Hybrid Beamforming for Massive MIMO Systems

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May 2017

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I would like to dedicate this thesis to my loving family for all their support.
Abstract

Massive multiple-input multiple-output (MIMO) technology is considered as one of the enabling technologies to scale up the data rates for the future communication systems. Traditional MIMO systems employ digital beamforming where each antenna element is equipped with one radio frequency (RF) chain. When the number of the antennas are scaled up, the cost and power consumption of massive MIMO systems also increase significantly. Recently, hybrid analog-and-digital beamformers have attracted a lot of attention as a cost effective approach to benefit from the advantages of massive MIMO. In hybrid structure, a small number of RF chains are connected to a large number of antennas through a network of phase shifters. The optimal hybrid beamforming problem is a complex nonconvex optimization due to the nonconvex constraint imposed by phase shifters. The overall objective of this thesis is to provide simple and effective hybrid beamforming solutions for narrowband point-to-point and multiuser massive MIMO scenarios.

Firstly, hybrid beamforming problem for a point-to-point communication system with perfect channel state information (CSI) is investigated, and an effective codebook based hybrid beamforming with low resolution phase shifters is proposed which is suitable for sparse scattering channels. Then, by leveraging the properties of massive MIMO, an asymptotically optimal hybrid beamforming solution as well as its closed-form formula will be presented. It will be shown that the proposed method is effective in both sparse and rich scattering propagation environments. In addition, the closed-form expression and lower-bounds for the achievable rates are derived when analog and digital phase shifters are employed.

Secondly, hybrid beamforming problem to maximise the total sum-rate for the downlink of multiuser MIMO is investigated, and an effective solution as well as its closed-form expression for this system is proposed. The presented solutions for the single-antenna and multi-antenna scenarios are shown to be effective as they can achieve a similar sum-rate as digital beamforming can reach. In addition, it is shown that the proposed technique with low-cost low resolution phase shifters at the RF beamformer demonstrates a comparable performance to that of a hybrid beamformer with an expensive analog beamformer.

Finally, two novel hybrid beamforming techniques are proposed to reduce the power consumption at the RF beamformer. Defining a threshold level, it is shown that half of the phase shifters could be turned off without a performance loss when the wireless channel matrix is modeled by Rayleigh fading. Then, we reduce the number of the phase shifters by using a combination of phase shifters and switches at the RF beamformer. The proposed methods can significantly reduce the power consumption as switches, in general, have lower power consumption compared to phase shifters. It is noted that the presented algorithms and the closed-form expressions of their performance are derived by using the asymptotic properties of the elements of the singular vectors for the rich scattering channel matrix.

Key words: Massive MIMO, hybrid beamforming, phase shifter selection.
Acknowledgements

I would like to thank all the people who helped me during this journey. Firstly, I would like to thank my supervisors Professor Mehrdad Dianati and Dr. Mir Ghoraishi for their kind support. The constructive discussions and feedbacks helped me to carry out my research. Moreover, without the support of my family, this journey would not have been possible. Finally, I would like to gratefully acknowledge the support provided by colleagues and staff at Institute for Communication Systems, home of 5G Innovation Centre.
Notations

The following notation is used throughout this thesis: \( \mathbb{R} \) and \( \mathbb{C} \) are the field of real and complex numbers. \( \mathbf{A} \) represents a matrix, \( \mathbf{a} \) and \( \mathbf{a}^* \) are a vector and its conjugate, respectively. \( \mathbf{a}_m \) is the \( m \)th column of \( \mathbf{A} \) and \( \mathbf{A}_{1:m} \) is a matrix containing the first \( m \) columns of \( \mathbf{A} \). \( A_{mn} \) and \( |A_{mn}| \) denote the \((m,n)\) element of \( \mathbf{A} \) and its magnitude. \( \text{diag}(\mathbf{A}_1, \mathbf{A}_2, \ldots, \mathbf{A}_K) \) is a diagonal matrix with \( \mathbf{A}_1, \mathbf{A}_2, \ldots, \mathbf{A}_K \) on its diagonal. \( \mathbf{A}^{-1}, \det(\mathbf{A}), \|\mathbf{A}\|, \mathbf{A}^T, \mathbf{A}^H, \text{trace}(\mathbf{A}) \) denote inverse, determinant, Frobenius norm, transpose, Hermitian and trace of \( \mathbf{A} \), respectively. \( \mathcal{R}(\mathbf{a}, \mathbf{A}) \) and \( \mathcal{C}(\mathbf{a}, \mathbf{A}) \) present a random vector of real and complex Gaussian distributed elements with expected value \( \mathbf{a} \) and covariance matrix \( \mathbf{A} \). Moreover, \( \mathbf{0}_{m \times 1}, \mathbf{1}_{m \times 1} \) and \( \mathbf{I}_m \) are a vector of \( m \) zeros, \( m \) ones and an \( m \times m \) identity matrix. Finally, \( f_\mathbf{A}(\mathbf{a}), F_\mathbf{A}(\mathbf{a}) \) and \( \mathbb{E}[\mathbf{A}] \) denote the the probability density function (pdf), cumulative distribution function (cdf) and expected value of \( \mathbf{A} \), respectively.
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Chapter 1

Introduction

The desire for higher data rates and more reliable communications has lead the engineers and researchers in telecommunications industry towards development of new technologies. This requires disruptive changes to the existing infrastructures and user equipment. One of the foreseen technologies for the fifth generation (5G) of the cellular communication systems is massive multiple-input multiple-output (MIMO) systems [1]. Massive MIMO can improve the performance by providing large spatial multiplexing and diversity gains. Moreover, small scale fading and noise effects vanish, and low complexity linear beamforming techniques, such as zero forcing (ZF) and matched filtering (MF), result in a near optimal performance in multiuser scenarios [2]. Despite the benefits of massive MIMO systems, implementation of such systems is expensive and requires high power consumption. Hence, this thesis aims to investigate some of the key challenges of beamforming in massive MIMO systems and propose low cost and simple solutions towards the implementation of such systems.

1.1 Scope and Objectives

In general, the hardware architecture of beamformers can be divided into three categories as digital, analog and hybrid beamformers. Digital beamforming provides the highest level of flexibility in implementing the beamforming algorithms as it allows for manipulating the phase and amplitude of the signals on each antenna element. In this
method, each of the antennas is connected to baseband through a dedicated radio frequency (RF) chain. Due to the high cost, complexity and power consumption of the RF chains, the implementation of massive MIMO systems with digital beamformers becomes a very expensive and challenging task [3]. The simpler and cheaper approach is analog beamformers where the phase array antenna is connected to a single RF chain. However, the main disadvantage of this method is that spatial multiplexing gains are not achievable.

Since phase shifters are cheaper and have lower power consumption compared to RF chains, hybrid beamforming is able to provide a tradeoff between the performance of analog and digital beamformers. In this approach, a small number of RF chains are connected to the large antenna array through a network of phase shifters [3–5]. The design of the optimal hybrid beamforming schemes is a challenging task due to the nonconvex constant modulus constraint imposed by the phase shifters [3–5]. As it will be discussed in the next chapter, this problem has attracted a lot of attention in academia and industry and it has been investigated by many researchers as in [3–22] and references therein. Due to the complex nature of the problem, the existing hybrid beamforming solutions are suboptimal and generally computationally expensive. Moreover, to the best of our knowledge, there are no closed-form expressions of the spectral efficiency when hybrid beamforming is applied. In addition to the nonconvex constraint of the optimization problem, most of the commercial phase shifters are digitally controlled and they have discrete resolution. Since hybrid beamformers require a large number of phase shifters, there are many possible combinations of beamforming weights to be optimized. Searching the discrete space of all the possible combinations of phases imposes a huge computational burden.

Motivated by the previously mentioned challenges, this thesis investigates hybrid beamforming problem in massive MIMO scenarios. The main objectives of this research are:

1. Designing a simple and asymptotically optimal hybrid beamformer for the point-to-point and multiuser scenarios.

2. Investigating low complexity solutions when low-resolution digital phase shifters are used.
3. Proposing new structures for hybrid beamformers to reduce the power consumption of the phase shifter network.

4. Deriving the closed-form expressions of spectral efficiency achieved by the proposed methods.

1.2 Contributions

In accordance with the listed challenges, the contributions of this thesis are summarized as:

1. A novel RF codebook and channel estimation for the point-to-point mmWave systems are proposed. This RF codebook enables analog and hybrid beamformers to steer the beam towards any desired direction when low resolution phase shifters, e.g. with 2 bits of resolution, are used. The advantage of the proposed channel estimation over state-of-the-art algorithms is that it provides a tradeoff between the estimation time and the transmit power at the initial stages.

2. An asymptotically optimal hybrid beamforming solution and its closed-form expression for the point-to-point systems is proposed. Moreover, it is shown that the performance of digital beamforming can be achieved by hybrid beamformers when the number of the RF chains are equal or greater than the number of the transmit streams. The proposed beamformer reaches a near optimal performance in both rich and sparse scattering channels which are suitable for sub-6 GHz and mmWave systems, respectively. Moreover, the closed-form expressions of the spectral efficiency are derived. These objectives are accomplished by investigating the properties of the elements of the singular vectors of the channel matrix in massive MIMO scenarios.

3. The performance of hybrid beamforming for the downlink of multiuser scenarios is investigated when the user devices are equipment with a single-antenna and multiple-antennas. In addition, the closed-form expressions of the precoder and the total sum-rates with respect to the performance of digital beamforming are
proposed. Analytical and simulation results indicate that the achievable rates are comparable with the performance of digital beamforming. It is noted that the proposed hybrid beamforming for the single-antenna and multiantenna scenarios rely on zero-forcing and block diagonalization, respectively.

4. A simple and heuristic rounding technique to calculate the hybrid beamforming matrix with digital phase shifters is proposed. In addition, a performance lower-bound is derived and it is shown that 3 bits of resolution per phase shifter suffices to almost achieve the performance of analog phase shifting.

5. In order to simplify the hardware structure of the phase shifters network, hybrid beamforming with subconnected structure is investigated. In this configuration, each RF chain is connected to a subset of antennas using a phased array. For the subconnected phase shifter network and under the assumption of rich scattering channels, firstly an asymptotically optimal hybrid beamforming technique is presented. Then, in order to reduce the power consumption that is associated with the phase shifters, two novel combinations of phase shifter and switch networks are proposed. In the first method, the performance of hybrid beamforming with a subconnected phase shifter network that is connected to a fully-connected switch network is evaluated. However, as fully-connected switches have high insertion losses and cross talk distortions in practice, we also investigate the performance of using simple binary switches. The simulation and analytical results indicate that the performance loss is less than 10% when binary switches are used, i.e. the number of the phase shifter is reduced to 50%.

The research carried out during this PhD resulted in the following publications:


**J2** S. Payami, M. Ghoraishi and M. Dianati, "Hybrid Beamforming with Reduced Number of Phase Shifters in Massive MIMO Systems", submitted to IEEE Transactions on Vehicular Technology Correspondence.

**C1** S. Payami, M. Shariat, M. Ghoraishi and M. Dianati, "Effective RF codebook de-


1.3 Overview of the Thesis

This thesis is organized as following: Chapter 2 discusses the background and the existing works in the context of hybrid beamforming. In chapter 3, hybrid beamforming for the point-to-point systems is presented. In chapter 4, hybrid beamforming for multiuser scenario with both single-antenna and multiantenna user equipment is discussed. In chapter 5, the performance of hybrid beamforming with switches and phase shifter is evaluated. Finally, the conclusions and future works are presented in chapter 6.
Chapter 2

Background and Literature Review

The fifth generation (5G) of the cellular communication systems are required to provide very high data rates to support the user demands for various range of applications. The capacity of a single-user communication system depends on the system bandwidth and signal-to-noise ratio (SNR). Transmitting a high power signal over a large bandwidth can result in a very high data rate. However, this is not possible due to high power consumption, inter-cell and intra-cell interference effects and government regulations. Hence, an efficient use of spectrum and assuring a reasonable SNR levels at the receiver is crucial in order to meet the promise of high data rates in 5G. Multiple-input-multiple-output (MIMO) systems have attracted a lot of attention over the last decade as they can improve the transmission reliability as well as the spectral efficiency. The system reliability, also known as diversity, is improved by transmitting the signal over multiple links. The chance of having multiple links at a deep fading point decreases as the number of the antennas increases. Hence, the probability of receiving the correct signal at the receiver increases. The spectral efficiency can be improved by spatial multiplexing techniques and transmitting multiple symbols over the same time and frequency slot by spatial filtering. This is achieved by precoding and combining techniques at the transmitter and receiver, respectively. The received SNR can be improved by focusing the transmitted signal towards a desired spatial direction which is called beamforming.

The total number of the transmitted symbols over a wireless MIMO channel is upper-limited by the minimum number of the base station antennas and the total number of
the antennas at the user side. The more antennas are available at the base station, the more users can be supported and higher data rates can be achieved. In addition, large antenna arrays can provide pencil-beams by applying beamforming techniques. Consequently, a big portion of the power can be focused towards the intended user which results in higher receive SNR as well as lower interference among the rest of the users.

The aforementioned advantages of equipping the base station with a large number of antennas, known as massive MIMO systems, has motivated many researchers in academia and industry to consider this technology as one of the major candidates for 5G. An overview of such systems and their advantages, challenges and application is presented in the next section.

2.1 Massive MIMO Systems

The massive MIMO term refers to a scenario that the number of the antennas at the base station is much larger than the number of the user equipment. The purpose of this technology is to scale up the benefits of conventional MIMO systems and act as an enabler for more energy and spectral efficient, secure and robust systems [23]. The performance of massive MIMO systems heavily relies on the availability of channel state information (CSI) at the base station. In order for the base station to acquire the CSI in massive MIMO scenario, it is commonly assumed that the system operates in a time-division duplex (TDD) manner. In this case, the base station estimates the channel during uplink and uses the same parameters in the downlink transmission. Frequency-division duplex (FDD) is a more challenging task in massive MIMO as the channel estimation time increases with the number of the transmit antennas. In addition, the users are required to feedback the estimated channel to the base station which increases the signaling overhead. Moreover, high mobility reduces the coherence time of the channel. Hence, channel estimation in downlink may not be practical, and TDD operation simplifies the system design. In the TDD scenario, the uplink and downlink transmissions will take place over the same frequency resource but at a different time slot. When the coherence time of the channel is large enough, the channel remains unchanged in both directions.
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and the base station can use the same estimated channel for precoding in downlink. Even under TDD assumption, however, the transceiver circuitry are not reciprocal, and hence calibration is necessary. It has been reported that the mismatch between uplink and downlink hardware changes slowly over the time and simple calibration methods can mitigate this problem [24].

When the base station has CSI of the users, then it can apply beamforming techniques to improve the spectral and energy efficiencies of the system. It is noted that the performance of the beamformers relies on the properties of the MIMO channel. In other words, depending on the channel behavior, a new beamformer design may be required. For example, the number of the multipath components (MPCs), the angular correlation and the geometry of the antenna arrays can play a significant role in the effectiveness of the beamformers. For the future cellular communication systems, it is expected that massive MIMO systems will be used both in sub-6 GHz and millimeter wave (mmWave) systems.

2.1.1 Sub-6 GHz systems

Massive MIMO for sub-6 GHz bands has been shown to be a promising approach as favorable propagation conditions are observed [2]. Favorable propagation refers to the scenario that the channel vectors from the base station to user equipment are orthogonal to each other which results in interference-free transmission when suitable beamforming techniques are applied. One of the main advantages of massive MIMO systems over conventional MIMO systems is that linear beamforming algorithms can achieve a near-optimal performance. Hence, complicated methods such as dirty paper coding (DPC) are not required to cancel the interference between the users and achieve the capacity of the channel. It is noted that favorable propagation applies to both line-of-sight (LoS) and non-LoS (NLoS) scenarios [25–27]. Moreover, only large scale fading will be the dominant factor in the channel and small scale fading effects will vanish [2]. The wireless channel at sub-6 GHz NLoS scenarios consists of many MPCs, and researchers have commonly assumed that the wireless channel from each base station antenna to each receive antenna is modeled by Rayleigh fading, however, this may not hold in real
Chapter 2. Background and Literature Review

2.1.2 MmWave systems

The sub-6 GHz frequency bands for cellular communication systems are becoming saturated due to the limited availability of spectrum at such frequencies. One way out is the exploitation of large chunks of vacant unlicensed or lightly licensed spectrum at mmWave frequencies for access and backhaul links. Such systems are expected to be also used in local and personal area networks, self-driving cars and autonomous robots [29].

A lot of research has been carried out to analyze and model the propagation behavior at mmWave channels, for example as in [30, 31] and references therein. High propagation losses, smaller number of MPCs compared to sub-6 GHz channels, sensitivity to signal blockage and severe penetration losses in indoor-to-outdoor scenarios are some of the main characteristics of mmWave channels [31, 32].

In order to employ millimeter wave technology in commercial systems, many challenges must be addressed [3, 31, 33, 34]. Beginning with signal propagation and channel properties, mmWave signals experience high attenuation over the wireless channel due to the nature of electromagnetic waves at such high frequencies [31, 32]. Hence, using high gain and directional antennas is crucial to compensate for the path loss. Directional antennas impact the observed effective channel at the baseband, for example, the delay spread of the channel will be reduced. This can potentially reduce inter-symbol interference and improve the overall system performance [35]. On the other hand, considering that the symbol time in mmWave systems is also small because of the large bandwidth, complicated equalization techniques can be required [32].

Unlike long term evolution (LTE), mmWave systems are not expected to support high mobility, however, the support for low mobility environments with moving users is inevitable. When directional antennas at the transmitter and receiver are employed, perfect beam alignment is necessary as the number of the dominant MPCs of the channel is small. Furthermore, the communication links need to be set up very fast and it has to be able to adapt to the location of the users. Hence, directional antennas with beam steering capability are necessary for mmWave systems.
One of the common techniques to produce a steerable narrow beam is employing antenna arrays. Given the short wavelength of the signals at mmWave frequencies, it is possible to place a large number of antennas within a tiny space and keep the form factor very small. It is noted that the directional transmission by antenna arrays requires a suitable beamforming algorithm. In the next section, an overview of beamforming techniques for massive MIMO systems will be discussed.

2.2 Beamforming in Massive MIMO Systems

In general, beamforming is defined as a type of spatial filtering technique to exploit the spatial properties of the signals from multiple sensors. For example, by manipulating the phase and amplitude of the signals from each sensor, beamforming can be performed such that the signals from a desired direction are added constructively or deconstructively. In this thesis, beamforming term is often used as a technique at both the transmitter and receiver to increase the received SNR. Whereas the terms precoding and combining are always used when the spatial filter is designed to achieve spatial multiplexing at the transmitter and the receiver. It is noted that it is common in the literature, as well as throughout this thesis, to interchangeably use the terms "beamforming" and "precoding/combining".

In conventional MIMO systems, each antenna element is connected to the baseband processor. This requires a dedicated mixer, analog-to-digital converter (ADC) or digital-to-analog converter (DAC), filters and amplifiers per antenna. The series of the components that connect the antennas to the baseband are called radio frequency (RF) chains. Hence, precoding and combining can be performed at the baseband by digital beamforming techniques where there is a full control over the phase and amplitude of the signals at/from each antenna element. Figure 2.1 shows the block diagram of a digital precoder with $N$ antennas and RF chains.

As the number of the antenna elements at the transceiver goes large, higher diversity and multiplexing gains are achievable and the channel matrix tends to have favorable conditions [2, 17]. Moreover, the total transmit power can be reduced as large beamforming gains are provided. It was also shown that linear precoding and combining techniques
such as matched filtering (MF) and zero-forcing (ZF) can result in an asymptotically optimal performance due to the favorable propagation properties [2]. However, RF chains are expensive and have a high power consumption [23], specially in mmWave systems that have a large bandwidth [3, 36]. Hence, cost and power consumption can become a prohibitive factor in applying digital beamforming in massive MIMO scenarios.

In order to reduce the number of the RF chains in MIMO systems with large arrays, hard and soft antenna selection techniques are proposed [4]. In the hard selection, the RF chains are connected to the antennas by a network of switches. Depending on the performance metric, e.g. maximizing the spectral efficiency, the best set of antennas are selected. The optimum performance is achieved by exhaustive search over different combination of the selected antennas. However, this is a combinatorial optimization problem and it imposes a high computational complexity. Hence, suboptimal approaches, based on convex optimization to maximize the spectral efficiency, are proposed in [37-40]. The drawback of hard antenna selection is that large beamforming gains cannot be achieved when the number of the antennas is significantly larger than the number of the RF chains due to the loss of array gain. In the soft antenna selection, the RF chains and the antennas are connected through a network of phase shifters [3, 4, 41]. In the next section, a summary of the state-of-the-art soft selection techniques, including analog beamforming and hybrid beamforming, is presented.
2.3 Soft Antenna Selection

In this approach, a network of phase shifters is used to connect the baseband to the antenna array. Then, the beamforming procedure can be divided into digital beamforming and RF beamforming. The RF beamforming is performed by using analog circuits by changing the phase of the signals on the antennas. Since the beamforming process is taking place in the RF domain and by using analog devices, the terms analog beamforming and RF beamforming are commonly used interchangeably in the literature as well as in this thesis. However, it is noted that the term "analog beamformer" in the literature also refers to the structure of Fig. 2.2 where the antenna array with $N$ elements is connected to a single RF chain. In this scheme, each antenna is equipped with a phase shifter. The structure of the analog beamformer in Fig. 2.2 is known as the simplest soft antenna selection technique.

By adjusting the phase of each element, it is possible to mitigate the interference and increase the SNR at the intended user. Analog beamforming is already being used for short range mmWave communications such as in IEEE 802.15 standard [42]. One of the problems associated with analog beamforming is the constant modulus constraint imposed by the phase shifters. Depending on the beamforming criteria, solving the optimization problem can become a challenging task [43]. This problem becomes even more challenging when the practical constraints such as quantized resolution of the phase shifters are taken into account which can cause a huge computational burden.

In [44–46], the beamforming weights are found via an iterative transmission algorithm where the transmitter and receiver must collaborate with each other. The disadvantage of these works is that the complexity and the training time increase as the number of
the antennas grows. In [47,48], an adaptive beamwidth analog beamforming protocol for indoor applications is proposed and it has been adopted by IEEE 802.15.3c. It consists of three stages as device-to-device linking, sector-level searching and beam-level searching. In order to perform each stage of the algorithm, a predefined codebook is proposed to reduce the search area. The proposed beamforming starts with a quasi-omni pattern radiation, then eventually the search space is narrowed down to sector level and finally beam level where the maximum array gain and directionality is achieved. The authors in [47] proposed that by simultaneously producing multiple beams with unique signatures the training time can be reduced. While the algorithms in [47,48] are suitable for indoor environments, a multilevel tree-structure beamforming using subarray techniques for outdoor wireless backhaul scenario is presented in [49,50]. In these works, the authors consider the impact of digital phase shifters and the beam misalignments due to the vibrations caused by wind. The disadvantage of [43–50] is that these algorithms are only designed for analog beamforming which can only support single-stream transmission, and spatial multiplexing gains cannot be exploited.

In order to provide a tradeoff between the performance and cost of the digital and analog beamformers, hybrid analog-and-digital beamformers have been proposed. In this structures, as shown in Fig. 2.3, a small number of RF chains are connected to a large number of antennas through a network of phase shifters. Two common configurations for the phase shifter network are subconnected and fully-connected structure [5,51]. In the subconnected structure, shown in Fig. 2.3a, each RF chain is connected to a subset of antennas where each antenna element is equipped with a single phase shifter. For the fully-connected structure, Fig. 2.3b shows that there is a connection from each RF chain to all of the antennas and a larger number of phase shifters is required. It can provide higher beamforming gain and spectral efficiency compared to the subconnected architecture. However, the fabrication of the fully-connected configuration is more difficult due to the required number of RF paths as well as high power consumption in the RF beamformer [5,51]. Hence, the subconnected structure is more suitable for practical application. In theoretical works, however, the fully-connected phase shifter network model is frequently used.

Similar to analog beamformers, the design challenges arise when the constant modulus
2.3. Soft Antenna Selection

(a) Subconnected structure, (b) Fully-connected structure,

Figure 2.3: Block diagram of hybrid beamforming structures.

property and discrete resolution of the phase shifters are considered. These constraints turn the optimization of the hybrid beamforming design into difficult nonconvex and combinatorial problem. In addition, as the overall performance the beamformer depends on the joint design of analog and digital beamformers, the design procedures are different from the traditional beamforming in MIMO systems. This is due the fact that traditional systems only relied on either analog or digital beamforming techniques, and not the combination of the two methods. The optimization problem with digital phase shifters becomes a combinatorial problem with a huge search space. For example, if phase shifters with 3-bits of resolution are used in a fully-connected configuration with 8 RF chains and 64 antennas, then $2^{3 \times 8 \times 64} = 2^{576}$ possible combinations of phases are possible.

It is noted that the hybrid beamforming design problem can be investigated based on various criteria in different scenarios. For example, maximization of spectral efficiency can be considered in point-to-point/multiuser, narrowband/wideband channels, with perfect/imperfect CSI, with joint/separate design the digital and analog beamformers. In the following, an overview of the existing works on hybrid beamforming is presented. It is noted that most of the available literature have focused on maximization of spectral efficiency in different scenarios subject to the constant modulus constraint imposed by the phase shifters.
2.3.1 Narrowband Point-to-Point Systems with Perfect CSI

In a point-to-point MIMO scenario, which is also called single-user MIMO, joint signal processing can be performed to generate/decode the transmit/received signals on/from the transmitter/receiver antennas. In other words, there is a full collaboration between the antennas at each side of the transmission. In a narrowband system, it is assumed that the channel does not vary over the frequency, and the transmission bandwidth is smaller than the coherence bandwidth of the channel. It is noted that the optimal transmission scheme with digital beamforming is based on the singular value decomposition (SVD) of the channel matrix. In this case, the optimal precoder is set according to the right singular vectors and waterfilling whereas the combiner is set based on the left singular vectors. In hybrid beamforming approach, it has been shown that the baseband precoder and the RF beamformer can be designed either jointly, as in [3, 8], or in two stages as in [9-11]. In the two stage design approach, the RF beamformer is calculated based on the channel matrix. Then, the baseband precoder takes the impact of the channel matrix and the RF beamformer into account.

Hybrid beamforming with a fully-connected structure for the narrowband point-to-point system was first studied in [4]. Initially, it was shown that when the number of the RF chains are twice larger than the number of the streams, then the performance of a fully-digital beamformer can be achieved. Afterwards, the optimal analog beamforming for the single-stream transmission with analog phase shifters was derived. In this scenario, the optimal RF beamformer at the transmitter and the receiver are directly derived based on the phase of the elements of the right and left singular vector corresponding to the strongest singular value of the channel matrix. Based on the simulation results, it was shown that hybrid and digital beamformers can achieve a similar spectral efficiency when multiple symbols are transmitted over both correlated and uncorrelated channels. In this case, the optimality of the hybrid beamformer and its performance closed-form expressions were not derived.

As it will be discussed, many researchers exploit the properties of the channel matrix and array geometry to design the hybrid beamformer [3, 12-15] and references therein. More specifically, such papers focus on the mmWave systems where the assumption of
the angular sparsity over the wireless channel holds. It is assumed that there is a small number of MPCs in the channel and the channel matrix between the transmitter and the receiver is close-to-singular. Moreover, the beamformer is designed for only uniform linear/planar arrays with half a wavelength antenna spacing.

When the number of the transmit and receive antennas are very large, the authors in [12] show that a simple beam steering towards the angle-of-arrival (AoA) and angle-of-departure (AoD) asymptotically achieves the performance of a fully digital beamforming. This is due to the convergence of the singular vectors of the channel matrix and the steering vectors towards the AoAs and AoDs in this special scenario. A joint design of analog and digital beamformers based on matching pursuit was proposed for the sparse channels in [3, 13]. In this method, firstly the singular vectors of the channel must be calculated. Then, the hybrid beamformer is derived by minimizing the Euclidean distance between the matrices containing the singular vectors and the weights of the hybrid beamformer. Considering that the calculation of the singular vectors is computationally expensive, the second round of computations can cause severe delays in practical systems. In addition, the spectral efficiency based on [3] significantly depends on the number of RF chains in the system and MPCs in the channel.

A close-to-optimal performance for both rich and sparse scattering channels was achieved by proposing an iterative algorithm based on approximating the nonconvex optimization with a series of convex problems [8]. The problem associated with such iterative algorithms is that the convergence time depends on the initial conditions. Hence, the processing time to calculate the parameters of the hybrid beamformer can become a prohibitive factor in real-time systems. Another hybrid beamforming algorithm that can achieve a close-to-optimal performance for both rich and sparse scattering channels was reported in [9–11]. In these works, the RF beamformer with per-antenna power constraint was iteratively calculated.

In [52], an iterative precoding scheme is proposed which provides a tradeoff between the multiplexing and beamforming gains. In this scenario, the hybrid beamformer achieves the capacity of the channel at low SNR and full multiplexing gain at high SNR. The proposed algorithm increases the spectral efficiency by deciding whether it is better to
use an additional subarray to increase the SNR or transmit a new stream. In [16], an iterative hybrid beamformer for the subarray structure is proposed. In this approach, the optimization problem is decomposed into multiple optimization problems where each subarray is treated separately from the rest. Then, successive interference cancellation (SIC) is used to remove the interference between the subarrays. In this method, the first subarray is adjusted so that it maximizes the data rate. Then, its interference effect on the second subarray is removed by SIC, and this algorithm continues till the last subarray is adjusted. The simulation results indicate that a near-optimal spectral efficiency can be achieved. Singh et al. [14] proposed a codebook based hybrid beamforming for the subconnected structure using the sparsity of the mmWave channels. It is noted that the codebook based approaches can be only used for a fixed scenario, i.e. special type of channel and fixed array geometry. When the parameters of the either channel or the array change, then new RF codebooks are required.

2.3.2 Narrowband Multiuser Systems with Perfect CSI

In this scenario, the base station communicates with multiple single-antenna or multi-antenna user equipment. The main idea of most of hybrid beamforming algorithms for multiuser scenarios is based on using analog beamforming to increase the SNR at each user, and then applying digital beamforming algorithms at the baseband, such as ZF, to mitigate the remaining interference, for example as in [17, 18].

The authors in [15] consider a mmWave downlink scenario where the base station and users are equipped with hybrid and analog beamforming architectures, respectively. This algorithm consists of two stages as i) the received power at the user terminals is maximized by an RF codebook based beam steering approach, ii) baseband precoder is designed such that the inter-user interference is further reduced. It was shown that a near-optimal performance is achieved and the performance is comparable to a fully-digital beamformer.

In [53], a hybrid beamforming technique for the subarray structure was proposed which is based on equal gain transmission constraint. In contrast to [15], the baseband precoder is designed at the first step, and the analog beamformer is iteratively calculated.
2.3. Soft Antenna Selection

Liang et al. in [19] propose a hybrid beamformer that almost achieves the performance of a fully-digital ZF in downlink scenario with single-antenna users. In the proposed approach, the analog beamformer is set according to equal gain transmission scheme to maximize the signal levels at the users. In other words, the phase shifters are set according to the phase of the conjugate transpose of the elements of the channel matrix. Then, ZF is used at the baseband to remove the residual interference.

The authors in [20] consider an uplink scenario with single-antenna user terminals. It is assumed that the channels from the base station to the users share some common scatterers which can lead to severe inter-user interference. The analog beamformer is derived according to the Gram-Schmidt method to reduce this interference. Then, the baseband combiner applies minimum mean square error (MMSE) on the effective channel. Malla et al. in [21] sets the objective function of the optimization problem as minimization of the transmit power level subject to (s.t.) the tolerable interference level at the user equipment. The analog beamformer is designed according to the equal gain transmission whereas the digital precoder is based on ZF and Perron-Frobenius theorem. When digital phase shifters are used, Zho et al. suggests that the phase shifters can be set by rounding the phase of the elements of the channel matrix to the nearest possible phase, depending on the resolution of the phase shifters [22]. Then, ZF or MF can be used at the baseband and a near-optimal performance was observed. Sohrabi et al. extends the works in [10, 11] to both point-to-point, and single-antenna multiuser scenarios [54]. Compared to digital beamforming, the heuristic solution by Sohrabi et al. achieves a similar spectral efficiency.

A hybrid beamforming algorithm for the downlink of a multiantenna multiuser scenario, suitable for mmWave systems, was proposed in [55]. Considering the achievable sum-rate of block diagonalization with a fully-digital beamformer as reference, the hybrid beamformer is designed based on weighted-sum mean square error and orthogonal matching pursuit. It was shown that the performance gap between digital and hybrid beamformers can be reduced when the number of the RF chains increases. Zhang et al. also considers a downlink multiantenna multiuser scenario in [56]. In this approach, it was shown that hybrid beamforming could almost achieve the performance of block diagonalization with digital beamforming. Moreover, the proposed hybrid beamformer
has a better performance at high SNR regime when it is based on the regularized block diagonalization, whereas the MMSE based solution has a better performance at lower SNRs. In [57], a multiantenna multiuser scenario with erroneous CSI and a hybrid beamformer with finite resolution phase shifters is investigated. Since some inter-user interference will remain in the system, an error floor for the bit error rate (BER) was observed. It was shown that by extending the null space of interference channels, the performance can be improved. In the following, an overview of the channel estimation techniques and the impact of partial CSI on hybrid beamforming will be presented.

### 2.3.3 Hybrid Beamforming with Partial CSI

Availability and quality of the channel state information plays a significant role in the performance of beamforming algorithms at the transmitter and the receiver. Whether the system is operating in FDD or TDD modes, there is a limited amount of resources dedicated to channel estimation. This results in erroneous estimation which in return degrades the beamforming performance. As mentioned before, channel estimation is a challenging task in both TDD and FDD massive MIMO scenarios.

In FDD operation, the transmitter sends a series of pilots over the channel. Then, the receiver estimates the channel and feeds back the information to the transmitter. In this case, the estimation error is associated with the presence of noise as well as the limited number of available feedback bits from the receiver to the transmitter. When the number of the transmit antennas increases, the signaling and estimation time may surpass the coherence time of the channel. In a TDD system, on the other hand, a calibration of the uplink and downlink circuits is necessary. In addition, the number of the orthogonal pilots is related to the number of the users, and hence the users in different cells that use the same pilot sequences will cause interference for each other [58].

When hybrid analog and digital structure is used, the aforementioned problems will get even more challenging as the number of the RF chains are significantly smaller than the number of the antennas. One major consideration in designing channel estimation techniques for hybrid beamformers is to minimize the channel estimation overhead on
the system which limits the effective spectral efficiency. Second, the estimation delay has an adverse impact on the performance due to the inherent variations of channel. This problem particularly arises in outdoor environments where the relative mobility of the nodes is typically higher than indoor environments. Third, in mmWave systems with a large bandwidth and high attenuation factor, the received SNR will become very low if beamforming is not used. In other words, the estimated channel at the baseband requires to take the impact of the RF beamformer into account. Further complications in the design of the channel estimation algorithms can also arise in the practical scenarios. As an example, the design of the beamformer relies on the geometry of the antenna array. However, this knowledge may not be available due to the impact of the user’s fingers that block the array [59].

Most of the existing works on channel estimation techniques with hybrid beamformers rely on the sparse nature of mmWave channels in the angular domain [3, 15, 48–50, 60–64]. As a result of the sparsity in the angular domain, the wireless channels at such frequencies can be modeled by AoA, AoD and the gain of each path. This also allows for using compressed sensing techniques to estimate the elements of the large channel matrix with a relatively small number of measurements with respect to the dimensions of the channel matrix [60]. In addition, the channel estimation techniques can be divided into closed-loop or open-loop depending on whether a feedback link is required or not [61]. In the closed-loop estimation, the receiver chooses the best indices from a predefined beamforming codebook and feeds them back to the transmitter as in [48, 50]. In such approaches, the choice of the RF codebook plays a significant role in the performance of the system. Large codebook size enables the generation of pencil beams with a sharp roll-off while it increases the feedback overhead. Hence, the impact of the RF codebooks have been investigated by many researchers as in [3, 15, 48–50] and references therein.

A simple RF codebook can be made by uniformly sampling the beam steering space [3, 15]. However, the disadvantage of this approach is that 6-7 bits of resolution per phase shifter is required for a desirable performance. High resolution phase shifters have high insertion losses, and hence RF codebooks with low resolution phase shifters are preferred in practice. Minimizing the mean square error (MSE) between the ideal
codebook and the generated beam patterns, an effective hybrid codebook is proposed in [62] by using orthogonal matching pursuit. It is noted that this codebook requires analog phase shifting capability and the overall performance significantly varies with the number of the RF chains.

A closed-loop multi-resolution channel estimation for the point-to-point system is proposed in [63] where the transmitter and receiver employ wide beams with low directivity gains at the early stages of channel estimation. A multi-step power allocation scheme is adopted where the transmit power is high when the beam is wider but it is gradually reduced as the beam becomes sharper. This algorithm requires exclusive feedback channel which has been addressed in [64]. High transmit power requirement at the initial stages of channel estimation, and the fact that the number of training steps scales with the number of the multi-path components are disadvantages of multi-resolution channel estimation techniques.

Alkhateeb et al. extends the channel estimation technique in [64] to a one-sided search which reduces the initial transmit power [65]. In addition, the need for a separate feedback link for the channel estimation can be diminished as the proposed approach is based on ping-pong iterations. Another open-loop CSI acquisition and hybrid beamforming technique, based on multi-grid orthogonal matching pursuit, is presented in [61]. It was shown that this algorithm outperforms conventional least squares method while requiring less computational complexity.

Hybrid beamformers can be used to reduce the channel estimation delay and signaling overhead by adjusting the RF beamformer according to slowly-varying parameters of the channel. More specifically, the dimensions of the channel matrix observed at the baseband processor is reduced to a smaller dimension by setting the RF beamformer according to the second-order statistics of the channel. This approach for the single-user MIMO systems was first used in [66]. For the multiuser scenario with single-antenna user terminals and a base station with a hybrid beamformer, a joint spatial multiplexing division multiplexing technique was proposed to reduce the downlink training and uplink feedback burden [67, 68]. In this approach, the user channels are grouped based on their covariance matrices. Then, the RF beamformer is designed such that the inter-group
interference is reduced. Finally, the baseband precoder separates the user signals in each group.

2.3.4 Wideband Hybrid Beamforming

All the previously mentioned works were focused on narrowband systems where the bandwidth of the signal is in the range of the coherence bandwidth of the channel. However, this assumption does not hold in real systems, and hence, wideband beamforming techniques are used in practice. In traditional wideband systems, firstly the signal and the channel are decomposed into subbands where the channel remains constant over each subband [69]. Then, based on the performance requirements, narrowband beamforming algorithms can be applied over each subband. One of the common wideband beamforming algorithms in communication systems is orthogonal frequency division multiplexing (OFDM) combined with MIMO techniques [70].

Current MIMO-OFDM systems require a fully-digital system where it is possible to control the phase and amplitude of the signal per subcarrier. Hybrid beamforming, however, the RF beamformer reduces the degrees of freedom for the wideband systems. Once the phase shifters are set, then they provide the same phase shift for all the subbands which makes the wideband hybrid beamforming problem more challenging. In this scenario, the objective function is commonly defined as maximization of the average spectral efficiency over all the subcarriers [18, 71–74].

In [73], a channel estimation algorithm and a single-stream transmission scheme for a mmWave point-to-point MIMO-OFDM system with subconnected structure was investigated. The beamforming is performed based on exhaustive search over a codebook of beamforming vectors. In [74], a hybrid beamforming solution for a point-to-point wideband mmWave scenario is proposed. This approach is based on designing RF codebooks for wideband analog beamforming and it applies Gram-Schmidt orthogonalization for hybrid precoding. In [71, 75], a downlink space-division multiple-access and orthogonal frequency-division multiple-access (SDMA-OFDMA) multiuser massive MIMO scenario is investigated. Aimed at maximizing the sum-rate, it was shown that frequency-domain scheduling is a crucial factor in determining the performance of hybrid beamformers.
Wideband hybrid beamforming still requires more research to find algorithms with low computational complexity. It is expected that a combination of hybrid beamforming design and frequency scheduling will be necessary for the future research [17,18].

2.4 Summary

Future generations of communications systems will exploit massive MIMO systems to provide a high spectral efficiency. However, the high cost and power consumption of the RF chains can become a prohibitive factor to commercialize such systems. Hence, hybrid beamformers have attracted a lot of attention in the research community. The performance of hybrid beamformers has been evaluated in various scenarios, e.g. point-to-point, multiuser, narrowband and wideband systems.

Compared to digital systems, channel estimation and beamforming for hybrid beamforming are more challenging due to the smaller number of RF chains as well as the nonconvex constraint imposed by the phase shifters. The existing algorithms for narrowband point-to-point and multiuser systems with perfect CSI can achieve a near-optimal performance, and the the achievable rates are comparable to digital beamformers. However, the closed-form expression of the hybrid beamformer and its performance performance remains as an open research problem. Especially, when digital phase shifters are used, the derivation of the closed-form bounds becomes more challenging due to the dependency of the optimization solutions on the numerical calculations. Furthermore, due to the computational delays associated with the derivation of the hybrid beamforming weights, the algorithms may not be suitable for practical systems depending on the application. Further references on hybrid beamforming can be found in the survey papers [17,18,41].
Chapter 3

Hybrid Beamforming for
Point-to-Point Systems

In this chapter, hybrid beamforming for the point-to-point MIMO systems will be investigated. In this scenario, which is also called single-user MIMO, both transmitter and the receiver are equipped with a hybrid beamformer. In general, the capacity of MIMO channels is achieved when the transmitter and receiver have full channel state information (CSI), and both of them are equipped with a fully-digital system. However, this requires a dedicated RF chain per antenna element. Digital beamforming, where each antenna element is equipped with a dedicated RF chain, can provide a higher degree of freedom to improve the system performance. Due to the complexity of mixed signal circuits and high level of power consumption, however, the implementation of a large number of RF chains can become very expensive. Alternatively, analog beamformers can be implemented with a single RF chain and a phased array antenna. Although they are cheaper to implement and to operate; analog systems provide a lesser degree of freedom in comparison to digital beamformers resulting in poorer performance. A third approach is to use a combination of the previous techniques, known as hybrid beamforming that can provide a trade-off between performance and cost. Hybrid beamforming consists of two major stages as

1. Channel estimation algorithm: depending on the hardware and the properties
of the wireless channel, a suitable channel estimation technique needs to be applied. For example, exhaustive search techniques for beam alignment in mmWave systems are the most power efficient methods as maximum array gains at the transmitter and receiver are used. However, such methods are not practical due to the long estimation time and limited coherence time of the channel. Hence, finding an efficient channel estimation algorithm that takes the impact of the system constraints, e.g. transmit power and coherence time, into account is crucial.

2. Beam generation: considering the hardware constraint, the beamformer must be capable of generating the desired beams for the proposed estimation algorithm. For example, the transmitter has to be able to generate variable beam-width towards desired angles considering the impact of the limited number of the RF chains, digital phase shifters and large number of the antennas. Finally, a suitable beamforming should be used to transmit data and achieve a high spectral efficiency.

After formally introducing the system model and the hybrid beamforming optimization problem in sections 3.2 and 5.1, section 3.3 sheds light on the seminal state-of-the-art hybrid beamforming and channel estimation approaches and their associated shortcomings [3, 64]. In order to alleviate the deficiencies of the existing works, and to facilitate real world implementations, the contributions of section 3.3 are as follows:

1. The state-of-art algorithms [3, 64] require phase shifters with 6-7 bits resolution. In addition, the algorithms are sensitive to the number of RF chains due to their RF codebook design. In this work, we show that by choosing an optimal RF codebook, 2-bit phase shifters can provide promising results with less sensitivity to the number of RF chains. This codebook could be employed for both beam switching and hybrid beamforming purposes. In addition, employing this codebook, it is possible to achieve maximum array gain in various directions.

2. We propose a low complexity channel estimation algorithm with limited number of training steps and transmit power. The training time of the proposed algorithm does not scale with the number of multi-path components unlike the recent
literature [64]. The proposed scheme is capable of estimating the channel angle of arrivals (AoAs) and angle of departures (AoDs) with a high accuracy and without exclusive feedback channel requirements.

The RF codebook based hybrid beamformers, however, are typically designed for a particular channel model. For example, the hybrid beamformer in section 3.3 and [3] are only suitable for mmWave systems. When perfect CSI is available, more general beamforming approaches, that are applicable to variety of channels, are investigated in [4, 8–11, 15, 54]. However, as it was discussed in chapter 2, the closed-form expression of the performance of hybrid beamforming algorithms in the literature is not available. In addition, most of the existing works have a relatively high computational complexity.

In this direction, section 3.4 presents an asymptotically optimal hybrid beamforming scheme, which has a relatively low complexity, and the closed-form expressions of the hybrid beamformer and the spectral efficiency for the point-to-point scenario in rich and sparse scattering channels are derived when the number of the antennas are asymptotically large. It is assumed that the rich and sparse scattering channels follow Rayleigh fading and geometry based models, perfect CSI is available at the transmitter and the number of the antennas are large. In section 3.4, the following issues will be discussed to derive and analyze the asymptotically optimal beamformer:

1. The performance of the digital beamformers is achievable when the number of the RF chains is two times larger than the number of the transmitted symbols.

2. In order to calculate the hybrid beamformer for the independent and identically distributed (i.i.d.) Rayleigh fading scenario when the number of the RF chains and symbols are equal, the distribution of the elements of the singular vectors of the large channel matrix are derived which, to the best knowledge of the authors, has not been previously reported. Based on this distribution, the asymptotically optimal hybrid beamforming schemes for both the point-to-point scenarios are derived.

3. It is shown that the asymptotically optimal RF beamforming matrix is achieved when the phase shifters are set according to the phase of the elements of the
singular vectors of the channel matrix. Then, the closed-form expressions of the spectral efficiencies achieved by the proposed hybrid beamformers are calculated.

4. When digital phase shifters are used, a simple but effective hybrid beamforming scheme is proposed and its performance lower-bound is derived.

The advantages of the proposed approach over the state-of-the-art are simplicity, low computational delays and asymptotically optimal behavior. As it will be shown, analytical and simulation results demonstrate that there is a good match between the closed-form expressions and simulation results. Moreover, the performance of the proposed scheme with phase shifters with more than 2-bits of resolution is similar to the hybrid beamformer with analog phase shifters.

3.1 System Model

In a point-to-point MIMO communication system, the transmitter and the receiver are equipped with $N_t$ and $N_r$ antennas, respectively. The transmitter sends a vector $\mathbf{s} \in \mathbb{C}^{K \times 1}$ of $K$ symbols to the receiver where $E[\mathbf{s}\mathbf{s}^H] = \mathbf{I}_K$. The transmit signal vector becomes $\mathbf{x} = \sqrt{\frac{P_t}{\Gamma_t}} \mathbf{F} \mathbf{P}^{1/2} \mathbf{s}$, where $P_t$ is the total transmit power, $\mathbf{P} \in \mathbb{R}^{K \times K}$ is a diagonal power allocation matrix with $\sum_{k=1}^{K} P_{kk} \leq 1$, $\mathbf{F} \in \mathbb{C}^{N_t \times K}$ is the precoder matrix and $\Gamma_t = \text{trace}(\mathbf{F}^H \mathbf{F})/K$ is a normalization factor such that $\|1/\sqrt{\Gamma_t} \mathbf{F}\|^2 = K$.

Let $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ and $\mathbf{y} \in \mathbb{C}^{N_r \times 1}$ denote the normalized channel matrix and the received signal vector. Assuming the noise vector at the receiver antennas $\mathbf{z} \sim \mathcal{CN}(0_{N_r \times 1}, \sigma_z^2 \mathbf{I}_{N_r})$ has i.i.d. elements with variance $\sigma_z^2$, the channel input-output relationship is expressed...
as \( y = Hx + z \). Applying the combiner matrix \( W \in \mathbb{C}^{N_r \times K} \) at the receiver, the input \( s \in \mathbb{C}^{K \times 1} \) to the detector is

\[
\hat{s} = \sqrt{\frac{P_t}{\Gamma_t \Gamma_r}} W^H H F P^{1/2} s + \sqrt{\frac{1}{\Gamma_r}} W^H z,
\]

(3.1)

where \( \Gamma_r = \text{trace}(W^H W) / K \) is a normalization factor such that \( \|1/\sqrt{\Gamma_r} W\|^2 = K \).

The block diagram of a point-to-point communication system with a hybrid beamformer is shown in Fig. 3.1. A hybrid beamformer consists of a baseband precoder \( F_B \in \mathbb{C}^{M \times K} \) connected through \( M \) RF chains to the RF beamformer \( F_RF \in \mathbb{C}^{N_t \times M} \) such that \( F = F_RF F_B \). The elements of the RF beamformer are either analog or digital \( B \)-bit resolution phase shifters as

\[
F_{RF} = \begin{pmatrix}
e^{j\theta_{11}} & e^{j\theta_{12}} & \cdots & e^{j\theta_{1M}} \\
e^{j\theta_{21}} & e^{j\theta_{22}} & \cdots & e^{j\theta_{2M}} \\
\vdots & \vdots & \ddots & \vdots \\
e^{j\theta_{N_t1}} & e^{j\theta_{N_t2}} & \cdots & e^{j\theta_{N_tM}}
\end{pmatrix},
\]

(3.2)

\( \forall \theta_{n,m} \in \Theta, n_t \in \{1, ..., N_t\}, m \in \{1, ..., M\} \),

where \( \Theta = [0, 2\pi] \) for analog phase shifters and \( \Theta = \{0, 2\pi/2^B, ..., (2^B - 1)2\pi/2^B\} \) for digital phase shifters. For the sake of the notation simplicity, throughout the thesis it is assumed that \( n_t \in \{1, ..., N_t\}, m \in \{1, ..., M\} \) and \( k \in \{1, ..., K\} \) and the number of the RF chains at the transmitter and receiver are equal to \( M \). Similar notation is used for the hybrid beamformer at the receiver as \( W = W_{RF} W_B \) where \( W_{RF} \in \mathbb{C}^{N_t \times M} \) and \( W_B \in \mathbb{C}^{M \times K} \). Finally, the system model (3.1) for the hybrid scenario becomes

\[
\hat{s} = \sqrt{\frac{P_t}{\Gamma_t \Gamma_r}} W_B^H W_{RF}^H H F_{RF} F_B P^{1/2} s + \sqrt{\frac{1}{\Gamma_r}} W_B^H W_{RF}^H z.
\]

(3.3)

In this thesis, we derive an asymptotically optimal hybrid beamformer for a narrowband system under rich and sparse scattering channels under the assumption that \( E[\|H\|^2] = N_t N_r \). Rayleigh fading with i.i.d. elements \( H_{n,m} \sim \mathcal{CN}(0,1) \) is employed to model the rich scattering channel.

A geometry based model with \( L \ll \min(N_t, N_r) \) multipath components is applied for the sparse scattering scenario. In this case, the channel matrix is expressed as [76]

\[
H = \sqrt{\frac{N_t N_r}{L}} \sum_{l=1}^{L} \beta_l a_r(\phi_l) a_t^H(\phi_1),
\]

(3.4)
where $\beta_l \sim \mathcal{CN}(0,1)$ is the multipath coefficient, $\phi_l$ and $\phi_r$ are angle-of-departure and angle-of-arrival of the $l$th multipath. Without loss of generality, it is assumed that $|\beta_1| \geq |\beta_2| \geq \ldots \geq |\beta_L|$. The steering vector $a_u(\phi_u), \forall u \in \{t, r\}$, for linear arrays is expressed as

$$a_u(\phi_u) = \frac{1}{\sqrt{N_u}}(1, e^{j2\pi d_u \cos(\phi_u)}, \ldots, e^{j2\pi d_u(N_u - 1) \cos(\phi_u)})^T$$

where $\phi_u \in [0, \pi]$, $\lambda$ is the wavelength and $d_u$ is the antenna spacing [76]. In the rest of this thesis, it is assumed that the transmitter and the receiver are equipped with linear arrays with $d_u = \lambda/2$.

### 3.2 Problem Statement

The optimal beamforming and power allocation for a fully-digital point-to-point system is achieved by singular value decomposition (SVD) and waterfilling. The SVD factorizes the channel matrix as $H = U \Sigma V^H$ where the columns of $V \in \mathbb{C}^{N_t \times N_t}$ and $U \in \mathbb{C}^{N_r \times N_r}$ are the right and left singular vectors of $H$, and the diagonal elements of $\Sigma \in \mathbb{R}^{N_r \times N_t}$ are the singular values of $H$. For a full-ranked $H$, the capacity of the MIMO channel at high SNR grows linearly with $\min(N_t, N_r)$ when $K = \min(N_t, N_r)$ streams are transmitted over the channel [77]. When $K < \min(N_t, N_r)$, the maximum achievable rates are derived by setting the combining and precoding matrices based on thin-SVD as $W_d = U_{1,K}$ and $F_d = V_{1,K}$ [4]. In this case, $\Gamma_t = \Gamma_r = 1$ and the capacity of a point-to-point system with $K$ streams over $H$ with Gaussian entries $s_k$ is [4]

$$C = \max I(s; \hat{s}) = \max_{\text{trace}(P) \leq 1} \log_2 \det \left( I_K + \frac{P_t}{\sigma_z^2} R_n^{-1} W_d^H H F_d P F_d^H H^H W_d \right)$$

$$= \max_{\sum_{k=1}^K P_{kk} \leq 1} \sum_{k=1}^K \log_2 (1 + P_t P_{kk} \sigma_{kk}^2 / \sigma_z^2)$$

where $I(s; \hat{s})$ is the mutual information between $s$ and $\hat{s}$, $R_n = \frac{1}{\Gamma_r} W^H W = \frac{1}{\Gamma_r} W_d^H W_d = I_K$, $\sigma_{kk}^2$ are the ordered eigenvalues of $H H^H$ and the optimal $P_{kk}$ is derived by waterfilling. In this case, the capacity growth at high SNR is proportional to $K$. It should be noted that if the channel is rank-deficient it is not possible to transmit more than
When the hybrid beamformers are used, the achievable rate is expressed as \[ R = \log_2 \det \left( I_K + \frac{\rho}{\Gamma_t \Gamma_r} R_n^{-1} W_B^H W_{RF}^H H F_{RF} F_B P F_{RF}^H F_B^H H W_{RF} W_B \right), \] (3.7) where \( \rho = \frac{P_t}{\sigma_i^2} \) is a measure of link SNR.

One of the main challenges of designing hybrid beamformers is the joint design of the RF beamformers and baseband precoders/combiners considering the constant modulus constraint on the phase shifters. Designing \( F_B, F_{RF}, W_B \) and \( W_{RF} \) to maximize (3.7) is a nonconvex problem and in general it is difficult to solve [3]. Due to the similarity between the hybrid beamformer matrices at the transmitter and the receiver, same design algorithms are applicable to both sides. Hence, the discussions and derivations during this thesis are mostly focused on the hybrid beamformer at the transmitter. In this case, it is desired to maximize the mutual information \( I(s; y) \) as

\[
(F_{\text{opt}}^B, F_{\text{opt}}^RF) = \arg \max_{F_B, F_{RF}} I(s; y) = \arg \max_{F_B, F_{RF}} \log_2 \det \left( I_N + \frac{\rho}{\Gamma_t} H F_{RF} F_B P F_{RF}^H F_B^H H \right),
\] (3.8)

subject to (s.t.) \( |F_{RF,nm}| = 1 \), where \( F_{\text{opt}}^B \) and \( F_{\text{opt}}^RF \) are the optimal baseband precoding and RF beamforming matrices.

A suboptimal joint baseband and RF design based on matching pursuit was proposed to solve (3.8) for a sparse scattering channel [3]. In addition, the authors in [64] employ this beamforming technique to generate the desired beam patterns for the channel estimation purposes. The disadvantage of this hybrid beamformer is that it requires phase shifters with 6-7 bits of resolution which results in lossy phase shifters in practice. Moreover, the problem with the multiresolution channel estimation technique of [64] is that very high transmit power is needed at the initial stages. This is due to the use of wide beams with small gains. In the next section, a novel RF codebook and channel estimation technique are proposed to resolve the problems associated with [3] and [64].
3.3 Hybrid Beamforming and Channel Estimation for MmWave Systems

An RF codebook consists of a predefined set of combinations of antenna phases to shape the beam towards special direction. Letting $F_{RF}$ and $W_{RF}$ denote precoder and combiner RF codebooks, the optimization in (3.8) can be reformulated as [3]

$$
(F_{opt}^{B}, F_{opt}^{RF}) = \arg\min_{F_{B}, F_{RF}} \|F_{d} - \sqrt{\frac{1}{I_{t}}} F_{RF} F_{B}\|,
$$

(3.9)

s.t. $F_{RF} \in F_{RF}$,

s.t. $|F_{RF,nm}| = 1$.

Similarly, the combiner is derived as

$$
\min \|W_{d} - W_{RF} W_{B}\|,
$$

(3.10)

s.t. $W_{RF} \in W_{RF}$.

The system performance reported in [3, 63] noticeably degrades as the phase shifters with lower resolution are employed. This is mainly due to the inefficient design of the RF codebook. To this end, we propose an RF codebook for ULAs with large number of antenna elements and low-resolution phase shifters. For a system with $N$ antenna elements and phase shifters with $B$-bits phase shifting resolution, there can be $2^{BN}$ possible RF beamforming vectors. As an example, assuming 32 antennas and 2-bit phase shifters in place, there will be $2^{64}$ possible beamforming vectors. Therefore, it will be impractical to search such a large space.

Let $\psi_n = (2\pi/\lambda) d \cos(\phi_n), \forall n \in \{1, ..., N\}$ and $-\pi \leq \phi_n, \psi_n \leq \pi$. It is noted that $\phi_n$ represents the steering angle that the beamformer will set its beam towards that direction. In the following, the RF codebook will be designed such that the phased array antenna is capable of transmitting and receiving in the direction

$$
\psi_n \in \{-\pi + \frac{\pi(n - 1)}{N} \mid n = 1, ..., N\},
$$

(3.11)

with maximum array gain. For simplicity and without loss of generality, in this section it is assumed that $N = N_t = N_r$. More accurately, the RF beamformer at the combiner
3.3. Hybrid Beamforming and Channel Estimation for MmWave Systems

should satisfy

\[
\mathbf{f}_{\text{RF},n}^{\text{opt}} = \arg\max_{\mathbf{f}_{\text{RF},n}} \| \mathbf{f}_{\text{RF},n}^H \mathbf{a}_i(\psi_n) \|^2, \tag{3.12}
\]

\[
[\mathbf{f}_{\text{RF},n}]_i \in \left\{ \frac{1}{N} \exp\left( \frac{j2\pi b}{2B} \right) \mid b = 0, \ldots, 2^B - 1 \right\}, i = 1, \ldots, N.
\]

Hence, the precoder’s RF codebook could be expressed as \( \mathcal{F}_{\text{RF}} = \{ \mathbf{f}_{\text{RF},n}^{\text{opt}} \mid n = 1, 2, \ldots, N \} \). Similarly, the combiner’s RF codebook could be designed such that \( \mathcal{W}_{\text{RF}} = \mathcal{F}_{\text{RF}} \). This codebook is equivalent to implementing discrete Fourier transform (DFT) matrix as the precoder and combiner instead with low-resolution phase shifters. Due to nonconvexity of the optimization problem and its inherent off-line nature, we employ GA algorithm as the optimization tool to solve (3.12).

To implement GA, initially, a random set of the phase shifter settings, called population, is generated. Each row of the matrix represents one phase shifter configuration. This population evolves over a number of generations until the radiated power in the desired direction is maximized. We employ a genetic algorithm as discussed in [43].

Fig. 3.2 shows the effect of the phase shifters’ resolution on the beam pattern for the case of \( N = 32 \) and \( B = 2 \). As shown, when the resolution of the phase shifters increases, the side lobe levels will diminish approaching the ideal scenario. However, it is worth noting that the desired main lobe gain is generally satisfactory, irrespective of the phase shifters’ resolution. In addition, Fig. 3.3 demonstrates that maximum array gain is equally achievable in any desired direction employing our proposed codebook.

Fig. 3.4 shows the impact of RF codebook on the performance of hybrid beamforming algorithm. The proposed codebook achieves near-optimal performance with simply 2-bit phase shifters. However, achieving similar performance requires 6 bits in resolution if the beamsteering space is equally divided [3].

### 3.3.1 Channel Estimation

A practical mmWave channel estimation algorithm should consider several trade-offs such as the number of the receiver RF chains, number of training steps and transmit power. In particular, if the receiver is equipped with multiple RF chains, then it can
Figure 3.2: Comparison of the phased arrays with different phase shifter resolution and GA parameters as $N = 32$, population = 100, generation = 400, selection rate = 50% and mutation rate = 1%.

Figure 3.3: Array gains towards different spatial directions $\psi_n \in \{-\pi + \pi(n - 1)/N | n = 1, ..., N\}$ with $w_{RF,n}$ based on the proposed codebook, $N = 32$, $B = 2$. 
simultaneously scan multiple angles reducing the training time. Although exhaustive search algorithms are power-efficient due to employing maximum array gain on both sides, they require large number of training steps. On the other hand, multi-resolution beamforming algorithms employ lower directivity gain at the early stages of channel estimation and narrow down the beam over multiple levels. Such a strategy was adopted in [63] as it can reduce the training time at the price of high transmit power. For example, more than 40 dBm transmit power is required for $\rho \leq -20$ dB and based on equations (42) and (43) in [63].

Therefore, we aim to design an alternative channel estimation algorithm with limited training steps and practical training power requirements. In this direction, we develop an enhanced one-sided search algorithm applicable to both analog and hybrid beamformers. The estimation algorithm consists of a two-stage handshake between $d_1$ and $d_2$ as outlined below. Fig. 3.5 schematically shows the channel estimation algorithm for $M = 4$, $N = 16$ and $L = 2$. 

Figure 3.4: Impact of the RF codebook on the performance of hybrid beamformer.
Stage I

At the first stage of channel estimation, only one antenna at the transmitter radiates an omnidirectional signal whereas the receiver will directionally scan the channel as it is shown in Fig. 3.5.a and Fig. 3.5.b. Since the receiver is equipped with $M = 4$ RF chains, it is able to simultaneously scan $M = 4$ directions per step, $k \in \{1, 2, ..., K\}$. $K = \lceil N/M \rceil$ measurements are needed to scan all $N = 16$ required directions. Let the subscript "p" represent the notation during the channel estimation. At step $k$, the measured vector $\hat{s}_p^{(k)}$ is calculated as

$$\hat{s}_p^{(k)} = \sqrt{\rho_p} W_p^{(k)^H} H F_p s_p + W_p^{(k)^H} z^{(k)},$$

(3.13)

where the combiner matrix $W_p^{(k)}$ is set as

$$W_p^{(k)} = \begin{cases} (w_{RF,(k-1)M+1}, ..., w_{RF,kM}), & k < \lceil N/M \rceil, \\ (w_{RF,(k-1)M+1}, ..., w_{RF,(k-1)M+\text{mod}(N,M)}), & k = \lceil N/M \rceil. \end{cases}$$

(3.14)

At step $K$, the processor concatenates all the measured vectors and combiner matrices into vector $\hat{s}_p = (s_p^{(1)^T}, s_p^{(2)^T}, ..., s_p^{(K)^T})^T$ and $\hat{W}_p = (W_p^{(1)^T}, W_p^{(2)^T}, ..., W_p^{(K)^T})^T$, respectively. As a result, vector $\hat{s}_p$ could be expressed as

$$\hat{s}_p = \sqrt{\rho_p} \hat{W}_p^H H F_p s_p + W_D^H \hat{z},$$

(3.15)

where $W_D = \text{diag}(W_p^{(1)}, W_p^{(2)}, ..., W_p^{(K)})$ and $\hat{z}$ is the concatenation of measured noise vectors $z^{(k)}$. Multiplying $\hat{s}_p$ by $(\hat{W}_p^H)^{-1}$

$$\tilde{s}_p = \sqrt{\rho_p} H F_p s_p + (\hat{W}_p^H)^{-1} W_D^H \hat{z}.$$  

(3.16)

It is observed that (3.16) has the form of simple array processing equation for AoA estimation. At this stage, a simple delay-and-sum AoA detection is sufficient to estimate AoAs from $\tilde{s}_p$ [78].

Stage II

At this stage, the role of the transmitter and receiver change, i.e. the device that acted as the receiver in the first stage will transmit the responding pilot signals in the
Figure 3.5: Exemplary channel estimation procedure for two MPCs (purple and brown arrows) stage I: a) d1 transmits omnidirectional b) d2 scans multiple directions, stage II: c) d2 transmits in the direction of its AoDs, d) d1 scans multiple directions.

direction of $\psi_{d2,l}$, $l = 1, ..., L$. This could be done by applying the hybrid precoding algorithm proposed in (3.9) as shown in Fig 3.5.c. Then, the receiver applies the stage I approach to detect its AoAs. After this handshake, one extra transmission is required to determine the relative values of channel gains. Once all the AoAs, AoDs and the relative path gains are derived, the channel matrix can be calculated at both sides according to (4.4).
3.3.2 Simulation results

The ergodic spectral efficiency is calculated over 2000 channel realizations will be presented in the following. We will investigate the effects of phase shifter resolution, initial transmit power as well as the number of RF chains and the number of transmit symbols on the performance of the algorithm. In order to gain an insight regarding the total amount of the transmit power, let $\rho = P/\gamma \sigma_z^2$ where $\gamma$ is the shadow fading coefficient in the channel. The path loss is calculated according to [31] assuming operation at 28 GHz:

$$\gamma[\text{dB}] = 72 + 29 \cdot 2 \log_{10}(r),$$

(3.17)

where $r$ is the distance in meters. It is assumed that the distance will vary between 5.6 m to 643 m for a multipath scenario. For the sake of simplicity, all paths have the same $\gamma$. Thermal noise power spectral density is -174 dBm/Hz and the system bandwidth is 100 MHz. Let $P_p$ dBm, $P_d$ denote the transmit power during estimation and data transmission. Then, the default system parameters are set as $P_p = 35$ dBm, $P_d = 20$ dBm, $B = 2$, $M = 4$, $N = 32$ and $L = K$; except a different value is explicitly mentioned. In addition, $\psi_{u,l}$ is uniformly distributed over $[-\pi, \pi]$.

The effect of the phase shifters’ resolution on the spectral efficiency of the hybrid system is shown in Fig. 3.6. It is observed that employing 2-bit phase shifters provides acceptable performance for $\rho \geq -40$ dB. Therefore, a higher resolution phase shifter is not generally required.

Fig. 3.7 shows how the initial transmit power affects the performance of the algorithm. It is observed that the performance of the algorithm can be improved at low $\rho$ regime by increasing the transmit power in the estimation phase. As shown, $P_p = 40$ dBm can provide very close performance to unconstrained SVD scenario, avoiding high power requirements.

Figure 3.8 presents the performance of the proposed channel estimation in the hybrid case when multiple streams are simultaneously transmitted. As shown, the estimation algorithm provides promising performance compared with unconstrained SVD, i.e. the optimal solution. The performance gap, however, widens at higher number of MPCs.
3.3. **Hybrid Beamforming and Channel Estimation for MmWave Systems**

![Graph](image)

**Figure 3.6:** Impact of the phase shifter control bits on spectral efficiency.

![Graph](image)

**Figure 3.7:** Impact of transmit power at the channel estimation stage on spectral efficiency.

Nevertheless, Fig. 3.9 shows that increasing the number of the RF chains can diminish the performance gap at higher $\rho$. 
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Figure 3.8: Impact of the number of the symbols on spectral efficiency in multi-stream scenario.

Figure 3.9: Impact of the number of the RF chains on spectral efficiency, $K = 3$. 
3.4 Hybrid Beamforming without RF Codebooks

Although the performance of the major state-of-the-art [3, 64] hybrid beamforming and channel estimation techniques are improved by introducing new RF codebook and estimation technique, these methods are only applicable to mmWave systems. For a more general channel, including Rayleigh fading, this optimization problem can be approximated as a convex problem and a joint iterative suboptimal solution was proposed in [8].

Another approach to design the hybrid beamformer is to calculate $F_{RF}^{opt}$ at the first step, and then derive the $F_{B}^{opt}$ for the effective channel $H_e = HF_{RF}$. Letting $x_B = F_B P^{1/2} s$, data-processing inequality indicates that

$$I(s; y) \overset{(a)}{\leq} I(x_B; y) \leq C. \quad (3.18)$$

where inequality (a) turns into equality when $F_B = V_e$ as $H_e = HF_{RF} = U_e \Sigma_e V_e^H$, and $P$ is derived by waterfilling. It could be concluded that max $I(s; y)$ only depends on the design of $F_{RF}$. In this case,

$$F_{RF}^{opt} = \arg \max_{F_{RF}} I(s; y)$$

$$= \arg \max_{F_{RF}} \log_2 \det \left( I_{N_e} + \frac{\rho}{\Gamma_t} HF_{RF} V_e P V_e^H F_{RF}^H H^H \right),$$

s.t. $|F_{RF,n,m}| = 1$,

where $\Gamma_t = \text{trace}(F_{RF} V_e V_e^H F_{RF}^H) / K = \text{trace}(F_{RF} F_{RF}^H) / K = N_t$. The two-stage design of $F_B$ and $F_{RF}$ has been previously studied in [9]-[11]. However, the spectral efficiency based on these works depends on numerical calculations and it is not possible to derive the closed-form expression of the performance. Based on the two-stage approach, a virtually optimal hybrid beamforming and the closed-form expression of the spectral efficiency for a point-to-point system with large number of antennas under two specific channel scenarios are presented in the following.

In this section, an asymptotically optimal hybrid beamformer that maximizes the achievable rate in (3.8) is presented. Initially, based on some basic properties of the elements of the singular vectors, it will be shown that analog phase shifters with $K = M/2$ can achieve the performance of digital beamformers. It is notable that the analysis presented for this scenario is a modification of the approach in [4]. Under this assumption
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the system is underperforming as the multiplexing gain is limited to $M/2$. In order to develop a hybrid beamforming algorithm that efficiently employs all the RF chains to transmit $K = M$ streams, some of the properties the singular vectors of $\mathbf{H}$ are investigated. Then, the hybrid beamforming solution for large antenna array systems with analog phase shifters are presented. For the case of $K < M < 2K$, a combination of the methods for $M = K$ and $M = 2K$, and its performance is discussed. When digital phase shifters are employed, a simple heuristic suboptimal solution and its performance lower-bound is presented. Finally, a discussion on the proposed method and a comparison with the state-of-the-art are provided at the end of the section.

3.4.1 Hybrid beamforming with analog phase shifters and $K = M/2$

Since $\mathbf{V}$ is a unitary matrix, $\mathbf{v}_k^H \mathbf{v}_k = \sum_{n=1}^{N_t} |V_{nk}|^2 = 1$ and $|V_{nk}| \leq 1$. Thus $|V_{nk}|$ is in the domain of the inverse cosine function. Hence,

$$|V_{nk}| e^{j \angle V_{nk}} = e^{j \angle V_{nk} \cos \left( \cos^{-1}(|V_{nk}|) \right)}$$

$$= \frac{1}{2} e^{j \angle V_{nk} + j \cos^{-1}(|V_{nk}|)} + \frac{1}{2} e^{-j \angle V_{nk} - j \cos^{-1}(|V_{nk}|)}$$

This means that two phase shifters and an adder at the RF are sufficient to produce $V_{nk}$ when $M = K$ RF chains and $2MN_t$ phase shifters are available. Alternative approach to the adders is employing $M = 2K$ RF chains and $2MN_t$ phase shifters. In this case, $1/\sqrt{\Gamma_t} \mathbf{F}_{\text{RF}}^{\text{opt}} \mathbf{F}_{\text{B}}^{\text{opt}} = \mathbf{F}_d$ is achieved by setting

$$F_{\text{RF},nk'}^{\text{opt}} = \begin{cases} e^{j \angle V_{nk} + j \cos^{-1}(|V_{nk}|)} & \text{for } k' = 2k - 1 \\ e^{j \angle V_{nk} - j \cos^{-1}(|V_{nk}|)} & \text{for } k' = 2k, \end{cases}$$

and $\mathbf{F}_{\text{B}}^{\text{opt}} = \frac{1}{2} \text{diag}(\mathbf{1}_{2 \times 1}, \ldots, \mathbf{1}_{2 \times 1})$ and $\Gamma_t = 1$. Hence, the maximum rate in (4.21) can be achieved with this design. In order to derive $\mathbf{F}_{\text{B}}^{\text{opt}}$ and $\mathbf{F}_{\text{RF}}^{\text{opt}}$ for $M = K$ scenario, further properties of the singular vectors are investigated in the following subsection.
3.4.2 Properties of the Channel Singular Vectors

The behaviors of the channel singular vectors for Rayleigh and geometry based models are presented in Theorem 1 and Proposition 1 in the following.

Theorem 1: If \( H_{n_r n_t} \sim \mathcal{CN}(0, 1) \) are i.i.d. and \( N_t \to \infty \), \( N_r \to \infty \), then the elements of the singular vectors of \( H \) are i.i.d and follow \( \sqrt{N_t} V_{n_t n_t'} \), \( \sqrt{N_r} U_{n_r n_r'} \sim \mathcal{CN}(0, 1) \), \( \forall n_t, n_t' \in \{1, \ldots, N_t\} \) and \( \forall n_r, n_r' \in \{1, \ldots, N_r\} \).

Proof: It is known that the left and right singular vectors of \( H = U \Sigma V^H \) are uniformly distributed on a complex \( N_t \)-hypersphere and a \( N_r \)-hypersphere with radius 1 [80]. As a result, \( \sqrt{N_t} v_{n_t} \) and \( \sqrt{N_r} u_{n_r} \) are uniformly distributed on the surface of a \( N_t \) and \( N_r \) dimensional hyperspheres with radius \( \sqrt{N_t} \) and \( \sqrt{N_r} \). Moreover, the coordinates of a randomly chosen point according to a uniform distribution on an \( N \)-hypersphere of radius \( \sqrt{N} \) are i.i.d. with \( \mathcal{CN}(0, 1) \) when \( N \to \infty \) [81]. Hence, the elements of \( \sqrt{N_t} v_{n_t} \) and \( \sqrt{N_r} u_{n_r} \) are i.i.d. with \( \mathcal{CN}(0, 1) \).

Remark: As far as the authors are aware, the distribution of the elements of the singular vectors of matrix \( H \), when \( H_{n_r n_t} \sim \mathcal{CN}(0, 1) \) for \( N_t \to \infty \) and \( N_r \to \infty \), has not been previously reported in the literature, although the pieces of the proof have been available for a long time and they have been studied by different researcher such as Love and Spruill [80, 81].

The real and imaginary parts of random variables with \( \mathcal{CN}(0, 1) \) are distributed as \( \mathcal{RN}(0, \frac{1}{2}) \) [76]. Hence, \( \sqrt{N_t} |V_{n_t k}| \) has a Rayleigh distribution with parameter \( \sigma_R = \frac{1}{\sqrt{2}} \) and its expected value is \( \sigma_R \sqrt{\frac{2}{\pi}} \) [82]. Fig. 3.10 shows that the Rayleigh distribution can provide a good approximation even for a finite \( N_t \in \{16, 64\} \). The properties of the sparse scattering channels are described in the following Proposition.

Proposition 1 [12]: For a geometry based channel model with \( N_t \to \infty \) and \( N_r \to \infty \), the relationship between the singular and steering vectors is expressed as \( v_l = a_t(\phi_{l t}) \) and \( u_l = a_r(\phi_{l r}) \), \( \forall l \in \{1, \ldots, L\} \).
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Figure 3.10: Comparison between the probability density function (PDF) of $\sqrt{N_t} |V_{n,k}|$ when it follows Rayleigh distribution with parameter $\sigma_R = \frac{1}{\sqrt{2}}$, simulation results for the PDF of $\sqrt{N_t} |V_{n,k}|$ for a Rayleigh fading channel over 1000 realizations with $N_t = N_r = 16$ and $N_t = N_r = 64$.

3.4.3 Hybrid Beamforming for $M = K$ Scenario

The proposed hybrid beamformer when $M = K$ is presented in the following Proposition.

Proposition 2: The asymptotically optimal solution $F_{RF}^{opt}$ and $W_{RF}^{opt}$ to the optimizations in (3.7), (3.8) and (3.19) for large $N_t$ and $N_r$ with $M = K$ and analog phase shifters is $F_{RF,nk}^{opt} = e^{j\angle V_{n,k}}$, $W_{RF,nk}^{opt} = e^{j\angle U_{n,k}}$. In this case, the baseband precoder and combiner matrices become $F_B = W_B = I_K$.

Proof: Defining the positive semidefinite matrices $Q, Q \in \mathbb{C}^{N_t \times N_t}$ as $Q = \frac{1}{N_t} F_{RF} F_B P F_B^H F_{RF}^H$
and $\tilde{Q} = V^H Q V$, the mutual information $I(s, y)$ is expressed as

$$I(s, y) = \log_2 \det \left( I_{N_t} + \rho HQH^H \right)$$

$$= \log_2 \det \left( I_{N_t} + \rho U\Sigma V^H Q V \Sigma^H U^H \right)$$

$$= \log_2 \det \left( I_{N_t} + \rho \Sigma \tilde{Q} \Sigma^H \right)$$

$$= \log_2 \det \left( I_{N_t} + \rho \Sigma^H \Sigma \tilde{Q} \right)$$

$$\leq \log_2 \left( \prod_{n_t=1}^{N_t} \left( 1 + \rho \sigma^2_{n_t} \tilde{Q}_{n_t} \right) \right), \quad (b) \quad (3.22)$$

where $\sigma^2_{n_t}$ are the diagonal elements of $\Sigma^H \Sigma$, and the inequality $(b)$ comes from linear algebra as for any positive semidefinite matrix $A \in \mathbb{C}^{N_t \times N_t}$, $\det(A) \leq \prod_{n_t} A_{n_t,n_t}$. If $\tilde{Q}$ is a diagonal matrix, then $(b)$ in (3.22) turns into equality. Hence, the objective is to design $F_{RF}$ and $F_B$ such that they can diagonalize $\tilde{Q}$. In order to analyze $\tilde{Q}$, we investigate the behavior of the elements of $G \in \mathbb{C}^{N_t \times K}$, define as $G = 1/\sqrt{N_t} V^H F_{RF}$, when $\frac{1}{\sqrt{N_t}} v^H_{n_t} f_{RF,k} = 0$ and $\frac{1}{\sqrt{N_t}} v^H_{n_t} f_{RF,k} \neq 0, \forall n_t \neq k$. In the first case that $\frac{1}{\sqrt{N_t}} v^H_{n_t} f_{RF,k} = 0 \forall n_t \neq k^*$, it could be easily shown that all of the elements of $G$ except the $G_{kk}$ become zero. Then, the last term in (3.22) can be written as

$$\log_2 \left( \prod_{n_t=1}^{N_t} \left( 1 + \rho \sigma^2_{n_t} \tilde{Q}_{n_t} \right) \right) = \log_2 \left( \prod_{k=1}^{K} (1 + \rho \sigma^2_{kk} \tilde{Q}_{kk}) \right). \quad (3.23)$$

On the other hand, if $F_B$ is a diagonal matrix, then $F_B P F_B^H$ will have the same property. As a result, $\tilde{Q} = G F_B P F_B^H G^H$ will also become a diagonal matrix since off-diagonal elements of $G$ are zero. In addition, in (3.19), it was discussed that $F_B$ should be a unitary matrix to maximize the spectral efficiency. As $F_B$ is a diagonal and a unitary matrix, it could be concluded that $|F_{B,kk}|^2 = 1$. In this case, $\tilde{Q}_{kk}$ becomes

$$P_{kk} |G_{kk}|^2 = \frac{P_{kk}}{N_t} |v^H_k f_{RF,k}|^2 = \frac{P_{kk}}{N_t} \sum_{n_t=1}^{N_t} V^*_{n_t k} \epsilon^{j \theta_{n_t k}} |V_{n_t k}|^2$$

$$\leq \frac{P_{kk}}{N_t} \sum_{n_t=1}^{N_t} |V_{n_t k}|^2, \quad \text{where the left hand side of (c) is maximized when all the elements of } v_k \text{ are added constructively. In other words, (c) in (3.24) turns into equality if } F_{RF} = F_{RF}^{opt.1} \text{ as}$$

$$F_{RF,n_t k}^{opt.1} = e^{j \angle V_{n_t k}}. \quad (3.25)$$
In the following, we analyze the impact of setting $F_{RF} = F_{RF}^{\text{opt}1}$ on the off-diagonal elements of $G$ for rich and sparse scattering channels. For the Rayleigh channel, Theorem 1 expresses that the elements of singular vectors of the channel matrix are zero-mean i.i.d. random variables and their phases are uniformly distributed over $[0, 2\pi]$. As a consequence of law of large numbers

$$
\lim_{N_t \to \infty} \frac{1}{\sqrt{N_t}} V_{n_t}^H f_{RF,k} = \lim_{N_t \to \infty} \frac{1}{\sqrt{N_t}} \sum_{n_t' = 1}^{N_t} \sqrt{N_t} V_{n_t' n_t}^* e^{i \angle V_{n_t' k}} = E[\sqrt{N_t} V_{n_t k}] = 0,
$$

for $n_t \neq k$. For the geometry based model, Proposition 1 states that the RF precoder in (3.25) becomes $f_{RF,k} = \sqrt{N_t} v_k$, hence $1/\sqrt{N_t} V_{n_t}^H f_{RF,k} = v_{n_t}^H v_k = 0, \forall n_t \neq k$. As a result, it could be concluded that all of the elements of $G$ except the diagonal elements become zero for both channels, when $F_{RF} = F_{RF}^{\text{opt}1}$. As a result, the choice of $F_{RF} = F_{RF}^{\text{opt}1}$ and a diagonal $F_B$, with $|F_{B, kk}|^2 = 1$ imposes (b) in (3.22) to turn into equality. Finally, $I(s, y)$ is maximized when the diagonal matrix $P$ is calculated based on waterfilling.

It could be easily shown that when the hybrid beamformer at the receiver is also considered, by applying a similar RF beamformer at the receiver, $\sqrt{\frac{1}{N_t} W_{RF}^H H F_{RF}^{\text{opt}}}$ becomes a diagonal matrix for both channels. In addition, $W_B$ will have a similar structure to $F_B$. Hence, $F_B = W_B = I_K$, $\Gamma_t = N_t$, $\Gamma_r = N_r$ is the capacity achieving hybrid beamformer for both channels. \hfill \Box

It was previously shown that for the geometry based channel models, (3.8) could be approximated by (5.3) which is equivalent to minimizing the Euclidean distance between $F_d$ and $1/\sqrt{N_t} F_{RF} F_B$ [3]. It is noted that the proposed RF beamformer of Proposition 2 can be alternatively derived by

$$
\text{minimize}_{F_{RF}} \| \sqrt{\frac{1}{N_t}} F_{RF} - F_d \|^2, \quad \text{s.t.} \quad |F_{RF, n_t k}| = 1.
$$

**Proof:** Since $V$ is a unitary matrix, $\| \sqrt{\frac{1}{N_t}} F_{RF} - F_d \|^2 = \| \sqrt{\frac{1}{N_t}} V^H F_{RF} - V^H F_d \|^2$. It could be easily verified that

$$
\| \sqrt{\frac{1}{N_t}} V^H F_{RF} - V^H F_d \|^2 \geq \| \sqrt{\frac{1}{N_t}} F_{RF}^H F_{RF} - I_K \|^2,
$$

s.t. $|F_{RF, n_t k}|^2 = 1$. \hfill (3.28)
The right hand side of the inequality can be reformulated as
\[
\min_{\mathbf{F}_{\text{RF}}} \sum_{k=1}^{K} \left| \frac{1}{N_t} \mathbf{f}_{d,k}^H \mathbf{f}_{\text{RF},k} - 1 \right|^2 + \sum_{k'=1}^{K} \sum_{k=1}^{K} \left| \frac{1}{N_t} \mathbf{f}_{d,k}^H \mathbf{f}_{\text{RF},k'} \right|^2, \quad (3.29)
\]
s.t. \(|\mathbf{F}_{\text{RF},nk}|^2 = 1\).

The cost function can be lower-bounded as
\[
\min \left( \sum_{k=1}^{K} \left| \frac{1}{N_t} \mathbf{f}_{d,k}^H \mathbf{f}_{\text{RF},k} - 1 \right|^2 + \sum_{k'=1}^{K} \sum_{k=1}^{K} \left| \frac{1}{N_t} \mathbf{f}_{d,k}^H \mathbf{f}_{\text{RF},k'} \right|^2 \right) \quad (3.30)
\]
\[
= \min \left( \sum_{k=1}^{K} \left| \frac{1}{N_t} \mathbf{f}_{d,k}^H \mathbf{f}_{\text{RF},k} - 1 \right|^2 \right) + \min \left( \sum_{k'=1}^{K} \sum_{k=1}^{K} \left| \frac{1}{N_t} \mathbf{f}_{d,k}^H \mathbf{f}_{\text{RF},k'} \right|^2 \right)
\]
\[
\geq \min \left( \sum_{k=1}^{K} \left| \frac{1}{N_t} \mathbf{f}_{d,k}^H \mathbf{f}_{\text{RF},k} - 1 \right|^2 \right) + \min \left( \sum_{k'=1}^{K} \sum_{k=1}^{K} \left| \frac{1}{N_t} \mathbf{f}_{d,k}^H \mathbf{f}_{\text{RF},k'} \right|^2 \right)
\]
\[
\geq \sum_{k=1}^{K} \min \left( \left| \frac{1}{N_t} \mathbf{f}_{d,k}^H \mathbf{f}_{\text{RF},k} - 1 \right|^2 \right) = \sum_{k=1}^{K} \left( \max \left( \left| \frac{1}{N_t} \mathbf{f}_{d,k}^H \mathbf{f}_{\text{RF},k} \right| - 1 \right) \right)^2, \quad (f)
\]
where \((f)\) comes from the fact that \(\left| \frac{1}{N_t} \mathbf{f}_{d,k}^H \mathbf{f}_{\text{RF},k} \right| \leq 1\). Hence, the last term in (3.30) is minimized if
\[
\max_{\mathbf{f}_{\text{RF},k}} \left| \frac{1}{N_t} \mathbf{f}_{d,k}^H \mathbf{f}_{\text{RF},k} \right|, \quad \text{s.t.} \ |\mathbf{F}_{\text{RF},nk}|^2 = 1, \quad (3.31)
\]
which is similar to (3.24) in the proof of Proposition 2. It was shown that \(\left| \frac{1}{N_t} \mathbf{f}_{d,k}^H \mathbf{f}_{\text{RF},k'} \right| = 0, \forall k \neq k'\) and \(\frac{1}{N_t} \mathbf{f}_{d,k}^H \mathbf{f}_{\text{RF},k}\) becomes a real and positive number when \(\mathbf{F}_{\text{RF},nk} = e^{j\angle \mathbf{F}_{\text{RF},nk}}\). Hence, \((e)\) and \((d)\) turn into equality, and the cost function in (3.29) is minimized. Finally, (3.28) turns into equality and \(\|\mathbf{F}_{\text{RF}} - \mathbf{F}_{\text{d}}\|^2\) is minimized. \(\square\)

In order to implement the hybrid beamformer of Proposition 2, the first \(K\) singular vectors and values of \(\mathbf{H}\) should be initially calculated. Then, each phase shifter at the transmitter and the receiver is directly set to the phase of the corresponding element in the right and left singular vectors, respectively. Considering the impact of the RF beamformers, the baseband precoder and combiner matrices are equal to an identity matrix. Finally, the optimal allocated power to each symbol is derived by waterfilling.

The performance of the proposed hybrid beamformer compared to \(C\) in (4.21) for \(M = K\) and Rayleigh fading channel is expressed in the following Proposition.
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Proposition 3: For large $N_t$ and $N_r$ and at high SNR regime, the difference between the maximum rate $C$ from (4.21) and the rate $R_C$ achieved by the beamforming scheme of Proposition 2 for a Rayleigh channel is expressed as

$$C - R_C = -2K \log_2 \left( \frac{\pi}{4} \right).$$

(3.32)

Proof: As a result of Theorem 1, $E[\sqrt{N_t} | V_{n,k} |] = E[\sqrt{N_r} | U_{n,k} |] = \frac{\sqrt{\pi}}{2}$, and hence,

$$\lim_{N_t \to \infty} \frac{1}{\sqrt{N_t}} | V_{k}^{\text{opt}}_{RF,k} | = \lim_{N_r \to \infty} \frac{1}{\sqrt{N_r}} | U_{k}^{\text{opt}}_{RF,k} | = \lim_{N_t \to \infty} \frac{1}{N_t} \sum_{n=1}^{N_t} | \sqrt{N_t} V_{n,k} | = \frac{\sqrt{\pi}}{2}.$$

(3.33)

Referring to the matrix $G = 1/\sqrt{N_t} V^{H} F_{RF}$ in the proof of Proposition 2, $G_{kk} = \sqrt{\pi}/2$ and $G_{n,k} = 0, \forall n \neq k$. Applying a similar RF beamformer at the receiver side, it could be easily verified that $R_n = 1/N_r W_{RF} H W_{RF} = I_K$. The spectral efficiency in (3.7) at high SNR becomes

$$R_C = \lim_{N_t \to \infty} \lim_{N_r \to \infty} \log_2 \det \left( \frac{\rho}{N_t N_r} R_n^{-1} W_{B}^{H} W_{RF}^{H} H F B F^{H} P_{RF} H W_{RF} W_{B} \right)$$

(3.34)

$$= \lim_{N_t \to \infty} \lim_{N_r \to \infty} \log_2 \det \left( \frac{\rho}{N_t N_r} W_{RF}^{H} U \Sigma \Sigma^{H} F_{RF} P_{RF}^{H} V \Sigma^{H} W_{RF} \right)$$

$$= \log_2 \det \left( \left( \frac{\pi}{4} \right)^2 \rho \Sigma^{H} P \right) = \sum_{k=1}^{K} \log_2 (\left( \frac{\pi}{4} \right)^2 \rho P_{kk} \sigma_{kk}^2)$$

$$= \sum_{k=1}^{K} \log_2 (\rho P_{kk} \sigma_{kk}^2) + 2K \log_2 \left( \frac{\pi}{4} \right),$$

where $\Sigma' = \text{diag}(\sigma_{1}^2, ..., \sigma_{K}^2)$. Considering that the first term in the last line is $C$ in (4.21), the Proposition is proved. □

Proposition 3 indicates that the spectral efficiency achieved by the digital beamformers is $0.7K$ bits/Hz/s more than the performance of the hybrid beamformers when the number of antennas are large and the channel is modeled by Rayleigh fading.

When $K < M < 2K$, two RF chains are used per symbol to transmit in the direction of the singular vectors corresponding to the first $M - K$ singular values of the channel. In this case, the hybrid beamformer of $M = 2K$ is used. For the remaining $2K - M$ symbols, the hybrid beamformer of Proposition 2 is used. That is $2(M - K)$ RF chains
are used to transmit \( M - K \) symbols and each of the remaining \( 2K - M \) symbols are transmitted on one of the remaining RF chains. For example, assuming that \( K = 3 \) and \( M = 5 \), the baseband precoder becomes \( \mathbf{F}_R^{opt} = \text{diag}(\frac{1}{2} \mathbf{1}_{2\times1}, \frac{1}{2} \mathbf{1}_{2\times1}, 1) \). Then, (3.21) is used to design the the RF beamforming vectors \( \mathbf{f}_{RF,1}, \mathbf{f}_{RF,2} \) according to \( \mathbf{v}_1 \), and \( \mathbf{f}_{RF,3}, \mathbf{f}_{RF,4} \) based on \( \mathbf{v}_2 \). Finally, \( \mathbf{f}_{RF,5} \) is adjusted based on \( \mathbf{v}_3 \) and Proposition 2.

Similar approach can be also applied at the receiver side. In a general scenario that \( K \leq M \leq 2K \), by following similar approaches as in the proof of Lemmas 2 and 3, it can be easily verified that (3.32) becomes

\[
C - R_C = -2(2K - M) \log_2(\frac{\pi}{4}).
\]

For example, letting \( K = 3 \) and \( M = 5 \), then \( C - R_C = -2 \log_2(\frac{\pi}{4}) \). It should be noted that adding an extra RF chain at each side can increase the spectral efficiency by \( - \log_2(\frac{\pi}{4}) \). However, this improvement will also increase the system cost, complexity and power consumption.

For a geometry based channel, the singular vectors and the steering vectors become equal and the proposed algorithm will be translated into steering the beams towards the channel multipath components as proposed in [12]. Following a similar approach as in the proof of Proposition 3, it can be easily shown that \( 1/\sqrt{N_t} \mathbf{a}_t(\phi_{i,k})^H \mathbf{f}_{RF} = 1 \) and \( C - R_C = 0 \). Hence, extra RF chains \( M - K > 0 \) will not improve the performance in such channels.

### 3.4.4 Digital Phase Shifters

Another challenge for designing hybrid beamformers is the discrete resolution of the phase shifters. When \( B \)-bit resolution phase shifters are employed, the search space for the optimum set of phases becomes \( 2^{BMN_t} \) which can be very large for large \( N_t \). As an example, when there are \( N_t = 64 \), \( M = 4 \) and 2-bit resolution phase shifters, there are \( 2^{512} \) possible phase combination which is computationally expensive to search in the real-time applications. One way out is the use a predefined set of phases known as RF codebooks [3]. The disadvantage of the RF codebooks is that they are usually designed for a fixed type of channel such as sparse channels. The alternative approach to design
the RF beamformer with discrete resolution phase shifters is rounding the phases as

\[
\theta_{n_t k}^d = \arg \min_{\theta_{n_t k}} |\angle F_{d,n_t k} - \theta_{n_t k}|,
\]

s.t. \(\theta_{n_t k} \in \{0, \ldots, (2^B - 1)2\pi/2^B\}\),

where \(\theta_{n_t k}^d\) is the phase of \(F_{RF,n_t k}\). The lower-bound on the rate loss with this design is provided in the following Proposition.

**Proposition 4**: The gap between \(R_C\) and the achievable rate \(R_D\) by the hybrid beamformer based on (4.27) with \(B\)-bit resolution digital phase shifters is bounded as

\[
R_C - R_D \leq -K \log_2 \left( \cos^4 \left( \frac{2\pi}{2^{B+1}} \right) \right). \tag{3.37}
\]

**Proof**: In the proof of Proposition 2, it was shown that the achievable rate depends on

\[
\frac{1}{\sqrt{N_t}} |v_k^H f_{RF,k}|. \quad \text{Letting } \delta_{n_t k} = \theta_{n_t k}^d - \angle F_{d,n_t k} \text{ where } \frac{-2\pi}{2^{B+1}} \leq \delta_{n_t k} \leq \frac{2\pi}{2^{B+1}},
\]

\[
\frac{1}{\sqrt{N_t}} |v_k^H f_{RF,k}| = \frac{1}{\sqrt{N_t}} \left| \sum_{n_t=1}^{N_t} |V_{n_t k}| e^{-j\angle V_{n_t k} e^{j\theta_{n_t k}^d}} \right| = \frac{1}{\sqrt{N_t}} \left| \sum_{n_t=1}^{N_t} |V_{n_t k}| e^{j\delta_{n_t k}} \right| \quad \tag{3.38}
\]

\[
= \frac{1}{\sqrt{N_t}} \left| \sum_{n_t=1}^{N_t} |V_{n_t k}| \left( \cos(\delta_{n_t k}) + j \sin(\delta_{n_t k}) \right) \right| \geq \frac{1}{\sqrt{N_t}} \left| \sum_{n_t=1}^{N_t} |V_{n_t k}| \cos(\delta_{n_t k}) \right| \geq \frac{1}{\sqrt{N_t}} \cos \left( \frac{2\pi}{2^{B+1}} \right) \sum_{n_t=1}^{N_t} |V_{n_t k}|.
\]

It could be shown that \(1/\sqrt{N_t} |v_k^H f_{RF,k}| = 0 \forall k \neq k'\) holds for both channel models. Following a similar approach as in the proof of Proposition 3, the rest of the proof is straightforward. \(\square\)

Proposition 4 indicates that hybrid beamformers with analog phase shifters can achieve maximum \(0.45K\) bits/Hz/s higher spectral efficiency compared to the scenario that digital phase shifters with \(B = 3\) are employed. As hybrid beamformers target the transmission of a small number of symbols, the gains achieved by using analog phase shifters are negligible at high SNR regime. In addition, the low cost and computational complexity of the proposed scheme in (4.27) makes it an effective approach for practical applications.
3.4.5 Discussion and Comparison with the State-of-the-Art

In this paper, the analytical discussions are focused on asymptotically large antenna arrays. This is in contrast to the works in [3, 8, 9, 11] where the analysis are presented for limited number of antennas. The advantages of considering asymptotically large arrays are two-fold. Firstly, it facilitates the analysis to derive the virtually optimal hybrid beamformer and the closed-forms for the achievable spectral efficiency. Secondly, as it will be shown in section VII, the simulation results indicate that the analysis for the asymptotically large array scenario provides a reliable estimate of achievable performance for scenarios with limited number of antennas.

One of the common approaches in the literature is to decompose the unconstrained thin-SVD based beamformer matrix into RF beamformer and baseband precoder matrices, [3, 8]. The computational complexity of the rank-\(M\) thin-SVD of \(H\) is \(O(N_t N_r M)\) for \(M \ll \sqrt{N_t N_r}\) [83]. The state-of-the-art hybrid beamformers that require a second round of computations to decompose \(F_d\) into \(F^{\text{opt}}_B\) and \(F^{\text{opt}}_{\text{RF}}\) can cause high computational delay and complexity [3]. An iterative algorithm can be used to solve the optimization problem in (3.8), however, the iterative algorithm renders a high computational cost and delay [8, 9, 11]. For example, the complexity of the hybrid beamformer in [10] is \(O(\max(N_t, N_r)^2 \min(N_t, N_r))\).

Compared to the state-of-the-art, the proposed hybrid beamformer of Proposition 2 is faster and it is virtually the optimal scheme for the systems with large \(N_t, N_r\) operating in Rayleigh and sparse channels. The computational complexity of the proposed scheme is equal to the complexity of rank-\(M\) thin-SVD as \(O(N_t N_r M)\). In addition, the closed-form expressions of the achievable rates are derived which to best of the authors’ knowledge was not previously reported.

In this following, the performance of the proposed hybrid beamforming schemes for the point-to-point system operating in rich and sparse scattering channels is evaluated by Monte-Carlo simulations. The performance metric is average spectral efficiency over 1000 independent channel realizations and it is assumed that \(M = K = 4\). In this paper, all the closed-form expressions were derived for the scenario that \(N_t, N_r \rightarrow \infty\). To obtain the appropriate assumption on the number of the antenna elements for the simulations,
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the behavior of the hybrid beamformer with respect to $N_t$ and $N_r$ is first analyzed. In the following, the superscript "A" is used to distinguish the analytical results of the Lemmas 2, 3 and 4 from the performance of the proposed schemes derived by the Monte-Carlo simulations. For example, the analytical spectral efficiency by the hybrid beamformer in Proposition 2 is expressed as $R^A_C = C - \Delta_C$ where $\Delta_C = \lim_{N_t,N_r \to \infty} C - R_C$. The performance of the hybrid beamformer of Proposition 2 with the closed-form expression of Proposition 3 are investigated with respect to the number of antennas, where it is assumed that $N_t = N_r$ and then $N_r = 8$ for $N_t \in \{8, 16, 32, 64, 128, 256, 512\}$. Figure 3.11 shows this performance $R^A_C$ compared to the simulation result, $R_C$, for the Rayleigh fading channel whereas Fig. 3.12 presents these for the geometry based model. It is observed that $R^A_C$ and $R_C$ converge for both channels when $N_t = N_r$ is large. For the Rayleigh fading channel, $R^A_C$ predicts slightly lower spectral efficiency compared to the results from simulations $R_C$ when $N_r = 8$ as shown in Fig. 3.11. On the other hand, it is observed from Fig. 3.12 that $R^A_C$ is always larger than $R_C$ for the geometry based model as for this channel $C = R^A_C$. 

Figure 3.11: $C, R_C, R^A_C$ when the number of the antennas varies, $\rho = 34$ dB and Rayleigh channel.
3.4. Hybrid Beamforming without RF Codebooks

Figure 3.12: $C$, $R_C$, $R_C^A$ when the number of the antennas varies, $\rho = 34$ dB and geometry based channel with $L = 5$.

Figure 3.13: Spectral efficiency achieved the hybrid beamformer with digital phase shifters based on (4.27) $R_D$, compared to the bound based on Proposition 4 $R_D^A$, $R_C$ and $C$ for Rayleigh channel.
Figure 3.14: Spectral efficiency achieved the hybrid beamformer with digital phase shifters based on (4.27) $R_D$, compared to the bound based on Proposition 4 $R^A_D$, $R_C$ and $C$ for geometry based channel with $L = 5$.

Figure 3.13 shows the performance of the hybrid beamformer with digital phase shifters, denoted as $R_D$, for a point-to-point system operating in rich scattering channel. It is observed that $R_C - R_D$ for $B = 2$ and $B = 3$ is 3.5 and 0.7 bits/s/Hz which is negligible compared to the high spectral efficiency achieved by large antenna arrays at high SNR. Hence, a simple rounding technique to set the discrete phases of the phase shifters with $B \geq 3$ can significantly simplify the calculations, and achieve a similar performance as analog phase shifters are employed. In addition, the lower-bound of the spectral efficiency based on Proposition 4, denoted as $R^A_D$, provides a good approximation when $B \geq 3$. For example, when $B = 3$, $R_D - R^A_D$ is 1.2 bits/s/Hz. Figure 3.14 presents a similar result for the sparse scattering channel.

Figure 3.15 and Fig. 3.16 show the performance of the proposed algorithm in Proposition 2 compared to the state-of-the-art [3, 8, 10] for Rayleigh and geometry based channels. It is observed that the algorithm of [3] is not applicable to the Rayleigh fading channel, although it has a very good performance for the sparse scattering channel.
3.4. Hybrid Beamforming without RF Codebooks

Figure 3.15: Spectral efficiency achieved by the proposed algorithm compared to the state-of-the-art [3, 8, 10] when the wireless channel follows Rayleigh fading.

Figure 3.16: Spectral efficiency achieved by the proposed algorithm compared to the state-of-the-art [3, 8, 10] when the wireless channel follows geometry based model with $L = 5$. 
The performance of the iterative algorithms of [8] and [10] is similar to the proposed scheme for both channels.

3.5 Summary and Conclusion

In this work, we derived a practical RF codebook, based on GA, for analog and hybrid beamforming schemes where the large phased array antenna is equipped with low-resolution phase shifters. As shown, the optimized codebook achieves a promising performance in this practical configuration as it can reach maximum array gain in any desired direction. In addition, we proposed a low complexity channel estimation algorithm which is based on an enhanced one-sided search and is capable of estimating AoAs and AoDs with a high accuracy, limited power and limited number of measurements. We also investigated the trade-offs between different design parameters such as the resolution of the phase shifters, number of the RF chains and the required power for the channel estimation phase.

Moreover, we derived the asymptotically optimal hybrid beamforming schemes to maximize the spectral efficiency for the point-to-point system with large antenna arrays, operating in rich and sparse scattering channels. The optimality of the solution was proved based on the properties of the singular vectors of the channel matrix. The elements of these vectors have a complex Gaussian distribution for Rayleigh fading model, and the singular vectors are equal to the steering vectors of the channel matrix for the geometry based model. In addition, we derived the closed-form expressions for the spectral efficiency when the proposed hybrid beamformer is used. It was shown that the performance of the hybrid beamformer, employing phase shifters with resolution more than 2-bits, can approach the performance of a similar system with analog phase shifters.
Chapter 4

Hybrid Beamforming for Multiuser Scenario

In the previous chapters, it was discussed that the wireless channels tend to have favorable conditions when the number of the base station antennas grows large. Using the asymptotic behaviors of the channel matrix, the asymptotically optimal hybrid beamformer for the point-to-point multiple-input multiple-output (MIMO) system was derived. In this scenario, the capacity of the channel is achieved when the transmission scheme is based on singular value decomposition (SVD) and waterfilling. In the multiuser scenario, however, calculation of SVD on both side is not feasible as there is no collaboration between the user equipment.

In general, the capacity of the broadcast channels is derived by dirty paper coding which is difficult to implement [77]. Hence, in practice the suboptimal linear precoding algorithms with low complexity such as zero-forcing (ZF) are preferred. It has been shown that the performance of ZF converges to optimal sum-capacity for the Rayleigh channel when the number of the base station antennas goes large [2].

In this chapter, the spectral efficiency achieved by the hybrid beamforming in multiuser scenarios over rich and sparse scattering channels will be investigated. Similar to the single-user case, the properties of the elements of the singular vectors of the channels will be applied to preserve the singular of values of the channel matrix. In the following,
Chapter 4. Hybrid Beamforming for Multiuser Scenario

the performance of the hybrid beamforming for the scenarios that user equipment have a single-antenna as well as multiple-antennas will be presented.

For the single-antenna case, the closed-form of the performance bound is presented. It is shown that the hybrid beamformer almost achieves the spectral efficiency of a digital system. Multiantenna multiuser hybrid beamforming is studied in [15]. However, it was assumed that each user is equipped with a single RF chains and an antenna array. Hence, multistreaming to that terminal is not possible. Equipping the user devices with multiple RF chains, a combination of hybrid beamforming and block diagonalization for mmWave systems was proposed [56,57]. For this scenario, firstly a modification of block diagonalization technique to enhance the SNRs at the receivers will be presented. In this method, the singular vectors of the channel matrix are used to reduce its dimensions while its singular values are preserved. Then, the RF beamforming technique of chapter 3 and block diagonalization are employed to include the phase shifter constraints and support multiantenna multiuser transmission. It is shown that the performance of the proposed hybrid technique is similar to the sum-rate achieved by traditional digital block diagonalization. The simulation results show that this technique can almost achieve the performance of a MIMO system where all the receiver antennas can collaborate in sharing channel state information (CSI) and signal processing. In addition, the performance of the system in both rich and sparse scattering channels is investigated. Finally, it is shown that the achievable sum-rate by the proposed algorithm with 3-bits resolution phase shifters is almost the same as its performance with analog phase shifters.

In the following, firstly the system model for the downlink multiuser scenario will be presented. Then, the performance of the hybrid precoding in downlink for the single-antenna and multiantenna scenarios are presented, respectively.

4.1 System Model

Consider a $K$-user downlink massive MIMO scenario. The number of the antennas at the base station and users are denoted as $N_t$ and $N_r$, respectively. Let $s_k \in \mathbb{C}^{N_r \times 1}$, $\forall k \in \{1, ..., K\}$, represent the vector of the symbols that the base station transmits to user $k$. 
The base station uses $F_k \in \mathbb{C}^{N_t \times N_r}$ to precode $s_k$ and create the transmit signal vector $x \in \mathbb{C}^{N_t \times 1}$ as

$$x = \sqrt{\frac{P_t}{\Gamma_t}} \sum_{k=1}^{K} F_k s_k,$$

(4.1)

where $\Gamma_t$ is a power normalization factor to keep the total transmit power to $P_t$. More precisely,

$$\Gamma_t = \mathbb{E} \left[ \text{trace} \left( F s s^H F^H \right) \right] = \mathbb{E} \left[ \text{trace} \left( F F^H \right) \right] / K,$$

(4.2)

where $F = [F_1 \ldots F_K]$. Transmitting $x$ over the narrowband wireless channel, the received signal vector $y_k \in \mathbb{C}^{N_r \times 1}$ at the antennas of user $k$ is

$$y_k = \sqrt{\frac{P_t}{\Gamma_t}} \sum_{k'=1}^{K} H_k F_k s_{k'} + z_k,$$

(4.3)

where $H_k \in \mathbb{C}^{N_r \times N_t}$ and $z_k \in \mathbb{C}^{N_r \times 1}$ are the channel matrix and receiver noise vector. It is assumed that the elements of $z_k \sim \mathcal{CN}(0, \sigma^2_z)$ are uncorrelated independent and identically distributed (i.i.d.) random variables where $\sigma^2_z$ is the noise variance.

At receiver $k$, the combiner matrix $W_k \in \mathbb{C}^{N_r \times N_r}$ is used to compensate the transmit precoding and channel effects. Then, vector $\hat{s}_k \in \mathbb{C}^{N_r \times 1}$, where $\hat{s}_k = W_k^H H_k x + W_k^H z_k$, is passed to the receiver detector.

In general, the performance of the beamformers depends on the availability of the knowledge of CSI at the transmitter. In order to evaluate the sum-rate achieved by our proposed beamformer and compare it to the capacity of the channel, it is assumed that perfect CSI is available at the transmitter.

In this work, the system performance is investigated over rich and sparse scattering channels. For rich scattering channel, we use Rayleigh fading model where the elements of $H_k$ are i.i.d. and $H_{knm} \sim \mathcal{CN}(0, 1)$. For the sake of simplicity, the sparse channel is modeled by geometry based model. It is assumed that the transmitter and the receivers are equipped with a uniform linear array. When there are $L$ multipath components (MPCs) from the transmitter to each receiver, the channel matrix is expressed as

$$H_k = \sqrt{\frac{N_t}{L}} \sum_{l=1}^{L} \beta_{kl} a_t(\phi_{t,kl})^H a_r(\phi_{r,kl}),$$

(4.4)

where $\beta_{kl} \sim \mathcal{CN}(0, 1)$ is the multipath coefficient, $\phi_{t,kl}$ and $\phi_{r,kl}$ are angle-of-departure and angle-of-arrival of the $l$th multipath for the channel between the base station and
the $k$-th receiver. The steering vector $\mathbf{a}_u(\phi_{u,kl})$, $\forall u \in \{t, r\}$, for linear arrays is expressed as

$$\mathbf{a}_u(\phi_{u,kl}) = \frac{1}{\sqrt{N_u}} (1, e^{\frac{2\pi d_u}{\lambda}} \cos(\phi_{u,kl}), ..., e^{\frac{2\pi d_u}{\lambda} (N_u - 1) \cos(\phi_{u,kl})})^T$$  \hspace{1cm} (4.5)$$

where $\phi_{u,kl} \in [0, \pi]$, $\lambda$ is the wavelength and $d_u = \lambda/2$ is the antenna spacing.

It is also assumed that the base station is equipped with a large array and performs hybrid beamforming. In addition, the users have a small number of antennas, and hence they apply digital combining. The block diagram of the hybrid beamformer of the base station is shown in Fig. 4.1.

Let $M = KN_r$ denote the number of the RF chains at the transmitter. Then, $\mathbf{F}_k$ is decomposed into a product of RF beamforming matrix $\mathbf{F}_{RF,k} \in \mathbb{C}^{N_t \times N_r}$ and baseband precoding matrix $\mathbf{F}_{B,k} \in \mathbb{C}^{N_r \times N_r}$ as $\mathbf{F}_k = \mathbf{F}_{RF,k} \mathbf{F}_{B,k}$. The RF beamformer consists of a network of phase shifters which their angles are chosen from the set $\Theta = [0, 2\pi]$ for analog phase shifting.

It is noted that $\Theta = \{0, 2\pi/2^B, ..., (2^B - 1)2\pi/2^B\}$ when digital phase shifters with $B$ bits of resolution are used. Finally, letting $\mathbf{F}_{RF} = [\mathbf{F}_{RF,1}, ..., \mathbf{F}_{RF,K}]$, the elements of $\mathbf{F}_{RF}$ are described as

$$F_{RF,nm} = 1/\sqrt{N_t} \exp(j\theta_{nm}), \theta_{nm} \in \Theta.$$  \hspace{1cm} (4.6)$$
4.2 Single-Antenna User Equipment

In this scenario, it is assumed that \( N_r = 1 \) and \( M = K \). The optimal sum-capacity of a downlink channel matrix \( H \) is derived by [84]

\[
C_{\text{sum}}(P_t, H) = \max_{\text{trace}(P) \leq 1} \log_2 \det \left( I_K + \frac{P}{\sigma^2} PHH^H \right),
\]

(4.7)

where \( P \) is the power allocation matrix. The capacity can be achieved by nonlinear DPC techniques, however, suboptimal linear algorithms are preferred due to lower complexity. In addition, due to the favorable conditions of the channel matrix in massive MMO scenarios linear precoding techniques such as ZF and MF show a close-to-optimal performance. For the hybrid structure, the vector of the received signals becomes \( y = \sqrt{\frac{P}{T}} H_F R_F B^H s + z \). In the following Proposition, the virtually optimal hybrid beamformer and its performance, achievable sum-rate with respect to the sum-rate capacity of \( H \) when the channel is modeled by Rayleigh fading, will be presented.

**Proposition 5:** The asymptotically optimal hybrid beamformer for the multiuser scenario with Rayleigh channel and in the high SNR regime consists of \( F_{\text{RF}}^{\text{opt}} \) from Proposition 2, and \( F_{B}^{\text{opt}} = (H F_{\text{RF}}^{\text{opt}})^{-1} \). In this case, the difference between the sum-capacity \( C_{\text{sum}} \) and the maximum achievable sum-rate \( R_{\text{sum}} \) at high SNR is

\[
C_{\text{sum}}(P_t, H) - R_{\text{sum}}(P_t, \frac{1}{\sqrt{T_t}} H_{\text{RF}} F_{\text{RF}}^{\text{opt}} F_{B}^{\text{opt}}) = -K \log_2 (\pi/4). \tag{4.8}
\]

**Proof:** Letting \( H = U \Sigma V^H \) and \( F_d = V_{1:K} \), the sum-rate capacity of a multiuser broadcast channel can be expressed as [84]

\[
C_{\text{sum}}(P_t, H) = \max_{\text{trace}(P) \leq 1} \log_2 \det \left( I_K + \frac{P}{\sigma^2} PHH^H \right) \tag{4.9}
\]

\[
= \max_{\text{trace}(P) \leq 1} \log_2 \det \left( I_K + \frac{P}{\sigma^2} PU \Sigma V^H V \Sigma^H U^H \right)
\]

\[
= \max_{\text{trace}(P) \leq 1} \log_2 \det \left( I_K + \frac{P}{\sigma^2} PU \Sigma_{1:K}^2 U^H \right)
\]

\[
= C_{\text{sum}}(P_t, HF_d),
\]

where the last two equalities comes from the fact that \( \Sigma \) has only \( K \) nonzero elements and \( \Sigma_{1:K}^2 = \Sigma V^H V \Sigma^H = \Sigma V^H F_d F_d^H V \Sigma^H \). As the singular vectors of the channel are
in the direction of the channel steering vectors, it can be easily concluded that the RF beamformer of Lemma 2 is virtually the optimal scheme for the sparse channel model.

For the Rayleigh channel employing $F_d$ as the RF beamformer is equivalent to relaxing the constant modulus constraint of the phase shifters. When $N_t \to \infty$, the performance of ZF beamformer with $F_{ZF} = H^H (HH^H)^{-1}$ converges to the sum-capacity [2], and the channel input-output relationship becomes:

$$ y = \sqrt{\frac{P_t}{\Gamma_t}} H H^H (HH^H)^{-1} s + z = \sqrt{\frac{P_t}{\Gamma_t}} s + z. \quad (4.10) $$

In this case, $\Gamma_t$ in (4.2) is

$$\Gamma_t = E \left[ \frac{1}{K} \text{trace}(F_{ZF} F_{ZF}^H) \right] \quad (4.11)$$

$$ = \frac{1}{K} E \left[ \text{trace} \left( H^H (HH^H)^{-1} [(HH^H)^{-1}]^H H \right) \right]$$

$$ = \frac{1}{K} E \left[ \text{trace} \left( HH^H (HH^H)^{-1} [(HH^H)^{-1}]^H \right) \right]$$

$$ = \frac{1}{K} E \left[ \text{trace} \left( (HH^H)^{-1} \right) \right] = \frac{1}{N-K},$$

as $E[\text{trace}((HH^H)^{-1})] = K/(N-K)$ for central complex Wishart matrices [85]. The spectral efficiency achieved by ZF is expressed as

$$ C_{\text{sum}}(P_t, H) = C_{\text{sum}}(P_t, \frac{1}{\sqrt{\Gamma_t}} HF_{ZF}) = K \log_2 (1 + \rho) \quad (4.12)$$

$$ = K \log_2 \left( 1 + \frac{P_t E[|s_k|^2]}{\Gamma_t \sigma_z^2} \right)$$

$$ = K \log_2 \left( 1 + \frac{P_t}{K \Gamma_t \sigma_z^2} \right),$$

where $E[ss^H] = 1/K I_K$ and $\rho$ is the received SNR at the user side.

In addition, by applying ZF to the effective channel $H_e = HF_d = U \Sigma_{1:K}$, the precoder matrix becomes $F_{ZF_e} = H_e^{-1} = \Sigma_{1:K}^{-1} U^H$. It should be noted that $F_d H_d = I_K$, and the rank of $H \in \mathbb{C}^{K \times N_t}$ is $K$ and hence $\Sigma$ has only $K$ nonzero elements. Then, the
normalization factor $\Gamma_t$ can be calculated as

$$
\Gamma_t = \frac{1}{K} \mathbb{E} \left[ \text{trace} \left( F_d F_{Z_F} F_{Z_F}^H F_d^H \right) \right] = \frac{1}{K} \mathbb{E} \left[ \text{trace} \left( F_{Z_F} F_{Z_F}^H \right) \right] \quad (4.13)
$$

$$
= \frac{1}{K} \mathbb{E} \left[ \text{trace} \left( \Sigma_{1:K}^{-1} U^H \Sigma_{1:K}^{-1} \right) \right] = \frac{1}{K} \mathbb{E} \left[ \text{trace} \left( \Sigma_{1:K}^{-2} \right) \right]
$$

$$
= \frac{1}{K} \mathbb{E} \left[ \text{trace} \left( (\Sigma \Sigma^H)^{-1} \right) \right]
$$

$$
= \frac{1}{K} \mathbb{E} \left[ \text{trace} \left( (\Sigma \Sigma^H)^{-1} U^H U \right) \right]
$$

$$
= \frac{1}{K} \mathbb{E} \left[ \text{trace} \left( (U \Sigma \Sigma^H U^H)^{-1} \right) \right]
$$

$$
= \frac{1}{K} \mathbb{E} \left[ \text{trace} \left( (HH^H)^{-1} \right) \right].
$$

As a consequence, $1/\sqrt{\Gamma_t} HF_{Z_F} = 1/\sqrt{\Gamma_t} HF_d F_{Z_F}$ and

$$
C_{\text{sum}}(P_t, H) = C_{\text{sum}}(P_t, \frac{1}{\sqrt{\Gamma}} H e F_{Z_F}). \quad (4.14)
$$

Hence, the asymptotically optimal hybrid beamforming scheme is derived when the constant modulus constraint at the RF beamformer is relaxed.

Since $K$ is fixed and $N_t \to \infty$, the array gain and therefore the received SNR grows large. Hence, the asymptotic behavior of MIMO channels at high SNR can be applied. In Theorem 3 of [86] and Theorem 2 of [87], it was shown that

$$
\lim_{\rho \to \infty} \left[ C(P_t, H) - C_{\text{sum}}(P_t, H) \right] = 0, \quad (4.15)
$$

where $C(P_t, H)$ is the capacity of the point-to-point system. Considering $C(P_t, H) = C(P_t, U^H H)$, it could be concluded that

$$
C_{\text{sum}}(P_t, U^H H) = C(P_t, U^H H) = C(P_t, H) = C_{\text{sum}}(P_t, H). \quad (4.16)
$$

Let $R_{\text{sum}}(P_t, \frac{1}{\sqrt{N_t}} H F_{RF})$ denote the achievable sum-rate of multiuser scenario when the constant modulus is taken into account. Similar to (4.16), it could be easily verified that

$$
R_{\text{sum}}(P_t, \frac{1}{\sqrt{N_t}} H F_{RF}) = R_{\text{sum}}(P_t, \frac{1}{\sqrt{N_t}} U^H H F_{RF})
$$

$$
= \max_{\text{trace}(P) \leq 1} \log_2 \det \left( I_K + \frac{P_t}{N_t \sigma_z^2} P \Sigma \Sigma^H F_{RF} F_{RF}^H \Sigma \Sigma^H \right), \quad (4.17)
$$
Now, the RF beamformer that maximizes $R_{\text{sum}}(P_t, \frac{1}{\sqrt{N_t}} U^H H F_{\text{RF}})$ is obtained by

$$F_{\text{RF}}^{\text{opt}} = \arg \max_{F_{\text{RF}}} R_{\text{sum}}(P_t, \frac{1}{\sqrt{N_t}} U^H H F_{\text{RF}}), \quad (4.18)$$

s.t. $|F_{\text{RF},n_k}| = 1$.

Similar to the proof of Lemma 2, $F_{\text{RF}}^{\text{opt}}$ that can diagonalize $P \Sigma V^H F_{\text{RF}} F_{\text{RF}}^H V \Sigma^H$ will also maximize $R_{\text{sum}}(P_t, \frac{1}{\sqrt{N_t}} U^H H F_{\text{RF}})$ in (4.17). On the other hand, in the previous chapter it is shown that $\frac{1}{\sqrt{N_t}} H F_{\text{RF}}^{\text{opt}} = \sqrt{\frac{\pi}{2}} U \Sigma V^H F_d = \sqrt{\frac{\pi}{2}} H_e$. Additionally, in (4.14) it was discussed that $F_{ZF_e}$ is asymptotically optimal for $H_e$. As a result,

$$R_{\text{sum}}(P_t, \frac{1}{\sqrt{N_t}} U^H H F_{\text{RF}}^{\text{opt}}) = \max_{\text{trace}(P) \leq 1} \log_2 \det \left( I_K + \frac{P_t}{N_t \sigma_z^2} P \Sigma V^H F_{\text{RF}} F_{\text{RF}}^H V \Sigma^H \right)$$

$$= \max_{\text{trace}(P) \leq 1} \log_2 \det \left( I_K + \frac{\pi P_t}{4} P \Sigma V^H F_d F_d^H V \Sigma^H \right)$$

$$= C_{\text{sum}}(\frac{\pi}{4} P_t, U^H H F_d) = C_{\text{sum}}(\frac{\pi}{4} P_t, \frac{1}{\sqrt{N_t}} H_e F_{ZF_e})$$

$$= K \log_2 (1 + \frac{\pi P_t}{4 K T_s \sigma_z^2}).$$

Hence, by letting the baseband precoder for the hybrid beamformer with constant modulus constraint as $F_{B}^{\text{opt}} = F_{ZF_e}$ combined with $F_{\text{RF}}^{\text{opt}}$ of Lemma 2, the asymptotically optimal hybrid beamformer is achieved. Finally, it could be easily verified that

$$C_{\text{sum}}(P_t, H) - R_{\text{sum}}(P_t, \frac{1}{\sqrt{N_t T_s}} H F_{\text{RF}}^{\text{opt}} F_{B}^{\text{opt}})$$

$$= \lim_{N_t \to \infty} K \log_2 \frac{1 + \frac{P_t}{K T_s \sigma_z^2}}{1 + \frac{\pi P_t}{4 K T_s \sigma_z^2}} = -K \log_2 (\pi/4).$$

□

The difference between right hand side of (3.32) and (4.8) is a scalar number 2. This factor comes from the fact that the transmitter and the receiver in the point-to-point system are equipped with hybrid beamformer, and the losses imposed by the RF beamformer should be counted at both sides. For the case of the geometry based channels, the proposed RF beamformer in Proposition 2 is still asymptotically the optimal beamformer as it is shown in the proof of Proposition 5. In order to achieve the maximum achievable rate, nonlinear precoding schemes should be used at the baseband precoder.
4.2. Single-Antenna User Equipment

Figure 4.2: Sum-rate achieved by ZF (digital beamforming) $C_{\text{sum}}$, the proposed hybrid beamformer for the multiuser scenario $R_{\text{sum}}$ and the bound based on Proposition 5 $R_{\text{sum}}^A$ for Rayleigh fading channel.

The performance of ZF baseband precoder for sparse channels and the multiantenna multiuser scenario considering the impact of imperfect CSI on the system performance is investigated in [15]. Under the assumption of single-antenna users, sparse channel, a base station with a linear array and ZF baseband precoder, the beamformer of Proposition 5 and the algorithm in [15] will result in the same performance. However, the hybrid beamformer in [15] is not applicable to Rayleigh channels due to employing a special RF codebook.

The RF beamformer of Proposition 5, however, is applicable to both rich and sparse channels, and it is adaptable to different scenarios. For the downlink multiuser scenario with large number of antennas at the base station, ZF has been shown as the asymptotically optimal beamforming scheme in Rayleigh channels. Fig. 4.2 shows the achievable sum-rates by ZF with a digital beamformer and the proposed hybrid beamformer, denoted as $C_{\text{sum}}$ as $R_{\text{sum}}$, when $N_t = 64$ and $M = K = 4$. The performance of the proposed hybrid beamforming scheme is evaluated by Monte-Carlo simulations.
over 1000 independent channel realizations. It is observed that the digital beamformer achieves 1.4 \text{bit/s/Hz} higher spectral efficiency than the hybrid beamformer as in Proposition 5.

4.3 Multiantenna User Equipment

In this scenario, it is assumed that the base station is transmitting multiple streams to each user. Block diagonalization techniques are commonly used for such scenarios \cite{56,88,89}. It is noted that the received signals from the antennas of each mobile device can be jointly processed. Hence, it is expected that block diagonalization techniques can achieve higher spectral efficiency compared to the single-antenna user equipment precoding methods. In addition, higher data rates can be achieved if all the received signals could be jointly processed.

In order to extend the hybrid beamforming techniques for the point-to-point MIMO and the single-antenna multiuser scenarios to the multiantenna multiuser case, firstly, a summary of the key points of the presented hybrid beamformers will be presented in the following. It is noted that these methods provide an upper-bound and a lower-bound for the sum-rate of the proposed algorithm for the multiantenna multiuser scenario. Then a review of block diagonalization will be presented. Finally, combining the key principals of the proposed hybrid beamformers and block diagonalization, a novel hybrid beamformer for the multiantenna multiuser scenario will be presented.

4.3.1 Performance upper-bound

When the received signal from all the antennas of all the users can be jointly processed, higher rates could be achieved compared to the scenario that users cannot collaborate. We choose this scenario as the performance upper-bound for our algorithm. It is noted that the joint processing of the received signals, is equivalent to a single-user scenario with $M = KN_r$ receive and $N_t$ transmit antennas. Let $\mathbf{H} \in \mathbb{C}^{M \times N_t}$, where $M \ll N_t$, as the channel matrix. When the precoding and combining matrices $\mathbf{F}$ and $\mathbf{W}$ are used,
the spectral efficiency for this scenario is \[77\]

\[ R_{u-b} = \log_2 \det \left( I_{N_r} + \frac{\rho}{\Gamma_t} R_n^{-1} W^H \Gamma F F^H H^H W \right), \] (4.21)

where \( R_n = W^H W \) and \( \rho = P_t / \sigma_z^2 \) is a measure of the SNR. The optimal transmission that achieves the channel capacity is singular value decomposition (SVD) based precoding and combining with waterfilling \[77\]. For the sake of simplicity, waterfilling will not be used in this paper. The achieved rate in equation (4.21) will be used as the performance upper-bound for our proposed algorithm.

The SVD of \( H \) is expressed as \( H = U \Sigma V^H \) where the columns of \( V \in \mathbb{C}^{N_t \times N_t} \) and \( U \in \mathbb{C}^{M \times M} \) are the right and left singular vectors of \( H \), respectively. Moreover, \( \Sigma \in \mathbb{R}^{M \times N_t} \) is a diagonal matrix that contains the singular values of \( H \). It could be easily verified that \( H \) and \( HV_{1:M} \) have equal singular values, and consequently, equal capacities.

The key idea of the hybrid beamformer for the point-to-point system is that the singular values of \( H \) are preserved to the feasible extent considering the constant modulus constraint of the phase shifters. This is achieved by setting \( F_{RF} \) as

\[ F_{RF,n,m} = e^{j \angle V_{n,m}}. \] (4.22)

The achievable rate by this technique is calculated by letting \( F = F_{RF} \) in equation (4.21).

### 4.3.2 Performance lower-bound

The lower-bound for the sum-rate of our proposed algorithm is the scenario where there is no collaboration between any of the receive antennas in the system. This is equivalent to the multiuser scenario with \( M = KN_r \) single-antenna users. Similar to the single-user MIMO case, the RF precoder for this scenario was designed such that it maps the channel matrix \( H \in \mathbb{C}^{M \times N_t} \) to the effective channel matrix \( H_{eff} \in \mathbb{C}^{M \times M} \) with smaller dimension, while the singular values are preserved as much as possible. It was shown that the optimal RF beamformer in this scenario is the same as (4.22). Then, different baseband precoding techniques such as ZF could be applied to the effective channel \( H_{eff} \).

We select the sum-rate achieved by the hybrid beamformer with ZF at the baseband
as the performance lower-bound for our proposed scheme. In this case, the achievable rate is

\[ R_{l-b} = M \log_2(1 + \frac{\rho}{\Gamma_t}). \]  

(4.23)

### 4.3.3 Multiantenna multiuser with block diagonalization

When the base station is equipped with a digital beamformer, block diagonalization is one of the commonly used approaches. In this technique, \( F_k \) is designed such that the signal for user \( k \) does not cause interference for the other users. To achieve this, the precoding matrix \( F_k \) is decomposed into \( F_k = T_k A_k \) where \( T_k \) cancels the inter-user interference and converts the problem into parallel single-user scenarios. Then, \( A_k \) and \( W_k \) are designed such that the rate of the single-user transmission is maximized. More precisely, \( T_k \) is designed such that it satisfies

\[ H_{k'}^T T_k = 0, \quad \forall k, k' \in \{1, ..., K\}, k \neq k'. \]  

(4.24)

To perform block diagonalization, the following procedure for designing \( T_k \) was proposed in [88–90]. Firstly, create \( \tilde{H}_k \) which includes all the channel matrices \( \mathbf{H}_{k'} \) of all the users except user \( k \) as

\[ \tilde{H}_k = [\mathbf{H}_1^T \ldots \mathbf{H}_{k-1}^T \mathbf{H}_{k+1}^T \ldots \mathbf{H}_K^T]^T, \quad \forall k \neq k'. \]  

(4.25)

Let \( \tilde{r} \) denote the rank of \( \tilde{H}_k \), then the SVD of \( \tilde{H}_k \) is \( \tilde{H}_k = \tilde{U}_k \tilde{\Sigma}_k \tilde{V}_k^H \) where \( \tilde{V}_{\tilde{r}+1:N_t} \) forms orthogonal basis vectors for the null space of \( \tilde{H}_k \). Hence, letting \( T_k = \tilde{V}_{\tilde{r}+1:N_t} \) results in \( \mathbf{H}_{k'}^T T_k = 0, \forall k, k' \in \{1, ..., K\}, k \neq k' \). The sum-rate achieved by block diagonalization is [88]

\[ R_{BD} = \sum_{k=1}^{K} \log_2 \left( \det \left( \mathbf{I}_{N_r} + \frac{\rho}{\Gamma_t} \mathbf{H}_k F_k F_k^H \mathbf{H}_k^H \right) \right). \]  

(4.26)

### 4.3.4 Proposed Hybrid Beamformer

In this section, we present an efficient algorithm which uses hybrid beamforming to improve the achievable sum-rate of digital block diagonalization in the massive MIMO scenarios.
4.3. Multiantenna User Equipment

The singular values of the channel matrix have a profound impact on the performance of the beamformers and the achievable rates. Hence, it is desired that the RF beamformer of the hybrid beamformer retain the singular values of the channel to the feasible extent considering the constraint imposed by the phase shifters. It is noted that the RF beamformer maps the channel matrix $\mathbf{H} \in \mathbb{C}^{M \times N_t}$ to $\mathbf{H}_{\text{eff}} \in \mathbb{C}^{M \times M}$. To this end, the proposed RF beamformer in previous sections aimed at mapping the channel matrix to a smaller dimension while the nonzero singular are preserved as much as possible. In fact, by relaxing the constant modulus constraint of the phase shifters and using the first $M$ right singular vectors, the nonzero singular values of the $\mathbf{H}$ and $\mathbf{H}_{\text{eff}}$ will be equal.

In order to preserve the nonzero singular values of the $\mathbf{H}$ while reducing the dimensions of $\mathbf{H}$, $\tilde{\mathbf{H}}$ and $\mathbf{T}_k$ in block diagonalization, our proposed hybrid beamformer consists of the following steps:

1. Let $\mathbf{H} = \mathbf{U} \Sigma \mathbf{V}^H$ as the SVD of $\mathbf{H} = [\mathbf{H}_1^T \ldots \mathbf{H}_K^T]^T$.

2. Select the first $M = KN_r$ right singular vectors of $\mathbf{H}$ and create the unconstrained RF beamformer matrix $\mathbf{F}_\mathbf{U}$ as

$$\mathbf{F}_\mathbf{U} = \mathbf{V}_{1:M}.$$ 

3. Set $\mathbf{F}_{\text{RF}}$ by applying the phase shifter constant modulus constraint to $\mathbf{F}_\mathbf{U}$ using equation (4.22) as

$$F_{\text{RF},nm} = e^{j\angle V_{nm}}.$$ 

4. Create $\tilde{\mathbf{H}}_{\text{eff},k} = [\mathbf{H}_{\text{eff},1}^T \ldots \mathbf{H}_{\text{eff},k-1}^T \mathbf{H}_{\text{eff},k}^T \ldots \mathbf{H}_{\text{eff},K}^T]^T$, where $\mathbf{H}_{\text{eff},k} = \mathbf{H}_k \mathbf{F}_{\text{RF}}$.

5. Calculate SVD of $\tilde{\mathbf{H}}_{\text{eff},k}$ as $\tilde{\mathbf{H}}_{\text{eff},k} = \tilde{\mathbf{U}}_{\text{eff},k} \tilde{\Sigma}_{\text{eff},k} \tilde{\mathbf{V}}_{\text{eff},k}^H$.

6. Set $\mathbf{T}_k \in \mathbb{C}^{N_t \times N_r}$ as

$$\mathbf{T}_k = \tilde{\mathbf{V}}_{\text{eff},k,(K-1)N_r+1:KN_r}.$$ 

7. Set $\mathbf{F}_{\mathbf{B},k}$ and $\mathbf{W}_k$ to the right and left singular vectors of $\mathbf{H}_{\text{eff},k} \mathbf{T}_k$. 

It is noted that this approach can be also used in massive MIMO systems with digital beamforming by skipping step 3. When digital phase shifters with $B$-bits of resolution are used, then we set the RF beamformer at step 3, according to
\[
\theta_{nm}^{\text{digital}} = \arg \min_{\theta_{nk}} |\angle F_{U_{nm}} - \theta_{nm}|, \quad (4.27)
\]
subject to $\theta_{nm} \in \{0, ..., (2^B - 1)2\pi/2^B\}$.

This technique simply rounds the phase of the elements of the RF beamformer with analog phase shifters to the nearest discrete phase. Finally, the sum-rate of the proposed method is derived by replacing $H_k$ in (4.26) with $H_{\text{eff},k} = H_k F_{\text{RF}}$.

In the following, the simulation results for the average sum-rate of the proposed algorithm over 1000 realizations are presented. The performance is compared to the sum-rate by block diagonalization with digital beamforming $R_{\text{BD}}$ in (4.26), ZF based hybrid beamforming $R_{\text{u-b}}^{\text{HB}}$ in (4.23), collaborating receivers with both digital beamforming $R_{\text{u-b}}^{\text{DB}}$ and hybrid beamforming $R_{\text{u-b}}^{\text{HB}}$ based on (4.21). In order to provide a comprehensive comparison between the beamformers, the sum-rate is evaluated under rich and sparse scattering channel assumptions. The simulations consider the impact of the number of the MPCs $L$, number of the transmitter antennas $N_t$ and phase shifter resolution $B$. It is assumed that the number of the users and their antennas are $K = 4$ and $N_r = 2$, respectively.

Figure 4.3 presents the spectral efficiency of the proposed technique for the sparse scattering channel using (4.4) where $L = N_r = 2$. It is observed that the proposed algorithm provides slightly lower sum-rate $R_{\text{proposed}}$ compared to block diagonalization with digital beamforming $R_{\text{u-b}}^{\text{DB}}$ and a much higher spectral efficiency compared to $R_{\text{u-b}}$. For the Rayleigh fading channel, Fig. 4.4 shows that our algorithm achieves higher performance compared to $R_{\text{u-b}}^{\text{HB}}$. However, Fig. 4.4 also indicates that $R_{\text{u-b}}^{\text{HB}}$ and $R_{\text{u-b}}^{\text{HB}}$ provide a tight bound where $R_{\text{u-b}}^{\text{HB}}$ denotes the lower-bound of the achievable sum-rate by the hybrid beamformer in section 4.3.2. Considering that ZF based hybrid beamformer achieves high spectral efficiency in this scenario, it can be advantageous to use this technique over the proposed algorithm. Figures 4.3 and 4.4 indicate that the performance of the presented beamformer and $R_{\text{u-b}}^{\text{HB}}$ can significantly vary depending on the number of the MPCs in the channel. Figure 4.5 shows that, irrespective of
4.3. Multiantenna User Equipment

Figure 4.3: Sum-rate by the proposed algorithm $R_{\text{proposed}}$, block diagonalization $R_{\text{BD}}$, ZF based hybrid beamforming $R_{\text{HB}}^{\text{u-b}}$, collaborating receivers with digital beamforming $R_{\text{DB}}^{\text{u-b}}$ and hybrid beamforming $R_{\text{HB}}^{\text{u-b}}$ for the geometry based model, $L = N_r = 2$, $K = 4$ and $N_t = 32$. 
the number of the MPCs, the proposed algorithm always has a better performance compared to zero-forcing based hybrid beamformer.

Figure 4.6 presents the impact of $N_t$ on the sum-rate. It is observed that the performance achieved by the proposed technique converges to its upper-bound $R_{\text{HB}}^{\text{u-b}}$ as $N_t$ goes larger. Finally, Fig. 4.7 shows the impact of digital phase shifters on the sum-rate of the presented algorithm. It is observed that with 3 bits of resolution and using equation (4.27), the sum-rate is almost the same as analog phase shifting.

4.4 Summary

In this chapter, hybrid beamforming for downlink of a multiuser scenario was investigated. Compared to a fully-digital system, it was shown that similar spectral efficiency can be achieved when the asymptotically optimal RF beamformer is set according to the phase of the elements of the right singular vectors of the channel matrix. It was discussed that such RF beamforming can almost preserve the singular values of the
4.4. Summary

Figure 4.5: $R_{\text{proposed}}$, $R_{\text{BD}}$, $R_{\text{HB}}^L$, $R_{\text{DB}}^L$, and $R_{\text{HB}}^R$ v.s. the number of the multipath components for the geometry based model, $\rho = 20$ dB, $N_r = 2$, $K = 4$ and $N_t = 32$.

Figure 4.6: $R_{\text{proposed}}$, $R_{\text{BD}}$, $R_{\text{HB}}^L$, $R_{\text{DB}}^L$, and $R_{\text{HB}}^R$ v.s. the number of the base station antennas for the geometry based model, $\rho = 20$ dB, $N_r = 2$, $K = 4$ and $L = 4$. 
Figure 4.7: $R_{u-b}^{DB}$ and $R_{u-b}^{HB}$ and $R_{\text{proposed}}$ achieved by the proposed technique with analog and digital phase shifters with $B = 2$ and $B = 3$ bits of resolution, Rayleigh fading, $\rho = 20$ dB, $N_r = 2$, $K = 4$ and $N_t = 32$.

channel matrix. Then, applying ZF or block diagonalization at the baseband results in almost the same sum-rate as the scenario that the base station is equipped with a fully-digital system.
Chapter 5

Hybrid Beamforming with Phase Shifters and Switches

Hybrid beamforming for large arrays can have high power consumption due to the massive number of the phase shifters. In this chapter two combinations of phase shifters and switches are proposed to alleviate this problem. In general, there are two types of phase shifter networks known as fully-connected and subconnected [5]. In the fully-connected structure, each RF chain is connected to all the antennas as in [3, 4]. It can exploit the full array gain, however, its power consumption can be very high due to the massive number of the phase shifters it requires [5, 7]. In the subconnected configuration, each RF chain is connected to a subset of antennas which results in simplicity of the circuits and lower cost, but also a lower spectral efficiency compared to the fully-connected system [5]. In general, the design of the optimal soft antenna selection schemes is a challenging task due to the nonconvex constant modulus constraint imposed by the phase shifters [3–5]. Compared to switches, phase shifters are not only more expensive, but also they have a higher power consumption [5, 7, 91–93]. For example, it was reported that phase shifters and switches at 2.4 GHz consume 28.8–152 mW and 0 – 15 mW, respectively [92, 93]. Similar to the phase shifter networks, switch networks have fully-connected and subconnected structures. Due to the large number of switches, fully-connected configuration has high hardware complexity, insertion losses as well as cross talk distortion [94]. Hence, subconnected configuration for the switch
network, such as binary switches, is preferred in practice despite providing less degrees of freedom in designing the antenna selector.

In order to reduce the power consumption of the soft antenna selection in massive MIMO systems, a combination of fully-connected switches and non-tunable phase shifters is proposed in [7]. The potential use of both hard and soft antenna selection techniques in massive MIMO systems motivates investigating new techniques to reduce the power consumption [3–5, 7, 40, 91].

In this chapter, two novel combinations of hard and soft antenna selection techniques are proposed and the achievable rates by these methods are evaluated. To this end, firstly the closed-form expression of spectral efficiency for a near optimal soft selection with subconnected structure is derived. Based on this approach and using a phase shifter selection technique, it is shown that the number of the phase shifters can be reduced to 50% without a performance loss. However, the proposed structure requires a fully-connected switch network which may not be suitable for practical applications. Hence, it is desirable to substitute the complex switch network with simpler structures, for example binary switches. The proposed method is able to achieve a similar performance as the soft selection can. Finally, the simulation results indicate that the asymptotic closed-form expressions of the spectral efficiency provide a good approximation of the performance for moderate number of antennas and phase shifters.

5.1 System Model

In this work, a narrowband single-cell multiuser scenario in downlink where the base station with $N$ omni-directional antennas serves $K$ single antenna users is considered. The wireless channel matrix $\mathbf{H} \in \mathbb{C}^{K \times N}$ follows an uncorrelated Rayleigh fading model with independent and identically distributed (i.i.d.) elements as $H_{kn} \sim \mathcal{CN}(0, 1), \forall k \in \{1, ..., K\}$ and $\forall n \in \{1, ..., N\}$. In this case, the relationship between the channel input vector $\mathbf{x} \in \mathbb{C}^{N \times 1}$ and output vector $\mathbf{y} \in \mathbb{C}^{K \times 1}$ is expressed as $\mathbf{y} = \mathbf{Hx} + \mathbf{z}$ where $\mathbf{z} \in \mathbb{C}^{K \times 1}$ is a vector with i.i.d. additive white Gaussian elements with $z_k \sim \mathcal{CN}(0, 1)$. It is assumed that the transmitter has perfect channel state information. A vector of $K$ symbols $\mathbf{s} \in \mathbb{C}^{K \times 1}$ with $\mathbf{E}[\mathbf{ss}^H] = \mathbf{I}_K$ are precoded using the precoding matrix
5.1. System Model

Then, the signal at the transmitter antennas is \( x = \sqrt{P} F s \) where \( P \) is the total transmit power per stream and \( \Gamma = \frac{\text{trace}(FF^H)}{K} \) is a power normalization factor. The sum-capacity of downlink channel is expressed as

\[
C(H, P) = \max_{\text{trace}(P) \leq 1} \log_2 \det \left( I_K + \rho PHH^H \right),
\]  

(5.1)

where \( \rho = P/\sigma_z^2 \), and \( P \) is a diagonal power allocation matrix. In massive MIMO systems, it has been shown that linear precoders such as zero-forcing (ZF) can achieve a close to optimal performance. Applying ZF precoding matrix \( H^H( HH^H)^{-1} \), the sum-rate becomes

\[
R_{ZF} = K \log_2 \left( 1 + \frac{P}{\Gamma_{ZF} \sigma_z^2} \right),
\]  

(5.2)

where \( \Gamma_{ZF} = \mathbb{E} \left[ \text{trace}((HH^H)^{-1}) \right]/K = 1/(N - K) \) is the power normalization factor for ZF precoder. To maximize multiplexing gain in the high signal-to-noise ratio (SNR) regime, it is assumed that \( M = K \) where \( M \) is the number of the RF chains.

Figure 5.1a presents the diagram of a fully-connected antenna selection structure where each RF chain is connected to all antennas through a network of switches or phase shifters. Depending on the performance metric, e.g. maximizing the spectral efficiency or diversity gain, the hard antenna selection chooses the best \( M \) out of \( N \) antennas using its switching network. The disadvantage of this approach is that the large array gains cannot be achieved when \( M \ll N \). In general, soft antenna selection techniques provide a better performance compared to hard selection [3, 4]. However, the fully-connected structure in Fig. 5.1a requires \( MN \) switches or phase shifters which becomes very large in massive MIMO scenarios [5]. This introduces high insertion losses and hardware complexity. Hence, the subconnected configuration, shown in Fig. 5.1b, is preferred in practice. The precoder matrix \( F = F_{\text{sub}}F_B \) for the structure of Fig. 5.1b consists of a block diagonal RF beamforming matrix \( F_{\text{sub}} \in \mathbb{C}^{N \times M} \) and a baseband precoder \( F_B \in \mathbb{C}^{M \times K} \). The RF beamformer has to be designed such that the spectral efficiency \( R_{\text{sub}} \) is maximized subject to \( F_{\text{sub},nm} = e^{j\theta_{nm}}, \forall \theta_{nm} \in [0, 2\pi], \quad n \in I_m = \{ \frac{N}{M} (m-1) + 1, ..., \frac{N}{M} m \}, \) otherwise \( |F_{\text{sub},nm}| = 0 \). In this case, \( \Gamma_{\text{sub}} = \frac{\text{trace}(F_{\text{sub}}F_{\text{sub}}^H)}{M} = N/M \).

In general, soft selection (hybrid beamforming) is a challenging task as the maximization of the spectral efficiency is a nonconvex problem due to the constant modulus constraint imposed by the phase shifters [3–5].
In the following, firstly the closed-form expression for an asymptotically optimal beamformer will be presented. In order to reduce the power consumption of the structure in Fig. 5.1b, it will be shown that the configuration of Fig. 5.1c can replace 50% of the phase shifters with switches and without a performance loss. Finally, Fig. 5.1d proposes a simpler structure for the complicated switch network with low-cost 1-out-of-$S$ switches where $S$ is the ratio of the number output-to-input ports.

### 5.2 Subconnected Structure with Phase Shifters

The singular value decomposition (SVD) of the channel matrix is denoted as $\mathbf{H} = \mathbf{U} \Sigma \mathbf{V}^H$, where $\mathbf{V} \in \mathbb{C}^{N \times N}$ and $\mathbf{U} \in \mathbb{C}^{K \times K}$ contain the right and left singular vectors.
5.2. Subconnected Structure with Phase Shifters

The diagonal matrix $\Sigma \in \mathbb{R}^{K \times N}$ includes the singular values of $H$. It is noted that $H = U \Sigma_{1:M} V_{1:M}^H$ as $H$ has only $M$ nonzero singular values. The statistical properties of the elements of $V$ when $N \to \infty$ was analyzed in chapter 3 and it was shown that

1. $\sqrt{N} V_{nn'} \sim \mathcal{CN}(0, 1)$, $\forall n, n' \in \{1, ..., N\}$ are i.i.d.,
2. $|\sqrt{N} V_{nn'}|$ is a Rayleigh variable with parameter $\frac{1}{\sqrt{2}}$,
3. $\sqrt{N} \mathbb{E}[||V_{nn'}||] = \sqrt{\pi}/2$.

In the high SNR regime, (5.1) is expressed as

$$R_{\text{sub}} = \log_2 \det \left( I_K + \frac{\rho P}{F_{\text{sub}}^H H_{\text{sub}} H_{\text{sub}}^H} \right)$$

$$\approx \log_2 \det \left( \frac{\rho P}{F_{\text{sub}}^H U \Sigma_{1:M} V_{1:M}^H F_{\text{sub}}^H} \right)$$

$$= \log_2 \det \left( \frac{\rho}{N} \frac{\Sigma_{1:M}^H U^H P U \Sigma_{1:M}}{V_{1:M}^H F_{\text{sub}}^H V_{1:M}} \right)$$

$$+ \log_2 \det \left( \frac{M}{N} \frac{V_{1:M}^H F_{\text{sub}}^H V_{1:M}}{G_{\text{sub}}^H G_{\text{sub}}^H} \right),$$

where the last equality is a result of $\det(AB) = \det(A)\det(B)$. Letting $G = \sqrt{M/N} V_{1:M}^H F_{\text{sub}}$, then $GG^H$ is positive semidefinite and $\det(GG^H) \leq \prod_{m=1}^M |G_{mm}|^2$ where the equality holds when $GG^H$ is a diagonal matrix. Hence, $F_{\text{sub}}$ that upper bounds $R_{\text{sub}}$ should i) diagonalize $GG^H$ which requires $M/N |v_{m}^H f_{\text{sub},m}|^2 = 0$ when $m \neq m' \forall m, m' \in \{1, ..., M\}$, ii) maximize the diagonal elements $G_{nm}$ of $GG^H$ where

$$G_{nm} = \frac{\sqrt{M} v_{m}^H f_{\text{sub},m}}{\sqrt{N}} = \sqrt{\frac{M}{N}} \sum_{n \in I_m} V_{nm}^* e^{j\theta_{nm}}.$$  

(5.4)

This term is maximized if $\theta_{nm} = \angle V_{nm}$ as discussed in chapter 3. When $\theta_{nm} = \angle V_{nm}$, then

$$\lim_{N \to \infty} \frac{M \sum_{n \in I_m} |\sqrt{N} V_{nm}|}{N \sqrt{M}} = \frac{\mathbb{E}[|\sqrt{N} V_{nm}|]}{\sqrt{M}} = \frac{\sqrt{\pi}}{2\sqrt{M}},$$

(5.5)

due to the law of large numbers. Similarly, $\lim_{N \to \infty} |\sqrt{N} v_{n}^H f_{\text{sub},n}| = 0, \forall n \neq n$ as $\mathbb{E}[|\sqrt{N} V_{nn}|] = 0$. Hence, (5.3) is maximized when

$$F_{\text{sub},nm} = \begin{cases} e^{j\angle V_{nm}} & \text{if} \ n \in I_m, \\ 0 & \text{if} \ n \notin I_m. \end{cases}$$

(5.6)
Chapter 5. Hybrid Beamforming with Phase Shifters and Switches

In this following, the spectral efficiency for the configuration of Fig. 5.1b will be calculated when (5.6) is used. From (5.5), it can be easily shown that \( \sqrt{\frac{M}{N}}F_{\text{sub}} = \sqrt{\pi/(2\sqrt{M})}U\Sigma_{1:M} \). Applying ZF to \( \sqrt{\frac{M}{N}}H_{\text{sub}} \) to cancel the interference between the users, the precoding matrix becomes \( F = F_{\text{sub}}F_B \) where \( F_B = (H_{\text{sub}})^{-1} \). Then, the power normalization factor becomes

\[
\Gamma = E\left[ \text{trace}\left( F_{\text{sub}}(H_{\text{sub}})^{-1}(F_{\text{sub}}H^H)^{-1}F_{\text{sub}}^H \right) \right]/M
\]

(5.7)

\[
= E\left[ \text{trace}\left( (F_{\text{sub}}^H H^H)^{-1}F_{\text{sub}}^H F_{\text{sub}}(H_{\text{sub}})^{-1} \right) \right]/M
\]

\[
= E\left[ \text{trace}\left( \left( \sqrt{\frac{M}{N}}F_{\text{sub}}H^H \right)^{-1} \left( \sqrt{\frac{M}{N}}H_{\text{sub}}^H \right)^{-1} \right) \right]/M
\]

\[
= \frac{4\pi}{\pi} E\left[ \text{trace}\left( (HH^H)^{-1} \right) \right] = \frac{4M}{\pi} \Gamma_{ZF},
\]

as \( \lim_{N \to \infty} F_{\text{sub}}^H F_{\text{sub}}/N = 1/MI_M \). Hence, the achievable sum-rate by the proposed hybrid beamformer is

\[
R_{\text{sub}} = M \log_2 \left( 1 + \frac{\pi P}{4M\Gamma_{ZF}\sigma_z^2} \right),
\]

(5.8)

when \( N \to \infty \). It is observed that in the high SNR regime, the fully-digital scheme results in \(-M \log_2(\frac{\pi}{4M})\) bits/Hz/s higher spectral efficiency compared to the hybrid beamforming with subarray structure.

Remark: The presented approach to derive (5.8) will be used in the rest of this paper. These steps can be summarized as

1. Diagonalization of \( GG_H \).

2. Use the i.i.d. and zero-mean properties of \( V_{nm} \) to conclude \( 1/\sqrt{T_{\text{sub}}|\sqrt{NV_{nm}^H}f_{\text{sub},m}|} \to 0, \forall m \neq n \).

3. Calculate \( \lim_{N \to \infty} 1/\sqrt{T_{\text{sub}}}|\sqrt{NV_{nm}^H}f_{\text{sub},m}| = E[|\sqrt{NV_{nm}}|] \).

4. Calculate the power normalization factor of the hybrid beamforming, as in (5.7), when the RF beamformer and baseband ZF precoder are combined.

5. Replace \( \Gamma_{ZF} \) in (5.2) with the power normalization factor from step 4.
5.3 Subconnected Phase Shifter Network - Fully-connected Switch Networks

The performance of the proposed soft selection for Fig. 5.1b depends on $|V_{nm}|$. It is noted that the phase shifters that are multiplied with smaller $|V_{nm}|$ have a relatively smaller contribution to the spectral efficiency. Moreover, turning off such shifters in Fig. 5.1b is equivalent to switching the corresponding antenna off. Thus, the structure of Fig. 5.1c is proposed to further reduce the number of the phase shifters by employing switch networks. By this means, the power consumption of the RF beamformer is reduced as switches require significantly smaller power to operate compared to phase shifters [7, 91]. The phase shifters in (5.6) corresponding to $|\sqrt{N}V_{nm}| \leq \alpha$ can be turned off, i.e. $F_{\text{sub},nm} = 0$, where $\alpha$ is a predefined threshold. We call this method of choosing phase shifter as phase shifter selection technique. Let $L$ denote the number of the phase shifters connected to each RF chain. This can be achieved by employing a fully-connected switch network and $ML$ phase shifters and $N/M$ antennas per RF chain.

Defining $V$ as an i.i.d. random variable with the same Rayleigh distribution as $|\sqrt{N}V_{nm}|$, then $\beta = f_V(\alpha \leq v) = \exp(-\alpha^2) = ML/N$ is a measure of the reduction in the number of the phase shifters. It is noted that $\alpha$ should be chosen carefully as $M$, $L$, $N$ are integer numbers. Let $F_{\text{SF}}$ denote the RF beamforming matrix for the subconnected phase shifters with fully-connected switch networks. Similar to (5.4), the received signal power is related to $\frac{1}{\sqrt{\Gamma_{\text{SF}}}} \tilde{\eta}_m f_{\text{SF},m}$ where $\Gamma_{\text{SF}} = 1/\sqrt{L}$. This term can be obtained as a function of $\alpha$

$$
\tilde{\eta}_m f_{\text{SF},m} = \lim_{N \to \infty} \frac{M \sum_{\forall n \in I_m} |\sqrt{N}V_{nm}^* F_{\text{SF},nm}|}{\sqrt{M f_V(\alpha \leq v))N}} = \frac{E[\tilde{V}]}{\sqrt{M f_V(\alpha \leq v))}}
$$

where $\tilde{V}$ is defined as

$$
\tilde{V} = \begin{cases} 
0 & \text{if } \sqrt{N}|V| \leq \alpha, \\
\sqrt{N}|V| & \text{if } \alpha < \sqrt{N}|V|,
\end{cases}
$$

where $E[\tilde{V}]$ is calculated in the following. Theorem 1 states that $\sqrt{N}|V_{nk}|$ follows a
Rayleigh distribution with parameter $\sigma_R$. As a result, the PDF of $\tilde{V}_{n,k}$ is expressed as

$$f_{\tilde{V}}(\tilde{v}) = \begin{cases} f_V(v)(\sqrt{N}[V| \leq \alpha)\delta(0), & \tilde{V} \leq \alpha, \\ \frac{v}{\sigma_R}e^{-\frac{v^2}{2\sigma_R^2}}, & \alpha < \tilde{V}. \end{cases} (5.11)$$

The expected value of $\tilde{V}$ is calculated as

$$E[\tilde{V}(\alpha)] = \int_{-\infty}^{\infty} \tilde{V}f_{\tilde{V}}(\tilde{v})d\tilde{V} = \int_{0}^{\alpha} \frac{\tilde{V}^2}{\sigma_R^2}e^{-\frac{\tilde{V}^2}{2\sigma_R^2}}d\tilde{V}$$

$$= \int_{0}^{\alpha} \frac{\tilde{V}^2}{\sigma_R^2}e^{-\frac{\tilde{V}^2}{2\sigma_R^2}}d\tilde{V} - \int_{0}^{\alpha} \frac{\tilde{V}^2}{\sigma_R^2}e^{-\frac{\tilde{V}^2}{2\sigma_R^2}}d\tilde{V}$$

$$= \sigma_R \sqrt{\frac{\pi}{2}} - \int_{0}^{\alpha} \frac{\tilde{V}^2}{\sigma_R^2}e^{-\frac{\tilde{V}^2}{2\sigma_R^2}}d\tilde{V}$$

$$= \sigma_R \sqrt{\frac{\pi}{2}} - \frac{\sqrt{\pi}}{2}\sigma_R \left( \frac{\alpha}{\sqrt{2}\sigma_R} - \frac{\alpha}{\sqrt{2}\sigma_R} e^{-\alpha^2/2\sigma_R^2} \right)$$

$$= \frac{\sqrt{\pi}}{2} + \alpha e^{-\alpha^2} - \frac{\sqrt{\pi}}{2}\text{erf}(\alpha), \quad (5.12)$$

where (a) and (b) are derived from [95]. In this case, (5.9) results in $1/\sqrt{L}\text{HF}_{SF} = E/\sqrt{Mf_V(\alpha \leq v)}[\tilde{V}]\text{US}_{1:M}$. When the impact of ZF at the baseband is considered, the achievable rate $R_{SF}$ is

$$R_{SF} = M \log_2 \left( 1 + \frac{(\frac{\sqrt{\pi}}{2} + \alpha e^{-\alpha^2} - \frac{\sqrt{\pi}}{2}\text{erf}(\alpha))^2}{Mf_V(\alpha \leq v)}\Gamma_{ZF}\sigma_z^2 P \right). \quad (5.13)$$

It is noted that (5.13) is a generalization of (5.8) as for $L = N/M$ (equivalently $\alpha = 0$), then $R_{SF} = R_{sub}$.

### 5.4 Subconnected Phase Shifter network - Subconnected Switch Networks

As it will be discussed in the next section, the performance of hybrid selection with a fully-connected switch network and $L = N/(2M)$ phase shifters is almost equal to the subarray structure. This is equivalent to 50% reduction in the number of phase shifters and significantly smaller power consumption. However, employing a fully-connected switch network requires a complex hardware with high insertion losses and crosstalk distortions. Hence, we evaluate the performance of the proposed hybrid beamformer
5.4. Subconnected Phase Shifter network - Subconnected Switch Networks

When subconnected switch networks are employed. In this structure, as shown in Fig. 5.1d, each phase shifter is connected to only one of the $S$ adjacent antennas. In other words, the $l$th, $l \in I_m$, phase shifter connected the $m$th RF chain is able to choose one of the antennas which its index is in $\mathcal{J}_q = \{(q-1)S + 1, ..., qS\}, \forall \mathcal{J}_q \subset I_m$ where $q \in \{1, ..., N/S\}$. Following a similar argument as for the phase shifter selection technique, the $l$th phase shifter will be connected the corresponding antenna element $\hat{n}$ according to $\hat{n} = \arg \max_{n \in \mathcal{J}_q} |V_{nm}|$. Let $N_m = \mathcal{J}_l \cap I_m$ be a set that contains $\hat{n}$, where its cardinality is $L$, and $V$ be a random variable that has the same distribution as $\max_{n \in \mathcal{J}_l} |\sqrt{N}V_{nm}|$.

The RF beamforming matrix $F_{SS} \in \mathbb{C}^{N \times M}$ for this scenario can be derived according to Algorithm 1. Since $N = MLS$ and $\Gamma_{SS} = L$, the received power at user side is related to

$$
\frac{\mathbf{v}_{m}^H \mathbf{f}_{SS} \sqrt{\Gamma_{SS}}}{\sqrt{\Gamma_{SS}}} = \lim_{N \to \infty} \frac{1}{\sqrt{\Gamma_{SS}N}} \sum_{n \in \mathcal{J}_m} \sqrt{N}V_{mn}^* F_{SS, nm} (5.14)
$$

$$
= \lim_{N,L \to \infty} \frac{1}{L \sqrt{MS}} \sum_{n \in N_m} |\sqrt{N}V_{mn}| = \frac{1}{\sqrt{MS}} \mathbb{E}[\hat{V}].
$$

In order to calculate $\mathbb{E}[\hat{V}] = \int_{-\infty}^{+\infty} f_{\hat{V}}(\hat{v}) \hat{v} d\hat{v}$, first we calculate $F_{\hat{V}}(\hat{v})$. Since $\hat{V}$ is the maximum of $S$ i.i.d. Rayleigh distributed elements when $N \to \infty$, then

$$
F_{\hat{V}}(\hat{v}) = F_{V}(v)^S = (1 - e^{-\hat{v}^2})^S
$$

(5.15)

where $F_{V}(v) = 1 - e^{-v^2}$ as $V$ follows Rayleigh distribution. Then, the pdf of $\hat{V}$ is calculated as

$$
f_{\hat{V}}(\hat{v}) = \frac{d}{d\hat{v}} (1 - e^{-\hat{v}^2})^S = 2S\hat{v}(1 - e^{-\hat{v}^2})^{S-1}e^{-\hat{v}^2}
$$

(5.16)

$$
= \frac{b}{\mathcal{H}} 2S\hat{v} e^{-\hat{v}^2} \sum_{s=0}^{S-1} \binom{S-1}{s} (-1)^s e^{-s\hat{v}^2}
$$

$$
= 2S\hat{v} \sum_{s=0}^{S-1} \binom{S-1}{s} (-1)^s e^{-(s+1)\hat{v}^2},
$$

where $(b)$ is the binomial expansion of $(1 - e^{-\hat{v}^2})^{S-1}$. The expected value of $\hat{V}$ is
Calculate the RF beamformer

1: \( F_{SS} = 0_{N \times M} \),
2: \( \text{for } m = 1 : M \text{ do} \)
3: \( \mathcal{N}_m = \emptyset \),
4: \( \mathcal{J}_m = \{ \frac{N}{M}(m - 1) + 1, ..., \frac{N}{M}m \} \),
5: \( \text{for } q = 1 : N/S \text{ do} \)
6: \( \mathcal{J}_q = \{ (l - 1)S + 1, ... , lS \} \),
7: \( \text{if } \mathcal{J}_q \subseteq \mathcal{I}_m \text{ then} \)
8: \( \hat{n} = \arg \max_{n \in \mathcal{J}_q} |V_{nm}| \),
9: \( F_{SS, \hat{n}m} = \exp j\angle V_{\hat{n}m} \),
10: \( \mathcal{N}_m \leftarrow \mathcal{N}_m \cup \{ \hat{n} \} \),
11: \( \text{end if} \)
12: \( \text{end for} \)
13: \( \text{end for} \)
14: Return \( F_{SS} \).

expressed as

\[
E[\hat{V}] = \int_{-\infty}^{+\infty} f_V(\hat{v})\hat{v}d\hat{v} = 2S \sum_{s=0}^{S-1} \left( \begin{array}{c} S - 1 \\ s \end{array} \right) (-1)^s \int_0^{+\infty} \hat{v}^2 e^{-(s+1)\hat{v}^2} d\hat{v}
\]

\[
= \sum_{s=0}^{S-1} \left( \begin{array}{c} S - 1 \\ s \end{array} \right) (-1)^s S \sqrt{\pi} \frac{1}{2(s + 1)^{3/2}}
\]

where \((c)\) results from \( \int_0^{+\infty} \hat{v}^2 e^{-(s+1)\hat{v}^2} d\hat{v} = \sqrt{\pi}/4(s + 1)^{3/2} \) [95]. As a result of (5.14) and (5.17),

\[
\frac{1}{\sqrt{L}}HF_{SS} = \frac{U \Sigma_{1:M} V_{1:M}^H F_{SS}}{\sqrt{L}}
\]

\[
= \sum_{s=0}^{S-1} \left( \begin{array}{c} S - 1 \\ s \end{array} \right) (-1)^s \sqrt{S\pi} \frac{1}{2\sqrt{M}(s + 1)^{3/2}} U \Sigma_{1:M}.
\]

The performance of the proposed system with ZF at the baseband can be derived following the steps in (5.7) and (5.8). In this case the spectral efficiency by the proposed
5.5 Simulation Results

In this section, computer simulations are used to evaluate the performance of the proposed antenna selection techniques for the subconnected structures shown in Fig. 5.1b to Fig. 5.1d. In addition, the closed-form expressions in (5.8), (5.13) and (5.19) will be examined when \( N \to \infty \) does not hold. Monte-Carlo simulations over 1000 realizations for \( M = K = 4 \) and \( \rho = 10 \) dB are used to assess the performance. Figure 5.2 shows the tradeoffs between the spectral efficiency and the total number of the phase shifters \( ML \) when \( N \) is fixed. In order to guarantee that the properties of massive MIMO are observed and the hybrid beamformer of (5.6) is close to optimal, \( N \) is set to a large number as \( N = 512 \). It is noted that the fully-connected switch network provides more flexibility between the number of the input and output ports which is

\[
R_{SS} = M \log_2 \left( 1 + \frac{\left( \sum_{s=0}^{S-1} \frac{(S-1)}{s} \frac{(-1)^s}{(s+1)^{3/2}} \right)^2 PS \pi}{4MTZF\sigma_z^2} \right). \tag{5.19}
\]

It is noted that (5.19) is a generalization of (5.8) as for \( S = 1 \), then \( R_{SS} = R_{\text{sub}} \).
not possible with 1-out-of-$S$ switches. When $ML/N = 0.75$, Fig. 5.2 indicates that the fully-connected switch networks with phase shifter selection provides slightly higher spectral efficiency compared to the the structure of Fig. 5.1b. In addition, compared to the scenario that each antenna has a phase shifter, the number of the phase shifters can be reduced to 50% without a performance loss when a fully-connected switch networks with $ML/N = 0.5$ is used. Figure 5.2 also shows that when a simple binary switch is used, i.e. $S = 2$, the loss of the achievable rate is less than 1 bits/Hz/s compared to soft selection with subconnected structure. It is observed that the proposed method with $S = 4$, or equivalently $ML = 128$ phase shifters, achieves around 93% of the spectral efficiency compared to the scenario that $ML = 512$. Figure 5.2 also shows that there is good match between the simulation results and the closed-form expressions of (5.8), (5.13) and (5.19) for various ratios of the number of inputs to outputs.

Figure 5.3 shows the impact of the number of the antennas on the accuracy of the closed-form expressions of spectral efficiency. It is assumed that the ratio of inputs to outputs is $ML/N = 0.5$. At $N = 32$, 12% error between the simulation results and (5.13) and (5.19) is observed. This is due to the fact that $L = N/2M = 4$ is small and, hence, the law of large numbers does not hold. When $L$ increases to 8, the
error between the simulations and analytical results reaches to around 3%. Figure 5.3 indicates that equations (5.8), (5.13) and (5.19) can provide a good approximation of the performance when $16 \leq L$. Finally, Fig. 5.4 presents the achievable rates by the proposed beamformer with binary switches when $\rho$ varies. Compared to the structure of Fig. 5.1b with $L = N/M$ phase shifters, it is observed that the performance loss due to the use of binary switches is almost negligible at the high SNR regime.

5.6 Summary

In this chapter, we investigated the performance of hybrid beamformers when the RF beamformer consists of a combination of subconnected phase shifter network with fully-connected/subconnected switch networks. The proposed beamforming methods and the closed-form expressions of their spectral efficiencies were derived based on the properties of the singular vectors of the channel matrix when the propagation environment is modeled by Rayleigh fading. Such structures reduce the power consumption of hybrid beamformers with phase shifters only as switches require significantly lower power to operate compared to the phase shifters. Specially, in massive MIMO systems where
the number of the phase shifters is large. It was shown that the fully-connected switch network provides slightly better performance compared to the subbonnected structure. However, due to the simplicity of the second approach and lower insertion losses and crosstalks, the subconnected structure is preferred in practice. Compared to hybrid beamforming with phase shifters only, the analytical and simulation results indicated that this method can achieve more than 92% of the spectral efficiency when the number of the phase shifters is reduced by 75%.
Chapter 6

Conclusions and Future Works

Massive MIMO with digital beamforming is one of the promising technologies to provide high data rates. However, such systems are costly and power hungry due to the large number of RF chains that are required to connect the antennas to baseband. Since phase shifters are significantly cheaper and have lower power consumption compared to RF chains, hybrid beamformers are considered as an enabling technology to achieve the promises of massive MIMO. However, the design of hybrid beamformers is a challenging task due to the nonconvex nature of the problem.

This thesis addressed some of the key challenges in the design of narrow-band hybrid beamformers with perfect CSI for massive MIMO systems. In following, firstly the summary and conclusions of this research will be presented. Then, considering the state-of-the-art survey on hybrid beamforming that was presented in chapter 2, future directions of this research will be introduced.

6.1 Summary and Conclusions

In this thesis, first we provided a comprehensive literature survey to identify the gaps in the existing works. To the best of our knowledge, there were no closed-form expressions for the hybrid beamformers as well as the spectral efficiency achieved by them. In addition, most of the works were focused on sparse scattering channels and they were
not applicable to other scenarios such as rich scattering channels. High computational complexity associated with most of the state-of-the-art techniques was another limiting factor for the implementation of hybrid beamformers in massive MIMO scenarios. Especially, the use of digital phase shifters with discrete resolution can result in a dramatically high complexity and loss of performance. Last but not the least, the power consumption associated with the massive number of phase shifters can be very high. To address these challenges, the contributions of this thesis are summarized as following.

In chapter 3, hybrid beamforming for point-to-point systems was investigated and the closed-form expressions of the precoder, combiner and the performance were presented. To this end, firstly we derived a practical RF codebook for analog and hybrid beamforming with low-resolution phase shifters for mmWave systems. As shown, the optimized codebook achieves a promising performance in this practical configuration as it can reach maximum array gain in any desired direction. Then, we proposed a low-complexity channel estimation algorithm which is based on an enhanced one-sided search and is capable of estimating AoAs and AoDs with a high accuracy, low power and small number of measurements. We also investigated the trade-offs between different design parameters such as the resolution of the phase shifters, number of the RF chains and the required power for the channel estimation phase.

The disadvantage of the RF codebook based hybrid beamformers is that they can only perform for special channel scenarios, e.g. mmWave channels. Hence, we derived an asymptotically optimal hybrid beamforming scheme to maximize the spectral efficiency for the point-to-point systems with large antenna arrays that are operating in rich and sparse scattering channels. The optimality of the solution was proved based on the properties of the singular vectors of the channel matrix. The elements of these vectors have a complex Gaussian distribution for Rayleigh fading model, and the singular vectors are equal to the steering vectors of the channel matrix for the geometry based model. In addition, we derived the closed-form expressions for the spectral efficiency when the proposed hybrid beamformer is used. It was shown that the performance of the hybrid beamformer, employing phase shifters with more than 2-bits of resolution, can approach the performance of a similar system with analog phase shifters.
In chapter 4, the hybrid beamformer for the point-to-point MIMO systems was extended to single-antenna and multiantenna multiuser scenarios. Mapping the channel matrix into a smaller dimension by using its singular vectors, the sum-rate of the effective channel was shown to be equal to the original channel. It was shown that the hybrid precoder almost achieves the same sum-rate by digital zero-forcing and block diagonalization. For the multiantenna user equipment and rich scattering channels, the proposed technique almost reaches the performance of a system that the received signals of all the users can be jointly processed. For the sparse scattering channels, this algorithm achieves a dramatically higher sum-rate compared to ZF. Moreover, 3 bits of resolution for the phase shifters is enough for the proposed algorithm to achieve the performance of analog phase shifting.

In chapter 5, two combinations of switches and phase shifters are introduced and investigated to reduce the power consumption of hybrid beamformers in massive MIMO systems. This is due to the fact that, in general, switches require significantly lower power compared to phase shifters. First, proposing a phase shifter selection technique, it was shown that the spectral efficiency can be increased while the power consumption is reduced. Simulation results indicated that spectral efficiency improves when up to 50% of the phase shifters are turned off. The proposed phase shifter selection scheme can be also applied to fully-connected phase shifters network.

Then, we investigated the performance of hybrid beamformers when the RF beamformer consists of a combination of subconnected phase shifter network with fully-connected/subconnected switch networks. The proposed beamforming methods and the closed-form expressions of their spectral efficiencies were derived based on the properties of the singular vectors of the channel matrix when the propagation environment is modeled by Rayleigh fading. Such structures reduce the power consumption of hybrid beamformers with phase shifters only as switches require significantly lower power to operate compared to the phase shifters. Specially, in massive MIMO systems where the number of the phase shifters is large. It was shown that the fully-connected switch network provides slightly better performance compared to the subbconnccted structure. However, due to the simplicity of the second approach and lower insertion losses and crosstalks, the subconnected structure is preferred in practice. Compared to hybrid
beamforming with phase shifters only, the analytical and simulation results indicated that this method can achieve more than 92% of the spectral efficiency when the number of the phase shifters is reduced by 75%. In future, we are aiming to analyze the energy efficiency of such structures.

6.2 Future Works

The hybrid beamforming methods investigated in this research were developed and evaluated under certain assumptions such as the availability of perfect CSI, narrowband systems, no RF impairments and ideal lossless hardware. However, in order to integrate hybrid beamformers into practical systems, the impact of these parameters should be investigated. Hybrid beamforming problem under more realistic scenarios requires solving many more research problems including imperfections that can degrade the performance. In this section, we only focus the research topics that are associated with channel estimation.

1. Hybrid beamforming with erroneous channel state information: In general, CSI acquisition for MIMO systems is a challenging task due to the presence of noise and the estimation errors caused by uncertainties in time division duplex (TDD) or frequency division duplex (FDD) modes. In this thesis, it was assumed that perfect CSI is available to the transmitter and receiver. However, this assumption can never be fulfilled in real systems due to the presence of noise. Investigating the performance of the proposed hybrid beamforming algorithms when channel estimation errors are introduced is the first step towards extending this research. This analysis provides information regarding the required accuracy of the estimated channel and necessary conditions for our hybrid beamformer to operate.

2. Channel estimation and calibration for hybrid beamformers in time division duplex systems: Channel estimation for hybrid beamformers is a challenging task as the number of the receive antennas is significantly larger than the number of the RF chains. Hence, it is necessary to include the impact of the RF beamformer into account as it can dramatically attenuate the effective channel that is observed
at baseband. Moreover, the base station uses the estimated channel in uplink for downlink transmission in TDD operation. However, the transceiver circuits have different behaviors in uplink and downlink and calibration techniques are necessary. To the best of our knowledge, such calibration techniques for hybrid beamformers in massive MIMO system are not investigated and they should be investigated as a future research area.

3. **System level analysis of the proposed channel estimation and hybrid beamforming:**

In real systems, the channel changes over the time and channel estimation must repeat after the coherence time of the channel is over. Investigating the tradeoffs between the tolerable estimation error, the signaling overhead and data transmission within the coherence time of the channel are necessary steps towards the implementation of hybrid beamformers. In other words, the target of this step is to optimize the signaling and data transmission time given the performance of the beamformer is directly related to the accuracy of the estimated channel.

4. **Hybrid beamforming with reduced CSI:** The signaling overhead for channel estimation increases with number of the transmit antennas. This becomes a huge burden for massive MIMO systems to operate in FDD mode. However, using the second order statistics of the channel and the location to set the RF beamformer, the signaling can be significantly reduced. For example, by grouping the users into clusters according to their location, the RF beamformer can increase the SNR of the users in each cluster. Then, baseband precoder can mitigate the interference between the users in each cluster by using accurate CSI of the effective channel. This approach can even simplify the channel estimation in uplink and TDD scenarios where the base station estimates the effective channel that includes the RF beamformer and the wireless channel.

5. **Hybrid beamforming with phase shifters and switches:** In this thesis, we proposed several combinations of phase shifters and switches to reduce the power consumption of hybrid beamformers. It was shown that such structures can result in a good performance under ideal conditions, e.g. when the insertion losses of phase shifters and switches were not included. Further analysis and evaluation of the
proposed methods are required to decide whether the proposed methods can be applied in real systems. For future research, the impact of the losses of the phase shifters and switches, the tradeoffs between power and spectral efficiencies need to be investigated.
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