Isomer building blocks and K-forbidden decays

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Abstract

High-K isomers are long-lived excitations of deformed atomic nuclei. Their structure is built from broken nucleon pairs that generate high angular momentum, K, along the nuclear symmetry axis. The partial conservation of this quantity leads to strong inhibition of electromagnetic decay, and hence to isomerism. The present work examines the hindrance factors for a range of multipole orders, with a focus on highly K-forbidden $E1$ transitions from multi-quasiparticle isomers in the $A \approx 170$ region of nuclei. Allowing for a general 10^4 inhibition of $E1$ transitions, there is good accord with other multipole orders. A key feature is that the inhibition declines for isomers that are more highly excited, relative to a rigid rotor of the same total angular momentum. Comparison is also made with K-forbidden $E1$ transitions in the quasicontinuum, and similar inhibition properties are found.

1. Introduction

It is now exactly one hundred years since Soddy hypothesised the existence of states in a given nuclide that are "different in their stability and mode of breaking up" [1], but it was almost another twenty years before the theoretical underpinning of von Weizsäcker [2] led to a proper understanding of the metastability of some excited nuclear states. The key aspect is that excited states can sometimes only decay internally by large changes in angular momentum (or spin) and decay energies can be small. Both of these effects conspire to make long half-lives for γ-ray emission.

The term "isomer" comes from chemistry, representing different physical arrangements of a given set of building blocks. In nuclei, it is the protons and neutrons that can be in different arrangements of their quantised orbits. When the only possible internal decay of an excited state involves a large spin change, then the state may be referred to as a "spin isomer". This is the most common form of nuclear isomerism, but there are also two other causes.

Another kind of isomer arises due to the vector nature of angular momentum, which applies to deformed nuclei with an axis of symmetry. The projection, K, of the spin on the symmetry axis is approximately conserved, and γ-ray transitions are called K forbidden if the change in K, ΔK, is greater than the angular momentum of the transition, λ. The degree of forbiddenness is defined as $\nu = \Delta K - \lambda$. A conceptual illustration is given in figure 1, where an $I = 8$, $K = 8$ state decays by a $\lambda = 1$ transition to an $I = 8$, $K = 0$ state (a member of the rotational band built on the ground state). The $\lambda = 1$ transition is seven-fold K-forbidden ($\nu = 7$) and its very existence shows that K is not strictly a good quantum number. Nevertheless, in the case of ^{180}Hf, for example, the half-life of the isomer is 5.5 hours, and the transition is inhibited by a factor of 10^{16} compared to the Weisskopf single-particle estimate [4, 5], i.e. the Weisskopf hindrance factor is $F_W = 10^{16}$. Evidently, the K quantum number has a large influence on the transition rate, and thus on the half-life of the "K isomer".

The third type of isomer is the "shape isomer", where a change in nuclear shape is required for internal decay. The rearrangement of nucleon orbits leads to inhibition of the corresponding γ-ray transition. This type of isomer includes fission isomers [9], where fission competes with (and may dominate) internal decay.

The half-life of a given isomer could depend on some combination of the above three mechanisms. For example, the $T_{1/2} = 31$ year, $K = 16$ isomer of ^{178}Hf shows both spin and K isomerism. It must decay by (at least) a $\lambda = 3$ transition [6], which has a low energy of 13 keV and $\nu = 5$. The K inhibition is evident from its Weisskopf hindrance factor of 10^9, but even without that, the $E3$ decay half-life would be approximately one second (which is long compared to the half-life of a typical $I = 16$ state). In this case, it is notable that electron conversion also has a large influence. The 13 keV, $E3$ transition has a total conversion coefficient of 1.3×10^7 [7]. If all the atomic electrons were to be removed (which is possible in a storage ring [8]) the half-life would increase from 31 years to about 150,000 years.

There is no definition of the minimum half-life that is required for an excited nuclear state to be called an isomer. For spin and K isomers in medium-mass and heavy nuclei, a few nanoseconds can be sufficient for recoil and electronic-timing techniques to separate isomers from the plethora of prompt radiation from beam-induced nuclear reactions. This provides excellent sensitivity for the detection of isomeric decays. However, shorter-lived states may also be referred to as isomers, so long as some decay inhibition is demonstrable. Examples include fission isomers in heavy nuclei with half-lives down to a few picoseconds [9], and shape isomers in light nuclei [10], such as the 1-ps first excited 0^+ state of ^{10}Be. At the other half-life extreme, only one isomer, in ^{180}Ta, is so long-lived ($> 10^{16}$ years [11]) that it is naturally occurring in the Earth's crust.

Recent isomer reviews [5, 12, 13, 14] give much experimental and theoretical information, and more-general aspects and applications have been discussed in less-specialised journals [15, 16, 17]. The present work builds

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on the analysis by Patel et al. [18] of highly K-forbidden $E1$ transitions from multi-quasiparticle isomers. The aim is to present a unified view of the K inhibition of γ-ray transition rates.

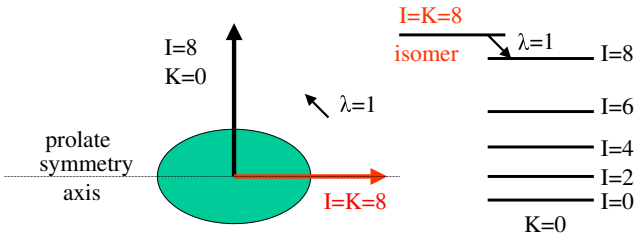

Fig. 1: Conceptual view of K-isomer decay, for a $\lambda = 1$, K-forbidden transition from a $K = 8$, $I = 8$ state to a $K = 0$, $I = 8$ state, from Ref. [3] with permission.

2. Building blocks

Each nucleon in a deformed, axially symmetric nucleus has a well defined angular momentum projection, Ω, on its symmetry axis [19]. In a nuclear ground state, there are at most two unpaired nucleons, as occurs in any odd-odd nuclide. If sufficient energy is available, neutron and proton pairs can be broken, forming multi-quasiparticle states, where the number of quasiparticles is equal to the number of unpaired nucleons. The K quantum number is the sum of Ω values of the individual quasiparticles: $K = |\Omega_1 \pm \Omega_2 \pm \Omega_3 ...|$. Experimentally, the K value of an isomeric state (in a deformed nucleus) is taken to be the same as the spin value of the isomer, though, in the absence of a significant half-life, more careful consideration would be needed [14].

Model calculations should, at the very least, be able to reproduce the spin, parity and excitation energy (to within two or three hundred keV) of any multi-quasiparticle isomer. Other discriminating observables include the magnetic dipole moment and the associated rotational band properties, such as its moment of inertia and $(\Delta I = 1)/(\Delta I = 2)$ γ-ray branching ratios [5, 13, 14].

Quantities derived from the bandhead half-life show strong K-value dependence, but this aspect is more subtle and depends substantially on systematic comparisons rather than model calculations. In particular, there is sensitivity to different K-mixing mechanisms. First, however, the dependence on decay multipole order and decay energy is typically allowed for by calculating, for any particular decay branch, the Weisskopf hindrance factor, and hence the reduced hindrance, $f_\nu = (F_W)^{1/\nu}$. This should ideally be independent of ν, in the sense that transitions with different ν should have approximately the same f_ν. Variations in f_ν can then be interpreted in terms of K-mixing mechanisms, such as rotational (Coriolis) mixing, loss of axial symmetry, and level-density effects [5, 13, 14, 20, 21].

In this work, two particular aspects will be discussed. First, the extra Weisskopf hindrance of $E1$ transitions is considered, providing support for the commonly used factor of 10^4. Second, using this factor and with a focus on isomers that involve at least four quasiparticles, $E1$ reduced hindrances are compared with $E2$ reduced hindrances, building on the recent observations of Patel et al. [18].

It is worth mentioning that only the deformed $A = 160 - 190$ region has K isomers that involve four or more quasiparticles with well defined decay radiations [5]. Therefore, all the data presented here are associated with this mass region. The reason for the favouring of $A \approx 170$ nuclides is that this is the most accessible high-mass region where both the neutron and proton Fermi surfaces are high, but not too high, in their respective shells [14], thus generating both strong deformation and high K values.

3. $E1$ hindrance factors

It is well known that Weisskopf hindrance factors are large for $E1$ transitions, even in the absence of K forbiddenness. For example, Löbner [22] refers to factors of 10^3 to 10^7. In their report on ^{180}Os, Venkova et al. [23] are perhaps the first to be explicit about the use of a factor of 10^4 to divide F_W before the reduced hindrance, f_ν, is calculated. An aspect of ^{180}Os which is intriguing in this respect, though not discussed by Venkova et al., is that the $K^\pi = 7^-$ isomer at 1928 keV decays by (among others) two $E1$, $I \rightarrow I-1$ transitions with different degrees of forbiddenness, ν. In the absence of the 10^4 division, the f_ν values differ by a factor of 3.3, while with the 10^4 division the "effective" f_ν values are only a factor of 1.5 different. The numerical quantities are given in table 1.

Other examples [5] where there are at least two $\nu \geq 4$ $(I \rightarrow I-1)$ $E1$ transitions in a given nuclide are also included in table 1. All (except ^{180}Os) have at least one decay from a multi-quasiparticle isomer, involving four or more quasiparticles. For each nuclide where the ν values differ, the initial (unattenuated) variation is reduced when effective values are calculated. (For the nuclides where the ν values are equal, *i.e.* ^{160}Sm with $\nu = 4$, and ^{174}Yb with $\nu = 6$, the *fractional* variation inevitably remains the same, whatever multiplication factor is applied.)

This lesser variation of f_ν, when effective values are used, is perhaps most clear when the four different ^{174}Hf data points are displayed graphically, as in figure 2. The variation by more than a factor of eight reduces to less than a factor of three. Nevertheless, there is seen to be a reduction in f_ν as ν increases – and this effect can be found consistently in the other cases in table 1.

The physical explanation, for the observation that f_ν decreases as ν increases, could have its origin in rotational K mixing. First, it should be noted that a larger-ν transition results in the populated state being further in spin from its bandhead (even though, with the restriction to $I \rightarrow I-1$ transitions, the absolute spin of the populated state is unchanged if the transitions come from the same isomer). It is therefore straightforward to see that larger ν is associated with more collective spin in the populated state, hence more Coriolis K mixing. However, there is not yet an appropriate framework to evaluate this quanti-

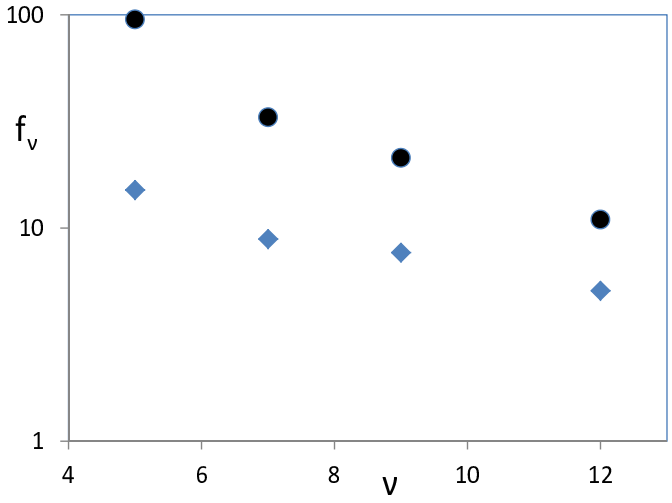

Fig. 2: Reduced hindrance, f_ν, for $E1$ ($I \rightarrow I-1$) decays with no attenuation (black circles) and effective values (blue diamonds) as a function of degree of forbiddenness, ν, from the ^{174}Hf four-quasiparticle isomer.

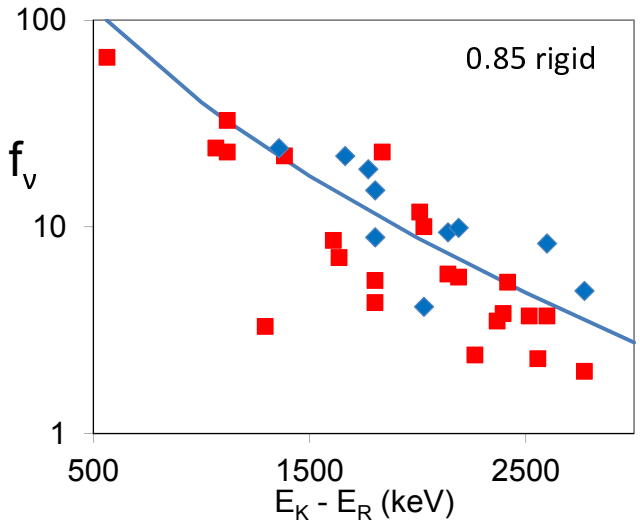

Fig. 3: Reduced hindrance, f_ν, for $E1$ decays (blue diamonds, effective f_ν values) and $E2$ and $E3$ decays (red squares) with forbiddenness, $\nu \geq 4$ from multi-quasiparticle isomers in even-even and odd-A nuclei, as a function of excitation energy relative to a rotor with 85% of the full rigid-body moment of inertia. Quasiparticle numbers range from four to nine ($I = 12 \rightarrow 57/2$). For odd-A nuclei, a pairing-gap energy of 0.9 MeV has been added. The full line represents the expected level-density dependence [24]. The figure is adapted from Refs. [13, 18].

tatively.

Notwithstanding complications of this kind, Patel et al. [18] analysed a broad range of data from multi-quasiparticle isomers, each involving at least four quasi-particles. They identified nine nuclides in the $A \approx 170$ region with strongly forbidden ($\nu \geq 4$) $E1$ decays, providing ten $E1$ ($I \rightarrow I-1$) decays (though the less-intense 340 and 539 keV transitions from the ^{174}Hf four-quasiparticle isomer were not included). Effective values cover the range, $f_\nu = 4 \rightarrow 24$. It was found (see figure 3) that the smaller f_ν values tend to be from isomers at higher excitation energies (relative to a rigid-rotor reference). This is very similar to the behaviour of $E2$ and $E3$ transitions, as discussed in the next section.

Although not specified here, the f_ν uncertainties are given by Kondev et al. [5]. They are typically smaller than the size of the data points of figure 3. Of greater risk to the interpretation could potentially be the K-value assignments, in each case taken to be equal to the bandhead spin. This can sometimes be poorly defined [14].

4. $E2$ and $E3$ hindrance factors

$E2$ and $E3$ decays from multi-quasiparticle isomers appear to be sensitive to the level density, in the sense that more highly excited isomers have lower reduced-hindrance values. This is physically intuitive, because close-lying states with the same spin and parity, but different K values, will have mixed-K wave-functions, and the amount of mixing will depend on the level density at that spin value. Following earlier work [24], where the level-density estimate involved a rotor subtraction with the full rigid-body moment of inertia, it was found [25] that a rotor reference with a moment of inertia that is 85% of the full rigid-rotor value gives a better f_ν ($E2$) correlation when a large range of angular momentum values is involved. The data from Refs. [5, 13] are shown on figure 3. It should be noted that the considerable fluctuations indicate the influence of other K-mixing mechanisms. In particular, the role of rotational (Coriolis) K mixing has been mentioned in the previous section, as well as being discussed elsewhere in more detail [13, 14].

It is striking to see that the $E1$ transitions shown in figure 3 have essentially the same magnitude and excitation-energy dependence as the $E2$ and $E3$ transitions, implying the same level-density dependence, and this is with the inclusion of the 10^4 division of F_W to obtain effective $E1$ reduced-hindrance values. Although Kondev et al. [5] warn that, for $E1$ transitions, the division of the Weisskopf hindrance by large factors "can cause confusion", there does seem to be justification, because the $E1$ transitions then follow the same excitation-energy dependence as $E2$ and $E3$ transitions, albeit with fluctuations.

5. Other multipole orders

There are, in fact, only two $E3$ data points in figure 3, for ^{177}Hf and ^{178}Hf, with $\{E_K - E_R$ (keV)$,f_\nu\}$ values of $\{1110,23\}$ and $\{564,66\}$ respectively. Furthermore, there is only one higher-multipole transition (not in the figure) with $\nu \geq 4$, which is an $M4$ transition in ^{178}Hf $\{564,72\}$, *i.e.* from the same $K^\pi = 16^+$ isomer as the $E3$ transition and with a very similar f_ν value.

In contrast, there are many highly K-forbidden $M1$ transitions. These appear to have, in general, slightly higher reduced-hindrance values, compared to $E2$ transitions, but typically the degree of $M1/E2$ mixing is unknown. (For the electric transitions, it is a reasonable assumption that there are at most only small admixtures of

Table 1: Nuclides with high-K isomers involving at least two $\nu \geq 4$, $E1$ ($I \rightarrow I-1$) transitions. For each is given its K^π, half-life, excitation energy, $E1$ γ-decay energy, degree of K forbiddenness and reduced hindrance [5, 18], as well as the effective reduced hindrance (final column).

	K^π	$T_{1/2}$ (ns)	E (keV)	E_γ (keV)	ν	f_ν	f_ν [a]
^{180}Os	7^-	26	1928	302	4	103	10
				1134	6	31	6.8
^{179}W	$35/2^-$	750	3348	625	12	8.6	4.0
	$21/2^+$	390	1632	567	6	57	12
^{175}Hf	$45/2^+$	1940	4636	291	4	219	22
	$19/2^+$	1100	1434	615	6	40	8.7
^{174}Hf	14^+	3700	3312	155	5	95	15
				340 [b]	9	21	7.7
				379	7	33	8.9
				539 [b]	12	11	5.1
^{174}Yb	14^+	55	3699	786	6	46	9.8
	7^-	256	1765	1239	6	38	8.1
^{164}Er	12^+	68	3378	555	4	93	9.3
	7^-	23	1985	1371	6	41	8.8
^{160}Sm	11^+	1800	2757	641	4	220	22
	7^-	120	1361	1128	4	183	18

[a] Effective values, with F_W divided by 10^4 before taking the ν^{th} root.
[b] Decays not included in figures 3 and 4.

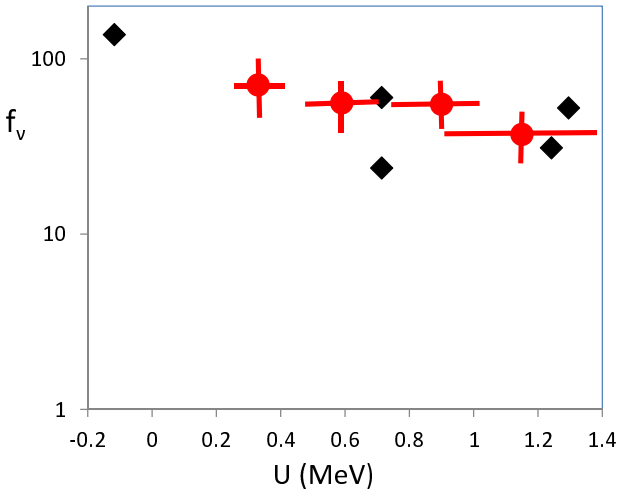

Fig. 4: Reduced hindrance, f_ν, for $E1$ decays from selected – see text – discrete isomers (black diamonds) and the quasicontinuum analysis of Vandone et al. [26] (red circles) with a factor of ten attenuation of the Weisskopf hindrance factors for both data sets. The values are shown as a function of the energy, U, relative to the corresponding yrast line. For the quasicontinuum data [26], the error bars on the energy scale refer to the interval considered for the averaging over the excited levels. Note that there is a negative energy for one discrete-isomer case (^{182}Hf) because its $K^\pi = 13^+$ isomer falls lower in energy than the ground-state band (assuming a smooth interpolation between the 12^+ and 14^+ members).

higher-multipole magnetic transitions, but it can be a poor assumption that $M1$ transitions have only small $E2$ admixtures.) Therefore, it is not possible to evaluate the $M1$ reduced-hindrance component in a consistent and quantitative manner. Nevertheless, apart from $M1$ transitions being excluded, the reduced hindrances of the other multipole orders behave in a remarkably consistent manner as a function of excitation energy.

The above comments apply to states in even-even and odd-A nuclides involving at least four quasiparticles. There are still challenges with odd-odd nuclides and lower quasiparticle numbers, where pairing may have greater influence. Further discussion can be found in Refs. [5, 13, 14]. For the multi-quasiparticle decays represented in figure 3, one of the next challenges is to quantify the the contributions of the collective rotational (Coriolis) and vibrational (γ-distortion) K-mixing effects in a consistent manner.

6. Comparison with quasicontinuum data

As part of a programme to study the transition from order to chaos with increasing excitation energy, the erosion of the K quantum at high angular momentum in ^{174}W has been studied by Vandone et al. [26] using quasicontinuum γ-ray spectroscopy. The quasicontinuum is made up from many $E1$ transitions, so that comparison with discrete $E2$ transitions depopulating isomeric states is not straightforward [13, 26]. However, now that the treatment of discrete $E1$ transitions has been clarified, as discussed above, a more direct comparison can be made with the quasicontinuum $E1$ transitions.

The analysis of Vandone et al. [26] uses the energy relative to the *yrast* line (the locus of lowest-energy states at each angular momentum). In the present work, therefore, the choice of $E1$ decaying isomers has been restricted to even-even nuclides where the yrast band is known up to at least the isomer's angular momentum, forming a subset of the cases analysed by Patel et al. [18], namely ^{164}Er ($K^\pi = 12^+$), ^{174}Yb ($K^\pi = 14^+$), ^{174}Hf ($K^\pi = 14^+$; two decay branches) and ^{182}Hf ($K^\pi = 13^+$).

Another significant feature is the choice of attenuation of the $E1$ Weisskopf hindrance factors. Vandone et al. used a factor of ten [27], rather than the 10^4 factor used in section 3 above. Although a smaller attenuation factor leads to a spread of effective f_ν values that is non-optimal, as outlined in section 3, the change in attenuation factor is simple to make. The result is shown in figure 4. In this way, a consistent analysis is obtained, with good accord between the discrete-isomer and quasicontinuum data sets; and the important role of level density in the determination of K-forbidden transition rates is supported.

7. Summary

The basic structure of high-K, multi-quasiparticle isomers has been described, and their K-forbidden electromagnetic decay rates have been discussed. The treatment of $E1$ transition rates by Patel et al. [18] has been further

developed. In addition, a realistic comparison with K-forbidden $E1$ transitions in the quasicontinuum has been achieved. The overall decline in hindrance with increasing excitation energy is supported.

Acknowledgement

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