Explicit model predictive control of active suspension systems

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Abstract: Within the development of modern chassis technologies significant industrial effort is put into the implementation of more and more advanced controllers, to enhance system performance without increasing hardware costs, and without significantly affecting the computational load associated with the real-time operation of the system. In this context, the paper presents explicit model predictive control (e-MPC) algorithms for active suspension systems. The explicit solution of the model predictive control problem via off-line multi-parametric quadratic programming (mp-QP) is proposed to overcome the well-known real-time capability issues related to the more conventional implicit model predictive control (i-MPC) framework. e-MPC reduces the on-line algorithm to a function evaluation, which replaces the computationally demanding on-line solution of the quadratic programming (QP) problem. The controller formulation is based on experimentally validated quarter car models of each corner of a case study Sport Utility Vehicle (SUV). The aim is to increase ride comfort by reducing the vertical acceleration of the sprung mass, especially at frequencies < 4 Hz. Assessments are carried out by means of 4-poster test rig experiments, considering typical road profiles. The proposed e-MPC implementations reduce the root mean square (RMS) value of the sprung mass acceleration by ~40% compared to the passive vehicle set-up for frequencies < 4 Hz, and by ~20% compared to a skyhook controller for the whole range of comfort-relevant frequencies. Very importantly, e-MPC also attenuates by 19% the medium-high frequency (> 4 Hz) acceleration issues typical of the skyhook algorithm applied to active suspension systems with relatively low-bandwidth actuators.

Keywords — Model predictive control, explicit solution, multi-parametric programming, active suspension, ride comfort
1 – Introduction

Semi-active and active suspensions with hydraulic actuators are widely used on production cars. The permanent challenge of improving ride comfort without increasing hardware costs requires the continuous enhancement of the system intelligence.

The skyhook algorithm is frequently used for primary ride improvement [1]. It is based on the introduction of a virtual damper between the sprung mass and a fixed surface, i.e., the ‘sky’. Skyhook can be actuated in full only through an active suspension system, since the vertical velocity of the sprung mass can have a different sign from the relative velocity between the sprung mass and the unsprung mass. In fact, a semi-active suspension system can only generate controllable forces that oppose the relative motion between the damper mounting points. The skyhook algorithm was extended for use on controllable dampers, by introducing a condition based on the sign of the ratio between the estimated velocity of the sprung mass and the estimated (or measured) relative velocity between the sprung mass and the unsprung mass [2]. If this is positive, ideal skyhook can be actuated, otherwise the damping coefficient of the controllable damper is set to its lowest possible value. A further common simplification of the skyhook principle for semi-active suspension systems consists of an on-off control of the damping coefficient of the actuator between a maximum value and a minimum value, depending on the sign of the velocity ratio. [3] presents an extended skyhook algorithm, in which the damper force is a linear combination of a contribution proportional to the vertical velocity of the chassis (skyhook term) and a contribution proportional to suspension deflection rate.

While skyhook reduces the vehicle body acceleration, the groundhook algorithm improves the unsprung mass dynamics, thus reducing the oscillations of the vertical tyre load, which are detrimental to vehicle handling performance [2]. The hybrid skyhook-groundhook controller (see [4-5]) reduces both the dynamic tyre force and body acceleration. [6-7] introduce the semi-active suspension balance logic, targeting a reduction of the sprung mass acceleration by cancelling the dynamic spring force through the damper force, which is possible only when they act in opposite directions. Alternatively, [8] proposes a form of groundhook blended with the balance logic. The acceleration-driven damping [9] is similar to the semi-active two-state approximation of the skyhook algorithm. The only difference
is that the shock absorber is deactivated when the body acceleration, rather than the speed, has opposite sign with respect to the suspension speed.

The linear quadratic regulator (LQR) / linear quadratic Gaussian (LQG) theory has been extensively applied to automotive active suspensions [10-16]. $H_\infty$ and $H_2/H_\infty$ suspension controller formulations have been implemented with the goal of satisfying specific performance criteria, while dealing with model uncertainties and parameter variations [17-20]. Similarly, active suspension systems based on sliding mode control are characterized by robustness and compensation capability of external disturbances and system parameter variations [21-23]. A number of papers deals with neural network implementations for suspension control [24-28].

Model predictive control is a promising option for controllable suspension systems. In particular, i-MPC, in which the optimization process is run on-line, requires significant computational power, which makes the practical implementations of such controllers for high bandwidth systems, including electronic suspensions, difficult. Moreover, the implicit solution cannot be formally analysed a-priori from the viewpoint of its stability and robustness. To the knowledge of the authors, most of the studies proposing i-MPC for electronic suspension systems are limited to simulation-based validations [29-35], with very rare exceptions such as [36], using a high-performance 300 MHz Alpha processor.

This paper discusses an active suspension system based on e-MPC (the theory of e-MPC is detailed in [37-38]). With e-MPC the optimisation problem is solved off-line, i.e., explicitly, which reduces the on-line algorithm to a function evaluation. As a consequence, e-MPC requires limited on-line computational power compared to i-MPC, while providing similar control performance and ability to handle constraints. On the other hand, the challenges of e-MPC are the increased design complexity and random-access memory (RAM) demand. e-MPC has already been implemented and, to some extent, experimentally validated on semi-active suspensions [39-42]. In all cases the simple two-mass quarter-car model was used for control system design. However, to the knowledge of the authors, e-MPC has not been proposed for fully active suspensions so far, nor model predictive control for active suspension has ever been implemented on automotive grade microprocessors. This gap is partially covered by this contribution, discussing e-MPC algorithms for active suspensions systems and
their experimental validation on a vehicle demonstrator, including performance comparison with a skyhook controller.

The paper is organised as follows. Section 2 describes the model for control system design. Section 3 explains the control system and mp-QP problem formulations. Finally, the explicit control law and the experimental results are presented in Section 4.

2 – Model for control system design

The quarter car model, depicted in Fig. 1, is used as a basis for control system design. The active suspension component is a hydraulic actuator, generating an ideal force input, \( u(t) \), without delays.

\[
\begin{align*}
\ddot{x}_1 + \frac{k_1}{m_1} \dot{x}_1 - \dot{x}_2 + c_1 (\dot{x}_1 - \dot{x}_2) + u &= 0 \\
\dot{m}_2 \ddot{x}_2 + k_1 (x_2 - x_1) + k_2 (x_2 - w) + c_1 (\dot{x}_2 - \dot{x}_1) + c_2 (\dot{x}_2 - \dot{w}) - u &= 0 
\end{align*}
\]

(1)

where \( m_1 \) and \( m_2 \) are the sprung and unsprung masses; \( k_1, k_2, c_1 \) and \( c_2 \) are the vertical suspension stiffness, the vertical tyre stiffness, the vertical damping coefficient associated with the passive suspension components, and the vertical tyre damping coefficient; \( x_1, x_2 \) and \( w \) are the vertical displacement of the sprung mass, the vertical displacement of the unsprung mass, and the vertical displacement of the road profile. The system can be converted into a continuous time state-space notation:
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) + EW(t) \\
y(t) &= Cx(t) + Du(t)
\end{align*}

(2)

where \( A, B, C \) and \( D \) are the system matrices and \( E \) is the disturbance matrix. The road input is represented by the column vector \( W(t) = [w \ w]^T \), where \( \dot{w} \) is the vertical velocity of the road at the tyre contact point.

e-MPC is based on a state feedback law. Hence, the controller performance strongly depends on the accuracy of the state estimates, and the appropriate selection of the states is very important. In the specific implementation of this study the estimates of \( x_1 \) and \( \dot{x}_1 \) are obtained through the band-pass filtering and mathematical integration of the vertical acceleration measurements of the vehicle body. The vertical displacement of the suspension system, \( x_1 - x_2 \), is estimated through the direct measurement of the active suspension actuator displacement, and consideration of the suspension installation ratio. The vertical suspension velocity, \( \dot{x}_1 - \dot{x}_2 \), is obtained through differentiation of \( x_1 - x_2 \) by using the hybrid smooth derivative method [43]:

\[\Delta L = \frac{a_0 \Delta L_0 + a_1 \Delta L_1 + a_2 \Delta L_2 - a_3 \Delta L_3 - a_4 \Delta L_4 + a_5 \Delta L_5}{b_0 \Delta t}\]

(3)

with

\[\Delta L_i = \Delta L[k - i], \quad i = 0,1,...,5\]

(4)

where \( \Delta L = x_1 - x_2 \), \( \Delta t \) is the time step, \( k \) is the current time, and \( a_i \) and \( b_i \) are positive constants. The method avoids the phase delay typical of low-pass filtering and direct differentiation.

The state-space formulation of the model can be re-written for the selected state vector \( x(t) = [x_1 \ \dot{x}_1 \ x_1 - x_2 \ \dot{x}_1 - \dot{x}_2]^T \) and output \( y(t) = \dot{x}_1 \) :

\[
\begin{align*}
\dot{x}(t) &= \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & -\frac{k_1}{m_1} & -\frac{c_1}{m_1} \\
k_2 & c_2 & \frac{k_1}{m_2} & -\frac{c_1}{m_2} \\
\frac{k_1}{m_2} & \frac{c_2}{m_2} & \frac{k_1}{m_2} & 1 \\
\end{bmatrix} x(t) + \begin{bmatrix}
0 \\
-\frac{1}{m_1} \\
0 \\
-\frac{k_2}{m_2} \\
\end{bmatrix} u(t) + \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
-\frac{k_2}{m_2} & -\frac{c_2}{m_2} \\
\end{bmatrix} W(t) \\
y(t) &= \begin{bmatrix}
0 & 0 & -\frac{k_1}{m_1} & -\frac{c_1}{m_1} \\
\end{bmatrix} x(t) + \begin{bmatrix}
-\frac{1}{m_1} \\
\end{bmatrix} u(t)
\end{align*}
\]

(5)

in which:
\[
\frac{1}{m_e} = -\left(\frac{1}{m_1} + \frac{1}{m_2}\right)
\]  

(6)

\(E\) represents the influence of the unknown disturbances \(w\) and \(\dot{w}\), and is neglected during the e-MPC design.

### 3 – Control system formulation

#### 3.1 – System prediction

The discrete state-space formulation of the vehicle model, discretized via zero order hold, is:

\[
\begin{align*}
{x}[k + 1] &= A_d x[k] + B_d u[k] \\
y[k] &= C_d x[k] + D_d u[k]
\end{align*}
\]  

(7)

Given the initial state, \(x[k]\), the initial control input, \(u[k]\), and the system (7), the output over the prediction horizon \(\hat{y}\) is calculated as:

\[
\hat{y} = \begin{bmatrix}
C_d A_d \\
C_d A_d^2 \\
\vdots \\
C_d A_d^p
\end{bmatrix}
\begin{bmatrix}
x[k] \\
\vdots \\
x[k]
\end{bmatrix}
+ \begin{bmatrix}
C_d B_d \\
\vdots \\
C_d B_d
\end{bmatrix}
\begin{bmatrix}
u[k] \\
\vdots \\
u[k]
\end{bmatrix}
\]  

(8)

which, more concisely, is:

\[
\hat{y} = \Lambda x[k] + \Theta_{u0} u[k] + \Theta_u \hat{u}
\]  

(9)

where:

\[
\hat{y} = \begin{bmatrix}
y[k + 1] \\
\vdots \\
y[k + p]
\end{bmatrix}, \quad \hat{u} = \begin{bmatrix}
u[k + 1] \\
\vdots \\
u[k + n]
\end{bmatrix}
\]  

(10)

\(n\) and \(p\) are the control horizon and prediction horizon, respectively. The states over the prediction horizon, \(\hat{x}\), are given by:
\[
\hat{x} = \Psi x[k] + \Omega_{u0} u[k] + \Omega_u \hat{u}
\]  
(11)

in which:
\[
\hat{x} = \begin{bmatrix} x[k+1] \\ \vdots \\ x[k+p] \end{bmatrix}
\]  
(12)

The matrices \(\Psi, \Omega_{u0}\) and \(\Omega_u\) are calculated from the system model (7).

3.2 – Objective function

The general goal of suspension design is the optimisation of ride comfort, suspension rattle space and road holding. The ride comfort improvement is achieved through the reduction of vehicle body acceleration levels, while limiting chassis motion as much as possible. Hence, a cost function penalising \(\ddot{x}_1, x_1, x_1 - x_2\) (the rattle displacement) and the control effort \(u\) is used in this study. The performance index \(J_{e-MPC}\) to be minimized is:

\[
J_{e-MPC} = \int_0^T \left( \frac{\rho_1}{n_1} \ddot{x}_1^2 + \frac{\rho_2}{n_2} (x_1 - x_2)^2 + \frac{\rho_3}{n_3} u^2 + \frac{\rho_4}{n_4} x_1^2 \right) dt
\]  
(13)

where \(\rho_i\) and \(n_i\) are the weighting and normalisation factors, and \(T\) is the period of observation, i.e., in this case the duration of the prediction horizon.

3.3 – mp-QP problem formulation

\(J_{e-MPC}\) is re-arranged to be consistent with the discretised system prediction formulation of Section 3.1. \(J_{e-MPC}\) is quadratic, and is used for the following minimisation problem:

\[
\begin{align*}
\min_{\hat{u}} \left( \hat{y}^T Q_1 \hat{y} + \hat{x}^T Q_2 \hat{x} + \hat{u}^T R \hat{u} \right) \\
\end{align*}
\]  
(14)

where \(Q_1, Q_2\) and \(R\) contain the factors \(\rho_i\) and \(n_i\) of (13). The insertion of (9) and (11) into (14) results in:

\[
\begin{align*}
\min_{\hat{u}} \left[ (x[k]^T A^T + u[k]^T \Theta_{u0}^T + \hat{u}^T \Theta_u^T) Q_1 (Ax[k] + \Theta_{u0} u[k] + \Theta_u \hat{u}) \\
+ (x[k]^T \Psi^T + u[k]^T \Omega_{u0}^T + \hat{u}^T \Omega_u^T) Q_2 (\Psi x[k] + \Omega_{u0} u[k] + \Omega_u \hat{u}) + \hat{u}^T R \hat{u} \right] \\
\end{align*}
\]  
(15)

By eliminating the terms not depending on \(\hat{u}\) and dividing by 2, (15) becomes:
min \ \frac{1}{2} \hat{u}^T (\Theta_u^T Q_1 \Theta_u + \Omega_u^T Q_2 \Omega_u + R) \hat{u} \\
+ \left(x[k]^T (\Lambda^T Q_1 \Theta_u + \Psi^T Q_2 \Omega_u) + u[k]^T (\Theta_{u0}^T Q_1 \Theta_u + \Omega_{u0}^T Q_2 \Omega_u)\right) \hat{u}
\tag{16}

Hence, the model predictive control formulation is represented by the following QP problem:

\min \ \frac{1}{2} \hat{u}^T H \hat{u} + x'[k]^T F \hat{u}
\tag{17}

where \( H \) is the Hessian matrix, and \( F \) includes the physical system parameters and the weighting and normalisation factors. \( x'[k] \) contains the initial states of the system as well as the initial actuator force.

A conventional i-MPC would execute an on-line optimisation at each time step \( \Delta t \) for a given value of \( x'[k] \), and the control law \( u = u(x') \) would be implicitly obtained by the QP solver. In the e-MPC case the optimisation is performed off-line. Given the global set of parameters, \( X \), the idea is to determine the set of feasible parameters \( X^* \) of all \( x' \in X \), for which a solution of the problem exists. The QP is solved for all possible values of \( x' \), which generates the explicit solution, \( u = u(x') \). The optimisation problem becomes an mp-QP problem, generally described as follows:

\min \ \frac{1}{2} \hat{u}^T H \hat{u} + x'^T F \hat{u} + \frac{1}{2} x'^T Y x'
\tag{18}

subject to:

\[ N \hat{u} \leq M_1 + M_2 x' \]
\tag{19}

where \( N, M_1 \) and \( M_2 \) are constant matrices. The constraints are typically related to actuator force and its rate. The last term in (18) is neglected, since it does not depend on \( \hat{u} \).

The solutions of the mp-QP problem are: i) the piecewise affine function \( U^*: X^* \to \mathbb{R}^s \), which associates the corresponding \( \hat{u} \) to each parameter \( x' \); and ii) the e-MPC law \( u(x') = [1 \ 0 \cdots 0] U^* \).

Hence, the explicit representation of the control action is a piecewise affine state feedback law defined on a polyhedral partition of the state-space:

\[ u(x') = \begin{cases} L_1 x' + p_1, & O_1 x' \leq S_1 \\ \vdots \\ L_4 x' + p_6, & O_6 x' \leq S_6 \end{cases} \]
\tag{20}

where \( L_i, p_i, O_i \) and \( S_i \) are constant matrices.
3.4 – Distributed controller

To reduce the off-line computational time and the on-line RAM requirements of the explicit solution, four quarter car controllers are used, i.e., one for each suspension system. The quarter car model is represented by four parameters in the mp-QP problem formulation, while a vehicle model including pitch dynamics would imply a greater number of parameters per mp-QP problem and more demanding RAM requirements.

To account for the static mass distribution of the vehicle and the different characteristics of the passive components of the front and rear suspensions, two different quarter car parameter sets are used for the front and rear suspension systems.

4 – Control system implementation and experimental evaluation

4.1 – Vehicle demonstrator

The developed controller was implemented and experimentally validated on a sport utility vehicle (SUV) demonstrator (see Fig. 2) with a hydraulic active suspension system, i.e., the Tenneco Monroe intelligent suspension – ACOCAR. The sensor set and valves are identical to those of the Tenneco CVSA2 (continuously variable semi-active system with two valves) suspension technology [44]. The ACOCAR actuators are pressurised by means of a pump, which allows inputting energy into the system and actively controlling the actuation forces.

![Fig. 2 – The SUV demonstrator on the 4-poster test rig.](image)
The SUV demonstrator was used to compare the experimental performances of the ACOCAR e-MPC implementations with those of:

- The passive set-up of the car. This was obtained by applying zero currents to the actuation valves, which represents the fail-safe state of the ACOCAR system, corresponding to a suspension tuning that is very close to the one of the passive version of the SUV in terms of ride comfort.
- A conventional production-ready skyhook algorithm for active suspensions, configured with high gains. The adopted skyhook damping coefficients for the heave, pitch and roll motions are respectively 10,000 Ns/m, 12,000 Nms/rad and 12,000 Nms/rad.

The comparison was carried out for the excitation profile of a typical ride comfort road, i.e., the Blauwe Kei road at the Ford Lommel proving ground in Belgium, which was reproduced by means of a Schenck Instron 4-poster test rig, exciting the SUV demonstrator.

4.2 – Controller implementation method

The first step of the e-MPC implementation process was the validation of the front and rear quarter car models for control system design. This was based on experimental results obtained for the passive set-up of the SUV demonstrator, measured on the 4-poster test rig. Then the mp-QP problems for the front and rear suspensions were solved with the multi-parametric toolbox 3 (MPT3) [45], for different sets of coefficients of the objective function \( J_{e-MPC} \). Finally, simulations of the implemented controllers with an experimentally validated vehicle model for control system assessment along the Blauwe Kei profile were used for the identification of the coefficients of \( J_{e-MPC} \) providing the most desirable e-MPC behaviours. The performance assessment was carried out with the same RMS-based performance indicators that will be reported in the following Table II (see Section 4.4).

In the control design phase a prediction horizon \( p = 5 \) and a control horizon \( n = 5 \) were adopted, with a controller sample time \( \Delta t = 10 \) ms. Each actuator force was constrained to \( \pm 5,000 \) N. In particular, at the completion of the process, two e-MPC settings, called e-MPC\(_1\) and e-MPC\(_2\) with objective function tunings \( J_{e-MPC_1} \) and \( J_{e-MPC_2} \), were considered for further experimental evaluation:

- e-MPC\(_1\). Compared to the skyhook, this setting focuses on the reduction of \( \ddot{x}_1 \) for frequencies \(< 4\) Hz, without excessively increasing the acceleration levels above that frequency. The latter specification is based on the experience of the company supporting this research, showing that
active suspensions with hydraulic actuators in parallel to the springs, as it is the case here, tend to reduce ride comfort at medium-high frequencies. This is caused by the typically limited actuator bandwidth (~8 Hz) and significant non-linearities.

- e-MPC$_2$. With respect to the skyhook, this setting targets similar performance around the resonance frequency of the sprung mass (1-1.5 Hz), and the reduction of the acceleration levels at frequencies $> 4$ Hz. In comparison with the e-MPC$_1$, the e-MPC$_2$ increases the penalty on $\ddot{x}_1$ by decreasing $n_1$ in $J_{e-MPC}$, and reduces the penalty on $x_1$ by increasing $n_4$.

The selected objective function coefficients for the e-MPC$_1$ and e-MPC$_2$ implementations are reported in Table I, and are the same for the front and rear suspension controllers.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$J_{e-MPC}$</th>
<th>$J_{e-MPC}$</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_1$</td>
<td>45</td>
<td>45</td>
<td>-</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>1</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>$\rho_3$</td>
<td>1</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>$\rho_4$</td>
<td>45</td>
<td>25</td>
<td>-</td>
</tr>
<tr>
<td>$n_1$</td>
<td>100</td>
<td>20</td>
<td>m$^2$/s$^3$</td>
</tr>
<tr>
<td>$n_2$</td>
<td>0.01</td>
<td>0.01</td>
<td>m$^2$/s</td>
</tr>
<tr>
<td>$n_3$</td>
<td>$10^5$</td>
<td>$10^5$</td>
<td>N$^2$/s</td>
</tr>
<tr>
<td>$n_4$</td>
<td>0.00005</td>
<td>0.0001</td>
<td>m$^3$/s</td>
</tr>
</tbody>
</table>

4.3 – The explicit solution of the e-MPC$_1$

This section discusses the explicit solution of the e-MPC$_1$. The control law, $u = u(x')$, consists of a set of functions with affine gains over 1217 polyhedral regions within the state-space. Fig. 3 shows the state-space partition, sliced at $x_1 - x_2 = 0$ and $\dot{x}_1 - \dot{x}_2 = 0$. In region 1 the control law varies as a function of $x_1$ and $\dot{x}_1$. In regions 2, 3, 4 and 5 the actuator force is saturated.

Fig. 3 also reports the simulation results for the front left SUV corner in the passive and e-MPC$_1$ set-ups, in the form of state trajectories for the Blauwe Kei road input. Interestingly, in the e-MPC$_1$ case the system operation is limited to one region, i.e., the first sub-partition bounded by:
The e-MPC state-space partition sliced at $x_1 - x_2 = 0$ and $\dot{x}_1 - \dot{x}_2 = 0$, with the simulated trajectories of the front left SUV corner with the passive and e-MPC set-ups on the Blauwe Kei road.

Although Fig. 3 depicts only a two-dimensional slice of the four-dimensional state-space partition, this behaviour was verified on the four-dimensional partition. As a consequence, on the specific road the control law could be replaced by the following single affine function of the states:

$$ u(x') = \begin{bmatrix} 1.1086 \cdot 10^5 & 0.0350 \cdot 10^5 & -0.0368 \cdot 10^5 & -0.0033 \cdot 10^5 \end{bmatrix} x' + \begin{bmatrix} 9.0949 \cdot 10^{-13} \end{bmatrix} $$

resulting in a significant reduction of the RAM requirements. Obviously, this would not be advisable during operation on more aggressive road profiles. The potential simplification of the e-MPC control
law, either through formal and systematic methods (see [46]) or the empirical observation of the most commonly used sub-partitions, will be the topic of future research.

The whole explicit e-MPC$_1$ solution from the MPT3 toolbox was uploaded on the dSPACE AutoBox rapid control prototyping unit of the SUV demonstrator, and required a total RAM capacity of 8 MB.

4.4 – Experimental results and comparisons

This section reports the experimental SUV results on the 4-poster test rig along the assessed mission profile. In particular, Fig. 4 and Fig. 5 plot the frequency response characteristics of the power spectral densities (PSDs) of the heave position and heave acceleration of the centre of gravity of the sprung mass for the four considered set-ups, i.e., passive, skyhook, e-MPC$_1$ and e-MPC$_2$. Table II shows the corresponding root mean square (RMS) values for the frequency ranges below and above 4 Hz, corresponding to the so-called primary ride and secondary ride, and for the 0-100 Hz ride comfort frequency spectrum.

In the 0-4 Hz frequency range the e-MPC$_1$ reduces the sprung mass heave acceleration by ~43% relative to the passive set-up, and by ~26% compared to the skyhook, which is a major primary ride enhancement. On the other hand, above 4 Hz the skyhook strategy increases the vertical acceleration by ~79% compared to the passive set-up. This phenomenon, which is well-known to the technical specialists of the industrial company supporting this research, is attributed to the limited actuation dynamics of the hydraulic active suspension system. In fact, simulations of the system response with actuators with better dynamic properties did not show such a trend. This behaviour brings a deterioration of the secondary ride. In the same frequency range (> 4 Hz) the e-MPC$_1$ shows an increase in the vertical acceleration level of ~65% compared to the passive set-up, which is an ~8% reduction of the secondary ride problem of the skyhook. On the 0-100 Hz frequency range, the e-MPC$_1$ reduces the skyhook vibration levels by ~11%.

The e-MPC$_2$ was implemented with the purpose of attenuating the secondary ride issues of the skyhook and e-MPC$_1$, while providing good primary ride performance. Fig. 5 shows that at approximately the resonance frequency of the sprung mass, i.e., at 1-1.5 Hz, the e-MPC$_2$ and the skyhook give origin to similar responses.
Fig. 4 – PSD of the heave displacement of the centre of gravity of the sprung mass for the passive, skyhook, e-MPC₁ and e-MPC₂ set-ups.

Fig. 5 – PSD of the heave acceleration of the centre of gravity of the sprung mass for the passive, skyhook, e-MPC₁ and e-MPC₂ set-ups.
The e-MPC₂ improves the skyhook acceleration performance by ~22% in the 0-4 Hz frequency range, and by ~19% above 4 Hz. Moreover, on the whole frequency range the e-MPC₂ produces lower acceleration levels than the e-MPC₁, which is expected given the increased penalty on \( \ddot{x}_1 \) in \( J_{e-MPC^2} \).

The conclusion is that for the given actuators the e-MPC₂ conjugates a significant enhancement of the primary ride, without an excessive penalisation of the secondary ride performance.

Table II – RMS values for the heave position and acceleration of the centre of gravity of the sprung mass for the passive, skyhook, e-MPC₁ and e-MPC₂ set-ups.

<table>
<thead>
<tr>
<th></th>
<th>RMS values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Passive</td>
</tr>
<tr>
<td>Heave position: 0 – 4 Hz (m)</td>
<td>0.0132</td>
</tr>
<tr>
<td>Heave position: 4 – 100 Hz (m)</td>
<td>0.0004</td>
</tr>
<tr>
<td>Heave position: 0 – 100 Hz (m)</td>
<td>0.0132</td>
</tr>
<tr>
<td>Heave acceleration: 0 – 4 Hz (m/s²)</td>
<td>1.01</td>
</tr>
<tr>
<td>Heave acceleration: 4 – 100 Hz (m/s²)</td>
<td>0.91</td>
</tr>
<tr>
<td>Heave acceleration: 0 – 100 Hz (m/s²)</td>
<td>1.36</td>
</tr>
</tbody>
</table>

5 – Conclusion

To the knowledge of the authors, for the first time this paper discussed the application of e-MPC to active suspension systems for passenger cars, mainly targeting primary ride improvements. Multi-parametric quadratic programming was used to solve control problem formulations based on quarter car models. The solution is represented by explicit control laws, based on state feedback. Hence, e-MPC brings a reduction of the computational requirements of the control system hardware with respect to i-MPC, since the on-line implementation consists of a function evaluation. The results show
significant benefits of the developed controllers with respect to a pre-existing skyhook algorithm. In fact, the e-MPC\textsubscript{1} and e-MPC\textsubscript{2} implementations reduce the vehicle body acceleration levels by \(~26\%\) and \(~22\\%), respectively, in the frequency range below 4 Hz, and by \(~8\%\) and \(~19\ \%)\, respectively, above 4 Hz. Future developments will focus on the systematic fine-tuning of the objective function for e-MPC design, and the assessment of the controllers on different actuation hardware.

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**References**


