High-K Structure in the $^{164}$Er Region

By

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A thesis submitted to the Physics Department at the University of Surrey for the degree of Doctor of Philosophy.

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June 2011
Abstract

Several multi-quasiparticle isomers have been observed for the first time in a gamma-spectroscopy experiment performed at the Australian National University Heavy Ion Accelerator Facility. A fusion-evaporation reaction using a 58 MeV, $^9$Be beam, incident on a $^{160}$Gd enriched target was used to populate nuclei in the vicinity of $^{164}$Er. Of specific interest was a known four quasiparticle (4QP) isomer in $^{164}$Er with an uncertain half-life. A measurement of the half-life, and the observation of high $\Delta K$ branches from the isomer was anticipated to extend our understanding of 4QP isomer decays outside of the $A=180$ isomer group, establishing whether the models derived for this region are applicable at $A=164$. A pulsed beam provided ideal conditions for the half-life measurement of all gamma-decaying levels observed using the CAESAR array, which consists of nine HPGe detectors and two LEPS detectors.

Twelve different isotopes with observable $\gamma$-ray transitions were detected. Of these, five are observed to possess new isomers: $^{161}$Dy, $^{162}$Dy, $^{163}$Ho, $^{162}$Er and $^{165}$Er. Half-life measurements and structural assignments for the new isomers are presented in this work. The $^{162}$Dy isomer, a 2QP structure decaying by a highly forbidden E2 transition, could be compared with other 2QP isomers that share the same decay characteristics. The $^{162}$Dy isomer agrees well with systematics correlating the reduced hindrance with the product of the valence neutron and proton numbers ($N_p N_n$) over an extended $N, Z$ range. Small deviations from $N_p N_n$ dependence are analysed for the 2QP isomer decays, and interpreted as arising from a weak dependence on the isomer excitation energy relative to the yrast line.

A precise measurement of the 4QP, $^{164}$Er isomer's half-life is made, and $\Delta K=12$ branches are found decaying to the ground state band. The reduced hindrance of these branches could be compared with a plethora of other 4QP and 5QP isomers that share the same decay characteristics. The $^{164}$Er isomer agrees well with systematics correlating the reduced hindrance with the isomer excitation energy relative
to the yrast line, a model that previous to this experiment had only been applied to the A~180 isomer group.

An analysis yielding an estimate for the K value of the s-band (K_s) in $^{164}$Er is performed. A negative $K_s$ is obtained for $^{164}$Er, necessitating a re-evaluation of the assumption of equality in the quadrupole moment of the s-band and ground state bands; a remarkable determination in a well-deformed nucleus. A larger quadrupole moment for the s-band in $^{164}$Er is calculated to reproduce positivity in the K value, and the theoretical interpretation of this increased deformation allows for the relative magnitude of s-band quadrupole moments and K values in other nuclides in the N=82→126 shell to be predicted.
I would like to thank my father who, in a world of beliefs and religions, never burdened me with an ideology beyond the desire to find my own path, stressing only the importance of a good education and a healthy lifestyle. To enter this world with such an advantage is something for which I will be eternally grateful.

I would like to thank my supervisor, Professor Phil Walker, whose excitement with the intricacies of nuclear structure has helped to fuel my own interest. Phil has always had time to answer my questions, whether they be silly questions, or questions that require a very detailed answer. This has been invaluable, as I have found discussions to be far superior to other forms of learning in extending the frontiers of my knowledge of physics.

I would like to thank my secondary supervisor, Dr. Zsolt Podolyak, whose wealth of expertise in many branches of physics supplemented that of my supervisor, opening other avenues of research to me.

I would like to thank the ANU research and support staff, in particular Professor George Dracoulis and Dr. Greg Lane, for their help in understanding the physics of the experiment this thesis is based upon, and for the work they put in to make the experiment happen.

I would like to thank the great many post-doctoral researchers and post-graduate students who have had time to answer my questions, Greg Farrelly, Steve Steer, Pete Mason, Nicky Patterson, Simon Brown, Nawras Al-Dahan, and Matt Reed to name some.

Belatedly, I would like to thank my MPhys supervisors at ORNL, Dr. Steve Pain and Dr. Kate Jones. Steve, who’s extensive knowledge of just about everything, allowed me to greatly improve my Master’s dissertation, and Kate, who made the transition to life in America much less scary than I imagined it would be, have taught me a great deal about research, making the PhD experience easier.

Finally, I would like to thank the Physics Department at the University of Surrey
who give young researchers the opportunity to travel the world to experimental
facilities and conferences, making personal travel in these countries less expensive
than it would otherwise have been. Many people of my age seek to travel the world,
partly out of curiosity and partly out of an innate urge to flee the nest. This degree
has provided the opportunity to satisfy that compulsion.
Publications
(excluding conference proceedings)

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T.P.D.Swan, P.M.Walker, Zs.Podolyak, M.W.Reed, G.D.Dracoulis, G.J.Lane, T.Kibedi, M.L.Smith

*Discovery of a nonyrast $K^\pi = 8^+$ isomer in $^{162}$Dy, and the influence of competing $K$-mixing mechanisms on its highly forbidden decay*

Co-author (latest to earliest)

Int.J.Mod.Phys. E20, 474 (2011)

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N.Winckler, M.Winkler, P.J.Woods, T.Yamaguchi

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A.Prochazka, W.Prokopowicz, B.Rubio, D.Rudolph, H.Schaffner, P.Strmen, I.Szarka,
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I. J. Cullen, A. Y. Deo, P. Detistov, Z. Dombradi, C. Domingo-Pardo, M. Doncel, F. Farinon,
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N. Kurz, S. Leoni, F. Molina, A. I. Morales, D. Montanari, A. Musumarra, R. Nicolini,
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S. Verma, S. J. Steer, P. Strmen, T. P. D. Swan, I. Szarka, J. J. Valiente-Dobon, P. M. Walker,
H. Weick, H. J. Wollersheim

Angular Momentum Population in Fragmentation Reactions

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J. A. Howard, M. S. Johnson, R. L. Kozub, J. F. Liang, R. J. Livesay, Z. Ma, B. H. Moazen,
P. D. O'Malley, C. D. Nesaraja, S. D. Pain, N. P. Patterson, S. V. Paulauskas, D. Shapira,
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Introduction

Nuclear physics is the exploration of a world beyond the limits of our senses, existing like an etheric fabric beneath the realm of the apparent. Yet unlike the mythical parallel realities of Lewis Carroll and James Barrie, our knowledge of the nucleus is borne from over a century of observation and experiment, and is an apposite testament to human curiosity and endeavour.

In 1844, Michael Faraday was the first to coin in literature the term “nucleus” to describe the centre of an atom [1]. Although, it wasn’t until 1909, when Ernest Rutherford performed his famous alpha-scattering experiment on gold foil, that we were truly able to understand the structure of the atom and the basic properties of the nucleus [2]. Thus, the science of nuclear physics was born, and the journey that began in Rutherford’s Manchester laboratory has since taken us to every recess of the universe, unearthing the secrets of stars, the distribution of elements in the cosmos and our own physical existence therein. The journey has given us a weapon for our own destruction, in the atomic bomb, but also the knowledge to perpetuate our innate drive for advancement to ends unimagined. This reckless human impulse to acquire knowledge, this evolutionary byproduct of the survival instinct, burns brightly like the ancestral stars in the minds of scientists, and perhaps answers the question of why one would choose to study the peculiarities of nuclear structure.

The advent of quantum theory in the 1920’s, and the discovery of the neutron
by Chadwick in 1932 [3], provided a platform for a model of nuclear substructure. In 1949, the ‘Shell model’ [4; 5] described the nucleus as a collection of protons and neutrons (nucleons) that are separated into discrete energy levels, or shells, within a potential well, with the number of particles in each shell determined by the Pauli exclusion principle. This model, with subsequent augmentations, still endures today, having been confirmed numerous times as a valid approximation of verity.

Much like a satellite in orbit around the Earth, nucleons orbiting the nuclear core possess angular momentum. When a nucleus becomes excited through a collision or nuclear reaction, the collective nucleus, or individual nucleons, will accommodate the absorbed energy together with additional angular momentum. When below the threshold for particle emission, the nucleus will lose excess energy by emitting γ rays. If the nuclear reaction is initiated in a controlled environment, one may observe these γ rays, which through their energy, intensity and decay rate, can tell us the post-reaction angular momentum of the nucleus, and how that momentum was accommodated.

Nucleons will occupy orbitals in pairs where their individual angular momenta cancel out. Where there are odd numbers of nucleons, or where a pair has been separated through a nuclear reaction, a ‘quasiparticle’ is the name given to the unpaired nucleon(s), resulting in a nuclear state with a highly localised accommodation of angular momentum. The angular momentum vector for a quasiparticle can be aligned with the rotation axis, or it can be aligned to varying degrees with a second axis, the symmetry axis, where the degree of alignment will be evident in the observed γ-ray spectrum. The K-quantum number is the factor expressing the degree of alignment with the symmetry axis, and like all quantum numbers, it is subject to conservation laws. For example, a de-excitation, or γ-ray decay, from a high-K state may involve a change in the angular momentum vector of 90° if the daughter state is characterised by collective rotation. In this example, the change in K would be equal to the K quantum number of the high-K state, requiring that the associated γ ray carry away all of the difference. This may be highly unlikely,
and yet such transitions do proceed, appearing with greater intensity than allowed. This reduced hindrance can result from a number of different nuclear effects. As the nuclear shape does not always have a unique symmetry axis, the conservation laws may be violated depending on the axial asymmetry of the nucleus. Furthermore, features of the nucleus, such as the Coriolis effect, and the mixing of two close lying quantum energy levels, may similarly dilute conservation laws to provide the observation of $\gamma$ rays that should normally be forbidden. The decay rate of these $\gamma$ rays embody how the forbiddenness hinders the decay.

These hindered, or metastable, quasiparticle states are called isomers, and they typically have a half-life exceeding one nanosecond. Isomers that are hindered specifically as a result of K-forbiddenness are called K-isomers, and these occur in regions of the isotopic chart where axial symmetry in the form of prolate or oblate deformation is prevalent. The best known isomer is the stable 75 keV, $K^0=9^-$ level in $^{180}$Ta [6]. A notable K isomer is the 2.4 MeV, $K^0=16^+$ level in $^{178}$Hf with a half-life of 31 years [7].

The decay rates of high angular momentum isomers in deformed nuclei can elucidate many nuclear structure effects, aiding in the creation of more precise models, and providing a greater understanding of the underlying physics. Indeed, the energies of quasiparticle states can typically be predicted to within a few hundred keV using models based upon the nuclear pairing force [8], although the unfactored presence of residual interactions prevents more unequivocal estimations. The prediction of isomer half-lives is much less precise, and despite the overview of forbidden decays offered here, understanding decay rates remains a considerable challenge [9], and lacks an appropriate theoretical framework. Nevertheless, such isomers, apart from their own structural interest, may be important more generally - in astrophysical environments, at the limits of stability, and controversially, for novel energy storage applications [10]. A larger body of data is needed, although the population of high-K isomeric states can be achieved through many reaction methods, including fusion evaporation, incomplete fusion, scattering, and fragmentation. For an
In order to extend our knowledge of isomer decay rates, an investigation of the $K^*=12^+$, 4-quasiparticle (4QP) isomer in $^{164}$Er was proposed, populating the isomer via the fusion evaporation method. This isomer, being one of only a few 4QP isomers known to exist outside of the $A=180$ isomer group, may offer special insight into the universality of competing models that have so far only been used to interpret the half-lives of isomers in the $A=180$ region [11]. The $K^*=12^+$, 4QP isomer in $^{164}$Er was discovered by Bark et al. in 1997 [12], although the experiment was not optimised for its detection, and only an estimation of the half-life was possible.

This work reports the results of an experiment at the Australian National University (ANU) Heavy Ion Accelerator facility that produced the $^{164}$Er isomer with higher yields than the Bark experiment, but crucially, with the use of a pulsed beam for a precise half-life measurement. As well as investigating the decay branches of the isomer, the aim was to better understand the nature of all $\gamma$-ray transitions in $^{164}$Er, and whether these can help explain the decay rate of the isomer through the K mixing mechanisms alluded to earlier.

A secondary aim was to investigate any other isomers within the $^{164}$Er nucleus, and within neighbouring nuclides, in the same manner. It was the fulfillment of this secondary aim that provided the greatest source of fascination. New isomers were discovered in $^{192}$Er (2QP), $^{161}$Er (3QP), $^{163}$Ho (3QP), $^{161}$Dy (1QP) and $^{162}$Dy (2QP) through the detection and interpretation of $\gamma$ rays. Associated rotational bands were observed for all but the $^{162}$Er isomer, and $\gamma$-ray intensity information was used to assign quasiparticle structures for all of the new isomers. The decay rate for the $^{162}$Dy isomer was compared with that for isomers in other $N=82\rightarrow126$ nuclei, leading to a statistical analysis of the different K mixing processes, and an estimation of their relative importance for highly K-forbidden, 2QP isomers.

With regards the primary aim, the half-life was precisely measured for the 4QP, $^{164}$Er isomer, and four new decay branches were observed, including high-$\Delta K$ branches to the ground state band. The decay rate of the $A=164$ isomer was
compared with that for 4QP and 5QP isomers in the A≈180 isomer group, leading to a better understanding of the universality of existing models.

Further findings include a precise half-life measurement for a known isomer in $^{163}$Ho with a previously unknown half-life, and the observation of a new $\gamma$-ray transition in $^{164}$Er at the band-crossing of the s-band with the ground state band. This transition completed the necessary band-crossing information required for the calculation of the $^{164}$Er, s-band K value and a subsequent estimation of the quadrupole moment.
Theory

SECTION 2.1

The nuclear shell model

Nuclear structure can be described in terms of a shell model [4; 5], which places protons and neutrons in over-laying shells from the core to the Fermi surface. The nucleons in outer shells are less bound, and are therefore more energetic than those in the inner shells. When a shell is full, a new shell at higher energy and larger radius is opened. The opening of a new shell corresponds to a jump in energy, and a filled shell represents a state of high stability. A filled shell contains a magic number of nucleons. This nuclear shell structure is reflected in nucleon separation energies in a similar way to electronic shell structure in atomic ionisation energies. Each shell consists of a group of discrete quantum states created by an attractive potential well, which is generated by the strong nuclear force. Each nucleon contributes to the potential, but as individual nucleon interactions would vastly complicate matters, they are approximated by a mean field potential.

The quantum states are first grouped by what is sometimes called the principal quantum number (N), an integer, which for any given orbital shape describes the size (and therefore the level energy) of the orbital. Secondly, each nucleon has an orbital angular momentum quantum number (l), and an intrinsic spin (s) of one half, which
2.2. THE DEFORMED SHELL MODEL

The deformed shell model accurately determines the properties of quantum states for nuclei close to the magic numbers, but its consistency expires for mid-shell nuclei, where deformation of the nuclear shape alters the energy level scheme. This is especially
evident in the shell model’s prediction of quadrupole moments, which deviate by more than an order of magnitude from the experimentally determined values. The deformed shell model, or Nilsson model \cite{14}, finds a more universal agreement with experiment. It applies a harmonic oscillator potential with additional factors relating to the ellipsoidal deformity of the nucleus and the empirical systematics of the energy levels for a particular shell. The potential takes the form:

\[
V_n = \frac{1}{2} m \left( w_x^2 x^2 + w_y^2 y^2 + w_z^2 z^2 \right) - Cl.s - D.I.I. \tag{2.2}
\]

The quantity \( m \) is the mass of the nucleon, \( x, y \) and \( z \) are the radial position of the nucleon, \( C \) and \( D \) are shell-specific experimentally determined constants, and \( w_x, w_y \) and \( w_z \) are oscillator frequencies related to the degree of deformation (\( \delta \)) for that axis:

\[
w_x^2 = w_y^2 = w_0^2 \left( 1 + \frac{2}{3} \delta \right) \quad \text{and} \quad w_z^2 = w_0^2 \left( 1 - \frac{4}{3} \delta \right). \tag{2.3}
\]

The quantity \( w_0 \), where \( \hbar w_0 = 41.A^{-1/3} \text{ MeV} \), corresponds to the oscillator frequency in a spherical nucleus (\( \delta = 0 \)). The degree of deformation can be equated with, \( \epsilon_2 \), the ellipsoidal deformation parameter:

\[
\epsilon_2 = \delta + \frac{1}{6} \delta^2 + \frac{5}{18} \delta^3, \quad \text{where} \quad \epsilon_2 = \frac{5}{4} \sqrt{\frac{\pi}{5}} \frac{\Delta R}{R_{\text{trc}}}, \tag{2.4}
\]

and where \( \Delta R \) is the difference between the semimajor and semiminor axes of the ellipse. The constant \( D \) in equation 2.2 determines the energy spacing of single particle levels, as a small \( D \) will reduce the spacing, making a squarer well. The constant \( C \) determines the strength of the spin-orbit interaction. Both constants are calculated for zero deformation within the chosen shell from the relevant experimental data.

The Nilsson model produces a relation between single-particle excitation energy and quadrupole deformation (\( \epsilon_2 \)) that may be plotted (see figures 2.1 and 2.2).
Each Nilsson orbit is defined by:

\[ \Omega^e [Nn_z \Lambda] . \]  
(2.5)

The parameter \( \Omega \) is the projection of the single-particle angular momentum onto the symmetry axis, \( \Lambda \) is the projection onto the symmetry axis of the single-
2.2. THE DEFORMED SHELL MODEL

Figure 2.2: Nilsson diagram for $82 \leq N \leq 126$ ($\varepsilon_4 = -\varepsilon_2^2/6$).

particle orbital angular momentum, $N$ is the principal quantum number, and $n_z$ is the number of oscillator nodes along the symmetry axis.

The degree of quadrupole deformation may be negative, which corresponds to an oblate shape rather than prolate (for positive deformation). An oblate shape results in a reduced energy for high-\(\Omega\) orbitals in comparison with prolate shapes. This is due to the greater core-interaction that an oblate shape offers to nucleons in this
configuration. This is demonstrated in the Nilsson diagrams in figures 2.1 and 2.2.

In the Nilsson model, only $\Omega$ and $\pi$ are good quantum numbers (are conserved), and orbits with the same values for these numbers repel each other.

\section*{Pairing and multi-quasiparticle excitations}

Pairing is an attractive force occurring between pairs of identical nucleons coupling to form $0^+$ states through having anti-parallel angular momentum vectors. The attractive pairing force lowers the energy of a nucleon, by an amount referred to as the pairing gap ($\Delta$). In even-even nuclei, the energy of the first two-quasiparticle state is therefore more than twice the pairing gap.

The pairing force also scatters pairs of nucleons into other orbitals, creating partial occupancies close to the Fermi surface, and mixing the wave functions of the $0^+$ pairs, enlarging the pair gap. If a single particle is excited into an unoccupied orbit, then this blocks that orbit from pair-scattering. This blocking is therefore an important factor in calculating the pair gap:

$$\Delta = G \Sigma_{i \neq i_j} U_i V_i.$$  \hfill (2.6)

The quantity $G$ is the pairing strength, which is greater for neutrons because protons experience a Coulomb repulsion. It is also inversely proportional to the mass, as larger masses contain orbitals with a larger average spatial distribution, resulting in a smaller interaction. The quantities $U_i$ and $V_i$ are the occupation values for holes and particles respectively. The parameter $i_j$ corresponds to a singly occupied orbital, and thus $i \neq i_j$ satisfies the blocking condition. The emptiness and occupation values take the form [15]:

$$U_i = \frac{1}{\sqrt{2}} \left[ 1 + \frac{(\epsilon_i - \lambda)}{\sqrt{(\epsilon_i - \lambda)^2 + \Delta^2}} \right]^{\frac{1}{2}}.$$ \hfill (2.7)
2.3. PAIRING AND MULTI-QUASIPARTICLE EXCITATIONS

\[ V_{\ell} = \frac{1}{\sqrt{2}} \left[ 1 - \frac{(\epsilon_{\ell} - \lambda)}{\sqrt{(\epsilon_{\ell} - \lambda)^2 + \Delta^2}} \right]^{\frac{1}{2}} . \]  

(2.8)

Once the pairing gap is determined, the quasiparticle excitation energy \( E_{\ell} \) can be calculated from the Nilsson single particle energy \( \epsilon_{\ell} \) and the Fermi energy \( \lambda \), defined as the energy when the occupation probability is 1/2 [15]:

\[ E_{\ell} = \sqrt{(\epsilon_{\ell} - \lambda)^2 + \Delta^2} \]  

(2.9)

For a multi-quasiparticle excitation, the total nuclear angular momentum projection on the symmetry axis \( K \) is defined as the sum of each nucleon's projection, \( K = \sum \Omega \).

Multi-quasiparticle excitations can be calculated using a blocking code based around the theory of superconductivity in electron pairs, developed by Bardeen, Cooper and Schrieffer (BCS) [16]. In these BCS calculations, only orbitals close to the Fermi surface are used, as bound orbitals far from the Fermi surface will have an occupation probability near to unity, and would require large energies to scatter to available orbits. The BCS blocking code for nuclei [8] essentially uses the same equations listed in this section. The code produces Nilsson single particle energies for the chosen nucleus, using these to calculate blocked quasiparticle excitation energies for protons and neutrons separately. The code then combines the two blocking calculations to form multi-quasiparticle states, that can consist of both neutrons and protons, providing their energy and \( K \)-projection. It is preferable with odd nuclei to adjust the Nilsson energies to fit any experimentally known one-quasiparticle excitation energies. For even numbers of neutrons or protons the code should be run for all neighbouring odd nuclei, with the fitted Nilsson energies being averaged to estimate their energy for an even number of nucleons.

The pairing strengths \( G_n \) and \( G_p \) must be chosen by the user, but can be estimated by fitting the energy of known multi-quasiparticle structures. Other inputs to the code are the proton number \( Z \), neutron number \( N \), quadrupole deformation...
tion parameter ($\varepsilon_2$), and the hexadecapole deformation parameter ($\varepsilon_4$). The code neglects to consider residual nucleon-nucleon interactions, which favour the anti-alignment of intrinsic spins for nucleons of the same type, and the alignment of the intrinsic spins for unlike nucleons, resulting in extra binding energy of the order of a few hundred keV.

**SECTION 2.4**

**Collective rotation**

There are two different modes of nuclear rotation: collective or non-collective. Non-collective rotation concerns the motion of individual nucleons separately from the core. Collective rotation concerns the motion of the nuclear core. Collective rotation dominates in well-deformed nuclei because deformed nuclei classically have large moments of inertia ($\mathcal{J}$) due to a non-spherical shape. Consequently, the rotational speed ($\omega$) is relatively slow for a given angular momentum ($I = \mathcal{J}\omega$). A slow rotational speed leads to the motion of nucleons remaining undisturbed in the deformed potential, favouring collective rotation. Non-collective rotation appears more generally in spherical nuclei, where a small number of nucleonic orbitals may contribute to a high angular momentum state through their orbital alignment.

The collective angular momentum ($R$) for an axially symmetric nucleus can be expressed in terms of the vector subtraction of the total angular momentum around the symmetry axis ($K$) from the total angular momentum of the nucleus ($I$):

$$R = \sqrt{I(I+1) - K^2}. \quad (2.10)$$

The rotational energy of a nucleus is:

$$E = \frac{\hbar}{2\mathcal{J}} [I(I+1) - K^2]. \quad (2.11)$$

A measure of nuclear collectivity is given by the ground state band excitation energy ratio of the $4^+$ level over the $2^+$ level, which, according to equation 2.11, should take a value of 3.33 in a rigid, rotating, even-even nucleus. Often well-deformed nuclei come close to this value, for example in $^{164}$Er: $E(4^+)/E(2^+) = 3.29$. 


For weakly deformed nuclei the ratio tends to be much smaller due to a higher $2^+$ level energy. In all cases, the nucleus does not behave like a rigid body because the $E(4^+)/E(2^+)$ ratios infer a moment of inertia that increases with energy or angular momentum. One can explain the phenomenon by thinking of the nucleus as possessing some degree of superfluidity [10]. The superfluidity is the result of nucleon pairing correlations, and the increase in the moment of inertia is the result of these pairing correlations being eroded by the Coriolis interaction. This decreases the superfluidity of the nucleus, forcing it to behave more like a rigid rotor. The Coriolis force must be large for weakly deformed nuclei with high collective angular momentum, and small in well-deformed nuclei, to produce the observed $E(4^+)/E(2^+)$ ratios.

**SECTION 2.5**

The Coriolis interaction and rotation alignment

The Coriolis interaction is an inertial force as it arises only in a rotating frame of reference. Tempestuous weather systems are a far cry from the femtoscopic world of the nucleus, but the principle of the Coriolis force remains the same in both. Nucleons moving relative to a rotating nucleus will be diverted in a direction perpendicular to their velocity.

Classically, the velocity and acceleration of an object in an inertial reference frame related to a rotational reference frame is:

$$v_{\text{in}} = v_{\text{rot}} + \omega \times r$$

$$a_{\text{in}} = \left( \frac{dv_{\text{in}}}{dt} \right)_{\text{rot}} + \omega \times v_{\text{in}}.$$

(2.12)

The quantity $\omega$ is the rotational velocity of the nucleus. The Coriolis force can be derived by substituting $v_{\text{in}}$ into the acceleration relation to give:

$$a_{\text{rot}} = a_{\text{in}} - 2\omega \times v_{\text{rot}} - \omega \times (\omega \times r).$$

(2.13)

The Coriolis acceleration is $2\omega \times v_{\text{rot}}$, whilst $\omega \times (\omega \times r)$ is the centrifugal acceleration.
2.5. THE CORIOLIS INTERACTION AND ROTATION ALIGNMENT

Thus, the Coriolis force increases for a high rate of collective rotation, and for large single particle velocity. In the quantum realm of the nucleus this equation translates as [17]:

\[
H_C = -\frac{\hbar^2}{2J} [(I \mp K)(I \pm K + 1)(j \mp \Omega)(j \pm \Omega + 1)]^{1/2}. \quad (2.14)
\]

A large total nuclear angular momentum \(I\) or large single particle angular momentum \(j\) will experience a large Coriolis force, whereas a large projection of total or single particle angular momentum onto the symmetry axis \(K\) or \(\Omega\) will experience a small Coriolis force.

It is also worth noting from equation 2.14 that a large moment of inertia will result in a small Coriolis force. This is evident in well-deformed nuclei, which classically have a small Coriolis force due to their slow rotational velocity for a given angular momentum. The large moment of inertia attributed with a slow rotation is what keeps the Coriolis force small for well-deformed nuclei, providing an explanation for the high \(E(4^+)/E(2^+)^\) ratios observed.

In particular, the Coriolis force is large with a large collective rotation, \(R = I - j\), because it becomes energetically favourable to reduce the rotation of the core by transferring that angular momentum to individual nucleons. This manifests as rotation alignment, where a pair of nucleons will switch from the typical antiparallel rotation around the core, to a parallel configuration in the direction of core rotation. Thus, the Coriolis force has a pair-breaking effect, which destroys the superfluidity of the nucleus described in the previous section, thereby increasing the moment of inertia. The increase in inertia reduces the level energy separation at the angular momentum at which alignment occurs (evident from equation 2.11) in a phenomenon known as back-bending. Specifically, the aligned and non-aligned configurations mix, introducing a high moment of inertia into the ground state configuration, causing a reduction in level energy separation. The alignment, \(L_a(I)\),
may be quantified using the equation [18]:

\[ I_x(I) = \sqrt{(I + 1/2)^2 - K^2} \tag{2.15} \]

although it is often plotted relative to the ground state band alignment, \( I_xg(I) \), as a function of the average angular frequency, \( \omega \), taking the form:

\[ i(\omega) = I_x(\omega) - I_xg(\omega) \tag{2.16} \]

where the ground state band alignment is defined as:

\[ I_xg(\omega) = \omega J_0 + \omega^3 J_1 \tag{2.17} \]

where \( J_0 \) and \( J_1 \) are Harris Parameters, with typical values for the \(^{164}\)Er region being \( J_0 = 37 \ h^2 \text{MeV}^{-1} \) and \( J_1 = 42 \ h^2 \text{MeV}^{-3} \) [12].

An estimate of equation 2.2 for the Nilsson model's single particle energy is [17]:

\[ H_P = A + B\beta \left[ \frac{3\Omega^2 - j(j + 1)}{4j(j + 1)} \right] \tag{2.18} \]

where \( H_P \) is the Hamiltonian (total energy) of a particle in the absence of rotation, and \( A \) and \( B \) are constants. This equation relates the deformation parameter of a nucleus (\( \beta \)) with the projection of a nucleon's angular momentum onto the symmetry axis (\( \Omega \)). For a given set of orbitals, with differing \( \Omega \), a large deformation will mean the orbitals are more spread out. Therefore, a large energy distribution of \( \Omega \) orbitals also coincides with a small Coriolis force.

**K isomers**

Nuclear isomers are metastable excited states in atomic nuclei. The fundamental characteristic of K isomers is more easily explained in spin-trap isomers, although K isomers may also be spin-traps.
Gamma-ray transition rates \( (T) \) between nuclear states are inhibited if the transition is low in energy or if it requires the emission of high multipolarity (\( \lambda \)) radiation [19]:

\[
T_{\gamma} \propto (E_{\gamma} R/\hbar c)^{2\lambda+1}. \tag{2.19}
\]

As transitions involving large changes in angular momentum have high multipolarity, the quasiparticle structures from which these transitions proceed are called spin-trap isomers if their transition rate exceeds a half-life of 1 ns. Thus, spin-trap isomers can be found in mid-shell areas of the Segrè chart where high-spin orbitals are available.

The \( K \) quantum number is defined as the projection of total nuclear angular momentum onto the symmetry axis of the nucleus, as shown in figure 2.3. The total single-particle angular momentum vector \( (j = l \pm s) \) is added to the collective angular momentum \( (R) \), which has its vector pointed along the rotation axis, to produce the total nuclear angular momentum \( (I) \). If a nucleon has a component of its angular momentum along the symmetry axis then this will give the nucleon a \( K \)-component \( (\Omega) \), which is the summed projection onto the symmetry axis of the nucleon’s angular momentum \( (A) \) and spin \( (\Sigma) \). The \( K \) quantum number for the nucleus is the sum of all the single particle values.

Thus, \( K \) isomers follow the same basic rule as spin-trap isomers, where transitions with a large change in angular momentum (along the symmetry axis in this case) are inhibited. Spherical nuclei have no unique symmetry axis, therefore, such large \( K \)-changing transitions require an axially symmetric deformed nucleus. On the Segrè chart, \( K \) isomers are found just above the mid-point between two closed shells, where axial deformation ensures \( K \) is well conserved (closed shells favour a spherical shape), and where high-\( \Omega \) orbitals are available.

However, unlike other quantum numbers, the \( K \) quantum number is not strictly conserved. Transitions where the \( K \) quantum number is violated by changes that are greater than the multipolarity \( (\lambda) \) of the decay radiation, are merely hindered rather
than completely forbidden. The degree of forbiddenness is defined as \( \nu = \Delta K - \lambda \). The degree to which forbidden K-transitions are hindered is stated relative to the Weisskopf estimate for the decay transition rate. The Weisskopf estimates are based on equation 2.19 [21]. The level of hindrance is then the experimental half-life divided by the Weisskopf estimate of the half-life:

\[
F_W = T_{1/2}^{\text{exp}} / T_{1/2}^W.
\] (2.20)

In an effort to show the effect of forbiddenness on the hindrance of the decay, the level of hindrance per degree of forbiddenness, or reduced hindrance is expressed as:

\[
f_\nu = F_W^{1/\nu}.
\] (2.21)

Much effort [22; 23] has been devoted to obtaining empirical values of \( f_\nu \), and it generally ranges between values of \( 2 < f_\nu < 100 \) [10]. Evidently, more factors than the degree of forbiddenness need to be accounted for in order to describe the hindered decays of K isomers.
2.6.1 **K mixing**

K mixing can often explain the range of $f_\nu$ values and the lack of conservation of the K quantum number. K mixing refers to the introduction of small admixtures of different K value states into the isomeric state, or its daughter state. This can allow apparently K-forbidden transitions to proceed without violating physical laws. There are many different ways in which K mixing can proceed.

As mentioned earlier, the Coriolis force can divert the motion of nucleons to form rotationally aligned states. Similarly, the Coriolis force may generate different K value states, preferentially in the daughter state of the isomeric decay. The daughter state will typically possess a large collective rotation due to fewer quasiparticles, and it will possess a (relatively) smaller $\Omega$ value. Therefore, the Coriolis force will preferentially introduce a high K-component into the daughter state, allowing for potentially forbidden transitions to conserve the K quantum number.

To see the effect of Coriolis mixing, it is worth re-visiting a previous argument. The Coriolis effect will increase the moment of inertia by inhibiting pairing. Therefore, by plotting reduced hindrance against the rate of increase in the (kinematic) moment of inertia ($J_1$), which is otherwise known as the dynamic moment of inertia, $J_2$, one can see how the Coriolis force affects the hindrance of K-forbidden decays. This relationship is shown in figure 2.4. It is clear that as the dynamic moment of inertia increases, signifying larger Coriolis effects, the reduced hindrance decreases, signifying greater K mixing.

Another method of K mixing can be attributed to level density effects. The higher the excitation of an isomer relative to a rigid rotor, the higher the density of states will be. Thus statistically, the level of K mixing will be higher due to greater local level density. This happens via low-K states, with the same spin and parity as the isomer, mixing with the isomeric state. As a result of this mixing, reduced hindrance becomes dependent on excitation energy relative to a rigid rotor, as shown in figure 2.5. A third method of K mixing arises if the K isomer decays through a number of intermediate asymmetric shapes, with no K-projection, in the
The Coriolis effect intensifies with decreasing neutron and proton number in the lower half of the $N=82\rightarrow126$ and $Z=50\rightarrow82$ shells due to the population of high-$j$, low-$\Omega$ orbitals. In contrast, greater axial asymmetry occurs with increasing neutron and proton number in the upper half of the shells, leading to an increase in $K$-mixing associated with $\gamma$-tunnelling, where $\gamma$ is the axial asymmetry parameter. The systematic effect of Coriolis and $\gamma$-induced $K$-mixing may surprisingly be characterised through a single variable, the product of the valence neutron and proton numbers, $N_pN_n$, at least for 2-quasiparticle (2QP) isomers decaying by E2 transitions [24]. The $N_pN_n$ product was introduced by Casten [25], and has proven to be useful in describing structural changes over extended regions [26]. The relationship between $f_\nu$ and $N_pN_n$ is shown in figure 2.6. The good correlation in figure 2.6 shows that $N_pN_n$ is a suitable representation of these two $K$ mixing mechanisms, but also that the two mechanisms are largely responsible for the reduced hindrance in these isomers.
2.7. ELECTRON CONVERSION

Other than the emission of $\gamma$ rays, an excited nucleus may decay through the competing process of electron conversion (often called internal conversion). In this scenario, the wavefunction of a low-lying electron penetrates the nucleus, allowing the nucleus to transfer the excitation energy of a decay transition to the electron by means of a 'virtual' photon. The electron will be emitted with a discrete energy, equivalent to the transition energy minus the binding energy of the electron. A higher-lying electron will occupy the vacant orbital, leading to the emission of an X-ray or Auger electron. The probability for a decay transition to proceed via electron conversion is defined as the conversion coefficient, $\alpha$, equivalent to:

$$\alpha = \frac{T_e}{T_\gamma}$$

(2.22)

where $T_e$ is the rate of conversion electrons and $T_\gamma$ is the rate of $\gamma$ ray emission. The conversion coefficient is observed to be high for a high atomic number ($Z$) due...
to an increased density of electrons. It is also high for low energy and/or high multipolarity transitions, as these will have longer \( \gamma \)-ray decay lifetimes (evident in equation 2.19), allowing for stronger competition from electron conversion. Indeed, for \( Z=68 \) (Erbium) electron conversion is the dominant decay path for \( E<160 \) keV (provided \( E > \) electron binding energy) and \( \lambda \geq 2 \) transitions. Furthermore, given that magnetic transitions are typically slower than electric transitions of the same multipolarity [29], conversion coefficients will compete better with, and be higher for, magnetic transitions.

As \( \alpha \) will vary significantly for different multipolarities [30], measurements of \( \alpha \) can be used to assign transition multipolarities by comparison with known multipole transitions, allowing for the spin and parity of parent levels to be determined. More generally, conversion coefficients are required in any calculation of the total intensity or half-life of a nuclear level, and are therefore required in the determination of reduced hindrances. For these purposes, theoretical predictions of \( \alpha \) can be acquired using the BrIcc code [31], which calculates conversion coefficients within a precision of \( \sim 1\% \) from experimentally measured values.

Electron intensities for conversion coefficient measurements are typically obtained using a Si(Li) particle detector within a superconducting solenoid. Electrons are measured simultaneously with \( \gamma \) rays to obtain \( \alpha \). A table of measured conversion coefficients is given in Ref. [32].

\[ \text{SECTION 2.8} \]

s and t bands

An s-band or t-band is a rotation-aligned, 2-quasiparticle band. It is defined as the first band to cross the ground state band (becoming yrast). The Coriolis interaction favours rotation aligned bands when the angular momentum (\( I \)) is sufficiently high, lowering them in energy relative to the ground state band for a given angular momentum. The residual interaction produces an s-band and a t-band which mix together to produce the observed crossing band. The s-band is formed when the
K-components (Ω) of the two nucleons are anti-parallel, resulting in $K = 1$. The t-band is deformation aligned, consisting of two nucleons with parallel Ω, resulting in a high K value. An example is given in figure 2.7 for $^{164}$Er where the observed s/t-band consists of two $i_{13/2}$ neutrons in $Ω = 5/2$ and $Ω = 7/2$ orbitals. The two bands mix together to produce the observed band with a K value that is dependent on the residual energy separation and the level of mixing between the two bands. If the t-band is yrast then the observed band will have a high K value, and this will be amplified if the level of mixing is low. The observed band is identified as an s-band or a t-band depending on which structure is dominant.

\[ K = Ω_x - Ω_y \quad \text{t-band} \]
\[ K = Ω_x + Ω_y \quad \text{s-band} \]

Figure 2.7: Example s and t-band structures in $^{164}$Er

A t-band is evident when a high-K contribution is indicated in the observed s-band. For example, an experiment to observe γ-ray transitions in $^{184}$Os by Shizuma et al [33], found that high-K bands were decaying to the s/t-band, suggesting a high-K, t-band structure, while strong mixing with the ground-state band suggested a low-K, s-band structure. Further work by Walker et al [34] helped to confirm this prediction with a $K = 8$ estimate by using the band-mixing analysis outlined in section 2.9.
K values for s-bands

It is difficult to determine the K value of s-bands as the bandhead is rarely well defined. It is often assumed that K=0 [35; 36], yet the Bohr-Mottelson collective model [37] involves the K quantum number in the reduced E2 transition probability, 
\[ B(E2, I \rightarrow I') = \frac{1}{16\pi} e^2 Q_0^2 (IK20 | I'K)^2, \]
allowing for consideration of the K variable. Typically, the residual interaction favours the formation of s-bands in the singlet configuration with \( K = 1 \), however deformation-alignment can result in a high K t-band as described in section 2.8. As we go up through the N=82–126 shell, the t-band structure may become energetically favourable at a particular threshold of K, dependent on the residual energy separation.

A way to measure K (thus determining the presence of a t-band) was devised by Walker [34]. The technique involves the measurement of relative E2 transition strengths at the band-crossing of the s-band and the ground state band (g-band). If s-band transition rates are enhanced relative to those in the ground-state band, the Walker method calculates a low K value for the s-band, assuming the quadrupole deformation \( (Q_0) \) of the two bands is equal. The Walker method found significant sensitivity to K values for even-even nuclides. However, the s-band rates may be enhanced beyond explanation through the K value alone, requiring consideration of other variables such as the quadrupole moment, although, for well-deformed nuclei \( (E_{41}^+/E_{21}^+ > 3) \), it is common to assume equality of the quadrupole moments.

The Walker method utilises a K-dependent two-band mixing analysis for the s-band and ground state band in order to replicate experimental transition rate ratios. The intensities of the 2 inband and 2 outband transitions at the band-crossing are required for the experimental transition rate ratios, which can be calculated from transition matrix elements for the quantum state [38]:
2.9. K VALUES FOR S-BANDS

\[ T_7^{EL} = \frac{8\pi (L + 1)e^2b^L}{L[(2L + 1)^2\hbar]^2} \left[ \frac{E_2}{hc} \right]^{2L+1} B(EL), \]  

(2.23)

where \( B(EL) \) is the electric transition matrix element for a given transition multipolarity, \( L \), \( T_7^{EL} \) is the photon transition probability, and \( b \) is \( 10^{-24} \) cm\(^3\). A \( B(E2) \) transition rate ratio involves rearranging the equation and cancelling the constants to produce:

\[ R[B(E2)] = \frac{B(E2)_{\text{out}}}{B(E2)_{\text{in}}} = \frac{I_{\text{out}}/E_{\text{out}}^5}{I_{\text{in}}/E_{\text{in}}^5}. \]  

(2.24)

The quantity, \( I \), is the relative intensity of the transition, which can be determined from the efficiency-corrected peak area on a gamma spectrum, and \( E \) is the energy of the transition in MeV. For the two mixing bands there are two \( R[B(E2)] \) ratios determined from four gamma-ray transitions.

The reduced \( E2 \) transition probability for the Bohr-Mottelson collective model (stated earlier in this section) provides a dependence on \( K \), allowing for a theoretical reproduction of the experimental \( B(E2) \) ratios by finding the correct combination of the \( s \)-band \( K \) value \( (K_s) \) and the mixing strength with the ground state band \( (V) \). For the ground state band, \( K = 0 \) is assumed. The theoretical \( B(E2) \) transition rate ratio is defined:

\[ R[B(E2)] = \frac{c\beta\delta + c\gamma}{c\beta\gamma - c\delta} \]  

(2.25)

where \( c \) is the ratio of the Clebsch-Gordon coefficients defined as:

\[ c = \left[ \frac{(I - K_1)(I - K_1 - 1)(I + K_1 - 1)}{I - K_2)(I - K_2 - 1)(I + K_2 - 1)} \right]^{\frac{1}{2}}, \]  

(2.26)

where \( K_1 \) is the \( K \) value of the state from which the transitions are proceeding, and \( K_2 \) is the \( K \) value of the state to which the out-of-band transition is decaying. Thus, \( c \) can take two values depending on the principal band chosen. Two \( R[B(E2)] \) ratios are therefore possible, allowing for a comparison with the experimental ratios. If one of the \( K \) values is unknown (the \( s \)-band \( K \) value), it can be adjusted until the theoretical and experimental ratios match.
The quantities, $\alpha$, $\beta$, $\gamma$, $\delta$, are wavefunction amplitudes that determine the contribution of each unperturbed state to the perturbed observed states, as described in the two band mixing analysis outlined by Casten [15]. The amplitudes are determined by calculating the unperturbed states from the measured observable states. For example, if there is strong mixing between the two $16^+$ levels and the two $14^+$ levels of the ground-state band and the $s$-band, then the two unperturbed $16^+$ levels may be calculated from the energies of the observed, perturbed levels $(E_{g16}, E_{s16})$ with the equation:

$$E_{16} = \frac{1}{2}(E_{g16} + E_{s16}) \pm \sqrt{(E_{g16} + E_{s16})^2 - 4V^2}. \quad (2.27)$$

The two unperturbed $14^+$ levels can be calculated from the observed levels $(E_{g14}, E_{s14})$ with the equation:

$$E_{14} = \frac{1}{2}(E_{g14} + E_{s14}) \pm \sqrt{(E_{g14} + E_{s14})^2 - 4V^2}. \quad (2.28)$$

The quantity $V$ is the mixing matrix element that determines the strength of mixing between the two bands. It can be no greater than half of the perturbed energy separation and is estimated as such. The estimations of $V$ and $K_s$ (either $K_1$ or $K_2$) are adjusted until the theoretical and experimental ratios match. The values of $\alpha$ and $\delta$ are the smaller of the two amplitudes at the $16^+$ and $14^+$ levels respectively, and are calculated using the equations:

$$\alpha = \frac{1}{\frac{R_{16}}{2} + \sqrt{\frac{R_{16}^2}{2} + R_{14}^2}} \quad (2.29)$$

$$R_{16} = \frac{\Delta E_{16}}{V}. \quad (2.30)$$

The quantity $\Delta E_{16}$ is the difference in energy of the calculated unperturbed $16^+$ levels.

$$\delta = \frac{1}{\frac{R_{14}}{2} + \sqrt{\frac{R_{14}^2}{2} + R_{16}^2}} \quad (2.31)$$
2.10. INTRINSIC G-FACTORS

\[ R_{14} = \frac{\Delta E_{14}}{V}. \]  

(2.32)

The quantity \( \Delta E_{14} \) is the difference in energy of the calculated unperturbed \( 14^+ \) levels. Finally \( \beta \) and \( \gamma \) are calculated using:

\[ \alpha^2 + \beta^2 = 1, \]  

(2.33)

\[ \gamma^2 + \delta^2 = 1. \]  

(2.34)

Some cases of the use of these equations are shown in Walker [34].

Large \( K \) values for the \( s/t \)-band can explain why high-\( K \) isomers decay into this band. A high-\( K \) \( t \)-band combined with a high mixing strength with the ground-band can explain why highly \( K \)-forbidden transitions to the ground state band can sometimes proceed with small levels of hindrance.

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**Intrinsic g-factors**

There are a number of different \( g \)-factors that act as proportionality constants relating the magnetic moment of a nuclear particle with its angular momentum or spin. The orbital angular momentum \( g \)-factor, \( g_i \), is 1 for protons and 0 for neutrons, reflecting the travel of a charged particle in an orbit. The spin \( g \)-factor, \( g_s \), is 5.586 for protons and -3.826 for neutrons [29], reflecting sub-nucleon structure.

The intrinsic \( g \)-factor, \( g_K \), depends on the accommodation of angular momentum in the nucleus. The projection of the spin and angular momentum, whether parallel or anti-parallel, or the degree to which they are aligned with the symmetry axis, will produce different intrinsic \( g \)-factors in nuclear states with the same total angular momentum. Thus, experimental measurement of the intrinsic \( g \)-factor for a rotational band can be useful, as one may compare it with the expected \( g_K \) for various orbital occupations, elucidating the correct quasiparticle structure for the measured rotational band.
Intrinsic g-factors may be measured experimentally from the $\Delta I = 1/\Delta I = 2$ branching intensities from levels within a rotational band that is built upon a quasi-particle excitation. In calculating the intrinsic g-factor, the branching intensity, $\lambda_b$, is first used to determine the multipole mixing ratio, $\delta$, where $\delta^2$ is the E2 intensity divided by the M1 intensity for the $\Delta I = 1$ transition ($T_1(E2)/T_1(M1)$). While the intensity of the $\Delta I = 2$ transition is assumed to be purely E2 ($T_2(E2) = T_2(E2)$), the intensity of the $\Delta I = 1$ transition includes both E2 and M1 components with an unknown multipole mixing ratio ($T_1 = T_1(E2) + T_1(M1)$). By rearranging the equation for the branching ratio, $\lambda_b$, in terms of $\delta$, we can derive the quadrupole admixture, $q$:

$$\lambda_b = \frac{T_2}{T_1} = \frac{T_2(E2)}{T_1(E2) + T_1(M1)} = \frac{T_2(E2)/T_1(M1)}{1 + \delta^2} = \frac{\delta^2}{1 + \delta^2} \frac{T_2(E2)}{T_1(E2)}$$

(2.35)

$$q = \frac{\delta^2}{1 + \delta^2} = \frac{T_1(E2)}{T_2(E2)\lambda_b}.$$  

(2.36)

The right side of equation 2.36 includes a ratio of E2 transition intensities that may be calculated through their conversion to reduced transition probabilities using equation 2.23. Using the B(E2) rotational model formula (see section 2.9) equation 2.36 becomes [39]:

$$q = \frac{\delta^2}{1 + \delta^2} = \frac{2K^2(2I - 1)}{(I + 1)(I - 1 + K)(I - 1 - K)} \left(\frac{E_1}{E_2}\right)^5 \lambda_b,$$

(2.37)

allowing $\delta$ to be determined. Here, $I$ is the angular momentum of the level from which the branching intensities are measured, $K$ is the K value of the band, and $E_1$ and $E_2$ are the energies (in MeV) of the $\Delta I = 1$ and $\Delta I = 2$ transitions respectively. The M1 reduced transition probability, $B(M1)$, while related to $\delta$ through equations 2.23 and 2.35, is also related to the intrinsic g-factor, $g_K$, in terms of its difference with the collective g-factor, $g_R$ [40]:
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\[ B(M1; I_f K \rightarrow I^I K) = \frac{3}{4\pi} e^2 | \langle I_1 1K0 | I_f K \rangle |^2 (g_K - g_R)^2 K^2, \quad (2.38) \]

where \( e \) is one unit of charge. Thus, once \( \delta \) has been determined, it may be equated with \( g_K \) using equation 2.39 [39] (see also [20]).

\[ | g_K - g_R | = \frac{0.93 Q_0 E_2}{\sqrt{I^2 - 10}}. \quad (2.39) \]

In equation 2.39, \( Q_0 \) is the intrinsic electric quadrupole moment (in e.b), given by the equation [29]:

\[ Q_0 = \frac{3}{\sqrt{5}} R_{ave} Z \beta_2 (1 + 0.16 \beta_2), \quad (2.40) \]

where \( Z \) is the atomic number, \( \beta_2 \) is the quadrupole deformation parameter, taken from [41] or calculated from the \( 2^+ \) excitation energy in the ground-state band, \( E(2^+) \), using Grodzins formula [42]:

\[ \beta_2 = \sqrt{\frac{1225 \text{ MeV}}{A^{7/3} E(2^+)}}, \quad (2.41) \]

Theoretically, the intrinsic \( g \)-factor for a quasiparticle structure, multiplied by the \( K \) value, is calculated from the sum along the symmetry axis of the spin (\( \Sigma \)) and orbital angular momentum (\( \Lambda \)) magnetic moments in units of nuclear magnetons [37]:

\[ g_K K = 0.6 \sum g_\Sigma \Sigma + \sum g_\Lambda \Lambda, \quad (2.42) \]

where \( \Sigma = 1/2 \), and its sign relative to \( \Lambda \) is dictated by \( \Sigma = \Omega - \Lambda \); quantities found in the Nilsson model description of an orbital. The experimental and theoretical methods are compared to find the best agreement between measurement and a candidate quasiparticle structure. In equating to the experimentally determined \[ | g_K - g_R | \), it is common to assume \( g_R = 0.3 \) [43].
Alternately, $g_R \sim Z/A$ can provide a good approximation of the collective g-factor, as this is proportional to the collective model magnetic moment corresponding to uniformly charged nuclear matter [44]. Deviations from this approximation of $g_R$ can be understood as due to either protons or neutrons contributing more to the motion of the nuclear core, altering the magnetic moment (and the proportionality constant $g_R$) up or down respectively. This can occur in rotational bands built upon quasiparticle excitations. As described in section 2.3 the quasiparticle occupation of nuclear orbitals will result in them being blocked from pair-scattering, reducing the pairing force. In section 2.4, a reduction in the pairing force is described as contributing to an increase in the moment of inertia, causing the nucleus to behave more like a rigid rotor. Therefore, if proton orbitals are blocked as a result of a quasiproton excitation, the nuclear moment of inertia will be generated to a greater extent by protons, leading to a large $g_R$. If neutron orbitals are blocked due to a quasineutron excitation, protons will contribute less to the moment of inertia, and $g_R$ will be small. Evidence for this correlation is provided in Ref.[45].
Experimental setup

SECTION 3.1

Reaction mechanism

The aim of this experiment is to observe excited $\gamma$-ray decaying states in $^{164}$Er and its neighbouring nuclides. In order to populate these nuclides at sufficiently high energy and angular momentum, a fusion, evaporation reaction mechanism was employed. This reaction, referred to as complete fusion (GF), involves the fusion of a projectile nucleus with a target nucleus to form an excited compound nucleus, which de-excites by the evaporation of neutrons until its energy is below the neutron separation energy. The residual nucleus remains sufficiently excited, leading to $\gamma$-ray emission.

3.1.1 Incomplete fusion reactions

At energies close to the Coulomb barrier, incomplete fusion will compete with complete fusion, and based on the specific reaction channel, will produce some nuclides with many times the yield of complete fusion. In an incomplete fusion reaction (ICF) the projectile breaks up in the Coulomb field of the target, with one of the fragments fusing with the target, and the other being emitted. Figure 3.1 shows the two reaction mechanisms. Typically, at projectile energies of less than 10 MeV/nucleon,
CF and ICF are the dominant reaction mechanisms [46].

<table>
<thead>
<tr>
<th>Projectile</th>
<th>Target</th>
<th>Compound Nucleus</th>
<th>Evaporation</th>
<th>Residual Nucleus</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^9\text{Be}$</td>
<td>$^{160}\text{Gd}$</td>
<td></td>
<td>$^{169}\text{Er}$</td>
<td>$^{164}\text{Er}$</td>
</tr>
<tr>
<td>$^5\text{He}$</td>
<td>$^{160}\text{Gd}$</td>
<td>$^{165}\text{Dy}$</td>
<td></td>
<td>$^{162}\text{Dy}$</td>
</tr>
</tbody>
</table>

Figure 3.1: Possible complete fusion (CF) and incomplete fusion reaction channels for a $^9\text{Be}$ projectile and a $^{160}\text{Gd}$ target.

The incomplete fusion reaction was recognised decades ago [48; 49], although its benefits have not been fully exploited. Where the heavier fragment from the break-up of the projectile fuses with the target, ICF reactions can be used to study relatively neutron-rich nuclides which cannot be populated using complete fusion reactions that employ stable beams. Furthermore, greater spin input to the residual nucleus may be achieved with an ICF reaction, as compared with a CF reaction with a projectile equivalent to the fusing fragment, increasing the probability of producing high-K isomeric states [47]. For example, in figure 3.2 the two reaction mechanisms are compared for the population of the band above a $K^\pi=15^-$ isomer [50] in $^{180}\text{Ta}$. The ICF reaction produces more of the high-K band, and to higher angular momentum. Similar results were found in the rotational band above the $K^\pi=16^+$ isomer [51] in $^{178}\text{Hf}$ where the $^{176}\text{Yb}(^9\text{Be},3\alpha n)$ ICF reaction produced higher spin input than the conventional $^{176}\text{Yb}(\alpha,2n)$ CF reaction.

The ICF reaction is not well understood, and nuclide production yield models
3.2. BEAM-TARGET COMBINATIONS

The possible beam-target combinations are based on the types of beam possible at the ANU accelerator facility, and the radioactive stability of the target material. The possible beams available for use are: $^7$Li, $^9$Be, $^{10,11}$B, $^{12,13}$C, $^{16,17,18}$O, $^{19}$F, $^{24,25,26}$Mg, $^{28,29,30}$Si, $^{31}$P, $^{32,33,34}$S, $^{35,37}$Cl, and $^{58}$Ni [54]. The beams with $Z>8$ cannot produce erbium isotopes as neutron rich as $^{164}$Er with a stable target. This leaves beams up to and including oxygen. The possible targets for combination with these beams depend on the $Z$ ($Z_{\text{Erbium}} - Z_{\text{Beam}}$), and stability. The stability of the targets was not available. However, ICF is often evident when substantial deviations from predicted CF yields are observed [52, 53].

**Figure 3.2:** Comparison of normalised spectra gated on a $16^- \rightarrow 15^-$ transition in $^{180}$Ta from the $^{176}$Yb($^{11}$B,α3n) incomplete fusion reaction (top) and from the $^{176}$Yb($^7$Li,3n) complete fusion reaction (bottom) [47]. The dots show known contaminants.
discovered from the NNDC website [55]. The possible combinations are:

\[
\begin{align*}
7^\text{Li} + ^{150}\text{Tb} &\rightarrow ^{164}\text{Er} + 2n, & 9^\text{Be} + ^{158}\text{Gd} &\rightarrow ^{164}\text{Er} + 3n, \\
9^\text{Be} + ^{160}\text{Gd} &\rightarrow ^{164}\text{Er} + 5n, & 12^\text{C} + ^{154}\text{Sm} &\rightarrow ^{164}\text{Er} + 2n, \\
13^\text{C} + ^{154}\text{Sm} &\rightarrow ^{164}\text{Er} + 3n, & 10^\text{O} + ^{150}\text{Nd} &\rightarrow ^{164}\text{Er} + 2n, \\
17^\text{O} + ^{150}\text{Nd} &\rightarrow ^{164}\text{Er} + 3n, & 18^\text{O} + ^{150}\text{Nd} &\rightarrow ^{164}\text{Er} + 4n.
\end{align*}
\]

The reactions with 2-neutron and 3-neutron products have too small a yield for

\(^{164}\text{Er}\) at beam energies above the Coulomb barrier. This leaves:

\[
\begin{align*}
9^\text{Be} + ^{160}\text{Gd} &\rightarrow ^{164}\text{Er} + 5n, \\
18^\text{O} + ^{150}\text{Nd} &\rightarrow ^{164}\text{Er} + 4n.
\end{align*}
\]

As the principal aim of the experiment is to produce and measure the \(^{12+}\) isomer in \(^{164}\text{Er}\), the angular momentum of the residual \(^{164}\text{Er}\) nucleus must be high enough to populate the isomer, and the production percentage of \(^{164}\text{Er}\) versus its contaminants must be high.

The statistical model PACE4 code [56] produced \(^{164}\text{Er}\) complete fusion yields, production percentages for nuclides in the region, and angular momentum population yields, as a function of beam energy for the two possible reactions. For an oxygen beam, PACE predictions are given in figure 3.3. It is clear that once the energy of the beam exceeds the Coulomb barrier the production percentage decreases. Conversely, the angular momentum of the residual nucleus increases, but once it reaches a high enough value to populate the isomer, the production percentage is below 60%. This is not ideal as there would be many impurities, but it would still yield valid, measurable results.

For the beryllium beam figures 3.4 and 3.5 give the cross sections, production percentages and most populated angular momenta as a function of beam energy. The cross section peaks at a beam energy of 57 MeV, coinciding with the energy for the highest production percentage of \(^{164}\text{Er}\) (\(>70\%\)). Furthermore, the angular momentum of the residual nucleus is high enough at this energy to populate the...
desired isomer. For these reasons a 58 MeV (57 MeV + beam energy loss in target) $^9$Be beam and a $^{160}$Gd target were used in the experiment.

Figure 3.3: PACE4 calculation for an oxygen beam.

Figure 3.4: PACE4 calculation for a beryllium beam ($^{164}$Er cross section only).
A highly enriched $^{160}$Gd target of thickness 4.10 mg/cm$^2$ was chosen for the experiment. It was turned 20° from an angle perpendicular to the beam axis giving an effective thickness of 4.36 mg/cm$^2$. A thin target will result in less energy loss (and less energy spread) of the beam on its passage through the target, meaning a more uniform selection and distribution of reaction products and angular momentum. A thick target will help ensure the reaction products stop within the target material, increasing the efficiency of detection from the CAESAR array surrounding the target. The chosen thickness is a compromise between these effects.

A recoiling erbium nucleus will have a kinetic energy of about 3.2 MeV due to conservation of momentum. The range of such a nucleus as calculated in LISE++ [57] is 0.5 mg/cm$^2$ in $^{160}$Gd. The energy loss of a 58 MeV $^9$Be beam particle in 4.36 mg/cm$^2$ of $^{160}$Gd is $\sim$ 2 MeV, resulting in the optimum reaction beam energy of 57 MeV at the centre of the target. Based on these calculations 8/9 of recoiling
3.4. BEAM CREATION AND ACCELERATION

Erbium nuclei will stop within the 4.36 mg/cm² target and the energy spread of the reacting beam particles will be ±1 MeV.

**SECTION 3.4**

**Beam creation and acceleration**

As tandem accelerators require negative ions for acceleration, the beryllium is molecularised, as a stable negative ion can be formed in no other way. The disadvantages of molecular beams are that, post-disassociation, the final atomic ion beam is reduced in energy and may be of poorer quality. These effects are minimised by binding the beryllium with the lightest possible atom, hydrogen [58]. Ammonia is used in a reaction to produce beryllium hydride, which is compressed into a hole in a metal cathode within the ion source. The heated ion source is injected with caesium vapour, which ionises easily (caesium is highly electronegative) causing the caesium atoms to collide with the beryllium hydride cathode. The beryllium hydride sputters from the cathode, picking up electrons removed from the caesium, forming the negatively charged $^9\text{BeH}^-$. The $^9\text{BeH}^-$ ions emerge from the ion source through electrostatic repulsion, are focussed using an Einzel Lens, and are magnetically steered through slits to collimate them into a narrow beam. An iris is used to control the intensity of the emerging beam, which is accelerated through a potential of 150 kV in the low energy area of figure 3.6. The low energy beam enters a 90° single focussing inflection magnet, which magnetically selects $^9\text{BeH}^-$ ions, as opposed to ions containing other beryllium isotopes. An electrostatic quadrupole then refocuses the beam for injection into the tandem accelerator.

The negative $^9\text{BeH}^-$ ions are attracted to the high voltage positive terminal of the ANU tandem pelletron accelerator [59], which is half-way down its 20 m chain length. Here they are stripped of electrons using a thin carbon stripper foil and a gas stripper. The stripper causes an effect called Coulomb explosion, where the electrons that bind the two atoms of the negative molecular ion are violently torn
away, causing the two atoms to experience atomisation and Coulomb repulsion. The now positive and atomised $^9$Be ions are accelerated away from the positive terminal to the base of the machine, acquiring an energy of 58 MeV. The tandem is operated at 13.5 MV to produce this energy, which is within its maximum operational voltage of 15.5 MV. A 90° double focussing analysis magnet selects ions of the correct charge state and energy. The beryllium charge state chosen was $4^+$ (fully stripped). The switching magnet then directs these ions into the CAESAR experimental area.


In figure 3.6 there is a beam buncher and a number of choppers which form the beam into focussed beam bursts to allow well-resolved gamma ray time measurements relative to an iterative beam-time event. The diagram in figure 3.7 helps to explain the effect on the beam of each.

![Diagram of beam structure and timing](image)

**Figure 3.7:** The effect of the buncher and choppers on the beam [60].

The low energy chopper removes 15/16 of the beam, leaving 107 ns sections of beam separated by a gap of 1605 ns. The buncher then applies a sinusoidal AC current to the beam sections that acts to accelerate ions at the rear of the section and decelerate ions at the front of the section, pushing them towards the centre of the section, and forming an intense concentrated beam burst. This AC current has an associated RF signal. A second chopper then removes the less intense tails of the beam section, and a third chopper, with a wider region for transmission, removes any remaining beam outside of the concentrated beam burst. The bunched and chopped beam bursts have a width of about 1 ns.

### SECTION 3.6

**HPGe gamma-ray detectors**

High purity germanium detectors are essentially semiconductor diodes. A semiconductor has a narrow gap between its valence and conduction bands, allowing the material's valence electrons to easily acquire enough energy to reach the conduction
3.6. HPGE GAMMA-RAY DETECTORS

band. A photon, incident on the semi-conductor material, may excite electrons into the conduction band. An electric field can then be used to sweep the electrons to a collecting electrode. The collected charge is amplified and converted to a voltage pulse of a size proportional to the energy of the photon. The energy difference between the valence band and the conduction band is called the band-gap, and a small band-gap is preferable as it means a small amount of energy is required to produce a charge carrier, resulting in large output signals and high energy resolution. Germanium is such a material, with a band-gap of 0.74 eV.

The germanium crystals in this experiment are n-type because they have a thin n-type dopant (boron) for the inner contact, and a lithium-diffused outer p-type contact. An electric field is achieved by applying a reverse biased voltage, causing the dopant and semiconductor materials to ionise, forming a depletion region. This removes any charge carriers, and electron-hole pairs created by an interacting photon will be swept up by the electric field.

Gamma rays are an extremely penetrating form of radiation, and require large depletion depths. This can only be achieved with extremely pure germanium, possessing impurities less than 1 part in $10^{12}$. Nevertheless, detection efficiency can drop substantially for high energy (penetrating) gamma rays. Large volumes of germanium are therefore favourable, and are made possible by using co-axial detectors. Rather than having a planar configuration that limits the volume due to the finite width of germanium crystals, a coaxial configuration has its electrodes along the length of the crystal. The coaxial germanium detector is a germanium cylinder with a smaller cylinder hollowed out from its core. The inner surface will always possess the electrode corresponding to the material's charge-type (n-type in this case), and the outer surface electrode will be fabricated from the opposite charge-type using a dopant [61]. This is because in this configuration the depletion region will grow inward under reverse bias, requiring a smaller voltage and producing a larger electric field in the outer regions of the detector (where most of the volume is). The crystal can be made to great lengths, allowing a large volume detector to be constructed.
The small band-gap of germanium requires the detector to be cooled to prevent noise from leakage current. Germanium detectors are placed in thermal contact with liquid nitrogen with the whole setup being housed within a vacuum chamber. One advantage of using HPGe detectors as opposed to Ge(Li) is that the detectors can be allowed to return to room-temperature when not in use.

Gamma-ray detectors are often Compton suppressed, meaning that Compton scattered events are removed through their detection and veto in a scintillation detector surrounding the gamma-ray detector. This scintillation detector is made from bismuth germanate (Bi$_4$Ge$_3$O$_{12}$) and is called a BGO shield. The BGO becomes excited through interaction with the Compton scattered gamma ray, causing the emission of light following its de-excitation. The light is detected in photo-multiplier tubes through electron emission and multiplication.

The Compton suppression is more efficient if gamma rays enter the side of the germanium, meaning it is favourable to turn the germanium detectors so their long edge is facing the array. The forward scattering Compton events are suppressed in this configuration meaning the Compton background at low energies is reduced. This configuration however takes up more space.

Charge collection will be poor in large volume co-axial detectors as it may take a long time for the charge to reach the electrode, resulting in a time delay and poor time resolution. Conversely, planar gamma-ray detectors have a finite size meaning a smaller volume. Thus, they are inefficient for high-energy gamma rays but more efficient at lower energies. These planar HPGe detectors are called LEPS detectors (Low Energy Photon Spectrometers). There is little need for Compton suppression on these detectors as the Compton cross section is smaller for lower energy gamma rays [62].
Gamma ray measurements were obtained using the CAESAR detector array [63]. The array consists of six co-axial, n-type, Compton suppressed, HPGe detectors mounted at angles of ±48°, ±97° and ±148° with respect to the beam axis, as shown in figure 3.8. The detectors are side-ways mounted to optimise Compton suppression. To better detect low energy gamma rays and X-rays, the CAESAR array includes two planar unsuppressed LEPS detectors at -90° and -135° out of plane. In addition to the CAESAR array, three larger, forward facing, co-axial, Compton suppressed HPGe gamma-ray detectors were used to increase the total efficiency of the array. They were placed out of plane at ±45° and +135°.

Figure 3.8: A schematic of the CAESAR array, taken from [64].
CAESAR electronics

A schematic of the electronics used in this experiment is shown in figure 3.9. Each detector contains a pre-amplifier which converts the energy-proportional charge pulse from the detector into a measurable voltage pulse. This energy signal is then sent to an Ortec 572 amplifier at point B, which shapes the pulse (shaping time 2μs) and further amplifies it with a gain of 20. The pulse, now in the volt range, is processed into a digital signal by ADCs and sent to the acquisition computer.

The Compton suppression of the planar gamma-ray detectors allows for the vetoing (at point G) of events where a coincidence occurs between the Compton suppression and the fast timing signal of the gamma-ray detectors. This removes events where the gamma ray has Compton scattered in the germanium and been detected in the BGO. The veto occurs in the PH758 Logic Unit. When not vetoed, the fast timing signal is sent through point A and stops the detector’s corresponding TDC (Time to Digital Converter) at point C. The TDCs were started in response to the beam’s RF signal. The fast timing signals are also passed to the coincidence electronics that determine if two or more signals over-lap within the chosen time window, and the TDCs are gated on this coincidence in order to only record γ−γ−t coincidences. The PH755 logic module ensures that only a valid coincidence sends a gate to the TDCs. In order to process γ−t events a TAC was used, which also starts in response to the beam’s RF signal at point H, and stops when receiving an un-vetoed fast-timing signal from the germanium detectors at point F. The TAC was replaced with a clock to record γ−t events over a longer period. At point D, single fast timing events can be collected solely, without coincidence conditions. The electronics following point D simulate the coincidence condition in order to allow the detection of single events.
3.8. CAESAR ELECTRONICS

The LEP detectors are unshielded and have only a TFA and CFD.

Figure 3.9: A schematic of the CAESAR array electronics, adapted from [65].
Data Analysis

The eleven detectors were energy and time gain-matched prior to data sorting. A total of $7.7 \times 10^8$ coincidence events were sorted into $4096 \times 4096$ channel matrices. On the $\gamma$-ray energy axis the CAESAR array was calibrated to 0.5 keV per channel for HPGe detectors, and 0.2 keV for LEPS detectors. The time axis was calibrated to 0.78125 ns per channel, but for the measurement of longer half-lives this was changed to 0.25 $\mu$s per channel (details in section 4.1). The energy and efficiency calibrations were made at the end of the experiment using $^{152}$Eu and $^{138}$Ba $\gamma$-ray emitting sources. Details of the efficiency calibration follow in section 4.3.

SECTION 4.1

Matrices

Table 4.1 gives a summary of the two-dimensional matrices created in the data sorting process. In the table, the second set of $\gamma$-time matrices was sorted for the purpose of measuring longer-lived isomers. In order to detect correlated events across isomeric states, two "early-delayed $\gamma-\gamma$" matrices (and their transpose) were produced from events with a time difference of 30-150 ns and 150-832 ns. Measurements in these matrices typically have low efficiency as the delayed components have widths of 120 and 682 ns, which may be narrow relative to the half-life of an isomer. No
unique analysis was made using the LEPS detectors, and the matrices produced from their recorded data are not mentioned here.

Table 4.1: Details of all of the matrices used in the analysis of data from this experiment.

<table>
<thead>
<tr>
<th>No.</th>
<th>Type (name)</th>
<th>Time range t</th>
<th>Coincidence condition</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>γ-time</td>
<td>0-1550 ns</td>
<td>γγ ± ≤150 ns</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>time-γ</td>
<td>0-1550 ns</td>
<td>γγ ± ≤150 ns</td>
<td>Transpose of matrix 1.</td>
</tr>
<tr>
<td>3</td>
<td>γ-time</td>
<td>0-150 μs</td>
<td>0.25 μs per channel.</td>
<td>In-beam range = 30 μs.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>In-beam is not sorted.</td>
</tr>
<tr>
<td></td>
<td>time-γ</td>
<td>0-150 μs</td>
<td></td>
<td>Transpose of matrix 3.</td>
</tr>
<tr>
<td>5</td>
<td>γ-γ</td>
<td>0-1550 ns</td>
<td>γγ ± ≤150 ns</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>out-of-beam γ-γ</td>
<td>30-300 ns</td>
<td>γγ ± ≤150 ns</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>out-of-beam γ-γ</td>
<td>300-1550 ns</td>
<td>γγ ± ≤150 ns</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>early-delayed γ-γ</td>
<td>0-1550 ns</td>
<td>γγ ± 30-150 ns</td>
<td>The first observed γ ray is labeled early, the second is labeled delayed.</td>
</tr>
<tr>
<td>9</td>
<td>delayed-early γ-γ</td>
<td>0-1550 ns</td>
<td>γγ ± 30-150 ns</td>
<td>Transpose of matrix 8.</td>
</tr>
<tr>
<td>10</td>
<td>early-delayed γ-γ</td>
<td>0-1550 ns</td>
<td>γγ ± 150-832 ns</td>
<td>The first observed γ ray is labeled early, the second is labeled delayed.</td>
</tr>
<tr>
<td>11</td>
<td>delayed early γ-γ</td>
<td>0-1550 ns</td>
<td>γγ ± 150-832 ns</td>
<td>Transpose of matrix 10.</td>
</tr>
</tbody>
</table>

† This gives the out-of-beam range on the time axis. Unless otherwise specified, the in-beam range consists of a 1 ns beam pulse.
Section 4.2

Measurement of peak areas

Radware [66] is a collection of programs that operate on the Linux operating system and are designed specifically for the analysis of gamma spectroscopy data. Within the radware package, the gf3 program is used to analyse γ-ray spectra from germanium detectors. It does this by fitting γ-ray peaks and outputting their energy and area. In gf3 there is the option to choose a peak-fitting function that specifies a Gaussian with a skewed Gaussian on the lower energy side, and a step function to increase the background on the lower energy side. As the CAESAR array has good charge collection and Compton suppression, the benefits of a skewed Gaussian and step function are negligible, thus, a purely Gaussian shape was chosen for the fitting equation. The gf3 program comes with a fitting command called autofit. This command will iteratively search a specified region of the spectrum for peaks, perform a least-squares fit, then search again for more peaks based on the difference from the fit. It will keep fitting and searching until no more peaks are found. It then takes the chosen fitting equation (a pure Gaussian) and applies it to the specified region of the γ-ray spectrum in the following way:

\[
y = \text{background} + \text{peak height} \times \exp \left[ -(x - c)^2 / \left(\frac{\text{FWHM}}{\sqrt{2 \ln 2}}\right)^2 \right]
\]  

(4.1)

where \( x \) is the channel number and \( c \) is the centroid of the peak in channels. The full width half maximum (FWHM) and the background are free parameters in autofit, although some constraints are utilised. The relative FWHM is fixed, meaning that for a fitted region, it increases proportionally with the energy according to the following equation:

\[
\text{FWHM} = w + \sqrt{9 + \frac{8E_\gamma}{1000}}
\]  

(4.2)
where \( w \) is a free parameter and a constant for the fit; typically \( w \sim 0.4 \). Such a relation with energy is physically true for \( \gamma \)-ray detectors, as high energy \( \gamma \) rays produce a large number of charge carriers within the detector material, resulting in large statistical deviations, and wide peaks. The background is given by the equation:

\[
\text{Background} = A + Bx + Cx^2
\]

(4.3)

where \( A \), \( B \) and \( C \) are constants and free parameters in the fit, and \( x \) is the channel number minus an offset. The end result is a fitted spectrum that looks like figure 4.1, reporting that auto-fit performed 33 iterations, assigned the background constants \( A \), \( B \) and \( C \), recognised that the fit parameters specified a pure gaussian equation with no step function (\( R=0 \) and \( \text{STEP}=0 \)), and used widths (FWHM) proportional with the energy according to equation 4.2.

The measurement of \( \gamma \)-ray intensities can provide important structural information for the nucleus. The relevant quantity in the measurement of \( \gamma \)-ray intensities is the area of the peak, which is included in the output of figure 4.1. However, this is not equivalent to the intensity, as \( \gamma \)-ray detector arrays have an energy-dependent efficiency that must be considered.

### Efficiency

After approximately 120 keV, gamma-ray detectors are typically less efficient, for reasons outlined in section 3.6. Therefore once a peak area has been measured, it is necessary to divide it by the energy efficiency of the CAESAR gamma-ray detector array in order to produce a meaningful relative intensity for the peak.

To determine the energy efficiency of the array, a \(^{152}\text{Eu}\) gamma emitting source was placed at its centre. A singles spectrum from the array was produced in gf3, as shown in figure 4.2 and the gamma-ray peak areas were fitted using the auto-fit
4.3. EFFICIENCY

33 iterations, Chisq/d.o.f. = 39.155
Background: $A = 655.2(11)$, $B = -0.1837(18)$, $C = -0.0002(0)$
Shape: $R = 0.0(0)$, Beta = 1.7488(0), Step = 0.0(0)

<table>
<thead>
<tr>
<th>position</th>
<th>width</th>
<th>height</th>
<th>area</th>
<th>centroid</th>
<th>energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>183.071(14)</td>
<td>3.648(4)</td>
<td>5246(42)</td>
<td>20375(159)</td>
<td>183.071(14)</td>
</tr>
<tr>
<td>2</td>
<td>370.393(3)</td>
<td>3.786(4)</td>
<td>1476(25)</td>
<td>5916(162)</td>
<td>370.393(3)</td>
</tr>
<tr>
<td>3</td>
<td>416.263(0)</td>
<td>3.739(4)</td>
<td>2177(62)</td>
<td>87967(308)</td>
<td>416.263(0)</td>
</tr>
<tr>
<td>4</td>
<td>435.09(3)</td>
<td>3.806(4)</td>
<td>1479(25)</td>
<td>5993(103)</td>
<td>435.09(3)</td>
</tr>
<tr>
<td>5</td>
<td>565.84(5)</td>
<td>3.885(4)</td>
<td>821(21)</td>
<td>3396(88)</td>
<td>565.84(5)</td>
</tr>
<tr>
<td>6</td>
<td>629.883(7)</td>
<td>3.823(4)</td>
<td>1849(74)</td>
<td>77212(289)</td>
<td>629.883(7)</td>
</tr>
<tr>
<td>7</td>
<td>816.22(11)</td>
<td>4.031(4)</td>
<td>774(37)</td>
<td>3320(159)</td>
<td>816.22(11)</td>
</tr>
<tr>
<td>8</td>
<td>820.476(10)</td>
<td>4.033(4)</td>
<td>1320(67)</td>
<td>57030(283)</td>
<td>820.476(10)</td>
</tr>
<tr>
<td>9</td>
<td>965.942(9)</td>
<td>4.127(4)</td>
<td>1104(55)</td>
<td>40530(233)</td>
<td>965.942(9)</td>
</tr>
<tr>
<td>10</td>
<td>1129.412(10)</td>
<td>4.205(4)</td>
<td>934(50)</td>
<td>41822(217)</td>
<td>1129.412(10)</td>
</tr>
<tr>
<td>11</td>
<td>1329.645(11)</td>
<td>4.265(4)</td>
<td>768(45)</td>
<td>35864(262)</td>
<td>1329.645(11)</td>
</tr>
<tr>
<td>12</td>
<td>1415.570(17)</td>
<td>4.360(4)</td>
<td>3993(33)</td>
<td>18534(151)</td>
<td>1415.570(17)</td>
</tr>
<tr>
<td>13</td>
<td>1493.86(3)</td>
<td>4.399(4)</td>
<td>1817(24)</td>
<td>8569(110)</td>
<td>1493.86(3)</td>
</tr>
</tbody>
</table>

Counts

Channel no.

Figure 4.1: Example spectrum showing the output of the autorfute routine.

The process was then repeated for a $^{133}$Ba source, as shown in figure 4.3, in order to observe more lower energy peaks. Only $^{133}$Ba $\gamma$ rays under 100 keV were measured. This is because the two spectra require normalisation, which introduces additional uncertainty. Below 100 keV this uncertainty was tolerated in exchange for extra data points. The region above 100 keV is overwhelmingly $^{152}$Eu, and the $^{133}$Ba data is not required.

The areas were divided by the known relative intensities of these gamma rays [67] to determine the relative (energy) efficiency of the array at the energy of the measured peaks, as shown in table 4.2.

With the efficiency of the array known at these particular energies, a plot was
4.3. EFFICIENCY

Figure 4.2: Spectrum for a $^{152}$Eu source. The $^{152}$Eu $\gamma$-ray energies are labeled in keV for the peaks that had their areas measured.

Figure 4.3: Spectrum for a $^{133}$Ba source. The $^{133}$Ba $\gamma$-ray energies are labeled in keV for the peaks that had their areas measured.
### Table 4.2: The process for producing relative efficiency by dividing the measured area by the known intensity (I).

<table>
<thead>
<tr>
<th>E(_{\gamma}) (keV)</th>
<th>Area</th>
<th>Error</th>
<th>I(_{\gamma}) [67]</th>
<th>Error</th>
<th>Efficiency</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{152})Eu</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40.00</td>
<td>1696889</td>
<td>1323</td>
<td>28255</td>
<td>958</td>
<td>60.06</td>
<td>2.04</td>
</tr>
<tr>
<td>45.68</td>
<td>611958</td>
<td>807</td>
<td>7088</td>
<td>239</td>
<td>86.34</td>
<td>2.91</td>
</tr>
<tr>
<td>121.78</td>
<td>2719346</td>
<td>1672</td>
<td>13620</td>
<td>160</td>
<td>199.66</td>
<td>2.35</td>
</tr>
<tr>
<td>244.69</td>
<td>544898</td>
<td>760</td>
<td>3590</td>
<td>60</td>
<td>151.78</td>
<td>2.55</td>
</tr>
<tr>
<td>344.28</td>
<td>1499986</td>
<td>1241</td>
<td>12750</td>
<td>90</td>
<td>117.65</td>
<td>0.84</td>
</tr>
<tr>
<td>411.12</td>
<td>103437</td>
<td>355</td>
<td>1070</td>
<td>10</td>
<td>96.67</td>
<td>0.96</td>
</tr>
<tr>
<td>443.98</td>
<td>137800</td>
<td>400</td>
<td>1480</td>
<td>20</td>
<td>93.11</td>
<td>1.29</td>
</tr>
<tr>
<td>778.90</td>
<td>404654</td>
<td>650</td>
<td>6190</td>
<td>80</td>
<td>65.37</td>
<td>0.85</td>
</tr>
<tr>
<td>867.39</td>
<td>123464</td>
<td>370</td>
<td>1990</td>
<td>40</td>
<td>62.04</td>
<td>1.26</td>
</tr>
<tr>
<td>964.13</td>
<td>397895</td>
<td>641</td>
<td>6920</td>
<td>90</td>
<td>57.5</td>
<td>0.75</td>
</tr>
<tr>
<td>1112.12</td>
<td>338492</td>
<td>590</td>
<td>6490</td>
<td>90</td>
<td>52.16</td>
<td>0.73</td>
</tr>
<tr>
<td>1408.01</td>
<td>439052</td>
<td>666</td>
<td>10000</td>
<td>30</td>
<td>43.91</td>
<td>0.15</td>
</tr>
<tr>
<td>(^{133})Ba</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30.91</td>
<td>191443</td>
<td>448</td>
<td>14032</td>
<td>369</td>
<td>13.64</td>
<td>0.36</td>
</tr>
<tr>
<td>35.22</td>
<td>103944</td>
<td>335</td>
<td>3379</td>
<td>89</td>
<td>30.76</td>
<td>0.82</td>
</tr>
<tr>
<td>53.16</td>
<td>38020</td>
<td>213</td>
<td>348</td>
<td>7</td>
<td>109.25</td>
<td>2.28</td>
</tr>
<tr>
<td>81.00</td>
<td>957315</td>
<td>991</td>
<td>5120</td>
<td>40</td>
<td>186.98</td>
<td>1.47</td>
</tr>
</tbody>
</table>

made of energy versus relative efficiency. The data points were fitted using the radware program ‘effit’, which applies the equation:

\[
\text{Efficiency} = EXP \left[ (A + Bx + Cx^2)^{(-G)} + (D + Ey + Fy^2)^{(-G)} \right]^{-1/G} \quad (4.4)
\]

where the letters \(A\) to \(G\) are constant parameters, \(x = \ln(E\gamma/100 \text{ keV})\) and \(y = \ln(E\gamma/1000 \text{ keV})\). The parameters \(E, F\) and \(G\) were adjusted manually to
provide the best fit possible in the region above 120 keV. This is because the fitting program placed too much importance on particular points, rather than the overall trend across all energies. Parameter values of $E = -0.88$, $F = 0.23$ and $G = 1.00$ were fixed before allowing effit to complete the fit, giving $A = 17.32$, $B = 11.52$, $C = 0.0$, and $D = 4.394$. The fit is shown in figure 4.4.

The relative efficiency fit equation provides an efficiency for all $\gamma$-ray energies and will be used in all intensity measurements in the form: `relative intensity = measured area / relative efficiency`. The uncertainty in the fit reaches a maximum of $\sim 3\%$, but is consistently at least $2\%$ across most of the energy range. Therefore an uncertainty in the relative efficiency of $3\%$ is adopted at all energies, and thus all measured intensities will have at least a $3\%$ uncertainty.
4.4. HALF-LIFE MEASUREMENT

SECTION 4.4

Half-life measurement

A number of $\gamma$-ray peaks appear in the out-of-beam matrices, inferring a half-life of longer than ~10 ns. To measure the half-life, the channel number of the peak of interest is noted in the out-of-beam matrix (as this is where longer-lived peaks are best resolved) before looking for it in the $\gamma$-time matrix. Figures 4.5 and 4.6 show the difference between the two matrices. A gate on a selected $\gamma$-ray peak in the $\gamma$-time matrix can then be used to project a time spectrum, as shown in figure 4.7. The time spectrum has a background subtraction consisting of the same number of channels as the $\gamma$-ray peak, as seen in figure 4.6.

![Figure 4.5: Raw (un-gated) $\gamma$ spectrum from the out-of-beam $\gamma-\gamma$ matrix in the 30–300 ns range, showing a well resolved peak at 555 keV between channels 1108 and 1112.](image1)

![Figure 4.6: Raw (un-gated) $\gamma$ spectrum from the $\gamma$-time matrix showing the same 555 keV peak from figure 4.5. Channels 1108-1112 are gated on with channels 1102-1106 subtracted as background to produce figure 4.7.](image2)

It becomes clear at this point whether the half-life can be measured, or whether it is contaminated with other delayed components. Clearly, figure 4.7 has a smooth exponential decay that must result from a clean gate. This is not always the case, and non-exponential curves can result from contamination and/or a weak intensity for the peak. In such circumstances, other techniques may be attempted to produce a measurable exponential, usually by reversing the process described here, gating...
Figure 4.7: Background subtracted time spectrum for the 555 keV peak from figures 4.5 and 4.6.

Instead on large slices of time in the $\gamma$-time matrix, projecting the $\gamma$ spectrum, and measuring the area of the $\gamma$-ray peak in each time slice (described in detail in the next chapter).

Once at this stage, however, the procedure is consistent. The $gf3$ spectrum, in the form of figure 4.7, is written to a data file and opened in the graphical analysis program, xmgrace [68]. Here the measurable region (where an exponential decay is clearly present) is fitted with the standard equation for an exponential decay:

$$y = A_0 \times \exp \left( \frac{-(x - a)}{A_1} \right)$$  (4.5)

where $x$ is the channel number, $a$ is the offset (the lowest channel in the region to be fitted), and $A_0$ and $A_1$ are free parameters of the fit. The resultant fitted spectrum is shown in figure 4.8.

In order to produce figure 4.8, the channel number had to be converted to nanoseconds at a rate of 1 channel = 0.78125 ns, with zero ns at 1460 channels. Superficially, the edges of the spectrum were removed, and the number of channels was divided by 20 (fewer bins) to produce fewer data points. The time required for the fit equation to divide by two is then defined as the half-life of the $\gamma$-ray peak.
In cases where one isomer decays into another, there may be two delayed components in the decay curve. This results in a non-exponential curve, but it can be fitted with the sum of two exponentials:

\[ y = A_0 \times \exp\left(-\frac{x-a}{A_1}\right) + A_2 \times \exp\left(-\frac{x-a}{A_3}\right) \] (4.6)

where \(A_2\) and \(A_3\) are the free parameters of the second delayed component. If one of the delayed components has a known half-life, then some of the parameters in the fit can be fixed, greatly improving the accuracy of the fit for the unknown delayed component.

**Section 4.5**

**Level scheme construction**

All \(\gamma\)-\(\gamma\) matrices may be used in level scheme construction through the identification of \(\gamma\)-ray coincidences. Looking back at the spectrum in figure 4.1, if we place a gate on the 208.1 keV transition in the \(\gamma\)-\(\gamma\) matrix, we produce the spectrum in figure 4.9(a), allowing us to confirm that the large peaks all belong to the most strongly populated (yrast) band from an Er isotope. Searching for these transitions on the
4.5. LEVEL SCHEME CONSTRUCTION

National Nuclear Data Center (NNDC) [55], shows that they belong to the ground state band of $^{164}$Er. Gating further up the band, on the 564.7 keV transition, we see structure from rotational bands that must be feeding into the ground state band (see figure 4.9(b)).

![Figure 4.9: (a) Coincidence spectrum for 208.1 keV γ rays. (b) Coincidence spectrum for 564.7 keV γ rays.](image)

By combining the γ-ray coincidence information with the correct combinations of γ-ray transition energies, we are able to create the level scheme shown in figure 4.10, which assigns all of the transitions labeled in figure 4.9 with a parent level energy and spin. Typically, intensity information is needed to assign the spin, but in this case the NNDC confirmed that all of these levels and transitions are already known. However, this method of γ-ray and subsequent level energy identification can be used to identify new regions of the level scheme as will be seen in the next chapter.

![Figure 4.10: Partial level scheme for $^{164}$Er showing the transitions identified in figure 4.9.](image)
Results

Using the techniques outlined in the previous chapter, γ-transition level schemes were constructed for 12 different nuclides. In all, 105 new transitions and 5 new isomers were discovered, consistent with a literature review of gamma spectroscopy experiments for the 12 nuclei, as well as the NNDC. The level schemes follow in order of ascending Z, then by N. Where new information for a particular nuclide has been obtained, this is summarised in the corresponding level scheme figure caption.

Figure 5.1: Level scheme for $^{155}$Gd.
Figure 5.2: Level scheme for $^{151}$Tb.

Figure 5.3: Level scheme for $^{162}$Dy.

Figure 5.4: Level scheme for $^{161}$Dy. The isomer is new. The 8 newly discovered transitions are shown with bold arrows. Intensities and identifying spectra for these transitions are in section 5.8.
Figure 5.5: Level scheme for $^{169}$Dy. The isomer is new. The 17 newly discovered transitions are shown with bold arrows. Intensities and spectra for the 341.4 and 685.7 keV transitions are shown in figures 5.6-5.7. All other new transitions are given in section 5.3.
Figure 5.6: Coincidence spectrum in the $\gamma-\gamma$ matrix for the 1018.5 keV transition in $^{163}$Dy, showing the newly discovered 341.4 keV transition (denoted by an asterisk). Contamination due to a 1018.2 keV transition in $^{163}$Er is also labeled.

Figure 5.7: Coincidence spectrum in the $\gamma-\gamma$ matrix for the 372.7 keV transition in $^{162}$Dy, showing the newly discovered 685.7 keV transition (denoted by an asterisk).

Table 5.1: Relative $\gamma$-ray branch intensities from the 2281.5 keV level (top) and the 4518.6 keV level (bottom) in $^{162}$Dy. The new transitions are denoted by an asterisk. The uncertainty in the $\gamma$-ray energies is typically 0.1 keV.

<table>
<thead>
<tr>
<th>$E_{\gamma}$ (keV)</th>
<th>$I_{\gamma}$</th>
<th>$I^p_{\gamma}$</th>
<th>$I^r_{\gamma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>341.4*</td>
<td>12 (2)</td>
<td>11(--9)</td>
<td></td>
</tr>
<tr>
<td>906.1</td>
<td>100 (3)</td>
<td>11(--10+)</td>
<td></td>
</tr>
<tr>
<td>685.7*</td>
<td>(100)</td>
<td>19(--18+)</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5.8: Level scheme for $^{162}$Dy.
Figure 5.9: Level scheme for $^{169}$Ho. The 2109.4 keV isomer is new. The 1506.4 keV isomer has a more precise half-life. The 7 newly discovered transitions are shown with bold arrows. The intensities and identifying spectra for these transitions are given in section 5.5.

Figure 5.10: Level scheme for $^{169}$Ho.

Figure 5.11: Level scheme for $^{168}$Er. The isomer is new.
Figure 5.12: Level scheme for $^{163}$Er. The 1 newly discovered transition is shown with a bold arrow. The intensity and spectrum for the 98.0 keV transition is shown in figure 5.13.
Although the $^{163}$Er isomer is known, the relative intensity of its decay branches are not well known. As the isomer is not discussed later in this work, and as the intensity information is required later to calculate the yield of the isomer, the measurement is presented here in figure 5.14 and table 5.3.

### Table 5.2: Relative γ-ray branch intensities from the 189.9 keV level in $^{163}$Er. The new transition is denoted by an asterisk.

<table>
<thead>
<tr>
<th>$E_\gamma$ (keV)</th>
<th>$I_\gamma$</th>
<th>$I^T_\gamma$</th>
<th>$I^F_\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>98.0*</td>
<td>5.6 (3)</td>
<td>9/2$^-$</td>
<td>7/2$^+$</td>
</tr>
<tr>
<td>105.2</td>
<td>18 (1)</td>
<td>9/2$^-$</td>
<td>7/2$^-$</td>
</tr>
<tr>
<td>189.9</td>
<td>100 (3)</td>
<td>9/2$^-$</td>
<td>5/2$^-$</td>
</tr>
</tbody>
</table>

### Table 5.3: Gamma-ray energy, relative γ-ray intensity and final angular momentum for branches from the $K^\pi=11/2^+$ isomer in $^{163}$Er.

<table>
<thead>
<tr>
<th>$E_\gamma$ (keV)</th>
<th>$I_\gamma$</th>
<th>$I^F_\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>323.7</td>
<td>100 (3)</td>
<td>9/2$^+$</td>
</tr>
<tr>
<td>255.0</td>
<td>92 (3)</td>
<td>9/2$^-$</td>
</tr>
<tr>
<td>244.6</td>
<td>97 (3)</td>
<td>11/2$^+$</td>
</tr>
<tr>
<td>196.9</td>
<td>78 (3)</td>
<td>13/2$^+$</td>
</tr>
<tr>
<td>123.8</td>
<td>20 (2)</td>
<td>11/2$^-$</td>
</tr>
</tbody>
</table>

Figure 5.13: Coincidence spectrum in the out-of-beam γ-γ matrix for the 255.0 keV transition in $^{163}$Er, showing the newly discovered 98.0 keV transition (denoted by an asterisk).

Figure 5.14: Gamma-ray spectrum generated from the early-delayed γ-γ matrix showing delayed coincidences with 171.3 or 193.7 keV early transitions in $^{163}$Er. Direct branches from the isomer are denoted by the hatched symbol, the newly discovered 98.0 keV transition is denoted by an asterisk. Contamination from $^{164}$Er is labeled.
Figure 5.15: Level scheme for $^{169}$Er. The 3378.0 keV isomer has a more precise half-life. The 40 newly discovered transitions are shown with bold arrows. Intensities and spectra for 18 of these transitions are shown in figures 5.16-5.24. All others are given in section 5.6.
Figure 5.16: Coincidence spectrum in the $\gamma$-$\gamma$ matrix for the 1150.2 keV transition in $^{164}$Er, showing the newly discovered 199.9, 327.4, and 344.8 keV transitions (denoted by an asterisk).

Figure 5.17: Coincidence spectrum in the $\gamma$-$\gamma$ matrix for the 1030.4 keV transition in $^{164}$Er, showing the newly discovered 353.9 and 366.9 keV transitions (denoted by an asterisk).

Figure 5.18: Coincidence spectrum in the $\gamma$-$\gamma$ matrix for the 296.9 keV transition in $^{164}$Er, showing the newly discovered 419.1 keV transition (denoted by an asterisk).

Figure 5.19: Coincidence spectrum in the $\gamma$-$\gamma$ matrix for the 792.1 keV transition in $^{164}$Er, showing the newly discovered 537.0 keV transition (denoted by an asterisk).

Table 5.4: Relative $\gamma$-ray branch intensities from levels in $^{164}$Er. Different levels are separated by a horizontal line. The new transitions are denoted by an asterisk. The uncertainty in the $\gamma$-ray energies is typically 0.1 keV.

<table>
<thead>
<tr>
<th>$E_{\gamma}$(keV)</th>
<th>$I_{\gamma}$</th>
<th>$E_{\m}(\text{keV})$</th>
<th>$I_{\m}$</th>
<th>$I_{\m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>172.3*</td>
<td>5.7 (7)</td>
<td>2875.5</td>
<td>14+</td>
<td>14+</td>
</tr>
<tr>
<td>355.6</td>
<td>82 (3)</td>
<td>2875.5</td>
<td>14+</td>
<td>12+</td>
</tr>
<tr>
<td>792.1</td>
<td>100 (3)</td>
<td>2875.5</td>
<td>14+</td>
<td>12+</td>
</tr>
<tr>
<td>118.8</td>
<td>31 (2)</td>
<td>1904.3</td>
<td>8-</td>
<td>7-</td>
</tr>
<tr>
<td>199.9*</td>
<td>5.0 (3)</td>
<td>1904.3</td>
<td>8-</td>
<td>7-</td>
</tr>
<tr>
<td>219.8</td>
<td>100 (6)</td>
<td>1904.3</td>
<td>8-</td>
<td>6-</td>
</tr>
<tr>
<td>419.1*</td>
<td>11.1 (4)</td>
<td>1904.3</td>
<td>8-</td>
<td>7+</td>
</tr>
<tr>
<td>289.2*</td>
<td>100 (3)</td>
<td>3641.9</td>
<td>14-</td>
<td>13-</td>
</tr>
<tr>
<td>562.3*</td>
<td>46 (9)</td>
<td>3641.9</td>
<td>14-</td>
<td>12-</td>
</tr>
<tr>
<td>303.0*</td>
<td>60 (6)</td>
<td>3945.0</td>
<td>15-</td>
<td>14-</td>
</tr>
<tr>
<td>592.3*</td>
<td>100 (3)</td>
<td>3945.0</td>
<td>15-</td>
<td>13-</td>
</tr>
<tr>
<td>277.5</td>
<td>34 (2)</td>
<td>2092.2</td>
<td>8-</td>
<td>6-</td>
</tr>
<tr>
<td>327.4*</td>
<td>6.0 (4)</td>
<td>2092.2</td>
<td>8-</td>
<td>7-</td>
</tr>
<tr>
<td>347.5</td>
<td>18 (2)</td>
<td>2092.2</td>
<td>8-</td>
<td>8+</td>
</tr>
<tr>
<td>546.8</td>
<td>100 (3)</td>
<td>2092.2</td>
<td>8-</td>
<td>7+</td>
</tr>
<tr>
<td>344.8*</td>
<td>&lt;3</td>
<td>2108.4</td>
<td>9-</td>
<td>7-</td>
</tr>
<tr>
<td>1084.2</td>
<td>100 (3)</td>
<td>2108.4</td>
<td>9-</td>
<td>8+</td>
</tr>
<tr>
<td>353.9*</td>
<td>5.5 (19)</td>
<td>2408.8</td>
<td>11-</td>
<td>9-</td>
</tr>
<tr>
<td>890.4</td>
<td>100 (3)</td>
<td>2408.8</td>
<td>11-</td>
<td>10+</td>
</tr>
<tr>
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<td>100 (3)</td>
<td>2422.1</td>
<td>10-</td>
<td>8-</td>
</tr>
<tr>
<td>366.9*</td>
<td>31 (1)</td>
<td>2422.1</td>
<td>10-</td>
<td>9-</td>
</tr>
<tr>
<td>443.9</td>
<td></td>
<td>2422.1</td>
<td>10-</td>
<td>9-</td>
</tr>
<tr>
<td>537.0*</td>
<td>16.8 (8)</td>
<td>3412.2</td>
<td>16+</td>
<td>14+</td>
</tr>
<tr>
<td>709.2</td>
<td>100 (3)</td>
<td>3412.2</td>
<td>16+</td>
<td>14+</td>
</tr>
<tr>
<td>549.0</td>
<td>100 (3)</td>
<td>2733.9</td>
<td>12+</td>
<td>10+</td>
</tr>
<tr>
<td>650.7*</td>
<td>33 (2)</td>
<td>2733.9</td>
<td>12+</td>
<td>12+</td>
</tr>
<tr>
<td>693.3*</td>
<td>100</td>
<td>5031.4</td>
<td>22-</td>
<td>20-</td>
</tr>
<tr>
<td>704.8*</td>
<td>100</td>
<td>5936.8</td>
<td>23+</td>
<td>21+</td>
</tr>
<tr>
<td>740.5*</td>
<td>100</td>
<td>6421.9</td>
<td>24-</td>
<td>22-</td>
</tr>
<tr>
<td>(767.5)*</td>
<td>100</td>
<td>6703.3</td>
<td>25+</td>
<td>23+</td>
</tr>
<tr>
<td>386.6</td>
<td>100 (3)</td>
<td>1745.3</td>
<td>8+</td>
<td>6+</td>
</tr>
<tr>
<td>730.5</td>
<td>81 (3)</td>
<td>1745.3</td>
<td>8+</td>
<td>8+</td>
</tr>
<tr>
<td>1130.9*</td>
<td>20 (1)</td>
<td>1745.3</td>
<td>8+</td>
<td>6+</td>
</tr>
</tbody>
</table>
Figure 5.20: Coincidence spectrum in the $\gamma$-$\gamma$ matrix for the 388.1 keV transition in $^{164}$Er, showing the newly discovered 172.3 keV transition (with *). Contamination from $^{163}$Er and $^{162}$Dy is also labeled.

Figure 5.21: Coincidence spectrum in the early-delayed $\gamma$-$\gamma$ matrix for the 139.8, 240.5 and 1370.6 keV transitions in $^{164}$Er, showing newly discovered transitions in the $K^\pi=7^-$ band (with *).

Figure 5.22: Coincidence spectrum in the $\gamma$-$\gamma$ matrix for the 533.8 keV doublet transition in $^{164}$Er, showing the newly discovered 650.7 and 1130.9 keV transitions (denoted by an asterisk).

Figure 5.23: Coincidence spectrum in the $\gamma$-$\gamma$ matrix for the 330.2, 379.5, 443.9, 515.6, 584.0 and 642.9 keV transitions in $^{164}$Er, showing the newly discovered 693.3 and 740.5 transitions (with *).

Figure 5.24: Coincidence spectrum in the $\gamma$-$\gamma$ matrix for the 432.0, 502.6, 499.6 and 571.9 keV transitions in $^{164}$Er, showing the newly discovered 704.6 and 767.5 keV transitions (with *).
Figure 5.25: Level scheme for $^{156}$Er. The 1823.0 keV isomer is new. The 32 newly discovered transitions are shown with bold arrows. Intensities and spectra for the 803.0 and 113.1 keV transitions are shown in figures 5.26-5.27. All others are given in section 5.4. It is not known which of the bands above the 1823.0 keV isomer is the isomer band, hence the floating position of the two bands.

Figure 5.26: Coincidence spectrum in the $\gamma-\gamma$ matrix for the 306.6, 384.0, 456.9, 583.2, and 637.0 keV ground band transitions in $^{156}$Er, showing the newly discovered 803.0 keV transition (with *).

Figure 5.27: Coincidence spectrum in the out-of-beam $\gamma-\gamma$ matrix for the 374.7 keV transition in $^{156}$Er, showing the newly discovered 113.1 keV transition (with *), and contamination from $^{162}$Dy.

Table 5.5: Relative $\gamma$-ray branch intensities from the 175.9 keV level (top) and the 5590.8 keV level (bottom) in $^{156}$Er. The new transitions are denoted by an asterisk.

<table>
<thead>
<tr>
<th>$E_\gamma$ (keV)</th>
<th>$I_\gamma$</th>
<th>$I^*_\gamma$</th>
<th>$I^*_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>98.6</td>
<td>12 (2)</td>
<td>9/2-</td>
<td>7/2-</td>
</tr>
<tr>
<td>113.1*</td>
<td>31 (2)</td>
<td>9/2-</td>
<td>7/2+</td>
</tr>
<tr>
<td>175.9</td>
<td>100 (5)</td>
<td>9/2-</td>
<td>5/2-</td>
</tr>
<tr>
<td>803.0*</td>
<td>(100)</td>
<td>53/2+</td>
<td>49/2+</td>
</tr>
</tbody>
</table>
For all isotopes, or residual nuclei, $X$, produced in the $^{160}\text{Gd}(^{8}\text{Be},Y)X$ reaction, the isotopic and isomeric yields have been measured from $\gamma-\gamma$ coincidence data. Details of this method will follow. In figure 5.28 the yield for each isotope, $X(\%)$, is given as a percentage of the total yield ($\sum X(\%) = 100$). Each square then contains a list of any isomers detected in the isotope, with the spin and parity of the isomer given in brackets, and the yield. This isomeric yield is included in the $X(\%)$ value shown for the isotope. Isomers constituted 5.8(2)% of the total yield.

![Figure 5.28: Yields for isotopes and isomers from the $^{160}\text{Gd}(^{8}\text{Be},Y)X$ reaction. The half-life of the ground states [55] is indicated by the colour of the square, where black squares are considered stable isotopes.](image-url)
The predictions given by PACE4 for this fusion evaporation reaction show isotopic yields quite different from what was measured (see figure 5.29). It is clear that PACE4 significantly under-estimates yields for incomplete fusion reactions, thereby over-estimating complete fusion yields. This is not surprising as PACE4 only considers complete fusion channels, and as explained in section 3.1, evidence for ICF is typically obtained from deviations of the measured yield from that of models such as PACE4. Accepting this shortfall, we can at least see consistency between the predicted yields of the three main erbium isotopes ($^{163,164,165}$Er) and what was measured, in terms of their ratio with one another.

<table>
<thead>
<tr>
<th>Z</th>
<th>N</th>
<th>A</th>
<th>events</th>
<th>percent</th>
<th>x-section (mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>68</td>
<td>96</td>
<td>166 Er</td>
<td>23</td>
<td>0.023%</td>
<td>0.314</td>
</tr>
<tr>
<td>67</td>
<td>99</td>
<td>166 Ho</td>
<td>1</td>
<td>0.001%</td>
<td>0.0136</td>
</tr>
<tr>
<td>68</td>
<td>97</td>
<td>165 Er</td>
<td>3712</td>
<td>3.71%</td>
<td>50.6</td>
</tr>
<tr>
<td>67</td>
<td>98</td>
<td>165 Ho</td>
<td>215</td>
<td>0.215%</td>
<td>2.93</td>
</tr>
<tr>
<td>66</td>
<td>99</td>
<td>165 Dy</td>
<td>1</td>
<td>0.001%</td>
<td>0.0136</td>
</tr>
<tr>
<td>68</td>
<td>96</td>
<td>164 Er</td>
<td>71490</td>
<td>71.5%</td>
<td>974</td>
</tr>
<tr>
<td>67</td>
<td>97</td>
<td>164 Ho</td>
<td>831</td>
<td>0.831%</td>
<td>11.3</td>
</tr>
<tr>
<td>68</td>
<td>95</td>
<td>163 Er</td>
<td>22374</td>
<td>22.4%</td>
<td>305</td>
</tr>
<tr>
<td>67</td>
<td>96</td>
<td>163 Ho</td>
<td>33</td>
<td>0.033%</td>
<td>0.45</td>
</tr>
<tr>
<td>66</td>
<td>97</td>
<td>163 Dy</td>
<td>3</td>
<td>0.003%</td>
<td>0.0409</td>
</tr>
<tr>
<td>68</td>
<td>94</td>
<td>162 Er</td>
<td>3</td>
<td>0.003%</td>
<td>0.0409</td>
</tr>
<tr>
<td>66</td>
<td>95</td>
<td>162 Dy</td>
<td>445</td>
<td>0.445%</td>
<td>6.07</td>
</tr>
<tr>
<td>65</td>
<td>97</td>
<td>162 Tb</td>
<td>3</td>
<td>0.003%</td>
<td>0.0409</td>
</tr>
<tr>
<td>66</td>
<td>95</td>
<td>161 Dy</td>
<td>850</td>
<td>0.85%</td>
<td>11.6</td>
</tr>
<tr>
<td>66</td>
<td>94</td>
<td>160 Dy</td>
<td>16</td>
<td>0.016%</td>
<td>0.218</td>
</tr>
<tr>
<td>TOTAL</td>
<td></td>
<td></td>
<td>100000</td>
<td>100%</td>
<td>1.36e+03</td>
</tr>
</tbody>
</table>

Figure 5.29: The isotopic yield predictions of PACE4 [56] for a 58 MeV beam with an energy loss in the target of 2 MeV.

### 5.1.1 Measuring the yields

All yields were measured in the $\gamma$-$\gamma$ coincidence data with a gate on a gamma-ray energy, showing a second gamma ray in coincidence that is exposed to most of the yield from that isotope. The peak area of this second gamma ray is measured and corrected for the efficiency and electron conversion coefficient of both the gated and measured peak energies to produce a relative yield. This process may be repeated several times with different gate/measurement combinations (especially in
odd nuclei) to produce a more precise, inclusive yield. The branching ratio for each measured $\gamma$ ray, as stated on the NNDC [55] or measured in this work, is also accounted for. If a gate/measurement combination is measured in an out-of-beam $\gamma$-$\gamma$ matrix, then a time factor is used to scale up the yield, e.g. a factor of 1550/1250 = 1.24 would be used when a yield in measured in the 300$\rightarrow$1550 ns matrix. This is an approximation as, to varying degrees depending on the half-life, more than 24% of the intensity will be lost in the 0$\rightarrow$300 ns proceeding the matrix.

For $^{159}$Gd, the isotopic yield was measured with the 173.5/75.4 keV, 4$^+$ $\rightarrow$ 2$^+/2^+$ $\rightarrow$ 0$^+$, gate/measurement. For $^{161}$Tb, the isotopic yield was measured with the 360.9/56.5 keV gate/measurement, compensating for its measurement in the 300$\rightarrow$1550 ns matrix, the branching ratio of the 417.4 keV level, and the population of the level in the beta decay from $^{161}$Gd (83.5%). For $^{160}$Dy, the isotopic yield was measured with the 197.0/86.9 keV, 4$^+$ $\rightarrow$ 2$^+/2^+$ $\rightarrow$ 0$^+$, gate/measurement.

For $^{161}$Dy, the isotopic yield was measured with the summed 296.9/118.8, 256.6/157.5, 167.5/100.4, 223.2/140.4, 231.1/74.4 keV gates/measurements, compensating for the branching ratio of each measured peak, as stated on the NNDC. For the $^{161}$Dy [11/2$^-$] isomer, the isomeric yield was measured with the summed 157.5/284.3, 175.6/284.3, 98.3/284.3 keV gates/measurements, compensating for their measurement in the 300$\rightarrow$1550 ns matrix, and for the branching ratio of the isomer as measured in this work.

For $^{162}$Dy, the isotopic yield was measured with the 185.1/80.8 keV, 4$^+$ $\rightarrow$ 2$^+/2^+$ $\rightarrow$ 0$^+$, gate/measurement. For the $^{162}$Dy [8$^+$] isomer, the isomeric yield was measured with the summed 185.1/504.3 keV gate/measurement, compensating for its measurement in the 300$\rightarrow$1550 ns matrix, and for the branching ratio of the isomer as measured in this work.

For $^{163}$Dy, the isotopic yield was measured with the summed 287.3/114.1, 248/167.5, 300.0/212.1, 238.3/160.8 keV gates/measurements, compensating for the branching ratio of each measured peak as stated on the NNDC. Due to additional $^{163}$Dy yield from the beta decay of $^{163}$Tb, the summed 494.4/390.0, 494.4/316.6 keV
gates/measurements were added, compensating for their measurement in the 300→1550 ns matrix, the branching ratio of the 884.5 keV level, and the population of the level in the beta decay from $^{163}$Tb (34.8%).

For $^{163}$Ho, the isotopic yield was measured with the summed 144.6/122.4, 309.8/122.4, 353.2/144.6 keV gates/measurements, compensating for the branching ratio of each measured peak as stated on the NNDC. For the $^{163}$Ho $[23/2]$ isomer, the isomeric yield was measured with the 140.2/340.2 keV gate/measurement, compensating for its measurement in the 300→1550 ns matrix, and for the branching ratio of the isomer and the 1769.2 keV level as measured in this work. For the $^{163}$Ho $[17/2^+]$ isomer, the isomeric yield was measured with the 155.2/973.8 keV gate/measurement, compensating for its measurement in the 30→300 ns and 300→1550 ns matrices, for the branching ratio of the isomer as measured in this work, for the branching ratio of the 532.6 keV level as stated on the NNDC, and finally by subtracting the yield of the $^{163}$Ho $[23/2]$ isomer.

For $^{164}$Ho, the isotopic yield was measured with the summed 135.2/115.2, 289.6/115.2, 327.9/135.2 keV gates/measurements, compensating for the branching ratios of the measured peaks. For $^{162}$Er, the isotopic yield was measured with the 227.6/102.1 keV, $4^+ \rightarrow 2^+/2^+ \rightarrow 0^+$, gate/measurement. For the $^{162}$Er $[7^-]$ isomer, the isomeric yield was measured with the 227.6/1359.6 keV gate/measurement, compensating for its measurement in the 30→300 ns and 300→1550 ns matrices, and for the branching ratio of the isomer as measured in this work.

For $^{163}$Er, the isotopic yield was measured with the summed 213.1/108.1, 217.5/126.9, 276.8/189.9, 320.4/236.2, 344.6/246.6, 295.9/396.0 keV gates/measurements, compensating for the branching ratio of each measured peak as stated on the NNDC. For the $^{163}$Er $[11/2^-]$ isomer, the isomeric yield was measured with the 255.0/189.9 keV gate/measurement, compensating for its measurement in the 300→1550 ns matrix, and for the branching ratio of the isomer and the 189.9 keV level as measured in this work.

For $^{164}$Er, the isotopic yield was measured with the 208.1/91.6 keV, $4^+ \rightarrow$
2^+ / 2^+ \rightarrow 0^+ \text{, gate/measurement. For the } ^{164}\text{Er} [12^+] \text{ isomer, the isomeric yield was measured with the 208.1/555.0 keV gate/measurement, compensating for its measurement in the 30→300 ns and 300→1550 ns matrices, and for the branching ratio of the isomer as measured in this work. For the } ^{164}\text{Er} [7^-] \text{ isomer, the isomeric yield was measured with the 208.1/139.5 keV gate/measurement, compensating for its measurement in the 30→300 ns and 300→1550 ns matrices, for the branching ratio of the isomer as measured in this work, and finally by subtracting the yield of the } ^{164}\text{Er} [12^+] \text{ isomer.}

For } ^{165}\text{Er}, \text{ the isotopic yield was measured with the summed 306.6/225.4, 306.0/205.0, 375.8/299.9, 336.9/259.6 keV gates/measurements, compensating for the branching ratio of each measured peak as stated on the NNDC. For the } ^{165}\text{Er} [19/2] \text{ isomer, the isomeric yield was measured with the 155.6/317.0 keV gate/measurement, compensating for its measurement in the 300→1550 ns matrix, and for the branching ratio of the isomer and the 1506.0 keV level as measured in this work. For the } ^{165}\text{Er} [11/2^-] \text{ isomer, the isomeric yield was measured with the 104.6/383.2, 175.9/374.7, 140.3/312.1, 77.3/473.3 keV gates/measurements as the branching ratios of the isomer on the NNDC were not precise enough, compensating for their measurement in the 300→1550 ns matrix, for the branching ratio of the gated transitions as stated on the NNDC, and finally by subtracting the 317.0 keV branch yield from the } ^{165}\text{Er} [19/2] \text{ isomer.}

SECTION 5.2

Isomers

Although the aim of the experiment was to observe isomers in } ^{164}\text{Er}, \text{ new isomers were discovered in } ^{161}\text{Dy, } ^{163}\text{Dy, } ^{165}\text{Ho, } ^{166}\text{Er and } ^{165}\text{Er. The following sections prove their existence, show half-life measurements, and use decay branch intensities to infer spin-parity assignments. Previously unknown half-lives for isomers in } ^{164}\text{Er and } ^{165}\text{Ho were measured. Unless stated otherwise, coincidence spectra are not background subtracted due to a reduced background in out-of-beam matrices.}
5.3. ISOMER IN $^{162}$DY

SECTION 5.3

Isomer in $^{162}$Dy

In the out-of-beam $\gamma-\gamma$ matrix, a gate on the $4^+ \rightarrow 2^+$ (185 keV) ground-state band transition yielded clear evidence for an isomer decaying by 504 keV and 380 keV $\gamma$-transitions within $^{162}$Dy (see figure 5.30). Other less intense decay routes were subsequently determined, for example, figure 5.31 shows two direct transitions to the ground-state band. In the early-delayed $\gamma-\gamma$ matrix, an in-beam requirement was used to identify transitions within the isomer band (see figure 5.32). This measurement had low efficiency as the delayed component had a width of 682 ns, compared with the 8.3(3) $\mu$s half-life of the isomer. The half-life was measured in the $\gamma$-time matrix with a background subtracted gate on 504 keV $\gamma$ rays, as shown in figure 5.33. Other time spectra gave consistent values.

Eleven branches directly from the isomer were identified. The relative intensities, determined in the out-of-beam $\gamma-\gamma$ matrix, are shown in table 5.6. The $\gamma$-ray energies of these branches places the isomeric state at 2188.1(3) keV. A partial level scheme with measured $\gamma$-ray transitions that are related to the isomer is shown in figure

Figure 5.30: Coincidence spectrum for the 185 keV transition in the out-of-beam $\gamma-\gamma$ matrix. The strongest direct decays from the isomer are denoted by an asterisk.
5.3. ISOMER IN $^{162}$DY

Figure 5.31: Summed coincidence spectra for the 185 keV and 283 keV transitions in the out-of-beam $\gamma - \gamma$ matrix, showing delayed transitions to the ground-state band.

Figure 5.32: Summed coincidence spectra for the delayed 504 keV and 1135 keV transitions, showing transitions which precede the isomer (denoted by an asterisk). Contaminant transitions from random coincidences in $^{162}$Dy are labelled with their energy, and those from $^{164}$Er are labelled Er.

5.3.4. The 504 keV and 380 keV transitions have been previously reported by Fields et al. [69] and Aprahamian et al. [70] although both works tentatively assigned the parent level as the (9$^-$) member of a $K^\pi = (6^-$) band not detected in the present work. As the level is now shown to be isomeric, the earlier assignments need to be
5.3. ISOMER IN $^{162}$DY

Figure 5.33: Time spectrum for 504 keV $\gamma$ rays. The fitted half-life is 8.3(3) $\mu$s.

Table 5.6: Relative intensity, final angular momentum, multipolarity, total conversion coefficient, hindrance factor, forbiddenness and reduced hindrance for the direct branches from the $^{162}$Dy isomer, based on a $K^\pi$=8$^+$ assignment. Uncertainty in the $\gamma$-ray energies is typically 0.1 keV for $E_\gamma < 1$ MeV and 0.2 keV above.

<table>
<thead>
<tr>
<th>$E_\gamma$ (keV)</th>
<th>$I_\gamma$</th>
<th>$I_\gamma (\sigma M)$</th>
<th>$\alpha^T$ [31]</th>
<th>$F_W$</th>
<th>$\nu$</th>
<th>$l_\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>146.4</td>
<td>1.6 (2)</td>
<td>$8^+$ (M1)</td>
<td>0.810</td>
<td>1.4x10$^8$</td>
<td>3</td>
<td>616(23)</td>
</tr>
<tr>
<td>238.6</td>
<td>1.5 (3)</td>
<td>$0^-$ (E1)</td>
<td>0.004</td>
<td>5.4x10$^6$</td>
<td>5</td>
<td>140(6)</td>
</tr>
<tr>
<td>248.0</td>
<td>0.4 (1)</td>
<td>$9^-$ (E1)</td>
<td>0.028</td>
<td>2.6x10$^{11}$</td>
<td>2</td>
<td>5.1(7)x10$^9$</td>
</tr>
<tr>
<td>300.3</td>
<td>4.0 (3)</td>
<td>$7^+$ (M1)</td>
<td>0.113</td>
<td>4.7x10$^6$</td>
<td>3</td>
<td>780(22)</td>
</tr>
<tr>
<td>341.8</td>
<td>0.9 (2)</td>
<td>$8^-$ (E1)</td>
<td>0.013</td>
<td>3.0x10$^{11}$</td>
<td>5</td>
<td>197(6)</td>
</tr>
<tr>
<td>380.2</td>
<td>65 (3)</td>
<td>$8^-$ (E1)</td>
<td>0.010</td>
<td>5.7x10$^0$</td>
<td>2</td>
<td>7.6(3)x10$^4$</td>
</tr>
<tr>
<td>435.4</td>
<td>1.3 (2)</td>
<td>$6^+$ (E2)</td>
<td>0.022</td>
<td>1.7x10$^6$</td>
<td>2</td>
<td>1312(116)</td>
</tr>
<tr>
<td>504.3</td>
<td>100 (5)</td>
<td>$7^-$ (E1)</td>
<td>0.005</td>
<td>8.7x10$^6$</td>
<td>2</td>
<td>33(3)x10$^8$</td>
</tr>
<tr>
<td>560.3</td>
<td>6.2 (4)</td>
<td>$7^-$ (E1)</td>
<td>0.004</td>
<td>1.8x10$^{11}$</td>
<td>5</td>
<td>179(1)</td>
</tr>
<tr>
<td>1266.5</td>
<td>0.6 (2)</td>
<td>$8^+$ (M1)</td>
<td>0.003</td>
<td>2.4x10$^{11}$</td>
<td>7</td>
<td>42.2(28)</td>
</tr>
<tr>
<td>1630.2</td>
<td>1.0 (3)</td>
<td>$6^+$ (E2)</td>
<td>0.001</td>
<td>1.7x10$^0$</td>
<td>6</td>
<td>18.5(14)</td>
</tr>
</tbody>
</table>

re-evaluated. In the present work, all of the other direct isomeric branches, as well as the transitions above the isomer, are newly observed. The other parts of the level scheme are well established [69; 70; 71].

Transition rates for the assigned decays of the isomer limit its spin and parity. Table 5.6 shows the experimental intensities for these decays. Transitions to the 6$^+$
levels are too strong to result from a spin 9 state, and the absence of transitions to either of the 6\(^{-}\) states rules out the possibility of a spin 7 state. Finally, an 8\(^{+}\) assignment is favoured over 8\(^{-}\), as the latter would require M2 transitions to the 6\(^{+}\) levels, which are not likely to compete with E1 and M1 decays. For example, isomers with an 8\(^{-}\) configuration in \(^{134}\)Nd [72] and \(^{180}\)Hf [73] decay with E1/M2 reduced intensity ratios of \(B_{\gamma}(E1/M2)=1564\) MeV\(^2\) and 462 MeV\(^2\) respectively, while for an 8\(^{-}\) isomer in \(^{162}\)Dy, \(B_{\gamma}(E1/M2)=3\) MeV\(^2\), making such an assignment unlikely (here, \(B_{\gamma}(E1/M2)=I_{\gamma}(E1)E_{\gamma}(M2)/I_{\gamma}(M2)E_{\gamma}(E1)\), with energies in MeV).

Therefore, with the usual assumption that the K value is equal to the band-head spin, we propose a \(K^{\pi}=8^{+}\) assignment for the \(^{162}\)Dy isomer.

Table 5.6 shows that the E1 transition rate to the 9\(^{-}\) level in the \(K^{\pi}=2^{-}\) band is greater than to the 9\(^{-}\) level in the \(K^{\pi}=5^{-}\) band. The levels are 20 keV apart, at 1940 keV and 1960 keV (see figure 5.34). As the transition rates are, as expected, higher to the 8\(^{-}\) and 7\(^{-}\) states in the \(K^{\pi}=5^{-}\) band, the rates to the 9\(^{-}\) levels are surprising. A simple two-band-mixing analysis [69] at the band-crossing (7\(\hbar \to 9\hbar\)), aiming to
replicate the experimental transition rates from the isomer to the 7− and 9− levels, produced a mixing strength of $V \sim 9$ keV. The analysis required a higher K for the daughter level of the more intense transition, necessitating an interchange of the 9− levels. On the basis of our analysis, the 1960 keV level is predominantly K=5, and the 1940 keV level is predominantly K=2. We therefore recommend re-arranging the levels between the two bands to coincide with their dominant configuration.

After applying this change and re-plotting the alignment from figure 6.2, the deviations from the $K^*=5^−$ fit at low angular frequency are reduced. Similarly, the hindrance factors ($F_w$) in Table 5.6 favour interchanging the 9− levels, as the 248.0 keV transition is more hindered whilst being less forbidden than the 228.6 keV transition.

Another feature seen in Table 5.6 and figure 5.34 is that the hindrances of the transitions to the $K^*=4^+$ band are unusually large. This perhaps necessitates a re-examination of the structure of that band, especially since the band-head, and hence the K value, is not well defined. However, in general the detailed understanding of $f_\nu$ values is poor when $\nu$ is small, and the contributions of competing K-mixing mechanisms are more apparent when considering large-$\nu$ decays to ground-state bands. This aspect will be discussed in more detail in the next chapter.

Isomers in $^{165}$Er

In the out-of-beam $\gamma-\gamma$ matrix, a gate on the $13/2^- \rightarrow 11/2^- (155.6$ keV) transition within the known $^{165}$Er isomer band ($T_{1/2} = 0.25\mu s$ [74]) yielded clear evidence for a second isomer, at higher energy, decaying by a 317.0 keV $\gamma$-transition into this band (see figure 5.35). In figure 5.36 a gate on the 317.0 keV $\gamma$-transition confirmed that the new isomer decays through an unknown band into the known isomer band. Other direct branches from the new isomer were detected at 1144.6 and 1050.6 keV (see figures 5.37 and 5.38). In figure 5.39 gates on the delayed 317, 800 and 624 keV peaks were summed within the “early-delayed $\gamma-\gamma$” matrix, indicating early
transitions from two separate bands feeding into the isomer.

Figure 5.35: Coincidence spectrum for the 155.6 keV transition in the out-of-beam $\gamma - \gamma$ matrix. Transitions from the newly discovered $^{165}$Er isomer are denoted by an asterisk. Known $^{165}$Er transitions are labeled with their energy. Random coincidences from $^{164}$Er and $^{162}$Dy are labeled Er and Dy respectively.

Figure 5.36: Coincidence spectrum for the 317.0 keV transition in the out-of-beam $\gamma - \gamma$ matrix. Transitions from the newly discovered $^{165}$Er isomer are denoted by an asterisk. Known $^{165}$Er transitions are labeled with their energy. Random coincidences from $^{164}$Er, $^{162}$Dy, and $^{161}$Tb (following beta decay from $^{161}$Gd) are labeled Er, Dy and $\beta^-$ respectively.

Figure 5.37: Coincidence spectrum for the 306.0 keV transition in the out-of-beam $\gamma - \gamma$ matrix. Decay from the $^{165}$Er isomer is denoted by an asterisk. Random coincidence in $^{162}$Dy is labeled Dy.

Figure 5.38: Background subtracted coincidence spectrum for the 259.6, 336.9, and 400.0 keV transitions in the out-of-beam $\gamma - \gamma$ matrix. Decay from the $^{165}$Er isomer is denoted by an asterisk.

As the 317 keV transition was highly contaminated by delayed transitions with similar energies and differing half-lives (from isomers in $^{162}$Dy, $^{164}$Er, and from beta-delayed transitions in $^{161}$Tb, $^{163}$Dy), the half-life of the isomer was measured within the $\gamma$-time matrix with a background subtracted gate on the 799.8 keV transition. A measurement of $T_{1/2} = 0.37(4)$ $\mu$s was determined from the fit in figure 5.40.
The transitions detected in the figures 5.35-5.39 were placed into a partial level scheme based on their energies and coincidences (see figure 5.41). These energies place the new isomer at 1823.0(3) keV. The early transitions in figure 5.39 could not be detected in coincidence measurements, however $\Delta I=2$ transitions allowed for sufficient verification of $\Delta I=1$ placement into the two bands that were constructed.

Table 5.7 contains the relative intensities for the newly discovered delayed transitions. Table 5.8 contains the relative intensities for the newly discovered early transitions.
Figure 5.41: Partial level scheme for $^{165}$Er showing the decay of the new isomer ($T_{1/2} = 0.37(4)$ μs). Energies are in keV. Spin and parity assignments for X and Y are tentative.

Table 5.7: Gamma-ray energy, relative γ-ray intensity, initial level energy, final angular momentum, and half-life for new delayed transitions in $^{165}$Er.

<table>
<thead>
<tr>
<th>$E_{γ}$ (keV)</th>
<th>$I_γ$</th>
<th>$E_i$</th>
<th>$I_f$</th>
<th>$T_{1/2}$</th>
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</thead>
<tbody>
<tr>
<td>317.0</td>
<td>100 (8)</td>
<td>1823.0</td>
<td>17/2$^-$</td>
<td>0.37(4)μs</td>
</tr>
<tr>
<td>1050.6</td>
<td>5 (1)</td>
<td>1823.0</td>
<td>17/2$^-$</td>
<td>0.37(4)μs</td>
</tr>
<tr>
<td>1144.6</td>
<td>17 (2)</td>
<td>1823.0</td>
<td>19/2$^+$</td>
<td>0.37(4)μs</td>
</tr>
<tr>
<td>188.2</td>
<td>13 (1)</td>
<td>1506.0</td>
<td>15/2$^-$</td>
<td></td>
</tr>
<tr>
<td>623.6</td>
<td>24 (2)</td>
<td>1506.0</td>
<td>15/2$^-$</td>
<td></td>
</tr>
<tr>
<td>799.8</td>
<td>62 (4)</td>
<td>1506.0</td>
<td>13/2$^-$</td>
<td></td>
</tr>
<tr>
<td>767.2</td>
<td>18 (2)</td>
<td>1317.8</td>
<td>11/2$^-$</td>
<td></td>
</tr>
</tbody>
</table>

transitions. The relative intensities for the branches out of the isomer are of little use in making a structural assignment. The strong 317 keV transition decays into an unknown level, while the two weaker branches have similar intensities and large
uncertainties, although perhaps the weaker of the two favours a higher multipole.

Table 5.8: Gamma-ray energy, relative intensity, initial level energy, initial angular momentum and final angular momentum for the two bands found to be feeding the new isomer in $^{165}$Er. Top: band X (see figure 5.41). Bottom: band Y (see figure 5.41). Asterisk denotes $\Delta l=2$ transitions.

<table>
<thead>
<tr>
<th>$E_\gamma$ (keV)</th>
<th>$I_\gamma$</th>
<th>$E_i$</th>
<th>$I_i^\pi$</th>
<th>$I_f^\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>101.8</td>
<td>40 (3)</td>
<td>1924.8</td>
<td>21/2$^-$</td>
<td>19/2$^-$</td>
</tr>
<tr>
<td>140.2</td>
<td>83 (8)</td>
<td>2065.0</td>
<td>23/2$^-$</td>
<td>21/2$^-$</td>
</tr>
<tr>
<td>168.8</td>
<td>100 (7)</td>
<td>2233.8</td>
<td>25/2$^-$</td>
<td>23/2$^-$</td>
</tr>
<tr>
<td>192.5</td>
<td>75 (6)</td>
<td>2426.3</td>
<td>27/2$^-$</td>
<td>25/2$^-$</td>
</tr>
<tr>
<td>213.2</td>
<td>61 (7)</td>
<td>2639.5</td>
<td>29/2$^-$</td>
<td>27/2$^-$</td>
</tr>
<tr>
<td>232.8</td>
<td>51 (5)</td>
<td>2872.3</td>
<td>31/2$^-$</td>
<td>29/2$^-$</td>
</tr>
<tr>
<td>249.3</td>
<td>45 (6)</td>
<td>3121.6</td>
<td>33/2$^-$</td>
<td>31/2$^-$</td>
</tr>
<tr>
<td>266.3</td>
<td>27 (6)</td>
<td>3387.9</td>
<td>35/2$^-$</td>
<td>33/2$^-$</td>
</tr>
<tr>
<td>(242.0)$^*$</td>
<td>19 (7)</td>
<td>2065.0</td>
<td>23/2$^-$</td>
<td>19/2$^-$</td>
</tr>
<tr>
<td>(309.0)$^*$</td>
<td>24 (8)</td>
<td>2233.8</td>
<td>25/2$^-$</td>
<td>21/2$^-$</td>
</tr>
<tr>
<td>361.3$^*$</td>
<td>26 (4)</td>
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<td>27/2$^-$</td>
<td>23/2$^-$</td>
</tr>
<tr>
<td>405.7$^*$</td>
<td>30 (10)</td>
<td>2639.5</td>
<td>29/2$^-$</td>
<td>25/2$^-$</td>
</tr>
<tr>
<td>446.0$^*$</td>
<td>25 (3)</td>
<td>2872.3</td>
<td>31/2$^-$</td>
<td>27/2$^-$</td>
</tr>
<tr>
<td>482.1$^*$</td>
<td>40 (7)</td>
<td>3121.6</td>
<td>33/2$^-$</td>
<td>29/2$^-$</td>
</tr>
<tr>
<td>515.6$^*$</td>
<td>34 (6)</td>
<td>3387.9</td>
<td>35/2$^-$</td>
<td>31/2$^-$</td>
</tr>
<tr>
<td>188.7</td>
<td>95 (6)</td>
<td>2011.7</td>
<td>21/2$^+$</td>
<td>19/2$^+$</td>
</tr>
<tr>
<td>205.1</td>
<td>66 (6)</td>
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<td>23/2$^+$</td>
<td>21/2$^+$</td>
</tr>
<tr>
<td>221.2</td>
<td>70 (10)</td>
<td>2438.0</td>
<td>25/2$^+$</td>
<td>23/2$^+$</td>
</tr>
<tr>
<td>237.0</td>
<td>46 (7)</td>
<td>2675.0</td>
<td>27/2$^+$</td>
<td>25/2$^+$</td>
</tr>
<tr>
<td>251.3</td>
<td>40 (6)</td>
<td>2926.3</td>
<td>29/2$^+$</td>
<td>27/2$^+$</td>
</tr>
<tr>
<td>265.4</td>
<td>40 (6)</td>
<td>3191.7</td>
<td>31/2$^+$</td>
<td>29/2$^+$</td>
</tr>
<tr>
<td>(488.3)$^*$</td>
<td>25 (6)</td>
<td>2926.3</td>
<td>29/2$^+$</td>
<td>25/2$^+$</td>
</tr>
<tr>
<td>(516.7)$^*$</td>
<td>32 (6)</td>
<td>3191.7</td>
<td>31/2$^+$</td>
<td>27/2$^+$</td>
</tr>
</tbody>
</table>
An isomer has been previously observed in $^{163}$Ho at 1506.4 keV, decaying by a 973.1 keV transition to the ground state band with $T_{1/2} \geq 15$ ns [75]. The half-life measurement was limited by the loss of residual nuclei exiting the detection system. The same transition was observed in this work with a slightly higher energy of 973.8 keV. A second much weaker branch from the same isomer was discovered at 581.1 keV (see figure 5.44).

In the out-of-beam $\gamma$-$\gamma$ matrix, a gate on the 974 keV transition yielded clear evidence for a second isomer, at higher energy, decaying by 340 and 183 keV $\gamma$-transitions into the known isomer band (see figure 5.42). In figure 5.43 gates on the delayed 340 and 974 keV transitions were summed within the "early-delayed $\gamma$-$\gamma$" matrix, indicating three new early transitions within the higher energy isomer band.

The level scheme of figure 5.9 shows the 7 new transitions observed in $^{163}$Ho.

**Figure 5.42:** Coincidence spectrum for the 973.8 keV transition in the out-of-beam $\gamma$-$\gamma$ matrix in the 300$\rightarrow$1550 ns range. Transitions from the new $^{163}$Ho isomer are denoted by an asterisk. Known $^{163}$Ho transitions are labeled with their energy. Random coincidences from $^{162}$Dy are labeled Dy.

**Figure 5.43:** Background subtracted early spectrum in coincidence with delayed 973.8 or 340.2 keV transitions in the "early-delayed $\gamma$-$\gamma$" matrix in the 150$\rightarrow$832 ns range. Transitions within the $^{163}$Ho isomer band are labeled with their energy. Random coincidences from $^{164}$Er are labeled Er.
The new isomer in $^{163}$Ho was the weakest observed in this experiment, necessitating a different approach to the measurement of its half-life. It was measured in the $\gamma$-time matrix with a number of background subtracted gates on the 973.8 keV transition. The time component of the $\gamma$-time matrix was sliced into 156 ns chunks, as shown in figure 5.45. For each slice the projected $\gamma$-ray spectrum provided a peak at 974 keV with an area that was measured in gf3, as shown in figure 5.46. These areas were then plotted as a function of time, and fitted to provide a half-life measurement, as shown in figure 5.47. A measurement of $T_{1/2} = 0.80(15)$ $\mu$s was achieved. The large uncertainty is due to the wide time slices, which were necessary because even with such a width, no more than 300 counts were observed in the peaks measured. As a result there are relatively few data points.

As the determination of the half-life involved the measurement of peak areas, it was convenient to use a strong peak with minimal contamination from other delayed components. The 974 keV peak was used because the 340 keV peak lies very close to a beta-delayed gamma ray in $^{161}$Tb at 338 keV. This approach is justified as the size of the 340 and 140 keV peaks in figure 5.42 is approximately the same as the 165 and 145 keV peaks, indicating that within the 300–1550 ns time range, nearly all of the intensity comes from the 2109.4 keV isomer. Furthermore, the early spectrum in figure 5.43 includes a gate on the delayed 974 keV transition, yet none of the
Figure 5.45: The total un-gated time spectrum for the $\gamma$-time matrix. The dotted lines show the slices of time that were gated on to produce the $\gamma$-ray energy spectra (a to h) shown in figure 5.46.

Figure 5.46: The $\gamma$-ray energy spectra (a to h) for the slices labeled (a to h) in figure 5.45. The energy of the measured peak followed by its area is given.
The question therefore arises as to whether the 1506.4 keV state is an isomer at all. Spectra over a shorter delayed range indicate it is a much shorter lived isomer than the new 2109.4 keV, $T_{1/2} = 0.80(15) \mu$s isomer. Figure 5.48 uses the same 974 keV gate as figure 5.42, but only includes coincidences detected in the shorter 30–300 ns range. We can see that the 340 keV and 140 keV peaks are smaller in comparison with the 165 and 145 keV peaks than they are in figure 5.42, proving a separate, shorter, delayed component for the 974 keV transition. In figure 5.49, an early spectrum using the same 974 keV gate as figure 5.43, but over a shorter time range, shows transitions from the known band above the 1506.4 keV isomer, proving that the 974 keV transition from the 1506.4 keV isomer is in delayed coincidence with the band above it.

The half-life of the 1506.4 keV isomer is clearly shorter than 300 ns, but measurements are difficult due to the weak intensity of the isomer (due to the small isotopic yield for $^{163}$Ho shown in figure 5.28) and the need to apply a second, stronger component to any half-life fit. However, the large apparent difference between the half-lives of the two isomers gives a reasonable chance for success.
Figure 5.48: Coincidence spectrum for the 973.8 keV transition in the out-of-beam $\gamma-\gamma$ matrix in the 30–300 ns range. Transitions within $^{163}$Ho are labeled with their energy. Random coincidences from $^{164}$Er are labeled Er.

Figure 5.49: Background subtracted early spectrum in coincidence with the delayed 973.8 transition in the 30–150 ns range. Transitions within $^{163}$Ho are labeled with their energy. Random coincidences from $^{164}$Er are labeled.

The half-life of the 1506.4 keV isomer was measured in the $\gamma$-time matrix with a background subtracted gate on 974 keV $\gamma$ rays. A measurement of $T_{1/2} = 41(11)$ ns was obtained from the fit in figure 5.50. The fit is the sum of two exponentials, representing the two isomers, one of which is set with a half-life of 800 ns.

This half-life agrees with the evidence of figure 5.42, which indicates a negligible contribution from the 1506.4 keV isomer after 300 ns. To further prove the validity of this half-life we can integrate the two exponentials in figure 5.50 within the limits of 30–300 ns to estimate the relative intensity from the two isomers during this time frame. We can then compare these intensities with figure 5.48, where the 340 and 183 keV peaks represent the intensity from the 2109.4 keV isomer within this time frame, and the 165 keV and 310 keV peaks represent the intensity from both isomers. Therefore, the combined intensity from both isomers versus the intensity from the 2109.4 keV isomer can be determined from two separate methods, as shown in table 5.9.

In the bottom half of table 5.9, the 2109 keV isomer intensity includes the intensity of the 183.3 keV transition, which is determined from its intensity relative to the 340 keV transition in the out-of-beam $\gamma-\gamma$ matrix in the 300–1550 ns
Figure 5.50: Time spectrum for 973.8 keV $\gamma$ rays. The fitted half-life for the 1506.4 keV isomer is 41(11) ns. The solid line is a fit to the data including the summed counts from both isomer decays.

Table 5.9: The intensity from both $^{153}$Ho isomers as a ratio of the intensity from the 2109 keV isomer. Two methods are compared for the 30–300 ns time range. Top: The integrated intensities from figure 5.50, assuming a 41 ns half-life for the 1506.4 keV isomer. Bottom: Intensities calculated from peak areas in figure 5.48.

<table>
<thead>
<tr>
<th>Isomer(s)</th>
<th>Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1506.4 keV isomer</td>
<td>981</td>
</tr>
<tr>
<td>2109 keV isomer</td>
<td>705</td>
</tr>
<tr>
<td>Both isomers</td>
<td>1686</td>
</tr>
<tr>
<td>Ratio (both/2109)</td>
<td>2.39</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Isomer(s)</th>
<th>Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>2109 keV isomer</td>
<td>71 (11)</td>
</tr>
<tr>
<td>Both isomers</td>
<td>150 (14)</td>
</tr>
<tr>
<td>Ratio (both/2109)</td>
<td>2.11 (40)</td>
</tr>
</tbody>
</table>

range, shown in table 5.10. There is reasonable agreement between the two ratios, supporting a half-life measurement of 41(11) ns for the 1506.4 keV isomer.
Two isomers are known in $^{164}$Er [12], one of which is a 2-quasiproton, $K^\pi=7^- (\pi7/2^-[523]\pi7/2^+[404])$ isomer with an energy of 1985 keV and a half-life of 23.0(12) ns, the other a 3378 keV, 4-quasiparticle, $K^\pi=12^+ ([7^-] \otimes \nu5/2^+[642]\nu5/2^-[523])$ combination of the 7$^-$ isomer with the known 5$^-$ excitation. The $K^\pi=12^+$ isomer was approximated to have $T_{1/2} \geq 170$ ns [12] based on the width of the GASP array [76] coincidence window. In this section, for the $K^\pi=12^+$ isomer, the half-life is determined to higher precision, two decay branches to new bands are found, and two new high-$\Delta K$ transitions to the ground state band are confirmed.

In figure 5.51, a gate on the 168.7 keV ($13^+ \rightarrow 12^+$) early transition above the $K^\pi=12^+$ isomer, showing $\gamma$-ray transitions in delayed coincidence, identified the known 555 keV branch from $K^\pi=12^+$ isomer, as well as two new branches at 156...
keV and 427 keV. Figures 5.52 and 5.53 show the out-of-beam coincidence spectra for these branches, identifying transitions in the daughter bands.

Figure 5.51: Coincidence spectrum for the early 168.7 keV transition showing transitions which proceed the K^+ = 12^+ isomer. Direct branches from the isomer are denoted by an asterisk.

Figure 5.52: Coincidence spectrum for the 156.4 keV transition in the out-of-beam γ–γ matrix. Delayed transitions identified as being from a new band are denoted by an asterisk.

Figure 5.53: Coincidence spectrum for the 427.3 keV transition in the out-of-beam γ–γ matrix. Delayed transitions identified as being from a new band are denoted by an asterisk.

Figure 5.54: Time spectrum for 555.0 keV γ rays. The fitted half-life is 68(2) ns.

The half-life for the K^+ = 12^+ isomer was determined in the γ-time matrix with a background subtracted gate on 555 keV γ rays, as shown in figure 5.54. A measurement of T_{1/2} = 68(2) ns was obtained, consistent with the half-life for the 427 keV branch. The 156 keV branch was contaminated with γ rays following the decay of the new ^{105}Er isomer, and did not provide a reliable half-life measurement.
Two new high-ΔK branches from the K\(^r=12^+\) isomer to the ground state band in \(^{164}\text{Er}\) were observed. A 1294.8 keV branch to the 12\(^+\) level was identified in the out-of-beam \(\gamma - \gamma\) matrix (see figure 5.55) and in the delayed-early \(\gamma - \gamma\) matrix (see figure 5.56). A tentative 1859.5 keV branch to the 10\(^+\) level was observed in the out-of-beam \(\gamma - \gamma\) matrix (see figure 5.57).

The new transitions were placed in the partial level scheme in figure 5.58. All \(\gamma\)-ray transitions into, within, and out of the K\(^r=(8^+)\) and K\(^r=(9^-)\) bands are discovered in this work, as well as the branches from the K\(^r=12^+\) isomer to the ground state band. All other transitions are known from previous works [12; 77; 78].
5.6. ISOMERS IN $^{164}$ER

Figure 5.58: Partial level scheme for $^{164}$Er showing the decay of the $K^\pi=12^+$ isomer.

Table 5.11: Gamma-ray energy, relative $\gamma$-ray intensity, final angular momentum, multipolarity, total conversion coefficient, hindrance factor, forbiddenness, and reduced hindrances. Direct branches from the $K^\pi=12^+$ and $K^\pi=7^-$ isomers are shown in the top and bottom subsections of the table respectively. For a complete level scheme, please refer to figure 5.15. The uncertainty in the $\gamma$-ray energies is typically 0.1 keV for $E_\gamma < 1$ MeV and 0.2 keV above.

<table>
<thead>
<tr>
<th>$E_\gamma$(keV)</th>
<th>$I_\gamma$</th>
<th>$I_\gamma'(\sigma\lambda)$</th>
<th>$a^T$</th>
<th>$F_W$</th>
<th>$\nu$</th>
<th>$f_\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>156.4</td>
<td>22 (2)</td>
<td>11$^-$ (E1)</td>
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</tr>
<tr>
<td>427.3</td>
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<td>11$^+$ (M1)</td>
<td>0.052</td>
<td>1.7x10$^6$</td>
<td>3</td>
<td>120</td>
</tr>
<tr>
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<td>100 (3)</td>
<td>11$^-$ (E1)</td>
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<td>7.7x10$^3$</td>
<td>4</td>
<td>9.4</td>
</tr>
<tr>
<td>1294.8</td>
<td>2.0 (3)</td>
<td>12$^+$ (M1)</td>
<td>0.003</td>
<td>5.0x10$^8$</td>
<td>11</td>
<td>6.2</td>
</tr>
<tr>
<td>(1859.5)</td>
<td>0.4 (2)</td>
<td>10$^+$ (E2)</td>
<td>0.001</td>
<td>5.3x10$^7$</td>
<td>10</td>
<td>5.9</td>
</tr>
<tr>
<td>139.8</td>
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<td>7$^-$ (M1)</td>
<td>1.095</td>
<td>1.9x10$^4$</td>
<td>1</td>
<td>18910</td>
</tr>
<tr>
<td>240.1</td>
<td>6 (1)</td>
<td>8$^+$ (E1)</td>
<td>0.032</td>
<td>9.4x10$^3$</td>
<td>4</td>
<td>9.8</td>
</tr>
<tr>
<td>240.5</td>
<td>181 (6)</td>
<td>6$^-$ (M1)</td>
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<tr>
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<td>6</td>
<td>9.2</td>
</tr>
<tr>
<td>1370.6</td>
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<td>6$^+$ (E1)</td>
<td>0.001</td>
<td>5.3x10$^5$</td>
<td>6</td>
<td>9.0</td>
</tr>
</tbody>
</table>
The relative intensities and hindrances for the direct branches from the two isomers are shown in table 5.11. The relative intensities for transitions involving the newly discovered $K^s=(8^+)$ and $K^s=(9^-)$ bands are shown in table 5.12.

Table 5.12: Gamma-ray energy, relative $\gamma$-ray intensity, initial level energy, initial angular momentum and final angular momentum for the new bands identified in $^{16}{}^4$Er; ordered by level spin from high to low. Top: proceeds the 156.4 keV transition. Bottom: proceeds the 427.3 keV transition. The uncertainty in the $\gamma$-ray energies is typically 0.1 keV for $E_\gamma < 1$ MeV and 0.2 keV above.

<table>
<thead>
<tr>
<th>$E_\gamma$(keV)</th>
<th>$I_\gamma$</th>
<th>$E_i$</th>
<th>$I_i^*$</th>
<th>$I_f^*$</th>
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<tbody>
<tr>
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<td>11 (1)</td>
<td>3221.6</td>
<td>11$^-$</td>
<td>10$^-$</td>
</tr>
<tr>
<td>462.3</td>
<td>1.8 (2)</td>
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<td>9$^-$</td>
</tr>
<tr>
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<td>4.4 (4)</td>
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<td>10$^-$</td>
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<tr>
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<td>9$^-$</td>
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<tr>
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<td>9$^-$</td>
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<td>8$^-$</td>
</tr>
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<td>0.8 (3)</td>
<td>2759.3</td>
<td>9$^-$</td>
<td>8$^-$</td>
</tr>
<tr>
<td>773.9</td>
<td>4.1 (5)</td>
<td>2759.3</td>
<td>9$^-$</td>
<td>7$^-$</td>
</tr>
<tr>
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<td>8 (2)</td>
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<td>11$^+$</td>
<td>10$^+$</td>
</tr>
<tr>
<td>366.6</td>
<td>5.4 (8)</td>
<td>2950.7</td>
<td>11$^+$</td>
<td>10$^-$</td>
</tr>
<tr>
<td>424.4</td>
<td>8.4 (9)</td>
<td>2950.7</td>
<td>11$^+$</td>
<td>9$^+$</td>
</tr>
<tr>
<td>203.7</td>
<td>5.9 (3)</td>
<td>2730.0</td>
<td>10$^+$</td>
<td>9$^+$</td>
</tr>
<tr>
<td>389.6</td>
<td>2.2 (2)</td>
<td>2730.0</td>
<td>10$^+$</td>
<td>8$^+$</td>
</tr>
<tr>
<td>185.9</td>
<td>7 (1)</td>
<td>2526.3</td>
<td>9$^+$</td>
<td>8$^+$</td>
</tr>
<tr>
<td>362.1</td>
<td>7.1 (5)</td>
<td>2526.3</td>
<td>9$^+$</td>
<td>8$^-$</td>
</tr>
<tr>
<td>355.0</td>
<td>10.9 (5)</td>
<td>2340.4</td>
<td>8$^+$</td>
<td>7$^-$</td>
</tr>
</tbody>
</table>
5.7. ISOMER IN $^{162}$Er

In the out-of-beam $\gamma-\gamma$ matrix, a gate on the $6^+ \rightarrow 4^+ \ (337.3 \text{ keV})$ ground-state band transition in $^{162}$Er yielded clear evidence for an isomer decaying by 1359.6 keV and 930.1 keV $\gamma$-transitions within this nucleus (see figure 5.59). The half-life of the isomer was measured in the $\gamma$-time matrix with a background subtracted gate on 1360 keV $\gamma$ rays. A measurement of $T_{1/2} = 88(16) \text{ ns}$ was determined from the fit in figure 5.60.

![Figure 5.59: Coincidence spectrum for the 337.3 keV transition in the out-of-beam $\gamma-\gamma$ matrix. Transitions in $^{162}$Er are labeled with their energy. Random coincidences from $^{164}$Er are labeled Er.](image)

![Figure 5.60: Time spectrum for 1359.6 keV $\gamma$ rays. The fitted half-life is 88(16) ns.](image)

The relative intensities, determined in the out-of-beam $\gamma-\gamma$ matrix, are shown in table 5.13. The $\gamma$-ray energies of these branches places the isomeric state at 2026.6(6) keV. A partial level scheme is shown in figure 5.61. The two branches from the isomer have been previously reported by De Boer et al. [79] and Janssens et al. [80] with both works assigning a spin of 7 to the parent level on the basis on angular distributions. Neither work reported the level as an isomer. All parts of the level scheme are well established in these previous works.
Table 5.13: Gamma-ray energy, relative $\gamma$-ray intensity, final angular momentum, multipolarity, hindrance factor, forbiddenness, and reduced hindrance for the isomer in $^{162}$Er, based on a $K^*=7^-$ assignment. The uncertainty in the $\gamma$-ray energies is typically 0.1 keV for $E_\gamma < 1$ MeV and 0.2 keV above.

<table>
<thead>
<tr>
<th>$E_\gamma$(keV)</th>
<th>$I_\gamma$</th>
<th>$I_\gamma^\gamma(\sigma \lambda)$</th>
<th>$F_W$</th>
<th>$\nu$</th>
<th>$f_\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>930.1</td>
<td>31 (4)</td>
<td>$8^+(E1)$</td>
<td>1.32$x10^5$</td>
<td>6</td>
<td>7.1</td>
</tr>
<tr>
<td>1359.6</td>
<td>100 (7)</td>
<td>$6^+(E1)$</td>
<td>1.27$x10^5$</td>
<td>6</td>
<td>7.1</td>
</tr>
</tbody>
</table>

The energy of the isomer, its half-life, and the energy and intensity of its decays are very similar to the $7^-$ isomer in $^{164}$Er. As the $^{164}$Er isomer was also detected, a number of comparisons can be made. Referring to tables 5.11 and 5.13, the intensity ratio and energies of the two high energy branches from the $7^-$ isomer to the ground state band are very similar in both $^{164}$Er and $^{162}$Er. The intensity ratio is identical within the uncertainty. Furthermore, despite the $7^-$ structure in $^{164}$Er primarily decaying to a 2-quasineutron $5^-$ band, as opposed to $^{162}$Er where the $5^-$ band was not detected, the reduced hindrance and half-life for the isomers are also very similar. We therefore adopt a $K^*=7^-$ assignment for the isomer.

SECTION 5.8

Isomer in $^{161}$Dy

Jungclaus et al. [81] identified a band above a $K^*=11/2^-$ band-head at 486 keV [82] in $^{161}$Dy. The experiment had no out-of-beam condition, and as no branches were found decaying from this level, it became a good candidate for an isomer. Furthermore, the same structure is an isomer in both $^{163}$Er and $^{165}$Er. In the out-of-beam $\gamma-\gamma$ matrix, gates on low-spin members of the $K^*=5/2^-$ and $K^*=5/2^+$
bands (see figure 5.4) identified delayed branches from the 486 keV level, indicating the level is an isomer. The clearest delayed spectrum, which also shows the relative intensities of all the isomer branches, was produced in the "early-delayed $\gamma-\gamma$" matrix by gating on a number of early transitions within the $K^*=11/2^-$ band (see figure 5.62).

It was necessary to prove that the peak labeled '217.6+217.8' is a doublet. Gating on this doublet produces the spectrum in figure 5.63. The 165.0 keV and 167.5 keV transitions shown to be in coincidence prove that the isomer decays to both the $13/2^+$ level of the $K^*=5/2^+$ band and the $11/2^-$ level of the $K^*=5/2^-$ band, as shown in the level scheme in figure 5.4. In figure 5.64 a gate on the strong, delayed 385.3 keV transition within the "early-delayed $\gamma-\gamma$" matrix showed the isomer band in early coincidence, indicating one new $\Delta I = 2$ transition.

![Figure 5.62: Coincidence spectrum for the early transitions showing transitions which proceed the $K^*=11/2^-$ isomer. Direct branches from the isomer are denoted by an asterisk. Random coincidences from $^{164}\text{Er}$ and $^{166}\text{Er}$ are labeled.](image)

![Figure 5.63: Coincidence spectrum for the delayed 217.6+217.8 keV transition in the out-of-beam $\gamma-\gamma$ matrix. The direct branch from the isomer is denoted by an asterisk.](image)

The relative intensities of the branches from the isomer, determined from figure 5.62, are shown in table 5.14. The $\gamma$-ray energies place the isomeric state at 485.7(3) keV. The relative intensity of the 217.8 keV branch was determined by gating on the 165.0 keV branch in the out-of-beam $\gamma-\gamma$ matrix, measuring the 217.8/220.3 keV intensity ratio, comparing this with the same ratio in figure 5.62, and attributing
the extra intensity in figure 5.62 to the 217.8 keV isomeric branch.

Table 5.14: Top: Gamma-ray energy, relative γ-ray branch intensity, final angular momentum, multipolarity, total conversion coefficient, hindrance factor, forbiddenness, and reduced hindrances for direct branches from the $K^x=11/2^-$ isomer in $^{161}$Dy. Middle and bottom: relative γ-ray branch intensity for the newly discovered 220.3 and 612.5 keV transitions in $^{161}$Dy. The uncertainty in the γ-ray energies is typically 0.1 keV.

<table>
<thead>
<tr>
<th>$E_\gamma$(keV)</th>
<th>$I_\gamma$</th>
<th>$I_\gamma''(\sigma\lambda)$</th>
<th>$\alpha^T$</th>
<th>$F_W$</th>
<th>$\nu$</th>
<th>$f_\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>165.0</td>
<td>6.3 (6)</td>
<td>11/2$^-$ (M1)</td>
<td>0.579</td>
<td>3.79 x 10$^6$</td>
<td>2</td>
<td>1947</td>
</tr>
<tr>
<td>217.8</td>
<td>4.1 (6)</td>
<td>13/2$^+$ (E1)</td>
<td>0.039</td>
<td>1.30 x 10$^5$</td>
<td>2</td>
<td>360</td>
</tr>
<tr>
<td>284.3</td>
<td>19 (1)</td>
<td>9/2$^-$ (M1)</td>
<td>0.130</td>
<td>6.42 x 10$^6$</td>
<td>2</td>
<td>2534</td>
</tr>
<tr>
<td>301.4</td>
<td>3.4 (5)</td>
<td>11/2$^+$ (E1)</td>
<td>0.017</td>
<td>4.14 x 10$^8$</td>
<td>2</td>
<td>643</td>
</tr>
<tr>
<td>382.6</td>
<td>13 (2)</td>
<td>7/2$^-$ (E2)</td>
<td>0.031</td>
<td>6.76 x 10$^9$</td>
<td>1</td>
<td>6764</td>
</tr>
<tr>
<td>385.3</td>
<td>100 (4)</td>
<td>9/2$^+$ (E1)</td>
<td>0.009</td>
<td>2.95 x 10$^4$</td>
<td>2</td>
<td>172</td>
</tr>
<tr>
<td>119.3</td>
<td>15 (2)</td>
<td>9/2$^-$ (M1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>217.6</td>
<td>100 (5)</td>
<td>7/2$^-$ (E2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>220.3</td>
<td>83 (5)</td>
<td>9/2$^+$ (E1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(612.5)</td>
<td>(100)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The low energy of the branches from the isomer made measurement of the half-life difficult, as most of the branches were contaminated with other delayed components. For example, the 284.3 keV branch is contaminated by the strong 282.9 keV, $4^+ \rightarrow 2^+$ ground band transition in $^{162}$Dy, while the 385.3 branch is contaminated by the 383.2 keV branch from the $^{163}$Er isomer. This contamination made it necessary to use the slices method of half-life measurement applied to a $^{163}$Ho isomer in section 5.5. In the 1550 ns out-of-beam setting (using 0.78125 ns per channel) slices of 156 ns were gated on, and the intensity of the 385.3 keV peak was measured in the corresponding $\gamma$-ray spectra. There was a lot of contamination at early times, meaning only four slices could be reliably measured, however, it became clear that the half-life was long enough to show up in the 150 $\mu$s out-of-beam setting (using 250 ns per channel) used to measure the $^{162}$Dy isomer. In this setting, the intensity of the 385.3 keV peak was measured in slices of 250 ns as well as 500 ns and 1000 ns. A summary of these measurements is given in figure 5.65.

![Figure 5.65: Half-life plots for the delayed 385.3 keV transition. Four measurements were taken using different widths of time gates.](image)
By using the same $\gamma$-energy gate in both the $30 \to 300$ ns and $300 \to 1550$ ns out-of-beam $\gamma-\gamma$ matrices, an isomer branch in coincidence can be shown in both slices of time. The half-life can be measured by comparing the intensity in the two matrices, and plotting them as areas under a decay curve. For the 284.3 keV branch, the 157.5, 175.6 and 98.3 keV transitions were each gated on to show the branch in coincidence within the two matrices. The area of the 284.3 keV peak was measured in both matrices from each of the three coincidences, providing three measurements of the half-life. A summary of these measurements is given in figure 5.66.

![Graph showing decay curve for different time regions]

Figure 5.66: Half-life of the 284.3 keV branch as measured from its area in two time regions when in coincidence with the 157.5 keV (top), 175.6 keV (middle), and 98.3 keV (bottom) transitions. In each case the peak areas are given under the exponential fit line.

This method could not be applied to the 385.3 keV branch, as only the 56.5 and 100.4 keV transitions are in coincidence, having high contamination and weak relative intensity respectively. The contamination when using the 56.5 keV gate is due to this energy over-lapping the $K_\beta$ X-ray energy for Er, meaning that the 383.2 keV contaminant peak from the $^{163}$Er isomer features prominently, as shown in figures 5.67 and 5.68. The fact that the 385.3 keV appears stronger than the 383.2 contaminant in the 300 $\to$ 1550 ns range, as compared with the 30 $\to$ 300 ns range, at least shows that it has a longer half-life. The half-life of the $^{163}$Er isomer (and therefore the 383.2 contaminant) is 0.58(10) $\mu$s [83], supporting the longer half-life measurements for the $^{161}$Dy isomer determined here. The validity of this technique was confirmed by using it to measure the half-life of $^{164}$Er and $^{165}$Er.
isomers within their quoted uncertainty, although the precision decreases quickly with the decreasing intensity of the isomer branch.

![Coincidence spectrum for the 56.5 keV transition in the out-of-beam $\gamma - \gamma$ matrix in the 30→300 ns range. The 385.3 keV branch from the $^{161}$Dy isomer is shown.](image1)

![Coincidence spectrum for the 56.5 keV transition in the out-of-beam $\gamma - \gamma$ matrix in the 300→1550 ns range. The 385.3 keV branch from the $^{161}$Dy isomer is shown.](image2)

As no measurement is more valid than another, the half-life for the $^{161}$Dy isomer was taken as the average of the 7 measurements obtained. As there is some spread in the measurements, a conservative estimate of the uncertainty was chosen based on the lower limit of the lowest measurement. The half-life of the isomer was determined to be 0.76(17) $\mu$s.
Discussion

SECTION 6.1

Intrinsic g-factors

Intrinsic g-factors may be determined from the relative intensity of $\Delta I=1$ and $\Delta I=2$ transitions from levels within rotational bands, using the method outlined in section 2.10. These may be compared with theoretically determined g-factors for candidate quasiparticle structures. Thus, rotational bands may be assigned quasiparticle structure, and therefore, a spin and parity for the band-head, on the basis of the greatest agreement between experiment and theory. Before assigning isomer bands, it is prudent to test the procedure on rotational bands with known quasiparticle structure, as shown in Tables 6.1 and 6.2.

There is a trend for the measured g-factors to be $\sim 30\%$ less than the theoretical estimate. The accuracy of the experimental values is reinforced by the g-factor measurements of the $K^*=11/2^-$ band, which show similar values ($\sim 0.39$) in three different isotopes. The same is true for the other source of comparison: the two isotopes observed with a $K^*=3/2^-$ band.
Table 6.1: Intrinsic g-factors (\(g_K - g_R\)) for a selection of observed rotational bands. Comparison of the experimental g-factor (subscript E) and the g-factor of the assigned quasiparticle structure (subscript T) is made. Assumes \(g_R = 0.3\) with intrinsic spin g-factors of +5.59 and -3.83 for protons and neutrons respectively, with a quenching factor of 0.6. Deformation calculated from the E\(^{2+}\) energy or taken from [41] for odd nuclei.

| K^\(\pm\)I | \(E_{\gamma}^{\Delta I=1}\) | \(E_{\gamma}^{\Delta I=2}\) | \(I_{\gamma}^{\Delta I=1}\) | \(I_{\gamma}^{\Delta I=2}\) | \(|g_K - g_R|_E\) | \(|g_K - g_R|_T\) | QP structure | \(\beta_2\) | \(Q_0\) | Energy (MeV) |
|----------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|--------|--------|----------------|
| 1\(^{162}\)Dy | 0.272 | 6.0 eb | 3/2^-;7/2 | 81.0 | 137.7 | 32.5(10) | 11.2(5) | 0.70(2) | \(\nu 3/2^-[521]\) | 1.07 |
| 1\(^{162}\)Dy | 0.326 | 7.3 eb | 4^+;7 | 135.3 | 253.4 | 0.71(6) | 1.09(9) | 0.29(3) | \(\nu 5/2^-[523]\) | 0.30 |
| 1\(^{162}\)Dy | 0.281 | 6.3 eb | 5^-;8 | 124.1 | 231.7 | 7.2(3) | 10.3(4) | 0.20(1) | \(\nu 5/2^-[642]\) | 0.76 |
| 1\(^{163}\)Ho | 0.282 | 6.4 eb | 7/2^-;13/2 | 135.2 | 250.4 | 15.8(15) | 7.6(11) | 0.20(1) | \(\nu 7/2^-[523]\) | 1.04 |
| 1\(^{163}\)Ho | 0.293 | 6.7 eb | 135.2 | 250.4 | 15.8(15) | 7.6(11) | 0.50(8) | \(\nu 7/2^-[523]\) | 1.04 |
6.1. INTRINSIC G-FACTORS

Table 6.2: Table 6.1 continued.

| $K^+;I$ | $E^A_{I=1}$ | $E^A_{I=2}$ | $I^A_{I=1}$ | $I^A_{I=2}$ | $|g_K - g_R|_E$ | $|g_K - g_R|_T$ | QP structure |
|---------|-------------|-------------|-------------|-------------|----------------|----------------|---------------|
| $^{163}$Er | $\beta_2=0.272$ | Q$_0=6.3$ eb |
| $3/2^-;7/2$ | 85.4 | 145.0 | 9.8(7) | 4.6(8) | 0.66(8) | $\nu 3/2^-[521]$ | 1.07 |
| $3/2^-;8/2$ | 110.6 | 196.1 | 9.9(7) | 10.3(6) | 0.73(5) | $\nu 3/2^-[521]$ | 1.07 |
| $11/2^-;15/2$ | 193.7 | 364.8 | 42.4(15) | 16.3(5) | 0.39(2) | $\nu 11/2^-[505]$ | 0.51 |
| $11/2^-;19/2$ | 234.4 | 449.2 | 20.0(10) | 18.1(11) | 0.47(2) | $\nu 11/2^-[505]$ | 0.51 |
| $11/2^-;21/2$ | 253.1 | 487.7 | 12.2(4) | 14.8(6) | 0.48(2) | $\nu 11/2^-[505]$ | 0.51 |
| $^{164}$Er | $\beta_2=0.301$ | Q$_0=7.0$ eb |
| $5^-;8$ | 118.8 | 219.8 | 92(3) | 317(11) | 0.09(2) | $\pi 5/2^+[642]\otimes\pi 5/2^-[523]$ | 0.30 |
| $5^-;10$ | 262.8 | 508.9 | 77(3) | 555(17) | 0.12(2) | $\pi 5/2^+[642]\otimes\pi 5/2^-[523]$ | 0.30 |
| $7^-;9$ | 199.9 | 378.8 | 37(2) | 62(3) | 0.51(2) | $\pi 7/2^+[404]\otimes\pi 7/2^-[523]$ | 0.70 |
| $7^-;10$ | 220.1 | 420.0 | 27.5(9) | 10.0(3) | 0.50(2) | $\pi 7/2^+[404]\otimes\pi 7/2^-[523]$ | 0.70 |
| $7^-;11$ | 238.9 | 458.9 | 37(2) | 21.1(6) | 0.46(2) | $\pi 7/2^+[404]\otimes\pi 7/2^-[523]$ | 0.70 |
| $^{165}$Er | $\beta_2=0.282$ | Q$_0=6.6$ eb |
| $11/2^-;15/2$ | 176.2 | 331.8 | 0.51(6) | 0.19(3) | 0.38(6) | $\nu 11/2^-[505]$ | 0.51 |

The theoretical $|g_K - g_R|_T$ values assume $g_R = 0.3$, but in section 2.10 a dependence of $g_R$ on the nucleonic configuration was described, indicating large $g_R$ values for quasiproton excitations, and small $g_R$ values for quasineutron excitations. It is possible that this dependence could cause the $\sim 30\%$ discrepancy between experiment and theory. In figure 6.1 the value of $g_R$ is altered to force agreement between $|g_K - g_R|_T$ and $|g_K - g_R|_E$ for each of the structures in tables 6.1 and 6.2.

There is a clear correlation between high $g_R$ values and quasiproton excitations. The 4QP, $^{164}$Er isomer, which includes both protons and neutrons, is a midpoint, beneath which all the low $g_R$ values correspond to quasineutron excitations. This is strong evidence that quasiparticle excitations will cause pair blocking, leading to an increase in the moment of inertia that is attributable to the particle involved in the excitation, thereby affecting the collective magnetic moment of the nucleus (and therefore the $g_R$ values) in the observed manner for neutrons and protons. It is interesting to note that there is no clear difference in the inferred $g_R$ for 1QP
and 2QP structures. This is perhaps due to the particle occupation probability decreasing for orbitals further from the Fermi surface (see equation 2.8), resulting in more pair blocking from the first quasiparticle. Thus, the second quasiparticle, in 2QP structures, will have less of a pair blocking effect, as the particle occupation probability for this orbital will be lower.

Beyond the well-understood differences mentioned, there is no major discrepancy between experiment and theory, and this exercise has confirmed the applicability of g-factors for the structural assignment of isomers, especially in cases where different candidate structures would produce vastly different g-factors.
6.2. BCS PAIRING STRENGTHS

BCS calculations, as described in section 2.3, are used to predict likely configurations for all of the multi-quasiparticle structures investigated in this chapter. In some cases the pairing strengths for neutrons and protons are fitted in conjunction with Nilsson single particle energy levels to provide exact energies for known broken pair excitations. In other cases the strengths are estimated. Table 6.3 summarises the pairing strengths used for the nuclei discussed in this chapter.

Table 6.3: Pairing strengths for neutrons ($G(n)$) and protons ($G(p)$). An asterisk denotes an estimated pairing strength. The † denotes a fit to a 3QP structure in $^{162}$Ho after a prior estimation for the proton pairing strength (more details in section 6.4).

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Neutrons</th>
<th>G(n)</th>
<th>Protons</th>
<th>G(p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{164}$Er</td>
<td>96</td>
<td>20.35</td>
<td>68</td>
<td>21.50</td>
</tr>
<tr>
<td>$^{162}$Dy</td>
<td>96</td>
<td>19.84</td>
<td>66</td>
<td>21.25*</td>
</tr>
<tr>
<td>$^{162}$Er</td>
<td>96</td>
<td>19.50†</td>
<td>67</td>
<td>21.25*</td>
</tr>
<tr>
<td>$^{165}$Er</td>
<td>97</td>
<td>19.85*</td>
<td>68</td>
<td>21.50*</td>
</tr>
</tbody>
</table>

SECTION 6.3

$^{162}$Dy isomer structural assignment

For the isomer discovered in $^{162}$Dy (see section 5.3), transition rates indicated an $8^+$ assignment. In order to support this evidence, intrinsic g-factors, alignment data and BCS calculations were utilised. The Fermi surface for $^{162}$Dy lies between the $5/2^-$[$523$] and $5/2^+$[$642$] neutron orbitals. The Nilsson model [38] and experimental single-particle energies for neighboring odd nuclei [84] indicate the close proximity of the $11/2^-$[$505$] neutron orbital, suggesting either an $8^+$ ($\nu11/2^-[505]\nu5/2^-[523]$)
or $8^- (\nu 11/2^- [505] \nu 5/2^+ [642])$ 2-quasineutron structure.

The alignment of the band above the isomer, shown in figure 6.2, favours the $5/2^- [523]$ rather than the $5/2^+ [642]$ neutron, as the alignment of the isomer band is significantly lower than the alignment of the $5/2^+ [642]$ neutron in $^{161}$Dy.

![Figure 6.2: Angular momentum alignment with the rotation axis for the isomer band, the $5^-$ band involving the two candidate $5/2$ orbitals, and single-particle values for those orbitals in $^{161}$Dy. All measurements are from this experiment. For the isomer band, $\Delta I = 1$ transitions were chosen to provide more data. Elsewhere, $\Delta I = 2$ transitions were chosen to minimise fluctuations. The Harris parameters are: $J_0 = 37 G^2$ MeV$^{-1}$; $J_1 = 42 G^4$ MeV$^{-3}$ [18].](image)

Intrinsic g-factors were extracted from $\gamma$-ray intensity measurements for the isomer band. Using the theoretical method in section 2.10, $g_K - g_R = -0.30$ is predicted for the $8^+$ configuration, and $g_K - g_R = -0.59$ for the $8^-$ configuration. The experimental values, summarised in Table 6.4 again favour the $8^+$ configuration.

In addition, multi-quasiparticle (MQP) calculations were performed with the BCS blocking code [8] to produce theoretical level energies for the two candidate configurations. The pairing energy for neutrons was adjusted to fit the $5^- (\nu 5/2^- [523] \nu 5/2^+ [642])$ band-head at 1486 keV. The MQP calculations predict the $8^+$ configuration only 12 keV away from the experimental energy. The $8^-$ configuration is predicted 226 keV above the experimental energy. The estimated uncertainty
Table 6.4: Intrinsic g-factors for the isomer band, assuming $K=8$, $g_R = 0.3$, and a quadrupole moment of $Q_0 = 7.3 \text{ eb}$. The intrinsic spin g-factors used were +5.4 and −3.8 for protons and neutrons respectively, with a quenching factor of 0.6.

| $L$, $K$ | $E_{\Delta I=1}^E$ | $E_{\Delta I=2}^E$ | $I_{\Delta I=1}^J$ | $I_{\Delta I=2}^J$ | $|g_K - g_R|$ |
|----------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 10,8     | 199.0           | 379.8           | 1.0(1)          | 0.51(8)         | 0.24$^{+4}_{-4}$ |
| 11,8     | 216.7           | 415.7           | 0.30(6)         | 0.30(7)         | 0.27$^{+8}_{-6}$ |

is 150 keV, based on the difference between experimental and predicted energies for the $K=4^+(\nu_5/2^-[523]\nu_3/2^-[521])$ band-head. Residual interactions were not included, but the pairing energy was chosen to better reproduce configurations that are favoured by residual interactions. If included, the $8^-$ configuration would be about 200 keV further from the experimental energy. All of these considerations support a $K^+=8^+$ assignment for the isomer.

### 163Ho Isomers Structural Assignment

For the isomer discovered in $^{163}$Ho (see section 5.5) as well as the 1506 keV isomer, intrinsic g-factors, alignment data and BCS calculations were considered for their structural assignment. For the 1506 keV isomer, the strong intensity of the 974 keV transition relative to the 581 keV transition (see table 5.10) favours a spin 17/2 configuration, while a spin 19/2 configuration is highly unlikely as the 974 keV transition would be required to have a higher multipolarity. BCS calculations reflect three possibilities for the 1506 keV isomer, all being 17/2 configurations (no 19/2 configurations were produced). These configurations are $17/2^+ (\pi_7/2^-[523]\nu_5/2^-[523]\nu_5/2^+[642])$, $17/2^- (\pi_7/2^+[404]\nu_5/2^-[523]\nu_5/2^+[642])$, and $17/2^- (\pi_7/2^-[523]\pi_7/2^+[404]\pi_3/2^+[411])$, referred to subsequently as X, Y and Z respectively. Intrinsic g-factors were extracted from the intensities in table 5.10 giving $|g_K - g_R| = 0.22^{+0.02}_{-0.03}$, allowing for comparison with the three candidate configurations. Configuration X has $|g_K - g_R| = 0.07$, Y has $|g_K - g_R| = 0.26$ and Z
has $|g_K - g_R| = 0.78$. The greatest agreement comes from configuration Y.

However, there are a number of reasons to doubt this agreement. First, the statistics are low, and contaminants could have skewed the $\Delta I = 1/\Delta I = 2$ intensity ratio. Indeed, Hojman et al. [75] found differing ratios, albeit higher up in the band. However, Hojman also had low statistics and suspected contamination. Second, as alluded to in section 6.2, the neutron pairing strength in the BCS calculations was chosen to fit the 1506 keV isomer to the energy of configuration X. Although the choice of configuration is arbitrary, the relative energies of the three candidate configurations are not arbitrary. Configuration Y was predicted 440 keV above the energy of X as a result of the one quasiparticle energy of the $\pi 7/2^+[404]$ proton [84], and Z was predicted 277 keV above the energy of X. The BCS code therefore predicts candidate X to be closest to yrast. One could reduce the neutron pairing strength to fit candidate Y to 1506 keV, but candidate X would still be more yrast, and the neutron pairing strength is already very low compared with the other nuclei in table 6.3. Third, and perhaps most importantly, Hojman et al. [75] found reasonable evidence in favour of configuration X based on the same structure structure in $^{165}$Tm [86]. In $^{165}$Tm, candidate X decays into the $\pi 7/2^- [523]$ ground state band with a 1066 keV E1 transition. In $^{163}$Ho, the 1506 keV isomer decays to the same level with a 974 keV transition. Fourth, based on the energy of $5^- (\nu 5/2^- [523] \nu 5/2^+ [642])$ structures in $^{164}$Er and $^{162}$Dy, the 1506 keV isomer's level energy agrees well with a $5^-$ configuration built upon the $\pi 7/2^- [523]$ ground state band, favouring candidate X. The $^{162}$Dy, $5^-$ band, level energy is 1486 keV, and for $^{164}$Er it is 1664 keV. Configuration Y requires a much lower energy $5^-$ excitation of 1506 $-$ 440 = 1066 keV, which is not favoured by these systematics.

The low energy $\Delta I = 1$ transitions in this band indicate high alignment and the presence of the $5/2^+ [642]$ neutron, further ruling out configuration Z. Based on all these considerations, we tentatively favour candidate X, agreeing with the $17/2^+$ ($\pi 7/2^- [523] \nu 5/2^- [523] \nu 5/2^+ [642]$) assignment of Hojman et al [75].

For the new 2109 keV isomer, the BCS calculations produce only two configura-
6.5. ISOMER STRUCTURAL ASSIGNMENT

For the isomer discovered in $^{165}$Er, discussed in section 5.4, the BCS blocking code [8] was used to predict theoretical level energies for possible multi-quasiparticle configurations. The proton pairing strength was chosen based on the fitted $Z=68$ value for $^{164}$Er. The neutron pairing strength was estimated from the range of values for neighbouring nuclei. The calculations predict a $19/2^+ (\pi7/2^- [523] \nu 5/2^- [523] \nu 7/2^+[404])$ configuration 70 keV above the experimental energy of the new isomer in $^{165}$Er. An alternative is the $19/2^- (\nu 5/2^- [642] \pi 7/2^- [523] \pi 7/2^+[404])$ configuration, which is predicted 117 keV above the experimental energy. These two configurations are both observed in the $^{163}$Er nucleus (see figure 6.3), and the similarities between the level schemes of $^{163}$Er and $^{165}$Er strongly support the BCS calculations that indicate a $19/2^+$ or $19/2^-$ configuration for the bands labeled X and Y in figure 5.41.

Intrinsic g-factors were extracted for band X, which has confirmed $\Delta I=2$ transitions (see top half of table 5.8 for intensities). These are summarised in table 6.5 and indicate that $|g_K - g_R| \sim 0.33$ for band X. The theoretical g-factor for the $19/2^+$ configuration was calculated to be $|g_K - g_R| = 0.56$, and for the $19/2^-$ configuration $|g_K - g_R| = 0.32$. The g-factors favour the $19/2^-$ configuration for band X.
6.5. $^{165}$Er ISOMER STRUCTURAL ASSIGNMENT

<table>
<thead>
<tr>
<th>$K^\pi = 19/2^+$</th>
<th>unknown</th>
<th>$K^\pi = 19/2^-$</th>
<th>$K^\pi = 11/2^-$</th>
</tr>
</thead>
</table>

Figure 6.3: Partial level scheme for $^{165}$Er showing four bands that bear remarkable similarities to structures proposed for $^{166}$Er. Energies are in keV.

Table 6.5: Intrinsic g-factors for the new early band with confirmed ΔI=2 transitions in $^{165}$Er. Calculations assume $K=19/2$, and $g_R = 0.3$ using a quadrupole moment of $Q_0 = 6.6$ eb [85]. The intrinsic spin g-factors used were +5.4 and −3.8 for protons and neutrons respectively, with a quenching factor of 0.6.

| $I$, $K$   | $|g_K - g_R|$  |
|------------|---------------|
| 27/2,19/2  | 0.31$^{+5}_{-3}$ |
| 29/2,19/2  | 0.33$^{+10}_{-7}$ |
| 31/2,19/2  | 0.39$^{+5}_{-4}$ |
| 33/2,19/2  | 0.33$^{+5}_{-6}$ |
| 35/2,19/2  | 0.31$^{+5}_{-7}$ |

As the $19/2^-$ and $19/2^+$ configurations both contain the $7^- (\pi 7/2^-[523] \pi 7/2^+[404])$ 2-quasiproton excitation coupled with either the $\nu 5/2^+[642]$ or $\nu 5/2^-[523]$ neutrons, there should be a considerable difference in the alignment of the two configurations, reflected by the difference in alignment for the 1-quasineutron $\nu 5/2^+[642]$ and $\nu 5/2^-[523]$ bands in $^{166}$Er. Figure 6.4 plots the alignment for these structures, confirming that band X is more aligned than band Y by approximately the same
amount that the $\nu 5/2^-[642]$ band is more aligned than the $\nu 5/2^-[523]$ band. This is strong evidence that band X contains the $\nu 5/2^+[642]$ neutron, resulting in a $19/2^-$ assignment. This agrees with the g-factor measurement. We therefore assign band X a band-head spin of $19/2^-$, and band Y a band-head spin of $19/2^+$.

Figure 6.4: Angular momentum alignment with the rotation axis for the two new bands, the $\nu 5/2^+[642]$ band, and the $\nu 5/2^-[523]$ band in $^{155}$Er. The Harris parameters are: $J_0=37 \ h^2\text{MeV}^{-1}$; $J_1=42 \ h^4\text{MeV}^{-3}$ [18].

It is not possible to assign the $19/2^+$ or $19/2^-$ band-heads as the isomer band. We can only state that both bands decay through the isomer, as both bands are in early coincidence with respect to the delayed isomeric transitions. It is highly likely that one of these bands is the isomer band, while the other decays into the isomer band-head through a low energy E1 transition. It is believed that this transition is swamped by the X-ray spectrum between 40 and 60 keV. There is some small evidence for this transition having an energy of 46.5 keV, but reliable coincidence measurements could not verify this.
In section 5.7 we adopted a $K^*=7^-$ assignment for the newly discovered isomer in $^{162}\text{Er}$. In order to confirm this assignment, the BCS blocking code \cite{8} was used to produce theoretical level energies for possible multi-quasiparticle isomer configurations, allowing a comparison to be made with the experimental energy. The pairing strength for neutrons was adjusted to fit the $4^+ (\nu 3/2^- [521] \nu 5/2^- [523])$ band-head at 1712 keV. In the absence of known 2-quasiproton structures in $^{162}\text{Er}$, the proton pairing strength was set to the value determined for $^{164}\text{Er}$ ($Z=68$), which is based on the $7^- (\pi 7/2^- [523] \pi 7/2^+ [404])$ band-head at 1986 keV. The blocking code predicted the same $7^-$ configuration in $^{162}\text{Er}$ at only 4 keV from the experimental energy of the isomer, strongly confirming the $K^*=7^-$ assignment. With earlier works \cite{79; 80} both assigning spin 7 for this level on the basis of angular distributions, other spin 7 configurations were considered, however the closest was 307 keV above the experimental energy. As the isomer was very weakly populated, no band was detected above the isomer, and g-factor and alignment calculations were not possible.

For the $K^*=12^+$ isomer in $^{164}\text{Er}$, and the newly discovered bands to which it decays (see section 5.6), intrinsic g-factors and BCS calculations were utilised to structurally assign the two bands and confirm the isomer assignment. As the two new bands are not isomers, the band-heads are not well defined. However, as both sets of out-band transitions populate the $K^*=7^-$ band, it is reasonable to predict $K \geq 7$. The absence of transitions to lower $K$ bands also favours this prediction. Furthermore, for the band populated by the 427 keV transition, the 355 keV out-band transition is the
most intense in the band, suggesting that the parent level is the band-head.

The BCS blocking code predicted three configurations with $7 \leq K \leq 12$. These are the $8^- (\nu5/2^+[642]\nu11/2^-[505])$ at 2366 keV, the $8^+ (\nu5/2^-[523]\nu11/2^-[505])$ at 2393 keV, and the $9^- (\nu7/2^+[633]\nu11/2^-[505])$ at 2658 keV. These calculations use a neutron pairing strength that was adjusted to fit the energy of the $5^- (\nu5/2^+[642]\nu5/2^-[523])$ excitation, and a proton pairing strength that was adjusted to fit the energy of the $K^*=7^- (\pi7/2^-[523]\pi7/2^+[404])$ isomer.

The band populated by the 427 keV branch, with an experimental band-head energy of 2340 keV, therefore finds good agreement with either spin 8 configuration. The band populated by the 156 keV branch, with an experimental band-head energy of 2759 keV, finds reasonable agreement with the 9$^-$ configuration.

Table 6.6 shows intrinsic g-factors, extracted from $\gamma$-ray intensity measurements, for the $K^*=12^+$ isomer band and the band populated by the 427 keV branch from this isomer. For the band populated by the 156 keV branch, the $\Delta I=1$ transitions were too heavily contaminated to obtain useful measurements. For the band populated by the 427 keV branch, theoretical calculations give $g_K - g_R = -0.30$ for the $8^+$ configuration and $g_K - g_R = -0.59$ for the $8^-$ configuration. The experimental value in table 6.6 strongly favours the $8^+$ configuration. We therefore tentatively assign this band $K^*=8^+$. 

As g-factors could not be obtained for the band populated by the 156 keV transition, other considerations were made. The difference between the intensity of the 156 keV transition and the intensity of the transitions feeding out of the band it populates infer a total conversion electron coefficient of $0.12^{+0.10}_{-0.12}$, strongly favouring an $E1$ multipolarity for the 156 keV transition. An $E1$ requires a conversion coefficient of 0.10, while an $M1$ requires 0.80, and an $E2$ requires 0.58 [31]. This however, only indicates a negative parity for the band, meaning the $8^-$ and $9^-$ configurations are both possibilities. The band-head is not well defined due to the out-of-band 774 keV transition being weaker than the 858 keV transition. Furthermore, the BCS predicted energies for the $8^-$ and $9^-$ configurations would be higher if residual inter-
actions were considered, and if a transition at the bottom of the band of \( \sim 200 \) keV were included in the level scheme, it could make the \( 8^- \) configuration more likely. However, as no such transition is indicated in the spectra, we tentatively assign this band \( K^+ = (9^-) \).

Intrinsic g-factors for the \( K^+=12^+ \) isomer band (see table 6.6) found strong agreement with Bark's predicted configuration [12] for this band (detailed in section 5.6), for which a theoretical value of \( |g_K - g_R| = 0.283 \) was obtained.

Table 6.6: Intrinsic g-factors for the \( K^+=12^+ \) isomer (top) and for the band populated by a 427 keV transition from this isomer (bottom). Energies are in MeV. Calculations assume \( K=8 \) for the new band, \( K=12 \) for the isomer band, and \( g_R = 0.3 \) using a quadrupole moment of \( Q_0 = 7.0 \) eb [85]. The intrinsic spin g-factors used were \( +5.4 \) and \( -3.8 \) for protons and neutrons respectively, with a quenching factor of 0.6.

| \( I, K \) | \( E_{\gamma}(M1) \) | \( E_{\gamma}(E2) \) | \( I_{\gamma}(M1) \) | \( I_{\gamma}(E2) \) | \( |g_K - g_R| \) |
|----------|-----------------|-----------------|-----------------|-----------------|------------------|
| 14,12    | 0.189           | 0.358           | 21.4(7)         | 2.6(2)          | 0.26$^{+2}_{-2}$  |
| 16,12    | 0.227           | 0.435           | 16.3(5)         | 5.6(3)          | 0.30$^{+2}_{-2}$  |
| 17,12    | 0.244           | 0.470           | 13.7(4)         | 7.1(3)          | 0.30$^{+2}_{-2}$  |
| 10,8     | 0.204           | 0.390           | 5.9(3)          | 2.2(2)          | 0.28$^{+3}_{-3}$  |

### SECTION 6.8

**The \( Q_0 \) and K of s-bands in \( N=82 \rightarrow 126 \) nuclei**

In section 2.9, the Walker method was outlined as a procedure for calculating the K value of an s-band. The method is dependent on intensity measurements of the two in-band and two out-band transitions observed at the band-crossing with the ground state band. The discovery of a 537.0 keV transition in \( ^{164}\text{Er} \) (see chapter 5) completed the set of four transitions required to determine the s-band K value. Figure 6.5 shows the relative \( \gamma \)-ray intensities for the four E2 transitions involved in the band-crossing, and the experimental transition rate ratios.
6.8. THE $Q_0$ AND $K$ OF S-BANDS IN $N=82\rightarrow126$ NUCLEI

![Diagram showing band crossing of s-band and g-band in $^{164}$Er.](image)

Figure 6.5: The bandcrossing of the s-band and g-band in $^{164}$Er. To the left of each arrow is the $\gamma$-ray energy, and to the right is the relative $\gamma$-ray intensity. The experimental E2 transition rate ratios, the level energies in parentheses (this does not denote uncertainty) and the level spins are given.

Following the procedure of section 2.9, the $K$ value of the s-band was calculated by reproducing these transition rate ratios, giving $K_s = -4.9$ and $V = 48.3$ keV. The procedure has been previously utilised for a number of well-deformed $N=82\rightarrow126$ shell nuclei [34]. Figure 6.6 shows the combined data.

![Plot showing $K_s$ value as a function of neutron number (N).](image)

Figure 6.6: A plot of s-band $K$ value ($K_s$) as a function of neutron number ($N$). Squares: existing data from Walker [34]. Circle: new data for $^{164}$Er. The line gives the maximum possible $K$ value, defined as $\Omega_1 + \Omega_2$. 
The negative $K$ value for $^{164}$Er requires that variables other than $K$ be considered as affecting the transition rates. The $s$-band rates appear to be enhanced, something that could result from a large relative quadrupole moment ($Q_0$) for the $s$-band. If the quadrupole moment of the two bands were not equal (as assumed in the Walker method), equation 2.26 becomes:

$$c = \frac{Q_0(1)}{Q_0(2)} \left[ \frac{(I - K_1)(I - K_1 - 1)(I + K_1 - 1)}{(I - K_2)(I - K_2 - 1)(I + K_2 - 1)} \right]^{\frac{1}{2}}. \quad (6.1)$$

For each transition rate ratio, the quadrupole moment of the band from which the transitions are proceeding is defined as $Q_0(1)$, so $K_1 = K_s$ and $Q_0(1) = Q_0(s)$ for $R[B(E2)]_s$.

A quadrupole moment for the $s$-band of $Q_0(s)/Q_0(g) = 1.12^{+0.05}_{-0.05}$ would require $K_s = 0$, sufficiently explaining the transition rate ratios. It is therefore reasoned that as $K_s \geq 0$, the quadrupole moment of the $s$-band in $^{164}$Er is at least 12% greater than that of the $g$-band. This is surprising as we expect shape stability for well-deformed nuclei, however, the similarly deformed $N=96$ nucleus, $^{162}$Dy, was calculated to have $Q_0(s)/Q_0(g) \approx 1.20$ [87], supporting the results of this work.

Large beta deformation (resulting in a large quadrupole moment) for $s$-bands relative to ground state bands at $N=96$ (for $^{164}$Er and $^{162}$Dy) could be explained in terms of the deformed shell model. At $N=96$ the two $i_{13/2}$ neutrons closest to the Fermi surface occupy $\Omega = 5/2$ and $\Omega = 7/2$ orbitals [84], resulting in a $K=6$ maximum. These orbitals that are in the lower half of the $i_{13/2}$ shell reduce in energy with increasing deformation, and once unpaired, could drive the nucleus to an energetically favourable, more highly deformed state.

It is therefore reasonable to theorise that unpaired high-$\Omega$ $i_{13/2}$ orbitals, which increase in energy with increasing deformation, would drive the nucleus to a less deformed state. There is some evidence for this theory. At $N=100$ the $\Omega = 9/2$ orbital is the lowest $\Omega$ orbital to increase in energy with increasing deformation. Nuclei with $N \geq 100$ are therefore expected to have less deformed $s$-bands. The $^{174}$W ($N=100$) nucleus is predicted by the Walker method to have $K_s \approx 8$ (see figure 6.6,
but the absence of direct transitions from a $K=12^+$ isomer to the $s$-band is difficult to reconcile with this prediction [88]. A relatively small quadrupole deformation for the $s$-band could explain this absence, as it would require a smaller $K$-value to explain the transition rate ratios. For example $Q_0(s)/Q_0(g) \approx 0.80$ requires $K_s \approx 2$, making the isomeric transitions highly forbidden.

For $^{184}$Os ($N=100$) the high-$\Omega$ orbitals, $\Omega = 9/2$ and $\Omega = 11/2$, form the $s$-band in this nucleus. The Walker method predicted $K_s \approx 8$, however this was inconsistent with a three-band mixing analysis [33], which predicted $K_s \approx 3.5$. If we assume $K_s = 3.5$ with the remaining discrepancy in the $B(E2)$ ratios accounted for by differing quadrupole moments, we get $Q_0(s)/Q_0(g) \approx 0.76$, consistent with the theory being proposed. Quantitatively, a relation in the form of equation 6.2 is implied:

$$Q_0(s) \propto Q_0(g) / (\Omega_1 + \Omega_2) \quad (6.2)$$

where $j=13/2$ for the $s$-bands being discussed in the $N=82\rightarrow126$ shell. However, it is immediately clear that this relation is insufficient for the lowest $\Omega$ orbitals, as it would predict huge quadrupole moments. As the theory concerns the energy gradient of the orbitals for a given deformation, one needs to evaluate the differential of the Nilsson equations for each $\Omega$ in the $i_{13/2}$ group. In other words, for each Omega: $dE_{s,\Omega}/d\epsilon_2 = f(\epsilon_2)$, where $f(\epsilon_2)$ is a function involving only the $\epsilon_2$ variable, is required. This task, and the evolution of the theory, are beyond the main focus of this thesis, but will undoubtedly be investigated by myself or by others in future publications.

Reduced hindrance systematics for 4QP and 5QP isomers in $N=82\rightarrow126$ nuclei

The reduced hindrances of the high-$AK$ branches from the $K^r=12^+$ isomer in $^{164}$Er (see table 5.11) can be compared with other 4QP isomers. Reduced hindrance
is well correlated with $E - E_R$ for 4QP and 5QP isomers in the $A=174 \rightarrow 184$ mass region, elucidating the importance of level-density K-mixing [89]. Deviations from the correlation are often explained by inferring the presence of Coriolis or axial asymmetry K-mixing.

With an $^{164}\text{Er}$ moment of inertia of $74 \, \hbar^2 \text{MeV}^{-1}$, calculated by scaling with $A^{5/3}$ from a $^{178}\text{Hf}$ moment of inertia of $85 \, \hbar^2 \text{MeV}^{-1}$, a rigid rotor energy of 1052 keV is obtained for $I=12$. This gives $E - E_R = 2.326$ MeV for the $K^+=12^+$ isomer. Figure 6.7 plots the reduced hindrance versus $E - E_R$ for the $E_2$ branch from the $K^+=12^+$, $^{164}\text{Er}$ isomer, alongside all known 4QP and 5QP isomers with $E_2$ or $E_3$ branches with a measured reduced hindrance.

The $^{164}\text{Er}$ isomer agrees well with the general trend of the data. The remarkably similar reduced hindrance of the $^{174}\text{Yb}$ isomer, which has an $E - E_R$ that is only 90 keV higher than $^{164}\text{Er}$, shows that using $E - E_R$ as a measure of level density K-mixing, is as valid at $A=164$ as it is in the $A=174 \rightarrow 184$ region.

A plot including the reduced hindrance of the $\Delta I=0$ branch (M1-assumed) from the $K^+=12^+$, $^{164}\text{Er}$ isomer, alongside all known 4QP and 5QP isomers with M1-assumed branches with a measured reduced hindrance, is shown in figure 6.8. Again, the agreement with the general trend and with $^{174}\text{Yb}$ is good. The overall correlation of the data in figure 6.8 is less impressive than figure 6.7, possibly due to variation in multipole mixing ratios.

The relatively close proximity of the isomer to the level density estimate of the reduced hindrance in figures 6.7 and 6.8 suggests that level density K mixing in the isomeric state is predominantly responsible for the reduced hindrance of the isomer. A much smaller contribution from other mixing effects is implied. This is consistent with the low K value predicted for the s-band in $^{164}\text{Er}$ (see section 6.8). An s-band with a high K-value (a t-band), that Coriolis mixes strongly with the ground state band, can decrease the reduced hindrance to well below the level density estimate. Similarly, a loss of axial symmetry in a rotation aligned s-band that mixes strongly with the ground state band can produce a smaller reduced
6.9. REDUCED HINDRANCE SYSTEMATICS FOR 4QP AND 5QP ISOMERS IN \( N = 82 \rightarrow 126 \) NUCLEI

Figure 6.7: Reduced hindrance values for \( \Delta K > 5 \), E2 and E3 transitions, updated from Ref.[89]. The circles represent 4QP isomers in even-even nuclides, and the squares are for 5QP isomers in odd-mass nuclides. The data are shown as a function of excitation energy relative to a rigid rotor, with a pairing energy of 0.9 MeV added for odd-mass nuclides. The curve represents a spin-dependent level-density estimate [90]. The open square is explained in the text.

These arguments have been applied to explain many of the very low \( f_r \) values in figure 6.7 [91; 92; 93; 94]. Indeed, the open square in figure 6.7 represents the reduced hindrance for a transition from the \( {^{179}}W \) isomer to a pure, un-mixed ground state band. The abnormally low \( f_r \) suggests mixing with the t-band in \( {^{170}}W \). The filled square represents the isomeric decay proceeding to the t-band. A 50% mixture of the two bands would produce a point precisely on the level density estimate [95].

In section 6.8 we calculated a mixing strength of \( V = 48.3 \) keV at the crossing the s-band with the ground state band in \( {^{164}}Er \). Assuming this mixing strength to
6.9. REDUCED HINDRANCE SYSTEMATICS FOR 4QP AND 5QP ISOMERS IN N=82→126 NUCLEI

Figure 6.8: Reduced hindrance values for ΔK>5, M1-assumed transitions. The circles represent 4QP isomers in even-even nuclides, and the squares are for 5QP isomers in odd-mass nuclides. The data are shown as a function of excitation energy relative to a rigid rotor, with a pairing energy of 0.9 MeV added for odd-mass nuclides. The curve represents a spin-dependent level-density estimate.

be consistent, and given that the 12+ levels in the two bands are 436.7 keV apart, the daughter level of the ΔI=0 branch from the K^*=12+, ^{104}Er isomer should be a relatively pure K=0 level. Together with the low K-value predicted for the s-band, this is good supporting evidence that Coriolis and axial asymmetry K-mixing have little effect on the reduced hindrance of the isomer, and that level density K-mixing in the isomeric state is the dominant mechanism.
For the \( K^\pi = 8^+ \) isomer in \(^{162}\)Dy, the E2, \( \nu = 6 \) transition (1639.2 keV) to the ground-state band has a reduced hindrance of \( f_\nu = 35 \) (see table 5.6). The relation between \( N_pN_n \) and reduced hindrance is shown in figure 6.9. The graph shows 2QP, E2 data from Walker et al. [24], with the addition of the \(^{162}\)Dy transition from this work, and a 694 keV transition from a recently discovered isomer in \(^{150}\)W [96]. The new data agree remarkably well with the existing trend. The previously known isomers are in \( Z = 70 \rightarrow 76 \) and \( N = 100 \rightarrow 108 \) nuclei. The \(^{162}\)Dy data point extends this to \( Z = 66 \) and \( N = 96 \), while \(^{190}\)W has \( N = 116 \).

Figure 6.9: Modified plot from Walker et al. [24]. The open circles represent the reduced hindrance of all E2, \( \nu > 4 \) transitions for 2QP isomers in the \( N = 82 \rightarrow 126 \) shell as a function of \( N_pN_n \). New data are shown for \(^{162}\)Dy with the filled square, and for \(^{190}\)W [96] with the filled circle. The fit-line is of the form \( g_\nu = A + \exp[(N_pN_n - 80)B/C] \) where \( A = 2.3, B = 1.873, \) and \( C = 3 \times 10^5 \).
The small yet significant deviations from the fit in figure 6.9 can be attributed to excitation energy effects. For example, the $^{162}$Dy isomer, which has an $f_{\nu}$ that is marginally below the fit, has an energy considerably above the yrast line, suggesting a reduction in $f_{\nu}$ through level-density induced K-mixing (see section 2.6.1 for a discussion of K mixing mechanisms). Figure 6.10 shows $f_{\nu}$ as a function of $E-E_R$ for the isomers being discussed.

In figure 6.10 the correlation with $E-E_R$ is poorest for $^{162}$Dy and $^{174}$Yb, which have the two largest $N_pN_n$ values, and $^{190}$W, which has the smallest $N_pN_n$ value. It is apparent that $N_pN_n$ dependent K-mixing is responsible for the poor correlation in figure 6.10, while the quality of the correlation in figure 6.9 establishes $f_{\nu}$ as being highly dependent on $N_pN_n$.

The small differences between the reduced hindrances and the fit line in figure 6.9, given by $f_{\nu} - g_{\nu}$ (where $g_{\nu}$ is the fit value), indicate the magnitude of the reduced hindrance that can be attributed to degrees of freedom other than $N_pN_n$.

The ratio $(f_{\nu} - g_{\nu})/f_{\nu}$ is the fraction of the reduced hindrance that this magnitude represents, a quantity that can be compared for all of the isomers being discussed.
Figure 6.11: The relative scatter from the fit in figure 6.9 as a function of $E-E_R$. The solid line is to guide the eye. Uncertainties are smaller than the data points.

As the correlation of $f_\nu$ with $N_pN_n$ is very good, it is reasonable to assume that the ratio removes all dependence on $N_pN_n$. Figure 6.11 shows this ratio as a function of $E-E_R$, identifying a small yet significant dependence on $E-E_R$, and confirming that it is partially responsible for the deviations from the $N_pN_n$ correlation. However, it is clear that this dependence is small in comparison to the $N_pN_n$ dependence. This suggests that the contribution of level-density K-mixing to $f_\nu$ is small for E2 decays from 2-quasiparticle isomers, in contrast to the situation for higher quasiparticle numbers [10; 9; 24; 90].

The $N_pN_n$ dependence, which incorporates $\gamma$-asymmetry and Coriolis effects, must be predominantly responsible for the K-mixing. As $E-E_R$ largely accounts for the residual deviations from the $N_pN_n$ fit, it appears that other considerations such as the isomer configurations are relatively unimportant. This is perhaps not surprising if the dominant K-mixing occurs within the daughter state of isomeric transitions. Nevertheless, the remaining scatter of data points in figure 6.11 implies that other degrees of freedom do indeed play a small role.
Conclusion

A fusion-evaporation experiment, with the purpose of observing delayed transitions in $^{164}$Er through $\gamma$-spectroscopy measurements, has revealed new isomers in $^{162}$Er (2QP), $^{165}$Er (3QP), $^{163}$Ho (3QP), $^{164}$Dy (1QP) and $^{162}$Dy (2QP). The $^{163}$Ho, $^{164}$Dy and $^{162}$Dy isomers were populated through incomplete fusion reactions, evident from the high production yields of these nuclei in comparison with the predictions of the PACE4 code. In all, 10 isomers were detected in 12 different isotopes, and the isomeric and isotopic yields were measured from $\gamma$-ray intensity measurements. Over 900 distinct $\gamma$-ray transitions, 105 of which are new to physics, were detected in the CAESAR array at the Australian National University, and placed within level schemes for the 12 isotopes. The new transitions were primarily decay branches or the associated rotational bands of the new isomers. Rotational bands were observed above all but the $^{162}$Er isomer. Gamma-ray branch intensity information, BCS blocking code predictions, and the intrinsic g-factors and alignment of the detected rotational bands, were used to assign candidate quasiparticle structures for all of the new isomers.

The new $^{162}$Er isomer was observed to decay by 2 $\gamma$-ray branches from a level energy of 2026.6(6) keV. A half-life of 88(16) ns was measured for the 1359.6 keV branch. The level and transition energies were previously known, with a spin of 7 assigned, although it had not been identified as an isomer. Similarity with the
K^r = 7^- isomer in ^{164}Er in terms of the level energy, transition energy, transition intensity, daughter levels, half-life and reduced hindrance for the new isomer in ^{162}Er allowed for a K^r = 7^- (π7/2^-[523]π7/2^+[404]), 2-quasiproton assignment. This was strongly supported by BCS calculations.

The new ^{163}Er isomer was observed to decay by 3 γ-ray branches from a level energy of 1823.0(3) keV. A half-life of 0.37(4) μs was measured for the 799.8 keV transition. Branch intensities to the daughter levels indicated a spin-parity of between 17/2 and 21/2. BCS calculations predicted a 19/2^+ and a 19/2^- configuration close to the observed isomer. The observation of both of these structures in the ^{163}Er nucleus supported the BCS predictions. Two rotational bands were observed above the ^{163}Er isomer, agreeing with the BCS predictions. The alignment and intrinsic g-factors of these bands allowed for their assignment as 19/2^+ and 19/2^-, although it could not be confirmed which band is built directly on the isomer.

The new ^{165}Ho isomer was observed to decay by 2 γ-ray branches from a level energy of 2109.4(3) keV. A half-life of 0.80(15) μs was measured for the 973.8 keV transition. Branch intensities indicated a spin 21/2 or 23/2 isomer on the basis of the higher energy transition having greater intensity, making a higher multipole unlikely. BCS calculations strongly supported a spin 23/2 assignment. The parity of the isomer is unclear as intrinsic g-factors and the alignment of the rotational band gave contradictory assignments. A lower energy, known isomer in ^{163}Ho, with an unknown half-life, and tentatively assigned K^r = 17/2^+ spin-parity, was found to have a half-life of 41(11) ns, as measured for the 973.8 keV transition. Branch intensities indicated a spin of 17/2 on the basis of greater intensity for the higher energy transition, and this was supported by BCS calculations. Quasiparticle excitation energy systematics in nearby nuclei, and BCS calculations supported a K^r = 17/2^+ assignment, although intrinsic g-factors indicated a negative parity. Given the poor statistics for the g-factor measurement, the tentatively assigned K^r = 17/2^+ configuration is supported.

The new ^{161}Dy isomer was observed to decay by 6 γ-ray branches from a level energy of 485.7(3) keV. A half-life of 0.76(17) μs was measured for the 385.3 and
284.3 keV branches. The level energy was previously known, as was the associated rotational band and the $K^*=11/2^-$ assignment.

The new $^{162}$Dy isomer was observed to decay by 11 $\gamma$-ray branches from a level energy of 2188.1(3) keV. A half-life of 8.3(3) $\mu$s was measured for the 504.3 keV branch. Branch intensities indicated an $8^+$ spin-parity on the basis of the spin-parity of the daughter levels, and the large transition strengths to $6^+$ levels. In support of this evidence, BCS calculations were performed, and the alignment and intrinsic g-factor of the rotational band above the isomer were measured. All strongly supported a $K^*$ = 8$^+$ ($\nu 11/2^-[505]\nu 5/2^-[523]$) 2-quasineutron configuration.

The transition rates of the branches from the $^{162}$Dy isomer to the 9$^-$ levels in the $K^*$ = 2$^-$ and $K^*$ = 5$^-$ bands favoured switching the levels between the two bands. This was indicated by a two-band mixing analysis as well as by hindrance factors for the two branches.

The reduced hindrance for the 1639.2 keV, E2 branch from the $^{162}$Dy isomer was determined to be 35. This value was compared with that for other highly K-forbidden ($\nu > 4$) 2QP isomers in $N=82$→$126$ nuclei that decay by E2 transitions, leading to a statistical analysis of the different K mixing processes, and an estimation of their relative importance. The analysis involved comparing small deviations from the well correlated relationship of $f_\nu$ with $N_pN_n$, and interpreting these as due to mechanisms other than Coriolis or $\gamma$-asymmetry K-mixing. The deviations showed a small correlation with $E-E_R$, indicating that level-density K-mixing plays a small role in reducing the hindrance of K isomers of this type. The remarkable correlation with $N_pN_n$ was seen as strong evidence that Coriolis and $\gamma$-asymmetry K-mixing are the dominant mechanisms in these isomers.

The observation of a new $\gamma$-ray transition, at 537.0 keV in $^{164}$Er, completed the set of four transitions required to perform an analysis that allows for the calculation of the K value of the s-band. The new transition proceeds from the 16$^+$ level of the ground band to the 14$^+$ level of the s-band and is one of four E2 transitions involved at the band-crossing of the two bands. The analysis found a negative K value for
the s-band as a result of greatly enhanced s-band transition rate measurements. The assumption of equality in the quadrupole moments for the two bands had to be revised to restore positivity. By factoring differing quadrupole moments into the analysis equations, it was calculated that the quadrupole moment of the s-band in $^{164}$Er must be at least 12% larger than that of the ground state band. This result found agreement with a work describing the s-band of the similarly deformed nucleus $^{162}$Dy. A theory was devised, speculating that unpaired high-$\Omega$ $i_{13/2}$ orbitals, which increase in energy with increasing deformation, would drive the nucleus to a less deformed state, while unpaired low-$\Omega$ $i_{13/2}$ orbitals would drive the nucleus to a more deformed state. The s-bands in $^{174}$W and $^{184}$Os were interpreted in this way, better reconciling measurement with prediction.

The half-life of the 4QP, $K^\pi=12^+$, $^{164}$Er isomer was measured to be 68(2) ns, disagreeing with the estimation of Bark et al. [12] who measured $T_{1/2} > 170$ ns. It is thought that as the Bark measurement was based on the width of a coincidence window, significant uncertainties could have been introduced. The $K^\pi=12^+$ structural assignment made by Bark was supported in this work by intrinsic g-factor measurements. Four new decay branches from the isomer were observed, two of which decay to new high-$K$ rotational bands. The other new branches are $\Delta K=12$ transitions to the ground state band, with reduced hindrances of $\sim 6$. The reduced hindrances were compared with that for high-$\nu$ 4QP and 5QP isomers in the $A\approx 180$ isomer group. A plot of reduced hindrance versus $E-E_R$ showed that the level density $K$ mixing model used to interpret decays in the $A\approx 180$ group is applicable at $A=164$. The proximity of the new $^{164}$Er data points to the level density estimate of reduced hindrance indicated that level density K mixing is the dominant mechanism in the reduced hindrance of the isomer, and that Coriolis and axial asymmetry K mixing play a much smaller role. This is supported by indications that the $K$-value of the s-band is low, and the mixing of the s-band with the ground state band is weak, features that could otherwise produce a relatively small reduced hindrance for the decay through the Coriolis and axial asymmetry K mixing mechanisms.
In this work, the statistical analysis used to interpret the reduced hindrance of highly K-forbidden 2QP isomers in the N=82→126 shell should prove useful to others wishing to gauge the contribution of the level density K mixing mechanism in these isomers, allowing for a better interpretation of isomer decay rates. The analysis of s-bands in the N=82→126 shell, and the prediction of a link between the $\Omega$-dependent energy gradient of rotation-aligned nucleons and quadrupole deformation, could, as has been demonstrated already for $^{174}$W, be used to interpret decay rates from isomers into states involving the s-bands of nuclei. The discovery of high-$\Delta K$ branches from a 4QP isomer in $^{164}$Er has reinforced the universality of an existing model that describes the decay rates of 4QP and 5QP isomers. Thus, it is believed that the aims set out in the introduction have been sufficiently met, and our understanding of isomer decay rates should improve as a result of this work. In addition, the discovery of 5 new isomers, and 105 new $\gamma$-ray transitions, should provide a wealth of additional data for the models and theories of others, allowing for more precise and wide-reaching predictions of isomer properties.
Bibliography


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