Breakup of $^8$B and $^8$Li

John Mortimer

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Department of Physics
School of Physics and Chemistry
University of Surrey

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Abstract

Fully three-body kinematical calculations have been developed for analysing the breakup of light, weakly bound two-body projectiles using several theoretical models. Parallel momentum distributions of the \( ^7\text{Be} \) fragments produced in the breakup of \( ^8\text{B} \) on Ag and Pb targets at 44 and 82 MeV/nucleon have been calculated. The asymmetry seen in these distributions due to E1/E2 interference is instructive in trying to understand the E1 and E2 contributions to the total cross section. The predicted E2 cross section in these reactions was found to be model dependent. Semi-classical first-order perturbation calculations show that an E2 strength of 70% that given by the Esbensen and Bertsch \( ^8\text{B} \) structure model reproduces the asymmetry observed in the measured distributions for a Pb target at 44 MeV/nucleon. Non-perturbative coupled discretised continuum channels (CDCC) calculations gave distributions with a greatly reduced asymmetry when compared with first-order calculations highlighting the presence of higher-order effects in the breakup process, which suppress the E1/E2 interference. Increasing the E2 amplitude by a factor of 1.6 in these calculations reproduced the asymmetry seen in the all measured distributions.

Comparisons between the CDCC and adiabatic methods have been made. The adiabatic method is a higher-order theory, like CDCC, but cannot be applied to projectiles where both fragments are charged. Therefore the breakup of \( ^{9}\text{Li} \) has been studied. Large discrepancies are seen between the results of the two methods. Approximations have been made in evaluating the DWBA post-form \( T \)-matrix. The results are similar to those of the adiabatic method, rather than prior-form DWBA calculations. These results reveal a post-prior disagreement in both first-order and higher-order theories.
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Chapter 1

Introduction

The properties of light, weakly bound nuclei have been studied in nuclear physics for a number of years. Of particular interest is the Coulomb breakup of these nuclei, which can be used to gain information about the inverse capture reactions that are important in nuclear astrophysics [1]. Charged particle capture reactions at stellar temperatures have very small cross sections due to the colliding particles not being able to overcome the Coulomb barrier at low energies. Therefore, measurement of these cross sections in the laboratory is difficult. However, the breakup reactions of light nuclei in the Coulomb field of a high $Z$ nucleus have significantly larger cross sections than the inverse capture reactions given a high enough projectile incident energy. Measurements of the projectile fragments emerging at extremely forward angles allows the interactions between the projectile fragments at low relative energies to be investigated.

1.1 History of $^8$B breakup experiments

One such capture reaction that has been the source of much interest is the proton capture reaction $^7$Be$(p, \gamma)^8$B. Determining the reaction rate at solar temperatures is relevant to the solar neutrino problem, as the neutrinos produced in the decay
of $^8\text{B}$ are a major contributor to the high energy neutrino flux from the sun. Several attempts have been made to measure the $^\text{7Be}(p, \gamma)^8\text{B}$ cross section directly, e.g. [2-6], at proton energies considerably higher than those found at solar temperatures which are typically 15-20 keV. The data from these experiments have then been extrapolated to low energies to extract the zero energy astrophysical $S$-factor, $S_{17}(0)$ [4,6-8]. Unfortunately, due to difficulties in measuring the capture cross section and the subsequent extrapolation, the value of $S_{17}(0)$ is not known to sufficient accuracy. Indirect studies of the $^\text{7Be}(p, \gamma)^8\text{B}$ cross section have been made [9-11] by performing Coulomb breakup experiments for $^8\text{B}$ on a lead target. In [9] the double differential cross section was measured as a function of the proton-$^\text{7Be}$ centre of mass scattering angle and relative energy for an incident projectile energy of 46.5 MeV/nucleon. A fit of the data was made using the semi-classical method of Ref. [12] to calculate the cross section. However, only E1 transitions in the excitation of $^8\text{B}$ were included in the calculations.

While E1 transitions completely dominate the $^\text{7Be}(p, \gamma)^8\text{B}$ capture reaction, E2 transitions may make a significant contribution to the breakup cross section depending on the kinematics of the breakup reaction. Thus, to measure the E1 strength accurately, the E2 contribution to the total breakup cross section must also be known. Predictions of the importance of the E2 transitions in the Coulomb breakup of $^8\text{B}$ using various different $^8\text{B}$ structure models have been made in Refs. [8,13-15]. In these references, the cross section was calculated as a function of proton-$^\text{7Be}$ centre of mass scattering angle and relative energy in an attempt to reproduce the data of Ref. [9]. However, these cross sections are not ideal to gauge the E2 strength as the E1 and E2 cross sections add incoherently in these observables, and thus the cross section is not particularly sensitive to E1/E2 interference.

A more appropriate observable is the parallel momentum distribution of the $^\text{7Be}$ fragment produced in the breakup of $^8\text{B}$. In this distribution, the different mul-
tipole cross sections interfere coherently, thus making it more sensitive to E1/E2 interference. Calculations and measurements of 7Be parallel momentum distributions from the breakup of 8B on high Z targets with beam energies in the range ~ 40 – 80 MeV/nucleon have shown a pronounced asymmetry in the shape of the distributions [15-20]. This asymmetry, caused by interference between E1 and E2 transitions, suggests there is a significant E2 contribution to the total breakup cross section. In [18,20] the asymmetry observed in the measured 7Be parallel momentum distribution for 8B incident lead target at 44 MeV/nucleon was reproduced by a semi-classical calculation with an E2 strength 70% that of the Esbensen and Bertsch 8B structure model [15]. However, the projectile excitation was treated to first-order in this calculation, thus higher-order effects in the breakup process were not taken into account. Higher-order calculations where the breakup reaction is treated as a time-dependent process [21] have shown the asymmetry of the 7Be distributions for 8B on gold at a 41 MeV/nucleon was reduced when compared with results of first-order calculations [15,17]. Therefore, higher-order effects could play a significant role in the breakup process and need to be considered when trying to determine the E2 contribution to the breakup cross section.

The breakup cross section of 8B on 58Ni at the much lower beam energy of 26 MeV has also been measured [22-24]. At this energy it was thought that the E2 contribution to the cross section would be more important than in the experiments with higher beam energies, and thus easier to measure. Several theoretical calculations for this reaction have been performed [25-28]. It was found that the data were not accurately described by first-order calculations but were consistent with the results of non-perturbative coupled discretised continuum channels (CDCC) calculations. In Ref. [28] the results of CDCC calculations reproduced the measured 7Be energy distributions of Ref. [24] using both the Kim [7] and Esbensen and Bertsch [15] structure models. However, the expected E1/E2 interference was not seen, instead the calculated distributions were almost symmetrical. This
is evidence, along with the failure of first-order calculations to reproduce the data, that higher-order effects are prominent in the breakup process at this beam energy, leading to the suppression of the E1/E2 interference. CDCC calculations have also given a good description of 6B breakup data from the higher energy experiments [20]. The asymmetry of the measured 7Be parallel momentum distributions for 6B on lead at 82 MeV/nucleon was reproduced accurately using the Esbensen and Bertsch structure model without any adjustment to the E2 strength.

Nuclear induced breakup could also contribute to the total breakup cross section. Calculations for the breakup 3B on 58Ni at 26 MeV have shown there are significant nuclear effects in the breakup process despite the sub-Coulomb incident energy [27]. The first-order calculations of Ref. [20] for 3B on silver at 44 MeV/nucleon fail to reproduce the widths of the parallel momentum distributions. This was suggested to be due to nuclear effects in the breakup process, which are not taken into account in these calculations. Despite the peripheral nature of the reactions, where the 7Be fragments were detected at very forward angles or a sub-Coulomb incident energy was used, nuclear interactions between the projectile and target were possible, due to the extended nature of the 3B wavefunction.

1.2 Analysis of Coulomb breakup in the current work

In the current work, the breakup the 3B on lead and silver targets at 44 and 82 MeV/nucleon is investigated. Fully three-body kinematical calculations have been developed enabling the calculation of the breakup triple differential cross section. Thus, predictions can be made for the measured 7Be parallel momentum distributions from experiments conducted at the National Superconducting
Cyclotron Laboratory (NSCL), MSU [20]. Several theoretical descriptions of the breakup process, as detailed in Chapter 2, are used in the calculations for comparison with the data. Chapter 3 contains the results of first-order calculations using semi-classical [29] and fully quantum mechanical distorted wave Born approximation (DWBA) [26] methods. In Chapter 4 the results of CDCC calculations are presented and compared with the first-order calculations together with the data as a means of analysing the importance of higher-order effects in each reaction. The contribution of nuclear induced breakup to the total cross section is also investigated.

In our preliminary calculations, we adopt a single-particle model of a proton interacting with an inert, spinless $^7$Be core for the $^8$B structure model. Thus, core excitations are ignored. Several single-particle structure models have been proposed for $^8$B [7, 15, 30–33] and a comparison of them is given in [34]. Here, a modified version of the Esbensen and Bertsch model [15] is used, the details of which are given in Section 3.1. Microscopic shell model calculations have shown that for a bound state of a $p_{3/2}$ proton coupled to the ground state of $^7$Be the spectroscopic factor is very close to unity [35, 36]. Other calculations where $^8$B is treated as a three-body system ($\alpha + ^3$He + p) have shown the spectroscopic factor for this configuration to be significantly lower at approximately 0.7 [37]. However, as we shall see in Section 3.1.3, little error is expected in assuming the proton is purely in a $p_{3/2}$ orbit and neglecting the spin of the core. The E2 contribution to the breakup cross section is studied by adjusting the E2 strength given by the Esbensen and Bertsch structure model, as in Refs. [18, 20], to reproduce the observed asymmetry in the measured parallel momentum distributions.

The CDCC calculations in Chapter 4 are compared with first-order calculations. To investigate further the accuracy of the CDCC technique as means of describing Coulomb breakup, comparisons with other all-order theories would be insightful. To this end, adiabatic [38] and non-perturbative time dependent [39] calcula-
tions are presented in Chapter 5 along with CDCC calculations. In the adiabatic method, the three-body scattering wave function appearing in the post-form breakup \( T \)-matrix is approximated by the solution to the three-body Schrödinger equation in the adiabatic limit. Unfortunately, this method is only applicable to reactions where the target interacts with only one of the projectile fragments and thus, the breakup of \(^8\text{B}\) cannot be studied. The mirror nucleus of \(^8\text{B}\) is \(^8\text{Li}\), which can also be thought of as a two-body projectile with a valence particle in a \(p_{3/2}\) orbital, except the valence particle is a neutron. Hence, the breakup of \(^8\text{Li}\) is investigated instead. Post-form DWBA calculations for the breakup of \(^8\text{Li}\) are also shown in Chapter 5. However, the post-form DWBA breakup \( T \)-matrix element cannot be evaluated exactly and thus approximations to the matrix element are used in the calculations. The results are shown together with prior-form DWBA and semi-classical calculations. This enables a post-prior comparison of first-order theories.
Chapter 2

Theoretical models for Coulomb breakup

In this chapter we develop the necessary formalism for analysing the breakup of a two-body projectile nucleus, $p$, with charge $Z_p$, mass $m_p$, and laboratory energy $E_{lab}$, incident on a target nucleus, $t$, of charge $Z_t$ and mass $m_t$. The projectile nucleus consists of a core particle, $c$, of mass $m_c$ and charge $Z_c$, and a valence particle, $v$, of mass $m_v$ and charge $Z_v$. We have in mind systems such as $^8$B in which the valence particle is a weakly bound nucleon with intrinsic spin $s_0 = 1/2$, with a single bound state. We are interested in calculating triple differential cross sections for the breakup of the projectile to $c+v+t$ final states, which can be used to compute three-body observables measured in experiments. If, for example, the energy of the emerging core particle is detected, then for a three-body final state, the valence particle energy is uniquely specified and the triple differential cross section has the form

$$\frac{d^3\sigma}{d\Omega_c d\Omega_v dE_c} = \frac{2\pi}{\hbar c} \frac{1}{(2\gamma_0 + 1)} \sum_{m \sigma} |T_{m\sigma}|^2 \rho(E_c, \Omega_c, \Omega_v). \quad (2.1)$$

Here, $\rho(E_c, \Omega_c, \Omega_v)$ is the three-body phase space or density of states factor and is derived using both relativistic and non-relativistic kinematics in Appendix A.
The projectile-target relative velocity, \( v_t \), is also calculated either relativistically or non-relativistically from \( E_{\text{lab}} \). \( T_{\text{mod}} \) is the three-body breakup \( T \)-matrix and shall be derived using several theoretical approaches to three-body breakup in the following sections.

### 2.1 Breakup of 2-body projectiles

![Diagram of three-body breakup](image)

Figure 2.1: Coordinate system adopted when considering a three-body breakup reaction for a two-body projectile, consisting of a core particle \((c)\) and valence particle \((v)\) incident on a target nucleus \((t)\).

Figure 2.1 shows the coordinate system adopted for this analysis, where \( \mathbf{R} \) is the position vector of the projectile centre of mass relative to the target and \( \mathbf{r} \) is the position vector of the valence particle relative to the core particle. The core-target separation can be written as \( \mathbf{R}_c = \mathbf{R} - \gamma_{\text{ct}} \mathbf{r} \), and \( \mathbf{R}_v = \gamma_{\text{tc}} \mathbf{R}_c + \mathbf{r} \), where \( \gamma_{\text{ct}} = m_v/(m_v + m_c) \) and \( \gamma_{\text{tc}} = m_t/(m_c + m_t) \). The projectile interacts with the target through effective interactions \( V_{\text{ct}}(\mathbf{R}_c) \), the core-target interaction, and \( V_{\text{vt}}(\mathbf{R}_v') \), the valence particle-target interaction, where \( \mathbf{R}_v' = \mathbf{R} + (1 - \gamma_{\text{ct}}) \mathbf{r} \), the valence particle-target separation. In general, \( V_{\text{ct}}(\mathbf{R}_c) \) and \( V_{\text{vt}}(\mathbf{R}_v') \) include both nuclear and Coulomb interactions, the nuclear interactions taking the form
of optical potentials fitted to elastic scattering data, wherever possible.

The initial state of the projectile is taken to be a bound state of the core and valence particles. The core, assumed here to be inert and spinless, is bound to the valence particle, with intrinsic spin $s_0$, in a state with relative orbital angular momentum $\ell_0$. Thus, the projectile initial state, $|i, m_0\rangle$, having total angular momentum $j_0 (j_0 = \ell_0 + s_0)$ with projection $m_0$, has the form

$$
|r|\langle i, m_0\rangle = \sum_{m_\ell, m_0} (\ell_0 m_\ell s_0 \sigma_0 |j_0 m_0\rangle Y_{\ell_0 m_0} (\hat{r}) \chi_{s_0 \sigma_0} \frac{u_{s_0 \ell_0 j_0} (r)}{r},
$$

(2.2)

where $\chi_{s_0 \sigma_0}$ is the valence particle internal wave function with spin projection $\sigma_0$.

The radial wavefunction, $u_{s_0 \ell_0 j_0} (r)$, in Eq. (2.2) satisfies the equation

$$
\left[-\frac{\hbar^2}{2\mu_{c\nu}} \left(\frac{d^2}{dr^2} - \frac{\ell_{\ell}(\ell_{\ell} + 1)}{r^2}\right) + V_{c\nu}^{s_0 \ell_0 j_0} (r)\right] u_{s_0 \ell_0 j_0} (r) = -\varepsilon_0 u_{s_0 \ell_0 j_0} (r).
$$

(2.3)

Here, $\varepsilon_0$, $\mu_{c\nu}$ and $V_{c\nu}^{s_0 \ell_0 j_0} (r)$ are the separation energy, reduced mass and binding potential of the core-valence system, respectively.

Upon interaction with the target nucleus, the projectile can be excited from its initial state to core-valence continuum states, $|k^{(+)} \sigma'\rangle$, which satisfy outgoing waves boundary conditions with incident wave vector $k$. Assuming $s_0 = 1/2$

$$
|r|k^{(+)} \sigma'\rangle = \sum_{\sigma} \phi^{(+)}_{s_0 \sigma'} (k, r) \chi_{s_0 \sigma}
$$

$$
= 4\pi \sum_{j m} \sum_{\ell m_\ell} \sum_{m_\ell} \delta^{\ell_0 s_0 \sigma_0} (j m) Y_{\ell m_\ell}^* (\hat{k}) \frac{u_{\ell j k} (r)}{k r}
$$

$$
\times \sum_{m_\sigma} (j m s_0 \sigma |j m\rangle Y_{\ell m_\ell} (\hat{r}) \chi_{s_0 \sigma}.
$$

(2.4)

Here $\sigma'$ labels the asymptotic spin projection of the valence particle. The $u_{\ell j k}$ are the continuum wave functions for wavenumber $k$ and partial wave $\ell$, and satisfy the equation

$$
\left[-\frac{\hbar^2}{2\mu_{c\nu}} \left(\frac{d^2}{dr^2} - \frac{\ell (\ell + 1)}{r^2}\right) + V_{c\nu}^{\ell j} (r) - E_k\right] u_{\ell j k} (r) = 0
$$

(2.5)
where $E_k = \hbar^2 k^2 / 2 \mu_{cv}$ and $V_{cv}^{ij}(r)$ is the core-valence continuum state potential. This potential can be chosen to be identical to the binding potential for all partial waves or may be $\ell$ and $j$ dependent. The $u_{ijk}$ have asymptotic normalisation

$$u_{ijk}(r) \to \sin(kr - \frac{1}{2} \ell \pi + \delta_{ijk})$$

as $r \to \infty$, (2.6)

where $\delta_{ijk} = \delta_{ijkl}^\text{Coul} + \delta_{ijkl}^\text{nuc}$, is the sum of the phase shifts due to the Coulomb and nuclear potentials. With this normalisation, the $u_{ijk}$ are real when $V_{cv}$ is real.

2.2 Semi-classical model

The following analysis of three-body breakup uses the (high energy) semi-classical model for Coulomb excitation as set out by Alder and Winther, [29]. Within this semi-classical model, the relative motion of the projectile centre of mass (c.m.) and target nucleus is treated classically, whereas the excitation, assumed to be pure Coulomb, is described quantum mechanically. Following the convention of [29], the projectile c.m. is assumed to travel along a straight line trajectory of impact parameter $b$, with constant velocity, $v_i$. The direction of $v_i$ is chosen to be the $z$-axis. A necessary condition for the classical description of the relative motion of the projectile and target to be valid is that

$$\eta = \frac{Z_p Z_t e^2}{\hbar v_i} \gg 1.$$  

(2.7)

and the projectile-target relative wave number $K$ is large enough so that $\lambda$ is small compared to distances over which the target potential varies significantly. If $\eta \gg 1$, the projectile is assumed to move in accordance with the equations of classical mechanics along its trajectory [40].

The Coulomb interaction between the projectile and the target is written

$$V = V(R) + \Delta V$$

(2.8)
where $V(R)$ is the monopole part of the Coulomb field which acts on the centre of mass of the projectile and $\Delta V$ is the part which causes the projectile excitation and breakup. The projectile excitation is treated in first-order perturbation theory. In using first-order perturbation theory, it is assumed that coupling between excited states is weak, so after the Coulomb field of the target excites the projectile from its initial state to a continuum state, no further excitation occurs. For a two-body projectile, $V = V_{cl}(R_c) + V_{vl}(R')$ and thus $\Delta V$ has the form

$$\Delta V(R, r) = Z_4 e^2 \left( \frac{Z_u}{|R + (1 - \gamma_{nc})r|} + \frac{Z_c}{|R - \gamma_{nc}r|} - \frac{Z_c + Z_u}{R} \right). \quad (2.9)$$

For a vector $\rho = R - r$ where $R > r$, one can write the following multipole expansion [41]

$$\frac{1}{\rho} = \sum_{\lambda \mu} \frac{4\pi}{2\lambda + 1} \frac{(r^\lambda / R^{\lambda+1}) Y_{\lambda \mu}(r)}{Y_{\lambda \mu}(R)}. \quad (2.10)$$

Assuming the projectile and target do not overlap during the reaction i.e, $R > r$ always, $\Delta V$ can be written as a multipole expansion and the Coulomb excitation amplitude can then be expressed in terms of these multipole matrix elements [15]

$$a_{m_0 \sigma'}(k, K) = \frac{1}{i} \sum_{\lambda \mu} \langle \sigma' | M_{\lambda \mu} | i, m_0 \rangle. \quad (2.11)$$

The above expression gives the Coulomb excitation amplitude, using first order perturbation theory and a semi-classical description of scattering, for exciting a two-body projectile in an initial state $|i, m_0\rangle$ to final state $|k^{(+)} \sigma'\rangle$ for a given final state core-valence particle relative wave vector, $k$, and projectile c.m. wave vector, $K$, in the projectile-target c.m. frame. The $F$-amplitudes in Eq. (2.11) carry the $K$ dependence and are given by [15]

$$F_{\lambda \mu} = i^{\lambda+\mu} Z_T e^{\gamma} \frac{16\pi}{\hbar v_i \gamma} \frac{(\omega / v_i)^\lambda G_{\lambda \mu}^\ast}{2\lambda + 1} \frac{K_{\mu} (\omega b / \gamma v_i) e^{-i\mu \phi_K}}{\sqrt{\lambda + \mu}! (\lambda - \mu)! \gamma v_i}. \quad (2.12)$$

The multipole operator, $M_{\lambda \mu}$, has the form

$$M_{\lambda \mu}(r) = e^\lambda r^\lambda Y_{\lambda \mu}(r) \quad (2.13)$$
where

\[ e_\lambda = Z_e e (\gamma_{\text{ee}})^\lambda + Z_u e (\gamma_{\text{eo}} - 1)^\lambda. \]  

(2.14)

In Eq. (2.12)

\[ \omega = \frac{E_k + \varepsilon_0}{\hbar}, \]  

(2.15)

thus \(h\omega\) is the excitation energy. Also

\[ \gamma = \frac{E_{\text{lab}}}{m_p c^2} + 1 \]  

(2.16)

takes into account the Lorentz contraction of the electromagnetic field where \(E_{\text{lab}}\) is the projectile incident energy in the laboratory frame, and

\[ \frac{v_i}{c} = \frac{\sqrt{\gamma^2 - 1}}{\gamma}. \]  

(2.17)

The impact parameter and c.m. scattering angle are related by

\[ b = \frac{Z_i Z_p e^2}{\mu_p v_i^2 \tan(\theta_K/2)}, \]  

(2.18)

where \(\theta_K\) is the angle of the projectile-target wave vector relative to the z-axis and \(\mu_p\) is the projectile-target reduced mass. The \(G_{\lambda\mu}\) factors for dipole and quadrupole excitations are

\[ G_{10} = G_{20} = G_{2\pm2} = \frac{1}{\gamma}, \quad G_{1\pm1} = 1, \quad G_{2\pm1} = \frac{1}{2} \left(1 + \frac{1}{\gamma^2}\right). \]  

(2.19)

The formulas used in Eqs. (2.11) and (2.12) are those of Ref. [15], which are essentially identical to the original expression for the excitation amplitude given in [29]. Only electric transitions are taken into account, which is a reasonable approximation for the projectile incident energies considered here.

Eq. (2.12) differs by a factor \(e^{-i\mu\theta_K}\) from the expression given in [15]. This is because in Ref. [15] the \(x\)-axis is chosen to be in the scattering plane of \(K\),
such that the azimuthal angle $\phi_K$ is zero. However, when considering three-body observables it is convenient to define the coordinate system with respect to the fixed positions of the detectors in the laboratory frame. For such a coordinate system $\phi_K$ is, in general, not zero, thus the amplitude's dependence on $\phi_K$ must be re-introduced by the factor $e^{-i\mu_\phi K}$.

Assuming straight line trajectories for the projectile is a reasonable approximation when considering large impact parameters. However, the expression for the Coulomb field in (2.8) contains a monopole part, $V(R)$, which acts on the projectile centre of mass. As $b \to \infty$, $V(R) \to 0$, but for finite impact parameters $V(R)$ will cause the path of the centre of mass of the projectile to deviate from a straight line trajectory. A better choice for the path is a classical Rutherford trajectory. To account for this, the value at which the $K_\mu$ Bessel function in (2.12) is evaluated can be changed [29] by making a modification to the impact parameter

$$b' = \left( b + \frac{\pi Z_p Z_q e^2}{2 \mu_p v_0^2 \gamma} \right).$$

(2.20)

We now evaluate explicitly the quantum mechanical matrix element in Eq. (2.11). Under time reversal, the term $\langle k^{-}\sigma'|r\rangle$ in the multipole matrix element in Eq. (2.11) is related to the final state by [42]

$$\langle k^{-}\sigma'|r\rangle = \sum_{\sigma} \phi_{\sigma\sigma'}^{(+)}(k, r)\chi_{50\sigma}^{-\dagger} = \sum_{\sigma} (-)^{\sigma'} \phi_{-\sigma-\sigma'}^{(-)}(-k, r)\chi_{50\sigma}^{-\dagger}.$$  

(2.21)

Hence the matrix element has the form

$$\langle k^{-}\sigma'|M_{\lambda\mu}(r)|i, m_0\rangle = \frac{4\pi}{k} \sum_{jm} \sum_{\ell m_\ell} (-i)^2 e^{i\theta_{\lambda\mu}} \langle \ell m_\ell s_0 \sigma'|j m \rangle Y_{\ell m_\ell}(k) R_{\ell j\lambda}(k) \times \langle (\ell s_0) jm | Y_{\lambda\mu}(r) | (\ell s_0) jm \rangle.$$  

(2.22)
where the two-body wavefunctions enter through the radial integrals,

$$
\mathcal{R}_{\ell j\lambda}(k) = e_{\lambda} \int_{0}^{\infty} u_{\ell j\lambda}(r)r^{\lambda}u_{\ell \lambda}(r) \, dr.
$$

(2.23)

Using the convention of Ref. [41] the matrix element \(\langle (\ell s_0)jm|Y_{\lambda}(\hat{r})|(\ell_0s_0)j_0m_0\rangle\) in Eq. (2.22) can be written in terms of a reduced matrix element as follows

\[
\langle (\ell s_0)jm|Y_{\lambda}(\hat{r})|(\ell_0s_0)j_0m_0\rangle = (j_0 m_0 \lambda \mu |jm\rangle \langle (\ell s_0)j||Y_{\lambda}(\hat{r})||(\ell_0s_0)j_0\rangle.
\]  

(2.24)

In general, for a spin \(s_0\), the reduced matrix element has the form

\[
\langle (\ell s_0)j||Y_{\lambda}(\hat{r})||(\ell_0s_0)j_0\rangle = (-)^{j - \lambda - s_0 + \ell_0} \frac{\lambda \hat{\lambda} \hat{\ell}_0}{\sqrt{4\pi}} W(\ell\ell_0jj_0; \lambda s_0)\langle \ell_0000|00\rangle.
\]  

(2.25)

Using (2.24) and (2.25), the three-body Coulomb breakup amplitude can then be written

\[
a_{m_0\sigma'}(k, K) = \frac{1}{i} \sum_{\lambda \mu} F_{\lambda \mu} \langle k(-\sigma')|M_{\lambda \mu}|i, m_0\rangle
\]

\[
= \frac{4\pi}{iK} \sum_{\lambda \mu} F_{\lambda \mu} \sum_{\ell j} \sum_{s_0} (-i)^{j - \lambda - s_0 + \ell_0} e^{i\lambda \hat{\lambda} \hat{\ell}_0 / \sqrt{4\pi}} W(\ell\ell_0jj_0; \lambda s_0)
\]

\[
\times Y_{\ell j}(\hat{k}) R_{\ell j\lambda}(k).
\]

(2.26)

Finally, the Coulomb breakup amplitude, \(a_{m_0\sigma'}(k, K)\), is related to the \(T\)-matrix, \(T_{m_0\sigma'}(k, K)\), in Eq. (2.1) as follows

\[
T_{m_0\sigma'}(k, K) = \sqrt{\left(\frac{d\sigma}{d\Omega}\right)_{Ruth}} \left(\frac{2\pi\alpha^2}{\mu pt}\right) a_{m_0\sigma'}(k, K)
\]

(2.27)

where the Rutherford cross section \((d\sigma/d\Omega)_{Ruth}\) has the form

\[
\left(\frac{d\sigma}{d\Omega}\right)_{Ruth} = \left(\frac{Z_1 Z_2 e^2}{\mu pt v_1^2}\right)^2 \frac{1}{4\sin^4(\theta_K/2)}.
\]

(2.28)
2.3 Coupled discretized continuum channels method

In the coupled discretized continuum channels (CDCC) method [43,44], the continuum for core-valence particle excited states with quantum numbers $i,j$, is divided into intervals, or bins, of width $\Delta k_i = [k_i - k_{i-1}]$, up to some maximum wave number $k_{\text{max}}$. For each $\alpha \equiv (i,l,j,s_0)$ bin, the core-valence relative motion wave function has the form

$$\phi^\alpha_\alpha(r) = \frac{1}{r} \sum_{m_\ell s_\sigma} (\ell m_\ell s_\sigma | j m_j) Y_{\ell m_\ell} (\hat{r}) \chi_{s_\sigma} u_\alpha(r). \quad (2.29)$$

The radial wavefunctions $u_\alpha(r)$ are a superposition of the continuum wave functions $u_{ljk}(r)$, as defined by Eqs. (2.4) and (2.6), over the momentum range $\Delta k_i$

$$u_\alpha(r) = \sqrt{\frac{2}{\pi N_\alpha}} \int_{k_{i-1}}^{k_i} g_\alpha(k) u_{ljk}(r) \, dk. \quad (2.30)$$

Here $g_\alpha(k)$ is the weight function and the normalisation constant $N_\alpha$ is defined as

$$N_\alpha = \int_{k_{i-1}}^{k_i} |g_\alpha(k)|^2 \, dk. \quad (2.31)$$

For a non-resonant continuum the weight functions are typically chosen to be $g_\alpha(k) = 1$, except for s-wave bin states where it is convenient to choose $g_\alpha(k) = k$ [28].

The three-body wavefunction for the c+n+t system with total angular momentum $J$ and projection $M$ can then be expanded as

$$\Psi_{J M}^{cd}(r,R) = \sum_{LMJLm} \phi^m_\alpha(r) (LM LJM | J M) Y_{LMJ} (\hat{R}) f_{\alpha L J}(R)/R. \quad (2.32)$$

The projectile-target radial wave functions $f_{\alpha L J}(R)$ are the solutions of the set of coupled equations
where $\epsilon_i$ is the average energy of the continuum bin $\alpha_i$, or $\epsilon_i = -\epsilon_0$ for the projectile bound state. The matrix elements $V^J_{\alpha L, \alpha' L'}(R)$ are the coupling interactions between the bin states

$$V^J_{\alpha L, \alpha' L'}(R) = \langle [L \otimes \phi_\alpha(r)]_J | V_{al}(R) + V_{st}(R') | [L' \otimes \phi_{\alpha'}(r)]_J \rangle. \quad (2.34)$$

When evaluating the coupling interactions, the projectile internal coordinate is truncated at some maximum radius $r_{\text{bin}}$ which is chosen to be sufficiently large so that $\langle u_\alpha | u_\alpha \rangle = 1$. The interactions $V_{al}(R)$ and $V_{st}(R')$ are expanded as Legendre multipole potentials

$$V_{aL}^{\lambda}(r, R) = \frac{1}{2} \int_{-1}^{+1} [V_{al}(R) + V_{st}(R')] P_{\lambda}(x) \, dx \quad (2.35)$$

where $\lambda$ is the multipole order and $x = \hat{r} \cdot \hat{R}$. The $V^J_{\alpha L, \alpha' L'}(R)$ can then be written as an expansion in $\lambda$, [25] and calculated up to some maximum multipole order.

Eq. (2.33) is solved, using the coupled channels code FRESCO [46], either exactly, if the coupled equations are not too numerous, or iteratively:

$$\left[ -\frac{\hbar^2}{2\mu_{pt}} \left( \frac{d^2}{dR^2} - \frac{L(L+1)}{R^2} \right) + \epsilon_i - E \right] f^{(n)}_{\alpha L J}(R)$$

$$= \sum_{\alpha' L' \neq \alpha L} i^{L'-L} V^J_{\alpha L, \alpha' L'}(R) f^{(n-1)}_{\alpha' L' J}(R) \quad (2.36)$$

with $n = 0, 1, 2, \ldots$. The $n^{th}$ order DWBA may be regarded as the solution of the $n^{th}$ iteration of (2.36) [25, 26]. Thus for $n = 1$, the first-order DWBA solution for the scattering amplitude can be calculated from the asymptotic $S$-matrix of the
solution \( f_{\alpha\ell}^{(1)}(R) \) to (2.36). For a suitably large \( n \) the coupled channels solution is given by the multistep DWBA results, either directly or by the method of Padé approximants [25].

The (two-body) scattering \( T \)-matrix is related to the scattering amplitude as follows

\[
T_{\alpha\ell}^{a}(K_{\alpha}) = -\frac{2\pi \hbar^{2}}{\mu_{\text{pt}}} \sqrt{\frac{K_{0}}{K_{\alpha}}} \tilde{a}_{\alpha}(K_{\alpha}). \tag{2.37}
\]

Here \( K_{0} \) is the incident wave number of the projectile in the projectile-target c.m. frame and \( \tilde{a}_{\alpha}(K_{\alpha}) \) is the (two body) scattering amplitude, summed over partial waves, for exciting the projectile in initial state \( j_{0}, m_{0} \) to a final bin state \( j, m \) with c.m. wave vector \( K_{\alpha} \) [28]. However, when calculating three body observables, we are interested in the \( T \)-matrix \( T_{\alpha\ell}^{a}(k,K) \). This \( T \)-matrix is for projectile breakup from an initial state \( j_{0}, m_{0} \) to a general three-body final state where each final state configuration has associated with it a specific projectile c.m. wave vector \( K \) and core-valence relative wave vector \( k \). To make the connection between \( T_{\alpha\ell}^{a}(K_{\alpha}) \) and \( T_{\alpha\ell}^{a}(k,K) \) we write the prior form breakup \( T \)-matrix as [28]

\[
T_{\alpha\ell}^{a}(k,K) = \langle k(-)\sigma', e^{iK_{R}} | V_{ct}(R_{c}) + V_{vt}(R_{v}) | \Psi_{R_{0}}^{CD}(r,R) \rangle. \tag{2.38}
\]

Here \( \Psi_{R_{0}}^{CD}(r,R) \) is the CDCC approximation to the three-body scattering wave function and is written as an expansion in the total angular momentum eigenstates \( \Psi_{JM}^{CD}(r,R) \) [45]. Inserting the complete set of bin states

\[
T_{\alpha\ell}^{a}(k,K) = \sum_{\alpha_{\ell}} \langle k(-)\sigma' | \phi_{\alpha_{\ell}}^{m} e^{iK_{R}} | V_{ct}(R_{c}) + V_{vt}(R_{v}) | \Psi_{R_{0}}^{CD}(r,R) \rangle. \tag{2.39}
\]

where the sum over \( \alpha_{\ell} \) is for all bin states containing the wave number \( k \). Now we see that the matrix elements \( \langle \phi_{\alpha_{\ell}}^{m} e^{iK_{R}} | V_{ct}(R_{c}) + V_{vt}(R_{v}) | \Psi_{R_{0}}^{CD}(r,R) \rangle \) in Eq. (2.39) are the scattering \( T \)-matrix elements \( T_{\alpha\ell}^{a}(k,K) \). Thus, on calculation of the overlap integral \( \langle k(-)\sigma | \phi_{\alpha_{\ell}}^{m} \rangle \) in Eq. (2.39), the breakup \( T \)-matrix has the form
\[ T_{\text{moo}}(k, K) = \frac{(2\pi)^{3/2}}{k} \sum_{\ell m} (-i)^{\epsilon}(\ell m_\sigma g_0' j m) e^{i\frac{\pi}{4}} \times Y_{\ell m}(k)g_\alpha(k)T_{\text{moo}}(\alpha, K) \] (2.40)

where

\[ T_{\text{moo}}(\alpha, K) = \exp(i[m_0 - m]\phi_K) \frac{T_{\text{moo}}(K)}{\sqrt{N_\alpha}}. \] (2.41)

Here, for each \( K \), the \( T_{\text{moo}}(K) \) are interpolated from the coupled channels solutions for the scattering matrix elements \( T_{\text{moo}}(K_\alpha) \) of Eq. (2.37), which are calculated on a grid of \( \theta_\alpha \) and \( K_\alpha \) values. As with the semi-classical model in the previous section, the breakup \( T \)-matrix has to be multiplied by the factor \( \exp(\phi_K) \) due to the coordinate system being defined with respect to detector positions rather than the plane of wave vectors \( K_\alpha \) and \( K_0 \).

### 2.4 Post-form breakup amplitudes

In the subsequent theoretical analysis of the breakup of a 2-body projectile, we assume that the projectile only interacts with the target through the core-target interaction, \( V_{ct} \) and that the valence particle-target interaction, \( V_{vt} \) can be neglected, [38].

For a three-body system of a projectile, consisting of a core and neutral valence particle, incident with momentum \( q_p \) in the centre of mass frame on a target nucleus, the scattering wave function, \( \Psi_{q_p\sigma_0}(r, R) \), satisfies the Schrödinger equation

\[ [T_R + V_{ct}(R - \gamma_{xc}r) + H_{sc} - E]|\Psi_{q_p\sigma_0}(r, R)| = 0. \] (2.42)

Here \( H_{sc} = T_r + V_{sc}(r) \) is the internal Hamiltonian of the projectile, \( T_r \) being the kinetic energy operator for the relative motion of the core and valence particle,
and \( V_{\text{ct}}(r) \) is the core-valence potential. \( T_R \) is the projectile-target kinetic energy operator and \( V_{\text{ct}}(R - \gamma_{\text{vc}}r) \) is the interaction between the core particle and the target.

### 2.4.1 Adiabatic approximation

The adiabatic approximation in the context of three-body breakup assumes the core-valence relative excitation energies are small when compared with \( E \), the incident energy of the projectile \([47]\). \( H_{\text{vc}} \) is then replaced by a constant energy in (2.42), where this energy is chosen to be \(-\varepsilon_0\), the projectile binding energy, as defined in Eq. (2.3). Therefore, in the adiabatic limit, the three-body Schrödinger equation takes the following form

\[
[T_R + V_{\text{ct}}(R - \gamma_{\text{vc}}r) - E_0] \Psi_{q^{\text{m}o}}^{(+)}(r, R) = 0
\]  

(2.43)

where \( E_0 = E + \varepsilon_0 \) and \( \Psi_{q^{\text{m}o}}^{(+)}(r, R) \) is the approximate three-body wave function.

The exact solution of Eq. (2.43) is \([38, 48]\)

\[
\Psi_{q^{\text{m}o}}^{(+)}(r, R) = \exp(i\gamma_{\text{vc}}q_{\text{p}}r)\chi_{q^{\text{m}o}}^{(+)}(R_{\text{c}})(r|\psi, m_0)
\]  

(2.44)

where \( \chi_{q^{\text{m}o}}^{(+)}(R_{\text{c}}) \) is a distorted wave for a projectile of mass \( \mu_{\text{pt}} = m_p m_t/(m_p + m_t) \), evaluated at the position of the core particle, and \( (r|\psi, m_0) \) is the projectile bound state, given in Eq. (2.2).

The exact post-form \( T \)-matrix for the breakup of a projectile with initial momentum \( q_{\text{p}} \) to a three-body final state with core particle momentum \( q_{\text{c}} \) and valence particle momentum \( q_{\text{v}} \) is \([38, 49]\)

\[
T_{m_0\sigma'\sigma}(q_{\text{c}}q_{\text{v}}q_{\text{p}}) = \langle \chi_{q_{\text{c}}}^{(-)}(R_{\text{c}})\chi_{q_{\text{v}}}^{(\sigma')}\chi_{q_{\text{p}}}^{(\sigma)}| V_{\text{ct}}(r)| \Psi_{q^{\text{m}o}}^{(+)}(r, R) \rangle
\]  

(2.45)

where \( \chi_{q_{\text{c}}}^{(-)}(R_{\text{c}}) \) is an in-going distorted wave for the core particle and \( \chi_{q_{\text{v}}}^{(\sigma')} \) is the valence particle spin wave function with spin projection \( \sigma' \). By writing
the projectile ground state as

$$\langle r | i, m_0 \rangle = \sum_{m_\sigma \sigma_0} (\ell_0 m_{\ell_0} s_0 \sigma_0 | j_0 m_0 ) \Phi_{m_\sigma \sigma_0}^{m_0}(r) \chi_{s_0 \sigma_0}^{m_0}$$  \hspace{1cm} (2.46)$$

where $\Phi_{m_\sigma \sigma_0}^{m_0}(r) = Y_{m_\sigma \sigma_0}(r) u_{m_0}(r)/r$, and replacing in (2.45) the solution of the exact three-body Schrödinger equation, $\Psi^{(+)}_{q_{\sigma_0} m_0}(r, R)$, with $\tilde{\Psi}^{(+)}_{q_{\sigma_0} m_0}(r, R)$, we obtain the following approximation for the post-form breakup $T$-matrix in the adiabatic limit

$$T^{AD}_{m_\sigma \sigma_0}(q_{\sigma_0} q_{\sigma}, q_{\sigma}) = \sum_{m_\sigma \sigma_0} (\ell_0 m_{\ell_0} s_0 \sigma_0 | j_0 m_0 )$$

$$\times \langle\tilde{\Psi}_{q_{\sigma_0} m_0}^{(+)}(r, R) \chi_{s_0 \sigma_0}^{m_0} | V_{c\nu} | \chi^{(+)}_{q_{\sigma_0}}(\bar{R}_c) \phi_{m_\sigma \sigma_0}^{m_0}(r) \chi_{s_0 \sigma_0}^{m_0} \rangle.$$  \hspace{1cm} (2.47)$$

As detailed in [38], the post-form is chosen for the breakup $T$-matrix due to limitations in the accuracy of the approximate scattering wave-function. The factor $\Phi_{m_\sigma \sigma_0}^{m_0}(r)$ means that $\tilde{\Psi}^{(+)}_{q_{\sigma_0} m_0}(r, R)$ will vanish exponentially as $r \to \infty$. Thus, $\tilde{\Psi}^{(+)}_{q_{\sigma_0} m_0}(r, R)$ will be inaccurate for large values of $r$. However, the presence of $V_{c\nu}$ in (2.47) means that values of $r$ where $\tilde{\Psi}^{(+)}_{q_{\sigma_0} m_0}(r, R)$ is inaccurate are not considered when calculating $T^{AD}_{m_\sigma \sigma_0}$, assuming that the valence-core potential has no Coulomb part and is purely a nuclear interaction. Since one of the initial conditions when writing the three-body Schrödinger equation, (2.42), is a neutral valence particle, this criterion is automatically met.

Here, the potential $V_{c\nu}$ is assumed to be central allowing Eq. (2.47) to be integrated over spin variables and upon summing over $\sigma_0$, $T^{AD}_{m_\sigma \sigma_0}$ reduces to

$$T^{AD}_{m_\sigma \sigma_0}(q_{\sigma_0} q_{\sigma}, q_{\sigma}) = \sum_{m_\sigma \sigma_0} (\ell_0 m_{\ell_0} s_0 \sigma_0 | j_0 m_0 ) \int d\bar{R}_c \int dr e^{-i q_{\sigma_0} R_{\sigma} \chi_{q_{\sigma_0}}^{(-)}(\bar{R}_c)}$$

$$\times V_{c\nu}(r) \langle \chi^{(+)}_{q_{\sigma_0}}(\bar{R}_c) \phi_{m_\sigma \sigma_0}^{m_0}(r) \rangle.$$  \hspace{1cm} (2.48)$$

Eq. (2.48) then factorizes when the variable of integration in the first integrand is changed from $R$ to $R_c$ (with unit Jacobian)

$$T^{AD}_{m_\sigma \sigma_0}(q_{\sigma_0} q_{\sigma}, q_{\sigma}) = \sum_{m_\sigma \sigma_0} (\ell_0 m_{\ell_0} s_0 \sigma_0 | j_0 m_0 ) \langle P_{\nu} | V_{c\nu}(r) | \phi_{m_\sigma \sigma_0}^{m_0} \rangle \langle Q_{\sigma_0} | \chi_{q_{\sigma_0}}^{(-)}(\bar{R}_c) \chi_{q_{\sigma_0}}^{(+)}(\bar{R}_c) \rangle.$$  \hspace{1cm} (2.49)$$
where $P_v = q_v - \gamma_{ee}q_p$ and $Q_v = \gamma_{ee}q_v$. The second factor in (2.49) contains the reaction dynamics and, assuming the core-target interaction is pure Coulomb, can be expressed in terms of the bremsstrahlung integral \cite{50,51}. The first factor in (2.49) contains the projectile structure information and is referred to as the vertex function \cite{38,49}. By expanding the plane wave $\exp(iP_v \cdot r)$ as follows

$$
\exp(iP_v \cdot r) = 4\pi \sum_{\ell m_\ell} i^\ell j_\ell (P_v r) Y_{\ell m_\ell}(\hat{r}) Y^*_{\ell m_\ell}(\hat{P}_v)
$$

and integrating over $\hat{r}$, the vertex function can be written

$$
\langle P_v | V_{cu}(r) | \phi_{\ell_0m_0}^{m_0} \rangle = (-i)^\ell \sqrt{4\pi} D_{\ell_0m_0}(P_v) Y_{\ell_0m_0}(\hat{P}_v)
$$

where

$$
D_{\ell_0m_0}(P_v) = \sqrt{4\pi} \int_0^\infty j_\ell (P_v r) V_{cu}(r) u_{\ell_0m_0}(r) r \, dr.
$$

Hence, taking the square modulus of $T_{m_0\sigma_0}^{AD}$ and summing over $m_0$ and $\sigma_0$, we are left with the following expression for use in calculating three-body cross sections

$$
\frac{1}{2j_0 + 1} \sum_{m_0\sigma_0} |T_{m_0\sigma_0}^{AD}|^2
$$

2.4.2 DWBA approach

A different approximation that can be applied when considering three-body breakup reactions is the distorted wave Born approximation (DWBA). In DWBA the three-body scattering wave function is approximated by

$$
\Psi_{q_mq_0}^{(+)}(r, R) \approx \sum_{m_0} \langle \ell_0 m_0 g_0 | j_0 m_0 \rangle \chi_{\ell_0}^{(+)}(R) \phi_{\ell_0m_0}^{n_0}(r) \chi_{g_0}^{(-)}
$$

where $\chi_{\ell_0}^{(+)}$ is the projectile distorted wave evaluated at the projectile centre of mass position, $R$. Inserting the DWBA scattering wave function into Eq. (2.45) leads to the following expression for the approximate $T$-matrix

$$
T_{m_0\sigma_0}^{DW}(q_m, q_0, q_0) = \sum_{m_0} \langle \ell_0 m_0 g_0 | j_0 m_0 \rangle \int dR \int dr \ e^{-i\mathbf{q}_m \cdot \mathbf{R}} \chi_{\ell_0}^{(+)}(R) \int dR' \int dr' e^{-i\mathbf{q}_0 \cdot \mathbf{R}'} \chi_{g_0}^{(-)}(R') \chi_{\ell_0}^{(+)r}(R') \chi_{g_0}^{(-)}(R) V_{cu}(r) | \phi_{\ell_0m_0}^{n_0}(r) |.
$$
The core distorted wave in (2.55) can be written as

$$\chi_{qc}^{(-)}(R_c) = e^{-i\gamma_{qc} Q \cdot r} \chi_{qc}^{(-)}(R),$$  \hspace{1cm} (2.56)

where $Q = -iV_R$. If treated exactly, then (2.55) cannot be factorised in a similar way to (2.47). However, following the method of [52], the magnitude of $Q$ has been approximated by

$$Q(R_D) = \sqrt{\frac{2\mu_{tc}}{\hbar^2}(E_c - V_{ct}(R_D))}$$  \hspace{1cm} (2.57)

where $\mu_{tc}$ is the reduced mass of the core and target, $E_c$ is the asymptotic kinetic energy of the core in the c.m. frame, and $V_{ct}(R_D)$ is the core-target potential evaluated at some distance, $R_D$, between the core and the target. Previously, this approximation has been referred to as the local momentum approximation [49,52]. A more appropriate term for this approximation, and the one we shall use here, is the global momentum approximation (GMA), as the momentum, $Q$, is evaluated at some core-target separation, $R_D$, and held constant for all $R_c$. The justification for using this approximation is stated to be that the magnitude of $Q$ is virtually constant over all core-target separations that are relevant to Coulomb breakup reactions [52]. If the direction of $Q$ is taken to be the same as the direction of the core momentum, then $Q$ can be replaced by $q_c$ in (2.56). Since $Q$ is no longer dependent on $R$ using this approximation, (2.55) can now be factorised

$$\psi^{DW}(:, q_c) = \sum_{m_{\ell_0}}(\ell_0 m_{\ell_0} \delta_{\ell_0} | f_{m_0}(\tau) | P_v | V_{ct}(r) | \phi_{m_0}(\tau) (Q_c; \chi_{qc}^{(-)}|\chi_{q_c}^{(+)}))$$  \hspace{1cm} (2.58)

which is identical to Eq. (2.49) except in this case the vector $P_v = \alpha q_c - \gamma_{qc} Q$, where $\alpha = 1 - \gamma_{tc} \gamma_{qc}$. Thus, the only operational difference between this approach and the adiabatic method when calculating triple differential cross sections, is the value of $P_v$ at which $D_{f_{m_0}}(P_v)$ is evaluated in Eq. (2.53).

Factorising (2.55) into two separate integrals can also be performed by changing, instead the argument of the projectile distorted wave, from $R$ to $R_c$, [49], i.e.

$$\chi_{q_p}^{(+)}(R) = e^{i\gamma_{qc} Q \cdot r} \chi_{q_p}^{(+)}(R_c).$$  \hspace{1cm} (2.59)
Then applying the GMA to $Q$, (2.55) can to be factorised in a similar way to Eqs. (2.49) and (2.58), with $P_v = q_v - \gamma_w Q$. In fact, $T^{DW}_{\text{moe}}$ is identical to (2.49) if $Q$ is chosen to be equal to $q_p$. 
Chapter 3

First-order perturbation theory calculations for the breakup of $^8$B

3.1 Structure model for $^8$B

In this section, the structure model used for $^8$B in the 1st order breakup calculations is described. It is based on the structure model used by Esbensen and Bertsch [15], but is simplified as outlined below.

3.1.1 Ground state of $^8$B

To a very good approximation, $^8$B consists of a valence proton weakly bound to an inert $^7$Be core by 0.137 MeV. Some uncertainty remains as to whether $^8$B has a proton halo, [17], but that is not important here. The important feature of $^8$B is that it can be treated as consisting of two distinct bodies weakly bound to each other, and it is these particles that are detected after breakup.

The ground state of $^8$B is a $2^+$ state with the dominant configuration a proton in a nodeless $p_{3/2}$ orbit coupled to the $3/2^-$ ground state of the $^7$Be core. In Refs. [15, 16] it is assumed that the $^8$B ground state can be completely described using
this configuration. Here, we adopt a modified version of the structure model of Refs. [15,16]. As shown in Fig. 3.1, the $p_{3/2}$ proton is bound to the $^7$Be core, assumed to be a spectator here, and so can be assumed to be spinless giving a $3/2^-$ ground state. Thus, we have an initial (bound) state $|i, m_0\rangle$, with orbital angular momentum $\ell_0 = 1$, spin $s_0 = 1/2$ and total angular momentum $j_0 = 3/2$ with projection $m_0$.

A spherical Woods-Saxon plus spin-orbit potential is used for the core-proton nuclear interaction

$$V^{t_0}_{\text{cv}} = \left( V_0 + V_{\ell s_0} \ell_0 \cdot s_0 \frac{1}{r} \frac{d}{dr} \right) f(r) \quad (3.1)$$

where

$$f(r) = \frac{1}{1 + \exp[(r - R)/a]} \quad (3.2)$$

The parameters used in $f(r)$ are $r_0 = 1.25$ fm, where $R = r_0 A^{1/3}$ and $a = 0.52$ fm. Using these parameters, a well depth of $V_0 = 48.09$ MeV reproduces the correct binding energy of 0.137 MeV if spin-orbit forces are ignored, i.e. $V_{\ell s_0} = 0$. Including the spin-orbit force by choosing $V_{\ell s_0} = 19.59$ MeV fm$^2$ as suggested in
[15,16], a well depth of $V_0 = 44.97$ reproduces the correct binding energy. The spin orbit force is included in the ground state structure model used in subsequent calculations although as shown in Section 3.1.3, this makes little difference in the multipole strength functions for $^8\text{B}$

### 3.1.2 Continuum states

![Diagram of possible excitations of the $^8\text{B}$ ground state to continuum states through E1 and E2 transitions](image)

Figure 3.2: Possible excitations of the $^8\text{B}$ ground state to continuum states through E1 and E2 transitions

When considering the Coulomb breakup of $^8\text{B}$ to first order, both E1 and E2 transitions are important for excitations from the ground state to excited states. Therefore, starting in a $p$-wave in the ground state, the valence proton in $^8\text{B}$ can be excited into $s$ and $d$ states through E1 transitions and $p$ and $f$ states through E2 transitions, see Fig. 3.2. No excited bound states exist for $^8\text{B}$, and with coupling to the more tightly bound $s$ state being Pauli blocked, all excitations are to the continuum.

The core-proton interaction for the final states is again modelled by a Woods-Saxon plus spin-orbit potential of the form shown in Eq. (3.1). In Refs. [15,16] the well depths, $V_0$, for the different partial waves are chosen to reproduce known resonances in the $p_{3/2}$ continuum waves. However, here we will use the ground
state potential parameters for the continuum states for all partial waves.

### 3.1.3 Multipole strength functions

Here, the dipole (E1) and quadrupole (E2) strength functions are calculated as a function of core-proton relative kinetic energy using the $^8$B structure model described above. Comparing them with the strength functions of Refs. [15,16] can then be used as a check for the simplified structure model used in this analysis. Given the final state asymptotic normalisation in (2.6), the multipole strength functions are defined as

$$\frac{d\mathcal{B}(E\lambda)}{dk} = \frac{2}{\pi} \sum_{\ell j} (2j + 1) \mathcal{R}_{\ell j\lambda}(k)^2 \frac{|\langle \ell s_0 \rangle f \langle Y_{\lambda}(\vec{r}) \rangle |^{2}}{2j_0 + 1}$$

(3.3)

where $\mathcal{R}_{\ell j\lambda}(k)$ are the radial integrals involving the core-proton wavefunctions as defined in (2.23).

Eq. (3.3) is expressed as a function of core-proton relative wave number $k$, whereas the dipole and quadrupole strengths in Refs. [15,16] are plotted as functions of relative energy, $E_{\text{rel}}$. The relationship between the two is as follows

$$\frac{d\mathcal{B}(E\lambda)}{dE_{\text{rel}}} = \frac{\mu_{\text{pp}}}{\hbar^2 k} \frac{d\mathcal{B}(E\lambda)}{dk}$$

(3.4)

Figure 3.3 shows dipole and quadrupole strength functions for the Coulomb breakup of $^8$B, plotted as a function of relative energy. The strength functions have been calculated without including any spin-orbit interactions (solid curve) and with the proton in a $p_{3/2}$ orbital (dashed) and $p_{1/2}$ orbital (dot-dashed) in the ground state. Very little difference is seen in E1 strength function in choosing the valence proton to be in a $p_{1/2}$ or $p_{3/2}$ orbital in the ground state. A more noticeable difference is seen in E2 strength function for the two different ground state configurations at relative energies of approximately 0 to 2 MeV. Even so,
the errors incurred by assuming the proton to be purely in a \( ^3p_{3/2} \) orbital in the ground state are only likely to be of the order of 1%. The plots in Fig. 3.3 compare well with those for the multipole strength functions in Refs. [15,16] where the spin of the core has been included in the calculations. Therefore, taking the core spin into account does not seem to be of great importance and is neglected here. There is a difference in the E2 strength function of Refs. [15,16] in that a peak at approximately \( E_{\text{rel}} = 0.6 \) MeV is not reproduced here. This peak is included in the E2 strength function of [15,16] to reflect a resonance in the \( 1^+ \) channel due to M1 transitions. However, at the beam energies considered here, M1 transitions are small compared to E1 and E2 transitions and can be neglected.
3.2 Results of semi-classical calculations

In this section, the results of first-order perturbation theory calculations using the semi-classical method described in Section 2.2 are used in an attempt to reproduce experimental data for the breakup of $^8$B on $^{208}$Pb at 44 and 82 MeV per nucleon. The data are from an experiment performed by Davids et al at the NSCL, MSU [18].

First-order semi-classical calculations, following the methods of Alder and Winther [29] and Baur, Bertulani and Rebel [12], have often been used to analyse the Coulomb breakup of $^8$B [8–10, 15–20]. In experiments involving the breakup of a two-body projectile such as $^8$B, it is common to express the measurements as the double differential cross section as a function of the relative energy of the emerging projectile fragments and the angle of the c.m. of the projectile fragments relative to the beam direction. Attempts have been made to reproduce the relative energy spectrum data for the breakup of $^8$B with first-order semi-classical calculations in order to extract the E1 strength, e.g. [8–10, 15]. However, the cross section is not particularly sensitive to E1/E2 interference for this observable as the amplitudes for the different multipoles add incoherently. Thus, it is difficult to make an accurate prediction of the E1 strength from the relative energy spectrum as the importance of the E2 contribution to the cross section is also uncertain.

A more appropriate observable to look at when trying to understand the E1 and E2 transition strengths in the breakup of $^8$B is the parallel momentum distribution of the $^7$Be core. For this observable, the E1 and E2 amplitudes add coherently, resulting in asymmetry in the distribution [15–20]. Figure 3.4 shows parallel momentum distributions of $^7$Be from the Coulomb breakup of $^8$B on $^{208}$Pb at 44 MeV per nucleon calculated using the first-order semi-classical method. In these calculations the E1 strength, as defined by the structure model described in Section 3.1, is held constant while the percentage of the total E2 amplitude
contributing to the overall breakup cross sections is increased from 0 to 100%. This is effectively the same as modifying the structure model to adjust the E2 strength. As the E2 contribution to the total cross section is increased the distribution becomes more asymmetric as well as shifting towards higher momenta. Therefore, the asymmetry of the parallel momentum distribution caused by the E1/E2 interference effects is useful in determining the E2 contribution to the breakup cross section.

Figure 3.4: Calculated parallel momentum distributions of $^7$Be from the breakup of $^8$B on Pb at 44A MeV, showing the effect of scaling the E2 amplitude. $^7$Be $\theta_{\text{max}} = 3.5^\circ$

Figure 3.5 shows parallel momentum distributions of $^7$Be fragments from the breakup of $^8$B at 44 MeV per nucleon measured for 3 different $\theta_{\text{max}}$ values. Here, $\theta_{\text{max}}$ is the maximum scattering angle, relative to the beam direction, of the $^7$Be fragments. The solid curves are the results from semi-classical calculations using the structure model given in Section 3.1. Spin orbit forces were included in the $^7$Be-proton potentials for these calculations, although this makes little difference to the shape of the distributions. The distributions calculated without any adjustment to the E2 strength do not describe the data accurately, in both magnitude and asymmetry of the distribution's peaks. Scaling the E2 amplitude
by 0.7 to reduce the asymmetry of the distributions, as suggested in Ref. [18], gives a better description of the data for all angular cuts as shown in the right hand graph of Figure 3.5. As well as reducing the E2 strength in the calculations, these distributions have been rescaled to bring their magnitude in line with the data.

![Parallel momentum distributions for the breakup of $^8$B on Pb at 44A MeV.](image)

Figure 3.5: Parallel momentum distributions of $^7$Be from the breakup of $^8$B on Pb at 44A MeV. The curves are the results of first-order semi-classical calculations with no E2 amplitude scaling (left) and the E2 amplitude scaled by 0.7 (right).

The parallel momentum distributions for the breakup of $^8$B on Pb at 82 MeV per nucleon calculated without an E2 strength adjustment also fail to reproduce the data accurately, as shown in Fig. 3.6. The effect of rescaling the E2 amplitude by 0.7 in these calculations can be seen in the right hand graph of Fig. 3.6. Again, the distributions are rescaled to bring the magnitudes in line with the data. However, in this case the asymmetry of the calculated distributions is not reduced sufficiently to match the slope of the peaks in the data. Thus, while reducing the E2 strength in the calculations can give a good description of one set of data, the semi-classical model does not give an accurate description of all $^8$B breakup data using a consistent E2 strength.

Another inconsistency between the data and theoretical predictions is the width of the distributions. This is particularly evident at the higher energy of 82 MeV.
per nucleon. At both 44 and 82 MeV per nucleon, the contribution of nuclear induced breakup to the cross section could be significant, even at the forward scattering angle being considered here. Since the semi-classical model only takes into account Coulomb interactions between the projectile and target, this could be a reason for the discrepancy between the predicted and measured distribution widths.

### 3.3 Relativistic effects

The semi-classical method described in Section 2.2 allows relativistic effects to be included when calculating the breakup amplitude. However, the calculations of parallel momentum distributions in the previous section are not completely relativistic as the three-body kinematics used in calculating the triple differential cross section were non-relativistic.

Appendix A shows how the triple differential cross section, and hence the parallel momentum distribution, is evaluated using both relativistic and non-relativistic
three-body kinematics. A comparison of $^7$Be parallel momentum distributions calculated with and without relativistic kinematics and breakup amplitude is shown in Figure 3.7. The solid curve is the result of a completely non-relativistic semi-classical calculation for $^8$B on Pb at 44 MeV. In this calculation, the $\gamma$ factor in Eq. (2.12) is taken to be unity, and the projectile-target relative velocity is evaluated using the classical relationship for kinetic energy and velocity instead of Eq. (2.17). The dashed curve is the result of a semi-classical calculation with relativistic effects included in both the calculation of the breakup amplitude and the three-body kinematics. Also shown is the distribution from Section 3.2 for $^8$B on Pb at 44 MeV per nucleon (dotted curve), where relativistic effects are included when calculating the breakup amplitude but not in the three-body kinematics.

![Figure 3.7: Parallel momentum distributions of $^7$Be from the breakup of $^8$B on Pb at 44A MeV. The curves are the results of first order semi-classical calculations.](image)

Including relativistic effects when calculating the triple differential cross section appears to alter the magnitude parallel momentum distributions but has no effect on the shape of the distribution. Repeating the calculations for $^8$B on Pb at 82 MeV per nucleon gives similar results for the parallel momentum distributions,
as shown in Figure 3.8. The increase in magnitude as a result of using relativistic three-body kinematics is greater than at 44 MeV per nucleon, as might be expected at the higher energy, but again, there appears to be little or no change in the shape of the distribution. The position on the $p_{\parallel}$ axis of the distributions calculated relativistically is shifted towards higher momenta than those calculated non-relativistically. This effect is greater in the results of calculations for 82 MeV/nucleon then 44 MeV/nucleon. However, for comparison here, the distributions have been recentered on the data.

![Figure 3.8: Parallel momentum distributions of $^7$Be from the breakup of $^8$B on Pb at 82$\text{A}$ MeV. The curves are the results of first order semi-classical calculations.](image)

Although the semi-classical method has been formulated so that a fully relativistic calculation can be performed, the other theoretical methods described in Chapter 2 have not. However, the results in this section show that even though including relativity in calculating parallel momentum distributions results in some change in the magnitude, the shape of the distributions is not altered. Since an understanding of the asymmetry of the measured distributions is our present aim and not reproducing the magnitude, we will continue to use non-relativistic calculations in the following sections when comparing to the data. The inclu-
sion of relativistic effects in all calculations, while not vitally important here, is a problem that may need to be addressed in the future, especially as higher beam energies will be used, following the NSCL coupled cyclotrons upgrade for example.

3.4 Results of 1st order DWBA calculations

The results in the previous section show that the first-order semi-classical model does not accurately describe the breakup of $^8$B using the structure model described in Section 3.1. This could be because the structure model is incorrect. However, the semi-classical model is only an approximate method for describing Coulomb breakup dynamics. Therefore, a more detailed calculation may describe the data to greater precision without any need for a change in the structure model.

This section shows the results from (first-order) DWBA calculations. Here, first-order DWBA is regarded as the first iteration of the coupled equations in Eq. 2.36, as described in Section 2.3 and Refs. [25,26]. This is a fully quantum mechanical approach, so no semi-classical approximation is made for the projectile-target relative motion. Nuclear interactions can also be included in the core-target and proton-target potentials. However, as with the semi-classical approach, the excitation of the projectile by the target is assumed to be a one-step process, thus higher-order effects are ignored.

Figure 3.9 shows the parallel momentum distributions of $^7$Be fragments from the breakup of $^8$B on Ag at 44 and 82 MeV/nucleon. The data are from experiments conducted by Davids et al at the NSCL [20] and the calculated distributions are the results of the DWBA calculations using pure Coulomb and Coulomb plus nuclear projectile-target interactions. The results from the semi-classical calculations are also included for comparison. In all the calculations, the structure
Figure 3.9: Parallel momentum distribution of $^7$Be from the breakup of $^8$B on Ag at 44 (left) and 82 (right) MeV/nucleon. The curves are the results of first order perturbation theory calculations using the semi-classical and DWBA methods.

model in Section 3.1 is used and the E2 amplitude has not been scaled. The calculated distributions are absolute predictions and have not been rescaled to fit the data.

For the Coulomb plus nuclear DWBA calculations, the $^7$Be-target and proton-target nuclear interactions are represented using optical potentials. The parameters used for the $^7$Be-target potential are those given by Cook to fit $^7$Li elastic scattering data [53] and the proton-target potential parameters are calculated using the Becchetti and Greenlees optical model for the elastic scattering of nucleons [54]. Both Figures show that the DWBA and semi-classical calculations give similar results when only Coulomb interactions are considered. There is a small difference in magnitude of the distributions calculated using the different methods and the asymmetry is slightly reduced in the DWBA distribution. Since the structure model of $^8$B used in both calculations is identical, this gives a first indication that differences in the magnitude and shape of the parallel momentum distributions can be dependent on the reaction model used as well as the
structure model input. Including nuclear interactions in the DWBA calculations increases the magnitude of the distributions further, and also widens the distributions, bringing them closer to the data at higher and lower momenta. Therefore, it appears that nuclear induced breakup makes a significant contribution to the breakup cross section even at the forward angles considered here and cannot be ignored when trying to reproduce the data accurately, as is done if using the semi-classical (Coulex) calculations.

Figure 3.10: Parallel momentum distribution of $^7$Be from the breakup of $^8$B on Pb at 44 (left) and 82 (right) MeV/nucleon. The curves are the results of first order perturbation theory calculations using the semi-classical and DWBA methods.

Figure 3.10 shows the parallel momentum distributions of $^7$Be fragments from the breakup of $^8$B on Pb at 44 and 82 MeV/nucleon. The results of the various calculations are similar to those shown in Fig. 3.9 for $^6$B on Ag. Including Coulomb and nuclear projectile-target interactions in the DWBA calculation again gives noticeable increases in cross section when compared with the results of the pure Coulomb calculation. The width of the distributions is increased when nuclear interactions are included in the calculations, although at 82 MeV per nucleon this increase in width is not enough to bring the distribution in line with the data.

Using the DWBA method and including nuclear interactions gives a better description of the data than the semi-classical method, especially for the 44 MeV
per nucleon data. The asymmetry of the distributions is reduced and their widths are also increased. Therefore, the DWBA method does take us someway toward being able to describe the data without needing to alter the E2 strength, as is done in the semi-classical calculations. However, the DWBA calculations are still not the most complete calculations that can be performed as the projectile excitation is only treated to first-order. The importance of higher-order effects in the breakup of $^8$B has not been clarified and this issue is the subject of the next chapter.
Chapter 4

CDCC calculations for the breakup of $^8\text{B}$

Both the semi-classical and DWBA descriptions of breakup in Chapter 2 treat projectile excitation in first-order perturbation theory. That is, the excitation of the projectile by the target is assumed to be a one step process, and that after the initial breakup of the projectile no further excitations occur. However, the Coulomb force is a long range interaction, so assuming no further projectile excitation after the initial excitation in a Coulomb dominated process may not be an appropriate approximation. Since the breakup reactions for $^8\text{B}$ we are considering here are Coulomb dominated processes the importance of higher order effects needs to be considered. Previous non-perturbative calculations for similar $^8\text{B}$ breakup reactions to those investigated here have indicated the presence of significant higher-order effects in the breakup process [15, 20]. Thus, the non-perturbative coupled discretised continuum channels (CDCC) method, described in Chapter 2, is used in this chapter to investigate the importance of higher-order effects in the breakup of $^8\text{B}$.
4.1 Model space for CDCC calculations

The model space parameters for the CDCC calculations are defined as follows. For all spin-parity assignments, \( J^\pi \), the continuum is discretised up to a maximum relative energy of \( \varepsilon_{\text{max}} = 10 \text{ MeV} \) with the number of bins for each \( J^\pi \) as follows: \( J^\pi = 1/2^+ \) has 20 bins, \( J^\pi = 1/2^-, 3/2^-, 5/2^+ \) have 10 bins and \( J^\pi = 5/2^-, 7/2^- \) have 5 bins. The bins are constructed so that they are evenly spaced in \( k \), the relative wave number, from \( k = 0 \) to \( k_{\text{max}} \). When constructing each bin state, the numerical integration over \( k \) in Eq. (2.30) uses 50 intervals from \( k_{i-1} \) to \( k_i \). Figure 4.1 shows the cross section for each \( J^\pi \) state as a function of bin energy for \(^8\text{B} \) on \( \text{Pb} \) at 44 MeV/nucleon. The plots show that 10 MeV is a high enough maximum energy for the continuum discretisation for each \( J^\pi \).

![Figure 4.1: Cross section for \(^8\text{B} \) on \( \text{Pb} \) at 44 MeV/nucleon as a function of bin energy for each spin-parity state](image)

Multipoles up to \( \lambda = 2 \) and a maximum radius of 60 fm for all bins is used for the radial integrations over \( r \) when evaluating the coupling interactions, \( V^{J,\lambda,\alpha L}_{\alpha L,\alpha'L'}(r) \). Using multipoles up to \( \lambda = 3 \) made little difference to the calculations for the incident energies used in the NSCL experiments and thus, E3 couplings are ne-
glected.

For the centre of mass motion of the projectile relative to the target, partial waves up to \( L = 10000 \) and values of \( R \) up to 1000 fm are used to compute the relative motion wavefunctions \( f_{\alpha' J}(R) \). Eq. (2.36) is solved in steps of 4 in the range \( L = 0 \) to \( L = 200 \), steps of 10 in the range \( L = 200 \) to \( L = 300 \), steps of 50 in the range \( L = 300 \) to \( L = 1000 \) and steps of 200 from \( L = 1000 \) to \( L = 10000 \). The size of the steps reflects the variations in cross section over the ranges of \( L \).

The DWBA calculations of the previous chapter have the same model space as the CDCC calculations, except the bin states are coupled to the ground state in first-order only.

### 4.2 Comparison with first-order calculations

![Figure 4.2](image)

**Figure 4.2:** Parallel momentum distribution of \(^7\)Be from the breakup of \(^8\)B on Ag at 44 (left) and 82 (right) MeV/nucleon. The curves are results of semi-classical, DWBA and CDCC calculations using the assumed \(^8\)B structure model of Chapter 3.

Figure 4.2 shows the \(^7\)Be parallel momentum distributions for the breakup of \(^8\)B on Ag at 44 and 82 MeV per nucleon and Figure 4.3 shows the \(^7\)Be parallel momentum distributions for the breakup of \(^8\)B on Pb, also at 44 and 82 MeV per
nucleon. In each figure, distributions calculated using semi-classical, DWBA and coupled discretised continuum channels (CDCC) methods are shown.

Figure 4.3: Parallel momentum distribution of $^7$Be from the breakup of $^8$B on Pb at 44 (left) and 82 (right) MeV/nucleon.

The calculated distributions are absolute predictions and have not been scaled to fit the data. In the DWBA and CDCC calculations, projectile-target nuclear interactions have been included as well as Coulomb interactions. The nuclear interactions used here are the same as for the DWBA calculations of the previous chapter where the $^7$Be-target interaction is that of Cook [53] for $^7$Li and the proton-target interaction is calculated using the nucleon optical potential model of Becchetti and Greenlees [54].

Both figures show that the asymmetry of the parallel momentum distributions calculated using the CDCC method is significantly less than for the first-order calculations, which is consistent with earlier calculations [15]. Since the $^8$B structure model used in the CDCC calculations is the same as in the first-order calculations, it appears that including higher-order effects in the calculation of $^7$Be parallel momentum distributions has the same effect as reducing the E2 strength. However, we see that this effective reduction of the E2 strength results in a larger reduction of the distributions' asymmetry than is needed to describe the data.
As with the DWBA calculations, the widths of the CDCC calculated distributions do not match the data at 82 MeV per nucleon. This could be due to projectile-target nuclear interactions being significant at this energy and inaccurate nuclear potentials being used in the calculations. The parameters for $^7$Be-target nuclear interaction used in the calculations have no energy dependence, they are either fixed or depend on the mass of the target. Since these parameters were originally chosen to fit $^7$Li elastic scattering data on various targets at total projectile incident energies of around 50 MeV, the potential may not be appropriate for the projectile energies considered here. Therefore, the CDCC calculations were repeated with a $^7$Be-target nuclear potential with parameters that are dependent on the projectile incident energy. This potential was constructed using the folding model [55] and is given by

$$V_F(R) = \int dr_1 \int dr_2 \rho_1(r_1)\rho_2(r_2)\psi_{NN}((R + r_1 - r_2)).$$

(4.1)

Here $R$ is the separation of the centres of mass of the two nuclei, $r_1$ is the internal coordinate of the target and $r_2$ is the internal coordinate of the projectile, in this case $^7$Be. The mass density distribution of $^7$Be, $\rho_2$, was chosen to have a Gaussian form using an rms radius 2.27 fm, calculated from the rms charge radius of $^7$Li given in [56]. Following [57], for an Ag target the mass density distribution, $\rho_1$, is of Woods-Saxon form and has an rms radius of 4.5 fm and a diffuseness of 0.515 fm. These parameters were estimated from the Woods-Saxon parameters given for various nuclei in [57]. Again following [55], the nucleon-nucleon interaction $\psi_{NN}((R + r_1 - r_2))$ has the Yukawa form

$$\psi_{NN}((R + r_1 - r_2)) = -(v + iw)\frac{\exp[-|R + r_1 - r_2|/\lambda]}{|R + r_1 - r_2|/\lambda}.$$  

(4.2)

For $^7$Be + Ag, $v$ and $w$ were chosen to be 40 MeV and 45 MeV respectively for 44 MeV per nucleon and 35 MeV for both $v$ and $w$ at 82 MeV per nucleon with a Yukawa range of $\lambda = 0.7$ fm. These values were estimated from the $v$ and $w$ values given in [57] for several elastic scattering reactions.
Figure 4.4 compares the results of CDCC calculations for the breakup of $^8\text{B}$ on Ag at 44 and 82 MeV per nucleon, using either the Cook or the folded nuclear potential for the $^7\text{Be}+\text{Ag}$ nuclear interactions. The results of CDCC calculations including only Coulomb projectile-target interactions are also shown. The figure shows that, as with the DWBA calculations, including nuclear interactions in the calculations changes the magnitude and width of the distributions, most noticeably at 44 MeV per nucleon. However, there is little sensitivity to which of the two nuclear potentials is used in the $^7\text{Be}+\text{Ag}$ interaction. Therefore the Cook potential is used for the $^7\text{Be}$-target nuclear interaction in all subsequent calculations.
4.3 Extraction of the E2 amplitude from CDCC results

As shown in Figures 4.2 and 4.3, the parallel momentum distributions calculated using the CDCC method have a greatly reduced asymmetry when compared with the results of first-order calculations. Now the slopes of these distributions are less than that of the data, so the E2 strength has to be increased for the calculated distributions to fit the data. Figures 4.5 and 4.6 show how scaling the E2 amplitude in CDCC calculations changes the asymmetry in the ⁷Be parallel momentum distributions. To scale the E2 amplitude in CDCC calculations, the matrix elements $V_{al,a'1'}$ given in Eq. (2.34) are multiplied by the appropriate factor. The calculated distributions have all been scaled to bring their magnitude in line with the data, enabling a comparison of how well the slope of each curve fits the data. From the figures it can be seen that a factor of 1.6 gives a good description of data.

It should be noted that the important part of this analysis is how accurately the slopes of the peaks in the calculated distributions describe the data. Reproducing the magnitude of the data with the calculations is not so important. This is because of the uncertainty in the $\theta_{max}$ values of the experimental data. Figure 4.7 shows the 'height' of the measured and calculated distributions, with various E2 scaling factors, plotted against $\theta_{max}$ for ³B+Ag at 44 and 82 MeV per nucleon. The 'height' is taken as the value of $d\sigma/dp_\parallel$ at the mid-point of the distributions. For the calculated distributions the mid-points are at parallel momentum values of $p_\parallel = \sqrt{2E_cm_c}$, where $m_c$ and $E_c$ are the incident energy and mass of the core respectively, and are 2.011 GeV/c at 44 MeV/nucleon and 2.745 GeV/c at 82 MeV/nucleon. However, due to uncertainties in the experimental beam energies, the mid-points of the measured distributions are not at these $p_\parallel$ values. Thus, the calculated distributions have to be shifted along the $p_\parallel$ axis to centre on
Figure 4.5: Parallel momentum distribution of $^7$Be from the breakup of $^8$B on Ag at 44A MeV. The curves are results of CDCC calculations with the E2 amplitude multiplied by different factors.

The intrinsic angular resolution for the measurement of the $^7$Be fragments given in [20] is approximately 0.1 degrees (0.2 mrad). So, for example, a decrease in $\theta_{\text{max}}$ from 2.0° to 1.9° in the CDCC calculation with E2 amplitude scaled by 1.6 at 44 MeV/nucleon, reduces the 'height' of the distribution from 1.27 to 1.175 b/GeV/c. Similarly, decreasing $\theta_{\text{max}}$ from 1.0° to 0.9° in the 82 MeV/nucleon calculation reduces the 'height' from 0.64 to 0.54 b/GeV/c. Therefore, due to the sensitivity of the distribution magnitudes to the value of $\theta_{\text{max}}$, the error in
Figure 4.6: Parallel momentum distribution of $^7\text{Be}$ from the breakup of $^8\text{B}$ on $\text{Pb}$ at 44A MeV. The curves are results of CDCC calculations with the E2 amplitude multiplied by different factors.

The experimental value of $\theta_{\text{max}}$ can account for the discrepancy in the magnitudes of the calculated and measured distributions at 44 MeV/nucleon. However, the discrepancy in the magnitudes of the calculated and measured distributions at 82 MeV/nucleon cannot be reconciled by an angular resolution of 0.1 degrees. An angular resolution of approximately 0.2 degrees is necessary for the calculated distributions to be within the limits of uncertainty of the measured distributions using the current $^8\text{B}$ structure model with the E2 amplitude scaled by 1.6.

As shown in Figures 4.5 and 4.6, scaling the E2 amplitude by a factor of 1.6 in the CDCC calculations for $^8\text{B}$ on Ag and Pb at 44 MeV/nucleon increases the asymmetry of the parallel momentum distributions sufficiently to match that seen in the measured distributions. However, this scaling factor has only been used to fit two sets of data. Any changes to the $^8\text{B}$ structure model need to be universal and should give results that fit all sets of data. Figures 4.8 and 4.9
show the $^7$Be parallel momentum distributions for $^8$B on Ag and Pb at 44 and 82 MeV/nucleon for all available angular cuts. In each figure the predictions from CDCC calculations with no scaling of the E2 amplitude are shown in the left hand graphs and the results from CDCC calculations with the E2 amplitude scaled by 1.6 are shown in the right hand graphs.

The magnitude of the distributions has been scaled to bring them in line with the data, highlighting the effect of increasing the E2 strength. The asymmetry observed in the data at 44 MeV/nucleon is not reproduced by the calculated distributions with no E2 scaling. For the calculated distributions where the E2 amplitude has been increased by a factor of 1.6, the asymmetry seen in the measured distributions is reproduced with the exception of $^8$B on Pb at 44 MeV/nucleon for the largest angular cut of $\theta_{\text{max}} = 3.5^\circ$. At 82 MeV the asymmetry seen in the measured distributions is reproduced with and without scaling the E2 amplitude. This seems to indicate that the CDCC calculations aren't particularly sensitive to changes in the E2 strength for this incident energy. Therefore,
Figure 4.8: Parallel momentum distribution of $^7$Be from the breakup of $^8$B on Ag at 44 (lower graph) and 82 (upper graph) MeV/nucleon. Curves in the left hand graphs are the predictions from CDCC calculations using the assumed structure model of Chapter 3. Curves in the right hand graphs are the predictions from CDCC calculations with the E2 amplitude multiplied by 1.6.

scaling the E2 amplitude by a factor of 1.6 in the CDCC calculations gives a consistent description of the asymmetry seen in the measured distributions for all data in this analysis. However, as noted previously, the widths of the distributions from the 82 MeV/nucleon calculations do not match the data. Although scaling the E2 amplitude by 1.6 in the CDCC calculations results in distributions with the correct asymmetry, to reproduce this increase in E2 strength by chang-
Figure 4.9: Parallel momentum distribution of $^7$Be from the breakup of $^8$B on Pb at 44 (lower graph) and 82 (upper graph) MeV/nucleon. Curves in the left hand graphs are the predictions from CDCC calculations using the assumed structure model of Chapter 3. Curves in the right hand graphs are the predictions from CDCC calculations with the E2 amplitude multiplied by 1.6.

Figure 4.10 shows the calculated $^7$Be energy distributions for the breakup of $^8$B on $^{58}$Ni at 26 MeV. The CDCC and DWBA calculations of Ref. [28] are depicted along with CDCC calculations where the E2 amplitude has been multiplied by 1.6.
Figure 4.10: Calculated $^7$Be energy distributions for the breakup of $^8$B on $^{58}$Ni at 26 MeV for the laboratory angles indicated. All calculations use the Esbensen and Bertsch $^8$B ground state structure model.

[58]. We see that asymmetry due to E1/E2 interference distributions seen in the parallel momentum distribution for the breakup of $^8$B at higher energies is also present in the DWBA calculations. In the CDCC calculations, the asymmetry is almost completely absent due to the suppression of the E1/E2 interference by the higher-order couplings. Increasing the E2 strength in the CDCC calculations does not alter the asymmetry of the distributions in this case. The magnitude of the distributions is greatly increased however. In Ref. [28] it is shown that the experimental data are well described by the CDCC calculations without any adjustments to the $^8$B structure input. Therefore, an increase in the E2 amplitude by a factor of 1.6 may not be appropriate in this case.
Chapter 5

Calculations for breakup of $^8\text{Li}$

In the previous Chapter the results of coupled discretised continuum channels (CDCC) calculations indicate that higher-order effects play a significant role in the breakup of $^8\text{B}$ on the targets and at the beam energies used in the experiments of Davids et al [20]. However, the CDCC calculations underestimate the asymmetry in the measured $^7\text{Be}$ parallel momentum distributions of Ref. [20] and it was shown that large changes to the $^8\text{B}$ structure model might be needed to fit the data. The CDCC calculations are the most complete calculations available for analysing these breakup reactions, in that they are fully quantum mechanical, include higher-order effects and can include fragment-target nuclear interactions. Within the CDCC method there are parameters such as the maximum wave number for the core-valence relative motion, the momentum bin width and maximum radius, the choice of values for which could affect the convergence of the calculations. Thus, it would be useful to compare the results of the CDCC calculations with other calculations that include higher-order effects. One such method is that described in Section 2.4, where the breakup cross section is calculated using the adiabatic approximation to the exact post-form breakup amplitude. Unfortunately the closed form expression for the three-body wave function exists only when one fragment interacts with the target, and thus this method cannot be
applied to the breakup of nuclei with a charged core and valence particle. Therefore, to compare the results of CDCC calculations with the results of the adiabatic method we have chosen to analyse the breakup of $^8\text{Li}$ which is another nucleus with a $p$-wave valence particle but, unlike $^8\text{B}$, the valence particle is a neutron.

### 5.1 Structure model for $^8\text{Li}$

The assumed structure model of $^8\text{Li}$ is similar to that of $^8\text{B}$. The ground state consists of a neutron in a nodeless $p_{3/2}$ orbit bound to a spinless $^7\text{Li}$ core. Spin-orbit forces are ignored and a Woods-Saxon potential of the form shown in Eq. (3.2) with $a = 0.52$ fm and $r_0 = 1.25$ fm is used for bound state core-neutron interaction. A well depth of $V_0 = 47.66$ MeV reproduces the observed binding energy of 2.033 MeV [59]. For the continuum states, the same Woods-Saxon parameters are used for the core-neutron interaction as for the bound state. This may not be a completely accurate structure model but the objective of this analysis is to be able to compare the results of CDCC and adiabatic calculations for the breakup of a nucleus with a valence particle in a $p$-wave orbital, rather than trying to describe experimental data. At present, there are no suitable data available for the breakup of $^8\text{Li}$.

### 5.2 Results of calculations for the breakup of $^8\text{Li}$

The observable we will calculate is the parallel momentum distributions of the core fragment, in this case $^7\text{Li}$, produced in the breakup reaction of interest. Figure 5.1 shows the results of our calculations for the breakup of $^8\text{Li}$ on Pb at 44 MeV/nucleon for three different angular cuts of the core angular distribution. All calculations include only Coulomb interactions between projectile and the
target. The asymmetry seen in the $^7$Be distributions calculated using the CDCC, DWBA and semi-classical methods is no longer evident in the corresponding $^7$Li distributions. This can be explained by looking at the expression for the electric multipole operators given in Eq. (2.13). For $^8$Li the $e_\lambda$ factors for the dipole and quadrupole operators are

$$e_1 = \frac{3e}{8}, \quad e_2 = \frac{3e}{64}$$

(5.1)

and for $^8$B they were

$$e_1 = \frac{-3e}{8}, \quad e_2 = \frac{53e}{64}.$$  

(5.2)

Therefore, E1 transitions are far more dominant in the Coulomb breakup of $^8$Li than in the breakup of $^8$B, resulting in far less E1/E2 interference in the $^8$Li case. The CDCC, DWBA and semi-classical calculations, which are all ‘prior-form’ calculations, give distributions which, although varying in magnitude, all have the same basic shape. However, the distributions calculated using the CDCC method are centred on higher $p_{//}$ value than the DWBA and semi-classical results. This is also seen in the $^7$Be distributions calculated using these three methods, although it is not obvious from the results presented in Chapter 4 as the distributions have been re-centred on the data in those graphs.

Also shown in Figure 5.1 are the results of calculations using the adiabatic method. This method gives significantly different results to the other three methods in that the distributions are not symmetric and the cross section is greatly reduced. The asymmetry of the distributions does become less prominent as $\theta_{\text{max}}$ is reduced from 2.0° to 1.0° but does not disappear completely. One feature of the results of adiabatic calculations that is common with the CDCC results is the shifting of the distributions towards higher momenta than the results of the first-order calculations, suggesting this could be a higher-order effect. Figure 5.2 shows
the results of calculations of the cross section as a function of another observable: The relative energy of the neutron and \(^7\)Li core. As with the parallel momentum distributions, the results of the CDCC and semi-classical calculations are similar whereas the curve calculated using the adiabatic method is quite different. The peak is at a higher energy and the magnitude in the region of approximately 0-3 MeV is considerably less than that of the other curves, although the curves do converge as they go towards higher relative energy. The additional points shown in Figure 5.2 are the results of a non-perturbative time-dependent calculation [60], the details of which are described in Ref. [39] where it is used to study the Coulomb breakup of \(^{11}\)Be. This method gives results which are similar to those of the CDCC and semi-classical calculations in both magnitude and the position of the peak.

The adiabatic method may be giving conflicting results to the other calculations due to the adiabatic approximation not being valid for this reaction. In the adiabatic approximation, the internal Hamiltonian \(H_{sc}\) is replaced with a constant energy in the three-body Schrödinger equation, Eq. (2.42). This constant is chosen to be the binding energy of the projectile. However, if the binding energy is not sufficiently small enough, when compared with the projectile incident
energy, the adiabatic approximation may not be valid. With this in mind, the calculations for the breakup of $^8$Li on Pb at 44 MeV/nucleon were repeated with the binding energy of $^8$Li reduced to 0.2 MeV. Figure 5.3 shows the results of these calculations. The magnitudes of the $^7$Li parallel momentum distributions have been greatly increased as would be expected by reducing the binding energy of the projectile and the distributions are also narrower. The distributions calculated using the DWBA and CDCC methods have a similar shape to the calculations using a binding energy of 2.033 MeV but now the distribution calculated using the adiabatic method is also symmetric. However, despite now being symmetric, the adiabatic calculated distribution is not simply different from the other calculated distributions by only a scaling factor as its width is (relatively) greater than that of the other distributions.

In the absence of experimental data for the breakup of $^8$Li, it cannot be said with certainty whether the adiabatic or CDCC calculations are incorrect. Although the results of CDCC calculations are closer to the results of the semi-classical and DWBA results than the results of the adiabatic calculations, this does not automatically mean that the results of the adiabatic method are incorrect. The expressions
for the $T$-matrix in the CDCC, DWBA and semi-classical methods are based on the prior-form of the breakup $T$-matrix whereas the adiabatic method uses the post-form, so it should not be surprising that the results from the CDCC, DWBA and semi-classical calculations are all similar.

Previous calculations using the adiabatic method to make predictions for the breakup of weakly bound nuclei such as deuterons [38] and $^{11}$Be [49], have given
good descriptions of the experimental data. Also, there is better agreement between the results of the adiabatic calculations and the other theoretical methods used in this analysis for these nuclei than for $^8$Li. For example, Figure 5.4 shows the $^{10}$Be parallel momentum distribution for the reaction $^{11}$Be+$\text{Ta}$ at 63 MeV/nucleon calculated using the semi-classical method and the adiabatic method. The semi-classical calculation has identical initial and final state potentials. Although the magnitudes are still significantly different, the two distributions are almost identical if the adiabatic distribution is scaled to bring its magnitude in line with that of the semi-classical distribution. However, the deuteron and $^{11}$Be have valence particles in an $s$-wave orbital whereas the valence neutron in $^8$Li is in a $p$-wave orbital and this appears to have a significant effect on the results of the different calculations. The CDCC method has already been shown in Chapter 4 to give results that reproduce, to a fair degree of accuracy, the experimental data from the breakup reactions of a nucleus with a $p$-wave valence particle, namely $^8$B. Therefore, it is likely that although the adiabatic method is reliable for projectiles with $s$-wave valence particles, it cannot be used for nuclei with $p$-state projectiles.

A possible reason for this is revealed by examining the form of the vertex functions, $D_{\ell_0j_0}$, for the different nuclei, shown in Figure 5.5. In the adiabatic method, the structure information of the projectile enters through the vertex function, so it is this part of the expression for the $T$-matrix that is sensitive to differences in the structure of nuclei. From Figure 5.5 we can see that the characteristics of the $D_{\ell_0j_0}$ for nuclei with $s$-wave valence particles are quite different to those with $p$-wave valence particles at small values of $P_\nu$. In the adiabatic method, $P_\nu$ is equal to $|q_v - \gamma_0 q_p|$, the momentum transferred to the valence particle in breakup.

For deuterons and $^{11}$Be, $D_{\ell_0j_0}$ has a maximum at $P_\nu = 0$ whereas for $^8$Li $D_{\ell_0j_0} = 0$ at $P_\nu = 0$ and increases to a maximum at $P_\nu \approx 1$. Therefore, the sensitivity of the
cross section to changes in $P_v$ for low momentum transfers is greater for $p$-state projectiles than $s$-state projectiles.

### 5.3 Post-form finite range DWBA calculations

In Section 2.4, another expression for the breakup $T$-matrix was derived by inserting the DWBA form of the three-body scattering wave function in the post-form $T$-matrix and applying the global momentum approximation (GMA) to either the entrance or final channel distorted wave. This expression is similar to the $T$-matrix derived using the adiabatic approximation to the three-body scattering wave function, the only difference being the precise expression for $P_v$ used in evaluating the vertex function. If the GMA is applied to the entrance channel distorted wave [49] then $P_v = q_v - \gamma_{sc} Q$ where $Q$ is the effective momentum of the projectile, $q_v$ is the final state valence particle momentum in the projectile-target c.m. frame and $\gamma_{sc} = m_v/(m_c + m_v)$. Applying the GMA to the final channel core distorted wave [52] gives $P_v = \alpha q_v - \gamma_{sc} Q$ where $Q$ is now the effective core momentum and $\alpha = 1 - \gamma_{tc} \gamma_{sc} (\gamma_{tc} = m_t/(m_c + m_t))$. However, in both cases it is not clear what choice should be made for either the magnitude
or direction of the effective momentum $Q$.

Figure 5.6 shows the $^7\text{Li}$ parallel momentum distributions for the breakup of $^8\text{Li}$ on Pb at 44 MeV/nucleon calculated using various choices for $Q$. The solid curves in the figure show the results of calculations where the vertex function is evaluated using $P_v = q_v - \gamma_{\text{pc}}Q$ with $Q$ equal to the projectile incident momentum, $q_p$ [49]. In this case the expression for the $T$-matrix is identical to that of the adiabatic approximation to the post-form $T$-matrix but is suggested to arise from the DWBA. The dashed curves are the result of choosing the magnitude $Q$ to be that of $q_p$ but taking the direction to be parallel to the direction $K$, the asymptotic momentum of the c.m. of the projectile fragments in the final state. The dotted curves are the results of evaluating the vertex function at $P_v = \alpha q_v - \gamma_{\text{pc}}Q$ with $Q = q_c$. This is the same choice of direction for the effective momentum as used in the calculations in [52] which are claimed to be finite range DWBA calculations. However, in [52] the magnitude of the effective core momentum is evaluated as that in a Coulomb potential at $R_D = 10$ fm in Eq. (2.57) rather than $R_D = \infty$, although it is stated that the magnitude of $Q$ remains constant for values of $R_D$ above 10 fm. The figures show that the different choices of effective momentum make only small differences to the parallel momentum distributions. Evaluating the vertex function using $P_v = \alpha q_v - \gamma_{\text{pc}}P_c$ gives distributions which are far more similar to the distributions calculated using the adiabatic method than the results of DWBA calculations using the prior-form $T$-matrix (Fig 5.1).

Such discrepancies between the post and prior calculations are not entirely unexpected. The region of the three-body scattering wave function which is important in evaluating the $T$-matrix in the post-form adiabatic method is quite different from that in the CDCC, prior-form DWBA and semi-classical methods. In the post-form $T$-matrix element, the presence of the core-valence potential ensures that only small values of $|r|$ are important in evaluating the integral over $r$,
assuming the valence particle is uncharged. This is desirable in the adiabatic method due to the stated inaccuracies in the adiabatic approximation to the three-body wave function at large values of |r|. The prior-form of the breakup $T$-matrix contains the core-target and valence-target interactions. Therefore, for a Coulomb dominated breakup reaction, any approximations to the three-body scattering wave function need to be valid over much larger regions of configuration space due to the long range of the Coulomb interaction between the target and projectile fragments.

We note that in the DWBA approximations to the post-form $T$-matrix, the GMA is applied to the projectile distorted wave in the DWBA wave function so that the $T$-matrix can be factorised into two separate integrals. Approximating the projectile distorted wave in this way for all values of $R$ means that the DWBA wave function now contains a factor $\exp(i\gamma\omega Q\cdot r)$ giving it a similar form to the adiabatic scattering wave function. Unlike the adiabatic approximation to the exact scattering wave function, which retains breakup components through its complex dependence on $r$, the DWBA wave function has vanishing overlaps with the projectile excited states. However, this form now clearly has non-vanishing overlaps with the projectile excited states which contradicts the definition of the

Figure 5.6: Parallel momentum distribution of $^7\text{Li}$ from the breakup of $^8\text{Li}$ on Pb at 44A MeV. See text for an explanation of calculations
DWBA wave function. Similarly, applying the GMA to the core distorted wave also introduces further inaccuracies into the DWBA post-form $T$-matrix. In this case, the core distorted wave is now evaluated at $R_c$ rather than $R_c$ and the expression in the left hand side of the matrix element is no longer a solution of the Hamiltonian describing the motion of the projectile fragments relative to the target in the absence of a core-valence interaction. Therefore, one would not expect the application of the GMA to either the core or projectile distorted wave to give a good approximation to the exact finite range DWBA $T$-matrix.

Previously, the results of post and prior calculations for the breakup of $s$-state nuclei have been compared [61]. The adiabatic method was shown to give almost identical results to those from calculation using the semi-classical approximation to the prior-form DWBA $T$-matrix in the zero range limit. In this limit, the adiabatic approximation to the post-form $T$-matrix reduced to that of the DWBA. The agreement therefore reflects rather accurate post-prior DWBA agreement in the zero-range limit. Since the zero-range approximation can only be applied to $s$-state projectiles, a similar post-prior comparison in the zero-range limit is not possible for $p$-wave projectiles. When including finite-range effects, the post and prior calculations do not agree for either $s$ or $p$-state projectiles, as is shown in this chapter.

The disagreement between the post and prior DWBA calculations shown in this chapter may not be entirely due to the inaccuracies introduced by using the GMA in the evaluation of the post-form $T$-matrix. One would not necessarily expect even the exact post and prior DWBA methods to give identical results, even though they do agree in the zero-range limit, due to the interaction being treated to first-order is different in each case. In the post-form DWBA matrix element, the core-valence interaction is treated to first-order, whereas in the prior-form DWBA, it is the interaction of the projectile fragments with the target that is treated to first order. The adiabatic and CDCC theories however are both non-
perturbative and thus, the differences between the results of these methods do not arise from treating the interactions to first-order. Since practical methods for evaluating the exact post-form DWBA T-matrix have not yet been developed, a final comparison between the results post-prior DWBA methods is not possible.

In summary, we have seen that the results of the adiabatic and CDCC calculations do not agree, despite both being non-perturbative theories. The disagreement is greater for p-state projectiles than for s-state projectiles, possibly due to the greater sensitivity of the vertex function to small changes in the magnitude of the momentum transfer, \( P_v \), at small values of \( P_v \), for p-state projectiles. The global momentum approximation (GMA) was used in evaluating the post-form DWBA T-matrix. Applying the GMA allowed the post-form DWBA T-matrix to be factorised in a similar way to the adiabatic post-form T-matrix. However, the nature of the GMA raised questions as to whether it is a good approximation to the post-form DWBA T-matrix. The GMA approach introduced a free parameter \( Q \), the effective momentum at which the vertex function is evaluated. Various choices of effective momentum were used in evaluating the T-matrix and gave similar results to the adiabatic method (Fig 5.6) whereas the prior DWBA method gave results similar to the (non-perturbative) CDCC calculations (Fig 5.1). In the zero-range limit, the post and prior DWBA methods give almost identical results. However, a post-prior DWBA comparison using a realistic finite-range core-valence potential was not possible as the exact post-form DWBA T-matrix cannot be evaluated.
Chapter 6

Summary and conclusions

The Coulomb breakup of $^8$B on Ag and Pb targets at 44 and 82 MeV/nucleon has been investigated using several theoretical descriptions of the Coulomb breakup process. A fully three-body description of reaction kinematics was developed enabling the calculation of the triple differential cross section. The parallel momentum distributions of the $^7$Be fragments produced in the breakup were studied in trying to find the E1 and E2 contributions to the breakup cross section. E1/E2 interference causes asymmetry in the distributions and this asymmetry is used in trying to understand the E1 and E2 transition strengths.

As an initial attempt to reproduce the measured $^7$Be parallel momentum distributions, first-order perturbation theory calculations were performed. A modified version of the Esbensen and Bertsch single particle $^8$B structure model was adopted for the calculations, with the ground state configuration taken to be a $p_{3/2}$ proton bound to a spinless $^7$Be core. Little error was expected in neglecting the $p_{1/2}$ proton configuration in the ground state and assuming a 100% $p_{3/2}$ state. It was found that first-order semi-classical calculations overestimated the asymmetry observed in the measured distributions. Reducing the E2 amplitude to 70% in the calculations reproduced the asymmetry of the measured distributions at 44 MeV/nucleon. However, calculations of the parallel momentum distributions at
82 MeV/nucleon with the same E2 adjustment still overestimated the asymmetry seen in the data. The discrepancy between the asymmetry seen in the calculated and measured distributions was far greater than any error due to assuming a purely $p_{3/2}$ proton configuration in the ground state.

Fully quantum mechanical (first-order) DWBA calculations of the $^7$Be parallel momentum distributions had asymmetry that was less than seen in the semi-classical calculations. Thus, using the DWBA method did go someway toward reproducing the data without having to alter the assumed structure model. Both the semi-classical and DWBA first-order calculations failed to reproduce the widths of the measured distributions using only pure Coulomb projectile-target interactions. The disagreement with the data was especially evident at 82 MeV/nucleon and therefore it was thought there may be significant nuclear induced breakup contributions to the cross section. Core-target and proton-target nuclear interactions can be included in the DWBA method as well as the Coulomb interactions. Including these interactions in the DWBA calculations gave an increase in magnitude and width of the parallel momentum distributions, although there was still some discrepancy between the widths of the calculated and measured distributions at 82 MeV/nucleon. Therefore, these results indicated nuclear induced breakup does make a significant contribution to the total breakup cross section in the reactions being investigated here.

The semi-classical and DWBA methods, in treating the projectile excitation to first-order, ignore higher-order effects that may be present in the breakup process. Coupled discretised continuum channels (CDCC) calculations were performed to study the importance of higher-order effects in the $^8$B breakup reactions being investigated here. The parallel momentum distributions calculated using the CDCC method had a greatly reduced asymmetry when compared with the first-order calculations, which is consistent with the results of previous higher-order calculations. Therefore, higher-order effects appear to play an important role
in the breakup process and suppress the E1/E2 interference. However, the suppression of the E1/E2 interference was such that the asymmetry seen in the measured distributions at 44 MeV/nucleon was underestimated. For the CDCC calculations to reproduce the observed asymmetry, it was found that an E2 amplitude 1.6 times that defined by the assumed structure model was needed. Again, this discrepancy between the measured and calculated distributions is far greater than the errors due to approximations made to the ground state configuration. The CDCC calculations at 82 MeV/nucleon described the asymmetry seen in the data for both Ag and Pb targets without any need for an adjustment to the E2 strength. Increasing the E2 amplitude by 1.6 in this case produced only a small change in the asymmetry of the calculated distributions and the asymmetry seen in the data was still well described.

Thus, we have a theoretical description of the breakup process that reproduces the asymmetry seen in all the available data sets for the breakup of $^8$B on Ag and Pb targets using a consistent $^8$B structure model (modified Esbensen and Bertsch model with the E2 amplitude multiplied by 1.6). However, to reproduce the increase in the E2 strength needed to describe the data by altering the $^8$B structure model would require large changes to the structure model that may not be physically justifiable. It was also seen that increasing the E2 amplitude by a factor of 1.6 in the CDCC calculations for the breakup of $^8$B on $^{56}$Ni at the much lower beam energy of 26 MeV, results in a considerable increase (up to a factor of 2) in the magnitude of the energy distributions. While there is no noticeable change in the asymmetry of the distributions, the magnitude of the data is overestimated. The data were previously well described without any E2 enhancement.

As a check, the CDCC method was compared with the adiabatic method, which also includes higher-order effects. Unfortunately, the adiabatic method can only be applied to the breakup of projectiles where one of the fragments is uncharged,
so a comparison with the CDCC calculations for the breakup of $^9$B was not possible. Instead, the breakup of $^8$Li on Pb at 44 MeV/nucleon was studied. Like $^8$B, $^8$Li is a weakly bound nucleus with the valence particle in a $p$-state, except the valence particle is a neutron instead of a proton. As with the $^7$Be distributions, the CDCC results were similar to those of the other 'prior-form' calculations i.e, the DWBA and semi-classical methods. The 'post-form' adiabatic calculations however, gave distributions which were markedly different in shape and magnitude to those calculated using the CDCC method.

In the absence of experimental data for the breakup of $^8$Li, it is not certain which of the two calculations is correct. However, since the CDCC method was reasonably successful in reproducing the breakup data for $^8$B, it is likely that the adiabatic method is incorrect in the case of $^8$Li. For the breakup of $s$-state projectiles where the adiabatic approximation is valid i.e, where the incident projectile energy is much greater than the core-valence excitation energies, good agreement has been seen between experimental data and adiabatic calculations. Also, the agreement between adiabatic and prior-form calculations, while not exact, is better for $s$-state projectiles than for $^8$Li. Therefore, it appears that the adiabatic method is applicable to the breakup of $s$-state projectiles but fails when considering non-$s$-state projectiles. The failure of the adiabatic method is thought to arise due to the characteristics of the vertex functions of non-$s$-state nuclei, through which the projectile structure information enters in the adiabatic method.

The global momentum approximation (GMA) was used in evaluating the post-form DWBA $T$-matrix. In this approach, the expression obtained for the breakup $T$-matrix in very similar to that of the adiabatic method, the only operational difference being the form of $P_\gamma$ appearing in the vertex function. Within the GMA the exact form of $P_\gamma$ is not specified and is a free parameter in this approach. Various different choices of $P_\gamma$ were used in calculating the $^7$Li parallel momen-
tum distributions for the breakup of $^8$Li on Pb at 44 MeV/nucleon. The results were all similar to those of the adiabatic calculations rather than the prior-form DWBA calculations. The nature of the GMA raised questions as to whether it is an appropriate approximation to make in evaluating the post-form DWBA $T$-matrix element. However, the disagreement between the post and prior DWBA calculations is not necessary entirely due to inaccuracies resulting from the GMA approach. In evaluating the post-form DWBA $T$-matrix, the core-valence interaction is treated to first order whereas in the prior-form of the DWBA $T$-matrix it is the fragment-target interactions that are treated to first-order. Thus, even the exact post and prior DWBA calculations may disagree.

6.1 Achievements

- A theoretical method was developed to calculate the triple differential cross sections for the breakup of weakly bound, two-body projectiles using the Alder and Winther (high energy) semi-classical model for Coulomb excitation.
- Parallel momentum distributions for the breakup of $^8$B on Pb and Ag at 44 MeV/nucleon were calculated using this method which were consistent with results of first-order semi-classical calculations in previous studies of this breakup reaction.
- Relativistic kinematics were used to derive an expression for the density of states in calculating the triple differential breakup cross section. Parallel momentum distributions were then calculated using a fully relativistic semi-classical method.
- All measured parallel momentum distributions from the NSCL $^8$B breakup experiments were compared with CDCC calculations for the first time. The asymmetry seen in the measured distributions was described by the CDCC calculations using a consistent $^8$B structure model of the Esbensen and Bertsch model with
the E2 strength multiplied by 1.6.

- The large discrepancies between the results of theories using post and prior forms of the breakup T-matrix for the breakup of p-state projectiles were highlighted for the first time.

### 6.2 Future work

The calculations of $^8$B breakup presented in this work incorporate a structure model where the ground state configuration of $^8$B is assumed for simplicity to consist of a $p_{3/2}$ proton weakly bound to an inert, spinless $^7$Be core. Calculations using a more detailed structure model, including state mixing in the ground state and core spin and excitation, could be performed. To gain further insight into the post-prior disagreement seen in both first-order and all-order theories, it would be useful to have experimental data for the breakup of a p-state projectile with a neutral valence particle such as $^8$Li with which to compare the theoretical calculations. Developing the practical methods needed to evaluate the exact post-form DWBA and all-order T-matrix elements would also allow a final comparison of the post and prior methods.
Appendix A

Triple differential cross sections

Quite generally, the cross-section for a three-body breakup reaction with particles emerging in momentum intervals \( \Delta p_v, \Delta p_c, \Delta p_t \) can be written

\[
\sigma = \frac{2\pi}{\hbar v_i} \int \frac{dp_v}{\Delta p_v} \int \frac{dp_c}{\Delta p_c} \int \frac{dp_t}{\Delta p_t} \frac{1}{(2j_0 + 1)} \sum_{m_o' \sigma'} \frac{|T_{m_o' \sigma'}|^2}{(2\pi \hbar)^6} 
\times \delta(p_f - p_i) \delta(E_f - E_i - Q).
\]  

(A.1)

Here \( T_{m_o' \sigma'} \) is the breakup \( T \)-matrix, calculated using the various methods described in Chapter 2, and \( v_i \) is the initial projectile-target relative velocity. The subscripts \( v, c \) and \( t \) refer to the valence, core and target particles respectively and \( i \) and \( f \) refer to the initial and final states. The triple differential cross section can be evaluated using either relativistic or non-relativistic three-body kinematics.

A.1 Non-relativistic three-body kinematics

The infinitesimal volumes in momentum space in Eq. (A.1) have the form

\[
dp = p^2 dp d\Omega \\
= mp dE d\Omega.
\]  

(A.2)
Thus, Eq. (A.1) can be rewritten as

$$
\sigma = \frac{2\pi}{\hbar v_i} \int dE_v d\Omega_v \int dE_c d\Omega_c \int d\mathbf{p}_c \frac{1}{(2j_0 + 1)} \sum_{m_{ao}} \frac{|T_{m_{ao}}|^2}{(2\pi \hbar)^6} \times m_o p_o m_c p_c \delta(p_f - p_i) \delta(E_f - E_i - Q). \tag{A.3}
$$

Integrating over the momentum of the (unobserved) target, \(p_t\), will eliminate the momentum conserving \(\delta\)-function, \(\delta(p_f - p_i)\), leading to the following constraint on the final state particle momenta

$$
p_f = p_{tot} = p_o + p_c + p_t = p_t. \tag{A.4}
$$

Thus, we can write the triple differential cross-section for detection of particle \(c, v\) in \(d\Omega_c, d\Omega_v\), and particle \(c\) in \(E_c \to E_c + dE_c\) as

$$
\frac{d^3\sigma}{d\Omega_c d\Omega_v dE_c} = \frac{2\pi}{\hbar v_i} \int dE_v \frac{1}{(2j_0 + 1)} \sum_{m_{ao}} \frac{|T_{m_{ao}}|^2}{(2\pi \hbar)^6} \times m_o p_o m_c p_c \delta(E_f - E_i - Q). \tag{A.5}
$$

where the final state particle momenta are now constrained by the relationship given in Eq. (A.4). The projectile-target relative velocity \(v_i\) is

$$
v_i = \sqrt{\frac{2E_p}{m_p}}. \tag{A.6}
$$

where \(E_p\) is the projectile incident kinetic energy in the laboratory frame and \(m_p\) is the projectile mass.

To evaluate the remaining integral in Eq. (A.5), we follow the method of Fuchs [62] and change the variable of integration from \(E_v\) to \(E_f\) using the Jacobian

$$
\frac{\partial E_f}{\partial E_v} = \frac{1}{m_c} \left( m_t + m_v - \frac{m_v (p_{tot} - p_c) \cdot p_v}{p_v^2} \right). \tag{A.7}
$$

Integrating the right hand side of Eq. (A.5) eliminates the energy conserving \(\delta\)-function, fixing the total final state kinetic energy at

$$
E_f = E_v + E_c + E_t = E_i + Q \tag{A.8}
$$

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and the triple differential cross section reduces to
\[
\frac{d^3\sigma}{d\Omega_a d\Omega_c dE_c} = \frac{2\pi}{7} \frac{1}{2j_0 + 1} \sum_{m_{\sigma'}} |T_{m_{\sigma'}}|^2 \rho(E_a, \Omega_a, \Omega_c). \tag{A.9}
\]
Here the density of states, \(\rho(E_c, \Omega_c, \Omega_v)\), is equal to [62]
\[
\rho(E_c, \Omega_c, \Omega_v) = \frac{\left[ m_v p_v m_c p_c \right]}{(2\pi \hbar)^6} \frac{\left[ \frac{\partial E_f}{\partial E_v} \right]}{\left[ \frac{\partial E_v}{\partial E_v} \right]} \bigg|_{E_f = E_v + Q} \\
= \frac{m_v p_v m_c p_c}{(2\pi \hbar)^6 (m_v m_c m_e m_l)}.
\]

The triple differential cross-section is calculated for a range of detected \(\theta\) and \(\phi\) angles and energies for the observed particle, in this case the core, and a range of detected angles for the second (valence) particle in the frame (either laboratory or c.m.) of interest. Therefore, for each \(\theta_v, \phi_v, \theta_c, \phi_c, E_c\) combination, the momentum of the core and the direction of the valence particle momentum is known. From Eqs. (A.4) and (A.8) the following expression is obtained to calculate the magnitude of the valence particle momentum
\[
E_i + E_{bind} = \frac{p_y^2}{2m_v} + \frac{p_e^2}{2m_e} + \left( \frac{p_{tot} - p_v - p_c}{2m_c} \right)^2
\]
where \(E_{bind}\) is the binding energy of projectile. \(E_{bind}\) is the equivalent of \(Q\) in the energy conserving \(\delta\)-function and is defined as a negative value. The total initial kinetic energy, \(E_i\), is equal to \(E_p\), in the laboratory frame, or \(m_e/m_{tot}E_p\) in the c.m. frame. All quantities in Eq. (A.11) are known except the magnitude of \(p_v\). Hence, by solving (A.11) for \(p_v\), the valence particle momentum can be found. The final state wave vector of the c.m. of the projectile fragments \(K\) and core-valence relative wave vector \(k\) can then be expressed in terms of the final state particle momenta as follows
\[
\hbar K = \hbar K = p_v + p_c - m_v + m_e \cdot p_{tot}
\]
where \(m_{tot} = m_v + m_e + m_c\), and
\[
\hbar k = \frac{m_e}{m_v + m_e} p_v - \frac{m_v}{m_v + m_e} p_c.
\]
A.2 Relativistic three-body kinematics

Relativistically, a particle with rest mass \( m \) and momentum \( p \) will have kinetic energy

\[
E = (p^2 c^2 + m^2 c^4)^{1/2} - mc^2. \tag{A.14}
\]

Thus, the infinitesimal volumes in momentum space in Eq. (A.1) now have the form

\[
dp = p^2 dpdΩ = \frac{p}{c^3} (p^2 c^2 + m^2 c^4)^{1/2} dEdQ. \tag{A.15}
\]

For this derivation, \( m_e, m_c \) and \( m_t \) will refer to the rest masses of the three particles and all energies will be kinetic rather than total energy. We also confine ourselves to the cross section in the laboratory frame.

Using the above expression for \( dp \), the triple differential cross section has the form

\[
\frac{d^3σ}{dΩ_vdΩ_c dE_c} = \frac{2π}{\hbar ν} \int dE_v \sum_{m_{00'}} \frac{1}{(2π\hbar)^6} \frac{|T_m|²}{c²} \frac{p_{v} (p_{v} c^2 + m_{v} c^4)^{1/2}}{\left(2j_0 + 1\right)} \times \frac{p_{e} (p_{e} c^2 + m_{e} c^4)^{1/2}}{c^2} \delta(E_f - E_i - Q). \tag{A.16}
\]

with the constraint of the final state particle momenta given in (A.4) and the projectile-target relative velocity defined as

\[
\frac{v_i}{c} = \frac{\sqrt{γ^2 - 1}}{γ}, \tag{A.17}
\]

where

\[
γ = \frac{E_p}{m_p c^2} + 1. \tag{A.18}
\]
As in the non-relativistic case, the integral in Eq. (A.16) is evaluated by changing the variable of integration from $E_y$ to $E_f$. To find the Jacobian used in this change of variable, we note that

$$\frac{\partial E_f}{\partial E_y} = \frac{\partial E_f}{\partial p_y} \frac{dp_y}{dE_y}, \quad (A.19)$$

and

$$E_f = (p_y^2c^2 + m_y^2c^4)^{1/2} + (p_c^2c^2 + m_c^2c^4)^{1/2}$$

$$+ \left[ (p_{tot} - p_y - p_c)^2c^2 + m_{tot}^2c^2 \right]^{1/2} - m_{tot}c^2. \quad (A.20)$$

Differentiating (A.20) with respect to $p_y$

$$\frac{\partial E_f}{\partial p_y} = \frac{p_y c}{(p_y^2 + m_y^2c^2)^{1/2}} - \frac{c(p_{tot} - p_y - p_c).p_y}{[(p_{tot} - p_y - p_c)^2 + m_y^2c^2]^{1/2}} \quad (A.21)$$

and $E_y$ with respect to $p_y$

$$\frac{dE_y}{dp_y} = \frac{p_y c}{(p_y^2 + m_y^2c^2)^{1/2}} \quad (A.22)$$

leads to the following expression for the Jacobian

$$\frac{\partial E_f}{\partial E_y} = 1 - \frac{(p_y^2 + m_y^2c^2)^{1/2}(p_{tot} - p_y - p_c).p_y}{p_y^2[(p_{tot} - p_y - p_c)^2 + m_y^2c^2]^{1/2}}. \quad (A.23)$$

Therefore, the expression for the triple differential cross section is the same as in Eq. (A.9) except the density of states $\rho(E_c, \Omega_c, \Omega_v)$, is now equal to

$$\rho(E_c, \Omega_c, \Omega_v) = \left[ \frac{1}{(2\pi\hbar)^6} \frac{p_y}{c^2} \left( p_y^2c^2 + m_y^2c^4 \right)^{1/2} \frac{p_c}{c^2} \left( p_c^2c^2 + m_c^2c^4 \right)^{1/2} \right]$$

$$\div \left[ 1 - \frac{(p_y^2 + m_y^2c^2)^{1/2}(p_{tot} - p_y - p_c).p_y}{p_y^2[(p_{tot} - p_y - p_c)^2 + m_y^2c^2]^{1/2}} \right]. \quad (A.24)$$

If $p_v << m_v c, p_c << m_c c$ and $(p_{tot} - p_v - p_c) << m_v c$, Eq. (A.24) reduces to

$$\rho(E_c, \Omega_c, \Omega_v) \approx \left[ \frac{p_v m_v p_c m_c}{(2\pi\hbar)^6} \right] \left[ 1 - \frac{m_v c (p_{tot} - p_v - p_c).p_v}{p_y^2 m_v c} \right] m_v p_y m_c p_c m_c$$

$$= \frac{m_v p_y m_c p_c m_c}{(2\pi\hbar)^6(m_v + m_c (p_{tot} - p_c).p_v/p_y^2)} \quad (A.25)$$
which is the non-relativistic expression for $\rho(E_c, \Omega_c, \Omega_v)$ given in Eq. (A.10).

The constraints placed on the final state particle momenta and energies by the energy and momentum conserving $\delta$-functions in (A.16) give the following expression

\[ E_i + E_{\text{bind}} = \left( p_i c^2 + m_v c^4 \right)^{1/2} + \left( p_c c^2 + m_c c^4 \right)^{1/2} + \left( p_{\text{tot}} - p_v - p_c \right)^2 c^2 + m_{\text{tot}} c^2. \]  

(A.26)

By solving the above equation for $p_v$ the valence particle final state momentum can be found. Here, the laboratory frame is chosen as the frame of interest. Hence, the initial total kinetic energy, $E_i$, is equal to the incident kinetic energy of the projectile, $E_p$.

The wave vectors $k$ and $K$ are connected to the final state particle momenta through the following relations

\[ \hbar K = p'_c + p'_v \]  

(A.27)

and

\[ \hbar k = \frac{m_v}{m_v + m_c} p'_c - \frac{m_c}{m_v + m_c} p'_v \]  

(A.28)

where $p'_c$ and $p'_v$ are the core and valence momenta in the centre of mass frame of the three-body system. However, if the values of $\theta_c, \phi_c, E_c, \theta_v, \phi_v$ are given in the laboratory frame, then $p_v$ and $p_c$ will be calculated in the laboratory frame. Therefore, to find $k$ and $K$ we need to find how $p_v$ and $p_c$ are related to $p'_c$ and $p'_v$. This is done by performing a Lorentz boost to the centre of mass frame of the three-body system i.e., the frame where the total momentum is zero. Then using this transform, $p_v$ and $p_c$ can be transformed to the c.m. frame.

From Eq. (A.4), the final state total momentum in the laboratory frame is equal to the incident momentum of the projectile. The laboratory frame is chosen so that the direction of the incident momentum of the projectile is along the $z$ axis.
Thus the Lorentz transformation from the laboratory to the c.m. frame for the total momentum 4-vector is

\[
\begin{pmatrix}
\frac{E_{cm}}{c} \\
0 \\
0 \\
0
\end{pmatrix} =
\begin{pmatrix}
\gamma_{cm} & 0 & 0 & -\beta_{cm}\gamma_{cm} \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
-\beta_{cm}\gamma_{cm} & 0 & 0 & \gamma_{cm}
\end{pmatrix}
\begin{pmatrix}
\frac{E_p}{c} + \frac{E_{bind}}{c} + m_{tot}c \\
0 \\
0 \\
0 \\
\end{pmatrix}.
\]

From the above matrix equation, \( \beta_{cm} \) is

\[
\beta_{cm} = \frac{p_{tot}}{\frac{E_p}{c} + \frac{E_{bind}}{c} + m_{tot}c} \quad (A.29)
\]

and given \( \beta_{cm}, \gamma_{cm} = (1 - \beta_{cm}^2)^{-1/2} \).

Using this transformation, the particle momenta in the c.m. frame of the 3-body system are

\[
\begin{pmatrix}
p_x' \\
p_y' \\
p_z'
\end{pmatrix} =
\begin{pmatrix}
p_x \\
p_y \\
\gamma_{cm} \left( p_z - \frac{p_{tot}(E/c + m)}{E_p/c + E_{bind}/c + m_{tot}} \right)
\end{pmatrix}.
\]

Thus, \( \mathbf{k} \) and \( \mathbf{K} \) can now be calculated.

### A.3 Parallel momentum distributions

For core particle parallel momentum distributions, we need to calculate the differential cross section \( \frac{d\sigma}{dp_{el/mm}} \), where \( p_{el/mm} \) is the component of core momentum along the beam axis, which in this case is the z-axis. Therefore

\[
\frac{d\sigma}{dp_{el/mm}} = \int d\Omega_y \int dp_{ex} \int dp_{ey} \left( \frac{d^3\sigma}{d\Omega_y dp_e} \right) . \quad (A.30)
\]
The limits of the momentum integrals are controlled by the measured aperture. In Eq. (A.30), the triple differential cross section $d^3\sigma/d\Omega_0 dp_c$ is defined in terms of a differential of core momentum, whereas in Eq. (A.9) it is defined as a differential in core energy. To find how the two are related, we note that

$$\int dp_c \left( \frac{d^3\sigma}{d\Omega_0 dp_c} \right) = \int dE_c d\Omega_0 \left( \frac{d^3\sigma}{d\Omega_0 d\Omega_0 dE_c} \right), \quad (A.31)$$

Therefore, using either the non-relativistic (A.2) or relativistic (A.15) expression for $dp$ in we find that

$$\frac{d^3\sigma}{d\Omega_0 dp_c} = \frac{1}{p_c m_c} \frac{d^3\sigma}{d\Omega_0 d\Omega_0 dE_c} \quad (A.32)$$

or

$$\frac{d^3\sigma}{d\Omega_0 dp_c} = \frac{c^2}{p_c(E_c + m_c c^2)} \frac{d^3\sigma}{d\Omega_0 d\Omega_0 dE_c} \quad (A.33)$$
Bibliography


[58] I. J. Thompson, private communication.


[60] Vladimir Melezhik, private communication.