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SPIN DEPENDENCE IN HEAVY ION INDUCED NUCLEAR REACTIONS.

BY

GORDON WINDHAM

A thesis submitted to the Faculty of Science at the
University of Surrey for the Degree of Doctor of
Philosophy.

Department of Physics,

University of Surrey,

June 1986.
Abstract

The interest of this work is to explore the spin dependent effects of projectile structure in nuclear heavy ion induced reactions. The projectiles are considered to consist of two clusters which remain inert, the intercluster wavefunctions are obtained by the use of the orthogonality condition model. Excitation to the low lying projectile states is included by the use of coupled channels calculations. The projectile excited states are included by allowing excitation in the inter cluster wavefunctions. Coupling to the excited states is performed by multipole terms arising from a single folding model, which is used consistently throughout the work. The effects of projectile excitation are considered in two areas, elastic and inelastic scattering, and transfer reactions.

It is found that the inclusion of the projectile excited states has a very strong effect on the spin dependent elastic observables, in particular the vector analysing powers. In contrast to earlier analysis it is found that projectile excitation plays a dominant role in reproducing the experimentally observed vector analysing powers for the elastic scattering of $^6\text{Li}$ from $^{16}\text{O}$ and $^{28}\text{Si}$ at 22.8 MeV. Projectile excitation is also seen to produce strong spin dependent effects in the elastic scattering of $^7\text{Li}$ from $^{120}\text{Sn}$ at 44 MeV and of $^{19}\text{F}$ from $^{28}\text{Si}$ at 60 MeV.

The inclusion of projectile excitation via CCBA calculations produces changes in the transfer cross section for the $^{28}\text{Si}(^{19}\text{F},^{16}\text{O})^{31}\text{P}(1/2^+,\text{g.s.})$ reaction. The observed changes can be understood in terms of the effects of spin dependence in the reaction path. The inclusion of projectile excitation however does not significantly improve the quality of the agreement with experimental data.
Acknowledgments

The work reported in this thesis was carried out in the Department of Physics at the University of Surrey. The financial support of the Science and Engineering Research Council (SERC) is gratefully acknowledged.

I wish to express my thanks to my supervisor, Professor R.C. Johnson, for his enthusiastic and patient guidance throughout the period of this research.

It is a pleasure to acknowledge the help of Dr. H.N. Nishioka and Dr. J.A. Tostevin, the former for introducing me to the ideas of projectile excitation, and the latter for assistance with computing and the later stages of this work. I should also like to thank the members of the Physics department for their help and encouragement, and for making my stay at Surrey such a happy one.

I should also like to thank my fiancee, Miss M.E. Sotheby, for her assistance in the final preparation of the manuscript, and her patience and support during its writing.
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References
Most of the work presented in this thesis has been disseminated via contributions to conferences. In addition to these contributions the work presented here has lead to the following publications appearing in the scientific literature.

Jo 83 : R.C. Johnson, H. Nishioka, J.A. Tostevin and G. Windham,

Proc. of the Int. Conf. on Heavy Ion Physics and Nuclear Physics, Catania, 1983.

Wi 84 : G. Windham, H. Nishioka, J.A. Tostevin and R.C. Johnson,


St 86 : E.J. Stephenson, C.C. Foster and G. Windham, Proc. Sixth


Chapter 1

1.1 Information from heavy ion reactions

The detailed study of nuclear reaction processes is responsible for our present knowledge of nuclear structure and nuclear interactions. Due to experimental limitations most of the early reaction data was obtained from reactions using light projectiles. More recently it has become possible to obtain sufficiently intense beams of heavier projectiles which has produced experimental data for reactions induced by a variety of heavy ions. The use of heavy ions as probes enables the examination of aspects of nuclear structure not possible with light projectiles. For example multi nucleon transfer reactions, which allow us to consider the extent to which nucleons correlate and form sub structures within a nucleus.

If either or both of the participating nuclei in a collision have non zero spins, observables in addition to the cross section may be obtained. In this thesis only reactions induced by projectiles with non zero spins on spinless targets will be considered. In order to experimentally determine spin dependent observables using a beam of nuclei in which the spins are randomly orientated, it is necessary to perform a double scattering experiment.

†[ This term is used to describe projectiles with a mass greater than an alpha particle ]
Such experiments suffer from the problem of low detected intensities. If however the spins of the incident nuclei within the beam are oriented in a non random manner, referred to as a polarized or aligned beam, a single scattering experiment is sufficient to determine spin sensitive observables. Recent advances in experimental techniques have led to the advent of polarized beams of heavy ions; the earliest example being for lithium isotopes [We 76] at center of mass scattering energies of about 20 MeV.

1.1.1 Magnitude of spin dependence

In a discussion of the spin dependence of heavy ion induced reactions simple considerations suggest that elastic scattering will involve little spin dependence in comparison with nucleon and light-ion scattering [Gl 79]. The spin dependence of a nucleon scattering from a spin zero target nucleus arises from the familiar \( \hat{l} \cdot \hat{s} \) spin orbit force. Thought of in terms of the individual nucleon nucleon interactions, the only non vanishing contribution to an \( \hat{l} \cdot \hat{I} \) interaction between an incident nucleus of spin \( \hat{l} \) and a spin zero target comes from the unpaired nucleons, which form a small fraction of the total number of nucleons \( A \). When such pair wise interactions are averaged over the ground state of the projectile a resultant \( \hat{l} \cdot \hat{I} \) interaction is produced. This interaction is smaller by a factor of \( n/A \) as compared to the nucleon target \( \hat{l} \cdot \hat{s} \) interaction, where \( n \) is the number of unpaired nucleons.

Experimental data obtained with polarized beams, together with less direct information about the spin dependence of the nucleus nucleus
interaction such as spin flip measurements [Du 79, Ta 81, Ch 83] and reaction asymmetry measurements [Wu 79] suggest that, contrary to these simple arguments, the spin orbit interaction for heavy ions scattering is not small. Rather they indicate that the magnitude of the nucleus nucleus spin dependence is comparable to that observed in the nucleon nucleus interaction. Since the qualitative arguments, which are supported by detailed folding folding model calculations [Am 76, Pe 78, We 76, Bo 79, Ni 84], fail to agree with the observed data there must exist, as part of the reaction process, other mechanisms that contribute a spin dependence in addition to that of the usual $\hat{I}_L$ force discussed above.

1.1.2 Projectile Structure Effects

An important property of heavy ions is that they possess internal structure; such ions are therefore capable of supporting excited states. There is convincing evidence [Bu 77, Lo 82] that many of the qualitative features of the nuclear structure of light nuclei can be understood if we consider the nuclei to consist of two or more elementary clusters. This cluster model provides a good description [Wi 77, Fu 80] of the projectiles dealt with in this thesis, namely $^6$Li, $^7$Li and $^9$F. These projectiles will be regarded as consisting of alpha + deuteron clusters for $^6$Li, alpha + triton clusters for $^7$Li, and $^6$O + triton clusters for $^9$F. All ground state and excited state wave functions for the projectiles will be obtained within the cluster model. For any heavy ion induced reaction in which the bombarding energy is sufficiently high transitions to these excited states may be caused as part of the direct reaction mechanism. In this thesis attention will be focused on the effects upon the calculated
reaction observables produced by coupling to such projectile excited states; in particular the spin dependence arising from such excitations will be considered. Within the simple cluster model picture, the mechanism of projectile excitation will be introduced by allowing excitation in the wave functions describing the relative motion between the clusters representing the projectile; the clusters taken to comprise the projectile remain inert throughout the interaction.

It has been shown \[ \text{Ni 84} \] that the spin dependent observables for the elastic scattering of \(^{6}\text{Li}\) and \(^{7}\text{Li}\) from \(^{56}\text{Ni}\) at center of mass energies in the range 13 to 20 MeV can not be reproduced using folded spin dependent forces alone. This work showed that the inclusion of projectile excitation was essential in reproducing, both in sign and magnitude, the spin dependent observables. The purpose of the present work is to apply the ideas presented in this reference to the study of other reactions in order to test their general validity \[ \text{Cl 79, Oh 82, To 83} \].

The large projectile mass in heavy ion induced reactions results in the associated de Broglie wavelengths being short in comparison with the sizes of the colliding nuclei, hence semi-classical ideas assume an important role in the description of such interactions. In the light of this, semi-classical ideas will be used in section 3.3 to offer a qualitative description of the spin dependence arising from the mechanism of projectile excitation. Semi classical arguments are convenient for visualising and describing, at least in general terms, observed reaction data. The quantal nature of nuclear reactions however requires that for a direct comparison with data detailed quantum mechanical calculations should be used.
1.1.3 Scope of this thesis

A number of different reactions can occur when two composite systems interact. The interest here is centered on reactions referred to as direct. A precise definition of which is rather difficult [Au 70, Sa 83]. A reaction is usually termed direct when there is good overlap between the wave functions of the incident and exit channels. In this situation the collision can occur quickly and with a minimum of rearrangement of the constituent nucleons. Inelastic projectile excitation may usefully be described as a direct reaction, provided one is careful about the manner in which the projectile is described. The effect of this excitation mechanism will be considered in detail for the following reactions, where the references refer to the experimental data for the reactions considered:

1) The elastic scattering of $^6$Li from $^{28}$Si and from $^{16}$O for the $^6$Li incident at 22.8 MeV, [We 76].

2) The elastic scattering of $^7$Li from $^{120}$Sn at 44.00 MeV, and the inelastic reaction $^{120}$Sn($^7$Li, $^7$Li*$^1/2^-$, 0.48 MeV) $^{120}$Sn, [Tu 85].

3) The elastic scattering of $^{19}$F from $^{28}$Si at 60.00 MeV, [Ku 77].

4) The three nucleon transfer reaction $^{28}$Si($^{19}$F, $^{16}$O)$^{31}$P leading to the residual $^{31}$P states ($1/2^+$, g.s.), ($3/2^+$, 1.27 MeV) and ($5/2^+$, 2.23 MeV), for the $^{19}$F incident at 60.00 MeV, [Ku 77].

Elastic scattering clearly satisfies the criteria of a direct reaction
since, except for the possibility of the reorientation of the spins of the participating nuclei, the reaction can occur without any rearrangement taking place.

There are a variety of ways in which the transfer of three nucleons can occur, the simplest of which is the transfer of the three nucleons as a correlated cluster in a one step process. The ejectile cross sections for reactions which are dominated by such direct stripping processes tend to be forward peaked; we may therefore be guided by experimental data as to the relative importance of direct mechanisms over compound events in which the ejectile has no "memory" of the incident conditions. Given that such a reaction is dominated by a direct process it is possible to describe the reaction within a formalism in which few degrees of freedom are involved. The most straightforward way is to use cluster representations for the projectile and residual nuclei, the reaction occurring as the transfer of a valence cluster, assumed to remain inert, between them. Multinucleon transfer reactions which are dominated by a direct one step reaction probe the extent to which the participating nuclei contain clusters as well defined groups.

Observables that may be measured using a beam of projectiles with spin \( \hat{I} \) are defined in section 1.2. The cluster model which will be used consistently throughout this work to describe the projectiles will be discussed in section 1.3. The formalism for elastic scattering, including the effects of projectile excitation will be developed in chapter 2. The folding model which will be used to obtain the potentials acting between the colliding nuclei will also be developed in this chapter. Chapter 3 will present the results of the elastic and inelastic reactions considered,
and will offer interpretations of the results. The models and formalism used to describe the transfer reactions via a single step will be developed in chapter 4. The effects of projectile excitation on the transfer reactions will be the subject of chapter 5. Finally, chapter 6 will present some concluding remarks on the role of projectile excitation in describing the spin dependence of nuclear heavy ion induced reactions.

1.2 Observables

In any direct reaction initiated by a polarized beam it is possible to measure simultaneously not only the cross section, as a function of angle between the incident and outgoing directions, but also the sensitivity to the incident beam polarization. The polarization state of a beam is most conveniently described in terms of the density matrix formalism [Fa 57], defined as an expansion in terms of normalised states

\[ \rho(I) = \sum_n P_n |n\rangle \langle n| \]  

with \( P_n \) being the probability for finding the state \( |n\rangle \). A collection of particles such as a beam is a mixture of pure states, and the density operator thus contains the statistical information about the relative probabilities of finding each of the pure states. The states \( |n\rangle \) can be expanded in terms of the complete set of states \( |I M_\pi\rangle \) as

\[ |n\rangle = \sum_{M_\pi} C^n_{M_\pi} |I M_\pi\rangle \]  

Hence the density matrix can be written

\[ \rho(I) = \sum_{M_\pi M'_\pi} \rho_{M_\pi M'_\pi} |I M_\pi\rangle \langle I M'_\pi| \]
The density operator may be used to obtain the expectation value of an operator through the relation

\[
\langle \sigma \rangle = \frac{\text{Tr} (\rho \hat{\sigma})}{\text{Tr} (\rho)}
\]

where \( \text{Tr} \) is the trace of the resulting matrix; in practice the incident density matrix is often normalised to unity. The polarization of the incident beam can be described in terms of the irreducible tensor operators \( \tau_{kq} \) which are defined through the relation

\[
\langle I M_x | \tau_{kq}(I) | IM_x \rangle = \hat{k} \langle IM_x | k_q | IM_x \rangle
\]

where \( \hat{k} \), known as "stat k" and equalling \((2k+1)^{1/2}\) will be used throughout this work. The \( \tau_{kq}(I) \) form a complete set of operators in \((2I+1)\) by \((2I+1)\) spin space. They are orthogonal, satisfying

\[
\text{Tr} (\tau_{kq} \tau_{k'q'}) = \hat{I}^2 \delta_{kk'} \delta_{qq'}
\]

By construction they transform under rotations as a tensor of rank \( k \). Their Hermitian conjugates satisfy

\[
\tau_{kq}^\dagger (I) = (-)^q \tau_{k-q} (I)
\]

In terms of these operators the density matrix can be expanded as

\[
\rho (I) = \hat{I}^{-2} \sum_{kq} t_{kq} \tau_{kq}^\dagger \text{Tr} [\rho (I)]
\]

where the \( t_{kq} \) are the tensor moments of the beam, they serve to specify its polarization, and may be obtained by inverting equation 1-9.

\[
t_{kq} = \frac{\text{Tr} (\rho (I) \tau_{kq} (I))}{\text{Tr} (\rho (I))}
\]
1.2.1 Analysing powers

The sensitivity of the outgoing cross section to the incident polarization may be described by the analysing powers of the reaction. The analysing powers $T_{kq}$, referred to the Madison convention coordinate system [Ba 71], with $z$ along $\hat{k}_n$ and $y$ along $\hat{k}_n \wedge \hat{k}_{out}$ are defined through

$$T_{kq} = \frac{\text{Tr} \left( f T_{kq} f^T \right)}{\text{Tr} \left( f f^T \right)}$$

1-11

Where the reaction amplitudes $f(\Theta)$ will be defined for the elastic and inelastic reactions in section 2.2, and for the transfer reaction in section 4.2. The $T_{kq}$ satisfy

$$T_{kq}^* = (-)^q T_{k-q}$$

1-12

The cross section measured by a spin insensitive detector for a polarized beam is defined as

$$\frac{d\sigma}{d\Omega} \bigg|_{RL} = \sigma_0(\Theta, \phi) \left\{ 1 + \sum_{kq} t_{kq} T_{kq}^* (I) \right\}$$

1-13

where the summations are over the ranges $1<k<2I$ and $-k<k$. The differential cross section $\sigma_0(\Theta, \phi)$ is the cross section for an unpolarized beam, defined through

$$\sigma_0(\Theta, \phi) = \frac{\text{Tr} \left( f f^T \right)}{\hat{I}^2}$$

1-14

The main interest of this work is concerned with the effects of first rank, vector, forces. There is a strong correlation between the rank of a force and the corresponding observable [Bo 71a]. The constraint of parity
conservation ensures that the first rank effects are described by the single, purely imaginary, quantity $T_{11}(\Theta)$. The first rank observable considered will be the real vector analysing power, $i T_{11}(\Theta)$. The constraints of time and parity invariance allow three independent second rank observables, defined within the Madison convention as $T_{20}$, $T_{21}$, and $T_{22}$. These second rank observables will be considered for the $^7\text{Li} + ^{130}\text{Sn}$ system for which data exists [Tu 85]. In order to compare the calculations with the available data the quantities $T_{20}$, $T_{21}$, and $^T T_{20}$ will be used where $^T T_{20}$ is defined through the relation [De 78, Zu 80, Mo 82].

$$^T T_{20} = -\frac{1}{2} T_{20} - \frac{1}{2} \sqrt{6} T_{22}$$

1.3 Cluster Model

Many of the quantitative features of nuclear structure can be understood by considering the nuclei to consist of clusters, for example [Wi 58, Ar 67, Ar 72, Ik 68, Fu 79, Ar 84]. A precise way of describing such a nucleus in which antisymmetrization of, and between, the clusters can be explicitly dealt with is the resonating group method, or RGM [Gi 75, Wo 75]. This is often reduced to the form [Bu 77, Fr 77]

$$\Lambda [T_c + V_c] |\phi_c> = E_c |\phi_c>$$

where $E_c$ is the separation energy of the clusters, $V_c$ is a local potential, often obtained from a folding model, and $T_c$ is given by

$$T_c = -\frac{\hbar^2}{2\mu} \nabla^2 \bar{r}$$

where $\bar{r}$ is the coordinate between the clusters. $\Lambda$ is a projection operator, which ensures that the nucleons within the clusters do not occupy states that are blocked by the Pauli exclusion principle.
1.3.1 Orthogonality Condition Model

The RGM formalism can be simulated by using a deep potential to bind the clusters and discarding the low lying states which approximate the blocked states, corresponding to the replacement of the RGM by the Orthogonality Condition Model, or OCM, [Sa 69, Sa 73, Fr 77]. The OCM may be obtained from the RGM theory by the use of the WKB approximation [Ao 83].

A characteristic feature of heavy ion reactions is the dominance of strong absorption of the incident ions. As a result of this the reactions tend to be dominated by impact parameters between the colliding nuclei that correspond to grazing collisions. These orbits are particularly sensitive to the long range parts of the interaction and hence to the structure in the surface regions of the participating nuclei. Within the cluster model the surface of the projectile corresponds to the tail of the intercluster wave function, the functional form of which is fixed by the separation energy and orbital angular momentum between the clusters. The normalisation of the tail however remains undetermined and will be dependent on the details of the nuclear interior. It can be deduced by considering the r.m.s. radius, which is dominated by large cluster separations, for the nucleus under consideration. Provided this quantity is in agreement with experimental data, the part of the projectile wave function to which the reactions being considered are most sensitive is well determined. The internal considerations necessary to fix the tail are accounted for by the OCM. This dependence on the tails of the intercluster wave functions also implies that the regions in which the OCM approximation is less valid, that is when the clusters overlap...
considerably, is not probed.

The OCM cluster model wave functions for the projectile nuclear states may thus be expressed in terms of their angular momentum properties as

\[ \Phi_{IM_{I}}(x_{c},x_{v},r,\sigma) = \varphi_{c}(x_{c}) \varphi_{v}(x_{v}) \sum (\ell m_{\ell} s m_{s} | IM_{I})(i) \varphi_{\ell}(r) Y_{\ell m}(\hat{r}) X_{sm_{s}}(\sigma) \]

where \( I, M_{I} \) are the projectile spin and its z component respectively; \( \ell \) is the intercluster orbital angular momentum with z component \( m_{\ell} \) and \( s, m_{s} \) are the valence cluster's spin and z component. \( (\ell m_{\ell} s m_{s} | IM_{I}) \) is a vector addition, or Clebsch Gordan coefficient [Br 71]. The intrinsic internal cluster wave functions \( \varphi_{c}(x_{c}) \) will be seen to be unimportant within the context of the models used to describe the reactions considered. The radial forms of the inter cluster radial wave functions, denoted by \( \varphi_{\ell}(r) \), thus remain to be determined, and are considered below for the projectiles studied.

The energy levels for the nuclear states considered are shown in figure 1.1, together with their angular momentum and parity assignments. Also shown are the relative orbital angular momenta between the clusters for each of the states considered, these values are discussed below for the nuclear states considered.
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<td>3/2^-</td>
<td>1.55</td>
</tr>
<tr>
<td>2</td>
<td>5/2^-</td>
<td>2.23</td>
</tr>
<tr>
<td>2</td>
<td>3/2^-</td>
<td>1.27</td>
</tr>
<tr>
<td>0</td>
<td>1/2^-</td>
<td>0</td>
</tr>
</tbody>
</table>

*Resonance states

Figure 1.1 Energy level diagram for the nuclei under consideration.
1.3.2 Lithium 6

The shell model prediction for $^6$Li is for four nucleons to be in $0s$ states and two in $1p$ states with respect to the centre of mass of the nucleus. The $^6$Li clusters are in $0s$ states with respect to their centres of mass. Within the oscillator model the energy of a nucleon, $i$, is equal to $(2n_i + l_i) \frac{\hbar \omega}{\hbar}$, where $\omega$ is the oscillator constant, $n_i$ is the number of nodes in the wave function, and $l_i$ is the orbital angular momentum of the $i$th nucleon. The energy of a nuclear state is thus given by

$$E = \sum_{i=1}^{n} (2n_i + l_i) \frac{\hbar \omega}{\hbar}$$

where $n$ is the number of nucleons in the nucleus. $^6$Li thus has two quanta of excitation arising from the two nucleons in the $p$ states. Since the clusters in $^6$Li have no internal excitation quanta the intercluster wave function will be required to have $2 \frac{\hbar \omega}{\hbar}$ excitation quanta. Within the cluster model these quanta are distributed in the same manner, according to

$$E = (2n + l) \frac{\hbar \omega}{\hbar}$$

where $n$ is the number of nodes in the intercluster wave function, and $l$ is the orbital angular momentum between them.

The ground state of $^6$Li has $J^\pi = 1^+$, and since the $\alpha$ cluster has $J^\pi = 0^+$ and the deuteron has $J^\pi = 1^+$ the intercluster wave function can be formed with $l = 0$ and $l = 2$. The $l = 2$ component is very small [Ni 84] and would be expected to contribute mainly second rank tensor forces, and hence will be ignored in these calculations. The ground state of $^6$Li is therefore considered to be in a pure $s$ state with one node. The intercluster wave function is then obtained by binding a deuteron to an $\alpha$
cluster, with the clusters represented only by their gross properties, such as charge and mass. The potential chosen to bind the clusters, appendix A, has a geometry similar to that of a microscopic calculation [Ha 67, Br 68, Ku 72]. The well depth is treated as a parameter, varied to produce the correct separation energy of the clusters. The radial function can most conveniently be obtained by defining

\[ U_{\ell}(r) = r \phi_{\ell}(r) \]

\( U_{\ell}(r) \) can be obtained as the solution of

\[ \left[ -\frac{\hbar^2}{2\mu} \left[ \frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} \right] + V_{\text{Bind}}(r) + V_{\text{Coul}}(r) - E_{\text{Sep}} \right] U_{\ell}(r) = 0 \]

by integrating out from the origin, in the presence of the potential and matching to a Whittaker function [Ab 70]. The commutation of the potentials \( V_{\text{Bind}}(r) \) and \( V_{\text{Coul}}(r) \) with \( r \) has been used in obtaining this equation.

\( ^6\text{Li} \) has in addition to its one bound state a triplet of \( \ell = 2 \) resonance states which, within the cluster model, can be viewed as a spin orbit split triplet. The radial form for these states can be obtained by solving the local Schrödinger's equation 1-16 for the scattering of a deuteron from an alpha nucleus at the resonance energy using real potentials only, formed as a sum of repulsive Coulomb and centrifugal terms together with an attractive well with the same geometry as the ground state potential.

\[ \left[ -\frac{\hbar^2}{2\mu} \left[ \frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} \right] + V_{\text{Coul}}(r) + V_{\text{Bind}}(r) - E_{\text{Res}} \right] U_{\ell}(r) = 0 \]

where \( \ell = 2 \) for the excited states considered and the Coulomb potential is defined in appendix A. The binding potential \( V(r) \), given in appendix A, is
short ranged in nature. Outside this potential 1-23 becomes
\[
\left[-\frac{k^2}{2\mu} \frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} \right] + V_{\text{Coul}}(r) - E_{\text{Res}} \right] U_{\ell}(r) = 0 \tag{1-24}
\]

The solutions of this differential equation are the Coulomb functions \( F_{\ell}(kr) \) and \( G_{\ell}(kr) \) [Ab 70].

These may be combined to form incoming and outgoing Coulomb waves \( I_{\ell}(kr) \) and \( O_{\ell}(kr) \) according to
\[
O_{\ell}(kr) = G_{\ell}(kr) + i F_{\ell}(kr) \tag{1-25}
\]
with the incoming waves being the complex conjugate of the outgoing ones.
\[
I_{\ell}(kr) = G_{\ell}(kr) - i F_{\ell}(kr)
\]

Asymptotically we require the condition that there are only outgoing wave components. The inclusion of the short ranged binding potential \( V(r) \) introduces a phase shift into the asymptotic solution
\[
U_{\ell}(r) = \cos \delta_{\ell} F_{\ell}(kr) + \sin \delta_{\ell} G_{\ell}(kr) \tag{1-26}
\]
The well depth of the attractive potential is then adjusted to give a phase shift \( \delta_{\ell} \) of \( \pi/2 \) for the \( l = 2 \) partial wave at the resonance energy, taken to be the mean energy of the resonant state. The radial form of these states resemble long range bound states with oscillating tails. To facilitate the evaluation of matrix elements and to give simply normalisable wave functions, the radial functions are smoothly matched on to exponentially decaying tails. Also, to avoid undue complexity in the subsequent calculations, the three excited states were taken to have the same radial parts.
1.3.3 Lithium

The ground (3/2⁻) and first excited state (1/2⁻, 0.48 MeV) of ⁷Li can be formed by taking the triton cluster, with \( s = 1/2 \), to be in orbit around the alpha core with \( l = 1 \). From equation 1-19 \( ^{7}\text{Li} \) has \( \xi (2n_t + 1_l) = 3 \) and since the alpha and triton clusters have zero excitation quanta the intercluster function is required to have one node. These states are both bound. The relative functions \( \phi_\xi (r) \) can be obtained as the solution of equation 1-22 according to the techniques outlined there. \( ^{7}\text{Li} \) has two resonance states, (7/2⁻, 4.63 MeV) and (5/2⁻, 6.68 MeV). These can be taken as \( l = 3 \) states. The radial functions \( \phi_\xi (r) \) were taken to be the same for both of these states, the functions being obtained in the same manner as the \( ^{6}\text{Li} \) resonance states. The binding potentials, given in appendix A, were chosen to represent microscopic calculations [Ha 67, Br 68], with the well depths treated as variational parameters.

1.3.4 Flourine

The ground state of \( ^{19}\text{F} \) has \( J^\pi = 1/2^+ \), since \( ^{16}\text{O} \) is \( 0^+ \) and a triton 1/2⁺ this state can be represented by a triton in an \( l = 0 \) state about the \( ^{16}\text{O} \) core. The contribution to the total energy of the \( ^{19}\text{F} \) ground state comes from the nucleons forming the \( ^{16}\text{O} \) core and from the three remaining nucleons which are in the sd shell above the \( ^{16}\text{O} \). Since the former are common to both descriptions, and the nucleons in the triton are in 0 s states, the relative wave function between the clusters is required to have 6πω excitation quanta. For an s state the relative wave function is thus required to have three nodes. The lowest lying excited states of \( ^{19}\text{F} \) are
(5/2^+, 0.20 MeV) and (3/2^+, 1.55 MeV), these are taken to be purely 1' = 2 states. From the constraint 2N + L = 6 these are required to be 2D states.

The relative wave functions can be obtained in the same manner as that used for the ground state of ^6 Li. The attractive well used to bind the cluster is given in appendix A, the geometry being chosen to reproduce the r.m.s. radius for the ground state and the depth being adjusted to give the correct separation energy of the triton cluster. The value obtained is in good agreement with that published elsewhere [Bu 77]. The same well, together with a small spin orbit potential reproduces the energy levels of the excited states under consideration, and gives good agreement with the observed electromagnetic transitions between these levels and the ground state [Bu 77].

1.3.5 Phosphorus 31

In order to perform the transfer reactions to be considered in chapters 4 and 5 the bound state wave functions for the residual nuclear states are required. Similar considerations to those of the ^14 F case lead to 2N + L = 6. Since the triton has zero excitation quanta the ground state (1/2^+) is taken as a 3 s state, and the two excited states, (3/2^+, 1.27 MeV) and (5/2^+, 2.23 MeV) as 2 d states. The bound state wave functions for ^31 P were obtained in the same manner as those for ^14 F. The functional form of the binding potential, and its parameters are listed in appendix A. The values chosen represent a "conventional" choice, and were selected so that the residual nuclear wave functions would be in agreement with
previous work on the transfer reactions [ Ku 77 ].
Chapter 2

2.1 Projectile excitation

With the assumption that the projectile consists of two well defined clusters which remain inert, the interaction of the projectile and target is a three body problem. Due to the presence of an infinite ranged Coulomb potential between each of the three bodies, and the large number of partial waves that contribute to heavy ion reactions, the problem cannot be solved directly by such techniques as Faddeev equations, for example [Ei 72] and references therein. Instead the method of Coupled Channels, see for example [Ch 58, Ja 70, Ho 71b, Ei 72, Sa 83], will be employed.

2.2 Initial Boundary conditions

Initially when the colliding nuclei are very far apart the wave function for the entire system can be written as the product

\[ \Psi_{TPIMz}(x_T, x_p, \vec{k}, \vec{R}) = \phi_T(x_T) \phi^p(x_p) \chi(\vec{k}, \vec{R}) \quad 2-1 \]

where the wave function is labelled by the incident conditions and the relative scattering function, in the absence of Coulomb forces, is a plane wave.

\[ \chi(\vec{k}, \vec{R}) = e^{i\vec{k}.\vec{R}} \quad 2-2 \]

The total wavefunction \( \Psi_{TPIMz}(x_T, x_p, \vec{k}, \vec{R}) \) is required to be a
solution of the time independent Schrödinger equation for the entire system.

\[(H-E)\Psi(x_T, x_p, \vec{R}, \vec{R}) = 0\]  \hspace{1cm} 2-3

where \(E\) is the total energy and the Hamiltonian \(H\) is a sum of the Hamiltonians for the two isolated nuclei, together with an interaction Hamiltonian between them.

\[H = H_T + H_p + H_{\text{int}}\]  \hspace{1cm} 2-4

The isolated nuclei will satisfy the equations

\[(H_T - E_T)\phi_T(x_T) = 0\]  \hspace{1cm} 2-5a
\[(H_p - E_p)\phi_p(x_p) = 0\]  \hspace{1cm} 2-5b

where \(E_{p(T)}\) is the binding energy for the ground state of the projectile (target). The interaction Hamiltonian consists of a relative kinetic energy operator and a potential energy term.

\[H_{\text{int}}(x_p, x_T, \vec{R}, \vec{R}) = -\frac{\hbar^2}{2\mu} \nabla^2 \frac{1}{R} + V_{\text{int}}(x_p, x_T, \vec{R}, \vec{R})\]  \hspace{1cm} 2-6

The effect of the interaction potential will be to introduce outgoing spherical waves. For clarity we first consider the case where there are no explicit inelastic excitation or spin dependent forces acting. In this case the interaction depends only on the radial separation of the colliding nuclei, and the wave function can be written, for large \(R\), as

\[\Psi^{(+)}_{T_p I M} (x_p, x_T, \vec{R}, \vec{R}) = \phi_T(x_T)\phi_p(x_p) \left[ e^{i\vec{k}\cdot \vec{R}} + f(\Theta, \phi) e^{iKR} \right]\]  \hspace{1cm} 2-7

The (+) sign indicates the choice of boundary conditions as an incident
plane wave plus outgoing spherical waves. The colliding nuclei however are
charged bodies. The Coulomb potential between them, being infinite in
range, alters the form of the asymptotic solution. Assuming however the
nuclei to be spherical charges the wave function can still be written in
the form of incident and scattered waves as, to order \((KR-\vec{R}_R)^{-1}\)

\[
\Psi^{(+)}(x_p, x_T, \vec{k}, \vec{R}) = \Phi_+^{(+)}(x_T) \Phi_0^{(T)}(x_p) \left[ e^{i(k\vec{r} + \eta \ln(KR - \vec{k}_R))} \right. \\
\left. \left\{ 1 + i \left( \frac{\eta^2}{LR - \vec{k}_R} \right) \right\} + \frac{f_{\text{Coul}}(\Theta)}{R} \right]
\]

where the Coulomb scattering amplitude \(f_{\text{Coul}}(\Theta)\) is

\[
f_{\text{Coul}}(\Theta) = \left( \frac{\eta}{2KR \sin^2(\Theta/2)} \right)^{1/2} e^{-i \eta \ln(\sin^2(\Theta/2))} + 2i \sigma_0
\]

where \(\eta\) is the Sommerfeld parameter

\[
\eta = \frac{Z_T Z_p e^2 \mu}{k^2 K}
\]

\(Z\) is the charge of the projectile (target), \(e\) the charge of a proton
and \(K\) the asymptotic wavenumber; \(K=(2\mu E)^{1/2}/\hbar\) with \(\mu\) the reduced mass of
the projectile, \(\mu=m_p m_T/(m_p+m_T)\) in atomic mass units. \(\sigma_0\) is the Coulomb phase
shift

\[
\sigma_L = \arg \left( 1 + L + i\eta \right)
\]

for \(L=0\), with \(\Gamma(z)\) the gamma function for complex argument [Ab 70].

### 2.2.1 Inclusion of Inelastic Channels

Although in isolation the two nuclei will be in their respective ground
states, the interaction between them may induce transitions to other
eigenstates $p', T'$ satisfying equations 2-5 a and b with eigenenergies $E_{p'}$, $E_{T'}$. There also exists the possibility of either of the two nuclei being broken up.

The total wave function of equation 2-1 may then be written as

$$
\psi_{TPIM_z}^{(4)} (x_p, x_T, K, R) = \sum_{T'P 'I'M'_{z}} \phi_{TPIM_z} (x_p) \phi_{I'M'M_z} (x_p) \chi_{I'M'_{z}M_z} (K, R)
$$

2-12

where the summations include the respective ground states, and include integrations over configurations corresponding to break up of the participants.

The summations over the projectile degrees of freedom have been separated for later convenience. The summation over $T'$ implicitly includes sums over any target spins. Asymptotically the wave function

$$
\psi_{TPIM_z}^{(4)} (x_p, x_T, K, R)
$$

has the incident conditions of 2.2, but supplemented by outgoing waves corresponding to the interacting nuclei being in the states $p', T'$. Neglecting, for now, the presence of Coulomb forces which may be included by the techniques indicated in the last section, the asymptotic wave function can be written as

$$
\psi_{TPIM_z}^{(4)} (x_p, x_T, K, R) = \phi_{TPIM_z} (x_p) e^{\frac{iK \cdot R}{\hbar}} + \sum_{M'_{z}} \phi_{TPIM_z} (x_p) \phi_{I'M'_{z}M_z} (x_p) \chi_{I'M'_{z}M_z} (K, R)
$$

2-13

The outgoing waves having been split into two parts, elastic and inelastic. The value of the asymptotic inelastic wavenumber $K'$ is given by

$$
\frac{\hbar^2}{2 \mu} \frac{K'^2}{2} = \frac{\hbar^2}{2 \mu} \frac{K^2}{2} - (E_{T'P'} - E_{TP})
$$

2-14

The presence of spin dependence introduces the possibility of changing the
The z component of the incident projectile spin. The scattering amplitude $f_{M_L}^{I,N_\uparrow}(\theta,\phi)$ is a $(2I+1)$ by $(2I+1)$ matrix in the spin space of the projectile. In the absence of any spin dependent interactions

$$f_{M_L}^{I,N_\uparrow}(\theta,\phi) = \sum_{M_{\uparrow \downarrow}} f^{I}(\theta) \chi_{M_{\uparrow \downarrow}}^{I}$$

2-15

The direct evaluation of equation 2-3 with the wave function of 2-11 is not tractable due to the many states that may be present in the summations. The essential approximation of the coupled channels method is to assume the summations present in 2-11 may be truncated to a sum over a small number of states that are expected on physical grounds to be strongly excited as part of the reaction mechanism. The effect of coupling to the states not explicitly included in the summation being accounted for by the replacement in equation 2-6 of $V_{\text{int}}(x_p,\mathbf{r}_n,\mathbf{r}_t)$ by an effective interaction. Since coupling to the states excluded from the sum will remove flux from the incident elastic channel this effective potential will be complex, the imaginary part causing absorption.

As attention is being focussed on the effects of excitation of the projectile, the sum will be taken to include only terms corresponding to some of the lowest lying projectile states, the explicit contribution from excited target states being ignored. With this restriction equation 2-13 becomes

$$\Psi^{(+)\nu}_{I,M_{\uparrow}}(x_p,\mathbf{r}_n,\mathbf{r}_t) = \phi_{\nu}(x_p) \sum_{I',M'_{\uparrow}} \phi_{I',M'_{\uparrow}}(x_p) \chi^{I'\uparrow}_{M'_{\uparrow}}(\mathbf{r}_n,\mathbf{r}_t)$$

2-16

where the excited projectile states are labelled only by their total spin, which can be taken to represent all of the properties of that state for the cases under consideration. Since events corresponding to target excitation
are taken into account via the effective interaction, this may conveniently be chosen so that equation 2-6 becomes

\[ H_{\text{int}}(x_p, x_T, \vec{K}, \vec{R}) = \frac{-\hbar^2}{2\mu} \nabla^2 \frac{1}{\vec{R}} + U_{\text{int}}(x_p, \vec{K}, \vec{R}) \]  

The possibility of rearrangement in which nucleons are transferred between the reactants cannot simply be included within this prescription. The rearranged nuclei are not eigenstates of equations 2-5 a & b and so an expansion in terms of these eigenstates would require a large number of terms. Also the rearranged states are not orthogonal to these states. The effects of rearrangement events as sources for a loss of flux from the incident elastic channel are included within the effective interaction \( U_{\text{int}}(x_p, \vec{K}, \vec{R}) \).

Using as an ansatz the wave function of equation 2-16, with the interaction of equation 2-17, equation 2-3 becomes

\[ \left[ H_p - (E - E_T) - \frac{\hbar^2}{2\mu} \nabla^2 \frac{1}{\vec{R}} + U_{\text{int}}(x_p, K, \vec{R}) \right] \sum \phi_{\text{im}}(x_p) \chi_{im}^{T} (\vec{K}, \vec{R}) = 0 \]  

### 2.2.2 Expansion in Partial Waves

In order to conserve total angular momentum when the projectile is excited a change in the spin of the projectile must be accompanied by a corresponding change in the relative orbital angular momentum between the participating nuclei. The total wave function for the system can be expanded in terms of eigenstates of the total, conserved, angular momentum \( J \) and its z component \( M_J \) as [ Sa 83 ]

\[ \Psi_{IM_i}^{(i)}(x_p, \vec{K}, \vec{R}) = \frac{4\pi}{KR} \sum (LM_L I M_i J M_J) Y^* (\vec{K}) \chi_{LM_L I}^{T} (K, R) Y_{LM_L}^{M_J} (\vec{R}, x_p) \]
where the sums run over $L,M,L',M',L'',M',J$ and $M_J$, and the second pair of labels $L'$ on the scattering function $\chi_{L',L'}^{J}(KR)$ indicate the boundary conditions of incident waves in the $L$-$I$ combination.

The $Y^{M_J}_{J(L'I')} (R,x_p)$ form an orthonormal basis with respect to integrations over the angles of $\hat{R}$ and the projectile coordinates

\[ \int dx_p \int d\hat{R} Y^{M_J}_{J(L'I')} (\hat{R},x_p) Y^{M_J}_{J(L'I')} (\hat{R},x_p) = \delta_{J,J'} \delta_{M_J,J'} \delta_{L,L'} \delta_{I,I'} \]

where $d\hat{R} = \sin(\theta) d\theta \, d\phi$. These spin angle functions are formed in terms of the orbital angular momentum and projectile spin eigenstates as

\[ Y^{M_J}_{J(L'I')} (\hat{R},x_p) = \sum \left( i \right)^{L'} (L'|L'I'I'|M_J) \Phi_{I'I'M_J} (x_p) Y^{M_J}_{J(L')}(\hat{R}) \]

where the order of coupling has been chosen as $J = L' + I'$ and the $(i)^{L'}$ factor has been included to give convenient properties under the action of the time reversal operator $\Theta$

\[ \Theta \, Y^{M_J}_{J(L'I')} (\hat{R},x_p) = (-)^{T-M_J} Y^{M_J}_{T-J(L'I')} (\hat{R},x_p) \]

Using 2-19 in 2-18 a set of coupled equations for the $\chi_{L'I',L'}^{J}(KR)$ for each $J$ can be obtained by multiplying on the left by $Y^{M_J}_{J(L')} (KR) Y^{M_J}_{J(L')} (\hat{R})$ and integrating over $d\hat{R}$, $dK$ and $dx_p$, to yield

\[ \left[ T_R^{L''} - (E - E_I^{L''}) + V_{\text{Coul}} (R) + V_{L''}^{J}(L'';L'';L'';I') (R) \right] \chi_{L'';L'';I''}^{J}(K'',R) = - \sum_{I'I''} V_{L''}^{J}(I';L'';I'') (R) \chi_{L'';L'';I}^{J}(K,R) \]

where $T_R^{L''}$ is the kinetic energy operator for the $L''$ th partial wave

\[ T_R^{L''} = -\frac{\hbar^2}{2\mu} \left[ \frac{d^2}{dR^2} - \frac{L''(L''+1)}{R^2} \right] \]

with $K'' = (2\mu(E-E_I^{L''}))^{1/2}$. Used here is the fact, proved in section 2.3.1,
that the interactions considered conserve total angular momentum $J$ and its $z$ component $M_J$. The $V_{jJ^i, l, l}(R)$ are given by

$$V_{jJ^i, l, l}(R) = \langle J(L', I') | U_{\text{int}}(x, L') | J(L, I) M_J \rangle$$

The projectile excitation is considered to arise only from the nuclear interaction. With the restriction that Coulomb excitation will not be considered, the Coulomb interaction between the colliding nuclei will be taken to be that of a uniformly charged body, the radial form of which is given in appendix A. Since this is a function only of the radial variable $R$ it commutes with the variables which have been integrated over, and so has been separated from those arising from the nuclear interactions. The potentials $V_{jJ^i, l, l}(R)$, which remain functions of the radial variable $R$, will be discussed in section 2.3. The absorptive parts of the potentials will reduce the magnitude of those outgoing waves for which there are incident waves, whilst the coupling potentials will introduce outgoing flux into channels with $I', L' \neq I, L$. The diagonal ($L = L'$ and $I = I'$) and coupling potentials are short ranged. Outside the range of these potentials the $\chi_{jJ^i, l, l}(KR)$ satisfy the uncoupled radial equations

$$\left[ (E - E_{\chi'}) - T'' - V_{\text{Coul}}(R) \right] \chi_{jJ^i, l, l}(K'', R) = 0$$

Which are simply the radial Coulomb equations of equation 1-24, with the asymptotic solutions $F_{L''}(KR)$ and $G_{L''}(KR)$. In the absence of any nuclear forces the $\chi_{jJ^i, l, l}(KR)$ remain finite if the physical solutions contain only the regular functions $F_{L''}(KR)$. It is convenient to choose the phase of the $\chi_{jJ^i, l, l}(KR)$, in the absence of any nuclear forces, as

$$\chi_{jJ^i, l, l}(K'', R) = e^{i\sigma''} F_{L''}(K'' R)$$
The presence of the potentials \( V_{L'_{\text{III}},L_{\text{III}}}^J(R) \) will introduce irregular components into the asymptotic solution. The asymptotic form of the
\[
\chi_{L'_{\text{III}},L_{\text{III}}}^J(K',R) \text{ can then be written as}
\]
\[
\chi_{L'_{\text{III}},L_{\text{III}}}^J(K',R) = \frac{i}{2} \left( \frac{V_v}{V_u} \right)^{1/2} e^{i\alpha \left[ I_{L'}(K'R) S_{L'L_{\text{III}}}^{*} + S_{L'L_{\text{III}}}^{J_{\text{III}}} O_v(K'R) \right]}
\]
where the expansion coefficients \( S_{L'L_{\text{III}}}^{J_{\text{III}}} \) contain all of the information on the departure from pure Coulomb scattering introduced by the nuclear potentials in the presence of the Coulomb potential, \( v (v') \) is the asymptotic velocity in the entrance (exit) channel.

The distorted waves of 2-18 are then given by
\[
\chi_{M'_{\text{II}},M_{\text{II}}}^J(K,R) = \frac{4\pi}{KR} \sum (L'M_{\text{II}}',M_{\text{II}}',J_{\text{II}}) (L'M_{\text{II}}M_{\text{II}}',J_{\text{II}}) \chi_{L'M_{\text{II}}}(K,R) Y_{L'M_{\text{II}}}^J(\hat{R})
\]
(2-29)
where the sums run over \( L',M_{\text{II}}',L,M_{\text{II}},J \), and \( M_{\text{II}} \). Substituting the asymptotic form of the \( \chi_{L'_{\text{III}},L_{\text{III}}}^J(K,R) \) into this equation the scattering amplitude, which multiplies the outgoing waves, arising from the nuclear potential is
\[
I_{M'_{\text{II}},M_{\text{II}}}^J(\theta,\phi) = \frac{2\pi}{iK} \sum (L'M_{\text{II}}',M_{\text{II}}',J_{\text{II}}) (L'M_{\text{II}}M_{\text{II}}',J_{\text{II}}) e^{i(\sigma_v' + \sigma_L')}
\]
\[
\left[ S_{L'L_{\text{III}}}^{J_{\text{III}}} - \delta_{L',L} \delta_{J',J} \right] Y_{L'M_{\text{II}}}^J(\hat{R}) Y_{L'M_{\text{II}}}^J(\hat{R})
\]
(2-30)
Allowing \( R \) to tend to \( \infty \) along the direction \( \hat{K}_{\text{out}} \) introduces the simplification \( m = 0 \), since
\[
Y_{LM}(0,\phi) = \left( \frac{2L+1}{4\pi} \right)^{1/2} \delta_{L0}
\]
(2-31)
Using this simplification, the scattering amplitude can be written as
\[
I_{M'_{\text{II}},M_{\text{II}}}^J(\theta) = \frac{1}{2iK} \sum (L'M_{\text{II}}',M_{\text{II}}',J_{\text{II}}) (L_{\text{II}}M_{\text{II}}',J_{\text{II}}) e^{i(\sigma_v + \sigma_L')}
\]
\[
\left\{ \begin{array}{c}
\hat{L}' \hat{L}' \left( \begin{array}{c}
L' \!-\! M_{\text{II}}' \\
L \!+\! M_{\text{II}}
\end{array} \right) \right\}^{1/2} \left[ S_{L'L_{\text{III}}}^{J_{\text{III}}} - \delta_{L',L} \delta_{J',J} \right] P_{L'M_{\text{II}}}(\theta)
\]
(2-32)
where \( P_{L}^{M_{L}'}(\cos(\theta)) \) is an associated Legendre function [ Ab 70 ].

The elastic scattering amplitude also requires a component which arises from Coulomb scattering.

\[
\hat{f}_{\text{II}}^{\text{M}_{L}^{I} M_{L}^{I}}(\theta) = \hat{f}_{\text{Nuclear}}^{\text{M}_{L}^{I} M_{L}^{I}}(\theta) + \hat{f}_{\text{Coul}}(\theta) \delta_{M_{L}^{I} M_{L}^{I}}
\]

This Rutherford scattering amplitude can be given in partial wave form as

\[
\hat{f}_{\text{Coul}}(\theta) = \frac{4\pi}{2iK} \sum_{L} \left( e^{2i\sigma_{L}} - 1 \right) \sum_{M} Y_{L M}^{*}(\hat{R}) Y_{L M}(\hat{R})
\]

In order to consider the evaluation of the matrix elements \( V_{L'I'I',LL}^{I,J}(R) \) it is convenient to first consider the case of no excitation. The coupling and excited state diagonal potentials can then be evaluated within the formalism developed. In the absence of projectile excitation the coupled equations 2-23 reduce to

\[
\left[ E - T_{R}^{'L''} - V_{\text{Coul}}(R) - V_{L'I'I',L'I'}^{J}(R) \right] \chi_{L''I''I''I'}^{J}(K,R) = \sum_{L''I''I''I''} V_{L''I''I''I''}^{J}(R) \chi_{L''I''I''I''}^{J}(K,R)
\]

There is still the possibility of coupling, via second rank tensor interactions, to other partial waves within the elastic channel [ Sa 60 ]. Parity and time invariance allow three such interactions.

\[
T_{R} = \left( \hat{\mathbf{P}} \cdot \hat{\mathbf{R}} \right)^{2} \quad 2-36a
\]

\[
T_{P} = \left( \hat{\mathbf{P}} \cdot \hat{\mathbf{P}} \right)^{2} - \frac{1}{3} \hat{\mathbf{I}}^{2} \quad 2-36b
\]

\[
T_{L} = \left( \hat{\mathbf{L}} \cdot \hat{\mathbf{I}} \right)^{2} + 2 \hat{\mathbf{L}} \cdot \hat{\mathbf{I}} - \frac{1}{3} \hat{\mathbf{I}}^{2} \quad 2-36c
\]

Within the context of the models used only the first of these, \( T_{R} \), acts in the \( ^7\text{Li} \) calculations. If these interactions are neglected the equations reduce further to the uncoupled equations...
The folding model was first proposed for deuterons \[ \text{(Wa 58)} \]. In the model the potential experienced by the deuteron is considered to be a sum of the potentials experienced separately by the two nucleons. The nucleons however form a bound structure and to allow for this the potentials are folded, that is they are multiplied by the deuteron nucleon density \( |\phi_d(r_d)|^2 \) and integrated over the nuclear volume. This produces the potential experienced by the deuteron. Within a cluster picture similar ideas can be applied to heavier composite structures, for a comprehensive overview see \[ \text{(Sa 79)} \].

The coordinate system used to describe the folding model is shown in figure 2.1. Regarding this figure, the potential experienced by the projectile can be obtained from the potentials between the target and the core and valence clusters. The core cluster and target are both spin zero, so the only interaction between these is central. For all projectiles considered the valence cluster carries an intrinsic spin, so other tensor interactions are possible. For \(^{19}\text{F}\) in which the valence cluster has spin 1/2 this is limited to a first rank, vector, spin orbit potential. The valence cluster in \(^6\text{Li}\) is a deuteron with spin 1, and hence the deuteron target interaction can include, in addition to the central and spin orbit interactions, second rank tensor forces. The interaction between the deuteron cluster and the target will be taken to consist only of central and spin orbit terms. Since the orbital angular momentum between the clusters in \(^6\text{Li}\) is zero the central potentials will generate only a
central potential between the \(^6\)Li projectile and the target. Also the deuteron target spin orbit potential will generate only a spin orbit potential between the \(^6\)Li and target. The valence cluster of \(^7\)Li, a triton, has spin 1/2 so that the interaction between this cluster and the target will be taken to possess central and spin orbit terms. Due to the non zero angular momentum of the triton cluster around the alpha core the central cluster target potentials will generate central and tensor potentials between the \(^7\)Li and target, and the spin orbit potential will generate central, spin orbit, second and third rank tensor forces between the \(^7\)Li and target.

The evaluation of the central terms \(U_{\text{Int}}(\vec{R}, \vec{r})\) will be considered in section 2.3.1, within a single channel framework. The valence target spin orbit potential will be considered in section 2.3.2. The formalism developed within these sections will be generalised in section 2.3.3 to include terms responsible for coupling to the excited states considered, and for the potentials that act within the excited states.
Figure 2.1 Coordinate system used for the folding model calculations.
2.3.1 Central Potentials

From the assumption of section 1.3.1 that the clusters remain inert during the reactions considered it follows there is no explicit dependence upon their internal coordinates. The effects of events corresponding to excitation and break up of the clusters are accounted for by the use of phenomenological cluster target optical potentials. The energy dependence of the interaction potential $U_{\text{int}}(x_p, \vec{R}, K)$ can be taken into account by using the phenomenological cluster target optical potentials at $m_{\text{cluster}}/m_{\text{projectile}}$ times the projectile incident energy. Hence the interaction potential can be written as

$$U_{\text{int}}(\vec{R}, \vec{r}) = U_{cT}(\vec{X}_{cT}) + U_{vT}(\vec{X}_{vT})$$

where the vector relations apparent from figure 2.1 can be used to express the cluster target vectors in terms of $\vec{R}$ and $\vec{r}$.

$$\vec{X}_{cT} = \vec{R} - \frac{m_c}{m_c + m_{vT}} \vec{r}$$ \hspace{1cm} 2-39a

$$\vec{X}_{vT} = \vec{R} + \frac{m_{vT}}{m_c + m_{vT}} \vec{r}$$ \hspace{1cm} 2-39b

In a standard manner the central part of the phenomenological optical potentials between the clusters and the target will be taken to depend only on the magnitude of the cluster target separation. Hence, for example

$$U_{cT}(\vec{X}_{cT}) = U_{cT}(\vec{X}_{cT})$$

Since the interaction potential $U_{\text{int}}(\vec{R}, \vec{r})$ does not depend on the internal cluster coordinates the integrations in equation 2-25 can be factored to yield,
the integrations over the cluster wave functions simply giving unity. The matrix element has been written in its most general form assuming the projectile to remain in its ground state, in particular no assumptions have been made about its overall rotational properties.

In order to carry out the integrations over r and R it is necessary to express the potentials $U(\vec{r},\vec{t})$ and $U_0(\vec{r},\vec{t})$ in terms of these coordinates. This can be done by expanding over a sum of multipoles in terms of a complete set of functions, the Legendre polynomials as

$$U(X) = \sum_k v_k(R,r) \hat{R}^2 P^k_0(\cos \hat{R})$$

The expansion coefficients $v_k(R,r)$ are obtained using

$$v_k(R,r) = \frac{1}{2} \int_{-1}^{1} P^k_0(\cos \hat{R}) U(\vec{R},\vec{r}) d(\cos \hat{R})$$

where $U(\vec{R},\vec{r})$ can be found using the vector relations 2-39 a & b, for example

$$U_{ct}(X_{ct}) = U \left( \left[ R^2 + \left( \frac{m_c}{m_c + m_r} \right)^2 r^2 - \frac{2m_c}{m_c + m_r} r R \cos \hat{R} \right]^\frac{1}{2} \right)$$

The Legendre polynomial of equation 2-42 can be expanded in terms of spherical harmonics in $\hat{R}$ and $\hat{r}$.

$$P^k_0(\cos \hat{R}) = 4\pi \hat{R}^{-2} \sum_q Y^*_q(\hat{R}) Y_q(\hat{r})$$

Using this multipole expansion the matrix element of equation 2-41 is

$$\langle J'(L' I') M'_J | U_{ct}(\vec{R},\vec{r}) | J(L I) M_J \rangle = \langle \phi_c | \phi_c \rangle$$

$$\langle \phi_{v'} | \phi_v \rangle \langle J'(L' I') M'_J | U_{ct}(\vec{R},\vec{r}) | J(L I) M_J \rangle $$

$$\sum_{l m} \sum_{l' m'} \sum_{l'' m''} \int d\hat{R} Y^{*}_{l m}(\hat{R}) Y^{*}_{l' m'}(\hat{R}) X^{*}_{l'' m''}(\hat{R}) \int d\hat{r} Y_{l'}^{*}(\hat{r}) Y_{l''}(\hat{r}) \int r^2 dr v^k(R,r) \int | \phi_c (r) |^2$$
where the sums run over \( J', M_J', k, q, M_k', M_M', M_{L'}, M_{L}, m_m, m_{m}', m_3 \) and \( m_{s'} \). The summation over the spin degree of freedom gives \( S_{m_3} S_{m_{s'}} \). The integration of the three spherical harmonics over \( \hat{r} \) gives

\[
\langle l m_m' k q | l m \rangle < l \| k \| l > \tag{2.47}
\]

Evaluating the reduced matrix element yields

\[
(\langle l m_m' k q | l m \rangle (l o k o | l o) \frac{\hat{R}}{\sqrt{4\pi}}) \tag{2.48}
\]

Similarly the integration over \( \hat{R} \) yields.

\[
(\langle L'M_{L'} k-q | l M_{L} \rangle (L'o k o | l o) \frac{\hat{R}}{\sqrt{4\pi}} \hat{L'}) \tag{2.49}
\]

Collecting the results, and defining

\[
\omega^R(R) = \int r^2 dr \nu^R(R,r) |\phi_k(r)|^2 \tag{2.50}
\]

gives the following expression for the matrix element

\[
\sum (\langle l m_m s m_s | l M_{M} I_M I_T \rangle (\langle l m_m' s m_s | l M_{M} I_M I_T \rangle (\langle l m_m' k q | l m_m \rangle

(\langle L_{L'} I_M I_T | J_{M'} I_{M'} \rangle (\langle L'M_{L'} I_M I_T | J_{M'} I_{M'} \rangle (\langle L_{M} R-q | l M_{M} \rangle

(\langle L'o k o | l o | l o k o | l o \rangle \frac{\hat{R}}{\sqrt{4\pi}} \hat{L'}) \omega^R(R) \tag{2.51}
\]

where the sum runs over the same quantum numbers as before, except \( m_{s'} \).

Performing the sums over \( m_m, m_m' \) and \( m_s \) the first three Clebsch Gordan coefficients can be contracted to form a Racah coefficient and a Clebsch Gordan coefficient \( [ Br 7 ] \). The matrix element becomes
where the remaining sums are over \( J', M_J, k, q, M_{\pi}, M_{\pi}', M_L \) and \( M_L' \). The last five of these sums can be performed to contract the last four Clebsch Gordan coefficients in this expression to form a Racah coefficient.

Performing the contraction the following expression for the matrix element is obtained:

\[
\sum_{k, q} \binom{L'}{J} \binom{L}{J'} \binom{L, L'}{L'} \binom{J'}{J} W(L, s, t, L') \delta_{J, J'} \delta_{M_J, M_J'}
\]

The Kronecker deltas arising from this contraction confirm that the total angular momentum and its z component are conserved, which had been previously assumed.

The projectiles taken to be purely \( l = 0 \) in their ground states allow, via the parity selecting Clebsch Gordan coefficient \((10k0|10)\), only \( k = 0 \). For \( ^7\text{Li} \), with \( l = 1 \), the values \( k = 0 \) and 2 are permitted. In both cases evaluating the values of the Racah and Clebsch Gordan coefficients with \( k = 0 \) leads to the expression

\[
<J(L'|J) M_J | U_{L4}(\hat{R}, \hat{\theta}) | J(L) M_J> = \sum_{J, L, \ell} \int_0^\infty r^2 d\rho \rho^2 (R, r) \phi_j(r)^2
\]
Hence the $k = 0$ component of the multipole expansion of the sum of the cluster target potentials contributes only to the central part of the projectile target interaction.

Evaluating explicitly the Racah and Clebsch-Gordan coefficients for $k = 2$ leads to the term

$$
\sum_{L \pm 2, L'} \int_0^\infty r^2 \, d r \, \nu^2 (R, r) \mid \phi_k (r) \mid^2
$$

so that the $k = 2$ multipole of the central parts of the cluster target interactions act as a tensor potential between the projectile and target, coupling partial waves that differ by 2 units of orbital angular momentum.

### 2.3.2 Spin orbit terms

The matrix element arising from the spin orbit potential between the valence cluster and the target is

$$
\langle J(LI) M_J \mid U_{so} (X_{vT}) \hat{L}_{vT} \cdot \hat{S}_{vT} \mid J(LI) M_J \rangle
$$

where

$$
\hat{L}_{vT} = -i \hbar \hat{X}_{vT} \wedge \nabla_{\nu T}
$$

The projectiles which are taken to have $l = 0$ in the ground state will be considered explicitly, the results for $^7$Li, with $l = 1$, will be quoted below. In order to evaluate the projectile target spin orbit potential it is necessary to express the operator $\hat{L}_{vT} \cdot \hat{S}_{vT}$ in terms of the orbital angular momentum between the projectile and target, and the total spin of
the projectile $\mathbf{I}$. Since in the ground state the valence cluster is assumed to be in a pure $l = 0$ state about the core the spin $\mathbf{I}$ of the projectile arises entirely from the intrinsic spin of the valence cluster, so that $\mathbf{I} = \mathbf{S}$.

The operator $\mathbf{L}_{\nu T}$ is defined between the valence cluster and the target, keeping their centre of mass fixed. Hence it is convenient to express the position of the core cluster with respect to this point rather than the centre of mass of the target alone.

From Figure 1.2

\[
\mathbf{R} = \frac{m_c}{m_c + m_{\nu r}} \mathbf{y} + \frac{(m_T + m_c + m_{\nu r}) m_{\nu r}}{(m_T + m_{\nu r})(m_c + m_{\nu r})} \times_{\nu T} \mathbf{x}_{\nu T} \tag{2-58a}
\]

\[
\mathbf{r} = \mathbf{y} + \frac{m_r}{m_T + m_{\nu r}} \mathbf{x}_{\nu T} \tag{2-58b}
\]

Expressing the differential operator in terms that operate on $\mathbf{r}$ and $\mathbf{R}$

\[\nabla^2_{\mathbf{X}_{\nu T}} = \frac{d^2 R}{d x^2_{\nu T}} \nabla^2_{\mathbf{R}} + \frac{d^2 R}{d x^2_{\nu T}} \nabla^2_{\mathbf{r}} \tag{2-59}\]

The weighting factors can be obtained from equations 2-58 a and b, so that

\[
\nabla^2_{\mathbf{X}_{\nu T}} = C_R \nabla^2_{\mathbf{R}} + C_r \nabla^2_{\mathbf{r}} \tag{2-60}\]

where

\[
C_R = \frac{(m_T + m_c + m_{\nu r}) m_{\nu r}}{(m_T + m_{\nu r})(m_c + m_{\nu r})} \tag{2-61a}\]

\[
C_r = \frac{m_T}{m_T + m_{\nu r}} \tag{2-61b}\]
Using 2-57 and 2-39 b, equation 2-56 becomes
\[
\hat{L}_{vT} = (-i\hbar) \left[ \hat{R} + \frac{m_e}{mc+\mu_v} \hat{p} \right] \wedge \left[ C_R \nabla_{\hat{R}} + C_R \nabla_{\hat{p}} \right]  \tag{2-62}
\]
Expanding the cross product gives
\[
\hat{L}_{vT} = (-i\hbar) \left[ C_R \hat{R} \wedge \nabla_{\hat{R}} + C_R \hat{R} \wedge \nabla_{\hat{p}} + \frac{m_e}{mc+\mu_v} \left[ C_R \hat{R} \nabla_{\hat{R}} + C_R \hat{p} \nabla_{\hat{R}} \right] \right]  \tag{2-63}
\]
The first term of 2-62 is simply $C_R \hat{R}$, giving
\[
\langle J(LI) M_J | \hat{L}_{vT} X_{vT} | J'(LI) M'_J \rangle  \tag{2-64}
\]
The spin orbit form factor $U_{s_o} (X_{vT})$ can be expanded as a sum of multipoles in the same manner as the central terms. Since the operator $\hat{L}$ is independent of $\hat{r}$ the integration over the angles of $\hat{p}$ gives the same result as the central case, namely $k = 0$ only. Equation 2-64 becomes
\[
\int_0^{\infty} r^2 dr |\phi_o(r)|^2 U_{s_o}(R,r) \langle J(LI) | \hat{L}_{vT} | L'I' \rangle \langle L'I' | J'(LI) \rangle  \tag{2-65}
\]
The term $V_{s_o}(R,r)$ is given by equation 2-42. The reduced matrix element with $\hat{L}, \hat{R}$ acting yields [ Br 79 ]
\[
\delta_{L'I'} \delta_{L'I} \delta_{J'J} (-)^{J-L-I} W(LLI;J) \left[ L(L+1)(2L+1)I(I+1)(2I+1) \right]^{1/2}  \tag{2-66}
\]
using the explicit value of the Racah coefficient gives
\[
\frac{1}{2} \left[ J(J+1) - L(L+1) - I(I+1) \right]  \tag{2-67}
\]
This relation can be obtained more simply by squaring the relation that expresses the coupling to form the total angular momentum $\hat{J} = \hat{L} + \hat{I}$, but equation 2-66 is required for later work. This purely numerical factor can be taken out of the integration over $\hat{R}$ in equation 2-64 to leave the orthogonality integral. The first term of 2-62 thus gives a contribution to the spin orbit potential between the projectile and target of
\[ V_{s_0}(R) \hat{\mathbf{L}} \cdot \hat{\mathbf{I}} = C_R \int_0^\infty r^2 \, dr \, |\phi_o(r)|^2 \, V_{s_0}^*(R, r) \, \hat{\mathbf{L}} \cdot \hat{\mathbf{I}} \]  

The second term in equation 2-62 is

\[ -i \kappa (\hat{\mathbf{r}} \wedge \nabla_{\mathbf{R}}^2) \]  

The operator \((\hat{\mathbf{r}} \wedge \nabla_{\mathbf{R}}^2) \hat{\mathbf{I}}\) can be expressed in terms of \(\hat{\mathbf{L}} \hat{\mathbf{I}}\), by expanding in terms of its spherical components as

\[ (\hat{\mathbf{r}} \wedge \nabla_{\mathbf{R}}^2) \hat{\mathbf{I}} = \sum_m (1m | -m | 00)(\hat{\mathbf{r}} \wedge \nabla_{\mathbf{R}}^2)_m \hat{I}_{-m} \]  

where the Clebsch Gordan coefficient ensures they are coupled to form a scalar, using the value of this coefficient

\[ (\hat{\mathbf{r}} \wedge \nabla_{\mathbf{R}}^2) \hat{\mathbf{I}} = \sum (-)^{|m|} \hat{I}_{-1} (\hat{\mathbf{r}} \wedge \nabla_{\mathbf{R}}^2)_m \hat{I}_{-m} \]  

The \(m\)th component of the cross product term can be obtained by the use of bipolar harmonics as

\[ (\hat{\mathbf{r}} \wedge \nabla_{\mathbf{R}}^2)_m = \frac{\sqrt{2}}{2} \sum_{\chi\mu} (1\chi | 1\mu | 0m) r_{\chi} \nabla_{\mu} \]  

The \(\chi\)th component of \(\hat{\mathbf{r}}\) can be written in terms of the spherical harmonics as

\[ r_{\chi} = \left( \frac{4\pi}{3} \right)^{1/2} \ Y_{1\chi} (\hat{\mathbf{r}}) \]  

The integral over the angles of \(r\) becomes

\[ \int d\mathbf{r} \ Y_{\psi o}^* (r) Y_{R\psi}^* (r) Y_{1\chi} (r) Y_{oo} (r) = \left( \frac{-1}{4\pi} \right)^{\psi} \delta_{R1} \delta_{\chi\Psi} \]  

so the sum over multipoles reduces to the \(k = 1\) term only. Collecting the remaining components of the operator \((\hat{\mathbf{r}} \wedge \nabla_{\mathbf{R}}^2) \hat{\mathbf{I}}\) together with the \(Y_{R\psi} (\hat{\mathbf{R}})\) from the multipole expansion we have
\[ \sum_m (-)^{1-m} \hat{r}^{-1} \frac{\sqrt{2}}{l^m} \sum_{\kappa \mu} |1 \kappa |_\mu | l m \rangle r \left( \frac{4\pi}{3} \right)^{\!\!\frac{3}{2}} Y_{1 \kappa} (\hat{r}) \nabla^2_{\mu} \] 2-75

This is simply
\[ \frac{r}{R} (\hat{r} \times \nabla_{\hat{r}}) \cdot \hat{r} = \frac{r}{R} \nabla \cdot \hat{r} \] 2-76

Using this result the contribution to the projectile target spin orbit potential from the second term in equation 2-62 is
\[ V_{So} (R) \nabla \cdot \hat{r} = \frac{m_c}{m_c + m_{t'}} \sum \int \frac{r^3}{R} \frac{\partial \langle \psi_o (R) \rangle^2}{\partial r} \langle J(L) | \nabla \cdot \hat{r} | J(L) \rangle \] 2-77

The remaining matrix element can be evaluated as for the first spin orbit term. The third term in the expansion 2-62 leads to
\[ -i \hbar (\hat{r} \times \nabla_{\hat{r}}) \] 2-78

Expanding the operator \((\hat{r} \times \nabla_{\hat{r}}) \cdot \hat{r}\) in the same manner as equation 2-70 gives
\[ (\hat{r} \times \nabla_{\hat{r}}) \cdot \hat{r} = \sum_m (-)^{1-m} \hat{r}^{-1} (\hat{r} \times \nabla_{\hat{r}})_m \cdot \hat{r} \] 2-79

where the mth component of \((\hat{r} \times \nabla_{\hat{r}}) \) is given by
\[ (\hat{r} \times \nabla_{\hat{r}})_m = \frac{\sqrt{2}}{l} \sum_{\kappa \mu} (1 \kappa |_\mu | l m \rangle \nabla_{\mu} \] 2-80

Inserting this expansion into the integration over \( \hat{r} \) yields
\[ \int dr r^2 \phi_o^*(r) \int dr Y_{\mu o}^*(\hat{r}) Y_{\kappa \mu} (\hat{r}) \sum_{\kappa \mu} (1 \kappa |_\mu | l m \rangle \nabla_{\mu} \phi_o (r) Y_{\mu o} (\hat{r}) \] 2-81

The \( \mu \)th component of the differential operator on the product \( \phi_o (r) Y_{\mu o} (\hat{r}) \) gives
\[ -\sqrt{3} (1 \mu 0 o | 1 \mu \rangle Y_{l \mu} (\hat{r}) \frac{d}{dr} (\phi_o (r)) \] 2-82

Using the value of the Clebsch Gordan coefficient this becomes
\[ Y_{l \mu} (\hat{r}) \frac{d}{dr} (\phi_o (r)) \] 2-83
Hence the integration over $\vec{r}$ is

$$\int r^2 \, dr \, \phi_0^*(r) \frac{d}{dr} \phi_0(r) \int d\hat{r} \left| Y_{l}^{\ell}(\hat{r}) \right|^2 Y_{kq}^{\ast}(\hat{r}) Y_{\nu \mu}(\hat{r})$$

The angular part of this integration picks out the $k = 1$ and $q = \mu$ components. Collecting the remaining parts of the operator ($\vec{R} \cdot \vec{v}_r$) we have

$$\sum_{\nu \mu} R \, Y_{\nu \mu}(\hat{R}) \, Y_{1 \mu}(\hat{R})$$

The sum yields zero, since this is simply $\vec{R} \cdot \vec{R}$, hence the third term does not contribute to the folded spin orbit potential. The operator in the fourth term of equation 2-62 is $\vec{I} \cdot \vec{T}$, and since $\vec{S} = \vec{I}$ this can be replaced by $\vec{I} \cdot \vec{S}$. The expectation value of this operator in the nuclear part of the matrix element is zero since the ground states are taken to be purely $l = 0$ states.

To summarise, considering single channel elastic scattering for the $l = 0$ projectiles leads to a central potential between the projectile and target given by

$$U_{\text{Central}}(R) = \int_0^{\infty} r^2 \, dr \left[ \phi_0^2(r) \right]^{\frac{1}{2}} \left[ U_{\text{ct}}(X_{\text{ct}}) + U_{\nu \nu}(X_{\nu \nu}) \right] d(\cos \hat{R})$$

and a spin orbit potential given by

$$U_{\text{So}}(R) = C_R \int_0^{\infty} r^2 \, dr \left[ \phi_0^2(r) \right]^{\frac{1}{2}} \left[ \frac{1}{2} U_{\nu \nu}(X_{\nu \nu}) d(\cos \hat{R}) - \frac{m_e}{m_e + m_v} \right]$$

$$\int_0^{\infty} \frac{r^2}{R} \, dr \left[ \phi_0^2(r) \right]^{\frac{1}{2}} U_{\nu \nu}(X_{\nu \nu}) P_l(\cos \hat{R}) d(\cos \hat{R})$$

Similar analysis [ Ni 84 ] for $^7$Li leads to the folded potentials

$$U_c(R) = \int_0^{\infty} r^2 \, dr \left[ \phi_0^2(r) \right]^{\frac{1}{2}} \left\{ \frac{A}{2(A+u)} \frac{r}{R} v_{\nu}(R,r) + \frac{A}{(A+u) \alpha} v_{\nu}^2(R,r) \right\}$$

$$U_{So}(R) = \int_0^{\infty} r^2 \, dr \left[ \phi_0^2(r) \right]^{\frac{1}{2}} \left\{ \frac{3}{3(A+u)} v_{\nu}^2(R,r) + \frac{v(A+\alpha)}{5A+u} v_{\nu}^2 - \frac{2c v(A+\alpha)}{5\alpha(A+u)} v_{\nu}^2 \right\}$$

$$U_T(R) = \int_0^{\infty} r^2 \, dr \left[ \phi_0^2(r) \right]^{\frac{1}{2}} \left\{ \frac{A}{2(A+u)} \frac{r}{R} v_{\nu}^2(R,r) + \frac{A}{2A(A+u)} v_{\nu}^2(R,r) \right\}$$
Where we note that there are two contributions to the $T_{\gamma}$ potential that act in the elastic channel, one arising from the $k = 2$ multipole component of the central potentials, and a smaller term from the cluster target spin orbit potential. Because the third rank observables are very small [Tu 85] and they have a negligible effect on the other observables [Ni 84], they will be neglected in the subsequent calculations.

### 2.3.3 Coupling Potentials

With the model for the interaction established we now return to the coupled equations and consider the evaluation of the matrix elements responsible for coupling between the projectile states, and those for scattering within the excited states.

The central terms of the interaction $U_{\Delta r} (\vec{R}, \vec{r})$ are much larger than the spin orbit term and hence the latter will be neglected in all but the ground state term considered previously. In order to evaluate 2-25 it is again convenient to expand the central interaction potentials as sums over multipoles. Using this expansion 2-25 can be written

$$V^{J}_{L'' I''; L I} (R) = \sum_{K} V^{J K}_{L'' I''; L I} (R)$$

where the $V^{J K}_{L'' I''; L I} (R)$ are given by

$$V^{J K}_{L'' I''; L I} (R) = \hat{L} \hat{I} \hat{I}'' \hat{L}'' \hat{K} (-)^{J-L''-I+I''-s+L'\prime} (k) ^{L-L''+L'\prime-I''}$$

$$(L \hat{K} | L'' 0) (L \hat{K} | L' 0) W(L'' L I'' I''; k J) W(L I'' I''; k s)$$

$$\int dr r^{2} \mathcal{V}^{J K} (R, r) \phi_{L'}^{*} (r) \phi_{L} (r)$$

2-92
The Racah coefficients again arise from the recoupling of the Clebsch Gordan coefficients. The parity selecting Clebsch Gordan coefficient imposes constraints on the value of k. For the projectile states considered with either \( l = 0 \) or \( 2 \) k must be an even multipole term. Further the terms that couple the \( l = 0 \) ground states to the \( l' = 2 \) excited states are limited to the \( k = 2 \) term only. Coupling between and within the excited states with \( l = l' = 2 \) is limited to \( k = 0, 2, 4 \). The Racah coefficient \( W(l\,l';l''\,l''';k\,s) \) is proportional to \( \delta_{ll'} \delta_{l'l''} \) for \( k = 0 \), hence only the \( k = 2 \) term can couple different states. The presence of terms other than \( k = 0 \) within the excited channels couple different partial waves within the excited channels in analogy with equation 2-35. For the ground state equation with \( l' = l = 0 \) equation 2-82 reduces to 2-76.

The \(^{7}\text{Li} \) case, with \( l = 1 \) in the ground and first excited states, and \( l = 3 \) in the second and third excited states differs slightly. The parity selecting Clebsch Gordan coefficient limits the coupling between the ground and first excited state to the \( k = 0 \) and 2 terms only. Just as in the \(^{6}\text{Li} \) case the \( k = 0 \) term does not act between the states because of the Racah coefficient. Coupling from the ground to the \( l = 3 \) states allow \( k = 2 \) and 4 terms, whilst \( k = 0, 2, 4 \) and 6 are permitted between states with \( l' = 3 \). The Racah coefficient again forbids coupling via \( k = 0 \) terms. The \( k = 6 \) terms are ignored in the present calculations due to computing code restrictions.

All of the folding model interactions were obtained using a Fortran computer code which is discussed, along with accuracy checks, in appendix C.
Chapter 3

3.1 Results and discussion

This section presents the results of the elastic and inelastic scattering calculations performed using the folding model potentials detailed in section 2.3. The coupled equations were solved numerically [Bu 63, Ta 65b]. All calculations presented here were performed using the coupled channels Fortran computer code CHUCK3 [Co]. The excited projectile states were included in the manner detailed in appendix D, the single elastic channel calculations for $^6$Li were checked against the code DDTP [To], excellent agreement being obtained. The coupled channels calculations for $^{19}$F were compared with the code FRESCO [Th], again the two calculations were in excellent agreement. The angular momentum coefficients in CHUCK3 for the channel coupling were checked by coupling to the ground state via a $T_\pi$ operator and comparing the results with those obtained from the code LINA [Tu], again good agreement was obtained. The predicted results for the elastic cross sections and vector analysing powers for the $^6$Li cases studied are presented in section 3.1.1. The results for $^7$Li + $^{119}$Sn elastic scattering and $^7$Li + $^{120}$Sn inelastic scattering are presented in section 3.1.2, and the results for $^{19}$F + $^{28}$Si scattering are presented in section 3.1.3.
3.1.1 \(^6\)Li + \(^{28}\)Si and \(^6\)Li + \(^6\)O elastic scattering at 22.8 MeV

Figure 3.1 shows the results of calculations for \(^6\)Li scattering from \(^{28}\)Si at 22.8 MeV. The dashed curves are the results of a single channel calculation. The solid curves are the results of a two channel calculation, and the dotted curves the results for a four channel calculation. The cross section data shown in this figure include a normalisation correction \([\text{Fi} 83]\). They are larger than the values originally shown \([\text{We} 76]\) by a factor of 2.27. The iT\(_n\) data are from \([\text{We} 76]\). This reference found that a single folding model spin orbit potential, together with phenomenological central potentials, was able to reproduce the experimental data. Since the analysis of this reference was performed using the incorrectly normalised cross section data, their fit to the vector analysing power was fortuitous.

A one channel folding model calculation shown by the dashed curves gives a very tiny magnitude for iT\(_n\), although reasonable values are obtained for \(\sigma/\sigma_n\). The results of the two channel calculation, shown by the solid curves, yields a vector analysing power with enhanced magnitude. The magnitude and phase of the predicted iT\(_n\) are now in much better agreement with the data, an improvement in the agreement is also seen in the cross section. It is usually possible to enhance the magnitude of the vector analysing power without increasing the strength of the spin orbit potential by changing the central potentials so that the cross section has larger peak to valley ratios. In the present case however, the amplitude of the oscillations in the cross section is actually decreased by the inclusion of channel coupling. It is clear therefore that a large
effective spin orbit potential has been produced by the inclusion of the excited channel. The results for the four channel calculation produce a vector analysing power which is slightly reduced in comparison to the two channel calculation, but the essential features are unchanged. The agreement with the cross section however has become poorer, particularly at larger angles.

Repeating the analysis of [We 76] for $^6\text{Li} + ^{16}\text{O}$ produced a folding model spin orbit potential which was a factor of 1.6 smaller than that presented in this reference. This is because, in [We 76], the deuteron target spin orbit potential was incorrectly multiplied by a factor of 2 [Am 83] and the factor $(A + 6)/(A + 2)$ present in equation 2-77 was taken as unity. Accordingly the magnitude of $iT_{11}$ obtained in a one channel calculation, using the phenomenological central potentials of [We 76], should be reduced by a factor of about 1.6. This reduction implies that the agreement between the data and calculations in this reference is not valid.

Together with the $^6\text{Li} + ^{28}\text{Si}$ results this implies that, contrary to the original conclusions of [We 76], the single folding model spin orbit potential is unable to reproduce the experimental data.

Figure 3.2 shows the results of calculations for $^6\text{Li} + ^{16}\text{O}$ scattering at 22.8 MeV. As in figure 3.1, the dashed curves correspond to a one channel calculation, the solid curves to a two channel calculation, and the dotted curves to a four channel calculation. Large channel coupling effects are evident in both the cross section and vector analysing power. In the two channels calculation $\sigma/\sigma_R$ is not reproduced well, in
particular the phase of the oscillations is opposite to the data, suggesting that the potential depths require adjustment. A multiplicative factor of 0.5 applied to all interactions present in the calculations was found to give a reasonable fit to the cross section data.

The results of calculations with this renormalisation factor included are shown in figure 3.3. The dashed curves again refer to a one channel calculation, the solid to a two channel calculation and the dotted to a four channel calculation. The renormalised two channels calculation increases the magnitude of the predicted $iT_{\|}$ and gives a much better description with the data, in comparison to the renormalised one channel calculation. The agreement with $\sigma/\sigma_R$ is also improved. The four channels calculation reduces the magnitude of $iT_{\|}$ a little from the two channel result, and improves the agreement with both $\sigma/\sigma_R$ and $iT_{\|}$ at angles larger than 60°, this same renormalisation factor of 0.5 was also found to be necessary in the case of $^6$Li + $^{58}$Ni elastic scattering [Ni 84] when a precise fit to the cross section was demanded. There is evidence that a normalisation of this type is associated with the effect of $^6$Li break up [Gl 80, St 80a, Sa 80, St. 80b, Co 81] calculations with the normalisation factor 0.5 were also performed for $^6$Li + $^{28}$Si scattering. The results are very similar to the calculations shown in figure 3.1, and are not shown here.

These results for $^6$Li scattering are therefore consistent with the general requirement of renormalisation for the folding model potential for $^6$Li scattering [Sa 79]. This renormalisation correction has recently been confirmed as arising from the effects of $^6$Li break up [Th 81, Sa 85].
Figure 3.1 Results for the elastic scattering of $^6\text{Li}$ from $^{28}\text{Si}$. 

$^{28}\text{Si}(^6\text{Li}, ^6\text{Li})^{28}\text{Si}, E_{^6\text{Li}} = 22.8 \text{ MeV}$
Figure 3.2 Results for elastic scattering of $^6$Li from $^{16}$O.
Figure 3.3 Results for elastic scattering of \(^6\text{Li}\) from \(^{16}\text{O}\) with a renormalisation factor of 0.5 included.
3.1.2 $^7\text{Li} + ^{120}\text{Sn}$ elastic and inelastic scattering

Figure 3.4 shows the predicted cross sections and vector analysing powers for the elastic scattering of $^7\text{Li}$ from $^{120}\text{Sn}$ at 44 MeV. Figure 3.5 shows the second rank tensor analysing powers, $T_{20}$, $T_{21}$, $T_{22}$, for the same elastic scattering. In both figures the dashed curves are the results for a one channel calculation, the solid curves correspond to a two channel calculation, and the dotted curves to a four channels calculation. The dash dotted curve for the vector analysing power in figure 3.4 is the result of a one channel calculation in which the tensor force has been neglected.

It is clear that, just as in the $^6\text{Li}$ cases studied in section 3.1.1, the inclusion of the projectile excited channels produces strong effects on the spin dependent observables. The inclusion of the second of the bound $1 = 1$ states clearly has a large effect on the spin dependent observables. The strongest effect being produced for $iT_{11}$. The inclusion of the two $1' = 3$ states reduces the magnitude of the predicted vector analysing power, but doesn't appreciably change the results for any of the other observables. From figure 3.4 it is clear that within a one channel calculation $iT_{11}$ is dominated by the effects of the tensor potential rather than the spin orbit potential. This is in keeping with the finding of Nishioka et al. [Ni 84] for $^7\text{Li}$ induced scattering from $^{58}\text{Ni}$.

Figure 3.6 shows the results for the inelastic scattering

$^{120}\text{Sn}(^7\text{Li},^7\text{Li})^{120}\text{Sn}$, with the outgoing $^7\text{Li}$ being in the $1/2^-$, $E_{c.m} = 0.48$
MeV state, induced by the $^7$Li incident at 44 MeV. The curves correspond to a two channel calculation. The prediction for the cross section and vector analysing power are in reasonable agreement with the experimental data, although the magnitude of the predicted cross section is a little small in comparison to the data. The inclusion of the two $1' = 3$ states, which is not shown here, was found to have a negligible effect on the inelastic observables.

The results for a two channel calculation using a renormalisation factor of 0.5 are shown in figures 3.14 and 3.15. In agreement with the general finding for the elastic scattering of lithium isotopes the agreement with the experimental cross section is improved.
Figure 3.4 Results for the cross section and vector analysing power for $^7\text{Li}$ scattering from $^{120}\text{Sn}$. 

$^{120}\text{Sn}(^7\text{Li},^7\text{Li})^{120}\text{Sn}, E_{^7\text{Li}} = 4.4 \text{ MeV}$
Figure 3.5 Results for the second rank observables for the elastic scattering of $^7$Li from $^{120}$Sn.
Figure 3.6 Observables for the inelastic scattering of $^7$Li from $^{120}$Sn.
3.1.3 $^3$H + $^{28}$Si elastic scattering

Figure 3.7 shows the results of calculations for the elastic scattering of $^3$H from $^{28}$Si at 60 MeV. The dashed curves are the results of a one channel calculation, the solid curves a two channels calculation and the dotted curves a three channels calculation. The inclusion of projectile excitation again clearly produces a strong effect on the $i\tau_n$. These results are in agreement with those obtained independently [Oh 84]. The largest effects are produced by coupling to the first excited state only, the inclusion of the second excited state reducing the magnitude of the predicted $i\tau_n$. This is consistent with the $^6$Li calculations.
Figure 3.7 Results for the elastic scattering of $^{19}\text{F}$ from $^{28}\text{Si}$.
3.2 Analysis of results

From the figures it is clear that coupling to the inelastic projectile excited states produces large effects, by back coupling, to the elastic channel. These effects which involve excitation and subsequent de-excitation are highly non local in nature. The non locality arises from the fact that de-excitation can occur at different values of the relative coordinates $\vec{R}$. These effects are incorporated automatically within the framework of the coupled channels calculations. The complexity of these calculations, however, obscures some of the underlying physics.

Two approaches will be considered to explain the observed results of the coupled channels calculations. Section 3.3 will present arguments based on semi-classical ideas to explain the observed effects of coupling to the projectile inelastic states. Section 3.4 will offer an explanation of these effects via a quantal description similar to that of the Feshbach formulation for treating inelastic events [Fe 62].

Section 3.5 is concerned with a quantitative description of the elastic observables in terms of a nearside-farside decomposition, with particular emphasis being placed on the vector and tensor analysing powers.
3.3 Semi-classical arguments

In order to be definite we shall consider the $^{17}$F + $^{28}$Si system. Due to the higher energy of this system, the large masses of the reactants and the large Sommerfeld parameter $\Lambda$, this is the most classical of those studied in this work. These ideas can be applied to the $^{7}$Li cases but increasing caution is required with the decrease in target mass and charge, and relative kinetic energy.

In order to clarify the arguments it is convenient to consider the potentials acting to be purely central. With the neglect of any spin dependent forces between the clusters and target the projectile spin $I$ and its $z$ component $M_z$ remain constant within the elastic channel. Within the cluster model used to describe the $^{17}$F projectile this implies that the triton's spin components $s, m_s$ are also conserved under the effect of the elastic channel forces. The potentials that couple different channels, and those that act within the excited channels are independent of the cluster spins. Hence any produced spin dependence arises from the central terms between the clusters and the target.

Treating the angular momentum vectors in a classical manner, the 1st excited state of $^{17}$F, with $l' = 2$ and $I' = 5/2^+$, has $l'$ and $s$ parallel. For a spinless target the total, conserved, angular momentum $J$ is formed as a sum of $L$, the relative orbital angular momentum between the colliding nuclei, and $I$, or $s$. Conservation of $J$ and $s$ implies that in the intermediate excited channel the relative angular momentum between the colliding nuclei, $L'$ is $L - 2$ if $J = L + I$ and $L + 2$ if $J = L - I$. Hence imposing the conservation of angular momentum there is a direct correspondence between $J$ and $L'$. This situation is shown pictorially in figure 3.8.
Figure 3.8 Classical vector addition for coupling to the first excited state.
The elastic scattering is dominated by a few partial waves near the grazing angular momentum, the lower partial waves being almost completely absorbed whilst the larger partial waves are prevented from contributing by the large centrifugal barrier. The centrifugal barrier increases as $L(L+1)$ with increasing $L$. Coupling to the inelastic channel with $L' = L - 2$ will thus be more effective than with $L' = L + 2$ because of the centrifugal barrier term acting in the excited channel. Hence, because of the kinematical link between $J$ and $L'$, coupling to the inelastic channel strongly depends on whether $J = L + I$ or $J = L - I$.

The subsequent return to the elastic channel will thus be fed more by those parts of the reaction which have gone via the $L' = L - 2$ path. The effect of inelastic excitation thus depends strongly on $J$ for a given $L$, that is it is spin dependent. For the $^6$F + $^8$Si case the spin dependence is limited to an $L^I$ term. The effect however is non local and highly $L$ dependent. Coupling to the first excited state of $^6$Li produces a similar set of arguments, but in general the effective spin dependence yields $L^I$ and second rank tensor forces. Similarly coupling to the $^7$Li excited states produces vector and second and third rank tensor forces. The $^7$Li results clearly show that the induced second rank forces are significantly smaller than the vector forces.

For a given intermediate channel, $I'$, the spin dependence arises because different intermediate angular momenta are coupled to the elastic channel with different strengths. These arguments suggest that spin dependence should arise from the inelastic excitation. They cannot however say anything about the expected magnitude, or even sign, of the induced spin orbit effect. Although the absolute sign of the effective spin orbit
potential produced by coupling to a given intermediate channel $I'$ is not fixed by these arguments, we can compare the effects of coupling to other excited states $I''$ with those of coupling to $I'$.

The discussion will again be limited to the $^{19}$F + $^{28}$Si system, and only central forces will be assumed to act in the elastic channel, with coupling between the ground and excited state $I''$ via the quadrupole term in the multipole expansion of these central potentials.

The same semiclassical assumptions employed in the arguments for coupling to the first, $I' = 5/2$, state can be used for coupling to the second, $I'' = 3/2$, excited state, as shown in figure 3.9. In this case, since the second excited state has $I'' = 3/2$, $I'$ must be anti parallel to $s$. Then, by the same set of arguments as before, the coupling is most effective for the $J = L - I$ route, and least for the $J = L + I$ path. Hence the net effective interaction is opposite to that for coupling to the first excited state.

Although the absolute sign of coupling to either of the excited states still cannot be fixed, these arguments show that coupling to the two $I' = 2$ excited states produces a spin dependence of opposite sign. This is clearly borne out by the coupled channels calculations. A ramification of this is that both members of a doublet with given $I'$ must be included. Calculations which do not include both members may lead to spuriously large predicted analysing powers. The same arguments may be applied to the cases studied with $^6$Li as a projectile.
Figure 3.9  Classical vector addition for coupling to the second excited state.
3.3.1 Degenerate Excited States

The further assumption is now made that the excited states which, within the cluster model, form a spin orbit split multiplet are degenerate in energy. We also assume that coupling to these states from the ground state is independent of $I'$. The effective spin orbit potential produced from the inclusion of the two states will then be equal and opposite, leading to no net effective spin orbit forces.

Figure 3.10 shows the result of a four channel calculation for $^6\text{Li} + ^{30}\text{Si}$. The calculation was performed with no spin orbit potential acting in the entrance channel, and with the energy of the three excited states being taken as 2.18 MeV. This figure clearly shows cancellation between the induced spin orbit effects caused by coupling to each of the excited states considered.

In practice the states are not degenerate, and there also exists the possibility of coupling to the elastic channel via multi step processes that pass through each of the excited states. All of these effects are automatically incorporated within the coupled channels calculations. The degenerate calculation results suggest that the induced spin dependent effects are largely dominated by the two step processes. The presence of multistep paths are responsible for the non zero $iT^\alpha_n$ in this figure.

In essence the vector spin dependence in the elastic channel is produced, within the cluster model, by the spin orbit splitting of the
excited projectile states. It is important to note that the cancellation occurs because the ground states of the $^9F$ and $^6Li$ nuclei are taken to be purely $l = 0$.

3.3.2 Effect of non-zero angular momentum between clusters

Similar arguments may be applied to the $^7Li + ^{120}Sn$ system. Coupling to the first excited state however produces an effective spin orbit potential which is opposite in sign to that produced by coupling to the first excited state of $^6Li$. This is because the triton spin is aligned with the inter cluster angular momentum in the ground state, and anti aligned in the first excited state. The summation over a multiplet of states with the same $l'$ no longer predicts a vanishing effective spin orbit potential.
Figure 3.10 Four channel calculation with the three excited states assumed degenerate in energy.
3.4 Quantum mechanical arguments

The spin dependence arising from coupling to the inelastic projectile states can be described by quantum mechanical arguments that complement the semi-classical ones of section 3.3. To be definite we shall again consider the $^9\text{F} + ^{28}\text{Si}$ system. The arguments used parallel those of the Feshbach [Fe 62] formulation using projection operator techniques.

If we initially consider coupling only to the first $^9\text{F}$ excited state we obtain the pair of coupled equations, in symbolic notation

$$\begin{align}
(E - T_{1} - V_{ii}(R)) \chi_{1}(\hat{R}, \hat{R}) &= V_{ij}(R) \chi_{2}(\hat{R}', \hat{R}') \quad 3.1a \\
((E - E_{2}) - T_{2} - V_{22}(R')) \chi_{2}(\hat{R}', \hat{R}') &= V_{21}(R) \chi_{1}(\hat{R}, \hat{R}) \quad 3.1b
\end{align}$$

where the $T_{i}$ are the 3 dimensional kinetic energy operators for the $i$th channel and $V_{ii}(R)$ is the diagonal optical potential, given for example by the folding model of section 2.3. The $V_{ij}(R)$ are the coupling potentials which have been treated in radial form in section 2.3.3.

Formally these equations can be reduced to an uncoupled equation for the elastic channel by transferring the operator on the left hand side of equation 3.1(b) to the right hand side

$$\chi_{2}(\hat{R}', \hat{R}') = \left[\left((E - E_{2}) - T_{2} - V_{22}(R') + i\epsilon\right) V_{21}(R')\right]^{-1} \chi_{1}(\hat{R}, \hat{R})$$

The additional term $+i\epsilon$ has been added to avoid the singularity at $H_{2} = E_{2}$, the + sign being chosen to give outgoing wave solutions for the
\( \chi_2(\hat{R}, \hat{R}'). \) Substituting this expression for \( \chi_2(\hat{R}, \hat{R}'). \) into 3-1(a) yields

the uncoupled equation for the elastic channel

\[
( E - T_1 - V_{\|}(R)) \chi_1(\hat{R}, \hat{R}) = V_{l2}(R) G_{l2}(\hat{R}, \hat{R}') V_{21}(R') \chi_1(\hat{R}, \hat{R}')
\]

3-3

This equation represents a formal solution of the coupled equations, but is not amenable for direct solution. The presence of the Green's function propagator

\[
G_{l2}(\hat{R}, \hat{R}') = [(E - E_a) - T_2 - V_{\perp}(R') + \mu \varepsilon]^{-1}
\]

3-4

makes this equation non local and energy dependent. The right hand side of equation 3-3 acts as a source term, adding an inhomogeneous term to the elastic equation.

\[
( E - T_1 - V_{\|}(R)) \chi_1(\hat{R}, \hat{R}) = V_{\text{eff}}(\hat{R}, \hat{R}') \chi_1(\hat{R}, \hat{R}')
\]

3-5

where

\[
V_{\text{eff}}(\hat{R}, \hat{R}') = V_{l2}(R) G_{e2}(\hat{R}, \hat{R}') V_{21}(R')
\]

3-6

3.4.1 Multipole expansion

The total solution of this equation can be expanded in partial wave terms as before, and is projected on to a relative state with definite \( J, \)

\( L'' \) to yield

\[
\begin{align*}
&\left[\begin{array}{c}
E - T_2 - V_{\text{eff}}(R) - <L'' I M_J | J_l(L'' I) M_J > \chi_J^{L'' I J L I} (K, R) \\
\end{array}\right]
\end{align*}
\]

3-7

The matrix of the effective potential \( V_{\text{eff}}(\hat{R}, \hat{R}') \) can be written, since the Greens function is invariant under rotations in total \( J, \) as
where the $V_{L',J; L'' I}^{R}$ are given by equation 2-82.

If the potential $V_{11} (R')$ of equation 3-1 is considered to be purely central, the Green's function $G_{L', I; L'' I}^{J} (R, R')$ is independent of $J$. The $k$th multipole component of $V_{L' I; L'' I}^{R} (R')$ is usefully rewritten as

$$V_{L' I; L'' I}^{R} (R') = (-)^{L' + I' - J} \frac{L - L'}{L} \hat{R} \hat{L} (L)(L', k)$$

where the $V_{I' I'' I}^{R} (R')$ are independent of the relative orbital angular momentum between the colliding nuclei, and are given by

$$V_{I' I'' I}^{R} (R') = (-)^{I' + s' - s'' - k} \frac{I' - I''}{I'} \hat{R} \hat{L} (L)(L', k)$$

In general the the effective potential $V_{L'' I}^{\text{Eff}} (R, R')$ can be expressed in terms of the irreducible tensor operators introduced in section 1.2 as

$$V_{L'' I}^{\text{Eff}} (R, R') = \sum_{k' L} C_{k'}^{+} (I) V_{k L I}^{\text{Eff}} (R, R')$$

Hence for reactions induced by $1^q F$ the effective potential is limited to central and vector potentials only. Using this expansion the matrix of
the effective potential can be written [Jo 77]

\[ \langle J(L''I') \| V_{\text{eff}}(\hat{R},\hat{R}') \| J(LI) \rangle = \sum_{k_I} \hat{\mathcal{R}}^{2} (-)^{I+L''-J} \hat{\mathcal{R}}_{I} W(L''II; k_I J) \]

\( (i) \hat{L}'' < L'' \| V_{\text{eff}} \| L > \langle J(L''I') \| \mathcal{T}_{RI} \| J(LI) \rangle \)

3-13

This equation can be inverted by using the completeness relation for the Racah coefficients, and noting that

\[ \langle J(L''I') \| V_{\text{eff}}(\hat{R},\hat{R}') \| J(LI) \rangle = \hat{\mathcal{R}}_{I} \]

3-14

to give

\[ \langle L'' \| V_{\text{eff}} \| L \rangle = \sum_{L'} (i) \hat{L}_{I} (-)^{I-L+R_{I}} \hat{\mathcal{R}}_{I} \frac{\hat{\mathcal{L}}_{I}^{2}}{L} \left( L''O L' 0 | k 0 \right) \]

\[ \left( L0L'0|k0\right) W(k_{I}RLL'; kL'') W(k_{I}R II'; kI) V_{II'}^{k}(R) G_{II'}(R, R') V_{II'}^{k}(R') \]

3-15

A general formula for the non local effective potential can be obtained by using equations 3-15, 3-13 and 3-10 and by performing the sum over \( J \), to yield

\[ \langle L'' \| V_{\text{eff}} \| L \rangle = \sum_{L'} (i) \hat{L}_{I} (-)^{I-L+R_{I}} \hat{\mathcal{R}}_{I} \frac{\hat{\mathcal{L}}_{I}^{2}}{L} \left( L''O L' 0 | k 0 \right) \]

3-16

for the \( k_{I} \)th rank potential arising from coupling to an inelastic channel \( I' \) via a rank \( k \) multipoles.

In the semiclassical arguments of the last section it was seen that the spin orbit effect arose from the differences in which the orbital angular momentum of \( L'' \) the intermediate channel are coupled. If the \( L' \) dependence of \( G_{II'}(R, R') \) is ignored, the sum in equation 3-16 can be performed using

\[ \sum_{L'} \hat{\mathcal{L}}_{I}^{2} \left( L''O L' 0 | k 0 \right) \left( L0L'0|k0\right) W(k_{I}RLL'; kL'') W(k_{I}R II'; kI) V_{II'}^{k}(R) G_{II'}(R, R') V_{II'}^{k}(R') \]

3-17
The second parity selecting Clebsch Gordan coefficient in this equation limits $k_\perp$ to even terms only, so that the $k_\perp = 1$ vector potential vanishes if the $L'$ dependence is neglected.

We can identify the non local, $L$ dependent, effective spin orbit potential arising from coupling to the inelastic channel $I'$ as

$$V_{So}^{\text{eff}}(R, R', L) = \hat{R}_{L} / \left[ L(L+1) I(I+1) \right]^{1/2} L || V_{L}^{\text{eff}} || L >$$ \hspace{1cm} 3-18

Hence for $k = 2$ quadrupole excitation the effective spin orbit potential is

$$V_{So}^{\text{eff}}(R, R', L) = \frac{1}{2} \sqrt{\frac{3}{10}} \frac{6 + I(I+1) - I'(I'+1)}{I(I+1) I'} \frac{1}{I'(I'+1)} \frac{1}{L} \sum_{L'} (-)^{I-I'} \left[ L(L+1) \right]^{1/2} \left( L_{0} L' 0 20 \right) W(12 L L'; 2L) V_{Li}^{(R)}(R) G_{L'L'}^{(R,R')} \phi_{L}^{(R')}$$ \hspace{1cm} 3-19

Using equation 3-10 this potential can be expressed as

$$V_{So}^{(R, R', L)} = \frac{1}{2} \sqrt{\frac{3}{10}} \frac{6 + I(I+1) - I'(I'+1)}{I(I+1) I'} \frac{1}{I'(I'+1)} \frac{1}{L} \sum_{L'} (-)^{2s} \sum_{L'} \left( L_{0} L' 0 20 \right)^{2} W(12 L L'; 2L) W(12 L L'; 2L) \phi_{L}^{(R)} G_{L'L'}^{(R,R')} \phi_{L}^{(R') \text{c}}$$ \hspace{1cm} 3-20

where

$$w_{c}^{(R)}(R) = \int_{0}^{\infty} r^{2} dr \phi_{c}(r) u^{(R)}(R, r) \phi_{L}(r)$$ \hspace{1cm} 3-21

and spin dependent forces in the entrance channel have been neglected.

The sign of the effective potential $V_{So}^{\text{eff}}(R, R', L)$ is thus determined by the factor $6 + I (I + 1) - I' (I'+1)$. Hence, considering $^{19} F_{1}$ coupling to the first excited state with $I' = 5/2$ will produce an opposite effect to that of coupling to the second excited state with $I' = 3/2$. 
3.4.2 Inclusion of other excited states

Coupling to more than one excited state can be considered within this framework. If the coupling between the excited state multiplets is ignored the effective potential is simply a sum of terms of the form 3-6, one for each of the excited states included.

\[
V^\text{Eff}(\mathbf{R}, R') = \sum_{I'} V_{I', I}(R) G_{I, I'}(\mathbf{R}, R') V_{I'}(R') \tag{3-22}
\]

Expressing this in partial wave form using the techniques of the last section the effective potential from coupling to the excited states is

\[
\langle L''\|V\|L\rangle = \sum_{L'} (-)^{k_I} \langle L''-L'|L\rangle \frac{\hat{L}^2}{\hat{L}^2} \langle L'0|L\rangle \langle L0|L'\rangle \sum_{I'} (-)^{I-I'} W(k_I' k I I' ; k I ) V_{I, I'}(R) G_{I', I'}(R, R') V_{I'}(R') \tag{3-23}
\]

If the excited states are considered to be degenerate in energy, the radial Green's function propagator is the same for all excited states \( I' \), but still depends on \( L' \). The radial Green's function may then be taken outside the sum over \( I' \). By using the explicit forms of the \( V_{I, I'}(R) \) from equation 3-11, the sum over \( I' \), which runs over the multiplet of states with \( l' = 2 \), can be performed analytically, to yield

\[
(-)^{I+L'+S+\frac{1}{2}} W(l' l k R_I j k R) W(l l II ; R_I S) \tag{3-24}
\]

The triangular relations present in the second Racah coefficient in this equation limits \( k_I \) to zero for projectiles with \( l = 0 \) in the ground state. This is the result previously obtained from the semi-classical arguments of section 3.3. The effective spin orbit potential vanishes if
the excited state multiplet is degenerate in energy. The effective spin orbit potential produced by projectile excitation arises, within the cluster model, from the presence of spin orbit and tensor forces between the clusters of the projectile.

3.5 Nearside/Farside Interpretation

Since the de Broglie wavelengths for the elastic scattering of heavy ions are short in comparison to their sizes, for example $1.1 \text{ fm}$ for the $^{14}\text{F} + ^{28}\text{Si}$ system, classical ideas assume an important role. Classically the scattering can be described by considering the nuclei to travel along well defined orbits [Po 59]. Heavy ion scattering is characterised by the presence of strong absorption, so that orbits in which the nuclei overlap considerably are almost completely prevented from contributing to the observables. It is the purpose of this section to use these ideas to interpret some of the observed results of section 3.1.

In order to see how these ideas may be used it is instructive to first consider a simplified system. Initially we assume the interaction between the colliding nuclei contains only central terms. With this restriction the projectile spin is constant, and equation 2-30 for the nuclear scattering amplitude reduces to

$$f_{M,M_X}^{m,M_X}(\theta) = \frac{1}{2i\hbar} \sum_L \hat{L}^1 e^{2i\theta L} (S_L - i) P_L(\cos \theta)$$

In the extreme limit of complete absorption of the lower partial waves the $S_L$ coefficients are zero for all $L$ less than a critical orbital angular momentum $L_c$, that is there are no outgoing wave components in equation 2-28 for $L$ less than $L_c$. Partial waves greater than $L_c$ correspond to
orbits in which the nuclei pass each other without any interaction occurring between them. The scattering process then arises only from grazing orbits with $L = L_C$. Quantum mechanically there is some spreading of this over simplistic view, however the scattering processes are dominated by a narrow region of angular momentum values centered around a critical value $L_C$. Since the value for the grazing angular momentum is large, for example 27 for the $^{19}$F + $^{28}$Si system, the Legendre polynomial in equation 3-25 may reasonably be replaced by its asymptotic value [Ab 70].

$$P_L(\cos \theta) \xrightarrow{L \to \infty} \left[ \frac{1}{2\pi L \sin \theta} \right]^{1/2} \left\{ \exp \left[ i \left( \left( L + \frac{1}{2} \right) \theta - \frac{\pi}{4} \right) \right] \right\}$$

A formula for the semiclassical scattering amplitude is then obtained by replacing $L$ by a continuous variable $l = L + 1/2$ and assuming that $S_L \to S(l)$, where $S(l)$ is a smooth function of $l$. The summation over the discrete variable $L$ is then replaced by an integration over $l$, leading to the following semiclassical expression for the scattering amplitude:

$$f(\theta) = \frac{1}{2i k (2\pi \sin \theta)^{1/2}} \int_0^{\infty} l^{1/2} dl e^{2iS(l)} \left\{ \exp \left[ i \left( l \theta - \frac{\pi}{4} \right) \right] + \exp \left[ -i \left( l \theta - \frac{\pi}{4} \right) \right] \right\}$$

For simplicity, the Coulomb phase has been dropped from this expression, although it may be incorporated without producing any fundamental differences. The $-1$ component has also been dropped since it contributes only to forward scattering, in which region the arguments of this section no longer hold.
Regions in which the phase of the integrand oscillate vigorously lead to cancellation. By the arguments of stationary phase the integral is dominated by regions in which the arguments of the exponentials are smooth and slowly varying functions of $l$. Equating the derivatives of the exponentials to zero gives the classical deflection function equations

$$2 \frac{d\delta(l)}{dl} = \pm \Theta$$

There are thus two components which contribute mainly to the semi-classical scattering amplitude, for a given observation angle $\Theta$. These may be identified with the two classical paths as shown in figure 3.11. They are denoted as the nearside and farside components.

Heavy ion elastic scattering in the presence of strong absorption is in principle dominated by the grazing angular momentum. In practice, however, the low partial waves are not completely absorbed and contribute to the scattering process. We may be guided by the limit in which the observed elastic cross section arises from one partial wave, or a very narrow band of partial waves, and use the asymptotic form of the Legendre polynomials. Quantities which retain the interpretation of nearside and farside components given to the two parts of the asymptotic form can be defined for all angular momentum. The Coulomb amplitude of equation 2-34 can also be written in terms of nearside and farside components.

The method for a complete description in terms of nearside and farside components within a fully quantal calculation was given by Fuller [Fu 73, Fu 74, Fu 75a, Fu 75b] who considered both the Coulomb and nuclear amplitudes. Although originally defined only for the spinless amplitude of
equation 3-25 the formalism may be generalised to deal with the full amplitudes of equation 2-33. We shall first consider the decomposition into travelling wave components of the nuclear scattering amplitude in section 3.5.1 whilst the Coulomb amplitude will be considered in section 3.5.2.

3.5.1 Legendre polynomial decomposition

It was shown by Fuller that the correct decomposition of the Legendre polynomials with the asymptotic form of equation 3-26 can be expressed as a linear combination of solutions of the associated Legendre's equation.

\[ Q_{LM}^{(\pm)}(\cos \Theta) = \frac{1}{2} \left[ P_{LM}(\cos \Theta) \mp \frac{2i}{\pi} Q_{LM}(\cos \Theta) \right] \quad 3-29 \]

Where the \( Q_{LM}(\cos(\Theta)) \) are the irregular solutions of the associated Legendre's equation, and the \( P_{LM}(\cos(\Theta)) \) are the regular solutions [Ab 70]. The \( Q_{LM}^{(\pm)}(\cos(\Theta)) \) can then be identified with the farside, and the \( Q_{LM}^{(+)}(\cos(\Theta)) \) with the near. Asymptotically these \( Q_{LM}^{(\pm)}(\cos(\Theta)) \) behave as

\[ Q_{LM}^{(\pm)}(\cos \Theta) \rightarrow \frac{(L)^{M}}{(2\pi \sin \Theta/2)^{1/2}} \exp \pm i \left\{ \left( L + \frac{1}{2} \right) \Theta \mp \frac{\pi}{4} - \frac{M \pi}{2} \right\} \quad 3-30 \]

which, for \( m = 0 \) is equation 3-26. The \( Q_{LM}^{(+)}(\cos(\Theta)) \) posses the important symmetry property

\[ Q_{LM}^{(-)}(-\cos(\Theta)) = Q_{LM}^{(+)}(\cos \Theta) \quad 3-31 \]

which ensures their analytic continuation around \( \Theta \) and 180 degrees.

The scattering amplitude of equation 2-33 can then be expanded in terms
of its nearside and farside components as

\[
\mathcal{F}_{N/F}(\theta) = \frac{i}{2\pi} \sum_{L'M'_{L}J_{M'}} (L'M'I'M_{I}\mid J_{M_{I}}) (LOIM_{I}\mid J_{M_{I}}) e^{i(\omega_{L} + \omega_{L'})} (-)^{M'_{L}} \left[ \frac{(L'-M'_{L})!}{(L'+M'_{L})!} \right]^{\frac{1}{2}} (S^{J_{F}-1})_{L'L}^{Q^{-1}_{L}L'M'_{L}} (\cos \theta)
\]

\[3-32\]

### 3.5.2 Coulomb decomposition

It might naively be expected that the repulsive nature of the Coulomb potential would lead only to a positive angle deflection, so that the Coulomb amplitude would be unchanged from that of equation 2-34 for the nearside, and absent for the farside. Indeed near \( \theta = 0 \), a singularity in the Rutherford scattering amplitude of equation 2-34, the nearside component of the Rutherford amplitude is virtually that of the full amplitude, whilst the farside component is extremely small, diminishing with increasing \( \eta \). At larger angles however the nearside component is reduced, and the farside component becomes comparable with it.

By insisting that the nearside and farside components of the Rutherford amplitude are equal at 180 degrees, and that they should have the same analytic form as the \( Q_{LM}(\theta) \), Fuller was able to decompose the Rutherford amplitude. The two branches can be expressed as

\[
\frac{f_{N}(\theta)}{f_{c}(\theta)} = (1 - e^{-\frac{3\pi\eta}{4}})^{-1} - \frac{i}{2\pi} \left[ \sin^{2} \left( \frac{\theta}{2} \right) \right]^{1+\eta} S(\theta)
\]

\[3-33\]

and

\[
\frac{f_{F}(\theta)}{f_{c}(\theta)} = (1 - e^{-\frac{3\pi\eta}{4}})^{-1} (-e^{-\frac{3\pi\eta}{4}}) + \frac{i}{2\pi} \left[ \sin^{2} \left( \frac{\theta}{2} \right) \right]^{1+\eta} S(\theta)
\]

\[3-34\]
where $S(\Theta)$ is given by

$$S(\Theta) = (1+i\eta)^{-1} F\left(\frac{1}{2}, 1+i\eta, 2+i\eta; \sin^2 \frac{\Theta}{2}\right)$$

$F(a,b,c;d)$ is the hypergeometric function [Ab 70] for complex argument, $f_c(\Theta)$ is the Rutherford amplitude given in equation 2-34, the other quantities being defined there.
Figure 3.11 Schematic representation of the classical nearside and farside contributions.
2.5.3 Decomposition of the scattering Amplitude

Sections 3.5.1 and 3.5.2 thus define the nearside and farside components of the nuclear and Coulomb scattering amplitudes of equation 2-33. The complete amplitude is the sum of these components, viz

\[ f_{M^\prime M} (\theta) = f_{M^\prime M}^F (\theta) + f_{M^\prime M}^N (\theta) \] 3-36

It must be emphasised that the nearside / farside decomposition is only interpretational; it does not correspond to physical observables. The decomposition however does give useful insights into some details of nuclear reactions, and does allow one to make predictions based on physical arguments.

The implications of the decomposition for the spin dependent observables can be seen most clearly in terms of the Booton Johnson [Bo 71a] decomposition of the scattering amplitude, which displays the underlying tensorial structure of the scattering amplitude \( f_{M^\prime M} (\theta) \). In general the square matrix \( f_{M^\prime M} (\theta) \) can be expanded in terms of the irreducible tensor operators \( \tau_{kq} \) introduced in section 1.2. From the expansion used in section 1.2, the matrix of scattering amplitudes can be written as

\[ f_{M^\prime M} (\theta) = \sum_{kq} \hat{k} (\text{IM}^\prime kq | \text{IM}^\prime) f_{kq} (\theta) \] 3-37

where \( k \) times the Clebsch Gordan coefficient is the value of \( \tau_{kq} \) evaluated in the \( | \text{IM}^\prime > \) basis.
Explicit formulae for the $f_{kq'}(\theta)$ can be obtained by reordering the coupling of equation 2-30 to give, within the Madison convention coordinate system,

$$f_{kq'}(\theta) = \frac{1}{2i^{k}} \sum_{L L'} (L' q L 0 | k q) (-)^{L-q'} L^{2n+1} \left[ \frac{(L-q')!}{(L+q')!} \right]^{n} S_{LL'}^{k} P_{L}^{q'}(\cos \theta)$$

Here the Kronecker delta $\delta_{q'ML'}$ arising from the Clebsch Gordan coefficient has been used implicitly to forge a link between the index of the associated Legendre polynomial and the index of the irreducible scattering amplitude. The $S_{LL'}^{k}$ are given by

$$S_{LL'}^{k} = \sum_{J} \frac{2J+1}{L+L'} (-)^{J-L'-1} W(LL' II; kJ) \left[ e^{i(\sigma_L+\sigma_L')} \right] [S_{LL'}^{J} - 1]$$

Within the Madison coordinate system the $f_{kq'}(\theta)$ satisfy the same symmetry relations as the $T_{kq'}(\theta)$ of section 1.2, in particular $f_{10}(\theta) = 0$ and $f_{11}(\theta) = -f_{-11}(\theta)$.

Since the spin dependent forces are much weaker than the central terms we may appeal to the arguments of perturbation theory to treat these forces. From first order perturbation theory there is a direct correspondence between the rank k of the $f_{kq'}(\theta)$ and the rank of the potentials present in the interaction between the colliding nuclei \[ Jo 70 \].

The interaction of a spin one projectile with a spinless target permits second rank tensor forces, which have been ignored in the calculations presented here, and second rank effects arising from terms quadratic in the spin orbit force. These quadratic terms are negligibly small in comparison to those that are linear in the spin orbit force.
Ignoring any tensor forces or terms quadratic in the spin orbit force, the vector analyzing power can be written as

$$i T_{\|} (\theta) = -2 \mathbb{I}_M \left( f_{\theta \theta} (\theta) f_{\theta \theta}^* (\theta) \right) / \sigma_0 (\theta) \quad 3-40$$

where $\sigma_0 (\theta)$ is given by

$$\sigma_0 (\theta) = \sum_{\ell \eta} | T_{\ell \eta} (\theta) |^2 \quad 3-41$$

This expression is, of course, exact for the spin $1/2$ $^1H$ F + $^{30}$Si system.

The results of the calculations presented here do not make this truncation and include the small second rank forces arising from channel coupling and the spin orbit potential acting quadratically. For the purpose of discussion, the arguments may be seen more clearly if these are neglected. Their inclusion produces no appreciable changes however.

The scattering of the spin $3/2$ $^7$Li projectiles allow vector, second and third rank tensor forces. Assuming again that terms acting quadratically can be ignored, the $T_{\ell \eta} (\theta)$ can be written as

$$T_{\ell \eta} (\theta) = 2 Re \left[ f_{\theta \theta} (\theta) f_{\eta \eta}^* (\theta) \right] / \sigma_0 \quad 3-42$$

In the nearside-farside decomposition, the $f_{\eta \eta} (\theta)$ have two components, arising from the replacement in equation 2-32 of the Associated Legendre polynomial by the $Q_{LM}^{(1)} (\theta)$, and the addition of $f_{\text{coul}}^{EF} (\theta)$ to the $k = q = 0$ term. The nearside-farside decomposition is most useful when one of the two central components $f_{\theta \theta} (\theta)$ is dominant. A major contribution to the $k = q = 0$ term comes from the decomposed Rutherford terms. The nearside branch is much larger than the far for small angles and large $\eta$ and $k$ values. The decomposition was performed by modifying the computer code.
3.5.4 Results

The results of performing this decomposition are shown in figures 3.12 to 3.16. Figure 3.12 shows the cross section and vector analysing power for a two channel $^6$Li + $^6$O calculation. The solid curve corresponds to the full amplitude, the dashed the nearside component and the dotted the farside. Figure 3.13 is the result of a two channel calculation for the elastic scattering of $^6$Li from $^{28}$Si. The solid cross section curve is again the full amplitude, the dashed the nearside component, and the dotted the far. The vector analysing power curves are discussed in more detail in section 3.5.5. Figures 3.14 and 3.15 show decomposed observables for $^7$Li + $^{120}$Sn scattering. The solid cross section curve is again the full amplitude, the dashed the nearside component, and the dotted the farside. The spin dependent observables are again discussed in the next section. The curves correspond to a two channels calculation, with a renormalisation factor of 0.5 applied to all of the interactions. Figure 3.16 shows the results of a two channels calculation for the elastic scattering of $^{18}$F from $^{28}$Si. The solid curve is again the full amplitude, the dashed the nearside component, and the dotted the farside.
Figure 3.12 Decomposed cross section and vector analysing power for the elastic scattering of $^6\text{Li}$ from $^{16}\text{O}$. 

$^{16}\text{O} (^6\text{Li},^6\text{Li})^{16}\text{O}, E_{^6\text{Li}} = 22.8 \text{ MeV}$
Figure 3.15 Decomposed cross section and vector analysing power for the elastic scattering of $^6\text{Li}$ from $^{28}\text{Si}$. 

$^{28}\text{Si}(^6\text{Li},^6\text{Li})^{28}\text{Si}, E_{^6\text{Li}} = 22.8 \text{ MeV}$
Figure 3.14. Decomposed cross section and vector analysing power for the elastic scattering of $^7\text{Li}$ from $^{120}\text{Sn}$. The calculations include a renormalisation factor of 0.5.
\( ^{120}\text{Sn} \rightarrow ^{7}\text{Li}, \rightarrow ^{7}\text{Li}\rightarrow ^{120}\text{Sn} \), \( E_{\text{Li}} = 44 \text{ MeV} \)

Figure 3.15 Decomposed second rank observables for the elastic scattering of \(^7\text{Li}\) from \(^{120}\text{Sn}\).
Figure 3.16 Decomposed observables for $^{19}$F.

Elastic scattering.
3.5.5 Discussion of results

Figures 3.13, 3.14, and 3.16 clearly show that for the heavier systems, $^6\text{Li} + ^{10}\text{Si}$, $^{18}\text{F} + ^{28}\text{Si}$ and $^7\text{Li} + ^{10}\text{Sn}$ the elastic cross sections, as ratios to Rutherford, are nearside dominated. Therefore we can reasonably replace $f_{oo}^\prime(\theta)$ by $f_{oo}^N(\theta)$ in equations 3-40 and 3-42. The nearside components of the scattering amplitude, both nuclear and Coulomb, asymptotically behave as $\exp(-iL\theta)$. Given this replacement, equation 3-40 for the vector analysing power can be written as

$$iT_{il}(\theta) = -2\text{Im}\left[f_{oo}^N(\theta)(f_{il}^{N*}(\theta) + f_{il}^{F*}(\theta))\right] / \sigma_o(\theta)$$  \hspace{1cm} (3-43)

The quantities plotted for the vector analysing power correspond to this equation, the "nearsidet" term being $f_{oo}^N(\theta)$ and the "farsidet" $f_{oo}^N(\theta)$ and the "farsidet" $f_{oo}^N(\theta)f_{il}^{F*}(\theta)$. Similarly equation 3.42 for the tensor analysing powers can be approximated by

$$T_{2q}(\theta) = 2\text{Re}\left[f_{oo}^N(\theta)(f_{2q}^{N*}(\theta) + f_{2q}^{F*}(\theta))\right] / \sigma_o(\theta)$$  \hspace{1cm} (3-44)

The difference in the appearance of the "nearsidet" and "farsidet" spin dependent observables can be easily understood. For simplicity we shall consider the classical case in which one $L$ value dominates the reaction. The $f_{ik}^{N/F}(\theta)$ contain the $L$ dependent phase factors of equation 3-30. In the term in which both of the $f_{ik}^{N/F}(\theta)$ are from the nearside, the phase factors cancel since one of them is complex conjugated in the definition of the vector and tensor analysing powers. In the cross terms involving the central nearside component and the spin dependent farsidet one, the phases add. The terms involving only nearside components thus have a smooth appearance, whilst the cross terms are oscillatory in nature, the period of
the oscillations being twice that of the contributing partial wave.

This decomposition is most useful for discussing the observed nature of spin dependent observables when one of the central $f'_{oo}(\theta)$ is dominant. This condition is easily satisfied by the cases studied with $^{28}$Si and $^{120}$Sn as targets. The comparable magnitude of the nearside and farside central components for the $^6$Li + $^6$O system, as indicated by the Fraunhofer type of cross section, render the consideration of its vector analysing power in these terms far less fruitful.

It might be thought that, since in the heavier ion cases $f'_{oo}(\theta)$ is much larger than $f_{oo}(\theta)$, the same ratio would apply to the vector and tensor $f_{kv}(\theta)$ terms. From the explicit calculations presented in figures 3-13 to 3-16 this is clearly not the case. The $f_{kv}(\theta)$ terms arise from the difference of amplitudes for different spin orientations, so that although the amplitudes may be very different in magnitude, their differences may be comparable. Also the spin orbit interaction acts mainly at a radial separation of the two nuclei at which the surfaces are just touching. From figure 3-11 the farside component has a trajectory, in a classical sense, in which this surface contact is maintained for a longer path than the near side component. The spin orbit potential is thus able to act for a greater part of the important section of the trajectory, and hence have a stronger effect.

The actual appearance of the vector analysing power and second and higher rank analysing powers depends subtly on the potentials, both central and spin dependent between the colliding nuclei. A general rule however, for a fixed spin orbit potential, may be discerned from the arguments
presented above. The farside $f^{F}_{00}(\theta)$ may be suppressed by the inclusion of a large amount of surface absorption. The appearance, or absence, of oscillations in experimentally observed analysing powers for a reaction in which the cross section is Fresnel in appearance can be used as a guide for optical model practitioners.
Chapter 4

4.1 Introduction to the transfer reactions

Due to experimental limitations it is not possible to obtain polarised beams of $^{19}$F projectiles. Hence the predicted analysing powers and the effects of the inclusion of projectile excitation for the $^{19}$F + $^{28}$Si system cannot be compared against data. It has however been suggested [Ku 77] that certain transfer reactions induced by $^{19}$F on $^{28}$Si require large spin orbit forces in the entrance channel in order to reproduce the experimental data. The particular reactions considered are the transfer of three nucleons from the incident $^{19}$F projectile to the $^{28}$Si target, $^{28}$Si($^{19}$F,$^{6}$O) $^{31}$P, leading to the residual $^{31}$P nucleus being in the $1/2^+$ ground state, $3/2^+$ $E_{x1} = 1.27$ MeV first excited state, and the $5/2^+$ $E_{x2} = 2.23$ MeV second excited state.

Since the mechanism of projectile excitation was shown in chapter 3 to generate an effective spin orbit potential between $^{19}$F and $^{28}$Si it is interesting to see whether this mechanism offers a means of explaining the experimental data.

The present chapter will be concerned with the development of the formalism and models required to describe the transfer reaction, in which an inert triton is transferred from the incident $^{19}$F projectile to the target in a single step. The reaction will be analysed using the Distorted Waves Born Approximation, or DWBA. Inclusion of the inelastic excited channels in the entrance partition will be deferred until chapter 5.
The transfer reaction will be described using the extreme cluster model previously employed, representing \(^{10}\text{F}\) as a triton bound to an \(^{6}\text{O}\) core, and the residual \(^{31}\text{P}\) nucleus as a triton bound to the target \(^{28}\text{Si}\), as was discussed in section 1.2. In order to maintain consistency with the preceding work, and to avoid the ambiguities inherent in optical potentials, the folding model will be used throughout the following analysis to generate the interaction potentials acting in the incident and outgoing channels.
Figure 4.1. Coordinate system used to describe the transfer reaction $A(a,b)B$.
4.2 Distorted Waves Born Approximation

Although lacking a rigorous theoretical basis, the DWBA offers a widely and successfully used prescription for describing direct nuclear reactions \([ \text{To 61, Au 64, Sa 64, Au 70} ] \). In analogy with equation 2-16 an ansatz form for the wave function describing the system can be written as

\[
\Psi(x_A, x_B, \vec{k}_a, \vec{k}_b, R_a, R_b) = U_A(\vec{k}_a, R_a) \phi_A(x_a) + U_B(\vec{k}_b, R_b) \phi_B(x_b)
\]

where the \( \phi_A(x_a) \) ( \( \phi_B(x_b) \) ) represent a product of the internal wave functions for the nuclei in the entrance (exit) channel. Because the incoming and outgoing channels correspond to different mass partitions of the available nucleons, the \( \phi_v(x_i) \) are not orthogonal to each other. Also the relative functions \( U_v(\vec{R}_i, \vec{R}_i) \), in contrast to the earlier work with inelastic channels, are not referred to the same coordinates. The coordinate system used to describe the rearrangement reaction is shown in figure 4.1. Ignoring for now the presence of Coulomb forces, which may be included in the same manner as indicated in section 2.2, the asymptotic forms of the \( U_v(\vec{R}_i, \vec{R}_i) \) are

\[
\begin{align*}
U_A(\vec{k}_a, R_a) & \rightarrow e^{i\vec{k}_a \cdot \vec{R}_a} + f_{AA}(\Theta_A) \frac{e^{iK_a R_a}}{R_a} \\
U_B(\vec{k}_b, R_b) & \rightarrow f_{BB}(\Theta_B) \frac{e^{iK_b R_b}}{R_b}
\end{align*}
\]

The ansatz wave function of equation 4-1 satisfies the time independent Schrödinger equation

\[
(E-H) \Psi(x_A, x_B, \vec{k}_a, \vec{k}_b, R_a, R_b) = 0
\]
The Hamiltonian can be written in the two equivalent forms:

\[
H = H_a + H_a' + T_{1a}^a + V_{x_{A}}(\vec{r}_{1}) + V_{b_{A}}(\vec{r}) \quad 4-5
\]

\[
H = H_b + H_b' + T_{1b}^b + V_{x_{B}}(\vec{r}_{1}) + V_{b_{B}}(\vec{r}) \quad 4-6
\]

appropriate for the entrance or exit channel respectively. The coupled reaction channels equations for the transfer reaction can be obtained in analogy with the derivation in chapter 2. Multiplying equation 4-4 to the left by \( \phi_{A}^{*}(x_{A}) \) or \( \phi_{B}^{*}(x_{B}) \) and integrating over the appropriate internal coordinates leads to the following equations:

\[
\int dx_{A} \phi_{A}^{*}(x_{A}) [E-H] U_{A}(\vec{r}_{A}, \vec{r}_{a}) \phi_{A}(x_{A}) = \int dx_{A} \phi_{A}^{*}(x_{A}) (H-E) U_{B}(\vec{r}_{b}, \vec{r}_{b}) \phi_{B}(x_{B}) \quad 4-7
\]

\[
\int dx_{B} \phi_{B}^{*}(x_{B}) [E-H] U_{B}(\vec{r}_{a}, \vec{r}_{b}) \phi_{B}(x_{B}) = \int dx_{B} \phi_{B}^{*}(x_{B}) (H-E) U_{A}(\vec{r}_{a}, \vec{r}_{a}) \phi_{A}(x_{A}) \quad 4-8
\]

where the integrations over \( dx_{A} \) and \( dx_{B} \) are performed with fixed \( \vec{r}_{a} \) and \( \vec{r}_{b} \). Using the appropriate forms of the Hamiltonians in these two equations, the following coupled equations can be obtained:

\[
\left[ (E-E_{a} - E_{A}) - T_{1a}^a - \langle A | V_{x_{A}} + V_{b_{A}} | A \rangle \right] U_{A}(\vec{r}_{A}, \vec{r}_{a}) = \int dx_{A} \phi_{A}^{*}(x_{A}) (H-E) U_{B}(\vec{r}_{b}, \vec{r}_{b}) \phi_{B}(x_{B}) \quad 4-9
\]

\[
\left[ (E-E_{b} - E_{B}) - T_{1b}^b - \langle B | V_{x_{B}} + V_{b_{B}} | B \rangle \right] U_{B}(\vec{r}_{b}, \vec{r}_{b}) = \int dx_{B} \phi_{B}^{*}(x_{B}) (H-E) U_{A}(\vec{r}_{a}, \vec{r}_{a}) \phi_{A}(x_{A}) \quad 4-10
\]
In analogy with the techniques used in section 3.4 the second equation can be formally solved to yield

\[ U_B(\vec{K}_b, \vec{R}_b) = \int d\vec{R}'_b \ G(\vec{R}'_b, \vec{R}_b) \int dx_g \ \phi^*_g(x_g) \]

\[ (H-E) \phi_A(x_A) \ U_A(\vec{R}_a, \vec{R}_a) \]

where \( G(\vec{R}'_b, \vec{R}_b) \) is the coordinate representation of the inverse of the operator appearing on the left hand side of equation 4-10.

\[ G(\vec{R}'_b, \vec{R}_b) = \langle \vec{R}_b' | \left( E - T^{bb} - \langle B | V_{bx} + V_{ba} | B \rangle + i\epsilon \right)^{-1} \vec{R}_b \rangle \]

The solution of equation 4-11 with the Green's function given in equation 4-12 is dealt with in many standard texts [Me 62, Ro 67, Au 70]. For large \( R_b \) the asymptotic form can be written as

\[ U_B(\vec{K}_b, \vec{R}_b) \rightarrow \left( -\frac{\mu_B}{2\pi \hbar^2} \right) \frac{e^{iK_b R_b}}{R_b} \int d\vec{R}'_b \ \chi_g^{(-)\ast}(\vec{K}_b, \vec{R}_b') \]

\[ \int dx_g \ \phi^*_g(x_g) \left[ H - E \right] \phi_A(x_A) \ U_A(\vec{R}_a, \vec{R}_a) \]

where \( \vec{R}_b \) has been allowed to tend to infinity along \( \vec{R}_b \), the ejectile's detected direction, and \( \mu_B \) is the reduced mass for the outgoing nucleus. \( \chi_g^{(-)\ast}(\vec{R}_b, \vec{R}_b) \) satisfies the relation

\[ \chi_g^{(-)\ast}(\vec{K}_b, \vec{R}_b) = \chi_g^{(+)\ast}(-\vec{K}_b, \vec{R}_b) \]

where \( \chi_g^{(+)\ast}(\vec{R}_b, \vec{R}_b) \) is the distorted wave for elastic scattering in the outgoing channel. It satisfies the homogenous equation for the outgoing channel, equation 4-10 with zero on the right hand side.

\[ \left( [E - E_b - E_b^{bb}] - T^{bb}_b \langle B | V_{bx} + V_{ba} | B > \right) \chi_g^{(+)\ast}(\vec{K}_b, \vec{R}_b) = 0 \]
By comparing equations 4-13 and 4-3 the transition amplitude can be identified as

\[ f_{BA} (\hat{K}_b, \hat{K}_a) = \left( -\frac{\mu_b}{2\pi k^2} \right) \int d\vec{R}_b \chi_B^{(-)} (\vec{K}_b, \vec{R}_b) \]
\[ \int dx_b \phi_B^* (x_b) (H-E) \phi_A (x_A) U_A (\vec{K}_a, \vec{R}_a) \]

4-16

It is conventional to work with a renormalised transition amplitude, defined by

\[ T_{BA} (\hat{K}_b, \hat{K}_a) = -\frac{2\pi k^2}{\mu_b} f_{BA} (\hat{K}_b, \hat{K}_a) \]

4-17

The transition amplitude is thus defined by

\[ T_{BA} (\hat{K}_b, \hat{K}_a) = \int d\vec{R}_b \int dx_b \chi_B^{(-)} (\vec{K}_b, \vec{R}_b) \phi_B^* (x_b) \]
\[ (H-E) \phi_A (x_A) U_A (\vec{K}_a, \vec{R}_a) \]

4-18

The Hamiltonian appearing in this expression can be written in the form of equation 4-6, and allowed to operate backwards on the product \( \chi_B^{(-)} (\vec{K}_b, \vec{R}_b) \phi_B^* (x_b) \). Since the bound state wave functions represented by \( \phi_b (x_b) \) fall off rapidly with increasing separation any surface terms that appear can be ignored. The operators \( H_b \) and \( H_B \) simply yield the eigen energies of the nuclear states. From equation 4-15

\[ \chi_B^{(-)} (\vec{K}_b, \vec{R}_b) (T_{bb} - E_{bb}^c) = -\chi_B^{(-)} (\vec{K}_b, \vec{R}_b) \phi_b (x_b) <B | V_{xb} + V_{ba} | B> \]

4-19

Equation 4-18 can be written

\[ T_{BA} (\hat{K}_b, \hat{K}_a) = \int d\vec{R}_b \int dx_b \chi_B^{(-)} (\vec{K}_b, \vec{R}_b) \phi_B^* (x_b) \]
\[ [V_{bx} + V_{ba} - <B | V_{xb} + V_{ba} | B>] \phi_A (x_A) U_A (\vec{K}_a, \vec{R}_a) \]

4-20
Within the limitations imposed by the adoption of the ansatz wave function this expression is still exact. In order to reduce this expression to the DWBA expression for the transition amplitude the following approximations are made.

1) Replace the function \( U_A (\vec{K}_a, \vec{R}_a) \) by the elastic scattering function for the entrance channel, \( \chi_A^{(+)} (\vec{K}_a, \vec{R}_a) \), which satisfies

\[
\left[ E_{\alpha A}^m - T_{\alpha A} - \langle A | V_{\alpha A} + V_{bA} | A \rangle \right] \chi_A^{(+)} (\vec{K}_a, \vec{R}_a) = \phi
\]

2) Replace the interaction that appears between the nuclear wave functions \( \phi_c (x_i) \) by the binding potential \( V_{b\alpha} (r_i) \).

The replacement of \( U_A (\vec{K}_a, \vec{R}_a) \) can be achieved by disregarding the right hand side of equation 4-9. The cross sections for the transfer reactions are fairly small, so that in a sense coupling between the entrance and exit channels is relatively weak. The effects of back coupling on the elastic channel, which is second order in this transition, would thus be expected to be very small. The first approximation is therefore easily justified. The second of these approximations however is not so easy to justify. Essentially the replacement assumes that

\[
V_{b\alpha} (r) - \langle B | V_{b\alpha} \left( -\vec{R}_b - \frac{A}{A+\alpha} \vec{r}_2 \right) + V_{b\alpha} \left( \vec{R}_b + \frac{\alpha}{A+\alpha} \vec{r}_2 \right) | B \rangle = \phi
\]

Although this may not seem valid, for example the coordinates of the two components differ, it will be justified a posteri. We may be guided by calculations using optical potentials to obtain the distorted waves in the entrance and exit channels. In this case the interaction that appears in
equation 4-20 is

$$V_{bx}(r_i) + V_{bA}^{opt}(r) - V_{bB}^{opt}(R_b)$$  \text{4.23}$$

In, for example, A(d,p)B reactions, where much of the development of the DWBA has been concentrated, the optical potentials are those between a proton and a nucleus with A nucleons, and between a proton and a nucleus with A+1 nucleons. The cancellation of the second two terms in equation 4-23 is thus reasonable. Using the same approximation it is possible to accurately describe a variety of heavy ion transfer reactions, so that although difficult to justify theoretically, the approximation at least provides a convenient and simple means of describing transfer reactions. The validity of this approximation has been considered in some detail [To 73, De 74, De 75, To 76, Na 76]. The conclusion of their work is that the transfer cross sections are largely unchanged by the inclusion of the full nuclear interactions. This result is reasonable since the evaluation of the matrix element involves an integration over $r_i$ and the potential $V_{bx}(r_i)$ and the function $\Phi_A(r_i)$ have a good spatial overlap. This term would thus be expected to contribute most to the overall strength of the matrix element responsible for coupling between the incident and exit channels.

Until now we have considered only nuclear potentials to be present. In analogy with equation 4-23 there are three Coulomb terms inside the matrix element

$$V_{bx}^{Coul}(r_i) + V_{bA}^{Coul}(r) - V_{bB}^{Coul}(R_b)$$  \text{4.24}$$

The references cited above find that the cancellation of these Coulomb potentials is not so
valid, but the change is largely just one of magnitude in the predicted cross sections, the inclusion increasing the cross section by a factor of about two. Since the concern of this present work is in the structure of the cross sections rather than their absolute magnitudes, and we are not considering Coulomb excitation, this is not relevant. The results presented later in this chapter indicate that the use of a consistent folding model and the assumption of the cancellation of equation 4-22, offers agreement as good as that obtained using optical potentials, with the assumption of the cancellation of the last two terms in equation 4-23.

Using these two assumptions the DWBA transition amplitude can be written as

\[
T_{BA} (\vec{K}_b, \vec{K}_a) = \int d\vec{R}_b \int d\vec{X}_b \chi_b^{(-)}(\vec{K}_b, \vec{R}_b) \phi_a^{*}(\vec{X}_b)
\]

\[
V_{bx} (\vec{r}_1) \phi_A (\vec{X}_A) \chi_A (\vec{K}_A, \vec{R}_A)
\]

4-25

The symbolic functions \(\phi_c(x_i)\) can be written in terms of their cluster model descriptions as

\[
\phi_A (x_A) = \phi_{1o} (x_o) \phi_{c} (x_c) \phi_{28s} (x_{28}) \phi_{e-o} (\vec{r}_1, \sigma_e)
\]

4-26

\[
\phi_B^{*} (x_B) = \phi_{1o}^{*} (x_o) \phi_{c}^{*} (x_c) \phi_{28s}^{*} (x_{28}) \phi_{e-o}^{*} (\vec{r}_2, \sigma_e)
\]

4-27

Integrating over the internal coordinates of the clusters, which are assumed inert, equation 4-25 can be written as

\[
T_{BA} (\vec{K}_b, \vec{K}_a) = \Theta (b_A: A) \int d\vec{R}_b \int d\vec{r}_2 \chi_b^{(-)}(\vec{K}_b, \vec{R}_b)
\]

\[
\phi_{e-28s} (\vec{r}_2, \sigma_e) V_{bx} (\vec{r}_1) \phi_{e-o} (\vec{r}_1, \sigma_e) \chi_{A}^{(+)} (\vec{K}_a, \vec{R}_a)
\]

4-28
which is adopted as the model used to describe the transfer reaction. The factor $\Theta(bx:Ax)$, called the spectroscopic factor, has been included [El 69, Cl 73]. This factor is a measure of the extent to which the participating nuclei are described by the wave functions used, the spectroscopic amplitude defined here includes any combinatorial factors that are present [Sa 83]. Spectroscopic amplitudes may be defined for the incident and exit channels separately as

$$\Theta(Tx) = \int dx_{-} \bar{\Phi}(x) \phi_{\text{Model}}(x)$$

where $\bar{\Phi}(x)$ is the actual many body wave function in the entrance or exit channel. The individual spectroscopic amplitudes cannot be measured, hence the overall factor

$$\Theta(bx:Ax) = \Theta(Ax) \Theta(bx)$$

is defined.

In order to evaluate equation 4-28, which is expressed in terms of its natural coordinates, it is convenient to reexpress the six-dimensional integral in terms of either $\vec{R}_a$ and $\vec{R}_b$, or $\vec{r}_1$ and $\vec{r}_2$. The arguments of the components of the integrand appear in a mixture of these variables. The first of the two choices will be adopted, namely $\vec{R}_a$ and $\vec{R}_b$. This choice makes it necessary to express the nuclear wave functions in terms of these variables. The transition amplitude can thus be written as

$$T_{\hat{b}A} (\hat{K}_b, \hat{K}_a) = \Theta(bx:Ax) J \int d\vec{R}_a \int d\vec{R}_b \chi^{(-)*}_A(\hat{K}_b, \hat{R}_b) \phi^{*}_b(\vec{R}_2, \sigma_2) V_{bx}(r_1) \phi^{(-)}_A(\vec{R}_1, \sigma_1) \chi^{(+)}_A(\hat{K}_a, \hat{R}_a)$$

where $J$ is the Jacobian for the transformation from integrating over $\vec{r}_2$ and
\[ J = \left[ \frac{aB}{x(a+A)} \right]^3 \]

Equation 4-31 thus defines the DWBA transition amplitude in general terms. This can be written in terms of the angular momenta of the system as

\[ T_{M_{I_b} M_{I_a}}^{M_{I_b} M_{I_a}} \left( \hat{R}_b, \hat{R}_a \right) = \sum T_{M_{I_b} M_{I_a} M_{I_b} M_{I_a}}^{M_{I_b} M_{I_a}} \left( \hat{R}_b, \hat{R}_a \right) \]

where the \( T_{M_{I_b} M_{I_a} M_{I_b} M_{I_a}} \left( \hat{R}_b, \hat{R}_a \right) \) are given by

\[ T_{M_{I_b} M_{I_a} M_{I_b} M_{I_a}}^{M_{I_b} M_{I_a}} \left( \hat{R}_b, \hat{R}_a \right) = \Theta \left( b \times A \right) \int \int d \hat{R}_a d \hat{R}_b \chi_{M_{I_b} M_{I_a}}^I \left( \hat{R}_b, \hat{R}_a \right) \left( I_{M_{I_b}} \left| V_{b \times A} \left( r \right) \right| I_{M_{I_a}} \chi_{M_{I_a}}^I \left( \hat{R}_a, \hat{R}_a \right) \right) \]

The transfer cross section measured by a spin insensitive device, for an unpolarised beam is then given by

\[ \frac{d\sigma}{d\Omega} = \frac{\mathcal{M}_b \mathcal{M}_A}{2\pi k^2} \frac{k_b}{k_a} \sum_{M_{I_b} M_{I_a}} \left| T_{M_{I_b} M_{I_a}} \left( \hat{R}_b, \hat{R}_a \right) \right|^2 \]

In order to solve equation 4-34 two distinct components need to be evaluated. The first is the determination of the scattering functions \( \chi_{M_{I_b} M_{I_a}}^I \left( \hat{R}_b, \hat{R}_a \right) \) for elastic scattering in the entrance and exit channels. These distorted waves, in partial wave form, can be obtained from the expressions in chapter 2. The second component is the evaluation of the nuclear matrix element that appears sandwiched between the two distorted scattering functions in equation 4-34.
The scattering functions will be considered in section 4.2.1. The nuclear matrix element, expressed in partial wave form and in terms of the coordinates \( \vec{R}_a \) and \( \vec{R}_b \), will be considered in sections 4.2.2 to 4.2.4. The complete scattering amplitude will then be given in section 4.2.5.

### 4.2.1 Partial wave expansion of the distorted waves

The partial wave decomposition of the distorted waves \( \chi_{M'_a M'_I}^z (\vec{R}_a, \vec{R}_b) \) has already been introduced in section 2.2.2. The expansions will merely be restated, in terms of the quantum numbers such as angular momentum, that are appropriate for this section of the work.

For the incident channel the distorted wave can be expanded as a sum over partial waves according to equation 2-29 as

\[
\chi_{M'_a M'_I}^{I_a} (\vec{R}_a, \vec{R}_b) = \frac{4\pi}{K_a R_a} \sum (L_a M_a I_a I_a I_a M_a J_a) (L_a M'_a I_a I_a M'_a J'_a) \\
(\ell) L_a \chi_{M'_a}^{J_a} (K_a R_a) Y_{L_a M_a} (\vec{K}_a) Y_{L_a M'_a} (\vec{R}_a)
\]

The sums running over \( L_a, M_a, M'_a, J_a \) and \( M'_a \). This form assumes that there are no tensor forces acting, and we have used the result of section 2.3.1 that the total angular momentum \( J_a \) and its z component \( M_{za} \) are conserved.

The asymptotic form of the \( \chi_{M'_a M'_I}^{J_a} (K_a, R_a) \) are given in equation 2-28 with \( L'' = L \) and \( I'' = I \). The partial wave components \( \chi_{M'_a}^{J_a} (K_a, R_a) \) satisfy the differential equations

\[
\left[ \frac{E - T_{R_a}}{R_a} - V_{\text{Coul}} (R_a) - V_{L_a I_a, I_a I_a} (R_a) \right] \chi_{M'_a}^{J_a} (K_a, R_a) = 0
\]

The potentials \( V_{L_a I_a, I_a I_a} (R_a) \) that act in these equations will be discussed in
The distorted waves for the outgoing $^6$O $+$ $^3$P channel can be expanded by using the symmetry relation

$$\chi_{M_{I_b}M_{I_b}^*}^{I_2}(\vec{k}_b, \vec{r}_b) = (-)^{M_{I_b} - M_{I_b}^*} \chi_{-M_{I_b}^* - M_{I_b}}^{I_2}(\vec{k}_b, \vec{r}_b)$$

and the parity relation

$$\gamma_{L_b M_{L_b}}^{I_2}(-\hat{R}_b) = (-)^{L_b} \gamma_{L_b M_{L_b}}^{I_2}(\hat{R}_b)$$

as

$$\chi_{M_{I_b}^* M_{I_b}}^{I_2}(\vec{k}_b, \vec{r}_b) = \frac{4\pi}{k_b R_b} \sum (L_b M_{L_b} I_b - M_{I_b} I_b) (L_b M_{L_b}^* I_b - M_{I_b}^* I_b) \gamma_{L_b M_{L_b}}^{I_2}(\vec{k}_b, \vec{r}_b)$$

$$(-)^{L_b} \gamma_{L_b M_{L_b}}^{I_2}(\vec{k}_b, \vec{r}_b) \gamma_{L_b M_{L_b}}^{I_2}(\hat{R}_b)$$

with the sums running over $L_b, M_{L_b}, M_{I_b}^*, J_b$ and $M_{I_b}$. The radial components of the distorted waves satisfy the analogues of equation 4-37.

$$[(E - Q - E_{I_b}) - T_{L_b}^{R_b} - V_{Coul}(R_b) - V_{I_b L_b}^{J_b}(R_b)] \chi_{L_b I_b}^{J_b}(K_b, R_b)$$

The asymptotic wavenumber for the final state under consideration is given by

$$\frac{k_b^2}{2\mu_b^*} = \frac{k_a^2}{2\mu_a} - Q - E_{I_b}$$

where $E_{I_b}$ is the excitation energy of the residual nuclear state under consideration and $Q$, the $Q$ value for the reaction, is the difference in binding energies of the triton in the incident and exit partitions. From appendix B, $Q$ has the numerical value $5.797$ MeV.
4.2.2 Nuclear matrix element

The matrix element in equation 4-34 acts as an effective interaction for the transition between the scattering states $\chi_{M_i a, M_a}^X (\vec{R}_a, \vec{R}_a)$ and $\chi_{M_i a, M_a}^X (\vec{R}_b, \vec{R}_b)$. Using the expressions in section 1-3 the matrix element can be written as

$$ (I_B M_i' | V_{bx} (r_i) | I_a M_i') = \int d\vec{x}_B \phi^*_l (r_i) \gamma_{l_1 m_1, l_2 m_2}^l (\vec{r}_2) \chi_{l_1 m_1, l_2 m_2}^l (\vec{r}_2) (i)^{l_1 - l_2}$$

$$V_{bx} (r_i) \phi^*_l (r_i) \gamma_{l_1, m_1} (r_i) \chi_{l_1 m_1, l_2 m_2}^l (\vec{r}_2) (l_1 m_1, l_2 m_2, I_a M_i') (l_1 m_1, l_2 m_2, I_B M_i')$$

4-43

In order to evaluate the integrals in equation 4-43 it is necessary to express this matrix element as a function of $\vec{R}_a$ and $\vec{R}_b$. In obtaining equation 4-43 the internal coordinates of the clusters have been implicitly integrated over.

We can express the fact that this matrix element remains a function of the coordinates $\vec{R}_a$ and $\vec{R}_b$ by writing

$$ (I_B M_i' | V_{bx} (r_i) | I_a M_i') = I_B M_i' ; I_a M_i' (\vec{R}_b, \vec{R}_a)$$

4-44

It is convenient to expand this matrix element as a sum over multipoles of the transferred orbital angular angular momentum.

4.2.3 Multipole decomposition of the nuclear matrix element

It is convenient to recouple the angular momenta present in equation 4-43 in terms of the orbital angular angular momentum that is transferred
during the reaction. Defining

\[ \vec{L} = \vec{I}_a - \vec{I}_b \ ( = \vec{I}_b - \vec{I}_a ) \]

the expansion may be performed according to

\[ \mathcal{I}_{\vec{I}_a \vec{M}_1^\prime \vec{I}_b \vec{M}'_b} (\vec{R}_b \vec{R}_a) = \sum L (-\vec{I}_a \vec{M}'_a (I_a \vec{M}'_a \vec{I}_a \vec{M}_1 | \vec{I}_a \vec{I}_b \vec{I}_b | L, M, L) \mathcal{G}^M_{\vec{L}_q \vec{I}_a \vec{I}_b} (\vec{R}_b \vec{R}_a) \]

where the \( \mathcal{G}^M_{\vec{L}_q \vec{I}_a \vec{I}_b} (\vec{R}_b \vec{R}_a) \) are defined by the inverse relation, which can be obtained by using the orthogonality properties of the Clebsch Gordan coefficients, as

\[ \mathcal{G}^M_{\vec{L}_q \vec{I}_a \vec{I}_b} (\vec{R}_b \vec{R}_a) = \sum (-\vec{I}_a \vec{M}'_a (I_a \vec{M}'_a \vec{I}_a \vec{M}_1 | \vec{I}_a \vec{I}_b \vec{I}_b | L, M, L) \mathcal{G}^M_{\vec{L}_q \vec{I}_a \vec{I}_b} (\vec{R}_b \vec{R}_a) \]

Recoupling the nuclear matrix element of equation 4-43 according to the coupling scheme of equation 4-45, the \( \mathcal{G}^M_{\vec{L}_q \vec{I}_a \vec{I}_b} (\vec{R}_b \vec{R}_a) \) can be written as

\[ \mathcal{G}^M_{\vec{L}_q \vec{I}_a \vec{I}_b} (\vec{R}_b \vec{R}_a) = \sum (-\vec{I}_a \vec{M}'_a (I_a \vec{M}'_a \vec{I}_a \vec{M}_1 | \vec{I}_a \vec{I}_b \vec{I}_b | L, M, L) \mathcal{W}(l_1, l_2, \vec{l}_1 \vec{l}_2 \vec{I}_b ; S, L) \]

\[ (l_1 l_2 l_1 - l_1 \vec{m}_1 ) U_l (r_1) V_{l_2} (r_1) U_{l_1} (r_1) Y_{l_1 m_2} (\vec{r}_1) Y_{l_2 m} (\vec{r}_2) \]

where \( s \) is the spin of the transferred triton cluster.

The \( \mathcal{G}^M_{\vec{L}_q \vec{I}_a \vec{I}_b} (\vec{R}_b \vec{R}_a) \) can be written as

\[ \mathcal{G}^M_{\vec{L}_q \vec{I}_a \vec{I}_b} (\vec{R}_b \vec{R}_a) = \sum B_{l_1 l_2}^{l_1 l_2} H^M_{\vec{L}_q \vec{l}_1 l_2} (\vec{R}_b \vec{R}_a) \]

The factor \( B_{l_1 l_2}^{l_1 l_2} \) has been implicitly incorporated into the overall spectroscopic factor \( \Theta(b_x a_x) \), and will not be considered further. The \( H^M_{\vec{L}_q \vec{l}_1 l_2} (\vec{R}_b \vec{R}_a) \) is a function of six variables, it can be expanded in terms of the spherical harmonics in \( R_a \) and \( R_b \) as

\[ H^M_{\vec{L}_q \vec{l}_1 l_2} (\vec{R}_b \vec{R}_a) = \sum \{ (l_1 l_2 \vec{m}_1) \vec{m}_a \vec{m}_b \} (\vec{R}_b) \star h_{l_1 l_2}^{l_1 l_2} (R_b, R_a) \sum \{ (l_1 l_1 \vec{m}_1) \vec{m}_a \vec{m}_b \} (\vec{R}_b) \star (l_1 l_2 \vec{m}_1 \vec{m}_2 | L, M, L, M, L) \]
where the $H_{\ell m}^{M}$ $(\hat{R}_{b}, \hat{R}_{a})$ has been defined so that it rotates as the conjugates of the spherical harmonics for later convenience.

### 4.2.4 Coordinate transformation

It is necessary to express the functions which appear in equation 4-49 as functions of their natural coordinates in terms of the channel coordinates $\hat{R}_{a}$ and $\hat{R}_{b}$. From figure 4.1 the coordinates present satisfy the relations

\[ \hat{r}_1 = s_1 \hat{R}_{a} + t_1 \hat{R}_{b} \]
\[ \hat{r}_2 = s_2 \hat{R}_{a} + t_2 \hat{R}_{b} \]

where

\[ s_1 = \frac{aB}{xT}, \quad t_1 = \frac{-bB}{xT} \]

and

\[ s_2 = \frac{A}{xT}, \quad t_2 = \frac{-B}{xT} \]

with

\[ T = (a + A) = (b + B) \]

This transformation can be performed by introducing the functions

\[ W_{\ell}(r) = \varphi_{\ell}(r) r^{-\ell} \]

and the solid spherical harmonics

\[ \gamma_{\ell m}(\hat{r}) = r^{\ell} \gamma_{\ell m}(\hat{r}) \]

There exists a transformation between solid spherical harmonics which are
functions of variables that are linked via a linear combination as in equation 4-51 [Mo 59]

\[
r^l \gamma_{lm}(\hat{r}) = \sum_{\lambda} \sqrt{4\pi \lambda} \gamma_{l-\lambda} \left( \frac{2l}{2\lambda} \right)^{\frac{1}{2}} (SR_a)^{l-\lambda} (tR_b)^\lambda (l-\lambda m-\mu l m) \gamma_{l-\lambda m-\mu} (\hat{R}_a) \gamma_{l m} (\hat{R}_b)
\]

where the sum over \( \lambda \) runs from zero to 1 and \( \binom{x}{y} \) is the binomial coefficient, defined by [Ab 70]

\[
\binom{x}{y} = \frac{x!}{y!(x-y)!}
\]

The \( \hat{r}_1 \) and \( \hat{r}_2 \) are expressed in terms of \( \hat{R}_a \) and \( \hat{R}_b \) and the angle between them using the vector relations 4-51 and equation 2-44. After some lengthy Racah algebra the \( h_{l_1 l_2 l_1}(R_b, R_a) \) can be written as [Ta 74a., Sa 83]

\[
h_{l_1 l_2 l_1} = \sum_{\lambda} (s_1 R_a)^{l_1-\lambda_1} (t_1 R_b)^{\lambda_1} (s_2 R_a)^{\lambda_2} (t_2 R_b)^{l_2-\lambda_2} (l_1)^{L+L_1+l_2} (-1)^{k} \hat{\lambda}_1 \hat{\lambda}_2 \left( \frac{2l_1}{2\lambda_1} \right)^{\frac{1}{2}} \left( \frac{2l_2}{2\lambda_2} \right)^{\frac{1}{2}} m_k(R_b, R_a) \hat{r}_1 \hat{r}_2
\]

\[
\sum_{\lambda_1 \lambda_2} \hat{\lambda}_1 \hat{\lambda}_2 \left( \lambda_1 \lambda_2 \rho \rho 0 \right) (\lambda_1 \lambda_2 \rho \rho 0) (l_1-\lambda_0 \lambda_0 \lambda_0 \rho) \left( l_2-\lambda_2 \rho \rho 0 \right) W(L_1 l_2 L_1 L_2 \lambda_1 \lambda_2 ; l K) \begin{pmatrix} l_1 & l_2 & l_1 & l_2 \lambda_1 \lambda_2 & l_1-l_2 \lambda_1 \lambda_2 \end{pmatrix}
\]

where

\[
\eta_k(R_b, R_a) = \frac{1}{2} \int P_k^l (\cos \hat{R}_a \hat{R}_b) W_{l_1}(r_1) W_{l_2}(r_2) V_{bx}(r_1)
\]
If $l_1$ and/or $l_2$ are zero then this expression simplifies considerably, however we maintain the general expression for later convenience.

4.2.5 Partial Wave expansion of the Transition Amplitude

A complete prescription for the evaluation of the transition amplitude in partial wave form may now be given, using the expressions given in the preceding three subsections. The transition matrix elements of equation 4-34 may be written as

$$T_{M'_b M_b; M'_a M_a} = \frac{\hbar \pi}{k_b k_a} \sum (i) I_a - L_a + L_b - L_\beta (-)^J_a + L_b - M'_a,$$

$$(I_a - M'_a M_a | J_a M_a) (L_a M'_a J_a M_a | J_a M_a) (L_b M'_b J_b M_b | J_b M_b) \int d\hat{R}_a Y_{L'_a M'_a} (\hat{R}_a) Y^*_{L M_a} (\hat{R}_a)$$

$$\int d\hat{R}_b Y_{L'_b M'_b} (\hat{R}_b) Y^*_{L M_b} (\hat{R}_b) \int d\hat{R}_a d\hat{R}_b \hat{R}_a \hat{R}_b X_{I_a I_a} (K_a, R_a) Y^*_{L_a M_a} (\hat{R}_a) Y_{L_b M_b} (\hat{R}_b)$$

where the sums run over $I_a, M_a, J_a, M_a, L_b, M_b, J_b, M_b, L_b, M_l, l_1$ and $l_2$. The $h_{L'_a L'_b} (R_a, R_b)$ are defined by equations 4-57 and 4-58. The integrations over the angles of $\hat{R}_a$ and $\hat{R}_b$ can be performed immediately to give $S_{L_a M_a} S_{L_b M_b} S_{L'_a M'_a} S_{L'_b M'_b}$. Many of the sums present in this expression are limited in range for the particular reactions considered. A lengthy part of the numerical evaluation of this expression is the evaluation of the double integral over the magnitudes of the vectors $\hat{R}_a$ and $\hat{R}_b$, particularly since these are required for all of the $I_a, L_b$ combinations
that are permitted by the conservation of total angular momentum. The ranges of the integrals are from zero to infinity for both variables. From equation 4-51a \( R_b \) can be written as

\[
R_b = \left[ \left( \frac{a}{b} \right)^2 R_a^2 + \left( \frac{x}{b^2 B} \right)^2 r_1^2 - \frac{2axT}{b^2 B} R_a r_1 \cos(R_a r_1) \right]^{1/2}
\]

Since the two functions that appear in the radial coordinate \( r \), the binding potential and the bound state wave function both tend rapidly to zero for large \( r_1 \) values, there are regions of \( R_a R_b \) space that contribute zero to the integral. Thus, without approximation, the limits of one of the variables present in the double integrations may be truncated to be near the value of the other variable. The region of \( R_a R_b \) space over which the integration is performed is indicated in figure 4.2.
Figure 4.2 The contributing area of $R_a R_b$ space.
4.3 Distorting potentials

Within the DWBA the distorting potentials in the incident and outgoing channels are taken to reproduce the elastic scattering in the two channels. The incident channel distorting potentials will be taken to be those used to describe the elastic scattering of $^9$F from $^{30}$Si at 60 MeV discussed in section 2.3. The real and imaginary central potentials are defined by equation 2.86. The spin orbit potential between the two nuclei is defined by equation 2.87. The cluster target potentials, taken as input for the folding procedure are discussed in section 2.3, and are defined in appendix B.

In order to maintain consistency the folding model will again be used to obtain the distorting potentials in the outgoing $^16$O + $^3$P channel. Since the primary interest is in entrance channel effects we shall adopt the simplest description for the outgoing channel. By comparing figures 2.1 and 4.1, $\hat{R}$, $\hat{R}$, can be identified with $\hat{R}$, and $\hat{r}$, $\hat{r}$, with $\hat{r}$. The situation differs in some respects from that for the incident channel however. In particular the spin in the outgoing channel is now carried by the residual nucleus. Available computing codes do not consider $L,I$ coupling between orbital angular momentum and target spin, hence any spin orbit forces present will be neglected. The channels which leave the residual nucleus in an excited $l = 2$ state also permit the possibility of reorientation terms that couple partial waves in the outgoing channel. These will also be neglected. The distorting potentials in the outgoing channel will thus be taken to consist of central terms only, namely real and imaginary nuclear
and real Coulomb. With the assumption that there are no spin dependent forces acting in the outgoing channel the z component \( M_{ix} \) is conserved and hence equation 4-59 simplifies slightly. The Coulomb potential between the \(^{16}\)O and \(^{31}\)P nuclei is taken as the standard form of appendix A, with \( Z_1 = 8 \) and \( Z_2 = 15 \).

In order to perform the folding model calculations the triton - \(^{16}\)O and \(^{28}\)Si - \(^{16}\)O cluster - target optical potentials are required. The relative kinetic energy between the \(^{16}\)O and \(^{28}\)Si clusters is reduced from that in the incident channel because of the change in binding energy of the triton cluster to the respective cores in the two channels, and the excitation energy for transitions leading to the \( l = 2 \) states of \(^{31}\)P. Despite this the same \(^{16}\)O - \(^{28}\)Si optical potential that was used as an input for the folding procedure in the entrance channel will be used again in the exit channel. The best optical model fit for the elastic scattering of tritons from \(^{16}\)O at an appropriate energy is that performed by Pullen et al [Pu 64]. Within the context of the present calculations their analysis has some undesirable features. Due to the difference in the masses of the clusters the folded potential is dominated by the lighter cluster's potential, particularly for large separations along the channel coordinate \( R_b \). All the real potentials between the triton and \(^{16}\)O and the triton and \(^{28}\)Si clusters used in this work have the common feature that they are fairly deep, that is about 50 MeV per triton nucleon. The analysis of [Pu 64] however yields a real triton - \(^{16}\)O potential that is shallow. The real part of the triton - \(^{16}\)O optical potential should ideally be similar to that which binds the two clusters in the entrance channel. In the absence of an imaginary potential between these clusters, the triton would experience a large and unphysical change in momentum whilst
transferring between these two potentials.

In order to obtain reasonable consistency for the potentials used the optical model search code ATHREE [Au 78] was used to obtain an optical model potential between the triton and \(^{16}\)O which had a real part with a depth of about 50 MeV per triton nucleon. The results of the search code, and the resulting optical model potentials are given in appendix B. The optical model search was made adopting starting values of \(V_0 = 150\) MeV, \(r_0 = 1.25\) fm and \(a_0 = 0.65\) fm for a Woods-Saxon potential defined in appendix A. These parameters were allowed to vary slightly in the search. The imaginary potential was treated as essentially arbitrary, and allowed to vary freely in order to reproduce the cross section of Pullen et al. A calculation using the optical potentials of \([\text{Pu 64}]\) was used as the input data to the search code. The resulting optical potential set was checked using the code CHUCK3. The two codes were in excellent agreement.
Figure 4.3 $1/2^+$ DWBA transfer cross section.
Figure 4.4 3/2⁺ DWBA transfer cross section.
Figure 4.5 \( 5/2^+ \) DWBA transfer cross section.
4.4 Preliminary Results and Discussion

The preceding sections offer a complete definition for describing the transfer reactions $^{16}\text{P}(^{28}\text{Si}, ^{31}\text{P})^{16}\text{O}$ leading to the final states $1/2^+ \text{g.s.}$, $3/2^+ 1.27 \text{MeV}$ and $5/2^+ 2.23 \text{MeV}$ for the $^{31}\text{P}$. The calculations were performed using the computer code TWOFNR [Ki] and the calculations were checked against the code FRESCO, good agreement being obtained. The results of these calculations are shown in figures 4.3 to 4.5 for the three final states considered. The spectroscopic amplitudes for the three curves were obtained by scaling the calculations to agree with the data for the first peaks. It is clear from these graphs that the theoretical curves are in very poor agreement with the experimental data. This suggests that there may be something wrong with the models or parameters used to describe these reactions. Since we wish to work consistently with the models that have already been used successfully for the elastic calculations the latter case will be considered. Although the overall interest is in the effects of projectile excitation its inclusion is unlikely to have a sufficiently strong effect on the transfer cross sections to overcome the poorness of the agreement with the experimental data.

Considering figures 4.3 to 4.5 the most obvious discrepancy between the calculations and the data is the appearance of a 'hump' with increasing angle for the $L = 0$ transfer. The shape of the calculated cross sections is reminiscent of the 'bell shaped' transfer cross sections obtained from light ion induced transfer reactions, rather than the sharply falling diffractive heavy ion type that is suggested by the data. There has been a
suggestion [Po 80] that the 'cosh' potential used to bind the triton in the projectile is not appropriate for use within the DWBA theory. In order to see why this may be so we can make the following arguments.

4.4.1 Plane wave Born Approximation

Due to the larger masses involved in heavy ion induced reactions the linear momenta of the reactants is increased in comparison to light ion induced reactions. There is thus an increase in the possible momentum transfer between the projectile and ejectile. If, for discussional purposes the distorting potentials are neglected, then the transition matrix elements of equation 4-34 reduce to

\[ T(R_b, R_a) = \int \Theta(b\alpha; A\alpha) \int d\hat{\vec{r}}_a d\hat{\vec{r}}_b e^{-i\hat{\vec{k}}_b \cdot \hat{\vec{R}}_b} \left( I_\beta M_{\tau_\beta} \mid V_b(r) \mid I_\alpha M_{\tau_\alpha} \right) e^{i\hat{\vec{k}}_a \cdot \hat{\vec{R}}_a} \]

where the spin degrees of freedom have been dropped since, in the absence of any spin dependent distorting potentials, they remain constant. The vector relations of equations 4-49 may be inverted to give

\[ \hat{\vec{R}}_a = \hat{\vec{r}}_2 - \frac{b}{a} \hat{\vec{r}}_1 \]

\[ \hat{\vec{R}}_b = \frac{a}{b} \hat{\vec{r}}_2 - \hat{\vec{r}}_1 \]

Equation 4-61 can be rewritten as

\[ T(R_b, R_a) = \int \Theta(b\alpha; A\alpha) \int d\hat{\vec{r}}_1 d\hat{\vec{r}}_2 e^{-i\hat{\vec{k}}_b \cdot \hat{\vec{r}}_2} \left( I_\beta M_{\tau_\beta} \mid V_b(r) \mid I_\alpha M_{\tau_\alpha} \right) e^{i\hat{\vec{k}}_a \cdot \hat{\vec{R}}_a} \]

where we have now chosen to perform the double integration with respect to \( \hat{\vec{r}}_1 \) and \( \hat{\vec{r}}_2 \). The Jacobian \( J' \) for this transformation is given by

\[ J' = 1 \]
The plane wave arguments can be manipulated by using equation 4-62 to give

\[ T(\hat{R}_b, \hat{R}_a) = J' \theta(bx; Ax) \int d\hat{r}_1 d\hat{r}_2 e^{i(\hat{R}_a - \hat{R}_b).\hat{r}_1} \]

\[ (I_b M_{I_b} | V_{bx}(r_1) | I_a M_{I_a}) e^{i(\hat{R}_b - \hat{R}_a).\hat{r}_2} \]

4-65

and, using the fact that for the \( L = 0 \) transfer both nuclei are in s states,

\[ (I_b M_{I_b} | V_{bx}(r_1) | I_a M_{I_a}) = \phi_0^{31}(r_2) Y_{oo}^{*}(\hat{r}_1) \chi_{s m_s}^{*}(\sigma_\ell) j_0(r_1) Y_{oo}(\hat{r}_1) \chi_{s m_s}(\sigma_\ell) \]

where

\[ j_0(r_1) = \phi_0^{19}(r_1) V_{bx}(r_1) \]

Defining the momentum transfers

\[ \hat{q}_1 = \hat{K}_a - \frac{A}{B} \hat{K}_b \]

\[ \hat{q}_2 = \hat{K}_b - \frac{b}{a} \hat{K}_a \],

4-68

equation 4-65 can be written in the factored form

\[ T(\hat{R}_b, \hat{R}_a) = J' \theta(bx; Ax) \int d\hat{r}_1 d\hat{r}_2 e^{i\hat{q}_1 \cdot \hat{r}_1} \phi_0^{31}(r_2) Y_{oo}^{*}(\hat{r}_1) \int d\hat{r}_2 \phi_0^{19}(r_1) Y_{oo}(\hat{r}_1) \]

4-69

The plane waves can be expanded as sums over partial waves, according to

\[ e^{i\hat{q} \cdot \hat{r}} = 4\pi \sum_{LM} (l)^L j_L(qr) Y_{LM}^{*}(\hat{q}) Y_{LM}(\hat{r}) \]

4-70

Using this expansion in equation 4-69 the transition amplitude can be written

\[ T(\hat{R}_b, \hat{R}_a) = J' \theta(bx; Ax) \int d\hat{r}_1 Y_{oo}^{*}(\hat{r}_1) Y_{LM}^{*}(\hat{q}_1) \int d\hat{r}_2 Y_{LM}(\hat{r}_2) \int d\hat{r}_1 r_1^2 \int d\hat{r}_2 r_2^2 \phi_0^{31}(r_2) j_L(q_2 r_2) \]

\[ \int d\hat{r}_1 Y_{oo}(\hat{r}_1) Y_{LM}(\hat{r}_1) \int d\hat{r}_2 r_1^2 \int d\hat{r}_1 r_1^2 j_0(r_1) j_L(q_1 r_1) Y_{LM}^{*}(\hat{q}_1) Y_{LM}^{*}(\hat{q}_1) \]

4-71
The integrations over the angles of $\hat{r}_1$ and $\hat{r}_2$ are simply the orthogonality integrals for the spherical harmonics. Using the coordinate system with $z$ along the incident direction, and $y$ along $\hat{r}_a \wedge \hat{r}_b$, the PWBA transition amplitude can be written as the product of two one dimensional Fourier transforms

$$T(\theta) = J'\theta(bx; Ax) \frac{4\pi}{q_1 q_2} \int dr_2 \sin(q_2 r_2) r_2 \phi_0(\hat{r}_2) \int dr_1 \sin(q_1 r_1) r_1 F_0(r_1)$$

The angular dependence is then contained within the momentum transfers $q_1(\theta)$ and $q_2(\theta)$.

The lack of range in momentum transfer implied by figure 4.3 suggests, via the properties of the Fourier transforms of equation 4.72, that the range of the function $F_0(r_1)$ is too long. The major difference between the 'cosh' potential and a traditional Woods Saxon type of potential is that the tail of the 'cosh' potential is much longer. As a result of this we will employ a radial wave function obtained by using a Woods Saxon well to bind the triton to the $^{16}O$ core. For comparative purposes the functions $r, V(r), \phi_0(r)$ for the 'cosh' and Woods Saxon wells are shown in appendix A, together with their Fourier transforms.

It was recognised in section 4.2, in which the spectroscopic factors were introduced, that the cluster model does not offer a complete description for the two nuclei $^{19}F$ and $^{31}P$ under consideration. The parameters for the Woods Saxon well, chosen to bind the clusters in the projectile were taken as those of [Ku 77]. These parameters, listed in appendix A, are conventional and make no attempt to reproduce the observed
properties of the $^{19}$F nucleus. The radial wave function $\phi_0(r_1)$ obtained using a 'cosh' potential is slightly longer in range than that obtained with a Woods Saxon potential. The largest effect on the range of the product $\phi_0(r_1)V(r_1)$ arises from the potential itself.

The potentials obtained from the folding procedure are not particularly sensitive to exact details of the radial components of the wave function. The folded potentials calculated with a radial function obtained from a Woods Saxon well are slightly shorter in range than those previously obtained using 'cosh' potentials. The elastic and coupled inelastic channels calculations were repeated using folded potentials obtained using Woods Saxon bound radial functions. The results, which will not be shown here, are essentially the same as those obtained using 'cosh' bound radial functions. There is however a slight reduction in the magnitude of the vector analysing power produced by the three channel calculation of about 4 per cent. The phase of the oscillations in the calculated vector analysing power are unchanged, and the discussion of the earlier results for $^{19}$F + $^{28}$Si scattering are still valid. The reduction in the magnitude of the vector analysing power arises from a reduction in the magnitude of the effective spin orbit potential, caused in turn by a reduction in the range of the coupling potentials.

A calculation to test the ranges used to perform the double integration over $R_a$ and $R_b$ was performed for the code TWOPNR. The calculation was performed with no Coulomb or distorting potentials present. The calculation was compared against a result obtained by evaluating equation 4-72 numerically. The two calculations were in excellent agreement.
4.4.2 Results with modified entrance wavefunction

Figures 4.6 to 4.8 show the results of calculations using the computer code TWOFNR [Ki] for the transfer reactions considered. The spectroscopic factors were obtained as before. The theoretical curves are now in much better agreement with the experimental data, in particular the fall off with increasing angle is reasonable. The phase and magnitude of the oscillations in the calculations and data however are not in agreement. It was precisely this problem of reproducing the observed phase that led [Ku 77] to the suggestion that the data required a large spin orbit force in the entrance channel. The results presented in figures 4.6 to 4.8 suggest that the models used to describe the reactions are reasonable. It is thus now appropriate to consider the effects of projectile excitation on the transfer cross sections.

The spectroscopic factors $\Theta(bx;Ax)$ are much larger than those obtained with the cosh potential. This supports the arguments of section 4.5.1, since the smaller spectroscopic factor for the cosh potential suggests that the coupling between the entrance and exit channels is too strong. This in turn implies that the product $V_{ed}(r_1)\phi(r_1)$ is too long. The spectroscopic factors obtained here are very similar to those of [Ku 77].
Figure 4.6 $1/2^+$ DWBA transfer cross section.
Figure 4.7 3/2$^+$ DWBA transfer cross section.
Figure 4.8 5/2$^+$ DWBA transfer cross section.
Chapter 5.

5.1 Inclusion of projectile excitation

The purpose of this chapter is to combine the work of chapters 2 and 4, and to see whether the mechanism of projectile excitation can offer an explanation of the observed data.

This may be achieved by allowing the folded potentials used in the entrance channel to induce transitions to the low lying inelastic $^{19}$F states. This is equivalent to allowing the distorting potentials in the entrance channel to become generalised distorting potentials. As in chapter 2 the inelastic transitions will be considered within the framework of coupled channels. The transitions between each of the $^{19}$F + $^{28}$Si channels considered and the exit channel will be treated as single step DWBA transitions. The whole problem will thus be solved by a coupled channels Born approximation, or CCBA, approach [Pe 64, Ta 65b, Ta 67, Du 68, As 69, Co 69, Ta 70, Ta 71, Ud 74]. It should be realised that the treatment is not simply a single step DWBA calculation with a modified distorting potential in the entrance channel including effective spin orbit forces defined by, for example, equation 3-20. Rather it is a sum of transitions between entrance partition states that are obtained as solutions of coupled channels calculations, and the final state under consideration.

Due to computing restrictions it was only possible to perform CCBA calculations for transitions leading to the ground state $^{31}$P (1/2$^+$, 0.0 MeV) + $^{16}$O final channel.
5.2 Derivation of the CCBA

Within the consistent framework of a cluster model description for the nuclei under consideration, and a folding model prescription for the generalised distorting potentials the CCBA can be derived in the same manner as the DWBA. An ansatz wave function of the form

\[ \Psi(x_A, x_B; \vec{K}_a, \vec{K}_b; \vec{R}_a, \vec{R}_b) = \sum_{\alpha} \phi_{\alpha}(x_A) u_{\alpha'}(\vec{K}_a, \vec{R}_a) + \phi_{\beta}(x_B) u_{\beta}(\vec{K}_b, \vec{R}_b) \]

is adopted, where the sum runs over all of the included entrance partition channels considered, including the elastic channel. Asymptotically the \( U_c(\vec{K}_c, \vec{R}_c) \) behave as

\[ U_{\alpha'}(\vec{K}_a, \vec{R}_a) = e^{i\vec{K}_a \cdot \vec{R}_a} \delta_{\alpha \alpha'} + \frac{e^{i\vec{K}_a \cdot \vec{R}_a}}{R_a} f_{\alpha'}(\vec{K}_a, \vec{R}_a) \]

\[ U_{\beta}(\vec{K}_b, \vec{R}_b) = \frac{e^{i\vec{K}_b \cdot \vec{R}_b}}{R_b} f_{\beta}(\vec{K}_b, \vec{K}_a) \]

The asymptotic value of \( K_{\alpha}' \) is given by

\[ \frac{\hbar^2}{2\mu_A} K_{\alpha}'^2 = \frac{\hbar^2}{2\mu_A} E_{\alpha}' \]

and \( K_b \) by equation 4-42, with \( E_{\alpha} = 0 \).

\[ \frac{\hbar^2 K_b^2}{2\mu_B} = \frac{\hbar^2 K_a^2}{2\mu_A} - Q \]

The ansatz wave function \( \Psi(x_A, x_B; \vec{K}_a, \vec{K}_b, \vec{R}_a, \vec{R}_b) \) satisfies the time independent Schrödinger equation

\[ (E-H) \Psi(x_A, x_B; \vec{K}_a, \vec{K}_b, \vec{R}_a, \vec{R}_b) = 0 \]

where the Hamiltonian can be written in the forms of equations 4-5 or 4-6. A set of coupled equations for the \( U_c(\vec{K}_c, \vec{R}_c) \) are obtained by multiplying
equation 5-6 to the left by a member of the expansion 5-1, and integrating over the corresponding internal coordinates. If N excited entrance partition states are included in the sum, the following N + 1 coupled equations are obtained.

\[
\left[ (E - \varepsilon'_A) - T_{AA}' - \langle A'|V_{xx} + V_{ba}|A'\rangle \right] \mathbf{U}_{AA}'(\mathbf{\tilde{r}}_a', \mathbf{\tilde{r}}_a) = \sum_{A''} \langle A'|V_{xx} + V_{ba}|A''\rangle \mathbf{U}_{AA}''(\mathbf{\tilde{r}}_a', \mathbf{\tilde{r}}_a)
+ \int \mathrm{d}x_A \mathbf{\phi}_A^*(x_A)(H-E) \mathbf{\phi}_B(x_B) \mathbf{U}_B(\mathbf{\tilde{r}}_b', \mathbf{\tilde{r}}_b) \quad 5-7
\]

\[
\left[ (E - Q) - T_{BB} - \langle B|V_{xx} + V_{ba}|B\rangle \right] \mathbf{U}_B(\mathbf{\tilde{r}}_b, \mathbf{\tilde{r}}_b) = \int \mathrm{d}x_B \mathbf{\phi}_B^*(x_B)(H-E) \sum_{A'} \mathbf{\phi}_{A'}(x_A) \mathbf{U}_{A'A}(\mathbf{\tilde{r}}_a', \mathbf{\tilde{r}}_a) \quad 5-8
\]

The sum over A'' in equation 5-7 excludes the A'' = A' term. The orthogonality of the \( \mathbf{\phi}_A(x_A) \) has been used in obtaining equation 5-7.

Changing the order of summation and integration in equation 5-8 leads to

\[
\left[ (E - Q) - T_{BB} - \langle B|V_{xx} + V_{ba}|B\rangle \right] \mathbf{U}_B(\mathbf{\tilde{r}}_b, \mathbf{\tilde{r}}_b) = \sum_{A'} \int \mathrm{d}x_B \mathbf{\phi}_B^*(x_B)(H-E) \mathbf{\phi}_{A'}(x_A) \mathbf{U}_{A'A}(\mathbf{\tilde{r}}_a', \mathbf{\tilde{r}}_a) \quad 5-9
\]

Formally solving this equation yields

\[
\mathbf{U}_B(\mathbf{\tilde{r}}_b, \mathbf{\tilde{r}}_b) = \sum_{A'} \int \mathrm{d}\mathbf{\tilde{r}}_b' G(\mathbf{\tilde{r}}_b', \mathbf{\tilde{r}}_b) \int \mathrm{d}x_B \mathbf{\phi}_B^*(x_B)(H-E) \mathbf{\phi}_{A'}(x_A) \mathbf{U}_{A'A}(\mathbf{\tilde{r}}_a', \mathbf{\tilde{r}}_a) \quad 5-10
\]

where \( G(\mathbf{\tilde{r}}_b', \mathbf{\tilde{r}}_b) \) is given by equation 4-12. Using the asymptotic form of this Green's function equation 5-10 becomes

\[
\mathbf{U}_B(\mathbf{\tilde{r}}_b, \mathbf{\tilde{r}}_b) = (\frac{-\mathbf{M}_B}{2\pi\mathbf{k}_b^2}) \frac{e^{ik_b\mathbf{R}_b}}{\mathbf{R}_b} \sum_{A'} \int \mathrm{d}\mathbf{\tilde{r}}_b' \int \mathrm{d}x_B \mathbf{\chi}_A(x_A) \mathbf{\phi}_B^*(x_B)(H-E) \mathbf{\phi}_{A'}(x_A) \mathbf{U}_{A'A}(\mathbf{\tilde{r}}_a', \mathbf{\tilde{r}}_a) \quad 5-11
\]
where $\chi^{(\cdot)\ast}_{b} (\vec{R}_b^\cdot, \vec{R}_b)$ was discussed in section 4.2. Comparing equations 5.11 and 5.3 together with the definition of equation 4-17 leads to the identification of the transition amplitude, exact within the limitations of the ansatz wave function, as

$$T_{BA} (\vec{R}_b^a, \vec{R}_a) = \left( \frac{-\mu_b}{2\pi \hbar^2} \right) \sum_{A'} \int dR_b^d d\chi_b \chi^{(\cdot)\ast}_{b} (\vec{R}_b^\cdot, \vec{R}_b)$$

$$\phi^{\ast}_b (\chi_b) (H - E) \phi_A (\chi_A) \sum_{A'} U_{A'A} (\vec{R}_a^{'}, \vec{R}_a)$$

5-12

The two approximations that were used to define the DWBA transition amplitude may be used again.

1) Neglect the term coupling the transfer channel back to any of the $^{1200}F^{\ast} + ^{28}\text{Si}$ channels.

2) Express the Hamiltonian in equation 5-12 in the form of equation 4-6, and assume that equation 4-22 is valid.

The first of these approximations replaces $U_{A'A} (\vec{R}_a^{'}, \vec{R}_a)$ by $\chi^{(\cdot)}_{A'A} (\vec{R}_a^{'}, \vec{R}_a)$, which satisfy the coupled equations

$$\left[ (E - E_{A'}) - T_{A'A} - \langle A' | V_{xA} + V_{bA} | A' \rangle \right] \chi^{(\cdot)}_{A'A} (\vec{R}_a^{'}, \vec{R}_a)$$

$$= \sum_{A''} \langle A' | V_{xA} + V_{bA} | A'' \rangle \chi^{(\cdot)}_{A''} (\vec{R}_a^{'}, \vec{R}_a)$$

5-13

which were discussed in chapter 2.

With these approximations the CCBA transition amplitude can be written as
This can be written as

\[ T_{BA} (\hat{R}_b, \hat{R}_a) = \sum_{A'} T_{BA'} (\hat{R}_b, \hat{R}_a) \]

that is as a sum of transitions between entrance partition states, and the final state. \( T_{BA'} (\hat{R}_b, \hat{R}_a) \) is given by

\[ T_{BA'} (\hat{R}_b, \hat{R}_a) = \int d\hat{R}_b d\alpha_B \chi_B^{(\gamma)} (\hat{R}_b, \hat{R}_b) \phi_B^* (x_B) V_{bx} (r_i) \phi_A^* (x_A) \chi_{\alpha' A} (\hat{R}_a, \hat{R}_a) \]

The inclusion of the inelastic excited states in the entrance partition thus has two main effects, in comparison with the DWBA formalism.

1) The inclusion of two step reactions, passing via inelastic channels to the final state.

2) The replacement of the distorted wave in the entrance channel by the solution of the coupled equations.

The calculation is shown schematically in figure 5.1. From figure 3.7 the cross sections for \(^{19}F + ^{29}Si\) obtained from single or coupled channels calculations differ very little. This however does not imply that the functions \( \chi_{\alpha A} (\hat{R}_a, \hat{R}_a) \) and \( \chi_{\alpha' A} (\hat{R}_a, \hat{R}_a) \) are the same. The calculations for the elastic observables are sensitive only to the asymptotic behaviour of these functions. Although from the earlier results the functions are obviously similar in this region, there is no reason to suppose that this is true at the shorter radii that the transfer cross sections are sensitive to.
The integral can, as before, be converted to an integration over the variables \( \hat{R}_a \) and \( \hat{R}_b \).

\[
T_{BA} (\hat{K}_b, \hat{K}_a) = \mathcal{J} \Theta \left( b x : A x \right) \sum_{A'} \int d\hat{R}_a d\hat{R}_b \chi^{(*)}_B (\hat{R}_b, \hat{R}_b) \phi^{*}_{d-\alpha_b} (r_1, \sigma_1) V_{ba} (r_1) \phi'_{d-\alpha_b} (r_1, \sigma_1) \chi^{(+)}_{A'A} (\hat{K}_a, \hat{R}_a)
\]

where the Jacobian \( \mathcal{J} \) is defined in equation 4-32, and the spectroscopic factor \( \Theta \left( b x : A x \right) \) has been defined as the overall factor for the CCBA transition, and again includes the combinatorial factors that arise from the explicit consideration of antisymmetisation of the nucleons. Equation 5-17 offers a prescription for obtaining the transition amplitude using the CCBA. In principle the coupled channels scattering functions \( \chi_{A'A} (\hat{K}_a, \hat{R}_b) \), obtained by the techniques of chapter 2, and equation 5-15 can be used together with the explicit form of equation 4-59 shown in section 4.2.5 to solve the problem. The general scattering and bound state functions can now be labelled more explicitly in terms of their spin properties as

\[
T_{BA} (\hat{R}_b, \hat{K}_a) = T_{M_I B M_I A} (\hat{R}_b, \hat{K}_a)
\]

where the \( T_{M_I B M_I A} (\hat{R}_b, \hat{K}_a) \) are given by

\[
T_{M_I B M_I A} (\hat{K}_b, \hat{K}_a) = T_{M_I' B M_I' A} (\hat{K}_b, \hat{K}_a)
\]

Exactly as in equation 4-33. The transition matrix this time is given by,

\[
T_{M_I B M_I' A} (\hat{R}_b, \hat{K}_a) = \mathcal{J} \Theta \left( b x : A x \right) \sum_{I_A} \int d\hat{R}_b d\hat{R}_a \chi^{(*)}_{M_B/M_A} (\hat{R}_b, \hat{R}_a) \chi_{I_A I_A (+)} (\hat{K}_a, \hat{R}_a)
\]

the analogue of equation 4-34. In order to solve this expression it is again convenient to reduce the six-dimensional integrals to sums over two
dimensional integrals by partial wave decompositions of the generalised distorted waves, and multipole decompositions of the nuclear matrix elements. The distorted waves are considered in section 5.2.1. With an obvious reinterpretation of the quantum numbers present, the decomposition of the nuclear matrix elements $\langle I_0 M_0 | V_{ba} (R_1) | I_0' M_0' \rangle$ are given in sections 4.2.2 to 4.2.4. The transition matrix of equation 5.20 is expressed, in partial wave form, in section 5.2.3.

5.2.1 Partial wave decomposition of the distorted waves

The decomposition of the distorted waves in the outgoing channel is given by equation 4-40. The radial functions $\chi_{L_a I_a}^{J_b} (R_b)$ being the solutions of equations 4-41. The entrance channel scattering functions can be expanded over partial waves according to equation 2-19. Redefining the quantum labels that appear in that section of this work the expansion may be performed according to

$$\chi_{L_a I_a}^{J_a} (R_a) = \sum (L_a' M_a I_a' M_a | I_m J_m) (L_a M_a I_a M_a | L_a M_a I_m J_m)$$

$$\chi_{L_a I_a}^{J_a} (R_a) \gamma_{L_a' M_a' I_a' M_a'} (R_a) \gamma^*_{L_a M_a I_a M_a}$$

where the sums run over $J_a, M_{a'}, I_a, L_a, M_a$ and $M_{a'}$. The $\chi_{L_a' M_a' I_a' M_a'} (R_a)$ satisfy the radial equations

$$[\left( E - E_a \right) - T_{R_a} - V_{coul} (R_a) - V_{l_a I_a}^{J_a} (R_a)] \chi_{L_a I_a}^{J_a} (R_a)$$

$$= \sum_{L_a'' I_a''} V_{l_a'' I_a''}^{J_a''} (R_a) \chi_{L_a'' I_a''}^{J_a''} (R_a)$$

5-22

The terms with $I_a'' = I_a$ and $L_a'' = L_a''$ are excluded from the right hand side of this expression. These equations can be solved by the numerical techniques
discussed in [Bu 63, Ta 65].

5.2.2 Partial wave decomposition of the T matrix

Using equations 5-20, 5-21 and 4-46 to 4-50 the transition matrix can be written as

\[
T_{M_I^J_b M_I^a M_I^b M_I^a}(\widehat{R}_b, \widehat{R}_a) = \mathcal{J}(bx:Ax) \left[ \prod_{k_b, k_a} \sum_{I_a^a} (-)^{I_a^a + I_b^a + I_a^b} \langle I_a^a, M_a^a | I_b^a, M_b^a \rangle \right] (L_b M_b, I_b M_b, J_b M_b) (L_a M_a, I_a M_a, J_a M_a) (L_a^a M_a^a, I_a^a M_a^a, J_a^a M_a^a)
\]

\[
(R_a, I_a^a) (R_b, I_b^b) \int dR_a dR_b R_b^* L_a^b \left( \vec{R}_b, \vec{R}_a \right) \right] L_a^b \left( \vec{R}_b, \vec{R}_a \right)
\]

where the unlabelled sum runs over \(L_a, M_a, I_a, M_a^a, I_a^a, M_a^a\) and \(M_b, J_b\), the analogue of equation 4-59. The orthogonality integrals over the angles of \(\vec{R}_a\) and \(\vec{R}_b\) present in the earlier expression having been explicitly performed. Although this expression is nearly identical in appearance with equation 4-59, the presence of the summation over \(I_a^a\) implies that many more of the time consuming radial integrals are required for its evaluation. In this expression \(h_{L_a^b L_a^a}^{L_b M_b} (R_b, R_a)\) is given by equations 4-57 and 4-58. Adopting the coordinate system with z along \(\widehat{R}_a\) and y along \(\widehat{R}_b\) to describe the reaction simplifies the angular dependence of this expression to

\[
Y_{L_b M_b}^* (\widehat{R}_b) Y_{L_a M_a}^* (\widehat{R}_a) = \frac{1}{\sqrt{4\pi}} \int_{-1}^{1} \int_{-1}^{1} \left[ \frac{(L_b - M_b)}{(L_b + M_b)} \right]^{1/2} P_{L_b M_b} (\Theta_b)
\]

The transition matrix for the CCBA calculation is thus completely defined.
Figure 5.1 Schematic representation of the CCBA calculations.
5.3 Results of the CCBA calculations

The matrix $T_{BA'}(\Theta)$, and hence the observables for the CCBA reaction, were obtained using the computer code FRESCO [Th]. This is the only code available to the author capable of performing the CCBA calculations in the full exact finite range used to treat the DWBA steps of the reaction paths. The full CCBA results presented here are thus unchecked against any other code. As mentioned in section 3.1 the coupled channels calculations for the $^{19}F + ^{28}Si$ system were checked between FRESCO and CHUCK, and found to be in excellent agreement for the elastic and inelastic observables. The comparison between the two codes was actually performed using a full CCBA calculation in FRESCO, hence the scattering functions $\chi_{A' A}(\vec{R}_1, \vec{R}_2)$ used in the CCBA calculation are in agreement with those obtained from CHUCK.

The DWBA results presented in figures 4.3 to 4.7 were checked between the codes TWOFNR and FRESCO. The single step DWBA transitions may thus be believed. These checks represent the most thorough that the Author was able to perform. Figures 5.2 and 5.3 show the results of CCBA calculations leading to the $^{31}P(1/2^+ \text{ g.s.}) + ^{16}O$ channel. Figure 5.2 is the result of a CCBA calculation including the ground and first excited states in the entrance $^{19}F + ^{28}Si$ partition. Figure 5.3 is the result using the ground and first two $^{19}F$ excited states in the entrance partition. The spectroscopic factors have been obtained, as before, by scaling the results to the experimental data at the first peak.
Figure 5.2 1/2+ CCBA transfer cross section with two channels in $^{19}$F+$^{28}$Si partition.
Figure 5.3 1/2⁺ CCBA transfer cross section with three channels in $^{19}$F + $^{28}$Si partition.
5.4 Discussion of the CCBA results

Repeating the CCBA calculations using 'cosh' potentials produces curves that are very similar to figure 4.3, these will not be shown. The 'cosh' results will be ignored in any further discussion.

Comparing figures 5.2 and 5.3 to 4.6 two main features can be observed.

1) There is a small, but significant effect produced by the inclusion of the excited states in the entrance partition, and

2) The net effect is to reduce the magnitude of the oscillations in the predicted transfer cross sections, whilst leaving the fall off with increasing angle largely unchanged.

The first feature is reasonable. Assuming the transfer between an excited $^{19}$F + $^{29}$Si channel to be comparable with that between the elastic and final channels, the two step contribution would be expected to be significantly smaller than the one step. This, of course, says nothing about the effect of replacing the entrance channel scattering function by the solution of the coupled equations.

The second observed effect can be understood in terms of the replacement of the elastic scattering function $\chi_A(\vec{R}_a)$ by the solution of the coupled equations $\chi_{A/A}(\vec{K}_a, \vec{R}_a)$. The results and discussion of chapter 3 indicate that the inclusion of the excited states
produces an effective, complex, spin orbit potential in the entrance channel. The arguments of the following section will show that the reduction in magnitude of the oscillations in the observed transfer cross section are consistent with the inclusion of a spin orbit force in the entrance channel.

Since only $L = 0$ transfers are being considered in this section of the work the semiclassical arguments of section 3.3 can be used to infer that the two step transition, passing through the entrance partition excited channel, introduces spin dependence into the reaction path. The final state shown in figures 3.8 and 3.9 can be viewed as the residual $^1\!{}^o\!{}^e\!{}^0\!{}^+{^3}\!^1\!{}^p\!{}^0$ channel, rather than the elastic entrance channel. The arguments of section 3.3 then follow, and the reaction path becomes spin dependent. There are some differences from the case in which the final channel is the elastic one however. In particular the coupling from to the final state is caused by matrix elements of the form of equation 4-46, rather than equation 2-92. Although coupling via the former are weaker the arguments still follow. The induced spin dependence would however be expected to be reduced in comparison to the case in which the final channel is the elastic one.

It is the presence of spin dependence produced within this reaction path that makes it necessary to perform the full CCBA calculation rather than, say, fitting phenomological spin dependent optical potentials to the coupled channels results in order to obtain a $\tilde{\chi}_{A}(\vec{R}_a,\vec{R}_a)$ that resembles the $\chi_{AA}(\vec{R}_a,\vec{R}_a)$. For the sake of simplicity the arguments of the next section assume that the effect produced by the inclusion of the entrance partition inelastic channels is simply spin dependence in the entrance channel. The
spin dependence arising from the passage through the inelastic channel to the final residual transfer channel being included in the overall effective spin orbit potential acting in the entrance channel.

5.4.1 Effect of a spin orbit force on the transfer cross section

The oscillatory nature of the observed transfer cross section can be understood in terms of nearside/farside interference, in analogy with the discussion of section 3.5 [St 73]. In principle the decomposition for the transfer cross section can be made by replacing the $P_{LM}(\cos(\theta))$ of equation 5-24 by the $Q_{LM}(\cos(\theta))$ discussed in section 3.5. However the identification in terms of orbits is not so clear [Jo 85, To 85]. To avoid pedantic discussion handwaving arguments will be used to describe to describe the transfer cross section in nearside/farside terms [Pr 74, Sh 74, To 85].

From figure 3.11 the transfer reaction can be viewed as occurring in three steps. 1) The interacting entrance partition nuclei moving along orbits, on either side of the target nucleus. 2) The transfer of the inert triton cluster between the participating nuclei. 3) The outgoing nuclei travelling along orbits on either side of the residual nucleus. The outgoing orbits will differ from the entrance ones, for a given side, because of the different charge distributions and distorting potentials acting in the two channels, and the different energies due to the $Q$ value of the reaction.

Just as in the elastic case the passage, particularly along orbits close enough that the transfer can occur, is altered between the two sides
for a particular spin orientation. The phase of the two waves differs depending upon which side of the nucleus they have travelled. So that when they re-meet along the detection direction the cancellation and reinforcement is less pronounced. Figures 5.2 and 5.3 are consistent with these arguments. The results for the elastic observables indicating that the two channel calculation produces the largest effective spin orbit force, consistent with the smallest oscillations in the corresponding transfer calculation. These arguments are in agreement with the finding of, for example [Ba 78], that the depths of the transfer cross oscillations are sensitive to details of the entrance channel spin orbit potential.

5.5 Comparison with earlier calculations

One of the motivations for performing the CCBA calculation was the suggestion [Ku 77] that the transfer reactions considered required a large spin orbit force in the entrance channel in order to reproduce the experimental data. A feature of the these earlier calculations is the central optical potentials used to obtain the distorted waves in the entrance and exit channels. These potentials are listed in appendix B for reference. The two sets A and B correspond to the spin independent and spin dependent sets of Kubono et al. respectively.

A striking feature of these potentials, apart from the large spin orbit potential in the entrance channel, is the different nature of the exit channel potentials. Set B has an imaginary component that is very short ranged, that is set B is an example of a surface transparent optical potential [Ba 75]. This surface transparency allows the far side waves to
travel around the residual nucleus without too much absorption, leading to large interference effects in the predicted cross section. As was discussed in section 5.4.1 a spin orbit potential has the effect of damping any oscillations that occur between the two branches. It is thus the author's belief that the most significant feature of the optical potentials previously used to describe these reactions is the surface transparent nature of the exit potentials. The large spin orbit potential is necessary only to damp the unacceptably large oscillations that occur when these potentials are used. These arguments have been borne out by calculations, which will not be presented here since they do not fit within the consistent framework of folding model calculations.
Chapter 6

6.1 Conclusions.

The fundamental objective of this thesis was to explore the spin dependent effects of projectile excitation in nuclear heavy ion reactions. Earlier work [Ni 84] had shown that the inclusion of inelastic projectile excited channels could explain differences in the observed vector analysing powers for the elastic scattering of $^6$Li and $^7$Li from $^{58}$Ni at about 20 MeV. This reference employed cluster model descriptions for the lithium nuclei, and included the projectile excited states via coupled channels calculations with all interactions being obtained from a single folding model.

These models have been applied consistently in this present work to study a variety of heavy ion induced nuclear reactions. The work presented here may be considered to fall into two main categories. 1) The description of the elastic and inelastic observables. 2) The analysis of the transfer reactions considered. These categories will be considered below individually.

6.2 Elastic and inelastic calculations

The models previously employed [Ni 84] were first applied to the scattering of $^6$Li from two lighter targets at an increased energy, 22.80 MeV. The decrease in the charge number of the target nuclei, and the increase in the scattering energy, make the present calculations more sensitive to details of the participating nuclei and the interactions
between them than the earlier calculations.

In contradiction to the original analysis of $^6\text{Li} + ^{16}\text{O}$ and $^6\text{Li} + ^{28}\text{Si}$ elastic scattering [ We 76 ] it was seen that the folded spin orbit potential was unable, within a consistent model, to reproduce the experimental vector analysing powers. The analysing powers for the elastic scattering of $^6\text{Li}$ were seen to be dominated by the effective spin dependence induced by the inclusion of the low lying projectile excited states.

The $^7\text{Li} + ^{120}\text{Sn}$ at 44.00 MeV results are in agreement with this finding; again projectile excitation plays an important role in reproducing the trends of the experimental spin dependent observables for the elastic scattering. The gross features of the inelastic $^7\text{Li} (1/2^-, E_x = 0.48 \text{ MeV})$ cross section and analysing power are reproduced well by the models employed, although the magnitude of the inelastic cross section is smaller than the experimental data by a factor of about two. The analysis given here considers excitation to arise only from nuclear coupling terms. Coulomb excitation may provide an explanation of the "missing" cross section, particularly in light of the discussion of section 4.2. Coulomb excitation was not considered due to the problems involved in handling the infinite ranged coupling potentials [ Lo 83 ]. It should however be realised that the inclusion of Coulomb excitation may well enhance the effective spin dependence, and that the elastic results might not be reproduced so well if it were included.

The $^{19}\text{F} + ^{28}\text{Si}$ results for scattering at 60.00 MeV show that the models used at lower energies with lighter projectiles are still applicable for
this system. The induced spin orbit effect is reduced in comparison with the lighter systems however. This is in accordance with the expectation that the magnitude of the induced spin dependent effects would decrease as the center of mass scattering energy becomes much larger than the excitation energies of the states considered.

Overall projectile excitation is clearly an important part of the reaction mechanism, and is applicable for a variety of situations in which the projectile is well described in terms of a cluster model.

6.3 Transfer Calculations

The predicted spin dependence produced by the projectile excitation mechanism for the $^{19}$F+$^{28}$Si system provides an opportunity to test the hypothesis [Ku 77] that the data from the $^{28}$Si($^{19}$F,$^{16}$O)$^{34}$P reactions require a large spin orbit potential in the entrance channel. The results of chapters 4 and 5 suggest that this hypothesis may not be true. The arguments presented in those chapters suggest that the data may be sensitive to exit channel effects, rather than entrance channel ones.

The results of chapter 5 do however indicate that the inclusion of projectile excitation produces a spin dependent effect on the transfer cross section. The agreement with the experimental data analysed here is not improved by the inclusion of projectile excitation however.

As experimental techniques, particularly the production of polarised heavy ion beams, continues to improve spin dependent transfer data should become available. Such data will then provide a sensitive test of the
induced spin dependence caused by inelastic events in the entrance channel.

6.3.1 Suggestions for future work

The folding model DWBA calculations agree fairly well with those obtained using set A of appendix B. This is quite significant since, apart from the arbitrary choice of parameters chosen to bind the triton to the $^{16}$O and $^{28}$Si cores, these are essentially parameter free calculations.

Differences are observed upon examining the spectra of the two nuclei $^{19}$F and $^{31}$P. $^{19}$F exhibits a well defined cluster structure, the excited states with $I^\pi = 5/2^+$, $3/2^+$ have eigen energies which are reasonable if they are viewed as members of a spin orbit split doublet, with $l = 2$. The spectrum of $^{31}$P however is different, in particular the two excited states, $5/2^+$ and $3/2^+$ are inverted with respect to $^{19}$F. It is known that $^{28}$Si, although $0^+$ in its ground state, is a well deformed nucleus [Sw 69, Ho 71b, Re 72, Ko 84].

These facts, together with the closing remarks of section 5.4.2 lead to the suggestion that the problem in reproducing the phase of the transfer cross section data lies not in the entrance channel, but rather in the exit channel.

A possible calculation to test this hypothesis would be a CCBA calculation, including the first two excited states in the exit partition, together with a better model for the residual $^{31}$P nucleus.
Appendix A

This appendix defines the functional forms of the potentials used throughout the preceding work, and gives the radial forms of the nuclear states considered. Within the literature there exist varying definitions for these potentials. The parameters listed in this appendix, and appendix B, are consistent with the definitions given below. In some cases the parameters have been adjusted from their original published values due to the differing definitions. All potential depths $V_0$ are in MeV, and radius parameters $r_0$, $a_0$ and $R_c$ are in fm.

The Coulomb potential between two charged spherical bodies is taken as

$$V_c(r) = \begin{cases} \frac{-Z_1Z_2e^2}{r} & r > R_c \quad \text{A-1a} \\ -\frac{Z_1Z_2e^2}{2R_c \sqrt{3 - \left(\frac{r}{R_c}\right)^2}} & r < R_c \quad \text{A-1b} \end{cases}$$

where $Z_1$ and $Z_2$ are the number of protons in each of the nuclei, $e$ is the proton charge and $R$ is the charge radius, often chosen to be approximately $0.8 \ A^{1/3}$, where $A$ is the mass number of the heavier nucleus.

The short ranged nuclear potentials used throughout the preceding work are defined as

Woods Saxon (WS)

$$V(r) = -V_0 \left[ 1 + \exp \left[ \frac{(r - r_0 A^{1/3})/a_0}{\alpha_0} \right] \right]^{-1} \quad \text{A-2}$$

Woods Saxon Derivative (WSD)
\[ V(r) = -V_o \exp \left[ \frac{(r-r_0 A^{1/3})}{a_o} \right] \left[ 1 + \exp \left[ \frac{(r-r_0 A^{1/3})}{a_o} \right] \right]^{-2} \]  

Thomas spin orbit (TSO)

\[ V(r) = -\frac{2}{a_o} V_o \exp \left[ \frac{(r-r_0 A^{1/3})}{a_o} \right] \left[ 1 + \exp \left[ \frac{(r-r_0 A^{1/3})}{a_o} \right] \right]^{-2} \]  

Cosh (CO)

\[ V(r) = \frac{-V_o \left[ 1 + \cosh \left( \frac{r_0}{a_o} \right) \right]}{\cosh \left( \frac{r}{a_o} \right) + \cosh \left( \frac{r_0}{a_o} \right)} \]  

Cosh spin orbit (CSO)

\[ V(r) = -V_o \sinh \left( \frac{r}{a_o} \right) \left[ 1 + \cosh \left( \frac{r_0}{a_o} \right) \right] \left[ \cosh (r/a_o) + \cosh (r_0/a_o) \right]^{-2} \]  

Table A1 lists the parameters that were used to define the potentials that bind the radial parts of the inter cluster wave functions for the nuclear states considered. An asterix denotes that the nuclear state is resonant, rather than bound, the 'separation' energy in this case is the scattering energy. All spin orbit depths and potential geometries are fixed, the depths \( V_o \) are obtained as variational parameters.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>State</th>
<th>Potl. Sep.energy</th>
<th>( V_o )</th>
<th>( r_o )</th>
<th>( a_o )</th>
<th>( R_c )</th>
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<td>( 1^+ ) (gs)</td>
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<td>77.34</td>
<td>1.197</td>
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<td>( ^6 \text{Li} )</td>
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<tr>
<td>Nucleus</td>
<td>State</td>
<td>Potl.</td>
<td>Sep. energy</td>
<td>V₀</td>
<td>r₀</td>
<td>a₀</td>
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</tr>
<tr>
<td>7Li</td>
<td>3/2⁻ (gs)</td>
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<td>93.46</td>
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<td>7Li</td>
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<td>7/2⁻ , 5/2⁻ (*)</td>
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<td>110.08</td>
<td>1.291</td>
<td>0.7</td>
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<td>1/2⁺ (gs)</td>
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<td>114.67</td>
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<tr>
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<td>WS</td>
<td>10.18</td>
<td>115.80</td>
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<tr>
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</tr>
<tr>
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<td>5/2⁺</td>
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<td></td>
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</table>
The radial forms of the functions $rU_l(r)$ are shown in the following figures.

Figure A.1.1 shows the ground state radial function for $^6$Li obtained with the parameters listed above. Figures A.1.2 and A.1.3 show the radial function taken to represent the three $^6$Li excited resonance states. A.1.2 shows the obtained resonance function and A.1.3 the radial function after matching to a decaying exponential tail. The wave functions are normalised to unity. Figure A.2.1 shows the ground state radial function, and figure A.2.2 the first excited state radial function of $^7$Li. Figures A.2.3 and A.2.4 show the resonant $^7$Li radial functions, before and after matching to a decaying exponential tail. Figures A.3.1, A.3.2 and A.3.3 show the ground, first and second excited state radial functions of $^{17}$F, obtained using the $\cosh$ potentials. Figures A.4.1 and A.4.2 show the product $rU_o(r)V_{\text{bind}}(r)$ for $\cosh$ and Woods Saxon binding potentials respectively. Figures A.4.3 and A.4.4 show their Fourier transforms, defined by equation 4-72, respectively.

Finally figure A.1.4 shows the ground state radial function for $^{31}$P.
Appendix B

The parameters that serve to define the cluster-target potentials, which are treated as inputs for the folding model code discussed in appendix C, are tabulated in this appendix. The functional forms are defined in appendix A. Due to the differing definitions for the radial dependences the values tabulated below differ in some cases from those found in the original references.

Table B

<table>
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<th>Projectile</th>
<th>Target</th>
<th>Scatt. Potential energy</th>
<th>type</th>
<th>$V_0$</th>
<th>$r_o$</th>
<th>$a_o$</th>
<th>Ref.</th>
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<td>8.0</td>
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<td>1.012</td>
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</table>

The optical potential between the triton and $^{16}$O in set B was obtained using the search code ATHISREE. Figure B.1 shows the result of an elastic calculation using this potential set. The data points, which were used as input to the search code, were obtained using set A from [ Pu 64 ].
Figure B.1 Comparison of optical potential sets A and B for the elastic reactions of a t from 16O.
Appendix C

The folding model potentials, given by equations 2-86, 2-87 for the $l = 0$ cases, and 2-88 to 2-90 for the $l = 1$ case, were obtained using a Fortran computer code, as were the coupling potentials, defined by equation 2-92. The folding was performed in coordinate space. In coordinate space the evaluation of the folded potentials requires the performance of a double integral for each of the resultant potential points required. The integrations are over the angle between the vectors $\mathbf{R}$ and $\mathbf{R}'$ of figure 2.1, and over the radial coordinate $r$. The integration over the angle between $\mathbf{R}$ and $r$ was performed by using a Gaussian integration method [Ab 70], since this is particularly appropriate for integrals bounded in the domain $-1$ to $1$. The number of terms in the expansion was varied to ensure convergence of the integral. The weights and abscissas for the integration were taken from [Ab 70]. The integration over the radial variable $r$ was performed using Simpson's rule, again the number of steps was varied in order to ensure convergence of the integrals. Although fairly inefficient, Simpson's rule offers an accurate prescription for evaluating integrals, provided that a sufficiently large number of steps are used. The University of Surrey computing unit possesses a powerful FPS120 B array processor, hence the integrations could be performed with a large number of steps. Accuracy, rather than efficiency, was strived for.

A variety of checks were performed upon the folding codes. The folded potentials obtained for the $^6\text{Li} + ^28\text{Si}$ system were checked against another folding program [Ni 84] which used a Gaussian expansion technique.
to perform the integrations. The results were found to be in good agreement.

The results were also tested explicitly. By integrating over \( dr \) in, for example, equation 2-86 the following relation can be obtained

\[
\int \frac{V_0(R) R^4 dR}{V_0(R) R^2 dR} = \frac{m_e^2 + m_p^2}{(m_e + m_p)^2} \int \frac{r^2 \phi(r)^2 dr}{r^2 \phi(r)^2 dr} + \int \frac{X_v^4 V(X_v) dX_v}{X_v^2 V(X_v) dX_v} + \int \frac{X_c^4 V(X_c) dX_c}{X_c^2 V(X_c) dX_c}
\]

This check was performed for all of the central projectile-target potentials obtained from the program, as a matter of course. Excellent agreement, usually to better than six significant figures, was obtained.

A similar check can be performed for the \( k = 2 \) multipole terms [Sa 62, Ma 75, Ma 77, Sa 79]

\[
\int R^2 V_2(R) dR = \int \frac{r^2 U(r)^2 dr}{r^2 U(r)^2 dr} + \int \frac{X_e^2 V(X_e) dX_e}{X_e^2 V(X_e) dX_e} + \int \frac{X_p^2 V(X_p) dX_p}{X_p^2 V(X_p) dX_p}
\]

Examples of the \( k = 2 \) multipole terms were checked, again excellent agreement was obtained.

Some examples of the folded potentials are given in the following figures. Figures C.1.1 to C.1.3 show the folded potentials for \(^6\)Li scattering from \(^{16}\)O, the curves are for real, imaginary and spin orbit potentials respectively. Figures C.2.1 to C.2.3 show the real, imaginary and spin orbit potentials for \(^6\)Li scattering from \(^{28}\)Si. Figures C.3.1 to C.3.3 again show the real, imaginary and spin orbit potentials between \(^{19}\)F and \(^{28}\)Si. The real and imaginary potentials between \(^{16}\)O and \(^{31}\)P, acting in the outgoing channel in the DWBA and CCBA calculations are shown in figures.
C.1.4 and C.2.4 respectively. Figures C.4.1 to C.4.3 show the potentials between $^7$Li and $^{120}$Sn. The curves correspond to real, imaginary, and spin orbit potentials, acting in the ground state. Figure B.4.4 shows a real $k = 4$ potential, acting within the resonant $I^\pi = 7/2^-$ channel. The corresponding imaginary $k = 4$ term is very similar, although reduced in magnitude. Figures C.5.1 and C.5.2 show the real and imaginary $k = 2$ terms, acting in the ground state $I^\pi = 3/2^-$ channel between $^7$Li and $^{120}$Sn. Figures C.5.3 and C.5.4 show real and imaginary $k = 2$ terms that couple the ground and second, $I^\pi = 7/2^-$, excited states of $^7$Li. They are very similar in appearance to the $k = 2$ terms that act in the ground state. Apart from the obvious fact that they are opposite in sign, two interesting features can be seen by considering these curves. The first is that the radial dependence of the imaginary potentials are almost equal, but the magnitude of the coupling potential is slightly reduced. This is because the overlap of the radial functions in the ground state is unity, and slightly reduced for the coupling terms. The second interesting feature is that the real terms show the same trend. This confirms that the real $k = 2$ term in the ground state is dominated by the component arising from the central term, rather than the spin orbit ones.

The shapes of the $k = 2$ and $k = 4$ terms shown in the last set of figures is typical of those that arise from all of the calculations.
C.1.1

C.1.2

C.1.3

C.1.4
Appendix D

The coupled channels results presented in chapter three were obtained using the computer code CHUCK3 [Co]. Although the code was originally written for target excitation, it can perform inelastic projectile excitation calculations by the careful interpretation of the angular momenta present. Inelastic projectile channels can be included by the use of the ICODE = 3 option.

The matrix elements responsible for coupling between, and within, the channels are given by equations 2-91 and 2-92. The computer code calculates some, but not all, of the angular momentum coefficients present in equation 2-92. Writing the coupling potentials, according to equation 2-91, as

\[
\langle J(L'I') M_J | V(R, r) | J(L'I) M_J \rangle = \sum_R V^{JR}_{LI, L'I'}(R) D-1
\]

The \( V^{JR}_{LI, L'I'}(R) \) can be written, in accordance with the program code documentation, as

\[
V^{JR}_{LI, L'I'}(R) = i^{L_L - L'_L} \left( \frac{\alpha}{2} \right)^{\frac{1}{2}} W(L'L'I'I; kJ) \beta \frac{\hat{r}}{R} W_{K}(R) D-2
\]

where

\[
W_{k}(R) = \int dr \ r^2 \ U^*_{\alpha'}(r) \ U^k(R,r) \ U_{\beta}(r) D-3
\]

Equation 10 of the computer code documentation has been used, with \( I_A = 0 \), \( I_b = 0 \), and \( j = 0 \), with \( l = s = k \). \([\alpha]\) and \([\beta]\) are the intrinsic parities of the nuclear states. Equation 2-92 can be rewritten in the factored form
The term inside the square brackets is calculated automatically within the code. The term inside the round brackets can then be identified with the factor \( \beta_k \) introduced in equation D-2. The code thus requires as inputs the factor \( \beta_k \) and the form factor \( W_k(R) \), calculated by the code discussed in appendix C, in order to perform the coupled channels calculations.

The definition of \( \beta_k \) given in equation D-4 has been obtained consistently using the definitions of phases provided in the CHUCK3 documentation. However, due to a possible problem with phases within the code the negative of this quantity is required, and hence \( \beta_k \) is defined as

\[
\beta_k = - \frac{\hat{\beta}_k^3}{\sqrt{4\pi}} \cdot (-)^{I-5-L} \cdot W(l' I' I; KS) \cdot \hat{\ell}' (l' 0 k 0 | l 0) \quad \text{D-5}
\]

As was mentioned in section 3.1, these coupling factors \( \beta_k \) have been checked by comparing outputs of CHUCK3 with those produced by using the code LINA [Tu]. These calculations clearly showed that the coupling coefficients are given by equation D-5, rather than D-4.
Appendix E

The nearside farside decomposition considered in section 3.5 was performed by modifying the subroutine XSECT of the computer code CHUCK3. The regular solutions of Legendre's equation, the \( P_{LM}(\cos(\theta)) \) are already utilised in this program. The phase convention differs to that used in section 3.5. The \( Q^{(\pm)}_{LM}(\cos(\theta)) \) were constructed according to equation 3-29, as

\[
Q^{(\pm)}_{LM}(\cos(\theta)) = \frac{1}{2} \left[ P_{LM}(\cos(\theta)) \mp \frac{2i}{\pi} Q_{LM}(\cos(\theta)) \right]
\]

the phase of the resulting \( Q^{(\pm)}_{LM}(\cos(\theta)) \) was then converted in order to be consistent with the internal definitions of CHUCK3. The \( Q_{LM}(\cos(\theta)) \) were checked against tabulated values [Ab 70] for a variety of \( L \) and \( M \) values. The functions were obtained by using explicit forms for the \( Q_{LM}(\cos(\theta)) \) for \( L = 0 \) and \( L = 1 \), and using a recursion relation to define the \( Q_{LM}(\cos(\theta)) \) for successive \( L \) values.

The nearside farside decomposition for the Coulomb amplitude was performed according to equations 3-33 to 3-35. An explicit formula for the hypergeometric function \( F(1,1+i\eta,\sin^2(\theta/2)) \) given in [Ab 70] was used, and the obtained values were checked against those tabulated within this reference, for a range of arguments. The formula involves the summation of an infinite number of terms. In practice a conditional test was performed within the summation loop, and the procedure was assumed to have converged when the addition of another term affected only the sixth significant figure, or lower. The function requires many terms for small \( \theta \)
values, particularly for large $\eta$ values, but converges very rapidly for $\Theta$ values larger than, say, $20^\circ$.

The subroutine XSECT, in which the modifications were performed, is the last step in the calculations of the elastic and inelastic observables. Therefore any errors introduced in this subroutine should only affect the final results to approximately the same extent.

The overall implementation of the decomposition was checked by comparing the results obtained using the modified code with some previously published results [Fu 75a,b, Hu 84]. The comparison could only be made by comparing the trends of the calculated nearside and farside curves against the published graphs. There was agreement between the trends produced by the modified version of CHUCK3 and the previously published work. The earlier work only considered the elastic scattering of spin zero particles, and hence strictly the check is only valid for $M = 0$. Despite this the agreement between these curves, and the explicit numerical checks of the functions $Q_{LM}(\cos(\Theta))$ and $S(\Theta)$, suggest that the decomposition has been correctly implemented.
References


Am 83 : H. Amakawa and K. -I. Kubo, Private communication.


Ar 70 : A. Arima, V. Gillet and J. Ginnochio,


Ar 72 : A. Arima, H. Horiuchi, K. Kubodera and N. Takigawa,

Adv. Nucl. Phys. 5(1972)

Ar 84 : A. Arima and J. Kubono, 'Treatise on Heavy Ion Physics',


Au 64: N. Austern, R.M. Drisco, E.C. Halbert and G.R. Satchler,

Phys. Rev. 113(1964)B3.

Au 70: N. Austern, 'Direct Nuclear Reaction Theories', Wiley,


Ba 75: A.J. Baltz, P.D. Bond, J.D. Garrett and S.K. Kahana,


Ba 71: Ed. H.H. Barschall and W. Saeberli, 'Polarization Phenomena in


Ba 78: B.F. Bayman, A. Dudek-Ellis and P.J. Ellis,


Bu 67 : B. Buck, H. Friedrich and C. Wheatley,


A327(1979)29.

Ch 83 : S. Chakravarti, P.J. Ellis, B.F. Bayman, and Q.K.K. Liu,


Ch 58 : D.M. Chase, L. Wilets and A.R. Edmonds,


Ch 73 : C. Chasman, S. Kahana and M.J. Scheider,

Cl 79: N.M. Clarke, M.D. Cohler and R.J. Griffiths,


Co: J.R. Comfort, University of Pittsburgh, computer code CHUCK3,

unpublished, via N.M. Clarke, Kings College.

Co 81: J. Cook, N.M. Clarke and R.J. Griffiths,


Co 82: J. Cook, N.M. Clarke, J. Coopersmith and R.J. Griffiths,


De 74: R.M. De Vries, G.R. Satchler and J.G. Cramer,


Du 79 : W. Dunnweber, P.D. Bond, C. Chasman and S. Kubono,


Fl 83 : D. Fick, Private communication.

Fo 84: T. Fortune, Private communication.

Fr 74: W.A. Friedman, K.W. McVoy and G.W.T. Shuy,


Phys. 61(1979)1629, and


Fu 80: H. Furutani, H. Kanada, T. Kaneko, S. Nagata, H. Nishioka,


- 180 -


Gi 75 : B. Giraud, J. Le Tourneux and E. Osnes,


Gl 80 : C. W. Glover, R.I. Cutler and K.W. Kemper,

Nucl. Phys. A341(1980)137, and

C.W. Glover, K.W. Kemper, L.A. Parks, F. Petrovich and


Gr 70 : I.J.R. Griffith, M. Irshad, O. Karban and S. Roman,


Ha 80 : R.A. Hardekopf, R.F. Haglund Jr, G.C. Ohlsen, W.J. Thompson,


Ho 71c: P.E. Hodgson, 'Nuclear Reactions and Nuclear Structure',

Ho 78 : P.E. Hodgson, 'Nuclear Heavy Ion Reactions',


Ho 71b: Y. Horikawa, Y. Torizuka, A. Nakaneda, S. Mitsunoba,

Hu 84 : M.S. Hussein and K.W. McVoy, Prog. in Part. and


Ja 70 : D.F. Jackson, 'Nuclear Reactions', Methuen and Co.,
London, 1970


Ku 77 : S. Kubono, D. Denhard, D.A. Lewis, T.K. Li, J.L. Artz, D.J. Weber, P.J. Ellis and A. Dudek-Ellis,


Th 81: I.J. Thompson, computer code FRESCO, unpublished.

Th 81: I.J. Thompson and M.A. Nagarajan,

To 61: W. Tobocman, 'Theory of Direct Nuclear Reactions',

Oxford University Press, London.

To 73: W. Tobocman, R. Ryan, A.J. Baltz and S.H. Kahana,


To 85: J.A. Tostevin, modified Computer code DDTP,

from R.P. Goddard.

To 85: J.A. Tostevin and R.C. Johnson, I.O.P. conference, 1985, and


To 76: K.S. Toth, J.L.C. Ford, G.R. Satchler, E.E. Gross, D.C. Hensley,


Tu 85: G. Tungate, computer code LINA, unpublished.


Wa 58: S. Watanabe, Nucl. Phys. 8(1958)484.


Zu 80: P. Zupranski, W. Dreves, P. Egelhof, K. -H. Mobius, E. Steffens, G. Tungate and D. Fick,