Integrated Structural and Electromagnetic Optimisation of Large Terrestrial and Space Antenna Structures

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Abstract

In this study, a novel multi-parameter overall situation optimisation method and mathematical model has been developed for use with terrestrial and space reflector antenna electro-mechanical systems and other metallic and polymer composite civil engineering structures. To satisfy extremely high design requirements, the proposed approach incorporates the objectives from various structural and electromagnetic (EM) performances of the system such as structural frequency, weight, stiffness, strength, reflector surface accuracy, antenna EM efficiency (gain), and radiation patterns at many working/loading cases simultaneously. The optimisation involves geometric and material design variables, and integrated design of composites and structural systems. Various terrestrial, launch and orbital working environments and loading cases which affect antenna performances have been included in the optimisation. These involve self-weight at different elevation attitudes, wind loading, random/dynamic loads and temperature distributions. Both truss and sandwich parabolic reflector panels with honeycomb core and carbon fibre laminate skins stiffened with composite ribs have been optimised.

The effects of structural deformation on antenna EM performances have been investigated, modelled and repeatedly analysed in the iterative optimum-seeking procedure. Optical ray tracing, spline function aperture field interpolation, geometric optics aperture integration, Zernike modes analysis and FFT techniques have been used to analyse the EM performances of distorted reflector antennas.

An important aspect of the work was the establishment of evaluation criteria in optimising engineering systems. A new method is presented, which can be used as a design review tool to assess the design quality of engineering systems. This systematic method quantitatively evaluates a design from multi-discipline and numerous points of view simultaneously for Pareto optimisation.

A general purpose optimisation program MOST (Multifactor Optimisation of Structures Technique) has been developed to implement the proposed approach. MOST has the ability to utilise ABAQUS as an analysis routine for linear and non-linear, static and dynamic structural analysis in the optimisation procedures. Examples are presented to demonstrate the capabilities of the optimisation methodology and MOST program system. These examples are: an 8m Cassegrain antenna system, a 3.6×2.6m composite space deployable reflector antenna structure, and two 4m low side-lobe off-set antenna systems (with composite structures). The optimisation results for these antennas show that the optimisation procedures succeed in that at all the working/loading cases the antenna performances have been greatly improved.
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To my family
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Chapter 1

Introduction

Reflector antennas have been used extensively in satellite communication, space exploration, earth observation, remote sensing and radar techniques. The tendency in today’s antenna technology has made antenna structural design increasingly become a critical key issue, because structural distortions can seriously affect the electromagnetic (EM) performances due to undesirable phase shift of the microwave energy. The requirements for extremely precise, large aperture size, light weight, high structural frequency, high EM efficiency, low sidelobe level and high reliability, large ground and space antenna systems have made it necessary to utilise carbon fibre reinforced plastic materials and to optimise the structural systems.

A rapidly increasing number of large aperture antennas are being used and designed, particularly at microwave frequencies, to meet a wide range of applications in both terrestrial and orbital environments. Because of the requirements for large size and extreme accuracy of these instruments, studies have shown that materials and structures drive the design of these systems.

A reflector can be large, but it is a precise and complicated structure and its design involves electromagnetics, microwave and antenna theories, structural mechanics, mechanical engineering, material science, and computational mathematics.

The performances of antennas are strongly related to the surface accuracy. Actual reflector surface undergoes significant changes with respect to its ideal surface, because of the elastic deformations due to the applied static and dynamic loads, and environmental conditions.

There are two main trends influencing the design of modern antenna reflectors, the first is an increase in the diameter of the reflector D, which is required for the purpose of
increasing the antenna gain $G$. This can be approximately expressed as a proportion
relation (Vorobei and Voitkov, 1990):

$$G \propto D^2/\lambda^2$$

where $\lambda$ is the wavelength of the EM radiation. The second trend is a reduction in the
standard deviation $\delta$ (RMS) of the reflecting surface from the theoretical form due to
external factors and manufacturing errors. Here, the ratio $\delta/D$ is very small, it reaches
about $10^{-5}$ to $10^{-6}$. Having these values for the ratio of the error of the functional
dimension to the maximum dimension is characteristic of the design of reflector antenna
structures.

Future needs will call for space antennas from 10 to approximately 500 m in diameter.
Values of $D/\lambda$ in the range 100 to 1000 are typical of communications antennas such as
the land mobile satellite. The microwave radiometer spacecraft has a $D/\lambda$ of
approximately $1 \times 10^3$ and the infrared wavelength large deployable reflector concept has
a $D/\lambda$ of about $1 \times 10^6$, (Greene, 1985). Size, surface accuracy, structural flexibility, the
effects of structural deformation on EM performances, and the structural interactions
with the control system present design challenges across the frequency spectrum, from
radio to near-infrared and optical frequencies similar to that of the Hubble Space
Telescope, but with a diameter five times as great, (Hearth and Boyer, 1985).

The trend of ground and space antenna applications is toward the utilisation of higher
and higher radio frequencies (millimetre and submillimetre wavelengths) where
efficiency loss and sidelobe degradation due to structural distortions can severely
penalise antenna performance. Deviation of a reflector surface from that of an ideal
paraboloid results in degradation of its radiation pattern. Design and analysis of large
precision antenna structures is a challenging task requiring simultaneous considerations
on surface and pointing accuracy, dynamics, mass, hygrothermal effects, composite
materials and eventually deployment kinematics. For ground antennas, performance
requirements are based on the need to maintain the accuracy of the antenna surface
profile and that of the subreflector position while subject to wind, temperature and
gravitational loads in any azimuth and elevation position; for space antennas, in addition
to the surface accuracy, severe stiffness, thermal and dynamic stability, weight,
packaging and launch requirements, are the major drivers of antenna structures
technology. The design can be solved by the development and application of strongly
integrated design and optimisation routines. Apart from this, thorough modelling on
structural, sandwich and material level also based on concise data bases is required to obtain sufficiently accurate distortion results. Large reflectors are very costly to build and test, and even ground testing is impossible to perform. Therefore, detailed analyses are required and these include prediction of structural deformations and their effects on EM performances where the strength of the signal depends on the deflected shape of the reflector. The deformations are introduced by various loading cases, for instance the cyclic heating on a space antenna structure during orbiting, as well as the different temperature distributions and extreme temperatures for different seasons such as the equinox and the solstice of winter and summer and at different times of day. Computational methods must be heavily relied upon during the design of reflector systems, but clearly, the computations must be accurate and reliable.

Difficulties arise, in many cases, in the design and manufacture of antenna reflectors, due to the extremely high severity of the surface accuracy and stiffness requirements. The surface-accuracy effects become more pronounced as the electromagnetic (EM) operating frequency increases. The higher the radio frequency of operation used, the more accurate the surface must be. The generally accepted rule-of-thumb figure is that the permissible root mean square (RMS) deviation is \( \lambda /32 \sim \lambda /16 \) (Blake, 1966) or later \( \lambda /100 \sim \lambda /20 \) (Greene, 1985), where \( \lambda \) is a wavelength of RF (Radio Frequency) to be used. Antennas working at frequencies between 200 ~ 300 GHz (wavelength between 1.5 ~ 1 mm) need a reflector surface accuracy of smaller than 0.05 mm RMS; if the working frequency range is 300 ~ 2000 GHz, the distortions in the reflector surface are required to be less than 0.008 ~ 0.01 mm RMS (Olver, et al., 1989). An example of a high precision ground reflector was the 10-m-diam Submillimetre Telescope which required surface accuracy of better than 0.025 mm RMS and a pointing precision of < 1 second under operational conditions, (Baars, 1992). For a space antenna to operate a 100m aperture at a frequency of 94 GHz, a simplified general requirement was that the RMS error of the aperture be maintained below 1/32 of a wavelength, i.e. RMS < 0.1 mm (at 94 GHz the wavelength is 3.2 mm), (Staubs, Chadwick and Woods, 1979). This allowable error must accommodate all manufacturing, assembly, environmental operations and material property variations including shape changes. Another example was a 20-m-diam space antenna, discussed by Anon, (1986), which was required for deep space measurements of infrared radio frequencies. This application required that the deflections in the reflector surface be no greater than 0.002 mm RMS (Mikulas and

In addition to structural performance considerations, the electromagnetic (EM) performances of antennas, which are directly related to the reflector surface accuracy, must be considered in the design of antenna structures. An example which shows that a distorted reflector surface degrades the EM performance is given in (Grantham, 1986).

The increasing demand for large precise antenna structures has give rise to active research in this field in the UK and overseas (NASA, ESA, Matra-Marconi, UK BAe and GEC-Marconi). The utilisation of a best fit paraboloid concept (Horne, and Barrett, 1969), optimum rigging angle method (Levy, 1971) and homologous design idea (Von Hoener, 1967) has theoretically made it possible to greatly reduce the difference between the deformed reflector surfaces and their ideal surfaces. The Jet Propulsion Laboratory in the USA is developing expensive adaptive structures (Wada, 1990) which use actuators and active structural members connected with control systems in response to internal or external stimulation. The action of these actuators is to lengthen and shorten the selected individual elements to which they are attached to actively modify geometric and physical structural characteristics to meet mission requirements. A research goal of NASA's ongoing research activities is to achieve surface accuracy for large segmented reflectors of the order of 0.01 ~ 0.02 mm RMS passively (Mikulas, 1991).

Science missions that require the structures with very strict accuracy tolerances have made extensive use of composite materials in backup structures and surface panels. Because of their unique combination of high specific stiffness and strength, good dimensional stability and high specific damping capacity, high-modulus graphite reinforced epoxies combined with a coefficient of thermal expansion close to zero make them particularly attractive for high-precise antenna structures for both space (Freeland, 1985; Mikulas and Collins, 1991) and ground (Baars, 1992) applications. The analysis, optimisation and fabrication of composite reflector panels are the most critical aspects in some antenna structural designs (Abt, 1989; and Cirese, 1990).

The analysis and design of composite structures always involves an original approach to the matter. This is not only due to the intrinsic character of the tailorability of the
materials, but also to the lack of wide and well established experience of design and data collection. In order to realise full benefits from the capacity of composite materials, integrated methodologies, as described by Saravanos and Chamis, (1990), for the optimal design of composite structures must be developed:

- multiple objectives to effectively represent the competing design requirement
- capability for tailoring the basic composite materials and/or laminates
- capability for shape optimisation
- design criteria based on the global static and dynamic response of the composite structure.

Civil engineering structures and in particular antenna panels would generally consist of a rib or a backing truss stiffened by means of a sandwich shell manufactured from composite laminates and metallised facesheet and honeycomb core. They should be analysed and optimised as either individual structural systems or as an element in a complete configuration. The laminates consist of stacked plies orientated in such a way as to give the laminate plates the desired effective mechanical, thermal-mechanical, and hygrothermal properties. In the optimisation procedure, classical laminated plate theory has been used to determine the plate behaviour resulting from the properties of its constituent plies and their orientations. The ply properties include various mechanical moduli, Poisson’s ratios, mechanical strengths, coefficient of thermal expansions, and hygrothermal coefficients.

Optimisation techniques provide designers and analysts with the potential to significantly improve designs and to reduce engineering time and costs. It is best described by Deming (1993) who says “Optimisation is the process of orchestrating the efforts of all components toward achievement of the stated aim. Optimisation is management’s job. Everybody wins with optimisation.” The history of structural optimisation dates back to the mid-1600s, when Galileo sought to find the optimal shape of a variable-depth beam (Morris, 1982). As structural analysis progressed, so did attempts to minimise weight. In the mid-1900s, minimum-weight structural design became more important with the development of aircraft. In the 1960s, the development of mathematical programming techniques as well as the finite element method made possible the emergence of structural optimisation as a potential design tool. The last three decades witnessed intensive research and development work in this field. Today,
structural optimisation is advancing rapidly in concert with advances in digital computing and numerical methods, however, it must also deal with the problems of multi-objective functions, constraints from multiple disciplines, and a large number of design variables. Optimisation methods are used on an assortment of problems, such as composite laminate design, vehicle shape design, and the ‘tuning’ of an analysis model to match test data (Flanigan, 1987; Cobb, Canfield and Liebst, 1996).

Since an antenna is a complicated electro-mechanical system, the design of antenna structures relates to two important areas, and these are:

a) structural performances, including structural frequency, weight, stiffness, strength, composite materials, hygrothermal effects and reflector surface accuracy,

b) EM performances, including antenna efficiency, pointing accuracy, radiation patterns, sidelobe level.

These structural and EM parameters should be considered simultaneously as objective functions or constraints in an optimisation design procedure. The design variables should be member sizes, node positions of the structures and the ‘tailorable variables’ of composite materials. The performance of structures vary according to the loads on the structures, viz., wind loading, temperature distributions and other random loads or dynamic loads, and in addition, for antennas, self-weight at different elevation angles must be considered. Therefore, antenna structural design is an electro-structural synthesis exercise of a multi-factor optimisation kind involving many design variables, objectives, constraints and loads.

The most important purpose of an antenna design is to obtain optimum EM performances and these depend largely on the details of the distortion throughout the reflector surface and the deformation of the sub-reflector/feed support structure. However, the difference between the specialist fields of study of EM and the structural design leads to a division of work along traditional EM and structural lines, i.e. structural design and EM performance are treated inappropriately as separate issues. Although some researchers are beginning to address control system optimisation and EM performance calculations in certain antenna structural design (Padula, et al., 1992; Grandhi, 1989; and Padula, et al., 1989), currently the surface root-mean-square (RMS) is still the only juncture of the structure and the EM design. However, RMS can only
be used to evaluate the gain-loss for a slightly distorted reflector by using Ruze's formulas (Ruze, 1966) based on a statistical concept, and the RMS is not a reliable indicator for other important performances for instance radiation beam shape, pointing accuracy and sidelobe level. Therefore, it is very important to incorporate EM performances in the analysis and optimisation of antenna structures.

Due to the complex nature of such an interdisciplinary multi-factor optimisation, in which the problems of multi-objective functions, constraints from multiple disciplines, a large number of design variables, and the combination of complicated loading cases must be dealt with, there is no unique optimisation method available. Many papers and books have been written on structural optimisation, but most of the methods use only one parameter such as the lowest material cost, lowest weight, fully stressed design or smallest displacement design as part or all of their objective function or design criterion in the selection of a structural arrangement. These optimisation criteria are not entirely applicable for the antenna structures. To date, only very limited structural performances and loading cases are taken into consideration in most optimisation procedures for antenna structures. These efforts are mostly limited to structural performances only instead of including EM performances. Moreover, in many cases, problems arise and the results obtained do not satisfy the various requirements of engineering practice, due to the limitation of the optimisation models. For instance, an optimised structure at a particular loading case may not be satisfactory for other loading cases, and a structure obtained by optimising a certain performance may be unsatisfactory when considering another performance. For instance, gravitational forces are usually important in reflectors that can be steered when they are pointing sometimes at the horizon and sometimes at the zenith or intermediate elevation angles. If a reflector shape is perfect at one of these angles, the loading at other angles may be such that it will deform greatly. These are the main outstanding problems to which this research tries to work out a solution.

The issues mentioned above are the main impetus of conducting this research. It is increasingly important, as here proposed, to investigate the effects of structural parameters and deformations on EM performances, and to develop a multi-factor optimisation model integrating various forms of structural and EM performances which are under various working environments and loading cases and, in addition, to find a satisfactory iteration solution procedure to the optimisation model.
The aims of this interdisciplinary research are to develop such an advanced structural optimisation method for antenna structures and other engineering structures, and to satisfy the ever more rigorous requirements of the construction and communications industry including space science and technology. This optimisation method, which is an extension of the conventional optimum design method, optimises a design from numerous points of view simultaneously. The procedure is based on improving structural performances (RMS surface error, mass, frequency, stiffness, strength) and also EM performances (antenna efficiency, gain, radiation pattern, and sidelobe level). This is unlike previous work, which is based on improving the RMS surface error only, thereby indirectly improving antenna EM performance. Based on a combination of functional requirements such as surface precision, area density, thermal/hygrothermal stability and workable state-of-the-art materials in practical hardware configurations, a spectrum of structural optimisations have been carried out, ranging from the optimisation of metal/CFRP backup structures to optimal tailoring of composite laminates of the panels.

The novel interdisciplinary multi-factor optimisation method (including mathematical model, algorithms and computer package) has been successfully developed and is available to optimise various antenna systems and other engineering structures and systems. Such an optimisation procedure would satisfy extremely high structural design requirements. The development of the systematic procedures for optimising the structures is a natural extension of the development of numerical analysis procedures. Since state equations in engineering design are usually multidisciplinary settings, this optimisation system is designed with new concepts based on the nature of engineering design which has considerably greater scope when compared with the analysis of an unique disciplinary state equation.

This research is significant because it addresses various parameters and their interrelations in the design of antenna structures with special emphasis on the effects of structural parameters on EM performances. The objective functions, constraints, design variables and loading cases in the optimisation model are derived from actual design specifications. These make the optimisation very practical. The optimisation method and computer program, as a general engineering optimisation tool developed in this research, can be used in the design of reflector antenna structures for terrestrial and
space applications and other engineering structures and systems, and its advanced nature and the potential for significant improvement of design will be illustrated.

The optimisations of whole backup structures, including their panels, have been carried out. Four different types of antennas have been satisfactorily optimised to illustrate the method and program. The optimisation for an 8m Cassegrain round paraboloid ground antenna structural-electromagnetic system considers 9 structural and electromagnetic objective functions, 20 structural design variables, and 7 loading cases simultaneously. Space thermal environment and both structural and material design variables are included in a 3.6×2.6m solid surface space deployable composite reflector antenna structural optimisation. Zernike mode analysis method (Searle and Humphrey, 1997), as a means of characterising highly correlated reflector surface distortions, has been utilised in the EM analysis of two 4m low side-lobe antennas with composite structures. All these optimisations are very successful. The optimisations change the structural design variables in such a way that the structural and EM performances approach the optimality criterion.

This thesis presents a description of the work and proposes, develops, and demonstrates an optimisation procedure for the design of antenna structural systems. The main feature that distinguishes this work from previous efforts is that the structures are optimised to satisfy explicit EM design requirements rather than implicit limits on RMS surface accuracy.
Chapter 2

The Characteristics and Requirements of Large Antenna Systems and Structures

2.1 Introduction

An antenna, which is an interface between a free-space electromagnetic wave and a guided wave, is a device for transmitting or receiving radio waves. There are many different types of antennas and many different variations on the basic types, but their mode of operation is essentially the same: a radio-frequency transmitter 'excites' electric currents in the conductive surface of the antenna and it radiates an electromagnetic wave. If the same antenna is used with a receiver, the converse process applies: an incident radio wave excites currents in the antenna surface which are conducted to the receiver. The ability of an antenna to work both ways as a receiver or a transmitter is termed the principle of reciprocity (Williamson, 1990).

A large reflector gives a narrower beam providing a higher gain (resolution) over a smaller area. This is an advantage for both receive and transmit. The surface of a reflector is defined typically to be a paraboloid. The function of the reflecting surface of an antenna during a receiving cycle is to collect microwave energy emanating from a distant source and redirect this to a focal collection point. A converse function is performed during a transmitting cycle. To perform successfully, the antenna must focus most of its radiated energy within a very narrow beam. This directivity gain is also a measure of sensitivity, in that signals received in the aperture are concentrated by a similar factor.

The antenna EM performance requirements lead to stringent requirements on the antenna surface shape. Deformations from the ideal geometry of the antenna structure include reflector surface distortions and relative motion between the feed and reflector (or reflectors). The deformations from the ideal surface are due both to surface...
inaccuracies resulting from manufacturing operations, such as residual displacement, and elastic deformations under applied service loads and environmental conditions, temperature and humidity, their sum must be lower than a very small fraction of the reflector’s characteristic length.

During operation cycles, deformations of the actual surface from an ideal paraboloid result in loss of efficiency (gain), a distorted beam pattern, and beam pointing errors, because of pathlength changes and undesirable phase shift of the RF energy. The deformations can also cause higher side lobe radiation levels (grating lobes) in isolated regions of the radiation pattern than those levels which would normally be expected. In fact, the closer the actual reflector’s geometry approaches the ideal, the better are the characteristics of microwave propagation. The undesired spreading of the beam and high sidelobe levels can result in false signals and increase susceptibility to jamming. In addition, overall attitude errors result in beam pointing error. A requirement exists also to limit the overall pointing error based on the translation and rotation of the substructures that form the antenna assembly. The beam itself must be pointed to within some fraction of a beamwidth of its intended direction (Lesieutre, 1985).

Deformations of the antenna profile are characterised in terms of RMS error between a best-fit paraboloid and the actual deformed surface. The RMS deviation of the surface is usually required to be less than some fraction of a wavelength. This surface accuracy has a first-order effect on the phasing of the reflected waves. How accurate the surface needs to be depends on its mission.

Constraints on eigenfrequencies are to be imposed in order to separate from the critical launcher and space craft-bus frequencies or from attitude control frequencies, the latter one being also relevant for terrestrial reflector antennas.

In general, the main structural constraints which drive the structural and mechanical design are: surface accuracy, mass, mass distribution, available envelope, RF transparency, approved materials, natural frequency requirements, stiffness requirements, strength requirements, geometric alignment and stability, safety, maximum operating temperature, thermal interface, mechanical interface, radiation resistance, mechanical environment, cost and schedule. It is rare to be able to satisfy the requirements dictated by the above constraints without some compromise.
The design requisites drive the materials, thermal, structural, and EM analytical models. The peculiar requirements of this kind of structures will be described in this chapter, particularly as regards the high precision antenna reflector.

2.2 The characteristics of antenna structures

2.2.1 Geometric parameters of antenna reflectors

Typical parabolic reflector geometry is illustrated in Figure 2.1 for axisymmetric and off-axis front-feed antenna configurations. The principal parameters in defining the reflector geometry are the focal length of the paraboloid, $f$, the reflector envelope such as the diameter, $D$, the location of the paraboloid focal point and the position of the principal axis of the paraboloid which passes through the focal point and the paraboloid vertex.

To function properly as an antenna reflector, the paraboloid must be properly positioned with respect to the antenna axis and the antenna feed phase centre. If not properly located, the antenna will not perform in an acceptable manner. The critical errors consist of linear deviation of the reflector focal point from the antenna feed location and angular deviation of the paraboloid principal axis relative to the antenna axis. This causes a net pointing error in the RF-axis and a defocus of the antenna system (Archer, 1979).

If a feed, which is located at the focal point $F$, is used to illustrate a reflector, see Figure 2.2, the most important requirements for the system are: (1) to collimate the secondary (reflected) rays, and (2) to keep

$$\overline{FA + AB} = \overline{FA_o + A_oB_o}.$$ 

In other words, any two rays, which are radiated from the feed at the same time and then reflected by the reflector, must reach the aperture plane in parallel fashion and simultaneously. Any difference of the ray path-length will cause the phase error of the RF waves, and in turn degrade the EM performance.
Let the point $A$ have co-ordinates $(x, y, z)$ and the point $F$ have co-ordinates $(0, 0, f)$. Then the requirement that $\overline{FA} + AB = \overline{FA_0} + A_0B_0$ becomes

$$\sqrt{(z-f)^2 + x^2 + y^2} + (a-z) = f + a$$

This reduces to

$$x^2 + y^2 = 4fz$$

which is the reflector surface equation of a paraboloid.

2.2.2 Ground antenna structures

Round paraboloid antennas are most extensively used in communication, radar and radio astronomy. The typical structural shape of the antennas is shown in Figure 2.3, which is a typical front feed reflector antenna. The antenna composes a central-located welded steel hub structure to which radial trusses are attached. It is well known that trusses possess high stiffness, and that well made trusses can provide a very precise framework for supporting reflective panel surfaces. Intercostal bracing members are positioned between the ribs to form a 3-dimensional space structure. Use is made of high-strength friction-grip bolts to eliminate connection slippage. The surface of the reflector is assembled from shaped panel assemblies to form a complete paraboloid. The panels are mounted on the supporting (backup) truss structure and checked for alignment relative to a best fit paraboloid, and then adjusted by means of their interface hardware with the support truss. The truss structure can be fabricated of metallic members with various cross-sections or thin-walled fibre-epoxy tubular members. Panel adjustment is provided via screwed connectors. Panel setting is by means of an optical template.

The antenna system is supported on a mount that provides for the excursion in elevation and also azimuth. The assembly is mounted on a pair of bearings with bearing-cell components attached to the rear face of the dish hub and the support fabrication. In order to aim at and follow the tracks of the target, the antenna pitch shaft can be turned by a servo-control mechanism which drives the pinion engaged with the big pitching gear wheel to change the angle of elevation over a 90 degree arc. Alternatively, the elevation motion can be provided via a recirculating ball leadscrew interposed between the central hub of the antenna and the elevation mount fabrication. The azimuth can be changed by pedestal which turn on the circular track. Also, the azimuth motion can be provided through a similar lead screw interposed between the top of the fixed support.
tube and the mount fabrication (Humphrey and Burrows, 1994). Generally, the antenna structure signifies the part with a pitching movement.

### 2.2.3 Space antenna structures

The most important present and near-future beneficial uses of space include communication, Earth observation, and space exploration. For these missions, one key element is the antenna.

Since a fairing payload envelope of a launching vehicle is not so large, major additional design considerations for space antennas are the mass and deployment of the structural systems. Both the mass and packaged volume of the antenna system are limited by the size and mass capabilities of launch and transport vehicles. E.g. ARIANE 5 has a maximum launchable diameter of 4.5 m (Olver, et al., 1989). This implies that large space antenna structures must be foldable and deployed after launch. Cargo bay stowing followed by in orbit deployment is a means of launching large, accurate structures into orbit. Hence, lightweight deployable antenna concepts have received considerable attention in the literature.

Many large space antennas have been developed for science applications. The proposed types of antenna structures are: deployable or erectable antennas, solid, mesh, membrane or inflatable surfaces, and truss, elastic rib or wire-system back-up structures, as described, for example, in (Campbell, et al., 1984; Freeland, Garcia and Iwamoto, 1984; Coyner, 1984; Furuya and Miura, 1991; Natori, et al., 1990; Fanning and Hollaway, 1993). Many concepts for large deployable antennas are possible by utilising some appropriate combinations of these elements, such as foldable rigid petals, mesh with deployable ribs, mesh on cable networks, a rigid surface with deployable truss, and an inflatable rigidising antenna. To prevent member buckling of a truss structure in space, members of the backup structures can be designed as thin-walled circular carbon-epoxy sections, and the sections can be filled with a very-low-density foam.

These antennas can be roughly classified into two groups: rigid-surface antennas and mesh surface antennas. Rigid-surface antennas can achieve high surface accuracy, but there are difficulties in their packaging method, because of the high package volume. On the other hand, mesh surface antennas have less accurate reflector surface than the
rigid surface type, because usually a mesh surface can not realise the positive curvatures in two directions (Natori, et al., 1990), but they can be packaged easily and compactly. Since the surface error should be smaller than about $0.02\lambda$ ($\lambda$ denotes wavelength) to obtain appropriate antenna gain, high-frequency antennas usually need rigid-surface reflectors, and relatively low-frequency antennas normally employ mesh surface reflectors (Mitsugi and Yasaka, 1990).

The ideal structural material for space antennas would have a zero thermal coefficient of expansion, high rigidity, and low specific weight. Hence, fibre matrix composite materials have been extensively used in the manufacture of space systems. Although high performance composites are expensive, the high launch cost of satellites, which is $2000-20000$ per kg (Tenney, Sykes and Bowles, 1985) or even $30000$ per kg (Russell, 1991) make a kilogram of mass saved worth much more than the extra cost in utilising composites.

It is anticipated that future space missions will involve antenna structures which are extremely large compared to those in use today, ranging in size from the state-of-the-art to kilometre size futuristic antennas. The size of the structural systems required for future missions and the gravitational and atmospheric effects will make it difficult, if not impossible, to test these systems on the ground to validate their service configuration (Wada and Garba, 1992). It is a major challenge to devise ways to accomplish these missions. A mission of extreme high complexity and resulting high cost dictates that the structures, which permit mission accomplishment, should be developed with high precision and efficiency. This will require that the structural mass transported to orbit should be reduced to a minimum (Bush and Heard, 1980).

### 2.3 Electromagnetic performance requirements

The most important property of an antenna is its radiation pattern. The radiation characteristics of an antenna as a function of direction are given by the antenna pattern. By the principle of reciprocity, this pattern is the same for both receiving and transmitting conditions in most circumstances.

This pattern is a graphical representation of the power radiated from the antenna in the different angular directions. The pattern may be expressed in terms of the field intensity (field pattern) or radiation intensity (power pattern). The latter is used most often.
typical antenna power pattern, which consists of a main lobe and a number of sidelobes, is shown in Figure 2.4 as a polar plot in linear units, and is shown in Figure 2.5 in rectangular co-ordinates on a dB scale. The latter presentation shows the minor-lobe structure in more detail. In most cases, the main lobe is the desired lobe and, thus, the sidelobes are the minor lobes. If the pattern is not symmetric, a three-dimensional diagram or contour plot will be required to show the pattern in its entirety. However, in practice, two patterns (one through the narrowest part of the lobe; the other, perpendicular to it through the widest part of the lobe) may suffice. These mutually perpendicular patterns through the main-lobe axis are called the principal-plane patterns and, based on their polarisations, sometimes are referred to as the E- and H-plane patterns.

An important parameter is sidelobe level, which is the ratio of the pattern value of a sidelobe peak to the pattern value of the main lobe peak and typically is expressed in dB. For example, in Figure 2.5, the first sidelobe level is at -24 dB. The power level of the sidelobes must be kept as low as possible for two main reasons. Firstly, they dissipate the output energy from the transmitter and secondly, they can be a major cause of noise and interference when the antenna is in its receiving mode.

Figure 2.4 also illustrate the half-power beamwidth (HPBW) which is another important numerical specification of the pattern. HPBW represents the angular width of the main lobe between the two points on the radiation pattern which are 3 dB below the main beam peak. The angle at the half-power level or HPBW is most commonly used and defined as

$$\text{HPBW} = |\theta_{\text{HP, left}}| + |\theta_{\text{HP, right}}|$$

(2.2)

where, $\theta_{\text{HP, left}}$ and $\theta_{\text{HP, right}}$ are angles at half-power levels of the main lobe down from the left and right to the peak, respectively (viz. from the vertical on the left and right sides of the peak value). The antenna HPBW can be different in different pattern cuts for asymmetric patterns.

The primary antenna performance requirements are related to maximum on-axis gain and antenna beam pointing. Antenna gain is a measure of the maximum signal intensity which is transmitted or received by the antenna from a standard source.
Imagine, for the purposes of illustration, that the space antenna is transmitting energy to a point on the earth's surface and that the principal objective is to maximise the amount of energy transmitted to the earth receptor. The energy radiated by a point source in space is evenly distributed over the surface of an imaginary sphere of radius $R$, where $R$ is the range from the satellite antenna to the receptor antenna on the earth's surface. The power received by the receptor antenna is defined by the expression

$$P_r = \frac{P_t}{4\pi R^2}$$

(2.3)

where $P_r$ is the power received in Watts/m$^2$; $P_t$ is the power transmitted in Watts; $R$ is the range separation in metre.

If now it is desired to increase the intensity of the energy density at the earth's surface, the satellite antenna may be provided with an increase in its gain. Gain is the power multiplication factor of an antenna which is the ratio of output power to input power of an amplifier. Gain is achieved in an antenna by controlling the distribution of energy in such a way that it lies in the most preferred direction and hopefully in no other. The advantage that accrues from this is illustrated in Figure 2.6. A narrower beam antenna of maximum gain ($G$) is now compared to the original case where the energy was distributed uniformly over the surface of a sphere. The uniform distribution case is the reference in antenna theory and has a reference gain value of unity ($G=1$). The gain achieved with an antenna is the improvement in a signal level, as compared to the reference case. Specifically, a gain of 2 indicates that the signal received on the face of the earth would be twice as high as that achieved if the energy from the satellite were uniformly distributed over the surface of a radiating sphere. For modern large precise antennas, the gains are required to be $1 \times 10^8$ to $1 \times 10^{10}$ (80 dB to 100 dB), where, gain is often expressed in decibels (dB), wherein $G$ in dB = $10 \log$ of the ratio (i.e. if $G =$ the ratio 2 as above, $G_{db}= 10 \log 2 = 3.0$ dB). The equation above may now simply be rewritten as

$$P_r = \frac{PG}{4\pi R^2}$$

(2.4)

This expression indicates that the amount of power received on the earth is controlled by the separation between the satellite antenna and the earth itself, as well as the gain.

The gain of the antenna may now be directly related to the physical properties of the antenna structure itself. For the moment, consider the case of a round aperture having a
diameter D. In this instance, the antenna gain is directly related to the diameter by the simple expression (Staubs, Chadwick and Woods, 1979)

$$G = \eta \left( \frac{\pi D^2}{\lambda} \right) = \eta \left( \frac{Df}{0.98} \right)^2$$

(2.5)

where, D is the diameter of reflector; \( \lambda \) is the wavelength; \( \eta \) is the antenna efficiency factor (product of all contributing efficiencies); and \( f \) is the operation frequency.

The above formula indicates that the gain of an antenna is directly proportional to the square of the reflector diameter and the square of increasing frequency. The efficiency factor (\( \eta \)) accounts for the normally encountered degrading factors in any antenna system, including, among many other effects, such as losses from surface accuracy, resistive, reflection, aperture distribution, and blockage.

Highly efficient systems have an efficiency of nearly 70 percent. Broad bandwidth systems have an efficiency of approximately 30 to 40 percent; moderate bandwidth systems of conventional performance generally have efficiencies in the 50 to 55 percent region. By specific exclusion, this efficiency factor does not include the impact of structural distortions (Staubs, Chadwick and Woods, 1979).

The defocus error adversely affects gain and also contributes to the pointing error. The effect of defocus on pointing varies with the antenna configuration geometry and must be examined on an individual basis. The maximum antenna performance is obtained when the target is aligned with the maximum-gain point of the antenna. It is difficult, if not impossible, to maintain exact target tracking. Therefore, an acceptable pointing error is usually specified by degrees away from boresight; this determines the positioning requirements for a given antenna. The distortions of an antenna structure will cause pointing error, because of the misalignment between the antenna electrical and mechanical boresights.

Pointing requirements vary with the RF frequency at which the antenna is operated. The basic pointing requirement is related to the half-power beam width which varies with frequency and aperture, as per the following relation for a circular aperture with a parabolic tapered feed illumination (Archer, 1979).

$$\theta_H = 1.273 \frac{\lambda}{D} \text{ radians}$$

(2.6)

where \( \lambda \) is the wavelength and D is the diameter of the circular aperture.
Nominal pointing requirements to attain maximum gain may be one-quarter of the half-power beam width. This would result in less than 5 percent gain loss. Only a portion of this requirement may be allocated to reflector performance, however, resulting in a reflector pointing requirement of one-fifth to one-tenth of the half-power beam width. For example, pointing error < 10λ/D degrees (Archer, 1979).

2.4 Antenna structural accuracy requirements

The main effects of antenna structural inaccuracies are the reduction of available gain and beam isolation and the increase of total sidelobe power. Antenna structures have special deformation requirements. In addition to strength, stiffness and dynamic characteristics, the reflector surface accuracy is the most important performance criterion. A convenient measure of surface accuracy for design is the δ value which is the RMS of half the difference in pathlength of microwave energy beam in travelling from a deformed reflector surface to the focus compared with the pathlength from a surface that is a perfect paraboloid. The complete reflector surface is equivalent to an infinite set of points; in practice the surface is replaced by a finite set of ‘target’ points which are taken to be a representative sample of the entire set. The nodes on the reflector surface of the model used in the finite element analysis are taken as the ‘target’ points.

Based on the approximate geometric optics theory and statistical analysis, Ruze (Ruze, 1966) produced an expression which describes the relationship among the antenna gain, the shortest working wavelength and the root mean square (RMS) value of deformation of the reflector structure, i.e. the generally accepted Ruze formula:

\[
\eta_s = \frac{G}{G_0} = e^{-\frac{4\pi \delta}{\lambda}^2}
\]  

(2.7)

where \( \eta_s \) is gain (efficiency) reduction factor; \( G \) is the actual antenna gains with reflector surface distortions; \( G_0 \) is the antenna gain with a perfect reflector contour; \( \delta \) is the RMS of half the difference in pathlength of the microwave energy beam travelled considering all the computation points on the reflector surface; and \( \lambda \) is the wavelength. This relationship is plotted in Figure 2.7, where it is seen the detrimental effect of the RMS surface error is proportional to the RF frequency at which the antenna operates. From the formula, it can be seen that antenna’s efficiency will sharply decline with increasing RMS value. For example, when \( \delta = \lambda/30 \), \( \eta_s \) is 83.9 %; and when \( \delta = \lambda/16 \), \( \eta_s \) is
only 54.1% which means that because of the deformation, the reflector is only as good as a much smaller but perfect reflector with only half the reflection area. Nominally, the maximum acceptable gain reduction due to RMS surface error is 0.25 dB, which corresponds to an RMS value of 0.02 \( \lambda \) (Archer, 1979). Although the Ruze formula is a statistical expression, is approximate and is based on an assumption of random, uniformly distributed aperture errors with small correlation intervals, to date it remains the reference standard and the criterion used in design of many antennas. Accordingly, the minimisation of the mathematical expression for this half-pathlength difference \( (\delta) \) for loadings is one of the main design objectives. In Table 2.1 a few examples of in use ground antennas with high accuracy are listed (Safak, 1990).

<table>
<thead>
<tr>
<th>No.</th>
<th>Agency/location</th>
<th>Diameter(m)</th>
<th>RMS (mm)</th>
<th>Diameter/RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Cal tech, Owens Valley</td>
<td>10.4</td>
<td>0.025</td>
<td>416000</td>
</tr>
<tr>
<td>2</td>
<td>Univ. of Massachusetts</td>
<td>13.7</td>
<td>0.100</td>
<td>137000</td>
</tr>
<tr>
<td>3</td>
<td>ESSCO</td>
<td>20.1</td>
<td>0.150</td>
<td>134000</td>
</tr>
<tr>
<td>4</td>
<td>Crimean RT-22</td>
<td>22.0</td>
<td>0.120</td>
<td>183000</td>
</tr>
<tr>
<td>5</td>
<td>NRAO, Charlottesville</td>
<td>25.0</td>
<td>0.070</td>
<td>357000</td>
</tr>
<tr>
<td>6</td>
<td>IRAM, Grenoble, Spain</td>
<td>30.0</td>
<td>0.090</td>
<td>333000</td>
</tr>
<tr>
<td>7</td>
<td>Nobeyama</td>
<td>45.0</td>
<td>0.200</td>
<td>225000</td>
</tr>
<tr>
<td>8</td>
<td>Effelsberg*</td>
<td>60.0</td>
<td>0.400</td>
<td>150000</td>
</tr>
<tr>
<td>9</td>
<td>Effelsberg</td>
<td>100.</td>
<td>0.730</td>
<td>137000</td>
</tr>
</tbody>
</table>

* Inner 60m part of the 100m Effelsberg antenna

The cost effectiveness and quality of microwave transmission from GEO satellites have made the orbiting reflectors an attractive means of providing long distance communication services (Hollaway, et al., 1991). Communication, remote sensing, radiometric, and surveillance satellites increasingly require large space reflector antennas to satisfy future mission requirements. Studies of these missions indicate that common requirements for these reflectors are high surface accuracy and high stiffness (high natural frequencies) for controllability.

In the past, many large deployable space reflectors are typically designed for some applications to operate at low frequencies where large surface errors can be tolerated without significant performance loss. However, the trend of current and future
advanced space applications is toward the utilisation of higher (above Ku-Band) frequencies where gain loss and sidelobe degradation due to surface error can severely penalise antenna performance. To meet future requirements, the large deployable reflectors will be required to achieve very high on-orbit surface accuracy.

Some of the predicted needs are characterised in Figure 2.8 given by Hedgepeth, (1982). The missions shown in the figure are seen to involve diameter-to-wavelength ratios of up to more than 100,000, with the majority centred around a ratio of 1000. For those missions for which the main beam must contain almost all the radiated energy, the emitted wave front must be accurate to $\lambda/25$. These missions include all the Earth-directed antennas in which sidelobe gain must be kept very low. Even in the cases wherein the on-axis gain is of primary importance, the RMS errors in the wave front are held to less than $\lambda/8$. These missions include outward-pointed antennas for which the sidelobe gain can be relatively large.

In a reflector antenna, the wave-front error (phase error) is very nearly twice the component of structural distortion normal to the reflector surface. Thus the surface error of a reflector antenna must be held to one-fiftieth of a wavelength for the low-sidelobe missions and one-sixteenth of a wavelength for the high-gain missions (Hedgepeth, 1982). Combining the foregoing relationships with the data in the Figure 2.8. yields the requirement on structural surface accuracy. Submillimetre radio astronomy, for example, requires an accuracy of 1 ppm (part per million) of the diameter. Those Earthward-pointed missions which have a diameter wavelength ratio of around 1000 require a surface accuracy of 20 ppm. At the other end, low-frequency radio astronomy allows the surface error to be as much as one-thousandth of the diameter.

The achievement of such accurate reflectors requires the development of very high precision and thermally stable support structures as well as high precision and stable panels to form the reflector surface.

Large deployable antennas require high-accuracy lightweight reflectors in order to satisfy the EM characteristics for ensuring both gain and beam pointing accuracy. Several factors contribute to on-orbit surface error, including surface geometry.
approximations, fabrication and adjustment tolerances, thermal and dynamic distortion due to the orbital environment, gravity effects, and deployment repeatability.

Beam pointing errors can be caused by attitude control errors, thermally introduced asymmetrical deformations of the reflector, movement of the feed and reflector relative to each other. It can be assumed that most of the pointing error associated with the antenna will be caused by relative movement between feed and reflector (Foldes and Dienemann, 1980).

2.5 Antenna structural dynamic requirements

High inherent structural stiffness, which is also characterised by structural natural frequencies, is required for both terrestrial and space antennas. In order to separate structural and servo control system frequencies and minimise active control needed to suppress structural vibrations excited by slewing torques and other dynamic disturbances, antenna structures have dynamic requirements. High stiffness also reduces deflections and reflector surface contour errors due to gravity effects during manufacture, adjustment (Dyer and Dudeck, 1986).

When attitude, surface accuracy, or pointing requirements for some missions of terrestrial and space antennas imply control with substantial bandwidth, control system primary resonant frequencies should be kept away from several of the low-frequency modes of the antenna structure (Nurre, et al., 1984). Structural frequencies are also required to avoid dynamic coupling between two subsystem/component frequencies. The controls designer would prefer structural frequency to increase with the diameter of an antenna, however, this does not happen. Thus the controls designer confronts a significant coupling with the system structural dynamics. Any inherent non-rigid characteristic of the structure can result in multiple natural modes of flexibility with frequencies within the controller’s bandwidth, be it attitude or surface. A dynamic interaction between the structural behaviour and the closed-loop control response can result and, in turn, lead to instability and loss of control. The significant frequency modes are required to be well above the controller’s operating bandwidth or frequency response range and therefore do not cause an interaction problem. However, large antenna structures may produce significant structural modes that fall within the controller’s bandwidth and, in this case, the situation has to include expensive distributing sensors and actuators throughout the structure, by employing modal control.
of the structural vibrations induced by such activities as Shuttle docking and separation, orientation manoeuvres, and management of momentum exchange systems. Therefore, it would be necessary to design antenna structures with several major natural frequencies away from control system resonant frequencies, so as to ensure that the design of the control system is sufficiently robust to assure that substantial tolerances can be accommodated in the structural model.

For some antenna structures, vibration-control is required, but the difficulties of the vibration-control extend across the frequency spectrum, even to optical systems. The Charles Stark Draper Lab (Hearth and Boyer, 1985) found that the method of control is a function of three quantities, these are:

- the diameter of the antenna
- the lowest significant structural frequency of the antenna structure (including appendages such as booms for the antenna feed)
- the electromagnetic wavelength of the antenna system.

Should the ratio of these three parameters \((D/f\lambda)\) exceed \(10^4\), then active vibration control is probably required. Thus when antennas are of relatively large diameters, low structural frequencies, and short operating wavelengths, vibration control will likely be required. The antenna diameter trades directly with the structural frequencies.

The characteristics and requirements of large antenna structures and systems discussed in this chapter will be taken into consideration in the following chapters. To satisfy these requirements, detailed antenna structural and electromagnetic analyses will be undertaken.
Fig. 2.1 Paraboloid reflector geometry

Fig. 2.2 A paraboloidal reflector with a feed
Fig. 2.3 An antenna structural sketch

1. feed, 2. reflector, 3. backup structure, 4. pitch shaft,
5. big pitching gear wheel, 6. pinion, 7. pedestal, 8. circular track

Fig. 2.4 Normalised antenna pattern in polar co-ordinates and linear power scale
Fig. 2.5 Normalised antenna pattern in rectangular co-ordinates in dB power scale

Fig. 2.6 Receiver/transmitter geometry
Fig. 2.7 Gain loss versus reflector contour RMS

Fig. 2.8 Large space antenna requirements
Chapter 3

Antenna Structural Analyses

3.1 Introduction

The analysis of antenna structures has to be carried out by the utilisation of reliable numerical techniques. Normally finite element method and computer codes are used for the analysis and verification. For antenna structures, the surface accuracy analyses (through best fit paraboloid calculations) and modal analyses (to evaluate natural frequencies and mode shapes) are generally the most significant. These analyses can be performed at various levels of detail, as in the case of analyses for preliminary design orientation, optimisation iteration, or very detailed verification analyses. In every case, the validity of the results of the finite element calculations is affected by many factors, related to the loading, structure and process idealisation/mathematical model and reliable material data.

3.2 Antenna structural static analysis

The relation between the static load vector \( \{ p \} \) and the displacement vector \( \{ u \} \) for an antenna structure with \( n \) degrees of freedom is given by

\[
[ K ] \{ u \} = \{ P \}
\]

where \([ K ]\) is the \( n \times n \) symmetric stiffness matrix of the structure, which is a system-level representation of the materials, dimensions, and mutual interconnections of the structure. The vector \( \{ P \} \) includes mechanical forces and torques that are applied to the structure. For thermal loading, the \( \{ P \} \) is derived from the temperature changes experienced and is proportional to the coefficient of thermal expansion (CTE) of the structure materials. Having the temperature changes and knowing the thermoelastic properties of the structure, the thermal distortions and stresses can be readily obtained.

The most obvious way of solving (3.1) is to derive the 'flexibility' matrix of the structure,

\[
\{ F \} = [ K ]^{-1}
\]

(3.2)
and hence apply the trivial multiplication

$$\{u\} = [F] \{P\}$$  \hspace{1cm} (3.3)

However, this would be a very inefficient way of solving the problem, because matrix inversion is a very time consuming process. In addition, \([K]\) is often well-banded, while its inverse, \([F]\), is usually fully populated. The alternative approach of matrix division or factorisation allows us to maintain a storage area that is not much larger than that required for \([K]\). In particular, we need only use the area which the band or 'sky-line'. Resolutions, with different right-hand-sides (load vectors), are reasonably economical because the main work relates to the factorisation.

The solution of the system of linear equations, (3.1), is the most time consuming part. On smaller computers, the choice of the equation solver is often decided by its memory requirement since \([K]\) is a large and sparse matrix for large-scale structures. In finite element analysis, the frontal solver (Irons, 1970) based on the Gaussian elimination approach is still the most widely used method. The solution to equation (3.1) gives the displacements in the structure, and equations (3.2) and (3.3) can be used to determine the displacement at a specified node point. Stresses are obtained by further manipulations of the \(\{u\}\) vector at the individual finite element level.

In analysing large antenna structural performance it is necessary to accurately characterise the reflector surface points. Any deviation from its ideal geometry causes the antenna performance to degrade. The most important aspect of the antenna structural static analysis is the evaluation of the antenna's accuracy. It involves the calculations for best fit paraboloid and RMS of the surface errors. This analysis will be discussed in section 3.8.

3.3 Antenna structural loading

Antenna performance relies, to a great extent, on the correct operation of the antenna reflector, and that, in turn, depends on the surface accuracy. The reflector surface undergoes significant changes with respect to its ideal shape, because of the elastic deformations caused by the applied static and dynamic loads, and environmental conditions.

The kinds of loads applied to antenna systems and associated support structures vary in an extremely wide range, depending mainly on the type of application, i.e. earth based,
naival, airborne or in space. They can be static or quasi-static, like those due to self-weight, superficial loading effects resulting from snow and ice, or steady manoeuvres, but the most critical are generally of a random or dynamic nature, like those related to wind, inertia, vibration and shock. Furthermore, there are loads due to the interaction between structural and environmental conditions such as temperature and humidity gradients and thermal cycles (Cirese, 1990). For example, as described in (Steinbach and Winegar, 1985), a 106" space reflector is required to maintain a shape that is within 3 mils RMS of the ideal shape, while subjected to a dynamic thermal environment of +150 to -265 degrees Fahrenheit. Furthermore, the undeployed antenna must tolerate 9 Gs acceleration plus vibration loads, and weigh approximately 35 pounds.

When wind effects can be considered as quasi-static in the structural performance assessment, a terrestrial antenna structure exposed to the wind loading should be sufficiently stiff and respond statically. In these conditions the deflections and stresses throughout the structure are determined from the integrated wind pressures distributed over the structure. When the structure responds dynamically, it is believed that the spectrum of the turbulent wind contain sufficient energy at a structural resonance to overcome the inherent damping and to excite the structure dynamically.

A number of considerations arise in the analysis and design of a terrestrial antenna structure to sustain wind loading: (1) ensured accuracy, the maximum load (or wind speed) at which electrical performance is within the required specification, (2) reduced accuracy, the maximum load (or wind speed) for degraded but acceptable antenna performance, (3) structural survival, the maximum load (or wind speed) at which the structure will not fail or it will not be necessary to stow the antenna in the fully elevated position thus reducing drag and pitching moments.

Generally, the above three maximum wind speeds are in the ranges of 15-20 m/s, 21-30 m/s and 50-60 m/s respectively. Extreme terrestrial climatic conditions are generated by storm mechanisms including large-scale depressions, hurricanes, thunderstorms. The most common basis used for design is the 1 in 50 years return wind with a probability of exceedance of 0.02 in any one year (Humphrey and Burrows, 1994).
External thermal environments in space, such as solar heating, tend to produce temperature gradients that result in irregular distortions of the antenna surface. Accordingly, it is essential that the analyst be able to predict the temperature and distortion profiles and resultant antenna performance in order to establish a satisfactory thermo/structural design. It is necessary to know the temperature gradient through the structure when creating the ‘Temperature’ set cards for structural analysis software used.

The temperature distributions (orbital) are inputs from the thermal analysis. The structural thermal deformation also depends on the extreme temperature deviations in-orbit from the mean manufacturing temperature of the structure. Support constraints, gravity loading, mount loading, and other static or dynamic loading can be accounted for in the analysis. The temperature loading on space antenna structures will be discussed in the next section.

3.4 Space thermal environment and temperature loading

Antenna structures must remain accurate in the presence of environmental effects after they are established in space. The environmental service conditions, mainly temperature and humidity, affect the precision of the shape, the dimensional stability and may involve the degradation of mechanical properties. It is assumed that materials will be available with the necessary dimensional stability in the vacuum, UV, and particulate radiation environment that exists in orbit. Furthermore, it is assumed that redundant design will be used to resist the deleterious effects of meteoroids. Similarly, the influence of load-induced distortions and the uncertainty in such distortions can be kept to acceptable limits by proper design, but there remains the ubiquitous effects of thermal strains.

The surface shape of a space reflector will be distorted by the thermal gradients across the panel. These distortions will be a function of the method of construction of the panels and backing structure together with the influence of the incoming thermal radiation. The thermal distortions across the panel take on a complicated pattern due to a combination of the panel design and its backing structure.

The environmental heat sources applied to the space structure are solar heating, earth emitted heating and earth reflected solar heating. Earth emitted heating and earth
reflected solar heating depend on altitude and orientation of the structure. The total incident heating rate $q$ (per unit area) on the structure is given by

$$ q = q_\text{s} + q_\text{e} + q_\text{a} $$

where $q_\text{s}$, $q_\text{e}$ and $q_\text{a}$ are the incident solar heating, incident earth emitted heating and earth reflected solar heating rates, respectively.

To assure satisfactory performance of the orbiting antennas, detailed analyses of structural integrity and stability are required. These analyses include prediction of structural deformations introduced by cyclic heating on the structure during the orbit. Deformations must be kept within design allowable tolerances.

During orbit, structural deformations and thermal stresses are produced due to environmental heating. To perform the structural analysis, the structural temperature distribution is needed to compute the thermally equivalent nodal forces. The structural temperature can be computed if the environmental heating is known.

Temperature distributions over the antenna surface can be determined by the use of close-form mathematical equations to assess the solar heat input and numerical techniques to solve the resultant thermal balance equations. These equations include definitions of incident solar, albedo, and earth energy; radiation to space; and internode radiation and conduction as applicable to the entire antenna structure. Thermal effect of the support structure on the antenna can be simulated by constructing a boundary-value problem with specific nodes of the antenna connected to the boundary support structure. These connections may involve either/both radiation and conduction coupling.

A typical equation can be solved using finite difference technique for transient time marching and Newton-Raphson iteration at each time step. The temperature distribution of the structure may be determined at each time step for the entire orbit in this manner.

In addition to the temperature profiles over the antenna surface, calculations can be made to assess the temperature differential through the thickness of the antenna structure. Because of the relatively high thermal conductivity of the honeycomb structure in the normal direction, the temperature differential is very small. An example
has shown that the temperature differential was only 3°F on the average (Florio and Josloff, 1968).

For skeletal antenna structures, for example, once the heating rate on the structural member has been determined, the structure temperature distribution at different orbital position can be computed. Basic type of heat transfer for a typical space structure element are member conduction and surface radiation. For a structure made from composite materials such as graphite epoxy, member heat conduction is small due to the low material thermal conductivity. Thus for composite materials, the temperature is nearly uniform along the member length. With this assumption, the governing differential equation for a structural member is

\[ \rho c V \frac{dT}{dt} + \sigma a_s A_e T^4 = a_s A_q q(t) \]  

(3.5)

where \( \rho \) is density, \( c \) is specific heat, \( V \) is the member volume, \( \sigma \) is the Stefan-Boltzman constant, \( a_s \) is surface emissivity, \( A_e \) is the element radiation area, \( a_s \) is the surface absorbtivity, \( A_q \) is the incident heating area, and \( q(t) \) is the incident heating rate per unit area.

The above differential equation has been used (Thornton, Dechaumphai and Pandey, 1985) to formulate an isothermal finite element via the method of weighted residuals. With this concept, temperatures for each member can be computed independently.

If a low earth orbit (LEO) is not feasible because of the high thermal environment, a high earth orbit may be necessary which may reduce the thermal distortions to an acceptable level.

The antenna structure to be analysed later is assumed to be in a geosynchronous earth orbit (GEO), oriented as shown in Figure 3.1 with the antenna surface pointing towards the earth. It is felt that the extremes of the thermal environment are characterised by three different 24 hour orbits, namely, the solstice (maximum declination), the equinox (maximum eclipse), and a declination of -4.33 degree where the spacecraft antenna receive the maximum solar insulation for the maximum time (Steinbach and Winegar, 1985).

In order to bracket the thermal environmental extremes, the thermal analysis usually considers two orbits (see Figure 3.2): one occurring during equinox (which includes a
72-min eclipse) and the other occurring during solstice (no eclipse). Although the temperature variations are similar for both orbits, the more extreme heating and cooling of the equinox orbit produces a more diverse thermal environment and greater thermal distortions (Farmer, et al., 1992). Consequently, the equinox orbit was selected as the worst case.

If the structure enters the earth’s shadow during the orbit, the heating on the structure is greatly reduced due to the absence of solar heating. The duration of the shadowing depends upon the altitude of the orbit. The heating on an area of a reflector (or a member) depends strongly on the orientation of the area (or the member) with respect to the solar vector and, consequently, may vary significantly from area to area (or member to member) and with time during the orbit.

Transient temperatures were shown to range from 150 to -180°C, depending on orbit position. The temperatures were strongly influenced by shadowing, particularly the shadow cast by the reflector surface. The effects of this shadow were evident throughout a large portion of both the equinox and solstice orbits. The shadow cast by the reflector was much more prominent than that cast by the Earth, such that results for both orbits were similar with only minor differences due to the presence of the Earth’s shadow (Farmer, et al., 1992).

### 3.5 Antenna structural dynamic analysis

The entire equations of motion include the inertia forces, damping forces, and time-dependent disturbances can be expressed, in matrix form, as follows

$$[M] \{\ddot{X}\} + [C] \{\dot{X}\} + [K] \{X\} = \{F(t)\}$$

where, $[M]$ is the mass matrix of the structure. It includes both the structural masses that resist the deformation as well as the masses derived from nonstructural elements; $\{\ddot{X}\}$ is the acceleration vector at the nodes; $[C]$ is the damping matrix; $\{\dot{X}\}$ is the velocity vector at the nodes; $[K]$ is the stiffness matrix of the structure; $\{X\}$ is the displacement vector at the nodes; $\{F(t)\}$ is the time history of the applied force or torque disturbance on the structure. If this forcing function changes very slowly relative to the natural frequencies of the structure, the acceleration and inertia terms become small in comparison to the elastic term ($[K] \{X\}$) and reduces the problem to a static one.
In the case of laminated composite structures, the stiffness, damping, and mass matrices, \([K], [C],\) and \([M]\) respectively, are synthesised utilising micromechanics, laminate, and structural mechanics theories representing the various material and structural scales in the composite structure.

The primary step in the dynamic (vibrational) analysis is the extraction of the natural frequencies and their associated mode shapes. The modal analysis utilises the eigenvalue/eigenvector determination technique by calculating the undamped free vibration equation. To do this, the homogeneous solution to the differential equations (3.6) can be obtained assuming negligible damping. Thus

\[
[M]\{\dot{X}\} + [K]\{X\} = \{0\} \tag{3.7}
\]

letting

\[
\{X\} = \{\Phi\}e^{\omega t}
\]

thus

\[
\{\dot{X}\} = -\omega^2 \{\Phi\}e^{\omega t}
\]

Substituting into equation (3.7), the eigenvalue system is obtained

\[
[K]\{\Phi\} = \omega^2 [M]\{\Phi\} \tag{3.8}
\]

where \(\omega\) and \([\Phi]\) are the eigenvalues and eigenvectors (or modal frequencies and mode shapes), respectively, associated with the free vibration of the model. The normal modes are, of course, orthogonal (a property responsible for many of the simplification strategies in the dynamics analyses).

It is quite common for only a few lowest natural frequencies and modes of structural system to be extracted. Therefore, the eigenvalues and eigenvectors are determined using Subspace Iteration technique (Bathe and Wilson, 1976) in this work. This technique is useful in that it can considerably reduce the cost of the analysis, when compared with other solution techniques, if large matrix bandwidths exist, as is frequently the case in structural problems.

### 3.6 Composite materials for antenna structures

Antenna structures that can accommodate the science requirements must be made from materials that are cost effective, lightweight (high specific stiffness), thermally and dimensionally stable in the terrestrial or space environment. The advances in materials technology have demonstrated that the application of composite materials has great potential to further the technology readiness of these mission concepts.

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3. Antenna Structural Analyses
The recent advances in composite techniques and the information on various composite systems currently in use has been given by Hollaway, (1993). The use of composite materials in the fabrication of structural systems significantly widens the range of possible designs open to the engineer. These new degrees of freedom offer the prospect of achieving marked advances in the development of high performance antenna structural systems. From the structure design standpoint, the composite materials can be arranged in such a way as to obtain the same stiffnesses and strengths as metallic structures at reduced weight, by virtue of the higher specific stiffness and strength, lower specific weight and, mainly, the possibility of tailoring the material in order to obtain the desired engineering properties. However, to realise this promise will require the rational and responsible management of these new degrees of freedom.

In order to provide the most reliable data about the material and structural properties, to give rise to original data bases, and to validate the results of the calculations and methodological choices, experimental studies and accurate characterisation of composite materials and honeycomb core at laminate and basic structure levels may be required.

The tailorability of composite materials considerably modifies the conventional approach to the structural design. A spectrum of design methods has been developed, ranging from basic to optimal tailoring of composite laminates and structures, in order to meet requirements for high performance and light-weight. In fact, the concepts of 'material' and 'structure', distinct and independent in the traditional design techniques, become intimately joined. To design a structure to be manufactured with composites involves design of the most suitable material, at lamina and laminate level, that acquires its own 'essence' related to that structure. As composite materials readily provide specific moduli, high stiffness-to-weight and strength-to-weight ratios, and tailorable anisotropic elastic properties, in the general cycle of design and optimisation of the structure, the design of the material plays a fundamental role. It can consist of two steps: (1) design of the lamina, by the evaluation of its properties starting from the constituents (micromechanics); (2) design of the laminate, by the evaluation of its properties from the lamina properties and stacking sequence (macromechanics).
Generally, as described by (Cirese, 1990), due to both the lack of simple and reliable models, and to the obvious opportunity to use products already available from the material supplies, the first step is not generally carried out, except probably for sensitivity analyses. So, this one, rather than a ‘design’, generally becomes a ‘characterisation’ stage, by means of experimental activities.

The macromechanics step, on the other hand, is the most important basic approach to the composite design and analysis, both as regards the material arrangement and the preliminary structure analysis. The classical theory of lamination, on which the methodology of this stage is based, constitutes a rather simple and powerful tool to evaluate the engineering properties of the laminate, with the possibility of optimising the stacking sequence of laminae of proper characteristics, and performing a preliminary strain and stress analysis.

Figure 3.3 and 3.4 (Fager, 1976) compare certain mechanical properties of various materials, Figure 3.3 shows the stiffness-to-density ratio of the graphite composite materials to metals. Only high-priced beryllium is competitive with the unidirectional graphite in stiffness. In general, beryllium will cost four to five times that of graphite structure. The true figure of the merit in material selection is the relationship of stiffness (E) divided by density (p) and thermal expansion (CTE). Figure 3.4 shows the dominating aspects of graphite on this basis. A poor characteristic is its low thermal conductance properties, which are similar to Invar (Fager, 1976). In a large space antenna, conductance is not critical if the local cross section can be minimised. Table 3.1 shows the properties of graphite-epoxy together with two other mostly used metals. An almost zero coefficient of thermal expansion and a stiffness-to-mass ratio of almost four times that of steel are the two factors that make graphite-epoxy a very attractive material for the precision structures.

Space antenna structures will experience temperature extremes in space environment which affects both the stiffness and the strength of composite materials. For example, resin-dominated properties (transverse tensile and compressive strength and stiffness) are reduced approximately 30% at 177°C. Mechanical property data for establishing the structural performance of composite materials at temperatures as low as -200°C are not extensive; however, sample data available indicate that mechanical properties of composite materials at very low temperatures generally exceed room-temperature
values. Thermal cycling of composite materials between low and high temperature extremes, (-212°C to +177°C) does not appear to alter these findings (Garibotti, Reck and Cwiertny, 1978).

Table 3.1 Material properties of graphite-epoxy composite and metals
(Akgul, Gerstle and Jonhson, 1992)

<table>
<thead>
<tr>
<th>material</th>
<th>density $\rho$ (kg/m$^3$)</th>
<th>maximum allowable stress $\sigma$ (MPa)</th>
<th>modulus of elasticity $E$ (MPa)</th>
<th>coefficient of thermal expansion $\alpha$ ($10^{-6}$/K)</th>
<th>$E/\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>steel</td>
<td>7850</td>
<td>140.6</td>
<td>210000</td>
<td>4.270</td>
<td>26.75</td>
</tr>
<tr>
<td>aluminium</td>
<td>2700</td>
<td>91.4</td>
<td>69600</td>
<td>8.430</td>
<td>25.78</td>
</tr>
<tr>
<td>graphite-epoxy*</td>
<td>1580</td>
<td>450.0</td>
<td>161736</td>
<td>0.073</td>
<td>102.36</td>
</tr>
</tbody>
</table>

* Fibre volume percentage of 62%.

It is to be emphasised that, for the specific applications of antenna structures in which the dimensional stability is a fundamental requirement and a design goal, the tailorable property of the material, allowing one to tune also the CTE, offers the possibility of obtaining much better performances. Structural materials which exhibit near-zero CTE are desirable from the standpoint of thermal deformation resistance.

Because the graphite fibres have such a high negative CTE, zero-CTE graphite/epoxy composites can possibly be designed by the proper selection of fibre orientation. In the case of metal matrix composites, having selected a matrix, similar foil, and a graphite fibre leaves an engineer with two free materials design parameters, fibre volume fraction and fibre orientation, which can be chosen to yield zero-CTE. Lesieutre, (1985), indicated that the achievement of zero-CTE in graphite composite typically requires ply angles of 25° with fibre volume fractions of 60%. If very high modulus pitch-based graphite filaments (> 100 Msi) are developed, zero-CTE can be achieved at reasonable fibre volume fractions, while maintaining a unidirectional orientation for maximum stiffness. Table 3.2 (Lesieutre, 1985) compare properties of various materials, including several zero-CTE designs.

Reported research on the damping of unidirectional composites and laminates (Saravanos and Chamis, 1990) has shown that the damping of composites is highly-tailorable and is primarily controlled by constituent parameters (fibre/matrix properties,
fibre volume ratio), and laminate parameters (ply angles/thicknesses, stacking sequence). The work suggested that properly designed composite structures can provide significant passive damping, and they may further improve the dynamic performance and fatigue endurance by attenuating undesirable elastic-dynamic phenomena such as structural resonances, overshooting, and long settling times.

Table 3.2 Properties of some materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Longitudinal CTE (ppm/C)</th>
<th>Longitudinal Modulus (Msi)</th>
<th>Density (pci)</th>
<th>Specific Stiffness (*10^8 inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al 6061</td>
<td>22.7</td>
<td>9.90</td>
<td>0.098</td>
<td>1.00</td>
</tr>
<tr>
<td>Mg AZ91C</td>
<td>25.2</td>
<td>6.50</td>
<td>0.065</td>
<td>1.00</td>
</tr>
<tr>
<td>GY70/934</td>
<td>0</td>
<td>23.0</td>
<td>0.056</td>
<td>4.11</td>
</tr>
<tr>
<td>*Al 58% P100</td>
<td>0</td>
<td>62.2</td>
<td>0.086</td>
<td>7.19</td>
</tr>
<tr>
<td>*Al 50% P140</td>
<td>0</td>
<td>75.0</td>
<td>0.088</td>
<td>8.48</td>
</tr>
<tr>
<td>*Mg 50% P100</td>
<td>0</td>
<td>53.3</td>
<td>0.072</td>
<td>7.43</td>
</tr>
<tr>
<td>*Mg 42% P140</td>
<td>0</td>
<td>62.5</td>
<td>0.071</td>
<td>8.84</td>
</tr>
</tbody>
</table>

* Unidirectional

Unlike metals and many other structural materials which are homogeneous and isotropic, composites are highly anisotropic in nature and, by definition, not homogeneous. The tailorability of the material is the key to a non-trivial approach to composites and their technologies, and constitutes the most stimulating feature from the researcher's point of view. In composite structures, the stiffnesses in the two principal material directions can vary considerably from each other, and the strengths can differ by several orders of magnitude. For composite materials the engineer has the ability to control the orthotropy to achieve the design goals.

As regards the manufacturing point of view, composite materials are particularly well suited to complex geometric shapes, such as double curvature antenna reflectors, because they are much easier to form than metallic materials, moreover allowing achievement of especially accurate surface finishes.

Such venerable guides to materials evaluation and selection as specific modulus, specific strength, and structural indices will continue to be valuable tools under certain circumstances. However, the structural performances depend, among other things, on
the material properties, and the acceptable behaviour will be required in several distinct load conditions. Modern structural concepts will often be statically indeterminate leading to significant behaviour interaction between the individual elements of the structural systems. The need to consider the influence of numerous material properties, multi-load conditions, and structural component interaction becomes particularly apparent when thinking in terms of tailoring a composite material by varying its composition and constituents.

It is thought that extending the structural synthesis approach to include composition and material design variables will provide a comprehensive and rational tool to assist the engineer in the intelligent exploitation of the potential offered by composite materials. On the other hand, it involves some ‘collateral effects’. Although the potential for weight saving and flexibility in the tailoring of material properties makes them attractive to the designer, the basic construction of composite materials also results in a substantial increase in the degree of complexity in the analysis and design. This complexity of composite structures represents an objective difficulty as regards the industrial operational practice and makes closed-form handbook solutions of structural behaviour (stress, deflections, mode shapes, etc.) far more difficult for the analyst, and numerical analysis methods are usually used. In fact, it constitutes a considerable complication with respect to the usual analysis and design process, because it generally requires a greater amount of knowledge and professional skill of the analysts/designers, to be applied to the necessity of much deeper investigative activities, as, for example, the need to resort to the computer in the very early stages of the design. Furthermore, design with composites requires a very high degree of interdisciplinary expertise, related to the fields of mechanics of materials, stress analysis and technology. Each of these specific fields, in turn, may involve more difficult problems than those related to the metallic materials.

Clearly, advanced composite materials possess many attractive characteristics and do have the potential to satisfy the performance requirements of large space structures. However, before the full potential of advanced composite materials can be fully realised, certain key behavioural characteristics must be determined or more investigated. These characteristics include the response of composite materials over long-duration of time to high- and low-temperature thermal ageing and to cyclic
thermal environments, as discussed previously, and additionally, response to ionisation (charging) radiation (Garibotti, Reck and Cwiertny, 1978).

### 3.7 Composite reflector panels analysis

A material/structure configuration of an antenna reflector must be designed to have an acceptable performance under a spectrum of design conditions. The usefulness of mesh surface reflectors is limited to wavelengths many times larger than the mesh spacing (Hedgepeth, 1986). Thus there is little discussion of application of mesh surface reflectors at wavelengths shorter than $K_u$ band. For high precise reflectors, which use shorter wavelengths, a solid or membrane surface must be employed. In this project, the work will be concentrated on the analysis and optimisation of solid surface reflectors.

It is worthwhile to note that the development of high accuracy CFRP antenna structures is an exciting possibility for various ground and space applications. Both the extremely small coefficient of thermal expansion and the relatively low weight (about 5 times less than steel for a comparable stiffness) render this material very suited for space applications.

The sandwich system is the most used structural solution for high accuracy light-weight antenna reflectors; these systems are manufactured by co-curing or secondary bonding. A honeycomb sandwich panel is a layered construction typically formed by bonding two thin face sheets to a thick core. The face sheets take the membrane and bending loads while the core resists the shear loads. It is clear that sandwich construction provides a very lightweight structural configuration for many load conditions. Likewise, it is well known that fibre reinforced composite materials provide one of the lightest weight material systems, due to their excellent specific strength. Thus, sandwich panels employing fibre reinforced composite material faces brings together the excellence of both the structural configuration and materials system. A typical antenna reflector consists of a combination of carbon fibre reinforced polymer matrix (CFRP), glass fibre reinforced polymer matrix (GFRP), aluminium alloy and titanium structural items. In order to get a high stiffness to mass ratio, honeycomb sandwich is used extensively, usually aluminium alloy or Kevlar honeycomb with carbon fibre faceskins, in several layers, to give acceptable thermal expansion characteristics.
Based on the type of loading, sandwich panel structures need to be designed to meet several different strength and local buckling requirements. These requirements, described by Kodiyalam, et al, (1994), include:

a) The face sheets must be sufficiently thick to withstand the tensile, compressive, and shear stresses.

b) The core must be designed with sufficient thickness and shear modulus to withstand the shear stresses from the loads and to prevent overall buckling of the sandwich panel.

c) The core cell size must be designed to prevent intercell buckling of the face sheets.

d) The compressive strength of the core must be adequate to prevent wrinkling of the face sheets.

When carbon composites are used together with Al honeycomb core, proper isolation must be provided to prevent galvanic corrosion. In general, it is obtained by insertion of one or two layers of glass/epoxy between the Al and C (Cirese, 1990). Besides, the high radio frequencies require the carbon fibre/epoxy reflecting surfaces to be metalised.

Starting from the initial choices of the materials for skins and core, and their physical properties, an index of the surface accuracy, typically the RMS value of the displacements from the ideal reflector surface, can be evaluated.

By increasing a reflector’s thermal stability, in-space contour and beam-pointing tolerance can be improved substantially, with corresponding gain improvement. In addition, a more than half weight reduction of existing metal systems can be achieved with graphite. Apart from the reflector structures, RF components such as filters, cavities, and oscillators can be built at one-fifth the weight of equivalent Invar components.

The surface shape of a main reflector will be distorted by the thermal gradients across the panels. These distortions will be a function of the method of construction of the panels and backing structure together with the influence of the incoming thermal radiation.
Generally, the reflector panels will consist of rib or backup truss stiffened sandwich shells with composite and metalised facesheet and honeycomb core. Using a deep truss as a support structure is clearly an attractive possibility for large reflectors whether the surface is composed of stiff panels attached to it after deployment (or even erection) in orbit. For the longer wavelengths, there are greater freedoms, but for the shorter ones, the surface must be stiff.

In this work, a composite space antenna structure with paraboloid reflector has been analysed and optimised. The construction of the primary reflector is a strong driver for the overall system design. Since the main reflector with the satellite is larger than the payload compartment of the carrier and a mesh reflector is not suitable for Ka-band applications, a deployable reflector with rigid panels has been chosen. The panels of the reflector are stiffened with composite sandwich ribs. The analysis of the structure will be detailed in section 3.10. The objective of the analysis work is to advance the technology for designing to specification and accurately predicting the performance of advanced structural-composite panels.

This reflector was chosen for the contour accuracy for a frequency range of (20/30 GHz) which requires 0.2 mm RMS and, in addition, already includes production and deployment errors as well as thermal deformations in orbit. Further more, weight was to be minimised, leading to the development of a new sandwich construction with extraordinary characteristics, these are:

- extremely good stiffness-to-weight ratio
- coefficient of thermal expansion less than $2\times10^{-5}$ k$^{-1}$
- sandwich area weight less than 1.8 kg/m$^2$.

The faces of the reflector panels and the ribs are fabricated with laminates. These laminates consist of stacked plies orientated in such a way to give the laminate plates the desired effective mechanical, thermal, and thermo-mechanical properties. The individual plies are 'composites' of long fibre filaments encapsulated unidirectionally in a matrix material giving them orthogonal material properties.

Special modelling problems in reflector (panel) analysis, relating to the evaluation of the surface accuracy, are (Cirese, 1990):
1) the necessity of refined meshes to match the precision levels required
2) the modelling of the composites and honeycombs
3) the difficulty of schematising the loads, for which, generally, realistic deterministic models can hardly be defined, mainly as regards temperature cycling, shock and vibration conditions, while envelopes are too conservative

In the current work, ABAQUS (Hibbitt, Karlsson and Sorensen Inc., 1994) is used to determine the panel behaviour resulting from the properties of its constituent plies and their orientations. Two layered elements, STRI35 and S4R5, offered by the ABAQUS finite element library, have been used. They are 3- and 4-node shell elements to model composite thick structures (Figure 3.5). These composite elements are well developed and complete enough to model thick advanced laminated fibre composite structures with each ply (or group of identical unidirectional plies) corresponding to a layer with a specific fibre direction and material properties. The assumptions on these elements can be summarised as:

1) Normals to the centre-plane are assumed to remain straight after deformation, but not necessarily normal to the centre-plane.
2) There is no significant stiffness associated with the rotation about the element normal axis, a nominal value of stiffness is present, however, to present free rotation at the node.
3) The bonding of layers is perfect (i.e. no slippage is considered).
4) Interlaminar shear stresses are based on the assumption that no shear is carried at the top and bottom surfaces of the element.

The ply input properties include various mechanical moduli, densities, Poisson’s ratios, mechanical strengths, coefficient of thermal expansions (CTE). All of these properties may be different in each of the principle ply orthogonal axes directions and can be a function of temperature. The laminate analysis becomes even more complex as the number of plies with their associated laminate orientations is increased (Helms, et al., 1989). In this work, the procedure of a standard finite element analysis that comprises a model generation, solution and results, is a support to optimise the objective functions. Further steps are necessary to optimise and to verify the results until the design solution
is obtained. The model geometry, material properties, applicable geometric constants, and boundary conditions and loadings must all be specified.

The optimisation of this structure, which will be discussed in Section 7.B, has been undertaken, using the program ‘MOST’ specially developed during the progress of this current work. The program can be used to determine the best stacking sequence of a laminated face to obtain the minimum weight for a given sandwich construction and given composite lamina properties.

Over the last few decades many efforts have been directed toward the structural analysis and optimisation of sandwich panels to insure structural integrity and/or attain minimum weight. Such studies are discussed by Freeland, 1985; Vinson and Handel, 1988; and Borri, Speranzini and Vetturini, 1992.

3.8 Best fit paraboloids (BFP) of deformed reflector surfaces

It is not the absolute displacement value of antenna surface nodes that affect the electromagnetic performance, but the relative deformation of surface nodes. If an antenna structure has only a rigid body movement, the electromagnetic performances are not affected as long as the feed is simply moved to its new focus. Therefore, a concept of best fit paraboloid (BFP) was proposed (Horne and Barrett, 1969). In practice, it is appropriate to consider the pathlength differences with respect to an alternative paraboloid that best fits the data. It can be imagined that a new paraboloid be fabricated for the deformed antenna reflector (real antenna surface) and the new paraboloid has vertex displacement, axis rotation and focal length change with respect to the original design paraboloid (nominal paraboloid). There are innumerable new paraboloids of this kind, but there is only one new paraboloid which makes the smallest RMS of the pathlength difference with respect to the deformed reflector. Such a new paraboloid is the BFP (see Figure 3.6) which has a new vertex and a new focus, and if the feed is adjusted to the new focus, the electromagnetic performance will be greatly enhanced.

The BFP surface represents the reflector surface in an average sense. When the surface error is zero the BFP surface reduces to the ideal or the design surface geometry. Generally, the RMS deviations of deformed reflector surface with respect to their BFP
is only about one tenths or less than the RMS deviations with respect to the original
design paraboloid (ODP).

The alternate paraboloid, BFP, which represents a paraboloid for which the RMS
deviation is minimum, is defined by a maximum of six 'homology' fitting parameters
relative to the distorted surface defined by the structural nodes. These six fit parameters
consist of the three translational vertex shift (u_A, v_A, and w_A) parallel to the co-ordinate
axes, rotations (\(\phi_x\) and \(\phi_y\)) about the X- and Y- axes, and a focal length change
parameter \(h\). For structures and loading symmetric about the Y-Z plane, the X-axis shift
and Y-axis rotation are zero. The focal change parameter is permissible if the antenna
has a dynamically adjustable focal point.

The overall quality of the distorted surface is assessed by calculating the RMS error
between the resulting reflector structural nodes (analytical or measured) describing the
distorted reflector contour and the idealised best-fit paraboloid. The RMS is an error
term representing the closeness of the deflected shape to a paraboloidal surface. The
magnitude of the contour deviation error is defined as the half-path-length errors seen
by the RF energies focused by the reflector.

In Figure 3.7, suppose \(x, y, z\) are co-ordinates with respect to axes OX, OY, OZ which
are the axes of reference for ODP, and \(x_i, y_i, z_i\) with respect to axes O_iX_i, O_iY_i, O_iZ_i
which are the axes of reference for BFP. The ODP has its vertex at O, focal axis OZ
and focal length \(f\), while the BFP has vertex at O_i, focal axis O_iZ_i and focal length \(f+h\).

The complete reflector surface is equivalent to an infinite set of points. In practice the
deformed reflector surface is replaced by a finite set of points (the structural analysis
points on antenna reflector surface) which are taken to be a representative sample of the
entire set, and its BFP can be defined by six parameters, i.e. \(u_A, v_A, w_A, \phi_x, \phi_y, h\). Once
these fit parameters are evaluated, the BFP is found. The equations to obtain the fit
parameters can be deduced as follows:

The equation to the ODP is

\[ x^2 + y^2 = 4f^2 \]  \hspace{1cm} (3.9)

and the equation to the BFP is

\[ x_i^2 + y_i^2 = 4(f+h)z_i \]  \hspace{1cm} (3.10)
Take \( u_a, v_a, w_a \) as the co-ordinates of \( O_i \) with respect to axes \( OX, OY, OZ \) and \( \phi_x, \phi_y, \phi_z \) as the rotations of \( O_iX_i, O_iY_i, O_iZ_i \), where \( u_a, v_a, w_a \), are the deviation of BFP vertex from ODP vertex along axes \( OX, OY, OZ \).

Since \( \phi_x, \phi_y, \phi_z \) are all small quantities compared with unity, ignoring the second order terms, the co-ordinate transformation equations are then
\[
\begin{align*}
x_i &= (x - u_a) - z\phi_y + y\phi_z \\
y_i &= (y - v_a) - x\phi_z + z\phi_x \\
z_i &= (z - w_a) - y\phi_x + x\phi_y \\
\end{align*}
\] (3.11)

Substituting equation (3.11) into equation (3.10) and ignoring the second order quantities, the equation to the BFP referred to axes \( OX, OY, OZ \) becomes
\[
\begin{align*}
x^2 + y^2 + 2yz\phi_x - 2xz\phi_y - 2x(u_a + 2f\phi_y) \\
-2y(v_a - 2f\phi_x) + 4f\phi_a - 4z(f + h) = 0 \\
\end{align*}
\] (3.12)

Suppose that a point \( A(x_0, y_0, z_0) \) on the ODP moves to \( B(x_0 + u, y_0 + v, z_0 + w) \) due to the structural loads, all co-ordinates being with respect to axes \( OX, OY, OZ \) (Figure 3.8). Since the point \( A \) is on the ODP,
\[
x_0^2 + y_0^2 = 4fz_0 \\
\] (3.13)

Point \( B \) will be on neither ODP (3.9) nor BFP (3.12). Because structural deformation is small, the normal direction cosines of point \( B \) to BFP (3.12) can be estimated by the normal direction cosines of point \( A \) to ODP (3.9), i.e.
\[
\begin{align*}
l &= \frac{-x_0}{2f(f + z_0)} \\
m &= \frac{-y_0}{2f(f + z_0)} \\
n &= \frac{-z_0}{f + z_0} \\
\end{align*}
\] (3.14)

Supposing that the normal to the BFP through point \( B \) intersect the BFP at point \( C \), let \( \Delta = BC \), and point \( C \) has co-ordinates
\[
\begin{align*}
x &= x_0 + u + l\Delta \\
y &= y_0 + v + m\Delta \\
z &= z_0 + w + n\Delta \\
\end{align*}
\] (3.15)

Since point \( C \) is on the BFP, substituting (3.15) in (3.12) and considering (3.13), again ignoring second order terms, the following expression is obtained for the normal deviation \( \Delta \).
\[
\Delta = \frac{1}{2f(f + z_0)} [x_0(u - u_a) + y_0(v - v_a) - 2f(w - w_a) \\
-2hw_0 + y_0\phi_x(z_0 + 2f) - x_0\phi_y(z_0 + 2f)] \\
\] (3.16)

For convenience sake, the following dimensionless quantities are introduced:

3. Antenna Structural Analysis
\[ X_0 = x_0/f, \quad Y_0 = y_0/f, \quad Z_0 = z_0/f, \quad U = u/f, \]
\[ V = v/f, \quad W = w/f, \quad U_A = u_A/f, \]
\[ V_A = v_A/f, \quad W_A = w_A/f, \quad H = h/f \]

Equation (3.16) becomes
\[ \Delta = \frac{f}{2(1 + Z_0)} \left[ X_0(U - U_A) + Y_0(V - V_A) - 2(W - W_A) \right. \]
\[ -2Z_0H + Y_0(2 + Z_0)\phi_x - X_0(2 + Z_0)\phi_y \]

If it is desired to weight the deviations according to the illumination and the influenced areas, one first determines a normalised weighting factor for each point at which calculations are made. Assuming that the area influenced by a defected point 'i' is \( \alpha_i \), and the illumination factor for the area is denoted by \( q_i \) (which is the power density with which the feed illuminates the reflector at angle \( \alpha \) relative to the axial power density), this weighting factor for a given point is:

\[ F_i = \frac{q_i\alpha_i}{\sum q_i\alpha_i} \]  

(3.19)

Using this factor, the weighted root square value of all \( \Delta_i \) is

\[ \Delta_d^2 = \frac{\sum \Delta_i^2\alpha_i q_i}{\sum \alpha_i q_i} \]  

(3.20)

where the summation \( \sum \) is for all the structural analysis points on the reflector surface.

Substituting (3.18) in (3.20), the equation (3.20) becomes

\[ \Delta_d^2 = \frac{1}{\sum \alpha_i q_i} \left\{ \frac{f^2}{4(1 + Z_0)} \left[ X_0(U - U_A) + Y_0(V - V_A) - 2(W - W_A) \right. \right. \]
\[ \left. \left. -2Z_0H + Y_0(2 + Z_0)\phi_x - X_0(2 + Z_0)\phi_y \right] \right. \}

(3.21)

Letting

\[ Q = \frac{\alpha_i q_i}{\sum \alpha_i q_i} \cdot \frac{1}{4(1 + Z_0)} \]  

(3.22)

and

\[ B = X_0U + Y_0V - 2W \]  

(3.23)

the equation (3.21) becomes

\[ \Delta_d^2 = \sum Qf^2 \left[ B - X_0U_A - Y_0V_A + 2W_A - 2Z_0H \right. \]
\[ \left. + Y_0(2 + Z_0)\phi_x - X_0(2 + Z_0)\phi_y \right]^2 \]  

(3.24)

The \( \Delta_d^2 \) is to be minimised with respect to the fit parameters \( U_A, V_A, W_A, \phi_x, \phi_y, H \), so these parameters can be obtained by partially differentiating with respect to these variables and equating to zero.
\[
\frac{\partial \Delta^2}{\partial U_A} = 0, \quad \frac{\partial \Delta^2}{\partial V_A} = 0, \quad \frac{\partial \Delta^2}{\partial W_A} = 0,
\]
\[
\frac{\partial \Delta^2}{\partial \phi_x} = 0, \quad \frac{\partial \Delta^2}{\partial \phi_y} = 0, \quad \frac{\partial \Delta^2}{\partial H} = 0
\]

the following six equations can be obtained
\[
\begin{align*}
\sum QX_0^2 & - 2\sum QX_0Y_0 + 2\sum QX_0Z_0 \\
\sum QX_0Y_0 & - 2\sum QY_0 + 2\sum QY_0Z_0 \\
\sum QX_0 & - 2\sum QZ_0 + 2\sum QZ_0 \\
\sum QX_0Y_0(2 + Z_0) & - 2\sum QY_0(2 + Z_0) + 2\sum QY_0Z_0(2 + Z_0) \\
\sum QX_0Z_0 & - 2\sum QZ_0 + 2\sum QZ_0 \\
\sum QX_0Y_0(2 + Z_0)^2 & - 2\sum QY_0(2 + Z_0)^2 + 2\sum QY_0Z_0(2 + Z_0)^2
\end{align*}
\]

\begin{pmatrix}
U_A \\
V_A \\
W_A \\
H \\
\phi_x \\
\phi_y
\end{pmatrix} = \begin{pmatrix}
\sum QBX_0 \\
\sum QBY_0 \\
\sum QB \\
\sum QBZ_0 \\
\sum QBY_0(2 + Z_0) \\
\sum QBX_0(2 + Z_0)
\end{pmatrix}

(3.26)

Once the six fit parameters are obtained by solving the set of linear equations (3.26), substituting them in (3.16), the normal deviation \( \Delta \) of all deflected points with respect to the BFP can be evaluated, and therefore the weighted root mean square value \( \Delta_0 \) is found.

A convenient measure of performance for design is the RMS of half the difference in pathlength of the microwave energy beam in travelling from a deformed reflector surface to the focal point compared with the pathlength from a surface that is a perfect paraboloid. The half difference in pathlength can be computed by modifying equation (3.22) as follows

\[
Q = \frac{1}{\sum a_i q_i} \cdot \frac{1}{4(1 + Z_0)^3}
\]

(3.27)

3.9 The analyses of an 8m ground antenna structure

Based on the preceding theoretical analysis and the formula derivation, a computer program has been developed, and the flow chart for this is shown in Figure 3.9. This program provides an effective antenna structural analysis tool contributing to the
development of a further program for antenna structural optimisation. A finite element method is used to determine structural deflection and stresses. The program is verified by comparing the results using this program with the results that can be obtained by running ABAQUS for the same structure.

In the analysis, after the structure is completely defined, the program follows the numerical steps of the finite element method of analysis, such as formulation of finite element matrices, numerical integration, assembly, and solution of the linear equilibrium equations and equations of motion. A computer algorithm was developed for the root-mean-square (RMS) error calculation for three-dimensional regression of the paraboloid surface. The calculation produces all six parameters representing the best fit paraboloids as well as the RMS surface errors at all working/loading cases considered in the analysis.

To illustrate the analysis, an 8-m-diameter Cassegrain paraboloid antenna structure has been analysed using the program.

The RF analysis requires that the reflector focal point be at the \((0, 0, f)\) point in space, and that the boresight face along the \(z\) direction. These requirements combine to give the required structure model orientation in three-space. The focal length, \(f\), is specified as 3m and an initial choice of subreflector diameter is of 0.6m. Figure 3.10 shows a cross-section of the Cassegrain design.

The 8m-diameter antenna structure in Figure 3.11 and 3.12 is a computational model of a practical reflector. It contains most of the essential features of typical reflector frameworks. The antenna assembly is typically modelled as a complete unit comprising a centralised hub modelled as an assembly of strong truss elements integral with the backing structure as an assembly of truss elements. The truss can also be fabricated from graphite/epoxy composites for high stiffness and low thermal expansion. Surface panels are modelled separately as they offer little in the way of stiffness to the overall assembly. Boundary conditions are introduced to simulate the structural constraints.

The kinematic nature of the interconnections between the antenna assembly and the mount permits the analysis to be carried out on the antenna in isolation. The elevation bearings fulfil different functions in that both bearings transmit radial loads whereas the
side load is transmitted through one side only. Thus no hyperstatic constraint forces are generated, and the major structural subunits are decoupled (Humphrey and Burrows, 1994).

The Cassegrain antenna structure is a combined structure consisting of a space bar framework of \( n \) one-dimensional steel bars and a rotating thin shell reflector. The reflector is made of thin aluminium plate with mass density \( \rho_f = 2730 \text{ (kg/m}^3\text{)}\). The thin shell reflector is divided into 12 lobes with the angular dimension of 30 degrees and 4 rings with different radii, and because it is installed in a piece-wise manner, the weight of the shell is considered as a load but did not contribute to the stiffness of the overall structure. The reflector backup structure (steel skeleton) is constructed by 12 identical radial truss beams (ribs) spaced at 30 degree increments. The radial beams are interconnected to each other by 4 circumferential beams (hoops) and many diagonal bracing members (oblique jackstays). Each of the skeleton bars is completely characterised by its area \( A \), length \( L \), elastic modulus \( E \), and density \( \rho \). The modulus and density for the skeleton members are \( E = 205.94 \text{ (GPa)} \) and \( \rho_t = 7850 \text{ (kg/m}^3\text{)}\) respectively. As shown in Figure 3.12, the total number of members in the computation model is 336, and the number of nodes is 96. The cross-sectional areas of all the members are incorporated into 28 groups (i.e. 28 different types of positions: 13 rib-member, 9 hoop-member, 6 diagonal-member). These 28 groups are further incorporated into 12 groups with four kinds of areas, 100, 150, 200 and 300 (mm\(^2\)) (see Table 3.3 and Figure 3.11). These 12 groups will be considered as individual design variables later in the optimisation procedure in chapter 6. An X-Y-Z Cartesian coordinates system with origin at the paraboloidal vertex is defined; the Z-axis is the focal axis and the focus length of the antenna is 3 m. The Y-Z plane is a symmetry plane and the gravity loading is resolved into components parallel to the Y and Z-axes. The lower chord nodes of the inner hoop beam are the fixed supports of the model. The plane of the supports is taken to be capable of rotation about the X-axis to vary the elevation attitude. The reflective surface is approximated by 48 nodes on the reflector surface which are involved for the RMS computation. The illumination weight factor used in the analysis (see equation (3.20)) is

\[
g = 1 - 0.766(r/R)^2 \tag{3.28}
\]

where \( R \) is the reflector radius.
Table 3.3 the element group of 8m antenna structure

<table>
<thead>
<tr>
<th>the groups of the bars</th>
<th>the cross-sectional areas (mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>c, g, l</td>
<td>100</td>
</tr>
<tr>
<td>b, d, e, f, h, j</td>
<td>150, 200</td>
</tr>
<tr>
<td>a, i, k</td>
<td>300</td>
</tr>
</tbody>
</table>

Because low natural frequencies will result in lower structural stiffness and higher vibrational amplitudes, the antenna structure is subjected to a modes analysis to determine the lowest five modal frequencies and corresponding mode shapes. Concentrated masses have been used in the modelling of the surface panels and backing structure. The results of the modal analysis give the lowest structural frequency to be equal to 9.162 Hz, and from Fig. 3.13 its corresponding mode shape is seen to be a twist mode. The other four low structural frequencies are 12.076 Hz, 12.076 Hz, 15.713 Hz and 15.713 Hz respectively, and their mode shapes are shown in Figures 3.14 ~ 3.17 to illustrate general vibration behaviour.

The structural analysis is performed under seven different working cases, considering seven different elevation angles spaced at 15 degree increments from 0 degree (horizon attitude) to 90 degree (zenith attitude). A best-fit paraboloid analysis is performed to determine RMS surface error and defocus from each analysis. The best-fit parameters which describe the defocus of the reflector at seven different elevation angles are listed in Table 3.4.

Table 3.4 Best-fit parameters at seven different working cases

<table>
<thead>
<tr>
<th></th>
<th>0°</th>
<th>15°</th>
<th>30°</th>
<th>45°</th>
<th>60°</th>
<th>75°</th>
<th>90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>u (m)</td>
<td>-7.68x10⁻⁸</td>
<td>.118x10⁻²</td>
<td>-2.58x10⁻⁸</td>
<td>-1.61x10⁻⁸</td>
<td>.388x10⁻⁸</td>
<td>.164x10⁻⁸</td>
<td>.380x10⁻⁸</td>
</tr>
<tr>
<td>v (m)</td>
<td>.554x10⁻²</td>
<td>.535x10⁻²</td>
<td>.480x10⁻²</td>
<td>.392x10⁻²</td>
<td>.277x10⁻²</td>
<td>.144x10⁻²</td>
<td>.358x10⁻⁸</td>
</tr>
<tr>
<td>w (m)</td>
<td>.576x10⁻¹⁰</td>
<td>.620x10⁻⁵</td>
<td>-1.20x10⁻⁴</td>
<td>-1.69x10⁻⁴</td>
<td>-2.07x10⁻⁴</td>
<td>-2.31x10⁻⁴</td>
<td>-2.39x10⁻⁴</td>
</tr>
<tr>
<td>θ₁ (rad.)</td>
<td>-1.25x10⁻²</td>
<td>-1.20x10⁻²</td>
<td>-1.08x10⁻²</td>
<td>-8.80x10⁻³</td>
<td>-6.23x10⁻³</td>
<td>-3.22x10⁻³</td>
<td>-5.15x10⁻⁹</td>
</tr>
<tr>
<td>θ₂ (rad.)</td>
<td>-1.15x10⁻⁸</td>
<td>.176x10⁻⁸</td>
<td>-.383x10⁹</td>
<td>-.238x10⁹</td>
<td>.578x10⁹</td>
<td>.244x10⁹</td>
<td>.566x10⁹</td>
</tr>
<tr>
<td>h (m)</td>
<td>.312x10⁻⁹</td>
<td>.142x10⁻³</td>
<td>.274x10⁻³</td>
<td>.387x10⁻³</td>
<td>.474x10⁻³</td>
<td>.529x10⁻³</td>
<td>.548x10⁻³</td>
</tr>
</tbody>
</table>

The results which include all the displacements of every node and all the stresses and inner forces of every member on the structure are obtained. In Table 3.5, a part of the main results are listed, where ΔD is the RMS deviation of deformed reflector surface...
measured with half of the difference in pathlength of the microwave energy beam travelled with respect to its BFP; $\Delta N_2$ is the RMS deviation of the deformed reflector surface, measured with normal (direction) deviation with respect to its ODP; $\sigma_{\max}$ is the maximum stress; and $\delta_{\max}$ is the maximum displacement. The mass of the backup structure is 515.67kg, and the mass of the assembly of the reflector shell is 604.62kg. The structural gravity centre in the Z direction is 0.478m from the paraboloidal vertex.

<table>
<thead>
<tr>
<th>$\Delta N_1$ (mm)</th>
<th>0°</th>
<th>15°</th>
<th>30°</th>
<th>45°</th>
<th>60°</th>
<th>75°</th>
<th>90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0557</td>
<td>0.0541</td>
<td>0.0493</td>
<td>0.0419</td>
<td>0.0330</td>
<td>0.0244</td>
<td>0.0203</td>
<td></td>
</tr>
<tr>
<td>$\Delta N_2$ (mm)</td>
<td>0.726</td>
<td>0.702</td>
<td>0.632</td>
<td>0.522</td>
<td>0.380</td>
<td>0.226</td>
<td>0.130</td>
</tr>
<tr>
<td>$\sigma_{\max}$ (M N/m²)</td>
<td>18.43</td>
<td>17.96</td>
<td>16.26</td>
<td>13.46</td>
<td>9.741</td>
<td>5.358</td>
<td>4.925</td>
</tr>
<tr>
<td>$\delta_{\max}$ (mm)</td>
<td>1.58</td>
<td>1.48</td>
<td>1.27</td>
<td>0.981</td>
<td>0.623</td>
<td>0.275</td>
<td>0.193</td>
</tr>
</tbody>
</table>

The structural distortions translate into excessive scattering of the short-wavelength signal and reduced efficiency of the antenna. The effects of the structural distortions on its antenna electromagnetic performances will be analysed in the Chapter 4. The assessment of both structural and electromagnetic performances for this antenna will be given in Chapter 5.

### 3.10 The analyses of a 3.6×2.6m composite space antenna structure

For the missions of this antenna, structural considerations relating to size, surface tolerances, thermal stability, pointing, and environmental disturbance attenuations represent the central difficulties.

The antenna is an offset system. The reflector has a projected aperture size of 3.6×2.6m and a highly accurate surface. The nominal surface is a section of a paraboloid having a focal length of 1.8m and offset by 0.4m from the paraboloid axis (Figure 3.18 and 3.19). The reflector dish is fabricated from a graphite composite honeycomb sandwich panel structure stiffened by a ribbed backing structure which is formed by a lattice of beams, also of honeycomb sandwich construction. Figure 3.20 shows the backside of the reflector. The backing structure consists of four main ribs. There are also many secondary ribs installed on a frame connecting those main ribs. All these ribs are

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3. Antenna structural analyses
assembled and bonded to the rear of the shell. These rib elements were connected to the appropriate surface grid points and were offset toward the rear of the reflector; the ribs can vary in height over the structure. In this analysis, all the ribs have a height of 0.1m.

The dish sandwich panel is a 0.01m thick aluminium alloy honeycomb core covered with graphite fibre reinforced epoxy (CFRP) face sheets. The sheets in both sides of the core are constructed with 0.0001m thick GFRE layers in a [0/90/45/-45] lay-up (4 plies of 0.1mm material at 0°, 90°, 45°, -45°). The front surface is coated with a metalised material to provide the required high radio frequency (RF) reflectivity and to minimise the temperature excursions and the resulting distortions of the panels. The rib sandwich panel is 0.02m thick aluminium alloy honeycomb core covered with GFRE face sheets in a [0/90/45/-45] lay-up with 0.0001m GFRE layers. The overall mass of the reflector is 18.6 kg. All the thermal and dynamic loads are carried by both the surface sandwich shell and backing structure (ribs). The total mass of the structure is 18.6 kg.

In its deployed position, the reflector has a four-point interface with the spacecraft, consisting of two antenna deployment mechanisms and two release assemblies mounted on the spacecraft. The backing structure provides suitable attachment point for the hold-down and release mechanisms. In the stowed state, there are two more interface points of the reflector with the spacecraft. These two points are located in the upper middle area of the reflector front surface (points A and B in Figure 3.19).

The structure is analysed to determine the distortions of the parabolic reflector surface, subject to solar heating in a synchronous orbit. The thermal environment in such an orbit is described by (Farmer, et al., 1992). Wide temperature variations are experienced by the structure, ranging from 115 to -250° C for the steady state and from 115 to -160° C for the transient analysis. The difference in temperature between the hottest and coldest elements at each orbit position also varies significantly, to a maximum of 190°C. As the average and minimum temperatures drop, the maximum temperature remains relatively constant. (This maximum is the hottest temperature in the structure, which does not necessarily occur at the same local area of the reflector for each orbit position.).

The influence of thermal strains on surface accuracy is complex and dependent to a great extent on detailed design. As the temperatures change throughout the orbit, the
structural elements expand or contract depending on their thermal expansion properties and the change in element temperature relative to its undeformed temperature (22°C for this analysis). The distortion of an element also depends on the distortions of the nearby elements (and, thus, their temperatures).

Concern has been expressed about the loss in antenna accuracy when entering and leaving the earth’s shadow. This can be characterised first as a major change in average temperature; the static consequences of this change are considered. The dynamic effects should be no more than twice those of the static effects (Hedgepeth, 1982). Secondly, the various parts of the structure may change the temperature at different rates. The worst case for the sun/antenna orientations result in extensive lateral surface temperature gradients and relatively small temperature gradients normal to the antenna surface. Care must, of course, be exercised to ensure that the thermal inertials of various important structural members do not differ widely enough to cause problems.

The thermal coefficients of expansion are specified for each of the elements based upon the average thermal coefficient of the face sheet and honeycomb materials of both the surface shell and the ribs from room temperature to the imposed temperature. This particular temperature distribution represents very high thermal gradients and a large temperature change from the ambient condition in the local area of the reflector surface.

The most cost effective way to analyse/design fibre composite structures is through the use of computer codes. An ABAQUS structural model was constructed of composite shell type elements. In the ABAQUS model with 133 nodes, the antenna is described by 198 three-side and four-side, irregular composite plate/shell elements (Figure 3.21). A mesh of STRI35 shell elements for the reflector surface was generated. In Figure 3.22, the normal directions of every element in the reflector surface are also shown. A framework of S4R5 elements was generated (see Figure 3.23) to represent the reflector backing structure (ribs).

The properties of surface shell elements and rib plate elements were taken from the physical structure of the sandwiches (composite face sheets and honeycomb core). The lay-up of these elements are assumed to be constant across the whole dish. The following composite physical properties are used in the analysis:

—face sheets: Graphite/Epoxy laminate
elastic moduli 
\[ E_1 = 289 \text{ GPa} \]
\[ E_2 = 6.1 \text{ GPa} \]
shear elastic moduli (needed to define transverse shear behaviour in shells)
\[ G_{12} = G_{13} = G_{23} = 4.21 \text{ GPa} \]
Poisson’s ratio \( \mu_s = 0.29 \)
density \( \rho_s = 1750 \text{ kg/m}^3 \)
Thermal expansion coefficients (needed to define orthotropic expansion)
\[ \alpha_{11} = -1.15\times10^{-6} / ^\circ\text{C} \]
\[ \alpha_{22} = \alpha_{33} = 36.2\times10^{-6} / ^\circ\text{C} \]
— aluminium alloy honeycomb core:
elastic moduli 
\[ E_1 = 200 \text{ MPa} \]
\[ E_2 = 200 \text{ MPa} \]
shear elastic moduli \( G_{12} = G_{13} = G_{23} = 140 \text{ MPa} \)
Poisson’s ratio \( \mu_s = 0.3 \)
density \( \rho_s = 32 \text{ kg/m}^3 \)
Thermal expansion coefficient
\[ \alpha = 22\times10^{-6} / ^\circ\text{C} \]

Values above from industrial firm (personal contact)

A linear-static structural analysis was performed to calculate the thermal distortions of the reflector structure and assess their effect on antenna performance. A simplification of the analysis was achieved by neglecting the anisotropy of the honeycomb cores. The resulting nodal displacements experienced by the structure at each of the orbit positions were obtained.

In the analysis, the average temperatures and temperature gradients in the whole structure at selected orbit positions are used to determine surface distortions, which are proportional to temperature difference. In addition to the average element temperatures shown, the structural model receives temperature gradient information at each elemental location. Thus, the bending moments due to these gradients are accounted for in the deflection analysis.

Three thermal patterns were investigated, these were uniform cooling, uniform heating and a transition temperature field (see Figure 3.24) resulting from reflector shadowing. Temperature ranges were estimated from previously published work (Farmer, et al., 1992; Florio and Josloff, 1968), and brief transients were not considered in this study.

3. Antenna Structural Analyses
Overall variations of $-180^\circ \text{C}$ to $+115^\circ \text{C}$ were used to estimate structural distortions due to thermal loads.

The reflector assembly has been analysed in the deployed and stowed configurations with thermal, constant acceleration and launch loads. The launch loads are simulated by giving accelerations in $x/y/z$ directions. These accelerations are $8 \text{Gs}/11.2 \text{Gs}/30 \text{Gs}$ (Prud’hon, Gautier and Flechais, 1994). Four different worst space loading cases are selected to give maximum and minimum absolute temperatures, maximum thermal gradient, and launch accelerations. These cases are:

Case 1: extremely low temperature ($-180^\circ \text{C}$).
Case 2: extremely high temperature ($+115^\circ \text{C}$).
Case 3: a temperature gradient distribution from 0 to $-180$ degree in the structure.
Case 4: stowed reflector in $8/11.2/30 \text{Gs}$ accelerations in $x/y/z$ directions in the launch case.

As discovered in the subsequent structural analyses, the worst-case temperature profile corresponded to an extremely low temperature and a condition of temperature gradients across the antenna surface. The calculated temperature profiles are a result of the steady-state analysis and, hence, they correspond to only one point in the earth synchronous orbit.

The parameters that most affect the thermal distortions of the antenna are shown to be the temperature distribution, the ‘face sheet material’ coefficient of expansion, and the details of structural and material design of the surface panel and ribs.

The results of the analyses are presented as displacement and Von Mises stress plots for the selected loading conditions. Figures 3.25 ~ 3.32 show the deformations and the stress contours in the structure for the above mentioned four loading cases. The reflector surface RMS deviations, maximum stresses, and maximum displacements of the structure at the selected four worse loading cases are listed in Table 3.6, where the RMS values are the deviations of deformed reflector surface measured with normal deviation with respect to its ODP (not BFP in this case). The curves of RMS, maximum stress, maximum displacement versus absolute temperature excursions from the ambient fabrication temperature are shown in Figure 3.33, where the range of the temperature is
from -180 °C to +120 °C with respect to different orbit positions and times. The figure shows the orbital variation in RMS surface error due to the thermal distortion of the structure. The figure illustrates the dependence of the RMS surface error on temperature characteristics.

Table 3.6 some static analysis results of a space reflector structure

<table>
<thead>
<tr>
<th></th>
<th>Case 1 (-180 °C)</th>
<th>Case 2 (+115 °C)</th>
<th>Case 3 Thermal gradient</th>
<th>Case 4 Launch case</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS (mm)</td>
<td>2.28</td>
<td>1.05</td>
<td>1.98</td>
<td>1.43</td>
</tr>
<tr>
<td>σ_{max} (M N/m²)</td>
<td>136.</td>
<td>62.7</td>
<td>136.</td>
<td>33.0</td>
</tr>
<tr>
<td>δ_{max} (mm)</td>
<td>4.88</td>
<td>2.25</td>
<td>4.27</td>
<td>4.75</td>
</tr>
</tbody>
</table>

An antenna structure must be stiff enough to avoid undesirable interaction with close-loop control system and to withstand the various disturbing forces without suffering unacceptable distortions. Ordinarily, the stiffness of a spacecraft structure is expressed in terms of its natural frequency. Normal modes analysis are performed entirely on the models. Both stowed and deployed conditions are analysed. The lowest four vibration frequencies of the structure in deployed and stowed configurations are listed in Table 3.7 and their corresponding mode shapes are shown in Figures 3.34 to 3.41.

Table 3.7 lowest four structural frequencies in deployed and stowed conditions

<table>
<thead>
<tr>
<th></th>
<th>First mode frequency (Hz)</th>
<th>second mode frequency (Hz)</th>
<th>third mode frequency (Hz)</th>
<th>fourth mode frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>deployed reflector</td>
<td>4.35</td>
<td>7.23</td>
<td>21.4</td>
<td>28.9</td>
</tr>
<tr>
<td>stowed reflector</td>
<td>24.3</td>
<td>35.0</td>
<td>56.7</td>
<td>67.1</td>
</tr>
</tbody>
</table>

In this chapter, the aspect of antenna structural analysis has been discussed. Two example antenna structures have been analysed. The surface distortions of the antennas caused by the loading cases and environmental conditions were obtained by the structural analysis. The results will be utilised to analyse the effects of structural distortions on antenna EM performances in the next chapter.
Fig. 3.1 Orientation of an antenna in earth orbit

Fig. 3.2 Description of equinox and solstice orbit
Fig. 3.3 Materials as a function of stiffness and density

Fig. 3.4 Materials as a function of stiffness, density and thermal stability
Fig. 3.5 Composite panel construction

Fig. 3.6 Best fit paraboloid
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Fig. 3.11 main sizes and element groups of the 8m antenna structure
Fig. 3.12 8m antenna structural sketch

Fig. 3.13 The first vibration mode of the structure
Fig. 3.14 The second vibration mode of the structure

Fig. 3.15 The third vibration mode of the structure
Fig. 3.16 The fourth vibration mode of the structure

Fig. 3.17 The fifth vibration mode of the structure
Fig. 3.18 An off-set reflector

Fig. 3.19 The size of the reflector
Fig. 3.20 Reflector geometry

Fig. 3.21 The structure model
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Fig. 3.22 The finite elements and their normal directions of the reflector surface

Fig. 3.23 The finite elements of the reflector backup framework
Fig. 3.24 Reflector isotherms for a sun angle

Fig. 3.33 RMS, maximum stress and displacement versus absolute temperature excursions
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Fig. 3.25 Reflector deformation at loading case 1

Fig. 3.26 Reflector deformation at loading case 2
3. Antenna structural analyses
Fig. 3.29 Reflector stress contours at loading case 1

Fig. 3.30 Reflector stress contours at loading case 2
Fig. 3.31 Reflector stress contours at loading case 3

Fig. 3.32 Reflector stress contours at loading case 4
Fig. 3.34 The first vibration mode of deployed reflector

Fig. 3.35 The second vibration mode of deployed reflector
Fig. 3.36 The third vibration mode of deployed reflector

Fig. 3.37 The fourth vibration mode of deployed reflector
Fig. 3.38 The first vibration mode of stowed reflector

Fig. 3.39 The second vibration mode of stowed reflector
Fig. 3.40  The third vibration mode of stowed reflector

Fig. 3.41  The fourth vibration mode of stowed reflector
Chapter 4

The Analyses of the Effects of Structural Whole-System Deformation on Antenna Electromagnetic Performances

4.1 Introduction

Deformations of antenna structures result in random errors in the surface and ray-path length. These errors affect antenna EM performances in a very complicated way. This complex issue is related to the type of antenna used, the distribution of errors, and many other factors. The simplest and currently most used method to evaluate the effect of surface distortions on EM performance is by using Ruze's formula (Ruze, 1966). The interrelationship between gain in the ideal case and that which is achieved practically for structural distortions is represented by the relationship

\[ G_d = G \cdot e^{-\frac{4\pi\delta}{\lambda}} \] (4.1)

where \( \delta \) is the RMS surface error. This can be rewritten in terms of reflector efficiency as

\[ \eta = e^{-\frac{4\pi\delta}{\lambda}} \] (4.2)

If statistically independent error sources are being dealt with, then \( \delta \) is the RSS (the square root of the sum of their squares) of the individual error sources. These are

1) \( \delta_s \), the surface manufacturing error
2) \( \delta_A \), the surface approximation error
3) \( \delta_T \), the structural loading induced surface error.

The relationship may be written as
\[ \eta = e^{-\frac{16\sigma^2}{\sigma^2}} \]  

where the errors \( \delta_s \) and \( \delta_T \) are a function of material properties and property variations.

However, Ruze's formula is only an approximate statistical expression which is based on the assumption of random, uniformly distributed aperture errors with small correlation intervals. Also, Ruze's formula can only be used to evaluate the gain-loss for a slightly distorted reflector (main reflector only, cannot consider the deformation of sub-reflector/feed support structure) and it is not a reliable indicator for other important performances for instance radiation beam shape and sidelobe levels. Therefore, to achieve an accurate solution for the effects of whole structural deformation on antenna various EM performances is a very important issue.

An effective numerical analysis procedure for combining structural analysis with EM aperture field analysis has been developed in this work. This is for the purpose of: (1) to account for the relation between details of the structural distortion and antenna EM performance, and (2) to estimate the degradation in antenna performance resulting from the surface deviations of a particular reflector at different loading/working cases. In this chapter, the methodology will be presented and the practicality of using this approach to deterministically calculate the effects of various structural deformations on various EM performances of the reflector antenna, will be demonstrated.

Reflector antennas are the most popular of the high-gain antennas. This is because of the relative simplicity of a reflector antenna as compared to the competing technology of large arrays requiring a complicated feed network and which in turn includes losses that reduce efficiency.

The electromagnetic waves that must be reflected vary in wavelength from a fraction of a micrometer (visible light) to nearly a meter (L-band, UHF). The nature of the reflecting surface and the precision to which it must be positioned are dependent primarily upon the wavelength, and upon the terminology used by practitioners to describe the reflection processes. For the lower wavelengths, the science of optics prevails, with the primary concerns being the specularity and efficiency of the reflection from the surface and the accuracy with which the rays impinging on the surface follow their required path through the optical system. On the other hand, at long wavelength, the much newer technology of radio frequency engineering is dominant, with the main
concern being paid to the magnitude and phasing of H- and E-field vectors at an aperture plane (Hedgepeth, 1986).

The utilisation of reflector antennas have lead to a long history of theoretical, numerical, and experimental research into their EM performance. Although methods of reflector EM analysis have been studied extensively in the past by the EM people, these methods account for changes in feed characteristics and reflector geometry but assume that the reflector have a perfect surface. The emphasis here is on the application to distorted reflectors.

The degradation in the gain of a reflector antenna due to surface deviations is a function of the distribution of the deviations over the reflector aperture, as well as of the illumination pattern of the feed. This is particularly true in the case of a systematic distortion of the surface, where the best-fit parabolic parameters change with orientation of the axial plane.

Dimensional errors in the structure affect the EM performances of the antenna by causing a distortion of the surface. The distortion primarily shifts the phase of the signal which is emitted, by reflection or direct radiation, locally from the surface. The detailed way in which the structural distortion changes the signal phase is dependent on the type of antenna. For a reflector, the important distortion is that normal to the surface.

Distorted antenna analysis requires ray tracing; i.e., the determination of the point of intersection of a ray (line) and the reflector surface. Each ray projected from the feed horn is reflected by the antenna by simply equating the angle of incidence with the angle of reflection. The reflected ray continues to the X-Y plane (the near field plane) where the amplitude and phase information are combined to give the near field pattern.

Three major techniques for analysing reflector antennas are geometrical optics/aperture integration (GO/AI), physical optics/aperture integration (PO/AI), and geometric theory of diffraction (GTD). In this study, the technique of GO/AI was used in computing radiation patterns of reflector antennas. The EM calculations are obtained by a geometrical optics projection of the feed radiation related from the antenna surface onto an aperture plane normal to the axis of the paraboloid. The phase of these fields is then
perturbed by the difference in the ray path length caused by the reflector surface distortion.

Fields in an aperture plane which is located in front of the reflector are determined by ray-tracing whereby rays are projected from the feed horn position to each point on a regularly spaced X-Y grid on the antenna surface (or surfaces) and then to the aperture plane. As noted in Figure 4.1, the rays will leave the reflector surface in a parallel fashion if they emanated in phase from the focal point of the parabolic reflector which had a perfect surface. The fields in the aperture plane are determined by applying the appropriate boundary conditions. These fields are then integrated to determine the radiation pattern. A double numerical Fourier transform of these perturbed fields yields the EM characteristics of the reflector antenna. For the smooth reflector (perfect) as shown, determination of the fields is straightforward.

The analysis of the effects of structural deformation on EM performance consists of three main subroutines. The functions of the three routines are:

a) to constitute a geometrical optic analyser which simulates the phase-amplitude RF signal in the near field plane;
b) to interpolate the phase errors in the aperture field by employing a curved surface spline function;
c) to perform a fast Fourier transform of the simulated near field phase-amplitude data, and thus determine the antenna gain and the far field pattern.

Therefore, instead of the nominal surface, the deformed reflector surfaces (real surfaces) are employed in the EM analysis. In the analysis, the gain-loss due to the blocking effects of sub-reflector and its supporting structure is also considered.

### 4.2 Cassegrain antenna and its geometric parameters

As shown in Figure 4.2, a common dual reflector antenna is composed of main reflector, sub-reflector and feed. The electromagnetic wave radiated from the feed centre is reflected to the main reflector by the sub-reflector, and then reflected into space by the main reflector. The spherical waves radiated from the feed are turned into plane waves on the antenna's aperture, via the reflection from the sub-reflector and main reflector both of which have appropriate shapes and contours.
There are many kinds of dual reflector antennas. The discussion will be developed with the most commonly used Cassegrain antennas as an example.

A standard Cassegrain antenna and its geometric parameters are shown in Figure 4.3 and Figure 4.4 respectively. The main reflector is a revolutionary paraboloid and the sub-reflector is a hyperboloid. The virtual focus of the hyperboloid coincides with the focus of the paraboloid in the point F and the feed is located at the real focus F' of the hyperboloid. From the geometric properties of the hyperboloid, it is known that, after reflection from the hyperboloid, the spherical waves radiated from the feed will look just like the spherical waves radiated from the focus of the paraboloid (the virtual focus of the hyperboloid). Then the spherical waves, reflected by the paraboloid, turn into plane waves and radiate out.

The geometric design of a Cassegrain antenna depends mainly on four parameters, f, fc, D and d. These parameters are determined by the EM designer based on the missions and user requirements.

The relations of geometric parameters of Cassegrain antenna are as follows:

\[ \tan\left(\frac{\psi_0}{2}\right) = \frac{D}{4f} \]  
(4.4)

\[ \frac{1}{\tan \psi_0} + \frac{1}{\tan \phi_0} = \frac{2f_c}{d} \]  
(4.5)

\[ \frac{\sin\left(\frac{\psi_0 + \phi_0}{2}\right)}{\sin\left(\frac{\psi_0 - \phi_0}{2}\right)} = \frac{2L_v}{c} \]  
(4.6)

\[ e = \frac{\sin\left(\frac{\psi_0 + \phi_0}{2}\right)}{\sin\left(\frac{\psi_0 - \phi_0}{2}\right)} = \frac{M + 1}{M - 1} \]  
(4.7)

\[ M = \frac{e + 1}{e - 1} = \frac{\tan \frac{\psi_0}{2}}{\tan \frac{\phi_0}{2}} = \frac{L_v}{L_w} \]  
(4.8)

where, \(M\) is the magnifying factor; \(e\) is the eccentricity of the hyperboloid \((e>1)\). The equivalent focal length is \(f' = f \times M\). It is one of the advantages of Cassegrain antennas.
that for $M>1$, Cassegrain antennas with short focal lengths can obtain the performances of the antennas with long focal lengths.

### 4.3 The differences of ray-path length caused by the deformation of sub-reflector supporting structure

Based on the geometric optics law, the ray-path and ray-path length from feed centre to aperture of deformed reflector can be deduced as follows:

As shown in Figure 4.5, the ray paths travelled by the electro-magnetic waves are composed of $L_1$, $L_2$, and $L_3$, starting from the feed centre $A$, via point $B$ of deformed sub-reflector and then the point $C$ of best fit paraboloid of deformed main reflector, to point $D$ of antenna aperture.

The ray path $L_1$, starting from point $A$, is satisfied by the equation

$$\frac{x-x_0}{l_0} = \frac{y-y_0}{m_0} = \frac{z-z_0}{n_0} \tag{4.9}$$

where $l_0$, $m_0$ and $n_0$ are the direction cosines of the $L_1$. Point $B$, as an intersection point of $L_1$ and sub-reflector, should be satisfied with the equations

$$\frac{n_0}{m_0} (y_1 - y_0) + z_0 = F_1 \left( \frac{l_0}{m_0} (y_1 - y_0) + x_0, y_1 \right) \tag{4.10}$$

$$x_1 = \frac{l_0}{m_0} (y_1 - y_0) + x_0 \tag{4.11}$$

$$z_1 = F_1 (x_1, y_1) \tag{4.12}$$

where

$$z = F_1 (x(\delta_x), y(\delta_y)) + \delta_z$$

$$= f - \left[ a^2 + b^2 + a \cdot \left\{ 1 + \left[ (x + \delta_x)^2 + (y + \delta_y)^2 \right] / b^2 \right\} \right] + \delta_z \tag{4.13}$$

is the deformed sub-reflector surface equation (for hyperboloid sub-reflector). The point $B$ can be found by solving the equations (4.10), (4.11) and (4.12). In the equations above:

- $f$ is the focal length,
- $a$ and $b$ are the real half axis and the virtual half axis respectively, and
- $\delta_x$, $\delta_y$ and $\delta_z$ are the displacements of the joining node of sub-reflector support bars in three co-ordinate directions obtained by structural analysis.
According to the reflection law (Snell law), the unit vector of the ray-path $L_2$ is

$$
\vec{I}_1 = -\vec{I}_0 + 2(\vec{I}_0, \vec{N}_1) \cdot \vec{N}_1
$$

(4.14)

where $\vec{I}_0$ and $\vec{N}_1$ are the unit vectors composed of direction cosines in $L_1$ and the normal direction of point $B$ respectively, i.e.

$$
\vec{I}_0 = (l_0, m_0, n_0)
$$

(4.15)

$$
\vec{N}_1 = (l_{N_1}, m_{N_1}, n_{N_1})
$$

(4.16)

We get from (4.14)

$$
\vec{I}_1 = (l_1, m_1, n_1) = (l_0(2m^2_{N_1} - 1), m_0(2m^2_{N_1} - 1), n_0(2n^2_{N_1} - 1))
$$

(4.17)

Replacing it into the straight line equation of $L_2$,

$$
\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}
$$

(4.18)

The equations are obtained which are entirely similar to the equations (4.10)–(4.12). In this case, the subscripts 0 and 1 in (4.10)–(4.12) are replaced with 1 and 2 respectively, and

$$
z = F_3(x, y) = \frac{2x(u_x + 2f\phi_x) + 2y(v_x - 2f\phi_x) - fw_x - x^2 - y^2}{2y\phi_x - 2x\phi_y - 4(f + h)}
$$

(4.19)

is the best fit paraboloid equation at the selected optimum elevation angle, where $u_x$, $v_x$, $w_x$, $\phi_x$, $\phi_y$, and $h$ are the best fit parameters obtained by structural analysis. The point $C$ can be obtained by solving the equations.

Again, by the Snell law, we have the unit vector $\vec{I}_2$ of the ray-path $L_3$

$$
\vec{I}_2 = -\vec{I}_1 + 2(\vec{I}_1, \vec{N}_2) \vec{N}_2
$$

(4.20)

and $L_3$ should be satisfied with the straight line equation

$$
\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}
$$

(4.21)

In the same way, we can obtain the equations which are entirely similar to the equations (4.10)–(4.12), but in this case, the subscripts 0 and 1 in (4.10)–(4.12) are replaced with 2 and 3 respectively, and

$$
z = F_3(x, y) = \text{Constant}
$$

(4.22)

is the antenna aperture equation. The point $D$, therefore, on the antenna aperture field can be obtained by solving the equations (4.10)–(4.12).

Therefore, the real ray-path length under the deformation of sub-reflector support structure is
\[ L = \sum_{j=1}^{3} L_j = \sum_{j=1}^{3} \left( (x_j - x_{j-1})^2 + (y_j - y_{j-1})^2 + (z_j - z_{j-1})^2 \right) \]  
\[ (4.23) \]

Consequently, for any point \( D_i \) \((i=1, 2, \ldots, K)\) on the aperture field, the phase difference because of the deformation of the sub-reflector support structure is

\[ \Delta_i = \frac{2\pi (L_i - L^*)}{\lambda}, \quad (i = 1, 2, \ldots, K) \]  
\[ (4.24) \]

where, \( K \) is the number of the discrete points on the aperture field; \( L^* \) is the ideal ray-path length without deformation of sub-reflector support structure; \( \lambda \) is the antenna working wavelength.

The generalised ray tracing method has been programmed and incorporated into the MOST program and executed in the analysis and optimisation procedure.

### 4.4 The differences of ray-path length caused by the deformation of main reflector supporting structure

Reflector Surface distortions are defined by displacements and surface slopes at the nodal points in the finite element model. The resultant displacements in the optical axis will be passed to the EM analysis model.

The pathlength geometry relationship is shown in Figure 4.6. Solid line V-G-C represents the original surface; the broken line represents the deflected surface. Target point \( G \) on the original surface is shown as deflected to point \( D \). An incident ray parallel to the focal axis is shown crossing the aperture plane at \( A \) and is reflected at \( D \) to the focal point. With respect to the original surface, an alternative ray is shown to cross the aperture plane at \( B \) and then to be reflected at \( C \) to the focal point along the path, C-D. Consequently, it can be seen that the pathlength difference for these two rays is the sum of the distance from \( E \) to \( C \) and from \( C \) to \( D \). The component of the deflection normal to the surface is indicated by dimension \( \text{`dn'} \). The half-pathlength difference of a typical target point can be expressed as

\[ \rho = \gamma_z \text{dn} \]

in which \( \gamma_z \) = the direction cosine of the surface normal with respect to the reflector focal axis.

In this analysis, the phase perturbation technique (Cockrell and Rudduck, 1985) is used. Figure 4.7 shows the ray paths for a focus fed distorted parabolic reflector. The true ray
paths are shown in Figure 4.7 (a). Rays that emanate from the focus are reflected in a
direction depending on the normal to the distorted reflector, and thus no longer leave
the reflector surface parallel. Figure 4.7 (b) shows the approximate ray paths used in the
phase perturbation technique in which the reflected rays are assumed to leave the main
reflector surface parallel. The phase perturbation is the difference between the path
length from the focus to the aperture plane via the smooth reflector (best fit paraboloid)
and the path length from the focus to the aperture plane via the distorted reflector.

As shown in Figure 4.8, the main reflector surface (best fit paraboloid) equation is
\( z = F_2(x, y) \). For the displacement from point C to point C' on the surface under
deformation of the structure, the difference of ray-path length caused can be computed
by using the method described below.

When antenna is at its zenith or horizon attitudes, assuming that point C moves to point
C' and the three co-ordinate components of the displacement are \( \delta_x, \delta_y \) and \( \delta_z \), the
difference of the half ray-path length, caused by this shift of C to C', should be the sum
of \( d_x, d_y \) and \( d_z \) which are the projections of \( \delta_x, \delta_y \) and \( \delta_z \) on \( L_2 \) direction. Figure
4.9 shows the contribution of \( d_x, d_y \) and \( d_z \) to ray-path length when \( \delta_x, \delta_y \) and \( \delta_z \)
are along positive directions of the three co-ordinates. The (a), (b) and (c) of the Figure
4.9 indicate respectively the contributions of the projections of displacement
components \( \delta_x, \delta_y \) and \( \delta_z \) on \( L_2 \) to ray-path length when antenna is at its zenith attitude
and \( \delta_x, \delta_y \) and \( \delta_z \) are along positive directions of the three co-ordinates and point C is at
different quadrants. The symbol '+' means positive projection (increasing ray-path
length) and '-' means negative projection (decreasing ray-path length). When \( \delta_x, \delta_y \) and
\( \delta_z \) are along negative directions of the three co-ordinates, the symbols in the Figure 4.9
will change. The (d), (e) and (f) of the Figure 4.9 have the same meaning as above
described, but for the horizon attitude.

In the static analysis of antenna structures, which generally assume small deformation,
linear and elastic systems and, in addition, Hooke law will apply to the structural
materials, so that there is a linear relation between displacement and self-weight load.
When the antenna is at any elevation angle \( \alpha \), the self-weight load \( P \) of the structure
can be decomposed into two components (as shown in Fig. 4.10):

\[
P_1 = P \cdot \sin \alpha, \quad P_2 = P \cdot \cos \alpha
\]  

(4.25)
i.e. axial (zenith) load and radial (horizon) load. Therefore, when the antenna is at any
elevation angle, any node displacement \( \{ \delta \}_\alpha \) on the main reflector surface can be
linearly expressed by the displacements of zenith \((\alpha=90')\) and horizon \((\alpha=0')\), i.e.

\[
\begin{pmatrix}
\delta_x \\
\delta_y \\
\delta_z \\
\end{pmatrix}
\alpha = \begin{pmatrix}
\delta_x \\
\delta_y \\
\delta_z \\
\end{pmatrix}
\alpha = 90' \\
\times \\
\times
\begin{pmatrix}
\cos \alpha \\
\sin \alpha
\end{pmatrix}
\]

(4.26)

In this case, the sum \( d_x + d_y + d_z \) of the projections of \( \{ \delta \}_\alpha \) on \( L_2 \) is the difference \( d \) of half
ray-path length when the antenna is at any elevation angle.

### 4.5 The interpolation of the phase errors on antenna
aperture

The differences of half ray-path length obtained are only the ones calculated at the
nodes on the main reflector surface of antenna structure, and the number of these
nodes, which are the upper chord nodes (the intersection nodes of radiation beams and
circular beams) of backup structure, are limited. However, the RF analysis requires that
the surface be defined on a very fine regular grid.

As shown in Figure 4.11, in antenna electrical analysis, the differences of ray-path
length must be taken at the points on the reflector surface which are corresponding to
the points of antenna aperture field. Therefore, the differences of the half ray-path
length of the structural nodes must be transformed into the ones of the points on the
reflector surface which are corresponding to the points of antenna aperture field. To
solve this problem, a unique interpolation algorithm is used to perform the
transformation by employing a curved surface spline function as described by Harder
and Desmarais, (1972).

A surface spline is a mathematical tool for interpolating a function of two variables. It is
based upon the small deflection equation of an infinite plate. The method was originally
developed for interpolating aircraft wing deflections and computing slopes for
aeroelastic calculations. The main advantages of the surface spline are that the co-
ordinates of the known points need not be located in a rectangular array and the
function may be differentiated to find slopes. The surface spline depends upon the
solution of a system of linear equations.
The method accounts for the actual ray-path length differences predicted by the finite element structural analysis and ray tracing technique. Using this spline interpolation, the values of the errors in the ray-path length are interpolated for points on the reflector surface that lie between the nodes on the reflector surface in the structure.

In this method, the spline function expressions are developed for the surface nodes in the structural model. A set of equations for the coefficients is generated. The resulting system of linear equations are solved by a modified Gaussian elimination technique. The solution to this system of equations yields a close-form expression for the ray-path length errors for any point on the fine-grid. This interpolation scheme is particularly suitable for the present antenna applications, as has been proved in practice.

The curved surface spline function can be regarded as the pure bending deformation of a plate of infinite extent. The differential equation relating bending deflections with loads acting on the plate is

\[ D \nabla^4 w = q \]  (4.27)

where \( D \) is the stiffness of the plate. Deflections are specified at \( n \) independent points \((x_i, y_i)\) \(i = 1, \ldots, n\). This requires point loads \( P_i \) at these \( n \) points. The values of these loads must be determined to give the specified deflections.

Instead of forcing the plate to pass through the \( n \) given points, on the assumption that elastic spring forces are applied to the plate and these forces are proportional to the difference between the desired data point and the smoothed interpolated surface, the curved surface spline function can be expressed as follows (Harder and Desmarais, 1972)

\[ w(x, y) = a_0 + a_1 x + a_2 y + \sum_{i=1}^{n} F_i \eta_i^2 \ln(\eta_i^2 + \varepsilon) \]  (4.28)

The equation (4.28) is developed by introducing polar co-ordinates \((x = r \cdot \cos \theta, y = r \cdot \sin \theta)\), using distributed loads instead of point loads and considering the conditions that surface spline function should become 'flat' a long distance from the applied loads. The loads approach the point loads as the parameter \( \varepsilon \) approaches zero. The resulting equations have been rearranged into a form useful for computation, where \( F_i = P_i/16\pi D \), and

\[ \eta_i^2 = (x_i - x)^2 + (y_i - y)^2 \]  (4.29)

This procedure produces a surface for which all derivatives exist everywhere.

4. The analyses of the effects of structural whole-system deformation on antenna electromagnetic performances
If $k_j$ is the elasticity constant with regard to point $j$, then the $n+3$ unknown quantities $F_i$ ($i=1, 2, \ldots, n$), $a_0$, $a_1$ and $a_2$ can be obtained by solving the group of equations as below

$$w_j = a_0 + a_1 x + a_2 y + \sum_{i=1}^{n} F_i r_i^2 \ln(r_i^2 + \varepsilon) + c_j F_j$$

$$(j = 1, 2, \ldots, n; \quad r_i^2 = (x_i - x_j)^2 + (y_i - y_j)^2; \quad c_j = \frac{16\pi D}{k_j})$$

and

$$\sum_{i=1}^{n} F_i = \sum_{i=1}^{n} x_i F_i = \sum_{i=1}^{n} y_i F_i = 0$$

Equations (4.31) are recognised as the equilibrium equations.

The matrix form of the equation system is

$$[A] \{x\} = \{b\}$$

where

$$A =
\begin{bmatrix}
c_1 & r_{12}^2 \ln(r_{12}^2 + \varepsilon) & \ldots & \ldots & \ldots & 1 & x_1 & y_1 \\
r_{12}^2 \ln(r_{12}^2 + \varepsilon) & c_2 & \ldots & \ldots & \ldots & 1 & x_2 & y_2 \\
\vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots \\
r_{1n}^2 \ln(r_{1n}^2 + \varepsilon) & \ldots & \ldots & r_{n-1,n}^2 \ln(r_{n-1,n}^2 + \varepsilon) & c_n & \ldots & 1 & 0 & 0 \\
1 & \ldots & \ldots & \ldots & 1 & 0 & 0 & 0 \\
x_1 & \ldots & \ldots & \ldots & x_n & 0 & 0 & 0 \\
y_1 & \ldots & \ldots & \ldots & y_n & 0 & 0 & 0
\end{bmatrix}$$

and

$$\{x\} = (F_1, F_2, \ldots, F_n, a_0, a_1, a_2)^T,$$

$$\{b\} = (W_1, W_2, \ldots, W_n, 0, 0, 0)^T.$$
In order to evaluate the surface on the fine-mesh spacing required by the RF analyser, the above process has been programmed in a computer code. To execute this interpolation, considerable information must be extracted from the structural analysis and ray tracing calculation. This information consists of the deviations of ray-path length on the reflector surface nodes and the co-ordinates of both structural surface nodes and RF analysis grid points. Then, this information is processed to yield the solution required.

4.6 The computation of far-field gain of antenna

The aperture integration methods GO/AI and PO/AI are the most popular reflector analysis methods because they avoid integration over a non-planar surface. In GO/AI analysis, geometrical optics (ray tracing) ideas are used to set up equivalent currents in a plane in front of the antenna (the aperture plane). In PO/AI analysis, the induced current on the reflector is approximated by an equivalent current related to twice the incident magnetic field intensity. The radiation integral is transformed into an integral in aperture plane co-ordinates.

It is not clear which analysis technique is better for analysing distorted reflectors. PO is potentially superior to GO because of its inclusion of axially directed surface currents (Rusch and Potter, 1970). However, some studies have shown that the overall improvement in the agreement to measured results offered by PO/AI over GO/AI for the distorted reflector is not significant (Smith and Stutzman, 1989; Rudge and Adatia, 1978).

Because the GO formulation is simpler than PO, the PO/AI method will be used in this study. Briefly, the GO/AI method (or aperture field method) is to find the reflection point on the reflector surface that satisfies reflection principle for the ray between the feed point and the aperture field point. Once the reflection point has been determined, the field at the aperture point can be determined by geometrical optics.

The GO approximation assumes that the resulting electric field radiated by the reflector system is

$$\vec{E} = -j k \eta e^{-j k r} \left( T_\theta \hat{\theta} + T_\phi \hat{\phi} \right)$$

(4.36)
where
\[
\tilde{F}(\theta, \phi) = \frac{1}{\eta} \iiint_{\text{aperture}} [\tilde{E}_a (\mathbf{r}) e^{j k \rho \cos \phi} \rho' d\rho' d\phi']
\] (4.37)

and the geometry of Figure 4.12 applies. Note that this integral is evaluated over the projected aperture of the reflector in the x'y' -plane. The aperture electric field \( \tilde{E}_a \) is obtained by assuming that the feed radiation reflects locally as a plane wave (i.e. ray tracing applies). This theory is general and the reflector can have any shape (Smith and Stutzman, 1989).

For a focused paraboloidal reflector the field at the aperture point is modified only by a phase reversal after reflection and added phase delay from the reflection point to the aperture point. For small distortions in the reflector surface the significant effect is a small displacement of the reflection point, accompanied by a change in the ray-path length. Obviously a good reflector surface will not deviate very much from its ideal surface, e.g., a perfect paraboloid; otherwise, the reflector's antenna performance will not be acceptable.

### 4.6.1 Aperture integration using FFT

An antenna aperture field integral method is used to compute antenna gain. The aperture field method assumes that there is a plane in front of the main reflector, called aperture plane. Based on the approximation of the aperture field method, the relation of the distributions between the far zone electric field and the aperture field is the two dimension Fourier integral. If the size of the antenna's aperture is far larger than the working wavelength, at point \( P(r_0, \theta, \phi) \) the far zone field can be written as
\[
E_p = \frac{j}{\lambda r_0} \iint_{\text{aperture}} f(x, y) \cdot e^{-jkr} \, dx \, dy
\] (4.38)

where \( f(x, y) \) is the distribution of the aperture field, as shown in Figure 4.13; \( \lambda \) is the working wavelength; \( k (=2\pi\lambda) \) is the wave number. For \( kr \gg 1 \), we have
\[
r = r_0 - (x \sin \theta \cos \phi + y \sin \theta \sin \phi)
\] (4.39)

Substituting equation (4.39) into (4.38), we get
\[
E_p = \frac{j}{\lambda r_0} e^{-jkr_0} F(\theta, \phi)
\] (4.40)

where
\[
F(\theta, \phi) = \iint_{\text{aperture}} f(x, y) \cdot e^{j k \sin \theta (x \cos \phi + y \sin \phi)} \, dx \, dy
\] (4.41)
is the function of the antenna radiation pattern, and the relation between it and the
distribution of the aperture field presents the form of the Fourier Transform.

In equation (4.41), let
\[ \sin u = \sin \theta \cos \phi, \quad \sin v = \sin \theta \sin \phi \]  
(4.42)
and for the observation point at the far zone field near the axis, as \( \sin \theta < 1 \), \( \sin u \) and \( \sin v \) can be approximately expressed by their independent variables \( u \) and \( v \) respectively. Therefore, equation (4.41) can be written as
\[ F(u, v) = \int \int f(x, y) \cdot e^{ik(x'y')} dx dy \]  
(4.43)
Taking the effects of phase errors caused by structural deformation into consideration, the far field gain function can be written as
\[ G(u, v) = \frac{k^2}{\pi} |I(u, v)|^2 \]  
(4.44)
where
\[ I(u, v) = \int \int T(x, y)e^{jL(x,y)}e^{ik(x'y')} dx dy \]  
(4.45)
where \( L(x, y) \) is the phase error caused by antenna structural deformation; \( T(x,y) \) is a tapering function and is defined as the electric field in the aperture plane normalised by square root of the input power \( P_{in} \) and free space impedance \( Z_0 \), i.e.
\[ T(x, y) = \frac{E(x, y)}{P_{in} \cdot Z_0} = \sqrt{\frac{G_f(\theta', \varphi')}{{4\pi(f^2 + \frac{r^2}{4f^2})}}} \]  
(4.46)
where \( G_f(\theta', \varphi') \) is the feed gain function (the illumination pattern of the feed). In most parabolic-reflector antennas, the illumination of the reflector is tapered so that the power density at the edge is of the order 10 ~ 15 dB lower than that at the centre. For Cassegrain antenna, based on the equivalent paraboloid method, the equivalent focal length is \( f^e = Mf \), where \( f \) is the paraboloid focal length and \( M \) is antenna magnifying factor.

### 4.6.2 Feed gain and the discussion about the tapering function
Antenna feed is a device designed to illuminate the surface of an antenna reflector when transmitting an RF signal, and to collect radiation reflected from the antenna when receiving. It is necessary to use feed gain in the computation of the gain of antenna far field. The amplitude of the projected ray is a function of the angle between the ray and the feed horn's boresight. This variation in illumination between the centre and the edge
of the reflector is known as the ‘edge taper’. The 8m antenna will have an edge taper of 15 dB.

If the sizes of the feed aperture, in pyramid horn feeds, as shown in Figure 4.14, are very much greater than the wavelength and, in addition, the open angle is small, the gain of feed far field can be approximately expressed as

\[ E - \text{plane: } E = E_0 \frac{\sin u_a}{u_a} \]
\[ H - \text{plane: } E = E_0 \frac{\cos u_a}{1 - \left(\frac{u_a}{\lambda}\right)^2} \]  \hspace{1cm} (4.47)

and

\[ u_a = \frac{\pi a}{\lambda} \sin \theta, \quad u_b = \frac{\pi b}{\lambda} \sin \theta \]  \hspace{1cm} (4.48)

where, \( a \) and \( b \) are the aperture size of rectangle feed along H plane and E plane respectively; \( \lambda \) is the working wavelength; \( E_0 \) is a feed constant.

In addition to the structural deformation, an antenna structure (except some off-set reflectors) has blockage effect, which will reduce the EM performance because of the blockage of propagation of the radio waves. This blockage comes from the sub-reflector and sub-reflector/feed supports (legs). The base of the legs must be attached to the main reflector, for example, for the 8m antenna the attachment is at about half its radius. The consequence of the attachment point is that there are two types of blockage loss. Firstly, plane wave blocking where the incoming waves strike the sub-reflector supports; and second, spherical wave blocking where the incoming waves are reflected by the outer part of the main reflector and then strike the supports. The influence of the second blockage is reduced by the amplitude taper across the main reflector. Calculations are made of the loss in efficiency due to the size and shape of the support. Typical sizes lead to a loss in efficiency about 5%, a value which is small by comparison with the reflector deformation losses at some loading cases.

Tapering functions are gradually changing and well behaved. A typical tapering function is shown in Figure 4.15. In the area with blockage of sub-reflector supporting systems, the functional value is zero.
In the computation of the aperture field integral, an engineering superposition method is used, which takes advantage of the linearity of the integral. In this method, the tapering function is broken into several parts and is written as unblocked tapering function plus several corrections, i.e.

\[ T(x, y) = T_u(x, y) - T_s(x, y) - \sum_{i=1}^{n} T_{th}(x, y) \]  

(4.49)

where the function \( T_u(x, y) \) is the tapering function in which the antenna would have no blockage; \( T_s(x, y) \) is the blockage function considering the sub-reflector blockage; \( T_{th}(x, y) \), (\( i=1, 2, ..., n \)), are the blockage function for sub-reflector supporting legs and \( n \) is the number of the legs. These functions have a zero value outside the area to which they refer and \( T_u(x, y) \) is the value inside this area. The contribution of each component function is calculated separately in the integral of equation (4.45) and then the results are totalled.

4.6.3. The computation of antenna’s gain and efficiency

A computer program, which is a EM performance solver for distorted reflectors, has been developed. Geometric-optics/aperture-integration has been used, which calculates the RF far-field pattern and provides the beam efficiency, gain, sidelobe level, and beamwidth at first null. The effects of the main reflector RMS surface deviations, deformation of sub-reflector/feed support structure and their blockages are included in the aperture integration, resulting in a total antenna system beam efficiency value. A brief flow chart is shown in Figure 4.16.

The antenna’s gain can be obtained by calculating the integral of equation (4.44). The calculation is composed of three main steps:

1) A lattice is created by dividing aperture field into parallel meshes; and the \( T(x,y) \) values of every point on the lattice is computed.
2) Based on the results of stiffness analysis of antenna structures, geometric ray tracing and aperture field interpolation, the phase errors of every point on the lattice is computed.
3) The computation of a two-dimensional discrete fast Fourier transformation (FFT) (Champeney, 1973) is carried out.

4. The analysis of the effects of structural whole-system deformation on antenna electromagnetic performance
After obtaining the antenna's gain $G$, the antenna's efficiency $\eta$ can be computed through following equation

$$\eta = \frac{100 G^2}{\pi D^2} \ %$$  \hspace{1cm} (4.50)

where $D$ is the diameter of main reflector.

### 4.7 The electromagnetic analysis of the distorted 8m antenna system

As an example, the 8m antenna is analysed using the analysis methods discussed in this chapter. The diameter of the main reflector is specified as 8m which means that it is 400 wavelengths in diameter at 15 GHz. The focal length, $F$, is determined by the illumination of the main reflector and factors associated with the sub-reflector supports. Consideration of the need for the antenna to be compact and the surface area of the main reflector to be a minimum indicates that an $F/D$ of 0.375 gives a good design.

The support structure of the antenna is described and analysed in Chapter 3. The working wavelength of the antenna is $\lambda = 0.02$ m, and focal length is $f' = 3$ m. The sub-reflector is a hyperboloid with radius 0.3 m and mass 15 kg, which is supported by three legs.

Based on the stiffness analysis of the antenna structure and the computation of the phase errors caused by the deformations of the structure at different working/loading cases, the antenna EM analysis is completed using GO/AI method. The obtained EM performances of the antenna system with structural distortions are: (1) antenna gains (in decibel) and efficiencies (in %) with surface deformations and blockages of sub-reflector and its supporting legs, (2) blocking gain losses (in dB) and phase-error gain losses (in dB), (3) radiation patterns, (4) main radiation beam widths at -3dB power, (5) sidelobe levels in U1 and U2 principal-plane patterns (U1 and U2 are mutually perpendicular patterns through the main-lobe axis). All the EM performances are computed at seven different elevation angles from horizon attitude to zenith attitude, including an ideal case (no structural deformation at all). The results are listed in Table 4.1.
Table 4.1 EM performances of 8m antenna system at seven working attitudes

<table>
<thead>
<tr>
<th></th>
<th>0°</th>
<th>15°</th>
<th>30°</th>
<th>45°</th>
<th>60°</th>
<th>75°</th>
<th>90°</th>
<th>no defo.</th>
</tr>
</thead>
<tbody>
<tr>
<td>antenna gain (dB)</td>
<td>58.72</td>
<td>58.78</td>
<td>58.94</td>
<td>59.16</td>
<td>59.39</td>
<td>59.55</td>
<td>59.60</td>
<td>59.61</td>
</tr>
<tr>
<td>antenna efficiency (%)</td>
<td>47.14</td>
<td>47.79</td>
<td>49.63</td>
<td>52.24</td>
<td>54.97</td>
<td>57.05</td>
<td>57.83</td>
<td>57.86</td>
</tr>
<tr>
<td>blockage gain loss (dB)</td>
<td>.3271</td>
<td>.3248</td>
<td>.3188</td>
<td>.3107</td>
<td>.3030</td>
<td>.2974</td>
<td>.2954</td>
<td>.2958</td>
</tr>
<tr>
<td>phase-error gain loss (dB)</td>
<td>.8902</td>
<td>.8300</td>
<td>.6659</td>
<td>.4434</td>
<td>.2227</td>
<td>.0612</td>
<td>.0021</td>
<td>.0000</td>
</tr>
<tr>
<td>mainlobe -3dB width (m deg.)</td>
<td>185.1</td>
<td>184.6</td>
<td>182.3</td>
<td>179.2</td>
<td>177.3</td>
<td>175.3</td>
<td>173.6</td>
<td>173.6</td>
</tr>
<tr>
<td>sidelobe area in pattern U1-plan</td>
<td>1280.</td>
<td>1279.</td>
<td>1273.</td>
<td>1265.</td>
<td>1255.</td>
<td>1248.</td>
<td>1245.</td>
<td>1229.</td>
</tr>
<tr>
<td>sidelobe area in pattern U2-plan</td>
<td>1153.</td>
<td>1147.</td>
<td>1130.</td>
<td>1090.</td>
<td>1065.</td>
<td>1077.</td>
<td>1073.</td>
<td>1071.</td>
</tr>
</tbody>
</table>

The EM radiation characteristics for the reflector antenna are illustrated in Figures 4.17 - 4.24 by a plot of radiated power density vs. the off-boresight angle θ (in radian, measured from the paraboloidal axis) which is scaled by multiplying π-D/λ. The plots are normalised to the maximum power density, referred to as the antenna gain G. The maximum sidelobe level (SLL) is defined as the highest level of the radiation pattern outside the main beam. All these patterns incorporate the effects of the antenna structural deformations.

These figures illustrate the effects of structural deformations on radiation patterns of the antenna. Figures 4.17 and 4.18 are the 3-D radiation patterns of the antenna when working at elevation angles 0 degree (horizon) and 90 degree (zenith). The vertical views of these 3-D radiation patterns are shown in Figures 4.19 and 4.20. It can be seen, from Figure 4.19, that the radiation pattern at elevation angle 0 degree is no longer symmetric because of the skew symmetry of the loading case; while from Figure 4.20, that the radiation pattern at elevation angle 90 degree is roughly symmetric because the loading of the antenna structure at this zenith position is basically symmetric.

Patterns for two orthogonal planes (U1 and U2) are presented. Figure 4.21 gives the radiation patterns in U1 plane at antenna elevation angles 0°, 30°, 60° and 90°. Fig 4.22 shows the radiation patterns in U2 plane at those angles. An ideal case where no structural deformation was supposed is also shown in the figures. For viewing them clearly, the radiation patterns in U1 and U2 planes at elevation angles 0° and 30° are shown in Figures 4.23 and 4.24.

4. The analyses of the effects of structural whole-system deformation on antenna electromagnetic performances
An antenna radiation pattern with high sidelobes and distorted shapes is particularly detrimental to the operation of an antenna. One of the most important performance parameters for an antenna is the amount of energy within the main beam (lobe) relative to the amount of energy in all other directions (sidelobes). The received power is an integral over all angular directions of EM emission arriving at the antenna. The emission received at each angle is weighted inside the integral by the relative power in the antenna radiation pattern for that angle. Therefore, low sidelobe levels in the radiation pattern suppress the influence of extraneous emissions. Conversely, large sidelobe levels can cause totally inaccurate readings. This happens, for example, if the antenna is pointed at areas of low emission, but side lobes are pointed at areas of high emission.

It can be seen from the figures that the radiation patterns for the undeformed antenna structure are symmetric, and the radiation patterns for the distorted antenna are no longer symmetric but become a function of the elevation angle. When the working elevation angles change (i.e. the structural loading conditions change), the radiation patterns will change. The change of elevation angles has more effect on U2-plane patterns than U1-plane patterns. When the elevation angles getting smaller, the distortions of the radiation patterns become worse. The smallest distortion of the radiation patterns occurs at antenna’s zenith attitude, and the biggest distortion occurs at horizon attitude.

In this chapter, a numerical method for combining structural analysis with EM aperture field analysis using FFT is given to determine the effects of structural deformation on EM performances. Instead of the nominal surface, the deformed reflector surfaces (real surfaces) are employed in the antenna EM analysis. Optical ray tracing, spline function aperture field interpolation techniques have been used to determine the difference of ray-path lengths and phase errors of all points on the deformed antenna aperture. Geometric optics aperture integration and fast Fourier transformation techniques have been used to analyse the EM performances of distorted reflector antennas. A connection between structural deformation and EM performances for different loading
cases is found therefore, including the gain-loss due to the blocking effect of subreflector and its supporting structure.

This chapter describes the analysis of the effects of structural deformation on antenna electromagnetic performances. This analysis will be repeatedly performed in an iterative optimum-seeking procedure for antenna structures. The optimisation method and procedure will be presented in the following chapters.
Fig. 4.1 Ray paths

Fig. 4.2 The geometric relationships of arbitrary dual reflector antenna

Fig. 4.3 Standard Cassegrain antenna
Fig. 4.4 The geometric parameters of Cassegrain antenna

Fig. 4.5 The ray-paths of deformed dual reflector antenna

Fig. 4.6 Deflection geometry of antenna surface
Fig. 4.7 True and approximate ray paths

Fig. 4.8 Main reflector deformation

Fig. 4.9 The contribution of displacement to ray-path length

4. The analyses of the effects of structural whole-system deformation on antenna electromagnetic performances
4. The analyses of the effects of structural whole-system deformation on antenna electromagnetic performances
Fig. 4.13 Aperture distribution and geometric parameters

Fig. 4.14 Pyramid horn feed

Fig. 4.15 The tapering function with blockage

4. The analyses of the effects of structural whole-system deformation on antenna electromagnetic performances
begin

input and print antenna geometric and EM parameters

get deformation information from structural analysis

surface interpolation by employing surface spline function

calculation of phase errors caused by structural whole system deformation

calculation of tapering function $T(x, y)$

discrete FFT calculation

the calculations of antenna gain, efficiency, radiation pattern, main beam width and sidelobe level

end

Fig. 4.16 EM performance analysis of distorted antennas

4. The analyses of the effects of structural whole-system deformation on antenna electromagnetic performances
Fig. 4.17 Antenna 3-D radiation pattern at elevation 0 degree

Fig. 4.18 Antenna 3-D radiation pattern at elevation 90 degree
Fig. 4.19 The vertical view of 3-D radiation pattern at elevation 0 degree

Fig. 4.20 The vertical view of 3-D radiation pattern at elevation 90 degree

4. The analyses of the effects of structural whole-system deformation on antenna electromagnetic performances
Fig. 4.21 The radiation patterns of U1 plane at different elevation angles

Fig. 4.22 The radiation patterns of U2 plane at different elevation angles

4. The analyses of the effects of structural whole-system deformation on antenna electromagnetic performances
Fig. 4.23 The radiation patterns of U1 and U2 planes at elevation 0 degree

Fig. 4.24 The radiation patterns of U1 and U2 planes at elevation 30 degree
Chapter 5

A Multi-factor Assessment Method for the Designs of Antenna Systems

5.1 Introduction

An antenna is a electro-mechanical interdisciplinary system and an antenna structure is a large complicated structure working at different environments and loading cases. An antenna's performance can be divided broadly into two parts, these are the structural and electromagnetic performances. The former comprises RMS deviation, stiffness, strength, stability, dynamic character and mass. The latter includes gain (efficiency), radiation patterns, sidelobe levels, and pointing accuracy etc. The performance of an antenna varies according to the loads on its structure, for example self-weight at different elevation angles, wind loading, temperature and other random loads. Therefore, antenna design is a complicated electro-structural synthesis problem involving a multi-factor assessment of many design variables, constraints and loads (Liu and Zeng, 1988; Zeng and Liu, 1987).

The complex electro-mechanical inter-relationships raises the question of how best to assess or appraise the design of such a system to yield an overall, quantitative performance index. New analytical and computational tools must be developed to assess the intricate effects of environment and loads on antenna structures and further on EM performance with increased confidence.

An antenna system design may be viewed as a complex, multi-level decision-making process involving an interrelated mix of quantifiable and non-quantifiable decisions. The interface between that portion of the structural design process that can be put on a strictly logical basis and that portion that is non-quantifiable is being addressed. Seeking to automate the quantifiable portion of the design process helps to distinguish the strictly logical decisions from the qualitative judgements.
A new systematic method for evaluating engineering design is presented here. This novel multi-factor design assessment procedure can be used to evaluate quantitatively the electro-mechanical design of antenna systems. The method is useful for the estimation of a global index of many performances of a design. The various performance parameters mentioned above are included and each parameter may be weighted according to importance. The evaluation procedure seeks to reveal weak spots in the performance of the system. It does this by reviewing the proximity of each calculated performance parameters (e.g. RMS) to the best possible expectation for various loading cases. Using the concept of parameter profiles (Liu and Thompson, 1996; Thompson and Goeminne, 1993), the procedure reviews, in a non-dimensional manner, the profile of the performance of antenna system parameters with respect to different loading cases. Certain overall performance indices are also obtained.

This method evaluates an antenna system design from structural performance, electromagnetic performance, and other points of view at many different working/loading cases simultaneously in addition to considering mass and cost. An overall performance is achieved as a function of many individual performance parameters which are obtained from structural and electrical analyses for a spectrum of loading cases. It allows a designer or chief engineer to make an objective judgement using predetermined weighting factors or to perform sensitivity studies by merely modifying the weighting factors.

To the extent that we can quantitatively formulate design synthesis problems and we can clearly define an overall index of the design for composite material antenna structural systems or any other complex systems, it should be possible to seek solutions using optimisation algorithms. This design synthesis concept provides a framework for formulating the quantifiable portion of a system design on which modern optimisation techniques can be brought to bear.

The feasibility of the method will be demonstrated in this chapter and the results of the structural and electromagnetic analyses of the 8m antenna are used to show how the evaluation procedure is applied. The numerical methods used in the structural and EM analyses are described previously. The results of the computational analyses and the evaluation are given.
5.2 Design assessment matrices

5.2.1 Performance data matrix (PDM)

The objective of the design assessment procedure is to bring out the characters of antenna systems. It is advantageous to apply quantitative analysis procedures as early as possible in a design to determine the merits and demerits of a system as it takes shape. The basis of the analysis method is a matrix of data which describes system performance under different working/loading cases of the antenna structure, see Table 5.1. The matrix, called the performance data matrix (PDM), is a schematic representation of a collection of data. The matrix lists every item of the loading cases considered and also every performance parameter relevant to the individual loading cases. The matrix is defined by the set of performance parameters $P_i$ included in the analysis at loading cases $C_j$ considered. Thus, data point $d_{ij}$ is the performance of the antenna with respect to performance $P_i$ at case $C_j$. All the data points of the matrix should be obtained by conventional structural and electromagnetic analyses of the antenna.

<table>
<thead>
<tr>
<th>$C_1$</th>
<th>$C_2$</th>
<th>...</th>
<th>$C_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>$d_{11}$</td>
<td>$d_{12}$</td>
<td>...</td>
</tr>
<tr>
<td>$P_2$</td>
<td>$d_{21}$</td>
<td>$d_{22}$</td>
<td>...</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$P_m$</td>
<td>$d_{m1}$</td>
<td>$d_{m2}$</td>
<td>...</td>
</tr>
</tbody>
</table>

Table 5.1 The performance data matrix

where
- $P_i$ = the $i$-th parameter describing the system performance
- $C_j$ = the $j$-th loading case of the antenna structure
- $d_{ij}$ = the data point

5.2.2 The limits and the best levels of performance parameters

The actual values of the performance parameters in Table 5.1 are of limited use by themselves. They are best considered with respect to the range of performance which may be expected of a parameter, i.e. an acceptable limit and a best performance that can be
expected from the antenna considered. Different performance parameters may have different acceptable limits. Some of them have upper limits or lower limits and some have both limits for different loading cases according to design requirements. Both the acceptable limits and the best level values are generally well known for the designers or can be determined from technical literature or from estimates by experienced designers. For example, for a middle-sized microwave antenna, the best level and acceptable limit with respect to the RMS deviation are about 0.025 mm and 0.15 mm respectively.

5.2.3 Parameter profile matrix (PPM)
The character of an engineering system is assessed by a review of the profile of the performance parameters at different loading cases, and with respect to the proximity of actual performance to the acceptable limit and the best level value of the performance.

An evaluation matrix called the parameter profile matrix (PPM) is used in the assessment method. The matrix, as shown in Table 5.2, is different from the performance data matrix in Table 5.1. The data point \( D_{ij} \) which is inserted into the evaluation matrix is a non-dimensional number in the range 0-10 which is determined by the closeness of the actual performance \( d_{ij} \) to the acceptable limit and the best level value of the performance.

\[
\begin{array}{cccc}
C_1 & C_2 & \ldots & C_n \\
D_{11} & D_{12} & \ldots & D_{1n} \\
D_{21} & D_{22} & \ldots & D_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
D_{m1} & D_{m2} & \ldots & D_{mn} \\
\end{array}
\]

The principle used for the derivations of data points \( D_{ij} \) is shown in Figure 5.1. If it is desired that a certain performance should be at a fixed value or in a limited value region, the case of Figure 5.1(b) will apply.
Best performance that can be expected — Score=10

Calculated performance d

Data point D
(0 < D < 10)

Lowest limit of acceptable performance — Score=0

Fig. 5.1(a) : Calculation of a data point where there is one acceptable limit

Upper limit of acceptable performance

Calculated performance

B

Range=20

Lower limit of acceptable performance

Date point D is the lower value of A and B (0 < D < 10)

Fig. 5.1(b) : Calculation of a data point when there are two acceptable limits

The calculation of the data point D\textsubscript{ij} for only one acceptable limit are as follows:

For the case of acceptable lower limit, the D\textsubscript{ij} is

\[ D_{ij} = \frac{d_{ij} - l_{ij}}{b_{ij} - l_{ij}} \times 10 \quad (5.1) \]

where, \(d_{ij}\) is the actual value of the performance taking from the matrix in Table 5.1; \(l_{ij}\) and \(b_{ij}\) are the lower limit and best level value respectively. Expression (5.1) is valid for \(l_{ij} < d_{ij} < b_{ij}\); for \(d_{ij} > b_{ij}\), \(D_{ij} = 10\), and for \(d_{ij} < l_{ij}\), \(D_{ij} = 0\).

The data point D\textsubscript{ij} for the cases of acceptable upper limit and double acceptable limits can be calculated in a similar way. For example, if a reflector surface RMS error (performance \(i\)) at an elevation of 60 degree (loading case \(j\)) is 0.02 mm and the corresponding highest limit of acceptable performance and best level values are 0.05 and 0.01 mm respectively, the data point D\textsubscript{ij} can be calculated as \(D_{ij} = 10 \times (0.05 - 0.02)/(0.05 - 0.01) = 7.5\).
This procedure is repeated for each performance parameter relevant for each loading case. The parameter profile matrix contains a combination of information about the antenna system, acceptable limits of performance, and the best performance values that can be expected. This combination of information results in new, valuable information which can be used to assess the antenna system and to form an optimisation mathematical model. The analysis of the matrix will be considered next.

5.3 Design evaluation principles — performance assessment by the analysis of the matrix

The information obtained from the parameter profile matrix makes it possible to evaluate the quality and serviceability of the system.

For each row and column, the mean and standard deviation (SD) for each parameter and loading case are calculated. The SD is a measure of the degree of the dispersion of the data around the mean. A well designed system should have a low SD and a high mean which is close to 10. The existence of high SDs signifies that the system will be likely to have significant problematic areas. Therefore, a high SD for a row indicates a very variable performance at different loading cases in the system for a particular parameter. A high SD for a column indicates that the system, at that loading case, will work close to an acceptable limit. The analyst may then decide if the design is satisfactory or if further engineering design work is required.

It is possible to analyse the system at a more advanced level. A parameter performance index (PPI) and a case performance index (CPI) can be defined:

\[
(PPI)_i = U_i \times n, \quad i = 1,2,...,m
\]  

(5.2)

where

\[
\frac{1}{U_i} = \sum_{j=i}^{n} \frac{1}{D_{ij}}
\]  

(5.3)
and $D_{ij}$ is the data point of the parameter profile matrix (see Table 5.2), and $n$ is the number of columns of the matrix. When a parameter $i$ is very vulnerable, i.e., some data points $D_{ij}$ of the parameter profile matrix are close to 0, the $U_i$ and $(PPI)_i$ will be close to 0.

Similarly,

$$(CPI)_j = V_j \times m, \quad j = 1, 2, \ldots n$$

where

$$\frac{1}{V_j} = \frac{1}{m} \sum_{i=1}^{m} \frac{1}{D_{ij}}$$

and $m$ is the number of rows of the matrix. When the antenna is vulnerable at a particular loading case $j$, then $V_j$ and $(CPI)_j$ will be close to 0.

The means, SDs, PPIs and CPIs give an overall performance rating for each system performance and each loading case respectively. A performance data point which is nearly 0 means that it is performing near its acceptable limit with respect to that particular parameter. The expression therefore has to be sensitive to low values of data points. The indices are calculated by summing the inverse of the data points to avoid the effect of any particularly low scores being hidden by high scores in other respects which is possible when only the mean is calculated. Simple multiplication is not used because the total score is then very prone to error in individual data points. The performance indices are brought into the range 0-10 no matter how many data points are used in each calculation for ease of analysis. This enables different parameters and loading cases to be compared in order to gain an overall perspective of the character of the system. Therefore, for each row and column, the mean, SD and index are calculated. The system may be reviewed by using this information as follows:

a) A comparison of PPIs will indicate if the system performs better with respect to some performances than others.

b) A comparison of CPIs will show if the system performs significantly better at some loading cases than others.

c) High mean values generally indicate that a good performance can be expected, should the design proceed to construction.
d) High SDs generally indicate that performances will be variable with respect to some parameters or at some working/loading cases.

By comparing the indices CPIs, the weakest loading case can be identified. Once this is given, it is possible to determine which performance parameter has the most influence on that weak behaviour. If this is given only for one item, then it is sufficient to search the particular column for the lowest performance data point. This will identify the parameter which performance needs to be improved.

5.4 Application of the 8m antenna analysis results to the evaluation procedure

This section shows how the method is applied to a design. To illustrate the evaluation procedure, the 8m antenna is analysed as an example. Seven different elevations are included in the analysis to rate antenna perform from horizon to zenith attitude and the following performance parameters of the antenna are taken into consideration:

1) antenna efficiency — is the most important electromagnetic performance which is calculated through an aperture field integral. In the analysis, the influence of the antenna structural distortion is considered.
2) RMS — is the RMS deviation of deformed reflector surface measured with half of the difference in path length of the microwave energy beam travelled with respect to its BFP (Best Fit Paraboloid) considering all the computation points on the reflector surface.
3) beam -3dB width — is a measure of the direction sensitivity of an antenna
4) side-lobe level — is the sensitivity in other directions outside the main beam or main lobe of an antenna radiation pattern
5) maximum displacement — is the maximum displacement of all the nodes in the antenna structure.
6) maximum stress — is the maximum stress of all the members in the antenna structure.
7) structural mass — is structural self-mass.
8) structural frequency — is structural fundamental frequency which is one important structural dynamic performance.
5.4.1 The establishment of system matrices

The performance data matrix obtained by the structural and electrical analyses for the 8m antenna is shown in Table 5.3.

Table 5.3 The performance data matrix for 8m antenna

<table>
<thead>
<tr>
<th>performance parameters</th>
<th>working/loading cases</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 deg.</td>
</tr>
<tr>
<td>antenna efficiency (%)</td>
<td>47.14</td>
</tr>
<tr>
<td>RMS error (mm)</td>
<td>0.0557</td>
</tr>
<tr>
<td>max. displacement (mm)</td>
<td>1.580</td>
</tr>
<tr>
<td>max. stress (MPa)</td>
<td>18.43</td>
</tr>
<tr>
<td>structural mass (kg)</td>
<td>515.7</td>
</tr>
<tr>
<td>-3 dB width (m deg.)</td>
<td>185.1</td>
</tr>
<tr>
<td>sidelobe area in pattern U1</td>
<td>1280.</td>
</tr>
<tr>
<td>sidelobe area in pattern U2</td>
<td>1153.</td>
</tr>
</tbody>
</table>

Of the performance parameters, antenna efficiency has lower limits, but beam -3dB width, side-lobe level, RMS, maximum displacement, maximum stress and structural mass have upper limits, and structural frequency has double limits.

For the 8m antenna, a matrix of codes of acceptable limits are listed in Table 5.4, where the numbers 0, 1 and 2 mean that the corresponding performance parameters have lower limits, upper limits and double limits respectively.

Table 5.4 The codes of acceptable limits for the 8m antenna

<table>
<thead>
<tr>
<th></th>
<th>0 deg.</th>
<th>15 deg.</th>
<th>30 deg.</th>
<th>45 deg.</th>
<th>60 deg.</th>
<th>75 deg.</th>
<th>90 deg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>antenna efficiency</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>RMS error</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

5. A multi-factor assessment method for the designs of antenna systems
As an example, we give the acceptable limits and the best level values for the antenna in Table 5.5. The requirements for the performance of an antenna vary with size, shape, working environment, manufacturing conditions, mission and working radio frequencies of the antenna. The data set presented is a typical set of the acceptable limits and the best level values for an antenna of the type analysed. For every pair of data, the upper one should be the best level values, upper limit or upper limit again respectively, and the lower one should be the lower limit, best level value or lower limit respectively, depending upon whether the performance parameter has lower, upper or double limits.

Table 5.5 The acceptable limits and the best level values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>0 deg.</th>
<th>15 deg.</th>
<th>30 deg.</th>
<th>45 deg.</th>
<th>60 deg.</th>
<th>75 deg.</th>
<th>90 deg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>antenna efficiency (%)</td>
<td>56.8</td>
<td>57.0</td>
<td>57.2</td>
<td>57.4</td>
<td>57.6</td>
<td>57.8</td>
<td>58.0</td>
</tr>
<tr>
<td>RMS error (mm)</td>
<td>0.080</td>
<td>0.075</td>
<td>0.070</td>
<td>0.065</td>
<td>0.060</td>
<td>0.060</td>
<td>0.060</td>
</tr>
<tr>
<td>maximum displacement (mm)</td>
<td>6.50</td>
<td>6.00</td>
<td>5.50</td>
<td>5.00</td>
<td>4.50</td>
<td>4.50</td>
<td>4.50</td>
</tr>
<tr>
<td>maximum stress (MPa)</td>
<td>90.0</td>
<td>90.0</td>
<td>90.0</td>
<td>90.0</td>
<td>90.0</td>
<td>90.0</td>
<td>90.0</td>
</tr>
<tr>
<td>structural mass (kg)</td>
<td>1100</td>
<td>1100</td>
<td>1100</td>
<td>1100</td>
<td>1100</td>
<td>1100</td>
<td>1100</td>
</tr>
<tr>
<td>structural frequency (Hz)</td>
<td>16.0</td>
<td>16.0</td>
<td>16.0</td>
<td>16.0</td>
<td>16.0</td>
<td>16.0</td>
<td>16.0</td>
</tr>
<tr>
<td>main beam -3dB width (m deg.)</td>
<td>206.3</td>
<td>204.3</td>
<td>202.0</td>
<td>200.0</td>
<td>198.0</td>
<td>196.0</td>
<td>194.0</td>
</tr>
</tbody>
</table>

3. A multi-factor assessment method for the designs of antenna systems 120
sidelobe area in pattern U1

<table>
<thead>
<tr>
<th></th>
<th>1350</th>
<th>1350</th>
<th>1340</th>
<th>1340</th>
<th>1330</th>
<th>1330</th>
<th>1320</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1230</td>
<td>1230</td>
<td>1230</td>
<td>1230</td>
<td>1230</td>
<td>1230</td>
<td>1230</td>
</tr>
<tr>
<td>sidelobe area in pattern U2</td>
<td>1270</td>
<td>1260</td>
<td>1250</td>
<td>1240</td>
<td>1230</td>
<td>1220</td>
<td>1210</td>
</tr>
<tr>
<td></td>
<td>1040</td>
<td>1040</td>
<td>1040</td>
<td>1040</td>
<td>1040</td>
<td>1040</td>
<td>1040</td>
</tr>
</tbody>
</table>

For the antenna, the parameter profile matrix is shown in Table 5.6. The nondimensional data in the matrix represents the proximity of the calculated performance to the limits and best level values of performance as described in Section 5.2.

Table 5.6 The parameter profile matrix

<table>
<thead>
<tr>
<th></th>
<th>0 deg.</th>
<th>15 deg.</th>
<th>30 deg.</th>
<th>45 deg.</th>
<th>60 deg.</th>
<th>75 deg.</th>
<th>90 deg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>antenna efficiency</td>
<td>3.88</td>
<td>3.86</td>
<td>4.67</td>
<td>6.15</td>
<td>7.91</td>
<td>9.36</td>
<td>9.84</td>
</tr>
<tr>
<td>RMS error</td>
<td>4.19</td>
<td>3.74</td>
<td>3.83</td>
<td>4.44</td>
<td>5.41</td>
<td>6.72</td>
<td>7.08</td>
</tr>
<tr>
<td>maximum displacement</td>
<td>8.56</td>
<td>8.46</td>
<td>8.54</td>
<td>8.83</td>
<td>9.34</td>
<td>9.99</td>
<td>10.0</td>
</tr>
<tr>
<td>maximum stress</td>
<td>8.84</td>
<td>8.89</td>
<td>9.10</td>
<td>9.45</td>
<td>9.91</td>
<td>10.0</td>
<td>10.0</td>
</tr>
<tr>
<td>structural mass</td>
<td>7.69</td>
<td>7.69</td>
<td>7.69</td>
<td>7.69</td>
<td>7.69</td>
<td>7.69</td>
<td>7.69</td>
</tr>
<tr>
<td>structural frequency</td>
<td>6.32</td>
<td>6.32</td>
<td>6.32</td>
<td>6.32</td>
<td>6.32</td>
<td>6.32</td>
<td>6.32</td>
</tr>
<tr>
<td>main beam -3dB width</td>
<td>5.97</td>
<td>5.88</td>
<td>6.35</td>
<td>7.17</td>
<td>7.68</td>
<td>8.27</td>
<td>8.88</td>
</tr>
<tr>
<td>sidelobe area in pattern U1</td>
<td>5.80</td>
<td>5.92</td>
<td>6.08</td>
<td>6.80</td>
<td>7.45</td>
<td>8.23</td>
<td>8.32</td>
</tr>
<tr>
<td>sidelobe area in pattern U2</td>
<td>5.11</td>
<td>5.14</td>
<td>5.72</td>
<td>7.52</td>
<td>8.70</td>
<td>7.94</td>
<td>8.08</td>
</tr>
</tbody>
</table>

On the first inspection of Table 5.6, it can be seen that performance parameter RMS has the lowest ratings. Therefore, the antenna design appears weakest in this respect and that the performance is worst at an elevation range of 15 ~ 30 degrees. Other observations are that the antenna design has a low efficiency rating at 0 ~ 30 degrees and that there are no concerns about the stress levels in the structure which is the most conservative aspect of the design.

5.4.2 System profile analysis

However, it is more informative to review the mean, standard deviation and parameter performance indices (PPIs) for the performance parameters across all the working/loading cases. These are given in Table 5.7.
### Table 5.7 System parameter profile analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>P.P.I.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antenna Efficiency</td>
<td>6.52</td>
<td>2.35</td>
<td>5.70</td>
</tr>
<tr>
<td>RMS Error</td>
<td>5.06</td>
<td>1.28</td>
<td>4.77</td>
</tr>
<tr>
<td>Maximum Displacement</td>
<td>9.10</td>
<td>0.63</td>
<td>9.06</td>
</tr>
<tr>
<td>Maximum Stress</td>
<td>9.46</td>
<td>0.48</td>
<td>9.43</td>
</tr>
<tr>
<td>Structural Mass</td>
<td>7.69</td>
<td>0.00</td>
<td>7.69</td>
</tr>
<tr>
<td>Structural Frequency</td>
<td>6.32</td>
<td>0.00</td>
<td>6.32</td>
</tr>
<tr>
<td>Main Beam -3dB Width</td>
<td>7.17</td>
<td>1.08</td>
<td>7.01</td>
</tr>
<tr>
<td>Sidelobe Area in Pattern U1</td>
<td>6.94</td>
<td>1.00</td>
<td>6.80</td>
</tr>
<tr>
<td>Sidelobe Area in Pattern U2</td>
<td>6.89</td>
<td>1.40</td>
<td>6.59</td>
</tr>
</tbody>
</table>

Inspection of Table 5.7 reveals more of the ‘character’ of the design. The structural performance parameters have high means and low standard deviations. Correspondingly the PPIs for these parameters are high. This means that the design is uniformly good for all working elevations with respect to displacement, stress, mass and vibration frequency.

The PPIs for efficiency and RMS are clearly lower than most other performance parameters. The antenna would appear to perform less well with respect to these parameters. Also the mean is low coupled with relatively high standard deviations. This indicates a variable performance across the range of working cases.

Turning to a review of the parameter profile matrix down columns, the mean, SD and case performance index (CPI) for each loading case is shown in Table 5.8.

### Table 5.8 Loading case profile analysis

<table>
<thead>
<tr>
<th></th>
<th>0 deg.</th>
<th>15 deg.</th>
<th>30 deg.</th>
<th>45 deg.</th>
<th>60 deg.</th>
<th>75 deg.</th>
<th>90 deg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>6.26</td>
<td>6.21</td>
<td>6.48</td>
<td>7.15</td>
<td>7.82</td>
<td>8.28</td>
<td>8.47</td>
</tr>
<tr>
<td>SD</td>
<td>1.68</td>
<td>1.75</td>
<td>1.62</td>
<td>1.40</td>
<td>1.32</td>
<td>1.24</td>
<td>1.25</td>
</tr>
<tr>
<td>CPI</td>
<td>5.81</td>
<td>5.70</td>
<td>6.05</td>
<td>6.84</td>
<td>7.58</td>
<td>8.09</td>
<td>8.28</td>
</tr>
</tbody>
</table>

5. A multi-factor assessment method for the designs of antenna systems

...
Inspection of Table 5.8 reveals that the lowest mean score (the most vulnerable loading case with respect to performance) is at an elevation of 15 degrees and the highest standard deviation is found also at this elevation. At 15 degrees elevation, the CPI is also lowest at 5.70, followed by scores of 5.81 and 6.05 at 0 and 30 degrees elevations respectively. At 45 degrees elevation, the CPI raises only to 6.84. Clearly then, the antenna will be likely to perform less well at elevation angles between 0 and 30 degrees.

Considering the analysis of performance profile indices, Table 5.7, and case profile indices, Table 5.8, it may be concluded that the weakest aspects of the antenna design are the efficiency and RMS when working at elevation between 0 and 30 degrees. Based on this conclusion, the designer may wish to revise the design and take specific, directed measures to effect improvement in the weak area identified.

5.5 Discussions and conclusions

When a large engineering system is being designed, it is not sufficient only to describe the requirements precisely and to apply the correct design methods in order to achieve quality, but it is necessary to analyse the design in its totality to determine the least efficient area in design.

A novel method has been presented, which can be used to produce an assessment of an engineering system with respect to the quality of design. This method considers and evaluates simultaneously a design from numerous points of view. The method is capable of analysing antenna system and other complicated engineering system with respect to different kind of performance and different kinds of variables. The method, which can be used as a design review tool, is rational, systematic, analytical and able to use quantifiable parameters.

At the heart of the method is the proximity of the level at which a system will perform in some respect to the level of performance which can reasonably be expected. This is similar in concept to a safety margin, that is, the closeness of the actual level of performance to a limit of performance. Thus, the method seeks to identify the 'weak spots' in a design. The intended use for the method is in a design review exercise or in an optimisation iteration procedure in which the overall quality of a design is being
assessed. Re-design of 'weak spots' may be carried out if necessary prior to manufacture.

The engineering synthesis approach can be employed to assess the sensitivity of optimum designs to perturbations in the load conditions and design variables, which will be discussed in the next chapter. This approach can provide a tool which can be employed to assess the merits of structural and material design for a specified structural task. Thus, engineers may be able to quantitatively specify goals, in terms of attainable combinations of engineering properties, towards which materials research and structural design should be directed.

In the design assessment procedure, matrixes are used to collect all relevant data. The matrix shape encourages a systematic approach to the design analysis. A parameter profile matrix is first analysed with respect to the individual parameters and the individual loading cases. The data points of the parameter profile matrix describe how far a device is working from its best level values and acceptable limits. This results in the parameter performance indices and loading case performance indices. From these indices, the system parameters and loading cases which are very vulnerable can be identified. The vulnerability is defined as a tendency to failure or to poor performance and is quantified by comparing the actual performance of a system with its best level values and acceptable limits of the system.

An 8m antenna electro-mechanical system is analysed by using the method to illustrate the feasibility and efficaciousness of the method. In addition to self-weight at different elevation angles which are calculated in this chapter, loading cases can include temperature, wind loading, other random loads, dynamic loads and their combinations (see Chapter 7 for more examples), and performance parameters can include structural reliability and more electromagnetic performances which are currently not considered in this chapter.

The antenna is treated as an engineering system, the overall performance of which is measured by different parameters. The system is first analysed with respect to the parameters which, collectively, describe the overall performance of the antenna. Then, in order to combine parameters, the calculated values are converted into scores using a linear relationship based on the actual performance to the nearest performance limit. Analysis of the scores is carried out with respect to each performance parameter for the range of loading cases of the antenna. An overall rating for each performance parameter
is obtained across the working/loading range. Similarly, for each loading case considered, the scores for each performance parameter are analysed to obtain an overall rating for each loading case. Thus, in a systematic manner the evaluation seeks to identify:

a) the parameters in which the performance is weakest and
b) the working/loading cases at which the antenna is least effective.

Simultaneous consideration of parameters and loading cases identifies the weakest aspects of the design.

In the case considered, the design may be analysed effectively without the aid of the method proposed. However, only a range of variables have been taken into account, the intention being to present the principles of the evaluation. Thus, the conclusions of the evaluation process may be checked readily. In a large system, a design review would involve many more variables. The detailed engineering analyses would be carried out by several engineers with different specialisms. The procedure proposed here brings together the separate analyses and combines them into a manageable design review procedure. Importantly, the results of individual analyses are combined in an attempt to obtain an overall perspective of a proposed design. The tendency to focus on particular aspects of design is avoided. The objective is to highlight those parameter/loading-case combinations which are the 'weak spots' of the design. Thus attention is drawn to significant areas where the antenna might under perform.

The evaluation process necessitates the establishment of performance limits which represent the best and worst performances that can be reasonably expected. The quantification of these limits is not easy. The argument may be put that this process is too difficult and that the evaluation will be subject to error if the limits are set incorrectly. However, to mount such an argument is to ignore the basic principles of evaluation. When any calculation is performed, the results only have value if they are judged against some criteria. In any design assessment, a judgement is made concerning the acceptability of a calculation which means that some limit of acceptability is used. Often design engineers make such judgements based on their experience. But what is experience? It is a knowledge of what is reasonable and what quantifies the assessment criteria and so requires quantitative judgements to be made based on experience. Therefore, when undertaking a design review using the method proposed, the designer must carefully and systematically: decide what are the acceptable

5. A multi-factor assessment method for the designs of antenna systems
performance limits of the antenna, analyse the proposed design with respect to the performance parameters for the range of working/loading conditions and combine the analysis with the acceptable performance limits.

This technique readily leads itself to the performance of many types of sensitivity studies. In the next chapter, the two kinds of indices PPI and CPI will be combined into a parameter/loading-case index, which indicates the highest influence on the overall system and the vulnerability (or superiority) of each individual parameter/loading-case combination. In the analysis procedure, both performance parameters and loading cases can, if desired, be weighted according to importance. An overall performance index (total score) can be obtained on the basis of the assessment method. The total score is an indicator of how well the design fulfils the desires associated with a given set of weighting factors, best level values and acceptable limits.

It may be concluded that a new systematic assessment method has been devised to analyse the quality of engineering systems such as antenna electro-mechanical systems. The evaluation uses quantified judgements of acceptable performance. The performance of the antenna is assessed at different loading cases for each performance parameter. The analyses are compared to performance limits in order to identify the proximity of the design to a limit of acceptable performance. In this way the ‘weak spots’ in a design may be identified in a design review exercise. The evaluation procedure can be used as the basis of a multi-factor optimisation problem incorporating structural and electromagnetic performance parameters.
Chapter 6

The Mathematical Model, Algorithm and Program of a Multi-factor Optimisation Method

6.1 Introduction

This chapter presents the development of an optimisation method which allows the optimisation of antenna structures and other engineering systems for optimal design based on multiple objectives and multiple loading conditions. Some recent advances and new techniques appeared in the last decade in general structural and system optimisation will be summarised in Section 6.3. A preliminary evaluation of the methodology on the optimisation of ground and space antenna structures will be presented in the following chapter. The results quantify the importance of structural factors in improving the antenna's structural and electromagnetic performances, and illustrate the suitability of this novel multi-factor optimisation method in the optimal design of such complicated structural systems.

With the advent of more powerful computers and the maturation of the finite element method, confidence has grown in the ability to predict the detailed performance of a structure. So, the desire has grown to improve the design in a systematic way toward the optimum. The need to reduce structural weight without compromising structural integrity is all important in airspace and aerospace applications, and much of the motivation behind the development of structural optimisation methods has been due to this factor. Furthermore, development has been assisted by making use of mathematical methods drawn from such fields as operation research and optimal control theory. The field of structural optimisation is blossoming into practical application.

A design process transforms the needs of the users, owners, and regulators into the detailed specification of a system, product, or object which has physical form and...
behaviour consistent with the needs of these customers. Design optimisation is the process of optimising the system to best suit the needs of the customers.

A structural optimisation process is fundamentally a two-step iterative procedure: structural analysis and structural redesign. The analysis is typically done using finite elements or other numerical methods, and a recurrence relation is used to redesign iteratively. It is concerned with achieving a structural design that minimises (or maximises) an objective function, usually weight, while also satisfying system constraints, such as stress or deflection limitations. Mathematically, the structural optimisation problem is stated as follows: Find \( X=(x_1, x_2, \ldots, x_n) \), the design variables, to minimise/maximise \( F(X) \), the objective, subject to that \( G(X) \leq 0 \), the inequality constraints, and \( H(X) = 0 \), the equality constraints, and \( X_L < X < X_U \), the side constraints. The main task in structural optimisation is determining the choice of the design variables, objectives, and constraints. Depending on the problem and the recurrence relationship, the process may converge to an optimum design. In structural optimisation, two fundamentally different approaches have been most commonly used (see Grooms, et al., 1990; Jan, 1986; Horimatsu and Kikuchi, 1993).

The first approach is called optimality criterion method (indirect approach). In this method, an optimality condition related to the behaviour of the structure is derived; the expectation is that when the structure is designed to satisfy this selected criterion, the objective function is automatically attains an optimum value. The selected criterion is generally intuitive but can also be mathematically defined. The optimality criterion is generally related to a set of non-linear equations and any method for solving these may be used to obtain an optimal solution. The fully stressed and uniform strength design is an example of early use of optimality criteria. Optimality criterion methods, like numerical optimisation methods, are iterative solutions and, also like numerical optimisation methods, impose constraints on the structure, such as allowable stress or maximum displacements. Methods using iterative schemes derived from the necessary conditions of optimality (Kuhn-Tucher conditions) are sometimes classed as optimality criterion methods even though they are purely numerical in nature.

The second approach is called mathematical programming methods (direct approach). These methods are applicable to a wide range of problems, of which structural optimisation represents only one particular application. In this approach, Non-linear
mathematical programming or sequence linear programming techniques are used to locate an optimum in a feasible design space. One starts with an engineering estimate of the optimum design, and a direction of travel (search) in the design space is then computed based on the local behaviour of objective and constraint functions. A small step taken along this direction moves the current design point to a new point which is close to the optimum. Finite element structural analyses form a mathematical basis on which numerical search procedures are used to progress iteratively to the optimum. The capability to deal with all types of objective and constraint functions makes these programming methods very versatile. Textbooks, (Zoutendijk, 1976; Beveridge and Schechter, 1970; Morris, 1982), provide lucid expositions of various algorithms used in programming techniques.

If the application range of a design system is limited in a single disciplinary problem for linear elastic or eigenvalue analysis of a structure, it is much more efficient to develop an optimisation system using the optimality criteria method to find the optimum. However, the optimality criteria method is highly dependent upon the nature of state equations in the design problem as well as a 'single' design constraint; furthermore, it is very difficult to extend in order to solve multi-disciplinary design optimisation problems (Horimatsu and Kikuchi, 1993). The method of mathematical programming, especially non-linear programming is most suitable for the system because the method is independent of the state equation and in most cases, problems have highly non-linear objective functions or constraints.

The majority of existing design optimisation is for stress analysis. However, a wide range of state equations must be involved in structural optimisation for complicated systems, and they might be simultaneous. Thus it is required in a design system that any kind of state equations can be dealt with using different analysis software; i.e. a design system must be open. If other discrete methods such as boundary element and finite difference methods are applied to solve state equations, we should still be able to incorporate with these different type analysis capabilities in a design optimisation system.

Generally speaking, optimisation has the following limitations:

6. The mathematical model, algorithm and program of a multi-factor optimisation method
1) With the exception of unimodal problems, optimisation cannot guarantee a global optimal unless an exhaustive search has undertaken (which may require a prohibitive amount of CPU time).

2) If the analysis program is not theoretically precise or if the design problem formulation is not accurately and adequately defined, the optimisation results may be misleading.

3) Optimisation will invariably utilise errors to yield mathematical design improvements.

Multiobjective optimisation has recently been acknowledged as an advanced design technique in structural optimisation (Grandhi, Bharatram and Venkayya, 1993; Eschenauer, Koski and Osyczka, 1990). Most of the real-world problems are multidisciplinary and complex, as there is always more than one important objective function in each problem. These design problems may be at least formally cast into a mathematical optimisation problem:

Find design variables

\[ X = (x_1, x_2, ..., x_n) \]

to optimise the structural performance

\[ F(X, S) = F(f_1(X, S), f_2(X, S), ..., f_m(X, S)) \]

subject to

\[ \{ g_j(X, S) \leq 0, h_i(X, S) = 0, S_k(X, S) = 0; \quad j = 1, ..., p, i = 1, ..., q, k = 1, ..., m \} \]

where \( S_k \) are the system equations. The system parameter vector \( S \) (displacements, frequencies, ...) has to be determined for a given design variable vector \( X \).

To accommodate many conflicting design goals, it is necessary to formulate the optimisation problem with multiobjective. The objective function \( F \) is a certain combination of some subobjectives as functions of the design variables \( x_1, ..., x_n \) such as cross-sectional areas, plate thicknesses, fibre angles, laminate parameters etc. A constrained multi-objective problem involving minimisation and/or maximisation of \( m \) objective functions is described in the following mathematical form:

\[ \min \{ f_1(X), f_2(X), ..., f_p \} \quad \text{and} \quad \max \{ f_{p+1}(X), f_{p+2}(X), ..., f_m(X) \} \quad (6.1) \]

One important reason for the success of the multiobjective optimisation approach is its property of allowing the designer to participate in the design selection process even after the formulation of the mathematical optimisation model.
The solution of a multi-criteria problem is a so called efficient or Pareto-optimal design where an improvement in one component of \( F \) can be achieved only by a worsening of at least one of the others. It also depends on the choice of weighting factors which might be hard to define at least at the beginning of the optimisation investigation to obtain a 'reasonable' Pareto-optimum (Baier and Helwig, 1985).

In many cases, reduction of duration of design and manufacturing processes becomes much more important than reducing the cost of raw material of a product while the required functionality is fulfilled, to reduce the overall cost and to lead success of a new product. In order to reflect this situation to research and development in design methodology, we must reconstruct the notion of structural optimisation. In the past most structural design optimisations tried to minimise the cost of raw material under certain constraints which are implied from mechanics and manufacturing requirement. But now, the most important matter is how easily certain design can be improved with minimal effort by design engineers rather than just considering minimisation of the cost (i.e. weight) of raw material. In other words it is becoming much more important to examine how the optimal design can be achieved (Horimatsu and Kikuchi, 1993).

Antenna structures must be designed in such a way that, under the design loads and environmental conditions, the displacements remain, in every case, in the elastic field, and their magnitude is properly kept within the limits compatible with the specified EM performances. Although there is a substantial literature in the field of optimum structural design, most frequently considered is design for minimum structural weight subject to primary behavioural constraints (such as stress, displacement, and bucking) and side constraints (such as fabrication requirements). Much of this is useful background, but has only indirect bearing on reflector structure design.

The design goal for a high precision antenna structure is to maximise its performance while satisfying constraints on static as well as dynamic displacements, stresses, eigenfrequencies, weight, deployment performance etc. From a mathematical perspective, many constraints would be the equations governing the electromagnetic behaviour, the structural behaviour, and any coupled electromechanical behaviour for the system. Other constraints could be limits for certain variables dictated by current technology. Finally, other constraints could arise from system factors such as the 

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6. The mathematical model, algorithm and program of a multi-factor optimisation method
project budget or management mindset. Typical performance criteria are shape errors such as RMS contour and pointing errors caused by manufacturing and environmental loads. The RF surface efficiency performance requirements tend to restrict the permissible structural deformations to considerably less than those for ordinary structures of corresponding dimensions. Stress constraints arise from launch loads for space reflectors or heavy wind loads for earth bound reflectors but usually are not severe. Consequently, a deflection-oriented design is required, with the result that stresses in relatively few of the structural members tend to become critical, even during nonoperational survival hurricane wind velocities (Levy and Melosh, 1973). Mass limitations are obvious for space structures and are imposed for earth bound systems to limit CFRP material costs. In order to separate structural frequencies from attitude/servo control system frequencies or critical launcher and space craft-bus frequencies, and minimise active control needed, constraints on eigenfrequencies are to be imposed. If applicable, proper and accurate deployment is essential. Typical design variables range from configurational and topological properties such as sandwich and backup structure design principle, to material selection, stiffness distribution and in the case of severe requirements also active elements to provide shape control and active damping (Baier and Helwig, 1985).

High surface accuracy results in improved EM performance. However, increases in surface accuracy and EM performance may typically result in structural mass addition. Therefore, an optimisation model including both the performance and structural mass as objectives is important. There is great interest in integrated structural and EM optimisation, but most research has focused on problems with simple models and simplified constraints implemented using special purpose software. Reported work on the optimisation of antenna structures is mostly limited on structural performances instead of combined structural and EM ones. Obviously, new methods are required to optimise these large antenna structures for low weight while meeting all structural, electromagnetic, operational, and safety requirements.

In application, design specification may impose specific constraints for different loading cases. For a structure with a single loading case, the optimum design for a certain objective function with some constraints can readily be found. However, the resulting optimum design may not satisfy the constraints for the structure in other loading cases.
and, more often than not, may even worsen other design objectives. These make the
optimisation impractical.

The design loading^t^5 to be considered for performance improvement constitute an infinite
set of loading cases. For space antennas, for example, the temperature distribution of
the structures in space environment will change greatly. For ground antennas, the
orientation of the gravity loading vector relative to the structure is changeable over a
continuous range of elevation attitudes. This loading, moreover, is a design-dependent
function of weight distribution of the structural members. Therefore, the loading system
for antenna structures is apparently an overwhelming obstacle to the optimisation
process.

All of the above mentioned items make it very difficult to optimise the design of an
antenna structure, as it was said in (Mikulas and Collins, 1991) that ‘no attempt is made
to optimise the reflectors because it is extremely difficult to establish an absolute
objective function’. However, the current work of this thesis gives a solution to these
complicated problems.

An approach for multi-objective structural optimum design under many constraints at
many loading cases has been developed. Based on generalised compound scaling
techniques, this multiobjective optimisation algorithm handles any number of objective
functions, similar to handling behaviour constraints. Pseudotargets are defined for the
objective functions at each iteration, to integrate them into a total pseudo objective. An
optimisation mathematical model has been found. This mathematical model converts
a constrained optimisation problem to an unconstrained one, and a new method for
multi-objective optimisation based on this technique. In the optimum design procedure,
these objectives, constraints and loading cases are considered simultaneously.

This optimisation method transforms the performance objective functions into a set of
goal functions in connection with loading cases. An envelope (cumulative goal function)
of all the set of functions is searched for an unconstrained maximum. Search toward the
maximum of the envelope function advances the design toward the compromise
constrained optimum. That optimum is reached in an iterative procedure, which updates
a set of behaviour and performance objective functions and their envelope function at
the outset of each iterative cycle. The constrained optimum it attains conforms to the

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classical Pareto-optimum definition. In multi-objective applications, the approach has the advantage of locating a compromise optimum without the need to optimise separately each individual objective function. The constrained to unconstrained conversion is described, following a description of the multiobjective problem. The conversion technique may be categorised as a ‘Sequential Unconstrained Minimisation Technique’ (SUMT) class method, but it does not require the use of a draw-down factor, unlike the classical procedure. Also the unconstrained function it uses to present the constrained problem at hand is defined over both the feasible and non-feasible domains, similar to an extended penalty function.

In this optimisation approach, the mathematical models of optimisation problems are solved with non-linear programming, and a modified conjugate directions search technique is utilised. In contrast to other multiobjective optimisation procedures, the method showed an ability to locate compromise optimum design without the expense of having to optimise individual objectives and to include as many loading cases as desired in a one-run of the optimisation procedure.

The above approach has been implemented in the computer program MOST which can employ ABAQUS as its structure analysis code. Examples will be presented to demonstrate the robustness of the method, and to illustrate the capability of MOST system in the optimum design of structures with multi-objective, multi-constraint and multi-loading-case.

6.2 The mathematical description of optimisation problems

A structural system is described by a set of quantities, some of which are viewed as variables during the design process. In formulating a structural optimisation problem, certain quantities are fixed at the outset and these will be called preassigned parameters. All those quantities describing the structural system that are not preassigned will be called design variables.

The form of a general mathematical programming problem is: Find the set of design variables \( X = (x_1, x_2, ..., x_n)^T \) that will

Minimise or maximise performance parameter

\[ F(X) \quad \text{Objective Function} \]

Subject to
\[ G_j(X) \leq 0, \quad j=1, p \]  
Inequality Constraints

\[ H_i(X) = 0, \quad i=1, q \]  
Equality Constraints

\[ x_i^L \leq x_i \leq x_i^U, \quad i=1, n \]  
Side Constraints

Where, \( n \) is the number of design variables, \( p \) is the number of inequality constraints, and \( q \) is the number of equality constraints. The number of inequality constraints tends to be high for many problems. For example, consider a truss design problem where each member has stress and buckling constraints. Also, all constraints may be calculated for multiple load conditions. \( X^L \) and \( X^U \) are lower and upper bounds on the design variables, respectively. A common use of lower bounds is to prevent the cross-sectional areas from going below zero. It would make no sense to design a member that had a negative cross-sectional area (Hansen and Vanderplaats, 1988).

The following constraints on structural behaviour must be satisfied:

1) Member stresses
\[
\sigma_i^L \leq \sigma_i \leq \sigma_i^U \tag{6.2}
\]
where \( \sigma_i^L \) is the maximum compressive stress and \( \sigma_i^U \) is the maximum tensile stress for member \( i \) under load condition \( j \).

2) Euler buckling
\[
\sigma_{bl} \leq \sigma_i \tag{6.3}
\]
where \( \sigma_{bl} \) is the Euler buckling compressive stress limit for member \( i \). For a truss member, it is taken as
\[
\sigma_{bl} = -K_iE_iA_i/L_i^2 \tag{6.4}
\]
where \( K_i \) is a constant determined from the cross-sectional geometry and \( E_i \) is the Young's Modulus of the material. Local crippling and system buckling constraints are not expressed here.

3) Limits on member sizes
For example, truss member cross-sectional areas are constrained within a range defined by
\[
A_i^{min} < A_i < A_i^{max} \tag{6.5}
\]
where \( A_i^{min} \) and \( A_i^{max} \) are the minimum and maximum member areas for the \( i \)-th design variable.

4) Linking of member-size design variables
A practical consideration in design is to limit the number of design variables with unique values (such as unique cross-sectional areas, unique plate thicknesses etc.) Hence, design variables may be linked in the design process as

$$A_k = A_i$$

(6.6)

where \( A_k \) is a dependent and \( A_i \) is an independent design variable.

5) Limits on co-ordinate design variables

Co-ordinates may be constrained during the optimisation as follows

$$x_j^- \leq x_j \leq x_j^+$$

(6.7)

where \( x_j^- \) and \( x_j^+ \) are lower and upper bounds on the location of joint \( j \).

6) Linking of co-ordinates variables

Co-ordinate variables are often linked to preserve the symmetry of a structure. The relationship between linked co-ordinates is

$$x_k = a_k + b_k x_i$$

(6.8)

where \( a_k \) and \( b_k \) are constraints; \( x_k \) is a dependent co-ordinate variable; and \( x_i \) is the independent co-ordinate variable.

The optimisation problem is non-linear when at least one of the above equations is non-linear. Typically, the structural optimisation problem is a non-linear problem whose solution requires an iterative process. In each design cycle the optimum design problem is solved iteratively by a combined analysis/optimisation procedure. The combination of finite element analysis together with optimisation method constitutes an effective and reliable approach for solving practical optimum design problems. The iterative process involves five major steps:

1) Development of an initial design
2) Analysis of the design
3) Sensitivity analysis
4) Redesign
5) Assessment of design convergence (Return to step 2 if necessary).

When the optimisation problem has multiple minima, the development of the initial design can have significant influence on the final optimal design configuration found. This dependence on the initial design for a solution is typical of numerical approaches
for the analysis of non-linear equations. The challenge is to find the initial design that yields the global minimum. The initial design also affects the number of solution search cycles required to converge on an optimal design. In fact, with the selection of some initial designs, an optimisation process may fail to converge (Grooms, et al, 1990).

Numerical structural analysis, step 2, is an important part of the systematic design process because it verifies that the design is within the feasible region (i.e. no constraints have been violated). Finite element analysis technology, used in numerical structural analysis, enables the designer to reliably and efficiently analyse highly complicated structures as accurately as he desires.

The evaluation of objective function, constraint functions, objective derivatives and/or constraint function derivatives with respect to design variables constitutes design sensitivity analysis. The calculation of these functions and derivatives poses a serious obstacle to some applications of structural optimisation with many design variables (>1000) because of the high computational costs associated with implementation.

The fourth step of the optimisation process is to synthesise a better design—one that improves the system's objective function. Using the appropriate optimiser is important since it will affect how many iterations of the overall design cycle are run before an optimum is reached and even many well-known optimisation methods may not be suitable for a specific optimisation problem. The most common form of the redesign process is

\[ X^{i+1} = X^i + \alpha S^i \]  \hspace{1cm} (6.9)

where \( i \) is the iteration number, \( S \) is a search direction vector, and \( \alpha \) is a scale that defines the step-length to move in direction \( S \) in the design space.

The final step of the iterative process assesses the convergence of the design. The convergence criteria might be any or all of several popular tests, such as the Kuhn-Tucher conditions, changes in objective function gradients, changes in design variables, or simply a limit on the number of iterations.

\[ 6. \text{ The mathematical model, algorithm and program of a multi-factor optimisation method} \]
6.3 Multi-objective optimisation, Pareto concept, multidisciplinary design optimisation, genetic algorithms and artificial neural networks

In the past decade, great progress is made and some new techniques appeared in the area of structural and system optimisation. This section will give a brief review in these respects.

6.3.1 Multi-objective optimisation and Pareto concept

Multi-Objective Optimisation extends optimisation theory by permitting multiple objectives to be ‘optimised’ simultaneously. It is known by various names which include Pareto optimisation, vector optimisation, efficient optimisation, multicriterion optimisation, and others. The solutions are referred to as Pareto optima, vector maxima, efficient points, and nondominated solutions. Multi-objective optimisation has been used in economics (Takayama, 1974) and management science (Evans, 1984) for years and has gradually crept in engineering (Wu, 1995).

Difficulty in defining a single objective function in many engineering design problems is a motivation for continuing interest in development of techniques for multiobjective optimisation applications (Sobieszanski-Sobieski, Dovi and Wrenn, 1988). A variety of techniques and applications of multiobjective optimisation have been developed over past few years. A summary of the progress in the field of multiobjective optimisation has been given by Grandhi, Bharatram and Venkayya, (1993). A earlier summary was given by Stadler, (1984). They inferred from their surveys that if one has decided that an optimal design is to be based on the consideration of several criteria, then the multicriteria theory (Pareto theory) provides the necessary framework. Structure, system, product and process optimisation are best viewed as a Pareto optimal process seeking a consensus in which many objectives are balanced so that the improvement of any single objective will result in a negative impact on at least one other objective. Such a system of objectives is said to be Pareto optimal at any point for which this is true. In addition, if the minimisation or maximisation is the objective for each criterion, then an optimal solution should be a member of the corresponding Pareto set. Only then does any further improvement in one criterion require a clear trade-off with at least one other criterion.
The basic statement for a general multiobjective (multicriteria) optimisation problem is:
Find the vector of design variables
\[ X = (x_1, x_2, \ldots, x_n)^T \]
that minimise and/or maximise a vector objective function
\[ F(X) = (f_1(X), f_2(X), \ldots, f_m(X))^T \]
over the feasible design space
\[ X \in X \subseteq \mathbb{R}^n \]
where the functional constraint set is
\[ X = \{X \in \mathbb{R}^n : X \in \Omega \subseteq \mathbb{R}^n, g(X) \leq 0, h(X) = 0\}. \]

When all of the constraint functions and the criterion functions are linear, then, of course, one has a linear multicriteria programming problem.

It is the determination of a set of nondominated solutions (Pareto optimum solutions or noninferior solutions) that achieves a compromise among several different, usually conflicting, objective functions. The Pareto optimal is stated in simple words as follows (Grandhi, Bharatram and Venkayya, 1993): 'A vector \( X^* \) is Pareto optimal if there exists no feasible vector \( X \) which could increase some objective function without causing a simultaneous decrease in at least one objective function.' This definition can be explained graphically. An arbitrary collection of feasible solutions for a two-objective maximisation problem is shown in Figure 6.1. The area inside of the shape and its boundaries are feasible. The axes of this graph are the objectives \( F_1 \) and \( F_2 \) which need to be maximised. It can be seen from the graph that the noninferior solutions are found in the portion of the boundary between points A and B. For multi-objective optimisation, the solution space may be viewed as a space of compromise solutions in which each objective could be improved, but if it was, it could be improved at the expense of at least one other objective. Any point of these constrained multiobjective maximum solutions has the property that one can not depart from it without either violating the constraint(s) or decreasing at least one of the objective functions — the classical definition of a Pareto-optimum. A Pareto optimal solution is not unique, but is a member of a set of such points which are considered equally good in terms of the vector objective. Thus, here arises the decision-making problem from which a partial or complete ordering of the set of nondominated objectives is accomplished by considering the preferences of the decision maker.
Fig. 6.1 Graphical interpretation of Pareto optimal

Usually the criteria, which conflict for an antenna structural design, are a measure of the accuracy of the surface, a weighted RMS value of the residual deviations from a best-fit deformed surface of the same type as the original surface, and the mass of the structure.

Pareto optimality serves as the basic multicriteria optimisation concept in virtually all of the previous literature. The main purpose of their work was to apply the multiobjective optimisation techniques to the selection of system parameters and to solve structural design optimisation problems. Radford et al., (1985) in their study explored the role of Pareto optimisation in computer-aided design. Rao, (1984) and Rao, (1987) treated several different problems mainly by either applying the methods in which the objectives are a priori fixed or using the goal-programming and the game theory approach. Usually, the Pareto-optimal set was determined by the linear weighting, the min-max, or the constraint methods. Using linear goal-programming techniques with successive linearisation, El-Sayed et al., (1989) demonstrated an algorithm for solving non-linear structural optimisation problems. A three-bar truss problem with uncertainty in load was solved by using goal-programming technique (Sandgren, 1989). Hajela and Shih, (1990) proposed a minimum variant of the global criterion approach to obtain solutions to multiobjective optimum design problems involving a mix of continuous, discrete, and integer design variables. Saravinos and Chamis, (1992) developed a design for lightweight, low-cost composite structures of an improved dynamic minimum variant of the global criterion approach. Tseng and Lu, (1990) proposed a mini-max multiobjective optimisation model for truss structural optimisation. Because of the complexity of a multiobjective optimisation, some researchers such as Sobieski, (1982)
and Barthelemy, (1989) have proposed to decompose the engineering design problem into a number of optimisation subproblems and then optimisation procedures can be applied to the decomposed design problem.

Most of the multiobjective optimisation techniques are based on how to elicit the preferences and determine the best compromise solution. Nearly all of the solution schemes used in multiobjective optimisation involve some sort of scalarisation of the vector optimisation problem. The vector problems are replaced by some equivalent scalar minimisation problem. Because the Pareto set is generally infinite, an additional use of scalarisation is the selection of a unique member of the Pareto set as the optimum for the vector optimisation problem. Usually, a problem is scalarised either by defining an additional supercriterion function or by considering the criteria sequentially. Many of the multiobjective optimisation methods require either a conversion to a single objective function by means of a composite function with judgmental ‘weight factors’, or separate optimisations for each objective followed by an additional ‘global’ optimisation to arrive at a suitable compromise.

Balachandran and Gero, (1987) have discussed the relative merits and demerits of three basic techniques used for generating noninferior solutions. These are the weighting method, the noninferior set estimation (NISE) method, and the constraint method. These methods also come under the category of nonpreference technique. However, some nonpreference techniques use preference techniques (e.g., weighting and NISE methods) concepts repeated over a number of different parametric values to generate the entire Pareto set. Still confronted with these Pareto solutions, the designer must choose among them by some other means (Grandhi, Bharatram and Venkayya, 1993).

6.3.2 Multidisciplinary design optimisation (MDO)

Multidisciplinary design optimisation (MDO) is a developing field of study that is concerned with how to optimally design and analyse systems composed of multiple disciplinary models that are coupled (Ballling and Wilkinson, 1997). Usually the design of such complex systems is performed by a team that is subdivided into groups associated with the disciplines. It is a part of the concurrent engineering technology that may well be an enabling technology for complex advanced systems (Sobieszczanski-Sobieski and Tulinius, 1992).
Several approaches started to appear in the last decade for formulating and solving MDO problems mainly in the area of aircraft industry. In the past, MDO approaches have been categorised as either hierarchic or nonhierarchic according to the types of systems to which they apply (Sobieszczanski-Sobieski, 1990; Balling and Sobieszczanski-Sobieski, 1996). In hierarchic systems, children disciplines are coupled only to parent disciplines and not to each other. Nonhierarchic systems are more general since no restrictions are placed on how disciplines are coupled.

Balling and Wilkinson, in their recent paper (1997), categorised MDO approaches into three groups: The first group consists of single level optimisation approaches, in which, optimisation is performed only at system level, and the role of the disciplines is limited to analysis and function evaluation (Hajela, Bloebaum, and Sobieszczanski-Sobieski, 1990; Haftka, Sobieszczanski-Sobieski, and Padula, 1992). The second group consists of collaborative optimisation approaches (Kroo, et al, 1994; Balling and Sobieszczanski-Sobieski, 1995). The third group consists of concurrent subspace optimisation approaches (Renaud, and Gabriele, 1993; Eason, et al, 1994). In these latter two groups, optimisation is performed at both the system level and within the disciplines. A major difference between these groups is that in concurrent subspace optimisation, each discipline attempts to satisfy its own constraints as well as approximations to the constraints of the other disciplines, whereas in collaborative optimisation, each discipline satisfies its own constraints and tries to match target values on coupling functions that are needed by other disciplines in the evaluation of their constraints.

Since the MDO is a developing field of study, the robustness and efficiency of the approaches developed in the last few years are not well understood. The proper selection of approach is vital to the efficient solution of MDO problems. The current usefulness and future development of MDO methodology is hampered by the lack of a common accepted and readily available test problems. Realistic test problems often involve cumbersome disciplinary software packages and results may be strongly influenced by interfacing details and the internal programming characteristics of the disciplinary analyses (Balling and Wilkinson, 1997). These may cloud the inherent behaviour of the MDO approaches and makes test problems difficult to reproduce. Therefore, further research is needed.
Researchers at NASA Langley Research Centre announced at the Sixth AIAA/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimisation in September 1996, that a prototype MDO test suite, for evaluation by the MDO community, would be constructed. Access to the test suite is now available through the Internet (Padula, 1997). The home page at the universal resource locator (URL) is:

http://fmad-www.larc.nasa.gov/mdob/

6.3.3 Genetic algorithms (GAs)

In recent years, genetic algorithms (GAs), inspired by the basic mechanism of natural evolution and based on the theory of biological evolution and adaptation, have been used in the field of the engineering optimisation. GAs are emerging as a viable tool for dealing with the problem of discrete design variables and provide the designer with multiple optima as against a single optimum solution. GAs are efficient global-search algorithms in which the iterative histories of structural optimisation problems are simulated by artificial evolution and adaptation. In GAs, the Darwinian survival-of-the-fittest theory is employed to yield the best or better characters among the old population (Goldberg, 1989), and a random information exchange is performed to create superior offspring. A GAs-based structural optimisation algorithm requires encoding of design variables as bit strings, evaluation of fitness of each string in the population (require a structural analysis), and population regeneration using genetic operators such as reproduction, crossover, and mutation. To simulate the biological evolution, GAs uses the fitness to represent the objective value of the optimisation, and artificial chromosomes to represent the design variables.

GAs are different from gradient-based mathematical-programming algorithms which, similar to local hill climbing, usually seek a solution in the neighbourhood of the starting point. If more than one local optimum exists (non-convex problem), the solution will depend on the choice of the starting point, and the global optimum may not be found. In contrast to the conventional search algorithms, basic GAs have the following characteristics (Adeli and Cheng, 1993):

1) All the genetic-algorithm operations work with finite-length binary strings (chromosomes) instead of real parameter sets, resulting in a finite point-search algorithm.

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2) GAs consider a group of points in the search space in every iteration other than a single point, called a population of points.

3) GAs use a random search based on the prior information to guide the search, instead of gradient search, so that the derivative information and step-size calculation are not necessary.

4) GAs must work in a bounded space for coding the parameters.

5) GAs are not hill-climbing algorithms. So-called local hill-climbing problems are eliminated in these algorithms. Therefore, the probability of being entrapped in a local minimum/maximum is reduced.

6) GAs algorithms can be used directly only for solving unconstrained optimisation problems. Therefore, in order to use GAs, a constrained optimisation problem must be transformed to an unconstrained one.

The advantage that GAs require only the function information guarantees that GAs can be applied to solve a large range of optimisation problems. By maintaining a population of well-adapted artificial chromosomes, GAs have more potential to obtain the global optimum solutions. The use of GAs as an approach to solving structural optimisation problems is reported in (Hajela, 1990; Jenkins, 1991; Deb, 1991; Rao, et al, 1991; Lu, et al, 1996). GAs have also been reported in solving composite structural optimisation problems. These include: laminate design problems (Callahan and Weeks, 1992; Ball, Sargent and Ige, 1993), sandwich structure optimisation problem (Kodiyalam, Nagendra and DeStefano, 1996), laminate stacking sequence design problem subject to buckling and strength constraints (Le Riche and Haftka, 1993), and the design optimisation problems of unstiffened and stiffened composite panels (Nagendra, Haftka and Gurdal, 1992; Nagendra, et al, 1994).

Despite the successful use of GAs in some optimisation problems, a major drawback of GAs is that they often require a high number of function evaluations and therefore require unacceptable large computer processing time for large-scale, finite element-based structural optimisation problems. In reducing the expensive computational cost of GAs, approximation procedures have been reportedly used, such as local least-squares approximations (Acikgoz and Kodiyalam, 1994). For the issue of reducing the prohibitively high amount of computational time of large structural optimisation, parallel and vector algorithms on a shared memory parallel machine (Adeli and Kalmal, 1992) and distributed GAs for optimisation on a cluster of workstations connected via a
local area network (LAN) (Adeli and Kumar, 1995) have been reported. To minimise the overall computational effort for large-scale design optimisation problems, alternate approximation concepts are still necessary.

6.3.4 Artificial neural networks (ANNs)

Artificial neural networks (ANNs). In recent years, have also been used to solve the engineering optimisation problems. ANNs consist of numerous artificial neurones, which are basic computing elements to model simply the processing mechanics of the biological neurones. ANNs can provide powerful computation ability which is derived from the non-linear functions of the neurones. In structural optimisation, ANNs are mainly used to model between the design variables and performances, such as the stresses and displacements of the structures. ANNs have been one of the effective tools for solving non-linear problems (Lu, et al, 1996).

ANNs have been used to model the optimum weight and optimum solution of a 10-bar structure, and the model is based on the ANNs between the design variables and the displacements of the structure (Berke and Hajela, 1993). A counter propagation neural network is used to model between the internal stresses and the design variables of 6- and 10- and spatial 28-bar structures (Szewczyk and Hajela, 1993). Lu, et al (1996), presents an approach to structural approximation analysis based on ANNs in the improved strategy for GAs, in order to reduce the expensive computational cost.

6.4 A new mathematical model for multi-objective, multi-load-case optimisation of complicated structural systems

The technique, presented in this thesis, is shown to have an intrinsic applicability to multiobjective and multidisciplinary optimisation. It makes it feasible to optimise complicated structures and systems considering many loading conditions simultaneously in a one-run. One of its primary benefits in the application is the elimination of the potentially expensive separate optimisations for each objective. In addition, through the constrained-to-unconstrained optimisation problem conversion, the optimisation method readily lends itself to GAs if preferred to use GAs.

Using the performance data matrix, described in section 5.2.1, all the analysis results can be collected. These results represent a variety of performance parameters of a
complicated structural system at various loading conditions. To optimise such a structural system signifies to improve these performance parameters (objectives) under all the loading cases considered and all the side constraints of the design variables.

Based on engineering judgement, the actual environment to which the structural system is exposed is replaced with several distinct sets of mechanical and thermal loads. Each set of loads is referred to as a load case and the several distinct load cases will be referred to as the loading system.

The stated problem is a non-linear programming, with a large number of objectives, variables and constraints. In real problems these objectives are usually antagonist functions — the Pareto concept. In mathematic-analytic terms, to optimise some of these functions may correspond the worse of the others. For this reason and owing to the complexity of the structure and of the material, it is not possible to find a simple closed-form relationship that includes these functions and the design variables. Moreover, at least theoretically some problems might occur for nonconvex optimisation problems which we usually have. So it is often more convenient and reliable to transform the multicriteria problem to a scalar one which will be described below.

Although the solution to such optimisation formulations is very complex, it should be explored because of the tremendous efficiencies possible. An unconventional more direct approach may be suggested whereby an optimum solution is obtained through the optimisation for these performance parameters.

In multiobjective optimisation, it is known that the Pareto-optimal set lies on the intersection of objective and constraint function contours (for most structural problems); hence, treating the constraint functions in a same way as the objective functions seems a reasonable approach. The difficulty in doing so is that the constraints have a target to satisfy and objective functions do not. However, constraints and objectives are mutually convertible. In practice, in most of the existing optimisation methods (in which only one objective can be optimised), in order to optimise one most important objective, other objectives are degraded as constraints. On the other hand, undoubtedly, any constraint functions have a preferred optimum (i.e. to be maximised, minimised, or keep in a specified value as close as possible). Therefore, any constraint function can be treated as an objective function.
In the optimisation model proposed, all the performance parameters, no matter whether they are considered as objectives or constraints, are collected into performance data matrix (Table 5.1). By introducing acceptable limits and best level-values for each performance, a parameter profile matrix (Table 5.2) can be founded (see Section 5.2.3). This procedure transforms every performance parameter into a set of goal functions in connection with loading cases. These goal functions are the elements of the parameter profile matrix. In this way, a goal system is established and it brings all the performance data into the range of 0-10. For every performance parameter, the goal is the same and its value is specified as 10. The goal functions represent closenesses to the predetermined targets (best level values of the performances). The closeness value for each parameter is an adjustable quantity related to the acceptable limit(s) and best level value of the performance. Hence, the original optimisation problem is converted to the problem of minimising the deviations between all these goal functions and their pseudotargets — quantitative value 10.

This is equivalent to minimising the distance between the performance \( P_i \) and its given best value \( P_i^* \), \( i = 1, \ldots, m \), that is:

\[
\min \| P_i - P_i^* \|, \quad i = 1, \ldots, m
\]  

(6.10)

From the parameter profile matrix (Table 5.2), a parameter performance index (PPI) and a case performance index (CPI) can be derived (as formulated in Section 5.2.3). It should be useful if a ranking could be derived of the parameter/loading-case combinations, i.e., cells from the parameter profile matrix. This ranking should give an estimation of the vulnerability (or superiority) with respect to overall system performance at all loading cases considered.

The PPI which is a measure of the vulnerability of each performance parameters and CPI which is a measure of the vulnerability of each loading case are now available from the analyses of the rows and the columns of the parameter profile matrix. These two have to be combined and the results should be a measure of the vulnerability of the particular parameter/loading-case combination. High vulnerability results in low indices and high superiority results in high indices. A single multiplication therefore seems most appropriate:

\[
S_{ij} = PPI_i \times CPI_j, \quad i = 1,2,\ldots,m; \quad j = 1,2,\ldots,n
\]

(6.11)

\[\Box\]
This way, a matrix consisting of parameter/loading-case vulnerability (or superiority) index is formed, the data points \( S_{ij} \) of which are in the range of 0–100. A ranking can be derived from the cells of the matrix. The ranking should give an estimation of the vulnerability with respect to all system performances at all loading cases considered.

The optimisation objective function should be an overall measurement of design quality of an antenna or other engineering systems. An overall performance index (OPI) is used to form the overall objective function. The overall performance index, which is a qualitative score, can be established for the system considering all the performances and all the loading cases. Mathematically, this is expressed as

\[
OPI = \frac{1}{m \times n} \sum_{i=1}^{m} \sum_{j=1}^{n} S_{ij}
\]  

(6.12)

Each performance parameter and loading case may be weighted according to importance if desired, and the data points are calculated as

\[
S_{ij} = W_{pi} \cdot PPI_i \times W_{cj} \cdot CPI_j, \quad i = 1,2,...,m; \quad j = 1,2,...,n
\]  

(6.13)

where the \( W_{pi} \) and \( W_{cj} \) are weighting factors in the range of 0 ~ 1 reflecting the preference for different parameters and different loading cases respectively. The given candidate designs can be evaluated using different sets of weighting factors.

The overall performance index in the weighted case is calculated by

\[
OPI = \frac{100}{m \times n} \sum_{i=1}^{m} \sum_{j=1}^{n} S_{ij}
\]  

(6.14)

The OPIs in the equations (6.12) and (6.14) are in a range of 0–100. They are overall measurements of design quality of an engineering system and can be regarded as assessment score marks of an engineering system. Either of them can be used as an overall objective function for the optimisation of the system. The objective function comprises all the \( m \) parameters considered under different total \( n \) loading conditions. This objective function specifies the deviations of all performance parameters from their goals and priorities for the achievement of each goal, in quantitative terms.

The overall objective function presented here is of great significance because it integrates all optimisation objectives with all design constraints in such a way that all
the system performances are treated as objectives in the optimisation and once some of
the performances are improved up to their best levels, these performances will
transform into constraints to be fixed to their best levels until all other performances
also reach their best levels or can not be improved any more (convergence).

The mathematical model of optimisation makes it possible to force those performances
(whatever are considered as objectives or constraints) which have not achieved their
best level values to approach these values. The nearer the performances are to their
acceptable limits, the more severe will be 'the punishments'. Taking advantages of
imposing different weights on both different performances and different loading cases
according to their importance as shown in equation (6.14), this mathematical model is
of great flexibility to design a structure with maximum satisfaction, which will approach
the requirements of the designers. In addition, the method also benefited by giving
different acceptable limits and best level values derived from actual design specifications
for various performances at various loading cases. An outstanding superiority of the
model is that it can ingeniously transform the constrained optimisation problem into an
unconstrained one by including constraints as the implicit functions of the overall
objective function. It is well known that the algorithms for unconstrained optimisation
are much more powerful than those for constrained one.

The objective function is established in structural optimisation as a means to replace
many objectives and performance constraints with a single cumulative function. This
constrained-to-unconstrained conversion technique replaces the constraint boundary
surfaces and the objective function surface in n-dimensional design space with a single
envelop surface constructed using the OPI functions.

The design variables used here are continuous properties of the structure such as plate
thickness, truss element diameters and properties of materials. Since shape design is an
important issue to be studied as structural optimisation, an optimisation system should
be able to deal with the shape optimisation of a structure as well as sizing optimisation.
Therefore, the co-ordinates of structures are also included in the design variable sets in
the optimisation.

Using presently available composite materials, with their actual physico-mechanical
properties, it is possible to optimise the structure of the reinforcement of the bearing

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layers. These materials offer the structural designers a wide range of new degree of freedom to think in terms of simultaneous optimisation of structural configuration and structural material. The material cost is a crucial factor, restricting in many cases the use of composite materials. Fibre reinforced composites are non-uniform materials, hence, weight minimisation does not correspond to material cost minimisation. The material cost can be represented by the average cost of fibres per unit area. The use of fibre cost as a measure of the total material cost is justified in view of the very high cost of fibre compared to the cost of matrix. In this optimisation model developed, design variables can incorporate the tailorable variables of composite materials (if they are used in the structures) such as ply angles, laminate thicknesses, fibre volume ratios, and shape parameters.

Side constraints of minimum member sizes are prespecified to preclude over-stress and buckling or adjusted during the process to prevent over-stress for sets of additional loadings. Additional constraints can be imposed to restrict the numbers of different member sizes and different co-ordinate variables for fabrication economy and simplification. This is enforced by assigning particular members and co-ordinate components of nodes to groups that are required to have common sizes and symmetrical structural shapes. In addition to the performance constraints of the systems such as by prescribing frequency spacing and weight budgets, the following constraints, which are described in section 6.2, are also included in the optimisation procedures: Member stresses, Euler buckling, Limits on member sizes, Linking of member-size design variables, Limits on co-ordinate design variables, Linking of co-ordinates variables. The constraints insure a viable design, for instance, by specifying minimum gauges and minimum buckling strength for truss members.

The optimisation problem stated above is complicated by the fact that the objective does not always have continuous first and second derivatives for some engineering problems. Consequently the problem has been tackled by means of numerical processes. However, the numerical calculation of the gradient and Hessian matrix may be costly if not impossible. Therefore, suitable non-linear programming methods and search techniques must be selected for the problems, such as zero-order methods for the unconstrained optimisation problems, conjugate directions search methods (Brent, 1973), optimal sequential search and approximation derivative methods, direct search

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and random complex methods, Powell-Fletcher's methods, and penalty function and SUMT class methods.

The objective function is a complicated non-linear function of design variables. Therefore, an iterative conjugate direction search algorithm is used to solve the non-linear programming problem by maximising the objective function shown in equation (6.14). An effective polynomial interpolation unidimensional search technique (Beveridge and Schechter, 1970) is also used in the optimisation algorithm to determine the distance in the search direction that will maximise the objective as much as possible. The upper and lower limit constraints of the design variables are imposed in the optimisation procedure. If all the following criteria are satisfied in the $j$-th iteration, the convergence of the optimisation is achieved:

$$\left| \frac{OPI_{j-1} - OPI_j}{OPI_{j-1}} \right| \leq \varepsilon; \quad \left| \frac{V_{i,j-1} - V_{i,j}}{V_{i,j-1}} \right| \leq \varepsilon, \quad i = 1, 2, \ldots, N$$

(6.15)

where $\varepsilon$ is a given small quantity, say 0.0001, which is used to control the convergence. $OPI_j$ is the objective in $j$-th iteration. $V_{ij}$ is $i$-th design variable in $j$-th iteration.

The structural optimisation begins with an initial structural model and initial values for each design variable which may or may not meet design constraints. Side constraints are also established for each design variable group. The procedure consists of the steps of analysis of a prior established starting design, sensitivity analysis, and the development of preferential values of the design variables. Performance sensitivity to each of the design parameters for each of the design conditions is determined. Changes in performance and structural weight are expressed in terms of the variations in the design parameters. The analysis step may be mathematically 'exact', but nonlinearities in the system response with respect to changes in the design variables make the development step approximate. Therefore, in common with most of the prevalent design procedures, these two steps are repeated iteratively to achieve the final design.

This technique is based on the search of the optimal values by means of the optimising of the all the objective functions, step by step, starting from a tentative value. Preferential size is determined for each design variable by finding the best value of objective determined within its alteration range. A new design is established as a composite of all these preferential sizes, and the process is repeated cyclically until it appears that no new preferential values can be found to improve the objective. The final

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design thus obtained is either the optimum or possibly is stranded upon a ridge of the design space.

For solutions of structural problems the optimisation method has been coupled with the ABAQUS finite element code which is used to determine system response variables as function of design variables. The technique has been demonstrated on various structural systems built of various types of finite elements and different materials, contributing to a wide range of mechanical properties and cost to the design objectives.

6.5 MOST — a general optimisation program for engineering designs

The requirements for extremely precise and powerful large antenna reflectors have motivated the development of a program for optimisation of antenna structures. A multidisciplinary structural optimisation computer program system, MOST (Multifactor Optimisation of Structures Technique), has been developed to accommodate and implement the optimisation methodology proposed here. Several algorithms contained in the programs are used during the optimisation. The MOST system uses finite element static and dynamic analysis, surface interpolation, RF aperture integration, Zernike modes analysis, system assessment, and optimisation techniques. The program system provides a quick and cost-effective link in the optimisation design process for antenna structures and other engineering systems. The effectiveness of the set of algorithms and the program for solving engineering optimisation problems can be verified by the successful optimisation practices as shown in the next chapter.

The complete system includes the following eight integrated programs (solution modules). These programs interact with ABAQUS and with each other under the control of the loop executive control program. In brief, the eight solution modules are:

1) structural static analysis (using ABAQUS or own program to determine the system distortions for specified loading cases)
2) structural dynamic analysis (using ABAQUS or own program)
3) reflector surface best fit calculations, (using the antenna surface topography solver to determine the best fit parabolic surface and minimum RMS surface error to match surface distortions)
4) surface deviation interpolation (using surface spline function method)
5) phase errors calculations (using geometric ray tracing)
6) geometric optics aperture integration (to determine the RF performance such as the far-field pattern, antenna gain, and beam efficiency of the distorted antenna)
7) system assessment analysis (using parameter profiles analysis)
8) system optimisation design (to evaluate constraint, to compute the sensitivity of direct responses with respect to the appropriate set of design variables, to generate and solve the approximate optimisation problem (including numerical optimiser), to check the convergence at the end of each complete system analyses.

The computer code, MOST, is a synergistic combination of the aforementioned solution modules, together with some pre-phase programs of design optimisation, input/output, model update, and interfacing programs. The pre-phase programs, which are not included in the iterative design cycle, generate tables and matrices that are independent of the design variable values and are executed only once for each run. It utilises the ABAQUS and the theory of laminated plate to build a comprehensive analysis/design capability for structural composites.

The MOST program, which can utilise ABAQUS as an analysis tool, has been developed to perform all the essential aspects of mechanics/analysis/optimisation of complex structural systems. The program is modular, and open-ended. It can handle a variety of complicated structural and multilayered fibre composite systems. It can also simulate the electromagnetic analysis of distorted antenna systems. This feature is specifically useful in optimising antenna systems. The program can account for various environment conditions and various combinations of loads. These features make MOST a powerful, cost-effective, and reliable tool to analyse/optimise structural systems including antenna systems.

The optimisation system control program is written in UNIX shell scripts. It can control optimisation flow and execute application programs in UNIX environment so that all function can be integrated in a system. The present system has a facility to deal with sizing and shape optimisation in the same manner, while the nature of static/dynamic, or linear/non-linear problems for analysis does not affect the system itself since analysis is assumed to be independent of the system. In this sense, it is a sufficiently flexible optimisation system integrating finite element analysis, EM analysis and optimisation.

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algorithm capability using the concept of open-ended software modules in UNIX operating system.

A rough sketch of the flow chart of the program is shown in Figure 6.2. First of all the model of initial design is created. The significant items of input are the structure geometry, member physical properties, boundary conditions, loading conditions, and member temperatures with the appropriate mean thermal coefficients of expansion. If composite materials are used, the laminate parameters and details of the fibre and matrix of each layer must be read from the user submitted input data. For an optimisation, the basic design variables, objective functions and constraints, and the operational conditions and loading cases to be analysed must be chosen. For antenna system optimisation, the electromagnetic parameters of the system must be included in the input data. A summary of the input data is printed out along with the input data echo.

![Flow chart of the program](image)

**Fig. 6.2** The flow chart of the program

Once the input data have been validated by checking the user data file to ensure that all the required data for optimisation are included, the optimisation loop will be executed as many times as there are models. In the main loop, the system calculates the values or
the sensitivities of performance functions, which are required by the optimiser. If the values are required, the geometric model is updated according to the current design variables, then finite element model and boundary conditions are created. The values of performance functions are calculated from the output file of the analysis and they are given to the optimiser.

Finite element analysis must be a module inside of the design system for structural optimisation. There are many well-developed sophisticated software programs for finite element analysis at present, and they are used daily in design practice. Thus it becomes rather ineffective to start writing all the program of structural analysis from scratch. It is better to develop a design system that can utilise these as intact existing modules so that it does not require additional effort to be familiar with analysis and optimisation modules.

ABAQUS is selected to perform structural static, dynamic and buckling analyses. ABAQUS is a widely used proprietary finite element structural analysis program of the Hibbitt, Karlsson & Sorensen, Inc. which is a large-scale general purpose reliable computer code. ABAQUS provides both static and dynamic and, in addition, both linear and non-linear analysis capabilities. In the MOST system, the results from the ABAQUS are used to perform the sensitivity analysis in the design optimisation modules; and optimisation results can be readily used in the ABAQUS model for additional structural analyses.

In order to communicate between the modules and to accommodate automated treatment of interdisciplinary data, Interfacing software and management routines have been written by using input and output data files to transfer, reformat and reduce, as necessary, the results between program modules. These routines also facilitate automated recycling within a single discipline, as well as through the complete processing cycle. Common files are also used so that the intermediate results of each analysis are retrievable. A complete optimisation can be performed in a single computer run, and potential changes in design can be quickly and easily evaluated using this interdisciplinary design analysis tool.

The key to a realistic integrated structural and electromagnetic optimisation of an antenna system is a sophisticated analytical modelling of the structure and EM system.
For antenna system optimisation, one of the key concerns is the co-ordination of different disciplinary programs used for analysis or design of structure alone or EM alone. While it is possible, as Padula, et al., (1992) pointed out, to integrate the S-E design in a single computer program, this would require enormous programming effort.

In the MOST system, the transfer of data between the structural and EM models is expedited by coding the reflector surface deformation interpolation program. It is a specific routine for this problem which constructs a deviation distribution for the EM model by interpolating geometrically among the sparse structural nodes and associates these deviations with the aperture field locations in the format of the EM model. After the interpolation, the phase error of each grid point of the aperture field is calculated and placed in a data set for submittal to the beam pattern analysis program for the evaluation of EM performance. When an EM analysis is required the structural program can be directed to output the data needed for as many loading cases specified.

The last part of the design optimisation is the set of optimisation algorithms to solve the optimal design problem that is formulated as a non-linear mathematical programming problem. Using the values and sensitivities of the performance functions at many loading cases, the optimiser will calculate the values of the design variables of the next trial design. This completes one computational (design) cycle. The optimisation process terminates when either the design converges to an optimum or the user specified maximum number of cycles is reached. After the optimisation is converged, the program exits from the main loop and outputs optimal design.

For a structural optimisation tool to be useful in an engineering design environment, it must be capable of solving practical structural design problems. It is clear that if the total numbers of design variables are in a reasonable range, and if computing speed is not so critical, this simple and effective design optimisation system can solve multidisciplinary optimisation problems in structural size, shape and material design. These have made the MOST program capable of solving design optimisation problems for a large class of three dimensional structures.

In general, this optimisation program consists of efficient schemes for iterative scaling, the linking of design variables, the formulations of recurrence relationships, and the choosing of active, passive, and side constraints.
Now that the software tools are mated together, we have an excellent multidisciplinary design optimisation tool. For antenna structures, when the proposed antenna design and the materials are changed structurally, the simulation models and material properties are altered to incorporate the change. Once the changes have been made, the complete simulation is re-run to determine the RF performance of the modified design.

The analyses and optimisations of an 8m ground antenna system, a 3.6×2.6m space reflector structure and two low side-lobe antenna systems under various environmental conditions have been performed by using the MOST optimisation system. These will be discussed in the next chapter.
Chapter 7

Integrated Structural Electromagnetic Optimisation of Antenna Systems

7.1 Introduction

Several example problems of design optimisation of antenna structures and systems have been solved to demonstrate the capability of the optimisation system developed. These preliminary and/or practical applications on antenna structures and systems illustrated that the proposed multi-objective optimisation, as opposed to single objective functions, simultaneously improved all objectives.

In the optimisation of space composite reflector structures, the structural response will depend upon numerous engineering material properties, which, in turn, can be properly tailored in order to fulfil the requirements related to the behaviour of the structure. Therefore, the design and optimisation iterations, rather than referring to a 'redistribution' of the material, as usually happens in the design with metallic materials, extend to the constitution of the material itself.

The present optimisation method is significant because it can use general purpose structural analysis code, and the objective and constraint functions are derived from actual design specifications. This optimisation approach significantly improves the design of antenna structures which are chosen as demonstration problems. The conventional approach to antenna structural design is to use RMS surface error as the sole criterion for improvement in the reflector surface geometry. The present research demonstrates an integrated structure-electromagnetic optimisation which incorporates realistic design and operational objectives and constraints for antenna structures. The advantages of including EM performance criteria are explored.
Part A: The optimisation of a terrestrial Cassegrain antenna system

7.2 Factors considered for the multidisciplinary optimisation of the antenna system

The first test problem chosen to demonstrate the optimisation method is the 8m dual-reflector antenna electro-mechanical system described in Chapter 3 and 4. The performance of this antenna has been analysed in Chapter 3, 4 and 5. The multidisciplinary performance optimisation of this ground based 8m antenna is an example for a more global system oriented application. The following EM and structural performances are included as objectives in the optimisation model for the antenna, these are: 1) structural mass, 2) reflector surface RMS errors, 3) structural fundamental frequency, 4) maximum stress and 5) maximum displacement in the structure, 6) antenna EM efficiencies (gain), 7) EM radiation main beam -3dB width, 8) radiation pattern sidelobe levels in principal-plane patterns U1 and U2 which are mutually perpendicular patterns through the main-lobe axis. For this large antenna application, these are the most important performance parameters. Therefore, including these parameters in the optimisation procedure will demonstrate the flexibility of the procedure for a wide class of large antenna applications.

In the context of an electro-mechanical system, the objective is to obtain the best electro-mechanical system performance, not just the best mechanical or the best electronic performance. Because the disciplines are coupled, the work must be co-ordinated at the system level so that an overall optimum design can be achieved for the system. The EM efficiencies need to be maximised; the structural frequency needs to be increased and kept to the given best value 11 Hz to satisfy the requirement of servo system, and the other performances (i.e. mass, RMS errors, stress, displacement, -3dB beam width, and sidelobes) need to be minimised. The solution is the set of values for the design variables which maximise and/or minimise, as desired, the values of the performance parameters (objectives).

In antenna structural engineering practice there is an essential distinction between the superimposed loads and the operation loads. Under the former, such as those extreme
loads caused by hurricane and earthquake, the design must be safe, and collapse must be guarded against, although the antenna accuracy and EM performances are not considered under these conditions. To the latter loads the structure should satisfy the requirements of surface accuracy and EM performance, and unsatisfactory behaviour under operational loads is guarded against.

For a ground antenna (satellite ground station antenna, radar or telescope etc.), the operational loading that causes antenna surface deformations consists of a deterministic gravity loading and some random components from wind, temperature, sonic boom and earthquake. The working cases considered for the optimisation are gravity loads which result from the self-weight and the change in direction of the weight vector relative to the structure with change in antenna elevation attitude, because for ground station antennas, gravity is omnipresent and tends to be the most significant component with respect to performances.

Therefore, optimal design of the structure to control gravity loading distortions is a logical and feasible approach to performance enhancement. We will consider the reflector structure design as a design for optimised performance through control of the gravity loading structural deflections.

The variation of all deformations arising solely from gravity forces (i.e. excluding wind, temperature, etc.) will be a function of the elevation angle of the antenna only. Seven working cases (antenna elevation angles from 0 to 90 degree in a 15 degree increment) are incorporated in the optimisation. More performances and more working/loading cases can also be incorporated in the optimisation model without additional difficulty if there are different design considerations and requirements for various structures.

The definition of loads for an antenna structure is an iterative process. The flexible-body loads are a function of weight which changes as the design modified. Therefore, the loads change as the design changes.

The design variables for this antenna optimisation are member sizes of all members in the structure (linked into 12 group of cross-sectional area variables) and backup structural geometric shapes (node co-ordinate variables). The design constraints described in Section 6.2 are imposed in the optimisation procedure.
To investigate whether internal forces are significant loads in the design of the truss members, the Euler buckling loads of pin-ended members can be considered in terms of the member's slenderness ratio $L/r$, which is a function of the length of the member $L$ and its radius of gyration $r$, as

$$P_e = \frac{\pi^2 EA}{(L/r)^2}$$  \hspace{1cm} (7.1)

where

$$r = \sqrt{I/A}$$

If the prescribed ratio of member design load to Euler load is denoted by $\gamma$, an expression for the required member $L/r$ can be written as

$$\left( \frac{L}{r} \right)_{req} = \pi \sqrt{\frac{EA\gamma}{P_i}}$$  \hspace{1cm} (7.2)

### 7.3 Analysis results and design assessment for optimisation of the original antenna system

The EM performances are obtained through EM field analysis and the effects of structural deformations on the EM performances are incorporated; and structural performances are obtained through finite element structural analysis and best fit calculation. The performance data matrix obtained by structural and EM analyses for the 8m antenna is shown in Table 5.3. The data in the matrix represent the performances of the original design of the antenna structure. It can be calculated from the RMS values given in Table 3.4, that the RMS values measured with respect to the nominal paraboloid at seven elevations are $\lambda/28$, $\lambda/29$, $\lambda/32$, $\lambda/38$, $\lambda/53$, $\lambda/88$ and $\lambda/154$ respectively, where $\lambda=0.02m$ for the antenna. This original antenna structure fully satisfies the general requirement mentioned in the Chapter 1 that the RMS should not be larger than a value in the range of $\lambda/32 \sim \lambda/16$; also, every performance is within the acceptable ranges (refer to Table 5.5), i.e. the original structure satisfies all design requirements. Therefore, it can be said that this original structure is a properly designed antenna structure. In order to demonstrate the effectiveness of the optimisation method, the structure is taken to be an original design to see what can be achieved through the optimisation for this properly designed structure.
In order to access the design quality of the antenna and to form an optimisation objective function, the acceptable limits and the best level values of the performances have to be considered. Antenna efficiency has a lower acceptable limit, and structural frequency has both a lower and an upper limit, while the RMS error, maximum displacement, maximum stress, structural mass, beam -3dB width, sidelobe levels in U1 and U2 planes have each an upper acceptable limit. These limits and the best level values for this particular antenna have been given in Table 5.5.

The parameter/working-case superiority (or vulnerability) indices (defined by Equation (6.11)) for the antenna are shown in Table 7.A.1. From this table, the calculation clearly identifies again the items which were already found to be weak in Section 5.4, and the weakest one is the RMS at about 15 degree elevation angle.

**Table 7.A.1 The parameter/case superiority indices**

<table>
<thead>
<tr>
<th></th>
<th>0 deg.</th>
<th>15 deg.</th>
<th>30 deg.</th>
<th>45 deg.</th>
<th>60 deg.</th>
<th>75 deg.</th>
<th>90 deg.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>antenna efficiency</strong></td>
<td>33.10</td>
<td>32.50</td>
<td>34.51</td>
<td>39.02</td>
<td>43.24</td>
<td>46.13</td>
<td>47.19</td>
</tr>
<tr>
<td><strong>RMS error</strong></td>
<td>27.69</td>
<td>27.19</td>
<td>28.88</td>
<td>32.65</td>
<td>36.18</td>
<td>38.60</td>
<td>39.48</td>
</tr>
<tr>
<td><strong>maximum displacement</strong></td>
<td>52.60</td>
<td>51.65</td>
<td>54.85</td>
<td>62.02</td>
<td>68.72</td>
<td>73.31</td>
<td>75.00</td>
</tr>
<tr>
<td><strong>maximum stress</strong></td>
<td>54.75</td>
<td>53.76</td>
<td>57.09</td>
<td>64.55</td>
<td>71.53</td>
<td>76.31</td>
<td>78.06</td>
</tr>
<tr>
<td><strong>structural mass</strong></td>
<td>44.64</td>
<td>43.82</td>
<td>46.54</td>
<td>52.62</td>
<td>58.31</td>
<td>62.21</td>
<td>63.64</td>
</tr>
<tr>
<td><strong>structural frequency</strong></td>
<td>36.71</td>
<td>36.04</td>
<td>38.28</td>
<td>43.28</td>
<td>47.96</td>
<td>51.16</td>
<td>52.34</td>
</tr>
<tr>
<td><strong>main beam -3dB width</strong></td>
<td>40.70</td>
<td>39.96</td>
<td>42.44</td>
<td>47.99</td>
<td>53.17</td>
<td>56.73</td>
<td>58.03</td>
</tr>
<tr>
<td><strong>sidelobe area in pattern U1</strong></td>
<td>39.50</td>
<td>38.79</td>
<td>41.19</td>
<td>46.57</td>
<td>51.61</td>
<td>55.06</td>
<td>56.32</td>
</tr>
<tr>
<td><strong>sidelobe area in pattern U2</strong></td>
<td>38.24</td>
<td>37.54</td>
<td>39.87</td>
<td>45.08</td>
<td>49.95</td>
<td>53.29</td>
<td>54.51</td>
</tr>
</tbody>
</table>

In this optimisation practice, the performances and loading cases are weighted according to importance in deriving OPIs. The weights for the performances and loading cases are shown in Tables 7.A.2 ~ 7.A.3 and in Figures 7.A.1 ~ 7.A.2 respectively. These weights are given according to requirements and some common particularities of antenna structures. For example, an antenna structure has an excess of strength provided that stiffness and surface accuracy requirements are satisfied; and also the displacements of non-surface nodes are of less importance. Therefore, the
performance weights given for maximum displacement and maximum stress are only 2.5% and 2% respectively in contrast to more important performances, efficiency, accuracy, mass and frequency, which are 33%, 17%, 28%, and 5.5% respectively. The weights for main beam -3dB width, sidelobe levels in U1 and U2 planes are all designated as 4%. It can be seen from Table 7.4 that a heavy weight of 30% is given to the elevation position of 60 degree. The reason for this partiality is that this particular antenna will mainly work around that elevation. There are less possibilities for the antenna to work at elevations 0, 15, 30 and 90 degree, so the weights for these working cases are only 3%, 7% and 10% respectively, much lighter than the weights in other cases.

Table 7.A.2 The weights of parameters for the 8m antenna

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Weights</td>
<td>0.33</td>
<td>0.17</td>
<td>0.025</td>
<td>0.28</td>
<td>0.055</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Table 7.A.3 The weights of loading cases for the 8m antenna

<table>
<thead>
<tr>
<th>Working/Loading Case</th>
<th>0 deg.</th>
<th>15 deg.</th>
<th>30 deg.</th>
<th>45 deg.</th>
<th>60 deg.</th>
<th>75 deg.</th>
<th>90 deg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weights</td>
<td>0.03</td>
<td>0.07</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
</tr>
</tbody>
</table>

The parameters and loading cases are weighted, as shown in Tables 7.A.2 and 7.A.3 respectively, according to the importance from an engineering judgement and Table 7.A.4 shows the parameter/working-case superiority (or vulnerability) indices.

Table 7.A.4 The weighted parameter/case superiority indices

<table>
<thead>
<tr>
<th></th>
<th>0 deg.</th>
<th>15 deg.</th>
<th>30 deg.</th>
<th>45 deg.</th>
<th>60 deg.</th>
<th>75 deg.</th>
<th>90 deg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>antenna efficiency</td>
<td>0.328</td>
<td>0.751</td>
<td>1.139</td>
<td>2.576</td>
<td>4.281</td>
<td>3.045</td>
<td>1.557</td>
</tr>
<tr>
<td>RMS error</td>
<td>0.141</td>
<td>0.324</td>
<td>0.491</td>
<td>1.110</td>
<td>1.845</td>
<td>1.312</td>
<td>0.671</td>
</tr>
<tr>
<td>maximum displacement</td>
<td>0.040</td>
<td>0.090</td>
<td>0.137</td>
<td>0.310</td>
<td>0.515</td>
<td>0.367</td>
<td>0.188</td>
</tr>
<tr>
<td>maximum stress</td>
<td>0.033</td>
<td>0.075</td>
<td>0.114</td>
<td>0.258</td>
<td>0.429</td>
<td>0.305</td>
<td>0.156</td>
</tr>
<tr>
<td>structural mass</td>
<td>0.375</td>
<td>0.859</td>
<td>1.303</td>
<td>2.947</td>
<td>4.898</td>
<td>3.484</td>
<td>1.782</td>
</tr>
</tbody>
</table>
The overall performance index OPI (see Section 6.4) for the original design of this antenna, which for the unweighted case is 48.65 and for the weighted case is 46.69, identifies that the antenna is well designed. This can also be seen from Tables 5.7 and 5.8, since most of the PPIs and CPIs (which should be in the range of 0 - 10) are in excess of 6, some are greater than 9.

The weighted performance index OPI is utilised to formulate the overall objective function for the optimisation. The design can also be evaluated using different sets of weighting factors according to different design considerations. The objective function provides a way to simultaneously consider many aspects of the design. Weighting factors ($W_j$) are used in conjunction with item scores ($S_j$) to provide an overall evaluation of a candidate design, but before a design can be evaluated, using these weighting factors, each parameter at each loading case must receive a performance score (i.e. $D_{ij}, i=1, 2, \ldots, m; j=1, 2, \ldots, n$. see Section 5.2.3).

### 7.4 Optimisation results of the 8m antenna

The design variables considered for the optimisation are:

1) a total of 12 grouped truss member cross-sectional areas

2) a total of 8 grouped geometric position variables which are the radiuses and heights of 4 circular beams (hoops). These co-ordinates variables involve 48 nodes in x, y and z directions.

A convergent iteration history of the optimisation is shown in Figure 7.A.3. It can be seen that the overall objectives, the weighted and unweighted OPIs, are greatly enhanced from original scores of 48.65 for the unweighted and 46.69 for the weighted OPIs, to the optimum scores of 86.08 for the unweighted and 87.22 for the weighted cases. Figure 7.A.4 ~ Figure 7.A.11 indicate that the optimisation procedure succeeds in improving all the antenna performances at almost all the working cases, and all these

<table>
<thead>
<tr>
<th>structural frequency</th>
<th>0.061</th>
<th>0.139</th>
<th>0.211</th>
<th>0.476</th>
<th>0.791</th>
<th>0.563</th>
<th>0.288</th>
</tr>
</thead>
<tbody>
<tr>
<td>main beam -3dB width</td>
<td>0.049</td>
<td>0.112</td>
<td>0.170</td>
<td>0.384</td>
<td>0.638</td>
<td>0.454</td>
<td>0.232</td>
</tr>
<tr>
<td>sidelobe area in pattern U1</td>
<td>0.047</td>
<td>0.109</td>
<td>0.165</td>
<td>0.373</td>
<td>0.619</td>
<td>0.441</td>
<td>0.225</td>
</tr>
<tr>
<td>sidelobe area in pattern U2</td>
<td>0.046</td>
<td>0.105</td>
<td>0.160</td>
<td>0.361</td>
<td>0.599</td>
<td>0.426</td>
<td>0.218</td>
</tr>
</tbody>
</table>
performances converge smoothly to their final values during the optimisation procedure.

Through the optimisation procedure, the following significant improvements in both structural and EM performances have been achieved (also see Table 7.A.5 and Table 7.A.6):

- the weakest antenna efficiency which is 47.1% when the antenna operates at 0 degree elevation is increased to 56.1% (at 0 degree);
- the worst RMS error of 0.0557 mm (at 0 degree) is reduced to 0.0276 mm (at 0 degree);
- the largest structural displacement (non-reflector-surface nodes) 1.58 mm (at 0 degree) is reduced to 0.77 mm (at 0 degree);
- the highest structural stress 18.4 MPa (at 0 degree) is reduced to 9.13 MPa (at 75 degree);
- the lowest structural natural frequency 9.16 Hz is increased to the given best level value of 11.0 Hz (if a higher best level value is given, the frequency can be higher);
- the widest -3dB width of the main lobe 185.1 (m deg.) (at 0 degree) is reduced to 177.3 (m deg.) (at 0 degree);
- the largest area, which is 1280 (at 0 degree), between the curve of all sidelobes and the line of -50dB in radiation pattern U1, is reduced to 1251 (at 90 degree);
- the largest area, which is 1153 (at 0 degree), between the curve of all sidelobes and the line of -50dB in radiation pattern U2, is reduced to 1081 (at 30 degree);
- and what is more significant is that all of those mentioned above are achieved in the case when the structural mass of 516kg is decreased to 364kg (a reduction of 30% from the original mass of 516kg).

Table 7.A.5 Performances of original antenna system at seven different working cases

<table>
<thead>
<tr>
<th>Performance Parameters</th>
<th>Working/Loading Cases</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 deg.</td>
</tr>
<tr>
<td>antenna efficiency (%)</td>
<td>47.14</td>
</tr>
<tr>
<td>RMS error (mm)</td>
<td>0.0557</td>
</tr>
<tr>
<td>Performance Parameters</td>
<td>Working/Loading Cases</td>
</tr>
<tr>
<td>----------------------------------------</td>
<td>-----------------------</td>
</tr>
<tr>
<td></td>
<td>0 deg.</td>
</tr>
<tr>
<td>antenna efficiency (%)</td>
<td>56.12</td>
</tr>
<tr>
<td>RMS error (mm)</td>
<td>0.0276</td>
</tr>
<tr>
<td>max. displacement (mm)</td>
<td>0.773</td>
</tr>
<tr>
<td>maximum stress (MPa)</td>
<td>7.724</td>
</tr>
<tr>
<td>structural mass (kg)</td>
<td>364.4</td>
</tr>
<tr>
<td>structural frequency (Hz)</td>
<td>11.00</td>
</tr>
<tr>
<td>-3 dB width (m deg.)</td>
<td>177.3</td>
</tr>
<tr>
<td>sidelobe area in pattern U1</td>
<td>1236.</td>
</tr>
<tr>
<td>sidelobe area in pattern U2</td>
<td>1049.</td>
</tr>
</tbody>
</table>

The convergence is obtained in such a way that not only all the performances are greatly improved, but also the performances at different working cases tend to be consistent which means that the optimised structure reduces to minimum the vulnerable performances and working cases. It should be mentioned that if the original structure is a poorly designed one, the improvements would be even greater.

7. Integrated structural electromagnetic optimisation of antenna systems — Part A
The iteration histories of mean, SD, PPI for each performance parameter and mean, SD, CPI for each loading case in the performance profile matrix are shown in Figure 7.A.12 ~ Figure 7.A.17. These figures show that all of the structural and EM characteristics steadily converge towards their desired values. The values of means, SDs, PPIs and CPIs for the optimised antenna system are listed in Tables 7.A.7 and 7.A.8. From these tables, a comparison can be made by referring to the values of the same parameters for the original antenna system which are listed in the Tables 5.7 and 5.8.

### Table 7.A.7 System parameter profile analysis of the optimised antenna

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>P.P.I.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Antenna Efficiency</strong></td>
<td>9.68</td>
<td>0.14</td>
<td>9.68</td>
</tr>
<tr>
<td><strong>RMS Error</strong></td>
<td>9.14</td>
<td>0.54</td>
<td>9.11</td>
</tr>
<tr>
<td><strong>Maximum Displacement</strong></td>
<td>9.93</td>
<td>0.10</td>
<td>9.93</td>
</tr>
<tr>
<td><strong>Maximum Stress</strong></td>
<td>10.00</td>
<td>0.01</td>
<td>10.00</td>
</tr>
<tr>
<td><strong>Structural Mass</strong></td>
<td>9.68</td>
<td>0.00</td>
<td>9.68</td>
</tr>
<tr>
<td><strong>Structural Frequency</strong></td>
<td>9.99</td>
<td>0.00</td>
<td>9.99</td>
</tr>
<tr>
<td><strong>Main Beam -3dB Width</strong></td>
<td>8.49</td>
<td>0.29</td>
<td>8.49</td>
</tr>
<tr>
<td><strong>Sidelobe Area in Pattern U1</strong></td>
<td>8.44</td>
<td>0.64</td>
<td>8.39</td>
</tr>
<tr>
<td><strong>Sidelobe Area in Pattern U2</strong></td>
<td>8.51</td>
<td>0.71</td>
<td>8.45</td>
</tr>
</tbody>
</table>

Inspection of Table 7.A.7 reveals that both EM and structural performance parameters have much higher means and much lower standard deviations and PPIs than the original design. This means that the design is remarkably improved for all loading cases with respect to all the objectives.

### Table 7.A.8 Loading case profile analysis of the optimised antenna system

<table>
<thead>
<tr>
<th></th>
<th>0 deg.</th>
<th>15 deg.</th>
<th>30 deg.</th>
<th>45 deg.</th>
<th>60 deg.</th>
<th>75 deg.</th>
<th>90 deg.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>9.51</td>
<td>9.41</td>
<td>9.18</td>
<td>9.21</td>
<td>9.26</td>
<td>9.35</td>
<td>9.31</td>
</tr>
<tr>
<td><strong>Standard Deviation</strong></td>
<td>0.54</td>
<td>0.61</td>
<td>0.77</td>
<td>0.76</td>
<td>0.80</td>
<td>0.79</td>
<td>0.85</td>
</tr>
</tbody>
</table>
Inspection of Table 7.A.8 reveals that the optimisation maximises the performance reliability and minimises the possibility for the antenna to perform unsatisfactorily at all loading cases considered.

For the optimised antenna system, the parameter profile matrix, the parameter/case superiority indices and weighted parameter/case superiority indices are listed in Tables 7.A.9, 7.A.10 and 7.A.11 respectively. By comparing the values in these tables with their corresponding values in Tables 5.6, 7.A.1 and 7.A.4 for the original antenna system, great improvement can be observed.

Table 7.A.9 The parameter profile matrix of the optimised antenna system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>0 deg.</th>
<th>15 deg.</th>
<th>30 deg.</th>
<th>45 deg.</th>
<th>60 deg.</th>
<th>75 deg.</th>
<th>90 deg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS error</td>
<td>9.04</td>
<td>8.63</td>
<td>8.53</td>
<td>8.73</td>
<td>9.19</td>
<td>9.86</td>
<td>10.00</td>
</tr>
<tr>
<td>Maximum displacement</td>
<td>9.96</td>
<td>9.94</td>
<td>10.00</td>
<td>10.00</td>
<td>10.00</td>
<td>9.90</td>
<td>9.70</td>
</tr>
<tr>
<td>Maximum stress</td>
<td>10.00</td>
<td>10.00</td>
<td>10.00</td>
<td>10.00</td>
<td>10.00</td>
<td>9.98</td>
<td>10.00</td>
</tr>
<tr>
<td>Main beam -3dB width</td>
<td>8.20</td>
<td>8.14</td>
<td>8.25</td>
<td>8.46</td>
<td>8.69</td>
<td>8.86</td>
<td>8.87</td>
</tr>
<tr>
<td>Sidelobe area in pattern U1</td>
<td>9.50</td>
<td>9.17</td>
<td>8.58</td>
<td>8.31</td>
<td>7.97</td>
<td>7.89</td>
<td>7.63</td>
</tr>
<tr>
<td>Sidelobe area in pattern U2</td>
<td>9.61</td>
<td>9.65</td>
<td>8.02</td>
<td>8.08</td>
<td>7.97</td>
<td>8.13</td>
<td>8.08</td>
</tr>
</tbody>
</table>

Table 7.A.10 The parameter/case superiority indices of the optimised antenna system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>0 deg.</th>
<th>15 deg.</th>
<th>30 deg.</th>
<th>45 deg.</th>
<th>60 deg.</th>
<th>75 deg.</th>
<th>90 deg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antenna efficiency</td>
<td>91.64</td>
<td>90.65</td>
<td>88.14</td>
<td>88.51</td>
<td>88.84</td>
<td>89.79</td>
<td>89.22</td>
</tr>
<tr>
<td>RMS error</td>
<td>86.27</td>
<td>85.34</td>
<td>82.98</td>
<td>83.32</td>
<td>83.63</td>
<td>84.53</td>
<td>83.99</td>
</tr>
<tr>
<td>Maximum displacement</td>
<td>94.03</td>
<td>93.01</td>
<td>90.44</td>
<td>90.81</td>
<td>91.15</td>
<td>92.14</td>
<td>91.55</td>
</tr>
<tr>
<td>Maximum stress</td>
<td>94.69</td>
<td>93.67</td>
<td>91.08</td>
<td>91.45</td>
<td>91.80</td>
<td>92.78</td>
<td>92.19</td>
</tr>
</tbody>
</table>
Table 7.A.11 Weighted parameter/case superiority indices of optimised antenna system

<table>
<thead>
<tr>
<th></th>
<th>0 deg.</th>
<th>15 deg.</th>
<th>30 deg.</th>
<th>45 deg.</th>
<th>60 deg.</th>
<th>75 deg.</th>
<th>90 deg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>antenna efficiency</td>
<td>0.91</td>
<td>2.09</td>
<td>2.91</td>
<td>5.84</td>
<td>8.79</td>
<td>5.93</td>
<td>2.94</td>
</tr>
<tr>
<td>RMS error</td>
<td>0.44</td>
<td>1.02</td>
<td>1.41</td>
<td>2.83</td>
<td>4.27</td>
<td>2.87</td>
<td>1.43</td>
</tr>
<tr>
<td>maximum displacement</td>
<td>0.07</td>
<td>0.16</td>
<td>0.23</td>
<td>0.45</td>
<td>0.68</td>
<td>0.46</td>
<td>0.23</td>
</tr>
<tr>
<td>maximum stress</td>
<td>0.06</td>
<td>0.13</td>
<td>0.18</td>
<td>0.37</td>
<td>0.55</td>
<td>0.37</td>
<td>0.18</td>
</tr>
<tr>
<td>structural mass</td>
<td>0.77</td>
<td>1.78</td>
<td>2.47</td>
<td>4.96</td>
<td>7.46</td>
<td>5.03</td>
<td>2.50</td>
</tr>
<tr>
<td>structural frequency</td>
<td>0.16</td>
<td>0.36</td>
<td>0.50</td>
<td>1.01</td>
<td>1.51</td>
<td>1.02</td>
<td>0.51</td>
</tr>
<tr>
<td>main beam -3dB width</td>
<td>0.10</td>
<td>0.22</td>
<td>0.31</td>
<td>0.62</td>
<td>0.93</td>
<td>0.63</td>
<td>0.31</td>
</tr>
<tr>
<td>sidelobe area in pattern U1</td>
<td>0.10</td>
<td>0.22</td>
<td>0.31</td>
<td>0.61</td>
<td>0.92</td>
<td>0.62</td>
<td>0.31</td>
</tr>
<tr>
<td>sidelobe area in pattern U2</td>
<td>0.10</td>
<td>0.22</td>
<td>0.31</td>
<td>0.62</td>
<td>0.93</td>
<td>0.63</td>
<td>0.31</td>
</tr>
</tbody>
</table>

Figure 7.A.18 and Figure 7.A.19 illustrate the change in structural design variables (all member sizes and the co-ordinates of all reflector low chord nodes). The change in design variables are detailed in Table 7.A.12. The structural shape before and after optimisation is shown in Figure 7.A.28 (for clearness, only part of the structure is shown). A structural sketch of the optimised 8m antenna is shown in Figure 7.A.29. It has been shown that sizing variables and geometry variables can be considered simultaneously when using this method. All the members in the optimised structure satisfy the given buckling constraints, i.e. the cross-sectional areas of all the members are larger than or equal to the given lower limits.
Table 7.A.12 Design variables in original and optimised designs

<table>
<thead>
<tr>
<th>Design variables</th>
<th>group 1 (cm²)</th>
<th>group 2 (cm²)</th>
<th>group 3 (cm²)</th>
<th>group 4 (cm²)</th>
<th>group 5 (cm²)</th>
<th>group 6 (cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before optimisation</td>
<td>3.0</td>
<td>1.5</td>
<td>1.0</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>After optimisation</td>
<td>4.85</td>
<td>0.29</td>
<td>0.40</td>
<td>0.33</td>
<td>0.55</td>
<td>0.87</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Design variables</th>
<th>group 7 (cm²)</th>
<th>group 8 (cm²)</th>
<th>group 9 (cm²)</th>
<th>group 10 (cm²)</th>
<th>group 11 (cm²)</th>
<th>group 12 (cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before optimisation</td>
<td>1.0</td>
<td>2.0</td>
<td>3.0</td>
<td>2.0</td>
<td>3.0</td>
<td>1.0</td>
</tr>
<tr>
<td>After optimisation</td>
<td>0.25</td>
<td>0.64</td>
<td>1.45</td>
<td>1.24</td>
<td>2.27</td>
<td>1.15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Design variables</th>
<th>group 1 (m)</th>
<th>group 1 (m)</th>
<th>group 1 (m)</th>
<th>group 1 (m)</th>
<th>group 1 (m)</th>
<th>group 1 (m)</th>
<th>group 2 (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before optimisation</td>
<td>4.0</td>
<td>3.1</td>
<td>2.0</td>
<td>0.8</td>
<td>0.3</td>
<td>0.6</td>
<td>0.8</td>
</tr>
<tr>
<td>After optimisation</td>
<td>4.10</td>
<td>3.00</td>
<td>1.78</td>
<td>1.26</td>
<td>0.02</td>
<td>0.77</td>
<td>0.96</td>
</tr>
</tbody>
</table>

The system performances before and after optimisation are shown in Table 7.A.5 and Table 7.A.6 respectively for comparison. Figure 7.A.20 ~ Figure 7.A.27 show the comparisons of each performance at different working cases before and after optimisation. The results should be compared to the given acceptable limits and best level values on each performance parameter at each working/loading case, and these limits and values are also shown in these figures. These graphs distinctly illustrate the level of the dramatic improvements in various antenna performances through the optimisation. In addition, the mass is reduced from the original structure 515.7 kg to the optimised structure 364.4 kg; and structural frequency is increased from the original 9.16 Hz to the given best value 11.0 Hz.

In the optimisation procedure the radiation patterns change for each step of the optimisation. Figure 7.A.30 and 7.A.31 show the 3-D radiation patterns of optimised antenna at elevation 0 degree (horizon-pointing) and 90 degree (zenith-pointing). The predicted relative power levers on any plane normal to paraboloid axis can be seen from the vertical view of the 3-D radiation pattern of optimised antenna. The vertical view of 3-D radiation patterns of optimised antenna at elevation 0 and 90 degrees are shown in Figures 7.A.32 and 7.A.33.
Figures 7.A.34 – 7.A.37 illustrate the change in antenna radiation patterns, when the antenna working at 0 degree through to 90 degree, as a result of the optimisation procedure. These figures represent relative power as a function of the off-boresight angle \( \theta \) scaled by multiplying \( \pi \cdot D \). Both the optimised patterns and original patterns are shown in these figures so that they may be compared. Since the energy contained in the side lobe region can be a significant source of error for an antenna system, minimising the levels of the radiation for side lobes is one of the primary objectives in the optimisation procedure. It can be seen that the sidelobe levels in both U1 and U2 planes are reduced, and the main beam of the radiation patterns are narrowed through the optimisation.

Figures 7.A.38 and 7.A.39 compare the antenna patterns after optimisation with the antenna patterns with respect to an ideal antenna system where there is no structural deformation at all. From these figures, it can be seen that the optimisation minimised the difference between the optimised patterns and the ideal case pattern.

The results of the modal analysis for the optimised antenna structure give the lowest structural frequency to be equal to 11.00 Hz. This value is the given target (best) value expected to be approached through the optimisation. From Figure 7.A.40, it can be seen that the corresponding mode shape is a twist mode. The other three low structural frequencies are 16.8 Hz, 16.8 Hz and 17.1 Hz respectively, and their mode shapes are shown in Figures 7.A.41 – 7.A.43 respectively to illustrate general vibration behaviour. A comparison for low structural frequencies can be made between original and optimised antenna structures (see Table 7.A.13).

<table>
<thead>
<tr>
<th>Table 7.A.13</th>
<th>The comparison of low frequencies between original and optimised structures</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>frequencies (Hz)</strong></td>
<td><strong>lowest</strong></td>
</tr>
<tr>
<td>original structure</td>
<td>9.16</td>
</tr>
<tr>
<td>optimised structure</td>
<td>11.00</td>
</tr>
</tbody>
</table>

It is anticipated that all the above-mentioned changes will have a significant effect on the improvement of antenna performance. A new system which has tremendous improvements on all the structural and EM performances (under almost all the loading
cases) has been achieved through the one-run optimisation procedure, even if the original structure is a 'properly designed' structure by experienced engineers.
Fig. 7.A.1 The weighted function for performance parameters

Fig. 7.A.2 The weighted function for different elevations

Fig. 7.A.3 The convergence history of overall objectives
Fig. 7.A.4 The iteration history of antenna efficiencies at different elevations

Fig. 7.A.5 The iteration history of reflector surface accuracies (RMSs) at different elevations
Fig. 7.A.6 The iteration history of maximum displacements at different elevations

Fig. 7.A.7 The iteration history of maximum stress at different elevations

7. Integrated structural electromagnetic optimisation of antenna systems — Part A 175
Fig. 7.A.8 The iteration history of structural mass and fundamental frequency

Fig. 7.A.9 The iteration history of mainlobe -3dB widths at different elevations
Fig. 7.A.10 The iteration history of sidelobe areas of radiation U1 patterns at different elevations

Fig. 7.A.11 The iteration history of sidelobe areas of radiation U2 patterns at different elevations
Fig. 7.A.12 The iteration history of the means of performances in the parameter profile matrix

Fig. 7.A.13 The iteration history of the standard deviations of performances in the parameter profile matrix
Fig. 7.A.14 The iteration history of the parameter performances indices

Fig. 7.A.15 The iteration history of the means of working/loading-cases in the parameter profile matrix
Fig. 7A.16 The iteration history of the standard deviations of working/loading-cases in the parameter profile matrix.

Fig. 7A.17 The iteration history of the working/loading-cases performance indices.
Fig. 7.A.18 The iteration history of design variables of member cross-sectional areas

Fig. 7.A.19 The iteration history of structural geometric design variables
Fig 7.A.20 The comparison of antenna efficiencies between original and optimised structures

Fig 7.A.21 The comparison of reflector surface accuracies (RMSs) between original and optimised structures
Fig 7.A.22 The comparison of maximum displacements in original and optimised antenna structures

Fig 7.A.23 The comparison of maximum stresses in original and optimised antenna structures
Fig 7.A.24 The comparison of structural masses and frequencies between original and optimised antenna structures.

Fig 7.A.25 The comparison of radiation pattern mainlobe -3dB width in original and optimised antenna structures.
Fig 7.A.26 The comparison of sidelobe areas in radiation pattern U1 plane in original and optimised antenna structures.

Fig 7.A.27 The comparison of sidelobe areas in radiation pattern U2 plane in original and optimised antenna structures.
Fig. 7.A.28 The comparison of the geometric shapes of antenna radial beams in original and optimised structures

Fig. 7.A.29 Optimised 8m antenna structural sketch
Fig. 7.A.30 Antenna 3-D radiation pattern of optimised structure at elevation 0 degree

Fig. 7.A.31 Antenna 3-D radiation pattern of optimised structure at elevation 90 degree
Fig. 7.A.32 The vertical view of 3-D radiation pattern of optimised structure at elevation 0 degree

Fig. 7.A.33 The vertical view of 3-D radiation pattern of optimised structure at elevation 90 degree
Fig. 7.A.34 The comparison of radiation patterns in original and optimised structures at elevation 0 degree

Fig. 7.A.35 The comparison of radiation patterns in original and optimised structures at elevation 30 degree
Fig. 7.A.36 The comparison of radiation patterns in original and optimised structures at elevation 60 degree

Fig. 7.A.37 The comparison of radiation patterns in original and optimised structures at elevation 90 degree
Fig. 7.A.38 The comparison of U1 plane radiation patterns in optimised structure at different elevation degrees

Fig. 7.A.39 The comparison of U2 plane radiation patterns in optimised structure at different elevation degrees
Fig. 7.A.40 The first vibration mode of the optimised structure

Fig. 7.A.41 The second vibration mode of the optimised structure
Fig. 7.A.42 The third vibration mode of the optimised structure

Fig. 7.A.43 The fourth vibration mode of the optimised structure
7.5 The optimisation problem

The advent of high performance composite materials offers the structural designer a wide range of new degrees of freedom. To obtain an efficient structural design, one can operate on the shape of the cross-section or on its thickness, but also on the angle-ply and on the number and sequence of the layers. With this increase in the number of alternative choices available to the structural engineer comes the responsibility for providing a rational basis for seeking simultaneous optimum designs of structural configuration and structural material. As a consequence, the designer has a greater control of the behaviour of the structure but, at the same time, he is faced with the problem of selecting the values of a great number of design variables. The possibility of achieving a design that efficiently meets multiple requirements, coupled with the difficulty in selecting the values of a large set of design variables, makes structural optimisation an important tool for the design of laminated composite structures.

An optimisation procedure for orbiting space antenna reflectors will be described in this Part. The procedure (using MOST program) employs standard finite element structural analysis (ABAQUS software) and optimisation techniques to predict a structural design which will improve antenna performance while minimising the structural mass and launch cost.

Fibre-reinforced thermosetting polymer composites are presently the preferred materials for antenna reflectors due to their advantageous properties such as light weight, high strength and stiffness, low coefficient of thermal expansion and in certain cases, radio frequency transparency. Laminate, sandwich configuration and rib shape in the structure are optimised in this application. Typically, laminate optimisation is more exposed to problems related to nonconvexity because of the trigonometric transformations involved in the system equations based on laminate plate theory. The problem consists of selecting the design variables (such as the ply, the laminate thickness, the laminate...
direction) in such a way that the corresponding values of the objective functions
determine an optimum trade-off situation.

Considerable work has been undertaken on the optimisation of this structure for several
performance parameters simultaneously. Thus a design was sought in which best
performance, maximum stiffness and minimum mass were obtained, while considering
the effects of temperature changes and launch case.

Minimising structural distortions caused by temperature changes encountered during
orbital flight are the most critical aspects in designing structural and material systems
for large space antenna structures. Surface profile accuracy is of prime importance for
performance of antenna reflectors for space communication satellites. Deviations of the
reflector shape from the ideal shape could cause a change of position to the reflector focal point, reduction in the peak antenna gain, and an increase in side-lobe level. The
surface RMS error

$$RMS = \left( \sum_{i=1}^{n} \Delta_i^2 / n \right)^{1/2}$$

provides an independent performance index against which distorted reflector antennas
can be compared. The deviations $\Delta_i$ were calculated in directions normal to the defined
profile. Thus, the RMS deviations from the design profile of the composite reflector
form one of the objective functions to be minimised in the design optimisation.

For the following reasons, lightweight space structures are desirable:
* to reduce launch and orbit transfer costs, e.g. reduce propellant mass for orbit change operation;
* to reduce torque and power required to slew and point on-orbit;
* to allow an increase in the mass for other subsystems.

It is of practical interest to design an antenna structure of reduced surface errors but of
less structural mass by finding an optimal stiffness distribution. Therefore, structural
mass is taken as one of the objectives which is to be minimised.

Structural fundamental frequencies at deployed and stowed conditions are also included
for optimisation. Being able to locate the natural frequencies of structures provides a
method for keeping the structure from being within certain critical ranges which may
give peak acceleration or an unwanted response due to resonance.
The reflector was constructed with honeycomb sandwich panel and ribs, shown in Figure 3.20 and 7.B.18. The structure is required to be optimised for mass, frequency, stiffness, strength and surface accuracy. Both the surface panel and ribs were modelled with carbon fibre sheets and aluminium honeycomb sandwiches. The local use of stiffening ribs improves the quality of the structure by avoiding the use of heavier face sheets over the entire structure. The design optimisation problem was required to minimise the structural mass, reflector surface RMS error, maximum displacement and maximum stress of the structure, and to increase structural frequencies at stowed (in the launch case) and deployed shapes to their given best level (the target) values. All of these topics are of great importance to the structural designer.

The optimisation studies to determine possible new fibre/matrix combinations as ply candidates and a new rib stiffening system. The design model included a total of 23 design variables representing structural configurations and composite materials in the sandwich panels and ribs. The design variables used for optimisation include the shapes of the ribs, and individual ply thicknesses of the honeycomb face sheets, ply orientations (fibre angles), and honeycomb-cores thicknesses, in both panels and ribs. These design variables provided the designer with more control to fine tune the structure and these sizing variables and geometry variables will be considered simultaneously in the optimisation.

The design of the complete structural system for surface accuracy, static and dynamic requirements is a complex task, and therefore, structured design methods are essential for rapid evaluation of configuration changes and for achieving optimal designs. In the case of multidisciplinary design and complex laminated composite material structures it is not always easy to use analytical optimisation methods.

The optimisation procedure is applied to a specific test problem detailed in Section 3.10.

7.6 Analysis of the original design of the reflector structure
This is a space reflector antenna made of carbon fibre composite sandwich panels (with Al honeycomb core) and backup ribs (also composite sandwich). This structure has 133 nodes, 198 laminate element, and 774 degrees of freedom. The detailed analysis of the
performances of the original design of the reflector structure under four different extreme loading cases is given in Section 3.10. The structural system is first analysed with respect to the parameters which, collectively, describe the overall performance of the antenna. The performance data matrix (PDM) obtained by the analysis is shown in Table 7.B.1.

Table 7.B.1 Performances of original structure at four different loading cases

<table>
<thead>
<tr>
<th></th>
<th>Case 1 (-180 °C)</th>
<th>Case 2 (+115 °C)</th>
<th>Case 3 Thermal gradient</th>
<th>Case 4 Launch case</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS A (mm)</td>
<td>2.28</td>
<td>1.05</td>
<td>1.98</td>
<td>1.43</td>
</tr>
<tr>
<td>Maximum stress</td>
<td>136.1</td>
<td>62.7</td>
<td>136.1</td>
<td>33.0</td>
</tr>
<tr>
<td>σ_max (M N/m²)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Structural mass (kg)</td>
<td>18.6</td>
<td>18.6</td>
<td>18.6</td>
<td>18.6</td>
</tr>
<tr>
<td>Structural frequency (Hz)</td>
<td>4.35</td>
<td>4.35</td>
<td>4.35</td>
<td>24.3</td>
</tr>
<tr>
<td>Maximum displacement</td>
<td>4.88</td>
<td>2.25</td>
<td>4.27</td>
<td>4.75</td>
</tr>
</tbody>
</table>

For this antenna structure, the structural frequency needs to be increased to the values of over 8 Hz for deployed shape and over 28 Hz for stowed shape in the launch case, and other performances (i.e. reflector surface RMS error, structural mass, maximum displacement and maximum stress) at all loading cases need to be minimised.

The acceptable limits and best level values for this antenna are given in Table 7.B.2. Depending upon whether the performance parameter has lower, upper or double limits, for every pair of performance data under the 4 cases, the upper one represents the best level, upper limit or upper limit value respectively, and the lower one represents the lower limit, best level or lower limit value respectively.

Table 7.B.2 The acceptable limits and the best level values

<table>
<thead>
<tr>
<th></th>
<th>Case 1 (-180 °C)</th>
<th>Case 2 (+115 °C)</th>
<th>Case 3 Thermal gradient</th>
<th>Case 4 Launch case</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS A (mm)</td>
<td>3.00</td>
<td>2.00</td>
<td>3.00</td>
<td>2.00</td>
</tr>
<tr>
<td>Maximum stress</td>
<td>200.0</td>
<td>100.0</td>
<td>200.0</td>
<td>200.0</td>
</tr>
<tr>
<td>Maximum stress</td>
<td>200.0</td>
<td>100.0</td>
<td>200.0</td>
<td>200.0</td>
</tr>
</tbody>
</table>
Using the information in Tables 7.B.1 and 7.B.2, the calculated values are converted into scores using a linear relationship based on the actual performance to the nearest performance limit. The parameter profile matrix (PPM) can be obtained by undertaking a performing performance proximity calculation. The data of the PPM are shown in Table 7.B.3.

<table>
<thead>
<tr>
<th>$\sigma_{\text{max}}$ (M N/m²)</th>
<th>60.0</th>
<th>60.0</th>
<th>60.0</th>
<th>60.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>structural mass (kg)</td>
<td>22.0</td>
<td>22.0</td>
<td>22.0</td>
<td>22.0</td>
</tr>
<tr>
<td>structural frequency (Hz)</td>
<td>8.00</td>
<td>8.00</td>
<td>8.00</td>
<td>28.0</td>
</tr>
<tr>
<td>maximum displacement</td>
<td>6.00</td>
<td>4.00</td>
<td>6.00</td>
<td>6.00</td>
</tr>
<tr>
<td>$\delta_{\text{max}}$ (mm)</td>
<td>0.20</td>
<td>0.08</td>
<td>0.40</td>
<td>3.00</td>
</tr>
</tbody>
</table>

**Table 7.B.3** The parameter profile matrix of the original antenna structure

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-180 °C</td>
<td>+115 °C</td>
<td>Thermal gradient</td>
<td>Launch case</td>
</tr>
<tr>
<td>RMS $\Delta$</td>
<td>2.42</td>
<td>4.78</td>
<td>3.47</td>
<td>5.16</td>
</tr>
<tr>
<td>maximum stress</td>
<td>4.56</td>
<td>9.33</td>
<td>4.54</td>
<td>10.00</td>
</tr>
<tr>
<td>structural mass</td>
<td>3.4</td>
<td>3.4</td>
<td>3.4</td>
<td>3.4</td>
</tr>
<tr>
<td>structural frequency</td>
<td>0.87</td>
<td>0.87</td>
<td>0.87</td>
<td>5.35</td>
</tr>
<tr>
<td>maximum displacement</td>
<td>1.92</td>
<td>4.47</td>
<td>3.10</td>
<td>4.18</td>
</tr>
</tbody>
</table>

Analysis of the scores is carried out with respect to each performance parameter for the range of loading cases of the antenna. An overall rating for each performance parameter is obtained across the working/loading range. Similarly, for each loading case considered, the scores for each performance parameter are analysed to obtain an overall rating for each loading case. Through the system parameter profile analysis and loading case profile analysis, the mean values, standard deviations and parameter performance indices (PPIs) and case performance indices (CPIs) can be obtained. Thus, in a systematic manner the evaluation identifies the parameters in which the performance is weakest and the working/loading cases at which the antenna is least effective. The PPIs for the performance across all the loading cases and CPIs for the loading cases across all the performance parameters are shown in Tables 7.B.4 and 7.B.5 respectively.
Table 7.B.4 System parameter profile analysis of the original antenna structure

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>P.P.I.</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS Error</td>
<td>3.96</td>
<td>1.08</td>
<td>3.62</td>
</tr>
<tr>
<td>Maximum Stress</td>
<td>7.11</td>
<td>2.57</td>
<td>6.19</td>
</tr>
<tr>
<td>Structural Mass</td>
<td>3.40</td>
<td>0.00</td>
<td>3.40</td>
</tr>
<tr>
<td>Structural Frequency</td>
<td>1.99</td>
<td>1.94</td>
<td>1.10</td>
</tr>
<tr>
<td>Maximum Displacement</td>
<td>3.42</td>
<td>1.00</td>
<td>3.06</td>
</tr>
</tbody>
</table>

Table 7.B.5 Loading case profile analysis of the original antenna structure

<table>
<thead>
<tr>
<th>Case</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>C.P.I.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1 (-180 °C)</td>
<td>2.64</td>
<td>1.26</td>
<td>1.93</td>
</tr>
<tr>
<td>Case 2 (+115 °C)</td>
<td>4.57</td>
<td>2.75</td>
<td>2.52</td>
</tr>
<tr>
<td>Case 3 (Thermal gradient)</td>
<td>3.08</td>
<td>1.21</td>
<td>2.20</td>
</tr>
<tr>
<td>Case 4 (Launch case)</td>
<td>5.62</td>
<td>2.30</td>
<td>4.93</td>
</tr>
</tbody>
</table>

Simultaneous consideration of parameters and loading cases will identify the weakest aspects of the design. Therefore, the two kinds of indices PPI and CPI should be combined into a parameter/loading-case index, which indicates the highest influence on the overall system and the vulnerability (or superiority) of each individual parameter/loading-case combination.

The parameter/working-case superiority (or vulnerability) indices (see Section 6.4) for the antenna are shown in Table 7.B.6. From this table, the calculation clearly identifies that the structural frequencies at both stowed and deployed shapes are very weak followed by structural maximum displacement and structural mass. It also identifies that the worst two loading cases are extremely low temperature (-180 °C) and a distribution of temperature gradient (case 3).
Table 7.B.6 The parameter/case superiority indices of the original antenna structure

<table>
<thead>
<tr>
<th></th>
<th>Case 1 (-180 °C)</th>
<th>Case 2 (+115 °C)</th>
<th>Case 3 Thermal gradient</th>
<th>Case 4 Launch case</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RMS Δ</strong></td>
<td>6.98</td>
<td>9.13</td>
<td>7.96</td>
<td>17.87</td>
</tr>
<tr>
<td>maximum stress</td>
<td>11.92</td>
<td>15.58</td>
<td>13.59</td>
<td>30.50</td>
</tr>
<tr>
<td>structural mass</td>
<td>6.55</td>
<td>8.56</td>
<td>7.47</td>
<td>16.76</td>
</tr>
<tr>
<td>structural frequency</td>
<td>2.12</td>
<td>2.77</td>
<td>2.41</td>
<td>5.42</td>
</tr>
<tr>
<td>maximum displacement</td>
<td>5.90</td>
<td>7.72</td>
<td>6.73</td>
<td>15.10</td>
</tr>
</tbody>
</table>

Using the sets of weight functions for performance parameters and loading cases shown in Tables 7.B.7 and 7.B.8 respectively, the weighted parameter/case superiority indices are obtained (Table 7.B.9).

Table 7.B.7 The weights of parameters for the antenna structure

<table>
<thead>
<tr>
<th>Performance Parameter</th>
<th>RMS Δ</th>
<th>maximum stress</th>
<th>structural mass</th>
<th>structural frequency</th>
<th>maximum displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weights</td>
<td>0.35</td>
<td>0.10</td>
<td>0.30</td>
<td>0.15</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Table 7.B.8 The weights of loading cases for the antenna structure

<table>
<thead>
<tr>
<th>Working/Loading Cases</th>
<th>Case 1 (-180 °C)</th>
<th>Case 2 (+115 °C)</th>
<th>Case 3 Thermal gradient</th>
<th>Case 4 Launch case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weights</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Table 7.B.9 The weighted parameter/case superiority indices of the original antenna structure

<table>
<thead>
<tr>
<th></th>
<th>Case 1 (-180 °C)</th>
<th>Case 2 (+115 °C)</th>
<th>Case 3 Thermal gradient</th>
<th>Case 4 Launch case</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RMS Δ</strong></td>
<td>0.61</td>
<td>0.80</td>
<td>0.70</td>
<td>1.56</td>
</tr>
<tr>
<td>maximum stress</td>
<td>0.30</td>
<td>0.39</td>
<td>0.34</td>
<td>0.76</td>
</tr>
<tr>
<td>structural mass</td>
<td>0.49</td>
<td>0.64</td>
<td>0.56</td>
<td>1.26</td>
</tr>
<tr>
<td>structural frequency</td>
<td>0.08</td>
<td>0.10</td>
<td>0.09</td>
<td>0.20</td>
</tr>
<tr>
<td>maximum displacement</td>
<td>0.15</td>
<td>0.19</td>
<td>0.17</td>
<td>0.38</td>
</tr>
</tbody>
</table>
The overall performance index (OPI) for the original design of this reflector structure is very low, and is only 10.05 for unweighted and 9.77 for weighted cases. This indicates that the design of the antenna structure is unsatisfactory and a full optimisation of the structural and material system should be carried out.

### 7.7 Optimisation results of the reflector structure

Four different space loading cases are considered simultaneously in the optimisation. They are extremely cold temperature, extremely hot temperature, a temperature gradient distribution from 0 to -180 degree in the structure, and stowed reflector in 8/11.2/30 Gs accelerations in x/y/z directions in the launch case.

The original design had a fundamental frequency of 4.35 Hz in deployed shape and 24.3 Hz in stowed shape with an overall mass of 18.6 kg. Other performances of the original design can be found in Table 7.B.1.

This optimisation problem can be solved quickly because evaluation for the RMS distortion is trivial compared to evaluating antenna radiation performance. After the optimisation iteration, a reflector structure which is much stronger, stiffer, lighter and more accurate than the original design was obtained. The convergence history of the optimisation is shown in Figure 7.B.1. The OPIs are greatly enhanced through the optimisation. The weighted OPI is increased from original score of 9.77 to the optimised score of 83.27 and the unweighted OPI is increased from original score of 10.05 to the optimised score of 80.80. Figures 7.B.2 - 7.B.5 illustrate that all the structural performances at all the loading cases have been successfully improved. The convergence history of the optimisation shows that the current procedure converges to a much better design than the original one based on the optimisation criteria.

The structural performances at all loading cases before and after optimisation are shown in Table 7.B.1 and Table 7.B.10 respectively, from which, the tremendous improvement can be seen. The significant improvement in the design, following the optimisation, is shown by the following:
1) in loading cases 1 (-180 °C), 2 (+115 °C) and 3 (thermal gradient):
   - The RMS errors are reduced from the original 2.28, 1.05 and 1.98 mm to the optimised 0.042, 0.019 and 0.285 mm respectively.
   - The maximum stresses are reduced from the original 136, 62.7 and 136 MPa to the optimised 130, 59.7 and 130 MPa respectively.
   - The maximum displacements are reduced from the original 4.88, 2.25 and 4.27 mm to the optimised 0.457, 0.211 and 1.13 mm respectively.

2) in loading case 4 (launch case):
   - The RMS error is reduced from the original 1.43 mm to the optimised 0.857 mm.
   - The maximum stress is reduced from the original 33.0 MPa to the optimised 19.8 MPa.
   - The maximum displacement is reduced from the original 4.75 mm to the optimised 2.99 mm.

3) in addition to the above
   - The final design has a total structural mass of 12.7kg, a reduction of 32% from the original mass of 18.6kg.
   - The structural fundamental frequency is increased from the original 4.35/24.3 Hz to the optimised 9.07/29.1 Hz for deployed/stowed shape.

Table 7.B.10 Performances of optimised structure at four different loading cases

<table>
<thead>
<tr>
<th></th>
<th>Case 1 (-180 °C)</th>
<th>Case 2 (+115 °C)</th>
<th>Case 3 Thermal gradient</th>
<th>Case 4 Launch case</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS Δ (mm)</td>
<td>0.042</td>
<td>0.019</td>
<td>0.285</td>
<td>0.857</td>
</tr>
<tr>
<td>maximum stress</td>
<td>130.0</td>
<td>59.7</td>
<td>130.0</td>
<td>19.8</td>
</tr>
<tr>
<td>σ_{max} (M N/m²)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>structural mass (kg)</td>
<td>12.7</td>
<td>12.7</td>
<td>12.7</td>
<td>12.7</td>
</tr>
<tr>
<td>structural frequency (Hz)</td>
<td>9.07</td>
<td>9.07</td>
<td>9.07</td>
<td>29.1</td>
</tr>
<tr>
<td>maximum displacement</td>
<td>0.457</td>
<td>0.211</td>
<td>1.13</td>
<td>2.99</td>
</tr>
<tr>
<td>δ_{max} (mm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The improvement between the original antenna and the optimised antenna is very significant. It can be seen that it has been possible to obtain a stress-safe design with two-thirds of the original mass and a substantially improved RMS. Figures 7.B.6 ~
7.B.9 provide a more complete comparison between the characteristics of the antenna before and after optimisation. These figures simply and clearly show the performance parameters before and after optimisation at all the loading cases considered. The best level values and the given acceptable limits on each performance parameter at each loading case are also shown in these figures.

Figures 7.B.10 ~ 7.B.15 illustrate the convergence histories for the mean values, SDs, PPIs and CPIs for all the performance parameters and all the loading cases considered in the optimisation. The values of means, SDs, PPIs and CPIs for the optimised antenna system are listed in Tables 7.B.11 and 7.B.12. From these tables, a comparison can be made by referring to the values of the same parameters for the original antenna system which are listed in the Tables 7.B.4 and 7.B.5.

Table 7.B.11 System parameter profile analysis of the optimised antenna structure

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>P.P.I.</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS Error</td>
<td>9.78</td>
<td>0.31</td>
<td>9.77</td>
</tr>
<tr>
<td>Maximum Stress</td>
<td>7.51</td>
<td>2.49</td>
<td>6.68</td>
</tr>
<tr>
<td>Structural Mass</td>
<td>9.30</td>
<td>0.00</td>
<td>9.30</td>
</tr>
<tr>
<td>Structural Frequency</td>
<td>10.00</td>
<td>0.00</td>
<td>10.00</td>
</tr>
<tr>
<td>Maximum Displacement</td>
<td>9.48</td>
<td>0.48</td>
<td>9.46</td>
</tr>
</tbody>
</table>

Table 7.B.12 Loading case profile analysis of the optimised antenna structure

<table>
<thead>
<tr>
<th>Case Description</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>C.P.I.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1 (-180 °C)</td>
<td>8.76</td>
<td>1.89</td>
<td>8.17</td>
</tr>
<tr>
<td>Case 2 (+115 °C)</td>
<td>9.78</td>
<td>0.27</td>
<td>9.78</td>
</tr>
<tr>
<td>Case 3 (Thermal gradient)</td>
<td>8.45</td>
<td>1.77</td>
<td>7.93</td>
</tr>
<tr>
<td>Case 4 (Launch case)</td>
<td>9.86</td>
<td>0.28</td>
<td>9.85</td>
</tr>
</tbody>
</table>

A significant improvement in design quality can be observed from the comparison. It can be seen that the structural performance parameters have much higher mean values
and PPIs and much lower standard deviations at all loading cases considered in the optimised structure than in the original design. Also, at each loading case the antenna will behave in such a way that all the performances have increased reliability and have decreased possibility of performing unsatisfactorily.

For the optimised reflector structure, the parameter profile matrix, parameter/loading-case superiority indices and weighted parameter/loading-case superiority indices are listed in Tables 7.B.13, 7.B.14 and 7.B.15 respectively. A more detailed comparison can be made by inspecting these results and comparing them with their corresponding values given in Tables 7.B.3, 7.B.6 and 7.B.9 which are for the original structure.

Table 7.B.13 The parameter profile matrix of the optimised antenna structure

<table>
<thead>
<tr>
<th></th>
<th>Case 1 (-180 °C)</th>
<th>Case 2 (+115 °C)</th>
<th>Case 3 Thermal gradient</th>
<th>Case 4 Launch case</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS Δ</td>
<td>9.93</td>
<td>9.95</td>
<td>9.24</td>
<td>10.00</td>
</tr>
<tr>
<td>max stress</td>
<td>5.02</td>
<td>10.00</td>
<td>5.01</td>
<td>10.00</td>
</tr>
<tr>
<td>mass</td>
<td>9.30</td>
<td>9.30</td>
<td>9.30</td>
<td>9.30</td>
</tr>
<tr>
<td>freq</td>
<td>10.00</td>
<td>10.00</td>
<td>10.00</td>
<td>10.00</td>
</tr>
<tr>
<td>disp</td>
<td>9.56</td>
<td>9.67</td>
<td>8.70</td>
<td>10.00</td>
</tr>
</tbody>
</table>

Table 7.B.14 The parameter/case superiority indices of the optimised antenna structure

<table>
<thead>
<tr>
<th></th>
<th>Case 1 (-180 °C)</th>
<th>Case 2 (+115 °C)</th>
<th>Case 3 Thermal gradient</th>
<th>Case 4 Launch case</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS Δ</td>
<td>79.80</td>
<td>95.50</td>
<td>77.49</td>
<td>96.24</td>
</tr>
<tr>
<td>max stress</td>
<td>54.57</td>
<td>65.31</td>
<td>52.99</td>
<td>65.81</td>
</tr>
<tr>
<td>mass</td>
<td>75.97</td>
<td>90.92</td>
<td>73.77</td>
<td>91.62</td>
</tr>
<tr>
<td>freq</td>
<td>81.69</td>
<td>97.76</td>
<td>79.32</td>
<td>98.52</td>
</tr>
<tr>
<td>disp</td>
<td>77.25</td>
<td>92.45</td>
<td>75.01</td>
<td>93.16</td>
</tr>
</tbody>
</table>
Table 7.B.15 The weighted parameter/case superiority indices of the optimised antenna structure

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(-180 °C)</td>
<td>(+115 °C)</td>
<td>Thermal gradient</td>
<td>Launch case</td>
</tr>
<tr>
<td>RMS Δ</td>
<td>6.98</td>
<td>8.36</td>
<td>6.78</td>
<td>8.42</td>
</tr>
<tr>
<td>maximum stress</td>
<td>1.36</td>
<td>1.63</td>
<td>1.32</td>
<td>1.65</td>
</tr>
<tr>
<td>structural mass</td>
<td>5.70</td>
<td>6.82</td>
<td>5.53</td>
<td>6.87</td>
</tr>
<tr>
<td>structural frequency</td>
<td>3.06</td>
<td>3.67</td>
<td>2.97</td>
<td>3.69</td>
</tr>
<tr>
<td>maximum displacement</td>
<td>1.93</td>
<td>2.31</td>
<td>1.88</td>
<td>2.33</td>
</tr>
</tbody>
</table>

For the optimised antenna structure, the above tables (7.B.10 ~ 7.B.15) show that the worst-case temperature profile corresponded to a condition of temperature gradients across the antenna surface and not to a situation of absolute temperature excursion from the ambient fabrication temperature.

The curves of the RMS, maximum displacement, and maximum stress in the optimised structure versus absolute temperature excursions from the ambient fabrication temperature are shown in Figures 7.B.37 and 7.B.38, where the range of the temperature is from -180 °C to +120 °C with respect to different orbit positions and times. To make a comparison, the curves for the same performance parameters but in the original structure are also shown in these two figures.

Table 7.B.16 shows the original design and the resultant optimal design of the multi-objective optimisation. A review of the solutions confirms the sizing of the members and plate thicknesses obtained by the optimisation. The determination of fibre orientation angles is an important subject in optimum design of composite materials. The fact that the design variables for the fibre angles in both composite surface panels and composite stiffening ribs did not alter throughout the optimisation process verified that the ±45° fibre angles for these components are the best choice. The iteration history of the design variables is shown in Figure 7.B.16 and 7.B.17. The optimised reflector geometry is shown in Figure 7.B.18. The changes in geometric shapes of the stiffening ribs in the original design and the optimised structure can be clearly observed by comparing the shapes in Figure 7.B.19 and Figure 7.B.20. The deformations and stress contours of the optimised reflector at all the loading cases considered are illustrated in Figures 7.B.21 ~ 7.B.28.
Table 7.B.16 Design variables in original and optimised designs

<table>
<thead>
<tr>
<th>Group</th>
<th>Design variables</th>
<th>Before optimisation</th>
<th>After optimisation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>thickness of each fibre layers in surface panels (cm)</td>
<td>0.01</td>
<td>0.005</td>
</tr>
<tr>
<td>2</td>
<td>thickness of each fibre layers in stiffening ribs (cm)</td>
<td>0.01</td>
<td>0.011</td>
</tr>
<tr>
<td>3</td>
<td>thickness of honeycomb core in surface panels (cm)</td>
<td>1.00</td>
<td>0.893</td>
</tr>
<tr>
<td>4</td>
<td>thickness of honeycomb core in stiffening ribs (cm)</td>
<td>2.00</td>
<td>1.935</td>
</tr>
<tr>
<td>5</td>
<td>ply orientations in the third and fourth layers of the skin sheets of the surface panels (degree)</td>
<td>±45.0</td>
<td>±45.0</td>
</tr>
<tr>
<td>6</td>
<td>ply orientations in the third and fourth layers of the skin sheets of the stiffening ribs (degree)</td>
<td>±45.0</td>
<td>±45.0</td>
</tr>
<tr>
<td>7</td>
<td>ribs' geometric variable (m)</td>
<td>-0.043</td>
<td>-0.193</td>
</tr>
<tr>
<td>8</td>
<td>ribs' geometric variable (m)</td>
<td>0.011</td>
<td>-0.114</td>
</tr>
<tr>
<td>9</td>
<td>ribs' geometric variable (m)</td>
<td>0.093</td>
<td>0.058</td>
</tr>
<tr>
<td>10</td>
<td>ribs' geometric variable (m)</td>
<td>0.194</td>
<td>0.191</td>
</tr>
<tr>
<td>11</td>
<td>ribs' geometric variable (m)</td>
<td>0.324</td>
<td>0.324</td>
</tr>
<tr>
<td>12</td>
<td>ribs' geometric variable (m)</td>
<td>0.478</td>
<td>0.456</td>
</tr>
<tr>
<td>13</td>
<td>ribs' geometric variable (m)</td>
<td>0.657</td>
<td>0.607</td>
</tr>
<tr>
<td>14</td>
<td>ribs' geometric variable (m)</td>
<td>0.861</td>
<td>0.797</td>
</tr>
<tr>
<td>15</td>
<td>ribs' geometric variable (m)</td>
<td>1.090</td>
<td>1.078</td>
</tr>
<tr>
<td>16</td>
<td>ribs' geometric variable (m)</td>
<td>1.378</td>
<td>1.378</td>
</tr>
<tr>
<td>17</td>
<td>ribs' geometric variable (m)</td>
<td>1.624</td>
<td>1.714</td>
</tr>
<tr>
<td>18</td>
<td>ribs' geometric variable (m)</td>
<td>0.207</td>
<td>0.295</td>
</tr>
<tr>
<td>19</td>
<td>ribs' geometric variable (m)</td>
<td>0.311</td>
<td>0.317</td>
</tr>
<tr>
<td>20</td>
<td>ribs' geometric variable (m)</td>
<td>0.440</td>
<td>0.452</td>
</tr>
<tr>
<td>21</td>
<td>ribs' geometric variable (m)</td>
<td>0.594</td>
<td>0.589</td>
</tr>
<tr>
<td>22</td>
<td>ribs' geometric variable (m)</td>
<td>0.774</td>
<td>0.773</td>
</tr>
<tr>
<td>23</td>
<td>ribs' geometric variable (m)</td>
<td>0.978</td>
<td>1.058</td>
</tr>
</tbody>
</table>

The modal analysis for the optimised antenna structure shows that the lowest structural natural frequencies for both the deployed and stowed cases are 9.07 Hz and 29.1 Hz respectively. These frequencies are increased from their original values of 4.35 Hz and 24.3 Hz, and satisfied the given target values which should be over 8 Hz and 28 Hz respectively. The lowest four structural frequencies and their vibration mode shapes of

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7. Integrated structural electromagnetic optimisation of antenna systems — Part B
the optimised reflector structure for both the deployed and stowed shapes are shown in Figures 7.B.29 ~ 7.B.36 to illustrate general vibration behaviour. For a comparison, Table 7.B.17 lists the low frequencies between original and optimised structures in deployed and stowed cases. The optimised structure produces significantly higher modal frequencies than the original structure. This is due to the relatively stiff nature of the optimised structure.

Table 7.B.17 The comparison of low frequencies between original and optimised structures

<table>
<thead>
<tr>
<th></th>
<th>deployed structure</th>
<th>stowed structure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>original</td>
<td>optimised</td>
</tr>
<tr>
<td>lowest</td>
<td>4.35</td>
<td>9.07</td>
</tr>
<tr>
<td>second lowest</td>
<td>7.23</td>
<td>9.35</td>
</tr>
<tr>
<td>third lowest</td>
<td>21.4</td>
<td>31.1</td>
</tr>
<tr>
<td>fourth lowest</td>
<td>28.9</td>
<td>34.6</td>
</tr>
</tbody>
</table>

The application of the method on this composite antenna structure appeared very encouraging. The results show that the method has the ability to locate compromise design satisfying all the constraints. The multi-objective optimisation was proved to be superior in optimising the competing requirements for high surface accuracy, low weight, high stiffness and strength, and resulted in significant simultaneous improvements in all objective functions under all loading cases considered.
Fig. 7.B.1 The convergence history of overall objectives

Fig. 7.B.2 The convergence history of reflector surface accuracies (RMSs) at different loading cases
Fig. 7.B.3 The convergence history of maximum displacements at different loading cases

Fig. 7.B.4 The convergence history of maximum stresses at different loading cases
Fig. 7.B.5 The convergence history of structural mass and fundamental frequencies

Fig. 7.B.6 The comparison of reflector surface accuracies (RMSs) between original and optimised antenna structures
Fig. 7.B.7 The comparison of the maximum displacements between original and optimised antenna structures

Fig. 7.B.8 The comparison of the maximum stresses between original and optimised antenna structures
Fig. 7.B.9 The comparison of structural masses and frequencies between original and optimised antenna structures

Fig. 7.B.10 The convergence history of the means of performances in the parameter profile matrix
Fig. 7.B.11 The convergence history of the standard deviations of performances in the parameter profile matrix

Fig. 7.B.12 The convergence history of the parameter performance indices
Fig. 7.B.13 The convergence history of the means of working/loading cases in the parameter profile matrix.

Fig. 7.B.14 The convergence history of the standard deviations of working/loading cases in the parameter profile matrix.
Fig. 7.B.15  The convergence history of the working/loading cases performance indices

Fig. 7.B.16  The convergence history of design variables
(thicknesses of layers and honeycomb cores)
Fig. 7.B.17 The convergence history of structural geometric design variables

Fig. 7.B.18 The optimised reflector geometry
Fig. 7.B.19 The original geometry of the reflector structure

Fig. 7.B.20 The optimised geometry of the reflector structure
DISPLACEMENT MAGNIFICATION FACTOR = 991.

Fig. 7.B.21 The deformation of the reflector structure at loading case 1

DISPLACEMENT MAGNIFICATION FACTOR = 2.152E+03

Fig. 7.B.22 The deformation of the reflector structure at loading case 2
Fig. 7.B.23 The deformation of the reflector structure at loading case 3

Fig. 7.B.24 The deformation of the reflector structure at loading case 4
Fig. 7.B.25 The stress contours of the reflector structure at loading case 1

Fig. 7.B.26 The stress contours of the reflector structure at loading case 2
Fig. 7.B.27 The stress contours of the reflector structure at loading case 3

Fig. 7.B.28 The stress contours of the reflector structure at loading case 4
Fig. 7.B.29 The first vibration mode of deployed reflector

Fig. 7.B.30 The second vibration mode of deployed reflector
Fig. 7.B.31 The third vibration mode of deployed reflector

Fig. 7.B.32 The fourth vibration mode of deployed reflector
Fig. 7.B.33 The first vibration mode of stowed reflector

Fig. 7.B.34 The second vibration mode of stowed reflector
Fig. 7.B.35 The third vibration mode of stowed reflector

Fig. 7.B.36 The fourth vibration mode of stowed reflector
Fig. 7.B.37 RMS, maximum displacement versus absolute temperature excursions before and after optimisation

Fig. 7.B.38 Maximum stress versus absolute temperature excursions before and after optimisation
Part C: Other examples (the low-sidelobe-orientated optimisation of the antenna systems for GEC-Marconi, UK)

The concept of using Zernike polynomials as a means of characterising highly correlated reflector surface distortions has been utilised in the EM analysis of these antennas. The method is based on the fact that some forms of distortion may be identified as being particularly damaging to some antenna EM performances such as beam sidelobe. Optimising the antenna structures to minimise the occurrence of these undesirable ‘modes’ may, therefore, be an indirect method of constraining sidelobe levels (in the absence of a rigorous EM analysis).

7.8 Zernike mode analysis of reflector distortions

Reflector surface errors can be classified as either random or systematic distortions. Random distortions may be treated statistically; their effects are well understood and they are generally controlled by specifying a RMS tolerance on allowable surface errors. However, it is the systematic (highly correlated) forms of error that are the most detrimental to the antenna sidelobe performance (Searle and Humphrey, 1997). Specific constraints must be imposed on systematic distortions if very low sidelobes are to be realised. By suppressing systematic surface distortions, the tolerable RMS error of the surface may be relaxed. This may avoid the situation of needing to specify a RMS tolerance which is unrealisable in practice.

To assess the sidelobes, the Zernike mode analysis method has been used at GEC-Marconi Research Centre (Searle and Humphrey, 1997). In this method, a range of analytic distortions were applied to the reflector surface and their impact on sidelobe levels noted. The distortions chosen were represented by Zernike polynomials, which are a set of orthogonal functions defined over a circular area. These functions are particularly useful because their orthogonality allows any arbitrary distortion to be...
expressed as a series of basic polynomials (or modes), i.e. to be represented as a superposition of these polynomials.

Mathematical definitions of Zernike polynomials may be found in the literature (see for example Born and Wolf, 1975); however the basic nature of each mode is best appreciated graphically. The polynomials comprise two main components - a radial variation, plus a periodic azimuthal variation. The form of these components is controlled by two integer indices \( n \) and \( m \). Not all combinations of \( n \) and \( m \) are possible; permitted values of \( n \) and \( m \) are restricted by the inequality \( n \geq m \), and by the requirement that the difference \( n - m \) must be an even number. Each mode is characterised by an oscillatory radial variation and a periodic azimuthal variation, which are determined by integer indices \( n \) and \( m \).

To provide a single numerical indication of likely electromagnetic performance, Zernike mode amplitudes present in a distorted surface are compared to specified tolerances. A 'Figure of Merit' (FoM) is then defined by summing the differences between actual mode amplitudes \( \alpha_{n,m} \) and their tolerance \( tol_{n,m} \), for those modes where the amplitude is greater than its tolerance, i.e.

\[
\Delta_{n,m} = \alpha_{n,m} - tol_{n,m}
\]

\[
FoM = \sum_{n,m} \Delta_{n,m} \quad \text{for} \quad \Delta_{n,m} > 0
\]

The relative increases in sidelobe level observed due to each Zernike mode are illustrated in Figure 7.C.1. It is clear that the antenna sidelobe performance will be far more tolerant to some forms of systematic reflector distortions than others. Modes with small values of \( n \) and \( m \) have little impact, but those with a radial index of around 6 or 7, combined with low order azimuthal variation, are particularly damaging.

Greater insight into the acceptability of the mechanical structure may be gained by analysing the predicted reflector distortions in terms of their component Zernike modes. Using random simulations of arbitrary, but highly correlated, reflector distortions, it has been found that suppressing the undesirable component modes leads to an increased probability of achieving low sidelobe performance. A typical example is shown in Figure 7.C.2, which represents the distortion of a 4m antenna under wind loading from the side at 60m/sec. While some modes occur with a relatively high magnitude, those identified as particularly damaging to the sidelobe performance (shown in Figure 7.C.1) have been largely avoided.
A Zernike mode analysis code has been developed in GEC-Marconi Research Centre at Chelmsford, UK. The analysis is based on the circle polynomials of Zernike. This code is developed to be interfaced with MOST program as a subroutine, which is called repeatedly during the optimisation procedures. The routine is formulated in that reflectors have a circular aperture when projected onto the x-y plane and the z axis is in the direction of the antenna beam. The routine is given a set of co-ordinates for points on the ideal reflector surface, and a corresponding set of displacements of those points for various loading conditions. Given tolerances for the acceptable magnitudes specified modes, the code will return a FoM which is a measure of the undesirable modes presented in the surface distortion. The FoM represents the extent of undesirable Zernike modes and this is a number that the structural optimisation should seek to minimise.

7.9 The Optimisation of a 4m preliminary composite reflector antenna structure

7.9.1 The structure, materials, loading cases and optimisation parameters
A 4m preliminary reflector antenna was constructed with sandwich parabolic reflector panels, honeycomb core and carbon fibre laminate skins stiffened with composite ribs and was utilised to illustrate and verify the optimisation method, program and procedure.

The effects of structural deformation on antenna EM performances (sidelobe levels) have been included and repeatedly analysed in the iterative optimum-seeking procedure. This was performed in an indirect way by incorporating a Figure of Merit derived from Zernike mode analysis of distorted reflector surfaces.

The antenna was an offset system and the reflector had an aperture size of 4m in diameter. The geometric nominal surface was a section of a paraboloid having a focal length of 2m. The reflector dish was fabricated from a carbon fibre composite honeycomb sandwich panel structure stiffened by a ribbed backing structure formed by
a lattice of beams, also of honeycomb sandwich construction. Figure 7.C.3 shows the backside of the reflector. All the ribs were assembled and bonded to the rear of the shell. These rib elements were connected to the appropriate surface grid points and were offset toward the rear of the reflector; the ribs can be varied in height over the structure. In this original structure, all the ribs have a height of 0.1m. The total mass of the structure is 27.8kg.

The dish sandwich panel is a 0.01m thick aluminium alloy honeycomb core covered with carbon fibre reinforced epoxy (CFRP) face sheets. The sheets on both sides of the core are constructed with 0.001m thick CFRP layers in a [0/90/45/-45] lay-up (4 plies of 0.1mm material at 0°, 90°, 45°, -45°). The rib sandwich panel is 0.02m thick aluminium alloy honeycomb core covered with CFRP face sheets in a [0/90/45/-45] lay-up with 0.0001m CFRP layers. All the loads are carried by both the surface sandwich shell and backing structure (ribs).

An ABAQUS structural model was constructed of composite shell type elements. In the ABAQUS model with 146 nodes, the antenna is described by 216 three-side and four-side, irregular composite plate/shell elements. A mesh of STRI35 shell elements for the reflector surface was generated. A framework of S4R5 elements was generated to represent the reflector backing structure (ribs).

The properties of surface shell elements and rib plate elements were taken from the physical structure of the sandwiches (composite face sheets and honeycomb core). The lay-up of these elements are assumed to be constant across the whole dish. The following composite physical properties are used in the analysis:

— face sheets: Carbon/Epoxy laminate  
  
  elastic moduli  
  \[ E_1 = 289 \text{ GPa} \]  
  \[ E_2 = 6.1 \text{ GPa} \]  
  
  shear elastic moduli (needed to define transverse shear behaviour in shells)  
  \[ G_{12} = G_{13} = G_{23} = 4.21 \text{ GPa} \]  
  
  Poisson’s ratio  
  \[ \mu_3 = 0.29 \]  
  density  
  \[ \rho_s = 1750 \text{ kg/m}^3 \]

— aluminium alloy honeycomb core:
elastic moduli \quad E_1 = 200 \text{ MPa} \\
\quad E_2 = 200 \text{ MPa} \\
shear elastic moduli \quad G_{12} = G_{13} = G_{23} = 140 \text{ MPa} \\
Poisson’s ratio \quad \nu = 0.3 \\
density \quad \rho_c = 32 \text{ kg/m}^3

Five loading cases were considered and included in the optimisation. These cases are (all the values in Pa):

(1) Symmetric pressures (Pa): \quad +X \quad +Y \quad -X \quad -Y \\
\begin{align*}
-1500 & \quad -1500 \\
-1500 & \quad -1500
\end{align*}

(2) Skew symmetric about OX: \\
\begin{align*}
0 & \quad -1500 \\
0 & \quad +1500
\end{align*}

(3) Skew symmetric about OY: \\
\begin{align*}
-1500 & \quad 0 \\
+1500 & \quad 0
\end{align*}

Loading Case A = (1), \quad \text{see Figure 7.C.4}
Loading Case B = (2), \quad \text{see Figure 7.C.5}
Loading Case C = (3), \quad \text{see Figure 7.C.6}
Loading Case D = 0.5 \times (1) + (2), \quad \text{see Figure 7.C.7}
Loading Case E = 0.25 \times (1) + (3), \quad \text{see Figure 7.C.8}

The boundary condition is the same for all loading cases, i.e. 3 fixed nodes are in 120 degree distribution at the backup ribs (intersections of radial beams and a circular beam).

A linear-static structural analysis was performed to calculate the distortions of the reflector structure. A simplification of the analysis was achieved by neglecting the anisotropy of the honeycomb cores. The resulting nodal displacements experienced by the structure at each of the loading cases were obtained and these distortions are used to assess their effect on antenna performance. Figures 7.C.9 ~ 7.C.13 show the z direction displacement contours at loading cases A, B, C, D and E.

The optimisation for the 4m solid surface composite reflector structure considers 3 structural and electromagnetic objective functions, 13 structural and material design variables, and 5 loading cases simultaneously. The optimisation changes the structural...
design variables in such a way that the structural and EM performances approach the optimum criterion.

7.9.2 The optimisation results for the preliminary antenna

The optimisation results for the antenna show that the optimisation procedures succeed in that all the working/loading cases considered, for both structural and EM performances of the antenna, have been greatly improved (including 28% reduction of the structural mass). The optimised structure is shown in Figure 7.C.14.

Table 7.C.1 lists the reflector performances of the original structure, and Table 7.C.2 lists the performances of the optimised structure. All these performances are evaluated at all five loading cases (A ~ E) considered. Comparing the two tables, significant performance improvement through the optimisation can be seen. The design variables for the original and optimised reflector structures are listed in Table 7.C.3.

<table>
<thead>
<tr>
<th>Table 7.C.1 Antenna performances before optimisation (original structure)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Performances</td>
</tr>
<tr>
<td>Figures of merit</td>
</tr>
<tr>
<td>Max. stresses (MPa)</td>
</tr>
<tr>
<td>Structural mass (kg)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 7.C.2 Antenna performances after optimisation (optimised structure)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Performances</td>
</tr>
<tr>
<td>Figures of merit</td>
</tr>
<tr>
<td>Max. stresses (MPa)</td>
</tr>
<tr>
<td>Structural mass (kg)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 7.C.3 Design variables (before and after optimisation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.</td>
</tr>
<tr>
<td>---------------------------------------------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
</tbody>
</table>
7.9.3 Alternative results for the preliminary antenna

There is a trade-off to be made between the antenna performances and the structural mass. If we allow the structural mass to be more than the value of 20kg, or if we give more weights to the performances and less weight to the structural mass in the optimisation, the performances can be improved even further. Table 7.C.4 gives a set of alternative results (the original performances are the same as shown in Table 7.C.1). The optimised structure is shown in Figures 7.C.15 and 7.C.16. The design variable variations are shown in Table 7.C.5.

<table>
<thead>
<tr>
<th>Performances</th>
<th>Loading A</th>
<th>Loading B</th>
<th>Loading C</th>
<th>Loading D</th>
<th>Loading E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figures of merit</td>
<td>0.0037</td>
<td>0.0083</td>
<td>0.0086</td>
<td>0.0096</td>
<td>0.0095</td>
</tr>
<tr>
<td>Max. stresses (MPa)</td>
<td>35.3</td>
<td>62.8</td>
<td>61.5</td>
<td>79.2</td>
<td>69.5</td>
</tr>
<tr>
<td>Structural mass (kg)</td>
<td>25.2</td>
<td>25.2</td>
<td>25.2</td>
<td>25.2</td>
<td>25.2</td>
</tr>
</tbody>
</table>
Table 7.C.5 Design variables (before and after optimisation)

<table>
<thead>
<tr>
<th>No.</th>
<th>Design Variables</th>
<th>Original</th>
<th>Optimised</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>the thickness of every carbon fibre composite layer</td>
<td>0.1000E-03</td>
<td>0.9147E-04</td>
</tr>
<tr>
<td></td>
<td>in the skins of reflector surface panel (m)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>the thickness of every carbon fibre composite layer</td>
<td>0.1000E-03</td>
<td>0.1221E-03</td>
</tr>
<tr>
<td></td>
<td>in the skins of the stiffening ribs (m)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>the thickness of the honeycomb of reflector surface panel (m)</td>
<td>0.1000E-01</td>
<td>0.1015E-02</td>
</tr>
<tr>
<td>4</td>
<td>the thickness of the honeycomb of the stiffening ribs (m)</td>
<td>0.2000E-01</td>
<td>0.2085E-01</td>
</tr>
<tr>
<td>5</td>
<td>the fibre angles in third and fourth layers in the skins of reflector surface panel (m)</td>
<td>±0.4500E+02</td>
<td>±0.4451E+02</td>
</tr>
<tr>
<td>6</td>
<td>the fibre angles in third and fourth layers in the skins of the stiffening ribs (m)</td>
<td>±0.4500E+02</td>
<td>±0.2846E+02</td>
</tr>
<tr>
<td>7</td>
<td>the z coordinate of the first (rim) nodes in the stiffening ribs (m)</td>
<td>0.4000E+00</td>
<td>0.4896E+00</td>
</tr>
<tr>
<td>8</td>
<td>the z coordinate of the second nodes in the stiffening ribs (m)</td>
<td>0.2472E+00</td>
<td>0.3370E+00</td>
</tr>
<tr>
<td>9</td>
<td>the z coordinate of the third nodes in the stiffening ribs (m)</td>
<td>0.1222E+00</td>
<td>0.1375E+00</td>
</tr>
<tr>
<td>10</td>
<td>the z coordinate of the fourth nodes in the stiffening ribs (m)</td>
<td>0.2500E-01</td>
<td>0.2737E-01</td>
</tr>
<tr>
<td>11</td>
<td>the z coordinate of the fifth nodes in the stiffening ribs (m)</td>
<td>-0.4440E-01</td>
<td>-0.2444E+00</td>
</tr>
<tr>
<td>12</td>
<td>the z coordinate of the sixth nodes in the stiffening ribs (m)</td>
<td>-0.8610E-01</td>
<td>-0.6486E-01</td>
</tr>
<tr>
<td>13</td>
<td>the z coordinate of the seventh (centre) node in the stiffening ribs (m)</td>
<td>-0.1000E+00</td>
<td>-0.1000E-01</td>
</tr>
</tbody>
</table>

7.10 The optimisation of a 4m low sidelobe composite reflector antenna satellite ground terminals

Only very brief description of the antenna and the results can be given here. The 4m low sidelobe antenna structure has 1128 nodes and 1067 elements (carbon fibre composite/steel shells and beams). The details of the structure, loading cases and boundary conditions may be obtained from GEC-Marconi Research Centre at Chelmsford, UK. The following three antenna EM and structural performance parameters were taken as the optimisation objectives (to be minimised):

a) Figure of Merit from Zernike mode analysis
b) Maximum stress in whole structure
c) Structural mass
Five loading cases, A ~ E, were considered in the optimisation. These loading cases are:

- **Loading Case A:** 60m/s wind load (normal)
- **Loading Case B:** 60m/s wind load (side)
- **Loading Case C:** 60m/s wind load (elevated)
- **Loading Case D:** self-weight load at 0 degree elevation angle
- **Loading Case E:** self-weight load at 90 degree elevation angle

Tables 7.C.6 and 7.C.7 show briefly the results of analysis for the original and final configurations of the reflector for all the loading cases (A ~ E) considered. A comparison can be made by comparing the values in these tables. The initial and final design information (design variables) is provided in Table 7.C.8.

### Table 7.C.6 Antenna performances of the original structure

<table>
<thead>
<tr>
<th>Performances</th>
<th>Loading A</th>
<th>Loading B</th>
<th>Loading C</th>
<th>Loading D</th>
<th>Loading E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figures of merit</td>
<td>9.68</td>
<td>4.47</td>
<td>5.88</td>
<td>0.058</td>
<td>1.14</td>
</tr>
<tr>
<td>Max. stresses (MPa)</td>
<td>131.</td>
<td>19.0</td>
<td>79.8</td>
<td>28.4</td>
<td>18.3</td>
</tr>
<tr>
<td>Structural mass (kg)</td>
<td>836.</td>
<td>836.</td>
<td>836.</td>
<td>836.</td>
<td>836.</td>
</tr>
</tbody>
</table>

### Table 7.C.7 Antenna performances of the optimised structure

<table>
<thead>
<tr>
<th>Performances</th>
<th>Loading A</th>
<th>Loading B</th>
<th>Loading C</th>
<th>Loading D</th>
<th>Loading E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figures of merit</td>
<td>6.23</td>
<td>3.24</td>
<td>3.74</td>
<td>0.033</td>
<td>0.67</td>
</tr>
<tr>
<td>Max. stresses (MPa)</td>
<td>112.</td>
<td>22.3</td>
<td>67.6</td>
<td>29.8</td>
<td>16.4</td>
</tr>
<tr>
<td>Structural mass (kg)</td>
<td>831.</td>
<td>831.</td>
<td>831.</td>
<td>831.</td>
<td>831.</td>
</tr>
</tbody>
</table>

### Table 7.C.8 Design variables of the 4m low sidelobe antenna

<table>
<thead>
<tr>
<th>No.</th>
<th>Design Variables</th>
<th>Original</th>
<th>Optimised</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>the thickness of carbon fibre composite layers (skins) of the reflector surface panel (mm)</td>
<td>0.50</td>
<td>0.632</td>
</tr>
<tr>
<td>2</td>
<td>the thickness of carbon fibre composite layers (skins) of the stiffening ribs (mm)</td>
<td>0.50</td>
<td>0.593</td>
</tr>
<tr>
<td>3</td>
<td>the thickness of the honeycomb of reflector surface panel (mm)</td>
<td>58.72</td>
<td>60.0</td>
</tr>
<tr>
<td></td>
<td>Description</td>
<td>Value 1</td>
<td>Value 2</td>
</tr>
<tr>
<td>---</td>
<td>------------------------------------------------------------------------------</td>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td>4</td>
<td>the thickness of the honeycomb of the stiffening ribs (mm)</td>
<td>48.72</td>
<td>46.30</td>
</tr>
<tr>
<td>5</td>
<td>the z coordinate of the edge of the stiffening ribs (mm)</td>
<td>-15.50</td>
<td>-100.0</td>
</tr>
<tr>
<td>6</td>
<td>the distance between the edge of ribs and the edge of the back box beam</td>
<td>169.1</td>
<td>50.02</td>
</tr>
<tr>
<td></td>
<td>(see Figure 7.C.17) (mm)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>the distance between the edge of ribs and the middle of the back box beam</td>
<td>299.5</td>
<td>435.0</td>
</tr>
<tr>
<td></td>
<td>(see Figure 7.C.17) (mm)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>the shell thickness of the steel shell set GLOBAL-S (mm)</td>
<td>3.0</td>
<td>2.859</td>
</tr>
<tr>
<td>9</td>
<td>the shell thickness of the steel shell set STEEL-20 (mm)</td>
<td>20.0</td>
<td>10.0</td>
</tr>
<tr>
<td>10</td>
<td>the shell thickness of the steel shell set STEEL-6 (mm)</td>
<td>6.0</td>
<td>3.0</td>
</tr>
<tr>
<td>11</td>
<td>the shell thickness of the steel shell set JACK-CLE (mm)</td>
<td>25.0</td>
<td>18.54</td>
</tr>
</tbody>
</table>

The original shape of the 4m low sidelobe antenna structure is shown in Figure 7.C.18 ~ 7.C.21. The undeformed and deformed structural shapes for the five loading cases are shown in figures 7.C.22 ~ 7.C.30 (in these figures, the deformations are exaggerated for the sake of comparison). The optimised structural geometry is shown in Figure 7.C.31 ~ 7.C.33.
Fig. 7.C.1 Zernike mode effects on sidelobes

Fig. 7.C.2 Zernike mode analysis of reflector surface distortions
Fig. 7.C.3 The original shape of a 4m preliminary composite antenna structure

Fig. 7.C.4 Loading case A
Fig. 7.C.5 Loading case B

Fig. 7.C.6 Loading case C
Fig. 7.C.7 Loading case D

Fig. 7.C.8 Loading case E
Fig. 7.C.9 The z direction displacement contour at loading case A

Fig. 7.C.10 The z direction displacement contour at loading case B
Fig. 7.C.11 The z direction displacement contour at loading case C

Fig. 7.C.12 The z direction displacement contour at loading case D
Fig. 7.C.13 The z direction displacement contour at loading case E

Fig. 7.C.14 The optimised structural of the 4m preliminary antenna
Fig. 7.C.15 The optimised structure

Fig. 7.C.16 The optimised structure
box beam

Fig. 7.C.17. Design variables V5, V6 and V7

dish

Fig. 7.C.18 The original structure of the low sidelobe antenna structure
Fig. 7.C.19 The original structure of the low sidelobe antenna structure

Fig. 7.C.20 The original structure of the low sidelobe antenna structure
Fig. 7.C.21 The original structure of the low sidelobe antenna structure

Fig. 7.C.22 The undeformed and deformed structural shape for the loading case A
Fig. 7.C.23 The undeformed and deformed structural shape for the loading case A

Fig. 7.C.24 The undeformed and deformed structural shape for the loading case B
Fig. 7.C.25 The undeformed and deformed structural shape for the loading case B

Fig. 7.C.26 The undeformed and deformed structural shape for the loading case C
Fig. 7.C.27 The undeformed and deformed structural shape for the loading case C

Fig. 7.C.28 The undeformed and deformed structural shape for the loading case D

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Fig. 7.C.29 The undeformed and deformed structural shape for the loading case E

Fig. 7.C.30 The undeformed and deformed structural shape for the loading case E
Fig. 7.C.31 The optimised structure of the 4m low sidelobe antenna

Fig. 7.C.32 The optimised structure of the 4m low sidelobe antenna
Fig. 7.C.33 The optimised structure of the 4m low sidelobe antenna
Chapter 8

Conclusions

An optimal design methodology for optimising the EM and structural performances of reflector antenna structures and systems based on multiple objectives and multiple working/loading cases has been proposed. The development of the integrated structural and EM optimisation procedure for large terrestrial and space antenna reflectors has been described. Finite element analysis, GO/AI EM analysis, and optimisation techniques were employed to design the reflector antenna structures for the purpose of improving both antenna EM and structural performances and reducing structural weight. The effects of metal and composite reflector structural deformations on antenna EM performances are incorporated. The design objectives include maximisation of antenna EM efficiency, structural stiffness, strength and fundamental frequencies, and minimisation of structural mass, reflector surface RMS errors and antenna sidelobe levels. The importance of the optimisation method also lies in that it overcomes the drawbacks of separating antenna design into two, structural and EM design, isolated tasks which ignores the significant connection between them.

A new technique for converting a constrained optimisation problem to an unconstrained one was demonstrated to be an useful tool in multi-objective applications. The technique transforms the objective functions into target goals, and the goal values for all objectives are adjustable quantities. The optimum can be searched for by effective unconstrained optimisation algorithms. Problems of the type are not only formulated and solved on a more global level but may be also specialised for decomposed smaller problems.

The quality of the optimum design can be improved by improving the finite element model of the structure, by increasing the number of discrete points used to describe the surface, by increasing the design variables, and by increasing the number of working/loading cases considered.
An aspect of the research was the analysis and optimisation of composite surface panels. The method has been presented herein for the design of three dimensional structures and thermally stable, lightweight structural composites, and can be used to solve a variety of practical structural optimisation problems.

The analysis, design and optimisation approaches for large precision antennas have been discussed. The task is presented as a multidisciplinary problem requiring the consideration of structure, material and EM performance for many working/loading responses more or less simultaneously. These problems and their interrelation are addressed with special emphasis on predicting the effects of structural deformations on antenna EM performance because of the design change. The design variables considered are member sizes, node (mesh) positions and the 'tailorable variables' of the composite materials. Operating environment loads, on which antenna performances depend, are specified for the optimisation. These loading cases can define:

- point loads, pressures and specified displacements on the structures,
- gravity loads at various elevation angles,
- wind loads at different directions to the antennas in different velocities,
- extreme temperatures from the manufacturing temperature, thermal gradient and temperature distributions in the structures,
- global loads resulting from accelerations,
- other random and dynamic loads.

By including the reflector surface errors with the aperture integration of the 'best-fit' surface, the EM analysis determines the far-field pattern, including the beam efficiency, and thus the antenna performance of the reflector antennas. Although all the antenna examples presented here have a paraboloid surface, the method can readily be extended to nonparaboloid reflector surfaces: spherical, planar, hyperbolic, elliptical, etc.

The development of high EM performance, large aperture size, light weight, and reliable parabolic reflector antenna structures for long life utilisation in both terrestrial and the environmental extremes of outer space is a continuing challenge to the antenna structural engineers. All of which were of particular interest in this study. The following specific items have been investigated in this project:

8. Conclusions
1) To evaluate the quality and serviceability of the structures, structural static, dynamic analyses and the calculation of the best fit paraboloids have been performed by using the finite element methods:

- to identify inservice performances under different working environments,
- to predict margins of safety under static and dynamic loads and shock conditions,
- to evaluate natural frequencies and mode shapes,
- to analyse thermal distortions within the structures when they are in low earth orbit (LEO) or geostationary earth orbit (GEO).

2) Numerical analysis has been performed to study the effects of composite panel parameters on antenna performances. The model is applied to sandwich parabolic reflector panels with aluminium alloy honeycomb core and carbon, Kevlar or glass/epoxy laminate skins. An optimisation model for panel design has been developed. Using available composite materials with their actual physico-mechanical properties, the optimisation model can be used to attain optimum EM and structural performances, as well as minimum weight and cost by optimising the structure of the reinforcement of the bearing layers. Various static and dynamic loading cases have been accounted for in the analysis and optimisation procedures.

3) EM performances of the antennas have been analysed by using geometrical optics/aperture integration (GO/AI) method. The emphasis here is on the application to distorted reflectors predicted by the finite element structural analysis. Instead of nominal surface, the deformed reflector surfaces (real surfaces) are employed in the antenna EM analysis. The phase of these aperture fields is perturbed by the difference in the ray path length caused by the reflector surface distortion. Using optical ray tracing and aperture field interpolation by employing a curved surface spline function, the difference of ray-path length of all points (say a lattice of $128 \times 128$) on the deformed antenna aperture can be obtained. A numerical method for combining structural analysis with EM aperture field analysis using fast Fourier transform (FFT) has been employed to determine the effects of structural deformation on EM performances.

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4) Zernike mode analysis of reflector distortions is used in the optimisation of antenna structures. The analysis is based on the circle polynomials of Zernike. The presence of particular modes may be related to the EM performance of antennas. Some forms of surface distortions are particularly damaging to EM performance. It is found that suppressing the undesirable component modes leads to an increased probability of achieving low sidelobe performance. Using the information of a set of co-ordinates for points on the ideal reflector surface, and a corresponding set of displacements of those points for various loading conditions, the analysis will produce a measure of the undesirable modes presented in the surface distortion. These undesirable modes are minimised by optimising the antenna structures.

5) A systematic method has been developed to evaluate quantitatively the design of antenna electro-mechanical systems. The considered performances include various antenna EM and structural performances such as antenna efficiency, gain, radiation pattern, main-lobe shape, sidelobe level, surface accuracy RMS error, maximum displacement, maximum stress, structural mass, structural frequency. Self-weight at different elevation attitudes and wind loads for ground antennas have been included in the analysis as well as the quasi-static loads at launch phase, temperature and thermal cycling loading at earth orbits for space antennas. The evaluation procedure seeks to reveal weak spots in the performances of the system. It does this by reviewing the proximity of each calculated performance to the best possible expectation and the acceptable limit for every working case of the structure. The means, the standard deviations and the evaluation indices of the antenna performances and of the loading cases are formulated and calculated to give a scientific quantitative evaluation for the system. An overall performance index is presented for the design. These indices form a foundation to establish the optimisation mathematical model.

6) A novel mathematical model and method of design for use on integrated structural electromagnetic optimisations of large antenna systems and whole reflector backup structures has been developed. The optimisation model is characterised by multi-objective, multi-variable and multi-working-case optimisation synthetic techniques. The interdisciplinary optimisation method has been used to carry out overall situation optimisation including most of the

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structural and EM performances under various working environments and loading cases. Cross-sectional areas, node co-ordinates, member size parameters and physical property variables of materials in the supporting metal/CFRP structures have been included as design variables. Non-linear mathematical programming techniques have been used to locate the optimal designs in a feasible design space.

7) A high volume of computer coding and verifying for corresponding numerical analysis and optimisation have been involved in this project. In addition, efficient data organisation and transfer among optimiser, structural finite element analysis and EM analysis codes is necessary for an efficient and reliable computer program. A general purpose optimisation program MOST (Multifactor Optimisation of Structures Technique) has been developed to implement the proposed approach. The developed MOST optimisation system possesses the flexibility and openness to integrate various different disciplinary analysis codes according to the nature of the optimisation problem and to the preference of an user. This engineering optimisation system has the ability to utilise some general-purpose structural analysis packages such as ABAQUS for linear and non-linear, static and dynamic structural analysis in the optimisation procedures, so that engineering structures and systems which can be analysed by these packages may then be optimised by the optimisation system. Interfacing codes have been designed to transfer the data and results, and an optimisation control program using UNIX Shell has been developed. The complete analysis has been repeatedly performed and potential changes in design can be quickly evaluated, so that the optimisation can be performed in a single computer run by using this interdisciplinaiy design and analysis tool.

8) The procedure described in this thesis is applicable to a wide variety of large space antenna concepts. The only assumption is that the reflector surface distortion is precisely known at a discrete number of surface locations and that the change in this distortion for a prescribed change in structural design can be predicted. The procedure is tested for a number of terrestrial and space antenna designs that have surface deformations caused by various working/loading cases. An 8m Cassegrain antenna and a 3.6×2.6m composite space reflector and two 4m off-set low sidelobe composite antennas have been optimised using the techniques. The results are given to show how the optimisation procedure is
applied and comparisons are made between original and optimum designs. The optimisation procedures for the examples show that each optimisation converges to a much better design than the original design.

In summary, the possibility of achieving a design that efficiently meets multiple requirements, together with the difficulty of seeking the values of a large number of design variables, makes structure optimisation an obvious and useful technology for the design of large complex antenna structures and systems. It is concluded that an integrated structural-electromagnetic optimisation procedure is highly desirable for the design of large space antennas. A mathematical optimisation procedure has been developed that improve antenna structural and electromagnetic performances while minimising structural weight. In short, the structural optimisation procedure provides the following:

- A systematic design process requiring minimum human intervention,
- Reduced design time,
- Optimising multiple objectives under multiple working/loading cases simultaneously,
- Suitable for the design of member size, structural geometry and material (including composites),
- The ability to deal with many design variables and constraints concurrently,
- Design improvement in both EM and structural performances,
- Feasible solutions to complex design problems.

From the example problems it is clear that this optimisation system can solve both multidisciplinary and multi-factor antenna structural/system optimisation problems by calling a general purpose finite element structural analysis program, an antenna electromagnetic analysis program, a system assessment program, and an optimisation code. The results indicate the benefit of including EM performance calculations in procedure for optimisation of large antenna reflectors.

The present system can handle most of optimisation problems in mechanical/structural design, except for the case that the speed of the process is too concerned (e.g. for ‘real-time’ problems). The described method has been developed into an in-house research code MOST. The developed software tool has enabled the completion of the antenna
structural/system design in an efficient and quick manner by minimising the manual handling of input and output data. The optimisation procedure is presently fully automated. Numerical results of the example problems described here illustrate the efficiency of the MOST program. With some modification, the method and computer program can be further developed towards a general engineering optimisation tool aiming at building better, stronger, lighter, safer, and less costly engineering structures and systems in less time.

The current extensive activity and growing capability in the structural optimisation area suggest that the extension of structural optimisation and associate techniques into the realm of structural design can be pursued with a reasonable expectation of success.

In this study, the author considers the following research and development as his original work and contributions:

- A method combining structural and EM analysis for predicting antenna structural designs, including an assessment method to evaluate quantitatively the design quality of antenna systems.
- A study into the effects of antenna structural distortions on EM performances by using the above method.
- A mathematical model for integrated structural and EM optimisation of antenna systems.
- A development of an iterative optimisation technique considering simultaneously many objectives, many constraints, many design variables and under many working/loading cases.
- A study into the applicability of the optimisation for both terrestrial and space antenna structures with composite materials.
- A low-sidelobe-orientated optimisation method which minimise undesirable Zernike modes to increase the probability of achieving low sidelobe performances by employing Zernike modes analysis of reflector distortions (The method has not, to the author’s knowledge, been previously reported).
- An optimisation program system and the corresponding interface programs which have been used and verified.
- An integrated optimisation of wider scope than reported previously has been carried out, in which many aspects of antenna optimisation have been presented.
The author recommends the following areas for future investigations:

- An extension of the analysis of the effects of structural distortions on EM performances by utilising other antenna EM analysis techniques, such as physical optics/aperture integration (PO/AI), and geometric theory of diffraction (GTD).
- An extension of the optimisation method to include structural and/or system reliability directly as one of the objectives for antenna structures.
- An investigation of the feasibility of incorporating GAs in the optimisation system.
- An extension of the optimisation method to include nonparaboloid reflector antennas.
- To undertake an intensive topological optimisation for antenna structures.
- To develop an user-friendly optimisation tool for large antenna structures based on the techniques.
- To explore other geometric shapes and composite materials for large reflector antenna structures.
- To further verify the optimisation technique over a wider range of materials and geometric parameters and under more different loading configurations, and for problems in other disciplines and more complex structural configurations.
References


