Reconstruction of Clinical Bremsstrahlung Spectra in the Range 4 to 30 MeV

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Abstract

Analytical and numerical models for reconstructing clinical bremsstrahlung spectra with maximum photon energies in the range 4 to 30 MeV from indirect measurements such as transmission or depth-dose curves are investigated and compared. The iterative and Laplace transform models of Huang et al and Archer and Wagner are extended to the energy region of interest and a further two models based on thin and thick target bremsstrahlung theory developed.

Calculated transmission curves for a set of 21 simulated and measured bremsstrahlung spectra covering a broad range of filtration conditions are used to assess each model's ability to represent photon spectra.

A model based on the Schiff expression for forward-directed thin-target bremsstrahlung differential in photon energy with added inherent filtration is shown to provide a promising method for reliably reconstructing megavoltage spectra in terms of 3 parameters. For this model, input data was reproduced to within 0.1 percent, this being the same order of magnitude as the accuracy of its generation and within expected uncertainties for measured input data.

Inclusion of a fourth parameter in the model is shown to allow the effective maximum photon energy present in the spectrum to be derived.

The considerations involved in the practical use of reconstruction models to derive clinical photon spectra from measured transmission and depth-dose data and likely applications of these spectra are discussed.
Acknowledgements

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I am greatly indebted to the late Professor Daphne F. Jackson, who despite illness and a heavy work load as Head of the Physics Department at Surrey, as well as her many other commitments, always made time for regular discussions and provided a great deal of encouragement in the early stages of the work.

I would like to thank Dr Nicholas M. Spyrou for taking over the supervision of the work and Dr Behain Ama‘ee for support through the latter stages of the project.

On the non-academic side, many thanks to my parents for their full support and much needed financial aid throughout these final years of my student career.
Chapter 1

Introduction

The spectral distribution of megavoltage x-ray beams used in radiotherapy departments is a fundamental quantity from which in principle all required information relevant to radiotherapy treatments could be determined. As well as providing the fullest description of a beam’s quality, it is an essential ingredient in the calculation of dose-distributions using convolution techniques [29,30,2,31,70,75,73,106,7] which have received much attention over the past few years. If measured on a regular basis, routine quality assurance protocols are provided with a quantitative measure of changes in output from linear accelerators due to processes such as target pitting or variation in incident electron energy. The effects of beam flattening filters on the photon spectrum and in particular it’s variation away from the beam central axis can also be established. Calculation of stopping power ratios for secondary electrons set in motion by incident photon beams [6] and mass-energy absorption coefficients needed for dosimetry and calibration procedures [34] also require knowledge of the distribution in energy of x-ray beams.

Direct measurement of megavoltage photon spectra is difficult due to the high dose-rates involved in radiotherapy treatments and the large range of high-energy photons, which can lead to pulse pile-up in and escape from the detector respectively. Corrections for these effects and also the energy dependence of the detector used, which are required in order to unfold the spectrum from measurements are far from trivial to estimate [43]. Despite these problems measurements have been obtained by several workers [68,69,32,42,43,66] with sodium iodide or germanium...
detectors either directly or by Compton spectrometry. The machine-time and equipment required for such measurements makes these techniques impractical in all but a small number of radiotherapy departments.

Monte-Carlo simulation has been widely used to determine the shape of clinical spectra [74,36,41,7]. This is possible only where full descriptions of machine-head construction are available, which is information rarely divulged by competing manufacturers and in any case does not provide the opportunity for comparison of output spectra over a period of time.

In the diagnostic energy region (up to approximately 0.3 MeV) the sharp decrease in total linear attenuation coefficients with increasing energy due to the dominance of the photoelectric cross-section has been used by many workers to derive information on the photon spectrum. Silberstein [98] first described how a number of narrow-beam transmission measurements through a suitable filter material could be used to derive the photon spectrum by the use of Laplace transformations. This approach was extended by Bell [20], Jones [60], Greening [46,48] and Archer and Wagner [9]. Other approaches include that of Dance [38], who represented diagnostic spectra as a superposition of known spectral shapes calculated by Birch and Marshall [22] and the application of neural networks by Boone [27,28,37].

In the low megavoltage region the use of Laplace transform models has been continued by Saylor [93] for 2 MeV spectra and more recently Huang et al [51] for 4 MeV spectra and Archer and Wagner [10] for maximum photon energies up to 25 MeV, although reliable results were only obtained up to 2 MeV. The use of iterative procedures to extract the photon spectrum from transmission measurements has also been investigated by Twidell [104] and Huang [52,53]. Piermattei [87] investigated the application of the model proposed by Huang for photon energies up to 40 MeV, concluding that reliable results could be obtained up to 10 MeV.

Depth-dose curves in water are also characteristic of the incident photon spectrum and have been used to derive spectra with maximum beam energies from 4 to 24 MeV by Ahnesjo and Andreo [3] who formed a reconstruction model from an approximate expression for thick-target bremsstrahlung production and Sauer and Neumann [91] for energies up to 16 MeV who represented depth-dose curves.
from a polyenergetic photon spectrum as a superposition of monoenergetic depth-dose curves. Nath and Schulz [80] derived 30 MeV spectra using photoactivation foils and applying orthonormal polynomial expansion to the measured reaction probability.

The measurement of transmission and depth-dose curves is relatively simple to carry out in any radiotherapy department to a high degree of accuracy and could therefore provide an accessible routine check on output spectra if a reconstruction model can be found which reliably represents megavoltage spectra over the full range of beam energies and filtrations of interest to radiotherapy. A complicating factor in the megavoltage region however is the reduction in differentiation between successive energies due to the slowly decreasing Compton cross-section and increased contribution of the pair-production cross-section to the total attenuation coefficient, making the task of deriving spectra more demanding on the method employed.

The aim of this project was therefore to investigate the possibility of deriving clinical megavoltage x-ray spectra from the indirect measurements of transmission and depth-dose curves by the use of analytical or numerical models. This has been achieved by comparing and extending where necessary previously published models which have been shown to be successful in the diagnostic and megavoltage energy range and by constructing new models more firmly based on the theoretical shape of clinical bremsstrahlung spectra in the energy region of interest.

In chapter 2 the principles of the analysis and details of the methods employed to compare various models are described along with outlines of the dosimetry considerations involved with the measurement of transmission curves and the relation of these measurements and those of depth-dose curves to the photon energy fluence. Chapters 3 and 4 are concerned with extending the region of application of the published models of Huang et al [52,53] and Archer and Wagner [9,10,11,12,13,14] which rely on iterative and Laplace transform methods respectively. Chapters 5 and 6 describe a more theoretical approach to the problem by considering analytical expressions for both thin and thick-target bremsstrahlung spectra respectively and forming reconstruction models by parametrizing such expressions. In chapter
7 results for each approach are compared and discussed. The practical considerations involved in applying a reconstruction model to measurements are discussed and applications of such models are suggested.
Chapter 2

Principles of Analysis and Generation of Input Data

The transmitted intensity of an x-ray beam through given thicknesses of filter material and the dose received in a water phantom at a given depth are characteristic of the incident beam. These quantities may be easily and accurately measured in a radiotherapy department and have been used for many years to serve as a check of x-ray beam quality and to provide depth-dose information for patient treatment planning. In order to derive the energy fluence of an incident x-ray beam, we must relate these measurements to the photon spectrum. In the following sections the dosimetry considerations, principles of the analysis procedure and details of input data generation are described.

2.1 Transmission Curves

The measurements discussed below are assumed to be made with an air-filled ionization chamber with air-equivalent walls, encased in water equivalent build-up material to achieve charged particle equilibrium (CPE). The general geometry under consideration is depicted in figure 2.1.

For photon beams with maximum energies up to approximately 2 MeV, the absorbed dose to water, $D_w(k)$, for mono-energetic photons of energy, $k$, may be
determined using an exposure-calibrated cavity chamber via [62,16,49]  

\[ D_w(k) = MN \left( \frac{W_a}{e} \right) \frac{[\mu_{en}(k)/\rho]_w}{[\mu_{en}(k)/\rho]_a} \]  

(2.1.1)

where \( M \) is the chamber reading, \( N \) the exposure calibration factor, \( W_a \) the average energy required to produce an ion pair in air and \([\mu_{en}(k)/\rho]_m\) the mass-energy absorption coefficient for photons of energy, \( k \), in material, \( m \) where the suffixes \( w \) and \( a \) refer to water and air respectively. If CPE is established, this dose can be expressed in terms of the incident photon energy fluence, \( \Psi(k) \), differential in photon energy as

\[ D_w(k) = \Psi(k)[\mu_{en}(k)/\rho]_w \]  

(2.1.2)

For a poly-energetic photon beam the measurements can then be expressed in terms of the incident spectrum as

\[ M = \int_0^E \Psi(k)[\mu_{en}(k)/\rho]_w \left( \frac{e}{W_a} \right) R(k) \, dk \]  

(2.1.3)
where $E$ is the maximum photon energy present in the spectrum and the function, $R(k)$, describes the energy dependence of the detector.

Equation 2.1.1 assumes that all secondary electrons entering the air-filled cavity are produced in the chamber wall material. At energies above 2 MeV, this assumption becomes less valid as contributions from the surrounding medium become larger. For this reason the concept of exposure is not used for photon energies above approximately 2 MeV [62,49,16]. If the assumption is made that the ionization produced in the cavity chamber is due to electrons generated in the surrounding build-up material only [62], then the dose, $D_w(k)$ due to mono-energetic photons of energy, $k$, may be expressed in terms of the measured charge per unit mass in the cavity, $J_a$ [49]

$$D_w(k) = J_a \left( \frac{W_a}{e} \right) s_{w,a}$$  \hspace{1cm} (2.1.4)

where $s_{w,a}$ is the stopping power ratio from water to air for the secondary electrons entering the chamber.

If CPE is established, the dose may again be represented in terms of the incident photon fluence by equation 2.1.2 and the reading, $M$, obtained by a chamber placed in a polyenergetic x-ray beam will then represent the quantity

$$M = \int_0^E \Psi(k) \frac{[\mu_{en}(k)/\rho]_w s_{a,w}}{[\mu_{en}(k)/\rho]_w} \times R(k) \exp\{-\mu_n(k)z_i\} \, dk$$  \hspace{1cm} (2.1.5)

In practice for the energy range of interest in this work, electrons set in motion in both the wall material and surrounding medium will contribute to the measured ionization produced in the chamber [34,67]. If $\beta$ denotes the fractional ionization due to electrons generated in the surrounding medium, the total dose is given by [97,49]

$$D_w(k) = J_a \left( \frac{W_a}{e} \right) \left\{ \beta s_{a,w} + (1 - \beta) \frac{[\mu_{en}(k)/\rho]_w}{[\mu_{en}(k)/\rho]_a} \right\}^{-1}$$  \hspace{1cm} (2.1.6)

From the above dosimetry considerations narrow-beam transmission measurements, $S(z_i)$, will be given by the expression

$$S(z_i) = \int_0^E \Psi(k) \left( \frac{e}{W_a} \right) \left\{ \beta s_{a,w} \left[ [\mu_{en}(k)/\rho]_w + (1 - \beta) [\mu_{en}(k)/\rho]_a \right] \times R(k) \exp\{-\mu_n(k)z_i\} \right\} \, dk$$  \hspace{1cm} (2.1.7)
where $x$ represents the thickness of a chosen filter material and $\mu_m(k)$ is the total linear attenuation coefficient for photons of energy, $k$, for this material. Since in this work we will be considering calculated transmission curves, we simplify the above expression by assuming $\beta = 1$ and represent the transmission curve as

$$S(x) = \int_0^\infty \Psi(k)R(k) \exp[-\mu_m(k)x] \, dk$$  

(2.1.8)

where the constant, $W_a/e$, has been excluded since the transmission measurements are normalized such that $S(0) = 1.0$ and the mass energy absorption coefficient, $[\mu_{en}/\rho]_a$, is absorbed into the function, $R(k)$. The stopping power ratio from air to water, $s_{a, w}$, is a slowly varying function for the energy range considered here [57] and so is taken to be unity.

If a filter material is selected whose total linear attenuation coefficient varies monotonically with energy, then by approximating equation 2.1.8 with a summation over a defined discrete array of energies, $k_j$, it is possible in principle to deduce the photon energy fluence from a set of transmission values by constructing a set of simultaneous linear equations;

$$S(x) = \sum_{j=1}^m a(i,j)\Psi(k_j)\Delta k$$

$$S(x_2) = \sum_{j=1}^m a(2,j)\Psi(k_j)\Delta k$$

$$\vdots$$

$$S(x_n) = \sum_{j=1}^m a(n,j)\Psi(k_j)\Delta k$$  

(2.1.9)

where $a(i,j) = R(k)\exp[-\mu(k_j)x_i]$ and $\Delta k$ is the spacing between energy values.

Writing equations 2.1.9 in terms of matrices we have

$$S_{n1} = a_{nn} \times \Psi_{m1}$$  

(2.1.10)

Provided $n \geq m$ equation 2.1.10 could in principle be solved for $\Psi$, however, errors in the calculation of the elements of matrix, $a$, due to uncertainties in the values of $\mu_m(k)$ makes the exact solution of equation 2.1.10 often unphysical and rarely possible as $a$ is often singular. A more sophisticated approach is therefore needed to ensure that physically realistic solutions are obtained.
2.2 Depth-dose Curves in water

In the preceding section, narrow-beam geometry was considered in order to minimize scatter contributions so that photon spectra may be derived using tabulated values of the total linear attenuation coefficient. The dimensions of the treatment room and linear accelerator itself will limit the experimental geometry which can be achieved and so may greatly restrict the accuracy with which narrow-beam transmission curves may be measured. In such cases, the measurement of depth-dose curves in a water phantom may be more easily carried out, using full scatter conditions to simulate the dose received by tissue. Figure 2.2 shows the geometry under consideration for depth-dose measurements.

Despite the ease of measurement, relating the absorbed dose at a point, \( p \), at a depth, \( z \), on the beam central axis to the photon energy fluence incident on the water phantom is very difficult to derive from first principles. The total dose at point, \( p \), results from electrons set in motion by photon interactions at many...
different points in the phantom, to calculate this dose we therefore need to describe both the transport of electrons and the photon distribution with energy throughout the phantom.

Following the method of Ahnesjo et al [2,3] the total energy released per unit mass (TERMA) at a point, $r$, in the phantom by primary photons of energy, $k$, is given by

$$ T(k, r) = \mu_w(k) \Psi(k, r) $$

(2.2.1)

where $\mu_w(k)$ is the total linear attenuation coefficient for water (expressed in cm$^2$ g$^{-1}$) and the primary photon energy fluence at point $r$, $\Psi(k, r)$, is related to the energy fluence incident on the phantom surface, $\Psi(k)$, through

$$ \Psi(k, r) = \Psi(k) \left( \frac{r_0}{r} \right)^2 \exp[-\mu_w |r - r_0|] $$

(2.2.2)

Where the quadratic term corrects for the inverse-square law decrease in fluence. The TERMA includes energy carried away by scattered photons, unlike the quantity KERMA (kinetic energy released per unit mass) [49] which describes only the kinetic energy imparted to charged particles. The total TERMA for a polyenergetic beam will then be given by

$$ T(r) = \int_0^E \mu_w(k) \Psi(k, r) \, dk $$

(2.2.3)

The dose deposited at point, $p$, due to interactions of primary photons of energy, $k$, at point, $r$, can be written

$$ D(k, p) = T(k, r) h(k, p - r) $$

(2.2.4)

where $h(k, p - r)$ is a point spread function (PSF) which describes the fraction of energy released at $r$ which is deposited at $p$, normalized such that

$$ \int_V h(k, r) \, d^3r = 1 $$

(2.2.5)

Calculation of this PSF requires Monte Carlo simulation and has been carried out by Ahnesjo et al [2,3] for photon energies over the range 0.1 to 50 MeV. The total dose at point $p$ will then be given by integrating over primary photon energies and

\[ \text{\ldots} \]
the total phantom volume, \( V \).

\[
D(p) = \int_V \int_0^E T(k, r) h(k, p - r) \, dk \, d^3r \tag{2.2.6}
\]

\[
= \int_V \int_0^E \Phi(k, r) K(k, p - r) \, dk \, d^3r \tag{2.2.7}
\]

\[
= \int_0^E \Phi(k, r_0) d(k, p) \, dk \tag{2.2.8}
\]

where \( K(k, p - r) \) is the PSF for dose transport used by Boyer and Mok [29,30, 31] and \( d(k, p) \) the dose per energy fluence function described by Ahnesjo and Andreo [3] which from equations 2.2.1 and 2.2.6 is seen to be related to the PSF, \( h(k, p - r) \), through

\[
d(k, p) = \int_V \mu_w(k) \exp[-\mu_w(k)|p - r|] h(k, p - r) \, d^3r \tag{2.2.9}
\]

Boyer and Mok [29] constructed the PSF, \( K(k, p - r) \), by considering individual contributions to the deposited dose at \( p \) from interactions of primary, single scattered and multiply scattered photons;

\[
K(k, p - r) = K_p(k, p - r) + K_s(k, p - r) + K_m(k, p - r) \tag{2.2.10}
\]

where the suffixes \( p \), \( s \) and \( m \) refer to primary, single scattered and multiply scattered photons respectively. The kernel describing the primary contribution being given by

\[
K_p(k, p - r) = \mu_w(k) f(k, p - r) \tag{2.2.11}
\]

where \( f(k, p - r) \) describes the fractional dose deposited at \( p \) due to electrons set in motion at \( r \) in a similar manner to the function, \( h(k, p - r) \), mentioned previously. Calculation of this kernel by Monte Carlo techniques has been studied by Mackie [70].

By using the dose per energy fluence function, \( d(k, p) \), we can represent a depth-dose curve defined at depths, \( z_i \), in terms of the incident photon energy spectrum, \( \Phi(k) \), in a similar manner to that obtained for transmission curves in the last section;

\[
D(z_i) = \int_0^E \Phi(k) d(k, z_i) \, dk \tag{2.2.12}
\]
\(\Psi(k)\) could therefore be derived in principle from a number of depth-dose measurements. Unfortunately, the similarity with the use of transmission curves includes the fact that a satisfactory solution for \(\Psi(k)\) cannot generally be obtained and so the use of analytical or numerical models is again required.

### 2.3 Spectrum Data-base

In order to compare various reconstruction models a data-base of measured and simulated spectra was gathered in an attempt to provide the full range of spectral shapes likely to be found across radiotherapy centres, covering the range of maximum photon energies from 4 to 30 MeV. Five sources of spectral data were used, providing geometries ranging from simple targets of tungsten or lead alone to full simulations of clinical linear accelerator heads including beam flattening filters used to ensure uniform dose along the beam profile.

Seven spectra with maximum photon energies 4, 6, 8, 10, 12, 16 and 19 MeV were simulated at the NPL [41,34] using the EGS4 Monte Carlo code [84,81]. These spectra represent the bremsstrahlung produced from electrons incident on tungsten targets of 3 to 5 mm thickness, lightly filtered with small thicknesses of either copper or tungsten. Exact details of target thickness and filtration conditions for each spectrum are given in table 2.1.

<table>
<thead>
<tr>
<th>Energy (MeV)</th>
<th>Target (mm W)</th>
<th>Filtration (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>3</td>
<td>3 Cu</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>3 Cu</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>4 Cu</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>5 Cu</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td>3 W</td>
</tr>
<tr>
<td>16</td>
<td>5</td>
<td>4 W</td>
</tr>
<tr>
<td>19</td>
<td>5</td>
<td>5 W</td>
</tr>
</tbody>
</table>

Table 2.1: Target thicknesses and additional filtration for NPL Monte Carlo simulations [41,34]

A two degree collection angle from the incident electron direction for photons radiating from the target was selected for all cases as this is comparable with clinical geometries. Smoother spectra would be obtained for larger collection angles, however, as well as increasing the proportion of low energy photons this would also
increase the proportion of annihilation photons collected, which are distributed equally in all directions and give rise to an uncharacteristic peak in the spectrum at 0.511 MeV.

Measurements of bremsstrahlung spectra resulting from incident electrons of maximum energies 10, 15, 20, 25 and 30 MeV have been made using a large NaI detector (20 cm diameter by 25 cm long) by Faddegon et al [42,43]. Targets of lead with thicknesses nominally 10 percent larger than the csda range [63,57] of the incident electrons were used. Actual thicknesses and electron csda ranges for the five spectra are given in table 2.2. In order to unfold the photon spectra from measurements, Monte Carlo simulations of the detector response were carried out.

<table>
<thead>
<tr>
<th>Energy (MeV)</th>
<th>Target thickness (mm Pb)</th>
<th>csda range (mm Pb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>6</td>
<td>5.404</td>
</tr>
<tr>
<td>15</td>
<td>8</td>
<td>7.008</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
<td>8.281</td>
</tr>
<tr>
<td>25</td>
<td>10</td>
<td>9.330</td>
</tr>
<tr>
<td>30</td>
<td>12</td>
<td>10.23</td>
</tr>
</tbody>
</table>

Table 2.2: Target thicknesses for measured spectra of Faddegon et al [42,43] and electron csda ranges in lead [57]

In the introduction it was suggested that if complete details of the machine head construction are available, then a full simulation of the expected photon energy spectrum for a given linear accelerator can be made. Mohan et al [74] have carried out such simulations for 4, 6, 15 and 24 MeV spectra from a number of linear accelerators manufactured by the Varian Corporation. Details of target thickness and additional filtering are given in table 2.3. A photon collection angle of 3 degrees was selected from Mohan et al's data for the same reasons mentioned for the simulated NPL spectra.

<table>
<thead>
<tr>
<th>Energy (MeV)</th>
<th>Target thickness (mm W)</th>
<th>Filtration (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1.07</td>
<td>1.52 Cu, 11.89 Pb/6% Sb</td>
</tr>
<tr>
<td>6</td>
<td>1.02</td>
<td>1.52 Cu, 15.04 Pb/6% Sb</td>
</tr>
<tr>
<td>15</td>
<td>0.64</td>
<td>7.92 Cu, 19.30 W</td>
</tr>
<tr>
<td>24</td>
<td>0.76</td>
<td>7.23 Cu, 83.06 Fe</td>
</tr>
</tbody>
</table>

Table 2.3: Target thicknesses and filtration details for simulated spectra of Mohan et al [74,36]
Compton spectrometry of 15, 20 and 25 MeV x-ray beams has been performed by Landry and Anderson [66] using a high purity Ge detector with a volume of 60 cm$^3$. All spectra were produced by a Sagittaire clinical accelerator with a 4.0 mm tungsten target. An aluminium scatterer placed in the beam enables photons interacting by the Compton effect to be measured at a carefully determined angle which is later used to convert the Compton-scattered spectrum to that incident on the scatterer. Additional filtration of the beam due to the experimental set-up was removed by calculation.

Levy et al [68,69] also used Compton-spectrometry to derive the photon spectra produced from a Mevatron VII (Applied Radiation Corporation) and Sagittaire linear accelerator. The Compton-scattered spectra from 8 MeV electrons incident on a 0.51 mm platinum target and 27 MeV electrons incident on a 3.0 mm tungsten target were measured using a NaI(Tl) crystal 12.7 cm in diameter and length. Beam flattening filters were present during the measurements for both spectra.

Comparisons of spectra from the different sources are shown in figures 2.3 and 2.4. The effect of the heavier filtration present in the spectra of Mohan et al [74, 36] as compared to the lightly filtered spectra from the NPL [41] and measured by Faddegon et al [42] is clearly shown in the 4, 6 and 15 MeV plots. Although not as marked, a similar difference in beam filtration is evident between the measured 20 and 25 MeV spectra of Landry et al [66] and Faddegon et al.

2.4 Generation of Transmission Data

The calculation of transmission curves is straightforward and can be achieved to a high degree of accuracy since the only uncertainty lies in the value of the total linear attenuation coefficient, $\mu_m(k)$, interpolated from tabulated data and the numerical evaluation of the integral in equation 2.1.8. Since the calculation of depth-dose curves requires the time-consuming simulation of dose per energy fluence, $d(p,k)$, values, only the reconstruction of energy fluence spectra by analysis of transmission data will be considered further. With the possible exception of the Laplace-transform approach to spectral reconstruction, which relies on fitting an analytical expression to the input transmission curve, the comparison between
Figure 2.3: Comparison of 4, 6, 8 and 10 MeV spectra from the NPL [41], Mohan et al [74], Faddegon et al [42,43] and Levy et al [69]
Figure 2.4: Comparison of 15, 20 and 25 MeV spectra from the NPL [41], Mohan et al [74], Faddegon et al [42, 43] and Landry et al [66].
models would be expected to be independent of whether transmission curves or depth-dose curves are used for the analysis as it is the representation of the photon energy fluence itself which is being assessed.

Transmission curves were calculated for the spectra described in the last section to provide input data to all models being investigated. Following the work of Huang et al [51,52,53] and Piermattel et al [87] transmission curves were defined by 25 thicknesses of filter material logarithmically spaced to provide transmission values of down to 0.1 percent of the initial beam intensity. For reconstruction models which fit either an assumed analytical spectral shape or discrete energy fluence values directly to the input transmission curve, the function, $R(k)$, of equation 2.1.8 was set to unity in the calculation of the input data, since any correction incorporated in the transmission curve calculation can as easily be accounted for in the particular model. For the Laplace transform method mentioned in the introduction however, this is not possible since the fitting in this case is to the Laplace transform of the energy fluence spectrum (which provides an analytical expression for the transmission curve). Any corrections required to the input transmission curve must therefore be absorbed by the assumed functional form of the energy fluence during the fitting and removed later. This feature could only be overcome if the function, $R(k)$, of equation 2.1.3 is known analytically and the Laplace transform of its product with the energy fluence function can be found.

Since the majority of spectra considered in this study have mean photon energies above 2 MeV, the function, $R(k)$, for the calculation of input data to the Laplace transform model was set equal to the mass energy absorption coefficient for water [54], $\mu_{m}/\rho_{w}$, for the dosimetry reasons outlined in section 2.1. Values of this coefficient corresponding to the energies defining the reconstructed spectrum were found by fitting a monotonicity-preserving Hermite polynomial interpolant [76] to the log of the data given in reference [54]. For energies above 20 MeV values were found by log-linear extrapolation [85]. Interpolated, extrapolated and tabulated values of $\mu_{m}/\rho_{w}$ for the energy range considered are shown in figure 2.5.

Look-up tables were constructed containing values of the total linear attenuation coefficient, $\mu_{m}(k)$, for likely filter materials covering photon energies from
Figure 2.5: Interpolated, extrapolated and tabulated [54] values of the mass energy absorption coefficient for water
0.044 to 30.0 MeV using Berger and Hubbell's program XCOM [21]. A monotonicity preserving Hermite polynomial [76] was again fitted to the log of attenuation coefficient values to allow interpolation to required energies. Evaluation of the attenuation integral of equation 2.1.8 was performed by finite difference formulae [78], an estimated value of the error in the integral evaluation due to the numerical procedure is returned. In all cases, this estimate was less than 0.1 percent.

2.5 Choice of Filter Material

As mentioned in section 2.1, in order for the energy fluence to be derived from transmission measurements the total linear attenuation coefficient for the filter material, $\mu_m(k)$, should be monotonic over the expected range of photon energies in the spectrum to be reconstructed. To optimize the analysis these $\mu_m(k)$ values must also provide the greatest differentiation between successive energy bins, in other words, the magnitude of $d\mu_m(k)/dk$ must be maximized. Figures 2.6 and 2.7 show values of $|d\mu_m(k)/dk|$ for possible filter materials covering a wide range of atomic number. Minima are given where the pair-production cross-section begins to increase the total attenuation coefficient against the decreasing Compton cross-section. The figures show that lead is only suitable for maximum beam energies below approximately 3 MeV, water or carbon should be used for all energies up to 30 MeV, unless the bulk of the energy fluence spectrum is expected to lie below approximately 1.5 MeV in which case aluminium or copper becomes more favourable.

For some reconstruction models it would be possible to employ mixed filter analysis [52,53], further optimizing the energy differentiation by changing between filter materials when indicated by the mean energy of the transmitted beam. Differentiating equation 2.1.8 with respect to thickness, $x$, we obtain the fluence-weighted mean total linear attenuation coefficient, $\bar{\mu}_m(k, x)$ of the transmitted beam for filter thickness, $x$;

$$
\frac{dS(x)}{dx} = - \int_0^E \mu_m(k)\Psi(k)\exp(-\mu_m(k)x)dk
$$

$$
= -\bar{\mu}_m(k, x)
$$

(2.5.1)
Figure 2.6: Variation of the slope, $d\mu_m(k)/dk$ of the total linear attenuation coefficient with energy for Al, Cu, Pb, $H_2O$ and C up to 5 MeV.
Figure 2.7: Variation of the slope, $d\mu_m(k)/dk$ of the total linear attenuation coefficient with energy for Al, Cu, Pb, H$_2$O and C above 5 MeV
hence the mean energy of the transmitted beam can be found for different filter thicknesses and the optimum filter chosen to continue the attenuation. By considering figures 2.6 and 2.7 we see that the introduction of the above procedure would at most only lead to changing between aluminium/copper and water/carbon filters for the photon energies of interest and since this would be expected to effect all models equally, it has not been employed in the generation of transmission curves. In the following chapters, unless otherwise stated, the filter material used for generation of input transmission data from the spectra described in section 2.3 was aluminium for maximum beam energies up to and including 6 MeV and water at higher energies.
Iterative Models

3.1 Background

In the previous chapter the principle of deriving a photon spectrum by solving equation 2.1.10 for a spectrum defined by \( m \) energy intervals from a number, \( n \geq m \), of transmission measurements was described. Although an exact solution cannot generally be obtained for real transmission measurements, the use of iterative techniques employing constraints on the possible spectral shapes allows approximate but physically realistic spectra to be obtained.

Using the notation of equation 2.1.9 we represent the transmission curve of the iterative model, \( S_H \), in the form

\[
S_H(x_i) = \sum_{j=1}^{m} \Psi(k_j) \Delta k \exp[-\mu(k_j)x_i]
\]

(3.1.1)

where \( \Delta k \) is the spacing in the energy array and \( \mu(k_j) \) is the total linear attenuation coefficient for the filter material for photons of energy, \( k_j \). The method then involves sequentially modifying the spectral elements, \( \Psi(k_j) \), re-calculating the transmission curve and comparing this with that of the spectrum which is to be reconstructed. If improved agreement is obtained, then the modification is retained and the process is repeated for the next spectral value. The basic procedure is outlined below;

1. Assign an initial guess for the \( m \) spectral elements, \( \Psi(k_j) \).

2. Calculate the transmission curve, \( S_H(x_i), i = 1 \) to \( n \) for this spectrum.
3. Calculate the deviation of the calculated transmission curve from the true values.

4. Set $i = 1$

5. Set $j = 1$

6. Modify the $j^{th}$ element of $\Psi(k)$ by comparing the $i^{th}$ calculated and required transmission values.

7. Re-calculate a transmission curve and deviations from true values for this modified spectrum.

8. If the results for the modified spectrum are improved, let this spectrum replace the model's current spectrum.

9. Set $j = j + 1$ and repeat the process from step 6.

10. Having covered all spectral components, set $i = i + 1$ and return to step 5.

11. Having covered all transmission values return to step 4 until no improvement is observed or a satisfactory solution has been obtained.

The iterative process is carried out moving from high energy elements of the spectrum and low transmission values to the low energy region and high transmission values.

Twidell [104] represented the photon spectrum by 20 rectangular components and modified the $j^{th}$ element using the quantity;

$$Q_i = \frac{S_T(x_i) - S(x_i)}{S(x_i)}$$ (3.1.2)

where $S_T$ is the model's transmission curve as defined in equation 3.1.1, giving the modified element, $\Psi'(k_j)$

$$\Psi'(k_j) = \Psi(k_j)(1 + Q_i/2)$$ (3.1.3)

The average percentage deviation, $Y$, from the true transmission values is then calculated to assess any improvement

$$Y = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{S_T(x_i) - S(x_i)}{S(x_i)} \right| \times 100$$ (3.1.4)
Transmission curves for the two 2 MeV spectra investigated were defined by 20 thicknesses of lead filters giving transmitted intensities down to 2 to 3 percent of the incident beam. Resulting reconstructed spectra reproduced the input data to within approximately 1 percent.

Huang et al [52,53] improved on this approach by approximating the transmission integral of equation 2.1.8 using Simpson's rule for 25 photon energy and transmission values, so that the model's transmission curve, $S_H$, is given by

$$S_H(x_i) = (\Delta k/3) \sum_{j=1}^{m} A_{i,j} \phi(k_j)$$

where

$$A_{i,j} = \alpha \exp[-\mu(j) x_i]$$

and

$$\alpha = \begin{cases} 1 & \text{for } j = 1 \text{ and } m \\ 4 & \text{for } j = 2, 4, ..., m - 1 \\ 2 & \text{for } j = 3, 5, ..., m - 2 \end{cases}$$

Spectral elements are modified simply through multiplying by the ratio of true and calculated transmission values, $S(x_i)/S_H(x_i)$ and full transmission curves compared by calculating the quantity $\chi^2$;

$$\chi^2 = \sum_{i=1}^{n} \left\{ \frac{S_H(x_i) - S(x_i)}{\gamma_i S(x_i)} \right\}^2$$

where $\gamma_i$ represents the uncertainty in the $i^{th}$ transmission measurement. A combination of lead and aluminium filters were used to maximise the slope of the total linear attenuation coefficient against photon energy to give the optimum energy resolution as described in chapter 2 for the 4 MV spectra investigated by the authors. Reconstructed spectra yielded transmission curves which agreed with the entered data to within 0.5 percent. Piermattei et al [87] investigated the application of Huang et al's model to photon spectra with energies up to 40 MV using calculated and measured transmission curves in aluminium. They concluded that reliable results are obtained for maximum beam energies up to 10 MV and that the slope of total linear attenuation coefficient against energy for aluminium is not
sufficiently steep at higher energies to allow spectra to be reconstructed. However, aluminium's greatest transparency for photons is at approximately 20 MeV, so for calculated transmission curves at least, we would expect the limit for aluminium filters to be nearer this energy. The use of different filter materials, such as water or carbon whose attenuation coefficients monotonically decrease beyond 40 MeV was not commented upon.

Sauer and Neumann [91] derived spectra for beams with maximum photon energies of 4, 8 and 16 MeV from measurements of depth-dose curves in water by assuming that depth-dose curves for polyenergetic spectra, $D(x_i)$, may be represented as a superposition of mono-energetic depth-dose curves, $D_j(x_i)$.

$$D(x_i) = \sum_{j=1}^{m} \Delta k \Psi(k_j)D_j(x_i) + C(x_i)$$  \hspace{1cm} (3.1.9)

where $D_j(x_i)$ is the depth-dose for a mono-energetic beam of energy, $k_j$, determined by Monte Carlo calculations and $C(x_i)$ is the dose due to contaminant electrons. An iterative procedure of a similar form to that of Twidell [104] was then used to derive the photon spectra, $\Psi(k_j)$.

3.2 Constraints

Despite the apparent simplicity of the iterative approach to solving the transmission integral 2.1.8 for the photon spectrum, in order to obtain a physically realistic spectrum, constraints must be applied to values taken by the spectral components, $\Psi(k_j)$. We would expect the spectral components of megavoltage spectra to increase monotonically from the low energy cut-off up to the modal energy and then to decrease monotonically to the maximum photon energy. For energy elements arranged in decreasing order this constraint becomes:

$$\Psi(k_j) \leq \Psi(k_{j+1}) \text{ for } j \leq j_{mod}$$

$$\Psi(k_j) \geq \Psi(k_{j+1}) \text{ for } j \geq j_{mod}$$  \hspace{1cm} (3.2.1)

where $j_{mod}$ represents the point of modal energy. This constraint in itself need not lead to a smooth spectral shape and so a smoothing constraint must also be
applied. Sauer and Neumann [91] arrived at a smoothed spectrum, $\Psi_s(k_j)$, via the expression

$$\Psi_s(k_j) = \sum_{l=0}^{p} g_l (\Psi(k_{j-l}) + \Psi(k_{j+l}))$$  \hspace{1cm} (3.2.2)

where $p$ represents the number of nearest neighbours considered and the coefficients, $g_l$, satisfy

$$\sum_{l=0}^{p} g_l = 0.5$$  \hspace{1cm} (3.2.3)

3.3 Construction of an Iterative Model

In order to analyse the success of the iterative approach in deriving clinical photon spectra from transmission measurements, a model based on that of Huang et al [52,53] was constructed incorporating the smoothing constraint of Sauer and Neumann [91]. A flow chart outlining the procedure is given in figure 3.1.

In the work of Huang et al, the maximum photon energy in the spectrum was not assumed to be known exactly and the first set of iterations were carried out to determine this energy value by setting the modal energy of the model’s spectrum to equal that of a mono-energetic source whose total linear attenuation coefficient in polystyrene equals that of the poly-energetic spectrum. In order to allow comparison with other approaches no such iteration was performed here and the maximum spectral energy was set to the known value for the spectra described in chapter 2.

The initial spectral values, $\Psi(k_j)$, are defined by a triangular distribution whose modal energy is set to one of the energy array elements, $k_j$, before the iteration procedure and is not allowed to change during iterations. The whole process is then repeated for different values of modal energy and final deviations from the required transmission values compared to determine the optimum value.

3.3.1 Application of Constraints

Having obtained a modified spectral element, $\Psi'(k_j)$, via

$$\Psi'(k_j) = \frac{S(x_i)}{S_H(x_i)} \Psi(k_j)$$  \hspace{1cm} (3.3.1)
Figure 3.1: Flow chart describing the iterative model used in this work, based on that of Huang et al [52]. $S$ is the input transmission curve (Chapter 2), $S^H$ is the iterative model's transmission curve and primes refer to modified spectral elements and transmissions. The integers $i$ and $j$ refer to filter thickness and energy array elements respectively. $\Psi^0$, the initial guess for the photon spectrum is given by a triangular distribution.
the monotonicity constraints of expression 3.2.1 are tested. Should either of these be violated, the modified element is reset to an acceptable value as shown below for a particular case;

$$j \leq j_{mod} \quad \text{and} \quad \Psi'(k_j) \leq \Psi(k_{j-1})$$

then

$$\Psi'(k_j) = \Psi(k_{j-1}) + \beta |\Psi'(k_j) - \Psi(k_{j-1})|$$

(3.3.2)

where $\beta$ was set to equal 0.25 so that the direction of modification as determined by equation 3.3.1 is maintained.

Smoothing is then carried out on the new spectrum by considering the next nearest neighbours (corresponding to $p = 1$ in expression 3.2.2)

$$\Psi_s(k_j) = 2g_0 \Psi(k_j) + g_1 (\Psi(k_{j-1}) + \Psi(k_{j+1})) \quad \text{For} \quad j \neq j_{mod}$$

$$\Psi_s(k_j) = 2g_0 \Psi(k_j) + g_1 \max[\Psi(k_{j-1}), \Psi(k_{j+1})] \quad \text{For} \quad j = j_{mod}$$

(3.3.3)

The second form of smoothing for $j = j_{mod}$ is required in order to ensure that the modal energy in the model's spectrum remains unchanged.

A smoothing factor, $2g_1 \times 100$, was defined ($g_0 + g_1 = 0.5$) and the effect that its value has on the model's results investigated. It was found that values within the range 0.05 to 0.5 percent gave similar results, whilst lower values gave insufficient smoothing of reconstructed spectra and higher values resulted in over-smoothing, reducing the flexibility of the model and consequently leading to dramatically increased maximum deviations from entered transmission curves. Figure 3.2 compares returned spectra for the 15 MeV spectrum of Mohan et al [74] for different values of the smoothing factor. Although returned spectra for values greater than 0.2 percent appear to be in good agreement with the original data, maximum deviations from entered transmission data given in table 3.1 are seen to be poor. A fixed value of 0.2 percent was therefore selected. Table 3.1 also shows how increasing the smoothing constraint restricts the model's flexibility, leading to a reduced number of major iterations, $r$, carried out before convergence is reached.

In order to test the model developed here against published work, program runs were carried out for two of the (nominally) 4 MeV spectra published by Huang et
Figure 3.2: Dependence of spectra reconstructed by the iterative model on degree of smoothing, given in terms of the smoothing factor, sf (percent).

<table>
<thead>
<tr>
<th>smoothing factor</th>
<th>r</th>
<th>max. deviation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1000</td>
<td>0.62</td>
</tr>
<tr>
<td>0.2</td>
<td>205</td>
<td>0.22</td>
</tr>
<tr>
<td>1.0</td>
<td>44</td>
<td>0.73</td>
</tr>
<tr>
<td>5.0</td>
<td>7</td>
<td>2.10</td>
</tr>
</tbody>
</table>

Table 3.1: Number of major iterations, r, required for convergence and maximum deviations from entered transmission data for different degrees of smoothing.
al [82] and for the 6 and 10 MeV spectra reconstructed by Piermattei et al [87]. Transmission curves were calculated as described in chapter 2 for entry into the program using a single filter material of either aluminium or water, rather than a combination of materials as used in Huang et al's work. Figure 3.3 shows the returned spectra which were defined by 25 energy array elements. Curves A and B in the first plot of figure 3.3 are the reconstructed spectra obtained for differing minimum and maximum energies which define the energy array, being set to 0.05 and 4.0 MeV respectively for curve A and 0.1 and 4.1 MeV for curve B, the latter being the values used by Huang et al. These small differences in the definition of the energy array elements are seen to produce significantly different returned spectra, although both curves reproduce entered transmission data to the same degree of accuracy.

3.4 Results

Iterative fitting to the input transmission curves for the megavoltage spectra described in chapter 2 was carried out with the maximum number, \( r_{\text{max}} \), of major iterations set to 1000. Following the approach of Huang et al [52,53] the photon energy spectrum was defined by 25 equally-spaced energy array elements. The procedure was terminated if no improvement in the \( \chi^2 \) value of expression 3.1.8 was obtained after each \( r^{th} \) iteration. Due to the restrictions imposed on possible spectral shapes by the application of smoothing constraints mentioned in the last section, convergence was reached in all cases, final \( r \) values lying in the range 10 to 350.

The accuracy of the final spectrum resulting from the iterations was assessed by calculating the model's transmission curve from equation 2.1.8 by numerical integration using finite difference formulae [78] in order to remove the uncertainty due to the use of Simpson's rule. Returned maximum deviations and modal energies are given in table 3.2.

The large number of computations required in order to obtain an iterative solution for the photon energy fluence leads to far longer program run times for this approach than for the Laplace transform and thin-target models described in
Figure 3.3: Comparison of reconstructed 4 MeV spectra of Huang et al [52] and 6 and 10 MeV spectra of Piermattei et al [87] with the iterative program of this work. Curves A and B in the first plot correspond to energy array elements defined by minimum and maximum energies of 0.05, 4.0 and 0.1, 4.1 MeV respectively.
<table>
<thead>
<tr>
<th>Energy (MeV)</th>
<th>max. dev. (%)</th>
<th>$k_{mod}$ (MeV)</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>NFL [41]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>.55</td>
<td>0.37</td>
<td>268</td>
</tr>
<tr>
<td>6</td>
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<td>200</td>
</tr>
<tr>
<td>8</td>
<td>.17</td>
<td>1.09</td>
<td>44</td>
</tr>
<tr>
<td>10</td>
<td>.10</td>
<td>1.34</td>
<td>17</td>
</tr>
<tr>
<td>12</td>
<td>.12</td>
<td>1.50</td>
<td>31</td>
</tr>
<tr>
<td>16</td>
<td>.14</td>
<td>2.09</td>
<td>22</td>
</tr>
<tr>
<td>19</td>
<td>.10</td>
<td>2.46</td>
<td>75</td>
</tr>
<tr>
<td>Faddegon et al [42,43]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>.10</td>
<td>1.75</td>
<td>61</td>
</tr>
<tr>
<td>15</td>
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<td>12</td>
</tr>
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<td>20</td>
<td>.11</td>
<td>2.59</td>
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</tr>
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<td>.11</td>
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<td>12</td>
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<td>30</td>
<td>.25</td>
<td>3.84</td>
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<tr>
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<td>1.37</td>
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<td>3.20</td>
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<tr>
<td>24</td>
<td>.21</td>
<td>5.08</td>
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<td>Landry et al [66]</td>
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</tr>
<tr>
<td>15</td>
<td>.44</td>
<td>2.68</td>
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</tr>
<tr>
<td>20</td>
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<td>10</td>
</tr>
<tr>
<td>25</td>
<td>.33</td>
<td>4.25</td>
<td>137</td>
</tr>
<tr>
<td>Levy et al [69,68]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>.34</td>
<td>2.40</td>
<td>340</td>
</tr>
<tr>
<td>27</td>
<td>.30</td>
<td>3.46</td>
<td>113</td>
</tr>
</tbody>
</table>

Table 3.2: Returned maximum percentage deviations, modal energies, $k_{mod}$ and number of major iterations, $r$, required for 25-point Huang model reconstructions.
chapters 4 and 5, typical run-times on a SUN Sparc 10 being of the order of 20 hours as opposed to around 1 minute.

The effect of increasing the number of elements was not found to provide any improvement in the accuracy to which entered transmission curves were reproduced (shown in table 3.3 for 49-point runs), although observable differences in reconstructed spectra were found, a greater number of elements generally resulting in a lower returned value for the modal energy, \( k_{\text{mod}} \). Figures 3.4 to 3.10 show that for lightly filtered spectra (NPL, Faddegon et al) this generally improved the observed agreement between plots of original and reconstructed spectra whilst for more heavily filtered spectra (Mohan et al) poorer determination of the modal energy than for 25-point runs was achieved. This observation may be explained by the fact that the modal energy for lightly filtered spectra lies at a relatively low energy and since equally spaced energy intervals are used this leads to poorer definition of the reconstructed spectrum around the region of maximum energy fluence. For heavily filtered spectra adequate definition is provided and the use of fewer energy array elements provides greater differentiation between successive energy intervals.

<table>
<thead>
<tr>
<th>Energy (MeV)</th>
<th>max. dev. (%)</th>
<th>( k_{\text{mod}} ) (MeV)</th>
<th>( r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPL [41]</td>
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<td>.21</td>
<td>.93</td>
<td>126</td>
</tr>
<tr>
<td>16</td>
<td>.20</td>
<td>1.43</td>
<td>286</td>
</tr>
<tr>
<td>Faddegon et al [42,43]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>.11</td>
<td>1.13</td>
<td>139</td>
</tr>
<tr>
<td>20</td>
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<td>Mohan et al [74]</td>
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<td>.13</td>
<td>1.29</td>
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<tr>
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<td>.30</td>
<td>1.98</td>
<td>9</td>
</tr>
</tbody>
</table>

Table 3.3: Returned maximum percentage deviations, modal energies, \( k_{\text{mod}} \) and number of major iterations, \( r \), required for 49-point Huang model reconstructions.

The use of Simpson's rule to approximate the transmission integral of expression 2.1.8 inevitably introduces errors which limit the accuracy with which the iterative model described can reproduce input transmission curves. For the case of 25 energy array elements, the error introduced in the calculation of transmission
Figure 3.4: 4, 6, 8 and 10 MeV NPL spectra [41] and iterative model reconstructions. Energy arrays are defined by 25 points unless otherwise stated.
Figure 3.5: 12, 16 and 19 MeV NPL spectra duane and iterative model reconstructions. Energy arrays are defined by 25 points unless otherwise stated.
Figure 3.6: 10, 15, 20 and 25 MeV spectra of Faddegon et al [42,43] and iterative model reconstructions. Energy arrays are defined by 25 points unless otherwise stated.
data from the initial triangular energy fluence distribution was found to be of the order of 0.25 percent. Simply increasing the number of energy elements used to define the photon spectrum will reduce this inherent source of error, but at the same time reduces the differentiation between successive energy intervals which is needed to guide the iterative process. In order to fully assess the iterative approach to spectral reconstruction, the Simpson’s rule approximation was replaced by more accurate numerical integration using finite difference formulae [78]. Although reducing the maximum deviation from input transmission data to some degree, no improvement in spectral plots was found.

Reconstructed spectra in figures 3.4 to 3.10 generally show unphysical peaks at the modal energy value, a feature which is likely to be dependent on the value assigned to the smoothing factor, $\alpha$, described earlier. The potential benefit obtained by increasing this parameter’s value to returned spectra is out-weighed by the decrease in accuracy with which the entered transmission curves would be reproduced.
Figure 3.8: 4, 6, 15 and 24 MeV spectra of Mohan et al [74] and iterative model reconstructions. Energy arrays are defined by 25 points unless otherwise stated.
Figure 3.9: 15, 20 and 25 MeV spectra of Landry et al [66] and iterative model reconstructions.
Figure 3.10: 8 and 27 MeV spectra of Levy et al [69,68] and iterative model reconstructions.
Chapter 4

Laplace Transform Models

4.1 Development

It has been known for many years that the attenuation integral of equation 2.1.8 can be represented as a Laplace transform (LT) by representing the energy fluence spectrum in terms of a function of the total linear attenuation coefficient, \( \mu(k) \). Silberstein [98] first applied this technique to derive kilovoltage spectra from their transmission curves in aluminium and copper. Representing the transmitted intensity of an x-ray beam through a thickness, \( x \), of filter as

\[
S(x) = \int_0^E \Psi(k) \exp[-\mu(k)x]dk
\]

(4.1.1)

where the terms have the same meaning as in equation 2.1.8, we make the substitution;

\[
f(\mu) = -\Psi(k) \frac{dk}{d\mu}
\]

(4.1.2)

giving

\[
S(x) = \int_{\mu_0}^\infty f(\mu) \exp[-\mu x]d\mu
\]

(4.1.3)

where \( \mu_0 \) is a constant representing the total linear attenuation coefficient for photons of energy equal to the maximum present in the spectrum, \( E \), and the coefficient for \( k = 0 \) is set to infinity. The function \( f(\mu) \), referred to as the pre-spectrum, is the representation of our true spectrum, \( \Psi(k) \), in \( \mu \)-space, and will be zero for
Figure 4.1: Translation of function, \(g(\mu)\), to represent the pre-spectrum, \(f(\mu)\).

\(\mu < \mu_0\). We aim to model \(f(\mu)\) by an analytical expression, this task is simplified if the chosen expression is related to the pre-spectrum by

\[
f(\mu) = g(\mu - \mu_0)
\]

(4.1.4)

Using this relation we introduce the cut-off of \(f(\mu)\) at \(\mu = \mu_0\) which corresponds to the high-energy cut-off in the energy representation by translating the modelling function, \(g\), from the origin (corresponding to \(\mu(\infty) = 0\)) as shown in figure 4.1.

Inserting the model's pre-spectrum into equation 4.1.3 we obtain

\[
S(x) = \int_{\mu_0}^{\infty} g(\mu - \mu_0) \exp[-\mu x] d\mu
\]

(4.1.5)

making the substitution \(\mu' = \mu - \mu_0\)

\[
S(x) = \int_0^{\infty} g(\mu') \exp[-(\mu' + \mu_0) x] d\mu'
\]

\[
= \exp[-\mu_0 x] \int_0^{\infty} g(\mu') \exp[-\mu' x] d\mu'
\]

\[
= \exp[-\mu_0 x] \mathcal{L}[g(\mu')]
\]

\[
= \exp[-\mu_0 x] h(x)
\]

(4.1.6)

where \(\mathcal{L}\) is the Laplace transform operator and \(h(x)\) is the Laplace transform of the modelling function, \(g(\mu')\). Hence the transmission curve, \(S(x)\), is given by the product of the Laplace transform of the modelling function and an exponential
term which represents the transmission curve for photons of energy equal to the maximum present in the beam. This general form for $S(x)$ has been adopted in almost all models developed since Silberstein’s original work [98,4] as it is logical to expect the transmission curve of a polyenergetic spectrum to become that of the highest energy photons as the thickness of attenuator is taken to infinity. The function, $h(x)$, is then seen to modify the exponential term to account for the lower energy photons in the spectrum.

The task of modelling photon spectra is then reduced to finding suitable functions, $h(x)$, which allow measured transmission curves to be accurately represented. The energy fluence spectrum, $\Psi(k)$, is then given by

$$\Psi(k) = -\frac{d\mu}{dk} \left\{ h(x) \exp[-\mu_0 x] \right\}$$

$$= -\frac{d\mu}{dk} g(\mu - \mu_0)$$

(4.1.7)

In his original work, Silberstein assumed the transmission curve could be represented by the expression

$$S_0(x) = \exp[-Ax - Bx^{1/2}]$$

(4.1.8)

and that the relationship between $k$ and $\mu$ could be approximated to

$$\mu = \mu_0 + \frac{\beta}{k^3}$$

(4.1.9)

where $\beta$ is a constant, to allow $d\mu/dk$ an analytical form. Using expressions 4.1.8 and 4.1.9, the integral of equation 4.1.3 can be solved [98], resulting in a photon energy fluence, $\Psi(k)$, of

$$\Psi(k) = (\mu - A)^{-3/2}(\mu - \mu_0)^{3/3} \exp[-D^2/(4(\mu - A))]$$

(4.1.10)

where numerical constants have been omitted. If we consider the form of equation 4.1.8 for large $x$, where the second term becomes negligible in comparison to the first, we arrive at the conclusion that parameter, $A$, must in fact be equal to $\mu_0$. This feature was first noticed by Bell [20]. Jones [60] extended Silberstein’s approach, after showing that equation 4.1.8 as it stands is not sufficient to describe transmission curves.
Table 4.1: Laplace-transform pairs. In Huang et al's model, parameter $A$ is not set equal to $\mu_0$, but is maintained as a parameter to be fitted to the measured transmission curve.

The most published LT model of recent years has been that of Archer and Wagner (AW) [9,10,11,12,13,14], which along with the published models of Silberstein [98], Jones [60], Saylor [93] and Huang et al [51] were compared in their ability to reconstruct the photon spectrum of a 4 MeV beam by Ahuja et al [4]. Their results clearly indicate that the model of Archer and Wagner (AW) provides the most accurate representation of spectra. Table 4.1 shows the various forms of function, $h(x)$, and choice of parameters for the models.

The appeal of the LT representation for transmission curves is the fact that fewer parameters can be used to define the resulting energy fluence spectrum than the previously described iterative methods. In a successful model, these parameters may then provide a useful method for beam characterization.

4.2 AW model

The AW model was derived by considering the theoretical shape of kilo-voltage transmission curves as described by Joseph [61]. In his derivation the unfiltered photon energy spectrum, $\Psi(k)$, is assumed to be given by

$$\Psi(k) \propto (k - E)$$ (4.2.1)
as suggested by Kramers [65]. The transmission curve for this approach, \( S_J(x) \), can then be represented in terms of equation 4.1.1 as

\[
S_J(x) = \int_0^E (k - E) \exp(-\mu(k)x)dk
\]  

(4.2.2)

By approximating the total attenuation coefficient, \( \mu(k) \), to equation 4.1.9, Joseph arrived at the asymptotic form for \( S_J(x) \)

\[
S_J(x) \rightarrow \frac{\exp(-\mu_0x)}{(\mu'_0x + c_1)(\mu'_0x + c_2)} \rightarrow \frac{\exp(-\mu_0x)}{x^2}
\]  

(4.2.3)

where \( c_1, c_2 \) and \( \mu'_0 \) are constants, \( \mu'_0 \) being the energy-dependent part of \( \mu \) as given in equation 4.1.9 corresponding to the maximum photon energy present in the beam. The above expression is seen to be of the same form as that of Archer and Wagner, whose transmission curve, \( S_{AW}(x) \), is given by

\[
S_{AW}(x) = \exp(-\mu_0x)h(x) = \exp(-\mu_0x)\left(\frac{ab}{(x + a)(x + b)}\right)^\nu
\]  

(4.2.4)

where \( a, b \) and \( \nu \) are constants. The photon energy fluence, \( \Psi_{AW}(k) \), in the AW model is then given by finding the inverse Laplace transform for \( h(x) \) given above from tables

\[
\Psi_{AW}(k) = -\frac{d\mu}{dk}g(\mu') = -\frac{d\mu}{dk}e^{\nu\mu}e^{-0.5\mu'(a + b)}I_s[0.5\mu'(a - b)]
\]  

(4.2.5)

where \( \mu' = \mu - \mu_0 \) as before and \( s = \nu - 0.5 \). \( I_s(t) \) is a modified Bessel function given, for \( s > 0 \), by [15]

\[
I_s(t) = i^s J_s(it) = \sum_{l=0}^\infty \frac{(t/2)^{s+2l}}{[l!(s+l)!]}
\]  

(4.2.6)

Figure 4.2 shows graphically the constituent functions describing \( \Psi_{AW}(k) \) and reveals information on the boundaries for parameters \( a, b \) and \( \nu \).

The main area of work reported for the AW model has been in the reconstruction of diagnostic spectra whose maximum photon energies are in the range 25 to 300 keV. In this energy range, characteristic radiation contributes a significant part of the spectrum and an empirical correction to allow its inclusion has been incorporated into the model by the authors [12]. The ability of the model to reconstruct spectra in the megavoltage range has not been fully investigated, although its performance using measured transmission curves from 6 and 25 MeV spectra.
was reported by the authors to be poor [10]. Since it was not clear whether these poorer results were due to limitations of the model or simply due to insufficient accuracy in the measurement of the transmission curves, a full theoretical assessment was carried out using the calculated transmission curves described in chapter 2.

Parameters $a$, $b$ and $\nu$ were found by minimizing the objective function

$$Q(a, b, \nu) = \sum_{i=1}^{n} \left\{ \frac{1 - \left[ \frac{ab}{(x+a)(x+b)} \right]^{\nu} \exp[-\mu_0 x_i]}{S(x_i)} \right\}^2$$  \hspace{1cm} (4.2.7)$$

where the summation is over $n$ filter thicknesses and $S(x_i)$ represents the calculated transmission value for the $i^{th}$ thickness. Minimization was carried out using a quasi-Newton algorithm [79]. The fitted values can then be substituted into expression 4.2.5 to yield the photon energy fluence spectrum. User-defined values for the minimum and maximum energies in the spectrum were used to calculate an energy array consisting of from 50 to 100 elements to define the final spectrum. Values of $d\mu/d\epsilon$ required by equation 4.2.5 were obtained from the Hermite polynomial interpolant [76] fit to the total linear attenuation coefficients described in chapter 2.
The implementation of the AW model was tested by reconstructing kilo-voltage spectra published by the authors. Due to the dosimetry considerations for photons with energies below 2 MeV outlined in chapter 2, the calculations of transmission curves in these cases differed slightly from the megavoltage case in that the function, \( R(k) \), of equation 2.1.8 was set equal to the mass energy absorption coefficient for air \([54]\), \([\mu_{en}(k)/\rho]_a\). The data-base of linear attenuation coefficients, \( \mu(k) \), for kilo-voltage runs consisted of 135 values covering the energy range 2 keV to 2 MeV. A check on the photon spectrum returned by the model was carried out by recalculating a transmission curve via expression 2.1.8 by numerical integration using finite difference formulae \([78]\). In this way, uncertainties in the estimated value of \( d\mu/dk \) could be assessed. Four diagnostic spectra with maximum photon energies, 45, 55, 65 and 80 keV, defined by parameter values appearing in reference \([9]\) were used to calculate theoretical transmission curves through aluminium for entry into the model. The maximum percentage deviations from input data and returned parameter values, compared with those published by Archer and Wagner \([9]\), are given in table 4.2.

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>Parameter</th>
<th>Published</th>
<th>This work</th>
<th>Max. dev. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>( \nu )</td>
<td>0.71692</td>
<td>0.73205</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>( a )</td>
<td>5.2495</td>
<td>5.6438</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( b )</td>
<td>0.99986</td>
<td>1.0103</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \mu_0 )</td>
<td>0.203</td>
<td>0.2018</td>
<td></td>
</tr>
<tr>
<td>65</td>
<td>( \nu )</td>
<td>0.97802</td>
<td>1.0031</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>( a )</td>
<td>8.3240</td>
<td>9.1861</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( b )</td>
<td>0.50462</td>
<td>0.58090</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \mu_0 )</td>
<td>0.252</td>
<td>0.2506</td>
<td></td>
</tr>
<tr>
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<td>( \nu )</td>
<td>0.9146</td>
<td>1.0787</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>( a )</td>
<td>6.5534</td>
<td>8.2058</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( b )</td>
<td>0.40346</td>
<td>0.40377</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \mu_0 )</td>
<td>0.396</td>
<td>0.3151</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>( \nu )</td>
<td>0.98314</td>
<td>1.0165</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>( a )</td>
<td>2.8749</td>
<td>3.3213</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( b )</td>
<td>0.22352</td>
<td>0.23402</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \mu_0 )</td>
<td>0.450</td>
<td>0.4466</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.2: Comparison of parameter values of reference \([9]\) with the present work and maximum percentage deviations from input transmission data.
Figure 4.3: Comparison of reconstructed exposure spectra of reference [9] by the AW model.

4.3 Results

The calculated transmission curves described in chapter 2 were used to assess the AW model's ability to reconstruct megavoltage spectra. Comparison of the kilovoltage spectra in figure 4.3 with the megavoltage spectra in figures 2.3 and 2.4 shows how the general shape of the spectra has changed in moving to higher energies. This change in spectral shape and the fact that less energy differentiation is possible at higher energies since the slope of $\mu(k)$ decreases with energy (figures 2.6 and 2.7), will be expected to reduce the accuracy to which megavoltage spectra can be reconstructed by this model.

To initiate the minimization procedure which determines the optimum parameter values, an initial guess for the solution is required. The procedure is then to use first and second derivatives of the objective function to indicate in which direction the solution is to be found. This procedure is only capable of finding the nearest, or local, minimum. Finding the true global minimum can never be assured and so the acceptance of a single, returned solution must be based on the accuracy to which it allows the input data to be reproduced. Returned parameter values for
the AW model were found to be very sensitive to the starting point in many cases, indicating that the objective function contained many minima.

Table 4.3 gives returned parameter values for all reconstructed spectra and values of the maximum percentage deviation of the model's transmission curves from the input data. The tables show a sudden and dramatic increase in the magnitude of parameter, \( a \), as the maximum spectral energy passes 8 MeV. This feature was found for all spectral sets and shows the presence of two distinct areas of solution in the minimization of the objective function. In fact, given that one solution has parameters \( a \) and \( b \) approximately equal, inspection of the model's attenuation curve (equation 4.2.4) suggests the presence of a second solution with \( a >> b \). If \( a_1, b_1 \) and \( \nu_1 \) are the returned parameter values of the first solution then by removing constant terms from expression 4.2.4 we have

\[
S_{AW}(x) \sim \frac{\exp[-\mu_0]}{(x + a_1)^n (x + b_1)^{\nu_1}} \sim \frac{\exp[-\mu_0 x]}{(x + b_1)^{\nu_1}}
\]

(4.3.1)

the final expression above can also be obtained if \( a_2 >> x \), in which case \( (x + a_2) \) can be removed as a constant and the expressions become equal for \( \nu_2 \sim 2 \times \nu_1 \). This is seen to be approximately true by comparing the parameter values returned for the 6 and 8 MeV spectra from the NPL.

Comparisons of reconstructed spectra with those used to calculate input data are shown in figures 4.4 to 4.10. As reported by Archer and Wagner [11] for diagnostic energies, values of parameter, \( \nu \), below about 0.7 lead to over-estimation of the energy fluence in the high energy portion of the reconstructed spectra, this effect can be seen in the reconstructed 30 MeV spectrum of Faddegon et al [42,43].

In general very good results were obtained, reconstructed spectra agreeing well with the corresponding originals and reproduction of input transmission data achieved to within 0.3 percent in most cases. The worst results were found for the spectra of Levy et al [69,68] where maximum deviations reached one percent and above. However, comparison of Levy's spectra with those from the other sources casts doubt on their validity and so the failure of the AW model to accurately reconstruct them may signify that expression 4.2.5 is a suitable functional form in
Table 4.3: Returned maximum percentage deviations and parameter values for the AW model.
Figure 4.4: 4, 6, 8 and 10 MeV NPL spectra [41] and AW model reconstructions.
Figure 4.5: 12, 16 and 19 MeV NPL spectra [41] and AW model reconstructions.
Figure 4.6: 4, 6, 15 and 24 MeV spectra of Mohan et al [74] and AW model reconstructions.
Figure 4.7: 10, 15, 20 and 25 MeV spectra of Faddegon et al [42,43] and AW model reconstructions.
which to express clinical photon energy fluence spectra.

The main area of discrepancy between the spectral shape of reconstructed and original spectra was found at the low energy limit, where $\Phi_{AW}(k)$ tended to overestimate the energy fluence, leading to a lower peak height for the returned spectra in the normalized plots. Modifications to the AW model to improve the reconstructions in this area are described in the next sections. The poorer maximum deviation found for Mohan et al's 4 MeV spectrum in comparison to other spectra from this source was expected to be due to the poor definition of the original spectrum, seen in figure 4.6. This was confirmed by cubic-spline smoothing the spectrum before calculating input transmission data. Landry et al's 15 MeV spectrum is seen to similarly be poorly defined in the low energy region (figure 4.9) which explains the higher maximum deviation found for this spectrum.

4.3.1 Weighting of Transmission Data

Transmission data for smaller thicknesses of filter material give information on the lower energy components of the x-ray spectrum, since these components are preferentially removed under filtration. The low energy region of the reconstructed spectrum may therefore be improved by weighting input transmission data in favour
Figure 4.9: 15, 20 and 25 MeV spectra of Landry et al [66] and AW model reconstructions.
Figure 4.10: 8 and 27 MeV spectra of Levy et al [69,68] and AW model reconstructions.

of the smaller thicknesses. A weighting function for the $i^{th}$ transmission point defined by

$$W_i = A^{(i-1)} \times W_1$$

with

$$A = \left[ \frac{1}{W_1} \right]^{1/(n-1)}$$

where $n$ is the total number of thicknesses, was applied in the objective function definition (equation 4.2.7). The weighting factor for the first transmission point, $W_1$, was user-defined and remaining factors calculated such that $W_n = 1$. Transmission data through water for the 8 MeV spectrum from the NPL and the 6 MeV spectrum of Mohan et al were weighted using the above function for $W(1)$ values ranging from 1 to 20. Reconstructed spectra did not show any improvement in the prediction of the low energy cut-off present in the original spectra.

4.3.2 Minimum Spectral Energy

In forming the LT representation of transmission curves (equations 4.1.1 to 4.1.6), we assumed that the lower limit of the integral can be taken as zero. The spectral
plots in chapter 2, however, show that most original spectra have a clearly defined minimum energy, if this energy can be incorporated into the AW model as a known constant or as an extra parameter, we may be able to improve the spectral reconstructions in the low energy region.

The slope of the transmission curve at \( x = 0 \) provides information on the effective energy, \( k_m \), of the beam. From equation 4.1.1 we have

\[
\frac{\partial S(x)}{\partial x} = - \int_0^\infty \mu(k)\Psi(k)\exp[-\mu(k)x]dk
\]

\[
= - \mu(k_m, x)
\]  (4.3.4)

For \( x = 0 \), \( \mu(k_m) \) therefore represents the fluence weighted average energy of the incident beam (since \( S(0) \) is normalized to unity). Higher order derivatives of equation 4.1.1 will simply have greater powers of \( \mu(k) \) appearing in the integral, by normalizing these integrals, we can obtain effective total linear attenuation coefficients, \( \mu_n(k_m) \), weighted by powers of \( \mu(k) \)

\[
\mu_n(k_m) = - \frac{\left( \frac{\partial^n S(x)}{\partial x^n} \bigg|_{x=0} \right)}{\left( \frac{\partial^{n-1} S(x)}{\partial x^{n-1}} \bigg|_{x=0} \right)}
\]

\[
= \frac{\int_0^\infty \mu(k)\Psi(k) \left[ \mu(k)^{n-1} \right] dk}{\int_0^\infty \left[ \Psi(k)\mu(k)^{n-1} \right] dk}
\]  (4.3.5)

Since \( \mu(k) \) decreases sharply with increasing photon energy, we would expect \( \mu_n(k_m) \) values to increase, gradually levelling off as the value corresponding to the effective minimum energy of the spectrum was reached. Although in principle, this approach could lead to an estimate for the minimum spectral energy, estimating high order derivatives from measured transmission curves is unlikely to be possible to achieve in practice, as only transmission values at discrete points are known.

If we cannot deduce a reasonable estimate of the minimum spectral energy, then another possibility is to simply incorporate it into the model as an extra parameter. This entails evaluating the attenuation integral of equation 4.1.5 with the upper limit now involving the linear attenuation coefficient corresponding to the minimum energy, \( \mu_m \). Substituting for the function, \( g(\mu') \), the integral becomes

\[
S(x) = \exp[-\mu_0 x] \int_0^{\mu_m-\mu_0} \left( \frac{2\mu'}{w} \right)^n I_s(w\mu')\exp[-(x + t)\mu'] d\mu'
\]  (4.3.6)
where \( t = (a + b)/2 \) and \( w = (a - b)/2 \). Substituting for the modified Bessel function using equation 4.2.6, we obtain

\[
S(x) = \exp[-\mu_0 x] \sum_{l=0}^{\infty} \frac{1}{l!(l+s)!} \left( \frac{w}{2} \right)^{2l} \int_0^\infty \mu^{\mu - \mu_0} \exp[-\mu' (x + t)] d\mu' \tag{4.3.7}
\]

we therefore wish to solve an integral of the form [45]

\[
\int_0^\beta \mu^\nu \exp[-\mu' \delta] d\mu' = \delta^{-(\nu + 1)} \gamma(\nu + 1, \delta \beta) \ \text{Re}(\nu) > -1 \tag{4.3.8}
\]

where \( \gamma[p, q] \) is the incomplete gamma function. The transmission curve is then represented by

\[
S(x) = \exp[-\mu_0 x] \sum_{l=0}^{\infty} \frac{1}{l!(l+s)!} \left( \frac{w}{2} \right)^{2l} (x + t)^{-2l+s+1/2} \times \gamma[2(l + s) + 1, (x + t)(\mu_m - \mu_0)] \tag{4.3.9}
\]

subject to \( s > 0 \). Substituting back for \( a, b \) and \( \nu \) we have

\[
S(x) = \frac{\exp[-\mu_0 x]}{(2x + a + b)^{2\nu}} \sum_{l=0}^{\infty} \left\{ \frac{1}{l!(l+\nu-1/2)!} \left[ \frac{a - b}{2(2x + a + b)} \right]^{2l} \times \gamma[2(l + \nu), (x + (a + b)/2)(\mu_m - \mu_0)] \right\} \tag{4.3.10}
\]

If the summation over \( l \) converges sufficiently rapidly, then the above form for the model's transmission curve could be used. However, when tests were carried out using typical \( a, b \) and \( \nu \) values it was found that more than ten terms in the series were needed to give within 5 percent accuracy, making the above form impractical. The possibility of allowing parameter, \( \nu \), to take values less than 0.5 was investigated (\( \nu \) now being limited to \( \nu > 0 \)) leading to a slightly different form for \( S \), similar results in convergence tests were found.
Chapter 5

Thin-target Models

In the development of analytical models to reconstruct x-ray spectra it is important to ensure that the model is sufficiently flexible to allow all likely spectral shapes to be represented but at the same time does not allow unphysical shapes to be returned. These requirements are best achieved by using a model which has a firm theoretical footing. This chapter is therefore concerned with the development of models directly from bremsstrahlung theory, which will be briefly described and assessed. A more complete description of the theory behind bremsstrahlung production is given in references [50,72,25].

5.1 Bremsstrahlung Theory

The bremsstrahlung cross section, $d\sigma$, is derived from the $S$ matrix element [72].

$$< p' k | S | p > = -2\pi i M \delta (E - E' - k)$$

(5.1.1)

where $| p >$ and $< p' k |$ represent the initial and final state vectors respectively, for electron momenta $p$ and $p'$ and total energies, $E$ and $E'$ (in units of $m_0c^2$). The delta function is present to ensure energy conservation for the transition and $M$ is the Feynman amplitude given by [101]

$$M = \int d^3r \psi_2 \alpha \cdot e^* \psi_1 exp[-ik \cdot r]$$

(5.1.2)

where $\alpha$ are the Pauli matrices [72], $\psi_1$ and $\psi_2$ are the wave functions of the incident and emitted electron respectively which satisfy the Dirac equation and the symbols...
\( \dagger \) and \( * \) refer to the Hermitian adjoint and complex conjugate respectively [95]. The geometry under consideration is shown in figure 5.1. \( e^{\text{exp}[-i \mathbf{k} \cdot \mathbf{r}] \text{ is the Fourier-}
\text{expanded spatial term of the vector potential, } A(r, t), \text{ e being a unit vector in the}
\text{direction of polarization of the emitted photon. In the electric dipole approximation}
\text{exp}[-i \mathbf{k} \cdot \mathbf{r}] \text{ is set to unity, which is justified provided the wavelength of the emitted}
\text{radiation is large compared to the dimensions of the system being considered [25].}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure5.1.png}
\caption{Momentum space geometry for bremsstrahlung production}
\end{figure}

Following the notation of reference [72], \( d\sigma \) is then given by

\[ d\sigma = \langle p'k | S | p \rangle^2 \frac{V d^3 k V d^3 p' V E}{(2\pi)^6} \left| \frac{p}{p'} \right| \]  

(5.1.3)

\[ = \frac{m^2}{(2\pi)^6 2\omega |p'|} M d^3 k d\Omega_e \]  

(5.1.4)

where \( d\Omega_e \) is the element of solid angle in the direction of the emitted electron and \( V \) is the cubic normalization volume [95]. The quantity

\[ \frac{V d^3 k V d^3 p'}{(2\pi)^6} \]

(5.1.5)

is the density of final states [72] and \( |p|/(VE) \) represents the incident electron flux. Expression 5.1.3 relates to given spin directions of the incident and final electrons and to a given polarization of the emitted photon. We therefore sum over final electron spins and photon polarizations and average over the initial electron spins.
In principle then, if the exact wave functions, \( \psi_1 \) and \( \psi_2 \), of the incident and final electrons were known equation 5.1.3 could be used to find an exact expression for bremsstrahlung radiation in the field of the nucleus. However, in practice this is not possible since the Dirac equation cannot be solved in closed form for an electron in a Coulomb field [24] and so approximate wave functions must be used. Use of the Born approximation results in the Bethe-Heitler bremsstrahlung cross section [23,50,64], differential in photon energy and photon and electron emission angles. Integrating over the final electron direction, \( d\Omega_e \), results in the Sauter formula [92,64]

\[
\frac{d\sigma}{dkd\Omega_k} = \frac{p}{k} \left\{ \frac{8\sin^2\theta(2E^2 + 1)}{p^2\Delta^4} - \frac{2(5E^2 + 2EE' + 3)}{p^2\Delta^2} - \frac{2(p^2 - k^2)}{Q^2\Delta^2} \right. \\
+ \frac{4E'}{p^2\Delta} \left[ \frac{4E\sin^2\theta(3k - p^2E)}{p^2\Delta^4} + \frac{4E^2(2E^2 + E'^2)}{p^2\Delta^2} \right] \\
+ \frac{2 - 2(7E^2 - 3EE' + E'^2)}{p^2\Delta^2} + \frac{2k(E^2 + EE' - 1)}{p^2\Delta} \left. \right\} \\
- \left( \frac{4\kappa}{p\Delta} \right) + \left( \frac{\kappa^Q}{pQ} \right) \left[ \frac{4}{\Delta^2} - \frac{6k}{\Delta} - \frac{2k(p^2 - k^2)}{Q^2\Delta} \right] \} \tag{5.1.6}
\]

where

\[ L = \ln \left[ \frac{EE' - 1 + pp'}{EE' - 1 - pp'} \right] \quad \kappa = \ln \left[ \frac{E' + p'}{E - p} \right] \quad \kappa^Q = \ln \left[ \frac{Q + p'}{Q - p'} \right] \]

\[ p = |p| \quad \Delta = E - p\cos\theta \quad Q^2 = p^2 + k^2 - 2pk\cos\theta \]

and \( \theta \) is the angle between the emitted photon and the initial electron directions. In the derivation of the above formula, the effect of screening of the nuclear Coulomb field by atomic electrons is neglected. It would be expected that the screening effect be least important for the higher photon energies of the bremsstrahlung spectrum, since this corresponds classically to the incident electron passing nearer to the nucleus and so inside the atomic electrons orbits. Screening effects may be estimated using the atomic form factor, \( F_e \), defined by [23,64]

\[
F_e(q, Z) = \int \rho(r) \exp[iqr] \, dr \tag{5.1.7}
\]

63
where $q = |q| = |p - p' - k|$ is the momentum transferred to the nucleus and $\rho(r)$ the electron charge distribution at radius $r$ from the nucleus. In the Born approximation, the matrix element, $M$, may be expressed as [64]

$$M \propto [F_n - F_e]$$  \hspace{1cm} (5.1.8)

$F_n$ being the nuclear form factor, which here can be assumed equal to unity since the screening correction will only be significant where the incident electron is at a distance of the order of the atomic radius from the nucleus. Hence the unscreened bremsstrahlung cross-section, $d\sigma/d\Omega_k d\Omega_e dk$, may be corrected for screening effects via the multiplicative factor $[1 - F_e]^2$.

Bethe and Heitler applied this screening correction for numerically calculated form factors to their cross section and integrating over photon and electron emission angles, expressed the bremsstrahlung cross section differential in photon energy, $d\sigma/dk$, as [23,64]

$$\frac{d\sigma}{dk} = \frac{1}{k} \left\{ \left( 1 + \frac{(E')^2}{E} \right) \left[ \frac{\phi_1(\gamma)}{4} - \frac{1}{3} \ln Z \right] - \frac{2E'}{3E} \left[ \frac{\phi_2(\gamma)}{4} - \frac{1}{3} \ln Z \right] \right\}$$  \hspace{1cm} (5.1.9)

where $\gamma = 100k/(EE'Z^{1/3})$ and $\phi_1(\gamma)$ and $\phi_2(\gamma)$ are screening functions defined by integrals over the atomic form factor [23,64,96]. These screening functions have been approximated to within 1-2% by Butcher and Messel [35,81] as

$$\phi_1(\gamma) = 20.867 - 4.409\gamma + 1.156\gamma^2$$

$$\phi_2(\gamma) = 20.029 - 2.625\gamma - 0.159\gamma^2$$  \hspace{1cm} (5.1.10)

for $\gamma \leq 1$ and

$$\phi_1(\gamma) = \phi_2(\gamma) = 21.12 - 4.184 \ln(1.36\gamma + 0.952)$$  \hspace{1cm} (5.1.11)

otherwise.

Schiff [94] applied an approximate screening correction to the Bethe-Heitler cross section differential in photon energy and photon and electron emission angles using the modified Coulomb potential of the Thomas-Fermi model:

$$\left( \frac{Ze}{r} \right) \exp[-r/a] \propto \frac{1}{Z^{1/3}}$$  \hspace{1cm} (5.1.12)
which has the corresponding form factor

$$F(q, Z) = \left\{ Z[1 + (aq)^2] \right\}^{-1}$$  \hspace{1cm} (5.1.13)

In his derivation, Schiff made the assumptions that the kinetic energies of the incident and final electrons and the emitted photon are all large compared to the electron rest mass ($E, E', k >> 1$) and that small photon emission angles only are considered ($\sin \theta = \theta$). Integrating over the final electron angle, $d\Omega_e$, leads to the result [64]

$$\frac{d\sigma}{dkd\Omega_k} = \frac{1}{k} \left\{ \frac{16y^2E'}{(y^2 + 1)^4E} - \frac{(E + E')^2}{(y^2 + 1)^2E^2} + \left[ \frac{E^2 + E'^2}{(y^2 + 1)^2E^2} \right] \right\} \ln M(y)$$  \hspace{1cm} (5.1.14)

$$y = E\theta \quad M(y) = \left\{ \left( \frac{k}{2EE'} \right)^2 + \left( \frac{2^{\frac{1}{2}}}{3^{\frac{1}{2}}(y^2 + 1)} \right)^2 \right\}^{-1}$$  \hspace{1cm} (5.1.15)

The above expression for $d\sigma/dkd\Omega_k$ reduces to that of Sommerfeld [99] for no screening by putting $Z = 0$ in the log term.

The expressions for bremsstrahlung production so far have relied upon the Born approximation for the description of initial and final electron wave functions. The validity of this approximation is dependent on the inequalities

$$\frac{2\pi Z}{137} \ll 1 \quad E, E' \gg 1$$  \hspace{1cm} (5.1.16)

Hence the Born approximation is expected to break down for high atomic number targets, low to moderate initial electron energies and in the high energy region of the resulting photon spectrum. Despite the restrictions imposed by the above inequalities, which are very likely to be violated in the consideration of x-ray spectra in the energy range of interest (4 to 30 MeV), use of the Born approximation results in relatively simple analytical expressions for bremsstrahlung cross sections and at worst can still be expected to give cross sections to the correct order of magnitude [64]. In their review paper, Koch and Motz [64] suggest that Born approximation formulae can be expected to be accurate to within 10 percent for initial electron energies, $E$, above 2 MeV and within a factor of two for energies
below 2 MeV. Correcting for deviations from the Born approximation (usually termed the Coulomb correction) for the energy range of initial electrons considered here relies on comparison with experiment [64,25]. At extreme relativistic energies (above about 50 MeV), the theory developed by Davies, Bethe, Maximon and Olsen [24,39,86] (DMBO), which takes the form of an additive correction to the cross section, can be applied. Figure 5.2 compares the predicted bremsstrahlung energy spectra produced by incident electrons with energies 1, 10 and 30 MeV incident on a tungsten target for the Schiff expression, the Bethe-Heitler expressions with and without screening correction and the tabulations of Seltzer and Berger [96]. It can be seen that the predicted thin-target photon spectrum from the Schiff expression agrees well with the more complicated screened Bethe-Heitler expression, both of these being in good agreement with the tabulations of Seltzer and Berger for electron energies above 10 MeV.

For initial electron energies up to 2 MeV, Tseng and Pratt [101,88,102,103] have calculated bremsstrahlung cross sections numerically by representing the electron wave functions as a series of partial waves. The sparse experimental measurements suggest that this approach yields cross sections to within a few percent, however for energies above about 2 MeV the number of partial waves required to maintain accuracy increases rapidly [96] making this method impractical. Seltzer and Berger [96] incorporated these calculations in their published tables of bremsstrahlung cross sections differential in photon energy for initial electron energies in the range 1 keV to 10 GeV over atomic numbers up to 100. Above 50 MeV the Bethe-Heitler expression differential in photon energy (equation 5.1.6 integrated over photon emission angle, $d\Omega$) is used with the high energy screening and Coulomb corrections of the DMBO theory. For initial electron energies between these two regions, cross sections are estimated by cubic-spline interpolation.

### 5.2 Construction of a thin-target model

In order to model the bremsstrahlung spectrum produced by a thick target of thickness, $t$, using the theory described in the last section, we assume that all photons are produced within a small depth, $\delta$, in the target, which is sufficiently
Figure 5.2: Comparison of cross section formulae for bremsstrahlung production in tungsten. Abscissa is $k\sigma/dk$ (MeV cm²) normalized by area.
small that the energy and angular distribution of the incident electrons in traversing \( \delta \) has a negligible effect on the bremsstrahlung spectrum produced. The final spectrum is then obtained by filtering this thin target photon spectrum for the target thickness. For typical clinical beam dimensions, the angle subtended by the target at the distance of treatment is the order of 1-5 degrees, we therefore consider bremsstrahlung produced in the forward direction. Putting \( \theta = 0 \) into the Schiff cross section differential in photon energy and angle (equation 5.1.14) we obtain [40]

\[
\frac{d\sigma}{dk}_{\theta=0} = \frac{1}{k} \left\{ \frac{-(E + E')^2}{E^2} + \left[ \frac{E'^2 + E'^2}{E^2} \right] \ln M(0) \right\}
\]

Neglecting constant factors, the photon energy fluence before filtering, \( \Psi_{\delta}(k) \), will be given simply by

\[
\Psi_{\delta}(k) = k \times \frac{d\sigma}{dk}_{\theta=0}
\]

Substituting for the energy of the final electron, \( E' = E - k \), in equation 5.2.1 then leads to \([1,40]\]

\[
\Psi_{\delta}(k) = 2 \left( 1 - \frac{k}{E} \right) \left( \ln \eta - 1 \right) + \left( \frac{k}{E} \right)^2 \left( \ln \eta - \frac{1}{2} \right)
\]

where \( \eta = \sqrt{M(0)} \). Assuming narrow-beam geometry, the thick target spectrum, \( \Psi_{t}(k) \), can then be represented by

\[
\Psi_{t}(k) = \Psi_{\delta} \exp[-\mu_{t}(k)t] \prod_{i=1}^{n} \exp[-\mu_{i}(k)t_i]
\]

where \( \mu_{t}(k) \) is the total linear attenuation coefficient for photons of energy, \( k \), in the target material and \( \mu_{i}(k) \) is the corresponding coefficient for material, \( i \), of thickness, \( t_{i} \), present in the machine head.

Desobry and Boyer [40] have shown that the above expression yields photon energy spectra in good agreement with a thick target expression involving more sophisticated approximations (described in the following chapter) where the machine head construction is known. It is therefore proposed to model clinical photon spectra by parameterizing the Schiff expression for forward-directed bremsstrahlung differential in photon energy and including inherent filtering.
Parameter values are determined by minimizing the objective function

\[ O(p_1, ..., p_m) = \sum_{i=1}^{n} \left\{ 1 - \frac{\int x_{g}(k, p_1, ..., p_m) \exp[-\mu_f(k)x_i] \, dk}{S(x_i)} \right\}^2 \]  

(5.2.5)

where \( p_j \) are parameters to be fitted to entered transmission data, \( S(x_i) \), \( x_{g} \) is the model's spectrum and \( \mu_f(k) \) is the total linear attenuation coefficient for photons of energy, \( k \), in the filter material.

The integral appearing in equation 5.2.5 is evaluated numerically [78] over the user-defined minimum and maximum photon energies and defines the model's transmission curve. The minimization procedure is therefore limited to the accuracy with which the integral can be estimated. For minimization algorithms in which the search direction is decided by calculating first and second derivatives, numerical errors may be magnified and lead to unreliable solutions. This latter concern was tested by comparing results for the quasi-Newton algorithm [79] used in the AW model with those for a one-dimensional minimization procedure based on golden section search [85] (Section 5.6), which requires objective function values only to guide the search. For the case of a single parameter in the thin target model representing the inherent filter thickness, identical results were obtained for each algorithm over a range of input transmission data and so no reason was found to use an alternative algorithm for this model.

5.3 Parameterization

The objective in selecting suitable parameters for the model's spectrum, \( x_{g} \), is obviously to enable accurate representation of clinical spectra for the minimum number, \( m \), of parameters. Results obtained as \( m \) is increased and the considerations involved in the selection of further parameters are given below.

5.3.1 1 and 2 Parameter Models

With only a single parameter, \( t_w \), representing the thickness of an inherent tungsten filter, the fitting procedure resulted in maximum deviations between the model's transmission curve and input data of on average two percent. This clearly demonstrates the advantage of using a functional form for the spectrum which has a firm
theoretical basis.

Comparison of Monte Carlo simulated spectra with predictions of equation 5.2.3 given in reference [40] show that the Schiff expression causes the photon energy fluence in the high energy region of the spectrum to be over-estimated. This is to be expected since in a thick target we would expect photons with energies approaching that of the incident electron to be produced at shallow target depths and therefore to be attenuated by almost the total target thickness. On the other hand, the production of lower energy photons will continue to a greater depth in the target and so these photons will generally undergo less attenuation by the target itself. The one-parameter thin-target model will therefore be expected to filter the low energy part of the spectrum too heavily.

Investigation of the terms in the Schiff expression 5.2.3 (figure 5.3) show that this may be partly corrected for through the addition of a second parameter, \( \alpha \), which weights the second term and is expected to take values less than unity. The model’s spectrum is then given by

\[
\Psi_s(k) = \left\{ \left(1 - \frac{k}{E}\right) \ln\varepsilon - 1\right\} + \alpha \left(\frac{k}{E}\right)^2 \ln\varepsilon - \frac{1}{2} \right\} \exp[-\mu_t t_w] \quad (5.3.1)
\]

This 2 parameter version of the model was found to produce much improved results, as seen from table 5.1. Excepting the results for the spectra of Levy et al, maximum deviations from input data were on average 0.3 percent and below 1 percent in all cases, although appreciable variation in these values for different spectra was found. The inherent filter material was varied over elements from copper to lead to assess the dependence on filter atomic number. In only one case (Mohan et al’s 24 MeV spectrum) was a result obtained which was a significant improvement on the use of a tungsten filter. Results obtained for the spectral measurements of Levy et al [69,68] are significantly poorer than the others. This feature was also found in the results for the AW model and casts further doubt on the validity of these spectra. Selected reconstructed spectra are shown in figure 5.4 and are seen to agree well with the originals, in particular the low energy cut-off is correctly predicted.
Table 5.1: Returned maximum percentage deviations and parameter values for 2 parameter thin-target model

<table>
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<tr>
<th>Energy (MeV)</th>
<th>max. dev. (%)</th>
<th>$t_w$ (g/cm²)</th>
<th>$\alpha$</th>
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Figure 5.3: Contributing terms to the unfiltered photon energy fluence \((k\sigma/dk)\) obtained from the Schiff forward-directed bremsstrahlung cross section.

5.3.2 3 Parameter Model

The over-estimation of energy fluence in the high energy region of reconstructed spectra described earlier was only crudely accounted for in the incorporation of parameter, \(\alpha\). This situation may be improved upon by introducing an energy dependent correction to the inherent filtration which has the effect of preferentially increasing photons of lower energy. Also, the modelling of machine-head filtration may be easily improved by either increasing the number of inherent filter materials, thereby making \(\Psi_s\) more closely resemble \(\Psi_t\) in equation 5.2.4, or by including the filter atomic number itself as a parameter (described in the next chapter). It was found that both forms of correction could be achieved in the following ways;

1. Incorporation of an additive term, \(\beta/k\), in the filtering exponential.

2. Addition of a carbon (or water) filter whose thickness, \(t_c\), may become negative.

For positive \(\beta\) and negative \(t_c\) the two approaches above are seen to be equivalent, since the total linear attenuation coefficient for carbon (and water) monotonically decreases with increasing energy over the entire energy range considered.
Figure 5.4: Comparison of original and reconstructed spectra for 2 parameter thin-target model.
The 3 parameter model then becomes:

$$\Psi_s(k) = \left( 1 - \frac{k}{E} \right) (\ln \eta - 1) + \alpha \left( \frac{k}{E} \right)^2 (\ln \eta - \frac{1}{2}) \right) \exp[\beta/k - \mu_c t_c]$$  (5.3.2)

for method 1, the exponential being replaced in method 2 by

$$\exp[-(\mu_c t_c + \mu_w t_w)]$$  (5.3.3)

where $\mu_c(k)$ is the total linear attenuation coefficient of carbon for photons of energy, $k$. Following the considerations outlined above, we expect the value of $\beta$ to be positive (or $t_c$ negative) in order to provide an increased energy fluence at lower photon energies. This will be particularly true for lightly filtered spectra where the effects of target filtration will dominate. For heavily filtered spectra these effects will become less dominant and a negative value of $\beta$ (positive $t_c$) may be more appropriate, to allow greater flexibility in the modelling of inherent filtration.

Figure 5.5 compares the arguments of the exponential in equations 5.3.2 and 5.3.3 for fitted parameter values corresponding to Faddegon et al’s 15 MeV spectrum.

Figure 5.5: Comparison of third parameter corrections for Faddegon et al’s 15 MeV spectrum. $f(k)$ represents the exponential terms in equations 5.3.2 and 5.3.3 for $\beta = 0.2844$ MeV in method 1 and $t_c = -7.567$ g/cm$^2$ in method 2.
## 5.4 3 Parameter Model Results

The selection of a set of parameter values to initiate the minimization procedure is simplified for the two versions of the 3 parameter model since all parameters can be easily interpreted; parameter $\alpha$ is a weighting factor expected to lie between 0 and 1, $t_w$ and $t_c$ are thicknesses (g/cm$^2$) of inherent tungsten and carbon filters respectively and $\beta$ (MeV) forms a corrective term to the inherent filtration. Sensible upper and lower bounds for parameter values can therefore also be set. Returned parameter values for different input transmission curves, shown in tables 5.2 and 5.3 for both methods, were found to be very stable against the initial starting point used suggesting that global minima had been found.

<table>
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<th>$\beta$ (MeV)</th>
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Table 5.2: Returned maximum percentage deviations and parameter values for method 1 of the 3 parameter thin-target model.

Maximum percentage deviations of the model's final transmission curves from the entered data were on average 0.03, which is of the same order of magnitude as
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<th>$t_\alpha$ (g/cm²)</th>
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<td>Levy et al [69,68]</td>
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<td>40.060</td>
<td>0.6692</td>
<td>-28.398</td>
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Table 5.3: Returned maximum percentage deviations and parameter values for method 2 of the 3 parameter thin-target model.
the uncertainty involved in the calculation of input data values from equation 2.1.8 by numerical integration [78]. Corresponding reconstructed spectra are shown in figures 5.6 to 5.12, which show how the excellent fitting to input transmission data is reflected by the close agreement between original and reconstructed spectra. The close agreement between reconstructed spectra for methods 1 and 2 in all cases confirms the equivalence of each approach in correcting the model's inherent filtration term in equation 5.3.1 for the under-estimation of energy fluence in the low energy region of the spectrum.

The fluctuations present in original spectra are the limiting factor in assessing the thin target model’s reconstructions, both in the plots of figures 5.6 to 5.12 and also in the effect these fluctuations may have on the calculated transmission curves. An attempt to resolve this problem was made by comparing results for cubic-spline [77] smoothed spectra for the most pronounced cases. Fluctuations in the high energy region of Faddegon et al’s 30 MeV spectrum [42,43] were suspected to be responsible for the poorer maximum deviation found for this spectrum relative to the other spectra from this source. This was confirmed by using a smoothed spectrum which reduced the returned maximum deviation from .07 to .02. Similar results were obtained for the NPL 8 MeV spectrum which contains an uncharacteristic drop in energy fluence around 2.5 MeV (maximum deviation reduced to 0.07) and Mohan et al’s 4 MeV spectrum.

It has been shown that the two methods presented for incorporation of a third parameter produce almost identical results. Since method 2 requires more initial information (specifically, the total linear attenuation coefficients for carbon), subsequent work referring to the 3 parameter thin-target model will be based on expression 5.3.2 for method 1.
Figure 5.6: 4, 6, 8 and 10 MeV NPL spectra [41] and 3 parameter thin-target model reconstructions.
Figure 5.7: 12, 16 and 19 MeV NPL spectra [41] and 3 parameter thin-target model reconstructions.
Figure 5.8: 10, 15, 20 and 25 MeV spectra of Faddegon et al [42,43] and 3 parameter thin-target model reconstructions.
Figure 5.9: 30 MeV spectrum of Faddegon et al [42,43] and 3 parameter thin-target model reconstructions.
Figure 5.10: 4, 6, 15 and 24 MeV spectra of Mohan et al [74] and 3 parameter thin-target model reconstructions.
Figure 5.11: 15, 10 and 25 MeV spectra of Landry et al [66] and 3 parameter thin-target model reconstructions.
Figure 5.12: 8 and 27 MeV spectra of Levy et al [69,68] and 3 parameter thin-target model reconstructions.
5.5  Practical Considerations

The input transmission curves used so far in testing different models have been chosen in order to provide the greatest amount of information on the incident spectrum. For practical measurements of transmission curves, the choice of filter material, number of thicknesses and maximum thickness may be restricted due to practical considerations such as availability, time required and achievable set-up geometries. In this section, the dependence of the spectral reconstructions obtained with the 3 parameter model on the input transmission curve is investigated.

5.5.1  Dependence on filter material

In the preceding section it was shown that the 3 parameter model reproduced input transmission curves through selected filter materials to within the order of accuracy in their generation. As a test of the uniqueness of these reconstructed spectra, further reconstructions for different choices of filter material were obtained for 9 selected spectra covering the full range of maximum photon energies. Filter materials whose total linear attenuation coefficients are not monotonic over the energy range of a particular spectrum were included in order to assess what effect this would have on spectral shapes returned.

In all but one case similar maximum deviations (\( \sim 0.03 \) percent) between the model's transmission values and input values were returned for all filter materials. The effect of the poorer deviation (0.2 percent) obtained for the NPL 4 MeV spectrum for transmission through water is clearly seen in the first plot of figure 5.13. For all other cases, close agreement between reconstructed spectra was obtained for filter materials whose attenuation coefficients were monotonic over the required energy range.

The plots in figures 5.13 to 5.15 reveal an important feature where the minimum in total linear attenuation coefficient for each filter material (shown in figures 2.6 and 2.7) coincides with the energy range for a given spectrum. For cases where this minimum occurs away from the mean spectral energy (NPL 10 MeV spectrum, 20 MeV spectrum of Faddegon et al, 25 MeV spectrum of Landry et al and 24 MeV spectrum of Mohan et al), reconstructed spectra are in very good agreement with
those returned for the optimum filter material, discussed in section 2.5. Only in cases where this minimum occurs in the region of the mean spectral energy (Mohan et al 15 MeV spectrum and 16 MeV NPL spectrum) is a significant deviation in spectral distribution observed.

This observation may be of great use in the practical measurement of transmission curves, especially for spectra with maximum photon energies above approximately 20 MeV, where rather than being limited to water or carbon filtering, aluminium or lead may be used, greatly reducing the thicknesses of filter required to obtain sufficiently low transmitted intensities.

5.5.2 Definition of Transmission Curves

Obtaining measured transmission curves through a given material may be made significantly easier if the number of filter thicknesses required to define the transmission curve is reduced or if the minimum transmission to be measured can be raised. The stability of spectral reconstructions by the 3 parameter model on these factors was assessed for selected spectra covering the range of energies from 6 to 25 MeV. It was found that reconstructed spectra may be produced with similar reliability to the previous definition of input transmission curves by defining the input data by 15 logarithmically-spaced filter thicknesses giving transmissions of down to only 10 percent of the incident beam intensity. Comparative plots are shown in figure 5.16.

In carrying out measurements the attempt must obviously be made to define the transmission curve as fully as possible in order to present the reconstruction model with the greatest amount of information on the incident spectrum. However, figure 5.16 shows that this task should be relatively easily achieved despite the restrictions of the set-up geometry available and uncertainty involved in measuring low transmitted intensities.

5.6 Effective Maximum Energy

It has been assumed so far that the maximum photon energy present in clinical spectra is given by the nominal electron energy, $E$. In practice there is some
Figure 5.13: Dependence of 3 parameter model spectral reconstructions on choice of filter material
Figure 5.14: Dependence of 3 parameter model spectral reconstructions on choice of filter material
uncertainty involved and in order to achieve the accuracy of reconstructed spectra found for the thin-target model in the last section we must allow the effective maximum photon energy, \( k_0 \), to be derived from input transmission data. Simply introducing this maximum energy as a fourth parameter was found to produce poor results, drastically increasing the time required for minimization and in many cases failing to satisfy some of the conditions for a minimum. This is to be expected since the objective function is not strongly dependent on the exact value of \( k_0 \), changes of up to a few percent having little effect on the success of the fit to input data. The approach was therefore taken to define a discrete set of allowed \( k_0 \) values lying within \( \pm 10\% \) of the user-defined nominal maximum energy, the exact number of which is dependent on the user-defined number of energy array elements. Calls to the minimization procedure for each value of \( k_0 \) can then be made and the optimum value found.

The variation in final objective function value, \( O(p_j) \), defined by equation 5.2.5 with \( k_0 \) was investigated for several spectra, and in each case was found to describe a single, global minimum, depicted in figure 5.17. These results indicated that rather than performing the minimization for each allowed value sequentially, the number of calls to the procedure could be reduced by the use of a separate, 1-
Figure 5.16: Dependence of 3 parameter model spectral reconstructions on the definition of input transmission curves.
dimensional minimization routine. The method used was based on the golden section search [85], in which the minimum is found from a triplet of points, $a$, $b$ and $c$ shown in figure 5.17 where $a$ and $b$ enclose the true minimum, and $O_c(p_j)$ lies below $O_a(p_j)$ and $O_b(p_j)$. In the present method, $b$ is set to 10 percent above the nominal energy, $E$, and $c$, $a$ to the calculated energy array elements nearest to the nominal energy and 10 percent below the nominal energy respectively. The optimum value for $k_0$ is then found by reducing the energy interval which bounds the minimum to new points, $a'$ and $b'$ given by

$$a' = f|c - a| + a$$
$$b' = f|b - c| + c$$

(5.6.1)

where $f$ is set arbitrarily to 0.4, since we expect the true minimum to lie nearer to the nominal energy than the boundaries. Values $a'$, $b'$ are then set to elements in the energy array and the corresponding objective function values determined. The triplet of points is then modified according to these values and the process repeated until $a$, $b$ and $c$ are adjacent points in the energy array in which case, $c$ is the optimum value of $k_0$.

![Figure 5.17: One-dimensional minimization for derivation of effective maximum photon energy, $k_0$.](image)

**5.6.1 Results**

The ability of the thin target model to derive the effective maximum photon energy present in a spectrum was investigated for selected measured and simulated spectra.
whose maximum energy is known exactly). Runs were performed for entered maximum energies equal to and nominally 5% above and below the known value to provide a full range of starting points. 50 energy array elements defined the reconstructed spectra, giving a resolution for \( k_0 \) of approximately 1%. Returned values of \( k_0 \) and final objective function values \((\times 10^{-7})\) are shown in table 5.4. The objective function value returned from the 3 parameter model, \( O_3(p_j) \), of the last section is included for comparison.

<table>
<thead>
<tr>
<th>Spectrum</th>
<th>( E ) (MeV)</th>
<th>( k_0 ) (MeV)</th>
<th>( O(p_j) )</th>
<th>( O_3(p_j) )</th>
<th>( \Delta% )</th>
</tr>
</thead>
<tbody>
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<td>3.9</td>
<td>500</td>
<td>40</td>
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<td>4.4</td>
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<td>10</td>
<td></td>
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<tr>
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<td>0.2</td>
<td>1.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.3</td>
<td>6.1</td>
<td>0.3</td>
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<td></td>
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<td>0.8</td>
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<tr>
<td></td>
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<td>1.0</td>
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<tr>
<td></td>
<td>10.5</td>
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<td>20.0</td>
<td>18.4</td>
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<td>Landry 25 MeV</td>
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<td>0.2</td>
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<td>0.2</td>
<td>2.8</td>
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<tr>
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<td>1.0</td>
<td>4.0</td>
<td>2.2</td>
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<td>29.7</td>
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Table 5.4: Returned effective maximum photon energies, \( k_0 \), for varying values of nominal maximum energy, \( E \). Objective function values for \( k_0 \) estimation, \( O(p_j) \), and from the 3 parameter model, \( O_3(p_j) \), are in units of \( 10^{-7} \) and \( \Delta\% \) is the percentage deviation from the known maximum photon energy.

Values of the objective function for \( k_0 \) fitting, \( O(p_j) \), are seen to be less in most
cases than those found in the previous section where the returned maximum energy differs from the quoted value by more than the available resolution. Figures 5.18 and 5.19 show comparisons of reconstructed spectra with and without fitting to the maximum photon energy for the cases where a significant difference in the returned value of $k_0$ was found. The variation of the exact value of $k_0$ is seen to have little effect on the general spectral shape. The apparently poor results obtained for Mohan et al's 4 MeV spectrum are likely to be due to the poor definition of the spectrum and the fact that the energy fluence is seen to be non-zero at the high energy limit (figure 5.18). This latter observation is also true for Mohan et al's 15 MeV spectrum where again the returned value of $k_0$ is greater than the true maximum energy, although in this case the plot in figure 5.18 shows very good agreement between original and reconstructed spectra and the objective function value, $O(p_f)$, is seen to be almost an order of magnitude lower than that obtained without fitting to the maximum energy.

Returned maximum energies appear to be independent of the entered nominal energy which supports the use of the thin-target model for maximum energy determination, since it shows that the incorporation of this discrete-valued parameter does not produce too much flexibility in the model which could lead to incorrect spectral shapes.

A clear example of the use of fitting to the effective maximum photon energy is shown by the inclusion of Levy et al's spectrum [68], whose nominal energy is reported by the authors to be 25 MeV. The fitted value of 26.4 MeV is seen to result in an improved reconstruction of this spectrum.

For the 30 MeV spectrum of Faddegon et al, the higher of the entered nominal maximum energies had to be lowered from 31.5 MeV to 30.5 MeV in order to bound the minimum of the objective function. The discrepancy observed for the returned maximum photon energy for this spectrum was removed by smoothing out the fluctuations in the high energy region, as for the 3 parameter model, shown in the final plot of figure 5.19.
Figure 5.18: Spectra resulting from fitting to effective maximum photon energy. $E_{nom}$ is the entered nominal maximum energy and $k_0$ the fitted value.
Figure 5.19: Reconstructed spectra from effective maximum photon energy fitting for Levy et al’s 27 MeV spectrum [68] and Faddegon et al’s 30 MeV spectrum [42, 43].
Chapter 6

Thick-Target Models

In the last chapter it was shown that by applying inherent filtering to Born approximation bremsstrahlung cross sections (which describe the production of thin-target bremsstrahlung), a reliable reconstruction model could be developed. This approach is extended in the present chapter by considering thick-target bremsstrahlung production in which the energy and angular distribution of electrons within the target has a significant effect on the resulting photon spectrum.

6.1 Approximations

The estimation of thick-target photon spectra in comparison to thin-target spectra is greatly complicated since even in considering only forward-directed bremsstrahlung we must now consider the production of photons from an electron fluence changing with depth in the target which is distributed in both energy and angle. Since calculation of the electron distribution itself would require Monte Carlo simulation, several approximations are necessary. Here we follow a combination of the methods of Andreo and Brahme [6,3] and Desobry and Boyer [40] making use of the following assumptions:

1. The distribution of electron energies within the target can be replaced by the mean electron energy at a given depth in the target

2. The reduction in electron fluence with depth due to scattering can be modelled by the mean electron scattering angle
3. The bremsstrahlung cross-section differential only in photon energy describes the bremsstrahlung produced.

4. Photon attenuation in the target can be approximated using the mass-energy absorption coefficient for the target material.

6.1.1 Mean electron energy at depth

The total stopping power for electrons in tungsten with kinetic energies from approximately 1 to 35 MeV varies almost linearly with energy, as shown in figure 6.1. The variation of electron energy, $E(z)$, with depth, $z$, in the target may therefore be modelled by the expression [40]

$$E(z) = [E(0) + a/b] \exp[-b z] - a/b$$  \hspace{1cm} (6.1.1)

where $a$ and $b$ are the intercept and slope respectively given by fitting a straight line to the stopping power data [57] in figure 6.1 and $E(0)$ is the incident electron energy. For the tungsten target assumed in this work, values of 0.9801 and 0.1356 were taken for $a$ and $b$ respectively. Figure 6.2 compares the predicted mean electron energies with target depth for the above expression and that of Brahme and Andreo [5,56], which is known to give a reasonable approximation to the mean energy over the first half of the electron's range and underestimate the true value at greater depths.

6.1.2 Electron dispersion

Desobry and Boyer [40] estimate the reduction in the forward-directed electron fluence, $\Phi(z)$, with depth in the target using the expression

$$\Phi(z) = \frac{\Delta^2}{\bar{\theta}^2(z)} = \frac{\Delta^2}{(E(z)/E(z)^2 \beta^4 X_0) \rho z + \bar{\theta}_0^2}$$  \hspace{1cm} (6.1.2)

$$\beta = [E(z)(E(z) + 2\mu)]^{1/2}/(E(z) + \mu)$$

where $\Delta$ represents a small angle about the forward direction which can be chosen to normalize the electron fluence to unity at zero target depth, $\bar{\theta}(z)$ is the mean-square angle of scattering [56], $E_s$ has a constant value of 21.2 MeV, $X_0$ is the
Figure 6.1: Total stopping power for electrons in tungsten [57]

Figure 6.2: Variation of mean electron energy with depth in tungsten target for 1, 10 and 30 MeV electrons.
Figure 6.3: Relative forward electron fluence, \(4\pi/\bar{\theta}^2(z)\), as a function of target depth, \(z/z_{\text{max}}\) for initial electron energies of 5, 10 and 20 MeV. The mean electron energy at the depth at which the distribution becomes fully dispersed \(4\pi/\bar{\theta}^2(z) = 1\) is indicated.

The rapid dispersion of electrons within the target means that the bremsstrahlung produced will be best described by the cross-section differential in photon
energy only. Since we are concerned here with constructing a physically realistic model of thick-target bremsstrahlung, the values of the bremsstrahlung cross-section are taken from the tabulations of Seltzer and Berger [96] rather than the corresponding Schiff expression obtained by integrating equation 5.1.14 over photon emission angles [94,40,64]. Interpolation between tabulated values is carried out by fitting a bicubic-spline [76] to Seltzer and Berger's data.

Correcting the electron fluence at depth in the target for dispersion from the forward direction results in a reduction in the relative number of low energy photons present in the final spectrum. With electron dispersion neglected, our model closely resembles that of Ahnesjo and Andreo [3] which the authors used to derive clinical spectra by fitting to calculated and measured depth-dose curves in water. Using the true target and filtration thicknesses given in chapter 2, spectra were calculated both with and without a dispersion correction and are compared in figure 6.4. The importance of such correction is seen to be essential in order to generate a physically realistic thick-target spectrum from the known target and filtration thicknesses, especially where the beam is lightly filtered as is the case for the NPL spectra. It should be pointed out however that in their paper Ahnesjo and Andreo state that the parameters they obtain in fitting to depth-dose curves should not be assumed to give information on the actual structure of the treatment head. The omission of electron dispersion is partly corrected for since their published parameter values given for the spectra of Mohan et al generally show a smaller target thickness and lower effective atomic number for the inherent filtration than the true values, which would combine to reduce the production of bremsstrahlung from electrons with energy far less than that of the incident beam and increase the proportion of high energy photons by reducing the pair-production cross-section in the inherent filter material.

6.1.3 Target attenuation of photons

Where the target thickness is comparable to or greater than the electron range, we may expect photon scattering and absorption in the target to make a significant contribution to the photon spectrum produced. As indicated above, the electrons
Figure 6.4: Effect of reduction in forward electron fluence due to electron dispersion on thick-target spectra calculated using known details of target and filtration conditions.
disperse rapidly in the target and consequently the photons produced would be expected to have a similar distribution. In this case, we must consider broad-beam geometry as photons will be scattered into as well as out of the forward direction. Target attenuation would therefore be expected to be best described using the mass-energy absorption coefficient for the target material rather than the total linear attenuation coefficient. Figure 6.5 shows a comparison between mass-energy absorption coefficients and total linear attenuation coefficients for tungsten over the energy range of interest. We see that use of the former quantity will be expected to result in a relative decrease in photon fluence at lower energies and an increase for photon energies above approximately 10 MeV.

Figure 6.6 compares 6 and 16 MeV NPL spectra calculated from a thick target expression using assumptions 1 to 3 where target filtration is either neglected or modelled using the mass-energy absorption coefficient or the total linear attenuation coefficient for tungsten. The target thicknesses and additional filtration for each spectra were taken from the tables in chapter 2. It is seen that as expected the use of mass-energy absorption coefficients provides better agreement with the true spectrum for these lightly filtered spectra. For more heavily filtered spectra, such as those of Mohan et al [74] (which also have smaller target thicknesses) tar-
get filtration was found to have a negligible effect on the resulting spectral shape, however, it's inclusion in a parameterized thick-target model is important in order to maintain the dependence of the model on target thickness when the electron range in the target material is exceeded.

### 6.2 Thick-target expression

With the four assumptions described in the last section, we may represent the photon energy fluence, $\Psi(k)$, produced by a tungsten target of thickness, $z_0$, and filtered by $n$ materials with atomic numbers, $Z_i$ and thicknesses, $t_i$ by the expression

$$
\Psi(k, z_0) = k \left\{ \prod_{i=1}^{n} \exp[-\mu_i(k)t_i] \right\} \int_0^{z_0} \left( \frac{d\sigma}{dk} \right)_{E(x)} \exp\left[-\left(\frac{\mu_{en}(k)}{\rho}(z_0 - z)\right)\right] \frac{dx}{E^2_p z/(E(x)^2\beta^4 X_0) + \theta^2_0}
$$

where $(\mu_{en}(k)/\rho)$ is the mass-energy absorption coefficient for photons of energy, $k$, in tungsten and $\mu_i(k)$ are the total linear attenuation coefficients for the inherent filter materials. Figures 6.4 and 6.6 show that where details of the target and inherent filtration are known, the above expression can be expected to provide a
good approximation to the true spectrum. The accuracy of the expression was quantified by comparing calculated transmission curves for the estimated and true spectra in the same way that reconstruction models have been assessed. Maximum deviations were found in general to be less than 10 percent for transmitted intensities down to 0.1 percent of the initial beam.

6.2.1 Parameterization

Having obtained a realistic expression for thick-target bremsstrahlung spectra, the next step is to parameterize equation 6.2.1 so that fitting to calculated transmission curves by minimizing an objective function similar to that of equation 5.2.5 can be carried out. Logical choices for parameters are the target thickness, $z_0$, the atomic number, $Z_f$ and thickness, $t_f$ of an inherent filter. Photon spectra are then described by the expression

$$\Psi_t(k) = \exp[-\mu_f(Z_f,k)t_f]\Psi(k, z_0)$$

(6.2.2)

where $\Psi_t(k)$ is the photon energy fluence and $\mu_f(k)$ is the total linear attenuation coefficient for photons of energy, $k$, in the inherent filter material.

A continuous distribution for $Z_f$ values was provided by Hermite polynomial interpolation [76] between the total linear attenuation coefficients of a data-base of 16 elements with atomic numbers ranging from 6 to 92.

6.3 Results

All three parameters in the thick-target model developed here represent physical quantities and so a sensible initial point for the minimization procedure and realistic upper and lower bounds on the parameter values may be set. It was found that the number of energy elements defining the reconstructed spectrum had to be increased from 50 (as for most previous models) to 100 in order to achieve sufficient accuracy in the numerical routines which provide the total photon spectrum by integrating over depth (incident electron energy) in the target. The greater complexity of this model in comparison to the thin-target model described previously and the need for larger energy and attenuation coefficient arrays resulted in far longer run-times
for parameter fitting, being of the same order as for the iterative model of Chapter 3.

Table 6.1 gives values of the fitted parameters and maximum deviations from the input transmission curve where the initial conditions have been set to average parameter values of \( z_0 = 0.3 \text{ cm}, t_f = 20 \text{ gcm}^{-2} \) and \( Z_f = 50 \). The ability of the thick-target model to reproduce input data is seen to vary greatly between spectra, with maximum deviations ranging from 0.01 percent for Faddegon et al’s 20 MeV spectrum to 42.0 percent for the 8 MeV spectrum of Levy et al. In all but one case (NPL 10 MeV spectrum) where it was possible to compare the deviations given in Table 6.1 with those resulting from the use of true parameter values as mentioned in section 6.2, it was found that the fitted values resulted in closer agreement with input transmission data.

Stability of the returned parameter values against the initial point used for minimization was investigated where appreciable difference between this point and the true values was apparent. In such cases the initial point was made equal to these true values. Only in the case of the NPL 10 MeV spectrum was an improvement in the solution found, reducing the maximum deviation from 12.6 to 2.3 percent. For other cases it was noticed that although solutions for differing initial points resulted in very similar deviations from the input transmission data, values of the effective atomic number, \( Z_f \), were seen to vary considerably, indicating that the objective function is not strongly dependent on this parameter.

The corresponding reconstructed spectra shown in figures 6.7 to 6.13 reflect the variation in accuracy with which the input transmission curves are reproduced by the thick-target model. It is seen that in general better reconstructions are given for higher energy spectra, which corresponds to an increased target and inherent filter thickness.

In 4 cases the inherent filter thickness is returned equal to zero, which despite the risk of resulting in an unphysical shape for the reconstructed spectrum, as is the case for the 4 MeV NPL spectrum (the returned target thickness also being close to zero), results in better reproduction of the input transmission data than the use of the true parameters. It is significant that out of the 4 such cases, 3
occur for the lightly-filtered NPL spectra, the fourth being for Levy et al's 8 MeV spectrum, whose spectral shape has been questioned in previous chapters. Under the situation of $t_f = 0$, the reconstructed spectra are defined simply by the target thickness, $z_0$.

In constructing the present model the attempt was made to describe the true process of thick-target bremsstrahlung production as closely as is reasonably possible. This appears to have over restricted the flexibility of the model making it incapable of representing all spectra used in this study to the degree of accuracy found for other models, although in 5 cases resulting maximum deviations were found to be of similar magnitude to those obtained for the 3 parameter thin-target model. By comparing returned parameter values with the true target thicknesses and filtration conditions, where known, it was found that in only one case (Mohak et al's 24 MeV spectrum) were parameter values returned which were in good agreement with the true values. It must therefore be concluded that even where reconstructed spectra agree well with original data, the parameters themselves may not be taken to represent the true geometries under which the beam is produced.
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Table 6.1: Returned maximum percentage deviations and parameter values for thick target model
Figure 6.7: 4, 6, 8 and 10 MeV NPL spectra [41] and thick-target model reconstructions.
Figure 6.8: 12, 16 and 19 MeV NPL spectra [41] and thick-target model reconstructions.
Figure 6.9: 10, 15, 20 and 25 MeV spectra of Faddegon et al [42,43] and thick-target model reconstructions.
Figure 6.10: 30 MeV spectrum of Faddegon et al [42,43] and thick-target model reconstruction.
Figure 6.11: 4, 6, 15 and 24 MeV spectra of Mohan et al [74] and thick-target model reconstructions.
Figure 6.12: 15, 20 and 25 MeV spectra of Landry et al [66] and thick-target model reconstructions.
Figure 6.13: 8 and 27 MeV spectra of Levy et al [69,68] and thick-target model reconstructions.
Chapter 7

Conclusions and Further Work

7.1 Summary of Results

In the preceding chapters a full range of approaches to the problem of reconstructing photon spectra from narrow-beam transmission curves have been applied, from iterative models which are constrained only by being forced to produce a physically realistic spectrum to a model firmly based on an approximate thick-target expression which takes into account the passage of electrons and photons through a transmission target and subsequent filtering materials. Here a comparison will be made between models, both in the accuracy with which entered transmission data were reproduced and the agreement achieved between original and reconstructed spectra.

Table 7.1 compares maximum deviations from entered transmission curves showing the 3 parameter thin-target model of chapter 5 to produce the least deviations in all cases (on average 0.03 percent), achieving up to an order of magnitude improvement over other models. This level of accuracy is of the same order as the uncertainty in transmission values calculated from the source spectra described in chapter 2 for entry into all models and is well within the likely accuracy of experimentally measured transmission curves, estimated by Huang et al [52] to be 0.15 - 0.5 percent.

The iterative model based on the work of Huang et al [52,53] and the Laplace transform model of Archer and Wagner [9] are seen to produce similar results, with
maximum deviations from input data being on average 0.2 - 0.3 percent. Solutions for these models were found to be strongly dependent on the initial values used to begin the minimization procedure, suggesting the presence of many local minima. The likelihood of determining the global minimum for these models could be greatly improved through the application of simulated annealing [85], however this would lead to a dramatic increase in computing time and has therefore not been pursued.

Results for the thick-target model of chapter 6 are seen to vary considerably between spectra, better results being obtained in general for heavily filtered beams with maximum beam energies towards the higher end of the range considered in this work, where maximum deviations are of similar magnitude to the thin-target model results.

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<td>27</td>
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Table 7.1: Comparison of maximum percentage deviations from entered transmission data for reconstruction models considered in this work. †deviation obtained after cubic-spline smoothing original spectrum (chapter 5), ‡deviation obtained after setting initial point for the minimization procedure to known parameter values (chapter 6).
A check on the reliability of reconstructions by the thin-target model can be made by comparing the corresponding results for the thick-target model where similar reproduction of input data was obtained. Figure 7.1 shows such a comparison for Mohan et al's 15 and 24 MeV spectra, Faddegon et al's 20 MeV spectrum and the 25 MeV spectrum of Landry et al. The excellent agreement obtained for these two quite different models supports the validity of the spectral shapes returned by the thin-target model for clinical megavoltage photon beams.

There is obviously a strong connection between the accuracy obtained in fitting to input transmission data and the agreement found between original and reconstructed spectra. Determining an upper limit for the maximum deviation at which point the reconstructed spectrum can be assumed to be a sufficiently close approximation to the true spectrum will depend on the required accuracy for intended applications and is also dependent on the uncertainties present in input data. Where acceptable reproduction of input data is achieved, the selection of a robust model which is restricted to describing realistic photon spectra provides some assurance that input data has been correctly related to the true photon spectrum. This feature will become important in the practical use of reconstruction models where dosimetry considerations and the effects of detector energy dependence and scatter contributions must be corrected for in the model.

### 7.2 Corrections for Practical Measurements

The thin-target model developed in this work has emerged as a promising method for determining clinical spectra from measured transmission curves by relating the measured transmissions, \( S(x) \), to the photon energy fluence, \( \Psi(k) \), via the equation presented in chapter 2;

\[
S(x)S(x) = \int_{0}^{\infty} \Psi(k)R(k) \exp[-\mu_f(k)x]dk
\]  

(7.2.1)

Where \( \Psi(k) \) is represented by expression 5.3.2. In order that the transition from theoretical to measured transmission curves be a smooth one, likely sources of error must be investigated and possible solutions sought. Some aspects to be considered are listed below;
Figure 7.1: Comparison of spectra reconstructed by the thin- and thick-target models.
1. Influence of detector energy-response

2. Presence of impurities in filter material

3. Contributions due to scattering in filters

4. Distribution in energy of electrons incident on target

Correction for a known detector energy response, $R(k)$, is easily incorporated into the model's transmission integral as described in chapter 2. The accuracy with which this function is known is a major factor in assigning uncertainties to the reconstructed spectrum.

Provided the relative proportions of impurities in the chosen filter material are known, correcting for their presence involves simply replacing the exponential term in equation 7.2.1 by

$$\exp\left\{\sum_{i=0}^{n} c_i \mu_i(k) x\right\}$$

(7.2.2)

where $c_i$ is the proportion by weight of the $i^{th}$ element present in the filter [58].

Large thicknesses of filter material will generally be required in order to define the transmission curve as fully as possible. For example, to attenuate a 10 MeV spectrum to ten percent of the incident intensity (which was shown in section 5.5 to be a sufficient minimum value) requires approximately 26 cm of aluminium, 30 cm of carbon (assuming $\rho_c = 2.26$) or 65 cm of water (these being the values for Faddegon et al’s 10 MeV spectrum). For such filter thickness, photons may be scattered into as well as out of the forward direction and so contribute to the measured transmission, the tabulated total linear attenuation coefficients so far used to construct the model’s transmission curves would therefore no longer be appropriate. The apparent decrease in effective attenuation coefficient with increasing filter thickness, which would result from increasing scatter contributions, may be investigated using Monte Carlo simulations for mono-energetic photons using typical set-up geometries. Corrections may then be incorporated into the model by replacing tabulated coefficients with calculated values.

In the development of the thin-target model it has been assumed that mono-energetic electrons are incident on the target. Where this energy is not known
accurately the extension of the model allowing an estimate of the maximum photon energy present in the spectrum to be derived (described in section 5.6) would be required. In cases where there is appreciable spread in incident electron energy, modifications to the model may be required if insufficient flexibility exists in its present form to account for this eventuality.

### 7.3 Spectral Reconstructions from Depth-dose Curves in Water

In carrying out a comparison of reconstruction models for theoretical data, narrow-beam transmission curves alone have been used because of the simple relationship between a photon spectrum and its theoretical narrow-beam transmission curve through a given filter material. Having established that the thin-target model is capable of accurately representing clinical spectra, we now consider its use for deriving spectra from measured depth-dose curves in water which may be a more easily obtained measurement in radiotherapy departments. As described in section 2.2, the dose at depth, $x$, on the beam central axis is related to the photon energy fluence, $\Psi(k)$, incident on the water phantom by

$$D(x) = \int_{0}^{E} \Psi(k) d(x, k) dk$$

(7.3.1)

where $E$ is the maximum photon energy present in the spectrum and $d(x, k)$ is the dose delivered per unit energy fluence by photons of energy, $k$, at depth, $x$ [2], or in other words, the depth-dose resulting at $x$ from monoenergetic photons of energy, $k$. If measured or calculated values of $d(x, k)$ for different depths and photon energies can be obtained with sufficient accuracy the photon energy spectrum, represented by the thin-target model, can be obtained from a measured depth-dose curve through minimizing the objective function

$$O(t_w, \alpha, \beta) = \sum_{i=1}^{n} \left\{ 1 - \frac{D(x_i)}{D_m(x_i)} \right\}^2$$

(7.3.2)

where $t_w$, $\alpha$ and $\beta$ are the thin-target model parameters defining the photon spectrum and $D_m$ is the measured depth-dose curve defined by $n$ depths. This approach to spectral reconstruction has been carried out by Ahnesjo and Andreo [3] by
representing the photon spectrum using a parameterized thick-target expression mentioned in section 6.1.2. Values of $d(x,k)$ were derived through Monte-Carlo simulations [2,3].

In applying this method to experimentally measured depth-dose curves, the effects of detector energy response and any other energy dependent factors must be included in equation 7.3.1 as for the treatment of transmission curves. An additional factor to be considered in using measured depth-dose curves in water is the contamination of the photon beam by electrons generated in the machine-head and in the intervening air, which increases the measured dose in the build-up region [33].

7.4 Applications of Reconstruction Models

7.4.1 Calculation of Dose-distributions

Calculation of central-axis depth-dose curves mentioned in the last section through Monte-Carlo simulation of energy-deposition kernels, $d(x,k)$, is easily extended to the calculation of full dose-distributions in tissue [29,30,2,31,73,75,7]. This development in radiotherapy treatment planning has received much attention since it provides the opportunity to calculate dose-distributions from first principles rather than by interpolating between measured beam profiles for different field sizes and depths in a phantom which allow only limited inter-comparison between different machines. In order that this approach to treatment planning be accessible, photon spectra from clinical linear accelerators must be known in all radiotherapy departments. Derivation of photon spectra through application of the thin-target model developed in this work would therefore have a direct use in routine calculation of dose-distributions for treatment planning.

7.4.2 Extension to Diagnostic Energies

The original work in x-ray spectral reconstructions was concerned with the diagnostic energy range. Having extended the ideas to the megavoltage region and obtained a very promising model for spectral representation we may return to di-
agnostic energies and examine whether or not the same expression still leads to improved results. The main modification to the present thin-target model will be in the inclusion of characteristic radiation in order to correctly represent diagnostic spectra. Such a correction may be applied along the lines of Archer and Wagner's [12] empirical correction to their Laplace transform model, using known characteristic x-ray contributions to the total energy fluence for different target materials.

In the construction of the thin-target model for megavoltage spectra the geometry under consideration was that of a transmission target, the subsequent theory and corrective terms applied could be altered if necessary to more closely reflect the production of bremsstrahlung from a reflection target present in diagnostic x-ray sets and also allow the target material to be specified. The Schiff expression for forward-directed bremsstrahlung itself is strictly only valid in the megavoltage region as described in section 5.1 and so the use of alternative expressions could be investigated.

7.4.3 Derivation of Electron Spectra

A related but more complex problem than the reconstruction of photon spectra is the derivation of electron beam spectra. The increased complexity being due to the high level of scattering of electrons in matter making analytical treatments of changes in the spectrum after traversing simple geometries extremely difficult. A simple technique for deriving electron spectra emerging from a treatment machine can be constructed by considering measured electron depth-dose curves in water and applying a method along the lines of Sauer and Neumann's [91] approach for deriving photon spectra. In this approach we represent the electron depth-dose curve of a polyenergetic beam of electrons, $D_p$, as a superposition of monoenergetic depth-dose curves

$$D_p(x_i) = \sum_{j=1}^{n} c_j D_j(x_i)$$  (7.4.1)

where $D_j$ represents the monoenergetic depth-dose curve for the $j^{th}$ energy element and $c_j$ is the weighting of this energy component. Extending the above expression
to a continuous distribution of energies we have

$$D_p(x_i) = \int_0^{E_0} \Phi(E) D(E, x_i) dE$$

(7.4.2)

where $\Phi(E)$ now represents the electron particle fluence differential in electron energy normalized to one incident electron. Values for $D(E, x)$ could be derived using Monte-Carlo simulation in a similar manner to that carried out for photon depth-dose curves [2,3,70,82]. Equation 7.4.2 resembles the representation of polyenergetic photon depth-dose curves and consequently a similar objective function to expression 7.3.2 can be used to determine the parameters of a suitable analytic expression describing the electron particle spectrum incident on the water phantom. If such an expression cannot easily be derived, an iterative approach to the derivation of the electron spectrum could be applied.

### 7.4.4 Calculation of Stopping Power Ratios and Quality Indices

The dosimetry considerations described in chapter 2 mentioned the use of electron stopping power ratios required in order to convert the dose absorbed in an air-filled cavity chamber to absorbed dose to water through the Bragg-Gray relation [49]. These stopping powers refer to the secondary electron spectrum crossing the chamber, resulting from the interactions of photons both in the water surrounding the chamber and in the chamber wall material (neglecting the production of secondary electrons in the cavity itself). Calculation of accurate stopping power ratios therefore requires knowledge of the incident photon spectrum.

The quality of clinical megavoltage photon spectra is widely represented as the ratio of absorbed dose at 20 and 10 cm depth in a phantom, $\text{TPR}_{10}^{20}$, the tissue phantom ratio [33] (TPR) being the dose at a given depth relative to that at a chosen reference depth. For known photon spectra, such dose ratios may be calculated using monoenergetic depth-dose data derived from Monte-Carlo simulations as discussed previously. Andreo and Brahme [6] have calculated stopping powers and TPR values for a number of published photon spectra using data for monoenergetic photons derived from Monte-Carlo simulations.

A further quantitative comparison between the spectra described in chapter 2 and reconstructions by the 3 parameter thin-target model is therefore possible.
by calculating stopping powers and TPR values. The program of Andreo [8] uses Monte Carlo simulated data to calculate the stopping power ratio from water to air, $S_{w,a}$, at calibration depths in a water phantom (5 cm for maximum photon energies up to 10 MeV, 7 cm up to 25 MeV and 10 cm otherwise) and TPR values for entered photon spectra. Results of such calculations are shown in table 7.2 and figure 7.2. Deviations in TPR values between original and reconstructed spectra were found to be on average 0.2 percent, differences in stopping power values being negligible.

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<td><strong>Mohan et al [74,36]</strong></td>
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<td><strong>Landry et al [66]</strong></td>
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<td><strong>Levy et al [69,68]</strong></td>
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Table 7.2: Calculated stopping power ratios from water to air, $S_{w,a}$, and tissue-phantom ratios, TPR$_{10}^{w}$, for original spectra and thin-target model reconstructions.

The data in figure 7.2 shows how the degree of inherent filtering affects the beam's quality as defined by dose ratios, the lightly filtered spectra simulated at the NPL and measured by Faddegon et al being of similar quality and therefore lying on approximately the same curve, whilst those of Mohan et al and Landry et
Figure 7.2: Quality index \( (TPR^2_{130}) \) and stopping power ratios from water to air, \( S_{w,a} \) for original spectra and thin-target model reconstructions.
al are seen to contain greater levels of filtration resulting in larger TPR values.

It follows from figure 7.2 that there is some uncertainty involved in selecting an appropriate stopping power value for a photon beam specified in terms of its TPR value, this being dependent on the degree of filtration present for the beams generation. If a close approximation to the full photon spectrum can be derived from measured transmission or depth-dose curves, this uncertainty is removed by calculating stopping powers directly. In this way, we may improve the description of clinical photon beams by specifying them in terms of quantities calculated directly from the photon energy fluence itself, such as stopping power ratios. A similar quantity may be seen from the discussion of photon dosimetry in chapter 2, where the absorbed dose to water was related to the charge per unit mass measured in a cavity chamber by

\[
D_w(k) = J_a \left( \frac{W_a}{e} \right) \left\{ \beta s_{a,w} + (1 - \beta) \frac{\mu_{en}/\rho}{\mu_{en}/\rho}_w \right\}^{-1}
\]

(7.4.3)

Calculation of the bracketed term on the right-hand side of the above expression for derived spectra may therefore provide a more suitable quantity for photon beam quality specification.
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