Numerical and Experimental Analyses of Large Composite Skeletal Satellite Systems.

by

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Abstract:

The World's ever increasing demand for communication capacity has been the catalyst for the development of a range of next generation satellite reflectors. This new generation are significantly larger than those currently in orbit. Their dimensions prohibit transportation into space in their operational configuration.

This thesis investigates the use of deployable tetrahedral trusses for the reflecting surface support structures of a 50 m diameter Land Mobile Communication System. A deployable structural system was selected ahead of other possible forms, as it satisfied the majority of the criteria of low part count, quick assembly times and an economical packaging volume thereby minimizing transportation and on-orbit fabrication costs. The composite material examined is formed from a polyethersulphone thermoplastic matrix reinforced with high strength, low modulus carbon fibres, C-PES.

The modal characteristics of a series of scaled sub-units of the proposed structure were examined. These units, manufactured from the two types of material considered, C-PES and Perspex, were subjected to a range of excitation functions. The C-PES units were thermally cycled under high vacuum to simulate the space environment. The accelerated thermal cycling of the composite structural units revealed that a shift in resonant frequency occurred, together with some surface cracking which could affect the long term stability of the material. The effects of surface pigmentation on the thermal response were also considered.

An algorithm has been developed which allows low cost materials to be used to predict the behaviour of geometrically similar units manufactured from the composite material.

The interaction between the predicted dynamic structural behaviour and the electrical performance of the satellite is also addressed and suggests that some form of active control system will be required if the maximum defocus parameter is not to be violated.
Many men fight for worldly goods
but,
few fight hard for knowledge.

To Barbara,

who has supported me fully throughout the years.
I owe her a great debt which I hope to repay.

(c) D.A.C.SPARRY 1992.
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1.0. Introduction.

The objective of the thesis is to demonstrate the principle of large deployable satellite reflector systems to provide land mobile communication facilities for Europe; specific design criteria have been examined by experimental and numerical methods. The experimental work is used to provide validation of the theoretical finite element models of the complete structure by making a step by step progression from small scale sub units of the structure through to near full size sub units, using both Perspex and a Carbon Fibre/Polyethersulphone (Carbon/PES) composite. The use of these two materials allows a study of the effect of material property changes on the dynamic behaviour of geometrically similar units.

The work considers a large tetrahedral truss structure. This configuration enables a near isotropic structural system to be obtained. The antenna is a high performance requirement structure, pointing accuracy, reflecting surface shape control, defocus and surface error parameters are tight, particularly as the structure is required to fold into an economic package for transport into space via either the Space Shuttle or Ariane. Once at Low Earth Orbit (LEO) it should be self deploying. This final requirement means that some technique for providing energy to the system must be incorporated into the structure, the most convenient position being at the structural joints of the system. An energy loaded joint has been developed for use with the structure under consideration (Hollaway 1987 [1]).

The aims of the work are to examine the suitability of the low cost material, perspex, for use in the dynamic testing of the structural models. To consider the implications of model size on the testing procedures adopted. To derive and validate a finite element modelling technique by using a series of scaled sub-units such that a confident estimation of the response of the proposed structure, which cannot be tested in a 1 G environment, can be made (Wijker 1991 [2]). To derive an experimental testing procedure for the ground based testing of a series of scaled sub-units manufactured from both a low cost material and the proposed
carbon reinforced thermoplastic in a series of test environments, including high vacuum with thermal cycling.

A polymer composite was selected for the material to form the truss. The material provides a high specific stiffness. The high technology thermoplastic polymer polyethersulphone reinforced by graphite fibres was chosen because of its advantages over the thermosetting systems particularly in respect to vacuum and combined irradiation and thermal cycling resistance.

The testing techniques employed on Earth are required to simulate, as closely as possible, the environment of space. This presents the researcher with a classical problem in the dynamic testing of structures. The model is supported in a 1-G environment whilst trying to simulate a 0-G situation. This requires that the structure has unconstrained boundary condition, clearly a situation that cannot be fully achieved but with careful implementation of a suitable suspension system the constraints can be minimized. This is particularly important when testing a joint dominated structure such as the tetrahedral truss system under consideration.

1.1 General Performance Requirements.

There has been a dramatic increase in the demands for communication capacity over the last decade which has resulted in the development of new technological solutions. The use of high altitude satellites to provide communication links is not new but, the concept of very large reflectors being used to provide communications for the mass market is still in its infancy. To provide near global coverage three satellites placed in a Geostationary Orbit (GEO) would be required, but a single reflector could provide coverage over large populated areas such as Europe. It is anticipated that satellite antennas will increase in size to form a new generation of Large Space Systems (LSS). This presents a problem with the transportation of these reflectors into orbit. The current capacity of the available launch vehicles limits the maximum dimensions
to around 18 m by 4 m (Wade 1986 [3]). Compact packaging is desirable as in many cases the volume, rather than the weight, dominates the transportation costs of these large lightweight reflector systems. A system which allows a reflector to be 'packaged' down into a payload that can be accommodated within the current launch vehicles is therefore required. There are a number of reasons for using very large reflectors as the space component of a communication system. The larger the reflector the narrower the beam it produces, which results in a high gain over a small area. This enhanced gain allows the use of smaller ground based transmitters, which is of particular use in the field of mobile communications. The narrow beam width allows the use of frequency re-use techniques which results in more traffic being carried for a given bandwidth. The large reflector also allows for shaping of the beams to take place thereby reducing overlap and hence interference. High pointing accuracy becomes an important requirement as the beam width narrows. By utilizing very large antennae in space, at GEO, the ground based components can be both small and of low cost, particularly as no tracking system is required as the satellite is always above the horizon.

Currently one of the largest satellite reflectors flying in GEO is the 9.14 m diameter parabolic reflector of the Applications Technology Satellite 6 (ATS-6) (Wales 1981 [4]). The diameter of the proposed antenna for this project is nearly six times this size at 50 m. The ATS series demonstrated the feasibility of many techniques that are common place today as well as the use of large antennae in space. More typical diameters are in the range of one to three metres but recent developments, including SAT-2, are increasing the size obtainable to between five and 15 metres (Berasconi 1989 [5], Takano 1989 [6]).

Many methods for achieving packaging and subsequent deployment or erection of the satellite have been suggested (Freeland 1978 [7], 1984 [8]). Large satellite reflectors can be described as members of one of the following three categories:
Deployable,
with three sub-classes of:
  Sequential deployment.
  Synchronous deployment.
  Assembly of deployable sub units.
  Erectable.
  Fabrication and assembly in orbit.

Each of these systems have a series of desirable and less desirable qualities associated with them (Mukulas 1985 [9]). Within each of these categories there are a number of different concepts of how to achieve the desired large reflector. Such techniques include structures being made up from trusses, both tetrahedral and box types, inflatable members and dishes, structures employing a series of members and cables and structures made up from a series of rigid panels (Card 1980 [10]).

Considering the erectable class of structures first, these types of structures have not been considered suitable for use in this project because of the very large number of members required to build up the truss. Their nature is such that they require a very large number of Extra-Vehicular-Activity (EVA) man-hours in order to construct the satellite. This combined with the type of truss being considered, which could present access difficulties, is felt to out weigh the considerable packing density advantage that they can possess. Some success have been achieved in this area particularly with the construction of beam sections formed from deep trusses. On-orbit tests using EVA techniques have provided some positive results, including the 15 m triangular truss tower constructed in 1985 (Klein 1986 [11]). The beam sections of the Space Station Freedom are to be formed from a 5 m deep truss constructed using the erectable technique (Mukulas 1990 [12]).
Space fabrication of the members forming the truss and their subsequent assembly whilst in orbit is possible. The characteristics of the thermoplastic composite, Carbon-PES, being considered for the project, would lend itself to an on-orbit manufacturing technique. However, it is not considered a feasible option at this time because of the complexity and high power requirements of such a production plant. For these reasons together with the potential for substantial EVA this class of large antenna systems have been rejected.

The deployable class of antenna was therefore considered. It was found that it satisfied the considerations of a low on-orbit part count and assembly time together with an acceptable launch package volume. This class of structures is not perfect for use in space because of the large number of joint mechanisms that such structures possess; these joints are required to function reliably in a hostile environment. This is an area which is currently receiving a considerable amount of research interest by other workers in the field. A number of researchers have advocated the use of one of the 'truss' systems proposed for large antenna concepts (Andersen 1985 [13]). The truss systems have a number of desirable attributes which lend themselves for use in large antennae, these can be summarized by the following points:

1. High structural stiffness due to the depth of the truss.
2. The large number of joints provide 'built-in' attachment points for connecting sub-systems.
3. The final truss can be easily sub-divided into smaller units which allows transportation difficulties to be readily overcome.
4. A very high degree of redundancy which will minimize the effect of the loss of one or more members due to unforeseen events.

The tetrahedral truss selected for this project is based upon the General Dynamics Convair (NASA 1982 [14]) system. The advantages of this system are; the higher packaging density which can be achieved over that of the box truss, proposed by Martin Marietta (NASA 1982 [15]) and the spatial diagonal stiffened
truss, (SDT) proposed by Fuji (Takamatsu 1991 [16]) and the considerable weight saved by using fewer members. The proposed structure, a 50 m diameter reflector is to be packaged into a cylindrical shape and then deployed by using energy loaded joints which are situated at the mid-points and ends of the top and bottom members. The proposed truss is suitable for a wide range of applications including earth resource sensing, telecommunications and astronomy as well as forming a multi-functional platform onto which modules could be mounted.

The characteristics of these three classes of large antenna reflectors are summarized in Table 2.1. The advantages of these various systems are summarized in Tables 2.2 and 2.3. A list of some of the various designs proposed, in each category, is given in Table 2.4.

1.2 Electrical Performance Requirements

Whilst it is not intended to give the theoretical background of the electrical performance requirements for the proposed reflector system references [17] to [22] (Rudge 1982 [17], Collin 1969 [18], Wright 1981 [19], Matthews 1976 [20], Miya 1986 [21], Mittra 1983 [22]) provide background information on the derivation of the following parameters:

- Frequency of operation: 1.3/1.5 GHz
- 50m diameter main reflector
- F/D ratio : 1.2/1
- 300 spot beams
- 5m sub-reflector
- Max member length : 2.25 m
- Max allowable surface error : 1.36 mm rms.
- Max linear/angular defocus within 100 mm sphere. (± 0.038°)
Typically in a tetrahedral truss reflector, a flexible radio frequency (RF) reflective surface, e.g., a woven gold plated molybdenum mesh, is held under tension in a parabolic form and supported by the truss. Any deviation away from the ideal parabolic surface results in a rapid fall off in the electrical performance of the antenna particularly as the frequency of operation increases. There are several possible causes for such deviations; the most important of which are the thermal and the dynamic distortions, the latter partly due to station keeping operations. Gravity gradients across the structure as well as fabrication tolerances also cause distortions in the system. Further constraints on the structure are radiation pattern purity and the need to maximize gain performance. In addition, the effect of aperture blockage caused by the feed system being positioned in front of the reflector and the dumb-bell effect caused by positioning two relatively massive bodies at either end of a long slender beam needed to be avoided. After considering these factors an offset gregorian type antenna was adopted. This type of antenna consists of a parabolic main reflector with a smaller hyperbolic sub-reflector and a separate feed system, Figure 1.1.

1.3 Material Performance Implications

To meet the electrical requirements the following mechanical properties of the material and of the structure are required:

**High stiffness:**
This reduces deflections and reflector surface contour errors due to gravity effects. It also minimizes the effects of structural vibrations caused during orientation (station keeping) manoeuvres and other dynamic disturbances.

**Good dimensional stability:**
This requires a very low coefficient of thermal expansion and a high thermal conductivity to reduce the deflections induced by the global and local thermal gradients experienced by the structure. This leads to reduced
RF scatter and a high gain which in turn allows for the use of lower power and smaller mobile earth stations.

**Rapid point and track capability:**
This requires that both the material and the structure as a whole have good damping capabilities, although it is possible to improve this by artificial means.

**Deployable/Erectable in space:**
This allows the structure to have a large aperture and to be of the offset type both of these attributes cater for the small earth stations envisaged. This is countered by the fact that three Space Shuttle flights would be required in order to assemble the structure at Low Earth Orbit (LEO) before it is transferred into its final operational orbit.

The development of new high technology carbon composite materials has meant that many of these requirements can now be largely met. However there are some areas of difficulty which have yet to be overcome. One particular area of concern is that of the operation or the design life of the antenna. At present with the cost of spacecraft being so high designers are reluctant to take risks with the electrical/computer control side of the antenna. This has meant that much of the technology presently flying is in fact positively antique by modern office standards, so whilst current papers are suggesting that structures should have a design life of 30 years it is questionable whether the electronics installed would still be useful in the next century. So whilst it is reasonable to design a space platform for that period this may not be the case for an antenna system. Refurbishment is a possibility and this may in fact take place particularly if developments in Space Tugs come to full fruition. The question of operational life also needs to be addressed in terms of the end of life power requirements of the antenna. Power is generated from the Sun's rays using solar arrays, these suffer degradation during their operational life so the arrays must be designed to allow for this.
Estimates vary on the likely losses but they may be as much as 50% over even a 10 year period.

Another area of particular concern when designing large space antennas is that of the structure's dynamic response. Very careful consideration must be given to the natural frequency of vibration of the antenna's components as well as its global response to any forcing frequency it might experience, e.g. manoeuvring the satellite into its final operational orbit. The use of a front fed reflector would aggravate the problem as the two masses would tend to oscillate around the axis of the main beam. A tripod system of support for the feed array could be employed but this would increase the effect of aperture blockage and/or RF interference caused by transmission paths from the power supplies. Damping characteristics also become vital as viscous damping is all but non-existent in space so good structural damping must be achieved. Active control of the relative positions of the two reflectors can also be used provided that the basic modes are not too low. For the structure under consideration a fundamental frequency in excess of 0.15 Hz is likely to be achievable which would allow such control systems to be employed.
### 1.4 Tables

<table>
<thead>
<tr>
<th>Class</th>
<th>Sub-Class</th>
<th>Characteristics</th>
<th>Initial/Orbit</th>
<th>Transfer Vehicle(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deployable</td>
<td>Synchronous</td>
<td>Simultaneous deployment of ALL structural elements.</td>
<td>GEO-GEO</td>
<td>STS + IUS. Ariane or Atlas.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>LEO-GEO</td>
<td>STS + low thrust</td>
</tr>
<tr>
<td>Sequential</td>
<td></td>
<td>Sequential deployment of modules/connected elements.</td>
<td>LEO-GEO</td>
<td>STS + low thrust</td>
</tr>
<tr>
<td>Modules &amp;</td>
<td>Two stages: Deploy</td>
<td>LEO-GEO</td>
<td>STS + low thrust</td>
<td></td>
</tr>
<tr>
<td>Assembly</td>
<td>Modules, Modules</td>
<td>assembly using EVA</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>assembled using EVA</td>
<td>to form final structure.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Erectable</td>
<td></td>
<td>Elements assembled using EVA</td>
<td>LEO-GEO</td>
<td>STS + low thrust</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fabrication in</td>
<td></td>
<td>Elements manufactured from raw materials. Elements</td>
<td>LEO-GEO</td>
<td>STS + low thrust</td>
</tr>
<tr>
<td>space</td>
<td></td>
<td>are assembled to form structure.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 1.1:** Characteristics of Classes of Large Antenna Reflectors.

<table>
<thead>
<tr>
<th>Class</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deployable Reflectors</td>
<td>Low Stowed Volume.</td>
<td>Complex with many moving parts.</td>
</tr>
<tr>
<td></td>
<td>Low Relative Mass.</td>
<td>Pointing accuracy</td>
</tr>
<tr>
<td></td>
<td>Surface Adjustable.</td>
<td>Surface accuracy</td>
</tr>
<tr>
<td>Inflatables</td>
<td>Very low stowed volume.</td>
<td>Difficult to cure evenly on orbit.</td>
</tr>
<tr>
<td></td>
<td>Very low relative mass.</td>
<td>Non adjustable surface.</td>
</tr>
<tr>
<td>Solid Reflectors</td>
<td>Pointing accuracy.</td>
<td>Max. dimensions</td>
</tr>
<tr>
<td></td>
<td>Surface accuracy.</td>
<td>set by launcher</td>
</tr>
<tr>
<td></td>
<td></td>
<td>High relative mass.</td>
</tr>
</tbody>
</table>

**Table 1.2:** Comparison of General Classes of Large Antenna Reflectors.
<table>
<thead>
<tr>
<th>Class</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Truss Based</td>
<td>Adjustable mesh. Feed offset easy. Relatively low mass High Stiffness.</td>
<td>Complex, large number of joints. May require multiple launches for very large apertures.</td>
</tr>
<tr>
<td></td>
<td>Easily adapted basic design. Very wide range of apertures available.</td>
<td></td>
</tr>
<tr>
<td>Umbrella Based</td>
<td>Adjustable mesh. Adaptable design.</td>
<td>Central support possible aperture blockage. Relatively high mass. Aperture range limited by stored length of central support.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wrap-Rib Based</td>
<td>Very large apertures</td>
<td>Mesh not adjustable.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hoop-column Based</td>
<td>Adjustable mesh Very large apertures</td>
<td>Centre feed only. Mission unique.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflatables</td>
<td>Very light weight Very low stored volume</td>
<td>Difficult to cure evenly whilst in-orbit. Reflective surface not adjustable.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solid</td>
<td>Pointing accuracy Surface accuracy</td>
<td>Maximum dimensions of Reflectors imposed by launcher. Relatively high mass.</td>
</tr>
</tbody>
</table>

Table 1.3: Comparison of Specific Classes of Large Antenna Reflectors.
<table>
<thead>
<tr>
<th>Classes</th>
<th>Concepts</th>
<th>Manufacturer/Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple Continuum</td>
<td>TRW Advanced Sunflower</td>
<td></td>
</tr>
<tr>
<td>Petal or Panel Structures</td>
<td>Precision Deployable Antenna.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Dornier Daisy Precision</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Deployable Antenna.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Lockheed High-Frequency</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Radial-Rib Antenna.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Deployable Solar Concentrator</td>
<td>Fuji Heavy Industries.</td>
</tr>
<tr>
<td></td>
<td>(DSC)</td>
<td></td>
</tr>
<tr>
<td>Frames, Members</td>
<td>LMSC Wrap-Rib Antenna.</td>
<td></td>
</tr>
<tr>
<td>and/or Stringers</td>
<td>JPL Radial-Rib Antenna.</td>
<td></td>
</tr>
<tr>
<td>Synchronous Deployment Trusses</td>
<td>GDC Paraboloidal Extendable Truss</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Antenna (PETA).</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fuji Heavy Industries Spatial</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Diagonal-Stiffened Truss (SDT)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Aerospatiale Parabolic Truss</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Antenna</td>
<td></td>
</tr>
<tr>
<td>Inflatable Membranes</td>
<td>Contraves Inflatable Space</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Rigidised Structures (ISRS)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ESA QUASAT.</td>
<td></td>
</tr>
<tr>
<td>Membrane</td>
<td>Kawasaki Heavy Industries (MIR)</td>
<td>Modularized Inflatable Reflector.</td>
</tr>
<tr>
<td>Spin-Stiffened Reflector.</td>
<td>LMSC Spin-Stiffened Membrane</td>
<td></td>
</tr>
<tr>
<td>Electrostatically Configured</td>
<td>MIT Electrostatically</td>
<td></td>
</tr>
<tr>
<td>Membrane</td>
<td>Figured Membrane Reflector.</td>
<td></td>
</tr>
<tr>
<td>Precision Panels</td>
<td>ARC Sequentially Deployed Sequential</td>
<td></td>
</tr>
<tr>
<td>Deployment Meshes</td>
<td>ARC Sequentially Deploying</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Truss with expanding mesh.</td>
<td>Martin Marietta Expandable</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Box Truss Reflector.</td>
</tr>
</tbody>
</table>

Table 1.4: Summary of Conceptual Class Designs.
<table>
<thead>
<tr>
<th>Classes</th>
<th>Concepts</th>
<th>Manufacturer/Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deployable Modules with Part Assembly</td>
<td>LMSC Deployable Module Concept (DCM).</td>
<td></td>
</tr>
<tr>
<td></td>
<td>GDC Deployable Cell Module.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>GDC MOD-PETA Concept.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>GDC Modular Extendable Truss Antenna (META).</td>
<td></td>
</tr>
<tr>
<td>Erectable</td>
<td>NASA Tapered Nestable Column Concept.</td>
<td></td>
</tr>
<tr>
<td>Fabrication &amp; Assembly on Orbit.</td>
<td>Marshall Space FC-Grumman Beam Builder.</td>
<td></td>
</tr>
</tbody>
</table>

Table 1.4 (cont.): Summary of Conceptual Class Designs.
Figure 1.1 General Arrangement of Proposed Satellite.
2.0 Review.

2.1 Introduction.

The ever increasing need for quicker, better, yet cheaper communications has led to the development of large space systems aimed at the mass communication markets. Such systems draw upon the knowledge of many specialist areas, in fact they have led to the development of whole new classes of specialisations being formed in structural, mechanical, material and electrical engineering. The development of these systems requires the use of many of the most sophisticated experimental and analytical techniques known to man.

2.2 Space Environment

The proposed structure, a skeletal satellite weighing around 4000 Kg and overall dimensions of 50 by 75 metres, would ultimately be placed in an equatorial geosynchronous orbit (GEO). However because of its size and complexity it would be initially assembled and tested at low earth orbit (LEO) prior to being transferred to GEO. This constraint of assembly at LEO and subsequent transfer to GEO has severe implications upon the material property requirements of any candidate material. As a result of this and the extended service life of the structure the development of in orbit non-destructive tests will be required as little experience of materials in the space environment currently exists (Forli 1990 [23]). The mass of the proposed structure would allow its injection into LEO in a single launch from either the Space Shuttle or the Ariane launcher. However, the structure's stowed volume, despite a relatively efficient packing system, is such that multiple (3+) launches would be required. Two of these would transport the structural components, whilst the third would inject the power supplies and signal conditioners into LEO. Currently the turn around times for the Shuttle are such that some components of the structure would be parked in LEO for some months prior to final assembly and subsequent transfer to GEO. It will therefore be necessary to take into account the loads applied to the members during the various stages of the structure's life in both its packaged and
deployed states. These various stages may be broken down into the following stages:

1) Launch loads whilst in a packaged form.
2) Deployment in a packaged state from the cargo bay into LEO.
3) Deployment at LEO of the skeletal structure
4) Fabrication of the satellite from its constituent parts.
5) Transfer to GEO.

and
6) Operational life including the likely final re-positioning out of a valuable GEO slot at the end of its useful life.

It will also be necessary to account for the effect that the particularly hostile environment at LEO, where the atomic oxygen concentrations are significantly higher than at GEO, has upon the material and its abilities to meet the imposed loads over a projected thirty year lifespan (Leger 1987 [24]). This lifespan may well be over ambitious as the likelihood that the electronics packages installed on the structure being operational and in a useful form over such an extended period must be open to debate. Re-furbishment of the structure could provide a means to prolong its useful life but this would require either EVA at GEO or some method of recovery to a lower orbit. Additionally it would require that some kind of modular connection be formed between the structure and the electrical packages, such a 'plug and socket' arrangement could prove to be a source of considerable electrical noise, especially considering the hostile environment.

The structural units undergo a series of loading conditions during their lifetime, the sources of the induced stresses expected in the members fall broadly into the following categories:

1) Packaging and stowage
2) Launch
3) Deployment and fabrication at LEO
4) Transfer to GEO
Within each of these main areas there are various factors which give rise to these induced stresses. The use of energy loaded mid-joints on the face members of the tetrahedral truss will require that some form of lateral restraint is applied to the structure in order to prevent its premature deployment. The structure will experience considerable self weight loads whilst on the Earth and yet have this load relieved during injection into LEO. Once at LEO the structure will undergo a shock loading induced by deployment. All these areas require detailed investigation for an actual structure. For this reason these areas have not been rigorously investigated at this stage. Thermally induced stresses obviously will also play a major role in any loading regime for a satellite. Such stresses can be continuously varying as is the case with self-shadowing or be periodic as in the case of the vernal and autumnal eclipses (Annandale 1986 [25]) in which very dramatic and rapid changes can occur. In LEO the orbital period is approximately 90 minutes and assuming a life expectancy of thirty years the structural material would receive 17,500 cycles. The temperature range during each cycle would be typically -60°C to +75°C depending upon the physical properties of the material. In GEO the typical temperature range would be between -150°C and 80°C, again dependent upon the physical properties of the material. The severe thermal shock that the material and structure are subjected to during such periods must be catered for, so that the dimensional stability of the antenna reflector system is sufficiently high for it to remain operational. Probably the most critical test of the dimensional stability of the structure would be when the sun's rays are parallel to the reflecting surface of the structure in which case a satellite of fifty metres width would develop a temperature difference of about 80°C.

At LEO the structure is exposed to a hostile combination of atomic oxygen (Leger 1982 [26], 1983 [27], 1984 [28]) and high energy particles imparting energies in the MeV ranges as well as impacts from various forms of space debris, both man made and natural. The atomic oxygen problem was not fully appreciated until the shuttle was utilised for launches and since then a facility has
been developed in the USA to specially simulate atomic oxygen and to research into its effects as there appears to be no correlation between particle energy and damage (Gregory 1985 [29], Peters 1986 [30]).

Further consideration needs to be given to solar activity levels as atmospheric effects can cause a decay in the orbit of a structure even at the relatively high LEO orbits between 400 and 500 kilometres (Sanders 1989 [31]). At this altitude evidence of attack by atomic oxygen was found on surface samples recovered from the Solar Maximum spacecraft in May 1984 (Fristrom 1985 [32]) only four years after the spacecraft had been launched into a 500 km orbit in February 1980. The levels of erosion varied between 0.5% and 30% of initial thickness. The Shuttle currently normally operates around 300 kilometres. It would be anticipated that deployment and assembly would take place during a period of solar quiescence. Transfer of the assembled structure to GEO would in itself exert stresses in the members. The design of a suitable propulsion method by which this transfer could be achieved has yet to be finalised. A brief outline of a possible alternatives is presented in the section on propulsion systems. Having successfully traversed the Van Allen belts during the transfer from the assembly orbit to GEO a further set of environmental conditions will apply. At GEO the structure will no longer be subjected to attack from atomic oxygen but it will be exposed to the effects of solar flares and the high energy electrons and protons, collision with which not only causes immediate local damage on the member but also secondary damage by the release of Bremsstrahlung radiation (Maiden 1985 [33]). At GEO the material of the satellite will be exposed to high energy particles and solar wind. The latter is a hydrogen plasma which is a mixture of positively charged atoms or molecules and negatively charged electrons travelling at speeds of 400km/s. The radiation environment at this orbit is primarily electrons and protons with minor particles such as cosmic rays, gamma rays, x rays, neutrons and higher z particles, all of which have negligible effects on the materials of the structure despite the very high energies that some of these rays possess (Muller 1989 [34]). Electrons in GEO have a broad energy range of 100KeV to 6MeV producing both surface and bulk property changes in materials up to a thickness
of 10mm; the energy range of the protons is up to 2MeV. These interact primarily through ionisation, which affects surface properties only (Maiden 1985 [33]). From the GEO dose-depth curve, the ionisation dose absorbed by a lightweight structure, such as an antenna, is $10^6$ rads for a thirty year life (Sykes 1985 [35]). The intensity of the solar wind is a more variable quantity and is dependent upon solar activity (Glasstone 1965 [36]). Such attacks can all significantly reduce the structural integrity of the satellite over an extended period of time. Extensive work has been carried out into methods of reducing some or all of these effects. The use of some inert fillers in the outer layer of the composite (King 1987 [37]) or the application of a fine metallic coating to the members (Hollaway 1990 [38]) have been suggested and tested (Whitaker 1985 [39]). Such treatments whilst adding to the cost of fabrication, considerably enhance the material's thermal characteristics and provide the necessary protection from atomic oxygen attack whilst in the assembly stage at LEO. (Pippin 1989 [40]) identifies the main requirements for long life structures in the more hostile LEO.

It is a well known fact that the Earth has a significant magnetic field. As a rough approximation this magnetic field is equivalent to the field that would be produced if a magnetic dipole, such as a bar magnet, of suitable strength was contained within the Earth's interior. The axis of this hypothetical magnet does not coincide with the Earth's geographical North and South poles, it is displaced some 400 Km from the Earth's centre. As a result the geomagnetic field in space is not symmetrical with respect to the Earth's surface. This feature coupled with the solar wind, the emission of solar plasma produces a very restricted geomagnetic field on the daylight side of the Earth, of the order of 10 Earth radii, with a much larger "tail" on the dark side. The discovery of the Van Allen Belts, (Van Allen 1959 [41]) areas of electrically charged particles, in 1958, following the launches of Explorer I and III was not wholly un-anticipated but was still a surprise to many. Many previous experiments (Stormer 1904 [42] and Alfven 1950 [43]) had been carried out to demonstrate that a magnet field can confine electrically charged particles. In an experimental simulation of the phenomenon, Bennett (Bennett 1938 [44]) allowed a stream of electrons to impinge on a sphere
with a magnet field resembling that of the Earth's. Some of the patterns produced bore a striking resemblance to that currently accepted for the Earth's radiation belt.

As a consequence of the magnetic field lines being distorted by the solar winds, particles can be released and this sets an upper altitude limit to the Van Allen belt. An increase in solar activity which leads to an increase in the density of the solar plasma (solar wind) can lead to compression of the upper normally undisturbed regions of the belt. As a consequence of this the Van Allen belts lie approximately 75° N and 75° S on the sunlit side of the Earth and between 70° N and 70° S on the dark side, i.e. a doughnut like, or toroidal, shape is formed.

It has become apparent from continued studies that there is actually only one radiation belt, but within this region both positively and negatively charged particles are trapped. However, when the energy distribution of the charged particles is considered it becomes obvious that there must be at least two distinct regions, one for the protons and one for the electrons. In comparing protons and electrons it is obvious that all other things being equal the gyromagnetic radius of a proton will be 1840 times that of an electron since this is the ratio of their masses. The gyromagnetic radius of a particle can be calculated from the formula (Glasstone 1965 [45]):

\[ R = \frac{MV_p C}{eB} \]

where

- \( M \) = mass of the particle
- \( V_p \) = velocity component perpendicular to the magnetic field lines.
- \( C \) = velocity of light in a vacuum.
- \( e \) = electric charge.
- \( B \) = magnetic field strength.
The maximum flux for these regions occurs at about 3,200 and 16,000 Km above the Earth's surface for protons and electrons respectively.

As far as can be ascertained the inner zone consisting of high energy protons is symmetrical about the Earth's axis, i.e. it is not affected by the solar winds. The energies of the protons in this inner zone lie within the range 20 to 40 MeV with a typical value being about 30 MeV. This corresponds with a particle velocity of $7.6 \times 10^9$ cm/s. However, it must be realised that the total number of very high energy particles is small and more over, they constitute a small proportion of the total number of protons present in the inner zone.

The outer zone consists of high energy electrons with a typical value of 1.6 MeV. The electron velocities are close to that of light, that is $3 \times 10^{10}$ cm/s. The composition and dimensions of this outer belt are, as has been stated, subject to marked fluctuations especially during the periods of exceptional solar activity. In between and surrounding these two areas of high flux there is a substantial region occupied by moderately energised protons and electrons (Davis 1962 [46]), hence the statement that there is only one belt around the Earth.

When the satellite is taken from LEO to GEO, sensitive areas such as the solar arrays and electronic equipment have to be protected from the effects of the high energy charged particles. Recent experimental data has shown that the electron plasma waves occurring locally around the Earth's bow wave are highly time variable and broad band in their nature (Treumann 1986 [47]). The choice of transportation method will have a direct effect upon the amount of damage that the structure could be expected to suffer during the orbit raising manoeuvre from LEO to GEO.

2.3 Choice of materials

The choice of materials for the manufacture of large satellite reflectors and space stations are limited, one of the main mechanical property
requirements is that they should possess low specific weight and from a physical point of view they are required to resist the hostile environment which has been discussed in the previous section. Currently aluminium and carbon/epoxy composite materials are used mainly as structural or semi-structural components for units of spacecraft. Both materials have advantages and disadvantages; the former has the advantage of a greater resistance against the bombardment of the high energy particles and attack of atomic oxygen but has the disadvantage of a relatively high specific weight. The latter has the advantage of lightness and the possibility of a refined material design procedure in that the material can be designed to provide a near zero coefficient of expansion in addition to achieving a high stiffness and strength in the required directions. However, the composite has not the same physical resistance to the above stated exposure that the aluminium possesses.

The introduction of the new generation of thermoplastic polymers during the last decade and their effects on engineering practise have been well documented (Cogswell 1989 [48], Leach 1986 [49]). These materials were hailed as the structural material of the 1990’s. Such thermoplastics include Polyethersulphone (PES) and Polyetheretherketone (PEEK). These polymers offer better impact toughness, an improved resistance to the effects of thermal cycling and vacuum and much better jointing characteristics than those of thermoset polymers (Beevers 1990 [50], Brown 1990 [51]). Most of the advantages that the thermoplastics possess over thermosetting polymer composites can be traced to the differences in their chemistry. Thermoplastic polymers consist of long chains of repeating molecular units (monomers) with a fixed chemical structure. This endows them with the characteristics of being fairly elastic and chemically inert. When subjected to heat, they soften and can be easily shaped or molded. Thermosetting polymers are chemically complex materials that cure into a solid resin by crosslinking of the chains to one another. Heat is used as a catalyst to the reaction process, during the course of which volatile fractions are liberated and can become entrapped within the matrix forming weak spots and finally outgassing under on-orbit conditions. The nature and the spread of such impact
damage is not yet as well understood as it is in metals, where the flaws can be more readily observed. Laminates, due to their construction, may well be flawed but the damage hidden, however this again is another area in which much research effort will be required.

2.4 Material Performance Requirements

The following mechanical properties of the material and the structure are required in order to meet the environmental and operational requirements of such systems:

i) **High stiffness;** this reduces deflections and reflector surface contour errors due to gravity effects. It also minimizes the effects of structural vibrations caused during orientation (station keeping) manoeuvres and other dynamic disturbances.

ii) **Good dimensional stability;** this requires a very low coefficient of thermal expansion and a high thermal conductivity to reduce the deflections induced by the global and local thermal gradients experienced by the structure. This leads to reduced RF scatter and a high gain which in turn allows for the use of lower power and smaller mobile earth stations.

iii) **Rapid point and track capability;** this requires that both the material and the structure as a whole have good damping capabilities, although it is possible to improve this by artificial means.

iv) **Deployable/Erectable in space;** this allows the structure to have a large aperture and to be of the offset type; both of these attributes cater for the small earth stations envisaged. This is countered by the fact that three or more Space Shuttle flights would be required in order to assemble the structure at Low Earth Orbit (LEO) before it is transferred into its final operational orbit.
In addition the material would be required to have minimal out-gassing levels and combine atomic oxygen, radiation and fatigue resistance to ensure structural integrity during the system's operational lifetime. Both of the aforementioned thermoplastics possess the above mentioned properties but to a greater or lessor extent. PEEK exhibits a lower absorption rate, and therefore will out-gas less than PES. However the out-gassing levels of PES lie well within the acceptable tolerance limits (Private 1988 [52], Haskins 1980 [53]). The carbon-PES material has been selected for the investigation because a fabrication process currently exists by which 25mm external diameter tubing can be readily manufactured, as opposed to the best that can currently be achieved using a carbon-PEEK prepreg which is a 50mm diameter tube (Curran 1988 [54]). The use of such large diameter members would result in a relatively inefficient stowed package as the cross-sectional area of the launch configuration is proportional to the square of the member diameter, for a given structure. This loss of usable volume would further restrict the overall dimensions that the deployed system could achieve. In both manufacturing techniques the film stacking process (Philips 1980 [55]) is used and the prepreg is wrapped around an expandable mandrel. The PEEK prepreg has proved to be too brittle to be wound around a tighter radii than that provided by a mandrel of at least 50mm diameter. The use of pultrusion techniques may in the future allow the production of smaller diameter members in either PES or PEEK, this would be particularly of benefit to the use of PEEK. However, it is almost certain that which ever thermoplastic composite is selected for use in space it will require a final coating of some description, the film stacking process may therefore prove to be the most suitable. PES would also benefit from it's lower processing temperature if an on orbit production facility became available.

2.5 Propulsion Systems

2.5.1 Introduction

Since the time that practical space flight began the cost of placing both man and machine in space has been enormous. This was because most of the
early efforts were concentrated on multi-stage expendable chemical propulsion units. This technology has been developed throughout the last decade and now reusable systems exist e.g. the Space Shuttle, and there is the promise of further developments during the present decade, e.g. Hermes.

Changing economics have dictated that such vast expenditures, which were previously tolerated can no longer be so, and if large scale industrialization (Proceedings 1976 [B1], Proceedings 1989 [B2]) of space is to be achieved then significantly cheaper methods of propulsion must be found. The re-usable Shuttle fleet have considerably reduced the costs in relation to the Earth to LEO transfer, however, a very large and increasing proportion of the envisaged missions will require transportation to GEO, or to highly elliptical orbits or escape trajectories. At present, work is being undertaken to develop systems capable of executing this task especially with reference to the very large satellite systems (LSS) under consideration, which present a series of conflicting requirements. There is some controversy as to the most appropriate type of propulsion system that should be employed for the near Earth orbit transfer missions, (Davis 1982 [56], Chase 1981 [57], Rehder 1974 [58]).

2.5.2 Theory of Rockets

It is not intended, nor is it within the scope of this thesis, to give an exhaustive account of the theory of rocket propulsion. Instead a basic outline will be given to enable the reader to grasp the principles behind rocket propulsion systems so that the effects experienced by the structures under examination may be better understood. There are many dedicated texts written on the subject and the reader is referred to these for a more detail explanation (Stulinger 1964 [59] and Fearn 1982 [60]).

From the conservation of momentum the following equation can be set up:
where \( M_R \) and \( V_R \) are the mass and the velocity of the rocket respectively and \( M_p \) and \( V_p \) are the mass and the velocity of the propellant exhaust.

In principle either the mass or the velocity of the propellant exhaust would be varied in order to obtain an increase in the velocity of the rocket. However, in practice these are generally fixed for a particular system. Different propulsion systems will give different exhaust velocities and obviously the higher this velocity the lower is the propellant mass required to obtain a given velocity change.

The mass of propellant required for a given mission has a very large effect on the overall mass of the rocket, hence the exhaust velocity is a very important quantity and is used to determine the specific impulse \((I_{sp})\) for a given propulsion unit.

\[
I_{sp} = \frac{\text{Thrust}}{\text{Propellant mass flow rate}} \quad \text{(seconds)}
\]

This value \( I_{sp} \) is used to indicate the relative merits of a propulsion system, in general a system with a 'high' \( I_{sp} \) is better than a system with a 'low' \( I_{sp} \), however, there are other factors to be taken into account. Since propellant is burnt continuously during an engine burn the overall mass of the rocket decreases in the following relationship:

\[
\frac{M_f}{M_o} = e^{-\frac{dV}{g I_{sp}}}
\]

where \( M_f \) = mass of the rocket after burn
\( M_o \) = mass of the rocket before burn
\( dV \) = velocity change
\( I_{sp} \) = specific impulse of the system
\( g \) = gravity constant
This value of $M_r$ is derived from two main components, the final delivered payload and the dry weight of the propulsion system. Again different propulsion systems will have different dry weights.

The thrust to weight ratio of a particular system must also be considered, to give reasonable acceleration a launch vehicle would require a ratio of about twice the local surface gravity. Hence for an orbit transfer from LEO to GEO the thrust to weight ratio can be considerably smaller than that required for the initial raising to LEO from the Earth. This allows the use of a larger number of potential propulsion techniques, which can be broadly categorised under the labels chemical systems or electrical systems.

### 2.5.2.2 Chemical Systems

There are essentially two types of chemical propellant available, either solid or liquid. Solid propellants have been used for many years and have probably reached a stage at which little performance improvement can be expected to be made. Considerable efforts were made into the investigation of such fuels in connection with their military applications during the 'Cold War' period. Such propellants characteristically have low $I_{sp}$ (typically 200-300 s), but they gain considerable favour in other ways. Their dry weight is low, around 10% of the propellant mass, they are considerably safer and simpler to store and handle than liquid propellants and they do possess a high density $I_{sp}$. One such orbit transfer vehicle under development is the Inertial Upper Stage (IUS) (Ketchum 1979 [61]) which is Shuttle compatible.

Liquid propellants can deliver higher $I_{sp}$ than solid fuels, e.g. the Shuttle main engines produce an $I_{sp}$ of 460 s. They also can be throttled to provide variable thrust and they can also be re-fuelled, a feature that is not available for solid propellants, making these systems re-usable, an important consideration in the future. The added complexity of allowing re-fuelling to take place increases the dry weight of such systems to typically 15% of the propellant mass. Liquid propellants
do have increased safety problems over that of solid fuels, e.g. the Shuttle disaster, Nov. 1984. It is also possible, by the use of an exotic cocktail of propellants to raise the \( I_{sp} \) e.g. \( O_2/H_2 \) achieves an \( I_{sp} \) of 500 s. An example of such a system is the Orbiter Transfer Vehicle (OTV) (Heald 1981 [62]) developed by NASA for a wide range of missions, including the transfer of very large satellites, manned and unmanned servicing at GEO as well as, and perhaps controversially, extraterrestrial nuclear waste disposal. These OTV's are based on the current Centaur system (\( I_{sp} \) 445 s) and are expected to be able to deliver to GEO payloads of more than double that achievable by an IUS system. The OTV is re-usable and may include an aero-braking device which will greatly increase it's return from GEO capabilities.

2.5.2.3 Electrical Systems

Electrical propulsion systems use electrical energy to accelerate a propellant to produce the thrust, as opposed to chemical systems which make use of thermal energy. The electrical power required can be generated from one of two sources, either from solar power, in which case the propulsion system is known as a Solar Electric Propulsion Stage (SEPS) or from a nuclear reactor generically known as Space Reactor Power Systems (SRPS) (Jaffe 1988 [63]). The most recent publicized launch of a nuclear powered satellite is the Sun probe Ulysses, which uses a nuclear generator containing 24 lbs of plutonium (Hawkes 1990 [64]).

The use of electric and electromagnetic forces allows the charged propellant atoms or ions to be accelerated to much higher velocities than that which may be achieved by combustion. Hence electrically powered systems can achieve \( I_{sp} \)'s of between 1000 and 20000 s (Fearn 1981 [65]).

Ion engines have been under development for over twenty years, and despite a chequered stop start development are being incorporated in plans for missions in the early 1990's (Martin 1989 [66]). SEP systems have been developed to use 30cm thrusters with an \( I_{sp} \) of around 3000 s. Larger thrusters developing a higher
thrust and an increased $I_{sp}$ are under development but will require the use of nuclear power to provide the required electrical power (Truscello 1984 [67]).

One of the propellants most frequently suggested for use in electric propulsion systems is mercury. It has a high density and is easily stored, however it does have several drawbacks not least of which is it's tendency to amalgamate easily with many metals and to coat optical devices. It is also necessary to vaporize the propellant before accelerating it's ions otherwise it would breakdown the high voltage insulation, shorting out the power supplies and causing an engine failure. It is because of these tendencies and requirements that the search for other propellants was started.

Interest is currently centred on Xenon as it is non-contaminating and non-reactive. It does not condense on spacecraft components so it simplifies the design of the power supplies as the need for heaters etc are removed. A wide range of papers on SRPS and nuclear technology are available (Proceedings [B3]).

The overall power requirements of the mission will largely dictate the type of electric propulsion system selected. It is envisaged that by the end of the century power units will be producing 1 MWe. Solar arrays are effectively limited in the power that they can deliver because the greater the power requirement, the larger their area and mass becomes. Even low power requirements of 100 KWe would require more than 2,300 m$^2$ of solar cells assuming an efficient system producing 45 W/Kg and assuming 0.95 Kg/m$^2$. The consequence of this are that the spacecraft have high moments of inertia, making them difficult to control and considerably increasing their complexity particularly in respect of the deployment of the solar arrays themselves.

Solar arrays can also suffer severe degradation whilst moving through the Van Allen belts, in which as much as 30-50% may be lost due to radiation damage (Weddell 1978 [68], Private 1989 [69]), however, it is possible to protect the solar arrays to reduce the damage sustained by increasing the thickness of the
protective glass layer but with a resultant loss of efficiency (Dailey 1982 [70]). The use of Galium-Arsenide (GaAs) solar cells as opposed to the more usually employed Silica cells would also reduce the effects due to radiation damage as GaAs solar cells can tolerate a higher density of defects due to radiation (Knechtil 1984 [71]).

There are two basic forms of nuclear propulsion available, firstly nuclear thermal rockets in which thermal energy is used to heat the propellant, frequently hydrogen because of its low molecular weight. This technology is fairly well understood as work started on a program in the late 1940's by the U.S. Air Force to develop a nuclear powered, supersonic, long-range bomber led to various spin off technologies. This work continued throughout the 1950's but was eventually shelved as the emphasis moved over to intercontinental ballistic missiles (ICBMs). One of the spin offs lead to work being carried out on a nuclear powered ICBM which was taken over by NASA for use in the Saturn V rocket. The rocket programme as a whole was ended in 1972 in favour of development of the Space Shuttle and because the earlier planned manned mission to Mars was cancelled. However several successful ground based test firings had been performed. Work was revived in the early 1980's and continues with the SP-100 program. Such systems with high thrust levels and a high $I_{sp}$ do offer considerable promise.

The other main type of nuclear powered propulsion system uses electrical power generated to accelerate ions in the same way that solar generated electrical systems do, albeit at greater thrust levels.

Nuclear reactors in space are a sensitive subject, particularly in the current climate of environmental friendliness (Buden 1984 [72]). Since the re-entry of the Soviet nuclear powered satellite Comos 954, guidelines for their use have been laid down which have performance implications on the Shuttle. The current US program SP-100 for the development and flight testing of a nuclear power system includes guidelines such that a system will be launched cold (sub-critical) and only operate in a 'nuclear safe' orbit. These are defined as orbits in which the natural
decay time of the orbit due to atmospheric drag is such that the reactor will be at a safe level of radio-activity before re-entry occurs. A typical nuclear safe orbit would be around 700 Km, this would reduce the Shuttle's payload capacity down from around 29,000 Kg to 20,000 Kg. This reduction in payload capacity is not likely to greatly influence the use of a nuclear powered "tug" since the proposed structures are volume constrained and not mass constrained. It is also probable, that as the number of orbital manoeuvring vehicles (OMVs) increases, the nuclear powered units would be allowed to descend to LEO to collect payloads for transfer to other orbits. The expected first flight of the SP-100 system is in the mid to late 1990's.

There is one further system that is arousing considerable interest in the field of orbit transfer and that is a hybrid system which combines a high $I_{sp}$, low T/W electric thruster with a low $I_{sp}$, high T/W nuclear thermal power system. One concept uses a nuclear reactor to heat hydrogen (high T/W,$I_{sp}$ of 650s) or alternatively to electrically accelerate hydrogen (low T/W,$I_{sp}$ of 5000s). The total energy of the reactor can be shared between either of the two systems to provide a continuously variable thrust and $I_{sp}$. Such hybrid systems would be ideal for a LEO to GEO transfer of the type envisaged for the system under investigation where comparatively short trip times, high dV's and low gravity loss is required. The ability to constantly vary the thrust and $I_{sp}$ would permit the deployment of slender booms at LEO prior to transfer to GEO. This would allow any deployment difficulties to be rectified in the relative ease of LEO, should they be required. Investigations are currently continuing within NASA into the possibilities of using antimatter-matter annihilation to produce exhaust velocities approaching the speed of light. Two techniques are being considered for transforming the energy from microwaves into the kinetic energy required. The microwave electrothermal (MET) process has already produced propellant velocities of 6 km/sec and by employing a magnetic nozzle to improve the constriction of the propellant flow the researchers hope to raise this to 20 km/sec in the very near future. The other technique under consideration, electron cyclotron resonance (ECR) may produce even higher propellant velocities, upto 100 km/sec (Lerner 1990 [73]).
There are several other types of potential propulsion systems under consideration, in fact competitions into the fastest round trips using a solar sail propulsion system have been suggested, and indeed it is a British design that is thought to be the most likely winner (The Times 1990 [74]).

2.6 Damping Considerations.

Damping is a subject which has a significant effect upon the behaviour of the structure under test, yet it is an area in which little knowledge exists and is in fact subject to the largest number of assumptions being made about it. The presence of damping leads to the dissipation of energy when materials are deformed. If a force was applied to a completely elastic material and then removed, it would start to oscillate about its neutral position due to the energy stored within the material. However, if a force were applied to a completely plastic material all the work would be dissipated and no displacement would occur when the force was removed. These two extremes define the states known as undamped and completely damped respectively. Obviously no material is absolutely perfectly elastic nor plastic, so only a certain amount of energy can be stored or dissipated (Castellani 1978 [75]). Again, it is clear that in a complex structure such as those proposed, the damping is not solely due to the plastic characteristics of the materials involved. Damping is frequently assumed to be linear in nature and viscous in behaviour, i.e. the forces due to damping are proportional to the velocity. The assumption that damping is both linear and viscous is convenient for a large number of engineering structures but two other non-linear forms can be employed. The first is Coulomb damping (Meirovitch 1986 [76]) which applies in the situation of components sliding over each other in an unlubricated state. The other is structural damping which is related to the hysteresis phenomenon associated with cyclic stress in elastic materials. This form of damping would be linked with the direction of the fibre orientation and the number of layers of fibre/matrix material present in the case of composite materials (Georgi 1984 [77]).
It is desirable for structures to possess sufficient damping such that their response to a given excitation is acceptable and within the tolerances that may be placed upon the system by other requirements, for example antenna pointing accuracy. There are two basic forms of structural vibrations, that is either steady state, in which the exciting force is produced by a constantly present action, or transient vibration caused by a short duration disturbance e.g. a collision between bodies.

Damping is the small fraction of energy that is lost during each cycle of a vibration of a structure. During this cycle energy is transferred between the structure’s kinetic and potential energy components. This dissipation of energy can be due to internal, external, or a combination of these two factors. Previous studies undertaken (Wada 1979 [78]) and (Wijker 1986 [79]) into the effects of air damping have shown a considerable change in the damping ratios obtained when structures were tested in air and then vacuum. The presence of air considerably reduces the resonances of the structures under test. A discussion of the effects upon a test structure can be found in Chapter 6, in which a structure was examined firstly under normal atmospheric conditions and secondly under vacuum with temperature cycling. Confirmation for these studies comes from the study undertaken on the actual and predicted response and damping characteristics of flight tested communication satellites. The results obtained from data from the Hermes communication satellite indicated that the predicted damping values were in error by factors in excess of three or more (Hearth 1985 [80]).

The importance of the suspension systems upon the behaviour of a body cannot be over emphasised, a poorly chosen system will greatly effect the response of the structure to an exciting force (Read 1984 [81]).

The effect of damping on the behaviour of structures is difficult to assess with a great deal of confidence as there is extensive evidence that the type of excitation technique used in the modal survey of a structure has an effect on the estimates of the damping values obtained. (Chen 1984 [82]).
The layers present in composites are assumed to adhere to the matrix such that no form of Coulomb damping or other form of slipage are considered for the damping contribution of the material (Lazan 1968 [83]). It is quite common, particularly in the case of composites that the capability of an element to dissipate energy is often dramatically reduced because the material selected to form the member is chosen for properties other than damping. To augment the energy dissipation present in the proposed structure the use of polymeric materials should be considered for inclusion within the various joints. This would be of particular benefit to the reduction of chattering of the sleeves present around the joints. Such materials have been used commercially at cryogenic temperatures (Cervenka 1980 [84]) but it is not known if they have been used in circumstances involving high vacuum. The benefit of the presence of joints in terms of damping augmentation is quite large. The presence of the interfaces of the joint which is required to maintain contact considerably increase the complexity of the analysis of a structural unit as it becomes important to consider not only the component materials and parts from which the structure is formed but also the energy dissipation caused by the slipage at the interface. The type of joint found in the proposed structure is a dry interface joint, which generally is subject to either a separation of the surfaces or a shearing effect. This shearing effect allows for considerable energy dissipation to occur but caution must be exercised in a design which maximizes this type of motion as serious interface damage can result which could generate fatigue cracking.

2.7 Antenna Systems.

2.7.1 Introduction.

An antenna is a device for radiating or receiving electromagnetic waves. It is the antenna that couples the signal to be transmitted into the propagation channel, receives the electromagnetic waves and feeds this information signal into the receiving equipment (Belrose 1989 [85]). Since the propagation channel is not loss free it is important to start out and end as efficiently as possible. There is very little fundamental difference between
transmitting and receiving antennas, so little in fact that they are commonly used for both purposes. As antennas form an integral part of the communication system and can have an over riding effect on the performance of such a system it is important to have a thorough understanding of the system as a whole and consider the effects of changes made in one area upon the other components within the package.

The most important properties possessed by antennas can be categorised into the following areas:

1) Polarization.
2) Radiation pattern
3) Power gain
4) Radiation resistance
5) Bandwidth
6) Effective aperture
7) Power transfer
8) Reciprocity.

2.7.2 Polarization.

Electromagnetic waves can be polarized in one of three ways, that is either linear polarized, circular polarized or elliptical polarized. Within each general type there are further possibilities dependant largely upon the required use of the signal. Linear polarized signals can be either vertically or horizontally polarized. In the first case this indicates that the $E$ vector is vertical, similarly for the second case the $E$ vector is horizontal. This $E$ vector describes the shape and orientation of the lows of the electric field vector as a function of time. A comprehensive study of polarization theory can be found in the reference (Collin 1969 [86]). Circular polarization is a combination of vertical and horizontal polarization. Both of the afore mentioned states are special cases of the general form of polarization which is referred to as elliptical polarization. Elliptical polarization is characterised by the following criteria;
The ratio of the major axis to the minor axis.

The tilt angle of the major axis to the reference direction.

The sense of rotation (either clockwise or anti-clockwise).

It is possible to express an elliptical polarization as the resultant of either two linear polarizations or of two circular polarizations.

The sense of rotation is a particularly useful tool to employ particularly in satellite reflectors as it is possible to use the same frequency for both transmission and reception, thereby economising both on the bandwidth occupied. This technique is know as frequency reuse and will be extensively employed in the proposed satellite to provide the 300 spot beams envisaged. However, something is never for nothing and so it is particularly important to keep cross-polarization effects to an absolute minimum (Collin 1985 [87]).

2.7.3 Radiation Pattern.

The most important property of an antenna is its radiation pattern, or more correctly its polar pattern. This pattern is a graphical representation of the power radiated from the antenna in the different angular directions. The plot can be obtained in the vertical and the horizontal planes. The application currently under investigation in which 300 spot beams will be provided requires the use of the so called pencil beam pattern in which most of the radiated energy is concentrated into a single direction, Figure 2.1. A typical pattern is shown and consists of a main lobe and a number of sidelobes. The power level of the near-in sidelobes, those occurring within 5-10° of the bore sight must be kept as low as possible for two main reasons, firstly, they dissipate the output from the transmitter and secondly, which includes the case under consideration they can be a major cause of noise and interference when the antenna is in its receiving mode. The figure also illustrates the half-power beamwidth (HPBW) which represents the angular width between the two points on the radiation pattern.
which are 3dB below the main beam peak. This can be determined from the following equation.

\[ \text{HPBW} = \frac{N \lambda}{D} \]  \hspace{1cm} 2.5

where  
- \(N\) = beamwidth factor dependant on the aperture illumination distribution.
- \(D\) = circular aperture diameter.
- \(\lambda\) = operating wavelength.

Generally speaking it is not possible to obtain the situation in which there is constant power distribution across the whole of the aperture in which case the antenna could be considered to be 100% efficient. A more realistic approach would consider that a tapered distribution would be achieved and that the antenna has an efficiency of 65%. The factor \(N\) varies between about 58 for a uniform distribution to about 75 for the tapered distribution case.

### 2.7.4 Power Gain.

The power gain of an antenna is normally defined in the direction of maximum radiation, for the case under investigation this would correspond with the boresight. The power gain provides very important information as to the ultimate performance of the system and it is closely associated with the transmitting power and the receiver sensitivity. Antenna size restrictions and weight will limit the maximum gain that can be achieved. For a circular aperture antenna the gain is derived from the following formula.

\[ G = 10 \log_{10} \left( n \left(\frac{\pi D}{\lambda}\right)^2 \right) \]  \hspace{1cm} 2.6

where  
- \(G\) is expressed in dBi.
- \(D\) = circular aperture diameter.
- \(\lambda\) = operating wavelength.
- \(n\) = antenna efficiency factor.

**Note:** Power Gain is the gain over a reference antenna.
2.7.5 Radiation Resistance.

This feature is of particular importance to the transmitting side of the antenna and is associated with the power radiated. In common with the basic laws of electricity the power radiated by an antenna is determined by $IR$, watts where $R$ is a fictitious resistance. Pointing determined the power radiated by an antenna in his vector theorem a comprehensive review of which may be found in (Connor 1986 [88]). The radiation resistance should be maximized as this maximizes the power radiated from the antenna. For the receiving part of the antenna it is the terminal impedance, defined as the ratio of voltage to current at its terminals which requires special attention. It is not necessary for the radiation resistance and the terminal impedance to be equal, although in some cases they are.

2.7.6 Effective Aperture.

The power received by an antenna is proportional to its collecting area, i.e. the larger the collecting surface the greater the received power. The design under consideration utilises this feature by having a very large main reflector some 50m diameter, thereby allowing the use of small low powered ground stations. The collecting area is referred to as the effective aperture $A_e$. It can be shown that the effective aperture can be calculated from the following relationship;

$$A_e = \frac{P_r}{P_d}$$  \hspace{1cm} 2.7

and similarly that,

$$A_o = \frac{G\lambda^2}{4\pi}$$  \hspace{1cm} 2.8
It is however important to consider that the effective aperture may also be associated with the physical aperture as is the case with a micro-wave horn, although it is generally less than this size.

2.7.7 Space Loss.

As electromagnetic waves traverse space and enter the atmosphere before being received by the ground station they experience a dramatic reduction in signal strength. This reduction is directly proportional to the square of the distance, in meters, between the transmitter and the receiver, so for geostationary communication satellites the losses are very large, typically around 190 dB for an operating frequency of 1.5 GHz (Douglas 1988 [89]). This very large reduction in signal strength is one of the main reasons why so much care and research is devoted to improving the efficiency of the antenna systems themselves and in particular the use of high gain antennas.

2.8 Vibration Testing.

All physical systems are continuous. That is to say that they are made up of an infinite number of 'particles' and hence an infinite number of degrees of freedom for each of their component parts. The term 'finite element' indicates that the mathematicians break down the continuous system into a discrete number of elements in order to perform a theoretical analysis of the structure. There is a need to perform experimental testing either to validate the finite element model by simple comparison of the measured parameters with those calculated, or to provide data for production structures.

The aims of the experimental tests undertaken for a particular project can be varied. They are dependent on the particular requirements for which the results of the individual models tested are going to be used. It is important to consider what is the required outcome of the test and how the results should be presented for whilst it can be said generally that the overall aim is to produce a
mathematical model of the structure, it is how this model is to be used that produces the differences.

The most common requirements are to use the measurement of vibration data to compare with those obtained from a finite element model. The purpose is to validate the theoretical model prior to using it to predict some set of responses for which it is impractical and/or impossible to test experimentally. Validation is considered acceptable if the major modes of vibration of the test and theoretical models can be corroborated. Hence for this application accurate measurement of the natural frequencies and a definition of the mode shapes associated with those frequencies are all that is required. It is generally very difficult to obtain estimates of the damping factors present when a theoretical analysis is performed so obtaining these experimentally is superfluous. However it is possible to incorporate such information into the theoretical model thereby enhancing it.

In many cases the comparison between experimental and theoretical models does not extend to the complex stage; theoretical systems are modelled to simplified experimental ones and the former are then extended to the prototype structures. An extension of the modelling process is that of correlation, in which the two sets of results are combined quantitatively, so as to assess the causes of the differences encountered. This requires that a more precise expression of the mode shapes is obtained than that required for purely pictorial assessment.

The use of such data allows a mathematical model of the experimental test to be built up. This allows for substructuring to take place, a process in which a smaller sub-section of the full structure is tested and then added to the remainder such that the total behaviour of the structure may be predicted. The requirements here are accurate data that contain information on the natural frequencies, modal damping and mode shapes with the important constraint that all the modes must be included simultaneously. All modes must be considered as even 'out of range' modes will influence the behaviour of a complex structure in a given frequency
Sub-structuring of finite element models of the antennas can also be performed. Several sub-structuring techniques have been developed in which the very large but sparsely occupied symmetrical matrices used to describe the global structure are modified and replaced by using the mass, stiffness and damping matrices of the sub components of the structure (Kaung 1985 [90], Hale 1984 [91]). Each sub-structure is analyzed separately and the results are combined using suitable constraints at the interfaces to model the global behaviour of the complete system.

A wide range of testing techniques have been developed over the past decades to determine the characteristics of structures. Such modal survey techniques may be divided into a whole series of methods and are usually known by the method of excitation that they employ. That is, transient, steady state, chirp etc. An evaluation of the various methods can be found in (Hamma 1976 [92]) The selection of the test method which is to be employed can be made by considering the following list:

- Linearity of the structure
- Expected modal behaviour
- Availability of test equipment and test duration
- Experience of the testing personnel
- End use of the test data. (Wada 1986 [93])

Further refinements can be made in that the mathematically expressed experimental data can be altered to predict the effect of modifications to the structure, as originally tested.

Similarly sub-structuring of the finite element models allows structural modifications to be performed without the need to re-analyze the entire structure; this is a considerable advantage. A comprehensive description of the sub-structuring technique with respect to both lightly and heavily damped systems can be found in Leung’s 1988 paper. (Leung 1988 [94]). Again it should be emphasised
that sub-structuring is not a panacea, for although improvements in convergence can be achieved by various methods (Bertram 1984 [95]) it appears that the method can only be successfully used in cases where the mode shapes of the modified structures can be adequately described by that of the original structure.

2.9 Vibration Control

2.9.1 Damping Control Methods

The ways in which the amplitude of the response of a large satellite to a disturbing motion may be reduced can be summarized under two categories, these are either active damping control or passive control. A considerable amount of research effort is being applied to these areas. In the past, structures were designed to sustain their structural requirements throughout the total operational life time in order to maintain the intended activity. Such 'passive structures' consisted of units fixed in a relative position, in space, under the applied external forces. The reduction of the responses due to the dynamic disturbances experienced by the structure are achieved by the damping inherent in the structure. The effective control of the flexible modes of such structures can result in significant improvements in a wide range of areas not least the dynamic performance (Jones 1979 [96]). Various techniques can be employed at the design stage in order to improve the dynamic response of a structure. Simple solutions can be adopted such as:

1.) Shifting the problematic eigenfrequency into a higher frequency range by changing the material properties or altering the layout of the basic structure.
2.) Minimizing the input forces.
3.) Increasing the generalised stiffness by, for example, altering the member cross-sections.

Such passive techniques have in the past proved unable to meet the requirements for the dynamical behaviour of some structures. The sizes and
complexity of the proposed structures will result in a reduction of the resonant frequencies of the flexible modes to limits normally associated with those for active control systems (Venneri 1986 [97]).

2.9.2 Attitude Control.

Currently the conventional design approach to spacecraft attitude control systems requires that a precise model of the dynamic characteristics of both the structure and its attendant sensors and actuators. The parameters that the controlling system is required to possess will then be some function of the total model parameters and in addition to those defining the closed loop performance requirements. It is therefore evident that the response of the control system will be susceptible to any modelling errors present in the system. As the physical sizes of spacecraft increase, the corresponding dynamic models will increase in complexity as the number of flexure modes increases. In this field, work is currently underway into the development of 20N thrusters for use in the Hermes attitude control system. Similar sized thrusters are proposed for use in satellite attitude and orbit control systems (Mathieu 1990 [98]).

2.10 Structural Testing.

Traditionally the structural testing of all spacecraft has involved the use of Earth based tests. Ideally development work should be carried out under the conditions that the structure is likely to experience, i.e. the space environment. However the costs and duration of such testing would be prohibitive. This situation has led to the development of a number of testing techniques which are aimed at overcoming the problems of testing large space structures, a summary of the problems is presented by (Hanks 1983 [99]).

The use of high flying aircraft following a preset flight path can produce a zero-gravity environment but the duration is quiet short, frequently less than 30
seconds per path and the effects upon the personnel involved in the tests are unpredictable. (Hanks 1983 [99] and Ikegami 1988 [100])

2.11 Finite Element Method.

As previously stated the size, complexity and the difficulty in ground testing future space structures makes it paramount that an efficient analytical method of determining the behaviour of such structures be found. One such method, which has gained wide acceptance, is the so called Finite Element Method (F.E.) which has some advantages over the finite difference and energy methods when used to model complex geometric forms and material properties, both of which are becoming increasingly common. The basic concept of the finite element method is that the structure to be analyzed is modelled as a number of elements, or pieces, connected together at a series of grid points. A continuous system or real structure is composed of an infinite number of the elements and grid points, whereas the model is composed of a finite number which is used to describe the behaviour of the real structure. It should be borne in mind that although the approximation is only made with describing the physical nature of the structure, the method used in the mathematical system is exact. The equations describing the behaviour of each element is calculated and the these are assembled together to form a set of global equilibrium equations of the structure. The set of simultaneous equations are solved by minimizing the energy associated with the model. The response of the structure is then determined from the results of these equations.

The type of element employed can have a direct effect on the accuracy of the results obtained. The assumption is made that the finite element method will converge towards a unique solution to the problem, asymptotically as the mesh density increases. However the shape functions of the element employed in the analysis will determine whether or not it deforms in the correct manner so as to model effectively those deformations of the system under examination.
There are two schools of thought on the methods to be employed to increase the accuracy of the finite element method. Firstly the "h" method which advocates an increase in the number of elements used, so called because in the Rayleigh-Ritz method this implies that the width 'h' of the finite element is reduced and, secondly the "p" method which promotes the use of higher order elements. Intuitively it would seem that increasing the number of elements, the mesh density, would result in an increase in the accuracy of the result (Keane 1989 [101]). For the skeletal spacecraft structures this is likely to be the case as their essentially beam-like-behaviour at lower modes of vibration would indicate that the use of higher order elements, would firstly be costly to perform and secondly would not significantly enhance the accuracy of the results. This is because the presence of mid point grids with their associated degrees of freedom would be largely inactive.

Other researchers have shown that increasing the order of the element employed in the analysis, particularly with respect to static analyses, has significantly improved results obtained for a given number of degrees of freedom (Babuska 1982 [102]) and in addition to which the rate of convergence can be higher than that of the "h" version (Meirovitch 1986 [103]).

The beam element can be shown to be nearly exact in its behaviour so the substitution of a shell element with its associated assumptions is likely to lead to a degradation in the accuracy of the results.

There are many other element types available with which the mesh can be formed. These include membranes, plates, shells and axi-symetric elements. Each of these element types has a series of sub types allowing a high degree of flexibility and choice for the engineer. Membrane elements are frequently used to analyse complex shell type structures, particularly at low frequencies (Petyt 1982 [104]). A derivative of this type of element is the iso-parametric element. Plate elements or plane stress/strain elements are also widely used to model the behaviour of thin plates provided that reduced integration techniques are
employed. Shell elements are widely employed as are axi-symmetric elements as many structures in the aerospace industry lend themselves to the use of such elements.

There are a number of factors which have been identified as influencing the ability of the finite element technique to accurately model the dynamic behaviour of a structure. The most important of these are the ability of the element selected to reflect the transverse shear deflection that the structure develops (Hibbitt 1989 [105]), and the ability to determine the effect of the rotary inertia, particularly with respect to the lowest modes of vibration, the others have already been mentioned. The distribution of mass within the model also has a significant bearing on the accuracy of the results. Finite Element programs generally employ either the lumped mass approach or the consistent mass approach.

There is considerable discussion about the merits of the two approaches. The idealization of the distributed mass into discrete lumped masses was originally proposed from physical consideration. However, the use of a consistent mass formulation would clearly appear to be more accurate. Indeed as far back as the mid sixties it has been indicated that the use of the 'consistent mass matrix' as opposed to a lumped mass model, particularly in cases of uniformly distributed stiffness and mass, was significantly improved (Archer 1963 [106]).
Figure 2.1: Typical Radiation Pattern Field.
3.0 Experimental Testing.

3.1 Introduction.

The experimental work carried out during this thesis has led to the development of an operating technique from which the natural frequencies and damping values, for a wide variety of structures in the 'free-free' condition, may be obtained. The models were tested in such a way that creep in the material was not significant. The rationale behind the experimental testing that has been undertaken is to provide a step by step approach to the problem of ground based testing of a composite satellite. A series of models increasing in complexity were tested using a number of different excitation devices and analysis techniques. This section is split into two parts, the first describes the model construction and background information to the testing procedures, whilst the second section describes the actual tests performed on the individual models.

3.2 Aims.

The aims of the experimental tests undertaken for the project are varied. They are dependent upon the particular requirements for which the results of the individual models tested are going to be used. It is important to consider the required outcome of the test and how the results could be presented. It can be said generally that the overall aim is to produce a mathematical model of the structure, it is how this model is to be used that produces the differences.

The approach that is generally adopted is to compare the experimental and numerical vibration solutions of a model system and then to use the numerical modelling technique to analyze a prototype structure, the experimental testing of which would be impractical. Validation is considered acceptable if the major modes of vibration of the test and theoretical models can be corroborated. Hence for this application accurate measurement of the natural frequencies and a definition of the mode shapes associated with those frequencies are all that is required. It is generally very difficult to predict estimates of the damping factors present when
a theoretical analysis is performed, consequently obtaining these experimentally may be superfluous. However, it is possible to incorporate such information into the theoretical model thereby enhancing it.

In many cases the comparison of experimental and theoretical models stops at the stage of simple comparison, with adjustments being made to the theoretical model to bring the results into closer agreement. An extension of this process is that of correlation, in which the two sets of results are combined quantitatively, so as to assess the causes of the differences encountered. This requires that a more precise expression of the mode shapes is obtained than that required for purely pictorial assessment.

The use of such data allows a mathematical model of the experimental test to be built up. This allows for sub-structuring to take place, a process in which a smaller sub-section of the full structure is tested and then added to the remainder such that the total structures behaviour may be predicted. The requirements here are accurate data that contain information on the natural frequencies, modal damping and mode shapes with the important constraint that all the modes must be included simultaneously. All modes must be considered as even 'out of range' modes will influence a complex structure's behaviour in a given frequency range.

3.3 Model Description.

3.3.1 Perspex.

The majority of the models employed in the tests were manufactured from perspex tube, having a 6 mm external diameter and a 1 mm wall thickness. A range of different model types were produced with member lengths varying from 130 to 1000 mm. The nodal points were made up from perspex discs, 20 mm in diameter and 10 mm thick. In-plane members of the models were embedded into the nodes whilst the out-of-plane members were held by a glue line to the node face. A typical configuration is shown in Figure 3.1. The models were manufactured so that a step by step build up of the components used to form the
proposed satellite structure was obtained. A second series of models was constructed using a larger diameter member so that modelling of the structural elements formed using the carbon-PES composite material could also be performed in perspex. A 25 mm external diameter and 2 mm wall thickness tube was obtained and used to form geometrically identical structural elements to those of the composite. The construction technique used in the formation of these units was the same as that used with the smaller diameter tubing, except that the jointing discs were considerably larger at 80 mm in diameter and 35 mm thick. The progression of models examined ranged from single units, pyramids and the basic building block of the main proposed truss. This basic building block consisted of a nine noded, twenty-one element tetrahedral unit. All these units were examined in both sizes of the perspex material. In addition to those essentially triangular based units used in the construction of the main and sub reflectors, models of the supporting arm structures were also constructed and tested. A summary of these models is shown in Table 3.1. Wooden falsework was used to aid the manufacture of the perspex skeletal systems to ensure that the correct member and joint alignment was achieved prior to bonding using TENSOL 70 cement (ICI 1991 [107]). Fixed joints were selected in preference to pinned ones to facilitate accurate analytical modelling of the member’s fixity. This also corresponded to the expected conditions in the proposed structure once deployment had been completed.

3.3.2 Carbon-PES Composite.

The composite used to form the various structural units used in the experimental tests was manufactured from a Polyethersulphone thermoplastic matrix and high strength, low modulus carbon fibres (T-300 Toray Industries Inc. "Toraycal"). The matrix was modified during the testing schedule to take account of excessive surface cracking that occurred after a structural unit had undergone an accelerated thermal cycling test programme. The first composite members consisted of tubes, with a 25 mm external diameter and a 1.8 mm ± 0.5 mm wall thickness. The 955 mm tubes were manufactured, using a carbon-PES pre-preg, by the film stacking process (Phillips 1980 [108]). Each member was formed using
five layers of the pre-preg interspersed with twelve films of neat PES. The stacking sequence for the composite was 0°/90°, about the main axis of the tube, for the pre-preg interleaved by 1/2/3/3/2/1 layers of the PES film. The composite member was wrapped around an expandable mandrel and then placed into one half of a 25 mm diameter split metal mould. The mould was then formed and transferred to a pre-heated press where a pressure in excess of 4 GPa was applied for a period of two hours.

The grade of PES used initially was 4200G but this was changed to a higher viscosity grade namely 4800G and the lay-up of the pre-preg modified to +10/-10/0/-10/+10 after the problems of surface cracking were experienced. These members were used to form the various structural units examined.

To facilitate the formation of a joint a cork end cap with a short, 2 mm diameter metal peg protruding is inserted into the end of the members. The members are then offered up to one another and the pegs are soldered together to ensure that the required geometry is maintained. A mass representative joint is formed in the shape of a truncated 100 mm diameter sphere by using a soft rubber mould. This joint was manufactured from an epoxy resin and randomly orientated chopped strand glass fibres. A pigment was added to the hardener so that uniform mixing of the products could be easily observed. This type of joint was chosen because it was anticipated that there would be a considerable amount of bending developed in the members of the skeletal units, particularly in their lower modes of vibration which form the area of greatest interest for this study. This is particularly likely to occur when the ratio of the member length to overall dimensions of the structural unit is high, as is the case with the skeletal units under consideration. The joint provides a high degree of rigidity, hence allowing the transfer of moments and facilities accurate FE analysis of the models. A summary of the models constructed and tested is given in Table 3.2.
3.4 Experimental Technique.

3.4.1 Initial Testing Procedure.

The first experimental tests were carried out on a simple tripod structure with member lengths of 200 mm, 6mm diameter and 1mm wall thickness. One end of each member was rigidly fixed into a 10 mm thick perspex block, providing an encastre end condition. Movements of this plate, at the base of the tripod arrangement, were prevented by clamps which rigidly held it to a substantial metal table.

Excitation of the structure was provided by means of an impact hammer. The structure was loaded as symmetrically as possible, the degree to which this was achieved was largely due to the skill of the operator. The time domain response was recorded on a storage oscilloscope and a force against time graph produced using an X-Y plotter. A peak hold voltmeter was used to provide the peak applied load. The structure under test was instrumented using two 60Ω foil strain gauges applied at a 20mm offset from the mid-points of the members. These gauges formed a quarter bridge network. The structure's time domain response was recorded on the twin beam storage oscilloscope and a strain against time graph produced. A peak strain meter indicated the peak applied strain for the particular member under investigation. A short description of the testing procedure and some typical results obtained from these experiments are presented in Appendix A.

The move from simple time domain free-decay plots using strain gauges as the detecting device to the use of a DataLab software package, AQUIRE (Datalab 1986 [109]) using accelerometers as the detecting device was very large. The multi channel transient recorder (DL1200) allowed readings from more than one response point to be recorded simultaneously. This was a major advance for two reasons. Firstly it allowed for greater repeatability to be obtained as one channel could be allocated as a reference and the results normalised to that channel. Secondly it allowed for determinations of results in the X-Y-Z planes instead of...
effectively one plane as had been the case previously because of the difficulty in obtaining a repeatable impact. This allowed for a much improved picture of the models response to an impact to be obtained. The DL1200 transient recorder uses a 12 Bit A/D converter to digitally store the results. The sampling rate was up to 0.5 MHz.

This improvement in the ability to detect the response behaviour of the various models allowed more complex structural units to be considered. Previously only single sections of the truss, or sections of the main support arm of the satellite had been tested. Now, due to the improvements in the testing procedure, models of parts of the main and sub-reflectors were built and tested in a near 'free-free' condition. The various perspex models were constructed using the node and member technique, with the member's being 200 mm in length. Initial testing showed that the fundamental frequency of the 9 node, 21 member basic building block was in the region of 170 Hz. This value was confirmed by using the hardware available in the Structures Department at RAE Farnborough. Two further models were also constructed; one represented the main reflector whilst the other formed the main support arm with a lumped mass to simulate the sub-reflector. Preliminary testing of these models indicated fundamental frequencies of around 210 Hz and 110 Hz respectively.

Several problems were found to occur when testing these models in a Free-Free condition. Since the forcing function was a pulse, the amplitude level of the energy applied to the structure was a function of the mass and the velocity of the hammer. This is due to the concept of linear momentum. The conservation thereof indicates the linear impulse that is applied. As it is difficult to control the velocity of the hammer, for although this is associated with the operator's skill and technique, it is usual to control the force level by varying the mass of the hammer. It is interesting to note at this point that the minimum mass available to us using the B&K hammer type 8202 (Bruel 1985 [110]) was 500 grams whilst the mass of a typical model was around 300 grams, definitely a case of using a sledge hammer to crack a nut. This led to a problem of signal truncation which was never really
completely eliminated. The frequency content of the energy applied to the model is determined by the stiffness of the contacting surfaces and to some extent by that of the mass of the hammer (Hewlett-Packard 1986 [111]). The stiffness of the surfaces determine the shape of the force pulse which then determines the frequency content. The harder the tip of the hammer the shorter the pulse duration and so the higher the frequency content of the impulse.

The maximum frequency, the cut-off frequency, excited by an impact from the hammer is defined by the -10 dB point of an FFT representation of the force component signal strength (Herlufsen 1985 [112]). The distribution of the frequencies within this range is governed by the relationship Sin(X)/X. Increasing the mass of the hammer results in a reduction in the cut-off frequency but an increase in the energy transferred to the lower end of the spectrum.

The impact hammer can have the tendency to introduce non-linearities into the test results for some structures. This is due to the relatively high crest factor, the ratio of the maximum to the average signal amplitude, that can be induced in this type of testing. Another feature of impact testing is that the levels of energy imparted to a structure can be relatively low, resulting in the possibility that responses may not be sufficiently excited to be detectable above the background noise levels. However, the impact hammer does have the advantage of not adding additional mass to the structure under test and once suitable analyzer levels have been selected the test duration can be quite short.

Initially all testing had been carried out using an instrumented impact hammer as the source. It was quickly realised that whilst impact testing had several advantages, namely relatively fast experimental set-up times allowing a trial and error approach to be adopted and a broad band of excitation frequencies it was not ideal for use on some of the models under test. This was because many of them had many closely spaced modes separated by relatively large dormant regions. Sampling requirements and the lack of anti-aliasing filters in the test equipment meant that the frequency resolution that was obtainable was frequently
much less than that desired. This led to the loss of important data. In some cases double impacts were not initially detected resulting in odd transfer functions being obtained.

3.4.2 Early Shaker Methodology.

To allow for greater resolution in the areas of interest a different type of excitation technique was required. This was provided by using a shaker driven by a sine wave generator. Initially the generator was set up on a frequency of interest as detected by the impact hammer technique and readings obtained for that frequency. This obviously allowed for much improved repeatability but due to a software fault averages were not able to be taken which would have helped to eliminate the presence of background noise which was proving to be a problem along with signal clipping when using the impact hammer technique. Experiments with a hand driven sine sweep were undertaken and proved to be quite encouraging. It was now possible to obtain a reasonable resolution over the frequencies of interest which assisted greatly in the determination of the structural damping using the 'Half-Power' (Ewins 1984 [113]) technique whereas previously the 'Log Decrement' (Craig 1981 [114]) technique had been used in the time domain responses. Some problems were experienced when the frequency was varied particularly if this involved a large bandwidth. This was due to the variation of the vibrator impedance and power factor with changing frequencies which resulted in the output current changing.

At this stage the section now had a testing ability which allowed it carry out both transient and sine swept tests on models in both an earthed and a free-free condition. The results obtained from these tests were compared with those obtained from a geometrically exact (within the tolerances of construction) finite element model with a mass and stiffness distribution similar to that of the physical structure under test. The structures under test were supported in such a way as to minimize the effect of the supports so that they could be excluded from the finite element model analysis. Tests were carried out to determine the rigid
body frequencies and the support system natural frequencies so that they again could be excluded. In almost all cases it was found that, including the boundary conditions imposed by, the support system had negligible effect on the F.E. results.

3.5 Experimental Analysis using the ENTEK Software.

3.5.1 Introduction.

The 'AQUIRE' software employed in the early tests had a number of quite serious weaknesses, including the inability to average a series of repeated tests. An alternative suite of software packages was purchased together with a considerably improved signal analyzer. The HP3562A digital signal analyzer which comprised the hardware side of the experimental testing equipment resulted in a considerable improvement in the repeatability of tests and the correlation with the parallel finite element analyses. This combined with the ENTEK (ENTEK 1991 [115]) software allowed for considerably improved data handling and processing over that which had been previously available. The hardware lent itself to two types of excitation; these are the shaker (vibrator) types (random noise, sine sweep, chirp, etc.) and the impact hammer.

Some difficulty was experienced when using the impact hammer to excite the smaller lightweight perspex models held in the free-free condition. The low mass and high stiffness structural systems do not perform well using an impact hammer. In order to obtain a reasonable signal to noise ratio such that a credible frequency response function could be obtained, it was vital to keep the signal input level at a value of at least half that of the selected range, (the in-built auto-range facility could not be used successfully as the signal level fell to zero between each successive strike which resulted in an overload occurring at the next impact). However, it should be stated that using the impact hammer provided a useful initial indication of the area of excitation interest; this technique was therefore frequently used prior to undertaking a modal analysis using the shaker method.
The shaker method which was used for the models employs two transducers, one a reference force transducer and the other a roving accelerometer. A force was applied at the reference location and the accelerometer was positioned in the x, y and z directions of all points of interest. The frequency response functions were then obtained for each position and direction such that a full modal analysis could be derived.

A short description of the digital signal analyzer and its associated components is given together with a brief introduction to the software subpackages.

3.5.2 Hardware Description.

The hardware used in the experimental testing procedure developed for the thesis consisted of a number of units which will be described in turn. The primary unit was an HP 3562A Digital Signal Analyzer. This unit provides a highly accurate dual-channel FFT analyzer combined with a series of internal source types allowing flexibility in the excitation method employed on a particular model. The microprocessor-based unit includes a wide range of measurement capabilities to complement the source types and a number of post-test analysis capabilities including curve fitting and frequency response synthesis. The operating characteristics of the analyzer allow dc to 100 kHz operation with an 80 dB range. The structural systems under consideration made use of the good low frequency capabilities of the analyzer as they could be characterized by their sub 250 Hz resonances.

The determination of the frequency response of a structural system and/or some of its components is likely to be a requirement of an experimental test. The DSA provides two separate techniques for determining the frequency response of the test specimen, firstly a Swept Sine Fourier Analysis (SFA) and secondly a Fast Fourier Transform Analysis (FFT). The SFA is a commonly employed measurement technique using a swept sine source and an integration process
which simulates a band-pass filter that tracks the input. The technique allows the gain and the phase angle of the system to be determined by measuring the fundamental components of the input signal and the system's response. It is implied that the frequencies of these fundamentals are the same. A whole series of these values are collected for different frequencies and used to build up the frequency response of the system. The technique relies upon the use of a very narrow bandwidth tracking filter. This filter minimizes the effects of non-linearities within the system. These may be caused by a dc offset, random noise and harmonic distortions.

To simulate the tracking filter the DSA uses a discrete Fourier Transform to calculate the energy within a narrow frequency range. This transform is calculated at several points during a sweep. The centre frequency of the transform analysis is kept constant with that of the swept sine source thereby simulating the very narrow bandwidth filter.

By comparison the FFT analysis is much more complex for instead of sweeping a single narrow band-pass filter in line with the swept sine source the FFT method uses a different form of the Transform to create, using integration, up to 800 adjacent and overlapping band-pass filters. These filters simultaneously measure the energy present over the entire frequency span thereby allowing for exceptionally high measurement speeds. The effect of non-linearities on the system, resulting in the presence of additional energies at different frequencies within the spectrum, are not filtered out directly using this measurement procedure but are instead removed by the use of an averaging technique of several repeated measurements. This assumes that these responses are not coherent with the input signal.

The advantage of employing a FFT measurement technique is that it allows the use of a very wide variety of excitation signal types. These may range from actual operating signal functions through swept sine waves to impulse function.
This flexibility contributes to the characterization of the behaviour of the system particularly if a non-linearity occurs.

The second main component was the Hewlett Packard Series 300 computer (Hewlett-Packard 1988 [116]) which was linked to a 100 Mbyte hard disc to allow for mass storage of data. The only modification made to the system was that a high speed graphics card was used so as to allow for the animation of the mode shapes of the structural unit under test.

The third main component of the hardware was that of the source type employed in the various tests. This was either an instrumented impact hammer, initially a B&K model 8202 (Bruel and Kjaer 1985 [110]) and later a PCB Piezotronics model 86B01 (PCB 1990 [117]), or an electromagnetic shaker, a Ling Dynamic Systems 200 series vibrator, (Ling 1985 [118]). This consists of a lightweight armature, formed from an epoxy bonded coil wound onto a laminated former, top and bottom spiders contained within the body. A permanent magnet is used which obligates the need for a field power supply. The vibrator operating range is from D.C. to 13 Khz., with a maximum throw of 2.5 mm. The maximum thrust level is 17.8 N using the natural cooling procedure adopted during the experimental investigations.

Various other periphery hardware was employed for the tests, these related to signal conditioning and pre amplification of the forcing function.

3.5.3 Software Description.

The software consisted of a series of inter-related packages which allowed the acquisition and storage of experimental test data, obtained by using a variety of transducers. This data could then be curve fitted, using a number of mathematical methods. The extraction of mode shapes from frequency response data, and the calibration of the mode shape coefficients was also available. The suite of packages also allow animation of the various mode shapes
such that a qualitative analysis may be undertaken and once scaling of these shapes using the modal mass and stiffness has been completed that a structural modification analysis could be performed.

A brief outline of the capabilities of each of the software packages is presented below. The software is written in HP-Basic and divided into a series of individual blocks or actions. This results in only parts of the program being called into memory at any one time and therefore can cause delays whilst new information is accessed from the mass storage units, in this case a hard disk drive. These delays are particularly noticeable with the graphical presentation of data as plots need to be recalculated each time the screen is refreshed. The package consists of five separate sections, some of which are interrelated, but others are stand alone modules:

...) **EMODAL:** This modal analysis program experimentally characterizes the dynamic behaviour of complex mechanical structures in terms of natural frequencies, damping and mode shapes.

...) **EMESH:** This interactive mesh generation program allows the rapid generation of a test structure’s geometry, in terms of the required format of coordinates and links, for modal analysis.

...) **EMDOF:** This program allows for the single and multiple-degree-of-freedom curve-fitting of experimental data to be made. This allows accurate extractions of the modal parameters from complex structures with coupled modes of vibration.

...) **ESMOD:** This program allows structural modifications to be undertaken. The program computes the effects that alterations in terms of mass and stiffness, at discrete points, would have on the dynamic behaviour of a structure.

...) **EFORCE:** This program allows the complex dynamic response of a structure, subjected to a set of arbitrary forcing functions, to be determined.
The facilities of all of these modules have been exploited during the course of this project. Some limitation have been discovered which have caused some problems. The most serious problem occurs in the modal analysis module. It is not possible to import data from the Dynamic Signal Analyzer into the module in a Log-Log format from a frequency response function. Nor is it possible to import the data in a Log-Linear format and then convert that format into the Log-Log format. The manufacturers were unable to assist and have since issued a warning regarding this matter as they do not guarantee that the number of data points per decade will accurately reflect the distribution of data within the spectrum. This results in the possibility of incorrectly determining the resonant frequency.

3.6 Experimental Testing Considerations.

3.6.1 System Linearity Assessment.

One of the general assumptions of a modal test is that the system under test is essentially linear over the range and the amplitude of excitation to which it will be subjected. This does not however preclude the testing of nonlinear systems, merely some of the stimulus types that may be employed. If the system is essentially linear then the choice of which combination of stimuli and analysis method to employ is simply a matter of speed as any combination will produce accurate results. It is important that an assessment of the system's linearity is made before a final selection of the excitation type and analysis method is made. A comparison between two or more tests using different excitation techniques should be made so as to determine if a non-linearity exists in the device under test. Should this prove to be the case a further series of tests may be required to characterize the non-linearity and thereby aid the selection of excitation function. A comparison between several of the more common types of excitation techniques are given in Table 3.3.

Note: To check the linearity of a system when using a single excitation method the usual procedure is to alter the level of the input force and check that doubling the input force doubles the response.
3.6.2 Theoretical Background

There are many references on the detailed mathematical background to this subject viz. (Brown 1979 [119], Simley 1984 [120], Ewins 1984 [121], Corelli 1988 [122]). The experimental procedure for characterising the system is outlined elsewhere and will not be repeated here. The experimental data obtained from the test is transformed into the frequency domain and is then subjected to a curve fitting procedure so that a mathematical representation of the experimental data is obtained. The mathematical data is then manipulated so that information on the displaced shape, damping, mass and stiffness associated with each resonance can be calculated. The curve-fitting procedure is based on equation 3.1 which states that there are a discrete number of modes of vibration (k) included in the response function. Clearly this would imply that a continuous (real) structure would have an infinite number of modes of vibration. This presents a problem as it is only practical to consider a finite number of these modes, thereby approximating the system. The modal parameters of the system are made up from the parameters of each mode, that is it's natural frequency, damping ratio or decay rate and any residue for each measurement location associated with that mode. We can further state that the frequency and the damping values of a mode are global, that is they will be the same for any position on the structure, this is not to be confused with the amplitude of a particular point of the structure which could of course vary. The residues, however, are dependent on the location of the measurement position and therefore will change from point to point on the structure.

\[ H_{ij}(s) = \sum_{r=1}^{n} \left( \frac{R_{ij}}{s - \rho_r} + \frac{R_{ij}^*}{s - \rho_r^*} \right) \]

where \( H_{ij}(s) \) = frequency response function of the response at point "i" due to the input at point "j".

- \( R_r \) = residue for mode "r", a complex number.
- \( \rho_r \) = pole for mode "r".
- \( s \) = Laplace operator.
- \( * \) = complex conjugate.
The basic assumption that modal analysis makes is that the structure is linear. For real structures this approximation can be considered valid provided that the structural damping ratio for the mode does not exceed 20 to 25% of critical. All the models which have been examined during the course of the experimental work were found to be very lightly damped, typically the damping ratios were around 2 or 3% for the perspex structural units and around half that for the composite units. A variety of curve-fitting methods are provided such that the best-fit can be achieved for the particular mode and thereby the closest approximations of the actual modal parameters of modal mass, damping and stiffness can be obtained.

The simplest procedure is the pick-peak or quadrature extraction method. This uses the imaginary part of a frequency response function and relies on the fact that the quadrature of a SDOF reaches its maximum at the natural frequency and similarly approaches zero as it moves away from the resonant frequency. It is possible therefore to use this feature so as to determine an individual mode from a set of relatively closely spaced modes. It is also possible to use the real part of a frequency response function, the coincident response can be used when dealing with a velocity/force function or with transmissibility functions. The program does not in fact 'curve-fit' when this procedure is employed as it simply uses the value of the FRF at the point chosen by the operator and places it into the parameter table for subsequent use in the displaced shape extraction process.

Two other procedures are available these are both based on the rational fraction polynomial. This first method performs SDOF curve-fitting to extract both the natural frequency and the complex residues of a mode. The damping ratio is obtained by solving a subset of the rational fraction technique which includes the residual terms present from the existence of modes which fall outside the immediate fit-range. The second method used is a similar MDOF technique in which the rational fraction polynomial is used to analytically represent the frequency response function or part thereof. However this leads to a particularly complex set of equations which would require a considerable amount of computing
time to solve. A solution to this has been found by assuming that the response function is symmetric about the resonance and thereby allowing the use of orthogonal rather than ordinary polynomials. The disadvantage of this assumption is that these orthogonal polynomials have to be repeatedly calculated in a recurring manner which results in a considerable increase in computer solution time. The use of matrix algebra as set out in Wei's work (Wei 1988 [123]) has reduced the computational time considerably and is used in the current curve-fitting procedure. The use of the rational fraction polynomial curve-fitting technique can produce very accurate results and whilst the solution time is longer than the other routines available it has been widely used in the experimental testing undertaken due to it's improved performance over the other techniques. It performs best when there are some well defined inflection points between closely spaced modes but nevertheless is capable of separating these modes should these not be present. The technique also appears to perform better than other algorithms if there is a significant amount of noise within the measurements, a situation which should of course be avoided if possible.

3.6.3 Suspension Systems.

The importance of the suspension systems upon the behaviour of a body cannot be over emphasised, a poorly chosen system will greatly effect the response of the structure to an exciting force.

The suspension system employed during the test procedure was one which emulated the 'free-free' condition. In order to achieve this condition, experiments were carried out into various types of suspension methods until the most satisfactory condition was obtained. The 6 mm diameter models up to 1000 mm in member length were suspended using 1.8 m of lightweight nylon cord connected to the apex of the pyramid by a lightweight elastic band (E=30 Nm) i.e. only one suspension point was employed. The larger models used a similar arrangement except that a lightweight steel spring was substituted for the elastic band. Cable suspension systems are frequently used as the support mechanism for ground
based vibration tests. Herr (Herr 1974 [124]) described the effect of a number of
different cable suspension systems on model vibrations whilst Hanks (Hanks 1983
[125]) showed that by adjusting the length of the suspension cables the
fundamental pendulum mode and the first structural mode can be separated.
Improved low frequency suspension techniques are constantly being sought e.g.
(Keinholz 1989 [126]). These support conditions resulted in a situation in which
the structures were constrained but in which the rigid body modes were of very
low frequency, typically in the range of 1 to 5 Hz depending on the model type.
These sorts of values indicated that the rigid body modes would have negligible
effect on the flexible modes which were for most models under test occurring at
much higher frequencies. A further advantage of employing a 'free-free' boundary
condition is that it eliminates the damping losses due to Coulomb friction that
would occur at joints.

The ideal test configuration for the evaluation of a spacecraft's structural
behaviour would be those experienced in a dedicated flight test but these are
rarely available so a number of other methods have been suggested. Fixed
supports, with base excitation, have been widely used so as to determine the
satellite's response to launch and injection loads (Steels, 1986 [127]). In
practice it is difficult to obtain complete fixity at a support as some degree of flexibility will
invariably exist. The effects of such flexibility within the supports on the dynamic
behaviour of the structure are difficult to determine accurately and will vary
depending upon the location of the supports. Read and Dean indicate that the
accuracy of any analyses is dependant on the accurate estimation of the level of
fixity that the restraints provide (Read 1978 [128]).

The resonances of the suspension system were categorised as being one of
three types, a pendulum type of motion of the complete system, as lateral
vibrations of the supporting cords, or as longitudinal extensions of the cords. The
free vibration of a pendulum is dependant upon it's effective length and is
independent of the mass supported. The second and third categories of vibrations
can be considered to be governed by the equation of motions of a vibrating string.
The longitudinal extension natural frequency is dependent upon the axial stiffness of the support system and the mass being supported. Both these types of suspension motion will impart more energy into the system being supported than the lateral string vibrations. If it is considered that the string is stretched between two points then a differential equation may be developed representing the displacement of the string from its neutral position. Substitution for the end conditions into this equation allows the development of the following equation from which the natural frequency of lateral vibration of the string may be determined. This equation is reproduced below.

$$f_n = \frac{n}{2L} \sqrt{\frac{T}{\rho}}$$

where $L$ is the effective length of the string,
$T$ is the tension in the string,
$\rho$ is the mass per unit length,
$n$ is the mode number.

Both these equations indicate that the maximizing the suspension length will reduce the natural frequency of vibration of the suspension system. The second equation indicates that reducing the tension in the supporting string will also reduce the frequency. One method of achieving this would be to use a large number of strings to support the model. The effectiveness of this approach would need to be considered in the light of the increase in restraint that might be applied to the model and the resultant change in resonances that could occur. For this reason the first approach has been adopted. That is, for each suspension system, to maximize the available suspension length for each support position.

3.6.4 The Fourier Transform.

Many applications involving the continuous Fourier transform rely on a digital computer for implementation, which leads to the uses of the discrete Fourier transform and hence the FFT. The discrete Fourier transform is of
interest largely because of its approximation to the continuous Fourier transform. The degree to which this approximation converges with the actual data is a function of the waveform being analyzed. The presence of extraneous noise on the waveforms needs to be reduced to a minimum. Considerably efforts were made to ensure that the input and response signals are as free from interference as possible.

3.6.5 Accelerometer Choice.

Accelerometers are frequently used to measure the accelerations of structural components. The commonly made assumption is that the motion occurs in the primary sensing axis direction of the accelerometer. This assumption is therefore based on one of two choices, either the direction of motion is known before it is measured or that the motion is assumed to be normal to the mounting surface. The assumption does led one into potential difficulties as in practice the direction of motion is often unknown. In this situation it is quite possible for the motion to occur perpendicular to the primary sensing axis of the accelerometer and thereby bring the cross-axis sensitivity of the transducer into play. Typically this cross-axis sensitivity is in the range of ±5% of that of the primary axis. One solution to this problem could be the use of a triaxial accelerometer which provides acceleration responses for three mutually perpendicular axes and thereby characterise the motion. However closer examination of the consequence of using such a device reveal that each of the three acceleration signals is contaminated by components of the other two orthogonal directions. This is inherent in the design of the triaxial accelerometer as the cross-axis sensitivity is much higher than uniaxis accelerometers. The effect of this is that there is a high probability that the motion predicted from the three signals will be seriously in error. It is for this reason that three orthogonal measurements were taken for each location point on the models under test. Each of these readings were taken using a uniaxial accelerometer with a low cross-sensitivity, thereby reducing the contamination from the components of the motion, if any, that was occurring in the mutually perpendicular directions. A recent paper by Han (Han 1990 [129]) has confirmed
the advisability of this method of testing, despite the attendant increase in testing time required.

Accelerometers are a popular transducer for general vibration analysis because they possess a number of important attributes not least of which is that they are accurate and rugged. The sensitivity of the accelerometer is largely dependant upon the size of the vibrating mass, the larger the mass the higher the output produced. However, there is a trade off in that the larger mass reduces the natural frequency of the accelerometer as a whole and thereby reduces its useful range. The output from accelerometers may be categorised as low-level but of high impedance which requires signal conditioning before being input into the signal analyzer. The most common method adopted for this signal conditioning is to use a charge amplifier, a description of which may be found in (B&K 1980 [130]). Other alternatives exist including voltage amplification and integrated circuit piezoelectric accelerometers which have an in-built signal conditioner and therefore only require a supply current. The use of signal conditioners also requires that low-loss low-noise cabling be used to connect the accelerometer to the signal conditioner. It has been observed from the large number of tests that have been carried out that the micro-dot cabling used in the experimental configuration can induce a response in the signal conditioning unit when that unit is operating on its most sensitive range. To counteract this response the cables must be kept stationary during the test. This would imply one of two causes. Firstly, that the shielding on the cables is not sufficient to counteract the effects of the wire moving in an electromagnetic field, such as might exist around the test equipment and secondly that the motion of the cable is causing shorting across the micro-dot connectors. These have been a source of some concern because they are very prone to internal breakages.

The key characteristics of piezoelectric accelerometers are that they offer a broad frequency bandwidth but are not really suitable for use below about 5 Hz because of the very low output voltages produced as a consequence of the low acceleration levels (Crawley 1986 [131]). Conversely, the large outputs at high
frequencies require the use of a signal analyzer with a wide dynamic range otherwise the lower frequency components will be masked. To make vibration readings below 5 Hz piezoresistive (Crawley 1987 [132]) or cantilever-beam accelerometers should be employed as these are capable of responding down to a direct current (dc). Again it should be emphasised that the response levels at these low frequencies is very small.

Accelerometers are particularly sensitive to the mounting technique. They should be securely fixed to the surface of the device under test by either a mechanical threaded mounting stud or a high quality adhesive tape. The use of uni-axial accelerometers in the test configurations undertaken required the use of a variety of mounting blocks to be firmly fixed to the device under test prior to attaching the accelerometer to the block. The addition of such blocks required that mass loading tests including the blocks be undertaken. The mass of the accelerometer should not be greater than ten times less than the equivalent mass of the point at which it is attached. If this guideline is not observed then the presence of the accelerometer will unduly influence the behaviour of the structure. This proved to be the case with the initial testing that was undertaken during the project were a combination of small lightweight models and relatively large accelerometer masses combined to cause inaccuracies in the responses recorded.

3.7 Experimental Description: Perspex Models.

3.7.1 Introduction.

The experimental testing of the perspex models took place over an extended period during which time experience was gained into the optimum test procedures to adopt. Each structural form tested possessed a set of individual characteristics which required that the experimental set-up, support conditions and excitation technique be adjusted and developed to suit the responses being observed. A description is provided for each set of tests, with particular emphasis being placed upon the differences that occurred either within the set or with other sets. The perspex material was found to be a suitable modelling material for most
configurations except for the very largest member lengths were deformation under self weight was found to become a problem if the models were not carefully supported. The modal survey of each structure was a time consuming exercise particularly as the number of points being examined grew with the complexity of the structure being investigated.

3.7.2 Single Member.

Two members were considered, each was a 25 mm diameter, 2 mm wall thickness perspex tube but with a different overall length. The first member was 0.958 metres in length whilst the other was 2.0 metres. The mass density of the material was determined as being 1192.4 Kg/m³. The testing procedure was identical in each case so only one will be described. The member was suspended in a simulated free-free condition from it’s fundamental bending node points by using a 4.2 metre long nylon cord and elastic suspension system. The member’s characteristic frequency response functions were then determined using two methods, firstly an instrumented impact hammer with a fixed transducer, and secondly using a fixed force transducer and shaker with a roving accelerometer. Table 3.4 contains information as to the analyzers experimental set-up.

A lightweight hammer and fixed transducer were used to determine the broad range frequency response functions of the member. Mass loading tests were performed which proved that the model was not susceptible to this form of error. The shaker test provided an opportunity to conduct both sine sweep and fixed sine examinations of the member. The same support conditions were used for these tests with the driving point being positioned 60 percent along the length of the beam. This position was to allow the development of the first three modes of vibration of the member. Table 3.5 contains the experimentally determined resonances for the 2.0 metre member using the swept sine technique. Figure 3.2 illustrates the frequency response function for a point on the member.
3.7.3 Pyramid Units: 6 mm diameter members.

A series of perspex pyramid were produced using the 6 mm diameter tubing with member lengths ranging from 200 mm up, in increments of 200 mm, to 1000 mm. The mass density of the material was determined as 1192.4 Kg/m³. A number of different suspension techniques were used to support these structures. Initially 400 mm long lightweight elastic bands were used to support each corner of the base of the 200 mm pyramid. This proved to be unacceptable as once impact had occurred the model proceeded to bounce excessively up and down thereby effectively masking the local deformations of the structure and making a visual appreciation of the mode shapes impossible. Examination of the frequency response functions of the model revealed the presence of these modes, hence allowing correlation with the finite element analyses. The suspension length for the other models was then increased to reduce their associated resonant frequencies, so as to maintain the separation between the structural and suspension frequencies. Table 3.6 contains the experimental set-up data for this series of test and Tables 3.7 to 3.11 contain the parameter table results for the various structural units.

3.7.4 Pyramid Units: 25 mm diameter members.

A set of three structural units were prepared and tested using a similar suspension system to that described for the later 6 mm perspex units. A lightweight nylon cord was attached in series with a lightweight spring to the apex of the model under test. The suspension length was varied between 1.0 and 3.2 metres. The resonances for the suspension system were calculated and the length adjusted so as to maintain the greatest separation between the structural and the suspension resonances. It was noted that the 2.0 metre member model suffered from the effects of deformation under its self-weight which caused some problems with the correlation between the experimental and finite element analyses. An alternative suspension system in which each joint of the 2.0 metre model was supported using the lightweight support system was tried and found
to alleviate many of the problems associated with this deformation. Table 3.12 contains the experimental set-up data for this series of models and Tables 3.13 to 3.15 contain the parameter table results for each individual structure.

3.7.5 Unit Building Block 6 mm diameter tubes.

A series of perspex unit building blocks were produced in the 6 mm diameter tubing and suspended in a free-free condition. Three models were considered ranging up in member length from 200 to 600 mm in 200 mm increments. These models were subjected to both impact and sine sweep excitations. The shaker was mounted in a near free-free condition by using a counter-balanced pulley system. The suspension methods used for these models consisted of a rigid metal frame, firmly anchored to the floor, from which a number of elasticated bands were hung. Three 450 mm bands connected the extremes of the major base triangle to the frame whilst the fourth connected the apex of the upper triangle to the frame. Figure 3.3 illustrates the suspension method. The frame size varied with the model size. Table 3.16 contains the experimental set-up data for all the models. These models were susceptible to the effects of mass loading due the presence of the 4 gramme accelerometer used in the initial investigations. This effect was much reduced for the 1 gramme transducer used in the later investigations for which the results are shown. Tables 3.17 to 3.19 contain the parameter tables for these units.

3.7.6 Unit Building Block 25 mm diameter members.

The 1000 mm member length model was suspended from the upper triangle joints using an elasticated nylon cord and lightweight stainless steel springs. The spring extension under load, was calculated to be 50% of the maximum allowable. The cord length was varied from 1.0 to 5.2 metres so as to assess the effect of the support vibration frequencies on the model. The effect on this particular model of the change of length was not significant indicating that the string resonances and rigid body modes did not lie with the range of interest.
for this model configuration. Calculation showed that the rigid body pendulum motion of the system varied between 2.0 and 4.6 Hz for the two extremes whilst the string vibration frequency, for longitudinal extension, varied between 2.3 and 1.0 Hz for the two cases. Table 3.20 contains the experimental set-up data whilst Table 3.21 contains the parameter table for the model. The shaker, used in the tests, was suspended in a near free-free condition using the counter-balanced pulley system. This system allowed both vertical and horizontal adjustments to be made.

3.7.7 Perspex Arm Section 6 mm diameter members.

These models were suspended in a rigid metal frame that was firmly anchored to the floor. Lightweight elastic bands, 450 mm in length, connected the apex of each bay to the frame. The models were subjected to both impact and sine sweep excitations. The shaker was mounted in a near free-free condition by using a counter-balanced pulley system and was connected to an end joint via the force transducer. This provided the axial excitations that were expected to dominate in this configuration.

The impact testing of this structure proved to be a problem as once impact had occurred the model proceeded to bounce excessively up and down thereby effectively masking the local deformations of the structure when the mode shape was produced. This was evident as the first three modes all appeared to be identical in nature. The problem was eventually resolved by using the lightweight impact hammer and transducers.

The initial swept sine testing of these structures revealed a series of closely spaced modes which appeared to be dominated by the presence of the transducer mass at the end of the model. An alternative position for the excitation point was adopted for the final modal test. A similar position was adopted for the larger near full sized arm sections tested later. Table 3.22 contains the experimental set-up information and Tables 3.23 to 3.24 contain the parameter table information.
3.7.8 Perspex Arm Section 25 mm diameter members.

This model represented a near full sized three bay section of the support arm. The maximum dimension of the members was 2.0 metres, an enforced restriction due to availability. This produced a model 4.2 metres in length and 1.89 metres tall. This model weighed 10 kilograms and was supported using four lightweight supports, one situated at the apex of each triangle. The supports were 4.78 metres in length and consisted of a lightweight nylon cord in series with a lightweight spring. Two spring positions were considered in the preliminary test sequence, the first was with the spring being positioned closest the fixed support point, whilst the second position involved the spring being positioned close to the attachment point of the model. The second position was adopted for convenience as no differences in the response of the structure, due to this change in spring position were detectable.

The output from the force transducer was particularly carefully monitored during the test sequence. Previously it had been assumed that the force detected at the force transducer was the force present in the structure. This is the case away from resonance but as the applied frequency of excitation approaches a structural resonance the feedback loop, which is used to apply a constant force to the structure, interacts with the shaker and so reduces the force applied. This results in a reduction in the force detected at the transducer and hence the trough that appears in the power spectrum for the transducer. The total force available for transmission through the structure is therefore a combination of the applied force and the inertia forces present in the structure due to it’s displaced positions for that particular mode. Tables 3.25 and 3.26 contain the experimental set-up and parameter table results respectively.
3.8 Carbon-PES Composite Structures.

3.8.1 Introduction.

A number of the perspex structures were duplicated using the composite material. The composite tubing was only available in lengths of 0.955 metres with a 25 mm external diameter. In general the models formed from the composite material were much less sensitive to the applied support conditions. This is due in part to the high specific stiffness of the material and to the experience gained with the perspex material. It proved not to be possible to obtain accurate estimates of some of the material constants for the composite material. Although efforts were made to ascertain the modulus of elasticity in the hoop direction and the minor Poisson's ratio by experimental means, the high stiffness of the material combined with the sample end effects were found to unduly influence the results. The results obtained for the material and a detailed account of the experimental set-ups may be found in the report by Hollaway and Thorne (Hollaway 1990 [133]).

3.8.2 Carbon-PES Composite Single Members.

The purpose of these tests was to establish the behaviour of the elementary component of the proposed structural configuration. The beam was excited using both impact and swept sine techniques. It was found that the high stiffness of the material resulted in small amplitudes of vibration occurring. A series of beams was examined starting with a 0.950 metre and continuing up to a 1.872 metre beam incorporating two glass epoxy end caps. A lapped central joint was used to join two single members together to form the longer element. This joint, 30 mm in length, was formed by reducing the wall thickness of two tubes, internally and externally respectively, by half and bonding these members together using a 15% solution of PES suspended in a solvent.

The impact tests used a lightweight hammer and fixed transducer to determine the broad range frequency response functions of the member. Mass
loading tests were performed which proved that the model was not susceptible to this form of error. The suspension system used in the tests was similar to that used with the perspex single member tests. The suspension length was 4.2 metres. The driving point for the shaker driven tests was positioned 60 percent along the length of the beam. This position allowed the development of the first three modes of vibration of the member. Tables 3.27 to 3.30 contain the experimentally determined resonances for the various members examined using the swept sine technique.

3.8.3 Carbon-PES Composite Triangles.

An additional set of composite models were constructed and examined in conjunction with the thermal cycling test series. The two dimensional structure occurs within the proposed tetrahedral system and allowed the examination of the 'in' and 'out' of plane modes expected to arise within the face elements. Two equilateral triangles were constructed from the lap jointed members. The members were joined at their ends by the 100 mm truncated sphere. The member lengths were identical to those used in the single member with end caps test described in the previous section. Two identical models were constructed so that a loaded and unloaded test could be undertaken in the thermal cycling chamber. The models were suspended using the lightweight suspension system previously described, with the overall length being 1.0 metres. This short suspension system was adopted because the working diameter available within the thermal chamber precluded the use of a longer system. A full modal test was only carried out on one of the units, the details of which are contained in Tables 3.31 and 3.32.

3.8.4 Carbon-PES Composite Pyramids

A series of composite pyramidal structures were built up to be geometrically identical to the 25 mm diameter perspex models examined. This provided an overlap for all three material/member diameter configurations at the 1.0 metre member length. The models were suspended using a 4.2 meter
lightweight suspension system attached to the apex of the triangular based pyramid. A purpose made steel spring was substituted for the elastic cord used in the perspex pyramid tests. Impact testing of these units revealed the general region of interest for which the swept sine tests were then performed. The excitation point was positioned 60 percent along the length of one of the base members, allowing the development of the lower modes of vibration of the structure. The mode shapes were characterised by the lack of motion that the end joints experienced. The high stiffness and rigidity of these structures configuration resulted in a particularly good set of results being obtained. Examination of the associated coherence function spectrums revealed very near unity for all tested points. Table 3.33 contains the details of the analyzer set-up for the two models and Tables 3.34 and 3.35 contain the parameter table details for the 1.0 and 2.0 metre structures respectively.

3.8.5 Carbon-PES Composite Unit Building Block.

The unit building block for the proposed satellite structure is composed of nine joints and twenty-one elements formed into the double layer tetrahedral truss configuration. Two structures were examined, one was manufactured from the initial material lay-up whilst the other was produced from the later refined lay-up and matrix material. Both of these structures were subjected to a pre thermal test chamber examination in the free-free condition. The models were supported by using three lightweight suspension systems attached to three of the units vertices. A stainless steel spring was used to provide the elasticity of the supports. The suspension length was again restricted, due to the available diameter within the thermal test chamber, to 1.0 metres. This produced a resonant frequency for lateral displacement of the supports of 78 Hz. This was outside the desired separation band for the support to structural response frequencies, but was not considered to be a major problem as the amount of energy transferred from the suspension system to the model would be negligible. The experimental mode shapes for these models were again characterised by the small deformations that the joints experienced for the lower modes of vibration.
The high stiffness and rigidity of these structures configuration resulted in a particularly good set of results being obtained. Examination of the associated coherence function spectra revealed, as with the previous set of models, very near unity for all tested points. Table 3.36 contains the details of the analyzer set-up for the two models and Tables 3.37 and 3.38 contain the parameter table details for the two different materials respectively.

3.8.6 Carbon-PES Composite Arm Section.

The three bay triangular arm section model was formed from a set of lap-jointed members and the truncated 100 mm sphere joints. The inter-bay diagonal members dimension for the model was 1.872 metres. This produced a total length of 3.98 metres for the three bays and a height of 1.76 metres. This was slightly smaller than the equivalent perspex model that was constructed. The support system consisted of four lightweight nylon cords and springs attached to the vertices of the four triangles of the model. The spring position was close to the model attachment point. The total model mass for the structure was 14.6 Kg. The excitation point for the structure was 60 percent along one of the horizontal members, lying along the main axis of the arm, in an end bay. Table 3.39 contains the experimental set-up details and Table 3.40 contains the parameter table results.
3.9. Tables.

<table>
<thead>
<tr>
<th>Model Type</th>
<th>200</th>
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<th>600</th>
<th>800</th>
<th>1000</th>
<th>2000</th>
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<tr>
<td>Single Member</td>
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<td></td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Pyramid (6)</td>
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<td>X</td>
</tr>
<tr>
<td>Pyramid (25)</td>
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<td></td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
</tr>
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<td>9/21 (6)</td>
<td>X</td>
<td>X</td>
<td></td>
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</tr>
<tr>
<td>9/21 (25)</td>
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<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Arm (6)</td>
<td>X</td>
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<td></td>
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<tr>
<td>Arm (25)</td>
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<td>X</td>
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Table 3.1: Summary of Perspex Models Tested.

<table>
<thead>
<tr>
<th>Model Type</th>
<th>200</th>
<th>400</th>
<th>600</th>
<th>800</th>
<th>1000</th>
<th>2000</th>
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</thead>
<tbody>
<tr>
<td>Single Member</td>
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<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Triangle</td>
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<td>Pyramid</td>
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<td></td>
<td></td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>9/21</td>
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<td>Arm</td>
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<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.2: Summary of Carbon-PES Composite Models Tested

<table>
<thead>
<tr>
<th>Function Type</th>
<th>S/N Ratio</th>
<th>Test Meas. Length</th>
<th>Controlled Freq.</th>
<th>Distortion Removed</th>
<th>Characterise Non-linearity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady State Sine</td>
<td>V.High</td>
<td>V.Long</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
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<tr>
<td>True Random</td>
<td>Fair</td>
<td>Short</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Fast Sine</td>
<td>High</td>
<td>Medium</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
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<tr>
<td>Impact</td>
<td>Low</td>
<td>V.Short</td>
<td>No</td>
<td>No</td>
<td>No</td>
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</table>

Table 3.3: Comparison of Common Excitation Functions.

Note: This method will show that a non-linearity exists but not what is causing the non-linearity, so it is better to put NO here.
### Table 3.4: Perspex Single Member: 25mm diameter members.

<table>
<thead>
<tr>
<th>Model Description</th>
<th>Source Type</th>
<th>No. of Points</th>
<th>No. of Blocks</th>
<th>Integ. Time</th>
<th>No. of Averages</th>
<th>Freq. L. U.</th>
<th>Resol. mHz.</th>
<th>Trans. No.</th>
<th>F.T.</th>
<th>Acc.</th>
</tr>
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<tbody>
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<td>0.958 m unit</td>
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<td>1</td>
<td>75ms</td>
<td>5</td>
<td>40 160</td>
<td>125</td>
<td>3156</td>
<td>4622</td>
<td></td>
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<td>2.000 m unit</td>
<td>S.S.</td>
<td>6</td>
<td>1</td>
<td>75ms</td>
<td>5</td>
<td>10 90</td>
<td>100</td>
<td>3156</td>
<td>4449</td>
<td></td>
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</table>

### Table 3.5: Perspex Single Member, Parameter Table: 25mm diameter members (2000 mm member length)

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq Value</th>
<th>%Damping Value</th>
<th>Peak Value</th>
<th>Phase Angle</th>
<th>Points in Fit</th>
<th>Generalized Stiffness</th>
<th>Generalized Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13.95</td>
<td>2.38</td>
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</tr>
<tr>
<td>2</td>
<td>38.38</td>
<td>3.04</td>
<td>2.86</td>
<td>-95</td>
<td>183</td>
<td>2.409E+05</td>
<td>4.143E+00</td>
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<tr>
<td>3</td>
<td>73.65</td>
<td>2.95</td>
<td>2.41</td>
<td>93</td>
<td>67</td>
<td>1.777E+06</td>
<td>8.298E+00</td>
</tr>
</tbody>
</table>

### Table 3.6: Perspex Pyramids: 6mm diameter members.

<table>
<thead>
<tr>
<th>Model Description</th>
<th>Source Type</th>
<th>No. of Points</th>
<th>No. of Blocks</th>
<th>Integ. Time</th>
<th>No. of Averages</th>
<th>Freq. L. U.</th>
<th>Resol. mHz.</th>
<th>Trans. No.</th>
<th>F.T.</th>
<th>Acc.</th>
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</thead>
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<tr>
<td>200mm Pyramid</td>
<td>S.S.</td>
<td>11</td>
<td>3</td>
<td>50ms</td>
<td>5</td>
<td>150 250</td>
<td>300</td>
<td>3156</td>
<td>1742</td>
<td></td>
</tr>
<tr>
<td>400mm Pyramid</td>
<td>S.S.</td>
<td>11</td>
<td>3</td>
<td>50ms</td>
<td>5</td>
<td>10 70</td>
<td>300</td>
<td>3156</td>
<td>1742</td>
<td></td>
</tr>
<tr>
<td>600mm Pyramid</td>
<td>S.S.</td>
<td>11</td>
<td>3</td>
<td>50ms</td>
<td>5</td>
<td>20 150</td>
<td>325</td>
<td>3156</td>
<td>1742</td>
<td></td>
</tr>
<tr>
<td>800mm Pyramid</td>
<td>S.S.</td>
<td>17</td>
<td>3</td>
<td>75ms</td>
<td>5</td>
<td>7 27</td>
<td>100</td>
<td>3156</td>
<td>4107</td>
<td></td>
</tr>
<tr>
<td>1.0 m Pyramid</td>
<td>S.S.</td>
<td>17</td>
<td>3</td>
<td>75ms</td>
<td>5</td>
<td>5 25</td>
<td>200</td>
<td>3156</td>
<td>3861</td>
<td></td>
</tr>
<tr>
<td>Mode No.</td>
<td>Freq</td>
<td>%Damping Value</td>
<td>Peak Value</td>
<td>Phase Angle</td>
<td>Points in Fit</td>
<td>Generalized Stiffness</td>
<td>Generalized Mass</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>-------</td>
<td>----------------</td>
<td>------------</td>
<td>-------------</td>
<td>---------------</td>
<td>-----------------------</td>
<td>-----------------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>205.3</td>
<td>3.33</td>
<td>1.38</td>
<td>-88</td>
<td>26</td>
<td>8.360E+06</td>
<td>5.024E+00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>206.8</td>
<td>6.48</td>
<td>1.88</td>
<td>-90</td>
<td>79</td>
<td>2.128E+07</td>
<td>1.260E+01</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>219.8</td>
<td>4.75</td>
<td>0.94</td>
<td>-87</td>
<td>30</td>
<td>1.137E+07</td>
<td>5.961E+00</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.7: Perspex Pyramid, Parameter Table: 6mm diameter members (200 mm member length)

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq</th>
<th>%Damping Value</th>
<th>Peak Value</th>
<th>Phase Angle</th>
<th>Points in Fit</th>
<th>Generalized Stiffness</th>
<th>Generalized Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>51.37</td>
<td>4.78</td>
<td>9.36E-02</td>
<td>-87</td>
<td>30</td>
<td>1.137E+06</td>
<td>1.124E+01</td>
</tr>
<tr>
<td>2</td>
<td>52.36</td>
<td>3.29</td>
<td>6.19E-02</td>
<td>91</td>
<td>28</td>
<td>9.937E+07</td>
<td>9.181E+02</td>
</tr>
<tr>
<td>3</td>
<td>57.14</td>
<td>3.68</td>
<td>2.96E-02</td>
<td>93</td>
<td>83</td>
<td>3.614E+06</td>
<td>2.804E+01</td>
</tr>
</tbody>
</table>

Table 3.8: Perspex Pyramid, Parameter Table: 6mm diameter members (400 mm member length)

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq</th>
<th>%Damping Value</th>
<th>Peak Value</th>
<th>Phase Angle</th>
<th>Points in Fit</th>
<th>Generalized Stiffness</th>
<th>Generalized Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>22.45</td>
<td>3.33</td>
<td>1.38</td>
<td>-88</td>
<td>26</td>
<td>1.279E+05</td>
<td>6.423E+00</td>
</tr>
<tr>
<td>2</td>
<td>22.84</td>
<td>2.48</td>
<td>1.88</td>
<td>-90</td>
<td>79</td>
<td>2.428E+05</td>
<td>1.179E+01</td>
</tr>
<tr>
<td>3</td>
<td>26.72</td>
<td>2.75</td>
<td>0.94</td>
<td>-87</td>
<td>30</td>
<td>1.737E+05</td>
<td>6.163E+00</td>
</tr>
</tbody>
</table>

Table 3.9: Perspex Pyramid, Parameter Table: 6mm diameter members (600 mm member length)
<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq</th>
<th>%Damping Value</th>
<th>Peak Value</th>
<th>Phase Angle</th>
<th>Points in Fit</th>
<th>Generalized Stiffness</th>
<th>Generalized Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12.55</td>
<td>2.56</td>
<td>3.515</td>
<td>-89</td>
<td>38</td>
<td>2.193E+05</td>
<td>3.526E+01</td>
</tr>
<tr>
<td>2</td>
<td>13.78</td>
<td>2.33</td>
<td>1.734</td>
<td>91</td>
<td>53</td>
<td>3.659E+05</td>
<td>4.881E+01</td>
</tr>
<tr>
<td>3</td>
<td>15.67</td>
<td>2.49</td>
<td>6.126</td>
<td>90</td>
<td>93</td>
<td>2.665E+05</td>
<td>2.749E+01</td>
</tr>
</tbody>
</table>

**Table 3.10: Perspex Pyramid, Parameter Table: 6mm diameter members (800 mm member length)**

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq</th>
<th>%Damping Value</th>
<th>Peak Value</th>
<th>Phase Angle</th>
<th>Points in Fit</th>
<th>Generalized Stiffness</th>
<th>Generalized Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.08</td>
<td>3.26</td>
<td>9.52</td>
<td>86</td>
<td>63</td>
<td>4.019E+03</td>
<td>1.558E+00</td>
</tr>
<tr>
<td>2</td>
<td>8.81</td>
<td>2.85</td>
<td>8.31</td>
<td>-89</td>
<td>57</td>
<td>5.217E+03</td>
<td>1.703E+00</td>
</tr>
<tr>
<td>3</td>
<td>9.95</td>
<td>2.68</td>
<td>6.51</td>
<td>-86</td>
<td>94</td>
<td>3.871E+03</td>
<td>9.904E-01</td>
</tr>
</tbody>
</table>

**Table 3.11: Perspex Pyramid, Parameter Table: 6mm diameter members (1000 mm member length)**

<table>
<thead>
<tr>
<th>Model Description</th>
<th>Source</th>
<th>No. of Points</th>
<th>No. of M.Blocks</th>
<th>Integ. No. of averages</th>
<th>Freq. L.</th>
<th>Resol. mHz.</th>
<th>Trans. No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>800mm Pyramid</td>
<td>S.S.</td>
<td>17</td>
<td>3</td>
<td>75ms</td>
<td>5</td>
<td>48 128</td>
<td>3156 4622</td>
</tr>
<tr>
<td>1.0 m Pyramid</td>
<td>S.S.</td>
<td>17</td>
<td>3</td>
<td>75ms</td>
<td>5</td>
<td>30 110</td>
<td>3156 3861</td>
</tr>
<tr>
<td>2.0 m Pyramid</td>
<td>S.S.</td>
<td>17</td>
<td>3</td>
<td>75ms</td>
<td>5</td>
<td>7 27 100</td>
<td>3156 4391</td>
</tr>
</tbody>
</table>

**Table 3.12: Perspex Pyramids: 25mm diameter members**
<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq</th>
<th>%Damping</th>
<th>Peak Value</th>
<th>Phase</th>
<th>Points in Fit</th>
<th>Generalized Stiffness</th>
<th>Generalized Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>57.41</td>
<td>4.05</td>
<td>8.55E-01</td>
<td>95</td>
<td>171</td>
<td>1.876E+06</td>
<td>1.442E+01</td>
</tr>
<tr>
<td>2</td>
<td>59.20</td>
<td>2.40</td>
<td>1.13E+00</td>
<td>-87</td>
<td>116</td>
<td>2.547E+06</td>
<td>1.841E+01</td>
</tr>
<tr>
<td>3</td>
<td>77.80</td>
<td>2.10</td>
<td>1.45E+00</td>
<td>-86</td>
<td>94</td>
<td>7.266E+06</td>
<td>3.041E+01</td>
</tr>
</tbody>
</table>

Table 3.13: Perspex Pyramid, Parameter Table: 25mm diameter members (800 mm member length)

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq</th>
<th>%Damping</th>
<th>Peak Value</th>
<th>Phase</th>
<th>Points in Fit</th>
<th>Generalized Stiffness</th>
<th>Generalized Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>36.57</td>
<td>3.26</td>
<td>4.52</td>
<td>93</td>
<td>27</td>
<td>1.936E+05</td>
<td>3.666E+00</td>
</tr>
<tr>
<td>2</td>
<td>38.81</td>
<td>2.65</td>
<td>5.61</td>
<td>88</td>
<td>37</td>
<td>4.997E+05</td>
<td>8.404E+00</td>
</tr>
<tr>
<td>3</td>
<td>52.57</td>
<td>2.68</td>
<td>3.51</td>
<td>91</td>
<td>48</td>
<td>7.041E+05</td>
<td>6.454E+00</td>
</tr>
</tbody>
</table>

Table 3.14: Perspex Pyramid, Parameter Table: 25mm diameter members (1000 mm member length)

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq</th>
<th>%Damping</th>
<th>Peak Value</th>
<th>Phase</th>
<th>Points in Fit</th>
<th>Generalized Stiffness</th>
<th>Generalized Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.17</td>
<td>4.02</td>
<td>2.21E+00</td>
<td>81</td>
<td>116</td>
<td>1.867E+04</td>
<td>5.629E+00</td>
</tr>
<tr>
<td>2</td>
<td>10.47</td>
<td>3.65</td>
<td>3.92E-01</td>
<td>81</td>
<td>31</td>
<td>1.513E+05</td>
<td>3.497E+01</td>
</tr>
<tr>
<td>3</td>
<td>11.92</td>
<td>4.91</td>
<td>4.97E-01</td>
<td>-97</td>
<td>76</td>
<td>1.149E+05</td>
<td>2.048E+01</td>
</tr>
</tbody>
</table>

Table 3.15: Perspex Pyramid, Parameter Table: 25mm diameter members (2000 mm member length)
<table>
<thead>
<tr>
<th>Model Description</th>
<th>Source type</th>
<th>No. of Points</th>
<th>No. of M.Blocks</th>
<th>Integ. time</th>
<th>No. of averages</th>
<th>Freq. L.</th>
<th>Resol. mHz.</th>
<th>Trans. No.</th>
<th>F.T. Acc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>200mm 9/21</td>
<td>S.S.</td>
<td>30</td>
<td>3</td>
<td>50ms</td>
<td>5</td>
<td></td>
<td>150</td>
<td>250</td>
<td>200</td>
</tr>
<tr>
<td>400mm 9/21</td>
<td>S.S.</td>
<td>30</td>
<td>3</td>
<td>50ms</td>
<td>5</td>
<td></td>
<td>35</td>
<td>75</td>
<td>200</td>
</tr>
<tr>
<td>600mm 9/21</td>
<td>S.S.</td>
<td>30</td>
<td>3</td>
<td>50ms</td>
<td>5</td>
<td></td>
<td>10</td>
<td>30</td>
<td>200</td>
</tr>
<tr>
<td>400mm 9/21</td>
<td>Imp.</td>
<td>30</td>
<td>1</td>
<td>50ms</td>
<td>5</td>
<td></td>
<td>0</td>
<td>400 bw375</td>
<td>3894</td>
</tr>
<tr>
<td>600mm 9/21</td>
<td>Imp.</td>
<td>30</td>
<td>1</td>
<td>50ms</td>
<td>5</td>
<td></td>
<td>0</td>
<td>400 bw375</td>
<td>3894</td>
</tr>
</tbody>
</table>

Table 3.16: Perspex Unit Building Block: 6mm diameter members.

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq Value</th>
<th>%Damping Value</th>
<th>Peak Value</th>
<th>Phase Angle</th>
<th>Points in Fit</th>
<th>Generalized Stiffness</th>
<th>Generalized Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>168.35</td>
<td>2.26</td>
<td>1.32</td>
<td>92</td>
<td>87</td>
<td>4.238E+05</td>
<td>3.788E-01</td>
</tr>
<tr>
<td>2</td>
<td>169.88</td>
<td>2.58</td>
<td>3.31</td>
<td>89</td>
<td>92</td>
<td>6.507E+05</td>
<td>5.711E-01</td>
</tr>
<tr>
<td>3</td>
<td>178.56</td>
<td>2.18</td>
<td>2.83</td>
<td>-86</td>
<td>74</td>
<td>7.481E+05</td>
<td>5.944E-01</td>
</tr>
</tbody>
</table>

Table 3.17: Perspex Unit Building Block, Parameter Table: 6mm diameter members (200 mm member length)

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq Value</th>
<th>%Damping Value</th>
<th>Peak Value</th>
<th>Phase Angle</th>
<th>Points in Fit</th>
<th>Generalized Stiffness</th>
<th>Generalized Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>41.85</td>
<td>2.18</td>
<td>0.781</td>
<td>-86</td>
<td>53</td>
<td>3.895E+05</td>
<td>5.633E+00</td>
</tr>
<tr>
<td>2</td>
<td>42.35</td>
<td>2.05</td>
<td>0.922</td>
<td>-84</td>
<td>45</td>
<td>4.131E+05</td>
<td>5.854E+00</td>
</tr>
<tr>
<td>3</td>
<td>44.59</td>
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<td>0.823</td>
<td>-91</td>
<td>71</td>
<td>6.814E+05</td>
<td>7.414E+00</td>
</tr>
</tbody>
</table>

Table 3.18: Perspex Unit Building Block, Parameter Table: 6mm diameter members (400 mm member length)
<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq Value</th>
<th>%Damping Value</th>
<th>Peak Value</th>
<th>Phase Angle</th>
<th>Points in Fit</th>
<th>Generalized Stiffness</th>
<th>Generalized Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17.87</td>
<td>2.42</td>
<td>3.82</td>
<td>-93</td>
<td>39</td>
<td>9.869E+04</td>
<td>7.828E+00</td>
</tr>
<tr>
<td>2</td>
<td>18.19</td>
<td>2.78</td>
<td>3.52</td>
<td>94</td>
<td>47</td>
<td>8.358E+04</td>
<td>6.399E+00</td>
</tr>
<tr>
<td>3</td>
<td>19.29</td>
<td>2.68</td>
<td>4.11</td>
<td>85</td>
<td>94</td>
<td>9.521E+04</td>
<td>6.481E+00</td>
</tr>
</tbody>
</table>

Table 3.19: Perspex Unit Building Block, Parameter Table: 6mm diameter members (600 mm member length)

<table>
<thead>
<tr>
<th>Model Description</th>
<th>Source type</th>
<th>No. of Points</th>
<th>No. of M.Blocks</th>
<th>Integ. No. of time. averages</th>
<th>Freq.</th>
<th>Resol.</th>
<th>L. U.</th>
<th>mHz.</th>
<th>Trans.No F.T. Acc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0 m 9/21 S.S.</td>
<td>50</td>
<td>3</td>
<td>75ms</td>
<td>5</td>
<td>20</td>
<td>50</td>
<td>150</td>
<td>3156</td>
<td>19293</td>
</tr>
</tbody>
</table>

Table 3.20: Perspex Unit Building Block: 25mm diameter members.

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq Value</th>
<th>%Damping Value</th>
<th>Peak Value</th>
<th>Phase Angle</th>
<th>Points in Fit</th>
<th>Generalized Stiffness</th>
<th>Generalized Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>29.95</td>
<td>3.61</td>
<td>10.31</td>
<td>89</td>
<td>83</td>
<td>1.617E+06</td>
<td>4.569E+01</td>
</tr>
<tr>
<td>2</td>
<td>30.29</td>
<td>3.27</td>
<td>11.42</td>
<td>-89</td>
<td>77</td>
<td>2.710E+06</td>
<td>7.483E+01</td>
</tr>
<tr>
<td>3</td>
<td>31.59</td>
<td>3.08</td>
<td>12.31</td>
<td>-88</td>
<td>59</td>
<td>2.719E+06</td>
<td>6.904E+01</td>
</tr>
</tbody>
</table>

Table 3.21: Perspex Unit Building Block, Parameter Table: 25mm diameter members.

<table>
<thead>
<tr>
<th>Model Description</th>
<th>Source type</th>
<th>No. of Points</th>
<th>No. of M.Blocks</th>
<th>Integ. No. of time. averages</th>
<th>Freq.</th>
<th>Resol.</th>
<th>L. U.</th>
<th>mHz.</th>
<th>Trans.No F.T. Acc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 Bay 200mm S.S.</td>
<td>43</td>
<td>3</td>
<td>50ms</td>
<td>5</td>
<td>100</td>
<td>300</td>
<td>200</td>
<td>3156</td>
<td>3861</td>
</tr>
<tr>
<td>8 Bay 200mm S.S.</td>
<td>43</td>
<td>3</td>
<td>50ms</td>
<td>5</td>
<td>60</td>
<td>160</td>
<td>150</td>
<td>3156</td>
<td>4107</td>
</tr>
</tbody>
</table>

Table 3.22: Perspex Triangular Arm Section: 6mm diameter members
<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq Value</th>
<th>%Damping Value</th>
<th>Peak Value</th>
<th>Phase Angle</th>
<th>Points in Fit</th>
<th>Generalized Stiffness</th>
<th>Generalized Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>113.0</td>
<td>3.26</td>
<td>9.52</td>
<td>88</td>
<td>63</td>
<td>3.809E+05</td>
<td>7.558E-01</td>
</tr>
<tr>
<td>2</td>
<td>124.5</td>
<td>2.85</td>
<td>8.31</td>
<td>91</td>
<td>57</td>
<td>5.738E+05</td>
<td>9.374E-01</td>
</tr>
<tr>
<td>3</td>
<td>132.1</td>
<td>2.68</td>
<td>6.51</td>
<td>-86</td>
<td>94</td>
<td>4.871E+05</td>
<td>7.071E-01</td>
</tr>
</tbody>
</table>

Table 3.23: Perspex Triangular Arm Section, Parameter Table: 3 Bay Unit.

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq Value</th>
<th>%Damping Value</th>
<th>Peak Value</th>
<th>Phase Angle</th>
<th>Points in Fit</th>
<th>Generalized Stiffness</th>
<th>Generalized Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>70.83</td>
<td>2.82</td>
<td>0.536</td>
<td>-93</td>
<td>69</td>
<td>2.519E+05</td>
<td>1.269E+00</td>
</tr>
<tr>
<td>2</td>
<td>97.78</td>
<td>2.19</td>
<td>0.438</td>
<td>-89</td>
<td>88</td>
<td>3.914E+05</td>
<td>1.037E+00</td>
</tr>
<tr>
<td>3</td>
<td>114.4</td>
<td>2.28</td>
<td>0.224</td>
<td>-96</td>
<td>102</td>
<td>9.831E+05</td>
<td>1.904E+00</td>
</tr>
</tbody>
</table>

Table 3.24: Perspex Triangular Arm Section, Parameter Table: 8 Bay Unit.

<table>
<thead>
<tr>
<th>Model Description</th>
<th>Source type</th>
<th>No. of Points</th>
<th>No. of M.Blocks</th>
<th>Integ. time</th>
<th>No. of Freq.</th>
<th>Resol. mHz.</th>
<th>Trans. No F.T.</th>
<th>Acc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 Bay 4.2m</td>
<td>S.S.</td>
<td>60</td>
<td>3</td>
<td>75ms</td>
<td>5</td>
<td>5</td>
<td>25</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3156</td>
</tr>
</tbody>
</table>

Table 3.25: Perspex Triangular Arm Section: 25mm diameter members

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq Value</th>
<th>%Damping Value</th>
<th>Peak Value</th>
<th>Phase Angle</th>
<th>Points in Fit</th>
<th>Generalized Stiffness</th>
<th>Generalized Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12.33</td>
<td>3.66</td>
<td>4.52</td>
<td>-93</td>
<td>84</td>
<td>2.923E+05</td>
<td>4.871E+01</td>
</tr>
<tr>
<td>2</td>
<td>12.81</td>
<td>3.85</td>
<td>4.31</td>
<td>-87</td>
<td>75</td>
<td>2.429E+05</td>
<td>3.741E+01</td>
</tr>
<tr>
<td>3</td>
<td>13.25</td>
<td>3.48</td>
<td>6.72</td>
<td>-88</td>
<td>94</td>
<td>4.681E+05</td>
<td>6.755E+01</td>
</tr>
</tbody>
</table>

Table 3.26: Perspex Triangular Arm Section, Parameter Table: 3 Bay Unit.
<table>
<thead>
<tr>
<th>Model Description</th>
<th>Source type</th>
<th>No. of Points</th>
<th>No. of M.Blocks</th>
<th>Integ. time</th>
<th>No. of averages</th>
<th>Freq. L. U.</th>
<th>Resol. mHz</th>
<th>Trans. No F.T.</th>
<th>Acc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.958 m unit</td>
<td>S.S.</td>
<td>6</td>
<td>1</td>
<td>75ms</td>
<td>5</td>
<td>100 1100</td>
<td>2500</td>
<td>3156</td>
<td>14158</td>
</tr>
<tr>
<td>0.990 m unit</td>
<td>Imp.</td>
<td>6</td>
<td>1</td>
<td>75ms</td>
<td>5</td>
<td>100 1100</td>
<td>b1250</td>
<td>3894</td>
<td>14158</td>
</tr>
<tr>
<td>1.080 m unit</td>
<td>Imp.</td>
<td>8</td>
<td>1</td>
<td>75ms</td>
<td>5</td>
<td>50 550</td>
<td>bw625</td>
<td>3894</td>
<td>14158</td>
</tr>
<tr>
<td>1.795 m unit</td>
<td>Imp.</td>
<td>8</td>
<td>1</td>
<td>75ms</td>
<td>5</td>
<td>0 400</td>
<td>bw250</td>
<td>3894</td>
<td>14158</td>
</tr>
<tr>
<td>1.966 m unit</td>
<td>Imp.</td>
<td>8</td>
<td>1</td>
<td>75ms</td>
<td>5</td>
<td>0 200</td>
<td>bw250</td>
<td>3894</td>
<td>14158</td>
</tr>
</tbody>
</table>

**Table 3.27: Carbon-PES Composite Single Member: 25mm diameter members**

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq</th>
<th>%Damping Value</th>
<th>Peak Value</th>
<th>Phase Angle</th>
<th>Points in Fit</th>
<th>Generalized Stiffness</th>
<th>Generalized Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>202.1</td>
<td>0.23</td>
<td>1.38</td>
<td>-88</td>
<td>76</td>
<td>3.153E+06</td>
<td>1.955E+00</td>
</tr>
<tr>
<td>2</td>
<td>534.7</td>
<td>0.21</td>
<td>1.88</td>
<td>-90</td>
<td>79</td>
<td>1.752E+07</td>
<td>1.552E+00</td>
</tr>
<tr>
<td>3</td>
<td>943.6</td>
<td>0.25</td>
<td>0.94</td>
<td>-87</td>
<td>80</td>
<td>2.437E+06</td>
<td>1.442E+01</td>
</tr>
</tbody>
</table>

**Table 3.28: Carbon-PES Composite Single Member, Parameter Table: 0.958 m.**

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq</th>
<th>%Damping Value</th>
<th>Peak Value</th>
<th>Phase Angle</th>
<th>Points in Fit</th>
<th>Generalized Stiffness</th>
<th>Generalized Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>188.8</td>
<td>0.25</td>
<td>1.467</td>
<td>89</td>
<td>67</td>
<td>1.706E+06</td>
<td>1.212E+00</td>
</tr>
<tr>
<td>2</td>
<td>502.5</td>
<td>0.19</td>
<td>1.255</td>
<td>-87</td>
<td>54</td>
<td>2.568E+07</td>
<td>2.576E+00</td>
</tr>
<tr>
<td>3</td>
<td>926.2</td>
<td>0.21</td>
<td>6.781</td>
<td>90</td>
<td>97</td>
<td>1.155E+08</td>
<td>3.410E+00</td>
</tr>
</tbody>
</table>

**Table 3.29: Carbon-PES Composite Single Member, Parameter Table: 0.990 m.**
<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq Value</th>
<th>%Damping Value</th>
<th>Peak Value</th>
<th>Phase Angle</th>
<th>Points in Fit</th>
<th>Generalized Stiffness</th>
<th>Generalized Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>56.97</td>
<td>0.23</td>
<td>1.587</td>
<td>-90</td>
<td>78</td>
<td>2.378E+05</td>
<td>1.855E+00</td>
</tr>
<tr>
<td>2</td>
<td>157.1</td>
<td>0.17</td>
<td>1.345</td>
<td>93</td>
<td>123</td>
<td>3.878E+06</td>
<td>3.980E+00</td>
</tr>
<tr>
<td>3</td>
<td>298.89</td>
<td>0.22</td>
<td>3.451</td>
<td>90</td>
<td>96</td>
<td>2.285E+07</td>
<td>6.478E+00</td>
</tr>
</tbody>
</table>

Table 3.30: Carbon-PES Composite Single Member, Parameter Table: 1.795 m.

<table>
<thead>
<tr>
<th>Model Description</th>
<th>Source Type</th>
<th>No. of Points</th>
<th>M.Blocks</th>
<th>Integ. time. averages</th>
<th>Freq. L. U.</th>
<th>Resol. mHz.</th>
<th>Trans. L. U.</th>
<th>Acc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat 2.0m unit</td>
<td>S.S.</td>
<td>9</td>
<td>2</td>
<td>75ms</td>
<td>5</td>
<td>15</td>
<td>35 100</td>
<td>3156</td>
</tr>
</tbody>
</table>

Table 3.31: Carbon-PES Triangular Section Unit: 25mm diameter members.

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq Value</th>
<th>%Damping Value</th>
<th>Peak Value</th>
<th>Phase Angle</th>
<th>Points in Fit</th>
<th>Generalized Stiffness</th>
<th>Generalized Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>22.20</td>
<td>0.22</td>
<td>1.587</td>
<td>91</td>
<td>34</td>
<td>5.671E+04</td>
<td>2.915E+00</td>
</tr>
<tr>
<td>2</td>
<td>26.56</td>
<td>0.21</td>
<td>1.345</td>
<td>89</td>
<td>23</td>
<td>7.448E+04</td>
<td>2.674E+00</td>
</tr>
<tr>
<td>3</td>
<td>29.38</td>
<td>0.19</td>
<td>1.451</td>
<td>90</td>
<td>46</td>
<td>1.172E+05</td>
<td>3.438E+00</td>
</tr>
</tbody>
</table>

Table 3.32: Carbon-PES Triangular Section Unit, Parameter Table: 2000 mm.

<table>
<thead>
<tr>
<th>Model Description</th>
<th>Source Type</th>
<th>No. of Points</th>
<th>M.Blocks</th>
<th>Integ. time. averages</th>
<th>Freq. L. U.</th>
<th>Resol. mHz.</th>
<th>Trans. L. U.</th>
<th>Acc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0 m Pyramid</td>
<td>S.S.</td>
<td>17</td>
<td>3</td>
<td>75ms</td>
<td>5</td>
<td>80</td>
<td>120 200</td>
<td>3156</td>
</tr>
<tr>
<td>2.0 m Pyramid</td>
<td>S.S.</td>
<td>17</td>
<td>3</td>
<td>75ms</td>
<td>5</td>
<td>20</td>
<td>60 200</td>
<td>3156</td>
</tr>
</tbody>
</table>

Table 3.33: Carbon-PES Composite Pyramids: 25mm diameter members.
<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq Value</th>
<th>%Damping Value</th>
<th>Peak Value</th>
<th>Phase Angle</th>
<th>Points in Fit</th>
<th>Generalized Stiffness</th>
<th>Generalized Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>116.06</td>
<td>0.23</td>
<td>1.278</td>
<td>-86</td>
<td>63</td>
<td>3.69E+07</td>
<td>6.96E+01</td>
</tr>
<tr>
<td>2</td>
<td>119.32</td>
<td>0.31</td>
<td>0.845</td>
<td>89</td>
<td>82</td>
<td>3.24E+07</td>
<td>5.77E+01</td>
</tr>
<tr>
<td>3</td>
<td>123.89</td>
<td>0.29</td>
<td>1.089</td>
<td>90</td>
<td>46</td>
<td>4.92E+07</td>
<td>8.13E+01</td>
</tr>
</tbody>
</table>

Table 3.34: Carbon-PES Composite Pyramids, Parameter Table: 1.0 m members.

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq Value</th>
<th>%Damping Value</th>
<th>Peak Value</th>
<th>Phase Angle</th>
<th>Points in Fit</th>
<th>Generalized Stiffness</th>
<th>Generalized Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>29.81</td>
<td>0.22</td>
<td>1.587</td>
<td>91</td>
<td>34</td>
<td>5.67E+04</td>
<td>2.91E+00</td>
</tr>
<tr>
<td>2</td>
<td>30.59</td>
<td>0.21</td>
<td>1.345</td>
<td>89</td>
<td>23</td>
<td>7.44E+04</td>
<td>2.67E+00</td>
</tr>
<tr>
<td>3</td>
<td>31.23</td>
<td>0.19</td>
<td>1.451</td>
<td>90</td>
<td>46</td>
<td>1.17E+05</td>
<td>3.43E+00</td>
</tr>
</tbody>
</table>

Table 3.35: Carbon-PES Composite Pyramids, Parameter Table: 2.0 m members.

<table>
<thead>
<tr>
<th>Model Description Type</th>
<th>Source</th>
<th>No. of Points</th>
<th>No. of M.Blocks</th>
<th>Integr. No. of time. averages</th>
<th>Freq.</th>
<th>Resol.</th>
<th>Trans.</th>
<th>No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0 m 9/21 S.S.</td>
<td>50</td>
<td>3</td>
<td>75ms</td>
<td>5</td>
<td>85</td>
<td>105</td>
<td>100</td>
<td>3156 3861</td>
</tr>
</tbody>
</table>

Table 3.36: Carbon-PES Composite Unit Building Block: 25mm diameter members

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq Value</th>
<th>%Damping Value</th>
<th>Peak Value</th>
<th>Phase Angle</th>
<th>Points in Fit</th>
<th>Generalized Stiffness</th>
<th>Generalized Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>103.88</td>
<td>0.38</td>
<td>6.455</td>
<td>-87</td>
<td>38</td>
<td>5.78E+06</td>
<td>1.35E+01</td>
</tr>
<tr>
<td>2</td>
<td>104.96</td>
<td>0.34</td>
<td>5.881</td>
<td>93</td>
<td>52</td>
<td>8.54E+06</td>
<td>1.97E+01</td>
</tr>
<tr>
<td>3</td>
<td>107.17</td>
<td>0.37</td>
<td>7.031</td>
<td>-89</td>
<td>44</td>
<td>4.77E+06</td>
<td>1.05E+01</td>
</tr>
</tbody>
</table>

Table 3.37: Carbon-PES Composite Unit Building Block, Parameter Table: Material 1.
### Table 3.38: Carbon-PES Composite Unit Building Block, Parameter Table: Material 2.

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq Value</th>
<th>%Damping Value</th>
<th>Peak Value</th>
<th>Phase Angle</th>
<th>Points in Fit</th>
<th>Generalized Stiffness</th>
<th>Generalized Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>98.72</td>
<td>0.28</td>
<td>3.337</td>
<td>-90</td>
<td>54</td>
<td>6.444E+06</td>
<td>1.675E+01</td>
</tr>
<tr>
<td>2</td>
<td>99.36</td>
<td>0.31</td>
<td>3.144</td>
<td>89</td>
<td>49</td>
<td>9.798E+06</td>
<td>2.514E+01</td>
</tr>
<tr>
<td>3</td>
<td>102.38</td>
<td>0.29</td>
<td>3.572</td>
<td>-88</td>
<td>71</td>
<td>3.462E+06</td>
<td>8.867E+00</td>
</tr>
</tbody>
</table>

### Table 3.39: Carbon-PES Composite Triangular Arm Section: 25mm diameter members

<table>
<thead>
<tr>
<th>Model Description</th>
<th>Source</th>
<th>No. of Points</th>
<th>No. of M.Blocks</th>
<th>Integ. No. of time, averages</th>
<th>Freq.</th>
<th>Resol. L. U. mHz.</th>
<th>Trans.No F.T. Acc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 Bay 3.998m S.S.</td>
<td>60</td>
<td>5</td>
<td>75ms</td>
<td>5</td>
<td>20</td>
<td>60 50</td>
<td>3156 14158</td>
</tr>
</tbody>
</table>

### Table 3.40: Carbon-PES Composite Triangular Arm Section, Parameter Table: 25 mm diameter members.

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq Value</th>
<th>%Damping Value</th>
<th>Peak Value</th>
<th>Phase Angle</th>
<th>Points in Fit</th>
<th>Generalized Stiffness</th>
<th>Generalized Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>39.86</td>
<td>0.26</td>
<td>2.511</td>
<td>88</td>
<td>67</td>
<td>1.823E+06</td>
<td>2.915E+01</td>
</tr>
<tr>
<td>2</td>
<td>40.97</td>
<td>0.29</td>
<td>2.239</td>
<td>89</td>
<td>86</td>
<td>1.639E+06</td>
<td>2.474E+01</td>
</tr>
<tr>
<td>3</td>
<td>42.38</td>
<td>0.21</td>
<td>3.044</td>
<td>-90</td>
<td>92</td>
<td>2.232E+06</td>
<td>3.148E+01</td>
</tr>
</tbody>
</table>

Table 3.40: Carbon-PES Composite Triangular Arm Section, Parameter Table: 25 mm diameter members.
3.10 Figures.

Figure 3.1 Typical Nodal Joint Configuration

2.0 m Perspex Member
Ref = 8Y+ Resp = 2Y-

Figure 3.2 Typical Frequency Response Function
Figure 3.3 Typical Support Configuration for Perspex Unit Building Block.
4.0 Analytical Modelling.

4.1 Introduction.

This section describes the theoretical work and its background using the computer software packages ABAQUS [134] and MSC/NASTRAN [135]. The formulation of finite element (FE) models which can be used to predict accurately the dynamic behaviour of the experimental structures is vital. The final proposed structure is such that it is not feasible to construct physically and test under 1 G conditions, so the use of a mathematical model is required (Wijker 1991 [136]). The FE models of the experimental structures are required to reveal the characteristics recorded during the modal tests. These include the accurate prediction of the various natural frequencies and a consistent mode shape. Great effort has been placed on the development of the initial models for each test unit with particular care being exercised on the distribution of mass and stiffness.

The finite element models also perform a useful pre-test function in that predictions can be made as to the optimal position of placement for the various transducers, as well as indicating the frequency bandwidth likely to be required. No allowances have been made for the suspension systems employed with the various experimental structures as tests indicated that they allowed the unit to attain a near unrestrained condition. Test runs were performed in which the presence of the lightweight suspensions were included into the analysis but these were later excluded as they had no noticeable effect upon the results.

The first part of this chapter describes the relevant parts of the two finite element packages used in the analyses, including the extraction procedures. A short section on the capabilities of the various element types used, or considered for use, in the analyses are also included. The second part contains a description of the various finite element models that have been formulated to describe both the perspex and the carbon-PES units. It should be noted that where the word 'mode 1' is used, it implies the first significant mode of the model after the rigid body modes (RBMs) of the unrestrained structure have been excluded. The
selection of the two FE packages was made for a variety of reasons but based upon the solution sequences and element formulations available as well as their availability.

4.2 ABAQUS.

4.2.1 Introduction.

As ABAQUS does not have any in-built limits on job size it is therefore capable of solving the large problems that are encountered. It incorporates a similar number of numerical analysis techniques and elements, to MSC/NASTRAN.

4.2.2 Static.

There are two solution procedures available within ABAQUS, both of which assume that the inertia forces are essentially zero, or only vary very slowly throughout the analysis. The first procedure is used when the response to a given load is required, whilst the second is utilised in the situation when a non-linear load-displacement response is present. This modified Riks method is an algorithm which allows effective solution of such cases.

In the linear case a choice is available in which either an automatic, based on a user specified tolerance, or user defined load increment is selected. For the non-linear case the loading is assumed not to change suddenly as in the case of a bifurcation.

4.2.3 Eigenvalue Extraction.

4.2.3.1 Real Eigenvalue Analysis.

ABAQUS contains several options for the linear case in which the eigenmodes of the system are examined. The eigenvalue extraction problem has
been comprehensively covered by many researchers in the past (Wilkinson 1965 [137]). The particular cases that arise from the use of finite elements can often be characterised by their very large but often narrow banded matrices and that usually only a small number of the eigenvalues are required. As in the case with MSC/NASTRAN the effect of damping is neglected during the eigenvalue extraction procedure and only symmetric matrices are considered, i.e. both $M$ and $K$ are symmetric and positive definite. Various authors have previously investigated the structural eigenvalue problem in respect of finite elements and a number of solution strategy methods have been proposed, with the Lanczos (Lanczos 1950 [138]) and the subspace iteration (Bathe 1972 [139]) methods appearing to be favoured by some.

As mentioned before it is quite common for only a few natural frequencies and modes of structural system to be extracted so ABAQUS uses a variant of the subspace iteration technique. This technique (Bathe 1971 [140]) is useful in that it can considerably reduce the cost of the analysis, when compared with other solution techniques, if large matrix bandwidths exist, as is frequently the case in structural problems. The intention of the method is to solve for the lowest 'p' eigenvalues and corresponding eigenvectors which satisfy the equation:

$$K\Phi = M\Phi\Lambda$$ 4.1

where $\Lambda = \text{diag } (\Lambda_i)$

$$\Phi = [\phi_1,...,\phi_p]$$

It should also be noted that the eigenvectors are also conform to the orthogonality conditions of (Bathe 1982 [141]):

$$\Phi^T K \Phi - \Lambda ; \quad \Phi^T M \Phi - I$$ 4.2

where $I$ is the identity matrix of order $p$ because $\Phi$ stores only $p$ eigenvectors. The technique redefines the size of the problem to more manageable proportions prior to solution. Hence the generalized eigenproblem becomes:
which are 3dB below the main beam peak. This can be determined from the following equation.

\[
\text{HPBW} = \frac{N \lambda}{D}
\]

where \( N \) = beamwidth factor dependant on the aperture illumination distribution.

\( D \) = circular aperture diameter.

\( \lambda \) = operating wavelength.

Generally speaking it is not possible to obtain the situation in which there is constant power distribution across the whole of the aperture in which case the antenna could be considered to be 100% efficient. A more realistic approach would consider that a tapered distribution would be achieved and that the antenna has an efficiency of 65%. The factor \( N \) varies between about 58 for a uniform distribution to about 75 for the tapered distribution case.

2.7.4 Power Gain.

The power gain of an antenna is normally defined in the direction of maximum radiation, for the case under investigation this would correspond with the boresight. The power gain provides very important information as to the ultimate performance of the system and it is closely associated with the transmitting power and the receiver sensitivity. Antenna size restrictions and weight will limit the maximum gain that can be achieved. For a circular aperture antenna the gain is derived from the following formula.

\[
G = 10 \log_{10} \left( n \left( \frac{\pi D}{\lambda} \right)^2 \right)
\]

where \( G \) is expressed in dBi.

\( D \) = circular aperture diameter.

\( \lambda \) = operating wavelength.

\( n \) = antenna efficiency factor.

Note: Power Gain is the gain over a reference antenna
where $K'$ and $M'$ are the reduced stiffness and mass matrices, of the structure. The mass matrix is frequently banded, in which case the half-bandwidth for the two matrices are the same. A banded mass matrix, obtained using a consistent mass analysis, is always positive definite, whereas a lumped mass matrix is only positive definite if the diagonal components are larger than zero. If the mass matrix is an identity matrix then this equation reduces to the standard eigenvalue problem, that is the eigenvalues and eigenvectors correspond to the frequencies squared and the vibration mode shape when unit mass is applied at each degree of freedom.

The optimum solution method for this generalised equation depends on a number of factors including the size, symmetry, and positive definiteness of the equations, and the number of eigenvalues required. However, the most common type of eigenproblem encountered is that of the 'standard' eigenproblem, or ones that can be reduced to the standard form. The main purpose of which is to allow the determination of the properties of the eigenvalues, eigenvectors and the generalised characteristic polynomials of the generalized eigenproblem from those of the standard form quantities (Bathe 1982 [142]). The secondary reason for performing the transformation is that a large number of solution strategies have been developed for the standard form.

The reduction of the reduced mass matrix $[M']$ from the generalized to the standard form is achieved in ABAQUS by utilizing a Cholesky decomposition into its lower triangular form, which is a special case of the more general case:

$$M' = SS^T$$

where
\[ S = L_M \]

where \( L_M \) is the lower triangular matrix.

The alternative method for the decomposition of mass matrix is the spectral technique which requires the solution of the complete eigensystem of that matrix. The Cholesky method is generally considered to be computationally more efficient than spectral decomposition because fewer operations are required to calculate \( L_M \) than the orthonormal eigenvectors and the diagonal eigenvalue matrix required in the spectral method.

ABAQUS solves the reduced mass and stiffness matrices in the sub-space by using the Householder (Strang 1976 [143]) with the Quarter Rotation technique. This method is considered to be very efficient in solving for the complete solution of a small eigenproblem, for which all eigenpairs must be found. The solution method assumes that the mass and stiffness matrices are positive definite and reduces the stiffness matrix to a tri-diagonal form by using the Cholesky decomposition of the mass matrix, as outlined above, and then by pre- and post-multiplying the stiffness matrix by the inverse of the lower and upper triangular matrices so that a new matrix of the following form is produced:

\[
[A] = [L_M']^{-1} [K'] [L_M']^{-1}
\]

where the matrix \([A]\) in this form is sometimes referred (Tauchert 1974 [144]) to as the inverse dynamical matrix.

ABAQUS then utilises the Quarter Rotation (QR) (Francis 1962 [145]) method, in an iterative procedure, to extract all the eigenpairs in the subspace. The QR procedure can be applied to the original stiffness matrix but to improve the efficiency of the solution, it is applied to the tri-diagonalized form of the matrix. The iterative algorithm is obtained by decomposing the tri-diagonal matrix...
For each iteration, by using an orthogonal (real and unitary) and an upper triangular matrix as follows:

\[
[T_n] = [Q_n] [R_n]
\]

where \([Q]\) is an orthogonal matrix,

\([R]\) is an upper triangular matrix.

and then applying,

\[
[T_{n+1}] = [R_n] [Q_n]
\]

where \([T]_{n+1}\) tends to \(\lambda\) and \(Q^1 ...Q^n\) tends to 0, as \(n\) tends to \(\infty\).

Hence, as the iteration progresses, the diagonal elements of \([T]^N\) tend to the eigenvalues which will be real and positive for structural systems. The convergence of the QR procedure is related to that of the inverse iteration technique. In this technique the rate of convergence of the results is related to the proximity of the base of the calculations to the values being extracted. ABAQUS utilizes this factor, and a numerical shifting facility is implemented internally to increase the convergence rates. After the convergence of each eigenvalue, which is very rapid in this algorithm due to the technique of internal shifting (ABAQUS [146]), the order of the tri-diagonal matrix is reduced by one. To calculate the eigenvectors associated with the eigenvalues the ABAQUS package utilizes the inverse power method.

To prevent ill-conditioning of the matrices, due to the absence of external restraints on the model, a numerical shift can be incorporated. This moves the starting point of the calculations away from the origin. A consequence of this is that the inclusion of a shift can be used to extract all the eigenvalues about a particular frequency of interest. This occurs because the eigenvalues are extracted in an order relating to how close they are to this new 'origin'.
It has been found that the unmodified version of the Householder method has been incorporated into the version of ABAQUS which is being used, therefore, the program is unable to consider systems containing singular mass matrices.

Once the eigenvalues have been calculated the eigenvectors are recovered. Pre-multiplication of the generalized eigenproblem by the transpose of the eigenvector matrix yields:

\[
[K_G] = [\Lambda] [M_G]
\]  \hspace{1cm} (4.9)

where:

\[
\]  \hspace{1cm} (4.10)

where:

\[
[\phi] = [\phi_1 \ldots \phi_n]
\]  \hspace{1cm} (4.11)

These matrices are frequently referred to as the generalized or modal stiffness and mass matrices of the structure, respectively.

Applying this transformation uncouples both \([K_G]\) and \([M_G]\) which only contain non zeros along the diagonal. Each mode \((\phi)_i\) represents a set of relative amplitudes and hence can be adjusted to make the generalized mass, \([M_G]_i\), for each mode have a preselected value. ABAQUS arbitrarily scales each eigenvector so that its maximum value is that of unity. This implies that a non-unit generalised mass for each mode is retained. This value for each mode is then used to calculate the modal participation factor which gives an indication of how strongly motion in a particular degree of freedom is present in each eigenvector, thus allowing determination of the type of motion occurring. Both of these values are used to calculate the modal effective mass which indicates the mass associated with each degree of freedom. The sum of these in any degree of freedom should
sum to the total model mass, providing that all the eigenvectors are included and that mass is not positioned at a kinematically restrained point.

4.3 MSC/NASTRAN.

4.3.1 Introduction.

This finite element program incorporates a number of numerical analysis techniques which have the capability of solving very large problems in an efficient manner. There are no inherent problem size limitations.

4.3.2 Static Analysis.

The solution procedure adopted for the static analysis uses a technique that allows for the same kind of reduction in the number of calculations required that exists with the more usual sub-structuring techniques. MSC/NASTRAN uses an active/passive column technique to eliminate internal variables before assembling the matrices. The matrix partitioning procedure which is available in the rigid formats allows the user to divide the degrees of freedom into two sets: the 'a' set which is retained and the 'o' set which is omitted in subsequent processing. This allows the 'o' set grid points in different parts of the structure to be uncoupled from each other, an example of this could be that grid points in the interiors of regions can be placed in the 'o' set whilst the grid points along the interfaces between such regions are retained for calculation in the 'a' set. The generation of the reduced stiffness matrix and the reduced load vector can then take place for each region. The procedure by which MSC/NASTRAN performs this sub-structuring can be summarized by the following three steps:

1. Each of the substructures is analyzed to produce a matrix which describes its behaviour at the boundary degrees of freedom.
2. These matrices are then combined in the appropriate manner to form a description of the structure. This set of matrices are then analyzed by MSC/NASTRAN.
3. The individual substructures are then analyzed again using the vector of the displaced components contained in the 'a'set which was determined in the main analysis.

This coupled with the grid point sequencing procedure which operates on either the complete problem or internal components of the individual super elements results in efficient looping and forward/backward substitution.

4.3.3 Eigenvalue Extraction.

The procedures available for solving the eigenvalue extraction problem may be divided into two groups, namely the transformation methods and the 'tracking' methods. In a transformation method the matrix is first transformed, whilst still retaining and preserving it's eigenvalues, into a special more easily handleable form, e.g. diagonal or tri-diagonal, from which the eigenvalues may be more easily extracted. By contrast in the tracking method the roots of the equations the eigenvalues, are extracted one by one using an iterative process applied to the original dynamic matrix. There are three algorithms that can be used for the evaluation of real eigenvalues in MSC/NASTRAN. These are:

- The Inverse Power Method with shifts (a root tracking method)
- The Givens Triangularization Method (a transformation method)
- The Modified Givens Method.

It is up to the user to select which of these will be used but factors affecting this selection include the number of eigenvalues and eigenvectors to be extracted, the formation of the matrices and the order of the system matrices. Previous studies have indicated that the tri-diagonal or Givens method when used with a reduced number of degrees of freedom in the 'a'set is the most efficient if there are a significant number of eigenvectors to be extracted, as can be the case with large problems. This is because the bulk of the calculations for the problem are carried out prior to the extraction of the first eigenvalue and so the method is not highly
dependant on the number of eigenvalues to be extracted. In contrast to this, the 'tracking' method is approximately linearly proportional in the amount and time of calculation required to the number of eigenvalues to be extracted. Hence the general guideline about the choice of eigenvalue extraction technique could be that for small numbers of eigenvalues the inverse power method should be employed.

It should also be noted that the triangulation method is not suitable for use in cases where buckling or complex eigenvalues are to be extracted. This is due to the restrictions that are placed on the form of matrix which may be handled. The procedure would require that the differential stiffness matrix be negative definite, which very rarely occurs. Normally in the theory of structures if an elastic structure is subjected to a set of forces, it is a necessary prerequisite that the 'local stiffness matrix' is positive definite if the equilibrium is to be established as stable (Bajpai 1985 [147]). Similarly the kinetic energy of the system is always positive except for the special case in which all the velocities are zero. The mass matrix is then said to be positive definite.

The analyses of real and complex eigenvalues are performed using two completely separate modules within MSC/NASTRAN. The solution for real eigenvalues makes use of the symmetric mass and stiffness matrices and uses the stiffness and differential stiffness matrices to compute the buckling modes. The solution module for the extraction of the complex eigenvalues is used to calculate the damped modes of vibration of a structure. Some of the restrictions placed on real eigenvalue extraction that is, the requirement that there are symmetric mass and stiffness matrices and that no damping of the motion is considered, are removed.

4.3.4 Real Eigenvalue Analysis.

For the case of free vibration modes real eigenvalue analysis is used to compute the results. The general form of the second order differential equation from which all the analysis types stem from is;
\[ [M]\{u''\} + [C]\{u\} + [K]\{u\} = \{P(t)\} + \{N\} + \{Q\} \]

where \([M]\) is the mass matrix.
\([C]\) is the damping matrix.
\([K]\) is the stiffness matrix.
\([P(t)]\) is a vector of time dependent forces.
\([N]\) is a vector of forces dependent on vectors \([u]\) and \([u']\).
\([Q]\) is a constraint forces vector.
\([u]\) is the displacement of the grid points vector.
\([u']\) is the vector of grid point velocities.
\([u'']\) is the vector of grid point accelerations.

The constraints of real eigenvalue analysis reduce this general equation by setting \([C]\), \([P(t)]\) and \([N]\) to null matrices and vectors respectively. This implies that no damping is present, no external forces are applied to the system and that there are no non-linear force or moment functions of unknown displacements. A slight restriction of MSC/NASTRAN is that it does not consider second time derivatives of unknown displacements. A further requirement is that the mass and stiffness matrices are both real and symmetric. The global stiffness matrix is formed by merging together the stiffness matrices for all the elements. This matrix is frequently singular. A non singular stiffness matrix is formed by eliminating from the displacement vector those components of motion which are zero, or are related to other components of motion in a linear fashion, these form the single and multiple point constraints applied to the structure, the rigid body motions and those eliminated by the use of either static or dynamic condensation after the application of the single and multi-point constraints.

Real eigenvalue analysis also assumes that all parts of a structure are vibrating sinusoidally with the same frequency and that they are in phase. By substitution into the general equation of these constraints the following condensed form is achieved.
which can be rearranged into the following form

$$[K - \lambda M] \{\phi\} = 0$$

where $\lambda = \omega^2$

The non-trivial solution to this equation occurs only for a discrete set of frequencies when the matrix $[K - \lambda M]$ is singular. These frequencies, the eigenfrequencies, have a corresponding vector $\{\phi\}$, the eigenvector. Each eigenfrequency and its associated eigenvector define a free vibration mode of the structure. It should be noted that the number of eigenvectors is less than or equal to the order of the displacement vector, it can only be less if there is a null row and column in the mass matrix.

MSC/NASTRAN incorporates both the Givens and the modified Givens tridiagonal solution methods. Both of these methods are based upon the standard eigenvalue problem. The differences between the two techniques can be summarised in that the modified Givens method redefines the eigenvalue used in the problem by incorporating a shift point prior to applying the Cholesky decomposition. The advantage that this achieves over the traditional method is that it can deal efficiently with a mass matrix that contains massless degrees of freedom, that is, a singular mass matrix. The Givens method requires a positive definite mass matrix but there are no restrictions on the stiffness matrix other than it must be symmetric. The technique handles the dense matrices that are frequently produced as a result of the various reduction methods that may be employed, but it does not take advantage of any scarcity of the matrices which may occur in small problems. The procedure adopted for this technique can be broken down into the following stages. First a Cholesky decomposition of the mass matrix is performed (Wilkinson 1965 [148]). The eigenvalue problem is then reduced to its standard form:
The \([J]\) matrix is then transformed using the Givens method (Wilkinson 1965 [149]) into a tri-diagonal matrix, prior to extraction of all of the eigenvalues using a modified QR algorithm. The corresponding eigenvectors are then calculated and transformed using inverse iteration.

The modified Givens method follows a similar procedure, except that instead of performing a Cholesky decomposition upon just the mass matrix, the decomposition occurs on the modified matrix;

\[
[K + \lambda A M] = [L] [L]^T
\]

where \(\lambda\) is an internally selected positive value used to optimize the eigenvalue extraction procedure.

The standard form of the eigenvalue problem then takes the following form:

\[
[J - \lambda I] \{w\} = 0
\]

where

\[
\lambda = \frac{1}{\lambda + \lambda_S}, \quad \{J\} = [L]^{-1} [M] [L]^{-1}, T
\]

The rest of the procedure is then the same as for the Givens method outlined above. A fuller explanation of the Cholesky decomposition is given in the sections on ABAQUS.
The final method available in MSC/NASTRAN for eigenvalue extraction is the inverse power method incorporating shifts. This method is particularly attractive if the matrices are sparsely populated and if only a small number of eigenvalues are required. The eigenvalue problem can be stated as follows:

\[ [K - \lambda M] \{u\} = 0 \]  \hspace{1cm} (4.19)

where \( \lambda = \lambda_0 + \Lambda \)

\( \lambda_0 \) is the shift point

The iteration algorithm is then

\[ [K - \lambda_0 M] \{w_n\} = [M] \{u_{n-1}\} \]  \hspace{1cm} (4.20)

where

\[ \{u_n\} = \frac{1}{C_n} \{w_n\} \]  \hspace{1cm} (4.21)

where \( C_n \) is equal to the value of the element of \( \{w_n\} \) with the largest absolute value.

Once the value of \( \{u_n\} \) has been calculated it is moved to the right-hand-side of the equation and the iteration resumes. The value of \( 1/C_n \) tends to \( \lambda_1 \), the shifted eigenvalue that is nearest to the shift point. It should be noted that the inclusion of the shift point in the method, enhances it's capabilities considerably over the standard inverse power method, particularly in the structural problems being considered where rigid body modes are encountered (MSC/NASTRAN 1983 [150]).

The recovery of the eigenvectors in MSC/NASTRAN is similar in some respects to that in ABAQUS. The use of the eigenvectors to transform the mass and stiffness matrices into a diagonal form is also employed. MSC/NASTRAN utilizes two methods for the normalization of the eigenvector. The first method
normalizes the eigenvector to the modal mass, i.e. $M_i = 1$, whilst the second follows the same pattern as in ABAQUS in that the largest value within the eigenvector is normalized to one.

The pyramidal and the 9/21 models under examination exhibit an interesting phenomenon in that being symmetrical they have triplets of modes. In addition due the nature of their support they possess rigid body modes corresponding to a frequency of zero hertz.

4.4 Dynamic Analysis using Modal Superposition.

The use of a linear dynamic analysis using the modal superposition technique is not new but it allows a valuable insight into the dynamic behaviour of the system under consideration. It is relatively inexpensive computationally speaking when compared with the direct integration methods. The basic concept of this analysis type is that the response of the structure is expressed by using a reduced number of eigenmodes obtained from the whole system. Normally only those eigenmodes that are close to the frequencies of interest are used, these are normally the lowest few frequencies in the cases being considered. Again the orthogonality of the eigenmodes decouples the system. The linear dynamic response of a structure under load can be described in terms of the contributions of the individual responses of its modes using the mode superposition method. A mode superposition analysis can be performed using experimentally obtained modal data as well as finite element generated data so it can be considered as either a viable alternative to, or a check of the FEM. ABAQUS contains a procedure which allows the analysis of the steady state harmonic response of the structure. The response of the structure to the forcing function is expressed in terms of the peak amplitudes and phase relationships as functions of frequency. This allows for comparisons between predicted FE results and those determined experimentally to be readily obtained. An advantage of the experimental mode superposition technique is that a description of the material properties of the
structure is not required, this information being implicitly contained in the modal parameters determined in the tests.

The equilibrium equations governing an undamped multi-DOF linear system subjected to an external forcing function can be written in matrix form as (Ottens 1979 [151]):

\[
[M] \{u\} + [K] \{u\} = \{F\}
\]

where \{u\} is the vector of displacements of the system,
\[F = [F(t)]\] is a vector of externally applied loads,
and \([M]\) and \([K]\) are the mass and stiffness matrices respectively.

This equation may be transformed from the physical to the modal coordinates by using a vector of modal or generalised coordinates \(q\) and the modal matrix \([\phi]\).

\[
\{u\} = [\phi] \{q\}
\]

Hence, by substituting this expression for the displacement vector \(\{u\}\) we obtain:

\[
\]

The pre-multiplication of each term of this equation by \([\phi]^T\) results in the following equations:

\[
[M_G] \{q''\} + [K_G] \{q\} = \{P_G\}
\]

where \(\{P_G\} = [\phi]^T \{F\}\) the vector of modal or generalized forces.
The above equations of motion are uncoupled by this transformation using the eigenvectors and hence the contribution of each mode can be evaluated separately as this is the equation of motion for mode \( n \) of a SDOF system.

This procedure can be used to obtain an independent SDOF equation for each mode of vibration of the structure. Hence, by utilizing the generalised coordinates, the equations of motion are transformed from a set of \( n \) simultaneous different equations, which are coupled by the off-diagonal terms in both the mass and stiffness matrices, into a set of \( n \) independent generalised coordinate equations. The response of the structure can therefore be obtained by solving for the dynamic response of each modal or generalised coordinate and then superimposing these using equation 19 to obtain the response of the structure in the original coordinates (Clough 1975 [152]).

Rearranging this equation by dividing each term by the modal mass matrix and making use of the relationship between the modal mass and the modal stiffness with the frequency of vibration the following relationship for the response of each mode is obtained:

\[
\{q''\}_i + [\lambda]_i\{q\}_i = \frac{\{P\}_i}{[M]_i} \tag{4.26}
\]

The general solution to the excitation of an undamped single DOF system is given by the combination of the complementary solution and the particular solutions. These take the form:

\[
q_i = A_1 \cos(\omega_n t) + A_2 \sin(\omega_n t) + (q_i)_p \tag{4.27}
\]

where \( A_1, A_2 \) are arbitrary constants determined from the initial conditions, and \( (q_i)_p \) is the particular solution for the mode.

These values are collected for each mode to find the total response of the structure and are then back-transformed using equation 19 as stated above.
In order to minimize the response of a structural unit to a given modal loading it can be seen that there is a requirement to reduce the value of $Q_{limax}$. This may be achieved by either maximizing the modal masses or the natural frequency of the modes. The former is generally not suitable for the space environment were un-necessary increases in the static mass of a structure should be avoided because of the transportation costs. The second case is usually adopted with the frequency being maximised by ensuring that the modal stiffness is maximized for each mode. This procedure usually results in, for a given stiffness, a minimum mass design for the structure. The problem can also be avoided if the external loading, expressed in normal coordinates, is zero for that mode.

The practise of mode superposition is usually done in one of three ways. The most conservative approach is to add numerically the responses of the modes, which yield the upper limit of the combined responses. This approach is reasonable in the cases where the contribution of the fundamental mode dominates the response of the structure. Alternative approaches have been suggested (Fertis 1973 [153]) in which a less conservative approach is used. One such method involves taking the sum of the fundamental mode and the square root of the sum of the squares of the higher modes. The advantage of this method is that it takes account of the case in which the contribution of the higher modes is considerable. The accuracy of the method is dependant upon the number of modal responses considered so some engineering judgement is required so as to select the appropriate number for individual models.

4.5 Direct Integration Dynamic Analyses.

An alternative solution technique to that of mode superposition are the direct integration methods. In these methods it is assumed that variations of displacements, velocities, and accelerations occur within a time interval, $\delta t$. These assumptions are then employed to solve the equilibrium equation at each discrete time interval, $\delta t$, in order to obtain the unknown variables of the system. There are two classes of this method the implicit and the explicit solution technique.
ABAQUS employs the simple central difference method for stress analysis problems. There are stability problems with the technique involving the time required for the elastic wave to cross the smallest element dimension in the model mesh (ABAQUS 1989 [154]) and as such are considered to be only conditionally stable. Implicit schemes remove this problem and one such method, the Hilber-Hughes-Taylor implicit scheme (Hilber 1978 [155]) for direct time integration, is used in ABAQUS for its dynamic analyses. The variables, outlined above, are assumed to change within each time step, $\delta t$. This process is similar to that found in the linear acceleration form of the Newmark implicit method, which is frequently used in practice (Bathe 1982 [156]). Although these methods satisfy slightly different equilibrium equations for each time step (ABAQUS 1987 [157]), the scheme can be regarded as a development of the Newmark method because the accuracy, stability, and cost of a solution procedure is related to the assumed changes of the variables within each time step in the problem (Bathe 1982 [158]).

In the Hilber-Taylor-Hughes scheme a single parameter, $\alpha$, is used to impose a controlled amount of numerical damping into the solution. At each increment, a balance of the forces at the beginning and end of the time step and a weighted average of the static forces at the beginning and end of the time step are solved for the system's accelerations. The solution is then completed by using the following Newmark integration formulae for the displacements and velocities of the system respectively:

$$U_{(t+\Delta t)} = U_{(t)} + \Delta t \dot{U}_{(t)} + \frac{\Delta t^2}{2} \left( \frac{1}{2} - \beta \right) \ddot{U}_{(t)} + \beta \ddot{U}_{(t+\Delta t)} \tag{4.28}$$

and

$$\dot{U}_{(t+\Delta t)} = \dot{U}_{(t)} + \Delta t \left( 1 - \gamma \right) \ddot{U}_{(t)} + \gamma \ddot{U}_{(t+\Delta t)} \tag{4.29}$$

with the $\alpha$ parameter of the Hilber-Hughes-Taylor method being related to variables $\beta$ and $\gamma$ used in the Newmark equations as follows:
\[
\beta = \frac{(1-\alpha)^2}{4}, \quad \gamma = \frac{1-\alpha}{2}
\]

with the value of \(\alpha\) varying between the limits of

\[-\frac{1}{3} \leq \alpha \leq 0\]

By altering the value of \(\alpha\) the amount of numerical damping introduced can be easily controlled. This numerical damping is valuable because it can be used to rapidly suppress the high frequency numerical noise that it generated as the time step changes. By setting the value of \(\alpha\) to its maximum of 0 results in no numerical damping being applied, and the integration scheme becomes a trapezoidal rule. Conversely, by allowing \(\alpha\) to equal its minimum of value -1/3 results in significant numerical damping of the calculated structural dynamic response. Numerical tests performed by the manufacturer indicate that a value for \(\alpha = -0.05\) causes rapid damping of these undesired responses whilst conserving the desired lower frequencies.

It should be noted that the cost of a direct integration analysis is directly proportional to the number of time steps required for the solution. This implies that the optimum value for the time step must be selected which provides the desired accuracy, but no more, otherwise the analysis will become unduly expensive. The use of higher order elements can be effective in implicit time integration in the analysis of dynamics problems. However, if the model is so large that it contains a large bandwidth it can become more efficient to use the lumped mass approach and reduce the time step.

Considering the above it appears that the mode superposition method should provide the greater accuracy of the two techniques because the higher, and so probably less accurately known, frequencies of the finite element are excluded from the analysis. However by considering the use of the unconditionally stable direct integration method it is possible to select a value of \(\delta t\), which assuming that
the analysis accurately predicts all the important frequencies, which allows accurate integration of the first few equations and thereby neglects the higher frequency responses of the system. Hence the choice of method is largely one of effectiveness and possibly cost.

4.6 ABAQUS Element Formulations.

The formulation of the elements chosen to model a structure determines the accuracy of the analyses to a very great extent. Elements embodying the desirable attributes for frequency extractions were selected from the ABAQUS library for the development of the models in this study. The suite of elements, whose compatibility is assured in the ABAQUS package, was also chosen with the view that they would be equally applicable to the analysis of both the physical structures surveyed experimentally and of the behaviour of the large skeletal systems subsequently investigated.

4.6.1 Beam Element.

The basic formulation of this element is the simplified Cosserat beam theory (Dupuis 1969 [159]). This beam element was selected in preference to others in the element library because it possesses transverse shear deflection modelling capabilities. The effect of this capability is that as the element length increases, in comparison with its cross-section dimension, the element behaviour changes from a 'shear beam' (Timoshenko 1956 [160]) to one in which the cross section is forced to remain normal to the defined beam main axis. The rotary inertia terms are included in the formulation of the elements mass matrix. The two node, linear form of this element, the 'B31' element, was chosen in preference to the three node, quadratic version which is also provided in the package. This was because the presence of the additional mid-element nodes, associated with the use of the higher order beam element, would substantially increase the size of the model, particularly in the case of the full structure, while not contributing significantly to the modal behaviour of these systems. Numerical tests performed
by the manufacturers, Hibbitt, Karlsson and Sorensen, Inc have shown that by lumping the mass, for this linear element, at each node results in a more accurate representation of the eigenvalues for a given mesh. All the first order elements in ABAQUS use 'lumped masses' which results in a diagonal mass matrix. For this reason it has not been possible to perform comparison tests using a consistent mass formulation for the element. Nevertheless, extensive use of the B31 element has been made throughout the investigation and it has been found to be highly accurate in the mesh densities in which it has been used.

4.7. MSC/NASTRAN Element Formulation.

The structural element library in MSC/NASTRAN consists of a wide range of elements which may be categorised as having either no, one, two or three dimensions. The elements considered for this analysis have come mainly from the first two categories. The none dimensional elements are those associated with mass etc.

4.7.1 BAR Element.

The element used in the MSC/NASTRAN models equivalent to that used in ABAQUS is the BAR element. This is a sub-class of the straight beam type elements. It does not allow for variation in the cross-section axially nor does it possess an off-set shear centre capability. The neutral axis of the BAR element is considered to coincide with the local x-axis. The element possess a transverse shear capability and allows for stress recovery at up to four points in the cross-section at either end. MSC/NASTRAN provides two methods for calculating the masses associated with elements. These are the lumped or consistent mass formulations. The lumped mass formulation is generally used unless a consistent mass matrix is specifically requested. The lumped mass method only associates masses with the translational degrees of freedom. The mass is then distributed to the nodal points and a portion assigned to each of the translational degrees of freedom. In the consistent mass (Meirovitch 1986 [161]) matrix approach the mass
matrix is defined by using a consistent energy formulation. The work done by concentrated inertia forces calculated using the consistent mass matrix is the same as the work done by the distributed inertia forces. Hence, by using the consistent mass matrix in conjunction with the accelerations of the node points the equivalent work inertia forces can be calculated. There is much controversy as to which method provide the most accurate results. It seems that the decision on which of the two formulations to employ depends upon the type of element being utilized and the type of analysis being performed (Haggenmacher 1980 [162]).

4.8 Element Mesh Density.

Recommendations as to the minimum number of elements necessary to model a beam-type structure in frequency extractions have been suggested by other researchers (Schaffer 1984 [163]). This estimation was based on the number of vibrational modes to be examined multiplied by a factor of ten. Consideration was given to these guidelines but parameter studies have shown that this degree of density is not required for the structures under consideration. The mesh density is dependent upon a number of factors including the type of analysis being performed and the type of results required. However, to maximize the benefits of the mesh chosen it is important that the elements are placed along the expected paths of development of these modes to ensure their adequate modelling. It is also possible to determine, using this empirical rule, an estimation of the mesh densities required to model a structure in both static and direct integration dynamic analyses. The above guideline has been used in to assist in determining a suitable mesh density for all the models generated during this research. It has been observed during this and other studies that, in contrast to the discretization of continua, the optimum element DOF frontwidth for skeletal systems does not necessarily increase with mesh refinement. Hence, considerations of DOF frontwidths have not placed undue constraints upon the number of elements utilized in the various models examined in this study.
4.9 Mass Distribution.

The effect of the modelling of the distribution of mass within a skeletal structure when performing eigenvalue extractions has been found to be quite sensitive. This effect has also been observed by other researchers (Bertram and Conrad 1986 [164]). Consequently, particular attention has been paid to this property when developing the FE representation of the various structures. In general the lumped mass formulation has been employed in preference to the consistent mass method. This is because of the types of elements selected for the analysis have been shown to yield the most accurate results when this method is used.

4.10 Pre/Post Processing.

During the course of this research it became necessary to develop a number of pre and post processing facilities. The pre-processor programs were developed as the version of ABAQUS available and the effects of the off-site running of the program did not provide a suitable model development facility. The various programs either generated and plotted the mesh or optimized it prior to submission to ABAQUS for solution. The post-processing programs were developed for although ABAQUS provides a post-processing facility, the constraints of running the program off-site required that the inevitably large data files associated with these skeletal models be transferred via the available networks. The post-processing programs either extracted data directly from the output files for direct examination or provided graphical interpretations of the results obtained from the different types of analysis performed. Comparisons between the finite element results and those obtained from experimental methods were also obtained using this post-processing programs.

The use of some of these pre- and post-processors was not required for the analyses performed using MSC/NASTRAN. The MSC/XL pre- and post-processor became available in the latter part of the research but prior to that much of the
data preparation and results processing were performed by using the SDRC/I-DEAS FEM package (I-DEAS 1991 [165]). A self-written conversion program was developed to convert a mesh output file from the MSC/XL program into a mesh input file for use with ABAQUS.

4.11 Solution Techniques.

ABAQUS uses the element frontal solution method (Irons 1970 [166]) to solve the systems of simultaneous equations presented in the analyses. A benefit of the frontal solution technique is that at any time only the DOF of the active elements are included in the system of equations being solved and so the resulting core requirements of the package are reduced, with an ensuing reduction in the cost of the analyses. MSC/NASTRAN uses a number of different solution sequences depending upon the type of dynamic analysis being employed.

It should be noted that no definitive method exists for reordering the elements to minimize the maximum DOF frontwidth encountered in a model. Both ABAQUS and MSC/NASTRAN internally optimize the wavefront prior to solution. Consequently, whilst some attention was paid to the optimization of the skeletal models analyzed in this study using one of a number of the element renumbering algorithms available (Sloan 1983 [167]), final optimization was performed by the internal numbering sequences.

4.12 Finite Element Model Analysis.

4.12.1 Introduction.

This section details the development and subsequent analysis of the numerous finite element models that were examined during the course of the project. The first section considers the perspex models which were considered to have isotropic material properties. It has been found to have a modulus of elasticity of perspex materials is dependent on the frequency at which it is being stressed (Read 1978 [168]). It was therefore necessary to determine the bandwidth
in which the various structures would be tested so that the material properties for this plastic could be calculated. For most models the lower modes of vibration of the perspex structures appeared within a bandwidth of 250 Hz. The material properties were therefore determined as being 4.5 GN/m$^2$ for its modulus of elasticity, $E$, and 1.71 GN/m$^2$ for its shear modulus, $G$. The mass density of perspex was measured as 1192.38 Kg/m$^3$.

The second section considers the finite element modelling of the structures manufactured from the carbon composite material. Following tests carried out on glass reinforced composites in which the material properties were considered to be isotropic and yet there had been no loss in correlation between the experimental and F.E. results a decision was made that for the purposes of the project the carbon composite was considered to have isotropic material properties. This simplified the analyses in general but was of considerable importance when the full-scale structure was analyzed. In house testing of the composite materials supplied resulted in the following values of 60.0 GN/m$^2$ for the Modulus of Elasticity, $E$, and 0.25 for the Poisson's ratio, being adopted for the FE models containing the original Carbon-PES composite material. The two constants were obtained from $E_1$ and $\mu_{12}$ measurements taken from static tensile tests on appropriately strain gauged tubular members (Hollaway 1988 [169]). These values were subsequently modified when the orientation of the fibre reinforcement and the matrix were changed following the cracking that had been experienced in the accelerated thermal cycling tests (Hollaway 1990 [170]). The values determined for this modified material were 58.0 GN/m$^2$ for the modulus of elasticity and 0.35 for the Poisson’s ratio. The mass density of the graphite reinforced composite was measured as 1382 Kg/m$^3$.

The mathematical models use a beam type element (B31 Abaqus library); this is a two node beam suitable for use in 3 dimensions. This element has been used in preference to the pipe elements available as the latter gives detail through the cross-section rather than along the length of the section. A similar element BAR is used in MSC/NASTRAN, this defines a simple two node/grid point beam
element. Such elements have their properties and behavioural variables continuously distributed along their length which is defined as the line joining two node/grid points. They are assumed to have a uniform cross-sectional area and material properties. The B31 element uses the Timoshenko beam theory and includes the elastic shear deformations.

4.13 Perspex Models.

4.13.1 Perspex Single Member.

The purpose of this analysis was to establish the minimum number of elements that would be required to model the behaviour of a single member. Several element types were examined for the single member case in order to establish the most economical but also suitable element for the project. A perspex member 2000 mm long and of 25 mm diameter was modelled in three ways. Initially a solid SB8R brick element was used to form a ring which was then repeated to form the required length. In the second case and third cases two and three noded beam elements were utilized, the B31 and the B34 elements respectively. A series of analyses were undertaken so as to determine the minimum number of elements required to model the beam’s behaviour. The number of elements for the solid element case ranged from 3840 to 480. The very size of this problem in its self lead to the conclusion that the solid element was not going to be a viable proposition for the analyses of even the pyramidal structures. The number of elements used in the analyses for the two different beam element types varied from 400 down to 4. Tables 4.1 to 4.3 contain the results for the cases of varying the number of elements for the three element types respectively. The comparison between using the minimum number of elements compatible with each type and classical bending equations is shown in Table 4.4. Metal tests carried in the laboratory also correlated very well with these frequencies determined from the F.E. analyses.
4.13.2 Perspex Pyramids.

This configuration consisted of six members so it was expected that considerable care would need to be exercised when modelling the joints contained within the perspex pyramid. Consequently, the cylindrical joints at the vertices of the structure were modelled using a number of short beam elements each having an enhanced stiffness and density so as to reflect the behaviour of the joint. Each member in the structure was discretized using six beam elements of equal length and this mesh allowed the model to accurately adopt the deflected shapes of the structure's lowest modes. The 6 mm tube members ranged up in member length from 200 mm to 1000 mm in 200 mm increments. In general the correlation between the experimentally determined results and those obtained using the F.E. analysis was good. The level of agreement increased as the model member lengths increased. A preliminary natural frequency extraction was performed on for the various models and the resulting mode shapes were used for reference in the associated experimental modal survey of the related perspex pyramids.

It has already been stated that the effect of the mass distribution within the F.E. models has a pronounced effect on the results. Hence, consideration was given to the inclusion of the force transducer and accelerometer masses within the finite element models. It was anticipated that the accuracy of the model, especially when considering the mode shape representations, would benefit from the inclusion of these masses in the description of each pyramid. Finite element models were set up to include the added masses of the transducers used in the experimental analyses. A second series excluding the added masses were also performed and these showed a marked change in the predicted results. This prompted a later, second, series of experimental tests in which the roving transducer was reduced from four grammes to one gram. The effect of the added mass was structure dependant, the smaller 6 mm diameter models showed marked changes in their mode shapes and natural frequencies. This occurred because the position of the point mass dominated the lower mode behaviour of these structures. In contrast the larger and therefore the heavier the model became, the lower was the effect
of the inclusion of this mass. This was particularly apparent with the 25 mm diameter members where the inclusion of the mass had no apparent effect on the mode shape or natural frequency. The natural frequencies extracted from both the preliminary and updated 6mm diameter member models are given in Tables 4.5 and 4.6. Tables 4.7 and 4.8 records these results obtained from the 25 mm diameter member pyramids respectively.

The position of the driving point for the modal test of the smallest perspex pyramid was shifted to a joint and the analysis repeated incorporating the transducer mass. The shifting effect on the natural frequencies was considerably reduced and the mode shapes again resembled those determined experimentally. Figures 4.1 to 4.3 show the first three vibrational modes developed by this refined model and compares them with the related experimentally derived mode shapes. Figures 4.4 to 4.6 show the first three vibrational modes developed by for the 1000 mm, 25 mm diameter perspex pyramid model and compares them with the related experimentally derived mode shapes.

4.13.3 Perspex Unit Building Blocks.

Extensive use of the modelling experience gained from previous perspex models was used in the analyses performed for the various nine noded, twenty-one membered tetrahedral truss units. Each member was modelled using six beam elements whilst the joints were modelled using a short beam with modified stiffness and density values. Alternative modelling techniques including the use of lumped masses were also investigated. The inclusion of the transducer mass within the F.E. models caused similar problems to those experienced with the small, 6 mm diameter member, pyramid models. Refined analyses utilizing the one gramme transducer produced similar changes in the predicted responses for these small units. The 25 mm diameter member model showed the same robustness to the change of the transducer mass that occurred with the pyramid models. The results for the refined models are given in Table 4.9. Figures 4.7 to
4.9 illustrate the comparison between the F.E. and the experimentally derived mode shapes for the 1000 mm, 25 mm diameter 9/21 perspex tetrahedral truss.

4.13.4 Perspex Triangular Arm Sections.

Three triangular arm sections, representing sub-units of the main supporting structure for the tetrahedral units, were analysed using the same modelling techniques employed for the other perspex models. A similar process, to that previously used, of reducing the number of beam elements required to model the structures' behaviour was adopted before the final mesh density was determined. Each member was meshed using six equal length beam elements; this level of discretization being deemed sufficient to accurately model the lower global mode shapes developed in the structure. The joints were also considered to have a much reduced influence on the global modes developed by the arm and as such were modelled by lumped masses at the intersection points, the change in the number of elements entering the end joints of the beams was accounted for by altering the masses appropriately.

The effect of including the transducer mass within the analyses was markedly different to that experienced with the small pyramid models. This was due to the transducer being positioned onto a joint and not near mid-member as had been the case with the pyramids. Similarly the 25 mm diameter member model was not affected by the change in transducer mass. The experimental modal survey correlated well with the analytical predictions, hence further refinement of these models was not considered necessary. Table 4.10 details the F.E. predicted natural frequencies for these models. Figures 4.10 to 4.12 illustrate the F.E. and experimentally derived mode shapes for the first three modes for the 4.2 m arm section. These are typical for this structural configuration.

Examination of the physical experimental model of the 25 mm diameter arm section indicated that some warping of the section had occurred. Although the warping was only relatively minor it prompted a further analysis to be
undertaken. This analysis examined the differences that occurred between the predicted response of the model when 'perfect' geometry as opposed to the actual geometry was considered. This is of particular interest as it would indicate the changes in the response that might be expected to occur if fabrication errors were present in the structure. A further benefit would be that guidelines might be established as to the acceptable tolerances, from the structural response viewpoint, that could be endured whilst still maintaining a valid analysis. The results of this analysis showed that a slight reduction in the resonant frequency occurred due to the warping of the members. Table 4.11 details the results. The effect upon the predicted mode shapes was found to be negligible so they are not reproduced here.


The F.E. description of these models followed a similar pattern to that used for the perspex single member analyses in which the number of equal length beam elements was reduced from 100 down to 4. This reduced number and the fact that the presence of the transducers were not taken into account in the description was based on experience gained from the perspex single member analyses. Again six beam elements of equal length were found sufficient to allow the model to deform into the first three transverse bending modes of the beam.

A more refined model, which took the accelerometer mass of 4 grammes into account, was generated and analyzed following the experimental modal survey of the beam. The results are not presented here as they follow the classical behaviour of a single member. The frequencies for the unrestrained preliminary and refined models are detailed in Table 4.12.

4.14.2 Carbon-PES Composite Lapped Single Member.

The manufacturing process currently being employed only allows member lengths of 1000 mm to be produced. However the anticipated member
lengths for the full structure would be approximately 2225 mm. Near full sized sub-units were to be produced using two members joined using a centrally located, 30 mm, lapped joint. Due to the nature of the construction of the joint it was assumed to develop a stiffness equal to that of the adjoining beams and so was not modelled in the analysis. The lapped composite beam was modelled in the same manner as the single unit to determine the minimum number of elements required to accurately model it's behaviour. Various member lengths were investigated, the results for the predicted natural frequencies of these are detailed in Table 4.12. The mode shapes extracted for these models are comparable to those for the single beam model and hence have not been plotted.

4.14.3 Carbon-PES Composite Lapped Single Member with End Caps.

A further set of analyses was undertaken so as to predict the responses for a lapped member incorporating mass representative joints at either end. The beam length was determined as the lumped mass centre to centre distance. This lumped mass of 0.565 Kg, determined from the experimental work, represented the joints found in the more complex composite structures. A more refined analysis, performed following the completion of the experimental modal survey, concentrated on modelling the effects of the end caps on the modal behaviour of the lapped beam. The length of the beam was taken as the distance to the outer faces of the end caps and the mass of the caps was distributed between those elements forming the caps. An assumption was made that each end cap formed a structural component of the beam and that it was at least as stiff as the composite material.

Figures 4.13 to 4.15. illustrate the comparisons between the F.E. and the experimentally determined mode shapes for the first three natural frequencies. The natural frequencies for these models are shown in Table 4.12.

The initial F.E. mesh used techniques determined from previously examined models of the single member with the end caps as the starting point. No account was made of the mid-point lap-joints of the triangle and the mass of the end caps was uniformly distributed onto the vertex elements. A comparison of the frequencies extracted from this initial model with those obtained experimentally from the modal survey indicated that distributing the mass of the vertices in this structure decreased the effective free length of the members. This resulted in a set of higher frequencies for most of the modes of the triangular structure. A revised mesh lumped the mass of the joints onto a single node, situated at the intersection of the centre lines of the members. This configuration was found to represent the behaviour of the triangle more accurately. A comparison of natural frequencies obtained from both these models is shown in Table 4.13.

4.14.5 Carbon-PES Composite Pyramids

Each of the members in the triangular-based pyramidal structure was discretized using six ABAQUS beam elements and its four end cap joints, following the experience gained from the triangular models, were modelled as lumped masses. No account was made for the mid-point lap-joint on each side of the pyramid. Both of the structures were considered to be sufficiently massive so as to allow the accelerometer mass to be neglected. Very good agreement exists between the frequencies extracted from the F.E. models and those determined from the modal analysis of the actual structures, hence, no further refinement was developed for these structures. Figures 4.16 to 4.18 illustrate the first three mode shapes for the F.E. and experimental 1000 mm model. Table 4.14 details the natural frequencies for the 1000 mm and 2000 mm member length models.
4.14.6 Carbon-PES Composite Unit Building Block.

The experience gained from the previous analyses was extensively used to form the mesh describing the twenty one member, nine noded structures. Each member was modelled by using six ABAQUS beam elements whilst the nine rigid end joints were modelled using lumped masses. Very good correlation was found to exist between frequency values extracted from these models and those detected experimentally in the modal survey of these double layer tetrahedral truss units. Consequently, no further development of the modelling technique was deemed to be necessary. Figures 4.19 to 4.21 illustrate the comparison between the mode shapes for the experimental and F.E. derived results. Table 4.15 contains details of the natural frequencies for these models.

Following the extraction of the natural frequencies of the 1000 mm 9/21 unit a further set of analyses were undertaken to assess the response of the structure to a sinusoidal forcing function, as experienced in the experimental procedure. The model was subjected to a loading of one Newton, at the equivalent mesh point to that of the shaker in the physical model, and the response determined at another mesh point corresponding to a physical model test point. Figures 4.22 and 4.23 illustrate the predicted and actual responses for two points on the 1000 mm model.

4.14.7 Carbon-PES Composite Triangular Arm Section.

Each member was modelled by using four equal length beam elements whilst the joints were modelled by using lumped masses. Very good correlation was found to exist between frequency values extracted from these models and those detected experimentally in the modal survey of these double layer tetrahedral truss units. Consequently, further development of the modelling technique was deemed unnecessary. Figures 4.24 to 4.26 illustrate the comparison between the mode shapes for the experimental and F.E. derived results. Table 4.16 contains details of the natural frequencies for these models.
4.15 Carbon-PES Composite Full Proposed Structure.

4.15.1 Introduction.

The proposed satellite reflector consists of a number of major structural components, the support arm system, the 50 metre diameter reflector, the 5 metre diameter sub-reflector and the various ancillary components including the solar arrays and feed systems. The configuration of the tetrahedral system is such that in order to form the required 50 metre diameter, a maximum dimension of 60 metres is required. Preliminary models assumed slightly enhanced material properties over that used in the other analyses. This situation arose because, at the time, the tube manufacturer was considering an improved compaction technique for the tubular members which would have resulted in a 1 mm wall thickness being achieved. This compares with the actual wall thickness achieved of 1.85 mm. Subsequent analyses have continued to assume this narrower wall thickness but the revised, more conservative, modulus of elasticity has been implemented.

The aims of the analyses were to assess and ascertain the lowest global deformation modes of the structures. For this reason each of the members within the structures were modelled by using a single beam element. The members were formed from Carbon-PES tubes of 25 mm external diameter and 2.25 metres in length. This mesh yielded the required global deformations. Studies were carried out on the main reflector, in isolation, to ascertain its natural frequencies and mode shapes. Table 4.17 indicates the natural frequencies for the reflector alone. The preliminary analyses did not include an allowance for the mid member joints that would be present in the actual structure. Table 4.18 details the results obtained for the initial analysis of the proposed structure. Modifications were made to the interfaces between the reflectors and the support arm which, together with the inclusion of the feed array support section, increased the fundamental to around 0.2 Hz. Current active control systems are capable of acting effectively upon structures with resonant frequencies of above 0.15 Hz (Marconi 1988 [171]). This frequency formed the goal for the study. The refined analyses included an

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allowance for a 200 gramme joint in each member. Subsequent investigations, carried out within the Research Group, into the physical characteristics of this joint have indicated that this provides an accurate estimate. The first four natural frequencies of the proposed structure without the solar arrays are given in Table 4.19. The first four mode shapes for this configuration are shown in Figures 4.27 to 4.30.

The solar array panels were modelled using a series of solid elements, the S8R element in the ABAQUS element library. The initial model utilized a series of S4R elements but these developed a set of inextensional modes. The S8R elements were substituted for the insufficiently restrained lower order type "S4R" four node elements in the ABAQUS analyses. The "Quad4" element was used in the associated MSC/NASTRAN analyses. The main reflector mass was calculated as 1876 Kg. The main supporting structure mass was 254 Kg whilst the sub-reflector contributed a further 35 Kg. The solar array size is a notional value based upon a first estimate of the power requirements. A mass of 0.73 tonnes was incorporated in the analyses for the solar array. The results for this modified configuration are shown in Table 4.20. Figures 4.31 to 4.34 show the associated mode shapes.

An additional analysis was performed in which the 6 DOF beam elements on the structure were replaced with 3 DOF bar elements. Marginally lower structure frequencies arose but the values did not differ by more than 0.1%, indicating that the degree of restraint of rotations was low and that axial member deformations were predominant in the development of the mode shapes in the structure. Table 4.21 compares the results for the two analyses.

The various components of the proposed satellite lent themselves to be considered as super-elements for the analysis procedure. In a conventional analysis all the elements are considered simultaneously during the particular solution technique that is being utilized. In a sub-structuring analysis, the model is divided into a set of collected elements. These smaller pieces of the structure,
referred to as sub-structures, are first solved as separate structures and then combined. A super-element can be considered as the mathematical equivalent of these sub-structures. Partial solution of these super-elements are obtained for their reduced stiffness and equivalent springs at the boundary points. The reassembly and solution procedure results in an identical solution to those obtained from the conventional solution. The criteria for a successful super-element analysis are that the super-elements must have linear equations; this allows for a valid reduction to occur, and secondly that the super-elements must be completely isolated from each other by the described boundary grid points. This is required because each super-element is reduced separately. The available version of ABAQUS did not possess this feature. This resulted in a long and computationally expensive analysis for the whole structure. In contrast MSC/NASTRAN with its super-element facility allowed greater flexibility particularly in respect of cost reductions for re-analysis, reduced high-speed memory and peripheral storage requirements and more convenient modelling and output control. A full description of the super-element capability of MSC/NASTRAN may be found the Reference 172 (MSC/NASTRAN 1982 [172]).

4.15.2 Discussion.

4.15.2.1 Main Reflector.

The fundamental mode shape of the truss is equivalent to bending occurring about the two perpendicular axes, x and y, Figure 4.35. This results in maxima occurring at four of the apexes of the truss. The maximum positive and negative displacements for this mode are 0.989 and -0.939 metres respectively. This corresponds to the equivalent of +4.95\(\lambda\) and -4.70\(\lambda\) and are found at diagonally opposite apexes of the truss. The maximum gradient between adjacent mesh support points occurs near to the centre of one of the edges of the truss, points 2622-2623. The gradient of -0.0681 corresponds to a change of 0.34\(\lambda\). The second mode shape is similar in form, in that bending occurs around two perpendicular axes but these no longer correspond with the x and y axes but with ones bisecting those axes. The resulting maxima, both positive and negative displacements for

Note \(\lambda\) refers to the signal wavelength.
this mode occur at the centre of opposite edges and at opposite apexes of the truss. The magnitude of the displacements are 1.004 and -0.774 metres respectively, Figure 4.36. This is equivalent to a displacement of +5.02 \lambda and -3.87 \lambda respectively. The maximum gradient between adjacent mesh support points is found near to one of the apexes of the truss, points 2264-2282. The gradient of 0.05746 corresponds to a change of 0.29 \lambda. Excitation of the third mode results in a significant change in the deformed shape adopted by the main reflector. This change in shape corresponds with a significant change in the resonant frequency of the structure, from 2.3689 Hz for the first mode, 2.3690 Hz for the second mode, to 4.0891 Hz for the third mode. The mode shape can best be described as a dish or dome shape, with a near stationary set of mesh support points being observed to form a ring at the mid points of the reflector, Figure 4.37. The maximum positive and negative displacements for this the third mode are 0.499 and -0.518 metres respectively. This corresponds to the equivalent of 2.5 \lambda and -2.59 \lambda and are found at the apexes and centre of the truss respectively. The maximum gradient between adjacent mesh support points occurs at points 2189-2167 which is approximately the half radius point of the reflector. The gradient, across these points, of -0.03665 corresponds to a change of -0.18 \lambda. The fourth mode shape arises from bending taking place about three axes, each bisecting a face, resulting in a nearly stationary central point of the reflector, Figure 4.38. The maximum positive and negative displaced positions occur at the apexes of the truss system and correspond to +1.000 and -0.999 metres respectively. This is equivalent to a change of ±5 \lambda. The maximum gradient for two adjacent points occurs at points 2027-2028 an apex of the truss. The value of -0.069 equates to a change of 0.346 \lambda. Tables 4.22 and 4.23 detail the results for the maximum displacement and adjacent point gradients respectively.

It is evident from examining these first four mode shapes that considerable deformations of the reflecting surface supported by the double layer truss can be expected, if resonance is allowed to occur. Whilst the quoted values for the deformations represent the maxima for a particular mode and in general occur at the periphery of the truss, it is clear that the allowable surface distortions \delta_m, of
1.36 mm will be exceeded. The third mode is of greatest concern as the radius of curvature of the reflector would be changing with a period of around 0.25 seconds. It should be noted that the resonant frequencies being considered here are for the reflector in isolation. These resonant frequencies are considerably above the fundamental frequency expected to be experienced for the structure as a whole.

4.15.2.2 Main Satellite Structure.

The proposed structure was examined in two stages, initially the full structure excluding the solar arrays and communications package was examined and secondly with these components included. This was done so that the effects of these panels may be easily seen and consideration be given to altering their position and/or construction. It was anticipated that the addition of the solar array panels would have a significant effect on the mode shape of the proposed structure. The communication package which is essentially a point mass concentrated at the corner of the main supporting arm was not thought to be likely to cause a dramatic change as the inclusion of the solar arrays. For this reason it was modelled as a set of planar elements.

The first four modes of the satellite without the solar arrays and communications package showed a significant reduction in the natural frequencies when compared to those obtained from the examination of the tetrahedral truss in isolation. The fundamental frequency dropped from 2.3689 to 0.07605 Hz., the resulting mode shape was a rotational displacement about the corner of the main supporting arm, Figure 4.27. The centre to centre displacement of the main and sub-reflectors was 0.896 metres which corresponds to a change of 4.48 \( \lambda \). The effect of this mode on the shape of the main reflector is small. The largest change in gradient between two adjacent mesh support points is 0.396 \( \lambda \). The second mode, Figure 4.28, sets up a bending and torsional displacement resulting in a maximum centre to centre displacement of 1.086 metres which is equivalent to 5.43 \( \lambda \). Similarly, the effect on the shape of the reflecting surface is very much smaller, the maximum gradient between adjacent points being 0.069 \( \lambda \). The third mode is
a torsional displacement about the upper corner of the main support arm, Figure 4.29. The centre to centre displacement is 0.296 metres or 1.48 λ. The effect of this mode on the overall shape of the truss is also considerably smaller than the previous two modes with the maximum gradient between adjacent mesh support points being 0.020 λ. The fourth mode results in bending about the mid-point of the main support arm, Figure 4.30. The maximum centre to centre displacement is 0.579 metres or 2.9 λ. The resulting shape change on the truss is also very small with the maximum gradient between two mesh support points being 0.022 λ. Table 4.24 summaries the results for the point to point displacement of the two reflecting surfaces. The surface distortions of the reflector are considerably less than in the previous case where the reflector was considered in isolation but the maximum defocus sphere, of 100 mm, is exceeded in all four of the modes considered. This problem may be alleviated by the use of active control methods to counter-act the effects of global movements.

The inclusion of the solar arrays and communications package was anticipated to have a dramatic effect on the modal behaviour of the satellite. It is clear from the results that their inclusion was not as great as might have been expected. This is largely due to the relatively small additional mass being added, even though this mass was distributed away from the centre of mass of the structure. The reduction in the fundamental frequency was 1.4% but there was a significant change in the mode shape. In particular the centre to centre displacement of the trusses was reduced from 4.48 λ to 2.16 λ with the main reflector remaining essentially planar. The effect on the second mode was similar with the resonant frequency reducing by 4.8%. The centre to centre displacement was also dramatically reduced from 5.43 λ to 0.46 λ with the truss again remaining essentially planar. The third resonance is very close in frequency to the previous mode but the displaced shape is significantly different with the centre to centre displacement rising to 2.81 λ but again the main reflector remains almost planar. The fourth mode results in a centre to centre displacement of 3.42 λ with the main reflector remaining planar but displaced. Comparison of these deformations with the displacements of the main reflector when considering the
previous case are clear. The inclusion of the additional panels has not altered the overall form of the displacements of the main reflector and so altering the position of the solar panels is not considered to be necessary. Table 4.25 summaries the results for the point to point displacement of the two trusses.

4.16 Model Size Effects.

The development of a reliable algorithm which models the effect of changes in member length on the resonant frequency of the various structural units under consideration was one of the aims of the project. Consideration was given to the simplest of the structural units examined, that is the single member. This structure was modelled as a beam. The beam represents one of the simplest structural elements available. By considering the classical starting point for a prismatic beam and assuming that small deflections occur it is possible to arrive at the moment acting at any section. This results in the classic beam equation:

$$EI \frac{d^2y}{dx^2} = M$$  \hspace{1cm} 4.32

The reader is referred to Thomson’s work for the derivation of the standard form of the equation for the natural frequencies of lateral vibration of beams.

$$\omega_n = \beta^2 \sqrt{\frac{q EI}{w}}$$

where w/g is the mass per unit length of the beam and $\beta$ depends on the boundary conditions of the problem.

This equation may be readily re-written into it’s most common form:

$$\omega_n = \left(\beta_n l_c \right)^2 \sqrt{\frac{EI}{m l^4}}$$

Note: This section has been altered at the request of the external examiner such that the equation is not derived here. This has resulted in Equations 4.33 to 4.46 being deleted. The text is continued on page 136.

Note: This section has been altered at the request of the external examiner such that the equation is not derived here. This has resulted in Equations 4.33 to 4.46 being deleted. The text is continued on page 136.
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The effect of simply changing the length of the member for the single beam case, retaining the same material and geometric properties can then be predicted from application of the following formula:

\[ \frac{f_{na}}{f_{nb}} = \frac{L_b^2}{L_a^2} \]  

Application of this equation to determine the results of the natural frequencies of the single beam reveals that the observations are correct. The formula applies for all end condition cases because the constant \( \frac{1}{k^2} \) divides out.
leaving the relationship dependant upon inverse of the square of the member length.

This formula applies to continuous beams. The structural forms examined in this thesis are formed from a series of members whose properties are identical. This feature combined with the analysis taking place in a free-free condition causes the structures to behave as some form of 'continuous member'. The boundary conditions within each structural type are the same, hence no allowance need be made for the member end conditions as they simply cancel each other out. Examination of the mode shapes for the various structural types reveals that the same displaced shape is adopted for each model. This combined with the relatively small displacements that the joints of each structural unit undergo in their lower modes of vibration, contribute to the 'continuous beam' type of behaviour.

In most cases the error is less than one percent but in one case it does rise to four percent. Table 4.26 shows the actual and predicted natural frequencies along with the percentage errors that the algorithm produces for the perspex pyramids. Table 4.27 shows the actual and predicted natural frequencies for the pyramid structures with a change of material, that is from perspex to the carbon-PES composite. Table 4.28 shows the actual and natural frequencies along with the percentage errors for the perspex 9/21 truss units.
### 4.6 Tables:

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>No. of Elements in mesh</th>
</tr>
</thead>
<tbody>
<tr>
<td>3840</td>
<td>1920 960 480</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Natural Frequency (Hz.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

**Table 4.1** Single Member Natural Frequency using 'S8R' element.

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>No. of Elements in mesh</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>100 20 10 6 4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Natural Frequency (Hz.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

**Table 4.2** Single Member Natural Frequency using 'B31' element.

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>No. of Elements in mesh</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>100 20 10 6 4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Natural Frequency (Hz.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

**Table 4.3** Single Member Natural Frequency using 'B34' element.
<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Classical</th>
<th>Solid 'S8R'</th>
<th>Beam 'B31'</th>
<th>Beam 'B34'</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14.18</td>
<td>14.18</td>
<td>14.19</td>
<td>14.21</td>
</tr>
<tr>
<td>2</td>
<td>39.05</td>
<td>39.10</td>
<td>39.09</td>
<td>39.16</td>
</tr>
<tr>
<td>3</td>
<td>76.63</td>
<td>77.09</td>
<td>77.39</td>
<td>77.69</td>
</tr>
</tbody>
</table>

**Table 4.4** Single Member Natural Frequency using the minimum number of each element types.

<table>
<thead>
<tr>
<th>Model Description</th>
<th>Natural Frequency (Hz.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200 mm members</td>
<td>(6 mm)</td>
</tr>
<tr>
<td>400 mm members</td>
<td>(6 mm)</td>
</tr>
<tr>
<td>600 mm members</td>
<td>(6 mm)</td>
</tr>
<tr>
<td>800 mm members</td>
<td>(6 mm)</td>
</tr>
<tr>
<td>1000 mm members</td>
<td>(6 mm)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Natural Frequency (Hz.)</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>95.43</td>
<td>180.23</td>
<td>192.43</td>
<td></td>
</tr>
<tr>
<td>31.60</td>
<td>51.03</td>
<td>73.60</td>
<td></td>
</tr>
<tr>
<td>15.83</td>
<td>22.85</td>
<td>24.18</td>
<td></td>
</tr>
<tr>
<td>8.83</td>
<td>11.79</td>
<td>13.13</td>
<td></td>
</tr>
<tr>
<td>6.51</td>
<td>8.59</td>
<td>10.76</td>
<td></td>
</tr>
</tbody>
</table>

**Table 4.5** Perspex Pyramids: Natural Frequency Responses (4 Gram Transducer).

<table>
<thead>
<tr>
<th>Model Description</th>
<th>Natural Frequency (Hz.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200 mm members</td>
<td>(6 mm)</td>
</tr>
<tr>
<td>400 mm members</td>
<td>(6 mm)</td>
</tr>
<tr>
<td>600 mm members</td>
<td>(6 mm)</td>
</tr>
<tr>
<td>800 mm members</td>
<td>(6 mm)</td>
</tr>
<tr>
<td>1000 mm members</td>
<td>(6 mm)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Natural Frequency (Hz.)</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>202.87</td>
<td>204.70</td>
<td>215.35</td>
<td></td>
</tr>
<tr>
<td>50.68</td>
<td>51.13</td>
<td>56.61</td>
<td></td>
</tr>
<tr>
<td>22.65</td>
<td>23.39</td>
<td>25.69</td>
<td></td>
</tr>
<tr>
<td>12.76</td>
<td>13.28</td>
<td>14.62</td>
<td></td>
</tr>
<tr>
<td>8.16</td>
<td>8.52</td>
<td>9.40</td>
<td></td>
</tr>
</tbody>
</table>

**Table 4.6** Perspex Pyramids: Natural Frequency Responses (1 Gram Transducer).
<table>
<thead>
<tr>
<th>Model Description</th>
<th>Natural Frequency (Hz.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>800 mm members</td>
<td>58.21</td>
</tr>
<tr>
<td>(25 mm)</td>
<td></td>
</tr>
<tr>
<td>1000 mm members</td>
<td>35.15</td>
</tr>
<tr>
<td>(25 mm)</td>
<td></td>
</tr>
<tr>
<td>2000 mm members</td>
<td>9.20</td>
</tr>
<tr>
<td>(25 mm)</td>
<td></td>
</tr>
</tbody>
</table>

**Table 4.7 Perspex Pyramids: Natural Frequency Responses (4 Gram Transducer).**

<table>
<thead>
<tr>
<th>Model Description</th>
<th>Natural Frequency (Hz.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>800 mm members</td>
<td>57.09</td>
</tr>
<tr>
<td>(25 mm)</td>
<td></td>
</tr>
<tr>
<td>1000 mm members</td>
<td>36.78</td>
</tr>
<tr>
<td>(25 mm)</td>
<td></td>
</tr>
<tr>
<td>2000 mm members</td>
<td>9.19</td>
</tr>
<tr>
<td>(25 mm)</td>
<td></td>
</tr>
</tbody>
</table>

**Table 4.8 Perspex Pyramids: Natural Frequency Responses (1 Gram Transducer).**

<table>
<thead>
<tr>
<th>Model Description</th>
<th>Natural Frequency (Hz.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>200 mm members</td>
<td>170.0</td>
</tr>
<tr>
<td>(6 mm)</td>
<td></td>
</tr>
<tr>
<td>400 mm members</td>
<td>42.65</td>
</tr>
<tr>
<td>(6 mm)</td>
<td></td>
</tr>
<tr>
<td>600 mm members</td>
<td>18.06</td>
</tr>
<tr>
<td>(6 mm)</td>
<td></td>
</tr>
<tr>
<td>1000 mm members</td>
<td>30.73</td>
</tr>
<tr>
<td>(25 mm)</td>
<td></td>
</tr>
</tbody>
</table>

**Table 4.9 Perspex Unit Building Block: Natural Frequency Responses.**
<table>
<thead>
<tr>
<th>Model Description</th>
<th>Natural Frequency (Hz.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>3 Bay 6 mm members</td>
<td>119.20</td>
</tr>
<tr>
<td>8 Bay 6 mm members</td>
<td>71.24</td>
</tr>
<tr>
<td>3 Bay 25 mm members</td>
<td>12.69</td>
</tr>
</tbody>
</table>

Table 4.10 Perspex Triangular Arm Section: Natural Frequency Responses.

<table>
<thead>
<tr>
<th>Model Description</th>
<th>Natural Frequency (Hz.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>3 Bay Perfect Geometry</td>
<td>11.40</td>
</tr>
<tr>
<td>3 Bay Actual Geometry</td>
<td>12.69</td>
</tr>
</tbody>
</table>

Table 4.11 Perspex Triangular Arm Section: Natural Frequency Responses for 'Perfect' and 'Actual' Geometries.

<table>
<thead>
<tr>
<th>Model Description</th>
<th>Natural Frequency (Hz.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>0.958 mm</td>
<td>201.39</td>
</tr>
<tr>
<td>0.990 mm</td>
<td>179.02</td>
</tr>
<tr>
<td>1.080 m (End Caps)</td>
<td>77.93</td>
</tr>
<tr>
<td>1.795 m</td>
<td>57.18</td>
</tr>
<tr>
<td>1.966 m (End Caps)</td>
<td>25.23</td>
</tr>
</tbody>
</table>

Table 4.12: Carbon-PES Composite Single Members.
<table>
<thead>
<tr>
<th>Model Description</th>
<th>Natural Frequency (Hz.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1.975 m</td>
<td>22.48</td>
</tr>
</tbody>
</table>

Table 4.13: Carbon-PES Composite Triangular Section Unit.

<table>
<thead>
<tr>
<th>Model Description</th>
<th>Natural Frequency (Hz.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1000 mm</td>
<td>119.32</td>
</tr>
<tr>
<td>2000 mm</td>
<td>30.65</td>
</tr>
</tbody>
</table>

Table 4.14: Carbon-PES Composite Pyramid Units.

<table>
<thead>
<tr>
<th>Model Description</th>
<th>Natural Frequency (Hz.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1000 mm Material 1</td>
<td>103.06</td>
</tr>
<tr>
<td>1000 mm Material 2</td>
<td>99.11</td>
</tr>
</tbody>
</table>

Table 4.15: Carbon-PES Composite Unit Building Block.

<table>
<thead>
<tr>
<th>Model Description</th>
<th>Natural Frequency (Hz.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>3.994 m Arm Section</td>
<td>40.55</td>
</tr>
</tbody>
</table>

Table 4.16: Carbon-PES Composite Triangular Arm Section.
<table>
<thead>
<tr>
<th>Model Description</th>
<th>Natural Frequency (Hz.)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main Reflector (50.0 m)</td>
<td></td>
<td>2.3689</td>
<td>2.3690</td>
<td>4.0690</td>
<td>4.7897</td>
</tr>
<tr>
<td>Discription</td>
<td>Torsional</td>
<td>Bending</td>
<td>Bending</td>
<td>B + T</td>
<td></td>
</tr>
</tbody>
</table>

**Table 4.17: Main Reflector Only.**

<table>
<thead>
<tr>
<th>Model Description</th>
<th>Natural Frequency (Hz.)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Structure (No arrays)</td>
<td></td>
<td>0.01495</td>
<td>0.08588</td>
<td>0.11827</td>
<td>0.29008</td>
</tr>
<tr>
<td>Discription</td>
<td>Arm Torsion</td>
<td>Torsional</td>
<td>Bending</td>
<td>Bending</td>
<td></td>
</tr>
</tbody>
</table>

**Table 4.18: Full Structure Initial Model (No Solar Arrays).**

<table>
<thead>
<tr>
<th>Model Description</th>
<th>Natural Frequency (Hz.)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Structure (No arrays)</td>
<td></td>
<td>0.07806</td>
<td>0.14694</td>
<td>0.25080</td>
<td>0.32847</td>
</tr>
<tr>
<td>Discription</td>
<td>Torsional</td>
<td>Bending</td>
<td>Arm Torsion</td>
<td>Bending</td>
<td></td>
</tr>
</tbody>
</table>

**Table 4.19: Full Structure Modified Model (No Solar Arrays).**

<table>
<thead>
<tr>
<th>Model Description</th>
<th>Natural Frequency (Hz.)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Structure</td>
<td></td>
<td>0.074994</td>
<td>0.139927</td>
<td>0.147219</td>
<td>0.259718</td>
</tr>
<tr>
<td>Discription</td>
<td>Torsional</td>
<td>Torsional</td>
<td>Bend. + T</td>
<td>Bending</td>
<td></td>
</tr>
</tbody>
</table>

**Table 4.20: Full Structure Model.**
<table>
<thead>
<tr>
<th>Model Description</th>
<th>Natural Frequency (Hz.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Full Structure</td>
<td>0.074994</td>
</tr>
<tr>
<td>Discription</td>
<td>(6 DOF)</td>
</tr>
<tr>
<td>Full Structure</td>
<td>0.074619</td>
</tr>
<tr>
<td>Discription</td>
<td>(3 DOF)</td>
</tr>
</tbody>
</table>

**Table 4.21: Full Structure Model Comparison of Element Degrees of Freedom.**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Natural Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Maximum Positive Disp. (m)</td>
<td>0.989</td>
</tr>
<tr>
<td>Maximum Negative Disp. (m)</td>
<td>-0.939</td>
</tr>
</tbody>
</table>

**Table 4.22: Main Reflector Displacements.**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Natural Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Maximum Gradient</td>
<td>0.34</td>
</tr>
</tbody>
</table>

**Table 4.23: Main Reflector Maximum Adjacent Point Gradients.**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Natural Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Displacement (m)</td>
<td>0.896</td>
</tr>
<tr>
<td>Displacement (λ)</td>
<td>4.48</td>
</tr>
</tbody>
</table>

**Table 4.24: Full Structure Model Centre to Centre Displacement of Reflecting Surfaces (No Solar Arrays).**
<table>
<thead>
<tr>
<th>Variable</th>
<th>Natural Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Displacement (m)</td>
<td>0.432</td>
</tr>
<tr>
<td>Displacement (λ)</td>
<td>2.16</td>
</tr>
</tbody>
</table>

Table 4.25: Full Structure Model Centre to Centre Displacement of Reflecting Surfaces.

<table>
<thead>
<tr>
<th>Model</th>
<th>Mode I</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Length</td>
</tr>
<tr>
<td></td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>400</td>
</tr>
<tr>
<td></td>
<td>600</td>
</tr>
<tr>
<td></td>
<td>800</td>
</tr>
<tr>
<td></td>
<td>1000</td>
</tr>
<tr>
<td>800</td>
<td>25</td>
</tr>
<tr>
<td>1000</td>
<td>25</td>
</tr>
</tbody>
</table>

Table 4.26: Comparison of Experimental, Finite Element and Algorithm derived Natural Frequencies for the Perspex Pyramid Units.
### Table 4.27: Comparison Finite Element and Algorithm derived Natural Frequencies for the Perspex and Carbon-PES Pyramid Units.

<table>
<thead>
<tr>
<th>Model</th>
<th>Material</th>
<th>Length/Diameter</th>
<th>Perspex 6mm</th>
<th>Perspex 25mm</th>
<th>C-PES 25mm</th>
<th>Algorithm P_F-P_u</th>
<th>Algorithm P_F-C_u</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>800</td>
<td>12.76</td>
<td>57.08</td>
<td>---</td>
<td>57.73</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>8.15</td>
<td>36.78</td>
<td>119.32</td>
<td>36.95</td>
<td>123.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2000</td>
<td>9.19</td>
<td>30.65</td>
<td>---</td>
<td>30.79</td>
<td></td>
</tr>
</tbody>
</table>

### Table 4.28: Comparison of Experimental, Finite Element and Algorithm derived Natural Frequencies for the Perspex 9/21 Truss Units.

<table>
<thead>
<tr>
<th>Model</th>
<th>Mode I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>Dia. Expt</td>
</tr>
<tr>
<td>200</td>
<td>6</td>
</tr>
<tr>
<td>400</td>
<td>6</td>
</tr>
<tr>
<td>600</td>
<td>6</td>
</tr>
<tr>
<td>1000</td>
<td>25</td>
</tr>
</tbody>
</table>
4.7 Figures

Figure 4.1: 200 mm Perspex Pyramid, F.E. v. Expt Mode Shape 1

Figure 4.2: 200 mm Perspex Pyramid, F.E. v. Expt Mode Shape 2

Figure 4.3: 200 mm Perspex Pyramid, F.E. v. Expt Mode Shape 3
Figure 4.4: 1000 mm Perspex Pyramid, 25 mm diameter, F.E. v. Expt. Mode Shape 1

Figure 4.5: 1000 mm Perspex Pyramid, 25 mm diameter, F.E. v. Expt. Mode Shape 2

Figure 4.6: 1000 mm Perspex Pyramid, 25 mm diameter, F.E. v. Expt. Mode Shape 3
Figure 4.7: 1.0 m Perspex 9/21 Unit Building Block, 25 mm diameter
F.E. v. Expt. Mode Shape 1

Figure 4.8: 1.0 m Perspex 9/21 Unit Building Block, 25 mm diameter
F.E. v. Expt. Mode Shape 2

Figure 4.9: 1.0 m Perspex 9/21 Unit Building Block, 25 mm diameter
F.E. v. Expt. Mode Shape 3
Figure 4.10: 4.2 m Perspex Triangular Arm Unit, 25 mm diameter
F.E. v. Expt. Mode Shape 1

Figure 4.11: 4.2 m Perspex Triangular Arm Unit, 25 mm diameter
F.E. v. Expt. Mode Shape 2

Figure 4.12: 4.2 m Perspex Triangular Arm Unit, 25 mm diameter
F.E. v. Expt. Mode Shape 3
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5.0 Thermal Investigation.

5.1 Introduction.

The effects of solar activity on the materials making up the large satellite structures under investigation will have a dramatic effect on their in service behaviour characteristics. This is due to the large differences that exist at the microscopic level between the coefficients of thermal expansion of the matrix resin and the fibres, which will result in substantial thermally induced stresses being set up (Hartung 1984 [173]). The open frame nature of many of the components of the proposed structures will result in a high degree of self shadowing occurring, further exasperating the problem particularly during the equinox periods. An investigation into the effects of thermal cycling upon a 9/21 unit truss fabricated from the carbon fibre polyethersulphone composite was undertaken in a 3.8 m diameter solar simulation chamber. The structure under test was cycled between 113 K (-160°C) and 298 K (+25°C) under a high vacuum, equivalent to an orbit of 300 Km. The structure was subjected to a dynamic excitation from which the response functions were obtained. The structure was also heavily thermocoupled so that the temperature gradient distribution could be mapped. A series of coupons used for tensile and compressive tests were also subjected to the same conditions. It should be noted that the material currently used in the tests has not been specifically engineered to have a zero coefficient of thermal expansion.

5.2 Test Chamber Description.

The chamber used for the series of tests carried out on the proposed materials and a unit building block of the main truss was made available at the RAE (Farnborough). The chamber is designed to simulate three of the main features of the space environment that the satellite will experience, these are the extreme vacuum, the cold heat sink effect of deep space and the heating effect of the sun. The 3.8 m diameter chamber is lined with a black painted aluminum shroud through which liquid nitrogen is circulated by pumps. In order to
compensate for the effects of the lamps used in the solar simulation there is a greater flow of liquid nitrogen through the door mounted shroud. The cold sink that this provides is equivalent to about 80 K (-190°C). This compares with the typically expected temperatures at LEO and GEO of 15 K and 4 K respectively (UoSAT 1991 [174] and Thorne 1990 [175]). The vacuum is achieved using a combination of a rotary pump followed by two cryogenic (RAE 1989 [176]) pumps which allow a pressure of \(10^{-7}\) atmospheres to be maintained during both the hot and the cold cycles. The heating effect of the sun is simulated using three xenon arc lamps, which provide the complete solar spectra for LEO (Thorne 1990 [177]). This configuration allowed for an illumination of up to 2 kW/metre\(^2\) over the 2.5 m focal plane centred on the 9/21 unit under test. This output is equivalent to 1.48 times the sun's intensity at LEO. For the purposes of the test an intensity of 1.352 kW/metre\(^2\), equivalent to one sun, was used. Figure 5.1 shows the lay out of the solar simulation chamber. Plate 5.1 shows the general experimental setup of the 9/21 unit in the solar simulation chamber. Plate 5.2 shows the 9/21 unit building block being suspended in the free-free condition using stainless steel springs.

Electrical connections into and out of the chamber were provided by a series of high vacuum penetration points mounted on the outer casing of the chamber. The decision to excite dynamically the structure required that a special penetration cap be formed to carry the shielded micro cables to and from the structure. This cap underwent extensive vacuum tightness tests, in the vacuum facility available within the section, prior to installation on the chamber. In addition to these cables, thermocouple, strain gauge and power supply cables were required for the tests, these were carried into the chamber using existing penetration points and caps.

5.3 Experimental Description.

5.3.1 Equipment.

The equipment used in the experimental monitoring of the thermal cycling tests consisted of the ENTEK software running on an HP300 series computer, and
the HP3562A Dynamic Signal Analyzer. The test specimens were temperature cycled 25 times between 113 K (-160°C) and 318 K (+45°C) under high vacuum. The temperature extremes were achieved by cycling on a two hour cold cycle followed by a one hour hot cycle, basis. This cycling period was selected so that the rate of change of a particular thermocouple was very small, that is, local equilibrium had been achieved during both the cold and hot cycles. The maximum total darkness period that the proposed satellite structure would experience at GEO is predicted as 72 minutes for 100 days in two 50 day periods (Annandale 1986 [178]). The extended two hour cold cycle was selected so as to compensate for the higher cold sink temperature and allow for local 'near equilibrium' to be established. It was not possible to increase the number of cycles achieved during the test series due to constraints on the availability of the environmental test chamber.

The specimens were monitored using type "T" copper-constantine thermocouples. The specimen was monitored every 10 minutes, throughout the 5 day test period. Thermocouple readings were obtained using a Schlemberger data recording unit which also provided calibration for the strain gauges employed on the first thermal tests. The extreme cold experienced in the chamber on the cold cycles prevented the use of a force transducer as the detecting transducer on the two channels system. Instead an accelerometer was used to detect the input forcing function present during the thermal cycling. Accelerometers were used to detect the response at a series of pre-determined positions on the truss. It was not possible to cover all points on the structure so a displaced shape for the structure could not be attained for this test configuration.

The extreme cold expected to occur during the test resulted in the selection of a solenoid as the excitation device rather than a shaker. This was due to a series of anticipated problems associated with the interaction between the structure and the excitation device. Firstly, the accurate positioning of the shaker with respect to the test structure, required for a dynamic test, would be difficult to maintain whilst they were undergoing such large temperature variations. Secondly, the large thermal mass that the shaker presented in comparison to that
of the member it was attached to, and the shielding that would be required was felt to be a serious weakness. Thirdly, the presence of various materials in the construction of the shaker would have adversely affected the operation of the cryogenic pumps which rely on the diffusion principle and are therefore highly susceptible to contamination from material outgassing components.

The operation of the chamber is such that there are a large number of materials which are not suitable for inclusion in the test specimen or its peripherals. A comprehensive list of such materials has been prepared by NASA (Campbell 1990 [179]) and by ESA (Jollet 1986 [180]). These lists includes a very large number of cleaning agents and materials normally used during the 'in air' testing carried out on the specimens. These requirements led to the adoption of PTFE covered microdot cables being used for the accelerometers and PTFE covered cables for all other internal electrical connections. Prior to placing in the environmental chamber the test specimens were thoroughly cleaned with an approved alcohol to remove any traces of mounting tape, marker pen etc. Various other test specimens, including carbon-PES plates and loaded tubes were included in the chamber for the thermal cycling test series. The results of these material based experiments are reported in detail elsewhere (Hollaway 1990 [181]).

5.3.2 Suspension System.

The physical dimensions of the environmental chamber were such that an alternative suspension system was developed to support the 9/21 truss unit. This system consisted of a 1.0 m equilateral triangle formed from aluminium angle section. Woven nylon cords connected to stainless steel springs then supported the truss from the three 'upper' node points. The stainless steel springs were chosen such that the working load was a third of the yield point load so as to avoid the possibility of a brittle failure occurring due to the extreme cold. Particular care was taken to ensure that all the cables connecting the various instruments on the structure were supported so as to reduce the drag and damping effects that might be induced. The resonances of this suspension system were determined prior to the
test and were found to satisfy the general requirements for not affecting the structural resonances.

5.4 First Thermal Test Series.

The first thermal cycling test consisted of the 9/21 tetrahedral truss unit instrumented with ten thermocouples, two accelerometers (Burchell 1991 [182]) and thirteen strain gauges (Showa 1977 [183]). It was however only possible to monitor ten of these gauges because of instrumentation difficulties experienced prior to and during the test. Figure 5.2 shows the positions of the various transducers for the first series of thermal tests. The 9/21 unit for this test cycle was made up from a batch of tubes using a 90-10 lay-up with 90% of the fibres lying in the longitudinal direction. The matrix material was the PES type 4200G.

Strain gauges, positioned at 120° around the circumference of the carbon-PES tubes, were included in the first test series as it was anticipated that it would be possible to calibrate them at a depressed temperature and obtain meaningful readings from them. The manufacturers stated that the range of operation for the gauges was greater than those expected to occur in the test series so no particular problems were anticipated. The gauges were bonded to the tubes using a cyanoacrylate adhesive. Figures 5.3, 5.4 and 5.5 illustrate the longitudinal strain variations recorded for different points on the truss. The interpretation of the strain readings obtained from the test series require careful consideration. The quarter bridge configuration using an external dummy gauge should allow the interpretation of the overall expansion or contraction of the tubes due to the temperature changes. It would appear that it is only possible to compare strains for a particular temperature within the overall range and not on a continuous basis. This would explain the very large changes of strain encountered which are due in part to the changes in resistance of the gauge wire as well as changes in material length. Their use in terms of determining the strains in the members due to the excitation is yet to be proved. In the event it transpired that this was not possible so they have been disregarded pending further investigations.
The use of an internal dummy gauge, bonded to a specimen of the carbon-PES composite would provide the temperature compensation required but as it would be subject to the same loading environment the strains due to the small changes of length associated with any variation in the coefficient of thermal expansion are very difficult to determine.

The first thermal cycling tests undertaken used a solenoid with a mechanical return spring. This proved to be less reliable than the two pole solenoid used in the later series of tests. Particular difficulties were experienced in the initial start up the solenoid at the beginning of the cycling period after the initial 5 hour pump down period when the temperature of the solenoid was around 133 K (-140°C). An exact figure for the solenoid temperature is not available due to the failure of the relevant thermocouple. This was somewhat of a disappointment as the mechanical return solenoid had undergone testing at below this temperature prior to installation in the test chamber. This testing had been carried out at atmospheric pressure by immersing a tube containing the solenoid into a bath of liquid nitrogen. It is therefore not possible to say with any accuracy what caused the initial start up problems encountered other than to put forward the possibility of some form of cold welding occurring during the initial pump down procedure. This hypothesis is presented because the problem was overcome when the current to the solenoid was substantially increased causing some local heating to occur.

The variations in temperature, due to the solar simulation, on the 9/21 truss are shown in Figures 5.6 through 5.10. These figures show the variation experienced for two complete cycles, which cover a six hour period during the middle of the 25 cycle test. The rates of change of temperature of thermocouple pairs 1-2 and 3-4 is very similar to that which would be expected from their positions along the face nearest the solar simulation source. As expected thermocouple 3 reaches a higher temperature than number 1 as it is situated 0.288 m closer to the light source. The upper temperature achieved by thermocouples, numbers 1 and 4 are virtually identical despite number 1 being
fully exposed to the effects of the solar simulation as opposed to number 4 which was situated on the self shadowed side of the lower front member. This illustrates the effect of not being in the focal plane of the lamps. It would be anticipated that whilst there would be a difference in the absolute values of thermocouple readings taken from such positions on the proposed satellite their range would be smaller. This assumes that no self-shadowing effects would be present.

The temperature extremes experienced over the truss are shown in Figure 5.11. This 65 K (65°C) difference was somewhat lower than expected from theoretical calculations. It is postulated that this might not be the greatest difference that the truss experiences but only the maximum that was recorded at the particular placements of the thermocouples. Consideration of the effect of the large thermal mass present in the chamber which would reduce the effect of the cold sink still further. This thermal mass is due to the presence of the rotating sting, normally used for supporting more conventional satellites. The sting was dismantled to its maximum extent but this still left in place a significant portion of its total mass. The sting casing is not provided with a nitrogen filled shroud and therefore will be at a higher temperature than the rest of the chamber which is covered by the shroud. The sting would also be subject to a significant variation in its temperature as the thermal cycling period progresses. The overall effect of the presence of the sting on the behaviour of the 9/21 unit truss is difficult to assess but it is likely that it would reduce the extremes to which the structure as a whole, and the members closest to the sting in particular, would be exposed. This might partially explain the lower than calculated temperature range experienced by the material. The fact that the upper temperature could not be achieved is of greatest concern. This value should have been attainable especially as it was known that the cold sink provided by the chamber was less than that of space. A possible explanation for the reason as to why this expected temperature was not achieved is that the lamps were not operating at one solar equivalent on the whole structure. The chamber set-up is such that the lamps are focused so as to produce an intensity of one solar equivalent at one plane within the chamber. The output from the lamps is monitored and controlled in a feedback loop, in this
focal plane. It is known that considerable parts of the structure fell outside this plane and as such were not subjected to the same one solar equivalent. Parts of the structure were placed in a position of shadow or partial shadow, which is a situation that would be experienced in space simply because of the large number of the inner members of a double layer skeletal structure. The emissivity and the absorptance of the composite would affect the predicted values for the temperature extremes as these values were determined assuming a carbon black body (Annandale 1986 [184]). Many other authors have also indicated that the predication of the extremes of temperature experienced by such structures are highly dependent upon these surface properties in addition to the material's bulk thermal conductivity and specific heat capacity (Mazzio 1983 [185], Hillesland 1985 [186], Sonoda 1984 [187], Sykes 1986 [188]). The effect of the white pigment within the end caps of the 9/21 unit must also not be discounted. Their effect might be to radiate/absorb heat more slowly than the black tubes and so reduce the extremes. Samples of the node surface materials were prepared and sent for emissivity and absorptance testing at RAE (F). The results of these two tests are shown in Table 5.1.

Further factors do affect the radiative energy potential of a surface. Previous researchers (Houchens 1972 [189]) have indicated that the roughness of the surface could influence the amounts of energy emitted and reflected relative to that of the constituent material as well as the spatial distribution of these energies, even if the effect on the overall heat transfer is relatively small. This factor may become significant with the large number of interacting radiative bodies present in the proposed satellites.

Some difficulty was experienced when the transducer wires were being supported. Their stiffness was such that it might influence the damping of the structure. They were supported at several points along their length in an attempt to reduce any such effects to a minimum. It is not possible to say if the measures undertaken were entirely successful over the full thermal cycle range.
Examination of the dynamic excitation of the composite unit tetrahedral truss reveals that there is a high degree of repeatability over the thermal cycling period. Figure 5.12 illustrates the relationship between the resonant frequency as determined by the pick peak method and the temperature of the member at the detection point. This illustrates that a near 1 Hz shift in frequency occurs during the cycling period. The repeatability of the cycling process is also clearly shown. One feature that should be addressed is that the data points for the graph were obtained at uniform time increments of thirty minutes. This was due to the collection of data from other points on the structure. This relatively large time interval results in an apparently coarse change of gradient within the data. This is particularly so as data were being collected at the times that the solar simulation lamps were either being shrouded or uncovered. Figure 5.13 illustrates the relationship between the resonant frequency and the cycle time. This also illustrates that a near 1 Hz shift in frequency occurs during the cycling period. The repeatability of the cycling process is also clearly shown. The change in the resonant frequency of the structure can be attributed to a number of factors. As the temperature falls the members become subjected to an axial load due to their contraction, the effect of this is to cause an increase in the resonant frequency of the unit. This is consistent with the theoretical study of continuous structures because as the temperature rises back towards that at which the unit was constructed the axial stresses reduce back towards their 'neutral' level. An alternative explanation for the change in resonant frequency may be found if a change in material stiffness is considered to occur. If this solution is accepted then a 3 percent rise in the modulus of elasticity would be required. It is likely that there are several factors contributing to the detected rise in the resonant frequency as the temperature falls. The effect of the return spring on the shaker contracting and stiffening as the temperature falls may also contribute to the phenomenon.

It was noticed that after the thermal cycling test series had been completed, severe cracking had occurred in the hoop direction of the outer films of the matrix. The exact reasons for these cracks are not known but it is thought that they were...
due to the interaction between the three outer layers of the PES matrix material under the thermal loading. The relatively low viscosity of the matrix medium also contributes to the formation of the cracks.

A 1798 mm single member was also included in the test series but a similar problem with the mechanical solenoid, to that which was experienced on the 9/21 tetrahedral truss unit, occurred. In this case it was not possible to correct the defect and the item was abandoned for the dynamic test series. It should be noted that the solvent used in joining the two sections together was still intact after the thermal cycling test series.

5.5 Second Thermal Test Series.

The second thermal cycling test consisted of the 9/21 tetrahedral truss unit instrumented with ten thermocouples and three accelerometers (Burchell 1991 [182]). The suspension system used to simulate the free-free condition was the same as that used in the first test series. Two 2.0 m triangular units were also included in the dynamic test configuration; one of these was loaded with 3 kg at its apex. Both these specimens were suspended using stainless steel springs so as to simulate the free-free condition. The same criteria for the working load of these springs to those used with the 9/21 unit were adopted. Figures 5.14 and 5.15 show the positions of the various transducers for the series of thermal tests. A further 1 m tube thermocoupled so as to detect the temperature gradient that exists through the wall thickness of the tube was also included in the test series. The previous test series had only considered the surface temperature of the tubing. The composite tubes used to manufacture the 9/21 tetrahedral unit for the second test series had a significantly different lay-up from those used in the first test series. The fibre orientation was +10/-10/0/+10/-10 degrees to the longitudinal axis with 10% of the fibres lying at right angles to this direction. The matrix material was PES type 4800G. This grade of material was used as its higher viscosity, over that of the 4200G, was adopted after consultations with the manufacturers, ICI. The type 4800G grade of PES has a lower end link to chain length ratio than the
4200G. This results in an increased viscosity of the material as the ends of the polymer chains can be considered to act as lubricants to the surrounding molecules. In addition to this the increased length of the polymer chain gives rise to a higher degree of chain entanglement, resulting in a further increase in the viscosity of the material. It was suggested that the longer polymer chain that occurred in the 4800G PES would eliminate or at least reduce the cracking that had been experienced with the previous matrix. The number of external layers of the PES was also reduced to one with the same aim in mind. Current thinking within the research group leans towards removing the outer matrix layer completely. This results in a rough surface to the tubing but removes the problem of having a layer with a completely different coefficient of thermal expansion, which over a long period of time is almost certain to peel off. A considerable amount of effort is being expended into investigations of other external final coatings (Hollaway 1990 [190]).

Emissivity and absorptance test samples of the two grades of the PES matrix were prepared and sent for analysis at RAE (F). The results of these tests are shown in Table 5.2.

Two of the thermocouples were positioned within nodes, one on the upper layer and one on the lower layer. The pigment used in the formation of these nodes was white and black respectively. It was anticipated that there would be a difference in the rates of change of temperature of these two nodes due to their surface properties. This effect would be altered by their respective positions within the environmental chamber.

The temperature variations experienced by the white and black nodes are represented by thermocouples 5 and 6 respectively. These values for the mid test cycle are shown in Figure 5.16. This indicates that the responses are virtually identical throughout the entire cycle, contrary to that which would be expected when considering the surface differences, recorded in Table 5.1. Closer inspection of the data reveals that the difference in the temperatures between the two nodes
never varies by more than 0.5°C throughout the cycle. The thermocouples were cast into the nodes at the time of construction and therefore give information on the internal node temperatures. These values may not be significantly affected by the surface characteristics of the nodes as some heat energy will be transferred into and out of the nodal points by the carbon-PES members. The black node joins three members, two of which are subjected to the full effects of the solar simulation and are not subject to shadowing. The white node joins five members, of which three members receive an uninterrupted view of the Xenon lamps. The effect of energy being transferred along the members on the internal temperature of the nodes is not known at this stage. The relative position of the two nodes within the environmental test chamber is known, with the black node being 0.288 m closer to the Xenon lamps than the white node. Both these nodes are positioned in front of the focal plane of the lamps and as such will be receiving less than the one solar equivalent. The conclusion that must be drawn from the results of the two thermocouples is that despite having different surface characteristics the internal temperatures of the nodes are subject to a variety of influences from the incoming members in addition to their geographical position within the test chamber. Data on the temperatures of the relevant members of the truss was not recorded for this test.

The 1 m tube was positioned such that it received the full effect of the solar simulation, but was subjected to the possible effects of the sting. The tube typically reached an 'equilibrium' temperature on the cold cycle of 126 K (-147°C). The temperature range experienced through the tube on the cold cycle was less than 1°C. Conversely on the hot cycle the leading edge of the tube reached 297.5 K (24.5°C). The inner surface also recorded this temperature. Comparison of the trailing edge values show that at the end of the cycle a 3.8°C temperature gradient still existed across the wall thickness. The inner face was at 219.9 K (18.9°C) whilst the outer face was at 287.1 K (14.1°C). The total change across the tube was therefore 10.4°C. Figure 5.17 illustrates the thermocouple readings for a complete three hour cycle mid way through the test series. Figure 5.18 shows the original and subsequent cycle for all four thermocouples.
Figure 5.19 illustrates the strain gauge configuration for the specimen tubes mounted on the loading frame which was positioned at the base of the chamber. The use of a dummy gauge outside the environmental test chamber illustrates the change in strain readings as the temperature is cycled, thereby making the determination of the changes in strain values due to any variation in the coefficient of thermal expansion very difficult. The alternative situation which could have been adopted, that is an internal dummy gauge, bonded to a specimen of the carbon-PES composite. This would provide the temperature compensation required but as it was subjected to the same loading environment the strains due to the small changes of length would be masked.

The two 2 m triangles were positioned between the 9/21 truss unit and the door. Both the triangles were thermocoupled but due to an unknown fault the thermocouple on the shadowed side of the rear most triangle, the yellow noded triangle, failed after the second cold cycle. This made the comparison of readings on that triangle impossible. The minimum temperature reached by the leading edge of the black noded triangle was 127°K (-146°C) whilst the trailing edge reached 124°K (-149°C). On the hot cycle the temperature range across the tube was 269°K (-4°C) on the leading edge to 227°K (-46°C) on the trailing edge.

The dynamic excitation of the 9/21 truss revealed some significant differences to the response behaviour exhibited in the first solar simulation test series. Figure 5.20 illustrates the relationship between the resonant frequency and the cycle temperature. This illustrates that a shift in frequency occurs during the cycling period. This shift is significantly smaller than that which occurred in the first solar simulation test. The change in material and in particular the change in the fibre orientations appears to result in a more stable member. The shift in frequency is about 0.5 Hz, which is equivalent to an increase in modulus of 1.4 percent. The enhanced stability, or reduction in the coefficient of expansion of the members due to the fibre orientation also reduces the change in axial stress. This factor also contributes to the reduced frequency shift. The effect of the return spring on the shaker contracting and stiffening as the temperature falls was
eliminated for this series of test due to the use of a two pole magnetic solenoid as the excitation device.

5.6 Third Thermal Test Series.

The third thermal test series consisted of the same 9/21 truss unit used in the previous test series being subjected to a further period of thermal loading. Figure 5.21 indicates the instrumentation positions for the third thermal cycling test series, it will be noted that there was a considerable increase in the number of thermocouples used in the third test series over the first and second tests. This was due to the experience gained from these first two thermal test series in which the analytical analysis assumed a uniform distribution of energy from the solar simulation which subsequently proved not to be the case, Figure 5.22 illustrate the temperature distribution from the third test series. All the elements were thermocoupled, some were thermocoupled in three places but the majority only at the mid-point. The purpose of the multiple thermocouples was to obtain information about the temperature gradients that exist along the length of a member, particularly those subject to partial shadowing from other members. This is a situation that will predominate within the full structure.

Pre and post chamber testing of the structure using both the solenoid and a shaker was undertaken to obtain modal data. The results of these tests indicated that the structure suffered no ill effects from the thermal cycling. It is evident from this that the cracking seen to occur in the members is only surface cracking. Despite a change in the grade of the PES matrix from 4200 to 4800 there was little change in the surface cracking of the tubular specimens tested in the first solar simulation test. However, those individual tubes tested in the Second Solar Simulation Test showed no signs of surface distress. Discussions with the manufacturers have indicated that no change of supply of the materials or in the methods of manufacture for this batch of tubes. The cracking is limited to the outer, matrix rich, region of the tube and does not extend through the outer prepreg. The tubes were made up from pre-pregs and sheets of PES with the polymer
sheets being added to give the required fibre percentage in the composite. It had been assumed that by positioning a sheet of PES on the surface of the outer prepreg it would provide a good surface finish to the tube. However it is now realised that this PES layer should be within the body of the composite to reduce the surface cracking to a minimum.

The strain gauge readings taken during the solar simulation tests require more time to analyze fully. However it can be said that the gauges cannot be used to measure strain in a comparable form for different temperatures. It would appear that the gauges are measuring a combination of temperature and strain values in the material as the temperature varies. The problems arise due to the positioning of the dummy gauge. The use of the internal dummy on the RAE facility shows how the readings vary as the temperature changes but makes measurement of changes due to stimulation difficult. The use of a dummy gauge bonded to carbon/PES composite positioned inside the chamber would perform temperature compensation but would, in doing so, mask any strain which was induced due to changes in length as a result of the temperature change.

Figure 5.23 illustrates the temperature cycles experienced by the white node and the mid points of the members which converge into the node. A similar graph, Figure 5.24, illustrates the cycle for the black node and it's connecting members. It can be seen from these graphs that the members connected to the black node experience a wider temperature range than those leading into the white node. This situation arises because of the relative positions of the members in the chamber, for although the black node is closer to the heat source, it is also lower in the chamber and therefore is unlikely to be receiving the same intensity as the white node and it's connecting members. The black node and it's connecting members will also be subject to lower temperatures due to their proximity to the heat sink. The important feature to consider when comparing the behaviour of the two nodes is that as the thermocouples are recording the internal temperature of the nodes, the relative emissivity and absorptance of the surfaces may well not be significant.
The dynamic analysis of the unit building block revealed some startling results. Figure 5.25 illustrates the relationship between the resonant frequency and the cycle temperature. This illustrates that whilst the now expected 1 Hz shift in frequency occurs during the cycling period this only applied to the first three resonances. A complete reversal was observed for the forth and fifth resonances along with a dramatic increase in the shift occurring. The likely cause of this effect is due to the effects of the thermal cycling that the unit had previously undergone. The surface cracking that had occurred during the previous thermal test series has resulted in a 2 percent reduction in the modulus of the members if the fundamental is considered rising to 9 percent for the higher modes. The effects of the change in modulus were only apparent at the reduced temperatures. Post chamber testing indicated only a negligible change in the response characteristics of the unit building block when compared with the pre chamber tests.

5.7 Gas Analysis and Vacuum Tests.

A series of proving tests were undertaken upon the constituent parts of the apparatus to be included in the thermal test series. Gas analysis of the accelerometer cables and the strain gauge bonding solvents were undertaken. In addition this apparatus was used to vacuum test the penetration point mouldings prior to fitting on the environmental chamber.

The damping problems associated with the PTFE co-axial cables which were used in the first thermal test series led to the screening of other cables for low outgassing performance and greater flexibility. The cable adopted was ex BICC designated RG 178 B/U to MIL-C-17D. The insulation consists of a thin PTFE core and FEP external cover which weighs 18 grams for a 2 m cable with terminations, compared with 27 grams for a comparable one of the original type. The cable may be readily formed into a helical spiral by suitable heat treatment, thus increasing the effective flexibility. These spirals were positioned at the ends used for connecting to the transducers.
The atomic spectra scan shown in Figure 5.26 indicated that there is a small volume of organic vapour with peaks at 68 and 92 AMU after heating at 393°K (120°C) for 30 minutes which is insignificant after a 3 hour soak, Figure 5.27. A further cycle was undertaken from room temperature, 295°K (22°C), to 393°K (120°C) which indicated that no further outgassing occurred and that there is a reduction in the total pressure to 5.5x10⁻⁶ torr, Figure 5.28. The background gas content of the apparatus with no sample present is shown in Figure 5.29.
5.8 Tables

<table>
<thead>
<tr>
<th>Pigment Colour</th>
<th>White</th>
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<td>0.987</td>
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<td>Absorptance</td>
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Table 5.1: Pigment Colour Emissivity/Absorptance Constants.

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<tr>
<th>Grade of PES</th>
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<th>4800</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emissivity</td>
<td>0.868</td>
<td>0.891</td>
</tr>
<tr>
<td>Absorptance</td>
<td>0.902</td>
<td>0.906</td>
</tr>
</tbody>
</table>

Table 5.2: PES Material Emissivity/Absorptance Constants.
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6.0 Discussion.

6.1 Introduction.

This chapter discusses the results obtained from the experimental and theoretical analyses undertaken. Each individual model type is discussed in a separate section and the conclusions that may be drawn are contained in the following chapter. The relative merits of the various analytical models used to describe the different structures are also discussed. The chapter includes further discussion of the results obtained from the parametric study of the modal behaviour of the full size satellite structure (Chapter 4) and from those obtained from the thermal investigations (Chapter 5). A separate section discusses the algorithm developed from which predictions of the natural frequencies of the various structural units can be made.

6.2 Perspex Structures.

6.2.1 Single Member.

The mode shapes for the first three modal frequencies obtained from the experimental and finite element models are plotted in Figures 6.1 to 6.3. These figures show that the single member behaves in the expected, classical, manner. The mode shapes indicate that bending occurs about the major axis of the member, with each successive mode indicating an increasing level of complexity of the deformation. The inclusion of masses, distributed over the central few node points, representing the mass and position of the proposed joint in its deployed state reveals a similar set of mode shapes but reduced frequencies for the first and third modes. This is due to the generalised mass for these modes and the areas of the tube that are in motion. For both of these modes the central region of the tube, where the joint is located, are at their maximum deflection. This causes the joint mass to become active and therefore results in the reduction of the frequency. Conversely, in the second mode the central region does not move, as it represents a nodal position for this mode shape. Hence the joint mass is only partially
mobilised resulting in only a very slight change in the resonant frequency. It should be noted that no change in the stiffness of the beam, due to the inclusion of the 'joint', was considered. This was justified on the grounds that the overall section around the joint would be reduced due to the nature of the joint's construction and operation. A detailed description of the nature and operation of the proposed joint is given in Reference 181 (Hollaway 1990 [181]). An increase in the stiffness of the section, due to the joint, would, to some extent, counteract the effects of the corresponding increase in mass.

The selection of the element type used for the mathematical analysis of this configuration is seen not to be critical. Any of the selected elements successfully predicted the behaviour of the beam. For this reason the element that just adequately described the behaviour of the beam was selected for use in the following analyses. It should be noted that due to symmetry of the members pairs of modes were predicted from the theoretical analyses because deformation occurs about orthogonal axes for each resonance.

6.2.2 Pyramids: 6 mm diameter members.

The first three mode shapes for the 1000 mm member length model, Figures 6.4 to 6.6 are illustrated as being typical of the deformations occurring in these systems. The shape is characterised by the relative inactivity of the vertices of the model. This explains why very good agreement between the experimental and finite element models may be obtained by using the lumped mass configuration for the joints. Utilization of a refined expression for the joint masses does not result in a significant improvement in the correlation. The geometric configuration of the structure is such that there is a high degree of symmetry. This is confirmed by the existence of multiple sets of modes in the finite element model. Triplets of modes were predicted from the theoretical analyses because deformation occurs about orthogonal planes for each resonance.
Consideration of the higher modes from the finite element models reveal the presence of frequencies corresponding to the fundamental mode of a single member of the appropriate lengths if they are considered to have fixed ends. This indicates that local member deformations within a three dimensional pyramidal system may be studied using this configuration. Expanding this to the wider field it might be expected that the unit building block for the tetrahedral system would also be a suitable configuration in which to study the local member deformations.

Examination of the frequency response functions for this class of structure indicate that the modal damping is typically less than 4% although a maximum value of 6.5% occurs in the smallest, 200 mm member length structure.

The effects of mass loading on this class of structure are clearly illustrated in Tables 4.5 and 4.6. The effect of the larger mass of the 4 gram transducer, which is equivalent to 13% of the total model mass of the 200 mm member length model is dramatic. The resonant frequencies recorded for the first three modes were 47, 12 and 10.5 percent lower respectively, than those obtained when using the smaller and lighter 1 gram transducer. The location of the transducer had a particularly marked effect upon the first resonance as the combination of the first mode of vibration of a beam and a near central position for the transducer resulted in a sharp reduction in the frequency of vibration. This effect is reduced as the ratio of transducer mass to structure mass decreases with the change from the 4 gram to 1 gram transducer. This is clearly illustrated by the differences recorded for the 1000 mm member length unit and the 200 mm member length unit. The resonant frequencies of the first mode differed by some twenty percent for the former, as compared to more than twice this value in the case of the later.

6.2.3 Pyramids 25 mm Perspex.

The behaviour of the 25 mm diameter member perspex pyramids was found to be very similar to that of the 6 mm structures. Again the mode shapes are characterised by the small displacements that occur at the vertices. The 2.0
metre member length pyramid initially caused correlation problems due to the
deviation under self-weight of the members. This illustrates the difficulties that
ground based testing of large lightweight structures poses. The situation is
analogous to attempting to support and test a large section of the proposed
structure in a 1 G environment, the structure would deform under its own weight
making accurate determinations of the behavior difficult to achieve. The effects
of axial loads on single members causes changes in their responses, the same
changes will occur in a continuous structure. The solution in this case was to
support each vertex of the pyramid by the lightweight suspension system thereby
reducing the stress levels within individual members. Again it should be noted
that due to symmetry of the structure triplets of modes were predicted from the
theoretical analyses because deformation occurs about orthogonal planes for each
resonance.

Examination of the frequency response functions for the larger diameter
members indicates that the modal damping is again typically less than 4%, with
the maximum value of 4.9% occurring in the largest, 2000 mm member length
structure. This is against the trend and may be associated with the deformations
due to self-weight of the members.

The effect of mass loading, illustrated by the results obtained in Tables 4.7
and 4.8, due to the presence of the transducers and mounting blocks was not found
to be significant in the 25 mm diameter member units. The 4 and 1 gram
transducer masses were 0.32 and 0.12% respectively of the total mass of the
smallest and lightest unit in this particular class, the 800 mm member length
unit.

6.2.4 Unit Building Block, 6 mm Perspex.

The behavior of the smaller diameter unit building blocks was similar in
form to that experienced with the pyramidal models. The mode shapes were
characterized by the lack of motion in the vertices of the structure. Similarly the
damping ratios were low, typically in the region of 2.5%, somewhat lower than experienced in the pyramidal structures, but of the same order of magnitude. The resonances were also closely spaced which tended to favour the use of the vibrator excited analysis technique rather than the impact method. The transient excitation method was used successfully to identify the regions of interest for the various units.

The degree of correlation between the experimentally and finite element derived results was good for these structures. The reasons being the same as those detailed in section 6.2.2., namely that the use of the lumped mass approach, in the finite element analysis, is adequate in the cases were there are small deformations of the vertices. In addition the enhanced restraints applied by the configuration would allow the bending moments experienced by the individual members to be transferred more efficiently thereby improving the comparison with the finite element analysis.

6.2.5 Unit Building Block, 25 mm Perspex.

The behaviour of the 25 mm diameter, 1000 mm member length unit building block was similar to that of the equivalent pyramidal model. The mode shapes were again characterized by the lack of motion in the vertices of the structure. Similarly the damping ratios were low, averaging 3.3%, slightly lower than experienced in the pyramidal structure, but of the same order of magnitude. The degree of fixity of the members that was achieved by the structural configuration and the relatively large joints allowed greater efficiency in the transfer of moments from member to member. The curvature of the members that occurs in the first few modes of the unit would indicate that the hoop and radial moduli of the material could have a significant effect upon the development of these modes. The use of perspex considered to be an isotropic material results in only a small loss of accuracy in the finite element model but may in part account for some of the differences experienced as the effect of locked in fabrication stresses within the perspex tubing was not considered.
6.2.6 Triangular Arm Sections, Perspex.

The behaviour of the three bay 25 mm diameter member perspex triangular arm section shows the beginnings of global deformations starting to influence the lower modes of the structure. The study of the effect of the assumed 'perfect' and actual geometries on the modal behaviour of the section revealed an important feature which had not previously been considered in detail, the effect of manufacturing defects. The actual structure was found to be warped which resulted in the end bays being rotated with respect to each other about the main axis of the arm section. The effect of this fabrication error was to raise the natural frequency of the unit by around eleven percent for the first three modes. This would imply that there were tensile stresses being carried by some of the members resulting in an increase in the stiffness of the elements. The effect on the mode shapes of the unit for the respective geometries was not found to be significant. The results obtained from this study are not sufficient in themselves to enable a set of guidelines on the degree of acceptable tolerances to be established. However, it is clearly an important subject that requires further detailed study.

The degree of correlation between the results derived from the experimental and the finite element techniques was found to be satisfactory. It is not clear whether the presence of an increased number of supports and hence constraints that have been applied to the physical model or the presence of the relatively large number of point masses in the finite element model resulted in the larger than expected differences in the results obtained for the smaller diameter member units. Some difficulty had been experienced with the experimental testing of these units as they were found to be susceptible to the effects of mass loading and the position of the transducers. In general it was found that the finite element meshes tended to over-estimate the resonances for this particular class of unit. This indicates that the constraints applied to the meshes may not have been of sufficient accuracy.
6.3 Carbon-PES Composite Structures.

6.3.1 Single Members Carbon-PES Composite.

The behaviour of the single member composite tubes without the mass representative end joints followed that of the classical theory and so will only be discussed briefly. The results tabulated in Tables 3.28 to 3.30 and Table 4.12 indicates that there is generally good agreement between the experimentally and finite element derived results for the fundamental modes of these elements. However there is poorer agreement when the second and third modes of vibration are considered. The finite element model over-predicts the resonant frequency in the case of the predominately uni-axially reinforced 0.958 m member whereas it under predicts the second mode of vibration by a relatively large 5.5% for the 0.990 m member which was reinforced using the ±10 degree weave. The reason for this discrepancy is not clear as the finite element model appeared to accurately reflect the subsequent mode, both in terms of resonant frequency and mode shape. The levels of damping experienced by these structural elements was found to be very low, typically in the order of 0.25% of critical. This feature characterised itself by the very sharp and well defined peaks that occurred in the frequency response functions.

The effect of the central joint in the 1.795 m element was found to be negligible. Its form of construction, a lap joint, meant that there was no increase in mass at the central point. Any increase in mass at this point could have led to inaccuracies arising in the odd-numbered modes. It is also clear that the length of the lap was sufficient to efficiently transfer the applied loads without excessive local deformations arising. This would indicate that there was little if any significant loss in stiffness of the member due to the use of a joint of this type. It should be noted that the finite element model did not explicitly take into account the presence of the lap joint.

The effect of the 100 mm diameter truncated spheres, which represented the main joint components, on the modal behaviour of the single elements was
significant. The reduction in the resonant frequency for the fundamental mode for the 0.990 m member with end caps (1.080 m total length) was by a factor of 2.3. The reduction was found to be of the same order for the lapped joint member with end caps. The effect on the subsequent modes was less marked, the reduction being of the order of 1.7 and 1.5 for the 1.966 m lapped joint member. The reductions for the shorter member were less consistent but of the same order. The use of an isotropic material property which in effect over-estimates the off axial stiffnesses of the elements does not appear to have significantly degraded the behaviour of the structural member as predicted by the finite element model. A contributory factor to the level of agreement might be attributed to the small deformations that occurred during the experimental testing which used an impact hammer as the excitation source. These small deformations would mitigate the effect of the over estimation of the hoop and radial stiffnesses of the elements.

6.3.2 Triangular Unit Carbon-PES Composite.

The behaviour of the triangular unit was dominated by it's relatively low inertia in it's out of plane direction. The lowest modes were developed in this direction consistent with the above statement. However it was found that the in-plane modes of vibration which resulted in smaller translational components being contributed by the end caps were more accurately modelled. A further contributory factor may have been that the degree of curvature of the members was smaller for these modes. This indicates that the isotropic material assumption made for the composite may have a larger influence than was originally postulated. The effect of the hoop and radial moduli of the composite particularly when the member undergoes a considerable degree of bending would appear to influence the degree of deformation. The isotropic material assumption ignores such effects.

6.3.3 Pyramids 25 mm Carbon-PES Composite.

The behaviour of the carbon composite pyramids follows a similar pattern to that of the perspex structures. The mode shapes are characterized by
the small displacements of the vertices again accounting for the agreement between the results when utilising the lumped mass configuration for the joint masses. The most important feature obtained from these models is that the isotropic description of the composite material used in the finite element adequately describes the behaviour of the anisotropic composite present in the experimental model. The finite element mesh used to describe the 2000 mm member length unit did not incorporate explicitly the presence of the mid-point lap joint of each member. The presence of the joint appeared to have little effect upon the correlation of the results obtained experimentally and from the finite element analysis, confirming the results obtained during the single member investigation (Section 6.3.1). Additional finite element steady state analyses of the composite structures, incorporating the damping ratios obtained from the modal analyses, reveal good agreement on a point for point basis for both frequency and peak value of the response.

6.3.4 Unit Building Block, 25 mm Composite.

The behaviour of the carbon composite unit building blocks follows a similar pattern to that of the equivalent perspex structure. The mode shapes were once again characterized by the small displacements of the vertices. The effect of the change in the material property specification was directly observed in this class of model. Following the first solar simulation test cycle the composite material fibre orientation was re-aligned in an attempt to reduce the degree of cracking that had been induced. The change in the grade of PES used in the matrix of the composite combined with the re-orientation resulted in a slight reduction in the apparent stiffness of the individual members. It is this reduction in the stiffness of the component members that resulted in the overall reduction in the resonant frequencies determined by the experimental and finite element analyses. The effect was linear with the near 4% reduction in the modulus being mirrored by a 4% reduction in the resonant frequency. Additional finite element steady state analyses of the composite structures were undertaken. These analyses which incorporated the damping ratios obtained from the experimental modal
analyses, also revealed that good agreement on a point by point basis for both frequency and peak value of the response could be expected, Figures 4.22 and 4.23. This degree of correlation indicates that for the cases in which the component members do not develop significant levels of curvature the isotropic description of the composite material used in the finite element models is valid. It adequately describes the behaviour of the anisotropic composite present in the experimental model even though the values were obtained from static tensile tests. The over estimation of the torsional stiffness components of the composite material's stiffness matrix would indicate that the degree of correlation between the experimental and finite element results would be expected to degrade as the radius of curvature of the members decreased. In addition it suggest that the material property values obtained from the static tests were of the correct order of magnitude as these are dependent upon temperature and frequency.

6.3.5 Triangular Arm Section, 25 mm Composite.

The construction of the 25 mm diameter member composite arm section was closely observed with a theodolite being used to accurately position the members. A detailed examination after construction revealed no significant deviations of either the members or the joints from their intended positions. For this reason the finite element model assumed the 'perfect' geometry case. It was observed that the finite element model over predicted the resonant frequencies of the lower modes of the unit. As discussed in Section 6.2.6 the effect of the applied constraints may not have been of sufficient accuracy to adequately model the actual physical supports. The observed mode shapes did not reveal that significant deformations of the members were occurring, which would have resulted in a loss of accuracy due to the use of an isotropic representation of the composite material. The behaviour of the eight bay unit indicated that the lower modes of vibration resulted in axial straining of the members rather than lateral deformations so the continued use of the isotropic approximation in justified.
6.4 Finite Element Model Quality.

Considerable effort was directed towards the development of the finite element modelling techniques used to describe the behaviour of the various structural members and units examined during the course of this thesis. This section describes the relative quality of the results obtained from these models when compared to the results obtained from the experimental analyses. Figures 6.7 to 6.11 show the comparison, for the first three modes, between the results obtained from the experimental and finite element analyses of the 6 mm diameter member perspex unit building blocks. A perfect finite element representation of the physical model would be exhibited by the two lines connecting the experimental and finite element result points, coinciding and having a slope of unity. This situation does not occur for any of the results presented here but nevertheless clear trends are established by these graphs. Even though the sample of three points is quite modest it can be seen that the scatter about the forty-five degree line is small, indicating that there is a reasonable correlation between the two sets of results. In general the finite element models appear to under predict the natural frequencies of the first four modes for this class of structure. One possible cause of this could be that the effect of the supports applied to the physical units was not as small as had been originally suggested. The finite element models assumed that a free-free condition existed, this may not have been the exact case but, tests made early in the analyses indicated that the effect of including a light flexible spring was negligible. This may suggest that an alternative source of error should be considered. The use of a constant value for the modulus of elasticity for the perspex may not be ideal when one is considering such a broad range of frequencies. However, the differences between the results are only of the order of a few percent and as such lie within the boundaries of normal modelling and measurement process differences.

On the simple structures with widely spaced modes it was relatively easy to identify the mode shapes for the two sets of results. The more complex structures with very closely spaced modes proved to be more challenging.
Particular care was give to the assessment of the mode shapes and it’s associated natural frequency. It would be expected from the very nature of finite elements that the two lines would diverge because the number of degrees of freedom for the finite element model would be considerably less than those of the physical structure. The presence of these extra constraints would result in a degrading of the representation as the mode number increases. Additionally the effect of the differences occurring between the real and complex mode shapes should not be ignored.

6.5 Damping Effects.

The effect that damping has on a structural unit will in part depend upon the configuration of that unit. A single member will be subjected to damping in relation to it’s material properties, the particular mode being excited and the effect of any fluid surrounding the unit. The effect on a complete structure will relate to these points and also to the interaction of the individual members forming the unit. The change in damping with the change of mode shape is generally more pronounced in a more complex structure as different modes will cause excitation of different parts of the structure, some of which will be more heavily damped than others. Material damping is known to be dependent upon the amplitude and frequency of the vibration. In general the amplitude of the forcing function was kept as low as possible in an attempt to avoid the production of non-linear effects. Some damping is derived from losses occurring at joints and connections which accounts for some of the enhanced damping experienced by complex structures over that experienced in simple units.

The effect of the surrounding air on the modal behaviour of the structural units being tested was more difficult to ascertain. Tests were performed on the composite unit building block in the thermal test chamber after the thermal cycling had been completed but prior to release of the vacuum. These tests were then repeated after the vacuum had been released. There was little change in the resonant frequencies derived from these tests but there appeared to be a small but
fairly consistent 3 to 4% change in the level of damping derived from the frequency response functions. The change, which indicated that a lower level of damping occurred when the structure was suspended in the vacuum, would appear to be attributable to the removal of the effects of air damping. The differences calculated were surprisingly large given that the composite unit building block possesses a relatively small surface area. However, these differences could be due to 'normal' experimental differences, but in view of their consistent nature this may considered too pessimistic.

6.6 Model Size Effects.

One of the aims of the work was to assess the effect of member length, for an individual structural configuration, on the resonant frequencies of that system. The intention was to develop a reliable algorithm from which changes in the resonant frequency of a unit could be predicted.

The beam represents one of the simplest structural elements available and the vibration of a continuous elastic beam forms the starting point for the development of the algorithm. Considering the classical mathematical starting point for a prismatic beam and assuming small deflections it is possible to derive the equation for the moment acting at any section. A similar derivation may be made for each type of boundary conditions. Examination of these equations reveals that the frequency of oscillation for the member is:

a) inversely proportional to the square of it's length.

b) proportional to the square root of the modulus and the moment of inertia of the beam.

c) inversely proportional to the square root of the cross-sectional area and the density of the beam.
The effect of simply changing the length of the member for the single beam case, retaining the same material and geometric properties can then be predicted from application of the following formula:

\[ \frac{f_{na}}{f_{nb}} = \frac{L_b^2}{L_a^2} \]  

6.1

Application of this equation to determine the results of the natural frequencies of the single beam reveals that the observations are correct. The formula applies for all end condition cases because the constant \( \lambda^2 \) divides out leaving the relationship dependent upon the inverse of the square of the member length.

This formula applies to continuous beams. The structural forms examined in this thesis are developed from a series of members which have identical properties. This feature, combined with the analysis which assumes a free-free condition, causes the structure to behave as some form of 'continuous member'. The boundary conditions for each structural type are the same. Hence no allowance need be made for the member end conditions as they cancel each other out. Examination of the mode shapes for the various structural types reveals that the same displaced shape is adopted for each model. This combined with the relatively small displacements that the joints of each structural unit undergo in their lower modes of vibration, contribute to the 'continuous beam' type of behaviour.

Application of equation 6.1 to the prediction of the natural frequencies of the structural units examined, reveals that there is, in general, good agreement between the actual and predicted results using this algorithm. Table 4.26 illustrates that this applies equally to the experimentally derived natural frequencies and to the finite element derived frequencies.
The full algorithm incorporating the modulus of elasticity, moment of inertia, cross-sectional area and density of the members making up the structure, can be applied so as to determine the natural frequencies of structural units consisting of elements of different sizes and materials. Extensive testing of the algorithm confirm that in all cases a resonance will occur within a few percent of the actual frequency of the structure, whether that is an experimentally derived value or a finite element prediction. In most cases the error is less than one percent but in one case it does rise to four percent. Ideally the algorithm would always under-predict the actual resonant frequency as this would result in a conservative situation which for control purposes is preferable to an algorithm which over predicts the result.

6.7 Parametric Study of the Proposed Structure.

A study of the viability of using a deployable structure as the space component of a Land Mobile Communications System was one of the main aims of the thesis. A series of finite element models have been developed which represent the various constituent parts of the structure. There are many factors which will influence the decision on whether or not a deployable structure should be used. One of these is the ability to produce a flexible joint which allows the structure to be folded for transportation into orbit and then successfully deployed. This problem has been addressed by a parallel study (Hollaway 1991 [191]) and so is not discussed here. It has been assumed that such a joint would act as a fixed joint once deployment had occurred. The electrical performance of the satellite can be significantly altered by the deformations that it experiences. Such deformations arise from a number of sources including on-orbit station keeping manoeuvres, thermal shock and impact from foreign bodies. The global deformations that occur in the structure are one source of error, whilst the deformations that occur on the surfaces of the reflectors are another. These two sets of deformations are best analyzed separately as they occur at distinctly different frequencies. The global deformations occur at significantly lower frequencies than the local surface distortions of the main and sub reflectors, this
is due to the relative stiffnesses of these components. The most significant global deformations occur when a combination of the transverse displacements are considered, whereas the most significant local deformations of the main reflector are found in the out of plane direction.

It is evident from examining the first four mode shapes of the truss in isolation that considerable deformations of the reflecting surface can be expected, if resonance is allowed to occur. Whilst the quoted values for the deformations represent the maxima for a particular mode and in general occur at the periphery of the truss, it appears that the allowable surface deformation, $\delta_{\text{max}}$, of 1.36 mm will be exceeded in each of the modes considered. Mode three is likely to be of greatest concern as the radius of curvature of the truss would be changing through $\pm900$ m during a period of around 0.25 seconds. The resonant frequencies being considered here are for the reflector in isolation. These frequencies are considerably above the fundamental frequency of 0.076 Hz. expected for the structure as a whole.

The addition of the main support arm, sub-reflector, solar arrays and the communication package significantly alters the modal properties of the tetrahedral truss. The additional components were added in two stages. The effect of the main support arm and sub-reflector was considered first and as anticipated, caused a marked change in the modal behaviour of the satellite. It was anticipated that the addition of the solar array panels would also have a significant effect on the mode shape of the proposed structure. The communication package which can be considered to act essentially as a point mass concentrated at the corner of the main supporting arm was thought to be unlikely to cause such a dramatic change as the inclusion of the solar arrays.

It is clear from the results that the inclusion of the solar arrays was not as great as might have been expected. This is largely due to the relatively small additional mass being added and the location of this mass. There was only a relatively small reduction in the fundamental frequency of 1.4% but there was a
significant change in the mode shape. The centre to centre displacement of the two reflectors was reduced from 4.48\(\lambda\) to 2.16\(\lambda\) with the main reflector remaining essentially planar, Figure 4.31. A similar effect was recorded for the second mode with the resonant frequency reducing by 4.8\% and the centre to centre displacement reducing from 5.43\(\lambda\) to 0.46\(\lambda\) with the truss again remaining essentially planar, Figure 4.32. There was a significant change in the mode shape for the third resonance. The centre to centre displacement of the two reflectors rises to 2.81\(\lambda\) from 1.48\(\lambda\). The main reflector remains essentially planar for this mode, Figure 4.33. The fourth mode results only a small change in the centre to centre displacements, up from 2.9\(\lambda\) to 3.42\(\lambda\) with the main reflector remaining planar but displaced, Figure 4.34. Comparison of these deformations with the displacements of the main reflector when considering the previous case which excluded the communication and solar array packages are clear. The inclusion of the additional panels has not altered the overall form of the displacements of the main reflector. However, the displacements experienced by the reflecting surfaces should they resonate would give cause for concern as in all cases the criteria set down for the allowable defocus and surface deformation, \(\delta_{w}\), were exceeded.

6.8 Thermal Investigation.

This section summarises the results obtained from the three thermal cycling tests that were performed on the triangular section and unit building blocks formed from the two types of composite material. The solar simulation tests performed on the unit building block manufactured from the composite materials provided some valuable results but also posed many more questions than they answered.

The effects of the thermal cycling on the dynamic properties of the carbon-PES unit building block indicated that the natural frequencies of the unit altered by about 1 Hz between the extremes of the temperature cycle. This variation could have significant effects upon the response of the proposed structure and hence it's performance. One possible effect would be to alter the order of the modes as well
as reducing the frequency at which they occur. The performance of the proposed structure would be significantly degraded if the threshold at which effective active control of the structure, used to reduce the effects of point to point deformations, is crossed.

The effects of the cracking that was observed to have taken place during the thermal cycling did not appear to significantly affect the performance of the structure. The effects due to the initial thermal test series on the unit building block were only observable whilst the unit was subjected to a thermal loading as the results from the pre and post chamber tests showed no significant changes in the modal behaviour of the unit. A more significant change was noticed during the subsequent thermal cycling of the same unit as there was a considerable reduction in the resonances detected across the extremes of the temperature cycle. This was particularly noticeable in the cases of modes three and above, Figures 5.20 and 5.24. It is thought likely that the effects of the cracking observed to have taken place after the first test series were responsible for this reduction. This has implication for the long term stability of such materials in the space environment.

The effect of surface pigmentation on the internal thermal response of two of the nodes on the unit was not as predicted. The differences in the emissivity and absorptance of the two pigmentations together with the different locations of the nodes were such that a greater variation would have been expected. However it is possible that a combination of factors, including the location of the white node closer to the thermal mass of the sting but nearer to the focal plane of the lamps, had a cancelling effect upon the surface characteristics. It is not possible at this stage to give a definitive answer to this point, as further experimentation will be required.

The instrumentation used within the chamber was found to be sensitive to the conditions. Some considerable difficulty was experienced in initially exciting the structural units despite their low damping properties, this was due to the effects of the temperature cycling on the shakers. The various metals used in their
construction had significantly different coefficients of thermal expansion which is believed to be largely responsible for the initial start-up problems. It was found that electric resistance strain gauges were not able to record the straining of specimens again because of the temperature cycling in the solar simulation chamber.

The temperature variations recorded across the unit building block were greater than those which had been previously predicted for a structure in space. There are several possible causes for this variation. It is thought that the influence of the various internal components of the chamber, in particular the mobile support arm mechanism, combined with the non-uniform exposure of the structural units to the solar simulation were responsible for this discrepancy. The differences in the thermal masses of the chamber’s internal components and the structural components under test was very large, which may have contributed to this phenomenon. Further examination of the causes of the change in resonant frequency of the various structures exposed to thermal excitation will be required. It is necessary to determine which of the following criteria or combination of these caused this variation to arise:

a) the effects of straining,

b) the coefficient of thermal expansion,

c) changes in other material properties,

or d) instrumentation variations at the reduced temperatures.

The results obtained from the thermal investigations indicate that the surface cracking experienced had little effect on the overall behaviour of the structural units which increases the confidence for using this composite material in deployable space structures particularly if the outer layer of the tubes is formed from the pre-impregnated material. It has not been possible to simulate the effects of atomic oxygen attack on the material and therefore it is recommended that a study of this and remedial measures, should they be necessary, in the form of coatings be studied to ascertain the long term stability of the composite material.
6.9 Figures.

Figure 6.1: Perspex Single Member: 25 mm diameter
F.E. v. Expt. Mode Shape 1

Figure 6.2: Perspex Single Member: 25 mm diameter
F.E. v. Expt. Mode Shape 2

Figure 6.3: Perspex Single Member: 25 mm diameter
F.E. v. Expt. Mode Shape 3
Figure 6.4: 1000 mm Perspex Pyramid, 6 mm diameter, F.E. v. Expt. Mode Shape 1

Figure 6.5: 1000 mm Perspex Pyramid, 6 mm diameter, F.E. v. Expt. Mode Shape 2

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Figure 6.7: Experimental v Finite Element Natural Frequency comparison for the 200mm 9/21 Unit Building Block.

Figure 6.8: Experimental v Finite Element Natural Frequency comparison for the 400mm 9/21 Unit Building Block.
Natural Frequency Expt. v. F.E.
600 mm 9/21 Unit Building Block.

Figure 6.9: Experimental v Finite Element Natural Frequency comparison for the 600mm 9/21 Unit Building Block.

Natural Frequency Expt. v. F.E.
800 mm 9/21 Unit Building Block.

Figure 6.10: Experimental v Finite Element Natural Frequency comparison for the 800mm 9/21 Unit Building Block.
Figure 6.11: Experimental v Finite Element Natural Frequency comparison for the 1000mm 9/21 Unit Building Block.
7. Conclusions.

7.1 Introduction.

To validate the finite element modelling technique to be used to predict the behaviour of the proposed satellite structure a series of experimental tests have been performed. These tests have been developed on a step by step basis from single members to near full sized elements of the proposed structure. During the course of these tests information has been gathered on the techniques that need to be employed if successful ground based tests are to be achieved. Two material types were considered and where possible geometrically identical units have been manufactured and tested, thereby continuing the step by step process. The experimental tests undertaken yielded information on the modal characteristics of the various structural units, i.e. their structural frequencies, mode shapes and damping ratios. Finite element models of each of the structural units were formed and refined so that an accurate representation of the physical models behaviour was obtained. The behaviour of these models accurately describes both the local member and global deformations that the physical units experience when subjected to excitations. All the aims of the project have been achieved and some additional work on the thermal behaviour of the composite material units has been undertaken.

7.2 Perspex Structural Units.

The progression towards increasing structural complexity of the perspex units has allowed an assessment of the effects of both local member and global deformations on the global modal behaviour to be obtained. By considering the change in member cross-section, the effects of the macroscopic structural influences are seen to diminish in importance as the size of the elements forming the structural unit increase up to a point at which the effects of gravity cause deformations under self weight to occur. The influence of the joint rigidity reduces as the complexity of the structural unit increases, this is particularly noticeable with the transition from the pyramid to the unit building block. The modal
behaviour of the units rapidly becomes dominated by global rather than local member deformations. The effect of local member deformations is still detectable for all configurations but their influence decreases with increasing complexity. The modal behaviour of structural forms becomes increasingly dominated by the units overall dimensions. This is observable in the transition from the three bay arm to the eight bay arm units. The effect of the transducer masses on the smaller units has been observed to be very large. A guideline for the ratio of transducer mass to structure mass of around 0.05 is recommended. These attributes have resulted in increasing correlation between the experimental and finite element analyses being obtained as the size of the units increases up to a point at which the effects of gravity cause deformations under self weight to occur.

The material constants used in the finite element analyses have been based on the results obtained from dynamic testing of perspex over a range of frequencies. The change in modulus over the range of frequencies to which the models were subjected was small and consequently a single value was adopted. The change in modulus appears to be dependant upon the levels of strain induced in the material. This would indicate that, at low frequencies where the induced strains would be higher, the modulus would change more rapidly. However, the experimental techniques employed, namely the transient and shaker driven tests, induced only low levels of straining in the members. The suspension systems adopted also tended to reduced the stresses in the elements. Hence the use of a single material constant was found to be satisfactory. This is illustrated by the excellent correlation obtained for the single beam case.

7.3 Perspex as a Modelling Material.

One of the reasons for selecting a modelling material is to reduce the cost of the experimental testing programme. However, the material chosen must achieve a reasonable representation of the behaviour of the actual material to be used if the test results are to be valid. An initial inspection of the material properties of perspex and the Carbon-PES composite would not indicate that they
were compatible; one is isotropic and the other orthotropic. The high specific stiffness and low damping characteristics of the composite are almost the opposite of those found in the perspex material. Nevertheless it has been observed from parametric studies that the stiffness characteristics of the large space systems are largely determined by the longitudinal stiffness of the members which are frequently uni-axially aligned. This results in the minimization of the differences between the materials and removes the requirement for an orthotropic modelling material and hence allows the use of perspex as a modelling medium.

The suitability of perspex as a modelling material increases as the complexity and size of the sub-structure units increases. However, good estimates of the resonant frequency and the mode shape may be obtained by using perspex as the modelling material even in the sparse pyramidal units. It is not suitable for determining the modal damping properties of the equivalent composite unit.

The use of solvent jointing techniques for both materials ensure that a consistent joint is obtained in each case. Such jointing cannot be guaranteed for other possible candidate materials, e.g. aluminium. The effect of the joints particularly in respect of the development of modes in the large structures is seen to be low, whereas in the sparse structural units their bending rigidity becomes more important. The good correlation that has been achieved between the finite element and the experimental analyses of the sub-structural units in both perspex and the carbon-PES composite indicate that perspex is a suitable material for modelling these structures.

7.4. Model Size Effects.

A reliable algorithm from which changes in the resonant frequency of a unit could be predicted has been derived. The effect of changing the length of the member for the single beam case, retaining the same material and geometric properties reveals that the dependence upon the square of the length is correct. The formula applies for all end condition cases because the constant \( \frac{1}{k} \) divides out
leaving the relationship dependent upon the inverse of the square of the member length. Within each model type examined the individual members material properties were identical. The boundary conditions within each structural type are the same. Hence, no allowance need be made for the member end conditions as they cancel each other out. Examination of the mode shapes for the various structural types reveals that the same displaced shape is adopted for each model. This combined with the relatively small displacements that the joints of each structural unit undergo in their lower modes of vibration, contribute to the 'continuous beam' type of behaviour.

The full algorithm incorporating the modulus of elasticity, moment of inertia, cross-sectional area and density of the members making up the structure, can be applied so as to determine the natural frequencies of structural units consisting of elements of different sizes and materials. The effect of changing the material property within a set was addressed when the composite material was changed following the first thermal cycling test. Extensive testing of the algorithm confirm that in all cases a resonance will occur within a few percent of the actual frequency of the structure, whether that is an experimentally derived value or a finite element prediction. In most cases the error is less than one percent but in one case it does rise to four percent. Ideally the algorithm would always under predict the actual resonant frequency as this would result in a conservative situation which for control purposes is preferable to an algorithm which over predicts the result.

7.5 Composite Structural Units.

Like many of the equivalent perspex sub-structural units investigated the composite units were not sufficiently large to allow modes dominated by global deformations to be developed; hence, no conclusions as to the suitability of the end joint adopted can be drawn. The pyramidal configuration was found to develop fixed end behaviour in it's members and as such might be a useful method of investigating the modal behaviour of such a member. The
composite models provided further valuable correlation opportunities for the development of the modelling technique used in the finite element analyses of the proposed structure.

The material constants for the composite system used in the finite element analysis were determined from experimental tests carried out within the Research Group. The stiffness of the composite is largely a function of the effect of the carbon fibres as the PES matrix largely just aligns the fibres. The constants determined from the tests were calculated on the basis of statics and hence there is room for justifiable criticism as the dynamic material constants were not used. The constants were determined from specimens prepared from flat sheets and not from tubes; it would have been difficult, although desirable, to have obtained the material constants from tests performed on the tubes. Examination of the cross-sectional areas indicated that the compaction was higher in the flat sheet material than in the tubes which could result in a discrepancy arising. The absence of other material property information required to describe the composite accurately meant that an isotropic representation of the composite material was adopted for the purposes of the finite element analyses. This was partly justified due to the fibre orientation adopted for the initial material. This isotropic representation has been shown to produce accurate estimates of the composite's modal behaviour particularly for its lower modes of vibration. It is clear that in modes where the contribution of the hoop and radial moduli becomes significant the isotropic representation would not reflect their contribution. This is not thought to be a serious weakness as the parametric studies of the proposed structure have shown that the lowest global modes of vibration are due to the axial straining of the elements.

In the cases of the sub-structural units it is believed that the potential loss of accuracy is considerably out-weighed by the simplification of the material description. It must also be borne in mind that the aim of the correlation is to model the behaviour of the full structure. Hence it is considered that the isotropic representation of the composite material which is predominately uni-axially
reinforced can be justified and indeed the good correlation, between the experimental and theoretical analyses, confirms this assumption.

7.6 Parametric Study of the Proposed Structure.

One of the aims of the project was to assess the viability of using a deployable structure as the space component of a Land Mobile Communications System. The electrical performance of the structure will be dependant to a large extent upon the deformations that it experiences. The excitations which cause these deformations to occur are due to normal on-orbit station keeping manoeuvres, thermal shock and impact from foreign bodies. Two distinct sets of deformations occur, the global deformations and the local surface deformations of the reflectors. These occur at distinctly different frequencies. The global deformations occur at significantly lower frequencies than those of the local surface distortions of the main and sub reflectors, this is due to the relative stiffnesses of these components. The most significant global deformations occur when a combination of the transverse displacements are considered, whereas the most significant local deformations of the main reflector are found in the out of plane direction.

Considerable deformations of the reflecting surfaces can be expected to arise, particularly if resonance is allowed to occur. Examination of the first four mode-shapes of the truss in isolation, that the allowable surface deformation, $\delta_m$, of 1.36 mm will be exceeded in each of the modes considered.

The addition of the main support arm, sub-reflector, solar arrays and the communication package had the anticipated effect of significantly altering the modal behaviour of the structure. The effect of the addition of the main arm and sub-reflector was much greater than the subsequent addition of the solar array panels and communication package. In all cases the criteria set down for the allowable defocus and the permitted surface deformation, $\delta_m$, was exceeded. This

Note: The displacements reported do not relate to a given force level hence it is not possible to say definitely if the criteria will be exceeded.
suggests that some form of active control of the structure will be required if these criteria are to be met.

7.7 Thermal Investigation.

The effect of the thermal tests on the dynamic properties of the carbon-PES unit building block was that it revealed that the natural frequencies of the unit altered by about 1 Hz between the extremes of the temperature cycle. This variation occurred for both material types investigated. If this variation occurred in an actual structure it would result in a significant change in the response of that structure. The effect on point to point displacements could be very large particularly if the threshold above which effective active control of the structure can occur is crossed. It will be necessary to carry out further more detailed investigations into this effect as it is not possible to give a definitive answer at this stage.

The cracking that was observed to have taken place during the thermal cycling does not appear to significantly alter the performance of the structure within a test sequence. The detection of these cracks in the members in the first composite unit building block resulted in significant changes to the materials and lay-up used to form the composite. Examination of the modified structure following the thermal tests revealed that some cracking had occurred in the outer PES layer. However, the creation of the cracks caused no significant changes in the modal behaviour of the unit during the first thermal test series. It is therefore concluded that the cracks were formed during the period in which the vacuum was being created in the chamber, prior to the dynamic tests being performed. A more significant change was noticed during the subsequent thermal cycling of the same unit as there was a 1.5 to 8.5% reduction in the resonances detected across the extremes of the temperature cycle, particularly with the third and above modes.

The effect of surface pigmentation on the internal thermal response of two of the nodes on the unit was different from that expected. Further experimentation
will be required to determine why the differences in the emissivity and absorptance of the two pigmentation did not result in a larger difference being recorded. However, it is probable that a combination of factors, including the location of the white node closer to the thermal mass of the sting but nearer to the focal plane of the lamps, had a cancelling effect upon the surface characteristics.

The instrumentation used within the chamber was found to be sensitive to the conditions. Some considerable difficulty was experienced in initially exciting the structural units despite their low damping properties, this was due to the effects of the temperature cycling on the shakers. It was found that strain gauges were not able to record the straining of specimens in the chamber.

The temperature variations recorded across the unit building block were greater than those which had been previously predicted for a structure in space. The differences in the thermal masses of the chamber’s internal components and the structural components under test was very large, which may have contributed to this phenomenon. The internal components of the chamber, in particular the mobile support arm mechanism also contributed to the discrepancy. The non-uniform exposure of the structural units to the solar simulation are also considered to have contributed to the differences. Further examination of the causes of the change in resonant frequency of the various structures exposed to thermal excitation will be required as the exact causes are currently unknown. The effect of the cracking that occurs in the composite material has important implications with respect to the integrity of any protective coatings that may be required.
8.0 Recommendations for Future Research.

The work undertaken during the course of this thesis has revealed several areas into which further research efforts should be concentrated. It has also become evident that this thesis has encompassed several areas that the European Space Agency intends to pursue. Their current technological research and development program for the next three years (TD(91) 1991 [192]) indicates that this thesis has addressed the areas outlined below with varying degrees of detail.

a) Deploying Truss Structures (TD(91) 1991 [193]).

b) Measurement of the vibration/damping/modulus characteristics of materials during exposure to a simulated space environment (TD(91) 1991 [194]).

c) Dynamics for large reflectors (TD(91) 1991 [195]).

d) Analysis/Design of Composite Material Systems (TD(91) 1991 [196]).

The most important area of further research is in the field of the thermal behaviour of the carbon-PES composite materials. The cracking that occurs in the composite material following thermal cycling must be more fully understood. The interaction at a microscopic level of the matrix and the fibres should be investigated in more detail so that the crack density may be reduced or eliminated. Some investigation into this area has been undertaken but further work needs to be performed so that the use of atomic oxygen resistant coatings may be effectively utilised. The dynamic behaviour of the material whilst subjected to thermal loading is also not yet fully understood. Further investigations into the behaviour of both the current fixed jointed structures and identical units using deploying joints needs to be undertaken. The effects of shock loading due to deployment should be considered together with a detailed study of the damping properties of the composite material and the structure as a whole.
The second major area for future research must lie in the use of active vibration suppression using a combination of intelligent materials that can compensate for externally applied loadings and optical fibres as sensors. The size and geometrical form of large space structures is such that the suppression of modes cannot be successfully achieved by the use of material properties alone. Some form of active control will be required. This implies that detection of motion/strain must be undertaken in addition to corrective action. Optical fibres could be easily incorporated into a composite material at the manufacturing stage. Light could then be passed around the system and used as a detection device. This would allow continuous measurements to be made over a large area. The use of intelligent materials, ones which are able to alter their properties in response to a given set of inputs could be used to reduce the deformations experienced by these structures. This combined with the use of conventional actuators may be used to overcome the excessive deformations that are currently seen to occur in the proposed structure. This investigation should be run in parallel with a detailed assessment of the damping behaviour of the composite material.

The third main area in which future research effort should be directed is in the effects of the orbit transfer of the deployed structures. The fabrication time involved with structures of this type would require that they remain in a hostile environment for some weeks prior to the orbit raising manoeuvre taking place. The use of electric propulsion techniques is recommended for structures of this type so as to reduce the applied loading to a minimum. The effects on the material of exposure to a hostile environment followed by a protracted orbit transfer have not previously been studied together, yet the implications this would have on the uses of composites in space are enormous.
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Appendix A

A.1 Initial Experimental Impact Procedure.

The initial experimental testing procedure was based upon using an impact hammer as the excitation device. The time domain data was processed using three items, a peak hold voltmeter, a twin beam storage oscilloscope and a peak hold strain meter. The peak hold voltmeter was used to provide the peak applied load. The structure under test was instrumented using two 60 Ω foil strain gauges applied 20 mm away from the mid-points of the members. These gauges formed a quarter bridge network. The structure’s time domain response was recorded on the twin beam storage oscilloscope and a strain against time graph produced. The peak strain meter indicated the peak applied strain for the particular member under investigation.

A.1.1 Tripod Structure.

Experimental tests were carried out on a tripod structure with leg member lengths of 200 mm and diameter 6 mm with a 1 mm wall thickness. One end of each member was rigidly fixed into a square plate which provided an encastrate end condition. This plate, 250 mm square and 10 mm thick was rigidly clamped to a substantial metal table to prevent movement of the base of the tripod arrangement. The structure was loaded as symmetrically as possible, the degree to which this was achieved was largely due to the skill of the operator. Various tip types were investigated to obtain the most satisfactory response from the structure. Figures A.1 and A.2 illustrate the nearly linear force against strain results obtained from a set of repeated impacts using two different tip types and striking surface materials. The repeatability of the results was dependant to some extent upon the expertise of the operator.

The impact of the hammer tip on the structure produces a force pulse which closely resembles a half-sine wave. This wave has a nearly uniform frequency
content up to a particular frequency after which there is a rapid reduction in the strength of the remaining frequencies present. This frequency is referred to as the cut-off frequency. There is a direct relationship between the duration of the half-sine wave and the cut-off frequency. The shorter the pulse duration the higher the frequency content of the impact. The relative stiffnesses of the contact materials and the masses involved also influence the frequency content of the impulse. The stiffer the materials the shorter the pulse and hence the higher the frequency content. Similarly the lower the mass of the hammer the higher the frequency content. In general it was found that the most satisfactory combination was the rubber tip and the perspex plug. This combination resulted in fewer miss-hits, greater uniformity in the loading of the members and concentrated the frequency content into the range of interest. A typical time domain response for both the load and the member are given in Figure A.3. The damping factor present in the unit was determined from the free decay of the vibration amplitudes.

The information derived from the force against time graph was used to produce the load variance for the finite element analysis of the structure under test. Table A.1 summaries the axial strain results obtained from the experimental and finite element analyses of the tripod structure for an equivalent 100 Newton load.

A.1.2 Four Bay Triangular Arm Section.

A second model was examined using the same technique of impact hammer and strain gauges. This model represented a section of the main support arm of the proposed satellite. The triangular section consisted of members 150 mm in length with a 6 mm diameter and 1 mm wall thickness. The inter-bay length members 150 mm. Strain gauges were attached at a 15 mm offset from the mid-points of the inter-bay members to measure the axial strains induced by an impact load striking an end node. Strain against time graphs were obtained for each strain gauge down one edge of the arm, Figure A.4. The graph clearly shows the time delay in the responses of each gauge as the distance between the impact
point and the gauge increases. Table A.2 summarizes the axial strain results obtained from the experimental and finite element investigations of this model for an equivalent 20 Newton load.

The axial strain results obtained from the finite element programs ABAQUS and MSC/NASTRAN compared well with the experimental results obtained from both of the models tested. In both cases the finite element analyses over estimated the axial strains by approximately 6.5%. MSC/NASTRAN was slightly more consistent than ABAQUS in this respect. It may be concluded that the finite element models predict, with sufficient accuracy the peak strains that were developed in the members when the systems were subjected to an impact load.

Work carried out using this time domain data is based on the assumption that the impulse response functions are a linear combinations of the system's eigenvalues. The accuracy of the time domain modal techniques is determined by the ability to separate the closely spaced modes and the simplicity of generating free-decay time functions; both of these points illustrated the requirement that a form of signal processing along the lines of a Fourier Transform would be required. It was shortly after this that the commercial package AQUIRE was obtained to perform this transformation.
### A.2 Tables.

<table>
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<th>NASTRAN</th>
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**Table A.1:** Comparison of Axial Strain for the Tripod Unit.

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<th>NASTRAN</th>
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</table>

**Table A.2:** Comparison of Axial Strain for the 4 Bay Arm Section.
A.3 Figures.

**Impact Hammer Load v. Strain Response**

**Metal Tip on Rubber Cushion**

![Figure A.1: Load v. Strain Response Metal Tip on a Rubber Pad.](image1)

**Rubber Tip on Perspex Plug**

![Figure A.2: Load v. Strain Response Rubber Tip on a Perspex Plug.](image2)
Figure A.3: Typical Tripod Unit Member Strain Response.

Figure A.4: Strain v Time Response of Strain Gauges along one edge of the 4 bay Triangular Arm Unit.