Channel Estimation and Optimal Pilot Signals for Universal Filtered Multi-carrier (UFMC) Systems

Lei Zhang*, Chang He**, Juquan Mao**, Ayesha Ijaz** and Pei Xiao**
*School of Engineering, University of Glasgow
Glasgow, G12 8QQ, UK, Email: lei.zhang@glasgow.ac.uk
**Institute for Communication Systems (ICS), University of Surrey
Guildford, GU2 7XH, UK, Email: {c.he, juquan.mao, a.ijaz, p.xiao}@surrey.ac.uk

***Abstract*—We propose channel estimation algorithms and pilot signal optimization for the universal filtered multi-carrier (UFMC) system based on the comb-type pilot pattern. By considering the least square linear interpolation (LSLI), discrete Fourier transform (DFT), minimum mean square error (MMSE) and relaxed MMSE (RMMSE) channel estimators, we formulate the pilot signals optimization problem by minimizing the estimation MSE subject to the power constraint on pilot tones. The closed-form optimal solutions and minimum MSE are derived for LSLI, DFT, MMSE and RMMSE estimators.**

I. Introduction

Universal filtered multi-carrier (UFMC) system provides *flexibility* to filter a subband with arbitrary bandwidth, enabling the system to adapt specific users or service types by adjusting the subband and filter parameters only [1], [2]. For example, a UFMC system may serve two types of services (e.g., tactile and machine type communications) in two subbands with different communications requirements and frame structures, but without generating significant inter-service-band-interference (ISBI) [3], [4] due to waveform's low out of band (OoB) emission. Such flexibility offers a major advantage of the UFMC over orthogonal frequency division multiplexing (OFDM), making it one of the most promising candidate waveforms for 5G systems and beyond. In addition, the time/frequency synchronization requirements in UFMC system can be relaxed comparing with OFDM system. On the other hand, UFMC inherits most of the advantages of OFDM, e.g., ease in the implementation of multi-antenna techniques and low complexity one-tap channel equalization algorithms.

However, the subband filtering operation for OoB emission reduction may cause different filter gain at different subcarriers in one UFMC symbol. This altered signal model may invalidate optimal channel estimation algorithms previously proposed for OFDM systems. For example, the polynomial interpolation based channel estimation algorithms in OFDM may not work properly due to the variation in filter gain across the subcarriers between pilot subcarriers. One can first normalize the filter gain at the pilot subcarriers and perform the conventional interpolation algorithms as in OFDM system, then compensate the filter gain back, as suggested by [5]. However, the three-step algorithm may not be optimal with the original pilot signal designed for OFDM system. To the best of the authors' knowledge, there is no solution in the literature that systematically designs the optimal channel estimation algorithms for UFMC systems.

Comb-type pilot pattern, as compared to block-type pilot arrangement, can more effectively track the fast changing channels [6] and has been used in many standards such as the 3rd Generation Partnership Project (3GPP) Long Term Evaluation (LTE). Note that the comb-type pilot pattern may be not the optimal due to the filter gain differences among the subcarriers. The joint optimal pilot signal and pilot pattern design could be significantly more complex than the OFDM system. In this correspondence, we will adopt the comb-type pilot pattern to focus on the pilot signals optimization problem only, and the block-type arrangement can be considered as a special case by setting the adjacent pilot tone distance to be 1.

For the comb-type pilot arrangement, various algorithms have been proposed by considering the trade-off between estimation accuracy and computational complexity. Minimum mean square error (MMSE) is the optimal estimator in terms of estimation MSE but with the highest numerical complexity. In addition, it requires the knowledge of channel correlation matrix and noise variance. Replacing channel correlation matrix by a known matrix (e.g., identity matrix or a diagonal matrix with exponential decaying elements), the relaxed MMSE (RMMSE) was proposed [7], [8]. While the least square with linear interpolation (LSLI) has significant computational complexity advantage comparing with other methods and has been widely used in multi-carrier systems, however, it suffers from error floor when the pilot density is insufficient. DFT based estimator has no interpolation error and exhibits better spectral efficiency than frequency domain linear interpolation [6].

In this correspondence, we propose optimal pilot design for channel estimation in UFMC systems by minimizing the MSE

subject to pilot power constraint. The optimization problems are carried out for LSLI, DFT, MMSE and RMMSE based estimators. The optimal pilot signals and the minimum MSE are derived analytically.

Notations: $\{\cdot\}^H$ and $\{\cdot\}^T$ stand for the Hermitian conjugate and transpose operation, respectively. We use $E\{\mathbf{A}\}$, trace$\{\mathbf{A}\}$ and $\{\mathbf{A}\}^\dagger$ to denote the expectation, trace and pseudo inverse of matrix $\mathbf{A}$. $\|\cdot\|_F$ refers to the Frobenius matrix norm. $\mathbf{I}_M$ and $\mathbf{0}_{M\times N}$ represent identity matrix of M dimension and $M \times N$ matrix with all of its elements being zero, respectively.

II. UFMC Signal Model and Channel Estimation Algorithms

We consider the case when the UFMC symbol carrying pilots is protected by sufficient guard interval (e.g., zero padding or cyclic prefixing) to eliminate the effect of inter-symbol-interference. At the receiver, after the DFT operation, the signal at the k-th subcarrier can be written as [9], [10]

$$Y_k = X_k F_k H_k + V_k, \quad 0 \le k \le K-1 \; , \tag{1}$$

where X_k, F_k, H_k and V_k are the transmitted signal, filter response, channel frequency response and the white Gaussian noise with zero mean and variance σ^2 at the k-th subcarrier, respectively. K is the total number of subcarriers. Note that the filter gain F_k could be *different* among K subcarriers due to the filter ramp-up and ramp-down effect, and the difference could be significant when the subband bandwidth is small [10]. However, the value of F_k at each subcarrier is *fixed* and it is only dependent on the subband filter parameters. Note that the UFMC signal model in (1) is a generalized expression of multi-carrier system and in the special case when $F_k = 1$, for $0 \le k \le K-1$, (1) boils down to the OFDM system.

For the comb-type pilot transmission, let us assume the $M+1$ pilot tones are uniformly inserted into the transmitted signal for channel estimation and L is the interval of the pilot subcarriers. The LS channel estimation at the pilot subcarrier can be written as

$$\begin{aligned} H_{mL}^{\text{LS}} &= \frac{Y_{mL}}{X_{mL}F_{mL}} \\ &= H_{mL} + \frac{V_{mL}}{X_{mL}F_{mL}}, \quad 0 \le m \le M \; . \end{aligned} \tag{2}$$

With linear interpolation, the channel estimation at the $(mL+l)$-th subcarrier is given by [1]

$$H_{mL+l}^{\text{LS}} = \frac{L-l}{L} H_{mL}^{\text{LS}} + \frac{l}{L} H_{(m+1)L}^{\text{LS}}, \quad 1 \le l \le L-1 \; . \tag{3}$$

[1]Note that we only consider the inner interpolation here, the interpolation at the edge subcarriers can follow the same derivation and the optimization problem is straightforward.

To describe the DFT and MMSE based channel estimation, we write the received signal at the pilot subcarriers in a matrix form as

$$\mathbf{y} = \mathbf{X}\mathbf{F}\mathbf{h}_f + \mathbf{v} \; , \tag{4}$$

where $\mathbf{y} = [Y_0, Y_L, \cdots, Y_{ML}]$. $\mathbf{X} = \text{diag}[X_0, X_L, \cdots, X_{ML}]$ and $\mathbf{F} = \text{diag}[F_0, F_L, \cdots, F_{ML}]$ are diagonal matrices. $\mathbf{v}$ is the noise vector with its k-th element being V_{kL}. $\mathbf{h}_f = [H_0, H_L, \cdots, H_{ML}]^T$ is the channel frequency response vector at the pilot subcarriers. Let us denote the L_{CH} taps channel impulse response as $\mathbf{h}_t = [h_t(0), h_t(1), \cdots, h_t(L_{CH}-1)]^T$. Then we have $\mathbf{h}_f = \bar{\mathbf{W}}_M \mathbf{h}_t$ with matrix $\bar{\mathbf{W}}_M$ obtained by taking the first L_{CH} columns of the M-point DFT matrix $\mathbf{W}_M$ with the element in i-th row and n-th column being $e^{-j2\pi ni/M}$.

Substituting $\mathbf{h}_f = \bar{\mathbf{W}}_M \mathbf{h}_t$ into (4), the channel impulse response can be estimated as $\mathbf{h}_t^{\text{DFT}} = (\mathbf{X}\mathbf{F}\bar{\mathbf{W}}_M)^\dagger \mathbf{y}$. Then the DFT based channel estimation can be written as

$$\mathbf{h}_f^{\text{DFT}} = \mathbf{W}\mathbf{D}_K (\mathbf{X}\mathbf{F}\bar{\mathbf{W}}_M)^\dagger \mathbf{y} \; , \tag{5}$$

where $\mathbf{W}$ is a K-point DFT matrix and $\mathbf{D}_K = [\mathbf{I}_{L_{CH}}; \mathbf{0}_{(K-L_{CH})\times L_{CH}}]$.

Similarly, the MMSE based channel estimation can be expressed as

$$\mathbf{h}_f^{\text{MMSE}} = \mathbf{W}\mathbf{D}_K \mathbf{R}_h \mathbf{Q}^H (\mathbf{Q}\mathbf{R}_h\mathbf{Q}^H + \sigma^2 \mathbf{I})^{-1} \mathbf{y} \; , \tag{6}$$

where $\mathbf{Q} = \mathbf{X}\mathbf{F}\bar{\mathbf{W}}_M$ and $\mathbf{R}_h = E\{\mathbf{h}_t \mathbf{h}_t^H\}$ is the channel impulse response correlation matrix.

III. Proposed Channel Estimation Algorithms and Optimal Pilot Signals

With given pilot signal $\mathbf{X}$ and known filter response $\mathbf{F}$, the channel can be estimated by using LSLI, DFT or MMSE based algorithms as shown in equation (2), (5) or (6), respectively. For OFDM systems, the equal power allocation (i.e., $|\mathbf{X}| = \sqrt{\frac{P}{M+1}} \mathbf{I}_M$ with P being the total power on the pilot tones) to all pilot tones will achieve the optimal performance in terms of estimation MSE [7]. For UFMC system, however, due to the filter response (i.e., $\mathbf{F}$) selectivity among the subcarriers, equal power allocation to the pilot tones is no longer the optimal solution. In the sequel, we will formulate and solve the optimization problems in terms of pilot signals by minimizing the MSE subject to the total power constraint on pilot tones, based on the aforementioned channel estimation criteria.

1) LSLI channel estimation: Based on the comb-type pilot pattern, the LSLI channel estimation has two steps with the first one to estimate the channel at the pilot subcarriers by LS method as described in (2), which suffers from the noise induced estimation error. The second step is the linear interpolation by using (3), which suffers from the modeling error. Therefore, the estimation error includes two parts. In

order to minimize the total MSE, we can formulate the following constrained optimization problem for LSLI based channel estimation as

$$\min_{\mathbf{X}} \quad \varepsilon^{\text{LSLI}} = \varepsilon^{\text{LS}} + \varepsilon^{\text{LI}}, \quad \text{s.t.} \quad \|\mathbf{X}\|_F^2 = P \; , \tag{7}$$

where

$$\varepsilon^{\text{LS}} = \frac{1}{K} E\|(\mathbf{XF})^{-1}\mathbf{v}\|_F^2 = \frac{1}{K}\sum_{m=0}^{M} \frac{\sigma^2}{|X_{mL}|^2 |F_{mL}|^2} \tag{8}$$

is the average LS estimation error per subcarrier contributed by the pilot subcarriers. ε^{LI} is the contribution from the other subcarriers than pilot subcarriers due to the interpolation error, which can be expressed as

$$\begin{aligned} \varepsilon^{\text{LI}} &= \frac{1}{K}\sum_{m=0}^{M-1}\sum_{l=1}^{L-1} E|H^{\text{LS}}_{mL+l} - H_{mL+l}|^2 \\ &= \alpha + \beta \; , \end{aligned} \tag{9}$$

where

$$\begin{aligned} \beta = \frac{\sigma^2(2L-1)(L-1)}{6LK}&\Big[\frac{1}{|F_0X_0|^2} + \frac{2}{|F_LX_L|^2} + \cdots , \\ &+ \frac{2}{|F_{(M-1)L}X_{(M-1)L}|^2} + \frac{1}{|F_{ML}X_{ML}|^2}\Big] \; . \end{aligned} \tag{10}$$

In addition

$$\alpha = \alpha_1 - \alpha_2 \tag{11}$$

with

$$\alpha_1 = \frac{5L-1}{3L}R_f(0) + \frac{L+1}{6L}[R_f(-L) + R_f(L)] \tag{12}$$

and

$$\begin{aligned} \alpha_2 &= \frac{1}{L-1}\sum_{l=1}^{L-1}\Big[\frac{L-l}{L}(R_f(l) + R_f(-l)) \\ &+ \frac{l}{L}(R_f(l) + R_f(-l))\Big] \end{aligned} \tag{13}$$

and $R_f(i) = E\{H(m+i)H(m)^*\}$ being the channel frequency response correlation factor [11]. Note that α is not related to the pilot signal $\mathbf{X}$, therefore, the optimization problem in (7) can be equivalently rewritten as

$$\min_{\mathbf{X}} \quad \tilde{\varepsilon}^{\text{LSLI}} = \varepsilon^{\text{LS}} + \beta, \quad \text{s.t.} \quad \|\mathbf{X}\|_F^2 = P \; . \tag{14}$$

By using $\|\mathbf{X}\|_F^2 = \sum_{m=0}^{M} |X_{mL}|^2$, we can solve the constrained optimization problem in (14) by formulating the following Lagrange cost function

$$\phi = \varepsilon^{\text{LS}} + \beta + \lambda\Big(\sum_{m=0}^{M-1} |X_{mL}|^2 - P\Big) \; , \tag{15}$$

where λ is the Lagrange multiplier. By setting $\partial\phi/\partial X_{mL} = 0$ for $m = 0, 1, \cdots, M$ and noting the power constraint function $\sum_{m=0}^{M} |X_{mL}|^2 = P$, we have the LSLI based optimal pilot signal for UFMC as

$$X^{\text{LSLI}}_{mL} = \frac{e^{j\varphi}}{\sqrt{B_m|F_{mL}|}}\sqrt{\frac{P}{\sum_{n=0}^{M}\frac{1}{B_n|F_{nL}|}}} \quad \text{for } m = 0, \cdots, M \tag{16}$$

with

$$B_m = \begin{cases} \sqrt{\frac{6L}{2L^2+3L+1}} & \text{if } m = 0 \text{ or } M \\ \sqrt{\frac{3L}{2L^2+1}} & \text{if } m = 1, 2, \cdots, M-1 \end{cases} \; . \tag{17}$$

φ can take an arbitrary value from the range $[0, 2\pi]$, which means that phase of the pilot signal can be arbitrary.

(17) implies that the pilot tones at the edges (i.e., $m = 0$ or M) have different contribution to the optimal solution from the one in the middle (i.e., $m = 1, 2, \cdots, M-1$). Indeed, the middle pilots are used twice for the interpolation while the are edges used only once.

Substituting (16) into (8) and (9) and summing them, we can obtain the average estimation MSE of LSLI per subcarrier as

$$\varepsilon^{\text{LSLI}} = \alpha + \frac{\sigma^2}{P}\Big(\sum_{m=0}^{M}\frac{1}{B_m|F_{mL}|}\Big)^2 \; . \tag{18}$$

When $F_{mL} = 1$, i.e., without subband filtering, the optimal solution (16) and minimum MSE (18) for UFMC will be boiled down to the OFDM system.

2) DFT-based channel estimation: Considering the power constraint on the pilot tones, the estimation error of equation (5) can be minimized by optimizing the pilot signal as

$$\begin{aligned} &\min_{\mathbf{X}} \quad \frac{1}{K}\|\mathbf{h}^{\text{DFT}}_f - \mathbf{h}^{\text{A}}_f\|_F^2 = \frac{1}{K}\text{trace}\|\mathbf{WD}_K(\mathbf{XFW}_M)^\dagger\mathbf{v}\|_F^2 \\ &\text{s.t.} \quad \|\mathbf{X}\|_F^2 = P \; , \end{aligned} \tag{19}$$

where $\mathbf{h}^{\text{A}}_f = [H_0, H_1, \cdots, H_{K-1}]^T$ is the actual channel frequency response vector for all subcarriers. The optimal solution for DFT based channel estimation as can be expressed as

$$X^{\text{DFT}}_m = \frac{e^{j\varphi}}{|F_m|}\sqrt{\frac{P}{\sum_{n=0}^{M}\frac{1}{|F_{nL}|^2}}} \quad \text{for } m = 0, 1, \cdots, M \; . \tag{20}$$

Proof: Note that $\text{trace}\|\mathbf{WD}_K(\mathbf{XF\bar{W}}_M)^\dagger\mathbf{v}\|_F^2 = \sigma^2 K\text{trace}[(\mathbf{\bar{W}}_M^H\mathbf{F}^H\mathbf{X}^H\mathbf{XF\bar{W}}_M)^{-1}]$. According to the Lemma 1 in [7], for a positive definite matrix $\mathbf{A}$ with its i-th diagonal element being $A_{i,i}$, the following inequality holds: $\text{trace}[(\mathbf{A})^{-1}] > \sum_{i=1}^{m}\frac{1}{A_{i,i}}$, this inequity holds only if $\mathbf{A}$ is diagonal. Therefore, we can design the pilot $\mathbf{X}$ for the positive definite matrix $\mathbf{\bar{W}}_M^H\mathbf{F}^H\mathbf{X}^H\mathbf{XF\bar{W}}_M$ to minimize the MSE. Since $\mathbf{F}^H\mathbf{X}^H\mathbf{XF}$ is a diagonal matrix, we can write its diagonal elements in a vector form as $\mathbf{u} = [|u_1|^2, |u_2|^2, \cdots, |u_M|^2]^T$. We further notice that $\mathbf{\bar{W}}_M$ comprises of the first L_{CH} columns of the M-point DFT matrix, hence, $\mathbf{\bar{W}}_M^H\mathbf{\bar{W}}_M = M\mathbf{I}_{L_{CH}}$. To make

$\bar{\mathbf{W}}_M^H\mathbf{F}^H\mathbf{X}^H\mathbf{X}\mathbf{F}\bar{\mathbf{W}}_M$ diagonal, the following equality must hold

$$\bar{\mathbf{W}}_M^H\mathbf{u} = [\mu; \mathbf{0}_{(L_{CH}-1)\times 1}] \tag{21}$$

with μ being an arbitrary non-zero value. Equation (21) implies that $\mathbf{u}$ is orthogonal to the DFT matrix $\bar{\mathbf{W}}_M^H$ from the second to the last columns, i.e., $\mathbf{u} = \mu\mathbf{1}_{M\times 1}$, i.e., all the diagonal element of matrix $\mathbf{F}^H\mathbf{X}^H\mathbf{X}\mathbf{F}$ should be identical, i.e, $|X_0F_0|^2 = |X_LF_L|^2 = \cdots = |X_{ML}F_{ML}|^2$. In addition, it satisfies the power constraint equation $\|\mathbf{X}\|_F^2 = P$, hence we obtain (20).

Using (20), we have $\bar{\mathbf{W}}_M^H\mathbf{F}^H\mathbf{X}^H\mathbf{X}\mathbf{F}\bar{\mathbf{W}}_M = M\sum_{n=0}^{M-1}\frac{P}{|F_{nL}|^2}\mathbf{I}_M$, which is a diagonal matrix and the minimum MSE of the DFT based estimation can be expressed as

$$\varepsilon^{\text{DFT}} = \frac{L_{CH}}{M}\cdot\frac{\sigma^2\sum_{n=0}^{M-1}\frac{1}{|F_{nL}|^2}}{P}\ . \tag{22}$$

3) MMSE-based channel estimation: Note that the MSE of the estimator (6) can be expressed as $(\mathbf{R}_t^{-1} + \frac{1}{\sigma^2}\bar{\mathbf{W}}_M^H\mathbf{F}^H\mathbf{X}^H\mathbf{X}\mathbf{F}\bar{\mathbf{W}}_M)^{-1}$ [6]. Considering the power constraint on the pilot tones, the estimation error of equation (6) can be minimized by optimizing the pilot signal $\mathbf{X}$ as

$$\begin{aligned} \min_{\mathbf{X}} \quad & \text{trace}(\mathbf{R}_t^{-1} + \frac{1}{\sigma^2}\bar{\mathbf{W}}_M^H\mathbf{F}^H\mathbf{X}^H\mathbf{X}\mathbf{F}\bar{\mathbf{W}}_M)^{-1} \\ \text{s.t.} \quad & \|\mathbf{X}\|_F^2 = P\ . \end{aligned} \tag{23}$$

Following the same derivation as for the DFT based algorithm, we have the optimal pilot signal

$$X_m^{\text{MMSE}} = \frac{e^{j\varphi}}{|F_{mL}|}\sqrt{\frac{P}{\sum_{n=0}^{M-1}\frac{1}{|F_{nL}|^2}}} \quad \text{for } m=0,1,\cdots,M\ . \tag{24}$$

Substituting (24) into the first equation of (23), we get the MSE of the MMSE based estimator as

$$\varepsilon^{\text{MMSE}} = \sum_{l=0}^{L_{CH}-1}\frac{\sigma^2R_t(l)\sum_{n=0}^{M-1}\frac{1}{|F_{nL}|^2}}{MPR_t(l) + \sigma^2\sum_{n=0}^{M-1}\frac{1}{|F_{nL}|^2}}\ , \tag{25}$$

where $R_t(l)$ is the l-th diagonal element of the channel impulse correlation matrix $\mathbf{R}_t$. Comparing with the DFT based estimation (20), the MMSE based estimator (24) has the same optimal pilot signal, i.e., $X_m^{\text{MMSE}} = X_m^{\text{DFT}}$. However, comparing with DFT based estimation MSE in (22), MMSE based algorithm achieves different estimation MSE in (25). Apparently, MMSE based algorithm has smaller estimation error since

$$\begin{aligned} \varepsilon^{\text{MMSE}} &\leq \sum_{l=0}^{L_{CH}-1}\left[(\sigma^2R_t(l)\sum_{n=0}^{M-1}\frac{1}{|F_{nL}|^2})/(MPR_t(l))\right] \\ &= \varepsilon^{\text{DFT}} \end{aligned} \tag{26}$$

4) RMMSE-based channel estimation: Though the MMSE based channel estimation algorithm outperforms other estimators, it requires the channel correlation matrix $\mathbf{R}_t$ and the noise power. However, the information is hard to obtain in some scenarios. To relax the requirements, [7], [8] proposed the RMMSE based algorithm by replacing the channel correlation matrix $\mathbf{R}_t$ in (6) by either power normalized identity matrix, i.e., $\frac{1}{L_{CH}}\mathbf{I}_{L_{CH}}$, or diagonal matrix $\mathbf{R}_{EP}$ with exponential delaying elements with its i-th diagonal element being $R_{EP}(i) = e^{-i\ln(2L_{CH})/L_{CH}}/\sum_{n=0}^{L_{CH}-1}e^{-n\ln(2L_{CH})/L_{CH}}$ [8]. Then we can obtain the RMMSE-ID and RMMSE-EP for UFMC systems as follows:

$$\mathbf{h}_f^{\text{MMSE-ID}} = \frac{1}{L_{CH}}\mathbf{W}\mathbf{D}_K\tilde{\mathbf{Q}}^H(\frac{1}{L_{CH}}\tilde{\mathbf{Q}}\tilde{\mathbf{Q}}^H + \frac{\sigma^2}{\rho_{ID}}\mathbf{I})^{-1}\mathbf{y}\ , \tag{27}$$

$$\mathbf{h}_f^{\text{MMSE-EP}} = \mathbf{W}\mathbf{D}_K\mathbf{R}_{EP}\tilde{\mathbf{Q}}^H(\tilde{\mathbf{Q}}\mathbf{R}_{EP}\tilde{\mathbf{Q}}^H + \frac{\sigma^2}{\rho_{EP}}\mathbf{I})^{-1}\mathbf{y}\ , \tag{28}$$

where $\tilde{\mathbf{Q}} = \mathbf{X}^{\text{MMSE}}\mathbf{F}\bar{\mathbf{W}}_M$. In addition, we introduce two parameters ρ_{ID} and ρ_{EP} in equation (27) and (28) to denote the noise power estimation error.

IV. Numerical results

In this section, we compare the optimal LSLI, DFT, MMSE, RMMSE-ID and RMMSE-EP channel estimators numerically for the UFMC systems. While the LSLI, DFT, MMSE based channel estimation error will be compared with the analytical results in (18), (22) and (25), respectively. In addition, we will specialize (25) to OFDM system with $F_{nL} = 1$ as benchmark.

We consider the total number of subcarriers $K = 1200$, which are split into 100 subbands with each subband containing 12 subcarriers. We adopt a finite impulse response (FIR) Chebyshev filter [1] with OoB emission level equal to -50 dB and the filter length $L_F = 60$ taps. The International Telecommunication Union (ITU) Urban Macrocell (UMa) and Urban Microcell (UMi) channel models are used.

Figure 1 (a) and (b) examine the performance of the proposed algorithms in terms of MSE with fixed pilot interval $L = 10$ for UMi and UMa channels, respectively. The pilot power is $P = 120$ (unity power for each pilot tone on average) and the SNR = $1/\sigma^2$. It can be seen that all of the simulated results match the analytical results perfectly. The LSLI based estimator shows the worst performance due to the modeling error (i.e., linear interpolation), and tends to show error floor in the high SNR region for the UMa channel. DFT and MMSE based estimators show significantly better performance than the LSLI estimator and there is no error floor in all scenarios. For the RMMSE-ID and RMMSE-EP estimators, $\rho_{ID} = \rho_{EP} = 1$, which show performance loss compared to MMSE estimator due to the channel correlation matrix mismatch. In addition, the MMSE estimator for OFDM

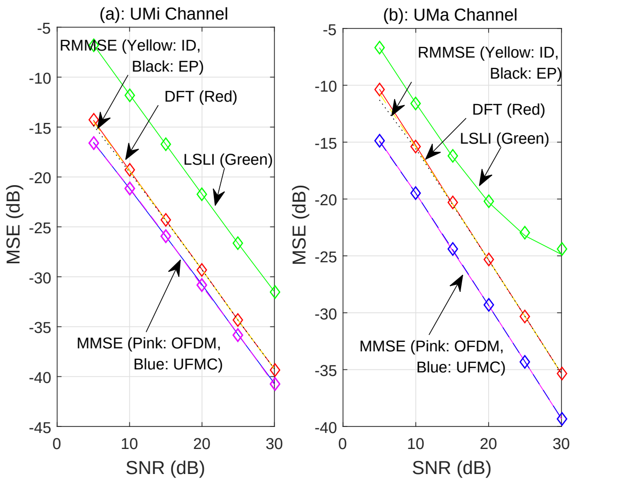

Fig. 1. Channel estimation MSE versus input SNR for both UMi and UMa channels. (Diamonds: Analytical results, Lines: Simulated results).

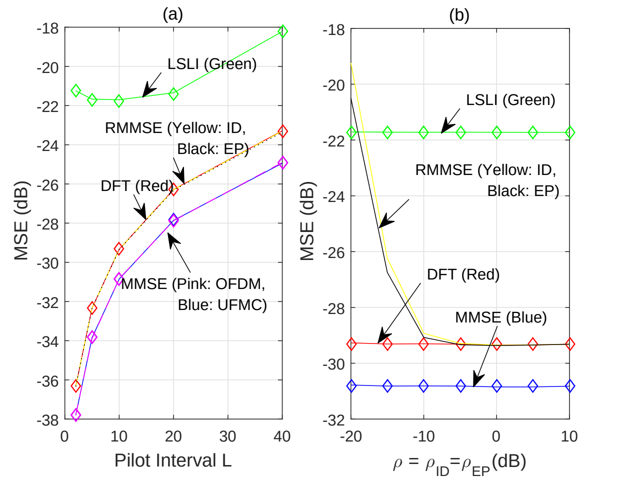

Fig. 2. Channel estimation MSE versus: (a) pilot interval and (b) noise power mismatch (with $\rho = \rho_{ID} = \rho_{EP}$) for UMi channel. (Diamonds: Analytical results, Lines: Simulated results).

system shows very slightly better performance to the UFMC systems for both channels.

Fig. 2 (a) shows the algorithms sensitivity to the pilot tones interval L for all algorithms in the UMi channel with SNR $=$ 20 dB. We can see that larger L (i.e., lower pilot density) results in a larger estimation error for DFT and MMSE based estimators, while the estimation error for LSLI estimator goes down and then up when L increases. The reason is when L increases from 2 to 10, more power is allocated to the pilot subcarriers, rendering the LS estimation more accurate. Meanwhile, the pilot distance increase caused LI error is minor, resulting in the overall MSE reduction. With L further going up, the pilot insufficiency caused LI error is significantly increased than the LS caused MSE reduction, leading to an overall increase in MSE.

Fig. 2 (b) examines the MSE performance of RMMSE channel estimators versus the noise power estimation error ρ_{ID} and ρ_{EP} in UMi channel, with fixed SNR $=$ 20 dB and $L = 10$, where LSLI, MMSE and DFT based algorithms are served as benchmarks. It can be seen that at lower ρ, the noise power mismatch caused performance loss is significant. This loss reduces sharply when ρ increases, and finally approaches the DFT based algorithms.

V. Conclusions

The work introduced in this correspondence establishes a framework for various channel estimations in future UFMC based 5G wireless systems. Based on LSLI, DFT, MMSE, RMMSE-ID and RMMSE-EP criteria, the optimization problems in terms of pilot signals subject to power constraint were formulated and solved by minimizing the estimation mean square error. Closed-form solutions and MSE are derived analytically and validated by simulations.

References

[1] V. Vakilian, T. Wild, F. Schaich, S. Ten Brink, and J.-F. Frigon, "Universal-filtered multi-carrier technique for wireless systems beyond LTE," in *IEEE Globecom Workshops (GC Wkshps)*, 2013, pp. 223–228.

[2] A. Ijaz, L. Zhang, M. Grau, A. Mohamed, S. Vural, A. U. Quddus, M. A. Imran, C. H. Foh, and R. Tafazolli, "Enabling massive IoT in 5G and beyond systems: PHY radio frame design considerations," *IEEE Access*, vol. 4, pp. 3322–3339, 2016.

[3] X. Zhang, M. Jia, L. Chen, J. Ma, and J. Qiu, "Filtered-OFDM - enabler for flexible waveform in the 5th generation cellular networks," in *2015 IEEE Global Communications Conference*, Dec 2015, pp. 1–6.

[4] L. Zhang, A. Ijaz, P. Xiao, and R. Tafazolli, "Multi-service System: An Enabler of Flexible 5G Air-Interface," *IEEE Communications Magazine*, pp. 1–9, 2017, to appear.

[5] X. Wang, T. Wild, F. Schaich, and S. ten Brink, "Pilot-aided channel estimation for universal filtered multi-carrier," in *IEEE Vehicular Technology Conference (VTC Fall)*, Sept 2015, pp. 1–5.

[6] S. Song and A. Singer, "Pilot-aided OFDM channel estimation in the presence of the guard band," *IEEE Transactions on Communications*, vol. 55, no. 8, pp. 1459–1465, Aug 2007.

[7] M. Biguesh and A. B. Gershman, "Training-based MIMO channel estimation: a study of estimator tradeoffs and optimal training signals," *IEEE Transactions on Signal Processing*, pp. 884–893, March 2006.

[8] S. Omar, A. Ancora, and D. T. M. Slock, "Performance analysis of general pilot-aided linear channel estimation in LTE OFDMA systems with application to simplified MMSE schemes," in *IEEE PIMRC*, Sept 2008, pp. 1–6.

[9] 5GNOW, "D3.2: 5G waveform candidate selection," Tech. Rep., 2014.

[10] L. Zhang, A. Ijaz, P. Xiao, A. Quddus, and R. Tafazolli, "Subband Filtered Multi-Carrier Systems for Multi-Service Wireless Communications," *IEEE Transactions on Wireless Communications*, vol. 16, no. 3, pp. 1893–1907, March 2017.

[11] J. Kim, J. Park, and D. Hong, "Performance analysis of channel estimation in OFDM systems," *IEEE Signal Processing Letters*, vol. 12, no. 1, pp. 60–62, Jan 2005.