Nuclear structure studies of the deformed neutron-deficient europium, samarium, and promethium isotopes.

by

Barry Duane Dmitri Pingleton

Thesis submitted for the degree of Doctor of Philosophy

Department of Physics, University of Surrey.

March 1991
Dedicated to my father George, "a reasonable man".
ABSTRACT

Nuclear dipole magnetic moments have been measured for highly neutron deficient light rare-earth nuclei using the technique of on-line low temperature nuclear orientation. A novel pulsed implantation technique has been used to set a limit of the relaxation times of both $^{139}$Eu and $^{139}$Sm isotopes in iron of < 3s.

The measured moments are compared to calculations using a particle plus triaxial rotor model, and the ground-state configurations are discussed. The large magnetic moment for the nucleus $^{139}$Eu enables the assignment of the $\{n5/2^{-}[532] \otimes n7/2^+[404]\}^-$ Nilsson two-quasiparticle configuration. The measured moments for both $^{139}$Eu and $^{139}$Sm are explained in terms of the weak coupling of the respective odd-nucleon to a strongly triaxial rotating core.
ACKNOWLEDGEMENTS

I am indebted to my supervisor, Phil Walker, for his continual help, encouragement, and not least patience, throughout the course of my PhD, and I am very grateful.

I thank Anu Bhagwat for her valuable contributions to both the experimental and data analysis aspects of the work, and for keeping my morale high during the darkest hours.

I thank Professor Ron Johnson for giving me the opportunity to study at Surrey, and for the offer of a research fellowship within the department whilst completing my studies; the SERC for providing me with financial support as an SERC student; the technical/scientific staff at the Daresbury Laboratory, especially Chris Leyland for his expert tuition on the operation of the dilution refrigerator; Irena Rikovska for helpful discussion and provision of the particle rotor codes used in this thesis; and all the collaborators from the universities of Manchester, Oxford, Surrey, and the Daresbury Laboratory, for their help in providing me with the data, and a happy environment to work in.

Special thanks are due to my mother for being my mother and for constantly feeding me on my visits home; Pete, Jon, Roy, Chris, and Sean, for buying me the occasional “few cheeky beers”, whenever they could find the excuse to do so; and Michelle and Lisa for not being physicists.

My heart felt thanks go to Stephanie and Marc, for their continual support and understanding, and for providing me with a happy home to escape to; life and I would not be the same without you.

Finally I would like to thank my father, for giving me the inspiration to study physics, and for being a physicist.
Chapter 4 Data Analysis

4.1 Introduction ................................................. 67
4.2 Spectrum analysis ............................................ 67
4.3 Experimental determination of $W(\theta)$ .................... 70
4.4 Temperature determination .................................... 75
4.5 Experimental determination of the LTNO parameters ............. 75
4.5.1 The hyperfine interaction .................................. 75
4.5.2 The angular distribution and de-orientation coefficients .... 77

Chapter 5 Hyperfine field determination for Eu, Sm, and Pm isotopes in iron

5.1 Introduction ..................................................... 80
5.2 Low temperature nuclear orientation of $^{142}$Eu .................. 80
5.2.1 Experimental procedure ...................................... 81
5.2.2 Experimental results ........................................ 82
5.3 Low temperature nuclear orientation of $A = 141$ isotopes ....... 93
5.3.1 Experimental procedure ..................................... 93
5.3.2 Experimental results for $^{141}$Eu ........................... 94
5.3.3 Experimental results for $^{141}$Sm .......................... 99
5.3.4 Experimental results for $^{141}$Pm .......................... 101
Chapter 6 Magnetic moments of the nuclei $^{139,138}\text{Eu}$, $^{139}\text{Sm}^{m}$ and $^{138}\text{Pm}$

6.1 Introduction .............................................. 105
6.2 Low temperature nuclear orientation of $A=139$ isotopes ........ 105
6.2.1 Experimental procedure .................................. 106
6.2.2 Experimental results for $^{139}\text{Eu}$ ...................... 109
6.2.3 Experimental results for $^{139}\text{Sm}^{m}$ .................... 113
6.3 Low temperature nuclear orientation of $A=138$ isotopes ........ 116
6.3.1 Experimental procedure .................................. 117
6.3.2 Experimental results for $^{138}\text{Eu}$ ...................... 117
6.3.3 Experimental results for $^{138}\text{Pm}$ ...................... 127

Chapter 7 Particle-Rotor Model Calculations 129

7.1 Introduction .............................................. 129
7.2 The particle-rotor model .................................... 129
7.2.1 The Nilsson modified harmonic oscillator potential ........ 130
7.2.2 The pairing interaction ................................... 136
7.2.3 The strong coupling basis ................................ 137
7.2.4 Electromagnetic moments and transition probabilities ...... 138
7.2.5 The unified model of odd-odd nuclei .................... 139
7.3 Particle rotor model calculations ........................... 140
7.3.1 The nucleus $^{139}\text{Eu}$ ................................. 141
7.3.2 The nucleus $^{139}\text{Sm}^{m}$ ............................... 147
7.3.3 The nucleus $^{138}\text{Eu}$ ................................. 151

Chapter 8 Summary .............................................. 158

Bibliography .................................................. 162
CHAPTER ONE

INTRODUCTION

Considerable experimental and theoretical effort has, over the past decade, been spent in order to understand both the shape and structure of nuclei far from stability. It is therefore perhaps a little surprising that nuclear physics lacks a coherent theoretical formulation that would permit the physicist to analyze and interpret all phenomena in a fundamental way; atomic physics has such a formulation in quantum electrodynamics, which permits calculations of several observable quantities to more than 6 significant figures. As a result, nuclear physics is very often discussed in a phenomenological way, using a different formulation to describe each type of phenomenon such as $\alpha$-decay, $\beta$-decay, nuclear reactions or fission. In place of a single unifying theory there are isolated areas of knowledge in a plethora of apparently unconnected observations.

Like many systems governed by the laws of quantum mechanics, the nucleus is a complicated object whose properties are much more difficult to characterize than those of macroscopic objects. The number of mutual interactions of the 57 nucleons in $^{57}$Co for example, could include 57! or about $10^{76}$ terms. It is therefore desirable to specify the overall characteristics of the nucleus as a whole.

To some degree it is possible however, to describe a nucleus by a relatively small number of parameters: electric charge, radius, mass, binding energy, angular momentum, parity, magnetic dipole and electric
quadrupole moments, and energies of excited states, these are the static properties of nuclei.

Much of what is known about nuclear structure comes from studying not the strong nuclear interaction of nuclei with their surroundings, but instead the much weaker electromagnetic interaction. The strong interaction establishes the distribution and motion of nucleons in the nucleus, and the experimentalist can probe that distribution using the electromagnetic interaction. This has the advantage that the electromagnetic fields have less effect on the motion of the nucleons than the strong force of the nuclear environment. Consequently any measurements do not seriously distort the object the experimentalist is trying to measure.

It has been shown that clues to the deuteron structure, and the existence of stable deformation in heavy nuclei, were deduced mainly from measurements of ground state dipole and quadrupole moments via the electromagnetic interaction. It would therefore seem appropriate to define precisely what is meant by the terms magnetic dipole and electric quadrupole moment.

Classically, the magnetic dipole moment $\mu$ arises from the motion of charged particles, and can be regarded as a way of characterizing a distribution of currents whose effect on the surroundings (on other moving charges) can be considered "magnetic". When describing the nucleus in the formalism of quantum mechanics, there is one important addition: the intrinsic angular momentum (spin) also contributes to the magnetic moment.

It is customary to assign to the charge and current distribution of the nucleus an electromagnetic multipole moment associated with each characteristic spatial dependence; the $1/r^2$ electric field arises from the
net charge, which can be assigned as the zeroth or monopole moment; the \(1/r^3\) electric field arises from the first or dipole moment; the \(1/r^4\) electric field arises from the second or quadrupole moment and so on. The magnetic multipole moments behave similarly, with the exception of the monopole moment; which appears not to exist.

Each electromagnetic multipole moment has a parity, determined by the behaviour of the multipole operator when \(r\) changes to \(-r\). The parity of electric moments is \((-1)^L\), where \(L\) is the order of the moment \((L = 0\) for monopole, \(L = 1\) for dipole, and \(L = 2\) for quadrupole etc.); for magnetic moments the parity is \((-1)^{L+1}\). When the expectation value of a moment is computed, an integral of the form \(\int \psi^* \mathcal{O} \psi \, dv\) must be solved where \(\mathcal{O}\) is the appropriate electromagnetic operator. The parity of \(\psi\) itself is not important because \(\psi\) appears twice in the integral. If however, \(\mathcal{O}\) has odd parity then the integrand is an odd function of the coordinates and must vanish identically. Thus all odd parity static multipole moments must vanish: electric dipole, magnetic quadrupole, electric octupole, and so on.

The monopole electric moment is just the net nuclear charge \(Ze\). The next nonvanishing moment is the magnetic dipole moment \(\mu\). The quantum mechanical expression for \(\mu\) may be written:

\[
\mu = \frac{e}{2m} \int \psi^*(r) \ell \psi(r) \, dv \quad (1.1)
\]

If the wave function corresponds to a state of definite \(l_z\), then only the \(z\) component of the integral is nonvanishing, and

\[
\mu_z = \frac{e}{2m} \int \psi^*(r) \ell_z \psi(r) \, dv \quad (1.2)
\]
\[ \mu_z = \frac{e\hbar}{2m} \ell_z \quad \text{with} \quad \ell_z = m_i \hbar \]  

What is observed in an experiment as the magnetic moment is defined to be the value of \( \mu_z \) corresponding to the maximum possible value of the \( z \) component of the angular momentum. The quantum number \( m_i \) has a maximum value of \( +\ell \), therefore \( \mu \) is

\[ \mu = \frac{e\hbar \ell}{2m} \]  

The quantity \( e\hbar/2m \) has the dimensions of a magnetic moment (\( \ell \) is a dimensionless quantum number) and is called a magneton. Putting in the proton mass for \( m \), we get a nuclear magneton \( \mu_n \):

\[ \mu_n = \frac{e\hbar}{2m} = 3.15245 \times 10^{-8} \text{ eV/T} \]  

Considering the intrinsic spin, which has no classical analogue, it is possible to extend our equation for \( \mu \):

\[ \mu = (g_1 \ell + g_s s) \mu_n / \hbar \]  

where the \( g \) factors \( g_1 \) and \( g_s \) account for the orbital and intrinsic contributions to \( \mu \). Their values can be adjusted as needed for individual particles: \( g_1 = 1 \) for protons and \( g_s \) is measured to be 5.5856912. For neutrons, which are uncharged, \( g_1 = 0 \), and \( g_s \) is measured to be -3.8260837.

In real nuclei, a modification to allow for the effects of all the nucleons has to be made:

\[ \mu = \sum_{i=1}^{A} \left[ g_{1,i} \ell_i + g_{s,i} s_i \right] \mu_n / \hbar \]
Unfortunately there is no single theory that allows the physicist to evaluate the above equation to calculate $\mu$ because the interactions between the nucleons are strong and the relative spin orientations are not sufficiently well known. However, in certain cases, it is possible to make simplifying assumptions, based on nuclear models. In the independent particle shell model, odd mass nuclei have $A-1$ nucleons coupled pairwise to zero spin, which do not contribute to $\mu$. For the remaining odd nucleon, the shell model theory gives the coupling of $l$ and $s$ to form $I$, which permits $\mu$ to be calculated. The effect of the "core" nucleons cannot however be neglected, and are assigned a "collective" $g$ factor usually designated $g_K$, so that

$$\mu = \left[ g_R \frac{I}{I} + \sum_i (g_{\ell, i} + g_{s, i}) \right] \mu_n / \hbar \quad (1.8)$$

where $I_c$ refers to the core and the sum is carried out over a few nucleons outside the core. If "pure" collective states are considered, with no odd nucleons, the collective model gives $g_R = Z/A$, the ratio of the nuclear charge to its mass.

When considering deformed nuclei however, it is advantageous to introduce a new quantum number $K$ into eq. 1.8, that is, the projection of the angular momentum of the unpaired nucleon onto the symmetry axis of the nucleus. Hence eq. 1.8 becomes:

$$\mu = \frac{1}{K} \left[ g_R + (g_K - g_R) \frac{K^2}{I(I+1)} \right] \mu_n / K \quad (1.9)$$

where $g_K = \frac{1}{K} \left[ \sum_i (g_{\ell, i} + g_{s, i}) \right] \quad (1.10)$
Inspection of eq 1.9 reveals the dependence of $\mu$ on both the magnitude of $\ell$ of the unpaired nucleon in addition to the coupling of $\ell$ with $s$ (whether $\ell$ and $s$ are parallel or antiparallel). Both of these parameters depend on the wave function of the unpaired nucleon, which is intimately connected with (perhaps governs) the shape of the nucleus as a whole.

One successful approach to parameterizing the shape of the nucleus was developed by A Bohr, B Mottelson, S G Nilsson, the basis of which is the assumption that the radius vector $R$ of the nucleus is given by

$$R = R_0 \left[ 1 + \sum_{\lambda \mu} \alpha_{\lambda \mu} Y_{\lambda \mu}^* (\theta \phi) \right]$$

(1.11)

where $\lambda$ describes the multipolarity of the shape ($\lambda = 2$: quadrupole, $\lambda = 3$: octupole, etc; see Fig. 1.)

Fig. 1. Nuclear shapes with quadrupole ($\lambda=2$), octupole ($\lambda=3$) and hexadecapole ($\lambda=4$) deformations.

$R$ defines the location of a point on the nuclear surface, and $R_0$ is the average radius of the nucleus. The $\alpha_{\lambda \mu}$ are the components of the surface collective tensor $\alpha^{(\lambda)}$ where $\lambda$ determines the multipolarity of the nuclear motion. The $\lambda = 0$ mode merely scales the nuclear volume, while the $\lambda = 1$
mode corresponds (for small deformations at least) to a trivial translation of the centre of mass. Therefore the $\lambda = 2$ quadrupole mode is the lowest order mode of interest. The $\lambda = 3$ terms correspond, in general, to higher lying excitations. Hence only quadrupole terms will be considered and the index $\lambda$ will be dropped.

Whenever the nucleus is described by a nonspherical shape, rotations can result besides the vibrational motion as is shown in Fig. 2:

![Rotational motion of a deformed nucleus](image)

Fig. 2 Rotational motion of a deformed nucleus (characterized by the rotational vector $\omega$). The internal degrees of freedom are described by $\beta$, $\gamma$, and the rotational degrees of freedom are described by the Euler angles $\theta_1$, $\theta_2$, $\theta_3$.

Here it is convenient to transform from the coordinates $\alpha_\mu$ to the 3 Euler angles and the two intrinsic variables for $\lambda = 2$, using

$$\alpha'_\lambda = \sum_{\nu} D^2_{\mu\nu}(\Omega) \alpha'_\nu$$  \hspace{1cm} (1.12)

where $D^2_{\mu\nu}$ is the Wigner D or rotation matrix in the new system. It is required that:

$$\alpha'_1 = \alpha'_{-1} = 0; \quad \alpha'_2 = \alpha'_{-2}$$  \hspace{1cm} (1.13)
if the new "intrinsic" axes are to be the principal axis of the nuclear shape. Without any loss of generality this transformation may be chosen such that:

\[ \alpha_0 = \beta \cos \gamma \]  
\[ \alpha_1 = 0 \]  
\[ \alpha_2 = \frac{1}{\sqrt{2}} \beta \sin \gamma \]

(1.14) \hspace{1cm} (1.15) \hspace{1cm} (1.16)

Hence the shape of the nuclear surface may be adequately described by the parameters \( \beta \) and \( \gamma \), if we assume the deformation of the nucleus is purely quadrupole in nature. The parameter \( \beta \) defines the major/minor axis ratio, for an axially symmetric nucleus, the parameter \( \gamma \) measures the departure from axial symmetry; \( \gamma = 0^\circ \) giving a prolate, \( \gamma = 60^\circ \) an oblate shape, therefore all shapes can be obtained within the sector \( (0^\circ, 60^\circ) \).

The potential energy \( V(\beta, \gamma) \) will also determine the nuclear dynamics in a major way, as can be seen in Fig. 3.

Fig. 3. Different shapes \( V(\beta, \gamma) \) in the \( \beta, \gamma \) sector corresponding to a spherical vibrator, a prolate rotor, a \( \gamma \)-soft vibrator and a triaxial rotor.
The neutron deficient light rare-earth nuclei are situated below the N = 82 closed shell, as shown in Fig. 4. Such neutron deficient nuclei with N<82 and Z>50 are located in a region where large quadrupole deformation (β>0.2) is expected. The importance of axial asymmetry is evident through, for example, the systematic appearance of low-lying $^{2}\Gamma = 2^{+}2$ γ-vibrational states. Considerable attention has been given recently to the in-beam study of high-spin states in this relatively unexplored region, but rather little is known about the low-lying structure and the associated Nilsson single particle configurations. Ragnarsson et al., in macroscopic-microscopic calculations around the neutron deficient Sm and Gd isotopes, have predicted stable prolate shapes except for the N = 76 isotones, where the γ-degree of freedom is expected to dominate the nuclear structure.

When trying to model experimental data for the measurement of the ground state moments, it would seem appropriate to choose a nuclear model which can accommodate both the β and γ degrees of freedom, in addition to the Coriolis effect of the rotating nucleus on the nucleonic motion. Such a model is available in the form of the particle-rotor-model and is described in detail in chapter seven.

The purpose of the research presented in this thesis is therefore, to investigate the interplay of the collective and independent motions of the nucleons within the light rare-earth nuclei, as reflected by their respective magnetic dipole moments. Hence a theoretical basis for an experimental technique to measure μ is given in chapter two.
Figure 4 Chart of the nuclides showing the light rare earth nuclei below the \( N = 82 \) closed shell.
2.1 Introduction

Both α and β decay may leave the final nucleus in an excited state. These excited states decay rapidly to the ground state through the emission of one or more γ-rays, which are photons of electromagnetic radiation. Gamma-rays have energies typically in the range 0.1 to 10 MeV, characteristic of the energy difference between nuclear states. The probability of emission of electromagnetic radiation, is broadly related to the angle θ between the expectation value <J> of the angular momentum vector of the nucleus, and the direction k in which the photons are observed.

If an ensemble of radiating nuclei have their J vectors randomly oriented, then <J> = 0, and the radiation is isotropic in space. In order to observe an anisotropic radiation pattern, the spatial distribution of the angular momentum vectors has to favour a definite direction, so that <J> ≠ 0. The intensity and polarization properties are then a function of the angles between the orientation axes, and the directions of observation k and the polarization P of the radiation.

When the radiation is observed with a detector which is sensitive to the polarization properties P of the radiation then the polarization distribution W(k,P) is measured. If the polarization is not observed then
by definition the directional distribution $W(k)$ of the radiation is measured. The term angular distribution is used to refer to both.

Oriented ensembles of nuclei may be prepared by three different methods:

1. Extranuclear static and dynamic orientation. (Low temperature orientation, optical pumping, radiofrequency methods).

2. Orientation by absorption of nuclear radiation of well defined direction and polarization. (Nuclear reactions, Coulomb excitation).

3. Orientation by observing a preceding emitted radiation in a well defined direction and with well defined properties. (Angular correlations, nuclear cascade radiation).

In Low Temperature Nuclear Orientation (LTNO), which forms the main subject of this work, orientation is achieved through the interaction of external fields with either the static magnetic dipole moment or the static electric quadrupole moment of the nucleus. At very low temperatures the energetically lower states, $m$-states in the case of magnetic interactions, become preferentially populated according to the Boltzmann distribution. Hence in thermodynamic equilibrium, various degrees of orientation are obtained depending on the relative magnitudes of the energy splitting of the nuclear substates and the randomizing thermal energy, $kT$.

The following sections will deal with how the angular distribution of gamma radiation is related to the various nuclear properties of interest and to the orienting interaction.

2.2 The angular distribution of radiation from an oriented source.
2.2.1 The description of mixed quantum states by density matrices.

A physical system is in a pure state if every value of a complete set of commuting observables has been measured. There is then maximum information known about the state under consideration which can then be described uniquely by a complete set of quantum numbers.

An angular distribution measurement does not result in a complete knowledge of the individual nuclei nor the individual quanta, only ensemble averages are observed. The statistical nature of the ensemble is involved and maximum information is not available. The quantum mechanical description of such mixed states requires incoherent superpositions of pure quantum states. Hence an ensemble cannot be described by state vectors, but instead described by a *density matrix* \[^{[PA53]}\].

When considering such a pure state \(|\Psi\rangle\), which by selecting a coordinate system in Hilbert space can be described by its projections onto all coordinate axes, it is possible to choose the complete orthonormal set \(|\phi\rangle\) where:

\[
|\Psi\rangle = \sum_{\phi} |\phi\rangle <\phi|\Psi\rangle \quad (2.1)
\]

and the transformation brackets \(<\phi|\Psi\rangle\) are unitary. The expectation value of any operator \(a\) in the state \(|\Psi\rangle\) is therefore

\[
<\Psi|a|\Psi\rangle = \sum_{\phi,\phi'} <\phi|\phi'\rangle <\phi'|a|\phi'\rangle <\phi'|\Psi\rangle <\Psi|\phi\rangle \quad (2.2)
\]

A mixed state consisting of an ensemble of several independent nuclei must then be described by an incoherent superposition of all the possible
pure states $|\Psi_n\rangle$, weighted according to the proportion of nuclei $g(n)$ in those pure states. The expectation value of the operator $a$ in such a mixed state is therefore given by the weighted average of the expectation values $\langle \Psi | a | \Psi \rangle$

$$<a> = \sum_{n=1}^{N} g(n) \frac{\langle \Psi_n | a | \Psi_n \rangle}{\sum_{n=1}^{N} g(n)}$$ (2.3)

It is convenient to normalise the weights $g(n)$ to unity:

$$\sum_{n=1}^{N} g(n) = 1$$ (2.4)

Substituting (2.2) into (2.3) and using (2.4):

$$<a> = \sum_{n} <\phi | n \rangle g(n) <\Psi_n | a | \Psi_n >$$

or alternatively

$$<a> = \sum_{\phi'\phi} <\phi | \rho | \phi' > <\phi' | a | \phi >$$ (2.6)

where

$$<\phi | \rho | \phi' > = \sum_{n=1}^{N} <\phi | n \rangle g(n) <\Psi_n | \phi' >$$ (2.7)

Hence the state of the ensemble is completely characterized by the density matrix $\rho$ whereas the matrix $A$, with elements $<\phi | a | \phi >$, depends only on the operator $a$ and the representation $|\phi\rangle$. With these definitions (2.6) may be rewritten in the form
\[ <a> = \text{Tr}(\rho A) \]  

(2.8)

The density matrix can be considered as a representation of the density operator

\[ \rho_{\text{op}} = \sum_{n=1}^{N} |\Psi_n> g(n) <\Psi_n| \]  

(2.9)

in the basis |\phi>. The density operator completely describes the state of the ensemble, whether it is mixed or pure.

2.2.2 The statistical tensors

In angular distribution problems the symmetries of the ensemble with respect to rotations are of interest and so it is natural to choose the basis states |\phi> to be eigenstates |\rho jm> of the angular momentum operator \( J^2 \) and its projection onto the z-axis, \( J_z \). The symbol \( \rho \) represents the other quantum numbers needed to specify the states uniquely. The matrix elements of the density operator in this representation are

\[ \rho(\rho_1 j_1 \rho_2 j_2) = \langle \rho_1 j_1 m_1 |\rho| \rho_2 j_2 m_2 > \]

\[ = \sum_n \langle \rho_1 j_1 m_1 |\Psi_n> g(n) <\Psi_n| \rho_2 j_2 m_2 > \]  

(2.10)

The grand density matrix \( \rho(\eta) \) which describes an ensemble of nuclei \( \eta \) in all their possible states is a square matrix with \((2j_1 + 1) + (2j_2 + 1) + \ldots \) rows and columns, where \( j_1 \) are the angular momenta of the different nuclear levels or of the different daughter nuclei and radiation fields into which the nuclei \( \eta \) may decay. The density matrix \( \rho(j_1 j_2) \) is just a
\((2j_1+1) \times (2j_2+1)\) submatrix of this grand matrix.

The angular momentum states \(\mid j_m \rangle\) transform under a rotation of the quantization axis from \(S(xyz)\) to \(S(x'yz')\) according to

\[
\mid j_m \rangle_{S} = \sum \mid j_m \rangle_{S} D^{j_m}_{m_m} (S \rightarrow S) \tag{2.11}
\]

where the D-matrices are defined in Brink and Satchler [BR68]. The elements transform under the same rotation according to

\[
\langle j_1 m_1 \mid \rho \mid j_2 m_2 \rangle_{S} = \sum D^{j_1 -}_{m_1 m_1} (S \rightarrow S) \langle j_1 m_1 \mid \rho \mid j_2 m_2 \rangle_{S}
\]

\[
\times D^{j_2 -}_{m_2 m_2} (S \rightarrow S). \tag{2.12}
\]

Making use of the Clebsh-Gordon series of D-matrices then

\[
\langle j_1 m_1 \mid \rho \mid j_2 m_2 \rangle = \sum \langle j_1 m_1 \mid \rho \mid j_2 m_2 \rangle_{S} (-1)^{m_2 - \bar{m}_2} (2\lambda + 1) \begin{pmatrix} j_1 & j_2 & \lambda \\
\bar{m}_1 & \bar{m}_2 & -q \end{pmatrix}
\]

\[
\times \begin{pmatrix} j_1 & j_2 & \lambda \\
\bar{m}_1 & \bar{m}_2 & -q \end{pmatrix} = \begin{pmatrix} \lambda \end{pmatrix} (S \rightarrow S). \tag{2.13}
\]

After multiplying both sides of this equation by the factor

\[
(-1)^{j_2 + \bar{m}_2} (2\lambda + 1)^{1/2} \begin{pmatrix} j_1 & j_2 & \lambda \\
\bar{m}_1 & \bar{m}_2 & -q \end{pmatrix}
\]

and summing over \(\bar{m}_1\) and \(\bar{m}_2\) (keeping \(q\) fixed) we find due to the orthogonality relations of the 3-j symbols,
The linear combinations of the elements of $\rho(j_1, j_2)$ enclosed in braces transform like spherical tensors of rank $\lambda$. Due to this simple transformation property it is more convenient to use these linear combinations than the matrix elements themselves. Hence it is possible to define the statistical tensors $\rho\lambda_q(j_1, j_2)$ to be

$$\rho\lambda_q(j_1, j_2) = \sum_{m_1, m_2} <j_1 m_1 | \rho | j_2 m_2> (-1)^{j_2 + m_2} (2\lambda + 1)^{1/2} \begin{pmatrix} j_2 & j_1 & \lambda \\ -m_2 & m_1 & q \end{pmatrix} \lambda_q - (S \rightarrow \bar{S})$$

(2.15)

This represents the vector relationship $\lambda = J_x - J_y$ and so the rank of the statistical tensor is an integer in the range $|j_2 - j_1|$ to $j_2 + j_1$.

2.2.3 Oriented nuclear states

If the interacting fields that cause the orientation of the ensemble have an axis of cylindrical symmetry, which in the case of LTNO is that defined by the direction of the external field, then the density operator (2.9) may be expressed in the form

$$\rho_{op}^{+I} = \sum_{n=I}^{+I} |\text{Im}> g(n) <\text{Im}|$$

(2.16)

where the eigenstates $|\text{Im}>$ of the nuclear spin operator $I_z$ are defined with respect to the symmetry $z$-axis. The corresponding density matrix $\rho(I)$
in the angular momentum representation \(|\tilde{m}\rangle\) chosen with respect to some representation-coordinate system \(\tilde{S}\) is

\[
\langle \tilde{m} | \rho | \tilde{m'} \rangle = \sum_n \langle \tilde{m} | \tilde{m} \rangle g(n) \langle \tilde{m} | \tilde{m'} \rangle.
\] (2.17)

If one chooses \(\tilde{S}\) to be the symmetry frame defined by the external field then the density matrix becomes diagonal

\[
\langle m | \rho | m' \rangle = g(m) \delta_{mm'}.
\] (2.18)

and the weighting factors \(g(m)\) are then simply the relative populations of the states \(|\tilde{m}\rangle\) in the ensemble.

An ordinary ensemble of nuclei with spin \(I\) at room temperature has in general no fixed orientation axis in space, hence it must be invariant with respect to three dimensional rotations. Thus the statistical tensors describing such a random ensemble must satisfy the relation

\[
\rho_{qq}^\lambda(I) \bar{s} = \sum_q \rho_{qq}^\lambda(I) \bar{s} D_{qq}(S \to \bar{s}) = \rho_{q0}^\lambda(I) \bar{s}
\] (2.19)

for any rotation \(S \to \bar{s}\). This is only possible if \(D\) is the unit matrix \((q = \bar{q} = \lambda = 0)\) or if \(\rho_{q0}^\lambda(I)\) vanishes. Hence the statistical tensor in this case is

\[
\rho_{q0}^0(I) = (2I+1)^{-1/2}
\] (2.20)

with corresponding density matrix

\[
\rho(I) = (2I+1)^{-1}1
\] (2.21)

expressing the fact that the population is uniformly distributed amongst the various \(|\tilde{m}\rangle\) states.
When orientation is achieved, unequal populations of the $|\text{Im} >$ states occur, giving rise to statistical tensors $\rho^\lambda_q(I)$ with $\lambda \neq 0$. With the axis of cylindrical symmetry, the z-axis, defined by the applied field it is required that the statistical tensors be invariant with respect to rotations through an arbitrary angle $\alpha$ about this axis. This rotation is represented by

$$D^\lambda_{qq}(\alpha, 0, 0) = e^{-i\alpha} \delta_{qq}$$  \hspace{1cm} (2.22)

and leads to the condition

$$\rho^\lambda_q(I) = \sum_q \rho^\lambda_q(I) D^\lambda_{qq}(\alpha, 0, 0) = \rho^\lambda_q(I) e^{-i\alpha} = \rho^\lambda_q(I).$$  \hspace{1cm} (2.23)

Since $\alpha$ is arbitrary this equation implies that $q = \tilde{q} = 0$. Hence with cylindrical symmetry the statistical tensors can be uniquely described by the series of parameters, the orientation parameters $B_\lambda^q(I)$ \cite{BL57}, where

$$B_\lambda^q(I) = (2I+1)^{1/2} \rho^\lambda_0(I)$$

$$= \sum (-1)^{l+m} [(2I+1)(2\lambda+1)]^{1/2} \begin{pmatrix} I & I & \lambda \\ -m & m & 0 \end{pmatrix} g(m).$$  \hspace{1cm} (2.24)

The orientation parameters are normalised to unity: $B_\lambda^q(I) = 1$.

The form of the interaction used in LTNO is now introduced explicitly. In principle this could be either the interaction of the nuclear static quadrupole moment with an applied electric field gradient, or that of the static nuclear magnetic dipole moment with an applied magnetic field. In this thesis only the latter interaction is considered. Using the direction of the $B$-field as the representation axis, the magnetic energy of the nuclear $m$-levels is
\[ E(m) = -\frac{\mu B}{I} m \]  \hspace{1cm} (2.25)

where \( \mu \) is the nuclear magnetic dipole moment. In accordance with the Boltzmann distribution the populations in thermodynamic equilibrium are

\[ g(m) = \langle m | \rho | m \rangle = e^{\beta m^2} / \sum_m e^{\beta m^2} \]  \hspace{1cm} (2.26)

with

\[ \beta = \frac{\mu B}{I k_T} \]  \hspace{1cm} (2.27)

Hence the statistical tensors for this magnetically oriented ensemble are given by

\[ \rho_\lambda^\lambda(I) = (2I+1)^{-1/2} B_\lambda^\lambda(I) \]

\[ = \left\{ \sum_m e^{\beta m^2} \right\}^{-1} \sum_m (-1)^{I+m} (2\lambda+1)^{1/2} \begin{pmatrix} I & I & \lambda \\ -m & 0 \\ m \end{pmatrix} e^{\beta m^2} \]  \hspace{1cm} (2.28)

Figure 2.1 illustrates the temperature dependence of the orientation parameters.

![Figure 2.1](image_url)

**Figure 2.1** The dependence of the orientation parameter \( B_\lambda^\lambda \) on \( \Delta m/T \), where \( \Delta m = \mu B/I k \). [ST77]
2.2.4 The efficiency matrices and tensors

The probability of finding a system described by the density matrix \( \rho \) in the state \( |\phi_n\rangle \) is seen from (2.7) to be given by

\[
P(\phi_n) = \langle \phi_n | \rho | \phi_n \rangle. \tag{2.29}
\]

Generally it is necessary to calculate this probability when the ensemble is observed by a measuring device which responds to the different pure states with different efficiencies. In direct analogy with the density matrix of the ensemble, such a detector must be described by a weighted incoherent superposition of the pure state response characteristics. If the efficiency for detecting the pure state \( |\phi_n\rangle \) is denoted by \( e_n \) then the result of a measurement with the ensemble in an arbitrary state is

\[
W = \sum_n e_n P(\phi_n) = \sum_n e_n \langle \phi_n | \rho | \phi_n \rangle \tag{2.30}
\]

or

\[
\bar{W} = \text{Tr}(\varepsilon \rho) \tag{2.31}
\]

where \( \varepsilon \) is the diagonal matrix with elements \( e_n \). By defining the efficiency operator

\[
\varepsilon = \sum_n |\phi_n\rangle \langle \varepsilon(n) | \phi_n \rangle \tag{2.32}
\]

so that the efficiency matrix is the efficiency operator represented in some convenient basis \( |m\rangle \).
\[ <m|\varepsilon|m'> = \sum_n <m|\phi_n> \varepsilon(n) <\phi_n|m'> \quad (2.33) \]

then it is possible to construct the efficiency tensor according to (2.15)

\[ \varepsilon^\lambda_q(j_1 j_2) - \sum_{m_1, m_2} <j_1 m_1|\varepsilon|j_2 m_2>(-1)^{j_2 + m_2}(2\lambda + 1)^{1/2} \begin{pmatrix} j_2 & j_1 & \lambda \\ m_2 & m_1 & q \end{pmatrix} \quad (2.34) \]

With these definitions it can be shown that \[ [^{ST75}] \] the result of a measurement with a measuring device characterized by efficiency tensor \( \varepsilon^\lambda_q(j_1 j_2) \) on an ensemble represented by the statistical tensor \( \rho^\lambda_q(j_1 j_2) \) is given by

\[ w = \sum_{j_1 j_2 \lambda q} \varepsilon^\lambda_q(j_1 j_2) \rho^\lambda_q(j_2 j_1) \quad (2.35) \]

Often, as in the case of LTNO, measurements are only made on the emitted radiation implying the existence of, but giving no information on, a final nuclear state. In such cases a summation over the final unobserved states is performed and the measurement is represented by the unit efficiency matrix. If these states have angular momentum \( j_1 \) then by analogy with (2.20) the corresponding efficiency tensor is

\[ \varepsilon^\lambda_q(j_1 j_2) = (2j_1 + 1)^{1/2} \delta_{j_1 j_2} \delta_{\lambda 0} \delta_{q 0} \quad (2.36) \]

Finally, the efficiency matrix for the detection of emitted electromagnetic radiation in helicity states \( |\varphi_\tau> \) is considered. Clearly the efficiency operator should be defined in the representation \( |\varphi_\varphi> \). Hence

\[ \text{22} \]
Where $Q(\kappa)$ is the response of the detector to the plane wave state of pure polarization $\kappa = \pm 1$. The efficiency matrix is therefore

$$<\phi_\tau|\epsilon|\phi_\tau'> = \sum_{\kappa} <\phi_\tau|\kappa\rangle Q(\kappa) <\kappa|\phi_\tau'>$$

(2.38)

and so for a detector which responds only to either positive or negative helicity states

$$\epsilon(+) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \text{or} \quad \epsilon(-) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

(2.39)

2.2.5 The time evolution of the final state density matrix

An ensemble of nuclei $\eta$, which at the time $t = 0$ is described by the density matrix $<I_{f}\epsilon_{m}|\rho|I_{f}\epsilon_{m'}>$, will ultimately decay into a final nuclear state $I_{f}$ by the emission of radiation with a definite momentum $p$. This final state is represented by the final state density matrix $<\phi_{f}\epsilon_{m}|\rho|\phi_{f}\epsilon_{m'}|\phi_{f}>$. This subsection will deal briefly with the derivation of this final state matrix. Starting with the time dependent Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} |\psi> = H |\psi>$$

(2.40)

whose general solution may be written

$$|\psi(t)> = U(t, t_0) |\psi(t_0)>$$

(2.41)
where $U(t, t_0)$ is the unitary time-evolution operator, then the time behaviour of the density operator is given by

$$\rho_{cp}(t) = \sum_n \langle \psi_n(t) | \langle \psi_n(t_0) | U(t, t_0) | \psi_n(t_0) \rangle \rangle$$

$$= U(t, t_0) \rho_{cp}(t_0) U^\dagger(t, t_0)$$

from (2.40) and (2.41) it is clear that $U(t, t_0)$ must also satisfy the Schrödinger equation

$$\frac{\partial}{\partial t} U(t, t_0) = H U(t, t_0)$$

(2.43)

The total Hamiltonian of the system is

$$H = H_0 + H_{\text{int}}$$

(2.44)

where $H_0$ is that part of the Hamiltonian which gives rise to the stationary states and $H_{\text{int}}$ induces transitions between the eigenstates of $H_0$. For the case of electromagnetic radiation \[2.45\]

Performing the unitary transformation to the Heisenberg representation leads to the operators

$$H_H(t) = e^{iH_0 t/\hbar} H_{\text{int}} e^{-iH_0 t/\hbar}$$

(2.46)

and

$$U_H(t, t_0) = e^{iH_0 t/\hbar} U(t, t_0) e^{-iH_0 t/\hbar}$$

(2.47)
The expression corresponding to (2.43) in this representation may be solved to give to first order

\[
U_H(t, t_0) = 1 - \frac{\hbar}{i} \int_{t_0}^{t} H_H(t') \, dt' + \ldots
\]  

(2.48)

Combining the second term in this expansion with the transformed analogue of (2.42) and using first order perturbation theory leads to

\[
\langle \psi | \mathbf{I}_m | \rho(t) \mathbf{I}_m' \psi' \rangle = \frac{\psi}{\mathbf{I}_m} \mathbf{I}_m' | \rho(0) \mathbf{I}_m' \psi' \rangle \int_0^t \left[ e^{\left(\frac{iH_0}{\hbar} t'\right)} H_{\text{int}} e^{\left(\frac{-iH_0}{\hbar} t'\right)} dt' \right] e^{\left(\frac{iH_0}{\hbar} t\right)} | \mathbf{I}_m' \psi' \rangle
\]

(2.49)

The statevector

\[
e^{\left(\frac{iH_0}{\hbar} t\right)} | \mathbf{I}_m' \psi' \rangle = | \mathbf{I}_m' \psi' \rangle
\]

(2.50)

is the time independent eigenvector of \( H_0 \) in the Heisenberg representation. Since the initial grand density matrix \( \rho(0) \) has only the elements \( \langle \mathbf{I}_m | \rho(0) | \mathbf{I}_m' \rangle \), then (2.49) can be written

\[
\langle \psi | \mathbf{I}_m | \rho(t) \mathbf{I}_m' \psi' \rangle = \langle \psi H_{\text{int}} \mathbf{I}_m | \rho(0) \mathbf{I}_m' \psi' \rangle \int_0^t \left[ e^{\frac{iH_0}{\hbar} t'} \mathbf{I}_m' e^{\frac{-iH_0}{\hbar} t'} \right] | \mathbf{I}_m' \psi' \rangle
\]

(2.51)

where the arguments of the exponential arise from the action of \( e^{\left(\frac{iH_0}{\hbar} t\right)} \) on \( \langle \psi | \mathbf{I}_m | \rho(0) \mathbf{I}_m' \psi' \rangle \) respectively. In accordance with the conservation of energy the integral is only appreciably different from zero if \( \hbar \omega = \hbar \omega_0 = E_1 - E_2 \). In the Heisenberg representation the
transition matrix elements of (2.51) are time independent

\[ \langle \rho(t) | \sum_{I_{f} = I_{f}'} \int_{I_{i} = I_{i}'} |j(r')| A_{pr}(r') \, dv' \rangle \]

(2.52)

\[ \sqrt{\frac{2 \pi}{\omega^{3}}} \int_{I_{f} = I_{f}'} |j(r')| e^{i \rho / \hbar} \, dv' |I_{i} m_{i} > \]

since the time dependence of \( j(r',t) \), namely \( e^{i(E_{f} - E_{i})t / \hbar} \), is exactly compensated by that of \( A(r,t) \), namely \( e^{i \omega t} \). By integrating over \( \omega \) and multiplying by the density of states, the last term in (2.51) results in a factor \( L^{3} \omega_{0}^{2} \text{tdt}/(2 \pi)^{2} \). This is correct for values of \( t \) much smaller than the mean life time \( 1/\lambda_{\gamma} \), where \( \lambda_{\gamma} \) is the total transition probability. For times larger than this, \( t \) should be replaced by the radioactive decay factor \( (1-e^{-\lambda_{\gamma} t})/\lambda_{\gamma} \). Hence the elements of the final state density matrix are

\[ \langle \rho(t) | \sum_{I_{f} = I_{f}'} |I_{i} m_{i} > \rangle \]

(2.53)

Introducing the absolute transition amplitudes \( \gamma(\pi L, I_{i} \rightarrow I_{f}) \) which are proportional to the multipole expansions of (2.52) and again using the Clebsh-Gordon series for the product of two D-matrices gives
&lt;\rho_{\tau m_f} | \rho(\omega) | \tau m'_f \rho_{\tau'} &gt; = \frac{d\Omega}{8\pi\lambda} \sum_{m} (2I_1+1)^{1/2} \sum_{L L'} \frac{1}{[(2I_1+1)(2L'+1)]^{1/2}}

\times (2\lambda+1) \left[ \begin{array}{cc} 1 & 1 \\ m_1 & M - m \\ \end{array} \right] \left[ \begin{array}{cc} 1 & 1 \\ m'_1 & M' - m'_1 \\ \end{array} \right] (2.54)

\times \left[ \begin{array}{cc} L & L' \\ \lambda & \lambda' \\ M - M & \mu \\ \end{array} \right] \left[ \begin{array}{cc} L & L' \\ \tau & -\tau' \\ \end{array} \right] \left[ \begin{array}{cc} m_1 & m'_1 \\ \end{array} \right] \left[ \begin{array}{cc} I & 1 \\ \end{array} \right] \left[ \begin{array}{cc} I & 1 \\ \end{array} \right] \left[ \begin{array}{cc} \rho(\omega) | I m_f & I m'_f \end{array} \right]

\times [\gamma(EL) + \tau \gamma(ML)] [\gamma^*(EL') + \tau' \gamma^*(ML') ] D^{\tau \gamma}_{q \mu} (e_z \rightarrow k)

for times $\gamma_\tau \sim \omega$. The diagonal elements of this final state density matrix expression give the absolute probability that a photon with helicity $\tau$ is emitted into the solid angle $d\Omega$ in the direction $p$ from the initial state $\rho(0)$ to form the final nuclear state $|I m_f>$.

From this expression the final state statistical tensor $\rho_{q f}^\lambda(I \tau')$ may be constructed in accordance with (2.15). This is a $2 \times 2$ matrix in the helicity space $\tau$, whose four elements in $|Im>$ space are

\begin{align*}
&<\tau | \rho_{q f}^\lambda(I \tau') | \tau' > = \frac{d\Omega}{8\pi\lambda} \sum_{\lambda_i \lambda_q \lambda_1 \lambda_1} (-1)^{\lambda_i - \lambda_q} (2\lambda + 1)^{1/2} \left( \frac{2I_1 + 1}{2I_1 + 1} \right)^{1/2} \\
&\times \left[ \begin{array}{cc} \lambda & \lambda \\ \lambda_i & \lambda_i \\ \end{array} \right] \rho_{q f}^\lambda(I \tau') D_{q \mu}^{\lambda \gamma} (e_z \rightarrow k) [\gamma(EL) + \tau \gamma(ML)]
\end{align*}

\begin{align*}
&\times [\gamma^*(EL) + \tau' \gamma^*(ML)] \frac{1}{F^{\lambda \lambda_i} (LL' I \tau I)} (2.55)
\end{align*}
where the $F^\lambda_{\lambda'}^{\lambda'}(LL'I_{1'}I_{0})$ are the generalized F-coefficients defined by

$$F^\lambda_{\lambda'}^{\lambda'}(LL'I_{1'}I_{0}) = (-1)^{L'+\lambda_{1}+\lambda_{0}+1}(2I_{0}+1)(2I_{1}+1)(2L+1)(2L'+1)$$

$$\times (2\lambda_{0}+1)(2\lambda_{1}+1)(2\lambda+1)^{1/2} \left( \begin{array}{cc} L & L' & \lambda \\ 1 & -1 & 0 \end{array} \right) \left( \begin{array}{c} I_{1} \ L \ I_{0} \\ I_{1} \ L' \ I_{0} \\ \lambda_{1} \ \lambda \ \lambda_{0} \end{array} \right)$$

(2.56)

If one of the oriented states $I_{0}$ or $I_{1}$ is random, so that either $\lambda_{0}$ or $\lambda_{1}$ is zero, then the generalized F-coefficients reduce to the ordinary F-coefficients $F^\lambda_{\lambda'}(LL'I_{1}I_{0})$.

2.2.6 The angular distribution of gamma radiation from LTNO sources

It has been shown that equation (2.55) is a general expression for the final state statistical tensor, derived only from the assumption of the invariance under three dimensional rotations. The conditions specific to nuclear orientation problems can now be introduced. Firstly, the statistical tensor of the initial state can be replaced by the orientation parameters (2.24). Secondly, the final nuclear state is not observed and so its efficiency tensor is given by (2.36). Only the radiation quanta are observed, in the direction $k$ using a detector described by the efficiency matrix $c$. Again it will be assumed that the observed quanta are of electromagnetic origin, although the formalism applies also to alpha and beta radiation. Thus using (2.35) the probability of detecting the photon is
\[ W(k) = \sum_{T} \langle \tau | \rho_{0}^{0} (I_{e}) | \tau' \rangle <\tau' | e | \tau > (2I_{e} + 1)^{1/2} \quad (2.57) \]

where \( \langle \tau | \rho_{0}^{0} (I_{e}) | \tau' \rangle \) is given by (2.55).

Considering the detection of circularly polarized radiation with helicity \( \tau \) by an ideal detector with efficiency matrix (2.39), then \( \tau = \tau' \) and so

\[ W(\theta, \tau) = \frac{d\Omega}{8\pi} \sum_{\lambda \gamma_{LL'}} \sum_{T} \lambda \nu_{L+L'} B_{\lambda} (I_{1}) F_{\lambda} (LL'I_{e} I_{1}) \]

\[ \times [\gamma^{*} (EL) + \tau \gamma (HL)] [\gamma^{*} (EL') + \tau \gamma (HL')] \ P_{\lambda} (\cos \theta) \]

where use has been made of the relation

\[ D^{\lambda*}_{00} (\phi, \theta, 0) = P_{\lambda} (\cos \theta) \quad (2.59) \]

The angles \( \phi, \theta \) are the polar angles of the observation direction \( k \) with respect to coordinate system \( xyz \) in which the initial state statistical tensor is represented. With the inclusion of the parity selection rules for electromagnetic transitions: \( L+L' \) even (odd) if both components are electric or magnetic (one component electric while the other magnetic),

the circular polarization distribution is obtained as

\[ W(\theta, \tau) = \frac{d\Omega}{8\pi} \sum_{\lambda} \lambda \nu B_{\lambda} (I_{1}) A_{\lambda} F_{\lambda} (\cos \theta) \quad (2.60) \]

where the angular distribution coefficients, \( A_{\lambda} \) are given by

\[ A_{\lambda} = \sum_{L \nu L'L'} \frac{F_{\lambda} (LL'I_{e} I_{1}) \gamma (\pi L) \gamma^{*} (\pi' L') / \sum_{\nu L} | \gamma (\pi L) |^{2}} {L_{LL'}^{\nu} \pi' \nu L' \nu} \quad (2.61) \]

with the normalization \( A_{0} = 1 \); and by definition
For two mixed multipoles $L$ and $L' = L + 1$ with amplitude mixing ratio $\delta = \gamma(\pi' L + 1)/\gamma(\pi L)$ the angular distribution coefficients can be written in the form

$$A_\lambda = \left( F_{\lambda}^{(LL_1 L_1)} + 2\delta F_{\lambda}^{(LL_1 +11 II_1)} + \delta^2 F_{\lambda}^{(L +1L+1II)} \right) / \left( 1 + \delta^2 \right)$$

Figure 2.2 shows an example of the $\delta$ dependence of the angular distribution coefficients $5/2 - 3/2$.

Figure 2.2 The dependence of the angular distribution coefficients $A_2, A_4$ on $Q$, where $Q = \delta^2 / \left( 1 + \delta^2 \right)$.

The directional distribution of gamma radiation is now obtained by summing (2.60) over both helicity states which yields a factor of $2(0)$ if $\lambda_2$ is even (odd). Hence
2.2.7 Unobserved intermediate radiations

Usually in LTNO experiments the detected gamma radiation quantum is preceded by a cascade of unobserved intermediate radiation. In order to construct the final state statistical tensor, the procedure outlined in subsection (2.2.5) must be performed in turn for each emitted photon. Since only the last of these is observed it is necessary to integrate the final state statistical tensor over solid angle for each unobserved quantum. In this way it can be shown that the final state statistical tensor \( \rho_{q_n}^{\lambda_n}(I_n) \) that describes the ensemble after the emission of an unobserved gamma radiation \( \gamma_n \) from an initial state \( \rho_{q_{n-1}}^{\lambda_{n-1}}(I_{n-1}) \) is obtained from the relation

\[
\rho_{q_n}^{\lambda_n}(I_n) = U_n(\gamma_n) \rho_{q_{n-1}}^{\lambda_{n-1}}(I_{n-1}) \delta_n \delta_{n-1} \delta_{q_n q_{n-1}} \quad (2.65)
\]

where the de-orientation coefficients, \( U_\lambda \), have been introduced, and are given by

\[
U_\lambda = \sum_{L,K} U_\lambda(I_I L) |\gamma(\pi L)|^2 / \sum_{L,K} |\gamma(\pi L)|^2 \quad (2.66)
\]

or in terms of \( \delta \)

\[
U_\lambda = \{U_\lambda(I_I L) + \delta^2 U_\lambda(I_I L+1)\} / (1 + \delta^2) \quad (2.67)
\]

with the de-orientation factor defined by
The de-orientation factors are normalized to unity: $U^0 = 1$. Hence, for a cascade of $n$ unobserved photons the final state statistical tensor is

$$
\rho_q^\lambda (I) = U_\lambda (\gamma_n) U_\lambda (\gamma_{n-1}) \ldots U_\lambda (\gamma_1) \rho_q^\lambda (I_0) \tag{2.69}
$$

In particular, the directional distribution of a gamma radiation $\gamma_{n+1}$ that is emitted from a state $I_n$, formed by the emission of $n$ unobserved quanta from an axially oriented state $I'_0$, is

$$
W(\theta) = \sum_{\lambda = \text{even}} B_{\lambda} (I_0) U_\lambda (\gamma_1) U_\lambda (\gamma_2) \ldots U_\lambda (\gamma_n) U_\lambda (\gamma_{n+1}) P_\lambda (\cos \theta)
\quad = \sum_{\lambda = \text{even}} B_{\lambda} (I_0) U_\lambda^{\text{cascade}} A_\lambda P_\lambda (\cos \theta) \tag{2.70}
$$

where the solid angle factor has been dropped for simplicity. If several different cascades are possible between the oriented level and the observed radiation then an effective de-orientation coefficient is obtained as the average of the $U_\lambda^{\text{cascade}}$, weighted by the appropriate branching intensity (which should be normalised to unity).

The formalism of this subsection also applies without modification to alpha, beta and electron capture decay. In the LTNO experiments to be described in this thesis, the gamma cascade resulting in the observed photon is preceded by the lepton quantum which initiates the nuclear decay. In this case the term $U_\lambda (\gamma_i)$ in (2.70) refers to the lepton field rather than to the electromagnetic field. The appropriate leptonic
de-orientation factor is calculated from (2.66) with $\gamma(L)$ understood to be the amplitude of the $L^{th}$ multipole component in the lepton field and $L$ the total angular momentum carried away by both the electron and neutrino.

In as much as the observed directional distribution in LNNO is dependent upon nuclear factors alone, (2.70) represents the final expression for its angular dependence and in principle is an infinite series. However the tensor nature of its coefficients, manifested in their dependence upon the $n-j$ symbols, imposes limitations upon its maximum rank $\lambda$. By inspection these can be seen to be:

$$B^\lambda_{I_0} = 0 \text{ if } \lambda > 2I_0$$

$$U^\lambda_{LI_1 I_f} = 0 \text{ if } \lambda > 2I_1 \text{ or } \lambda > 2I_f$$

$$A^\lambda_{L' I_1 I_f} = 0 \text{ if } \lambda > L + L' \text{ or } \lambda > 2I_1.$$  \hspace{1cm} (2.71)

Since the gamma transition probability greatly reduces as the multipole order increments, transitions with $L > 2$ are experimentally rare. Hence in the majority of cases and in the absence of further selection rules the directional distribution series may be truncated to

$$W(\theta) = 1 + B^2_2 A^2_2 P_2(\cos\theta) + B^4_4 A^4_4 P_4(\cos\theta)$$ \hspace{1cm} (2.72)

where use has been made of the fact that the zero rank coefficients are normalized to unity.

2.3 Non-nuclear contributions to the angular distribution in LNNO

In the derivations of the previous section only the "nuclear"
contributions to the angular distribution have been considered. Attention is now given to those aspects of the LTNO system which are "non-nuclear" in origin.

2.3.1 The solid angle correction factors

The directional distribution of (2.70) applies only to detection of nuclear radiation at a single angle $\theta$. Owing to the finite solid angle subtended by the detector, the observed angular distribution will differ somewhat from this ideal distribution expected for "point" detectors. The proper interpretation of precision angular distribution measurements must therefore depend on the accurate knowledge of this difference.

To perform this correction it is necessary to integrate (2.70), weighted by the detector efficiency, over the subtended solid angle. In the case of an axially symmetric detector centered along the radius vector with polar angle $\theta_0$, using this radius vector as the initial line, it is possible to introduce the polar angle $\beta$, where $\theta = \theta_0 + \beta$, such that the detector subtends a right cone of semi-angle $\Delta \beta$ at the origin. In this coordinate system the useful absorption of gamma radiation incident onto the detector is proportional to

$$\left(1 - e^{-\tau(\gamma)x(\beta)}\right)$$

(2.73)

where $\tau(\gamma)$ is the full energy absorption coefficient and $x(\beta)$ is the path length through the active volume of the detector.

The measured directional distribution will therefore be given by
Separating out the angular terms from this expression it is seen that the quantities of interest are the integrals

\[
I_\lambda = \int P_{\lambda}^\theta (\cos(\theta_0 + \beta)) \left\{ 1 - e^{-\tau(\gamma)x(\beta)} \right\} \sin \beta \, d\beta \, d\phi. \tag{2.75}
\]

Following the method of Rose\(^{[ROS53]}\) the spherical harmonic addition theorem

\[
P_{\lambda}^\theta (\cos(\theta_0 + \beta)) = P_{\lambda}^\theta (\cos \theta_0) P_{\lambda}^\lambda (\cos \beta) + \ldots. \tag{2.76}
\]

is applied, where the ellipsis indicates azimuthal terms. These do not contribute to the integration over \(\phi\) and so (2.75) reduces to

\[
I_\lambda = 2\pi \int_0^\Delta P_{\lambda}^\theta (\cos \theta_0) \int_0^{\Delta \beta} P_{\lambda}^\lambda (\cos \beta) (1 - e^{-\tau(\gamma)x(\beta)}) \sin \beta \, d\beta \tag{2.77}
\]

Introducing the solid angle correction factors, \(Q_{\lambda \theta}^\lambda\), defined by

\[
Q_{\lambda \theta}^\lambda \frac{P_{\lambda}^\theta (\cos \theta_0)}{P_{\lambda}^\lambda (\cos \theta_0)} = \frac{I_\lambda}{I_0} \tag{2.78}
\]

enables the corrected directional distribution (2.74) to be written in the form

\[
\bar{W}(\theta) = \sum_{\lambda} B_{\lambda} U_{\lambda} Q_{\lambda \theta}^\lambda P_{\lambda}^\theta (\cos \theta_0) \tag{2.79}
\]

The evaluation of the integrals \(I_\lambda\) requires a knowledge of both the detector geometry and the functional form of absorption coefficients \(\tau(\gamma)\). These are calculated from the theoretical photoelectric, Compton and pair production cross sections of the detecting material using Monte Carlo simulations \(^{[ST57]}\). By this method correction factors have been calculated
by Yates for Na(I) detectors, and by Camp and Van Lehn, and Krane for Ge(Li) detectors. The correction factors used in the present work are calculated using the method of Krane [KR72].

Typical dependences of the solid angle correction factors on gamma ray energy are illustrated in figure 2.3

![Figure 2.3 Dependence of $Q_k$ on gamma-ray energy for a Ge(Li) detector 5cm away from the source, and dimensions Length=3.2cm, and radius = 1.8cm. [KR72]](image)

2.3.2 The magnetic hyperfine interaction

In subsection 2.2.3 the orienting interaction was considered to be that between the static nuclear magnetic (electric) moment with an externally applied magnetic field (electric field gradient). In the
magnetic case, the degree of nuclear orientation is determined largely by
the parameter $\beta$ of (2.27). From (2.26) it can be seen that an appreciable
alignment is characterized by $\beta \approx 1$. For a nucleus with a gyromagnetic
ratio of one and taking $5\text{mK}$ as a lower limit to the continuously
attainable temperature of the present generation of $^3\text{He}-^4\text{He}$ dilution
refrigerators, then a magnetic field of at least $15\text{T}$ would be required for
successful orientation. The attainment of such a high field whilst
simultaneously maintaining low temperatures is not easily achieved. Hence
the "brute force" method, although simple in principle and applicable to
all types of nuclei, is not always practical.

Most of the successful orientation experiments have made use of the
hyperfine interaction in solids to avoid the need for such high values of
$B_{app}$. All experiments described in this thesis have been performed on
atomic nuclei implanted into a polycrystalline iron host. The origins of
the magnetic hyperfine field $^{[ST87]}$ will now be discussed.

In the case of a free atom, an electronic current density $J_e \, d\tau$ at a
point $r$ in a coordinate system centred at the nucleus will produce a
magnetic field at the nucleus:

$$B = \frac{\mu_0}{4\pi} \frac{r \times J_e}{r^3} \, d\tau \quad (2.80)$$

Writing $J_e \, d\tau = v \, dq$, where $v$ is the velocity of the charge element $dq$ and
noting that the electronic angular momentum is $l = n_e \, r \times v$, then for each
electron $i$

$$\mathbf{B}_i = -\frac{\mu_0}{4\pi} 2\mu \frac{l}{\langle r^{-3} \rangle} \quad (2.81)$$

Performing the summation over $i$, the contributions from closed subshells
disappear and in the LS coupling limit

\[ B_L = \frac{\mu_0}{4\pi} 2\mu_B \langle r^{-3} \rangle \]  \hspace{1cm} (2.82)

Similarly the electron spin will also give rise to a magnetic field, but since there is now a finite probability of electrons with non-zero spin existing at the nucleus it is necessary to consider separately the contributions from electrons "inside" and "outside" of the nucleus. Electrons outside the nucleus give rise to a simple dipole sum field

\[ B_s = -\frac{\mu_0}{4\pi} g_s \mu_B \sum_i \left[ 3(s_i \cdot \hat{r}) \hat{r} - s_i \right] \langle r^{-3} \rangle \]  \hspace{1cm} (2.83)

while those inside give rise to the Fermi contact term, which can be written in the LS coupling limit as

\[ B_c = -\frac{\mu_0}{4\pi} \frac{8\pi}{3} g_s \mu_B |\psi_s(0)|^2 \mathbf{S}. \]  \hspace{1cm} (2.84)

In the solid state there are two mechanisms which give rise to such a contact field. In a ferromagnet the s-d exchange interaction between the magnetic 3d-electrons and the s-like conduction electrons removes the degeneracy of the the \textit{s} and \textit{d} conduction bands. This leads to a net surplus of spin-up electrons and consequently a "conduction electron polarization" Fermi contact term of the form

\[ B_F = -\frac{\mu_0}{4\pi} \frac{8\pi}{3} g_s \mu_B \left| \frac{1}{2} \right|^2 \mathbf{S}. \]  \hspace{1cm} (2.85)

Secondly, due to the overlap between the polarized conduction electrons of the magnetic host with the inner s-electrons of the non-magnetic impurity, it is possible for filled orbitals with spin parallel to the net spin to have a different density distribution from those same orbitals with
antiparallel spin. The result of such exchange correlation effects, which for example can be incorporated into Hartree-Fock calculations by relaxing the condition that the radial wave-function is independent of the spin orientation, is that a closed s-subshell can exhibit a net spin without losing its spherical symmetry. These core s-electrons then contribute to the hyperfine field via an "exchange core polarization" contact term of the form (2.85). Such contact terms are important for all systems with non-zero resultant spin. However their significance is greatest for d-electron atoms for which in many cases the strong ligand field present in the lattice serves to quench the orbital contribution. Further, in a system of cubic symmetry such as iron, the "outside" spin term vanishes.

These mechanisms give rise to the large magnetic fields, typically $10 - 1000T$ for impurities in an iron host, required at the nucleus. It only remains to apply a modest external field $B_{\text{app}}$ to polarize the host lattice and produce the axis of quantization. Fields of only $0.7T$ are sufficient to almost completely (>99%) align the domains in iron. The effective magnetic field at the nucleus is therefore taken to be $|B_{\text{hyp}} + B_{\text{app}}|$.

The hyperfine interaction experienced by an impurity atom is of course highly sensitive to the nature of the surrounding lattice. It is not necessarily the case that the sites occupied by impurity atoms in the host lattice are unique and so they will be subject to a range of different hyperfine fields. A static LTNO experiment detects only the average hyperfine interaction for the system. If all the possible implantation lattice sites are labelled $i$, with associated magnetic hyperfine fields $B_{\text{hyp}}^i$ and population probabilities $f_i$, then the observed directional distribution of gamma radiation in a static LTNO experiment will be
\[ W(\theta) = \sum_{\lambda i} f^i \langle B_{\lambda i}^i \rangle U_{\lambda i} Q_{\lambda i} P_{\lambda i} (\cos \theta) \quad (2.86) \]

or in truncated form

\[ W(\theta) = 1 + \sum_{i} f^i \left\{ B^i_{\lambda i} U_{\lambda i} Q_{\lambda i} P_{\lambda i} (\cos \theta) + B^i_{\lambda i} U_{\lambda i} Q_{\lambda i} P_{\lambda i} (\cos \theta) \right\} \quad (2.87) \]

where use has been made of the fact that \( \sum f^i = 1 \). These are the final forms of the directional distribution expressions and will be used throughout this thesis.

Finally the magnetic hyperfine anomaly is discussed. This does not influence the form of the angular distribution formula, but is included here for the sake of completeness.

Once determined for one isotopic implant of a given element, the hyperfine field being atomic in origin should be the same for all other members of that isotopic chain which are implanted under similar conditions. However, implicit in the Fermi contact term (2.84) is the assumption of a point nucleus. In reality the nucleus is an extended body implying that (2.84) should be integrated over the nuclear volume and that an "inside" orbital contribution should also be considered since electrons of non-zero orbital angular momentum exist within the nucleus. This provides a nucleus dependent perturbation to the hyperfine interaction energy \(-\mu \cdot B\), which can be considered as a nucleus dependent correction to an otherwise atomic hyperfine field. The hyperfine anomaly is related to the fractional change in the hyperfine interaction when the nuclear perturbation is "turned on" and typically being only \(-1\%\) is neglected in this thesis. Therefore in the remainder of this thesis it is assumed that
the magnetic hyperfine field as determined for one nucleus may be applied without correction to all other nuclei in the same isotopic chain.

2.3.3 The nature of the lattice site occupation

In the experiments described in subsequent chapters of this thesis, samples were prepared by implanting the radioactive nuclei emerging from an isotope separator at energies of ~60 keV into the iron host matrix. Monte Carlo simulations [BB76] show that the entrant nucleus produces a series of primary lattice recoils which can go on to initiate independent subcascades. The resulting vacancies and interstitials typically cover a volume of order \((100 \text{ lattice spacings})^3\). The local temperatures produced in these cascades can reach 5000K and last for about 10ps [AV88], during which time a degree of self-annealing can occur. Frequently this leaves the implant in an undamaged area of the lattice, with residual vacancies and interstitials several atomic spacings away. Hence, provided that the implanted dose is not so great that the entire lattice is significantly damaged, it can be expected that most of the implanted nuclei will come to rest in undamaged substitutional sites. This critical dose may be taken to be of order \(10^{14}/\text{cm}^3\) or ~1 atomic percent. By comparison a typical LTNO dose is only of order 0.001 - 0.1 atomic percent.

The actual fraction of nuclei which occupy substitutional sites and also the nature of any alternative sites depends on their "solubility" in the host lattice. For "soluble" implants it is reasonable to make the simple assumption that the system contains only two sites, the
substitutional site with magnetic hyperfine field $B_{\text{sub}}$ and associated population $f$, and a second site with zero field and population $1-f$, corresponding to incompletely implanted activity on the surface of the lattice. In this regime the truncated directional distribution is simply

$$W(\theta) = 1 + f \left\{ B_2 (B_{\text{sub}}) U_{2} Q_{2} P_{2} (\cos \theta) + B_4 (B_{\text{sub}}) U_{4} Q_{4} P_{4} (\cos \theta) \right\}$$

(2.88)

For insoluble implants the situation is potentially very different. If the implanted ion is much smaller than the host then the lattice will contract locally and will therefore trap interstitials. In the reverse situation, implants will tend to accumulate vacancies. Thus, provided that the enthalpy of solution is sufficient to bind the interstitials or vacancies, the favoured sites will not necessarily be substitutional. This will be especially the case if the temperature is above 140K or 200K, the temperature at which interstitials and vacancies respectively achieve full mobility in iron $^{[78]}$. Hence it is expected that the proportion of insoluble implants in substitutional sites will be larger for cold implantation ($<4K$) than for room temperature implantation. In general therefore, insoluble implantations will exhibit a substitutional site and one or more sites associated with interstitials or vacancies, together with sites experiencing small or zero net hyperfine fields. In this complicated regime it is necessary to retain the general directional distribution expression (2.87).

2.4 Electric quadrupole orientation

In addition to the magnetic hyperfine field it is also possible to
exploit the large electric field gradients in solids to achieve orientation and hence a measurement of the nuclear electric quadrupole moment \[^{188}\text{Ha}^\text{27}\]. This can be done only in a host lattice of non-cubic symmetry where the quadrupole interaction is non-vanishing. Also the requirement of a uniquely defined direction for the electric field gradient necessitates the use of a single crystal host.

Since iron is a cubic lattice the electric interaction disappears for implants occupying substitutional sites. For non-substitutional sites this symmetry is destroyed and a composite magnetic and electric interaction must be considered. In a polycrystalline lattice the contribution to the total orientation arising from the electric quadrupole interaction is greatly reduced since the principal axis of each microcrystal is randomly oriented in space. Nevertheless, the integrated effect of the individual electric interactions leads to an attenuation of the observed orientation when compared to that expected from the magnetic interaction alone. The magnitude of this attenuation depends upon the relative strengths of the magnetic and electric interactions as well as the temperature and oriented spin \[^{188}\text{Ha}^\text{78}\].

The experiments described in this thesis were all performed on nuclei implanted into polycrystalline iron foils. A significant proportion of nuclei (of order 50%) came to rest in non-substitutional lattice sites. However, the resulting electric quadrupole attenuations are expected to be small, of order a few percent. On the other hand, the iridium sample used as a nuclear orientation thermometer, was melted with iron leading to an almost complete occupation of substitutional sites. With full cubic symmetry thus obtained, the electric quadrupole interaction vanishes completely.
Since the role of the electric quadrupole interaction is either small or nonexistent it is not considered in this work.

2.5 Nuclear spin-lattice relaxation

To conclude this survey of the principles of LTNO the situation that arises when the assumption of thermodynamic equilibrium fails will be considered. In such cases it is necessary to include the time dependence of the approach to equilibrium into the calculation of the orientation parameters \([KL87]\). It will be assumed that the spin system is highly dilute so that impurities are coupled only to the host lattice and mutual interactions within the nuclear spin system are small. In this limit the nuclei relax independently and so cannot be described by a temperature. If this relaxation proceeds via an exchange of energy with the degenerate Fermi gas formed by the conduction electrons of the (metallic) host lattice then the spin-lattice interaction is given by the Hamiltonian

\[
H_{S-L} = A I \cdot S = A I S_z + \frac{1}{2} (S_+ I_- + S_- I_+) \tag{2.89}
\]

where \(I\) is the nuclear spin operator and \(S\) the effective lattice spin operator which is related to the orbital and spin operators of the conduction electrons. Transitions induced by the non-diagonal elements link states differing by one unit in the magnetic quantum number. The corresponding transition probabilities, derived from Fermi's golden rule are

\[
W_{m,m+1} = \frac{\Delta E}{2kC_k} \left[ \frac{([I(I+1) - m(m+1)]}{e^{\Delta E/kT} - 1} \right] \tag{2.90}
\]
Here $\Delta E$ is the magnetic substate energy splitting and $C_k^*$, the Korringa constant, is a system dependent constant arising from the integral over electron states. Under the conditions that the nuclear spin system is weakly coupled to the lattice and has a much smaller heat capacity, it can be shown that the spin density matrix remains diagonal at all times [SH60]. These diagonal elements, which correspond to the sublevel populations $p_m$, obey a gain-loss equation of the form

$$\frac{dp_m}{dt} = \sum_n (W_{mn}p_n - W_{nm}p_m)$$

or in matrix notation

$$\frac{dp}{dt} = Rp$$

where $p$ is a $(2I+1)$ dimensional column vector and $R$ is the time dependent relaxation matrix. Under the equivalence transformation $D$, where $D_{mn} = \delta_{mn} e^{-S/m/2kT}$, the matrix $R^* = D^*RD$ becomes symmetric. Hence (2.93) is solved by the diagonalisation of $R^*$ which leads to

$$p(t) = DUe^{Kt^{-1}D^{-1}}p(0)$$

where $U$ and $K$ are the matrices of the eigenvectors and eigenvalues of $R^*$ respectively. Thus the time evolution of the populations is a multi-exponential function which is completely determined by the Korringa constant and the initial populations $p_m(0)$. 

\[ W_{m+1,m} = W_{m,m+1} + \frac{\Delta E}{2kC_k^*} [I(I+1) - m(m+1)]. \] (2.91)
Experimentally it is convenient to define a single effective time constant $\tau_{\text{slr}}$ to approximate the time dependence of (2.94) such that

$$\ln \left[ \frac{B_2(t = \infty) - B_2(0)}{B_2(t = \infty) - B_2(\tau_{\text{slr}})} \right] = 1$$  \hspace{1cm} (2.95)

The initial populations $B_2$ could be either semi-oriented, if they were produced by decay from an oriented precursor, or random. For the case of interest to the present work the parent state is produced directly by on-line implantation so that the latter assumption is appropriate and $B_2(0) = 0$. A simple empirical estimate for $\tau_{\text{slr}}$ can then be made from

$$\tau_{\text{emp}} = \min (\tau_{\text{htl}}, \tau_{\text{lrl}})$$  \hspace{1cm} (2.96)

where

$$\tau_{\text{htl}} = \frac{4}{3} \frac{C_k}{T} \quad \text{and} \quad \tau_{\text{lrl}} = \frac{3.3}{(I + \frac{1}{2})} \frac{C_k}{\beta T}$$  \hspace{1cm} (2.97)

The labels $\text{htl}$ and $\text{lrl}$ refer to the high and low temperature limits, as described by the conditions $\beta << 1$ and $\beta >> 1$ respectively. Apart from the region where the two limits overlap, i.e. where $\beta$ is of order 1, the agreement with $\tau_{\text{slr}}$ as deduced from an exact calculation of (2.94) is good.

Provided that the relaxation proceeds via electron-hole pair excitations and magnetic dipole interactions (neglecting hyperfine anomalies) then it is possible to derive the scaling property

$$\left( \frac{\mu}{I} \right)^2 C_k = \text{constant}$$  \hspace{1cm} (2.98)

for different isotopes of a given element in the same host. Hence, $C_k$
values and hence relaxation times may be estimated for a whole range of isotopes provided that at least one nucleus of the same element has been measured.

The significance of relaxation in LTNO comes when the radioactive decay is considered. If \( t_{rel} \), which can vary from \( 10^{-3} \) to \( 10^4 \) s for impurities in iron, is significant in comparison with the nuclear lifetime, then an appreciable number of nuclei decay from incompletely oriented states. This results in a temperature dependent reduction of the observed angular distribution anisotropy via an attenuation in the orientation parameters.

In the case of on-line LTNO the situation is further complicated by the steady stream of incoming warm nuclei. Sublevel populations representing the relaxing and populating states and allowances for the radioactive decay of the relaxing sublevels must then be included into (2.93). On the assumption that \( N \) nuclei are implanted per second and that these are equally distributed among the relaxing sublevels then the modified analogue of (2.92) may be written as

\[
\frac{dp_n}{dt} = \sum_n \left( W_{nm} p_n - W_{mn} p_m \right) - \frac{\ln 2}{t_{1/2}} p_m + \frac{N}{(2I+1)}
\] (2.99)

The sublevel populations will then attain secular equilibrium, given by the condition \( dp/dt = 0 \). From this the sublevel populations and hence the orientation parameters may be deduced.
3.1 Introduction

This chapter will briefly outline the experimental apparatus and methods used to measure the ground state magnetic moments of the neutron deficient europium, samarium and promethium isotopes. The practical aspects of radioactive beam production, and transport, and low temperature nuclear orientation will be discussed, as an introduction to the experimental results presented in chapters five and six.

3.2 Radioactive beam production and transport

3.2.1 The Nuclear Structure Facility at Daresbury

The Nuclear Structure Facility (NSF) is situated within the Daresbury Laboratory, Warrington, Cheshire, England, and provides a wide variety of ion beams which are used by UK and overseas experimenters in nuclear structure physics.

The NSF houses a 20MV tandem Van de Graaff generator, which consists of an ion source, an evacuated beam tube, a steel pressure vessel filled with $SF_6$ gas, an insulating column and centre terminal, as shown schematically in figure 3.1.
Figure 3.1 Schematic view of the 20 MV tandem accelerator at Daresbury.
Figure 3.2 Floor plan of the experimental areas at Daresbury.
The singly charged negative ions produced by the ion source, are accelerated by a 500 KV supply through a 90° bending magnet into the pressure vessel. Once inside the vessel, the negatively charged ions are accelerated towards the positively charged centre terminal, where they are stripped of many of their electrons by collisions in a thin carbon foil. The positively charged ions are then accelerated away from the centre terminal to ground, and out of the vessel through a rotating 90° bending magnet into one of 3 experimental areas shown in figure 3.2.

The charge is brought onto and taken off the centre terminal by the laddertron, a continuous loop of metal rungs moving at 10ms⁻¹. It can produce very stable voltages of up to 20MV.

The energy gain of the beam passing through the carbon stripper foil can be quite dramatic. For ⁴⁸Ti charge state 1⁻, the energy before hitting the carbon foil would be 20.5MV x 1. After passing through the foil, losing 11 electrons, it will gain 10 x 20MV going to ground. In effect the beam can have its energy increased by a factor of 10 or so, just by the act of passing through the carbon foil. The beam from the vertical tandem is then deflected through 90° and focussed by a magnetic quadrupole doublet onto the target of the DOLIS on-line ion source.

3.2.2 The isotope separator

At Daresbury, the isotope separator (DOLIS) on-line to the 20 MV NSF Tandem Van de Graff accelerator, can produce a variety of stable or radioactive beams. The DOLIS layout with its beam lines is shown in figure 3.4, a detailed description has been given by Grant et al. [GR87]. The NSF beam passes through an appropriate target placed outside the separator ion...
source and the heavy ion fusion reaction products are stopped in the "catcher" of the ion source, after passing through a thin tungsten window. An essential requirement here, is that the time for the release of the radioactive products from the catcher and their ionization should be short so that the limitation on the accessible half life is not too severe.

Two types of ion source are operational at Daresbury, a FEBIAD and a thermal ion source \[^{[KI81]}\]. Both these sources operate at high temperatures (above 2000\(^\circ\)C) and are shown schematically in figure 3.3(a) and 3.3(b). The secondary beam from the ion source is extracted and may be accelerated up to 100 keV. The 30 kV extractor backs off the 100 kV accelerating supply so that the secondary beam energy does not depend on the extractor voltage. In practice, the ions are usually accelerated by only 60 kV. Mass separation is achieved by a 60\(^\circ\) bending magnet with a resolution of 1 part in 700. The alignment of the ion source and the focal plane axes may be viewed through the magnet vacuum chamber.

![Diagram of FEBIAD and Thermal ion sources]

Figures 3.3a and 3.4b. Schematic diagrams of the FEBIAD and Thermal ion sources at used at Daresbury.
The beam optics is flexible and good transport conditions are obtained by X-Y steering plates, an einzel lens and an electrostatic quadrupole triplet situated after the ion source position. Beyond the focal plane is the electrostatic switchyard system which can direct the separated beams to three different beam lines (figure 3.4). One of the beam lines leaves the switchyard on the axis of the focal plane chamber, and the other two are at ±40° to the central line.

The central beam line leads into the the $^3$He-$^4$He dilution refrigerator used for the LTNO experiments. A $5^\circ$ bend is introduced on this line before the beam enters the dilution refrigerator to enable reduction of thermal radiation and neutral beam heating of the system. The right-hand line is dedicated for collinear laser spectroscopy, and the left-hand line is utilised for $\gamma$-$\gamma$ and e-$\gamma$ spectroscopy. One of the interesting features of the switchyard is simultaneous beaming to the
left-hand and central beam lines (of masses A-2 and A respectively at A = 140).

3.3 The $^3$He-$^4$He dilution refrigerator

3.3.1 The $^3$He-$^4$He cooling process

The operation of the $^3$He-$^4$He dilution refrigerator relies on the special properties of the $^3$He-$^4$He mixture at low temperatures. Figure 3.5 shows the phase diagram of $^3$He-$^4$He mixtures at saturated vapour pressures. At the coexistence curve (T ≤ 0.87K) the liquid spontaneously separates into two components; one of the phases being rich in $^4$He, and the other, rich in $^3$He, floats on the former. Below 0.1K the concentrated (upper) $^3$He phase becomes almost pure, whereas even at temperatures near to absolute zero the $^4$He (lower) rich phase still contains about 6.4% of $^3$He atoms, at equilibrium. Because of its zero nuclear spin and superfluid properties, liquid $^4$He is, below 0.5K, both thermally and hydrodynamically inert. It is a quantum mechanical ground state and has zero entropy. The

![Phase diagram of $^3$He/$^4$He mixtures, where $x$ is the $^3$He concentration.](image)

Figure 3.5 Phase diagram of $^3$He/$^4$He mixtures, where $x$ is the $^3$He concentration. [LO74]
lighter $^3$He, in contrast, with its nuclear spin $I = 1/2$ obeys Fermi-Dirac statistics and thus behaves differently: It has a high heat capacity and large entropy. Through the act of pumping on the dilute phase, $^3$He is preferentially removed by virtue of its higher vapour pressure. In order to maintain the $^3$He-$^4$He equilibrium concentration, $^3$He is drawn across the phase boundary from the concentrated phase. This process can therefore be considered as the "evaporation" of $^3$He atoms from the concentrated phase into the "vacuum" provided by the dilute phase, hence leading to cooling. A full discussion of the properties of $^3$He-$^4$He mixtures is given by Lounasmaa [1074].

3.3.2 The dilution refrigerator design

The principle of the dilution refrigerator was first suggested by London in 1951. The first cryostat of this type was built by Das De Bryn, Ouboter and Taconi in 1965, and in recent years temperatures of a few mK are easily reached. A schematic view of the main features of a conventional $^3$He-$^4$He dilution unit is shown in figure 3.6. In stable operation, the concentrated-dilute phase boundary lies in the mixing chamber, which is the coldest part of the refrigerator. The lower half of the mixing chamber, which contains the dilute phase, is connected to the still via a series of heat exchangers. The still is pumped by a large diffusion pump backed by a sealed rotary pump, preferentially removing $^3$He across (~ 95%) despite its low (~ 7%) concentration in the dilute phase. As has already been noted, this leads to the migration of $^3$He across the phase boundary and consequently cooling. For optimal circulation rates an electric heater maintains the still at 0.8K. The $^3$He emerging from the
rotary pump exhaust re-enters the cryostat via liquid nitrogen cold traps which remove any air that may have leaked into the system. Once inside the cryostat, the $^3$He is pre-cooled to 1.2K by contact with the pumped $^4$He pot and then liquefied in the condenser. A flow impedance keeps the pressure of the $^3$He sufficiently high for condensation to occur. The $^3$He is further cooled to 0.8K by contact with a wrap around heat exchanger on the still and then returns to the mixing chamber via a concentric tube continuous heat exchanger and four sintered silver step heat exchangers, where cooling takes place by contact with the outgoing $^3$He from the pumped

Figure 3.6 Dilution refrigerator operating principle.
dilute phase. The efficiency with which the pre-cooling process is performed is one of the limitations on the minimum attainable temperature.

The conventional cryostat design external to the dilution unit shown in figure 3.7, consists of the Inner and Outer Vacuum Chambers (IVC and OVC; both evacuated to pressures $< 10^{-6}$ Torr) which enclose the main $^4$He bath. Within the OVC lies a radiation shield maintained at 77K by a liquid nitrogen cooled jacket. The IVC contains the dilution unit, the main helium bath on its outer face acting as a 4.2K heat shield. Finally, in thermal contact with the dilution unit there is the 1.2K $^4$He pot (fed either continuously or intermittently by the main bath), the 0.8K heat shield (attached to the still) and the 25mk heat shield.

The experimental samples and thermometers are soldered to a copper cold finger screwed into the base of the mixing chamber.

Figure 3.7 Schematic diagram of $^3$He/$^4$He cryostat, showing 4K side access. [GN87]
3.3.3 The operation of the dilution unit

In attaining the base temperature in a continuous mode, a balance must be struck between the extra cooling induced by increasing the rate at which $^3$He crosses the phase boundary, and the consequent additional heating caused by the greater volume of returning gas and lower heat exchange efficiency at higher circulation rates. For the Daresbury On-Line Isotope Separator Cryogenic On-Line Device (DOLIS-COLD) refrigerator, the optimum $^3$He circulation rate is ~ 400 μmoles s$^{-1}$. With the 4K baffle of the side access tube shut (see subsection 3.3.4) this can lead to a base temperature of ~ 8mK.

Experimentally in LTNO it is important to be able to vary the temperature of the refrigerator in some way. In principal this can be done by adjusting the power output of the still heater, hence changing the circulation rate. However, this is a rather uncertain and unreliable method. Greater control is achieved by adding known quantities of heat directly into the mixing chamber via a thermally attached resistor. If this power input is denoted by $Q$ then it can be shown $^{[L074]}$ that

$$Q = C \left( T_{mc}^2 - T_{base}^2 \right)$$

(3.1)

where $T_{mc}$ is the actual temperature of the mixing chamber and $T_{base}$ the temperature which would be achieved in the absence of $Q$. From a knowledge of the parameters $C$ and $T_{base}$ it is possible to select the required temperature by supplying the appropriate current to the mixing chamber resistor. For the DOLIS-COLD refrigerator typical values for these parameters are
\[ c = 0.019 \mu W/K^2 \quad \text{and} \quad T_{\text{base}} = 8\text{mK} \]

### 3.3.4 The cooled side access

In an on-line experiment the dilution unit has three extra heat loads placed upon it in addition to the usual heat leaks. These are caused by both the thermal radiation from the incoming beam line, the kinetic energy of the 60 keV mass separated beam itself, and the radioactive decay energy deposited. This heat load must be kept at the \( \mu W \) level if temperatures of order 10mK are to be maintained. To this end the refrigerator is supplied with a one metre long side access tube, the inner copper wall of which is cooled to 4K by thermal contact with the refrigerator's main helium bath.

One metre from the cold finger there is a variable diameter iris at 77K and at 40cm a fixed aperture lead plug, maintained at 4K by contact with the wall of the side access tube. These not only act as radiation baffles, but also prevent scattered beam from striking the target.

The detectors around the refrigerator (see subsection 3.6) are shielded against any activity accumulated on the lead plug by an 8cm wall of lead bricks. As beam transmission rates of \( > 90\% \) are consistently achieved, this level of background radiation is never very great, (also, any activity on the plug is much further from the detectors than the target itself).

Finally, at 7cm from the sample position, there is a closeable 4K radiation baffle which has been modified to allow ion current measurements. This can be fully closed for off-line work or for beam diagnostics.
3.4 Sample preparation and polarization

In this work the host lattices have all been polycrystalline iron foils of at least 99.998% purity. These are prepared by cold rolling an iron sheet down to a thickness of ~ 0.1mm and then annealing under an atmosphere of dry hydrogen at a temperature of ~ 800°C for 24 hours. Finally, just before use, the foil is chemically etched in a solution of 54 vol % H₃PO₄ (80%), 36 vol % H₂O₂ (30%), 0.5 vol % distilled water and 1.5 vol % butoxyethanol [VI81].

The nuclear orientation thermometers used in this thesis, namely $^{57}$CoFe, and $^{192}$IrFe were prepared by high temperature diffusion of the radioactive isotope in iron [MA87]. After etching, > 99% of nuclei in full substitutional lattice sites can be consistently achieved.

The host foil and thermometer are soldered onto a solid, high purity, copper cold finger which screws into a copper adapter attached to the mixing chamber of the dilution unit. The DOLIS refrigerator has a top loading facility which enables the cold finger to be changed whilst the refrigerator is running. This process which typically takes ~ 3 hours, is often of vital importance to an on-line experiment where the target foil may need to be changed to remove unwanted longer lived activities built up in the sample during implantation.

The loaded sample and thermometer lie at the centre of a 1.5T superconducting split coil magnet, which is used to polarize the magnetic domains in the foils. Typically a field of ~ 0.7T is applied, which provides not only magnetic saturation but also ensures that the solder is not superconducting and hence a good thermal contact to the cold finger is
achieved. The field produced by the magnet has a homogeneity of 1 % over the sample volume.

3.5 The Mini-Beam Line

The Mini-Beam Line is a general purpose line which in the past has mainly been used for electron conversion studies, allowing the measurement of gamma ray multipolarities and facilitating the search for E0 transitions.

The beam line terminates in two cubes. The mass separated beam enters the upper cube directly and is collected on a moveable 6.4 mm wide cassette tape. Activity can therefore be transported down to the lower cube. In this way longer lived isotopes can be studied in the absence of shorter lived varieties and vice-versa. Both cubes have re-entrant covers to allow the detectors to be positioned <4cm away from tape for coincidence measurements. Electron spectroscopy is carried out using a mini-orange magnetic filter in conjunction with a cooled Si(Li) detector. The electrons are focussed onto the face of the Si(Li) detector (which is shielded from direct gamma rays by a lead plug in the axis of the magnet) while positrons are defocussed and therefore not detected.

Spectroscopic studies which can be performed on the mini-beam line include:
1. $\gamma - \gamma$ coincidence
2. $\gamma - \beta^-$ coincidence
3. conversion electron spectroscopy
4. half-life measurements
3.6 Data acquisition

For the work described in this thesis gamma rays have been observed using either hyperpuregermanium or Ge(Li)detectors, which combine reasonable photopeak efficiency (either ~ 25%, or ~ 80% relative to a 3" x 3" NaI detector at 25cm from the source) with adequate resolution (typically ~ 2.4 keV at 1333 keV). For low temperature directional distribution measurements these are positioned axially (θ=0) and equatorially (θ=π/2) with respect to the polarization axis and detect gamma events in singles mode. The Daresbury configuration allows for two axial and two equatorial detectors to be used. The detector to source distance is ~ 8cm. A block diagram illustrating singles mode data acquisition is shown in figure 3.8.

![Block diagram illustrating singles mode data acquisition.](image)

In order to correct for dead time losses in the system see (subsection 4.3) a pulse generator is connected to the test input of the detector pre-amplifier. The pulser frequency is normally set to ~ 10 Hz.
and has an amplitude such that it is placed at a convenient position at the high energy end of the gamma ray spectrum. The pre-amplifier signals are amplified and directed into standard Analogue to Digital Converters. Once the signals have passed through the ADC they are processed by the GEC event manager, and recorded into 4k spectra for subsequent analysis.

### 3.7 Thermometry

In any nuclear orientation experiment accurate measurement of the low temperature is of the utmost importance. The natural approach for such measurements is to capitalise upon the LTNO effect itself. An experimentally viable LTNO thermometer needs to satisfy several important conditions, namely:

1. the hyperfine interaction is well known and a 100 % occupation of full substitutional sites in the host lattice can be relied upon,
2. the decay scheme is simple with accurately known $U_{\Lambda \Lambda}$ coefficients,
3. the nuclear spin relaxation time is short.

Further experimental considerations of less thermometric importance include long half lifes and small radioactive heating. A nuclear orientation thermometer subject to the above constraints will therefore, by the use of equation (2.88), have a known (or calculable) temperature dependence of gamma ray directional distribution. Hence the measurement of gamma ray distribution will yield the temperature of the thermometer (which is assumed throughout this thesis to be in thermodynamic equilibrium with the host lattice).
The useful temperature range of a thermometer can be defined in terms of the sensitivity function $\frac{\partial W(\theta, T)}{\partial T}$. The maximum of the sensitivity function (for axial measurements) is denoted $T_{\text{max}}$. The upper and lower limits of the useful temperature range, $T_u$ and $T_l$, define the region where the sensitivity function is greater than 5 %. These properties for some of the most frequently used LTNO thermometers are shown in table 3.1 \[^{[587]}\]. Not surprisingly, there is little difference between $T_{\text{max}}$ and the temperature for which the Boltzmann parameter $\beta$ (2.27) is one.

The experiments described in this thesis were performed on neutron deficient light rare-earth nuclei, where the magnitude of the hyperfine fields are expected to be large for an iron host. Consequently the temperature at which these nuclei become fully oriented is sometimes expected to be unusually high ($\sim 40\text{mK}$) necessitating the use of a thermometer with good sensitivity at relatively high temperatures ($\sim 200\text{mK}$). Therefore a relatively novel thermometer in the form of $^{192}\text{IrFe}$, was used in conjunction with a conventional $^{57}\text{CoFe}$ thermometer, for the directional distribution measurements described in chapters five and six. The maximum of the sensitivity function for $^{192}\text{IrFe}$ is calculated to be 38mK, with a useful temperature range of 7mK to 200mK for the 468keV $\gamma$-ray transition. The decay schemes for $^{57}\text{Co}$ and $^{192}\text{Ir}$ are shown in figures 3.9 and 3.10 respectively.

The accuracy of this method of thermometry has previously been put to the test by a direct comparison of a $^{60}\text{CoCo(hcp)}$ thermometer against a Josephson junction noise thermometer \[^{[586]}\]. In the temperature range 10-50mK, the results show only a 0.5% difference in the implied temperatures from the two devices. Hence the measurement of temperature by the LTNO method with errors in the region of 1-2% is relatively...
straightforward and unambiguous.

![Diagram](image)

Figure 3.9 γ-ray decay scheme following the β-decay of $^{57}$Co.

<table>
<thead>
<tr>
<th>Thermometer</th>
<th>$E_γ$ (keV)</th>
<th>$βT$ (mK)</th>
<th>$T_L$ (mK)</th>
<th>$T_{max}$ (mK)</th>
<th>$T_u$ (mK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{54}$MnFe</td>
<td>834.8</td>
<td>-9.2</td>
<td>1.8</td>
<td>7.2</td>
<td>51</td>
</tr>
<tr>
<td>$^{54}$MnNi</td>
<td>834.8</td>
<td>-13.1</td>
<td>2.6</td>
<td>10.4</td>
<td>74</td>
</tr>
<tr>
<td>$^{57}$CoFe</td>
<td>136.5</td>
<td>-14.2</td>
<td>2.9</td>
<td>12.4</td>
<td>90</td>
</tr>
<tr>
<td>$^{57}$CoNi</td>
<td>136.5</td>
<td>-5.9</td>
<td>1.2</td>
<td>5.2</td>
<td>37</td>
</tr>
<tr>
<td>$^{60}$CoCo (hcp)</td>
<td>1173.2</td>
<td>-6.1</td>
<td>1.3</td>
<td>6.9</td>
<td>50</td>
</tr>
<tr>
<td>$^{60}$CoFe</td>
<td>1173.2</td>
<td>-8.0</td>
<td>1.8</td>
<td>9.0</td>
<td>66</td>
</tr>
<tr>
<td>$^{60}$CoNi</td>
<td>1173.2</td>
<td>-3.3</td>
<td>0.8</td>
<td>3.7</td>
<td>27</td>
</tr>
<tr>
<td>$^{166}$HoHo (hcp)</td>
<td>810.3</td>
<td>137</td>
<td>32</td>
<td>137</td>
<td>750</td>
</tr>
</tbody>
</table>

Table 3.1 The useful temperature ranges of the more commonly used LTNO thermometers.

[MA87]
Figure 3.10 $\gamma$-ray decay scheme following the $\beta$-decay of $^{192}\text{Ir}$.

References: [78a, 78b]
CHAPTER FOUR

DATA ANALYSIS

4.1 Introduction

As has been shown in chapter 2, the angular distribution of gamma radiation from an oriented source is the combined effect of a great many properties of the nuclear system. This chapter will deal with the methods by which these nuclear properties (hyperfine interaction, oriented spin, daughter spins and multipole mixing ratios) are extracted from experimental measurements.

4.2 Spectrum analysis

Once acquired, the gamma ray spectra are analysed using the Daresbury GEC peak analysis program. This package offers two methods of extracting the gamma ray peak areas from these spectra. The first of these, integrates the counts in the region of interest and subtracts a background linearly interpolated between lower and upper background regions. An example of such a peak integration, with its associated region defining markers, is shown in figure 4.1. Markers 3 → 4 define the lower background region, 1 → 2 the region of interest and 5 → 6 the upper background region.

The second method, performs a multiple Gaussian peak fit, allowing unresolved multiplets to be analyzed. This method invokes a least squares
fit to the function

\[ y(x) = b_0 + b_1 x + \sum_{p=1}^{N} A_p \exp \left\{ -\frac{1}{2} \left( \frac{x - c_p}{\sigma_p} \right)^2 \right\} \]  \hspace{1cm} (4.1)

Figure 4.1 Example of a non peak fitting peak integration.

where \( x \) is the spectrum channel number. The parameters \( b_{0,1} \) define the background, while \( c_p \) and \( \sigma_p \) are the centroids and Full Width Half Maxima of the \( N \) peaks labelled by \( p \). Initial guesses for \( c_p \) are required. These are provided by the placement of markers \( p1 \) and \( p2 \), the markers \( r1 \) and \( r2 \) being used to define the beginning and end of the region to be fitted. These are illustrated for the case of a doublet in figure 4.2.

Since the peak shapes need not necessarily be truly Gaussian, the non
peak fitting procedure is used in preference to the Gaussian fit, unless the use of the fit is forced by the inadequate resolution of multiplet peaks.

Frequently the background under the peak of interest is discontinuous. In these cases, the 122 keV peak of $^{57}$Co being a good example, the behaviour of the background under the peak is not clearly defined. The GEC spectrum analysis program allows for this by enabling peak markers to be saved into a file, enabling peak markers to be transferred from one spectrum to another. Thus although both the peak integrating routines described above

![Figure 4.2 Example of a double Gaussian fit for peak integration.](image)

may subtract a discontinuous background in a non-unique way, this is to a large extent compensated by the fact that each peak and its associated background is treated consistently in all spectra.
4.3 Experimental determination of $W(\theta)$

In chapter 2 the directional distribution $W(\theta)$ of gamma rays from an LTNO source (2.86) was derived. For a particular gamma ray this can be related to the observed peak area $N_\gamma(\theta)$ by the expression

$$N_\gamma(\theta) = W_\gamma(\theta) I_\gamma \varepsilon_\gamma \Omega D$$

(4.2)

where $I_\gamma$ is the gamma ray intensity per parent decay and $D$ denotes the number of parent decays during the counting period. The detector is assumed to have a gamma ray detection efficiency $\varepsilon_\gamma$ and to subtend a solid angle $\Omega$ at the source.

The parameter $D$ may be evaluated explicitly for the case where the sample decays according to the usual exponential decay law, with decay constant $\lambda$. Thus for a spectrum whose counting period is of length $t_{\text{real}}$ and commences at time $t_{\text{start}}$

$$D = \int_{t_{\text{start}}}^{t_{\text{start}} + t_{\text{real}}} e^{-\lambda t} \, dt$$

$$= e^{-\lambda t_{\text{start}}} (1 - e^{-\lambda t_{\text{real}}})$$

(4.3)

where constant coefficients have been neglected for simplicity. It is clear that the factors $D$ depend on the counting period of the spectrum. However, it is necessary to distinguish between the period for which the ADC was actually available for the processing of signals, $t_{\text{live}}$, and the
total counting period, $t_{real}$. Since in the course of registering a signal
the ADC is unable to receive further data, these two times will differ by
an amount equal to the dead time, $t_{dead}$. Typically this constitutes 5-25%
of the counting time, depending on the count rate. This can be accounted
for by introducing a corrective factor, $t_{live}/t_{real}$. Hence
\[ D = t_{live} R \]  
(4.4)

where the rate $R$ is given by
\[ R = \frac{e^{-\lambda t_{start}}}{t_{real}} (1 - e^{-\lambda t_{real}}). \]  
(4.5)

Experimentally it is necessary to have some means of determining the
live time, and the real time of the data acquisition system as a whole.
This is achieved by the pulser method\cite{8070}, in which an artificial peak
is introduced into the measured gamma ray spectrum by a stable pulse
generator, whose signals are fed into the pre-amplifier of the detector
(see section 3.6). These undergo the same pulse processing as the nuclear
events and are subject to almost the same counting losses. Therefore the
area of the pulser peak divided by the constant repetition rate yields the
live time.

However, there is a systematic error associated with such a
correction. Namely the fact that the pulser signals come more or less
evenly spaced in time and hence they are able to interact with the gamma
ray signals, which are randomly distributed in time, but not with
themselves. As a result the pulser signals can only correct for the
partial gamma ray deadtime, not the total dead time to which they
themselves contribute. For the most part these errors can be avoided if
the pulser frequency is sufficiently low that the ratio of its dead time 
to that of the gamma ray is kept below 10% \[ \text{[W71]} \].

If \( p \) denotes the pulser peak area and \( f \) its repetition rate, then 
from (4.4) one obtains

\[
D = \frac{p}{f} R \quad (4.6)
\]

Generally peak areas from several spectra \( j \) acquired under similar 
conditions are summed. Hence

\[
\sum_{j} \frac{[N/p]_j \gamma}{\Omega} = \frac{1}{\Omega} \sum_{j} \frac{R_j}{\Omega} \sum_{j} R_j \quad (4.7)
\]

In order to eliminate the pulser repetition rate, relative intensity,
efficiency and solid angle terms, "warm" counts are taken at a 
sufficiently high temperature, usually \( \sim 1K \), so that the directional 
distribution becomes isotropic \((W(\theta) = \text{a constant})\). Therefore by taking 
the "cold" to "warm" ratio and rearranging

\[
W(\theta) = \frac{\sum_{j} \frac{[N/p]_j}{\Omega} (\theta)}{\sum_{j} R_j} \times \frac{\sum_{k} R_k}{\sum_{k} \frac{[N/p]_k}{\Omega} (\theta)}
\]

\[
= \sum_{\lambda_i} f_{\lambda} \lambda (B_{\lambda} \Gamma)(U, A, Q, P, \cos \theta) \quad (4.8)
\]

where the subscript \( \gamma \) has been dropped for simplicity and the labels \( j \) and 
\( k \) are understood to refer to summation over "cold" and "warm" spectra 
respectively. From this expression, the fundamental measurement in LTNO, 
the anisotropy, can be defined to be \((W(\theta) - 1)\%\). 

72
So far only one specific mode of source decay has been considered. If the source is fed by one or more decaying parent nuclei, then \( D \) can be derived (from 4.3) in an identical manner using instead a multi-exponential integrand. Where the source is produced continuously on-line, the explicit time dependence of the integrand is unknown as a result of the variable production rate. However, the arguments leading to (4.3), (4.6), (4.7), and (4.8) are still valid, even though \( R \) is unknown.

LTNO experiments generally detect the gamma radiation directional distribution at two angles, axial \((\theta = 0)\) and equatorial \((\theta = \pi/2)\). The data acquisition procedure is such that the counting period begins and ends simultaneously in all detectors. Corresponding axial and equatorial spectra will therefore have common values of \( R \). Applying (4.8) to the ratio of axial and equatorial directional distributions, with the condition that the summations over the two directions are identical, yields

\[
\frac{W(0)}{W(\pi/2)} = \frac{\sum_{j} \{(N/p)_{j}(0)\}}{\sum_{k} \{(N/p)_{k}(0)\}} \times \frac{\sum_{k} \{(N/p)_{k}(\pi/2)\}}{\sum_{j} \{(N/p)_{j}(\pi/2)\}}
\]

\[
= \frac{\sum_{\lambda}^{L} c_{\lambda} \Lambda (B_{\lambda})^{(1)}}{\sum_{\lambda}^{L} c_{\lambda} \Lambda (B_{\lambda})^{(0)}}
\]

(4.9)

In this way the observed gamma ray peak areas (with pulser normalisation) can be directly related to the theoretical angular distribution without making any assumptions as to the nature of the source decay.

The terms on the right hand sides of (4.8) and (4.9) are, in the absence of any previous knowledge, multiparameter expressions. Assuming the truncated directional distribution (2.87) leads to just two unknown
parameters, \( \sum f_i B_i U_2 A_2 \) and \( \sum f_i B_i U_A A_4 \). In order to calculate these separately it is necessary to retain the two independent axial and equatorial anisotropies (4.8), thus allowing the separation of the two terms by Gaussian elimination. In this way and in the case where the axial and equatorial solid angle correction factors are identical one obtains

\[
\sum_{i} f_i B_i (E_i^{\text{hyp}}) U_{2,i} A_{2,i} Q_{2,i} = \frac{1}{7} \left\{ 3(W(0) - 1) - 8(W(\pi/2) - 1) \right\} \tag{4.10}
\]

and

\[
\sum_{i} f_i B_i (E_i^{\text{hyp}}) U_{4,i} A_{4,i} Q_{4,i} = \frac{4}{7} \left\{ (W(0) - 1) + 2(W(\pi/2) - 1) \right\} \tag{4.11}
\]

When the series is confined to \( \lambda = 2 \) by any of the conditions of (2.71), or if a priori knowledge allows the two parameters to be related, then there is no loss of information incurred by considering only axial/equatorial ratios.

Finally, it should be noted that the R factors can be removed from (4.8) without calculation by normalizing to a gamma ray peak that is known to exhibit no anisotropy. Presumably this would be by virtue of it having arisen from a level of spin zero or one half, and it must belong to the nucleus under consideration in order to achieve the decay correction. In this way, separate axial and equatorial anisotropies may be obtained even in the case of on-line sources. However, since a suitable transition may either not be available or may be statistically poor, this is not universally applicable unlike the axial/equatorial method.
4.4 Temperature determination

The temperature is determined from the anisotropy of an isotope whose gamma-rays have well known orientation coefficients. Since these thermometers are invariably off-line sources, for which the $R$ factors are always calculable, the temperatures can be extracted from either axial, equatorial, or axial/equatorial anisotropies. A more complete discussion of thermometry sources is given in section 3.7.

4.5 Experimental determination of the LTNO parameters

4.5.1 The hyperfine interaction

The extraction of the hyperfine interaction generally proceeds from a least squares fit to the temperature dependence of the experimental anisotropies. If the anisotropies are denoted by $E_1$ with errors $\sigma_1$ they will correspond to the inverse temperature $1/T_1$. If the appropriate angular distribution function (axial, equatorial, or axial/equatorial) to be fitted to these data is $E(1/T_1, \alpha_m)$ which depends on the parameters $\alpha_m$, then the reduced $\chi^2$ quality of fit is defined as

$$ \chi^2 = \sum_1^{1/2} \frac{1}{\sigma_1^2} \left( E(1/T_1, \alpha_m) - E_1 \right)^2. \quad (4.12) $$

Optimum values for the parameters $\alpha_m$ are then found such that $\chi^2$ is minimised in the parameter space. The error $\sigma(\alpha_m)$ on the parameter is
determined from the method outlined by Cline and Lesser [CL70], which includes corrections due to correlated errors. In this method the statistical quantity $S(a_m)$ is introduced and corresponds to the $\chi^2$ per degree of freedom ($\chi^2(N-p)$, where $N$ is the number of independent measurements and $p$ the number of parameters being determined). Figure 4.3 shows a typical dependence of the statistic $S$ on a parameter $a$ for data fitted to an arbitrary function.

The value $S_L$, which determines the error in the parameter $a_m$ shown in figure 4.3, is calculated using

$$S_L^{-\beta} = S(\bar{a}) \left\{ 1 + \frac{p}{N-p} F(p,N-p,1-\beta) \right\} \quad (4.13)$$

![Graph showing the variation of $\chi^2$ per degree of freedom with $a$.]

Figure 4.3 Error determination of a parameter $\alpha$ determined by regression analysis, using the variation of the $\chi^2$ per degree of freedom with $\alpha$. [CL70]

where $F(p,N-p,1-\beta)$ is the statistical $F$ distribution, for the $100(1-\beta)$ confidence level. The $F$ coefficient is approximately unity for $N$ large and
\[ \beta = 0.317 \] which is the confidence level corresponding to one standard deviation.

In the case of directional distributions, the parameters \( a \) refer to the \( f_i \), the hyperfine interaction (electric and magnetic), the \( U_\lambda \) and the \( \Lambda_\lambda \) (or the multipole mixing ratio \( \delta \)). Hence by performing a fit to the temperature dependent data the hyperfine interaction may be deduced. In principle, since the orientation parameters depend on both the Boltzmann parameter \( \beta \) and the oriented spin \( I_o \), it is also possible to deduce \( I_o \) in the fitting procedure. In practice the presence of more than one lattice site with non-zero hyperfine interaction, or even the relative effects of the second and fourth rank terms, often have a greater influence on the shape of the temperature dependence and so the more subtle shape differences due to \( I_o \) may be lost.

### 4.5.2 The angular distribution and de-orientation coefficients

Once the hyperfine interaction and hence the orientation parameters are known, then the \( U_\lambda \) and \( \Lambda_\lambda \) coefficients can be calculated from (4.10), (4.11), (and also from (4.9) if only second rank terms are present). They are also obtained, if sometimes less reliably where both second and fourth rank terms occur, as by products of the above fitting procedure. In fact if only axial/equatorial ratios are available then this latter method is the only way in which the second and fourth rank terms can be separated if both are present.

Since neither the \( U_\lambda \) nor the \( \Lambda_\lambda \) coefficients are temperature dependent they cannot be distinguished experimentally. In the case where the source is "soluble" in the host matrix (2.88) the same is also true of
and so experimentally it is only possible to determine the product \( fU_A \). The \( U_A \) can only be calculated separately if the factor \( f \) can be determined from a gamma-ray with known \( U_A \) coefficients. If the source is "insoluble" then the more general directional distribution formula (2.87) must be invoked. Here the \( f_i \), by virtue of their association with the \( B(B^\perp) \) coefficients, scale not only the magnitude of the anisotropy but also its temperature dependence. Thus the parameters \( f_i \) are determined directly in the fitting process and the \( U_A \) can be obtained immediately.

Further analysis depends on the knowledge of certain details in the decay scheme of the daughter:

1. If the \( U_A \) coefficients can be calculated theoretically for a certain level according to (2.67) and (2.69), then the \( A^\perp \) coefficients can be deduced for all gamma rays arising from this level. From these it is possible to calculate any one of the properties \( \delta, I_f, I_i \) using (2.63), provided that the other two are known. Occasionally, if the \( A^\perp \) coefficients are consistent with a transition of pure multipolarity, then it is possible to deduce either one of the initial or final level spins knowing only the other.

2. If the \( U_A \) coefficients of a level are not calculable, then in order to deduce them experimentally one relies on the knowledge of the \( A^\perp \) coefficients for one of the daughter transitions. The \( U_A \) coefficients thus derived can, if sufficient information is known, be used to determine any one of \( \Delta\beta, \delta^\perp, I_i, I_f \) and the relative intensities of the feeding transitions (see subsection 2.2.7). Further analysis then proceeds using the method outlined in 1.

In this way it is possible to extract a great deal of useful
spectroscopic information about the daughter nucleus provided that certain
details of the decay scheme are known. Without being privy to such
details, however, the analysis of LTNO data frequently proceeds little
further than the extraction of the hyperfine interaction.
CHAPTER FIVE

HYPERFINE FIELD DETERMINATION FOR EU, SM, AND PM ISOTOPES IN Fe.

5.1 Introduction

As discussed in chapter one, the relative importance of both the independent and collective motions of the nucleons within the nucleus may be deduced from the magnitude of the ground state magnetic moment. The experiments described in the following chapter have therefore been performed on the neutron deficient europium, samarium, and promethium isotopes, using the technique of on-line low temperature nuclear orientation, in order to measure their ground state magnetic moments, and hence investigate the associated nuclear structure.

In order to extract a value for the magnitude of the ground state magnetic moment, it requires that the value of the hyperfine field of the nucleus is known or can be measured. The experiments described in this chapter therefore attempt to address this problem, using the technique of LTNO for nuclei with a known value for \( \mu \).

5.2 Low temperature nuclear orientation of \( ^{142}\text{Eu} \)

The magnitude of the isomeric magnetic moment of \( ^{142}\text{Eu} \) has been measured previously, by the technique of collinear laser spectroscopy to be \( 2.98 \mu_N \)\(^{[AHE85]} \). The motivation to study the nucleus \( ^{142}\text{Eu} \) arose however, from the necessity to measure the magnitude of the hyperfine field of europium isotopes in iron. As discussed in chapter 2, the value
of the hyperfine field for any isotope of a particular element should be a constant. A measurement of the hyperfine interaction of $^{142}\text{Eu}$ in an iron host will therefore yield a universal value for the hyperfine field of all isotopes of Eu in iron, at least for those implanted under the same conditions, that is at very low temperatures (as opposed to room temperature annealed).

5.2.1 Experimental procedure

In order to choose the appropriate beam species and energy, with an appropriate target, CASCADE calculations were performed to establish the maximum cross-section for the production of $^{142}\text{Eu}$ from a selection of possible fusion reaction product processes. Allowance had to be made however for the energy loss of the incident beam as it passes through the target foil (typically of order $3\text{mgcm}^{-2}$), placed outside the ion source of the isotope separator, as described in subsection 3.2.2. The choice of the fusion reaction for the production of $^{142}\text{Eu}$ is therefore given by eq. 5.1:

$$220\text{MeV} \quad ^{48}\text{Ti} + ^{98}\text{Mo} \rightarrow ^{146}\text{Gd}^{*} \rightarrow ^{142}\text{Eu} + \text{p3n}$$

Once ionised the recoil products were extracted from the thermal ion source, accelerated to $60\text{keV}$, mass separated using a $60^\circ$ bending magnet, and the $A=142$ radioactive beam implanted into an iron foil soldered to the copper cold finger of the $^3\text{He}/^4\text{He}$ dilution refrigerator.

A polarizing field of 0.7 tesla was applied across the iron foil within the dilution refrigerator using a split coil superconducting
magnet, and both $^{57}$Co and $^{192}$Ir thermometers were used to monitor the temperature of the implanted nuclei. Four hyperpure Ge detectors (two axial, two equatorial to the polarizing field direction) were placed around the dilution refrigerator in order to measure the $\gamma$-ray anisotropy following the $\beta$-decay of $^{142}$Eu.

In order to obtain a normalised set of anisotropies, the $\gamma$-ray anisotropy when the nuclei are "warm" (i.e. when the emitted radiation is isotropic) was measured by stopping the circulation of the dilution refrigerator, resulting in a rise in temperature of the nuclei to $\sim 1K$.

The temperature dependences of $\gamma$-ray anisotropy following the $\beta$-decay of the $A=142$ implants were then recorded using the four hyperpure Ge detectors for successive counting periods of twenty minutes. Periodically the temperature of the implanted nuclei was changed by varying the power to the resistive heater thermally attached to the mixing chamber of the dilution refrigerator, using values ranging between $0 \rightarrow 600\mu W$, corresponding to a temperature range of between $8 \rightarrow 1000mK$.

5.2.2 Experimental results.

A typical $\gamma$-ray singles spectrum obtained over a 20 minute counting period, for one of the 80% efficient (relative to a 3" by 3" NaI) hyperpure Ge detectors for $A=142$ nuclei at $\sim 1K$ is shown in figure 5.1.

As described in chapter three, the use of $^{57}$Co comelting into an iron lattice provides a convenient and accurate way of measuring absolute temperatures below $100mK$. The solubility of $^{192}$Ir in an iron lattice cannot however be relied upon, and consequently the fraction of Ir implants experiencing the full substitutional hyperfine field may be less than
Fig 5.1 γ-ray singles spectra for mass A=142 implanted into the dilution refrigerator, collected for an axial detector at T=1K.
100%. A temperature dependence of γ-ray anisotropy following the β-decay of $^{192}$Ir was therefore measured using the $^{57}$Co thermometer, and is shown in fig 5.2. The strength of the hyperfine interaction for $^{192}$Ir in Fe is known from the literature to be $282.2 \mu T$, and the $U_A A_e$ coefficients are easily calculable from the decay scheme to be: $U_2 A_2 = -0.3814$ and $U_4 A_4 = -0.1537$. Hence a fit to the data shown in fig 5.2 using the two site model described in section 2.3 yields a value for $\lambda$ (the fraction of Ir implants experiencing the full substitutional field) of 99.6(7)%, with a $\chi^2$ of 1.31.

Fig 5.2 Temperature dependence of the 468keV γ-ray anisotropy following the β-decay of $^{192}$Ir.
The high value for the fraction $f$ for Ir implants in Fe (effectively 100%) made the use of the $^{192}$IrFe thermometer in conjunction with the $^{57}$CoFe ideal for the measurement of temperatures for all the experiments described in this and the following chapter. However, the difference in sensitivity functions between $^{192}$Ir and $^{57}$Co for temperature determination, coupled with the relative strengths in activity, resulted in the use of the Ir thermometer for temperature measurements above 30mK, and the Co thermometer for temperatures below 30mK (using the 136keV $\gamma$-ray following $\beta$-decay).

Figure 5.3 shows the $\gamma$-ray decay scheme following the $\beta$-decay of $^{142}$Eu. The spin and parity of the metastable state quoted tentatively in brackets to be $7^-$ has been measured directly by the technique of collinear laser spectroscopy \cite{AH85} to be $8^-$, and this value has been used to calculate the $U_A$ coefficients.

Figures 5.4a and 5.4b show the temperature dependences of $\gamma$-ray anisotropy following the $\beta$-decay of $^{142}$Eu$^m$ for the 557keV transition, for both pairs of Ge detectors (each pair comprising one axial and one equatorial detector with respect to the external polarizing field). The calculation of the $U_A$ coefficients for this transition is relatively straightforward, due to the transition appearing in the "high spin" part of the decay scheme where the $\beta$-feeding is strong and direct, and the feeding of the transition level from preceding $\gamma$-rays is simple and weak. Consequently the $U_A$ coefficients may be calculated (assuming the beta transition is an allowed Gamow-Teller $\Delta I = 1$ transition for $8^- \rightarrow 7^-$) to be:

- $U_2 = 0.9068$ ; $U_4 = 0.7113$. Assuming a pure $E1$ transition for the $5^- \rightarrow 4^+$ $\gamma$-ray the $A_A$ coefficients may be calculated to be: $A_2 = 0.2944$ ; $A_4 = 0.0$. 

86
Figure 5.3 γ-ray decay scheme following the β-decay of $^{142}$Eu. [LE78]

A fit to the experimental data presented in figures 5.4a and 5.4b thus proceeds rather simply (due to the $\lambda=4$ terms vanishing) yielding the following fitted values for the strength of the hyperfine interaction ($\mu.B$) and $f$:

<table>
<thead>
<tr>
<th>Pair</th>
<th>$\mu.B$ ($\mu_n$T)</th>
<th>$f$ (%)</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pair 1</td>
<td>360 (25)</td>
<td>44.5 (2.0)</td>
<td>1.39</td>
</tr>
<tr>
<td>Pair 2</td>
<td>302 (23)</td>
<td>46.7 (2.4)</td>
<td>3.58</td>
</tr>
</tbody>
</table>
Taking a weighted average of the values obtained from above, the strength of the hyperfine interaction of $^{142}$Eu in Fe can be deduced from the 557keV $\gamma$-ray to be $331(35)\mu_T$. Taking the value for $\mu_z^{[^{142}\text{Eu}]}$ of $2.98\mu_n$ it is possible to derive the magnitude of the hyperfine field of EuFe to be $111(12)T$. This value differs from that quoted $[^{RA85}]$ in the literature of $B_{\text{hf}}(\text{EuFe}) = 148.2 \pm 0.9 T$, derived from Mössbauer experiments. LTNO can measure only an average field, unlike Mössbauer measurements, where individual components of the $B_{\text{hf}}$ can be identified. During LTNO experiments, radiation damage following implantation remains unannealed during the measurement, hence the vacancies associated with the impurity cannot be excluded. This leads to sites of lower symmetry, thereby contributing to the difference in the $B_{\text{hf}}$ measured. Hence the LTNO measured value for $B_{\text{hf}}$ will be used throughout this thesis.

Figures 5.5a and 5.5b show the temperature dependence of the $\gamma$-ray anisotropy for the 1023keV transition, again following the $\beta$-decay of $^{142}\text{Eu}^m$, for both detector pairs 1 and 2. Similarly figures 5.6a and 5.6b show the $\gamma$-ray anisotropy for the 768keV transition, following the $\beta$-decay of both the ground and metastable states of $^{142}\text{Eu}$. Inspection of the decay scheme presented in figure 5.3 reveals that the complex nature of the $\gamma$-ray feeding to the low lying $\gamma$-ray transitions such as the 1023 and 768keV lines, makes exact calculation of the $U_A$ coefficients impossible. In addition the 768keV transition, being fed by two differently oriented parents ($^{142}\text{Eu}^m$, and $^{142}\text{Eu}^g$) will present further problems when trying to extract its $U_A$ coefficients in particular. The statistical quality of the data presented in figures 5.5 and 5.6 is superior to that shown in figure 5.4, yet the confidence in extracting an accurate value for the strength of the hyperfine interaction is reduced, due to the uncertainty in the
Figure 5.4a Temperature dependence of the 557keV γ-ray anisotropy following the β-decay of $^{142\text{m}}$Eu for detector pair 1.

Figure 5.4b Temperature dependence of the 557keV γ-ray anisotropy following the β-decay of $^{142\text{m}}$Eu for detector pair 2.
evaluation of the $U_\Lambda$ coefficients. This apparent anomaly illustrates the importance of selecting a suitable $\gamma$-ray transition, for the extraction of nuclear properties from LTNO experiments, striking a balance between observed $\gamma$-ray intensity, and absence of ambiguity in the calculation of the $U_\Lambda$ coefficients.

Assuming the spins and parities presented in brackets in figure 5.3 are correct, and neglecting $\beta$-feeding branches of less than 4%, and the $\beta$-feeding of the ground state (spin/parity $1^+$), approximate values for the $U_\Lambda$ coefficients may be deduced. These are presented below, together with the fitted values for the strength of the hyperfine interaction and fraction of implants in substitutional sites obtained for both the 1023keV and 768keV transitions:

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>$\mu B(\mu T)$</th>
<th>$f(%)$</th>
<th>$U_\Lambda$</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1023</td>
<td>404 (27)</td>
<td>42 (1)</td>
<td>-0.3696</td>
<td>1.68</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.1539</td>
<td></td>
</tr>
<tr>
<td>768</td>
<td>395 (25)</td>
<td>43 (1)</td>
<td>-0.3663</td>
<td>1.57</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.1530</td>
<td></td>
</tr>
</tbody>
</table>

It is clear from the data obtained for mass $A=142$, that detector pair 1 gave consistently higher values for the strength of the hyperfine interaction than detector pair 2. This systematic bias may be the result of the beam spot incident on the cooled Fe foil shifting during the course of the measurement. However, taking a weighted average of the two values was hoped, in part, to remove the statistical bias.
Figure 5.5a Temperature dependence of the 1023keV γ-ray anisotropy following the β-decay of $^{142}$Eu, for detector pair 1.

Figure 5.5b Temperature dependence of the 1023keV γ-ray anisotropy following the β-decay of $^{142}$Eu, for detector pair 2.
Figure 5.6a  Temperature dependence of the 768keV γ-ray anisotropy following the β-decay of $^{142}$Eu, for detector pair 1.

Figure 5.6b  Temperature dependence of the 768keV γ-ray anisotropy following the β-decay of $^{142}$Eu, for detector pair 2.
5.3 Low temperature nuclear orientation of $A = 141$ isotopes.

The value of the hyperfine field of promethium isotopes in an iron host has not been previously measured. In order to determine the magnitude of the ground state magnetic dipole moment of promethium isotopes using the technique of LTNO, it is required that some means of measuring, or at least estimating that field is undertaken.

The nucleus $^{141}$Pm like $^{143}$Pm and $^{147}$Pm has a ground state spin and parity of $5/2^+$, arising from the strongly coupled odd proton to the rotating core. The ground state magnetic dipole moments of the nuclei $^{143}$Pm and $^{147}$Pm have been measured using LTNO and Mössbauer techniques [GR63],[BA70] to be $3.78(50)$ and $3.55(10)$ respectively. If a similar value is assumed for $^{141}$Pm then a measurement of the strength of the hyperfine interaction of $^{141}$Pm in an iron host will lead to a realistic estimate of its hyperfine field.

Measurements of the strength of the hyperfine interaction of $^{141}$Eu and $^{141}$Sm in iron, will additionally serve to calibrate the hyperfine field of these isotopes in Fe, since both of their magnetic moments have again been measured using the technique of collinear laser spectroscopy [AH85],[MI87].

5.3.1 Experimental procedure

The experimental procedure adopted for the experiments described in this section was similar to that presented in subsection 5.2.1 with the following exceptions:
CASCADE calculations were performed, and the following combination of beam energy and species with target was chosen to optimise the production of $^{141}\text{Eu}$ from the possible fusion reaction products:

$$210\text{MeV} \quad ^{48}\text{Ti} + ^{96}\text{Mo} \rightarrow ^{144}\text{Gd}\ ^* \rightarrow ^{141}\text{Eu} + p2n \quad (5.2)$$

Once implanted into the Fe host the $^{141}\text{Eu}$ nuclei with a half life of 40s, decay into $^{141}\text{Sm}^{m}$ and $^{141}\text{Sm}^{g}$ with half lifes of 22.5 and 10.2 minutes respectively, and then into $^{141}\text{Pm}$ with a half life of 20.9 minutes. The relative shortness of halflife of these isotopes was advantageous, since it made the measurement of their respective strengths of hyperfine interaction possible during the same experiment.

5.3.2 Experimental results for $^{141}\text{Eu}$

A typical $\gamma$-ray singles spectrum obtained over a 20 minute counting period, for one of the 25% efficient hyperpure Ge detectors for $A=141$ is shown in figure 5.7. Figure 5.8 shows the $\gamma$-ray decay scheme following the $\beta$-decay of $^{141}\text{Eu}^{m,g}$ into $^{141}\text{Sm}^{m,g}$. Inspection of this decay scheme reveals two good $\gamma$-ray transitions for the extraction of the strength of the hyperfine interaction for $^{141}\text{Eu}$ in Fe, namely the 384.5 and 382.9keV transitions. Both these $\gamma$-rays correspond to the decay of the $3/2^+$ level to either a $3/2^+$ or $1/2^+$ state, where the $A_4$ (and hence the $\lambda=4$) terms are equal to zero. A fit to the experimental data presented in figures 5.9 and 5.10 for detector pair one, yields the following fitted values for the strength of the hyperfine interaction and the product $\bar{U}_2A_2$.
Figure 5.7 \( \gamma \)-ray singles spectrum for mass \( A=141 \) implanted into the dilution refrigerator for an axial Ge detector at \( T \sim 1 \)K.
Figure 5.7, continued.
Figure 5.8: 3-γ-ray decay scheme following the β-decay of 141Eu.

[Diagram showing the decay scheme with labels and energies such as 2.950, 1.656, 1.356, etc., and decay modes like β⁺EC, β⁻EC, and γ-rays.]
Figure 5.9 Temperature dependence of the 382.9keV $\gamma$-ray anisotropy following the $\beta$-decay of $^{141}$Eu$^{m+g}$.

Figure 5.10 Temperature dependence of the 384.5keV $\gamma$-ray anisotropy following the $\beta$-decay of $^{141}$Eu$^{m+g}$. 
Due to poor resolution in detector pair 2 (of order 2.5 keV) the proximity in energy of the two gamma rays (1.6 keV) made peak integration statistically poor, despite the use of the Gaussian doublet fitting routine described in chapter 4.

Using the literature value of $3.494(8)\mu_n$ for the ground state magnetic dipole moment of $^{141}\text{Eu}$, and taking a weighted value for the measured strength of the hyperfine interaction presented above, it is possible to derive a value for $B$ (the hyperfine field) of $129(17)$ T, which compares favourably with that obtained from the $^{142}\text{Eu}$ measurement.

5.3.3 Experimental results for $^{141}\text{Sm}$

Figure 5.11 shows the $\gamma$-ray decay scheme following the $\beta$-decay of $^{141}\text{Sm}^{m+g}$ into $^{141}\text{Pm}$. Inspection of the decay scheme reveals that the 777 keV $\gamma$-ray transition is fed only by the decay of $^{141}\text{Sm}^m$ and has a high branching ratio (-99%), and so becomes an obvious candidate for the extraction of the hyperfine interaction during the LETNO experiment. Since the transition is from $11/2^+ \to 7/2^+$ the multipolarity can be assumed to be pure $E2$ with known $A_\lambda$ coefficients. The $U_\lambda$ coefficients for the transition are fairly straightforward to estimate if it is assumed that the $\beta$-feeding to the levels 2.1190 and 2.0916 MeV above the ground state are allowed Fermi or Gamow-Teller in nature $\Delta_{L\text{F}} = 0$ or 1, implying $U_2A_2 = -0.35(3)$ and $U_4A_4 = -0.13(4)$. A fit to the experimental data presented in figure
Figure 5.11 γ-ray decay scheme following the β-decay of \(^{141}\text{Sm}\)
5.12 for the 777keV transition yields a value for $\mu B$ and $\xi$ of $306(30)\mu_n T$ and $54(3)\%$ respectively, with a $\chi^2$ of 0.73. Assuming the value of $\mu = 0.83(2)\mu_n$ measured \cite{OP87} using the technique of collinear laser spectroscopy, it is possible to deduce the value of $B$ for Sm in Fe to be $369(37)T$, which is in broad agreement with the value quoted \cite{RA85} in the literature of $314(35)T$.

![Figure 5.12](image.png)

**Figure 5.12** Temperature dependence of the 777keV $\gamma$-ray anisotropy following the $\beta$-decay of $^{141}$Sm.

### 5.3.4 Experimental results for $^{141}$Pm

Figure 5.13 shows the $\gamma$-ray decay scheme following the $\beta$-decay of $^{141}$Pm into $^{141}$Nd. Inspection of the decay scheme reveals the 1223keV $\gamma$-ray \cite{Z87} arising from a $5/2^+$ to $3/2^+$ transition, and may be assumed to have an
admixture of M1/ E2. Due to the complex nature of the γ-ray feeding from levels of unknown spin and parity to this level, the \( U \) coefficients may not be calculated exactly. Therefore a fit to the experimental data presented in figure 5.14 proceeds by varying the parameters \( \tilde{U}_{A} \) and \( \mu B \). Despite the strength of the feeding to the \( 5/2^+ \) level, the quality of the data is poor, resulting in a fitted value of the strength of the hyperfine interaction of 1444(352)\( \mu T \). Assuming a value of \( \mu = 3.55(10) \) from the systematics of the ground state magnetic moments of odd \( A \) Pm isotopes, then a value of the hyperfine field of Pm isotopes in iron can be deduced to be 406(100)\( T \).

The relatively high value for the hyperfine field of Pm isotopes in iron, in addition to the other light rare-earth isotopes studied, clearly makes thermometry in this region problematic. Although many Nuclear Orientation thermometers have been successfully used for the measurement of very low temperatures (\( \sim 20\) mK), the choice for the relatively high temperatures (\( \sim 100\) mk) required in the light rare earth isotopes is very limited.
Figure 5.13 γ-ray decay scheme following the β-decay of 141Pm.

[Diagram of the γ-ray decay scheme following the β-decay of 141Pm]
Figure 5.14 Temperature dependence of the 1223keV γ-ray anisotropy following the β-decay of $^{141}\text{Pm}$. 
CHAPTER SIX

MAGNETIC MOMENTS OF THE NUCLEI $^{139,139*}$Eu, $^{139}$Sm, AND $^{138}$Pm.

6.1 Introduction

The experiments described in this chapter were performed on the neutron deficient europium, samarium and promethium isotopes, in order to measure their respective ground-state dipole magnetic moments, using the technique of low temperature nuclear orientation. The motivation to measure the moments of these nuclei arose due to the reported triaxiality of the nuclei in this region, and the sensitivity of the value of $\mu$ to the underlying nuclear structure.

6.2 Low temperature nuclear orientation of A=139 isotopes.

Having established a value for the hyperfine field of Eu and Sm isotopes in Fe from measurements presented in the previous chapter, a measurement of the strength of hyperfine interaction of A=139 isotopes will yield the value of the magnetic moment $\mu$. However, in order to extract a value of $\mu$ from static nuclear orientation measurements, the value of the relaxation time (the time the nuclear implants reach thermodynamic equilibrium with the host lattice) must be small compared with the half life of the oriented parent nucleus. Therefore the experiments described in this section were performed to both measure $\mu$ for $^{139}$Eu and $^{139}$Sm* ($^{139}$Sm* having a ground state spin/parity = $1/2^-$ gives no $\gamma$-ray anisotropy) and estimate the respective relaxation times.
6.2.1 Experimental procedure.

In order to optimise the production of $^{139}$Eu CASCADE calculations were performed, and the following combination of beam energy and species with target was chosen:

$$220\text{MeV} \ ^{48}\text{Ti} + ^{94}\text{Mo} \rightarrow ^{142}\text{Gd}^* \rightarrow ^{139}\text{Eu} + p2n \quad (6.1)$$

Once implanted into the Fe host thermally attached to the copper cold finger of the dilution refrigerator, the $^{139}$Eu nuclei with a half life of 22s decay into $^{139}$Sm$^{m\#}$. In order to extract a value for $\mu$, temperature dependencies of $\gamma$-ray anisotropy were obtained following the same method outlined in subsection 5.2.1. To measure the spin lattice relaxation time of $^{139}$Eu and $^{139}$Sm$^m$, a novel pulsed implantation technique was used. The mass A=139 ions were implanted into an iron foil maintained at ~10mK. In order to build up the source, the activity was implanted for 10s and the counts in the axial and equatorial Ge detectors were collected for more than three half lives, at an interval of 5s, before implanting again. During each of the 5s intervals the counts were stored in 2k routed spectra. This cycle was repeated for a period of a few hours to obtain adequate statistics.

In an attempt to isolate the relaxation time corresponding to $^{139}$Sm from that of its parent $^{139}$Eu, CASCADE calculations showed the following nuclear reaction would preferentially produce $^{139}$Sm directly:

$$238\text{MeV} \ ^{48}\text{Ti} + ^{96}\text{Mo} \rightarrow ^{144}\text{Gd}^* \rightarrow ^{139}\text{Sm} + 2p3n \quad (6.2)$$
Figure 6.1 $\gamma$-ray singles spectra for mass $A=139$ implanted into the dilution refrigerator, collected for an axial detector at $T=1\text{K}$. 
Figure 6.1 continued.
N.B.

\[ \mu = 6.3(8) \]

see your notes.
6.2.2 Experimental results for $^{139}$Eu.

A typical $\gamma$-ray singles spectrum obtained over a 20 minute counting period, for one of the 80% efficient hyperpure Ge detectors for $A=139$ is shown in figure 6.1. Figure 6.2 shows the $\gamma$-ray decay scheme following the $\beta$-decay of $^{139}$Eu into $^{139}$Sm$^{m+q}$. Inspection of the decay scheme reveals the difficulty in choosing a suitable $\gamma$-ray transition above the $11/2^-$ isomer with known $U_\lambda$ coefficients. However, a fit to the experimental data presented in figures 6.3a and 6.3b for the 497 keV $\gamma$-ray transition for both detector pairs, yields the following values for the product $\mu_B$ and $f$, (assuming the relaxation time is zero):

<table>
<thead>
<tr>
<th>Detector Pair</th>
<th>$\mu_B(\mu_nT)$</th>
<th>$f(%)$</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>665(59)</td>
<td>44(2)</td>
<td>1.12</td>
</tr>
<tr>
<td>2</td>
<td>740(78)</td>
<td>37(2)</td>
<td>2.16</td>
</tr>
</tbody>
</table>

Using the value for the hyperfine field of Eu in Fe obtained following the orientation of $^{142}$Eu (see section 5.2), the value for the ground state magnetic dipole moment of $^{139}$Eu is readily calculated to be $6.3(8)\mu_n$.

In order to calculate the spin lattice relaxation time for $^{139}$Eu in Fe, the temperature limit for the system must be determined, by comparing the interaction temperature $T_{\text{int}}$ with the lattice temperature $T_L$, where the interaction temperature is defined by eq 6.2.
Figure 6.2 γ-ray decay scheme following the β-decay of $^{139}$Eu.

Figure 6.3a Temperature dependence of the 497keV γ-ray anisotropy following the β-decay of $^{139}$Eu for detector pair 1.
Figure 6.3b Temperature dependence of the 497keV γ-ray anisotropy following the β-decay of $^{139}$Eu for detector pair 2.

$$T_{\text{int}} = \frac{g_n \mu_n B_{\text{hf}}}{k} \quad (6.2)$$

Adopting a value of $g_n = 1.18$ for $^{139}$Eu, from systematics of odd A Eu moments, and using the value of $B_{\text{hf}}$ derived experimentally (section 5.2), it is possible to estimate the interaction temperature to be of order 50mK. When $T_{\text{int}}$ is compared to the lattice temperature, which was of order 10mK, it can be shown that our system lies in the low temperature limit regime, where the relaxation time $T_1$ is given by eqn 6.3.

$$T_1 = \frac{C_k}{I T_{\text{int}}} \quad (6.3)$$
The time dependent anisotropy for the 497 keV \(^{139}\text{Eu}\) \(\gamma\)-ray, for \(T_1\) at ~ 10mK, in an external field of 0.7T is shown in figure 6.4. Using eqn 6.4 the calculated values of \(W(\theta, t)\) for different values of \(T_1\), assuming \(W(\theta, t=0) = 0\), are also shown. A comparison of the saturation values of the anisotropy between our time and temperature dependent data shows that the former gives a higher value, from which it seems that the implanted \(^{139}\text{Eu}\) relax to the lattice temperature in a finite time. However, since the measurements were made under different experimental conditions, it is possible that the different anisotropies result from different Fe foil qualities. From figure 6.4 it is only possible to estimate the upper limit of \(T_1\) to be ~ 3s.

\[
W(\theta, t) = W(\theta, \infty) = W(\theta, t=0) - W(\theta, \infty) e^{-t/T_1} \quad (6.4)
\]

Figure 6.4 Time-dependence of 497keV \(\gamma\)-ray anisotropy following the \(\beta\)-decay of \(^{139}\text{Eu}\), shown with calculated time dependencies for \(T_1 = 1, 2, 3\) s.
For \[ I \frac{C_k}{k} = T_1 \frac{I}{T_{\text{int}}} \] (6.5)

From eqn 6.5 it is possible to determine a limit for the Korringa constant for EuFe of \( C_k \leq 0.8sK \). The upper limit of \( C_k \) would imply that the relaxation is an order of magnitude slower than the predicted value \([ST71],[ST85] \). It must however be noted that most of the predictions are based on simple spin-relaxation by the magnetic interaction between the nuclei and the conduction electrons. Since in rare-earth atoms the outermost f electrons are screened, and hence indirectly coupled to the conduction electrons, the actual SLR mechanism may be complicated \([KL87] \), leading to slower relaxation.

To date no such measurements for rare-earths in iron have been made and consequently our experiments were undertaken to investigate if any corrections for the SLR time is required in the LTNO of \(^{138}\text{Eu} \) \( (t_{1/2} = 12s, g_n \sim 0.9) \) in Fe. Considering the empirical relation \( g_n^2 C_k \text{ } \gamma = \text{ constant} \), for different isotopes in the same host, it can be concluded that little or no correction for the value of \( \mu \) determined for \(^{138}\text{Eu} \) using LTNO has to be made.

### 6.2.3. Experimental results for \(^{139}\text{Sm} \)

Inspection of the decay scheme shown in figure 6.2 shows the 190 keV \( \gamma \)-ray originates from the 11/2\(^{-}\) metastable state of \(^{139}\text{Sm} \). If it is assumed that the relaxation time of the metastable state is shorter than its half life \((-11s)\), then the nucleus will fully re-orient implying that the \( U_A \) coefficients will equal unity. Since the transition corresponds to a 11/2\(^{-}\) state going to a 5/2\(^{+}\) state the multipolarity of the transition
may be assumed to be pure E3, with known $A_\lambda$ coefficients.

A fit to the experimental data presented in figures 6.5a and 6.5b (which now has to include $A=6$ terms) for the 190keV transition for both detector pairs yields the following values for the strength of the hyperfine interaction and $f$:

<table>
<thead>
<tr>
<th>Pair</th>
<th>$\mu_B(\mu_n T)$</th>
<th>$f(%)$</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pair 1</td>
<td>345(23)</td>
<td>20.6(7)</td>
<td>1.40</td>
</tr>
<tr>
<td>Pair 2</td>
<td>310(25)</td>
<td>20.3(6)</td>
<td>1.59</td>
</tr>
</tbody>
</table>

Assuming the relaxation time of Sm in Fe is zero, the value for the dipole magnetic moment of $^{139}$Sm can be deduced to be $1.1(2)\mu_n$ (using the literature value for $B_{hf}$ of Sm in Fe). The estimated moment of $^{139}$Sm from systematics [RA88] is opposite in sign to that of the parent $^{139}$Eu. With both Eu and Sm having positive hyperfine fields in Fe [RA88], it would be expected that the initial population of the $^{139}$Sm will not be in thermal equilibrium. The partial relaxation of the population would show up in the time-dependent anisotropy measurements, which are shown in figure 6.6. The observed relaxation of the 190keV transition did not however show any such expected behaviour. From the data, again only an upper limit of the effective relaxation time could be set to be $\leq 3$s, which is similar to its parent. These results can be explained in terms of the interaction temperature of the $^{139}$Sm in Fe system. Since the $T_{int}$ of $^{139}$Sm is estimated to be $\sim 16$mK, which is close to $T_L$ (-10mK), the relaxation phenomena lies in the intermediate temperature limit and the spin lattice relaxation time could be fast. In our measurement however,
Figure 6.5a: Temperature dependence of the 190keV $\gamma$-ray anisotropy following the $\beta$-decay of $^{139}$Sm for detector pair 1.

Figure 6.5b: Temperature dependence of the 190keV $\gamma$-ray anisotropy following the $\beta$-decay of $^{139}$Sm for detector pair 2.
Figure 6.6 Time—dependence of the 190keV γ-ray anisotropy following the β-decay of $^{139}\text{Sm}$. The nucleus $^{138}\text{Eu}$ has recently been the focus of many nuclear structure experiments \cite{RE86}, \cite{LY88}, which have included both β-decay and "in beam" work. However, little is known about the underlying nuclear structure, in particular the ground state spin and parity and deformation.

the relaxation of the $^{139}\text{Sm}$ seems to be restricted mostly by the parent $^{139}\text{Eu}$. Hence it can be safely assumed that integral LTNO measurements of any Sm isotopes with comparable $g_n$ and $t_{1/2} > 3s$, will not be significantly affected by relaxation effects.

6.3 Low temperature nuclear orientation of $A=138$ isotopes.
The experiments described in this section therefore attempt to address
this problem using the technique of LTNO, to measure $\mu$.

Owing to the short half life of $^{138}\text{Eu}$ (12s) and its daughter $^{138}\text{Sm}$
(180s), the nucleus $^{138}\text{Pm}$ may also be studied, and a value of it's ground
state magnetic moment deduced.

6.3.1. Experimental procedure.

In order to optimise the production of $^{138}\text{Eu}$ CASCADE calculations
were performed, and the following combination of beam energy and species
with target was chosen:

$$230\text{MeV} \ 48\text{Ti} + 94\text{Mo} \rightarrow 142\text{Gd}^* \rightarrow 138\text{Eu} + p3n. \quad (6.7)$$

Once implanted into the Fe host thermally attached to the copper cold
finger of the dilution refrigerator, temperature dependences of $\gamma$-ray
anisotropy following $\beta$-decay were obtained, following the same method
outlined in subsection 5.2.1.

6.3.2 Experimental results for $^{138}\text{Eu}$.

A typical $\gamma$-ray singles spectrum obtained over a 20 minute counting
period, for one of the 25% efficient hyperpure Ge detectors for $A=138$ is
shown in figure 6.7. Figure 6.8 shows the $\gamma$-ray decay scheme following the
$\beta$-decay of $^{138}\text{Eu}$ into $^{138}\text{Sm}$. Inspection of the decay scheme again reveals
the difficulty in choosing a suitable $\gamma$-ray transition with known $U_A$
coefficients. However an estimate of the $U_A$ coefficients may be made if
Figure 6.8 γ-ray singles spectra for mass $A=138$ implanted into the dilution refrigerator collected for an axial detector at $T=1K$. 
all likely values of spin, and parity are taken for the relevant unassigned nuclear levels. These are shown in table 6.1 for the 686keV $6^+ \rightarrow 4^+$ transition, as the product $U_{A\pi}^\lambda$ (where the transition is assumed pure E2 with known $A_\pi$ coefficients). $I_{Eu}^\pi$ corresponds to the ground state spin and parity of $^{138}$Eu and, $I_{Sm}^\pi$ corresponds to the spin and parity of the level 2.5086 MeV above the ground state (marked A on figure 6.9) of $^{138}$Sm.

The range of possible values for $U_{A\lambda}^\pi$ presented in the table is fortunately quite small, hence the average value for $U_{2\lambda}^A$ and $U_{4\lambda}^A$ of 0.3626 and 0.1618 respectively, may be used with some confidence, without losing too much in terms of the underlying physics. A good degree of accuracy arises due to the high spin of both the parent nucleus and the $\gamma$-decaying state (in contrast to the $^{138}$Pm case which follows). Similarly an estimate of the $U_{A\lambda}^\pi$ for the 545 and 347keV $\gamma$-ray transitions can be made, if somewhat less reliably due to the ambiguity associated with the sidefeeding to these levels from states of unknown spin/parity and direct $\beta$-feeding.

![Diagram of the $\gamma$-ray decay scheme following the $\beta$-decay of $^{138}$Eu.]

Figure 6.9 $\gamma$-ray decay scheme following the $\beta$-decay of $^{138}$Eu.
Table 6.1 Possible range of values for the $U_{A\lambda}$ coefficients for the $686$ keV transition following the $\beta$-decay of $^{138}$Eu.

Hence a fit to the experimental data presented in figures 6.10a and 6.10b for the temperature dependence of the $686$ keV $\gamma$-ray anisotropy yields the following values for the strength of hyperfine interaction and fraction of implants in substitutional sites:

<table>
<thead>
<tr>
<th>$I_{Eu}^\pi$</th>
<th>$I_{Sm}^\pi$</th>
<th>$U_{A2,\min}$</th>
<th>$U_{A2,\max}$</th>
<th>$U_{A4,\min}$</th>
<th>$U_{A4,\max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6^-$</td>
<td>$6^+$</td>
<td>-0.3711</td>
<td>-0.3541</td>
<td>-0.1766</td>
<td>-0.1470</td>
</tr>
<tr>
<td>$6^-$</td>
<td>$7^+$</td>
<td>-0.3871</td>
<td>-0.3645</td>
<td>-0.1837</td>
<td>-0.1467</td>
</tr>
<tr>
<td>$6^-$</td>
<td>$8^+$</td>
<td>-0.3850</td>
<td>-0.3685</td>
<td>-0.1807</td>
<td>-0.1522</td>
</tr>
<tr>
<td>$6^-$</td>
<td>$5^-$</td>
<td>-0.3861</td>
<td>-0.3696</td>
<td>-0.1821</td>
<td>-0.1536</td>
</tr>
<tr>
<td>$7^+$</td>
<td>$5^-$</td>
<td>-0.3811</td>
<td>-0.3811</td>
<td>-0.1728</td>
<td>-0.1728</td>
</tr>
<tr>
<td>$7^+$</td>
<td>$6^+$</td>
<td>-0.3816</td>
<td>-0.3750</td>
<td>-0.1734</td>
<td>-0.1642</td>
</tr>
<tr>
<td>$7^+$</td>
<td>$6^-$</td>
<td>-0.3816</td>
<td>-0.3816</td>
<td>-0.1734</td>
<td>-0.1734</td>
</tr>
<tr>
<td>$7^+$</td>
<td>$7^+$</td>
<td>-0.3850</td>
<td>-0.3768</td>
<td>-0.1788</td>
<td>-0.1669</td>
</tr>
<tr>
<td>$7^+$</td>
<td>$8^-$</td>
<td>-0.3828</td>
<td>-0.3752</td>
<td>-0.1753</td>
<td>-0.1646</td>
</tr>
</tbody>
</table>

$\mu_B(\mu_n^T)$ $\xi(\%)$ $\chi^2$

Pair 1 $594(55)$ $59(3)$ 1.01
Pair 2 $551(50)$ $51(3)$ 1.15
Figure 6.10a Temperature dependence of the 686keV γ-ray anisotropy following the β-decay of $^{136}$Eu for detector pair 1.

Figure 6.10b Temperature dependence of the 686keV γ-ray anisotropy following the β-decay of $^{138}$Eu for detector pair 2.
Figure 6.11a Temperature dependence of the 545keV γ-ray anisotropy following the β-decay of $^{138}$Eu for detector pair 1.

Figure 6.11b Temperature dependence of the 545keV γ-ray anisotropy following the β-decay of $^{138}$Eu for detector pair 2.
Figure 6.12a Temperature dependence of the 347 keV \( \gamma \)-ray anisotropy following the \( \beta \)-decay of \(^{138}\)Eu for detector pair 1.

Figure 12b Temperature dependence of the 347 keV \( \gamma \)-ray anisotropy following the \( \beta \)-decay of \(^{138}\)Eu for detector pair 2.
The decay scheme of $^{138}$Eu shows that the 545 and 347keV $\gamma$-rays directly follow the 686keV transition, and if the band is considered in isolation should have the same $U_A A_A$ coefficients (assuming they are pure $E2$). However, from the decay scheme it is apparent that there is some degree of side-feeding from the neighbouring $\gamma$-vibrational band, which may have an effect on the $U_A A_A$ coefficients. Inspection of the relevant branching ratios from $\gamma$-ray spectroscopy experiments \cite{RB86} shows the degree of sidefeeding is small (-15%) and so has been neglected in this work. Therefore using the same $U_A A_A$ for the 545 and 347keV as those calculated for the 686keV transition when fitting the experimental data presented in figures 6.11a, 6.11b, 6.12a and 6.12b, yields the following values of $\mu B$ and $\ell$ for $^{138}$Eu implanted in iron:

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>$\mu B (\mu T)$</th>
<th>$\ell (%)$</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pair 1</td>
<td>545</td>
<td>681(35)</td>
<td>46(2)</td>
</tr>
<tr>
<td></td>
<td>347</td>
<td>782(59)</td>
<td>30(2)</td>
</tr>
<tr>
<td>Pair 2</td>
<td>545</td>
<td>665(58)</td>
<td>46(3)</td>
</tr>
<tr>
<td></td>
<td>347</td>
<td>753(80)</td>
<td>31(3)</td>
</tr>
</tbody>
</table>

Due to the large errors for $\mu B$ from the 545 and 347keV measurements (probably arising due to the ambiguity in assigning proper $U_A A_A$ coefficients) a final value of $\mu$ was taken using the 686keV data adopting a value for $B$ of 111(12)T from the experiments outlined in the previous chapter, and is calculated to be $5.3(7)\mu_n$.

In addition to extracting a value of $\mu$ from temperature dependent
data of γ-ray anisotropy, spectroscopic information may also be obtained, in particular the mixing ratio δ, as discussed in chapter 4. Table 6.2 shows the γ-ray anisotropy of several transitions in the decay scheme following the β-decay of $^{138}$Eu into $^{138}$Sm. The data shown was collected at an average temperature of 15.6mK, for a collection time of ~2 hours. However, due to the poor statistical accuracy of the data, and the large error in the determined value of $\mu$, extraction of spectroscopic information will not be presented in this work.

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>$\varepsilon(T)$% Pair 1</th>
<th>$\varepsilon(T)$% Pair 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>338</td>
<td>-19.6 (6.5)</td>
<td>-7.5 (7.8)</td>
</tr>
<tr>
<td>347</td>
<td>-27.0 (1.0)</td>
<td>-29.7 (1.1)</td>
</tr>
<tr>
<td>399</td>
<td>-1 (4)</td>
<td>+2 (5)</td>
</tr>
<tr>
<td>545</td>
<td>-45 (1.2)</td>
<td>-44 (1.3)</td>
</tr>
<tr>
<td>571</td>
<td>-24 (10)</td>
<td>-41 (9)</td>
</tr>
<tr>
<td>649</td>
<td>-41 (2.5)</td>
<td>-38 (3)</td>
</tr>
<tr>
<td>653</td>
<td>-43 (3.6)</td>
<td>-48 (3.5)</td>
</tr>
<tr>
<td>686</td>
<td>-50 (1.5)</td>
<td>-46 (1.4)</td>
</tr>
<tr>
<td>706</td>
<td>-45 (7)</td>
<td>-54 (5.6)</td>
</tr>
<tr>
<td>737</td>
<td>-56 (2.3)</td>
<td>-58 (2.4)</td>
</tr>
<tr>
<td>745</td>
<td>-38 (4.3)</td>
<td>-37 (4.5)</td>
</tr>
<tr>
<td>775</td>
<td>-39 (3)</td>
<td>-38 (4.2)</td>
</tr>
<tr>
<td>851</td>
<td>-57 (6)</td>
<td>-48 (9)</td>
</tr>
</tbody>
</table>

Table 6.2 γ-ray anisotropies following the β-decay of $^{138}$Eu at ~15mK.
6.3.3 Experimental results for $^{138}\text{Pm}$.

Figure 6.13 shows the $\gamma$-ray decay scheme following the $\beta$-decay of $^{138}\text{Pm}$ into $^{138}\text{Nd}$. The ambiguity associated with many of the spin assignments again made exact calculation of the $U_\lambda$ coefficients untenable. Therefore a fit to the temperature dependence of $\gamma$-ray anisotropy presented in figure 6.14 for the 521keV transition proceeded by varying the parameters $\mu B$, $U_\lambda$, and $\xi$. Hence the value of $\mu B$ was determined to be $1277(48) \mu_n T$, and $\xi = 24(1)\%$. Using the experimentally determined value of $B$ for $\text{PmFe}$ of $406(100) T$, the ground state magnetic moment of $^{138}\text{Pm}$ may be estimated to be $3.2(9) \mu_n$.
Figure 6.14  temperature dependence of the 521keV \( \gamma \)-ray anisotropy following the \( \beta \)-decay of \(^{138}\text{Pm}\).
CHAPTER SEVEN

PARTICLE-ROTOR MODEL CALCULATIONS.

7.1 Introduction

Having experimentally determined the magnetic dipole moments of $^{139,138}$Eu, and $^{139}$Sm, a nuclear model is required to extract some form of understanding of the underlying nuclear structure. This chapter will briefly review the particle-rotor model of the nucleus, and describe the calculations performed using this model, in order to interpret the data presented in the previous chapter. Due to the tentative nature of the magnetic moment assignment for $^{138}$Pm, no such calculations have been performed.

7.2 The particle-rotor model.

The many body Hamiltonian of an odd mass nuclear system can be approximated by

$$H = H_{\text{core}}(\alpha) + H_{\text{part}}(\bar{\alpha}) + H_{\text{int}}(\alpha)$$

(7.1)

where $H_{\text{core}}$ is the collective Hamiltonian for the even-even core, $H_{\text{part}}$ is the single particle Hamiltonian in an adiabatic field corresponding to some fixed set $\bar{\alpha}$ of the surface collective variables, and $H_{\text{int}}$ takes into account the effect of the odd particle on that part of the core field not already included in $H_{\text{part}}$. The collective motion can be restricted to a
purely rotational form leading to the particle-rotor Hamiltonian

\[ H = \sum_k \frac{1}{2} \mathbf{R}_k^2(\phi, \theta, \psi) + H_{\text{part-int}}(\lambda, \phi, \theta, \psi) \quad (7.2) \]

where \( \mathbf{R} \) is the angular momentum of the core taken with respect to the intrinsic axes \( k \). By diagonalising (7.2) in the strong coupling basis states (subsection 7.2.3), which are not themselves eigenstates of the core, the contributions from the particle-rotation interaction are transformed from \( H_{\text{part}} \) and into \( H_{\text{core}} \). Thus, expressing \( \mathbf{R} \) as the vector difference between the total angular momentum \( I \) and the single particle contribution \( j \), then (7.2) becomes

\[ H = \sum_k \frac{1}{2} \mathbf{R}_k^2(\phi, \theta, \psi) - 2I_k(\phi, \theta, \psi) j_k + j_k^2 + H_{\text{part}}(\lambda) \quad (7.3) \]

In this representation the particle-rotation coupling emerges in the form of a Coriolis term \( \sum I \cdot j / J \) and a recoil term \( \sum j^2 / 23 \). The nature of this Hamiltonian will be discussed in more detail after the deformed single particle solutions have been derived.

7.2.1 The Nilsson modified harmonic oscillator potential.

The basis for most discussions of the nuclear shell model lies in the self-consistent Hartree-Fock one-body potential calculated from the nuclear two-body interactions. Perhaps the most realistic of these, together with the spin-orbit term, is the Woods-Saxon potential, with constant surface diffuseness \( [Dv79] \)

\[ V_{\text{WS}} = V(r) + V_{\text{LS}} + V_{\text{coul}} \quad (7.4) \]
with \[ V(r) = -V_0 / \left(1 + \exp[\text{dist}(r,a)/a]\right) \] (7.5)

where \(a\) is the surface diffuseness and \(\text{dist}(r,a)\) denotes the distance (taken to be negative within the surface) between the radius vector \(r\) and the nuclear surface defined by (1.11). The spin-orbit potential \(V_{LS}\) takes the form

\[ V_{LS} = s \cdot V V_{so}(r) \times p \] (7.6)

where \(s\) and \(p\) denote the nucleon spin and linear momentum operators and \(V_{so}\) is an appropriate radial function which need not necessarily be identical to the potential \(V(r)\). The Coulomb potential \(V_{coul}\) enters only for protons and is generated by a charge \((Z-1)e\) uniformly distributed within the nuclear volume.

However, the Woods-Saxon potential suffers from the disadvantage that its matrix elements cannot be solved analytically. In this respect a far better choice is the Nilsson modified harmonic oscillator potential [NI55]

\[ V_{mho} = V_{osc} + V_{corr} + V_{LS} \] (7.7)

The shape of the harmonic oscillator term

\[ V_{osc} = \frac{1}{2} m \sum \omega_k^2 \omega_k'^2 \] (7.8)

is defined by the quadrupole parameters \(\delta\) and \(\gamma_0\), which are related to the oscillator frequencies \(\omega_k\) through the equations
Table 7.1 Typical $l, s$ and $l^2$ strength parameters $K$ and $\mu$, of the modified oscillator potential. Values are given for both protons and neutrons in the various oscillator shells.

\[
\omega_k^2 = \omega_0^2 \left( \delta + \frac{4}{3} \delta \cos \left( \frac{\gamma_\delta}{3} - \frac{2\pi}{3} k \right) \right)
\]  

(7.9)

$V_{\text{corr}}$ is a corrective term which to some degree bridges the differences between $V_{\text{osc}}$ and (7.5). Since the Woods-Saxon potential has greater depth than $V_{\text{osc}}$ close to the nuclear surface, high orbital angular momentum states, which have correspondingly large rms radii, are depressed relative to those in an oscillator potential. Therefore, the high $l$ oscillator states may be adjusted by the addition of a term

\[
V_{\text{corr}} = -\kappa \mu \hbar \omega_0 l^2
\]  

(7.10)

where $\kappa$ and $\mu$ are constant parameters. The spin-orbit term may be calculated from (7.6) using the assumption that the functional form of $V_{so}$ is identical to that of the oscillator potential $V_{\text{osc}}$. Thus in the limit
of spherical symmetry

\[ V_{LS} = -2\kappa\hbar\omega l.s \quad (7.11) \]

This form is generally retained even when spherical symmetry is violated. Values for the parameters \( \kappa \) and \( \mu \) are frequently quoted in the literature. A typical example, illustrating the oscillator shell dependence is given in table 7.1 \(^{[RA64]}\)

With the introduction of the "unstretched" coordinates defined by \( \xi_k = \sqrt{\frac{m\omega}{\hbar}} \), the modified harmonic oscillator Hamiltonian may be written

\[
H_{\text{mho}} = \frac{1}{2} \hbar\omega \left( \delta, \gamma_0 \right) \left( -V_j^2 + r^2 \right) - \kappa\hbar\omega \left( 2l.s + \mu(l^2 - \langle l^2 \rangle) \right)
\]

\[
-\frac{1}{2} \hbar\omega \left( \delta, \gamma_0 \right) r^2 \frac{4}{3} \sqrt{\frac{4\pi}{5}} \delta \left\{ \cos \gamma_0 Y_{20} + \frac{1}{\sqrt{2}} \sin \gamma_0 (Y_{22} + Y_{2-2}) \right\} \quad (7.12)
\]

The diagonalization of the Hamiltonian matrix is performed in one of the harmonic oscillator basis sets: \( |Nj\Omega > \) (coupled), \( |Nj\Lambda \Sigma > \) (decoupled) or \( |Nn_3\Lambda \Sigma > \) (cylindrical), where \( N \) is the principal oscillator quantum number, \( n_3 \) the number of quanta along the symmetry axis, and \( \Lambda, \Sigma, \Omega \) are the quantum numbers of \( l_3, s_3, \) and \( j_3 \) respectively.

Generally the Hamiltonian must be diagonalised numerically. Here the basis chosen for this purpose are the coupled wavefunctions \( |Nj\Omega > \). For the case of axial symmetry \( j_3 \) commutes with the total Hamiltonian and consequently \( \Omega = \Lambda + \Sigma \) is a good quantum number. At intermediate deformations the off-diagonal matrix elements of \((7.8)\) are still sufficiently small that the asymptotic quantum numbers \( |Nn_3\Lambda\Omega > \) remain
determined from the method outlined by Cline and Lesser [CL70], which includes corrections due to correlated errors. In this method the statistical quantity $S(a_m)$ is introduced and corresponds to the $X^2$ per degree of freedom ($X^2(N-p)$, where $N$ is the number of independent measurements and $p$ the number of parameters being determined). Figure 4.3 shows a typical dependence of the statistic $S$ on a parameter $a$ for data fitted to an arbitrary function.

The value $S_L$, which determines the error in the parameter $a_m$ shown in figure 4.3, is calculated using

$$S_L^ {1-\beta} = S(\bar{a}) \left\{ 1 + \frac{p}{N - p} F(p, N - p, 1 - \beta) \right\}$$  \hspace{1cm} (4.13)$$

![Figure 4.3](image)

Figure 4.3 Error determination of a parameter $a$ determined by regression analysis, using the variation of the $X^2$ per degree of freedom with $a$. [CL70]

where $F(p,N-p,1-\beta)$ is the statistical F distribution, for the 100(1-$\beta$) confidence level. The F coefficient is approximately unity for $N$ large and
Fig 7.1 Nilsson diagram for protons.
Fig 7.2 Nilsson diagram for neutrons
essentially preserved and are generally used to label orbitals in nuclear spectroscopy, as shown in figures 7.1 and 7.2.

7.2.2 The pairing interaction

In addition to the average one-body shell model potential there acts a relatively short range residual force between the nucleons which energetically favours states in which the nucleons are pairwise coupled to states of zero total angular momentum. From BCS theory it is possible to derive the following expression for the combined single particle one-body and pair Hamiltonian

$$ H(A) = \sum_1 \varepsilon_i (a^+_i a^+_i + a^-_i a^-_i) - \Delta \sum_1 (a^+_i a^-_i + a^-_i a^+_i) - \lambda \hat{N} $$

(7.13)

where $\varepsilon_i$ represents the energy eigenvalues of the one-body Hamiltonian, $a^+_i$ and $a^-_i$ are creation operators for a single particle state $|i\rangle$, and its time reversed state respectively. $\hat{N}$ is the particle number operator, $\Delta$ the pairing gap, and $\lambda$ the Fermi level. Performing the unitary transformation to the quasiparticle operators $\alpha^+_i$ and $\alpha^-_i$ [

$$ a^+_i = U^i_{\alpha^+_i} + V^i_{\alpha^-_i} \quad a^-_i = U^i_{\alpha^-_i} - V^i_{\alpha^+_i} $$

(7.14)

where $U^2 + V^2 = 1$, and with a suitable choice of $U^i$ and $V^i$, namely
(7.15)

\[
\begin{aligned}
\left\{ \frac{U_i}{V_i} \right\} &= \sqrt{\frac{1}{2} \left\{ 1 \pm \frac{e_i - \lambda}{\sqrt{(e - \lambda)^2 + \Delta^2}} \right\}} \\
\end{aligned}
\]

the Hamiltonian becomes diagonal in the quasiparticle representation, hence

\[
H(\Delta) = \sum_i \left\{ 2V_i^2(e_i - \lambda) - 2\Delta U_i V_i \right\} + \Delta^2 + \sum_i E_i (\alpha_i^+ \alpha_i + \alpha_i^{-}\alpha_i^{-})
\]

(7.16)

where \( G \) is a constant characterizing the residual two-body force, and \( E_i = \sqrt{(e_i - \lambda^2) + \Delta^2} \) has been introduced, as the quasiparticle energy.

To conclude, the effects of the pairing interaction can be incorporated into the one-body Hamiltonian by replacing the single particle energies \( e_i \) by the corresponding single quasiparticle energies \( E_i \) and by introducing the appropriately transformed single particle operators into the subsequent calculations.

7.2.3 The strong coupling basis

Having derived the deformed single particle solutions it is now possible to diagonalize the total Hamiltonian of the odd mass system. When dealing with the particle-(rigid) rotor Hamiltonian it is convenient to use the strong coupling basis so that the matrix calculation may be performed entirely in the intrinsic frame. Neglecting pairing, the unsymmetrized strong coupling basis states may be written

\[
| IMK_1 > = \sqrt{\frac{2I + 1}{8\pi^2}} D_{MK}^{\dagger} \chi_1
\]

(7.17)
where $M$ and $K$ are the projections of the total angular momentum (both core and single particle) onto the laboratory and intrinsic axes respectively and $\chi_i$ is the single particle wavefunction.

With the inclusion of pairing and some rearrangement the Hamiltonian (7.3) becomes

$$
H = \frac{1}{4} \left\{ \frac{1}{5^1} \right\} \left\{ I_1^2 - I_3^2 + j_1^2 - j_3^2 - (I_+ j_+ + I_- j_-) \right\}
$$

$$
+ \frac{1}{6} \left\{ \frac{1}{5^1} \right\} \left\{ I_1^2 + I_3^2 + j_1^2 + j_3^2 - 2(I_+ j_+ + I_- j_-) \right\}
$$

$$
+ \frac{1}{23^1} (I_3 - j_3)^2 + \sum_i E_i \alpha_i \alpha_i^* \tag{7.18}
$$

where in the last term the constant energy of the quasiparticle vacuum has been neglected. The required matrix elements for this Hamiltonian are given explicitly by Larsson et al [LA78].

6.2.4 Electromagnetic moments and transition probabilities.

The electromagnetic $M1$ and $E2$ multipole operators must be expressed in the intrinsic frame of reference, when using the Hamiltonian 7.18, and are given by the expressions

$$
M(M1;\mu) = \mu \sqrt{\frac{3}{4\pi}} \sum_{\nu} D_{1\nu}^1 \left\{ g_1 I_1^\nu + (g_1 - g_R) j_1^\nu + (g_s + g_1) s_1^\nu \right\} \tag{7.19}
$$

$$
M(E2;\mu) = \sum_{\nu} \delta_\nu e^{2} \mu_0 \mu_2 \gamma_2^\nu + \frac{3}{4\pi} E_B \sum_{\nu} \frac{r^2}{\mu_0} \left\{ \frac{p^2 \cos \gamma}{\mu_2} + \frac{1}{\sqrt{2}} \mu_2 \sin \gamma \right\} \tag{7.20}
$$

138
In this case $M(E2)$ is expressed in terms of the macroscopic quadrupole moment of the core, written here to first order in the $\beta, \gamma$ coordinates. From these operators can be calculated the reduced electromagnetic transition probabilities $B(\pi \lambda, I_1 \rightarrow I_f)$, defined by

$$B(\pi \lambda, I_1 \rightarrow I_f) = \frac{1}{2I_1 + 1} |<I_f| |M(\pi \lambda)| |I_1>|^2$$  \hspace{1cm} (7.20)

and also the static moments defined by

$$\mu_I = (III|II) \sqrt{\frac{4\pi}{3}} |<I| |M(M1)| |I>|$$ \hspace{1cm} (7.21)

$$eQ_I = (III20|II) \sqrt{\frac{16\pi}{5}} |<I| |M(E2)| |I>|$$ \hspace{1cm} (7.22)

Finally, with the magnetic and electric reduced matrix elements expressed in units of $\mu_n$ and $eb$ respectively, the $E2/M1$ mixing ratio is given by

$$\delta = \sqrt{\frac{0.7}{E_{if}}} \frac{<I| |M(E2)| |I>}{<I| |M(M1)| |I>}$$ \hspace{1cm} (7.23)

where $E_{if}$ is the energy of the transition in MeV.

7.2.5 The unified model of odd-odd nuclei.

Following on from the Hamiltonian (7.1) of the odd mass system, that of an odd-odd nucleus must treat the coupling of at least two quasiparticles to the collective core. Hence

$$H = H_{core}(\alpha) + \{H_{part}(\bar{\alpha}) + H_{int}(\alpha)\}^{pn} + V_{pn}$$ \hspace{1cm} (7.24)
where the terms in braces are taken twice, once for the odd proton and once for the odd neutron. The residual proton-neutron interaction is represented by the term $V_{pn}$. Only the particle-rotor limit of this Hamiltonian will be considered. With the core spin written as $R = I - j_p - j_n$, the particle-rotor Hamiltonian becomes [RA88]

$$H = \frac{1}{4} \left\{ \frac{1}{3} + \frac{1}{2} \right\} \left\{ I_3^2 - I_3^2 + j_p^2 + j_n^2 - j_{3p}^2 + j_{3n}^2 - j_p j_n + j_{-p} j_{-n} + j_{-p} j_{+n} \right. $$

$$\left. - I_{+}(j_{+p} + j_{-n}) + I_{-}(j_{+p} + j_{+n}) \right\}$$

$$+ \frac{1}{8} \left\{ \frac{1}{3} + \frac{1}{2} \right\} \left\{ I_{+}^2 + I_{-}^2 + j_{+p}^2 + j_{-n}^2 + j_{+n}^2 + j_{-n}^2 + 2(j_{+p} j_{+n} + j_{-p} j_{-n}) \right. $$

$$\left. - 2I_{+}(j_{+p} + j_{+n}) - 2I_{-}(j_{-p} + j_{-n}) \right\}$$

$$+ \frac{1}{2} \left\{ I_{3} - j_{3p} - j_{3n} \right\}^2 + \sum_{p} E_{p}^{+} \alpha_{p}^{+} \alpha_{p} + \sum_{n} E_{n}^{+} \alpha_{n}^{+} \alpha_{n} + V_{pn} \right\} \right\}$$

(7.25)

It then remains to multiply onto both the proton and neutron single particle angular momentum operators the appropriate pairing factor, $U_{i\bar{i}} + V_{i\bar{i}}$. Finally, the electromagnetic moments and transition probabilities are calculated from the suitably generalized operators of subsection 7.2.4.

7.3 Particle rotor model calculations.

The particle-rotor calculations performed in this section were facilitated by the provision of a computer code based on that described in the literature [LA78] and modified to include odd-odd nuclei by I. Ragnarsson. The code comprises three main subprograms, the first of which
calculates the single particle energies and various wavefunctions in a modified oscillator (Nilsson) potential, and was mainly designed to provide input for the subsequent particle rotor calculations. The second subprogram performs the diagonalisation of the particle-rotor Hamiltonian with single particle wavefunctions expressed in the deformed scheme. The third subprogram calculates the electromagnetic transition probabilities $B(E2)$ and $B(M1)$, and the static nuclear moments.

Nilsson ($\kappa, \mu$) parameter sets for the proton $N = 4, 5,$ and 6 harmonic oscillator shells have been modified for the $A = 120 - 140$ mass region by fitting an extensive set of bandhead energies, and are used for almost all the calculations described in the following sections. Table 7.2 therefore shows the modified values for $\kappa$ and $\mu$.

<table>
<thead>
<tr>
<th>$N$</th>
<th>$\kappa$</th>
<th>$\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.070</td>
<td>0.48</td>
</tr>
<tr>
<td>5</td>
<td>0.056</td>
<td>0.54</td>
</tr>
<tr>
<td>6</td>
<td>0.054</td>
<td>0.52</td>
</tr>
</tbody>
</table>

Table 7.2 Modified set of Nilsson parameters for the $A = 120-140$ mass region for protons.

7.3.1 The nucleus $^{138}\text{Eu}$.

The spin and parity of $11/2^-$ has been assigned to the ground-state of $^{138}\text{Eu}$ [LO85]. On this state a band is built with energy spacings which resemble the yrast band of $^{138}\text{Sm}$, and can be understood in the framework
of the rotation-aligned coupling scheme and implies a prolate deformation of the nuclei involved. Such decoupled bands built on the $11/2^+$ state in odd-proton nuclei, are known to occur in this region and are well interpreted within a particle-plus-triaxial-rotor-model \[HE75\].

From the lifetime of the $2^+$ state of $^{138}\text{Sm}$ \[LU85\] the nuclear deformation parameter $\beta = 0.24(2)$ has been extracted, and is slightly larger than that expected using the empirical relation of Grodzins given by eqn 7.26, which predicts a deformation of $\beta = 0.19$ (from the energy of the yrast $2^+$ state). The deformation is larger than that in the corresponding nucleus $^{150}\text{Sm}$ (six neutrons above the $N=82$ closed shell), and is probably due to the relevance of the $Z=64$ closed shell for nuclei with $N=82-88$.

\[
\varepsilon_2 = \sqrt{\frac{1224}{A^{7/3} E_1^{2+} (\text{MeV})}}
\] (7.26) #

The existence of a second low lying $2^+$ state is a common feature of the nuclei in this region and it is interpreted by assuming either a nucleus is "soft" with respect to the quadrupole non-axial $\gamma$-deformation, or a rigid triaxial nucleus. For a rigid triaxial nucleus it is possible to estimate the asymmetry parameter $\gamma$ from the excitation energy of the second $2^+$ state given by the empirical eqn 7.27 \[LA78\], and is estimated to be of order $27^\circ$ for $^{138}\text{Sm}$.

\[
\gamma = \frac{1}{3} \sin^{-1} \left[ \frac{9}{8} \left\{ 1 - \left( \frac{E_2^{2+}/E_1^{2+} - 1}{E_2^{2+}/E_1^{2+} + 1} \right)^2 \right\} \right]
\] (7.27)

* Use has been made of the Nilsson eccentricity parameter $e_2$ (defined in terms of the major/minor axis ratio) and is to first order $\approx 0.95\beta$. \[JB90\]
With this information about the "core" of $^{139}$Eu it is possible to fit the experimentally determined value for the ground-state magnetic moment $\mu$, and the corresponding energies of the ground state band using the particle-rotor model, using the procedure described below.

Inspection of the standard Nilsson diagram for protons reveals that the Fermi-surface for protons at $Z = 63$ lies around the $5/2 - [532]$ Nilsson orbital for deformations ranging from $\beta = 0.2 \rightarrow 0.3$, and arises from the $h_{11/2}$ oscillator shell. In order to construct the strong coupling basis states for the subsequent Coriolis mixing calculations (the Nilsson states), the first part of the particle-rotor code was run in the "stretched" co-ordinate system for $^{139}$Eu, with negative parity Nilsson states for the $h_{11/2}$ oscillator shell, in addition to neighbouring $K = 1/2$ ($1/2[p1/2]$, and $1/2[f7/2]$) orbitals which are expected to mix strongly in the strong coupling basis.

The moment of inertia parameters needed in the second part of the code in order to calculate the influence of the Coriolis force on the single particle energies are calculated using the formula

$$\mathcal{I}(K) = 4B^2 \varepsilon_y (\sin(\gamma + 2K\pi/3))^2$$

where

$$B = \hbar^2 A^{7/3} / 1225.$$  \hfill (7.28)

The pairing strength parameter $G$ described by eqn 7.15 is set using

$$G = \{(G_{NO} + G_{N1}) (N-Z)/A\} (\text{protons/neutrons})$$ \hfill (7.29)

where $G_{NO} = 19.2$ and $G_{N1} = 7.4$, and are standard parameters for $Z$ and $N$.
greater than 60. All the single particle matrix elements produced before Coriolis mixing are multiplied by a constant factor $\chi$, which is conventionally set at $\sim 0.8$.

In order to calculate the ground-state magnetic moments the single particle states produced by the second part of the code are used as input for the third part of the code. The scaling factor for the internal spin g-factor is set to 0.7 at this stage, in order to obtain an effective g-factor rather than that for "free" nucleons.

Having set most of the fixed parameters as described above, it is then possible to vary the deformation parameters $\beta$ and $\gamma$, in addition to the Coriolis attenuation factor $\chi$, running the program with one parameter set at a time. Once agreement with the rotational-band properties had been established, the value of the calculated ground-state moment was then compared to that measured by experiment as a final check.

Table 7.3 shows the composition of the Nilsson orbitals before Coriolis mixing near the Fermi-surface for protons. These are calculated using the Zhang parameter set (see table 7.2) using deformation parameters $\varepsilon_2 = 0.27$ and $\gamma = 29^\circ$.

Following Coriolis mixing of the Nilsson basis-states, using a $\chi$ attenuation parameter of 0.95, the ground-state spin parity for $^{139}$Eu is calculated to be $11/2^-$ in agreement with experiment. The composition of the $11/2^-$ state (which is calculated as the bandhead of a $\langle K \rangle \sim 3/2$ rotational band) is shown in table 7.4 as probability amplitudes as a function of the Nilsson orbitals (N) and K.
N.B. $\mu_{100} = 6.5(\pm 2)$. 
### Table 7.3 Composition of the single particle (parity -) Nilsson proton orbitals (N, with energy E(MeV)) expressed as a probability amplitude for the corresponding basis-states $\Omega [\text{m} \text{n} \text{l}]$, for the nucleus $^{139}\text{Eu}$, $\varepsilon_z = 0.27$, and $\gamma = 29^\circ$. Probability amplitudes $< 0.09$ are left as a blank.

Hence a calculation of $\mu$ corresponding to the ground-state obtained above, yields a value of $\mu = 6.44\mu_N$ which is in close agreement with that measured experimentally of $6.3(8)\mu_N$. Using the same parameters given above, a good fit to the energy of the first few members of the ground state rotational band is observed, as shown in table 7.5. The values of $\varepsilon_z$ and $\gamma$ derived in order to fit the ground state band are relatively high compared to that of the core $^{138}\text{Sm}$ ($\varepsilon_z = 0.24$, and $\gamma ~ 25^\circ$), and illustrate the importance of the Coriolis force upon the unpaired nucleon.
and the consequent impact that has on the shape of the nucleus as a whole. However, the possibility of a variable moment of inertia as the spin is increased, has not been included in these calculations, limiting the possibility of a more complete description of the nuclear structure of \(^{139}\text{Eu}\). Hence it is hoped that with the inclusion of such a 2nd order interaction it would be possible to obtain a better fit to the rotational bands at higher spins.

The nucleus \(^{139}\text{Eu}\) has therefore been shown to exhibit two of the major features of nuclear structure that are commonly seen in the deformed rare-earth nuclei, with \(N < 82\). These are the tendency for the nuclei to adopt axially asymmetric shapes in their ground states, and the weak coupling of the unpaired nucleon to the rotating core. It is the latter feature which accounts for the \(I^\pi\) of the ground state being taken from the harmonic oscillator quantum number \(j\), rather than the asymptotic quantum number \(\Omega\) (the projection of \(j\) onto the symmetry axis).
Table 7.5 Particle-rotor model fit to the low spin members of the $K=3/2$ ground-state rotational band for $^{139}$Eu, using $\epsilon_2=-0.27$ and $\gamma=29^\circ$.

<table>
<thead>
<tr>
<th>$I^N$</th>
<th>Measured energy (keV)</th>
<th>Calculated energy (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$11/2^-$</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$15/2^-$</td>
<td>322.8</td>
<td>325.6</td>
</tr>
<tr>
<td>$19/2^-$</td>
<td>877.0</td>
<td>881.3</td>
</tr>
<tr>
<td>$23/2^-$</td>
<td>1589.7</td>
<td>1627.9</td>
</tr>
<tr>
<td>$27/2^-$</td>
<td>2406.2</td>
<td>2582.9</td>
</tr>
</tbody>
</table>

7.3.2 The nucleus $^{139}$Sm.

The spin and parity of $1/2^+$ has been assigned to the ground-state of $^{139}$Sm. An isomeric state has also been observed with $T_{1/2} = 11\text{s}$ with spin and parity $11/2^-$ [BA90]. However, since spin 1/2 states give no $\gamma$-ray anisotropy in LTNO experiments, the purpose of the calculations presented in this subsection was to attempt to fit the bandhead moment and band structure for the isomeric-state, 457keV above the ground-state.

It has been shown [ME75] that for $N \leq 82$ and $Z \geq 50$ the high-$j$ $h_{11/2}$ subshell, due to the position of the Fermi-surface, becomes accessible to both proton and neutron excitations. As a result the odd $A$ nuclei in this mass region show collective structures built on the $h_{11/2}$ single particle states of proton or neutron structure. As has been illustrated in the previous subsection, for odd $Z$ nuclei with $N \leq 78$, the similarity of the observed bands to the ground state sequence in the neighbouring even-even nuclei suggest a rotational aligned character based on the $h_{11/2}$ Nilsson
orbital. Odd-neutron nuclei in contrast show $\Delta I=1$ sequences built on the $11/2^-$ neutron state, as shown in Fig 7.3. The significant difference in

![Diagram showing 11/2- band structure for the N=77 isotones.](image)

the nucleus $^{139}\text{Sm}$ is that the $15/2^- (I+2)$ state is pushed below the $13/2^- (I+1)$ state changing the level pattern observed in the known odd-$N$ nuclei with $N \leq 77$. This observation has been interpreted as arising from a soft $^{140}\text{Sm}$ core, which is easily polarized by the valence particles, giving rise to an oblate shape in contrast to the prolate shape observed in the nucleus $^{141}\text{Eu}$ (which shares the same core, coupled to an odd proton rather than the odd neutron).

Particle plus triaxial-rotor model calculations were therefore performed in a similar manner to that described in subsection 7.3.1, with the important exception that negative values for the deformation parameter $\varepsilon_2$ were used to fit the low lying nuclear structure. The Nilsson basis states calculated near the Fermi-surface for negative parity neutrons were again calculated using the Zhang parameter set, using deformation parameters $\varepsilon_2 = -0.20$ and $\gamma = 27.5^0$ and are shown in table 7.6.
Table 7.6 Composition of the single particle (parity -) Nilsson neutron orbitals \((N, \text{ with energy } E\text{(MeV)})\) expressed as a probability amplitude for the corresponding basis-states \(\Omega [\mathfrak{J}\hbar\Lambda]\), for the nucleus \(^{139}\text{Sm}\), \(\varepsilon_z = 0.2\), and \(\chi = 27.6^\circ\). Probability amplitudes < 0.08 are left blank.

Following Coriolis mixing of the Nilsson basis-states, using a \(\chi\) attenuation parameter of 1.0, the spin and parity for \(^{139}\text{Sm}\) is calculated to be \(11/2^-\) in agreement with existing data. The composition of this highly mixed metastable state (which is calculated as the bandhead of the \(K = 5/2\) rotational band) is shown in table 7.7 as probability amplitudes as a function of the Nilsson orbitals \((N)\) and \(K\).

Hence a calculation of \(\mu\) corresponding to the meta-stable state obtained above, yields a value of \(\mu = 1.41\mu_N\), which is broadly in agreement with that measured experimentally of \(1.1(2)\mu_N\). Using the same parameters as those given above, a good fit to the energy of the low-lying nuclear
Table 7.7 Composition of the $I = 11/2^-$ meta-stable state of $^{139}$Sm, in terms of the Nilsson orbitals $N$.

<table>
<thead>
<tr>
<th>$N$</th>
<th>$I=1/2$</th>
<th>$3/2$</th>
<th>$5/2$</th>
<th>$7/2$</th>
<th>$9/2$</th>
<th>$11/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>0.089</td>
<td>0.047</td>
<td>0.025</td>
<td>0.011</td>
<td>0.006</td>
<td>0.002</td>
</tr>
<tr>
<td>15</td>
<td>0.041</td>
<td>0.033</td>
<td>0.081</td>
<td>0.118</td>
<td>0.146</td>
<td>0.026</td>
</tr>
<tr>
<td>16</td>
<td>0.092</td>
<td>0.211</td>
<td>0.173</td>
<td>0.052</td>
<td>0.171</td>
<td>0.021</td>
</tr>
<tr>
<td>17</td>
<td>0.078</td>
<td>0.334</td>
<td>0.152</td>
<td>0.348</td>
<td>0.202</td>
<td>0.017</td>
</tr>
<tr>
<td>18</td>
<td>0.039</td>
<td>0.255</td>
<td>0.523</td>
<td>0.413</td>
<td>0.119</td>
<td>0.008</td>
</tr>
</tbody>
</table>

levels above the isomeric state is obtained, as illustrated in table 7.8. The values of $\varepsilon_2$ and $\gamma$ needed to force the $15/2^-$ state below the $13/2^-$ in energy, confirms the suggestion [RA90] that $^{139}$Sm exhibits oblate deformation in addition to the strongly triaxial shape observed in many of the nuclei studied in this mass region.

<table>
<thead>
<tr>
<th>$I^N$</th>
<th>Measured energy (keV)</th>
<th>Calculated energy (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$11/2^-$</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$15/2^-$</td>
<td>589.6</td>
<td>590.1</td>
</tr>
<tr>
<td>$13/2^-$</td>
<td>607.6</td>
<td>606.0</td>
</tr>
<tr>
<td>$17/2^-$</td>
<td>1321.1</td>
<td>1391.7</td>
</tr>
<tr>
<td>$19/2^-$</td>
<td>1412.0</td>
<td>1655.3</td>
</tr>
</tbody>
</table>

Table 7.8 Particle-rotor model fit to the low spin members of the $K=5/2^+$ band for $^{139}$Sm, using $\varepsilon_2$=0.2 and $\gamma$=27.5°.

The exclusion of the variable moment of inertia from the calculations
described above is again reflected in the difference in the calculated and experimentally measured excitation energies at higher spins. However, the ability to fit the isomeric magnetic moment and spin with relative ease, suggests that the particle-rotor model used is good enough to extract the dominant single particle configurations, and hence band head spins in the cases of both strongly coupled, and weakly coupled nuclear systems.

7.3.3 The nucleus $^{138}$Eu.

A spin and parity of $7^+$ has been tentatively assigned to the ground state of $^{138}$Eu, on the basis of measured log ft values [86]. The speculated components of the $7^+$ state were thought to be mainly $\nu 9/2^-[514]\omega 5/2^-[532]$ and $\nu 11/2^-[505]\omega 3/2^-[541]$, due to the position of the Fermi-surface for protons and neutrons on the standard Nilsson diagrams for $Z=63$ and $N=75$. In addition, Liang et al [88] following in beam work, have also suggested a ground state spin parity of $7^+$ from systematics of the $N=75$ isotones, and measurements of various $B(M1)/B(E2)$ values for their measured yrast band.

Included in the work by Cizewski and Gülmez [86] concerning the disappearance of the $Z=64$ shell gap for $N<82$, is a calculation of the Nilsson levels for neutrons for the $A=135$ mass region. Inspection of the Nilsson diagram (shown in fig 4) reveals that the Fermi-surface for $N=75$ comprises a range of both positive and negative parity Nilsson levels. Consequently any particle-rotor model calculation should allow for the possibility of both negative and positive parity Nilsson configurations.
Fig. 7.4: Nilsson diagram for Neutrons for \( A=135 \) mass region.

Employing the simple asymptotic formula for calculating \( \mu \) given by eqn 1.9 (assuming no Coriolis mixing) for the various proton and neutron Nilsson wavefunctions near the respective Fermi-surfaces, gives some insight into the most likely proton/neutron configuration, and are therefore given in table 7.9.

The asymptotic calculations shown in table 7.9 show that the suggested \( \hbar_{11/2} \odot \hbar_{11/2} \) configurations (labelled a, c, e in the table) give low values for \( \mu \) when compared with the value obtained experimentally of 5.3(7)\( \mu_N \). However, the \( \hbar_{11/2} \) proton coupled to the \( g_{7/2} \) neutron configuration (labelled b) is calculated to give a value of \( \mu \) closer to that obtained experimentally. Particle-rotor model calculations have
therefore been performed to include positive parity proton, and positive
and negative parity neutron configurations, in order to investigate the
effect of Coriolis mixing on the magnitude of the calculated moments, in
an effort to try and establish the most likely ground-state configuration,
and consequently ground state-spin of the nucleus $^{138}$Eu. The small value
of the moment estimated for the $\pi 5/2^+ [413] \otimes \nu 9/2^- [514]$ configuration
(0.67$\mu_n$) made the need to include positive parity proton Nilsson orbitals
in the particle-rotor calculations negligible, and consequently were not
performed.

In contrast to the calculations performed for a single nucleon
coupled to an even-even core presented in the previous two sections, the
extensive cpu time required to model an odd-odd mass system limited any
detailed excursions into the effect of $\gamma$-deformation on the low-lying
band-structure. In addition, the limited availability of data concerning
the ground-state band, tentatively assigned with a band head of $I^\pi = 7^+$

<table>
<thead>
<tr>
<th>$\pi \Omega([\Sigma z])$</th>
<th>$\nu \Omega([\Sigma z])$</th>
<th>$I^\pi$</th>
<th>$\mu (\mu_n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5/2^- [532]^a$</td>
<td>$9/2^- [514]$</td>
<td>$7^+$</td>
<td>2.45</td>
</tr>
<tr>
<td>$5/2^- [532]^b$</td>
<td>$7/2^+ [404]$</td>
<td>$6^-$</td>
<td>4.38</td>
</tr>
<tr>
<td>$5/2^- [532]^c$</td>
<td>$11/2^- [505]$</td>
<td>$8^+$</td>
<td>2.54</td>
</tr>
<tr>
<td>$5/2^+ [413]^d$</td>
<td>$9/2^- [514]$</td>
<td>$7^-$</td>
<td>0.67</td>
</tr>
<tr>
<td>$3/2^- [541]^e$</td>
<td>$11/2^- [505]$</td>
<td>$7^+$</td>
<td>1.63</td>
</tr>
</tbody>
</table>

Table 7.9 Asymptotic calculation for the ground state moment and spin
for $^{138}$Eu corresponding to the proton-neutron configurations near the
Fermi-surface.
from systematics [L188], made comparison of ground-state band energies to experiment vulnerable to misinterpretation.

Preliminary calculations assuming axial symmetry for the nucleus $^{138}$Eu were therefore performed using similar parameters to those described in section 7.3.1, using a Coriolis attenuation parameter of $1.0, \gamma = 0^\circ$, and varying the quadrupole deformation parameter $\varepsilon_2$. Table 7.10 shows the results of such calculations, giving the ground-state moment, energy, dominant $\Omega$, and nucleon configurations, for $\varepsilon_2$ spanning a range of $0.2 \rightarrow 0.3$, for the negative parity proton and neutron configurations.

<table>
<thead>
<tr>
<th>$\varepsilon_2$</th>
<th>$\pi \Omega [\hbar N^2 \Lambda]$</th>
<th>$\nu \Omega [\hbar N^2 \Lambda]$</th>
<th>$I$</th>
<th>$E$(keV)</th>
<th>$\mu (\mu_B)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.20</td>
<td>$5/2^-[532]$</td>
<td>$9/2^-[514]$</td>
<td>7</td>
<td>155.6</td>
<td>3.34</td>
</tr>
<tr>
<td>0.20</td>
<td>$3/2^-[541]$</td>
<td>$9/2^- [514]$</td>
<td>7</td>
<td>0.0</td>
<td>4.23</td>
</tr>
<tr>
<td>0.24</td>
<td>$5/2^- [532]$</td>
<td>$9/2^-[514]$</td>
<td>7</td>
<td>0.0</td>
<td>3.53</td>
</tr>
<tr>
<td>0.27</td>
<td>$5/2^- [532]$</td>
<td>$9/2^- [514]$</td>
<td>7</td>
<td>203.4</td>
<td>2.90</td>
</tr>
<tr>
<td>0.30</td>
<td>$5/2^- [532]$</td>
<td>$9/2^- [514]$</td>
<td>7</td>
<td>196.8</td>
<td>2.58</td>
</tr>
</tbody>
</table>

Table 7.10 Particle-rotor calculation of ground-state moment, and nucleon configurations for $^{138}$Eu as a function of $\varepsilon_2$, choosing negative parity proton and neutron orbitals.

Examination of the results presented in table 7.10, show that for the case of axial symmetry, no agreement may be found between the measured value for the ground-state magnetic moment of $^{138}$Eu of $5.3(7)\mu_N$, and those calculated using the particle rotor model. The calculated moment for the ground-state at a deformation of $\beta = 0.2$ although only just over one-standard deviation away from the measured value, assumes a deformation considerably smaller than that of its nearest neighbours. However, when
\[ -B \to \pi^+ = 6^- / \mu = 2.8 \pm 0.2 (\text{MeV}) \text{ (from ) } 6^+ \]
choosing positive parity neutron orbitals, good agreement is obtained for a wide range of $\varepsilon_2^*$, as shown in table 7.11.

<table>
<thead>
<tr>
<th>$\varepsilon_2$</th>
<th>$\pi \Omega [\pi n, \Lambda]$</th>
<th>$\nu \Omega [\nu n, \Lambda]$</th>
<th>$I$</th>
<th>$E$ (keV)</th>
<th>$\mu (\mu_n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.20</td>
<td>$3/2^- [541]$</td>
<td>$1/2^+ [400]$</td>
<td>5</td>
<td>0.0</td>
<td>7.07</td>
</tr>
<tr>
<td>0.20</td>
<td>$3/2^- [541]$</td>
<td>$7/2^+ [404]$</td>
<td>6</td>
<td>283.4</td>
<td>5.73</td>
</tr>
<tr>
<td>0.24</td>
<td>$5/2^- [532]$</td>
<td>$7/2^+ [404]$</td>
<td>6</td>
<td>0.0</td>
<td>5.41</td>
</tr>
<tr>
<td>0.27</td>
<td>$5/2^- [532]$</td>
<td>$7/2^+ [404]$</td>
<td>6</td>
<td>0.0</td>
<td>4.98</td>
</tr>
<tr>
<td>0.30</td>
<td>$5/2^- [532]$</td>
<td>$7/2^+ [404]$</td>
<td>1</td>
<td>0.0</td>
<td>0.73</td>
</tr>
<tr>
<td>0.30</td>
<td>$5/2^- [532]$</td>
<td>$7/2^+ [404]$</td>
<td>6</td>
<td>14.3</td>
<td>4.71</td>
</tr>
</tbody>
</table>

Table 7.11: Particle rotor calculation of ground-state magnetic moment, and nucleon configurations for $^{138}$Eu as a function of $\varepsilon_2^*$, choosing negative parity proton, and positive parity neutron orbitals.

Since axial asymmetry has been shown to be a prevalent feature of the nuclei in the light rare-earth region, a limited series of calculations were performed as above with the inclusion of $\gamma = 25^\circ$, and $\varepsilon_2 = 0.24$ (values similar to that of the even-even core, namely $^{136}$Sm). However, for the calculations involving negative parity neutron orbitals, the effect of axial asymmetry results in the $I^\pi = 7^+$ level shown in table 7.10 losing both its ground-state and bandhead status. The newly calculated ground-state has $I^\pi = 6^+$, with a calculated moment of $2.82\mu_n$, which is very low when compared to the experimental value. When considering positive parity neutron orbitals, the effect of axial asymmetry again results in the $I^\pi = 6^-$ level shown in table 7.11 losing both its ground-state and bandhead status. The corresponding ground-state $I^\pi$ is calculated to be $4^-$ which is not consistent with the logft values and
spin/parity assignments measured following $\beta$-decay, as shown in figure 6.9.

It therefore appears that an asymmetry parameter of less than 25° is required in order to force the particle-rotor calculations to give a ground-state configuration consistent with existing data. However, assuming axial symmetry, the magnitude of the experimentally determined value for $\mu$, enables the unique assignment of the $\{52^+\} [404]$ Nilsson two-quasiparticle configuration.

Employing the relation $\Omega = A + \Sigma$, where $\Sigma$ is the projection of the intrinsic spin of the nucleon on the symmetry axis, it becomes clear that the proton has its intrinsic spin aligned with its angular momentum, and the neutron against. The strong coupling of the odd proton and neutron to the deformed ($e_2 \sim 0.24$) rotating core, therefore appears to force the anti-parallel coupling of the proton and neutron intrinsic spin-projections in the ground-state, contrary to the Gallagher-Moszkowski coupling rules, which predict the $K = 1$ coupling would have a lower energy.

In order to investigate the relative importance of the Coriolis force upon the energies of the two possible couplings of the odd proton and neutron, particle-rotor model calculations were performed for the nucleus $^{138}$Eu using $e_2 = 0.24$, and $\gamma = 0^\circ$, and varying $\chi$, the Coriolis attenuation parameter in the range $0.0 \rightarrow 1.0$. Table 7.12 shows the results of such calculations, and show that for $\chi > 0.65$ the parallel coupling ($K = 6$) of the odd proton and odd neutron is energetically favoured over the antiparallel ($K = 1$) coupling. Since a $\chi$ parameter of $\sim 1.0$ is typically used in the particle-rotor model, it can be postulated that even with the inclusion of a residual proton-neutron interaction
(which can be of order 200keV \(^{[58]}\)), the calculated ground state of \(^{138}\)Eu could remain with spin and parity \(6^-\). However, even if the \(6^-\) state is not the ground state, it would be difficult to tell the difference experimentally, since electromagnetic transitions between the \(6^-\) and \(1^-\) states would be strongly hindered. There is however little evidence \([R86]\) of a low spin contribution for the \(\beta\)-decay to the low spin states of \(^{138}\)Sm.

### Table 7.12

<table>
<thead>
<tr>
<th>(\chi)</th>
<th>0.0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy (I^\pi=6^-) (keV)</td>
<td>105</td>
<td>96</td>
<td>69</td>
<td>24</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Energy (I^\pi=1^-) (keV)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>38</td>
<td>108</td>
</tr>
</tbody>
</table>

Hence, in contrast to the calculations performed for the odd-mass nuclei \(^{139}\)Eu, and \(^{139}\)Sm, where the magnitude of the measured ground-state magnetic moment gave information regarding the deformation of the nucleus, the calculations performed for \(^{138}\)Eu provide information on the ground-state odd nucleon configurations, and hence, the ground-state spin and parity. In conclusion, and in the absence of further spectroscopic data for the ground-state band of \(^{138}\)Eu, a tentative spin-parity of \(6^-\) may be assigned, based on the \(\{\pi5/2^- [532] @ \nu7/2^+ [404]\}\) configuration.
CHAPTER EIGHT

SUMMARY

The initial aims of the research presented in this thesis can be broadly generalised into two major areas. The first of these was to undertake systematic measurements of the low lying nuclear-structure in the light rare-earth region, in particular the measurement of ground-state magnetic dipole moments utilizing the technique of low temperature nuclear orientation. The second, and perhaps more pertinent aim, was to reconcile the experimental data with calculations performed using a nuclear model, recognised as being able to cope with both odd-\(A\), and odd-odd nuclei, namely the particle-rotor model.

Neutron-deficient Eu, Sm, and Pm nuclei in the mass region \(A = 138 \rightarrow 142\) were produced following heavy-ion fusion reactions, employing the 20MV Tandem accelerator at Daresbury. Temperature dependencies of \(\gamma\)-ray anisotropy following \(\beta\)-decay enabled hyperfine-field calibrations for isotopes of Eu, Sm, and Pm in an iron host lattice, as shown in table 8.1.

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Hyperfine Interaction ((\mu_T))</th>
<th>Hyperfine Field ((T))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{142})Eu</td>
<td>331(35)</td>
<td>111(12)</td>
</tr>
<tr>
<td>(^{141})Sm</td>
<td>306(30)</td>
<td>369(37)</td>
</tr>
<tr>
<td>(^{141})Pm</td>
<td>1444(352)</td>
<td>(406(100))</td>
</tr>
</tbody>
</table>

Table 8.1 Measured values of the strength of the hyperfine interaction, and hyperfine field for Eu, Sm, Pm isotopes in an iron host.
Using a novel pulsed implantation technique, measurements of the time
dependencies of γ-ray anisotropy following the β-decay of the nuclei
\(^{139}\text{Eu}\) and \(^{139}\text{Sm}\) were undertaken, in order to estimate the relaxation time
of these isotopes in iron. A limit on the relaxation time of < 3s was
measured for both \(^{139}\text{Eu}\) and \(^{139}\text{Sm}\), indicating that little correction had
to be made for subsequent integral LTNO measurements.

Once the solid-state aspects of the research had been investigated,
as described above, the nuclear structure measurements could proceed with a
good degree less ambiguity. Measurements of γ-ray anisotropy as a function
of temperature were thus performed on the nuclei \(^{139}\text{Eu}\), \(^{139}\text{Sm}\), \(^{138}\text{Eu}\), and
\(^{138}\text{Pm}\) in order to extract their respective ground-state magnetic moments.

Table 8.2 shows the measured values of the magnetic moments for the
isotopes studied, where the tentative value for \(^{138}\text{Pm}\) arises from the
assignment of the hyperfine field being made from systematics (see
subsection 5.3.4).

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Dipole Moment (\mu (\mu_N))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{139}\text{Eu})</td>
<td>6.3(8)</td>
</tr>
<tr>
<td>(^{139}\text{Sm})</td>
<td>1.1(2)</td>
</tr>
<tr>
<td>(^{138}\text{Eu})</td>
<td>5.3(7)</td>
</tr>
<tr>
<td>(^{138}\text{Pm})</td>
<td>(3.2(9))</td>
</tr>
</tbody>
</table>

Table 8.2 Experimental measurements of the magnetic dipole moment for the
isotopes studied in this work.

In order to understand the underlying nuclear structure arguments
leading to the observed values for the dipole magnetic moments, particle-rotor model calculations were performed for all the isotopes shown in table 7.2, with the exception of $^{138}\text{Pm}$, whose value was tentative. In the case of $^{139}\text{Eu}$, extensive calculations were performed in order to obtain good agreement with the measured energies of the low-lying members of the ground-state rotational band, and magnetic moment. Good agreement was achieved, requiring prolate deformation ($\epsilon_2 = 0.27$), and axial asymmetry ($\gamma \sim 29^\circ$), the ground-state configuration being based on a weakly coupled $h_{11/2}$ proton. Similarly, calculations performed for the nucleus $^{139}\text{Sm}$ were also able to obtain good agreement with the low-lying isomeric rotational band properties, and dipole moment, requiring oblate deformation $\epsilon_2 = -0.2$, and $\gamma = 27.5^\circ$, and based on weak coupling of the odd $h_{11/2}$ neutron.

The large magnetic moment measured for $^{138}\text{Eu}$ enables the assignment of the $\{\pi 5/2^{-} \otimes \nu 7/2^{+} \}$ Nilsson two-quasiparticle configuration, following particle rotor calculations assuming axial symmetry. The strong Coriolis coupling of the odd proton and odd neutron to the deformed ($\epsilon_2 = 0.24$) rotating core appears to force the anti-parallel coupling of the proton and neutron intrinsic spin-projections in the ground-state, contrary to the normal Gallagher-Moszkowski coupling rules. The difficulty in obtaining good agreement with the magnitude of the dipole moment when using an asymmetry parameter similar to that of $^{139}\text{Eu}$ ($\sim 25^\circ$) may indicate the nucleus $^{138}\text{Eu}$ exhibits a lesser degree of triaxiality than its nearest neighbours.

It is hoped that future in-beam spectroscopic experiments will be performed with the aim of unambiguously identifying the ground-state band of $^{139}\text{Eu}$, in order to clarify the associated nucleon configuration and
deformation. However, there still remains a great deal of information accessible to the technique of LTNO. Dedicated experiments for the extraction of δ's have clearly been shown to be feasible. Further relaxation time-measurements with shorter beam pulse times, could provide accurate determinations for the two isotopes studied, where only a limit < 3s could be set.

In conclusion, the technique of low temperature nuclear orientation by virtue of the interaction between nuclear moments and the electromagnetic fields of a solid-state environment, has been shown to be a very powerful tool for probing the nuclear structure of nuclei far from stability, in addition to a broad base of solid state phenomena.
[BA90]. D. Bazzacco et al., Z. Phys. A 335 (1990) 363
[BL57]. R.J. Blin-Stoyle and M.A. Grace, in Encycl. of Phys Vol XLII (Springer-Verlag, 1957)
[BO58]. N.N. Bogoliubov, Soviet Phys. JETP 7 (1958) 41
[BO70]. H.H. Bolotin et al., Nucl. Instr. and Meth. 83 (1970) 1
[FA53]. U. Fano, Phys. Rev. 90 (1953) 577
[GN87]. V.R. Green et al., Nucl. Inst. and Meth. 26B (1987) 482
[GR63]. R.W. Grant and D.A. Shirley, Phys. Rev. L30 (1963) 1100
[GR87]. I.S. Grant et al., Nucl. Inst. and Meth. 26B (1987) 95
[KI81]. R. Kirchner et al., Nucl. Instr. and Meth. 186 (1981) 295

[KR72]. K. S. Krane, Nucl. Instr. and Meth. 98 (1972) 205


[LE78]. C. M. Lederer, V. S. Shirley, Table of Isotopes (John Wiley & Sons, Inc. 1978)


[LU85]. S. Lunardi et al., Z. Phys. A 321 (1985) 177


[ME61]. A. Messiah, Quantum Mechanics (North Holland, 1961)


[PU76]. F. Puhlhofer, GSI report (1976)


[RA89]. P. Raghavan, Atomic Data and Nuclear Data Tables 42 (1989) 189


[SO80]. R. J. Soulen and H. Marshak, Cryogenics 7 (1980) 408


[ST71]. N. J. Stone in Hyperfine Interactions in Excited Nuclei (eds
Goldring and Kalish, Gordon and Breach, 1971) 237


[VI81]. D. Visser, Ph.D. Thesis (University of Groningen, 1981)


[WI71]. M. Wiernik, Nucl. Instr. and Meth. 96 (1971) 325


