Security of Distance-Bounding: A Survey

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Distance bounding protocols allow a verifier to both authenticate a prover and evaluate whether the latter is located in his vicinity. These protocols are of particular interest in contactless systems, e.g., electronic payment or access control systems, which are vulnerable to distance-based frauds. This survey analyzes and compares in a unified manner many existing distance bounding protocols with respect to several key security and complexity features.


Additional Key Words and Phrases: Information Security, Cryptography, Contactless, Relay attacks, Distance fraud, Mafia fraud, Terrorist fraud, Distance bounding, Proximity check

1. INTRODUCTION AND STATE-OF-THE-ART

1.1. From Relay Attacks to Evolved Distance-based Frauds

As explained in [Hancke et al. 2009], the basic concept of a relay attack was first described by Conway [Conway 1976] in 1976, in a scenario referred to as the "Chess Grandmaster Problem". In this scenario, any player could play against two Grandmasters by challenging both of them to a game of chess by post. The player would then simply forward the move received from one Grandmaster to the other, effectively making them playing against each other. This results in the player either winning one match, or earning a draw in both matches. Desmedt, Goutier, and Bengio [Desmedt et al. 1988] extended this concept to security protocols in 1987, with an attack on the Fiat-Shamir protocol [Fiat and Shamir 1986; Feige et al. 1987] they named "mafia fraud". In general, a protocol is seen to be executed between a party making a claim, the prover, and a party verifying this claim, the verifier. The mafia fraud involves a malicious third party who aims to convince the verifier that he is the legitimate prover. To start, the third party simply takes all the messages sent by the verifier and forwards these to the prover. As the messages are legitimate, the prover believes he is
communicating with the legitimate verifier. The prover then generates a valid response which the third party forwards to the verifier. Upon receiving this response, the verifier is convinced that he is communicating with the legitimate prover and the attack succeeds. A variant of the mafia fraud, denoted by terrorist fraud, is an attack in which the prover colludes with the adversary to deceive the verifier, and was subsequently proposed by Bengio et al. [Bengio et al. 1991]. In practice, this involves a prover sharing protocol information, other than key material, with a third-party in such a way that he allows this third-party to convince the verifier that he is the legitimate prover without having to relay all the verifier’s messages.

Even though the mafia fraud could be classified as a special type of man-in-the-middle attack, there are fundamental differences between these attacks. In man-in-the-middle attacks, the third party actively modifies messages between the verifier and the prover, and in general the attack is made possible through a security vulnerability in the protocol. In other words, man-in-the-middle attacks can be mitigated with conventional security mechanisms. In mafia frauds, the third party is passive and simply relays messages. He does not need to perform any further logical attack on the messages or the protocol sequence and in fact the third party does not even need to know what he is relaying. The protocol and security mechanisms are irrelevant as the attacker just relays the entire message generated by the legitimate parties, regardless of their content, thereby ensuring that both the verifier and the prover always receive a valid message. Conventional security mechanisms are therefore not an effective countermeasure.

Brands and Chaum early proposed the idea of using so-called distance-bounding protocols [Brands and Chaum 1993] to mitigate mafia fraud. In addition to mafia fraud, Brands and Chaum also considered the possibility of distance fraud. Distance fraud involves a fraudulent prover that wants to convince the verifier that he is closer than he really is. Most recently, a new fraud termed distance hijacking was proposed [Cremers et al. 2012]. In this case, a fraudulent prover takes advantage of a protocol executed between an honest prover and the verifier. The fraudulent prover selectively uses parts of this protocol instance to convince the verifier that he is at a distance, at which some other honest prover resides, which differs from the actual distance of the dishonest prover to the verifier.

1.2. Practical Attacks

The frauds discussed above are of practical significance when considering real-world system security. For example, mafia frauds are especially relevant in access control and payment systems. An RFID door access system might authenticate an access token by transmitting a challenge, e.g., a nonce, and then checking whether the cryptographic response, constructed with the token’s key, is valid. In such a case, an attacker can present a proxy-token, a device under the attacker’s control that emulates a token, to the door reader. At the same time his accomplice has a proxy-reader, a reader under the attacker’s control, which is used to communicate with a legitimate token. This can be done in a covert manner, e.g., holding the reader against the token holder’s pocket while he is outside the premises. The attacker’s proxy-token gets the challenge from the door reader and transmits it to the accomplice’s proxy-reader. The latter sends the challenge to the legitimate token. The proxy-reader thus obtains the valid response, which is transmitted to the proxy-token and then sent to the door reader. The door reader is now convinced that the token it is communicating with is the legitimate token and opens the door. A practical mafia fraud of this nature was first demonstrated by Hancke [Hancke 2006], using a built-for-purpose proxy-token and relaying radio channel with an effective range of 50 meters, alongside a modified off-the-shelf reader for the purpose of proxy-reader. Francillon, Danev, and Çapkun have also practically demonstrated the feasibility of mafia frauds against remote keyless entry systems in modern cars [Francillon et al. 2011].

Similarly, payment systems are also vulnerable to mafia fraud. An attacker could convince a customer to insert his payment card into a proxy-reader, perhaps to pay for a low-value item sold to the customer by the attacker. The attacker’s accomplice, in the meantime, purchases a high-value item and inserts his proxy-card into the merchant’s reader. The high-value transaction is then conducted, via the proxy devices, with the legitimate payment card. The proxy-reader only displays the low value amount for customer approval, who thinks that he is authorising the transaction by entering his PIN on the proxy-reader. This PIN is transmitted to the accomplice and it is entered into the merchant’s reader, which then verifies the PIN through the relay setup with the legitimate card. As a causality the customer ends up paying for the attacker’s item. This attack scenario was implemented against the “Chip and Pin” card payment system in the United Kingdom by Drimer and Murdoch [Drimer and Murdoch 2007], and illustrates that mafia frauds can be a serious threat even when systems use strong cryptography and two-factor authentication. The implementation of near-field communication (NFC) in mobile phones has potentially decreased the complexity of implementing mafia fraud. An NFC-enabled mobile phone can act as a token and a reader, so it can either act as a proxy-
token or proxy-reader, while offering multiple options with regards to communication channels for relaying messages. The potential use of NFC devices in mafia frauds is documented by Kfir and Wool [Kfir and Wool 2005], and a practical mafia fraud using NFC-enabled mobile phones has already been demonstrated by Francis et al. [Francis et al. 2010]. Some additional attack scenarios and a discussion on the practical implementation of mafia frauds can be found in [Hancke et al. 2009; Francis et al. 2013].

Real-time location systems (RTLS) are increasingly used to track high-value assets and people. A RTLS relies on the fact that the physical relation between reference nodes, with fixed and known locations, and the target can be estimated. If these estimates are somehow modified by an attacker then the overall localisation process will be adversely affected and the location of the target could be misrepresented. Capkun and Hubaux [Capkun and Hubaux 2006] have shown that in the case of trilateration, and the principle extends to multilateration, a target located outside a triangle of reference nodes cannot prove that it is inside the triangle without shortening the distance measured to at least one of the reference nodes. Similarly, a node located inside the triangle cannot prove it is at a different location without decreasing the measured distance to at least one of the nodes. This means that a fraudulent prover, wishing to misrepresent his own location, must perpetrate distance fraud against at least one reference node. In practice, distance fraud is relatively simple in certain RTLS systems. For example, if the distances are estimated using received signal strength, then an attacker could selectively attenuate or amplify his communication with a specific reference node. Some practical distance-fraud strategies enabling a fraudulent prover to decrease the round-trip-time of his responses are discussed in [Hancke and Kuhn 2008] and [Chulow et al. 2006].

Finally, relay attacks are particularly relevant in the field of digital rights management (DRM), although this issue is rarely discussed in the literature. For example, a provider may refuse to deliver a content to the customer if the latter is not in a clearly defined location, as stated for example in [Nikov and Vaulclair 2008; Abbadi and Mitchell 2007; Camp 2007].

1.3. Countermeasures to Relay Attacks

Mafia fraud is remarkably effective against conventional cryptographic mechanisms. To detect this fraud, we need to look beyond the data exchanged and incorporate the physical context of the interaction between verifier and prover into the protocol. To do so, several methods were proposed to enhance authentication protocols. Desmedt was the first to introduce solutions to counter the mafia fraud. In [Desmedt 1988], he proposed to sign the prover GPS coordinates. In a second proposal [Beth and Desmedt 1990], the notion of timed message exchanges was introduced. Using precise timing, Beth and Desmedt managed to detect the little girl fraud via the delay introduced by the relay. Desmedt, alone in a first time, then later along with Beth, was the first to remark that counting mafia fraud implied relying on physical properties (localization, or timing) rather than only depending on the cryptographic parts. This observation yields to several propositions to measure this physical property. Among them, distance bounding protocols were the most promising counterpart measure. To bound the distance between a prover and a verifier, several methods exist. Distance bounding protocols can be built on the Received Signal Strength (RSS) [Bahl and Padmanabhan 2000], by measuring the Angle-of-Arrival (AoA) [Ghavami et al. 2004], the noise level [Choudary and Stajano 2011], the physical property of the communication channel [Stajano et al. 2010], the ambient environment [Halevi et al. 2013; Uriena and Piramuthu 2014], or the measure of the Time Of the Flight (ToF).

The RSS, and the AoA methods are usually discarded due to implicit security flaws. Indeed, an adversary can by increasing its signal strength or building special antenna deceive these measurements [Cheung 2004]. Methods based on the noise level or on channel properties work in theory. However, they are not practical to implement.

ToF methods are more reliable, and often used to evaluate the distance d between two parties by calculating \( d = \frac{c \cdot p}{1 + \frac{c}{t_p}} \), where \( c \) is the propagation speed of signals on the medium of the communication channel and \( t_p \) is the one-way propagation time between the transmitter and the receiver [Hancke and Kuhn 2005]. An attacker committing mafia fraud will unavoidably increase the time that the message takes to travel between the prover and verifier. Even simply forwarding and transmitting messages increases the ToF. Measuring this time and checking for unexpected delay in a response is therefore recognised as a feasible method for detecting mafia fraud [Beth and Desmedt 1990]. ToF distance estimation comprises both Time-of-arrival (ToA) or round-trip-time (RTT) approaches. ToA requires both a verifier and prover to share a synchronised, high-precision clock and only the propagation time of a single message is measured. For example, the verifier sends a challenge \( \text{chall} \) to the prover, and records the time \( t_0 \) it was sent. The prover records the time \( t_0 + t_p \) the challenge was received and responds with the authenticated message \( (t_0 + t_p, \text{chall}) \). If both the prover and the verifier are trusted, this protocol is effective in detecting mafia frauds. However, it is vulnerable to
distance fraud as the prover can simply decrease the value of \( t_0 + t_p \) in the response to appear closer. From a practical perspective, both the prover and the verifier might not have a synchronised precise clock, e.g., an RFID reader could have such a precise clock but a tag not.

1.4. Distance-Bounding Based on RTT

Both these issues can be addressed using an RTT distance-estimation approach. In RTT, the verifier measures the time \( t_m \) from the moment he has sent a challenge to the moment the response is received. The verifier is therefore completely in control of the measurement and he is also the only entity that requires a precise clock. In this case the verifier can estimate the distance \( d = c \cdot (t_m - t_d) / 2 \), where \( t_m \) is the round-trip-time, equal to \( 2 \cdot t_p + t_d \), and \( t_d \) is the time the prover takes to calculate the response. For example, the verifier sends a challenge \( \text{chall} \) at \( t_0 \), which the prover receives at \( t_0 + t_p \). The prover sends a response back at \( t_0 + t_p + t_d \) and this is received at \( t_v = t_0 + 2 \cdot t_p + t_d \), allowing for the RTT to be calculated as \( t_m = t_v - t_0 \). The fraudulent prover can no longer directly influence the measurement, as is the case with ToA, but he could try to send its response earlier than he receives the challenge. To prevent this, a protocol must be designed in such a way that the response depends on the challenge, i.e., \( r = f(\text{chall}) \), so that the prover has to wait for the challenge before responding. This response function \( f \) also determines the length of the processing time \( t_d \), which must be minimal and deterministic, to achieve an accurate estimate. The response function must therefore be of minimal complexity and should be processable in a short and predictable time.

Distance-bounding protocols are closely linked to aspects of the physical communication channel, a side effect of requiring accurate timing measurements. The channel on which the challenges and responses are to be transmitted must therefore be chosen in such a way that it does not adversely affect the security of the protocol or the accuracy of the distance estimated. Conventional communication channels have been shown to be unsuitable for secure distance-bounding protocols, due to the possibility that an attacker could exploit the latency introduced in these channels by error-resistant measures, such as framing/integrity data and filters in transceivers [Chulow et al. 2006; Hancke and Kuhn 2008]. In practice, building a distance-bounding channel is a hard problem. Even if we only considered the distance estimation requirements, a timing measurement error of 1 ns could result in a distance estimation error of approximately 30 cm, and measuring the RTT to this level of accuracy is not feasible in systems often suggested to benefit from distance bounding. Implementing suitable channels is still an open research question, although there are several proposals already described and practically demonstrated in the literature [Reid et al. 2007; Rasmussen et al. 2009; Rasmussen and Capkun 2010; Hancke 2010; Drimer and Murdoch 2007]. In 2006, Chulow et al. [Chulow et al. 2006] proposed four principles for implementing a secure channel for timed challenge-response exchanges:

1. Use a communication medium with a propagation speed as close as possible to the physical limit, i.e. speed of light.
2. Use a communication format in which only a single symbol is transmitted as challenge of response.
3. Minimize the length of this symbol, or the time taken to decide the value of the symbol.
4. Design the protocol such that it copes with errors during the challenge-response exchange.

These principles have historical significance, as this work was the first to look at the security implications of the underlying implementation of the exchange channel. However, there is a growing opinion that these principles, aiming for theoretical security, are not fully achievable in practice. As such it is perhaps better to consider the intentions behind the principles’ definition, which helps us understand potential security threats and evaluate the effectiveness of a channel used for distance bounding, rather than considering these as hard conditions for secure distance bounding. The first principle advises against the use of channels with a relatively low propagation speed as this would allow an attacker to use a faster channel to relay the communication and not be detected. For example, if distance bounding is conducted across a sound channel the attacker can execute an undetectable relay attack using wired or radio communication. The second principle advises against sending multiple challenges and responses during a single timed exchange, and against the transmission of any additional information even for purposes of error detection or formatting, e.g. any parity bits, cyclic redundancy checks (CRC), headers/trailers or start/stop bits. In both cases it is shown that a dishonest prover could exploit such exchanges to correctly send a reply earlier than what is expected from a honest prover adhering to the channel rules. The nature of the attack depends on the format of the message but the general idea is that the dishonest prover can calculate and prepare the response before the entire challenge message is received, thus shortening the response time compared to a honest prover waiting for the entire message. The third principle advises that the decision as to the value
of the symbol should be made as quickly as possible. If the symbol modulation/encoding is such that the entire symbol must be received the symbol period must be minimised or the receiver should determine the value early on in the symbol period. This is meant to protect against early detect/late send relay attacks, where the attacker can take advantage of the duration between the start of the symbol and when the receiver actually determines its value. For example, when using non-return to zero (NRZ) coding the receiver usually samples the symbol after \( t_s/2 \), where \( t_s \) is the symbol period, which allows for the maximum tolerance to data clock differences between the sender and receiver. If the attacker can sample the symbol at \( t_s/10 \), he has \( 4 \cdot t_s/10 \) to relay its value and transmit it to the receiver. In such a case, there will be no detectable delay in the communication and distance bounding would be ineffective. To minimise the amount of time available to the attacker the receiver must therefore make its decision as early as possible during the symbol period. The fourth principle, taking into account that principles two and three would not allow for conventional error detection/correction measures and reduces the receiver’s tolerances for reliably decoding of data, advises that the protocol cannot expect that the exchange channel will have no communication errors and that this has to be taken into consideration elsewhere in the system.

1.5. Protocol Evolution
Distance-bounding protocols are based on the Round-Trip-Time (RTT) of challenge-response messages, and are essentially meant to detect any unexpected delay in the prover’s response inherently caused by the messages being relayed over a larger distance by a third party [Hancke et al. 2009]. To effectively achieve this goal, protocols must meet some simple requirements to obtain an accurate propagation time measurement, as explained in the previous section: the response and challenge must be single bits, the response must be dependent on the challenge and the time taken to calculate the response must be minimal and predictable. There are a number of protocols that aim to implement distance-bounding but do not adhere to these requirements, e.g., [Beth and Desmedt 1990; Waters and Felten 2003; Nikov and Vaucclair 2008]. However, they are not capable of providing accurate distance estimates because of the variation in the time taken to calculate the response, which makes them unsuitable for many use cases. For example, the time taken by a smart token to encrypt a message or perform a digital signature differs each time. If such a token usually takes 100 ms to calculate a response and if there is even a 0.1% variation, this results in a RTT variation of 0.1 ms and hence a 30000 km distance estimate error. This paper only considers protocols proposals adhering to the prescribed requirements for distance-bounding.

In a distance-bounding protocol, not all exchanged messages are subject to round-trip-time measurements. The protocol can be divided into three distinct phases: setup, exchange, and verification. During the setup phase, the verifier and the prover exchange some initial information and determine the cryptographic material used during the rest of the protocol. During the exchange stage, the verifier measures the round-trip-time of the challenge-response pairs. The validity of the responses and the distance-bound is checked during the verification stage. The setup and verification phases are commonly referred to as the “slow” phases, while the exchange phase is referred to as the “fast” phase, due to the nature of the communication during these phases. The slow phase uses a conventional channel while the fast phase requires a special channel.

The first distance-bounding protocol was proposed by Brands and Chaum [Brands and Chaum 1993]. This protocol, based on Beth and Desmedt’s [Beth and Desmedt 1990] idea that RTT can detect mafia fraud, bounds the distance between the parties by measuring the RTT of single-bit challenges and responses. During the setup phase, the prover cryptographically commits to a random string that he will use to calculate the responses using an XOR operation. During the verification stage the prover signs a message containing the challenges received and the response sent. The protocol achieved an optimal \( \left( \frac{1}{2} \right)^n \) resistance against both mafia and distance fraud, where \( n \) is the number of challenge-response exchanges. This concept formed the basis for numerous protocols, whose evolution is represented on Figure 1.

There are four direct descendents of Brands and Chaum’s protocol: [Peris-Lopez et al. 2010; Rasmussen and Capkun 2010; Capkun et al. 2003; Hancke and Kuhn 2005], each of which improved Brands and Chaum in its own way. Peris-Lopez et al. [Peris-Lopez et al. 2010] propose that cryptographic puzzles should be used to provide privacy in distance-bounding protocols. Rasmussen and Capkun protocol [Rasmussen and Capkun 2010] is based on XOR and a comparison function, and has the benefit that the prover does not need to demodulate the signal to answer to the verifier’s challenges. The MAD protocol proposed by Capkun et al. [Capkun et al. 2003] allows for mutual distance-bounding between the two parties. This protocol was enhanced by the MAD protocol of Singelée and Preneel [Singelée and Preneel 2007], which added bit-error resilience to MAD by using error correcting codes. The Hancke and Kuhn’s protocol [Hancke and Kuhn 2005], originally designed to be used in the RFID environment and is thus optimised for execution time and
Fig. 1: Distance-bounding evolution

minimal prover complexity, uses pre-computation, instead of a commitment step, during the setup phase in such a way that no additional messages need to be transmitted during the verification stage.

Hancke and Kuhn’s protocol has two issues: it does not take terrorist attacks into account, and achieves a sub-optimal performance-security trade-off with respect to the mafia and distance fraud of \( (\frac{3}{4})^n \). Subsequently, numerous proposals based on the pre-computation method used by Hancke and Kuhn were proposed in an effort to improve its performance. Bussard and Bagga’s protocol [Bussard and Bagga 2005] and all its descendants introduce resistance to terrorist fraud. These protocols are based on Bussard and Bagga’s idea that the prover long-term secret is incorporated into the pre-computed response options in such a way that if the prover revealed all the options his accomplice would also get the prover’s key. This therefore discourages the prover to participate in a terrorist fraud, but the cost is a complex proof-of-knowledge operation. Its descendants aim to achieve the same functionality but with decreased computational complexity. Reid et al. [Reid et al. 2007] so improves the computational efficiency but the fraud resistance is \( (\frac{1}{2})^n \) in comparison to Bussard and Bagga’s \( (\frac{3}{4})^n \). Tu and Piramuthu’s protocol [Tu and Piramuthu 2007] proposes a protocol compounded by a succession of fast and slow phases. However, this protocol suffers from several vulnerabilities, discussed in [Kim et al. 2008; Munilla and Peinado 2008b], that reveal the secret to an eavesdropper during a legitimate protocol run. The “swiss-knife” protocol [Kim et al. 2008] fixes the poor mafia fraud resistance problem by adding a third phase to Reid et al.’s protocol, and it also provides mutual authentication. In [Peris-Lopez et al. 2009], the authors claim that they found an attack on this protocol based on nonce repetitions, and thus propose the Hitomi variation. However, if the assumption is made that nonces repeat then Hitomi suffers, to a lesser extent, of a similar flaw. A further variation of the swiss-knife protocol [Avoine et al. 2011] explicitly introduces secret-sharing to counter the terrorist fraud, and studies the best settings in which to use it. The paper also explains several vulnerabilities found in previous protocols.
designed to mitigate terrorist fraud. Avoine and Tchamkerten’s protocol [Avoine and Tchamkerten 2009] introduces binary trees to compute the prover answers during the exchange phase, and succeeds in improving the mafia fraud resilience of the pre-computation protocol to almost $(\frac{1}{3})^n$. Indeed, various graph structures can be used instead of a tree structure. The interest of cyclic and $q$-partite graphs has been demonstrated in [Trujillo-Rasua et al. 2010] and [Mauw et al. 2016], respectively. Finally, Trujillo et al. [Trujillo-Rasua et al. 2014] show that precomputation-based protocols can also deal with noise without sacrificing security.

Munilla and Peinado [Munilla et al. 2006; Munilla and Peinado 2008a] initiated a new branch of the Hancke-Kuhn pre-computation family. They proposed to communicate during the exchange phase using binary symbols, 0 and 1, and also an additional “nothing” state. MUSE [Avoine et al. 2009] is a generalization of this idea by relaxing the number of possible states used during the exchange phase. Kim and Avoine’s protocols [Kim and Avoine 2009; 2011] enhance the attack detection mechanism. Its descendant [Yum et al. 2011] uses the detection mechanism as means to also provide mutual authentication. Finally, Kardaq et al. [Kardaq et al. 2011] introduce the use of PUFs in Hancke and Kuhn’s protocol and claim the protocol now resists to the terrorist fraud.

1.6. Provable Security

Most distance-bounding protocols have been analyzed without a formal approach. Instead, generic best-known attacks are usually adapted to the specific features of the protocol at hand, which has led to unsound analyzes and unfair comparisons. Examples are the protocols proposed in [Munilla et al. 2006]; [Tu and Piramuthu 2007], and [Yum et al. 2011], whose flaws are explored in [Avoine et al. 2011], [Munilla and Peinado 2008b], and [Avoine and Kim 2013], respectively. The first comprehensive formalization for analyzing distance-bounding protocols was proposed by Avoine et al. [Avoine et al. 2011]: this is not a provable security formalism, but it is a framework that can describe attack-scenarios in a unitary fashion, and thus offer a systematic manner of computing upper-bounds on the probabilities of typical attacks in distance-bounding and its variants. This unified framework [Avoine et al. 2011] defines the following important objects: the prover model (depending on the tampering-resistance of the prover, it can be either black-box or white-box); the prover’s computing capabilities (e.g., whether the prover can exploit latencies between the slow and fast phases); and the attacker’s strategies (e.g., pre-ask, post-ask, and early-reply). Details of this formalism are provided in Section 2.

Recent efforts have been made on proving security for distance-bounding [Dürholz et al. 2011; Boureanu et al. 2013a; 2013c; 2013b; Fischlin and Onete 2013b; Vaudenay 2013]. However, this is still a very young field that needs to overcome three main, inter-dependent challenges: (i) the introduction of sound communication, network and adversarial models that capture the notion of time-of-flight, (ii) the definition of clear and rigorous specifications of the classical frauds (i.e., formal definitions of these frauds that can be proven to hold or to be refuted within the model), and (iii) formal security proofs based on cryptographic assumptions. To illustrate for instance the difficulty of the third challenge, [Boureanu et al. 2012] proved that many protocols fall short in having their security based on the pseudorandom function (PRF) assumption of some underlying primitive.

The first formalism in this direction was put forward by Dürholz et al. [Dürholz et al. 2011]. The authors formalize the impossibility of illegitimate yet sufficiently fast round-trip communications using the notion of tainted sessions; to encode timing-restrictions, tainted sessions only allow certain flows of communication. Then, a protocol is said to be secure if no adversary executing it with tainted sessions can violate its security properties. The model comprises a formalisation of all the classical frauds and provides several (partial) security proofs for some protocols [Dürholz et al. 2011; Fischlin and Onete 2013a]. This formal model is a step in the right direction towards provably secure distance-bounding.

Another line on provably secure distance-bounding, which builds on the model by Dürholz et al., is in [Fischlin and Onete 2013b]. One addition therein is proposing a distance-bounding protocol that uses not one but two secret keys, i.e., one for the un-timed and one for the timed phase. This bypasses the aforementioned problem of using just the PRF-assumption to argue the security of (one-key) distance-bounding.

In [Boureanu et al. 2013a; 2013c], the authors provide a rather general model that captures the notion of concurrency (i.e., allowing adversaries to interact with many provers and verifiers, sometimes with the same keys). Their notions of distance and mafia frauds additionally capture the one of distance hijacking [Cremers et al. 2012] and impersonation [Dürholz et al. 2011], respectively. Furthermore, their definition for terrorist-fraud is more general than the notion of terrorist fraud adopted in this manuscript: after the initial collision, the possible threats to protect against may be stronger in [Boureanu et al. 2013a; 2013c] (e.g., MiM in
formalisations of the distance-bounding threats framework by Avoine et al. [Avoine et al. 2011]. This framework does not repose on the security/insecurity of distance-bounding provers and the attackers in order to best classify attack strategies, not exist.

and Onete 2013b] and the model inspired by interactive proofs in [Boureanu et al. 2013a] that suits the provable security of the SKI schemes [Boureau et al. 2015], or less realistic (like the SimTF formulation for terrorist-fraud resistance in [Dürholz et al. 2011]), or might be too general (like aforementioned collusion-fraud resistance in [Boureau et al. 2013a] that suits the provable security of the SKI schemes [Boureau et al. 2015]), or less realistic (like the SimTF formulation for terrorist-fraud resistance in [Dürholz et al. 2011]) in which the dishonest prover and the adversary are not allowed to communicate during the fast rounds). On the one hand, when we fix the model, we can nonetheless see that some of these definitions imply one another (StrongSimTF in [Fischlin and Onete 2013b] implies SimTF in [Dürholz et al. 2011], and soundness in [Vaudenay 2013] implies collusion-fraud resistance in [Boureau et al. 2013a; 2015]). To this end, Vaudenay took collusion-fraud resistance further into a notion of soundness [Vaudenay 2013] akin to similar expressions in interactive proofs.

Thirdly, one can argue that some of these formal definitions for TF resistance might yield too strong a requirement, disproving security all-throughout (like the SimTF formulation of terrorist-fraud resistance in [Dürholz et al. 2011]), or might be too general (like aforementioned collusion-fraud resistance in [Boureau et al. 2013a] that suits the provable security of the SKI schemes [Boureau et al. 2015]), or less realistic (like the SimTF formulation for terrorist-fraud resistance in [Dürholz et al. 2011]) in which the dishonest prover and the adversary are not allowed to communicate during the fast rounds). On the one hand, when we fix the model, we can nonetheless see that some of these definitions imply one another (StrongSimTF in [Fischlin and Onete 2013b] implies SimTF in [Dürholz et al. 2011], and soundness in [Vaudenay 2013] implies collusion-fraud resistance in [Boureau et al. 2015]), for certain parameters). On the other hand, even in such a fixed model, other definitions remain however incomparable (e.g., GameTF and StrongSimTF in [Fischlin and Onete 2013b]), underlying further the unsettlement of formalising terrorist-fraud resistance even within one and the same formalism.

Last but not least, formal comparisons between the session-based model in [Dürholz et al. 2011; Fischlin and Onete 2013b] and the model inspired by interactive proofs in [Boureau et al. 2015; Vaudenay 2013] do not exist. In the absence of a formal proof aligning the two models and their security definitions, it appears that SimTF resistance in [Dürholz et al. 2011] is equivalent to the notion of terrorist-fraud resistance in [Boureau et al. 2013b] and that GameTF resistance in [Fischlin and Onete 2013b] is equivalent to collusion-fraud resistance in [Boureau et al. 2013a] (for some parameters).

Similar discussions apply—of course—to the formalisations of threats other than terrorist-fraud in the aforementioned formalisms. As such, formal relations between the existing formal models for distance-bounding and their formal definitions of security is an avenue of future research.

Due to such differences between the formal models, we decided to carry out our analyses in the general framework by Avoine et al. [Avoine et al. 2011]. This framework does not repose on such fine-grained formalisations of the distance-bounding threats\(^1\), but instead it formalises classes of interactions between the provers and the attackers in order to best classify attack strategies, towards an unitary approach to assessing the security/insecurity of distance-bounding.

\(^1\)For instance, the popular take on TF resistance by reduction to impossible protection against the “trivial vulnerability” is not attainable in the “white box model for TF” from [Avoine et al. 2011], whilst some of the formal expressions for TF resistance summarised above would be.

8
1.7. Contributions
This article provides an in-depth security comparison of many existing distance bounding protocols. After the introduction in Section 2 of the notation and the methodology used in this article, the next twelve sections present several important published distance bounding protocols. Each section presents in a unified way the considered protocol and its security analysis. Those who are not familiar with the presented protocols will be able to consult Appendix A, which provides thorough descriptions of the twelve protocols. Section 15 presents the comparison methodology and results. The article also includes Appendix B, which discusses about variants and extensions that can be applied to most of the considered protocols.

2. ANALYSIS METHODOLOGY AND NOTATIONS
This paper analyzes twelve distance-bounding protocols using a unique methodology, based on the distance-bounding framework published in [Avoine et al. 2011]. Table I contains the unified notations used throughout the paper. Each protocol description is divided into 3 steps, namely initialization, protocol, and final phase, and includes a table that summarizes the protocol parameters. Protocols consist of slow phases that are not time-constrained, and fast phases where the verifier measures the round-trip times of exchanged messages. The fast phases are identified with a left square bracket. Anything in the bracket is repeated n times, except if stated otherwise. Each protocol description is followed by a security analysis according to the template provided in Section 2.2. The properties and performance are analyzed according to Section 2.3 and 2.4.

2.1. Fraud definitions
A distance bounding protocol is a process whereby a party (known as verifier) is assured (i) of the identity of a second party (know as prover) (ii) that the prover is located in his close vicinity (known as neighborhood). Four frauds against distance bounding are usually considered, impersonation, distance, mafia, and terrorist frauds [Avoine et al. 2011], which are introduced below.

- **Impersonation.** An impersonation fraud is an attack where an adversary acting alone purports to be a legitimate prover.

- **Distance fraud.** A distance fraud is an attack where a dishonest prover purports to be in the neighborhood of the verifier. He cheats without help of other entities located in the neighborhood.

- **Mafia fraud.** A mafia fraud is an attack where an adversary defeats a distance-bounding protocol using a man-in-the-middle between the verifier and an honest prover located outside the neighborhood.

- **Terrorist fraud.** A terrorist fraud is an attack where an adversary defeats a distance-bounding protocol using a man-in-the-middle between the verifier and a dishonest prover located outside of the neighborhood under the following circumstances. The dishonest prover actively helps the adversary to maximize her current attack success probability, but without giving her any advantage for future man-in-the-middle attacks. (In such attacks, the man-in-the-middle (MiM) would attempt to pass the distance-bounding protocol as a valid prover/tag which the MiM does not represent/possess.)

Note that protocols that are known to suffer from a key-recovery attack are not analyzed in this article. This includes Tu and Piramuthu’s protocol [Tu and Piramuthu 2007] whose flaws are discussed in [Kim et al. 2008; Munilla and Peinado 2008b], Reid et al.’s protocol [Reid et al. 2007] broken in [Avoine et al. 2011; Mitrokotsa et al. 2010], and Hitomi whose vulnerabilities are described in [Sohizadeh Abyaneh 2011]. While [Bay et al. 2012] points out a key recovery attack on Bussard and Bagga’s protocol [Bussard and Bagga 2005], this protocol is kept in this analysis because the attacks presented in [Bay et al. 2012] could be applied to other protocols and designers must be aware of their existence to avoid them. Note also that the length of the long-term secret keys of the parties, the length of the signatures (when appropriate), and the length of the nonces are assumed to be large enough, such that exhaustive search and replay attack are not relevant. Finally, the pseudo random functions used in the protocols are assumed to be without design flaws, i.e., no trapdoor pseudo random functions, like those discussed in [Bourreau et al. 2012].

Another type of fraud, known as distance hijacking, has recently been introduced in [Cremers et al. 2012]. The fraud considers a dishonest prover who aims to convince a verifier that he is located within the verifier’s neighborhood, abusing for that some other provers who are indeed in the verifier’s neighborhood. For example, a dishonest prover can reach his goal by hijacking the fast phase of a distance-bounding protocol executed between an honest (closer) prover and the verifier. Conceptually, distance hijacking can be placed between distance fraud and terrorist fraud. Unlike terrorist fraud, where a dishonest prover colludes with another attacker, distance hijacking considers a dishonest prover who interacts with (abuses) other honest provers.
Prover and Verifier

\( P, I_D \) Prover, Prover identity
\( V, I_D \) Verifier, Verifier identity
\( N_V, N_P \) Nonces sent by verifier and prover, respectively.

Rounds

\( n \) Number of rounds in the fast phase
\( i \) Index of the current round

Secrets

\( K \) Long-term secret key shared by prover and verifier
\( K_e, K_d \) Public/Private keys for Encryption/Decryption
\( K_s, K_v \) Private/Public keys for Signature/Verification

Time

\( \Delta t_i \) Round Trip Time (RTT) measured during round \( i \)
\( \delta_{\text{max}} \) Threshold on the round-trip time (typically, there is a round failure if \( \Delta t_i > \delta_{\text{max}} \))

Challenges and Responses

\( c_i \) Challenge sent by the verifier in round \( i \)
\( c'_i \) Challenge received by the prover in round \( i \)
\( r_i \) Response sent by the prover in round \( i \)
\( r'_i \) Response received by the verifier in round \( i \)

Registers

\( R^0, R^1 \) Main registers
\( Z^0, Z^1, \ldots \) Additional registers, when needed.

\( H \) Crypto function output \( \{\text{hash, encrypt, ...} \} \), usually viewed as a register, e.g., \( H = h(N_V, N_P) \)

Sizes

\( \sigma \) Size of the signature, commitment, or MAC (in the slow phase)
\( \iota_P, \iota_V \) Size of ID \( P \) and ID \( V \). If \( |ID_P| = |ID_V| \) then the value is simply denoted \( \iota \) (bits)
\( \kappa \) Size of \( K \) (bits)
\( \delta_P, \delta_V \) Size of the random nonces \( N_V \) and \( N_P \). If \( |N_V| = |N_P| \) then the value is simply denoted \( \delta \) (bits)

Errors

\( \epsilon_X \) Number of errors of type \( X \), e.g., \( \epsilon_C, \epsilon_R, \epsilon_T \)
\( \epsilon_{\text{max}} \) Threshold on the number of errors

Functions

\( dH(\,\,\,) \) Hamming distance
\( H(\,\,\,\,\,) \) Hamming weight
\( \text{Sign}_{K_s}(\,\,\,) \) Signature function with private key \( K_s \)
\( \text{Verif}_{K_v}(\,\,\,\,\,) \) Public-key signature verification function with public key \( K_v \)
\( \text{Commit}(\,\,\,\,\,) \) Commitment function
\( \text{Open}(\,\,\,\,\,) \) Open commitment function
\( h_{K_s}(\,\,\,\,\,) \) Cryptographic hash function
\( h_{K}(\,\,\,\,\,) \) Cryptographic hash function keyed with the secret key \( K \)
\( MAC_{K}(\,\,\,\,\,) \) Message authentication code keyed with the secret key \( K \)
\( \text{E}_{K}(\,\,\,\,\,) \) Encryption function keyed with the secret key \( K \)
\( D_{K}(\,\,\,\,\,) \) Decryption function keyed with the secret key \( K \)

Misc

\( \mathbf{E}(\,\,\,) \) Mathematical expectation
\( \xi_{\mathbf{E}} \) Randomly and uniformly picked in \( \mathbf{E} \)
\( \xi_{\mathbf{R}} \) Randomly and uniformly picked in \( \mathbf{R} \)
\( || \) Concatenation of words (possibly 1-bit words)
\( p \) Number of runs of the cryptographic function, in the analyzes
\( \rho \) Prime numbers
\( \mu_X \) Probability of event \( X \)
\( w \) Hamming weight, e.g., \( w = H(x) \)

Table I: Notations

Unlike distance fraud that only involves a dishonest prover and a verifier, distance hijacking also involves other honest provers. These seemingly subtle differences have significant consequences, e.g., the countermeasures proposed against terrorist fraud strictly depend on the fact that the dishonest prover needs to share data with another attacker. In fact, the protocols BC [Brands and Chaum 1993], MAD [Capkun et al. 2003], and RC [Rasmussen and Capkun 2010] are not resistant against hijacking fraud according to [Cremers et al. 2012]. The version of RC presented in Section 10 comes from [Rasmussen 2011]. This is a version that has been modified to be resilient to distance hijacking. Cremers et al. provide in [Cremers et al. 2012] a clear analysis.
of existing protocols that resist to the hijacking fraud. Vaudenay analyzes additional protocols in [Vaudenay 2015]. We consequently refer the reader to these articles to get more information about distance hijacking.

2.2. Security

The analyses usually performed in distance-bounding do not provide a security proof, but state the resistance of a protocol given a clearly defined scenario, which includes the type of fraud, but also the adversary’s capabilities and strategies, described below and summarized in Table II.

Table II: Attack scenarios

<table>
<thead>
<tr>
<th>Fraud</th>
<th>Prover Model</th>
<th>Prover Computing Capability</th>
<th>Adversary Strategy</th>
<th>Success Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impersonation</td>
<td>(1)</td>
<td>(1)</td>
<td>(4)</td>
<td>(P_{\text{Imp}})</td>
</tr>
<tr>
<td>Mafia</td>
<td>(1)</td>
<td>(1)</td>
<td>pre-ask, post-ask</td>
<td>(P_{\text{MF</td>
</tr>
<tr>
<td>Distance</td>
<td>black-box</td>
<td>(2)</td>
<td>pre-ask &amp; early-reply</td>
<td>(P_{\text{DF</td>
</tr>
<tr>
<td></td>
<td>white-box</td>
<td>single run, multiple run</td>
<td>post-ask &amp; early reply</td>
<td>(P_{\text{DF</td>
</tr>
<tr>
<td>Terrorist</td>
<td>black-box</td>
<td>(2)</td>
<td>(3)</td>
<td>(P_{\text{TF</td>
</tr>
<tr>
<td></td>
<td>white-box</td>
<td>single run, multiple run</td>
<td>early-provide</td>
<td>(P_{\text{TF</td>
</tr>
</tbody>
</table>

(1) The prover does not cheat in such a fraud: considering his capabilities is consequently not relevant.
(2) Considering the prover’s computing capabilities is not relevant in the black box model.
(3) This case is equivalent to the mafia fraud.
(4) No strategy is defined in [Avoine et al. 2011] for the impersonation.

- **Prover model.** Depending on the tamper-resistance of the prover, two models are defined: black-box and white-box models. In the black-box model, the prover cannot observe nor tamper with the execution of the algorithm. In the white-box model, the prover has full access to the implementation of the algorithm and a complete control over the execution environment, as detailed in [Avoine et al. 2011].

- **Prover computing capabilities.** The prover computing capabilities may affect the security of the protocol when considering distance and terrorist frauds in the white box model, given that the prover is also the attacker in such frauds. For example, in HK protocol, the prover may exploit a latency between the slow and fast phases to generate registers with a low Hamming distance [Avoine et al. 2011].

- **Adversary strategies.** The framework [Avoine et al. 2011] points out that three relevant adversary’s strategies should be considered when analyzing a distance-bounding protocol: pre-ask, post-ask, and early-reply strategies. In the pre-ask strategy, the adversary relays the first slow phase between the verifier and the prover, then executes the fast phase with the prover before the verifier starts it. In the post-ask strategy, the adversary relays the first slow phase, then executes the fast phase with the verifier without involving the prover. The adversary then queries the prover with the correct challenges received during the fast phase. This strategy is meaningful when the protocol is completed with a second slow phase used to check that the challenges received by the prover are correct. In the early-reply strategy, the adversary anticipates the replies to make them arrive on time, which is particularly relevant with distance fraud. No strategy for the terrorist fraud is defined in [Avoine et al. 2011]. We introduce here the early-provide strategy: in this strategy, the adversary located inside the neighborhood, first relays the slow phase to the prover. The latter then provides to the adversary some information to help him to improve his success probability during the fast phase with the verifier. Finally, the adversary relays the final slow phase, if any.

**Remark 2.1 (Circle analysis).** A prover located outside the neighborhood of the verifier but not too far may receive some challenges while the protocol is still running. When the rounds of the fast phase are independent, this late information is useless. However, the adversary may use this information to increase her success probability when the rounds are not independent. Consequently, when analyzing the resistance of a protocol against distance and terrorist frauds the area the prover is located should be considered. However,
in all the analyzed protocols, either this scenario is not relevant due to the round independency, or the calculation of the success probability is still an open problem.

Remark 2.2 (Multiple-execution). The framework also points out that some information could leak when the protocol is executed several times. Typically, this case occurs when the prover and the verifier generate two registers without involving randomness from the prover. None of the protocols analyzed in the paper are known to suffer from this weakness. Consequently, it is not explicitly addressed in the analysis.

2.3. Properties

The protocol properties considered in the paper are described below and summarized in Table III. Note that the type of data exchanged during the fast phase is usually binary. This is the case for all protocols considered in this analysis, except the one discussed in Section 14.

- **Adaptiveness.** Indicates whether the protocol provides an adjustable trade-off between resistance to mafia and distance frauds.
- **Mutual authentication.** Indicates whether the protocol provides mutual authentication. Note that, mutual authentication does not imply mutual distance-bounding: while the identity proof is bilateral in that case, the distance proof is unilateral in all the analyzed protocols.
- **Second slow phase.** Indicates whether there is a second slow phase in the protocol after the fast phase.
- **Independence of the rounds.** Indicates whether each expected response during the fast phase depends on the current challenge only.

2.4. Performance

The protocol performances considered are described below and summarized in Table IV.

- **Cryptographic primitives.** Type of cryptographic primitives needed to be implemented on the prover side: cryptographically-secure pseudo-random number generator, hash, encryption, commitment, and signature. Hash functions and ciphers are actually aggregated into a single category that is denoted *symmetric primitive*.
- **Exchanged bits (slow phase).** Number of exchanged bits during the slow phase(s).
- **Exchanged bits (fast phase).** Number of exchanged bits during the fast phase.
- **Memory consumption.** Amount of memory that is needed during the entire fast phase by the prover.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adaptiveness</td>
<td>Yes/No</td>
<td>Cryptographic primitives</td>
<td>Type</td>
</tr>
<tr>
<td>Mutual authentication</td>
<td>Yes/No/Optional</td>
<td>Exchanged bits (slow phase)</td>
<td>bits</td>
</tr>
<tr>
<td>Second slow phase</td>
<td>Yes/No</td>
<td>Exchanged bits (fast phase)</td>
<td>bits</td>
</tr>
<tr>
<td>Independence of the rounds</td>
<td>Yes/No</td>
<td>Memory consumption</td>
<td>bits</td>
</tr>
</tbody>
</table>

Table III: Properties Table IV: Performance
3. BRANDS AND CHAUM’S PROTOCOL (1993)
In 1993, Brands and Chaum designed several distance-bounding protocols [Brands and Chaum 1993]. This analysis focuses on their protocol (Algorithm 1) that mitigates both mafia and distance frauds.

Algorithm 1: Brands and Chaum’s Protocol

<table>
<thead>
<tr>
<th>Verifier (prover’s public key $K_v$)</th>
<th>Prover (prover’s private key $K_s$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commit($m_1</td>
<td>\ldots</td>
</tr>
<tr>
<td>Pick $c_i \in {0, 1}$ and Start Timer</td>
<td>Stop Timer</td>
</tr>
<tr>
<td>$r_i$ $\leftarrow$</td>
<td>$\text{Open(Commit)}, \text{Sign}_{K_s}(c_1</td>
</tr>
<tr>
<td>Check $r_i$ and $\Delta t_i \leq t_{\text{max}}$ for $1 \leq i \leq n$, then Verify Sign$_{K_s}$</td>
<td></td>
</tr>
</tbody>
</table>

3.1. Impersonation
Assuming that the signature scheme is secure, impersonating the prover can only be done by sending a randomly selected correct signature. Such a naive attack has the success probability: $\Pr_{\text{Imp}} = (1/2)^n$. However, while nonce-based replay attacks are not addressed in this paper (Section 2.1), a challenge-based replay attack should be considered. Indeed, if the challenges sent by the verifier are used twice, then the adversary can reuse the same $m_i$’s and thus has the correct $\text{Open(Commit)}$ and $\text{Sign}_{K_s}(c_1||r_1|\ldots||c_n||r_n)$. After eavesdropping one execution before the attack, the success probability becomes $\Pr_{\text{Imp}} = (1/2)^n$.

3.2. Mafia Fraud
- **Pre-ask strategy.** The adversary gets the commitment and queries the prover with random bits ($c_i$) during the fast phase. The adversary then receives the responses ($r_i$) and the final signature. With this information, the adversary simply computes $m_i = c_i \oplus r_i$, then sends the valid responses to the verifier during the fast phase, and finally the commitment and the signature during the second slow phase. However, the signature received from the prover is not valid for this protocol run, except if the challenges sent by the adversary to the prover and the challenges sent by the verifier to the adversary are the same. The success probability of this strategy is the probability of guessing the challenges correctly: $\Pr_{\text{MF}\mid\text{pre}} = \left(\frac{1}{2}\right)^n$ [Avoine et al. 2011].

<table>
<thead>
<tr>
<th>$n$</th>
<th>Number of iterations in the fast phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>Lower bound for the size of the commitment and the signature ($t &gt;&gt; n$)</td>
</tr>
<tr>
<td>$t_{\text{max}}$</td>
<td>Threshold of the round-trip time</td>
</tr>
<tr>
<td>Commit</td>
<td>Secure commitment function that outputs $t$ bits</td>
</tr>
<tr>
<td>Sign$_{K_s}$</td>
<td>Signature function whose private key is $K_s$</td>
</tr>
</tbody>
</table>

- **Post-ask strategy.** The adversary must predict the correct responses to be sent to the verifier during the fast phase without any assistance. We thus have: $\Pr_{\text{MF}\mid\text{post}} = \left(\frac{1}{2}\right)^n$.

3.3. Distance Fraud (White Box)
- **Early-reply strategy with one run.** Given that the adversary must predict the current challenge correctly beforehand, her success probability is: $\Pr_{\text{DF}\mid\text{WB}(1)\mid\text{early}} = \left(\frac{1}{2}\right)^n$ [Avoine et al. 2011].
- **Early-reply strategy with $p$ runs.** No cryptographic function is used to compute registers, contrary to Hancke and Kuhn’s approach. This fact trivially yields: $\Pr_{\text{DF}\mid\text{WB}(p)\mid\text{early}} = \Pr_{\text{DF}\mid\text{WB}(1)\mid\text{early}}$.
- **Circle strategy.** Rounds being independent, the circle analysis offers no benefit to an adversary.
3.4. Distance Fraud (Black Box)

- **Pre-ask combined with early-reply strategy.** With the pre-ask strategy, the adversary learns all the possible answers. However, she does not know the challenges, so when she sends her answers in advance, two cases occur: a) the verifier uses the same challenge as she did with the verifier. Therefore she always succeeds, b) the verifier picks another challenge and she has sent an incorrect answer. Hence, the success probability of this strategy is: \( Pr_{DF|BB|pre&early} = \left( \frac{1}{2} \right)^n \).

- **Post-ask combined with early-reply strategy.** Given that the adversary must commit during the first slow phase, she cannot just answer randomly during the fast phase and she will therefore need to predict the responses expected by the verifier. Hence we have: \( Pr_{DF|BB|post&early} = \left( \frac{1}{2} \right)^n \).

- **Circle strategy.** We previously stressed that the circle analysis is worthless for this protocol.

3.5. Terrorist Fraud (White Box)

This protocol is not designed to resist to the terrorist fraud in the white box model. Indeed, the prover without revealing his secret \( K \), is able to provide to his accomplice the commitment and the signature which are required to succeed. Consequently: \( Pr_{TF|WB} = 1 \) [Kim et al. 2008].


In 2003, Čapkun, Buttyán, and Hubaux introduced MAD [Čapkun et al. 2003], a protocol that works quite similarly to the BC protocol [Brands and Chaum 1993], but provides mutual authentication. Although denoted by \( P \) and \( V \), the two parties act as both prover and verifier during the execution of the protocol (Algorithm 2). The notations used in [Čapkun et al. 2003] are kept in the description below.

**Algorithm 2: MAD Protocol**

<table>
<thead>
<tr>
<th>Prover (secret K)</th>
<th>Verifier (secret K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Pick } r' \in_R [0,1]^d ) and ( r \in_R [0,1]^n )</td>
<td>( \text{Pick } s' \in_R [0,1]^d ) and ( s \in_R [0,1]^n )</td>
</tr>
<tr>
<td>( h(r</td>
<td></td>
</tr>
<tr>
<td>( \alpha_1 = r_1 )</td>
<td>( \beta_1 = s_1 \oplus \alpha_1 )</td>
</tr>
<tr>
<td>( \text{Start Timer} )</td>
<td>( \text{Start timer} )</td>
</tr>
<tr>
<td>( \text{Stop Timer} )</td>
<td>( \text{Stop timer} )</td>
</tr>
<tr>
<td>( \alpha_i = r_i \oplus \beta_{i-1} )</td>
<td>( \beta_i = s_i \oplus \alpha_i )</td>
</tr>
<tr>
<td>( \text{Start Timer} )</td>
<td>( \text{Start timer} )</td>
</tr>
<tr>
<td>( \text{Stop Timer} )</td>
<td>( \text{Stop timer} )</td>
</tr>
<tr>
<td>( \alpha_n = r_n \oplus \beta_{n-1} )</td>
<td>( \beta_n = s_n \oplus \alpha_n )</td>
</tr>
<tr>
<td>( \text{Start Timer} )</td>
<td>( \text{Start timer} )</td>
</tr>
<tr>
<td>( \text{Stop Timer} )</td>
<td>( \text{Stop timer} )</td>
</tr>
<tr>
<td>Compute ( s_i = \alpha_i \oplus \beta_i ) for ( 1 \leq i \leq n ) and ( \mu_P = \text{MAC}_K(Id_P</td>
<td></td>
</tr>
<tr>
<td>( r'</td>
<td></td>
</tr>
<tr>
<td>Verify ( h(s</td>
<td></td>
</tr>
<tr>
<td>Check ( \Delta t_i \leq t_{max} ) for ( 1 \leq i \leq n )</td>
<td>Check ( \Delta t_i \leq t_{max} ) for ( 1 \leq i \leq n )</td>
</tr>
</tbody>
</table>
4.1. Impersonation

The basic way to impersonate the prover is to generate the random numbers $r$ and $r'$, and to complete the first slow phase and the fast phase. The adversary must then guess the output of the MAC function in the second slow phase. The probability of a correct guess is: $P_{\text{Imp}} = \left(\frac{1}{2}\right)^n$.

4.2. Mafia Fraud

Without loss of generality, we assume that the adversary seeks to impersonate $P$ against $V$.

- **Pre-ask strategy.** To succeed in the mafia fraud, the output of the MAC function in the second slow phase needs to be valid. Since the adversary cannot compute this value, she needs to ensure that $P$ sends the correct output of the MAC function to $V$. This will only be the case if the adversary has guessed the values $s_i$ correctly during the pre-ask stage. Hence: $P_{\text{MF}}|_{\text{pre}} = \left(\frac{1}{2}\right)^n$ [Capkun et al. 2003].

- **Post-ask strategy.** Similarly, the adversary needs to ensure that $P$ sends the correct output of MAC$_K$. This will only be the case if she guessed all correct $r_i$ values in advance: $P_{\text{MF}}|_{\text{post}} = \left(\frac{1}{2}\right)^n$ [Capkun et al. 2003].

4.3. Distance Fraud (White Box)

Without loss of generality, we assume that $P$ wants to perform a distance fraud (the distance fraud success probability of $V$ is equal to the one of $P$).

- **Early-reply strategy with one run.** The responses $\alpha_i$ are computed by XORing the values of the responses $r_i$, which are completely controlled by the adversary, and the challenges $\beta_i$. The latter are uniformly distributed, and the values $\alpha_i$ inherit the same statistical distribution. So even if the adversary fully controls her hardware, the best strategy is to guess the challenges $\beta_i$ in advance. We have thus: $P_{\text{DF}(\text{WB})|\text{early}}^\dagger = \left(\frac{1}{2}\right)^n$.

- **Early-reply strategy with $p$ runs.** Similarly to Algorithm 1, no cryptographic function is used to compute registers, and so: $P_{\text{DF}(\text{WB})|\text{early}}^\dagger = P_{\text{DF}(\text{WB})|\text{early}}^\ast$.

- **Circle strategy.** The rounds of the MAD protocol are not independent, so the circle analysis should be applied. Assuming that $P$ knows the challenges of the first $i-1$ rounds, but not the challenge $\beta_i$, $P$ must compute the response $\alpha_{i+1}$ without knowing $\beta_i$ in order to perform a successful distance fraud. However, the rounds are dependent and the following equation holds: $\beta_i = s_i \oplus \alpha_i$. Therefore, $P$ can compute the response $\alpha_{i+1}$ as follows: $\alpha_{i+1} = r_{i+1} \oplus s_i \oplus \alpha_i$. In this equation, everything is known except the value $s_i$, which is uniformly distributed. Consequently, $P$ has only probability of $1/2$ to compute $\alpha_{i+1}$ correctly. As a result, $P$ does not gain any advantage by knowing the challenges and responses of the previous rounds.

4.4. Distance Fraud (Black Box)

We assume that the fraudulent party that performs the distance fraud is $P$.

- **Pre-ask combined with early-reply strategy.** The best strategy consists in guessing the $n$ challenges $\beta_i$. By querying itself in advance, the prover learns the values $r_i$, and computes the responses $\alpha_i$. The adversary uses these responses in the early-reply strategy. They are correct when the values $\beta_i$ were guessed correctly and consequently the MAC computed by the prover in the second slow phase is correct as well. If one of the challenges is guessed incorrectly, the prover will compute incorrect values $s_i$, and MAC$_K(.)$ will be wrong. The distance fraud success probability is: $P_{\text{DF}(\text{BB})|\text{pre&early}} = \left(\frac{1}{2}\right)^n$.

- **Post-ask combined with early-reply strategy.** The adversary has no information on the bits $r_i$. The best strategy is to send $n$ random responses $\alpha_i$. In each round, the adversary has a $1/2$ probability of being successful. This occurs when both $r_i$ and $\beta_i$ are guessed correctly, or when both guesses were wrong. When one of these values is correct and the other one is incorrect, the response of the adversary will be wrong. As a result, the distance fraud success probability is: $P_{\text{DF}(\text{BB})|\text{post&early}} = \left(\frac{1}{2}\right)^n$.

- **Circle strategy.** The rounds of the MAD protocol are not independent, so the circle analysis can be applied. However, as already demonstrated in the white box case, $P$ does not gain any advantage by knowing the challenges and responses of the previous rounds.

4.5. Terrorist Fraud (White Box)

This protocol is not designed to resist to the terrorist fraud in the white box model. Indeed, the prover without revealing her secret $K$, gives her accomplice the output of the commitment, and the values $\alpha_i$ and $r_i$, or $\beta_i$ and $s_i$. After the fast phase, the accomplice gives the observed values $s_i$ or $r_i$ to the prover, who
can then computes the MAC. This output is then sent back to the accomplice, who finally forwards it to the verifier. Hence: \( Pr_{\text{DF}\mid \text{WB}} = 1 \) [Kim et al. 2008].

5. HANCKE AND KUHN’S PROTOCOL (2005)

In 2005 Hancke and Kuhn published the first distance-bounding protocol [Hancke and Kuhn 2005] (Algorithm 3) clearly dedicated to RFID. The protocol relies on the original ideas of Desmedt et al. [Desmedt et al. 1988; Bengio et al. 1991] but is different from Brands and Chaum’s work [Brands and Chaum 1993] in the sense that Hancke and Kuhn’s protocol does not have any final signature after the fast phase.

Algorithm 3: Hancke and Kuhn’s Protocol

<table>
<thead>
<tr>
<th>Verifier (secret ( K ))</th>
<th>Prover (secret ( K ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V \mid \text{Pick } V \in {0,1}^s )</td>
<td>( P \mid \text{Pick } P \in {0,1}^s )</td>
</tr>
<tr>
<td>( H = h(K, V, P) )</td>
<td>( H = h(K, V, P) )</td>
</tr>
<tr>
<td>( R^p = H_{i+1} | H_{i+2} | \ldots | H_n )</td>
<td>( R^p = H_{i+1} | H_{i+2} | \ldots | H_n )</td>
</tr>
<tr>
<td>( R^p = H_{i+1} | H_{i+2} | \ldots | H_n )</td>
<td>( R^p = H_{i+1} | H_{i+2} | \ldots | H_n )</td>
</tr>
<tr>
<td>( \text{Pick } c_i \in {0,1} ) and Start Timer</td>
<td>( \text{Pick } c_i \in {0,1} ) and Start Timer</td>
</tr>
<tr>
<td>( \text{Stop Timer} )</td>
<td>( \text{Stop Timer} )</td>
</tr>
<tr>
<td>( \text{Check correctness of } r_i ) ( \Delta r_i \leq \max ) for ( 1 \leq i \leq n )</td>
<td>( \Delta r_i \leq \max ) for ( 1 \leq i \leq n )</td>
</tr>
</tbody>
</table>

5.1. Impersonation

The common attack consists in guessing all the answers during the fast phase: \( Pr_{\text{Imp}} = \left( \frac{1}{2} \right)^n \).

5.2. Mafia Fraud

- **Pre-ask strategy.** We have: \( Pr_{\text{MF}\mid \text{pre}} = \left( \frac{1}{2} \right)^n \) [Hancke and Kuhn 2005].
- **Post-ask strategy.** This protocol does not contain any second slow phase and the first slow phase consists of nonce exchanges only. As per Section 2 we have: \( Pr_{\text{MF}\mid \text{post}} = Pr_{\text{Imp}} \).

5.3. Distance Fraud (White Box)

- **Early-reply strategy with one run.** We have: \( Pr_{\text{DF}\mid \text{WB}(1)\mid \text{early}} = \left( \frac{1}{2} \right)^n \) [Trujillo-Rasua et al. 2010].
- **Early-reply strategy with \( p \) runs.** The formula expressing the attack success probability for this strategy was originally presented in [Avoine et al. 2011], but it contained a typing error. The correct formula is:

\[
Pr_{\text{DF}\mid \text{WB}(p)\mid \text{early}} = \frac{1}{2^{pn}} \cdot \left( \sum_{i=0}^{i=n-1} \left( \frac{1}{2} \right)^i \cdot \left( \sum_{j=i}^{j=n} \binom{n}{j} \right)^p - \left( \sum_{j=i+1}^{j=n} \binom{n}{j} \right)^p \right) + \left( \frac{1}{2} \right)^n .
\]

- **Circle strategy.** Rounds being independent, the circle analysis offers no benefit to an adversary.

5.4. Distance Fraud (Black Box)

- **Pre-ask combined with early-reply strategy.** With the pre-ask strategy, the adversary learns half of the possible responses. However, she does not know the challenge, so when she sends her responses in advance, two situations can occur: 1) the verifier asks her the same challenge that she asked the prover and therefore her response is correct, 2) the verifier sends a different challenge in which case she succeeds if the two possible responses were the same, i.e., her response is the same as the alternative response, and fails if the possible responses are different. Hence, the distance fraud success probability is: \( Pr_{\text{DF}\mid \text{BB}\mid \text{pre&early}} = \left( \frac{1}{2} \right)^n \)

- **Post-ask combined with early-reply strategy.** This protocol does not contain any second slow phase and the first slow phase consists of nonce exchanges only. As per Section 2 we have: \( Pr_{\text{DF}\mid \text{BB}\mid \text{post&early}} = Pr_{\text{Imp}} \).

- **Circle strategy.** We previously stressed that the circle analysis is worthless for this protocol.
5.5. Terrorist Fraud (White Box)

This protocol is not designed to resist to the terrorist fraud in the white box model. Indeed, the prover is able to provide to his accomplice the two registers required to successfully execute the protocol, without revealing his secret $K$: $Pr_{TF|WB} = 1$ [Kim et al. 2008].


Bussard and Bagga published the DBPK-Log protocol (Algorithm 4), which is a distance-bounding protocol based on a proof of knowledge and a commitment scheme [Bussard and Bagga 2005].

Algorithm 4: DBPK-Log Protocol

<table>
<thead>
<tr>
<th><strong>Verifier</strong> (prover’s public key $y$)</th>
<th><strong>Prover</strong> (prover’s private key $x$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pick $R_0 \in {0,1}^n$</td>
<td></td>
</tr>
<tr>
<td>Compute $R_1 = E_y(x) = x - R_0 \mod (p-1)$</td>
<td></td>
</tr>
<tr>
<td>$C_i(R_0) = \text{Commit}(R_0)$</td>
<td>$C_i(R_1) = \text{Commit}(R_1)$</td>
</tr>
<tr>
<td>$\forall i \in [0, n - 1]: C_i(R_0) = \text{Commit}(R_0)$</td>
<td>$\forall i, C_i(R_0) \leftarrow PK_{((x, v): z = \Omega(x, v) \land y = \Gamma(x))}$</td>
</tr>
<tr>
<td>$c_i$ and Start Timer</td>
<td></td>
</tr>
<tr>
<td>$r_i$</td>
<td></td>
</tr>
<tr>
<td>$c_i \rightarrow r_i$</td>
<td></td>
</tr>
<tr>
<td>Check $r_i = R_1^{c_i}$</td>
<td></td>
</tr>
</tbody>
</table>

6.1. Impersonation

In [Bussard and Bagga 2005], the authors described a statistical key recovery attack. They established the success probability of this attack: $Pr_{Imp} = (1/2)^{-4m'}$.

6.2. Mafia Fraud

- **Pre-ask strategy.** The adversary must pass the final slow phase to defeat the protocol. Forging the Open function is definitely not the best option. Instead the adversary should try to send the correct challenges to the prover during the pre-ask attack, and then relay the final slow phases. Her success probability with such a strategy is: $Pr_{MF|pre} = (1/2)^n$.

- **Post-ask strategy.** Due to the presence of a complex second slow phase, the post-ask strategy is as good as the pre-ask strategy against DBPK-Log. Indeed, when the adversary correctly guesses the responses expected by the verifier, she can apply the post-ask strategy and bypass the second slow phase. Thus, she needs only to succeed in the fast phase, which yields the success probability: $Pr_{MF|post} = (1/2)^n$.

6.3. Distance Fraud (White Box)

- **Early-reply strategy with one run.** The adversary can search for a random $R_0$ that minimizes the Hamming distance between $R_0$ and $R_1$. Denoting $a$ as the Hamming distance between $R_0$ and $R_1$ ($a = d_H(R_0, R_1)$), we have: $Pr_{DF|WB(1)|early} = (1/2)^a$. This points out that the analysis provided in [Bussard and Bagga 2005] under-evaluates the success probability of the adversary because the white box model is not considered.

  **Example 6.1.** Let us consider the safe prime $p = 59$ ($q = 29$ and $n = 6$) and $x = 27$. If $R_0 = 32$, then $d_H(R_0, R_1) = 1$ and so the success probability is $(1/2)$.

- **Early-reply strategy with p runs.** Running the pseudo-random generator for choosing $R_0$ once, or several times, has no impact in the protocol security: the malicious prover can choose an appropriate value for $R_0$ in order to maximize its success probability in the distance fraud. Hence: $Pr_{DF|WB(p)|early} = Pr_{DF|WB(1)|early}$. 

- **Circle strategy.** Rounds being independent, the circle analysis offers no benefit to an adversary.
6.4. Distance Fraud (Black Box)

- **Pre-ask combined with early-reply strategy.** The adversary must succeed in the second slow phase and has the same success probability as the Mafia Fraud. We then have: $\Pr_{DF|BB|pre&early} = (1/2)^n$.

- **Post-ask combined with early-reply strategy.** The adversary must succeed in the fast phase without the knowledge of the challenge. Then, the prover is queried by the adversary to gain information for the final slow phase. Similar to the mafia fraud, the success probability is: $\Pr_{DF|BB|post&early} = (1/2)^n$.

- **Circle strategy.** We previously stressed that the circle analysis is worthless for this protocol.

6.5. Terrorist Fraud (White Box)

- **Early-provide strategy with one run.** This protocol is designed to resist to terrorist fraud in the white box model. Indeed, the prover cannot reveal $R^0$ and $R^1$ without exposing the key, making him able to provide only $R^0$ or $R^1$ to the external adversary. Note that the prover cannot try to optimize the Hamming distance between $R^0$ and $R^1$ as in the distance fraud. Indeed the knowledge of one register and the procedure to choose it reveals some information on the key.

  **Remark 6.2.** The probability of terrorist fraud calculated in [Bussard and Bagga 2005] is lower than the one provided here. Indeed, the authors considers that the final slow phase cannot be relayed by the adversary.

- **Early-provide strategy with p runs.** Similarly to the distance fraud strategy with early provide one run, we have $\Pr_{TF|WB}(p) = \Pr_{TF|WB}(1)$.

  ▶ **A Dedicated Distance Fraud and Terrorist Fraud.** We describe here a distance fraud from [Bay et al. 2012]. The key idea is that a malicious prover could select $R^0 \approx \frac{x}{2} \mod (p - 1)$. That is, if $x$ is even, he takes $R^0 = \frac{x}{2}$ and gets $R^1 = R^0$. Otherwise, he takes $R^0 = \frac{x + 1}{2}$ and get $R^1 = R^0 \pm 1$ so that $R^0$ and $R^1$ differ in their least significant bit only. He can then run the protocol normally. We note that $R^0 = R^1$ except for one single round. So, the answers to the received challenges do not depend on it, except in one round. By sending the answer before the challenge arrives, the malicious prover can succeed in an early-reply strategy to run a distance fraud with a success probability larger than $\frac{1}{2}$.

  The paper [Bay et al. 2012] also describes a terrorist fraud. The idea of the attack is that the malicious prover starts the protocol but does not give the commit values. Instead, he computes $z$ and discloses it to the adversary through an early-provide strategy. The adversary will commit to random bits for $R^0$ and $R^1$ except for round $i = 1$. Then, he guesses the value $c_1$ and commit to a random bit for $R^1_{i-1}$. Finally, the commit value for $R^1_{i-1}$ is adjusted so that the equation $z = \prod_{i=1}^{n-1}(c_i(R^0_i)(R^1_{i}))^{2^{-i}} \mod p$ holds. Clearly, the adversary can answer all challenges (if his guess for $c_1$ is correct), since he knows the bits he committed to. Next, he can get the help of the malicious prover to run the PK protocol through the lazy phase. Due to the zero-knowledge property of the PK protocol, this leaks no information about $x$. This attack works with probability $\frac{1}{2}$ (due to the guess of $c_1$).

  Finally, the paper [Bay et al. 2012] proposes some man-in-the-middle attacks against variants of this protocol which are not using public-key cryptography, i.e., where PK is not used and $x$ is shared.


Munilla and Peinado introduced in [Munilla et al. 2006; Munilla and Peinado 2009] the concept of void challenges as a tool to improve distance-bounding protocols. These void challenges can also be used to decrease the mafia fraud success probability when applied to Hanke and Kuhn’s protocol [Munilla and Peinado 2008a], which is the case analysed in this section. Thus, for this protocol (Algorithm 5), the challenges can be 0, 1 or void, where a void challenge means that no challenge is sent. Void challenges are used to detect a mafia fraud using the pre-ask strategy.

7.1. Impersonation

The adversary must guess the responses to the non-void challenges and the signature. Hence:

$$\Pr_{Imp} = (1 - \frac{Pr_f}{2})^n \cdot (\frac{1}{2})^{3n}$$

7.2. Mafia Fraud

- **Pre-ask strategy.** The calculation of the success probability of the pre-ask strategy is:
Algorithm 5: Munilla and Peinado’s Protocol

Verifier
(secret $K$)

Pick a random $N_V$

$H = h(K, N_V, N_p)$

$Z = H_1 \| H_2 \| \ldots H_n$

$R^0 = H_{n+1} \| H_{n+2} \| \ldots H_{2n}$

$R^1 = H_{2n+1} \| H_{2n+2} \| \ldots H_{3n}$

If $Z_i = 1$: Pick $c_i \in \{0, 1\}$ and Start Timer

Stop Timer $\leftarrow h(K, R^0, R^1)$

Check correctness of $r_i$ and $\Delta t_i \leq t_{\text{max}}$

for $1 \leq i \leq n$ and verify $h(K, R^0, R^1)$

Prover
(secret $K$)

Pick a random $N_P$

$H = h(K, N_V, N_P)$

$Z = H_1 \| H_2 \| \ldots H_n$

$R^0 = H_{n+1} \| H_{n+2} \| \ldots H_{2n}$

$R^1 = H_{2n+1} \| H_{2n+2} \| \ldots H_{3n}$

If $c_i \neq \text{void}$ and $Z_i = 1$ then $r_i = R^0$;
Else if $c_i = \text{void}$ and $Z_i = 0$ then send no response; Else abort the protocol

$\Pr_{\text{MF|pre}} = \begin{cases} 
(1 - pf)^n & \text{if } pf < 4/7 \\
\left( \frac{3}{4} \right)^n & \text{if } pf \geq 4/7
\end{cases}$ [Avoine et al. 2011]

Note that $\Pr_{\text{MF|pre}}$ calculated in [Avoine et al. 2011] and provided above is an approximation of the real value. Indeed, once the adversary is detected by the device, she does not receive any useful information any more. However, she can still guess the correct answers to be sent to the verifier. Given that being detected by the device forces the adversary to guess the final signature, this case is nevertheless negligible (Section 2.1). Note that the security of the signature depends on the length of $3n$ though, which implies that this case should not be disregarded when $n$ is particularly small.

- **Post-ask strategy.** The adversary must predict the correct responses to the non-void challenges. We have:

\[ \Pr_{\text{MF|post}} = \left( 1 - \frac{pf}{2} \right)^n \] [Avoine et al. 2011]

Remark 7.1 (Best strategy). The best strategy is post-ask when $pf < 4/5$, and pre-ask when $pf > 4/5$.

7.3. Distance Fraud (White Box)

- **Early-reply strategy with one run.** When the challenge is not void, the adversary can correctly respond to the verifier with probability $1$ if $R_i^0 = R_i^1$, and with probability $1/2$ if $R_i^0 \neq R_i^1$. Consequently:

\[ \Pr_{\text{DF|WB(1)|early}} = \left( 1 - \frac{pf}{4} \right)^n \] [Avoine et al. 2011]

- **Early-reply strategy with $p$ runs.** This strategy is efficient against the protocol if the verifier sends his nonce first. This weakness can be easily fixed though. The success probability is provided in [Avoine et al. 2011]:

\[ \Pr_{\text{DF|WB(p)|early}} = \left( 1 - pf + pf \cdot \left( 1 - \frac{1}{2} \cdot \frac{E(d_H(v^0, v^1))}{n} \right) \right)^n \]

where $E(d_H(R^0, R^1))$ is the expected minimum Hamming distance between $R^0$ and $R^1$ for the non-void challenges after the hash function is run $p$ times with a different $N_P$. We have: $\lim_{p \to \infty} \Pr_{\text{DF|WB(p)|early}} = 1$.

- **Circle strategy.** Rounds being independent, the circle offers no benefit to an adversary.

7.4. Distance Fraud (Black Box)

- **Pre-ask combined with early-reply strategy.** With the pre-ask strategy in the black box model, the adversary carries out an attack similar to the mafia fraud but on its own device: $\Pr_{\text{DF|BB|pre&early}} \approx \Pr_{\text{MF|pre}}$. The approximation is due to a small difference in the two frauds as explained hereafter. As long as the adversary is not detected by the device, she has the same strategy (and same probability of success) in both mafia
fraud and distance fraud. In particular, she no longer receives useful information from the device once she
is detected. However, in the mafia fraud, she can still determine whether or not a round contains a void
challenge when communicating with the verifier, as she does not have time to get this information in the
distance fraud. The difference is however negligible because she has to guess the final signature in both cases.

- **Post-ask combined with early-reply strategy.** In this strategy, the adversary definitely obtains the correct
final signature. However, she does not know when a void challenge or a non-void challenge is expected.
Therefore, if the probability of a non-void challenge is lower (resp. higher) than 2/3 then her best strategy
is to keep quiet (resp. try to guess every response, with probability 1/2).

$$\Pr_{\text{DF|BB|post&early}} = \begin{cases} 
(1 - p_f)^n & \text{if } p_f < \frac{2}{3} \\
\left(\frac{p_f}{2}\right)^n & \text{if } p_f \geq \frac{2}{3} 
\end{cases}$$  \[\text{[Avoine et al. 2011]}\]

- **Circle strategy.** We previously stressed that the circle analysis is worthless for this protocol.

**Remark 7.2.** In this case, the pre-ask strategy has the best success probability.

### 7.5. Terrorist Fraud (White Box)

This protocol is not designed to resist to the terrorist fraud in the white box model. Indeed, the prover,
without revealing his secret $K$, is able to provide to his accomplice the two registers required to successfully
complete the protocol. We so have: $\Pr_{\text{TF|WB}} = 1$ [Avoine et al. 2011].

### 7.6. Published Attacks

A technique to reduce the required memory [Munilla et al. 2006] consists in using only one $(n+1)$-bit register,
where the responses are selected from the two edges. However, [Avoine et al. 2011] demonstrated that this
technique opens the door to an attack where the adversary queries in advance the two values of the edges.

### 8. KIM, AVOINE, KOEUNE, STANDAERT AND PEREIRA’S PROTOCOL (2008)

Kim, Avoine, Koeune, Standaert and Pereira introduced a protocol in [Kim et al. 2008] known as the Swiss-
knife distance-bounding protocol\(^2\) (Algorithm 6). We only consider in this analysis the case where $T = 1$,
that is when the protocol is not noise-resilient.

#### 8.1. Impersonation

The attacker could impersonate the prover by guessing all the answers during the fast phase and $T_B$ in the
second slow phase. To succeed the adversary would need to guess $\sigma + n$ bits. Therefore, it is better for
the adversary to guess $K$ which size is $\sigma$. Consequently, we have: $\Pr_{\text{Imp}} = \left(\frac{1}{2}\right)^\sigma$.

#### 8.2. Mafia Fraud

- **Pre-ask strategy.** The success probability of the pre-ask strategy is $\Pr_{\text{MF|pre}} = \left(\frac{1}{2}\right)^n$ [Kim et al. 2008].

- **Post-ask strategy.** The adversary must guess the responses expected in the fast phase: $\Pr_{\text{MF|post}} = \left(\frac{1}{2}\right)^n$.

#### 8.3. Distance Fraud (White Box)

- **Early-reply strategy with one run.** As the prover can access to the internal state of the registers, she knows
the content of two the registers. If $v_0^i = v_1^i$, she always responds correctly, otherwise she has to guess
the correct answer with probability $\frac{1}{2}$. Hence, $\Pr_{\text{DF|WB(1)|early}} = \left(\frac{1}{2}\right)^n$.

- **Early-reply strategy with $p$ runs.** The Swiss-knife Protocol generates only one register. Hence, multiple-run
of PRF does not increase the adversary success probability: $\Pr_{\text{DF|WB(p)|early}} = \Pr_{\text{DF|WB(1)|early}}$.

- **Circle strategy.** Rounds being independent, the circle analysis offers no benefit to an adversary.

\(^2\)Like Swiss-army knives used during WWII, the Swiss-knife protocol is a multi-purpose tool. The authors claim their protocol
"resists against both mafia fraud and terrorist attacks, reaches the best known false acceptance rate, preserves privacy, resists
to channel errors, uses symmetric-key cryptography only, requires no more than 2 cryptographic operations to be performed
by the tag, can take advantage of precomputation on the tag, and offers an optional mutual authentication" [Kim et al. 2008].
Algorithm 6: Swiss-knife Protocol

Verifier
(secret K, constant C)

Pick N_V \in (0,1)^d
Pick a random D s.t. \mathcal{H}(D) = n

N_V, D

Prover
(secret K, identifier ID, constant C)

Pick N_F \in (0,1)^d
a = f_K(C, N_F)
Z^0 = a, Z^1 = a \oplus K

For i = 1 to n:

j = index of the next “1” in the binary representation of D. B_i^0 = Z_i^0 \oplus R_i^0 = Z_i^1

8.4. Distance Fraud (Black Box)
- **Pre-ask combined with early-reply strategy.** The adversary sends her own challenges to the prover in advance. To obtain the correct signature in the second slow phase, the challenges sent by the adversary must be the same as the challenges sent by the verifier. We consequently have: \text{Pr}_{DF|BB|pre\&early} = \left(\frac{1}{2}\right)^n.
- **Post-ask combined with early-reply strategy.** The adversary has to correctly guess the response in each round. Hence: \text{Pr}_{DF|BB|post\&early} = \left(\frac{1}{2}\right)^n.
- **Circle strategy.** We previously stressed that the circle analysis is worthless for this protocol.

8.5. Terrorist Fraud (White Box)
- **Early-provide strategy with one run.** \text{Pr}_{TF|WB(1)} = \left(\frac{1}{2}\right)^n \text{ [Kim et al. 2008].}
- **Early-provide strategy with p runs.** For the same reason as Section 8.3: \text{Pr}_{TF|WB(p)} = \text{Pr}_{TF|WB(1)}.

8.6. Published Attacks
Peris-Lopez et al. proposed a passive full disclosure attack on the Swiss-knife RFID distance-bounding protocol [Peris-Lopez et al. 2009]. However, their assumption is not correct: they assume that the size of the secret key (K) and random nonces (N_V and N_F) are equal to n (number of iterations in the fast phase) and n is insecurely short, for example 32 bits or less in the Swiss-knife protocol. Based on this assumption, they assert that the Swiss-knife protocol is insecure. The authors of the Swiss-knife RFID distance-bounding protocol never claimed that their protocol is secure when the size of the long-term key and random nonces are so short. Under this assumption, all the distance-bounding protocols can be broken.

9. AVOINE AND TCHAMKERTEN’S PROTOCOL (2009)
The protocol (Algorithm 7) introduced by Avoine and Tchamkerten in [Avoine and Tchamkerten 2009] is a generalization of Hancke and Kuhn’s protocol that is more secure in terms of mafia and distance frauds.

9.1. Impersonation
To impersonate a legitimate prover one needs to guess the c authentication bits and the n replies of the fast phase. Hence: \text{Pr}_{Imp} = \left(\frac{1}{2}\right)^{c+n}.$

9.2. Mafia Fraud
- **Pre-ask strategy.** \text{Pr}_{MF|pre} = 2^{-n}(\frac{d}{2} + 1)^{\frac{d}{2}} = 2^{-d}(\frac{d}{2} + 1)^d \text{ with } n = d\ell \text{ [Avoine and Tchamkerten 2009].}


### 9.3. Distance Fraud (White Box)

- **Early-reply strategy with one run.** The analysis of the distance fraud probability in the case of the tree-based protocol is very similar to the analysis of the Poulidor protocol (Section 11) that is provided in [Trujillo-Rasua et al. 2010]. Unfortunately, this analysis only yields rough upper bounds. To find such an upper bound on the adversary success probability for distance fraud for the tree-based protocol, Theorem 3 available in [Trujillo-Rasua et al. 2010] is used. This theorem is related to Poulidor but the only difference between Poulidor and the tree-based protocol is that the latter creates a full tree as graph. Therefore, the distance fraud success probability of the tree-based protocol is upper bounded by:

\[
\frac{1}{2} \left( \frac{1}{2^n} + \sqrt{\frac{1}{2^{2n}} - \frac{4}{2^n} + 4q} \right) \quad \text{where} \quad q = \prod_{i=1}^{i=n} \left( \frac{1}{2} + \frac{1}{2^{i+1}} \sum_{k=0}^{k=2n-1} (A^i[0,k])^2 \right).
\]

The authors of [Trujillo-Rasua et al. 2010] define \(A^i[0,k]\) for a tree, considering that the nodes in the tree are labeled between 0 and \(2^n - 1\) using a breadth-first algorithm, then:

\[
A^i[0,k] = \begin{cases} 
1 & \text{if } 2^i - 1 \leq k < 2^{i+1} - 1, \\
0 & \text{otherwise},
\end{cases}
\]

and finally:

\[
q = \prod_{i=1}^{i=n} \left( \frac{1}{2} + \frac{1}{2^{i+1}} \right).
\]

- **Early-reply strategy with \(p\) runs.** Similar to the Poulidor case (Section 11), this strategy makes sense for this protocol but so far neither \(P_{DF|WB(1)|early}\) nor \(P_{DF|WB(p)|early}\) have been calculated.

- **Circle strategy.** Although it makes sense to consider the circle analysis for this protocol, the calculation of the distance fraud success probability in this scenario is also an open problem. Indeed, when the number of circles is greater than \(n\), this problem is as hard as the calculation of \(P_{DF|WB(1)|early}\).

### 9.4. Distance Fraud (Black Box)

- **Pre-as combined with early-reply strategy.** \(P_{DF|BB|pre&early} = 2^{-n}(\frac{3}{2}+1)^2\) [Avoine and Tchamkerten 2009]

- **Post-as combined with early-reply strategy.** Note that the post-as strategy will not allow the adversary to gain any information, i.e., \(P_{DF|BB|post&early} = \left(\frac{1}{2}\right)^{c+n}\).

- **Circle strategy.** Since we are in a black box setting, the prover does not have access to the labeling of the trees, hence the circle strategy yields probability of success of \(\left(\frac{1}{2}\right)^n\).

### 9.5. Terrorist Fraud (White Box)

This protocol is not designed to resist to the terrorist fraud in the white box model. An attacker can reveal the tree node labelization to an accomplice who so successfully passes the fast phase with \(P_{TF|WB} = 1\).
10. RASMUSSEN AND ČAPKUN’S PROTOCOL (2010)

The protocol (Algorithm 8) was introduced by Rasmussen and Čapkun and originally appeared in [Rasmussen and Čapkun 2010]. In this paper we consider the updated version that appeared in [Rasmussen 2011].

Algorithm 8: RC Protocol

<table>
<thead>
<tr>
<th>Verifier</th>
<th>Prover</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pick a random $N_V$</td>
<td>Pick a random $N_P$</td>
</tr>
<tr>
<td>Start Timer</td>
<td>Measure delay $n$</td>
</tr>
<tr>
<td>Stop Timer</td>
<td>From signal extract $N_V'$</td>
</tr>
<tr>
<td>From channels extract $N_P'$</td>
<td>From signal extract delay $n'$</td>
</tr>
<tr>
<td>Verify ($\Delta t, n = n'$, $N_V' = N_V, N_P' = N_P, \text{Sign}(M)$)</td>
<td>$M = \text{Commit}(N_P, ID_P)</td>
</tr>
</tbody>
</table>

10.1. Impersonation

We assume here that key size and nonce size are large enough to ensure that the probability of a key-recovery attack and a replay attack are negligible. The easiest manner to impersonate a prover is by forging the final signature. The success probability of this attack is: $\Pr_{\text{Imp}} = \frac{1}{2}^\sigma$.

10.2. Mafia Fraud

- **Pre-ask strategy.** In order to implement a mafia fraud using a pre-ask strategy an attacker has to guess the nonce $N_V$ of the verifier. Otherwise the final signature will not be valid. So, $\Pr_{\text{MF}|\text{pre}} = \frac{1}{2}^\delta_V$.

- **Post-ask strategy.** An attacker wishing to execute a mafia fraud attack must guess all the bits of the prover’s nonce in order to be able to reply correctly. Thus, $\Pr_{\text{MF}|\text{post}} = \frac{1}{2}^\delta_P$.

10.3. Distance Fraud (White Box)

- **Early-reply strategy with one run.** A malicious prover wishing to execute a distance fraud attack must guess all the bits of the verifier’s nonce to reply correctly. Hence: $\Pr_{\text{DF}|\text{WB}(1)|\text{early}} = \frac{1}{2}^\delta_V$ [Rasmussen 2011].

- **Early-reply strategy with $p$ runs.** The concept of round does not exist in this protocol, therefore: $\Pr_{\text{DF}|\text{WB}(p)|\text{early}} = \Pr_{\text{DF}|\text{WB}(1)|\text{early}}$.

- **Circle strategy.** The concept of rounds does not exist in this protocol.

10.4. Distance Fraud (Black Box)

The security of this protocol does not depend on a well behaved prover. Consequently black-box success probabilities are the same as in the white-box model.

10.5. Terrorist Fraud (White Box)

This protocol is not designed to resist to the terrorist fraud in the white box model. Indeed, the prover without revealing his secret $K$, is able to provide his accomplice with $N_P$, which is required to successfully execute the fast phase. Hence, $\Pr_{\text{TF}|\text{WB}} = 1$.


Poulidor, the graph-based distance-bounding protocol (Algorithm 9) designed by Trujillo-Rasua, Martin, and Avoine [Trujillo-Rasua et al. 2010], uses specific node and edge dependencies in the tree of the AT
11.3. Distance Fraud (White Box)

Also, let:

\[
\begin{bmatrix}
H_1 & \cdots & H_{I_d} = \text{hash}(K, N_P, N_V)
\end{bmatrix}
\]

Fill the graph:

\[
\begin{align*}
&\text{for } i = 0 \text{ to } 2n - 1:
&\qquad k_i = H_{i+2n+1} \\
&\qquad q_i = H_{i+1}
\end{align*}
\]

\[
\begin{array}{c}
\text{Pick } c_i \in \mathbb{R} \{0,1\} \\
\text{Start Timer} \\
\text{Stop Timer}
\end{array}
\]

Move from \( q_{i_1} \) to \( q_{i_1+1} \)

if \( r_i \neq q_{i_1+1} \) then abort the protocol

Check that \( \Delta t_i \leq t_{\text{max}} \) for \( 1 \leq i \leq n \)

\[
\begin{array}{c}
\text{Verifier} \\
\text{(secret } K) \\
\text{Pick a random } N_V \in \mathbb{R} \{0,1\}^4 \\
H_1 \cdots H_{I_d} = \text{hash}(K, N_P, N_V) \\
\text{Fill the graph:} \\
\text{for } i = 0 \text{ to } 2n - 1:
&\qquad k_i = H_{i+2n+1} \\
&\qquad q_i = H_{i+1}
\end{array}
\]

\[
\begin{array}{c}
\text{Prover} \\
\text{(secret } K) \\
\text{Pick a random } N_P \in \mathbb{R} \{0,1\}^4 \\
H_1 \cdots H_{I_d} = \text{hash}(K, N_P, N_V) \\
\text{Fill the graph:} \\
\text{for } i = 0 \text{ to } 2n - 1:
&\qquad k_i = H_{i+2n+1} \\
&\qquad q_i = H_{i+1}
\end{array}
\]

11.1. Impersonation

We assume that nonce size and key size are large enough to ensure the negligibility of the success probability for the key recovery and the replay attack. The common manner to impersonate a prover is by guessing all the answers during the fast phase. Hence, we have: \( \Pr_{\text{Imp}} = \left( \frac{1}{2} \right)^n \).

11.2. Mafia Fraud

\( \circ \) Pre-ask strategy. Let \( g(i, j, k) = \frac{1}{2} + \frac{1}{2^{2n-1}} \sum_{t=0}^{t=2n-1} (A^{i-t}[1, t]A^{i-t}[2, t] + A^{j-k}[2, t]A^{j-k}[1, t]) \) where \( A \) is the adjacency matrix of the graph which represents the graph-based protocol [Trujillo-Rasua et al. 2010]. Also, let:

\[
f(i, j, k) =
\begin{cases}
1 & \text{if } j < k \text{ and } i = j, \\
\frac{1}{2} & \text{if } j < k \text{ and } i \neq j, \\
\frac{1}{2} & \text{if } j > k \text{ and } i < k, \\
g(i, j, k) & \text{if } j \geq k \text{ and } i \geq k.
\end{cases}
\]

We then have: \( \Pr_{\text{MF[pre]}} = \sum_{k=1}^{k=n} \frac{1}{2^k} \prod_{j=k}^{j=n} \max(f(1, j, k), \ldots, f(n, j, k)) = \frac{1}{2^n} \) [Trujillo-Rasua et al. 2010]

\( \circ \) Post-ask strategy. This protocol does not contain any second slow phase and the first slow phase consists of nonce exchanges only. As per Section 2 we have: \( \Pr_{\text{MF[post]}} = \Pr_{\text{Imp}} \).

11.3. Distance Fraud (White Box)

\( \circ \) Early-reply strategy with one run. \( \Pr_{\text{DF[WB(1)]early}} \) is upper bounded by [Trujillo-Rasua et al. 2010]:

\[
\frac{1}{2} \left( \frac{1}{2^n} + \frac{1}{2^{2n-1}} - \frac{4}{2^n} + 4q \right)
\]

where

\[
q = \prod_{i=1}^{i=n} \left( \frac{1}{2} + \frac{1}{2^{2n-1}} \sum_{k=0}^{k=n-1} (A^i[0, k])^2 \right)
\]

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protocol [Avoine and Tchamkerten 2009] – which then can alternatively be represented by an acyclic graph. Poulidor benefits from a lower memory requirement compared to the AT protocol. Security is also reduced.
Computing in a similar way than in [Trujillo-Rasua et al. 2010], we find the following relation for \( A^i[0,k] \):
\[
A^i[0,k] = \begin{cases} 
\binom{k}{i} & \text{if } i \leq k \leq 2i, \\
0 & \text{otherwise},
\end{cases}
\]
and finally:
\[
q = \prod_{i=1}^{\frac{n-1}{2}} \left( \frac{1}{2} + \frac{n}{2^{i+1}} \right).
\]

Remark that finding an exact value for \( \Pr_{\text{DF|WB(1)|early}} \) is an NP-hard problem [Trujillo-Rasua 2013].

- **Early-reply strategy with** \( p \) **runs**. This strategy makes sense for this protocol, but so far, neither has been computed \( \Pr_{\text{DF|WB(1)|early}} \) nor can be computed \( \Pr_{\text{DF|WB(p)|early}} \).

- **Circle strategy**. Although it makes sense to consider the circle analysis for this protocol, the calculation of the distance fraud success probability in this scenario is also an open problem. Indeed, when the number of circles is greater than \( n \), this problem is as hard as the calculation of \( \Pr_{\text{DF|WB(1)|early}} \).

### 11.4. Distance Fraud (Black Box)

- **Pre-ask combined with early-reply strategy**. With the pre-ask strategy, the adversary may learn the values of a walk in the graph. Note that, this is exactly the same knowledge obtained for an adversary attempting to perform a mafia fraud attack by using the pre-ask strategy. However, contrary to the mafia fraud attack, the adversary does not receive any challenge from the verifier when she is performing a distance fraud attack. We consequently have \( \Pr_{\text{DF|BB|pre&early}} \leq \Pr_{\text{MF|pre}} \). The equality of this equation holds when the adversary actually receives every challenge before sending its corresponding response, i.e., when the adversary is in the close vicinity of the verifier. Therefore, for this protocol the circle strategy makes sense. The closer to the verifier the adversary is, the higher her probability of success is, but it is still upper-bounded by \( \Pr_{\text{MF|pre}} \).

- **Post-ask combined with early-reply strategy**. \( \Pr_{\text{DF|BB|post&early}} = \Pr_{\text{Imp}} \).

- **Circle strategy**. As explained above, the circle strategy makes sense for this protocol. Nevertheless, the adversary’s success probability by using this strategy is upper-bounded by \( \Pr_{\text{MF|pre}} \).

### 11.5. Terrorist Fraud (White Box)

This protocol is not designed to resist to the terrorist fraud in the white box model. Indeed, the prover is able, without revealing his secret \( K \), to provide his accomplice with the graph required to successfully pass thorough the protocol. Hence: \( \Pr_{\text{TF|WB}} = 1 \).

### 12. KIM AND AVOINE’S PROTOCOL (KA2) (2011)

Kim and Avoine introduced in 2009 a distance-bearing protocol with mixed challenges [Kim and Avoine 2009], namely challenges known and challenges unknown in advance by the prover. Challenges known in advance allow the prover to help the verifier to detect an attack, but these challenges also allow the prover to perform a distance fraud. Kim and Avoine improved their protocol in 2011, yielding a new variant known as KA2 [Kim and Avoine 2011], which is analyzed in this section (Algorithm 10).

#### 12.1. Impersonation

Guessing the fast phase answers is enough to impersonate the prover: \( \Pr_{\text{Imp}} = \left( \frac{1}{2} \right)^n \).

#### 12.2. Mafia Fraud

- **Pre-ask strategy**. \( \Pr_{\text{MF|pre}} = \left( \frac{1}{2} \right)^{n-\alpha} \left( \frac{1}{2} \right)^{\alpha} + \alpha \left( \frac{1}{2} \right)^{n+1} \) [Kim and Avoine 2011].

- **Post-ask strategy**. This protocol does not contain any second slow phase and the first slow phase consists of nonce exchanges only. As per Section 2 we have: \( \Pr_{\text{MF|post}} = \Pr_{\text{Imp}} \).

#### 12.3. Distance Fraud (White Box)

- **Early-reply strategy with one run**. \( \Pr_{\text{DF|WB(1)|early}} = \left( \frac{1}{2} \right)^{n-\alpha} \) [Kim and Avoine 2011].

- **Early-reply strategy with** \( p \) **runs**. The success probability in case of early-reply strategy with \( p \) runs of the pseudo-random function is provided in [Avoine et al. 2011]:
\[
\Pr_{\text{DF|WB(p)|early}} = \frac{1}{2^{(n-\alpha)}} \cdot \sum_{i=0}^{\frac{j=n-\alpha-1}{2}} \left( \frac{1}{2} \right)^i \cdot \left[ \sum_{j=1}^{\frac{j=n-\alpha}{j}} \left( \frac{n-\alpha}{j} \right)^p - \sum_{j=\frac{n+1}{j+1}}^{\frac{j=n-\alpha}{j}} \left( \frac{n-\alpha}{j} \right)^p + \frac{1}{2} \right]^n.
\]

- **Circle strategy**. Rounding being independent, the circle analysis offers no benefit to an adversary.
12.4. Distance Fraud (Black Box)

- Pre-ask combined with early-reply strategy. As with the Mafia fraud with pre-ask strategy, the success probability is $\Pr_{DF|BB|pre&early} = \Pr_{MF|pre}$.
- Post-ask combined with early-reply strategy. This protocol does not contain any second slow phase and the first slow phase consists of nonce exchanges only. As per Section 2 we have: $\Pr_{DF|BB|post&early} = \Pr_{Imp}$.
- Circle strategy. We previously stressed that the circle analysis is worthless for this protocol.

12.5. Terrorist Fraud (White Box)

This protocol is not designed to resist to the terrorist fraud in the white box model. The prover can give the registers to his accomplice to successfully pass the protocol: $\Pr_{TF|WB} = 1$.


Yum, Kim, Hon and Lee created a distance-bounding protocol with mutual authentication [Yum et al. 2011].

13.1. Impersonation

The only known way to succeed at the impersonation consists of guessing all the answers during the fast phase, which leads to the probability $\Pr_{Imp} = \left(\frac{1}{2}\right)^n$.

13.2. Mafia Fraud

- Pre-ask strategy. Avoine and Kim proposed a new attack that yields a higher adversary success probability [Avoine and Kim 2013]. Their attack depends on the probability of finding $D_i$’s, $\Pr_{D}$, which varies according to the system parameters. Following this attack, the probability of the mafia fraud is:

$$\Pr_{MF|pre} = \left\{ \begin{array}{ll}
\left(\frac{2}{3}\right)^n + \sum_{i=1}^{n} \left(\frac{2}{3}\right)^{i-1} \cdot \left(\frac{2}{3}\right)^{n-i}, & \text{if } \Pr_D = 1 \\
\left(\frac{2}{3}\right)^n + \sum_{i=1}^{n} \left(\frac{2}{3}\right)^{i-1} \cdot \left(\frac{1}{3}\right)^{n-i}, & \text{if } \Pr_D = \frac{1}{2}
\end{array} \right. \quad \text{[Avoine and Kim 2013]}$$

- Post-ask strategy. This protocol does not contain any second slow phase and the first slow phase consists of nonce exchanges only. As per Section 2 we have: $\Pr_{MF|post} = \Pr_{Imp}$.

13.3. Distance Fraud (White Box)

- Early-reply strategy with one run, $\Pr_{DF|WB(1)|early} = \left(\frac{7}{8}\right)^n$ [Yum et al. 2011].
Algorithm 11: YKHL Protocol

User A
(secret $K$)

Pick $N_A \in_R [0,1)^d$ $\rightarrow$ $N_A$

$H = h(K, N_A, N_B)$
$D = H_{i+1} || H_{i+2} || \ldots || H_{i+n}$
$R^0 = H_{i+n+1} || H_{i+n+2} || \ldots || H_{i+3n}$
$R^1 = H_{i+n+1} || H_{i+n+2} || \ldots || H_{i+3n}$

Case I: $D_i = 0$
Pick $c_i \in \{0,1\}$
Start Timer $\rightarrow$ $\epsilon_i$
Stop Timer $\leftarrow \epsilon_i$
If $r_i \neq R^0_{c_i}$ or collision is detected,
If $r_i \neq R^1_{c_i}$ or collision is detected,

Case II: $D_i = 1$
$r_i = \begin{cases} R^0_{c_i} \text{ if } c_i = 0 \\ R^1_{c_i} \text{ if } c_i = 1 \end{cases}$
If collision is detected,
$A$ enters into the protection mode.

Check correctness of $r_i$'s and
$\Delta t_1 \leq t_{\text{max}}$ for Case I

User B
(secret $K$)

Pick $N_B \in_R [0,1)^d$ $\rightarrow$ $N_B$

$H = h(K, N_A, N_B)$
$D = H_{i+1} || H_{i+2} || \ldots || H_{i+n}$
$R^0 = H_{i+n+1} || H_{i+n+2} || \ldots || H_{i+3n}$
$R^1 = H_{i+n+1} || H_{i+n+2} || \ldots || H_{i+3n}$

Check correctness of $r_i$'s and
$\Delta t_1 \leq t_{\text{max}}$ for Case II

○ Early-reply strategy with $p$ runs. An attacker who impersonates $B$ wins when $D_i = 1$ or $R^0_{c_i} = R^1_{c_i}$. Indeed, when $D_i = 1$, the prover sends a challenge to the verifier ($A$) and so trivially wins the round. When $D_i = 0$, the roles are inverted. To win a round, the prover must send his response in advance. When $R^0_{c_i} = R^1_{c_i}$, the potential answers are the same and the prover definitely wins. Running the cryptographic function $p$ times allows the prover to find $D$ with a higher Hamming weight than the average one, and $R^0$ and $R^1$ with a lower Hamming distance. In conclusion, the probability of success is higher with the YKHL Protocol than with the HK protocol, where the prover wins only when $R^0_{c_i} = R^1_{c_i}$. The probability of success can be calculated by considering $\Pr(X = x) = \binom{n}{x} \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{n-x} / 2^{2^n}$ instead of $\Pr(X = x) = \binom{n}{x} / 2^n$ in [Avoine et al. 2011].

○ Circle strategy. Rounds being independent, the circle analysis offers no benefit to an adversary.

13.4. Distance Fraud (Black Box)

○ Pre-ask combined with early-reply strategy. A distance fraud with the pre-ask strategy is similar to a mafia fraud in this case. Hence: $\Pr_{TF|BB|\text{pre&early}}(\text{post&early}) = \Pr_{TF|\text{pre}}(\text{post&early})$.

○ Post-ask combined with early-reply strategy. This protocol does not contain any second slow phase and the first slow phase consists of nonce exchanges only. As per Section 2 we have: $\Pr_{TF|BB|\text{post&early}}(\text{post&early}) = \Pr_{TF|\text{imp}}$.

○ Circle strategy. We previously stressed that the circle analysis is worthless for this protocol.

13.5. Terrorist Fraud (White Box)

This protocol is not designed to resist to the terrorist fraud in the white box model. Indeed, the prover can give the registers to his accomplice to successfully pass the protocol: $\Pr_{TF|\text{WB}} = 1$.

13.6. Published Attacks

Avoine and Kim demonstrated in [Avoine and Kim 2013] that the security of YKHL protocol is far below what is claimed in [Yum et al. 2011], and could be even worse than HK protocol in terms of mafia fraud.
14. SKI PROTOCOLS (2013)
In [Boureanu et al. 2013c; 2013b; 2013a], the authors introduced a series of protocols called SKI. These protocols are presented in Algorithm 12.

Algorithm 12: The SKI Protocols

<table>
<thead>
<tr>
<th>Verifier (secret K)</th>
<th>Prover (secret K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pick a ∈ ℤ_q^n, L ∈ ℒ, and N_v ∈ ℤ</td>
<td>Pick N_v ∈ {0, 1}^q</td>
</tr>
<tr>
<td>M = a + f_L(N_v, N_v, L)</td>
<td>a = M - f_K(N_v, N_v, L), K' = L(K)</td>
</tr>
<tr>
<td>Pick c_i ∈ {1,...,t}</td>
<td>r_i = F(c_i, a_i, K'_i)</td>
</tr>
<tr>
<td>Start Timer</td>
<td>Halt</td>
</tr>
<tr>
<td>Stop timer</td>
<td>Otherwise</td>
</tr>
<tr>
<td>Check the correctness of r_i and</td>
<td>( δ_t \leq t_{\text{max}} ) for at least ( n - r ) rounds</td>
</tr>
<tr>
<td>( i \in {1,\ldots,t} )</td>
<td>( a_{\text{out}} V )</td>
</tr>
<tr>
<td>Out</td>
<td></td>
</tr>
</tbody>
</table>

14.1. Impersonation
The only known way to perform an impersonation consists in guessing all the answers during the fast phase. Thus, \( \Pr_{\text{Imp}} = \left( \frac{1}{q} \right)^n \).

14.2. Mafia Fraud
- **Pre-ask strategy.** Using this strategy, the adversary is able to obtain one set of answers from the prover before executing the fast phase with the verifier. Without loss of generality, we assume that the adversary obtains \( \{F(1, a_1, K'_1), \ldots, F(1, a_t, K'_t)\} \), i.e., the answers corresponding to the challenges \( c_i \)'s equal to 1. Hence, at each rounds two cases occur: (a) the verifier’s challenge is 1 and she knows the answer, this happens with probability \( 1/t \), or (b) the verifier’s challenge is not 1, thus she has to guess the answer, and succeeds with probability \( 1/q \). Thus, the rounds independence yields to:

\[
\Pr_{\text{MF|pre}} = \left( \frac{1}{t} \cdot 1 + (1 - \frac{1}{t}) \cdot \frac{1}{q} \right)^n = \left( \frac{q + t - 1}{qt} \right)^n.
\]

For SKI_{\text{pre}}, this is \( \left( \frac{2}{3} \right)^n \).

- **Post-ask strategy.** This protocol does not contain any second slow phase and the first slow phase consists of nonce exchanges only. As per Section 2 we have: \( \Pr_{\text{MF|post}} = \Pr_{\text{Imp}} \).

14.3. Distance Fraud (White Box)
- **Early-reply strategy with one run.** Using this strategy, the adversary has to send her answers in advance. Due to the similarity between Hancke and Kuhn’s protocol and SKI’s protocol, the adversary applies a similar strategy to maximize her success probability. At each rounds she answers the most probable value among the possible registers. The most probable answer for a given round is the one that appears the most within the set \( \{F(1, a_1, K'_1), F(2, a_1, K'_1), \ldots, F(t, a_1, K'_t)\} \) of possible answers for this round. In order to compute the adversary success probability, let define the following events:

- \( \mathcal{W} \): the adversary provides the correct answer to the verifier at a given round.
- \( \mathcal{B}_j \): \( j = \max \{X_i\} \),

where \( X_i \) is the number of appearance times of the \( l \)-th element from \( \mathbb{F}_q^t \) among the set \( \{F(1, a_1, K'_1), F(2, a_1, K'_1), \ldots, F(t, a_1, K'_t)\} \). We then trivially have: \( \Pr(\mathcal{W}) = \sum_{j=1}^{t} \Pr(\mathcal{W}|\mathcal{B}_j) \Pr(\mathcal{B}_j) \), with \( \Pr(\mathcal{W}|\mathcal{B}_j) = \frac{1}{q} \). Thus, using the above equation, we deduce: \( \Pr(\mathcal{W}) = E \left( \max_{1 \leq i \leq q} \{X_i\} \right) \cdot \frac{1}{q} \). The tricky task
consists in computing $\mathbb{E}(\max_{1 \leq i \leq q} \{ X_i \})$. This is done below for the SKI protocol configurations suggested in [Boureanu et al. 2013a].

- $q = 2$, and $t = 2$: $\mathbb{E}(\max_{1 \leq i \leq q} \{ X_i \}) = \frac{3}{2}$, and $\Pr(W) = \frac{3}{4}$.
- $q = 2$, and $t = 3$: $\mathbb{E}(\max_{1 \leq i \leq q} \{ X_i \}) = \frac{9}{4}$ and $\Pr(W) = \frac{3}{4}$.
- $q = 2$, and $t = 4$: $\mathbb{E}(\max_{1 \leq i \leq q} \{ X_i \}) = 3$ and $\Pr(W) = \frac{3}{4}$.
- $q = 4$, and $t = 3$: $\mathbb{E}(\max_{1 \leq i \leq q} \{ X_i \}) = \frac{15}{8}$ and $\Pr(W) = \frac{5}{8}$.

Finally, the independence of the rounds provides $\Pr_{\text{DF}(\text{WB}(1))|\text{early}} = (\Pr(W))^n$. For SKI$_\text{pro}$, this is $(\frac{3}{4})^n$.

- Early-reply strategy with $p$ runs. This strategy does not make sense against these protocols. Indeed, since the prover does not have the verifier’s nonce before he sends its, he cannot compute several outputs of the pseudo-random function.

- Circle strategy. Rounds being independent, the circle analysis offers no benefit to an adversary.

14.4. Distance Fraud (Black Box)

- Pre-ask combined with early-reply strategy. A distance fraud with the pre-ask strategy is here similar to a mafia fraud. Hence: $\Pr_{\text{DF}(\text{BB})|\text{pre}&\text{early}} = \Pr_{\text{MF}|\text{pre}}$.

- Post-ask combined with early-reply strategy. This protocol does not contain any second slow phase and the first slow phase consists of nonce exchanges only. As per Section 2 we have: $\Pr_{\text{DF}(\text{BB})|\text{post}&\text{early}} = \Pr_{\text{Imp}}$.

- Circle strategy. We previously stressed that the circle analysis is worthless for this protocol.

14.5. Terrorist Fraud (White Box)

- Early-provide strategy with one run. Using this strategy, the adversary obtains register(s) before the start of the fast phase. First, note that the setting in which $t' = t = q = 2$ (i.e., SKI$_\text{Lite}$) does not resist against terrorist fraud. Second, to compute the success probability in the other cases, let denote $k$, the number of registers given by the prover to the adversary. As stated in [Avoine et al. 2011], the insurance that no information could leak, is furnished by the following equality: $k = t - 2$.

Once the adversary gets the $t - 2$ registers, she starts the fast phase with the verifier. Two cases occur, (a) the verifier asks an answer coming from one of the $t - 2$ known registers. Thus, the adversary definitely knows the correct answer. Or (b) the verifier asks her an answer coming from one the two unknown registers, and she has to guess the correct answer. The adversary consequently succeeds with probability $\frac{1}{q}$. Given the rounds are independent, we finally have:

$$\Pr_{\text{TF}(\text{WB}(1))} = \left( \frac{t - 2}{t} \cdot 1 + \frac{2}{t} \cdot \frac{1}{q} \right)^n = \left( \frac{qt + 2(1 - q)}{q^t} \right)^n.$$  For SKI$_\text{pro}$, this is $(\frac{2}{3})^n$.

- Early-provide strategy with $p$ runs. This strategy does not make sense against these protocols. Indeed, since the prover does not have the verifier’s nonce before he sends its answers, he cannot compute several outputs of the the pseudo-random function.

15. PROTOCOL COMPARISON

This section provides a summary of the analyses done in Sections 3 to 14. It then provides two approaches to compare the protocols: the first one consists of charts, while the second one is based on the concept of clusters. The charts depict the variation of a single parameter regarding another one, e.g., the mafia fraud success probability as a function of the number of rounds. The second approach introduces clusters of protocols sharing common security resistances and properties. It is worth remarking that a similar comparison approach based on decision theory has been recently published in [Avoine et al. 2015]. The findings of that work do not contradict ours; indeed every protocol found relevant there is also considered relevant here.

15.1. Summary of Properties and Performances

Table VI presents the properties and performances of every protocol analyzed through Sections 3 to 14. The description of the properties is provided in Section 2. Table VII and VIII summarize which cryptographic
building blocks are used and which properties are expected by each considered protocol. Greyed cells in Table VIII contain results already known, while other cells contain values provided by this survey.

On Table VIII, we can see that only two protocols do not have any attack with probability 1: Swiss-knife and SKI. In fact, there exists two other protocols which are not in this table: TDB [Avoine et al. 2011] (on which SKI is based) and the protocol by [Fischlin and Onete 2013b] (which is based on Swiss-knife).

15.2. Chart-based Comparison

Figure 2 depicts the mafia fraud success probability as a function of the number of rounds. The relative positions between the curves remain unchanged when the number of rounds increases, except for the tree-based protocol (with \( \ell = \sqrt{n} \)), which suffers from a step effect due to its structure. Several behaviors are observed in the figure: BC, for example, has a success probability of \((1/2)^n\), which is the optimal case, while HK, by contrast, has a \((3/4)^n\) success probability. The extreme case would be probability equals to 1 but no protocol falls into this category. Other intermediate behaviors are also present: protocols whose associated probability is not \((1/2)^n\) but tends to \((1/2)^n\) when \(n\) is large enough (e.g., \(AT(\sqrt{n})\)), protocols whose associated probability is between \((1/2)^n\) and \((3/4)^n\), and finally those whose associated probability is between \((3/4)^n\) and 1. These categories are summarized in Table IX.

Figure 3 represents the distance fraud success probability as a function of the number of rounds. The best ones without final slow phase have dependent rounds (see Section B.4), namely the tree-based protocol (with \(l = 1\) and with \(l = \sqrt{n}\)). Note that the tree-based protocol and Poulidor do not have close formula to express the associated distance fraud success probability: only a maximum bound is known, which makes the comparison with other protocols quite unfair.

<table>
<thead>
<tr>
<th>Protocol</th>
<th>Adaptiveness</th>
<th>Mutual Authentication</th>
<th>Second Phase</th>
<th>Independence of the Rounds</th>
<th>Exchanged Bits during Slow Phase</th>
<th>Exchanged Bits during Fast Phase</th>
<th>Memory</th>
</tr>
</thead>
<tbody>
<tr>
<td>BC</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>(2\ell + n)</td>
<td>(2\ell)</td>
<td>(2\ell)</td>
</tr>
<tr>
<td>MAD</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>(2(\ell + \sigma + \ell))</td>
<td>(2n)</td>
<td>(2n)</td>
</tr>
<tr>
<td>HK</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>(2\ell + \ell\sigma)</td>
<td>(2n)</td>
<td>(2n)</td>
</tr>
<tr>
<td>MP</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>(2\ell + \delta + \ell\sigma)</td>
<td>((2(n - \ell\tau_{total})))</td>
<td>(2n)</td>
</tr>
<tr>
<td>Swiss-knife</td>
<td>No(*)</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>((\delta + \ell\sigma + \ell\sigma + \ell\tau))</td>
<td>(2n)</td>
<td>(2n + 2\ell)</td>
</tr>
<tr>
<td>Tree-based</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>(\ell\ell_{if} \geq 2)</td>
<td>(2\ell + c)</td>
<td>((2\ell^{1/2} - 2))</td>
</tr>
<tr>
<td>RC</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>(\ell\ell)</td>
<td>(\ell\ell)</td>
<td>(\ell\ell)</td>
</tr>
<tr>
<td>Poulidor</td>
<td>No(*)</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>(\ell\ell)</td>
<td>(\ell\ell)</td>
<td>(\ell\ell)</td>
</tr>
<tr>
<td>KAT</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>(\ell\ell)</td>
<td>(\ell\ell)</td>
<td>(\ell\ell)</td>
</tr>
<tr>
<td>YKHL</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>(\ell\ell)</td>
<td>(\ell\ell)</td>
<td>(\ell\ell)</td>
</tr>
<tr>
<td>SKI_{p2o}</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>(\ell\ell)</td>
<td>(\ell\ell)</td>
<td>(\ell\ell)</td>
</tr>
</tbody>
</table>

(*) See Section 11 for a refined analysis about the adaptiveness.

Figure 4 presents the memory needed to store intermediate values during the execution of the protocol, including the registers. The memory consumption is expressed as a function of the number of rounds. The curves can be classified into three categories: linear curves, affine curves which are not linear due to a fixed overhead, and non-affine curves. In the latter case, which includes the tree-based protocol (with \(l = \sqrt{n}\)), the memory consumption is prohibitive. The overhead appears in the protocols that end with a final slow phase (Swiss-knife, MAD, and RC). In such a phase, the value of the challenges, or commitments used in the first slow phase, are usually stored all along the protocol execution because they are required for the final cryptographic operations. A final slow phase is consequently a handicap for implementations.
Table VII: Cryptographic building blocks

<table>
<thead>
<tr>
<th>Protocol</th>
<th>PRNG</th>
<th>Sym. Primitive</th>
<th>Commitment</th>
<th>Signature</th>
</tr>
</thead>
<tbody>
<tr>
<td>BC</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>MAD</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>HK</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>MP</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Swiss-knife</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Tree-based</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>RC</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Poulidor</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>KA2</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>YKHL</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>SKI</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Table VIII: Adversary success probabilities

<table>
<thead>
<tr>
<th>Protocol</th>
<th>Imp</th>
<th>Mafia</th>
<th>Distance</th>
<th>Terrorist</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>pre-ask</td>
<td>post-ask</td>
<td>early-reply</td>
<td>pre &amp; early</td>
</tr>
<tr>
<td>BC</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
</tr>
<tr>
<td>MAD</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
</tr>
<tr>
<td>HK</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
</tr>
<tr>
<td>MP</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
</tr>
<tr>
<td>Swiss-knife</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
</tr>
<tr>
<td>Tree-based</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
</tr>
<tr>
<td>RC</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
</tr>
<tr>
<td>Poulidor</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
</tr>
<tr>
<td>KA2</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
</tr>
<tr>
<td>YKHL</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
</tr>
<tr>
<td>SKI</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
<td>$\left(\frac{1}{2}\right)$</td>
</tr>
</tbody>
</table>

Table IX: Parameters and their values

<table>
<thead>
<tr>
<th>$\mathcal{M}_a$</th>
<th>$\mathcal{T}$</th>
<th>$\mathcal{D}$</th>
<th>$\mathcal{L}$</th>
<th>$\mathcal{B}$</th>
<th>$\mathcal{E}$</th>
<th>$\mathcal{M}_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value 1</td>
<td>$\mathcal{M}_a = \left(\frac{1}{2}\right)^n$</td>
<td>$t = \left(\frac{1}{2}\right)^n$</td>
<td>$d = \left(\frac{1}{2}\right)^n$</td>
<td>$\mathcal{I} = \text{NO}$</td>
<td>$\mathcal{B} = \text{YES}$</td>
<td>$\mathcal{E} = 2 \cdot n + \text{cost}$</td>
</tr>
<tr>
<td>Value 2</td>
<td>$\mathcal{M}_a = \left(\frac{1}{2}\right)^n$</td>
<td>$\lim \mathcal{T} = \left(\frac{1}{2}\right)^n$</td>
<td>$\lim \mathcal{D} = \left(\frac{1}{2}\right)^n$</td>
<td>$\mathcal{I} = \text{NO}$</td>
<td>$\mathcal{B} = \text{YES}$</td>
<td>$\mathcal{E} = 3 \cdot n + \text{cost}$</td>
</tr>
<tr>
<td>Value 3</td>
<td>$\left(\frac{1}{2}\right)^n &lt; \mathcal{M}_a &lt; \left(\frac{1}{2}\right)^n$</td>
<td>$\left(\frac{1}{2}\right)^n &lt; t &lt; \left(\frac{1}{2}\right)^n$</td>
<td>$\left(\frac{1}{2}\right)^n &lt; d &lt; \left(\frac{1}{2}\right)^n$</td>
<td>$\mathcal{I} = \text{YES}$</td>
<td>$\mathcal{B} = \text{YES}$</td>
<td>$\mathcal{E} = 4 \cdot n$</td>
</tr>
<tr>
<td>Value 4</td>
<td>$\mathcal{M}_a = \left(\frac{1}{2}\right)^n$</td>
<td>$t = \left(\frac{1}{2}\right)^n$</td>
<td>$d = \left(\frac{1}{2}\right)^n$</td>
<td>$\mathcal{I} = \text{YES}$</td>
<td>$\mathcal{B} = \text{YES}$</td>
<td>$\mathcal{E} = 4 \cdot n$</td>
</tr>
<tr>
<td>Value 5</td>
<td>$\left(\frac{1}{2}\right)^n &lt; \mathcal{M}_a &lt; 1$</td>
<td>$\left(\frac{1}{2}\right)^n &lt; t &lt; 1$</td>
<td>$\left(\frac{1}{2}\right)^n &lt; d &lt; 1$</td>
<td>$\mathcal{I} = \text{YES}$</td>
<td>$\mathcal{B} = \text{YES}$</td>
<td>$\mathcal{E} = 4 \cdot n$</td>
</tr>
<tr>
<td>Value 6</td>
<td>$\mathcal{M}_a = 1$</td>
<td>$t = 1$</td>
<td>$d = 1$</td>
<td>$\mathcal{I} = \text{YES}$</td>
<td>$\mathcal{B} = \text{YES}$</td>
<td>$\mathcal{E} = 4 \cdot n$</td>
</tr>
</tbody>
</table>

Figure 5 represents the mafia fraud success probability as a function of the distance fraud success probability, with the number of rounds $n$ equal to 36. The figure clearly shows that the protocols are more resistant to mafia fraud than to distance fraud. Several reasons could probably explain this phenomenon. In particular, the original objective of distance bounding was – early in the nineties – to protect authentication protocols against relay attacks. The distance fraud was then a side effect of distance bounding. It has been only recently, when the need to protect geolocalisation applications against distance frauds arose, especially when the prover is a mobile device, that distance fraud has started to be considered seriously.

15.3. Cluster-based Comparison

Comparing distance bounding protocols is quite a tricky task given the large number of parameters that can be considered. A given protocol $P_1$ can be better in terms of resistance against mafia fraud than another
equal to (and, to illustrate the concept, it is clearly better for a protocol to use single bit exchanges than ternary messages, etc. For each parameter, the values can be ranked: (Value 6) \prec (Value 5) \prec (Value 4) \prec (Value 3) \prec (Value 2) \prec (Value 1), where (Value i) \prec (Value j) means that (Value j) is better than (Value i) with respect to (Value j).

Note that in Table IX, no value is given to the constant. Since we are interested in how the number of exchanged bits scales with the number of rounds, the actual value of the constant does not really matter.

Fig. 2: \( \Pr_{MF|pre} \)

Fig. 3: \( \Pr_{DF|WB(1)early} \)

Fig. 4: Memory consumption

Fig. 5: \( \Pr_{MF|pre} \) vs. \( \Pr_{DF|WB(1)early} \) (n=36)
than (Value i) or, said differently, (Value j) is more convenient than (Value i) when implementing a distance bounding protocol. The parameters and their values are provided in Table IX.

The configuration of a protocol is an element of the cartesian product $\mathcal{M}_a \times \mathcal{T} \times \mathcal{D} \times \mathcal{L} \times \mathcal{B} \times \mathcal{E} \times \mathcal{M}_e$. All the possible configurations can be deduced from Table IX, and the configuration of each protocol presented in this work is provided in Table X. Note that there is no total order relation in $\mathcal{M}_a \times \mathcal{T} \times \mathcal{D} \times \mathcal{L} \times \mathcal{B} \times \mathcal{E} \times \mathcal{M}_e$, but a configuration $(\text{ma}, \text{d}, \text{t}, \text{i}, \text{b}, \text{e}, \text{me})$ is better than a configuration $(\text{ma}', \text{d}', \text{t}', \text{i}', \text{b}', \text{e}', \text{me}')$ if it is better for every considered parameter: $\text{ma}' < \text{ma}$, $\text{d}' < \text{d}$, ..., $\text{me}' < \text{me}$. As a consequence, there exist several best configurations in $\mathcal{M}_a \times \mathcal{T} \times \mathcal{D} \times \mathcal{L} \times \mathcal{B} \times \mathcal{E} \times \mathcal{M}_e$.

A cluster is a set (possibly empty) of protocols which have the same configuration $(\text{ma}, \text{d}, \text{t}, \text{i}, \text{b}, \text{e}, \text{me})$ or a configuration that is better than this one. A cluster is said better than another one if its configuration is better. The total number of clusters is large; it is actually equal to the cardinality of $\mathcal{M}_a \times \mathcal{T} \times \mathcal{D} \times \mathcal{L} \times \mathcal{B} \times \mathcal{E} \times \mathcal{M}_e$, which is $6^7 \cdot 2^2 \cdot 3^2 = 7776$. 5774 clusters are empty, meaning that no protocol matches the configuration of these clusters. The remaining 2002 non-empty clusters still represent a large amount of information, which is difficult to condense in a paper. To further reduce this information, only best configurations are kept, those whose clusters are not empty.

This process can be easily automated. A hierarchy of clusters is built and the best cluster of every branch is kept. After performing this operation, only 5 clusters remain: {Poulidor}, {Swiss-Knife}, {SKIhamir}, {RC}, and {BC, MAD}. Four of the remaining clusters are actually singletons, which means that among all the published protocols, none of them are equivalent with respect to the seven considered parameters. In the remaining cluster, {BC, MAD}, BC and MAD are equivalent since the mutual authentication is not considered in the configurations. We can also raise that, given constraints on memory, probabilities, etc. the best known protocol to be used belongs to these 6 finalists.

It is finally interesting to compare these 6 finalists with the distance-bounding evolution provided in Figure 1. A protocol that is not a finalist should not necessarily be blamed: most of them have been useful at some point and led to more evolved protocols. However, protocols published today should be new finalists in the cluster-based comparison, possibly after considering additional parameters in the comparison.

### Table X: Protocol configurations

<table>
<thead>
<tr>
<th>Protocols</th>
<th>$\text{ma}$</th>
<th>$\text{d}$</th>
<th>$\text{t}$</th>
<th>$\text{i}$</th>
<th>$\text{b}$</th>
<th>$\text{e}$</th>
<th>$\text{me}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BC</td>
<td>$(\frac{1}{2})^i$</td>
<td>$(\frac{1}{2})^i$</td>
<td>1</td>
<td>Y</td>
<td>Y</td>
<td>$3 \cdot n + \text{cst}$</td>
<td>linear</td>
</tr>
<tr>
<td>MAD</td>
<td>$(\frac{1}{2})^i$</td>
<td>$(\frac{1}{2})^i$</td>
<td>1</td>
<td>Y</td>
<td>Y</td>
<td>$3 \cdot n + \text{cst}$</td>
<td>linear</td>
</tr>
<tr>
<td>HK</td>
<td>$(\frac{1}{2})^i$</td>
<td>$(\frac{1}{2})^i$</td>
<td>1</td>
<td>N</td>
<td>Y</td>
<td>$2 \cdot n + \text{cst}$</td>
<td>linear</td>
</tr>
<tr>
<td>MP $p_f = 0.5$</td>
<td>$(\frac{1}{2})^i$</td>
<td>$(\frac{1}{2})^i$</td>
<td>$&gt; (\frac{1}{2})$ and $&lt; 1$</td>
<td>1</td>
<td>Y</td>
<td>$\geq 4 \cdot n$</td>
<td>linear</td>
</tr>
<tr>
<td>MP $p_f = 0.75$</td>
<td>$(\frac{1}{2})^i$</td>
<td>$(\frac{1}{2})^i$</td>
<td>$&gt; (\frac{1}{2})$ and $&lt; 1$</td>
<td>1</td>
<td>Y</td>
<td>$\geq 4 \cdot n$</td>
<td>linear</td>
</tr>
<tr>
<td>Swiss-Knife</td>
<td>$(\frac{1}{2})^i$</td>
<td>$(\frac{1}{2})^i$</td>
<td>$(\frac{1}{2})^i$</td>
<td>1</td>
<td>Y</td>
<td>$3 \cdot n + \text{cst}$</td>
<td>affine</td>
</tr>
<tr>
<td>Tree-Based $\ell = \sqrt{n}$</td>
<td>$\lim_{n \to +\infty} = (\frac{1}{2})^n$</td>
<td>$\lim_{n \to +\infty} = (\frac{1}{2})^n$</td>
<td>1</td>
<td>N</td>
<td>Y</td>
<td>$3 \cdot n + \text{cst}$</td>
<td>non affine</td>
</tr>
<tr>
<td>Tree-Based $\ell = 1$</td>
<td>$\lim_{n \to +\infty} = (\frac{1}{2})^n$</td>
<td>$\lim_{n \to +\infty} = (\frac{1}{2})^n$</td>
<td>1</td>
<td>N</td>
<td>Y</td>
<td>$3 \cdot n + \text{cst}$</td>
<td>non affine</td>
</tr>
<tr>
<td>RC</td>
<td>$(\frac{1}{2})^i$</td>
<td>$(\frac{1}{2})^i$</td>
<td>1</td>
<td>Y</td>
<td>N</td>
<td>$2 \cdot n + \text{cst}$</td>
<td>affine</td>
</tr>
<tr>
<td>Poulidor</td>
<td>$\lim_{n \to +\infty} = (\frac{1}{2})^n$</td>
<td>$\lim_{n \to +\infty} = (\frac{1}{2})^n$</td>
<td>1</td>
<td>N</td>
<td>Y</td>
<td>$2 \cdot n + \text{cst}$</td>
<td>linear</td>
</tr>
<tr>
<td>KA2 $\alpha = n$</td>
<td>$\lim_{n \to +\infty} = (\frac{1}{2})^n$</td>
<td>1</td>
<td>1</td>
<td>N</td>
<td>Y</td>
<td>$2 \cdot n + \text{cst}$</td>
<td>linear</td>
</tr>
<tr>
<td>KA2 $\alpha = \frac{n}{2}$</td>
<td>$(\frac{1}{2})^i$ and $&lt; (\frac{1}{2})^i$</td>
<td>$(\frac{1}{2})^i$ and $&lt; 1$</td>
<td>1</td>
<td>N</td>
<td>Y</td>
<td>$2 \cdot n + \text{cst}$</td>
<td>linear</td>
</tr>
<tr>
<td>YKHL</td>
<td>$(\frac{1}{2})^i$ and $&lt; 1$</td>
<td>$(\frac{1}{2})^i$ and $&lt; 1$</td>
<td>1</td>
<td>N</td>
<td>Y</td>
<td>$2 \cdot n + \text{cst}$</td>
<td>linear</td>
</tr>
<tr>
<td>SKIhamir</td>
<td>$(\frac{1}{2})^i$</td>
<td>$(\frac{1}{2})^i$ and $&lt; (\frac{1}{2})^i$</td>
<td>$(\frac{1}{2})^i$</td>
<td>N</td>
<td>N</td>
<td>$\geq 4 \cdot n$</td>
<td>linear</td>
</tr>
<tr>
<td>SKIpro</td>
<td>$(\frac{1}{2})^i$ and $&lt; (\frac{1}{2})^i$</td>
<td>$(\frac{1}{2})^i$ and $&lt; (\frac{1}{2})^i$</td>
<td>$(\frac{1}{2})^i$</td>
<td>N</td>
<td>N</td>
<td>$\geq 4 \cdot n$</td>
<td>linear</td>
</tr>
<tr>
<td>SKI</td>
<td>$(\frac{1}{2})^i$ and $&lt; (\frac{1}{2})^i$</td>
<td>$(\frac{1}{2})^i$ and $&lt; (\frac{1}{2})^i$</td>
<td>$(\frac{1}{2})^i$</td>
<td>N</td>
<td>N</td>
<td>$\geq 4 \cdot n$</td>
<td>linear</td>
</tr>
</tbody>
</table>

### 16. CONCLUSION

Distance bounding authentication protocols represent a new class of protocols aiming to thwart distance-based attacks whose feasibility is rendered possible by emerging technologies. This survey provides a thorough state-of-the-art of existing protocols and introduces refined security analyses. The comparisons made provide designers with new means to evaluate their performance in a unified manner according to several security
and resource parameters. It may be worthwhile pointing out that the provided cluster-based comparison can easily be modified to reflect specific practical considerations and/or to include other protocols. Finally, we are aware that attacks other than those considered in this paper might exist. Addressing provable security of distance bounding protocols is therefore a challenge for future research.

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Online Appendix to: Security of Distance-Bounding: A Survey

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A. DESCRIPTION OF THE PROTOCOLS


In 1993, Brands and Chaum designed several distance-bounding protocols [Brands and Chaum 1993]. This analysis focuses on their protocol (Algorithm 1) that mitigates both mafia and distance frauds.

1 Initialisation. The prover should own a signature public/private key.

2 Protocol. The prover randomly generates n commitment bits $m_i \in \{0, 1\}$ and the verifier randomly generates n challenge bits $c_i \in \{0, 1\}$ ($i = 1 \ldots n$). The prover commits on $m_1|\ldots|m_n$ and sends this commitment to the verifier. Then, a phase of n rapid bit exchanges starts. In each round, the verifier starts his timer and sends $c_i$ to the prover, who replies with $r_i = c_i \oplus m_i$. Upon receiving the response bit the verifier stops its timer. Finally, the prover concatenates $c_i$ and $r_i$, signs the 2n bits result, $\text{Sign}_k(c_1|\ldots|c_n|r_1|\ldots|r_n)$, and sends it to the verifier together with the opening of the commitment.

3 Final decision. Upon reception of the signature, the verifier concatenates the 2n bits $c_i$ and $r_i$, and verifies the received signature, the commitment, the measured $\Delta t_i$'s and whether $r_i = c_i \oplus m_i$ for $i = 1 \ldots n$.


In 2003, Capkun, Buttyán, and Hubaux introduced MAD [Capkun et al. 2003], a protocol that works quite similarly to the BC protocol [Brands and Chaum 1993], but provides mutual authentication. Although denoted by P and V, the two parties act as both prover and verifier during the execution of the protocol (Algorithm 2). The notations used in [Capkun et al. 2003] are kept in the description below.

1 Initialisation. Prior to the protocol execution, the two parties (P and V) agree on the security parameters and functions described in Table XI, and a common secret key $K$.

2 Protocol. In the first slow phase, P and V generate two random numbers $r$ and $s$ respectively and send a commitment to the other party on the two random numbers $h(r||s')$ and $h(s||s')$ respectively. During the fast phase the following steps are repeated n times:

- P sends the bit $\alpha_i$ to V, where $\alpha_1 = r_1$ and $\alpha_i = r_i \oplus \beta_{i-1}$ for $i > 1$;
- V sends the bit $\beta_i = s_i \oplus \alpha_i$ to P.

In the second slow phase, P retrieves the bit sequence s by assuming that $s_i = \alpha_i \oplus \beta_i$ for every $i \in \{1, 2, \ldots, n\}$ and computes $\mu_P = \text{MAC}_K(ID_P||ID_V||r_1||s_1||\ldots||r_n||s_n)$ using the secret key $K$. 

Similarly, $V$ computes the bits $r_1 = \alpha_1$ and $r_i = \alpha_i \oplus \beta_{i-1}$ for $i > 1$, with which $V$ computes $\mu_V = \text{MAC}_K(ID_V || ID_P || s_1 || r_1 || \ldots || s_n || r_n)$. Finally, $P$ and $V$ open the commitment sent in the first slow phase by transmitting $r'$ and $s'$, and exchange the values $\mu_P$ and $\mu_V$.

- **Final Decision.** The users $P$ and $V$ accept each other’s entity only if:
  - the $n$ responses of the fast phase are correct,
  - the commitment that was sent in the first slow phase is correctly opened in the second slow phase and corresponds to the bit sequence ($r$ or $s$) exchanged during the fast phase,
  - the output of the MAC function is correct, and
  - the time constraint $\Delta t_i \leq t_{\text{max}}$ is met for $i \in \{1, 2, \ldots, n\}$ and some threshold $t_{\text{max}} > 0$.

### Table XI: Parameters and functions (Algorithm 2)

<table>
<thead>
<tr>
<th>$n$</th>
<th>Number of iterations in the fast phase, which is also the size of the random numbers $r$ and $s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>Size of the random numbers $r'$ and $s'$</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Size of the secret key $K$</td>
</tr>
<tr>
<td>$t_{\text{max}}$</td>
<td>Threshold of the round-trip time</td>
</tr>
<tr>
<td>$\text{MAC}_K$</td>
<td>MAC function keyed with $K$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Output size of the MAC function</td>
</tr>
<tr>
<td>$h$</td>
<td>Collision-resistant one-way hash function used to compute the commitment</td>
</tr>
</tbody>
</table>


In 2005 Hancke and Kuhn published the first distance-bounding protocol [Hancke and Kuhn 2005] (Algorithm 3) clearly dedicated to RFID. The protocol relies on the original ideas of Desmedt et al. [Desmedt et al. 1988; Bengio et al. 1991] but is different from Brands and Chaum’s work [Brands and Chaum 1993] in the sense that Hancke and Kuhn’s protocol does not have any final signature after the fast phase.

- **Initialization.** Prior to the protocol execution, the legitimate prover and the verifier agree on the security parameters and functions described in Table XII, and a common secret key $K$.

- **Protocol.** During the slow phase, the verifier sends to the prover a nonce $N_V$ and the prover sends to the verifier a nonce $N_P$. Both the prover and the verifier then use the pseudo-random function $h$ and the secret key $K$ in order to generate two $n$-bit sequences $R_0$ and $R_1$. For each of the $n$ rounds of the fast phase, the verifier generates and sends a random challenge bit $c_i$, and the prover replies instantly with a one-bit response that is either $R_0^i$ or $R_1^i$, selected by the value of $c_i$.

- **Final Decision.** The verifier accepts the prover’s identity only if the $n$ responses of the fast phase are correct while meeting the time constraint $\Delta t_i \leq t_{\text{max}}$, $i \in \{1, 2, \ldots, n\}$, for some threshold $t_{\text{max}} > 0$.

### Table XII: Parameters and functions (Algorithm 3)

<table>
<thead>
<tr>
<th>$n$</th>
<th>Number of iterations in the fast phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>Size of the secret key $K$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Size of nonces $N_V$ and $N_P$</td>
</tr>
<tr>
<td>$t_{\text{max}}$</td>
<td>Threshold of the round-trip time</td>
</tr>
<tr>
<td>$h$</td>
<td>Hash function whose output size is $2n$</td>
</tr>
</tbody>
</table>


Bussard and Bagga published the DBPK-Log protocol (Algorithm 4), which is a distance-bounding protocol based on a proof of knowledge and a commitment scheme [Bussard and Bagga 2005].
 Initialization. Prior to protocol execution, the prover and the verifier agree on the security parameters and functions described in Table XIII. A trusted authority then chooses and publishes the following values: \( p \), a large safe prime such that \( p = 2q + 1 \) with \( q \) a large prime; \( g \), a generator of \( \mathbb{Z}_p^* \); and \( h \), a random value in \( \mathbb{Z}_p^* \). Once done, the prover selects a secret \( x \in \mathbb{Z}_{p-1} \setminus \{ q \} \) and the trusted authority then needs to create and publish a certificate for his public key \( y = g^x \).

 Protocol. The prover \( P \) possesses a private key \( x \) which is an odd secret, randomly chosen in \( \mathbb{Z}_{p-1} \setminus \{ q \} \), whose corresponding public key is \( y = g^x \mod p \). The prover picks a random one-time key \( R^0 \in \{ 0,1 \}^n \), and encrypts his private key \( x \) with \( R^0 \), using the encryption scheme \( E \), i.e., he gets \( R^1 = E_{g^0}(x) = x - R^0 \mod (p-1) \). The prover then commits to each bit of \( R^0 \) and \( R^1 \) independently using the Commit function (see Remark A.1). Then, for each of the \( n \) rounds of the fast phase, the verifier generates and sends a random challenge bit \( c_i \) and the prover replies instantly with a 1-bit response that is either \( R^0_i \) or \( R^1_i \), selected by the value \( c_i \). A second slow phase then starts, where the prover allows the verifier to open the commitment of each bit \( R^0_i \), for each challenge \( c_i \) that has been sent in the previous phase.

 Final Decision. The verifier checks the timing and verifies that the received values correspond to the committed ones (Open function is described in the original paper). A verification protocol is finally executed between \( P \) and \( V \) using a proof of knowledge. Note that [Bussard and Bagga 2005] only states that “at the end of distance-bounding stage, the verifier \( V \) is able to compute an upper bound on the distance to \( P \).”

<table>
<thead>
<tr>
<th>Table XIII: Parameters and functions (Algorithm 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
</tr>
<tr>
<td>( m )</td>
</tr>
<tr>
<td>( m' )</td>
</tr>
<tr>
<td>( E )</td>
</tr>
<tr>
<td>Commit</td>
</tr>
<tr>
<td>Open</td>
</tr>
<tr>
<td>( PK((x, v) : z = \Omega(x, v) \land y = \Gamma(x)) )</td>
</tr>
</tbody>
</table>

Remark A.1 (Commitment). The suggested Commit function works as follows: (a) a value \( h \) is randomly chosen in \( \mathbb{Z}_p^* \); (b) the values \( v_{R^0,i} \) and \( v_{R^1,i} \), \( \forall i \in \{0,\ldots,N-1\} \), are randomly chosen in \( \mathbb{Z}_{p-1} \); (c) \( C_i(R^0) = g^{R^0_i}h^{v_{R^0,i}} \mod p \) and \( C_i(R^1) = g^{R^1_i}h^{v_{R^1,i}} \mod p \).


Munilla and Peinado introduced in [Munilla et al. 2006; Munilla and Peinado 2009] the concept of void challenges as a tool to improve distance-bounding protocols. These void challenges can also be used to decrease the mafia fraud success probability when applied to Hancke and Kuhn’s protocol [Munilla and Peinado 2008a], which is the case analysed in this section. Thus, for this protocol (Algorithm 5), the challenges can be 0, 1 or void, where a void challenge means that no challenge is sent. Void challenges are used to detect a mafia fraud using the pre-ask strategy.

 Initialization. Prior to the protocol execution, the prover and the verifier agree on the security parameters and the functions described in Table XIV, and a common secret key \( K \).

 Protocol. During a first slow phase, the verifier sends to the prover a nonce \( N_V \) and the prover sends to the verifier a nonce \( N_P \) (Remark A.2). They both then use the pseudo-random function \( h \) and the secret key \( K \) to generate three \( n \)-bit sequences: \( R^0 \), \( R^1 \) and \( Z \). The values \( R^0 \) and \( R^1 \) are, as in Hancke and Kuhn’s protocol, the responses to the challenges, while \( Z \) defines which challenges are void. In the fast phase, the verifier sends random challenges \( c_i \) when \( Z_i = 1 \), and the prover instantly replies with 1-bit responses \( r_i \) that are either \( R^0_i \) or \( R^1_i \), depending on the value of \( c_i \); \( r_i = R^0_i \). If the prover receives a challenge for an interval where \( Z_i = 0 \), he assumes that the system is being attacked and aborts the protocol. Finally, the prover sends \( h(K, R^0, R^1) \) in a final slow phase to confirm that no adversary has been detected.

 Final Decision. The verifier accepts the prover as genuine only if the final signature is correct and all the responses \( r_i \) are correct and timely: \( \Delta t_i \leq t_{\max}, i \in \{ 1, 2, \ldots, n \} \), for some threshold \( t_{\max} > 0 \).

Remark A.2. The paper does not specify whether the verifier or the prover sends its nonce in first.
Table XIV: Parameters and functions (Algorithm 5)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>Number of iterations in the fast phase (it coincides with the length of vectors $R^0$, $R^1$, and $Z$)</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Size of the secret key $K$ (not defined in the original paper)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Size of random numbers $N_V$ and $N_P$ (not defined in the original paper)</td>
</tr>
<tr>
<td>$t_{\text{max}}$</td>
<td>Threshold of the round-trip time</td>
</tr>
<tr>
<td>$p_f$</td>
<td>Probability of an interval being non-void (optimal value $p_f = 4/5$, and practical value $p_f = 3/4$)</td>
</tr>
<tr>
<td>$h$</td>
<td>Hash (or pseudo-random) function whose output size is $3\kappa$</td>
</tr>
</tbody>
</table>

Remark A.3. During the fast phase, $2(n - n_{\text{void}})$ bits are exchanged, where $n_{\text{void}}$ is the number of void challenges for the protocol run. Given the average number of void challenges, namely $n(1 - p_f)$, the average number of exchanged bits is $2np_f$.


Kim, Avoine, Koeune, Standaert and Pereira introduced a protocol in [Kim et al. 2008] known as the Swiss-knife distance-bounding protocol$^4$ (Algorithm 6).

- **Initialization.** Prior to the protocol execution, the legitimate prover and the verifier agree on the security parameters and functions described in Table XV, a system-wide constant $C$ known to the verifier and the prover, and a common secret key $K$.

- **Protocol.** During the first slow phase, the verifier chooses a nonce $N_V \in \mathbb{R}\{0, 1\}^\delta$ and a random binary vector $D$ with Hamming weight $n$ and length $\sigma$. Intuitively, $D$ corresponds to a mask pointing to the positions on which the prover will be questioned during the fast phase. He transmits $N_V$ and $D$ to the prover. The prover chooses a nonce $N_P \in \mathbb{R}\{0, 1\}^\delta$ and computes $a := f_K(C, N_P)$. The prover then computes two registers using its permanent key $K$ as follows: $Z^0 := a$ and $Z^1 := a \oplus K$. He finally prepares the possible answers by extracting the relevant parts of $Z^0$, $Z^1$ according to the mask $D$, building the $n$-bit vectors $R^0$ and $R^1$. The prover ends the slow phase transmitting $N_P$ to the verifier. During the fast phase, the verifier generates and sends a random challenge bit $c_i$, and the prover replies instantly with a 1-bit response that is either $v^0_i$ or $v^1_i$, selected by the value of $c_i$. After $n$ iterations, the prover computes $T_B := f_K(c'_1, \ldots, c'_n, ID, N_V, N_P)$ and transmits $T_B$ and the challenges $c'_1, \ldots, c'_n$ received during the fast phase. The verifier performs a search over its database until he finds a pair $(ID, K)$ and computes $R^0, R^1$. If mutual authentication is expected, the verifier computes $T_A := f_K(N_P)$, sends it to the prover who checks its correctness.

- **Final Decision.** The authentication succeeds if and only if $err_C + err_R + err_T < T$.

Table XV: Parameters and functions (Algorithm 6)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>Number of iterations in the fast phase</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Size of the output of $f$ and consequently size of the secret key $K$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Size of nonces $N_V$ and $N_P$</td>
</tr>
<tr>
<td>$t_{\text{max}}$</td>
<td>Threshold of the round-trip time</td>
</tr>
<tr>
<td>$f$</td>
<td>Threshold of tolerable errors</td>
</tr>
<tr>
<td>$h$</td>
<td>Pseudo-random function whose output size is $\sigma$</td>
</tr>
</tbody>
</table>


The protocol (Algorithm 7) introduced by Avoine and Tchamkerten in [Avoine and Tchamkerten 2009] is a generalization of Hancke and Kuhn’s protocol that is more secure in terms of mafia and distance frauds.

- **Initialization.** Prior to the protocol execution, the legitimate prover and the verifier agree on the security parameters and functions described in Table XVI, in addition to a common secret key $K$.

$^4$Like Swiss-army knives used during WWII, the Swiss-knife protocol is a multi-purpose tool. The authors claim their protocol "resists against both mafia fraud and terrorist attacks, reaches the best known false acceptance rate, preserves privacy, resists to channel errors, uses symmetric-key cryptography only, requires no more than 2 cryptographic operations to be performed on the tag, can take advantage of precomputation on the tag, and offers an optional mutual authentication." [Kim et al. 2008].
Protocol. It consists of a slow authentication phase followed by a fast proximity check phase. Both phases have their own security parameters: the credential size $c$ for the authentication and the number of bit exchanges $n$ between the prover and the verifier during the fast phase.

Authentication. The verifier sends a nonce $N_{V'}$ to the prover, in the form of a uniformly random bit-string of size $\delta$. The prover then generates a $\delta$-bit nonce $N_P$ and, based on $N_V$ and $N_P$, computes a keyed-hash value $h_K(N_{V'}, N_P)$ whose output is a string of at least $c + \ell \cdot (2^{d+1} - 2)$ bits where $d, \ell \geq 1$ are such that $d \cdot \ell = n$. The prover sends to the verifier both $N_P$ and the first $c$ bits of $h_K(N_{V'}, N_P)$ denoted $[h_K(N_{V'}, N_P)]^c$.

Proximity Check. Using the subsequent $q = \ell \cdot (2^{d+1} - 2)$ bits of the hash value $h_K(N_{V'}, N_P)$, denoted by $[h_K(N_{V'}, N_P)]^{d+q}_{c+1}$, the prover and the verifier label $\ell$ full binary trees of depth $d$ as follows (see Figure 6 for an example). The left and the right edges of each tree are labeled 0 and 1 respectively, and each node of each tree, except the root, is associated with the value of a particular bit in $[h_K(N_{V'}, N_P)]^{c+q}_{c+1}$ in a one-to-one fashion. This labeling is possible since each tree has $2^{d+1} - 2$ nodes (excluding the root), which gives a total of $\ell \cdot (2^{d+1} - 2)$ nodes to be labeled.

An $n$-round fast bit exchange between the verifier and the prover proceeds using the trees: the edge and the node values represent the verifier’s challenges and the prover’s replies, respectively. At each step $i \in \{1, 2, \ldots, n\}$ the verifier generates a challenge in the form of a randomly uniform bit $c_i$ and sends it to the prover. Now let $j \geq 1$ be such that $(j - 1)(2^{d+1} - 2) + 1 \leq i < j(2^{d+1} - 2)$. Upon receiving $c_i$, the prover replies $r_i$, which corresponds to the value of the node in the $j$-th tree whose edge path from the root is given by $c_{(j-1)(2^{d+1} - 2) + 1}c_{(j-1)(2^{d+1} - 2) + 2}\ldots c_i$. The example illustrated by Figure 6 uses the following parameters: $n = 6$, $\ell = 2$, and $d = 3$. The sequence of challenges is $(1, 1, 0, 0, 1, 0)$, which corresponds to the two thick edge paths in the trees starting with the tree on the left. The corresponding sequence of replies is $(1, 1, 1, 0, 1, 0)$. Note that each reply $r_i$ is a function of at most $d$ previous $c_j$'s. Finally, for all $i \in \{1, 2, \ldots, n\}$, the verifier measures the time interval $\Delta t_i$ between the instant $c_i$ is sent until the instant $r_i$ is received.

Final Decision. The verifier accepts the prover’s identity only if the $c$ authentication bits are correct and if the $n$ replies of the fast phase are correct while meeting the time constraint $\Delta t_i \leq t_{\max}, i \in \{1, 2, \ldots, n\}$, for some threshold $t_{\max} > 0$.

Fig. 6: Two decision trees of depth 3, i.e., $\ell = 2$, $d = 3$

Remark A.4. The case $n = d \cdot \ell$ is the maximum situation where all $d$ replies of the $\ell$-th tree are used. We impose this constraint only to have somewhat simpler performance expressions. It is easy to see that this constraint can be replaced by $d \cdot \ell \geq n$, which is the situation where the last tree is only partly used.

Remark A.5. When $d = 1$ and $\ell = n$, the fast phase of the protocol is similar to the one of HK Protocol.


The protocol (Algorithm 8) was introduced by Rasmussen and Čapkun and originally appeared in [Rasmussen and Čapkun 2010]. In this paper we consider the updated version that appeared in [Rasmussen 2011].

Initialization. Prior to the protocol execution, the legitimate prover and the verifier agree on the security parameters, the functions described in Table XVII, and a common secret key $K$.

---

To do this, one sequentially assigns the bit values of $[h_K(N_{V'}, N_P)]^{c+q}_{c+1}$ to all the nodes of each tree, starting with the lowest level nodes, moving left to right, and moving up after assigning the nodes of the current level.

Poulidor, the graph-based distance-bounding protocol (Algorithm 9) designed by Trujillo-Rasua, Martin, and Avoine [Trujillo-Rasua et al. 2010], uses specific node and edge dependencies in the tree of the AT

Table XVI: Parameters and functions (Algorithm 7)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>Number of iterations in the fast phase</td>
</tr>
<tr>
<td>κ</td>
<td>Size of secret key</td>
</tr>
<tr>
<td>δ</td>
<td>Size of nonces $N_V$ and $N_P$</td>
</tr>
<tr>
<td>c</td>
<td>Credential size</td>
</tr>
<tr>
<td>ℓ</td>
<td>Number of trees ($d$ and $ℓ$ satisfy $d \cdot ℓ = n$)</td>
</tr>
<tr>
<td>$t_{\text{max}}$</td>
<td>Threshold of the round-trip time</td>
</tr>
<tr>
<td>$h_K$</td>
<td>Keyed-hash function whose output size is a bit string of size at least $c + ℓ \cdot (2^{d+1} - 2)$</td>
</tr>
</tbody>
</table>

○ Protocol. The prover starts the protocol by picking a fresh (large) nonce $N_P$. The prover then commits (using for example a hash) on $N_P$ and its identity. This commitment is not keyed. The prover now activates its distance-bounding hardware and set the output channel according to the opposite of the first bit of the nonce $N_P$. From this moment on, any signal that the prover receives on channel $C_0$ will be reflected on the output channel that is set. However, the prover does not start switching between output channels yet.

Upon receiving the commitment, the verifier picks a fresh (large) nonce $N_V$ and prepares to initiate the distance-bounding phase, in which it will measure the distance bound to the prover. The verifier starts a high precision clock to measure the (round trip) time-of-flight of the signal, $Δt$, and begins to transmit his nonce $N_V$ on channel $C_0$. From this point on, the verifier also listens on the two reply channels $C_1$ and $C_2$ and keeps listening on the two channels until he either receives the expected response from the prover or until he detects an error and aborts the protocol.

As soon as the prover receives (and, in parallel demo adulates) the first bit of $N_V$ on $C_0$, he starts switching reply channels according to the bits of his nonce $N_P$. When the first few bits are being demodulated, the prover is still reflecting the input (challenge) bits and the switching of the channels is not started yet (i.e., the prover does not start sending back $N_P$ yet). This function, used by the prover to form its reply to the verifier, is called “Challenge Reflection with Channel Selection” (CRCS). The demodulation of the bits is not done within the distance-bounding hardware (called the distance-bounding extension), but is done in the prover’s regular radio. A possible implementation of the distance-bounding extension (i.e., of CRCS) using analog mixers is described in [Rasmussen 2011]. It is not important how long it takes for the prover’s radio to demodulate the first bits since the prover does not need to begin to switch the output channels within any predefined time as long as the prover keeps track of the delay $n$. The delay represents the time taken by the prover to react to the incoming signal, i.e., to switch its circuit to transmit the first bit of its answer. The switching starts within the duration of $N_V$, and allows the transmission of $N_P$. The first part of $N_V$ could even be known and constitute a public and fixed-length preamble, upon the detection of which the prover would start switching the channels (i.e., would start sending $N_P$).

When the prover starts sending $N_P$, he sends the bits of $N_P$ with a fixed frequency (e.g., every 100ms) by switching channels depending on the value of the current bit. In each interval, the prover reflects back several bits of $N_V$ and a single bit of $N_P$. The bit of $N_P$ is encoded in the choice of the reply channel. The prover also receives in parallel the verifier’s challenge nonce (i.e., $N_V$) on channel $C_0$ using his regular radio.

When the verifier has sent all the bits of his nonce, he waits for the prover to complete the reflection of the signal and then both the prover and verifier disable their distance-bounding extensions. The verifier can then use an auto-correlation detector like the ones used in GPS receivers to determine the exact time of flight, $Δt$, of the reflected signal. This can also be done during the distance-bounding phase, i.e., in parallel to the analog distance-bounding circuit. Finally, the prover sends a signed message compounded by the commitment sent during the first slow phase, the delay $n$, his nonce $N_P$, and the verifier’s identity and nonce.

○ Final Decision. The verifier accepts the prover’s identity only if the bits of $N_P$ were sent within the same time duration, these bits match with those he received in the final message of the prover, the reflection of $N_V$ through the channel switch was correct, the signature in the final message is correct, the delay $n$ he computed match with the prover’s one $n$ (including in the final message), and finally that the round-trip time is below the time threshold $t_{\text{max}}$. 
protocol [Avoine and Tchamkerten 2009] – which then can alternatively be represented by an acyclic graph. Pouildor benefits from a lower memory requirement compared to the AT protocol. Security is also reduced.

- **Initialization.** Prior the protocol execution, the legitimate prover and the verifier agree on the security parameters and functions described in Table XVIII, and a common secret $K$.

### Table XVIII: Parameters and functions (Algorithm 9)

<table>
<thead>
<tr>
<th>$n$</th>
<th>Number of iterations in the fast phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>Size of the secret key $K$</td>
</tr>
<tr>
<td>$\delta_P$ and $\delta_V$</td>
<td>Size of nonces $N_P$ and $N_V$ respectively</td>
</tr>
<tr>
<td>$t_{\text{max}}$</td>
<td>Threshold of the round-trip time</td>
</tr>
<tr>
<td>$H$</td>
<td>Hash function whose output size is $2n$</td>
</tr>
</tbody>
</table>

Fig. 7: Graph when $n = 4$

- **Protocol.** During the slow-phase, both the verifier and the prover build a directed graph $G$. The proposed graph requires $2n$ nodes $\{q_0, q_1, \ldots, q_{2n-1}\}$, and $4n$ edges $\{s_0, s_1, \ldots, s_{2n-1}, \ell_0, \ell_1, \ldots, \ell_{2n-1}\}$ such that, $s_i$ ($0 \leq i \leq 2n-1$) is an edge from $q_i$ to $q_{(i+1) \mod 2n}$ and $\ell_i$ ($0 \leq i \leq 2n-1$) is an edge from $q_i$ to $q_{(i+2) \mod 2n}$.

- **Initialization.** Prior to the protocol execution, the legitimate prover and the verifier agree on the security parameters and functions described in Table XIX, along with a common secret $K$.

### Table XVII: Parameters and functions (Algorithm 8)

| $\delta_V$ | Size of the verifier’s challenge nonce $N_V$ |
| $\delta_P$ | Size of the prover’s nonce $N_P$ |
| $\sigma$ | Lower bound for the size of the commitment and the signature |
| $t_{\text{max}}$ | Threshold of the round trip time |
| *Commit* | Secure commitment function that outputs $\sigma$ bits. |
| *Sign* | Signature function whose output size is $\sigma$ |

Kim and Avoine introduced in 2009 a distance-bounding protocol with mixed challenges [Kim and Avoine 2009], namely challenges known and challenges unknown in advance by the prover. Challenges known in advance allow the prover to help the verifier to detect an attack, but these challenges also allow the prover to perform a distance fraud. Kim and Avoine improved their protocol in 2011, yielding a new variant known as KA2 [Kim and Avoine 2011], which is analyzed in this section (Algorithm 10).

- **Initialization.** Prior to the protocol execution, the legitimate prover and the verifier agree on the security parameters and functions described in Table XIX, along with a common secret key $K$. 


Kim and Avoine introduced in 2009 a distance-bounding protocol with mixed challenges [Kim and Avoine 2009], namely challenges known and challenges unknown in advance by the prover. Challenges known in advance allow the prover to help the verifier to detect an attack, but these challenges also allow the prover to perform a distance fraud. Kim and Avoine improved their protocol in 2011, yielding a new variant known as KA2 [Kim and Avoine 2011], which is analyzed in this section (Algorithm 10).

- **Initialization.** Prior to the protocol execution, the legitimate prover and the verifier agree on the security parameters and functions described in Table XIX, along with a common secret key $K$. 

### Table XIX: Parameters and functions (Algorithm 10)

| $K$ | Size of the secret key $K$ |
| $N_V$ | Size of the verifier’s nonce $N_V$ |
| $N_P$ | Size of the prover’s nonce $N_P$ |
| $\ell$ | Threshold of the round-trip time |

*Graph requires $2n$ nodes $\{q_0, q_1, \ldots, q_{2n-1}\}$, and $4n$ edges $\{s_0, s_1, \ldots, s_{2n-1}, \ell_0, \ell_1, \ldots, \ell_{2n-1}\}$ such that, $s_i$ ($0 \leq i \leq 2n-1$) is an edge from $q_i$ to $q_{(i+1) \mod 2n}$ and $\ell_i$ ($0 \leq i \leq 2n-1$) is an edge from $q_i$ to $q_{(i+2) \mod 2n}$. Figure 7 depicts the graph when $n = 4$. In order to build $G$, the verifier sends a nonce $N_V$ to the prover, and the latter sends a nonce $N_P$ to the verifier. From these values, and the secret $K$, they compute $H = h(K, N_P, N_V)$ and set up a graph $G$ as follows: the first $2n$ bits are used to value the nodes while the remaining bits are used to value the edges $s_i$ ($0 \leq i \leq 2n-1$), and finally $\ell_i = s_i \oplus 1$ ($0 \leq i \leq 2n-1$). After agreeing on the graph, the fast phase begins. This phase consists of $n$ stateful rounds numbered from 0 to $n-1$. Initially $q_{p_0} = q_{v_0} = q_0$, but in the $i$-th round $P$’s state and $V$’s state are represented by the nodes $q_{p_i}$ and $q_{v_i}$ respectively. Upon reception of the $i$-th challenge $c_i$, $P$ moves from the node $q_{p_i}$ to $q_{p_{i+1}}$ in the following way: $q_{p_{i+1}} = q_{(p_{i}+i) \mod 2n}$ if $s_i$ is labeled with $c_i$, otherwise $q_{p_{i+1}} = q_{(p_{i}+i) \mod 2n}$. Finally, the prover sends as response $r_i$ the bit-value of the node $q_{p_{i+1}}$. Upon reception of the prover’s answer $r_i$, the verifier stops his timer, and computes $\Delta t_i$, i.e., the round trip time spent for this exchange. Besides this, $V$ moves to the node $q_{v_{i+1}}$ using the challenge $c_i$ (as the prover did but from the node $q_{v_i}$) and checks if $q_{v_{i+1}} = r_i$.

- **Final Decision.** The verifier accepts the prover’s identity only if $n$ responses of the fast phase are correct and the time constraint $\Delta t_i \leq t_{\text{max}}, i \in \{1, 2, \ldots, n\}$, for some threshold $t_{\text{max}} > 0$. 
Protocol. The verifier sends the prover a nonce $N_V$ and the prover sends the verifier a nonce $N_P$. They then use the pseudo-random function $h$ and the secret key $K$ to generate a $2n$-bit sequence $D||R^0||R^1$.

During the first $\alpha$ rounds, the verifier sends predefined 1-bit challenges $c_i$. In every round, the prover sends a 1-bit response that is $R^0_i$ if $c_i = D_i$. Otherwise, he sends random answers until the end of the fast phase.

During the remaining $n - \alpha$ rounds, the verifier sends random 1-bit challenges $c_i$. In every round, the prover sends a 1-bit response that is $R^c_i$, or he sends random answers until the end of the fast phase if a problem ($c_i \neq D_i$) was detected during the first $\alpha$ rounds.

Final Decision. The verifier accepts the prover’s identity only if $n$ responses of the fast phase are correct, while also meeting the time constraint $\Delta t_i \leq t_{\text{max}}$, $i \in \{1, 2, \ldots, n\}$, for a threshold $t_{\text{max}} > 0$.

Table XIX: Parameters and functions (Algorithm 10)

<table>
<thead>
<tr>
<th>$n$</th>
<th>Number of iterations in the fast phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>Size of the secret key $K$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Size of nonces $N_V$ and $N_P$</td>
</tr>
<tr>
<td>$t_{\text{max}}$</td>
<td>Threshold of the round-trip time</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Number of predefined rounds</td>
</tr>
<tr>
<td>$h$</td>
<td>Pseudo-random function whose output size is $2n$</td>
</tr>
</tbody>
</table>


Yum, Kim, Hong and Lee created a distance-bounding protocol with mutual authentication [Yum et al. 2011].

Initialization. Prior to the protocol execution the users $A$ and $B$ agree on the security parameters and functions described in Table XX, in addition to a common secret key $K$.

Protocol. The protocol consists of a slow phase where two nonces ($N_A$ and $N_B$) are exchanged, and a fast phase where challenge bits $c_i$ and response bits $r_i$ are exchanged. In the slow phase, the users compute three $n$-bit sequences, $D, R^0,$ and $R^1$ using a pseudo-random function applied to $N_A$ and $N_B$. In the $i$-th round of the fast phase, each user acts as a prover or a verifier according to the “direction bit” $D_i$. When $D_i = 0$, $A$ sends a random challenge bit $c_i$ and $B$ answers with $R^c_i$, i.e., the $i$-th bit of the register $R^c$. When $D_i = 1$, $B$ sends a challenge and $A$ responds. If the received response bit is incorrect, the recipient moves to a “protection mode”: he sends random bits for all subsequent rounds. Each user also checks that no collision occurred in the round, that is, the two users did not talk or remain silent simultaneously.

Final Decision. $A$ accepts $B$ as legitimate only if the responses of the fast phase are correct and meet the time constraint $\Delta t_i \leq t_{\text{max}}$ for Case I, for some threshold $t_{\text{max}} > 0$. So does $B$ for Case II.

Table XX: Parameters and functions (Algorithm 11)

<table>
<thead>
<tr>
<th>$n$</th>
<th>Number of iterations in the fast phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>Size of the secret key $K$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Size of nonces $N_A$ and $N_B$</td>
</tr>
<tr>
<td>$t_{\text{max}}$</td>
<td>Threshold of the round-trip time</td>
</tr>
<tr>
<td>$h$</td>
<td>Pseudo-random function whose output size is $3n$</td>
</tr>
</tbody>
</table>

A.12. SKI Protocols (2013)

In [Boureanu et al. 2013c; 2013b; 2013a], the authors introduced a series of protocols called SKI. These protocols (presented in Algorithm 12) are described as follows.

Initialization. Prior to the protocol execution, the legitimate prover and the verifier agree on the security parameters and functions described in Table XXI, and a common secret $K$.

Table XXI: Parameters and functions (Algorithm 12)

<table>
<thead>
<tr>
<th>$n$</th>
<th>Number of iterations in the fast phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>Size of the secret key $K$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Size of nonces $N_A$ and $N_B$</td>
</tr>
<tr>
<td>$t_{\text{max}}$</td>
<td>Threshold of the round-trip time</td>
</tr>
<tr>
<td>$h$</td>
<td>Pseudo-random function whose output size is $3n$</td>
</tr>
</tbody>
</table>
Table XXI: Parameters and functions (Algorithm 12)

| n | Number of iterations in the fast phase |
| t | Size of the challenges domain          |
| t’ | Security parameter                     |
| q | Power of a prime number                |
| κ | Size of the secret key K                |
| δ | Size of the nonces N_P and N_V          |
| t_{max} | Threshold of the round-trip time        |
| f | Pseudo random function whose output size is t’ n elements of F_q |
| x | Maximum number of incorrect rounds      |

- **Protocol.** During the slow phase, the prover first generates a nonce N_P, and sends it to the verifier. The verifier then generates its nonce N_V along with a = (a_1, ..., a_t) (a_i ∈ F_q) where F_q is the finite fields of order q and the authors of [Boureanu et al. 2013a] employ in concrete examples q = 2 and a mapping L ∈ L, where L is defined below. Using its nonce and the prover’s nonce, he computes f_k(N_P, N_V, L) and XORs it with a, in order to obtain the mask M. Finally, the verifier sends N_V, L, and M to the prover. Using these two values and its nonce, the prover computes a and K’ = L(K).

Then the n-round fast phase begins. In each round, the verifier picks a challenge c_i ∈ {1, ..., t} at random. Then, he starts a timer and sends c_i to the prover. Upon reception of the challenge, the prover first checks whether c_i belongs to {1, ..., t}. If c_i ∉ {1, ..., t}, the protocol stops. If c_i ∈ {1, ..., t}, the prover computes its answer, r_i = F(c_i, a_i, K’), where the function F is presented in more details below. The prover then sends its answer back to the verifier. Once the verifier received r_i, he stops its timer and stores ∆t_i, the round trip time of the round i, as well as r_i.

As discussed below, the SKI protocol is specified with another set L = L_{best} containing all functions L_{μ}, for μ ∈ F_q defined by L_{μ}(K) = (μ · K, ..., μ · K) i.e., L_{μ}(K) is the n-bit vector in which all bits are set to the dot product of μ and K.

- **Final Decision.** The protocol succeeds if there are at least n − x rounds i for which r_i is correct and ∆t_i ≤ t_{max}. The verifier then outputs a message Out_V, denoting the success or failure of the protocol.

**Remark A.6.** With respect to the mapping in L introduced along with SKI, note that usual distance-bounding protocols would employ L = L_{classic} i.e., the set containing a single function L which is the identity function. Thus, in those case, L(K) = K (imposing further that κ = n). The value of x also introduced along with SKI is used to tolerate some level of noise in the time-critical exchanges. However, introducing this tolerance brings a new type of terrorist fraud, as it will be discussed in Section B.5. The purpose of L = L_{best} is precisely to defeat this attack. But, to compare with other protocols, our analyses below assume x = 0 and L = L_{classic}.

**Remark A.7.** The function F is essential for the SKI protocols. Using a different function leads to different protocol security achievements. Specifically, the authors mainly refer to the efficient cases of q = 2, t = 2, and F(1, a_i, K’_i) = (a_i), F(2, a_i, K’_i) = (a_i), and F(3, a_i, K’_i) = K’_i + (a_i), where K’_i ∈ GF(2), (a_i) ∈ GF(2), i = 1, 2. Generally speaking, this response function, denoted F_{xor}, can be given as follows:

F_{xor}(c_i, a_i, K’_i) = K’_i + (a_i)c_i + (a_i)c_i + (a_i)c_i, where c_i ∈ {1, ..., t}, K’_i ∈ GF(q), q ≥ 2, (a_i) ∈ GF(q), j ∈ {1, ..., l − 1}, and l_k is 1 if R is true and 0 otherwise.

The authors actually consider two variants SKI_{pro} with t = 3 and SKI_{lite} with t = 2, namely SKI_{lite} never uses the c_i = 3 challenge. Other cases (treated separately) are summarized as follows:

- **SKI_{pro}:** defined by the response-function F_{xor} above, with q = 2, t = 4, t’ = 3, i.e., F(c_i, a_i, x_i) = (a_i)c_i for c_i ∈ {1, 2, 3} and F(4, a_i, K’_i) = K’_i + (a_i)c_i + (a_i)c_i + (a_i)c_i, where (a_i) ∈ GF(2);
- **SKI_{lite}:** defined by a variant of response-function based on the Shamir secret sharing, with q = 4, t = 3, t’ = 2, i.e., F(c_i, a_i, K’_i) = K’_i + (a_i)c_i + (a_i)c_i for c_i ∈ GF(4)^*; with (a_i) ∈ GF(4). Here, c_{i,e} denotes a one-to-one mapping from {1, 2, 3} to GF(4)^*.

While SKI_{pro} can be presented as a variant of the TDB protocol proposed in [Avoine et al. 2011] and SKI_{lite} is very similar to the Hancke and Kuhn protocol [Hancke and Kuhn 2005], other variants of F can be suggested, yielding different SKI protocols. These functions have to respect the requirements provided in [Boureanu et al. 2013a]. These are informally summarized in Remark A.8.
Remark A.8 (Requirements for the function $F$ and the set $\mathcal{L}$; (see [Boureu et al. 2013a] for details)).

The $F$ function must comply to the following conditions, in order to ensure security, as stated in Section 1.6.

1. For any $c_i$, $F(c_i, \cdots)$ must be $GF(q)$-linear and non-degenerate in the $a_i$ part.
2. For any two values $c_i$ and $c'_i$ of the $i$-th challenge and for any $a_i$, $F(c_i, a_i, K'_i)$ and $F(c'_i, a_i, K'_i)$ give no information about $K'_i$.
3. For any $a_i$, one can compute $K'_i$ from the table of the map $c_i \mapsto F(c_i, a_i, K'_i)$.
4. For any $K'_i$, the largest preimage of $c_i \mapsto F(c_i, a_i, K'_i)$ must be small, on average over $a_i$.

The third requirement above is used for resistance to terrorist fraud. Note that SKI_{life} does not satisfy it, so it does not resist to terrorist fraud. The requirement on $\mathcal{L}$ is that given a source generating some $(L, L(K) + \epsilon)$ for $L \in \mathcal{L}$ uniformly distributed and $\epsilon$ of “small” Hamming weight, and arbitrary distribution, then $K$ can be reconstructed.

B. VARIANTS AND EXTENSIONS

Appendix B presents generic improvements that can be applied on distance-bounding protocols.

B.1. MUltiState Enhancement: MUSE

Although location-based authentication services that measure the round trip time of entire data packets have been proposed [Waters and Felten 2003], most of the distance-bounding protocols are based on the measurement of the round trip time of 1-bit messages. Munilla and Peinado [Munilla et al. 2006; Munilla and Peinado 2008a] initiated a new family of protocols that use an additional third state during the fast phase. Although binary data are still exchanged during that phase, Munilla and Peinado suggest to use void challenges. These void challenges, which means that no challenge is sent, are used to authenticate the verifier, reducing thus the success probability of a pre-ask strategy.

MUSE is a generalization of this idea proposed by Avoine, Floerkemeier, and Martin [Avoine et al. 2009], where the number of possible states used during the fast phase can be still larger: the authors indeed extend the concept of void challenges to $p$-symbols where $p \geq 2$. Using $p$-symbols is a generic technique that reduces the number of rounds during the fast phase. Algorithm 13 describes MUSE-3 HK, which is the 3-symbol variant of HK. In MUSE-3 HK, $H = h(K, N_V, N_P)$ is used to fill up three registers $R^0_j (j = 0, 1, 2)$ that each contains $n$ 3-symbols $\{(S_{j+1} + \cdots + S_{j+n})\}$. When considering the mafia fraud against MUSE-3 HK, the success probability is $Pr_{\text{MUSE}}(p) = \left(\frac{2}{3}\right)^n$, which is better than the 3-symbol protocol of Munilla and Peinado [Munilla et al. 2006; Munilla and Peinado 2008a]. Note that to be able to easily generate and store $p$-symbols ($p > 2$) on prover side the authors suggested to encode challenges and responses on $\lfloor\log_2(p)\rfloor$ bits.

![Algorithm 13: Hancke and Kuhn’s Protocol with MUSE-3](image)

B.2. PUF-based protocols

Kardaş, Kiraz, Bingöl, and Demirci introduce in [Kardaş et al. 2011] two novel distance-bounding protocols based on Physically Unclonable Functions (PUFs). A PUF is defined as an unclonable function embedded in a physical structure that is easy to implement but practically impossible to duplicate, even given the exact manufacturing process definitions. The output of the function is obtained as a result of inherent physical properties such as delays of gates and wires in a circuit, variations in the temperature and supply voltage.
The unclonability of the function is guaranteed by these physical processes, and some mechanisms (e.g., Fuzzy Extractors) are used to ensure the determinism. Since PUF’s behave as a random function (if one assumes that all the physical properties cannot be predicted), without having the actual PUF circuit it is hard to predict the outputs as given the inputs. Moreover, their intrinsic structure yields resistance against tampering since physically tampering will most likely change its physical structure.

The authors define a strong adversary model in which the adversary has access to volatile memory of the prover, namely an RFID tag. PUF functions are used to prevent an adversary from obtaining the long-term secrets and clone the tags. The main idea is that long-term secrets are not stored in the memory of the prover but they are reconstructed from pre-secrets using a PUF circuit during each protocol execution.

The first protocol proposed by Kardag et al. is described in Algorithm 14. They use two different long-term keys $K$ and $L$ which are consecutively generated as outputs of the PUF function. Note that $K$ and $L$ never appear in the volatile memory at the same time. First, $K$ is constructed by using PUF, and then completely deleted from the memory after being used as a key of PRF function. Then similarly, $L$ is generated and deleted after generation of registers. Hence, whenever an adversary tampers the tag she can only obtain one of the keys, under the assumption that the structure of the PUF circuit has been destroyed after the attack thus PUF cannot be re-evaluated anymore. The authors state that since the adversary cannot retrieve all the long-term keys, she can only perform the attack in black-box model.

Given that the success probability of mafia and terrorist frauds remains high, namely $(3/4)^n$, the authors introduce an extended protocol with a final signature that reaches $(1/2)^n$ against these frauds.

**Algorithm 14:** Kardag et al.’s protocol based on PUF without final signature

- **Verifier** (secret $K, L$)
  - Pick $N_V \in \{0, 1\}^n$
  - $N_V$ → $N_P$, $v, R^v$, $R^v' = f_L(N_P, N_V)$
  - If $v' \neq v$ then abort
  - Pick $c_i \in \{0, 1\}$ and Start Timer
  - $c_i$ → $r_i$
  - Check correctness of $r_i$’s and $\Delta t_i \leq t_{\text{max}}$ for $1 \leq i \leq n$

- **Prover** (pre-secret $G_1, G_2$)
  - Pick $N_P \in \{0, 1\}^n$
  - $K = PUF(G_1)$
  - $T = f_K(N_P, N_V)$
  - Delete $K$
  - $L = PUF(G_2)$
  - $v, R^v, R^v' = f_L(T)$
  - $|v| = |R^v| = |R^v'| = n$
  - Delete $L$

### B.3. Threshold Distance-Bounding Protocol to Defeat Terrorist Fraud

Many distance-bounding protocols are subject to terrorist fraud as the long-term key cannot be retrieved in practice from the information needed to successfully pass the protocol. Avoine, Lauradoux, and Martin in [Avoine et al. 2011] suggest that a secret-sharing scheme, possibly based on threshold cryptography can be used to thwart terrorist fraud. In their proposal, the authentication material consists of $p$ shares of a $(p, k)$ threshold scheme: if the prover reveals any combination of $k$ shares to the adversary, the long-term secret leaks. By contrast, gathering strictly less than $k$ shares reveals no information about the secret.

To illustrate this, the authors describe a variant of HK, which they call TDB (Threshold Distance-Bounding), where the responses to the challenges are generated using a threshold scheme. This protocol differs from HK in the way the registers are generated during the slow phase: after the nonce exchange, verifier and prover use their shared secret $K$ to compute a $p \times n$ matrix $R$ over a group $G$. The matrix $R$ is used to respond to the challenges as follows. The verifier requires the prover during the $i$-th round the value $r_{c_i}$ in $R$ ($c_i$-th row and $i$-th column). The challenges consequently consist of $\lceil \log_2 p \rceil$ bits and the responses
of $|\log_2 (|G|)|$ bits. The calculation of $R$ is such that the knowledge of any combination of $k$ elements of a given column reveals a coordinate of the key.

$$R = \begin{pmatrix} r_{1,1} & \cdots & r_{1,n} \\ \vdots & \ddots & \vdots \\ r_{p,1} & \cdots & r_{p,n} \end{pmatrix}$$

The authors introduce in [Avoine et al. 2011] three classes of adversaries: i.) BD-ADV or blind-adversary, who does not learn whether the protocol succeeds, ii.) RE-ADV or result-adversary, who can observe if the protocol succeeds and, iii.) RD-ADV or round-adversary, who has the capability of observing the result of each round. They then analyze the resistance of their approach when facing each of these adversaries, according to the parameters $p$ and $k$. The parameter $p$ is actually critical regarding mafia fraud, while $k$ impacts the probability of a successful terrorist fraud.

For BD-ADV, the maximum number of elements of a column of $R$ which can be safely given to the adversary is $k-1$. As a result, and for this adversary, TDB implemented with $(p,2)$ threshold scheme is secure against terrorist fraud (this probability coincides with that for the mafia fraud) for any $p \geq 2$.

When the other adversaries are considered, the post-ask strategy must be analyzed. These adversaries can learn two elements of each column of $R$ for each protocol round, modifying all the challenges $c_i$ received from the verifier and sending the modified versions $\tilde{c}_i$ to the prover; i.e., $\forall i \; c_i \neq \tilde{c}_i$. If a round succeeds, then $\tilde{r}_{ci} = r_{ci}$. The BD-ADV can do this on all rounds in parallel, while RE-ADV is limited to a single round per attack. So, TDB should be used with $k \geq 3$ if we want to protect the key against those stronger adversaries. On the other hand, the prover should give to the adversary at most $k-2$ shares at each round (and not $k-1$ as when BD-ADV was analyzed). Thus, in the context of RE-ADV and BD-ADV, to be secure against terrorist fraud attack, schemes $(p,3)$ for any $p \geq 3$ should be used.

The authors also describe a variant, called TTDB, that reduces the number of systems of shares computed. Whereas a column of $R$ is used only once in TDB, the same column is used $q$ times in TTDB. TTDB actually differs from TDB on three points: i.) The size of prover’s answers; TTDB works on vectors of $q$ coordinates in $G$, and therefore the responses of the prover are elements in $G^q$. ii.) The matrix computation; each distinct column is repeated $q$ times in the matrix. The overall number of rounds is kept constant $n$, and consequently there are only $n/q$ distinct columns in $R$. The resulting $p \times n$ matrix $R$ over $G^q$ is defined by:

$$\begin{pmatrix} r_{1,1} \cdots r_{1,1} \\ \vdots \vdots \\ r_{p,1} \cdots r_{p,1} \end{pmatrix}$$

Finally: iii.) when working on a given distinct column of $R$, the challenges $c_i$ are not allowed to be repeated.

The results show that TTDB is a generalization of TDB for the terrorist fraud. For BD-ADV, TTDB is secure when $q = k - 1$. Stronger adversaries, with the post-ask strategy, can recover at most $2q$ shares for round. Therefore $(p,2q+1)$ threshold schemes should be used, and the prover, when colluding with the adversary, should only reveal $q$ shares. For these values, TTDB is also secure against terrorist fraud.

**B.4. Previous-Challenge Dependent Protocols**

Previous-challenge dependent distance-bounding protocols are analyzed by Kara, Kardaş, Bingöl, and Avoine in [Kara et al. 2010]. They focus on the low-cost distance-bounding protocols having bitwise fast phases and no final signature. As for the classification, they introduce the notion of $k$-previous challenge dependent ($k$-PCD) protocols where each response bit depends on the current and the $k$ previous challenges. First, the authors analyze the case $k = 0$, that is when each response bit depends on the current challenge only, and the case $k = 1$. They show that the latter provides a better security than the former one and propose a natural extension to transform 0-PCD protocols into 1-PCD protocols. This modification consists in a simple polynomial arithmetic operation to compute the responses.

The authors show that mafia fraud and distance fraud are correlated by providing trade-off curves between the security levels of these two attacks. They give the theoretical security bounds for two classes: 0-PCD and 1-PCD. The authors thus claim that protocols can be designed to enforce the mafia or distance fraud resistance, but not both at the same time, without increasing the memory needs. For $k = 0$ they find that $P_{TFM}(R) + P_{TFD}(R) \geq 3/2$, where $P_{TFM}(R)$ and $P_{TFD}(R)$ are the maximum probabilities for an adversary of correctly guessing one bit response for mafia fraud and distance fraud respectively. As a consequence of
this result, one can conclude that protocols with \( k = 0 \) cannot attain the ideal security against distance fraud, i.e., \( \Pr_{\text{DF}}(R) = 1/2 \), without being totally vulnerable against mafia fraud; and also that the security of mafia fraud cannot be better than 3/4.

The optimal security limit for mafia fraud and the trade-off curve for protocols with \( k = 1 \) turn out to be \( \Pr_{\text{MF}}(R) \geq 5/8 \) and \( \Pr_{\text{DF}}(R) \geq 5/4 \) respectively, and therefore it lies below that the previous one for \( k = 0 \). Thus, the ideal security level against distance fraud can be reached with \( \Pr_{\text{MF}}(R) \geq 3/4 \).

Finally, the authors apply the natural extension to HK for improving distance fraud resistance in one case, and for improving mafia fraud resistance in the other case\(^6\).

The authors leave as an open question to construct trade-off curves for \( k \geq 2 \), but they conjecture that the security should be enhanced when \( k \) is increased.

**B.5. Distance bounding over noisy channels**

Distance-bounding protocols are conducted over noisy wireless *ad hoc* channels. The fast phase consists, for the most part, of single bits sent between the prover and the verifier. Due to the unreliability of the channel, the communicating parties might receive erroneous bits during this phase. Being robust to relatively high bit-error rates is a desirable property for a distance-bounding protocol.

There are two main approaches in the literature to make distance-bounding protocols noise-resilient, both requiring to increase the number of rounds during the fast phase.

The first and easiest approach to deal with noise is to allow up to \( x \) incorrect responses during the fast phase: the distance-bounding protocol succeeds if at least \( (n − x) \) bit-responses sent by the prover are correct. This technique can be easily applied when the correctness of each of the \( n \) responses can be verified independently, which is the case for most distance-bounding protocols.

The second approach consists of using an error correcting code. It can be applied on many protocols but it is particularly useful in protocols where one single bit error does not allow the verifier to check the correctness of the other rounds (e.g., in BC protocol). The idea is to apply an \((n, k)\) error correcting code on a bitstring of length \( k \), which is used by the prover to compute the responses in the fast phase (e.g., to compute a XOR of the \( i \)-th bit of this bitstring and the challenge). The error correcting code is constructed in such a way that it can correct at least \( x \) bit errors. By applying this code to the bitstring, its length increases to \( n \) bits. These \( n \) bits are then used in the fast bit exchange phase. After this phase, the verifier applies the error correcting code to compute and verify the original bitstring of \( k \) bits. Note that only the parameter \( k \) has an influence on the security, in contrast to \( n \).

Note that most distance-bounding protocols can be easily made noise-resilient by applying one of the two approaches. The second approach can be used by BC and MAD protocols, while the first approach can be easily applied on most other distance-bounding protocols. However, it seems harder to make DBPK-Log, Tree-based, Poulidor, and RC protocols noise-resilient.

When implementing a noise-resilient distance-bounding protocol, it is of the utmost importance to accurately estimate the bit error rate expected during the fast phase. If the estimation on the number \( x \) of expected bit errors is lower than the actual bit error rate then the *false rejection ratio* is significant, meaning that some honest provers are not accepted by the verifier. However, a high \( x \) affects the security level of the protocol in a negative way: an attacker can guess some responses wrongly, and blame it on the noise. Consequently, when analyzing the security properties of a noise-resilient distance-bounding protocol, it is typically assumed that no noise is present during the fast phase, but the verifier allows up to \( x \) bit errors. This is the worst case scenario. The success probability of an attacker depends on \( x \). This often makes the analysis more complex and the comparison of various distance-bounding protocols difficult. Noise resilience has consequently not been considered in the analyses of the protocols provided in Sections 3 to 14.

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\(^6\)Note that there is a typo in [Kara et al. 2010], where it should be \( y_i^k \oplus y_{k-1}^{l-1} \) instead of \( y_i^k \oplus y_{k-1}^{l-1} \).