}_y \quad \cdots \quad (20)
\]

It will be shown later how the value of the correction term, i.e., the extent of back mixing obtained independently of Eqns. (19) and (20).
II. Differential equations of a true differential contactor model.

To satisfy this condition there must be a continuous change in concentrations of solute in each phase along the length of the contactor in a counter-current system. For simplicity if we consider the countercurrent flow of an organic and aqueous phase, transfer of mass can occur in three ways (i) by convection, (ii) by longitudinal mixing and (iii) by interfacial mass transfer. It is important to appreciate the fact that the signs preceding each of the terms representing the conditions in the contactor depend upon the direction of flow of the phases relative to the ends of the column or contactor. It is felt that this point is insufficiently emphasised in the literature.

a. Cases I - (organic > water)

Consider a situation described by Fig. 3.2 where x and y represent the organic and aqueous phases respectively. Solute is being extracted from the organic phase into water as the two immiscible solvents flow countercurrently through a contactor. Longitudinal mixing occurs in a direction against the concentration gradient in respective of phase. It can be seen from Fig. 3.2 that for the organic phase the direction of
Fig. 2.1 Differential Contact Model For Sp.gr. of organic phase Larger than that of water phase.

Fig. 2.3 Differential Contact Model For Sp.gr. of Water phase Larger than that of organic phase.
longitudinal mixing is the same as the direction of phase flow, whereas for the aqueous phase the longitudinal mixing occurs in a direction opposite to the phase flow. As indicated in the present case under consideration the organic phase enters at the top of the column \((z = h)\), and the aqueous phase enters at the bottom \((z = 0)\).

Taking a material balance over a differential slice \(dz\), we have:

**Organic phase:**

Inlet \(V_x \cdot (x+dx) + \left\{ -E_x \cdot \frac{d(x+dx)}{dz} \right\} \)

Outlet \(V_x \cdot x + \left\{ -E_x \frac{dx}{dz} \right\} + (Ka) \cdot x \cdot (x-x_e) \cdot dz \)

**Aqueous phase:**

Inlet \(V_y \cdot y + \left\{ -E_y \cdot \frac{d(y+dy)}{dz} \right\} + (Ka) \cdot y \cdot (x-x_e) \cdot dz \)

Outlet \(V_y \cdot (y+dy) + \left\{ -E_y \cdot \frac{dy}{dz} \right\} \)

Under steady state conditions there is no accumulation in the differential slice so that from the above equations, we have:

**Organic phase:**

\[ V_x \cdot (x+dx) + \left\{ -E_x \cdot \frac{d(x+dx)}{dz} \right\} = V_x \cdot x + \left\{ -E_x \cdot \frac{dx}{dz} \right\} + (Ka) \cdot x \cdot (x-x_e) \cdot dz \quad \ldots \ldots \ldots \quad (21) \]

**Aqueous phase:**

\[ V_y \cdot y + \left\{ -E_y \cdot \frac{d(y+dy)}{dz} \right\} + (Ka) \cdot y \cdot (x-x_e) \cdot dz = V_y \cdot (y+dy) + \left\{ -E_y \cdot \frac{dy}{dz} \right\} \quad \ldots \ldots \ldots \quad (22) \]
Expanding equations (21) and (22), we have:

**Organic phase**

\[ E_x \frac{d^2x}{dz^2} - V_x \frac{dx}{dz} + (Ka)_x \cdot (x-xe) = 0 \quad \cdots \quad (23) \]

**Aqueous phase**

\[ E_y \frac{d^2y}{dz^2} + V_y \frac{dy}{dz} - (Ka)_x \cdot (x-xe) = 0 \]

As \((Ka)_x \cdot (x-xe) = (Ka)_y \cdot (ye-y)\), then for the aqueous phase we have:

\[ E_y \frac{d^2y}{dz^2} + V_y \frac{dy}{dz} - (Ka)_y \cdot (ye-y) = 0 \quad \cdots \quad (24) \]

**b. Case II \((f_{\text{water}} > f_{\text{organic}})\)**

This is represented by Fig. 3.3. Here the organic phase enters at the bottom of the column \((z=0)\) and the aqueous phase at the top \((z=h)\).

Carrying out a material balance over a differential slice \(dz\), we have:

**Organic phase:**

\[
\begin{align*}
\text{Inlet} & \quad V_x \cdot x + (-E_x \frac{dx}{dz}) \\
\text{Outlet} & \quad V_x (x+dx) + (-E_x \frac{d(x+dx)}{dz}) + (Ka)_x \cdot (x-xe) \quad \cdots \quad (25)
\end{align*}
\]

Under steady state equating equations (25) and (26), we have

\[ E_x \frac{d^2x}{dz^2} - V_x \frac{dx}{dz} - (Ka)_x \cdot (x-xe) = 0 \quad \cdots \quad (29) \]
Aqueous phase:

\[
\begin{align*}
\text{Inlet: } & \quad V_y (y+dy) + (-E_y \frac{dy}{dz}) + (ka)_y \cdot (ye-y)dz \ldots \quad (27) \\
\text{Outlet: } & \quad V_y \cdot y + (-E_y \frac{d(y+dy)}{dz}) \ldots \quad (28)
\end{align*}
\]

Under steady state equating equations (27) and (28), we have

\[
\frac{E_y}{y} \frac{d^2y}{dz^2} + V_y \frac{dy}{dz} + (ka)_y \cdot (ye-y) = 0 \quad \ldots \quad (30)
\]

III Differential equation based on Fick's Law

Fick (54) first proposed the formulation of diffusion rates. He observed that for a given temperature and pressure the rate of transfer appeared to be proportional only to the concentration gradient. He thereby established the relationship known as Fick's law, which for unidirectional diffusion flow may be written as:

\[
J = -E \frac{\partial c}{\partial z}
\]

Where \( J \) is the rate of material transfer per unit area and \( \frac{\partial c}{\partial z} \) is the concentration gradient.

Consider an infinitesimal column height interval \( dz \) of the contactor. As mentioned previously, during steady state of each phase within the column the mass transfer can be considered as occurring:

1. by convection, \( M_c \)
ii. by longitudinal mixing, $M_L$.

iii. by the interfacial mass transfer, $M_i$.

Now, the amount of $M_c$, $M_L$ and $M_i$ are defined as $V \cdot dc$, $-E \frac{d(dc)}{dz}$ and $(Ka)_y \cdot (y_e - y)$ respectively. The sign of each mass transfer term is dependent on its transfer direction. Consider $M_c$ and $M_L$ are positive, if the transfer direction is from the bottom to the top of the column. $M_i$ is positive, if the mass transfer is from the other phase into the phase and vice versa.

With reference to Fig. 3.4, the mass transfer within the contactor can $- (\rho_{organic}/\rho_{water})s$:

a. Case I - $(- (\rho_{organic}/\rho_{water})$)

i. Aqueous phase:

Due to the phase flow upward, the organic phase flows downward and the solute is transferred from the organic phase to water, the concentration gradient of the phase increases with increasing height of the column (positive direction), we have:

$$M_c = V_y \cdot dy$$

and

$$M_i = + (Ka)_y \cdot (y_e - y)dz$$

The longitudinal mixing is in the opposite direction to
Fig. 3.4
Differential contact model I

Fig. 3.5
Differential contact model II
the concentration gradient, thus we have;

\[ M_L = -J = + E_y \frac{d(dy)}{dz} \]

The mass balance within the phase of an infinitesimal column height interval \( dz \) will be;

\[ M_i = M_c + M_L \]

or

\[ (Ka)_y \cdot (ye-y) dz = V_y \cdot dy + E_y \frac{d(dy)}{dz} \]

or,

\[ (Ka)_y \cdot (ye-y) = V_y \frac{dy}{dz} + E_y \frac{d^2y}{dz^2} \]

Therefore,

\[ E_y \frac{d^2y}{dz^2} + V_y \frac{dy}{dz} - (Ka)_y (ye-y) = 0 \quad \ldots \ldots \quad (24) \]

ii. For organic phase

For organic phase, we have;

\[ E_x \frac{d^2x}{dz^2} - V_x \frac{dx}{dz} + (Ka)_x (xe-x) = 0 \quad \ldots \ldots \quad (23) \]

b. Case II - (organic < \( f \) water).

With reference to Fig. 3.5 the mass transfer within the contactor can be considered as follows:

i. Aqueous phase.

Due to the phase flow downward, the organic phase flow upward and the solute is transferred from organic phase to
aqueous phase, the concentration gradient of the phase is in the oppositional direction of column height (negative direction) hence, we have:

\[ M_c = -Vdc = -V_y dy \]

\[ M_l = + (Ka)_y (ye-y) dz \]

The longitudinal mixing is in the opposite direction to the concentration gradient, thus we have:

\[ M_L = J = -E_y \frac{d(dy)}{dz} \]

The mass balance within the phase of an infinitesimal column height interval \( dz \) will be:

\[ M_i = M_c + M_L \]

or

\[ (Ka)_y (ye-y) dz = -V_y dy - E_y \frac{d(dy)}{dz} \]

or

\[ E_y \frac{d^2y}{dz^2} + V_y \frac{dy}{dz} + (Ka)_y (ye-y) = 0 \quad ... \quad (30) \]

ii. Organic phase:

\[ M_c = +Vdc = V_x dx \]

\[ M_l = -(Ka)_x (xe-x) dz \]

\[ M_L = +J = -E_x \frac{d(dx)}{dz} \]

The mass balance within the phase of an infinitesimal column height interval \( dz \) will be:
\[ M_i = M_c + M_L \]

or

\[ -(K_a)_x (x - x_e) \, dz = V_x \, dx - E_x \frac{d(dx)}{dz} \]

or

\[ E_x \frac{d^2x}{dz^2} - V_x \frac{dx}{dz} - (K_a)_x (x - x_e) = 0 \ldots (29) \]

IV. Differential equation based on Miyauchi et al. (14, 53).

Smoot and Babb and Sleicher (55) have used the equation developed by Miyauchi (14, 53) which is given below. It will be shown that this corresponds to case II in the preceding development.

Consider an element of phase x in the differential contactor shown schematically in Fig. 3.6 where a solute is being extracted from a rich organic phase into water as the two immiscible phases flow counter-currently through the contacting device. The equation of conservation of mass can be written in its most general form as:

\[
\begin{align*}
\text{(Net rate of change of mass with time)} &= \text{(Net generation of mass in an element of phase } x) + \text{(Net loss due to diffusion from the element)} - \text{(Net loss due to convection from the element interface)} \\
\text{element) - (Net losses due to mass transfer across the element) } \\
\end{align*}
\]

\[ \ldots \ldots \ldots \ldots (31) \]
Fig. 3.6 Mass transfer mechanisms
For the case where there is no generation of mass and where transfer of mass by molecular diffusion may be assumed to be negligible compared to that transport by eddy diffusion.

Eqn. (31) can be written in vector notation as:

\[
\frac{\partial c_x}{\partial t} = \left( \text{div}(-E_x \text{grad } c_x) \right) - \text{div}(V_x c_x) - \phi(c_x) \quad \ldots (32)
\]

Where, \( c_x \) = solute concentration of phase \( x \)

\( t \) = time.

\( V_x \) = mass mean velocity of phase \( x \)

\( E_x \) = longitudinal mixing coefficient of solute in phase \( x \).

\( \phi(c_x) \) = interfacial mass transfer term.

If \( V_x \) and \( E_x \) are assumed to be constant, Eqn. (32) can be simplified to:

\[
\frac{\partial c_x}{\partial t} = E_x \left( \text{div } \text{grad } c_x \right) - (V_x \text{grad } c_x) - \phi( c_x )
\]

In cylindrical coordinates this equation becomes:

\[
\frac{\partial C_x}{\partial t} = E_x \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial C_x}{\partial r} \right) + \frac{1}{r} \frac{\partial C_x}{\partial \theta} + \frac{\partial C_x}{\partial z} \right) - \left( U_r \frac{\partial C_x}{\partial r} + \frac{U_{\theta}}{r} \frac{\partial C_x}{\partial \theta} + U_z \frac{\partial C_x}{\partial z} \right) - \phi(C_x) \quad (3.3)
\]

Where \( r, \theta, z \) are cylindrical coordinates, \( V_r, V_\theta, V_z \) are components of the vector \( V_x \).

Since the experimental measurement were conducted at
steady state condition with the flow confined to the z direction and with the radial and angular concentration gradient negligible, Eqn. (33) may be simplified to

\[ E_x \frac{d^2 c_x}{dz^2} - V_z \frac{dc_x}{dz} - \delta(c_x) = 0 \quad \ldots \ldots \ldots (34) \]

By assuming the existence of eddy diffusion in both phases, the Eqn. (34) can be written for both y (aqueous) and x (organic) phase as:

For organic phase:

\[ E_x \frac{d^2 x}{dz^2} - V_x \frac{dx}{dz} - (k_a)_x (x - x_e) = 0 \quad \ldots (29) \]

For aqueous phase:

\[ E_y \frac{d^2 y}{dz^2} + V_y \frac{dy}{dz} + (k_a)_y (y_e - y) = 0 \quad \ldots (30) \]
29

\textbf{V. Examination of the validity of the differential equation}

\[ E_y \frac{d^2 y}{dz^2} + V_y \frac{dy}{dz} - (ka)_y(y_e - y) = 0 \]

As explained in the derivation of this equation, the sign preceding each term is dependent upon the phase flow direction. It is important to establish that the signs used are correct, otherwise incorrect values for \( E_y \) will result. The following proof is of general interest as well as establishing the validity of the equation for the present experimental conditions.

A Solatron "274" Analog System Computer has been used for this purpose. The results from the computer show very clearly:

\begin{enumerate}
  \item The equation is stable in computer.
  \item The concentration profile obtained by the analog computer is very nearly the same as the one obtained from the experimental data in the laboratory. (Ref. to Fig. A6.6).
  \item By using the equation, as \( E \) value increases, \( y \) value decreases, i.e. the concentration difference decreases and vice versa. They coincided with the eddy diffusion theory. (Ref. to Fig. A6.5).
  \item \( E \) value is very sensitive with the shape of concentration profile.
\end{enumerate}
The other signs have been examined by the computer. They are shown unstable in the computer with the present experimental data.

The computer programme, its computing procedure and an example of the calculation are briefly described in Appendix 6.

VI Longitudinal mixing.

A simple approach to the design of liquid-liquid separation processes is to assume piston flow in both phases. However this assumption is acceptable in very few cases only and in the majority of types of extraction equipment it is not true even approximately. Extensive work on spray, packed, pulse and rotary disc columns operating such as to produce high mass transfer rates has shown that back or longitudinal mixing is appreciable. Aris et al (56, 57, 58); Bischoff et al (59); Swiff (60); Reman (61); Vermeulen (14) and Westerterp (62) refer specifically to the problem of longitudinal mixing. Kramers (63) has shown that in packed beds the ratio of packing to column diameter is critical and that longitudinal mixing is a function of the hydrodynamic flow pattern induced by the packing arrangements. This can be compared with the mixing in tubes which is due to a combination of liquid velocity and eddy diffusion.
The effect of longitudinal mixing is to lower separation efficiency. The efficiency decreases as mixing causes greater deviation from piston flow. Fig. 3.7 is typical of the situation arising from longitudinal mixing. It can be seen that in both phases, there is a reduction in solute concentration driving force at any position in an extractor due to back mixing. There is also a change in the concentration gradients.

In Fig. 3.8 is shown typical equilibrium and operating conditions. It is seen that at a given y value longitudinal mixing produces a reduction in \( x - x_e \) the organic phase driving force so that a greater column height is required to achieve the same separation.

As the degree of longitudinal mixing increases so the operating curve approaches the equilibrium curve and the column becomes less efficient.

Many investigators (13, 64, 65, 66, 67, 68, 69) have represented longitudinal mixing by an eddy diffusion coefficient and have proposed experimental techniques for its measurement. The various techniques used will be discussed later under residence time studies.
**Fig 37** CONCENTRATION PROFILE OF AQUEOUS AND ORGANIC PHASE

**Fig 38** MUTUAL RELATIONSHIP OF CONC. X AND Y ALONG THE COLUMN
VII. Mean Residence time study.

a. Distribution of residence time.

In any real situation the flow patterns in a container or vessel are generally non-ideal. Channelling and recycling of fluid and the existence of stagnant pools alter the residence time of elements of fluid in the system. The theory of residence time has been presented in a logical manner by vanckwarts (70); Levelspiel and Smith (67). It is intended to present a summary of this theory only.

The time taken by an element of fluid to pass from inlet to outlet in a flow system is known as its residence time or "age". All elements do not necessarily take the same time of passage so that there is a range of distribution of residence time. This can be expressed as frequency distribution function \( I \) and \( E \). \( I \) is such that \( I \theta \) represents the fraction of material with age between \( \theta \) and \( \theta + d\theta \) in the vessel. Again \( E \) is defined in such a way that \( Ed\theta \) is the fraction of material in the exit stream with age between \( \theta \) and \( \theta + d\theta \). Here \( \theta \) refers to dimensionless time.

From residence time theory

\[ t = \frac{V}{v} = \text{mean residence time} \]

\[ \Theta = \frac{t}{t} = \frac{t}{\tau} = \frac{vt}{V} = \text{reduced time (dimensionless)} \]

\[ I = t \cdot I(t) \]
b. **C-Curve and F-curve.**

Consider a quantity of tracer $Q$ injected instantaneously (Pulse tracer injection) into the entering stream of a vessel which has active volume $V$ of fluid in the vessel and volumetric flow rate $v$. The fraction of concentration of tracer in the exit stream at any instant time is $C$. The total fraction of tracer in the total out-flow at any time $t$ after the injection is $F$. $F$ can also be defined as the fraction of the exit fluid concentration when a step or jump tracer input having concentration $C_0$ is imposed on the fluid stream entering the vessel.

The C curve and F curve are on a dimensionless basis, and give the relationship between $C$ and $F$ and time.

Typical $C$ and $F$ curves are shown in Figs. 3.9 and 3.10 respectively. The total area under the C curve is unity or

\[ \int_0^\infty C \cdot d\theta = 1 \]

The deviation of flow of liquid in a vessel from ideality can be measured from either the C curve or F curve.
Delta-function or pulse tracer input signal

\[ C = \frac{c}{c_0} \]

Area = 1

Tracer output signal or C curve

Fig. 3.9 The C curve

Figure

Step function tracer input signal

\[ F = \frac{c}{c_0} \]

Tracer output signal or F curve

Fig. 3.10 The F curve

Consider steady state flow of fluid through a vessel and a step function of tracer introduced into the fluid entering the vessel. Suppose that the tracer is simply a second fluid introduced into the vessel at the time $t=0$ in place of the original flow fluid. Then at any time $t$ ($t > 0$), a material balance for the vessel gives:

\[
\text{(rate of tracer input)} = \text{(rate of tracer output)} + \text{(rate of tracer accumulation within vessel)}
\]

OR

\[
\text{(flow rate of second fluid into vessel)} = \text{(flow rate of fluid out of vessel)} \cdot \left( \frac{\text{fraction of second fluid in exit stream}}{\text{volume of vessel}} \right) + \frac{d}{dt} \left( \frac{\text{volume of vessel}}{\text{volume of vessel}} \right)
\]

Or in symbols:

\[
v = v \cdot F + \frac{d}{dt} \left[ v \int_0^\Theta I.d\Theta \right]
\]

Dividing by $v$ and noting $V/v = t$ and $d \left( t/t \right) = d\epsilon$, we have

\[
l = F + I
\]

Also noting that at time $\Theta$

\[
\text{(fraction of second fluid in exit stream)} = \text{(fraction of fluid in exit stream younger than } \epsilon \text{)}
\]
We have:

\[ F = \int_{\theta} E \, d\theta \]

Similarly for a pulse tracer input, we can show that

\[ C = E \]

Thus, summarising the relationship between the \( C \), \( F \), \( I \) and \( E \) curves at \( \theta \) is

\[ F + I = 1 \]

\[ F = \int_{\theta} E \, d\theta = 1 - I = \int_{\theta} C \, d\theta \]

\[ C = E = \frac{dF}{d\theta} = -\frac{dI}{d\theta} \]

d. Measurement of the deviation of fluid in a vessel from ideality

Both \( C \) and \( F \) curves can be used to measure the deviation of flow of fluid from piston flow, where elements of the fluid entering move with equal velocities in parallel paths through the system, and perfect mixing, where the exit stream has the same composition as the main body of fluid in the system.

In the \( F \)-curve, hold back is defined as magnitude of deviation from piston flow and segregation is defined as departure from perfect mixing. In the \( C \)-curve, mean residence time and longitudinal dispersion coefficient \( D \) are used to measure the deviation.
(i) **Hold-back**

The significance of hold back $H$, may be seen from a consideration of the following system.

Let the colour of the inflowing fluid suddenly change from white to red. The hold back is then equal to the fraction of volume of the fluid in the vessel which is white at the time when a volume of red fluid equal to the volume of the vessel has passed through. The hold back $H$, is measured as the area under the $F$-curve between $\theta = 0$ to $\theta = 1$. This is the area in Fig. 3.11, thus:

$$H = \int_{\theta=0}^{\theta=1} F \, d\theta$$

$H = 0$, when piston flow. The greater the hold back value of the system, the greater the active volume needed when compared with piston flow.

(ii) **Segregation.**

Segregation is used to measure the deviation of a system from perfect mixing. In the $F$ curve the segregation value may be obtained by superimposing the $F$ curve for a completely mixed system onto the curve for the system under examination.
Fig. 3.11  HOLD BACK

A

F

1.0

1 - A

0.0

0

1.0

\frac{A}{V}

\rightarrow
For a perfectly mixed system, the internal and external residence time distribution functions are identical, thus,

$$ I = E $$

From the relationship of $I$, $E$, $C$ and $F$ curves, we have

$$ I + F = 1 \text{ or } I = 1 - F $$

And

$$ dF = E \, d\theta $$

Hence,

$$ dF = I \, d\theta = (1 - F) \, d\theta, \text{ or } dF/(1 - F) = d\theta $$

$$ \ln(1 - F) = -\theta $$

$$ F = 1 - \exp^{-\theta} \quad (36) $$

Equation (36) is the equation of the $F$ curve for perfect mixing.

With reference to Fig. 3.12 the departure of a system from perfect mixing is obtained by the size of the shaded area ($A_1 = A_2$). Since the area between each curve and the line $F = 1$ is unity, the two shaded portions have the same area $A_1 = A_2$. The segregation can be simply defined as the area between the $F$ curve under examination and the curve for perfect mixing up to the point $\theta = 1$ when the curves cross.
Fig. 3.12 Segregation

\[ F = 1 - \exp \left( -\frac{vt}{V} \right) \]
There are two sets of parameters called the moments of the
distribution associated with every age distribution function.
One set are the moments taken about the origin, and the other
are the moments taken about the mean. The interesting property
of the moments of a distribution is that they can completely de­
fine the distribution. They can be used to compare distribution
without comparing the actual curves themselves.

Normally, the two moments will be used to define the dis­
tribution. The moment about the origin is commonly called the
mean or centroid of the distribution. It is the location para­
meter of the distribution. In the C-curve, we define the moment
as mean residence time, which has been dealt with previously

\[ \bar{t} = \frac{\sum t \cdot E(t)}{\sum E(t)} \]  

(37)

The second moment about the mean, commonly called the
variance measures the spread of the distribution about the
mean and is equivalent to the square of the radius of gyration
of the distribution. It is defined for a continuous distribution
as

\[ \sigma^2 = \frac{\int_0^\infty (t - \bar{t})^2 E(t) dt}{\int_0^\infty E(t) dt} \]  

(38)

When applied to C curves, the denominator \(\int_0^\infty E(t) dt\) will
be unity. Hence the variance of C curve is
If the time scale is measured in seconds, the variance will thus be in sec.\(^2\). The equation for the variance may also be written in a dimensionless form by dividing through the square of the mean residence time (\(\bar{t}\)) as,

\[
\sigma^2 = \left( \frac{\sigma}{\bar{t}} \right)^2 = \int_0^\infty (\theta - 1)^2 E(\theta) \, d\theta \quad (40)
\]

where \(\theta\) is the reduced time.

It is interesting to note that the mean residence time in the dimensionless scale is unity i.e. \(\theta = 1\)

f. The relationship between exit age distribution function and its variance.

One function which appears to fit the residence time exit age distribution quite well is the function:

\[
C(\theta) = \alpha \cdot \theta^k \, e^{-\beta \theta} \quad (41)
\]

The normalised distribution function obeys the three following conditions:

(1) The area under the curve is unity, i.e.

\[
\int_0^\infty C(\theta) \, d\theta = 1
\]
(2) The mean residence time is unity.

i.e.,

\[ \int_0^\infty \theta \cdot c(\theta) \, d\theta = 1 \]

(3) The dimensionless variance of the distribution is defined as:-

\[ \chi^2 = \int_0^\infty (1 - \theta)^2 \cdot c(\theta) \, d\theta \]

Each of these conditions are applied in turn to equation (41) to determine \( \alpha, \beta \) and \( \gamma \), we have:

\[
C(\theta) = \frac{\frac{1 - \sigma^2}{\sigma^2} - \frac{\theta}{\sigma^2}}{\sigma^2 \cdot \Gamma\left(\frac{1}{\sigma^2}\right)}
\]

(42)

This equation (42) has been referred to by Foss and Gerster (71) and by Thomas and Campbell (72). The details of the derivation of the equation are shown in Appendix 1.

g. Relationship between the longitudinal dispersion coefficient and variance of the residence time distribution function.

The following mathematical treatment is based on the work of Levenspiel and Smith (67) who considered a fluid flowing \( v \text{ ft}^2/\text{sec.} \) through a pipe which stretches to infinity in either direction. The fluid enters the experimental section of this pipe
at \( x = 0 \) and leaves at \( x = L \). The total volume of this section being \( V \) ft\(^3\).

Now at time \( t = 0 \), rapidly inject amount of \( Q \) ft\(^3\) of tracer into the entering fluid, if the flow rate and degree of mixing are independent of radical position, the slug of tracer will move downstream at an average velocity \( U = \frac{vL}{V} \) ft/sec. and its location at time \( t \) will be \( x = Ut \). With these conditions, the mixing of tracer with the surrounding fluid is given by Fick's law of diffusion.

\[
\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} \tag{43}
\]

If the tracer is injection by a delta-function input into the fluid at rest at \( x = 0 \), the solution of Eqn. 4.3 which gives the tracer distribution, a function of \( t \) and \( x \) as;

\[
C = \frac{A}{t^{\frac{1}{2}}} \cdot e^{-x^2/4Dt} \tag{44}
\]

The derivation of Eqn. (44) is shown in Appendix 2.

The total amount of substance, \( M \) diffusing in a cylinder of infinite length and unit cross-section is given by

\[
M = \frac{Q}{V/L} = \int_{-\infty}^{\infty} C \cdot dx \tag{45}
\]

In Equation 44, let \( n^2 = \frac{x^2}{4Dt} \)

We, have;

\[
dx = 2(Dt)^{\frac{3}{2}}dn
\]

Substituting Eqn. 44 into Eqn 45, we have;
Substituting Eqn. (47) into Eqn. (44)

\[ C = \frac{M}{2\sqrt{\pi}D} \cdot e^{-x^2/4Dt} \]  

But \( M = \frac{Q}{v} \), thus

\[ C = \frac{QL}{2V\sqrt{\pi}Dt} \cdot e^{-x^2/4Dt} \]  

However, the fluid is not at rest but is moving down the pipe. Thus, \( x = L - ut \) can be substituted into Eqn. (49) giving the solution as

\[ C = \frac{QL}{2V\sqrt{\pi}Dt} \cdot e^{-(L-ut)^2/4Dt} \]  

Rearranging Eqn. (50) becomes:

\[ \frac{CV}{Q} = \frac{1}{2\sqrt{\pi}v\frac{t}{V}} \cdot \frac{D}{UL} - \frac{(1 - \frac{V}{V})}{4\left(\frac{V}{V}\frac{D}{UL}\right)} \]
CV/Q is the concentration of tracer measured in terms of the initial concentration of injected tracer $C_0$, thus:

\[
\frac{CV/Q}{C_0} = C = C/C_0 \quad (52)
\]

From residence time theory, $vt/V = \theta$. Thus, Eqn. (52) gives the tracer distribution as a function of C-curve and it will become

\[
C = \frac{C}{C_0} \cdot \frac{1}{\sqrt{(\pi \theta \frac{D}{UL})}} \cdot e^{-(1-\theta)^2/4\theta\left(\frac{D}{UL}\right)} \quad (53)
\]

The variance of a function $f(x)$ can be defined as

\[
\sigma^2 = \int_{a}^{b} x^2 f(x) \, dx - \left[ \int_{a}^{b} x f(x) \, dx \right]^2 \quad (54)
\]

In applying to Eqn. (53) where $x$ corresponds to $\theta$, $f(x)$ is the function governing its variation and corresponds to $C$. The integral from $a$ to $b$ is the domain of the variable $x$, namely $0$ to $\infty$ for $\theta$. Thus, the variance of Eqn. (53) becomes:

\[
\sigma^2 = \int_{0}^{\infty} \theta^2 \left( \frac{1}{2\sqrt{\pi\frac{D}{UL}\theta}} \right) \cdot e^{-(1-\theta)^2/4\theta\left(\frac{D}{UL}\right)} \, d\theta
\]

\[
- \left[ \int_{0}^{\infty} \theta \left( \frac{1}{2\sqrt{\pi\frac{D}{UL}\theta}} \right) \cdot e^{-(1-\theta)^2/4\theta\left(\frac{D}{UL}\right)} \, d\theta \right]^2 \quad (55)
\]
Let \( \frac{D}{U_L} = k \), \( \frac{1}{4k} = \lambda \), and \( \theta = p^2 \).

Eqn. (55) can be simplified to

\[
\sigma^2 = 2\left(\frac{D}{UL}\right) + 8 \left(\frac{D}{uL}\right)^2
\]  \hspace{1cm} (56)

The detail of the simplification will be discussed in Appendix 2.
CHAPTER 4.

SCOPE OF EXPERIMENTAL STUDY.

I. Choice of liquid system.

To some extent the choice of liquid-liquid system was limited by the need for safety in the college laboratory. On this basis the system carbon tetrachloride-water with acetic acid as solute was chosen as being safe. The system offers other advantages:

(1) Carbon tetrachloride is difficult to disperse in water compared with other systems such as MIBK (methyl isobutyl ketone).

(2) The system shows little tendency to form stable emulsions.

(3) The acetic acid can be readily determined by titration.

(4) Carbon tetrachloride is used extensively as a solvent.

(5) The liquids are cheap and readily available.

The equilibrium relationship have to be established and this will be described. The important physical properties of the system are listed in Table 4.1.

II. Pilot plant experiments.

These will be divided into two parts:

1. A study of mass transfer rates and efficiency of separation and their relationships to speed of oscillation or rotation, flow rates and ratios, and flooding characteristics.
### Table 4.1

**Important Physical Properties of the System.**

<table>
<thead>
<tr>
<th></th>
<th>Water</th>
<th>Acetic acid</th>
<th>Carbon tetrachloride</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Formula</strong></td>
<td>$\text{H}_2\text{O}$</td>
<td>$\text{CH}_3\text{COOH}$</td>
<td>$\text{CCl}_4$</td>
</tr>
<tr>
<td><strong>Formula wt.</strong></td>
<td>18.016</td>
<td>60.032</td>
<td>153.84</td>
</tr>
<tr>
<td><strong>Sp. gr.</strong></td>
<td>1.00</td>
<td>1.049</td>
<td>1.595</td>
</tr>
<tr>
<td><strong>Melting pt., °C</strong></td>
<td>0</td>
<td>16.7</td>
<td>-22.6</td>
</tr>
<tr>
<td><strong>Boiling pt., °C</strong></td>
<td>100</td>
<td>118.1</td>
<td>76.8</td>
</tr>
<tr>
<td><strong>Solubility in 100 parts water</strong></td>
<td>-</td>
<td>Infinite</td>
<td>0.08</td>
</tr>
<tr>
<td><strong>Form</strong></td>
<td>Liquid</td>
<td>Liquid</td>
<td>Liquid</td>
</tr>
<tr>
<td><strong>Colour</strong></td>
<td>Colourless</td>
<td>Colourless</td>
<td>Colourless</td>
</tr>
</tbody>
</table>
2. A study of longitudinal mixing by a dye injection technique using residence time theory.

III. Equilibrium.

In the case of two immiscible liquids, the concentration of a third component or solute soluble in each phase at equilibrium is represented by the distribution law;

\[ y_e = mx \]

where \( y_e \) = solute concentration in the extract phase.
\( x \) = solute concentration in the raffinate phase.
\( m \) = the distribution coefficient.

This linear relationship is usually true for dilute solution only and in most cases \( m \) is not constant. It becomes necessary to establish experimentally the curve relating \( y_e \) and \( x \).

IV. Experimental determination of equilibrium values.

The measurement of solute concentration both in water and carbon tetrachloride was carried out by an Electronic Automatic Titrameter (Model 24, Electronic Instruments Ltda.). This titrator is made up from two parts, a control unit and a titration unit.
(1) Control unit:

The control unit is in effect an electronic pH meter, in which the output is used to control a set of relays, instead of producing an indication on a meter, a small meter is, however, included to give a HIGH-ZERO-LOW indication, principally for adjustment and calibration purpose.

The main control is the end-point setting dial, mounted in the centre of the sloping front panel. This dial is calibrated in pH units from 3 to 11, and in millivolts from +400 to -800 it controls a backing-off potential so arranged that when the required end-point is reached, the algebraic sum of the backing-off potential and the input potential is zero. This results in turn in a zero output, which allows the main control relay to release, so releasing the subsidiary relay, switching off the apparatus and lighting the completion warning lamp.

(2) Volumetric Titration Unit.

The volumetric titration unit, as shown in Fig. 4.1 consists of a stand carrying a burette, a tap unit, a beaker support, and an assembly containing a motor-driven stirrer, clips to hold the electrodes, and the doublewound selenium that operates valves of the tap unit. The tap has three
Fig. 4.1 AUTOMATIC VOLUMETRIC TITRATION UNIT

- Side arm for filling burette
- Burette
- Automatic tap
- Rubber seal
- Knurled ring for locking tap
- Clamping screw for adjusting height
- Stirrer motor drive
- Leads to control unit
- pH electrodes
- Glass stirrer in chuck
- Rupture stop
- Adjustable blaker stand
positions, fast, slow and stop. The burette which has graduations for 50 ml, fits by means of a ground glass joint into the top of the tap unit.

**Technique.**

About 50 ml of carbon tetrachloride and an equal amount of distilled water were placed in each of 250 ml beaker, different quantities of acetic acid were added to each beaker. The contents were vigorously stirred for an hour and then allowed to settle. A 10 ml sample was taken from each layer in the beaker and their acid concentration was determined by titration against 0.1N KOH. All titrations were running in duplicate.

**Results.**

The experimental equilibrium results are given in Table 4.2 and plotted in Fig. 4.2. Also included are some values obtained by J.B. Lewis (73) for comparison.

**V. Correlation of equilibrium results.**

As can be seen from Fig. 5.5 the results can be correlated by a curve. The equation of the "best" curve was obtained by the method of least squares. The computer programme used is given in Appendix 4.
Table 4.2 Distribution of acetic acid between H₂O and CCl₄

<table>
<thead>
<tr>
<th>( y_e )</th>
<th>( x )</th>
<th>( y_e )</th>
<th>( x )</th>
<th>( y_e )</th>
<th>( x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.0</td>
<td>0.12</td>
<td>409.5</td>
<td>27</td>
<td>13.4</td>
<td>0.52</td>
</tr>
<tr>
<td>19.5</td>
<td>0.18</td>
<td>419</td>
<td>29</td>
<td>24.5</td>
<td>0.23</td>
</tr>
<tr>
<td>30.0</td>
<td>0.42</td>
<td>420</td>
<td>29.5</td>
<td>59</td>
<td>0.8</td>
</tr>
<tr>
<td>38.5</td>
<td>0.65</td>
<td>427.5</td>
<td>30.0</td>
<td>79.5</td>
<td>1.75</td>
</tr>
<tr>
<td>46.5</td>
<td>0.88</td>
<td>445</td>
<td>32.5</td>
<td>122</td>
<td>4.00</td>
</tr>
<tr>
<td>86.5</td>
<td>1.92</td>
<td>446.5</td>
<td>32.5</td>
<td>162.5</td>
<td>5.9</td>
</tr>
<tr>
<td>96.5</td>
<td>2.35</td>
<td>467</td>
<td>37.5</td>
<td>206</td>
<td>7.5</td>
</tr>
<tr>
<td>191.0</td>
<td>7.0</td>
<td>475</td>
<td>40.5</td>
<td>246</td>
<td>10.3</td>
</tr>
<tr>
<td>202</td>
<td>8.5</td>
<td>506.5</td>
<td>45.0</td>
<td>263</td>
<td>14.3</td>
</tr>
<tr>
<td>223</td>
<td>8.6</td>
<td>507</td>
<td>45.0</td>
<td>317.5</td>
<td>16.8</td>
</tr>
<tr>
<td>288</td>
<td>14.0</td>
<td>515</td>
<td>47</td>
<td>368</td>
<td>21.25</td>
</tr>
<tr>
<td>331.5</td>
<td>15.5</td>
<td>515</td>
<td>47</td>
<td>399.2</td>
<td>28.3</td>
</tr>
</tbody>
</table>

Where

\( y_e \) = acetic acid in \( H₂O \) phase, gm/l.

\( x \) = acetic acid in \( CCl₄ \) phase, gm/l.
Fig 4.2 Acetic Acid Equilibrium Distribution Curve

$Y_e =$ acetic acid in $H_2O$ phase gm/L
$x =$ acetic acid in $CCl_4$ phase gm/L

$z =$ present experimental data

○ Data from J. B. Lewis
The equation established is as follows:

\[ y_e = 16.4226 + 35.7437x - 1.967727x^2 + 0.07789x^3 - 0.00156x^4 + 0.000012x^5 \]

This empirical equation satisfactorily correlates experimental equilibrium values except at very low concentrations of acetic acid in carbon tetrachloride.

VI. Experimental programme.

The experiments were conducted to investigate the efficiency of an Oscillating Baffle Column (OBC) in the liquid-liquid extraction process by using carbon tetrachloride – acetic acid – water system. The number of transfer units of the system for the OBC depends largely on the oscillating speed and phase flow. Different oscillating speeds up to flooding point of the system with different flow rates range 16.7 l/hr to 30 l/hr were investigated. A comparison was made of the OBC and RDC with mass transfer runs and mean residence time runs were also done. The rotating speed of RDC up to 1,600 rpm with different flow rates range 25 l/hr to 80 l/hr.

Preliminary studies mainly for calibration purposes were performed during and after the construction of the apparatus.

The experiment programme is conveniently divided into phases:
1. Mass transfer study.

2. Investigation of the longitudinal mixing.

3. Investigation of the continuous phase residence time distribution in the extracting column.

I. Mass transfer study.

This study is concerned with the effect of operational condition on the mass transfer of acetic acid (solute) from carbon tetrachloride to water in O.B.C. and R.D.C.

The operating oscillating speed in O.B.C. up to 264 Osc/min with 16.7 l/h flow rate and 1600 r.p.m. with 30 l/h for the system has been done.

Equilibrium data for the system has been determined by experiment.

An equilibrium curve and an empirical equation of the curve have been done by using an I.C.T. Ferranti "Sirius" 1602 computer.

The following measurements were made in each operating condition run:-

a. Baffle speed.

b. Organic phase concentration inlet and outlet of column.
c. Aqueous phase concentration along the column as shown in Fig. 5.2.
d. Organic and aqueous phases flow.
e. The measured number of transfer units. \( \bar{n}_{oy} \) height of a transfer \( H_{oy} \) unit and overall mass transfer coefficient \( (K_a)_{av} \) in water phase.
f. The \( (K_a)_{av} \) values in each 6" column high along the column.
g. The concentration profile of water phase.

2. Investigation of the longitudinal mixing.

The purpose of this investigation is to work out the longitudinal mixing (eddy diffusion) in the different operating conditions. The actual meaning of the longitudinal mixing is equal to the product of longitudinal coefficient with the 2nd derivative of concentration gradient i.e. \( E \frac{d^2y}{dz^2} \) in water phase and \( E \frac{d^2x}{dz^2} \) in organic phase.

The longitudinal mixing coefficient has been determined by concentration profile and tracer stimulus-response techniques.

The differential equations of a true differential contact model have been developed and studied by several methods from the theoretical consideration. The differential equation of the aqueous phase in present work has been examined the validity by using Solatron "274" Analog System Computer.

4. Investigation of the continuous phase residence time distribution in the extracting column.

Liquid flowing through an extractor will seldom, if ever, travel from inlet to outlet without mixing in the longitudinal direction. Usually mixing is incomplete and a concentration gradient in the liquid phase ensures. Mixing of the liquid phases cause some liquid to residence of other portions of liquid. The column efficiency will be affected by the distribution of the liquid residence time thus produced.

Thus knowledge of the factors which affect the liquid residence time distribution and thus the extent of liquid mixing in an extractor is going to be of great importance in predicting plant performance.
The mean residence time and a dispersion model with variance of the exit age distribution function are measured by the dye stimulus response technique to determine the extent of the liquid mixing in the column.
CHAPTER 5.
Description of pilot plant.

The plant is designed in such a way that different types of extractor columns can be investigated using the same flow system. In other words it becomes necessary only to replace one column by another for investigation to proceed. The following description therefore applies quite generally and the detail of the column is described respectively.

I. General apparatus.

The general apparatus is shown in Fig. 5.1 and the details of the flow system is shown in Fig. 5.2.

The liquids are stored in 40 gallon stainless steel tanks (2 and 4). Two ½ h.p. stainless steel centrifugal Worthington Simpson DFC-42 pumps (20a and 20b) were used to circulate and pump the carbon tetrachloride and water to head tanks (15a and 15b), each having a capacity of 2 litres and being placed 16 ft about floor level. In this way, a constant head of liquid is maintained to rotameters (14a and 14b). The flow rate of each phase was measured by the appropriate rotameter and controlled by a stainless steel needle valve attached to the bottom of each rotameter.

Two stainless steel external separators (1a and 1b) each having a capacity of 18.7 litres were connected with the
Fig. 5.1 General layout
<table>
<thead>
<tr>
<th></th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Main column</td>
</tr>
<tr>
<td>1a</td>
<td>Upper external separator</td>
</tr>
<tr>
<td>1b</td>
<td>Bottom external separator</td>
</tr>
<tr>
<td>2</td>
<td>Carbon tetrachloride - acetic acid storage tank</td>
</tr>
<tr>
<td>3</td>
<td>Stainless steel mixer</td>
</tr>
<tr>
<td>4</td>
<td>Water storage tank</td>
</tr>
<tr>
<td>5</td>
<td>Carbon tetrachloride after being extracted storage tank</td>
</tr>
<tr>
<td>6</td>
<td>Automatic absorptiometer</td>
</tr>
<tr>
<td>7</td>
<td>0-5 mv potentiometric recorder</td>
</tr>
<tr>
<td>8</td>
<td>0-3 second timer</td>
</tr>
<tr>
<td>9</td>
<td>Magnetic relay</td>
</tr>
<tr>
<td>10</td>
<td>Gear box unit</td>
</tr>
<tr>
<td>11</td>
<td>Dye storage cylinder</td>
</tr>
<tr>
<td>12</td>
<td>Magnetic valve</td>
</tr>
<tr>
<td>13</td>
<td>Automatic titrimeter unit</td>
</tr>
<tr>
<td>13a</td>
<td>Interface lever sight glass</td>
</tr>
<tr>
<td>20</td>
<td>Pumps</td>
</tr>
<tr>
<td>30</td>
<td>12 v. D.C. battery</td>
</tr>
</tbody>
</table>
water and carbon tetrachloride outlet from the column. They were used to separate the phases. All pipes lines were of polythene tubes. A 1 h.p. Mitchell stainless steel mixer (3) was inserted in the carbon tetrachloride storage tank (2). By running this propeller mixer, a homogeneous mixture could be ensured.

The driving shaft of the internal baffle passed through a packed gland at the upper end of the column and was supported by a lower bearing at the bottom of the column. The unit was driven by a ½ h.p. single phase, 250 V electric motor, through a Kopp variator and reducing gear box.

A dye storage cylinder (11) with a magnetic valve (12) was fitted at water inlet. A compressed air line was connected to the cylinder. The amount of dye being injected was controlled either by means of the cylinder pressure or by the time of opening of a magnetic valve. A light absorption meter was installed at the water outlet from the column for continuous measurement of dye concentration.

It is necessary to maintain a liquid-liquid interface at the bottom of the extraction column and for this purpose a sight glass (13a) was provided.

In the experimental work, two column were examined:
i. Oscillating Baffle Column (O.B.C.).


The construction and method of operation of these columns are described.

II. Oscillating Baffle Column.

i. Column.

The main column shown diagramatically in Fig. 5.3 was constructed from glass sections with stainless steel fittings. Each glass section was 6 inches long and with an inside diameter 2.75 inches. The whole unit consisted of 6 glass section with stainless steel spacers, which were fitted with $\frac{3}{8}$ inch sample cocks. Stainless steel headers were fitted at top and bottom of column for connection to the external separators.

The column was designed with sufficient flexibility to permit easy replacement of baffle by another baffle in which size, shape, pitch of holes etc. could be verified.

ii. The internal baffle.

The internal baffle is shown in Fig. 5.4. It is made of stainless steel and has a length of 2'6'' with $\frac{1}{8}'''$ diameter holes on $\frac{3}{8}$ square pitch in all four blades. The detail of the baffle is shown as follows.
Fig. 5.3 The column
<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>length of the baffle</td>
<td>2' 6&quot;</td>
</tr>
<tr>
<td>number of blades</td>
<td>4</td>
</tr>
<tr>
<td>angle of blades</td>
<td>90°</td>
</tr>
<tr>
<td>blade thickness</td>
<td>1/16&quot;</td>
</tr>
<tr>
<td>diameter of hole</td>
<td>0.25&quot;</td>
</tr>
<tr>
<td>pitch of hole</td>
<td>0.375 ins. square</td>
</tr>
<tr>
<td>number of holes in each blade</td>
<td>158</td>
</tr>
<tr>
<td>percentage of hole area in baffle</td>
<td>25.8%</td>
</tr>
<tr>
<td>diameter of driving shaft</td>
<td>½&quot;</td>
</tr>
</tbody>
</table>

iii. Driving force mechanism for the oscillating system.

The drive unit consists of a ½ h.p. 250 V single phase motor, a hopp variator and a 10 : 1 reducing gear box. The output speed is controlled by the variator which is precalibrated. Connected to the reducing gear box is a disc which in turn is connected by means of a rod to a bell-crank mounted on the top of the baffle shaft. The movement of the connecting rod activated by the rotating disc gives an oscillating movement to the bell-crank. The amplitude of the oscillation can be changed by varying the length of the connecting rod. In the present experiment the amplitude of oscillating was fixed at 0-45°.

III. Rotating disc contactor.

In order to make a comparison of the extracting efficiency of Rotating disc contactor (RDC) with Oscillating
Fig. 54: The internal baffle of the OBC.
baffle column (OBC), it is necessary to use the same column as in the oscillating baffle column arrangement, the only difference being in the baffle and the method of driving device.

i. **Internal baffle.**

The baffle is shown diagramatically in Fig. 5.5 and its detail is as follows:

- Diameter of rotor disc: 2 ins.
- Disc thickness: 1/16 ins.
- Outside diameter of stator ring: 2.7 ins.
- Inside diameter of stator ring: 2.2 ins.
- Height of compartment: 1 inch.
- Number of compartments: 30
- Diameter of driving shaft: ½ ins.

The stator rings, rotor disc and shaft were all made of stainless steel. The stator rings which fitted exactly inside the column were supported by 3 thin stainless steel rods distributed evenly around the column inner circumference i.e. 120°. In order to fix the stator rings in place and to maintain an even stator space, the support rods and the stators were appropriately slotted at set intervals and then tightened with nuts on both sides. It is important that the rods be of small diameter (less than ½" ) so that they do not
Fig. 5.5 The internal baffle of the RDC.
interfere with the flow patterns established by the rotating disc and also that they do not occupy too much of the extracting space. The end of the stator support rods were fitted into the bottom column's header.

The central rotor shaft consists of a \( \frac{1}{2} \)" diameter stainless steel rod specially fabricated so that it is dynamically balanced. The discs are not directly attached to this shaft but are sandwiched between pairs of concentric stainless steel sleeves (\( \frac{3}{8} \)" I.D. tube). End connection on shaft are used to compress the sleeve system and in this way the discs are held fast between the sleeves. The discs are evenly spaced in such a way that each disc lies at the centre of each column compartment of section. The central shaft is supported by bearing at top and bottom of the column the bearing being of P.T.F.E.

ii. **Driving force for shaft in RDC.**

The method of driving and the gear box arrangement are as already described for the oscillating system. However, in this case the shaft of the reducing gear box was directly coupled to the top of the baffle shaft by a rubber connector. The variator output was precalibrated.

IV. **Discussion of mode of operation of OBC and RDC.**

Reman (31) and Hung (74) consider the dispersed
phase flow patterns in a rotating disc extraction column as shown in Figs. 5.6, 5.7 and 5.8. The dispersed phase is broken off into droplets by the means of disc's centrifugal forces. The droplets of dispersed phase are circulated in the small compartment (as shown in Figs. 5.6 and 5.8). In this case, it may be considered to increase the eddy diffusion of phases and decrease the phase separating efficiency, or the dispersed phase will form a thin film attached at the interfaces of stator ring and rotor disc (as shown in Fig 5.7). It will decrease the interface area of dispersed phase and also decrease its extracting efficiency.

A continuous oscillating perforated blade baffle may be achieved to overcome these difficulties in RDC. The flow pattern of the baffle column is shown as Fig. 5.9 and can be considered as follows:

When the perforated baffle moves in a clockwise direction, there is a tendency to move the liquid with it. This is counteracted to a certain extent by slip of the liquid through the hole in the baffle. It will happen again, when the baffle is reversed in an anticlockwise direction. Occasionally, turbulence will be induced in the liquid during the time over of oscillating which helps to break up the dispersed phase, thus the oscillating of the baffle "combs" cause the liquid
Fig. 5.6 Flow pattern predicted by Reman

Fig. 5.7 Flow pattern observed by Kung
(Rotor speed < 300 Ft/m)

Perforated baffle
Dispersed phase droplets formation direction

Fig. 5.8 Flow pattern observed by Kung
(Rotor speed > 300 ft/m)

Baffle movement direction

Fig. 5.9 Flow pattern of Oscillating baffle column
phase to produce a minimal quantity rotating in the column and breaking up small drops of dispersed phase and increasing the interfacial area between the liquid phases, therefore giving a greater efficiency of liquid extraction.

It is proposed to compare the OBC and RDC in the experimental work to be described later. In this way an assessment can be made of the efficiency and reliability of the OBC.
CHAPTER 6.

Experimental technique.

I. General technique.

The calibration curves for the rotameters are given in Figs. 6.1 and 6.2 and were obtained by direct collection of liquid over a time interval 1 to 3 minutes. Several tests were made at each flow rate to ensure reproducibility. The shaft speed for both OBC and RDC were varied by means of the Kopp variator which was fitted with a reference index controller. A tachometer was used to measure the spindle speed after the reducing gear box each speed being related to an index number. The speed calibration charts are given in Figs. 6.3 and 6.4.

The method of calibration for the end point of the automatic titrameter is described in the Electronic Instruments Booklet (75) and will not be detailed here. The calibration graph is given in Fig. 6.5. The end point was at $PH = 7.6$.

II. Sampling

In all cases it was required to take continuous samples of the carbon tetrachloride feed to the column to determine its acetic acid content. As shown in Fig. 6.6 there is a series of equally spaced sample points along the column. Samples were continuously withdrawn from each of these sample points during the experimental run. It is important that (l)
Fig. 6.1
Calibration of organic phase rotameter
Fig. 6.2
Calibration of water phase rotameter
Fig. 6.3 Calibration of O.B.G. Baffles speed

Kopp Vacuum Reading

140 120 100 80 60 40 20 0

140 160 180 200 220 240 260 280

Baffles speed, rpm/min.
FIG. 6: Neutralisation curve of 0.1 N-acetic acid (100 ml) with 0.1 N- Sodium Hydroxide
DIMENSION AND SAMPLE COCK ARRANGEMENT OF COLUMN
only continuous phase be withdrawn and (2) that a small quantity of sample per unit time is taken so that there is no interference with the flow pattern and operation of the column.

Exclusion of drops from the samples was achieved by means of the insertion of a fine stainless steel gauze pad into the sample point as shown in Fig. 6.7. The samples are collected in small flasks from which the liquid can be pipetted at intervals to determine the acid content by means of the automatic titrarameter.

Fig. 6.7 Sampling cock.
The rate of withdrawal was restricted to 5 ml/min by adjustment of the cock.

Sufficient time was allowed for the system to come to a steady state with sample withdrawal in operation before the samples were considered for measurement. This usually takes 20-30 minutes of operation after a change in flow conditions.

III. Mass transfer runs.

The carbon tetrachloride previously saturated with water and the water phase saturated with carbon tetrachloride were fed to the column at a temperature of 21 to 23°C. The continuous phase was an aqueous solution of acetic acid. Mass transfer of acetic acid occurred from the carbon tetrachloride to the water. The heavy phase (carbon tetrachloride) fed to the top of the column descended and was dispersed into droplets by the action of the mechanical agitation.

An interphase was maintained at the bottom outlet by adjustment of the outlet valve and visual observation of the sight glass. A bottom separation provided a means of separation of any entrained water from the carbon tetrachloride. The latter was returned to the storage tank. This liquid contains a different amount of acetic acid from that originally in the tank but when a steady state is reached the actual acetic acid concentration of the liquid to the column is measured.
titrimetrically. The water leaves the top of the column and any small entrainment of carbon tetrachloride is separated in an overhead separator. Any separated carbon tetrachloride is returned continuously to the column.

To start a run the column is filled with water and then the phase flow adjusted to the required values. Steady operation is achieved in 30 to 40 minutes and the achievement of the steady state is established by taking a series of mass balances around the column. In general mass balances were accurate to approximately 3%.

The system behaved well and it was not difficult to attain the steady state. Results were reproducible as shown by the mass balances.

IV. **Residence time experiments.**

i. **Apparatus and technique.**

The residence time experiments were restricted to the continuous water phase. The method used was the injection of a pulse tracer at the water inlet to the column. The system used is shown in Fig.6.8. As in all such studies it was necessary to examine carefully three aspects viz.

(1) Tracer choice.

(2) Measurement and choice of inlet signal.

(3) Measurement of outlet signal.
Figure 5: Residence-time measuring equipment
1. Tracer.

It is important that the tracer be immiscible with carbon tetrachloride and highly soluble in water. It should be so markedly hydrophilic that other materials are not likely to be stained by the solution. It should be readily detectable in the present case by means of a photocell.

Nigrosine is an intense almost black dye soluble in water which satisfies all the conditions. Concentrated solution of Nigrosine in water were used as tracer in the present experiments.

2. Inlet signal.

Refer to Figs. 5.1 and 5.2, a saturated solution of Nigrosine in water is stored in a pressurised vessel (11). The magnetic valve (12) allows the pulse dye solution to water stream for a definite time interval. The length of this interval is determined by an electrical timer (8) operated by a 24 volt D.C. battery (30). The timer is also connected through a relay (9) to a pen recorder. In this way a record is made on a chart the instant tracer enters the stream. In this way the recorder traces a line for the definite time interval corresponding to tracer injection. The dye storage vessel is pressurised with air and it is important that this
pressure remains constant during the period of injection of the dye solution. On optimum time interval has to be determined experimentally whereby sufficient dye is used to produce a reasonable outlet tracer and also too high pressure can not be used otherwise it produces a secondary flow effect in the system. It was found that 0.2 to 0.4 second at pressure of 28 - 30 psig were suitable. This time interval is compared with a residence time of approximately 800 seconds in the system and is therefore acceptable.

3. Outlet signal.

This is the variation with time of tracer concentration in the outlet aqueous phase from the column and is taken as close to the column as possible. After consideration of several techniques, it was decided to use a Hilger & Watts Automatic Absorptiometer which was coupled to the potentiometric recorder some details of the absorptiometer system are given below.

4. Absorptiometer.

A general layout of this instrument is given in Figs. 6.9, 6.10 and 6.11. The unit comprises a lamp (12 V), pair of prism and lens filters. The inclusion of appropriate filters improved the sensitivity of the meter for the particular dye solution used. By use of a reference cell and
Fig. 6.9 The absorption meter
A and G  
B and F  
C and E  
D  
H, I, P and Q  
J  
K  
L and M  
N  
O  
R and S  
T  
U  
V  
W and X  
Y  

45° prisms  
Collimating lenses  
Heat absorbing filters  
Lamp  
Windows  
Programming filter  
Calibrating filter  
Colour filter  
Reference absorption cell  
Sample absorption cell  
Barrier layer photocells  
Selector switch  
Recorder  
Galvanometer  
Set full scale control  
Set zero control
FIG. 6. II Simple Automatic Absorption Meter

Circuit of Measuring section
a cell through which Nigrosine dye solution passes continuously
the photocells, a potential difference is set up and is
transferred to the Honeywell and Brown potentiometric recorder
0-5 mv.

ii. Residence time experimental run.

For a given set of steady state conditions in the
apparatus the optimum tracer pulse size was determined
experimentally. The outlet signal was chosen of sufficient
strength to be recorded accurately and yet still remain within
the linear range of the detector.

Duplicate tracers of the outlet signal were established
in each case to ensure reproducibility.
CHAPTER 7.

Experimental results of mass transfer in the Oscillating Baffle Column.

Results were determined for the concentration of acetic acid in the continuous aqueous phase under different operating conditions. These results are given in Table 7.1. As can be seen a wide range of conditions were examined to establish clearly the mode of operation of the contactor. Baffle speed from 35 to 264 osc/min, flow rates from 16.7 to 80 l/hr., and flow ratios from R = 0.5 to 4 show the comprehensiveness of the study.

From these results values for \((N_{oy})_M\), \((Ka)_{av}\) and \((H_{oy})_M\) can be calculated as shown in the following section. In addition values these parameters were obtained along the column.

I. Method of calculation of \((N_{oy})_M\), \((Ka)_{av}\) and \((H_{oy})_M\).

1. Evaluation of \((N_{oy})_M\).

To evaluate \((N_{oy})_M\), it is necessary to integrate the function \(dy/(ye-y)\) between the terminal condition \(y_B\) and \(y_T\). This was achieved by applying Simpson's rule and numerical integration which lends itself to direct computation by the Sirius Computer.

Corresponding value of \(x, y,\) and \(ye\) have to be
<table>
<thead>
<tr>
<th>Baffle speed</th>
<th>( V_y ) 1/hr</th>
<th>( V_x ) 1/hr</th>
<th>( y ) gm/l</th>
<th>( x ) gm/l</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( y_1 )</td>
<td>( y_2 )</td>
<td>( y_3 )</td>
<td>( y_4 )</td>
</tr>
<tr>
<td>35</td>
<td>35</td>
<td>35</td>
<td>6.33</td>
<td>6.80</td>
</tr>
<tr>
<td>35</td>
<td>45</td>
<td>45</td>
<td>9.70</td>
<td>10.20</td>
</tr>
<tr>
<td>35</td>
<td>60</td>
<td>60</td>
<td>6.60</td>
<td>7.02</td>
</tr>
<tr>
<td>35</td>
<td>75</td>
<td>75</td>
<td>7.41</td>
<td>7.90</td>
</tr>
<tr>
<td>71.8</td>
<td>20</td>
<td>20</td>
<td>7.72</td>
<td>8.60</td>
</tr>
<tr>
<td>71.8</td>
<td>30</td>
<td>30</td>
<td>6.52</td>
<td>7.51</td>
</tr>
<tr>
<td>71.8</td>
<td>45</td>
<td>45</td>
<td>7.70</td>
<td>9.00</td>
</tr>
<tr>
<td>71.8</td>
<td>60</td>
<td>60</td>
<td>7.33</td>
<td>8.13</td>
</tr>
<tr>
<td>71.8</td>
<td>75</td>
<td>75</td>
<td>5.30</td>
<td>8.10</td>
</tr>
<tr>
<td>105.90</td>
<td>20</td>
<td>20</td>
<td>2.75</td>
<td>8.00</td>
</tr>
<tr>
<td>105.90</td>
<td>30</td>
<td>30</td>
<td>3.78</td>
<td>10.79</td>
</tr>
<tr>
<td>105.90</td>
<td>60</td>
<td>60</td>
<td>4.00</td>
<td>11.50</td>
</tr>
<tr>
<td>105.90</td>
<td>75</td>
<td>75</td>
<td>1.15</td>
<td>6.00</td>
</tr>
<tr>
<td>175.40</td>
<td>20</td>
<td>20</td>
<td>1.50</td>
<td>4.25</td>
</tr>
<tr>
<td>175.40</td>
<td>30</td>
<td>30</td>
<td>2.10</td>
<td>7.20</td>
</tr>
<tr>
<td>175.40</td>
<td>45</td>
<td>45</td>
<td>1.30</td>
<td>5.20</td>
</tr>
<tr>
<td>175.40</td>
<td>60</td>
<td>60</td>
<td>2.82</td>
<td>9.33</td>
</tr>
<tr>
<td>175.40</td>
<td>75</td>
<td>75</td>
<td>3.30</td>
<td>10.00</td>
</tr>
<tr>
<td>222</td>
<td>20</td>
<td>20</td>
<td>0.645</td>
<td>0.21</td>
</tr>
<tr>
<td>Baffle speed</td>
<td>( v_y ) 1/hr</td>
<td>( v_x ) 1/hr</td>
<td>( y ) gm/l</td>
<td>( x ) gm/l</td>
</tr>
<tr>
<td>--------------</td>
<td>----------------</td>
<td>----------------</td>
<td>-------------</td>
<td>-------------</td>
</tr>
<tr>
<td></td>
<td>( y_1 )</td>
<td>( y_2 )</td>
<td>( y_3 )</td>
<td>( y_4 )</td>
</tr>
<tr>
<td>234</td>
<td>20</td>
<td>20</td>
<td>0.12</td>
<td>0.24</td>
</tr>
<tr>
<td>239.6</td>
<td>20</td>
<td>20</td>
<td>0.23</td>
<td>0.49</td>
</tr>
<tr>
<td>171.3</td>
<td>16.7</td>
<td>16.7</td>
<td>0.90</td>
<td>1.81</td>
</tr>
<tr>
<td>210</td>
<td>16.7</td>
<td>16.7</td>
<td>0.77</td>
<td>1.40</td>
</tr>
<tr>
<td>222</td>
<td>16.7</td>
<td>16.7</td>
<td>0.25</td>
<td>0.61</td>
</tr>
<tr>
<td>248</td>
<td>16.7</td>
<td>16.7</td>
<td>0.34</td>
<td>0.61</td>
</tr>
<tr>
<td>264</td>
<td>16.7</td>
<td>16.7</td>
<td>0.09</td>
<td>0.41</td>
</tr>
</tbody>
</table>
### Table 7.1-3 Mass-transfer O.B.C. continued

<table>
<thead>
<tr>
<th>Baffle speed Osc/m</th>
<th>$V_y$ 1/hr</th>
<th>$V_x$ 1/hr</th>
<th>$y_{1}$</th>
<th>$y_{2}$</th>
<th>$y_{3}$</th>
<th>$y_{4}$</th>
<th>$y_{5}$</th>
<th>$y_{6}$</th>
<th>$y_T$</th>
<th>$x_T$ gm/l</th>
<th>$x_B$ gm/l</th>
</tr>
</thead>
<tbody>
<tr>
<td>R = 0.5</td>
<td>33.33</td>
<td>66.67</td>
<td>16.70</td>
<td>13.00</td>
<td>17.55</td>
<td>17.50</td>
<td>17.55</td>
<td>17.51</td>
<td>17.32</td>
<td>19.58</td>
<td>10.57</td>
</tr>
<tr>
<td>71.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R = 4</td>
<td>30</td>
<td>20</td>
<td>0.50</td>
<td>0.98</td>
<td>1.50</td>
<td>1.66</td>
<td>2.00</td>
<td>2.25</td>
<td>2.30</td>
<td>19.57</td>
<td>9.66</td>
</tr>
<tr>
<td>R = 3</td>
<td>75</td>
<td>25</td>
<td>0.45</td>
<td>0.63</td>
<td>1.05</td>
<td>1.50</td>
<td>1.70</td>
<td>2.00</td>
<td>2.01</td>
<td>13.46</td>
<td>6.04</td>
</tr>
<tr>
<td>R = 2</td>
<td>66.67</td>
<td>33.33</td>
<td>1.05</td>
<td>1.80</td>
<td>2.10</td>
<td>2.55</td>
<td>3.00</td>
<td>3.18</td>
<td>3.20</td>
<td>14.03</td>
<td>5.78</td>
</tr>
<tr>
<td>R = 1</td>
<td>50</td>
<td>50</td>
<td>4.10</td>
<td>5.00</td>
<td>5.68</td>
<td>6.35</td>
<td>6.55</td>
<td>7.10</td>
<td>7.05</td>
<td>14.16</td>
<td>6.99</td>
</tr>
<tr>
<td>R = 0.5</td>
<td>33.33</td>
<td>66.67</td>
<td>12.25</td>
<td>13.25</td>
<td>13.95</td>
<td>14.70</td>
<td>15.20</td>
<td>14.70</td>
<td>15.20</td>
<td>14.20</td>
<td>7.19</td>
</tr>
<tr>
<td>105.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R = 4</td>
<td>80</td>
<td>20</td>
<td>0.15</td>
<td>0.50</td>
<td>0.98</td>
<td>2.00</td>
<td>2.80</td>
<td>3.20</td>
<td>3.35</td>
<td>13.35</td>
<td>5.49</td>
</tr>
<tr>
<td>R = 3</td>
<td>75</td>
<td>25</td>
<td>0.15</td>
<td>0.75</td>
<td>1.50</td>
<td>2.50</td>
<td>3.75</td>
<td>4.15</td>
<td>4.55</td>
<td>13.36</td>
<td>5.36</td>
</tr>
<tr>
<td>R = 2</td>
<td>66.67</td>
<td>33.33</td>
<td>0.45</td>
<td>1.00</td>
<td>1.90</td>
<td>3.75</td>
<td>4.99</td>
<td>5.93</td>
<td>5.78</td>
<td>15.12</td>
<td>5.80</td>
</tr>
<tr>
<td>R = 1</td>
<td>50</td>
<td>50</td>
<td>1.00</td>
<td>3.00</td>
<td>5.20</td>
<td>9.00</td>
<td>10.90</td>
<td>12.00</td>
<td>12.00</td>
<td>13.12</td>
<td>5.94</td>
</tr>
<tr>
<td>R = 0.5</td>
<td>33.33</td>
<td>66.67</td>
<td>5.10</td>
<td>10.05</td>
<td>17.00</td>
<td>22.00</td>
<td>25.00</td>
<td>27.20</td>
<td>27.04</td>
<td>16.93</td>
<td>3.34</td>
</tr>
<tr>
<td>175.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R = 4</td>
<td>30</td>
<td>20</td>
<td>0.01</td>
<td>0.05</td>
<td>0.25</td>
<td>1.10</td>
<td>2.50</td>
<td>4.25</td>
<td>4.39</td>
<td>16.93</td>
<td>0.29</td>
</tr>
<tr>
<td>R = 3</td>
<td>75</td>
<td>25</td>
<td>0.02</td>
<td>0.40</td>
<td>0.80</td>
<td>1.50</td>
<td>3.00</td>
<td>5.50</td>
<td>5.48</td>
<td>16.93</td>
<td>0.36</td>
</tr>
<tr>
<td>R = 2</td>
<td>66.67</td>
<td>33.33</td>
<td>0.08</td>
<td>0.15</td>
<td>1.20</td>
<td>3.00</td>
<td>5.00</td>
<td>8.50</td>
<td>8.24</td>
<td>16.93</td>
<td>0.28</td>
</tr>
<tr>
<td>R = 1</td>
<td>50</td>
<td>50</td>
<td>0.15</td>
<td>1.20</td>
<td>3.50</td>
<td>7.00</td>
<td>13.00</td>
<td>18.10</td>
<td>18.40</td>
<td>17.75</td>
<td>0.40</td>
</tr>
<tr>
<td>R = 0.5</td>
<td>33.33</td>
<td>66.67</td>
<td>1.45</td>
<td>4.00</td>
<td>8.10</td>
<td>16.18</td>
<td>27.50</td>
<td>34.80</td>
<td>35.10</td>
<td>17.46</td>
<td>0.22</td>
</tr>
</tbody>
</table>
Fig. 7.1 O.B.C. Concentration Profile
35 osc./min
Constant flow rate ratio run

- 25 g/l
- 35 g/l
- 45 g/l
- 60 g/l
- 75 g/l

z in. (from the bottom of the column)
Fig. 7.2 O.B.C. Concentration Profile
35 osc./min.
Constant through-put run

R = 4
R = 3
R = 2
R = 1
R = 0.5

y in. (from the bottom of the column)
Fig. 7.3 O.B.C. Concentration Profile
71.8 osc./min.
Constant flow rate ratio run

\[
\begin{align*}
&\text{\textbullet} \quad 20 \text{ l/h} \\
&\text{\textcircled{}} \quad 30 \text{ l/h} \\
&\text{\textdagger} \quad 45 \text{ l/h} \\
&\text{\texttimes} \quad 60 \text{ l/h} \\
&\text{\textasteriskcentered} \quad 75 \text{ l/h}
\end{align*}
\]
Fig. 7.4 O.B.C. Concentration Profile
71.8 osc./min.
Constant through-put run

\[ R = \frac{\text{A}}{\text{R}} \]

\[ R = 0.5 \]

\( Z \text{ in. (from the bottom of the column)} \)
Fig 7.5 O.B.C. Concentration Profile
105.9 osc./min.
Constant flow rate ratio run

- 20 g/hr
- 30 g/hr
- 45 g/hr
- 60 g/hr
- 75 g/hr

y in. (from the bottom of the column)
Fig. 7.6 0.B.C. concentration profile
105.9 osc/min
Constant through-put run.

- ▼ R = 4
- △ R = 3
- ○ R = 2
- × R = 1
- × R = 0.5

z inch. (from the bottom of the column)
Fig. 7.7 O.B.G. concentration profile

175.4 osc/min
constant flow rate ratio run

- 20 l/h
- 30 l/h
- 45 l/h
- 60 l/h
- 75 l/h

z inch (from the bottom of the column)
Fig. 7.8 O.B.C. Concentration Profile
175.4 osc./min.
Constant through-put run

\[ R = 4 \]
\[ R = 3 \]
\[ R = 2 \]
\[ R = 1 \]
\[ R = 0.5 \]

\( z \) in. (from the bottom of the column) →
Fig. 7.9 O.B.C. Concentration Profile
16.7 gal/hr.
Constant flow rate ratio run

- 222 osc./min.
- 210 osc./min.
- 171.3 osc./min.
Fig. 7. 100 B.C. Concentration Profile
16.7 lb/hr
Constant flow rate ratio run

- 248 osc/min.
- 264 osc/min.

z in. (from the bottom of the column)
Fig. 7.11a.b.c. Concentration Profile
20 g/hr
Constant flow rate ratio run

- ○ 234 osc./min.
- × 222 osc./min.
- △ 239.6 osc./min.

\[ \gamma \text{ in. (from the bottom of the column)} \]
established at equal intervals of $y$ between $y_B$ and $y_T$ under
the given operating conditions. The procedure of the
calculation is as follows:

1. In the present case $3\frac{1}{4}$ equal intervals of $y$ were taken
between $y_B$ and $y_T$. Each interval was sufficiently small to
ensure reasonable accuracy in integration.

2. The value $x$ corresponding to each value of $y$ under
operating conditions can be found directly from the operating
equation

$$x = y + x_B$$

3. The equilibrium value of $y$ i.e. $y_e$ corresponding to
each $x$ value can be obtained either from the graph Fig. 4.2
or from the empirical equation Eqn. (58).

4. $1/(y_e - y)$ vs. $y$ can now be tabulated.

5. From Simpson's rule, we have, quoting the general

\[
\int_{y}^{y+n\alpha} w \, dy = \frac{\alpha}{3} \left[ w_0 + 4(\sum w_i) + w_n \right] + \\
\sum_{0}^{n-2} \beta \, w_i
\]

where, $\beta = 1, 1\frac{1}{4}, 2, 1\frac{1}{4}, 2, \ldots, 2, 1\frac{1}{4}, 1$

$\alpha$ = integration interval

6. Putting $W = \frac{1}{y_e - y}$, and $\alpha = \frac{y_T - y_B}{3\frac{1}{4}}$ the value of
the integral can be obtained directly from the computer.
The programme and an example of such a calculation are seen in Appendix 5.

ii. Calculation of \((ka)_{av}\).

Having established value of \((N_{oy})_M\) from the mean column height \(h\) and flow rate \(V_y\) from Eqn. 10 and the \((N_{oy})_M\) values in the same operation on the computer values of \((ha)_{av}\) were recorded.

iii. Evaluation of \((H_{oy})_M\).

From Eqn. 14 this results also directly from the computer. Values for \((N_{oy})_M\), \((ka)_{av}\) and \((H_{oy})_M\) are given for the operation condition in Table 7.2.

II. Variation of \(ka\) along the column.

In order to establish \(ka\), it is necessary to obtain experimentally values for \(y\) vs. \(z\). As only 6 sample points were used it was possible to obtain with a limited but not unreasonable accuracy values of \((ka)_{av}\) corresponding to each of five sections (each section is therefore 6" long) along the column. This has the virtue of reflecting any marked change in \(ka\) over the column even though the true point values are indeterminable with the present arrangement.

The method of calculation of \((ka)_{av}\) in each section is as described previously for \((ka)_{av}\).
### Table 7.2: The Results of the measured $N_{OY}$, $Ka$ and $H_{OY}$

<table>
<thead>
<tr>
<th>Baffle speed &lt;br&gt;cm/s</th>
<th>$V_y$ l/hr</th>
<th>$V_x$ l/hr</th>
<th>Results &lt;br&gt;$N_{OY}$</th>
<th>$Ka \times 10^{-5}$</th>
<th>$H_{OY}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>25</td>
<td>25</td>
<td>0.044</td>
<td>10.2</td>
<td>58.22</td>
</tr>
<tr>
<td>35</td>
<td>35</td>
<td>35</td>
<td>0.038</td>
<td>12.2</td>
<td>68.01</td>
</tr>
<tr>
<td>35</td>
<td>45</td>
<td>45</td>
<td>0.045</td>
<td>18.6</td>
<td>57.37</td>
</tr>
<tr>
<td>35</td>
<td>60</td>
<td>60</td>
<td>0.028</td>
<td>15.4</td>
<td>92.64</td>
</tr>
<tr>
<td>35</td>
<td>75</td>
<td>75</td>
<td>0.032</td>
<td>21.9</td>
<td>81.29</td>
</tr>
<tr>
<td>71.8</td>
<td>20</td>
<td>20</td>
<td>0.039</td>
<td>17.2</td>
<td>62.11</td>
</tr>
<tr>
<td>71.8</td>
<td>30</td>
<td>30</td>
<td>0.035</td>
<td>9.5</td>
<td>74.85</td>
</tr>
<tr>
<td>71.8</td>
<td>45</td>
<td>45</td>
<td>0.042</td>
<td>27.2</td>
<td>65.53</td>
</tr>
<tr>
<td>71.8</td>
<td>60</td>
<td>60</td>
<td>0.039</td>
<td>21.4</td>
<td>66.51</td>
</tr>
<tr>
<td>71.8</td>
<td>75</td>
<td>75</td>
<td>0.039</td>
<td>27.2</td>
<td>65.53</td>
</tr>
<tr>
<td>105.9</td>
<td>20</td>
<td>20</td>
<td>0.062</td>
<td>11.3</td>
<td>42.00</td>
</tr>
<tr>
<td>105.9</td>
<td>30</td>
<td>30</td>
<td>0.065</td>
<td>18.1</td>
<td>39.90</td>
</tr>
<tr>
<td>105.9</td>
<td>45</td>
<td>45</td>
<td>0.069</td>
<td>28.5</td>
<td>37.56</td>
</tr>
<tr>
<td>105.9</td>
<td>60</td>
<td>60</td>
<td>0.072</td>
<td>39.6</td>
<td>36.06</td>
</tr>
<tr>
<td>105.9</td>
<td>75</td>
<td>75</td>
<td>0.071</td>
<td>49.1</td>
<td>36.34</td>
</tr>
<tr>
<td>175.4</td>
<td>20</td>
<td>20</td>
<td>0.078</td>
<td>14.4</td>
<td>32.95</td>
</tr>
<tr>
<td>175.4</td>
<td>30</td>
<td>30</td>
<td>0.117</td>
<td>32.4</td>
<td>21.99</td>
</tr>
<tr>
<td>175.4</td>
<td>45</td>
<td>45</td>
<td>0.115</td>
<td>47.8</td>
<td>22.40</td>
</tr>
<tr>
<td>175.4</td>
<td>60</td>
<td>60</td>
<td>0.107</td>
<td>58.9</td>
<td>24.21</td>
</tr>
<tr>
<td>175.4</td>
<td>75</td>
<td>75</td>
<td>0.127</td>
<td>37.7</td>
<td>20.33</td>
</tr>
<tr>
<td>222</td>
<td>20</td>
<td>20</td>
<td>0.251</td>
<td>46.3</td>
<td>10.28</td>
</tr>
</tbody>
</table>
Table 7.2 The measured $N_{oy}$, $Ka$ and $H_{oy}$ -- O.B.C. Run

<table>
<thead>
<tr>
<th>Baffle speed</th>
<th>$V_y$ 1/hr</th>
<th>$V_{x_1}$ 1/hr</th>
<th>Results</th>
<th>$N_{oy}$</th>
<th>$Ka \times 10^{-5}$</th>
<th>$H_{oy}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>234</td>
<td>20</td>
<td>20</td>
<td>0.322</td>
<td>54.20</td>
<td>8.27</td>
<td></td>
</tr>
<tr>
<td>239.6</td>
<td>20</td>
<td>20</td>
<td>0.533</td>
<td>89.60</td>
<td>6.87</td>
<td></td>
</tr>
<tr>
<td>171.3</td>
<td>16.7</td>
<td>16.7</td>
<td>0.187</td>
<td>26.21</td>
<td>14.24</td>
<td></td>
</tr>
<tr>
<td>210</td>
<td>16.7</td>
<td>16.7</td>
<td>0.261</td>
<td>36.51</td>
<td>10.22</td>
<td></td>
</tr>
<tr>
<td>222</td>
<td>16.7</td>
<td>16.7</td>
<td>0.332</td>
<td>46.55</td>
<td>7.78</td>
<td></td>
</tr>
<tr>
<td>248</td>
<td>16.7</td>
<td>16.7</td>
<td>0.452</td>
<td>63.42</td>
<td>5.89</td>
<td></td>
</tr>
<tr>
<td>264</td>
<td>16.7</td>
<td>16.7</td>
<td>0.756</td>
<td>106.10</td>
<td>3.52</td>
<td></td>
</tr>
<tr>
<td>Battle Speed (m/s)</td>
<td>$V_y$ (1/hr)</td>
<td>$V_x$ (1/hr)</td>
<td>Results</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------------------</td>
<td>--------------</td>
<td>--------------</td>
<td>---------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$N_{oy}$</td>
<td>$Ka_{x10^-5}$</td>
<td>$H_{oy}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35 R= 4</td>
<td>80</td>
<td>20</td>
<td>0.007</td>
<td>5.2</td>
<td>366.54</td>
<td></td>
</tr>
<tr>
<td>35 R= 3</td>
<td>75</td>
<td>25</td>
<td>0.009</td>
<td>6.3</td>
<td>285.06</td>
<td></td>
</tr>
<tr>
<td>35 R= 2</td>
<td>66.67</td>
<td>33.33</td>
<td>0.015</td>
<td>9.4</td>
<td>169.42</td>
<td></td>
</tr>
<tr>
<td>35 R= 1</td>
<td>50</td>
<td>50</td>
<td>0.030</td>
<td>13.6</td>
<td>87.43</td>
<td></td>
</tr>
<tr>
<td>35 R= 0.5</td>
<td>33.33</td>
<td>66.67</td>
<td>0.062</td>
<td>19.0</td>
<td>41.60</td>
<td></td>
</tr>
<tr>
<td>71.3 R= 4</td>
<td>80</td>
<td>20</td>
<td>0.008</td>
<td>5.9</td>
<td>319.71</td>
<td></td>
</tr>
<tr>
<td>71.3 R= 3</td>
<td>75</td>
<td>25</td>
<td>0.009</td>
<td>6.4</td>
<td>279.54</td>
<td></td>
</tr>
<tr>
<td>71.3 R= 2</td>
<td>66.67</td>
<td>33.33</td>
<td>0.015</td>
<td>9.1</td>
<td>173.74</td>
<td></td>
</tr>
<tr>
<td>71.3 R= 1</td>
<td>50</td>
<td>50</td>
<td>0.030</td>
<td>13.8</td>
<td>86.38</td>
<td></td>
</tr>
<tr>
<td>71.3 R= 0.5</td>
<td>33.33</td>
<td>66.67</td>
<td>0.064</td>
<td>19.6</td>
<td>40.48</td>
<td></td>
</tr>
<tr>
<td>105.9 R= 4</td>
<td>80</td>
<td>20</td>
<td>0.013</td>
<td>9.9</td>
<td>192.26</td>
<td></td>
</tr>
<tr>
<td>105.9 R= 3</td>
<td>75</td>
<td>25</td>
<td>0.017</td>
<td>11.9</td>
<td>150.04</td>
<td></td>
</tr>
<tr>
<td>105.9 R= 2</td>
<td>66.67</td>
<td>33.33</td>
<td>0.024</td>
<td>14.5</td>
<td>102.92</td>
<td></td>
</tr>
<tr>
<td>105.9 R= 1</td>
<td>50</td>
<td>50</td>
<td>0.049</td>
<td>22.5</td>
<td>52.84</td>
<td></td>
</tr>
<tr>
<td>105.9 R= 0.5</td>
<td>33.33</td>
<td>66.67</td>
<td>0.126</td>
<td>33.6</td>
<td>20.50</td>
<td></td>
</tr>
<tr>
<td>175.4 R= 4</td>
<td>80</td>
<td>20</td>
<td>0.029</td>
<td>21.2</td>
<td>39.73</td>
<td></td>
</tr>
<tr>
<td>175.4 R= 3</td>
<td>75</td>
<td>25</td>
<td>0.036</td>
<td>24.9</td>
<td>71.60</td>
<td></td>
</tr>
<tr>
<td>175.4 R= 2</td>
<td>66.67</td>
<td>33.33</td>
<td>0.056</td>
<td>34.5</td>
<td>45.95</td>
<td></td>
</tr>
<tr>
<td>175.4 R= 1</td>
<td>50</td>
<td>50</td>
<td>0.115</td>
<td>52.9</td>
<td>22.47</td>
<td></td>
</tr>
<tr>
<td>175.4 R= 0.5</td>
<td>33.33</td>
<td>66.67</td>
<td>0.253</td>
<td>77.5</td>
<td>10.22</td>
<td></td>
</tr>
</tbody>
</table>
Fig 7. $H_{oy}$ vs. oscillating speed in O.B.C.
constant flow rate ratio run

- $20$ $1/hr$
- $16.7$ $1/hr$
### Table 7.2: Ka value along the column -- O.B.C. Run

<table>
<thead>
<tr>
<th>Oscillating speed</th>
<th>$V_y$ (Osc/min)</th>
<th>$V_x$ (L/hr)</th>
<th>Ka value in section $x \times 10^{-3}$ (from bottom of the column)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$s - 1$</td>
</tr>
<tr>
<td>35</td>
<td>25</td>
<td>25</td>
<td>1.8</td>
</tr>
<tr>
<td>35</td>
<td>35</td>
<td>35</td>
<td>3.3</td>
</tr>
<tr>
<td>35</td>
<td>45</td>
<td>45</td>
<td>3.9</td>
</tr>
<tr>
<td>35</td>
<td>60</td>
<td>60</td>
<td>4.6</td>
</tr>
<tr>
<td>35</td>
<td>75</td>
<td>75</td>
<td>6.6</td>
</tr>
<tr>
<td>71.8</td>
<td>20</td>
<td>20</td>
<td>2.6</td>
</tr>
<tr>
<td>71.8</td>
<td>30</td>
<td>30</td>
<td>4.5</td>
</tr>
<tr>
<td>71.8</td>
<td>45</td>
<td>45</td>
<td>7.8</td>
</tr>
<tr>
<td>71.8</td>
<td>60</td>
<td>60</td>
<td>8.6</td>
</tr>
<tr>
<td>71.8</td>
<td>75</td>
<td>75</td>
<td>14.9</td>
</tr>
<tr>
<td>105.9</td>
<td>20</td>
<td>20</td>
<td>17.4</td>
</tr>
<tr>
<td>105.9</td>
<td>30</td>
<td>30</td>
<td>33.1</td>
</tr>
<tr>
<td>105.9</td>
<td>45</td>
<td>45</td>
<td>48.4</td>
</tr>
<tr>
<td>105.9</td>
<td>60</td>
<td>60</td>
<td>79.1</td>
</tr>
<tr>
<td>105.9</td>
<td>75</td>
<td>75</td>
<td>91.8</td>
</tr>
<tr>
<td>175.4</td>
<td>20</td>
<td>20</td>
<td>17.2</td>
</tr>
<tr>
<td>175.4</td>
<td>30</td>
<td>30</td>
<td>51.5</td>
</tr>
<tr>
<td>175.4</td>
<td>45</td>
<td>45</td>
<td>57.7</td>
</tr>
<tr>
<td>175.4</td>
<td>60</td>
<td>60</td>
<td>88.2</td>
</tr>
</tbody>
</table>
### Table 7.5 Ka value along the column—O.B.C. Run

<table>
<thead>
<tr>
<th>Oscillating speed</th>
<th>$V_y$ (1/hr)</th>
<th>$V_x$ (1/hr)</th>
<th>Ka value in section $x \times 10^{-5}$ (from bottom of the column)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>s-1</td>
</tr>
<tr>
<td>175.4</td>
<td>75</td>
<td>75</td>
<td>136.4</td>
</tr>
<tr>
<td>222</td>
<td>20</td>
<td>20</td>
<td>19.6</td>
</tr>
<tr>
<td>234</td>
<td>20</td>
<td>20</td>
<td>8.9</td>
</tr>
<tr>
<td>239.6</td>
<td>20</td>
<td>20</td>
<td>10.3</td>
</tr>
<tr>
<td>243</td>
<td>16.7</td>
<td>16.7</td>
<td>11.1</td>
</tr>
<tr>
<td>264</td>
<td>16.7</td>
<td>16.7</td>
<td>6.8</td>
</tr>
<tr>
<td>210</td>
<td>16.7</td>
<td>16.7</td>
<td>9.9</td>
</tr>
<tr>
<td>35</td>
<td>30</td>
<td>20</td>
<td>3.7</td>
</tr>
<tr>
<td>R= 4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R= 3</td>
<td>75</td>
<td>25</td>
<td>4.3</td>
</tr>
<tr>
<td>R= 2</td>
<td>66.66666666</td>
<td>73.33333333</td>
<td>3.1</td>
</tr>
<tr>
<td>R= 1</td>
<td>50</td>
<td>50</td>
<td>5.0</td>
</tr>
<tr>
<td>R= 0.5</td>
<td>33.33333333</td>
<td>66.66666666</td>
<td>2.0</td>
</tr>
<tr>
<td>71.3</td>
<td>30</td>
<td>20</td>
<td>6.9</td>
</tr>
<tr>
<td>R= 4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R= 3</td>
<td>75</td>
<td>25</td>
<td>7.4</td>
</tr>
<tr>
<td>R= 2</td>
<td>66.66666666</td>
<td>73.33333333</td>
<td>11.1</td>
</tr>
<tr>
<td>R= 1</td>
<td>50</td>
<td>50</td>
<td>8.3</td>
</tr>
<tr>
<td>R= 0.5</td>
<td>33.33333333</td>
<td>66.66666666</td>
<td>5.2</td>
</tr>
<tr>
<td>105.9</td>
<td>30</td>
<td>20</td>
<td>7.7</td>
</tr>
<tr>
<td>R= 4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 7.5 Ka value along the column—B.O.C. Run

<table>
<thead>
<tr>
<th>Oscillating speed Osc/m</th>
<th>V_y 1/hr</th>
<th>V_x 1/hr</th>
<th>Ka value in section x 10^3 (from bottom of the column)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>s - 1</td>
<td>s - 2</td>
<td>s - 3</td>
</tr>
<tr>
<td>105.9 R = 3</td>
<td>11.1</td>
<td>13.2</td>
<td>13.9</td>
</tr>
<tr>
<td>R = 2</td>
<td>9.9</td>
<td>13.9</td>
<td>21.6</td>
</tr>
<tr>
<td>R = 1</td>
<td>22.4</td>
<td>24.5</td>
<td>25.6</td>
</tr>
<tr>
<td>R = 0.5</td>
<td>51.7</td>
<td>42.7</td>
<td>30.8</td>
</tr>
<tr>
<td>175.4 R = 2</td>
<td>8.8</td>
<td>20.1</td>
<td>23.8</td>
</tr>
<tr>
<td>R = 3</td>
<td>40.7</td>
<td>17.9</td>
<td>23.1</td>
</tr>
<tr>
<td>R = 2</td>
<td>31.4</td>
<td>31.5</td>
<td>36.2</td>
</tr>
<tr>
<td>R = 1</td>
<td>46.1</td>
<td>45.1</td>
<td>57.0</td>
</tr>
<tr>
<td>R = 0.5</td>
<td>62.3</td>
<td>59.9</td>
<td>67.8</td>
</tr>
</tbody>
</table>
Fig. 1: Ka value along the column
35 osc/min
Constant through-put run

\[ R = 4 \]
\[ R = 3 \]
\[ R = 2 \]
\[ R = 1 \]
\[ R = 0.5 \]

\[ K_a \times 10^3 \text{ sec}^{-1} \]

\[ R = 1 \]
Constant flow rate ratio run
\[ 25 \text{ l/h} \]
\[ 35 \text{ l/h} \]
\[ 45 \text{ l/h} \]
\[ 60 \text{ l/h} \]
\[ 75 \text{ l/h} \]

3 in. (from the bottom of the column)
Fig. 7256. B.C. $K_a$ value along the column

71.8 osc./min.

Constant flow rate ratio run

- 20 l/h
- 30 l/h
- 45 l/h
- 60 l/h
- 75 l/h

$K_a \times 10^{-5}$ sec.

3 in. (from the bottom of the column)
Fig. 7.60 B.C. Ka value along the column
71.8 osc./min.
Constant through-put run

- R = 4
- R = 3
- R = 2
- R = 1
- R = 0.5

KAX10^5 sec.^-

3 in. (from the bottom of the column)
<table>
<thead>
<tr>
<th>Flow Rate (ocf/min)</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant Flow Rate Ratio Run</td>
<td>0.126</td>
<td>0.3</td>
<td>0.45</td>
<td>0.6</td>
<td>0.75</td>
<td>0.9</td>
<td>1.0</td>
<td>1.1</td>
<td>1.2</td>
<td>1.3</td>
<td>1.4</td>
</tr>
</tbody>
</table>

**Figure:** Graph showing the relationship between flow rate and constant flow rate ratio run.
Fig. 7. O.B.C. $K_a$ value along the column

105.9 Osc/min.

Constant throughput run

- R = 4
- R = 3
- R = 2
- R = 1
- R = 0.5

$K_a \times 10^5 \text{ sec}^{-1}$

2 in. (from the bottom of the column) →
Fig. 7-29 O.B.C. $K_b$ value along the column

175.4 osc./min

Constant flow rate ratio run

- $20$ l/h
- $30$ l/h
- $45$ l/h
- $60$ l/h
- $75$ l/h

$K_b \times 10^5$ sec.

2 in. (from the bottom of the column)
Fig. 7: 300 B.C. Ka value along the column

175.4 osc./min.

Constant throughput run

- R = 4
- R = 3
- R = 2
- R = 1
- R = 0.5

Kₐ x 10⁵ sec.

3 in. (from the bottom of the column)
Fig. 7:3/O.B.C. K_a value along the column
16.7 l/h
constant flow ratio run

- 248 osc/min
- 210 osc/min
- 222 osc/min
- 264 osc/min

\[ K_a \times 10^{-5} \text{ sec}^{-1} \]

\[ \frac{Z \text{ in. (from the bottom of the column)}}{32} \]
Fig. 2.3
O.B.C. Ka value along the column constant flow rate ratio 20 l/h run

- 2.22 osc./min.
- 2.34 osc./min.
- 2.394 osc./min.

Kₐ x 10⁻⁵ sec⁻¹

3 in. (from the bottom of the column)
The results are given in Table 7.5 and Figs. 7.24 to 7.31 and an example and computer programme is given in Appendix 5.

III. Experimental values for eddy diffusion coefficient based mass transfer runs.

In the theoretical section, a differential equation was developed for the present case of transfer of solute (acetic acid) from the organic phase (carbon tetrachloride) to water.

Three methods were used to calculate $E_y$ from Eqn. (24) using the experimentally determined value of $V_y$, $y$, $y_e$, $x$, $\kappa_a$ and $z$. These methods are described below:

Method I.

The sample points are arranged along the column in equal intervals. It is possible therefore to use a numerical differentiation method to evaluate $\frac{d^2y}{dz^2}$ and $\frac{dy}{dz}$. It has been shown by Scarborough (76) that

$$
\left( \frac{dy}{dz} \right)_n = \frac{1}{2s} \left( y_{n+1} - y_{n-1} \right) \ldots \ldots (59)
$$

$$
\left( \frac{d^2y}{dz^2} \right)_n = \frac{1}{s^2} \left( y_{n+1} - 2y_n + y_{n-1} \right) \ldots \ldots (60)
$$

where $s =$ height of each section.

Substituting Eqns. (59) and (60) into the differential Eqn. (24), we have:

$$
(E_y)_n = \frac{(\kappa_a)y_n (y_e - y_n) V_y}{2s} \left( y_{n+1} - y_{n-1} \right) \ldots (61)
$$
The variation of \( k_a \) along the column has been established experimentally as previously described. The results are summarised in Table 7.5 and Figs. 7.24 to 7.31.

The experimental data from three consecutive sample points are required for the evaluation of \( \frac{d^2y}{dz^2} \). It will be appreciated that the bottom and top sample points cannot therefore be included. Thus \( n \) is limited to 2, 3, 4 and 5.

The results are given in Table 7.7. An example and the computer programme are given in Appendix 5.

**Method II.**

The concentration of the inert phases at the top and bottom of the column undergo rapid changes which are not representative of the changes occurring in the column. When analysing the column conditions it is therefore necessary to consider the three central sections only.

The method of calculation used is briefly described as follows:

The differential equation for the aqueous phase is:

\[
E_y \frac{d^2y}{dz^2} + V_y \frac{dy}{dz} - k_a y (ye-y) = 0 \quad \ldots \ldots \quad (24)
\]

Rearranging Eqn. (24), we have:

\[
E_y \frac{1}{ye-y} \frac{d^2y}{dz^2} + V_y \frac{1}{ye-y} \frac{dy}{dz} - k_a y = 0 \quad \ldots \ldots \quad (62)
\]
or, \[ \frac{z_{n+1}}{y e^{-y}} \frac{d^2 y}{dz^2} \left( \frac{z_{n+1}}{y e^{-y}} \right) \left( \frac{z_{n+1}}{z_n} \right) = 0 \]

therefore,

\[ \int_{z_n}^{z_{n+1}} (ka) \, dz - \int_{z_n}^{z_{n+1}} \frac{dy}{y e^{-y}} \]

\[ = E_y \quad \ldots \ldots \quad (63) \]

The denominator of Eqn. 63 can be integrated by parts as follows:

If \( f(x) = u \cdot v \), then \( d(f(x)) = u dv + v du \)

or,

\[ \int u \, dv = uv - \int v \, du \]

Let \( u = \frac{1}{y e^{-y}} \) and \( dv = \frac{d^2 y}{dz^2} \, dz \), then we have,

\[ du = -\frac{1}{(ye-y)^2} \left( \frac{dy}{dz} - \frac{dy}{dz} \right) \, dz, \quad \text{and} \quad v = \frac{dy}{dz} \]

\[ = -\frac{1}{(ye-y)^2} \left( \frac{dy}{dy} - 1 \right) \frac{dy}{dz} \, dz \]
Hence, the denominator in Eqn. (63) will be:

\[ \frac{1}{ye-y} \left[ \frac{dy}{dz} \right]_{z_n}^{z_{n+1}} + \int_{z_n}^{z_{n+1}} \frac{1}{(ye-y)^2} \left( \frac{dy}{dz} \right)^2 \left( \frac{dy}{dy} - 1 \right) dz \]

Therefore:

\[
\int_{z_n}^{z_{n+1}} (Ka) y dz - V_y \int_{z_n}^{z_{n+1}} \frac{dy}{ye-y} - \frac{1}{ye-y} \left[ \frac{dy}{dz} \right]_{z_n}^{z_{n+1}} + \int_{z_n}^{z_{n+1}} \frac{1}{(ye-y)^2} \left( \frac{dy}{dz} \right)^2 \left( \frac{dy}{dy} - 1 \right) dz = E_y \ldots (64)
\]

Application of experimental results.

Experimental conditions.

<table>
<thead>
<tr>
<th>Column section</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>z_n</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>z_{n+1}</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>z_2 = 6&quot;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>z_3 = 12&quot;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>z_4 = 18&quot;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>z_5 = 24&quot;</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The calculation is performed in the following steps:

(a) Evaluation of \( \int_{z_n}^{z_{n+1}} Ka_y dz \).

The values of \( Ka_y \) between the above values of \( z_n \) and \( z_{n+1} \) can be obtained from Table 7.5 and Figs. 7.24 to 7.31. Using Simpson's rule (interval of 1 inch), \( \int_{z_n}^{z_{n+1}} Ka_y dz \) can be evaluated for section 2, 3, and 4.
(b) Evaluation of \[ \int_{y_n}^{y_{n+1}} \frac{dy}{ye-y} \, \frac{dy}{y_n} \].

This method of obtaining the value of this integral has already been described in the section on \( N_{o_y} \) (p 96). Values were obtained for sections 2, 3 and 4.

(c) Evaluation of \( \frac{dy}{dz} \).

This is obtained by a numerical differentiation using a central difference method. The technique is fully described in a standard text (76). The following equation has been established.

\[
\left( \frac{dy}{dz} \right)_z = \frac{1}{h} \left( \frac{\Delta y_1 + \Delta y_0}{2} - \frac{1}{3} \frac{\Delta^3 y_{-2} + \Delta^3 y_{-1}}{2} \right)
\]

Between \( z_1 = 6'' \) and \( z_4 = 24'' \), intervals of 1 inch were considered and the values of \( \frac{dy}{dz} \) obtained by the above equation at each interval.

(d) Evaluation of \( \frac{dye}{dy} \).

From the empirical polynomial equilibrium curve equation, we have:

\[ y_e = a + bx + cx^2 + dx^3 \quad \cdots \cdots \quad (65) \]

The operating condition is defined by Eqn. (4) i.e. \( x = y + x_B \)

Substituting Eqn. (4) into (65) we have:

\[ y_e = a' + b'y + c'y^2 + d'y^3 \]
Therefore,

\[ \frac{dy}{dy} = b' + 2c'y + 3d'y^2 \]

(e) Evaluation of \[ \int_{z_n}^{z_{n+1}} \frac{1}{(ye-y)^2} \left( \frac{dy}{dz} \right)^2 \left( \frac{dy}{dy} - 1 \right) dz \]

From the above the value of the function is obtainable at each 1 inch interval of height of column. Using Simpson's rule the integral can be evaluated.

The result of calculation of \( E_y \) in each of the section 2, 3 and 4 using equation 64 and method II as described above are given in Table 7.7. An example and computer programme is given in Appendix 5.

**Method III.**

A single value of \( E_y \) is calculated over the three sections \( z=6'' \) to \( z=24'' \). This may be achieved by using the same technique as described in method II with a change of limits as follows:

\[ E_y = \frac{\int_{z_2}^{z_5} (ka)y \, dz - V_y \cdot \int_{z_2}^{z_5} \frac{dy}{(ye-y)}}{\int_{z_2}^{z_5} \frac{1}{(ye-y)} \left( \frac{dy}{dz} \right)^2 + \int_{z_2}^{z_5} \frac{1}{(ye-y)^2} \left( \frac{dy}{dy} \right)^2 \left( \frac{dy}{dy} - 1 \right) \, dz} \]

The result of the calculations are given in Table 7.7.
<table>
<thead>
<tr>
<th>Oscillating speed</th>
<th>$V_y$ L/hr</th>
<th>$V_x$ L/hr</th>
<th>$E$ - value By Method 1</th>
<th>$E$ - value By Method 2</th>
<th>$E$ - value By Method 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>25</td>
<td>25</td>
<td>-0.0000966</td>
<td>0.000187</td>
<td>0.0002542</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0004241</td>
<td>-0.000232</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0015239</td>
<td>-0.006252</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0025255</td>
<td></td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>35</td>
<td>35</td>
<td>0.0000755</td>
<td>0.000920</td>
<td>0.0002392</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0001412</td>
<td>-0.000354</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0007726</td>
<td>-0.001119</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0007422</td>
<td></td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>45</td>
<td>45</td>
<td>0.0002223</td>
<td>0.000747</td>
<td>0.0027382</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0155366</td>
<td>0.010149</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0019924</td>
<td>0.001599</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0165752</td>
<td></td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>60</td>
<td>60</td>
<td>0.0006822</td>
<td>0.017566</td>
<td>0.0021122</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0013366</td>
<td>0.00766</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0001205</td>
<td>-0.007176</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0039903</td>
<td></td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>75</td>
<td>75</td>
<td>0.0004432</td>
<td>0.002170</td>
<td>0.0012593</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0002207</td>
<td>0.001108</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0005642</td>
<td>0.003419</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0013199</td>
<td></td>
<td></td>
</tr>
<tr>
<td>71.3</td>
<td>20</td>
<td>20</td>
<td>-0.0000707</td>
<td>-0.000319</td>
<td>0.0001525</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0002532</td>
<td>0.000313</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.000351</td>
<td>0.000079</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0000063</td>
<td></td>
<td></td>
</tr>
<tr>
<td>71.8</td>
<td>30</td>
<td>30</td>
<td>-0.0001028</td>
<td>0.000452</td>
<td>0.0005722</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0005493</td>
<td>0.000534</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0003763</td>
<td>0.000698</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0005400</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baffle speed Osc/m</td>
<td>$V_y$ 1/hr</td>
<td>$V_x$ 1/hr</td>
<td>E - value</td>
<td></td>
<td></td>
</tr>
<tr>
<td>------------------</td>
<td>-----------</td>
<td>-----------</td>
<td>----------</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>By Method 1</td>
<td>By Method 2</td>
<td>By Method 3</td>
</tr>
<tr>
<td>71.8</td>
<td>45</td>
<td>45</td>
<td>0.002816</td>
<td>0.0019754</td>
<td>0.0012865</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0017320</td>
<td>0.0009407</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0004396</td>
<td>0.0002795</td>
<td></td>
</tr>
<tr>
<td>71.8</td>
<td>60</td>
<td>60</td>
<td>-0.0000625</td>
<td>0.001024</td>
<td>0.0006394</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.000525</td>
<td>-0.0000329</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0002669</td>
<td>0.000851</td>
<td></td>
</tr>
<tr>
<td>71.8</td>
<td>75</td>
<td>75</td>
<td>0.0049749</td>
<td>0.002380</td>
<td>0.0022352</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0004681</td>
<td>0.003282</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0009864</td>
<td>0.001620</td>
<td></td>
</tr>
<tr>
<td>105.9</td>
<td>20</td>
<td>20</td>
<td>-0.0003225</td>
<td>-0.000244</td>
<td>-0.0014810</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.000557</td>
<td>-0.000993</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.002857</td>
<td>-0.00190</td>
<td></td>
</tr>
<tr>
<td>105.9</td>
<td>30</td>
<td>30</td>
<td>0.000748</td>
<td>0.00278</td>
<td>-0.0001991</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0006330</td>
<td>-0.000700</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0014802</td>
<td>-0.001089</td>
<td></td>
</tr>
<tr>
<td>105.9</td>
<td>45</td>
<td>45</td>
<td>-0.0000075</td>
<td>0.0002453</td>
<td>-0.00002234</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0000396</td>
<td>-0.000646</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0001762</td>
<td>0.000282</td>
<td></td>
</tr>
<tr>
<td>105.9</td>
<td>60</td>
<td>60</td>
<td>0.0015279</td>
<td>0.000079</td>
<td>-0.0013486</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0017338</td>
<td>-0.004109</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0046580</td>
<td>-0.004615</td>
<td></td>
</tr>
<tr>
<td>105.9</td>
<td>75</td>
<td>75</td>
<td>0.0007451</td>
<td>0.004256</td>
<td>0.0022764</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0011514</td>
<td>0.001620</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0002555</td>
<td>0.000678</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0005103</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baffle speed Osc/m</td>
<td>V_y 1/hr</td>
<td>V_x L/hr</td>
<td>E-value</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------------------</td>
<td>---------</td>
<td>----------</td>
<td>---------</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>By Method 1</td>
<td>By Method 2</td>
<td>By Method 3</td>
</tr>
<tr>
<td>175.4</td>
<td>20</td>
<td>20</td>
<td>-0.0029634</td>
<td>0.000937</td>
<td>0.0012234</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0161709</td>
<td>0.001162</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.000552</td>
<td>0.000510</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0002165</td>
<td></td>
<td></td>
</tr>
<tr>
<td>175.4</td>
<td>30</td>
<td>30</td>
<td>-0.0018050</td>
<td>-0.000294</td>
<td>0.0000636</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.001019</td>
<td>-0.000469</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.000837</td>
<td>0.000363</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0001716</td>
<td></td>
<td></td>
</tr>
<tr>
<td>175.4</td>
<td>45</td>
<td>45</td>
<td>-0.00197652</td>
<td>0.006351</td>
<td>0.0018365</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.001148</td>
<td>0.001961</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0031983</td>
<td>0.005795</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0003444</td>
<td></td>
<td></td>
</tr>
<tr>
<td>175.4</td>
<td>60</td>
<td>60</td>
<td>-0.0026348</td>
<td>-0.000560</td>
<td>0.0003397</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.001318</td>
<td>0.001600</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.001109</td>
<td>0.000487</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0001209</td>
<td></td>
<td></td>
</tr>
<tr>
<td>175.4</td>
<td>75</td>
<td>75</td>
<td>-0.0001789</td>
<td>0.003645</td>
<td>0.0024126</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0031082</td>
<td>0.003602</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0010361</td>
<td>0.000432</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0009683</td>
<td></td>
<td></td>
</tr>
<tr>
<td>222</td>
<td>20</td>
<td>20</td>
<td>-0.0004531</td>
<td>0.000490</td>
<td>0.0002603</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0003040</td>
<td>0.000288</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0000813</td>
<td>0.000450</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0001955</td>
<td></td>
<td></td>
</tr>
<tr>
<td>234</td>
<td>20</td>
<td>20</td>
<td>-0.0003153</td>
<td>0.000053</td>
<td>0.000112</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0003352</td>
<td>0.000071</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0003078</td>
<td>0.000120</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0003388</td>
<td></td>
<td></td>
</tr>
<tr>
<td>239.6</td>
<td>20</td>
<td>20</td>
<td>0.0007205</td>
<td>0.000670</td>
<td>0.0003435</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0006552</td>
<td>0.000364</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0001412</td>
<td>0.000662</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0003218</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 7.7 continued

<table>
<thead>
<tr>
<th>Baffle speed Osc/m</th>
<th>$V_y$ 1/hr</th>
<th>$V_x$ 1/hr</th>
<th>E-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>By Method 1</td>
</tr>
<tr>
<td>210</td>
<td>16.7</td>
<td>16.7</td>
<td>-0.0001019</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0001422</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0000490</td>
</tr>
<tr>
<td>222</td>
<td>16.7</td>
<td>16.7</td>
<td>-0.0001442</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0001259</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0000727</td>
</tr>
<tr>
<td>248</td>
<td>16.7</td>
<td>16.7</td>
<td>-0.0002928</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0002437</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0005007</td>
</tr>
<tr>
<td>264</td>
<td>16.7</td>
<td>16.7</td>
<td>0.0003336</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0002312</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0001265</td>
</tr>
<tr>
<td>35 R=4</td>
<td>30</td>
<td>20</td>
<td>-0.0010283</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0022299</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0076282</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0024834</td>
</tr>
<tr>
<td>75 R=3</td>
<td>75</td>
<td>25</td>
<td>-0.0002163</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0003694</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0008222</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0046575</td>
</tr>
<tr>
<td>66.67 R=2</td>
<td>33.33</td>
<td>33.33</td>
<td>-0.0038936</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0006014</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0029332</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0018078</td>
</tr>
<tr>
<td>50 R=1</td>
<td>50</td>
<td>50</td>
<td>-0.0007673</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0027749</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0007109</td>
</tr>
<tr>
<td>Baffle speed Osc/min</td>
<td>$V_y$ 1/hr</td>
<td>$V_x$ 1/hr</td>
<td>E-value</td>
</tr>
<tr>
<td>---------------------</td>
<td>-----------</td>
<td>-----------</td>
<td>---------</td>
</tr>
<tr>
<td>35 R= 0.5</td>
<td>33.33</td>
<td>66.67</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>71.8 R= 4</td>
<td>80</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R= 3 R= 3</td>
<td>75</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R= 2 R= 2</td>
<td>56.67</td>
<td>33.33</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R= 1 R= 1</td>
<td>50</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R=0.5 R=0.5</td>
<td>33.33</td>
<td>66.67</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>105.9 R= 4</td>
<td>80</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R= 3 R= 3</td>
<td>75</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baffle speed Osc/min</td>
<td>V_y</td>
<td>V_x</td>
<td>E-value</td>
</tr>
<tr>
<td>----------------------</td>
<td>-----</td>
<td>-----</td>
<td>---------</td>
</tr>
<tr>
<td></td>
<td>1/hr</td>
<td>1/hr</td>
<td>By Method 1</td>
</tr>
<tr>
<td>105.9 R= 3</td>
<td>75</td>
<td>25</td>
<td>-0.0017027</td>
</tr>
<tr>
<td>R= 2</td>
<td>66.67</td>
<td>33.33</td>
<td>0.0001943</td>
</tr>
<tr>
<td>R= 1</td>
<td>50</td>
<td>50</td>
<td>-0.0002647</td>
</tr>
<tr>
<td>R= 0.5</td>
<td>33.33</td>
<td>66.67</td>
<td>0.0011418</td>
</tr>
<tr>
<td>175.4 R= 4</td>
<td>80</td>
<td>20</td>
<td>-0.0020704</td>
</tr>
<tr>
<td>R= 3</td>
<td>75</td>
<td>25</td>
<td>-0.0186376</td>
</tr>
<tr>
<td>R= 2</td>
<td>66.67</td>
<td>33.33</td>
<td>0.0012216</td>
</tr>
<tr>
<td>R= 1</td>
<td>50</td>
<td>50</td>
<td>-0.0003137</td>
</tr>
<tr>
<td>Baffle speed Osc/min</td>
<td>$V_y$ (1/hr)</td>
<td>$V_x$ (1/hr)</td>
<td>E-value</td>
</tr>
<tr>
<td>---------------------</td>
<td>--------------</td>
<td>--------------</td>
<td>---------</td>
</tr>
<tr>
<td>175.4 R= 0.5</td>
<td>33.33</td>
<td>66.67</td>
<td>By Method 1:&lt;br&gt;-0.000930&lt;br&gt;-0.0069357&lt;br&gt;-0.0011122&lt;br&gt;0.0024149</td>
</tr>
</tbody>
</table>
An example and the computer programme are shown in Appendix 5.

IV. Experimental results for mean residence time.

(Oscillating Baffle Column)

In Figs. 7.33 and 7.34 are shown some actual recorder tracers of dye concentration in the aqueous phase at the column outlet as measured by the absorptiometer.

These results were obtained for both single and two phases flow. Values for the mean residence time, variance and eddy diffusion coefficients were obtained from the recorded $E(t)$ vs. $t$ curve.

The methods for obtaining the values of $\bar{t}$, and $D$ is as follows:

1. Mean residence time $\bar{t} = \frac{E(t)}{\sum E(t)}$ ...... (37)

2. $\sigma^2 = \frac{\sigma^2}{t^2}$

   $= \left( \frac{\frac{E(t)}{E(t)} t^2 - \left( \frac{E(t)}{E(t)} \right)^2 \right)}{t^2}$ ... (54)

3. $\sigma^2 = 8 \left( \frac{D}{UL} \right)^2 + 2 \left( \frac{D}{UL} \right)$,

   or $\frac{D}{UL} = \frac{1}{8} \left( \sqrt{8\sigma^2 + 1} - 1 \right)$ ...... (56)

Equation (37) has been derived on page 43, Eqns. (40) and (54) on pages 44 and 48 and Eqn. (56) on page 49.

In order to evaluate equal intervals of 10 seconds were taken between $t=0$ and $t=1000$ seconds and equations (37), (54) and (56) above used to compute $\bar{t}$, $\sigma^2$ and $D$ on the
Fig. 43 The actual recorder traces of dye concentration in the aqueous phase at OBC single phase run.
Fig. 7.44 The actual recorder traces of dye concentration in the aqueous phase at OBC two phases run.
Sirius Computer. The results are given in Table 7.9 and Figs. 7.45 and 7.46.

An example of the computer programme is shown in Appendix 5.
## Table 7.9 Mean Residence Time Study Results
### O.B.C. Two Phases Run

<table>
<thead>
<tr>
<th>Baffle speed Osc/m</th>
<th>$V_Y$ l/hr</th>
<th>$V_X$ l/hr</th>
<th>Mean Residence Time sec.</th>
<th>$\sigma$</th>
<th>D/UL</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>30</td>
<td>30</td>
<td>472.143</td>
<td>0.586775</td>
<td>0.173281</td>
<td>0.002918</td>
</tr>
<tr>
<td>35</td>
<td>45</td>
<td>45</td>
<td>376.453</td>
<td>0.510662</td>
<td>0.156682</td>
<td>0.004755</td>
</tr>
<tr>
<td>35</td>
<td>50</td>
<td>50</td>
<td>362.834</td>
<td>0.396689</td>
<td>0.130365</td>
<td>0.004390</td>
</tr>
<tr>
<td>35</td>
<td>60</td>
<td>60</td>
<td>252.5027</td>
<td>0.584631</td>
<td>0.172832</td>
<td>0.006984</td>
</tr>
<tr>
<td>35 (R=4)</td>
<td>80</td>
<td>20</td>
<td>193.541</td>
<td>0.502592</td>
<td>0.155087</td>
<td>0.008356</td>
</tr>
<tr>
<td>35 (R=3)</td>
<td>75</td>
<td>25</td>
<td>252.142</td>
<td>0.438220</td>
<td>0.140335</td>
<td>0.007089</td>
</tr>
<tr>
<td>35 (R=2)</td>
<td>66.67</td>
<td>33.33</td>
<td>251.979</td>
<td>0.458014</td>
<td>0.144957</td>
<td>0.006508</td>
</tr>
<tr>
<td>35 (R=1)</td>
<td>50</td>
<td>50</td>
<td>362.834</td>
<td>0.396689</td>
<td>0.130365</td>
<td>0.004390</td>
</tr>
<tr>
<td>100</td>
<td>30</td>
<td>30</td>
<td>538.910</td>
<td>0.238268</td>
<td>0.088095</td>
<td>0.001780</td>
</tr>
<tr>
<td>100</td>
<td>45</td>
<td>45</td>
<td>396.165</td>
<td>0.224721</td>
<td>0.074580</td>
<td>0.002881</td>
</tr>
<tr>
<td>100</td>
<td>50</td>
<td>50</td>
<td>383.354</td>
<td>0.229670</td>
<td>0.085550</td>
<td>0.002881</td>
</tr>
<tr>
<td>100</td>
<td>60</td>
<td>60</td>
<td>275.942</td>
<td>0.193675</td>
<td>0.074580</td>
<td>0.003014</td>
</tr>
<tr>
<td>100 R=4</td>
<td>80</td>
<td>20</td>
<td>235.825</td>
<td>0.145516</td>
<td>0.058887</td>
<td>0.003173</td>
</tr>
<tr>
<td>100 R=3</td>
<td>75</td>
<td>25</td>
<td>259.398</td>
<td>0.179004</td>
<td>0.058887</td>
<td>0.003533</td>
</tr>
<tr>
<td>100 R=2</td>
<td>66.66</td>
<td>33.33</td>
<td>298.380</td>
<td>0.244020</td>
<td>0.058887</td>
<td>0.004030</td>
</tr>
<tr>
<td>100 R=1</td>
<td>50</td>
<td>50</td>
<td>383.354</td>
<td>0.229670</td>
<td>0.085550</td>
<td>0.002881</td>
</tr>
<tr>
<td>150</td>
<td>30</td>
<td>30</td>
<td>478.266</td>
<td>0.152325</td>
<td>0.061187</td>
<td>0.001236</td>
</tr>
<tr>
<td>150</td>
<td>45</td>
<td>45</td>
<td>337.580</td>
<td>0.175325</td>
<td>0.068754</td>
<td>0.002084</td>
</tr>
<tr>
<td>150</td>
<td>50</td>
<td>50</td>
<td>279.677</td>
<td>0.158664</td>
<td>0.063369</td>
<td>0.002134</td>
</tr>
<tr>
<td>150</td>
<td>60</td>
<td>60</td>
<td>236.823</td>
<td>0.121800</td>
<td>0.050642</td>
<td>0.002046</td>
</tr>
<tr>
<td>150 R=4</td>
<td>80</td>
<td>20</td>
<td>227.614</td>
<td>0.074436</td>
<td>0.032891</td>
<td>0.001772</td>
</tr>
</tbody>
</table>
### Table 7.9 continued

<table>
<thead>
<tr>
<th>Baffle speed Osc/m</th>
<th>V_y 1/hr</th>
<th>V_x 1/hr</th>
<th>Mean Residence Time, sec.</th>
<th>$\sigma^2$</th>
<th>D/UL</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>150 R=3</td>
<td>75</td>
<td>25</td>
<td>220.076</td>
<td>0.097131</td>
<td>0.041633</td>
<td>0.002103</td>
</tr>
<tr>
<td>150 R=2</td>
<td>66.67</td>
<td>33.33</td>
<td>231.9352</td>
<td>0.120487</td>
<td>0.050178</td>
<td>0.002253</td>
</tr>
<tr>
<td>150 R=1</td>
<td>50</td>
<td>50</td>
<td>279.6765</td>
<td>0.158864</td>
<td>0.063369</td>
<td>0.002134</td>
</tr>
</tbody>
</table>
### Table 7.9-1: Mean Residence Time Study Results

**Q.B.C. Single phase Run**

<table>
<thead>
<tr>
<th>Baffle Speed</th>
<th>( V_y ) L/hr</th>
<th>( V_x ) L/hr</th>
<th>Mean Residence Time, sec.</th>
<th>( \sigma^2 )</th>
<th>D/UL</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>30</td>
<td>-</td>
<td>528.305</td>
<td>0.081839</td>
<td>0.035795</td>
<td>0.000723</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>-</td>
<td>342.581</td>
<td>0.058600</td>
<td>0.026493</td>
<td>0.000803</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>-</td>
<td>250.292</td>
<td>0.040012</td>
<td>0.018619</td>
<td>0.000752</td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>-</td>
<td>198.840</td>
<td>0.034895</td>
<td>0.016315</td>
<td>0.000827</td>
</tr>
<tr>
<td>100</td>
<td>30</td>
<td>-</td>
<td>538.910</td>
<td>0.078068</td>
<td>0.036093</td>
<td>0.000834</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>-</td>
<td>391.560</td>
<td>0.082570</td>
<td>0.036078</td>
<td>0.001093</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>-</td>
<td>268.723</td>
<td>0.070057</td>
<td>0.031145</td>
<td>0.001259</td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>-</td>
<td>209.655</td>
<td>0.073529</td>
<td>0.032532</td>
<td>0.001643</td>
</tr>
<tr>
<td>150</td>
<td>30</td>
<td>-</td>
<td>505.972</td>
<td>0.124952</td>
<td>0.051760</td>
<td>0.001046</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>-</td>
<td>344.842</td>
<td>0.110013</td>
<td>0.046396</td>
<td>0.001406</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>-</td>
<td>262.880</td>
<td>0.090355</td>
<td>0.039071</td>
<td>0.001579</td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>-</td>
<td>209.540</td>
<td>0.074673</td>
<td>0.032985</td>
<td>0.001666</td>
</tr>
</tbody>
</table>
Fig. 7. D vs oscillating speed in O.B.C. single phase run

- 75 l/hr
- 60 l/hr
- 45 l/hr
- 30 l/hr

Oscillating speed — osc/m —

D x 10^{-3} ft/sec

0.8

0.4

0

30 50 70 90 110 130 150
Fig. 2.6 D vs oscillating speed in O.B.C.
two phases constant flow rate ratio run

- □ 30 l/hr
- × 45 l/hr
- ● 50 l/hr
- ▼ 60 l/hr

Oscillating speed — osc/m
V. Discussion.

The experimentally determined values of \((N_{oy})_M\), \((ka)_y\), and \((Ho_y)_M\) in Table 7.2 show a dependence on the liquid flow rate, phase flow ratio and the oscillating baffle speed. Of these three variables, the oscillating speed is the more important as shown by the effect on the extraction efficiency as reflected by \((N_{oy})_M\). For flow rates up to 80 to 90% of flooding conditions it can be seen at all oscillating speeds that the larger the speed the higher the extraction efficiency. The higher the speed the greater the tendency to flooding and consequently the flow rate has to be decreased with increasing speed. It is thus seen that a compromise has to be reached between the efficiency of extraction and the load.

The effect of the oscillating speed on \(ka\) can be seen from Table 7.5 and from Figs. 7.24 to 7.32. The \(ka\) values are larger at the bottom of the column at low oscillating speed than at the top, but the reverse is true at high oscillating speed. The reason for this phenomenon can be explained as follows:

1. At low oscillating speeds the dispersion of the organic phase into droplets is only moderately effective being progressive down the column. It follows that the
interfacial area increases progressively down the column. $K_a$ will increase in value down the column.

2. At high oscillating speed ($>175.4$ osc/min) the baffle is so efficient that the organic phase is well dispersed into droplets immediately and further dispersion is small as we progress down the column. It follows that "a" the interfacial area is large initially and then becomes nearly constant along the column. The rate of mass transfer is therefore high at the top of the column i.e. $K_a$ is large. As we descend the column the extraction becomes more difficult, and there is no marked increase in interfacial area to aid the transfer process. Whether $K_a$ increases or decreases down the column will therefore depend very much on the effect of physical conditions on $K$ as we proceed down the column. In practice it was found that at high speed $K_a$ decreases down the column that "a" is likely to remain constant then $K$ decreases down the column at high speed.

**Values of $D$:**

The values of $D_y$ gives an indication of performance even dissociated from the term $E_y$.

It can be seen from Fig. 7.46 that an increase in speed results in a lowering in value of $D = E_y$ at constant flow rate. At high flow rates the effect is most marked and is less significant at low flow rates.
While this not a direct measure of the change in back mixing; a significant fall in the values of $D_y = E$ will almost certainly be associated with a lowering of the back mixing.

The smaller the back mixing the larger will be the efficiency of separation. If Fig. 7.47 is inspected it will be seen that there is a very large change in $H_{oy}$ with increasing in oscillating speed at constant flow rate e.g. 170 osc/min - 264, $H_{oy} = 2.4$ - 3.52 ft. The smaller the magnitude of $H_{oy}$ the more efficient is the column in effecting separation.

Although a direct relation between $H_{oy}$ and $E_y = D$ is not established it can be seen from Fig. 7.47 and Table 7.10 that clearly $H_{oy}$ is a function of $E_y$ and when $E_y$ is large $H_{oy}$ is small and vice versa.

Table 7.10  O.B.C. constant flow rate ratio 60 l/hr run.

<table>
<thead>
<tr>
<th>Oscillating speed osc/min</th>
<th>$H_{oy}$</th>
<th>$E_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>92.64</td>
<td>0.006894</td>
</tr>
<tr>
<td>71.8</td>
<td>66.51</td>
<td>0.003014</td>
</tr>
<tr>
<td>100.0</td>
<td>36.06</td>
<td>0.002046</td>
</tr>
<tr>
<td>105.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>150.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>175.4</td>
<td>24.21</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 7. 

$H_{oy}$ vs. oscillating speed

(OCB constant flowrate ratio 60 l/h, run)
Experimental results of mass transfer in the Rotating Disc Contactor.

In order to make a comparison of the performance of the Oscillating Baffle Column with a Rotating Disc Contactor, it was necessary to repeat the experiments already carried out.

To make the comparison valid and to remove the difficulties of scale-up the RDC unit was established by simply removing the OBC baffle from the column and replacing it by the lateral baffles and the central disc core as already described the system water-acetic acid-carbon tetrachloride was used.

The speed of rotation of the disc was in the range 400-1,600 rpm, flow rates 16.7 - 80 l/hr. and flow ratios $R = 0.5, 1, 2, 3, 4$.

$(N_{oy})_M, (H_{oy})_M, (Ka)_y, Kay$ variation along the column, $Ey$ (by three methods) and mean residence time studies were carried out. The results obtained are as follows:

I. Evaluation of $(N_{oy})_M, (H_{oy})_M$ and $(Ka)_y$.

In Table 7.3 are given the experimental values of $y$ at the top and bottom of the RDC, also along the column at five equally spaced sample points. ($y$ refers to the concentration of acetic acid in the water phase) The
<table>
<thead>
<tr>
<th>Baffle speed (rpm)</th>
<th>$V_y$ (1/hr)</th>
<th>$V_x$ (1/hr)</th>
<th>$y_{1}$</th>
<th>$y_{2}$</th>
<th>$y_{3}$</th>
<th>$y_{4}$</th>
<th>$y_{5}$</th>
<th>$y_{6}$</th>
<th>$y_T$</th>
<th>$x_T$</th>
<th>$x_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>16.7</td>
<td>16.7</td>
<td>0.27</td>
<td>0.46</td>
<td>0.11</td>
<td>1.06</td>
<td>1.63</td>
<td>2.40</td>
<td>2.41</td>
<td>4.05</td>
<td>1.56</td>
</tr>
<tr>
<td>400</td>
<td>30</td>
<td>30</td>
<td>0.39</td>
<td>0.42</td>
<td>0.63</td>
<td>0.92</td>
<td>1.42</td>
<td>2.23</td>
<td>2.24</td>
<td>3.83</td>
<td>1.44</td>
</tr>
<tr>
<td>400</td>
<td>45</td>
<td>45</td>
<td>0.22</td>
<td>0.41</td>
<td>0.60</td>
<td>0.89</td>
<td>1.37</td>
<td>2.20</td>
<td>2.24</td>
<td>3.72</td>
<td>1.42</td>
</tr>
<tr>
<td>540</td>
<td>50</td>
<td>50</td>
<td>0.69</td>
<td>1.68</td>
<td>2.93</td>
<td>4.45</td>
<td>7.02</td>
<td>14.95</td>
<td>14.64</td>
<td>20.82</td>
<td>7.11</td>
</tr>
<tr>
<td>540</td>
<td>60</td>
<td>60</td>
<td>1.33</td>
<td>1.86</td>
<td>2.91</td>
<td>4.44</td>
<td>7.22</td>
<td>12.30</td>
<td>12.35</td>
<td>19.36</td>
<td>7.01</td>
</tr>
<tr>
<td>540</td>
<td>75</td>
<td>75</td>
<td>0.73</td>
<td>1.78</td>
<td>2.85</td>
<td>4.82</td>
<td>7.22</td>
<td>12.84</td>
<td>12.69</td>
<td>19.36</td>
<td>6.99</td>
</tr>
<tr>
<td>700</td>
<td>30</td>
<td>30</td>
<td>1.63</td>
<td>2.78</td>
<td>4.73</td>
<td>7.15</td>
<td>10.61</td>
<td>15.29</td>
<td>15.30</td>
<td>22.93</td>
<td>6.54</td>
</tr>
<tr>
<td>700</td>
<td>45</td>
<td>45</td>
<td>1.24</td>
<td>2.96</td>
<td>4.73</td>
<td>7.28</td>
<td>10.48</td>
<td>15.27</td>
<td>15.74</td>
<td>23.50</td>
<td>6.17</td>
</tr>
<tr>
<td>700</td>
<td>50</td>
<td>50</td>
<td>1.14</td>
<td>2.78</td>
<td>4.73</td>
<td>7.29</td>
<td>10.85</td>
<td>16.05</td>
<td>16.31</td>
<td>23.56</td>
<td>6.41</td>
</tr>
<tr>
<td>700</td>
<td>60</td>
<td>60</td>
<td>1.08</td>
<td>2.69</td>
<td>4.53</td>
<td>6.90</td>
<td>10.05</td>
<td>15.01</td>
<td>15.30</td>
<td>21.45</td>
<td>5.58</td>
</tr>
<tr>
<td>700</td>
<td>75</td>
<td>75</td>
<td>1.56</td>
<td>2.94</td>
<td>4.83</td>
<td>7.17</td>
<td>10.39</td>
<td>15.94</td>
<td>16.02</td>
<td>22.15</td>
<td>6.14</td>
</tr>
<tr>
<td>960</td>
<td>16.7</td>
<td>16.7</td>
<td>1.46</td>
<td>2.72</td>
<td>4.73</td>
<td>7.68</td>
<td>11.64</td>
<td>15.53</td>
<td>15.81</td>
<td>16.15</td>
<td>0.97</td>
</tr>
<tr>
<td>960</td>
<td>30</td>
<td>30</td>
<td>0.75</td>
<td>1.82</td>
<td>3.51</td>
<td>6.12</td>
<td>10.07</td>
<td>15.02</td>
<td>15.26</td>
<td>16.13</td>
<td>1.11</td>
</tr>
<tr>
<td>960</td>
<td>50</td>
<td>50</td>
<td>0.82</td>
<td>2.21</td>
<td>4.13</td>
<td>6.38</td>
<td>11.01</td>
<td>17.32</td>
<td>17.66</td>
<td>19.35</td>
<td>1.80</td>
</tr>
<tr>
<td>972</td>
<td>30</td>
<td>30</td>
<td>1.28</td>
<td>2.84</td>
<td>5.39</td>
<td>9.17</td>
<td>14.95</td>
<td>19.85</td>
<td>20.02</td>
<td>21.53</td>
<td>1.49</td>
</tr>
<tr>
<td>972</td>
<td>45</td>
<td>45</td>
<td>0.95</td>
<td>2.37</td>
<td>4.53</td>
<td>7.83</td>
<td>13.32</td>
<td>22.18</td>
<td>22.59</td>
<td>24.43</td>
<td>1.86</td>
</tr>
<tr>
<td>972</td>
<td>60</td>
<td>60</td>
<td>1.02</td>
<td>1.69</td>
<td>3.05</td>
<td>5.04</td>
<td>8.13</td>
<td>13.58</td>
<td>14.35</td>
<td>15.80</td>
<td>1.27</td>
</tr>
<tr>
<td>972</td>
<td>75</td>
<td>75</td>
<td>1.25</td>
<td>2.15</td>
<td>3.31</td>
<td>6.13</td>
<td>9.02</td>
<td>14.16</td>
<td>14.28</td>
<td>15.41</td>
<td>1.01</td>
</tr>
<tr>
<td>R=0.5</td>
<td>33.33</td>
<td>66.67</td>
<td>3.53</td>
<td>6.67</td>
<td>10.85</td>
<td>18.17</td>
<td>26.05</td>
<td>37.08</td>
<td>39.48</td>
<td>20.28</td>
<td>1.73</td>
</tr>
<tr>
<td>R= 2</td>
<td>66.67</td>
<td>33.33</td>
<td>0.39</td>
<td>0.98</td>
<td>1.89</td>
<td>3.26</td>
<td>5.31</td>
<td>8.71</td>
<td>8.98</td>
<td>19.51</td>
<td>1.58</td>
</tr>
<tr>
<td>R = 3</td>
<td>75</td>
<td>25</td>
<td>0.25</td>
<td>0.67</td>
<td>1.27</td>
<td>2.19</td>
<td>3.62</td>
<td>5.76</td>
<td>6.22</td>
<td>19.06</td>
<td>1.39</td>
</tr>
<tr>
<td>Baffle speed</td>
<td>$V_y$ (l/hr)</td>
<td>$V_x$ (l/hr)</td>
<td>$y_{gm/l}$</td>
<td>$x_{gm/l}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>--------------</td>
<td>--------------</td>
<td>--------------</td>
<td>------------</td>
<td>------------</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_i = 2.2$</td>
<td>30</td>
<td>20</td>
<td>$y_1$ 0.14</td>
<td>$x_T$ 13.93</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1380</td>
<td>16.7</td>
<td>16.7</td>
<td>$y_2$ 0.46</td>
<td>1.18</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1180</td>
<td>20</td>
<td>20</td>
<td>$y_3$ 0.84</td>
<td>3.42</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1130</td>
<td>30</td>
<td>30</td>
<td>$y_4$ 1.42</td>
<td>1.42</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1130</td>
<td>40</td>
<td>40</td>
<td>$y_5$ 2.28</td>
<td>2.28</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1300</td>
<td>16.3</td>
<td>16.3</td>
<td>$y_6$ 3.43</td>
<td>3.43</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1300</td>
<td>20</td>
<td>20</td>
<td>$y_T$ 3.44</td>
<td>3.44</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1300</td>
<td>30</td>
<td>30</td>
<td>13.93</td>
<td>1.18</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1300</td>
<td>40</td>
<td>40</td>
<td>21.30</td>
<td>0.06</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1300</td>
<td>50</td>
<td>50</td>
<td>20.29</td>
<td>0.11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1400</td>
<td>20</td>
<td>20</td>
<td>25.50</td>
<td>0.12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1400</td>
<td>30</td>
<td>30</td>
<td>25.50</td>
<td>0.12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1600</td>
<td>20</td>
<td>20</td>
<td>25.50</td>
<td>0.12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fig. 12 R.O.C. Concentration profile
400 rev/min
Constant flow rate - ratio run

- 16.7 l/h
- 30 l/h
- 45 l/h

Z in. (from the bottom of the column)
Fig/3R.D.C. Concentration Profile
540 rev./min
Constant flow rate ratio run

\[ \frac{\text{Vol}}{\text{Vol}} \]

- 50 \( \text{L/h} \)
- 60 \( \text{L/h} \)
- 75 \( \text{L/h} \)

\( \text{q in. (from the bottom of the column)} \)
Fig. 14 R.D.C. Concentration Profile
700 rev./min.
Constant flow rate ratio run

- 30 l/h
- 50 l/h
- 45 & 60 l/h
Fig. 7.15 R.O.C. Concentration Profile
700 rev./min.; 75 g/h
Constant flow rate ratio run

$\text{H} \quad \text{in. (from the bottom of the column)}$
Fig 7.16 R.D.C. Concentration Profile
960 rev./min
Constant flow rate ratio run

- 16.7 l/h
- 30 l/h
- 50 l/h

\[ \frac{\text{g/ml}}{\text{in. (from the bottom of the column)}} \]
Fig. 7. R.O.C. Concentration profile
972.2 rev/min.
Constant flow rate ratio run

- 30 g/l
- 75 g/l

z m. (from the bottom of the column)
Fig 7. Liquid D.C. Concentration Profile
972.2 rev/min.
Constant flow rate ratio run

\[ \begin{align*}
\text{\(0\)} & \quad 45 \text{ l/h} \\
\text{\(\times\)} & \quad 50 \text{ l/h} \\
\text{\(\square\)} & \quad 60 \text{ l/h}
\end{align*} \]
Fig 7.19 R.D.C. Concentration Profile
972.2 rev/min.
Constant through-put run

○ R = 4
× R = 3
△ R = 2
□ R = 0.5

\( z \) in. (from the bottom of the column)
Fig. 28. R.D.C. Concentration Profile
1180 rev./min.
Constant flow rate ratio run

- 16.7 l/h
- 20 l/h
- 30 l/h
- 40 l/h
Fig. 7. R.O.C. Concentration Profile 1300 rev./min. Constant flow rate ratio run

- 16.8 g/l
- 20 g/l
- 30 g/l
Fig. 7. R.D.C. Concentration Profile
1400 rev./min.
Constant flow rate ratio run

- 16.7 g/l
- 20 g/l
- 30 g/l

3 in. (from the bottom of the column)
Fig. 7.23 R.D.C. Concentration Profile
1600 rev/min.
Constant flow rate ratio run

○ 16.7 l/h
▲ 20 l/h
<table>
<thead>
<tr>
<th>Buffer speed</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.167</td>
<td>16.7</td>
<td>16.7</td>
<td>0.023</td>
<td>3.7</td>
</tr>
<tr>
<td>0.3</td>
<td>30</td>
<td>30</td>
<td>0.023</td>
<td>6.4</td>
</tr>
<tr>
<td>0.45</td>
<td>45</td>
<td>45</td>
<td>0.059</td>
<td>24.6</td>
</tr>
<tr>
<td>0.667</td>
<td>60</td>
<td>60</td>
<td>0.060</td>
<td>27.8</td>
</tr>
<tr>
<td>0.85</td>
<td>85</td>
<td>85</td>
<td>0.060</td>
<td>33.1</td>
</tr>
<tr>
<td>1.09</td>
<td>109</td>
<td>109</td>
<td>0.060</td>
<td>38.3</td>
</tr>
<tr>
<td>1.33</td>
<td>133</td>
<td>133</td>
<td>0.060</td>
<td>43.5</td>
</tr>
<tr>
<td>1.67</td>
<td>167</td>
<td>167</td>
<td>0.060</td>
<td>48.8</td>
</tr>
<tr>
<td>2.0</td>
<td>20</td>
<td>20</td>
<td>0.060</td>
<td>54.1</td>
</tr>
<tr>
<td>2.33</td>
<td>233</td>
<td>233</td>
<td>0.060</td>
<td>59.4</td>
</tr>
<tr>
<td>2.67</td>
<td>267</td>
<td>267</td>
<td>0.060</td>
<td>64.7</td>
</tr>
<tr>
<td>3.0</td>
<td>30</td>
<td>30</td>
<td>0.060</td>
<td>70.0</td>
</tr>
<tr>
<td>3.33</td>
<td>333</td>
<td>333</td>
<td>0.060</td>
<td>75.3</td>
</tr>
<tr>
<td>3.67</td>
<td>367</td>
<td>367</td>
<td>0.060</td>
<td>80.6</td>
</tr>
<tr>
<td>4.0</td>
<td>40</td>
<td>40</td>
<td>0.060</td>
<td>85.9</td>
</tr>
<tr>
<td>4.33</td>
<td>433</td>
<td>433</td>
<td>0.060</td>
<td>91.2</td>
</tr>
<tr>
<td>4.67</td>
<td>467</td>
<td>467</td>
<td>0.060</td>
<td>96.5</td>
</tr>
<tr>
<td>5.0</td>
<td>50</td>
<td>50</td>
<td>0.060</td>
<td>101.8</td>
</tr>
<tr>
<td>5.33</td>
<td>533</td>
<td>533</td>
<td>0.060</td>
<td>107.1</td>
</tr>
<tr>
<td>5.67</td>
<td>567</td>
<td>567</td>
<td>0.060</td>
<td>112.4</td>
</tr>
<tr>
<td>6.0</td>
<td>60</td>
<td>60</td>
<td>0.060</td>
<td>117.7</td>
</tr>
<tr>
<td>6.33</td>
<td>633</td>
<td>633</td>
<td>0.060</td>
<td>123.0</td>
</tr>
<tr>
<td>6.67</td>
<td>667</td>
<td>667</td>
<td>0.060</td>
<td>128.3</td>
</tr>
<tr>
<td>7.0</td>
<td>70</td>
<td>70</td>
<td>0.060</td>
<td>133.6</td>
</tr>
<tr>
<td>7.33</td>
<td>733</td>
<td>733</td>
<td>0.060</td>
<td>138.9</td>
</tr>
<tr>
<td>7.67</td>
<td>767</td>
<td>767</td>
<td>0.060</td>
<td>144.2</td>
</tr>
</tbody>
</table>

Table 7.4, The results of the measured Noy, Ka and Hoy
### Table 7.4 The Results of the measured $N_{oy}$, $K_{a}$ and $H_{oy}$

#### R.D.C. Run continued

<table>
<thead>
<tr>
<th>Baffle speed rpm</th>
<th>$V_y$ l/hr</th>
<th>$V_x$ l/hr</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$N_{oy}$</td>
</tr>
<tr>
<td>$972 , R = 4$</td>
<td>80</td>
<td>20</td>
<td>0.020</td>
</tr>
<tr>
<td>1180</td>
<td>16.7</td>
<td>16.7</td>
<td>0.175</td>
</tr>
<tr>
<td>1180</td>
<td>20</td>
<td>20</td>
<td>0.146</td>
</tr>
<tr>
<td>1180</td>
<td>30</td>
<td>30</td>
<td>0.161</td>
</tr>
<tr>
<td>1180</td>
<td>40</td>
<td>40</td>
<td>0.160</td>
</tr>
<tr>
<td>1300</td>
<td>16.8</td>
<td>16.8</td>
<td>0.156</td>
</tr>
<tr>
<td>1300</td>
<td>20</td>
<td>20</td>
<td>0.182</td>
</tr>
<tr>
<td>1300</td>
<td>30</td>
<td>30</td>
<td>0.199</td>
</tr>
<tr>
<td>1400</td>
<td>16.8</td>
<td>16.8</td>
<td>0.161</td>
</tr>
<tr>
<td>1400</td>
<td>20</td>
<td>20</td>
<td>0.206</td>
</tr>
<tr>
<td>1400</td>
<td>30</td>
<td>30</td>
<td>0.218</td>
</tr>
<tr>
<td>1600</td>
<td>16.8</td>
<td>16.8</td>
<td>0.237</td>
</tr>
<tr>
<td>1600</td>
<td>20</td>
<td>20</td>
<td>0.218</td>
</tr>
</tbody>
</table>
Fig. 7 $H_{ov}$ vs. rotating speed in R.D.C.
constant flow rate ratio run

- 20 l/hr
- 30 l/hr
concentrations were obtained using the automatic titrimeter.

The method of calculating \( (N_{oy})_M \) from the results has been described (Chapter 7, page 96). Calculated values are given in Table 7.4.

II. \( k_{ay} \) variation along the column.

The method of calculation \( k_{ay} \) for each section is as carried out for OBC in page 112.

The results of the calculation for RDC are given in Table 7.6. The variation of \( k_{ay} \) along the column is shown in Figs. 7.32 to 7.41.

III. \( E_y \) and \( (E_y y' + V_y y' - k_{ay}(y_e - y)) = 0 \)

Three methods were used in examining the performance of the Oscillating Baffle Column as described p.132 to p.137. The same method were used to interpret the RDC results. The values of \( E_y \) obtained from the mass transfer studies are given in Table 7.8.

IV. Mean residence time studies.

The experimental technique of dye injection has been described (p.145 to p.148). The theoretical equations used to find the variance \( \sigma^2 \), and D/UL are given in pages 33 to 49.

The results of the calculations for RDC for single phase are given in Table 7.10.1 and for two phases flow in Table 7.10.2.
<table>
<thead>
<tr>
<th>Rotating speed</th>
<th>$V_y$</th>
<th>$V_x$</th>
<th>$s-1$</th>
<th>$s-2$</th>
<th>$s-3$</th>
<th>$s-4$</th>
<th>$s-5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>400 rpm</td>
<td>16.7</td>
<td>16.7</td>
<td>1.6</td>
<td>2.3</td>
<td>2.8</td>
<td>4.4</td>
<td>4.8</td>
</tr>
<tr>
<td>60</td>
<td>30</td>
<td>30</td>
<td>2.3</td>
<td>3.7</td>
<td>4.9</td>
<td>9.2</td>
<td>9.3</td>
</tr>
<tr>
<td>45</td>
<td>45</td>
<td>4.0</td>
<td>5.0</td>
<td>8.0</td>
<td>10.5</td>
<td>13.5</td>
<td></td>
</tr>
<tr>
<td>540 rpm</td>
<td>50</td>
<td>50</td>
<td>11.0</td>
<td>11.0</td>
<td>17.0</td>
<td>23.0</td>
<td>52.5</td>
</tr>
<tr>
<td>60</td>
<td>60</td>
<td>7.6</td>
<td>11.6</td>
<td>16.6</td>
<td>23.7</td>
<td>47.0</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>75</td>
<td>13.0</td>
<td>15.0</td>
<td>24.0</td>
<td>34.7</td>
<td>59.9</td>
<td></td>
</tr>
<tr>
<td>700 rpm</td>
<td>30</td>
<td>30</td>
<td>3.5</td>
<td>9.6</td>
<td>14.2</td>
<td>16.1</td>
<td>19.8</td>
</tr>
<tr>
<td>45</td>
<td>45</td>
<td>15.3</td>
<td>16.2</td>
<td>20.7</td>
<td>24.8</td>
<td>30.4</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>10.1</td>
<td>13.4</td>
<td>24.6</td>
<td>28.2</td>
<td>35.7</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>60</td>
<td>22.1</td>
<td>22.5</td>
<td>23.5</td>
<td>33.3</td>
<td>41.2</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>75</td>
<td>23.7</td>
<td>23.1</td>
<td>33.5</td>
<td>41.0</td>
<td>50.1</td>
<td></td>
</tr>
<tr>
<td>960 rpm</td>
<td>16.7</td>
<td>16.7</td>
<td>12.3</td>
<td>10.5</td>
<td>12.8</td>
<td>12.4</td>
<td>13.1</td>
</tr>
<tr>
<td>30</td>
<td>30</td>
<td>17.1</td>
<td>19.0</td>
<td>23.9</td>
<td>23.0</td>
<td>24.4</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>25.1</td>
<td>30.2</td>
<td>36.7</td>
<td>39.5</td>
<td>50.0</td>
<td></td>
</tr>
<tr>
<td>972.2 rpm</td>
<td>30</td>
<td>30</td>
<td>19.9</td>
<td>21.7</td>
<td>24.0</td>
<td>25.3</td>
<td>26.7</td>
</tr>
<tr>
<td>45</td>
<td>45</td>
<td>25.4</td>
<td>29.4</td>
<td>33.7</td>
<td>45.3</td>
<td>56.8</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>20.8</td>
<td>28.2</td>
<td>33.6</td>
<td>39.0</td>
<td>46.7</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>60</td>
<td>25.0</td>
<td>30.5</td>
<td>37.4</td>
<td>44.7</td>
<td>53.6</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>75</td>
<td>32.3</td>
<td>43.4</td>
<td>49.5</td>
<td>55.1</td>
<td>65.2</td>
<td></td>
</tr>
</tbody>
</table>
Table 7.6 Ka value along the column - R.D.C. Run continued

<table>
<thead>
<tr>
<th>Rotating speed (rpm)</th>
<th>$V_y$ (1/hr)</th>
<th>$V_x$ (1/hr)</th>
<th>Ka value in section $\times 10^{-5}$ (from bottom of the column)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>s - 1</td>
<td>s - 2</td>
<td>s - 3</td>
</tr>
<tr>
<td>972.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R = 0.5</td>
<td>33.33</td>
<td>66.67</td>
<td>37.0</td>
</tr>
<tr>
<td>R = 2</td>
<td>66.67</td>
<td>66.67</td>
<td>16.5</td>
</tr>
<tr>
<td>R = 3</td>
<td>75</td>
<td>25</td>
<td>10.5</td>
</tr>
<tr>
<td>R = 4</td>
<td>80</td>
<td>20</td>
<td>9.9</td>
</tr>
<tr>
<td>1180</td>
<td>16.7</td>
<td>16.7</td>
<td>13.8</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>20</td>
<td>21.2</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>30</td>
<td>37.2</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>40</td>
<td>42.7</td>
</tr>
<tr>
<td>1300</td>
<td>16.8</td>
<td>16.8</td>
<td>12.2</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>20</td>
<td>13.7</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>30</td>
<td>41.3</td>
</tr>
<tr>
<td>1400</td>
<td>16.8</td>
<td>16.8</td>
<td>14.0</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>20</td>
<td>17.8</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>30</td>
<td>58.5</td>
</tr>
<tr>
<td>1600</td>
<td>16.8</td>
<td>16.8</td>
<td>24.6</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>20</td>
<td>32.5</td>
</tr>
</tbody>
</table>
Fig. 23 R.D.C. Ka value along the column
400 rev/min
Constant flow rate ratio run

- 167 l/h
- 30 l/h
- 45 l/h

\( \frac{\text{in.}}{\text{sec}^2} \) (from the bottom of the column)
Fig. 734 R.O.C. $K_a$ value along the column
540 rev/min
Constant flow rate ratio run
- 60 l/h
- 50 l/h
- 75 l/h

$K_a \times 10^{-5}$ sec$^2$

$\frac{z}{8}$ in. (from the bottom of the column)
F=197.35 R.O.C. Kα value along the column
700 rev/min
Constant flow rate ratio run

- 30 l/h
- 45 l/h
- 50 l/h
- 60 l/h
- 75 l/h

\(\frac{\text{3 in. (from the bottom of the column)}}{131}\)
Fig. 36 R.D.C. Ka value along the column
960 rev/min.
Constant flow rate ratio run

- \( x \) 16.7 l/h
- \( o \) 30 l/h
- \( A \) 50 l/h
Fig. 37 R.D.C. $K_a$ value along the column
972.2 rev/min.
Constant flow rate ratio run

$K_a \times 10^{-5}$ sec$^{-1}$

30 l/h
50 l/h
45 l/h
60 l/h
75 l/h

$z$ in. (from the bottom of the column)
Fig. 7.38 R.D.C. $K_a$ value along the column
972.2 rev./min.
Constant throughput run

- $R = 4$
- $R = 3$
- $R = 2$
- $R = 0.5$

$K_a \times 10^{-5}$ sec$^{-1}$

$z$ in. (from the bottom of the column)
Fig. 39 R.D.C. Ka value along the column
1180 rev./min.
Constant flow rate ratio run

\( K_a \times 10^5 \text{ sec}^{-1} \)

- \( 16.7 \text{ l/hr} \)
- \( 20 \text{ l/hr} \)
- \( 30 \text{ l/hr} \)
- \( 40 \text{ l/hr} \)

3 in. (from the bottom of the column)
Fig. 40: R.D.C. K_a value along the column.

1800 rev./min

Constant flow rate ratio run

- 16.7 l/h
- 20 l/h
- 30 l/h

K_a \times 10^5 \text{ sec}^{-1}

\( z \) in. (from the bottom of the column)
Fig. 7.41 R.D.C. $K_a$ value along the column
1400 rev/min.
Constant flow rate ratio run

- $16.7$ l/h
- $20$ l/h
- $30$ l/h

$k_a \times 10^5$ vs. $y$ in. (from the bottom of the column)
Fig. 7.42 R.D.C. $k_a$ value along the column
1600 rev/min.
Constant flow rate ratio run

$\times$ 16.8 l/hr
$\circ$ 20 l/hr

$z$ in. (from the bottom of the column)
Table 7.8  
**E-value results calculated from the differential equation — R.D.C. Run**

<table>
<thead>
<tr>
<th>Baffle speed rpm</th>
<th>( V_y ) 1/hr</th>
<th>( V_x ) 1/hr</th>
<th>E-value By Method 1</th>
<th>E-value By Method 2</th>
<th>E-value By Method 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>16.7</td>
<td>16.7</td>
<td>-0.0001503</td>
<td>0.000239</td>
<td>-0.000172</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0003454</td>
<td>-0.0001648</td>
<td>-0.0005714</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0001648</td>
<td>0.000161</td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>30</td>
<td>30</td>
<td>-0.0001688</td>
<td>0.000239</td>
<td>0.00033236</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.00003097</td>
<td>0.000634</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0000553</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0001763</td>
<td></td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>45</td>
<td>45</td>
<td>0.0011863</td>
<td>0.0003119</td>
<td>0.00101038</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0003234</td>
<td>-0.0005714</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0014664</td>
<td>0.0004539</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0002330</td>
<td></td>
<td></td>
</tr>
<tr>
<td>540</td>
<td>50</td>
<td>50</td>
<td>-0.0078396</td>
<td>0.002002</td>
<td>0.00135142</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0002755</td>
<td>0.001274</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0000694</td>
<td>0.001027</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0010338</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>60</td>
<td>60</td>
<td>-0.0003342</td>
<td>0.000117</td>
<td>0.00027461</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0007691</td>
<td>0.000233</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0012516</td>
<td>0.000321</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0010692</td>
<td></td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>75</td>
<td>75</td>
<td>-0.0001555</td>
<td>0.002132</td>
<td>0.00202677</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0001323</td>
<td>0.001159</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0002334</td>
<td>0.002656</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0005281</td>
<td></td>
<td></td>
</tr>
<tr>
<td>700</td>
<td>30</td>
<td>30</td>
<td>0.0007215</td>
<td>0.000672</td>
<td>0.00017533</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0002169</td>
<td>-0.000507</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0006716</td>
<td>0.000332</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0000364</td>
<td></td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>45</td>
<td>45</td>
<td>-0.0001554</td>
<td>0.000233</td>
<td>0.00031475</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0000309</td>
<td>0.000094</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0004037</td>
<td>0.000487</td>
<td></td>
</tr>
<tr>
<td>Baffle speed rpm</td>
<td>$V_y$ 1/hr</td>
<td>$V_x$ 1/hr</td>
<td>E-value By Method 1</td>
<td>E-value By Method 2</td>
<td>E-value By Method 3</td>
</tr>
<tr>
<td>------------------</td>
<td>-----------</td>
<td>-----------</td>
<td>----------------------</td>
<td>----------------------</td>
<td>----------------------</td>
</tr>
<tr>
<td>700</td>
<td>50</td>
<td>50</td>
<td>0.0032752</td>
<td>0.001925</td>
<td>0.00110331</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0005601</td>
<td>0.000588</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0002933</td>
<td>0.001221</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0001904</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>60</td>
<td>60</td>
<td>0.0002387</td>
<td>0.000428</td>
<td>0.00054752</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0002193</td>
<td>0.000339</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0003833</td>
<td>0.000734</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0004503</td>
<td></td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>75</td>
<td>75</td>
<td>0.0070595</td>
<td>0.004958</td>
<td>0.00214647</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0012733</td>
<td>0.001190</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0000530</td>
<td>0.001967</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0009962</td>
<td></td>
<td></td>
</tr>
<tr>
<td>960</td>
<td>16.7</td>
<td>16.7</td>
<td>0.0003501</td>
<td>0.000255</td>
<td>0.00009380</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0000031</td>
<td>-0.000313</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0000707</td>
<td>0.000403</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0001307</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>30</td>
<td>30</td>
<td>0.0006963</td>
<td>0.000201</td>
<td>-0.00022254</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0003472</td>
<td>-0.001056</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0006725</td>
<td>0.000016</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0001035</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>50</td>
<td>-0.0001786</td>
<td>-0.000331</td>
<td>0.00020825</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0007639</td>
<td>-0.000444</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.000633</td>
<td>0.001562</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0001231</td>
<td></td>
<td></td>
</tr>
<tr>
<td>972.2</td>
<td>30</td>
<td>30</td>
<td>-0.0023671</td>
<td>-0.000788</td>
<td>-0.00041538</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0023103</td>
<td>-0.000651</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0010512</td>
<td>0.000046</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0006707</td>
<td></td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>45</td>
<td>45</td>
<td>-0.0001367</td>
<td>-0.000296</td>
<td>0.00040392</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0002024</td>
<td>0.001124</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0000174</td>
<td>0.000442</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.0004558</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baffle speed (rpm)</td>
<td>$V_y$ 1/hr</td>
<td>$V_x$ 1/hr</td>
<td>E-value</td>
<td>E-value</td>
<td>E-value</td>
</tr>
<tr>
<td>-------------------</td>
<td>------------</td>
<td>------------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td>972.2</td>
<td>50</td>
<td>50</td>
<td>-0.0000784</td>
<td>-0.001060</td>
<td>-0.00030431</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>60</td>
<td>-0.000147</td>
<td>-0.000921</td>
<td>0.00036023</td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>75</td>
<td>-0.0012292</td>
<td>-0.002027</td>
<td>-0.00080821</td>
</tr>
<tr>
<td>1180</td>
<td>16.62</td>
<td>16.62</td>
<td>0.0006894</td>
<td>0.000597</td>
<td>0.00035048</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>20</td>
<td>0.0005262</td>
<td>0.000483</td>
<td>0.0013575</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>30</td>
<td>0.0006222</td>
<td>0.001600</td>
<td>0.00050039</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>40</td>
<td>0.0004122</td>
<td>0.001170</td>
<td>0.00052653</td>
</tr>
<tr>
<td>1300</td>
<td>16.8</td>
<td>16.8</td>
<td>-0.0002578</td>
<td>-0.000161</td>
<td>-0.00009461</td>
</tr>
<tr>
<td>Baffle speed rpm</td>
<td>( V_y ) 1/hr</td>
<td>( V_x ) 1/hr</td>
<td>E-value</td>
<td></td>
<td></td>
</tr>
<tr>
<td>------------------</td>
<td>----------------</td>
<td>----------------</td>
<td>---------</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>By Method 1</td>
<td>By Method 2</td>
<td>By Method 3</td>
</tr>
<tr>
<td>1300</td>
<td>16.8</td>
<td>16.8</td>
<td>0.0000763</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>20</td>
<td>-0.0001878</td>
<td>0.000003</td>
<td>-0.0001153</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>20</td>
<td>-0.000200</td>
<td>-0.000431</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>20</td>
<td>-0.0001983</td>
<td>0.000017</td>
<td></td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>30</td>
<td>-0.0001262</td>
<td>0.000036</td>
<td>0.0001393</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>30</td>
<td>-0.0001331</td>
<td>-0.000071</td>
<td></td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>30</td>
<td>0.0001828</td>
<td>0.000518</td>
<td></td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>30</td>
<td>0.0001603</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1400</td>
<td>16.8</td>
<td>16.8</td>
<td>-0.0002001</td>
<td>0.000022</td>
<td>0.00020103</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>20</td>
<td>-0.0000001</td>
<td>-0.000111</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>20</td>
<td>0.0000875</td>
<td>0.000444</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>20</td>
<td>0.0000694</td>
<td>0.000543</td>
<td></td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>30</td>
<td>-0.0001012</td>
<td>-0.000043</td>
<td>0.00013173</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>30</td>
<td>-0.0001294</td>
<td>0.000207</td>
<td></td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>30</td>
<td>-0.0000726</td>
<td>0.000148</td>
<td></td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>30</td>
<td>-0.0002924</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1600</td>
<td>16.8</td>
<td>16.8</td>
<td>-0.0001274</td>
<td>-0.000476</td>
<td>0.00015564</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>20</td>
<td>-0.0001291</td>
<td>0.000415</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>20</td>
<td>-0.0000332</td>
<td>0.000397</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>20</td>
<td>-0.0001274</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The table continues with more data for different baffle speeds and velocities.
Some actual recorder tracers of dye concentration in the water phase for this measurement are shown in Figs. 7.48 and 7.49.
Fig. 7.48 The actual recorder traces of dye concentration in the aqueous phase at the RDC single phase run.
Fig. 7.49 The actual recorder traces of dye concentration in the aqueous phase at the RDC two phases run.
# Table 7.10-1 Mean Residence Time Study Results
## R.D.C. Single phase Run

<table>
<thead>
<tr>
<th>Baffle speed rpm</th>
<th>( V_y ) l/hr</th>
<th>( V_x ) l/hr</th>
<th>Mean Residence Time, sec.</th>
<th>( \sigma^2 )</th>
<th>D/UL</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>30</td>
<td>-</td>
<td>557.395</td>
<td>0.087210</td>
<td>0.037869</td>
<td>0.000765</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>-</td>
<td>367.561</td>
<td>0.094843</td>
<td>0.040772</td>
<td>0.001236</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>-</td>
<td>254.068</td>
<td>0.065221</td>
<td>0.029200</td>
<td>0.001180</td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>-</td>
<td>212.448</td>
<td>0.058412</td>
<td>0.026415</td>
<td>0.001334</td>
</tr>
<tr>
<td>700</td>
<td>30</td>
<td>-</td>
<td>548.023</td>
<td>0.111011</td>
<td>0.046760</td>
<td>0.000945</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>-</td>
<td>373.957</td>
<td>0.094737</td>
<td>0.040732</td>
<td>0.001234</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>-</td>
<td>307.907</td>
<td>0.125553</td>
<td>0.051972</td>
<td>0.002100</td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>-</td>
<td>234.905</td>
<td>0.086289</td>
<td>0.037515</td>
<td>0.001799</td>
</tr>
<tr>
<td>1000</td>
<td>30</td>
<td>-</td>
<td>538.018</td>
<td>0.137288</td>
<td>0.056069</td>
<td>0.001133</td>
</tr>
<tr>
<td>1180</td>
<td>30</td>
<td>-</td>
<td>505.569</td>
<td>0.132919</td>
<td>0.054554</td>
<td>0.001102</td>
</tr>
<tr>
<td>1400</td>
<td>30</td>
<td>-</td>
<td>460.563</td>
<td>0.160985</td>
<td>0.064072</td>
<td>0.001295</td>
</tr>
<tr>
<td>Baffle speed rpm</td>
<td>( V_y ) l/hr</td>
<td>( V_x ) l/hr</td>
<td>Mean Residence Time sec.</td>
<td>( \sigma^2 )</td>
<td>D/UL</td>
<td>D</td>
</tr>
<tr>
<td>------------------</td>
<td>----------------</td>
<td>----------------</td>
<td>--------------------------</td>
<td>----------------</td>
<td>-------</td>
<td>---</td>
</tr>
<tr>
<td>400 R=4</td>
<td>80</td>
<td>20</td>
<td>208.640</td>
<td>0.074633</td>
<td>0.032969</td>
<td>0.001776</td>
</tr>
<tr>
<td>R=3</td>
<td>75</td>
<td>25</td>
<td>217.045</td>
<td>0.074956</td>
<td>0.033096</td>
<td>0.001672</td>
</tr>
<tr>
<td>R=2</td>
<td>66.67</td>
<td>33.33</td>
<td>290.124</td>
<td>0.109087</td>
<td>0.046058</td>
<td>0.002068</td>
</tr>
<tr>
<td>R=1</td>
<td>50</td>
<td>50</td>
<td>317.270</td>
<td>0.093678</td>
<td>0.040332</td>
<td>0.001358</td>
</tr>
<tr>
<td>700 R=4</td>
<td>80</td>
<td>20</td>
<td>216.716</td>
<td>0.085728</td>
<td>0.037299</td>
<td>0.002010</td>
</tr>
<tr>
<td>R=3</td>
<td>75</td>
<td>25</td>
<td>232.797</td>
<td>0.105029</td>
<td>0.044587</td>
<td>0.002252</td>
</tr>
<tr>
<td>R=2</td>
<td>66.67</td>
<td>33.33</td>
<td>263.960</td>
<td>0.111529</td>
<td>0.046948</td>
<td>0.002108</td>
</tr>
<tr>
<td>R=1</td>
<td>50</td>
<td>50</td>
<td>324.335</td>
<td>0.123591</td>
<td>0.051278</td>
<td>0.001727</td>
</tr>
<tr>
<td>1000</td>
<td>30</td>
<td>30</td>
<td>516.451</td>
<td>0.147082</td>
<td>0.059419</td>
<td>0.001201</td>
</tr>
<tr>
<td>1180</td>
<td>30</td>
<td>30</td>
<td>434.645</td>
<td>0.143296</td>
<td>0.058131</td>
<td>0.001175</td>
</tr>
<tr>
<td>1400</td>
<td>30</td>
<td>30</td>
<td>339.900</td>
<td>0.112655</td>
<td>0.047357</td>
<td>0.000957</td>
</tr>
</tbody>
</table>
Chapter 9.  
Comparison of experimental results of OBC and RDC

All the experimental work carried out on the OBC was repeated on the RDC in order to make a comparison of the performance of the oscillating baffle column.

Refer to Tables 7.2 and 7.4 and Figs. 7.47, 7.50 and 7.51. If we choose a flow rate condition such as $V_y = V_x = 16.7 \text{ l/hr}$, then the maximum speed possible before flooding sets in is seen to be 264 osc/min for the OBC and 1,600 rpm for RDC. The efficiency under these conditions are reflected in the values of $H_{oy}$, (i) RDC $H_{oy} = 10.91 \text{ ft}$ and (ii) OBC $H_{oy} = 3.52 \text{ ft}$. Clearly the efficiency of separation achieved by the oscillating baffle column is far superior to that of the RDC at the same duty.

From Figs. 7.50 and 7.51 and Tables 7.2 and 7.4 consider the following:

<table>
<thead>
<tr>
<th>speed</th>
<th>$H_{oy}$</th>
<th>$V_y = V_x$</th>
<th>remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>RDC</td>
<td>1180</td>
<td>16.10</td>
<td>near flooding point</td>
</tr>
<tr>
<td>RDC</td>
<td>1180</td>
<td>14.75</td>
<td>well below flooding</td>
</tr>
<tr>
<td>RDC</td>
<td>1600</td>
<td>10.91</td>
<td>near flooding</td>
</tr>
<tr>
<td>OBC</td>
<td>264</td>
<td>3.52</td>
<td>near flooding</td>
</tr>
<tr>
<td>OBC</td>
<td>248</td>
<td>5.89</td>
<td>well below flooding</td>
</tr>
</tbody>
</table>

It can be seen that for RDC 1180 - 1600 rpm 16.3 l/hr there is a change in efficiency of 14.7 to 10.91 i.e. 40% approx. If we now
take the same change in efficiency for the OBC than $H_{OY} = 3.52$ ft near flooding and a 40% change gives $H_{OY} = 5.0$ which corresponds to a speed of approx. 252 osc/min. This is a change in speed of only $(264 - 252)/264$ or 5%, whereas the change in speed of RDC to give the same efficiency change at the same through-put is $(1600 - 1180)/1600$ or 25%. In addition it is observed that for RDC in going from 1180 to 1600 rpm there is a change in the maximum acceptable duty of i.e. $V_y = V_x =$ maximum approx. 40 - 20 l/hr i.e. a 50% decrease. In the case of OBC in going from 252 osc/min to 264 there is no change in the maximum acceptable flow rate of 16.7, so the flow rate is unaffected with change in the speed. It is seen therefore that:

i. The efficiency at the higher velocity is much more sensitive to speed for OBC than RDC.

ii. The OBC through-put is not so affected as for RDC when the speed is increased to give the same change in efficiency.

From the residence time study (refer to Tables 7.9 and 7.10 and Figs. 8.1 and 8.3) In the OBC, decreased oscillating speed from maximum down to 100 osc/min gives increased mean residence time in all constant flow rate ratio runs. Increasing mean residence time from Fig. 8.3 gives decrease in efficiency.

At the low oscillating speed ($100 - 175$ osc/min) the change of mean residence time with speed is seen to be either constant or slightly decreases depending on the accuracy of measurement. It is
Fig. 3.1 mean residence versus oscillating speed in OBC
constant flow rate ratio runs.

Fig. 3.2 mean residence time versus rotating speed in RDC
30 l/hr constant flow rate ratio run.
Fig. 8.4 $H_{oy}$ versus mean residence time in RDC 30 l/hr constant flow rate ratio run.

Fig. 8.3 $H_{oy}$ versus mean residence time in OBC 30 l/hr constant flow rate ratio run.
also found (refer to Fig. 8.3) that at the critical speed, if the mean residence time increases slightly, there is a large increase in \( H_{0y} \) or very rapid decrease in efficiency. This is due to the marked effect of oscillating speed on the interfacial area at low speed.

The same kind of relationship applies to RDC as shown in Figs. 8.2 and 8.4.

It is soon that when we are in a region of high mean residence time the changes in interfacial area, number of drops and drop size are less significant in changing the mean residence time of the continuous phase than at high speed.
CHAPTER 10

Summary and conclusion.

Theory.

The mode of operation of a new oscillating baffle contactor has been described. Its performance compared with a well established type, the Rotating Disc Contactor has been considered.

The general theory of mass transfer in continuous flow systems has been considered, and differential equations established for different operating conditions. It has been shown how the "true" and "measured" number of transfer units are related to each other.

The differential equations relating back-mixing, convection and mass transfer depend upon whether the density of the organic phase is greater or less than that of the extract phase. Each case has been treated separately and the differential equation was obtained in three ways:

1. By the use of mass balance.
2. By the use of Fick's second law equation.
3. From Miyauchi's general vector equation based on mass balance.
It is important to establish for a given set of boundary condition whether the signs in the differential equation (24) are correct. A separate study was made using an analog Computer and experimental data to establish the correct signs for a given equation. The validity of an equation was judged by the successful correlation of the results. It was possible to prove beyond doubt that a given equation described the transfer process occurring in the experiment under given conditions of operation.

The general theory of residence time has been discussed and it has been shown how the variance is related to the eddy diffusion coefficient.

**Experiment.**

Equilibrium values for the system carbon tetrachloride–acetic acid and water were obtained by using automatic pH meter.

Extraction of acetic acid from carbon tetrachloride by water in counter current flow has been carried out using both the oscillating baffle column and the rotating disc contactor. In each case the column height and diameter was fixed and the flow rates, flow ratio, and
oscillating or rotating speeds varied. Concentration profiles along the column in the water have been established and also values of $N_{ox}$, $K_a$ and $E$.

A dye pulse injection technique was made to find the mean residence time, $C$ curve and $D$ value.

**Calculation.**

Value of $dy/dz$, $N_{ox}$, $H_{oy}$, $K_a$ and $E$ were calculated using numerical analysis in some cases the Sirius and Analog Computer.

Values of variance of $C$ curve and longitudinal mixing coefficient have been established from the mean residence time.

**Longitudinal mixing coefficient.**

The significance of longitudinal mixing in the calculation of separation efficiency has been described. In order to allow for this back-mixing a "correction term" has to be introduced as shown in Eqns. 19 and 20. This correction term is a function of the eddy diffusivity, $E_y$, and of the second derivative of the concentration gradient along the column, $y''$. In order to determine the extent of back mixing under any operating condition it is therefore to be able to assign a value to $E_y$.

Two method were used to established $E_y$. In the first
method the differential equation \( 2h \) was solved to obtain \( E_Y \) and in the second method a dye injection technique was employed. The results of both method are given in Tables 7.7 and 7.9.

The mathematical solution of the differential equation \( 2h \) has not been previously reported. It can be seen from Table 7.7 that the values of \( E_Y \) obtained by method 2 and 3 are comparable but \( E_Y \) tends to be rather unstable. This may be due to the dependence of \( E_Y \) to so many variables e.g. \( y'' \), \( y' \), \( y \) and \( K_a \) and to the fact that \( y'' \) tends to be unstable. Nevertheless the solution of the equation gives a better insight into the problem.

The values for \( E_Y \) using the dye injection technique and the variance are given in Table 7.9. The values of \( E_Y \) obtained by this method are much more stable than those obtained from the solution of the differential equation. It will be observed from Tables 7.7 and 7.9 that the \( E_Y \) values obtained from the dye injection technique and the mathematical solution to the differential equation are comparable where they have meaning. Some examples are given below:
<table>
<thead>
<tr>
<th>Method</th>
<th>oscillating speed</th>
<th>( V_y = V_x )</th>
<th>( D )</th>
<th>comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>dye inject. 150</td>
<td>50</td>
<td>0.002134</td>
<td></td>
<td>stable</td>
</tr>
<tr>
<td>Math. Eqn. 175</td>
<td>45</td>
<td>0.001836</td>
<td></td>
<td></td>
</tr>
<tr>
<td>dye inject. 100</td>
<td>60</td>
<td>0.003014</td>
<td></td>
<td>stable</td>
</tr>
<tr>
<td>Math. Eqn. 105</td>
<td>75</td>
<td>0.00227</td>
<td></td>
<td></td>
</tr>
<tr>
<td>dye inject. 150</td>
<td>30</td>
<td>0.001236</td>
<td></td>
<td>unstable</td>
</tr>
<tr>
<td>Math. Eqn. 175</td>
<td>30</td>
<td>0.0000636</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It can be seen that in certain cases both at high and low speeds the \( E_y \) values obtained from the mathematical solution to the differential equation are much too small and it can only be attributed to some inherent instability in the function. It is more likely that the \( E_y \) values obtained from the dye injection method are the more acceptable.

Comparison of results for the OBC and RDC.

With reference to Chapter 9 in page 198, it is clearly shown that the efficiency of separation achieved by the OBC is far superior to that of the RDC at the same duty. The efficiency at the higher velocity is much more sensitive to
speed for OBC than RDC, and also the OBC throughput is not so affected as for RDC when the speed is increased to give the same change in efficiency.

The construction and performance of OBC in liquid extraction are quite simple and satisfactory and all the operating conditions are steady and easy to control. With reference to all the experimental results, there is little doubt that the OBC will be an excellent extractor when working at optimum conditions.

**Scope for further work.**

In the present work, it was found that the oscillating baffle speed has the greatest effect on the separating efficiency of the OBC. The shape of the baffle, size of the perforated baffle and its oscillating amplitude can be studied in further work. An experimental investigation can be check whether an alteration of the baffle will result in a greater feed rate, and higher possible oscillating speed—thus resulting in greater efficiency.
**NOMENCLATURE.**

- **A**
  - Cross-section area of column, ft\(^2\).

- **a**
  - Interfacial area, ft\(^2/\)ft\(^3\).

- **c**
  - Concentration of solute, g/l.

- **C**
  - Concentration of tracer in the fluid (dimensionless).

- **C\(_0\)**
  - Initial concentration of tracer in fluid (dimensionless).

- **C(t)**
  - Concentration of tracer material in the exit stream at any time \(t\).

- **C(\(\theta\))d\(\theta\)**
  - C-curve.

- **D**
  - Eddy diffusion coefficient, ft\(^2/\)sec.

- **E(t)**
  - External residence time frequency distribution function.

- **E(t)dt**
  - The exit age distribution function, it is the fraction of material having ages between \(t\) and \((t+dt)\) at the moment of leaving the extractor.

- **E_x**
  - Longitudinal mixing coefficient based on raffinate phase.

- **E_y**
  - Longitudinal mixing coefficient based on extract phase.

- **F(t)**
  - Total fraction of tracer in total outflow at any time \(t\) after the injection.

- **F(\(\theta\))d\(\theta\)**
  - F-curve.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>((H_{ox}), (H_{oy}))</td>
<td>Overall height of a transfer unit on x and y phases.</td>
</tr>
<tr>
<td>((H_{ox})<em>M, (H</em>{oy})_M)</td>
<td>Measured overall height of a transfer unit based on x and y phases respectively, ft.</td>
</tr>
<tr>
<td>((H_{ox})<em>p, (H</em>{oy})_p)</td>
<td>Apparent overall height of a transfer unit based on x and y phases respectively, ft.</td>
</tr>
<tr>
<td>(h)</td>
<td>Height of column, ft.</td>
</tr>
<tr>
<td>(H)</td>
<td>Hold-back.</td>
</tr>
<tr>
<td>(I(t))</td>
<td>Internal residence-time frequency distribution function.</td>
</tr>
<tr>
<td>(I(t)dt)</td>
<td>Internal age distribution function, it is the fraction of material within the extractor having age between (t) and ((t+dt)).</td>
</tr>
<tr>
<td>(J)</td>
<td>Rate of mass transfer per unit area due to diffusion.</td>
</tr>
<tr>
<td>(K_x, K_y)</td>
<td>Overall mass transfer coefficient based on x and y phases respectively, ft/sec.</td>
</tr>
<tr>
<td>(L)</td>
<td>Length of pipe, ft.</td>
</tr>
<tr>
<td>(M)</td>
<td>Total amount of tracer diffusing in a cylinder of a definite length per unit cross-section area, (M/\text{L}^2).</td>
</tr>
<tr>
<td>(M_c)</td>
<td>Mass transfer by convection.</td>
</tr>
</tbody>
</table>
Mass transfer by interfacial surfaces.

Solute distribution coefficient.

Number of overall transfer units based on x-phase and y-phase respectively.

Measured overall number of transfer units based on x-phase and y-phase respectively.

Apparent overall number of transfer units based on x-phase and y-phase respectively.

Peclet number, \( \frac{uL}{D} \) dimensionless.

Volume of tracer of unit concentration which would correspond to the actual amount of tracer introduced into the fluid, \( \text{ft}^3 \).

Flow ratio \( \frac{V_y}{V_x} \).

Section of column.

Height of section.

Residence time of age of material, sec.

Mean residence times, sec.

The average flow velocity, ft/sec.

Mass mean velocity, ft/sec.

Volumetric fluid flow rate, \( \text{ft}^3/\text{sec} \).
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V )</td>
<td>Volume of vessel, ( \text{ft}^3 ).</td>
</tr>
<tr>
<td>( V_x )</td>
<td>Superficial velocity based on ( x )-phase, ( \text{ft/sec} ).</td>
</tr>
<tr>
<td>( V_y )</td>
<td>Superficial velocity based on ( y )-phase, ( \text{ft/sec} ).</td>
</tr>
<tr>
<td>( z )</td>
<td>Distance from the bottom of the column, ( \text{ft} ).</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Dimensionless time, given by ( t/t ).</td>
</tr>
<tr>
<td>( \sigma^2_t )</td>
<td>Variance of residence time distribution, ( \text{sec}^2 ).</td>
</tr>
<tr>
<td>( \sigma^2 )</td>
<td>Dimensionless variance.</td>
</tr>
<tr>
<td>( \rho' )</td>
<td>Density.</td>
</tr>
</tbody>
</table>

**Subscripts.**
- \( B \): Bottom of the column.
- \( e \): Equilibrium.
- \( x \): Organic (raffinate) phase.
- \( y \): Aqueous (extract) phase.
- \( T \): Top of the column.
REFERENCES.


2. Treybal-Liquid extraction.

3. Murphy, N.F., J.E. Lastovia, and A.E. Skrzec;
A.I.Ch.E.Journal 2, 541 (1956).

4. Murdoch, R., and H.R.C. Pratt;

5. Treybal, R.E., and L.T. Work;

6. Fleming J.F., and H.F. Johnson;

7. Johnson, A.I., G.W. Minard,; C.J. Huang, J.H. Hansuld
and V.M. McNamara; A.I.Ch.E.Journal 3, 161, (1957).


10. Licht, W., and Conway, J.B.;


33. Stainthorp, E.P., and Sudall, N.;
34. Sudall, N.;
35. Reman, G.H. and J.G. Van Der Tusse.
37. Oldshue, J.Y., and J.H. Rushton;
38. Scheibel, E.G.;
   A.I.Ch.E. Journal 2, 74, (1956).
39. Scheibel, E.G.;
40. Vandijck, W.J.D.;
41. Sege, G., and F.W. Woodfields;
42. Jealous, A.C.; and E. Lieberman;
43. Thornton, J.D. et al.
44. Barson, Norman and G.J. Beyer; 
45. Mar, B.W., and Babb, A.L.; 
46. Smoot, L.D.; 
47. Smoot, L.D., and Ball A.L.; 
48. Podbielniak, W.J.; 
49. Coutor, C.; 
50. Eiseblohr, H.; 
51. Bird, R.B., W.L. Stewart, and E.N. Lightfoot; 
52. Sherwood & Pigford; 
53. Miyauchi, T.; 
55. Sleicher, et al;

56. Aris, R;

57. Aris, R;

58. Aris, R. and Amundson, N.R.;
A.I.Ch.E.Journal 3, 280 (1957).

59. Pischeff, K.B.;

60. Swiff, W.H. and Burger, L.K.;


63. Kramers, H.;

64. Deisler, P.F., Jr., and Wilhelm, R.H.;

65. Strand, D.A. and C.J. Geankoplis;
66. Boach, E.A. and White; 

67. Levenspiel and Smith; 

68. Carberry, J.J., and R.H. Bretton; 

69. Cairns, E.J. and Prausnitz, J.M.; 

70. Damkowerts, F.V.; 

71. Foss, A.S., J.A.Gerster and R.L.Pigford; 

72. Thomas W.J. and Campbell. 
Institution of Chemical Engineers 1966, (in press).

73. Lewis, J.B., 

74. Kung, E.Y. and Beckmann, R.B.; 
A.I.Ch.E. Journal 1, 319 (1961).

75 Electronic Instrument Booklet; 
Model 24 Automatic Titrineter, 
Electronic Instruments Ltd. Surrey, England.

76. Scarborough, J.B.; 
Oxford University Press.
The relationship between C curve and its variance.

The relationship i.e.

\[ C'(\Theta) = \frac{\Theta \left( \frac{1}{\sigma^2} - \frac{\Theta}{\sigma^2} \right)}{\sigma^2 \Gamma\left(\frac{1}{\sigma^2}\right)} \]  \( \text{(42)} \)

can be derived as follows;

Let the residence time exit age function be

\[ C'(\Theta) = \alpha \Theta^\beta e^{-\gamma \Theta} \]  \( \text{(A.1)} \)

The normalised distribution function obeys the three following conditions:

1. The area under the curve is unity, i.e.
   \[ \int_0^\infty C(\Theta) \, d\Theta = 1 \]  \( \text{(A.2)} \)

2. The mean age of the exit stream is unity, i.e.
   \[ \Theta_c = \int_0^\infty C(\Theta) \, \Theta \, d\Theta = 1 \]  \( \text{(A.3)} \)

3. The dimensionless variance of the distribution function is defined as
   \[ \gamma^2 = \int_0^\infty (1 - \Theta)^2 \, C(\Theta) \, d\Theta \]  \( \text{(A.4)} \)

Each of these conditions are applied in turn into Eqn. (A.1) to determine \( \alpha, \beta \), and \( \gamma \).
Applying condition (1),
\[ \int_0^\infty C(\theta) \, d\theta = \int_0^\infty \alpha \, \theta^\beta \, e^{\gamma \theta} \, d\theta \]  \hspace{1cm} (A.5)

Now, Eqn. (A.5) is the definition of a Gamma function, the general equation of which is
\[ \Gamma(n) = \int_0^\infty t^{n-1} \, e^{-t} \, dt \]  \hspace{1cm} (A.6)

By replacing \( \gamma \) by \( -\gamma \), Eqn. (A.5) becomes
\[ \int_0^\infty C(\theta) \, d\theta = \int_0^\infty \alpha \, \theta^\beta \, e^{-\gamma \theta} \, d\theta \]

Let \( \delta \theta = \phi \), \( \delta d\theta = d\phi \)
\[ \int_0^\infty C(\theta) \, d\theta = \int_0^\infty \alpha \frac{\phi^\beta}{\delta} \, e^{-\gamma \phi} \, d\phi = \frac{\alpha}{\delta^{\beta+1}} \int_0^\infty \phi^\beta \, e^{-\gamma \phi} \, d\phi \]
\[ = \frac{\alpha}{\delta^{\beta+1}} \cdot \Gamma(\beta+1) \]
\[ = \frac{\alpha}{\delta^{\beta+1}} \cdot \Gamma(\beta+1), \text{ or } \frac{\alpha}{(-\gamma)^{\beta+1}} \cdot \Gamma(\beta+1) = 1 \]  \hspace{1cm} (A.9)

Applying condition (2)
\[ \int_0^\infty \theta \, C'(\theta) \, d\theta = 1 = \int_0^\infty \alpha \, \theta^{\beta+1} \, e^{\gamma \theta} \, d\theta \]
\[ = \frac{\alpha}{(-\gamma)^{\beta+2}} \cdot \Gamma(\beta+2) = 1 \]

Applying condition (3)
\[ \sigma^2 = \int_0^\infty (\theta-\bar{\theta})^2 \, C'(\theta) \, d\theta \]
\[ = \int_0^\infty C'(\theta) \, d\theta - 2 \int_0^\infty \theta \, C'(\theta) \, d\theta + \int_0^\infty \theta^2 \, C'(\theta) \, d\theta \]
\[ = 1 - \frac{\alpha}{\delta^{\beta+3}} \cdot \Gamma(\beta+3) \]  \hspace{1cm} (A.11)

\[ A1.2 \]
\begin{align*}
\text{Now, } \Gamma(\beta+3) &= (\beta+2) \Gamma(\beta+2) \\
&= (\beta+2)(\beta+1) \Gamma(\beta+1) \quad (A.12) \\
&= (\beta+1) \Gamma(\beta+1) \quad (A.13)
\end{align*}

Applying Eqns. (A.12) and (A.13) into Eqns. (A.9) and (A.10)

\begin{align*}
\frac{\alpha}{(-\gamma)^{\beta+1}} \Gamma(\beta+1) &= 1 = \frac{\alpha}{(-\gamma)^{\beta+2}} \Gamma(\beta+2) \\
&= \frac{\alpha(\beta+1)}{(-\gamma)^{\beta+1}} \Gamma(\beta+1) \\
&= \gamma = \beta+1. \quad (A.14)
\end{align*}

Combining Eqns. (A.11) and (A.12)

\begin{align*}
\sigma^2 + 1 &= \frac{\alpha}{(-\gamma)^{\beta+2}} \Gamma(\beta+3) \\
&= \frac{\alpha}{(-\gamma)^{\beta+2}} (\beta+2) \Gamma(\beta+2) \quad (A.15)
\end{align*}

Substituting for \(-\gamma\) from Eqn. (A.14)

\begin{align*}
\sigma^2 + 1 &= (\beta+2)/(\beta+1) \\
\text{Or}
\beta \sigma^2 + \beta + \sigma^2 + 1 &= \beta + 2 \\
\beta (\sigma^2 + 1 - 1) &= 2 - 1 - \sigma^2 \\
\beta &= (1 - \sigma^2)/\sigma^2 \\
\therefore \quad \beta &= (1 - \sigma^2)/\sigma^2 \quad (A.16)
\end{align*}
Now

\[ \gamma = \beta + 1 \]

\[ = 1 + (1 - \sigma^2) / \sigma^2 \]

\[ \therefore \gamma = -1 / \sigma^2 \]  \hspace{1cm} \text{(A.17)}

From Eqn. (A.9)

\[ \alpha = \frac{(-\gamma)^{\beta + 1}}{\Gamma(\beta + 1)} = \frac{1 / \sigma^2}{\Gamma(1 / \sigma^2)} \]  \hspace{1cm} \text{(A.18)}

Substituting for \(\alpha, \beta\) and \(\gamma\) into Eqn. (A.1), we have:

\[ C'(\theta) = \frac{\left(\frac{1}{\sigma^2}\right)^{1/\sigma^2} \left(1 - \sigma^2\right) / \sigma^2 - \theta / \sigma^2}{\Gamma(1 / \sigma^2)} \]

\[ = \frac{\theta \left(1 - \sigma^2\right) / \sigma^2 - e^{-\theta / \sigma^2}}{(\sigma^2)^{1/\sigma^2} \Gamma(1 / \sigma^2)} \]

\[ = \frac{\theta \left(1 - \sigma^2\right) / \sigma^2 - e^{-\theta / \sigma^2}}{\sigma^2 / \sigma^2 \Gamma(1 / \sigma^2)} \]
APPENDIX 2.

The solution of Fick's second law.

It can be shown by dimensional analysis that the solution of Fick's second law i.e.
\[ \frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} \]  
(A)

is obtained as:
\[ C' = \frac{A}{\sqrt{t}} \exp\left(\frac{-x^2}{4Dt}\right) \]  
(B)

The derivation is lengthy, however, it will be shown by a differential method that Eqn. (B) is the solution of Eqn. (A).

Differentiating Eqn. (B) with respect to t, we have:
\[ \frac{\partial C'}{\partial t} = A \left[ e^{-\frac{x^2}{4Dt}} \cdot \left(-\frac{1}{2}\right) t^{-3/2} + \frac{1}{\sqrt{t}} \left(\frac{x^2}{4Dt^2}\right) e^{-\frac{x^2}{4Dt}} \right] \]
\[ = A e^{-\frac{x^2}{4Dt}} t^{-3/2} \left[-\frac{1}{2} + \frac{x^2}{4Dt} \right] \]
\[ = \frac{A}{2} e^{-\frac{x^2}{4Dt}} t^{-3/2} \left[ \frac{x^2}{2Dt} - 1 \right] \]  
(C)

Differentiating Eqn. (B) with respect to x, we have:
\[ \frac{\partial C'}{\partial x} = \frac{A}{\sqrt{t}} \left(\frac{-2x}{4Dt}\right) e^{-\frac{x^2}{4Dt}} \]  
(D)

Differentiating Eqn. (D) with respect to x again:
\[ \frac{\partial^2 C'}{\partial x^2} = \frac{A}{\sqrt{t}} \left[ \left(\frac{2x}{4Dt}\right)^2 e^{-\frac{x^2}{4Dt}} - \frac{2}{4Dt} e^{-\frac{x^2}{4Dt}} \right] \]
\[ = \frac{A}{2D} e^{-\frac{x^2}{4Dt}} t^{-3/2} \left[ \frac{x^2}{2Dt} - 1 \right] \]  
(E)
Multiply Eqn. (E) by (D), we have
\[
\frac{\partial^2 C'}{\partial x^2} = \frac{A}{2} e^{-\frac{x^2}{4Dt}} t^{-3/2} \left[ \frac{x^2}{4Dt} - 1 \right] = \frac{\partial C'}{\partial t}
\]

Hence Eqn. (B) is the solution to the Fick's second law.
APPENDIX 3.

The relationship between variance \((\sigma^2)\) and D/UL from a Gamma Function.

From Eqn. (55), we have:

\[
\sigma^2 = \int_0^\infty \frac{1}{2\pi \left(\frac{D}{UL}\right)\Theta} \cdot e^{-\frac{(1-\Theta)^2}{4\left(\frac{D}{UL}\right)\Theta}} \cdot d\Theta - \left[ \int_0^\infty \frac{1}{2\sqrt{\pi \left(\frac{D}{UL}\right)\Theta}} \cdot e^{-\frac{(1-\Theta)^2}{4\left(\frac{D}{UL}\right)\Theta}} \cdot d\Theta \right]^2
\]

\[= f(x_1) - [f(x_2)]^2\]

Let \(D/UL = k\); and \(-\frac{1}{4k} = \lambda\).

We have:

\[f(x_1) = \int_0^\infty \frac{1}{2\sqrt{\pi \left(\frac{D}{UL}\right)\Theta}} \cdot e^{\frac{(1-\Theta)^2}{4\left(\frac{D}{UL}\right)\Theta}} \cdot d\Theta\]

\[= \int_0^\infty \frac{1}{2\sqrt{\pi k \Theta}} \cdot e^{\frac{\lambda (1-\Theta)^2}{4\left(\frac{D}{UL}\right)\Theta}} \cdot d\Theta\]

\[= \frac{1}{2\sqrt{\pi k}} \int_0^\infty \Theta^{3/2} \cdot e^{-\lambda \left(\Theta + \frac{1}{\Theta} - 2\right)} \cdot d\Theta\]

Let \(f(x_3) = \int_0^\infty \Theta^{3/2} \cdot e^{-\lambda \left(\Theta + \frac{1}{\Theta} - 2\right)} \cdot d\Theta\)

and \(p^2 = \Theta\)

Then,

\[f(x_3) = \int_0^\infty p^4 \cdot e^{-\lambda \left(p - \frac{1}{p}\right)^2} \cdot dp\]

Let \(p = e^\omega\)
\[ p \to 0 , \quad w \to -\infty \]
\[ p \to +\infty , \quad w \to +\infty \]

and

\[ \frac{1}{p} = e^w - e^{-w} = 2 \sinh w \]

\[ f(x_3) = 2 \int_{-\infty}^{+\infty} e^{5w - 4\lambda \sinh^2 w} \, dw \]

\[ = 2 \left[ \int_{-\infty}^{0} e^{5w - 4\lambda \sinh^2 w} \, dw + \int_{0}^{+\infty} e^{5w - 4\lambda \sinh^2 w} \, dw \right] \]

\[ = 2 \left[ \int_{0}^{\infty} e^{-5w - 4\lambda \sinh^2 w} \, dw + \int_{0}^{\infty} e^{5w - 4\lambda \sinh^2 w} \, dw \right] \]

\[ = 2 \int_{0}^{\infty} (e^{5w} + e^{-5w}) e^{-4\lambda \sinh^2 w} \, dw \]

\[ = 4 \int_{0}^{\infty} e^{-4\lambda \sinh^2 w} \cdot \cosh 5w \, dw \]

\[ = 4 \int_{0}^{\infty} e^{-4\lambda \sinh^2 w} \cdot \cosh w (\cosh 4w + \sinh^2 w + \cosh 2w) \, dw \]

\[ = 4 \int_{0}^{\infty} e^{-4\lambda \sinh^2 w} \cdot \cosh w [(8\sinh^2 w \cosh^2 w + 1) + 4 \sinh^2 w (2 \sinh^2 w + 1)] \, dw \]
Let $\sinh 2y = y$ and $y^2 = t$

We have:

$$f(x_3) = 4 \int_0^\infty e^{-4\lambda y^2} \left[ 8y^2(1+y^2)+1+4y^2(2y^2+1) \right] dy$$

$$= 2 \int_0^\infty e^{-4\lambda t} \left( 16t^{3/2}+12t^{1/2}+t^{-1/2} \right) dt$$

$$= 2 \left[ \int_0^\infty e^{-4\lambda t} \cdot 16t^{3/2} dt + 12 \int_0^\infty e^{-4\lambda t} \cdot t dt + \int_0^\infty e^{-4\lambda t} \cdot t^{-1/2} dt \right].$$

But $\lambda = \frac{1}{4k}$, we have $4\lambda t = \frac{t}{k}$

Let $\frac{t}{k} = 4$, or $t = 4k$

We have:

$$f(x_3) = 2 \left[ k^{5/2} \cdot 16 \int_0^{3/2} e^{-s} ds + k^{3/2} \int_0^{1/2} e^{-s} ds + \right.$$

$$+ \left. \int_0^{1/2} e^{-s} ds \right]$$

$$= 2 \left[ k^{5/2} \cdot 16 \left( \Gamma \left( \frac{3}{2} \right) \right) + 12k^{3/2} \Gamma \left( \frac{3}{2} \right) + k^{1/2} \Gamma \left( \frac{1}{2} \right) \right]$$

$$= 2 \left[ k^{5/2} \cdot 16 \cdot \frac{3}{2} \cdot \frac{1}{2} \sqrt{\pi} + 12k^{3/2} \cdot \frac{3}{2} \cdot \frac{1}{2} \sqrt{\pi} + k^{1/2} \cdot \sqrt{\pi} \right]$$

$$= 2 \left( \pi k \right)^{1/2} \left[ 12k^2 + 6k + 1 \right]$$

$$\therefore f(x_1) = \frac{1}{2\sqrt{\pi k}} f(x_3)$$

$$= \frac{1}{2\sqrt{\pi k}} \cdot 2 \left( \pi k \right)^{1/2} \left[ 12k^2 + 6k + 1 \right]$$

$$= 12k^2 + 6k + 1$$

A 3.3
Similarly, we have:

\[ f(x_2) = 1 + 2k \]
\[ = f(x_1) - f(x_2)^2 \]
\[ = (12k^2 + 6k + 1) - (1 + 2k)^2 \]
\[ = 12k^2 + 6k + 1 - (1 + 4k + 4k^2) \]
\[ = 6k^2 + 2k \]

\[ \therefore \sigma^2 = 8 \left( \frac{D}{UL} \right)^2 + 2 \frac{D}{UL} \]
APPENDIX 4.

THE BEST CURVE FITTING FOR THE EQUILIBRIUM CURVE.

"The method of the least squares" is used for the purpose. It is used to seek an approximating function which will come as close as possible to the points of the experimental data and still retain a predetermental characteristic. The principle of the least squares states that the representation of the data is that which makes the sum of the squares of the residuals a minimum.

Let us consider the empirical equilibrium curve function to be a polynomial, i.e.

\[ Y_e = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 \ldots a_n x^n \quad (A.1) \]

From experimental work, we have \( R \) sets of \((Y_e, x)\) value, where \( R \geq n \). If we substitute the sets of value into Eqn. \((A.1)\), and by the principle of "The method of least squares", we have:

\[
\begin{bmatrix}
R & x_k & x_k^2 & \ldots & x_k^n \\
 & x_k & x_k^2 & \ldots & x_k^{n+1} \\
 & & x_k^2 & \ldots & x_k^{n+2} \\
 & & & \ldots & \ldots \\
 & x_k^n & x_k^{n+1} & \ldots & x_k^{2n} \\
\end{bmatrix}
\begin{bmatrix}
a_0 \\
a_1 \\
a_2 \\
\vdots \\
a_n \\
\end{bmatrix}
=
\begin{bmatrix}
y_{e_k} \\
x_k y_{e_k} \\
x_k^2 y_{e_k} \\
\vdots \\
x_k^n y_{e_k} \\
\end{bmatrix}
\]

Matrix A: \( A_{4.1} \) Matrix B Matrix C
Where $k = 1, 2, 3, \ldots \infty$

Where Matrix $A$ is a symmetric matrix, and matrices $B$ and $C$ are column matrices.

Hence the value of $a_0, a_1, a_2, \ldots, a_n$ can be obtained from the matrix $B$ and was obtained as follows:

$$(B) = (A)^{-1} (C)$$

The number of terms in the polynomial function is chosen in the way which the function has least error variance. The followed computer programme is so designed that it can compute the polynomial function from 1st. order to 5th order. The maximum points which are allowed were 50 sets.

The computer programme and its diagram are shown as follows.
Read in data

Set up matrix of variable x
i.e.
\[
\begin{pmatrix}
 x & x^2 & x^3 & \cdots \\
 x & x^2 & x^3 & \cdots \\
 x & x^2 & x^3 & \cdots \\
 x & x^2 & x^3 & \cdots \\
\end{pmatrix}
\]
according to order required

Transpose \( \{X\} \) matrix to give \( \{X\}^T \)

Multiply transposed \( \{X\}^T \) by \( \{X\} \) to give summation required for least squares fit

Multiply transposed \( \{X\} \) by \( \{Y\} \) to give summation for least squares i.e.
\[
\begin{pmatrix}
 x & y \\
 x^2 & y \\
 x^3 & y \\
 \vdots & \vdots \\
\end{pmatrix}
\]

Invert \( \{X\}^T \{X\} \) and multiply by \( \{X\}^T \{Y\} \) to give coefficient matrix

Print coefficients

Evaluate calculated y values as follows
\[
\{Y\}_{\text{calc.}} = \{X\} \{\text{coeff.}\}
\]
also
\[
\{\text{Errors}\} = \{Y\}_{\text{exp}} - \{Y\}_{\text{calc.}}
\]

Calculate error variance
\[
= \sqrt{\left( \frac{\{Y\}_{\text{exp}} - \{Y\}_{\text{calc.}}}{N - \text{polynomial order}} \right)^2}
\]

Print error variance

End
EMPirical POLYNOMIAL EQUATION FITTING FOR EQUILIBRIUM CURVE OF ACETIC ACID BETWEEN WATER AND CARBON TETRACHLORIDE.

\[ v = 1507 \]

\[ T_5 \]

\[ Jv_5 \]

\[ \text{ALL} \]

\[ L \]

\[ S615-14 \]

\[ S616-15 \]

\[ S617-16 \]

\[ T_5 \]

\[ Jv_1 \]

\[ v1 = \text{TAPEB*} \]

\[ v51 = \text{TAPE*} \]

\[ n10 = n0 \]

\[ n8 = 1 \]

\[ n10 \]

\[ nii = 0 \]

\[ 1 \]

\[ n9 = n9 + 1 \]

\[ nii = nii + 1 \]

\[ \rightarrow 2, nii = n10 \]

\[ \rightarrow 3, n8 = 1 \]

\[ nii = 0 \]

\[ 4 \]

\[ n9 = n9 + 1 \]

\[ nii = nii + 1 \]

\[ \rightarrow 4, nii = n1i \]

\[ \rightarrow 3, n8 = 3 \]

\[ nii = 0 \]

\[ 5 \]

\[ n9 = n9 + 1 \]

\[ nii = nii + 1 \]

\[ \rightarrow 5, nii = n10 \]

\[ \rightarrow 3, n8 = 3 \]

\[ nii = 0 \]

\[ 6 \]

\[ n9 = n9 + 1 \]

\[ nii = nii + 1 \]

\[ \rightarrow 6, nii = n10 \]

\[ \rightarrow 3, n8 = 4 \]
\[ \begin{align*}
\text{if } i &= 0 \\
v(102+n9) &= v(1+n1i)\times v(i+n1i) \\
v(102+n9) &= v(102+n9)\times v(102+n9) \\
v(102+n9) &= v(102+n9)\times v(i+n1i) \\
n9 &= n9 + 1 \\
n1i &= n1i + 1 \\
\rightarrow & 7, n1i = n10 \\
n12 &= n8 + 1 \\
n0 &= 2 \\
n1 &= 102 \\
n3 &= 402 \\
n4 &= n10 \\
n5 &= n12 \\
\rightarrow & SR14 \\
n1 &= 402 \\
n2 &= 102 \\
n3 &= 702 \\
n4 &= n12 \\
n5 &= n10 \\
n6 &= n10 \\
n7 &= n12 \\
\rightarrow & SR15 \\
n1 &= 402 \\
n2 &= 51 \\
n3 &= 738 \\
n4 &= n12 \\
n5 &= n10 \\
n6 &= n10 \\
n7 &= 1 \\
\rightarrow & SR15 \\
n0 &= 0 \\
n1 &= n12 \\
n2 &= n12 \\
n3 &= 702 \\
n4 &= 744 \\
\rightarrow & SR16 \\
n1 &= 744 \\
n2 &= 738 \\
n3 &= 780 \\
n4 &= n12 \\
n5 &= n12 \\
n6 &= n12 \\
n7 &= 1 \\
\rightarrow & SR15 \\
\end{align*} \]
\begin{verbatim}
11 = 102
n2 = 780
n3 = 786
n4 = ni0
n5 = ni2
n6 = ni2
n7 = 1
\rightarrow SRi 5

n0 = 1
n1 = 51
n2 = 786
n3 = 836
n4 = ni0
n5 = 1
\rightarrow SRi 4

TEXT
POLYNOMIAL ORDER
PRINTn8, 4000

TEXT
POLYNOMIAL ORDER COEFFICIENTS
PRINTv780, 1005
PRINTv781, 2005
\rightarrow 9, n8 = 1
PRINTv782, 2005
\rightarrow 9, n8 = 2
PRINTv783, 1005
\rightarrow 9, n8 = 3
PRINTv784, 2005
\rightarrow 9, n8 = 4
PRINTv785, 2005

9 ) n0 = 2
n1 = 836
n2 = 886
n3 = ni0
n4 = 1
\rightarrow SRi 4

n1 = 886
n2 = 836
n3 = 936
n4 = 1
n5 = ni0
n6 = ni0
n7 = 1
\rightarrow SRi 5
\end{verbatim}
\begin{verbatim}
PRINT v939, 3088
PRINT v940, 4088
+i2, n8=1
PRINT v941, 4088
+i2, n8=2
TEXT
PRINT v942, 3088
+i2, n8=3
PRINT v943, 4088
+i2, n8=4
PRINT, v944, 4088
+i2) n8=n8+1
TEXT
ERROR VARIANCE
PRINT v938, 4108
+i10, n8=6
(→o)
*****
\end{verbatim}
Appendix 5.

Examples of calculation and I.C.T. Ferranti "Sirius" 1602

Autocode computer programme.
AN EXAMPLE ILLUS TRATING THE COMPUTATION OF MEASURED NOY, KA AND HOY

1. OPERATING CONDITIONS
   O.D.C. 264 OSC/MIN, 16.7 L/HR CONSTANT FLOW RATE RATIO RUN

2. DATA
   \[ V_y = 16.7 \text{ L/HR} = 0.003970 \text{ FT/SEC} \]
   \[ X_B = 0.010 \text{ G/L} \]
   \[ Y_f = 19.27 \text{ G/L} \]
   \[ Y_B = 0.0 \text{ G/L} \]
   \[ \Delta Y = \Delta Y = (19.27 - 0)/34 = 0.567 \text{ G/L} \]

3. OPERATING LINE EQUATION
   \[ X = Y + X_B \]
   \[ Y = (19.27 - 0)/34 \times 0.567 \text{ Q/L} \]

4. COMPUTING
   \[ Y \times X = Y + X_B \]
   \[ YE \quad YE - Y \quad 1/(YE - Y) \]

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>0.010</td>
<td>0.660</td>
<td>0.660</td>
<td>1.515</td>
</tr>
<tr>
<td>0.567</td>
<td>0.577</td>
<td>36.178</td>
<td>35.611</td>
<td>0.028</td>
</tr>
<tr>
<td>1.134</td>
<td>1.144</td>
<td>58.637</td>
<td>57.504</td>
<td>0.017</td>
</tr>
<tr>
<td>1.700</td>
<td>1.710</td>
<td>77.461</td>
<td>75.761</td>
<td>0.013</td>
</tr>
<tr>
<td>2.267</td>
<td>2.277</td>
<td>93.981</td>
<td>91.714</td>
<td>0.011</td>
</tr>
<tr>
<td>2.834</td>
<td>2.844</td>
<td>108.252</td>
<td>105.418</td>
<td>0.009</td>
</tr>
<tr>
<td>3.401</td>
<td>3.411</td>
<td>121.361</td>
<td>117.961</td>
<td>0.008</td>
</tr>
<tr>
<td>3.967</td>
<td>3.977</td>
<td>133.520</td>
<td>129.553</td>
<td>0.008</td>
</tr>
<tr>
<td>4.534</td>
<td>4.544</td>
<td>144.882</td>
<td>140.348</td>
<td>0.007</td>
</tr>
<tr>
<td>5.101</td>
<td>5.111</td>
<td>156.107</td>
<td>151.006</td>
<td>0.007</td>
</tr>
<tr>
<td>5.668</td>
<td>5.678</td>
<td>166.804</td>
<td>161.337</td>
<td>0.006</td>
</tr>
<tr>
<td>6.234</td>
<td>6.244</td>
<td>177.053</td>
<td>170.818</td>
<td>0.006</td>
</tr>
<tr>
<td>6.801</td>
<td>6.811</td>
<td>187.413</td>
<td>180.611</td>
<td>0.006</td>
</tr>
<tr>
<td>7.368</td>
<td>7.378</td>
<td>196.518</td>
<td>189.150</td>
<td>0.005</td>
</tr>
<tr>
<td>7.935</td>
<td>7.945</td>
<td>205.148</td>
<td>197.314</td>
<td>0.005</td>
</tr>
<tr>
<td>8.501</td>
<td>8.511</td>
<td>214.281</td>
<td>205.780</td>
<td>0.005</td>
</tr>
<tr>
<td>9.068</td>
<td>9.078</td>
<td>223.252</td>
<td>214.184</td>
<td>0.005</td>
</tr>
<tr>
<td>9.635</td>
<td>9.645</td>
<td>232.030</td>
<td>223.395</td>
<td>0.004</td>
</tr>
<tr>
<td>10.202</td>
<td>10.212</td>
<td>239.965</td>
<td>229.763</td>
<td>0.004</td>
</tr>
<tr>
<td>10.769</td>
<td>10.779</td>
<td>247.324</td>
<td>236.574</td>
<td>0.004</td>
</tr>
<tr>
<td>11.335</td>
<td>11.345</td>
<td>254.834</td>
<td>243.499</td>
<td>0.004</td>
</tr>
<tr>
<td>11.902</td>
<td>11.912</td>
<td>261.945</td>
<td>250.432</td>
<td>0.004</td>
</tr>
<tr>
<td>12.469</td>
<td>12.479</td>
<td>269.512</td>
<td>257.043</td>
<td>0.004</td>
</tr>
<tr>
<td>13.036</td>
<td>13.046</td>
<td>276.211</td>
<td>263.175</td>
<td>0.004</td>
</tr>
<tr>
<td>13.603</td>
<td>13.612</td>
<td>283.581</td>
<td>269.799</td>
<td>0.004</td>
</tr>
<tr>
<td>14.169</td>
<td>14.179</td>
<td>290.149</td>
<td>275.980</td>
<td>0.004</td>
</tr>
<tr>
<td>14.736</td>
<td>14.746</td>
<td>296.705</td>
<td>281.969</td>
<td>0.004</td>
</tr>
<tr>
<td>15.303</td>
<td>15.313</td>
<td>302.939</td>
<td>287.637</td>
<td>0.003</td>
</tr>
<tr>
<td>15.869</td>
<td>15.879</td>
<td>308.794</td>
<td>292.425</td>
<td>0.003</td>
</tr>
<tr>
<td>16.436</td>
<td>16.446</td>
<td>314.730</td>
<td>298.393</td>
<td>0.003</td>
</tr>
<tr>
<td>17.003</td>
<td>17.013</td>
<td>320.622</td>
<td>303.619</td>
<td>0.003</td>
</tr>
<tr>
<td>17.570</td>
<td>17.580</td>
<td>326.045</td>
<td>308.473</td>
<td>0.003</td>
</tr>
<tr>
<td>18.136</td>
<td>18.146</td>
<td>331.818</td>
<td>313.682</td>
<td>0.003</td>
</tr>
<tr>
<td>18.703</td>
<td>18.713</td>
<td>337.346</td>
<td>318.424</td>
<td>0.003</td>
</tr>
<tr>
<td>19.270</td>
<td>19.280</td>
<td>342.628</td>
<td>323.358</td>
<td>0.003</td>
</tr>
</tbody>
</table>

4. RESULTS

\[ (N_{OY}) \quad (KA) \quad (HOY) \]

0.756 \hspace{1cm} 0.010619 \hspace{1cm} 3.521
\[ v = 1507 \]

\[ T_5 \]

\[ JV_5 \]

\[ ALL \]

\[ L \]

\[ S630 - 30 \]

\[ T_5 \]

\[ JV_1 \]

**TEXT**

The measured (NoY), average KA and (HoY) for constant flow rate ratio runs.

**TEXT**

<table>
<thead>
<tr>
<th>NO. OF RUN</th>
<th>(NoY)</th>
<th>(KA)</th>
<th>(HoY)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ v_{501} = \text{TAPE91} ]</td>
<td>[ n_3 = 1 ]</td>
<td>[ 8 ]</td>
<td>[ v_{92} = \text{TAPE4} ]</td>
</tr>
<tr>
<td>[ v_3 = v_{93} = v_{92} ]</td>
<td>[ v_4 = 34 ]</td>
<td>[ v_4 = v_3/v_4 ]</td>
<td>[ n_i = 0 ]</td>
</tr>
<tr>
<td>[ v_{163} = v_{92} ]</td>
<td>[ x_0 ]</td>
<td>[ v(164 + n_1) = v(163 + n_1) + v_4 ]</td>
<td>[ n_i = n_i + 1 ]</td>
</tr>
<tr>
<td>[ n_i = 0 ]</td>
<td>[ n_i \neq 34 ]</td>
<td>[ n_i = 0 ]</td>
<td>[ n_i = n_i + 1 ]</td>
</tr>
<tr>
<td>[ n_1 = 198 ]</td>
<td>[ n_1 = 98 ]</td>
<td>[ n_1 = 35 ]</td>
<td>[ n_2 = 0 ]</td>
</tr>
<tr>
<td>[ x_2 ]</td>
<td>[ v_1 = 0 ]</td>
<td>[ n_0 = 0 ]</td>
<td>[ n_0 = 497 ]</td>
</tr>
<tr>
<td>[ v_{497} = 35 ]</td>
<td>[ v_{498} = 35 ]</td>
<td>[ v_{499} = 0 ]</td>
<td>[ v_{500} = 0.5 ]</td>
</tr>
<tr>
<td>[ v_0 = v(198 + n_2) ]</td>
<td>[ \rightarrow SRa0 ]</td>
<td>[ v(128 + n_2) = v_1 ]</td>
<td>[ n_2 = n_2 + 1 ]</td>
</tr>
<tr>
<td>[ n_2 = 35 ]</td>
<td>[ n_2 = 35 ]</td>
<td>[ n_i = 0 ]</td>
<td>[ n_i = 0 ]</td>
</tr>
<tr>
<td>[ x_3 ]</td>
<td>[ v(233 + n_1) = v(128 + n_1) - v(163 + n_1) ]</td>
<td>[ v(233 + n_1) = 1/v(233 + n_1) ]</td>
<td>[ n_i = n_i + 1 ]</td>
</tr>
<tr>
<td>[ n_i = 35 ]</td>
<td>[ n_i = 35 ]</td>
<td>[ n_i = 35 ]</td>
<td>[ n_i = 35 ]</td>
</tr>
</tbody>
</table>
\begin{align*}
    & u_5 = 0 \\
    & u_5 = u_5 + u(234 + n_1) \\
    & n_1 = n_1 + 2 \\
    & -1.5; n_1 = 34 \\
    & n_1 = 0 \\
    & v_6 = 0 \\
    & u_6 = 0 + u(235 + n_1) \\
    & n_1 = n_1 + 2 \\
    & -1.5; n_1 = 33 \\
    & u_5 = u_5 \times 4 \\
    & v_6 = v_6 \times 2 \\
    & v_7 = v_7 + v_2 \times 7 \\
    & v_7 = v_7 + v_5 \\
    & v_7 = v_7 + v_6 \\
    & v_7 = v_7 \times v_4 \\
    & v_7 = v_7 / 3 \\
    & v_9 = n_3 \\
    & \text{PRINT} v_9, 3300 \\
    & \text{PRINT} v_7, 4323 \\
    & v_{10} = 31 \\
    & v_{10} = v_{10} / 13 \\
    & v_{95} = v_{95} \times 0.03531 \\
    & v_{95} = v_{95} / 3600 \\
    & v_{11} = 23 \\
    & v_{11} = v_{11} / 7 \\
    & v_{11} = v_{11} \times v_2 \times 75 \\
    & v_{11} = v_{11} \times v_2 \times 75 \\
    & v_{11} = v_{11} / 576 \\
    & v_{95} = v_{95} / v_{11} \\
    & v_8 = v_7 \times v_9 \\
    & v_8 = v_8 / v_{10} \\
    & \text{PRINT} v_8, 4226 \\
    & v_6 = v_{10} / v_7 \\
    & \text{PRINT} v_6, 4143 \\
    & \text{PRINT} v_9, 3066 \\
    & n_3 = n_3 + 1 \\
    & -8, n_3 \geq 200 \\
    & (\rightarrow 0)
\end{align*}
AN EXAMPLE ILLUSTRATING THE COMPUTATION OF KA ALONG THE COLUMN

1. OPERATING CONDITIONS
O.B.C. 264 OSC/MIN, 16.7 L/HR CONSTANT FLOWRATE RATIO RUN

2. DATA
VY = 16.7 L/HR = 0.003970 FT/SEC
Y1 = 0.09  Y2 = 0.41  Y3 = 1.13  Y4 = 3.08
Y5 = 7.42  Y6 = 19.25
XB = 0.01

OPERATING LINE EQUATION
X = Y + XB = Y + 0.01

3. COMPUTING AND RESULTS

<table>
<thead>
<tr>
<th>NO OF SECTION</th>
<th>(KA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000173</td>
</tr>
<tr>
<td>2</td>
<td>0.000134</td>
</tr>
<tr>
<td>3</td>
<td>0.000182</td>
</tr>
<tr>
<td>4</td>
<td>0.000225</td>
</tr>
<tr>
<td>5</td>
<td>0.000358</td>
</tr>
</tbody>
</table>
THE MEASURED (NoY), AVERAGE KA AND (HoY) FOR CONSTANT FLOW RATE RATIO RUNS BY SECTION

\[ V_{501} = \text{T A P E 91} \]
\[ n_3 = 1 \]
8) \[ V_{92} = \text{T A P E 8} \]
\[ V_{10} = 6.125 \]
\[ V_{10} = V_{10/12} \]
\[ V_{92} = V_{92} \times 0.03531 \]
\[ V_{11} = 23 \]
\[ V_{11} = V_{11}/7 \]
\[ V_{11} = V_{11} \times 2.75 \]
\[ V_{11} = V_{11}/576 \]
\[ V_{92} = V_{92}/V_{11} \]
\[ V_{92} = V_{92}/3600 \]

FLOW RATE
PRINT \[ V_{92}, 4226 \]

NO OF SECTION (NoY) (KA) (HoY)

\[ n_4 = 0 \]
5) \[ V_4 = (95+n_4) - V(94+n_4) \]
\[ V_4 = V_4/34 \]
\[ n_1 = 0 \]
\[ V_{163} = V(94+n_4) \]
10) \[ V(164+n_1) = V(163+n_1) + V_4 \]
\[ n_1 = n_1 + 1 \]
\[ \rightarrow 10, n_1 = 34 \]
\[ n_1 = 0 \]
11) \[ V(198+n_1) = V(163+n_1) + V_{93} \]
\[ n_1 = n_1 + 1 \]
\[ \rightarrow 11, n_1 = 35 \]
\[ n_2 = 0 \]

A 5.5
1.2) \( v_1 = 0 \)
\( n_0 = 0 \)
\( n_1 = 49.7 \)
\( v_{49.7} = 35 \)
\( v_{498} = 91 \)
\( v_{499} = 0 \)
\( v_{500} = 0.5 \)
\( v_0 = v(198+n_2) \)
\( \rightarrow SR_{10} \)
\( v(128+n_3) = v_1 \)
\( n_2 = n_2 + 1 \)
\( \rightarrow 12, n_2 = 35 \)
\( n_1 = 0 \)
\( 13) v(233+n_1) = v(128+n_1) - v(163+n_1) \)
\( v(233+n_1) = 1/v(233+n_1) \)
\( n_1 = n_1 + 1 \)
\( \rightarrow 13, n_1 = 35 \)
\( n_1 = 0 \)
\( v_5 = 0 \)
\( 14) v_5 = v_5 + v(234+n_1) \)
\( n_1 = n_1 + 2 \)
\( \rightarrow 14, n_1 = 34 \)
\( n_1 = 0 \)
\( v_6 = 0 \)
\( 15) v_6 = v_6 + v(235+n_1) \)
\( n_1 = n_1 + 2 \)
\( \rightarrow 15, n_1 = 32 \)
\( v_5 = v_5 x 4 \)
\( v_6 = v_6 x 2 \)
\( v_7 = v_233 + v_267 \)
\( v_7 = v_7 + v_5 \)
\( v_7 = v_7 + v_6 \)
\( v_7 = v_7 v_4 \)
\( v_7 = v_7 / 3 \)
\( v_9 = n_4 \)
\( v_9 = v_9 + 1 \)
\( v_8 = v_7 v_9 \)
\( v_8 = v_8 / v_10 \)
PRINTv9, 3300
PRINTv7, 4146
v6 = v10 / v7
PRINTv8, 4146
PRINTv6, 4146
n4 = n4 + 1
\( \rightarrow 5, n_4 = 5 \)
\( n_3 = n_3 + 1 \)
\( \rightarrow 8, n_3 = 50 \)
\( \rightarrow 0 \)
AN EXAMPLE ILLUSTRATING THE COMPUTATION OF EY BY METHOD ONE

1. OPERATING CONDITIONS
C.B.C. 264 OSC/MIN, 16.7 L/HR CONSTANT FLOWRATE RATIO RUN

2. DATA

\[ V_Y = 16.7 \text{ L/HR} = 0.003970 \text{ FT/SEC} \]

\[
\begin{align*}
(KA)_2 &= 0.000670 \\
(KA)_3 &= 0.000200 \\
(KA)_4 &= 0.000430 \\
(KA)_5 &= 0.000257
\end{align*}
\]

\[
\begin{align*}
X_B &= 0.01 \\
Y_1 &= 0.09 \\
Y_2 &= 0.41 \\
Y_3 &= 1.13 \\
Y_4 &= 3.08 \\
Y_5 &= 7.40 \\
Y_6 &= 19.25
\end{align*}
\]

OPERATING LINE EQUATION

\[ X = Y + X_B = Y + 0.01 \]

3. COMPUTING

<table>
<thead>
<tr>
<th>RUN</th>
<th>( X )</th>
<th>( Y ) (1ST.D.)</th>
<th>( Y ) (2ND.D.)</th>
<th>( V_{XY} ) (1ST.D.)</th>
<th>( KAX ) (YE-Y)</th>
<th>( E_{XY} ) (2ND.D.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>27.69</td>
<td>0.019</td>
<td>1.535</td>
<td>0.004</td>
<td>0.004556</td>
<td>0.000512</td>
</tr>
<tr>
<td>3</td>
<td>58.50</td>
<td>2.616</td>
<td>4.731</td>
<td>0.010</td>
<td>0.011474</td>
<td>0.001092</td>
</tr>
<tr>
<td>4</td>
<td>114.04</td>
<td>6.148</td>
<td>9.097</td>
<td>0.034</td>
<td>0.025533</td>
<td>0.001151</td>
</tr>
<tr>
<td>5</td>
<td>196.98</td>
<td>15.840</td>
<td>28.903</td>
<td>0.063</td>
<td>0.048760</td>
<td>-0.014119</td>
</tr>
</tbody>
</table>

4. RESULTS

<table>
<thead>
<tr>
<th>RUN</th>
<th>( S )</th>
<th>( E_Y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.0003336</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.0003312</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.0003365</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-0.0004885</td>
<td></td>
</tr>
</tbody>
</table>
BACK MIXING COEFFICIENT CALCULATION FOR CONSTANT FLOW RATE
RATIO RUNS BY MEIHOU NO. ONE

\( v = 1507 \)
\( T_5 \)
\( S_{620} = 20 \)
\( T_5 \)

\[ \text{NO OF RUN} \]
\[ \text{PRINT} v_8 = 4180 \]
\[ v_8 = \text{TAPE} 12 \]
\[ v_{30} = 3600 \]
\[ v_{30} = 38.3 x v_{30} \]
\[ v_{1} = 3.75 \]
\[ v_{1} = v_{31}/12 \]
\[ v_{1} = v_{31} x v_{31} \]
\[ v_{1} = v_{31} / 38 \]
\[ v_{1} = v_{31} x 32 \]
\[ v_{1} = v_{31} x v_{30} \]
\[ v_{8} = v_{8}/v_{31} \]

\[ \text{FLOW RATE} \]
\[ \text{PRINT} v_8 = 4336 \]
\[ n_3 = 0 \]
\[ 2) v(30+n_3)=v(15+n_3)+V_{13} \]
\[ n_3 = n_3 + 1 \]
\[ \rightarrow 3, n_3 \equiv 4 \]
\[ n_3 = 0 \]
\[ 3) v_1 = 0 \]
\[ n_0 = 0 \]
\[ n_1 = 96 \]
\[ v_{96} = 1 \]
\[ v_{97} = 91 \]
\[ v_{98} = 0 \]
\[ v_{99} = 0.5 \]
\[ v_0 = v(30+n_3) \]
\[ \rightarrow \text{SR} 20 \]
\[ v(35+n_3)=v_1 \]
\[ n_3 = n_3 + 1 \]
\[ \rightarrow 3, n_3 \equiv 4 \]

A 5.8
\[ v_{49} = 6.125 \]
\[ v_{49} = v_{49}/1.2 \]
\[ v_{49} = v_{49}/2 \]
\[ v_{51} = v_{49}v_{49} \]

\[
\begin{array}{cccc}
Z & Y & YE & KA \\
\hline
n2=0 \\
5) v61 = n2 \\
v61 = v61 + 2 \\
PRINT v61, 3020 \\
PRINT v(15+n2), 4082 \\
PRINT v(35+n2), 4082 \\
PRINT v(9+n2), 4047 \\
n2 = n2 + 1 \\
\rightarrow 5, n2 ≠ 4 \\
\end{array}
\]

\[
\begin{array}{cccc}
Z & Y(1ST.D) & Y(2ND.D) & VX Y(1ST.D) \quad KAX(YE-Y) \quad EX Y(2ND.D) \quad E \\
\hline
n2 = 0 \\
4) v40 = v(16+n2) - v(4+n2) \\
v40 = v_{40}v_{40}v_{50} \\
v41 = v8xv40 \\
v42 = v(35+n2) - v(15+n2) \\
v43 = v_{42}v_{42}(9+n2) \\
v44 = v(16+n2) + v(4+n2) \\
v44 = v44 - v(15+n2) \\
v44 = v44 - v(15+n2) \\
\rightarrow 7, v44 = 0 \\
v44 = v44xv51 \\
v45 = v43 - v41 \\
v46 = v45/v44 \\
v47 = n2 \\
v47 = v47 + 2 \\
PRINT v47, 3040 \\
PRINT v40, 4063 \\
PRINT v44, 4083 \\
PRINT v41, 4083 \\
PRINT v43, 4086 \\
PRINT v45, 4083 \\
PRINT v46, 4047 \\
7) n2 = n2 + 1 \\
\rightarrow 4, n2 ≠ 4 \\
n3 = n3 + 1 \\
\rightarrow 1, n3 ≠ 50 \\
(\rightarrow 0)
\end{array}
\]
AN EXAMPLE ILLUSTRATING THE COMPUTATION OF EY BY METHOD TWO

1. OPERATING CONDITIONS
   O.B.C. 264 OSC/MIN, 16.7 L/HR CONSTANT FLOWRATE RATIO RUN

2. DATA
   KA VALUE AT EACH INCH COLUMN HEIGH (FROM Z = 6 TO Z = 24)
   0.000000 0.0000727 0.0001787 0.0001841 0.0001898 0.0001950
   0.0002000 0.0002052 0.0002100 0.0002152 0.0002200 0.0002252
   0.0002301 0.0002372 0.0002400 0.0002440 0.0002490 0.0002530
   VY = 16.7 L/HR = 0.003970 FT/SEC
   X = Y + XB = Y
   OPERATING LINE EQUATION
   Y VALUE AT EACH INCH COLUMN HEIGH (FROM Z = 4 TO Z = 26)
   0.0020 0.0028 0.0033 0.0041 0.0050 0.0060 0.0072 0.0087 0.0105 0.0120
   0.0139 0.0163 0.0191 0.0224 0.0262 0.0309 0.0357 0.0415 0.0481 0.0559
   0.0651 0.0755 0.0870

3. COMPUTING
   INTEGRATING KA VALUE FOR A 6 INCH COLUMN HEIGH SECTION
   S2 = 0.000093 S3 = 0.000108 S4 = 0.000122
   VXNOY VALUE FOR A 6 INCH COLUMN HEIGH SECTION
   S2 = 0.000076 S3 = 0.000081 S4 = 0.000113

   Z   DYE/DY-i   1/YE-Y   DY/DZ   A   B
   6   23.07994   0.045282   0.740000   0.042790   0.033506
   7   24.002867   0.056625   1.040000   0.031970   0.038111
   8   21.918535   0.030398   1.130000   0.025861   0.034347
   9   21.825066   0.037023   1.300000   0.041703   0.035190
   10  21.713242   0.039332   1.610000   0.032325   0.038532
   11  21.573917   0.049010   2.040000   0.039220   0.042632
   12  21.407474   0.018281   1.970000   0.027766   0.036011
   13  21.269375   0.016748   1.960000   0.024918   0.032823
   14  21.093240   0.015139   3.580000   0.032181   0.039052
   15  20.870539   0.013582   3.120000   0.037490   0.042374
   16  20.623165   0.012183   3.650000   0.040780   0.044460
   17  20.370005   0.010989   4.220000   0.043714   0.046373
   18  19.980265   0.009951   5.140000   0.052297   0.051148
   19  19.575413   0.008994   5.690000   0.051274   0.051178
   20  19.106435   0.008229   6.290000   0.051337   0.051765
   21  18.665102   0.007516   7.420000   0.058058   0.055777
   22  18.11542   0.006869   8.580000   0.062909   0.058932
   23  17.471050   0.006259   10.200000   0.071215   0.063846
   24  16.734993   0.005702   11.790000   0.075644   0.067235

   S   1ST. TERM   2ND. TERM   3RD. TERM   4TH TERM
   2  0.000076   0.000092   0.002505   0.015601
   3  0.000081   0.000108   0.015135   0.017846
   4  0.000113   0.000122   0.016832   0.029862

4. RESULTS
   EY(S2) = 0.000910   EY(S3) = 0.000799   EY(S4) = 0.000201
\[ \text{TEXT} \]
\[ \text{BACK MIXING COEFFICIENT CALCULATION FOR CONSTANT FLOW RATE} \]
\[ \text{RATIO RUNS BY METHOD NO. TWO.} \]
\[ \text{FLOW RATE} \]
\[ \text{PRINT} v_{50}, 4166 \]
\[ n_1 = 0 \]
\[ 4) v(120+n_1) = v(54+n_1) + v_{51} \]
\[ v(139+n_1) = v(120+n_1) \times v(120+n_1) \]
\[ v(158+n_1) = 0.9483666 \times v(120+n_1) \]
\[ v(139+n_1) = 0.01221 \times v(139+n_1) \]
\[ v(158+n_1) = v(158+n_1) + v(139+n_1) \]
\[ v(158+n_1) = 22.39865 + v(158+n_1) \]
\[ n_1 = n_1 + 1 \]
\[ \rightarrow 4, n_1 = 19 \]
\[ n_2 = 0 \]
\[ a) v_1 = 0 \]
\[ n_0 = 0 \]
\[ n_1 = 497 \]
\[ v_{497} = 19 \]
\[ v_{498} = 91 \]
\[ v_{499} = 0 \]
\[ v_{500} = 0.5 \]
\[ v_0 = v(120+n_2) \]
\[ \rightarrow SR_{20} \]
\[ v(81+n_2) = v_1 \]
\[ n_2 = n_2 + 1 \]
\[ \rightarrow a, n_2 = 19 \]
\[ n_1 = 0 \]
\[ 3) v(100+n_1) = v(81+n_1) - v(54+n_1) \]
\[ v(100+n_1) = v(100+n_1) \]
\[ n_1 = n_1 + 1 \]
\[ \rightarrow 3, n_1 = 19 \]
\[ n_1 = 0 \]
\[ 5) v(180+n_1) = v(53+n_1) - v(52+n_1) \]
\[ n_1 = n_1 + 1 \]
\[ \rightarrow 5, n_1 = 23 \]
\[ n_1 = 0 \]
\[ 6) v(202+n_1) = v(181+n_1) - v(180+n_1) \]
\[ n_1 = n_1 + 1 \]
\[ \rightarrow 6, n_1 = 21 \]
\[ n_1 = 0 \]
\[ 7) v(223+n_1) = v(202+n_1) - v(202+n_1) \]
\[ n_1 = n_1 + 1 \]
\[ \rightarrow 7, n_1 = 20 \]
\[ n_1 = 0 \]
\[ 8) v_2 = v(181+n_1) + v(182+n_1) \]
\[ v_3 = v(223+n_1) + v(224+n_1) \]
\[ v_2 = v_2/2 \]
\[ v_3 = v_3/12 \]
\[ v_2 = v_2 - v_3 \]
\[ v_2 = v_2\times12 \]
\[ v(243+n_1) = v_2 \]
\[ n_1 = n_1 + 1 \]
\[ \rightarrow 8, n_1 = 19 \]

\[ \text{TEXT} \]

\[ n_4 = 0 \]
\[ n_5 = 0 \]
\[ 10) v_4 = v(60+n_4) - v(54+n_4) \]
\[ v_4 = v_4/34 \]
\[ n_1 = 0 \]
\[ v_263 = v(54+n_4) \]
\[ 11) v(264+n_1) = v(263+n_1) + v_4 \]
\[ n_1 = n_1 + 1 \]
\[ \rightarrow 11, n_1 = 34 \]
\[ n_1 = 0 \]
\[ 13) v(398+n_1) = v(363+n_1) + v_5 \]
\[ n_1 = n_1 + 1 \]
\[ \rightarrow 13, n_1 = 35 \]
\[ n_2 = 0 \]
\[ 13) v_1 = 0 \]
\[ n_0 = 0 \]
\[ n_1 = 497 \]
\[ v_{497} = 35 \]
\[ v_{498} = 91 \]
\[ v_{499} = 0 \]
V500 = 0.5
V0 = V \left( 100 + n1 \right)
\rightarrow SR30
V \left( 100 + n1 \right) = V1
n1 = n1 + 1
\rightarrow I13, n2 = 35
n1 = 0
I4 \left( V \left( 368 + n1 \right) = V \left( 333 + n1 \right) - V \left( 263 + n1 \right) \right)
V \left( 433 + n1 \right) = V \left( 433 + n1 \right) / V \left( 368 + n1 \right)
V5 = 0
I5 \left( V5 = V5 + V \left( 434 + n1 \right) \right)
V \left( 333 + n1 \right) = V \left( 334 + n1 \right) + 2
\rightarrow I5, n1 = 34
n1 = 0
V6 = 0
I6 \left( V6 = V6 + V \left( 435 + n1 \right) \right)
V \left( 368 + n1 \right) = V \left( 368 + n1 \right) + 2
\rightarrow I6, n1 = 32
V5 = V5 \times 4
V6 = V6 \times 2
V7 = V \left( 433 + n1 \times 3 \right)
V7 = V7 + V5
V7 = V7 + V6
V7 = V7 \times V4
V7 = V7 / 3
V8 = V7 \times V5\times 50
V9 = V8 \times 2
V \left( 10 + n5 \right) = V7
V \left( 13 + n5 \right) = V8
V \left( 16 + n5 \right) = V9
PRINT V7, 3086
PRINT V8, 4106
PRINT V9, 4106
n4 = n4 + 6
n5 = n5 + 1
\rightarrow I10, n5 = 3
n1 = 0
I7 \left( V \left( 20 + n1 \right) = V \left( 100 + n1 \right) \times V \left( 100 + n1 \right) \times V \left( 158 + n1 \right) \right)
V \left( 20 + n1 \right) = V \left( 20 + n1 \right) \times V \left( 243 + n1 \right)
V \left( 20 + n1 \right) = V \left( 20 + n1 \right) \times V \left( 243 + n1 \right)
V \left( 600 + n1 \right) = V \left( 100 + n1 \right) \times V \left( 243 + n1 \right)
V \left( 100 + n1 \right) = V \left( 100 + n1 \right) \times V \left( 243 + n1 \right)
V \left( 100 + n1 \right) = V \left( 100 + n1 \right) \times V \left( 243 + n1 \right)
\rightarrow I17, n1 = 19
TEXT

N DYE-1/DY 1/YE-Y DY/DZ A
\( n_i = 0 \)

9) \( v_5 = n_i \)

\( v_5 = v_5 + 1 \)

PRINT \( v_5 \), 3100

PRINT \((158 + n_i)\), 4066

PRINT \((100 + n_i)\), 4086

PRINT \((243 + n_i)\), 4066

PRINT \((20 + n_i)\), 4086

PRINT \((600 + n_i)\), 4086

\( n_i = n_i + 1 \)

\( \rightarrow 9, n_i \neq 19 \)

\( n_i = 0 \)

\( n_2 = 0 \)

18) \( v_619 = v(20 + n_2) + v(26 + n_2) \)

\( v_620 = v(21 + n_2) + v(23 + n_2) \)

\( v_620 = v_620 + v(25 + n_2) \)

\( v_620 = 4xv620 \)

\( v_621 = v(22 + n_2) + v(24 + n_2) \)

\( v_621 = 2xv621 \)

\( v_621 = v_621 + v_620 \)

\( v_621 = v_621 + v_619 \)

\( v_621 = v_621 / 36 \)

\( v(623 + n_i) = v_621 \)

\( v(625 + n_i) = v(636 + n_2) - v(600 + n_2) \)

\( v(628 + n_i) = v(616 + n_i) / 2 \)

\( n_2 = n_2 + 6 \)

\( n_i = n_i + 1 \)

\( \rightarrow 18, n_i \neq 3 \)

TEXT

1ST T 2ND T 3RD T 4TH D

\( n_i = 0 \)

9) \( \text{PRINT} (13 + n_i), 3086 \)

PRINT \((628 + n_i)\), 4086

PRINT \((625 + n_i)\), 4086

PRINT \((622 + n_i)\), 4086

\( n_i = n_i + 1 \)

\( \rightarrow 9, n_i \neq 3 \)

TEXT

BACK MIXING COEFFICIENT

\( n_i = 0 \)

30) \( v_700 = v(13 + n_i) - v(638 + n_i) \)

\( v_701 = v(625 + n_i) + v(622 + n_i) \)

\( v_700 = v_700 / v_701 \)

PRINT \( v_700, 3306 \)

\( n_i = n_i + 1 \)

\( \rightarrow 30, n_i \neq 3 \)

\( n_3 = n_3 + 1 \)

\( \rightarrow 1, n_3 \neq 50 \)

\( \rightarrow 0 \)
AN EXAMPLE ILLUSTRATING THE COMPUTATION OF EY BY METHOD THREE

1. OPERATING CONDITIONS

26.4 OSC/Min, 16.7 L/HR CONSTANT FLOWRATE RATIO RUN

2. DATA

KA VALUE AT EACH INCH COLUMN HEIGH (FROM Z = 6 TO Z = 24)

<table>
<thead>
<tr>
<th>Z</th>
<th>DYE/DY-I</th>
<th>1/YE-Y</th>
<th>DY/DZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>22.077994</td>
<td>0.045282</td>
<td>0.740000</td>
</tr>
<tr>
<td>7</td>
<td>22.002867</td>
<td>0.036653</td>
<td>1.040000</td>
</tr>
<tr>
<td>8</td>
<td>21.918535</td>
<td>0.030398</td>
<td>1.130000</td>
</tr>
<tr>
<td>9</td>
<td>21.826066</td>
<td>0.027073</td>
<td>1.300000</td>
</tr>
<tr>
<td>10</td>
<td>21.713224</td>
<td>0.023932</td>
<td>1.610000</td>
</tr>
<tr>
<td>11</td>
<td>21.573917</td>
<td>0.020901</td>
<td>2.040000</td>
</tr>
<tr>
<td>12</td>
<td>21.407474</td>
<td>0.016281</td>
<td>1.970000</td>
</tr>
<tr>
<td>13</td>
<td>21.269375</td>
<td>0.016748</td>
<td>1.960000</td>
</tr>
<tr>
<td>14</td>
<td>21.095240</td>
<td>0.015139</td>
<td>2.580000</td>
</tr>
<tr>
<td>15</td>
<td>20.876539</td>
<td>0.013582</td>
<td>3.120000</td>
</tr>
<tr>
<td>16</td>
<td>20.623165</td>
<td>0.014183</td>
<td>3.650000</td>
</tr>
<tr>
<td>17</td>
<td>20.327005</td>
<td>0.010989</td>
<td>4.220000</td>
</tr>
<tr>
<td>18</td>
<td>19.989265</td>
<td>0.009951</td>
<td>5.140000</td>
</tr>
<tr>
<td>19</td>
<td>19.576413</td>
<td>0.008994</td>
<td>5.690000</td>
</tr>
<tr>
<td>20</td>
<td>19.160345</td>
<td>0.008229</td>
<td>6.290000</td>
</tr>
<tr>
<td>21</td>
<td>18.665102</td>
<td>0.007516</td>
<td>7.420000</td>
</tr>
<tr>
<td>22</td>
<td>18.111142</td>
<td>0.006869</td>
<td>8.580000</td>
</tr>
<tr>
<td>23</td>
<td>17.471050</td>
<td>0.006349</td>
<td>10.200000</td>
</tr>
<tr>
<td>24</td>
<td>16.734693</td>
<td>0.005702</td>
<td>11.790000</td>
</tr>
</tbody>
</table>

VY = 16.7 L/HR = 0.003970 FT/SEC

OPERATING LINE EQUATION

X = Y + XB = Y + 0.01

Y VALUE AT EACH INCH COLUMN HEIGH (FROM Z = 8 TO Z = 26)

<table>
<thead>
<tr>
<th>Z</th>
<th>DYE/DY-1</th>
<th>1/YE-Y</th>
<th>DY/DZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.000270</td>
<td>0.000322</td>
<td>0.033723</td>
</tr>
<tr>
<td>9</td>
<td>0.000270</td>
<td>0.000322</td>
<td>0.033723</td>
</tr>
<tr>
<td>10</td>
<td>0.000270</td>
<td>0.000322</td>
<td>0.033723</td>
</tr>
<tr>
<td>11</td>
<td>0.000270</td>
<td>0.000322</td>
<td>0.033723</td>
</tr>
<tr>
<td>12</td>
<td>0.000270</td>
<td>0.000322</td>
<td>0.033723</td>
</tr>
<tr>
<td>13</td>
<td>0.000270</td>
<td>0.000322</td>
<td>0.033723</td>
</tr>
<tr>
<td>14</td>
<td>0.000270</td>
<td>0.000322</td>
<td>0.033723</td>
</tr>
<tr>
<td>15</td>
<td>0.000270</td>
<td>0.000322</td>
<td>0.033723</td>
</tr>
<tr>
<td>16</td>
<td>0.000270</td>
<td>0.000322</td>
<td>0.033723</td>
</tr>
<tr>
<td>17</td>
<td>0.000270</td>
<td>0.000322</td>
<td>0.033723</td>
</tr>
<tr>
<td>18</td>
<td>0.000270</td>
<td>0.000322</td>
<td>0.033723</td>
</tr>
<tr>
<td>19</td>
<td>0.000270</td>
<td>0.000322</td>
<td>0.033723</td>
</tr>
<tr>
<td>20</td>
<td>0.000270</td>
<td>0.000322</td>
<td>0.033723</td>
</tr>
<tr>
<td>21</td>
<td>0.000270</td>
<td>0.000322</td>
<td>0.033723</td>
</tr>
<tr>
<td>22</td>
<td>0.000270</td>
<td>0.000322</td>
<td>0.033723</td>
</tr>
<tr>
<td>23</td>
<td>0.000270</td>
<td>0.000322</td>
<td>0.033723</td>
</tr>
<tr>
<td>24</td>
<td>0.000270</td>
<td>0.000322</td>
<td>0.033723</td>
</tr>
</tbody>
</table>

3. COMPUTING

INTEGRATING KA VALUE FOR A 18 INCH COLUMN HEIGH SECTION (FROM Z=6 TO Z=24) = 0.00032183

VxNoY VALUE FOR A 18 INCH COLUMN HEIGH = 0.00027000

4. RESULTS

EY = 0.00053726
BACK MIXING COEFFICIENT CALCULATION FOR CONSTANT RATIO RUNS BY METHOD THREE

\[ v_{50} = \text{TAPE91} \]
\[ m_3 = 1 \]
\[ 1) v_{31} = \text{TAPE44} \]
\[ v_{50} = v_{50} \times 0.03531 \]
\[ v_{80} = 0.30 \]
\[ v_{80} = v_{80} / 7 \]
\[ v_{80} = v_{80} x 2 + 75 \]
\[ v_{80} = v_{80} / 576 \]
\[ v_{50} = v_{50} / v_{80} \]
\[ v_{50} = v_{50} / 3600 \]

FLOW RATE
PRINT \[ v_{50}, 4166 \]

\[ n_1 = 0 \]
\[ 4) v(130+n_1) = v(54+n_1) + v_{51} \]
\[ v(139+n_1) = v(130+n_1) \times v(130+n_1) \]
\[ v(158+n_1) = 0.9483636 \times v(130+n_1) \]
\[ v(139+n_1) = 0.00298 \times v(139+n_1) \]
\[ v(158+n_1) = 0.75865 + v(158+n_1) \]
\[ n_1 = n_1 + 1 \]
\[ n_4, n_1 = 19 \]
\[ n_3 = 0 \]

\[ 2) v_1 = 0 \]
\[ n_0 = 0 \]
\[ n_1 = 497 \]
\[ v_{497} = 19 \]
\[ v_{498} = 91 \]
\[ v_{499} = 0 \]
\[ v_{500} = 0.5 \]
\[ v_6 = v(120+n_2) \]
\[ v(81+n_2) = v_1 \]
\[ n_2 = n_2 + 1 \]
\[ n_2 = 19 \]
\[ ni = 0 \]

3) \[ v(100+ni) = v(81+ni) - v(54+ni) \]

\[ v(100+ni) = 1/v(100+ni) \]

\[ ni = ni + 1 \]

-3, \[ ni = 19 \]

\[ ni = 0 \]

5) \[ v(180+ni) = v(53+ni) - v(53+ni) \]

\[ ni = ni + 1 \]

-5, \[ ni = 22 \]

\[ ni = 0 \]

6) \[ v(103+ni) = v(101+ni) - v(100+ni) \]

\[ ni = ni + 1 \]

-6, \[ ni = 21 \]

\[ ni = 0 \]

7) \[ v(203+ni) = v(203+ni) - v(203+ni) \]

\[ ni = ni + 1 \]

-7, \[ ni = 20 \]

\[ ni = 0 \]

8) \[ v3 = v(181+ni) + v(182+ni) \]

\[ v3 = v(203+ni) + v(204+ni) \]

\[ v3 = v2/2 \]

\[ v3 = v3/13 \]

\[ v3 = v2 - v3 \]

\[ v3 = v3 \times 13 \]

\[ v(243+ni) = v3 \]

\[ ni = ni + 1 \]

-8, \[ ni = 19 \]

\[ v15 = 0 \]

\[ v15 = v31 + v49 \]

\[ ni = 0 \]

\[ v16 = 0 \]

23) \[ v16 = v16 + v(32+ni) \]

\[ ni = ni + 3 \]

-23, \[ ni = 18 \]

\[ ni = 0 \]

\[ v17 = 0 \]

31) \[ v17 = v17 + v(33+ni) \]

\[ ni = ni + 3 \]

-21, \[ ni = 16 \]

\[ v15 = v15 + v16 \]

\[ v15 = v15 + v17 \]

\[ v15 = v15/36 \]

\[ A 5.17 \]
INTEGRAL VALUE OF KA

PRINT\textit{vis}, 4448

\begin{align*}
\text{NoY} & \quad \text{VXNoY} & \quad (KA)AV. \\
v_4 & = v_7 - v_5 & v_4 & = v_4/34 \\
n_1 & = 0 & \text{\texttt{v263}} & = v_5 \\
(1) & \quad v(264+n_1) = v(263+n_1) + v_4 & n_1 & = n_1 + 1 \\
\quad \Rightarrow & \quad n_1 & = 34 \\
\text{\texttt{v263}} & = v_5 \\
(2) & \quad v(298+n_1) = v(263+n_1) + v_5 & n_1 & = n_1 + 1 \\
\quad \Rightarrow & \quad n_1 & = 35 \\
n_2 & = 0 \\
(3) & \quad v_1 = 0 & n_0 & = 0 \\
n_1 & = 397 & v_{497} & = 35 \\
v_{498} & = 91 & v_{499} & = 0 \\
v_{500} & = 0.5 & v_0 & = v(298+n_2) \\
\Rightarrow & \quad \texttt{SR20} & v(333+n_2) & = v_1 \\
n_2 & = n_2 + 1 & \quad \Rightarrow & \quad n_2 & = 35 \\
n_1 & = 0 \\
(4) & \quad v(368+n_1) = v(333+n_2) - v(263+n_1) & n_1 & = n_1 + 1 \\
v(433+n_1) & = 1/v(368+n_1) & \quad \Rightarrow & \quad n_1 & = 35 \\
n_1 & = 0 \\
v_5 & = 0 \\
(5) & \quad v_5 = v_5 + v(434+n_1) & n_1 & = n_1 + 2 \\
\quad \Rightarrow & \quad n_1 & = 34 \\
n_1 & = 0 \\
v_6 & = 0 \\
(6) & \quad v_6 = v_6 + v(435+n_1) & n_1 & = n_1 + 2 \\
\quad \Rightarrow & \quad n_1 & = 32
\end{align*}
<table>
<thead>
<tr>
<th>1ST. TERM</th>
<th>2ND. TERM</th>
<th>3RD. TERM</th>
<th>4TH. TERM</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRINT v8, 31 46</td>
<td>PRINT v15, 41 26</td>
<td>PRINT v700, 41 26</td>
<td>PRINT v69, 41 46</td>
</tr>
<tr>
<td>v) v700 + v619</td>
<td>v702 - v15</td>
<td>v703 = v702 / v701</td>
<td></td>
</tr>
</tbody>
</table>

**TEXT**

BACK MIXING COEFFICIENT

PRINT v703, 4608

n3 = n3 + i

-1, n3 ≥ 100

(=0)
AN EXAMPLE ILLUSTRATING THE COMPUTATION OF MEAN RESIDENCE TIME

1. OPERATING CONDITION
   O.R.C. 100 OSC/MIN, 45 L/HR TWO PHASES CONSTANT FLOW RATE RATIO RUN

2. DATA

<table>
<thead>
<tr>
<th>T(SEC)</th>
<th>X</th>
<th>T(SEC)</th>
<th>X</th>
<th>T(SEC)</th>
<th>X</th>
<th>T(SEC)</th>
<th>X</th>
<th>T(SEC)</th>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>0000</td>
<td>00.00</td>
<td>02.00</td>
<td>27.90</td>
<td>04.40</td>
<td>19.70</td>
<td>06.60</td>
<td>06.90</td>
<td>08.80</td>
<td>02.80</td>
</tr>
<tr>
<td>0010</td>
<td>00.00</td>
<td>02.30</td>
<td>29.10</td>
<td>04.50</td>
<td>18.90</td>
<td>06.70</td>
<td>06.55</td>
<td>08.90</td>
<td>02.60</td>
</tr>
<tr>
<td>0020</td>
<td>00.00</td>
<td>02.40</td>
<td>30.00</td>
<td>04.60</td>
<td>18.20</td>
<td>06.80</td>
<td>06.30</td>
<td>09.00</td>
<td>02.40</td>
</tr>
<tr>
<td>0030</td>
<td>00.00</td>
<td>02.50</td>
<td>30.80</td>
<td>04.70</td>
<td>17.50</td>
<td>06.90</td>
<td>06.05</td>
<td>09.20</td>
<td>02.30</td>
</tr>
<tr>
<td>0040</td>
<td>00.00</td>
<td>02.60</td>
<td>31.30</td>
<td>04.80</td>
<td>16.80</td>
<td>07.00</td>
<td>05.90</td>
<td>09.30</td>
<td>02.21</td>
</tr>
<tr>
<td>0050</td>
<td>00.00</td>
<td>02.70</td>
<td>31.50</td>
<td>04.90</td>
<td>16.20</td>
<td>07.10</td>
<td>05.80</td>
<td>09.30</td>
<td>01.99</td>
</tr>
<tr>
<td>0060</td>
<td>00.10</td>
<td>02.80</td>
<td>31.50</td>
<td>05.00</td>
<td>15.60</td>
<td>07.30</td>
<td>05.70</td>
<td>09.40</td>
<td>01.71</td>
</tr>
<tr>
<td>0070</td>
<td>00.50</td>
<td>02.90</td>
<td>31.40</td>
<td>05.10</td>
<td>15.00</td>
<td>07.30</td>
<td>05.60</td>
<td>09.50</td>
<td>01.55</td>
</tr>
<tr>
<td>0080</td>
<td>01.00</td>
<td>03.00</td>
<td>31.10</td>
<td>05.20</td>
<td>14.25</td>
<td>07.40</td>
<td>05.50</td>
<td>09.60</td>
<td>01.33</td>
</tr>
<tr>
<td>0090</td>
<td>01.90</td>
<td>03.10</td>
<td>30.50</td>
<td>05.30</td>
<td>13.60</td>
<td>07.50</td>
<td>05.35</td>
<td>09.70</td>
<td>01.24</td>
</tr>
<tr>
<td>0100</td>
<td>03.10</td>
<td>03.20</td>
<td>30.05</td>
<td>05.40</td>
<td>13.70</td>
<td>07.60</td>
<td>05.40</td>
<td>09.80</td>
<td>01.05</td>
</tr>
<tr>
<td>0110</td>
<td>04.80</td>
<td>03.30</td>
<td>29.20</td>
<td>05.50</td>
<td>13.00</td>
<td>07.70</td>
<td>04.80</td>
<td>09.90</td>
<td>01.05</td>
</tr>
<tr>
<td>0120</td>
<td>06.70</td>
<td>03.40</td>
<td>28.40</td>
<td>05.60</td>
<td>12.45</td>
<td>07.80</td>
<td>04.55</td>
<td>10.00</td>
<td>00.99</td>
</tr>
<tr>
<td>0130</td>
<td>08.80</td>
<td>03.50</td>
<td>27.60</td>
<td>05.70</td>
<td>11.90</td>
<td>07.90</td>
<td>04.30</td>
<td>10.10</td>
<td>00.77</td>
</tr>
<tr>
<td>0140</td>
<td>11.35</td>
<td>03.60</td>
<td>26.80</td>
<td>05.80</td>
<td>10.40</td>
<td>08.00</td>
<td>04.15</td>
<td>10.20</td>
<td>00.60</td>
</tr>
<tr>
<td>0150</td>
<td>13.80</td>
<td>03.70</td>
<td>25.90</td>
<td>05.90</td>
<td>09.90</td>
<td>08.10</td>
<td>04.00</td>
<td>10.30</td>
<td>00.55</td>
</tr>
<tr>
<td>0160</td>
<td>16.30</td>
<td>03.80</td>
<td>25.10</td>
<td>06.00</td>
<td>09.35</td>
<td>08.20</td>
<td>03.90</td>
<td>10.40</td>
<td>00.33</td>
</tr>
<tr>
<td>0170</td>
<td>18.70</td>
<td>03.90</td>
<td>24.20</td>
<td>06.10</td>
<td>08.90</td>
<td>08.30</td>
<td>03.70</td>
<td>10.50</td>
<td>00.29</td>
</tr>
<tr>
<td>0180</td>
<td>21.00</td>
<td>04.00</td>
<td>23.30</td>
<td>06.20</td>
<td>08.50</td>
<td>08.40</td>
<td>03.50</td>
<td>10.60</td>
<td>00.11</td>
</tr>
<tr>
<td>0190</td>
<td>23.00</td>
<td>04.10</td>
<td>22.40</td>
<td>06.30</td>
<td>08.00</td>
<td>08.50</td>
<td>03.40</td>
<td>10.70</td>
<td>00.12</td>
</tr>
<tr>
<td>0200</td>
<td>24.90</td>
<td>04.20</td>
<td>21.50</td>
<td>06.40</td>
<td>07.60</td>
<td>08.60</td>
<td>03.20</td>
<td>10.80</td>
<td>00.06</td>
</tr>
<tr>
<td>0210</td>
<td>26.50</td>
<td>04.30</td>
<td>20.60</td>
<td>06.50</td>
<td>07.25</td>
<td>08.70</td>
<td>03.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. COMPUTING
   SUMMATION X = 1208.35
   SUMMATION XXT = 478706.00
   SUMMATION XXT² = 33264230.00

4. RESULTS
   MEAN RESIDENCE TIME = 396.165
   VARIANCE = 0.324731
   DISPERSION NUMBER = 0.084082
   D = 0.003548

A 5.21
MEAN RESIDENCE TIME, VARIANCE, DISPERSION CALCULATIONS.

\[ \text{MEAN RESIDENCE TIME} \]
\[ v_9 = \frac{v_7}{v_5} \]
PRINT \( v_9, 4166 \)
TEXT

\[ \text{VARIANCE OF C-CURVE} \]
\[ v_{10} = \frac{v_8}{v_5} \]
\[ v_{11} = v_9 \times v_9 \]
\[ v_{10} = v_{10} - v_{11} \]
\[ v_{10} = v_{10} / v_9 \]
PRINT \( v_{10}, 4166 \)
\[ v_{15} = 3600 \]
\[ v_{15} = 28.32 \times v_{15} \]
\[ v_{16} = 2.75 \]
\[ v_{16} = v_{16} / 12 \]
\[ v_{16} = v_{16} \times v_{16} \]
\[ v_{16} = v_{16} / 28 \]
\[ v_{16} = v_{16} \times 22 \]
\[ v_{16} = v_{16} \times v_{15} \]
\[ v_{17} = v_{3} / v_{16} \]

TEXT
FLOW RATE =
PRINT \[ v_{17}, 4226 \]

TEXT

DISPERSION NUMBER

\[ v_{18} = v_{10} / 2 \]
\[ v_{19} = 8 \times v_{10} \]
\[ v_{19} = v_{19} + 1 \]
\[ v_{19} = \sqrt{v_{19}} \]
\[ v_{19} = v_{19} - 1 \]
\[ v_{20} = v_{19} / 8 \]
PRINT \[ v_{20}, 4226 \]
\[ v_{21} = 34 \]
\[ v_{21} = v_{21} / 12 \]
\[ v_{22} = v_{21} \times v_{20} \]
\[ v_{22} = v_{22} \times v_{17} \]

TEXT
D- VALUE

PRINT \[ v_{22}, 3166 \]
\[ n_{II} = n_{II} + 1 \]
\[ \rightarrow n_{II}, n_{II} = 100 \]

\[ \text{(-o)} \]
\[ \text{xxxxx} \]
AN EXAMPLE OF EXAMINING THE VALIDITY OF EQUATION
BY USING SOLATRION "274" ANALOG SYSTEM COMPUTER.

Rearranging the Eqn. (24) as

\[ E \frac{d^2y}{dz^2} + V_y \frac{dy}{dz} + Ka_y - Ka_{yd} = 0 \]

Or

\[ E \frac{d^2y}{dz^2} = Ka_y dz - Ka_y - V_y \frac{dy}{dz} \]

The column height "z" will be treated as independent variable

time t of the computer. The general diagram for Eqn. (24) is

shown as Fig. A6.1.

Fig. A6.1
Let $\alpha_1, \alpha_2$ and $\alpha_3$ be scale factors for $y$, $y'$ and $y''$, then we have the normalised scale equation as:

$$\frac{1}{\alpha_3} E(\alpha_3 y'') + \frac{V_y}{\alpha_2} (\alpha_2 y') + \frac{K_3}{\alpha_1} (\alpha_1 y) = K_3 y_e$$

Or

$$E(\alpha_3 y'') + \frac{\alpha_3}{\alpha_2} V_y (\alpha_2 y') + \frac{\alpha_3}{\alpha_1} K_3 (\alpha_1 y) = K_3 y_e \alpha_3$$

Now the computer diagram for Eqn. (24) will be:

![Computer Diagram](image-url)

Fig. A6.2
Both $K_a$ and $y_e$ are functions of $z$ and are approximated as straight lines and are generated as ramps. The boundary conditions at $z=0$ are:

$$y(0) = y'(0) = y''(0) = 0$$

Hence there are no initial conditions on the amplifiers. The final unscaled computer diagram for the problem is:

![Diagram](image)

where - Mult is multiplier, $P_1, P_2, \ldots, P_7$ are pot.

**Fig. A6.3**

A. 6.3
2. Calculation procedure.

\( y_T \), the maximum value of \( y \) was used as guide for computing. The maximum values of \( y' \) and \( y'' \) were estimated. Some preliminary values for potentiometers obtained and computer diagram scaled. This model was then put on the computer and final pot setting and scaling done after preliminary run.

During computation of final scaled model value of \( y \) was monitored. When \( y \) reached the given maximum value, the computation was stopped. \( y, y' \) and \( y'' \) were plotted on the function plotter. The plotter arm was driven by the ramp from the computer and the ramp was so adjusted that plotter moved 10" during the computing time of the problem, thus 10" on the x axis of the graph corresponded to the height of the extracting column.

Variation in the \( E \) value (from 10% to -10%) was achieved by varying the input pots of the summer, keeping other pots as they were. In computer increasing the input resistance to summer effectively decreased \( E \) value and vice versa.

3. Examples of computing.

O.B.C., 264 osc/min. 11.7 l/hr. was used as the example in computing.

A data.

\[ \begin{align*}
V_y &= 16.71/\text{hr.} = 0.004 \text{ ft/sec} \\
X_B &= 0.01 \text{ g/l} \\
E &= 0.0006 \text{ft/sec}^2
\end{align*} \]
The value varies from 0.0 to 230 for \( z \) from 0.0 to 2.5 ft. the slope of the line equals to \( \frac{230}{2.5} \), or 92.0, thus we have \( y_e = 92.0z \).

Ka value varies from 0.0 to \( 30 \times 10^{-5} \), for \( z \) from 0.0 to 2.5 ft. thus, we have \( Ka = 12 \times 10^{-5}z \).

Now the equation becomes:

\[
0.0006y'' + 0.004y' - 2 \times 10^{-5}z (92.0z - y) = 0
\]

Or, \( y'' + 6.666y' - 0.2z(92.0z - y) = 0 \)

The new slope of \( Ka \) vs. \( z \) is \( 0.2z \).

B. Scaling.

Scaling is used to estimate maximum value of variables \( y, y'' \) and \( y' \) and calculates scale factors \( \alpha_1, \alpha_2, \text{and } \alpha_3 \), so that the variables represented by voltages in analog computer and near 100 volts to make the computer work at the maximum efficiency and accuracy.

The \( y_{\text{max}} = 18.75 \), say 20.

\( \omega_n \) = The natural frequency of the process.

\( \omega_n = 0.2 = 0.445 \), say 0.5
Estimated \( y_1'_{\text{max}} \) = \( \frac{W_0 \times y_{\text{max}}}{2} \) = 0.5 \times 20/2 = 5.0 say 10

Estimated \( y_2''_{\text{max}} \) = \( \frac{W_0^2 \times y_{\text{max}}}{2} \) = \( \frac{20 \times 0.5^2}{2} \) = 2.5 say 5.0

\((K_a \cdot y_e)_{\text{max}}\) = \( \frac{(230 \times 30 \times 10^{-5})}{0.0006} = 115\) say 200.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Est. Max. Value</th>
<th>Scale</th>
<th>Scale Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>20</td>
<td>5v/ft.</td>
<td>5</td>
</tr>
<tr>
<td>( y' )</td>
<td>10</td>
<td>10v/ft/sec.</td>
<td>10</td>
</tr>
<tr>
<td>( y'' )</td>
<td>5</td>
<td>20v/ft/sec²</td>
<td>20</td>
</tr>
<tr>
<td>( K_a \cdot y_e )</td>
<td>200</td>
<td>( \frac{1}{2} ) v/ft.</td>
<td>( \frac{1}{2} )</td>
</tr>
</tbody>
</table>

\( \frac{\Delta L_2}{2} = 5/10 = 0.5 \quad \frac{\Delta L_3}{2} = 10/20 = 0.5 \)

\( \frac{\Delta L_3}{2} \times V_y = (20 \times 8.66)/10 = 17.32 \)

\( \frac{\Delta L_3}{2} = 20/5 = 4.0 \quad \frac{\Delta L_3}{4} = 20/10 = 40 \)

**C. Generation of ramps:**

Since \( y_e \) ramp is much greater than \( K_a \) ramp, ramp \( y_e \) is divided by 10 and \( K_a \) ramp multiplied by 10. The whole equation is divided by 100 to take off the factor of 100 introduced due to multiplier (multiplier gives output \( \frac{v_1 \cdot v_2}{100} \)).
The computer results showed that the terms $y''$ and $y'$ were bigger than $y$ and over 100 volts, hence the problem was rescaled.

As the problem took few seconds to complete in order to be able to use x-y plotter for recording variable, the problem had to be slowed down (problem time slowed i.e. 15-45 seconds for x-y recorder available). The problem was slowed down 2 times by halving the gain on integrator (i.e. halving the potentiometer).

**D. New scaling and final computing diagram.**

\[
\begin{align*}
\lambda_1 &= 5; \quad \lambda_2 = 5; \quad \lambda_3 = 10; \quad \lambda_4 = \frac{1}{2} \\
\frac{\lambda_2}{\lambda_3} &= \frac{5}{10} = 0.5; \quad \frac{\lambda_1}{\lambda_2} = \frac{5}{5} = 1; \quad V_y \cdot \frac{\lambda_3}{\lambda_2} = 6.66 \times \frac{10}{5} = 13.32 \\
\lambda_3 &= 10/2 = 20; \quad \frac{\lambda_3}{\lambda_1} = \frac{10}{5} = 2
\end{align*}
\]
Fig. 46.5 Concentration profile with variation of $E_y$
for OBG, 264 ccs/min, 16.7 l/m constant flowrate ratio run

- $E_y$ 10% decreased
- $E_y = 0.0006 \text{ ft/s}^2$
- $E_y$ 10% increased

$y$ cm/l

2 ft (from the bottom of the column)
Fig. 46.1 Examination the conc. profile by using Solatron "247" System Analog Computer

--- Conc. profile from experimental data

--- Conc. profile from differential Eqn.

by using the Computer