THE INITIAL FLOW IN A SHOCK TUBE

A Thesis submitted to the

University of London for the degree of

Doctor of Philosophy

by


Mechanical Engineering Department
University of Surrey
Abstract

The well-known idealised theory of the simple shock-tube predicts values of the various flow properties, many of which are in good accord with experimental observations particularly for cases involving only weak shocks.

Important departures from the above predictions can occur however, both in the flows at considerable distances from the diaphragm and those in its immediate vicinity. The former are mainly attributable to the finite opening-time of the diaphragm while the latter arise chiefly as a result of the boundary layer which develops at the walls of the tube; both these mechanisms are ignored in ideal shock-tube theory.

The present work is concerned principally with the initial stages of the flow and its chief aim is to determine the influence of the dynamic behaviour of the diaphragm on the flows developing in the tube. An experimental and theoretical study has been made of the static and dynamic behaviour of shock-tube diaphragms; the information obtained from this has been incorporated into a computer-formulation of the initial flow in the tube.

Experimental measurements have been made which provide confirmation of many of the computed results, particularly in regard to shock trajectories, shock-formation distances and pressure histories in the vicinity of the diaphragm during the opening process.
Acknowledgements

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No list of this sort would be complete without particular mention of the contribution made by the author's friend and colleague Dr. Lyn Davies of the Royal Aircraft Establishment, Farnborough, and Visiting Senior Lecturer in the Mechanical Engineering Department. Dr. Davies gave invaluable advice and assistance culminating in a full proof-reading of the thesis, as a result of which many improvements were made.

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**Notation**

a - Sonic velocity
A - Sonic velocity ratio, area
D - Diameter; distance of hole in light-mask from clamped edge of diaphragm
e - Specific internal energy
E - Internal energy ratio
f - Stress
D - Function of x, θ and u (Equation 7.12)
G - Mach-number function \( \frac{2/(\gamma - 1) + M^2}{\alpha} \)
K - Ratio of transverse diaphragm-groove elongation to distance from clamped edge.
L - Half-width of shock-tube
m - Moment on element of diaphragm petal
M - Mach-number; moment on complete diaphragm petal
N - Dimensionless slope of characteristic line \( \frac{1}{a_n} \frac{dx}{dt} \)
p - Static pressure
P - Pressure ratio—Riemann variable \( \frac{2/(\gamma - 1)}{\alpha} a + u \)
Q - Riemann variable \( \frac{2/(\gamma - 1)}{\alpha} a - u \)
r,R - Radius
S - Factor expressing influence of deflection of diaphragm on bursting strength, \( \frac{4}{L/\delta + \delta/L} \)
t - Thickness; time
T - Temperature
u,v - Velocity components in x and y directions respectively
w - Width of diaphragm groove
x,y - Cartesian co-ordinate axes
Greek Symbols

\( \alpha = \frac{(\gamma + 1)}{(\gamma - 1)} \)
\( \beta = \frac{2\gamma}{(\gamma - 1)} \)
\( \gamma \) - Specific heat ratio
\( \Gamma \) - Ratio of \( \gamma \)-values
\( \delta \) - Axial displacement of diaphragm surface
\( \tau \) - Shock-strength perturbation due to boundary layer action
\( \theta \) - Angle
\( \rho \) - Density
\( \tau \) - Dimensionless time \((a_4 t/L)\)
\( \phi \) - Angle of rotation of diaphragm petal
\( \phi, \psi \) - Constants in simple-harmonic petal-folding equation
\( \frac{(\theta - \phi)}{(\theta_0 - \phi)} = \cos \psi t \theta - \phi \)
\( \omega \) - Angular velocity

Special Notation

helium/air - denotes the combination of driver/driven gases
\( X_{i,j} \) - dimensionless ratio e.g. of flow properties, thermodynamic variables \( x_i/x_j \)
**Subscripts**

1, 2, 3, 4 - Relating to regions in the x-t plane representation of ideal shock-tube flow

13 - Taken along a P-wave between points 1 and 3
23 - Taken along a Q-wave between points 2 and 3
b - Due to bending stresses
d - Relating to diaphragm material or diaphragm exit-region
e - Effective value
f - Value taken at the shock formation point
g - Due to gas pressure
i - Inertia
ij - Ratio of i th to j th parameter
P, Q - Value taken along P or Q wave
R - Value taken at the point R or radius r
S - Value taken at the point S
ts - Ratio of temporal to spatial quantity
0 - Initial value
1D - Based on one-dimensional theory

**Superscripts**

- - Mean value
\(\wedge\) - Maximum value
\(*\) - Critical value
CHAPTER 1
INTRODUCTION

1.1. The Simple Shock Tube.

The shock tube is a device by means of which gas flows of short duration may be produced. In its simplest form it consists of a straight uniform duct divided into two compartments by a thin diaphragm. One of the compartments, the driven section is filled with gas at low pressure while the other compartment contains gas at a higher pressure. Removal of the diaphragm, either by piercing to induce rupture or by increasing the pressure in the driver section until spontaneous bursting occurs, produces a flow in which quasi-steady regions are separated by waves of finite amplitude.

The wave motion in the driven section is basically compressive and culminates, at a short distance from the diaphragm, in a plane normal shock travelling at supersonic speed into the undisturbed gas ahead of it. The energy required to sustain this motion is transferred from the high-pressure gas stored in the driver section by means of an expansion or rarefaction wave, the head of which travels at the undisturbed sonic velocity into the driver gas.

A zone of demarcation exists between the compressed driven-section gas and the expanded driver gas which in an idealised form is represented as a plane interface or contact surface. The static pressure and gas particle velocity are respectively equal on either side of this contact surface but the density, temperature and indeed the gases themselves may be different.
The non-instantaneous formation of the primary normal shock in a shock tube is a result of the finite opening time of real diaphragms for which the inertia and, for metal diaphragms, the bending resistance of the roots of the torn material are jointly responsible.

Existing experimental measurements of this quantity (1.1, 1.2)\(^*\) indicate values of up to \(10^{-3}\) s. Such times are not negligible in the context of shock-tube flows since during this time, a strong shock may travel a distance of several metres which is comparable with the total length of the driven section. The geometry of the diaphragm at any instant during the opening process, must influence the local flow properties and this influence is transmitted by wave motion to the flows in the driver and driven sections.

The trajectory of the shock for an appreciable portion of its motion along the driven section, and the corresponding properties of the flow in the driver and driven sections may thus be influenced by the motion of the diaphragm.

A knowledge of the trajectory of the shock and of the properties of the flows in the driver and driven sections is of considerable importance in all practical applications of the shock tube, the most important of which are outlined in Section 1.2.

No coherent theoretical analysis exists at present of the influence of the diaphragm on the development of the flows in the driver and driven sections and the main aim of the present work has been to provide such an analysis, and to compare its predictions with relevant experimental data.

* Numbers in parentheses relate to references.
1.2. Historical Development and Current Applications

The basic diaphragm-type shock tube has evolved little since the initial experiments of Vielle in 1899 (1.3) but developments in photographic techniques made after some three decades by Cranz and Schardin (1.4) and later by Payman and Shepherd (1.5) established its potentialities as a research tool. This later work paved the way for the considerable growth in the utilisation of the shock-tube which occurred during the Second World War and its aftermath. A comprehensive review of progress in these formative years of shock-tube research appears in the work of Glass and Hall (1.6.)

Currently the shock-tube has a variety of applications most of which exploit its facility for generating gas flows of high enthalpy even though these are necessarily of only short duration.

In the field of aeronautics, one of the principal means of producing flows in the hypersonic régime is the reflected shock tunnel (1.7.) in which the shock-wave system in a simple shock-tube is used to produce gas of high enthalpy which is then expanded through a nozzle to the required velocity. Although running times are short, a considerable degree of expansion of the gas is possible giving hypersonic Mach numbers while avoiding the liquefaction problems which can arise for example when blow-down facilities are used for this purpose.

Applications also arise in chemical research and in high-temperature gas-physics, by virtue of the facility of the shock-tube for producing very rapid heating of a sample of gas or vapour.
Prior to the introduction of the shock-tube, the gas or vapour was heated by contact with a hot surface, with consequent wall effects and finite time scale, and with an upper limit to the temperatures being imposed by the melting point of the surface material. In contrast, the shock-tube gives uniform and almost instantaneous heating of the gas or vapour over a range of temperatures extending far beyond the previous limits.

This greatly facilitates the study of chemical reactions initiated by high temperatures and also provides conditions appropriate to the study of internal energy exchanges in gas molecules. Following a rapid increase in the translational energy of the molecules caused by the arrival of a strong shock, these molecular energy exchanges can occur between other degrees of freedom and processes of rotational and vibrational relaxation and even electronic excitation and ionisation may arise.

An ingenious variant, the single-pulse shock-tube (1.6) facilitates the estimation of high-temperature chemical reaction rates. In this device, the end wall of the driver-section of a conventional shock-tube is replaced by a diaphragm which separates the driver-section initially from a large evacuated vessel. The enforced rupture of the second diaphragm at a known time interval after the rupture of the main diaphragm produces a rarefaction which quenches the reaction initiated by the shock and subsequent analysis indicates the amount of reaction and products.
A comprehensive account of chemical and physical applications of the shock-tube has been given by Gaydon and Hurle (1.9.) and also by Toennies and Greene (1.10).

A more recent development has been the use of shock-tubes to investigate the motion of shock waves at discontinuities in ducts, simulating the effects of explosions in industrial pipeline systems(1.11, 1.12).

For these and all other practical applications of the shock-tube, a knowledge of the true trajectory and of the flow properties produced from given initial conditions is a basic necessity.

In chemical studies for example the kinematics of the shock and, where appropriate, the quenching rarefaction are of crucial importance in determining the dwell-time of the high-temperature pulse and the opening processes of the two diaphragms involved have a direct influence on this dwell time.

In the field of hypersonic shock-tunnel technology a reliable prediction of the shock formation distance associated with a given diaphragm would be helpful at the design stage of a new tunnel. White's experiments (1.1) have shown, for example that the formation of a shock at $M_s = 10.0$ in air can occupy a distance in excess of 40 feet; uniform reservoir conditions at entry to the nozzle are obtainable only when the length of the driven section of the shock-tube exceeds the shock formation distance.
In the absence of a suitable analytical treatment of the shock trajectory and flow properties in terms of known initial conditions, a measure of empiricism is currently necessary in practical shock-tube usage. The present work is aimed at replacing this empiricism with an analytical approach and the relevance of this to all practical applications of shock-tube constitutes the main justification for the project.
1.3. Ideal Shock-Tube Flow

The idealised shock-tube theory developed independently by Schardin (1.15) and Taylor, working in association with Payman (1.5) is well documented in standard literature (1.6, 1.13, 1.14). It postulates that the diaphragm is removed instantaneously on bursting, that the driver and driven-section gases are inviscid and perfect and that no mixing or heat-transfer occur between them. Heat losses to the tube walls are also ignored. The resulting flow is illustrated in Fig 1.1 on axes of time vs distance along the tube, using the notation adopted by Glass and Hall (1.6). In this notation any wave travelling to the right i.e. in the positive x-direction relative to the gas ahead of it is termed a P-wave and any wave travelling to the left relative to the gas ahead of it is termed a Q-wave. This terminology is adopted for the remainder of the present work. The idealised shock-tube flow thus consists of a P-shock, a centred Q-rarefaction and a contact surface separating fields of quasi-steady flow as shown in Fig. 1.1.

The primary shock, on arrival at the end wall of the tube undergoes normal reflection and travels back towards the diaphragm station. In so doing it increases still further the pressure and temperature of the gas from region 2 which has already been processed by the primary shock and a state of very high enthalpy can be produced in the stationary gas in region 5, Fig.1.1.
When the reflected shock encounters the oncoming contact surface it crosses the latter and enters region 3, continuing as a Q-shock, but a P-wave is reflected from the intersection point which may be an expansion, a Mach wave or a shock depending on whether the quantity $E_{32} - (\alpha_3 + P_3)/(\alpha_2 + P_2)$ is greater than, equal to or less than zero. (Appendix 1.)

The optimum condition in the context of reflected-shock hypersonic tunnel operation, in the interests of achieving maximum running time with uniform stagnation conditions in region 5 Fig.1.1 is that a Mach wave be reflected from the intersection. This gives a negligible disturbance to the hot "reservoir" gas adjacent to the end wall of the tube and is referred to as the "tailoring" condition.
1.4. Flow in Real Shock-Tubes

1.4.1. Introduction

In view of the simplicity of the idealised shock-tube theory many of its predictions are in remarkably good agreement with actual observations over a wide range of initial gas conditions. Nevertheless, discrepancies do arise in practice which are due to the omissions implicit in the simplifying assumptions discussed in Section 1.3.

For example, although the maximum observed shock velocity for a given case, may be closely approximated by simple theory it is found in practice that this velocity is maintained for only a relatively small proportion of the total travel of the shock. A considerable amount of data on this subject has been amassed over the past three decades and an extensive bibliography is listed in standard literature (l.6). General agreement exists that, following an initial period of fairly rapid acceleration in the vicinity of the diaphragm, the shock proceeds to decelerate slowly in its subsequent motion along the tube (Fig. 2).

The finite distance travelled by the shock before it reaches full strength is attributable to the non-instantaneous opening of the diaphragm, while the slow deceleration of the fully-formed shock is due to the growth of a boundary layer at the tube walls. These two mechanisms are the principal causes of departure from ideal shock tube behaviour and while a considerable amount of research effort has been devoted to the study of shock-tube boundary layer effects, surprisingly few attempts have been made to account for the influence of real diaphragm opening on the flow, none of which is universally
applicable to the prediction of the flow properties from a
general set of initial gas conditions and diaphragm properties.

The existing attempts at a solution to this problem are
outlined briefly in Section 1.4.2 and discussed in more detail in
Chapter 8.

Also outlined in Section 1.4 are those aspects of non-ideal
shock-tube behaviour on which the motion of the diaphragm and its
influence on the initial flow in the shock-tube have a direct
bearing.
1.4.2 Diaphragm Opening Effects.

The influence of the diaphragm, though acknowledged in descriptions of the initial flow in the shock-tube, was omitted from early analyses of the problem. For example, in a semi-empirical approach (1.16) the observed motion of the leading portion of the contact zone, obtained from a series of photographs taken at different time-delays was treated as that of a piston emitting compression waves into the driven section. The analogy with the classical accelerated-piston problem is unhelpful in the present context however since even if the mass of the piston is neglected, a finite force must be applied to it in order to balance that due to the pressure difference between the forward face, which emits compressions, and the rear face which emits expansions. This force also provides the input of energy in the form of mechanical work which is transferred through the wave system, to the gas particles.

A contact surface in contrast, is characterised by mechanical equilibrium i.e. equality of static pressure and velocity on both sides and is therefore entirely passive in the sense of energy transfer or of the application of axial load to the flow.

The diaphragm itself is essentially the only active mechanical element present and all flow disturbances transmitted along P- and Q-waves must originate here.
The analysis formulated by White (1.1) assumes that the primary shock is formed as a result of a coalescence of a train of isentropic P-compression waves which, for simplicity, are considered to meet at a single point in the x-t plane (Fig. 1.3). The "focussing" of the isentropic waves at this point gives rise to an instantaneously-formed shock and a reflected Q-expansion wave and contact surface are also produced. This model allows calculation of the primary shock strength on an alternative basis to that of ideal theory and while agreement with the experimental results obtained by White is inferior to that of ideal theory at moderate shock strengths, these positions are reversed at high shock strengths.

However White's theory does not incorporate any aspect of the influence of the diaphragm on the flow, the focussed train of P-compressions being a convenient artifice which is unconnected in any formal manner with the diaphragm opening process.

The work of Kireyev (1.17) however, established a link between the instantaneous degree of opening of the diaphragm and the flows in the driven and driver sections of the tube, by means of a quasi-steady 1-dimensional treatment of the flow in the diaphragm region.

Certain aspects of this model are open to question; for example, as discussed further in Chapter 3, the assumption of one-dimensional plane flow in the diaphragm region is unrealistic particularly during the early stages of opening and may be replaced by an almost equally simple and more realistic radial-flow model.
Also, the flow leaving the diaphragm exit is assumed to pass through an isentropic steady expansion in attaining the full tube cross-sectional area, but calculations show (Chapter 8) that when a realistic representation of the diaphragm opening history is used in conjunction with this assumption, the initial P-wave propagating along the driven section becomes an expansion, which implies that the driven gas particles travel towards the diaphragm!

Nevertheless the overall emphasis of the approach is that of the linking of the flow in the driver and driven sections with the instantaneous flow in the diaphragm region itself and as such constitutes the first attempt at a comprehensive solution to the initial flow problem in a shock-tube.

The more recent work of Ikui, Matsuo and Nagai (1.18) ignores the contribution made by Kireyev and reverts instead to a variation of White's approach. The shock formation process is represented as a coalescence, not of a single "focussed" train of isentropic P-compressions, but of a series of such trains, each coming to a focus exactly on the shock trajectory (Fig. 1.4). However, a reflected Q-wave and a contact surface are produced at each focussing point and a complex interaction field is set up as incoming trains of P-waves intersect existing waves and contact surfaces, and the values of the local flow parameters are accordingly modified. This difficulty is avoided in White's much simpler model using single-point coalescence but is present in the formulation of Ikui et al and has been omitted from their calculations.
Moreover, in common with White's analysis, the motion of the diaphragm has not been taken into account and the time-scale of the emission of the coalescing wave trains is undefined. Finally, the descriptive representation of the formation process represents the contact surface in the role of a piston (Fig.1.4) a point discussed earlier in this section.

The most recent approach to the shock formation problem is due to Satofuka (1.19) who used a two dimensional numerical analysis, formulated for computer solution, to determine the flow properties. This type of analysis constitutes the repeated solution of the equations of fluid motion for a two-dimensional network of cells covering the flow-field and is described in Chapter 8.

Among the principal findings produced by Satofuka's work are that while the shock is initially curved, it becomes practically plane after having travelled for only about 3 tube diameters though it attains full strength only after at least 10 diameters more. However the velocity distributions calculated at various time intervals after the opening of the diaphragm indicate that the flow is virtually one-dimensional at some 5 diameters from the diaphragm and the two-dimensional treatment of the flow beyond this point could be replaced by a one-dimensional approach.

The motion of the diaphragm is represented as a series of step-changes of area, based on a linear time-dependence rather than on a more realistic representation of the motion of an actual diaphragm though the opening time itself is based on measured values.
However, the most serious obstacle to the use of the two-dimensional numerical approach is that, on account of the averaging process necessary for the calculation of flow properties in each cell, it is not applicable to cases of dissimilar driver/driven gas combinations and is therefore ruled out in the context of strong shock operation which invariably calls for the use of a light driver-gas used in conjunction with a driven-section gas of greater molecular weight.

Because of the deficiencies which exist in all present analyses of the initial flow problem in the shock tube, the need remains for a formulation which given the initial gas conditions and diaphragm properties, can predict the performance of the tube, giving details of the flow properties in various regions and also of the primary shock trajectory.

The analysis developed in Chapter 9 is aimed at fulfilling all these requirements but in common with all previous analyses of the problem it assumes that the gases are inviscid and that they obey the perfect-gas laws.

Deviations must therefore occur from this more realistic analysis which is essentially applicable only to the region adjacent to the diaphragm. Viscous attenuation effects are certain to influence the shock formation process when this extends over an appreciable proportion of the total tube length and these effects are discussed in sections 1.4.3. and 1.4.4.
The non-ideal behaviour of real gases is also omitted from the present analysis. The use of Mollier charts gives the information necessary to solve the shock wave equations at high shock-strengths and indeed a computer programme already exists giving shock tube performance subject to idealised diaphragm opening conditions (1.20).
1.4.3. Shock Attenuation

When the primary shock has attained its maximum velocity at a short distance from the diaphragm, it proceeds to decelerate slowly throughout its subsequent motion (Fig. 1.2). This gradual attenuation process is directly attributable to the growth of a boundary layer (Fig. 1.5) at the tube walls in the quasi-steady regions 2 and 3. (Fig.1.1).

In Mirels' formulation (1.21) of the process of shock attenuation, the boundary layer is envisaged as a mechanism by which mass is removed from the inviscid core-region of the flow. The resulting perturbations are equivalent to those produced by a transverse velocity at the boundaries of the flow, and appear as a train of rarefactions which overtake and coalesce with the primary shock thus weakening it. These rarefactions also produce an increase in the velocity of the contact surface.

On this basis, since the boundary-layer thickness at any point in the flow increases continuously as the shock recedes farther along the tube, the running time in state 2, Fig. 1.1. should first increase to a maximum, then decrease progressively to zero. In actual measurements however, (1.22, 1.23, 1.24) the running time available at any station, which is the time interval between the arrival of the shock and the arrival of the front of the contact region, is found to remain practically constant and the subsequent motion of the shock is practically uniform.
Both results are, of course, at variance with the predictions of ideal theory in which the running time increases continuously in proportion to the overall length of the tube.

Studies of the maximum running-time in a shock-tube involve situations in which the boundary layer accumulates an appreciable thickness and in this context, Miresl's perturbation analysis becomes inappropriate. An alternative formulation based on Duff's "leaky piston" analogy (1,22) has been devised (1.23, 1.24, 1.31) in which the problem is reduced to a steady state model in shock-fixed co-ordinates (Fig. 1.6) and it is then apparent that the boundary layer is a mechanism by which leakage of hot test-gas occurs from region 2 to the cold region 3.

The net mass-flow into region 2 is then the difference between the mass flow entering through the shock front and that passing, in the boundary layer, through the front of the contact region. The maximum running time is approached asymptotically as this leakage outflow from region 2 increases and finally becomes equal to the total inflow through the primary shock.

In calculations based on this model, it is assumed that the shock moves with uniform velocity and that the density in region 2 is constant. However, agreement with measured values of running time is obtained only when an empirical matching process is carried out (1.24) as a modification to the ideal shock trajectory.
This process is necessitated by the non-instantaneous nature of the shock formation process and involves the calculation of an effective origin for an ideal instantaneously formed shock.

A realistic estimate of the actual shock trajectory in this region based on an analysis taking into account the finite opening time of the diaphragm might replace the empiricism necessary at present in this context.
1.4.4 Effects of Viscosity on the Reflected Shock

When the primary shock reflects from the end wall of the shock tube and travels into the gas in region 2 (Fig. 1.1.) it encounters non-uniform incident flow conditions characterised by an inviscid central core-region and a boundary layer forming at the tube walls. Axial variations of the flow properties can occur as a result of the perturbations imparted by this boundary layer (1.21.) and also as a result of the non-instantaneous diaphragm opening process. As a further departure from ideal conditions, the interaction of the reflected shock with the boundary layer can produce bifurcation of the portion of the shock adjacent to the wall (1.25) Fig. 1.7.

In the mathematical formulation of this effect proposed by Mark (1.26) the boundary layer is treated as being stationary relative to the tube walls, and of uniform static pressure and density. When the stagnation pressure in this layer, with velocity reckoned in shock-fixed co-ordinates, becomes less than the static pressure in region 5, the boundary-layer gas is unable to pass through the shock and is trapped in a growing "bubble" which causes a steady increase in the height of the bifurcation point, an effect observed in detailed studies of the bifurcation phenomenon (1.25, 1.27).

A more recent analysis using a realistic boundary layer velocity profile based on the theory of Mirels has been presented by Davies (1.28) in which the conditions necessary to give bifurcation and the influence of the phenomenon on the flow in the vicinity are discussed.
The "vortex layer" sketched in Fig. 1.7. arises because of the difference in velocities in regions 5 and 5'. In region 5 the gas is at rest having passed through the normal portion of the reflected shock. In region 5' however, the gas has traversed the two oblique shocks forming the bifurcated portion and because these induce a smaller velocity change than does the adjacent normal shock, the flow in region 5' has a finite velocity directed towards the end wall of the tube. While this effect is relatively unimportant when the reflected shock is passing through the hot gas in region 2, after interaction with the contact surface the reflected shock passes into the cold driver-gas region 3 and as shown by Davies, if bifurcation persists under these circumstances, it provides a mechanism by which cold gas is transported to the end wall to arrive much earlier than does the contact surface as a whole. Such an effect has been observed experimentally (1.29) and has implications of the utmost seriousness in the context of reflected-shock hypersonic tunnel operation. A basic requirement is to obtain the longest possible duration of uniform "reservoir" conditions in order to allow adequate time for flow establishment and the contamination caused by the arrival of cold driver gas effectively terminates the useful portion of the run.

From the foregoing sections it is clear that an understanding of the mechanisms associated with the interaction of the reflected shock with the flow in region 2 and 3 (Fig.1.1) is a necessary pre-requisite to the calculation of tailoring conditions for shock-tunnel operation. At present a complete formulation of all the viscous effects mentioned, taking into account the influence of the non-instantaneous diaphragm opening is not available and the experimental approach is used in the attainment of the required conditions.
1.4.5. Contact Surface Effects

An analysis by Taylor (1.30) of the behaviour of a fluid interface separating regions of different densities shows that instability can result when the interface is given an acceleration normal to its surface and directed towards the side of greater density.

Markstein (1.32) has applied this analysis to the idealised contact surface in a shock tube and has shown that instability can occur after interaction with the reflected shock when initial waviness is present in the form of the contact surface and when the density in the hot-flow region 2 exceeds that in region 3.

Such an occurrence would provide a possible explanation for the contamination and early cooling of the test gas in a hypersonic shock tunnel. Calculations have been performed which show that the tendency towards instability increases with shock Mach number and "neutral - stability" Mach numbers corresponding to equal densities on either side of the contact surface have been evaluated for various driver/driven gas combinations. In practice these are very similar to those which tend also to produce bifurcation thus making an experimental resolution of the two effects rather difficult, though conditions have been suggested by Davies (1.28) which would allow a separate investigation of the two phenomena.

A major weakness of the explanation of driver-gas contamination in shock tunnels based on contact surface instability is that it relies on the existence, in the early stages of the flow, of a density interface.
It is clear from examination of relevant schlieren photographs (Ref. 1.25 and Plate 9.1) that the region in question is one of highly turbulent mixing which, even if initially approximating to a density interface, would rapidly develop into a smeared profile and while this would not eliminate completely the possibility of transport of discrete masses of cold driver-gas, by longitudinal buoyancy forces, the corresponding axial pressure- gradients would diminish in proportion to the increased length of the mixing region. Levine (1.33) has recently estimated, on the basis of a consideration of these buoyancy forces, the maximum velocity with which a discrete element of fluid from the cold-gas region behind the contact surface might travel forwards relative to the contact surface, and has termed this the "mixing velocity", associating it with the mixing process in the contact zone. Interferometric measurements reported in the same paper may be interpreted as evidence for contact-surface instability, at high shock Mach numbers, in that for conditions giving a large value of the mixing velocity, the test-gas sample, region 2 in Fig. 1.1., could disappear completely, the primary shock being followed immediately by a cold mixing region.

However, Duff (1.22) has earlier explained the disappearance of the hot test-gas region in cases of low initial driven-gas pressure \( p_1 \) on the basis of calculations showing that the cold boundary layer adjacent to the tube walls is, by virtue of its relatively high density, capable of absorbing the entire quantity of gas entering the region through the shock. Under these circumstances the driver gas advances to fill the space behind the shock. Levine's experiments were carried out at values of \( p_1 \) in the range 1.0 to 2.0 torr but no estimate was made of the relative importance of the boundary-layer mass in this context.
The importance of the influence of the properties of the contact surface on the flow in the shock-tube is emphasised in the foregoing discussion. Analyses featuring the contact surface almost invariably assume the idealised form of interface but evidence exists (1.25, 1.32) that in real shock-tube flows, a mixing region of finite width replaces the idealised contact surface.

Even if mixing between the driver and driven gases is discounted, the driver gas which emerges from the partially open diaphragm must form a region of non-uniform flow properties in the driven section, and the length of this region must relate directly to the opening time of the diaphragm.

The estimation of this length and of the local values of the flow properties is one of the products of an analysis of the shock-formation process and the substitution of such a model of the flow in this region, in place of the idealised interface used in current analyses of contact-region instability might yield more realistic results. It could also replace the idealised interface in the calculation of tailoring conditions in hypersonic shock-tunnels.

1.5. Objectives and Basic Approach of the Present Work.

The primary objective of the present work is to obtain a clearer understanding of the processes by which the non-instantaneous opening of the diaphragm in a simple shock-tube influences the motion of the shock and expansion waves and the properties of the flows in the driver and driven sections.
A necessary pre-requisite to the above is an accurate knowledge of the motion of real diaphragms and in Chapters 2 and 3 existing experimental and theoretical work in this area is discussed and a more realistic alternative analysis to the plane one-dimensional model used by Kireyev is proposed, for the calculation of the flow in the diaphragm regions itself.

The discussion is limited throughout to the case of metal diaphragms which by virtue of the greater tensile strength of the basic material, are applicable over a much wider range of pressure than is the case for non-metallic diaphragms.

An account is given in Chapter 4 of the present experimental work on the static strength of diaphragms of various materials. The use of these results in conjunction with a simple theoretical model of the deflected form of the diaphragm at the bursting point, gives a method of predicting the bursting pressure, from a knowledge of the tensile strength of the material.

Experimental measurements of the opening times of shock tube diaphragms are reported in Chapter 5 and comparison between these results and theoretical predictions are made in Chapter 6.

On the basis of known information concerning the motion of real shock-tube diaphragms a method-of-characteristics analysis is presented in Chapter 7, which tests the validity of the assumption of quasi-steady flow used in the determination of the motion of the diaphragm.
Since the indications from this confirm the general validity of the quasi-steady analysis, following a review of existing analyses of the shock-formation problem in Chapter 8, an alternative analysis is presented in Chapter 9, based on the quasi-steady diaphragm flow model. This seeks to combine the most realistic features of existing analyses while avoiding their less desirable aspects.

A summary is given in Chapter 10 of the results of calculations based on this analysis in comparison with existing experimental data, current experimental results and with results from other theoretical analyses.
CHAPTER 2.
Simplified Calculations of Diaphragm Motion

2.1. Diaphragm Failure Mechanism

Under the applied pressure-differences commonly used in shock-tube operation, metal diaphragms undergo plastic deformation over a considerable proportion of their total area. For the small thickness which are normal in practice, bending stresses are negligible compared with the "membrane" tensile stress produced at the bursting pressure (Section 3.1.). For unscribed diaphragms, this stress is equal to the ultimate tensile stress of the material at the point where cracks first form.

It is common practice to cut or scribe grooves in metal diaphragms in order to ensure that these cracks propagate in a controlled manner and transform the stressed membrane into separate "petals" which then fold about their clamped edges until they contact the tube walls.

The presence of grooves also ensures that the bursting pressure is reduced when compared with that of ungrooved diaphragms and this in turn reduces the likelihood of fragmentation of the diaphragm and the consequent risk of damage to windows, probes, etc in the low-pressure channel.
2.2: Idealized Minimum Opening Time

Some previous calculations of diaphragm petal motion, (1.2, 2.1, 2.2) have assumed, as an approximation, that the processes described in Section 2.1. take place without any reduction in the pressure applied to the high-pressure face of the petal. The pressure at the low-pressure face has not been included in this type of analysis.

The diaphragms used in the present square-section shock tube were milled with diagonal grooves which produced four identical triangular petals on bursting (Fig. 2.1.) Taking the shock tube internal cross section to be \(2L \times 2L\), the moment about the clamped edge of each petal, due to gas pressure forces is, on the above basis

\[
M_g = \frac{p_4 L^2}{3}
\]  

2.1

The moment of inertia of a petal about the clamped edge is

\[
M_i = \frac{\rho_d t_d L^4}{6}
\]  

2.2

The bending resistance \(M_b\) has been assumed (1.2, 2.1, 2.2) to be the yield-moment of an ideally-plastic prismatic cantilever beam of cross-section dimensions equal to those of the petal roots.

Thus:

\[
M_b = f_d t_d^2 L / 2
\]  

2.3

The equation of motion of the petals then becomes

\[
\theta = \frac{(2p_4 / t_d - 3f_d t_d / L^2)}{\rho_d L}
\]  

2.4

Assuming the petals rotate through an angle of \(90^\circ\), the opening time on the basis of equation 2.4 is:

\[
t_o = \left\{p_d t_d L^3 / (2p_4 L^2 - 3f_d t_d^2)\right\}^{1/2}
\]  

2.5

and if edge bending resistance is neglected, this becomes:

\[
t_o = \left\{\pi p_d t_d L / 2p_4\right\}^{1/2}
\]  

2.6
2.3 Discussion of Existing Idealised Theories

The analysis described in Section 2.2 overlooks the reduction in pressure difference which must occur during the petal folding process, as the rarefaction wave travelling into the driver gas becomes stronger, and the compression waves strengthen in the low-pressure channel. Nevertheless it is useful, particularly when the petal bending resistance is ignored, in indicating the ideal minimum opening time for any given diaphragm material. This time is unattainable in practice but it is shown in Section 6.2.2 that it provides a useful guide value which is of the same order as the actual opening time.

The analysis presented in Section 2.2 however is somewhat simpler than that used by Simpson et al (1.2) in which the pressure difference applied to the diaphragm petals, though assumed constant has been applied not to the petal area, but to its projection on a plane normal to the shock-tube axis. The implication of this is that the load bearing area decreases continuously with time. The moment about the clamped edge due to gas pressure in this case is

\[ M_g = p_n L^3 \cos^2 \theta / 3 \]

( cf equation 2.1 )

This approach has the effect of increasing the diaphragm opening times as compared with the predictions of equation 2.5.

Specifically, if the root bending resistance of the petals is ignored, the expression relating petal angle to elapsed time after the initiation of bursting is:

\[ t = (\rho_d r_d L / 2 p_n)^{1/3} \int_{\theta_0}^{\theta} \frac{d\theta}{(\sin \theta / 2 + \theta)^{1/2}} \]
The corresponding expressions for the case of a constant pressure difference applied to the full petal area are respectively

\[ t = 0.5 \left( \frac{\rho_d L}{p_t} \right) \int_0^\theta \theta^{-\frac{1}{2}} \, d\theta \]  

\[ t_o = 1.25 \left( \frac{\rho_d L}{p_t} \right)^{\frac{1}{2}} \]  

Thus, equation 2.9 overestimates the idealised minimum opening time by over 10%. The analysis by Drewry and Walenta (2.2) assumes arbitrarily that the pressure loading on the diaphragm petals decreases sinusoidally with petal rotation, but in other respects resembles the approach presented in Section 2.2.

Comparison of results based on this idealised theory is made with those of two more realistic analyses and with experimental measurements of diaphragm petal motion in Sections 3.7 and 6.3.
CHAPTER 3.

3.1 Analysis of the Flow in the Diaphragm Region

3.1.1 Kireyev’s Quasi-Steady Analysis

The approach used by Kireyev (1.17) constitutes an advance on the idealised treatment of the problem discussed in Chapter 2 in that it gives a time-varying pressure distribution on the diaphragm petal which is calculated on the basis of an analysis of the flow. It also points to a coherent solution of the initial flow problem in the simple shock tube in which the flows upstream and downstream of the diaphragm are instantaneously linked to the diaphragm-region flow.

An unsteady expansion wave is assumed to propagate into the driver gas, accelerating the particles and increasing their stagnation enthalpy. Unlike the Q - rarefaction envisaged in ideal shock-tube theory, this expansion wave is not necessarily centred and its pulses may be represented on an x-t diagram Fig. 3.1 as a series of lines originating at different points on the time axis.

The flow through the diaphragm region itself is assumed to be quasi-steady and one-dimensional.

The assumed form of the flow boundaries is that of a convergent duct of square cross-section, the plane of each wall representing one of the diaphragm petals (Fig. 3.2).

It is further assumed that no fluid passes through the spaces between adjacent petals the entire flow being assumed to emerge at the critical speed of sound $a^*$ through the end of the duct coincident with the petal tips.
This implies that the shock-pressure is less than the critical pressure $p^*$ in the driver-gas emerging from the diaphragm exit.

The assumption of critical conditions at the petal tips fixes the instantaneous strength of the unsteady expansion and so links this to the diaphragm petal rotation throughout the opening process.

### 3.1.2 Flow properties behind the Unsteady Expansion

For the wave-pulse emitted from the point B in Fig. 3.1 and passing through A, the slope $N$ may be expressed in terms of the undisturbed sonic velocity $a_4$ in the driver gas i.e.

$$N = x/(a_4(t - t_0))$$  \[3.1\]

The flow properties are constant along any such pulse in a simple wave and may be expressed conveniently in terms of $N$. Suffix "R" refers to conditions along the line AB (Fig. 3.1) and also instantaneously to the gas particles entering the convergent diaphragm-duct region (Line xx in Fig. 3.3) At the instant $t_0$ (Fig. 3.1) the mass-flow into the diaphragm region is given by the standard result (1.6):-

$$\rho_\infty u_R / \rho_0 a_4 = (1 - (1 + N)/a_4)^2/(\gamma - 1)$$

$$\times 2(1 + N)/(\gamma + 1)$$  \[3.2\]
and the stagnation temperature ratio is:

\[ \frac{T_{cr}}{T_4} = \left[ 1 + 0.5 \left( \frac{\gamma_4 - 1}{\gamma_4 + 1} \right) \frac{(N + 1)}{(1 - \beta_4 \gamma_4 N)} \right] \]

\[ \times \left[ 1 - \frac{(1 + N)}{\alpha_4} \right]^2 \]

The critical speed of sound is therefore obtained from:

\[ \frac{a^*}{a_4} = \frac{2}{\gamma_4 + 1} \left( \frac{\gamma_4 - 1}{2} \right)^{\frac{1}{2}} \]

Equating mass flows at the diaphragm-region entry and exit respectively (Fig. 3.3)

\[ \frac{\rho u}{\rho a^*} = (1 - \cos \theta)^2 \]

And using the isentropic flow assumption for the convergent duct-flow, together with equation 3.4 gives:

\[ \frac{\rho^*}{\rho_4} = \left( \frac{2}{\gamma_4 + 1} \right)^{\frac{1}{\gamma_4 - 1}} \left( \frac{\gamma_4 - 1}{2} \right)^{\frac{1}{2}} \]

The relationship between \( N \) and \( \theta \) is then obtained from equations 3.2, 3.5 and 3.6 in the form:

\[ (1 - \cos \theta)^2 = (1 + N)^{\frac{1}{2}} \frac{(\gamma_4 - 1)(N/2)}{(\gamma_4 - 1)} \]

All flow properties may then be determined as functions of the instantaneous petal-angle \( \theta \).
3.1.3 Pressure Distribution and Moment on Diaphragm Petal

The mass flow at section \( y-y \) Fig.3.3 is given by:

\[
\rho_u (1 - (y/L)\cos\theta)^2 = \rho_{u_R} = \rho^a a^a (1 - \cos\theta)^2
\]

The energy equation gives:

\[
p/2\beta_4 \rho + u^2/2 = \alpha_4 a^a^2/2
\]

Then using the isentropic flow equation and equations 3.8 & 3.9 the static pressure on the petal at section \( YY \) is given by:

\[
\alpha_4 (p/p^*)^2/\gamma^* - 2/(\gamma^* - 1) (p/p^*)^2\alpha_4 \beta_4 =
((1 - \cos\theta)/(1 - (y/L)\cos\theta))^b
\]

Kireyev's analysis assumes zero pressure in the forward face of the petal and the moment on the complete petal due to gas pressure forces is then

\[
M_g = 2\rho u L^3 (p^*/p_0) \int_0^1 (p/p^*)(1 - (y/L))(y/L) \, \text{d}(y/L)
\]

The gas pressure moment may thus be calculated from equation 3.11 for any value of petal angle since \( p^*/p_0 \) is obtainable for any \( \theta \) value from equations 3.6 and 3.7 while \( p/p^* \) is determined as a function of \( y/L \), from equation 3.11

3.1.4 Equation of Motion of the Diaphragm Petals

The moment of inertia of a petal is given in equation 2.2 and the bending resistance, assuming the diaphragm behaves as a simple prismatic cantilever, is given in equation 2.3.
The equation of motion for the diaphragm petal is thus

\[
\ddot{\theta} = \frac{(3/\rho d L)((4p^*/t_d) \int_0^1 (p/p^*)(1 - y/L)(y/L)d(y/L) - \frac{f_d t_d}{L^2})}{\rho d t_d L^3/6} \tag{3.12}
\]

A computer solution of equation 3.12 has been evolved which gives the opening history of the diaphragm in the form of a curve of \( \theta \) vs time.

Results are presented in Sections 3.7 and 6.3 where comparison is made with other theoretical analyses and with experimental results.

3.2.1 Discussion of One-Dimensional Quasi-Steady Diaphragm Flow Analysis

The quasi-steady one-dimensional approach to the problem of unsteady flow in rigid ducts involving discontinuous area-changes is well known (3.1 to 3.5). As in the analysis described in Section 3.1, the steady-flow conservation equations are applied across the discontinuity for short steps of time, in order to match together the regions of unsteady flow separated by the discontinuity.

Problems treated in this way include the impingement of shocks and isentropic pressure waves on orifices, sudden enlargements, sudden contractions and gauzes in ducts.

Experimental measurements, for example, of pressure-history are in good accord with the theoretical calculations provided the measurements are made far enough from the discontinuity to ensure that conditions are substantially one-dimensional (3.1 and 3.2)
The fundamental assumption underlying the above approach is that at the discontinuity in area, the axial length dimensions are sufficiently small to give spatial rates of change of the flow properties which greatly exceed the corresponding temporal rates of change.

In the present problem the axial extent of the diaphragm region is least immediately after the start of folding, and at this time, the rate of folding is also at its lowest.

These factors tend respectively to maximise the spatial rate of change of any flow property and to minimise the temporal rate of change. The quasi-steady assumption is thus likely to be least in error at this stage.

The assumption of one-dimensional plane flow is conversely at its most unrealistic at the start of the folding process when the diaphragm region is most sharply convergent, a condition likely to lead to appreciable variations in flow properties across the cross section. As the opening process continues, the petal folding rate and the axial length of the diaphragm region increase. Both factors are detrimental to the assumption of quasi-steady flow but the diaphragm region becomes progressively less sharply convergent and the assumption of one-dimensional plane flow becomes therefore more realistic.
The objects of the present analysis of flow in the diaphragm region are twofold, the first being the calculation of the opening time of the diaphragm, which is required in order to fulfill the second objective, that of determining the variation with time of the flow properties at the diaphragm exit region in order to provide the initial values for a shock formation analysis (Chapter 9).

In the context of opening-rate calculations, errors in the values of flow properties due to the shortcomings of the quasi-steady assumption are likely to increase as folding proceeds, but simultaneously the pressure-difference across the petals decreases, thereby diminishing the influence of such errors on the petal motion. Such considerations are not applicable to the calculation of flow properties at the diaphragm exit-region which might possibly be subject to considerable errors as a result of the one-dimensional quasi-steady approximation. It is therefore appropriate to substantiate the descriptive arguments presented above with a numerical estimate of validity of this flow model.

3.2.2 Numerical Assessment of Departures from One-Dimensional Quasi-Steady Flow in the Diaphragm Region

In order to isolate the effects of the respective assumptions of one-dimensional and quasi-steady flow, two separate analyses are discussed.

In Section 3.3 an estimate is made of the maximum variation in flow properties occurring over a cross section of a tapering duct, in which the flow is assumed to be steady. The variation is expressed as a function of the duct taper.
As a result of the information obtained from this analysis, an alternative quasi-steady theory is developed in Section 3.4 and calculations of diaphragm opening-rates and times based on this analysis are compared with experimental results in Section 6.3.

On the basis of the information on diaphragm opening rates obtained from the quasi-steady analysis and confirmed experimentally, an estimate is made in Chapter 7 of the influence of the motion of the diaphragm petals on the properties of the flow contained within the diaphragm region, and especially on the flow at the exit from the diaphragm which is linked analytically with the shock formation process in Chapter 9.

3.3 Comparison between Steady Plane One-Dimensional and Radial Flow.

3.3.1 Radial Flow Analysis

An approximate assessment of the effect of tapering flow boundaries on the validity of the one-dimensional plane flow analysis used by Kireyev (1.17) may be made by calculating the maximum variation in the flow properties occurring over a plane cross section of the duct shown in Fig. 3.4. The flows upstream and downstream of the duct are assumed to be plane one-dimensional but within the duct itself, conditions are assumed constant along any arc of fixed radius and short transition regions are assumed to couple the plane and radial portions. It is also assumed that the static pressure is low in the region downstream of the duct so that critical conditions are attained in the vicinity of the duct exit though not in the radial portion of the flow since this would call for an infinite local Mach-number gradient.
This may be demonstrated as follows:

Combining the energy-conservation equations of mass and stagnation enthalpy for steady radial flow gives the well known result:

\[ \frac{r}{r^*} = \left( \frac{1}{M} \right) \left( \frac{2}{(\gamma - 1) + M^2} \right)^{\alpha/2} \]  \hspace{1cm} (3.13)

relating Mach number \( M \) to radius \( r \); putting

\[ (2/(\gamma - 1) + M^2)/\alpha = G(M) \]

and differentiating equation 3.13 gives:

\[ \frac{dM}{dr} = \left( \frac{1}{r^*} \right) G^{\alpha/2} / (G - 1/M^2) \]  \hspace{1cm} (3.14)

From equation 3.13, \( r = r^* \) when \( M = 1 \) but equation 3.14 indicates that \( dM/dr = \infty \) when \( M = 1 \).

In order to avoid this difficulty, it is assumed in the analysis, that the radial flow leaving the duct exit passes through a short transition section involving steady isentropic expansion, to form a parallel one dimensional jet at the critical conditions. The area of cross-section of this jet is taken to be that of the plane aperture of the duct exit normal to the shock-tube axis. The "vena contracta" effect in the jet is assumed to be negligible, a hypothesis which is supported, in the present context, by the evidence from schlieren photographs of the jet emerging from a folding two-flap diaphragm, which show that the flow contraction in this region is very small even for low diaphragm pressure ratios (Plate 9.1).

With conditions fixed at one control section of the duct the entire flow is determinate in terms of the boundary geometry. For example, the Mach number may be determined in terms of the distance \( y \) along the inclined wall (Fig. 3.4).
The continuity equation, on the basis of the above assumption is given for the arc PQ, Fig. 3.4 as:

\[ \rho u (L \sec \theta - y) (\pi/2 - \theta) = \rho^* a^* \ L (1 - \cos \theta) \]  

Combining equation 3.15 with the energy equation for steady adiabatic flow gives the local Mach number \( M \) as:

\[ (1/M) G'_{y/2} = (1 - (y/L) \cos \theta) (\pi/2 - \theta)/(1 - \cos \theta) \cos \theta \]  

3.3.2 Transverse Variation in Mach Number

The difference in Mach number between two points A and B Fig 3.5 lying on a plane normal section of the flow intersected by planes OA and OB subtending an angle \( \delta \phi \) with OB inclined at \( \phi \) to the axis is:

\[ \delta M = (dM/dr) r \cdot d\phi \cdot \cot \phi \]

Substituting for \( r \) and \( dM/dr \) from equations 3.13 and 3.14, respectively:

\[ \delta M = G'_{y} / (MG - 1/M) \]

Differentiating, \( d/dr (\delta M) = d/dM (\delta M) (dM/dr) \)

\[ \therefore d/dr (\delta M) = (MG - 1/M)^{-2} G'_{y} (MG - 1/M) G^{-1} \cdot M dG/dM \]

\[ = G - 1/M^2 \] (dM/dr)

For \( M < 1 \) as in the present case \( G < 1 \) and \( MG - 1/M < 0 \).

Also \( dG/dM > 0 \) \( \therefore d/dr (\delta M) < 0 \).
The transverse variation in Mach number at any plane normal cross-section is therefore greatest where $r$ is least.

In the duct shown in Fig. 3.4 this maximum difference occurs between point $R$, and the point $S$ lying on the downstream limit of the radial portion of the flow. The Mach number at $S$ is obtained from equation 3.16 by putting $y/L = 1.0$ i.e.

$$\left(\frac{1}{M_s}\right)\left(\frac{G_s}{C_g}\right)^{\alpha/2} = (\pi/2 - \theta)/\cos \theta$$

The corresponding result for point $R$ is found by putting

$$\frac{y}{L} = (1 - \csc \theta + \cot \theta)/\cos \theta$$

3.19

3.3.3 Plane One-Dimensional Flow

The well known counterpart of equation 3.13 for one-dimensional flow, relating local Mach number to the duct cross-sectional area, gives, for the Mach number $M_{1D}$ at the section $S$, Fig. 3.4:-

$$\left(\frac{1}{M_{1D}}\right)\left(\frac{G_{1D}}{C_g}\right)^{\alpha/2} = \csc \theta$$

3.20

the static pressures corresponding to the Mach numbers $M_S$, $M_R$ and $M_{1D}$ being given respectively by the equations

$$p_S/p^* = (\cos \theta / (M_s(\pi/2 - \theta)))^{1/\alpha \beta}$$

3.21

$$p_R/p^* = (\cos \theta \sin \theta / (M_R(\pi/2 - \theta)))^{1/\alpha \beta}$$

3.22

$$p_{1D}/p^* = (\sin \theta / M_{1D})^{1/\alpha \beta}$$

3.23

assuming isentropic flow through the duct.
3.3.4 Comparison of Numerical Results based on Radial and Plane Flow Theories

The ratios $M_s/M_{1D}$ and $M_R/M_{1D}$ are plotted over a range of petal angles for monatomic and diatomic gases respectively in Fig. 3.6. It is clear from these curves that appreciable departures from ideal one-dimensional behaviour occur at conditions representing petal angles of less than about 70°. At this point the value of $M_s$ is 10% greater and that of $M_R$ 2% less than the plane one-dimensional flow value - the corresponding discrepancies at $\theta = 45^\circ$ being approximately +50% and -10% respectively. These values show little dependence on $\gamma$, the $M_R/M_{1D}$ curves in particular being virtually coincident throughout the range of $\theta$.

The corresponding ratios of $p_s/p_{1D}$ and $p_R/p_{1D}$ are plotted in Fig. 3.7 which indicates a greater degree of uniformity across the flow cross-section than in the case of the Mach-number curves.

3.3.5 Conclusions

The comparison between a one-dimensional analysis of the steady flow in a convergent duct and a radial analysis of the same case made in Sections 3.3.1. and 3.3.2. indicates that at a flow cross section in the vicinity of the duct exit, the Mach number near the walls may be appreciably lower than that on the duct centre-line; although the corresponding variation in static pressure is less, the net mass-and momentum-fluxes calculated on the basis of the one-dimensional flow theory may be significantly in error. Thus although the pressure loading on diaphragm petals calculated on the basis of the analysis given in Section 3.1 may be realistic, the use of the same theory to predict the flow conditions at the diaphragm exit, in the context of a shock formation analysis (Chapter 9) may lead to significant errors.
An alternative analysis has therefore been developed (Section 3.4) which discards the assumption of one-dimensional plane flow and makes use of a more plausible representation of the diaphragm geometry.

3.4 Radial Diaphragm Flow Analysis

3.4.1 Geometrical Considerations

In the present analysis as in that of Kireyev (1.17) it is assumed that the driver-gas particles are first set in motion by an unsteady Q-rarefaction wave. This flow then enters the diaphragm region and Fig. 3.8 shows the alignment of the petals at a particular instant during the folding process. Assuming equal rotation of all four petals, the lines "ac", "bd", "gh" and "ef" are axes of symmetry.

The flow cross-section may be divided into 8 similar triangular elements of which "gbo" is typical, and the determination of the flow properties as a function of time, for such an element constitutes a solution of the entire diaphragm flow region.

If viscosity is neglected, this basic element becomes a prismatic channel of triangular cross-section, spanned at its downstream end by an inclined flat plate "gib" representing a portion of the diaphragm. The fluid particle paths through "o" and "b" are parallel to the shock-tube axis and, by symmetry, fluid particles entering the diaphragm region in the plane through "go" and "ob" remain respectively in these same planes as they traverse the region. If the assumption is made that the velocity component normal to "go" is small at all intermediate stations between "o" and "b", a strip-theory representation is obtained in which the flow is constrained in planes parallel to "go" and to the shock-tube axis.
The flow pattern in any strip such as "g'o" Fig 3.9 is geometrically similar to that of any parallel strip and following the analysis discussed in section 3.3.1 is assumed to be radial i.e., to have flow properties which are constant along any arc of fixed radius, and to emerge from the diaphragm region at a high subsonic Mach number, forming a parallel jet at critical conditions just downstream of the diaphragm-region exit. Vena contracta in this region is ignored as discussed in Section 3.3.1.

The present analysis also includes an allowance for the effect on the flow properties of the deflection developed by the diaphragm prior to bursting.

3.4.2 Initial Petal Angle

When pressurized near to their bursting pressure, the diaphragms used in the present work bulge appreciably into the low-pressure channel. This effect is greatest in ductile diaphragm materials and has been measured experimentally, (Section 4.5 and 4.7).

The form of the flow boundaries is determined, at the initial stage of opening, by this effect and the total rotation required to give full opening becomes less than 90°.

An approximate allowance has been made for this by assuming that the diaphragm petals remain flat under pressurization, but that they also stretch and rotate about their clamped edges until the axial displacement of the centre of the diaphragm attains the appropriate experimentally-determined value. The petals at the instant of failure then begin to rotate from an initial angle θ₀ Fig. 3.9.
3.4.3 Relation between Petal Angle and Strength of Primary Expansion

The instantaneous mass-flow into the strip "g'o" Fig. 3.8 is equated to the corresponding mass-flow in the jet downstream of the diaphragm, assuming the static pressure here to be less than \( p^* \) as in Section 3.1.1

\[
\rho_R u_N \ell = \rho^* a^* \ell (1 - \cos \theta / \cos \theta_0)
\]

As in the case of one-dimensional plane flow,

\[
\rho_R u_N / \rho_u a_u, \quad \rho^* / \rho_4, \quad \text{and} \quad a^* / a_u
\]

are expressible in terms of the tail-slope \( N \) of the primary expansion through equations 3.2 3.4 and 3.6 which, together with equation 3.24 give for the present radial flow case:

\[
1 - \cos \theta / \cos \theta_0 = (1 + N)(1 - (\gamma_4 - 1)N/2)^2/(\gamma_4 - 1)
\]

\[
/((\gamma_4 - 1)N^2/2 + 1)\alpha_4/2
\]

which corresponds to equation 3.7 in the one-dimensional case,

3.4.4 Pressure Distribution Along the Petals

The mass-flow across a line of radius \( r \), which intersects the diaphragm at distance \( y \) from the clamped edge(Fig. 3.9) is given by:

\[
\rho u (\ell \sec \theta - y) (\pi/2 - \theta) = \rho_R u_N \ell = \rho^* a^* \ell (1 - \cos \theta / \cos \theta_0)
\]
And using the energy equation together with the equation for isentropic flow and equation 3.26 gives the result

\[ \alpha_4 \left( \frac{p}{p^*} \right)^{2/\gamma_4} - \frac{2}{(y_4 - 1)} \left( \frac{p}{p^*} \right)^{2\alpha_4 \beta_4} = \frac{(1 - \cos \theta / \cos \theta_0 \cos \theta)}{(1 - (y/L) \cos \theta)(\pi/2 - \theta)} \]

which gives the static pressure \( p \) at a distance \( y \) from the clamped petal edge.

The critical sonic velocity is given at the angle \( \theta \) by equations 3.4 and 3.26 and from this information all the flow properties at any radius within the diaphragm flow region may be calculated.

3.4.5. Moment due to Gas Pressure

Because of the geometrical similarity between all flow strips, the flow properties in the region adjacent to the rearward face of the petal are constant along lines radiating from the shock-tube corners (point "b" Figs 3.9 and 3.10, in which point "g" and "j" also correspond).

The moment about edge "gb" due to pressure forces on the element \( d\phi \) is

\[ M_g = \frac{2}{3} \left( \frac{p^*}{3} \right) \left( \frac{p}{p^*} - 1 \right) (L \sec \phi)^2 \cdot L \tan \phi \ d\phi \]

From Fig 3.9

\[ g'o' = g'b' = L \]

And for the complete petal the moment is

\[ M_g = 2/3 \ L^3 \ p^* \int_0^\pi (p/p^* - 1)(y/L) \ d(y/L) \]

\[ 3.29 \]
which corresponds to equation 3.11 in the plane one-dimensional analysis. The equation of petal motion based on equation 3.29 is given in Section 3.6.

3.5 Bending Resistance of Tapered Diaphragm Petals

A minor modification to the yield bending-moment analysis given in Section 3.1 is required since this analysis treats the petal as a prismatic cantilever. The petals of diaphragms in square-section shock-tubes are markedly non-prismatic having a considerable taper in plan-form and since the stress component normal to the tapered edges is zero, the bending resistance being the moment about the neutral axis of the forces due to these stresses, is modified accordingly.

The analysis of tensile stress concentrations in axially-loading tapered bars, as presented in standard literature (3.6) assumes a radial stress distribution (Fig. 3.11) and taking a yield-stress of constant magnitude \( f_d \) across the clamped edge, this gives a maximum bending moment of:

\[
0.5 t_d^2 f_d L \int_0^1 (1 + x/L \cos \theta_o)^{-2} d(x/L) \quad 3.30
\]

which, when integrated, gives a bending resistance:

\[
M_b = 0.5 t_d^2 f_d L \ln(\cos \theta_o + (1 + \cos^2 \theta_o)^{\frac{1}{2}}) \quad 3.31
\]
The logarithmic term in equation 3.35 is the factor by which the prismatic - beam yield moment must be multiplied in order to allow for the taper.

Values of this factor for a representative range of $\theta_0$ values are given in Table 3.1 which indicates that prismatic beam theory can overestimate the bending resistance by up to 17%.

A discussion of the effects of bending resistance on petal motion is included in Section 3.7 and in this context such a discrepancy is significant only in the case of the thickest diaphragms used in the present work.

Table 3.1

<table>
<thead>
<tr>
<th>$\theta_0$ (Degrees)</th>
<th>Logarithmic Taper Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.88127</td>
</tr>
<tr>
<td>5</td>
<td>0.87865</td>
</tr>
<tr>
<td>10</td>
<td>0.87082</td>
</tr>
<tr>
<td>15</td>
<td>0.85706</td>
</tr>
<tr>
<td>20</td>
<td>0.83806</td>
</tr>
</tbody>
</table>
3.6 Equation of Motion

The moment of inertia of the diaphragm petals about the clamped edge is modified as compared with the value obtained from equation 2.2 because of the difference in petal form caused by initial stretching prior to failure, an effect ignored in the formulation of equation 2.2.

The result for the deflected flat petals discussed in Section 3.4.2 is:

\[ M_i = \rho_d t_d L^4/6 \cos^3 \theta \]

3.32

In the above equation the mass of the petal has been taken to be equal to that of the undeflected petal of equation 2.2 i.e. \( \rho_d t_d L^2 \), only the radius of gyration being altered by the stretching process.

The gas-pressure loading is given by equation 3.29, and the edge bending resistance by equation 3.31.

The equation of motion then becomes:

\[
\ddot{\theta} = \cos^3 \theta_0 \left( \rho_d t_d L^4 \right) \left[ 4L^3 p^* \int_0^{\sec \theta_0} (p/p^* - 1)(y/L) \, dy/L \right] \\
- 3 t_d^2 f_d L \ln(\cos \theta_0 + (1 + \cos^2 \theta_0)^{1/2})
\]

3.33

The results from a computer solution of equation 3.33 are given in Section 3.7 where comparison is made with corresponding values based on the theory given in Section 3.1.
3.7 Theoretical Calculations of Petal Loads and Petal Motion

3.7.1 Moment on Diaphragm Petals Due to Gas-Pressure

A comparison between the moment values calculated on the basis of Kireyev's one-dimensional plane-flow theory, and the corresponding results based on the present radial flow analysis is shown in Fig. 3.12. The theoretical values are expressed as ratios of the moment assumed in the simple idealised theory in which the initial driver-pressure acts at the rear surface and the pressure on the forward face is neglected.

The moments are plotted to a base of petal angle and over the entire range, the plane-flow values exceed those of the radial-flow analysis and remain finite even when the diaphragm is fully open. The latter is due to the assumption of zero pressure at the forward face. The radial-flow theory, by contrast, incorporates the assumption that the pressure acts on the forward face so that the overall pressure difference vanishes when the diaphragm becomes fully open.

In order to assess the effect of the static pressure at the forward face of the petal and to compare the two theories on a more uniform basis, a third set of moment values was computed using the plane-flow analysis in conjunction with the assumption of a static pressure of acting on the forward face.

The results based on this analysis are also shown in Fig. 3.12 and are much closer to those of the radial flow theory than to those of the unmodified plane-flow theory. The moment on the diaphragm petals is thus strongly influenced by the choice of assumption concerning the pressure at the forward face and to omit, a priori, any consideration of this effect is unjustifiable.
3.7.2 Influence of Initial Petal-Angle on Moment due to Gas-Pressure

In Section 3.4.2 an initial petal angle $\theta_0$ was introduced to represent the deflection of the diaphragm prior to failure. The influence of this parameter on the moment applied to the diaphragm by gas pressure during opening is shown in Fig. 3.13 and 3.14. Moment values calculated for a representative range of initial petal angles are shown in Fig. 3.13 for the case of radial flow and in Fig. 3.14 for the unmodified plane-flow case.

The results show that a significant variation in moment is attributable to this effect, an initial deflection $\delta / L = 0.4$ for example giving a 20% increase in moment at the start of folding in the case of plane-flow theory and over 15% for radial flow.

Furthermore, since the total angle of rotation required for full opening is reduced by an increase in $\theta_0$, it is clear that given two diaphragms of equal strength and mass but different ductilities, the one which develops the larger initial deflection is likely to open the more rapidly.

3.7.3 Effect on Petal Motion of Pressure reduction during Opening

The idealised constant-pressure approach (Section 2.2) to the calculation of petal motion necessarily leads to shorter opening time when compared with an analysis allowing for pressure reduction at the driver side of the diaphragm during opening. The theoretical variation of petal angle with time for freely-hinged petals having densities equal to those of aluminium, copper and stainless-steel respectively is shown in Fig. 3.15. The time values for all three cases are expressed as ratios of the idealised opening times based on constant pressure theory (Section 2.2) and in this way the differences in bursting pressure and density are normalised.
The effect of edge bending resistance on the petal motion has been omitted.

Only the differences in initial angle $\theta_0$ prevent the curves for all three materials from coinciding - indeed the curves for stainless steel and aluminium are virtually coincident, since their initial angles are almost identical.

The main conclusion however is that the opening time is increased by up to 40% in comparison with the ideal value when the effects of pressure reduction are allowed for.

3.7.4 Effect of Bending Resistance on Calculated Petal Motion

The effect of adding realistic values of the petal edge bending resistance calculated according to the analysis discussed in Section 3.5, to the equation of motion of the petals Section 3.6 is shown in Fig. 3.16 to 3.18. These represent the results for diaphragms made from aluminium, copper and stainless steel as in Section 3.7.3, all three being in thicknesses which give the same nominal bursting pressure. The time scale is non-dimensional as in Fig.3.15.

In the case of aluminium the inclusion of the edge bending resistance has a marked effect on petal motion, increasing the opening time by over 50% in comparison with the freely-hinged case and by over 100% in comparison with the idealised constant-pressure theory.

For the other two materials the effect, though present, is practically negligible.
The required thickness of the aluminium diaphragms for the
given bursting pressure is 3 times that of the copper and 6 times
that of the stainless steel diaphragms. Moreover, equation 3.35
indicates that the bending resistance is proportional to the
product of the yield stress of the material and the square of petal
thickness. Thus, although the yield stress of the stainless steel is
some 3 times greater than that of aluminium and because that of
copper is slightly less than that of the aluminium sample, it is
clear that the bending resistance of the aluminium petals is an
order of magnitude greater than those of the other two materials.

The curves of petal angles vs. time shown in Fig. 3.16 to
3.18 are compared in Chapter 6 with experimental results obtained
as described in Chapter 5.
CHAPTER 4.

Mechanical Properties of Shock-Tube Diaphragms

4.1 Introduction

The two diaphragm properties which have the greatest effect on the flow in the shock tube are the bursting pressure, which directly determines the initial pressure in the driver gas; and the opening time which has been shown to influence the shock trajectory (1.1, 1.2).

The deflected shape of the petals at the instant of failure also influences the initial flow in the tube since this determines the form of the moving flow boundaries through which the driver-gas accelerates during the opening process.

A theoretical stress analysis of the diaphragm, giving the deflected form and specifying the bursting pressure would therefore be of considerable value in defining the initial conditions for an analysis of the petal motion. One possible approach to this problem might make use of the existing theory of the behaviour of thin plates. As formulated at present however, this deals with levels of loading and resulting deflections which are sufficiently small to allow the plate to retain its integrity as a structural member.

The basis of elastic plate-bending theory (4.1 and 4.2) is similar to that of the simple bending beams in that the deflections are assumed to be sufficiently small for the tangent of the surface slope at any point to be approximately equal to the slope itself.
On this basis a relationship may be derived between the deflected shape of the plate and the local bending moments in two mutually perpendicular directions and hence the local stress in the material may be calculated.

The influence of shear stress is neglected in this simple theory and stretching of the neutral surface of the plate is also ignored. Application of such a theory is restricted to cases in which the maximum plate deflection is small compared with the plate thickness.

Cases involving somewhat larger deflections may be analysed by means of the theory which takes into account the stiffening influence of the stretching of the neutral surface (4.1 to 4.4). This treat the effect of the "membrane" stresses associated with this stretching in a similar manner to that of the tension in a stretched string, and for values of plate deflection greater than approximately 1£ the thickness (4.2), membrane stresses predominate over bending stresses. Since the central portions of shock-tube diaphragms commonly deflect through 40x the basic thickness, before bursting (Section 4.5) although neither of the elastic-bending theories is applicable at such large deflections, a membrane model of the diaphragm should provide a realistic basis for at least a first-order solution of the diaphragm failure problem.

However, no analysis is available at present, by means of which the deflected form of a pressurised membrane stretched across an aperture of arbitrary form may be calculated. On the contrary, the experimentally measured form of pressurised soap bubbles has been used by G.L.Taylor (4.5) to estimate the distribution of shear stress in shafts, of a variety on non-circular cross sections under torsion.
The equations relating the deflection of the membrane to the applied pressure-difference are of the same form as those relating the shear-stress in the shaft to the applied torque, an analogy first noted by Prandtl (4.6).

In view of the analytical difficulties of the membrane problem, the strength and deflection under load of the shock-tube diaphragms used in the present work have been determined experimentally. A dimensionless strength parameter is evolved in Section 4.4 by means of which the bursting-pressure values for diaphragms of a given material and thickness measured in the present 2 in square shock tube may be used to predict corresponding values for square diaphragms of any size and thickness.

4.2 Diaphragm Bursting Characteristics

Two of the most important criteria to be met by shock-tube diaphragms are that they should split cleanly into constituent petals after failure and that these should remain attached to the shock tube walls at their clamped edges in order to avoid damage to windows and instrumentation which might result from the tearing away of fragments.

Repeatability is another desirable feature, in that successive diaphragms manufactured to the same specification are required to reproduce their natural bursting pressure within close limits.

The influence of variations in \( p_4 \) on the flow in the tube may be illustrated by considering the ideal shock pressure ratio produced by an air/air combination with a nominal \( P_{41} \) of 10\(^4\).
In such a case, a variation of only 5% in $p_i$ is sufficient
to produce a 1% change in $P_{21}$.

Speed of opening is a further important factor, the process
being required to occupy the shortest possible time in order to
approximate to the ideal case of infinite acceleration of the
shock at the beginning of the run.

Published data on the characteristics of metal diaphragms
relate to shock-tubes of circular cross-section (4.7 to 4.10)
and also to square-section tubes (1.1, 2.2 and 4.11 to 4.14).

In the absence of a suitable dimensionless presentation
however, it is difficult to apply the above results to shock-
tubes in general. One objective in the present work has therefore
been to present the results of diaphragm bursting tests in a
dimensionless form applicable to a variety of different diaphragm
metals, used in square-section shock-tubes of any size (4.13)

4.3 Grooving of Diaphragms

The basic form of the petals is determined by the configuration
of the grooves which channel the cracks along preferred directions
when the diaphragm bursts. The grooves also lower the bursting
pressure as compared with that of an ungrooved plate, thus
limiting the stresses at the clamped edges of the diaphragm and
reducing the likelihood of fragmentation of the petals after
bursting.

Grooving of metal diaphragms is a common practice, the
cutting techniques ranging from simple manual scribing (4.7) to
the milling of an accurately indexed groove with a cutter of known
tip-radius (1.1, 4.14).
A forming process using a special press-tool has been described (1,2) and a variation on this method making use of a diamond glass cutting wheel has also been reported (4,15). Forming processes are suitable however only for relatively soft materials whereas machining is applicable to a wider range of materials.

The method used in the present work has been to clamp the diaphragms against a flat-ground plate by means of a vacuum-chuck arrangement and to cut the grooves with a saw-type milling cutter of 1/16 in thickness having a square tooth-profile. This produces flat-bottomed grooves of sensibly constant form and no significant variation in bursting pressure has been observed with prolonged usage of the cutter. The flat-bottomed groove greatly facilitates measurement of the effective thickness $t_e$ (Fig. 4.1). A cruciform groove pattern has been used in all the present tests.

4.4.1 Idealised Bursting Pressure

In order to construct a simple idealised model of the diaphragm it is assumed that at the instant of failure the surface of each diaphragm petal has a cylindrical form and that the metal remaining in the grooved portion linking the petals develops a membrane stress equal to the full ultimate tensile stress of the material, along its entire length. On this basis, the grooved portions may be considered as parts of a thin-walled pressurised sphere of radius $r$ (Fig. 4.1) and the idealised bursting pressure then becomes

$$p_{l} = 2 \frac{\hat{f} t_e}{r}$$  

4.1
Geometrical considerations indicate that

\[ r' = \frac{\delta}{2}(1 - \left(\frac{L}{\delta}\right)^2) \]

Thus

\[ \frac{p_i}{t_e} = \frac{S}{L} \]

Where

\[ S = \frac{4}{L/\delta + \delta/L} \]

Table 4.1 gives values of \( S \) for representative values of \( \delta/L \) which taken in conjunction with equation 4.3 indicates that the ductility of the material has an important influence on the bursting strength.

<table>
<thead>
<tr>
<th>( \delta/L )</th>
<th>( S )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.396</td>
</tr>
<tr>
<td>0.2</td>
<td>0.768</td>
</tr>
<tr>
<td>0.3</td>
<td>1.10</td>
</tr>
<tr>
<td>0.4</td>
<td>1.38</td>
</tr>
<tr>
<td>0.5</td>
<td>1.60</td>
</tr>
<tr>
<td>0.6</td>
<td>1.76</td>
</tr>
</tbody>
</table>

4.4.2 Experimental Comparison

The bursting characteristics of diaphragms made from materials listed in Table 4.2 were investigated.

**Table 4.2**

<table>
<thead>
<tr>
<th>Material</th>
<th>Thickness (S.W.G)</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stainless Steel</td>
<td>28</td>
<td>302 - annealed</td>
</tr>
<tr>
<td>Copper</td>
<td>26</td>
<td>Soft annealed</td>
</tr>
<tr>
<td>Aluminium</td>
<td>28</td>
<td>&quot;</td>
</tr>
<tr>
<td>Aluminium</td>
<td>20</td>
<td>Half Hard</td>
</tr>
<tr>
<td>Aluminium</td>
<td>18</td>
<td>&quot;</td>
</tr>
</tbody>
</table>
The annealed copper and aluminium were of burster-disc quality while the other materials were of ordinary commercial grades.

For many materials, it has been found (4.12) that the range of groove depths giving reliable petalling lies between 25% and 75% of the basic sheet thickness and the present diaphragms were all manufactured within these limits.

The centre deflection of the diaphragms was measured in the shock-tube for a range of pressures up to the bursting point. The displacement of the centre of the diaphragm was transmitted to a dial gauge and simultaneously to a displacement transducer, by means of a thin copper wire attached to the diaphragm and passed through a curved conduit inserted through the shock-tube wall. Pressures were measured using a calibrated Bourdon gauge.

The gas supply to the driver section was regulated to give a very small rate of increase in pressure when the burst was imminent as indicated by relatively large changes in the dial-gauge reading. The displacement transducer output was recorded on the camera of an oscilloscope set to a low sweep-rate.

The bursting of the diaphragm invariably broke the fine copper wire and produced a discontinuity in the output signal from the transducer which facilitated the measurement of the displacement at the bursting point.

Similar tests were carried out on ungrooved diaphragms though these were not burst; the maximum pressure applied to a sample of a given material was the bursting pressure of the strongest grooved specimen.
In addition to the measurements of diaphragm strength and centre-deflection at failure, simple, tensile tests were performed on standard flat specimens, several of which were tested for each of the materials listed in Table 4.2 in order to obtain values of the ultimate tensile stress and the stress-strain relationship to the point of failure.

4.4.3 Results and Discussion

The results were plotted as a dimensionless bursting-pressure factor against dimensionless effective thickness.

The bursting pressure factor \( \phi \) is defined as the ratio of the actual bursting pressure to the ideal value, equation 4.3, i.e.

\[
\phi = p_a L / (\hat{f} \hat{t}^{k/3})
\]

The measured results over the range of \( t_e / t \) shown in Fig. 4.2 indicate that \( \phi \) is almost independent of \( t_e / t \) for the materials tested, but is less than the ideal value of unity; \( \phi = 0.86 \pm 5\% \) based on a simple arithmetic mean.

The simple theory ignores the decrease in tensile stress at the grooved sections in the vicinity of the clamped edges and is therefore liable to overestimate the true bursting pressure. Nevertheless, the near-constancy of all the experimental \( \phi \) values facilitates the prediction of diaphragm bursting pressures from a knowledge of the ultimate tensile stress of the material and the centre-deflection at failure.
The influence of the shape-factor $S$ (equation 4.2) in producing uniformity in the value of $\phi$ is emphasised by Fig. 4.3 in which the results show in Fig. 4.2 are plotted, again in dimensionless form, but with the factor $S$ omitted. In place of the single curve produced previously, the results now form a separate curve for each different material.

Reverting to the results shown in Fig. 4.2, the $\pm 5\%$ variation in the value of $\phi$ reflects the differences in shape and stress-distribution displayed by the different materials, which are ignored in the simple theory. Moreover, in the tensile tests carried out to determine the value of $\phi$ for each material, a variation of at least 5% occurred between the strength of various samples cut from different parts of the same sheet of material. A variation of up to $\pm 10\%$ also occurred in the measured value of $\delta$ for all the materials tested but individually measured values were used in all calculations of $\phi$. Errors incurred in the measurement of $\delta$ and $p_i$ are further small contributory factors but the variation in $\phi$ alone is sufficient to account for the observed scatter in the $\phi$ values.

4.4.4 Practical Determination of the Shape-Factor $S$

In Fig. 4.4 the centre-deflection of an unscribed copper diaphragm is plotted to a base of applied pressure-difference, and superimposed on this curve are the values of centre-deflection at failure of individual copper diaphragms having a variety of groove depths. The trend shown by these points is typical of the behaviour of each of the ductile material investigated in the present tests, in indicating that the presence of the grooves in the scribed diaphragm though effective in controlling the bursting pressure has no significant influence on the centre deflection.
The values of $\delta$ required for the calculation of $S$ in equation 4.2 for a diaphragm with grooves of a given depth may therefore be obtained from measurements on an unscribed diaphragm. The latter is simply clamped in place in the shock-tube, pressurised to a level equal to the designed bursting pressure of the scribed diaphragm and may then be removed from the tube to facilitate the measurement of $\delta$, e.g. using a surface plate and vernier height-gauge.

The release of the applied pressure-difference has a negligible effect on $\delta$, the total relaxation movement being of the same order as the plate thickness.

This approach is inappropriate in the case of less ductile materials such as half-hard aluminium for which the centre-deflection of a grooved diaphragm differs appreciably from that of an ungrooved diaphragm of the same thickness at the same pressure-loading. Under these circumstances, direct measurement of $\delta$ is necessary for each different groove depth.

4.5 Conclusion

The bursting pressure of diaphragms made from a variety of materials may be plotted on a single curve (Fig. 4.2) in the form of a dimensionless strength parameter, provided the geometrical shape of the deflected diaphragm is taken into account.

A simple theoretical model gives an approximation for the bursting pressure which is consistent for all the materials tested in the present work, over the working range of groove depths for satisfactory shock-tube operation. A method has been developed by which the bursting pressure of the diaphragm made from any ductile metal may be predicted, without performing bursting tests.
CHAPTER 5.
Experimental Studies of the Motion of Diaphragm Petals.

5.1. Methods of Determining the Petal Motion

Existing attempts to determine the motion of shock-tube diaphragm petals have been based on two different lines of approach. One of these depends on a time-resolved measurement of the total quantity of light transmitted from an external source, through the aperture of the opening diaphragm.

The other method relies on photography of the moving diaphragm using either a high-speed cine camera or spark illumination in conjunction with a still camera.

The approach used by White (1.1.) was of the light-transmission variety. A light source was mounted near a small window at the end of the driven section and a photomultiplier was used to measure the illumination at the side wall of the driven section and also to provide the initial triggering signal for the oscilloscope sweep.

The diaphragms concerned were of stainless steel in thicknesses ranging from 0.01 to 0.035 in, the shock-tube cross section being 3\(\frac{1}{4}\) in square. A cruciform groove was used to assist petalling and the results indicated an opening time of 600 \(\mu\)s.

The present work and also that of Campbell et al (4.10) has shown however, that the total quantity of light transmitted through the aperture of a bursting diaphragm increases relatively slowly near the start of opening. The system adopted by White for triggering the oscilloscope can therefore give rise to a considerable delay in the initiation of the sweep resulting in underestimation of the true opening time.
Drewry and Walenta (2.2) also used a light-transmission technique in measuring the opening times of stainless steel diaphragms in a combustion-driven shock tube. No external light-source was required since the hot combustion gases provided adequate illumination of the diaphragm region, and a photodiode inserted into the flow a few tube diameters downstream from the diaphragm was used to indicate the aperture area. A microphone attached to the outer wall of the tube near the diaphragm station was used to detect the impact of the diaphragm petals and the opening times indicated by the two independent measuring systems were said to be in good agreement, though no details were given of the system used for triggering the timing apparatus.

More recently Simpson et al. (1,2) used a light transmission method to measure diaphragm opening times in a 2in x 3in shock-tube in which the light source was situated in the driver-section and a photomultiplier was used to view the end of a perspex rod inserted into the driven section close to the diaphragm station (Fig. 5.1.) The system was first calibrated statically using a series of partially open diaphragms of known projected aperture area. From these measurements and from the results of subsequent shock-tube runs, it became apparent that a significant amount of light reached the photomultiplier via reflections from the tube walls and that this quantity varied with changes in the flow pattern and its associated refractive-index gradients.

The same problem was present even when the light was focussed on the perspex sensing probe and must also have been present to some extent in all previous work of this type based on light-transmission through the diaphragm.

Simpson et al. minimised the difficulty by reducing the amount of reflected light incident on the measuring apparatus.
This reduction was accomplished by the use of a long hood extending from the lamp housing towards the diaphragm station and by the use of a parallel light beam. The width of this beam however, according to the diagram reproduced in Fig. 5.1 from Ref. 1.2 is less than that of the shock-tube and the diaphragm petals pass out of the range of measurement before making contact with the tube walls.

Uppard and Mead (5.1) used a photographic technique to investigate the motion of rubberized textile diaphragms in a shock-tube with an internal cross-section of 18in x 30in. A tungsten-filament bulb in the atmospheric driven-section was used to illuminate the diaphragm and a high speed ciné camera viewed the diaphragm through a side window. The light source was expended on each run.

Campbell et al. (4.10) carried out spark photography of opening diaphragms in a circular-section shock-tube in addition to performing a light-transmission study similar to those of White (1.1) and Simpson et al (1.2).

Photographs were taken of unscored aluminium and cruciform-scored copper diaphragms. These indicated that the initial part of the opening process could occupy a disproportionately large fraction of the total opening time particularly in the case of ruptures induced artificially by piercing with a sharp-pointed plunger.

More recently Cheng, Dannenberg and Stephens (5.2) used an image-converter camera in conjunction with an astronomical telescope to view the diaphragm through a glass window in the end wall of a shock-tube. The driven section of the tube was 37ft in length and the telescope served as a telephoto lens. Triggering of the timing apparatus was based on light transmission through the diaphragm aperture and delays of up to $7.0 \times 10^5$ S following the decay of the arc-current used to heat the driver-gas were reported.
5.2 The Present Experimental Work on Diaphragm Petal-Motion

5.2.1 Triggering of Timing Apparatus

An essential requirement in the context of timing measurements made to determine the motion of the diaphragm is the generation of a triggering signal at the start of the petal motion, for the purpose of initiating the sweep of the timing apparatus.

The danger inherent in such a study is that unless conditions are carefully controlled, a significant delay may occur in the emission of a suitable triggering signal, following the rupture of the diaphragm. Taken in conjunction with an idealised constant-pressure analysis of the opening process (Section 2.2) this can lead to a spurious agreement between experiment and theory.

Several different triggering systems were therefore investigated in the present work, the first being of the light-transmission variety, in which the beam of a 1-milliwatt laser was passed through the glass end-plate of the driver section of the shock-tube so as to produce intense illumination of a region at the centre of the diaphragm where the first cracks occur. A photomultiplier was set up to receive light scattered from the walls of the driven section but was shielded against direct light from the laser.

An insulated pin mounted flush with the tube wall and held initially at a small negative potential was earthed by contact with one of the petals when the diaphragm became fully open; this method of detecting the attainment of full opening was used in all the present studies of petal motion.

The results obtained using the laser/photomultiplier system are described in Section 6.3.
The present programme of experimental work included spark photography of bursting diaphragms using a still camera with open shutter. In this context however, the use of a triggering system involving the passage of light from a continuous source through the diaphragm region is likely to result in "fogging" of the film. As an alternative therefore, a wire-breakage method was used, in which a short length of 36 S.W.G. copper wire was attached across the diaphragm at the intersection of the grooves, using epoxy-resin glue (Fig. 5.2). The active length of the wire was equal to the groove width and the free end of the wire was earthed to the surface of the diaphragm. Breakage of the wire produced an output voltage by opening a simple resistive D.C. Circuit.

In addition to its use as a trigger generator, the wire-breakage technique was used to investigate the early motion of the petals following rupture. For this purpose, several individual wires were attached across a diaphragm groove at various distances from the centre (Fig. 5.2) and the breakage times measured.

In order to interpret the breakage of a given wire in terms of a petal-rotation and finally as elapsed time following the start of rotation it is necessary to assume a theoretical model of the failure mechanism of the diaphragm. The model discussed in Section 2.1 forms the basis of the present interpretation, with the additional assumption that the cracks which separate the diaphragm into petals attain their full length before any appreciable folding of the petals occurs.
Theoretical calculations of petal angles as functions of time may be performed on the basis of this model, given information concerning the ultimate extension of the copper wire, and of the transverse static elongation of the diaphragm grooves at the point of failure as illustrated in Plate 5.1.

A full account of the measurement of these quantities and of the analysis of the relationship between the breakage time of a wire and its position along the diaphragm groove is given in Appendix 2. Results in the form of curves of petal rotation required to cause wire breakage as a function of wire position are shown.

The object of the present multiple wire-breakage tests was to verify the assumption of a negligible crack-propagation time and for this purpose, the theoretical values of wire breakage time for various wire positions (Appendix 2) are compared with the corresponding experimental values in Section 6.2.

Even on the basis of the above idealisation however, the theoretical analysis indicates that petal rotations of more than 20° following diaphragm rupture may be necessary in order to ensure the breakage of the first wire. This in turn implies, on the basis of idealised constant-pressure opening theory (Section 2.2) that because of the relatively slow initial motion, almost half the total opening time may elapse before any triggering signal is emitted.

Therefore although the multiple wire-breakage technique remains a viable basis for investigating the early motion of the petals, an alternative approach is required for the generation of the initial triggering signal.
The final system devised for this purpose makes use of a simple electrical contact-claw, this being a length of 32 S.W.G. steel piano-wire mounted in an insulated bush set in the shock-tube wall Fig. 5.3. The claw is initially manipulated so that it bears against the centre of the diaphragm with sufficient load to ensure electrical contact even when the maximum centre-deflection is attained. When the diaphragm ruptures, the ensuing folding process begins with a very large angular acceleration by virtue of the large initial hydrostatic pressure loading. The contact claw by contrast carries a negligible resultant pressure-load and therefore a large relative angular acceleration arises between diaphragm petal and contact claw which results in an early loss of contact and the generation of a prompt triggering signal (Appendix 3)

Results obtained with the use of this device are presented in Section 6.1 in comparison with those obtained by the use of the other triggering system.

5.2.2 Measurement of Diaphragm Opening Rates by Light Transmission

An important consideration in the tests to determine the opening rates of diaphragms is the avoidance of the assumption of strict proportionality between the instantaneous aperture area of the diaphragm and the total quantity of light transmitted through the aperture. Simpson's experiments, for example show that appreciable scattering of the light-beam occurs in the vicinity of the diaphragm.

In the approach adopted in the present work a parallel light beam from a concave mirror is passed across the shock-tube axis through small horizontal slit windows mounted in the walls and aligned on the centre-line (Fig. 5.4) These windows are situated in the driven section immediately adjacent to the diaphragm and the one nearest the
light source is covered with a metal masking plate having five
pin-holes spaced at regular intervals along its centre line. The
light traversing the tube is thus in the form of 5 parallel horizontal
filaments which on emerging from the test section are brought to a focus
by a second concave mirror and passed into a photomultiplier.

When the diaphragm bursts and folds towards the tube walls one
or other of the side petals intercepts each light-filament in turn
producing a step-change in the output signal from the photomultiplier
which is recorded on an oscilloscope camera.

From a knowledge of the position of each light-filament its
interception by the diaphragm may be interpreted in terms of an
angle of rotation and a record of petal angle vs time is thus
obtained.

This system, in common with those of White and Simpson et al
is open to the criticism that gas-dynamic disturbances can give rise
to displacements of the light filaments so leading to uncertainties
in the interpretation of position data.

However the path of the incident light between the window and
the diaphragm petal is very much shorter than are the light paths
of systems in which the light passes along the tube axis e.g. from
windows in the driven section (1.1) or from a lamp mounted inside
the driver (1.2) and this gives less likelihood of local beam
rotations becoming integrated along the light path into significant
displacements.

Nevertheless, as a check on the effectiveness of this method
comparison has been made, with results obtained using an alternative
technique based on spark photography of the diaphragm (Sections 5.2.3
and 5.2.4).
A discussion of the errors inherent in the use of the present system is given in Appendix 4.

The driver gas for all the shock-tube runs was oxygen-free nitrogen and the channel gas was atmospheric air.

The diaphragm materials tested were aluminium, copper and stainless steel.

All examples were grooved to a depth of between 25% and 50% of their basic thickness and detailed results are given in Section 6.3.

5.2.3 Spark Photography of the Bursting Diaphragm

The motion of the diaphragm was also investigated by taking spark photographs using a standard N.P.L. type spark light source (5.3) fired at a variety of time delays following rupture of the diaphragm.

Initial attempts to obtain silhouette pictures of the aperture by passing a parallel beam from a concave mirror along the tube axis through glass end-plates in the driver and low-pressure sections proved only partially successful, a blurred and distorted image being obtained (Plate 5.2).

Better results were obtained when the spark light output was used to illuminate the high-pressure side of the diaphragm using the arrangement shown in Fig 5.5. Here a small portion of the glass end-window in the driver section is used to admit the light from the spark. This is focused on a small circular plane mirror aligned to turn the beam through 90° into the tube where it then diverges to illuminate the diaphragm surface.

A standard S.L.R. camera with telephoto lens and extension tubes is mounted near the window and receives light from the diaphragm through the major unobscured portion of the window.
Sample photographs are shown in Plate 5.3 and results in the form of values of petal angle at various times are compared with the results of other experiments and with theoretical analysis in Section 6.3.

5.2.4. Multiple-Exposure Spark Photography

Using the optical arrangement shown in Fig. 5.5 the method described in Section 5.2.3 was extended by the use of a multi-spark light-source system with a repetition rate of up to 10 KHz. This gave up to 5 sparks at equal time intervals during the diaphragm opening process and when used in conjunction with a suitable pattern painted on the diaphragm surface provided multiple exposure pictures of the diaphragm petals on a stationary film.

Only aluminium diaphragms were used, since the other materials used in the present work gave opening times too short to allow more than 2 sparks to be fired with the maximum 10 KHz repetition rate imposed by the spark unit.

Various painted patterns were used on the diaphragm surface in an attempt to indicate clearly the positions of the tips and edges of the petals in each exposure - best results being obtained when a matt-white central square was set in a matt-black background.

Initial difficulties were experienced when using the standard hemispherical electrodes of the N.P.L. type spark gap, in holding the position of the spark channel fixed at high flashing rates. This difficulty resulted in erratic variations in the level of illumination of the diaphragm but was overcome by the use of a guided spark channel (5.4) which was used in obtaining all the photographs from which useful data was obtained.
Sample photographs are shown in Plate 5.4.

The spark firing times were controlled by feeding the output from a free-running square-wave oscillator set at 10 KHz through a gate circuit to an amplifier which in turn, drove the grid of the hydrogen thyratron valve used to pulse the energy stored in a bank of high voltage capacitors, to the spark channel.

The trigger signal for the gate was obtained from the contact claw (Section 5.2.1) and this also triggered the sweep of an oscilloscope on which the spark firing times were recorded, by means of a photomultiplier.

It was necessary to determine the spark times since although the time interval between successive firings was fixed by the oscillator, the first spark could occur at random during the 100 μs following the receipt of a triggering signal from the diaphragm.

In order to determine the influence of variations in the initial gas conditions on the motion of the diaphragm, a series of runs was performed using helium as driver in place of nitrogen, and in addition to using an initial driven section pressure of 1 atmosphere, several runs were carried out using a P1 value of 1.0 torr.
CHAPTER 6

Experimental Results of Petal-Motion Studies

6.1 Trigger-Signal Generation

In the comparative tests between the methods of trigger-signal generation, described in Section 5.2.1, the performance of the contact-claw system proved superior to those of both the light-transmission and wire-breakage arrangements.

Plate 6.4 illustrates the comparison between all three methods applied to a single diaphragm. All the oscilloscope traces are swept simultaneously at 100 μs/cm, the time-base triggered by the contact-claw.

The uppermost trace shows the photomultiplier output which gives a negative-going signal from an increased light input. More than 100 μs have elapsed, following the initiation of the sweep, before any sensible change in photomultiplier output occurs and a corresponding delay would result were this used as the trigger source.

The delay is even longer for the wire-breakage signal which is displayed on the second trace; over 160 μs elapse before the breakage of the first wire occurs, though a second wire positioned 5 mm farther along the groove breaks at 240 μs.

The third trace is the output from the wall-contact pin indicating a fully open diaphragm.

The almost simultaneous signals evident on the second trace indicate that the broken ends of the signal wires have again made earth contact on meeting the tube walls.

Because of its simplicity in use and its superior performance compared with the light transmission and wire-breakage methods, the contact-claw arrangement was used as trigger source for all subsequent shock-tube runs in the present work.
6.2 Early Motion of the Diaphragm

The results of the tests, described in Section 5.2.1 to study the early motion of the petals, are shown in Figs. 6.1 to 6.3. These give the elapsed times after the start of folding, for the breakage of the signal wires glued across the groove between a pair of adjacent petals, as a function of the distance of the wire location from the tip of the petal. The theoretical curves on each figure are based on values of petal rotation to cause wire failure derived from the analysis discussed in Appendix 3, used in conjunction with the theoretical curves of petal motion, including the effects of edge bending resistance, shown in Figs. 3.15 to 3.17.

In general, the measured wire-breakage times tend to exceed the calculated values, even those based on the assumption of maximum wire elongation to failure and minimum stretching of the petal grooves before failure.

For aluminium and stainless-steel diaphragms, the experimental and theoretical results are in reasonable accord for wire positions up to nearly 40% of the distance along the groove but the results diverge at greater distances. For copper diaphragms however the discrepancy is greater and the results even for wires near the diaphragm centre fall outside the area enclosed by the idealised scatter band.

A possible explanation for the above discrepancy is that the motion of the cracks which travel along the diaphragm grooves occupies a time which is not negligible compared with the total folding time. The gas-pressure forces would therefore, in the early part of the motion, be resisted not merely by the edge bending moment,
but by a moment about the edges due to the forces required to enable the tearing process to continue and such forces have been neglected in the present, and previous analyses.

The influence of this effect on the overall motion of the petals is considered in Section 6.3.

In an attempt to obtain closer agreement with experimental findings, the results shown in Figs. 6.1 to 6.3 were re-plotted using measured values of petal angle (Fig. 6.4 to 6.6) in the derivation of wire-breakage times, in place of the theoretical values used previously.

The discrepancy was diminished by this means for the three diaphragm materials tested, but over much of the range of wire positions, wire breakage continues to occur later than predicted.

This supports the explanation based on finite crack-propagation time since breakage time of the wire, especially at appreciable distances from the centre, would be influenced to a greater extent by a delayed parting of the groove edges than would the interception of the light filaments used in the measurement of petal folding rate (Section 5.2.2). This point is illustrated by Fig. 6.7 which shows schematically a possible diaphragm configuration, at an instant a short time after rupture, before the cracks have attained their ultimate length. Signal wires attached at the outer extremities of the grooves would remain unbroken, while sufficient petal folding would have occurred to give interception of the light filaments.

6.3 Comparison of Theoretical Opening Rates with Values Measured by Light-Transmission Technique

Calculated values of petal angle vs time, based on the quasi-steady flow analyses both plane and radial are shown in Figs. 6.8 to 6.10 in comparison with results based on the photomultiplier
measurements described in Section 5.2.2.

Kireyev's plane-flow theory is seen to underestimate the opening time considerably when compared both with the experimental results and with the results of the present theory. The predictions of the latter are in fair agreement with the experimental points for aluminium and stainless steel diaphragms.

The experimental results for the three different materials are now compared separately with the predictions of the present radial-flow theory.

**Aluminium Diaphragms**

The results in Fig. 6.8 show that the measured opening rate is lower, in the early stages of opening, than is predicted by the theory but later it increases and the theoretical opening time is very close to the measured value. This accords with the idea of a sizeable crack propagation time giving increased resistance to motion in the early stages. Evidence to support this view comes from the single-spark photographs (Plate 5.3) which indicate that the cracks between adjacent petals do not travel along the entire groove length before folding begins.

The results also suggest that the theoretical analysis underestimates the true gas-pressure loading, the closeness of the opening time values being to some extent fortuitous.

**Stainless-Steel Diaphragms**

The experimental points for stainless steel diaphragms also lie close to their corresponding theoretical curves in the early part of the motion but later diverge the experimental opening rate becoming less than the theoretical value. Nevertheless the final measured opening times are within 10% of the theoretical predictions.
Copper Diaphragms

The behaviour of copper diaphragms is in accordance with the indications obtained from the wire-breakage curves Fig. 6.1 in that an appreciable retardation is evident, early in the opening process and although in the later stages the experimental opening rate is in good agreement with the theoretical rate, the measured opening times are up to 25% greater than the theoretical values.

Summary

Overall, the radial-flow theory of Section 3.4 predicts the behaviour of the petals more accurately than do the idealised theory of Section 2.2, or Kiryeyev's plane-flow theory. However the experimental results suggest that for some materials notably copper, the time taken for the cracks which travel along the diaphragm grooves to convert the stress-bearing membrane into separate petals may not be negligible as is assumed in theoretical analyses.

6.4 Opening Rate Measurements from Spark Photographs

Samples of single-spark and multiple-spark photographs are shown in Plates 5.3 and 5.4. Despite normal precautions in focussing the camera, the quality of these is uniformly low. A possible explanation for this may be that the gas-dynamic disturbances in the vicinity of the diaphragm and in the primary expansion cause sufficient local variation in the refractive index of the gas to de-focus the subject. In spite of these shortcomings however, the photographs served their basic purpose in indicating the petal angles at various times and these results are plotted in Fig. 6.11.

Also shown are the corresponding radial-theory curve, and the experimental results from the light-transmission measurements.
Only aluminium diaphragms were investigated in the present tests since these had a nominal opening time greater than 600 μs which allowed the taking up to 4 satisfactory exposures with the limiting 10 KHz repetition rate of the present spark unit. The stainless-steel and copper diaphragms opened in 400 μs or less and for these materials a higher flashing rate would be required in order to give an adequate number of exposures.

However, the main objective of the photographic tests was to verify the measurements made with the photomultiplier system and the results for a single diaphragm material are sufficient for this purpose.

As shown in Fig. 6.11 the close proximity of the photographic results, both single-shot and multiple exposure, to the photomultiplier results provides mutual confirmation of both sets of results subject to the limitations imposed by errors in measurement (Appendix 4).

6.5 Effect of Driver-Gas Atomicity on Opening-Rate

According to the quasi-steady analyses of flow in the diaphragm region, the pressure distribution on the petal surface is a function only of the specific heat ratio of the driver for a given value of \( p_4 \).

Calculated values of petal angle vs time for aluminium diaphragms in conjunction with both monatomic and diatomic driver gases are shown in Fig. 6.12. Superimposed on this graph are the results taken from multi-spark photographs of the opening diaphragm, using helium as driver-gas.
The theoretical curves are in close mutual agreement but the experimental points lie closer to the diatomic-gas curve at low values of $\theta$. However as the opening process nears completion, the experimental points tend towards the monatomic-gas curves.

The discrepancy between experiment and theory for the monatomic gas is never greater than that occurring in the case of steel diaphragms using air as driver-gas (Fig. 6.10) and indeed is less than that found in the case of copper diaphragms (Fig. 6.9) and is therefore again attributable to the finite crack-propagation time.

In general therefore the influence of the atomicity of the driver-gas on the motion of the diaphragm petals both as predicted by the radial-flow theory and as measured experimentally is small.

6.6 Effect of Diaphragm Pressure-Ratio on Opening Petals

The results of the photomultiplier-based measurements of the opening rate of aluminium diaphragms, shown in Fig. 6.8 cover a range of driver-pressures from 10 bar to 3.8 bar abs.

Over the corresponding small range of diaphragm pressure ratios, atmospheric air having been used as the driven gas throughout, no significant effect attributable for example to variations in the strength of the wave system in the tube is either predicted by the theoretical analysis or evident in the experimental results.

The only restriction on the use of the quasi-steady choked-orifice approach to the flow through the diaphragm region is that the shock pressure $p_2$ should not exceed the final driver-gas critical pressure $p^*$. 

The results of the tests carried out at much greater $P_{41}$ values attained by using a driven-section pressure of 1 torr (Section 5.2.4) are shown in Fig. 6.13 and again the conclusion is that no significant
effect due to diaphragm pressure ratio is apparent, the results lying amongst those taken at other pressure ratios.

This result suggests that since the static pressure down-stream of the diaphragm has been shown (Section 3.7.1) to have an important influence on the opening rate, the pressure in this region must be independent of the initial pressure in the driven section. The implication is that the former is dominated by the conditions in the stream of driver-gas emerging from the diaphragm exit. This finding accords with one of the assumptions made in formulating the radial flow theory (Section 3.4.1) that the static pressure at the forward face of the diaphragm is equal to \( p^* \).

6.7 Linear Moment Variation

The variation with petal angle of the theoretical moment on the diaphragm petal due to gas-pressure loads is shown in Fig. 3.12 the pressure distribution being based on the present radial flow analysis. Over much of the range of petal angles, a straight line provides a good approximation, the corresponding expression for the moment \( M_g \) being:

\[
M_g = p_n L^3 (a - b \theta)
\]

where \( a \) and \( b \) are dimensionless constants.

The advantage of such an approximation is the simplification it allows in the calculation of petal motion.

Putting \( A = p_n L^3 a \) and \( B = p_n L^3 b \) and using equation 6.1, the equation of motion for a diaphragm petal may be written:
\[ \dot{\theta} = \frac{A - B\theta - M_b}{M_i} \quad \text{Equation 6.2} \]

where \( M_b \) = edge bending resistance
and \( M_i \) = moment of inertia about the clamped edge

The solution to 6.2 which is of simple harmonic form is:

\[ \theta = (\theta_0 - \phi) \cos \psi t + \phi \quad \text{Equation 6.3} \]

where \( \phi = (A - M_b)/B \)
and \( \psi = (B/M_i)^{1/2} \)

The values of the constants \( A \) and \( B \) may be chosen in a number of ways the choice being dependent on the area in which the best approximation is sought.

For example, if a high degree of accuracy is required in the early part of the motion the value of \( A \) might be made equal to the "exact" initial moment as calculated from radial-flow theory and the value of \( B \) could then be adjusted to give the correct value of total opening time.

Again, if the area enclosed by the "exact" and approximate moment lines were made equal, the net work input to the diaphragm would be the same in both cases and the final kinetic energy and angular velocity of the petals would be predicted exactly by the approximation.

A basic requirement for the present purpose however was to obtain an approximation which accurately reproduces the motion of the petals over as large a range as possible for the purpose of calculating initial values of the flow properties which are required in the solution of the flow in the shock-formation region.
The values of $A$ and $B$ were therefore chosen so as to make the total opening time and the time when half open, predicted by the linear approximation, identical with the corresponding "exact" values.

If $\theta_\frac{1}{2}$ and $t_\frac{1}{2}$ are the petal angle and time respectively at which the diaphragm is half open, and $t_0$ is the "exact" opening time, the constants $\phi$ and $\psi$ and the corresponding values of $A$ and $B$ are determined from the equations:

$$\theta_\frac{1}{2} = (\theta_0 - \phi) \cos \psi t_\frac{1}{2} + \phi$$

$$\pi/2 = (\theta_0 - \phi) \cos \psi t_0 + \phi$$

A comparison between the approximate and "exact" values of time at various angles is given in tables 6.1 to 6.3 which indicates that the greatest discrepancy is approximately 2.0 µs. For experimental comparisons, if it is assumed that the trace recorded by an oscilloscope camera can be read to the nearest $\frac{1}{2}$ mm, this represents a resolution limit of 5 µs for a sweep-rate of 100 µs/cm which is typical in the present context and the error attributable to the linear approximation is thus well within this limit.
Table 6.1

Aluminium Diaphragms

Bursting Pressure 14.0 bar (nominal)

Thickness 18 S.W.G.

te/t = 0.5

\( \phi = 1.0836 \) radian

\( \psi = 3383.85^{-1} \)

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<th>&quot;Exact&quot; Time (( \mu s ))</th>
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### Table 6.2

**Copper Diaphragms**

**Bursting Pressure** 14.0 bar (nominal)

**Thickness** 26 S.W.G.

\[ te/t = 0.5 \]

\[ \phi = 1.6643 \text{ Radian} \]

\[ \psi = 3306.85^{-1} \]

<table>
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<tr>
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Table 6.3

Stainless-Steel Diaphragms

Bursting Pressure 14.0 bar (nominal).

Thickness 34 S.W.G.

t_e/t = 0.4

φ = 1.7265 radian

ψ = 4629.88\(^{-1}\)

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CHAPTER 7
Unsteady Flow in the Diaphragm Region

7.1 Introduction

All quasi-steady analyses of the flow in the diaphragm region (Sections 3.1 and 3.4) are based on the assumption that the motion of the physical boundaries has no effect on the flow properties; for fixed initial gas conditions, the pressure distribution and moment on a moving diaphragm petal at a given angle are assumed to be identical with those on the corresponding stationary petal.

The diaphragm region is represented in Kireyev's analysis for example, as a straight tapered duct of square cross-section and on this basis, the opening process becomes simply a step-wise reduction of the taper, with stagnation-enthalpy, mass-flow, etc., fixed at each step (Fig. 3.3).

This artifice, while simplifying considerably the calculation of gas loads on the folding diaphragm petals, overlooks the essentially unsteady nature of the flow in this region.

In the real flow, the driver-gas particles are set in motion by the pulses of a Q-expansion wave which originate at the downstream end of the duct.

The effect of this wave system may be to introduce significant spatial variations in the stagnation enthalpy and mass-flow rate within the duct, in violation of the main assumptions of the quasi-steady analysis.

The Q-waves which emerge from the diaphragm region at the upstream end become transformed by wall reflections into the primary one-dimensional unsteady expansion.

However, the flow within the tapering duct itself is both three-dimensional and unsteady and these factors together with the motion of the boundaries present a formidable analytical problem.
Nevertheless, as the convergence of the flow boundaries diminishes, the internal flow must become increasingly one-dimensional. An indication as to the orders of magnitude of wave-induced perturbations in the various flow properties during the later stages of opening might thus be obtained on the basis of a one-dimensional method-of-characteristics solution. An analysis of this type, which differs from the quasi-steady analysis (Section 3.1 and 3.4) in incorporating an allowance for the influence of spatial and temporal changes in the duct cross-sectional area has therefore been formulated. (section 7.2).

The use of this type of approach in the analysis of wave-motion in ducts of fixed taper is well documented (7.1, 7.2 and 7.3) though no information appears to exist relating to ducts with time-varying wall geometry.

Hertzberg and Kantrowitz (7.1) investigated the motion of shocks through a convergent duct with a 15.5° taper and Kahane, Warren, Griffiths and Marino (7.2) conducted a similar research in a 10° duct. In both cases the isentropic flow-field behind the shock was analysed by a one-dimensional method-of-characteristics approach and the results in the form of shock trajectories in the 15.5° duct and density distributions in the 10° duct were in good accord with corresponding experimental measurements.

More recently Nevis (7.3) studied the motion of centred expansions through supersonic nozzles of hyperbolic contour attached to a large reservoir and found good agreement between one-dimensional method-of-characteristics analysis and experiment.
The results of these studies while providing justification for the use of this type of theoretical analysis in the present context give no indication as to the maximum value of taper at which a one-dimensional treatment is appropriate.

Aside from limitations associated with departures from one-dimensional behaviour at large values of taper, already discussed in the steady-flow context (Section 3.3) a further practical limitation is imposed in unsteady flow, by the construction of the characteristics network as discussed in Section 7.2.3.

The spatial rate of change of duct cross-sectional area may be shown (equation 7.1) to contribute to changes $\delta P$ and $\delta Q$ in the Riemann variables between adjacent wave elements and such changes must be kept small in the interests of accuracy.

For a case in which the initial petal angle $\theta_0$ is zero, the spatial gradient of area becomes infinite at $\theta = 0$ since the convergent duct degenerates into a flat plate.

The one-dimensional theories, both steady and unsteady then become inappropriate. The method-of-characteristics approach is therefore used only after the diaphragm has attained a finite degree of opening, initial values of the flow properties being computed from quasi-steady theory.

The basic equations of motion, together with details of the solution are given in Section 7.2 while in Section 7.3 a comparison is made between results derived from the unsteady analysis and those of quasi-steady theory.

The time scale of the solution is determined basically by the motion of the diaphragm. The initial computations were performed using data for aluminium diaphragms and assuming air as driver-gas.
Detailed results were computed for this case and are discussed in Section 7.3.

In order to assess the effect on the unsteady flow analysis of variations in the diaphragm opening-rate, several of the preceding results were re-computed using data appropriate to stainless-steel diaphragms; the opening times of the latter are approximately half those of aluminium diaphragms of the same bursting strength.

Finally, the influence of the driver-gas properties, notably sonic velocity was investigated by a repetition of the above calculation assuming helium as driver.

7.2.1 Equations of One-Dimensional Unsteady Flow with Area Change

The standard equations (Ref. 1.6, 1.14 and 7.4) giving the variation with time of the Riemann variables P and Q in a duct of variable area, assuming unsteady, one-dimensional flow are:

\[
dP, dQ = (du)_p, Q \pm \frac{2}{\gamma - 1} (da)_p, Q = \pm \frac{\dot{A}}{A} \frac{\partial (u\dot{A} + \dot{A})}{\partial t} dt, p, Q  
\]

where suffixes P and Q refer to conditions along curves having slopes in the x-t plane given respectively by:

\[
\frac{dx}{dt}_p = u + a \quad 7.2
\]
\[
\frac{dx}{dt}_q = u - a \quad 7.3
\]

Assuming the diaphragm-region to be represented as a tapering square-section duct (Fig. 7.1) as in Kireyev's quasi-steady analysis, the cross-sectional area at section XX is given by:
\[ \frac{A_x}{A} = (1 - \frac{x}{L} \cot \theta)^2 \] 7.4

and on differentiating, the spatial rate-of-change is given by:

\[ \frac{1}{A_x} \frac{\partial A_x}{\partial (x/L)} = -\frac{2}{L} \cot \theta \frac{1-x}{1-\frac{x}{L} \cot \theta} \] 7.5

and the temporal derivative is given by:

\[ \frac{1}{A_x} \frac{\partial A_x}{\partial \tau} = \left( \frac{x}{2 L \csc^2 \theta} \right) \frac{d\theta}{d\tau} \text{ where } \tau = \frac{A t}{L} \] 7.6

the times being non-dimensionalised, for convenience, with respect to the transit-time of an undisturbed acoustic wave across a duct of length L, as discussed further in Section 7.3.3.

The ratio of temporal to spatial rate of change of area at any petal angle is then given by:

\[ R_{ts} = \frac{-2 x}{\sin 2\theta} \frac{d\theta}{L d\tau} \] 7.7

Assuming a realistic relationship between petal-angle and time (equation 6.5 with \( \theta_0 = 0 \))

\[ \frac{d\theta}{d\tau} = \psi' \phi \left( \frac{\theta (2-\theta)}{2} \right) \] 7.8

Where \( \psi' = \frac{L \psi}{a_0} \)
Then equations 7.7 and 7.8 give the result:

\[ R_{ts} = -\frac{2}{\sin 2\theta} \frac{x}{L} \psi' \phi \left( \frac{\theta (2-\theta)}{\phi} \right)^2 \]  

Tables 6.1 to 6.3 indicate that the values of \( \phi \) and \( \psi' \) are of order unity and it follows from equation 7.9 that except at very small values of \( x \) or \( \theta \), the temporal rate-of-change of duct area is not negligible in comparison with the spatial rate of change. Indeed as \( \theta \) tends towards 90°, \( R_{ts} \) tends to infinity since although the temporal rate-of-change remains finite (equation 7.6) the spatial rate-of-change tends to zero.

These results conflict with the assumption, implicit in the quasi-steady approach that the temporal derivatives of the flow variables are negligibly small compared with the spatial derivatives. This in turn underlines the desirability of obtaining an estimate of the magnitudes of the unsteady-flow effects.

The method of approach used for this purpose in the present context, involves the construction of a characteristics network for the flow in the diaphragm region, using equations 7.5 and 7.6 in conjunction with equation 7.1 to account for the effects of changes in the duct geometry.

7.2.2 Determination of Characteristics Network

The basic unit-process in the solution of unsteady wave-flow problems involves the determination of positions in the \( x-t \) plane and flow-properties in the \( a-u \) plane for points such as 3, Fig. 7.2 from a knowledge of the positions and local flow properties of two adjacent points 1 and 2 which must be on different characteristic lines.
The location of 3 is given by the intersection of the appropriate physical characteristics through 1 and 2. The changes in the various flow properties occurring between points such as 1 and 3 or 2 and 3 are determined from equation 7.1 which when expressed in non-dimensional form and with the area-change terms given in equations 7.5 and 7.6 substituted, becomes:

\[
d\left(\frac{u}{a_4}\right)_P^Q = \pm \frac{d(a/a_4)_P^Q \pm a}{a_4} \left[ -\frac{u}{a_4} \left(\frac{2\cot \theta}{1-x \cot \theta}\right) + \frac{\left(\frac{2L \cosec^2 \theta}{L}\right) \frac{d\theta}{d\tau}}{1-x \cot \theta} \right]
\]

\[7.10, \ 7.11\]

In finite-difference form as applied to the case shown in Fig. 7.2 the above becomes:

\[
\frac{2}{\gamma_4-1} \Delta(a/a_4)_1^3 + \Delta(u/a_4)_1^3 = (\delta_1^3/A_1^3) \left[ F(x,0,u)_1^3 \right] \times \Delta T_1^3 = \Delta P_1^3 \quad 7.12a
\]

\[
\frac{2}{\gamma_4-1} \Delta(a/a_4)_2^3 - \Delta(u/a_4)_2^3 = (\delta_2^3/A_2^3) \left[ F(x,0,u)_2^3 \right] \times \Delta T_2^3 = \Delta Q_2^3 \quad 7.12b
\]

Where \(F(x,0,u) = \frac{2\cot \theta (u/a_4)}{1-x \cot \theta} - \frac{\left(\frac{2L \cosec^2 \theta}{L}\right) \frac{d\theta}{d\tau}}{1-x \cot \theta}\)

and \(F(x,0,u)_1^3 = F(\_1^3) + F(\_3^3)\)

In the practical solution (1.14) as a crude first approximation the second-order terms on the R.H.S. of equations 7.12a and 7.12b are neglected. This allows the determination of tentative values of the flow properties at 3, \(a_3\)', and \(u_3\)', from:

\[
\frac{2}{\gamma_4-1} a_2 - \frac{2}{\gamma_4-1} u_2 = \frac{2}{\gamma_4-1} a_3' - \frac{2}{\gamma_4-1} u_3'
\]

\[7.13a\]
With these provisional results, the approximate physical characteristic curves may be computed from 1 to 3, and from 2 to 3. Fig. 7, with mean slopes $a_{13} + \bar{u}_{13}$ and $\bar{u}_{23} - a_{23}$ respectively, giving the point $3'$. The terms on the R.H.S. of equations 7.12a and 7.12b — which express the influence of the motion of the duct on the flow variables are then calculated using values $\frac{d\theta}{dt}$ obtained from the quasi-steady diaphragm opening theory (Section 6.3). This in turn gives new values $a_3''$ and $u_3''$ which allow a revised estimate to be made of the position of point 3 in the physical plane. The process continues until the changes in the position of 3 and in the values of the flow variables becomes less than some prescribed limit.

7.2.3 Specification of Initial Values

A value of 0.2 has been quoted, in standard literature on wave-diagram construction (7.4) as a desirable maximum value for the terms $\frac{\bar{a}^2}{A} F$ $\Delta t$ on the L.H.S. of equation 7.12. These terms are equal to the differences in the Riemann variables between adjacent waves and since the basic process is one of the representation of a curvilinear network by a series of straight lines, excessive distortion of the pattern occurs when the discontinuous changes in slope between adjacent line-elements become large. Alternatively, if the value of the $F$ terms in equation 7.12 became very large then a correspondingly small $\Delta t$ would be required to keep the step-change $\frac{\bar{a}}{A} F \Delta t$ moderate. This in turn would call for an excessively large number of steps in the computation.
Equations 7.5 and 7.6 indicate that such a difficulty would arise if the present technique were used at small values of $\theta$ since as $\theta \to 0$ the R.H.S. of equation 7.1 tends to infinity. An accompanying difficulty is that the flow tends to depart increasingly from one-dimensional behaviour as $\theta$ becomes small.

A hybrid analysis has therefore been used in which the initial stage of the opening process has been computed on a quasi-steady basis, and the flow variables at a particular value of petal-angle used as initial data for the unsteady analysis applied to the remainder of the opening process.

Two basic considerations apply to the choice of petal angle at which the transition is made. On the one hand, too early an application of the unsteady analysis could result in excessively large values of $\Delta P$ and $\Delta Q$ between adjacent characteristics. Conversely, if application of the unsteady analysis were delayed until the flow became effectively one-dimensional on the basis of the steady-flow criterion discussed in Section 3.3.1, it is apparent from Fig. 3.6 that little of the opening process would remain for analysis, with the result that insufficient time would be available for any unsteady-flow effects to be integrated into sensible changes.

A trial analysis was therefore performed with the diaphragms aligned arbitrarily at a petal angle of approximately 45° as a starting point.

A convenient point close to this value was taken by using a 40% diaphragm opening based on petal angle, i.e.,

$$\frac{\theta - \theta_0}{\theta_2 - \theta_0} = 0.4$$
The values of \( u \) and \( a \) at eleven equally spaced points along the \( x \)-axis at the time value corresponding to the above petal-angle were used as the initial data.

The basic operation in the solution is the determination of \( \Delta P_{13} \) and \( \Delta Q_{23} \) at each wave intersection (equations 7.12a and b) within the body of the solution. In addition, details of the wave reflection conditions at the upstream and downstream ends of the diaphragm-duct are required, as discussed in Section 7.2.4.

The complete analysis was programmed for computer solution, and an X-Y plotter used to draw the resulting wave diagrams, samples of which are shown in Figs. 7.5 to 7.7.

### 7.2.4 End-Reflection Conditions

While the determination of the characteristics network for the bulk of the \( x-t \) plane representation is based on the interaction procedures described in Sections 7.2.1 to 3, there remains the problem of specifying the condition to be met when waves arrive at the upstream or downstream boundaries of the duct.

#### Upstream Boundary Reflection

This case is illustrated in Fig. 7.3 and involves the determination of the flow properties at point 3 which is the intersection of the Q-wave from point 2, with the slope-discontinuity at the upstream end of the diaphragm region.

Conditions are known at the point 2 and equation 7.12b provides one relationship between \( a_3 \) and \( u_3 \). The equation expressing the invariance of \( P \) in the driver section provides the second relationship.

\[
\frac{2}{\gamma_4-1} = \frac{2}{\gamma_4-1} \frac{a_3}{a_4} + \frac{u_3}{a_4}
\]

7.14
and \( a_3 \) and \( u_3 \) are then determined. The location of point 3 on the \( x-t \) diagram is found in the normal way from a knowledge of the co-ordinates of the point 2, i.e.,

\[
\frac{\Delta(x)_{23}}{\Delta(a_3, t)_{23}} = \frac{u_2 + u_2 - a_3 - a_2}{2a_0}
\]

Downstream Boundary Reflection

The curve in the \( x-t \) plane representing the path of the down­
stream boundary is based on values of \( x \) and \( t \) calculated for the
tips of the diaphragm petals. (Section 3.7).

The problem of specifying conditions along this envelope
involves typically, the determination of flow properties at the
point 3, Fig. 7.4, at which the envelope intersects the P-wave
from point 1. The relationship between \( u_3 \) and \( a_3 \) is given by
equation 7.12a.

A second relationship was provided initially by the assumption
that the Mach-number at the exit plane of the duct is unity. This
in turn implies a static pressure lower than \( p_3 \) in the region
downstream of the diaphragm and directly parallels the choking
assumption in steady flow.

With these two equations, the values of \( u_3 \) and \( a_3 \) become known
and the point 3 is fully determined.

Initial calculated results however, indicated that in the
later stages of opening, the Mach-number just upstream of the exit
becomes slightly greater than 1.0 and the assumption of sonic flow
at the exit is therefore unduly restrictive.

This was then modified so as to allow the flow to become
supersonic at the exit-plane.
In evaluating flow properties at a point such as 3 in Fig. 7.4 from known values at the point 1, if the Mach-number $M_1$ is less than unity then the assumption that $M_3 = 1$ is retained. If however, $M_1 > 1.0$ it is assumed that $M_3 = M_1$.

The alternative form of the wave diagram, constructed on this basis is shown in Figs. 7.5 to 7.7 which indicate that in the area in which supersonic flow develops the adjacent pairs of points of the same type as 1 and 3, Fig. 7.4 are physically very close and a Mach-number distribution evaluated near the end of the opening process shows only a moderate rate of change of Mach-number. These two factors tend to justify the revised treatment of the downstream boundary intersection.

7.3 Results and Discussion

The principal objective in performing the present unsteady-flow analysis was that of determining the order of magnitude of the influence on the flow properties, of the motion of the duct boundaries. The completed wave diagrams Figs. 7.5 to 7.7 serve partly as a check on the calculations in that they would reveal excessively large intervals in $P$ or $Q$ between adjacent wave-pairs as abrupt changes in slope. Initial computations showed the intervals to be small everywhere except for the pair of points situated at the downstream end of the duct.

In this region, the rates of change of the flow properties with distance along the duct have their largest values; $AQ$ here exceeded the recommended upper limit (7.4) by an order of magnitude. This produced a disproportionately large space between the corresponding pair of adjacent $Q$-waves and the diagram was therefore re-computed with appropriate sub-division of the large $Q$-interval, the corrected results being shown in Figs. 7.5 to 7.7.
The time-scale on each diagram is non-dimensionalised with respect to the opening-time of the corresponding diaphragm.

The major portion of the present results derives from calculations in which air is assumed as driver-gas, and aluminium as diaphragm material.

A discussion of this is given in Section 7.3.1 while in Section 7.3.2 and 7.3.3 selected results are presented, relating to stainless-steel diaphragms and helium driver respectively.

7.3.1 Aluminium Diaphragms – Air as Driver-Gas

The Q-waves which originate at the downstream boundary of the duct in Fig. 7.5 are seen to spread apart to an appreciable extent as they travel upstream and are thus identified as forming a rarefaction wave of finite amplitude.

The P-waves on the other hand, most of which originate as reflections of the Q-waves from the upstream boundary remain almost parallel, thus indicating that the P-disturbance is of very small amplitude.

The variation of P and Q along the P-wave entering the diaphragm (Fig. 7.5) at C on the upstream boundary and leaving at D on the downstream boundary is shown in Fig. 7.8. For a rigid uniform duct the value of P along such a wave would remain constant and any departure from this condition in the present context is attributable to spatial and temporal variations in the duct cross-section.

The indication in Fig. 7.8 is that although the P-wave intersects a train of Q-waves across which Q varies appreciably, the variation in P itself is small.

The influence of the variation in the duct geometry is thus shown to be insufficient to cause major changes in the Riemann variables.
This is confirmed by similar results (Fig. 7.9) for a Q-wave entering the wave-diagram (Fig. 7.5) at A and leaving at B.

The overall change in Q is somewhat larger in this case since the Q-wave AB traverses the region adjacent to the exit-plane earlier in the opening process than does the P-wave CD and it is in these early stages of opening that the influence of the motion of the duct is greatest. This is shown by Fig. 7.10 in which the disturbance term on the R.H.S. of equation 7.12a is plotted against petal-angle, for the points such as 1-3 Fig. 7.4 on the P-wave element nearest the exit.

The ratio of the temporal to the spatial contributions to the above disturbance term is shown in Fig. 7.11. This indicates an exponential rise in the above ratio which becomes very large as the diaphragm approaches full opening.

This result requires careful interpretation however, in that both terms at that stage are very small, but the spatial term tends to zero while the temporal term remains finite.

Comparisons are shown in Figs. 7.12 to 7.16 between the values of various flow properties determined on the basis of quasi-steady flow theory and corresponding values calculated from the present unsteady analysis.

In Figs. 7.12 to 7.14 the abscissae represent distance along the duct axis, values of Mach-number, static pressure and stagnation temperature being plotted at two different values of time corresponding respectively to 60% and 90% opening of the diaphragm based on petal-angle.

As shown in Fig. 7.5, the 60% open case corresponds to a time shortly after the arrival at the upstream end of the duct, of the
first Q-rarefaction pulses emitted from the exit plane in the unsteady régime. Differences apparent at this stage between the quasi-steady and unsteady results are seen to diminish both with increasing time, the results being in appreciably better agreement for the 90%-open case, and also with increasing distance along the duct, the results in both cases being in closest accord at the exit-plane.

In addition to the comparisons of the spatial variation of flow properties calculated on the basis of quasi-steady flow theory with those of unsteady flow theory, the temporal variation at a particular point in the duct is shown in Figs. 7.15 to 7.19. The point chosen is the exit plane and the flow properties are plotted to a base of petal-angle this being physically more informative than time in the present context.

In all cases, the unsteady results diverge initially from the quasi-steady values as the transition occurs between the two different methods of flow analysis. As opening proceeds however, this tendency diminishes and the curves remain almost parallel for the remainder of the petal movement.

The similarity between the two sets of results supports the conclusions drawn from the results shown in Figs. 7.8 to 7.14 that no gross changes in the flow properties arise as a result of the motion of the flow boundaries at the rate appropriate to the opening of aluminium diaphragms.

7.3.2. Stainless-Steel Diaphragms - Air as Driver-Gas

The effect on the flow-properties, as determined by the unsteady analysis, of the more rapid motion of stainless-steel diaphragms compared with aluminium, is illustrated in Figs. 7.15 to 7.17.
The values of all the flow properties computed for stainless-steel diaphragms depart to a greater extent from the appropriate quasi-steady curves than do those for aluminium diaphragms. This greater discrepancy reflects the increased value of the temporal area-derivatives for the stainless-steel diaphragm, as compared with those of aluminium.

Of the flow properties considered, static pressure is the one most affected (Fig. 7.17). However in the context of a quasi-steady analysis of the flow in the region downstream of the diaphragm, which forms the main purpose of the determination of the flow in the diaphragm exit, (Section 9.3) the mass-flow and stagnation enthalpy are of chief importance. The latter as indicated in Figs. 7.15 and 7.16 are comparatively little affected by the diminished time-scale of the stainless-steel diaphragms.

7.3.3 Aluminium Diaphragms - Helium Driver

The results in Figs. 7.18 and 7.19 indicate that unsteady flow effects are of negligible importance in the context of a helium driver. The characteristic time $T_{\text{ch}}$ which represents the transit time of an acoustic wave travelling in undisturbed driver-gas along the maximum length of the diaphragm-duct is considerably smaller for helium than for air. When expressed as a multiple of $L_{\text{ch}}$, the dimensionless opening time of a given diaphragm is appreciably greater for helium than for air and the resulting flow approximates more closely to the quasi-steady model. The wave-diagram for helium driver, Fig. 7.7 shows that several complete wave-passes occur along the length of the duct during the diaphragm opening time; this situation contrasts with that of the stainless-steel diaphragms using air as driver, as shown in Fig. 7.6 in which the tail of the
Q-wave originating at the point P fails to arrive at the upstream boundary until the diaphragm is fully open. In the latter case, the opening time is not small compared with \( \frac{L}{a_h} \) and the flow departs from the quasi-steady model to a much greater degree than occurs for a helium driver.

**7.4 Conclusion**

The general conclusion derived from the present results is that the quasi-steady approach is least in error in the context of slow-opening diaphragms and driver-gases of high sonic velocity.

The greatest discrepancy between quasi-steady and unsteady flow results occurred in the case of stainless-steel diaphragms with air as driver-gas. Even here however no gross departures occurred in the mass-flow and stagnation-temperature results and advantage may therefore be taken of the much greater simplicity of the quasi-steady model in calculating the flow-properties at the diaphragm exit. These in turn are used in Section 9.3 in the determination of initial values for a method-of-characteristics solution of the shock-formation problem.

Fundamentally, the absence of major departures from the predictions of quasi-steady theory in most of the flow properties, even when rapid movement of the flow boundaries occurs, is attributable to the small overall length of the duct. This in turn implies that the times during which the waves remain within the duct are short, giving correspondingly small values of \( \int \frac{\delta P}{\delta t} \) and \( \int \frac{\delta Q}{\delta} \) taken along the appropriate wave-paths.
A measure of justification for the use of a quasi-steady analysis to link two unsteady flow-fields separated by a short area discontinuity (3.2 to 3.6) is provided by these results particularly where the geometry of the discontinuity is fixed, since under these circumstances, the temporal terms in equations 7.12a and 7.12b disappear and the disturbances in the Riemann variables are even smaller than in the present case.
CHAPTER 8

Theoretical Analyses of the Shock-Formation Process

8.1 Introduction

The quantity of literature devoted to the theoretical calculation of the shock formation process is very small compared with that available on other shock-tube topics. Only four attempts appear to have been made to formulate the problem in more detail than is incorporated in ideal shock tube theory and as such, White (1958) and Ikui et al. (1970) evolved theories based on very similar lines in representing the shock formation process as a result of the coalescence of a train of isentropic compression waves. No account was taken of the influence of the diaphragm in formulating these theories, but Kireyev (1960) attempted to relate the motion of the diaphragm to the accompanying flow in the tube. Satofuka (1970) used a two-dimensional computer-oriented finite-difference approach to evaluate the flow in the tube and linked the flow development to the step-wise opening of an idealised diaphragm.

A review of these four analyses is given in the present chapter, Kireyev's work being explored in some detail in view of the promise it offers, of providing a coherent solution to the complete flow problem rather than a piecemeal approach to one isolated aspect as in the case of the work of White and Ikui et al.

8.2.1 White's Formation-From-Compression Analysis

The analysis formulated by White (1.1) in which the shock is assumed to form as a result of the coalescence of a train of isentropic compression waves, appears to be the earliest advance on simple ideal shock-tube theory in the calculation of real shock-tube performance. For simplicity, White assumed that the compression waves meet at a single point in the tube, at which the shock is
assumed to become fully formed instantaneously.

In order to satisfy the physical boundary conditions, a rarefaction wave must be reflected from the shock formation point and a contact surface is also required, separating the hot shocked gas from the cooler gas which has experienced only isentropic changes in passing through the incident compressions and reflected expansion.

This shock-formation model is illustrated on a \( p, x, t \) diagram in Fig. 8.1. The \((p, u)\) -plane representation of the process (Fig. 8.2) contrasts the respective properties of isentropic compression and shock waves and indicates that a reflected expansion is necessary to give mechanical equilibrium in states "2" and "3". For any given driver/driven gas combination and \( P_{I} \) value, curves of this type may be used in the calculation of shock pressure-ratios and hence of all the accompanying flow properties.

Real-gas effects were included in White's calculations relating to high primary shock-strengths in air and argon; equilibrium gas tables and charts were used for this purpose (8.1, 8.2). White's curves of shock Mach-number vs diaphragm pressure ratio for various gas-combinations are shown in Fig. 8.3 and 8.4. Corresponding experimental results are included and the comparison is discussed in Section 8.2.4.

8.2.2 Mixing in the Contact Region

An important aspect of real shock-tube behaviour which is ignored in ideal shock-tube theory is that of mixing between driver and driven gases in the contact region.

White has shown, in a qualitative analysis, that the effect of using a combination of driver/driven gases having different
specific heats is to produce a volume-change in the gas mixture forming the contact region. For cases in which an appreciable temperature-difference exists across the contact region, this volume-change is positive when the specific heat of the driven gas is less than that of the driver, and vice versa.

For example, a helium/air combination initially at room temperature and having \( P_{i1} = 10^6 \) should give a 20\% net volume increase for a mixture composed initially of two equal volumes of driver and driven gases separated by an ideal contact surface.

This volume increase would be associated with the emission of right- and left- moving compression waves which would tend respectively to strengthen the primary shock and weaken the primary expansion. However in order to estimate the magnitude of such an effect, it would be necessary to know the rate of mixing and in the absence of such information, the above figure is useful only as an indication of the trend of the effect on the flow properties.

The experimental values of shock Mach-number for various values of \( P_{i1} \) plotted in Figs. 8.3 and 8.4 relate to a variety of driver/driven gas combinations which includes the cases in which mixing at the contact region should cause respectively an increase, a decrease and no change in the shock Mach-number as calculated on the basis of an ideal contact surface. The results are discussed in Section 8.2.4.

8.2.3 Shock Formation Distance

White proposed a simple method of estimating the distance travelled by the primary shock before it attains its full strength. For a given value of \( P_{i1} \) the shock Mach-number is calculated on the basis of ideal theory and the shock is assumed to attain its maximum velocity \( a_{1M} \) at the beginning of the diaphragm opening
process. The length of the formation region is then taken to be the distance travelled by this ideal-theory shock before being overtaken by an acoustic $P$-wave emitted from the diaphragm, at the instant it becomes fully open. The acoustic wave traverses three separate regions: (Fig. 8.5).

(a) the unsteady centred rarefaction which extends into the driven section at all except very low values of $P_{d1}$,
(b) the quasi-steady region 3 of expanded driver-gas and
(c) the quasi-steady region 2 of shocked driven-gas.

As a further simplification the speed of the acoustic $P$-wave in the unsteady $Q$-rarefaction is taken to be the mean of the speeds at the diaphragm station and at the tail of the rarefaction. The formation distance on this basis becomes directly proportional to the diaphragm opening time.

Shock formation distances were calculated for three different cases and the results are compared with experimental measurements and discussed in Section 8.2.4.

8.2.4. Results and Discussion of White's Analysis

The shock-tube performance curves ($P_{d1}$ vs $M_{d}$ Figs. 8.3 and 8.4) show for a variety of gas-combinations that White's theory predicts shock strengths greater than those of ideal theory for diaphragm pressure ratios of order $10^3$ and greater.

At $P_{d1}$ values less than about $10^3$, this tendency is reversed.

For the combinations of helium/air and hydrogen/argon in which the driver and driven gases initially have similar specific heats*, thus minimising the contact-surface mixing effects discussed in Section 8.2.2., the experimental points tend to follow the ideal-theory curves for shock Mach-numbers less than 10 approximately.

* molal values.
For higher values of $M_s$ the points depart increasingly from the ideal-theory curves and approach the curves based on White's theory. One of the consequences however, of increasing the shock strength for values of $M_s$ in excess of 7 in air is to increase the specific heat through dissociation of the $O_2$ molecules followed at higher shock strength, by electronic excitation and finally ionisation; the latter is also responsible for an increase in the specific heat of argon at $M_s$ values exceeding 9.

On the basis of White's qualitative analysis of the influence on shock-strength of mixing at the contact surface, this increase in the driven-gas specific heat accords with the increase in the measured values of shock strength compared with those of ideal theory at high $P_{41}$ values.

The same trend is apparent in the helium/air and hydrogen/argon results; in the former case, contact-region mixing should increase the shock strength while for the latter, the reverse should occur and the experimental results confirm both these predictions.

The rate of increase of $M_s$ with $P_{41}$ as obtained from the experimental points exceeds the predictions both of ideal-theory and White's analysis. Nevertheless, for all the gas combinations shown in Figs. 8.3 and 8.4 except helium/nitrogen, the ideal-theory curve gives a close approximation to the experimental data for $P_{41}$ values up to $10^5$. Moreover such improvements in agreement as accrue from the use of White's theory at higher $P_{41}$ values stem, on the evidence cited above, from contact region mixing coupled with changes in the specific heats rather than from any mechanism envisaged in White's analysis.
Table 8.1

<table>
<thead>
<tr>
<th>Gases</th>
<th>Diaphragm Pressure Ratio</th>
<th>Shock Mach Number</th>
<th>Theoretical Formation Distance (ft)</th>
<th>Mean Experimental Formation Distance (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>He/N2</td>
<td>2500</td>
<td>6.0</td>
<td>39.0</td>
<td>7.0</td>
</tr>
<tr>
<td>H2/N2</td>
<td>2500</td>
<td>8.8</td>
<td>49.8</td>
<td>*</td>
</tr>
<tr>
<td>H2/N2</td>
<td>334</td>
<td>6.0</td>
<td>19.8</td>
<td>*</td>
</tr>
</tbody>
</table>

* No experimental results presented.

White's simple method of calculating shock-formation distances has been applied to three different initial gas conditions using his measured value of diaphragm opening time and the results are given in Table 8.1. In the only case for which comparison may be made, the experimental and theoretical values differ by a considerable margin.

Essentially the formation-distance theory is a simple first-order analysis which ignores the influence of the diaphragm on the flow development. For example the geometry of the partially-open diaphragm must exert an important influence on the strength of the primary expansion. If the flow is assumed to be sonic at the portion of minimum cross-sectional area then at the tail of the unsteady expansion in the driven section, the Mach-number is always less than unity. White's p-x-t diagram Fig. 8.1 relates to a case of this type in that the tail of the primary expansion travels upstream into the driver gas even when the diaphragm is fully open.

This implies a very low $P_{41}$ value however; in most practical cases of shock-tube operation the driver gas expands to a supersonic speed downstream of the diaphragm. Typical x-t diagrams of
such cases show the tail of the primary expansion travelling downstream in the driven section, and this degree of driver-gas expansion would certainly be necessary at all the $P_{h1}$ values in Figs. 8.3 and 8.4.

Despite its importance as an early attempt to obtain a more realistic model of shock-tube flow than that of ideal theory, White's work leaves several aspects unexplored providing scope for alternative analyses.

8.3.1 Kireyev's Analysis of the Initial Flow in a Shock-Tube (1.17)

The basic strategy underlying Kireyev's approach is that of linking the unsteady flow regimes upstream and downstream of the diaphragm with a quasi-steady analysis of the flow in the diaphragm region itself.

The portions of this analysis relating to the flow of driver-gas through the primary expansion and the convergent diaphragm-duct have been discussed in Section 3.1 in connection with the calculation of diaphragm opening times. For the remainder, downstream of the diaphragm, Kireyev used a method-of-characteristics approach.

In the determination of initial values for the characteristics solution, the flow is assumed to emerge from the diaphragm region at the critical sonic velocity subsequently passing through an isentropic steady expansion to attain the full tube cross-sectional area at some recovery plane downstream of the diaphragm. The diaphragm area-vs-time relationship used in Kireyev's calculations was linear however, which is surprising in view of the detailed analysis he applied to the calculation of opening rates and times for real diaphragms.
In order to allow a fuller exploration of the potentialities of Kireyev's overall approach, and in view of the lack of detailed information given in his rather short paper, the analysis has been repeated in the present work (Section 8.3.2) and its applicability to real diaphragms investigated (Section 8.3.3).

8.3.2. Analysis of Flow Downstream of the Diaphragm

Kireyev applied the equations of conservation of mass and stagnation-enthalpy to the flow of driver-gas emerging from the diaphragm exit and assuming a steady isentropic expansion to a recovery plane 3 downstream of the diaphragm (Fig. 8.6), the Mach-number here, $M_3$, is given in terms of the area-enlargement ratio $R$ ($R > 1$) by the equation:

$$1 + \left(\frac{Y-1}{2}\right)M_3^2 = \left(\frac{Y+1}{2}\right)(M_3R)^{2/\gamma}$$  \hspace{1cm} 8.1

From a knowledge of the Mach-number, the remaining flow properties in region 3 are obtained using the appropriate equation of isentropic duct-flow in one-dimension.

The values of these properties depend only on the diaphragm aperture-area for a given driver-gas provided $p^* \gg p_2$, (Fig. 8.6) which is shown in Chapter 9 to be the case for most practical shock tube operation. The static pressure and velocity in region 2 however, are obtained via the Rankine-Hugoniot equation from initial data in region 1.

The matching of pressure and velocity in states 3 and 2 is effected in Kireyev's analysis by means of a $Q$-expansion wave which, for supersonic flow in state 3, is convected downstream.
Using the Rankine Hugoniot equations to relate the changes in static pressure and particle velocity across the shock, together with the corresponding isentropic-wave results for the Q-expansion and imposing the condition that \( u_2 = u_2' \) gives the result:

\[
u_1 - u_1 + 2a_3/(\gamma_4-1) \left[ 1 - \left( \frac{P_{21}}{P_{31}} \right)^{\gamma_4} \right] - a_1(P_{21}-1) \left( \frac{\gamma_1(\beta_1(\alpha_1P_{21}^2+1))^{1/2}}{\gamma_1(\beta_1(\alpha_1P_{21}^2+1))^{1/2}} \right) = 0 \quad 8.2
\]

from which the shock pressure-ratio \( P_{21} \) may be determined.

The remaining flow properties in states 2 and 2' may then be calculated using the Rankine Hugoniot relations from state 1 or the isentropic equations from state 3 as appropriate.

Equation 8.2 should however be replaced by equation 8.3 if the solution of the former gives a value of \( p_2 \) greater than \( p_3 \), as is likely for example when \( R \) is very large near the start of opening, giving a very large \( M_3 \) coupled with a small \( p_3 \). The Q-wave in such a case becomes a shock and the equation corresponding to 8.2, obtained by applying the Rankine Hugoniot relation between regions 3 and 2 in addition to 1 and 2 is:

\[
u_1 - u_3 + a_1(P_{21}-1) \left( \frac{\gamma_1(\beta_1(\alpha_1P_{21}^2+1))^{1/2}}{\gamma_1(\beta_1(\alpha_1P_{21}^2+1))^{1/2}} \right) + a_3(P_{31}/P_{31}-1) \left( \frac{\gamma_3(\beta_3(\alpha_3P_{21}^2/P_{31}+1))^{1/2}}{\gamma_3(\beta_3(\alpha_3P_{21}^2/P_{31}+1))^{1/2}} \right) = 0 \quad 8.3
\]

8.3.3 Discussion of Kireyev's Analysis

The wave-diagram relating to the flow downstream of the diaphragm which is given in Kireyev's paper is shown schematically in Fig. 3.1. The diaphragm aperture-area is assumed to increase in a series of steps of constant amplitude giving a mean linear increase with time; at each step-change in aperture area, the P and Q waves produced are both convected downstream and are incorporated in the wave-diagram construction as shown.
However if the crude linear approximation to the aperture area is replaced by Kireyev's own more realistic model developed as a substantial portion of his paper, a much less satisfactory situation results.

This apparent paradox results from the extremely small apertures which are produced in the early part of the motion of real diaphragms. The assumption of isentropic expansion of the driver-gas from these small areas to the full shock-tube cross section leads to very low pressures in region 3 Fig. 8.6; this in turn calls for Q-shocks of such strength as to travel in an upstream direction despite the high Mach-numbers in region 3.

However, an upstream-moving Q-shock is impossible to incorporate in the wave-diagram shown in Kireyev's paper.

The variation with $P_{m1}$ of the slope of the above Q-shock is shown in Fig. 8.7 for helium/air and air/air combinations. The curves relate to the first stage in a four-step approximation to the opening of diaphragms made of aluminium and stainless-steel, being respectively the slowest and fastest-opening types used in the present work.

For virtually the whole of the range of $P_{m1}$ values shown in Fig. 8.7 the Q-shock slope is negative making its influence impossible to incorporate in the wave diagram (Fig. 3.1).

Substitution of the present radial-flow representation of the aperture area (Chapter 3) in place of Kireyev's model brings about some improvement (Fig. 8.8) but an appreciable range of $P_{m1}$ values remains for which the Q-wave slope is negative.

To add to this difficulty, calculations show that when realistic area-time relationships are assumed, the P-wave generated
at the first stage of a 4-step opening model becomes an expansion. This result, running quite contrary to physical reality, gives an initial flow of driven-gas directed towards the diaphragm!

Because of the failure of the isentropic-enlargement analysis to predict a plausible flow configuration for all practicable values of $P_{i1}$ when realistic diaphragm behaviour is assumed, an alternative approach is necessary; an important requirement in the formulation of such an approach is that of avoiding the prediction of a region of very high Mach-number just downstream of the diaphragm when the aperture area is small. This factor is a major unresolved difficulty in Kireyev's analysis though the problem is alleviated by the use of the linear area-time relationship.

8.4.1 Multi-Stage Shock-Formation Analysis

Ikui, Matsuo and Nagai (1.18) proposed a shock-formation analysis deriving essentially from White's earlier formulation but differing from the latter in replacing the single coalescing wave-group by several similar wave groups, each coalescing at a different point in the x-t diagram. It is assumed that a weak shock originates at the coalescence-point of the first group and that each successive group coalesces exactly on the path-line of the shock. The maximum shock strength is therefore not produced instantaneously since the arrival of each successive wave group on the path of the shock produces an increment in its velocity.

Experiments have shown (1.1, 1.2, 1.16) that the shock accelerates over a finite distance and the multi-stage formation model is more realistic in this respect than White's single-stage formation model.
8.4.2 Shock-Strength Calculations using Multi-Stage Analysis

An x-t diagram illustrating the multi-stage analysis is shown in Fig. 8.9 in which the number of stages is limited, for simplicity to two. More stages were incorporated into the actual calculations, but no account was taken of the effect of the Q-waves and contact surfaces arising from the coalescence points.

The first step in the evaluation involves the arbitrary selection of a point c lying on the curve c; this curve represents changes in static pressure and velocity induced by an isentropic P-compression, while c is the isentropic-expansion curve starting from some fictitious pressure p, which is less than p. Next, the point e, e' is obtained as the intersection of the curve representing an isentropic Q-expansion from conditions c with the corresponding curve for a P-shock from initial driven-gas conditions 1. The pressure and velocity in field e and e' (Fig. 8.9) are thus determined.

A similar procedure is adopted for the determination of the second step, with the simplifying assumption that the reflected Q-wave between c and e' and of the contact-surface ee' have a negligible effect on the second incident wave-group.

Points s and w in Fig. 8.9c represent the final states of the shocked gas on the basis of ideal theory and White's single-stage compression theory respectively.

The relative positions of s and w for fixed conditions 3 and d depend on the ratio of the slopes:

\[
\lambda = \frac{\text{slope } 3-s}{\text{slope } d-w}
\]

and

\[
\frac{(dp)}{(du)}_{3-s} \quad \frac{(dp)}{(du)}_{d-w}
\]
Ikui et al show on the basis of the equations for the changes in \( p \) and \( u \) induced by isentropic Q-waves that the magnitude of the slope-ratio \( \lambda \) depends on the values of \( P_{\alpha 1} \), \( \Gamma_{\alpha 1} \) and \( A_{\alpha 1} \). A large \( P_{\alpha 1} \) value and a gas combination for which \( P_{\alpha 1} \) exceeds unity combine to give shock strengths, based on White's theory which are greater than those of ideal theory; this tendency diminishes however for \( A_{\alpha 1} \) values greater than 1.0.

An extension of the above argument shows that the multi-stage model invariably predicts shock-strengths greater than those of White's single-stage analysis. The multi-stage results show similar trends to those of White's theory however, when compared with corresponding results based on ideal theory.

The point 22' (Fig. 8.9) represents the final state of the shocked gas on the basis of a 2-step analysis while \( v \) is the corresponding point for an infinite number of stages, convergence occurring at 50 steps (1.18).

Comparisons between ideal shock-tube theory and the formation-from-compression theories of White and Ikui et al are shown in Figs. 8.10 to 8.12. Also included are the results of the present theoretical analysis of shock-tube flow discussed in Chapter 9, and those of Satofuka's analysis (Section 8.5.4). Perfect gas behaviour is assumed for the various combinations chosen, which cover a range of \( A_{\alpha 1} \) and \( \Gamma_{\alpha 1} \) values, and experimental data is also included.

The ordinates of Figs. 8.10 to 8.12 represent the ratios of the appropriate calculated or measured shock Mach-numbers at given values of \( P_{\alpha 1} \) to the corresponding ideal-theory Mach-numbers, the latter being used as abscissas. This mode of presentation allows
comparisons with results for tubes having an area-change at the diaphragm in addition to tubes of uniform section.

The overall conclusion arising from these results is that for the Mach-number ranges shown, the multi-stage analysis tends to overestimate the shock-strength at high shock Mach-numbers. This is particularly pronounced in the hydrogen/argon results for which although \( \Gamma_{b1} \) is less than 1.0 and \( A_{b1} \) is greater than 4.0, \( P_{b1} \) is large and this factor is dominant. Although \( A_{b1} \) is less in the case of helium/air, here \( \Gamma_{b1} \) exceeds 1.0 and again the multi-stage theory predicts larger shock Mach-numbers than are found in practice. For both gas combinations White's theoretical results are in better agreement with experimental values than are those of the multi-stage model.

For the air/air combination the standard deviation of the experimental results is 3% which is comparable with the maximum difference of 4.5% between the respective predictions of ideal theory and multi-stage theory for the range of experimental Mach-numbers shown in Fig. 8.10.

For \( \Gamma_{b1} < 1.0 \) there appears to be no advantage in discarding ideal shock-tube theory for the straightforward calculation of maximum shock strengths though for more detailed information, for example an estimate of the shock formation distance, a more realistic analysis is required.

8.4.3. Shock Formation Distance

Ikui et al developed a dimensional argument relating the various parameters influencing the formation distance of the shock \( x_F \) which culminates in the expression:
The terms $f$ and $p_4-p_1$ are intended to represent the opening time of the diaphragm and the inclusion of the terms $f$ and $p_4-p_1$ is discussed in Section 8.4.4. However, Ikui et al. omitted to establish any functional relationship between the various parameter-groups and this precluded the calculation of formation distance though they claim that their analysis can provide such a relationship. They explained, descriptively how different diaphragm opening times can produce different ultimate shock strengths from the same initial conditions, the difference between the ideal-theory shock-strength and the actual value being expressed as the sum of two terms:

(a) $\pi_1$ arising out of the difference between the multi-stage process and the instantaneous ideal-theory formation as discussed in Section 8.4.2, and

(b) $\pi_2$ representing the decrease in shock-strength caused by boundary-layer growth in the flow behind the shock.

The Mirels shock-tube boundary layer theory (1.21) was cited as a suitable basis for calculating $\pi_2$ but no results were presented.

Ikui et al. compared the flows produced by diaphragms with long and short opening times respectively. They showed that the flow associated with the shorter diaphragm opening time is characterised by the stronger initial shock but the final shock strengths, when the formation processes are complete and $\pi_1 = 0$, should be equal.
This of course pre-supposes that the attenuation process produces equal decreases in shock strength during the formation period in both, but no experimental comparisons were made.

8.4.4 Discussion of Multi-Stage Shock-Formation Analysis

The chief advantage of the multi-stage model over White's single-stage shock-formation analysis is that it embodies the concept of a gradual acceleration of the shock over a finite distance in place of the instantaneous formation associated with a single-point coalescence. In common with White's analysis however, it fails to establish a quantitative link between the motion of the diaphragm and the emission of the compression waves which coalesce to form the shock. As a result, the time-scale of the formation process is undefined.

Furthermore although dimensional analysis is often useful when a new problem is under consideration, it constitutes a first-stage approach and is limited, in its capabilities, to suggesting significant dimensionless groupings of the relevant independent variables.

The functional relationship between these groups remain to be determined either experimentally or on the basis of further analysis. Such an analysis, linking the motion of the diaphragm to the shock formation process and hence indirectly to the formation distance, is already embodied in the work of Kireyev which therefore, in large measure anticipates this aspect of the work of Ikui et al.

The inclusion of both the ultimate tensile stress of the diaphragm material and the bursting pressure in the list of relevant independent variables is surprising since in the case
of natural rupture, the bursting pressure and U.T.S. are not independent, while alternatively if enforced rupture is used, the U.T.S. becomes irrelevant.

A more promising approach to this aspect of the problem might perhaps have sought to separate the static portion of the diaphragm burst, in which the U.T.S. undoubtedly plays a major role, from the dynamic portion in which the U.T.S., except for having determined \( p_n \), is unimportant; the yield stress which determines the root bending resistance of the petals is the predominant strength parameter during folding.

A further criticism of the multi-stage analysis is that whereas in White's single-stage model only one reflected rarefaction is produced by the coalescence of the single-compression group (Fig. 8.1) a train of reflected expansions must arise in the multi-stage analysis and because the strengthening shock gives rise to an increasing entropy-change, a contact region or entropy layer must also arise in the formation process. Ikui et al in omitting this effect from their calculations cite Ref. 1.14 as giving support to such a procedure. Certainly the literature in question states that in the formation of a weak shock by an overtaking process involving weaker individual shocks, entropy changes may be ignored, and uniform states assumed behind the shocks (2, 3, 4, etc., in Fig. 8.13).

However it goes on to state that when shocks of appreciable strength are involved such a procedure is unjustifiable and Fig. 8.14 indicates a typical case which might arise when a series of coalescent compression overtake a shock, giving rise to reflected expansions and contact surfaces which interact with the succeeding
incident compressions. Such effects are considered in the present work, in a computer-formulation of the flow in the shock-formation region as discussed in Chapter 9.

Finally an inconsistency arises in connection with the x-t diagram (Fig. 8.9) with which Ikui et al illustrate the shock formation process. This portrays the contact surface between the driver and channel-gas as the primary source of all wave-motion; the trains of P-compressions are shown as originating here as are the Q-expansions. The latter are depicted as travelling for a short distance upstream along the driven section before traversing the region of the opening diaphragm and continuing along the driver section. Despite the gross non-uniformities in flow properties likely to be encountered in such areas of the shock-tube flow, these Q-waves are shown as straight lines in Fig. 8.9. Clearly, no consideration has been given to the influence of the diaphragm in this context. In fact Fig. 8.9 is not relevant to the shock tube problem but is almost precisely the wave-diagram associated with the classical accelerated-piston problem (1.6, 1.13, 7.4) with a contact surface substituted for the piston path.

Unlike such a piston however, to which an external force must be applied to support the load due to the pressure difference resulting from the acceleration, the contact surface in a shock tube is characterised by equality of pressure and particle velocity in the adjacent gas fields on both sides.

In the case of shock-tube flows the diaphragm itself is the only medium, in the inviscid case, by which an axial force may be imparted to the gas; all primary wave-motion should originate at
the diaphragm station. This factor was taken into consideration in Kireyev's shock formation analysis, and also incorporated in the present analysis (Chapter 9).

8.5 Two-Dimensional Analysis of the Shock Formation Process

8.5.1 The fluid-in-Cell Technique

A novel approach to the analysis of the shock-tube problem was used by Satofuka (1.19) in the form of a two-dimensional numerical method known as the Fluid in Cell or F.L.I.C. technique (8.4, 8.5). This is one of a number of computational developments which have evolved over almost two decades (8.5 to 8.7) in parallel with improvements in computing facilities.

All are basically finite-difference methods for the numerical solution of the systems of differential equations associated with multi-dimensional unsteady flow problems. The existence of several such schemes implies that no single one is universally applicable to these problems.

The FLIC technique is an outgrowth of work by several authors (8.5 and 8.8 to 8.11) in which the basic computational scheme involves the division of the field of flow into a network of rectangular cells, for each of which, the Euler equation in two dimensions, is solved repeatedly at time intervals which are small in comparison with some characteristic time for the flow in question.

A typical shock-tube flow evaluation might use 400 cells in the x-direction and 20 cells in the y-direction.

The boundary cells are arranged to have special properties; the solid walls are reflective in the sense of giving zero normal velocity component, while the upstream and downstream flow cross-sections allow the local flow to pass through undisturbed.
For two-dimensional unsteady flow, the equation of mass-conservation may be expressed as:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} \left( \rho u \right) + \frac{\partial}{\partial y} \left( \rho v \right) = -\frac{\partial p}{\partial x}$$ \hspace{1cm} 8.5

The Euler equation has the two components:

$$\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} \hspace{1cm} \text{and} \hspace{1cm} 8.6$$

$$\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} \hspace{1cm} 8.7$$

The energy equation may be expressed as:

$$\rho \left[ \frac{\partial}{\partial t} \left( \frac{\partial}{\partial x} \left( \frac{\partial e}{\partial x} \right) \right) \right] + \frac{1}{2} (u^2 + v^2) = -\left( \frac{\partial p}{\partial x} u + \frac{\partial p}{\partial y} v \right) \hspace{1cm} 8.8$$

The first phase in the computations makes use only of terms involving pressures, in determining the forces on a cell.

Static pressures are computed for all cells, on the basis of density and specific internal energy values which are either determined from previous calculations or specified as initial data.

The gradients of the calculated pressures are then used to determine tentative values of the x- and y-components of velocity with the convective terms omitted from the left-hand side of equations 8.6 and 8.7 expressed in finite-difference form.

The new velocity components used in conjunction with the previous values allow the calculation of tentative new specific internal energies for each cell by means of equation 8.8 again without the convective terms on the left-hand side.
In the second phase of the calculations, the convective terms are included in equations 8.6, 8.7 and 8.8 to account for transport effects on the cell density, momentum and energy. The mass-flows across the cell boundaries are computed and the nett mass stored, and hence the local fluid density at the end of the current time step, are determined.

New velocity and specific-energy values may then be calculated, based on the new densities and finally, a new set of pressure values is obtained to complete the set of data required for the evaluation of the next cycle of results.

8.5.2 Diaphragm Opening

The diaphragm opening process in Satofuka's analysis is treated in the manner of an orifice-plate undergoing successive step-increases in area. The lateral distance-steps are made equal to the basic mesh-width of the flow network and are assumed to occur at equal intervals of time. These time intervals are selected so as to give realistic values of the total opening time, which Satofuka based on the experimental findings of Campbell et al (4.10).

The form of the relationship between aperture-area and time is therefore linear in the case of two-dimensional flow and parabolic for an axi-symmetric case.

8.5.3 Results from the F.L.I.C. Analysis

8.5.3a Velocity Distribution

For the case of axi-symmetric flow in a cylindrical shock-tube, Satofuka plotted the computed values of axial and radial velocity components against axial distance x and radius r (Fig. 8.15) at three different times very near the start of the opening process.
The axial velocity distributions indicate clearly the presence of a jet emerging from the diaphragm aperture, in accordance with experimental observations (Plate 9.1). The radial velocity distributions show that as distance from the diaphragm increases, the flow tends to adopt a one-dimensional form; the shock though initially curved becomes almost plane at a distance of some 3 hydraulic diameters from the diaphragm.

8.5.3b Pressure Distribution

Further evidence of the change in form of the shock from curved to plane is provided by the computed pressure distributions (Fig. 8.16) at various times in the vicinity of the shock front. The latter is invariably depicted as a smeared profile extending axially over several cell lengths, but the form of the profile shows little change with distance beyond approximately 3 tube diameters from the diaphragm. The pressure and velocity results referred to above relate to a diaphragm pressure-ratio $P_{41} = 1,000$.

8.5.3c Shock-Formation Distance

Values of the above quantity were determined from the FLIC analysis for diaphragm opening-times ranging from 100 μs to 500 μs and the results (Fig. 8.17) show that for a given diaphragm pressure ratio, the shock-formation distance increases with diaphragm opening times. The final shock Mach-number though higher than predicted by ideal theory, is independent of opening time.

The form of the relationship linking the diaphragm aperture-area with time was found to have only a small effect on the formation process. This follows from the comparison between the curves
of shock Mach-number vs distance for two cases of equal diaphragm pressure-ratio and opening time, one in axi-symmetric flow, giving a parabolic area-time relationship, and the other in two-dimensional flow, for which the area-time relationship is linear in the present context (Section 8.5.2).

The two-dimensional case gives the larger aperture area at all times except at the extremities of the opening process and the corresponding shock formation distance is marginally the shorter.

8.5.3d Shock-Strength

Maximum shock Mach-numbers were calculated for an air/air combination at diaphragm pressure-ratios of 10, 100 and 1000 and are shown in Fig. 8.10 in comparison with other theoretical and experimental results.

For $P_{41}$ = 10, $M_3$ = 1.6, the result based on two-dimensional theory coincides with that of ideal theory. As $P_{41}$ increases however the two-dimensional results exceed the corresponding ideal-theory values to an increasing degree. The maximum discrepancy of 5% nevertheless, remains moderate in the context of a 3% standard deviation in the experimental results.

8.5.4 Discussion of Two-Dimensional Shock-Formation Analysis

Several important features of the flow near the diaphragm are revealed by the numerical technique embodied in Satofuka's work, in a manner which could not be achieved by a one-dimensional approach. The jet emerging from the diaphragm exit is well defined and the progressive change from a curved to a plane shock is also apparent.

A minor difficulty concerns the form of the shock profile which is "smeared" over several cells and in addition appears to be oscillatory rather than flat-topped in the manner of the ideal normal shock.
An improvement in this area might result from the use of a different variant of the numerical method. No specific reason was advance by Satofuka for the use of the FLIC method in preference to any of the others mentioned in Section 8.5.1. There appears at present to be no definitive assessment of all the various available schemes; such a work would necessitate a formidable computational effort. However, Emery (8.12) has made a few such comparisons (8.8 to 8.11) and cites the scheme of Lax and Wendroff (8.10) as giving superior spatial and temporal resolution to those of other methods, if at the expense of a greater coding effort.

This results from the more detailed method used for calculating average values of the various cell-quantities which includes the effects of local gradient terms derived in turn from the properties of the surrounding cells.

The Rusanov scheme (8.11) is recommended as giving a satisfactory compromise between the complexity and extensive coding effort of the Lax-Wendroff scheme and the more rudimentary Lax scheme in that it gives adequate spatial and temporal resolution in return for a moderate coding effort.

The use of the Rusanov scheme, in preference to the FLIC method which is based on that of Lax, might therefore improve on the representation of the fine details such as the jet emerging from the diaphragm, and the profile of the shock itself.

Although coverage of the entire flow field was maintained for a period considerably greater than the diaphragm opening time, no results were presented for the flow in the vicinity of the diaphragm at times greater than about 10% of the opening time.
In the interests of obtaining a fuller understanding of the flow during this important formative stage, some more information in the form of pressure and velocity distributions at times extending at least to the full opening time of the diaphragm would have been valuable.

Despite the merits of the two-dimensional numerical approach, it suffers in its present form, from the major disadvantage of being restricted to flows involving a single gas by virtue of the averaging process necessary in the calculation of the fluid properties for each cell. For this reason, it is not applicable to practical cases in which high shock Mach-numbers occur since such conditions are generally obtained by the use of dissimilar driver/driver-gas combinations.

It might however be used to advantage in providing a fine coverage of the flow in the diaphragm region itself in order to produce initial values for a one-dimensional treatment of the flow at some distance from the diaphragm.

To a large extent the potentialities of a two-dimensional method are wasted when used in an essentially one-dimensional situation and a more efficient use of computer time might result from such a hybrid scheme.

8.6 Summary of Existing Theoretical Analyses of Shock Formation

The four theoretical analyses of the shock formation process described in Sections 8.2 to 8.4 have been formulated with a view to improving upon ideal shock-tube theory. Despite their more realistic representation of the formation process than that of the latter, all suffer from certain deficiencies as detailed in preceding sections.
For example, none of the methods may be considered as superseding ideal shock-tube theory in the prediction of shock strength; neither does any of the methods make possible the estimation of shock formation distance from a knowledge of the initial gas states and the diaphragm properties. Indeed none succeeds in establishing an analytical relationship between the motion of the diaphragm and the flow in the tube. Even in Kireyev's analysis which perhaps comes closest to providing a comprehensive solution of the initial-flow problem, the realistic diaphragm area-time relationship is discarded in favour of a linear model.

Therefore, the need remains for a theoretical analysis which, even if giving no advantage over ideal theory in the calculation of shock-tube performance, takes account of the influence of the diaphragm in determining the initial development of the flow.
CHAPTER 9
The Present Computer-based Analysis of the Shock Formation Process

9.1 Introduction

The basic objective in the present analysis has been to predict the properties of the initial flow in a shock-tube from a knowledge of the initial gas conditions and of the diaphragm properties.

The deficiencies in all existing analyses of this problem are detailed in Chapter 8. The most promising features of Kireyev’s solution (Section 8.3) have however been incorporated into the present analysis. These involve the use of a quasi-steady treatment of the flow in the diaphragm region in conjunction with a method-of-characteristics approach to the flow in the shock-formation region of the driven section.

Kireyev’s method of linking the two flows has however been shown in Section 8.3 to be viable only when an unrealistic linear diaphragm aperture-area variation with time is assumed, and this feature is discarded in the present model in favour of a pseudo-shock model discussed in detail in Section 9.4.4.

Also, Kireyev’s plane one-dimensional representation of the flow in the diaphragm region has been shown in Sections 3.3 and 3.7 to be less realistic than the present radial-flow model which therefore supplants it in the present analysis.

However, the method of characteristics is retained for the solution of the flow problem in the shock formation region. The use of this approach in the solution of problems involving unsteady flow in ducts is well documented (1.14, 7.4, 9.1). Standard techniques based on desk calculations and conventional draughting procedures have been evolved for the construction of the resulting wave diagrams.
The present problem involves the generation of a series of compressive P-waves which coalesce in the formation region and eventually produce a single shock. This process gives rise to reflected expansions and contact surfaces which in turn produce multiple interactions with successive incident elements. Where such elements are of finite strength, the wave-diagram rapidly attains considerable complexity and the effort needed to obtain a solution by manual computation can become excessive.

A computer programme has therefore been evolved in the present work, which incorporates algorithms for the determination of initial values of the flow properties in the region immediately downstream of the opening diaphragm, and the subsequent evaluation of the complex wave interaction processes in the shock formation region.

The unsteady flow which develops downstream of the diaphragm is computed on the basis of the "method of fields" in which the waves and contact surfaces are treated as lines of discontinuity separating quasi-steady fields in which the flow properties are assumed constant.

In the alternative "method of waves" formulation the equations of motion are solved at the intersections of a mesh of wave-lines, between which the flow properties are assumed to vary continuously.

Since shock trajectories generally fail to coincide with the mesh-points of the wave diaphragm, complex interpolative procedures are necessary to determine the local flow properties and the method of fields, in avoiding this difficulty offers a clear advantage.

As a simplifying measure, all compression waves in the present analysis are treated as shocks which is justifiable on the basis of actual measurements of wave speed (1.16). These have shown that
even at the earliest stages of the formation process, the speed of propagation of the head of the compression system exceeds the undisturbed sonic velocity in the driven section. This implies that shocks of finite strength are present in the region very close to the diaphragm, and to attempt to distinguish in this context, between isentropic and dissipative régimes is unnecessary.

The omission of isentropic compressions from the present computation scheme reduces significantly the amount of coding effort since it decreases the number of different types of possible wave and contact-surface interaction (Appendix 1).

A résumé of these interactions as included in the present analysis, is given in Section 9.2 and in Section 9.3 the main features of the solution scheme, as applied to the bulk of the unsteady flow downstream of the diaphragm, are presented.

Finally in Section 9.4 details are given of the analysis applied to the flow of driver-gas leaving the diaphragm exit region, by means of which the initial values of the flow properties as required for the main unsteady-flow analysis, may be determined.

9.2 Wave and Contact-Surface Interactions

Interaction between the various line-elements of the wave diagram can occur either as a result of the coalescence of two disturbances travelling the same direction, or as the collision of a pair travelling in opposite directions.

9.2.1 Coalescence Cases

The fields associated with a typical coalescence are shown in Fig. 9.1. The line-elements separating fields A and B may be a shock, an isentropic expansion or a contact surface in the present context, while the overtaking element may be a shock or an expansion.
Both transmitted and reflected waves may be shocks or expansions depending on the incident combination, while a contact surface may separate fields D and E. Table 9.1 sets out all the possible combinations of overtaking and overtaken elements and specifies the nature of the resulting transmitted and reflected waves, a contact surface also being produced in each case.

Detailed analysis of each interaction is given in Appendix 1, including sets of conditions, calculable from the initial data for fields A, B and C which indicate which combination of transmitted and reflected waves occurs, in cases where several possibilities exist.

For example the overtaking of a contact surface by a shock could result in the reflection of a shock, an expansion or a Mach-wave depending solely on the initial conditions in fields A, B and C, (Fig. 9.1).

<table>
<thead>
<tr>
<th>Overtaking Element</th>
<th>Overtaken Element</th>
<th>Transmitted Wave</th>
<th>Reflected Wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shock</td>
<td>Shock</td>
<td>Shock</td>
<td>Expansion</td>
</tr>
<tr>
<td>Expansion</td>
<td>Shock</td>
<td>Shock</td>
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<tr>
<td>Shock</td>
<td>Expansion</td>
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<tr>
<td>Shock</td>
<td>Contact Surface</td>
<td>Shock</td>
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</tr>
<tr>
<td>Shock</td>
<td>Contact Surface</td>
<td>Shock</td>
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<tr>
<td>Expansion</td>
<td>Contact Surface</td>
<td>Expansion</td>
<td>Expansion</td>
</tr>
<tr>
<td>Expansion</td>
<td>Contact Surface</td>
<td>Expansion</td>
<td>Shock</td>
</tr>
</tbody>
</table>
Collision Cases

The fields associated with a typical collision are as shown in Fig. 9.2 and in Table 9.2 the possible combinations of all participating elements are listed.

Contact surfaces are produced in all cases except that of the collision of two expansions when identical conditions are produced in states D and E Fig. 9.2., all processes involved being isentropic.

Table 9.2. (Elements numbered as in Fig. 9.2)

<table>
<thead>
<tr>
<th>Incident Element 1</th>
<th>Incident Element 2</th>
<th>Transmitted Wave 3</th>
<th>Reflected Wave 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shock</td>
<td>Shock</td>
<td>Shock</td>
<td>Shock</td>
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<tr>
<td>Shock</td>
<td>Expansion</td>
<td>Shock</td>
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<td>Expansion</td>
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<tr>
<td>Expansion</td>
<td>Shock</td>
<td>Expansion</td>
<td>Shock</td>
</tr>
<tr>
<td>Shock</td>
<td>Contact Surface</td>
<td>Shock</td>
<td>Shock</td>
</tr>
<tr>
<td>Shock</td>
<td>Contact Surface</td>
<td>Shock</td>
<td>Expansion</td>
</tr>
<tr>
<td>Expansion</td>
<td>Contact Surface</td>
<td>Expansion</td>
<td>Expansion</td>
</tr>
<tr>
<td>Expansion</td>
<td>Contact Surface</td>
<td>Expansion</td>
<td>Shock</td>
</tr>
</tbody>
</table>

Details of the analysis of all the above interactions are included in Appendix 1.

The solution of both collision and coalescence cases is effected by means of a single procedure in the programme (Appendix 8).

Strip Analysis of the Flow-Field

A typical region within the body of the present wave diagram is shown in Fig. 9.3. The flow field is treated as a series of strips separated by adjacent Q-waves. The properties of
the complete flow field are arranged in the two-dimensional arrays identified by coefficients $m, n$, where "$m$" denotes the order of the Q-wave lying to the left of any individual field, $m$ increasing with distance $x$, and "$n$" denotes the order along each Q-wave in which the field occurs, "$n$" increasing with time.

The line-element $AB$ (Fig. 9.3) separates the $(m,n)$ - field from the $(m,n+1)$ - field and the $x$- and $t$- co-ordinates are specified such that $(x_{m,n}, t_{m,n})$ denotes the point $A$.

9.3.1 P-Wave/Contact-Surface Coalescences

Each Q-wave in the body of the wave-diagrams arises as a result of the coalescence of two adjacent line-elements advancing from the preceding Q-wave. In Fig. 9.4 which shows a typical portion of such a strip, the 6th Q-wave is shown to arise from the coalescence of the $5,2$ and $5,3$ line-elements. The basic problem is that of determining a new set of data for the fields along this 6th Q-wave from a knowledge of similar data for the 5th Q-wave.

For any set of line-elements traversing a strip of the flow field, only one adjacent pair can coalesce; all the elements which follow collide with the Q-wave reflected from the coalescence. A systematic approach is used in the present formulation, to determine the true coalescence for each strip. This begins with the assumption that a coalescence takes place between the first pair of line-elements with suitable slopes e.g., $(5,2)$ and $(5,3)$ in Fig. 9.4.

Computation then proceeds on the assumption that elements $(5,4), (5,5)$ etc., collide with the Q-wave reflected from this first coalescence. At each step however the test is made of whether any adjacent pair of elements such as $(5,6)$ and $(5,7)$ form a coalescence before colliding with the Q-wave.
Where this occurs, the new coalescence i.e., the point P in Fig. 9.4 becomes the origin of a revised 6th Q-wave the 6-points already calculated being subsequently associated with a later Q-wave.

9.3.2 Q-Wave Coalescence

In addition to the coalescences occurring between P-wave pairs or P-waves and contact surfaces, the possibility arises of coalescences between the Q-waves themselves. For example, a Q-expansion may be overtaken by a neighbouring Q-shock, two neighbouring Q-shocks may coalesce, or a Q-expansion may overtake a Q-shock.

Where a coalescence occurs, for example, between the 5th and 6th Q-waves (Fig. 9.5) the fields in the neighbourhood of the coalescence i.e., (6,8), (6,9) and (6,10) are computed by the standard intersection procedure but this operation, and the evaluation of all subsequent fields along the 6th Q-wave, must make use of data from the 4th Q-wave which must therefore be in current on-line storage.

9.3.3 Termination of Strip Analysis

The analysis of each strip of the flow field is terminated either as a result of the coalescence of the bounding Q-wave pair or by their arrival at an upper time boundary illustrated in Fig. 9.6. The location of this boundary is prescribed so as to allow the primary shock to become fully formed. Until this process is complete however, the possibility remains that a Q-wave may be produced which interacts with earlier Q-waves and invalidates sections of the existing data as for example the data for fields 5,10, 5,11 etc. (not shown in Fig. 9.5) is rendered obsolete by the 5/6 Q-coalescence.
Furthermore, any given Q-coalescence may itself become invalidated by changes in the flow properties induced by a Q-coalescence which occurs earlier in time, but which is computed later in the analysis since it arises at a greater distance from the origin (Fig. 9.6).

Because of this basic uncertainty of the step-by-step solution, the whole of the data for each field is retained in on-line storage until the computations are complete.

9.3.4 Elimination of Weak P-Waves and Contact Surfaces

A feature of this present step-by-step analysis of the flow-field is that the number of data points can almost double between successive strips in the computation. For example, the collision of each P-wave or contact surface from a given Q-wave with the one ahead of it gives rise to both a P-wave and a contact surface, so doubling the total number of line elements at each collision; since some ten flow properties are placed in on-line storage following the solution of each flow-field, the successive doubling of the number of flow-fields of which there are typically ten at the first-strip would rapidly lead to an excessive core-storage requirement at least for a modest computing facility.*

However, across many of the waves and contact surfaces produced in the interaction processes, the flow variables change by only a small amount; in order to obtain efficient computer utilisation, such elements, across which the changes in the flow properties are less than a prescribed threshold limit, are ignored in the subsequent computations. The choice of this threshold limit has been based on an investigation of the influence of the value of the latter on the final computed results of shock-strength and formation distance. (Appendix 9).

* (48K words in the present case).
The strength of each P-wave produced in the present computations is assessed on the basis of the relative change in pressure between the adjacent fields which it separates; pressure and sonic velocity are the two independent thermodynamic variables featured in the present analysis and the former undergoes the greater change across any given P-wave.

For the contact surface, the sonic velocity change is used as the strength criterion since pressure remains unchanged.

9.4 Initial Values of Flow Properties for Wave-Diagram Construction

9.4.1 General Considerations

The initial stage in the computation of a wave-diagram for the shock formation process involves the determination of the flow properties in a set of quasi-steady fields adjacent to the line $x = 0$, and separated by waves and contact surfaces representing discontinuous changes in the flow properties (Fig. 9.7).

The use of the "field" method in the computation of the bulk of the flow field necessitates similar treatment of the diaphragm opening process. The continuous analytical relationship between the aperture-area and the time (Section 3.5) is therefore represented as a series of step-changes spaced at equal time-intervals, the quasi-steady aperture area at any stage being assumed equal to the mean for the corresponding time interval.

A four-step model of the opening process is illustrated in Fig. 9.7 in which, typically, 9 discrete fields arise, which are separated by a series of forward-going line-elements representing weak shocks and ideal contact surfaces.
In order to determine the values of the flow properties in each of the fields shown in Fig. 9.7 it is necessary to formulate a theoretical model of the process by which the flow of driver-gas emerging from the diaphragm region attains a state of mechanical equilibrium with the flow of channel gas farther downstream.

A quasi-steady analysis as used to determine the applied pressure-loading on the diaphragm petals (Chapter 3) may also be used to calculate the state of the gas emerging from the diaphragm exit at any given value of the aperture area. Such an analysis has been shown in Chapter 7 to give very similar values of the flow properties at the diaphragm exit to those obtained from an unsteady analysis which includes the effects of temporal and spatial variations in the geometry of the diaphragm region.

The flow is assumed to attain critical conditions on emerging from the diaphragm exit and is then assumed to pass first through an upstream-facing unsteady expansion and then to enter a region of highly turbulent flow in which a transition occurs from supersonic to subsonic flow, coupled with a continuing process of enlargement until the driver-gas stream completely fills the shock-tube cross-section. The formulation of this model, based on observations in the form of spark-schlieren photographs of the flow downstream of the diaphragm as discussed in Section 9.4.3.

By the use of such an approach the influence of the motion of the diaphragm is introduced into the analysis; this factor is of prime importance since it determines the time-scale of the shock-formation process by controlling the total period over which the initial gas-dynamic disturbances are imparted to the flow.
9.4.2 Mechanical Equilibrium between Driver and Channel-Gas

The static pressure and velocity of the stream of driver-gas emerging from the diaphragm region have been expressed in Section 3.1 as functions of the instantaneous petal angle. Provided the flow in the diaphragm exit-region remains choked, the properties in this region are independent of conditions farther downstream.

At a short distance downstream of the diaphragm however, the driver-gas stream must expand to the full shock-tube cross-sectional area and must also attain equality of velocity and static-pressure respectively, with the driven section gas ahead of it.

A variety of assumptions may be made as to the manner in which this state of mechanical equilibrium is attained. For example, Kireyev used a steady-isentropic-expansion model as discussed in Section 8.2.2 but was unable to use a realistic model of the diaphragm opening process because of the difficulties described in Section 8.2.5 and an alternative to Kireyev's steady-isentropic-expansion model must be sought.

As a prelude to the formulation of a theoretical model of the flow in this region, spark schlieren photographs were taken in a 2 inch shock tube of the flow immediately downstream to a two-flap diaphragm at various stages of the opening process.

The experimental details are given in Section 10.1. It is appropriate to consider only the results in this section, which are shown in Plate 9.1.

9.4.3 Flow Patterns in the Diaphragm Exit-Region

Almost all the photographs (Plate 9.1) show a central jet emerging from the diaphragm exit, exhibiting the characteristic diamond pattern associated for example, with steady supersonic nozzle exhaust flow.
The region surrounding the jet appears on all the photographs to be one of highly turbulent flow which has extended to the tube walls when the opening process is approximately half completed. This turbulent zone also spreads axially with time and ahead of it, on the early photographs, curved waves are visible which almost certainly form part of the compression system which induces changes in pressure and velocity in the channel-gas so as to ensure mechanical equilibrium between this and the expanded driver-stream.

Although the flow and indeed the geometry of the boundaries are time-varying, the appearance of the jet is similar to that of a steady-flow equivalent, a point discussed further in Section 10.3.2. What appears to be the inviscid core region tapers appreciably as in the case of a pseudo-shock in supersonic viscous pipe-flow (9.2) and the surrounding turbulent region grows in width until the entire cross-section is filled with turbulent flow. The transition in flow properties across such a "pseudo-shock" region is solved by application of steady-flow equations of conservation of mass, momentum and stagnation enthalpy, to a control volume surrounding the complex tapering interaction region; no attempt is made to resolve the fine details of the flow, such as the intersection of oblique shock and Prandtl-Meyer expansions, within this region.

The treatment is therefore identical with that of a stationary plane normal shock although the axial length of a pseudo-shock can extend for several tube diameters (9.2).

A similar approach has been used in the present analysis to the problem of determining the transition in the flow properties across the "sudden enlargement" region downstream of the diaphragm exit. Details of the present analysis are given in Section 9.4.4, but before this topic is discussed it is appropriate to consider two further deductions which arise from the schlieren photographs.
The taper in the central core region referred to above, could alternatively be interpreted as a "vena contracta" effect in the jet boundaries.

The transverse components of momentum flux at the diaphragm exit are greatest however when the petals are most sharply convergent and even at this stage, the photographs show only a slight convergence in the vicinity of the diaphragm exit, followed by a slight divergence. Vena contracta is therefore not the principal cause of the convergence which occurs much farther downstream.

A further alternative cause of the latter might be the motion of the diaphragm itself. At the instant in time which coincides with the taking of a given photograph, the flow currently situated at the downstream end of the driver gas jet emerged from the diaphragm exit at a time when the width of the latter was less than that which appears on the photograph. This effect would produce tapering of the boundaries, the degree of which would depend on the relative velocities of the local flow and of the tips of the diaphragm petals.

However, even at the very earliest stages of opening when the angular velocity of the petals is small and their inclination is such that the transverse component of the petal tip velocity is almost zero, a pronounced jet convergence occurs. Moreover, just upstream of this convergence, the flow boundaries actually diverge slightly, thus indicating that petal motion is not the major cause of the subsequent jet convergence.

On the basis of the preceding argument, jet mixing is the dominant mechanism in the flow downstream of the diaphragm. A "pseudo-shock" analysis as described above, has therefore been used to calculated the transition in flow properties across the area-enlargement region through which the driver-gas passes.
9.4.4 Pseudo-Shock Analysis of Flow Enlargement Downstream of Diaphragm

Fig. 9.8 shows schematically the assumed one-dimensional flow configuration in the region downstream of the diaphragm exit. The flow of driver gas emerging from the diaphragm exit at the current critical sonic conditions enters an unsteady Q-expansion in which it accelerates to a supersonic velocity before entering the pseudo-shock region in which the area enlargement and the transition to subsonic flow occur.

The strength of the unsteady expansion is selected so as to give mechanical equilibrium i.e. equality of static pressure and velocity between the expanded driver-gas in region 3 and the shocked gas in region 2.

Application of the steady-flow equations of conservation of mass, momentum and stagnation enthalpy to the flow across the pseudo-shock region gives the result:

\[ \frac{u_3}{a_d} = L - (L^2 - M^2)^{1/2} \]

where

\[ L = \left( \gamma u_d / a_d + a_d / A_e u_d \right) / (\gamma + 1) \]

\[ M = (2 + (\gamma - 1)(u_d / a_d)^2) \gamma / (\gamma + 1) \]

And \( A_e \) is the ratio of diaphragm exit-area to tube cross-section. The use of the negative sign in Equation 9.1 gives subsonic flow in region 3, coupled with an entropy increase whereas supersonic flow and a decrease in entropy result from the use of a positive sign.

The sonic velocity in region 3 may be obtained from equation 9.1 and the steady-flow energy equation:
\[
a_s/a_d = \left( \frac{\gamma_4 - 1}{2} \right) \left( \frac{u_d^2 - u_s^2}{a_d^2} \right) + 1
\]

9.4

the static pressure in region 3 is obtained from:

\[
\frac{p_3}{p_d} = \frac{\rho_3 a_3^2}{\rho_d a_d^2} = \left( \frac{u_d}{u_3} \right) \left( \frac{a_3}{a_d} \right)^2 A_e
\]

9.5

For the P-shock separating states 1' and 2' the static pressure and particle velocity changes are related by:

\[
\frac{p_2'}{p_2} = 1 + \gamma_1 (\gamma_1 + 1) H^2 + \gamma_1 H \left( \left( \frac{\gamma_1 - 1}{4} H \right)^2 + 1 \right)^{\frac{1}{2}}
\]

9.6

where \( H = \frac{u_2 - u_1}{a_1} \)

For the initial step in the solution, \( \gamma_1 \) and \( a_1 \) are properties of the undisturbed driven-section gas; for all subsequent steps, \( \gamma_1 \) in equation 9.6 becomes \( \gamma_4 \) as the P-shock travels initially into expanded driver-gas.

At each stage of the opening process, the conditions of compatibility in regions 2' and 3 Fig. 9.8 are:

\[
u_2' = u_3
\]

9.7

\[
p_2' = p_3
\]

9.8

The changes in the flow properties induced by the Q-expansion are given by:

\[
\left( \frac{2}{\gamma_4 - 1} \right) a_d + u_d = \left( \frac{\gamma_4 + 1}{\gamma_4 - 1} \right) a^*
\]
An iterative procedure is used to determine the strength of the unsteady expansion required to produce, from arbitrary initial driver/driven-gas states, a compatible set of static pressure and gas-velocity values, in accordance with equation 9.7.

9.4.5 Residual Pressure-Difference at Full Opening

On the basis of the "pseudo-shock" enlargement analysis (Section 9.4.4) even when the diaphragm becomes fully open, the static pressure in the exit region appreciably exceeds the maximum shock-induced pressure in the driven-section gas. Fig. 9.9 shows, on the basis of ideal shock-tube theory, the comparison between the minimum critical pressure at the diaphragm exit and the shock-pressure, for a range of diaphragm pressure ratios.

Further expansion of the driver-gas is therefore necessary and were the pseudo-shock analysis to be applied throughout the opening process, a strong centred Q-expansion would be necessary, accompanied by a strong P-shock and contact surface (Fig. 9.7) in order to produce compatibility. No physical effect exists however with which such a model might be associated and the artifice would be somewhat unrealistic.

A hybrid approach has therefore been used in the present computations in which the initial steps in the diaphragm opening process have been computed by means of the pseudo-shock analysis, while the later steps have been determined on the basis of Kireyev's steady-expansion model.

By this means, the residual pressure-difference at full opening is eliminated and the final unsteady expansion of the driver-gas is included in the construction of the wave diagram.
The early unsteady expansion however is not included since at this stage the driver gas which has emerged into the driven section occupies only a small fraction of the tube cross-section.

A schematic representation of a typical set of initial-value fields on which the subsequent computations for the wave-diagram are based is shown in Fig. 9.10.

Results in the form of shock trajectories, formation distances, maximum shock Mach-numbers and pressure histories at various stations in the tube have been computed on the basis of the present theoretical analysis and these are compared with corresponding experimental values in Chapter 10.

A listing of the completed programme together with a typical print-out of results are given in Appendix 8 in which several examples of computer-drawn wave-diagram are also included.
Experimental Verification of Shock-Tube Initial-Flow Analysis

10.1 Present Experimental Studies of Shock-Tube Flow

10.1.1 Determination of Shock Trajectories

The motion of the shock in the formation region was investigated using the 2 in square section shock-tube described in Appendix 6. Runs were carried out for a variety of initial diaphragm pressure ratios, keeping the driver-pressure fixed.

The influence of variations in the diaphragm opening rate was investigated by using diaphragms made from three different materials which gave opening times ranging from some 650 µs (aluminium) to less than half of this value (stainless-steel).

Twelve thin-film resistance thermometers spaced at intervals along the driven section Fig. A,6 were used as shock detectors. The outputs from the detectors were fed to individual trigger amplifiers manufactured to a design similar to that used by Bernstein (10.1).

The output from each trigger amplifier was connected via a decoupling diode to a pulse generator which was used to provide a blanking signal on an oscilloscope trace. The oscilloscope used for timing measurements was modified to give a raster display and the time-duration of the blank caused by the arrival of the shock at one of the detector stations was made longer than the flyback time of the time-base.

In this way it was possible to obtain a typical sweep-rate of 20 µs/cm while making measurements over a period of at least 5.0 ms without losing any detector signals in the flyback portion of the trace.
The initial driven-section pressures ranged from 1 torr to 1 atmosphere, the highest vacua being determined to better than 1% by means of the decantation method described in Appendix 7. Vacua in the region of 10 torr were measured using a proprietary absolute-pressure gauge* calibrated against a McLeod gauge whilst driver pressures were measured on a calibrated Bourdon gauge. Typical results in the form of maximum shock Mach-numbers and trajectories are shown in Figs. 10.1 to 10.6 and discussed in Section 10.2.

10.1.2 Pressure Measurements

In addition to the shock-trajectory measurements, pressure histories were determined at two transducer stations in the shock-tube. One of these was located in the driver-section, 6 in upstream of the diaphragm; the other was positioned in the driven section just downstream of the petal tips of a fully open diaphragm (Fig. A.6.1).

The pressure transducers were of standard piezo-electric type†, their outputs being fed via charge amplifiers to an oscilloscope. Filmed records of the traces from the latter were converted to graphs by the use of a programmable calculator‡‡ equipped with peripherals in the form of a digitizer and an X-Y plotter.

The results are shown in Figs. 10.9 to 10.15 and 10.17 to 10.23 and discussed in Section 10.3.1.

* Wallace and Tiernan 0-200 torr.
† Kistler Models 701A and 701S.
‡‡ Hewlett Packard 9810.
10.1.3 **Schlieren Study of Diaphragm Exit-Region Flow**

Several single-shot spark schlieren photographs were taken of the flow emerging from opening diaphragms (Plate 9.1).

A special window-section was available for the shock-tube which allowed coverage of a flow-field extending to the diaphragm clamping flange.

The diaphragms used were all of aluminium and were initially scored with an H-groove (Fig. 10.16) which channelled the cracks so as to produce a burst pattern in the form of two flaps. The latter are visible on all the photographs and had the combined effects of producing a basically two-dimensional flow field as appropriate to the present schlieren study, while eliminating damage to the windows which would have occurred had the more orthodox cruciform groove-pattern been used (Fig. 2.1). Photographs were taken over a range of values of time-delay following the start of folding, the latter being detected by means of the contact claw (Section 5.2.1).

Certain details of the flow in the diaphragm exit region as revealed by these photographs have been incorporated into an analytical model of the initial flow in the shock-tube as discussed in Sections 9.4.3. and 9.4.4.

10.2.1 **Theoretical and Experimental Shock Trajectories**

Samples of shock trajectories determined in the present experiments are shown in Figs. 10.1 to 10.6 in comparison with corresponding calculations based on the present theoretical analysis (Chapter 9).

The latter were determined on the basis of a six-stage model of the diaphragm opening process. The values of the flow properties at the first stage were based on the pseudo-shock model
those of the remainder were determined from the isentropic-expansion scheme (Section 8.3.2).

This configuration was chosen on the basis of an investigation into the effect on the final results, of varying the numbers of pseudo-shock and isentropic-expansion steps as described in Appendix 9. The influence of another arbitrary parameter, the minimum strength limit for P-waves and contact surfaces retained in the solution, is also discussed in Appendix 9 and its value was optimised in the present calculations.

The gas-combination concerned in the results shown in Figs. 10.1 to 10.5 is air/air with a P_{41} value of 10^9.

The points represent the local values of Mach-number of the leading shock in the group of coalescing waves at the head of the formation region. Ordinates represent the Mach-numbers of the leading shock expressed as ratios of the corresponding ideal-theory shock at the same P_{41} value; abscissae represent distances from the diaphragm.

The results, shown in Fig. 10.1 are representative of all the present trajectory calculations in that they accurately predict the experimental trajectory at very short distances from the diaphragm; at intermediate distance, theory underestimates the measured shock Mach-numbers by up to 10% but at the full formation distance, accuracy again improves and the maximum shock Mach-number is well predicted.

More significantly perhaps, the theoretical formation distance agrees well with the measured values.

The results in Figs. 10.2 to 10.4 relate to runs at a fixed P_{41} value obtained using the three different diaphragm materials.
They provide confirmation of the qualitative findings discussed above in relation to Fig. 10.1, and also demonstrate that diaphragms having a long opening-time (aluminium) give formation distances appreciably in excess of those of faster-opening diaphragms (stainless-steel and copper). The theoretical formation distances for the three materials substantially agree with the corresponding experimental values.

Similar concurrence between experiment and theory is indicated in the results shown in Figs. 10.4 to 10.6. These relate to runs in which various initial $P_{i1}$ values were used, but in conjunction with a single diaphragm-type with a nominally fixed opening-time.

Over the limited range of $P_{i1}$ values covered, the formation-distance appears to increase slowly with $P_{i1}$.

Confirmation of this trend is provided by White's measurements of shock-formation distances for helium/air and hydrogen/argon combinations covering a wide range of $P_{i1}$ values.

These results are illustrated in Fig. 10.7 in conjunction with corresponding calculations based on the present theory.

The experimental results for the two different gas combinations form separate bands and the theoretical results lie well within the scatter of the corresponding measurements. The present theory therefore gives a reliable estimate of the shock formation distance over a wide range of ultimate shock Mach-numbers which is relevant, in the case of the helium/air combination, to shock-tunnel design.

10.2.2 Maximum Shock Mach-Numbers

Maximum values of shock Mach-number calculated on the basis of the present theory for three different gas-combinations are shown in Figs. 8.10 to 8.12.
Corresponding results based respectively on the theories of White and Ikui et al are also included, together with appropriate experimental points all of which are plotted to a base of shock Mach-number based on ideal theory.

For the air/air case Fig. 8.10 indicates that over the range of Mach-numbers covered, differences between all the theoretical results including those of ideal theory are within the scatter of the experimental points.

A similar trend is apparent for the other gas combinations in respect of ideal theory and also of the present theory. However, the theory of Ikui et al is seen to overestimate the shock Mach-number both for the helium/air and especially for the hydrogen/argon results.

White's theory also overestimates the shock Mach-number for the hydrogen/argon case except at the upper end of the range covered. Here however, ionisation of the argon is increasing with shock Mach-number and the resulting increase in specific heat would produce enhanced shock strength on the basis of the contact-region mixing model discussed in Section 8.2.2.

The overall conclusion from the results shown in Figs. 8.10 to 8.12 is that whilst ideal shock-tube theory is incapable of providing detailed information concerning such topics as shock formation distance or the influence of diaphragm opening rate, it remains the simplest method of obtaining a reliable estimate of the maximum shock-strength.
10.2.3 Computer-Drawn x-t Diagrams and Print-out

Typical examples of computer-drawn x-t diagrams for various cases of shock-tube flow are shown in Appendix 8. These are based on data evaluated in accordance with the solution scheme described in Chapter 9; a typical print-out of numerical results is also included.

In each case the trajectories, in the x-t plane, of the primary shock and contact region begin to separate only when the shock-formation process is almost complete. The running time at any appropriate station is accordingly diminished in comparison with the value based on ideal theory.

Ackroyd (1.24) has made measurements of the running time at stations close to the diaphragm and has used such measurements as a basis for determining an apparent origin in the x-t plane for a corresponding ideal-theory shock. The displacement of this apparent origin along the x- and t-axes allows for the finite shock-formation distance. This, in turn, allows a more realistic interpretation of running-time measurements made at greater distances from the diaphragm (1.24).

The present computed results provide a basis for calculating the co-ordinates of such a revised origin (Fig. A.8.1) or alternatively, the running times at various stations may be obtained direct from the x-t diagram.

Another aspect of real shock-tube flows reproduced in the present results is the finite width of the contact region. In Fig. A.8.1 for example which relates to a helium/air combination with $P_{ha} = 10^5$ and assumes an aluminium diaphragm, the contact region eventually attains a width of several tube diameters. Such a model might for example
be used in preference to the ideal-theory interface in more
detailed calculations of the stability of the contact region,
or of the tailoring condition in shock tunnels.

The diagrams shown in Figs. A.8.3 to 5 relate to an air/air
combination with \( P_{e1} = 10^6 \). Aluminium, copper and stainless-steel
are the respective diaphragm materials assumed and the results
illustrate the influence of differences in opening time on the
overall length of the shock-formation region.
10.3 Pressure Histories

10.3.1 Rarefaction Wave Pressures

In the present theoretical analysis, the strength of the primary Q-rarefaction travelling along the driver-section is related at each instant during the diaphragm opening period to the current value of petal angle. Provided the shock-induced pressure $p_2$ in the driven section is less than the minimum critical pressure $p^*$ in the driver gas, the unsteady expansion in the latter should ultimately accelerate the gas particles to the sonic velocity this value being attained at the instant of full diaphragm opening.

The minimum value of $p^*$ exceeds $p_2$ by a considerable margin for most practical shock-tube operation and on this basis, the flow in the driver section should be quite independent of driven-gas conditions, though the diaphragm opening-rate should control the rate at which the strength of the unsteady expansion increases.

In Fig. 10.8 a comparison is shown between theoretical pressure histories for various diaphragms at the transducer station 6 in upstream of the diaphragm calculated using the present analysis.

The full curves relate to aluminium, copper and stainless-steel diaphragms respectively, while the dotted curve shows corresponding results based on ideal theory, implying instantaneous diaphragm opening.

The influence of the motion of the diaphragm on the pressure history is clearly illustrated; the rates of change of pressure for all three curves based on realistic diaphragm behaviour are appreciably lower than are those based on ideal theory.
Pressure varies most rapidly in the case of stainless-steel diaphragms which open in the shortest time, and least rapidly in the case of aluminium diaphragms which have the longest opening time, the results for copper being again intermediate between the above extremes.

However, the overall pressure levels between which the variation occurs are almost identical for the three diaphragm materials and should remain so on the basis of theory, irrespective of the initial diaphragm pressure ratio.

The experimental results substantially confirm all these predictions. Figs. 10.9 to 10.15 show typical pressure transducer records superimposed on the individual theoretical curves for aluminium, copper and stainless-steel diaphragms respectively, for an air/air combination. The experimental curves are in good accord with corresponding theoretical predictions which in turn are clearly distinguished on the basis of diaphragm material as already shown in Fig. 10.8. The experimental results in Figs. 10.9 to 10.11 relate to a fixed value of $P_{41}$; those in Figs. 10.11 to 10.13 cover a range of $P_{41}$ values and relate to a single diaphragm material. As anticipated by theory, the measured pressure histories are independent of $P_{41}$.

A repetition of the above investigation using a helium/air gas combination produced similar concurrence between theory and experiment (Figs. 10.14 and 10.15).

However, a tendency exists for the measured pressures to exceed the theoretical values by a small amount towards the end of the period of measurement. Indeed the transducer records
show a tendency to level-off here at approximately 40% of the driver pressure whereas the theoretical curves continue to show a slow asymptotic decrease.

This effect, though very slight, does suggest that the degree of expansion occurring in the driver-gas upstream of the diaphragm is less than is assumed in the theory. Further support for this conclusion is provided by the pressure histories in the vicinity of the diaphragm exit as discussed in Section 10.3.2.

10.3.2 Pressure Histories in Diaphragm Exit-Region

The exercise of making pressure measurements in the immediate vicinity of the diaphragm was at first envisaged as being essentially exploratory, and likely to yield information only up to the moment of impact of the diaphragm petals with the tube walls. It was anticipated that the severe local vibrations excited by this event might produce excessive noise in the transducer output, thus rendering unusable the succeeding portion of the trace.

However, although a high-frequency noise component appears on the oscilloscope trace, originating at a time consistent with that of wall impact, the noise becomes attenuated relatively rapidly and useful information is obtainable from the remainder of the trace.

The results shown in Figs. 10.17 to 10.19 relate to three different values of \( P_{d1} \) all obtained using stainless-steel diaphragms; those shown in Figs. 10.19 to 10.21 were obtained using three different diaphragm materials but keeping \( P_{d1} \) fixed.

On the basis of the present theory, the local static pressure in the region immediately downstream of the diaphragm should increase progressively from \( p_1 \) at the start of opening to the
minimum critical pressure $p^*_{\text{min}}$ when the diaphragm becomes fully open. At this stage the flow of driver gas should have attained a Mach-number of unity at entry to what previously formed the tapering diaphragm region but which ideally, at full opening becomes simply an unobstructed portion of the tube.

The measured pressures however, attain values between 20% and 40% greater than $p^*_{\text{min}}$, the discrepancies being unrelated to the value of $P_0$ or to the type of diaphragm used.

Moreover, the pressure, in all cases, increases steadily with time in the period following the opening of the diaphragm, but levels off rather rapidly near the end of the period of measurement. Similar results occurred for a Helium/Air combination (Figs. 10.22 & 23).

The period immediately following the opening phase is one in which the flow in the vicinity of the diaphragm ceases to be time-dependent and attains a quasi-steady régime. One of the important features of the latter illustrated in the final picture of the fully open diaphragm in Plate 9.1 is the normal shock standing just upstream of the diaphragm exit. A shock of this form invariably appeared on photographs of the fully-open diaphragm.

Most of the schlieren records as mentioned previously exhibit features usually associated with steady underexpanded jets despite the time-varying nature of the present flow.

For example, Prandtl-Meyer-type expansions appear to originate at the sonic throat, the latter being formed by the petal tips in the early stages of opening. These expansions reflect from the jet boundaries as compressions which, on the present schlieren records form dark regions terminated quite abruptly by sharp straight boundaries suggesting oblique shocks.
As the opening process nears completion, the origin of the initial expansion is seen to travel upstream. At this time, the entry to the diaphragm becomes the principal flow constriction and the "throat" is located here rather than at the petal tips. Simultaneously the compression region resulting from the reflection of the expansion from the jet boundary also travels upstream; the motion of this compression region across the transducer face could explain the abrupt stabilisation in pressure in the region of the petal tips as indicated on the transducer records Figs. 10.17 to 10.21 and could also culminate in the establishment of the normal shock within the diaphragm region. An unexplained aspect of the results however is the time lag between the initial wall impact of the diaphragm petals and the establishment of the final steady pressure level.

It is reasonable to associate the attainment of full diaphragm opening with the establishment of the final quasi-steady flow pattern since no primary wave motion can arise after this time.

The only mechanism likely to postpone the attainment by the diaphragm petals of their ultimate state of rest against the tube walls is that of petal bounce. This phenomenon has been analysed theoretically in Appendix 5, the main conclusion of which is that a significant period of bouncing motion can occur.

For example, with both copper and stainless-steel diaphragms, assuming a coefficient of restitution based on angular velocity, of only 20% the overall period of diaphragm motion is approximately
10 ms. This value of time is at least double the calculated opening time (Chapter 3) and coincides approximately with that at which the transducer records reach their final steady level. The evidence thus suggests a link between petal bounce and the time-lag referred to above.

In general, whilst the details revealed in the schlieren records are of considerable value in yielding physical insight into the flow, the pseudo-shock analysis is essentially independent of the complex wave phenomena apparent in the photographs.

Even the stationary normal shock which forms ultimately in the diaphragm region is argued in Section 10.3.3 to be of little significance in the context of the present shock formation analysis. Further evidence for this lies in the basic agreement between theoretical and experimental results described in Sections 10.2.2, 10.2.3 and 10.3.1 despite the omission of any allowance for this shock from the theory.

The hypothesis postulating the occurrence of the shock as the culmination of the upstream movement of a compression-wave system assumes the diaphragm to be in the final stages of opening. Consideration is now given to the flow problem which arises when the diaphragm becomes stationary.

With the diaphragm in its fully open position and with the petals at rest, it is assumed that the primary unsteady expansion has attained its full strength and that the flow emerges at near-sonic velocity from the tail of this expansion. Such a flow would be accelerated to sonic conditions in the constriction caused by the side-clearance strips of the two-flap diaphragm (Fig. 10.16).
The quasi-steady flow emerging from this constriction would expand further to the supersonic state in the enlargement region downstream; this short region of supersonic flow would be terminated by a normal shock which would occur at a point where the re-expansion of the flow cross-section is reduced e.g. by the curvature of the folded petals.

Although this explanation relates to diaphragms of the two-flap variety, a similar qualitative argument is applicable to the four-petal types used in the determination of pressure histories.

The folded petals again restrict the flow cross-section while the taper of their triangular plan-form allows the re-expansion in which the flow becomes supersonic.

A pressure transducer located just downstream of the petal tips would, on the basis of this argument, experience the static pressure downstream of the shock. As a numerical example, this pressure would be 25% greater than \( p_{\text{min}} \) for a diaphragm producing an 8% diminution in cross-sectional area when fully open. A diaphragm 0.04 in thick with flat petals lying flush with the walls would accomplish this, the resulting pressure increase being sufficient to account for the discrepancy between the theoretical and measured pressures.

Although the aluminium diaphragms used in the present runs are of even greater thickness than the 0.04 in mentioned above, those of copper and stainless-steel are less. However when pressurised prior to bursting both copper and stainless-steel diaphragms adopt a considerable curvature which is not eliminated on impact with the walls.
Despite their small thickness therefore, the diaphragms of both these materials constrict the flow appreciably even when fully open and the occurrence of the stationary attached shock is equally likely in such cases.

10.3.3 Influence of Incomplete Diaphragm Opening On Flow Properties

The implications of incomplete diaphragm opening in the context of the present theoretical analysis of the shock formation process may be assessed by means of the present quasi-steady radial analysis of the flow in the diaphragm region (Section 3.4); this analysis may be used to calculate the flow properties at entry to the diaphragm region as functions of the instantaneous petal angle.

Typical results are shown in Fig. 10.24 which indicates that static pressure is influenced to a much greater extent by the constriction than are the mass flow or the stagnation enthalpy. The very slight increase in measured static pressure as compared with the theoretical value just upstream of the diaphragm (Section 10.3.1) while supporting the diaphragm-constriction hypothesis also suggests that the corresponding perturbations in the mass-flow and stagnation enthalpy are very small indeed; moreover the latter two quantities form the basis for calculations of the initial values of the flow properties used in the construction of the formation-region wave diagram. It therefore follows that the results of this analysis are little influenced by incomplete diaphragm opening at least within the 10° range shown in Fig. 10.2.2 which appreciably exceeds the values observed in the present diaphragms.
A further consideration is that the final stages in the opening process have been computed on the basis of isentropic flow theory which overlooks the occurrence of shock waves. However, the incident Mach-number in the numerical example discussed in Section 10.3.3 is 1.33, and the corresponding entropy change \( \frac{\Delta s}{c_v} \) is less than 1%. The pressure in the shocked gas in most practical shock-tube operation is considerably less than \( p^*_{\text{min}} \) which itself is at least 20% below the pressure at the diaphragm exit. Considerable further expansion of the gas is therefore called for, the initial stages of which must be of the steady variety in order to accelerate the subsonic flow emerging from the stationary shock to a Mach-number of at least unity. This in turn is necessary in order to prevent the head of the Q-expansion in the driven section from travelling upstream. The steady pressure level indicated at the diaphragm-exit transducer is evidence that this does not occur.

Basically therefore the additional flow mechanisms introduced by the imperfectly opened diaphragm produce almost identically similar flows to those of the ideally-open diaphragm at appreciable distances from the latter. In spite of the considerable local influence particularly on the static pressure therefore, the overall effect on the flow is small.
CHAPTER II

Conclusions

1. The flow in a real shock tube exhibits properties which depart significantly from the predictions of ideal shock-tube theory, both in the immediate vicinity of the diaphragm and at appreciable distances downstream.

   The finite opening time of the diaphragm is responsible for the departures occurring near the diaphragm; those which occur in the region farther downstream are attributable to the growth of a boundary layer at the tube walls.

   Both effects are significant in the context of practical shock-tube usage and the present research has been aimed principally at a study of the former.

2. The acquisition of detailed knowledge of the static and dynamic behaviour of bursting diaphragms is a necessary prelude to any attempt to assess the influence of the diaphragm opening process on the development of the flow in the tube.

3. Diaphragms pressurised to their bursting point behave structurally as thin membranes in uniform tension; bending stresses are negligible. The current state of membrane theory precludes a full analysis of the failure of diaphragms for tubes of square cross-section and the problem has been examined experimentally in the present work.

4. The bursting strength and deflection at failure have been investigated for several diaphragm materials and the results have been expressed as dimensionless bursting-strength factor. This takes into account the deflected shape of the diaphragm and facilitates the prediction of bursting pressures for square diaphragms made from ductile materials.
5. Several existing attempts to predict the dynamic behaviour of opening diaphragms incorporate the assumption that the full driver pressure $p_d$ acts on the upstream side of the petals throughout the opening process.

One more realistic theory allows for a pressure reduction upstream linking an analysis of the flow emerging from the tail of the primary unsteady expansion with a plane quasi-steady treatment of the flow in the diaphragm region.

However this theory ignores the pressure at the downstream face of the petal, and in common with all existing analyses, assumes that at the bursting point, the diaphragm is an undeflected flat plate.

6. The present work has shown that the plane one-dimensional treatment of a flow similar to that in the diaphragm region leads to inaccuracies in the prediction of pressure distribution for example, along the flow boundary representing the petal when the latter is in the early stages of opening. It was therefore necessary in the present context to develop an alternative analysis. This incorporates a realistic initial petal-angle at failure, and also includes a bending resistance at the petal roots appropriate to a sharply-tapered, rather than a prismatic cantilever.

The principal findings derived from this analysis are:

(a) The initial petal-angle has a significant effect on the moment on the petal due to gas pressure.

(b) The use of a theoretical model which allows for the decrease in pressure upstream of the diaphragm can increase the predicted opening time by up to 50% compared with that of a simple constant-pressure model.
The effect of the edge bending resistance is of great significance in the case of a low-strength material such as aluminum which requires a relatively thick diaphragm (e.g. $8 \times t$ that of a steel diaphragm of equal bursting pressure); coupled with effect (b), this can give calculated opening times 100% greater than those of freely hinged diaphragms subjected to constant pressure loading.

The accurate determination of the opening time and motion of diaphragm petals requires the generation of a prompt electrical signal at the start of opening, in order to trigger the timing apparatus. Existing methods of trigger-signal generation rely on light transmission through the aperture and have been shown in the present work to give a signal output which is subject to a delay of up to 50% of the actual opening time. They are therefore quite unsuited to the making of accurate measurements of the opening time.

A triggering device has been developed in the present work in the form of a simple wire contact-claw which avoids the excessive delay associated with the former method.

The early motion of the diaphragm petals has been investigated by measuring the breakage times of a series of thin wires glued at intervals across the grooves between adjacent pairs of petals. The results indicate a retardation in the early petal motion consistent with the requirement of a small but finite propagation time for the initial cracks which transform the pressurised membrane into separate petals. This effect is most marked for copper among the materials covered in the present work.
Existing methods of determining the folding rate of diaphragm petals are based on the use of an analogue signal representing the diaphragm aperture area on the basis of the total quantity of light transmitted through it. Previous work has shown this procedure to be liable to errors due to light-scattering produced by flow disturbances in the tube.

A digital method has been developed in the present work which renders such measurements independent of light-scattering.

Multiple-exposure pictures have been taken of opening diaphragms using a repetitive spark light-source and the opening-rate results derived from these are in good agreement with those based on light-transmission measurements. The results of a previous plane-flow analysis underestimate by some 40% the measured opening times, but corresponding results based on the present radial-flow theory agree well with the measured values.

The present radial flow theory indicates that the motion of the diaphragm is little influenced either by the driver-gas atomicity or by the diaphragm pressure ratio. The experimental results substantiate these predictions.

A simple linear approximation for the relationship between the moment due to gas-pressure on the diaphragm and the petal angle, produced excellent agreement with opening rates calculated using the corresponding "exact" moment-vs-angle relationship.

A comparison was made between the values of the flow properties in the diaphragm region calculated using a quasi-steady analysis and corresponding results derived from an unsteady-flow method-of-characteristics analysis; the latter included an allowance for the influence on the flow properties of the rate of change of the flow cross section produced by the motion of the diaphragm petals.
The main conclusion from this order-of-magnitude study is that despite the rapid motion of the flow boundaries, the influence of temporal rates of change of area is generally secondary compared to that of the spatial gradients. No gross discrepancies arise between the results from the different analyses and the overall influence of the motion of the physical boundaries on the flow within the diaphragm region is small. Advantage may therefore be taken of the much greater simplicity of the quasi-steady analysis in all calculations relating to the flow in the diaphragm region, without introducing significant errors.

This finding lends support to the existing practice of using a quasi-steady analysis to link two unsteady flow-fields separated by a short area discontinuity particularly in cases where the geometry of the latter is fixed, e.g., the reflection of pressure waves from an orifice in a pipeline.

The quasi-steady and unsteady analyses of the flow in the diaphragm region are in closest accord in the context of a driver-gas of high sonic velocity and a diaphragm with a long opening time; the dimensionless time-scale of the opening process is maximised under these circumstances.

The existing theoretical analyses of the flow in the shock-formation region are deficient in several aspects:

(a) White's single-stage model which assumes shock formation from the simultaneous coalescence of an isentropic compression, tends to overestimate the shock strength, and in ignoring the motion of the diaphragm, is incapable of defining the time scale of the flow or of predicting the shock formation distance.
The multi-stage formation model of Ikui et al derives from that of White and is subject to the same limitation. It embodies only a superficial attempt to solve the wave-interaction problem in the shock formation region; reflected waves originating here are completely overlooked, and the resulting maximum shock strengths are excessively over-estimated. The physical x, t-plane representation of the shock-tube problem featured in the work of Ikui et al is that of the flow induced by an accelerating piston. This analogy is of questionable validity in the context of shock-tube flow.

Kireyev's analysis incorporates an allowance for the influence of the opening of the diaphragm, the time scale of the shock formation process being linked analytically with the diaphragm opening time. However the method-of-characteristics approach used to compute the flow in the formation region becomes unworkable at the initial-value stage in Kireyev's analysis, when a realistic diaphragm area vs time relationship is incorporated.

A two-dimensional Fluid-in-Cell analysis of shock tube flow reveals departures from one-dimensional behaviour in the vicinity of the diaphragm, but indicates for the case treated, that conditions become substantially one-dimensional within five diameters downstream. The use of the two-dimensional approach to this one-dimensional flow leads to the unnecessary duplication of calculations. Furthermore the method is incapable of representing shock waves other than as smeared profiles extending over at least three cells. Finally, owing to the averaging process carried out in calculating the flow properties for each cell, no two adjacent cells may contain dissimilar gases. The method is therefore inapplicable to the calculation of practical shock-tube flows involving dissimilar combinations of driver/driver-gases.
A computer-based analysis of the shock-formation problem has been developed in the present work; the main features of this analysis are as follows:

(a) A quasi-steady radial-flow analysis is used to calculate the flow variables in the diaphragm region.

(b) A method-of-characteristics approach is used to compute the unsteady flow in the shock-formation region.

(c) Shock waves may be accommodated in the solution, and both incident and reflected shocks and expansions and contact surfaces are catered for.

(d) Realistic diaphragm motion is used to define the time-scale of the solution.

(e) The analysis is not restricted to a single gas; computations may be performed for any combination of driver/driven-gases.

(f) The portion of the analysis relating to the matching of static pressures and velocities respectively, between the expanded driver-gas emerging from the diaphragm exit, and the existing flow in the driven section was formulated in the light of information obtained from schlieren observations of the flow in this region.

18. The following conclusions emerged from the schlieren observations of the flow immediately downstream of an opening two-flap diaphragm:

(a) A supersonic jet of driver-gas emerges from the exit of the diaphragm. Despite the unsteady nature of the flow in this region its physical appearance resembles that of viscous supersonic pipe flow.
The combined effects of vena-contracta and the transverse outward movement of the petal tips have only a small influence on the form of the jet boundaries.

A normal shock forms in the quasi-steady flow between the petal tips of the fully open diaphragm.

A system of curved waves appears on the early photographs in the region ahead of the driver-gas jet; this wave system is seen to be undergoing a process of multiple interactions and reflections from the tube walls. This process has been discussed by Henshall (11.1) and leads eventually to the production of the plane normal primary shock.

The results from the present shock formation analysis are generally in good accord with corresponding experimental findings.

Calculated values of maximum shock Mach-number agree well over a wide range of shock strengths and for several different driver/driven-gas combinations, with experimental values measured both in the present work and previously.

Shock trajectories calculated from the present theory give shock formation distances for a variety of gas-combinations and shock strengths which are confirmed by experimental findings both from the present work and that of White (1.1).

The theory predicts that fast-opening diaphragms give shorter shock formation distances than are obtainable using diaphragms which open more slowly; it further predicts that the maximum shock Mach-numbers are independent of the diaphragm opening time.

Experimental finding substantiate both these predictions.
For diaphragms of a given opening time, the theory indicates that shock formation distances should increase with the maximum shock strength; this prediction also is supported by the present experimental results.

Pressure measurements made in the driver section in the present work, showed the development of the primary unsteady rarefaction wave to be appreciably influenced by the motion of the diaphragm.

Pressure-histories predicted from the present theory agreed closely with corresponding measured results for each different diaphragm material. The results for the various different diaphragm materials were well differentiated both in theory and experiment. All the curves relating to the rarefactions produced in association with real diaphragm motion differed significantly from that of the centred rarefaction in ideal shock-tube theory.

Pressure measurements made just downstream of the diaphragm, showed that values appreciably higher than those predicted in the present theory are attained after the opening of the diaphragm, the final high pressure level being attained abruptly rather than asymptotically.

The pressure increment may be attributed to the presence of the stationary normal shock, shown on the schlieren records, just upstream of the transducer station. This shock appears to form from a compression region which travels upstream in the final phases of the diaphragm opening process and the transit of this region across the transducer face could explain the final abrupt change as the pressure reaches its steady level.
22. A simple first-order analysis has indicated the possibility that the petals of the present diaphragms may bounce for several cycles following the initial wall contact and this effect could be responsible for the delay observed on the diaphragm transducer records between the initial wall contact of the petals and the establishment of the final steady pressure level.

23. The present quasi-steady radial analysis of the flow in the diaphragm region has been used to calculate the flow properties at petal angles less than 90°, in order to simulate the effects on the flow of the stationary protrusions constituted by the curved petals of fully open diaphragms.

The results indicate that although the local static pressure at entry to the diaphragm region is appreciably influenced, the stagnation enthalpy, mass-flow and combined static- and momentum-pressures, \((p + pu^2)\) are virtually unaffected.

The latter three quantities form the basis for calculation of the flow properties at exit from the diaphragm and hence for the subsequent shock-formation calculations and it therefore follows that neither incomplete diaphragm opening nor petal bounce, within the practical range of amplitudes covered in the analysis, have any appreciable effect on the shock-formation process.

24. The present work has shown that on the basis of a one-dimensional model of the unsteady flow in the shock-formation region coupled with a quasi-steady radial analysis of the flow through a realistic representation of the diaphragm region, many aspects of the initial flow in a shock tube may be predicted reliably.
These include the shock-formation distance and the maximum shock Mach-number; the influence on these parameters of the motion of the diaphragm has been established.

On this basis, the main objective of the present work has been achieved.
CHAPTER 12

Suggestions for Further Work

1. The present schlieren studies of the flow downstream of the diaphragm indicated that much of the flow in this region is highly turbulent. The driver and driven gases must therefore mix continuously in the contact region. Such mixing could exert a significant influence on the maximum shock strength but no quantitative allowance for this effect is possible at present since the rate and extent of the mixing are unknown. A study of this effect aimed at furnishing such information, might be based on the use of a gas-sampling probe as developed by Jaques (12.1), applied to flows involving dissimilar driver/driven-gas combinations.

2. Real-gas effects and the influence of boundary-layer growth are two other mechanisms omitted from the present, and other analyses of the shock-formation problem. Both may become significant for flows involving very strong shocks for which formation distances in excess of 100 tube diameters are commonplace.

   Analyses allowing for both effects might be incorporated for example into a computer solution similar to that developed in the present work.

3. All work produced to date, both analytical and theoretical, on the shock-tube initial-flow problem relates to the use of metal diaphragms. Non-metallic diaphragms, for example those made of cellophane, might have different opening times from those of metal diaphragms having the same bursting pressure. Also, the shredded form of the torn cellophane diaphragm would differ markedly from the petalled form of a burst metal diaphragm and a
study might be directed towards determining the influence of this difference on the turbulence level and the mixing-rate in the flow downstream of the diaphragm.

4. The single-shot spark-schlieren pictures of the flow downstream of an opening diaphragm were necessarily obtained on different runs of the tube. Whilst they revealed several important details of the flow, a more coherent assessment of the flow might be obtained by tracing its development cinematically during a single run.

An inexpensive high-speed camera system is being developed, partly for this purpose, at present as a continuation of the project. The aim is to obtain some 20 pictures at a taking-rate of at least 20K pictures per second.
REFERENCES


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4.3 Turner, C.E. Introduction to Plate and Shell Theory. Longmans.


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5.3 North, R.J. A short-duration Argon-jet Spark Light-source System. NPL Aero Note 1008 (1962).

5.4 Edwards, D.C. and Lim, L.E.N. A Guided Spark-Channel for Use at High Repetition Rates. (To be published).


Fig 1.1
FLOW REGIONS IN IDEAL SHOCK TUBE THEORY

Fig 1.2
REALISTIC SHOCK TRAJECTORY
Fig 1.3
WHITE'S SHOCK FORMATION SCHEME

Fig 1.4
SHOCK FORMATION SCHEME of IKUI et al.
Fig 1.5
SHOCK TUBE BOUNDARY LAYER

Rarefaction

Fig 1.6
BOUNDARY LAYER VELOCITIES in
SHOCK-FIXED COORDINATES

Fig 1.7
BIFURCATION of REFLECTED SHOCK
Fig 2.1
UNBURST and BURST DIAPHRAGMS
Fig 3.1
KIREYEV'S SHOCK FORMATION SCHEME

Fig 3.2
KIREYEV'S IDEALISED DIAPHRAGM REGION
Fig 3.3  PLANE ONE-DIMENSIONAL FLOW THROUGH DIAPHRAGM REGION

Fig 3.4  RADIAL FLOW THROUGH TAPERED DUCT

Fig 3.5
Fig 3.6
Maximum Transverse Variation in Mach No.
in a Convergent Duct (taper=0)
Fig 3.7
Maximum Transverse Pressure Variation in One Dimensional Flow
RADIAL FLOW ANALYSIS

Fig 3.8 Geometry of Diaphragm

Fig 3.9 Notation
NORMAL PLAN of HALF-PETAL

Fig 3.10

RADIAL BENDING STRESS DISTRIBUTION

Fig 3.11
Variation of Dimensionless Gas Pressure Moment with Petal Angle

Plane Flow Analysis

Modified Plane Flow Analysis

Radial Flow Analysis

Initial Petal Angle $\theta_0 = 0$
Fig 3.13

Variation of Dimensionless Gas Pressure Moment with Petal Angle

Radial Flow Analysis
Fig 3.14

Variation of Dimensionless Gas Pressure Moment with Petal Angle

Plane Flow Analysis
DIAPHRAGM PETAL-ANGLE vs TIME

\[ t_i = \text{Ideal Opening Time} \]

No bending resistance

Fig. 3.15

Graph showing petal-angle vs time for ideal diaphragm and materials such as aluminium, stainless steel, and copper.
DIAPHRAGM PETAL-ANGLE vs TIME

$t_i = $Ideal Opening Time

Fig. 3.16
Aluminium Diaphragms

$I$ - Ideal diaphragm
$F$ - Radial flow - freely hinged
$B$ - Radial flow - edge bending resistance
DIAPHRAGM PETAL-ANGLE vs TIME

t_i = Ideal Opening Time

Fig. 3.17
Stainless Steel Diaphragms

I - Ideal diaphragm
F - Radial flow - freely hinged
B - Radial flow - edge bending resistance
DIAPHRAGM PETAL-ANGLE vs TIME

$t_i =$ Ideal Opening Time

Fig. 3.18
Copper Diaphragms

I - Ideal diaphragm
F - Radial flow - freely hinged
B - Radial flow - edge bending resistance
IDEALISED DIAPHRAGM GEOMETRY
BURSTING PRESSURE FACTOR $\phi$ (eqtn. 4.4) vs EFFECTIVE THICKNESS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Material</th>
<th>Condition</th>
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<tr>
<td>□</td>
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<td>Annealed</td>
</tr>
<tr>
<td>○</td>
<td>Copper</td>
<td></td>
</tr>
<tr>
<td>▽</td>
<td>Aluminium</td>
<td></td>
</tr>
<tr>
<td>□</td>
<td></td>
<td>Half-hard</td>
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</table>

- Fig. 4.2
Fig. 4.3

UNMODIFIED BURSTING PRESSURE FACTOR $\frac{2p_4L}{\hat{t}_e}$ vs EFFECTIVE THICKNESS
Fig. 4.4

VARIATION of DIAPHRAGM CENTRE DEFLECTION with BURSTING PRESSURE

Curve for ungrooved diaphragms

Bursting values for grooved diaphragms
Fig. 5.2
DIAPHRAGM WITH SIGNAL WIRES

- Wires
- Glue Spots
- Silver Paint

Fig. 5.3
DIAPHRAGM CONTACT CLAW

- Contact wire
- Insulating bush
- Diaphragm
OPTICAL ARRANGEMENT FOR DETERMINING THE MOTION OF DIAPHRAGM PETALS

----- Light Filaments

--- Diagram ---
Fig. 5.5.

OPTICAL ARRANGEMENT FOR DIAPHRAGM PHOTOGRAPHY

- Light Source
- Diaphragm
- Plane Mirror
- Camera
THEORETICAL PETAL ANGLES FOR WIRE BREAKAGE
vs DISTANCE FROM CENTRE

Copper Diaphragms

Aluminium Diaphragms

Stainless Steel Diaphragms
Fig. 6.1

SIGNAL-WIRE BREAKAGE TIME vs DISTANCE FROM CENTRE

\( t_0 \) = Diaphragm Opening Time

Curves based on theoretical \( \Theta \) vs t relationship

Copper Diaphragms

Idealised scatter band

<table>
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<th>Experiment Symbol</th>
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<td>204</td>
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<tr>
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<td>205</td>
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</table>
Fig. 6.2

SIGNAL-WIRE BREAKAGE TIME vs DISTANCE FROM CENTRE

$t_0$ = Diaphragm Opening Time

Curves based on theoretical $\Theta$ vs $t$ relationship

Aluminium Diaphragms

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<tr>
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<tr>
<td>△</td>
<td>13.3</td>
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<tr>
<td>×</td>
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Fig. 6.3

SIGNAL-WIRE BREAKAGE TIME vs DISTANCE FROM CENTRE

t₀ = Diaphragm Opening Time

Curves based on theoretical θ vs t relationship

Stainless Steel Diaphragms

Idealised scatter band

<table>
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<tr>
<th>Experiment</th>
<th>Symbol</th>
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Fig. 6.4

SIGNAL-WIRE BREAKAGE TIME vs DISTANCE FROM CENTRE

t₀ = Diaphragm Opening Time

Curves based on theoretical θ vs t relationship

Copper Diaphragms

<table>
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<th>Symbol</th>
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<td>×</td>
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</table>
Fig. 6.5
SIGNAL-WIRE BREAKAGE TIME vs DISTANCE FROM CENTRE

\[ t_0 = \text{Diaphragm Opening Time} \]

Curves based on theoretical \( \theta \) vs \( t \) relationship

Aluminium Diaphragms

Idealised scatter band

<table>
<thead>
<tr>
<th>Experiment Symbol</th>
<th>( p_4 ) (bar)</th>
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<tbody>
<tr>
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<td>×</td>
<td>20.8</td>
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</table>
SIGNAL-WIRE BREAKAGE TIME vs DISTANCE FROM CENTRE

\[ t_o = \text{Diaphragm Opening Time} \]

Curves based on theoretical \( \Theta \) vs \( t \) relationship

Stainless Steel Diaphragms

<table>
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<td>△</td>
<td>18.0</td>
</tr>
<tr>
<td>×</td>
<td>14.7</td>
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Fig. 6.7
DIAPHRAGM WITH INCOMPLETE PETAL FORMATION
Fig. 6.8
DIAPHRAGM PETAL ANGLE vs TIME
\( t = \text{Ideal opening time} \)

Aluminium Diaphragms
Air/Air \( p_1 = 1 \text{ bar} \)

<table>
<thead>
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<th>( t_i (\mu\text{s}) )</th>
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Fig. 6.9
DIAPHRAGM PETAL ANGLE vs TIME
$t = \text{Ideal opening time}$

Copper Diaphragms
Air/Air $p_1 = 1 \text{ bar}$

Kireyev's Plane-Flow Theory
Present Radial-Flow Theory

<table>
<thead>
<tr>
<th>Symbol</th>
<th>$p_4$ (bar)</th>
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Fig. 6.10
DIAPHRAGM PETAL ANGLE vs TIME
$t_i=$Ideal opening time

Stainless Steel Diaphragms

Air/Air $p_1=1$ bar

Kireyev's Plane Flow Theory
Present Radial Flow Theory

Experiment

<table>
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Fig. 6.11
DIAPHRAGM PETAL-ANGLE vs TIME

$t_i$ = Ideal Opening Time
Comparison between Spark Photograph and Light-Transmission Measurements
Aluminium Diaphragms

- Spark Photograph
- Light Transmission

Present Radial Theory
DIAPHRAGM PETAL-ANGLE vs TIME

$t_i$ = Ideal Opening Time

Influence of Driver-Gas Atomicity on Opening Rate.

Aluminium Diaphragms

Curves based on Radial Theory

Fig. 6.12
Fig. 6.13

DIAPHRAGM PETAL-ANGLE vs TIME

$t_i$ = Ideal Opening Time

Influence of Diaphragm Pressure Ratio on Opening Rate

---

**Legend:**
- +: Present
- ×: Radial Theory

**Table:**

<table>
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<th>Symbol</th>
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<tr>
<td>+</td>
<td>$10^4$</td>
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<td>×</td>
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Fig. 7.1
DIAPHRAGM DUCT GEOMETRY

Fig. 7.2
TYPICAL WAVE INTERSECTION
in x-t DIAGRAM
Fig. 7.3

UPSTREAM DUCT-BOUNDARY
WAVE-REFLECTION

Fig. 7.4

DOWNSTREAM BOUNDARY
REFLECTION
Fig. 75
WAVE DIAGRAM FOR UNSTEADY FLOW in the DIAPHRAGM REGION
DIAPHRAGM MATERIAL — ALUMINIUM
DRIVER GAS — AIR

QUASI STEADY RÉGIME
Fig. 7.6
WAVE DIAGRAM FOR UNSTABLE FLOW in the DIAPHRAGM REGION
DIAPHRAGM MATERIAL — STAINLESS STEEL
DRIVER GAS — AIR

QUASI STEADY RÉGIME

DISTANCE (X/L)

TIME (T/T₀)

0.400
0.600
0.800
1.000

0.000
0.200
0.400
0.600
0.800
1.000
Fig. 7.7

WAVE DIAGRAM FOR UNSTEADY FLOW in the DIAPHRAGM REGION

DIAPHRAGM MAT: ALUMINIUM

DRIVER GAS: HELIUM

QUASI STEADY RÉGIME
VARIATION OF RIEMANN VARIABLES ALONG A Q-WAVE
WITH DISTANCE ALONG DIAPHRAGM REGION

Direction of Wave Motion

P

Q

P, Q

x/L

0

25

50

75

1.0

0 4.0

4.0

5.0
VARIATION OF RIEMANN VARIABLES ALONG A P-WAVE WITH DISTANCE ALONG DIAPHRAGM REGION

Direction of Wave Motion

Fig. 7.9
VARIATION of TEMPORAL DERIVATIVE $\Delta P$ at DIAPRAGM EXIT $\Delta \tau$ with PETAL ANGLE.

$\Delta P$ & $\Delta \tau$ calculated by numerical approximation.
Fig. 7.11

RATIO TEMPORAL/SPATIAL AREA-CHANGE (Eqtn. 7.12) vs PETAL ANGLE

Ratio

Quasi Steady Flow

\[ \text{Ratio} \] vs \( \theta(\degree) \)
VARIATION of MACH NUMBER with DISTANCE ALONG DUCT AXIS

Quasi steady
Unsteady

Mach Number

60%
Open

90%
Open

0.5
0
0.25
0.50
0.75
1.0

y/L
VARIATION of STATIC PRESSURE with DISTANCE ALONG DUCT AXIS

- Quasi Steady
- Unsteady

Pressure Ratio $\frac{p}{p_4}$

Distance along Duct Axis $\frac{x}{L}$
VARIATION of STAGNATION TEMPERATURE RATIO
(with Quasi Steady) along DUCT AXIS

Temp. Ratio: \( \frac{T_{ou}}{T_{og}} \)

Fig. 7.14
EXIT PLANE STAGNATION TEMP. RATIO
vs
PETAL ANGLE

Quasi-steady Flow

$\frac{T_0}{T_4}$

- Quasi-steady
- Unsteady - Aluminium Diaph
- Unsteady - S' Steel

$\theta (^{\circ})$
EXIT PLANE MASS-FLOW RATIO

Mass Flow Ratio
\[ \frac{\rho u A_e}{\rho_4 a_4} \]

\[ \theta (^\circ) \]

- Quasi Steady
- Unsteady-Aluminium Diaph
- Steel

Quasi Steady Flow
STATIC PRESSURE RATIO at EXIT

Pressure Ratio $\frac{p}{p_4}$

Quasi Steady
Unsteady Aluminium Diaphr
" " S'Steel

$\theta (°)$

Fig. 7.17
EXIT-PLANE STAGNATION TEMPERATURE RATIO

Helium Driver

Quasi Steady Flow

Quasi Steady

Unsteady

\[ \frac{T_0}{T_4} \]

\[ \theta (^\circ) \]

30 45 60 75 90
EXIT-PLANE MASS-FLOW RATIO
VS
PETAL ANGLE

Mass Flow Ratio
$\frac{\rho u A_e}{\rho_4 c_4}$

Quasi Steady Flow
Quasi Steady
Unsteady
Fig. 8.1

$p_x, t$-DIAGRAM for WHITE'S SHOCK-FORMATION ANALYSIS

(From Ref. 1.1)

Suggested model of shock-tube flows as depicted by a $(p, x, t)$-diagram.
A graphical method for determination of the strength of the shock wave formed by steepening of an isentropic compression wave; $\gamma_1 = 1.4$. 

**Diagram for White's Shock-Formation Analysis**

(From Ref. 1.1)
**SHOCK-TUBE PERFORMANCE CURVES**

(From Ref. 1.1)

**Fig. 8.3** Observed and predicted shock-tube performance for helium/nitrogen and hydrogen/nitrogen. Experimental points indicate the maximum strength observed in an experiment. Compression waves due to contact zone mixing would enhance the shock strength for helium/nitrogen.

**Fig. 8.4** Observed and predicted shock-tube performance for helium/argon and hydrogen/argon. Expansion waves due to contact zone mixing would weaken the shock in hydrogen/argon experiments.
Fig. 8.5

WHITE'S APPROXIMATION for
SHOCK FORMATION DISTANCE (Ref. 1.1)

- Diagram showing the relationship between time and distance with labeled points:
  - To: Diaphragm opening time
  - Xf: Shock formation distance
  - Acoustic Wave
  - Lines 1, 2, 3, and 4 representing different stages or conditions in the process.
KIREYEVS STEADY-EXPANSION FLOW MODEL (Ref. 1.17)

Flow Regions:

4-R Unsteady Q-expansion
R-* Steady expansion
*3
3-2' Unsteady Q-wave (exp. or shock)
2'-2 Contact surface
2-1 P-shock
Fig. 8.7

KIREYEV'S SHOCK FORMATION ANALYSIS
Slope of First Q Wave vs Diaphragm Pressure Ratio.
Plane Flow Analysis

- - - - - Aluminium Diaph
- - - - - Stainless Steel

Wave Slope $\frac{dt}{dx}$

Helium/Air

Air/Air
Fig. 6.8

KIREYEV'S SHOCK FORMATION ANALYSIS
Slope of First Q Wave vs Diaphragm Pressure
Radial Flow Analysis

Wave Slope
\[ \frac{dt}{dx} \]

- Aluminium Diaph
- Stainless Steel

Air / Air

Helium / Air
MULTI STAGE SHOCK FORMATION ANALYSIS
(Ref. 1.18)

(a) $p, x, t -$ Diagram

(b) $x, t -$ Diagram

(c) $u, p -$ Diagram
MAXIMUM SHOCK MACH NUMBERS
Theoretical and Experimental Results

**Air/Air**

- Theory
  - White (1.1)
  - Ikui et al. (1.18)
  - Present theory
  - Satofuka (1.19)

**Experiment**

- Glass et al. (1.6)
- Ikui et al. (1.18)
- Present expt.
MAXIMUM SHOCK MACH NUMBERS
Theoretical and Experimental Results
Helium/ Air

--- Theory
-- White (1.1)
---- Ikui et al. (1.18)
----- Present theory

Experiment
  ○ Jones (8.13)
  ◯ N.P.L. (Unpubl.)
MAXIMUM SHOCK MACH NUMBERS
Theoretical and Experimental Results

Hydrogen Argon

Theory

--- White (11)

--- Ikui et al. (1.18)

--- Present theory

Experiment

○ White (1.1)
Fig. 8.13
COALESCENCE of WEAK SHOCKS

Fig. 8.14
COALESCENCE of STRONG SHOCKS
Velocity Distribution near the diaphragm, $P_{in}=100$ and $r_p=496.3$ 

- Velocity distributions at $t=2.5$, $t=5.0$, and $t=10.0$. 

- Velocity distributions near the diaphragm, $P_{in}=100$ and $r_p=496.3$. 

- $t = 2.5$, $t = 5.0$, and $t = 10.0$. 

$u$ - velocity component 
$x/D$ - normalized distance 
$t/D$ - normalized time 
$P_{in}$ - initial pressure 
$r_p$ - radius parameter 

I - cell number along computing mesh 
J - cell number across computing mesh 

Diaphragm open at $t = 300$ (500 µs) 
MEMSTP: Time interval for 6-step diaphragm opening.
Fig. 8.16

TWO DIMENSIONAL SHOCK FORMATION ANALYSIS
(Ref. 1.19)

Pressure Distribution

Shock front positions at various radial locations: (a) \( P_d = 100, r_{op} = 264.7 \mu \text{sec} \); (b) \( P_d = 100, r_{op} = 496.3 \mu \text{sec} \); (c) \( P_d = 1000, r_{op} = 264.7 \mu \text{sec} \).

I - cell number along computing mesh
J - "  "  "  " across "  "

Diaphragm open at \( t = 300 \) (500 \( \mu \text{sec} \))

MEMSTP = Time interval for 6-step diaphragm opening.
TWO DIMENSIONAL SHOCK FORMATION ANALYSIS
(Ref. 1.19)

Shock Trajectories

Variation of shock Mach numbers with distance along shock tube axis.

I - cell number along computing mesh
J - " across "
Diaphragm open at t = 300 (500 μs)
MEMSTP = Time interval for 6-step diaphragm opening.
Fig. 9.1.
TYPICAL P-WAVE COALESCENCE

Fig. 9.2.
TYPICAL WAVE COLLISION
TYPICAL STRIP of WAVE DIAGRAM between ADJACENT Q-WAVES

**Fig. 9.3.**

TYPICAL PORTION of WAVE DIAGRAM (Schematic)

**Fig. 9.4.**
COALESCEENCE of TWO Q-WAVES

Fig. 9.5.

COALESCEENCE c' INVALIDATED by COALESCEENCE c

Fig. 9.6.
Fig. 9.5.
COALESCEENCE of TWO Q-WAVES

Fig. 9.6.
COALESCEENCE c' INVALIDATED by COALESCEENCE c
Fig. 9.7.

INITIAL FLOW REGIONS of WAVE DIAGRAM
(Pseudo-shock analysis)

\[ A_t \text{ -- Full cross-section of shock-tube} \]
\[ \text{--- Mean area line} \]
\[ t_0 \text{ -- Diaph" opening time} \]

\[ a \text{ to } i \text{ -- Initial flow regions} \]
\[ \text{--- --- Contact surface} \]
Fig. 9.8.

PSEUDO-SHOCK MODEL of FLOW 
DOWNSTREAM of DIAPHRAGM

Flow Regions

4-R Unsteady Q-expansion
R-\* Steady expansion
\*-d Unsteady Q expansion
d-3 Pseudo-shock
3-2' Contact surface
2'-1' P-shock
1'-2 Contact surface (from preceding step)
2-1 P shock
Fig. 9.9

Ideal-theory shock pressure ratio $P_{24}^*$ and minimum critical pressure ratio $P_{\text{min}}^*/P_4$ vs $\rho_2/\rho_4$ for Air/Air and Helium/Air.
INITIAL FLOW REGIONS of WAVE DIAGRAM
(Combined pseudo-shock & isentropic expansion model)

- $t_0$: Diaphragm opening time
- $P$: Shock
- $Q$: Expansion
- Contact surface
**SHOCK TRAJECTORIES**

Gas Combination — Air - Air
Diaphragm Material — Aluminium

<table>
<thead>
<tr>
<th>Experiment Symbol</th>
<th>$P_{4i} \times 10^{-4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>1.075</td>
</tr>
<tr>
<td>□</td>
<td>1.038</td>
</tr>
<tr>
<td>△</td>
<td>-940</td>
</tr>
</tbody>
</table>

Present Theory ($P_{4i} = 10^4$)

$M_S$ Actual shock Mach No
$M_{SI}$ Ideal-Theory result

Maximum $M_S$ (Present Theory)
Fig. 10.2. Shock Trajectories

Gas Combination — Air Air
Diaphragm Material — Aluminium

Maximum \( M_s \)
(Present Theory)

<table>
<thead>
<tr>
<th>Experiment Symbol</th>
<th>( P_{41}\times10^{-3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \circ )</td>
<td>0.956</td>
</tr>
<tr>
<td>( \square )</td>
<td>0.979</td>
</tr>
<tr>
<td>( \triangle )</td>
<td>1.059</td>
</tr>
</tbody>
</table>

\( M_s \) Actual shock Mach No
\( M_{SI} \) Ideal-Theory result

Distance (ft)
Fig. 10.3.

SHOCK TRAJECTORIES

Gas Combination — Air Air Diaphragm Material — Copper

Maximum $M_s$ (Present Theory)

Present Theory $P_4 \times 10^3$

$M_s$ Actual shock Mach No

$M_{SI}$ Ideal-Theory result

Experiment $P_4 \times 10^3$

Symbol $1280$

$1261$

$1.69$

Distance (ft)

$M_s/M_{SI}$
SHOCK TRAJECTORIES

Gas Combination — Air Air
Diaphragm Material — Stainless Steel

Maximum $M_S$ (Present Theory)

<table>
<thead>
<tr>
<th>Experiment Symbol</th>
<th>$P_{41} \times 10^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<tr>
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<td>1.211</td>
</tr>
<tr>
<td>△</td>
<td>1.258</td>
</tr>
</tbody>
</table>

$M_S$ Actual shock Mach No
$M_{SI}$ Ideal-Theory result

Distance(ft)
SHOCK TRAJECTORIES

Gas Combination — Air—Air
Diaphragm Material — Stainless Steel

Maximum $M_S$
(Present Theory)

<table>
<thead>
<tr>
<th>Experiment Symbol</th>
<th>$P_{41} \times 10^{-4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>○</td>
<td>1.266</td>
</tr>
<tr>
<td>□</td>
<td>0.947</td>
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<tr>
<td>△</td>
<td>1.00</td>
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</tbody>
</table>

$M_S$ Actual shock Mach No
$M_{SI}$ Ideal-Theory result
SHOCK TRAJECTORIES

Gas Combination — Helium Air
Diaphragm Material — Stainless Steel

Maximum $M_S$ (Present Theory)

<table>
<thead>
<tr>
<th>Experiment Symbol</th>
<th>$P_{41} \times 10^{-4}$</th>
<th>Present Theory ($P_{41} = 10^4$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$M_S$ Actual shock Mach No.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$M_{SI}$ Ideal-Theory result</td>
</tr>
<tr>
<td>○</td>
<td>1.218</td>
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<tr>
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<tr>
<td>△</td>
<td>1.392</td>
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</table>
SHOCK-FORMATION DISTANCE vs MAX SHOCK MACH NUMBER.

Fig. 107: Plot of shock formation distance ($x_f(\text{ft})$) versus Mach number ($M_s$) with data points for different gases: Helium/Air, Hydrogen, and Argon. The graph compares present theory and experimental data.
RAREFACTION PRESSURE HISTORY
(Present Theory)

Pressure Ratio \( \frac{P}{P_4} \)

Time \( \frac{\alpha_4 t}{x_{\text{trans}}} \)

- Aluminium Diaphragms
- Copper
- Stainless Steel

Arrival of wave-head
RAREFACTION PRESSURE HISTORY

Diaphragm Material: Aluminium
Gas Combination: Air/Air

\[ P_{41} = 1.1 \times 10^4 \]

![Diagram showing pressure ratio versus time with annotations for theoretical and measured traces, and definitions for arrival of wave-head and distance of transducer from diaphragm.](image)
RAREFACTION PRESSURE HISTORY

Diaphragm Material: Copper  
Gas Combination: Air/Air

\[ p_{41} = 1.40 \times 10^4 \]

![Graph showing pressure ratio over time.]

- Ideal S/T Theory
- Present Theory
- Transducer Trace

\[ x_{trans} \] = Distance of transducer from diaphragm

Arrival of wave-head

Time \( \frac{\alpha_4 t}{x_{trans}} \)
RAREFACTION PRESSURE HISTORY

Diaphragm Mat = Stainless Steel
Gas Combination = Air/Air

\[ P_{41} = 1.37 \times 10^4 \]

Ideal S/T Theory ——
Present Theory ————
Transducer Trace ————
\[ x_{trans} = \text{Distance of transducer from diaphragm} \]

Pressure Ratio \( P/P_{41} \)

Arrival of wave-head

Time (\( a_4 t / x_{trans} \))
RAREFACTION PRESSURE HISTORY

Diaphragm Mat\textsuperscript{1} - Stainless Steel
Gas Combination - Air/Air

\[ P_{41} = 1.53 \times 10^3 \]

**Graph:**
- **Ideal S/T Theory**
- **Present Theory**
- **Transducer Trace**

**Equation:**
\[ x_{\text{trans}} = \text{Distance of transducer from diaphragm} \]

**Axes:**
- **Pressure Ratio** \( \frac{P}{P_4} \)
- **Time** \( \frac{\alpha_4 t}{x_{\text{trans}}} \)

**Labels:**
- **Arrival of wave-head**
RAREFACTION PRESSURE HISTORY

Diaphragm Mat: Stainless - Steel
Gas Combination - Air/Air

\[ P_{41} = 15.4 \]

Fig.10.13

Ideal S/T Theory ————
Present Theory ————
Transducer Trace ————

\[ x_{\text{trans}} = \text{Distance of transducer from diaphragm} \]

Arrival of wave-head
RAREFACTION PRESSURE HISTORY

Diaphragm Material: Stainless Steel
Gas Combination: Helium / Air

$P_{41} = 1.11 \times 10^4$

![Graph showing pressure ratio ($P/P_0$) versus time ($\alpha_{41} t/x_{trans}$).](image)

- Ideal S/T Theory
- Present Theory
- Transducer Trace

$x_{trans} = $ Distance of transducer from diaphragm

Arrival of wave-head
RAREFACTION PRESSURE HISTORY

Diaphragm Material: Aluminium
Gas Combination: Helium / Air

\[ P_{41} = 1.09 \times 10^4 \]

- Ideal S/T Theory
- Present Theory
- Transducer Trace

\( x_{\text{trans}} \) = Distance of transducer from diaphragm

Arrival of wave-head
Fig. 10.16.

TWO-FLAP DIAPHRAGM
PRESSURE HISTORY at DIAPHRAGM STATION

Diaphragm Mat. — Stainless Steel
Gas Combination — Air/Air

\[ P_{41} = 1.37 \times 10^4 \]

Transducer Output Trace

\[ \frac{P}{P_4} \]

Diaphragm petals strike tube wall
PRESSURE HISTORY at DIAPHRAGM STATION

Diaphragm Mat! — Stainless Steel
Gas Combination — Air/Air

\[ P_{41} = 1.53 \times 10^3 \]

Transducer Output Trace

\[ \frac{p}{p_4} \]

Time (\( s \times 10^3 \))

Diaphragm petals strike tube wall
PRESSURE HISTORY at DIAPHRAGM STATION

Diaphragm Matl. — Stainless Steel
Gas Combination — Air/Air
\( P_{41} = 15.38 \)

Transducer Output Trace

---

Pressure Ratio
\( \frac{P}{P_4} \)

Time (s\( \times 10^3 \))

- Diaphragm petals
- strike tube wall
PRESSURE HISTORY at DIAPHRAGM STATION

Diaphragm Matl. — Aluminium
Gas Combination — Air/Air

\[ P_{41} = 18.12 \]

Transducer Output Trace

- **Diaphragm petals**
- **strike tube wall**
PRESSURE HISTORY at DIAPHRAGM STATION

Diaphragm Material — Copper
Gas Combination — Air/Air

$P_{41} = 18.81$

Transducer Output Trace

*Diaphragm petals strike tube wall*
PRESSURE HISTORY at DIAPHRAGM STATION

Diaphragm Matl. — Aluminium
Gas Combination — Helium / Air

\[ P_{41} = 1.11 \times 10^4 \]

Transducer Output Trace

![Graph showing pressure ratio over time with annotations](image)

- Diaphragm petals strike tube wall
PRESSURE HISTORY at DIAPHRAGM STATION

Diaphragm Mat — Copper
Gas Combination — Helium / Air

$P_{41} = 1.45 \times 10^4$

Transducer Output Trace

Pressure Ratio $\frac{p}{p_4}$

Diaphragm petals strike tube wall

Time ($s \times 10^3$)
VARIATION of STATIC PRESSURE, MASS-FLOW RATE and STAGNATION TEMPERATURE with PETAL ANGLE (Radial Flow Analysis)

Flow Property Ratios

\[
\frac{p}{p_{\text{min}}} \quad \frac{T_0}{T_{0\text{min}}} \quad \frac{pu}{(pu)_{\text{min}}} \]

Petal Angle (degrees)
Fig. A.1.1.

COALESCENCE of TWO SHOCKS

s - Shock
r - Rarefaction
M - Mach wave
c.s. - Contact surface

Fig. A.1.2

COLLISION of TWO SHOCKS
Fig. A.1.3
COLLISION of TWO RAREFACTIONS

Fig. A.1.4
COLLISION of SHOCK with RAREFACTION

r - Rarefaction
s - Shock
c.s. - Contact surface
Fig. A.1.5.

RAREFACTION OVERTAKING SHOCK

- r - Rarefaction
- s - Shock
- cs - Contact surface
Fig. A.1.6.

SHOCK OVERTAKING RAREFACTION

r - Rarefaction
s - Shock
cs - Contact surface
SHOCK/CONTACT-SURFACE INTERACTION

- Fig. A.1.7 -

r or s / 107 /

r - Rarefaction
s - Shock
cs - Contact surface
RAREFACTION / CONTACT-SURFACE INTERACTION

- r - Rarefaction
- s - Shock
- c.s. - Contact surface
TRANVERSE GROOVE ELONGATION vs DISTANCE FROM CENTRE

Aluminium Diaphragms

Upper and lower bounds on elongation used in wire breakage calculations

---

Transverse Groove Elongation (%)

<table>
<thead>
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<th>$t_e/t$</th>
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</thead>
<tbody>
<tr>
<td>O</td>
<td>.75</td>
</tr>
<tr>
<td>△</td>
<td>.50</td>
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<tr>
<td>▽</td>
<td>.25</td>
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</tbody>
</table>

---

Diaphragm
TRANSVERSE GROOVE ELONGATION vs DISTANCE FROM CENTRE

Copper Diaphragms

Upper and lower bounds on elongation used in wire breakage calculations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>( \frac{t_e}{t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>.75</td>
</tr>
<tr>
<td>△</td>
<td>.50</td>
</tr>
<tr>
<td>▽</td>
<td>.25</td>
</tr>
</tbody>
</table>
TRANSVERSE GROOVE ELONGATION vs DISTANCE FROM CENTRE

Stainless Steel Diaphragms

Upper and lower bounds on elongation used in wire breakage calculations

Transverse Groove Elongation (\%)

\( O - t_e/t \times 0.5 \)

Diaphragm

Fig. A.2.3
STRETCHING of SIGNAL WIRE due to DIAPHRAGM DISTORTION and PETAL ROTATION

a. Diaphragm before pressurisation.
b. " at bursting point.
c. " wire breakage angle.

PQ Initial wire length
P'Q' Length at failure of diaphragm
P'Q' Ultimate length
Fig. A 2.5.

PETAL ROTATION $\phi$ TO BREAK SIGNAL WIRE vs DISTANCE of WIRE from PETAL TIP

Aluminium Diaphragms

<table>
<thead>
<tr>
<th>Groove elong (%)</th>
<th>Wire elong (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>40</td>
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<tr>
<td>30</td>
<td>30</td>
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</table>

Copper Diaphragms

<table>
<thead>
<tr>
<th>Groove elong (%)</th>
<th>Wire elong (%)</th>
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</thead>
<tbody>
<tr>
<td>16</td>
<td>40</td>
</tr>
<tr>
<td>8</td>
<td>30</td>
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</tbody>
</table>

Stainless Steel Diaphragms

<table>
<thead>
<tr>
<th>Groove elong (%)</th>
<th>Wire elong (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>40</td>
</tr>
<tr>
<td>8</td>
<td>30</td>
</tr>
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</table>
Fig. A.4.1.
DIAPHRAGM PETAL GEOMETRY

Fig. A.4.2.
UNCERTAINTY in PETAL DIMENSIONS
Fig. A.6.

TYPICAL CROSS-SECTION of SHOCK-TUBE

Not to scale

All dimensions are inches

P.T. Pressure Transducer
S.D. Shock Detector

POSITIONS of PRESSURE TRANSDUCERS and SHOCK DETECTORS
TRANVERSE GROOVE ELONGATION OF ALUMINIUM DIAPHRAGMS

BEFORE PRESSURISATION — PARALLEL GROOVES

AFTER PRESSURISATION — TAPERED GROOVES
PRELIMINARY SILHOUETTE PICTURES OF BURSTING ALUMINIUM DIAPHRAGMS AT VARIOUS TIME DELAYS.

Time Delay 200 μs

Time Delay 400 μs
PLATE 5.3

SPARK PHOTOGRAPHS OF BURSTING ALUMINIUM DIAPHRAGMS.

Thickness 18 s.w.g. \( t_e/t = 0.5 \)
Nominal Bursting Pressure 8.5 bar.

Time Delay Zero

Time Delay 200 \( \mu s \)

Time Delay 400 \( \mu s \)
PLATE 5.4

BURSTING OF ALUMINIUM DIAPHRAGM

Thickness 18 s.w.g. \( \frac{t_e}{t} \) 0.5
Bursting Pressure 14.2 bar
Oscilloscope Sweep Rate 100 \( \mu \text{s/cm} \)

Laser Output
Signal Wire
Wall Contact

Breakage of 1st wire
" 2nd "
" 3rd "
Wall Contact
PLATE 5.5

MULTIPLE EXPOSURE PHOTOGRAPHS of the BURSTING of ALUMINIUM DIAPHRAGMS

Thickness 18 s.w.g. $t_e/t_0 = 0.5$
Nominal Bursting Pressure 14.2 bar
Spark Flashing Rate 10 K.Hz
Plate 9.1

Schlieren Photographs of the Flow Emerging from a Folding 2-Flap Aluminium Diaphragm
APPENDIX I

Analysis of Wave-Contact-Surface Interaction in Unsteady Flow

A.1.1 Coalescence of Two Shock-Waves

When two adjacent normal shocks are travelling in the same direction, the relative flow behind the leading wave is subsonic. The following shock, travelling at supersonic speed relative to the flow ahead of it must therefore eventually overtake the leading shock, provided no additional disturbances are introduced.

This problem was considered by Bull, Fowell and Henshaw (A.1) who showed that although a qualitative representation of the flow in the $u, p$ plane (Fig. A.1.1) indicates that three wave configurations are possible:

(a) A transmitted shock, reflected expansion and contact surface;
(b) A transmitted shock and a reflected Mach wave;
(c) Transmitted and reflected shocks and a contact surface;

in fact only cases (a) and (b) are possible for realistic values of $\gamma$.

The configuration shown in Fig. A.1.1 relates to P-shocks but the analysis applies equally to Q-shocks.

From the Rankine-Hugoniot relations the velocity change across the leading shock is given as

$$\frac{u_2 - u_1}{a_1} = D \times G(P_{21}, \gamma_1)$$  \hspace{1cm} A.1.1

where $D = +1$ for a P-shock, $-1$ for a Q-shock and

$$G(P_{21}, \gamma_1) = \frac{P_{21} - 1}{(\gamma_1(\gamma_1 P_{21} + 1)^{\frac{1}{\gamma_1}})}$$

Similarly, for the overtaking shock:

$$\frac{u_3 - u_2}{a_2} = D \times G(P_{32}, \gamma)$$  \hspace{1cm} A.1.2
Thus, the particle velocity in region 3 is given by:

\[
(u_3 - u_1)/a_1 = D \cdot (G(P_{21}, \gamma) + G(P_{32}, \gamma))F(P_{21}, \gamma)
\]  \hspace{1cm} A.1.3

where \( F(P_{21}, \gamma) = ((\alpha + P_{21})P_{21}/(\alpha P_{21} + 1))^\frac{1}{\gamma} \)

and the corresponding result for region 5 is:

\[
(u_5 - u_1)/a_1 = D \cdot G(P_{43}P_{32}P_{21}, \gamma)
\]  \hspace{1cm} A.1.4

Let \((u_3 - u_1)/(u_5 - u_1) = R\)

Then if \(R > 1\), the reflected wave is a shock, and \(R < 1\) implies a reflected rarefaction.

If \(R = 1\), a Mach wave is reflected and \(u_5 = u_3\). Inserting the condition \(P_{43} = 1\) and \(u_5 = u_3\) into equations A.1.3 and A.1.4 gives, for a reflected Mach wave:

\[
((P_{21} - 1)(P_{32} - 1)(P_{32}P_{21} - 1))^2 \cdot \gamma(\alpha^3 - 2\alpha^2 + \gamma(1 - 4(1 + 1/P_{21})/P_{32})
- 4/(P_{32}^2P_{21})) = 0
\]  \hspace{1cm} A.1.5

Any given set of values of \(p_1\) and \(p_2\) yields a critical value of \(\gamma\) as a solution of the cubic equation A.1.5.

It is found that only for \(\gamma > 5/3\) can a reflected shock occur (i.e. \(R > 1\)). For all realistic values of \(\gamma\), the reflected wave is invariably an expansion.

The change in particle velocity across the reflected expansion is given by:

\[
(u_1 - u_3)/a_3 = D \times H(P_{43}, \gamma)
\]  \hspace{1cm} A.1.6

where \(H(P_{43}, \gamma) = \frac{2}{\gamma - 1} (1 - P_{43}^{\gamma})\)

Equating the particle velocities in fields 4 and 5 (Fig. A.1.2) gives:

\[
(F(P_{32}, \gamma)H(P_{43}, \gamma) + G(P_{32}, \gamma))F(P_{21}, \gamma) + G(P_{21}, \gamma)
- G(P_{43}P_{32}P_{21}, \gamma) = 0
\]  \hspace{1cm} A.1.7
Equation A.1.7 may be solved for $P_{12}$ from a knowledge of the static pressures in fields 1, 2 and 3.

The Rankine-Hugoniot relations then yield the remaining properties for field 5, while the isentropic flow relationship applied across the rarefaction gives the sonic velocity in field 4.

A.1.2 Collision of Two Shocks

No ambiguity arises in this case, in that both of the emergent waves are shocks. A contact surface is also produced unless the temperature behind the transmitted shocks are identical.

For the incident shocks, (Fig. A.1.2)

\[
\frac{u_1 - u_2}{a_2} = G(P_{12}, \gamma) ; \frac{u_3 - u_2}{a_2} = - G(P_{32}, \gamma)
\]

A.1.8

and for the transmitted shocks,

\[
\frac{u_4 - u_3}{a_3} = G(P_{43}, \gamma) ; \frac{u_5 - u_1}{a_1} = - G(P_{51}, \gamma)
\]

A.1.9

Equating the particle velocities in regions 4 and 5 gives:

\[
G(P_{12}, \gamma) + G(P_{32}, \gamma) = G(P_{51}P_{12}P_{23}, \gamma)F(P_{32}, \gamma)
\]

\[
+ G(P_{51}, \gamma)F(P_{12}, \gamma)
\]

which may be solved for $P_{51}$, from a knowledge of the static pressures in regions 1, 2 and 3; the remaining properties in fields 4 and 5 are then determined, using the Rankine-Hugoniot relations.

A.1.3 Collision of Two Rarefactions

Both transmitted waves are rarefactions in this case and since entropy is constant in the neighbourhood of both incident and transmitted waves, no contact surface is produced.
Across the incident waves, (Fig. A.1.3) we have respectively:

\[ \frac{u_1 - u_2}{a_2} = -H(P_{t2}, \gamma); \frac{u_3 - u_2}{a_2} = H(P_{s2}, \gamma) \]  

A.1.10

while across the transmitted waves the corresponding results are:

\[ \frac{u_4 - u_4}{a_3} = -H(P_{t3}, \gamma); \frac{u_4 - u_1}{a_1} = H(P_{m}, \gamma) \]  

A.1.11

Eliminating all particle velocities, the static pressure in region 4 is obtained from:

\[ P_{4} = P_{1}^{2} + P_{3}^{2} - 1 \]  

A.1.12

\[ u_4 \] is then obtained from equation A.1.11 and \( a_4 \) from the isentropic relation applied between 1 and 4.

A.1.4 Collision of Shock and Rarefaction

This interaction illustrated in Fig. A.1.4 invariably produces a transmitted rarefaction, a transmitted shock and a contact surface.

The solution proceeds on similar lines to those of the interactions already described and culminates, on equating the velocities in fields 4 and 5, in the relationship:

\[ P_{3}^{2} G(P_{51}P_{13}, \gamma) + H(P_{32}, \gamma) - H(P_{51}, \gamma)/F(P_{12}, \gamma) \]

\[ - G(P_{12}, \gamma) = 0 \]  

A.1.13

Provided the shock occurs between regions 1 and 2 and the rarefaction between 2 and 3, equation A.1.13 is applicable to both P-shock/Q-rarefaction and Q-shock/P-rarefaction cases.

A.1.5 Overtaking of a Shock by a Rarefaction

Since the relative flow behind the shock is subsonic it is possible for a wave travelling in this region at the local sonic velocity to overtake the shock. The wave system resulting from this interaction may be any of the following four possibilities:
(a) Shock transmitted - rarefaction reflected;
(b) Shock transmitted - shock reflected;
(c) Rarefaction transmitted - shock reflected;
(d) Rarefaction transmitted - rarefaction reflected.

This apparent ambiguity is explicable on the basis of an assessment of the relative magnitudes of the slopes of the shock and isentropic-wave polars respectively in the u-p plane.

From equation A.1.1 the slope of the shock-polar at the point 2 Fig. A.1.5 is given by:

\[
\frac{du}{dp}_2 = \frac{a_1}{2\gamma^2 p_1} = \frac{(\alpha P_{21} + 2 + \alpha)}{(\alpha P_{21} + 1)^{3/2}}
\]

Similarly, for the rarefaction-wave polar

\[
\frac{du}{dp}_2 = \frac{a_2}{\gamma p_2}
\]

The relative magnitudes of the two slopes depend on \( P_{21} \) which is part of the initial data. The variability of \( P_{21} \) for different examples gives rise to the four possible wave systems listed above and illustrated in Fig. A.1.5a to d. These are now analysed in detail.

A.1.5a Shock Transmitted-Rarefaction Reflected

On equating the velocities in regions 4 and 5, Fig. A.1.5a the equations for the incident and reflected waves may be combined to give the result:

\[
G(P_{21}, \gamma) + F(P_{21}, \gamma)(P_{32} B (2 - P_{43} B - 1)B^{-1} - G(P_{43} P_{32} P_{21}, \gamma) = 0
\]

which may be solved for \( P_{43} \).

The first derivative with respect to \( P_{43} \) of the L.H.S. of equation A.1.16 is:
The maximum value of $P_{43}$ consistent with a reflected rarefaction is not greater than unity. It follows from equations A.1.16 and A.1.17 that a necessary condition for a reflected rarefaction is:

$$G(P_{21}, \gamma) + F(P_{21}, \gamma)(P_{42}^{\frac{3}{2}} - 1) - G(P_{32}P_{21}, \gamma) < 0$$

a Mach wave being reflected when the L.H.S. is identically zero.

When expressed in terms of $P_{51}$ in place of $P_{43}$, equation A.1.16 becomes:

$$G(P_{21}, \gamma) + F(P_{21}, \gamma)(P_{32}^{\frac{3}{2}}(2 - P_{51}^{\beta}P_{13}^{\beta}) - 1) - G(P_{51}, \gamma) = 0$$

The first derivative of the L.H.S. with respect to $P_{51}$ is:

$$- F(P_{21}, \gamma)P_{42}^{\frac{3}{2}}P_{13}^{\beta}P_{51}^{\beta} - (\alpha P_{51} + 2 + \alpha)$$

which is also negative.

The minimum value of $P_{51}$ consistent with a transmitted shock is unity.

Inserting this condition into equation A.1.19 gives as a necessary condition for a transmitted shock the result:

$$G(P_{21}, \gamma) + F(P_{21}, \gamma)(P_{32}^{\frac{3}{2}}(2 - P_{13}^{\beta})) - 1) > 0$$

a Mach wave being transmitted when the L.H.S. is identically zero.

In general, $a_4 \neq a_5$ and a contact surface separates the two fields.
The analyses of cases b, c and d (section A.1.5) proceed on similar lines and lead to equations corresponding to A.1.16, for the determination of $P_{43}$ together with simultaneous pairs of conditions corresponding respectively to A.1.18 and A.1.21 for the existence of the appropriate reflected and transmitted waves. The details of this analysis are omitted and only the results referred to above are summarised for cases b, c and d.

### A.1.5b Shock Transmitted - Shock Reflected

$P_{43}$ (Fig. A.1.5b) is determined from the relation:

\[
G(P_{21}, \gamma) + F(P_{21}, \gamma) (P_{32}^2 (1 - (P_{43} - 1)\beta)^{-1} - (P_{43} + 1)^{-1}) - 1) - G(P_{32}P_{21}, \gamma) = 0 \quad \text{A.1.22}
\]

The condition for a reflected shock is:

\[
G(P_{21}, \gamma) + F(P_{21}, \gamma) (P_{32}^2 - 1)\beta^{-1}G(P_{32}P_{21}, \gamma) > 0 \quad \text{A.1.23}
\]

and that for a transmitted shock:

\[
G(P_{21}, \gamma) + F(P_{21}, \gamma) (P_{32}^2 (1 - G(P_{43}, \gamma)) - 1) > 0 \quad \text{A.1.24}
\]

### A.1.5c Rarefaction Transmitted - Shock Reflected

$P_{43}$ (Fig. A.1.5c) is determined from:

\[
G(P_{21}, \gamma)\beta^4 + F(P_{21}, \gamma) (P_{32}^2 (1 - G(P_{43}, \gamma)\beta^4) - 1) - (P_{43}P_{21})^\beta + 1 = 0 \quad \text{A.1.25}
\]

The condition for a reflected shock is:

\[
G(P_{21}, \gamma)\beta^4 + F(P_{21}, \gamma) (P_{32}^2 - 1) - (P_{43}^\beta - 1) > 0 \quad \text{A.1.26}
\]

and for a transmitted rarefaction:

\[
G(P_{21}, \gamma)\beta^4 + F(P_{21}, \gamma) (P_{32}^2 (1 - G(P_{43}, \gamma)\beta^4) - 1) < 0 \quad \text{A.1.27}
\]
A.1.5d Transmitted and Reflected Rarefactions

$P_{43}$ (Fig. A.1.5d) is determined from the relation:

$$G(P_{21}, \gamma)\beta^2 + F(P_{21}, \gamma)(P_{32} \beta (2 - P_{43} \beta) - 1) - (P_{43}P_{32})^\beta + 1 = 0$$  \hspace{1cm} A.1.28

The condition for a transmitted rarefaction is:

$$G(P_{21}, \gamma)\beta^2 + F(P_{21}, \gamma)(P_{32}\beta (2 - P_{43}\beta) - 1) \leq 0$$  \hspace{1cm} A.1.29

and for a reflected rarefaction:

$$G(P_{21}, \gamma)\beta^2 + F(P_{21}, \gamma)(P_{32}\beta - 1) - P_{32}^\beta + 1 \leq 0$$  \hspace{1cm} A.1.30

Provided the incident shock occurs between regions 1 and 2 and the rarefaction between 2 and 3, equations A.1.17 to 30 are equally applicable to the overtaking of P and Q-wave pairs.

A.1.6 Overtaking of a Rarefaction by a Shock

As in the case of the overtaking of a shock by a rarefaction, any one of four different combinations of reflected and transmitted waves may result from the present interaction. These combinations are identical with those already described in section A.1.5 and the individual cases are now summarised briefly.

A.1.6a Shock Transmitted - Rarefaction Reflected

$P_{43}$ (Fig. A.1.6a) is determined from:

$$P_{21}^\beta G(P_{32}, \gamma) - H(P_{21}, \gamma) + P_{21}^\beta F(P_{32}, \gamma)H(P_{43}, \gamma)$$
$$- G(P_{43}P_{32}P_{21}, \gamma) = 0$$  \hspace{1cm} A.1.31

The conditions for a reflected rarefaction and transmitted shock are respectively:

$$P_{21}^\beta G(P_{32}, \gamma) - H(P_{21}, \gamma) - G(P_{32}P_{21}, \gamma) \leq 0$$  \hspace{1cm} A.1.32

and

$$P_{21}^\beta G(P_{32}, \gamma) - H(P_{21}, \gamma) - F(P_{32}, \gamma)(\frac{P_{32}^\beta - P_{21}^\beta}{\gamma^\beta}) > 0$$  \hspace{1cm} A.1.33
A.1.6b Shock Transmitted - Shock Reflected

$P_{43}$ (Fig. A.1.6c) is determined from the equation:

$P_{21} G(P_{32}, \gamma) - F(P_{32}, \gamma) G(P_{43}, \gamma) - H(P_{21}, \gamma)$

- $G(P_{43}P_{32}P_{21}, \gamma) = 0$  \hspace{1cm} A.1.34

The conditions for reflected and transmitted shocks are respectively:

$P_{21} G(P_{32}, \gamma) - H(P_{21}, \gamma) - G(P_{32}P_{21}, \gamma) > 0$  \hspace{1cm} A.1.35

and

$P_{21} G(P_{32}, \gamma) + F(P_{32}, \gamma) G(P_{32P_{21}}, \gamma) - H(P_{21}, \gamma) > 0$  \hspace{1cm} A.1.36

A.1.6c Rarefaction Transmitted - Shock Reflected

$P_{43}$ (Fig. A.1.6c) is determined from the equation:

$P_{21} G(P_{32}, \gamma) - F(P_{32}, \gamma) G(P_{43}, \gamma)$

- $H(P_{21}, \gamma) + H(P_{43P_{32}P_{21}}, \gamma) = 0$  \hspace{1cm} A.1.37

The conditions for the reflected shock and transmitted rarefaction are respectively:

$P_{21} G(P_{32}, \gamma) - H(P_{21}, \gamma) + H(P_{32P_{21}}, \gamma) > 0$  \hspace{1cm} A.1.38

and

$P_{21} G(P_{32}, \gamma) + F(P_{32}, \gamma) G(P_{32P_{21}}, \gamma) - H(P_{21}, \gamma) < 0$  \hspace{1cm} A.1.39

A.1.6d Transmitted and Reflected Rarefactions

$P_{34}$ (Fig. A.1.6d) is determined from the equation:

$P_{21} (H(P_{43}, \gamma) F(P_{32}, \gamma) + G(P_{32}, \gamma))$

- $H(P_{21}, \gamma) + H(P_{43P_{32}P_{21}}, \gamma) = 0$  \hspace{1cm} A.1.40
The conditions for reflected and transmitted rarefactions are respectively:

\[ P_{21}^B G(P_{32}, \gamma) - H(P_{21}, \gamma) + H(P_{32}P_{21}, \gamma) < 0 \quad A.1.41 \]

and

\[ P_{21}^B G(P_{32}, \gamma) - H(P_{21}, \gamma) - F(P_{32}, \gamma) \left( \frac{P_{23}^B - P_{21}^B}{\gamma} \right) < 0 \quad A.1.42 \]

Provided the incident rarefaction occurs between region 1 and 2 and the shock between 2 and 3, equation A.1.31 to 42 are equally applicable to overtaking P- and Q-wave pairs.

### A.1.7 Interaction of a Shock-Wave with a Contact Surface

In this case, the transmitted wave is invariably a shock as shown in the following analysis. However, the reflected wave may be a shock, a rarefaction or a Mach wave depending upon the initial gas states in regions 1 and 2 (Fig. A.1.7).

The two different cases are now analysed, the equations given being applicable to both P and Q incident shocks.

### A.1.7a Shock Transmitted - Rarefaction Reflected

Applying the usual relationships between particle velocity and pressure-change across the various waves, and equating the velocities in regions 4 and 5 (Fig. A.1.7a) gives the result:

\[ G(P_{32}, \gamma_2) + F(P_{32}, \gamma_2)H(P_{43}, \gamma_2) - \left( \frac{\gamma_1}{\gamma_2} \right)^{1/2} \left( \frac{\gamma_1}{\gamma_2} \right)G(P_{43}P_{32}, \gamma_1)(E_{12})^{1/2} = 0 \quad A.1.43 \]

where \( E_{12} = (C_{v_1} T_1)/(C_{v_2} T_2) \)

The first derivative of equation A.1.43 with respect to \( P_{43} \) is:

\[ -\beta_2^{1/2} F(P_{32}, \gamma_2)P_{43}^{\beta_2-1} - P_{32}(E_{12})^{1/2} \left( \frac{\alpha_1 P_{43}P_{32} + 2 + \alpha_1}{2(\alpha_1 P_{43}P_{32} + 1)} \right)^{3/2} \]

which is negative.
The maximum value of $F_{t}$ compatible with a reflected rarefaction is $1.0$.

$$E_{12} \geq \frac{\alpha_{1}P_{32} + 1}{\alpha_{2}P_{32} + 1}$$  \hspace{1cm} \text{A.1.44}

For the special case of equal values of $\gamma$ on both sides of the contact surface, a rarefaction is reflected when $T_{1} > T_{2}$.

It may similarly be shown on expressing equation A.1.43 in terms of $P_{51}$, that a necessary condition for a transmitted shock is:

$$F(P_{32}, \gamma_{2})G(P_{13}, \gamma_{2}) + G(P_{32}, \gamma_{2}) > 0$$  \hspace{1cm} \text{A.1.45}

All terms on the L.H.S. of equation A.1.45 are positive and a shock is therefore invariably the transmitted wave.

A.1.7b Shock Transmitted - Shock Reflected

Using an approach similar to that of section A.1.7a, equating the velocities in regions 4 and 5 (Fig. A.1.7b) gives:

$$F(P_{32}, \gamma_{2})G(P_{43}, \gamma_{2}) - G(P_{32}, \gamma_{2}) + \left(\gamma_{1}\beta_{1}^{\frac{1}{2}}/\gamma_{2}\beta_{2}^{\frac{1}{2}}\right)G(P_{43}P_{32}, \gamma_{1}) E_{12}^{\frac{1}{2}} = 0$$  \hspace{1cm} \text{A.1.46}

From which the necessary condition for a reflected shock is:

$$E_{12} \leq \frac{\alpha_{1}P_{32} + 1}{\alpha_{2}P_{32} + 1}$$  \hspace{1cm} \text{A.1.47}

and, on expressing equation A.1.45 in terms of $P_{51}$, the necessary condition for a transmitted shock is:

$$F(P_{32}, \gamma_{2})G(P_{13}, \gamma_{2}) - G(P_{32}, \gamma_{2}) \leq 0$$  \hspace{1cm} \text{A.1.48}

For an incident shock, $P_{13} < 1.0$ and $P_{32} > 1$ and equation A.1.47 therefore indicates that the transmitted wave is invariably a shock.
A.1.8 Interaction of a Rarefaction Wave with a Contact Surface

The analysis of this interaction is essentially similar to that of the shock/contact surface interaction and the transmitted wave is invariably a rarefaction; the reflected wave may be a rarefaction or a shock and the two cases are summarised briefly.

A.1.8a Rarefaction Transmitted - Shock Reflected

The value of $P_{43}$ (Fig. A.1.8a) is determined from the equation:

$$
\left( \frac{\beta_1}{\beta_2} \right) \left\{ P_3 \beta_2 \left( 1 - G(P_{43}, \gamma_2) \beta_2 \gamma_2 \right) - 1 \right\} \\
+ \left( \gamma_1 - 1 \right) \frac{E_{12}}{2} H(P_{43}P_{32}, \gamma_1)/2 = 0
$$

from which the necessary condition for a reflected shock is:

$$
\left( \frac{\beta_1}{\beta_2} \right) \left( P_3 \beta_2 - 1 \right) / \left( P_2 \beta_1 - 1 \right) \geq \frac{E_{12}}{2}
$$

A.1.49

Expressing equation A.1.46 in terms of $P_{51}$ gives the necessary condition for a transmitted rarefaction as:

$$
\left( \frac{\beta_1}{\beta_2} \right) \left\{ P_3 \beta_2 \left( 1 - G(P_{43}, \gamma_2) \beta_2 \gamma_2 \right) - 1 \right\} \leq 0
$$

A.1.50

A.1.51

and since $P_{43} > 1.0$ for an incident rarefaction, equation A.1.51 indicates that the transmitted wave is invariably a rarefaction.

A.1.8b Rarefaction Transmitted - Rarefaction Reflected

The value of $P_{43}$ (Fig. A.1.8b) is determined from the equation:

$$
H(P_{32}, \gamma_2) - P_{32} \beta_2 \left( H(P_{43}, \gamma_2) \right) \\
- H(P_{43}P_{32}, \gamma_2) \left( \frac{\beta_1}{\beta_2} \right) \left( \frac{\gamma_1}{\gamma_2} \right) E_{12} = 0
$$

A.1.52
from which, the condition for a reflected rarefaction becomes:

\[
\frac{1}{2} \left( \frac{\beta_1}{\beta_2} \right) \left( \frac{P_{32} \beta_2^2 - 1}{P_{32} \beta_1^2 - 1} \right) < E_{12}^{1/2}
\]

Expressing equation A.1.52 in terms of \( P_{31} \) gives the necessary condition for a transmitted rarefaction as:

\[
P_{32} < 1.0
\]

Since this is necessarily the case for an incident rarefaction, the transmitted wave is invariably a rarefaction.
APPENDIX 2

Initial Motion of the Diaphragm Petals

A.2.1 Introduction

The breakage of a wire attached across a diaphragm groove indicates basically that the local relative movement between adjacent petals has exceeded the ultimate elongation of the wire. This relative movement is the summation of a static displacement occurring prior to failure of the diaphragm and a displacement caused by the rotation of the petals about different axes.

The interpretation of the breakage of the wire in terms of a petal rotation and, finally as an elapsed time following the start of rotation must be based on a theoretical model of the mechanism of failure of the diaphragm. In addition to this theoretical model, information concerning the ultimate elongation of the wire and the static extension of the grooved portion of the diaphragm at failure are required in order to estimate the petal angles at the instant of wire breakage.

The idealised theoretical model of the diaphragm opening process (section 2.2) is based on the assumption that when the diaphragm ruptures, cracks travel along the whole of the grooved portion before any appreciable rotation of the petals occurs. The only resistance to petal motion, on this basis is the bending moment at the clamped edge.

The simple theoretical analysis based on this model is given in section A.2.4 and details are given in sections A.2.2 and A.2.3 of the techniques used in the measurement of the transverse static extension of the diaphragm grooves and the ultimate elongation of the wire respectively.
An examination of the validity of the assumption of instantaneous petal formation at the beginning of the opening process was made by measuring the breakage time of a series of signal wires attached across the diaphragm grooves at a range of distances from the centre (Fig. 5.2).

These tests were performed on diaphragms made of aluminium, copper and stainless steel respectively, and having groove depths ranging from 25% to 75% of the basic thickness.

Comparison between the idealised predictions of wire breakage time and the measured results is made in section 6.2.

A.2.2 Transverse Groove Elongation

The transverse extension of the grooved portion of the diaphragm was determined by measurements made from photographs taken during pressurisation to failure. The construction of the shock tube (Appendix 6) in lengths not greater than 2 ft 6 in made possible the siting of a camera less than 3 ft from the diaphragm. The diaphragm was viewed through a window in the end wall of the driven section using the lens and plane-mirror arrangement shown in Fig. 5.5., a tungsten-filament bulb being substituted for the spark light source.

The diaphragms were painted matt white and the groove edges remained almost uncoated, showing clearly on the photographs (Plate 5.1) as narrow strips which facilitate the measurement of groove widths. Groove depths of 25%, 50% and 75% respectively of the total diaphragm thickness were used. The nominal groove width before pressurisation was \(\frac{1}{16}\) in and the true width of the groove edge strips was estimated from the enlarged photographs (Plate 5.1) to be 0.005 in. Assuming that the groove width could be measured to within half the width of the edge,
strip the measured groove strain values were subject to a
tolerance of ± 4% and corresponding bands are shown in Figs.
A.2.1 to A.2.3.

The results indicate that for the three materials tested,
a band of ± 4% about the mean value of elongation at the
centre of the diaphragm captures nearly all the experimental
points for the two ductile materials, annealed copper and stainless
steel. The results relating to a value of $\frac{e}{t}$ of 0.25 for all
three materials lie outside this band but diaphragms of this
specification frequently fail to open fully and were therefore
not used for shock-tube runs in the present work.

In view of the relatively large scatter evident in the
experimental results, the straightforward calculation of a mean
elongation is somewhat artificial and instead, the upper and
lower bounds of the ± 4% band were used respectively in the cal­
culation of minimum and maximum petal rotations to produce wire
breakage.

The relevant analysis is given in section A.2.4 and the
elongation values used are summarised in Table A.2.

Table A.2

<table>
<thead>
<tr>
<th>Material</th>
<th>$\frac{e}{t}$</th>
<th>Central Elongation (%)</th>
<th>Standard Deviation (%)</th>
<th>Upper-Bound Central Elongation (%)</th>
<th>Lower-Bound Central Elongation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annealed Copper</td>
<td>0.5 &amp; 0.75</td>
<td>12.0</td>
<td>3.0</td>
<td>16.0</td>
<td>8.0</td>
</tr>
<tr>
<td>Half-Hard</td>
<td>0.5 &amp; 0.75</td>
<td>25.0</td>
<td>3.0</td>
<td>29.0</td>
<td>21.0</td>
</tr>
<tr>
<td>Stainless Steel</td>
<td>0.5*</td>
<td>11.0</td>
<td>2.0</td>
<td>15.0</td>
<td>7.0</td>
</tr>
</tbody>
</table>

* Only 50% groove-depths used - machining scatter excessive at 25% owing
to small basic thickness of plate.
A.2.3 Elongation of Signal Wire

The elongation of the signal wire at failure was determined by gluing several samples with the adhesive used in the diaphragm tests, across a 1/16 in wide gap between a pair of 16 S.W.G. steel plates, and drawing the plates slowly apart while measuring continuously the distance between them. The wire was viewed under a microscope in order to facilitate detection of the onset of necking prior to failure.

One of the plates was held stationary while the other was drawn away by a micrometer screw which also indicated the net elongation at failure.

The mean value of elongation obtained from the above tests was 35.5% ± 5%.

A.2.4 Idealised Wire-Breakage Calculation

For the purpose of calculating the relative movement between adjacent diaphragm petals, it is assumed that under pressurisation, they adopt a circular cylindrical cross section (Fig. A.2.4) and also that the elongation of the grooved portion of the petal increases linearly with distance along the groove from the clamped edge, in accordance with the observation presented in Figs. A.2.1 to A.2.3.

At the limit of static stretching, just prior to failure of the diaphragm, the length of the wire which initially spanned the distance PQ Fig. A.2.4, has increased to P'Q'. The corresponding transverse displacements PP' and QQ' are then given respectively as Kr_p and Kr_q as defined in Fig. A.2.4.
The components of the distance \( P'Q' \) are thus:

\[
\begin{align*}
(a) & \quad w_o / \sqrt{2} + Kr_Q \\
(b) & \quad Kr_p \\
(c) & \quad \delta_p - \delta_Q
\end{align*}
\]

Assuming a rigid-body rotation \( \phi \) of all four petals to the position of wire breakage, the corresponding final components of the wire lengths \( P''Q'' \) are as follows:

\[
\begin{align*}
(a) & \quad \sqrt{2} w_o + r_Q (1 - \cos \phi (1 - K)) + \delta_Q \sin \phi \\
(b) & \quad r_p (1 - \cos \phi (1 - K)) + \delta_p \sin \phi \\
(c) & \quad (\delta_p - \delta_Q) \cos \phi + w_o (1 - K) \sin \phi
\end{align*}
\]

For the circular arc profile shown in Fig. A.2.4,

\[
\delta (2R - \delta) = r^2 
\]

\[
R = \frac{1}{2} \left( \frac{r + \delta}{r} \right)^2
\]

Also \( \delta_p = \delta - R (1 - \cos \beta) \) and \( R \sin \beta = r - r_p \)

\[
\sin \beta = 2 \left( 1 - \frac{r_p}{r} \right) \frac{r/\delta + \delta/r}{r/\delta + \delta/r}
\]

\[
\cos \beta = \left( \frac{1 - 4 \left( 1 - \frac{r_p}{r} \right)^2}{(r/\delta + \delta/r)} \right)^\frac{1}{2}
\]

Hence

\[
\delta_p = \frac{\delta}{2} \left( \frac{1}{2} \left( \frac{r}{\delta} \right)^2 + \left( \left( \frac{\delta}{2} \left( \frac{1}{2} + \left( \frac{r}{\delta} \right)^2 \right) \right)^2 - (r - r_p)^2 \right) \right)^\frac{1}{2}
\]

\[
\delta_Q = \frac{\delta}{2} \left( \frac{1}{2} \left( \frac{r}{\delta} \right)^2 + \left( \left( \frac{\delta}{2} \left( \frac{1}{2} + \left( \frac{r}{\delta} \right)^2 \right) \right)^2 - (r - r_Q)^2 \right) \right)^\frac{1}{2}
\]

Equation A.2.2 and A.2.3 were used to compute the angle \( \phi \) at which the overall length \( P''Q'' \) becomes equal to the maximum length.
at failure (section A.2.3). The computations were performed for a range of initial wire positions spaced along the groove. A lower bound for the condition of wire breakage was determined by assuming a realistic minimum elongation of the signal wire at failure and a maximum static elongation of the diaphragm grooves (Figs. A.2.1. to A.2.3.).

An upper bound was determined by taking the opposite of each of the above criteria.

The results are given in Fig. A.2.5 as curves of petal rotation $\phi$ required to produce wire breakage, against wire-distance $x$ from the petal tips.

In section 6.2. the same values of petal rotation are transposed into elapsed time values on the basis of a theoretical calculation of the diaphragm motion, and results in the form of wire breakage times for different wire positions are compared with experimental values in Fig. 6.1 to 3.
APPENDIX 3

Motion of the Contact-Claw Trigger Device

The effectiveness of the simple contact-claw arrangement, described in Section 5.2.1 and shown in Fig. 5.3, depends basically on the large difference between the angular accelerations of the diaphragm petals and of the claw itself. This difference produces an early loss of contact between the two elements, resulting in the prompt emission of a triggering signal.

Considering the wire as a cantilever of circular cross-section, diameter d, the maximum root bending-moment is $\frac{d^3 f_w}{6}$, where $f_w$ is the yield-stress of the wire. The maximum possible angular acceleration of the wire arises when the contact load between claw and diaphragm petal is sufficient to produce the yield bending-moment at the root.

The moment of inertia of the claw about the fixed end is $\pi \rho_w d^3 L^2/12$, where $\rho_w$ is the wire density and L the length.

The maximum angular acceleration of the wire is therefore:

$$\dot{\theta}_w = \frac{2f_w d}{\pi \rho_w L^3}.$$

Taking representative values of $f_w$, $\rho_w$ and d for 32 S.W.G. steel piano-wire, with a length L equal to the half-width of the shock-tube, gives a value of $\dot{\theta}_w$ of order $10^5$ radian/s$^2$.

The slowest-opening aluminum diaphragms discussed in section 3.6 had an initial angular acceleration in excess of $10^7$ radian/s.

The difference between the angular accelerations is therefore of order $10^7$ radian/s which is sufficient to produce a gap of 0.25 mm for example, after a period of order 10 $\mu$s; such a gap
is sufficient to open the low-voltage D.C. circuit used for trigger generation and the delay involved is not excessive in the context of timing measurements extending over at least 300 µs.
APPENDIX 4

Errors in the Measurement of Petal Motion

A.4.1 Light-Transmission Measurement

The values of petal angle shown in Fig. 6.8 to 10 were obtained from the photomultiplier measurements described in Chapter 5 in which a petal of radius \( r \) (Fig. A.4.1) intercepts a light-filament from a hole at a distance \( d \) from the diaphragm clamping charge.

The value of \( \delta \) is influenced by the centre deflection \( \delta \) developed by the diaphragms which have been measured for all the materials used (Chapter 4) in separate tests, though not during the actual timing runs. These were subject to a scatter of up to 10% but even assuming this maximum value in conjunction with the centre deflection of the most ductile material tested, the maximum variation in the value of \( \delta \) is less than 1%.

A more serious error could result however from the uncertainty in the path taken by the crack which separates an adjacent pair of petals. When burst diaphragms are removed from the tube, it is found that the strip of material nominally of \( 1/16 \) in width which links adjacent petals prior to bursting is distributed at random around all four petals (see for example Plate 5.3 Nos 3, 4, 5). The extremes which could occur in the present context are illustrated in Fig. A.4.2. On the one hand the entire width of both strips remains attached to the petal and on the other, both strips have torn away.

To assume the worst case, if the entire strip should remain attached to one or other of the side petals, then since it is impossible to determine which of these petals actually intercepts the light filaments, an uncertainty arises in the value of \( \delta \).
which, is shown in Fig. A.4.2 to be:-

\[ \Delta \lambda = \pm w \cos \pi/n, \text{ i.e. } \Delta \lambda = \pm 0.0442 \text{ in.} \]

The angle \( \theta \) is determined from: -

\[ \theta = \arcsin \left( \frac{d}{\lambda} \right) \]

and the net error in \( \theta \) due to error in \( d \) and \( \lambda \) is given by:

\[ |\Delta \theta| = \left| \frac{\partial \theta}{\partial d} \Delta d + \frac{\partial \theta}{\partial \lambda} \Delta \lambda \right| \]

\[ = \frac{\Delta d/d + \lambda \Delta \lambda/\lambda}{\left( \frac{\lambda}{d} \right)^2 - 1} \]

A.4.1

The greatest value of \( \Delta \theta \) occurs in conjunction with the smallest value of \( \lambda/d \) which for the results in Fig. 6.8 to 6.10 arises when an aluminum diaphragm, for which \( \lambda = 1.016 \), is used in conjunction with a light-mask (Fig. 5.10) for which the filament-hole farthest from the diaphragm clamping face is at a distance of 0.993 in giving \( \lambda/d = 1.024 \).

The value of \( \Delta d \) is small in comparison with \( \Delta \lambda \), the hole-spacing in the mask being determined by a milling-machine traverse which is indexed to 0.0005 in. Thus from equation A.4.1 substituting for \( \Delta \lambda \) and \( \lambda/d \), the maximum value of \( \Delta \theta \) is 11.6° for a petal-angle approaching 80°. At the minimum value of \( d \), \( d = 0.25 \) in this decreases to \( \Delta \theta = \pm 0.6° \).

A.4.2 Spark Photograph Measurements

In the determination of petal motion from the spark photographs (Plates 5.3 and 5.4) the petal angle is calculated in each case by measuring the distance \( x \), Fig. A.4.1 knowing the corresponding distance \( y \) for the point \( P \), this being either the tip of the petal or any other convenient sighting point visible on all the exposures in the case of the multiple-spark pictures.
The petal angle is calculated from the expression:

\( \theta = \arccos \left( \frac{x}{y} \right) \)

Thus the error in \( \theta \) resulting from errors in \( x \) and \( y \) is given by:

\[
\Delta \theta = \frac{\partial \theta}{\partial x} \Delta x + \frac{\partial \theta}{\partial y} \Delta y
\]

\[
= \frac{\Delta x y - y \Delta y}{(y/x)^2 - 1} \]

\[ \text{A.4.2} \]

The poor quality of the pictures leads to uncertainties in the location of the various sighting points used, and values of \( x \) and the initial \( y \) are subject to an estimated error of \( \pm 0.03 \) in.

At the start of the motion the value of \( \Delta \theta \) is infinite since \( x = y \) in equation A.4.2, the petal movement being effectively undetectable. However, taking the case of aluminium diaphragms for which the value of \( y/x \) remains closest to unity for a given petal angle, after a rotation of 10° for a point at the petal tip the denominator of equation A.4.8 attains a value of 0.415 and for \( \Delta x = \Delta y = 0.03 \), \( \Delta \theta = \pm 8.6° \).

This decreases progressively as \( \theta \) increases and for example at \( \theta = 80° \), \( \Delta \theta = \pm 2° \).

The two different methods of determining the motion of the diaphragm petals thus complement each other in that the photomultiplier technique is the more accurate at small petal angles while the photographic method gives the greater accuracy at large petal angles.
A.4.3 Timing Errors

Timing measurements for both methods were made from standard oscilloscope camera records, the oscilloscope having been calibrated against a square-wave crystal oscillator.

Assuming that the photographic traces could be read to the nearest quarter-division (¼ mm) for a sweep-rate of 100 µs/cm this represents an error of 5 µs. For the fastest-opening stainless-steel diaphragms (Fig. 6.10) the value of the ideal opening time was typically 300 µs approximately and this timing error amounts to less than 2% (0.017 on the dimensionless-time axes of Figs. 6.8 to 10.)
APPENDIX 5

Analysis of Petal Bounce

The analysis of diaphragm opening given in Section 6.4 is based on a linear variation of gas-pressure moment with petal angle:

\[ M_g = A - B\theta \]

The angular acceleration during opening then becomes:

\[ \Theta = \frac{(A - B\theta - M_b)}{M_i} \]

where \( M_b \) is the root bending moment of the petal and \( M_i \) is the moment of inertia.

Equation A.5.1 is simple-harmonic with coulomb damping provided by the bending resistance, and has the solution:

\[ \Theta = (\Theta_o - \phi) \cos \psi t + \phi \]

where \( \Theta = \Theta_o \) at \( t = 0 \),
\[ \phi = \frac{(A - M_b)}{B} \] and \( \psi = \left( \frac{B}{M_i} \right)^{\frac{1}{2}} \)

The petals first make contact with the tube walls at a time \( t_o \) given by:

\[ t_o = \frac{1}{\psi} \arccos \left( \frac{\pi/2 - \phi}{\Theta_o - \phi} \right) \]

The angular velocity at this instant is:

\[ \dot{\Theta}_w = \psi (\Theta_o - \phi) \sin \psi t_o \]

It is assumed that the petals rebound from the walls with an angular velocity \(-k\dot{\Theta}_w\), \( k \) being a coefficient of restitution based on angular velocity.

The equation for the outward motion is:

\[ \Theta = (A - B\theta + M_b)/M_i \]
which has the solution:

\[ \theta = K_1 \sin \psi t + K_2 \cos \psi t + \phi' \]  \hspace{1cm} A.5.5

where \( \phi' = (A + M_b)/B \)

\( K_1 \) and \( K_2 \) being determined from equations A.5.3 and A.5.4.

The use of equations A.5.3 to A.5.5 allows the determination of the amplitudes and periods of successive portions of the decaying motion.

The results given in tables A.5.1 to A.5.3 relate to diaphragms of stainless-steel, copper and aluminum respectively and indicate for a range of values of \( k \), the times of wall contact and also the petal angles and times at which the motion is arrested.

The main conclusion arising from these results is that the motion is strongly influenced by the value used for the coefficient of restitution. For both copper and stainless-steel diaphragms, the petals bounce for more than twenty cycles when a value of 1.0 is chosen for \( k \), but for \( k = 0.6 \) and \( k = 0.1 \), the number of cycles reduces to 5 and 2 respectively and the petals come eventually to rest lying flush with the tube walls.

For aluminum diaphragms however the root bending resistance is appreciably higher than for the former two materials, and the motion is invariably arrested following the first bounce. When the petal comes to rest in the initial rebound, the rootresistance is already greater than the opening-moment due to gas pressure and the diaphragm remains in this partially-open position.

Again however the coefficient of restitution is of considerable importance in determining the final value of the petal angle.

(Table A.5.3)
Table A.5.1.

Copper Diaphragms (2 in Square)

Coefficient of Restitution 1.0

<table>
<thead>
<tr>
<th>Bounce No.</th>
<th>Wall Contact Time (ms.)</th>
<th>Time at Min. Petal Angle (ms.)</th>
<th>Minimum Petal Angle (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.466</td>
<td>0.906</td>
<td>23.0</td>
</tr>
<tr>
<td>2</td>
<td>1.37</td>
<td>1.81</td>
<td>28.1</td>
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<tr>
<td>3</td>
<td>2.27</td>
<td>2.27</td>
<td>33.1</td>
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<tr>
<td>4</td>
<td>3.16</td>
<td>3.60</td>
<td>38.1</td>
</tr>
<tr>
<td>5</td>
<td>4.05</td>
<td>4.48</td>
<td>43.1</td>
</tr>
<tr>
<td>6</td>
<td>4.94</td>
<td>5.35</td>
<td>47.9</td>
</tr>
<tr>
<td>7</td>
<td>5.80</td>
<td>6.22</td>
<td>52.7</td>
</tr>
<tr>
<td>8</td>
<td>6.66</td>
<td>7.07</td>
<td>57.4</td>
</tr>
<tr>
<td>9</td>
<td>7.52</td>
<td>7.91</td>
<td>61.9</td>
</tr>
<tr>
<td>10</td>
<td>8.35</td>
<td>8.73</td>
<td>66.3</td>
</tr>
<tr>
<td>11</td>
<td>9.16</td>
<td>9.55</td>
<td>70.4</td>
</tr>
<tr>
<td>12</td>
<td>9.95</td>
<td>10.3</td>
<td>74.4</td>
</tr>
<tr>
<td>13</td>
<td>10.7</td>
<td>11.1</td>
<td>78.0</td>
</tr>
<tr>
<td>14</td>
<td>11.5</td>
<td>11.7</td>
<td>81.2</td>
</tr>
<tr>
<td>15</td>
<td>12.1</td>
<td>12.4</td>
<td>83.9</td>
</tr>
<tr>
<td>16</td>
<td>12.7</td>
<td>13.0</td>
<td>86.1</td>
</tr>
<tr>
<td>17</td>
<td>13.2</td>
<td>13.4</td>
<td>87.7</td>
</tr>
<tr>
<td>18</td>
<td>13.7</td>
<td>13.8</td>
<td>88.7</td>
</tr>
<tr>
<td>19</td>
<td>14.0</td>
<td>14.1</td>
<td>89.3</td>
</tr>
<tr>
<td>20</td>
<td>14.3</td>
<td>14.3</td>
<td>89.4</td>
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Coefficient of Restitution 0.6

<table>
<thead>
<tr>
<th>Bounce No.</th>
<th>Wall Contact Time (ms.)</th>
<th>Time at Min. Petal Angle (ms.)</th>
<th>Minimum Petal Angle (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.466</td>
<td>0.878</td>
<td>53.4</td>
</tr>
<tr>
<td>2</td>
<td>1.32</td>
<td>1.68</td>
<td>73.8</td>
</tr>
<tr>
<td>3</td>
<td>2.09</td>
<td>2.35</td>
<td>84.4</td>
</tr>
<tr>
<td>4</td>
<td>2.68</td>
<td>2.83</td>
<td>88.6</td>
</tr>
<tr>
<td>5</td>
<td>3.03</td>
<td>3.10</td>
<td>89.8</td>
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</table>

Coefficient of Restitution 0.2

<table>
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<th>Bounce No.</th>
<th>Wall Contact Time (ms.)</th>
<th>Time at Min. Petal Angle (ms.)</th>
<th>Minimum Petal Angle (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.466</td>
<td>0.758</td>
<td>82.1</td>
</tr>
<tr>
<td>2</td>
<td>1.12</td>
<td>1.18</td>
<td>89.7</td>
</tr>
</tbody>
</table>
Table A.5.2.

Stainless-Steel Diaphragms (2 in Square)

Coefficient of Restitution 1.0

<table>
<thead>
<tr>
<th>Bounce No.</th>
<th>Wall Contact Time (ms.)</th>
<th>Time at Min. Petal Angle (ms.)</th>
<th>Minimum Petal Angle (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.319</td>
<td>0.623</td>
<td>17.9</td>
</tr>
<tr>
<td>2</td>
<td>0.939</td>
<td>1.24</td>
<td>23.3</td>
</tr>
<tr>
<td>3</td>
<td>1.56</td>
<td>1.85</td>
<td>29.5</td>
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<tr>
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<td>2.16</td>
<td>2.46</td>
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</tr>
<tr>
<td>5</td>
<td>2.76</td>
<td>3.06</td>
<td>41.6</td>
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<tr>
<td>6</td>
<td>3.36</td>
<td>3.64</td>
<td>47.4</td>
</tr>
<tr>
<td>7</td>
<td>3.95</td>
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<td>8</td>
<td>4.53</td>
<td>4.79</td>
<td>58.5</td>
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<tr>
<td>9</td>
<td>5.09</td>
<td>5.35</td>
<td>63.6</td>
</tr>
<tr>
<td>10</td>
<td>5.62</td>
<td>5.88</td>
<td>68.5</td>
</tr>
<tr>
<td>11</td>
<td>6.15</td>
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</tr>
<tr>
<td>12</td>
<td>6.65</td>
<td>6.85</td>
<td>77.0</td>
</tr>
<tr>
<td>13</td>
<td>7.11</td>
<td>7.30</td>
<td>80.5</td>
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<tr>
<td>14</td>
<td>7.54</td>
<td>7.70</td>
<td>83.4</td>
</tr>
<tr>
<td>15</td>
<td>7.90</td>
<td>8.05</td>
<td>85.7</td>
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<td>16</td>
<td>8.24</td>
<td>8.35</td>
<td>87.3</td>
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<tr>
<td>17</td>
<td>8.50</td>
<td>8.59</td>
<td>88.4</td>
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<tr>
<td>18</td>
<td>8.70</td>
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<td>8.90</td>
<td>8.94</td>
<td>89.5</td>
</tr>
<tr>
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<td>9.00</td>
<td>9.05</td>
<td>89.7</td>
</tr>
<tr>
<td>21</td>
<td>9.10</td>
<td>9.12</td>
<td>89.8</td>
</tr>
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</table>

Coefficient of Restitution 0.6

<table>
<thead>
<tr>
<th>Bounce No.</th>
<th>Wall Contact Time (ms.)</th>
<th>Time at Min. Petal Angle (ms.)</th>
<th>Minimum Petal Angle (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.319</td>
<td>0.596</td>
<td>51.2</td>
</tr>
<tr>
<td>2</td>
<td>0.896</td>
<td>1.13</td>
<td>73.8</td>
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<td>1.39</td>
<td>1.54</td>
<td>84.8</td>
</tr>
<tr>
<td>4</td>
<td>1.74</td>
<td>1.82</td>
<td>88.7</td>
</tr>
<tr>
<td>5</td>
<td>1.93</td>
<td>1.97</td>
<td>89.7</td>
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Coefficient of Restitution 0.2

<table>
<thead>
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<th>Bounce No.</th>
<th>Wall Contact Time (ms.)</th>
<th>Time at Min. Petal Angle (ms.)</th>
<th>Minimum Petal Angle (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.319</td>
<td>0.499</td>
<td>82.4</td>
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<tr>
<td>2</td>
<td>0.718</td>
<td>0.754</td>
<td>89.8</td>
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Table A.5.3

Aluminium Diaphragms (2 in Square)

<table>
<thead>
<tr>
<th>Coefficient of Restitution</th>
<th>1.0</th>
<th>0.6</th>
<th>0.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bounce No.</td>
<td>Wall Contact Time (ms.)</td>
<td>Time at Min. Petal Angle (ms.)</td>
<td>Minimum Petal Angle (degrees)</td>
</tr>
<tr>
<td>1</td>
<td>0.636</td>
<td>0.902</td>
<td>69.3</td>
</tr>
<tr>
<td>(Motion arrested on 1st bounce).</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.636</td>
<td>0.825</td>
<td>81.4</td>
</tr>
<tr>
<td>(Motion arrested on 1st bounce).</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.636</td>
<td>0.709</td>
<td>88.9</td>
</tr>
<tr>
<td>(Motion arrested on 1st bounce).</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX 6

A.6.1 Design and Construction of the 2 in Square Shock-Tube

The experiments described in Chapters 4, 5 and 10 were performed in a 2 in square-section shock-tube, designed as part of the present project. The tube has an overall length of 30 ft and is composed of individual sections 2 ft 6 in long which are bolted together by means of end-flanges.

The typical cross-section is shown in Fig. A.6.1 and is basically similar to that of an existing wave-interaction tube (1.16).

However, a thin-walled liner, in the form of a 16 S.W.G. stainless-steel extrusion is included in the present tube. A filler of cold-setting expoxy-resin glue is used between the liner and the cladding walls which are of hot-rolled black mild-steel plate. The liner extends beyond the cladding walls and is sealed into the end-flanges by the expoxy-resin glue.

Bolted construction is used throughout and adjacent tube sections are sealed at the end-flanges with standard neoprene rubber '0' rings having a safe working pressure of 100 bar.

Each length of tube is mounted on two saddles, one bolted to each end-flange. The saddles rest on short lengths of rail carried on a rigid steel table which supports the tube at working height. The saddles are independently adjustable for height at their outer ends and allow each tube section to be moved vertically, and also rotated about three axes for alignment purposes.
A.6.2 Tube Assembly

The individual tube-sections were aligned along the axis of the mounting table with the aid of a parallel light-beam from a concave mirror. Target plates having four machined slits 0.020 in wide were clamped to the tube ends; the slits were arranged parallel and adjacent to the tube walls and the mounting saddles were adjusted until even illumination was observed at each slit, thus ensuring that the tube axes were aligned straight to within 0.02 in in 30 ins.

Mismatch of the tube cross-sections at the ends, due to manufacturing errors, was measured to within 0.0005 in by means of a dial-gauge for which a special extension was made. The greatest step in the wall at any junction was less than 0.003 in. Rotational misalignment between adjacent tubes in the longitudinal sense was virtually eliminated by the use of the dial-gauge extension mechanism, traversed along the small step between adjacent walls; this allowed the step size to be set to a constant value, using the saddle adjustment.

The above was subject to the limit imposed by lack of squareness of the tube cross-section (0.0005 in on the 2 in face width).
APPENDIX 7

Vacuum Measurement

The vacua used in the shock-tube runs at the highest Mach-numbers attained in the present tests were measured by means of a commercially available Pirani gauge calibrated against a McLeod gauge. Difficulty was experienced in obtaining consistency between the readings of the two different gauges at vacua in the region of 1.0 torr.

For the purposes of comparing experimental shock strengths with the results of corresponding calculations, e.g. in order to determine shock Mach-numbers to within 1% at $M_a = 5.0$ in air, it is necessary to know $P_{hi}$ to within 6%. At the 1.0 torr level the resolution limit of the Pirani gauge used in the present work was estimated to be approximately 10%.

A simple alternative method was used to pre-determine vacuum levels down to 1.0 torr, within $\pm 1.0\%$.

The tube was first exhausted to its ultimate vacuum (30 micron Hg $\pm 10\%$) and the contents of a small auxiliary flask of known volume was released into the driven section. The flask volume was fixed so as to give a driven-section pressure of 1 torr when its contents of dry gas at atmospheric pressure are decanted into the tube. The ratio of flask-to-tube volumes was determined to better than 1% by pressurising the flask with a small air compressor to a known pressure measured on a mercury manometer equipped with a travelling microscope; the pressure change, on releasing the compressed air into the shock tube was measured on an inclined-limb water manometer.
The ratio of the two volumes was 1:790 ± 0.8, and the uncertainty of ± 3 micron Hg in the initial ultimate vacuum became only 0.3% of the initial pressure $p_1$ in the driven tube, even at its lowest value of 1 torr.

In actual operation, the leak rate of this system including outgassing following a pump-down time of about 30 min was found to be 15 micron Hg per min. This was allowed for in the final estimation of driven-tube pressure by measuring the time interval between the dumping of the contents of the flask, and the bursting of the diaphragm, a typical value being 20s.
A.8.1 Summary of Principal Statements:-

Statements

4-9 Declaration of all parameters used in programme.

9-14 Procedure to compute pressure ratio across isentropic expansion given the particle velocities ahead of and behind the wave, and γ for the gas.

14-24 Procedure to compute pressure ratio across a shock given the particle velocities ahead of and behind the wave and γ for the gas.

24-73 Procedure to determine by repeated bisection the root of the equation \( F_1(x) = F_2(x) \) in the vicinity of the point \( x = x_0 \).

73-95 Procedures to determine the ratios of various flow properties across a normal shock.

97-105 Procedure to determine the arc-cosine of a parameter \( x \).

105-114 Procedure to determine the effective cross-section area of the diaphragm region according to either Kireyev's plane one-dimensional flow theory or the present radial-flow analysis.

114-146 Procedure to determine the mean cross-sectional flow area of the diaphragm region during the time interval TL to TU.

146-173 Procedure to determine the static pressure discrepancy between the expanded driver gas and the shocked gas ahead of it at a given step in the pseudo-shock flow enlargement analysis (Chapter 9).

173-216 Procedure to compute initial-value conditions for the diaphragm region flow on the basis of the pseudo-shock enlargement analysis.

216-243 Procedure to determine the root of a function of \( x \) in the neighbourhood of \( x = x_1 \) using the Newton-Raphson technique.
Procedure to eliminate excessively weak waves or contact surfaces from the computation scheme.

Procedure to identify and solve any of the wave/contact-surface interactions described in Appendix 1.

Procedure to locate the co-ordinates \(x_3t_3\) of the point of intersection of two lines with slopes \(s_1\) and \(s_2\) passing respectively through points \((x_1t_1)\) and \((x_2t_2)\).

Reading-in statements for data values of all appropriate parameters.

Print statements to identify gas-combination.

Calculation of effective area of flow cross section at diaphragm exit and of corresponding flow variables both of diaphragm exit and at tail of unsteady expansion.

Calculation of initial values of flow properties at the earliest stages of diaphragm opening using the pseudo-shock approach.

Calculation of initial values of flow properties at the later stages of diaphragm opening using the steady isentropic-enlargement model.

Print statement to output all appropriate initial-value flow properties.

Identification of earliest possible coalescence arising from a given set of P-waves and contact surfaces in the field between an adjacent Q-wave pair.

Up-dating process, necessary when coalescence does not occur at the P-wave head.

Additional steps necessary when coalescence involves an initial-value field.

Computation of flow properties in fields generated by coalescence.
839-845 Calculation of co-ordinates of collision between a Q-wave and either a P-wave or contact surface.
846-876 Revision of data necessary if collision predicted to occur upstream of diaphragm exit.
877-899 Process necessary in the event of a collision between two adjacent Q-waves.
899-927 Computation of flow properties in field generated by Q-wave coalescence.
928-941 Process to detect the invalidation of an existing P-wave coalescence by one occurring later in time.
942-959 Computation of flow properties in fields generated by collision between Q-wave and P-wave or contact surface.
960-969 Termination of computations of a given strip between adjacent Q-waves on arrival at upper time boundary.
970-978 Replacement of any adjacent pairs of contact surfaces lying close together in the same gas, by a single interface.
979-997 Enlargement of tolerance value for weak waves and contact surfaces if computations fail to converge to a fully formed shock within the assigned core-store limits.
998-1008 Restart of computations on next strip if incomplete.
1009-1048 Elimination of any invalidated Q-wave coalescences and computation of total number of fields associated with each given Q-wave pair.
1049-1114 Output statements.
1114-1129 Restart calculations if further cases to be evaluated.

A.8.2 Programme Listing.
.Compiler by XALT MK

...
```
JADT1 = (TU - TL) / MM
Tj = TL + DT; H1 = 1
JB: Tj = T + DT;
A(i) := FLAR(T, THETA0, PHI, PSI, A4, KIR)
IF IN < S THEN
BEGIN N1 = N + 1 ; GOTO JB ;
END
IF TU > T8 && 8 THEN
BEGIN N1 = N + 10 ; BJ = I ; I := 0 , 0 ;
GOTO JA ;
END
AREA = 1 / (TU - TL) ;
END;
PROCEDURE ERRUR (MG, P3, U3, A3, K4, K1, RTO, U1, A1, P1, P6, A6, DELTA);
BLOCK 15
VALUE MG, U3, A3, K4, K1, RTO, U1, A1, P1, P3;
REAL MG, U3, A3, K4, K1, RTO, U1, A1, P1, P6, A6, DELTA, P3;
BEGIN REAL PS1, C4, AQ3, P3, L, M, H;
BE4, MD4, P64, P14, A64;
C4 := 2 / (K4 - 1) ; BE4 := (K4 - 1) * 0.5 / K4;
AQ3 := (C4 + U3 / A3) / (C4 + MD4) ; PQ3 := AQ3 * (1 / BE4) ;
IF ABS (RTO - 1) < 0.2 THEN
BEGIN
B0Q1 := MQ;
P641 := 1, 0;
A641 := 1, 0;
P61 := P64 * PQ3 * P31;
GOTO JJ;
END
L1 := (K4 + K4 + RTO / MQ) / (K4 + 1) ;
M1 := (2 * (K4 + 1) * MQ + MQ) / (K4 + 1) ;
MB1 := L * SORC (L * M); 
P601 := Max2 (1 + MQ + MQ + MQ) / C4) / (RTO + MQ);
P61 := P60 * P3 * P31 ; AQ3 := SORC (P60 * MQ * RTO / MQ) ;
The1 := (MQ + AQ3 + A3 * U3) / A1;
P51 := (K1 + K1 (K1 + 1) * H + H / 4 + K1 * 
SORT (1 + (K1 + 1) * H + H / 4 + K1 * 
SORT (c(K1 + 1) * H + H / 4 + H / 2 + H * H) * P1) ;
DELTA := P61 * P51 ;
P01 := U0, 5 * (P01 + P51) ; AD1 := A6Q + AQ3 * A3;
END;
PROCEDURE ENL (P1, U1, A1, S1, K1, RTO, 
```
174  P3, U5, A3, K6,
174  P5, U5, A5, S5, K5,
174  P6, U6, A6, S6, KO, LAB;
175  VALUE P1, U1, A1, K1, RTU,
175  P3, U5, A3, K6,
176  REAL P1, U1, A1, S1, K1, RTO,
176  P3, U5, A3, K6,
176  REAL P3, U5, A3, K6,
176  P5, U5, A5, S5, K5,
176  P6, U6, A6, S6, K6;
177  LABEL LAB;
178  BEGIN REAL R4, M5, AL1, BE1, DELTA;
179  INTEGER IO;
180  REAL ARRAY I, ER[1:3];
181  AL1 = (K1 + 1) / (K1 + 1); BE1 = (K2 + 1) + 0.5 / K11
182  M5 : = U3 / A3;
184  JD11: "DO 1" 4011
185  FOR T = 1, 2, 3 DO
186  BEGIN I0; I0 = 0.0; ERROR (MR, P3, U3, A3, K4, I1,
187  RT0, U1, A1, P1, P6, A6, DELTA);
188  END;
189  IF I[3] > 0.5 THEN
191  END;
192  IF ER[1] + ER[3] > 0.0 THEN
194  END;
195  IF ER[1] + ER[2] > 0.0 THEN
196  BEGIN I[1] = I[2];
197  END;
200  P5 = P6 / S5;
201  U5 = U6 + A1 * P(K1, P6 / P1);
202  A5 = A1 * P(AL1, P5 / P1);
203  K5 = K1 * K6 = K4 * S1 = UI + A1 * MACH(AL1, BE1, P5 / P1);
204  S6 = IF ABS(RTO = 1) >= 2 THEN 0 ELSE U6 = AD;
205  END;
206  PROCEDURE INER HA(F, F, X, X1, DELTA);
207  BLOCK 17
208  VALUE X1, DELTA;
BEGIN "COMMENTS NO CONT SURF OR P WAVE"
S41=0; Y1=1; K51=K1;
P51:=P1; U51:=U1; A51=A1
GOTO 'EXIT'

PROCEDURE INT(U1, A1, P1, S1, K1,
BLOCK 19

U2, A2, P2, S2, K2;
U3, A3, P3, S3, K3;
U4, A4, P4, S4, K4;
U5, A5, P5, S5, K5;
U6, A6, P6, S6, K6;
VALUE
U1, A1, P1, S1, K1;
U2, A2, P2, S2, K2;
REAL
U1, A1, P1, S1, K1;
U2, A2, P2, EA, K2, EP, S2;
U3, A3, P3, S3, K3;
U4, A4, P4, S4, K4;
U5, A5, P5, S5, K5;
U6, A6, P6, S6, K6;

INTEGER D, Y;
BEGIN "REAL"
P21, P32, P63, A1, A2, R1, BE1, BE2, P51, ERR;
P, P0, P0, P12, S12, B1, B12, E, S, H, DELTA;
L1, L2, L3, A, K, 12, R12;
INTEGER D, SH12, EX12, CS12, SH23, EX23, FIRST;
P21 = P21/ P1; P32 = P32/ P2; P51 = P51/ P2;
AL1 = (K1+1)/ (K1-1); AL2 = (K2+1)/ (K2-1);
BE1 = (K1+1)/ (K2+1); BE2 = (K2+1)/ (K2* K2);
RO1 = SQRT(1+1); R21 = SQRT(1+2);
E1 = PD(U1, P21); F1 = P21+BE1*PA(AL1, P32);
G1 = PA(AL1, P21); H1 = PB(AL1, P21)+ PD(BE1, P32);
D:= IF U1=1 THEN 1 ELSE 1;
FIRST:= 1; ERR:= &<=3;
L1 = PB(AL1, P32)/ RB1; L2 = PB(AL2, P32)/ RB2;
SH12:= IF (B1=1 OR 1=3) AND ((S2=1) OR 0) AND (ABS(S1-U1)>=6))
' OR (B=2 AND (P1=2) GETL AND 1;ABS(S1-U2)><K=6)
THEN 1 ELSE 0;
EX12:= IF (C=1 OR B=3) AND ((P1>P2) AND (ABS(S1-U1)><K=6))
'TOR (B=2 AND (P2>P1) OR AND (ABS(S1-U2)><K=6)


(E+F*P21*TBE1*PA(AL1,PP)*PB(AL1,P32)/
PB(AL1,PP)) LE1D THEN
'GOTO'RE IF'B'=1 THEN ETS' ELSE'STER'1
'IF'(E+F=PO(BE1,PP)) LE1D AND
(C+F*(P21*TBE1=1/P32+TBE1)*)LE1D THEN
'GOTO' STER'
'E N D'1
'IF'C'S12=1 AND (B=1 OR B=2) THEN
'BEG I N' COMMENT CONTACT SURFAC E INTERACTIONS1
C = PA(AL2,P32)
RE12 = SRT(K2*K2=1/(K1*K1=1))*/A1/A2
IF'SH23=1 AND' (RE12=SRT((AL1*
P32+1)/(AL2*P32+1))*GE0) THEN'
'GOTO' STER'
'IF'B'=1 THEN' STER'ELSE' ETS'R;1
'IF'SH23=1 AND' (RE12=SRT((AL1*
P32+1)/(AL2*P32+1))*LE0) THEN'
'GOTO' STER'
'IF'E X23=1 AND' (RB2=RE12 RB1=1
(1=P32+TBE2)/(1=P32+TBE1))*LE0) THEN'
'GOTO' STER'
'IF'E X23=1 AND' (RB2=RE12 RB1=1
(1=P32+TBE2)/(1=P32+TBE1))*GE0) THEN'
'GOTO' STER'
'IF'B'=1 THEN' ETS'R ELSE' STER';
'E N D'1
'IF'E X23=1 AND' SH12=1 AND' B=2 THEN'
'GOTO' STER';
'IF'SH23=1 AND' EX12=1 AND' B=2 THEN'
'GOTO' ETS'R;
'IF'SH12=1 AND' SH23=1 AND' B=2 THEN'
'GOTO' ETS'R;
'IF'E X12=1 AND' EX23=1 THEN' GOTO' STER';
ST E1; FIRST1=0;
STER;
'IF'B'=2 OR 'B'=3 THEN'
'BEG I N' SOLVE SHUCKP(A3,U6,U3,K3); P1/P3=EXP(A1,U6,U1,K1),
U6; U3; ERK; FINISH';
S61=U51=U6; K51=K1; K01=K2;
P51=P61=0.5*(P5*SHUCKP(A3,U6,U3,K3)+P1*EXP(A1,U6,U1,K1)=1));
'IF'FIRST1=1 THEN' BEGIN'
'IF'P6>P3+AND'P>P1 THEN' GOTO' STER1;
'IF'P6<P3+AND'P1>P5 THEN' GOTO' STER1;
IF P6 < P3 AND P7 > P1 THEN GOTO IETS1
END1;
A5 = A1 * (P5 / P1) + IES1;
A6 = A3 * PB(AL2, P0 / P3);
S5 = U, 5 * (U1 * U5 - A1 - A2);
S3 = U3, A5 * MACH(AL2, BE2, P6 / P3);
GOTO IEX;
END1;

SOLVE (SHOCKP(A1, U6, U1, K1), P3 / P1 * EXP(A3, U6, U3, K3, = D), U6, U1, ERR, FINISH)
U3 = U6, K3 = K3, K4 = K2;
P5 = P6 = U, 5 * (P3 * SHOCKP(A3, U6, U3, K3) - P1 * SHOCKP(A1, U6, U1, K1))
IF FIRST = 1 THEN BEGIN
IF P5 < P1 AND P0 < P3 THEN GOTO IETS1;
IF P0 < P3 AND P7 > P1 THEN GOTO IETS1;
IF P5 < P1 AND P0 < P3 THEN GOTO IETS1;
END1;

A5 = A1 * PB(AL1, P0 / P1);
A6 = A3 * PB(AL2, P0 / P3);
P4 = P1; U4 = U1, A4 = A1; K4 = K1;
S4 = U4; A4 * MACH(AL1, BE1, P5 / P4);
S5 = U5;
S6 = U5 * (U6 * U3 * A6 - A3);
GOTO IEX;
STS1: FIRSTt = 0;

IF B = 1 THEN
BEGIN
SOLVE (SHOCKP(A1, U6, U1, K1), P3 / P1 * SHOCKP(A3, U6, U3, K3),
U6, U3, ERR, FINISH);
P5 = P6 = U, 5 * (P3 * SHOCKP(A3, U6, U3, K3) - P1 * SHOCKP(A1, U6, U1, K1))
IF FIRST = 1 THEN BEGIN
IF P5 < P1 AND P0 < P3 THEN GOTO IETS1;
IF P0 < P3 AND P7 > P1 THEN GOTO IETS1;
IF P5 < P1 AND P0 < P3 THEN GOTO IETS1;
END1;

TERM;
\begin{verbatim}
390  U5 := U6; K5 := K1; K0 := K2;
393  A5 := A1*P3(AL1, P3/P1);
394  A6 := A3*P3(AL2, P6/P3);
395  IF 'B = 2' THEN;
396  BEGIN;
397  S3 := U3 + A3*MACH(AL2, BE2, P6/P3);
398  S3 := U1 - A1*MACH(AL1, BE1, P5/P1);  
399  GOTO[I]JEX;
400  END;
401  S4 := U1 + A1*MACH(AL1, BE1, P5/P1);
402  S5 := U5;
403  S6 := U3 - A3*MACH(AL2, BE2, P6/P3);
404  P4 := P1; U4 := U1; A4 := A1; K4 := K1;
407  GOTO[I]JEX;
408  FIRST := 0;
409  ETER = SULVE(EXP(A1, U0, U1, K1, D), P3/P1*EXP(A3, U6, U3, K2, D));
410  U6 := U3; ERR = FINISH;
413  P5 := P6 := U.5*(P5*EXP(A3, U6, U3, K2, D)*P1*EXP(A1, U6, U1, K1, D));
414  IF FIRST = 1 AND;
415  P6 = P3*AL1; P5 = P1*EXP(A3, U6, U3, K2, D)*P1*EXP(A1, U6, U1, K1, D));
416  A5 := A1*(P5/P1)*THEE1;
417  A6 := A3*(P6/P3)*THEE2;
417  IF 'B = 2' THEN;
418  BEGIN;
419  S6 := U6;
420  IF FIRST = 1 THEN;
421  IF P6 > P3 AND P5 < P1 THEN;
422  GOTO[I]STE1;
423  END;
424  S3 := (U6 + U3 + A6 + A3) * 0.5;
425  GOTO[I]JEX;
426  END;
427  IF FIRST = 1 THEN;
428  IF P6 > P5 AND P5 > P1 THEN;
429  GOTO[I]ETS1;
430  END;
431  S4 := (U1 + U5 + A1 + A5) * 0.5;
432  S5 := U5;
433  S6 := (U3 + U6 - A3 + A0) * 0.5;
434  P4 := P1; U4 := U1; A4 := A1; K4 := K1;
438  GOTO[I]JEX;
438  ETS1; FIRST := 0;
440  ETSR;
\end{verbatim}
```
440 'IF' B=2'OR' B=3' THEN'
440 ' BEGIN' SOLVE (EXP(A1, U6, U3, K2, D), P1/P3=SHOCKP(A1, U1, U6, K1),
441 U6, U3, ERR, FINISH);
442 S41 = U51 = U61 = K51 = K61 = K21
443 P01 = P61 = U5 * (P3*EXP(A1, U6, U3, K2, D) + P1*SHOCKP(A1, U1, U6, K1));
444 ' IF' FIRST1' THEN' ' BEGIN' 
446 ' IF' P6>P3 AND' P0>P1' THEN' ' GOTO1'ST11
448 ' IF' P6<P3 AND' P0<P1' THEN' ' GOTO1'EST11
449 ' IF' P6>P3 AND' P0<P1' THEN' ' GOTO1'STE11
450 ' END1';
451 A51 = A1*P(e A1, P2/P3); 
452 A61 = A3*P6/P3*THEP;
453 S11 = U11 = A1*MACh (AL1, BE1, P5/P1);
454 S31 = U5 * (U3*U6*+A3+A0);
455 'GOTO1'JEX
456 ' END1';
457 SOLVE (EXP(A1, U6, U1, K1, D), P3/P1*SHOCKP(A3, U3, U6, K2);
458 U6, U3, ERR, FINISH);
459 S51 = U61 = K11 = K21
460 P51 = P61 = U5 * (P1*EXP(A1, U6, U1, K1, D) + P3*SHOCKP(A3, U3, U6, K2));
461 ' IF' FIRST1' THEN' ' BEGIN' 
463 ' IF' P5>P3 AND' P0>P3' THEN' ' GOTO1'ST11
465 ' IF' P5<P3 AND' P0<P3' THEN' ' GOTO1'EST11
467 ' IF' P5>P3 AND' P0<P3' THEN' ' GOTO1'STE11
468 ' END1';
469 A51 = A1* (P5/P1) THEP;
470 A61 = A3*PB (AL2, P0/P3);
471 P41 = P1*U41 = U11 A41 = A11 K41 = K11
472 S41 = U5 * (U4*U5+A4+A5);
473 S51 = U51
474 S61 = U31 = A3*MACh (AL2, BE2, P6/P3);
475 'GOTO1'JEX
476 JEX;
478 ' IF' B=1' THEN' 
479 ' BEGIN' FILTER (P41, U41, A41, S41, K41;
480 P5, U51, A51, K51
481 P6, U61, A61, K61
482 P5, U5, A5, S4, K5, V, E, EP)
483 ' GOTO1'FIN
484 ' END1';
485 FILTER (P3, U3, A3, S3, K3;
486 P6, U61, A61, K61
487 P5, U51, A51, K51
488 P6, U61, A61, S3, K6, V, E, EP)
```
```
FIN; END;

PORTOCHECK; LOC(X1, T1, S1, X2, T2, S2, X3, T3);

VALUE; X1, T1, S1, X2, T2, S2;

REAL; X1, T1, S1, X2, T2, S2, X3, T3;

BEGIN; IF S1 & S2 THEN;
    BEGIN; T1 = X1 * X T1 * X2 * T2 * S1; GOTO 1;
    END;
    IF S2 & S3 THEN;
    BEGIN; T3 = X1 * X T1 * X2 * T2 * S1; GOTO 1;
    END;
    PRINT; T1 = (X2 = X1 * S1 * T1 + S2 * T2) / (S1 + S2);
    X3 = X1 * S1 * (T3 + 77)

END;

J: END;

COMB; READ; CASE; = 1;
A1A1; K1; READ; A1; READ; K1; READ; K4; READ
C1; = 0.5 * (K1 + 1); C4; = 0.5 * (K4 + 1);
THETA: = READ; PHI; READ; PSI; READ
A41; = READ; LAST; = READ;
TOTAL; = 1;
AL; = (K1 + 1) / (K1 + 1); AC; = (K4 + 1) / (K4 + 1);
BE1; = (K1 + 1) / (K1 + 1); BE4; = (K4 + 1) * 0.5 / K4;
RF; = READ; RF; = READ; LIM; = READ;
STEP; = READ; RX; = READ; NIS; = READ; CARD; = READ
AA6; = A; = NIS; FOR N = 4 STEP 1 UNTIL 5000 DO;
    BEGIN; FOR L = 1 STEP 1 UNTIL N = 0 DO;
        BEGIN; PC[N, L] = IAM[N, L]; IRN[N, L] = 0;
        RP[N, L] = RQ[N, L]; RA[N, L] = 0;
        RS[N, L] = RK[N, L]; RX[N, L] = 0;
        RT[N, L] = RSQ[N, L] = 0;
    END;
    FOR N = 4 STEP 1 UNTIL 5000 DO;
    IF N = 4000 DO;
        BEGIN; IN[N] = IR[N]; IV[N] = IL[N];
        IY[N] = 1; ID[N] = 0;
        RXC[N] = RTC[N] = 0;
    END;
    RT[0, 1] = NSTEP; RX[0, 1] = 0; RK[0, 1] = K4;
    RX[A, 1] = RT[A, 1] = 0; UIMAX[A, 1] = 1;
    RTD = ARCOS((C1; 5707963 + PHI) / (THETA + PHI)) * A1 / PSI;
    JF;
    NEWLINE(7);
    PRINT(A41, 0, 4);
    PRINT(A41, 0, 4);
```
PRINT(LIMA,0,4);   PRINT(LIMP,0,4);  RNDLINEC(T);  
D1=DMAX1=NIS+1;  LTD1=1;  IFD=1 THEN;  
BEGIN RSLLD:D=SR:=1;  GOTO JMK; 
END;  
RSLLD:D=SR:=1/AKEA(KTD[D],RTD[D]=1), PHI,PSI,THETA0,A1,KIR);  
JLS;  
NERA0,5*(K4+1)*(1R*SR)+2/4)=1-0.5*(K4-1)*MR*M;  
(K4-1)*((MR+SR)*T2/AL4)/MR*HR),SR,0.5,&5);JMR;  
RADLD:=(ZT(K4+1)*MR*R/AL4)+0.5*(1.0.5*(K4-1)*MR)*A;  
RP[D]=CRADLJ/A41*(T1/BE4)*P41;  
RP[D]=1/((1+(K4-1)*0.5*MR)*T1/BE4)*P41;  
RSLD:=(RP[D]+P41*THE*A41*RUR[D]=MR*RAR[D]);  
RSLLD:=(RUR[D]+RSLJ);  
IF D>1 THEN;  
BEGIN D1=D-1; GOTO JMK; 
END;  
D:=DMAN: NIS;  
N:=1;  
JPS:Z=N+2; X:=N+1;  
EMR(RE[NJ],RU[LM],R[LM],R[N],R[M],N],RSD[D], 
RPL,D],RADLD],K4, 
RP[D],RU[LM],RU[M],R[M],X],R[M],X], 
RP[I]1,C],RU[M],Z],R[LM],Z],R[M],Z],FINISH);  
PH;=ABS[RP[N],N]-RP[M],X])2/(RP[M],N]+RP[M],X)];  
AU;=ABS[RM],Z]-R[M],X])2/(R[M],Z]*R[M],X);  
AU;=ABS[RU[M],N]+RU[M],X])2/(RU[M],N]+RU[M],X)];  
KJ;=ABS[RM],X]-PK[M],Z)];  
IF p<LI M' AND'AU< LIMA AND'KU<6 THEN'  
BEGIN 'COMMENT'Authority OUTPUT=pT;  
IF D>NIS+1 THEN 'COMMENT'NEXT STEP PS-SHOCK  
BEGIN D1=D-1;'GOTO JPS  
END;  
'COMMENT'NEXT STEP ISENT EXPI INMAX[M]=N1,'GOTO JIE  
END;  
IF P>LI M' AND'AW< LIM AND'KU<6 THEN'  
BEGIN 'COMMENT'Pressure SHOCK ONLY  
RX1[N]=0,01;  
RTLM,N]=RTD[D];  
IF D>NIS+1 THEN'  
BEGIN 'COMMENT'NEXT STEP PS-SHOCK  
D1=D-1; N1=X;
A5

1:52-
WRITE TEXT ("'AC2S') \( \text{M1 (12S)} \) \( \text{N1 (12S)} \) \( \text{P1 (12S)} \) \( \text{U1 (12S)} \)

FOR \( N = U \) STEP 1 UNTIL \( \text{INMAX} = 1000 \)

BEGIN FOR \( I = 1 \) STEP 1 UNTIL \( \text{INMAX} = 1000 \)

PRINT \( \text{R1} [N, 0, 4] \) \( \text{R1} [N, 0, 4] \) \( \text{R1} [N, 0, 4] \)

IF \( N = \text{INMAX} \) THEN

BEGIN WRITE TEXT ("'130S'") \( \text{IPC} [M, N] = 2, 0 \)

GOTO LDV

END

PRINT \( \text{R1} [M, N] = 0, 4 \) \( \text{R1} [M, N] = 0, 4 \) \( \text{R1} [M, N] = 0, 4 \)

END

END

C1 = EXIT \( \text{IP} = J1 = \text{EIP} = 0 \)

I1 = L = 1

N1 = U1 = 1

JST = L = \( \text{INMAX} [\text{M} * 1] \)

X1 = 0, 0; TA = RT1 [N + 1, L - 1]; GOTO JD1;

JEIG = N + 1;

BEGIN G1 = G1 + 1; GOTO CP

END

BEGIN ABS (RT1 [N] = RT1 [G1] < N + 6 OR

RS1 [N] = RS1 [G1] < N + 6)

BEGIN N1 = N1 + 1; GOTO CP

END Luc \( \text{RX} [M, N] = \text{RT} [M, N] = \text{RS} [M, G] = \text{XB} = \text{TB} \)

BEGIN U1 = M + 1; U1 = 0;

IF \( N + 1 \) THEN

BEGIN COMMENT RETRACE XM1; N1 = N; X1 = XB; TA = TB;

LB: A1 = 0; FOR \( V1 = 1 \) STEP 1 UNTIL \( \text{NU} = 1001 \)

IF \( \text{RS} [M, V] < 6 \) THEN \( \text{A1} = A1 + 1 \) \( \text{U1} = M + 1, 10 \)

BEGIN \( \text{PC1} [U1] = \text{NU} = A1 \)

KA: Z1 = N1; X1 = U1; X1 = U1; SQ = \( \text{RU} [M, N] = \text{RA} [M, N] \)

L1: IF \( \text{RARS} (\text{RS} [M, Z]) < N + 6 \) THEN

BEGIN Z1 = Z1 + 1; A1 = A1 + 1

IF \( Z1 = 0 \) THEN GOTO JM1

GOTO 91

END

END

END


RX [U, W] = RX [M, N]; IPC [U, W] = 0;


```
815 ' OR (ABS(RU[M,W]=RULZJ) > LIM* (ABS(RU[M,W]) + ABS(RU[U,Z])) * 0,5)
816 ' THEN IPC(U,Z)+1
817 ' N:G+1;L:1+Y;
818 ' IAX(U,LJ)=MIRNU[L,J]:N;
819 ' IF L'GE'NN=2 THEN 'GOTO OVN
820 'END;
821 'GOTO JPS
822 'END;
823 'IF (INAX [M]>2 THEN
824 'BEGIN N=0; XA=XB; TA=TSF;
825 'EXIT: '1 'GOTO DUL;
826 'END;
827 'IF (INAX [M]) < 8 THEN
828 'BEGIN N=N+1; RSQ[N,N]:RSQ[N,N+1];
829 'INAX [N]:N; GOTO 'NEW STRIP;
830 'END;
831 'JZ:
832 'IF (X N S < C A N THEN
833 'BEGIN 'CONDIF I NTEP % WITH %P % DATA;
835 'END;
836 'FA:
837 'IF (F D [F] < T THEN
838 'BEGIN 'DIF = D-1 'GOTO 'FA;
839 'END;
840 'INAX [M]=N;PC[I, [N]]:0;
841 'IF 'U = NIS THEN
842 'BEGIN 'I=0; A=LI; M=H+1; X=A+1; Y:=A+2; I:=I+1 'GOTO 'EP;
843 'END;
844 'IF 'D > NIS THEN
845 'BEGIN 'M+I=N+1;
846 'GOTO 'JPS;
847 'END;
848 'WRITE TEXT ("('SC') 'INTERFERES WITH INITIAL DATA")
849 'IF 'DMAX > ? THEN
850 'BEGIN 'WRITE TEXT ("('SC') 'LARGE DMAX2") 'GOTO 'FINISH;
851 'END;
852 'KIS:=NIS+1 'GOTO 'AA;
853 'END;
855 'AND 'X B < RX[U,H] THEN;
856 'BEGIN 'COMMENT: 'WAVE COALESCENCE; C:=C+1;
```
IF N = PC1(M) THEN
BEGIN V := 1; U := N1.
J Vij
IF ABS( (RX, M, N) / (RT, M, N1) ) = RS(W, V) THEN
BEGIN V := V + 1; GOTO J V i j.
END.
COMMENT' V=LINE ALIGNS WITH M, N
I := I + 1;
C := C + 1;
U := M + 1; N := V.
IAME +1, L) := M1N[M+1, L] := N;
GOTO J B 1.
TEST: 'IF' (XH- X) / (TB- TA) > (X H- RX(M, N+1) / (TB- RT(M, N+1)) THEN
BEGIN R := N1; GOTO TEST;
END.
S1 := RX(M, N) / RT(M, N); S1 := RX(M, N+1) / RT(M, N+1); S1, XA, TA, RSQ(M, N), PCR(M, C), RTC(C); A := IAM[M+1, N] + IR := IPCC[M+1, N]; V := K (I) + L + N; X := L + 2.
INT(K1[A1, R1, RA1, R1, PR1, X], S1, RX[M, N], RX[M, N+1], XA, TA, RSQ[M, N], PCR[M, C], RTC(C)); R := RX[M, N] / RT[M, N]; R := RX[M, N+1] / RT[M, N+1]; IF (ABS(RP[M, N] - RP[M, N+1]) > 0.5) THEN CPC[M, N] := 1.
1037 'BEGIN' 'IF' 'N' '=' 'X' 'THEN' 'GOTO' 'AAI';
1039 'END';
1040 'IF' 'B' '<' 'CMAX' 'THEN' 'GOTO' 'AAH';
1041 'INMAX' '(' 'X' ')' '=' '0';
1042 'AAH': 'IF' 'X' '=' '0' 'THEN' ';
1043 'BEGIN' 'X' '=' 'X' '=' '1'; 'B' '=' '0'; 'GOTO' 'AAH';
1044 'END';
1046 'END';
1047 'END';
1048 'IF' 'C' '<' 'CMAX' 'THEN' 'GOTO' 'JCl';
1049 'FIN';
1050 'WRITE' '(' '(' 'A' 'C' 'Z' ')' ',' '(' 'A' 'S' ')', '
1051 'FROM' 'P' '(' 'M', 'N' ')', '
1052 'IF' 'P' '(' 'M', 'N' ')' '<' '8' '-' '6' 'THEN' 'GOTO' 'FIT';
1053 'FOR' 'N' '=' '1' 'STEP' '1' 'UNTIL' 'INMAX' '(' 'M' ')'; 'DO'
1054 'BEGIN' 'PRINT' '(' '(' '2', '0' ')', 'PRINT' '(' '(' 'N', '2', '0' ')', '
1055 'PRINT' '(' 'P' '(' 'M', 'N' ')', '0', '4' ')', 'PRINT' '(' 'R' '(' 'M', 'N' ')', '0', '4' ')', '
1056 'PRINT' '(' 'C' '(' 'M', 'N' ')', '0', '4' ')', 'PRINT' '(' 'R' '(' 'M', 'N' ')', '0', '4' ')', '
1057 'PRINT' '(' 'T' '(' 'M', 'N' ')', '0', '4' ')', 'PRINT' '(' 'T' '(' 'M', 'N' ')', '0', '4' ')', 'NEWLINE' '(' '1' ')', 'INMAX' '=' 'M';
1058 'END';
1059 'IF' 'N' '=' 'M' 'THEN'
1060 'BEGIN'
1061 'N' '=' 'N' '+' '1'; 'GOTO' 'REN';
1062 'END';
1064 'FIT';
1065 'IF' 'CARD' 'D' '=' '1' 'THEN'
1066 'BEGIN' 'SELECT' 'OUTPUT' '(' '3' ')';
1067 'BEGIN' 'PRINT' '(' 'M' 'MAX', '2', '0' ')', 'PRINT' '(' 'C' 'MAX', '2', '0' ')', 'PRINT' '(' 'C' 'MAX', '2', '0' ')', '
1068 'NEWLINE' '(' '1' ')', 'FOR' 'N' '=' '1' 'STEP' '1' 'UNTIL' 'INMAX' '(' 'M' ')'; 'DO'
1069 'BEGIN' 'PRINT' '(' 'INMAX' '(' 'M' ')', '2', '0' ')', 'NEWLINE' '(' '1' ')', 'FOR' 'N' '=' '1' 'STEP' '1' 'UNTIL' 'INMAX' '(' 'M' ')'; 'DO'
1070 'BEGIN' 'PRINT' '(' 'P' '(' 'M', 'N' ')', '0', '8' ')', 'PRINT' '(' 'R' '(' 'M', 'N' ')', '0', '8' ')', 'PRINT' '(' 'R' '(' 'M', 'N' ')', '0', '8' ')', 'PRINT' '(' 'I' '(' 'M', 'N' ')', '2', '0' ')', 'PRINT' '(' 'I' '(' 'M', 'N' ')', '2', '0' ')', 'PRINT' '(' 'I' '(' 'M', 'N' ')', '2', '0' ')', 'NEWLINE' '(' '1' ')', 'END';
1072 'END';
1073 'END';
1074 'END';
1075 'END';
1076 'END';
1077 'END';
1078 'END';
1079 'END';
1080 'END';
1081 'END';
1082 'END';
1083 'END';
1084 'END';
I BEGIN PRINT(IL[N],2,0); PRINT(IR[N],2,0); PRINT(IR[N],2,0); PRINT(IR[N],2,0); PRINT(IR[N],2,0); NEWLINE(1); TEND; FOR IR[N]=7 STEP 1 UNTIL 1 MAX DO;
BEGIN PRINT(RA[N],0,8); PRINT(RA[N],0,8); PRINT(RA[N],0,8); NEWLINE(1); TEND;
PRINT(RP[0,1],0,4); SELECT OUTPUT(0);
FINISH;
IF TOTAL<LAST THEN;
BEGIN;
FOR M=0 STEP 1 UNTIL 1 NN DO;
BEGIN FOR IR[N]=7 STEP 1 UNTIL 1 NN DO; RP[M,N]=0;
END;
TOTAL=TOTAL+1; GOTO'RENEW;
END;
IF 'CASE<COMB THEN;
BEGIN CASE=1; GOTO'AAA;
END;
END;
END;
END;
END;
END;
END;
END;
CLOSE;
END;
END;
END;
END;
END;
END;
END;
END;
END;
END;
END;
END;
ENDDO;
LENGTH 17542
NO OF BUCKETS USED 360
SENDTO FILE EXTENDED = SIZE NOW 400 BUCKETS
COMPILED #M31D  EC
A.8.3 Sample of Output

Gas-combination   -  helium/air
Diaphragm Pressure-ratio - $10^6$
Diaphragm Material  -  aluminium

Notation for Output Parameters:

M,N - Identifying integers for array of fields (Section 9.3)
P - Static pressure ratio
U - Gas particle velocity ratio
A - Sonic velocity ratio
S - Slope of P-wave or contact surface
K - Specific heat ratio
X - Distance co-ordinate ($x$)
T - Time co-ordinate ($a_1t$)
A.8.4 Samples of Computer-Drawn Wave Diagrams
HELIUM/AIR
ALUMINIUM DIAPHRAGM
P41 = 10,000
P-AND.Q-WAVES
CONTACT-SURFACES
Fig.A.8.1.
HYDROGEN/ARGON
ALUMINIUM DIAPHRAGM
P41 = 10,000
P- AND Q-WAVES
CONTACT-SURFACES

Fig. A.8.2.
Fig A.8.3

ALUMINIUM DIAPHRAGM

P41 = 10,000

P-AND Q-WAVES

CONTACT SURFACES

UPPER TIME BOUNDARY

TIME (MS)

DISTANCE (FT)
AIR/AIR
COPPER DIAPHRAGM
P41 = 10,000
P-AND Q-WAVES
CONTACT-SURFACES

Fig. A.8.4.
APPENDIX 9

The Influence of Computing Approximations on the Calculated Shock Trajectories

Samples of shock trajectories computed on the basis of the present analysis (Chapter 9) are shown in Figs. 10.1 and 2. The points represent the local values of the Mach-number of the leading shock in the group of coalescing waves at the head of the formation region. The ordinates represent the Mach-numbers expressed as ratios of the corresponding ideal-theory Mach-numbers at the same $P_{4,1}$ value; the abscissae represent distances from the diaphragm. A six-stage model of the diaphragm opening process was used, with values appropriate to an aluminium diaphragm. The initial values of the flow properties at the first stage of opening were determined using the pseudo-shock analysis (section 9.4.4); those of the remaining stages were calculated using the isentropic area-enlargement scheme (section 8.3.2). The gas-combination concerned is air/air with a $P_{4,1}$ value of $10^4$.

As discussed in Section 9.3.4, all P-waves and contact surfaces weaker than a prescribed minimum strength-threshold are discarded from the computations. The influence on the computed shock trajectories of the value chosen for this limit is illustrated in Fig. A.10.1. The value concerned was varied from 2.5% to 10% in successive computation, these values being chosen arbitrarily in the first instance.

At 2.5%, the solution failed to converge to give the final strong shock within the present computer storage limitations. However the results for 5%, 7.5% and 10% limits are shown in Fig. A.10.1 from which the main conclusion is that subject to
being sufficiently large to ensure convergence, the precise value of the limit exerts only a small influence on the final results.

A small discrepancy occurs between the results for a 10% limit and those of the smaller threshold values at distances greater than approximately 50% of the formation-distance although the final shock strengths and formation distances are virtually identical in all three cases.

In view of this small divergence at the largest limit, the procedure adopted in all subsequent computations of this type has been to use the minimum practicable strength threshold, the latter being determined by trial and error for each different gas combination and $P_{\text{m}}$ value.

Another important factor in the present analysis is the stage at which the transition is made from the pseudo-shock to the isentropic area-enlargement model for the calculation of initial values of the flow variables. The influence on the final results of the relative numbers of pseudo-shock and isentropic-enlargement stages was investigated for several gas combinations and $P_{\text{m}}$ values and typical results are shown in Fig. A.10.2. This relates to similar initial gas conditions and diaphragm material to those of the previous example (Fig. A.10.1), the number of pseudo-shock stages ranging in the present investigation from one to three.

The overall conclusion from this exploratory analysis is that the ratio of the number of isentropic stages to that of the pseudo-shock stages influences the theoretical trajectory to a very
limited extent only. The maximum shock Mach-numbers for example, are virtually identical in all cases; the predicted formation distance is fractionally the greatest for the case a single pseudo-shock stage. This model would for example, give a greater margin of safety in the calculation of the minimum required length of the driven-section of a shock-tunnel. It also gives a slightly less abrupt initial increase in $M_s$ which is more in keeping with the measured results and was therefore adopted for all subsequent calculations.

By way of a physical justification for this artifice the argument may be advanced that whilst at very small diaphragm openings, appreciable dissipation of mechanical energy occurs in the "sudden enlargement" region downstream of the diaphragm, this effect must diminish progressively as opening proceeds, the expansion becoming correspondingly closer to an isentropic process.