Frequency Synchronisation for OFDMA Wireless Cellular Networks

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To my parents
Abstract

Orthogonal frequency-division multiple-access (OFDMA) has been adopted for next-generation broadband satellite and terrestrial wireless standards due to its robustness to the channel frequency-selectivity. In an OFDMA-based communication system, due to the presence of carrier-frequency-offset (CFO) that is induced by time-varying Doppler shift and oscillators' mismatch, the orthogonality of subcarriers cannot be guaranteed. Amongst the factors that cause phase-mismatch, CFO is one of the most challenging parameters to counteract, which gives rise to inter-carrier interference (ICI) and multiuser interference (MUI). In the last decade, this topic has received intensive investigations for single-user orthogonal frequency-division multiplexing (OFDM) or downlink OFDMA and not many works have been reported so far on the CFO estimation for uplink OFDMA which is the subject of research in this thesis.

The major contributions of this thesis are in three folds summarised as follows.

- First, a blind CFO estimation scheme was proposed for a linearly precoded (LP)-OFDMA system. The major idea was to take advantage of time correlation induced by the linear precoder, which offered a second-order moments-based blind CFO estimation.

- The second scheme was motivated by the fact that the best linear unbiased estimator (BLUE) that is used for CFO estimation is very sensitive to the MUI. The objective of this scheme was to improve the robustness of BLUE in multiuser scenarios by proposing three appropriately designed training sequences.

- The third scheme was based on a training-based iterative concatenated CFO estimation and compensation algorithm.

Our results showed that firstly, the class of training sequence designs could offer close performance to that of the single-user equivalent scenario. Secondly, the iterative algorithm showed significant improvement in CFO estimation and overall system performance after only one iteration. Finally, the blind scheme outperformed state-of-the-art blind methods in both single-user and multiuser scenarios.
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List of Acronyms

3GPP Third generation partnership project
AMMSE Approximate MMSE
APFE Alternating projection frequency estimator
AU Asynchronous user
AWGN Additive white Gaussian noise
BER Bit-error-rate
BLUE Best linear unbiased estimator
BS Base-station
CFO Carrier-frequency-offset
CP Cyclic prefix
CRLB Cramér-Rao lower bound
CSI Channel-state-information
CS Cyclostationarity
DA Data-aided
DAB Digital audio broadcasting
DFT Discrete-Fourier transform
DUFE Divide-and-update frequency estimation
DVB Digital video broadcasting
EAO Even-adjacency-order
ECM Conditional-expectation maximisation
FDMA Frequency-division-multiple-access
FER Frame-error-rate
FFT Fast Fourier transform
GCAS General carrier assignment scheme
IBI Inter-block-interference
ICI Inter-carrier-interference
ISI Inter-symbol-interference
LMMSE Linear MMSE
LOS Line-of-sight
LP Linearly precoded
LS Least-square
LTE Long-term evolution
MCRLB Modified Cramér-Rao lower bound
MD Multi-dimensional
MIMO Multiple-input multiple-output
ML  Maximum likelihood
MMSE  Minimum-mean-square-error
MSE  Mean-squared-error
MU-DM  Multi-user data-mapping
MUI  Multi-user interference
NDA  Non-data-aided
NU  New user
OAO  Odd-adjacency-order
OFDM  Orthogonal-frequency-division-multiplexing
OFDMA  Orthogonal frequency-division multiple-access
OTS  Orthogonal training sequence
pdf  Probability-density-function
PIC  Parallel interference cancellation
QoS  Quality of service
QPSK  Quadrature-phase-shift-keying
RU  Reference user
SAGE  Space-alternating generalized expectation-maximization
SCA  Schmidl and Cox algorithm
SCTS  Self-interference canceling training sequence
SINR  Signal-to-interference-plus-noise-ratio
SIR  Signal-to-interference ratio
SNR  Signal-to-interference ratio
SUE  Single-user equivalent
SVD  Singular-value-decomposition
TDMA  Time-division-multiple-access
TDTC  Time-domain training cyclostationarity
UPDP  Uniform power distribution based precoder
WiMax  Worldwide Interoperability for Microwave Access
WLAN  Wireless local area network
WLPIC  Weighted linear parallel interference cancellation
ZF  Zero-forcing
List of Symbols and Notations

\( A \): A matrix of the super-imposition of IFFT, channel, CFO, FFT, frequency-mapping and de-mapping units
\( \overline{A} = A^H A \)
\( \overline{A} \): Phase shifting matrix

\( a_{uv} \): Received signal after CP removal

\( B_m^{(k)} \): A matrix of circularly shifted data symbols on the \( m^{th} \) block of \( k^{th} \) user

\( b_{uv} \): Frequency-mapped precoded vector

\( c \): Light speed in free space

\( c_m^{(k)} \): CFO compensation vector affecting the \( m^{th} \) block of \( k^{th} \) user

\( c_m^{(k)} \): CFO modeling vector affecting the \( m^{th} \) block of \( k^{th} \) user

\( D_N(x) \): An \( N \times N \) diagonal matrix with vector \( x \) on its diagonal

\( D(X) \): A vector of all diagonal elements of matrix \( X \)

\( D_H \): Diagonal channel matrix

\( D \): Order of Kronecker product

\( d(k) \): An estimate of \( k^{th} \) user distance from the base-station

\( d_u \): A vector of CFO window operator

\( E \{ \cdot \} \): Expectation of the enclosed term

\( F \): An \( N \times N \) Fourier transform matrix

\( f^{(k)} \): Non-normalized CFO

\( G_{ev} \): Channel equaliser matrix for even-indexed blocks

\( G_{od} \): Channel equaliser matrix for odd-indexed blocks

\( g_{I_{m}} \): CFO induced ICI term at sample-level

\( H \): Circulant channel matrix

\( h_{ij} \): \( i^{th} \) element of channel impulse response vector

\( h \): Channel frequency response

\( h \): Channel impulse response

\( I_M \): An \( M \times M \) identity matrix

\( I_N \): An \( N \times N \) identity matrix

\( I_{N+2m} \): MUI term at sample-level

\( K \): Total number of active users

\( k \): User-index

\( L \): Number of subcarriers per user

\( L_h \): Channel length
$L_1$: Design parameter in the OTS-BLUE

$M$: Number of OFDM blocks per frame

$m$: Number of sub-block pairs in a frame

$m$: OFDM sub-block pair-index

$N$: Total number of subcarriers

$N_g$: CP length

$N_T := N + N_g$

$N$: Number of training blocks

$N_r$: Maximum correlation-lag in the BLUE estimator

$N$: Number of asynchronous users in TDTC-BLUE

$N_s$: Number of synchronous users in TDTC-BLUE

$n$: time-domain sample-index

$P_i$: $i^{th}$ diagonal element of matrix $P$

$P$: Real diagonal matrices used in the precoder $\Theta$

$P^{d}_m$: Probability density function of $\tilde{\mu}$ given $\epsilon$

$Q^{(k)}$: Diagonal matrix used to extract $k^{th}$ user's auto-correlation function (in time-domain)

$R_{\text{th}}$: Channel covariance matrix

$r_{m,n}$: Received $k^{th}$ user frequency-domain signal including MUI and additive noise

$r$: Received time-domain signal at sample-level

$r$: Frequency-domain received signal at sample-level

$r$: Frequency-domain received signal that is nulled out from synchronous users

$\{s^{(k)}\}$: $k^{th}$ user stream of data symbols

$s^{(k)}$: $k^{th}$ user vector of data symbols on $\rho^{th}$ block

$s_m$: A vector of super-imposition of $s_{2m}$ and $s_{2m+1}$

$T_s$: Sampling period

$\text{tr}(\cdot)$: Sum of diagonal elements of the enclosed matrix

$t$: Training symbol at sample-level

$t^{(k)}$: Training symbols vector of $k^{th}$ user

$U$: Unitary matrix used in the precoder $\Theta$

$U^{(k)}$: Diagonal matrix of propagation time-delay

$\nu$: Additive noise at sample-level

$\tilde{\nu}$: Frequency-domain noise at sample-level

$W^{(k)}$: Subcarrier-mapping unit for the $k^{th}$ user at sample-level

$W$: Diagonal filtering matrix used to filter out the synchronous users (in f-domain)

$X$: Denotes a matrix (i.e., any upper-case boldface symbol)

$X_m$: Denotes an $M \times M$ matrix

$X_{P \times M}$: Denotes an $P \times M$ matrix

$X_i$: $i^{th}$ diagonal element of matrix $X$

$x$: Precoded vector of data symbols with $i$ and $m$ being the sub-block index and pair-index respectively

$\hat{x}$: Denotes a vector (i.e., any lower-case boldface symbol)

$\hat{z}$: An estimate of the variable $z$

$\hat{z}$: The output of the CP-OFDM modulator at sample-level

$z$: Channel equalised signal at sample-level
$\alpha$ : Scalar parameter used in precoder $\Theta$
$\beta$ : Scaling factor in SCTS-BLUE
$\Gamma$ : Diagonal matrix of CFO-induced phase-shifts
$\Delta$ : Design parameter for the mitigation of propagation time-delay in CFO estimation
$\epsilon$ : CFO attenuation factor in [1]
$\Theta$ : Precoder matrix
$\Theta_1, \Theta_2$ : Upper and lower halves of $\Theta$
$\eta_{(k)}^{(m)}$ : Additive noise vector present on the $k^{th}$ user-band
$\lambda$ : Rate-factor in the exponential distribution
$\mu^{(k)}$ : Integer part of the propagation-delay
$\bar{\mu}$ : Second-order moments about $r_{km}$
$\xi_{km}$ : Total MUI term caused by all users on the odd-adjacency-order on $m^{th}$ sub-block of $m^{th}$ pair
$\xi_{km}$ : Total MUI term caused by all users on the even-adjacency-order on $m^{th}$ sub-block of $m^{th}$ pair
$\xi_A$ : A set of user indices order with entries of 1 for an asynchronous user and 0 for a synchronous user
$\sigma_x^2$ : Variance of the random-variable $x$
$\tau$ : Threshold of subcarrier distance in banded ICI matrix
$\tau^{(k)}$ : Propagation-delay for the $k^{th}$ user
$\Phi^{(k)}$ : Frequency-mapping matrix
$\psi^{(k)}$ : Set of all subcarriers belonging to $k^{th}$ user
$\Omega$ : Estimation noise matrix in the second-order moments based CFO estimator in the SUE scenario
$\Omega_{MUI}$ : Estimation noise matrix that is only caused by the MUI
$\omega_{km}$ : Additive noise vector
$\bar{\omega}$ : Sum of diagonal elements of $\Omega$
$\omega_k$ : Smoothing function for the BLUE estimator at sample-level
($\cdot$)* : Denotes complex conjugate of the enclosed parameter
($\cdot$)' : Denotes matrix transpose
($\cdot$)'* : Denotes Hermitian transpose
($\cdot$)^{-1} : Denotes matrix inverse operation
$| \cdot |$ : Absolute value of the enclosed complex scalar
$|| \cdot ||$ : Euclidean norm of the enclosed vector
$[\cdot]$ : Integer floor
$[-]$ : Integer ceiling
List of Publications

Journal Papers


Conference Papers


Chapter 1

Introduction

1.1 Background

Orthogonal frequency-division multiple-access (OFDMA) has been adopted by next-generation broadband wireless standards (e.g., third generation partnership project-long term evolution (3GPP-LTE) in Europe and Worldwide Interoperability for Microwave Access (WiMax) in US) due to its robustness to the channel frequency-selectivity. OFDMA possesses a number of favourable features such as increased robustness to narrow-band interference and capability of providing different requirements of multimedia communications from the data-rate and quality-of-service (QoS) points of view.

In an OFDMA-based communication system, in the ideal scenario, the overlapping subcarriers are orthogonal and therefore there is no interference induced. However in practical scenarios, due to the presence of carrier-frequency-offset (CFO) that is induced by time-varying Doppler shift and oscillators’ mismatch, the orthogonality

\[ \text{Doppler shift (Doppler effect)} \] is a shift in the frequency of a wave that is received by an observer due to the velocity of the observer relative to the transmitter. To explain it in better words in the context of CFO synchronisation, assume first, there is a line-of-sight (LOS) between the base station (BS) and the user of interest and second, the user of interest receives signals from the BS while on move, i.e. the user’s velocity is not zero. In this case the frequency of the received signal at user terminal is higher (than the transmitted wave) when the user approaches the BS, is the same when passing by the BS and is lower when receding. In the case of multipath this phenomenon holds for each path separately.
1.1. Background

of subcarriers is destroyed\(^2\). This interference can either be induced by the subcarriers within a user-band with respect to each other, in which case it is called inter-carrier-interference (ICI), or it can be due to interference of one user to other user-bands, which is called multiuser-interference (MUI) [2],[3]. Other factors that destroy the orthogonality of subcarriers are sampling-clock frequency discrepancies and also the time-delay caused by multipath propagation. From the above two timing-parameters, the former can be incorporated into the channel and compensated through channel equalization techniques and the latter one can be mitigated by using cyclic-prefix (CP). As a result, amongst the parameters that cause phase-mismatch, CFO is the most challenging one that gives rise to ICI and MUI and hence degrades the overall system performance significantly [3].

Coarse frequency synchronisation techniques (e.g., [4], [5]) can effectively reduce the CFO, normalized by subcarrier spacing, to a fractional number whose absolute value is smaller than 0.5. However, this fractional CFO causes ICI and MUI. Fine CFO synchronisation techniques are often employed to reduce the impact of the fractional CFO. Throughout this thesis, it is assumed coarse frequency synchronisation is already performed on the received signal through a variety of existing methods in the literature. Therefore the fine frequency synchronisation will be considered throughout this thesis. Provided sufficiently accurate CFO knowledge, there are two main methods to perform the fine CFO synchronisation, i.e., pre-compensation (e.g., [6]) and post-compensation (e.g., [1]-[8]). In the pre-compensation method, the base station estimates the CFO and feeds back the estimate to a mobile station. Then, the mobile station pre-compensates the CFO at the transmitter side. This method can effectively mitigate the CFO-induced interference but requires additional feedback of the estimated CFO from the base station (BS) to the mobile station. Post-compensation is an alternative scheme that performs CFO compensation at the receiver side (i.e., at the BS). This approach does not require any feedback of the CFO knowledge, but underperforms the pre-compensation approach

\(^2\)It should be noted that the phase noise also introduces a shift in the frequency domain of a signal and hence has contribution towards CFO. However, in comparison with time-varying Doppler shift and also the oscillators' mismatch, the value of phase noise can be neglected. In general sense, phase noise is the representation of the fast and random fluctuations in a waveform's phase that is caused by the time-domain instabilities.
1.2 Motivation and Objective

in terms of the CFO-induced interference mitigation.

The CFO estimation is the key to improve the performance of CFO synchronisation. State-of-the-art CFO estimation approaches can be classified into two categories: data-aided (DA) and non-data-aided (NDA) approaches (see [9]). DA schemes exploit well-designed training sequence or pilot pattern to perform robust and accurate CFO estimation (e.g. [6], [4], [10]-[22]). Alternatively, many NDA (blind) approaches have been proposed so far that do not exploit any training sequence and solely rely on the advanced signal processing techniques for CFO estimation (e.g. [23]-[40]).

1.2 Motivation and Objective

In the last decade, frequency synchronisation has received intensive investigations for single-user orthogonal-frequency-division-multiplexing (OFDM) or OFDMA downlink. Those contributions include NDA approaches that exploit statistical properties of received signals such as cyclostationarity introduced by cyclic prefix (CP) [40] and pulse-shaping filter [26], power spectral density on virtual subcarriers [27], second or higher-order stationarity [37],[38] and DA approaches that employ training sequences (e.g., [4],[18]). The NDA approaches can improve spectral efficiency for systems such as digital audio broadcasting (DAB) and digital video broadcasting (DVB) that conduct continuous signal transmission. The DA approaches can offer robust and accurate CFO estimation in most of communication systems. Recently, emerging wireless networks such as mesh networks and ad-hoc networks call for multiuser CFO estimation. This has motivated a number of research activities towards multiuser CFO estimation for OFDMA uplink. Major achievements so far include those NDA approaches proposed for OFDMA with guard bands [37], subcarrier interleaving [35] and DA approaches such as maximum-likelihood (ML) and best linear unbiased estimator (BLUE) [6]-[9]. However most of the existing approaches work under particular assumptions. Our work is mainly motivated by the fact that most of the proposed approaches for frequency synchronisation in single-user OFDM or OFDMA downlink fail in multiuser scenarios due to MUI.
1.3. Major Contributions

The objective of our work is to offer robust CFO estimation in multiuser scenarios,

I. by relaxing some of the assumptions made in the literature.

II. that shows improved performance in comparison with the existing approaches.

1.3 Major Contributions

The major contributions of this thesis are considered to be in three folds.

I. First, a novel method for blind CFO estimation in linearly precoded (LP)-OFDMA uplink is proposed. The investigation starts from the single-user equivalent (SUE) scenario presented in [6] where active users in the network are categorized into a number of reference (synchronised) users (RUs) and a new (asynchronous) user (NU). The major idea is to take advantage of time correlation induced by the linear precoder, which offers a second-order moments-based blind CFO estimation. The precoder design is carefully performed in terms of CFO identifiability, estimation accuracy and overall system performance. In the multiuser scenario, where all users can be misaligned in frequency domain, the proposed CFO estimator is capable of mitigating a considerable portion of interference from neighbouring users through exploitation of a novel time-frequency multiuser data-mapping (MU-DM) scheme. To demonstrate the MUI-resilience feature of proposed scheme, theoretical analysis is performed through derivation of approximate minimum-mean-square-error (MMSE) in both SUE and multiuser scenarios. It is shown that by exploitation of the proposed MU-DM scheme, the approximate multiuser MMSE is very close to that of SUE scenario. Simulation results show that the proposed approach outperforms state-of-the-art approaches in both SUE and multiuser scenarios.

II. The second proposed method is motivated by the fact that the BLUE estimation in OFDMA systems is very sensitive to MUI. The objective of this approach is

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*SUE scenario refers to the case in which only the user of interest requires synchronisation and all other users have already aligned themselves with BS references.
1.3. **Major Contributions**

to improve the robustness of BLUE in multiuser scenarios by proposing three appropriately designed training schemes. The first design borrows the idea about orthogonal training sequences (OTS) for multi-channel estimation in the synchronous multiple-input and multiple-output (MIMO)-OFDM. The contribution is to propose an efficient combination of OTS and BLUE in various cases of the asynchronous OFDMA. The second design is named time-domain training cyclostationarity (TDTC) approach, which exploits the fact that the interleaved subcarrier assignment introduces signal cyclostationarity in the time-domain. The last design is named self-interference cancelling training sequence (SCTS). It is shown that the OTS-BLUE approach offers single asynchronous-user equivalent performance in optimistic cases, but suffers significant performance degradation in the presence of synchronous users. As a complement, both the TDTC-BLUE and the SCTS-BLUE can offer single asynchronous-user comparable performance even if synchronous users are present. The latter approach is more robust to the MUI.

III. Finally an iterative concatenated approach for BLUE-based CFO estimation and compensation is proposed. In this approach, the CFO values of different users are first estimated from the received decomposed signal using BLUE estimator. The CFOs effects are then compensated based on the estimated values and also taking advantage of an iterative parallel interference cancellation (PIC) scheme. A novel contribution of this method is to introduce an iterative concatenated estimation and compensation algorithm for suppressing the residual interference due to imperfect CFO estimates. The introduced method is successful where the estimator converges after only one iteration with a significant improvement in estimation accuracy. Moreover, very close bit-error-rate (BER) performance result is achieved to that of systems with perfect knowledge or without CFOs particularly at practical signal-to-noise-ratios (SNRs).
1.4 Thesis Organisation

The rest of thesis is organised as follows. In chapter 2, the existing approaches for frequency synchronisation in OFDMA are reviewed. A novel blind CFO estimation approach for LP-OFDMA is introduced in chapter 3. A class of training designs for robust BLUE CFO estimation in OFDMA is presented in Chapter 4. An iterative approach for CFO estimation and compensation in OFDMA is proposed in chapter 5. Finally, Chapter 6 draws the conclusions and future work.
Frequency synchronisation can be regarded as a two-step process, carrier-frequency-offset (CFO) estimation and CFO compensation. In this chapter, the existing approaches in CFO estimation are categorised into data-aided (DA) and non-data-aided (NDA) schemes. Once CFOs are estimated, the next step is to compensate for their effects. There are two main approaches in the literature for CFO compensation, i.e. pre-compensation and post-compensation. In what follows first, an introduction will be given on some of the particular terms and terminologies used later on this chapter. Next, a detailed study on the existing methods for CFO estimation and compensation is provided.

2.1 Introduction to some basic concepts

2.1.1 OFDM and OFDMA

Orthogonal frequency-division multiplexing (OFDM) is a digital multi-carrier modulation scheme that exploits a large number of closely-spaced orthogonal subcarriers for data transmission (see Fig. 2.1 where \( s^{(k)} \) is the \( k^{th} \) user data-stream). This modulation scheme has a particular architecture both at the transmitter and receiver explained as follows. At the transmitter, first the source data-stream is serial-to-parallel converted
where the number of symbols in each batch is assumed to be equal to the number of subcarriers. In this case, each batch is called an **OFDM block**. In the next stage, each block goes through an inverse discrete Fourier transform (IDFT) unit, which has the same number of points as the number of symbols per block, is used for modulating the data symbols. Moreover, in the cyclic prefix (CP)-OFDM, after IDFT, a number of time-domain samples, which is referred to as CP, is copied from the tail of corresponding time-domain block and is added at the preamble of that block. The length of CP is assumed to be large enough to mitigate the timing offset and also the channel image from one block to the adjacent one. In this way CP mitigates the inter-symbol interference (ISI). At the receiver, first CP is discarded and then the received signal is passed on to the discrete Fourier transform (DFT) unit for symbols demodulation. In multiuser scenarios, OFDM can be used with a number of multiple-access schemes such as TDMA, FDMA, etc.

Alternatively if the total number of adjacent subcarriers are divided into a number of

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**Figure 2.1:** Overview of the CP-OFDM transmitter and receiver set-up.
groups and each group is assigned to one user, this would form up a new multiple-access scheme that is called Orthogonal frequency-division multiple-access (OFDMA). In other words, OFDMA is a multiple-access scheme that divides the total bandwidth into a number of sub-channels and allocates them to a number of users for simultaneous data transmission (see Fig. 2.2).

![Diagram](image)

**Figure 2.2:** Overview of the OFDMA transmitter and receiver set-up.

### 2.1.2 Carrier frequency offset (CFO)

CFO is the superimposition of time-varying Doppler shift and the oscillators' mismatch. The Doppler shift (Doppler effect) is a shift in the frequency of a wave that is received by an observer due to the velocity of the observer relative to the transmitter. To explain it in better words in the context of CFO synchronisation, assume first, there is a line-of-sight (LOS) between the base station (BS) and the user of interest and second, the user of interest receives signals from the BS while on move, i.e. the user's velocity is not zero. In this case the frequency of the received signal at user terminal is higher (than
the transmitted wave) when the user approaches the BS, is the same when passing by the BS and is lower when receding. In the case of multipath this phenomenon holds for each path separately. The oscillators’ mismatch refers to the frequency difference that exists between the oscillators of the mobile station and the BS. The oscillators’ frequency difference can be due to fabrication issues, environment’s temperature, etc.

In the presence of CFOs (belonging to different asynchronous users), the orthogonality of subcarriers can no longer be maintained. The CFOs manifest themselves as inter-carrier interference (ICI) within a user-band and multiuser interference (MUI) to other users and therefore can severely deteriorate the overall performance of a communication system. The overview of three adjacent subcarriers in two scenarios, i.e. with and without CFOs, is shown in Fig. 2.3. In this figure, for the sake of clarity, it is assumed each subcarrier belongs to one user. In this figure, the subcarrier-spacing in the ideal scenario, i.e. when subcarriers are orthogonal, is denoted by $f_0$. It can be observed that in the absence of CFOs, the subcarriers and hence the users are orthogonal in the frequency-domain ($\Delta f = f_0$). However, in the presence of CFOs, the subcarriers' orthogonality is damaged, which results in the occurrence of ICI and MUI ($\Delta f_1 \neq \Delta f_2 \neq f_0$). The primary consideration of this thesis is to estimate $\Delta f$ for all asynchronous users and to compensate for the CFOs effects based on the estimated values.

### 2.1.3 Iterative process

In general sense, an iterative process is an algorithm which exploits a repeated cycle of operations to calculate one or more desired parameter(s). In an iterative process there are normally one or more signals that are updated in each iteration, from which the desired parameter is calculated. An iterative process is desired to be convergent. This means the calculated value becomes closer to the actual value by increasing the number of iterations. Moreover, in algorithms which have practical implications, the number of iterations that are required for convergence is an important factor.

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1. This can exactly be the case for an interleaved subcarrier allocation scheme.
2. It is assumed each asynchronous user has a distinctive CFO.
2.1. Introduction to some basic concepts

2.1.4 Best linear unbiased estimator (BLUE)

In general sense, the best linear unbiased estimator (BLUE) is an estimator which has the following characteristics.

C1) It only exploits linear operations in the estimation process.

C2) It introduces a zero-mean estimation error.

C3) Its coefficients are optimised for the minimisation of estimation error variance.

2.1.5 Single-user equivalent (SUE) scenario

In the context of synchronisation, based on the Morelli’s synchronisation policy (see [6]), the single-user equivalent (SUE) scenario is referred to the scenario where the
active users in the network can be grouped into a number of reference (synchronised) users (RUs) and a single new (asynchronous) user (NU). According to [6], synchronised users are the ones whose timing and frequency synchronisation is accomplished at the transmitter side, i.e. the mobile station prior to uplink transmission. Therefore at the receiver, they do not cause interference to other users in both time and frequency domains. On the other hand, if the base station receives data symbols from a particular user that is misaligned in the time or frequency-domain, that user is referred to as an asynchronous user.

2.1.6 Correlation

The correlation function reflects the measure of linear similarity or dependency between two or more parameters. In one classification, depending on the signals whose similarity is being measured, the correlation is divided into **autocorrelation** and **cross-correlation**. The autocorrelation function \( r_{xx}(m_1, m_2) \) is the measure of similarity of a signal at the time instances \( m_1 \) and \( m_2 \). Mathematically it can be calculated through

\[
r_{xx}(m_1, m_2) = E\{x(m_1)x(m_2)\} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} x(m_1)x(m_2)f_{X(m_1),X(m_2)}(x(m_1),x(m_2))dx(m_1)dx(m_2)
\]

where \( f_{X(m_1),X(m_2)}(x(m_1),x(m_2)) \) is the joint probability density function (pdf) of the enclosed variables (see [45], page 65 for further explanations on autocorrelation function). The cross-correlation has the same mathematical expression as autocorrelation but is between the \((m_1)^{th}\) sample of signal \( x \) and \((m_2)^{th}\) sample of a second signal \( y \). In the context of CFO estimation, by assuming the communication channel has slow variations over a number of consecutive samples or blocks, the CFO can be estimated by calculating the autocorrelation function of those consecutive samples or blocks.

2.2 CFO estimation

The CFO estimation is the key process in performing CFO synchronisation. DA approaches exploit periodic transmission of well-designed training sequence to perform
robust CFO estimation. On the other hand, NDA approaches perform CFO estimation by solely relying on advanced signal processing techniques. NDA approaches can improve spectral efficiency for systems such as digital audio broadcasting (DAB) and digital video broadcasting (DVB) that conduct continuous signal transmission. In what follows, the CFO estimation approaches initially proposed for single-user orthogonal-frequency-division-multiplexing (OFDM) or orthogonal-frequency-division-multiple-access (OFDMA) downlink systems are reviewed. A number of these approaches are later on extended from single-user to multiuser scenario. Moreover, there are a number of approaches directly proposed for multiuser scenarios which are discussed in the following sections.

2.2.1 DA approaches in CFO estimation

In this section two main categories of DA CFO estimators, namely known as maximum-likelihood (ML) and correlation-based estimators, are considered and a number of main existing schemes in each category are reviewed.

ML based CFO estimation

One important class of DA approaches relies on the ML algorithm\(^3\) for CFO estimation. The exact solution to the optimum point of the likelihood function is too complex in practical scenarios as it necessitates a search over multi-dimensional (MD) space spanned by CFOs, timing offsets and also channel responses of all asynchronous users. There are a number of approaches in the literature to reduce this complexity.

\(^3\)ML estimation is a well-known estimation algorithm recommended and analysed by R. A. Fisher. This algorithm is aimed at the estimation of one or more parameters assuming the underlying statistical properties (e.g., the probability density function) of the observation signal is available. In the context of CFO estimation, the received signal that includes additive white Gaussian noise (AWGN) is the observation signal. The unknown parameters are channel responses, CFOs and timing offsets of the multiple asynchronous users. The likelihood function is formed based on the Gaussian distribution of the additive noise. The likelihood function is maximised in the multi-dimensional space spanned by the above parameters and the values that give rise to this maximisation are the results of the ML estimator.
As the first approach in this class, the one proposed by Pun et al. [41] is considered. This approach is proposed for joint estimation of CFOs, timing offsets and channel responses. As the use of general ML algorithm introduces significant complexity, the authors introduced a sub-optimum scheme based on alternating projection frequency estimator (APFE) for CFO estimation. In this approach it is assumed cyclic-prefix (CP) is sufficiently large so that the two-way propagation-delays and channel response duration are accommodated within the CP length. By fixing the channel response, CFOs are the only variables in the likelihood function. However maximising the likelihood function over the multiple-CFO space is still too cumbersome particularly for large number of asynchronous users. To counteract, by exploiting the APFE algorithm, the multi-dimensional search is replaced by a number of iterative mono-dimensional search. In this way, the CFOs estimation becomes an iterative process. Within each iteration, there are a number of cycles that updates each user's CFO. The iterations are repeated until convergence that, according to the authors, is achieved after two iterations. The initial values can either be set to zeros or alternatively the single-user best linear unbiased estimator (BLUE) CFO estimation approach can be used. Once CFOs are estimated, timing-offsets can be estimated to shorten the CP length. Finally, having estimated CFOs and timing offsets, channel responses are estimated by maximising the likelihood function. The advantages of this approach are its applicability to general carrier assignment scheme (GCAS) and joint estimation of a number of parameters through the likelihood functions. The disadvantages include higher computational complexity as compared to other approaches such as [35] and [6] and also making some restrictive assumptions in the estimation process. An example of such assumptions is fixing CFOs in the likelihood function to be able to reduce the number of dimensions.

Another ML-based approach for joint estimation of CFOs, channel responses and data detection is proposed in [42]. As it was the case for the previous approach, in this approach the complexity of ML is reduced through an iterative process. In each iteration, in the first step, the received superimposed signals at base-station (BS) are first decomposed by exploitation of the space-alternating generalized expectation-maximization (SAGE) algorithm. In the next step, each user's signal is passed to the conditional-expectation maximisation (ECM) algorithm. This approach is similar to parallel inter-
ference cancellation (PIC) algorithm where in each iteration, the interference is generated and removed from the received signal to improve the performance. The advantages of this approach are considered as follows. Firstly, as the SGAE and ECM algorithms are both performed at the receiver, the CFOs-induced interference is suppressed at the receiver and hence there is no requirement of feeding back the estimated CFOs to the mobile-stations for pre-compensation (e.g., see [6]). Secondly, this approach is suitable to be applied to GCAS. The main disadvantage of this approach is higher computational complexity than some other similar CFO synchronisation schemes (e.g. see [1], [35]).

A third ML-based approach relies on importance sampling to reduce the MD search [15]. In this approach, a closed-form solution for finding the global optimum point is obtained by replacing the maximisation operation with the multi-dimensional integration together with the exploitation of importance sampling. As it was the case for the previous two ML-based approaches, this approach is suitable for GCAS. Moreover, its complexity is less than APFE and its CFO estimation performance is close to the CRLB.

A fourth ML-based approach is proposed by Zeng and Leyman that relies on a special design for the pilot block [43]. This approach turns the MD optimization problem to approximated and independent one-dimensional (1-D) optimization problems. The main idea is to design a pilot sequence that guarantees any two active subcarriers, i.e. the subcarriers that are used for data transmission, are far enough from each other. These two subcarriers can either belong to one user or two different users. To achieve such a pattern, the authors have proposed to use a considerable number of null-subcarriers. Using such a special training design results in simplification of the optimization problem and also a fast algorithm that finds the closed-form for CFOs estimation at the price of considerable null-subcarrier usage.

In 2009, another iterative ML-based approach was proposed by Wang et al. [17], where in each iteration, CFO estimation is carried out in two steps, primitive and divide-and-update CFO estimation (DUFE). In the primitive step, in the $(i + 1)^{th}$ iteration, by

\footnote{For the definition of OFDM block, see Chapter 2, page 8, paragraph 1, line 2.}
assuming all users have the same residual CFOs (i.e., the difference between the actual 
CFO and the estimated CFO in the $i^{th}$ iteration), an ML algorithm is performed over 
a 1-D space. In the DUFE step, by defining a local search-range for each user and 
a number of test operations, the CFOs are further adjusted. In comparison with the 
APFE, the DUFE-based approach has lower complexity at the price of slight CFO 
estimation accuracy degradation.

Finally, another iterative ML-based approach was proposed by Sun and Zhang [44]. In 
this approach, two training blocks are required for CFOs estimation. The main idea is 
to first remove phase-rotation and interference induced by the CFOs of all users on the 
two consecutive blocks and then minimize the difference between these two blocks. The 
initial CFO estimates are set to zero, and after each iteration a new set of estimates 
are generated that are then used for interference suppression. This approach shows 
convergence after two iterations and slightly outperforms APFE for signal-to-noise-
ratios (SNRs) larger than 6 dB. In terms of computational complexity, each iteration 
has to deal with a 1-D search for finding the optimum solution, i.e. all CFO values. 
The main drawback of this approach is its poor CFO estimation performance at low 
SNRs (i.e. < 6 dB) which is due to existence of local minima in the cost function.

Correlation-based CFO estimators

A second class of DA approaches relies on the autocorrelation of consecutive samples or 
blocks of the received signal. A correlation-based CFO estimation scheme was proposed 
by Mengali and Morelli [46] for burst digital transmission systems. This estimator 
exploits the correlation over a number of consecutive received samples. This scheme 
offers very close CFO estimation performance to Cramér-Rao lower bound (CRLB) at 
SNRs as low as approximately 0 dB, by assuming the timing is ideal. Moreover its 
estimation range is limited to 20% of the subcarrier-spacing.

One of the earliest methods in this category for an OFDM system was proposed by 
Schmidl and Cox [18]. In this method, a joint estimation of timing and CFO is con-
sidered. The timing offset is estimated within one OFDM symbol by searching for two 
identical parts. In the next step, having estimated the timing offset, CFO is estimated
2.2. CFO estimation

by finding the correlation between two consecutive OFDM blocks. In the single-user scenario, this method offers very close performance to the CRLB.

Based on the Schmidt and Cox algorithm (SCA) [18], Morelli and Mengali modified their own previous method proposed for burst digital transmissions [46] for CFO estimation in OFDM applications [47]. The major difference between this method and SCA is dividing the training symbol into \( L > 2 \) identical parts as opposed to two identical parts in SCA. In this way the correlation is performed over \( L \) identical parts within a single OFDM block. As stated by the authors, such a scheme effectively results in the BLUE for CFO estimation. In this scheme, the CFO estimation range can be increased by increasing \( L \). However increasing \( L \) introduces more randomness to the correlation function, from which CFO information is extracted, which in turn causes less accurate CFO estimates. Moreover this scheme has higher complexity in comparison with SCA.

In 2004, Morelli extended the BLUE approach proposed in [47] to OFDMA systems [6]. However, his main underlying assumption is all other existing users in the network have already aligned and synchronised themselves with BS references and only the new entering user to the network requires synchronisation. Such an assumption will be referred to as single-user equivalent (SUE) scenario in the sequel. Based on this assumption, CFO estimation is carried out by finding the correlation between adjacent training OFDM blocks where a minimum of 2 blocks is necessary. However, he has shown that, increasing the number of training blocks to 4 results in significant improvement in CFO estimation accuracy. This method offers comparable performance to the modified CRLB (MCRLB) in SUE scenario even at SNRs as low as 0 dB.

2.2.2 NDA approaches in CFO estimation

As an alternative approach to DA CFO estimation schemes, many blind approaches have been proposed to improve the spectral efficiency particularly for continuous-transmission systems such as DAB and DVB (e.g. [23]-[40]). In this section, the main existing NDA CFO estimation approaches proposed for OFDM systems, i.e. where MUI is not present, are first considered. A number of these approaches are extended from single-user to multiuser scenario that are discussed later in this section.
Null subcarrier based CFO estimators

One class of blind CFO estimators for OFDM based systems takes advantage of null subcarriers (e.g. [27]-[29]). One of the earliest approaches in this class was proposed by Liu and Tureli et al. [28],[29]. In this approach, a number of null subcarriers are exploited to measure the CFO-induced energy leakage from information-bearing tones into the null-space. The estimation accuracy of this approach is comparable to the CRLB at the price of considerable computational complexity introduced by the grid-search.

Another null-subcarrier based CFO estimation approach was proposed by Ma et al. [27]. This approach relies on the same methodology as the ones proposed by Tureli et al. (see [28] and [29]). However the major contribution of this approach is to investigate the relationship between the placement of null-subcarriers and the performance of CFO estimator in the presence of channel nulls. The authors have shown that placing the null-subcarriers in a consecutive manner can result in a covariance matrix for the received signal that does not uniquely determine the CFO, i.e. the identifiability issue. It is also shown that the CFO identifiability can be guaranteed by exploiting three ways, 1) a judicious non-consecutive null-subcarrier placement 2) making the location of null-subcarriers related to the OFDM block index (i.e., subcarrier hopping) 3) combining subcarrier hopping with consecutive null-subcarrier placement. Alongside its satisfactory CFO estimation performance in a single-user system, a notable drawback of this approach is its high complexity [27] introduced by the grid-search.

Cyclostationarity based CFO estimators

Another class of blind estimators in an OFDM system relies on the cyclostationarity (CS) of the received signal that is either induced by the cyclic prefix (CP) [23],[24] or pulse-shaping filter [25]. These approaches are low-complex but sensitive to channel frequency selectivity.

As one of the earliest CP-based CFO estimation approaches in OFDM systems, the one proposed by Beek et al. [23] is considered. This approach relies on an ML-based
2.2. CFO estimation

algorithm, implemented on $2N + L$ consecutive samples of the received OFDM signal where $N$ is the number of subcarriers and $L$ is the CP length. It is assumed this sequence of samples contains one complete $(N + L)$-sample OFDM block. As this approach does not rely on training symbols, the main idea is to exploit the cyclostationarity observed on the first and last $L$ samples of an OFDM block through second-order statistics, i.e. the correlation function. In the next stage by performing ML-based estimation on the first and last $L$ samples and by having the knowledge about the correlation function, CFO and timing-offset can be estimated. The CFO estimation performance of this approach is satisfactory in the presence of additive-white-Gaussian-noise (AWGN) channels but error-floors are observable in case of frequency selective channels for SNRs $> 10$ dB. Moreover, the CFO estimation performance is a direct function of the CP length.

Based on the work proposed by Beek et al., Lashkarian and Kiaei [24] proposed a class of CP-based estimators for CFO estimation in an OFDM system with flat-fading channel. In this approach, ML estimator is considered for which the authors introduce a new probability density function (pdf). The new pdf takes into account the entire range of possible timing-offsets. Such a pdf globally characterises the estimation of the timing-offset and CFO. In this way, the authors claim that the estimator proposed by Beek et al. is a suboptimal approach as it only considers a limited range for the timing-offset. This gives rise to a considerable timing-offset estimation error with a probability of $L/(L + N)$ with $N$ and $L$ being the number of subcarriers in an OFDM block and CP length respectively. In the estimation of timing-offset, the new closed-form of ML estimator offers satisfactory estimation of timing-offset over the entire range of timing-offset, i.e. $[1, N + L]$. However the ML estimator in [23] fails for timing-offsets larger than $N$ which can lead to inaccurate estimates of CFO in joint estimation of timing-offset and CFO. In 1999, the CP-based approach was extended from the single-user scenario to the multiuser scenario through employment of filter-bank [40]. The channel is assumed to be frequency selective. The multiple-access scheme considered in this approach is a hybrid of time-division-multiple-access (TDMA) and frequency-division-multiple-access (FDMA). To form such a hybrid multiple-access scheme, the available spectrum is subdivided into bands of adjacent subcarriers (FDMA) and within each
2.2. CFO estimation

band a TDMA scheme is used. In this way, users are separated both in time and frequency. As this approach relies on the cyclostationarity of the CP, a filterbank is applied to the time-domain received signal to partially separate the subbands allocated to different users. However, this type of filtering, even by assuming the filters are ideal breakwall filters, does not completely separate the users' signals due to the presence of CP and also multiple CFOs. In the next stage, an ML estimator of timing-offsets and CFOs is applied to each filtered subband. The main drawback of this approach is its sensitivity to the number of subcarriers in each subband. A smaller number of subcarriers within each subband, i.e., a narrowband signal has correlation patterns between time samples out of the CP. This correlation causes distortion to the pairwise correlation between CP and the corresponding samples at the tail of OFDM block.

Another blind joint timing-offset and CFO estimation approach based on cyclic correlation is proposed by Park et al. [25]. This approach is based on the cyclostationarity of the received OFDM signal induced by the pulse-shaping filter by exploiting the fact that the timing-offset and CFO appear as cyclic-correlation phase. This method does not require channel information for estimating the timing-offset and CFO and shows almost identical performance as another cyclostationarity (CS)-based approach proposed by Bolcskei [26] which requires channel information.

Second or higher-order moments based CFO estimators

Another class of NDA approaches relies on the second or higher-order statistics for CFO estimation. In 2006, a blind CFO estimation approach for OFDM systems was proposed by Roman et al. [38] based on the diagonality of the received signal autocovariance matrix. This approach exploits the fact that the autocovariance matrix of the received f-domain OFDM signal has a diagonal form in the absence of CFO. However, in the presence of CFO, this diagonality is destroyed due to the correlation between subcarriers induced by the CFO. Based on this idea, a cost function is defined which aims at minimizing the energy of off-diagonal elements of the autocovariance matrix. The closed-form for the cost function is shown to attain a single-minimum over a CFO range of \([0, 1]\). This approach shows a comparable performance to the CRLB at low
SNRs, however the performance starts departing from the CRLB at moderate and high SNRs (i.e. > 10 dB). As the cost-function has a sinusoidal close-form, the authors claimed their approach is computationally efficient. However, in terms of estimation accuracy and convergence rate, 2000 OFDM blocks are required for achieving an MSE below $10^{-4}$. This is a limiting factor in terms of frame size in an OFDM system. Moreover, in an OFDMA-based system, as the received signal has undergone multiple CFOs, diagonalizing the autocovariance matrix has to be performed with respect to multiple CFOs which introduces a significant complexity.

As a second approach in this class, Yao and Giannakis proposed a Kurtosis-Type based approach through measure of the non-Gaussianity of fourth-order received signal moments [37]. The key idea behind this approach is to exploit the fact that a linear combination of random variables has a closer distribution to Gaussian in comparison with the original variables unless the source random variables are Gaussian themselves or the linear combination is trivial (see [48]). Applying this into the scenario of interest results in a higher Gaussianity of the received signal that has undergone CFO than a CFO-free scenario subject to having independent and non-Gaussian source symbols. To measure the non-Gaussianity of the received signal, the authors exploited a Kurtosis-type metric and later on derived a sinusoidal closed-form for CFO estimation. This approach shows a better CFO estimation performance than CP and CS-based estimators in an OFDM system. However, assuring an acceptable CFO estimation accuracy in multiuser environments is subject to the presence of sufficient number of null-subcarriers between neighbouring users.

Multiple signal classification based CFO estimator

In 2004, a multiple signal classification-type approach was proposed to offer reliable multiuser CFO estimation for an interleaved OFDMA system [35]. The major idea is to exploit the special periodic structure within an OFDMA block that is induced by the interleaved subcarrier assignment scheme. The CFOs are estimated uniquely by maximising a cost function. The cost function is derived by finding the singular value decomposition (SVD) of the received signal autocovariance matrix. This ap-
2.3. CFO compensation techniques

Once CFOs are estimated, their effects need to be compensated to mitigate/suppress the CFO induced inter-carrier-interference (ICI) and MUI. In the context of CFO synchronisation, one of the most important differences between broadcast and multiple-access channels is how to perform CFO compensation. In case of broadcast or downlink communication, the CFOs are normally estimated at the receivers, i.e. mobile stations. Based on the estimated values, CFOs are adjusted at the mobile stations too. On the other hand, in case of uplink, CFOs are usually estimated at the BS. In this case, since compensating for one user’s CFO introduces interference to other users, all signals arriving at the BS should be pre-aligned with respect to BS references. This means a pre-compensation stage is required to be accomplished at the mobile stations before uplink transmission. Nevertheless, there are a number of approaches in the literature.
for post-compensation of CFOs, i.e. compensation at the BS. However, as mentioned above, the use of post-compensation schemes necessitates the exploitation of ICI and MUI suppression algorithms at the receiver for the purpose of users' decomposition. In what follow, the above two CFO compensation approaches are introduced in more details.

2.3.1 Pre-compensation scheme

As an effective way to mitigate the CFOs-induced interference, pre-compensation scheme is adopted in several CFO synchronisation approaches (e.g., [6], [40]). In this scheme, after adding the CP, each mobile station pre-rotates the t-domain signal phase based on the received CFO estimate from the BS. In this way, the phase-rotation due to CFO is pre-compensated for each user separately before sending the signal through the communication channel.

In this scheme, for each user, a control channel is embedded in the downlink over which the control data based on the estimated CFOs are fed back to the mobile stations. By exploitation of these data, each user can then align its transmitted signals in accordance with the BS references. The downlink control channel is set up during the initial phase of the connection and apart from CFO, other control parameters such as which subbands, time-slots and transmission powers must be used for the uplink transmission, can be conveyed in this channel. Therefore, the main requirement of the pre-compensation scheme is the control channel. Alongside its favourable feature in mitigating the interference, the CFO feedback requirement introduces additional signalling load to the communication system.

2.3.2 Post-compensation scheme

CFOs post-compensation approaches are proposed in the literature to suppress the CFOs-induced ICI and MUI (e.g., see [1], [49], [50]). Compensating for CFOs effects at the BS is a challenging task even though perfect CFOs knowledge is available. This is mainly due to the fact that the received signal at the BS is the superimposition of all
users' signals that have undergone independent CFOs. In such a scenario, separation of different users is not straightforward and normally calls for advanced signal processing techniques. In the following part, a number of main approaches proposed for this purpose are reviewed.

An interference cancellation scheme is introduced by Huang and Letaief which exploits an iterative algorithm for restoring the transmitted signals [1]. The main assumption of this scheme is that all the users’ signals arriving at the BS have passed a coarse synchronisation stage at the mobile stations so that all CFOs are limited to be within 20% of subcarrier spacing. This approach is implemented as follows.

At the first stage, a user decomposition is performed on the received f-domain signal by choosing the subcarriers belonging to a user. This is followed by a simple f-domain CFO compensation process for each user through circular convolution. The circular convolution is performed due to the fact that the data symbols are multiplied by CFO vector in the t-domain and the equivalent process in the f-domain is circular convolution of the corresponding f-domain vectors. The resultant vectors are known as the initially compensated received signal for each user. In the next stage, the MUI is generated by performing circular convolution between the initially compensated vector and the CFO vector. By removing the MUI from the received f-domain signal, the last stage is to perform CFO compensation for each user as in the initial stage. It is important to note that as CFOs introduce energy leakage from one user-band to another, it is assumed the output signals from each stage of the above operations are passed through a filter to make the out-of-band samples zero. This approach can be regarded as a PIC scheme as it performs interference cancellation iteratively. However, in contrast to conventional PIC, it does not perform iterative detection or decoding to restore the original signal. It is shown the proposed iterative approach requires 2 iterations to converge and the complexity introduced by circular convolutions can be reduced. This approach performs well in cancelling the interference induced by CFOs of up to 20% of subcarrier spacing. In other words, by using this approach, normalized CFOs with the absolute values upper-bounded by 0.2 are tolerable. As a numerical example, consider the following case. The authors have shown that by defining a vector for CFO values for 4 users as $\epsilon^{(1)} = 0.1$ $\epsilon^{(2)} = 0.1$ $\epsilon^{(3)} = 0.05$ $\epsilon^{(4)} = 0.05$, when the
CFO attenuation factor $\eta = 1$ or $\eta = 2$, the frame-error-rate (FER) is the same and approximately equals to $6.5 \times 10^{-3}$ at SNR= 16 dB. Whereas, by increasing the CFO attenuation factor to $\eta = 5$, the FER degrades to $5 \times 10^{-2}$ for the same SNR value.

In 2007, Cao et al. proposed an approach for orthogonal spectral signal construction in generalised OFDMA uplink [49]. In this approach, Least-squares (LS) and minimum-mean-square-error (MMSE) criteria are used to reconstruct the orthogonal spectral signals from the OFDMA blocks that are received under the effect of CFOs-induced interference. By assuming perfect CFOs knowledge is available at the BS, closed-forms are provided for both LS and MMSE algorithms. However, the proposed closed-forms involve significant computational complexity particularly for an OFDMA system with large number of subcarriers, e.g. $N = 2048$ subcarriers as in IEEE 802.16 [51].

To counteract, the authors proposed a banded-matrix approximation approach. The approximation is based on the fact that the CFO-induced interference from the $\hat{m}^{th}$ subcarrier has significant effect on a number of nearby subcarrier only (see Fig. 3 in [49]). Therefore in practice, a threshold value $\tau$ can be defined such that for a subcarrier group $\{m : |m - \hat{m}| > \tau\}$, the interference induced by $\hat{m}^{th}$ subcarrier on the $m^{th}$ subcarrier can be assumed as the residual interference and be incorporated into additive noise. Based on this setting, there is a trade-off between computational complexity and system performance through selecting an appropriate value for $\tau$. The authors have shown that increasing $\tau$ causes increased signal-to-interference-plus-noise-ratio (SINR). It is also shown that in case of having $N = 2048$, by setting $\tau = 15$, comparable SINR to that of a full interference matrix is observed, i.e. less than 1 dB difference at SNR= 15 dB. Full interference matrix corresponds to the case where for any arbitrary $\hat{m}$ and $m$, the interference from $\hat{m}^{th}$ subcarrier to the $m^{th}$ subcarrier is taken into account.

Moreover, in terms of normalised MSE (NMSE) of the reconstructed orthogonal signals, comparable results are observable to that of a synchronous system for both LS and MMSE algorithms ($\approx 3$ dB difference at SNR= 25 dB). In terms of bit-error-rate (BER), the proposed LS and MMSE algorithms show comparable performance to that of a synchronous system and an approximately 4 dB performance loss is observed for
An interference-cancellation approach was proposed by Manohar et al. which suppresses the MUI through exploitation of a multi-stage weighted linear PIC (WLPIC). In this approach, at the first stage, a t-domain CFO compensation is performed on the received superimposed signal for each user separately followed by \( K \) discrete Fourier transform (DFT) operations where \( K \) is the number of active users. In the second and third stages, the MUI is estimated and scaled by some optimum weights. The reason for scaling the MUI is to compensate for poor channel conditions, e.g. low SNR and high interference that give rise to inaccurate MUI estimation. The optimum weights, which are stage-dependent, are obtained by maximising the corresponding signal-to-interference-ratio (SIR) at the output of each stage. It is shown that the proposed approach outperforms the one proposed by Huang and Lataief [1] when the difference between users’ CFOs are small. However, when all CFOs are small (i.e., within 20% of subcarrier-spacing), Huang and Lataief’s approach outperforms this approach.

In 2008, Hsu and Wu proposed a zero-forcing (ZF) algorithm for CFO compensation [52]. In this approach, the high complexity introduced by the matrix inversion in zero-forcing algorithm is replaced by an iterative matrix-inversion process. In the iterative process, the special structure of the CFO-induced ICI matrix is exploited and the Newton’s method is efficiently implemented by fast-Fourier transforms (FFTs). It is shown that the matrix-inversion complexity can further be reduced by using an interleaved subcarrier assignment scheme or a banded ICI matrix (as used in [49]). It is also shown that this approach shows a close performance to that of a direct ZF (i.e., the result of direct ICI matrix inversion) after 3 iterations. Moreover, in comparison with a banded-ZF and under the same computational complexity loads, this approach clearly outperforms the banded-ZF. However severe BER degradations are observable for near-far power ratios larger than 0 dB between a new user and all other existing users (having equal powers) for both this approach and the direct ZF.
2.4 Summary

In this chapter a full review was made on the main existing approaches for frequency synchronisation in OFDMA. It was shown that the process of frequency synchronisation can be divided into CFO estimation and CFO compensation sub-processes. The CFO estimation approaches were initially categorised into two broad groups, i.e. DA and NDA. Within the category of DA approaches, two CFO estimation techniques were introduced first, based on the simplified ML and second, by relying on the correlation of consecutive training data blocks. On the other hand, a number of approaches in the NDA category was reviewed, such as null-subcarrier-based, CP and CS-based, second or higher order moments-based and deterministic structure-based CFO estimators. Once CFOs were estimated, their effects could either be compensated at the transmitter, i.e. pre-compensation scheme or at the receiver, i.e. post-compensation scheme. The existing approaches that exploited each of the above compensation schemes were reviewed. Due to the appearance of CFO-induced interference, post-compensation scheme required some interference cancellation algorithms which introduces extra computational complexity.

As discussed earlier in this chapter, the existing NDA CFO estimation techniques are either not designed for multiuser scenario or they make some particular assumptions to enable their CFO estimator in the multiuser environment. Examples of such assumptions include allocating some virtual subcarriers to each user or having an interleaved subcarrier assignment scheme. Therefore, in Chapter 3, an NDA CFO estimation scheme is proposed that not only is particularly designed for the multiuser environment, but also relaxes some of the assumptions made in the state-of-the-art schemes.

Moreover, it was shown in this chapter the BLUE approach offers low-complex and sub-optimum CFO estimation performance but is very sensitive to the multiuser interference (MUI) (see Chapter 4 for details). This fact can be a good motivation for proposing two approaches to increase the robustness of BLUE CFO estimator in multiuser environments. The first approach combines BLUE with three appropriately designed training sequences (see Chapter 4) to offer very close or comparable performance to SUE in different scenarios. In Chapter 5, an iterative concatenated CFO
estimation and compensation algorithm is proposed, which significantly improves the CFO estimation performance of the BLUE estimator after only one iteration.
Chapter 3

Blind CFO Estimation for LP-OFDMA

3.1 Introduction

The conventional orthogonal-frequency-division-multiple-access (OFDMA) has shortcomings such as high peak-to-average power ratio (PAPR) (e.g., see [53]) and the loss of frequency diversity particularly for the uncoded information source. It has been shown in the literature (e.g., [54]-[58]) that combining OFDMA with the frequency-domain linear precoding (LP) technique can reduce the PAPR and recover a portion of frequency diversity gain. This kind of combined air-interface technique is referred to as LP-OFDMA, which has received increasing interests for future wireless standards such as LTE-advance. Recently, Lin and Phoong have proposed a class of channel-independent precoders which minimized the bit-error-rate (BER) [59]. Examples of those precoders include orthogonal matrices such as the Walsh-Hadamard matrix and the discrete Fourier matrix. Based on their work, Lin and Petropulu proposed an optimal linear precoder, which offered the minimum blind channel estimation error and the asymptotically minimum BER [60].

As a member of OFDMA family, LP-OFDMA is also very sensitive to the carrier-frequency-offsets (CFOs). This chapter presents a novel blind CFO estimation approach, which is specially designed for the LP-OFDMA system. Major contributions of this work are in four folds:
1) This work starts with the single-user equivalent (SUE) scenario introduced in [6] where active users in the network are grouped into a number of reference (synchronised) users\(^1\) (RUs) and a new (asynchronous) user\(^2\) (NU). The proposed method relies on the frequency-domain linear precoding of sub-blocks. The linear precoder is a non-redundant, non-unitary complex matrix that is applied to each pair of successive sub-blocks in each user-band\(^3\). The major idea is exploiting the time correlation induced by linear precoder to design a second-order moments-based CFO estimator. Computer simulations show that the proposed approach outperforms state-of-the-art approaches in the SUE scenario.

2) The linear precoder is carefully designed in terms of CFO identifiability, CFO estimation accuracy and overall system performance.

3) In the multiuser scenario, where all users can be misaligned in the frequency domain, the proposed CFO estimator is capable of mitigating a considerable portion of interference from neighbouring users through exploitation of a novel time-frequency multiuser data-mapping (MU-DM) scheme. Simulation results show that by taking advantage of proposed MU-DM scheme, the second-order moments-based estimator significantly outperforms existing blind approaches in multiuser scenario.

4) To demonstrate the multiuser interference (MUI)-resilience feature of proposed approach, theoretical analysis is performed through derivation of approximate minimum-mean-square-error (MMSE) in both SUE and multiuser scenarios. It is shown that by the exploitation of proposed MU-DM scheme, the approximate multiuser MMSE is very close to that of SUE scenario (around \(1 \times 10^{-5}\) MMSE difference observed for signal-to-noise ratio (SNR) > 5 dB).

\(^1\)According to [6], synchronised users are the ones whose timing and frequency synchronisation is accomplished at the transmitter side, i.e. the mobile station. Therefore at the receiver, they do not cause interference to other users in both time and frequency domains.

\(^2\)Based on the Morelli's definition (see [6]), if the base station receives data symbols from a particular user that is misaligned in the time or frequency-domain, that user is called an asynchronous user.

\(^3\)This precoder follows the general structure proposed in [60].
3.2 LP-OFDMA uplink system

Consider the uplink of an LP-OFDMA system with a total number of \( N \) subcarriers and \( K \) active users. Each user is assigned \( L = \lfloor N/K \rfloor \) subcarriers for simultaneous data transmission. As depicted in Fig. 3.1, the data stream of \( k^{th} \) user, \( \{ s^{(k)} \} \), after serial to parallel conversion at \( p^{th} \) block is given by

\[
s^{(k)}_p = \left[ s^{(k)}_p(0), s^{(k)}_p(1), \ldots, s^{(k)}_p(L-1) \right]^T,
\]

where \( 0 \leq p \leq M - 1 \) and \( M \) is the number of OFDM blocks per frame. A \( 2L \times 1 \) super-vector is formed by

\[
s^{(k)}_m = \begin{bmatrix} s^{(k)}_{2m} \\ s^{(k)}_{2m+1} \end{bmatrix}, \quad 0 \leq m \leq \lfloor \frac{M-1}{2} \rfloor - M-1
\]

and precoded it with a \( 2L \times 2L \) full-rank matrix \( \Theta \)

\[
\hat{x}^{(k)}_{2m} = \begin{bmatrix} \Theta_1 \\ \Theta_2 \end{bmatrix} s^{(k)}_m
\]

\[
h^{(k)}_{2m} = \Phi^{(k)} x^{(k)}_{2m}
\]

where \( \Theta_1, \Theta_2 \) are the upper and lower halves of \( \Theta \) each with a size of \( L \times 2L \). The precoded super-vector is then split (row-wise) to two halves (denoted by \( x^{(k)}_{2m} = \Theta_i s^{(k)}_m \)).
3.2. LP-OFDMA uplink system

The vector $x_{u,n}$ is extended by insertion of $N - L$ zeros at place of other users subcarriers to generate an $N \times 1$ vector

$$\Phi^{(k)}_{m,n} \triangleq \begin{cases} 1, & \text{if } m \in \Psi^{(k)}, n \in \{0, \ldots, L - 1\} \\ 0, & \text{otherwise} \end{cases}, 0 \leq m \leq N - 1, 0 \leq n \leq L - 1 \tag{3.4}$$

and $\Psi^{(k)} = \{(k - 1)L + 1, (k - 1)L + 2, \ldots, kL\}$ is the set of all subcarriers allocated to $k^{th}$ user with $1 \leq k \leq K$. The received signal, after cyclic-prefix (CP) removal, is given by

$$a_{m} = \sum_{n=1}^{K} e^{j2\pi n(m-1)/N_T} \Gamma^{(a)} H^{(a)} F^{N_T} b^{(a)}_{m} + \omega_{m} \tag{3.5}$$

where $F$ represents an $N \times N$ Fourier transform matrix with $F_{m,n} = (N)^{-1/2} \exp(-j2\pi nm/N)$ and $0 \leq n, m \leq N - 1$. Moreover, $N_T = N + N_g$ and $N_g$ is the CP length. $H$ is an $N \times N$ circulant channel matrix,

$$H_{m,n} = h_{(m-n) \mod N}, 0 \leq m, n \leq N - 1, \tag{3.6}$$

with its elements being the discrete channel impulse response samples $\{h_i\}_{i=0}^{L_H-1}$ where $L_H - 1$ is the channel length. The channel additive noise is denoted by $\omega_{m} \sim CN(0, \sigma^2 N)$. The frequency offset matrix of $k^{th}$ user is defined by $\Gamma^{(a)} = D_N \left(e^{j2\pi n/N}\right)$, $0 \leq n \leq N - 1$ where $e^{(a)}$ represents $n^{th}$ user CFO. Finally, users data decomposition is performed through FFT and user-specific subcarrier selection, i.e. frequency de-mapping. Therefore, the $k^{th}$ user's received signal can be expressed as

$$\begin{align*}
\mathbf{r}^{(k)}_{u,m} &= \left(\Phi^{(k)}\right)^T F a_{m} \\
&= e^{j2\pi n(2m+1)/N_T} \left(\Phi^{(k)}\right)^T \Gamma^{(a)} H^{(a)} F^{N_T} b^{(k)}_{m} \\
&+ \left(\Phi^{(k)}\right)^T F \sum_{q=1, q \neq k}^{K} e^{j2\pi n(2m+1)/N_T} \Gamma^{(a)} H^{(a)} F^{N_T} b^{(q)}_{m} \\
&+ \left(\Phi^{(k)}\right)^T F \omega_{m} = \eta^{(k)}_{u,m} \tag{3.7}
\end{align*}$$
3.3 CFO Estimation in The SUE Scenario

where the first term on the right hand side (RHS) is pure signal term of $k^{th}$ user, the second term is MUI and the last term is additive noise present on $k^{th}$ user sub-band.

Remark 1: In the above system description, the timing offset is assumed smaller than CP length, so that it will appear as a linear phase distortion in the frequency domain. In this case, the timing offset can be incorporated into channel frequency response.

Remark 2: According to the above system model, even and odd-indexed sub-blocks can be precoded with different precoding matrices i.e., $\Theta_1$ and $\Theta_2$. Therefore, at the receiver, the de-precoding process requires limited signalling (i.e., 1-bit) of precoding matrix indexes. However, by exploitation of frame synchronisation, signalling is not required. This is because by knowing the beginning of each frame, de-precoding can be carried out by applying $\Theta^{-1}$ to each received pair of consecutive sub-blocks.

3.3 CFO Estimation in The SUE Scenario

The proposed CFO estimation is performed by taking advantage of time correlation induced by the linear precoder $\Theta$ and through the second-order moments of the received signal. The received pair of precoded sub-blocks is expressible as below (by removing the MUI term and dropping the user index in (3.7))

$$r_{i,m} = e^{\frac{j2\pi((m+1)/N-1)}{\sigma_\epsilon^2}} A x_{i,m} + \eta_{i,m}$$

where $A$ is an $L \times L$ matrix given by

$$A = \Phi^T \Gamma \Phi \Sigma \Phi^T \Phi.$$  

Performing the second-order moments about $r_{i,m}$ results in

$$\tilde{\mu} = tr \left( \mathcal{E} \{r_{2,m} r_{1,m}^*\} \right) = e^{\frac{j2\pi i m}{N}} \sigma_\epsilon^2 tr \left( A \Theta_2 \Theta_1^T A^H \right).$$  

Through a careful precoder design in Section 3.4, $tr \left( A \Theta_2 \Theta_1^T A^H \right) > 0$. Then, the CFO, $\epsilon$, can be estimated via

$$\hat{\epsilon} = \frac{N \angle \tilde{\mu}}{2\pi N_T}$$
where $\angle \bar{\mu}$ denotes the argument of the $\bar{\mu}$ in radians. The CFO estimation range can be obtained through $-\pi \leq \angle \bar{\mu} \leq \pi$. Applying this inequality to (3.11) leads to $\frac{N}{2N_T} \leq \hat{\varepsilon} \leq \frac{N}{2N_T}$. For example by having $N = 64$ and $N_T = 8$, the CFO estimation range is $|\hat{\varepsilon}| \leq 0.44$. The CFO identifiability will be addressed in Section 3.4.

In practice, the ensemble correlation in (3.10) is replaced by the following sample average on finite blocks

$$
\bar{\mu} = \frac{1}{M} \sum_{m=0}^{M-1} \text{tr}\left(r_{2,m} r_{1,m}^T\right)
= e^{-\frac{2\pi n m}{N}} \frac{\sigma^2 \text{tr}\left(\mathbf{A}^T \mathbf{O}_2 \mathbf{O}_1^T \mathbf{A}^T\right) + \omega}{\beta}
$$

where $\omega$ is the estimation noise due to the sample average (defined in Appendix A). In the following sections, the performance analysis will be mainly based on (3.12).

### 3.4 The Precoder Design

In this section, the required properties of precoder matrix $\Theta$ that allow for the signal power preservation, CFO identifiability, CFO estimation and overall system performance are discussed.

#### 3.4.1 General Properties of The Precoder

The precoder $\Theta$ is a complex and non-unitary matrix which satisfies the following conditions.

C1) As shown in (3.10), the precoder should meet the condition $\text{tr}\left(\mathbf{A}^T \mathbf{O}_2 \mathbf{O}_1^T \mathbf{A}^T\right) > 0$ to enable the proposed CFO estimator. This condition indicates that firstly the above term cannot be zero as it causes zero-phase. Secondly, it cannot be a complex or negative-real value due to giving rise to phase ambiguity.

C2) To be able to recover the precoded information blocks, $\Theta$ has to be a full-rank matrix. If the precoder is not of full-rank, it causes inter-symbol-interference (ISI) in de-precoding process.
3.4. The Precoder Design

C3) Minimizing CFO estimation error and BER are conflicting issues. In a CFO-free case, smaller BER results is achieved by having a uniform eigenvalue distribution for $\Theta$. However, on the other hand, for higher CFO estimation accuracy, high correlation between consecutive OFDM sub-blocks, i.e. a non-uniform eigenvalue distribution, is required. This requirement would cause BER degradation due to induced inter-block interference (IBI) in de-precoding process. Therefore, there is a trade-off between CFO estimation and BER performance which needs to be taken into account when designing the precoder. (For more details see Section 3.4.4 and also [60]).

C4) To preserve the power of precoded symbols, the precoder is required to satisfy

$$\text{tr} (\Theta_1 \Theta_1^T) = \text{tr} (\Theta_2 \Theta_2^T) = L.$$  

This will be known as power preservation condition hereafter.

In order to fulfil the above criteria, the following linear precoder is proposed

$$\Theta = \begin{bmatrix} U & 0 \\ \Theta_1 \\ & \ldots \\ \Theta_2 \\ \text{PU} & \hat{P}U \end{bmatrix} \tag{3.13}$$

where $U$ is an $L \times L$ unitary matrix with $|U_{m,n}| = \frac{1}{\sqrt{L}} \ (0 \leq m, n \leq L - 1)$, and $P$ and $\hat{P}$ are $L \times L$ real diagonal matrices. This precoder is the generalization of the one proposed by Lin and Petropulu in [60].

3.4.2 MSE Criterion in The Precoder Design

The main concentration of this section is on the mean-square-error (MSE) criterion, as one of the primary objectives of precoder design. This is to improve the CFO estimation performance and is performed by minimizing the SUB-MMSE averaged over Rayleigh fading channels. The theoretical result given in [Appendix A, (15)], i.e. $\overline{\text{MMSE}_{\text{SUB}}(\epsilon)}$, is the approximate (A) MMSE for each channel realization. Section 3.6 will show that the AMMSE averaged over Rayleigh fading channels fits well with the simulation results. Therefore, it would be reasonable to perform the precoder design based upon the average AMMSE. Unfortunately, the closed-form of average AMMSE, i.e. 

$$\overline{\text{AMMSE}^{\text{SUB}}(\epsilon)}$$
3.4. The Precoder Design

\( \mathcal{E}(\text{MMSE}_{SU}(\varepsilon)) \), is still an open mathematical problem to this date. Therefore, the objective of precoder design reduces to minimizing an upper bound of the average AMMSE.

Utilizing the Jensen's inequality and the AMMSE (Appendix A, equation 15), the following upper bound of average AMMSE is derived as

\[
\begin{align*}
\mathcal{E}(\text{MMSE}_{SU}(\varepsilon)) & \leq \frac{1}{M} \left( \frac{N}{2\pi N_T} \right)^2 \left( \frac{\mathcal{E}(\| \hat{h} \|^2)}{\text{SNR}} + \frac{L}{2\text{SNR}^2} \right) \\
& \leq \frac{1}{M} \left( \frac{N}{2\pi N_T} \right)^2 \frac{\text{SNR} \mathcal{E}(\| \hat{h} \|^2) + \frac{L}{2\text{SNR}^2}}{\left( \sum_{i \in \Psi} P_i \right)^2} \\
& \leq \frac{1}{M} \left( \frac{N}{2\pi N_T} \right)^2 \frac{\text{SNR} \text{tr}(R_{hh}) + \frac{L}{2\text{SNR}^2}}{\left( \sum_{i \in \Psi} P_i \sigma_h^2 \right)} ,
\end{align*}
\]

where \( \hat{h} \) is the \( i \)th diagonal element of diagonalized channel matrix \( \hat{D}_H = \Phi \hat{F} \Phi_H \), \( \hat{D} = \Phi^T \hat{D}_H \Phi \) and SNR = \( \frac{\sigma_n^2}{\sigma^2} \). Utilizing the inequality \( \mathcal{E}X^2 \geq (\mathcal{E}X)^2 \), the upper-bound in (3.14) can be further loosened as

\[
\mathcal{E}(\text{MMSE}_{SU}(\varepsilon)) \leq \frac{1}{M} \left( \frac{N}{2\pi N_T} \right)^2 \frac{\text{SNR} \mathcal{E}(\| \hat{h} \|^2) + \frac{L}{2\text{SNR}^2}}{\left( \sum_{i \in \Psi} P_i \right)^2} \frac{\sum_{i \in \Psi} P_i \mathcal{E}(\| \hat{h} \|^2)}{\text{SNR}} + \frac{L}{2\text{SNR}^2} ,
\]

where \( R_{hh} \) is the channel covariance matrix and \( \sigma_h^2 \) is the channel variance. Utilizing the Cauchy inequality \( \left( \sum_{i \in \Psi} P_i \sigma_h^2 \right)^2 \leq \left( \sum_{i \in \Psi} P_i \right) \left( \sum_{i \in \Psi} \sigma_h^2 \right) \), the upper bound (3.16) reaches its minimum when \( P_i \triangleq \alpha \) is constant with respect to the index \( i \) with \( |\alpha| \leq 1 \). Therefore \( P = \alpha I_L \) and based on power preservation condition, \( \hat{P} = \sqrt{1 - \alpha^2} I_L \). By employing the above matrices \( P \) and \( \hat{P} \), (3.13) is referred to as uniform power distribution based precoder (UPDP) in the sequel and is given by

\[
\Theta = \left[ \begin{array}{cc} \mathbf{U} & 0 \\ \alpha \mathbf{U} & \sqrt{1 - \alpha^2} \mathbf{U} \end{array} \right] .
\]

3.4.3 BER Criterion in The Precoder Design

By assuming an MMSE-type receiver, based on block decision feedback mechanism, and also letting \( D(\cdot) \) denote decision making function over finite-signal alphabet, even-indexed blocks are recovered through

\[
\hat{s}_{2m} = D \left( G_e \mathbf{A} U \hat{s}_{2m} + G_e \eta_{2m} \right) , 0 \leq m \leq M - 1
\]
3.4. The Precoder Design

where

\[ G_e = U^H A^H \left( \sigma_{\eta L}^2 + AA^H \right)^{-1}. \]  

(3.19)

The odd-indexed blocks are detected after removing the effect of even-indexed blocks

\[ \hat{s}_{2m+1} = D \left[ G_o \left( r_{2m+1} - \alpha AU \hat{s}_{2m} \right) \right] \]  

(3.20)

where

\[ G_o = \sqrt{1 - \alpha^2} U^H A^H \left( \sigma_{\eta L}^2 + (1 - \alpha^2) AA^H \right)^{-1}. \]  

(3.21)

For sufficiently large SNR and M, the decision making function generates the transmitted data for even-indexed blocks, i.e., \( \hat{s}_{2m} = s_{2m} \). On the other hand, after removing the effect of even-indexed blocks, odd-indexed blocks can also be realized as the ones precoded the unitary matrix \( U \)

\[ \hat{s}_{2m+1} = D \left( \sqrt{1 - \alpha^2} G_o AU \hat{s}_{2m+1} + G_o \eta_{2m+1} \right). \]  

(3.22)

Based on the above discussion and in a synchronised system, UPDP results in even-indexed blocks to be precoded by the unitary matrix \( U \). Moreover, by assuming the scalar term \( \sqrt{1 - \alpha^2} \) can be absorbed by diagonal channel matrix, the odd-indexed blocks can also be seen as the ones precoded by unitary matrix \( U \). As shown in [59], by assuming an MMSE-type receiver and for quadrature-phase-shift-keying (QPSK) signalling, a class of channel-independent unitary-precoding matrices (e.g., \( U \)) is optimal in terms of BER in a synchronised system.

3.4.4 Parameter \( \alpha \) in Joint Consideration of MSE and BER

Based on the discussions made in Section 3.4.2, for the UPDP-based estimator, it holds

\[ \text{MMSE}_{SUB}(\epsilon) = \frac{1}{M\alpha^2} \left( \frac{N}{2\pi N_T} \right)^2 \frac{1}{\text{SNR}} \left\| \hat{h} \right\|^2 + \frac{L}{2\text{SNR}^2} \left\| \hat{h} \right\|^4. \]  

(3.23)

It can be observed that as \( |\alpha| \to 1 \) (i.e., its maximum possible value), \( \text{MMSE}_{SUB} \) is minimized with respect to \( \alpha \). However, this will give rise to the worst BER. This is due to induced IBI at the output of de-precoder for odd-indexed blocks (see (3.20)). Conversely, as \( |\alpha| \to 0 \) (i.e., a uniform eigenvalue distribution for UPDP), IBI \( \to 0 \), which results in the best BER performance in a synchronised system, but \( \text{MMSE}_{SUB}(\epsilon) \to +\infty. \)
Therefore \( \alpha \) should be chosen in a way to maintain both MSE and BER performances. As it is shown in [60], for a synchronised system, a precoder with similar structure to the UPDP results in minimum channel estimation error and asymptotically minimum BER by setting \( \alpha = 1/\sqrt{2} \). As shown in Section 3.6, by choosing the same value for \( \alpha \), the exploitation of UPDP results in satisfactory CFO estimation and comparable BER performance to a synchronous system.

Remark 3: It is shown in the literature (e.g., see [61]) that combining OFDM with linear precoding can effectively reduce the PAPR. However, as discussed above, the precoder considered here is designed in terms of MSE and BER and hence its effect on the PAPR reduction is out of the scope of this chapter.

3.5 CFO Estimation in the Multiuser Scenario

So far, the main underlying assumption was only one NU is present, over the acquisition period, in the network and all other \( K - 1 \) active users have already aligned and synchronised themselves with BS references. In this section, it is assumed all users can simultaneously be misaligned in the frequency domain. In order to offer high robustness to MUI, the second-order moments-based CFO estimation is performed after exploitation of a novel time-frequency MU-DM scheme. The functionality of MU-DM scheme will be evaluated in two ways. First, by investigating its impact on CFO estimation. Second, by deriving the approximate multiuser MMSE in two cases, i.e. with and without the exploitation of this scheme.

3.5.1 Time-frequency MU-DM Scheme

The proposed scheme is implemented through circular shifting of all sub-blocks (by one place) belonging to every other interfering user. To be able to have a simple model, first, the interfering users are divided into two groups, the ones with odd-adjacency-order (OAO) and the ones with even-adjacency-order (EAO). By assuming \( k \) is the user of interest's index, all the indexes belonging to users with OAO are collected in the set \( \Xi = \{(k \pm 1), (k \pm 3), \ldots \} \). For example, \( (k \pm 1) \) denotes the indexes of two interfering users,
Figure 3.2: Left: Overview of four adjacent users without any displacement. Right: The same four adjacent users after implementation of the proposed time-frequency MU-DM scheme. Each pair of precoded sub-blocks has a distinctive pattern.

both with adjacency orders of one. It indicates these interfering users are on the most adjacent frequency-bands to the $k^{th}$ user-band. Similarly, $\Xi^c = \{(k \pm 2), (k \pm 4), \ldots\}$ is the complement of $\Xi$ and contains the indexes of all users with EAO. The overview of four adjacent users after exploitation of MU-DM scheme is sketched in Fig. 3.2. As an example of users with OAO, the $(k - 1)^{th}$ user is considered. As shown, $0^{th}$ pair which consists of $x_{1,0}^{(k-1)}$ and $x_{2,0}^{(k-1)}$ is displaced by one place along the time direction. In the same way, all pairs belonging to $(k - 1)^{th}$ user are displaced by one place. For the last pair, i.e. $(M - 1)^{th}$ pair, displacement has caused a circular shift for the $2^{nd}$ sub-block $x_{2,M-1}^{(k-1)}$. For the users with EAO, $(k + 2)^{th}$ user is chosen as an example to be considered here. As shown, all pairs and underlying sub-blocks are in their original place without any displacement. As a result, the exploitation of MU-DM scheme causes all
pairs belonging to users with OAO to be displaced whereas the ones with EAO remain unaffected. The proposed MU-DM scheme can mathematically be expressed as

\[ x_p^{(u)} = x_p^{(u)} \quad \text{where} \quad \begin{cases} u - k : \text{odd} & \text{if} \quad u \neq k, \quad 1 \leq u, k \leq K \\ \tilde{\rho} = \left( (\rho + 1) \mod \frac{M}{M} \right), \quad 0 \leq \rho, \tilde{\rho} \leq \frac{M}{M} - 1 \end{cases} \] (3.24)

In multiuser scenario, to be able to model the \( \tilde{u} \)-th user CFO-induced energy-leakage into the \( k \)-th user-band, (3.9) is modified to

\[ A^{(u),(k)} = (\Phi^{(k)})^T F T^{(u)} H^{(u)} F \tau \Phi^{(u)} . \] (3.25)

By exploitation of circular shifting in (3.24) and having defined \( A^{(u),(k)} \), (3.7) can be rewritten as (see Fig. 3.2)

\[ \eta_{1,m}^{(k)} = e^{j2\pi \left ( \frac{(2m + 1) \eta_{1,m}}{N} \right )} A^{(k)} X_{1,m}^{(k)} \]

\[ + \sum_{\nu \in \Xi^{(u)}} e^{j2\pi \left ( \frac{(2m + 1) \eta_{1,m}}{N} \right )} A^{(u),(k)} X_{1,m}^{(u)} \]

\[ = \xi_{1,m}^{(u)} \quad \text{(users with OAO)} \]

\[ + \sum_{\nu \in \Xi^{(v)}} e^{j2\pi \left ( \frac{(2m + 1) \eta_{1,m}}{N} \right )} A^{(v),(k)} X_{1,m}^{(v)} \]

\[ = \xi_{1,m}^{(v)} \quad \text{(users with EAO)} \]

\[ + \eta_{1,m}^{(k)} \] (3.26)

where \( \nu \) is the complement of \( \nu \), and sub-blocks displacement for users with OAO leads to \( \tau(\nu, m) = ((m + \nu - 2) \mod M) \).

### 3.5.2 Impact of MU-DM on CFO Estimation

In this section, it is shown that by the exploitation of proposed MU-DM scheme, the CFOs induced interference of all users with OAO is mitigated in the process of \( k \)-th user CFO estimation. Consider (3.10) and the term \( tr \left ( \mathbb{E} \left \{ r_{1,m}^{(k)} \right \} \right ) \) after plugging in the equivalent sum of terms from (3.26). The above ensemble correlation can be expanded to 16 terms. The first term, \( e^{j2\pi \left ( \frac{(2m + 1) \eta_{1,m}}{N} \right )} \), is the desired term, from which \( k \)-th user CFO is estimated. Moreover, 14 terms are
zero due to zero-correlation of independent signals. This independence of signals is sometimes due to user-index difference such as
\[
e^{j2\pi s(k)(2m+1)N_0/N} \text{tr} \left( \mathbb{E} \left\{ \left( A^{(k)} x_{2,m}^{(k)} \right) \left( \xi_{1,m}^{(u)} \right)^H \right\} \right) = 0, \tag{3.27}
\]
or due to cross-correlation of data and additive noise e.g.,
\[
e^{j2\pi s(k)(2m+1)N_0/N} \text{tr} \left( \mathbb{E} \left\{ \left( A^{(k)} x_{2,m}^{(k)} \right) \left( \eta_{1,m}^{(k)} \right)^H \right\} \right) = 0, \tag{3.28}
\]
or due to cross-correlation of data signals with the same user-index but different pair-indexes, an example of which is
\[
\text{tr} \left( \mathbb{E} \left\{ \left( \xi_{2,m}^{(u)} \right) \left( \xi_{1,m}^{(u)} \right)^H \right\} \right) = \text{tr} \left( \mathbb{E} \left\{ \left( \xi_{1,r(2,m)}^{(u)} \right) \left( \xi_{2,r(1,m)}^{(u)} \right)^H \right\} \right) = 0 \tag{3.29}
\]
or finally it can be due to cross-correlation of two independent additive noise signals
\[
\text{tr} \left( \mathbb{E} \left\{ \left( \eta_{2,m}^{(k)} \right) \left( \eta_{1,m}^{(k)} \right)^H \right\} \right) = 0. \tag{3.30}
\]
The only non-zero term, out of 16 terms, is induced by users with EAO and does not fall into any categories above
\[
\text{tr} \left( \mathbb{E} \left\{ \left( \xi_{2,m}^{(w)} \right) \left( \xi_{1,m}^{(w)} \right)^H \right\} \right) \neq 0. \tag{3.31}
\]
As a consequence, the proposed scheme mitigates the interference, i.e. estimation noise, induced by half of the users, i.e. \((K-1)/2\) users, including the ones on the most adjacent sub-bands.

### 3.5.3 Approximate Multiuser MMSE

The objective of this section is to demonstrate the MU-DM scheme offers a CFO estimation performance that is comparable to that of SUE scenario. This is performed through the analysis of approximate multiuser MMSE. The approximate multiuser MMSE can be derived in the same way as approximate SUE-MMSE (see Appendix A). The only difference is in this scenario, the estimation noise defined in (3.12) needs to take into account the effect of MUI. For this purpose, define
\[
\tilde{\sigma}_{\text{MUI}} \overset{\Delta}{=} \text{tr} \left( \Omega_{\text{MUI}} \right), \tag{3.32}
\]
where the subscript \( MUI \) implies the contribution of MUI in estimation noise and from (3.26), \( \Omega_{MUI} \) can be calculated as

\[
\Omega_{MUI} = \frac{1}{M} \sum_{m=0}^{M-1} \left( e^{\frac{j2\pi(k)(2m+1)N_T}{N}} A^{(k)} \Theta_2 \bar{z}_m^{(k)} + \xi_{2,m} \right) + \eta_{1,m}^{(k)} \right) = \xi_{1,m}^{(k)} \right),
\]

(3.33)

The variance of \( \bar{\omega}_{MUI} \) is calculated through

\[
\sigma^2_{\bar{\omega}_{MUI}} = \frac{1}{L} tr \left( \mathcal{E}_{\bar{\omega},\eta} \left( \Omega_{MUI} \Omega_{MUI}^* \right) \right)
\]

(3.34)

where \( \mathcal{E}_{\bar{\omega},\eta} \) denotes the expectation over subscripted parameters. The first-order derivative of \( \sigma^2_{\bar{\omega}_{MUI}} \) as required in (Appendix A, equation 9) is calculated through

\[
\frac{\partial \left( \sigma^2_{\bar{\omega}_{MUI}} \right)}{\partial \xi^{(k)}} = \frac{1}{L} \mathcal{E}_{\bar{\omega},\eta} \left( \frac{\partial \Omega_{MUI}}{\partial \xi^{(k)}} \Omega_{MUI}^* + \Omega_{MUI} \frac{\partial \Omega_{MUI}^*}{\partial \xi^{(k)}} \right)
\]

(3.35)

where

\[
\frac{\partial \Omega_{MUI}}{\partial \xi^{(k)}} = \frac{1}{M} \sum_{m=0}^{M-1} e^{\frac{j2\pi(k)(2m+1)N_T}{N}} \left( \frac{j2\pi(2m+1)N_T}{N} A^{(k)} + \frac{\partial A^{(k)}}{\partial \xi^{(k)}} \right) \Theta_2 \bar{z}_m^{(k)} \xi_{1,m}^{(k)}
\]

\[
+ \frac{1}{M} \sum_{m=0}^{M-1} e^{\frac{-j2\pi(k)(2m+1)N_T}{N}} \xi_{2,m} \left( \frac{j2\pi(2m+1)N_T}{N} A^{(k)} + \frac{\partial A^{(k)}}{\partial \xi^{(k)}} \right) \right)
\]

(3.36)

Therefore the total estimation noise and its first-order derivative can be expressed as

\[
\sigma^2_{\bar{\omega}_{tot}} = \sigma^2_{\bar{\omega}} + \sigma^2_{\bar{\omega}_{MUI}}
\]

(3.37)

and

\[
\frac{\partial \left( \sigma^2_{\bar{\omega}_{tot}} \right)}{\partial \xi^{(k)}} = \frac{\partial \left( \sigma^2_{\bar{\omega}} \right)}{\partial \xi^{(k)}} + \frac{\partial \left( \sigma^2_{\bar{\omega}_{MUI}} \right)}{\partial \xi^{(k)}},
\]

(3.38)

respectively. By plugging (3.37) and (3.38) in (Appendix A, equation 9), the closed-form for approximate multiuser MMSE can be obtained. In what follows, the impact of MU-DM scheme on \( \sigma^2_{\bar{\omega}_{MUI}} \) is analysed.
Approximate multiuser MMSE without MU-DM scheme

Without exploitation of MU-DM scheme, all the interfering terms belonging to both users with OAO and EAO, i.e. $\xi_{1,m}$ and $\xi_{2,m}$ are considerable in (3.33). Therefore, for this case, $\Omega_{MUI}$, $\sigma^2_{\text{MUI}}$ and $\frac{\beta(\sigma^2_{\text{MUI}})}{\delta e^{(k)}}$ can be calculated from (3.33), (3.34) and (3.35) respectively without any change. However, taking the ensemble average in (3.34), (3.35) necessitates going through a tedious derivation and therefore in Section 3.6 it is assumed it can be replaced by sample average over finite terms.

Approximate multiuser MMSE with MU-DM scheme

As discussed in Section 3.5.2, by the exploitation of MU-DM scheme, all the terms belonging to users with OAO are negligible in comparison with the term belonging to users with EAO (see equations (3.27)-(3.31)). Therefore, for sufficiently large $M$, (3.33) can be simplified to

$$\Omega_{MUI}^{DM} = \frac{1}{M} \sum_{m=0}^{M-1} \xi_{2,m} \left( \xi_{1,m}^{(v)} \right)^{\kappa},$$

from which,

$$\left( \sigma^2_{\text{MUI}} \right)^{DM} = \frac{\sigma^2_{\text{MUI}}}{L} \text{tr} \left( \left( \sum_{v \in \mathbb{Z}} e^{\frac{j2\pi e^{(v)} N_p}{N} A^{(v),(k)} P (A^{(v),(k)})^{\kappa}} \right) \left( \sum_{v \in \mathbb{Z}} e^{\frac{j2\pi e^{(v)} N_T}{N} A^{(v),(k)} P^H (A^{(v),(k)})^{\kappa}} \right) \right). \quad (3.40)$$

Moreover, as $\Omega_{MUI}^{DM}$ is independent of $e^{(k)}$, $\frac{\beta(\sigma^2_{\text{MUI}})}{\delta e^{(k)}} = 0$.

3.6 Simulations and Discussions

Computer simulations were used to evaluate the performance of proposed CFO estimator in terms of MSE, i.e. $\mathcal{E}\left\{ \left| e^{(k)} - e^{(k)} \right|^2 \right\}$, and BER. An LP-OFDMA system is considered with $N = 64$ subcarriers and a CP length $N_p = 8$. The information-bearing symbols are randomly drawn from a QPSK set with the equal-probability. There are $K = 4$ active users in the system and each one is assigned $L = 16$ subcarriers. Second
user is the user of interest. All users' CFOs are random values uniformly distributed in \([-0.4, 0.4]\). The communication channel is frequency-selective slow Rayleigh fading with the power delay profile \(E\{\mid h(\ell)\mid^2\} = \sum_{\ell=0}^{4} e^{-\ell/5}, \ell = 0, \ldots, 4\), which is widely used in the literature (e.g. [37]). The user velocity is assumed to be 3 Km/h which is a typical setting for pedestrian environment. It is assumed all users' channels are statistically independent. Provided perfect channel state information (CSI) at the receiver, the linear MMSE method is used for channel equalization. The results were obtained by averaging over 10,000 independent channel realizations. Unless otherwise mentioned, CFO estimation is carried out over 20 and 100 OFDM blocks in a frame. By exploiting the information about CFO estimates on downlink feedback channel, a time-domain pre-compensation is performed for each user separately. The linear precoder has the form of (3.17) with \(U\) being the Walsh-Hadamard matrix and \(\alpha = \frac{1}{\sqrt{2}}\). The performance of the proposed method is compared with several state-of-the-art approaches, i.e., CP-based [40], CP-modified [24], virtual (null)-subcarrier [27] and Kurtosis-based [37] estimators. For the virtual-subcarrier based method, there are 3 null-subcarriers placed at the end of each user-band (as also assumed in [27]). As the existence of reference users violates the CFO estimation process for some state-of-the-art approaches, the performance of these approaches in single-user scenario will also be evaluated where all reference users keep silent.

3.6.1 Experiment 1: SUE and single-user Scenarios

The objective of this experiment is to evaluate the performance of proposed CFO estimator in SUE and single-user scenarios. The MSE performance of different blind CFO estimators in single-user scenario is plotted in Fig. 3.3. It is observed that the proposed approach outperforms all other approaches for the case of SNR < 22 dB in an LP-OFDMA system. The error floor that is observed for the proposed approach is due to the difference of \(\sigma_0^2, \sigma_s^2\) (see Appendix A, equations (2),(3)). The early error floor appeared for the Kurtosis-based estimator can be explained as follows. As pointed
3.6. Simulations and Discussions

![Figure 3.3](image)

Figure 3.3: Performance of different blind CFO estimators in single-user scenario.

out in [37], the main underlying assumptions for the Kurtosis-based estimator to work are \textit{independence} and \textit{non-Gaussianity} of the transmitted data symbols. When data symbols are precoded, firstly the precoded symbols are not any more independent due to the correlation induced by the precoder. Secondly, linear precoding indeed results in each precoded symbol to be a weighted summation of uniformly distributed symbols (e.g., QPSK). In this way, the precoded symbols follow an approximately Gaussian distribution due to central limit theorem. As the required conditions for the transmitted symbols do not hold in a linearly precoded system, Kurtosis-based estimator does not offer a satisfactory performance. As a result, in spite of great performance in an OFDMA system, this estimator would require some modifications to be applicable to LP-OFDMA systems.

In Fig. 3.4 the MSE performance of different blind CFO estimators is evaluated in SUE scenario. As shown, the null-subcarrier based approach is sensitive to the presence of RUs. The reason for this observation can be explained as follows. In this approach, a time-domain grid-search is required to find the candidate CFO that minimizes the power spectral density over the null-space. In the single-user scenario, as all RUs keep
3.6. Simulations and Discussions

Figure 3.4: Performance of different blind CFO estimators in SUE scenario.

silent, virtual subcarriers only contain the CFO-induced energy-leakage of NU. However in the SUE scenario, the grid-search performs time-domain CFO compensation on the super-imposed signal (of all users). This causes the RUs data to leak into the null-space. The CP-based and CP-modified approaches are also affected by the presence of RUs. This is due to the fact that the filter-bank that is exploited by these approaches cannot offer perfect users separation. However, it can be observed that the proposed approach is not affected by RUs and offers identical performance to single-user scenario.

Fig. 3.5 illustrates the BER performance of the system that employs different CFO estimators in single-user and SUE scenarios. The following observations are in order. Firstly, for the proposed scheme, the curves corresponding to single-user and SUE scenarios are coincided. Secondly, in the single-user scenario, a remarkable phenomenon is that, despite better CFO estimation performance by the null-subcarrier approach for SNR $> 22$ dB, its corresponding BER performance is worse than the proposed approach for that SNR range. This is because the employment of null subcarriers introduces minor change to the air-interface structure (the size of linear precoder) that slightly affects the overall system performance. Thirdly, in the SUE scenario, due to unsatisfactory estimation performance of existing approaches, early error floors are
3.6. Simulations and Discussions

Figure 3.5: BER Performance of different blind CFO estimators in both single-user and SUE scenarios. For the proposed approach, the curves corresponding to single-user and SUE are coincided.

observable (e.g., > 15 dB). Fourthly, the proposed approach outperforms all state-of-the-art approaches in both scenarios. Moreover, it shows a very close performance to the CFO-free system for SNRs up to 25 dB. For higher SNRs, small performance degradation can be observed, e.g. 1 dB at SNR= 30 dB. Finally, from the slope of BER curves for the proposed scheme, it can be observed that, the frequency-diversity order is 2 as result of linear precoding.

Fig. 3.6 illustrates a three-dimensional (3-D) view of $\sigma^2_\delta$ as a function of different CFO values and SNRs. The aim of this figure is to show that $\sigma^2_\delta$ varies negligibly with respect to CFO over all SNRs. (See Appendix A for details)

3.6.2 Experiment 2: Multiuser Scenario

The primary objective of this experiment is to evaluate the robustness of different blind CFO estimators to MUI. The MSE performance is plotted in Fig. 3.7. It is shown that
3.6. Simulations and Discussions

Figure 3.6: $\sigma_0^2$ as a function of different CFO values and SNRs.

Figure 3.7: Performance of different blind CFO estimators in multiuser scenario. For the comparison purpose, SUE performance of proposed approach is also provided.
the MSE of all existing approaches is severely degraded in presence of MUI by the appearance of early error floors (e.g., > 5 dB). The proposed approach shows a high resistivity to MUI, which is evident by its comparable performance to SUE scenario.

The BER performance is plotted in Fig. 3.8. The existing approaches show significant sensitivity to CFO-induced MUI by the manifestation of early error floors (i.e., > 10 dB). However, the proposed approach shows approximately identical performance to the synchronous system for small and moderate SNRs (i.e., up to 20 dB) and comparable performance to that at higher SNRs.

Fig. 3.9 illustrates the MSE convergence rate of different blind CFO estimators. It is observed that the proposed method significantly outperforms all other approaches for any number of blocks and at all SNRs. More importantly, due to the MUI-resilience feature of proposed approach, it enjoys higher estimation accuracy by increasing the number of blocks. However, this property cannot be observed in other approaches.

Finally, Fig. 3.10 illustrates the theoretical and simulation results for the estimation performance of proposed approach. The number of blocks participated in CFO estimation is 20. The theoretical results are based on the derivation of approximate SUE-MMSE (see Appendix A, equations (8), (9)), approximate multiuser MMSE without MU-DM through (3.34), (3.35) and finally approximate multiuser MMSE with MU-DM scheme through (3.39), (3.40). The left sub-plot illustrates the theoretical results. It can be observed that the MU-DM scheme offers a CFO estimation performance that is very close to SUE scenario i.e., around $1 \times 10^{-5}$ MMSE difference observed for SNR $> 5$ dB. However, without the exploitation of MU-DM, CFO estimation performance degrades significantly. This justifies that the MU-DM scheme is capable of offering an MUI-resilient approach for CFO estimation in LP-OFDMA uplink. The right sub-plot illustrates both theoretical and simulation results in SUE scenario and multiuser scenario with MU-DM. The following phenomena are in order. First, the theoretical results show close performance to the simulation results in both SUE and multiuser scenarios with a small difference. The difference is due to approximation made in modeling $\tilde{w}$ and $\tilde{w}_{MU}$ as Gaussian processes (see Appendix A, equations (1), (2)). Second, as it was the case for the theoretical results, simulation results also confirm that the
3.6. Simulations and Discussions

Figure 3.8: BER Performance of different blind CFO estimators in multiuser scenario. For the comparison purpose, SUE performance of proposed approach is also provided.

Figure 3.9: Performance of different blind CFO estimators with respect to the number of blocks participated in estimation in multiuser scenario.
Figure 3.10: Left: Approximate MMSE results in SUE and multiuser scenarios with and without MU-DM scheme. Right: Estimation performance comparison between SUE scenario and multiuser scenario with MU-DM.

MU-DM scheme is capable of mitigating a considerable amount of MUI. To benchmark the performance of proposed CFO estimator, the modified Cramér-Rao lower bound (MCRLB), as the most common bound in synchronisation theory, is also plotted [27]. The considerable gap observed between the MCRLB and MSE is due to two reasons. First, the MCRLB does not deterministically depend upon transmitted symbols. Second, the average performance over Rayleigh fading channels requires sufficient number of data symbols and channel realizations (see [27] for details).

Remark 4: Further simulations are performed by replacing the assumed 5-tap channel with an 8-tap frequency selective Rayleigh fading channel as in C2-NLOS scenario (see [62]). In terms of MSE of estimated CFOs, identical results to that of 5-tap channel were observed. This can be justified by using Equation 3.10, as the channel matrix is multiplied by its Hermitian and its effect is averaged out. Moreover, in terms of BER, by assuming the CP length is increased to mitigate both channel image and any possible timing-offset, the BER curves corresponding to 5-tap channel and 8-tap channel are
3.6. Simulations and Discussions

coincided.
3.7 Summary

In this chapter, a novel blind CFO estimation approach for LP-OFDMA uplink was presented. By taking advantage of time correlation induced by linear precoder, the proposed approach offered a second-order moments-based blind CFO estimation. The linear precoder was carefully designed to fulfill the requirements for CFO identifiability, CFO estimation accuracy and BER performance. In multiuser scenario, to mitigate a considerable portion of MUI, a novel MU-DM scheme was proposed. The MUI-resilience feature of MU-DM scheme was evaluated through derivation of approximate MMSE in both SUE and multiuser scenarios. It was shown that, by the exploitation of MU-DM scheme, the approximate multiuser MMSE is very close to that of SUE scenario. Simulation results have shown two important achievements. First, as it was also shown via theoretical analysis, the proposed MU-DM scheme offers very close CFO estimation performance to that of SUE scenario. Second, the proposed approach outperforms existing blind CFO estimation approaches in both SUE and multiuser scenarios.
A Class of Training Designs for Robust
BLUE CFO Estimation in OFDMA

4.1 Introduction

The focus of this chapter is on the best linear unbiased estimator (BLUE) approach supported by Morelli's synchronisation policy [6], where carrier-frequency offsets (CFOs) are compensated at the transmitters. Throughout this chapter, unless otherwise is specified, it is assumed the timing synchronisation is ideal for all users. This work is motivated by the fact that the BLUE approach can offer low-complexity and sub-optimum performance in the single asynchronous-user scenario, but is very sensitive to mutual interference between asynchronous users (AUs). Surely, one can employ the maximum likelihood (ML) approach to offer the optimum performance in multiple asynchronous-users scenarios. However, the ML approach has to simultaneously deal with many unknown parameters such as CFOs, timing offsets, and channel parameters, and thus suffers significant complexity in the case of many asynchronous users [6]. Therefore, the work about improving the robustness of BLUE in multiple asynchronous-users scenarios has a practical implication.

Major contribution of this chapter is to combine the BLUE approach with three appropriately designed training schemes so as to offer low-complex and sub-optimum CFO
estimation in multiple asynchronous-users scenarios. The first training design borrows the idea about orthogonal training sequences (OTS) originally proposed for multi-channel estimation in the synchronous multiple-input and multiple-output orthogonal-frequency-division-multiplexing (MIMO-OFDM) [63]. However, combination of the OTS and the BLUE is not straightforward. This is because the training orthogonality is easily destroyed by factors such as interference from synchronised users and the propagation time-delay. The first problem is solved by employing a simple zero-forcing (ZF) method, and the second problem by employing two delay-awareness approaches. The second training design is named time-domain training cyclostationarity (TDTC) approach, which exploits the fact that the interleaved subcarrier assignment introduces signal cyclostationarity in the time-domain. The third design combines the self-interference canceling training scheme (SCTS) proposed in [64] with the BLUE. The analytical and simulation results show that the OTS-BLUE approach offers single asynchronous-user equivalent performance in optimistic cases, but suffers significant performance degradation in the presence of synchronised users. As a complement, both the TDTC-BLUE and the SCTS-BLUE can offer single asynchronous-user comparable performance when synchronised users are present. However, the TDTC-BLUE is sensitive to the placement order of asynchronous users. The SCTS-BLUE shows a more robust performance in the presence of multiuser interference (MUI).

4.2 System Model and Problem Formulation

4.2.1 OFDMA Uplink

Consider a cyclic-prefix orthogonal-frequency-division-multiple-access (CP-OFDMA) system with $N$ subcarriers and accommodating a maximum of $K$ active users. The block diagram of such a system is referred to [6]. Specifically, each user transmits information-bearing symbols $\{s_t^{(k)}\}$ over $L = [(N)/(K)]$ adjacent subcarriers, where $k$ stands for the user index ($0 \leq k \leq K - 1$), $t$ for the symbol index, and $[.]$ for the integer floor. Prior to transmission, each user first groups those symbols into $L \times 1$
blocks
\[ s_n^{(k)} = [s_{nL}^{(k)}, s_{nL+1}^{(k)}, \ldots, s_{nL+L-1}^{(k)}]^T \]
and then maps \( s_n^{(k)} \) onto the corresponding subcarriers for instance the following set of subcarrier index
\[ \Psi(k) = \{kL, kL + 1, \ldots, kL + L - 1\} \]
where \( T \) stands for the matrix transpose, and \( n \) for the block index \((0 \leq n \leq M - 1)\).

The output of CP-OFDM modulator is expressible as
\[ \hat{x}_{nN_T+i}^{(k)} = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} \tilde{s}_{n,m}^{(k)} e^{j2\pi(mN+N_g)} \]
where
\[ \tilde{s}_{n,m}^{(k)} \triangleq s_{nN+m-kL}^{(k)} \sum_{l=0}^{L-1} \delta(m-kL-l) \]  
\( \delta(\cdot) \) denotes the Dirac-Delta function, \( N_g \) the CP length, \( N_T = (N_g + N) \), \( m \) is the frequency-domain subcarrier index \((0 < m < N - 1)\) and \( i \) the time-domain sample index \((0 < i < N_T - 1)\).

Then, the signal \( \hat{x}^{(k)} \) goes through a communication channel with the impulse response \( h^{(k)} \) where \( h^{(k)} = [h_0^{(k)}, h_1^{(k)}, \ldots, h_{L_h-1}^{(k)}]^T \), where \( L_h < N_g \) denotes the upper bound of channel length, and experiences the CFO \( f^{(k)} \) as well as the propagation delay \( \tau^{(k)} \). The received signal is the superposition of multiuser signals with the following discrete-time equivalent form [64]
\[ f_{nN_T+i} = \sum_{k=0}^{K-1} e^{j2\pi f^{(k)}(nN_T+i)} y_{nN_T+i-\mu(k)}^{(k)} + v_{nN_T+i} \]  
where
\[ y_{nN_T+i-\mu(k)}^{(k)} \triangleq \sum_{\ell=-\infty}^{\infty} \hat{x}_{\ell}^{(k)} h_{nN_T+i-\ell-\mu(k)}^{(k)} \]
v denotes the Gaussian noise with zero mean and variance \( \sigma_v^2 \), \( e \) the normalized version of \( f \), i.e. \( e = f NT_s \) \( (T_s \) stands for the sampling period), and \( \mu = [\tau/(T_s)] \) the integer part of propagation delay \( \tau \). The fractional part of \( \tau/(T_s) \) is incorporated into the channel parameter \( h \) (see [6] for the reason).
4.2. System Model and Problem Formulation

4.2.2 Morelli's Synchronisation Policy and Channel Equalization

This work is based on Morelli's synchronisation policy presented in [6], where the uplink receiver estimates CFOs and propagation delays, and then feeds them back to transmitters for conducting transmitter-side compensation. It is assumed, but only here, that all active users are well synchronised, i.e. \( \epsilon^{(k)} = 0 \) and \( \mu^{(k)} = 0, \forall k \). The receiver discards the CP part and performs discrete Fourier-transform (DFT) on the residual part \( \tilde{r}_{nN+i}, i=N_0, N_0+1, \ldots, N_0+N-1 \), to obtain the frequency-domain version of received signal as below

\[
\tilde{r}_{nN+m} = \sum_{k=0}^{K-1} \tilde{s}_{n,m}^{(k)} \tilde{h}_{m}^{(k)} + \tilde{\nu}_{nN+m}, \quad 0 \leq m \leq N - 1
\]

where \( \tilde{h}_{m}^{(k)} = \sum_{l=0}^{L_k-1} h_{l}^{(k)} \exp\left(-j\frac{2\pi l m}{N}\right) \) is the channel frequency response, and \( \tilde{\nu} \) the corresponding noise. Provided the knowledge of \( \tilde{h}_{m}^{(k)} \), first the linear minimum-mean-square-error (LMMSE) channel equalization can be applied as below

\[
z_{nN+m}^{(k)} = \frac{\tilde{r}_{nN+m}^{(k)} W_{m}^{(k)} h_{m}^{(k)}}{|\tilde{h}_{m}^{(k)}|^2 W_{m}^{(k)} + \sigma_{\nu}^2} + \tilde{\nu}_{nN+m}
\]

and then conduct detection on \( z_{nN+m}^{(k)} \), where * is the conjugate, and \( W_{m}^{(k)} \) the user filtering defined in 4.2.

4.2.3 CFO Estimation with Single Asynchronous-User

Consider a scenario where an asynchronous user with the user index \( k_0 \in \{0, 1, \ldots, K - 1\} \) sends training symbols \( t^{(k_0)} \) over its own user band for estimation of \( \epsilon^{(k_0)} \), while synchronised users are sending information-bearing symbols. The frequency-domain version of received signal is expressible as (see [65] for inter-carrier interference (ICI) analysis)

\[
\tilde{r}_{nN+m} = \sum_{k \neq k_0}^{K} \tilde{s}_{n,m}^{(k)} \tilde{h}_{m}^{(k)} + \sum_{\text{synch. users}} e^{j2\pi t^{(k_0)}(nN_0+m) / N} \tilde{\Gamma}_{m}^{(k_0)} W_{m}^{(k_0)} + \tilde{\nu}_{nN+m}
\]

where

\[
\tilde{\Gamma}_{m}^{(k_0)} = \sum_{\ell=0}^{N-1} e^{j2\pi t^{(k_0)} \ell / N} \tilde{h}_{\ell}^{(k_0)} t^{(k_0)}_{\ell} \tilde{\epsilon}_{\ell-m}
\]
4.2. System Model and Problem Formulation

4.2.1. CFO Estimation

The CFO estimation includes the following two steps:

Step 1: Multiply \( \tilde{r}_{nN+m} \) with \( W_m(k_o) \) to filter out those synchronised users, i.e.

\[
\tilde{r}_{nN+m}^{(k_o)} = W_m(k_o) \tilde{r}_{nN+m} = e^{j2\pi(k_o)(nN+m)} W_m(k_o) + \bar{v}_{nN+m}
\]  

(4.10)

(4.11)

where \( \bar{v}_{nN+m} \triangleq W_m(k_o) \tilde{r}_{nN+m} \).

Step 2: Perform the ML estimation by solving the following cost function

\[
\varepsilon^{(k_o)} = \arg\min_{n=0}^{N-1} \sum_{m=0}^{N-1} \left| \tilde{r}_{nN+m}^{(k_o)} - e^{j2\pi(k_o)(nN+m)} W_m(k_o) \right|^2
\]  

(4.12)

where \( \tilde{N} \) is the number of training blocks. Solving (4.12) requires joint estimation of multiple parameters \((\varepsilon^{(k_o)}, \mu^{(k_o)}, \bar{r}^{(k_o)})\). However, (4.12) does not lead to a closed-form, and thus the ML approach requires a high-complexity searching over all possible \((\varepsilon^{(k_o)}, \mu^{(k_o)}, \bar{r}^{(k_o)})\) [6].

Alternatively, the BLUE approach originally proposed in [46] and later on extended to OFDMA in [6] can be employed to gain the low-complexity and sub-optimum performance. Specifically, the BLUE approach is implemented as follows:

S1: Calculate the autocorrelation of \( \tilde{r}_{nN+m} \) as

\[
c_i = \frac{1}{\tilde{N} - i} \sum_{n=1}^{\tilde{N} - i} \sum_{m=0}^{N-1} \left( r_{(n-i)N+m} \left( \tilde{r}_{(n-i)N+m}^{(k_o)} \right)^* \right), \quad 1 \leq i \leq \tilde{N} - 1
\]  

(4.13)

where \( \eta_i \) is the interference plus noise term after the autocorrelation.

S2: Perform the CFO estimation by following the algorithm presented in [46]

\[
\varepsilon^{(k_o)} = \frac{N}{2\pi NT} \sum_{i=1}^{\tilde{N}} w_i \angle (c_i c_{i-1}^*)
\]  

(4.14)

where \( \angle \) stands for the angle, \( c_0 \triangleq 1 \),

\[
w_i = \frac{3((\tilde{N} - i)(\tilde{N} - i + 1) - \tilde{N}(\tilde{N} - \tilde{N}))}{N(4N^2 - 6\tilde{N}N + 3N^2 - 1)}.
\]  

(4.15)
The BLUE approach achieves the following minimum mean-square-error (MMSE) for $\bar{N} = (\bar{N})/(2\bar{N})$ (see [47])

$$\text{MMSE}(e^{(k_o)}) = \frac{3N^2\bar{L}_{o,2}^2}{2\pi^2\bar{N}(\bar{N}^2 - 1)\bar{N}^2} \sum_{m=0}^{N-1} \left| \Gamma_m^{(k_o)} \right|^2 W_m^{(k_o)}.$$ (4.16)

The estimation range is limited by $|e^{(k_o)}| < (N)/(2\bar{N})$. The performance difference between BLUE and ML can be observed by comparing (4.16) with the modified Cramer-Rao lower bound (MCRLB) presented in [6].

### 4.2.4 Problem Formulation in Multiple Asynchronous-Users Scenario

Consider an OFDMA system accommodating $K_o (> 1)$ asynchronous users with the set of user-index $\Psi \subset \{0, 1, \cdots, K - 1\}$. The frequency-domain received signal $\tilde{r}_{nN+m}$ is expressible as

$$\tilde{r}_{nN+m} = \sum_{k \in \Psi} s_{n,m}^{(k)} \tilde{h}_{n,m}^{(k)} + \sum_{k \in \Psi} \left( e^{j2\pi x^{(k)}(nN_T+N_o)m/N} \Gamma_m^{(k)} W_m^{(k)} \right) + \tilde{v}_{nN+m}. \quad (4.17)$$

Those synchronised users can be filtered out by multiplying $\tilde{r}_{nN+m}$ with the term $\sum_{k \in \Psi} W_m^{(k)}$ to get

$$\tilde{r}_{nN+m}^{(\Psi)} = \sum_{k \in \Psi} \left( e^{j2\pi x^{(k)}(nN_T+N_o)m/N} \Gamma_m^{(k)} W_m^{(k)} \right) + \tilde{v}_{nN+m}^{(\Psi)}. \quad (4.18)$$

where $\tilde{v}_{nN+m}^{(\Psi)} = \sum_{k \in \Psi} W_m^{(k)} \tilde{v}_{nN+m}$. Then, the ML approach can be applied here by solving the following cost function [41]

$$\min \left| \tilde{r}_{nN+m}^{(\Psi)} - \sum_{k \in \Psi} \left( e^{j2\pi x^{(k)}(nN_T+N_o)m/N} \Gamma_m^{(k)} W_m^{(k)} \right) \right|^2. \quad (4.19)$$

However, solving 4.19 requires joint estimation of $(e^{(k_o)}, \mu^{(k_o)}, \tilde{h}_{n,m}^{(k_o)})$ for $K_o$ users, and thus suffers significant complexity particularly for a large $K_o$.

Applying the BLUE approach here will meet a problem that asynchronous users are not orthogonal in the frequency-domain due to the ICI. For example, use (4.10) to filter out other users and perform the BLUE on the $k_o^{th}$ asynchronous user. Due to the
mutual interference between asynchronous users, (4.10) will not lead to (4.11) but the following form

\[ l_{nN+m}^{(k_o)} = e^{\frac{j2\pi (k_o)(\alpha_{N+m}+\alpha_o)}{N}} \Gamma_m^{(k_o)} W_m^{(k_o)} + \mathcal{I}_{nN+m} + \eta_n^{(k_o)} \tag{4.20} \]

where \( \mathcal{I}_{nN+m} \) is the MUI term with the following expression

\[ \mathcal{I}_{nN+m} = \sum_{k \in \Psi, k \neq k_o} e^{\frac{j2\pi (k_o)(\alpha_{N+m}+\alpha_o)}{N}} \Gamma_m^{(k_o)} W_m^{(k_o)}. \tag{4.21} \]

Suppose \( \mathcal{I}_{nN+m} \) to be approximately Gaussian [65], the BLUE approach 4.13-4.15 applied on 4.20 yields the following approximate MMSE

\[ \text{MMSE}(\hat{\epsilon}^{(k_o)}) = \frac{3N^2 (\sigma_m^2 + \sigma_2^2)}{2\pi^2 N(N^2 - 1)N^2 \sum_{m=0}^{N-1} |\Gamma_m^{(k_o)}|^2 W_m^{(k_o)}} \tag{4.22} \]

where \( \sigma_2^2 \) is the power of MUI with the following form

\[ \sigma_2^2 = \sum_{m=0}^{N-1} \sum_{k \in \Psi, k \neq k_o} |\Gamma_m^{(k_o)}|^2 W_m^{(k_o)}. \tag{4.23} \]

This result shows the impact of MUI on the CFO estimation performance for the approach 4.13-4.15. For the case of having large CFOs (e.g. \( \epsilon > 0.1 \)) and several adjacent asynchronous users, the power of MUI \( \sigma_2^2 \) becomes considerably large, and thus causes considerable performance degradation to the CFO estimation. In order to improve the robustness of BLUE to the MUI, three training approaches are proposed which are presented in the following sections.

4.3 The OTS-BLUE Approach

4.3.1 An Optimistic Scenario

Consider a multiuser scenario where all active users have CFOs and \( \mu^{(k)} = 0, \forall k \). In this case it holds \( \mathcal{I}_{nN+m} = \eta_n^{(k)} \), i.e., \( \Psi = \{0, 1, ..., K-1\} \). In order to clarify the technical presentation in this section, first (4.18) is rewritten into the following matrix form (see [66],[67])
4.3. The OTS-BLUE Approach

\[ \bar{r}_n = \sum_{k \in \Psi} \left( F H^{(k)} F^\dagger \epsilon_{(k)} \bar{\alpha}^{(k)} \right) e^{j2\pi n \epsilon_{(k)}} + \bar{v}_n \]  

(4.24)

where

\[ \bar{r}_n = \left[ r_{(N)}, r_{(N+1)}, \ldots, r_{(N+N-1)} \right]^T \]  

(4.25)

\[ t^{(k)} = \left[ t_0^{(k)}, t_1^{(k)}, \ldots, t_{N-1}^{(k)} \right]^T \]  

(4.26)

\[ \Gamma^{(k)} = D \left\{ \left[ 1, e^{j2\pi n \epsilon_{(k)}}, \ldots, e^{j2\pi n (N-1) \epsilon_{(k)}} \right]^T \right\} \]  

(4.27)

\( H^{(k)} \) is an \( N \times N \) circulant channel matrix whose first row is given by \( [h_0^{(k)}, 0, \ldots, h_{L_h-1}^{(k)}, \ldots, h_1^{(k)}] \), \( F \) is an \( N \times N \) Fourier transform matrix with \( F_{n,m} = (N)^{-1/2} \exp\left(-j2\pi nm/N\right) \) and \( 0 \leq n, m \leq N-1 \). Moreover, \( \bar{v}_n \) is the noise vector and \( \dagger \) denotes the matrix Hermitian transpose.

The BLUE approach as in (4.13) and (4.14) can be followed by first calculating the autocorrelation of \( \bar{r}_n \)

\[ C_t = \frac{1}{N_L-1} \sum_{n=1}^{N_L-1} \bar{r}_n \bar{r}_{n-i} \]  

(4.28)

where \( C_t \) is an \( N \times N \) autocorrelation matrix and \( \hat{\eta}_t \) is the corresponding noise matrix.

In the next step, the idea of OTS presented in [63] for the purpose of multichannel estimation can be utilised, by defining

\[ t^{(k)} = \left[ 1, e^{-j2\pi L_t k/N}, \ldots, e^{-j2\pi L_t (N-1) k/N} \right]^T \]  

(4.29)

so that the vector \( \alpha^{(k)} \) becomes

\[ \alpha^{(k)} = \Gamma^{(k)} \bar{\alpha}^{(k)} \left[ \left( h^{(k)} \right)^T 0_{N-L_h} \right]^T \]  

(4.30)

where \( 0_L \) stands for an \( L \times 1 \) vector with all elements being zero, \( L_t \geq L_h \) for the offset and \( \bar{P} \) for an \( N \times N \) shifting matrix, given by

\[ \bar{P} = \begin{bmatrix} 0_{N-1}^T & 0 \\ I_{N-1} & 0_{N-1} \end{bmatrix} \]  

(4.31)
The $\mathbf{I}_{N-1}$ for an $(N-1) \times (N-1)$ identity matrix. By exploitation of $\mathbf{t}^{(k)}$ defined in (4.29), one can easily justify the following result:

$$\text{Tr} \left\{ \mathbf{\alpha}^{(k_1)} \left( \mathbf{\alpha}^{(k_2)} \right)^* \right\} = \begin{cases} 0, & k_1 \neq k_2 \\ \| \mathbf{\alpha}^{(k_1)} \|_2^2, & k_1 = k_2 \end{cases}$$

(4.32)

where $\text{Tr} \{ \cdot \}$ stands for the sum of diagonal elements and $\| \cdot \|$ is the Euclidean norm of the enclosed vector. Eqn (4.32) indicates that the asynchronous users are orthogonal in $\alpha$-space and hence the CFO for each user can be estimated individually by employing

$$\mathbf{\xi}^{(k)} = \frac{N}{2\pi N_T} \sum_{i=1}^{N} \epsilon_i \left( \text{Tr} \left\{ \mathbf{Q}^{(k_i)} \mathbf{C}_i \right\} \text{Tr} \left\{ \mathbf{Q}^{(k_i)^*} \mathbf{C}_{i-1}^{*} \right\} \right)$$

(4.33)

where $\mathbf{C}_i \triangleq \mathbf{F}^H \mathbf{C}_i \mathbf{F}$ and $\mathbf{Q}^{(k)}$ is an $N \times N$ diagonal matrix used to extract the the desired user's autocorrelation matrix. Its mathematical expression is given by

$$\mathbf{Q}^{(k)} = \mathbf{D}^{k_{Lt}} \begin{bmatrix} \mathbf{I}_{Lt} & \mathbf{0}_{Lt \times (N-Lt)} \\ \mathbf{0}_{(N-Lt) \times Lt} & \mathbf{0}_{(N-Lt) \times (N-Lt)} \end{bmatrix}$$

(4.34)

Following the MMSE analysis in [47], the MMSE for the $k^{th}$ user's estimated CFO value is given by

$$\text{MMSE} \left( \mathbf{\xi}^{(k)} \right) = \frac{3N^2 L_t \sigma^2}{2\pi^2 \bar{N} \left( \bar{N}^2 - 1 \right) \text{Tr} \left\{ \mathbf{\alpha}^{(k)} \| \mathbf{\alpha}^{(k)} \|_2^2 \right\}$$

(4.35)

The OTS-BLUE approach requires the number of asynchronous users to be no larger than $\lfloor N/L_t \rfloor$, otherwise the user orthogonality would not hold (see [63]). In what follows two schemes are proposed to overcome the factors that can damage the OTS orthogonality.

4.3.2 Cases in The Presence of Synchronous Users

The OTS-Design requires the training sequences to spread over all subcarriers (see (4.29)). However, in the presence of synchronous users, data symbols belonging to synchronous users would be superimposed with training symbols belonging to asynchronous users. In this scenario, the orthogonality loss occurs between training sequences.

The matrix form of $\mathbf{r}_n$ can be expressed as

$$\mathbf{r}_n = \sum_{k \in \Psi} \mathbf{W} \mathbf{F}^{(k)} e^{j2\pi (k \tau_0 + N_k)n/N} + \mathbf{v}_n$$

(4.36)
where $\mathbf{W}$ is an $N \times N$ diagonal filtering matrix to null out the synchronous users. Its $i^{th}$ diagonal element is one if the corresponding subcarrier belongs to an asynchronous user. In this case, the autocorrelation of $\mathbf{r}_n$ becomes

$$
C_i = \frac{1}{N-\hat{T}} \sum_{n=1}^{N-\hat{T}} \sum_{k_1 \in \Psi} \sum_{k_2 \in \Psi} \mathbf{W} \mathbf{F} \mathbf{\alpha}^{(k_1)} \left( \mathbf{\alpha}^{(k_2)} \right)^\mathbf{H} \mathbf{F}^H \mathbf{W}^H
$$

where $\mathbf{\eta}_i$ is the corresponding noise matrix. By defining a matrix $\mathbf{Q} = \sum_{k \in \Psi} \mathbf{Q}^{(k)}$, one can easily justify $\mathbf{Q} \mathbf{\alpha}^{(k)} = \mathbf{\alpha}^{(k)}$. Therefore (4.37) can be rewritten as

$$
C_i = \frac{1}{N-\hat{T}} \sum_{n=1}^{N-\hat{T}} \sum_{k_1 \in \Psi} \sum_{k_2 \in \Psi} \mathbf{W} \mathbf{F} \mathbf{Q} \mathbf{\alpha}^{(k_1)} \left( \mathbf{\alpha}^{(k_2)} \right)^\mathbf{H} \mathbf{Q}^H \mathbf{F}^H \mathbf{W}^H
$$

First, a matrix $\mathbf{G} = \mathbf{W} \mathbf{F} \mathbf{Q}$ is defined whose rank is given by $\text{Rank}(\mathbf{G}) = \min (\text{Rank}(\mathbf{W}), \text{Rank}(\mathbf{Q}))$. Suppose $\text{Rank}(\mathbf{G}) = \text{Rank}(\mathbf{Q})$, then the following zero-forcing operation on $\mathbf{C}_i$ can be performed

$$
\mathbf{C}_i = \mathbf{G}^\dagger \mathbf{C}_i \left( \mathbf{G}^\dagger \right)^\mathbf{H}
$$

$$
= \frac{1}{N-\hat{T}} \sum_{n=1}^{N-\hat{T}} \sum_{k_1 \in \Psi} \sum_{k_2 \in \Psi} \mathbf{\alpha}^{(k_1)} \left( \mathbf{\alpha}^{(k_2)} \right)^\mathbf{H} \mathbf{e}^{-j2\pi \left( (\mathbf{\epsilon}^{(k_1)}, \mathbf{\epsilon}^{(k_2)}) \mathbf{n} + N_\mathbf{F} + (\mathbf{\epsilon}^{(k_2)}, N_\mathbf{F}) \mathbf{n} \right) / N} + \mathbf{\eta}_i
$$

where $\mathbf{\alpha}^{(k)} = \mathbf{G}^\dagger \mathbf{G} \mathbf{\alpha}^{(k)}$ with the following property

$$
\text{Tr} \left\{ \mathbf{\alpha}^{(k_1)} \left( \mathbf{\alpha}^{(k_2)} \right)^\mathbf{H} \right\} = \text{Tr} \left\{ \mathbf{\alpha}^{(k_1)} \left( \mathbf{\alpha}^{(k_2)} \right)^\mathbf{H} \right\},
$$

$\mathbf{\eta}_i$ is the corresponding noise and $^\dagger$ stands for pseudo-inverse. Due to the property (4.40), by replacing $\mathbf{C}_i$ with $\mathbf{\hat{C}}_i$ in (4.33), CFO for each asynchronous user can be estimated individually.

Remark 1: The zero-forcing OTS-BLUE requires the condition $\text{Rank}(\mathbf{G}) = \text{Rank}(\mathbf{Q})$ or more precisely $L_\mathbf{L} \leq L$. Otherwise CFO identifiability cannot be guaranteed.
4.3. The OTS-BLUE Approach

Remark 2: The MMSE performance of the $k^{th}$ user can be calculated by the following analysis in [47],

$$\text{MMSE} \left( e^{(k)} \right) = \frac{3N^2c^2\text{Tr}\left\{ \left( G^*G \right)^T \right\}}{2\pi^2 N \left( N^2 - 1 \right) N \alpha^{(k)} ||\alpha^{(k)}||^2}.$$  \hspace{1cm} (4.41)

In [68], it is shown that $\text{Tr}\left\{ \left( G^*G \right)^T \right\}$ is related to the singular-value distribution of $G^*G$. It reaches its minimum value, i.e. $\min \left( \text{Tr}\left\{ \left( G^*G \right)^T \right\} \right) = L$ when $G^*G$ is a diagonal matrix. However this condition requires those “1” elements to be uniformly distributed on the diagonal of $W$ and $Q$, which is not always the case in OFDMA systems. Moreover, $\text{Tr}\left\{ \left( G^*G \right)^T \right\}$ will be considerably large for some cases where those singular-values have significant difference. This is called the “singularity” problem in the OTS-BLUE approach. As a complement, two other training designs i.e. TDTC and SCTS will be proposed in the following sections which, in the presence of multiple synchronous and asynchronous users, offer reliable CFO estimation performance.

4.3.3 Cases in The Presence of Propagation Delay

Propagation delay is another factor that can damage the orthogonality of OTS. By incorporating the propagation delay $\mu^{(k)}$ (whose unit is assumed to be in samples), the frequency domain received signal $r_n$ can be expressed as

$$r_n = \sum_{k \in \Psi} WU^{(k)}F \alpha^{(k)} e^{\frac{j2\pi k n}{N}} + \eta_n$$ \hspace{1cm} (4.42)

where $U^{(k)} = \text{Diag}\left\{ 1, e^{\frac{j2\pi k}{N}}, \ldots, e^{\frac{j2\pi k (N-1)}{N}} \right\}$. In this case, the autocorrelation matrix $C_i$ in (4.37) becomes

$$C_i = \frac{1}{\hat{N} - \hat{\mu}} \sum_{n=0}^{\hat{N} - 1} \sum_{k_1 \in \Psi} \sum_{k_2 \in \Psi} WU^{(k_1)} F \alpha^{(k_1)} \alpha^{(k_2)} \left( U^{(k_2)} \right)^* W^* e^{\frac{j2\pi}{N} (k_1 - k_2) n (N + \mu)} + \hat{\eta}_i$$ \hspace{1cm} (4.43)

where $\hat{\eta}_i$ is the corresponding noise. Utilizing the Fourier decomposition

$$F^*U^{(k)}F = \left( F^T \right)^{\mu^{(k)}}$$ \hspace{1cm} (4.44)
(4.38) can be rewritten as
\[
C_i = \frac{1}{N - i} \sum_{n=i}^{N-1} \sum_{k_1 \in \mathcal{K}} \sum_{k_2 \in \mathcal{K}} \frac{\mathbf{W}_F \left( \mathbf{P}^T \right)^{\mu(k_1)} \alpha(k_1) \alpha(k_2) \mathbf{H} \left( \left( \mathbf{P}^T \right)^{\mu(k_2)} \right)^H \mathbf{F}^H \mathbf{W}^H}{e^{-j2\pi \left( \sum_{n=1}^{N_1} - \sum_{n=N_1}^{N_2} \right) \rho_i + \rho_2 + \rho_3}} + \tilde{\mathbf{n}}_i
\]
(4.45)
where \( \tilde{\mathbf{n}}_i \) is the corresponding noise matrix. One can easily justify
\[
\text{Tr} \left\{ \left( \mathbf{P}^T \right)^{\mu(k_1)} \alpha(k_1) \alpha(k_2) \mathbf{H} \left( \left( \mathbf{P}^T \right)^{\mu(k_2)} \right)^H \right\} = \text{Tr} \left\{ \alpha(k_1) \alpha(k_2) \mathbf{H} \right\}
\]
subject to two conditions, \( \mu(k_1) = \mu(k_2) \) and \( \mu(k) \leq L_t - L_n, \forall k \). In other words, the orthogonality of OTS is not affected under those conditions. However, in practice, the propagation delay may be larger than \( L_t - L_n \) and also \( \mu(k_1) \neq \mu(k_2) \), so that the estimator in (4.33) cannot offer a reliable CFO estimation performance. In what follows two delay-awareness approaches are proposed to improve the reliability of OTS-BLUE with two conditions: C1) \( 1 \leq \mu(k) \leq L_t - \Delta \) and C2) \( L_t \leq (LK_o - \Delta) \) where \( \Delta \) is the offset with \( 1 \leq \Delta \leq L_n - 1 \).

Subspace-Based CFO Estimation

The main idea of this approach is depicted in Fig. 4.1. As mentioned earlier, \( L_n \) is the maximum channel length of all users and \( L_t \) is a design parameter and its value is chosen in a way to keep the orthogonality of users in the time-domain. Without the propagation delay, users are orthogonal in the \( \alpha \)-space. \( \Delta \) indicates the statistical knowledge of the propagation delay for all users and \( L_t - \Delta \) is the upper-bound for the propagation delay of any user. Therefore, in the presence of propagation delay, the \( (k-1)^{th} \) user has at most \( L_t - \Delta \) components leaking to the \( \alpha(k) \)-subspace (see 4.24 for the definition of \( \alpha \)) where \( L_t - L_n \) components are zero. Therefore there are at least \( L_t - L_n + \Delta \) components in \( \alpha(k) \) that are not interfered by the \( (k-1)^{th} \) user which enable the OTS-BLUE approach. Assume to have the knowledge about \( \Delta \), the CFO estimation is performed as follows:

S1) Perform the autocorrelation and zero-forcing operation as in (4.38),(4.39).
4.3. The OSTS-BLUE Approach

Figure 4.1: Illustration of the $\alpha$-space with and without propagation-delay.

S2) Form a diagonal matrix

$$Q^{(b)} = F^{kL_t} \begin{bmatrix} 0_{(L_h-\Delta)\times(L_h-\Delta)} & 0_{(L_h-\Delta)\times(L_t-L_h+\Delta)} & 0_{L_t\times(N-L_t)} \\ 0_{(L_t-L_h+\Delta)\times(L_h-\Delta)} & I_{(L_t-L_h+\Delta)\times(L_t-L_h+\Delta)} & 0_{(N-L_t)\times(N-L_t)} \\ 0_{(N-L_t)\times L_t} & 0_{(N-L_t)\times(N-L_t)} & 0_{(N-L_t)\times(N-L_t)} \end{bmatrix}$$ (4.47)

to extract those non-interfered components from the $\alpha^{(k)}$-subspace for CFO estimation, i.e.

$$\hat{\epsilon}^{(k)} = \frac{N}{2\pi N_T} \sum_{t=1}^{N} w_t < \text{Tr} \left\{ Q^{(k)} \tilde{C}_t \right\} \text{Tr} \left\{ Q^{(k)} \tilde{C}^{*}_{t-1} \right\} > ,$$ (4.48)

The performance and reliability of this CFO estimation is related to the offset $\Delta$ corresponding to maximum propagation delay. Detailed performance and reliability analysis of this estimator will be provided in Section 4.6.
4.4. The TDTC-BLUE Approach

Location-Aided CFO Estimation

The main idea of this approach is based on an assumption that the uplink receiver knows the geographical location of each user. This assumption is reasonable for a communication network assisted by advanced positioning function [69],[70]. In this case the uplink receiver can estimate the propagation delay for the $k^{th}$ user by employing

$$\hat{\mu}^{(k)} = \left\lfloor \frac{\hat{d}^{(k)}}{cT_s} \right\rfloor$$

where $\hat{d}^{(k)}$ denotes the distance between the $k^{th}$ user and the uplink receiver, $c$ is the light speed in the free space and $\lfloor \cdot \rfloor$ stands for integer ceiling. The estimate $\hat{\mu}^{(k)}$ would be more accurate if a line-of-sight (LOS) exists between the $k^{th}$ user and the uplink receiver. When $\hat{\mu}^{(k-1)}$ is obtained, by replacing $\Delta$ in (4.47) with $\Delta^{(k)} = L - \hat{\mu}^{(k-1)}$, CFO estimation can be carried out via (4.48).

4.4 The TDTC-BLUE Approach

The main idea behind this approach is to exploit time-domain cyclostationarity of the received training blocks that is induced as follows. Asynchronous users, with the user-index set $\Psi = \{k_1, k_2, \ldots, k_{\mathcal{N}}\}$, spread their training symbols over the whole bandwidth in the same way as explained in Section 4.3.1. Whereas, synchronous users, with the user-index set $\bar{\Psi} = \{\bar{k}_1, \bar{k}_2, \ldots, \bar{k}_{\mathcal{N}}\}$, transmit their information-bearing symbols in interleaved fashion. In this way, each synchronous user shows a periodic structure, with the period $L$, in the received time-domain signal. By assuming there are $K = \mathcal{N} + \mathcal{N}$ intervals each with $L$ samples, those intervals that correspond to the elements of $\bar{\Psi}$ only contain the synchronous users. By measuring the synchronous data on those intervals and removing them from all time-samples, the BLUE CFO estimation can then be performed on the intervals that correspond to the elements of $\Psi$ for each asynchronous user individually. To be able to implement the proposed TDTC-BLUE approach, the following steps are required.
4.4. The TDTC-BLUE Approach

S1) Starting with the matrix form of the received time-domain signal

\[ r_n = \sum_{k \in \mathcal{V}} \alpha^{(k)} e^{j2\pi \left( e^{(k)} + n^{(k)} \right)(n,N_T+N_d)} \frac{1}{N} \]

\[ + \sum_{k \in \mathcal{V}} \Gamma^{(k)} H^{(k)} F \mathcal{A}^{(k)} e^{j2\pi \left( e^{(k)} + n^{(k)} \right)(n,N_T+N_d)} \frac{1}{N} + v_n \]  

(4.50)

where \( v_n \) is the time-domain noise vector and \( \mathcal{A}^{(k)} \) is an \( N \times 1 \) vector of interleaved information-bearing symbols belonging to \( k^{th} \) synchronous user. Its mathematical expression is given by

\[ \mathcal{A}^{(k)} = \mathcal{e}_n^{(k)} \otimes \mathcal{I}_K(k) \]

(4.51)

where \( \mathcal{I}_K(k) \) denotes the \( k^{th} \) column of \( \mathcal{I}_K \) and \( \otimes \) represents the Kronecker multiplication.

S2) Define

\[ \tilde{y}^{(k)} = \mathcal{Q}_2^{(k)} \mathcal{Q}_1^{(k)} r_n , \forall k \in \mathcal{V} \]

(4.52)

where \( \tilde{y}^{(k)} \) is an \( L \times 1 \) vector, \( \mathcal{Q}_1^{(k)} \) is an \( N \times N \) matrix given by

\[ \mathcal{Q}_1^{(k)} = \mathcal{P}^{kL} \begin{bmatrix} I_L & 0_{L \times (N-L)} \\ 0_{(N-L) \times L} & 0_{(N-L) \times (N-L)} \end{bmatrix} \]

(4.53)

and \( \mathcal{Q}_2^{(k)} \) is an \( L \times N \) matrix formed from those rows of \( \mathcal{I}_N \) that correspond to the elements of \( \mathcal{A}^{(k)} \).

S3) Define a phase-shifting matrix \( \mathcal{A} \) with a size of \( \mathcal{N}L \times \mathcal{N}L \), expressed as

\[ \mathcal{A} = \begin{bmatrix} A_{k_1,c_1} & A_{k_2,c_1} & \cdots & A_{k_{\mathcal{N}},c_1} \\
A_{k_1,c_2} & A_{k_2,c_2} & \cdots & A_{k_{\mathcal{N}},c_2} \\
\vdots \\
A_{k_1,c_{\mathcal{N}}} & A_{k_2,c_{\mathcal{N}}} & \cdots & A_{k_{\mathcal{N}},c_{\mathcal{N}}} \end{bmatrix} \]

(4.54)

where \( k_i \) is the \( i^{th} \) synchronous user index, and \( k_1 \leq c_i \leq k_{\mathcal{N}} \) represents the \( i^{th} \) period of corresponding synchronous user. The sub-matrices forming up the matrix \( \mathcal{A} \) are defined as

\[ A_{k_i,c_i} = \mathcal{D} \left( e^{j2\pi k_i(n-1)} \right) \begin{bmatrix} (c_i - 1)L + 1 \leq n \leq c_i L \\ 1 \leq c_i \leq K \end{bmatrix} \]

(4.55)
where $D(\cdot)$ represents a diagonal matrix of the enclosed vector and $A_{k_i, c_i}$ has a size of $L \times L$.

S4) Form a super-vector of received time-domain signal on the time-intervals that only contain synchronous users

$$
\hat{y}_{sup} = \begin{bmatrix}
\tilde{y}(k_1) \\
\tilde{y}(k_2) \\
\vdots \\
\tilde{y}(k_N)
\end{bmatrix}_{N_L \times 1}
$$

(4.56)

S5) Calculate the phase-removed vector through

$$
\hat{X} = \hat{A}^{-1}\hat{y}_{sup}
$$

(4.57)

where $\hat{X}$ is an $N_L \times 1$ vector.

S6) Define a secondary phase-shifting matrix $\hat{A}_{sec}$ with a size of $N_L \times N_L$ expressed as

$$
\hat{A}_{sec} = \begin{bmatrix}
A_{k_1, c_1} & A_{k_2, c_1} & \cdots & A_{k_N, c_1} \\
A_{k_1, c_2} & A_{k_2, c_2} & \cdots & A_{k_N, c_2} \\
\vdots & \vdots & \ddots & \vdots \\
A_{k_1, c_N} & A_{k_2, c_N} & \cdots & A_{k_N, c_N}
\end{bmatrix}
$$

(4.58)

where in (4.58), $k_1 \leq c_i \leq k_N$. In this way, $\hat{A}_{sec}$ contains the phase-shifting sub-matrices of synchronous users over the cycles of asynchronous users. In the next step $\hat{A}_{sec}$ can be used to give appropriate phase-shifts to the phase-removed periodic synchronous data $\hat{X}$ over the time-intervals belonging to asynchronous users.

S7) Find the super-imposition of all synchronous users on asynchronous users' intervals and remove it from the initial time-domain signal

$$
\hat{y} = \hat{y}_{sup} - \hat{A}_{sec}\hat{X}
$$

(4.59)
4.5 The SCTS-BLUE Approach

Having nullified the asynchronous users' time intervals from synchronous users' data, BLUE CFO estimation can now be performed in the same way as in (4.28) and (4.33) on the signal \( \tilde{y} \).

Remark 3: In the TDTC-BLUE approach, it is assumed that \( L_t > L + \mu \) where \( |\mu| \triangleq \max \{ |\mu^{(k_1)}|, |\mu^{(k_2)}|, \ldots, |\mu^{(k_N)}| \} \). In this way asynchronous users' symbols are kept in their own time-intervals and hence will not give rise to imprecise measurement of synchronous users' data.

Remark 4: To be able to remove the phase-shifts from synchronous users' data in (4.35) and hence fully remove synchronous data from asynchronous time-intervals, \( \hat{A} \) has to be a full-rank matrix. As it is observable from (4.55), the rank of \( \hat{A} \) depends on the position of synchronous users on the band-width. For example, by defining a set of asynchronous users indexes as \( \xi_A \triangleq [1, 1, 0, 0] \), which indicates 2nd and 3rd users are synchronous whereas 0th and 1st are asynchronous, (4.55) will cause \( \hat{A} \) to be a full-rank matrix. However by defining \( \xi_A \triangleq [0, 1, 0, 1] \), \( \hat{A} \) is not of full-rank any more and hence synchronous data cannot be fully removed from asynchronous users' time intervals. In what follows, another training pattern design is introduced that is more robust to MUI irrespective of the position of asynchronous users.

4.5 The SCTS-BLUE Approach

The ICI cancellation feature of this approach is due to the training pattern design. For the training pattern design, there are two primary assumptions: A1) subcarriers within each asynchronous user-band are clustered into \( L_c \) groups where each group consists of two or three adjacent subcarriers and A2) within each cluster, the channel fades are very close. In what follows, based on the number of consecutive subcarriers that are
clustered, two training schemes are proposed. In the first training pattern i.e. SCTS-I, each cluster consists of two adjacent subcarriers, whereas in the second training pattern i.e. SCTS-II, there are three adjacent subcarriers within each cluster.

4.5.1 SCTS-I

The training symbols within $i^{th}$ cluster belonging to $k^{th}$ user ($k \in \Psi$) obey the following criterion

$$I_{2l}^{(k)} = -I_{2l+1}^{(k)}.$$  

The inter-AU-interference can be calculated by applying A1) and A2) as well as (4.8) and (4.21) to obtain

$$I_{mN+m} = \sum_{k \in \Psi, k \neq k_0} e^{\frac{j2\pi m(k)(mN_1+N_2)}{N}} \sum_{l=0}^{L-1} e^{\frac{j2\pi l(N_2)}{N}} I_{2l}^{(k)} I_{2l+m}^{(k)}$$

where

$$I_{2l}^{(k)} = g_{2l-m} e^{\frac{j2\pi l(k)}{N}} g_{2l+1-m}.$$  

The training scheme proposed in (4.61) is not new. It was initially proposed by Zhao and Haggman (see [71]) for the purpose of ICI self-canceling data-transmission in AWGN downlink channel. One of the major results stated in [71] is

$$\left|\frac{g_{2l-m} - g_{2l+1-m}}{g_{2l-m}}\right|^2 \approx 10^{-2}.$$  

In this context, the above idea of training pattern design is borrowed for the purpose of CFO estimation.

To evaluate the inter-AU-interference canceling capability of the training pattern (4.61), a baseline is required to be set, i.e. the training symbol is pseudo-random and statistically independent with zero mean and constant amplitude $\sigma_l$. For the general case,

1The pseudo randomness of the training symbol is a reasonable assumption as it is used in many practical communication systems. The receiver has the full knowledge of the training symbols and can easily make the training symbols within a cluster to have same amplitude but different signs by adjusting the phase.
4.5. The SCTS-BLUE Approach

i.e. without exploitation of training scheme in (4.61), the inter-AU-interference power on the \( m \)th subcarrier is calculated through

\[
\mathcal{E} \{ |Z_{n,m+n}|^2 \} = \sigma_t^2 \sum_{k \in \Psi, k \neq k_0} \sum_{\ell \in \Lambda(k)} |h_{\ell}^{(k)}|^2 |g_{\ell-m}|^2
\]

\[\approx 2 \sigma_t^2 \sum_{k \in \Psi, k \neq k_0} \sum_{l=0}^{L_{\ell}-1} |h_{2l}^{(k)}|^2 |g_{2l-m}|^2 \tag{4.65}\]

where \( \mathcal{E} \{ \cdot \} \) stands for expectation. Equation (4.66) is based on the assumption A(2) and also \( |g_{2l-m}| \approx |g_{2l+1-m}| \) (see [71]). Using the training pattern (4.61), the training symbols within a cluster are fully correlated whereas the cross-cluster correlation is zero. Therefore, the inter-AU-interference for (4.62) can be expressed as

\[
\mathcal{E} \{ |Z_{n,m+n}|^2 \} = \sigma_t^2 \sum_{k \in \Psi, k \neq k_0} \sum_{\ell=0}^{L_{\ell}-1} |h_{2l}^{(k)}|^2 |\tilde{g}_l|^2. \tag{4.67}
\]

By assuming the timing offsets are zeros for all AUs, i.e. \( \mu(k) = 0, k \in \Psi \), (4.63) leads to \( \tilde{g}_l = g_2l-m - g_{2l+1-m} \). Therefore (4.64) can be used to compute the ratio

\[
\frac{\text{(4.67)}}{\text{(4.66)}} \approx 5 \times 10^{-3}. \tag{4.68}
\]

The ratio in (4.68) shows the significant interference cancellation capability of this training pattern. However, in the presence of timing offsets, it holds

\[
|\tilde{g}_l| = \left| g_{2l-m} - g_{2l+1-m} + g_{2l+1-m} \left( 1 - e^{-\frac{j2\pi \mu(k)}{N}} \right) \right| \tag{4.69}
\]

\[
\leq \left| g_{2l-m} - g_{2l+1-m} \right| + \left| g_{2l+1-m} \right| \left| 1 - e^{-\frac{j2\pi \mu(k)}{N}} \right| \tag{4.70}
\]

\[
\approx \left| g_{2l-m} \right| \left( 0.1 + \left| 1 - e^{-\frac{j2\pi \mu(k)}{N}} \right| \right) \tag{4.71}
\]

where (4.71) is derived using (4.64). It is observed that \( \nu \mu(k) \) is a monotonically increasing function for \( \mu(k) > 0 \). (4.67) is upper-bounded by

\[
\mathcal{E} \{ |Z_{n,m+n}|^2 \} \leq \nu \mu(k) \sigma_t^2 \sum_{k \in \Psi, k \neq k_0} \sum_{\ell=0}^{L_{\ell}-1} |h_{2l}^{(k)}|^2 |g_{2l-m}|^2. \tag{4.72}
\]

Therefore, in the presence of timing offsets, the ratio in (4.68) is upper-bounded by

\[
\frac{\text{(4.67)}}{\text{(4.66)}} \leq \frac{\nu \mu(k)}{2}. \tag{4.73}
\]
4.5. The SCTS-BLUE Approach

For example, by having $\bar{\mu}/N = 1/64$, the above ratio is upper-bounded by

$$\frac{(4.67)}{(4.66)} \leq 0.0196. \quad (4.74)$$

By comparing the ratios in (4.68) and (4.74), it is observed that the timing offset can affect the interference canceling capability of the proposed training scheme. However, some performance improvement can still be observed for small timing offsets such as $\bar{\mu}/N = 1/64$.

4.5.2 SCTS-II

In this training scheme, every three adjacent subcarriers are clustered by obeying

$$\frac{\sqrt{2}}{2} a_{3l}^{(k)} = -\sqrt{2} a_{3l+1}^{(k)} = \frac{\sqrt{2}}{2} a_{3l+2}^{(k)}. \quad (4.75)$$

By performing similar analysis as in (4.62), the inter-AU-interference term is calculated through

$$I_{nN+m} = \sum_{k \in \Psi, k \neq k_o} e^{j2\pi m(k)(nN+2N)} \sum_{l=0}^{L-1} e^{j2\pi l} \bar{a}^{(k)}_{3l} \tilde{g}_l. \quad (4.76)$$

where

$$\tilde{g}_l = \frac{\sqrt{2}}{2} g_{3l-m} - \sqrt{2} e^{j2\pi m(k)/N} g_{3l+1-m} + \frac{\sqrt{2}}{2} e^{j2\pi l} g_{3l+2-m}. \quad (4.77)$$

Another important result that is given in [71] is the following upper-bound

$$\left| \frac{\frac{\sqrt{2}}{2} g_{3l-m} - \sqrt{2} g_{3l+1-m} + \frac{\sqrt{2}}{2} g_{3l+2-m}}{\frac{\sqrt{2}}{2} g_{3l-m}} \right|^2 \leq 10^{-3}. \quad (4.78)$$

For the general case, i.e. without the training pattern in (4.75), the inter-AU-interference power on the $m^{th}$ subcarrier can be expressed as

$$\mathcal{E} \{ |I_{nN+m}|^2 \} \approx 3 \sigma_i^2 \sum_{k \in \Psi, k \neq k_o} \sum_{l=0}^{L-1} \left| h_{3l}^{(k)} \right|^2 \left| g_{3l-m} \right|^2 \quad (4.79)$$

where in (4.79) it is assumed the channel fadings within each cluster are close and also $|g_{3l-m}| \approx |g_{3l+1-m}| \approx |g_{3l+2-m}|$ (see [71]). The inter-AU-interference for the case of training pattern in (4.75) is calculated by

$$\mathcal{E} \{ |I_{nN+m}|^2 \} = \sigma_i^2 \sum_{k \in \Psi, k \neq k_o} \sum_{l=0}^{L-1} \left| \bar{a}^{(k)}_{3l} \right|^2 \left| \tilde{g}_l \right|^2. \quad (4.80)$$
4.5. The SCTS-BLUE Approach

By assuming perfect timing synchronisation for all asynchronous users, the interference canceling capability of the proposed training scheme can be evaluated by calculating the following ratio (see (4.78))

\[
\frac{(4.80)}{(4.79)} \leq 1.667 \times 10^{-4}
\]  

(4.81)

It is observed that the training pattern that consists of weighted combination of every three consecutive subcarriers shows stronger interference canceling capability than the one consists of two, introduced in section 4.5.1. When timing offsets are not zero, \( |\tilde{g}_l| \) can be represented as

\[
|\tilde{g}_l| = \left| \frac{\sqrt{2}}{2} g_{3l-m} - \sqrt{2} g_{3l+1-m} + \sqrt{2} g_{3l+1-m} \left( 1 - e^{\frac{j2\pi n(l)}{N}} \right) 
+ \frac{\sqrt{2}}{2} g_{3l+2-m} - \sqrt{2} g_{3l+2-m} \left( 1 - e^{\frac{j2\pi n(l)}{N}} \right) \right|
\]  

(4.82)

\[
\leq \left| \frac{\sqrt{2}}{2} g_{3l-m} - \sqrt{2} g_{3l+1-m} + \frac{\sqrt{2}}{2} g_{3l+2-m} \right|
\approx 10^{-3/2} \times \frac{\sqrt{2}}{2} |g_{3l-m}|
\]  

(4.83)

As it was the case for \( \nu (\mu(k)) \), \( \bar{\nu} (\mu(k)) \) is also a monotonically increasing function for \( \mu(k) > 0 \). Equation (4.80) is upper-bounded by

\[
\mathcal{E} \{ |I_{tN+m}|^2 \} \leq \bar{\nu}^2 (\bar{\mu}) \sigma^2 \sum_{k \in K, k \neq k_0} \sum_{l=0}^{L_c-1} \left| h_{3l}^{(k)} \right|^2 |g_{3l-m}|^2.
\]  

(4.85)

Therefore,

\[
\frac{(4.80)}{(4.79)} \leq \frac{\bar{\nu}^2 (\bar{\mu})}{3}.
\]  

(4.86)

By applying an example of \( \bar{\mu} / N = 1/64 \), (4.86) leads to

\[
\frac{(4.80)}{(4.79)} \leq 0.0159.
\]  

(4.87)
By comparing (4.87) and (4.74), it can be observed that SCTS-II outperforms SCTS-I in terms of the robustness to small timing offsets.

Remark 5: It is not difficult to see that SCTS-I and SCTS-II are special cases of the following general form

$$t^D = \beta t^{P-1} \otimes [1 - 1]^T$$

where $\beta$ is a scaling factor, the superscript $D$ is for the order of Kronecker product, and $t^0 = [1 - 1]^T$. For example, SCTS-I is the case of $t^0$, and SCTS-II the case of $t^1$ ($\beta = (\sqrt{2})/(2)$). In practice, the assumption $A2$ does not hold for a large $D$ ($> 2$), therefore analysing the interference-canceling capability of the general form is very challenging. Fortunately, the simulation results provided in Sec. 4.6 show that SCTS-I and SCTS-II can offer satisfactory performance for the CFO estimation.

4.6 Simulation Results

Computer simulations were used to evaluate the performance of proposed CFO estimator in terms of MSE, i.e. $\mathbb{E}\{|e(k) - \epsilon^{(k)}|^2\}$ where it is assumed $k = 2$, and bit-error-rate (BER). An OFDMA system is considered with $N = 64$ subcarriers and a CP length $N_g = 8$. The information-bearing symbols are randomly drawn from a quadrature-phase-shift-keying (QPSK) set with the equal-probability. Unless otherwise specified, there are $K = 4$ active users in the system and each one is assigned $L = 16$ subcarriers. All users' CFOs are random values uniformly distributed in $[-0.4, 0.4]$. Two scenarios are considered here for the propagation delay $\mu$. In the first scenario, minimum propagation delay for all asynchronous users is considered where $\mu_{\text{min}}$ is an $N \times 1$ vector whose entries are random values, uniformly distributed in $[1 - L_t - \Delta_{\text{max}}]$. In the second scenario, moderate propagation delays that are random values with uniform distribution in $[1 - L_t - \Delta_{\text{mod}}]$ is considered where $\Delta_{\text{mod}}$ corresponds to $\Delta_{\text{mod}} = [\Delta_{\text{max}} + \Delta_{\text{max}}/2]$. The communication channel is frequency-selective slow Rayleigh fading with the power delay profile $\mathbb{E}\{|h(\ell)|^2\} = e^{-\ell/\alpha} / \sum_{\ell=0}^{\infty} e^{-\ell/\alpha}$, $\ell = 0, ..., 4$, which is widely used in the literature (e.g. [37]). Throughout the simulations, it is assumed $L_t = L_h$. The results were obtained by averaging over 10,000 independently generated multiuser channel realiza-
4.6. Simulation Results

tions. Provided perfect channel state information (CSI) at the receiver\(^2\); the linear MMSE method is used for channel equalization. Throughout the simulations, each AU transmits 4 training blocks at the beginning of each frame for the purpose of CFO estimation. By exploiting the information about CFO estimates on downlink feedback channel, a time-domain pre-compensation is performed for each user separately. For the proposed approaches, all MSE and BER results are corresponding to the multiuser scenario. The baseline of comparison for CFO estimation is the single-user performance of BLUE approach proposed by Morelli [6] where optimum pilot sequence is employed (see (4.29)).

Fig. 4.2 illustrates the CFO estimation performance of the proposed OTS scheme in optimistic scenario with minimum propagation-delay. It is observed that, in the absence of propagation-delay, the OTS scheme offers identical performance to the baseline. This is because in this case users' data do not undergo MUI in the \(\alpha\)-space (see Fig. 4.1). By assuming all users experience minimum propagation-delay, it is evident that the subspace-based approach fully mitigates the impact of minimum propagation-delay. Similarly, by exploitation of location-aided approach that generates exact, i.e. error-free, estimation of propagation-delays, the impact of propagation-delays in CFO estimation is fully mitigated. However, the OTS scheme shows sensitivity to the estimation-error of propagation-delay introduced by the location-aided approach. It can be observed that the existence of one-sample error or two-sample error deteriorate the CFO estimation performance quite considerably. However the performance is still comparable to the baseline.

Fig. 4.3 represents the performance of the OTS scheme in optimistic scenario where all users have undergone moderate propagation-delays. It can be observed that both subspace and location-aided (with no estimation error) approaches show very comparable performance to the baseline. However, the existence of propagation-delay estimation error in location-aided approach causes significant performance loss, particularly for the case of two-sample error. In this figure, as the users experience moderate propagation-delays, a larger number of samples are affected by the MUI in \(\alpha\)-space. Therefore,

\(^2\)This simulation condition allows us to examine the impact of CFO estimation error on the BER performance without the influence of channel estimation errors.
4.6. Simulation Results

Figure 4.2: Performance of the OTS in optimistic scenario with minimum propagation delay.

For mitigation of MUI, a larger number of samples are discarded. This would in turn influence the amplitude of autocorrelation matrix and hence the CFO estimates are not as accurate as the case in Fig. 4.2.

In Figs. 4.4 and 4.5 the performance of zero-forcing algorithm with different order of placement for asynchronous users is examined in two cases i.e., $K = 4$ and $K = 16$ where in both cases $L = 16$. As the distribution of 1 elements on the diagonal of the matrices $W$ and $Q$ is directly related to the order of placement of asynchronous users, the proposed zero-forcing algorithm does not show satisfactory performance when $\xi_A = [1,1,0,0]$ or $\xi_A = [1,1,1,1,0,0,0,0,0,0,0,0,0,0,0]$ due to non-uniform distribution of 1 elements. However when this distribution is more uniform, i.e., $\xi_A = [0,1,0,1]$ or $\xi_A = [0,0,0,1,0,0,1,0,0,1,0,0,1,0,0,1]$, comparable CFO estimation performance to the baseline is achievable. Moreover, as the matrix $G^T G$ has a more uniform singular-value distribution for $K = 16$ than $K = 4$, better CFO estimation performance is achieved for the former case.
4.6. Simulation Results

The performance of the TDTC approach with different order of asynchronous users is plotted in Figs. 4.6, 4.7, 4.8 and 4.9. As mentioned earlier (see Remark 4) the rank of matrix $\mathbf{A}$ is directly related to the structure of $\xi_A$. Fig. 4.6 and Fig. 4.7 verify that when $\mathbf{A}$ is a full-rank matrix, synchronous users' data can fully be removed from the asynchronous time-intervals and therefore CFO estimation can be accomplished without the interference of synchronous users. Moreover, it can be observed that the presence of moderate propagation delay has a more notable negative impact on the CFO estimation than that of minimum delay. Fig. 4.8 and Fig. 4.9 show that in the case of having $\xi_A = [0, 1, 0, 1]$, due to having a half-rank for the matrix $\tilde{\mathbf{A}}$ and hence the presence of residual synchronous data on the asynchronous time-intervals, CFO estimation performance is influenced that is evident by the appearance of error-floors.

Figure 4.3: Performance of the OTS in optimistic scenario with moderate propagation delay.
4.6. Simulation Results

Figure 4.4: Performance of Zero-Forcing approach with different order of placement for asynchronous users. Minimum propagation delay is considered.

Figure 4.5: Performance of Zero-Forcing approach with different asynchronous users' placement order. Moderate propagation delay is considered.
4.6. Simulation Results

Figure 4.6: Performance of the TDTC scheme with $\xi_A = [1, 1, 0, 0]$ and minimum propagation delay.

Figure 4.7: Performance of the TDTC scheme with $\xi_A = [1, 1, 0, 0]$ and moderate propagation delay.
4.6. Simulation Results

Figure 4.8: Performance of the TDTC scheme with $\xi_A = [0, 1, 0, 1]$ and minimum propagation delay.

Figure 4.9: Performance of the TDTC scheme with $\xi_A = [0, 1, 0, 1]$ and moderate propagation delay.
4.6. Simulation Results

Figs. 4.10 and 4.11 evaluate the performance of SCTS-I and SCTS-II training schemes in two cases. First, when all users are asynchronous and second, when half of the users are asynchronous. When all users are asynchronous, SCTS-II outperforms SCTS-I due to its higher interference-canceling capability as discussed in section 4.5.2. Moreover, as timing-offset introduces the same linear phase-shift on training-blocks, its impact has been cancelled out in the process of calculating autocorrelation matrix $C_i$ in the BLUE approach. When half of the users are asynchronous, the cases that correspond to $\xi_A = [0, 1, 0, 1]$ outperform the ones with $\xi_A = [1, 1, 0, 0]$. This is because in the former case the neighbouring users to the user of interest are synchronous.

Figure 4.10: Performance of SCTS-I and SCTS-II schemes where all users are asynchronous.

Fig. 4.12 represents the BER performance of the OTS in optimistic scenario and TDTC training scheme with $\xi_A = [1, 1, 0, 0]$. It can be observed that both training sequences offer very close performance to the CFO-free scenario for low and moderate SNRs (i.e., $< 20$ dB). However, their performance starts degrading for the high SNRs (i.e. $> 20$ dB) due to residual CFOs where 2 dB loss is observable at SNR= 30 dB.

Fig. 4.13 illustrates the BER performance of the TDTC training scheme with $\xi_A = $
4.6. Simulation Results

Figure 4.11: Performance of SCTS-I and SCTS-II schemes where half of the users are asynchronous.

Figure 4.12: BER Performance of the OTS in optimistic scenario and the TDTC scheme with $\xi_A = [1, 1, 0, 0]$. 
4.6. Simulation Results

It can be observed that, although in this case the matrix $\mathbf{A}$ is not of full-rank, the TDTC training scheme still offers very close performance to the CFO-free case for SNRs up to 20 dB. This is because its CFO estimation performance has met a certain accuracy level to assure reliable BER performance (see [27]). It can also be observed that, in the presence of moderate time-delays, the location-aided approach introduces an error floor where the estimation of time-delay is accomplished with two-sample error.

Finally in Fig. 4.14 the BER performance of SCTS training sequence is evaluated where only the results for the worst case are provided, i.e. all users are asynchronous. It is evident that the performance is very close to the CFO-free scenario for low and moderate SNRs (i.e. < 20 dB). For higher SNRs the impact of residual CFOs causes performance degradation, e.g. a 2 dB loss at SNR = 30 dB.

Remark 6: The simulations were also performed to examine the effect of an 8-tap frequency-selective Rayleigh fading channel as in C2-NLOS scenario (see [62]). It was observed that both MSE and BER results for an 8-tap channel are coincided with
4.6. Simulation Results

Figure 4.14: BER Performance of SCTS-I and SCTS-II schemes. All users are asynchronous.
the corresponding curves in case of a 5-tap channel subject to having extended CP to accommodate both channel image and timing-offset, i.e. an inter-symbol interference (ISI)-free scenario.

4.7 Summary

In this chapter a class of training sequence designs for robust BLUE CFO estimation in OFDMA uplink was presented. This work was motivated by the fact that the BLUE estimator that is employed for CFO estimation is very sensitive to the MUI. The major contribution of this chapter was to combine BLUE CFO estimator with three appropriately designed training schemes to offer a low-complex and sub-optimum CFO estimation approach in multiuser scenarios. The OTS was introduced as the first training design to be exploited with the BLUE estimator. Despite offering single-user performance in optimistic scenarios, the orthogonality of OTS could be destroyed by two factors, first the presence of synchronous users and second the propagation time-delay. To counteract, the first factor was resolved by the zero-forcing algorithm and the second one by employing two delay-awareness approaches. It was shown that the delay-awareness approaches can mitigate the propagation-delays impact on the CFO estimation subject to having sufficiently accurate knowledge of propagation-delays. On the other hand, it was shown that the zero-forcing approach does not always offer a promising performance due to the singularity problem. As a complement, two other training schemes were proposed. The TDTC scheme facilitated a feasible way to remove the synchronous data symbols from asynchronous time-intervals. However its performance was closely related to the order of asynchronous users on the bandwidth. The SCTS scheme showed a solid and reliable performance regardless of the order of asynchronous users and the presence of propagation-delays.
Iterative concatenated CFO estimation and compensation in OFDMA

5.1 Introduction

In this chapter, a new approach that performs joint carrier-frequency-offsets (CFOs) estimation and compensation at the base station (BS) is proposed. In this way, two important requirements are satisfied jointly, i.e. sufficiently accurate CFOs knowledge at the BS and also well-suppressed inter-carrier-interference (ICI) and multiuser interference (MUI) through an iterative concatenated CFOs estimation and compensation algorithm. The users received signals at the output of fast-Fourier-transform (FFT) block enter a best linear unbiased estimator (BLUE)-based CFO estimator followed by an iterative PIC unit. The signal at the output of PIC is partially synchronised based on the initial estimated values and can be regarded as a secondary signal with residual CFOs equal to the estimation errors. The interference as a result of CFO estimation errors can be suppressed if the resultant signal is sent back to the estimation and compensation units again. Interestingly, the simulations show that after only one iteration, estimator reaches the steady-state with a significant improvement in the estimated values. Moreover, simulation results verify the proposed scheme achieves comparable bit-error-rate (BER) results to a synchronous system or the one with ideal
5.2. OFDMA uplink system description

5.2.1 OFDM signal model

Consider the uplink of an orthogonal-frequency-division-multiple-access (OFDMA) system with a total number of $N$ subcarriers and $K$ active users. Each user is assigned

$$L = \left\lfloor \frac{N}{K} \right\rfloor \text{ (for integer floor) subcarriers for simultaneous data transmission.}$$

There are $M$ orthogonal-frequency-division-multiplexing (OFDM) blocks in each frame. The stream of $k^{th}$ user data after serial to parallel conversion at the $m^{th}$ block is given by

$$s_m^{(k)} = \begin{bmatrix} s_m^{(k)}(0), s_m^{(k)}(1), ..., s_m^{(k)}(L-1) \end{bmatrix}^T, 0 \leq m \leq M - 1$$

which is mapped to a set of subcarriers allocated to $k^{th}$ user

$$b_m^{(k)} = \Phi^{(k)} s_m^{(k)} , 1 \leq k \leq K$$

where $\Phi^{(k)}$ is an $N \times L$ frequency mapping matrix whose element on $n^{th}$ row and $\ell^{th}$ column is given by

$$\Phi_{n,\ell}^{(k)} = \begin{cases} 1, & n \in \Psi^{(k)} \\ 0, & \text{otherwise} \end{cases}$$

For the subband allocation scheme $\ell = n - (k-1)L$ and for the interleaved scheme $\ell = \frac{n-(k-1)}{L}, 0 \leq n \leq N - 1, 0 \leq \ell \leq L - 1$. $\Psi^{(k)}$ is the set of all subcarriers allocated to $k^{th}$ user. At this point the data blocks are modulated over the allocated bandwidth. Modulating the OFDM symbols belonging to a block on a set of overlapping and orthogonal subcarriers in this way is equivalent to taking an $N$-point inverse Fourier transform from the corresponding OFDM-block

$$b_m^{(k)} = F^T b_m^{(k)}$$

where $F$ is an $N \times N$ Fourier transform matrix

$$F_{n,\ell} = \frac{1}{\sqrt{N}} e^{-j2\pi(n\ell)/N}, 0 \leq n, \ell \leq N - 1.$$
Before sending the signal through the communication channel, for mitigation of the inter-symbol interference (ISI) that is induced by channel delay-spread, each OFDM block is cyclically extended. The Cyclic Prefix (CP) whose length is not less than the channel length, is padded at preamble of each block

\[
\bar{b}_m^{(k)} = \begin{bmatrix}
0_{N_g \times (N-N_g)} & I_{N_g} \\
I_N & =T_{cp}
\end{bmatrix} \bar{b}_m^{(k)}
\]

(5.6)

where \( T_{cp} \) is an \( N_T \times N \) CP appender matrix, \( I_N \) is an \( N \times N \) identity matrix, \( 0 \) is an all zero matrix with its size at the subscript and \( N_g \) denotes the CP length. At the receiver, the time-domain super-imposed signal, after elimination of CP, can be expressed as

\[
a_m = \sum_{k=1}^{K} e^{j2\pi \epsilon_m^{(k)} N_T} \Gamma(e^{(k)}) B_m^{(k)} h^{(k)} + \omega_m
\]

(5.7)

where \( \epsilon^{(k)} \) is the normalized CFO of \( k^{th} \) user, \( N_T = N + N_g \), \( h^{(k)} \) is \( k^{th} \) user channel impulse response, \( \omega_m \sim CN(0, \sigma_w^2 I_N) \) is the receiver additive noise and,

\[
\Gamma(e^{(k)}) = D \left( a_w^{(k)} \right)
\]

(5.8)

with \( d_w \) being an \( N \times 1 \) vector of CFO window operator given by

\[
d_w^{(k)}(n) = e^{j2\pi \epsilon_m^{(k)} n}, 0 \leq n \leq N - 1.
\]

(5.9)

\( B_m^{(k)} \) is an \( N \times L_h \) data symbols matrix given by

\[
B_m^{(k)}(n, \ell) = \bar{b}_m^{(k)}(n-\ell), 0 \leq n \leq N - 1, 0 \leq \ell \leq L_h - 1
\]

(5.10)

and \( L_h \) is the channel length. Finally by performing users data decomposition through FFT and user-specific subcarrier selection

\[
r_m^{(k)} = \left( \Phi^{(k)} \right)^T F a_m.
\]

(5.11)

(5.11) can be rewritten as

\[
r_m^{(k)} = e^{j2\pi \epsilon_m^{(k)} N_T} \left( \Phi^{(k)} \right)^T \bar{D}_m^{(k)} h^{(k)}
\]

\[+ \left( \Phi^{(k)} \right)^T F \sum_{u=1, u \neq k}^{K} e^{j2\pi \epsilon_m^{(u)} N_T} \Gamma(e^{(u)}) B_m^{(u)} h^{(u)}
\]

\[+ n_m^{(k)}
\]

(5.12)
where,

\[ \bar{D}_m^{(k)} = \text{FP}(e^{(k)})B_m^{(k)}. \]  

(5.13)

In (5.12), the first term is the pure signal term of \( k \)-th user, second term is MUI and the last term is the receiver additive noise spectrum present in \( k \)-th user-band

\[ \eta_m^{(k)} = (\Phi^{(k)})^T F \omega_m. \]

(5.14)

### 5.2.2 Iterative parallel interference cancellation

In this section, by assuming perfect CFOs knowledge is available at the receiver, an iterative parallel interference cancellation (PIC) algorithm is implemented. As the CFO window vector is multiplied by the signal in time domain, it has to be compensated by circular convolution in frequency domain. Therefore, the initial CFOs compensation stage can be performed as follows [72]. First, set \( j = 0 \) as the counter of iterative PIC algorithm,

\[ \hat{r}_m^{(k), (j)} = A^{(k)} \left[ \tilde{A}^{(k)} r_m \odot c_m^{(k)} \right] \]

(5.15)

where \( \odot \) denotes circular convolution operation, \( \hat{r}_m = F a_m \) is an \( N \times 1 \) vector of superimposed f-domain signal and \( \tilde{A}^{(k)} \) is an \( N \times N \) diagonal matrix with the elements on its diagonal

\[ \tilde{A}^{(k)}_{m,n} = \begin{cases} 1, & n \in \Psi^{(k)} \\ 0, & \text{otherwise} \end{cases}, \quad 0 \leq n \leq N - 1. \]

(5.16)

\( c_m^{(k)} \) is an \( N \times 1 \) CFO compensation vector which, in ideal case, is based on perfect CFOs and is expressed as

\[ c_m^{(k)} = e^{-j2\pi n_m N} F d_m^{(k)}. \]

(5.17)

Having completed the initial stage of CFO compensation, the users signals are ready to be passed to the iterative PIC algorithm [1]. In this method other users' interfering signals are first modelled by circular convolutions and then removed from the received signal. The interference-removed signal is then compensated for each user in a similar fashion as the initial stage above. This iterative algorithm can mathematically be expressed as
5.3 Iterative CFO estimation and compensation

5.3.1 Training-based CFOs estimation

In this section, it is assumed all users can be misaligned along frequency direction simultaneously and hence a training-based estimation process (BLUE) is performed user by user. Consider the following block correlations

\[ C_{m-k'}^{(k)} = \frac{1}{\hat{N} - k'} \sum_{m=k'+1}^{\hat{N}} \left( \hat{r}_{m-k'}^{(k)} \right) \left( \hat{r}_{m}^{(k)} \right)^* , \quad 1 \leq k' \leq \hat{N} - 1 \]  \hspace{1cm} (5.20)

where \( \hat{N} \) is the maximum block correlation-lag and \( \hat{N} \) is the total number of training blocks. Now, define the following phases

\[ \alpha_{k'}^{(k)} = \zeta \left\{ C_{k'}^{(k)} \left( C_{k'-1}^{(k)} \right)^* \right\} \]  \hspace{1cm} (5.21)

where \( C_0 \) is equal to one by default. \( k^{th} \) user CFO can be calculated as ([6])

\[ \hat{\varepsilon}_{k'}^{(k)} = \frac{N}{2\pi N_T} \sum_{k'=1}^{\hat{N}} w(k') \alpha_{k'}^{(k)} \]  \hspace{1cm} (5.22)

where \( w(k') \) is a smoothing function which only depends on number of training blocks and maximum correlation-lag and hence could be pre-computed

\[ w(k') = \frac{3(\hat{N} - k')(\hat{N} - k' + 1) - \hat{N}(\hat{N} - \hat{N})}{\hat{N}(4\hat{N}^2 - 6\hat{N} + 3\hat{N}^2 - 1)} . \]  \hspace{1cm} (5.23)

---

loop: \( j = j + 1 \)

\[ \hat{r}_{m}^{(k),(j)} = \sum_{u=1, u \neq k}^{K} \hat{r}_{m}^{(u),(j-1)} \odot c_m^{(u)} \]  \hspace{1cm} (5.18)

\[ \hat{r}_{m}^{(k),(j)} = \sqrt{\hat{A}^{(k)}} \hat{r}_{m}^{(k),(j)} \odot c_m^{(k)} \]  \hspace{1cm} (5.19)

go back to loop

where, \( c_m \) is the CFO window vector affecting the \( m^{th} \) block, given by \( c_m^{(u)} = (c_m^{(u)})^* \).

As shown in [1], CFOs with a variance more than 0.0036 cause severe performance degradation. This shows the importance of sufficiently accurate CFOs knowledge exploited in the iterative PIC process. In the next section, an algorithm is proposed to achieve the required accuracy iteratively.
The variance of $\hat{e}$ for the case of single misaligned-user ($k^{th}$ user here) is minimized when the maximum block correlation-lag is half of the number of pilot blocks, i.e. $N = \bar{N}/2$ (see [6])

$$\text{var}_{\min} \left\{ e^{(k)} \right\} = \frac{3\sigma^2}{2 \left( \frac{\pi N_e}{N} \right)^2 \left\| D^{(k)} h^{(k)} \right\|^2 \bar{N} \left( \bar{N}^2 - 1 \right)}$$

(5.24)

where $\sigma^2$ is the variance of the receiver noise. In presence of MUI, by having separate estimation and PIC units, the BLUE estimator cannot offer an acceptable performance. To counteract this issue, an algorithm is introduced in next section which suppresses the MUI iteratively and hence improves the estimator performance.

5.3.2 Iterative concatenated CFOs estimation and compensation

A loop can be formed by concatenating the BLUE estimator and the PIC units. The overall layout of this algorithm is depicted in Fig. 5.1.

![Figure 5.1: Iterative concatenated CFOs estimation and compensation.](image)

The iteration index for this concatenated algorithm, which is known as outer loop in the
5.3. Iterative CFO estimation and compensation

sequel, is denoted by the superscript \((i)\). Similarly, the introduced iterative PIC unit will be known as *inner loop* with the iteration index \((j)\) which will also appear at the superscript of the signals. In following equations, where the signals are used in upper-case and without their block subscript \(m\), it means consecutive vectors (OFDM-blocks) of a frame are placed together to form a corresponding signal matrix. For example the super-imposed f-domain signal will be an \(N \times M\) matrix, which at the \(i = 0\) iteration is denoted by \(\hat{R}^{(0)}\). To start the outer and inner loops, first set \(i = 0\), and then start the outer loop by

**Outer loop:**

\[
\tilde{e}^{(k),(i)} = \frac{N}{2\pi N_T} \sum_{k'}^{N} w(k') \alpha^{(k),(i)}_{k'}.
\]  

(5.25)

For \(i = 0\), \(\alpha^{(k),(i)}_{k'}\) is directly calculated from the received pilot blocks and for \(i > 0\), it will be computed using partially synchronised training blocks that are fed back from previous iteration. Based on the CFO estimates of all users, initial CFO compensation followed by PIC can be performed as follows. First set the counter for the iterative PIC as \(j = 0\). The initial CFO compensation is performed by

\[
\hat{R}^{(k),(j),(i)} = \bar{A}^{(k)} \left[ \bar{A}^{(k)} \hat{R}^{(i)} \odot C^{(k),(i)} \right], 1 \leq k \leq K
\]  

(5.26)

where \(C'\) is an \(N \times M\) CFOs compensation matrix whose \(m^{th}\) column is calculated through (5.17). In the next stage the inner loop is performed as

**inner loop:** \(j = j + 1\),

\[
R^{(k),(j),(i)} = \hat{R}^{(i)} - \sum_{u=1, u\neq k}^{K} \hat{R}^{(u),(j-1),(i)} \odot \hat{C}^{(u),(i)}
\]  

(5.27)

where \(\hat{C} = C'^{**}\) is the CFO window matrix based on the estimated CFOs. The final stage of the inner loop is performed as

\[
\hat{R}^{(k),(j),(i)} = \bar{A}^{(k)} \left[ \bar{A}^{(k)} R^{(k),(j),(i)} \odot C^{(k),(i)} \right], 1 \leq k \leq K
\]  

(5.28)

go back to *inner loop*

\(i = i + 1\),
go back to *outer loop*.
It should be noted that in the next outer loop iteration and when estimating the CFOs, \( r_m^{(k)} \) and \( r_{m-k'}^{(k)} \) in (5.20) are replaced by the pilot blocks in \( \hat{\mathbf{R}}^{(k)},(\tau),(t-1) \). After the 0\(^{th}\) iteration, the signal is partially synchronised based on the initial CFO estimates. The resultant signal could be assumed as a secondary signal with another set of CFOs being equal to the estimation errors. By sending the signal back to the estimation and compensation units, the residual values of CFO are counteracted in the next iteration.

In terms of convergence, as shown in [1], the PIC algorithm that is exploited in the inner loop here requires 2 iterations for convergence. Moreover, the outer-loop is terminated when a particular threshold \( \delta \) is reached. In Sec. 5.4 it is shown that after only one outer-loop iteration, the estimator will converge to its steady-state value with a significant improvement in the CFO estimates.

As it can be observed from (5.24), when the term inside the norm becomes vanishingly small (i.e. \( ||D_{m,h}|| \approx 0 \)), MMSE tends to infinity and hence CFOs cannot be identified. The CFOs estimation range in multiuser scenario will be \( |e^{(h)}| < \frac{N}{2N_T} \). For example by having \( N = 64 \) subcarriers and \( N_g = 8 \), \( |e^{(h)}| < 0.44 \). It would be interesting to benchmark the single-user CFO estimation performance of the proposed scheme with the modified Cramér-Rao lower bound (MCRLB), as the lower bound of estimator variance. The MCRLB is calculated through (see [6],[7])

\[
\text{MCRLB}(e) = \frac{3}{2\pi^2} \frac{N^3\sigma^2}{||B_m h||^2 (NN_T)^3} \tag{5.29}
\]

where \( \frac{||B_m h||^2}{N} \) is signal energy of the new user per received sample and is independent of CFO. In Sec. 5.4, MCRLB will be employed to evaluate the estimator performance in the SUE scenario.

5.4 Simulation Results

Consider an OFDMA system with \( N = 64 \) subcarriers per OFDM symbol and QPSK modulation scheme. There are \( K = 4 \) active users in the system and each one is assigned \( L = 16 \) subcarriers. All users' CFOs are random values uniformly distributed in \([-0.2, 0.2]\). The communication channel is frequency-selective slow Rayleigh fading
whose power-delay profile is set according to the $C2-NLOS$ model \[62\]. It is assumed perfect knowledge of channels is available at the receiver (BS) and all users' channels are statistically independent. For each user a convolutional coding scheme with a rate of $1/2$ and constraint length of $5$ is used. In the simulations each frame consists of $\mathcal{M} = 14$ OFDM blocks, $\mathcal{N} = 4$ of which are pilot blocks. It is assumed the channels do not vary considerably within each frame but significant variations are observable from one frame to another. In each frame a $10 \times 32$ block-bit interleaver is employed. As mentioned earlier, it is assumed the data symbols do not suffer any timing-related discrepancies and hence CFOs are the only cause of phase mismatch. For the outer loop of the proposed concatenated algorithm, zero and one iterations and within which, for the inner loop, two iterations are considered.

The MSE of estimated CFOs versus number of outer loop iterations for subband and interleaved subcarrier assignment schemes is shown in Fig. 5.2. It can be observed that by exploiting the proposed method, significant performance improvement is achieved after only one iteration. It can also be seen that the amount of MSE improvement decreases as SNR increases. This is due to the fact that at higher SNRs, the induced interference of residual CFOs becomes a more influential factor for the estimator.

Fig. 5.3 illustrates the MSE versus SNR for zero and one outer-loop estimation iterations. The MSE of the SUE scenario together with the MCRLB are also drawn for comparison purposes. In case of SUE, the MSE shows a close behaviour to the MCRLB at all SNRs. Therefore the CFO BLUE estimator performs well as a sub-optimum scheme in the SUE scenario. On the other hand, in the multiuser scenario, by increasing the SNR, the estimator performance cannot be improved. This is why the BLUE estimator cannot be directly applied to the uplink OFDMA. However the proposed iterative algorithm offers significant MSE improvement after only one iteration for both subcarrier assignment schemes. The error-floor appeared for the proposed scheme after 1 outer-loop iteration can be explained as follows. As mentioned earlier, within each iteration of inner-loop, the MUI is reconstructed based on the CFO estimates of previous outer-loop and then removed from the received signal. The MUI-removed signal is then passed on to the outer-loop for a new set of CFO estimates. Therefore the accuracy of the reconstructed MUI and CFO estimates are inter-related.
5.4. Simulation Results

Figure 5.2: MSE vs. number of estimation iterations for subband and interleaved allocation schemes.

Figure 5.3: MSE vs. SNR for zero and one estimation iterations.
Once the reconstructed MUI reaches a certain accuracy level, the CFO estimation accuracy cannot be improved. This figure shows that such a phenomenon occurs for SNRs > 22dB.

In Fig. 5.4 the overall system performance in terms of frame-error-rate (FER) against SNR is drawn. The performance of the system after 0th iteration is extremely undesirable with the appearance of an early error floor. This is equivalent to having separate estimation and iterative PIC units (e.g. [1]). For SNR values of up to 22dB, the proposed algorithm can achieve very close performance to that of systems with ideal or without CFOs after only one iteration.

![Graph showing FER vs. SNR for subband and interleaved allocations.](image)

Figure 5.4: FER vs. SNR for subband and interleaved allocations.

**Remark 1:** Further simulations were performed to evaluate the performance of the proposed scheme under different physical channels. It was observed that by using the same 5-tap channel as in Chapters 3 and 4 and under sub-band subcarrier assignment scheme, both MSE and BER results are coincided with the results achieved for the 8-tap channel in C2-NLOS scenario.
5.5 Summary

In this chapter a new iterative concatenated CFOs estimation and interference cancellation algorithm for uplink OFDMA was proposed. In one hand, it was shown that the BLUE CFO estimator, which enjoys a close estimation accuracy to the MCRLB in the SUE scenario, is not directly applicable to the multiuser scenario due to the MUI. On the other hand, it was shown that the proposed iterative algorithm achieves very close performance results to that of systems with perfect knowledge or without CFOs particularly at practical range of SNRs. Also since a high performance gain is achieved after only one estimation iteration, the computational complexity is not a stringent issue.
Conclusions and Future Work

6.1 Conclusions

In this thesis the important issue of frequency synchronisation for orthogonal-frequency-division-multiple-access (OFDMA) wireless cellular networks was investigated. It was shown that frequency synchronisation is performed through two sub-processes, carrier-frequency-offset (CFO) estimation and compensation. Firstly, a full review was made on the main existing schemes for CFO estimation. The existing CFO estimation schemes were divided into data-aided (DA) and non-data-aided (NDA) estimators. The DA schemes exploited the periodic transmission of well-designed pilot symbols for CFO estimation and only required a few blocks. On the other hand, NDA schemes performed CFO estimation by solely relying on advanced signal processing techniques and normally required a large number of blocks for statistical convergence.

Secondly, the existing schemes for CFO compensation were reviewed and classified into two broad categories, i.e. pre-compensation and post-compensation. State-of-the-art approaches have shown that pre-compensation scheme is particularly successful in mitigating inter-carrier-interference (ICI) and multiuser interference (MUI) at the receiver. The only price that has to be paid by using this scheme was the overhead of additional signalling for feeding back the estimated CFOs. The post-compensation scheme is proposed and used in the literature as an alternative method. As the received
signal is the superimposition of all users, this scheme was required to be accompanied with the interference cancellation techniques.

Thirdly, a blind CFO estimation scheme for linearly precoded (LP)-OFDMA was proposed. The motivation for adopting the LP-OFDMA air-interface was the fact that the conventional OFDMA has some shortcomings such as high peak-to-average-power-ratio (PAPR) and lack of frequency-diversity gain particularly in uncoded systems. It was shown, through both analytical and simulation results, that with a careful precoder design and also by exploitation of a novel time-frequency multiuser data-mapping (MU-DM) scheme, the CFO estimation performance in a multiuser scenario is close to that of single-user equivalent (SUE) scenario. Moreover the proposed scheme outperformed all state-of-the-art approaches in both SUE and multiuser scenarios.

Fourthly, motivated by the fact that the best linear unbiased estimator (BLUE) CFO estimator, as a low-complex and sub-optimum scheme, is very sensitive to the MUI, it was proposed to combine the BLUE estimator with a class of training sequences which were namely known as orthogonal training sequence (OTS)-BLUE, time-domain training cyclostationarity (TDTC)-BLUE and self interference cancelling training scheme (SCTS)-BLUE. It was shown that the OTS-BLUE results in a CFO estimation accuracy that is identical to that of SUE scenario in optimistic cases. The TDTC-BLUE offered a SUE-comparable estimation performance but showed sensitivity to the order of placement for the synchronous users. However the SCTS-BLUE showed high resilience to the MUI irrespective of the synchronous users' order.

Lastly, as opposed to the previous two proposed schemes where CFO estimation and compensation are performed at the receiver and transmitter respectively, an approach was introduced which performed a joint CFO estimation and compensation at the receiver. Such a frequency synchronisation policy relaxes the requirement of additional signalling raised in CFO pre-compensation scheme. In this approach a double-layered iterative concatenated CFO estimation and compensation algorithm was introduced. It was shown that this iterative algorithm converges after only 1 iteration with a significant improvement in CFO estimation accuracy and overall system performance. Therefore, the computational complexity, as a major concern in iterative algorithms, was not a
6.2 Future work

To this end, some further works can be considered in the context of CFO estimation and compensation in the uplink OFDMA. These works can either be extensions to the proposed schemes or they can be new and are summarised as follows.

Firstly, the blind method in Chapter 3 was proposed for a particular air-interface i.e., LP-OFDMA. It would be interesting if this work can be extended to the general case in OFDMA.

Secondly, for the class of training sequences proposed in Chapter 4, Morelli’s synchronisation policy was considered. Morelli’s synchronisation policy categorises the active users in the network into synchronised and asynchronous users. Synchronised users are the ones whose CFO is pre-compensated at the transmitter. For a general case, i.e. without consideration of Morelli’s synchronisation policy, the above definition for synchronised users would not hold any more. This calls for an updated system model and a new CFO estimation scheme.

Thirdly, the iterative concatenated CFO estimation and compensation algorithm only works when the CFOs are not larger than 20% of subcarrier spacing as the inner-loop that performs interference cancellation cannot handle the MUI induced by larger CFOs. Therefore a new interference cancellation scheme is required that performs well even in the presence of CFOs as large as 50% of subcarrier spacing.

Fourthly, as the virtual (null) subcarriers are usually exploited in the practical communication systems, a null-subcarrier based approach can be proposed for estimation of multiple CFOs. Such an approach can be regarded as an extension to the earlier null-subcarrier based schemes proposed for single-user CFO estimation. However, in the multiuser scenario, dealing with the complexity introduced by the multi-dimensional (MD) search becomes a challenging task.

Lastly, the orthogonal feature of the subcarriers in an ideal OFDMA can be exploited
to propose an auto-regressive based approach that treats the received non-orthogonal subcarriers and retrieves their orthogonality.
Appendix A: Derivation of approximate SUE-MMSE

Start from the estimation noise \( \bar{\omega} \) as appeared in (3.12) and define

\[
\bar{\omega} \triangleq \text{tr} (\Omega)
\]

where \( \Omega \) is the estimation noise matrix with a size of \( L \times L \) and is calculated through

\[
\Omega = e^{-\frac{\Omega^2}{N}} \left( \Theta_2 (\hat{\sigma}_g^2 - \hat{\sigma}_s^2) \Theta_1^H \Lambda^H \right) + \frac{1}{M} \sum_{m=0}^{M-1} \left( e^{-\frac{2\pi (2m+1)K}{N}} \Theta_3 \bar{s}_m \eta_{1,m}^H + e^{-\frac{2\pi mK}{N}} \eta_{2,m} \bar{s}_m \Theta_1^H \Lambda^H + \eta_{2,m} \eta_{1,m}^H \right),
\]

and the approximate variance of source symbols

\[
\hat{\sigma}_s^2 = \frac{1}{2ML} \text{tr} \sum_{m=0}^{M-1} \bar{s}_m \bar{s}_m^H.
\]

As the information symbols are i.i.d., based on the central limit theorem, \( \bar{\omega} \) is an approximately Gaussian process for sufficiently large \( M \) (see [65]) with zero-mean and variance \( \sigma_{\bar{\omega}}^2 \). To find this variance, first calculate

\[
\text{tr} \left( \mathbb{E} [\Omega^H] \right) = \frac{L}{M} \sigma_n^2 \sigma_s^2 \text{tr} (\Theta_2 \Theta_2^H \Lambda^H) - \frac{1}{M} \sigma_s^2 \text{tr} \left( \begin{array}{c} \Theta_2 \Theta_1^H \Lambda^H \Lambda \Theta_1 \Theta_2^H \Lambda^H \end{array} \right)
+ \frac{1}{M} \text{tr} \left( \mathbb{E} \left( \Theta_3 \bar{s}_m \bar{s}_m^H \Theta_1^H \Lambda \Theta_1 \bar{s}_m \bar{s}_m^H \Theta_2^H \Lambda^H \right) \right)
+ \frac{L}{M} \sigma_n^2 \sigma_s^2 \text{tr} (\Theta_1^H \Lambda^H \Lambda \Theta_1) + \frac{L^2}{M} \sigma_n^4.
\]
from which $\sigma_\omega^2$ is given by

$$
\sigma_\omega^2 = \frac{1}{L} \text{tr} \left( E \{ \Omega \Omega^* \} \right) \\
= \frac{2}{M} \sigma_\eta^2 \sigma_\xi^2 \text{tr} (\bar{A}) - \frac{1}{ML} \sigma_\eta^2 \text{tr} (\mathbf{P} \mathbf{A} \mathbf{P}^* \bar{A}) \\
+ \frac{1}{ML} \text{tr} (\mathbf{A} \Theta_b E \left\{ \tilde{s}_m \tilde{s}_m^* \Theta_1^* \bar{A} \Theta_1 \tilde{s}_m \right\} \Theta_2^* \mathbf{A}^*) + \frac{L}{M} \sigma_\eta^4. \tag{5}
$$

The expectation on the RHS of (5) can be expressed as (see Appendix 6.2)

$$
E \left\{ \tilde{s}_m \tilde{s}_m^* \Theta_1^* \bar{A} \Theta_1 \tilde{s}_m \right\} = \sigma_\eta^4 \bar{B} \tag{6}
$$

where for diagonal elements, $\bar{B}_{p,p} = \text{tr}(B)$ and for off-diagonal elements, $\bar{B}_{p,q} = B_{p,q}$ with $0 \leq p, q \leq 2L - 1$. The probability density function (pdf) of $\bar{\omega}$ (as appeared in (3.12)) is given by

$$
p_{\bar{\omega}} (\bar{\omega} | \mathbf{e}) \approx \frac{1}{\pi \sigma_\omega^2} \exp \left( - \frac{|\bar{\omega}|^2}{\sigma_\omega^2} \right). \tag{7}
$$

The Cramér-Rao lower bound (CRLB) is calculated via (see [74])

$$
\text{CRLB}(\mathbf{e}) = \frac{1}{E \left\{ \left( \frac{\partial \ln p (\bar{\omega} | \mathbf{e})}{\partial \mathbf{e}} \right)^2 \right\}}. \tag{8}
$$

The first-order derivative of the log-likelihood function is calculated through

$$
E \left\{ \left( \frac{\partial \ln p (\bar{\omega} | \mathbf{e})}{\partial \mathbf{e}} \right)^2 \right\} \approx \frac{E \left\{ \frac{\sigma_\omega^2}{\sigma_\omega^2} \right\}}{\sigma_\omega^2} \\
+ \frac{(\partial \sigma_\omega^2/\partial \mathbf{e})^2}{\sigma_\omega^2} \left( E \left\{ |\bar{\omega}|^4 \right\} - \sigma_\omega^4 \right) \\
- 2 \frac{(\partial \sigma_\omega^2/\partial \mathbf{e})}{\sigma_\omega^2} \left[ E \left( \frac{\partial |\bar{\omega}|^2}{\partial \mathbf{e}} |\bar{\omega}|^2 \right) - E \left( \frac{\partial |\bar{\omega}|^2}{\partial \mathbf{e}} \right) \right] \sigma_\omega^2, \tag{9}
$$

where the first-order derivative of $|\bar{\omega}|^2$ is given by

$$
\frac{\partial}{\partial \mathbf{e}} |\bar{\omega}|^2 = \frac{\mathbf{e} \cdot j2\pi N_T}{N} \left( \frac{p_j \bar{\omega} N_T}{N} - \frac{\partial \bar{p}}{\partial \mathbf{e}} \right) \\
- \frac{\mathbf{e} \cdot j2\pi N_T}{N} \left( \frac{p_j \bar{\omega} N_T}{N} + \frac{\partial \bar{p}}{\partial \mathbf{e}} \right) + \frac{\partial \mathbf{e}}{\partial \mathbf{e}} \cdot \frac{\partial (p)^2}{\partial \mathbf{e}}. \tag{10}
$$
The proof of $E\{|\tilde{\omega}|^4\} = 2\sigma^4_\omega$ is provided in Appendix 6.2. Moreover,

$$|\tilde{\omega}|^2 = |\tilde{\mu}|^2 - \tilde{\mu}\tilde{p}e^{-\frac{j2\pi n N_T}{N}} - \tilde{\mu}^*\tilde{p}e^{\frac{j2\pi n N_T}{N}} + (\tilde{p})^2$$

and as it can be observed from (3.12), $E\tilde{\mu} = \tilde{p}e^{\frac{j2\pi n N_T}{N}}$. In (11), the first-order derivative of $\tilde{A}$ can be calculated through

$$\frac{\partial \tilde{A}}{\partial \varepsilon} = \left(\frac{\partial A^\kappa}{\partial \varepsilon}\right)A + A^\kappa\left(\frac{\partial \Lambda}{\partial \varepsilon}\right)$$

where

$$\frac{\partial \Lambda}{\partial \varepsilon} = \Phi^T F D_n \Gamma F^\kappa D_H \Phi$$

and $D_n = D(\begin{bmatrix} 0 & j2\pi /N & j4\pi /N & \ldots & j2(N-1)\pi /N \end{bmatrix})$.

To able to have a neater form for approximate SUE-MMSE, two approximations are made. First, $\sigma^2_s = \sigma^2_\omega$ which is valid for sufficiently large $M$. This equality will cause the second and third terms on the RHS of (5) to cancel out each other. Second, as shown in Fig. 3.6, $\sigma^2_\epsilon$ varies negligibly with respect to CFOs over all SNRs. This causes the second and third terms on the RHS of (9) to become negligible and $\tilde{p} \approx \sigma^2_s \sum_{i\in \Psi} |\tilde{h}_i|^2$.

Therefore the further loosened approximate SUE-MMSE, $\overline{\text{MMSE}}_{\text{SUE}}(\epsilon)$, can be given by

$$\overline{\text{MMSE}}_{\text{SUE}}(\epsilon) = \left(\frac{N}{2\pi N_T}\right)^2 \frac{\sigma^2_\omega}{2\tilde{p}^2}$$

$$= \frac{1}{M} \left(\sum_{i\in \Psi} P_i |\tilde{h}_i|^2\right)^2 \left(\frac{\sum_{i\in \Psi} |\tilde{h}_i|^2}{\sigma^2_s} + \frac{\sigma^4_L}{2\sigma^4_s}\right).$$

(15)
Appendix B: Proof of (6) in Appendix A

Start by defining

\[ Z \triangleq SBS \]  \hspace{2cm} (16)

where \( S = \bar{s}_m \bar{s}_m^T \). Next perform element-wise analysis on \( Z \)

\[
Z_{p,q} = \sum_{i=0}^{2L-1} \sum_{j=0}^{2L-1} S_{p,i} B_{i,j} \bar{s}_{j,q}
= \sum_{i=0}^{2L-1} \sum_{j=0}^{2L-1} [\bar{s}_m]_p^* [\bar{s}_m]_j [\bar{s}_m]_{j,q}^* B_{i,j}
\]

where \([\bar{s}_m]_p^*\) is the \( p \)th element of vector \( \bar{s}_m \). For diagonal elements

\[
\mathcal{E}(Z_{p,p}) = \sum_{i=0}^{2L-1} \sum_{j=0}^{2L-1} \mathcal{E}([\bar{s}_m]_p^* [\bar{s}_m]_j [\bar{s}_m]_{j,q}^*) B_{i,j}
= \mathcal{E} ([\bar{s}_m]_p^* [\bar{s}_m]_p) B_{p,p}
\]

\[
+ \sum_{q=0,q\neq p}^{2L-1} \mathcal{E} ([\bar{s}_m]_p^* [\bar{s}_m]_q [\bar{s}_m]_{q,p}^*) B_{q,q}
= \sigma_a^4 \sum_{p=0}^{2L-1} B_{p,p} \cdot \text{tr}(B)
\]

For off-diagonal elements, out of \((2L)^2\) terms to be summed up to produce \( \mathcal{E}(Z_{p,q}) \), there is only one non-zero term

\[
\mathcal{E}(Z_{p,q}) = \mathcal{E} ([\bar{s}_m]_p^* [\bar{s}_m]_q) B_{p,q} = \sigma_a^4
\]

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Appendix C: Derivation of $\mathcal{E} \{ |\bar{\omega}|^4 \}$ in Appendix A, Eqn. (9)

By defining $X \triangleq |\bar{\omega}|^2$, it holds

$$\mathcal{E} \{ |\bar{\omega}|^4 \} = \mathcal{E} \{ X^2 \}.$$  \hfill (20)

The expected value of an arbitrary function of $X$, $g(X)$, is calculated through

$$\mathcal{E} \{ g(X) \} = \int_{-\infty}^{+\infty} g(x)f(x)dx$$  \hfill (21)

where $f(x)$ is the pdf of random variable $X$. $|\bar{\omega}|$ has a Rayleigh distribution, hence $X$ follows an exponential distribution (see [75]), i.e. $f(x) = \lambda e^{-\lambda x}$ where $\lambda$ is the rate-factor and depends on the variance of $X$. By setting $g(x) = x^2$

$$\mathcal{E} \{ X^2 \} = \int_{0}^{+\infty} x^2 \lambda e^{-\lambda x} dx$$

$$= \left[ e^{-\lambda x} \left( -x^2 - \frac{2}{\lambda}x - \frac{2}{\lambda^2} \right) \right]_{x=0}^{x=+\infty}$$

$$= \frac{2}{\lambda^2}.$$  \hfill (22)

On the other hand, the variance of $X$ is calculated through

$$\text{var}(X) = \mathcal{E} \{ X^2 \} - [\mathcal{E} \{ X \}]^2,$$  \hfill (23)

which, after plugging in the equivalent terms, leads to

$$\mathcal{E} \{ |\bar{\omega}|^4 \} = \frac{2}{\lambda^2} = 2\sigma_0^4.$$  \hfill (24)
References


References


