In Orbit Calibration of Satellite Inertia Matrix and Thruster Coefficients

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To my wife, who supported and encouraged me through the hardship during my PhD study with her considerate care and considerable sacrifice and to my mother, brothers and sisters, who gave me their selfless love and care.
Abstract

In this research study, several new in-orbit algorithms are proposed to improve the performance of Attitude Determination and Control System (ADCS) by estimating the inertia matrix and calibrating the cold gas thruster system of the UoSAT-12 spacecraft. Computer-based simulation models will be constructed using MATLAB and SIMULINK in order to evaluate the expected performance.

The first focus is on the identification of the satellite inertia matrix. A new algorithm based on a Recursive Least Square (RLS) estimation technique is proposed for in-orbit use to estimate the inertia matrix (moments and products of inertia parameters) of a satellite. To facilitate this, one attitude axis is disturbed using a reaction wheel whilst the other two axes are controlled to keep their respective angular rates small. Within a fraction of an orbit three components of the inertia matrix can be accurately determined. This procedure is then repeated for the other two axes to obtain all nine elements of the inertia matrix. The procedure is designed to prevent the build up of momentum in the reaction wheels, whilst keeping the attitude disturbance to the satellite within acceptable limits. It can also overcome potential errors introduced by unmodeled external disturbance torques and attitude sensor noise.

The second focus is on a new algorithm for in-orbit use to calibrate thruster coefficients for thrust level and alignment, using three reaction wheel actuators. These algorithms will ensure robustness against modeling errors. The algorithms assume no prior knowledge of the thruster parameters and only an initial guess of the inertia matrix. It is proposed that this calibration can be used during normal mission conditions when the satellite is stabilised.

The final goal of this research study was to apply the proposed algorithms in real-time. Firstly, the thruster calibration algorithm was tested on an air-bearing table. Finally, both thruster calibration and moment of inertia algorithms were tested using data generated by UoSAT-12 while in orbit. The practical estimation results proved the feasibility of proposed algorithms.
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List of Symbols

Coordinate frames:

$X_B, Y_B, Z_B$  Satellite body coordinate axes
$X_O, Y_O, Z_O$  Orbit-referenced coordinate axes
$X_R, Y_R, Z_R$  Inertial-referenced coordinate axes

Operators:

$c$  Cosine function
$s$  Sine function
$\times$  Cross product of two vectors
$\| \|$  Amplitude of a vector
$E$  Expectation function
$V$  Gradient operator vector
$\| \|$  Absolute value of a scalar
$\frac{d}{dt}$  Derivative function with respect to time
$\hat{\cdot}$  Estimated value

Orbit and space environment:

$e$  Orbit eccentricity
$p_o$  Atmospheric density
$V$  Magnitude of spacecraft’s velocity vector
$\hat{V}$  Unit velocity vector
$A_p$  Total projected area of spacecraft
$c_p$  Vector between centre of mass and centre of pressure
$a$  Semi-major axis
$V_p$  Spacecraft’s velocity at perigee
$\alpha$  Orbit angle as measured from the ascending node
$V_a$  Spacecraft’s velocity at apogee
$d_o$  Solar radiation constant
i  Orbit inclination
i_s  Inclination angle of sun
q  Reflection factor
R  Geocentric position vector length
\vec{R}  Unit geocentric position vector

**Satellite attitude and rate parameters:**

\[ \phi \] Roll angle
\[ \theta \] Pitch angle
\[ \psi \] Yaw angle

\[ A \] Attitude transformation matrix
\[ A_{ij} \] Component of matrix A at row i and column j

\[ q \] Attitude quaternion with respect to orbital coordinates
\[ q_1, q_2, q_3, q_4 \] Attitude quaternion components (orbit-referenced)

\[ \Phi \] Euler rotation angle
\[ e \] Euler axis vector in orbit referenced coordinates
\[ e_{ax}, e_{ay}, e_{az} \] Euler axis components in orbit referenced coordinates

\[ q_e \] Attitude quaternion error
\[ q_{xe}, q_{ye}, q_{ze} \] Attitude quaternion error components

\[ q_{vec} \] The vector formed by the first three components of quaternion error

\[ q_{ref} \] Reference quaternion vector

\[ q_c \] Attitude quaternion command
\[ q_{1c}, q_{2c}, q_{3c}, q_{4c} \] Attitude quaternion command components

\[ \omega^I \] Inertially referenced body angular rate vector
\[ \omega_x, \omega_y, \omega_z \] Inertially referenced body angular rate components

\[ \omega^O \] Orbit referenced angular rate vector
\[ \omega_{ox}, \omega_{oy}, \omega_{oz} \] Orbit referenced angular rate components

\[ \omega_o \] Orbital angular rate vector
\[ \omega_o \] Mean orbital angular rate
\[ \Omega \quad \text{Angular rate matrix in attitude kinematic equation} \]

\[ \Lambda(q) \quad \text{Quaternion-related matrix in attitude kinematic equation} \]

\[ L \quad \text{Total satellite angular momentum vector} \]

\[ \omega^B \quad \text{Body relative angular rate vector in any reference coordinate} \]

\[ \omega_{Rx}, \omega_{Ry}, \omega_{Rz} \quad \text{Body relative angular rate components in any reference coordinate} \]

**Values of satellite moment of inertia (MOI):**

\[ I \quad \text{Moment of inertia tensor} \]

\[ I_{xx}, I_{yy}, I_{zz} \quad \text{Principal MOI components} \]

\[ I_{xy}, I_{xz}, I_{yz} \quad \text{Off-diagonal components of MOI} \]

**Satellite torques:**

\[ N_{ex} \quad \text{Torque acting on a satellite} \]

\[ N_x, N_y, N_z \quad \text{Torque components} \]

\[ N_D \quad \text{External disturbance torque vector} \]

\[ N_{dx}, N_{dy}, N_{dz} \quad \text{External disturbance torque components} \]

\[ N_{GG} \quad \text{Gravity-gradient torque vector} \]

\[ N_{gxx}, N_{gxy}, N_{gzz} \quad \text{Gravity-gradient torque components} \]

\[ N_M \quad \text{Magnetic torque vector} \]

\[ N_{mx}, N_{my}, N_{mz} \quad \text{Magnetic torque components} \]

\[ N_T \quad \text{Thruster torque vector} \]

\[ N_{Tx}, N_{Ty}, N_{Tz} \quad \text{Thruster torque components} \]

\[ N_{Aero} \quad \text{Aerodynamic torque vector} \]

\[ N_{solar} \quad \text{Solar radiation torque vector} \]

\[ N_{solarx}, N_{solary}, N_{solarz} \quad \text{Solar radiation torque components} \]

**Magnetorquer/Magnetometer and magnetic field parameters:**

\[ B \quad \text{Magnetic field vector in the satellite body coordinates} \]

\[ B_x, B_y, B_z \quad \text{Magnetic field components} \]
$B_e$ Geomagnetic field vector in the local orbital coordinates
$B_{ex}, B_{ey}, B_{ez}$ Geomagnetic field components in the local orbital coordinates
$M$ Magnetic control dipole vector of magnetorquers
$M_d$ Vector geomagnetic strength
$\Psi$ Geomagnetic field strength matrix
$M_{coll-max}$ Maximum magnetic dipole moment of magnetometer

**Reaction wheel parameters:**

$h$ Reaction wheel angular momentum vector
$h_x, h_y, h_z$ Reaction wheel momentum components
$N_w$ Reaction wheel torque vector
$N_{lex}, N_{ley}, N_{lez}$ Reaction wheel torque components
$N_{w-max}$ RW maximum torque vector

**Cold gas thruster parameters**

$p_1$ Minimum Pulse of the RCS
$p_2$ Maximum Pulse of the RCS
$e_1$ Minimum attitude dead band
$e_2$ Maximum attitude dead band
$I_{sp}$ Thuster specific impulse
$t_{min}$ Minimum thruster firing time in PWM
RL Thruster rise lag time
RT Thruster rise time
DPW Thruster drive pulse width
RPW Thruster response pulse width
FL Thruster fall lag time
FT Thruster fall time
DPA Thruster drive pulse amplitude
RPA Thruster response pulse amplitude
F Thrust vector
\( F_x, F_y, F_z \) Components of thrust

\( r \) Vector distance of the thruster from the centre of mass

\( r_x, r_y, r_z \) Components of distance of the thruster from the centre of mass

\( \alpha \) Azimuth angle

\( \beta \) Elevation angle

\( \Delta x, \Delta y, \Delta z \) Thruster torque arm

\( T_{Y+}, T_{Y-} \) Positive and negative yaw torques respectively

\( T_{R+}, T_{R-} \) Positive and negative roll torques respectively

\( T_{p+}, T_{p-} \) Positive and negative pitch torques respectively

\( T_{av+}, T_{av-} \) Positive and negative orbit torques respectively

\( T_Y, T_R, T_P \) Yaw, roll and pitch torques respectively

\( \tilde{T}_Y, \tilde{T}_R, \tilde{T}_P \) Normalised yaw, roll and pitch torques respectively

\( \Delta t_i \) Ratio between the thruster on-time and the sampling time for thruster i

**Control system plus estimator parameters:**

\( k_m \) A constant scalar gain for cross-product law

\( \xi \) Damping factor of the second order system

\( t_s \) Settling time of the second order system

\( \omega_n \) Undamped natural frequency of the second order system

\( K_p, K_d \) Control gain matrix for quaternion controller

\( k_p, k_d \) Positive gain scalars for quaternion controller

\( e \) Control error vector

\( e_{band} \) Error hysteresis band

\( K_1, K_2 \) Control gain matrix for Bang-Bang controller

\( k_1, k_2 \) Positive gain scalars for Bang-Bang controller

\( \lambda \) Forgetting factor in RLS algorithm

\( z \) Measurement vector

\( x \) Estimated parameter vector

\( v \) System noise vector

\( e \) Identification error vector
\( \mu \) Step size for LMS algorithm

\( \mathbf{H} \) Information matrix

\( J_{ld} \) Cost function

\( p_{\text{max}} \) Largest eigenvalues

\( \mathbf{P} \) Measurement noise covariance matrix

\( V(\theta,k) \) Cost function in RLS

\( \mathbf{K}(k) \) Updating gain for RLS algorithm

\( \theta \) Parameter vector for RLS calibration

\( \phi \) Regression vector for RLS calibration

\( \mathbf{v}_{\text{meas}} \) Sensor measurement vector in body coordinates

\( \mathbf{v}_{\text{orb}} \) Modelled measurement vector in orbit coordinates

\( \tau \) Time constant

\( \Delta t \) Discrete sampling time

\( \delta \) System bandwidth

\( k_a \) Scalar gain for acceleration estimator

\( I_{AB} \) Moment of inertia of the air-bearing rotation table

\( \omega_{AB} \) Angular speed of the air-bearing rotation table

\( I_w \) Moment of inertia of the reaction wheel around its rotation axis

\( \omega_w \) Angular speed of the reaction wheel

\( N_d \) External disturbance torque including air friction, aerodynamic drag, etc

\( \theta_{AB} \) Measured rotation angle of the air-bearing table

\( \omega_{\text{req}} \) Reference angular speed for the air-bearing table

\( \mathbf{A}_c \) Thruster calibration matrix

\( a_{ij} \) Elements of thruster calibration matrix

\( \mathbf{N}_c \) Calibration torque vector

\( \mathbf{T}_c \) Thruster torque command vector

\( T_p \) Positive thruster torque command

\( T_n \) Negative thruster torque command

\( N_{w-\text{max}} \) Reaction wheel maximum torque vector
<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>ACS</td>
<td>Attitude Control System</td>
</tr>
<tr>
<td>ADCS</td>
<td>Attitude Determination and Control System</td>
</tr>
<tr>
<td>AODCS</td>
<td>Attitude and Orbit Determination and Control System</td>
</tr>
<tr>
<td>CMGs</td>
<td>Control Moment Gyros</td>
</tr>
<tr>
<td>CoG</td>
<td>Centre of Gravity</td>
</tr>
<tr>
<td>CoP</td>
<td>Centre of Pressure</td>
</tr>
<tr>
<td>DCM</td>
<td>Direction Cosine Matrix</td>
</tr>
<tr>
<td>EKF</td>
<td>Extended Kalman Filter</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>IGRF</td>
<td>International Geomagnetic Reference Field</td>
</tr>
<tr>
<td>IMUs</td>
<td>Inertial Measurement Units</td>
</tr>
<tr>
<td>LEO</td>
<td>Low-Earth Orbit</td>
</tr>
<tr>
<td>LMS</td>
<td>Least Mean Square</td>
</tr>
<tr>
<td>LS</td>
<td>Least Square</td>
</tr>
<tr>
<td>LSE</td>
<td>Least Square Error</td>
</tr>
<tr>
<td>MOI</td>
<td>Moment of Inertia</td>
</tr>
<tr>
<td>MT</td>
<td>Magnetorquer</td>
</tr>
<tr>
<td>PD</td>
<td>Proportion and Derivative</td>
</tr>
<tr>
<td>PID</td>
<td>Proportion, Integration and Derivative</td>
</tr>
<tr>
<td>POI</td>
<td>Products of Inertia</td>
</tr>
<tr>
<td>PRBS</td>
<td>Pseudo Random Binary Sequence</td>
</tr>
<tr>
<td>PWM</td>
<td>Pulse-Width-Modulation</td>
</tr>
<tr>
<td>qEKF</td>
<td>Quaternion Extended Kalman Filter</td>
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<tr>
<td>RCS</td>
<td>Reaction Control System</td>
</tr>
<tr>
<td>RW</td>
<td>Reaction Wheel</td>
</tr>
<tr>
<td>RLS</td>
<td>Recursive Least Square</td>
</tr>
<tr>
<td>SSTL</td>
<td>Surrey Satellite Technology Limited</td>
</tr>
<tr>
<td>Thruster i</td>
<td>Thruster i</td>
</tr>
<tr>
<td>WLS</td>
<td>Weighted Least Square</td>
</tr>
<tr>
<td>ZOH</td>
<td>Zero Order Hold Circuit</td>
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Chapter 1

1. Introduction

1.1 Research Background

The motion of a rigid body in space is specified by its position, velocity, attitude and angular rates. The first two quantities are related to the translational motion of the centre of mass of the spacecraft which is called space navigation, (or orbit determination and control). The last two parameters are concerned with the rotational motion of the body of the spacecraft about the centre of mass and is the subject of what is called attitude determination and control.

The attitude of a spacecraft is its orientation in space. Attitude analysis may be divided into determination, prediction, and control. Attitude determination is the process of computing the orientation of the spacecraft relative to either an inertial reference or some object of interest such as the Earth. Computation of the orientation of spacecraft with respect to some reference, requires several types of sensors on the spacecraft and sophisticated data processing procedures. The accuracy limit is usually determined by a combination of processing procedures and spacecraft hardware. Attitude prediction is the process of forecasting the future orientation of the spacecraft by using the dynamic models to extrapolate the attitude history. In this case the limiting features are the knowledge of the applied and environmental torques and the accuracy of the mathematical model of the spacecraft dynamics and hardware. Attitude control is the process of orienting the spacecraft in a specified, predetermined direction. It consists of two areas; attitude stabilisation which is the process of maintaining an existing orientation and attitude manoeuvre control, which is the process of controlling the orientation of the spacecraft from one attitude to another.

Since the external (or environmental) disturbing torques can never be eliminated, some form of attitude determination and control is required for nearly all spacecraft. For
engineering or flight-related functions, attitude determination is required only to provide a reference for control. Attitude control is required to avoid solar or atmospheric damage to sensitive components, to control heat dissipation, to point directional antennas and solar panels (for power generation) and to orient thrusters used for orbit manoeuvres. The attitude requirement for spacecraft payloads is more varied and often more stringent than the engineering requirements. Payload requirements, such as antenna orientation, may involve attitude determination, attitude control, or both. Conventionally, a spacecraft is categorised by the procedure by which it is stabilised such as a spin-stabilised spacecraft or a three-axis stabilised spacecraft.

The purpose of the Attitude Determination and Control Subsystem (ADCS) is to stabilise the spacecraft in a desired attitude despite the external disturbance torques acting on it. Stabilisation and control can be accomplished via multiple techniques. These include gravity-gradient, magnetic, pure-spin, dual-spin, one-axis bias momentum, and three-axis stabilisation. The ADCS itself can be grouped into three distinct sections: (a) attitude sensors, (b) actuators, and (c) control logic/control computers. Attitude sensors come in several varieties, including sun sensors, earth-horizon sensors, star sensors, magnetometers, and inertial measurement units (IMUs). There are also several types of actuators, including reaction wheels, momentum wheels, control-moment gyros (CMGs), electromagnetic torquers, and thrusters. Each stabilisation, sensing, and control technique has its own advantages and disadvantages. The optimum combination of stabilisation and control techniques depend largely on the spacecraft system performance requirements imposed on the ADCS, and to a lesser extent, constraints imposed by other satellite subsystems (including the payload).

1.1.1 Attitude Determination

The goal of attitude determination is to determine the orientation of the spacecraft relative to either an inertial reference frame or a frame referenced to some specific object of interest, such as the Earth. To do this we need the following [Charalambos, 1999]:

- First of all we must have available one or more reference vectors, i.e. unit vectors in known directions relative to the spacecraft (commonly used reference vectors are the
Earth's magnetic field, and the unit vectors in the direction of the Sun, a known Star, or the centre of the Earth).

- Given a reference vector, an attitude sensor measures the orientation of that vector (or some function of the vector) in the frame of reference of the spacecraft body.
- Having done this for two or more vectors, we may compute the orientation of the spacecraft relative to these vectors.

The commonly used reference vectors and control torques are described in Table 1.1

**Table 1.1 Attitude reference sources and control torques**

<table>
<thead>
<tr>
<th>Reference sources</th>
<th>Sun, Magnetic field, Inertial space, Earth, Stars</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control actuators</td>
<td>Gas thrusters, Magnetic coils, Gravity gradient, Reaction/Momentum wheels</td>
</tr>
</tbody>
</table>

The spacecraft attitude can be determined by either deterministic methods or by utilising algorithms that combine kinematic and dynamic models with sensor data. However, all deterministic approaches fail when only one vector measurement is available (e.g., magnetometer data only). Attitude estimation algorithms utilise attitude kinematic and dynamic models of spacecraft, and subsequently can estimate the attitude of a spacecraft using pairs of measurements and reference vectors such as the geomagnetic field vector.

Commonly, to model or predict the time evolution of the attitude, two basic methods are employed:

- Dynamic modelling.
- Gyro modelling.

Dynamic modelling (see Fig. 1.1) consists of integrating both the dynamic and kinematic equations of motion using analytical or numerical models of the torque for gyroless ADCS. Gyro modelling (see Fig. 1.2) consists of using rate sensors or gyroscopes to replace the dynamic model such that only the kinematic equations need to be integrated [Wertz, 1989].
Attitude determination is not the primary focus of this thesis, but the impact of attitude estimation will be taken into consideration by adding sensor noise to the estimated satellite states. In Chapter 3 a brief introduction about the attitude determination of UoSAT-12 using a quaternion based extended Kalman filter will be introduced. The focus of this research is to improve the performance of ADCS by estimating the inertia matrix and to calibrate the cold gas thruster system for the UoSAT-12 spacecraft.
1.1.2 Attitude Control

Attitude determination provides the information needed for attitude control. Attitude control is the process of changing the orientation of spacecraft. It roughly comprises two areas:

- Attitude stabilisation: maintaining an existing orientation,
- Attitude slew manoeuvre: controlling the spacecraft from one attitude to another.

However, the two requirements are not totally distinct. For example, the stabilisation of a satellite with one axis towards the Earth implies a continuous manoeuvre relative to its inertial orientation. The control accuracy typically depends on the actuators and control algorithms.

The limiting factor for attitude control is typically the performance of the actuator hardware and control software. Although with autonomous control systems, it may also be the accuracy of the orbit or attitude information.

An attitude control system is both the process and the hardware by which the attitude is controlled. In general, an attitude control system consists of the following four major components (as shown in Fig. 1.3):

- Attitude sensors
- Control logic
- Attitude actuators
- Vehicle dynamics

An attitude sensor locates known reference targets such as the Sun or the Earth’s centre to determine when control is required, what torques are required, and how to generate them. The attitude actuator is the mechanism that supplies the control torques. Control systems can be classified as either an open loop system in which the control process includes human interactions (e.g. attitude data from the attitude sensors is analysed, and a control analyst occasionally sends command to the spacecraft to activate the control hardware), or a closed loop feedback system in which the control process is entirely electrical or computer controlled (e.g. attitude sensors sends attitude data to an on-board computer which determines the attitude and then activate the control hardware). Further there are
two types of attitude control mechanisms: active attitude control in which continuous
decision making and hardware operation is required (the most common sources of torques
for active control systems are gas jets, electromagnetic coils, and reaction wheels) and
passive attitude control which makes use of environmental torques to maintain the
spacecraft orientation (gravity gradient and permanent magnets are common passive
attitude control methods).

1.1.3 Literature Survey

There has been very little written on the subjects of in-orbit thruster calibration and in-
orbit moment of inertia estimation. In contrast, there is a very rich literature in the fields
of in-orbit attitude determination and in-orbit identification of spacecraft structural modes.

In the field of on-line thruster calibration Prickett and Hoang [1992] addressed in-orbit
thruster calibration applied to estimating fuel usage for the prediction of spacecraft life
expectancy. The accuracy of the satellite life calculation depends on the modelling of the
satellite's reaction control system (RCS). The RCS is a life-limiting factor for many
satellites. To determine the remaining orbital life of the RCS at any given point in the
satellite mission, it is essential to determine how much propellant remains onboard, as well
as how efficiently that propellant will deliver an impulse to the vehicle during projected
future RCS operation. Four important aspects of RCS propulsion modelling especially
relevant to satellite life calculations were evaluated, refined, and tailored to the spin-
stabilised HS 376 series communications satellite. The first aspect is the thruster level specific impulse (the RCS thruster-level efficiency of converting propellant into impulse). Thruster acceptance test data was used in combination with qualification test data to characterise specific impulse over a range of RCS operating conditions. The second aspect, thruster plume-induced RCS performance degradation, was evaluated using source flow plume modelling. The degradation was found to increase from approximately 6% at satellite beginning-of-life to approximately 10% at end-of-life. The final two aspects are alternate methods of determining onboard propellant status, namely, propellant bookeeping and propellant remaining computations using a detailed gas law approach. These two aspects were developed and then evaluated based on flight data generated during the STS recovery of two HS 376 satellites. The bookeeping method exhibited accuracies ranging from 0.4 to 3.8% for the four half-systems evaluated, while the gas law method demonstrated accuracies between 1.6% - 16.4% using single-point telemetry data.

Dodds and Milne [1988] presented a method for automatic in-orbit estimation of a critical parameter of a thruster’s impulse transfer characteristic. This method facilitates optimal control with regard to thruster life-time, fuel consumption and pointing accuracy, despite uncertainty in the thruster performance. A type of parameter estimator is developed which allows a thruster attitude control system to maintain an ideal limit cycle in the face of uncertainties in the thruster characteristics. The estimator facilitates automatic, in-orbit adaptation of the attitude control system to slow changes in the thruster characteristic, thereby maintaining optimal performance. Attention is restricted to a single, isolated axis. The parameters to be determined are the intercepts on the characteristic of each thruster relating the actual thruster firing to that demanded. The differences between the demanded and actual thruster responses will affect the performance of the attitude control system significantly. A state estimator (see Fig. 1.4) embodies a real time model of the spacecraft dynamics, which is driven by the spacecraft demanded thruster impulse. Differences between the response of this model and the corresponding response of the real spacecraft dynamics may then be detected. The thruster characteristics are estimated with the assumption that the real time model is well matched to the true dynamics. The error in the dynamic response of the spacecraft may then be compensated for by introducing a calculated mismatch between the control signals applied to the spacecraft dynamics $u$ and
its real-time model in the state estimator \( \hat{u} \). The state plus thruster disturbance estimator transfer function can be written as follows:

\[
T.F = \frac{e}{\Delta u} \quad \text{where} \quad \Delta u = u - \hat{u}
\]

\[ (1.1) \]

Figure 1.4 Single axis spacecraft attitude control system with state estimator

The result from the parameter estimator is capable of operating with large initial mismatches between the real thruster characteristic and those assumed.

Wittig et al. [1990] measured the micro-acceleration caused by thruster firings on a communication satellite. A set of three orthogonally arranged micro-accelerometers were installed on ESA's large communication satellite OLYMPUS. Using these accelerometers translation could be measured. The measurement of linear acceleration was performed by measuring the displacement of a damped spring-mass system using a differential capacitance transducer.

The aim of this experiment was to characterise and observe the behaviour of the different mechanisms in space and to get measurements of the vibration levels caused by thruster firing which are of relevance for the design of optical communication payloads. The vibration levels can not be characterised precisely enough during ground testing. Knowledge of the level of micro-vibrations on a spacecraft is important for the design of the tracking control loop of an optical communication system.
Parvez [1990] computed the disturbance torques resulting from the impingement plumes of thruster on the solar array using data available from an operational GSTAR satellite. These disturbance torques vary as a function of the array plane orientation with respect to the thruster location. The plume impingement results in disturbance torques along the satellite roll, yaw and pitch axes. There is also a disturbance torque from sources that cannot be calibrated on the ground, such as spacecraft mass mismatch, thruster misalignment, and thrust level mismatch (not considered). All of these disturbance torques have to be countered by the attitude control system. The duty cycle data for yaw, roll and pitch control thrusters as well as the change in momentum level corresponding to the solar array positions are available from spacecraft telemetry. This data are used to determine the disturbance levels during spacecraft manoeuvres. The duty cycles on thrusters can be converted into net yaw, roll and pitch disturbance, since the control torque available from each thruster is known.

Tahk et al. [1991] estimated the pitch and roll misalignment of the primary lift thruster of a kinetic energy weapon vehicle using an extended Kalman filter. One of the major concerns in attitude control of a kinetic energy weapon vehicle is that error sources such as the thrust misalignment of thrusters may induce excessive parasitic torques on the vehicle. These torques may result in significant attitude perturbations that are undesirable for target tracking. Test flight data obtained from a kinetic energy weapon was used to estimate the thruster misalignments using an extended Kalman filter with some modifications to test data reduction requirements. Specifically, an adaptive estimation scheme was used to handle the difficulties in estimation caused by uncertain timing of the thrust pulses. Inputs to this estimator consisted of angular rate and translational acceleration measurements provided by an onboard inertial reference unit (IRU) and the known thruster valve commands. The IRU contains three rate gyros and three accelerometers. The control strategy used in the test vehicle was to relay onboard IRU measurements to the ground via a telemetry link. A ground computer then computed control commands for the thruster in terms of thruster valve commands. These are then uplinked to the vehicle to control the thruster firings. This method estimates the thrust misalignments for one thruster at a time.

Wiktor [1996] addressed a procedure to determine the true relationship between the commanded and actual force output of a set of thrusters using a Kalman filter technique.
for the Gravity Probe B satellite. The developed estimation method accounts for unknown disturbance force, time varying thruster coefficients and thruster biases. The method assumes that to calibrate thrusters, calibration forces must be generated in all possible output directions of the thruster system. This means that independent calibration moments must be generated about the yaw, pitch and roll axes and translation forces must also be generated along three mutually perpendicular axes. The calibration forces are generated by independently movable masses inside the spacecraft. The estimation method assumes zero mean disturbance torques.

A Kalman filter estimates the average value of the thruster coefficients. Since the average value does not change with time it can be modelled by the following state equation:

\[ \dot{x}(k) = 0 \]  

(1.2)

By incorporating the measurement equation,

\[ \dot{x}(k+1) = \dot{x}(k) + K(k)\left(F_c(k) - T_c(k)x(k)\right) \]  

(1.3)

where,

- \( K(k) \) = a Kalman gain matrix
- \( F_c(k) \) = calibration force vector
- \( T_c(k) \) = commanded force vector
- \( x(k) \) = unknown thruster coefficients vector

In the field of on-line inertia matrix identification, Jasim [1998] developed a feedback adaptive control algorithm that achieved large-angle tracking of velocity and attitude commands in spite of inertia uncertainty. This control algorithm requires three mutually perpendicular torque inputs. The control law is globally valid, that is, free of singularities, and has the form of a sixth-order proportional-integral compensator which does not require knowledge of the inertia or centre of mass of the spacecraft. Furthermore, periodic command signals using his adaptive tracking algorithm are used to identify the spacecraft inertia matrix.
If the dynamic equation of motion of the spacecraft is given by:

\[ I \dot{\omega} = -\omega \times I \omega + u \]  

(1.4)

The idea is to assume that the adaptive feedback control law is not a function of inertia matrix \( I \) and has the form:

\[ \hat{\alpha} = f(\hat{\alpha}, \omega, q) \quad (1.5) \]

\[ u = g(\hat{\alpha}, \omega, q) \quad (1.6) \]

where,

\( \hat{\alpha} = \) the adjustable parameter vector

\( u = \) the required control torque vector to adjust the adaptive parameter \( \hat{\alpha} \).

Rewrite Eq. (1.4) in the form

\[ I \dot{\omega} = f(\omega)x + u \]  

(1.7)

where,

\[ f(\omega) = -\omega \times \sigma(\omega) \]

\[ \sigma(\omega) = \begin{bmatrix} \omega_x & 0 & 0 & \omega_y & 0 & \omega_z \\ 0 & 0 & \omega_x & 0 & \omega_z & 0 \\ 0 & \omega_y & 0 & \omega_z & 0 & \omega_x \end{bmatrix} \]

\[ x = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} & I_{yx} & I_{yz} & I_{zx} \end{bmatrix}^T \]

The adaptive control law given by Eqs. (1.5) and (1.6) is a sixth-order proportional-integral compensator. The state \( \hat{\alpha} \) represents adjustable parameters that, under certain conditions, converge to the vector \( x \) (which contains the sixth-elements of the inertia matrix). Eq. (1.6) represents the mechanism for adjusting these parameters. The state \( \hat{\alpha} \) is consequently termed the adaptive parameter. Although the time derivative of the adaptive parameter converges to zero as \( t \to \infty \), \( \hat{\alpha} \) does not necessarily converge. Under the control law given by Eqs. (1.5) and (1.6) the inertia matrix can be identified using a periodic command such that \( \omega \to 0 \) and \( q \to 0 \), which implies \( \alpha \to x \).
E. Bergman [1990] described an algorithm to determine the mass and centre of mass of a rigid spacecraft using torque-producing actuators such as control-moment gyros or reaction wheels, and commonly available sensors, e.g. rate gyros and accelerometers using a Kalman filter technique for the Space Station. Complex spacecraft with evolving configurations such as the NASA Space Station or those carrying diverse payloads such as the Space Shuttle must be controlled over a wide range of mass properties. Currently, spacecraft such as the Space Shuttle require careful estimation of component mass properties as well as prediction and/or measurement of consumables. Such a process is tedious at best and susceptible to a number of error sources. For a vehicle such as the Orbit Maneuvering Vehicle, which must operate with many different payloads, not all of which have known mass properties, so it is not always possible to accurately predict vehicle plus payload mass distributions. Efficiency of operation will increase with increased estimation accuracy of the mass properties. The inertia estimation method assumes that the spacecraft is a rigid body and that the inertia matrix does not change with time it also neglects the gyroscopic terms in the dynamic equation. The control torques are generated by using control moment gyros. The idea is to move one CMG gimbal at a time comparing the predicted and measured angular rate change due to the gimbal motion to estimate the inverse of the inertia matrix using a Kalman filter technique.

Neglecting the gyroscopic term and integrating Eq. (1.4) gives (assuming the moment of inertia is constant):

\[ \dot{\omega} = \mathbf{I}^{-1} \mathbf{h} \]  \hspace{1cm} (1.8)

where \( \mathbf{I}^{-1} \) is the inverse of the inertia matrix and can be written as

\[ \mathbf{I}^{-1} = \begin{bmatrix}
-1
-1
-1
-1
-1
-1
-1
-1
-1
\end{bmatrix}
\]

Rewrite Eq. (1.8) in the form

\[ \mathbf{\omega}(k) = f(\omega)\mathbf{x}(k) \]  \hspace{1cm} (1.9)

where,

\[ \mathbf{x} = \begin{bmatrix}
-1
-1
-1
-1
-1
-1
-1
-1
\end{bmatrix}^T \]
Identification of the inverse of the inertia matrix can be formulated as a Kalman filter as follows: Let $\hat{x}$ be the vector to be estimated. Assuming the inertia matrix does not change with time, then the state equation can be written as in Eq. (1.2), by incorporating the measurement equation:

$$\dot{x}(k+1) = \hat{x}(k) + K(k)(\omega_n(k) - \omega_p(k))$$ \hspace{1cm} (1.10)

where,

$K(k)$ = is a $6 \times 3$ Kalman gain matrix

$\omega_n(k)$ = measured angular rate vector

$\omega_p(k)$ = predicted angular rate

and the predicted angular rate is updated as given by Eq. (1.9)

1.1.4 ADCS on UoSAT-12

The present generation of smaller, lighter and cheaper spacecraft require an accurate attitude control to provide pointing capability. On-line calibration of the attitude control hardware is often necessary to satisfy this high accuracy ADCS requirement. If these systems are not properly calibrated in-orbit, significant attitude control errors can result. For example, spacecraft equipped with thrusters can present significant disturbance torques as well as large control coupling torques if the attitude control thrusters of a spacecraft are not calibrated properly. Accurate calibration of the thrusters on the ground prior to flight, is limited by various factors. It is also well known that the MOI of a satellite is measured before launch using ground equipment. This equipment is very expensive especially if the products of inertia are also to be determined. Furthermore, the mass properties of spacecraft may be uncertain or may even change due to fuel usage and articulation after launch. Knowledge of the moment and products of inertia are critical when designing an attitude control system to provide rapid acquisition, tracking and pointing capabilities, while the equations that govern large-angle maneuvers are coupled and nonlinear. Control system designs must consider the nonlinear dynamics and changes
in mass distribution. The proposed research in this thesis will focus on estimating the satellite inertia matrix and calibration of satellite thrusters in-orbit for small satellites. An example to be used during this study will be UoSAT-12, which is the first low-cost mini-satellite constructed by Surrey Satellite Technology Limited (SSTL) at the University of Surrey. UoSAT-12 has a full three-axis attitude determination and control capability. UoSAT-12 was launched on the 21st of April 1999 into a 650 km circular, 64.5° inclination orbit. The main objectives of this mission were to demonstrate the mini-satellite bus and to enhance the payload technology at SSTL. The UoSAT-12 attitude determination and control system comprises of magnetometers, rate gyroscopes, horizon, star and sun sensors, magnetorquers, cold-gas thruster, and 3-axis reaction wheel systems. The three-axis stabilised UoSAT-12 also carries a multitude of remote sensing, on-board data handing and communication experiments. Some physical parameters for UoSAT-12 are given in Table 1.2. The satellite is expected to maintain an Earth pointing attitude to an accuracy of 0.5 degrees, with an experimental target of 0.1 degrees for the benefit of Earth observation payloads and communication antennas [Steyn et al, 1999].

Table 1.2 Physical parameters of UoSAT-12

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal orbit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>semi-major axis</td>
<td>7028</td>
<td>km</td>
</tr>
<tr>
<td>eccentricity</td>
<td>1.92 e-3</td>
<td>degree</td>
</tr>
<tr>
<td>inclination</td>
<td>64.5</td>
<td></td>
</tr>
<tr>
<td>Physical structure</td>
<td></td>
<td></td>
</tr>
<tr>
<td>height</td>
<td>1200</td>
<td>mm</td>
</tr>
<tr>
<td>diameter</td>
<td>1100</td>
<td>mm</td>
</tr>
<tr>
<td>weight</td>
<td>320</td>
<td>kg</td>
</tr>
<tr>
<td>Moment of inertia stowed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>with a GG boom</td>
<td></td>
<td></td>
</tr>
<tr>
<td>X-axis</td>
<td>40.45</td>
<td>kg m²</td>
</tr>
<tr>
<td>Y-axis</td>
<td>42.09</td>
<td>kg m²</td>
</tr>
<tr>
<td>Z-axis</td>
<td>41.36</td>
<td>kg m²</td>
</tr>
</tbody>
</table>

Throughout the research, a model of the UoSAT-12 will be adopted, in order to test the newly proposed algorithms during simulation. The outline structure of UoSAT-12 is shown in Fig. 1.4.
1.1.5 Thesis Outline

An outline of the thesis is as follows:

Chapter 2 provides an introduction to attitude control. The relevant material on attitude definitions and dynamics are briefly reviewed. The UoSAT-12 ADCS is then summarised. The external disturbance torque model acting on UoSAT-12 is presented. A model of the Earth's magnetic field using single dipole model for UoSAT-12 is discussed. Finally, momentum management and the implemented attitude controllers on UoSAT-12 are discussed.

Chapter 3 presents a complete analysis of two recursive algorithms based on least square techniques. A brief introduction to attitude determination on UoSAT-12 using a quaternion based extended Kalman filter (qEKF) is introduced. Finally, a recursive acceleration estimator is proposed to estimate the satellite acceleration vector from the estimated angular rates. The proposed estimator requires accurate angular rate measurements and a dynamic model.

Chapter 4 describes the newly developed algorithms for in-orbit thruster calibration using three reaction wheel actuators. These algorithms will be robust against modelling errors. The algorithms assumes no knowledge of the thruster parameters and only knowledge of the inertia matrix. It is proposed that the calibration procedure can be used during normal
mission conditions when the satellite is nominally stabilised. An analysis and performance comparison of the two methods to calibrate the thrusters in orbit is made. Two estimation techniques are used (Recursive Least Square and Least Mean Square) and two torque profiles for the reaction wheels are used to disturb the satellite during calibration. The best techniques are then chosen to calibrate the UoSAT-12 thrusters in orbit. Numerical simulations illustrate the successful identification of the thruster parameters in spite of non-zero mean disturbance torques and sensor noise. The estimation algorithm could be applied in real-time onboard a LEO nadir pointing satellite in order to improve the attitude control performance.

Chapter 5 describes a technique based on recursive least square algorithm for in-orbit inertia matrix identification. This is a general procedure to identify the inertia matrix for any spacecraft equipped with three reaction wheel actuators. The idea is to disturb one axis using a Bang-Bang reaction wheel controller to determine one principal moment of inertia. The other two axis are controlled using quaternion feedback wheel controllers to determine the corresponding two products of inertia. This experiment will then be repeated for the other two axes to obtain all nine elements of inertia matrix. A Bang-Bang nonlinear controller is preferred to disturb the satellite attitude. This controller will avoid any build up of attitude errors and wheel momentum. Due to the low computation requirements of these algorithms, both the control and estimation scheme can be easily applied onboard satellites to accurately estimate the inertia matrix.

Chapter 6 presents some experimental test results. Firstly, the thruster calibration algorithm was tested on an air-bearing table. Secondly, the thruster calibration algorithms were tested using in-orbit data generated by UoSAT-12 to calibrate both the yaw and delta-V thrusters. Finally, MOI calibration algorithms were tested using real data generated by UoSAT-12. The practical estimation results prove the feasibility of the proposed algorithms.

In Chapter 7 the conclusions for this research study are discussed and some suggestions concerning the future work will be given.

Finally the Appendices list all the obtained results during the simulations.
Chapter 2

2. Attitude Dynamics and UoSAT-12 ADCS Review

2.1 Introduction

A spacecraft in orbit always needs to stabilise the attitude against the external disturbance torques acting on it. Attitude control usually needs to be autonomous or semi-autonomous. On UoSAT-12 the available actuators are reaction/momentum wheels, thrusters and magnetic torquing. A mixture of attitude estimation and control algorithms is needed: these take the sensor measurements as inputs, compute the attitude and rates of the satellite, and then send commands to the actuators to maintain or stabilise that attitude, or direct the satellite to a new attitude.

A wide range of attitude control concepts has been proposed over the years and several have practical application. They can be classified as active, passive, and semi-passive procedures. The active approach applies deliberate control procedures. The passive and semi-passive systems, on the other hand, exploit the environmental forces for stabilisation and control. The previous generation of UoSATs exploited the passive gravity gradient torque. A substantial amount of literature has studied the technical problems of ADCS in many different areas. The topics include:

- Attitude dynamics
- Development of sensors and actuators
- Attitude determination algorithms both deterministic and stochastic estimation methods
- Control algorithms (from classic PID controller to modern applied control theories)

The recent tendency is to build smaller, lighter and cheaper spacecraft. The present generation of spacecraft requires accurate attitude control to provide acceptable pointing capabilities. On-line calibration of the attitude control hardware is often necessary to
satisfy this high accuracy ADCS requirement. If these systems are not properly calibrated in-orbit, a significant attitude control error can result.

To begin with, this chapter presents the equations and the theory from which the simulation model of the UoSAT-12 spacecraft will be developed. A mathematical description of the satellite’s attitude dynamics and kinematics based on the quaternion is presented. The external disturbance torques acting on UoSAT-12 are modelled. Moreover, this chapter provides the definitions of the coordinate systems used throughout the thesis, Earth’s magnetic field, momentum dumping and finally the controllers implemented in UoSAT-12 to estimate the inertia matrix and satellite’s thrusters are presented.

2.2 Coordinate Systems

In this thesis, various coordinate systems, will be used to describe the orientation of the spacecraft during dynamic modelling. Three coordinates frames were chosen to model the dynamics of UoSAT-12: an inertial reference frame, orbit reference frame, and body frame.

2.2.1 The Orbit Reference Frame

The first coordinates, labelled $X_o, Y_o, Z_o$ are defined as shown in Fig. 2.1, this is called the orbit reference frame and has its origin centred in the spacecraft centre of mass. The $Z_o$ axis is defined in the nadir direction (i.e. towards the centre of the Earth), the $Y_o$ axis in the orbit anti-normal direction (the orbit normal is defined by a right-hand role) and the $X_o$ axis to complete the orthogonal set. Although this frame is defined with its origin in the centre of mass, it rotates around the orbital plane and, it is not fixed in the body of the satellite. Therefore, the $Z_o$ axis will always be nadir pointing.
2.2.2 The Inertial Reference Frame

Next reference coordinates, labelled $X_R, Y_R, Z_R$ will be defined for an inertial reference frame. The origin is located within the centre of the Earth, with $Y_R$ in the orbit anti-normal direction, similar to $Y_o$, the $Z_R$ axis is in the same direction as the Earth’s geometric north pole and the $X_R$ axis will complete the orthogonal set. This reference frame is used primarily to calculate the latitude and longitude of the satellite’s centre of mass as it moves along its orbit.

![Coordinate system](image)

Figure 2.1 Coordinate system

2.2.3 The Body Frame

A third set of coordinates, labeled $X_B, Y_B, Z_B$ are defined as shown in Fig. 2.1 and called the body frame. The origin of this coordinate frame is centred within the spacecraft centre of mass. The body frame is defined such that it is fixed in the satellite’s body and thus will be used to determine the satellite’s orientation with respect to the orbit reference frame.
2.3 Attitude Representation

There are several representations to describe the orientation of spacecraft. Euler angle representation is clear for geometrical interpretation, particularly for small rotations. Euler angles are also often presented as the input and output parameters during attitude calculations. Moreover, Euler angles are useful for finding a closed-form analytic solution to the equations of motion in several simple cases. However, Euler symmetric parameters (quaternion) representation is commonly used in numerical computation. Since there is no singularity and no trigonometric functions are required, which may increase the computation time. This representation is not obvious for physical interpretation.

2.3.1 Euler Angles

In common with boats and aircraft the orientation of a spacecraft can be defined by three angles (roll, pitch, and yaw). These angles are obtained from a sequence of right hand positive rotations from a reference \( X_R, Y_R, Z_R \) frame to a \( X_B, Y_B, Z_B \) set of spacecraft body axes. There are 12 possible sequences of rotations, which can be expressed using Euler angles. One example is a 2-1-3 sequence rotation as shown in Fig. 2.2. The first rotation is a pitch about the reference \( Y_R \) axis, this defines a pitch angle \( \theta \). The second rotation is a roll about the intermediate \( L \) axis, this define a roll angle \( \phi \). The last rotation is a yaw about the body \( Z_B \) axis, this define a yaw angle \( \psi \). The attitude matrix, \( A \), which transforms an arbitrary vector from the reference \( X_R, Y_R, Z_R \) coordinates to the spacecraft body \( X_B, Y_B, Z_B \) coordinates can be expressed as:

\[
A = \begin{bmatrix}
  c \psi \cos \theta + s \psi \sin \phi \sin \theta & s \psi \cos \phi - c \psi \sin \phi \cos \theta & -s \psi \sin \phi - c \psi \cos \phi \cos \theta \\
-s \psi \cos \theta + c \psi \sin \phi \sin \theta & c \psi \cos \phi + s \psi \sin \phi \cos \theta & s \psi \sin \phi + c \psi \cos \phi \cos \theta \\
  c \psi \sin \theta & -s \psi \cos \theta & c \psi \cos \theta
\end{bmatrix}
\]  
(2.1)

Where,
\( \phi = \) roll angle \hspace{1cm} \( \theta = \) pitch angle \hspace{1cm} \( \psi = \) yaw angle
\( c = \) cosine function \hspace{1cm} \( s = \) sine function
Figure 2.2 Definition of 2-1-3 euler angle rotation

For an earth-pointing satellite such as UoSAT-12, the attitude matrix $\mathbf{A}$ transforms the vector from local-vertical-local-horizontal (LVLH) referenced coordinates (i.e. orbital coordinates) to satellite body coordinates (see Fig.2.3). The rotation angles $\psi$, $\theta$ and $\phi$ are respectively called yaw, pitch, and roll angles.

Figure 2.3 Definition of euler angles

The geometrical explanation of the three-parameter Euler-angle representation as in Eq. (2.1) is apparent. However, the kinematic equations for Euler angles involve nonlinear and computationally expensive trigonometric functions, and the angles become undefined
for some rotations, which can cause problems in Kaman filtering applications. In view of these difficulties, attitude coordinates are generated by integration of the quaternion kinematic differential equations on UoSAT-12. With the aid of a non-linear transformation, a corresponding set of readily understood Euler angles can present the attitude to the ground-station operators.

### 2.3.2 Euler Symmetric Parameters

According to Euler's theorem any finite rotation of a rigid body can be expressed as a rotation through one angle ($\Phi$) about a fixed axis ($e$). Therefore, the transformation attitude matrix $A$ can be obtained by the rotating angle $\Phi$ about the fixed axis $e$. The Euler symmetric parameters $q_1, q_2, q_3, q_4$ in terms of angle $\Phi$ and rotation axis $e$ are given by:

$$
q_1 = e_{os} \sin \left( \frac{1}{2} \Phi \right) \\
q_2 = e_{oy} \sin \left( \frac{1}{2} \Phi \right) \\
q_3 = e_{oz} \sin \left( \frac{1}{2} \Phi \right) \\
q_4 = \cos \left( \frac{1}{2} \Phi \right)
$$

where,

$q = [q_1, q_2, q_3, q_4]^T = \text{attitude quaternion vector with respect to orbital coordinates}$

$e = [e_{ox}, e_{oy}, e_{oz}]^T = \text{euler vector in orbital referenced coordinates}$

$\Phi = \text{rotation angle around the Euler vector}$

The four Euler symmetric parameters are not independent, but satisfy the constraint,

$$q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1 \quad (2.3)$$

The attitude matrix Eq. (2.1) is expressed in term of Euler symmetric parameters as,
\[
A = \begin{bmatrix}
q_1^2 - q_2^2 - q_3^2 + q_4^2 & 2(q_1q_2 + q_3q_4) & 2(q_1q_3 - q_2q_4) \\
2(q_1q_2 - q_3q_4) & -q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2(q_2q_3 + q_1q_4) \\
2(q_1q_3 + q_2q_4) & 2(q_2q_3 - q_1q_4) & -q_1^2 - q_2^2 + q_3^2 + q_4^2
\end{bmatrix}
\] (2.4)

This expression contains no trigonometric functions which require time-consuming computation, and it can easily be referenced to the orbit coordinate system.

From Eqs. (2.1) and (2.4) it is now possible to establish a relation between the two representations. If the quaternion representation is used, the respective pitch, roll and yaw angles can be calculated as:

\[
\begin{align*}
\theta &= \arctan\left[\frac{A_{31}}{A_{33}}\right] \\
\phi &= \arcsin\{-A_{23}\} \\
\gamma &= \arctan\left[\frac{A_{13}}{A_{22}}\right]
\end{align*}
\] (2.5)

If the Euler angle representation is known, the DCM of Eq. (2.1) can be used to calculate the quaternion parameters:

\[
q_4 = \frac{1}{2} [1 + A_{11} + A_{22} + A_{33}]^{0.5} \quad \text{then,} \\
q_1 = \frac{1}{4q_4} [A_{23} - A_{32}], \quad q_2 = \frac{1}{4q_4} [A_{31} - A_{13}], \quad q_3 = \frac{1}{4q_4} [A_{12} - A_{21}]
\] (2.6)

### 2.4 Attitude Dynamics

The motion of a spacecraft presents two dynamic aspects of interest. Classical dynamics allows, under certain general conditions, for the motion of a body to be treated as the combination of two motions: a translational motion of the centre of mass and a rotation of the body about the centre of mass. The theory of attitude control generally considers only the second effect and ignores the first. The application of any force can only be interpreted as the resultant torque that would exist around the centre of mass and ignores any change to the translational velocity.
The equations of motion of a spacecraft can be divided into two parts: The dynamic equations of motion and kinematic equations of motion. The dynamic equations of motion express the relationship between the spacecraft body angular rate and the applied torque. These are necessary for dynamic simulations and for attitude prediction, whenever gyroscopic measurements of the angular rate is unavailable. The kinematic equations of motion are a set of first-order differential equations expressing the relationship between the attitude parameters and the rate [Wertz, 1989].

2.4.1 Dynamic Equations of Motion

The basic equation of attitude dynamics relates the time derivative of the angular momentum vector, \( dL/dt \), to the external torque, \( N_{ex} \) known as Euler’s equations of motion. This equation is given by Euler equations, as (assuming a fixed inertia tensor)

\[
dL/\,dt = N_{ex} - \omega'x \times (I \omega') = I \omega''
\] (2.7)

Where,

- \( \omega'x = [\omega_x \omega_y \omega_z]^T \) = inertielly referenced body angular rate vector
- \( I = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} \) = moment of inertia tensor of spacecraft (MOI)
- \( N_{ex} = \begin{bmatrix} N_x \\ N_y \\ N_z \end{bmatrix}^T \) = external torque vector including active control torques generated by thrusters and magnetorquers, and environmental disturbance torques

If the spacecraft is equipped with flywheels and thrusters, the dynamic equations derived above can still be used. Including the influence of the gravity gradient torque, thruster torque and reaction wheel angular momentum, the dynamic equation in the body-fixed frame can be expressed as:

\[
I \omega'' = N_{ex} + N_{D} + N_{M} + N_T - \omega'x \times (I \omega'x + h) - h
\] (2.8)
where,
\[
\mathbf{h} = \begin{bmatrix} h_x & h_y & h_z \end{bmatrix}^T = \text{reaction wheel angular momentum vector}
\]
\[
\mathbf{N}_{CG} = \begin{bmatrix} N_{srx} & N_{sry} & N_{sz} \end{bmatrix}^T = \text{gravity-gradient torque vector}
\]
\[
\mathbf{N}_D = \begin{bmatrix} N_{dx} & N_{dy} & N_{dz} \end{bmatrix}^T = \text{external disturbance torque vector}
\]
\[
\mathbf{N}_{M} = \begin{bmatrix} N_{mx} & N_{my} & N_{mz} \end{bmatrix}^T = \text{applied torque vector by 3-axis magnetorquers}
\]
\[
\mathbf{N}_T = \begin{bmatrix} N_{tx} & N_{ty} & N_{tz} \end{bmatrix}^T = \text{applied torque vector by 3-axis thrusters}
\]

2.4.2 Kinematic Equations of Motion

2.4.2.1 Attitude Kinematic Equation in Quaternion

The kinematic equations are defined as the rate of change of the attitude matrix with time. The rate of change of the quaternion is given by

\[
\dot{\mathbf{q}} = \frac{1}{2} \Omega \mathbf{q} = \frac{1}{2} \Lambda(\mathbf{q}) \omega^o_B
\]

where,
\[
\Omega = \begin{bmatrix}
0 & \omega_{oz} & -\omega_{oy} & \omega_{ox} \\
-\omega_{ox} & 0 & \omega_{oz} & -\omega_{oz} \\
\omega_{oy} & -\omega_{oz} & 0 & \omega_{ox} \\
-\omega_{oz} & -\omega_{oy} & -\omega_{ox} & 0
\end{bmatrix}
\]

\[
\Lambda(\mathbf{q}) = \begin{bmatrix}
q_4 & -q_3 & q_2 \\
q_3 & q_4 & -q_1 \\
-q_2 & q_1 & q_4 \\
-q_1 & -q_2 & -q_3
\end{bmatrix}
\]

Where,
\[
\omega^o_B = \begin{bmatrix} \omega_{ox} & \omega_{oy} & \omega_{oz} \end{bmatrix}^T = \text{body angular velocity vector referenced to orbital coordinates.}
\]
The angular body rates referenced to the orbit coordinates can be obtained from the inertially referenced body rates by using the transformation matrix \( A \):

\[
\omega_b^O = \omega_b^I - A \omega_e
\]  

(2.12)

If we assume the satellite in a near circular orbit with average orbital angular rate \( \omega_o \), then \( \omega_o = \begin{bmatrix} 0 & -\omega_o & 0 \end{bmatrix}^T \) is a constant rate vector.

Using the attitude matrix from Eqs. (2.1) or (2.4), Eq. (2.12) becomes:

\[
\begin{align*}
\omega_{ox} &= \omega_o + \omega_e A_{12} \\
\omega_{oy} &= \omega_o + \omega_e A_{22} \\
\omega_{oz} &= \omega_o + \omega_e A_{32}
\end{align*}
\]  

(2.13)

When the quaternion is used directly in the control algorithms, it will be convenient to define an error quaternion. The error quaternion will be the quaternion difference between the current quaternion and the commanded quaternion. It can be represented by [Wie, 1989], [Steyn, 1995]:

\[
\begin{bmatrix}
q_{1e} \\
q_{2e} \\
q_{3e} \\
q_{4e}
\end{bmatrix} =
\begin{bmatrix}
q_{4e} & q_{3e} & -q_{2e} - q_{1e} \\
-q_{3e} & q_{4e} & q_{1e} - q_{2e} \\
q_{2e} - q_{1e} & q_{4e} - q_{3e} \\
q_{1e} & q_{2e} & q_{3e} & q_{4e}
\end{bmatrix}
\begin{bmatrix}
q_1 \\
q_2 \\
q_3 \\
q_4
\end{bmatrix}
\]  

(2.14)

Where,

\[
q_e = \begin{bmatrix} q_{1e} & q_{2e} & q_{3e} & q_{4e} \end{bmatrix}^T = \text{attitude quaternion error vector}
\]

\[
q_c = \begin{bmatrix} q_{1c} & q_{2c} & q_{3c} & q_{4c} \end{bmatrix}^T = \text{commanded quaternion vector}
\]

**2.4.2.2 Attitude Kinematic Equation in 2-1-3 Euler Angles**

For the 2-1-3 Euler angle sequence, the kinematic equations can be derived by using a spacecraft referenced angular velocity vector \( \omega_b^S \) as follows:
\[
\dot{\phi} = \omega_R \cos \psi - \omega_R \sin \psi \\
\dot{\theta} = (\omega_R \sin \psi + \omega_R \cos \psi) \sec \phi \\
\dot{\psi} = \omega_R + (\omega_R \sin \psi + \omega_R \cos \psi) \tan \phi
\] (2.15)

Where,
\[
\omega_R = [\omega_{Rx} \quad \omega_{Ry} \quad \omega_{Rz}]^T = \text{body relative angular velocity in any reference coordinate frame}
\]

It can be seen that the dynamics of these angles are not independent. A change in one angle will couple to the other angles. Another important point is that this representation (2-1-3) has singularity when the roll angle equals 90 degrees.

### 2.5 ADCS of UoSAT-12

The structure of the UoSAT-12 ADCS is shown in Fig. 2.4.

![UoSAT-12 ADCS block diagram](image)

**Figure 2.4 UoSAT-12 ADCS block diagram**
2.5.1 Attitude Sensors

UoSAT-12 comprises of a wide range of sensors for attitude determination and a multi-channel GPS receiver for onboard orbit determination. The GPS receiver will also be used as an experimental attitude determination sensor with an expected accuracy of about 1 degree. A set of three-axis flux gate magnetometers are used to measure the geomagnetic field vector in the satellite's body coordinates. These measurements will be employed to determine the magnetorquer control vector and, in combination with the International Geomagnetic Reference Field (IGRF) model, to estimate the full attitude and angular rate vectors of the satellite in an extended Kalman filter. Four 2-axis (azimuth and elevation) sun sensors measure the sun vector angle to a high accuracy. A precise 2-axis infrared horizon sensor measures small nadir angles and so can be used during nominal nadir-pointing control, or for small pitch and roll off pointing. The most accurate attitude measurement can be obtained from a dual set of opposite-looking star sensors. Table 2.1 lists all the sensors used on UoSAT-12 and their characteristics [Steyn, 1998, 1999].

Table 2.1 Attitude determination sensors on UoSAT-12

<table>
<thead>
<tr>
<th>Supplier</th>
<th>Magnetometer</th>
<th>Sun Sensors</th>
<th>Horizon Sensor</th>
<th>Star Sensor</th>
<th>Rate Gyro</th>
<th>GPS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supplier</td>
<td>SSTL (2)</td>
<td>SSTL (4)</td>
<td>Servo MiDES</td>
<td>SSTL</td>
<td>BEI</td>
<td>SSTL</td>
</tr>
<tr>
<td>Quantity</td>
<td>3 units</td>
<td>4 x 2-axis</td>
<td>1 x 2-axis</td>
<td>2 unit</td>
<td>1 unit</td>
<td>1 unit</td>
</tr>
<tr>
<td>Range</td>
<td>±60 μTesla</td>
<td>±50°</td>
<td>±5.5°</td>
<td>14.4° x 19.2°</td>
<td>±5° / sec</td>
<td></td>
</tr>
<tr>
<td>Accuracy</td>
<td>30 nTesla (3σ)</td>
<td>0.2° (3σ)</td>
<td>0.06° (3σ)</td>
<td>0.02° (3σ)</td>
<td>0.02° (3σ)</td>
<td>1° (16)</td>
</tr>
<tr>
<td>Power</td>
<td>&lt;0.8 W</td>
<td>&lt;0.1 W</td>
<td>2.8 W</td>
<td>4 W</td>
<td>1.4 W</td>
<td>5-7 W</td>
</tr>
</tbody>
</table>

2.5.2 Attitude Actuators

Attitude actuators of UoSAT-12 consists of three-axis magnetorquers, reaction/momentum wheels and cold-gas thrusters, listed in Table 2.2.
2.5.2.1 Magnetorquers

Twelve magnetorquer coils are mounted in UoSAT-12 to give some level of backup and to generate full 3-axis magnetic dipole control moment. A dual polarity current pulse width control method is used to provide the required average level of magnetic moment per sample period. The magnetorquers are used for:

- Detumbling of the body angular rates after ejection from the launch vehicle;
- Momentum management of the reaction/momentum wheels;
- Nutation damping during spin stabilisation;
- Libration damping and yaw spin control after deployment of a backup gravity gradient boom.

Magnetorquers can be designed to provide momentum management on a low Earth orbiting spacecraft. Dipole moments generated by the magnetorquer interact with the Earth's magnetic field to generate small torques on the spacecraft. Since the magnetic torque is always orthogonal to the local magnetic field vector, it is not possible to generate instantaneously a required torque direction as demanded by a full 3-axis control system. However, in the course of an orbit the direction of the vector may change and it may be possible to generate the required torque on average during the course of an orbit. A consistent and reasonable strength vector is available only in LEO orbits.

<table>
<thead>
<tr>
<th>Supplier</th>
<th>Magnetorquers</th>
<th>Reaction/momentum wheels</th>
<th>Cold-gas thrusters</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSTL</td>
<td>12</td>
<td>SSTL (2)</td>
<td>SSTL &amp; Polyflex</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ithaco (1)</td>
<td></td>
</tr>
<tr>
<td>Quantity</td>
<td>3</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Range</td>
<td>±15 Am²</td>
<td>±4 Nms, ±5000rpm</td>
<td>±0.035 Nm</td>
</tr>
<tr>
<td></td>
<td>±0.015 Nm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Power</td>
<td>20 W maximum</td>
<td>2.8-14.6 W</td>
<td>3 W</td>
</tr>
<tr>
<td>Operation</td>
<td>PWM</td>
<td>Speed controlled</td>
<td>PWM</td>
</tr>
<tr>
<td>Accuracy</td>
<td>20 msec minimum pulse</td>
<td>±1 rpm</td>
<td>&gt;10 msec pulse</td>
</tr>
</tbody>
</table>
2.5.2.2 Reaction/Momentum Wheels

Three momentum/reaction wheels are installed in a three-axis configuration to enable full control of the attitude and angular momentum of the satellite. Reaction wheels are essentially torque motors with high-inertia rotors. They can spin in either direction. Roughly speaking one wheel provides for the control of one axis. A minimum of three wheels is needed for full 3-axis control. Momentum wheels are wheels with a nominal spin rate above zero. Their aim is to provide a nearly constant angular momentum. This momentum provides gyroscopic stiffness to two axes, while the motor torque may be controlled to precisely point around the third axis.

In sizing the wheels, it is important to distinguish between cyclic and secular disturbances, and between angular momentum storage and torque authority. For three-axis control systems, cyclic torques build up cyclic angular momentum in the reaction wheels, because the wheels are providing compensating torques to counteract these disturbances. We typically size the angular momentum capacity of a reaction wheel (limited by its saturation speed) to handle the cyclic storage during an orbit without the need for frequent momentum dumping. The secular torques and our total storage capacity then define how frequently the angular momentum must be dumped. The torque capability of the wheels usually is determined by slew requirements or the need for control authority above the peak disturbance torque in order for the wheels to maintain the required pointing accuracy [Larson et al, 1992].

The wheels are used for the following control functions on UoSAT-12 [Steyn, 1999]:

- Full 3-axis pointing and slow slew manoeuvres during imaging;
- Nadir, sun or inertial pointing of the payloads by using angular momentum stiffening;
- Fast spin-up or spin down of the satellite body;
- Cancellation of the disturbance torque caused by the propulsion system during orbit control;
- Calibration of the thrusters and the moment of inertia tensor.
2.5.2.3 Thrusters

Thusters produce a force or torque by expelling mass. A three-axis cold-gas thruster system is mounted onboard the satellite to generate the relatively large torques for fast attitude control. It is used to [Steyn, 1999]:

- Do agile attitude control;
- Manoeuvre the spacecraft over large angles;
- Dump momentum from the reaction/momentum wheels;
- Control the spin rate;
- Control nutation.

The advantage of thrusters over other sources of torque is high controllability. Thrusters can yield very high precision using small accurate thrusters if their mechanical configuration is well defined. An obvious disadvantage is their consumption of propellant. Once that has gone, there is no more control possible. A more subtle disadvantage is that their plumes may impinge on the spacecraft, contaminating surfaces and camera lenses. If the structure of a thruster is not well constructed, the output force from thrusters can be variable. This will generally cause a problem in achieving precise and accurate attitude control when employing thrusters.

2.6 Disturbance Torques

Disturbance torques acting on the spacecraft are due to [Shrivastava, 1983]: gravity gradient, solar pressure, the earth magnetic field and aerodynamic drag. Detailed derivation of the models used to compute those torques are given in [Wertz, 1989]. Disturbances are affected by the spacecraft's geometry, orientation, and mass properties. These torques for the UoSAT-12 satellite are computed using the assumption that (1) the satellite will fly at an altitude 650 km and 65° inclination, (2) the satellite has the shape of Fig 2.4 with a diameter and height of 1200 and 800 mm respectively and (3) the total mass of 320 kg is uniformly distributed. Due to the relative high atmospheric density at altitudes up to 800 km, the aerodynamic drag is the dominant disturbance torque. All the significant disturbance torques, which tends to disturb the satellite attitude, will be introduced next.
2.6.1 Aerodynamic Torque

Aerodynamic torque is caused by the atmospheric drag acting on the satellite. It can be quite significant, especially at low altitudes. From Wertz [1989], we can use the following simplified results: aerodynamic pressure is directly proportional to the air density, and the square of the relative air velocity. The major assumption leading to this result is that any surface exposed to the velocity direction of the spacecraft completely absorbs the momentum of an incoming colliding particle. The aerodynamic disturbance torque vector on a spacecraft structure can then be obtained by the cross product of the aerodynamic pressure vector on the total projected area and the vector from the centre of mass to the centre of pressure of the total structure.

\[ \mathbf{N}_{\text{aero}} = \rho_a V^2 A_p [\mathbf{c}_p \times \bar{V}] \]  

(2.16)

Where,
\( \rho_a \) = atmospheric density
\( V \) = magnitude of spacecraft's velocity vector
\( \bar{V} \) = unit velocity vector
\( A_p \) = total projected area of spacecraft
\( \mathbf{c}_p \) = vector between centre of mass and centre of pressure

For a spacecraft structure such as UoSAT-12 (see Fig. 2.4), the magnitude can approximately be estimated as presented in Table 2.3 [Currie, 1999], [Cowey, 2000].

Table 2.3 UoSAT-12 parameter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>7000</td>
<td>km</td>
</tr>
<tr>
<td>e</td>
<td>0.0026</td>
<td></td>
</tr>
<tr>
<td>( V_p )</td>
<td>7550.65</td>
<td>m/s</td>
</tr>
<tr>
<td>( V_o )</td>
<td>7511.49</td>
<td>m/s</td>
</tr>
<tr>
<td>( \rho_a ) (650 km)</td>
<td>\begin{align*} \text{Mean} &amp;= 4.73 \times 10^{-14} \ \text{Max.} &amp;= 4.77 \times 10^{-13} \end{align*}</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>( \mu )</td>
<td>3.986005 \times 10^{-5}</td>
<td>kg m$^3$/s$^2$</td>
</tr>
<tr>
<td>( A_p )</td>
<td>1.18</td>
<td>m$^2$</td>
</tr>
</tbody>
</table>
From the results in Table 2.4 it is clear that the aerodynamic disturbance torque at perigee will have the greatest influence. Although the torque profile over the full orbit will have some non-regular shape, it will not change much from orbit to orbit. It can therefore be modelled as a periodic waveform.

Table 2.4 Aerodynamic disturbance torque (Nm) on UoSAT-12

<table>
<thead>
<tr>
<th>Y-axis</th>
<th>$\rho_a$ (mean)</th>
<th>$\rho_a$ (max)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Perigee</td>
<td>Apogee</td>
</tr>
<tr>
<td></td>
<td>8.27 x 10^-2</td>
<td>8.1 x 10^-2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Z-axis</th>
<th>$\rho_a$ (mean)</th>
<th>$\rho_a$ (max)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Perigee</td>
<td>Apogee</td>
</tr>
<tr>
<td></td>
<td>1.9 x 10^-7</td>
<td>1.57 x 10^-7</td>
</tr>
</tbody>
</table>

Figure 2.5 Geometrical structure of UoSAT-12
2.6.2 Solar Radiation Pressure

Torque due to solar radiation pressure is caused by a difference in the satellite’s centre of pressure in the sun direction and its centre of gravity. While in the sun, solar radiation on the satellite will create a net torque about the centre of gravity. On an Earth orbiting satellite, these disturbances are cyclic over an orbit and are a function of the spacecraft’s reflectivity. The solar radiation torque can be calculated using the following equation:

\[ N_{solar} = d c_p \]  \hspace{1cm} (2.17)

With,

\[ d = d_o A_p (1 + q) \cos(i_s) \]

Where,

\( d_o = \) solar radiation constant (1358 Wm\(^{-2}\))
\( q = \) reflection factor (0.6 worst case)
\( i_s = \) angle of incidence of the sun.

Figure 2.6 shows solar radiation pressure torque acting on UoSAT-12 spacecraft. The magnitude of these worst case torques are all of the order of \(10^{-7}\) which is therefore at least 25 times smaller than the average aerodynamic disturbance torque, and its influence can be ignored.
2.6.3 Gravity Gradient Disturbance

The gravity gradient disturbance is a torque experienced by a low Earth orbiting spacecraft. This disturbance is created by the unsymmetric mass distribution of the spacecraft, causing a slight difference in the gravity forces acting on the body. The result is a torque around the spacecraft centre of mass. The gravity gradient torque is expressed as defined in Eq. (5.22). For an elliptical orbit, the magnitude will be inversely affected by the cube of the distance from the orbital position to the geocentric point. In case of UoSAT-12, the principal MOI for all axes are nearly equal, so we can neglect the influence of this torque compared to the aerodynamic disturbance torque.

2.7 Earth's Magnetic Field

The Earth's magnetic field can be characterised by a magnetic dipole such as that produced by a current loop or a sphere of uniform magnetisation. The magnetic field can more accurately be expressed mathematically by a spherical harmonic model, the so-called IGRF (International Geomagnetic Reference Field) model [Wertz, 1989]. Due to secular drift and magnitude decrease of the geomagnetic field, the coefficients of the IGRF model are updated every 5 years and supplied with secular variation terms. For the purpose of simulation, a first order dipole model [Rodden, 1984] will be used to represent the geomagnetic field vector. This dipole vector can be expressed as,

\[ B = \nabla \left[ \frac{\vec{R} \cdot \vec{M}}{R^3} \right] = \frac{\vec{M}}{R^3} \left[ 1 - 3\vec{R} \vec{R}^T \right] \]  

(2.18)

Where,
- \( \nabla \) = vector gradient operator
- \( R_i \) = geocentric position vector length
- \( \vec{R} \) = unit geocentric position vector
- \( \vec{M} \) = vector geomagnetic strength of dipole
- \( 1 \) = identity matrix
In orbital coordinates, the model is expressed as,

\[
\mathbf{B} = \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \frac{M_L}{R_i^3} \begin{bmatrix} \sin i \cos \alpha \\ -\cos i \\ 2\sin i \sin \alpha \end{bmatrix}
\] (2.19)

Where,

\(i\) = orbit inclination

\(\alpha\) = orbit phase angle as measured from the ascending node

The advantage of this model representation is that it gives the field in terms of simple trigonometric functions of the inclination \(i\) and orbital phase \(\alpha\). From this model it can be calculated that the low earth orbit (LEO) UoSAT-12 \((i = 65^\circ\) and average altitude \(\approx 650\) km) present a small constant \(B_x\) component of \(-9.7 \mu T\), a maximum \(B_y\) component of \(20.8 \mu T\) over the equator with \(B_x\) zero, and a maximum \(B_z\) component of \(41.6 \mu T\) over the polar region with \(B_x\) zero. The geomagnetic field vector, therefore, rotates inertially twice per polar orbit, almost within the orbital plane as shown in Fig. 2.7.

Figure 2.7 Earth magnetic field of dipole model in orbital coordinates
2.8 Momentum Dumping

2.8.1 Introduction

Any reaction wheel 3-axis stabilised satellite must employ a momentum management algorithm to restrict the wheel momentum within allowable limits. Momentum build-up naturally occurs due to the influence of external disturbance torques, for example, the torques due to passive gravity gradient, aerodynamic and solar forces, and active control torques from thrusters and magnetorquers (MT). These disturbances to the body of an attitude-controlled satellite cause an accumulation of momentum on the reaction wheels. The added momentum may cause saturation of the reaction wheel speed. Moreover, the existence of large angular momentum in the satellite causes control difficulties when attitude controllers are implemented, because the momentum provides the satellite with unwanted gyroscopic stability. Therefore, the management of three-axis reaction wheel momentum is required in order to counteract the influence of persistent external disturbance torques. A cheap and effective means of active unloading of this momentum is making use of magnetorquers, to force the wheel speed back to nearly zero speed.

2.8.2 Magnetic Unloading of the Reaction Wheels

Magnetorquers generate magnetic dipole moments whose interactions with the Earth’s magnetic field produce the torques necessary to remove the excess momentum. The magnetic torque vector can be expressed as the cross product of the magnetic dipole moment \( \mathbf{M} \) of the magnetic coils with the geomagnetic field strength \( \mathbf{B} \) in the body frame:

\[
\mathbf{N}_M = \mathbf{M} \times \mathbf{B} = \Psi(t)\mathbf{M}
\]  

(2.20)

Where,

\( \mathbf{M} = \) magnetic dipole control moment vector
The magnetic field $B$ in the body coordinates can be modelled by:

$$B = AB_{\gamma}$$  \hspace{1cm} (2.22)

Where,

$B_{\gamma}$ = geomagnetic field vector in the local orbital coordinates from an IGRF model.

The cross-product law algorithm was proposed to dump the extra momentum from the reaction wheel and can be written as:

$$M = k_{\alpha} (h \times B) / \|B\|$$  \hspace{1cm} (2.23)

Where,

$k_{\alpha}$ = a scalar gain

### 2.8.3 Determination of the Unloading Control Gain $k_{\alpha}$.

The torque produced by the magnetorquers change with orbit position, because the components of the Earth’s magnetic field in the orbit reference coordinates depend strongly on the orbit parameters. Moreover, the control law is expressed by Eq. (2.23) under the assumption that the magnetic dipole moment $M$ is always perpendicular to the Earth’s magnetic field $B$. Hence, an analytic procedure to obtain the correct value of $k_{\alpha}$ does not seem to be feasible. Several simulations with different $k_{\alpha}$ must be performed until an acceptable excess momentum remains, with magnetic torquer coils ($M_{\text{coil-max}} = 15 \text{ Am}^2$). In our solution, the gain constant ($k_{\alpha} = 40$) was optimized for the best simulation results.
2.9 Attitude Controller Implemented on UoSAT-12

2.9.1 Introduction

We present two feedback control laws that are used to provide appropriate control regimes during estimation of the inertia matrix and thruster parameters. The satellite is assumed to have a rigid body. The angular velocity vector of the satellite ($\omega_b^o$) is measured or estimated accurately. Its attitude ($q$) is estimated or measured via the quaternion Eq. (2.9). Consequently, the state vector of the satellite ($\omega_b^o$ and $q$) is accurately known though subject to sensor noise. The control laws use this state vector to control the satellite’s attitude.

2.9.2 Quaternion Feedback Controller

The globally stable quaternion PD feedback control law of [Wie, 1989] was modified to be implemented on UoSAT-12 as an orbit referenced pointing control law. This controller consists of linear error-quaternion feedback, with linear and nonlinear body-rate feedback terms to compensate for the gyroscopic coupling torques. The error quaternion is defined as the quaternion difference between the current quaternion and the commanded quaternion Eq. (2.14). The control torque vector is represented as

$$\mathbf{N}_{w/T} = \omega'_b \times (I \omega'_b + h) - k_p \omega_b^o - k_d \mathbf{q}_{wc}$$  \hspace{1cm} (2.24)

Where,

- $k_p, k_d =$ positive gain scalars
- $\mathbf{N}_{w/T} =$ applied torque vector of 3-axis reaction wheels or thrusters
- $\mathbf{q}_{wc} = [q_{1e} \quad q_{2e} \quad q_{3e}] =$ vector part of error quaternion

The controllers gains $k_p, k_d$ are chosen to satisfy certain requirements that will be explained next. The dynamic model in Eq. (2.8) can be approximated in one axis by [Wie et. al, 1989]:

2-23
where,

\[ \phi = \text{rotation angle} \]

Under the assumption of \( \phi \ll \) (small), we may approximate \( \sin(\phi) \approx \frac{\phi}{2} \). Therefore, Eq. (2.25) further simplifies as:

\[ \ddot{\phi} + k_d \dot{\phi} + k_p \frac{\phi}{2} = 0 \]  \hspace{1cm} (2.26)

Eq. (2.26) is a common linear second-order equation, the PD gain scalars \( k_p \) and \( k_d \) can be determined by properly choosing the damping ratio \( \xi \) and the undamped natural frequency \( \omega_n \) to satisfy:

\[ k_p = 2\omega_n^2 \quad \text{and} \quad k_d = 2\xi\omega_n \]

In order to avoid a high overshoot in the step response of the system, the damping ratio \( \xi \) can be chosen between 0.7 ~ 1. The undamped natural frequency \( \omega_n \) should be carefully selected to give a reasonable 5 % settling time \( t_s = \frac{3}{\xi\omega_n} \). If \( \xi \) is chosen to be smaller, the overshoot of the system will be too high. Usually the acceptable overshoot range is 0 ~ 15 %, \( \xi = 0.7 \) will generate a 5 % of overshoot. If \( \xi \) is chosen to be too big, the settling time becomes much longer. In this thesis, a 1-second sampling time is used for most demonstrations during simulation. Due to the consideration of reaction wheel saturation and the characteristics of the external disturbance, a 180 second settling time was finally chosen for the simulations. The value of the parameters is set to be \( \xi = 0.707, \omega_n = 0.024, k_p = 0.0011 \) and \( k_d = 0.033 \). The more detailed explanation of how to choose \( \xi \) and \( \omega_n \) adequately is beyond the scope of this thesis and can be found in reference [Wie et al, 1989].
2.9.3 A Bang-Bang Feedback Controller

A Bang-Bang controller is implemented using a linear PD feedback method to control a two level nonlinear control law. The switching function is determined by using a linear feedback equation and a hysteresis band and can be summarised by the following equation:

\[
\begin{align*}
    e &= K_1 q_{rec} + K_2 \omega_2^2 \\
    N_{WT} &= \begin{cases} 
        -N_{max} & \text{for } e \geq e_{\text{band}}, \text{ with } e \text{ increasing} \\
        N_{max} & \text{for } e \leq -e_{\text{band}}, \text{ with } e \text{ decreasing}
    \end{cases}
\end{align*}
\]

(2.27)

Where,

- \(K_1\) and \(K_2\) = controller gains
- \(e\) = control error
- \(e_{\text{band}}\) = error hysteresis band.

This error band is tuned to adjust the level of the wheel momentum indirectly (and the error quaternion). When reducing the error band, the torque output switches at a higher frequency and the wheel momentum build-up is reduced. An increase in the error band has the opposite effect. The controller gains \(K_1\) and \(K_2\) adjust the slope of the switching function, this also controls the stability of the non-linear limit cycle and the magnitude of the error quaternion. The controller gains \(K_1 = k_1 I\) and \(K_2 = k_2 I\) are optimised in simulation by trial and error.

The controller switches between two control torques (one for positive and one for negative). Fig. 2.9 explains how the Bang-Bang controller works Eq. (2.27). Suppose we
have a certain attitude and rate at point \( t_0 (\phi, \dot{\phi}) \), a positive wheel torque (negative satellite torque) is used to enable a trajectory from point \( t_0 \) until the switching line (point \( t_1 \)) is reached, when the controller switches to a negative wheel torque to enable a trajectory from point \( t_1 \) until a switching line (point \( t_2 \)) is reached. The controller switches again to a positive wheel torque to enable a trajectory from point \( t_2 \) until switching line (point \( t_3 \)) is reached, then the controller switches to a negative wheel torque to enable a trajectory from point \( t_3 \) until switching line (point \( t_4 \)) is reached. Finally the controller switches continuously between points \( t_3 \) and \( t_4 \) on the switching lines within the error band.

![Figure 2.9 Phase plane path of a bang-bang control system with linear PD feedback](image-url)

**Figure 2.9** Phase plane path of a bang-bang control system with linear PD feedback
3. Estimation Techniques

3.1 Estimation Algorithms

There are many different types of estimation algorithms that can be used during this study. A widely used estimation cost criterion is the mean square error. The cost function used in the estimation algorithms must be defined. The cost function uses the error when the actual output of the algorithm is compared with a desired output.

The performance of these different algorithms is based on a number of factors. One important factor is the algorithm's rate of convergence. This describes how quickly an algorithm can converge to the optimum solution. A fast rate of convergence also allows for better tracking of quickly changing parameters. Another performance measure is the adjustment error. This indicates how close the estimation criterion, such as the mean-square error, comes to the theoretical optimal value. The smaller the difference, the more accurate the data estimates. The computational complexity is another issue to consider when examining estimation algorithms. Some algorithms require many calculations per iteration while others require only a small amount.

Two of the most common estimation algorithms that use the mean-square error cost function are the Least Mean Squares (LMS) and the Recursive Least Squares (RLS) algorithms. LMS is the most widely used estimation algorithm because of its simplicity, but has a relatively poor performance with noisy data. The RLS algorithm, on the other hand, is more complex than the LMS algorithm, but provides a much faster rate of convergence under such conditions. Both of these algorithms will be discussed in detail below.
3.1.1 Least-Mean Square Algorithm

The least mean square (LMS) filter method is a single-step identification technique, which resembles a RLS filter, yet it is considerably less complex in its implementation. The LMS approach begins by assuming that corrections to the estimate should be made proportional to the square of the identified error. The steepest descent approach refers to this minimisation process [Widrow, 1995]. A useful graph to explain the steepest descent approach for a one-dimensional system is shown in Fig. 3.1.

![Figure 3.1 One dimensional view of a LMS gradient](image)

In this figure, the estimate of the parameter, $x$ is plotted on the horizontal axis and the square of the corresponding identification error $\varepsilon$ is plotted on the vertical axis to form a bucket shaped curve around the value of $x$ having the minimum identification error. It can be seen that given the slope of the identification error at any arbitrary starting value of $x$, the amount of correction to be made to the estimate is proportional to the negative of the slope of the identification error.

$$x(k + 1) = x(k) - \mu \left[ \frac{\partial \varepsilon^2}{\partial x} \right]$$  (3.1)

This states that, the correction to the estimate is proportional to and opposite to the gradient of the squared estimation error $\varepsilon^2$ with respect to $x$. The identification problem is now to find a good value of $\mu$ and the gradient. If $\mu$ is too large, the identification process may become divergent, and if too small, it may produce very slow convergence. The single-step LMS filter is not only computationally a fast identification method [Jacklin, 1998], but is also easy to implement, because it needs only one tuning parameter.
If the linear least-squares problem involves using a set of measurements, $z$ which are linearly related to the unknown quantities $x$ by the expression

$$z = Hx + v$$

(3.2)

where $v$ is a vector of measurement noise. The identification problem is to determine $\hat{x}$, given knowledge of the information matrix $H$ and the measured output $z$ that minimises the sum of the squares of the elements of $z - H\hat{x}$. The goal is to find an estimate of the unknown $x$, which is denoted by $\hat{x}$ given by the vector difference equation

$$\varepsilon = z - H\hat{x}$$

(3.3)

Since the $x$ will typically vary with the satellite operating conditions e.g. fuel consumption, CoG, thrust variation, etc., the method used for identification of the vector $\hat{x}$ should be an on-line method which can track or estimate the $\hat{x}$ based on the information contained in the most recent information matrix $H$ and measured output $z$. The LMS filter method makes no assumption other than that the measurement error have a normal distribution about the mean broadband measurement noise. The approach used by the LMS (and basically also by other identification methods used) is to find the estimate of $\hat{x}$ which minimises the errors produced between the measured output $z$ and the estimated output $H\hat{x}$ in Eq. (3.3). The approach used by the LMS method is to minimise the sum of the squares of the error (the error is raised to an even power so that positive and negative errors are treated uniformly). By taking the dot product of the error vector, a scalar identification index $J_{id}$ can be formed as

$$J_{id} = \varepsilon^T\varepsilon$$

(3.4)

Recall that the vector inner product generates the sum of squares of a vector. Thus, we wish to minimise the scalar function $J_{id}$, where

$$J_{id} = (z - H\hat{x})^T(z - H\hat{x})$$

(3.5)

This type of quadratic error equation is common to all the other methods of system identification e.g. Kalman Filter and RLS (special case of Kalman filter) [Morris, 1999].
To find the value of $\hat{x}$ which minimise the quadratic performance index, Eq. (3.5), the partial derivative with respect to $\hat{x}$ is computed

$$J_{lp} = z^T z - \hat{x}^T H^T z - z^T H \hat{x} + \hat{x}^T H^T H \hat{x}$$

$$\frac{\partial J_{lp}}{\partial \hat{x}} = -2 H^T z + 2 H^T H \hat{x}$$

(3.6)

Note: in the case of LS (least square) or WLS (weighted least square), the partial derivative is set to zero (to find the minimum). The LSE (least square error) estimate of $\hat{x}$ is

$$\frac{\partial J_{lp}}{\partial \hat{x}} = 0$$

$$\hat{x} = (H^T H)^{-1} H^T z$$

(3.7)

Equation (3.6) can be rewritten in the form:

$$\frac{\partial J_{lp}}{\partial \hat{x}} = -2 H^T (z - H \hat{x})$$

(3.8)

By substitution from Eqs. (3.8) into (3.1) and replace $x$ by $\hat{x}$ which give

$$\hat{x}(k + 1) = \hat{x}(k) + 2 \mu H^T (z - H \hat{x})$$

(3.9)

Which is a remarkable computationally efficient algorithm for recursive identification of $\hat{x}$, in which $\mu$ is a diagonal matrix. Eq. (3.9) is the LMS filter for system identification. It is similar to equations of RLS (or Kalman) with the updating gain replaced by $2\mu H^T$.

To analyse the convergence properties of the LMS filter, the expected value of Eq. (3.9) is taken as:

$$E[\hat{x}(k + 1)] = E[\hat{x}(k)] + 2 \mu E[H^T z] - 2 \mu E[H^T H \hat{x}]$$

(3.10)

Then, by defining

$$E[H^T z] = \Phi_{Hz}$$

(3.11)
\[ E[\mathbf{H}^T \mathbf{H}] = \Phi_{HH} \]

Equation (3.10) may be written

\[
E[\hat{x}(k+1)] = E[\hat{x}(k)] + 2\mu \Phi_{Hx} - 2\mu E[\hat{x}(k)]\Phi_{HH} \tag{3.12}
\]

\[
E[\hat{x}(k+1)] = E[\hat{x}(k)](I - 2\mu \Phi_{HH}) + 2\mu \Phi_{Hx} \tag{3.13}
\]

From Eq. (3.13) it can be seen that as long as the absolute value of the eigenvalues of the \([I - 2\mu \Phi_{HH}]\) are less than 1, the algorithm is stable. Thus the theoretical stability range for the \(\mu\) (diagonal elements of \(\mu\)) gain elements is

\[
0 < \mu < \frac{1}{\lambda_{\text{max}}} \tag{3.14}
\]

Where \(\lambda_{\text{max}}\) is the largest eigenvalue of the information covariance matrix \(\Phi_{HH}\). Values of \(\mu\) near the \(\frac{1}{\lambda_{\text{max}}}\) will cause rapid adaptation, but will also be more prone to tracking random noise disturbance. Good values of \(\mu\) are ones that result in convergence at a sufficiently rapid rate, yet do not track noise signals too closely.

### 3.1.2 Recursive Least Square Algorithm

The LMS algorithm is a simple algorithm that is easy to implement and has a low computational complexity. However, the algorithm requires a very large convergence time, especially when the eigenvalue spread of the covariance matrix is large. In instances where fast convergence is needed, it is necessary to use estimation algorithms that are more complex than the LMS algorithm. The recursive least squares (RLS) algorithm is another estimation algorithm that uses the mean-square error as its cost function Eq. (3.16). It is more complex than the LMS algorithm, but converges much more quickly.

The derivation of the RLS algorithm is too long and cumbersome to be included in this thesis. For a detailed description of the algorithm see [Marco, 1994]. The basic idea behind the RLS algorithms is to find the minimum of a deterministic sum of squared errors by using the method of least squares to obtain an initial estimate and upon receiving new data to use this information to update the estimate.
Denoting by \( \mathbf{0}(t) \) a matrix of unknown parameters, consider the following system

\[
\mathbf{z} = \mathbf{\phi}^T \mathbf{\theta} + \mathbf{v}
\]  

(3.15)

where \( \mathbf{z} \) is the observed measurement, \( \mathbf{\phi} \) is the regression vector, \( \mathbf{\theta} \) is the unknown parameter matrix and the model of Eq. (3.15) is also called a regression model.

The error function to be minimised in RLS is generally defined as an exponentially weighted sum of square error and is given by:

\[
V(\theta, k) = \frac{1}{2} \sum_{t=1}^{k} \lambda^{k-t} \mathbf{e}_t^2
\]  

(3.16)

where \( \lambda \) is a positive constant generally chosen close to one and the error to be minimised can be written as:

\[
\mathbf{e} = \mathbf{z} - \mathbf{\phi}^T \mathbf{\theta}
\]  

(3.17)

The full RLS algorithm will be given following the nomenclature of Åström, [1989]

- Compute the regression vector \( \mathbf{\phi}(k) \) and the error \( \mathbf{e}(k) \) from Eq. (3.17)
- Compute the update gain vector

\[
\mathbf{K}(k) = \mathbf{P}(k-1)\mathbf{\phi}(k)[\lambda + \mathbf{\phi}^T(k)\mathbf{P}(k-1)\mathbf{\phi}(k)]^{-1}
\]  

(3.18)

Where, \( \mathbf{P} \) is defined as the covariance matrix of the regression vector \( \mathbf{\phi}(k) \)

- Update the parameter vector

\[
\mathbf{\theta}(k) = \mathbf{\theta}(k-1) + \mathbf{K}(k)\mathbf{e}(k)
\]  

(3.19)

- Update the covariance matrix

\[
\mathbf{P}(k) = (\mathbf{I} - \mathbf{K}(k)\mathbf{\phi}^T(k))\mathbf{P}(k-1) / \lambda
\]  

(3.20)
Eqs. (3.18) and (3.20) both use the constant, $\lambda$, in their calculation. This constant is known as the forgetting factor of the algorithm. It determines how much past measurements will affect the current estimate. It is usually set to 1 for time-invariant parameters and a value less than, but close to 1, for time-variant parameters, usually in the range $0.8 < \lambda < 1$. The smaller the forgetting factor, the faster the tracking. However, if $\lambda$ is made too small, it can make the algorithm unstable. For this reason it is usually kept larger than 0.8.

The recursive Eqs. (3.19) and (3.20) are initialised with given (deterministic) vector $\theta(0)$ and matrix $P(0) = P^T(0) > 0$ respectively.

Processing the error to remove low frequency components and outlying values further modifies the above general RLS procedure. To improve the robustness of the RLS algorithm, the error can be modified by a non-linear saturation function [Steyn, 1995] as follows:

$$f\{e(k)\} = \frac{e(k)}{1 + b|e(k)|}$$  \hspace{1cm} (3.21)

The constant $b$ is defined such that the function is still linear for normal values of $e(k)$, while decreasing the influence of large outliers.

### 3.2 Attitude Determination

The attitude of UoSAT-12 was estimated using a quaternion based extended Kalman filter (qEKF), see Fig. 3.2. This filter uses measurement vectors (in the body frame) from all the attitude sensors and by combining them with corresponding modelled vectors (in a reference frame), it estimates the attitude and angular rate values of the satellite [Steyn, 1999]. A 7-element discrete state vector to be estimated, is defined as:

$$x = [\omega^T_B (k) \hspace{1cm} q^T (k)]^T$$  \hspace{1cm} (3.22)
With \( \alpha_i \) the inertial referenced angular rate vector and \( q \) the orbit referenced quaternion vector. The attitude sensors (magnetometer, sun, horizon etc.) will be used to determine the attitude of the satellite relative to the orbital frame. When using magnetic field data: a GPS receiver or an orbital propagator is used to obtain the position of the satellite. Using this position data, a model of the geomagnetic field, the International Geomagnetic Reference Field (IGRF) model, computes the geomagnetic \( B \)-field in orbit coordinates. On the other hand, the magnetic \( B \)-field is also measured by the 3-axis magnetometer in body coordinates. The attitude can then be solved from these two vectors over time.

The innovation value used in the EKF is the vector difference between the measured body referenced vector and a modelled orbit referenced vector, (see Fig. 3.3), transformed to the body frame by the estimated attitude transformation matrix.

\[
e(k) = \mathbf{v}_{\text{meas}}(k) - \mathbf{A}(q(k))\mathbf{v}_{\text{orb}}(k)
\]  

(3.23)

Where,

\[
\mathbf{v}_{\text{meas}}(k) = \mathbf{B}_{\text{meas}}(k) / \|\mathbf{B}_{\text{meas}}(k)\| \\
\mathbf{v}_{\text{orb}}(k) = \mathbf{B}_{\text{orb}}(k) / \|\mathbf{B}_{\text{orb}}(k)\|
\]

for the magnetometer measurement and the IGRF modelled vector.
The same method is applied for all the attitude sensors, to supply a measurement and modelled vector pair to the filter. Various measurement noise covariance values are used in the Kalman filter to put different weighting factors to the less accurate (e.g. magnetometer) and the more accurate (e.g. star sensor) innovations. The highest attitude measurement accuracy is obtained from a dual set of opposite looking star sensors. The sensors supply star measurement vectors and matched star catalogue vectors at a rate of once per second to an attitude and rate estimation filter.

3.3 Acceleration Estimator

In order to accurately estimate the spacecraft inertia matrix and cold gas thruster coefficients, the dynamics of the satellite must be known. Modelling the measurement equations requires knowledge of the spacecraft acceleration states which form part of the dynamic equations. Neglecting the acceleration term in the measurement equation leads to incorrect estimates. Unfortunately, there is no sensor to directly measure the angular acceleration on UoSAT-12. In order to estimate the angular acceleration vector for the satellite at each time step, a particular form of a recursive estimator is needed. Such an estimator can be used based on the following assumptions:

- The dynamics of the satellite can be modelled.
- Measurements of the satellite’s angular rate with direct or indirect information can be made.
The performance and some physical insight in this acceleration estimator can be obtained by considering a single axis control rotation. For example, if we take a single inertially referenced rotation around the body X-axis, the dynamic equation can be written as:

\[ I_{xx} \ddot{\phi} = N_{tot} \quad (3.24) \]

Where,
\[
\dot{\phi} = \omega_x = \text{inertial angular rate in X-axis} \\
\ddot{\phi} = \dot{\omega}_x = \text{angular acceleration in X-axis} \\
N_{tot} = \text{total torque applied to X-axis}
\]

If \( \omega_x \) is measured using attitude sensors, we can obtain \( \dot{\omega}_x \) (see Fig. 3.4) using the first order estimator given by:

\[
\dot{\omega}_x = \frac{N_{tot}}{I_{xx}} + k_o (\omega_{\text{mean}} - \dot{\omega}_x) \quad (3.25)
\]

Where,
\[
\omega_{\text{mean}} = \text{measured or estimated angular rate in X-axis by an attitude sensor or EKF} \\
k_o = \text{scalar gain} \\
I_{xx} = \text{moment of inertia of X-axis} \\
\dot{\omega}_x = \text{means estimate value}
\]

The second term in Eq. (3.25) is the error term, proper selection the value of \( k_o \) will minimize the difference between the measured and estimated angular rate.

![Figure 3.4 First order acceleration estimator](image)
By taking the Laplace Transform of Eq. (3.25) gives:

\[
s\dot{\omega}_x(s) = \frac{N_{snt}(s)}{I_{xx}} + k_a (\omega_{smn}(s) - \dot{\omega}_x(s))
\]

\[
\dot{\omega}_x(s) = \frac{1}{s + k_a} \frac{N_{snt}(s)}{I_{xx}} + \frac{k_a}{s + k_a} \omega_{smn}(s)
\]  \hspace{1cm} (3.26)

This equation represents a first order estimator, the value of \(k_a\) will be chosen to place the estimator pole at \(s = -\delta\) where \(\delta = \text{bandwidth of the system.}\) The time constant of the system is given by:

\[
\tau = \frac{1}{k_a}
\]  \hspace{1cm} (3.27)

For small values of \(k_a\), which correspond to a pole on the real axis in the left half plane (LHP) near the origin, the time constant is large and the system is a slow-responding system. On the other hand, for large \(k_a\), which correspond to a pole on the real axis far into the LHP, the time constant is small and it is a quick-responding system.

In the discrete form, Eq. (3.25) can be written in the form:

\[
\dot{\omega}_{plat}(k) = Q(k) + k_a (\omega_{smn}(k) - \dot{\omega}_x(k))
\]  \hspace{1cm} (3.28)

Where,

\[
Q(k) = \frac{N_{snt}(k)}{I_{xx}}
\]

\(\dot{\omega}_x(k)\) can be obtained by integrating \(\dot{\omega}_{plat}(k)\) and given by:

\[
\dot{\omega}_x(z) = \frac{\Delta t}{z - 1} \dot{\omega}_{plat}(z)
\]  \hspace{1cm} (3.29)

By substituting from Eqs. (3.29) into (3.28)
This equation represents a first order estimator, the value of \( k_a \) will be chosen to place the estimator poles within a unit circle. For stability, the value of \( k_a \) can be obtained as:

\[
0 < k_a \leq \frac{1}{\Delta t}
\]  

(3.31)

The estimator is stable for all positive values of \( k_a \), up to \( \frac{1}{\Delta t} \), which corresponds to a pole inside the unit circle, near the origin where the system is a quick-responding system. On the other hand, for small \( k_a \), which corresponds to a pole inside the unit circle near unity, it is a slow responding system.
Chapter 4

4. On Orbit Thruster Calibration for UoSAT-12

4.1 Introduction

Thrusters or gas jets produce torque by expelling mass. They can be adopted to control attitude, manoeuvre spacecraft over large angles, adjust orbits, dump extra momentum from a momentum wheel, reaction wheel or control moment gyro, control the spin rate and control nutation of momentum bias.

Spacecraft equipped with thrusters can present significant disturbance torques as well as large control coupling torques if the attitude control thrusters of a spacecraft are not calibrated properly. Thruster calibration is also important to accurately estimate and preserve fuel usage to maximise the spacecraft's life. The on-line calibration of attitude control hardware is also necessary to satisfy the high accuracy requirement sometimes needed for ADCS. Accurate calibration of the thrusters on the ground prior to flight is limited by various factors. Among these is the difficulty of maintaining a suitable vacuum in a vacuum chamber during thrusters' calibration while continuously discharging gas from the thrusters. Commonly, it is difficult to predict and establish during ground testing the exact temperature and pressure conditions under which the thrusters will operate while in orbit. The output direction of the thrust will not necessarily be concentric with the nozzle. Measuring the true direction and magnitude parameters of thruster force are very difficult on the ground especially considering plume impingement, which will further affect the net direction of a thruster force output. If they are not properly identified in orbit, significant attitude control errors can result.

Modern estimation theory and system identification techniques provide the possibility to implement on-board calibration procedures for attitude actuators. Although this will increase the software complexity of the on-board computer, it will provide better knowledge about the ADCS performance after launch in order to improve the future
design of accurate attitude controllers. The great advantage of cold gas thrusters is that they can provide large, instantaneous torques at any instant during the orbit. If we need accurate attitude control then a calibrated thruster is needed at all times. This chapter presents an analysis and performance comparison of two methods to calibrate the thrusters in orbit. Two methods of estimation techniques are used (Recursive Least Square and Least Mean Square) and two different types of reaction wheel torque (PRBS and Bang-Bang) are used to disturb the satellite attitude in order to compare the two methods to choose the best techniques to calibrate thrusters in orbit.

4.2 The Cold Gas System of UoSAT-12

4.2.1 Introduction

A cold gas propulsion system is used for attitude and orbit control onboard a small satellite. Cold gas thrusters are the simplest way of achieving thrust. Such a system consists firstly of a tank containing pressurised gas. The energy of the cold gas propulsion system comes from the potential energy contained in the high-pressure gas [Roland, 1995].

![Figure 4.1 Cold-gas thruster system architecture](image)
A typical system configuration is shown in Fig. 4.1. A working fluid, e.g. compressed nitrogen, is stored at high pressure (nominally > 200 bar). It is then regulated down to some operating pressure (around 10 bar). The nozzle is normally integral to the control valve. Opening the valve releases the gas to be expelled from the nozzle producing the thrust. Thrust levels for cold gas systems are practically limited by the maximum operating pressure of the control valve. Typically thrusters operate at levels below 1 Newton. A similar system was implemented on UoSAT-12, the first minisatellite designed and manufactured at SSTL.

4.2.2 Cold Gas System Architecture

The structure of the cold gas system for the minisatellite UoSAT-12 is shown in Fig. 4.2. Nitrogen for the cold gas system is stored in three tanks to provide a total volume of 27 litres at a storage pressure of about 200 bar. The tanks are filled with nitrogen by using a single fill/drain valve. The pressure of the tanks is monitored by the tank pressure transducer, which is designed for pressures of up to 300 bars [Semmler, 1997].

Assuming that nitrogen can flow from the tanks to the accumulators, the gas has to pass a filter of 7-micron size. In order to restrict the amount of gas that flows to the accumulators, the Lee Visco jet limits the nitrogen gas flow [Sellers, 1996].

Two solenoid valves are implemented in the system to control the filling of the two accumulators that operate at a pressure of about 5 bars. Although one solenoid valve is sufficient, an additional one is applied for redundancy. Due to the fact that the mechanical errors are estimated to be higher than the expected electrical ones, the solenoid valves are connected in series as mechanical errors usually cause the valve not to close properly. The result of an electrical failure would normally lead to valves not to be energised. Since an uncontrollable gas flow from the tanks to the accumulators would cause a disastrous effect on the satellite because the accumulator is designed for a maximum pressure of 13 bar, a relief valve is incorporated in the system. The safety valve opens at an accumulator pressure of 12 bar with respect to the ambient pressure (the ambient pressure for the LEO is almost zero). Eight cold-gas thrusters will be used for attitude control and two thrusters for orbit control.
Figs. 4.2 to 4.7 show the cold gas system hardware on UoSAT-12. Table 4.1 shows the most important parameters (from an AODCS view point) of the propulsion system on UoSAT-12.

Figure 4.2 UoSAT-12 cold-gas thruster system architecture
Table 4.1 Cold gas system parameters

<table>
<thead>
<tr>
<th>Performance Parameter</th>
<th>N₂ Cold Gas</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel Mass</td>
<td>7.1</td>
<td>kg</td>
</tr>
<tr>
<td>Specific Impulse $I_{sp}$</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>Delta V (adjust orbit velocity)</td>
<td>14.0</td>
<td>ms⁻¹</td>
</tr>
<tr>
<td>Fuel Lifetime</td>
<td>42 000</td>
<td>sec</td>
</tr>
<tr>
<td>Pulses per thruster (minimum)</td>
<td>20 000</td>
<td></td>
</tr>
<tr>
<td>Minimum Pulse</td>
<td>&lt;0.02</td>
<td>sec</td>
</tr>
<tr>
<td>Average Thrust</td>
<td>0.15</td>
<td>N</td>
</tr>
</tbody>
</table>

Fig. 4.4 shows the location of thrusters on UoSAT-12. The thrust arm of the pitch, roll and yaw thrusters to the CoM of UoSAT-12 is approximately 0.44 meter each, this gives a torque of 66 milli-Nm for attitude control. The Z-axis (yaw) control thrusters will, however, always be fired in an opposing pair to give a pure rotation without any translation forces, so the Z-axis thruster torque will be 132 milli-Nm per dual pulse. The X/Y-axis (roll and pitch) thrusters will be fired as single units for short periods of time and will therefore present a small translation disturbance to the orbit.

Figure 4.3 UoSAT-12 propulsion system
Figure 4.4 Location of UoSAT-12 thrusters

Figure 4.5 SpaceUoSAT-12 space facing facet
Figure 4.6 Three way thruster assembly

Figure 4.7 Two way thruster assembly
4.3 Reaction Thruster Attitude Control

4.3.1 Introduction

Reaction thrusters used in attitude control are mostly activated in a pulsing mode. The author is not aware of any linear, continuous reaction thrust controllers. Unlike other actuators, such as reaction wheels or control moment gyros, the thruster output consists of binary states: on or off. This fact somehow complicates the analytical treatment of attitude control systems when used as torque controllers. Proportional thrusters, whose fuel valves open a distance proportional to the commanded thrust level, are not employed much in practice. Mechanical considerations prohibit proportional valve operation, largely because of drift particles that can prevent complete closure for small valve openings. Fuel leakage through valves consequently causes opposing thruster firings and a large wastage of fuel. Pulse modulation techniques have been developed that fully open and close the fuel valves, while producing a nearly linear duty cycle. In general, pulse modulators produce a pulse command sequence to the thruster valves by adjusting the pulse width. Most attitude control laws calculate the actuator control torques to be applied about the body axes. In Section 4.3.5 we show how to transform the command control torque about the body axes to reaction thruster pulses. This transformation is made complicated for two reasons:

1. Reaction thrusters are not linear actuators, since the level of output thrust is mostly constant. Consequently, the equivalent torque that the thruster will produce depends on the time period during which the thruster is activated.
2. A thruster is capable of producing an one-signed torque only. In order to achieve a torque about the same axis with the opposite sign, a different thruster must be activated about the same axis in the opposite direction.

These two factors complicate the algorithm that transforms a body torque command into thruster pulse commands.

The performance of an attitude control system using propulsion torque controllers is strongly influenced by the placement and specification of the reaction thrusters. In general, six thrusters are needed to allow attitude manoeuvres in space, although some
sophisticated systems claim to achieve the same space manoeuvres with only four thrusters, strategically located on the satellite body [Marcel, 1997]. For various practical reasons, six or more thrusters are mostly necessary to complete a reaction control system.

The level of torque that a reaction thruster can apply about a satellite axis depends not only on its thrust level, but also on the torque-arm length about the axis. This statement suggests that correct thruster use depends primarily on its location in the satellite, and also on its direction relative to the satellite body axes. The location and direction of the thrusters are also influenced by the location of other subsystems and the solar panels. Fig. 4.4 shows a potential arrangement of the UoSAT-12 satellite thrusters. First, it is clear that they provide both positive and negative control torques about each of the satellite's body axes. Thrusters \( Th1, Th3, Th4 \) and \( Th6 \) apply pure positive and negative torques about the \( Z \) body axis. The \( X/Y \)-axis \( (Th2, Th5, Th7 \) and \( Th8) \) thrusters apply positive and negative torques for short periods of time, which presents a small transitional disturbance to the orbit.

4.3.2 The Pulse Width Modulator (PWM)

The output of the pulse-width modulator device is the thruster pulse width command (Fig. 4.8). A zero order hold (ZOH) device models this discrete signal to the thrusters. The value \( p_1 \) in Fig. 4.9 represents the minimum pulse width of the system; the dead zone is

![Figure 4.8 Attitude controller using thruster](image-url)
directly proportional to the attitude control dead band. The value $p_2$ represents the maximum pulse of the reaction control system (RCS), it is related to the discrete control system sampling time.

![Figure 4.9 Pulse Width Modulator (PWM)](image)

Note that the minimum pulse width will be a function of spacecraft parameters: e.g. the spacecraft inertia and thrust level. These parameters tend to change over time; as a result, the minimum pulse width may vary as well. Knowledge of the spacecraft properties is therefore required to estimate the minimum thruster pulse width.

### 4.3.3 Performance Parameter

An ideal thruster produces a constant, known torque on the spacecraft instantaneously in response to a thruster command signal. A real thruster, however, differs from this ideal in the following respects:

The parameters to characterise the drive pulse fed to the thruster and the subsequent thrust response is shown graphically in Fig. 4.10. They define various command and response terms associated with a thruster reaction control system in which a half-square wave command pulse is issued.

There is usually a lag time associated with both the rising and falling edges of the thrust pulse, relative to the rising and falling edges of the commanded signal. In a real reaction
control system, there can also be oscillation associated with the rising and falling edges of
the thrust pulse. The rise lag (RL) in Fig. 4.10 is defined as the elapsed time from the
leading edge of the drive (or control) pulse to the initiation of measurable thrust (i.e.
opening of the propellant valve). This is due to the exponential rise of solenoid current to
a value sufficient to open the valve. This is a measure of the electromechanical efficiency
of the actuator.

![Diagram showing control system and response times](image)

**Figure 4.10 Command and response transient performance definition**

Where,
- **RL** = rise lag time
- **RT** = rise time
- **DPW** = drive pulse width
- **RPW** = response pulse width
- **FL** = fall lag time
- **FT** = fall time
- **DPA** = drive pulse amplitude
- **RPA** = response pulse amplitude

The response pulse width (RPW) is defined as the base-to-base time of the response thrust
profile, which equals the elapsed time from thrust initiation to thrust termination for an
actuator responding to the minimum amplitude half-square wave command signal required
to initiate measurable thrust. The RPW is a useful measure of the absolute shortest
impulse that can be produced by a reaction control system.
If one views rise lag as electromechanical propagation delay, then one might expect the response pulse to terminate in a time interval equal to the drive pulse width (DPW) following initiation of measurable thrust from the actuator. In fact, this proves not to be the case. The difference in elapsed time from initiation of measurable thrust to the point at which thrust commences to decrease minus the DPW, constitutes a parameter known as fall lag (FL).

The steady state thrust may differ from that assumed by up to 10 %, due to manufacturing tolerance, this is indicated in Fig. 4.10 as thrust mismatch.

### 4.3.4 Calculating the Torque Components of a Single Thruster

If the thrust vector is \( \mathbf{F} \), then the torque about the centre of mass (CoM) of the spacecraft is given by:

\[
\mathbf{N}_x = \mathbf{r} \times \mathbf{F}
\]

Where,

\[
\mathbf{r} = \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix} = \text{vector distance of the thruster from the centre of mass}
\]

\[
\mathbf{F} = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}^T = \text{thrust level vector}
\]

The torque components applied by a thruster about each axis are a function of the thruster's location and direction, denoted in Fig. 4.11 by the azimuth and elevation angle \( \alpha \) and \( \beta \) respectively. Suppose that initially \( \mathbf{F} \) is in the direction of \( X_g \). After two rotations first about the \( y \) axis of the thruster by elevation angle \( \beta \), and then about the \( z \) axis of the thruster by azimuth angle \( \alpha \), we find that the components along the body axes are

\[
F_x = F \cos \alpha \cos \beta \\
F_y = F \sin \alpha \\
F_z = F \cos \alpha \sin \beta
\]
The position vector $\mathbf{r}$ of the thruster can be expressed as:

$$
\mathbf{r} = r_x \hat{i} + r_y \hat{j} + r_z \hat{k} 
$$

(4.3)

Figure 4.11 Thrust direction of single thruster

By substituting from Eqs. (4.2) and (4.3) into Eq. (4.1) the thruster torque components are:

$$
N_T = \begin{bmatrix}
N_{T_x} \\
N_{T_y} \\
N_{T_z}
\end{bmatrix} = \begin{bmatrix}
r_y \sin \beta \cos \alpha - r_z \sin \alpha \\
r_z \cos \beta \cos \alpha - r_y \cos \alpha \sin \beta \\
r_x \sin \alpha - r_y \cos \alpha \cos \beta
\end{bmatrix}
\begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z
\end{bmatrix} = F
$$

(4.4)

Eq. (4.4) establishes the equivalent torque arms $\Delta x, \Delta y, \Delta z$ of the thrust $\mathbf{F}$ about the body axes.
4.3.5 Torque Command Versus Thruster Activation Time

The principal of pulse width modulation will be used in UoSAT-12 to transform the torque commands into time activated pulses for the relevant thruster. The UoSAT-12 thrusters are arranged so that they provide the necessary torques for attitude control about the three body principal axes (8 thrusters) as well as the necessary thrust for orbit control (2 thrusters). Thrusters 7 and 8 provide the positive and the negative pitch control torques (respectively) about the \( Y_b \) body axis; thrusters 2 and 5 provides positive and negative roll control torques (respectively) about the \( X_b \) axis, thruster 1 and 4 provide the positive yaw control torques about the \( Z_b \) body axis, thruster 3 and 6 provide the negative yaw control torques about the \( Z_b \) body axis and thrusters 9 and 10 provides the positive and the negative control torque (respectively) to the orbit. Formally

\[
\begin{align*}
T_{r+} &= Th_1 + Th_4 \\
T_{r-} &= Th_3 + Th_6 \\
T_{r+} &= Th_2 \\
T_{r-} &= Th_5 \\
T_{p+} &= Th_7 \\
T_{p-} &= Th_8 \\
T_{\Delta y +} &= Th_9 \\
T_{\Delta y -} &= Th_{10}
\end{align*}
\]

Here the + and - signs indicate the sign of the produced torques about the body axes, and \( Th_i \) denotes thruster \( i \). In order to simplify the analysis, the thrusters are located symmetrically about the body axes, with equal arms about the same axes. The torque of each thruster depends on the thrust level \( F \) and also on the torque arms \( \Delta x, \Delta y, \Delta z \). Eq. (4.4). Then we can define the torque components about the body axes as follows:

\[
\begin{align*}
T_r &= [Th_1 + Th_4 - Th_3 - Th_6]F\Delta z \\
T_r &= [Th_2 - Th_5]F\Delta x \\
T_r &= [Th_7 - Th_8]F\Delta y \\
T_{\Delta y} &= [Th_9 + Th_{10}]F\Delta y
\end{align*}
\]

Since the produced torques depend on the thrust level and also on the torque arm, the average torque provided during a sampling time depends upon the time that the thrusters are on, relative to the sampling time \( \Delta t \). If we normalised the body torque Eq. (4.6) to
\[ \bar{T}_y = \frac{T_y}{F \Delta z} \]
\[ \bar{T}_R = \frac{T_R}{F \Delta x} \]
\[ \bar{T}_p = \frac{T_p}{F \Delta y} \]
\[ \bar{T}_{\Delta V} = \frac{T_{\Delta V}}{F \Delta y} \]  

and define \( \Delta t_i \) as the ratio between the thruster on time and the sampling time for thruster \( \text{Th}_i \). Thus Eq. (4.6) can be written in the following form:

\[ \begin{bmatrix} \bar{T}_y \\ \bar{T}_R \\ \bar{T}_p \\ \bar{T}_{\Delta V} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \Delta t_1 \\ \Delta t_2 \\ \Delta t_3 \\ \Delta t_4 \\ \Delta t_5 \\ \Delta t_6 \\ \Delta t_7 \\ \Delta t_8 \\ \Delta t_9 \\ \Delta t_{10} \end{bmatrix} \]  

(4.8)

Because the matrix of Eq. (4.8) is not square, it does only have a right pseudoinverse:

\[ \begin{bmatrix} \Delta t_1 \\ \Delta t_2 \\ \Delta t_3 \\ \Delta t_4 \\ \Delta t_5 \\ \Delta t_6 \\ \Delta t_7 \\ \Delta t_8 \\ \Delta t_9 \\ \Delta t_{10} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{T}_y \\ \bar{T}_R \\ \bar{T}_p \\ \bar{T}_{\Delta V} \end{bmatrix} \]  

(4.9)
4.3.6 Relationship between Commanded and Actual Torque for UoSAT-12

The location of the thruster positions onboard UoSAT-12 is illustrated in Fig. 4.4. The position of each thruster and its force direction vector are assumed known. For the ideal alignment case (i.e. azimuth and elevation angles $\alpha$ and $\beta$ respectively are set to zero in Eq. 4.2), this means that the thruster vector is parallel to body axis. If the thruster’s $i$ position is known in the body coordinates as $\mathbf{r}_i$, then the torque of each thruster with force direction vector $\mathbf{F}_i$ can also be written as: (similar to Eq. 4.1)

$$\mathbf{N}_{ti} = \mathbf{r}_i \times \mathbf{F}_i = \begin{bmatrix} N_{Txi} \\ N_{Tyi} \\ N_{Tzi} \end{bmatrix} = \mathbf{a}_c T_i$$  \hspace{1cm} (4.10)

where

$$\mathbf{a}_c = \begin{bmatrix} a_{xi} & a_{yi} & a_{zi} \end{bmatrix}^T = \text{thruster i calibration coefficients}$$

$$T_i = \text{expected (ideal) thruster i torque magnitude}$$

$$\mathbf{a}_e = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$  \hspace{1cm} (4.11)

Each of the components of $\mathbf{w}$ in Eq. (4.11) is a function of either scale factor variation $\Delta s$ or angle variations $\Delta \alpha$, $\Delta \beta$.

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} \Delta s \\ \Delta \alpha \\ \Delta \beta \end{bmatrix}$$  \hspace{1cm} (4.12)
The variation in scale factor $\Delta s$ is defined as the amount that the magnitude of $a_c$ varies from unity, $\Delta s = \|a\| - 1$. The significance of Eq. (4.12) is that it allows the variations in the thruster coefficients $w$ to be expressed in terms of parameters $\Delta s$, $\Delta \alpha$ and $\Delta \beta$, which have physical meaning. The variation in thruster scale factor $\Delta s$ can be the result of variations in temperature, pressure or specific impulse. The angle variations $\Delta \alpha$ and $\Delta \beta$ can be a result of thruster misalignment, non-concentric flow through the thruster nozzle, or plume impingement.

The relationship between the commanded torque and the resulting torque applied on UoSAT-12 from the 10 cold-gas thrusters can be written as:

$$
\begin{bmatrix}
T_{T_x} \\
T_{T_y} \\
T_{T_z}
\end{bmatrix}
= \begin{bmatrix}
[N_{T_x}] \\
[N_{T_y}] \\
[N_{T_z}]
\end{bmatrix}
= \begin{bmatrix}
a_{x1} & a_{x2} & a_{x3} & a_{x4} & a_{x5} & a_{x6} & a_{x7} & a_{x8} \\
a_{y1} & a_{y2} & a_{y3} & a_{y4} & a_{y5} & a_{y6} & a_{y7} & a_{y8} \\
a_{z1} & a_{z2} & a_{z3} & a_{z4} & a_{z5} & a_{z6} & a_{z7} & a_{z8}
\end{bmatrix}
\begin{bmatrix}
T_{T_x} \\
T_{T_y} \\
T_{T_z}
\end{bmatrix}
$$

Rewrite Eq. (4.13) in matrix form as follows:

$$
N_T = A_c T_c
$$

(4.14)

Where,

$N_T = 3 \times 1$ vector composed of three components of thruster torque

$T_c = 8 \times 1$ expected thruster torque command vector magnitude on UoSAT-12

$A_c = 3 \times 8$ thruster configuration matrix which contain information on the thrust direction and scaling values
4.4 On Orbit Thruster Calibration for UoSAT-12

4.4.1 Preliminaries

The dynamical model of a satellite, using reaction wheels as internal torque actuators and thrusters as external torque actuators, is:

\[ I\dot{\omega}_b = N_D + N_T - \omega_b \times (I\omega_b + h) - \dot{h} \]  

(4.15)

Where,

\[ \omega_b = [\omega_x \quad \omega_y \quad \omega_z]^T \] = inertially referenced body angular velocity vector

\[ I = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} \] = moment of inertia tensor of spacecraft (MOI)

\[ h = [h_x \quad h_y \quad h_z]^T \] = reaction wheel angular momentum vector

\[ N_D = [N_{d1} \quad N_{d2} \quad N_{d3}]^T \] = external disturbance torque vector

\[ N_T = [N_{T1} \quad N_{T2} \quad N_{T3}]^T \] = applied torque vector from 3-axis thrusters

Wiktor 1996 had previously used estimation techniques to calibrate the thrusters of a stabilised satellite in orbit (see Section 1.1.3). This approach used changing mass balance to provide calibration torques and was not able to work in the presence of significant disturbance torques. The new method developed at SSTL uses knowledge of a calibration torque (generated using reaction wheel actuators on UoSAT-12) whilst the attitude is controlled using the gas thrusters.

The in-orbit thruster calibration algorithms implemented in this thesis differ from Wiktor's idea in the following:
1. Two types of estimation technique are used (Recursive Least Square and Least Mean Square) to calibrate the thruster coefficients.

2. Two different types of reaction wheel torques profiles (Bang-Bang and PRBS) are used to disturb the satellite attitude.

3. An analysis and performance comparison of two methods are used to calibrate the thruster coefficients using either one or three experiments.

4. Non-zero mean aerodynamic disturbance torques are used to test the newly implemented estimation algorithms instead of zero mean disturbances.

5. Finally, three criteria will be used to select the best estimation methods and types of reaction wheel torque in order to compare the two methods to choose the best techniques to estimate the thruster coefficients.

### 4.4.2 Calibration Techniques

Figure 4.12 summarises the general in-orbit thruster calibration scheme. A known disturbance torque $N_{w}(k)$ is applied to the spacecraft using reaction wheels controlled by a Bang-Bang control law Eq. (2.27), whilst the attitude controller Eq. (2.24) sends commands $T_r(k)$ to the thrusters (generating torques $N_r(k)$) to compensate for both the known wheel torque $N_{w}(k)$ and also any unknown external disturbance torques $N_d(k)$.

The in-orbit thruster calibration procedure below the dotted line in Fig. 4.12 uses the known disturbance torque $N_{w}(k)$ and the resulting measured satellite state to calculate the calibration torque $N_c(k)$ acting on the satellite. This calibration torque is equal to the sum of the known and the unknown disturbance torques and can be obtained by rearranging Eq. (4.15),

$$N_c = I\omega_g + \omega_g \times (I\omega_g + h) - N_w = N_d + N_r \quad (4.16)$$

Where,

$$N_c = \begin{bmatrix} N_x \\ N_y \\ N_z \end{bmatrix} = \text{calibration torque vector}$$

$$N_w = -h = \text{reaction wheel torque vector}$$
If the satellite attitude is maintained, this torque is being exactly cancelled by the thruster torque given by Eq. (4.14), so to estimate $A^c$, rewrite Eq. (4.14) as a state estimation problem with the measurement equation as,

$$N_c = \hat{A}_c T_c + v$$ (4.17)

Where $v$ is the effect of measurement noise plus the external disturbance torques and $\hat{A}_c$ is the estimated value of the true thruster coefficient matrix $A_c$.

The objective of the thruster calibration procedure is to estimate the mean value $\hat{A}_c$ of the true thruster coefficient matrix $A_c$, which should not change with time, so they can be modeled by the state equation

$$a_{ji}(k + 1) = a_{ji}(k)$$ (4.18)

Where $a_{ji}$ is an element of the estimated thruster coefficient matrix $\hat{A}_c$. The thruster calibration problem can now be stated as follows: given state equation (4.18) together with the measurement equation (4.17), estimate the mean values of the thruster coefficients $\hat{A}_c$.

![Figure 4.12 On-orbit thruster calibration block diagram](image-url)
4.4.3 Calibration Methods

This section presents an analysis and performance comparison of two methods to calibrate the thrusters in orbit. First of these is to disturb the satellite by using 3-reaction wheel actuators, while the attitude is controlled using thrusters to determine all thruster coefficients by using one experiment. The other one is to disturb one axis using one wheel while this axis is controlled using thrusters. The other two axes are controlled using the remaining two wheels to determine the thruster coefficients for the disturbed axis. This method requires three different experiments to determine all thruster coefficients, one for each axis.

Two methods of parameter estimation are used (Recursive Least Square and Least Mean Square) and two different types of reaction wheel torque (PRBS and Bang-Bang) are used to disturb the satellite's attitude in order to compare the two methods to choose the best technique to estimate the thruster coefficients. Three criteria will be used to select the best parameter estimation method and type of reaction wheel torque. These criteria are speed of convergence, estimation accuracy and fuel consumption. The first two quantities are calculated according to the minimum root mean square error before and after convergence, i.e. during the first and last 1000 seconds of the simulation time. The last quantity is obtained by integrating the thruster torque over time. The flowchart in Fig. 4.13 explains the steps of comparison. The root mean square error is calculated as follows:

\[(r\text{ms})_{\text{error}} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (\hat{a}_{ji} - a_{ji})^2}\] (4.19)

Where
\[
\hat{a}_{ji} = \text{estimated value of thruster coefficients}
\]
\[
a_{ji} = \text{true value of thruster coefficients}
\]

To illustrate both methods, a set of three thrusters were used to identify a 3 x 3 calibration matrix \( A_c \). Rewrite Eq. (4.14), which gives
\[ \mathbf{N}_T = \begin{bmatrix} N_{Tx} \\ N_{Ty} \\ N_{Tz} \end{bmatrix} = \begin{bmatrix} a_{x1} & a_{x2} & a_{x3} \\ a_{y1} & a_{y2} & a_{y3} \\ a_{z1} & a_{z2} & a_{z3} \end{bmatrix} \begin{bmatrix} T_{cx} \\ T_{cy} \\ T_{cz} \end{bmatrix} \]

(4.20)

Where,
\( N_T \) = thruster torque vector
\( A_c \) = configuration matrix
\( T_c \) = thruster command vector

The goal of the thruster calibration is to choose the best method, parameter estimation technique and type of reaction wheel torque input. Then to identify these \( 3 \times 3 \)
coefficients during normal mission conditions, when the satellite is stabilised, in spite of sensor noise and external disturbance torques.

4.4.3.1 Determination of Thruster Coefficients by Using One Experiment

It is possible to determine all thruster coefficients in one experiment using the RLS estimation method. By substitution of Eq. (4.20) into Eq. (4.15) the dynamic equations of motion can be described when the satellite is disturbed by 3-reaction wheel actuators, while the attitude is controlled by using quaternion feedback thruster controllers. This can be written as:

\[
I_{xx}\dot{\omega}_x = I_{xx}\dot{\omega}_x + I_{xy}\dot{\omega}_y - \omega_y\omega_z(I_{yy} - I_{yz}) - \omega_x\omega_y I_{yx} + \omega_y I_{xy} + \omega_z I_{zy} + \omega_x I_{xz} - \omega_y I_{yx} - \omega_z I_{yz} - \omega_y h_x - \omega_z h_y - \dot{h}_x + a_{x1}T_{cx} + a_{x2}T_{cy} + a_{x3}T_{cz} \quad (4.21.a)
\]

\[
I_{yy}\dot{\omega}_y = I_{yy}\dot{\omega}_y + I_{yx}\dot{\omega}_x + \omega_x\omega_z(I_{xx} - I_{xz}) + \omega_x\omega_y I_{yx} + \omega_z I_{xz} - \omega_x I_{xy} - \omega_z h_x - \omega_y h_y + \dot{h}_y + a_{y1}T_{cx} + a_{y2}T_{cy} + a_{y3}T_{cz} \quad (4.21.b)
\]

\[
I_{zz}\dot{\omega}_z = I_{zz}\dot{\omega}_z + I_{yz}\dot{\omega}_y + \omega_x\omega_y(I_{yy} - I_{yz}) - \omega_y\omega_z I_{zy} - \omega_x I_{yz} - \omega_y h_x - \omega_z h_y - \dot{h}_y + a_{z1}T_{cx} + a_{z2}T_{cy} + a_{z3}T_{cz} \quad (4.21.c)
\]

Three separate RLS estimation algorithms are needed to estimate all nine elements of thruster coefficients. The algorithm is a recursive implementation of the least squares minimisation technique (see Chapter 3). The error vector to be minimised can be written as:

\[
e(k) = N_e(k) - \hat{A}_e(k)T_e(k) \quad (4.22)
\]

Where,

\[
\hat{A}_e(k) = \begin{bmatrix} a_{x1} & a_{x2} & a_{x3} \\ a_{y1} & a_{y2} & a_{y3} \\ a_{z1} & a_{z2} & a_{z3} \end{bmatrix} = \text{estimated thruster coefficient matrix}
\]
\[
T_c(k) = \begin{bmatrix}
T_{ex} & T_{ey} & T_{ez}
\end{bmatrix}^T = \text{thruster controller output vector}
\]

\[
N_c(k) = \begin{bmatrix}
N_{ex} & N_{ey} & N_{ez}
\end{bmatrix}^T = \text{calibration torque is given by}
\]

\[
N_{ex} = I_{xx} \dot{\omega}_x - I_{yx} \dot{\omega}_y - I_{xz} \dot{\omega}_z + \omega_y \omega_z (I_{yx} - I_{yy}) + \omega_x \omega_z I_{zx} - \omega_y \omega_x I_{xy} - \omega_x^2 I_{xx} \quad (4.23.a)
\]

\[
+ \omega_z^2 I_{zz} - \omega_x h_y + \omega_y h_z + \dot{h}_x
\]

\[
N_{ey} = I_{yy} \dot{\omega}_y - I_{yx} \dot{\omega}_x - I_{yz} \dot{\omega}_z + \omega_x \omega_z (I_{yx} - I_{xx}) - \omega_y \omega_z I_{xy} - \omega_x^2 I_{yx} + \omega_z^2 I_{zz} + \omega_x h_y - \omega_y h_x + \dot{h}_y \quad (4.23.b)
\]

\[
N_{ez} = I_{zz} \dot{\omega}_z - I_{xz} \dot{\omega}_x - I_{yz} \dot{\omega}_y - \omega_x \omega_y (I_{yx} - I_{xz}) + \omega_y \omega_z I_{zy} - \omega_x \omega_z I_{xz} + \omega_z^2 I_{zz} - \omega_x^2 I_{xz} + \omega_y h_x - \omega_z h_x + \dot{h}_z \quad (4.23.c)
\]

If the inertia matrix is diagonal i.e. \( I = \text{diag}(I_{xx}, I_{yy}, I_{zz}) \), then the error vector problem can be divided into three standard scalar least square parameter estimation problems and each is given by:

\[
\varepsilon_i(k) = y_i(k) - \varphi^T(k) \theta_i(k) \quad i = x, y, z \quad (4.24)
\]

Where,

\[
y_x(k) = N_{wx}(k) + I_x \dot{\omega}_x + (I_{xx} - I_{yy}) \omega_y \omega_x + \omega_y h_z - \omega_z h_y
\]

\[
y_y(k) = N_{wy}(k) + I_y \dot{\omega}_y + (I_{yy} - I_{xx}) \omega_x \omega_y + \omega_x h_z - \omega_z h_x
\]

\[
y_z(k) = N_{wz}(k) + I_z \dot{\omega}_z + (I_{zz} - I_{xx}) \omega_x \omega_z + \omega_x h_y - \omega_y h_x
\]

and,

\[
\varphi(k) = T_c(k) = \begin{bmatrix}
T_{ex} & T_{ey} & T_{ez}
\end{bmatrix}^T
\]

\[
\theta_x(k) = [a_{x1} \quad a_{x2} \quad a_{x3}]
\]

\[
\theta_y(k) = [a_{y1} \quad a_{y2} \quad a_{y3}]
\]

\[
\theta_z(k) = [a_{z1} \quad a_{z2} \quad a_{z3}]
\]
4.4.3.2 Determination of Thruster Coefficients by Using Three Experiments

This method disturbs one axis at a time using a reaction wheel, while the corresponding axis is controlled using thrusters. The other two axes are controlled using the remaining two wheels to determine the corresponding three elements of the thruster coefficient matrix. This procedure will then be repeated for the other two axes to determine the other six elements of the thruster coefficient matrix. For example, disturbing the X-axis using the X-wheel, while this axis is controlled using the X-thrusters. The Y/Z axes are controlled using the other two wheels. Substituting Eq. (4.20) into Eq. (4.15), the dynamic equations of motion can be written as:

\[
I_{xx} \ddot{\omega}_x = I_{xx} \dot{\omega}_x + I_{xy} \dot{\omega}_y - \omega_y \omega_z (I_{xz} - I_{yx}) - \omega_x \omega_z I_{yz} + \omega_x \omega_y I_{yx} + \omega^2 y I_{xy} - \omega^2 z I_{yz} + \omega_z h_x - \omega_y h_z = a_{xl} T_{ex}
\]

(4.25.a)

\[
I_{yy} \ddot{\omega}_y = I_{yy} \dot{\omega}_y + I_{yx} \dot{\omega}_x + \omega_x \omega_z (I_{xz} - I_{yx}) + \omega_y \omega_z I_{yz} + \omega^2 z I_{zx} - \omega^2 y I_{xy} - \omega_x \omega y I_{yx} - \omega_y h_x + \omega_z h_z - \dot{h}_y + a_{yl} T_{ey}
\]

(4.25.b)

\[
I_{zz} \ddot{\omega}_z = I_{zz} \dot{\omega}_z + I_{zy} \dot{\omega}_y + \omega_x \omega_y (I_{xy} - I_{yx}) + \omega_x \omega z I_{zx} - \omega_y \omega z I_{yz} + \omega^2 z I_{zx} - \omega^2 x I_{xy} - \omega_x h_y + \omega_y h_z - \dot{h}_z + a_{zl} T_{ez}
\]

(4.25.c)

In this case three separate RLS estimation algorithms are needed to estimate three elements of the thruster coefficient matrix. Consequently nine RLS estimators will be required to estimate all the nine elements of the thruster coefficient matrix. The error to be minimised is similar to Eqs. (4.22) and (4.24) with:

\[
\hat{A}_e(k) = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix} = \text{estimated thruster coefficient matrix}
\]

\[
T_e(k) = \begin{bmatrix}
T_{ex} \\
T_{ey} \\
T_{ez}
\end{bmatrix} = \text{thruster controller output vector}
\]

\[
N_e(k) = \begin{bmatrix}
N_{ex} \\
N_{ey} \\
N_{ez}
\end{bmatrix} = \text{calibration torque}
\]

\[
\varphi(k) = T_e(k) = \begin{bmatrix}
T_{ex} \\
T_{ey} \\
T_{ez}
\end{bmatrix}^T
\]

\[
\theta(k) = \begin{bmatrix}
a_{xl} \\
a_{yl} \\
a_{zl}
\end{bmatrix}, \quad \theta(k) = \begin{bmatrix}
a_{y1} \\
a_{y2} \\
a_{z1}
\end{bmatrix}, \quad \theta(k) = \begin{bmatrix}
a_{x1} \\
a_{x2} \\
a_{z2}
\end{bmatrix}
\]

4-25
4.5 Simulation Results

The UoSAT-12 satellite in a low Earth-orbit is used as an example during these simulations. The simulation parameters are given in Table 4.2. Simulation tests were implemented to investigate the performance of the proposed thruster estimation algorithms. The simulation program consisted of four basic parts: (1) Modelling the satellite dynamics and kinematics, (2) Computation of control command using thrusters, (3) Estimation of satellite angular acceleration and (4) Simulation of the identification methods used to estimate the thruster coefficients (Fig. 4.14). For more detail see Fig. C.4 in Appendix C.

![Simulation Diagram](image)

Figure 4.14 UoSAT-12 simulator during thruster calibration
Table 4.2 Simulation parameters

<table>
<thead>
<tr>
<th>Moment of inertia tensor</th>
<th>$I = \begin{bmatrix} 40.45 &amp; -0.2 &amp; -0.2 \ -0.2 &amp; 42.09 &amp; 0.4 \ -0.5 &amp; 0.4 &amp; 41.36 \end{bmatrix}$ kgm$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orbital parameters</td>
<td>Orbital rate = $2\pi / 6000$ rad/sec</td>
</tr>
<tr>
<td></td>
<td>Inclination $i = 65^\circ$</td>
</tr>
<tr>
<td></td>
<td>Orbital Period = 100 min</td>
</tr>
<tr>
<td>Sample time</td>
<td>1 sec</td>
</tr>
<tr>
<td>Reaction wheel</td>
<td>Maximum torque = 0.015 Nm</td>
</tr>
<tr>
<td></td>
<td>Maximum Momentum = 4 Nms</td>
</tr>
<tr>
<td></td>
<td>Moment of inertia = 0.0077 kgm$^2$</td>
</tr>
<tr>
<td></td>
<td>Maximum speed = 5000 rpm</td>
</tr>
<tr>
<td>Quaternion controller gain</td>
<td>$5% \ t = \frac{3}{\zeta \omega_n} = 180$</td>
</tr>
<tr>
<td></td>
<td>$k_p = 0.0079$ (proportional control gain)</td>
</tr>
<tr>
<td></td>
<td>$k_d = 0.0888$ (derivative control gain)</td>
</tr>
<tr>
<td>Bang-Bang controller gain</td>
<td>$k_1 = 1$ (proportional control gain)</td>
</tr>
<tr>
<td></td>
<td>$k_2 = 2.3$ (derivative control gain)</td>
</tr>
<tr>
<td></td>
<td>$e_{\text{band}} = 0.02$ rad</td>
</tr>
<tr>
<td>High pass filter</td>
<td>$2\text{mHz} = \text{cut off frequency}$</td>
</tr>
<tr>
<td>RLS parameters</td>
<td>$\lambda = 0.992$</td>
</tr>
<tr>
<td></td>
<td>$P(0) = 1e2$</td>
</tr>
<tr>
<td></td>
<td>$A_c(0) = \begin{bmatrix} 1 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 1 \end{bmatrix}$</td>
</tr>
<tr>
<td>LMS parameters</td>
<td>$\mu = 1e1$</td>
</tr>
<tr>
<td></td>
<td>$A_c(0) = \begin{bmatrix} 1 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 1 \end{bmatrix}$</td>
</tr>
</tbody>
</table>
During simulations, we used an unknown external disturbance torque (Fig. 4.15). Note that the level of disturbance torque used during these simulations is of $O(10^{-4})$ while the worst case disturbance torque acting on UoSAT-12 is of $O(10^{-6})$ (see Chapter 2). This was implemented to ensure that the estimation process is robust against external disturbance torques. Simulations were performed using MATLAB and SIMULINK. Both estimation methods (RLS and LMS) were implemented using a full simulation of satellite dynamics, sensors and environmental models. As explained in Section 4.4.3, the simulation results are divided into many combinations in order to choose the best method and reaction wheel torque profile. The estimation methods are compared against parameter accuracy, fuel consumption and speed of convergence. The flowchart in Fig. 4.13 explains the comparison steps. Tables A.1, A.2 and A.3 summarise all the obtained results during the simulations.

Simulation results are shown in appendix A and are introduced in the first page of that appendix. These results are referred to in the discussion that follows.

Figs 4.16 to 4.19 show the results when disturbing the satellite using full torque generated by the three reaction wheel actuators and the attitude is controlled by using the quaternion feedback thruster controllers to identify the $3 \times 3$ calibration matrix $A_c$ using one experiment. Figs 4.16 and 4.17 illustrate the performance of the RLS thruster calibration algorithm when using the two reaction wheel torque profiles: PRBS and Bang-Bang (Figs. A.1 and A.5 in Appendix A respectively). The advantage of disturbing the attitude using the PRBS torque over the Bang-Bang torque is the limitation of build-up of wheel momentum, this is due to that the PRBS torque are generated using open loop form while the Bang-Bang are generated using closed loop form (depends on attitude information). During the PRBS disturbance the wheel momentum does not exceed 2 Nms (compare Figs A.2 and A.6 in Appendix A). The Euler angles do not exceed 5°, which means the estimation can be done during normal mission conditions when the satellite is stabilised (Figs A.3 and A.7 in appendix A). Figs 4.16 and 4.17 illustrate the convergence of thruster parameters using the RLS estimation method. It is clear from these figures that the thruster coefficients converge to their true values in less than 1000 seconds. The parameter variation after convergence is very small around the true values and it is better
in the case of the PRBS compared to the Bang-Bang reaction wheels inputs. The fuel consumption is the same in the two cases (see Tables A.1 and A.2 in Appendix A).

Figs 4.18 and 4.19 illustrate the thruster parameter convergence using the LMS estimation method. It is clear from these figures that the convergence rate of the thruster coefficients for the PRBS reaction wheel input is better than the Bang-Bang reaction wheel torque input. It is also clear that the thruster coefficient estimation accuracy is best for the PRBS reaction wheel torque. From previous simulation result it is better to disturb the satellite using the PRBS reaction wheel torque: it gives faster convergence and the best accuracy. According to the flowchart in Fig. 4.13, the next step will be to compare the results of the both the estimation methods (RLS and LMS) using the PRBS as reaction wheel torque input. Figs. 4.16 and 4.18 illustrate the convergence rate of the thruster coefficients. It is clear from these figures that the thruster coefficients converge to their true values and the parameter variation after convergence is very small around their true values in both cases. The speed of convergence for the RLS method is faster compared to the LMS method especially when estimating the thruster coefficients in the Z-axis. The reason being that the disturbance torque in the Z-axis is greater compared to the other axes (Fig 4.15), one of the great advantages of the RLS estimation method is a robustness against external disturbance torques compared to the LMS estimation method (see Chapter 3). From previous simulation result, it is better to disturb the attitude using a PRBS reaction wheel torque and to estimate the thruster coefficients using the RLS estimation method.

Figs 4.20 to 4.25 show the simulation results when the satellite is disturbed using one reaction wheel actuator, while this axis is controlled using the thrusters. The other two axes are controlled using the remaining two wheels. Three coefficients of the calibration matrix $A_c$ are then identified. A total of three experiments are required (one for each axis) to estimate all nine elements of the calibration matrix $A_c$.

Figs 4.20 and 4.21 illustrate the performance of the RLS method when disturbing the X-axis using the X-wheel and controlling this axis using thrusters. The Y/Z axes are controlled using quaternion feedback wheel controller to identify $a_{x1}, a_{y1}, a_{z1}$. The reaction wheel momentum (see Fig. A.14 in Appendix A) is close to saturation (the max. saturation
momentum is 4 Nms for UoSAT-12) when using the Bang-Bang reaction wheel torque, this is due to the Bang-bang reaction wheel torque are generated using closed loop form. The speed of convergence, estimation accuracy, and fuel consumption are almost the same in the two cases. The results obtained are summarised in Table A.2 Appendix A.

Figs 4.22 to 4.25 illustrate the performance of the RLS method for the other two axes to estimate the remaining six parameters. It is clear from these figures that it is best to disturb the attitude using a PRBS disturbance and to estimate the thruster coefficients using the RLS estimation method. The final step is to compare all the obtained estimation, e.g. methods, disturbance torque and number of axes. Table 4.3 compares the results between the single axis versus combined axes thruster calibration procedures. The combined procedure is clearly more time efficient (three times faster), but not as accurate as the single axis procedure.
Table 4.3 Comparison between estimation procedures

<table>
<thead>
<tr>
<th>Estimation procedure</th>
<th>Single</th>
<th>Combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimation time</td>
<td>3/2 orbit</td>
<td>1/2 orbit</td>
</tr>
<tr>
<td>No of RLS estimation</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>Fuel consumption (Nms)</td>
<td>160.67</td>
<td>172.4</td>
</tr>
<tr>
<td>(Integrate torque over time)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Speed of convergence</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(RMS error first 1000 sec)</td>
<td>0.0194</td>
<td>0.031</td>
</tr>
<tr>
<td>Estimation accuracy</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(RMS error last 1000 sec)</td>
<td>9.9e-4</td>
<td>9.4e-4</td>
</tr>
</tbody>
</table>

Figure 4.15 External disturbance torque
Figure 4.16 Estimated thruster coefficients for the PRBS torque and a RLS estimation method.

Figure 4.17 Estimated thruster coefficients when using a Bang-Bang torque and a RLS estimation method.
Figure 4.18 Estimated thruster coefficients when using a PRBS torque and a LMS estimation method.

Figure 4.19 Estimated thruster coefficients when using a Bang-Bang torque and a LMS estimation method.
Figure 4.20 Estimated thruster coefficients when disturbing the X-axis with a PRBS torque

Figure 4.21 Estimated thruster coefficients when disturbing the X-axis with a Bang-Bang torque
Figure 4.22 Estimated thruster coefficients when disturbing the Y-axis with a PRBS torque

Figure 4.23 Estimated thruster coefficients when disturbing the Y-axis with a Bang-Bang torque
Figure 4.24 Estimated thruster coefficients when disturbing the Z-axis with a PRBS torque

Figure 4.25 Estimated thruster coefficients when disturbing the Z-axis with a Bang-Bang torque
4.6 Conclusion

Various autonomous in-orbit thruster calibration techniques were discussed, analysed and compared in this chapter. The algorithms assume no knowledge of the thruster parameters and only knowledge of the inertia matrix. It is proposed that the calibration be used during normal mission conditions when the satellite is stabilised.

Complete analysis and performance comparisons of two methods to calibrate the thrusters in orbit are presented using two estimation techniques (Recursive Least Square and Least Mean Square) and two different types of reaction wheel torques (PRBS and Bang-Bang). The preferred technique will be to disturb the satellite using a PRBS reaction wheel torque profile, and apply the RLS estimation method with a combined axes procedure to determine all thruster coefficients at once. This technique is better in terms of speed of convergence, estimation accuracy and calibration time (minimising use of thruster propellant fuel). Numerical simulations illustrate the successful identification of the thruster parameters in spite of non-zero mean external disturbance torques and sensor noise. These calibration techniques can be applied in real-time on board satellite with thrusters to improve the attitude and orbit control performance.
Chapter 5

5. On Orbit Moment of Inertia Estimation for UoSAT-12

5.1 Introduction

The recent tendency is to build smaller, lighter and cheaper spacecraft. The present generation of spacecraft requires accurate attitude control to provide pointing capabilities. On-line calibration of the attitude control hardware is often necessary to satisfy this high accuracy ADCS requirement to accommodate changing mass distributions. If these systems are not properly calibrated in-orbit, a significant attitude control error can result.

UoSAT-12 is a low-cost 320 kg minisatellite built by Surrey Satellite Technology Ltd. It is a technology demonstrator for high performance attitude control and orbit maintenance for a future constellation of earth observation satellites. The satellite uses a 3-axis reaction wheel configuration and a cold gas thruster system to enable precise and fast control of its attitude. Magnetorquer coils assist the wheels mainly for momentum dumping. Ten cold-gas thrusters can be used in various combinations for both attitude and orbit control and a single N₂O resisto-jet is used exclusively for orbit maintenance.

One of the key features of Surrey’s low-cost approach to satellite engineering is the replacement of tight performance requirements and expensive ground calibration campaigns with in-orbit calibration and adaptation. This has been applied extensively to the UoSAT-12 attitude control system, for estimation of the moments of inertia. In this chapter, we present a Recursive Least Square RLS method for use in orbit to estimate the inertia matrix (moments and products of inertia parameters) of a satellite. To facilitate this, one attitude axis is disturbed using a reaction wheel whilst the other two axes are controlled to keep their respective angular rates small. Within a fraction of an orbit three components of the inertia matrix can be accurately determined. This procedure is then repeated for the other two axes to obtain all nine elements of the inertia matrix. The procedure is designed to prevent the build up of momentum in the reaction wheels whilst keeping the attitude disturbance to the satellite within acceptable limits. It can also
overcome potential errors introduced by unmodeled external disturbance torques and attitude sensor noise. The results of simulations are presented to demonstrate the performance of the technique.

5.2 Motivation to Measure the Moment of Inertia (MOI)

The MOI of simple shapes may be calculated by well-known methods. However, reducing complex shapes or compound objects to an assemblage of simple objects and summing the moments of inertia can lead to large errors. It is more practical and faster to accurately determine the MOI of complex objects or of objects with varying density by direct measurement. Measuring MOI directly has these advantages [Wiener, 1998]:

Greater Accuracy - Typical errors in calculated MOI can range to over 30% due to simplifying the part shape, or making assumptions about average density. If hanging torsion rod pendulums are used to measure MOI, large errors result from multiple mode oscillations.

Cost Savings - Measurements can generally be made in a small fraction of the time required for exact MOI calculations. Cost savings in engineering time alone can quickly pay for the instrument. Furthermore, calculations do not account for manufacturing variations.

Quality Assurance - Military and industrial specifications frequently set limits on MOI, where these parameters are critical to the performance of rockets, and spacecrafts.

5.3 Calculating the Moment of Inertia

Moment of inertia is similar to inertia, except it applies to rotation rather than linear motion. Inertia is the tendency of an object to remain at rest or to continue moving in a straight line at the same velocity. Inertia can be thought of as another word for mass. Moment of inertia is, therefore, rotational mass. Unlike inertia, MOI also depends on the distribution of mass in an object. So two bodies of the same mass may possess different moments of inertia. The greater the distance the mass is from the centre of rotation, the greater the moment of inertia. Values for moment of inertia can only be positive, just as
mass can only be positive. Units of moment of inertia (or product of inertia) are mass x distance^2.

A rigid body can be considered a system of particles in which the relative positions of the particle do not change. MOI, sometimes called the second moment, for a point mass see Fig. 5.1 around any axis is given by:

\[ I = mr^2 \]  

(5.1)

Where,
- \( I \) = moment of inertia (MOI)
- \( m \) = mass of element
- \( r \) = distance from the point mass to the reference axis

![Figure 5.1 MOI of a point mass about line xy](image)

For a number of point masses or distributed mass, Eq. (5.1) can be written as:

\[ I = \sum_i m_i r_i^2 \]  

(5.2)

Only one axis is necessary to define moment of inertia. Although any axis can be chosen as a reference, it is generally desirable to choose the principal axis (axis passing through the centre of gravity and oriented such that the product of inertia about this axis is zero). If a reference axis has to be chosen to calculate the moment of inertia of a complex shape, choose an axis of symmetry to simplify the calculation. This axis can later be translated to another axis if desired; using the Parallel Axis Theorem.
Based on Eq. (5.2), one can obtain the moment of inertia of a rigid body shown in Fig. 5.2 [Wie, 1998]

\[
I = \sum m_i R_i^2 \\
= \sum m_i (|r_i| \sin \phi_i)^2 \\
= \sum m_i |r_i \times R_o|^2 \\
= \sum m_i (r_i \times R_o)^T (r_i \times R_o) \tag{5.3}
\]

Where,

\( r_i \) = position of the particle \( i \)

\( R_o \) = unit vector of the axis of rotation

Note here that the axis of rotation passes through the local reference frame, the \( XYZ \) system. Let
\[
\mathbf{r}_i = x_i \mathbf{i} + y_i \mathbf{j} + z_i \mathbf{k} = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}
\]

(5.4)

and

\[
\mathbf{R}_o = \cos \alpha \mathbf{i} + \cos \beta \mathbf{j} + \cos \gamma \mathbf{k} = \begin{bmatrix} \cos \alpha \\ \cos \beta \\ \cos \gamma \end{bmatrix}
\]

(5.5)

Where,

\[
\cos \alpha, \cos \beta \text{ and } \cos \gamma = \text{the direction cosines of the vector } \mathbf{R}_o \text{ to the } \text{XYZ system.}
\]

Substituting from Eqs. (5.4) and (5.5) into (5.3) leads to

\[
I = \sum_i m_i (\mathbf{r}_i \times \mathbf{R}_o)^T (\mathbf{r}_i \times \mathbf{R}_o)
\]

\[
= \sum_i m_i \begin{bmatrix} 0 & -z_i & y_i \\ z_i & 0 & -x_i \\ -y_i & x_i & 0 \end{bmatrix} \begin{bmatrix} \cos \alpha \\ \cos \beta \\ \cos \gamma \end{bmatrix}^T \begin{bmatrix} 0 & -z_i & y_i \\ z_i & 0 & -x_i \\ -y_i & x_i & 0 \end{bmatrix} \begin{bmatrix} \cos \alpha \\ \cos \beta \\ \cos \gamma \end{bmatrix}
\]

\[
= \sum_i m_i \begin{bmatrix} y_i \cos \gamma - z_i \cos \beta \\ z_i \cos \alpha - x_i \cos \gamma \\ x_i \cos \beta - y_i \cos \alpha \end{bmatrix} \begin{bmatrix} y_i \cos \gamma - z_i \cos \beta \\ z_i \cos \alpha - x_i \cos \gamma \\ x_i \cos \beta - y_i \cos \alpha \end{bmatrix}
\]

\[
= \sum_i m_i [(y_i \cos \gamma - z_i \cos \beta)^2 + (z_i \cos \alpha - x_i \cos \gamma)^2 + (x_i \cos \beta - y_i \cos \alpha)^2]
\]

\[
= I_{xx} \cos^2 \alpha + I_{yy} \cos^2 \beta + I_{zz} \cos^2 \gamma + 2I_{xy} \cos \alpha \cos \beta + 2I_{xz} \cos \alpha \cos \gamma + 2I_{yz} \cos \beta \cos \gamma + 2I_{zx} \cos \gamma \cos \alpha
\]

(5.6)

Where,

\[
I_{xx} = \sum_i m_i (y_i^2 + z_i^2), \quad I_{yy} = \sum_i m_i (z_i^2 + x_i^2), \quad I_{zz} = \sum_i m_i (x_i^2 + y_i^2)
\]

\[
I_{xy} = I_{yx} = -\sum_i m_i x_i y_i
\]

(5.7)

\[
I_{xz} = I_{zx} = -\sum_i m_i x_i z_i
\]

\[
I_{yz} = I_{zy} = -\sum_i m_i y_i z_i
\]
$I_{xx}$, $I_{yy}$ and $I_{zz}$ are called moments of inertia while $I_{xy}$, $I_{yx}$, $I_{xz}$, $I_{zx}$ and $I_{yz}$ and $I_{zy}$ are products of inertia. For a rigid body, the relative position of the particles do not change and one can write Eq. (5.7) as:

$$
I_{xx} = \int (y^2 + z^2) \, dm, \quad I_{yy} = \int (x^2 + z^2) \, dm, \quad I_{zz} = \int (x^2 + y^2) \, dm
$$

$$
I_{xy} = I_{yx} = -\int xy \, dm,
I_{xz} = I_{zx} = -\int xz \, dm,
I_{yz} = I_{zy} = -\int yz \, dm
$$

(5.8)

It is clear that from Eq. (5.6), the actual moment of inertia of a rigid body about an axis of rotation is a function of not only the moments and products of inertia for a given reference frame but also the orientation of the axis of rotation, $\alpha$, $\beta$ and $\gamma$. Thus, it would be more accurate to say that the moment of inertia of a rigid body reflects the mass distribution within the body with respect to the axis of rotations.

When the shape and the density distribution of the rigid body is precisely known, one can use Eq. (5.8) to compute the moments and products of inertia. Otherwise, it is difficult to compute them through integration. Rather, the moment of inertia must be measured directly from the object.

### 5.4 Parallel Axis Theorem

This parallel axis theorem is used very frequently when calculating the MOI of a spacecraft [Wiener, 1998]. The MOI of each component in the spacecraft is first measured or calculated around an axis through its CoG, and the parallel axis theorem is then used to determine the MOI of the total vehicle with these components mounted in their proper location. If we determine the MOI of the object about the axis $x_u$ rather than the axis $x$, through the CoG, then the value can be determined using the parallel axis theorem see Fig. 5.3:

$$
I_{xa} = I + d^2 m
$$

(5.9)
Where the offset \( d \) is the distance from the CoG of the component to the principal axis of the spacecraft.

\[ I_{x_a} = m \left( R^2 + d^2 \right) \]

**Figure 5.3 Parallel axis theorem**

### 5.5 MOI Measurement

There are a wide variety of MOI measuring instruments available today. The choice of which one to use depends on the accuracy required, the degree of automation required, and budgetary restriction. There are two methods which can be used to measure MOI.

- Hanging the object from a torsion pendulum (see Fig. 5.4).
- Oscillating the object on a torsion rod

There are a number of practical problems involved in hanging most test articles from a torsion pendulum. Where do you attach the torsion pendulum? How do you hang the whole system? How do you calibrate the device, and what do you do to correct for the change in calibration when the weight of the test object stretches the torsion pendulum? Modern MOI instruments consists of an inverted torsion pendulum which oscillates in a rotational sense and a means of measuring the exact period of oscillation of the torsion pendulum [Wiener, 1998]. Fig. 5.5 shows UoSAT-12 during MOI tests.
5.5.1 Hanging Torsion Pendulum

Although hanging torsion pendulums are not accurate enough for satellite and missile measurements, they are useful for measuring the MOI of an aircraft. Fortunately, the MOI tolerance of an aircraft is not critical and the resulting accuracy is acceptable.

This method consists of hanging an object from a torsion pendulum, twisting it to start it oscillating, and then timing the period of oscillation. Although it sounds like a simple device, the structure required to support the upper end of the torsion rod can be very expensive, and some accurate means is required to time the period of oscillation. One problem with the hanging torsion pendulum method is that the object swings from side to side and rocks up and down rather than rotating smoothly about an axis, making it difficult to acquire accurate time period data. It is essential that the centre of gravity of the object be aligned horizontally with the centre of the torsion pendulum. Otherwise, the moment the object is released, there will be a couple generated and the motion of the pendulum will be sideways as well as torsional. There is a serious practical problem when measuring heavy objects using this method: How do you attach the object and adjust its position so the CoG is in the centre of the pendulum?

A single hanging (steel) torsion rod has a torsional stiffness \(K\), which is proportional to the diameter. The equation of motion for this pendulum is given by:
\[ I = KT_p^2 \] \hspace{1cm} \text{(5.10)}

Where,
\( K \) = torsion rod stiffness
\( T_p \) = period of oscillation.

5.5.2 Inverted Torsion Pendulum Theory

Modern MOI instruments use inverted torsion pendulums, because these instruments are so accurate and easy to use. The test object is mounted on a gas bearing rotary table, so that friction is very low. A torsion rod provides a rotational spring. The object is twisted slightly and released. The period of oscillation is measured using an optical proximity sensor in combination with digital electronic circuitry and a quartz crystal oscillator. The period of oscillation is related to the moment of inertia of the entire oscillating system by the relationship [Wiener, 1998]:

\[ I = CT_p^2 \] \hspace{1cm} \text{(5.11)}

Where,
\( C \) = calibration constant
\( T_p \) = period of oscillation.

In order to establish the value of the calibration constant \( C \) of the instrument, MOI calibration standards are measured. MOI calibration standards are precision weights of simple geometry, known mass and known physical dimensions. The calibration procedure is identical with the procedure for measuring the moment of inertia of an object of unknown MOI except that in the computation the inertia is a known quantity and the value of the calibration constant is the unknown, which must be solved for. Because the air bearing supports the weight of the object these instruments are linear over a wide range of test part weight and moment of inertia. Only a single calibration measurement is required to establish the value of the calibration constant used for all measurements.
5.6 Product of Inertia

Consider the homogeneous balanced cylinder to which two equal weights have been attached 180° apart, and spaced equidistant along the length from the CoG of the cylinder. The addition of these weights will not alter the CoG of the cylinder and the cylinder remains statically balanced. However, if we spin this cylinder about the vertical $z$ axis, then centrifugal force acts through the two weights and produces a couple. If the cylinder is mounted on bearings, then this couple causes a sinusoidal force to be exerted against the bearings as the cylinder rotates. If the cylinder is spinning in space, then the axis of rotation of the cylinder shifts to align itself to a condition where the centrifugal forces are equalised (i.e., it shifts toward the unbalance weights slightly). This couple moment which occurs when the object is spinning is called the product of inertia see Fig. 5.6.

Basically, the product of inertia (POI) is a measure of dynamic unbalance. POI is expressed in the same units as moment of inertia. Values for product of inertia can be either positive or negative. The POI is derived by multiplying the incremental masses by two different distances see Eq. (5. 8) for POI calculation.
5.7 Measuring the Product of Inertia

The product of inertia is generally measured using a spin balance machine (see Fig. 5.7). In this type of machine, the object is rotated at a speed of about 100 RPM and the reaction forces against the upper and lower spindle bearings are measured. POI is then calculated automatically by the machine on-line computer using formulae that involve the vertical spacing between the upper and lower bearings and the height of the object above the mounting surface of the machine.

Figure 5.7 Spin balance machine
An object such as a satellite with extended solar panels cannot be measured using the spin method because of the large non-repeatable errors which are introduced by the entrained and entrapped air and turbulence. In these instances, POI can be determined by making a series of MOI measurements with the object oriented in six different positions [Kurt, 1992]. Product of inertia can then be calculated using formulae, which involve the rotation angles of the different fixture positions.

5.7.1 MOI Method of Measuring the POI

This method uses a torsion pendulum to determine the POI by making use of the relationship between the POI and MOI of an object. Special fixtures must be constructed to move the object to a number of positions while keeping both the object and the fixture CoG near the centre of oscillation. The MOI of the object is measured in each orientation and then used to determine the POI of the object. The calculation is quite complex, so an on-line computer is used. For the general case, the total number of MOI measurements needed for POI calculations is nine, three in each of three mutually perpendicular planes. If the intersections of these planes are selected to be coordinate axes, then the MOI about each of these axes will be common to two planes, thus reducing the total number of measurements to six: Three about the X, Y and Z axes, and three about axes at 45 degrees between the X-Y, Y-Z and Z-X axes [Kurt, 1992].

The coordinate system for the MOI method (Fig. 5.8) has its origin (O) at the test part CoG. The axes will be designated X, Y and Z passing through the CoG. If the test part were fixed so that it could be rotated through an angle C about a horizontal axis (i.e. the Z axis) and MOI measured about numerous axes in the X-Y plan, including the X and Y axes, the MOI would be found to vary sinusoidally. If the angle C ranges over 180 degrees, the maximum and minimum values of MOI can be seen in Fig. 5.9. The axes about which the maximum and minimum MOIs are measured are the principal axes. For all other axes, the MOI \( I_{AR} \) about an axis (A) in the X-Y plane at an angle C from the +X axis, and the POI \( P_{AR} \), are related through the equation:
\[ I_{Ax} = I_{xy} \sin^2 C + I_{xx} \cos^2 C - P_{xy} \sin 2C \]  

(5.12)

Solving this equation for \( P_{xy} \) forms the basis for the MOI method of POI determination

\[ P_{xy} = \frac{I_{xx} \cos^2 C + I_{yy} \sin^2 C - I_{Ax}}{\sin 2C} \]  

(5.13)

The equation used to calculate the POI in the X-Y plane when A is at 45 degrees is given by:

\[ P_{xy} = \frac{I_{xx} + I_{yy}}{2} - I_{Ax} \]  

(5.14)

Similarly, the POI for the Y-Z and Z-X planes would be calculated from MOI data about axes in those planes such that

\[ P_{xz} = \frac{I_{xx} + I_{zz}}{2} - I_{Ax} \]  

(5.15)

and

\[ P_{yz} = \frac{I_{yy} + I_{zz}}{2} - I_{Ay} \]  

(5.16)
5.8 On-Orbit Estimation of Inertia Matrix

In this section, we present a new method for identifying the spacecraft inertia matrix. The way the calibration torque is generated depends on the specific actuators installed on the spacecraft. UoSAT-12, for example, has three reaction wheels. Using accurate wheel speed measurements and knowledge of the wheel moment of inertia, the reaction wheels’ torques are accurately known and can be used to estimate the inertia matrix (moments and products of inertia parameters) in-orbit. Disturbing a specific attitude axis using a reaction wheel Bang-Bang controller does this. The other two axes are controlled using a quaternion feedback controller to control their reaction wheels to keep their respective angular rates small. This procedure is then repeated for the other two axes to obtain all nine elements of the inertia matrix.

5.8.1 Preliminaries

The dynamical model of an earth pointing satellite, using reaction wheels as internal torque actuators and magnetorquers as external torque actuators, is given by:
\[ I\dot{\omega}'_B = N_{GG} + N_D + N_M - \omega_B \times (I\dot{\omega}'_B + h) - \dot{h} \] (5.17)

Where,
\[ \omega'_B = \left[ \begin{array}{c} \omega_x \\ \omega_y \\ \omega_z \end{array} \right] \text{ = inertiely referenced body angular rate vector} \]
\[ I = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} \text{ = moment of inertia tensor of spacecraft (MOI)} \]
\[ h = \begin{bmatrix} h_x \\ h_y \\ h_z \end{bmatrix} \text{ = reaction wheel angular momentum vector} \]
\[ N_D = \begin{bmatrix} N_{dx} \\ N_{dy} \\ N_{dz} \end{bmatrix} \text{ = external disturbance torque vector} \]
\[ N_M = \begin{bmatrix} N_{mx} \\ N_{my} \\ N_{mz} \end{bmatrix} \text{ = applied torque vector by 3-axis Magnotorquers} \]

and, [Wertz, 1989]
\[ N_{GG} = \frac{3GM_\oplus}{R_s^3} \left[ R_s \times (I_i R_s) \right] \] (5.18)

Where,
\[ R_s = \text{geocentric position vector length} \]
\[ GM_\oplus = \text{earth’s gravitationa lconstant} \]
\[ R_s = \text{geocentric position vector of the origin of the body reference frame} \]

The complete set of dynamic equations of motion Eq. (5.17) can then be written in component form as:

\[ I_{xx} \ddot{\omega}_x = N_{dx} + N_{mx} + N_{ggx} + I_{xx} \dot{\omega}_x + I_{xx} \ddot{\omega}_x + \omega_y \omega_z (I_{yy} - I_{yz}) + \omega_z \omega_y (I_{zz} - I_{yz}) + \\
+ \omega_x \omega_y I_{xy} + \omega_y^2 I_{xy} - \omega_z^2 I_{yz} + \omega_z \omega_y h_y - \omega_y h_z - h_x \]
(5.19.a)

\[ I_{yy} \ddot{\omega}_y = N_{dy} + N_{my} + N_{ggy} + I_{yy} \dot{\omega}_y + I_{yy} \ddot{\omega}_y + \omega_x \omega_z (I_{xx} - I_{xz}) + \omega_z \omega_x I_{xy} - \\
- \omega_x \omega_y I_{xy} + \omega_x^2 I_{xx} + \omega_z^2 I_{xz} - \omega_x h_x + \omega_y h_z - h_y \]
(5.19.b)

\[ I_{zz} \ddot{\omega}_z = N_{dz} + N_{mz} + N_{ggz} + I_{zz} \dot{\omega}_z + I_{zz} \ddot{\omega}_z + \omega_x \omega_y (I_{xx} - I_{yy}) + \omega_y \omega_x I_{xy} + \\
+ \omega_x \omega_z I_{xz} - \omega_z^2 I_{xy} + \omega_z^2 I_{xy} - \omega_y h_x + \omega_y h_z - h_z \]
(5.19.c)
For a circular orbit \( (R_e \text{ constant}) \) and using Kepler's third law, which relates the cube of the semimajor axis to the square of the orbital period \( T \) (we may ignore the mass of the secondary), we can also write:

\[
GM_e = \frac{4\pi^2}{T^2} = \omega_o^2 \quad \text{(Square of the orbital rate)}
\]  

The reaction wheel dynamics for identical wheels aligned to the body principal axes, can be written as:

\[
\dot{\omega}_i = I_{wi} \omega_i = N_{wi}
\]

With,

\[
I_{wi} = \text{wheel-i moment of inertia}
\]

\[
\omega_i = \text{wheel-i angular rate}
\]

\[
N_{wi} = \text{wheel-i torque}
\]

The gravity gradient torque components can be written as:

\[
N_{ggx} = 3\omega_o^2 \left[ \left( I_{xx} - I_{yy} \right) A_{23} A_{33} - I_{yx} A_{13} A_{33} - I_{yx} A_{23}^2 + I_{yx} A_{13} A_{23} + I_{yx} A_{13}^2 \right]
\]

\[
N_{ggy} = 3\omega_o^2 \left[ \left( I_{yy} - I_{zz} \right) A_{13} A_{33} + I_{yy} A_{23} A_{33} + I_{yy} A_{23}^2 - I_{yy} A_{13} A_{23} - I_{yy} A_{13}^2 \right]
\]

\[
N_{ggz} = 3\omega_o^2 \left[ \left( I_{zz} - I_{xx} \right) A_{13} A_{23} - I_{yx} A_{23}^2 - I_{yx} A_{23} A_{33} + I_{yx} A_{13} A_{23} + I_{yx} A_{13} A_{33} \right]
\]

Where, \( A_i \) are the components of \( A \) (attitude transformation matrix) that transforms any vector from the referenced orbital to spacecraft body coordinates. Eq. (5.19) describes the complete dynamics of UoSAT-12 including gravity gradient torque, external disturbance torque, magnetorquers torque (momentum dumping) and all products of inertia.

5-16
5.8.2 Reaction Wheel Controllers

During MOI calibration, the attitude of UoSAT-12 will be controlled as follows:
The attitude in two axes using a quaternion feedback controller will be kept close to zero
two wheel controller). The third axis will be disturbed using the remaining wheel
according to a non-linear Bang-Bang control law with a certain error band in order to
avoid a build up of the wheel momentum (and the error quaternion).

5.8.2.1 Two Wheel Controller

The UoSAT-12 reaction wheel controller (two wheel controller) will be implemented
using a quaternion feedback control law Eq. (2.24). This controller is slightly modified in
order to apply it in two axes only. The quaternion feedback wheel controller with
cancellation of gyroscopic torque is given by:

\[ N_w = \omega_p^T \times (I \omega_p + h) - K_B \omega_p^2 - K_p q_z \]  \hspace{1cm} (5.23)

The PD control gain matrix \( K_p = k_p I, \ K_B = k_B I \), where \( k_p, k_B \) are positive scalars to be
properly determined according to Wie et al.[1989].

Depending on which two axes need control, from \( N_{wx}, N_{wy}, N_{wz} \) the control torques are
chosen.

5.8.2.2 Bang-Bang Controller

UoSAT-12 will be disturbed using one wheel according to a Bang-Bang nonlinear
controller with a certain error band to limit wheel momentum build-up and avoid an
increase in the error quaternion Eq. (2.27), this controller is slightly modified in order to
apply it in one axis. The control law can be summarised by the following set of equations:

\[ e_{w3} = k_1 q_{w3} + k_2 \omega_3 \]

\[ N_{w3} = \begin{cases} 
- N_{max} & \text{for } e_{w3} \geq e_{band}, \text{with } e_{w3} \text{ increasing} \\
N_{max} & \text{for } e_{w3} \leq -e_{band}, \text{with } e_{w3} \text{ decreasing}
\end{cases} \]  \hspace{1cm} (5.24)

Where,
\[ k_1 \text{ and } k_2 = \text{controller gains} \]
\[ e_{w3} = \text{control error} \]
\[ e_{\text{band}} = \text{error hysteresis band} \]

Note that subscript 3 refers to the disturbed axis x, y or z.

### 5.8.3 MOI Estimation Block Diagram

Figure 5.10 summarises the general in-orbit inertia matrix estimation scheme using reaction wheel actuators. A known disturbance torque \( N_{w3}(k) \) is applied to the spacecraft at time \( k \) as a result of the Bang-Bang control law, using the third wheel. This torque together with unknown external disturbance torques \( N_o(k) \) and the output from the two-wheel controller \( N_{w2}(k) \) acts on the satellite. The in-orbit moment of inertia calibration procedure (below the dotted line in Fig. 5.10) uses the known disturbance torque \( N_{w3}(k) \), the output from the two wheel controller \( N_{w2}(k) \) and the resulting satellite state to calculate the calibration torque \( N_c(k) \). This calibration torque is used in the measurement equation (5.25) for the RLS algorithm to estimate the inertia matrix \( \hat{\mathbf{I}} \) via a suitable high pass filter (see Paragraph 5.8.4 on high pass filter design).

\[
N_c(k) = f(I, \omega^i_\dot{h}, \dot{\Omega}^i_\dot{h}, N_M, h, N_w)
\]  

(5.25)

![Figure 5.10 On-orbit inertia matrix estimation block diagram](image-url)
5.8.4 High Pass Filter Design

Aerodynamic drag acts as a low frequency disturbance torque influencing the satellite states (angular rate, quaternion), which are used by the attitude controller to control the satellite with corresponding reaction wheel torque and momentum. All these disturbance terms form part of the RLS error, but can be filtered out using a second order Butterworth high pass filter during MOI calibration. This is the best location for the high pass filter, because the error contains all the parameters affected by the low frequency disturbance torques (reaction wheel torque, wheel momentum, angular rate and angular acceleration). During the moment of inertia estimation, the reaction wheel torque period generated by the Bang-Bang controller is approximately within the range of 200 to 300 seconds, which corresponds to a frequency range between 3.33 and 5 milli-Hz. Then the high pass filter cut-off frequency must be designed below these range of frequencies; see Table 5.1 for the value used during simulation. Without this filtering the estimates will not converge to the true values.

5.8.5 Reduced Equations of Motion

By disturbing one axis using the corresponding reaction wheel, the principal moment of inertia can easily be estimated while keeping the angular rate disturbances in the other two axes close to zero using the quaternion feedback controllers. When combining the dynamic equations of the latter two axes, we can estimate the products of inertia. This is a new method of MOI estimation in-orbit.

A problem for nadir pointing satellites is that the inertial angular rate in the Y-axis (pitch) is non-zero and equal to the orbital rate. It is therefore recommended first to disturb the Y-axis and control the X and Z-axes, keeping the angular rates in these axes close to zero. This will reduce the cross coupling in the dynamic equations between the Y-axis and the other two axes, so that the products of inertia terms when estimating $I_{yy}$, $I_{xy}$ and $I_{xy}$ are negligible. The second step will be to disturb the X or Z-axis and to control the remaining two axes.
The dynamic model of an Earth-pointing satellite using 3-axis reaction wheels as internal torque actuators, and magnetorquers as external torque actuators, is given in Eq. (5.19). It is clear from these dynamic equations that it is better to estimate the principal moment of inertia terms by disturbing the satellite using a small level of reaction wheel torque and momentum. This decreases the effect of the products of inertia in the dynamic equations, but ensures sufficient torque and momentum to counteract the influence of unknown external disturbance torques. Conversely, when estimating the products of inertia it is better to disturb the satellite using full reaction wheel torque and momentum in order to increase the influence of the products of inertia in the dynamic equations.

The presence of external disturbance torques tends to cause wheel momentum drift. Therefore, management of reaction wheel momentum is required in order to counteract the influence of persistent external disturbance torques. On UoSAT-12 an external torque using magnetorquers is applied for wheel momentum management. This also ensures that the satellite angular rate values in the two controlled axes are kept close to zero.

5.8.5.1 Disturb Y-axis and Control X and Z axes

Neglecting small terms, the reduced dynamic equations of motion Eq. (5.19) describe the effect of disturbing the Y-axis using the reaction wheel Bang-Bang controller Eq. (5.24) and controlling the X and Z axes using quaternion feedback reaction wheel controllers Eq. (5.23), (with momentum dumping) and can be written as:

\[ I_{xx} \dot{\omega}_x = N_{mx} + I_{xy} \dot{\omega}_x + \omega_y^2 I_{yx} + \omega_y h_y - \omega_y h_c - \dot{h}_c \]  
(5.26.a)

\[ I_{yy} \dot{\omega}_y = N_{my} - \omega_z h_x + \omega_x h_z - \dot{h}_y \]  
(5.26.b)

\[ I_{zz} \dot{\omega}_z = N_{mz} + I_{yz} \dot{\omega}_y - \omega_y^2 I_{yx} - \omega_y h_y + \omega_y h_c - \dot{h}_c \]  
(5.26.c)

Two separate RLS estimations are needed for each axis: One to estimate the principal moment of inertia \( I_{yy} \), and the other to estimate the products of inertia \( I_{yx} \) and \( I_{zy} \).

For the first of these, rewrite Eq. (5.26.b) in the form
\[ N_{cy}(k) = I_{yy} \dot{\omega}_y \]  

(5.27)

Where,

\[ N_{cy}(k) = N_{cy} - \omega_x h_z + \omega_z h_x - \dot{h}_y \]  

(5.28)

\( N_{cy}(k) \) is the calibration torque required to estimate \( I_{yy} \).

Equation (5.27) acts as the measurement equation for the RLS estimation to estimate \( I_{yy} \).

The error to be minimized can be written as:

\[ e_y(k) = N_{cy}(k) - \hat{I}_{yy} \dot{\omega}_y \]  

(5.29)

Where the \( \hat{\cdot} \) is used to denote an estimated parameter.

For the second RLS to estimate \( I_{xy} \) and \( I_{yx} \) adding, Eqs. (5.26.a) and (5.26.c) and rearranging gives:

\[ N_{cxp}(k) = \hat{I}_{xy} (\dot{\omega}_y - \omega_x^2) + \hat{I}_{yx} (\dot{\omega}_y + \omega_x^2) \]  

(5.30)

Where, \( N_{cxp}(k) \) is the calibration torque given by:

\[ N_{cxp}(k) = \hat{I}_{xx} \dot{\omega}_x + \hat{I}_{zx} \dot{\omega}_z - N_{mx} - N_{xz} - \\
\quad - h_y (\omega_x - \omega_z) - \omega_y (h_x - h_z) + \dot{h}_x + \dot{h}_z \]  

(5.31)

and \( \hat{I}_{xx}, \hat{I}_{zx} \) can be obtained from an initial estimate.

Equation (5.30) also acts as the measurement equation for the RLS estimation to estimate \( I_{xy} \) and \( I_{yx} \). The error to be minimized can be written as:

\[ e_{xp}(k) = N_{cxp}(k) - \hat{I}_{xy} (\dot{\omega}_y - \omega_x^2) - \hat{I}_{yx} (\dot{\omega}_y + \omega_x^2) \]  

(5.32)

Both error equations (5.29) and (5.32) are high pass filtered to remove the effects of low frequency disturbance torques.
5.8.5.2 Disturb X-axis and Control Y and Z axes

Similarly, the procedure for the Y-axis are repeated for the X and Z axes, the reduced dynamic equations of motion when disturbing the X-axis using the reaction wheel Bang-Bang controller and controlling the Y and Z axes using quaternion feedback reaction wheel controllers can be written as:

\[
I_{sx} \dot{\omega}_x = N_{mx} + \omega_y^2 I_{xy} + \omega_z h_z - \omega_y h_x - \dot{h}_x \quad (5.33.a)
\]

\[
I_{sy} \dot{\omega}_y = N_{my} + I_{yx} \dot{\omega}_x + I_{yx} \dot{\omega}_x + \omega_x \omega_y (I_{yz} - I_{yx}) - \\
- \omega_x \omega_y I_{xy} - \omega_x^2 I_{yx} - \omega_x h_x + \omega_y h_y - \dot{h}_y \quad (5.33.b)
\]

\[
I_{sz} \dot{\omega}_z = N_{mz} + I_{zx} \dot{\omega}_x + I_{zx} \dot{\omega}_x + \omega_x \omega_z (I_{zx} - I_{xz}) + \\
+ \omega_x \omega_z I_{xz} - \omega_z^2 I_{zx} + \omega_z I_{zx} - \omega_y h_y + \omega_y h_z - \dot{h}_z \quad (5.33.c)
\]

Two separate RLS estimations are needed for the X-axis: One to estimate the principal moment of inertia \( I_{sx} \), and the other to estimate the products of inertia \( I_{yx} \) and \( I_{zx} \). When previously disturbing the Y-axis, the estimated values of \( \hat{I}_{yx} \), \( \hat{I}_{xy} = \hat{I}_{yx} \) and \( \hat{I}_{zy} = \hat{I}_{yz} \) have been obtained so these estimated values can be used when disturbing the X-axis. By comparing the results of the estimated values of \( I_{xy} \) from the Y-axis and the estimated values of \( I_{yx} \) from the X-axis, the correctness of the estimation can be determined \( (\hat{I}_{yx} = \hat{I}_{xy}) \). The average values of the two estimated products of inertia can be calculated to obtain a better result.

For the first of these rewrite Eq. (5.33.a) in the form

\[
N_{ix}(k) = I_{sx} \dot{\omega}_x \quad (5.34)
\]

Where,

\[
N_{ix}(k) = N_{mx} + \omega_y^2 \hat{I}_{xy} + \omega_z h_z - \omega_y h_x - \dot{h}_x \quad (5.35)
\]

\( N_{ix}(k) \) is the calibration torque required to estimate \( I_{sx} \).
Equation (5.34) acts as the measurement equation for the RLS estimation to estimate $I_{xx}$.

The error to be minimised can be written as:

$$e_x(k) = N_{ex}(k) - \hat{I}_{xx}\omega_x$$  \hspace{1cm} (5.36)$$

The second RLS measurement equation to estimate $I_{yx}$ and $I_{tx}$ is obtained by adding Eqs. (5.33.b) and (5.33.c) and after rearranging gives:

$$N_{cp}(k) = \hat{I}_{yx}(\omega_x + \omega_x^2) + \hat{I}_{zx}(\omega_x - \omega_x^2)$$  \hspace{1cm} (5.37)$$

Where, $N_{cp}(k)$ is the calibration torque given by:

$$N_{cp}(k) = \hat{I}_{yy}\dot{\omega}_y + \hat{I}_{zz}\dot{\omega}_z - N_{yy} - N_{zz} - \hat{I}_{xy}\dot{\omega}_y + \hat{I}_{yx}\dot{\omega}_x - \omega_x\omega_y(\hat{I}_{xx} - \hat{I}_{yy}) - \omega_x\omega_z(\hat{I}_{xx} - \hat{I}_{zz}) + \omega_y^2\hat{I}_{xy} + \omega_z\omega_y\hat{I}_{yz}$$

$$- \omega_z(h_x - h_y) - h_x(\omega_y - \omega_z) + \dot{h}_y + \dot{h}_z$$  \hspace{1cm} (5.38)$$

The error to be minimised can be written as:

$$e_{xp}(k) := N_{cp}(k) - \hat{I}_{yx}(\omega_x + \omega_x^2) - \hat{I}_{zx}(\omega_x - \omega_x^2)$$  \hspace{1cm} (5.39)$$

Both error equations (5.36) and (5.39) are high pass filtered to remove the effects of low frequency disturbance torques.

**5.8.5.3 Disturb Z-axis and Control X and Y axes**

The reduced dynamic equations of motion when disturbing the Z-axis using the reaction wheel Bang-Bang controller and controlling the X and Y-axes using quaternion feedback reaction wheel controllers can be written as:

$$I_{xx}\ddot{\omega}_x = N_{xx} + I_{xx}\dot{\omega}_x - \omega_y\omega_z(I_{zz} - I_{yy}) + \omega_y^2I_{zy} - \omega_z^2I_{yx} + \omega_yh_y - \omega_zh_z - \dot{h}_z$$  \hspace{1cm} (5.40.a)$$
Two separate RLS estimations are needed for the Z-axis: One to estimate the principal moment of inertia $I_{zz}$, and the other to estimate the products of inertia $I_{zx}$ and $I_{zy}$. When previously disturbing the X- and Y-axes, the estimated values of $\hat{I}_{zx}$, $\hat{I}_{zy}$, $\hat{I}_{xy}$, $\hat{I}_{yx}$, $\hat{I}_{yz}$, and $\hat{I}_{zy}$ have been obtained so these estimated values could be used when disturbing the Z-axis. All products of inertia are estimated now by disturbing two axes (Y and X). By comparing the results of the estimated products of inertia, the correctness of the estimation can be determined.

For the first of these rewrite Eq. (5.40.c) in the form

$$N_{xz}(k) = I_{xz} \dot{\phi}_z$$  \hspace{1cm} (5.41)

Where,

$$N_{xz}(k) = N_{xz} - \omega_x \omega_z \hat{I}_{zx} - \omega_y \hat{I}_{xy} - \omega_z \hat{I}_{zy} + \omega_y h_z - \dot{h}_z$$  \hspace{1cm} (5.42)

$N_{xz}(k)$ is the calibration torque required to estimate $I_{xz}$.

Equation (5.41) acts as the measurement equation for the RLS estimation to estimate $I_{xz}$.

The error to be minimized can be written as:

$$e_z(k) = N_{xz}(k) - \hat{I}_{xz} \dot{\phi}_z$$  \hspace{1cm} (5.43)

The second RLS measurement equation to estimate $I_{zx}$ and $I_{zy}$ is obtained by adding Eqs. (5.40.a) and (5.40.b) and after rearranging gives:

$$I_{xz} \dot{\phi}_z = N_{xz} - \omega_x \omega_z I_{zx} - \omega_y \omega_z I_{xy} - \omega_z \omega_z I_{zy} - \omega_x h_x + \omega_y h_z - \dot{h}_z$$  \hspace{1cm} (5.40.c)

$$I_{zx} \dot{\phi}_z = N_{zx} - \omega_y \omega_z I_{zy} - \omega_x \omega_z I_{xy} - \omega_z \omega_z I_{zy} - \omega_y h_y + \omega_x h_z - \dot{h}_z$$  \hspace{1cm} (5.40.b)
\[ N_{\text{exp}}(k) = \hat{I}_{xx}(\dot{\omega}_z + \omega_z^2) + \hat{I}_{yx}(\dot{\omega}_z - \omega_z^2) \]  \hspace{1cm} (5.44)

Where, \( N_{\text{exp}}(k) \) is the calibration torque given:

\[ N_{\text{exp}}(k) = \hat{I}_{xx}\dot{\omega}_x + \hat{I}_{yx}\dot{\omega}_y - N_{mx} - N_{my} + \omega_x\omega_z(\hat{I}_{xz} - \hat{I}_{yy} - \omega_y\omega_z\hat{I}_{xy}) - \omega_z(h_{xz} - h_{yz}) - h_z(\omega_x - \omega_y) + \hat{h}_x + \hat{h}_y \]  \hspace{1cm} (5.45)

The error to be minimised can be written as:

\[ e_y(k) = N_{\text{exp}}(k) - \hat{I}_{xx}(\dot{\omega}_z + \omega_z^2) - \hat{I}_{yx}(\dot{\omega}_z - \omega_z^2) \]  \hspace{1cm} (5.46)

Both error Eqs. (5.43) and (5.46) are high pass filtered to remove the effects of low frequency disturbance torques.

### 5.8.6 Recursive Least Square (RLS) Implementation

A RLS calibration algorithm based on real time parameter estimation is proposed for improved convergence and accuracy. The algorithm is a recursive implementation of the least squares minimisation technique. From Eqs. (5.29), (5.32), (5.36), (5.39), (5.43) and (5.46) the error to be minimised can be written as in standard least square parameter estimation problems:

\[ e_i(k) = y_i(k) - \varphi_i^T \theta_i(k) \hspace{1cm} i = x, y, z, x_p, y_p, z_p \]  \hspace{1cm} (5.47)

Where,

\[ y_x(k) = N_{cx}(k) \hspace{1cm} y_y(k) = N_{cy}(k) \hspace{1cm} y_z(k) = N_{cz}(k) \]

\[ y_{xp}(k) = N_{cxp}(k) \hspace{1cm} y_{yp}(k) = N_{cyp}(k) \hspace{1cm} y_{zp}(k) = N_{czp}(k) \]

\[ \varphi_x(k) = \dot{\omega}_x \hspace{1cm} \varphi_y(k) = \dot{\omega}_y \hspace{1cm} \varphi_z(k) = \dot{\omega}_z \]

\[ \varphi_{xp}^T(k) = \begin{bmatrix} \dot{\omega}_x + \omega_x^2 \\ \dot{\omega}_x - \omega_x^2 \end{bmatrix}^T \hspace{1cm} \varphi_{yp}^T(k) = \begin{bmatrix} \dot{\omega}_y + \omega_y^2 \\ \dot{\omega}_y - \omega_y^2 \end{bmatrix}^T \hspace{1cm} \varphi_{zp}^T(k) = \begin{bmatrix} \dot{\omega}_z + \omega_z^2 \\ \dot{\omega}_z - \omega_z^2 \end{bmatrix}^T \]
The full RLS algorithm for any of the three parameters estimation problems is given in Paragraph 3.1.2. The forgetting factor \( \lambda \) is a constant such that \( \lambda \leq 1 \) to introduce time varying weighting of the data. This will ensure that any MOI changes will be tracked.

During the simulation tests, different values of \( \lambda \) (0.9-1) and covariance matrix \( P_i(0) \) were applied in order to obtain the best conversion results. Smaller initial \( P_i \) elements decrease the initial convergence performance and larger elements increase the initial parameter variance. The forgetting factor \( \lambda \) was chosen to minimise the RMS error (see Fig. 5.11) calculated from the final 3000 second parameter estimates. To improve the robustness of the RLS algorithm, the error can be modified by a non-linear function as follows:

\[
    f[\varepsilon_i(k)] = \frac{\varepsilon_i(k)}{1 + b|\varepsilon_i(k)|}
\]

(5.48)

The constant \( b \) is defined such that the function is still linear for normal values of \( \varepsilon_i(k) \), but it will reduce the influence of large outliers. Table 5.1 summarises the RLS parameters with best conversion results.

![Figure 5.11 Forgetting factor versus (rms)error in case of estimating \( I_{xx} \).](image-url)
5.9 Application Example

Simulation tests were implemented to investigate the performance of the proposed moment of inertia (MOI) estimation algorithms presented above. The UoSAT-12 satellite in a low Earth-orbit is used as an example during these simulations. The simulation parameters are given in Table 5.1. During simulations an unknown external disturbance torque (see Fig. 5.13) and sensor noise are added to the measurements of the state vectors \( \mathbf{q} \) and \( \omega_\text{n} \). Uniformly distributed measurement noise within the range of \(-10^{-4}\) to \(10^{-4}\) rad/sec was added to each component of the angular rate. Uniformly distributed measurement noise within the range of \(-0.1\) to \(0.1\) was added to the vector part of the error quaternion. The levels of the sensor noise are chosen according to SSTL experience.

Table 5.1 Simulation parameters

<table>
<thead>
<tr>
<th>Moment of inertia tensor</th>
<th>( \mathbf{I} = \begin{bmatrix} 40.45 &amp; -0.2 &amp; -0.5 \ -0.2 &amp; 42.09 &amp; 0.4 \ -0.5 &amp; 0.4 &amp; 41.36 \end{bmatrix} ) kgm(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orbital parameters</td>
<td>Orbital rate = ( \frac{2\pi}{6000} ) rad/sec</td>
</tr>
<tr>
<td></td>
<td>Inclination ( i = 65^\circ )</td>
</tr>
<tr>
<td></td>
<td>Orbital Period = 100 min</td>
</tr>
<tr>
<td>Sample time</td>
<td>1 sec</td>
</tr>
<tr>
<td>Reaction wheel</td>
<td>Maximum torque = 0.015 Nm</td>
</tr>
<tr>
<td></td>
<td>Maximum Momentum = 4 Nms</td>
</tr>
<tr>
<td></td>
<td>Moment of inertia = 0.0077 kgm(^2)</td>
</tr>
<tr>
<td></td>
<td>Maximum speed = 5000 rpm</td>
</tr>
<tr>
<td>Momentum dumping gain</td>
<td>( K_m = 50 )</td>
</tr>
<tr>
<td>Quatierion controller gain</td>
<td>( 5% , t_\alpha = \frac{3}{\zeta \omega_\text{n}} = 180 )</td>
</tr>
<tr>
<td></td>
<td>( k_p = 0.0079 ) (proportional control gain)</td>
</tr>
<tr>
<td></td>
<td>( k_d = 0.0888 ) (derivative control gain)</td>
</tr>
<tr>
<td>Bang-Bang controller</td>
<td>( k_1 = 1 ) (proportional control gain)</td>
</tr>
</tbody>
</table>
\[ g^2 = 2.3 \text{ (derivative control gain)} \]
\[ e_{\text{rand}} = 0.02 \text{ rad} \]

**High pass filter**

2mHz = cut of frequency

**RLS parameters**

<table>
<thead>
<tr>
<th>Axis</th>
<th>Off- diagonal</th>
<th>Diagonal</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>X-axis</strong></td>
<td>( \lambda = 0.998 )</td>
<td>( \lambda = 0.997 )</td>
</tr>
<tr>
<td></td>
<td>( P(0) = 1e4 )</td>
<td>( P(0) = 1e5 )</td>
</tr>
<tr>
<td></td>
<td>( I_y(0) = 0 )</td>
<td>( I_y(0) = 40 )</td>
</tr>
<tr>
<td><strong>Y-axis</strong></td>
<td>( \lambda = 0.996 )</td>
<td>( \lambda = 0.997 )</td>
</tr>
<tr>
<td></td>
<td>( P(0) = 1e4 )</td>
<td>( P(0) = 1e5 )</td>
</tr>
<tr>
<td></td>
<td>( I_y(0) = 0 )</td>
<td>( I_y(0) = 40 )</td>
</tr>
<tr>
<td><strong>Z-axis</strong></td>
<td>( \lambda = 0.995 )</td>
<td>( \lambda = 0.996 )</td>
</tr>
<tr>
<td></td>
<td>( P(0) = 1e4 )</td>
<td>( P(0) = 1e5 )</td>
</tr>
<tr>
<td></td>
<td>( I_y(0) = 0 )</td>
<td>( I_y(0) = 40 )</td>
</tr>
</tbody>
</table>

Where, \( i, j = x, y, z \quad i \neq j \)

The RLS algorithm was implemented in a full simulation program of the UoSAT-12 satellite's dynamics, kinematics, sensor noise and disturbance torques. Simulation tests were implemented to investigate the performance of the proposed MOI estimation algorithms. The UoSAT-12 simulator block diagram during inertia matrix identification is shown in Fig. 5.12 (for more detail see Fig. C.3 in Appendix C).

Figs. 5.14 to 5.20 show the results of the MOI estimation algorithm using the full reaction wheel torque (15 mNm) and a significant momentum (approximately 1.5 Nms peak-to-peak) in order to increase the influence of the products of inertia in the dynamic equations. One experiment is needed using two separate RLS algorithms to estimate both the principal and products of inertia for each axis.

The external disturbance torques were applied to investigate the convergence of the MOI parameters and the robustness performance of the RLS algorithms against these unmodelled disturbances. Momentum dumping is active during these simulations to prevent momentum build-up on the reaction wheels.
Figs. 5.14 to 5.18 illustrate the performance of the RLS algorithms when disturbing the Y-axis first (as already explained) using a Bang-Bang nonlinear controller Eq. (5.24). The X- and Z-axes are controlled using a quaternion feedback control law Eq. (5.23) to estimate $I_{yx}$, $I_{xy}$, and $I_{yx}$ Eqs. (5.29) and (5.32). It is clear from these figures that the convergence of the RLS estimated parameters was achieved in approximately 1000 seconds in case of the moment of inertia $I_{yy}$ and approximately 3000 seconds in case of the products of inertia $I_{yx}$ and $I_{xy}$. The parameter variation after conversion was also very small around the true value (Fig. 5.18) with an error of 0.1% rms for the principal moment of inertia ($I_{yy}$) and better than 1% rms for the products of inertia ($I_{xy}$, $I_{yx}$) as in Table 5.2. It is also clear from these figures that no saturation in the wheel torque and momentum occur (see Figs. 5.14 and 5.15 respectively). The wheel momentum can drift towards saturation due to the external disturbance torques (see Fig. 5.15) but thus is prevented by implementing a magnetorquer momentum management controller. The Euler angles are also small in the controlled roll and yaw axes (not exceeding $\pm 0.7^\circ$) and approximately $\pm 35^\circ$ in the disturbed pitch axis (Fig. 5.17). The wheel momentum is contained to keep the angular rates in the controlled axes as small as possible (Fig. 5.16) in order to estimate the products of inertia more accurately. Compare these results with the case where no external disturbance torque or momentum dumping control was simulated as shown in Appendix B.

Fig. 5.19 illustrates the performance of the RLS algorithms when disturbing the X-axis next (we can also disturb the Z-axis next). Similarly the Y- and Z-axes are controlled using a quaternion feedback control law to estimate $I_{xx}$, $I_{zx}$, and $I_{zx}$ Eqs. (5.36) and (5.39). When previously disturbing the Y-axis, the estimated values of $\hat{I}_{yy}$, $\hat{I}_{xy} = \hat{I}_{yx}$ and $\hat{I}_{xy} = \hat{I}_{yx}$ have been obtained so these estimated values could be used when disturbing the X-axis. It is clear from this figure that the convergence of the RLS estimation algorithms was achieved in approximately 1000 seconds when estimating $I_{xx}$ and approximately 3500 seconds when estimating $I_{zx}$ and $I_{zx}$. The parameter variation after was also very small around the true value (Fig. 5.19) with an error better than 0.1% rms for the principal moment of inertia $I_{xx}$ and better than 1% rms for the products of inertia $I_{zx}$, $I_{zx}$ (Table 5.2).
Finally Fig. 5.20 illustrates the performance of the RLS algorithms when the Z axis is disturbed. The X- and Y-axes are controlled to estimate $I_{xx}$, $I_{xy}$ and $I_{yy}$ Eqs. (5.43) and (5.46). When previously disturbing the X- and Y-axes, the estimated values of $\hat{I}_{xx}$, $\hat{I}_{yy}$, $\hat{I}_{xy} = \hat{I}_{yx}$, $\hat{I}_{yx} = \hat{I}_{xy}$ and $\hat{I}_{zz} = \hat{I}_{zz}$ have been obtained so these estimated values can be used when disturbing the Z-axis. Two experiments are then sufficient to estimate all products of inertia but for investigative purpose it is also useful to obtain two separate estimates of these product of inertia. It is clear from this figure that the convergence of the RLS estimation algorithms was similar to that of the previous two axes. The parameter variation after conversion was also very small around the true value Fig. 5.20 with an error of 0.1% rms for the principal moment of inertia $I_{xx}$ and better than 1% rms for the products of inertia $I_{zz}$.

Simulation results are shown in appendix B and are introduced in the first page of that appendix. These results are refered to in the discussion that follows.

Table B.1 (see Appendix B) summarises all the results obtained from disturbing the Y, X and Z axes respectively, for different values of the weighting factor $\lambda$, using full reaction wheel torque (15 mNm) and a large wheel momentum (approximately 1.5 Nms peak-to-peak). Tables 5.2 and 5.3 summarises the optimum values of $\lambda$ obtained which gives the minimum rms error (the values of $\lambda$ were chosen similar to Fig. 5.12, see Appendix B for the optimum values of $\lambda$). The estimated values of the principal moment of inertia converged to their true values with an error of 0.1% rms, and the estimated values of the products of inertia converged to their true values with an error better than 1% rms. It is also clear that the average values of the inertia matrix are close to the true values (compared to Table 5.1). We can also take the average values of pairs of the products of inertia to further improve the estimation results.
Table 5.2 MOI parameters conversion

<table>
<thead>
<tr>
<th>(rms)error</th>
<th>without external disturbance torques and magnetorquers (momentum dumping)</th>
<th>with external disturbance torques and magnetorquers (momentum dumping)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$I_{xx}$</td>
<td>$I_{yx}$</td>
</tr>
<tr>
<td>X- axis</td>
<td>0.000379</td>
<td>0.00441</td>
</tr>
<tr>
<td>Y- axis</td>
<td>$I_{xy}$</td>
<td>$I_{yy}$</td>
</tr>
<tr>
<td></td>
<td>0.00258</td>
<td>0.000493</td>
</tr>
<tr>
<td>Z- axis</td>
<td>$I_{xz}$</td>
<td>$I_{yz}$</td>
</tr>
<tr>
<td></td>
<td>0.002318</td>
<td>0.004284</td>
</tr>
</tbody>
</table>

Table 5.3 MOI average values

<table>
<thead>
<tr>
<th>MOI average values (kgm$^2$)</th>
<th>without external disturbance torques and magnetorquers (momentum dumping)</th>
<th>with external disturbance torques and magnetorquers (momentum dumping)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X- axis</td>
<td>$I_{xx}$</td>
<td>$I_{yx}$</td>
</tr>
<tr>
<td></td>
<td>40.45</td>
<td>-0.2036</td>
</tr>
<tr>
<td>Y- axis</td>
<td>$I_{xy}$</td>
<td>$I_{yy}$</td>
</tr>
<tr>
<td></td>
<td>-0.1976</td>
<td>42.09</td>
</tr>
<tr>
<td>Z- axis</td>
<td>$I_{xz}$</td>
<td>$I_{yz}$</td>
</tr>
<tr>
<td></td>
<td>-0.5014</td>
<td>0.4041</td>
</tr>
</tbody>
</table>
Figure 5.12 UoSAT-12 simulator during estimation of inertia matrix
Figure 5.13 Disturbance torque

Figure 5.14 Reaction wheel torque in case of inertia estimation (Y-axis) with disturbance
Figure 5.15 Reaction wheel momentum in case of inertia estimation (Y-axis) with disturbance

Figure 5.16 Angular rates in case of inertia estimation (Y-axis) with disturbance
Figure 5.17 Euler angles in case of inertia estimation (Y-axis) with disturbance

Figure 5.18 Inertia estimation (Y-axis) with disturbance
Inertia Coefficients: True (dashed), Estimated (solid)

Figure 5.19 Inertia estimation (X-axis) with disturbance

Inertia Coefficients: True (dashed), Estimated (solid)

Figure 5.20 Inertia estimation (Z-axis) with disturbance
5.10 Conclusion

The MOI estimation algorithms discussed in this chapter can be easily applied to any spacecraft equipped by three reaction wheel actuators to estimate the inertia matrix. A Bang-Bang nonlinear controller is used to disturb the satellite attitude. This controller will avoid any build up in the error quaternion and wheel momentum. A quaternion PD feedback controller controls the other two axes. This controller will ensure robustness against modelling errors and external disturbances. Due to the low computation requirements of these algorithms, both the control and estimation scheme can be easily implemented on-board satellites to accurately estimate the inertia matrix.

Simulation results have been given to illustrate the merits of the proposed estimation algorithms. The conclusions drawn from the simulation tests were: 1) The newly proposed MOI estimation algorithms could be suitable in practice to estimate the inertia matrix. 2) The estimation algorithms proposed, will estimate the inertia matrix parameters to accuracies better than 1% rms even during the presence of external disturbances and measurement errors.
Chapter 6

6. Practical Experimental Results

6.1 Introduction

In the last two chapters two estimation algorithms have been presented which can calibrate the cold gas thrusters and the satellite inertia matrix in orbit. Extensive simulations have been carried out to investigate the feasibility of the proposed estimation algorithms. The final goal of this research is to apply the estimation algorithms in practice onboard a satellite. Practical demonstrations will provide a more rigorous test. In this chapter, we are going to present several experimental tests to demonstrate:

- A thruster calibration algorithm on an air-bearing rotary table.
- Thruster calibration algorithms onboard UoSAT-12.
- Satellite moment of inertia estimations onboard UoSAT-12.

6.2 Thruster Calibration Algorithm on an Air-Bearing Table

6.2.1 Introduction to the Air Bearing and Experimental Hardware

An air-bearing table provide the capability of rotation, around one axis without significant friction. It is often used to test the dynamic characteristics and performance of a model satellite control system during the pre-launch experimental test phase on the ground. The satellite is fixed to a table floating on air, which allows nearly frictionless rotation. The rotation freedom depends on the mechanical structure. The air bearing used for this experiment only has one rotational degree of freedom. The bearing is in a spindle shape with air supplied under pressure to lift the table and the external loads. Due to a lack of contact between the rotating table and stationary platform, air bearings offer several significant advantages.
- Low friction
- High accuracy of motion
- Zero wear
- Small disturbance torques

The following equipment was used during the thruster calibration test on the air-bearing table: One SSTL experimental reaction wheel was used to deliver a known disturbance torque. A pair of PWM thrusters was implemented, one for ESAT and the other one for UoSAT-12 to feedback compensate and rotate the air-bearing table clockwise and anticlockwise respectively. A Sun sensor provided measurements of the rotation angle and angular rate (by differentiation) of the air-bearing table. The structure of the experimental platform used during the experiment is shown in Fig. 6.1.

Figure 6.1 Air-bearing table and experimental hardware for thruster calibration
6.2.2 Dynamics of the Air-Bearing Table and the Estimation Algorithm

Since the air-bearing table in this experiment rotates around a single axis, the approximate dynamics of the table rotation is given by:

\[ I_{AB} \dot{\omega}_{AB} = -I_{w} \dot{\omega}_{w} + N_T + N_d \]  \hspace{1cm} (6.1)

Where
\[ I_{AB} = \text{moment of inertia of the air-bearing rotating table} \]
\[ \omega_{AB} = \text{angular speed of the air-bearing rotating table} \]
\[ I_{w} = \text{moment of inertia of the reaction wheel around its rotation axis} \]
\[ \omega_{w} = \text{angular speed of the reaction wheel} \]
\[ N_d = \text{external disturbance torque including air friction, aerodynamic drag, etc.} \]
\[ N_T = \text{applied torque by thruster} \]

Based on the thruster calibration algorithm discussed in Chapter 4, the relationship between the commanded and actual torque output of the thrusters is given by:

\[ N_T = \begin{bmatrix} a_{11} & a_{22} \end{bmatrix} \begin{bmatrix} T_{cp} \\ T_{cn} \end{bmatrix} \]  \hspace{1cm} (6.2)

where
\[ a_{11} = \text{unknown thruster coefficient of UoSAT-12 thruster} \]
\[ a_{22} = \text{unknown thruster coefficient of ESAT thruster} \]
\[ T_{cp} = \text{thruster torque (0 = on and 1 = off) command for UoSAT-12 thruster} \]
\[ T_{cn} = \text{thruster torque (0 = on and 1 = off) command for ESAT thruster} \]

Define,
\[ N_c = I_{AB} \dot{\omega}_{AB} + I_{w} \dot{\omega}_{w} \]  \hspace{1cm} (6.3)
Equation (6.3) acts as a measurement equation for the RLS procedure to estimate the thruster coefficients $a_{11}$ and $a_{22}$. The error to be minimised is given by: (substitute Eqs.(6.2) into (6.1))

$$\varepsilon(k) = N_r(k) - a_{11}T_{cn} - a_{22}T_{cn}$$  \hspace{1cm} (6.4)

The thruster controller is working in PWM mode and only one thruster is active at a time. Two separate RLS procedures are required to estimate $a_{11}$ and $a_{22}$.

6.2.3 Experimental Results

The block diagram for this experiment is given in Fig. 6.2.

![Block diagram of the experiment on air-bearing table](image)

Figure 6.2 The block diagram of the experiment on air-bearing table

The parameters for the experiment are listed in Table 6.1. During the test, the reaction wheel was accelerated and decelerated between limits of $\pm 300$ rpm to follow a predetermined reference speed, see Fig. 6.4. The PD feedback gains for the thruster controller were chosen for a settling time of 25 seconds. Fig. 6.3 shows the measured rotation angle of the air-bearing table during the test. The resulting thruster calibration
result is shown in Fig. 6.5. Note that the two thrusters are fired and parameters identified
alternately during the experiment. It is clear from this figure that the thruster parameters
have settled in approximately 500 and 700 seconds for the UoSAT-12 and ESAT thrusters
respectively. The estimated thruster coefficients are approximately 1.28 for the ESAT
thruster and 0.95 for the UoSAT-12 thruster.

Table 6.1 Experimental parameters for the Thruster calibration

<table>
<thead>
<tr>
<th>Sample time</th>
<th>1 second</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated moment of</td>
<td>$I_{AB} = 5.11 \text{ kg.m}^2$</td>
</tr>
<tr>
<td>inertia of the air-bearing table</td>
<td></td>
</tr>
</tbody>
</table>
| Reaction wheel       | Moment of inertia: $I_w = 8 \times 10^{-3} \text{ kg.m}^2$
|                      | Maximum torque: $N_{w\text{-max}} = 0.015\text{Nm}$
|                      | Maximum speed: 5000rpm |
| Thruster torque      | Minimum firing time in PWM: $t_{\text{min}} = 10 \text{ msec}$
|                      | Maximum nominal torque: $N_f = 0.066\text{Nm}$ |

Figure 6.3 Rotation angle measurement of the air bearing table
Figure 6.4 The reference and measured rotation speed of the reaction wheel

Figure 6.5 Estimated thruster coefficients
6.3 Practical Test Onboard UoSAT-12

The SSTL’s 320 kg mini-satellite UoSAT-12 provides the capability for 3-axis attitude determination and control. It operates in a variety of modes using different combinations of sensors and actuators. The active attitude control actuators of UoSAT-12 consist of three-axis magnetorquers, reaction wheels and PWM cold-gas thrusters. For in-orbit thruster calibration and in-orbit moment of inertia estimation as discussed in Chapters 4 and 5, three reaction wheels are required to calibrate both the thruster coefficients and inertia matrix. However, the Z-axis reaction wheel of UoSAT-12 has failed. Therefore, slightly modified calibration procedures are needed to test the proposed estimation algorithms onboard UoSAT-12. All the estimation results were obtained by using the estimated angular rates from an Extended Kalman Filter with a 5 second sampling time.

6.3.1 Thruster Calibration Onboard UoSAT-12

In this section, we present the identification of thruster coefficients using real data generated onboard UoSAT-12, to calibrate both the yaw thrusters (4 thrusters) and delta-V thrusters (2 thrusters).

Fig. 4.4 shows the location of the thrusters for UoSAT-12. The thrust arm of the pitch, roll and yaw thrusters (8 cold gas) to the COG of UoSAT-12 is approximately 0.44 meter each, this gives a torque of 66 milli-Nm for attitude control. The Z-axis (yaw) control thrusters will, however, always be fired in an opposing pair to give a pure rotation without any translation forces, so the Z-axis thruster torque will be 132 milli-Nm per dual pulse. The X/Y-axis (roll and pitch) thrusters will be fired as single units for short periods of time and will therefore present a small translational disturbance to the orbit.

6.3.1.1 Yaw Thruster Calibration

The procedure for calibration is based on the theory outlined in Chapter 4. A pair of thrusters are used to generate an attitude control rotation of the satellite around the yaw-axis using a Bang-Bang thruster controller. The X and Y axes are controlled using a quaternion feedback reaction wheel controller to identify the \(3 \times 2\) coefficients of the Z-thrusters:
\[
N_T = \begin{bmatrix}
a_{xyp} & a_{xyp} \\
a_{xyp} & a_{xyp} \\
a_{xyp} & a_{xyp}
\end{bmatrix}
\begin{bmatrix}
T_{x} \\
T_{y} \\
T_{z}
\end{bmatrix}
\] (6.5)

Where,

- \( N_T = \) thruster torque vector
- \( T_{x} = \) positive yaw maximum torque level
- \( T_{y} = \) negative yaw maximum torque level
- \( A_c = 3 \times 2 \) thruster coefficients matrix

The parameters for the experiment are listed in Table 6.2. Fig. 6.6 displays the wheel momentum during the experiment. It is clear from this figure that the drift in the Y-wheel momentum is greater than the drift in the X-wheel momentum. This means that the cross disturbance torque due to the firing of the Z-thrusters is greater in Y-axis than in X-axis. The Z-axis thrusters are fired for short periods (a pulse width of 100 msec) using the Bang-Bang controller. Fig. 6.9 shows the Z-thruster torque plotted against time during the experiment. The Euler angle (see Fig. 6.7) does not exceed ±6° in the disturbed axis. The angular rates are shown in Fig. 6.8. It is clear from this figure that the slope of the positive angular rate in the Z-axis is faster than the negative angular rate slope. This means that the produced torque due to positive thruster firings is greater than the torque during negative thruster firings. The estimated thruster coefficients using the RLS estimation method are shown in Fig. 6.10. We observe from this figure that the mean values of the thruster coefficients converge to the following values at 1200 sec:

\[
\begin{bmatrix}
a_{xyp} & a_{xyp} \\
a_{xyp} & a_{xyp} \\
a_{xyp} & a_{xyp}
\end{bmatrix} = \begin{bmatrix}
0.02 & 0.03 \\
-0.05 & 0.1 \\
1.03 & 0.72
\end{bmatrix}
\] (6.6)

We can conclude from the estimated thruster coefficients that the cross disturbances in the Y-axis is greater than in the X-axis. The main coefficient of the positive Z-thruster firings is greater than the negative one as suggested by. This confirms the shapes seen for the Z-axis wheel momentum and angular rate. The RLS has successfully estimated the thruster coefficients.
Figure 6.6 Reaction wheel momentum when estimating the coefficients of the yaw thrusters on UoSAT-12

Figure 6.7 Euler angle when estimating the coefficients of the yaw thrusters on UoSAT-12
Figure 6.8 Angular rates when estimating the coefficients of the yaw thrusters on UoSAT-12

Figure 6.9 Thruster torque when estimating the coefficients of the yaw thrusters on UoSAT-12
Figure 6.10 Estimated yaw thruster coefficients on UoSAT-12

Table 6.2 Parameters for UoSAT-12 thruster calibration

<table>
<thead>
<tr>
<th>Sample time</th>
<th>5 seconds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment of inertia of</td>
<td></td>
</tr>
<tr>
<td>UoSAT-12</td>
<td>( I = \begin{bmatrix} 40.45 &amp; 0 &amp; 0 \ 0 &amp; 42.09 &amp; 0 \ 0 &amp; 0 &amp; 40.36 \end{bmatrix} \text{ kg m}^2 )</td>
</tr>
<tr>
<td>Reaction wheel</td>
<td>Z-axis wheel failed</td>
</tr>
<tr>
<td></td>
<td>Maximum torque output = 0.015 Nm</td>
</tr>
<tr>
<td></td>
<td>Moment of inertia of Y-wheel = 0.0077 kgm^2</td>
</tr>
<tr>
<td></td>
<td>Moment of inertia of X-wheel = 0.008 kgm^2</td>
</tr>
<tr>
<td></td>
<td>Maximum speed = 5000 rpm</td>
</tr>
<tr>
<td>Cold-gas thruster</td>
<td>Maximum torque = 0.066 Nm</td>
</tr>
<tr>
<td></td>
<td>Minimum firing time in PWM = 10 msec</td>
</tr>
<tr>
<td>Orbital parameters</td>
<td>Orbital rate = ( 2\pi / 6000 \text{ rad/sec} )</td>
</tr>
<tr>
<td></td>
<td>Inclination ( i = 65^\circ )</td>
</tr>
<tr>
<td></td>
<td>Orbital Period = 100 min</td>
</tr>
</tbody>
</table>
### Attitude determination

**Estimation algorithm:** Extended Kalman Filter
**Measurement source:** 3-axis magnetometers and Sun sensor

### Quaternion controller gain

- \( 5\% = \frac{3}{\zeta \omega_n} = 180 \)
- \( k_p = 0.0011 \) (proportional control gain)
- \( k_d = 0.033 \) (derivative control gain)

### Bang-Bang controller gain

- \( k_i = 1 \) (proportional control gain)
- \( k_d = 2.3 \) (derivative control gain)
- \( e_{band} = 0.02 \text{ rad} \)

### RLS parameters

- \( \lambda(0) = 0.95 \)
- \( P(0) = 1e4 \)

### 6.3.1.2 Delta-V Thruster Calibration

The pitch axis is disturbed by using a Bang-Bang reaction wheel controller for the Y-wheel \( (N_{max} = 2 \text{ mNm}) \), while this axis is controlled using a pair of delta-V thrusters, to identify the \( 3 \times 2 \) coefficients matrix. The X-axis is controlled using the X-wheel, while the Z-axis is controlled using the calibrated yaw thrusters from the previous experiment.

\[
N_T = \begin{bmatrix}
  a_{\Delta v_p} & a_{\Delta v_n} \\
  a_{\Delta v_p} & a_{\Delta v_n} \\
  a_{\Delta v_p} & a_{\Delta v_n}
\end{bmatrix}
\begin{bmatrix}
  T_{\Delta v_p} \\
  T_{\Delta v_n}
\end{bmatrix}
\]

(6.7)

Where,
- \( N_T \) = applied torque vector by thruster
- \( T_{\Delta v_p} \) = positive delta-V maximum torque level
- \( T_{\Delta v_n} \) = negative delta-V maximum torque level
- \( A_c \) = \( 3 \times 2 \) thruster coefficients matrix

The parameters for the experiment are the same as listed in Table 6.2. Fig. 6.11 illustrates the wheel momentum during the experiment. The drift in the wheel momentum for the Y-axis is due to the cross disturbances caused by the Z-thrusters. As is clear from Fig. 6.12
the pitch angle does not exceed $\pm 3^\circ$. The angular rate in the Y-axis is not triangular in shape (Fig. 6.13), this is due to the limited bandwidth of the feedback delta-V thruster controller when the Y-axis is disturbed with the Y-wheel. Fig. 6.14 shows the thruster torque during the experiment. The estimated thruster coefficients using the RLS estimation method are shown in Fig. 6.15. We observe from this figure that the mean values of the thruster coefficients converge approximately to the following:

\[
\begin{bmatrix}
a_{x_{\Delta v}} & a_{y_{\Delta v}} \\
a_{y_{\Delta v}} & a_{y_{\Delta v}} \\
a_{z_{\Delta v}} & a_{z_{\Delta v}}
\end{bmatrix} = \begin{bmatrix}
0.008 & -0.009 \\
0.567 & 0.55 \\
0.006 & -0.003
\end{bmatrix}
\]

For $N_f = 68$ mNm \hspace{1cm} (6.8)

We can conclude from the estimation results that the cross disturbances are very small.

Figure 6.11 Reaction wheel momentum when estimating the coefficients of the delta-V thrusters on UoSAT-12
Figure 6.12 Euler angle when estimating the coefficients of the delta-V thrusters on UoSAT-12

Figure 6.13 Angular rates when estimating the coefficients of the delta-V thrusters on UoSAT-12
Figure 6.14 Thruster torque when estimating the coefficients of the delta-V thrusters on UoSAT-12

Figure 6.15 Estimated delta-V thruster coefficients for UoSAT-12
6.3.2 MOI Estimation Onboard UoSAT-12

The procedure followed in this experiment is based on the theory in Chapter 5. The MOI estimation is based on a procedure to disturb the satellite axis in which it is hoped to determine the corresponding principal MOI, controlling the other two axes to determine the two products of inertia. Due to the failure of the Z-axis reaction wheel on UoSAT-12, and the small disturbance torques (calculated in Chapter 2) acting on UoSAT-12, only one axis was calibrated. The MOI estimation procedure was tested onboard UoSAT-12 by disturbing the Y-axis using a Bang-Bang wheel controller, whilst the X-axis was controlled using a quaternion feedback wheel controller. The Z-axis is not controlled. All the equations derived in Section 5.8.5.1 are used during the onboard test, except for the Z-axis wheel torque and momentum which are set to zero in the dynamic equations (i.e. \( \dot{h}_z = h_z = 0 \)). The experiment parameters are the same as listed in Table 6.2. The maximum wheel torque in the Y-axis is 10 milli-Nm and the maximum wheel momentum is approximately 1 Nms during the experiment (see Figs. 6.16 and 6.17 respectively). The maximum pitch angle is approximately ±50° (see Fig. 6.18). The angle and the angular rate in the uncontrolled axis (Z-axis) is kept small, this is due to the small external disturbance torque acting on UoSAT-12 (see Figs. 6.18 and 6.19 respectively). The estimated MOI and products of inertia using the RLS estimation method are shown in Fig. 6.20. It is clear from this figure that the estimated principal moment of inertia \( \hat{I}_{yy} \) converges to its measured value after approximately 1500 seconds with a mean value of \( \hat{I}_{yy} = 42.03 \text{ kgm}^2 \) (compare with the pre-launch measured value from Table 6.2). The variation after convergence is very small around the measured value. The mean values of the products of inertia \( \hat{I}_{xy}, \hat{I}_{yx} \) have converged to 0.22 and -0.09 kgm² respectively.
Figure 6.16 Reaction wheel torque when disturbing the Y-axis to estimate the moment and products of inertia for UoSAT-12

Figure 6.17 Reaction wheel momentum when disturbing the Y-axis to estimate the moment and products of inertia for UoSAT-12
Figure 6.18 Euler angles when disturbing the Y-axis to estimate the moment and products of inertia for UoSAT-12.

Figure 6.19 Angular rates when disturbing the Y-axis to estimate the moment and products of inertia for UoSAT-12.
Figure 6.20 Estimated moment and products of inertia when disturbing the Y-axis

Table 6.3 Analysis of the Practical Results

<table>
<thead>
<tr>
<th>Term</th>
<th>Average value</th>
<th>Standard deviation</th>
<th>Standard deviation as % of mean value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_{x2p} a_{x2n} )</td>
<td>0.0107</td>
<td>0.0301</td>
<td>40 14</td>
</tr>
<tr>
<td>( a_{y2p} a_{y2n} )</td>
<td>-0.0746</td>
<td>0.1093</td>
<td>17 14</td>
</tr>
<tr>
<td>( a_{z2p} a_{z2n} )</td>
<td>1.0215</td>
<td>0.7173</td>
<td>0.567 0.93</td>
</tr>
<tr>
<td>( a_{x2p} a_{x2n} )</td>
<td>0.0062</td>
<td>-0.00027</td>
<td>61 68</td>
</tr>
<tr>
<td>( a_{y2p} a_{y2n} )</td>
<td>0.8176</td>
<td>0.8134</td>
<td>1.87 1.89</td>
</tr>
<tr>
<td>( a_{z2p} a_{z2n} )</td>
<td>0.0062</td>
<td>-0.00027</td>
<td>0.34 0.63</td>
</tr>
<tr>
<td>( I_{xy} )</td>
<td>-0.0489</td>
<td>0.0224</td>
<td>45</td>
</tr>
<tr>
<td>( I_{xy} )</td>
<td>41.96</td>
<td>0.0257</td>
<td>0.06</td>
</tr>
<tr>
<td>( I_{zz} )</td>
<td>0.2258</td>
<td>0.0104</td>
<td>4.6</td>
</tr>
</tbody>
</table>
Analysis of the in-orbit results are summarised in Table 6.3. It is clear from the table that the ratios between the standard deviation and the mean value of the estimated parameters are very small in case of estimation of the main components of thruster coefficients (i.e. $a_{z\tau}$, $a_{z\theta}$, $a_{y\phi}$ and $a_{y\\theta\\alpha}$) and the principal MOI $I_{yy}$, whilst this ratio is larger when estimating the cross coupling components. This is due to the larger effective signal to noise ratio when estimating the small rates and accelerations due to the cross-coupling components. The estimation results can be improved by increasing the signal to noise ratio. This can be done either by using direct measurement from rate gyro and accelerometer sensors to estimate the satellite angular rates and acceleration respectively or by increasing the level of the disturbance torques to increase the measured values. The disadvantage of increasing the level of disturbance torque is that this will consume more fuel and perturb the satellite attitude more.

### 6.4 Conclusion

In this chapter, several experimental tests were presented to demonstrate the theory presented in chapters four and five. At first the thruster calibration algorithm was tested on an air-bearing table. One SSTL experimental reaction wheel was employed to generate an open-loop bang-bang control torque. Two Polyflex thrusters, one for ESAT and the other one for UoSAT-12 were calibrated using a PD feedback method to control the air-bearing table against the disturbance torque caused by the reaction wheel. The results obtained from the estimated thruster coefficients prove the principal of the proposed calibration algorithm. Secondly, thruster calibration algorithms were tested using in-orbit data generated by UoSAT-12. The calibration is based on a RLS estimation method. Two different tests were carried out to calibrate the yaw and delta-V thrusters. In the first test to calibrate the yaw thrusters (4 thrusters), the Z-axis was disturbed using a Bang-Bang controller for the Z-thrusters. The X/Y axes were controlled using wheels, to determine a total of six coefficients for the four thrusters. In the second test the delta-V thrusters were calibrated (2 thrusters). The Y-axis was disturbed using an open-loop Bang-Bang Y-wheel torque disturbance while this axis was feedback controlled using the two delta-V thrusters. The X-axis was controlled using a wheel. The Z-axis was controlled using the calibrated...
thrusters to determine a total of six coefficients for the two delta-V thrusters. The estimated thruster coefficients proved the practical application of the calibration method. Finally, MOI calibration algorithms were tested using in-orbit data generated by UoSAT-12. The calibration procedure is based on a RLS estimation method. The Y-axis was disturbed using the Y-wheel to estimate the principal MOI of the Y-axis. The X-axis was controlled using the X-wheel and the Z-axis was not controlled. The estimation results proved that the principal MOI converged to its pre-launch measured value in approximately 1500 seconds with small subsequent fluctuations around the true value. The mean value of the products of inertia also settled at specific values. The use of a smaller sampling time, full 3-axis reaction wheel actuation and more accurate rate sensors (e.g. rate gyroscopes) would improve the estimation performance.
Chapter 7

7. Conclusions

Several new calibration techniques for Earth orbiting satellites are presented in this thesis. The research was directed mainly to the development of autonomous parameter estimation algorithms to improve the performance of ADCS. The novel techniques are the use of new in-orbit estimation algorithms to calibrate cold gas thrusters and estimate the satellite inertia matrix for the LEO minisatellite UoSAT-12. Due to the accuracy of wheel speed measurements and wheel moment of inertia knowledge, the reaction wheel torques are accurately known and can be used to estimate the satellite inertia matrix and the satellite's thruster coefficients. The technique also includes a recursive estimator to estimate the satellite angular acceleration from noisy angular rates and the use of a high pass filter to remove the influence of low frequency disturbance torques. Most of the results are general and can be applied to any spacecraft equipped with three reaction wheel actuators. The new results were extensively tested by computer simulation. The in-orbit practical results provide confidence in the proposed algorithms.

A summary of the significant contributions from this research study follow:

7.1 High Pass Filter Design

In most of the estimation methods in literature, the errors in the system model are assumed to be represented by a zero-mean Gaussian process with known covariance. However, in practice the process noise is often not zero-mean. The assumption of a zero-mean Gaussian error process in such cases can lead to biased state estimates. A high pass filter was designed (see Chapter 5) to filter the RLS error and thus remove the effects of non-zero mean aerodynamic disturbance torques. Results using this high pass filter indicate that the modified recursive least square algorithm (RLS) successfully estimates the inertia matrix elements of a spacecraft in spite of the non-zero-mean aerodynamic disturbance torque.
7.2 Acceleration Estimator Design

In order to accurately estimate the spacecraft inertia matrix and cold gas thruster coefficients, the dynamics of the satellite must be known. Modelling the measurement equations requires knowledge of the spacecraft acceleration states which form part of the dynamic equations. During stabilisation, when the attitude controller is able to keep the satellite’s angular rate close to zero, the acceleration term has no significant effect when calculating the measurement equation and can be set to zero. In the case of moment of inertia estimation (see Chapter 5) the angular rate in the disturbed axis is high and changes linearly with time, consequently the acceleration term has a significant effect on the measurement equation. Neglecting the acceleration term in the measurement equation leads to incorrect estimates. Unfortunately, there is no sensor to directly measure the angular acceleration on UoSAT-12. This research study proposes the use of a new simple first order recursive feedback estimator (see Chapter 3) to estimate the angular acceleration from the angular rates (measured or estimated). Computer simulations to test the acceleration estimator have been performed as presented in Chapters 4, 5, and 6.

7.3 In-Orbit Thruster Calibration Algorithm

A cold gas propulsion system is sometimes used for attitude and orbit control onboard small satellites. The main advantage of cold gas thrusters during attitude maneuvers is that they can provide large, instantaneous torques at any instant during an orbit. Spacecraft equipped with thrusters can present significant disturbance torques as well as large control coupling torques if the attitude control thrusters are not calibrated properly. The quality of an attitude control system using propulsion torque controllers is strongly influenced by the following: thrust level, torque arm and thrust direction relative to the satellite body axis. Accurate calibration of the thrusters on the ground prior to flight is limited by a variety of factors. If thruster coefficients are not properly identified in orbit, significant attitude control errors can result. To satisfy high accuracy ADCS requirements calibrated thrusters are needed at all times.

In Chapter 4, a novel estimation algorithm is presented which will calibrate the cold gas thrusters in-orbit during normal mission conditions, when the satellite is stabilized. This
method requires knowledge of a calibration or known disturbance torque (generated using
the reaction wheel actuators) whilst the attitude is controlled using the thrusters. The
algorithm assumes no initial knowledge of the thruster parameters and knowledge only of
the inertia matrix. Different estimation methods and reaction wheel torque profiles were
used to choose the best techniques to calibrate the thrusters in-orbit. Numerical
simulations illustrate the successful identification of the thruster parameters in spite of
non-zero mean disturbance torque and sensor noise. The estimation algorithm could be
applied in real-time on board a LEO nadir pointing satellite in order to improve the attitude
control performance.

7.4 In-Orbit Inertia Matrix Identification

The recent tendency is to build smaller, lighter and cheaper spacecraft. On-line calibration
of the attitude control hardware is often necessary to satisfy a high accuracy ADCS
requirement and can be used to replace expensive ground test equipment. The MOI of
simple shapes may be calculated by well-known methods. When the shape and the density
distribution of the rigid body is poorly known or difficult to model, it is difficult to compute
the moment and products of inertia through numerical integration (see Chapter 5).
Therefore, the moment and products of inertia must be measured directly using the object.
Generally, the MOI of a satellite is measured using ground equipment. Such equipment is
very expensive especially if the products of inertia are also to be measured.

In Chapter 5, a novel in-orbit moment of inertia (MOI) estimation algorithm using
Recursive Least Square (RLS) algorithm is presented which will identify the inertia matrix
of a satellite. The MOI identification algorithm is a completely new way to estimate the
inertia matrix (moment and products), not previously stated or implemented elsewhere.
This is a general procedure to identify the inertia matrix of any spacecraft equipped with
three reaction wheel actuators and momentum dumping using magnetorquers. This
procedure is designed to prevent the build up of momentum on the reaction wheels whilst
keeping the attitude disturbance to the satellite within acceptable limits. It can also
overcome potential errors introduced by unmodeled external disturbance torques and
attitude sensor noise. The results of simulations are presented in Chapter 5 to demonstrate
the performance of this technique. Due to the low computation requirements of these algorithms, both the control and estimation scheme can be easily applied on-board LEO satellites to accurately estimate the inertia matrix.

### 7.5 Practical Tests

Besides the theoretical treatments, this thesis presents the results of practical tests using the proposed estimation algorithms. In Chapter 6, the thruster calibration algorithm was tested firstly on an air-bearing table using a reaction wheel and two cold gas thrusters. Next, two different tests were carried out to calibrate the yaw and delta-V thrusters using in-orbit attitude control data generated by UoSAT-12. The estimated thruster coefficients were proved to work successfully in practice after calibration of the UoSAT-12 thrusters. Finally, MOI calibration algorithms were tested using in-orbit attitude control data generated by UoSAT-12 to estimate one principal MOI and two products of inertia. The estimation result converged to the measured pre-launch principal MOI with an error of 0.1 \% rms.

### 7.6 Summary

In this thesis, two novel autonomous methods for in-orbit thruster calibration and in-orbit moment of inertia estimation, are proposed, studied in detail by simulation and evaluated through in-orbit implementation tests.

In the thruster calibration method, a repetitive torque disturbance (generated by a reaction wheel) is exerted on the stabilised satellite, the cold-gas thrusters are then used by an on-board control system to stabilise the satellite attitude. By recording the resulting known attitude disturbances, reaction wheel and the thruster controller outputs, a recursive least square or least mean square algorithm can be used to process the recorded data to estimate the practical thruster parameters.

The results obtained compare well with the results of another approach used by SSTL to calibrate thruster parameters [Steyn, 2001]. In this approach one thruster is commanded to
fire for a short time to produce a pulse of external torque disturbance, the resultant transient response of the satellite attitude and the internal reaction wheel control system are recorded and later analysed in the ground station by a batch filter algorithm which provides an estimate of the thruster parameters.

In the moment of inertia method, one wheel is commanded to produce external torque disturbance, whilst the other two wheels are used in an on-board control system to stabilise the other two axes. By recording the resulting attitude disturbances and the wheel controller outputs, a recursive least squares algorithm can be used to process these recorded data to estimate three elements of the inertia matrix.

Practical trials have shown that both the thruster calibration and the moment of inertia estimation algorithms work well on-line in orbit producing results with an acceptable accuracy and have been shown to be feasible for practical engineering application in future satellite missions.

7.7 Future Work

The noise level on satellite rate and acceleration measurements can be seen to be a major limitation on the estimation of both thruster coefficients and inertia cross coupling terms. Any steps which improve these measurements such as use of accelerometer or rate gyros should improve the quality of estimation and are worthy of further investigation.

The Z-axis wheel of UoSAT-12 was not functioning by the time the practical tests of this work were in progress. It would be useful to carry out a full set of inertia identification experiments in all three axes of a fully functioning satellite to investigate the technique further.

A further technique identifying all the MOI matrix in a single experiment has been proposed and investigated [El-Bordany, 2001]. Given a satellite with a fully functioning ADCS this technique should be investigated practically too.
References


Appendix A

A. Thruster Calibration Results for UoSAT-12

This appendix lists some of the simulation results obtained to investigate the performance of the proposed thruster calibration algorithms presented in Chapter 4. The UoSAT-12 satellite in a low Earth-orbit was used as an example during these simulations. As explained in Section 4.4.3 the simulation results are divided into many combinations in order to choose the best method and reaction wheel torque. Figs A.1 to A.8 show the results when disturbing the satellite using full torque generated by the three reaction wheel actuators and the attitude is controlled by using the quaternion feedback thruster controllers to identify the 3×3 calibration matrix \( A_e \) using one experiment. Figs A.9 to A.23 show the simulation results when the satellite is disturbed using one reaction wheel actuator, while this axis is controlled using the thrusters. The other two axes are controlled using the remaining two wheels. Three coefficients of the calibration matrix \( A_e \) are then identified. A total of three experiments are required (one for each axis) to estimate all nine elements of the calibration matrix \( A_e \). Tables A.1 to A.3 summarises all the obtained results during the simulation.
Figure A.1 PRBS reaction wheel torque

Figure A.2 Reaction wheel momentum for the PRBS torque
Figure A.3 Euler angles for the PRBS torque

Figure A.4 Angular rates for the PRBS torque
Figure A.5 Bang-Bang reaction wheel torque

Figure A.6 Reaction wheel momentum when using a Bang-Bang torque
Figure A.7 Euler angles when using a Bang-Bang torque

Figure A.8 Angular rates when using a Bang-Bang torque
Figure A.9 Reaction wheel torque when disturbing the X-axis using PRBS torque

Figure A.10 Reaction wheel momentum when disturbing the X-axis using a PRBS torque
Figure A.11 Euler angle when disturbing the X-axis with a PRBS torque

Figure A.12 Angular rates when disturbing the X-axis with a PRBS torque
Figure A.13 Reaction wheel torque when disturbing the X-axis with a Bang-Bang torque

Figure A.14 Reaction wheel momentum when disturbing the X-axis with a Bang-Bang torque
Figure A.15 Euler angles when disturbing the X-axis with a Bang-Bang torque

Figure A.16 Angular rates when disturbing the X-axis with a Bang-Bang torque
Figure A.17 Reaction wheel torque when disturbing the Y-axis with a PRBS torque

Figure A.18 Reaction wheel momentum when disturbing the Y-axis with a PRBS torque
Figure A.19 Euler angles when disturbing the Y-axis with a PRBS torque

Figure A.20 Angular rates when disturbing the Y-axis with a PRBS torque
Figure A.21 Reaction wheel torque when disturbing the Y-axis with a Bang-Bang torque

Figure A.22 Reaction wheel momentum when disturbing the Y-axis with a Bang-Bang torque
Figure A.23 Euler angles when disturbing the Y-axis with a Bang-Bang torque

Figure A.24 Angular rates when disturbing the Y-axis with a Bang-Bang torque
Figure A.25 Reaction wheel torque when disturbing the Z-axis with a PRBS torque

Figure A.26 Reaction wheel momentum when disturbing the Z-axis with a PRBS torque
Figure A.27 Euler angles when disturbing the Z-axis with a PRBS torque

Figure A.28 Angular rates when disturbing the Z-axis with a PRBS torque
Figure A.29 Reaction wheel momentum when disturbing the Z-axis with a Bang-Bang torque

Figure A.30 Reaction wheel momentum when disturbing the Z-axis with a Bang-Bang torque
Figure A.31 Euler angles when disturbing the Z-axis with a Bang-Bang torque

Figure A.32 Angular rates when disturbing The Z-axis with a Bang-Bang torque
Table A.1 Comparison of thruster coefficients for different inputs and estimation methods (see Paragraph 4.4.3.1)

\[
(rms)_{error} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (\hat{a}_i - a_i)^2}
\]

Diagonal = \[\frac{\sum_{i=0}^{n} a_{ij}}{3}\]

Off-Diagonal = \[\frac{\sum_{i<j} a_{ij}}{6}\]

<table>
<thead>
<tr>
<th></th>
<th>First 1000 sec</th>
<th>Last 1000 sec</th>
<th>First 1000 sec</th>
<th>Last 1000 sec</th>
<th>X-axis</th>
<th>Y-axis</th>
<th>Z-axis</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>RLS+</td>
<td>0.0274</td>
<td>0.0018</td>
<td>0.0301</td>
<td>0.0030</td>
<td>59.28</td>
<td>57.46</td>
<td>50.72</td>
<td>167.46</td>
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<tr>
<td>PRBS</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LMS+</td>
<td>0.0324</td>
<td>0.0017</td>
<td>0.0424</td>
<td>0.0021</td>
<td>59.77</td>
<td>57.47</td>
<td>55.16</td>
<td>172.4</td>
</tr>
<tr>
<td>PRBS</td>
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<td></td>
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</tr>
<tr>
<td>RLS+</td>
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<td>0.0087</td>
<td>0.0409</td>
<td>0.0090</td>
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<td></td>
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<tr>
<td>BB</td>
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<tr>
<td>LMS+</td>
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<td>0.0084</td>
<td>0.0554</td>
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</tbody>
</table>

Fuel consumption during a half orbit (Nms)
(The fuel consumption are calculated by integrating the thruster torque over time)
Table A.2 Comparison of thruster coefficients for different inputs (see Paragraph 4.4.3.2).

\[
(rms)_{error} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (\hat{a}_i - a_i)^2}
\]

<table>
<thead>
<tr>
<th>X- Axis</th>
<th>Coefficient</th>
<th>First 1000 sec</th>
<th>Last 1000 sec</th>
<th>Fuel consumption during a half orbit (Nms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RLS+</td>
<td>$a_{11}$</td>
<td>0.0244</td>
<td>0.0008</td>
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<td>PRBS</td>
<td>$a_{21}$</td>
<td>0.0314</td>
<td>0.0010</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$a_{31}$</td>
<td>0.0509</td>
<td>0.0012</td>
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</tr>
<tr>
<td>RLS+</td>
<td>$a_{11}$</td>
<td>0.02616</td>
<td>0.0005</td>
<td>57.19</td>
</tr>
<tr>
<td>BB</td>
<td>$a_{21}$</td>
<td>0.0322</td>
<td>0.0010</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$a_{31}$</td>
<td>0.05217</td>
<td>0.00149</td>
<td></td>
</tr>
</tbody>
</table>

| Y- Axis | | | |
|---------| | | |
| RLS+    | $a_{12}$    | 0.0432         | 0.00057       | 54.34                                    |
| PRBS    | $a_{22}$    | 0.0091         | 0.00113       |                                          |
|         | $a_{32}$    | 0.0167         | 0.00137       |                                          |
| RLS+    | $a_{12}$    | 0.0438         | 0.00111       | 55.78                                    |
| BB      | $a_{22}$    | 0.0099         | 0.00508       |                                          |
|         | $a_{32}$    | 0.0160         | 0.0020        |                                          |

| Z- Axis | | | |
|---------| | | |
| RLS+    | $a_{13}$    | 0.03095        | 0.00073       | 50.5                                     |
| PRBS    | $a_{23}$    | 0.0131         | 0.00077       |                                          |
|         | $a_{33}$    | 0.0247         | 0.00106       |                                          |
| RLS+    | $a_{13}$    | 0.03327        | 0.00122       | 51.74                                    |
| BB      | $a_{23}$    | 0.00994        | 0.00148       |                                          |
|         | $a_{33}$    | 0.0311         | 0.00179       |                                          |
Table A.3 Comparison of thruster coefficients convergence in terms of RMS error (see Paragraph 4.4.3.1)

\[
(rms)_{error}(A_c) = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}
\]

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<th>Method</th>
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<th>Last 1000 sec</th>
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<td>0.0019 0.0039 0.0028</td>
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<tr>
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<td>0.0038 0.0018 0.0022</td>
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<td>0.0281 0.0159 0.0302</td>
<td>0.0021 0.0033 0.0018</td>
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<tr>
<td>LMS+</td>
<td>0.0231 0.0307 0.0809</td>
<td>0.0013 0.0010 0.0006</td>
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<tr>
<td>PRBS</td>
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<td>0.0034 0.0018 0.0022</td>
</tr>
<tr>
<td></td>
<td>0.0486 0.0371 0.0488</td>
<td>0.0019 0.0032 0.0015</td>
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<td>0.0100 0.00714 0.0043</td>
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<td>0.0078 0.0071 0.0134</td>
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<tr>
<td></td>
<td>0.0378 0.0241 0.0481</td>
<td>0.0117 0.0098 0.0088</td>
</tr>
<tr>
<td>LMS+</td>
<td>0.0403 0.0775 0.0716</td>
<td>0.0037 0.0022 0.0026</td>
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<tr>
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<td>0.0488 0.0296 0.0298</td>
<td>0.0082 0.0112 0.0125</td>
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<tr>
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<td>0.0596 0.0448 0.0717</td>
<td>0.0122 0.01196 0.0102</td>
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</tbody>
</table>
Appendix B

B. Moment of Inertia

This appendix lists some of the simulation results obtained to investigate the performance of the proposed moment of inertia (MOI) estimation algorithms presented in Chapter 5. The UoSAT-12 satellite in a low Earth-orbit was used as an example during these simulations. The simulation results are divided into two sections. Figs. B.1 to B.15 illustrate the performance of the RLS estimation algorithm without external disturbance torques and momentum dumping when disturbing the Y, X and Z axes respectively. Figs. B.16 to B.23 illustrate the performance of the RLS estimation algorithm with external disturbance torques when disturbing the X and Z axes respectively. This appendix also includes calculation of the moment of inertia of a typical shape.

B.1 Calculating MOI of Typical Shapes

In computation of the moment of inertia, one can replace the summation shown in Eq. (5.2) of the inertia tensor by integration over the body

\[ I = \int r^2 \, dm \]  

(B.1)

B.1.1 Thin Rod

Applying Eq. (5.8) to a thin uniform rod shown in Fig. B.1. The MOI of the rod about the Y-axis is given by:

\[ I_{yy} = \int_0^L r^2 \, dm = \int_0^L x^2 \, (\rho dx) \]

\[ = \rho \int_0^L x^2 \, dx = \rho \left[ \frac{x^3}{3} \right]_0^L = \frac{1}{3} \rho L^3 = \frac{1}{3} mL^2 \]  

(B.2)
Since
\[ dm = \rho \, dx \]
\[ m = \rho \, L \]

![Figure B.1 MOI of thin rod](image)

**B.1.2 Circular Ring**

The MOI of the uniform circular ring shown in Fig. B.2 about the Z-axis (the symmetry axis) is given by:

\[
I_{zz} = \int_0^{2\pi} r^2 \, dm = \int_0^{2\pi} r^2 (\rho \, dl)
\]

\[ = \int_0^{2\pi} r^2 (\rho r \, d\theta) = \rho r^3 \int_0^{2\pi} d\theta = 2\pi \rho r^3 = mr^2 \]

![Figure B.2 MOI of a circular ring](image)

Since
\[ dm = \rho \, dl \]
\[ dl = r \, d\theta \]
\[ m = 2\pi \rho r \]
Where $dl$ = the length of the arc formed by $d\theta$. Eq. (B.3) is also applicable to a circular cylinder.

**B.1.3 Circular Disc**

A uniform circular disc of radius $R$ can be considered as cascaded uniform circular rings as shown in Fig B.3. Thus, from Eq. (B.3), the MOI about the $Z$-axis (the symmetry axis) becomes:

$$I_z = \int_0^R 2\pi \rho r^3 \, dr$$

$$= 2\pi \rho \left[ \frac{r^4}{4} \right]_0^R = \frac{1}{2} \pi \rho R^4 = \frac{1}{2} mR^2$$

(B.4)

Since $m = \pi \rho R^2$

Eq. (B.4) can be used in computing the MOI of a circular bar about its longitudinal axis as well.

![Figure B.3 MOI of a circular disc](image)

**B.1.4 Sphere**

A uniform sphere can be considered as cascaded uniform circular discs as in Fig B.4. Thus, from Eq. (B.4), the MOI about the vertical axis (Z-axis) is:
\[ l_z = \int_{-R}^{R} \left[ \frac{1}{2} \pi \rho y^4 \right] dz \]
\[ = \int_{-R}^{R} \frac{1}{2} \pi \rho \left( R^2 - z^2 \right)^2 dz = \frac{1}{2} \pi \rho \int_{-R}^{R} \left[ R^4 - 2R^2 z^2 + z^4 \right] dz \]
\[ = \frac{1}{2} \pi \rho \left[ R^3 z - \frac{2}{3} R^2 z^3 + \frac{1}{5} z^5 \right]_{-R}^{R} = \frac{8}{15} \pi \rho R^5 = \frac{2}{5} mR^2 \] (B.5)

Since
\[ m = \frac{4}{3} \pi \rho R^3 \]

Figure B.4 MOI of a sphere
B.2 Moment of Inertia Estimation Results for UoSAT-12

Figure B.1 Reaction wheel torque in case of inertia estimation (Y-axis) without disturbance

Figure B.2 Reaction wheel momentum in case of inertia estimation (Y-axis) without disturbance
Figure B.3 Angular rates in case of inertia estimation (Y-axis) without disturbance

Figure B.4 Euler angles in case of inertia estimation (Y-axis) without disturbance
Figure B.5 Inertia estimation (Y-axis) without disturbance

Figure B.6 Reaction wheel torque in case of inertia estimation (X-axis) without disturbance
Figure B.7 Reaction wheel momentum in case of inertia estimation (X-axis) without disturbance

Figure B.8 Angular rates in case of inertia estimation (X-axis) without disturbance
Figure B.9 Euler angles in case of inertia estimation (X-axis) without disturbance

Figure B.10 Inertia estimation (X-axis) without disturbance
Figure B.11 Reaction wheel torque in case of Inertia Estimation (Z-axis) without disturbance

Figure B.12 Reaction wheel momentum in case of Inertia Estimation (Z-axis) without disturbance
Figure B.13 Angular rates in case of Inertia Estimation (Z-axis) without disturbance

Figure B.14 Euler angles in case of Inertia Estimation (Z-axis) without disturbance
Figure B.15 Inertia estimation (Z-axis) without disturbance

Figure B.16 Reaction wheel torque in case of inertia estimation (X-axis) with disturbance
Figure B.17 Reaction wheel momentum in case of inertia estimation (X-axis) with disturbance

Figure B.18 Angular rates in case of inertia estimation (X-axis) with disturbance
Figure B.19 Euler angles in case of inertia estimation (X-axis) with disturbance

Figure B.20 Reaction wheel torque in case of inertia estimation (Z-axis) with disturbance
Figure B.21 Reaction wheel momentum in case of inertia estimation (Z-axis) with disturbance

Figure B.22 Angular velocity in case of inertia estimation (Z-axis) with disturbance
Figure B.23 Euler angles in case of inertia estimation (Z-axis) with disturbance

Table B.1 Comparison of MOI parameters conversion for different values of forgetting factor $\lambda$ with disturbance torque and momentum dumping

<table>
<thead>
<tr>
<th>X-AXIS</th>
<th>$I_{xx}$</th>
<th>$I_{yy}$</th>
<th>$I_{zz}$</th>
<th>$I_{xy}$</th>
<th>$I_{xz}$</th>
<th>$I_{yz}$</th>
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<td>$\lambda = 0.9$</td>
<td>40.45</td>
<td>-0.1942</td>
<td>-0.5049</td>
<td>0.004612</td>
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<td></td>
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<td>$\lambda = 0.92$</td>
<td>40.45</td>
<td>-0.1952</td>
<td>-0.5047</td>
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<td>$\lambda = 0.95$</td>
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<td>( \lambda ) = 0.994</td>
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<tr>
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</tr>
</tbody>
</table>

**Y-AXIS**

<p>| ( \lambda ) = 0.9 | ( I_{xy} ) | ( I_{yy} ) | ( I_{yz} ) | ( I_{xy} ) | ( I_{yy} ) | ( I_{yz} ) |
| ( \lambda ) = 0.92 | -0.1948 | 42.09 | 0.3947 | 0.03059 | 0.0181 | 0.02625 |
| ( \lambda ) = 0.95 | -0.1947 | 42.09 | 0.3947 | 0.0238 | 0.01405 | 0.02086 |
| ( \lambda ) = 0.98 | -0.1941 | 42.09 | 0.3937 | 0.01708 | 0.00837 | 0.01613 |
| ( \lambda ) = 0.993 | -0.1937 | 42.09 | 0.393 | 0.01302 | 0.003437 | 0.01408 |
| ( \lambda ) = 0.994 | -0.1954 | 42.09 | 0.3946 | 0.008945 | 0.001953 | 0.01097 |
| ( \lambda ) = 0.995 | -0.1957 | 42.09 | 0.3949 | 0.00833 | 0.00132 | 0.01035 |
| ( \lambda ) = 0.996 | -0.196 | 42.09 | 0.3952 | 0.007576 | 0.001202 | 0.009538 |
| ( \lambda ) = 0.997 | -0.1937 | 42.09 | 0.3949 | 0.006692 | 0.001096 | 0.008529 |
| ( \lambda ) = 0.998 | -0.1957 | 42.09 | 0.3949 | 0.00892 | 0.000892 | 0.009043 |</p>
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<th>$I_{xy}$</th>
<th>$I_{xz}$</th>
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<th>$I_{xy}$</th>
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<th>$I_{zz}$</th>
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Appendix C

C. UoSAT-12 Simulator Using MATLAB and SIMULINK

The results presented in this thesis were obtained with a simulator that implements the dynamics and kinematics of the satellite using MATLAB and SIMULINK. This simulator are modelled based on the equations that have already been described in this thesis. The UoSAT-12 satellite in a low Earth-orbit was used as an example during these simulations. The simulator calculates the attitude of UoSAT-12 as it travels around its orbit. The orbit are generated prior to the attitude simulation.

The satellite dynamics are modelled using Euler's equations for rigid body motion under the influence of internal and external torques. The torques considered are those generated from a magnetorquer, reaction wheels and cold gas thruster system. Within the simulator it is observed that the use of the quaternion in the kinematics description is justified, as it allows for a singularity free model. However, the attitude is always expressed in Euler angles, as they provide a better physical interpretation of the satellite attitude.

The simulation parameters are given in Tables 4.2 and 5.1. Several estimations and control strategies were implemented and simulated in a realistic environment using a full simulation of the satellite dynamics, sensors and environmental models. The UoSAT-12 reduced control capabilities due to the restricted nature of its actuators are also taken into consideration. During simulations unknown external disturbance torques and sensor noise are added to the measurements of the state vectors $q$ and $\omega$. Uniformly distributed measurement noise within the range of $-1e^{-4}$ to $1e^{-4}$ rad/sec was added to each component of the angular rate. Uniformly distributed measurement noise within the range of $-0.1$ to $0.1$ was added to the vector part of the error quaternion. The levels of the sensor noise are chosen according to the SSTL experience.

Figs. C.1 to C.4 explain the UoSAT-12 simulator during MOI estimation and thruster calibration. Fig C.1 explains modelling of the UoSAT-12 dynamics, kinematics and control command using wheels and thrusters. Fig. C.2 explains the UoSAT-12 momentum
dumping simulator using a cross-product law during MOI estimation algorithms. The geomagnetic field vector are modelled using a first order dipole model. Fig. C.3 explains the UoSAT-12 MOI simulator when disturbing the Y-axis using a Bang-Bang wheel controller and controlling the attitude using the X and Z wheels to determine three elements of inertia matrix. Fig. C.4 explains the UoSAT-12 simulator during thruster calibration, when disturbing the satellite's attitude using a Bang-Bang reaction wheel controller and controlling the attitude using the thruster controller to determine all thruster coefficients.

Figure C.1 Satellite system simulator
Figure C.2 Momentum dumping simulator

Figure C.3 UoSAT-12 simulator during MOI estimation
Figure C.4 UoSAT-12 simulator during thruster calibration