THE EFFECTS OF PARAMETER ERRORS IN FIELD ORIENTED CONTROL OF INDUCTION MACHINES.

by

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Field oriented control is an established technique for rapid control of torque in induction motors. The controller tracks the orientation of the rotor flux, which rotates at synchronous frequency. The component of stator current in phase with this flux (known as the "flux current"), can be used to maintain a constant flux. Under these conditions, torque is directly proportional to the quadrature component of stator current, or "torque current".

It has not proved cost-effective to measure either the rotor flux orientation or the motor torque directly. However both can be estimated from a combination of voltages and/or currents and position (or speed). The standard mathematical model uses the resistances and inductances of the motor equivalent circuit. These parameters may vary with temperature, motor operating speed and load. The underlying cause, range and timescale of these variations is examined, along with techniques for tracking the changes on-line. Detailed off-line characterisation results are presented for the test motor, in order to determine how accurately the parameters can be identified in practice.

A number of standard torque and flux estimators have been analysed and implemented. Experimental results are presented for a 5.5kW motor drive system. Parameter errors and delays within the controller, which cause an error in the orientation of the stator currents, are shown to affect the motor performance. The motor is incorrectly fluxed, which may reduce its efficiency and peak torque capability in the steady state. In addition, any change in demanded torque is coupled into the flux current, exciting the natural response of the motor. This is characterised by damped oscillations at slip frequency, decaying at a rate determined by the rotor time constant. The implications for closed loop torque and speed control are discussed.
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To my husband.
1. INTRODUCTION

Field oriented control is a technique which gives good dynamic control of torque from a standard industrial motor. The method was first developed by Blaschke /1972/, based on generalised machine theory and using the Park's transformation between stationary and rotating reference frames. However it did not find widespread application until Leonhard /1985/ used microprocessor and power switching technology to implement the technique. Products are now available commercially, and are usually called "Field oriented" or "Vector" drives, although these terms are sometimes applied rather loosely. At present, parameter errors in the controller can limit the performance of some vector drives, as investigated in this thesis.

1.1 Scalar control

The majority of industrial, variable-speed applications which utilise induction motors, employ what might be called scalar control. For example, the voltage/frequency ratio of the inverter is controlled but there is no consideration given to the orientation of the flux. Under a such a control scheme, the induction motor is supplied with three-phase, near-sinusoidal voltages (probably synthesised by pulse-width-modulation from an inverter). The motor rotates at a speed which differs from the frequency supplied to the motor by a load-dependent slip term, (typically about 5% at rated load).

The airgap flux level is approximately determined by the voltage amplitude. This voltage is normally pre-programmed as a function of frequency, for example as a "straight line" for a constant torque load, or "square law" for pump and fan applications, usually with a low speed boost. The flux level, together with the motor current rating, determines the peak torque capability and motor efficiency. Where precise speed regulation is not required, the supply frequency can be increased or decreased to adjust the speed. Various algorithms exist to estimate the required frequency correction with load. This correction depends on the motor characteristics, and on running conditions such as temperature and speed. Slip compensation may be difficult to set up and can lead to instability.
Speed feedback can be used to improve the accuracy of speed regulation, but with one reservation: rapid adjustments in the voltage and frequency supplied to the motor excite its natural response, introducing torque and flux transients. These typically last for several hundred milliseconds, destabilising the speed loop. The transient can be reduced, by limiting the rate of change of voltage and frequency, but this reduces the bandwidth of the speed controller. Hence the speed loop bandwidth is unlikely to be better than 1Hz.

Other scalar control methods can also be used. For example, speed control can be achieved by adjusting the slip frequency (the difference between supply frequency and rotor speed) and stator current magnitude. Speed feedback is required. The technique gives more accurate control of the steady-state airgap flux level and the control law for adjustment of frequency with load is better defined. This gives a more stable frequency compensation, but does not improve the dynamic response. Further information about scalar controllers can be found in standard textbooks (for example /Bose 1986/ and /Murphy 1988/). Yawamura /1980/ and Payn /1991/ analyse the dynamic response in detail.

1.2 Vector control

In contrast to scalar control, vector control actively suppresses the flux transient. It does this by explicitly forcing the motor flux (normally, the flux in the rotor) to a constant level. This requires continuous tracking of the instantaneous flux orientation, i.e. the peak of the spatial flux distribution. Two stator current components are controlled; the first maintains the magnitude of the flux, whilst the second (in quadrature) gives the desired torque. If these currents are oriented correctly with respect to the instantaneous flux, and certain other conditions are satisfied, independent control of torque and flux becomes possible. The two names for this control algorithm, "vector" and "field oriented", stress that both the magnitude and orientation of the stator current are important for transient-free control.

As with the dc machine, the torque bandwidth in a vector drive is set by the bandwidth of the current controller. This is limited by the inductance and resistance of the windings, the rating of the power converter and the motor back emf. Nevertheless an airgap torque bandwidth of 1kHz (although the mechanical system is unlikely to follow this) and speed bandwidth of
perhaps 50Hz can be achieved. This improvement in dynamic performance is at the cost of a more complex controller. In practice this means a faster, higher-resolution micro-processor and A/D and D/A converters, possibly more accurate current sensing and signal conditioning, and a shaft-mounted sensor for speed or position feedback.

Where a higher performance is required, a separately excited dc machine has normally been used, up till now. In specific high-performance applications (for example robotics, machine tools, aerospace) other options include permanent magnet brushless motors and switched reluctance motors. Field oriented control now enables the induction motor to compete in the lower end of this high-performance servo drive market. There is an on-going debate about the benefits of each motor type for particular applications, in terms of cost, performance and reliability.

1.3 A guide to this thesis

Field oriented control uses a model of the induction motor which is outlined in Chapter 2. This represents the motor in terms of equivalent circuit components: resistances, inductances and turns ratio. The relationship between the electromagnetic variables (voltage, current, flux and torque) can then be defined. A further equation describes the mechanical system.

Chapter 3 shows how the motor model simplifies, when vector terms are expressed with respect to the rotor flux orientation. The component of stator current in phase with the rotor flux (known as the "flux current"), can be used to maintain a constant flux. If this is done, then torque is directly proportional to the quadrature component of stator current, or "torque current". The overall controller structure is introduced. In fact, there has been a proliferation of subtly different control algorithms, according to the target application. The rationale behind these is also reviewed in chapter 3.

The assumptions underlying the motor model are explored in chapter 4. The equivalent circuit elements change with temperature, speed and load. Unless the controller can track these changes, it will end up using incorrect parameter values. The cause, range and timescale of expected parameter variations are discussed, as well as parameter tracking techniques. At
present, parameter errors limit the performance of field oriented controllers and complicate
the set-up procedures. Detailed off-line characterisation results for the test motor are given
in appendix 4, in order to determine how accurately the parameters can be identified in
practice.

The effects of controller errors are discussed in chapter 6. Firstly, the steady-state
performance degradation is shown for two commonly-used algorithms, /Nordin 1985/, /Du
1991/. The algorithms are then analysed under transient torque conditions, for two non-ideal
cases: incorrect parameter values and controller delays. The analysis is supported by
experimental results from a 5.5kW test system. Both controllers were implemented and tested
for steady-state and transient sensitivity to parameter errors. This required independent
implementation and validation of torque and flux estimators in order to measure the motor
performance. The estimators are examined in detail in chapter 5.

From a knowledge of the transient response, the stability and bandwidth of the overall torque
or speed control system can be investigated. The effect of the degraded transient response is
analysed for a typical PI speed controller in chapter 7.

The aim of the thesis has been to provide the information and techniques to decide what level
of initial motor characterisation is necessary, and what steps (if any) need to be taken to
correct for parameter errors on-line, for a given performance requirement. The thesis
concludes with an assessment of the significance of parameter errors in field oriented control.

Except where otherwise indicated, this thesis represents the author's own original work. In
particular, the analysis of the control algorithms for a change in demanded torque and the
experimental results for this and for the effects of controller delays in chapter 6, extend the
work by Nordin /1985/. The investigations into harmonic effects and core losses (appendices
1 and 2) are also new. Chapters 2 to 4 form a subject review and chapter 5 provides
necessary experimental support for chapter 6. Whilst the majority of the test system was
designed and built by the author, the controller hardware used bought-in cards, and the power
electronics was developed in collaboration with colleagues.
2 DYNAMIC MODEL OF THE INDUCTION MACHINE

In order to control the torque in an induction machine, some form of model of its behaviour is required. This chapter starts with a physical description of the relationship between currents, voltages, fluxes and torque in the motor. The resulting model can be shown to be equivalent to the generalised machine theory representation of the induction machine. The following chapter then investigates possible control algorithms, based on this model.

The stator is wound to give a sinusoidally-varying mmf distribution. The spatial distributions of current density, mmf and flux in the induction machine are expressed by Leonhard /1985/ in terms of space vectors. The vector notation introduced in this chapter, provides a convenient way of representing the magnitude and phase of the maximum of a sinusoidal distribution.

The analysis below makes a number of assumptions about the machine, which limit the validity of the motor model /Leonhard 1985/. Chapter 4 explores some of these simplifications in more detail, and looks at ways of compensating for changes in the model.

a) The windings are assumed to be sinusoidal, infinitely thin, and located on the surface of the stator and rotor. This ignores space harmonics and deep bar effects.
b) Iron losses are ignored.
c) The magnetic circuits are assumed to be linear. In practice, saturation will occur and is normally represented by making the inductances a function of current.
d) Skew, end effects and slotting effects are also neglected.

2.1 Stator current vector

For a single phase, the distribution of current density around the circumference of the machine due to the current in the winding, is assumed to be sinusoidal. As the current varies with time, the current distribution will vary in amplitude but will remain sinusoidal, as shown in Fig 2.1. (The negative sign has been chosen to follow Leonhard /1985/).
Fig 2.1 Phase 'a' stator current distribution.

\[ J_{sa}(\alpha,t) = -\frac{N_s}{2r}i_{sa}(t)\sin(\alpha) \]  

For a three-phase motor, the total current density at position \( \alpha \) on the stator circumference is the sum of the contributions from the three windings, 120° electrical angle apart.

\[ J_s(\alpha,t) = -\frac{N_s}{2r}[i_{sa}(t)\sin(\alpha) + i_{sb}(t)\sin(\alpha - \frac{2\pi}{3}) + i_{sc}(t)\sin(\alpha - \frac{4\pi}{3})] \]

A current vector can be defined in terms of unit vectors, oriented along the winding axes, and the currents in those windings:

Fig 2.2 Stator current vector

\[ i_s(t) = i_{sa}(t)\hat{a} + i_{sb}(t)\hat{b} + i_{sc}(t)\hat{c} \]
In terms of the current vector, equation 2.2 simplifies to:

\[
J_s(\alpha,t) = \frac{N_s}{2} \frac{1}{2j} \left[ e^{-j\alpha} i_s^a(t) - e^{j\alpha} i_s^b(t) \right] \]

The current vector is proportional to the maximum current density, but leads its position by 90°. For a balanced, three-phase, sinusoidal supply, the vector has a constant magnitude and rotates at the supply frequency.

The current distribution has associated sinusoidal mmf. For a path as shown in Fig 2.3, the mmf arising from phase 'a' is given by the total phase 'a' current enclosed.

**Fig 2.3 MMF due to phase 'a' current.**

![MMF due to phase 'a' current](image)

\[
\text{mmf}_{sa}(\Theta, t) = \int J_s(\alpha, t) r d\alpha
\]

Again the contributions of the three windings may be summed. Equation 2.6 expresses the mmf in terms of the current vector.

\[
\text{mmf}_s(\Theta, t) = \int J_s(t) r \quad d\alpha = \frac{N_s}{2} \mathcal{R} \left[ i_s^a(t) \int e^{-j\alpha} \quad d\alpha \right] = N_s \mathcal{R} (i_s e^{-j\theta})
\]

Maximum mmf occurs at a position aligned with the current vector. The latter can therefore be thought of as the mmf vector, indicating the instantaneous position and magnitude of the total mmf contributed by all the stator windings.
2.2 Rotor current vector

The rotor can be represented in the same way as the stator, with respect to its own winding axes. The rotor current vector is defined as:

\[ i_r(t) = i_n(t) \hat{a}_r + i_p(t) \hat{b}_r + i_c(t) \hat{c}_r \]  

However, since the rotor is moving, the unit vectors associated with the rotor windings will also rotate (Fig 2.4) with respect to the stationary stator-referenced unit vectors. The instantaneous displacement is given by \( e(t) \).

Fig 2.4 Rotor current vector.

Mathematically, this rotation is described by a multiplication by \( e^{kle} \). Equation 2.8 describes the current vector in its new reference frame; the superscripts \( s \) and \( r \) refer to the vector as it is seen from the stator, or from a point which is rotating with the rotor, respectively.

\[ ^s_i_r = i_r e^{kle} \]  

The displacement is related to the rotor's angular velocity. If the motor has more than one pole pair ("pp"), then there will be a gearing effect between electrical and mechanical angles. For simplicity, a two pole motor has been considered in this analysis.
The short circuited rotor of a squirrel cage machine is made up of equally spaced conductors rather than a sinusoidal winding. Nevertheless, it can be represented as three sinusoidal windings system since the rotor current distribution remains sinusoidal, and could have been produced by equivalent windings.

2.3 Flux vector

Flux is related to current via the mmf and magnetic circuit. The magnetic field varies with the geometry of the machine, and will be different in the teeth, back iron and airgap. A full solution requires a finite element analysis of the magnetic circuits of the machine. However, estimation of flux density can be simplified by assuming that the magnetic circuit is dominated by the airgap. The total field is the sum of the fields set up by the stator and rotor currents (denoted as $B_s$ and $B_r$ respectively), neglecting saturation (Fig 2.5). Each of these comprises a main component (which crosses the airgap) and a leakage component (which does not). The net fields at the stator surface, in the airgap and at the rotor surface are $B^s$, $B_g$ and $B_r$, respectively. $\chi$ is the coupling factor; it is determined by the magnetic circuits in the machine and for energy conservation, it must be the same from rotor to stator and from stator to rotor, respectively. $l_g$ is the airgap distance.

Fig 2.5 Relationship between stator and rotor fields.
The magnetic circuits in the machine ensure that the magnetic field is directed radially across the airgap. Hence in this analysis, the vector notation is used to represent the sinusoidal variation in flux density around the circumference of the machine, at the stator and rotor surfaces, and in the airgap, as shown in Fig 2.6.

The flux linking a single stator turn with its axis located at angle $\lambda$ (as shown in Fig 2.7) is simply related to the flux density at that angle:

$$\Phi_s(\lambda) = \int_{\frac{\lambda}{2}}^{\frac{\lambda + \pi}{2}} B_s(\theta) r l d\theta = 2lrB_s(\lambda)$$  \hspace{1cm} 2.11
Total flux linkage sums the flux linking each turn, over all the turns in the winding. The turns distribution for phase 'a' has already been described when introducing the sinusoidal current distribution. Summing over the phase gives:

\[
\psi_{sa} = \int \Phi_s(\lambda) d\frac{N}{2} \sin \lambda
\]

Equation 2.12 can be simplified by introducing the mutual inductance \( L_m \), and total stator and rotor inductances, \( L_s \) and \( L_r \). These are constant terms (in the simple linear model), related to the airgap size, degree of leakage and stator and rotor turns ratios.

\[
L_s = \frac{3N_s^2}{8l_g} \pi \mu_0 \quad L_r = \frac{3N_r^2}{8l_g} \pi \mu_0 \quad L_m = \frac{3\chi N_s N_r}{8l_g} \pi \mu_0
\]

Flux linkage simplifies to:

\[
\psi_{sa} = \frac{1}{3} [L_s \Phi_s(i_s) + L_m \Phi_s(i_r)]
\]

The flux linking the other stator windings is similar to equation 2.14, except that the limits of integration are shifted by \( 2\pi/3 \) and \( 4\pi/3 \) for the 'b' and 'c' windings respectively. The resulting flux linkage is therefore also shifted.
For example, for phase 'b':-

\[
\psi_{sb} = \frac{1}{3} \left[ L_s \Re(\hat{i}_s(t)e^{-j\frac{2\pi}{3}}) + L_m \Re(\hat{\tilde{i}}_s(t)e^{-j\frac{2\pi}{3}}) \right]
\]

The flux linking the rotor windings can be calculated in the same way to give similar results. If the rotor flux linkage is viewed from the rotor, the stator current will be rotated by -\(\epsilon(t)\). This will give for the phase 'a' rotor winding:

\[
\psi_{ra} = \frac{1}{3} \left[ L_s \Re(\hat{i}_s(t)) + L_m \Re(\tilde{i}_s(t)) \right]
\]

where \(\tilde{i}_s = i_s e^{-j\epsilon}\)

Just as three time-varying currents in sinusoidally-distributed windings give a mmf distribution whose peak magnitude and phase are given by a current vector \(i_s\), the flux linkages in the three stator windings can be described as arising from a flux vector \(\psi_s\). The rotor flux vector can be defined as:-

\[
\begin{align*}
\hat{s}_s(t) &= \psi_{sa}(t) \hat{a}_s + \psi_{sb}(t) \hat{b}_s + \psi_{sc}(t) \hat{c}_s \\
\tilde{r}_s(t) &= \psi_{ra}(t) \tilde{a}_s + \psi_{rb}(t) \tilde{b}_s + \psi_{rc}(t) \tilde{c}_s
\end{align*}
\]

Equations 2.13-2.17 above have shown that flux is related to the rotor and stator current distributions:

\[
\begin{align*}
\psi_s &= L_s \hat{I}_s + L_m I_r \\
\psi_r &= L_m I_r + L_m \hat{I}_s
\end{align*}
\]

The flux vector gives the instantaneous position and magnitude of the total flux distribution. This flux contains components arising from both stator and rotor currents. The flux linkage weights this flux by the distribution of turns in the winding. The equation holds true in an arbitrary reference frame, provided that all terms are transformed into the new reference frame.

2.4 Voltage vector

The third key variable in the machine is the voltage. This is related to the current and flux
linkage by a resistive drop and back emf term:-

\[ v_{sa} = i_{sa} R_s + \frac{d\psi_{sa}}{dt} \]
\[ v_{ra} = i_{ra} R_r + \frac{d\psi_{ra}}{dt} \]

If these equations are combined for all three phases the voltage vectors can be defined as:-

\[ v_s = v_{sa}(t) \hat{a}_s + v_{sb}(t) \hat{b}_s + v_{sc}(t) \hat{c}_s \]
\[ v_r = v_{ra}(t) \hat{a}_r + v_{rb}(t) \hat{b}_r + v_{rc}(t) \hat{c}_r \]

and shown to equal:-

\[ v_s = \frac{d\psi_s}{dt} \]
\[ v_r = \frac{d\psi_r}{dt} \]

The first equation above has been expressed as viewed from the stator. The second can be transformed to a stator reference frame. However, because the rotor is moving with respect to the stator, this introduces an extra term in the expression for rate of change of flux linkage, due to the relative motion.

\[ v_r = j \omega \psi_r - j \omega \psi_r \]

The voltage vector does not have a physical meaning in the same way as the current and flux vectors and is most easily understood as a set of winding equations.

### 2.5 Three to two phase transformation

It is possible to create the same current distribution in the machine by using two windings, that are 90° displaced, instead of the three at 120° apart. Details can be found in a standard textbook, for example /Murphy 1988/. In fact the flux and current distributions in the machine could have been produced by two, three or more windings, suitably displaced and supplied with appropriate currents. It simplifies the analysis to convert from a three winding system to the equivalent two winding one. The two axes are usually referred to in the literature as direct (d) and quadrature (q), but α and β may also be used for the stationary windings.
The transformation between three and two axes used throughout this thesis follows Leonhard /1985/. The relationship between the two systems is given below, as illustrated in Fig 2.8. The magnitude of the space vector is preserved in the transformation from the three axes to two axes.

**Fig 2.8 Three phase to two phase transformation.**

![Three phase to two phase transformation diagram](image)

\[ x_a = \frac{2}{3} x_q \quad x_b = -\frac{1}{3} (x_q + \sqrt{3} x_d) \quad x_c = -\frac{1}{3} (x_q - \sqrt{3} x_d) \]

\[ x_d = \frac{x_a - \frac{1}{2} (x_b + x_c)}{2} \quad x_q = \frac{\sqrt{3}}{2} (x_c - x_b) \]

The alternative transformation (as used by Bose /1986/ and Novotny /1986/) keeps the length of the unit axes the same in the two and three axes case. This does not preserve the magnitude of the space vector: the amplitudes of flux, current and voltage distributions in two axis notation are only 2/3 of their true values. An additional scaling factor must be introduced to compute power and torque. Both forms of transformation exclude the zero sequence term, assuming that the three-phase currents sum to zero. This is valid provided that the motor has an isolated neutral.

Equation 2.24 gives the two axis form of the stator and rotor voltage equations. These equations and their initial conditions uniquely define the electromagnetic circuits of the machine. These can be shown to be equivalent to the generalised machine theory representation of the induction machine. Provided that the motor speed is constant, the
equations are linear and standard control theory can be applied to design a controller for the motor. In practice, the motor speed is not constant, but the equations can still be treated as linear if the speed varies slowly. The validity of this assumption is discussed in section 4.2.

\[
\begin{align*}
\frac{d^2 \psi_{sq}}{dt^2} + 2\omega_s \frac{d \psi_{sq}}{dt} + s^2 R_s \psi_{sq} &= i_{sq} \\
\frac{d^2 \psi_{sd}}{dt^2} + 2\omega_s \frac{d \psi_{sd}}{dt} + s^2 R_s \psi_{sd} &= i_{sd} \\
\frac{d \psi_{rq}}{dt} + \omega_s \psi_{rd} &= 0 \\
\frac{d \psi_{rd}}{dt} + \omega_s \psi_{rq} &= 0
\end{align*}
\]

The motor can also be modelled by a pair of coupled equivalent circuits (Fig 2.9). The circuit equations are identical to the differential equations above.

**Fig 2.9 Induction motor dynamic equivalent circuit.**

In the steady state, these circuits can be shown to reduce to the familiar steady-state equivalent circuit (/Bose 1986/ and Fig 2.10). \(\omega_s\) is the supply frequency and the slip \(s\), is given by:

\[
s = \frac{\omega_e - \omega_r}{\omega_e}
\]
2.6 Torque production in the machine.

The equations above have related motor voltages, currents and fluxes. The aim is to control terminal voltages and/or currents, in order to get the desired torque. Hence some relationship between the electromagnetic variables and torque is required. The force on a current carrying conductor in a magnetic field is given by the vector product of magnetic flux density and current. In the induction motor the magnetic circuit ensures that field and current are mutually perpendicular. Maximum torque is produced when the maximum of the sinusoidal field distribution arising from the stator current and crossing into the rotor coincides with the maximum of the rotor current density distribution (or vice versa).
Expressions for magnetic field and current density have been introduced above. Torque is given by the product of magnetic field and current acting over the diameter of the machine and summed around its circumference.

\[ T_e = rl^2 \int_0^{2\pi} B_\alpha(\alpha)J_c(\alpha) \, d\alpha \]  

Leonhard /1985/ shows that this simplifies to the vector cross product of rotor flux and stator current.

\[ T_e = \frac{2}{3} L_m \bar{\Phi}(i_{\bar{r}}) \]
\[ = \frac{2}{3} L_m [i_{sd}i_{rd} - i_{ad}i_{rd}] \]

The same relationship is derived from power balance arguments by Vas /1990/. His argument uses:

\[ \text{Shaft output power} = [\text{input power} - \text{copper losses} - \text{rate of change of stored energy}] = \text{torque} \times \text{angular velocity} \]

He then defines his terms as:

\[ \text{Power in} = \frac{2}{3} [\Re(v_{\bar{s}})] \]
\[ \text{Copper losses} = \frac{2}{3} [\Re(i_{s}i_{r})R_s + \Re(i_{s}i_{r})R_r] \]
\[ \text{Stored energy} = \frac{2}{3} [\Re(\psi_{\bar{s}}) + \Re(\psi_{\bar{r}})] \]

He substitutes the stator and rotor voltage equations (2.21), into the above, to get:

\[ T_e = \frac{2}{3\omega_r} \Re\left[ \left( v_s - R_s i_s - \frac{d\Phi_s}{dt} - \frac{d\Phi_r}{dt} i_r \right) i_r^* \right] - \Re\left[ \left( R_r i_r + \frac{d\Phi_r}{dt} i_r \right) i_r^* \right] \]
\[ = \frac{2}{3\omega_r} \Re\left[ (-j\omega_r \psi_{\bar{s}}) i_r \right] \]

This is equivalent to equation 2.27 above, provided that the motor is linear. However neither derivation considers losses due to hysteresis or eddy currents, so the expression for torque will be an overestimate. Chapter 4 discusses these effects.
In the induction machine the currents and fluxes are inter-related. It equally valid to compute torque from the action torque on the rotor or the reaction on the stator. From the relationships above between currents and fluxes (equation 2.19), torque may be expressed in a number of forms. Chapter 3 assesses which form is easiest to incorporate into a controller.

\[ T_e = k(\psi_{sd}^d\psi_{sq}^q - \psi_{sq}^d\psi_{sd}^q) \]
\[ = k(\psi_{rd}^d\psi_{sq}^q - \psi_{rq}^d\psi_{sq}^q) \cdot \frac{L_m}{L_r} \]
\[ = k(\psi_{rd}^d\psi_{sq}^q - \psi_{rq}^d\psi_{sq}^q) \cdot \frac{L_m}{L_sL_r-L_m^2} \]

2.31

where \( k = \frac{2pp}{3} \)

2.7 Change of reference frame

The discussion above has looked at the machine variables as they would appear from a position fixed to the stator or rotor. One of the techniques of field orientation is to refer everything to the rotor flux, as this simplifies the direct and quadrature components of the rotor equation.

The equations can be expressed in their most general form by choosing an arbitrary rotating viewpoint. With respect to a point which is instantaneously at angle \( \theta_1 \), and rotating at a speed \( \omega_1 \), all other vectors are rotated by an angle \(-\theta_1\). As with the change from rotor to stationary reference frame, (section 2.2) the change to an arbitrary reference frame can be represented mathematically by a simple rotation (Fig 2.12 and equation 2.32).

\[ s_{x_q} = x_q \cos \theta_1 + x_d \sin \theta_1 \]
\[ s_{x_d} = -x_q \sin \theta_1 + x_d \cos \theta_1 \]

2.32

or \[ s_x = x e^{i\theta_1} \]
If the reference point is chosen to rotate or (be orientated) with the rotor flux vector, then the torque equation simplifies to the form given in equation 2.33. This is illustrated in Fig 2.13. Although the rotor flux vector has been chosen as the reference viewpoint here, chapter 3 examines the use of other vectors for simplifying the torque equation.

**Fig 2.13 Use of a change of reference frame to simplify the torque equation.**

\[
T_e = k \left( \psi_r i_{sq} - \psi_{rq} i_{sd} \right) \quad T_e = k|\psi| i_{sq}^1
\]

2.15
2.8 Equation of motion

Section 2.7 above has shown that torque is related to the electromagnetic variables by a non-linear equation. The resulting speed is determined by the inertia, friction and windage of the machine and load, and by the load torque:

\[ T_e(t) = T_L(\theta, \omega, t) + J\dot{\omega} + D\dot{\omega} + k_f \omega^2 + F \]

where \( D \) is the viscous damping, \( k_f \) is the fan coefficient and \( F \) is the friction.

2.9 Summary

This section has introduced a dynamic model of the induction machine. The model uses the concept of space vectors to represent rotating electromagnetic fields. Four linear first order equations relate currents, voltages and fluxes. These can be shown to be equivalent to the generalised machine theory representation of the induction machine. Torque is produced by the interaction of current and flux and can be calculated from any combination of current and flux in an arbitrary reference frame. Calculation of torque can be simplified by the choice of reference frame. These choices will be reflected in the controller design and account for many of the differences between the various field oriented control schemes. An equation of angular motion relates shaft torque and motor speed.
3 FIELD ORIENTED CONTROL

Torque is produced by the interaction of the stator currents with the rotor flux (or vice versa). To control the torque it is necessary to control the magnitude of both the stator current and rotor flux and also the angle between. This chapter examines the orientation of the stator currents with respect to the rotor flux (known as field orientation). A simple control algorithm links the desired values of torque and flux to components of the stator current. The implications of a practical implementation of this are explored and compared with alternative control strategies.

3.1 Ideal rotor-flux-oriented control.

The previous chapter derived a five-equation model of the machine, using a vector notation to represent the spatial distribution of currents and fluxes in the machine. Rotor-flux-oriented control simplifies this model by considering the motor from a reference point which is moving with the rotor flux vector.

\[ ^1 \psi_{rq} = 0 \]  \hspace{1cm} \text{(3.1)}

In addition, the flux transient response can be suppressed by keeping the magnitude of the rotor flux constant.

\[ ^1 \psi_{rd} = |\psi_r| = \text{constant} \]  \hspace{1cm} \text{(3.2)}

The superscript $^1$ denotes that the variables have been transformed into the rotor-flux-oriented reference frame.

The equations above are known as the field-oriented conditions /Lorenz 1986/. The conditions can be applied to the rotor voltage equation, with the rotor current expressed in terms of stator current and rotor flux.
The motor equations, under field-oriented conditions, simplify to:

\[
\psi_s = L_m i_{sd}
\]

\[
T_e = \frac{2pp}{3} \frac{L_m}{L_r} \frac{\psi_s}{i_{sq}}
\]

\[
\omega_1 = \frac{1}{\tau_r} i_{sq} + \omega_r
\]

Where \( \tau_r \) is the rotor time constant, \( L_r/R_r \).

The equations show that for a constant magnitude of rotor flux, motor torque is proportional to the q component of stator current. Rotor flux is set by the d component of stator current. The final equation gives the rate of change of rotor flux angle, \( \omega_1 \). Fig 3.1 shows how the block diagram of the rotor equation is simplified if the field-oriented conditions are satisfied.

**Fig 3.1** Block diagram of the rotor equation in stationary and field oriented reference frames.
The control problem reduces to one of /Acarnley 1990a/: 

a) locating the rotor flux reference vector,  
b) keeping the component of current aligned with the flux vector constant (this is the direct axis current $i_{sd}$),  
c) calculating the component of current in quadrature to the flux ($i_{sq}$), to give the desired torque,  
d) transforming the stator currents to the stationary reference frame,  
e) forcing the required currents into the machine.  

The complete controller structure (shown in Fig 3.2. and equations 3.4) is an inverted model of the motor, followed by a mapping from the rotor-flux-orientated to the stationary reference frame. The motor torque follows the quadrature component of stator current, as in a dc machine. Hence the dynamic response of the motor is only limited by the bandwidth of the stator current controller. The transient response of the flux has been suppressed.

Fig 3.2 Rotor flux oriented controller.

\[
\begin{align*}
  i_{sd}^* &= \frac{|\psi_r^*|}{L_m} \\
  i_{sq}^* &= \frac{1}{k_t} \frac{T_e}{|\psi_r^*|} \\
  \text{where } k_t &= \frac{3PP}{2} \frac{L_m^2}{I_r} 
\end{align*}
\]  

If the direct component of stator current ($i_{sd}$) is allowed to vary then the rotor flux lags stator...
current, as shown in equation 3.5 and Fig. 3.3. This introduces a lag into the torque, causing a deterioration in dynamic response. The time constant of this lag, \( \tau_n \), is typically 100ms for a few kW machine, extending up to a second in a 100kW motor. (s is the Laplace operator).

Fig 3.3 Rotor flux oriented controller with varying flux.

\[
|\psi_r(s)| = \frac{L_m i_{sd}(s)}{s\tau_r + 1}
\]

\[
\omega_1 = \frac{L_m i_{sd}}{\tau_r |\psi_r| + \omega_r}
\]

### 3.2 Problems with the simple controller

The effectiveness of the controller depends on how accurately it models the motor, and how sensitive it is to sensor noise. The problems with this simple control scheme are:

1. Identifying the orientation of the rotor flux vector,
2. Impressing the desired current into the machine,
3. Setting the correct torque and flux gains.

The most significant of these is the identification of the orientation of the rotor flux vector. Errors in the reference frame transformation will couple torque and flux control. If this occurs, then changes in demanded torque will result in changes in flux and transient-free control of torque cannot be achieved. The extent of the orientation error will determine the degree of cross-coupling.

In contrast, errors in the two paths relating flux to flux current and torque to torque current will simply result in scaling errors. In open-loop torque control, this means that the actual
operating point will differ from the setpoint. If there is an outer control loop forcing the torque to its set value (as occurs in speed control), then the motor will be operate with incorrect current and flux levels, for example being overfluxed (incurring higher core losses), or underfluxed (with higher torque currents for a given torque, hence higher copper losses).

The overall performance of the controller also depends on the current control. If the current control is not ideal, and the field-oriented condition of constant rotor flux is not maintained, then cross-coupling between current and flux will occur. The relationship between stator voltages, stator currents and rotor fluxes can be used to minimise current error, but this increases the controller complexity from second to fourth order.

3.3 Detection of the rotor flux orientation.

The rotor flux vector can be found by:-

a) direct measurement of flux,
b) indirect estimation from the demanded torque and flux,
c) direct estimation from terminal and shaft measurements.

3.3.1 Direct Measurement

Early schemes placed Hall sensors or search coils in the airgap of the machine to measure the flux, but these methods proved unreliable. This was partly because of limitations with the sensors. Hall-effect sensors experience thermal drift and have a limited temperature range. Search coils measure back emf, so are not effective at low frequencies. Rotor-mounted sensors of either type require slip rings or telemetry to transfer the data to the controller. The measurements can be made in the airgap instead, but need to be compensated to derive the rotor flux from the airgap value. In any case, sensors mounted inside the machine are undesirable, because the machine then becomes non-standard.

Direct measurements also proved unsuccessful because the induction machine does not conform to the ideal model presented in chapter 2. The sensors detect localised distortions in
fields, depending, for example, on whether they are mounted above a tooth or a slot.

3.3.2 "Indirect", "feedforward", or "slip calculator".

Indirect, flux-vector control uses equation 3.3 above, taken from the field-oriented conditions, and integrates to find the required orientation of rotor flux. If the current control is ideal, then the actual currents may be replaced by their demanded values in the above expression. A block diagram representation is shown in Fig 3.4.

Fig 3.4 Block diagram of slip calculator algorithm.

\[
\theta_1 = \int (\omega_r + \frac{1}{\tau_r} \frac{i_{sq}}{i_{ad}}) dt
\]

3.6

The control method is simple to implement and requires only one sensor (although in practice current sensing will be used in the current controllers). Chapter 6 examines the sensitivity of this controller to parameter errors. It also has the following drawbacks:

a) In practice, actual currents will lag demanded currents. There is no corrective feedback in the controller.
b) Errors in the slip calculation (because of errors in the rotor time constant), will give rise to decoupling errors.
3.3.3 "Direct" estimation

The alternative is to estimate rotor flux from some combination of easily-measurable variables. Such estimation techniques are termed "direct", because they are based on direct measurements, (usually some combination of motor currents, voltages, speed and position) rather than demanded values. Chapter 5 investigates four flux estimators in detail.

Although derived from direct measurement, these estimators rely on equation-based models, relating measured variables to motor fluxes. Hence the accuracy of the estimation depends on the accuracy of the models, and particularly of the motor parameters.

3.4 Current control

Field oriented control is essentially a current control strategy. The most suitable power converter would therefore be an ideal current source. No power converter gives this characteristic, but both a voltage source inverter and a current source inverter can be made to approximate a current source. The voltage source inverter is normally preferred because it has a high switching frequency, commutation is independent of the load, and relatively high bandwidths can be achieved (approximately 1kHz). This experimental work used a 20kHz voltage source inverter, with three independent ramp-comparator current controllers, because at below base speed this gave high bandwidth current control. The performance of the current controller will limit the torque bandwidth of the motor. This section explores current control strategies.

3.4.1 The power converter

Any inverter falls far short of an ideal current source in a number of ways:

a) Finite bandwidth.
b) Limited number of switch states.
c) Finite switching frequency.
d) Interlock delays and non-ideal switching.
e) Current and power constraints.

In a voltage source inverter, the difference between the motor back-emf and dc-link voltage defines the maximum voltage available to drive current into the stator windings. The stator can be treated as a resistive and inductive load with an associated time constant. The actual current lags the demanded value, limiting the bandwidth of the current controller, particularly at high speeds. There is also a large-signal, slew-rate limit.

The inverter cannot provide infinitely-variable voltage below the dc-link limit, but is constrained by the switch positions and switching frequency of the inverter. In the simple case (with ideal switches), where one, and only one switch of the each inverter leg is closed, each winding will be connected to one or other rail of the dc link. This gives eight possible combinations of switches. Fig. 3.5 shows the voltage vectors and relates these to the switch positions. Intermediate voltages have to be approximated by switching rapidly between the nearest available states. The closeness with which the desired voltage can be approximated depends on the switching frequency of the inverter. In a current source inverter, an analogous situation exists with the current vector.

Fig 3.5 Available voltage switch states.

Finite switching times and inverter dead-times (due to overlap protection) introduce errors into the output. These may be approximately characterised and compensated for, but increase the complexity of the controller.

Device ratings constrain the current, voltage and power handling capabilities of the power
3.4.2 The current control algorithm

A number of current control algorithms are introduced below. The hysteresis current controller provides effective control of current for motors up to approximately 20 kW, at moderate supply frequencies (less than 100 Hz). It is simple to implement and rugged to parameter variations. However, at higher powers and higher speeds more attention needs to be paid to the design of the current controller, because of the limited switching frequency of the power semiconductors. Voltage control is more complex but can give optimised current control at low switching frequencies. The inherent simplicity of the look-ahead schemes, combined with their ability to account for the finite switch states, makes them a promising option, but only at high switching frequencies.

3.4.2.1 Hysteresis or bang-bang current control

In hysteresis current control (Fig 3.6), the three motor line currents are controlled by individual regulators, which determine the switch position of the respective inverter legs. The regulator compares measured and desired currents to produce an error signal. If the error exceeds a specified upper limit, then the bottom switch of the inverter leg is enabled, connecting the motor line to the negative, dc rail. This tends to reduce the current. Conversely, if the error is less than a specified lower (negative) limit, then the inverter leg is switched to the positive, dc rail. The width of the hysteresis band between the two limits determines the average switching frequency. Hysteresis current control can be implemented with a small number of analogue components. It gives an acceptable performance at low speeds, but is unable to follow the demanded signal at high speeds, where the back-emf reduces the effective voltage available to force current into the motor.
3.4.2.2 Ramp comparator or current-regulated PWM.

The experimental system used three independent ramp comparators to generate inverter switching signals, as shown in Fig 3.7. A PI controller acts on the current error to produce a voltage demand. This is compared with a sawtooth to generate the switch control signal. This technique has the advantage of a defined switching rate. Because it is effectively a sampled system it can be analysed in terms of maximum current error and delay.
3.4.2.3 Voltage control

Voltage control is based on the relationship between stator voltages and currents derived in section 2, which can be used to map from desired stator currents to a demanded stator voltage vector. The voltage source inverter approximates this by a time-average of the nearest available voltage vectors. The current control can be implemented in either a stationary or rotating reference frame, because the controller acts at the motor terminals (i.e. the stationary reference frame), whereas the desired currents are initially derived in the rotor-flux reference frame. Fig 3.8 shows the relationship between stator currents, rotor fluxes and stator voltages in both reference frames.

Fig 3.8 Comparison of the relationship between stator currents and voltages in the stationary and rotor flux reference frames.

![Diagram showing the relationship between stator currents and voltages in stationary and rotor flux reference frames.]

Fig 3.8 can be used to design the current controller, but needs to overcome the following problems:

a) The direct paths (i_d to v_d and i_q to v_q) contain a differential term, which will be noise-sensitive and may saturate the controller. The stationary-referenced model also includes the differential of flux.

b) The rotor flux referenced model is cross-coupled; d axis currents influence q axis voltages and vice versa.

3.11
Fig 3.9 shows a modified closed loop form of the current controller, with decoupling compensation. This is similar to the current control in a dc motor. PID gains can be found by standard design rules. The controller has a first-order lag, with a time constant determined by the closed loop transfer function. The relationship between the decoupling term and the output shows the same first-order lag. Hence if the decoupling compensation is incorrect (or if the controller saturates, because the demanded voltage exceeds the available dc-link voltage) a step change in torque will feed through into the flux and reflect back into the torque.

Fig 3.9 PID current control.

The controller design may be formalised by using the state-space form of the machine equations. Either fluxes or currents may be chosen as the state variables. A common choice is rotor fluxes and stator currents, but since only the stator current is directly measurable, some form of estimation is required for the rotor flux. The controller itself will take the form of a matrix multiplication of feedback gains and states. The gains will have been calculated in advance according to standard control algorithms using either pole placement or optimisation techniques.
3.4.2.4 "Look-ahead" or predictive control.

The methods above, all produce a desired reference voltage and then approximate this by the average of the nearest available switch states. An alternative approach is to start from the existing motor state, and estimate the stator currents that would result from applying each voltage vector over a constant switching interval. The voltage which best approximates the desired current is selected. This scheme is referred to as 'predictive' or 'look-ahead'. It can result in a very simple algorithm, almost entirely in look-up tables, but requires a reasonably high update frequency. It is also possible to work directly with torque and flux, and to specify selection rules which keep the chosen flux constant but maximise the torque response.

3.5 Alternative approaches to dynamic torque control.

In order to get around the problems of rotor flux sensing and accurate current control, it may be more effective to look at the alternatives for high bandwidth torque control. A wide variety of subtly different control strategies have evolved which aim to reduce sensitivity to parameter variations or measurement noise and improve performance for a particular application. The trade-offs associated with each can be analysed and the control strategies can be reduced to a relatively small set of variations on a theme.

3.5.1 Choice of reference frame for torque calculation

In rotor flux oriented control (section 3.1 above), the reference frame was chosen to be aligned with the rotor flux in order to simplify the torque equation. Equation 2.31 expresses torque in a variety of ways in terms of stator, rotor or airgap flux vectors and stator or rotor current vectors. Any one of these vectors may be chosen as the reference and used to simplify the associated torque calculation.

It is normal to use stator current as one variable, as it is available at the motor terminals for measurement and control. This vector cannot be used as the reference vector, since the q component of the current would then be constrained to zero by the field oriented conditions, removing a degree of freedom.
This leaves the alternatives of computing torque as the vector product of stator current with:

a) Rotor flux, in the rotor flux reference frame,
b) Stator flux in the stator flux reference frame,
c) Airgap flux in the airgap flux reference frame.

In addition there are sometimes advantages in working in the stationary reference frame with a combination of stator flux and current, even though the torque equation is more complex /Takahashi 1989/.

These schemes can be compared by taking the motor equations in an arbitrary reference frame and applying the field oriented conditions, for the chosen reference vector /Ho 1988/. Fig 3.10 compares the resulting motor models, for a current-controlled and voltage-controlled motor. The figure indicates that the flux and torque are only naturally decoupled for the case of a current fed machine in the rotor flux reference frame.

A range of current control schemes were discussed in section 3.4. For a current fed machine, rotor-flux orientation is the natural choice. For a voltage control scheme, the choice of reference frame depends on the accuracy of the reference-frame transformation and the sensitivity to parameter variation and measurement errors. This is a function of the method used to detect the chosen reference flux. Since the accuracy may change with operating point, the choice of reference frame must be selected to give the required degree of control for the particular application.
Fig 3.10 Comparison of motor models in the stator, rotor and airgap flux reference frames.
3.5.2 Choice of reference frame for other calculations

If a rotating reference frame is used for the torque control algorithm, then at some stage the reference currents or voltage must be transformed into the stationary reference frame, to provide switching signals to the inverter. Feedback terms may either be transformed into the rotating reference frame or the entire feedback loop may be implemented in the stationary reference frame.

The main factors determining the choice of reference frame are:

- a) Effects of incorrect field angle - which will introduce errors at each reference frame transformation,
- b) Controller update frequency - which would normally be set to an order of magnitude above the desired bandwidth.

In the steady state, stator and rotor flux and current vectors all rotate at synchronous speed and hence appear as dc quantities, with respect to the synchronous reference frame. The update interval is only determined by the desired controller bandwidth. In a stationary frame, the required update-frequency must be significantly above the maximum rotor electrical speed (i.e. the mechanical speed multiplied by the number of pole pairs), since fluxes and currents appear as sinusoidal quantities.

3.5.3 Choice of measured variables

The choice of sensing is related to the above choices of reference frame, flux angle estimation and voltage or current control algorithm. Together they determine the degree of control that can be achieved across the operating range of the motor /Stefanovic 1986/.

- a) Torque and flux demands plus rotor position or speed.
- b) Stator current plus rotor speed or position.
- c) Stator currents and voltages.
- d) Stator flux and stator current and/or speed or position.
e) Stator voltage and speed or position.

The first option gives a simple algorithm, but one that is sensitive to errors in rotor time constant and current control. It does not even save on current sensors since these are almost certainly required for the current control loops. The second option, which makes use of the available current feedback, is much less sensitive to current control errors and fits naturally with a current controlled implementation. For both algorithms, speed sensing is better at high speeds, and position sensing at low speeds.

The use of stator currents and voltages is attractive, since no shaft-mounted sensor is required. It is not even necessary to measure output voltages, because these can be derived from the demanded inverter switch states and dc-link voltage. However, the flux computation is particularly sensitive to measurement noise and stator resistance at low speed. The method is more suitable for strategies which directly control the voltage. The use of search coils is intrusive, but eliminates the dependence on stator resistance.

Although theoretically possible, the final option is sensitive to all motor parameter variations and is essentially open loop. Hence no commercial implementation has been produced.

Additional measurements, beyond the combinations listed above, provide cross-checks which may be used to track parameter variations or correct observer outputs. They may also provide feedback or decoupling to a voltage or current control loop.

3.5.4 Wider control issues

With the possible exception of the predictive torque controller, all the schemes discussed so far use field orientation methods to derive reference stator currents. Complex control methods may be applied to the design of the current controller and identification of the field angle, but the torque control algorithm is a simple open loop inverse motor model. This may be because the torque control is nested inside a wider control loop (speed or position) which can compensate for any non-linearities but there is scope for putting the torque compensation inside the motor controller.
Two obvious developments would be to close the torque and flux control loops and to boost torque gain in the field weakening region to compensate for reduced flux. However neither variable is directly measurable, hence any estimate is sensitive to both parameter and measurement errors. Murata /1990/ uses a full order observer to estimate both quantities for feedback to PI controllers. Lorenz /1990/ uses a model reference adaptive system to adjust slip gain as a function of torque error.

3.6 Summary

This chapter has introduced the principles of rotor flux oriented control. By forcing a constant magnitude of rotor flux, and orienting the controller with the instantaneous position of the rotor flux vector, a simple torque control algorithm results, which is analogous to a separately excited dc machine. The armature and field currents are equivalent to the quadrature and direct axis components of the stator current.

* Torque is proportional to the quadrature component of stator current.
* Flux is proportional to the direct axis component of stator current (which must be held constant).

The complete controller structure, shown in Fig 3.3, has three key elements:

* Calculation of desired stator currents,
* Conversion from the field oriented reference frame to a rotating reference frame,
* Injection of the desired currents into the machine.

The remainder of this section has explored the options for rotor flux angle estimation and current control. Because neither operation is straightforward, other reference frames have also been considered. The final algorithm depends on the appropriate choice of reference frame, orientation angle identification and current or voltage controller, for the target application.
4 COMPENSATION FOR SHORTCOMINGS IN THE MOTOR MODEL

Chapter 2 introduced a model of the induction motor and listed the simplifying assumptions which the model incorporates. This was used in chapter 3 to design a controller. Since the controller performance can only be as good as the underlying model, this chapter investigates the significance of some of the assumptions.

The accuracy of the motor model can be improved by taking into account higher-order effects, but this destroys its inherent simplicity. Alternatively, the control algorithm in chapter 3 can be retained, provided that the values of the motor parameters are continuously updated. In practice, resistances will change with temperature, inductances will saturate, and the motor speed will vary. The "correct" value can be found by direct measurement, by inference from the behaviour of the electromagnetic circuits, or by some form of model. This chapter examines the cause of parameter variations, and looks at methods of tracking these changes.

To get a practical implementation, further simplifying assumptions are necessary. If the parameters are time-varying then more care must be taken in applying standard control theory. This is why the chapter also looks at the magnitude and timescales of parameter variations to ensure that the techniques used in the controller are valid.

4.1 Effects neglected by the motor model

4.1.1 Space harmonics

Space harmonics occur because the motor winding is not sinusoidal, but is made up of discrete conductors. Appendix 1 repeats the analysis of chapter 2, but extends it to include harmonic components as well as the fundamental winding. The key results are:-

a) There is a harmonic component of torque at 6n times the rotor frequency. This is actually the sum of contributions from a positive sequence (6n+1) and negative sequence (6n-1) harmonic. (n is a positive integer).

b) There is also a harmonic component to the rotor flux which varies with
rotor position. It is not possible to decouple torque and flux control simply through control of fundamental $i_{sn}$ and $i_{sw}$.

c) The harmonic components have only a small effect on torque, since they are at near-unity slip. For the test motor, the most significant harmonic (the 7th), is estimated to contribute a torque component of only 0.2% of rated torque.

The main effects are on motor losses (seen indirectly in resistance changes due to heating) and measurement noise.

4.1.2 Deep bar effects

Deep bar effects occur because the rotor conductor has a finite thickness below the surface of the rotor. At high frequencies, the current is concentrated near the outer surface and the effective impedance increases. This is the result of proximity effects, which cause a change in inductance across the bar; in large machines, the skin effect may also be significant (the skin depth is approx 10mm for copper). Machine designers exploit this, by shaping the rotor bar to increase the impedance for direct-on-line starts.

DeDoncker /1987/ presents a double cage model of the machine, which may be used for either deep bar or double cage motors. Because there are now two components of rotor flux, additional decoupling terms are required to maintain fast dynamic control of torque. However this problem tends to apply to large machines (greater than 0.5MW) or to retro-fit motors.

If the slip frequency is available in the controller, it should also be possible to use a look-up table to adjust resistance as a function of frequency. Machine manufacturers are able to compute this data for common bar shapes. This could be an area for further research. However, the preferred solution is to choose a simple bar geometry.

4.1.3 Core losses

Core losses become significant at high speeds and high fluxes, where they represent a few percent loss in torque. Neglecting core losses results in an error in the rotor flux estimation.

4.2
Appendix 2 analyses this for the slip calculator algorithm. This gives an error in flux angle of 2.5°, for the experimental motor, at full load and base speed and voltage. The angle error is sufficient to cause a significant reduction in full load torque of the order of 5 to 10%.

Existing models of core losses (e.g. Udayagiri 1989), are not suitable for real time control and do not cover transient effects. Compensation for core loss could be an area for further research.

4.1.4 Saturation

Saturation results in a change in the relationship between flux and mmf (or associated voltage and current) in the motor, that can be represented as a change in inductance. The saturation characteristics are a function of the machine geometry and material B/H curve. Stator and rotor fluxes depend on a combination of stator and rotor mmfs and may interact, as shared flux paths saturate. A complete saturation model would look at the relationship between each current and each flux. In practice, main flux depends primarily on magnetising current, and stator and rotor leakage fluxes on their respective stator and rotor currents.

4.1.4.1 Steady state

In the steady state, two important effects result from saturation of the main and leakage paths respectively.

a) Magnetising flux saturation reduces the available torque/amp.

As the magnetising flux path saturates, the magnetising voltage will clamp, limiting the rotor current. Further increases in supply voltage are all be dropped across the stator resistance. This increases the stator copper losses, but with little or no increase in torque. There is a compromise point where small increases in torque are offset by the penalties of a higher supply voltage, reduced efficiency and higher stator temperatures. For the nominal supply voltage and frequency, the magnetic circuits of a standard induction machine
are designed with this trade-off in mind.

b) Leakage flux saturation improves the dynamic response.

The time constant for changing stator current at constant rotor flux is given by:

\[
\frac{\sigma L_s}{R_s + (\frac{L_m}{L_r})^2 R_r} \quad \frac{L_l + L_l}{R_s + R_r}
\]

Under field oriented control, the maximum achievable torque bandwidth is equal to the current bandwidth. Therefore the dynamic torque response will improve with leakage path saturation.

Field oriented control aims to hold one flux (usually the rotor) constant. This limits the variation in airgap flux, and so magnetising inductance variations will be small. If the flux is increased (for extra transient torque), or reduced (for high speeds) a look-up table can be used to compensate both the magnetising inductance and the rotor time constant /Levi 1989/.

Magnetising inductance is normally defined by the relationship between airgap flux and magnetising current. In practice, for low leakage machines, it is adequate to adjust magnetising inductance as a function of rotor flux, /Vas 1990/,/Sumner 1993/, as shown in equation 4.2 below. In rotor flux oriented control, the magnitude of this flux is controlled, which significantly simplifies the design of the look-up table.

\[
\begin{align*}
|\Psi_r| &= |\Psi_r - \Psi_{rd}| = \Psi_{rd} \\
L_m &= \frac{1}{i_{rd}}
\end{align*}
\]

4.1.4.2 Transient.

In a saturated machine, changes in flux give two terms to the back emf, as shown in equation 4.3. The additional \(\frac{dL}{dt}\) term will be present in the rotor and stator voltage equations.
Vas /1990/ shows that in the stator voltage equation, an additional decoupling term is required to separate d and q axis equations. This applies if the current control is implemented in a rotating reference frame.

In the rotor voltage equation, the additional term only appears in the d axis equation (in the rotor flux reference frame), since the q axis is controlled to be zero. Additional d axis stator current is required to control the flux during a transient.

\[
\frac{\psi}{Ld} = \frac{di}{dt} + \frac{\psi}{Lq} = 0
\]

This condition only applies to rapidly varying magnetising current. Normally, in field oriented control, the magnetising current is deliberately maintained constant. Equation 4.4 shows a first order lag, which filters out rapid changes in stator current from the flux response.

Vas /1990/ also considers high leakage machines, where the magnetising flux is sufficiently different from the rotor flux, for the approximation of equation 4.3 above to be invalid. In this case, he shows that additional decoupling is also required in the rotor equations. However the degree of cross-coupling is small, and can usually be neglected.

4.1.5 Thermal effects

The temperature of the motor varies considerably with operating point. Rotor and stator resistances reflect this change according to equation 4.5. The maximum stator temperature is given by the insulation class, as defined in /BS4999 part 101/. An 80° rise in stator temperature (insulation class B) corresponds to an increase in stator resistance of 30%. In a machine with external air cooling, the rotor temperature would typically be up to 25° above stator temperature, corresponding to a resistance increase of 40% in an aluminium rotor. Machines with internal oil or water cooling may see more dramatic resistance increases of up to 100%.
\[ R(T) = R_o [1 + \alpha (T - T_o)] \]
\[ \alpha_{Al} = 40 \times 10^{-4}, \quad \alpha_{Cu} = 39 \times 10^{-4}, \quad @ 25^\circ C \]

Thermal models of induction machines are extremely complex. DeDonker /1986/ represents the machine as a series of heat sources, heat sinks, conduction paths and storage elements. Say /1988/ gives expressions for some of these elements. One problem is that many of the heat transfer coefficients are empirical and hard to characterise accurately. Even the power sources are difficult to define in practice: motor losses include higher order effects excluded from the dynamic model and are distributed throughout the stator and rotor, windings and core. The resulting heat flow models are impractical for real-time control.

A crude thermal model is given in Fig. 4.1. If the steady state temperature can be predicted or measured experimentally, an under-estimate of the thermal time constant is given by the time for the rotor to heat up to its steady state value if all the losses remain in the rotor. This would give a value of 30s for a 7.5 kW machine. Leonhard /1985/ estimates a lumped rotor and stator time constant of 10 to 60 mins, depending on machine size.

Because the thermal time constant of the rotor is significantly longer than the electrical time constants it is feasible to track rotor resistance via its effect on the dynamic behaviour of the machine. Direct measurement or thermal modelling are impractical.

Stator temperature measurement and resistance compensation using equation 4.5 is possible. Stator mounted temperature sensors (usually thermistors, although thermocouples are also available) are accepted motor protection device and could also be used for controller compensation.
HEAT TRANSFER EQUATIONS

(i) Through the conductor :-

\[ \frac{\partial \theta}{\partial t} = -\frac{\lambda}{\rho c_v} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \theta}{\partial r} \right) \right] + \frac{P_{in}}{\rho c_v} \]

(ii) Across an air metal boundary, (time varying):-

\[ \frac{\partial \theta}{\partial t} = \frac{P_{in} - P_{out}}{\rho c_v} \]

(iii) Across an air metal boundary (steady state):-

\[ P_{in} = P_{out} = 2\pi rhl \Delta \theta \]

where \( r \) is the radius, \( l \) the length, \( P_{in} \) the input power, \( h \) the heat transfer coefficient across a surface of temperature differential \( \Delta \theta \) and \( \lambda \) is the heat transfer coefficient through the conductor.

(iv) In the steady state, with uniform losses in the rotor only, the rotor temperature is:-

\[ \theta_r = \frac{P_{in} r}{2} \left[ \frac{1}{r_h r_r} + \frac{1}{r_g r_h} + \frac{1}{r_s r_h} + \frac{1}{\lambda} \frac{r}{r_g} \right] + \theta_s \]

(v) The initial rate of rotor temperature rise is:-

\[ \frac{d\theta}{dt} = \frac{P_{in}}{\rho c_v} \]
4.2 Speed variations

Field oriented control does not assume constant speed. However, in all but the simplest of schemes, speed appears as a parameter in the current control or flux sensing algorithms. As long as the speed is slowly varying in comparison with the electrical system, it can be treated as constant over the timescales of the control algorithms. However, the design of the controller must allow for the expected range in speed, by some way of measuring or estimating speed, and altering the controller gains or offsets. If the speed varies rapidly, design for controller stability becomes complex.

For a simple, damped, inertial load, the mechanical time constant is given by the ratio of inertia to damping coefficient. This is typically many times slower than the electrical time constant.

4.3 On-line parameter tracking

If the parameters change significantly during operation, then on-line adjustment of parameter values may be required. Parameter tracking has formed the subject of extensive research, with important contributions by /Garces 1980/, /Leonhard 1985/ and /Lorenz 1990/. More recently Sumner /1993/ has demonstrated a range of commissioning techniques. Krishnan /1991/ reviews approaches to parameter identification.

Sudden changes in the controller coefficients can introduce unwanted fluctuations in the motor torque and flux (/Du 1991/ and chapter 6). Hence the parameter tracking algorithm must be designed to avoid large sudden changes in value. Normally this is achieved by filtering, which can be computationally intensive, and limits the bandwidth (and convergence time) of the tracking algorithm. As discussed above, resistance changes with temperature are amenable to tracking techniques, since the timescales are slow. Magnetising inductance changes occur more quickly:- chapter 3 identified a first order lag between the rotor flux and the flux component of stator current, determined by the rotor time constant. Although identification techniques may be used, a look-up table is more common.
Hence on-line parameter tracking normally applies to rotor resistance or rotor time constant, although stator resistance identification may also be used. Recent research has also concentrated on combined speed and parameter estimation, for example [Minami 1991], as there are considerable commercial advantages in eliminating the motor shaft-mounted speed sensor.

4.3.1 Parameter identification from specific tests.

Parameter tracking algorithms may be intrusive or passive. In the former, a test signal is injected in order to identify a specific parameter. This has the advantage that the identification conditions can be controlled (for example to representative levels of flux and slip, or to isolate specific parameters), but care needs to be taken to avoid undue disturbance to the motor.

Signal injection techniques were used to characterise the test motor (detailed in Appendix 4). Although the tests were carried out off-line, and at zero speed, both the dc injection (to identify stator resistance), and the high frequency injection (to identify combined leakage inductance and resistances for current loop tuning) can also be used on-line.

Torque and flux decoupling can also be tested by modulating the d axis current. This technique provides an error signal which can be used to correct the rotor time constant in the slip calculator algorithm, without specifically calculating its value. Leonhard /1985/ argued that if a test signal of sufficiently high frequency and zero mean was added to the demanded flux current, it should have no effect on the resulting rotor flux, because of the filter action of the first order lag between flux and stator flux current (section 3.2). If there was a decoupling error, the flux current would be coupled into the torque and corresponding disturbances in torque and speed would occur. Leonhard /1985/ proposed a pseudo random binary sequence test signal, and correlation techniques for highly sensitive filtering, to minimise the amplitude of the test signal. DaCosta Branco /1991/ presents experimental results for this scheme. The technique is critically dependent on high bandwidth speed sensing to detect the test signal.
4.3.2 Parameter identification from passive measurements.

Passive techniques may be based entirely on measurements of speed or position, current and voltage, but are more likely to compare the demanded performance based on the controller settings with the measured response. In general, where there is more prior information available, the tracking algorithm becomes simpler, but more sensitive to errors in this prior information.

4.3.2.1 Model reference adaptive systems.

Model reference adaptive control is normally used to force a process to follow the desired behaviour of the model. The difference between the output of the process and that of the model is compared and used to adjust the control (normally in a non-linear way) until the desired performance is achieved.

In parameter tracking algorithms, model reference adaptive systems take the form of two independent estimators (for example rotor flux). The first estimator (the model) does not depend on parameter of interest and is assumed to be correct. The estimator (the process) includes the parameter of interest, so will only give a correct prediction if the parameter value is correct. Any error between the two predictions can therefore be used to adjust the parameter value until both estimators agree.

The independent estimators may be implemented entirely from passive measurements, with
no knowledge of the control strategy. In practice, because the field oriented controller actively controls either currents or voltages, and estimates at least one motor flux angle, the computation may be considerably simplified by utilising some of this information.

Model reference adaptive systems normally include a non-linear, time-varying, feedback element. The parameter correction is usually implemented with a PI controller, but the parameter is then used in the estimator in a non-linear way. This makes stability analysis difficult, although the Lyapunov stability criterion may be applied, /Astrom 1989/. However even if the stability of the algorithm can be demonstrated, this gives no indication of the convergence dynamics, which must be verified by simulation. The PI controller also acts as a low-pass filter, effectively smoothing out disturbance and measurement noise.

The adaption mechanism can use any variable which can be derived both from measurements on the motor and from an alternative method that is strongly dependent on the parameter to be tracked. The error needs to be single valued with the parameter to be tracked. Garces /1980/ uses a comparison of reactive power, as shown in Fig 4.3(a). Sumner /1993/ has extended this (fig 4.3(b)), to remove sensitivity to magnetising inductance and also looks at an adaption mechanism for estimating stator resistance.

Model reference schemes have also been developed, based on motor torque /Lorenz 1990/, and rotor flux /Sugimoto 1985/. Rowan /1991/ provides a useful comparison of five model reference adaptive systems for rotor time constant estimation, in terms of:

a) whether the scheme will converge for all loads,
b) whether the speed of convergence will change with load,
c) sensitivity to measurement errors,
d) sensitivity to errors in other parameters,
e) changes in the above sensitivities with operating frequency.
Model reference techniques are widely covered in the literature and have been incorporated into commercial drive products. However, many algorithms are only valid over a limited speed and load range. For example, a scheme which compares estimates of torque for rotor time constant tracking (for example /Lorenz 1990/), will be ineffective at light loads because both estimators go to zero at no load. The majority of schemes are only valid in the steady state, because simplifying steady state approximations have been made, but the scheme by Sumner /1993/ is an exception.
4.3.2.2 Motor parameter solvers.

Most model reference adaptive systems are based on equations that could have been used directly, to solve for one or more motor parameter. The adaption mechanism reduces the numerical complexity and filters the result. Nevertheless, a number of researchers have used model fitting techniques taken from standard control theory to compute motor parameters online. A typical identification algorithm would inject a test signal or sequence of signals into the system and measure the system response. This gives a set of inputs and corresponding outputs which can be used find the best-fit parameters for a system model of an assumed form (probably using some form of minimum-least-squares fit). In practice, because the motor parameters vary, a recursive-least-squares fit is used, with weighting factors to give most significance to more recent data.

In the case of motor parameter solvers, the motor model is well-defined (as outlined in chapter 2). The controller outputs and motor terminal measurements are normally used in place of a special test signal. Unfortunately the motor model is in terms of intermediate motor states (i.e. the fluxes) which are not directly measurable, but need to be estimated using the very parameter values that need to be tracked. The Kalman filter is a particular example of a closed loop estimator, which is specifically optimised for noise rejection, since the feedback gain is continuously adjusted to minimise random errors.

There is enough information in the motor model to estimate rotor time constant as well as rotor fluxes, and a number of researchers have used extended Kalman filters for combined flux and parameter estimation (for example /Atkinson 1990b/). However this involves small signal linearisation, and is computationally intensive. Atkinson proposes a look-up table of feedback gains to reduce the computational overhead.

However there is insufficient information to uniquely solve for all motor parameters at once. Minami /1991/ uses the differential of voltage and current terms to increase the information available. Pre-filtering is necessary to make measurement of the differential terms possible in a noisy motor environment and the results are considerably improved by prior knowledge of stator resistance. Klaes /1993/ compares measurements at two different loads. Because
different parameters dominate at the different loads, the accuracy of the remaining parameters is less critical. However the identification process is intrusive, requiring changes to the load.

4.4 Discussion

In this chapter, the parameter values in the model of the motor (equation 2.24) have been shown to be non-constant. In the case of motor speed, the variation with time can be either measured directly, or modelled (provided that the load dynamics are known). Saturation effects can be predicted, although initial characterisation of the magnetising curve is required. Frequency-related changes in rotor resistance variations could theoretically be modelled as a function of slip frequency (at least in the steady state). However changes in resistance with temperature cannot be predicted accurately, so must be inferred. Much research effort has been directed at on-line tracking of the rotor resistance (or the rotor time constant). The alternative to explicit parameter estimation, is to use a more complex controller, which is less sensitive to parameter errors.

Where parameters are slowly-varying compared with the time constants of the motor, adjustments can be gradual, ensuring that any undesirable transient response is suppressed. Normally speed and resistance are both treated as constant over the sample period of the controller. In contrast, the timescale of inductance changes is much faster. However section 4.1.4 showed that the additional decoupling terms can generally be neglected, if the rate of change of magnetising current is limited.

There has been considerable interest in ensuring that estimates of motor speed, rotor resistance, and magnetising inductance are correct. For large machines, deep bar effects are significant enough for commercial products to have been marketed. Effects of incremental inductance, space harmonics, slotting and skewing are only second order, and are usually neglected. Core loss is also usually neglected, but as shown in appendix 2, this can cause a significant decoupling error. The consequences for torque and flux are analysed in chapter 6.
5. ESTIMATION OF MOTOR TORQUE AND FLUX

The experimental work that follows investigated the differences between actual and demanded torque and flux respectively, for a range of algorithms and as a function of parameter variations. Neither torque nor flux can be measured satisfactorily under transient conditions with standard laboratory equipment. A number of estimators were implemented and tested, to provide a means of measuring torque and flux to about 1kHz bandwidth. This chapter investigates three flux estimation methods and four torque estimation methods.

Rotor flux was estimated from:-

   a) Line currents and speed ("i-θ" or "rotor flux" estimator),
   b) Stator voltages and currents ("v-i or "stator flux" estimator),
   c) Flux sensors on the stator.

The "full observer" is also considered in this chapter.

Each flux estimator can also be used for torque estimation, since torque is the vector product of current and flux. Chapter 2.6 showed that any combination of rotor, stator or airgap fluxes with stator or rotor currents can be used to compute torque, provided that the appropriate scale factor is used. In the experiments, torque was computed from the three flux estimators above, and also from:-

   d) Rotor currents (via slip rings).

Table 5.1 compares these estimators and highlights the parameters each incorporates. The underlying equations for torque and flux are identified. The key features of any estimator are the error dynamics (whether the estimator converges to the correct value and how long this takes), and the robustness to model and measurement errors. This chapter investigates the theoretical and measured performance in more detail and discusses the practical implementation. This was necessary, in order to characterise the estimators before they were used to measure motor performance.

5.1
Table 5.1 Comparison of estimators

<table>
<thead>
<tr>
<th></th>
<th>Rotor flux i-ω</th>
<th>Stator flux v-ι</th>
<th>Flux sensor</th>
<th>Rotor current</th>
<th>Full observer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flux Equation</td>
<td>rotor (5.1)</td>
<td>stator (5.17)</td>
<td>stator (5.17)</td>
<td>-</td>
<td>both (5.25)</td>
</tr>
<tr>
<td>Torque (ψ_r x i_r).κ</td>
<td>ψ_s x i_s</td>
<td>ψ_s x i_s</td>
<td>(i_r x i_s).L_m</td>
<td>(ψ_r x i_r).κ</td>
<td>(ψ_r x i_r).κ</td>
</tr>
<tr>
<td>Flux magnitude</td>
<td>τ_r, L_m</td>
<td>R_p, σL_p, 1/κ</td>
<td>σL_p, 1/κ</td>
<td>-</td>
<td>R_p, σL_p, R_p, κ, τ_r</td>
</tr>
<tr>
<td>parameters</td>
<td>τ_r</td>
<td>R_p, σL_p</td>
<td>σL_p</td>
<td>-</td>
<td>R_p, σL_p, R_p, κ, τ_r</td>
</tr>
<tr>
<td>Flux angle parameters</td>
<td>τ_r, L_m, κ</td>
<td>R_s</td>
<td>-</td>
<td>L_m</td>
<td>R_s, σL_p, R_p, κ, τ_r</td>
</tr>
<tr>
<td>Torque parameters</td>
<td>τ_r, L_m, κ</td>
<td>R_s</td>
<td>-</td>
<td>L_m</td>
<td>R_s, σL_p, R_p, κ, τ_r</td>
</tr>
<tr>
<td>Dynamic response</td>
<td>Converges</td>
<td>Steady-state error</td>
<td>Steady-state error</td>
<td>Instantaneous</td>
<td>Converges</td>
</tr>
</tbody>
</table>

where κ=L_m/L_r.

5.1 "i-ω" or "rotor flux" estimator.

This estimator directly computes rotor flux. It uses measured currents and speed ("i" and "ω"), in the stationary reference frame, based on the rotor voltage equation (equation 5.1 and fig 5.1).

\[ \Psi_r = \int (A\Psi_r + Bt_i) dt \quad 5.1 \]

where

\[ A = \begin{bmatrix} -1 & -\omega_r \\ -\tau_r & -\omega_r \\ \omega_r & -1 \end{bmatrix}, \quad B = \frac{L_m I_s}{\tau_r}, \quad \Psi_r = [\Psi_{r q}, \Psi_{r d}]^T \quad 5.2 \]

5.2
The model is the equivalent to the slip calculator (chapter 3.3.2), except that measured currents are used in place of demanded values and it is implemented in the stationary reference frame. This takes account of imperfect current control and incorrect field orientation. However it requires more calculations, because the signals are superimposed on the supply frequency, rather than being dc values in the steady state. Verghese /1988/ has investigated this estimator in some detail, with and without closed-loop feedback.

a) Error dynamics.

With correct parameters and no measurement errors, the difference between the estimated rotor flux and the true value gives the error dynamics (the equation is shown in differential form and the stationary reference frame is assumed in this and subsequent equations):

\[
\frac{d\Delta \Psi_r}{dt} = A\Delta \Psi_r, \quad \Delta \Psi_r = \Psi_r - \Psi_r
\]

Hence the error dynamics are described by matrix A which has eigenvalues \(-(1/\tau_r) + j\omega_r\) and \(-((1/\tau_r) - j\omega_r\). The estimator converges to its correct value with exponential decay envelope set by the rotor time constant and an oscillatory component at the rotor speed. There is no steady-state error.
b) Effects of parameter and measurement errors.

In the presence of parameter and measurement errors, the error dynamics become:

$$\frac{d\Delta \Psi_r}{dt} = \Delta \Psi_r + (\hat{A} - A)\Psi_r + (\hat{B} - B)i_r + \delta$$  \hspace{1cm} (5.4)

Where $\delta$ is the measurement noise, and the estimator now uses estimated parameter matrices (shown by the $\hat{}$) instead of the true values.

The error dynamics are still given by the eigenvalues of $A$, with the same time constant $\tau$, and frequency $\omega_s$. However the estimated flux also depends on the operating point (in terms of current and flux), and the extent of the parameter and measurement errors. If these errors are non-zero there will be a residual steady-state error.

In the steady state (i.e. constant magnitude sinusoidal current, constant speed and load), the rotor flux estimator has the solution:

$$\Psi_r = L_m \left[ \frac{1}{1 + (\omega_s \hat{\tau}_r)^2} \begin{bmatrix} 1 - \omega_s \hat{\tau}_r \\ \omega_s \hat{\tau}_r \\ 1 \end{bmatrix} \right]$$  \hspace{1cm} (5.5)

Hence the ratios of actual to estimated torque and flux can be found as given below. If the estimated flux is used for the vector rotation in a vector control algorithm, the same relationship results between actual and demanded flux and torque, respectively, as will be demonstrated in chapter 6.

$$T_e = k_i \frac{\tau_r}{L_m} \left[ 1 + (\omega_s \hat{\tau}_r)^2 \right]$$  \hspace{1cm} (5.6)

$$\hat{T}_e = \hat{k}_i \frac{\hat{\tau}_r}{\hat{L}_m} \left[ 1 + (\omega_s \hat{\tau}_r)^2 \right]$$  \hspace{1cm} (5.6)

$$|\Psi_r| = L_m \sqrt{\frac{1 + (\omega_s \hat{\tau}_r)^2}{1 + (\omega_s \tau)^2}}$$  \hspace{1cm} (5.7)

$$|\hat{\Psi}_r| = \hat{L}_m \sqrt{\frac{1 + (\omega_s \hat{\tau}_r)^2}{1 + (\omega_s \tau)^2}}$$  \hspace{1cm} (5.7)

$$\text{arg}(\Delta \Psi_r) = \text{atan}(\omega_s \hat{\tau}_r) - \text{atan}(\omega_s \tau_r)$$  \hspace{1cm} (5.8)

The expressions above show a simple linear variation of torque and flux with magnetising inductance error, (although in practice both the torque constant and the rotor time constant

5.4
will also change if the magnetising inductance changes, for example because of saturation. The estimated rotor flux angle is directly related to rotor time constant, but the effects on torque and flux are non-linear.

c) Use of closed-loop feedback.

The speed of convergence of the estimator can be improved with closed-loop feedback. The general principle is to find a variable that can be measured directly and can also be predicted from the estimator. The difference can be fed back to correct the estimator. Verghese (1988) initially proposed using stator voltage for corrective feedback, which can be estimated from:

\[
\dot{\psi}_s = \frac{L_m}{L_r} \left( \frac{d\psi_r}{dt} + \sigma L_s \frac{di_s}{dt} + i_s R_s \right)
\]

The resulting estimator becomes:

\[
\psi_r = \int \left[ A\psi_r + Bi_s + K(\dot{\psi}_s - v_s) \right] dt
\]

and the revised error dynamics are:

\[
\frac{d\Delta\psi_r}{dt} = (1 - K \frac{L_m}{L_r})^{-1} A \Delta\psi_r
\]

The response has been modified by the term \((1 - K\frac{L_m}{L_r})^{-1}\). K can be selected to give the desired response.

In practice this closed-loop correction is difficult to implement effectively, because of the differential terms in the estimated voltage. Verghese used a two-stage estimator with an auxiliary variable (z):

\[
z = \left( 1 - K \frac{L_m}{L_r} \right) \psi_r + Ko L_s j_s
\]

The estimator becomes:

\[
z = \int \left[ A\psi_r + Bi_s - K(\dot{\psi}_s - i_s R_s) \right] dt
\]

\[
\psi_r = \left( 1 - K \frac{L_m}{L_r} \right)^{-1} \left[ z - Ko L_s j_s \right]
\]
Fig 5.2 shows the open-loop estimator, with the simple closed-loop form around it and the more complex two-stage form. The effect of the closed-loop feedback on sensitivity to parameter and measurement errors can be found from a sensitivity analysis, for the particular closed-loop implementation, using the same techniques as above. In general, a closed-loop gain which gives faster convergence, will amplify parameter and measurement errors. Because of this, in the experiments that follow, the open-loop form of the estimator was used, since the main concern was measurement accuracy and not speed of response.

Fig 5.2 "i-ω" estimator in open loop, closed loop and modified closed loop forms.

(d) Implementation

The estimator was implemented in software; hence a discrete approximation to the continuous form above (equation 5.1) was required. The discretisation assumed constant speed and current across a sample interval (250μs). This is a reasonable approximation for speed:- at rated torque, with the motor inertia alone, the motor will change speed by only 0.1% of rated speed in this time. The change in current may be more extreme:- if the full dc link voltage is applied across the windings for a complete sample period, the current may change by as much as 30% of rated rms current. Nevertheless the estimator was tested in simulation and experiment and found to be reliable.
With the assumptions above, the differential equation was solved for the sample instants:

$$\Psi_r[n+1] = \Psi_r[n] + (e^{aT} - 1)(\Psi_r[n] - A^{-1}Bt[n])$$

(where \([n]\) denotes the \(n\)th sample and \(T\) the sample period).

In practice the matrix exponential was expanded as a series and truncated after the second term.

e) Derivation of speed from position

Speed was derived from regular sampled position information using the approximation:

$$\omega[n+1] = \frac{\theta[n+1] - \theta[n]}{\Delta T}$$

A rolling average of the past five samples of speed was then used.

This algorithm has known limitations due to the discrete nature of the position information, Payn 1991. For the encoder used (2 channels 6000 lines), the resolution is limited to one count in 150 at 1500rpm, reducing to 1 count in 5 at 50 rpm, for a sample time of 250\(\mu\)s. There are also inherent delays of half a sample period for the initial speed computation and two and a half for the rolling average, due to the use of old information in the speed calculation. The delays degrade the bandwidth of the speed measurement. Alternative techniques have formed the subject of detailed investigations by colleagues, so were deliberately excluded from this thesis, Payn 1994.

f) Computation of flux angle

The rotor flux angle was required in later experiments for control purpose. The algorithm required sine and cosine terms to perform the vector rotation. These were computed as:

$$\cos(\theta) = \frac{\Psi_{rd}}{|\Psi_r|}, \quad \sin(\theta) = \frac{\Psi_{rd}}{|\Psi_r|}$$

To avoid computation of a square root and division the reciprocal of the flux was computed as a binomial expansion.
5.2 "v-i" and flux sensor algorithms - "Stator flux" estimators

These estimators first compute the stator flux as the integral of the back-emf. Back-emf can be found from the terminal voltage less the resistive drop ("v-i"), or from search coils in the stator. The stator flux is then compensated for leakage to find the rotor flux. Fig 5.3 shows the estimator structure.

**Fig 5.3 "v-i" estimator**

\[
\begin{align*}
\Psi_s &= \int (v_s - i_s R_s) \, dt \\
\Psi_r &= \frac{L_r}{L_m} [\Psi_s - \sigma L_s i_s]
\end{align*}
\]

a) Error dynamics

Because the computation of the stator flux is a pure integration, the stator flux error dynamics are described by:

\[
\frac{d\Delta \Psi_s}{dt} = 0
\]

In other words, any initial error in estimated flux will continue without correction. The estimator is shown in block diagram form in Fig 5.4. Although very simple, there is no feedback path, and so no correction mechanism; the estimator will not converge.
b) Sensitivity to parameter errors and measurement noise.

In the presence of measurement and parameter errors, the error dynamics become:

\[
\frac{d\Delta \Phi_s}{dt} = -i_s(\hat{R}_s - R_s) + \delta
\]

Hence parameter and measurement errors integrate up indefinitely, without correction.

Stator resistance errors contribute a load-dependent, magnitude and phase error, if back-emf is estimated from terminal voltage and current (not applicable to flux sensors). Main flux errors (\(L_m/L_w\)) give a simple scaling error to the rotor flux magnitude and do not affect the phase. Leakage flux errors (\(\sigma L_a\)) give a load-dependent phase error to the rotor flux, but do not affect the estimated torque. However one advantage of this estimator is that the algorithm does not include the rotor time constant.

The estimator needs to be implemented with some care at low speeds. In these conditions, the signal-to-noise ratio is poor, because there is very little back-emf. The resistive drop is similar in magnitude to the terminal voltage. The resulting flux estimate is sensitive to measurement noise and stator resistance errors. Small constant errors can integrate up, leading to eventual saturation of the integrators. The resulting flux estimates are distorted.

c) Compensation for convergence problems.

The most common approach is to use a dc roll-off on the integrator to prevent saturation and to high-pass filter the result. This removes problems due to unknown initial conditions and analogue circuit offsets and drift. However it also prevents operation of the estimator at low
speeds, because the flux estimates are attenuated by the filter. This increases the sensitivity to noise of the estimated flux angle. Care needs to be taken in the design of the filter to ensure that the phase shift does not contribute to orientation errors.

This is a significant disadvantage. Because the estimator does not work down to zero speeds and loads, it cannot be used in vector control schemes which require flux to be maintained in the motor at zero speed (for example for a fast torque response and torque holding).

It is also possible to identify and compensate for dc offset and parameter errors. For example /Sul 1989/ argues that the stator flux should be sinusoidal, so the integral over a finite number of periods should average to zero. Any error can be used to correct for dc offsets. However this is only true in the steady state (unless a stator flux oriented control scheme has been implemented). There is no discussion about how the correction is made; for example subtracting a dc offset computed as a rolling average over a long time period would simply be another form of high-order high pass filter.

Stator resistance can be identified by direct measurement (for example by injection of a dc voltage and measurement of the resulting dc component of current /Sumner 1993/ ), or thermal changes can be predicted from thermal sensors in the stator. Alternatively, changes in stator resistance can be inferred from the behaviour of the current controller /Sumner 1993/ or from more complex parameter identification schemes /Minami 1991/. Hence stator resistance compensation would appear to be a viable option.

Vergheese /1988/ uses corrective feedback to ensure convergence, based of stator current estimation. The differential term in the expression above is not a problem since it is available prior to integration of the back-emf:

\[
\dot{i}_s = -\frac{d\Psi_s}{dt} - A\dot{i}_r
\]

5.20

The estimator becomes:

\[
\Psi_r = \int \left[ \frac{L_r}{L_m} (v_s - i_s R_s + \alpha L_s \frac{di_s}{dt}) + K(i_s - i_r) \right] dt
\]

5.21

5.10
This has error dynamics:

\[ \frac{d\Delta \psi_r}{dt} = \frac{r_r}{L_m} \left[ 1 - \frac{r_r}{L_m} K \right]^{-1} K A \Delta \psi_r \] 5.22

The estimator now includes a correction mechanism. However dependence on rotor time constant has been re-introduced, and the estimator has become more complex.

d) Implementation

Back-emf was derived from terminal voltage (less the resistive drop) and also from search coils in the stator. The measurements were used to compute stator flux. Both estimators were implemented in software, and incorporated digital, high-pass filters. Trapezoidal integration was used, as this was found to be more stable than forwards or backwards Euler in simulation. The integral (i) of input (x), from sample (n), was approximated as:

\[ i[n+1] = i[n] + \frac{1}{2} (x[n] + x[n+1]) \Delta T \] 5.23

The high-pass filter was designed in the s domain as a third order Butterworth, and then mapped to the discrete domain by replacing the integration stages with the trapezoidal integration approximation as above. The filter was designed with a 0.025Hz cut-off, in order to get less than 4° phase shift at 1Hz. Fig 5.5 shows the magnitude and phase response of the filter.

In practice, this filter was not adequate. Although the dc level was attenuated a significant dc offset remained. This was removed by post processing, to ensure that the average flux over a finite number of cycles summed to zero. The offset resulted from limited dynamic range in the processor; because of the huge difference between the sample frequency and the filter corner frequency, the filter coefficients were tiny (approx 1e-5), compared with the input (approx unity), leading to accumulated round-off errors. A better solution would have been a higher order filter, with a flatter phase response to the corner frequency. This would have permitted a higher cut-off frequency and more realistic filter coefficients.
5.3 Torque estimation from rotor currents

The experimental system used a wound rotor, slip-ring motor. This gave access to the rotor currents, providing an independent measure of the rotor state, which is not available in conventional squirrel cage machines. Torque was computed from the vector cross product of rotor and stator currents. There is no inherent bandwidth limit in the torque computation, and both currents can be measured directly.

(a) implementation details

With a slip ring motor, the rotor currents measured were actual values, rather than the stator referred values of the standard equivalent circuit. The measurements were therefore scaled by the turns ratio which was found from standard tests (appendix 3).

The rotor currents were also measured in the rotor reference frame, i.e. at slip frequency. To transform them to the stationary reference frame, the currents were rotated by the rotor position. The rotor position was measured via an encoder. The absolute position of the rotor
winding axes with respect to the encoder index marker was found by a series of experiments. In each case the rotor was locked at a known position (as measured by the encoder with respect to the index marker) and a pulse of current injected into the stator q axis winding. Both the stator current and the currents coupled into the d and q axes rotor windings were recorded. The relative coupling into d and q axes was plotted as a function of position. The axes were located when the rotor and stator q axes coincided. Stator referred rotor currents were then computed as follows, using sine/cosine look-up tables.

\[
\begin{align*}
i_{rq}^r &= i_{rq} \cos(\theta - \theta_o) + i_{rd} \sin(\theta - \theta_o) \\
i_{rd}^r &= i_{rd} \cos(\theta - \theta_o) - i_{rq} \sin(\theta - \theta_o)
\end{align*}
\]

(where \(\theta\) is the rotor position and \(\theta_o\) is the position of the winding axis)

This method of torque computation is sensitive to the magnetising inductance. A look-up table was used to account for saturation. Magnetising inductance was given as a function of magnetising voltage, derived from the no load test (appendix 4). Magnetising current could also have been used as this is the vector sum of stator and rotor currents, which were both measured.

### 5.4 Full observer

The algorithms above can be combined so that the complete motor model is calculated, as shown in fig 5.6 and equation 5.25. This gives more information that can be fed back to correct the estimate. However it is more complex to implement, and still depends on the accuracy of the parameters used. The full observer may be expressed in terms of any two out of four of the stator and rotor fluxes and stator and rotor currents. A normal choice is stator currents (which can be compared with measured values) and rotor fluxes (required for field orientation). Measurements of terminal voltage and current and rotor speed are required.
This estimator was not implemented, because it was more complex so took longer to compute. In simulation, the same discretisation technique was used as for the rotor flux estimator above. It is potentially more robust than the rotor and stator flux observers, since it is based on more information about the motor, (and requires more measurements). However the difference is not so great in closed loop.

Vergheese /1988/ examined convergence and error dynamics. As with the previous estimators, the feedback gains can be used to determine the convergence rate. They can be designed using pole placement techniques. However a fixed feedback gain will give a different response at different speeds, as the natural response of the motor changes with speed. /Acarnley 1990b/ proposed adjusting the feedback gains as a function of speed, although he was concerned with noise rejection, rather than convergence times.
5.5 Experimental procedure.

The aim of the experiments was to verify the implementation of the above estimators, to test their sensitivity to errors in estimator parameters, and to check their ability to track rapid changes in torque.

5.5.1 Steady-state tests

Steady-state measurements were performed, to look at the discrepancy between torque and flux predictions. The different estimators were compared with measured values where available and with each other. The effect of using incorrect parameter values in the estimators was investigated.

The motor was run under vector control (using the slip calculator algorithm). Demanded torque and flux levels were set up. The load was provided by a dc dynamometer which was controlled to run at constant speed. Torque was measured with a shaft-mounted, strain gauge transducer. The microprocessor system was used both to control the induction motor and to measure stator voltages and currents, rotor currents, the output of the flux sensors and the rotor position. Details of the test system are given in appendix 3. Flux estimation was computed on-line. Corrections (for example for offsets and saturation) were added in a post-processing stage.

At the end of the experiment, the inverter outputs were inhibited and the decay of flux was monitored in order to measure the rotor time constant (see appendix 4). Steady-state values of terminal voltages and currents, together with nominal stator resistance and stator leakage parameters were used to compute magnetising voltage. The magnetising curve derived from the no load test was then used to find magnetising inductance.

The experiments were carried out at 1000 and 500 rpm, with less detailed tests at 50 and 1500 rpm, in order to look at effects of speed variations. The experiments were repeated with the same motor conditions, but the parameter values used for the calculations in the estimators were adjusted as follows:-
i) nominal values resulting from the characterisation exercise (table A4.3),
ii) 150% and 66.7% of nominal rotor resistance,
iii) 120% and 83.3% of nominal magnetising inductance.

5.5.2 Transient tests

The ability to track changes in torque was tested, by introducing a moderate step increase in demanded torque, at a predefined speed. The machine was again operated under vector control. It was run to a pre-set speed under simple P-only, closed-loop speed control. The applied torque was controlled using the dynamometer load. Steady-state conditions prior to the step, were recorded as before, for use in offset adjustment. The speed loop was then opened and replaced with a constant torque demand, and the response of the estimators was monitored. During this time, the motor accelerated, due to the increased output torque. After a short time, the speed loop was closed again and the controller forced the motor back to its set speed. Fig 5.7 shows the controller structure first in system form, and then in terms of the ideal transfer function.

Fig 5.7 Controller structure used for transient experiments.

![Controller structure](image)

Fig 5.8 shows a typical demanded torque profile. The shaft torque was similar to this, but differed in detail, for a number of reasons. There were imperfections in the control of both the induction motor (PWM inverter with indirect vector control) and the dc dynamometer.
(thyristor controller with a 3.3ms update rate). Hence the induction motor torque was not a true step, and the dynamometer torque was not exactly constant. The mechanical system (a two-inertia system with resilient coupling as detailed in appendix 3), also modified the torque at the induction motor shaft. Hence the tests show the response of the estimators to a torque transient, but this is only an approximation to a true step response.

Fig 5.8 Typical demanded torque profile for transient tests.

<table>
<thead>
<tr>
<th>Torque/Nm</th>
<th>Full torque</th>
<th>Torque step</th>
<th>Load torque</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time/s</td>
<td>Accelerate to set speed</td>
<td>At speed</td>
<td>Step applied</td>
</tr>
</tbody>
</table>

The experiments were carried out at 500 and 1000 rpm. The parameter values used by the estimator were changed, as in the steady state. The aim of the tests was to investigate the ability of the estimators to follow rapid changes in torque.

5.6 Steady-state results

Steady-state torque estimates have been plotted against shaft torque, as measured by the inline strain gauge transducer. Because rotor flux was not measured directly, plots of rotor flux can only compare one estimator output with another. In all the steady-state results, flux magnitude estimates have been normalised by the estimate of rotor flux from the v-i estimator with nominal parameters. Angle estimates have been plotted against the estimated rotor flux angle from this estimator.

Whereas the experiments covered the range from no load to rated torque, the flux was only
investigated at close to its demanded value, since this is the region where the controller operates.

5.6.1 "v-i" estimator

Fig 5.9 shows estimated torque against measured torque, using the "v-i" estimator (section 5.2) to calculate torque. Results at 500rpm and 1000rpm have been combined. The figure compares computations with gross errors in the parameter values used by the estimator (rotor time constant and magnetising inductance). There is good agreement between estimated and measured torque and no apparent sensitivity to either speed or parameter variations. This is as expected from table 5.1.

The rotor flux from the "v-i" estimator is investigated in fig 5.10 (flux magnitude) and fig 5.11 (flux angle). Both figures demonstrate the lack of sensitivity to parameter variations. In fig 5.10 the results have been normalised by the estimate with ideal motor parameters. In fig 5.11, the results are plotted against the estimate with ideal parameters. These plots confirm that the estimated rotor flux magnitude shows only slight parameter dependence (+/-1%) for significant variations in magnetising inductance (+/-20%). The rotor flux angle is virtually independent of parameter errors.
Fig 5.9 Torque estimation as a function of parameter errors, for the v-i estimator.

Torque from the v—i estimator

Fig 5.10 Rotor flux magnitude as a function of parameter errors, for the v-i estimator.
Fig 5.11 Rotor flux angle as a function of parameter errors, for the v-i estimator, compared with the estimate with nominal parameters.

However results at 50rpm (Fig 5.12) show that the estimator is less reliable at low speeds, although exact details depend on the implementation. In the experiments, the offset compensation was less effective at low frequencies. This severely distorted the flux magnitude. The effect on the plot of steady-state torque is less severe, as the offset causes a ripple in the estimated torque at supply frequency. This is averaged out in the steady-state plot.

Because of this degradation at low speeds, the estimator was not tested for variations in stator resistance. However, these would also be expected to become more significant at low speeds.
Fig 5.12 Degradation of torque and flux estimates at low speed with the v-i estimator.

Torque from the v-i estimator, 50rpm

Rotor flux from the v-i estimator, 50rpm
5.6.2 Estimation from flux sensors

The estimate of torque, derived from the flux sensors (section 5.2), showed good agreement between estimated and measured torque, with results similar to Fig 5.9. This is in spite of the fact that there is some discrepancy between estimated rotor flux in the two cases. Fig 5.13 shows the rotor flux magnitude as estimated from the flux sensors, normalised by the rotor flux value derived from the stator flux estimator. Fig 5.14 compares the estimated rotor flux angle in both cases. The magnitude plot is consistently low, and the angle is consistently too large, getting worse with increasing load.

This was thought to be because the search coils measure something closer to airgap than stator flux. This discrepancy would not affect the torque computation, since the stator leakage flux does not contribute to the torque. Figs 5.13 and 5.14 also show comparisons of rotor flux magnitude and angle, with a modification to the way rotor flux is calculated. This is related to airgap flux by:

$$\Psi_r = \frac{L_r}{L_m} \Psi_g + L_{ir} I_r$$  \hspace{1cm} 5.26

The modified algorithm shows reduced discrepancy between the two estimators. The remaining errors can probably be attributed to errors in the value of stator resistance used by the stator flux estimator.

The plots include the estimates with a +/-20% error in magnetising inductance and +/-50% error in rotor time constant and at two different speeds (500rpm and 1000rpm). As with the stator flux algorithm, this algorithm is not sensitive to speed or parameter variations.

Details of the flux sensor design and calibration are given in appendices 3 and 4.
Fig 5.13 Effect of modifying the flux sensor algorithm for rotor flux magnitude estimation.

![Graph showing rotor flux magnitude (flux sensors) vs. measured torque (Nm). The graph compares modified algorithm, original algorithm, and ideal results.]

Fig 5.14 Effect of modifying the flux sensor algorithm for rotor flux angle estimation.

![Graph showing rotor flux angle (from flux sensors) vs. angle from v-i estimator (rads). The graph compares modified estimate, original estimate, and ideal results.]

5.23
5.6.3 "i-ω" estimator.

Fig 5.15 shows the relationship between measured and estimated shaft torque, for the "i-ω" estimator (section 5.1). The results have been combined for speeds of 500 and 1000rpm, since there were no significant differences with speed. The effect of using incorrect parameter values in the estimator computations is apparent. Predicted torque is also shown; this has been computed from equation 5.6, using measured slip, but the same incorrect parameter values as in the estimator. The plots show generally good agreement between measured and estimated torque using nominal parameter values in the estimator. When the estimator parameters are incorrect, the estimated torque differs from the measured value, but the error is close to that predicted by equation 5.6. This confirms the implementation of the estimator.

In fact the estimated torque is consistently slightly lower than predicted. This is because the nominal magnetising inductance was also slightly low.

Fig 5.16 shows the relationship between measured shaft torque and estimated rotor flux, as a function of parameter errors. Again, results at 500 and 1000rpm showed no significant differences, so have been combined. The predicted flux is based on equation 5.7, and also uses measured slip, but the same incorrect parameters as in the estimator calculations. The plots show the expected trends with errors in rotor time constant and magnetising inductance; the latter is independent of load, whereas the former becomes more extreme with increasing load. Again, the estimated flux magnitude is slightly lower than predicted at all speeds, because of the error in nominal magnetising inductance.

The estimated rotor flux angle (Fig 5.17) confirms the effects of parameter errors as predicted by equation 5.8; the angle does not show any clear trend with magnetising inductance, but does vary with rotor time constant.
Fig 5.15 Estimated torque as a function of parameter errors, from the "i-ω" estimator.

Torque from the i-ω estimator with $\tau_r$ errors

Torque from the i-ω estimator with $I_m$ errors
Fig 5.16 Rotor flux magnitude as a function of parameter errors, from the "i-ω" estimator.

Rotor flux from the i-ω estimator
with $\tau_F$ errors

Rotor flux from the i-ω estimator
with $L_m$ errors
Fig 5.17 Rotor flux angle as a function of parameter errors from the "i-ω" estimator, compared with the angle from the "v-i" estimator.

### Rotor flux angle from the i-ω estimator with $\tau_f$ errors

- **Nominal**
- 58.7% $\tau_f$
- 100% $\tau_f$
- Predicted (nominal $\tau_f$)
- Predicted (63.3% $\tau_f$)
- Predicted (100% $\tau_f$)

### Rotor flux angle from the i-ω estimator with $I_m$ errors

- **Nominal**
- 120% $I_m$
- 63.3% $I_m$
- Ideal
5.6.4 Rotor current.

Torque was also estimated from rotor currents (section 5.3). This technique was found to give reasonable agreement with measured values once variations in magnetising inductance due to saturation were accounted for. Fig 5.18 compares estimated and measured torque, using firstly nominal magnetising inductance, and then the true value (found from the magnetising voltage and a look-up table of the magnetising curve).

The estimator was used as a primary indication of motor torque, because it proved effective across the speed range (unlike the stator flux estimator and flux sensors), and independent of rotor time constant variations (unlike the rotor flux estimator).
Fig 5.18 Estimated against measured torque, based on the rotor current, showing the effects of main flux saturation.

Torque estimate from rotor current, nominal parameters

Torque estimate from rotor current, allowing for main flux saturation
5.6.5 Summary of steady-state results

The results show good agreement between the predicted and actual behaviour of the estimators. This confirms the implementation of the estimators. Where incorrect parameter values were used in the estimators, there were some discrepancies between the output of the different estimators and between the estimated and measured values. This indicates a sensitivity to parameter errors. However these errors matched the predictions.

Torque estimation from rotor current showed good agreement with measured torque, provided that the magnetising inductance value, used in the calculations was correct. The magnetising inductance was adjusted, using the measured stator voltage and current to estimate magnetising voltage, and then using a look-up table, derived from the no-load test. This method of torque estimation was used as a basis for evaluating controller performance in chapter 6.

The "i-\omega" estimator showed significant parameter dependence. The torque and flux errors varied as predicted in section 5.1, as a function of parameter errors in the estimator. However it proved stable, and showed no significant variation with speed.

The stator flux estimators ("v-i" and flux sensors) showed much reduced sensitivity to errors in rotor time constant and magnetising inductance, as expected. However there was a marked deterioration at low speeds.

The experiments showed a load-dependent phase shift between the flux estimate from the flux sensors, and that from the terminal voltage and current. This was attributed, at least in part, to the difference between the flux linked by the stator winding and by the search coils.
5.7 Transient results

The aim of the transient tests was to establish how well the estimators could track changes in torque. Fast changes in torque could not be measured adequately by the in-line torque transducer and the exact shape of the torque transient was not known (as discussed in section 6.5.2). The results below compare the estimators against each other.

For all estimators, the torque and flux were recorded in steady state before and after the transient. The initial and final values agreed with the steady-state results in section 5.6 above.

Fig 5.19 compares typical torque estimation results from "v-i" and "rotor current" estimators. The plot is for a demanded change in torque, as detailed in section 5.5.2 and Fig 5.8. Apart from initial glitches following the step increase and decrease in torque, the estimates compare well during the torque transient. The estimators have not been compensated for magnetising inductance saturation, which would account for the slight scaling error.

The trace for the "v-i" estimator in fig 5.19 actually shows filtered torque. The unprocessed results contain considerable ripple at the supply frequency. This is due to dc offsets in the flux as discussed in section 5.2. The data was post-processed with a 3rd order notch filter to reduce the ripple, but the filter oscillates, following step changes in input. Fig 5.20 compares the raw and filtered torque and flux estimates. These confirm that the glitches are an artefact of the filter and can be ignored.

The torque estimate from flux sensors is virtually identical to the result for the "v-i" estimator. Fig 5.21 shows, that these two estimators give similar rotor flux angle and magnitude estimates, apart from the slight discrepancies also apparent in the steady state (section 5.6.2) (a consistent scaling error in magnitude and a load-dependent angle offset).

The "v-i", "rotor current" and "flux sensor" estimators all show the same shape of transient. This is not a perfect step, but has been rounded by the effects of a non-ideal controller and load. In each of these cases, the estimator uses rapid feedback from the motor, which is not directly controlled.
Fig 5.22 compares the output of the "i-ω" to that of the rotor current estimator. This estimator uses only the rotor currents and motor speed. Over the timescales of the current transient, the latter is virtually constant. Hence the estimated torque reflects the step of current imposed on the motor, and hence predicts the ideal torque step, rather than what is actually achieved in the motor. This makes the estimator unreliable as a measure of transient performance.

The results confirm that the estimators are able to follow a torque step with sufficient bandwidth to be suitable for measuring the transient performance of the field oriented control algorithms investigated in the following chapter. The agreement between the "v-i", "stator flux" and "rotor current" confirms that the slip frequency ripple on the results is real, and not a measurement artifact.

Fig 5.19 Comparison of "v-i" and "rotor current" torque estimates during a torque transient.
Fig 5.20 Effect of filter on the output of the "v-i" estimator.

Effect of filter on estimated torque
"v-i" algorithm

Effect of filter on estimated rotor flux
"v-i" algorithm
Fig 5.21 Rotor flux magnitude and angle estimates during a torque transient for the "v-i" and "flux sensor" algorithms.

Estimated rotor flux magnitude
"v-i" and "flux sensor" algorithms

Estimated rotor flux angle
"v-i" and stator flux algorithms
Fig 5.22 Comparison of "i-ω" and "rotor current" torque estimates.

Torque from "i-ω" and "rotor current" estimators

- "i-ω" estimator
- Rotor current estimator

5.8 Discussion

The analysis and investigations reported in this chapter are not novel, but were necessary to validate the techniques used to evaluate controller performance in the following chapter. Because of this, the emphasis of the work has been on measurement accuracy, rather than online rejection of noise or speed of response. The former has formed the focus of much research (for example/Acarnley 1990b), utilising Kalman filters to improve noise rejection (a closed-loop estimator with feedback gains optimised for rejection of random noise).

Nevertheless the performance of standard flux estimator algorithms is of considerable interest, because rotor flux angle estimation is a prerequisite of vector control. There is very little in the literature about the performance of these estimators under transient loads. This thesis focuses on the accuracy to which parameters need to be known, for an acceptable response from standard estimators rather than the design of novel algorithms.

The results show that the "i-ω" estimator, whilst simple to implement, and stable, shows the
expected sensitivity to parameter variations. The experiments investigated gross parameter errors of up to 50%; this is about the maximum range expected if no on-line parameter tracking is carried out. For errors of up to approximately 10%, the performance of the estimator is probably acceptable, especially if the controller operates in closed loop. The "v-i" estimator, although subject to some implementation difficulties, is considerably less parameter-sensitive, except at low speeds. It is quite common to use a combination of the two techniques, to cover the speed range and compensate for parameter variations.
Section 3 showed that field oriented control can give simple, decoupled control of torque and rotor flux. This provides a fast torque response, limited only by the bandwidth of the current controller. However if there is an error in the implementation, coupling between the torque and flux is re-introduced. Consequently a change in demanded torque will excite a transient response in both the flux and the torque. The transient will depend on the natural response of the motor and the nature of the exciting signal.

Fig 6.1 Block diagram of a vector controller, showing desired, actual and estimated torque and flux.

Fig 6.1 shows an overview of the vector controller presented in chapter 3.1. From this figure, the agreement between actual and demanded torque and flux will depend on:

(a) The torque and flux current gains.
(b) The flux angle estimator
(c) The current controller.

(b) and (c) will also depend on the quality of the feedback and the details of the actual
In this chapter, popular algorithms for vector control have been implemented, and the demanded and actual motor performance has been compared. Because transient torque and flux are not directly measurable, the actual motor performance has been obtained indirectly, using the estimators outlined in chapter 5. The following chapter examines the effects of steady-state and transient errors on the performance of the drive system.

6.1 Analysis of the motor torque and flux response to parameter errors.

6.1.1 The slip calculator algorithm

The slip calculator algorithm was introduced in chapter 3.3.2, and is shown in block diagram form in Fig 3.3. The algorithm assumes ideal current control and constant rotor flux. If both conditions occur, then slip is proportional to a ratio of torque and flux currents. The rotor flux angle is found by adding the slip to the measured rotor speed to get the synchronous frequency, and integrating this to get the angle.

a) Steady state

Krishnan /1984/, Nordin /1985/ and DeDoncker /1991/ have all investigated the effects of parameter errors on steady state motor performance for the feedforward slip calculator algorithm. The analysis is reproduced below.

Section 3.3.2 showed that if the rotor flux magnitude is kept constant, then the motor equations can be simplified, (in the true rotor flux reference frame).

\[
\begin{align*}
\dot{\psi}_{rq} &= 0 \\
\dot{\psi}_{rd} &= L_{in}^r \cdot i_{rd} = \text{const} \\
T_e &= k_{v}^2 \frac{1}{L_{in}^r} \psi_{rd} \\
\omega_s &= \frac{\dot{\psi}_{rq}}{\tau_r i_{rd}}
\end{align*}
\]
The controller inverts this model, in what it estimates to be the rotor flux reference frame. In the steady state, both reference frames rotate at synchronous frequency. With ideal current control, the magnitude of the stator current will be the same in both reference frames. However if there is an error in the motor parameters, there will be an error in orientation angle, resulting in errors in the motor torque and flux. Fig 6.2 shows a vector diagram of the demanded and actual stator torque and flux currents, and also defines the torque angle $\delta$. (The * indicates a demanded variable).

**Fig 6.2 relationship between actual and estimated flux orientation**

\[
\tan \delta = \frac{i_{sd}}{i_{sq}}
\]

\[
\tan \delta^* = \frac{i_{sd}^*}{i_{sq}^*}
\]

The relationship between demanded and actual torque and flux can be found by the trigonometric relationships as shown in Fig 6.3 and equation 6.3. Similar equations using the estimated rather than the true value of $\tau$, can be written for the controller.
Fig 6.3 Trigonometrical relationships between currents, slip and flux angle

\[ i_{sq} = |i_s| \sin(\delta) = |i_s| \frac{\omega_s \tau_r}{\sqrt{1 + (\omega_s \tau_r)^2}} \]  

\[ i_{sd} = |i_s| \cos(\delta) = |i_s| \frac{1}{\sqrt{1 + (\omega_s \tau_r)^2}} \]

6.3

The relationship between slip and motor currents can be used in the torque equations. These relationships are the same as for the rotor flux estimator (chapter 5.1); this is not surprising, since the underlying equations are the same. (The \(^\wedge\) indicates an estimated value).

\[ \frac{T_e}{T_e^*} = \frac{k_t \tau_r}{k_t^* \dot{\tau}_r} \frac{L_m}{L_m^*} \frac{[1 + (\omega_s \dot{\tau}_r)^2]}{[1 + (\omega_s \tau_r)^2]} \]

6.4

and

\[ \frac{\psi_{rd}}{\psi_{rd}^*} = \frac{L_m}{L_m^*} \frac{1 + (\omega_s \dot{\tau}_r)^2}{1 + (\omega_s \tau_r)^2} \]

6.5

Fig 6.4 plots the ratio of actual to demanded torque and flux as a function of estimated rotor time constant for different slips. The estimated rotor time constant is normalised by the actual value. The flux angle error is also shown for different slips. The graphs are for the test motor, whose parameters are given in appendix 4, and for an extreme range of rotor time constant variations.
Fig 6.4 Variation of actual to demanded motor performance as a function of slip.

The figure shows that the flux error varies strongly with slip (and hence load). At no load, there is no torque current to couple into the flux, hence no flux error. However, the coupling of flux current into the torque, creates a small amount of retarding torque, even at no load. At approximately 1/3 rated slip, the changes in torque and flux currents balance, making the torque independent of estimated rotor time constant, in spite of errors in the flux magnitude and angle. As the load increases further, the machine will operate at higher than desired torque and flux, for an overestimate of rotor time constant.

Fig 6.5 plots the ratio of actual to demanded torque and flux again, but as a function of magnetising inductance error. In practice, an error in magnetising inductance may also introduce errors the rotor time constant \( \tau_r \), but for clarity, fig 6.5 assumes an ideal rotor time constant. Hence the resulting performance is independent of slip and to a first approximation, the actual motor torque follows the actual flux, and is lower than desired for an overestimate of magnetising inductance.
Fig 6.5 Variation of actual to demanded motor performance with estimated magnetising inductance.

\[ \beta = \text{ratio of estimated to actual magnetising inductance} \]
\[ \text{The estimated flux angle does not depend on } \beta \]

Fig 6.6 Variation of motor performance with estimated rotor leakage inductance as a function of slip

\[ \gamma = \text{ratio of estimated to actual rotor leakage inductance} \]
\[ \delta = \text{flux angle error (degrees)} \]

Fig 6.6 shows the effect of an error in rotor leakage inductance. This introduces a corresponding small error in rotor time constant. Hence the trends are similar to those of fig 6.6.
6.4, for small errors in rotor time constant. The resulting torque and flux is relatively insensitive even to dramatic changes in leakage inductance.

If the error in machine parameters causes an error in the flux, this will change the operating point of the machine, moving it either further into, or out of saturation. This will change the magnetising inductance, introducing further parameter errors. Hence the characteristics above will be modified by saturation. Nordin /1985/ shows the effects of saturation for two different machines.

In practice, the motor is unlikely to be run open loop. In closed-loop speed control, the outer speed loop will force the motor torque to balance the load. It is possible to eliminate slip, and redraw the above plots in terms of load torque. Nordin /1985/ also shows curves in terms of load torque for two different machines.

The results of detuning are discussed in chapter 7; there may be changes in efficiency, available torque per amp and running temperature.

b) Transient response

Du /1991/ and Garces /1986/ both include some analysis of the transient performance of the slip calculator algorithm. Both papers present the results in terms of sensitivity of torque and flux to changes in motor parameters; although this is useful, it is equally important to look at sensitivity to changes in demanded or load torque for a constant (but erroneous) parameter estimate. Garces’ analysis is extended below to cover the response to a change in demanded torque. The analysis in table 6.1 below presents all variables in the controller reference frame. Ideal current control is assumed as before.
### Table 6.1 Comparison of true motor equations and controller model

<table>
<thead>
<tr>
<th>Motor Equation</th>
<th>Controller Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{d}{dt} \Psi_r = A \Psi_r + B i_s$</td>
<td>$\frac{d}{dt} \Psi_r^* = \hat{A} \Psi_r^* + \hat{B} i_s^*$</td>
</tr>
<tr>
<td>$A = \begin{bmatrix} \frac{1}{\tau_r} &amp; -\omega_s \ +\omega_s &amp; -\frac{1}{\tau_r} \end{bmatrix}$; $B = \frac{L_m I}{\tau_r}$</td>
<td>$\hat{A} = \begin{bmatrix} \frac{1}{\hat{\tau}_r} &amp; -\omega_s \ +\omega_s &amp; -\frac{1}{\hat{\tau}_r} \end{bmatrix}$; $\hat{B} = \frac{\hat{L}_m I}{\hat{\tau}_r}$</td>
</tr>
<tr>
<td>$\Psi_r = [\Psi_{rq}, \Psi_{rd}]^T$</td>
<td>$\Psi_r^* = [0, \Psi_{rd}]^T = \text{constant}$</td>
</tr>
<tr>
<td>$i_s = i_s^*$</td>
<td>$i_{rq} = T e^* / \kappa_s \Psi_{rd}^*$</td>
</tr>
</tbody>
</table>

Note that the above controller equations can be used to define the required slip $\omega_s$ and the demanded flux current $i_{rd}^*$.

The error between actual and demanded rotor flux can be found from the difference between motor and controller equations:

$$\frac{d}{dt} \Delta \Psi_r = A \Delta \Psi_r + \Delta A \Psi_r^* + \Delta B i_s + B \Delta i_s$$

$$\Delta \Psi_r, \Delta \Psi_r^* - \Psi_r$$

where:

$$\Delta A = \left(\frac{1}{\tau_r} - \frac{1}{\hat{\tau}_r}\right) I, \quad \Delta B = \frac{\hat{L}_m}{\hat{\tau}_r} \frac{L_m I}{\tau_r}$$
Equation 6.14 can be re-written as:

\[
\frac{d}{dt}(\Delta \Psi_r) = A \Delta \Psi_r + \Psi_{rd}^* \tag{6.16}
\]

Equation 6.16 describes a second order system whose natural response is described by the matrix A. The eigenvalues of this matrix are \((1/\tau) + j\omega_s\), \((1/\tau) - j\omega_s\). Hence the rotor time constant determines the envelope of the exponential decay, and oscillations occur at slip frequency, with respect to the controller reference frame. The effect of reference frame is discussed later.

The excitation is given by terms involving the demanded rotor flux, the slip and the parameter errors. It is of interest in three cases.

a) Transient response to changes in rotor time constant.

For changes in rotor time constant, at constant slip and with no error in magnetising inductance, equation 6.16 can be written in Laplace form.

\[
\Delta \Psi_r(s) = (sI - A)^{-1} \left[ \begin{array}{c} \Delta \tau_c(s) \\ 0 \end{array} \right] \tag{6.17}
\]

\[
\Delta \tau_c = \dot{\phi}_r - \tau_r
\]

The matrix inversion is:

\[
(sI - A)^{-1} = \begin{bmatrix}
\frac{s + \frac{1}{\tau_r}}{\tau_r} & -\omega_s \\
\omega_s & \frac{s + \frac{1}{\tau_r}}{\tau_r}
\end{bmatrix} \frac{1}{1 + \frac{(s + \frac{1}{\tau_r})^2 + \omega_s^2}{\tau_r}} \tag{6.18}
\]
The flux and torque responses to a step change in rotor time constant are:

\[
\Delta \psi_r = k \psi_{rd} \begin{bmatrix} -a(t) \\ -\omega_s \tau_r b(t) \end{bmatrix}
\]

\[
T_e = \frac{3ppL_m}{2L_r} (\psi_{rd})^2 [\omega_s \tau_r (1+k\omega_s \tau_r b(t)) - ka(t)]
\]

where:

\[
a(t) = 1 - e^{-\frac{t}{\tau_r}} [\cos(\omega_s t) - \omega_s \tau_r \sin(\omega_s t)]
\]

\[
b(t) = 1 - e^{-\frac{t}{\tau_r}} [\cos(\omega_s t) + \frac{1}{\omega_s \tau_r} \sin(\omega_s t)]
\]

\[
k = \frac{\Delta \tau_r \omega_s}{(1 + (\omega_s \tau_r)^2)}
\]

Fig 6.7 shows the flux and torque responses to a step change in estimated rotor time constant as a function of slip. Plots are shown at half and full rated slip (i.e., 3% and 6%). Nominal motor parameters have been used, for the experimental motor detailed in appendix 4. The final values of the ratios of demanded to actual flux magnitude and torque, respectively, agree with equations 6.4 and 6.5 above.

As equation 6.14 suggested, the frequency excited is the slip frequency, and the rotor time constant determines the exponential envelope describing the damping. The magnitude depends both on the size of the step (\(\Delta \tau_r\)) and the load (related to \(\omega_s\)). If there is no torque current, however bad the rotor time constant error, there can be no coupling of torque current into the flux and no transient response excited.
Fig 6.7 Torque and flux response to a step change in estimated rotor time constant, as a function of slip.
The analysis above assumes constant slip, predicting the response of the motor to changes in estimated rotor time constant. This is useful in designing a parameter tracking algorithm; if the parameter is changed, the natural response of the motor will be excited. Because the response is proportional to the excitation (i.e. the size of the step change) the response can be suppressed by ensuring that the changes are made slowly, in small increments.

**Fig 6.8 Response to a step change in rotor time constant (a) at constant torque and (b) at constant slip**

![Graph showing response to a step change in rotor time constant](attachment:image.png)

However, in practice, the analysis is too simplistic, because if the estimated rotor time constant changes, demanded slip will also change. Fig 6.8 shows simulation results, comparing the response at constant slip (as described by the analysis above), to that at constant torque. Because of the change in rotor time constant, the response at constant torque introduces a change in torque current, and hence a change in slip. This can be seen in both the frequency and final values of the torque and flux responses.
b) Transient response to changes in magnetising inductance.

For changes in magnetising inductance, at constant slip, and with no error in rotor time constant, equation 6.14 becomes, in Laplace form:-

\[
\Delta \psi_r = \frac{\Delta L_m(s)}{\tilde{L}_m} (sI-A)^{-1} \left[ \frac{\omega_s}{\Omega} \right] \]

\[
\Delta L_m = \dot{\tilde{L}}_m - L_m
\]

The response to a step change in magnetising inductance is:-

\[
\Delta \psi_r = \psi_{rd}^* \frac{\Delta L_m}{\tilde{L}_m} \left[ e^{-\frac{t}{\tau_r}} \sin \omega_s t \right] \left( 1-e^{-\frac{t}{\tau_r \cos \omega_s t}} \right)
\]

\[
T_a = \frac{3pp}{2} \frac{L_m}{L_r} (\psi_{rd}^*)^2 \omega_s \tau_r \left[ 1 - \frac{\Delta L_m}{\tilde{L}_m} b(t) \right]
\]

Fig 6.9 shows the flux and torque responses to a step change in actual magnetising constant as a function of slip. Again the final values for the ratios of actual to demanded flux magnitude and torque agree with equations 6.4 and 6.5 above. In the steady state, the system settles with zero q axis flux and a simple scaling error in d axis flux. There is no orientation error.

The analysis again above assumes constant slip, but now predicts the response of the motor to changes in actual, rather than estimated magnetising inductance. It is also simplistic, as in practice, the rotor time constant would change if the magnetising inductance changed. Hence the exact transient response would be more complex than that predicted above.
Fig 6.9 Torque and flux response to a step change in magnetising inductance, as a function of slip.
c) Transient response to changes in demanded torque.

For step changes in demanded torque (i.e. changes in slip), with incorrect machine parameters a small signal analysis is necessary. Equation 6.14 can be written as:

\[
\frac{d}{dt} x_o = A_o x_o + \left[ \begin{array}{c} k_q \omega_s \\ k_d \end{array} \right]
\]

where:

\[
x = \Delta \Psi_r
\]

\[
k_q = \left[ 1 - \frac{L_m \dot{\Psi}_r}{\dot{L}_m \tau_r} \right] \Psi_{rd}
\]

\[
k_d = \frac{1}{\tau_r} \left[ 1 - (L_m / \dot{L}_m) \right] \Psi_{rd}
\]

For small deviations, this becomes:

\[
\frac{d}{dt} (x_o + \Delta x_o) = (A_o + \Delta A)(x_o + \Delta x_o) + \left[ \begin{array}{c} k_q (\omega_s + \Delta \omega_s) \\ k_d \end{array} \right]
\]

The difference is:

\[
\Delta x = (sI - A_o)^{-1} \left[ k_q x_{q0} - \Delta \omega_s \right] \Delta \omega_s
\]

This has the same natural response as before. Full solution of the response to a step change in slip gives:

\[
\Delta x(t) = \frac{\Delta \omega_s(t) \tau_r}{1 - (\omega_s \tau_r)^2} \left[ a_0(t)(k_q - x_{q0}) - \omega_s \tau_r b_0(t)x_{q0} \right]
\]
Fig 6.10 Torque and flux response to a step change in slip, with an existing rotor time constant error.
Fig 6.10 shows the predicted response for both a step increase and a step decrease in slip, with an estimated rotor time of (a) 66.7% and (b) 150% of its true value. The analysis above shows that errors in the motor parameters which lead to incorrect orientation, result in a failure to maintain decoupled torque and flux. Hence a step change in demanded torque (and hence slip) excites a transient response in the flux. This response is proportional to the change in torque. It is also dependent on the initial decoupling error (a function of the parameter errors) and the initial slip. Hence a large transient response will only occur if there is both a high initial slip, and significant parameter errors.

The small signal linearisation above, only gives an approximate response. As torque and flux change, the true response deviates from the prediction. Fig 6.11 compares simulation results with the predicted response of Fig 6.10(a).

**Fig 6.11 Comparison of small signal analysis and simulation results for a step change in slip, with an existing rotor time constant error.**

All of the analytical results above are in the controller reference frame. This does not affect the computed values of torque or flux magnitude and angle, but does change the d and q axis components of flux. To a stationary observer, they will be modulated at the supply frequency.
6.1.2 "i-ω" controller.

The "i-ω" flux estimator was introduced in section 3.3.3 and investigated in some detail in 5.1. The estimated rotor flux orientation can be used to decouple torque and flux, for control purposes. Since this estimator is based on the same equations as the slip calculator algorithm (although in a different reference frame), the response of the "i-ω" algorithm to parameter errors should be the same as that of the slip calculator; this is demonstrated below. However there is now no explicit relationship between desired torque and flux currents and slip. Instead, the controller reference frame is defined by the estimated rotor flux angle.

\[ \omega_s = \frac{d\theta}{dt} - \omega_r \]

\[ \theta = \text{atan} \left( \frac{\psi_{rq}}{\psi_{rd}} \right) \]

The estimator equations (chapter 5.1) simplify in the controller reference frame to give:-

\[ \frac{d}{dt} L_m i_r + \frac{1}{L_m} \psi_{rd} r = L_m i_r \]

With ideal current control, demanded currents can be substituted for actual values and related to desired torque and flux. Equation 6.32 integrates to:-

\[ \psi_{rd} = \psi_{rd}^* + ke^{-dt/\tau_r} \]

Hence the estimated flux will always converge to the demanded value. Equation 6.31 can be used to explicitly link slip to stator torque and flux currents as before:
The motor and controller equations are identical for both algorithms, and parameter sensitivity arguments apply equally to both. Differences will only occur if the current control is not ideal, or in the details of the implementation. This was verified by simulation; the responses to a step in slip (Fig 6.10) were found to overlay exactly for the slip calculator and "i-ω" algorithms.

6.2 Analysis of the motor torque and flux response to controller delay.

Delays in the controller are a source of controller error. These delays can be due to:

a) phase lag in the current controller (if analogue),
b) a finite power switching frequency (typically less than 20kHz),
c) phase lag in sensor signal conditioning circuits, especially filters,
d) sampling and data conversion delays,
e) computation delays.

Appendix 2 shows that neglecting core losses introduces a phase shift between estimated and true rotor flux angle. The analysis below can also be applied to assess the effects of core loss.

In the experimental system, the total delay lag was measured to be approximately two samples, (0.5ms). This was the result of a single sample computation delay, because the updated current demand was output synchronously with initiation of the next sample period. As well as this, a comparison of demanded and actual currents as monitored by the microprocessor system showed a further sample delay, the accumulated result of delays in the current controller, power electronics, sensors and feedback circuits.

At 50 Hz, a 0.5ms delay represents a significant phase lag of 1/40th cycle or 9°. This results in an orientation error in flux and torque currents, as shown in Fig 6.12. The delay ΔT causes
an orientation error of $\Delta \theta = \omega_c \Delta T$. The effect of the orientation error is outlined below.

**Fig 6.12 Effect of controller delay on torque and flux currents.**

Using small angle approximations for sine and cosine, the true motor torque and flux currents are:

$$
\begin{align*}
    i_s &= i_{sq}^* - i_{sd}^* \Delta \theta \\
    i_d &= i_{sd}^* + i_{sq}^* \Delta \theta
\end{align*}
$$

In the steady state, the flux current is larger than desired because of cross-coupled torque current, and the motor will be increasingly overfluxed with increasing load. The slip will be lower than expected. The torque current is smaller than desired. The net effect on torque will depend on both the torque and flux currents; it will reduce at light loads, and increase at high loads as shown in table 6.2.
Table 6.2 comparison of actual and demanded steady state motor performance in the presence of controller lag.

<table>
<thead>
<tr>
<th>Demanded</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_e^* = k_i i_d^* i_{sq}^* )</td>
<td>( T_e = k_i i_{sd}^* i_{sq}^* [1 - (\Delta \theta)^2] )</td>
</tr>
<tr>
<td>( \psi_{rd}^* = L_m i_{rd}^* )</td>
<td>( \psi_{rd} = L_m (i_{sd}^* + i_{sq}^* \Delta \theta) )</td>
</tr>
<tr>
<td>( \omega_s^* = \frac{1}{\tau_r} \frac{i_{sq}^<em>}{i_{sd}^</em>} )</td>
<td>( \omega_s = \frac{1}{\tau_r} \frac{i_{sq}^* - i_{sd}^* \Delta \theta}{i_{sd}^* + i_{sq}^* \Delta \theta} )</td>
</tr>
</tbody>
</table>

Under transient conditions a change in demanded torque will introduce a step in the flux current as well as in the torque current, exciting the flux transient. The error in the flux can be described by equation 6.14. With current relationships given by equation 6.35 above and ideal parameter values, this simplifies to:

\[
\frac{d}{dt} \Delta \Psi_r = A \Delta \Psi_r + B \left[ -i_{sd} \Delta \theta \right]
\]

The above equation has the same characteristic form, so will have the same frequency and time constant as the earlier analysis for parameter errors. The driving function depends on the flux and torque currents and on the size of the angle error. Because the angle error is relatively small, the size of the transient will also be small.
6.3 Test procedures.

6.3.1 Steady-state tests

The controllers were investigated in the steady state, for sensitivity to parameter errors and delays. The procedure was similar to the one for characterising the estimators (section 5.5). The load was provided by a dc dynamometer which was controlled to run at constant speed. Constant levels of demanded torque and flux were defined. The motor was run in torque-control mode, using the control strategy to be tested. Torque was measured with a shaft-mounted, strain-gauge transducer. The microprocessor system was used to measure motor performance, to estimate torque and flux on-line, using the estimators from chapter 5 and to log the results. Further corrections (for example for offsets and saturation) were added in a post-processing stage. The results compare demanded and actual (or estimated) torque and flux.

As in the tests on the estimators, at the end of each experiment on the controllers, the inverter outputs were inhibited and the decay of flux was monitored, in order to measure the rotor time constant. The magnetising inductance was found from the magnetising voltage, which was estimated from terminal measurements.

As before, the experiments were carried out at 1000 and 500 rpm, with less detailed tests at 50 and 1500rpm. The experiments were repeated with the same motor conditions, but the parameter values used for the calculations in the controllers were adjusted in the same way as previously used for the estimators:-

a) nominal values resulting from the characterisation exercise (table A4.3),

b) 150% and 66.7% of nominal rotor resistance,

c) 120% and 83.3% of nominal magnetising inductance.
6.3.2 Transient tests

The transient response of the controllers was tested in two ways: response to a step in demanded torque and response to a step in the parameter values used for computation.

To achieve the step in demanded torque, the machine was again operated under the desired control algorithm, but in speed-control mode. It was run to a pre-set speed under simple P-only, closed-loop speed control, and the applied torque was controlled using the dynamometer load. Steady-state conditions prior to the step were recorded. The speed loop was then opened and replaced with a constant torque demand, and the torque and flux monitored, using the estimators of chapter 5. Since the motor accelerated, due to the increased output torque, after a short time it was necessary to close the speed loop again to restore the set speed. The experiments were carried out at 500 and 1000 rpm. The parameter values used by the controller were changed, as in the steady state. These tests were identical to those on the dynamic response of the estimators (section 5.5.2).

A further set of tests was carried out to investigate the response to a change in the value of rotor time constant used in the controller, with constant torque and flux demands. The induction motor was operated in torque control mode, with constant torque and flux demands. Speed was controlled by the dc dynamometer. This was the same as for the steady-state tests. However, a step change in rotor time constant was then introduced. The motor response was monitored by the microprocessor system as before. Both a step increase and a step decrease in rotor time constant were investigated.

6.4 Steady-state results

The results that follow show the measured motor performance with the control algorithms as described above. Torque was measured directly, using an in-line strain gauge transducer. Slip was found from the difference between supply frequency (measured with a frequency meter) and rotor speed (measured with an optical tachometer). The rotor flux was estimated from the stator flux algorithm, using nominal parameter values.
The motor performance was measured over a range of load and speed, firstly with nominal motor parameters in the controller and then with parameter errors. Gross errors in controller parameters were used, so that the trends could be clearly identified, and any effects due to small differences between nominal and true motor parameter values could be neglected.

6.4.1 Slip calculator controller.

Fig 6.13 shows measured torque against demanded torque for the slip calculator algorithm, as a function of parameter value. The upper trace combines results at 50, 500 and 1000rpm; the lower trace shows results at 1500rpm. The dotted lines show the expected torque, based on equation 6.5, for errors in the controller parameter values (assuming that the true motor parameters correspond to their nominal values). There is a non-linear relationship between torque and rotor time constant error; for example, overestimating the rotor time constant results in too low torque at light loads, but too high torque at rated load.

Measured torque corresponds well with the predicted value, except at high loads, with an overestimate of rotor time constant. The effect is marked at 1500rpm. There are two reasons for this, which are not allowed for in the prediction:-

a) The overestimate in rotor time constant causes the motor to run at a higher than intended motor flux. The resulting saturation changes the true motor parameters.

b) The prediction assumes ideal current control, but in practice, the current control deteriorates as the back emf increases, (i.e. at high speeds). This is exacerbated by a high switching frequency and relatively long interlock time, which prevents the inverter reaching 100% duty.

This degradation at high speeds is also seen in the flux. Figs 6.14 and 6.15 show the magnitude and phase of the estimated rotor flux, as a function of rotor time constant errors in the controller. There is some discrepancy between expected and estimated rotor flux at all speeds, because the nominal magnetising inductance is lower than the true value in the

6.24
machine. This leads to higher than expected flux (based on equation 6.6). Nevertheless the
trends are as predicted:- for example, too high a rotor time constant in the controller results
in a load-dependent increase in flux and a smaller than expected flux angle. At higher speeds
the controller is unable to sustain high flux levels for the reasons given above; at 1500rpm
both the torque and the flux are significantly lower than expected.

The implementation of the slip calculator is confirmed by Fig 6.16, which shows that the
controller sets up the expected motor slip in all cases.

Figs 6.17 to 6.19 repeat the results as a function of magnetising inductance errors. Fig 6.17
shows that measured torque is linearly related to demanded torque; the magnetising
inductance error simply changes the scale factor between the two. The plots again show a
consistent over-estimate of flux compared with the predicted value, because both the
controller and the calculation of the predicted value assume a nominal magnetising inductance
which is slightly too low. For the same reason the plots also show a consistently small rotor
flux angle, compared with the expected value. Nevertheless the plots confirm the expected
general trends:- an over-estimate of magnetising inductance gives both reduced torque and
flux, with a small increase in the rotor flux angle.

The steady-state results confirm the analysis in section 6.1.2, and also the implementation of
the controller.
Fig 6.13 Measured torque as a function of rotor time constant for the slip calculator controller.
Fig 6.14 Rotor flux magnitude as a function of rotor time constant error, for the slip calculator algorithm.

Fig 6.15 Rotor flux angle as a function of rotor time constant error, for the slip calculator algorithm.
Fig 6.16 Measured slip as a function of rotor time constant error, for the slip calculator algorithm.

![Graph showing measured slip as a function of demanded torque](image)

**Slip calculator algorithm**

**Measured slip v demanded torque**

- Predicted (Nominal \(\tau_r\))
- Predicted (86.7% \(\tau_r\))
- Predicted (150% \(\tau_r\))

- Nominal
- 86.7% \(\tau_r\)
- 150% \(\tau_r\)

Fig 6.17 Torque as a function of magnetising inductance, for the slip calculator algorithm.

![Graph showing torque as a function of magnetising inductance](image)

**Slip calculator algorithm**

**Measured v demanded torque**

- Predicted (Nominal \(L_m\))
- Predicted (120% \(L_m\))
- Predicted (83.3% \(L_m\))

- Nominal
- 120% \(L_m\)
- 83.3% \(L_m\)

500rpm
1000rpm

500rpm
1000rpm

500rpm
1000rpm
Fig 6.18 Rotor flux magnitude as a function of magnetising inductance, for the slip calculator algorithm.

![Rotor flux magnitude vs demanded torque graph]

Fig 6.19 Rotor flux angle as a function of magnetising inductance, for the slip calculator algorithm.

![Rotor flux angle vs demanded torque graph]
6.4.2 "i-ω" controller.

Figs 6.20 and 6.21 show plots of measured torque and slip, as a function of demanded torque, for the "i-ω" control algorithm. In each case the dotted lines show the expected trends, based on equations 6.4 and 6.34. The upper graphs in each figure show poor agreement between prediction and measurement at 1000rpm; the discrepancy between prediction and measurement worsens at with increasing speed.

Closer examination of the controller identified an approximately 2 computation-cycle delay in the controller, which contributed an orientation error of 2ωΔT. This corresponds to phase shift of 9° at 1500rpm (Δθ in table 6.2). The lower traces in Figs 6.20 and 6.21 show the same measured points as above, but modify the predicted trends to account for the angle error, using the expressions in table 6.2. The prediction also includes effects of changes in magnetising inductance due to saturation, since one effect of the orientation error is to change the flux from its demanded value. The figures reduce the discrepancy, although the predictions could be improved further, for example by including effects of changes in rotor time constant.

Conversely, in Figs 6.22 to 6.24 the predictions are for the ideal controller, with no delay, and the controller has been compensated, by adding a phase advance in the vector rotation stage. For simplicity, the phase advance was computed using the measured rotor speed, rather than the supply frequency, since speed was already available in the controller. Small angle approximations for sine and cosine were used to implement the phase advance.

Figs 6.22 to 6.24 show reasonable agreement between the predicted values and the measured results. The measurements confirm the analysis in 6.12, relating to the rotor flux controller and 6.2, relating to the effects of phase lag.

Implementation delays also occurred in the slip calculator controller, but they were less significant. This is because:-

a) The slip calculator is effectively open loop. It is not affected by delays in
sensing and data conversion, reducing the total delay to only one sample.

b) The controller integrates the slip to compute a slip angle correction which is added to the rotor position. This is a relative correction, not an absolute angle. An initial error on the integrator may result in a difference between the intended flux orientation and what is actually established, but the stator current will be oriented relative to the true resulting flux.

Figs 6.22 to 6.24 also show the effects of rotor time constant errors on torque, flux magnitude and angle for the "i-ω" controller (with phase advance). These agree closely with the same measurements for the slip calculator algorithm (compare Figs 6.13-6.15). Investigations into the effects of magnetising inductance errors for the "i-ω" algorithm with phase advance also gave similar results (not shown) to those for the slip calculator algorithm. This confirms that both algorithms degrade in the same way. The choice of algorithm will depend on the implementation details and particularly the current control strategy.

In contrast, the stator flux estimator introduced in chapter 5 shows very different behaviour with minimal degradation with rotor time constant and magnetising inductance errors. This offers a real alternative in terms of parameter sensitivity.
Fig 6.20 Measured v demanded torque as a function of rotor time constant, for the "i-ω" controller, showing the effect of delays.
Fig 6.21 Measured slip as a function of rotor time constant, for the "i-ω" controller, showing the effect of delays.

"i-ω" algorithm, 1000rpm
Measured slip, no compensation

\[
\begin{align*}
\text{Predicted (Nominal } \tau_r) \\
\text{Predicted (66.7\% } \tau_r) \\
\text{Predicted (150\% } \tau_r) \\
\end{align*}
\]
- Nominal parameters
- 66.7\% } \tau_r
- 150\% } \tau_r

Demanded torque (Nm)

-- Demanded torque (Nm) --

"i-ω" algorithm, 1000rpm
Predictions adjusted for delay

\[
\begin{align*}
\text{Predicted (Nominal } \tau_r) \\
\text{Predicted (66.7\% } \tau_r) \\
\text{Predicted (150\% } \tau_r) \\
\end{align*}
\]
- Nominal parameters
- 66.7\% } \tau_r
- 150\% } \tau_r

Demande torque (Nm)
Fig 6.22 Measured vs demanded torque as a function of rotor time constant error, for the "i-ω" controller with phase advance.

![Torque as a function of τ_r errors](image)

Fig 6.23 Rotor flux magnitude as a function of rotor time constant for the "i-ω" controller with phase advance.

![Rotor flux magnitude as a function of τ_r errors](image)
6.5 Transient results

The results that follow show estimated torque and rotor flux against time. Torque was estimated from the vector product of rotor and stator currents (chapter 5.5). The estimate has not been compensated for magnetising inductance saturation, so may show scaling errors, but has sufficient bandwidth to track changes in torque effectively. Rotor flux was estimated from the "v-i" estimator (section 5.2). A notch filter has been used to attenuate ripple at the supply frequency, as detailed in chapter 5.7. The filter introduces a burst of ringing following a step in input; this should therefore be ignored in the flux plots that follow.

The results in Figs 6.25 and 6.26 compare the two control algorithms, showing the torque and flux response respectively to a step change in demanded torque, as a function of initial load. Nominal parameter values have been used in the controllers. The "i-\(\omega\)" algorithm includes phase compensation. A 10Nm step increase in demanded torque was applied at time zero, and removed after 500ms. The speed controller was then reasserted, applying braking torque to
counteract the acceleration which had occurred in the previous 500ms.

The results shown are for 500rpm. Tests at 1000rpm showed a similar response to the step increase in torque. They differed only once the step was removed, because the increased windage losses at the higher speed changed the action of the speed controller.

The response to a step in demanded torque, with nominal motor parameters, is almost identical for both controllers. The flux remains virtually constant throughout, but the torque shows a oscillatory component at slip frequency. This becomes increasingly significant at higher loads. The oscillation was also visible on the torque estimate derived from the "v-i" and "flux sensor" estimators, so was judged to be a real effect and not measurement noise.

Section 6.2 above predicts a damped oscillation of this form on both the torque and flux, but only if there is an error in the controller parameters which introduces a decoupling error. This suggests that the nominal rotor time constant in the controller did not reflect the true value in the motor.

Figs 6.27 to 6.30 explore the effects of controller parameter errors on torque and flux, for the same demanded torque profile. The results show partial agreement with expected trends. As before, the torque results show a slip frequency component, with a magnitude that increases with increasing load. This response was only expected for rotor time constant errors. The flux plots are not clear enough to positively confirm a similar transient in the flux magnitude. However if the rotor time constant is incorrect, the step change in load results in a change in the flux level. In contrast, for magnetising inductance errors, the torque step does not affect the flux. This was as predicted in section 6.4.

The above figures show results for 500rpm for the slip calculator algorithm. Results for the rotor flux algorithm at this speed, and for both algorithms at 1000rpm were very similar. This again was as expected.
Fig 6.25 Torque response to a step change in reference; comparison of algorithms with nominal parameters.

Response to a step in reference torque
Slip calculator algorithm

Response to a step in reference torque
"i-ω" algorithm
Fig 6.26 Flux response to a step change in reference; comparison of algorithms with nominal parameters.
Fig 6.27 Torque response to a step change in reference; slip calculator algorithm with an error in the rotor time constant.

**Torque response at 66.7% \( \tau_r \)**

(500rpm)

**Torque response at 150% \( \tau_r \)**

(500rpm)
Fig 6.28 Flux response to a step change in reference; slip calculator algorithm with an error in the rotor time constant.

Flux response at 68.7% $\tau_r$
(500rpm)

Flux response at 150% $\tau_r$
(500rpm)
Fig 6.29 Torque response to a step change in reference; slip calculator algorithm with an error in the magnetising inductance.
Fig 6.30 Flux response to a step change in reference; slip calculator algorithm with an error in magnetising inductance.
Figs 6.31 and 6.32 show the torque and flux responses to a step change in the value of rotor time constant used by the controller, as a function of demanded load (held constant during the step in the controller rotor time constant). The step was applied at time zero, and maintained for the duration of the plot. The results of both a step increase and a step decrease in rotor time constant are shown. The results shown are for the slip calculator controller at 500rpm; the "i-ω" algorithm gave a similar result.

As expected, both the torque and flux change, settling to final values which agree with the steady state measurements of the previous section. Again, a slip frequency component is visible on the torque, with an amplitude that increases with load, but is not positively identifiable on the flux. Nevertheless this tends to confirm the analysis of section 6.4.
Fig 6.31 Torque response to a step change in rotor time constant; slip calculator algorithm.

Response to a step increase in $\tau_r$
(Slip calculator, 500rpm)

Response to a step decrease in $\tau_r$
(Slip calculator, 500rpm)
Fig 6.32 Flux response to a step change in estimated rotor time constant; slip calculator algorithm.

Response to a step increase in $\tau_r$
(slip calculator, 500rpm)

Response to a step decrease in $\tau_r$
(slip calculator, 500rpm)
6.6 Summary

This chapter has analysed two, standard, field oriented control algorithms. Expressions have been derived for the effects on motor torque and flux, both in the steady state and following a change in demanded torque, if there are parameter errors in the controller. The effect of changing the parameter values used by the controller has also been considered. A simple delay within the controller has been shown to cause torque and flux errors. In each case, although the driving function differs, the same inherent motor response is excited. This is a function of the slip frequency and rotor time constant.

The experimental work showed good agreement between prediction and measurement in the steady state, once compensation for controller delays was provided. Transient results showed the excitation of the natural response of the motor, as expected.
Field orientation provides a means of decoupling the control of torque and flux in an induction motor, but if the underlying model is incorrect, the motor performance degrades as cross-coupling is re-introduced. As chapter 6 showed, both the steady state operation and the dynamic response are affected.

In deciding whether to use a field oriented induction motor (with or without parameter compensation), the effects of any detuning become important. In this chapter the significance of parameter errors on the overall system performance is investigated. The reasons for using field oriented control are considered for ideally-tuned vector control, and then reassessed in the light of torque and flux changes due to parameter errors, as identified in chapter 6.

The motor is only one part of the motion control system, which will also include the mechanical couplings and driven load, as well as feedback and control. Typically, the motor would be used as a torque actuator, in order to control the speed or position of the load. The overall system response will depend on the characteristics of each element. These can then be combined using standard control theory (for example, transfer function analysis in the s or z domain, pole plots, Bode or Nyquist diagrams) to determine the overall response. In this chapter, the transfer function developed in chapter 6 is combined with a simple speed control system, to evaluate the implications of parameter errors on overall system stability and bandwidth. Hence an informed decision about whether a vector controlled induction motor, (with or without on-line parameter tracking) is appropriate for a given application, can be made.

7.1 Significance of parameter errors on steady-state speed control.

In the steady state, vector control does not give any inherent performance benefits over a correctly-commissioned scalar controller such as those outlined in chapter 1.1. If the motor can be set to operate at the same voltage and frequency under both control strategies, then the same performance will result in the two cases.
In terms of steady-state performance, vector control is only advantageous because of its secondary features:

i) Explicit control of flux

There are a number of reasons for adjusting the motor flux level. It may be decreased at high speed to reduce the back emf and hence increase the speed range (field weakening), or increased temporarily at lower speeds to boost the peak torque capability, (provided that the thermal limits of the machine are not exceeded). Khater /1987/ discusses optimisation strategies to maximise torque/amp over the speed range of the motor, within the constraints set by the inverter voltage and current ratings. The flux may also be decreased at light loads, to improve system efficiency.

All of the above strategies can be implemented in a scalar controller, but may be more complicated to set up.

ii) Explicit control of torque

One key advantage over a scalar controller is that the vector controller is able to maintain the flux and control the torque (including zero torque) at zero speed. Thus it offers torque holding at zero speed, a faster start-up (since there is no delay whilst the flux is established) and maximisation of starting torque. These advantages must be weighed against one significant disadvantage - the need for position (or speed) feedback - although one manufacturer now claims to have eliminated the position sensor.

In fact, it is theoretically possible to maintain the flux at zero speed, even in scalar control, but it is difficult to achieve in practice.

Explicit torque control also allows the use of a standard position/speed controller for motion control systems, whatever type of the motor is used (for
example brushed dc, brushless or induction motor).

iii) Set-up parameters have a physical meaning and are not specific to one speed and load.

Drive system commissioning procedures have improved significantly. Nevertheless, the compensation factors in scalar controllers often appear as fudge-factors which do not relate directly to the motor, but must be set by trial and error, and may only apply over part of the operating range. By contrast, in a vector controller the motor parameters have a physical meaning, so can be measured, or taken from manufacturer’s data. This should make them generally applicable, although, as chapter 4 discussed, they may vary with speed and load.

The effects of de-tuning on steady-state performance with a vector algorithm were summarised by equations 6.4 and 6.5. The plots in Figs 6.4 to 6.6 showed the effects on torque and flux for extreme variations in rotor time constant and more realistic variations in magnetising and leakage inductances. The figures show significant deviations in flux and torque compared with the desired value. This may result in changes in efficiency, torque per amp, peak torque capability and running temperature. In addition, Garces /1986/ discusses two scenarios.

i) For an underestimate of rotor time constant the machine will be under-fluxed. To get the same torque as in the correctly tuned case, higher stator currents will be required, as the machine runs at higher slip, but the back-emf will be reduced. The magnetising inductance may increase slightly, as the machine comes out of saturation, but this will be balanced by an increase in rotor resistance due to heating caused by the increased copper losses.

ii) For an overestimate of rotor time constant the machine will be over-fluxed. To get the same torque as in the correctly tuned case, it will operate at a higher terminal voltage and lower stator currents than intended. This may increase net losses at high speeds and light loads (where iron losses dominate).
In these conditions, increased rotor resistance due to core heating and decreased magnetising inductance due to saturation, will aggravate the error in rotor time constant.

Hence the control of efficiency, torque/amp, and other operating characteristics is probably no better for a detuned vector controller than for a scalar controller. However the key features of torque and flux control at zero speed, and direct control of torque will be retained, albeit with some scaling errors.

7.2 Significance of parameter errors on dynamic control of torque and speed.

Chapter 3 showed that "Vector" control offers an enhanced dynamic performance compared with the "scalar" type speed controllers introduced in chapter 1, because the natural response of the motor is not excited. However this response is re-introduced in the detuned case. The example below uses a simple proportional-plus-integral, speed controller and simple damped inertial load in order to assess the effect of detuning. The approach can also be applied to more complex loads or different controller structures.

Fig 7.1 Block diagram of a PI speed controller, showing system elements and Laplace description.

Fig 7.1 shows a typical PI speed controller. In the case of ideal vector control, the motor
transfer function can be represented as a simple gain \( K_p \). Thus the open loop transfer function becomes:

\[
G(s) = \frac{K_p K_a (K_d J)(1+s \tau_i)}{s(\tau_i / \tau_m)(s \tau_m + 1)} \tag{7.1}
\]

Where the closed loop transfer function is:

\[
\frac{\omega}{\omega_{\text{ref}}} = \frac{G(s)}{1+G(s)} \tag{7.2}
\]

With non-ideal vector control, the motor transfer function changes to \( F_y(s) \), changing the open loop transfer function to:

\[
G(s) = \frac{K_p K_a [F_y(s) J](1+s \tau_i)}{s(\tau_i / \tau_m)(s \tau_m + 1)} \tag{7.3}
\]

The analysis in chapter 6 gave expressions for small signal rotor flux variations in Laplace form as a function of changes in slip (equation 6.28). Demanded torque or torque current can be substituted for slip. However the speed controller above requires the transfer function of actual to demanded torque, and this is a non-linear function of rotor flux. Hence there is no simple way of obtaining an exact transfer function for \( F_y(s) \). The analysis below is again a small signal analysis, neglecting the product of difference terms. For a small change in reference torque from steady state, the change in actual torque can be expressed as:

\[
\Delta T_e = T_e - T_{e\text{o}} = k [i_{sd} \Delta x_q - i_{sq} \Delta x_d + \Delta i_{sq} (\psi_{rd} - \Delta \psi_{rd} - \Delta x_d)] \tag{7.4}
\]

(where \( x \) is the change in rotor flux and the subscript \( \text{o} \) denotes the initial steady state value).

Neglecting the product of difference terms, and substituting for \( x(s) \) from equation 6.28, gives a transfer function in torque:

\[
\frac{\Delta T_e}{\Delta T_{e\text{o}}} = \frac{s^2 [1+\kappa] - s ([1-\kappa] \frac{\Delta \tau_r}{\tau_{\text{r,o}}} + \omega_{\text{so}}^2 [1+\kappa \frac{\Delta \tau_r^2}{\tau_{\text{r,o}}^2}]}{s^2 + \omega_{\text{so}}^2 \frac{1}{1+(\omega_{\text{so}} \tau_r)^2}} \tag{7.5}
\]

Where
The effect of neglecting the product of difference terms is demonstrated in Fig 7.2. The plots compare the computed torque response to a step in reference torque, firstly neglecting (trace (a)) and then including the difference term (trace (b), which can be computed analytically for the special case of a step in reference torque). The latter response was also compared with the torque predictions of chapter 6 (trace (c)). The analysis is similar to the transfer function stated by Nordin /1985/.

**Fig 7.2 Predicted response to a step in reference torque.**

![Graph showing normalized torque response](image)

**Fig 7.3 shows a root locus plot for the ideal speed controller and compares this with root loci examples for the detuned controller, for both an over- and under-estimate of rotor time constant. The root locus has been obtained by substituting equation 7.5 into equation 7.3. In the detuned controller, two additional pole-zero pairs have been introduced. The open-loop pole locations are determined the rotor time constant and slip frequency. The value of the real part of the zero is close to the rotor time constant and the imaginary part is lower than the**
Fig 7.3 Root locus plots showing the effect of rotor time constant errors on the speed loop.

a) no error

b) 50% of true $\tau_r$

c) 150% $\tau_r$
Fig 7.4 Zero locations as a function of slip and rotor time constant errors.

Zero locations, as a function of $\tau_r$
For a range of slip

Zero locations, as a function of slip
For a range of $\tau_r$
slip frequency. Fig 7.4 shows how the zero locations of the test motor move, as a function of rotor time constant error and slip.

The root locus shows that for small errors, the additional pole-zero pairs virtually cancel. They do not cause instability, and barely distort the remainder of the root locus plot, which is dominated by the mechanical system and PI gains. Under some conditions, there may even be an improvement in the speed of response. Fig 7.5 shows the time response for the three cases shown in Fig 7.3. The gain has been set for critical damping in with correct motor parameters. The difference in time response is dominated by the changes in gain caused by the de-tuning.

Fig 7.5 Speed loop step response as a function of estimated rotor time constant.

7.3 Summary

The effects of parameter errors on transient response have been shown to be insignificant for many applications. This is because the amplitude of the cross-coupled signals into the torque and flux currents is relatively low; in control theory terms, the driving function is small. In
fact the most significant implications are for steady state operation. If the system is to operate close to its limits (either in terms of the inverter power rating or motor thermal rating) it is important to look closely at how much variation in operating point can be tolerated.
8. CONCLUSIONS

The effects of parameter errors on motor performance have been investigated for two standard field oriented control algorithms: the slip calculator and the "i-ø" estimator. Gross parameter errors were introduced in the controller to represent the worst case that could be expected, assuming correct initial values in a normal industrial environment, with no subsequent correction for changes in speed, load or temperature. Chapter 6 presented results showing the effects on torque and flux, both during a transient and in the steady state. Chapter 7 discussed the likely implications for torque and speed control.

8.1 Steady-state torque and flux

Significant discrepancies between demanded and actual steady-state torque and flux can result from controller parameter errors. To a first approximation, magnetising inductance errors give rise to proportional scaling errors in flux and torque but do not affect the independent control of torque. In contrast, rotor time constant errors produce load-dependent errors in the torque and flux and also introduce coupling between the two. For the control algorithms studied, the motor performance is not sensitive to errors in leakage parameters and stator resistance.

The consequences of these steady-state errors depend on the application, the details of the machine design and the inverter power rating. These will determine the motor losses, running temperature and the torque, as a function of flux. For example, if the flux is too high, the voltage may be too low to achieve the desired current, reducing the peak torque. There may also be an increase in total losses, and hence running temperature. In general, steady-state errors can be avoided by closed loop control /Hung 1990/, provided that the torque and flux feedback is derived from estimators that are not sensitive to parameter errors.

Controller delays can also give significant and predictable steady-state errors. However the correction detailed in 6.3 restores an acceptable steady-state performance.
8.2 Transient torque and flux

Undesirable torque and flux transients can be excited in the motor, by changes in demanded torque. These occur when the controller is incorrectly decoupled, for example due to parameter errors or controller delays. The sudden change introduced into the flux current excites a flux transient. The magnitude of the response depends on the degree of coupling, which in turn depends on both the initial load and the size of the parameter errors. It also depends on the size of the torque change. Sudden changes in controller parameter values should be avoided, as these can excite undesirable transients as well.

The transient response which is excited, is the natural response of the motor, characterised by a damped oscillation at slip frequency, decaying with a time constant equal to the rotor time constant. In practice, the effect is relatively small, even for sizeable parameter errors, because the excitation is relatively small. It is only significant for high loads, and large parameter errors.

8.3 Closed-loop torque and speed control.

The effect of any parameter errors which introduce coupling between the torque and flux, is to introduce two additional pole zero pairs into the system transfer response. The real part of these pairs is dominated by the rotor time constant. The imaginary part is a function of slip frequency and hence of load. The location of the zeros also depends on parameter errors.

Because the pole and zero are close together, they have a relatively small influence on the closed loop torque or speed response. In some cases they can even speed up the response.

8.4 Requirements for parameter compensation.

This work has taken two standard field oriented control algorithms. These are simple algorithms, which can be implemented relatively easily, with only simple corrections. The schemes have been shown to be remarkably tolerant to parameter errors, under dynamic conditions. However there can be a significant steady state error. The dominant parameters
are the rotor time constant and magnetising inductance. The latter can be characterised off-line and corrected via a look-up table. Relatively simple tracking schemes exist if adjustment of the former is required. Realistically, parameters can be followed to accuracies of about +/-5%

Hence for some of applications, a simple control algorithm is more than adequate. Where control of motor flux levels is important, magnetising inductance compensation should be considered and simple rotor time constant tracking may also be required. If a more complex algorithm is required, this should be weighed up against the options offered by other motor types (for example permanent magnet brushless motors or conventional dc).

8.5 Further work

Chapter 3 introduced the main elements of any vector control algorithm:

a) the identification of the flux angle,

b) the control of motor currents,

c) the choice of reference frame,

d) the derivation of feedback.

In this thesis, two control algorithms have been taken. Both address the identification of the flux angle, with ideal current control and rotor flux orientation. Considerable research effort has concentrated on developing practical current controllers, especially at higher powers, where the power semiconductors limit the switching frequency. Control of voltage, rather than impressed stator currents is also popular, because it can potentially reduce the amount of precision analogue circuits. The effect of non-ideal current (or voltage) control would be an obvious extension of this work. The simplest model would be a first-order lag, but more complex schemes, including parameter errors, should also be considered.

A second extension would be to investigate "v-i" flux estimator, which can also be used to derive the rotor flux orientation angle for control purposes. This is a popular choice in low-cost systems, since it does not require speed or position feedback. It is also insensitive to rotor time constant errors. However practical implementations require high-pass filtering to eliminate distortion due to offsets and drift at the integrators. This introduces a phase shift,
which will affect the steady-state and dynamic response of the controller.

The two algorithms investigated utilise speed feedback (derived from position). The effects of parameter errors becomes more significant in the new "shaft sensorless" drive products, which attempt to eliminate position feedback. Because of the reduced amount of information available to the controller, it relies more heavily on the motor model and hence is more sensitive to parameter errors, including stator resistance. This needs to be quantified, again, both in terms of steady-state and transient performance, if drive system users are to make informed decisions about what type of controller and motor to specify for a given application.
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10. NOTATION

10.1 Style
x \quad \text{Scalar variable.}
x \quad \text{Vector variable.}
X \quad \text{Matrix variable.}
x(s) \quad \text{Laplace transform of } x(t).
\begin{align*}
  x_a, x_b, x_c & \quad \text{Components of } x \text{ in } 3 \text{ axis } (a,b,c) \text{ co-ordinate system.} \\
  x_d, x_q & \quad \text{Components of } x \text{ in } 2 \text{ axis } (d,q) \text{ co-ordinate system.} \\
  x_r, x_s, x_g & \quad \text{Value of } x \text{ in the rotor, stator and airgap, respectively.} \\
  x_m & \quad \text{Magnetising component of } x.
\end{align*}
^\hat{x} \quad \text{Estimated value.}
^* \quad \text{Demanded value.}

10.2 Symbols
A \quad \text{observer state feedback matrix.}
\begin{align*}
  \hat{a}_r, \hat{a}_s & \quad \text{unit vectors aligned on the rotor and stator } a \text{ phase winding axis.} \\
  B & \quad \text{observer input matrix.} \\
  B & \quad \text{magnetic flux density (T).} \\
  \hat{b}_r, \hat{b}_s & \quad \text{unit vectors aligned on the rotor and stator } b \text{ phase winding axis.} \\
  c_v & \quad \text{specific heat capacity (J kg}^{-1} \text{/C).} \\
  \hat{c}_r, \hat{c}_s & \quad \text{unit vectors aligned on the rotor and stator } c \text{ phase winding axis.} \\
  D & \quad \text{viscous damping (Nm s).} \\
  F & \quad \text{friction (Nm).} \\
  F & \quad \text{force (N).} \\
  h & \quad \text{boundary heat transfer coefficient (W m}^2 \text{/C).} \\
  I & \quad \text{identity matrix.} \\
  i & \quad \text{current (A).}
\end{align*}
J current density (A m$^{-1}$).
J inertia (kg m s$^{-2}$).
j $(-1)^{1/2}$.
K observer feedback matrix.
$k_f$ fan constant (Nm s$^{-3}$).
$k_t$ torque constant.
$L_m$ magnetising inductance (H).
$L_{l_r}, L_{l_s}$ rotor and stator leakage inductances (H).
$L_r, L_s$ rotor and stator inductances (H).
l length (m).
$l_g$ airgap distance (m).
mmf magnetomotive force (A turns).
N number of turns.
n harmonic number.
P power (W).
pp pole pairs.
$R_m$ magnetising resistance (Ω).
$R_r, R_s$ rotor and stator resistances (Ω).
r radius.
s slip.
s Laplace variable ($\sigma+j\omega$).
$T_{e}, T_i$ electrical and load torque (Nm).
$T, \Delta T$ sample time (s).
t time (s).
v voltage (V).

$\alpha, \theta, \lambda$ angles.
$\alpha$ temperature coefficient of resistivity.
$\Delta$ difference.
$\delta$ torque angle.
$\delta$ measurement noise vector.
$\varepsilon$ rotor position.
\( \theta \) temperature \((^\circ \text{C})\).

\( \lambda \) heat transfer coefficient through a material \((\text{W m}^{-1} / ^\circ \text{C})\).

\( \mu_0 \) permeability of air \((4\pi \times 10^{-7} \text{ H m}^{-1})\).

\( \rho \) resistivity \((\Omega \text{ m})\).

\( \sigma \) machine leakage factor.

\( \tau \) time constant.

\( \Phi \) magnetic flux \((\text{Wb})\).

\( \chi \) magnetic coupling factor.

\( \psi \) magnetic flux linkage \((\text{Vs})\).

\( \omega \) angular velocity \((\text{rad s}^{-1})\)

\( \omega_s \) supply frequency \((\text{rad s}^{-1})\)

\( \omega_r \) rotor angular velocity \((\text{rad s}^{-1})\)

\( \omega_s \) slip frequency \((\text{rad s}^{-1})\)

\( \Re \) Real part

\( \Im \) Imaginary part.
APPENDIX 1 - EFFECT OF WINDING HARMONICS

A1.1 Relationship between currents, mmf and fluxes

In the analysis below, the machine is assumed to have an \( n \)th harmonic winding, in series with the ideal sinusoidal winding. Winding harmonics on both the stator and rotor are considered.

Stator winding 'a' contributes a current density which is the sum of the harmonic and the fundamental. \( \phi \) is the displacement between these two stator windings.

\[
J_{sa}(\alpha,t) = \frac{-i_{sa}(t)}{2r} [N_s \sin(\alpha) + N_{sn} \sin(n\alpha + \phi)]
\]  

A1.1

For three phases, 120° displaced, the fundamental term in the above equation is still given by equation 2.3. However the stator current vector, defined in equation 2.4, represents the fundamental mmf only. The harmonic component of the current distribution can be calculated in the same way:

\[
J_{sn}(\alpha) = -\frac{N_{sn}}{2r} [i_{sa} \sin(n\alpha + \phi) + i_{sb} \sin(n(\alpha - \frac{2\pi}{3}) + \phi) + i_{sc} \sin(n(\alpha - \frac{4\pi}{3}) + \phi)]
\]  

A1.2

This goes to zero for triplen harmonics. Otherwise:

\[
J_{sn}(\alpha) = -\frac{N_{sn}}{2r} \sum_{n} i_{sc} e^{j(n\alpha + \phi)}
\]  

A1.3

[for \( n = 4,7,10,13,16,19\) (forward or positive sequence)]

\[
J_{sn}(\alpha) = \frac{N_{sn}}{2r} \sum_{n} i_{sc} e^{j(n\alpha + \phi)}
\]  

A1.4

[for \( n = 2,5,8,11,14,17\) (backward or negative sequence).]

The analysis below is for non-triplen, positive sequence harmonics.

Chapter 2 computed the mmf from the current distribution, as the sum of the current enclosed. This now includes a harmonic component.

\[
mmf_{sa}(\Theta,t) = \int_{0}^{\pi} (J_{sa}(\alpha,t) + J_{sn}(\alpha,t)) d[\alpha]
\]  

A1.5

A1.1
The harmonic term can be calculated separately. For the forward sequence, the harmonic mmf is:

\[ mmf_{sn}(\theta, t) = \frac{N_s}{2n} \Re \{ i_s e^{j(\theta + \phi)} [e^{-j\omega n} - 1] \} \]  

A1.6

This equation is zero for even harmonics, but for odd, non-triplen, positive sequence harmonics, gives:

\[ mmf_{sn}(\theta, t) = \frac{N_s}{n} \Re \{ i_s e^{j(\theta + \phi)} \} \]  

A1.7

The rotor mmf can be represented in the same way as the stator, with respect to its own winding axis. \( \xi \) is the displacement between the fundamental and harmonic rotor windings. By analogy with the stator mmf above, the rotor mmf at an angle \( \theta' \) from the rotor winding axis will be:

\[ mmf_n(\theta', t) = \frac{N_s}{n} \Re \{ i_r e^{-j(\theta' + \xi)} \} \]  

A1.8

Chapter 2 showed that the rotor current vector can be redefined in terms of equivalent currents in windings aligned with the stator current axes. The displacement between the rotor and stator winding axes, \( e(t) \), relates the angles \( \theta' \) in the rotor winding and \( \theta \) in the stator winding:

\[ \theta' = \theta - e \]  

A1.9

Hence, with respect to the stator axes, the harmonic rotor mmf is given by:

\[ mmf_{r}\theta(\theta, e, t) = \frac{N_s}{n} \Re \{ i_r e^{-j(\theta - (a - 1)e + \xi)} \} \]  

A1.10

The magnetic field is a linear superposition of stator and rotor fundamental and harmonic components. For example, the field at the stator surface is:-

\[ B_s = \frac{\mu_0}{2l_s} \left[ mmf_{s1} + mmf_{s2} + \chi mmf_{r1} + \chi mmf_{r2} \right] \]  

A1.11
The flux linking a single turn can be calculated from equation 2.11. The harmonic component of the stator flux linking a single turn is:

\[
\Phi_{sn} = \frac{N_{sn}r l_1 \mu_0}{2 l_n} \int [\Re(i_x e^{-j(n\lambda + \phi)}) + \Re(i_y e^{-j(n\lambda - (n-1)e+\xi)})] d\theta \\
= \pm \frac{N_{sn} r l_1 \mu_0}{2 l_n} [\Re(i_x e^{-j(n\lambda + \phi)}) + \Re(i_y e^{-j(n\lambda - (n-1)e+\xi)})] \\
= \pm \frac{2 l_r}{n} B_{sn}(\lambda)
\]
(positive for n=4k+1, negative for n=4k-1).

Total flux linking winding 'a' is given by the sum of flux linking the entire winding distribution, which has both fundamental and harmonic components.

\[
\psi_{sa} = \int [\Phi_{sf}(\lambda) + \Phi_{sh}(\lambda)] [\frac{N_s \cos(\lambda) + N_{sn} \cos(n\lambda + \phi)}{2r}] d\lambda \\
= \int \left[ \frac{L_{sn}}{3\pi} [i_x^* + i_y^* + \Re(i_z e^{-j2(n\lambda + \phi)})] d\lambda \\
\pm \int \left[ \frac{L_{mn}}{3\pi} [i_x e^{j\gamma} \cdot i_y^* e^{-j\gamma} + \Re(i_z e^{-j2(n\lambda + \phi)})] d\lambda \\
= \pm \frac{1}{3} \left( L_{sn} [i_x^* + i_y^*] + L_{mn} [i_x e^{j\gamma} \cdot i_y^* e^{-j\gamma}] \right)
\]

where:

\[
L_{sn} = \frac{3N_{sn}^2 r l_1 \mu_0}{8nl_g}, \quad L_{mn} = \frac{3xN_{sn} N_{rn} r l_1 \mu_0}{8nl_g}, \quad \gamma = (n-1)e+\phi+\xi
\]

This can be extended to three phases. Using the fundamental rotor and stator flux vectors, as defined in equation 2.17, the relationship between the fundamental component of flux linkage
and current vectors is unchanged. However there is an additional harmonic term to flux.

\[
\psi_{sn} = L_{sn} \bar{s} e^{j(n-1)e} e^{j(\phi - \xi)}
\]

The harmonic component of the rotor flux can be found by analogy to the stator, if all vectors are referred to their local reference axes:

\[
\psi_{sn} = L_{sn} \bar{s} e^{j(n+1)e} e^{j(\phi - \xi)}
\]

If all vectors are stator referenced, the rotor flux is:

\[
\psi_{rn} = L_{sn} \bar{s} e^{-j(n+1)e} e^{j(\phi - \xi)}
\]

Similar analysis with the negative sequence terms gives expressions where the sign of the exponent is reversed, and in the rotor terms \((n-1)e\) is replaced by \((n+1)e\). For example, mmf and flux linkage become:

\[
mmf_{rn}(\theta, e, \xi) = \frac{N_{rn}}{n} B_{sn}(\bar{s}, \xi, e^{j(n-1)e} e^{j(\phi - \xi)})
\]

In summary, because the magnetic circuits are linear, fluxes are linearly related to currents and the relationship between fundamental flux and current vectors is unchanged. However the mmf distribution contains a harmonic component which is introduced into the magnetic field and hence into the flux linkage. This will give additional terms in the voltage equations.

### A1.2 Torque calculation

Leonhard defines torque as the interaction of the flux arising from the stator, at the rotor surface, with the rotor current distribution. The component of flux arising from the rotor currents cannot interact with the rotor currents to produce torque. It is equally valid to look at the converse, i.e. the flux arising from the rotor and the stator current distribution. Again, the analysis is for odd, non-triplen, positive sequence harmonics.
Products of fundamental flux density and harmonic current density (or vice versa) integrate to zero. Total torque is:

\[ T_e = \frac{2}{3} \left( L_{ml} \left( i_x^p \dot{i}_x - i_x \dot{i}_p \right) + L_{mn} \left( i_x^p \dot{i}_y - i_x \dot{i}_n \right) \right) \]

In vector terms this can be written as:

\[ T_e = \frac{2}{3} \left[ L_{ml} \left( i_x^p \dot{i}_x - i_x \dot{i}_p \right) + L_{mn} \left( i_x^p \dot{i}_y - i_x \dot{i}_n \right) \right] \]

Negative sequence harmonics are similar except that the exponent term is \( +j[(n+1)e + \phi - \xi] \).

The first term is the torque that would be produced in the absence of any winding harmonics. The second term varies with \( (n-1)e \) or \( (n+1)e \) as the motor rotates. This produces pulsating 6th, 12th, 18th...harmonic torques on a pure sinusoidal supply. However the amplitude of the harmonic torque goes as \( 1/n \), and also depends on the amplitudes of both the stator and rotor winding harmonics. The machine would normally be wound to give no more than 5% of fundamental for any harmonic.

Normally in vector control, decoupled control of torque can be achieved by keeping the rotor flux constant and orienting the controller with the rotor flux axis. However in this case there is a time varying angular displacement between the harmonic rotor flux distribution and the fundamental, hence they cannot both be decoupled at the same time.
A1.3 Voltage calculation

The terminal voltage can still be determined from the winding 'IR' drop, plus back-emf. However since the flux now contains a harmonic component, the harmonic must also be present in the stator voltage if the machine is supplied with only fundamental current. Conversely there must be a harmonic in the stator current, if the machine is fed from a sinusoidal voltage. Thus a spatial harmonic has introduced time variations into the rotor and stator currents and voltages. This can be seen more clearly from the motor voltage equations below. Again the analysis is for the odd, non-triplen, positive sequence terms.

\[
\begin{align*}
\dot{V}_s &= i \dot{I}_s + \frac{d\Psi_{s1}}{dt} + j\omega_1 \frac{d\Psi_{sn}}{dt} \\
0 &= i \dot{I}_r + \frac{d\Psi_{r1}}{dt} + j\omega_1 \frac{d\Psi_{rn}}{dt}
\end{align*}
\]

In an arbitrary reference frame, rotating at angular velocity \(\omega_1\), these equations become:

\[
\begin{align*}
\dot{V}_s &= i \dot{I}_s + \frac{d}{dt} [\Psi_{s1} + j\omega_1 \Psi_{sn}] + j\omega_1 \frac{d}{dt} [\Psi_{r1} + j\omega_1 \Psi_{rn}] \\
0 &= i \dot{I}_r + \frac{d}{dt} [\Psi_{r1} + j\omega_1 \Psi_{rn}] + j\omega_1 \frac{d}{dt} [\Psi_{r1} + j\omega_1 \Psi_{rn}]
\end{align*}
\]

and in terms of the current vectors:

\[
\begin{align*}
\dot{V}_s &= i \dot{I}_s + \frac{d}{dt} (L_{s1} + L_{sn}) \frac{dI_s}{dt} + (L_{m1} + L_{mn} e^{jn\gamma}) \frac{dI_s}{dt} \\
&\quad + j\omega_1 (L_{s1} + L_{sn}) I_s + j\omega_1 L_{m1} I_s + j\omega_1 + j[n-1] \omega_1) L_{mn} I_s e^{jn\gamma} \\
0 &= i \dot{I}_r + \frac{d}{dt} (L_{r1} + L_{rn} e^{j(\gamma-\xi)}) \frac{dI_r}{dt} + (L_{m1} + L_{mn} e^{-j(n-1)\xi}) \frac{dI_r}{dt} \\
&\quad + j\omega_1 (L_{r1} + L_{rn} e^{j(\gamma-\xi)}) I_r + j\omega_1 L_{m1} I_r + j[\omega_1 - n\omega_1 - j(n-1)\xi] L_{mn} I_r e^{j(n-1)\xi}
\end{align*}
\]

For negative sequence components, \(\gamma = [(n-1)\xi + \phi - \xi]\) becomes \(\gamma' = [(n+1)\xi + \phi - \xi]\). In the stator voltage equation \((n-1)\omega_1\) becomes \(-(n+1)\omega_1\), and in the rotor voltage equation \(-n\omega_1\) becomes \(n\omega_1\).

These equations show a component of the voltage which varies with rotor position (and hence with time). There is no simple way of simplifying these equations through the control of stator currents, since there are insufficient degrees of freedom to control both the fundamental and harmonic components.
A1.4 Analysis of test motor.

The stator winding pattern of the test motor is as shown in Fig A1.1, (based on data from the motor rewind company, and verified by inspection).

**Fig A1.1 Motor winding patterns**

<table>
<thead>
<tr>
<th>Stator winding pattern</th>
<th>Delta wound, 36 slots, 4 poles, pitch 1-8.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rotor winding pattern</th>
<th>Delta wound, 24 slots, 4 poles, pitch 1-6.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

From this winding pattern, the current distribution in the motor can be built up as shown in Fig A1.2a. The figure assumes that phase 'a' carries 1 pu current, and phases 'b' and 'c' both carry -1/2 pu current. Fig A1.2b shows the results of summing the current distribution over a pole pitch. The resulting distribution, which is proportional to mmf, is near-sinusoidal. This is confirmed by a fourier series expansion. Figs A1.3a and A1.3b show the resulting magnitude spectrum associated with Figs A1.2a and A1.2b respectively.

Ignoring even and triplen harmonics (since these do not contribute to torque) the most significant harmonic is the 7th, which has a magnitude of 11.6% of fundamental in the winding pattern. The resulting mmf is significantly attenuated at 1.7%. Assuming a similar magnitude of 7th harmonic in the rotor, the resulting ripple torque would be less than 0.2% of fundamental. This is insignificant, compared with other sources of torque error.
Fig A1.2 Current and mmf distributions for the test motor.

![Current and mmf distributions](image)

- a) Current distribution
- b) Mmf distribution

x axis - angular position, 0 to 1 pole pitch
y axis - normalised current density
y axis - normalised mmf

Fig A1.3 Current distribution and mmf spectrum

![Current distribution and mmf spectrum](image)

- a) Current distribution
- b) Mmf distribution

x axis - harmonic number (1-40)
y axis - normalised current (dB)
y axis - normalised mmf (dB)
APPENDIX 2 - EFFECT OF CORE LOSSES

In the steady state, the significance of neglecting core losses can be evaluated by using the steady state equivalent circuit with and without the inclusion of a magnetising resistance ($R_m$, Fig A2.1), and comparing the phase of the rotor flux in each case.

**Fig A2.1 Induction motor steady state equivalent circuit**

At rated voltage and frequency, and approximately rated slip, the rotor flux was calculated for the test motor, as shown in Table A2.1. The phase refers to the orientation of the stator current with respect to the rotor flux. The resulting flux angle estimate in the controller is $2.5^\circ$ too large, if core losses are neglected. The flux magnitude is barely affected.

**Table A2.1 comparison of rotor flux, with and without core loss.**

<table>
<thead>
<tr>
<th>rotor flux</th>
<th>magnitude - $V_s$</th>
<th>phase - $^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>with core loss</td>
<td>1.15</td>
<td>61.22</td>
</tr>
<tr>
<td>without core loss</td>
<td>1.16</td>
<td>63.67</td>
</tr>
</tbody>
</table>

In the steady state (constant flux and torque demands, constant slip), the effect of this angle error can be found by looking at the relationship between demanded and actual currents in the controller reference frame. This is the same analysis as for the effect of controller delays (chapter 6.3). Fig A2.2 shows that for ideal current control, the desired effect would have been for the rotation from controller to stationary co-ordinates to be the inverse of the rotation in the motor (from stationary to true rotor flux oriented co-ordinates) with no resultant
rotation. With an angle error $\Delta \theta = \theta_1 - \theta_2$, there is also a net rotation by $\Delta \theta$.

Fig A2.2 Field oriented controller with non-ideal flux angle estimation.

Hence the motor currents are:

\[
\begin{align*}
    i_{sd} &= i_{sd}^* \cos(\Delta \theta) + i_{sq}^* \sin(\Delta \theta) \\
    i_{sq} &= i_{sq}^* \cos(\Delta \theta) - i_{sd}^* \sin(\Delta \theta)
\end{align*}
\]  

Using small angle approximations for sine and cosine, the resulting torque error becomes:

\[
\frac{\Delta T_e}{T_e} \approx \frac{i_{sq}^*}{i_{sd}^*} \Delta \theta
\]

Assuming that the torque current is typically two to three times the magnetising current at full load, this controller error causes a reduction in torque of approximately 1.5 to 2.7 times $\Delta \theta$, or 6 to 11% of rated torque for the test motor. This is significant. Compensation for the effects of core losses should form the subject of further research.
APPENDIX 3 - EXPERIMENTAL SYSTEM

This appendix provides a brief description of the experimental system and measurement methods. All the experiments were performed on the test system shown in Fig A3.1 and outlined below. Circuit diagrams of the power electronics, sensing and control circuits can be found in Figs A3.2 to A3.14.

Fig A3.1 Block diagram of experimental system

i) Induction motor.

The test motor was a 7.5 hp wound-rotor induction motor. Rating plate details are reproduced below. The motor was manufactured by Brook Crompton Parkinson, but subsequently rewound by Warsopps to an unknown specification. Hence the rating plate details and manufacturer's test details could not be treated as accurate.

A wound-rotor motor differs from a standard cage induction motor, but was chosen because the rotor current could be used to provide an additional measure of controller performance.

A3.1
The motor was instrumented with flux sensing coils. Thermocouples were attached to the stator slots, end windings, back iron and frame.

**Table A3.1 Motor rating plate**

<table>
<thead>
<tr>
<th>Serial No.</th>
<th>S5GVP815</th>
<th>Stator voltage</th>
<th>400 V (delta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power</td>
<td>7.5 hp</td>
<td>Stator current</td>
<td>13 A</td>
</tr>
<tr>
<td>Speed</td>
<td>1420 rpm</td>
<td>Rotor voltage</td>
<td>235 V (star)</td>
</tr>
<tr>
<td>Frequency</td>
<td>50 Hz</td>
<td>Rotor current</td>
<td>16 A</td>
</tr>
<tr>
<td>Phase</td>
<td>3</td>
<td>Insulation</td>
<td>Class E</td>
</tr>
</tbody>
</table>

**ii) Loading rig.**

A 22kW dc dynamometer loading rig was used to provide a constant load for steady-state tests. For dynamic tests, the induction motor could be run with the dynamometer unpowered, acting as an inertial load only. Table A3.1 below lists the mechanical system parameters; these give a lowest critical frequency of 103Hz.

**Table A3.2 Loading rig mechanical system parameters.**

<table>
<thead>
<tr>
<th>Induction motor inertia</th>
<th>Dynamometer inertia</th>
<th>Coupling stiffness</th>
<th>Resonant frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>kg m²</td>
<td>kg m²</td>
<td>Nm rad⁻¹</td>
<td>Hz</td>
</tr>
<tr>
<td>0.055</td>
<td>0.60</td>
<td>2.1x10⁴</td>
<td>103</td>
</tr>
</tbody>
</table>

**iii) Inverter.**

An in-house experimental inverter was used. This was rated at 415V, 100A peak, giving a continuous rating well above that of the motor, of approximately 25kW, at a switching frequency of 20kHz. Circuit diagrams are shown in Figs A3.2 to A3.4. The inverter had been designed to provide a test facility for the development of inverter power circuits and control
strategies. Unfortunately the prototyping methods used made it fragile, and the absence of snubbing introduced significant interference in control and sensing circuits.

iv) Current controller.

The current controller was also an in-house prototype, providing a standard implementation of current regulated PWM (for example /Garces 1986/). Circuit details are shown in Figs A3.5 to A3.6. Separate error amplifiers were used on each line current, with no attempt to optimise switching in the overmodulation range. Each error signal was then compared with a 20kHz ramp signal to generate a fixed switching frequency output. This analogue controller provided fast control of current. However it was not synchronised with the digital system, hence switching noise appeared on sampled signals.

v) Sensing.

Table A3.2 lists the available feedback signals. Interface circuits are shown in Figs A3.7 to A3.9. These signal-conditioning circuits buffered and scaled the raw sensor signals. Three-phase to two-phase conversion and analogue integration could be performed in analogue, although in practice the integration was implemented numerically in the signal processor. Anti-aliasing filtering was performed by a switched capacitor filter chip. 8th order filters were allowed for, but in practice, 4th order Butterworth filters, with a 2kHz cut-off were used, in conjunction with a 4 kHz sampling frequency. This was found to be acceptable, because the main frequency components were at the switching frequency (a decade higher at 20 kHz), and at low order harmonics of the supply frequency, (2kHz represents the 40th harmonic of motor base frequency).

In addition a digital input/output board interfaced with an encoder. This board was also designed to generate inverter switching signals from a set of voltage vectors and to provide a general interface for digital input/output. These features were bench tested, but were not used in the experimental work. The circuit diagram is shown in Fig A3.10.

Full calibration was performed manually by comparing captured signals with measurements.
from conventional laboratory instruments. Test routines allowed offset and gain verification of the signal conditioning circuits: a 1V peak ramp signal was generated by the digital signal processor system and fed back into the analogue inputs. Correlation techniques were used to determine offset and gain. This was used to compensate for zero errors.

Flux was measured using three, full pole-pitch coils in the stator winding. Because of the arrangement of stator slots, the sense coils were displaced by 90 electrical degrees from the stator winding axes. Four turns were used, to give a stator turns ratio of 70.5, and a peak voltage of 8.3V for a stator voltage of 415V rms. The scaling of the flux sensors was fine-tuned by comparing voltage measurements and flux sensors, following disconnection of the machine when running at full speed and no load. With zero stator current (during the decay of the back emf), the terminal voltage is the same as the airgap voltage, so both signals should be identical.

**Table A3.3 Sensor details**

<table>
<thead>
<tr>
<th>Signal</th>
<th>Sensor</th>
<th>bandwidth</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator current</td>
<td>Hall effect (LEM) sensors</td>
<td>100kHz</td>
<td>1%</td>
</tr>
<tr>
<td>Rotor current</td>
<td>Hall effect (LEM) sensors</td>
<td>100kHz</td>
<td>1%</td>
</tr>
<tr>
<td>Stator voltage</td>
<td>Differential voltage probe (SI9000)</td>
<td>10MHz</td>
<td>1%</td>
</tr>
<tr>
<td>Stator flux</td>
<td>Search coil (4 turns)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Position</td>
<td>Optical encoder, 6000 lines</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

vi) Controller.

The controller was based around a Texas instruments TMS320C30 digital signal processor (DSP). This is a 32 bit floating-point processor. The TMS320C30 was chosen for ease of development, both because of its floating point capability and because high level compilers were available. Most of the code was written in C.
The development system for the DSP consisted of a processor board with memory expansion, two analogue input/output boards and a digital bus interface, all produced by Loughborough Sound Images. This provided eight simultaneously-sampled analogue input channels, followed by two 12-bit multiplexed A to D converters which interfaced with the signal conditioning cards. Four 12-bit analogue output channels, also synchronised, were available; three channels were used as current demands for the current controller. The digital bus interface was used to read encoder information. The development system was mounted in a personal computer (IBM AT). The software was developed on the PC and downloaded to the target. Following an experiment, results could be transferred back to the PC for analysis.

vii) Instrumentation.

The following instrumentation was used to measure motor performance in the laboratory.

Table A3.4 Instrumentation details

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Instrument</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power, voltage current, phase angle</td>
<td>Infratek Digital Wattmeter; Cambridge ac test sets (moving iron, two wattmeter method).</td>
</tr>
<tr>
<td>Voltage</td>
<td>Keighley multimeter.</td>
</tr>
<tr>
<td>Current</td>
<td>Tektronix current probe.</td>
</tr>
<tr>
<td>Time domain signals</td>
<td>Lecroy oscilloscope.</td>
</tr>
<tr>
<td>Speed</td>
<td>Compact optical tacho.</td>
</tr>
<tr>
<td>Torque</td>
<td>IML torque transducer (strain gauge)</td>
</tr>
<tr>
<td>Frequency</td>
<td>Hewlett Packard spectrum analyser</td>
</tr>
<tr>
<td></td>
<td>Feedback frequency meter</td>
</tr>
<tr>
<td>Resistance</td>
<td>Valhalla micro-ohmmeter</td>
</tr>
</tbody>
</table>
Fig A3.2 Circuit diagram of the inverter, sht 1 of 3.
Fig A3.3 Circuit diagram of the inverter, sht 2 of 3.
Fig A3.4 Circuit diagram of the inverter, sht 3 of 3.
Fig A3.5 Circuit diagram of the current controller, sht 1 of 2.
Fig A3.6 Circuit diagram of the current controller, sht 2 of 2.
Fig A3.7 Circuit diagram of the sensor interface, sht 1 of 3.
Fig A3.8 Circuit diagram of the sensor interface, sht 2 of 3.
Fig A3.9 Circuit diagram of the sensor interface, sht 3 of 3.
Fig A3.10 Circuit diagram of the digital interface board, sht 1 of 4.
Fig A3.11 Circuit diagram of the digital interface board, sht 2 of 4.
Fig A3.12 Circuit diagram of the digital interface board, sht 3 of 4.
Fig A3.13 Circuit diagram of the digital interface board, sht 4 of 4.
APPENDIX 4 - MOTOR CHARACTERISATION

The aim of the experimental work was to investigate the effects of detuning on the motor’s dynamic performance. It was necessary to measure the true motor parameters in order to estimate the extent of the detuning. Conventional characterisation methods (for example no load, locked rotor, full load) have been compared with the results of specific tests (for example step and ramp response) in order to validate the latter. These tests were repeated immediately following each experiment, to determine motor parameters.

A4.1 Standard tests

The motor was characterised for a 50 Hz, sinusoidal supply using standard tests (for example /Say 1988/).

A4.1.1 Turns ratio

The stator was supplied from a three phase variac. The rotor was open-circuit and stationary. Stator and rotor terminal voltages were measured for a range of supply voltages up to rated voltage. This was repeated with the variable-voltage supply connected to the rotor and the stator open-circuit. From these measurements, two ratios of stator to rotor voltage, k1 and k2 respectively, were found. From the equivalent circuit model of the motor, it can be shown that:

\[ n = \sqrt{\frac{k_1 k_2 L_r}{L_s}} \]  

A4.1

For a split between rotor and stator leakage, in proportion to the ratio of stator-referred stator and rotor resistances, the equation above gave a turns ratio of 1.61 +/- 1%.
A4.1.2 Stator resistance.

Stator resistance was measured at standstill, using an accurate resistance meter. This instrument was also used to record resistance at the end of no load, locked rotor and part load tests, from which temperature rise could be inferred, as detailed in /BS4999 part 101/.

A4.1.3 No load test.

The standard no load test /Say 1988/ was carried out over a range of supply voltages, up to rated voltage. The motor was coupled to a dynamometer, which was adjusted to match the rotor speed to the measured supply frequency. Thermocouple measurements of stator temperatures were recorded throughout the test.

A4.1.4 Locked rotor test.

The locked rotor test /Say 1988/ was performed with the shaft turning at low speed, (less than 30 rpm), controlled by a dynamometer. The shaft was rotated, in order to provide a sliding contact between the slip rings and brushes, to prevent brushgear damage and provide a more representative brush drop. The test was carried out for a range of voltages up to rated supply current, and stator temperatures were again measured throughout.

A4.1.5 Part load tests.

The motor was supplied at rated voltage and allowed to stabilise at an operating point, which was set by a dynamometer load. Measurements of torque, speed and input voltage, current and power were repeated at 25%, 50%, 75% and full rated load.
A4.1.6 Inertia

Inertia was measured by the two wire method according to BS5000 Part 60. This gave an inertia of 0.055 kgm\(^2\)/2%, which agreed well with calculations based on rotor dimensions. Assuming a packing density of 65% for the copper in the slots, the calculated inertia was 0.061 kgm\(^2\). The dynamometer inertia was estimated from rotor dimensions as 0.78 kg m\(^2\).

A4.1.7 Friction and windage.

Run-down tests were performed for the induction motor alone, and again when coupled to the dynamometer /Say 1988/. Fig A4.1 shows measured speed against rate of change of speed, and the best fit straight line. The figure also shows speed against time, to show how the results of the curve fit compare with measured speed.

Fig A4.1 Friction and windage characterisation for the induction motor from a run down plot.
The straight line fit was of the form:-

\[ \int \frac{d\omega}{dt} = -(k_d\omega + k_f) \]  

The gradient and intercept were used to find the ratios of damping coefficient and friction to inertia. The absolute values of the damping coefficient, friction and inertia can be separated using the friction and windage loss. The loss term was calculated from the no load test, which was repeated for the induction motor alone, and again when coupled to the dynamometer, but without the dynamometer energised. Results for decreasing supply voltage were extrapolated back to find the input power at zero voltage. This corresponds to the friction and windage losses /Say 1988/. The results below compare mechanical system values from the different tests.

**Table A4.1** comparison of the results of inertia calculations, and friction and windage tests.

<table>
<thead>
<tr>
<th></th>
<th>Induction motor alone</th>
<th>Induction Motor and dynamometer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculated inertia: rotor dimensions (kg m²)</td>
<td>0.061</td>
<td>0.84</td>
</tr>
<tr>
<td>Measured inertia (kg m²)</td>
<td>0.055</td>
<td>-</td>
</tr>
<tr>
<td>Calculated inertia: run-down test (kg m²) +/-20%</td>
<td>a) 0.052</td>
<td>a) 0.60</td>
</tr>
<tr>
<td></td>
<td>b) 0.045</td>
<td>b) 0.55</td>
</tr>
<tr>
<td>Friction and windage (W)</td>
<td>72.5</td>
<td>240</td>
</tr>
<tr>
<td>Damping/inertia (rad s⁻¹)</td>
<td>a) 2.9x10⁻³</td>
<td>a) 6.6x10⁻³</td>
</tr>
<tr>
<td></td>
<td>b) 17.1x10⁻³</td>
<td>b) 5.4x10⁻³</td>
</tr>
<tr>
<td>Friction/inertia (rad s⁻²)</td>
<td>a) 16.7</td>
<td>a) 3.45</td>
</tr>
<tr>
<td></td>
<td>b) 15.6</td>
<td>b) 3.37</td>
</tr>
</tbody>
</table>

(a) was with clockwise and (b) was with anticlockwise rotation.
There is reasonable agreement between calculation and measured inertia for the induction motor alone. The results from the run-down test show some variation with direction of rotation, probably because of the brush contacts; the accuracy of the results is affected by the extrapolation. The calculated combined inertia is only approximate, since the dynamometer rotor was not removed for accurate dimensional measurements.

A4.2 Calculation of equivalent circuit values

Initial estimates of total equivalent circuit leakage inductance and total resistance were obtained from the locked rotor test. The no load test gave initial estimates of magnetising inductance and core loss, as a function of magnetising voltage, assuming that the stator resistance equalled its dc value. These initial estimates were refined in an iteration process, reproduced in table A4.2 below. The iteration process assumed:

a) The dc stator resistance measurement was an underestimate of its 50 Hz value, due to proximity (and possibly skin) effects.
b) The 50 Hz rotor resistance derived in the locked rotor test was likely to be an overestimate of its normal (i.e. slip frequency) value, both due to proximity effects and to rotor heating during the test.
c) Magnetising inductance would decrease with magnetising voltage due to saturation.
d) Core losses would increase with magnetising voltage.
e) Rotor resistance and leakage inductance values computed from the locked rotor test were relatively insensitive to magnetising inductance and magnetising resistance errors.
f) Magnetising resistance and main inductance values computed from the no load test were relatively insensitive to leakage inductance and rotor and stator resistance errors.
Table A4.2 Summary of the iteration process.

<table>
<thead>
<tr>
<th>INITIAL VALUES</th>
</tr>
</thead>
<tbody>
<tr>
<td>By measurement: stator resistance: $R_s$</td>
</tr>
<tr>
<td>Locked rotor test: rotor resistance: $R_L = \text{Real} \left{ \frac{v}{i_s} \right} - R_s$</td>
</tr>
<tr>
<td>leakage inductance: $X_L = \text{Imag} \left{ \frac{v}{i_s} \right}$</td>
</tr>
<tr>
<td>No load test: magnetising reactance: $X_m = \text{Real} \left{ \frac{v}{i_s} \right}$</td>
</tr>
<tr>
<td>magnetising resistance: $R_m = 1 \text{M} \Omega$</td>
</tr>
</tbody>
</table>

50:50 split between stator and rotor leakage: $\alpha = 0.5$

1. Set $R_s, R_L, X_m, \alpha$.

2. Read the no load results: line voltage, line current, input power, speed, frequency and temperature. Compute the phase angle and slip; adjust resistances for temperature.

3. Solve equation A4.3 for sets of $X_m(v_g)$, $R_m(v_g)$ using the Mathcad iterative solver (needs initial guesses for $X_m, R_m$). Create look-up table of magnetising curve.

4. Read the locked rotor results: line voltage, line current, input power, frequency, temperature. Compute the phase angle; adjust resistances for temperature.

5. Solve equation A4.3 for sets of $R_L, X_L$ using the Mathcad iterative solver (needs initial guesses for $R_L, X_L$). Find the average values.

6. Compare with the previous values of $R_L, X_L$. Adjust $\alpha$ to reflect the split in stator and rotor resistances. Repeat from step 1.

\[ v_g = v_s - i_s (R_s + j\alpha X_L) \]

\[ i_s = \frac{1}{v_g} + \frac{1}{R_m + \frac{R_L}{s} + jX_m(1-\alpha)} \]

\[ A4.3 \]

\[ A4.6 \]
The best fit parameters are listed in table A4.3.

Table A4.3 Measured machine parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator resistance (at 25°C)</td>
<td>2.37 Ω</td>
</tr>
<tr>
<td>Rotor resistance (at 25°C)</td>
<td>3.465 Ω</td>
</tr>
<tr>
<td>Magnetising resistance</td>
<td>2000 Ω</td>
</tr>
<tr>
<td>Magnetising inductance - at rated load.</td>
<td>0.372 H</td>
</tr>
<tr>
<td>Stator leakage inductance</td>
<td>0.015 H</td>
</tr>
<tr>
<td>Rotor leakage inductance</td>
<td>0.022 H</td>
</tr>
<tr>
<td>Rotor time constant</td>
<td>113 ms</td>
</tr>
<tr>
<td>Leakage factor</td>
<td>0.092</td>
</tr>
<tr>
<td>Inertia</td>
<td>0.055 kgm²</td>
</tr>
<tr>
<td>Viscous damping</td>
<td>1.6x10⁻⁴ N s rad⁻¹</td>
</tr>
<tr>
<td>Friction</td>
<td>0.46 N</td>
</tr>
</tbody>
</table>

Fig A4.2 shows the main flux magnetisation curve (derived from the no load test and iteration process). Leakage terms were treated as constant up to rated supply current and rated load.
A4.3 Verification of parameter values.

The parameters values in table A4.3 above, together with the magnetising curve, where appropriate, were used to compute current, phase angle and torque for the conditions of the part load tests. Fig A4.3 compares measured and predicted values of stator current magnitude and phase and shaft torque, for three cases.

a) Constant parameters, (including temperature effects).
b) Including main path saturation.
c) Including main path saturation and core losses.

The current and torque plots show the error between estimated and actual values, normalised by the rated full load value. The figure provides a means of judging the accuracy of the parameter values.

The simplest constant parameter model predicts current to within 2% of rated current and torque to within 5% of rated torque. The error in current magnitude is greatest at light loads, where saturation effects are most significant. Modelling main path saturation reduces the current error.
Fig A4.3 Comparison of estimated and measured motor performance using three sets of parameters.
The error in phase angle is not sensitive to saturation effects, but does change as core losses are accounted for; however it is not clear whether this improves the estimate. This may be because model of core loss was inaccurate. The torque estimate is not sensitive to saturation effects or core loss. However it is at the limits of the measurement accuracy of the characterisation exercise.

Fig A4.4 shows the effect of repeating the iteration process with changes in rotor resistance values of up to 10% at rated voltage and frequency. Errors in predicted torque and current increase with load, and are approximately proportional to errors in rotor resistance. The predicted torque and current are much more tolerant to errors in leakage inductance and stator resistance.

Stator temperature measurement can be used to compensate both stator and rotor resistances. Apart from under stall conditions, the rotor temperature is unlikely to differ by more than 25\(^\circ\) for an externally blown induction motor. This would give perhaps 10% error in rotor resistance.

With saturation compensation, and accurate initial characterisation, it should be possible to predict all other parameter values to a few percent. Hence the accuracy of the motor performance as predicted by the equivalent circuit model depends largely on the accuracy of the rotor resistance estimate.

This defines the accuracy requirements of on-line parameter tracking, compared with an approach of accurate initial characterisation, combined with stator temperature measurement and adjustment of magnetising inductance to allow for main flux saturation.
Fig A4.4 Effect of rotor resistance on estimated motor performance.
A4.4 Parameter identification tests.

The motor equivalent circuit parameters change with motor operating point and environmental conditions. The aim of this thesis has been to look at the effects of parameter errors on the dynamic response of the motor under field oriented control. Hence it was necessary to identify the true parameter values for each experiment. Specific tests were used to automatically capture the motor parameters at the end of each experiment. Parameter identification is also used for automatic commissioning in some commercially available products. Hence the accuracy of these tests is of some interest.

The tests imposed defined conditions on the motor, for example by injecting voltage or current signals into the motor. This isolated particular parameters, simplifying the identification process. The principles are introduced below and the results are summarised in table A4.4.

A full dynamic equivalent circuit model in the induction motor was introduced in chapter 2 in conventional T-I form. Fig A4.5 shows an alternative but exact form of the equivalent circuit, /Yawamura 1980/. In this form, three separate current paths can be identified and isolated with an appropriate test signal: for dc, high frequencies and circulating current.
A4.4.1 Stator resistance from dc injection

The inductive terms in Fig A4.5 have zero impedance to dc, hence there is no dc component to the back emf. The circuit equations are fully decoupled to dc, and can be readily solved, even when the motor is rotating and fully fluxed.

\[ v_{dc} = i_{dc} R_s \]  

With an inverter, a dc offset can be added to the normal reference voltage prior to generation of the PWM switching signals. However dc injection will cause plug breaking, resulting in braking torque and increased rotor losses. If the motor is rotating, the dc component needs to be of short duration (approx 1s) and low amplitude. This requires careful design of the signal conditioning circuits to minimise offset and drift.

Stator resistance was found from dc voltage injection at standstill. Using an inverter, the dc voltage was approximated by a constant duty cycle pulse train. Fig A4.6 shows the injected voltage and resulting current at zero speed, as measured by a storage oscilloscope.
Fig A4.6 PWM voltage and resulting current for dc resistance and high frequency time parameter estimation

Scaling:

<table>
<thead>
<tr>
<th>x axis</th>
<th>Time</th>
<th>20μs/box</th>
</tr>
</thead>
<tbody>
<tr>
<td>y axis (upper trace)</td>
<td>Voltage</td>
<td>200V/box</td>
</tr>
<tr>
<td>y axis (lower trace)</td>
<td>Current</td>
<td>5A/box</td>
</tr>
</tbody>
</table>
A4.4.2 Combined resistance and leakage inductance from high frequency injection.

At high frequencies the inductive terms in Fig.A4.5 are high impedance and the magnetising flux path can be ignored. The signal can take the form of either a voltage step /Schierling 1988/, or of a high frequency sinusoid. A PWM voltage can be thought of as a series of voltage steps, but also contains higher harmonics, which can be used to identify parameter values in the high frequency path /Green 1993/. Rotor skin effects may need to be accounted for.

At zero speed, the response of the motor to a voltage step is:

\[ v_{hf} = \sigma L_s \frac{di_{hf}}{dt} + i_{hf} \left( R_s + \frac{(L_m R_s)}{L_r} \right) \]  

A4.5

To a first approximate the solution simplifies to:

\[ \sigma L_s = \frac{v_{hf}}{\Delta i} \]  

A4.6

voltage removed:

\[ \tau = -\frac{\sigma L_s}{\left( \frac{(L_m R_s)}{L_r} + R_s \right)} \ln \left( \frac{\Delta i}{i_o} \right) \]  

A4.7

Care needs to be taken in interpreting the resulting parameter values (\( \sigma L_s \) and \( a^2 R_s + R_s \); \( a = \frac{L_m}{L_r} \)). If the motor magnetic circuit operates into saturation, then the resulting inductance estimates will be small signal values appropriate to the particular speed, load and flux level. Because of skin effect, the resistance estimates may differ considerably from their slip frequency values. Hence the result may be unsuitable for flux angle estimation (which ignores frequency effects and saturation), but remains valid for design of the current controller, (which will operate at PWM switching frequencies in any case). If the split between high frequency rotor and stator resistance is known, the tests may used to identify the change in rotor temperature. This can then be used to compensate the resistance value in the controller for temperature changes.
The high frequency resistance and inductance terms were found from the current response to
a series of voltage steps, again at zero speed. These terms dominate the design of the current
loop. Fig A4.6 also shows the high frequency components of the applied voltage and resulting
current, superimposed on the dc voltages and currents.

In practice, the step response was not used for parameter identification because of the high
bandwidth required. During the experiments, a sampling frequency of 4kHz was used. Hence
it was not possible to accurately measure a response with a time constant of milliseconds. The
alternative - a higher sampling rate and filter cut-off frequency, followed by digital filtering
and re-sampling was too computationally intensive.

**A4.4.3 Rotor time constant from open circuit conditions**

With the motor open circuit, the envelope of the decay of the back-emf gives the rotor time
constant. The back-emf is:

\[ v_{ad} = \frac{L_m}{L_r} \psi_{ro} \sin(\omega_f t)e^{-\frac{t}{\tau_r}} \]

\[ v_{aq} = \frac{L_m}{L_r} \psi_{ro} \cos(\omega_f t)e^{-\frac{t}{\tau_r}} \]  

A4.8

Fig A4.7 shows typical experimental results. Fig A4.7a plots the terminal voltages as
measured by the DSP and uses the magnitude of the voltage vector to extract the exponential
envelope. Fig A4.7b shows a best straight line fit on the logarithm of the voltage magnitude.
Fig A4.7c shows the variation of rotor time constant with terminal voltage, and compares it
with the results of the no load test.
Fig A4.7 Measured back emf, for rotor time constant identification.
The slope of the logarithmic fit gives the rotor time constant. As the magnetic circuit comes out of saturation, the rotor time constant will change. Therefore the test could be used to identify the shape of the saturation curve, as a function of magnetising voltage.

The open circuit test was found to give a good repeatable measure of rotor time constant, with a straight line fit. Attempts to derive a saturation curve were unconvincing. The test is intrusive, and therefore only appropriate to initial commissioning not on-line tracking.

A4.4.4 Combination of parameters from current ramp.

Irisa /1985/ applied a shaped current pulse to the motor at zero speed, to verify a range of motor parameters. The test is intrusive, and requires zero speed and closed loop current control, so applies to initial commissioning rather than on-line tracking. The circuit equations can be solved to find the voltage that arises from the injected current:

Current ramp:

\[ is(t) = \frac{I}{T} t \]  \hspace{1cm} A4.9

\[ v_s(t) = \frac{I}{T} \left[ R_s t + \sigma L_s + \left( \frac{L_m}{L_r} \right)^2 (1 - e^{-\frac{t}{\tau}}) \right] \]

Constant current:

\[ is(t) = I \]

\[ v_s(t) = IR_s - k e^{-\frac{t-t_0}{\tau}} \]  \hspace{1cm} A4.10

\[ k = \frac{L_m^2 I}{L_r T (1 - e^{-\frac{t}{\tau}})} \]
The following motor parameters can be derived:

(a) from the initial voltage step,
\[ \alpha L_s = v_{\text{step}} \frac{T}{I} \]  

(b) from the steady state voltage,
\[ R_s = \frac{V_{\text{dc}}}{I} \]

(c) The exponential time constant \( \tau_m \).

(d) From the scaling of the exponential decay \((V_i - V_{\text{dc}})\).
\[ \frac{L_m^2}{L_r} = (v_1 - v_{\text{dc}}) \frac{T}{I} (1 - e^{-\frac{t}{\tau_r}}) \]

Fig A4.8 shows typical experimental results, as measured on an oscilloscope. The test was repeated over two different timescales, to pick out the parameters which can be identified from the first the ramp and then the constant current stages of the test. Figs A4.9 and A4.10 show the results as processed by the DSP.

The test proved useful for identifying stator resistance, and approximating leakage inductances. As the traces indicate attempts to estimate magnetising current and rotor time constant were inaccurate and were not representative of normal operating conditions.
Fig A4.8 Measured but unprocessed results of ramp current injection test.

Scaling:

<table>
<thead>
<tr>
<th>x axis</th>
<th>Time</th>
<th>5ms/box</th>
</tr>
</thead>
<tbody>
<tr>
<td>y axis (upper trace)</td>
<td>Voltage</td>
<td>200V/box</td>
</tr>
<tr>
<td>y axis (lower trace)</td>
<td>Current</td>
<td>5A/box</td>
</tr>
</tbody>
</table>
Fig A4.9 Results of ramp current injection test as processed by the DSP, showing an exponential curve fit.

Fig A4.10 Results of ramp current injection test as processed by the DSP, showing the details of the initial step.
A4.5 Comparison of results.

The parameter values obtained from the standard tests are compared with the results of the parameter identification tests in table A4.4 below.

Table A4.4 Comparison of motor parameters by standard tests and specific signal injection tests.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Standard test</th>
<th>Injection test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator Resistance: $R_s / \Omega$</td>
<td>2.37</td>
<td>(a) 3.4</td>
</tr>
<tr>
<td>High frequency inductance: $\sigma L_s / mH$</td>
<td>35.6</td>
<td>(b) 20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(d) 36</td>
</tr>
<tr>
<td>High frequency resistance: $R_s + (L_m/L_s)^2R_r / \Omega$</td>
<td>5.46</td>
<td>(d) 5.9-7.8</td>
</tr>
<tr>
<td>Rotor time constant: $\tau_r / \text{ms}$</td>
<td>113</td>
<td>(c) 105-120</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(d) 75</td>
</tr>
</tbody>
</table>

Where the injection tests were (a) dc voltage injection, (b) high frequency injection (voltage step), (c) decay of back emf and (d) current ramp.

In general, the standard characterisation procedures gave more repeatable results, with a better measurement accuracy. This was, at least in part, because the former test used a 50Hz sinusoidal supply whereas the latter used signal injection from a voltage source inverter. Voltage measurements on the inverter used a transducer scaled for approximately 600V peak, to measure average dc values of a few tens of volts. Hence the accuracy of voltage measurements was poor for the dc and high frequency voltage tests. The experimental system controlled voltage indirectly, via the current loops; if direct control of the inverter switches had been implemented, the precise switch timing information could have been used to improve the voltage measurement.

Table A4.4 shows that the decay of the back emf gave a useful measure of the rotor time constant. The current ramp test was effective for total leakage inductance identification.
However there was considerable uncertainty about the high frequency resistance estimation. None of the tests really addressed magnetising inductance.

Some consideration should be given as to what the parameter value is to be used for. In the field oriented controller, the 50Hz parameter value is required for estimating the orientation and for torque and flux estimation; the PWM condition is more appropriate for current loop tuning.

The results of the characterisation exercises suggest that it is reasonable to estimate motor parameter values to an accuracy of about 5% with standard sinusoidal tests. With care, specific signal injection tests can approach this accuracy. On-line versions of the tests would need to be very carefully designed to match or improve on this accuracy. The field oriented controller must be able to tolerate at least this degree of parameter uncertainty.
APPENDIX 5 - SIMULATION SOFTWARE

Simulation results throughout this thesis were prepared using the "Psie" simulation environment. Where appropriate, modules from ERA's "Drivesim" library were incorporated.

Psie is a block-structured simulation language, produced by Bosa Automatisering, Delft, Holland. Simulation models are formed by linking a series of functional blocks, whose output is updated every time step. The blocks may be linear or non-linear, for example integrator, limiter, counter/timer, hysteresis block. Standard maths functions, logical operators and look-up tables are also permitted. A range of integration algorithms is available. State event detection allows accurate modelling of switched systems, even when the switching is not synchronised with the update time-steps.

The Drivesim library contains motor, power converter, controller and load modules which have been tested against analytical predictions and experimental results. The library was produced by ERA Technology, Leatherhead, Surrey.