To my parents and Alessandra,
unique and unequalled in their essence.
DECLARATION OF AUTHORSHIP

I hereby declare that the contents of this thesis except where specific reference is made to the work of others, are original and have not been submitted in whole or in part for consideration for any other degree or qualification in this, or any other university.

This thesis is my own work and contains nothing which is the outcome of work done in collaboration with others, except where specifically acknowledged.

Daniele Addari
27th September 2016
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It has been a fantastic journey and I am glad I had you all as my companions.
The term microvibrations generally refers to accelerations in the order of micro-gs and which manifest in a bandwidth from a few Hz up to say 500-1000 Hz. The need to accurately characterise this small disturbances acting on-board modern satellites, thus allowing the design of dedicated minimisation and control systems, is nowadays a major concern for the success of some space missions. The main issues related to microvibrations are the feasibility to analytically describe the microvibration sources using a series of analysis tools and test experiments and the prediction of how the dynamics of the microvibration sources couple with those of the satellite structure.

In this thesis, a methodology to facilitate the modelling of these phenomena is described. Two aspects are investigated: the characterisation of the microvibration sources with a semi-empirical procedure which allows derivation of the dynamic mass properties of the source, also including the gyroscopic effect, with a significantly simpler test configuration and lower computational effort compared to traditional approaches; and the modelling of the coupled dynamics when the source is mounted on a representative supporting structure of a spacecraft, including the passive and active effects of the source, which allows prediction of the structure response at any location.

The methodology has been defined conducting an extensive study, both experimental and numerical, on a reaction wheel assembly, as this is usually identified as the main contributory factor among all microvibration sources. The contributions to the state-of-the-art made during this work include: i) the development of a cantilever configured reaction wheel analytical model able to reproduce all the configurations in which the mechanism may operate and inclusive of the gyroscopic effect; ii) the reformulation of the coupling theory which allows retrieving the dynamic mass of a microvibration source over a wide range of frequencies and speeds, by means of the experimental data obtained from measurements of the forces generated when the source is rigidly secured on a dynamometric platform and measurements of the accelerations at the source mounting interface in a free-free suspended boundary condition; iii) a practical example of coupling between a reaction wheel and a honeycomb structural panel, where the coupled loads and the panel response have been estimated using the mathematical model and compared with test results, obtained during the physical microvibration testing of the structural panel, showing a good level of agreement when the gyroscopic effect is also taken into account.
CONTENTS

CHAPTER 1: INTRODUCTION
1.1 Microvibration Generation ........................................ 4
1.2 Aims and Objectives .................................................. 5
1.3 Elements of Novelty .................................................... 7
1.4 Outline of the Thesis .................................................. 9

CHAPTER 2: LITERATURE REVIEW ...................................... 12
2.1 History of Microvibrations on Satellites .......................... 12
2.2 Open Issues in Microvibration Characterisation, Analysis and Control 15
  2.2.1 Characterisation .................................................... 16
  2.2.2 Frequency-based Structural Analysis Methodologies .......... 18
  2.2.3 Control and Mitigation .......................................... 21
2.3 Microvibration Sources .............................................. 21
2.4 RWA Dynamics and Analysis ........................................ 24
  2.4.1 Reaction Wheel Assemblies ..................................... 24
  2.4.2 Nature of RWA Disturbances ................................... 25
  2.4.3 RWA Structural Modes .......................................... 27
  2.4.4 Traditional RWA Microvibration Analysis Methodologies .... 29
  2.4.5 RWA Dynamic Coupling ....................................... 34
2.5 Receivers ............................................................... 36
2.6 Summary ............................................................... 38
Chapter 3: RWA Analytical Model

3.1 Energy method ............................................. 40
3.2 RWA Equations of Motion ................................. 41
  3.2.1 RWA Schematic ........................................ 41
  3.2.2 RWA Kinetic Energy ................................... 44
  3.2.3 RWA Potential Energy ................................ 47
  3.2.4 RWA Damping Representation ......................... 51
  3.2.5 RWA Imbalanced Model ............................... 53
3.3 Hard-mounted Boundary Configuration .................... 55
3.4 Free-free Boundary Configuration .......................... 59
3.5 Summary ................................................... 62

Chapter 4: RWA Microvibration Model Validation 63

4.1 RWA and Test General Features ............................ 63
4.2 Signal Processing .......................................... 65
4.3 Harmonic Response ........................................ 67
4.4 Hard-mounted Microvibration Testing ....................... 71
  4.4.1 Hard-mounted Test Setup ............................... 71
  4.4.2 Hard-mounted Test Results ............................. 72
4.5 Free-free Microvibration Testing ........................... 80
  4.5.1 Free-free Test Setup .................................. 80
  4.5.2 Free-free Test Results ................................ 81
4.6 RWA Model Verification .................................... 86
  4.6.1 RWA State-space Model and Harmonic Excitation .... 88
  4.6.2 RWA Analytical Structural Modes ...................... 89
4.7 Summary ................................................... 92

Chapter 5: Dynamic Mass of a RWA 96

5.1 Mathematical Formulation ................................... 97
  5.1.1 Iterative Process ...................................... 100
5.2 Numerical Analysis ................................................. 102
  5.2.1 Influence of the Gyroscopic Effect on the Dynamic Mass Behaviour 103
  5.2.2 Complete Dynamic Mass Response ......................... 106
5.3 Direct Accelerance Measurements ................................ 106
  5.3.1 Accelerance Test Setup ...................................... 109
  5.3.2 Accelerance Test Data ...................................... 112
5.4 Empirical Dynamic Mass Measurement Method ................... 114
5.5 Broadband Noise Influence on the Dynamic Mass Results ...... 118
5.6 Summary ........................................................... 123

CHAPTER 6: COUPLED ANALYSIS .................................. 124
  6.1 Theory of Coupling ................................................. 124
    6.1.1 Benchmark Example ........................................ 128
  6.2 RWA-FFSM Coupled Microvibration Testing ..................... 133
    6.2.1 RWA-FFSM Test Setup ..................................... 133
    6.2.2 RWA-FFSM Test Results ................................... 133
  6.3 RWA-Panel Coupled Microvibration Testing ..................... 137
    6.3.1 RWA-Panel Test Setup ..................................... 137
    6.3.2 RWA-Panel Test Results ................................... 139
  6.4 Numerical Computation .......................................... 139
    6.4.1 Source Modelling Approaches .............................. 143
    6.4.2 RWA-FFSM Analysis ....................................... 144
    6.4.3 RWA-Panel Analysis ....................................... 146
  6.5 Summary ........................................................... 149

CHAPTER 7: CONCLUSIONS ........................................... 150
  7.1 Summary of the Work Conducted ................................ 150
  7.2 Main Achievements .............................................. 152
  7.3 Future Work ...................................................... 153

REFERENCES .................................................................. 155
<table>
<thead>
<tr>
<th>Appendix</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Tait-Brian Transformation Matrices</td>
<td>173</td>
</tr>
<tr>
<td>B</td>
<td>RWA System Matrices</td>
<td>175</td>
</tr>
<tr>
<td>C</td>
<td>Signal Processing Techniques</td>
<td>182</td>
</tr>
<tr>
<td>D</td>
<td>Test Equipment and Results Derivation Process</td>
<td>188</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

1.1 Schematic representation of microvibration propagation on-board spacecraft ........................................................................................... 2
1.2 Line of sight jitter on a detector plane after 5 seconds (ECSS, 2013) .................................................................................................................. 3
1.3 Loss of image quality due to microvibration artefacts (ECSS, 2013) ............................................................................................................... 3
1.4 Dynamic behaviour and interaction with regard to frequency range ............................................................................................................ 6
1.5 Flowchart showing the thesis layout .................................................................................................................................................. 11

2.1 Angular resolution of remote sensing satellite from 1970 to 2010 (Toyoshima et al., 2003). Note the Hubble Space Telescope has been added to the original figure ...................................................................................................................... 13
2.2 Macro-oscillations at the target area produced by micro-fluctuation of the instrument mounting interface .......................................................................................................................... 15
2.3 Schematic of a semi-empirical approach for microvibration sources modelling ............................................................................................ 17
2.4 Traditional dynamic mass measurement of a RWA: (a) z-axis measurement and (b) x-axis measurement (Zhang et al., 2013) ......... 18
2.5 Frequency response of the five spacecraft of the Rapid-Eye constellation (Remedia et al., 2015c) .............................................................................. 19
2.6 Comparison of the FE analysis prediction with test results applying the Craig-Bampton method (Remedia et al., 2015b) ......................... 20
2.7 Schematic cross-sectional view of RWA components and configurations: 1-flywheel, 2-motor, 3-rigid shaft, 4-ball bearings, 5-housing, 6-flexible components; (a) symmetric; (b) cantilever with rigid connection and (c) cantilever with flexible connection ......................................................................................... 25
2.8 Flywheel imbalance: (a) static and (b) dynamic .......................... 26
2.9 Structural modes of a mid-span RWA: (a) axial; (b) radial translational and (c) radial rotational ................................. 28
2.10 Structural modes of a cantilever-configured RWA: (a) axial and (b) combined radial translational and rotational .......................... 29
2.11 Campbell diagram of a RWA: (a) mid-span configured RWA and (b) cantilever configured RWA ................................. 30
2.12 Spike identification and broadband excitation derivation: (a) raw and filter signal and (b) spike identification (Zhang et al., 2014) .......................... 32
2.13 ESA reaction wheels characterisation facility (Wagner et al., 2012) .................. 34
2.14 Response at a receiver location from various implementation of the input (ECSS, 2013) ................................. 37
2.15 SDO jitter analysis using an integrate modelling approach (Liu et al., 2007) ................................. 37
3.1 Tait-Bryan angles definition and transformations between each coordinate frame by means of a y-x-z rotation sequence .......................... 40
3.2 Reaction wheel assembly schematic cross-section view: 1-flywheel (mass and inertia), 2-motor holder (flexible component), 3-motor shaft (massless and rigid), 4-motor body (included in the modelling of the flywheel), 5-wheel-base (mass and inertia) .......................... 42
3.3 Reaction wheel assembly schematic ................................. 43
3.4 RWA model in XZ-plane ................................. 48
3.5 RWA model in YZ-plane ................................. 50
3.6 RWA hard-mounted configuration schematic ................................. 57
3.7 RWA free-free configuration schematic ................................. 60
4.1 Reaction wheel assembly used during the whole test campaign .......................... 65
4.2 Background noise compared to the measurement response at 660 rpm (11 Hz): a) axial acceleration in free-free boundary condition and b) axial force in hard-mounted boundary condition .......................... 66
4.3 Total RMS value of $F_{hm,y}$ in both time and frequency domains .......................... 67
4.4 Seismic mass used as interface between the RWA and the shaker facility 68
4.5 Harmonic response test setup: a) $x_w$-axis; b) $y_w$-axis and c) $z_w$-axis 69
4.6 Campbell diagram of the cantilever configured RWA used for testing superimposed onto higher and bearing imperfections harmonics (dotted grey and magenta lines, respectively) 70
4.7 RWA hard-mounted microvibration test setup: (a) $x_w$-axis; (b) $y_w$-axis and (c) $z_w$-axis 73
4.8 Reference clock superimposed to a force sensor signal, during $z_w$-axis testing, as the flywheel operates at 600 rpm 74
4.9 PSD waterfall plots of measured forces and moments: (a) $F_{hm,x}$; (b) $F_{hm,z}$ and (c) $M_{hm,y}$ 75
4.10 Spectral maps of measured forces and moments: (a) $F_{hm,x}$; (b) $F_{hm,z}$ and (c) $M_{hm,y}$ 76
4.11 Zoomed spectral maps to show UK power line and rocking FW: (a) $F_{hm,z}$ and (b) $M_{hm,y}$ 77
4.12 RWA symmetry assessment from hard-mounted forces measurements: (a) fundamental harmonics of $F_{hm,x}$ and $F_{hm,y}$; (b) force cone and (c) force cone top view 79
4.13 Total RMS value for $F_{hm,y}$ and $F_{hm,z}$ 80
4.14 RWA free-free microvibration test setup: (a) schematic showing accelerometers direction of measurement and (b) detail of accelerometers location and coordinate system 82
4.15 PSD waterfall plots of measured accelerations: (a) $\ddot{x}_{mp}$; (b) $\ddot{z}_{mp}$ and (c) $\ddot{\phi}_{mp}$ 83
4.16 Spectral maps of measured forces and moments: (a) $\ddot{x}_{mp}$; (b) $\ddot{z}_{mp}$ and (c) $\ddot{\phi}_{mp}$ 84
4.17 Zoomed view of the rocking BW and FW from the free-free microvibration testing 85
4.18 Total RMS value for $\ddot{x}_{mp}$ and $\ddot{z}_{mp}$ 86
4.19 RWA symmetry assessment from free-free acceleration measurements: (a) fundamental harmonics of \( \ddot{x}_{mp} \) and \( \ddot{y}_{mp} \); (b) acceleration cone and (c) acceleration cone top view ........................................ 87

4.20 RWA analytical response compared to test results at 1200 rpm using a high mass imbalance: a) high imbalance hard-mounted test configuration and b) force \( F_{hm,x} \) ................................................................. 88

4.21 Complete analytical RWA harmonic excitation inputs: a) \( F_{s,xw} \) PSD waterfall plot; b) \( F_{s,zw} \) spectral map; c) \( F_{s,zw} \) PSD waterfall plot; d) \( F_{s,zw} \) spectral map; e) \( M_{s,yw} \) PSD waterfall plot and f) \( M_{s,yw} \) spectral map ................................................................. 90

4.22 Analytical forces and accelerations derived from the RWA state space model: (a) \( F_{hm,x} \); (b) \( \ddot{x}_{mp} \); (c) \( F_{hm,z} \); (d) \( \ddot{z}_{mp} \); (e) \( M_{hm,y} \) and (f) \( \ddot{\phi}_{mp} \) ................................................................. 91

4.23 Simulated structural modes in the hard-mounted boundary condition superimposed to spectral maps of experimental data: (a) \( F_{hm,x} \); (b) \( F_{hm,z} \) and (c) \( M_{hm,y} \) ................................................................. 93

4.24 Simulated structural modes in the free-free boundary condition superimposed to spectral maps of experimental data: (a) \( \ddot{x}_{mp} \); (b) \( \ddot{z}_{mp} \) and (c) \( \ddot{\phi}_{mp} \) ................................................................. 94

5.1 Iterative process to evaluate the dynamic mass from measurements of hard-mounted forces and free-free accelerations .................................................. 101

5.2 Comparison between dynamic mass elements obtained using the traditional and the novel methodologies whilst the flywheel spins at 2400 rpm: (a) element \( D_{rwa,11} \); (b) \( D_{rwa,44} \) and (c) \( D_{rwa,15} \) ............ 104

5.3 Effect of the gyroscopic effect on the dynamic mass response: (a) element \( D_{rwa,11} \); (b) element \( D_{rwa,33} \) and (c) element \( D_{rwa,44} \) ............ 105

5.4 Effect of the gyroscopic effect on the off-diagonal elements of dynamic mass: (a) element \( D_{rwa,12} \); (b) element \( D_{rwa,14} \) and (c) element \( D_{rwa,45} \) ............ 107

5.5 Evolution of the dynamic mass response as function of speed: (a) element \( D_{rwa,11} \); (b) element \( D_{rwa,15} \) and (c) element \( D_{rwa,45} \) ............ 108
5.6 Axial translational DoF dynamic mass coefficient response as function of speed ........................................... 109
5.7 Direct accelerance measurement test setup: (a) $A_{mp,11}$ and $A_{mp,15}$; (b) $A_{mp,33}$ and $A_{mp,44}$ and (c) $A_{mp,66}$ ............................................. 110
5.8 Direct accelerance measurement, accelerometer locations and force and moment directions: (a) $xy$-plane and (b) $xz$-plane ........... 111
5.9 RWA FE model ............................................. 113
5.10 RWA dynamic mass response comparison between FE model and direct accelerance measurements at static in the axial DoF ........... 113
5.11 Dynamic mass element $D_{rwa,11}$ behaviour at low speeds of rotation 116
5.12 Comparison between the predicted and experimental dynamic mass at 1320 rpm: (a) element $D_{rwa,33}$ and (b) element $D_{rwa,44}$ (Addari et al., 2016) ............................................. 117
5.13 Comparison between the noise spectral contribution from the hard-mounted and free-free tests: (a) element $\Phi_{11}$; (b) element $\Phi_{33}$ and (c) $\Phi_{44}$ ............................................. 119
5.14 Effect of different noise inputs on the calculation of the RWA dynamic mass at static: (a) element $D_{rwa,11}$; (b) element $D_{rwa,33}$ and (c) element $D_{rwa,44}$ ............................................. 121
5.15 Effect of different noise inputs on the calculation of the RWA dynamic mass at 1800 rpm: (a) element $D_{rwa,33}$; (b) element $D_{rwa,44}$; (c) element $D_{rwa,15}$; (d) element $D_{rwa,12}$; (e) element $D_{rwa,14}$ and (f) element $D_{rwa,45}$ ......... 122
6.1 Schematic of two counter-rotating systems mounted on platforms with stiffness $k$ ............................................. 125
6.2 Schematic of a source-supporting structure system used as a benchmark example for coupled analysis (ECSS, 2013) .................. 128
6.3 Disturbance force applied at the source and resulting force at support ............................................. 129
6.4 Response at the receiver by applying $\Phi_{hm}$ at the mounting location of the source or $\Phi_{inp}$ at the source itself ................................. 130
6.5 Schematic of the different approaches that are implemented in this section: (a) source not included in the model; (b) source represented as a lumped mass and (c) source included in the model by means of the coupling theory ............................................................. 131
6.6 Response at receiver implementing all the different approaches discussed in this section ............................................................. 132
6.7 Coupled RWA-FFSM test setup, including accelerometers configuration ............................................................. 134
6.8 PSD waterfall plot of output data from coupled RWA-FFSM experimental test: (a) coupled axial force $\Phi_{cp,33}$; (b) coupled moment $\Phi_{cp,55}$ and (c) structure response $\Phi_{out,11}$ ............................................................. 135
6.9 Spectral maps of output data from coupled RWA-FFSM experimental test: (a) coupled axial force $\Phi_{cp,33}$; (b) coupled moment $\Phi_{cp,55}$ and (c) structure response $\Phi_{out,11}$. The RWA-stiff support structural modes are superimposed and represented in black solid lines .... 136
6.10 Structural panel used for coupling analysis ............................................................. 137
6.11 Coupled RWA-panel test setup, including accelerometers configuration ............................................................. 138
6.12 PSD waterfall plot of output data from coupled RWA-panel experimental test: (a) coupled axial force $\Phi_{cp,33}$; (b) coupled moment $\Phi_{cp,55}$ and (c) structure response at accelerometer T2 location $\Phi_{out,T2,33}$ ............................................................. 140
6.13 Spectral maps of output data from coupled RWA-panel experimental test: (a) coupled axial force $\Phi_{cp,33}$; (b) coupled moment $\Phi_{cp,55}$ and (c) structure response at accelerometer T2 location $\Phi_{out,T2,33}$. ............................................................. 141
6.14 Supporting structure FE models: (a) FFSM and (b) sandwich panel 142
6.15 Structural panel dynamic mass of axial DoF element, $D_{ss,33}$, computed as a function of the frequency only ............................................................. 142
6.16 Cases analysed: (a) no source; (b) source as lumped mass, \( D_{rwa} |_{\omega=0,\Omega=0} \); (c) source dynamic mass, \( D_{rwa} (\omega) |_{\Omega=0} \) and (d) source dynamic mass including the gyroscopic effect, \( D_{rwa} (\omega, \Omega) \). \( \Phi_{hm} \) represents the PSD of the loads measured in a hard-mounted boundary condition.

6.17 Comparison of the interface RWA-stiff platform coupled dynamics derived from test results and the implementation of the four source modelling approaches for an angular speed of 1200 rpm (20 Hz): (a) coupled axial force \( |\Phi_{cp,33}| \); (b) coupled moment about RWA y-axis \( |\Phi_{cp,55}| \) and (c) support radial linear acceleration \( |\Phi_{out,11}| \).

6.18 Comparison of the interface RWA-stiff platform coupled dynamics derived from test results and the implementation of the four source modelling approaches for an angular speed of 2520 rpm (42 Hz): (a) coupled axial force \( |\Phi_{cp,33}| \); (b) coupled moment about y-axis \( |\Phi_{cp,55}| \) and (c) support radial linear acceleration \( |\Phi_{out,T2}| \).

D.1 Force sensors configuration adopted during axial hard-mounted test:
(a) top view and (b) \( y_w z_w \)-plane.

D.2 Schematic configuration of the force sensors used during microvibration hard-mounted testing in the \( x_w \)-axis.

D.3 Schematic configuration of the force sensors used during microvibration hard-mounted testing in the \( y_w \)-axis.

D.4 Accelerometers schematic configuration used during free-free microvibration testing: (a) \( x_w z_w \)-plane and (b) \( y_w z_w \)-plane. The green arrows indicate the positive direction of measurement of the corresponding accelerometer.
LIST OF TABLES

2.1 Typical internal microvibration sources on a modern high-stability spacecraft ........................................... 23
4.1 Mechanical Properties of the Thermoplastic Material ............... 64
4.2 RWA model parameters obtained from the harmonic response test campaign .................................................. 68
4.3 RWA model parameters derived from the RWA hard-mounted (blocked) and free-free microvibration testing ................. 95
6.1 Benchmark example model parameters .................................. 128
6.2 FRAC calculated for the RWA-FFSM coupled system responses at \( \Omega = 1200 \) rpm .......................................................... 146
6.3 FRAC calculated for the RWA-panel coupled system responses at \( \Omega = 2520 \) rpm .......................................................... 147
A.1 Transformation matrices from inertial frame to body frame ........... 173
A.2 Transformation matrices from body frame to inertial frame .......... 174
B.1 Compact form of the damping coefficients for the complete RWA system ......................................................... 175
B.2 Compact form of the stiffness coefficients for the complete RWA system ......................................................... 176
B.3 Compact form of the damping coefficients for the free-free RWA system ......................................................... 178
B.4 Compact form of the stiffness coefficients for the free-free RWA system ......................................................... 178
B.5 Compact form of the damping coefficients for the free-free RWA-stiff platform system ......................................................... 180
B.6 Compact form of the stiffness coefficients for the free-free RWA-stiff platform system ......................................................... 180
C.1 Relationship between the parameters implemented in the data acquisition .................................................................................. 183
D.1 List of the load cells used for force measurements .................................................................................................................. 188
D.2 List of the accelerometers used for acceleration and accelerance measurements ............................................................................... 189
D.3 Distance between force sensors and configuration CoM ........................................................................................................... 190
D.4 Orthogonal distance between the force sensors and the RWA mounting interface ........................................................................... 192
D.5 Distance between accelerometers in the free-free test configuration 195
NOMENCLATURE

Roman Symbols

$A$  amplitude of harmonics excitations

$a$  power spectral density matrix of an acceleration vector

$a,b,c$  wheel assembly first rotation frame

$C$  damping matrix

$c$  damping coefficient

$d$  shaft length

$f$  disturbances vector

$G$  gyroscopic matrix

$h$  distance from the flywheel-to-wheel-base suspension system to the wheel-base CoM

$I$  inertia tensor/unit matrix

$I$  moment of inertia

$i$  imaginary unit

$K$  stiffness matrix

$k$  spring stiffness coefficient

$L$  lagrangian
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
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<tr>
<td>$l$</td>
<td>imbalance mass moment arm</td>
</tr>
<tr>
<td>$M$</td>
<td>mass matrix</td>
</tr>
<tr>
<td>$M$</td>
<td>mass/generalised moment</td>
</tr>
<tr>
<td>$m$</td>
<td>generic mass/mass imbalance</td>
</tr>
<tr>
<td>$n$</td>
<td>total number of harmonics</td>
</tr>
<tr>
<td>$O$</td>
<td>centre of mass of specific part as mentioned in the thesis/coordinate origin</td>
</tr>
<tr>
<td>$Q$</td>
<td>work done</td>
</tr>
<tr>
<td>$q$</td>
<td>generalised coordinate/complex coordinates</td>
</tr>
<tr>
<td>$R$</td>
<td>coordinate system rotation matrix</td>
</tr>
<tr>
<td>$r$</td>
<td>imbalance mass radius</td>
</tr>
<tr>
<td>$s$</td>
<td>displacement vector in the inertial frame</td>
</tr>
<tr>
<td>$T$</td>
<td>transformation matrix</td>
</tr>
<tr>
<td>$T$</td>
<td>kinetic energy</td>
</tr>
<tr>
<td>$t$</td>
<td>generalised time</td>
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<td>$U$</td>
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<td>$u$</td>
<td>translational displacement vector</td>
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<tr>
<td>$v$</td>
<td>translational velocity vector</td>
</tr>
<tr>
<td>$v$</td>
<td>distance from the wheel-base CoM to the wheel-base-to-ground suspension system</td>
</tr>
<tr>
<td>$W$</td>
<td>broadband noise excitation</td>
</tr>
<tr>
<td>$\ddot{x}$</td>
<td>acceleration vector</td>
</tr>
<tr>
<td>$XYZ$</td>
<td>displacement in the RWA inertial frame</td>
</tr>
</tbody>
</table>
xyz displacement in the RWA body frame/general displacement

x’y’z’ wheel assembly second rotation frame

**Greek Symbols**

α generic angle

χ complex coordinate in the rotational DoF

Δ variation in space due to a rotation

Ω flywheel angular speed

ω angular velocity vector

ω reaction wheel assembly natural frequency

Φ power spectral density matrix of a force vector

θ, φ, ψ rotations about three RWA orthogonal axes x, y and z respectively

υ complex coordinate in the translation DoF

**Superscripts**

a axial degree of freedom

H Hermitian or conjugate and transpose

r rotational degree of freedom

T transpose

t translational degree of freedom

* conjugate

**Subscripts**

0 initial condition
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>axial degree of freedom</td>
</tr>
<tr>
<td>b</td>
<td>wheel-base</td>
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<tr>
<td>cp</td>
<td>coupled boundary condition</td>
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<tr>
<td>crit</td>
<td>critical speed</td>
</tr>
<tr>
<td>fis</td>
<td>free-free configuration</td>
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<tr>
<td>hm</td>
<td>hard-mounted configuration</td>
</tr>
<tr>
<td>i</td>
<td>number of harmonics considered in each DoF</td>
</tr>
<tr>
<td>j</td>
<td>j-th generalised coordinate</td>
</tr>
<tr>
<td>m</td>
<td>mass imbalance</td>
</tr>
<tr>
<td>mp</td>
<td>reaction wheel mounting point</td>
</tr>
<tr>
<td>r</td>
<td>rotational degree of freedom</td>
</tr>
<tr>
<td>s</td>
<td>system</td>
</tr>
<tr>
<td>ss</td>
<td>supporting structure</td>
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<tr>
<td>t</td>
<td>translational degree of freedom</td>
</tr>
<tr>
<td>w</td>
<td>flywheel/wheel assembly</td>
</tr>
<tr>
<td>xz</td>
<td>xz-plane</td>
</tr>
<tr>
<td>yz</td>
<td>yz-plane</td>
</tr>
<tr>
<td>z</td>
<td>axial degree of freedom</td>
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</tbody>
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**Acronyms**

- **AOCS**: Attitude on-Orbit Control System
- **AS**: Amplitude Spectrum
- **ATP**: Acquisition, Tracking and Pointing System
<table>
<thead>
<tr>
<th>Acronym</th>
<th>Full Form</th>
</tr>
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<tbody>
<tr>
<td>BW</td>
<td>Backward Whirl</td>
</tr>
<tr>
<td>CoM</td>
<td>Centre of Mass</td>
</tr>
<tr>
<td>DC</td>
<td>Direct Current</td>
</tr>
<tr>
<td>DoF</td>
<td>Degree of Freedom</td>
</tr>
<tr>
<td>DoF</td>
<td>Degree of Freedom</td>
</tr>
<tr>
<td>EoM</td>
<td>Equation of Motion</td>
</tr>
<tr>
<td>ESA</td>
<td>European Space Agency</td>
</tr>
<tr>
<td>FE</td>
<td>Finite Element</td>
</tr>
<tr>
<td>FFSM</td>
<td>Free Floating Seismic Mass</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transformation</td>
</tr>
<tr>
<td>FMCS</td>
<td>Full Monte Carlo Simulations</td>
</tr>
<tr>
<td>FRAC</td>
<td>Frequency Responisne Assurance Criterion</td>
</tr>
<tr>
<td>FRF</td>
<td>Frequency Response Function</td>
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<tr>
<td>FW</td>
<td>Forward Whirl</td>
</tr>
<tr>
<td>GOCE</td>
<td>Gravity Field and Steady-State Ocean Circulation Explorer</td>
</tr>
<tr>
<td>HST</td>
<td>Hubble Space Telescope</td>
</tr>
<tr>
<td>JWST</td>
<td>James Webb Space Telescope</td>
</tr>
<tr>
<td>MWA</td>
<td>Momentum Wheel Assembly</td>
</tr>
<tr>
<td>OICETS</td>
<td>Optical Inter-Orbit Communications Engineering Test Satellite</td>
</tr>
<tr>
<td>PSD</td>
<td>Power Spectral Density</td>
</tr>
<tr>
<td>RMS</td>
<td>Root Mean Square</td>
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<tr>
<td>RMS</td>
<td>Root Mean Square</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>RWA</td>
<td>Reaction Wheel Assembly</td>
</tr>
<tr>
<td>SDO</td>
<td>Solar Dynamics Observatory</td>
</tr>
<tr>
<td>SEA</td>
<td>Statistical Energy Analysis</td>
</tr>
<tr>
<td>SFEM</td>
<td>Stochastic Finite Element Method</td>
</tr>
<tr>
<td>SIM</td>
<td>Space Interferometry Mission</td>
</tr>
<tr>
<td>SOHO</td>
<td>Solar and Heliospheric Observatory</td>
</tr>
<tr>
<td>STEREO</td>
<td>Solar Terrestrial Relations Observatory</td>
</tr>
<tr>
<td>TF</td>
<td>Transfer Function</td>
</tr>
<tr>
<td>TPF</td>
<td>Terrestrial Planet Finder</td>
</tr>
<tr>
<td>WA</td>
<td>Wheel Assembly</td>
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</table>
INTRODUCTION

Spacecraft microvibrations and related issues have interested aerospace engineers and researchers for more than forty years. Nowadays, due to a significant improvement and advancing of the technology on-board spacecraft, they have become a major concern in the design of either imaging, communication or scientific missions which are characterised by high level of stability platform requirements. For instance, space astronomy and Earth observation missions most often demand fine pointing accuracies. Missions such as object tracking and targeting or laser interspace/space-ground communication can also pose challenging environments for the mitigation of microvibrations (Foster et al., 1995). Thereby, a significant amount of research is nowadays conducted to investigate the issues related to microvibrations on-board satellites; for instance, 24 papers concerning microvibrations characterisation, analysis and mitigation were presented at the “13th European Conference on Spacecraft Structures, Materials & Environmental Testing” in 2014, in contrast to only 6 papers delivered in the 5th edition of the conference in 1998 (Remedia, 2015). In order to clarify the terminology adopted in this work, we refer to microvibrations as mechanical-induced vibrations with amplitudes in the range of micro-g’s ($\mu g$) acting in a range of frequency from a fraction of Hz up to 1 kHz (ECSS, 2013).

Microvibrations are typically induced by some on-board satellite equipment, in particular rotary mechanisms such as Wheel Assemblies (WA), either Reaction Wheel Assemblies (RWA), or Momentum Wheel Assemblies (MWA) (Bialke, 1996), cryo-coolers (Tomaru et al., 2004), antenna and solar pointing mechanisms (Meza et al., 2005). Other possible sources of microvibration are thrusters and switches (Zhang et al., 2009). Of the various sources, however, RWAs and MWAs
are generally regarded as the most important (Babuska et al., 2004; Bialke, 2011; Fausz et al., 2009; Miller et al., 2007). This study therefore focuses on the characterisation of these mechanisms. The source-induced disturbances are transmitted through the satellite structure towards the payloads or sensitive on-board instruments, in this context referred to as receivers, negatively affecting their correct functioning. Furthermore, due to coupled dynamics between the source and the supporting structure (Elias et al., 2003), estimates of the microvibration effects are even more challenging (Takahara et al., 2006). The propagation of microvibrations throughout the spacecraft structure can be schematically represented as in Figure 1.1. Source-induced microvibration disturbances may present a hazard to the receivers, significantly deteriorating their performance. For instance, the line of sight jitter on a detector plane with respect to time can be observed in Figure 1.2. The centroid jitter may result in a loss of quality of the image acquired by the camera on-board a satellite, as depicted in Figure 1.3. Here, the first picture presents notable microvibrations, the second refers to the same picture after the application of some corrective measures, finally the third picture displays no perturbation during acquisition.

In a space engineering context, microvibration management can be arranged in three macro-categories:

- generation: the key questions are “how are microvibrations produced?” and “how can these disturbances be analytically represented?”. A considerable amount of publications which deal with sources of microvibrations, in particular their mathematical modelling, can be found in the literature and will be discussed later;
Figure 1.2: Line of sight jitter on a detector plane after 5 seconds (ECSS, 2013)

Figure 1.3: Loss of image quality due to microvibration artefacts (ECSS, 2013)
• analysis: the research is here concentrated in investigating the propagation of the source-induced disturbances throughout the structure towards sensitive equipment (i.e. receivers). A considerable amount of work on the development of prediction methods in the specific areas of applications has been carried out;

• control and mitigation: emphasis is given to the study of how microvibrations can be controlled and/or limited. With the purpose to achieve the appropriate level of stability, an either active, semi-active, or passive control may be required and implemented.

1.1 Microvibration Generation

In order to answer the questions posed in the previous section, in terms of microvibration generation, a comprehensive knowledge of the spacecraft subsystems (structure, avionics, propulsion, attitude control, etc.) is required. For instance, some familiarity with the noise produced by electronic switches, thermal clanks associated with thrusters or mechanical vibrations due to a non-uniform mass distribution of a RWA is needed to formulate equations and develop models which are valid and reliable representations of the microvibration sources. These would, then, allow the assessment of how severe microvibrations are for spacecraft payloads and sensitive instruments.

In these terms, the first issue is to quantify/predict the perturbation generated by the equipment. The disturbance forces can be either estimated by numerical analysis or retrieved by tests. The mathematical models should be able to represent, at least, the main physical characteristics of the disturbance source and to simulate the induced disturbances in its operative conditions. However, some equipment may display a complex behaviour that is difficult to mathematically represent from first principles. Therefore, experimental tests are performed and the data used to improve the analytical model including those features otherwise not easily representable. Nevertheless, this is not always possible due to cost and schedule constraints.

When dealing with rotary mechanisms, a second issue arises from the definition of the device inertial and mechanical properties, in other terms its mass, stiffness and damping attributes, over the range of frequency where microvibrations are a major concern (say from 1 Hz to 1000 Hz). The issues in estimating the internal dynamics of a microvibration source are created by a variety of reasons (e.g. non-linearity of bearing support or manufacturing imperfections that are difficult to
quantify and model). In the context of spacecraft mechanisms, the accelerance measurement method proposed by Zhang et al. (2013) provides good experimental estimate of the source internal dynamics when the source is not set in operation (i.e. the RWA is not spinning). Nevertheless, this approach presents challenges in terms of test setup. Moreover, it becomes unreliable when the mechanism is functioning. Therefore, a method which is able to tackle these problems, facilitating the test configuration and the retrieval of the source speed and frequency dependent mass, stiffness and damping properties, requires to be developed.

An example of the dynamic behaviour of a RWA with regard to the frequency range is shown in Figure 1.4. The three sensors display an increase in noise level at frequencies above 200 Hz and uncertainties in the disturbance forces.

The ultimate issue refers to the quantification/prediction of the response at a spacecraft sensitive location when this is subjected to microvibration excitations. Currently, the most common approach is to carry out experiments grounding the source on a dynamometric table (e.g. Kistler table) and measure the loads at the source interface whilst the source is in operation (Masterson, 1999), and subsequently apply these data as input to the spacecraft Finite Element (FE) model to predict the induced effects at, for example, the payload. This concept is, however, flawed because grounded source-microvibrations measurements are not representative of the real environment in which the source will operate. In fact, a further complication arises due to the coupling between the dynamics of the source and those of the structure, dramatically changing the behaviour of both the subsystems. The source dynamic theory developed by Masterson (1999) and Elias (2004) and further coupling with the structure (Remedia et al., 2015b; Zhang et al., 2013) has been implemented appositely to undertake this study. The procedure has been extended to include the internal dynamics of the source when this is in operation. In order to obtain accurate outcomes, their exact contributions both in the frequency and speed ranges of operation have to be derived.

1.2 AIMS AND OBJECTIVES

The analysis and modelling of satellite “macro”- and microvibration, where “macro” denotes disturbances with significantly high amplitude of vibrations generally associated with a spacecraft launch, represent a challenging subject due to the complexity in the construction of accurate models, which is further worsened by the introduction of uncertainties and non-linearity due to some of the micro-g phenomena. Satellite microvibration analysis and control is now a predefined step in almost all modern high precision satellite design. As microvibrations are critical
Multi-body dynamics
Spacecraft structural dynamics
Fundamental harmonics
Higher order harmonics and bearing noise
Source first few modes of vibration
Background noise

Figure 1.4: Dynamic behaviour and interaction with regard to frequency range

To the mission success (ECSS, 2013), a significant amount of analyses are carried out to address these issues. The analyses are, however, often unsatisfactory (long, expensive and at times inaccurate), hence a faster and more reliable methodology for the characterisation of microvibration sources would provide a significant boost to the managing of the interaction between the microvibration source and the spacecraft structure.

The research programme was initially sponsored by Satellite Services B.V. Space and Ground Systems UK and Technology Strategy Board. The project “Next Generation Reaction Wheel Development” consisted of two stages: an initial stage to familiarise with issues and tools and design and qualify a new RWA concept based on a cantilever configured soft-suspended RWA; this was followed by a research plan, which was the core of the current research programme, and had the objective to develop a methodology to characterise the microvibrations produced by a RWA, and allow the construction of models which better correlate with the dynamics of flight operational hardware. Emphasis was given to the microvibrations produced by a cantilever RWA configuration as this is a typical configuration used for space applications.

To achieve the aforementioned goals, the research activity set and accomplished a series of objectives:
• design and prototyping a reaction wheel for micro and nano satellites. This consisted of the architectural design and the vibrational analysis and testing of the wheel as well as detailed design of the critical components;

• investigate the microvibration disturbances induced by the RWA through a test campaign in order to determine the broadband excitation input model characteristic of the RWA;

• derive a mathematical model for the microvibrations emitted by the cantilever RWA configuration suitable for the widest range of boundary conditions;

• develop a methodology for accurately retrieving the RWA model parameters as function of the frequency and of the speed of operation;

• implement all the aforementioned objectives to fulfil the aim of predicting the coupled response in locations of interest without the necessity to perform an experimental campaign;

• verify the predictions against test results for the RWA mounted on a real supporting structure.

1.3 Elements of Novelty

The state-of-the-art in the assessment of the contributors to RWAs’ microvibration requires a practical experimental method to characterise these by test, which provides likely in-orbit vibration spectra with a high degree of confidence. This need was recognized by the European Space Agency (ESA) which issued an invitation to tender for a specific study entitled “Microdisturbance Sources and Characterisation” which proved highly useful to a number of current and future ESA missions and will also be of great benefit to RWAs providers to assist in the improvement of their wheels.

More specifically the contributions to the state-of-the-art that have been made by this research activity are:

• the hard-mounted cantilever-configured RWA model developed by Zhang & Aglietti (2011) was extended to include the gyroscopic effect and reproduce all the configurations in which the RWA will operate, including free-free and

∗ESA Intended Invitation to Tender, 12.1EC.10, http://emits.sso.esa.int/emits/owa/emits_iitt.show_iitt?actref=12.1EC.10&user=Anonymous
coupled boundary conditions. An experimental campaign, where the RWA-induced disturbances and the RWA accelerations were measured, was also conducted to validate the analytical predictions against test results. The model shows to correctly capture the RWA dynamics over a wide frequency band and range of operative speeds, taking into account also the gyroscopic effects (discussed in chapters 3 and 4). In addition, the theory of coupling was extended to include the gyroscopic effects in the modelling of the source dynamics and development of guidelines concerning its practical application. An experimental campaign installing the RWA on either a Free Floating Seismic Mass (FFSM) or honeycomb panel has been performed giving indication of the level of accuracy that can be achieved with this kind of analyses (articled in chapter 6);

- the investigation of an alternative method for the derivation of the dynamic mass of a RWA through a re-formulation of the theory of coupling aiming to facilitate the experimental campaign and reduce the computational effort in the derivation process. In this method, the measurements of the loads when the RWA is hard-mounted on a dynamometric platform and the accelerations that are produced at the RWA mounting interface when the mechanism is running free-free were opportunely combined. The method gives good predictions and estimates throughout the frequency and speed spectra. A study of the uncertainties related to the innovative dynamic mass measurement method through a numerical simulation using the RWA mathematical model was also conducted. In particular, the effect of the broadband noise has been examined and measures to tackle this problem proposed (discussed in chapter 5);

- definition of the approach to be pursued when a source of microvibration is acting on a spacecraft/supporting structure. The methodology gathers all the previously mentioned contributions to produce reliable and comprehensive predictions of the microvibration environment at specific locations on the spacecraft/supporting structure over the frequency and speed ranges of interest (discussed in chapter 6).

Part of the work presented in this thesis has been published and discussed in the following journal publications:


and presented in 2 conferences (Addari et al., 2014a) and (Addari et al., 2014b).

1.4 OUTLINE OF THE THESIS

The contents of this thesis are illustrated in the flowchart in Figure 1.5.

In chapter 2 a literature review on the microvibration issues is presented. The common RWA microvibration analysis methods are introduced and flaws in their practical application discussed. In particular, issues in the derivation of the RWA dynamic mass in the device full speed range of operation are outlined. The dynamic interaction between a source and a spacecraft is also investigated.

In chapter 3 the cantilever-configured RWA mathematical model is developed by means of an energy method. The Equations of Motion (EoM) are derived including the excitation due to flywheel mass imbalance, imperfections in the motor bearing system and broadband noise. Various boundary conditions in which the RWA operates are considered.

In chapter 4, the microvibration measurement procedure is defined and tests are carried out to characterise the RWA in both the hard-mounted and free-free boundary conditions. The experimental data is used to build a trustworthy RWA analytical model based on the EoM formulated in chapter 3. Processes for the simulation of harmonic and broadband excitations are also introduced.

In chapter 5 the issue concerning the retrieving of the RWA dynamic mass is tackled by the introduction of an innovative dynamic mass measurement procedure. RWA dynamic mass measurements were conducted in the full range of the RWA operative speeds, as to provide a comprehensive characterisation of the RWA internal dynamics including the gyroscopic effect. The iterative process is outlined and guidelines to improve the accuracy of the results presented.

In chapter 6 the dynamic interaction between a source and a supporting structure is examined through an extensive test campaign where the RWA was em-
bedded onto a FFSM at first and, subsequently, on a honeycomb panel. The computed numerical predictions are estimated with the use of the coupling theory, which was extended to include the gyroscopic effect in the RWA internal dynamics, and compared to the experimental outcomes.

In chapter 7 conclusions of the work presented in this thesis are summarised and possible future work is finally inferred.
Figure 1.5: Flowchart showing the thesis layout
CHAPTER 2

LITERATURE REVIEW

This chapter aims to describe the issues related to microvibrations, in particular emphasising on the state-of-the-art of the methods currently used to characterise the disturbances generated by the on-board mechanisms and how the internal dynamics of the microvibration sources are implemented to fully represent the interaction with a spacecraft structure in a microvibration environment. Moreover, a review of the analysis methods to compute predictions in all the spectra where microvibrations act is given.

2.1 HISTORY OF MICROVIBRATIONS ON SATELLITES

The importance of microvibrations in satellite engineering, and in particular the desire to develop methods to deal effectively with this phenomenon, has started, roughly, in the early 1980s, mainly triggered by the requirements of the Hubble Space Telescope (HST) project. Since then, the interest in microvibration analysis has risen as consequence of modern space systems, in particular those carrying the most advanced optical instruments, demanding extreme stringent pointing requirements and platform stability which pose challenging environment for the control and mitigation of microvibrations on-board satellite. Figure 2.1 illustrates the improved sharpness of images as result of increased angular resolution of optical payloads on some typical remote sensing satellites launched between 1970 and 2010.

A few examples of missions concerned by microvibration issues on the last three decades are listed below:

- HST: launched in 1990 as a collaboration between NASA and ESA, the HST featured the most demanding line of sight jitter requirements ever associated
Figure 2.1: Angular resolution of remote sensing satellite from 1970 to 2010 (Toyoshima et al., 2003). Note the Hubble Space Telescope has been added to the original figure.

with a spacecraft pointing system. For instance, the image stability for periods as short as 10 seconds and up to 24 hours would not exceed the 0.007 arcseconds RMS (Root Mean Square) requirement (Blair & Vadlamudi, 1988; Davis et al., 1986; Hasha, 1987);

- Solar and Heliospheric Observatory (SOHO): launched in 1995, SOHO offered the first full coverage of the coronal mass ejections of the sun, providing dramatic information on the effect the coronal mass ejections have on the technological world. The Attitude on-Orbit Control System (AOCS) objective was to limit the peak dynamic jitter as low as 0.5 arcseconds. The SOHO structural modelling, validation and testing including the identification of the main disturbance sources and the prediction of the peak dynamic jitter were carried out both at unit and spacecraft levels (Laurens et al., 1997a);

- Optical Inter-Orbit Communications Engineering Test Satellite (OICETS): The first bi-directional optical link used for both data and command transmission. The Acquisition, Tracking and Pointing System (ATPS) had to control a laser beam, traveling from the ARTEMIS payload OPALE (Optical Payload for Intersatellite Link Experiment) at distance of more than 36000 km, with an angular accuracy of a few micro radians. In order to assess the microvibration environment and verify the ATPS performance,
an on-ground microvibration test was carried out suspending the OICETS satellite as to represent a free-free boundary condition. Results showed an incremental residual tracking error smaller than 0.2 µ-radians (Jono et al., 2002; Stark & Stavrinidis, 1994; Toyoshima et al., 2003, 2010);

- Solar-C: it represents the next generation of Japanese solar physics satellite, in collaboration with United States and United Kingdom, following the predecessor Hinode (Solar-B) solar mission which brought unprecedentedly high quality observations of the Sun. The satellite carried telescopes whose pointing resolution was lower than 1/100,000 degree. In order to achieve this performance, a systematic approach was developed, including sophisticated microvibration transmissibility (from the source of microvibrations to the telescopes) tests as well as accurate measurements of the disturbance levels (Katsukawa et al., 2010; Takahara et al., 2004, 2006; Yoshida et al., 2004);

- Terrestrial Planet Finder (TPF): this mission aimed to detect habitable planets and life beyond Earth within the habitable zone of stars. In order to spot extra-solar planets, the on-board instrument required a contrast ratio stability of $2.0e-11$ which posed a stringent specification of 4 milli-arcseconds pointing stability and 5 nm jitter of the optics under mechanical disturbances. The mission has been recently cancelled. (Blaurock et al., 2005; Dewell et al., 2005);

- James Webb Space Telescope (JWST): expected to be launched in 2018, this is the largest cryogenic (40K) space telescope ever built and is planned to carry near and mid-infrared (1 µm - 10 µm) instruments for imaging and spectroscopy. It will require structures that preserve an out of plane level of stability in the order of 30 nm under dynamic and thermal loading while functioning at cryogenic temperature (Bagnasco et al., 2012; Clampin, 2012; Hyde et al., 2004; Reynolds et al., 2004);

- SPOT 4: launched in 1998 and active till 2013, SPOT-4 is considered a 2nd generation SPOT-series satellite of CNES, France. Among the different payloads on-board the satellite, PASTEC (un PASsager TECHnologique de SPOT 4) was the most important in terms of microvibrations. It was a technology demonstration passenger payload to study the orbital environment and carries two instruments, MEDY and MicroMEDY, which aim was to measure the satellite dynamics and characterise the in-flight vibration and microvibration environments, with a resolution up to 10 µg. The in-orbit data showed that resonance frequencies are slightly higher than those mea-
sured on ground whereas damping ratios are lower, in particular in the range of the first global modes (Le Duigou, 1998).

A significant amount of other missions where microvibrations have been an issue can be found in the literature: the SPOT family (Betermier et al., 1992; De Gaujac et al., 1991; Le Duigou, 1998), OLYMPUS (Dyne et al., 1993; Tunbridge, 1993), GOCE (Pavarin et al., 2008; Wacker et al., 2005), Solar Dynamics Observatory (SDO) (Blaurock et al., 2008; Lemen et al., 2012), Space Interferometry Mission (SIM) (Grogan & Laskin, 1998; Miller et al., 2001; Neat, 2003), STEREO (Eyles et al., 2009). At present, also the low cost end of the market, such as the microsatellites SSTL300-S1 (Richardson et al., 2014), Skybox (Desmet et al., 2012), GeoEye (Podger, 2012) and WorldView (Padwick et al., 2010), display challenging high-sensitive vibration instruments (i.e. cameras with a ground resolution toward or less than 1 meter). The line-of-sight of the instrument may be severely affected due to micro-displacements of the equipment mounting interface and therefore, a dramatic reduction of the instrument performance may occur, as depicted in Figure 2.2.

![Diagram of macro-oscillations](image)

Figure 2.2: Macro-oscillations at the target area produced by micro-fluctuation of the instrument mounting interface

### 2.2 Open Issues in Microvibration Characterisation, Analysis and Control

In chapter 1, the issues related to microvibration management were divided in three main areas of study: characterisation, analysis and control/mitigation. In
this section, a brief description of the state of the art in each subject is provided, highlighting the areas where improvements and further research are required.

2.2.1 Characterisation

The complete modelling of a source of microvibration relies on the understanding of the mechanism’s internal dynamics and the detailed description of its induced disturbances. The modelling of a device which introduces vibrational energy to a system (i.e. spacecraft) is typically derived from two different approaches: the analytical approach and the empirical approach. de Weck & Miller (1999) provided a definition of the two different procedures:

- analytical approach: the source of microvibration is physically modelled assigning realistic values to the representative parameters of the model;
- empirical approach: the source of microvibration is generated analysing experimental test data from which the dynamic disturbances of the source are obtained.

For microvibration analysis at satellite system level, that is to accurately describe the interaction of the source with the spacecraft structure, the two approaches need to be combined and refer one to the other. This methodology is known as the semi-empirical approach; a schematic representation of the procedure is illustrated in Figure 2.3.

This methodology has been widely accepted in the space industry and is largely used in source microvibration analysis. Masterson et al. (1999) showed the development of analytical and empirical models of the induced disturbances generated by a mid-span configured RWA in a hard-mounted boundary condition. Subsequently, the harmonic excitation parameters were extracted from test results and these were then implemented in the analytical model, allowing the construction of a more representative and more realistic semi-empirical model (Masterson et al., 2002).

A traditional approach to evaluate the mechanical source-induced microvibration disturbances is to mount the mechanism on a dynamometer (e.g. Kistler table) and measure the forces transmitted at the interface. This configuration is analytically reproduced by connecting the source to an infinitely rigid support (i.e. hard-mounted boundary condition, sometime also referred to as "blocked"). The outcomes are subsequently applied to the spacecraft supporting structure. In practice, however, the hard-mounted boundary condition is not representative of the connection between the source and its supporting structure on a spacecraft,
which is a flexible body hence with well defined internal dynamics. In addition, when a source is mounted on a spacecraft, its disturbances excite the spacecraft structure, which in turn excites the source itself, and so forth, producing coupled dynamics between the two flexible bodies (Elias, 2001; Elias et al., 2003).

An adequate mathematical model to represent a microvibration source is thereby necessary to reproduce the coupled dynamics when the source is mounted on its supporting structure. Masterson et al. (1999) observed that the coupled dynamics strongly depend on the internal dynamics of the source, which can be represented in terms of dynamic mass (or apparent mass, or its inverse, the accelerance). As defined by Ewins (2003), the dynamic mass is a complex, frequency dependent ratio between a load (force or moment) imparted on a body and its resulting co-located acceleration (linear or angular). Zhang et al. (2012) and Zhang et al. (2013) developed a procedure to experimentally measure the source mass, stiffness and damping properties of a RWA, also including the gyroscopic effect. The method proved correct when the mechanism was not operating whereas poor results were obtained when the device was set in motion. The complexity of the test configuration, shown in Figure 2.4, and the eventual interaction between the loads applied by the mini-shakers and those produced by the functioning of the RWA were identified as the main causes for the unsatisfactory outcomes.

A large number of projects have been conducted on RWA-induced microvibrations and on their influence on the satellite payload performance analysis. However, the development of higher performance payload (thus stringent structure...
stability) pushes further RWA designs toward low microvibration emissions and improvements in modeling capabilities to reliably simulate the dynamics of the physical hardware. RWA-induced microvibrations and characterisation of their internal dynamics are the main concerns in this work. A substantial review of the research conducted towards their complete modelling will be presented in the rest of this chapter.

2.2.2 Frequency-based Structural Analysis Methodologies

With reference to the frequency range within which microvibrations may occur, the common practice is to distinguish three different frequency areas:

- Low-frequency range: it includes the first few modes of the structure
- High-frequency range: region where high modal density is displayed
- Mid-frequency range: everything amidst the low and high frequencies

The FE analysis is a well-established, and industry accepted, structural analysis tool and a considerable amount of literature can be found (Budde et al., 1980;
Kamesh et al., 2010; Zienkiewicz et al., 2005). Nevertheless, it is able to provide good predictions only in the low-frequency range, where the first few resonances are recorded. At high frequencies, finer meshes are required and this would lead to large and unnecessary computational effort. In addition, Kompella & Bernhard (1993) have observed and demonstrated that even structures resembling the same line process, may present dissimilar dynamic response characteristics as frequency increases. The same was observed during the SSTL Rapid-Eye test campaign, where five nominally identical spacecraft displayed dissimilar behaviour, as shown in Figure 2.5.

![Figure 2.5: Frequency response of the five spacecraft of the Rapid-Eye constellation (Remedia et al., 2015c)](image)

For these reasons, at high frequencies, where the structure experiences a high modal density (that is a large number of modes actually participate in the response), stochastic approaches, rather than deterministic, are more suitable for the representation of the real behaviour of a structure. In this context, the Statistical Energy Analysis (SEA) has been successfully used. SEA formulation describes a complex structural assembly as a network of subsystems that exchange energy. Each subsystem is described in terms of vibrational energy and various methods have been developed in order to evaluate it (Cremer & Heckl, 1988; Lyon & De-Jong, 1995). The literature offers numerous missions where the SEA method was applied, displaying good agreement with the experimental results (Hwang, 2002; Larko & Hughes, 2008). Although both FE analysis and SEA methods are reliable where specified assumptions are met, their application in the mid-frequency range provides poor results. Therefore, different analysis techniques must be applied in
order to obtain acceptable predictions. One of the methods used to address the
problem in the mid-frequency range is the hybrid FE-SEA method, where some of
the structural components are treated as FE models whereas others as SEA meth-
ods (Shorter & Langley, 2005a,b). In order to include the uncertainties about
stiffness and material properties in the modelling of spacecraft structures, the
Stochastic Finite Element Method (SFEM) represents the most powerful tool to
describe the behaviour of a structure in the mid-frequency range (Stefanou, 2009).
In particular, the Full Monte Carlo Simulation (FMCS) is the simplest method
for treating the response variability calculation in the framework of SFEM. This,
however, requires a significant amount of computational effort and therefore, a
reduction method which gives results as accurate as the FMCS but at a fraction
of the computational time is implemented. Remedia et al. (2015c) proposed a
variant to the component mode synthesis for the computation of the transfer func-
tions between source and receivers where the spacecraft structure is divided in a
series of subsystems which are reduced using the Craig-Bampton method; the nat-
ural frequencies and the modal participation factors of the reduced subsystems are
then randomised according to the uncertainties relative to that subsytem. The
approach was subsequently applied to the satellite SSTL-300-S1 showing good
predictions and correlations with the test results, as depicted in Figure 2.6.

![Figure 2.6: Comparison of the FE analysis prediction with test results applying the Craig-Bampton method (Remedia et al., 2015b)](image-url)
2.2.3 Control and Mitigation

The last step in the management of microvibration emission is represented by the development of a methodology to control and limit the source-induced disturbances acting on the payload of a spacecraft. Generally, microvibrations cannot be controlled or reduced by the satellite AOCS because it has an upper controllable frequency limit of a few Hz, whereas microvibrations generally occur at higher frequencies (Oh et al., 2006). For this reason, a considerable amount of damping solutions have been proposed throughout the years. The most common solution lies in the use of passive dampers, which are characterised by functioning without the application of a power supply. In this category, due to their low cost, lightweight and simplicity, visco-elastic dampers typically represent the first option chosen by space companies. Nevertheless, these are limited by their mechanical and thermal properties (Demerville, 2013; Richardson et al., 2014). Other passive dampers used for space applications are: D-Struts (Davis et al., 1995) and shunted piezoelectric transducers (Hagood & von Flotow, 1991; Moheimani, 2003). To overcome the limitations of passive dampers, active dampers represent a valuable solution and have been widely used for space applications (e.g. voice coil actuators are generally installed in unidirectional strut and then implemented in a six strut configuration (Agrawal, 2009), or the application of piezoelectric patches attached to the structure where an electric field is applied to produce a stretching or shrinking of the patches which counteract the vibrational modes of the structure (Aglietti et al., 1997, 2000)). These devices display significantly high attenuation performance and allow full control of the damper but, on the other hand, require considerable amount of power and may also introduce instabilities. Moreover, Shimizu et al. (2008) proved that these methodologies are not sufficient yet in thoroughly isolating the payloads from the spacecraft structure. Thereby, current research aims to investigate the mechanical coupling between sources and spacecraft bus in order to improve the design of damping devices and minimise payload pointing errors (Yoshida, 2011).

2.3 Microvibration Sources

The identification process of potential microvibration sources typically occurs during the assessment of the performance requirements in the system design phase. The microvibration disturbances acting in orbit on a spacecraft can be arranged into external and internal perturbations. The former originates from the interaction of the spacecraft with the space environment including atmospheric drag, vari-
ations of the Earth gravity and magnetic fields, etc. (Dyne et al., 1993; Shangchun & Cao, 2012). These disturbances are generally characterised by continuous quasi-steady fluctuations (that is the duration of a cycle could be hours). In addition, micro-meteoroids and debris impacts are other possible external microvibration sources where the physics of the disturbance is an intermittent transient structural vibration (ECSS, 2013).

Internal disturbances, on the other hand, derive from devices embedded in the spacecraft. These include the AOCS, the propulsion system, the structure system, etc. which produce more important disturbances due to their micro-g amplitude and broadband nature (Zhao, 2006). These internal perturbations are typically generated by the operating of fast moving mechanisms such as RWAs and MWAs (Bialke, 2011; Fujita et al., 2002), control moment gyros (Luo et al., 2013), pointing systems (Blaurock et al., 2008; Bourkland et al., 2007) and cryo-coolers (Clapp et al., 2002; Tomaru et al., 2004). Moreover, sudden stress release due to thermal clank phenomena of structural joints and sensors (Ingham et al., 2000; Kim & McManus, 2001), sensor electrical noise (Comolli & Saggin, 2010) and the crackling/buckling of multi-layer insulation due to thermal heating cycles in the eclipse entry and exit process can also introduce microvibration disturbances (Deutsch & Grillenbeck, 2008).

A further classification of the disturbance sources takes into account whether the perturbations are harmonic or transient in nature. The disturbances produced by harmonic sources generally evolve for longer periods of time. For instance, the load spectrum of a RWA consists of several sinusoidal signals whose frequencies are multiples or fractions of the fundamental harmonic. However, it should be noted that both higher and lower frequency harmonics are not always integer multiples of the fundamental one. Laurens & Decoux (1997a) provided analytical expressions to identify the harmonic fraction at which a disturbance due to bearing imperfection occurs.

External disturbances are influential in the multi-body dynamics region and can be counterbalanced by the AOCS (Oh et al., 2006), whereas internal disturbances are dominant as the frequency increases. Nevertheless, in some instances, the disturbances generated by other mechanisms such as antenna pointing mechanisms, solar drivers and cryo-coolers can prevail over those produced by RWAs (Bailllon & Valli, 1996; Laurens et al., 1996; Sudey & Schulman, 1985). A list of typical internal microvibration sources on-board a modern high-stability spacecraft is presented in Table 2.1.
Table 2.1: Typical internal microvibration sources on a modern high-stability spacecraft

<table>
<thead>
<tr>
<th>Sub-system</th>
<th>Source</th>
<th>Physics</th>
<th>Signal Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avionics</td>
<td>RWA/MWA</td>
<td>mechanical vibration</td>
<td>continuous</td>
</tr>
<tr>
<td></td>
<td>magnetic torquer</td>
<td>thermal clank</td>
<td>single event</td>
</tr>
<tr>
<td></td>
<td>star/earth/sun sensors</td>
<td>clank phenomena</td>
<td>continuous</td>
</tr>
<tr>
<td></td>
<td>thruster</td>
<td>structural vibration</td>
<td>continuous</td>
</tr>
<tr>
<td></td>
<td>regulator/switch</td>
<td>mechanical vibration</td>
<td>continuous</td>
</tr>
<tr>
<td></td>
<td>tank storage</td>
<td>thermal clanks</td>
<td>continuous</td>
</tr>
<tr>
<td></td>
<td>pipe work</td>
<td>thermal clanks</td>
<td>single event</td>
</tr>
<tr>
<td>Structure</td>
<td>structural joints</td>
<td>thermal cycles</td>
<td>single event</td>
</tr>
<tr>
<td></td>
<td>sandwich panels</td>
<td>thermal cycle</td>
<td>single event</td>
</tr>
<tr>
<td>Thermal Control</td>
<td>heaters</td>
<td>electro-magnetic force</td>
<td>continuous</td>
</tr>
<tr>
<td></td>
<td>cryo-cooler</td>
<td>mechanical vibration</td>
<td>continuous</td>
</tr>
<tr>
<td></td>
<td>multi-layer insulation</td>
<td>crackling/buckling</td>
<td>single event</td>
</tr>
<tr>
<td>Communication</td>
<td>antenna pointing</td>
<td>mechanical vibration</td>
<td>single event</td>
</tr>
<tr>
<td></td>
<td>mechanism</td>
<td>thermal clanks</td>
<td>single event</td>
</tr>
<tr>
<td>Power system</td>
<td>solar array</td>
<td>mechanical vibration</td>
<td>continuous</td>
</tr>
<tr>
<td></td>
<td>harness</td>
<td>electro-magnetic force</td>
<td>continuous</td>
</tr>
<tr>
<td>Other</td>
<td>shutter (optics)</td>
<td>mechanical vibration</td>
<td>single event</td>
</tr>
<tr>
<td></td>
<td>electrical noise</td>
<td>structural vibration</td>
<td>continuous</td>
</tr>
<tr>
<td></td>
<td>instruments</td>
<td>mechanical vibration</td>
<td>continuous</td>
</tr>
<tr>
<td></td>
<td>(receivers)</td>
<td>thermal clanks</td>
<td>single event</td>
</tr>
</tbody>
</table>
2.4 RWA Dynamics and Analysis

Initially in chapter 1 and subsequently in section 2.3, the disturbances induced by RWAs were regarded as commonly the most significant and therefore, they have received the majority of attention throughout the years. For this reason, in this thesis, a relevant literature review is dedicated to these mechanisms. The fundamentals of RWAs and the microvibration analysis methodologies are introduced. The theoretical models and investigation approaches of previous RWAs are presented. In addition, the open issues and the difficulties encountered in the current study techniques are discussed. Finally, the importance of the coupling effect between a RWA and its supporting structure is also discussed.

2.4.1 Reaction Wheel Assemblies

Reaction wheel assemblies are mainly used for spacecraft control attitude, vibration compensation and orientation of solar arrays (Fukuda et al., 1986). RWAs operate on the principle of conservation of angular momentum of a closed system; by accelerating about one axis, reaction wheels force a spacecraft to rotate in the opposite direction about the same axis, thus conserving the total angular momentum of the system. Typical RWAs consist of a rotating flywheel mounted on a shaft supported by bearings (mechanical or magnetic) and driven by a brushless DC motor. Consequently, the sub-assembly is encased in a housing. Generally, the flywheel’s mass is concentrated at the outer edge of its diameter, maximizing its mass moment of inertia about the spin axis and providing sufficient torque authority over the spacecraft. Common flywheel configurations of a RWA are either symmetrical (flywheel at mid-span of the shaft) or cantilevered (flywheel at one end of the shaft), as illustrated in Figure 2.7. Although the sub-systems are different, they evince similar dynamic behaviour, except that the two flexural modes (lateral rocking and translational) are coupled for the cantilevered type, but well defined and separate for the symmetrical design (Bialke, 1996).

Nominally, RWAs have zero speed and may be accelerated or decelerated either forward or backward, predominantly up to 4000 rpm, to generate reactive torques utilised, mainly, to control the attitude of the satellite. MWAs generally spin at a high mean speed (typically between 5000 and 10000 rpm) to produce momentum bias and stability to the spacecraft (Fortescue et al., 2003). Both wheel types are often used in conjunction with external torquers, such as thrusters, and are especially useful when the spacecraft needs to be rotated by fractions of a degree and to maintain pointing without the consumption of on-board propellant (Kennedy,
1962). Note that the common practice is to install a combination of three or four RWAs on the spacecraft, thereby their relative position may also influence the vibrations transmitted to the spacecraft (Fortescue et al., 2003). In this thesis, microvibrations induced by a single RWA are considered.

2.4.2 Nature of RWA Disturbances

In Table 2.1, the microvibrations induced by a RWA are classified as harmonic and continuous occurring at well defined ratios of the angular frequency of a spinning flywheel (Elias & Miller, 2002). Wang & Hu (2009) and Bialke (2011) distinguished the disturbances as function of their nature and described them whether they depended on the flywheel mass imbalance, the bearing irregularities or motor imperfections. Moreover, Liu et al. (2008) observed that a low level broadband vibrational spectrum is always present.

Imbalance is a condition where a rotor inertia axis does not coincide with the rotor axis of rotation. The imbalance can be broken down in static imbalance and dynamic imbalance:

- static imbalance: the axes are parallel but the wheel’s centre of gravity is misaligned with respect to the spin axis (e.g. caused by a point mass at a certain radius from the centre of rotation, as shown in Figure 2.8(a)). The resulting disturbance is a radial force on the wheel, produced by the centripetal force acting on the spinning imbalance. Because the imbalance is fixed to the rotating frame of the wheel, the force appears sinusoidal in a fixed reference frame. Static imbalance involves resolving primary forces into one plane and adding a correction mass in that plane only.
dynamic imbalance: the axes cross each other creating a misalignment of the flywheel’s principal axis and the rotation axis (e.g. two equal masses placed symmetrically about the centre of gravity but positioned at 180 degrees from each other, as illustrated in Figure 2.8(b)). The rotor can be in static balance, i.e. no eccentricity of the centre of gravity, but when the rotor turns it creates a rotating moment. In Figure 2.8(b), the dynamic imbalance is illustrated as two lumped masses placed opposite each other radially and at an axial offset from each. Similar to the static imbalance force, the moment caused by a dynamic imbalance appears sinusoidal from a fixed reference frame. The imbalance can be corrected by taking vibration measurements with the rotor spinning and adding correction masses in two planes.

Note that static and dynamic imbalances might be present together generating disturbance forces and moments.

Disturbances originating from irregularities in the bearing geometry or malfunction of the motor can induce an abnormal level of the noise produced by the RWA. Following, a series of bearing and motor related issues which may lead to RWA microvibration disturbances is given:

- disturbances caused by the RWA motor bearings depend on the wheel technology, i.e. whether conventional ball bearings or highly sophisticated magnetic bearings are considered (Morales et al., 2003). For conventional mechanical ball bearings wheels, irregularities in the ball, races and cages lead to off-design contact between the components thus creating non-linear force and torque disturbances (Harsha & Kankar, 2004; Sinou, 2009) which occur
at sub- and higher-harmonics of the flywheel’s angular speed (Laurens & Decoux, 1997b);

• lubrication degradation over life can induce low-frequency perturbations and increased noise (Bialke, 2011);

• motor cogging and torque ripples manifest in permanent magnet brushless DC motors generating disturbances especially at low speeds of rotation or when the motor reverses spin direction. The former arises from the magnetic interaction between stator slots and rotor permanent magnets, affecting the smooth rotation of the rotor and leading to an increment in the noise level. The latter refers to the change in motor torque with respect to the angular position (Bialke, 2011);

• although bearing friction is present in the entire operational spectrum of a rotating mechanism, its importance is mainly relevant at low speed. For instance, when the direction of spin of a RWA requires to be changed, a disturbance can be generated by a discontinuity in acceleration due to the change in the relative signs of the friction and motor torque (Bialke, 2011).

The harmonic and broadband excitations can be linearly superimposed to allow the generation of the complete RWA input model and the prediction of RWA-induced microvibrations taking into account also the RWA internal dynamics (Liu et al., 2008).

2.4.3 RWA Structural Modes

The structural modes of a RWA depend largely on the configuration of the RWA. Typically, torque vibrations are ignored due to the flywheel angular speed domination, i.e. the flywheel spin speed driven by the motor is significantly larger than the perturbation angular speed in the torque DoF. Subsequently, a RWA can be described as a five Degrees of Freedom (DoFs) system which consists of one single DoF in axial translation, two DoFs in each radial (or in-plane) translations and two DoFs in each radial (or out-of-plane) rotations. This leads to five dominant structural modes which, due to symmetry about the axial axis, reduce to three modes referred to as the axial translation mode, the radial translational mode (here also referred to as lateral) and the radial rotational (here also referred to as rocking) mode, respectively. The structural modes for a mid-span-configured RWA and a cantilever-configured RWA are illustrated in Figures 2.9 and 2.10, respectively. The three structural modes of an axisymmetric RWA are decoupled
from each other. In contrast, for a cantilever RWA configuration, although the axial translational mode remains uncoupled, the two radial modes are combined into a mixed radial mode in each radial DoF; thereby, it is no longer possible to refer to them separately (Genta, 2005).

The structural modes of a RWA can be influenced by the speed at which the flywheel spins. For a mid-span-configured RWA, the rocking mode splits in two whirls as speed increases due to the gyroscopic precession of the spinning flywheel (Brar & Bansal, 2004). The initial resonance frequency (measured in static condition, that is zero rpm) diverges in a backward (or precession) whirl and a forward (or nutation) whirl. The former decreases in frequency as speed increases whereas the latter grows as speed increases (Muszynska, 2005; Swanson et al., 2005). On the contrary, both the axial and the radial translational modes are speed-independent hence the natural frequencies remain constant.

On the other hand, due to the combined radial translational and rotational modes, the structural dynamics of a cantilever-configured RWA are significantly affected by the gyroscopic effect. Similarly to the mid-span case, the frequencies associated with the radial translational and rotational modes split in backward and forward whirls generating a total of four speed-dependent structural modes whilst the axial translational mode continues to be uncoupled and speed-independent.

A graphical representation of the behaviour of the structural modes of a RWA as function of both frequency and speed is given in Figure 2.11 in terms of a Campbell diagram. Note that the frequency drift (how the structural mode evolves with speed) strongly depends on the ratio between the polar and transverse moments of inertia of the rotor hence on its geometry and shape (Yoon et al., 2013). In practice, the coupled motions in radial DoFs make the overall modeling process
Figure 2.10: Structural modes of a cantilever-configured RWA: (a) axial and (b) combined radial translational and rotational

of a cantilever-configured RWA considerably more complicated than that for a conventional mid-span-configured RWA.

2.4.4 TRADITIONAL RWA MICROVIBRATION ANALYSIS METHODOLOGIES

Preliminary studies on RWA microvibration modelling principally aimed to analyse the RWA harmonic characteristics adopting an empirical approach (Bosgra & Prins, 1982; Hasha, 1986; NASA, 1976). A first model was proposed by Melody (1995), who described the disturbances as a series of discrete and superimposed harmonics. The microvibrations induced by flywheel mass imbalance, bearing and motor local imperfections were subsequently identified and modelled. For instance, Bialke (1996, 1997) provided microvibration test results for the ITHACO family RWAs including bearing stiction and motor torque ripple. Similar works were carried out by Laurens & Decoux (1997a,b) and Laurens et al. (1997b) on a different RWA. It was observed that the disturbances due to bearing local imperfections (e.g. waviness of either inner or outer race) occur at a fraction of the fundamental harmonic (frequency at which the flywheel is spinning). In similar works, Harsha & Kankar (2004) and Harsha (2005, 2006) provided a detailed model of the bearing irregularities as function of race waviness and number of balls including their non-linear effects. Time domain expressions of the harmonic response in closed-form in radial DoFs were published by Oh & Rhee (2002) and Li & Dai (2005). The latter also included friction in the bearing system. Subsequently, the works were extended including an energy compensation method to improve the test accuracy and a semi-empirical model was developed (Sun et al., 2006; Zhao, 2006). The harmonic response in closed-form was derived for each DoF and its simulation
LITERATURE REVIEW

Figure 2.11: Campbell diagram of a RWA: (a) mid-span configured RWA and (b) cantilever configured RWA
presented in the frequency domain by Kim et al. (2010) and Shin et al. (2010). If the disturbance amplification due to their interaction with the structure natural modes is the dominating effect on the attitude accuracy of a spacecraft, amplification coefficients can be introduced to represent the effect of the modal frequency of the structure to the RWA-induced disturbance avoiding the derivation of an analytical expression of the EoM (Zhang et al., 2010). The dynamic behaviour of a cantilever-configured RWA using an unconventional soft-suspension system was published by Zhang & Aglietti (2011). The EoMs in each DoF were derived and validated using an opportually designed measurement system. Finally, it was also concluded that this configuration displays considerably lower emissions at high frequency compared to typical rigidly supported RWAs.

Issues related to RWA broadband excitation modeling were first examined by Liu et al. (2008), and it was observed that a pure analytical model would not be able to accurately reproduce a broadband noise. Nevertheless, it can be assumed that the broadband perturbations as function of speed can be expressed in a polynomial form combining both an analytical shape function of vibration and experimental data (Blaurock, 2009). In addition, a method for the modelling of the broadband noise from a raw signal was developed by (Zhang et al., 2014). The approach is based on the significant difference in frequency spectrum amplitude of the broadband excitations compared to that associated with either the fundamental harmonic or fractions of it. The method involved the application of an energy variation approach to identify spikes in the original signal and the design of band-stop filters to remove them.

A relevant aspect concerning the microvibration analysis is also the development of dedicated measurement platforms and test methods (Santiago-Prowald et al., 1998). The relevant factors to take into account when carrying out a test campaign were identified by Collins (1996) whose conclusion was “The basic requirement of any test is to measure exactly what is required and nothing else”. Generally speaking, the most common approaches to evaluate the interface loads produced by a disturbance source (Laurens & Dupuis, 1995) are:

- direct force measurements: the forces and torques are taken at the interface between the specimen and a rigid and massive foundation. The main advantages of this method lie in its simplicity and adaptability;

- indirect computation from combined acceleration and force measurements: the foundation is replaced by a previously measured impedance of the specimen supporting structure in the post-processing analysis. This allows obtaining more representative boundary conditions (i.e. on-orbit configuration) at
Figure 2.12: Spike identification and broadband excitation derivation: (a) raw and filter signal and (b) spike identification (Zhang et al., 2014)
the cost, however, of complex and demanding preliminary impedance measurements.

Numerous test benches based on the direct approach were developed in the 90s and air-spring suspended marbles were introduced in place of the rigid foundation (Eckert et al., 1993; Laurens & Guillaud, 1994). Similar works by Dupuis et al. (1996) and Galeazzi et al. (1996) should also be noted. The characterisation of a typical RWA ball bearing system at low frequencies was successfully accomplished by using an opportune designed air floating disturbance detector (Taniwaki & Kanazawa, 2001; Taniwaki & Ohkami, 2003; Taniwaki et al., 2007). Custom vibration measurement platforms have nowadays the majority of attention as they allow creating adaptable and portable systems. For instance, Heimel (2011) published a complete harmonic analysis of a RWA using a customised test bench. Furthermore, ESA developed a test facility which main functional requirements were to be compatible with the variety of RWAs manufactured in Europe and to be able to detect the six DoFs of the induced interface forces and moments with amplitudes between 10 mN to 200 N and 2 mNm to 20 Nm, respectively, in a range of frequency spacing from 5 Hz to 1000 Hz (Wagner et al., 2012). The concept was based on the operative principle of a Kistler table and was composed of four three-DoF load cell units connected on one side to an interface table where the specimen would be mounted and on the other to a ground-isolation system, as shown in Figure 2.13. The measurement test bench used in this study was presented by Zhang et al. (2012), with the requirement to develop a non-expensive, compact and simple platform yet able to provide a good level of accuracy.

In a typical RWA microvibration analysis, the loads produced by a spinning flywheel are assessed in a hard-mounted boundary condition, in which the RWA is mounted and secured on a rigid surface, opportunely isolated from any disturbance generated by the ground, and its interface is restrained to prevent any motion. As the flywheel spins, a set of load cells measure the resulting loads at the interface. For instance, Figure 2.13 illustrates the test facility used at ESA for the characterisation of wheel assemblies-induced disturbances. The dynamic models developed under such boundary condition are called the RWA hard-mounted microvibration models.

The current practice in the space industry is to use the resulting data from the hard-mounted configuration measurements as direct inputs for the satellite microvibration analysis (Liu et al., 2008; Ponslet, 2000). This approach is, however, flawed, unless the wheel is actually modelled, due to the poor representation that the hard-mounted boundary condition provides of the real environment in which the RWAs will operate when installed on a spacecraft. Therefore, with the
purpose to improve the quality of the predictions of the actual loads at the RWA-structure interface, RWA-structure coupled analysis methodologies require to be implemented.

2.4.5 RWA Dynamic Coupling

As already stated in section 2.2.1, the dynamics when a RWA is assembled with its supporting structure significantly differ from those observed in a hard-mounted boundary condition. Studies conducted by Elias et al. (2003) showed that the vibrations of a spacecraft structure, induced by the operating of a microvibration source, perturb the microvibration source itself thus creating coupled effects. Therefore, the hard-mounted model is not a valid representation of the in-orbit dynamics and alternative methods, which would provide a trustworthy prediction of the coupled dynamics occurring when a microvibration source acts on a spacecraft structure, require to be investigated.

The modern practice is to apply the loads experimentally measured in a hard-mounted configuration to the FE model of the spacecraft structure at the location where the source is installed, and representing the latter as a lumped mass (de Weck & Miller, 1999; Liu et al., 2008; Marucchi-Chierro & Galeazzi, 1995). Although this method is valid for sources whose modal frequencies are significantly higher than the frequency range of interest, it displays poor results when the dynamics of the source are defined in a range of frequency where the interac-
tion between the disturbance harmonics and the source resonances is important. Thereby, these “passive” effects (the presence of a source with its own dynamics will affect those of the spacecraft even if the source is not in operation) have to be considered and implemented together with the “active” effects (i.e. flywheel spinning and generating disturbances due to mass imbalance and bearing irregularities). The latter has been thoroughly discussed in section 2.4.4. The passive effects can be reproduced using either a detailed FE model of the source, although the modelling of rotary mechanisms such as RWAs can be quite challenging (Zhang & Aglietti, 2011; Zhang et al., 2012), or data from an experimental campaign.

A first attempt to model the RWA-structure dynamics was conducted by Masterson (1999), where both the RWA and the structure were considered as rigid bodies with internal flexibilities. Moreover, it was observed that the dynamic mass, or accelerance, at both RWA and structure driving points (location where the source and the structure interface with each other) varied as functions of frequency and were crucial parameters in the analysis of coupled microvibration dynamics. Based on this study, Elias (2001) and Elias & Miller (2002) elaborated an empirical methodology for the measurement of the RWA driving point in a zero-speed condition (e.g. flywheel not spinning, static condition). The test consisted of a six-DoF load cell for the measurement of the reaction loads and a series of accelerometers for the measurement of accelerations, all placed at the RWA driving point. The predicted coupled microvibrations including the RWA static accelerance displayed significant improvement with respect to the standard one. The gyroscopic effect, however, was not included in the RWA static accelerance hence unsatisfactory predictions were obtained over the speed spectrum. The issue was tackled by Elias et al. (2003) and Basdogan et al. (2007) and analytical expressions of the RWA accelerance in a dynamic condition (flywheel spinning at a constant speed different from zero) were derived from the RWA hard-mounted microvibration model. The formulation, however, only considered mass and inertia properties of the flywheel neglecting the stiffness and damping values of the suspension system. Coupled microvibration analyses were also conducted by Narayan et al. (2008) and Zhao et al. (2009), but for RWA platform design only.

All the aforementioned methods referred to a RWA hard-mounted microvibration model, however the RWA accelerance should be evaluated in a free-free boundary condition. In addition, stiffness and damping of the suspension system have a significant impact in the dynamic accelerance derivations and therefore should not be ignored. Zhang et al. (2013) developed an experimental method where the RWA was suspended using elastic cables in order to reproduce a free-free boundary condition (i.e. zero-g environment) and a set of eight accelerometers
was used to measure the response due to the application of unit forces and moments at the RWA driving point. The experiment was also conducted in a dynamic condition hence taking into account the gyroscopic effect. The outcomes showed a substantial improvement in the characterisation of the internal dynamics of RWAs and the subsequent implementation of the RWA dynamic accelerance, which also included the effect of both stiffness and damping of the suspension system, in the prediction of coupled microvibrations displayed an enhanced level of quality and correlation over the standard methods. Nevertheless, the complexity of the test configuration makes the application of this method a real challenge. Moreover, the accelerations measured at the RWA driving point in the dynamic condition are affected by the the disturbances associated with a spinning flywheel and, therefore, do not represent the effects of the RWA internal dynamics only. Thereby, a filter to erase these effects or an alternative methodology which restrains the effects of these disturbances need to be developed.

In chapter 13 of ECSS (2013), an example of the different approaches to estimate the structure response of a multi-body system due to the action of a microvibration source is described and the fundamental outcomes are reported here in terms of Figure 2.14 and comprehensively discussed in chapter 6 of this thesis. The exact response can be computed by solving the system EoMs. Predictions, on the other hand, can be calculated considering different approaches: source included in the system, source represented as a lumped mass and source internal dynamics included in the model. Among all the approaches, only the method which takes into account the source mass, stiffness and damping features is able to reproduce the exact response at the system output location.

2.5 Receivers

Microvibration analysis at spacecraft system level are often accomplished via an “integrated modelling and simulation”, where the potential internal and external sources are integrated in one framework and analysed in the time domain in a one-off computation (Briggs et al., 2006; Hyde et al., 2004). This approach allows the accurate simulation and prediction of the performance of payloads and/or sensitive instruments on-board a spacecraft in operative configuration (i.e. in-orbit operative configuration) and is considered to be the mainstream for future large space projects. An example of jitter analysis implementing an end-to-end analysis process is illustrated in Figure 2.15.

The “integrated modelling and simulation” is generally associated with the analysis of the dynamic performance for spacecraft requiring high stability which
Figure 2.14: Response at a receiver location from various implementation of the input (ECSS, 2013)

Figure 2.15: SDO jitter analysis using an integrate modelling approach (Liu et al., 2007)
can be either expressed in terms of pointing stability in arcseconds or instrument line of sight jitter, which is defined as the oscillation of an instrument’s line of sight over a specific period of time (Woodard, 1998). For remote sensing or space observation missions, the sensitive instruments are represented by the on-board telescopes and cameras, which for modern spacecraft may demand a platform pointing stability in the range of 0.05-1 arcseconds (Bagnasco et al., 2012; Liu et al., 2008; Podger, 2012; Takahara et al., 2006).

2.6 SUMMARY

An overview of the most severe sources of microvibration acting on a modern spacecraft has been given in this chapter. Among these, RWAs are considered one of the most important sources of microvibration and, as such, have received the majority of attention throughout the years. From the literature review, it is clear how the hard-mounted microvibration analysis is the most common approach for the characterisation of RWAs. This, however, has been shown to fail in reproducing the actual dynamics when a microvibration source is installed on a spacecraft. Thereby, in the last two decades, researchers have focussed on the investigation of the interaction between a RWA and its supporting structure, both numerically and experimentally. Methods to derive the source internal dynamics were also introduced. Nevertheless, the literature lacks a reliable and cost effective approach to compute the RWA dynamic mass over a wide range of frequencies and operative speeds, thus to include also the gyroscopic effect in the RWA characterisation. Therefore, a measurement procedure which allows the retrieval of the RWA dynamic mass, facilitating the test campaign and reducing the computational effort yet maintaining a high level of quality, is required. This would improve the understanding of the RWA behaviour in operative conditions and help in future spacecraft design and analysis.
RWA Analytical Model

In chapter 2, it was mentioned that the cantilever configured RWA can be considered as a multi DoF system with mass, stiffness and damping features whose motion can be described by specific EoM. In this chapter, the derivation process of the EoM, which define the dynamics of a cantilever configured RWA with two suspension systems when the flywheel presents mass imbalances, will be described, based on an energy (or Lagrangian) method. Under linear assumptions, analytical expressions of kinetic energy, potential energy and work done can be derived for the flywheel mass balanced case and subsequently for the mass imbalanced case. Finally, the linearised EoM of a mass imbalanced flywheel RWA are derived for both hard-mounted and free-free suspended RWA boundary conditions. In order to clarify the terminology adopted in this thesis, ”suspension” refers to the link between the flywheel and the rest of the mechanism, ”hard-mounted” and ”free-free” configurations refer to the boundary conditions in which the RWA operates, and in particular:

- hard-mounted is the case when a RWA is rigidly connected to the ”ground” with no motion at the RWA interface with the supporting structure;

- free-free indicates the configuration for which a RWA is hung with elastomers to represent a free floating situation.

In section 3.1, an introduction of the energy method will be given. This will be followed, in section 3.2, by the derivation of the kinetic energy, potential energy and work done associated with a cantilever configured RWA. Finally, the EoM for the hard-mounted and free-free boundary conditions will be defined in sections 3.3 and 3.4, respectively.
3.1 Energy Method

In this context, energy method refers to the series of energy principles in classical mechanics which provide the relationships between displacements, properties of the structure and structural loads by means of the energy or work done by internal and external forces. In particular, the principle of virtual displacements will be applied. Assume a mass balanced flywheel as a rigid disk whose Centre of Mass (CoM) O is free to rotate about any axes, and apply a conventional y-x-z rotation, as illustrated in Figure 3.1. Tait-Bryan angles (Krey & Owen, 2007) can be used to describe the rotations of a rigid body and to relate one coordinate frame to another. The definitions and notations used for Tait-Bryan angles are similar to those consider for classic Euler angles. The sole difference lies in that Tait-Bryan angles represent rotations about three distinct axes whereas classic Euler angles use the same axis for both the first and third elemental rotations.

![Figure 3.1: Tait-Bryan angles definition and transformations between each coordinate frame by means of a y-x-z rotation sequence](image)

In Rotordynamics, the Z-convention is commonly adopted due to coincidence of the Z-axis with the shaft pointing direction. Assume that the ground-fixed inertial frame, XYZ, and the body-fixed frame, xyz, co-occur at the origin O before rotation starts. The first rotation frame abc is formed by rotating the rigid body about the Y-axis by an angle $\phi$. Secondly, the rigid body is rotated by an angle $\theta$ about the a-axis, defining the second rotation frame, x'y'z'. Finally, the
rotation $\psi$ about the $z'$-axis defines the body-fixed frame, $xyz$. In addition, it represents the torque DoF of the flywheel. The three rotation angles thus defined describe the complete rotation of a rigid body about its CoM. The general forms of the transformation matrices between frames are given in appendix A. Lagrange’s equations are applied to derive the EoMs of the RWA model. Hazewinkel (1997) provides a general form of the Lagrange’s equations:

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_j} \right) - \frac{\partial L}{\partial q_j} = Q_j$$  

(3.1)

where $q$ is the generalised coordinate, $j$ is the $j$-th generalised coordinate, $Q$ is the generalised force (including dissipation) and $L$ is the lagrangian which is defined as the difference between the total kinetic energy $T$, and the total potential energy $U$, of the system. Using the dot to represent the first derivative of the generalised coordinate with respect to time $t$, the lagrangian can be expressed as:

$$L (\dot{q}_1, \ldots, \dot{q}_j; q_1, \ldots, q_j) = T (\dot{q}_1, \ldots, \dot{q}_j; q_1, \ldots, q_j) - U (q_1, \ldots, q_j)$$  

(3.2)

To derive the fully mass imbalanced RWA model, two combined cases are considered:

- kinetic energy, potential energy, and work done obtained from a mass balanced flywheel;
- kinetic energy derived by introducing a mass imbalance.

### 3.2 RWA Equations of Motion

#### 3.2.1 RWA Schematic

The RWA disturbance model was initially developed by Zhang et al. (2011) and subsequently re-elaborated by Addari et al. (2014b) and Addari et al. (2016) using an energy approach to derive the generalised EoMs using Equations (3.1) and (3.2). A schematic cross-section view of a RWA cantilever-configured is illustrated in Figure 3.2 showing how the components are generally arranged and is used as reference for the schematic model shown in Figure 3.3. The latter is able to capture the ten DoF modes of the RWA (five with respect to the flywheel and five with respect to the wheel-base) and also the gyroscopic effect due to the spinning
flywheel. In addition, it is able to represent the hard-mounted boundary condition as well as the free-free boundary condition.

![Figure 3.2: Reaction wheel assembly schematic cross-section view: 1-flywheel (mass and inertia), 2-motor holder (flexible component), 3-motor shaft (massless and rigid), 4-motor body (included in the modelling of the flywheel), 5-wheel-base (mass and inertia)](image)

The flywheel is modelled as a rigid disk with mass $M_w$, torsional inertia $I_{r,w}$, and inertia with respect to the spin axis $I_{z,w}$. In addition, it is connected by a massless and rigid shaft of length $d$ to the suspension system. Therefore, the mass and inertia of the motor can be included in the modelling of the flywheel thus to treat the two components as a single rigid element. The wheel-base is modelled as a rigid disk of mass $M_b$ and radial moment of inertia $I_{r,b}$.

The dynamic mass imbalance can be modelled as a point mass $m$, placed at radius $r$ on the flywheel and distance $l$ from the shaft. The point mass creates radial forces and moments when the flywheel spins whose magnitude is proportional to the radial distance from the mass imbalance to the shaft axis (the radial distance and the flywheel radius are not necessarily the same, despite this being the case for the model in Figure 3.3).

The flexible components in this system are the suspension system that connects the flywheel and the wheel-base (denoted with subscript ”w”) and the suspension system that connects the wheel-base to the ground (denoted with subscript ”b”). The former refers to the motor holder in Figure 3.2 which acts as a spring-damper system and it is where the RWA flexibility is concentrated. The latter, on the other hand, represents the connection of the RWA to an eventual platform or supporting structure. The wheel-base-to-ground suspension system is able to represent the hard-mounted boundary condition when the spring stiffness values tend to infinite and the free-free boundary condition when the spring stiffness values are considered zero.

The RWA is assumed axisymmetric about its shaft. Consequently, the linear
springs stiffness, $k_{t,w}$ and $k_{t,b}$, are the same in the two radial translation DoFs, as well as the two torsional springs stiffness $k_{r,w}$ and $k_{r,b}$, the two linear dashpot damping coefficients $c_{t,w}$ and $c_{t,b}$, and the two torsional dashpot damping coefficients $c_{r,w}$ and $c_{r,b}$. On the other hand, in the axial translation DoF, $k_{z,w}$ and $k_{z,b}$, and $c_{z,w}$ and $c_{z,b}$, represent the axial springs stiffness and the axial dashpot damping coefficients, respectively. The generalised Lagrangian coordinates in the
RWA Analytical Model

RWA model are ten: \(x_w, y_w, z_w, \theta_w, \phi_w, x_b, y_b, z_b, \theta_b\) and \(\phi_b\) shown in Figure 3.3, whereas \(\psi_w\) and \(\psi_b\) are not considered due to the assumption of flywheel steady speed rotation and, consequently, domination over angular speed perturbation in torque DoF.

3.2.2 RWA Kinetic Energy

The total kinetic energy is the sum of the translational kinetic energies and the rotational kinetic energies of the balanced flywheel and wheel-base and of the kinetic energy associated with an eventual non uniform distribution of the flywheel mass.

In Figure 3.3, the flywheel coordinate system is defined as \(x_w y_w z_w\) with origin \(O_w\) at the flywheel CoM. \(\theta_w, \phi_w\) and \(\psi_w\) are the corresponding rotations of the flywheel about its coordinate system. Note that in torsional DoF, the flywheel mass perturbation is significantly smaller compared to the flywheel angular speed and, therefore, neglected. For a mass balanced flywheel the geometric centre of the flywheel coincides with its CoM and the flywheel axis of symmetry co-occurs with the spin axis. The translational displacement vector \(u_w\) of the flywheel centre of mass can be expressed as:

\[
\mathbf{u}_w = \begin{bmatrix}
x_w \\
y_w \\
z_w
\end{bmatrix}
\]

(3.3)

Subsequently, differentiating Equation (3.3) with respect to time, the translational velocity vector \(v_w\) is derived:

\[
\mathbf{v}_w = \begin{bmatrix}
\dot{x}_w \\
\dot{y}_w \\
\dot{z}_w
\end{bmatrix}
\]

(3.4)

Finally, the translational kinetic energy of the balanced flywheel is obtained:

\[
T_w = \frac{1}{2} \mathbf{v}_w^T M_w \mathbf{v}_w
\]

(3.5)

where the superscript \(T\) denotes the transpose of a vector.

With reference to Figures 3.1 and 3.3, the angular velocity vector, \(\omega_w\), of the flywheel CoM is defined as the sum of three angular velocities acting in different directions: \(\dot{\phi}_w\) about the Y-axis, \(\dot{\theta}_w\) about the a-axis, and \(\dot{\psi}_w\) about the \(z_w\)-axis. Due to steady state rotation, it can be assumed \(\psi_w = \Omega\), where \(\Omega\) is the con-
stant angular speed of the flywheel. Opportunely applying the coordinate system rotation matrices, \( R_{abc \rightarrow x'y'z'} \) and \( R_{x'y'z' \rightarrow xyz} \), given in appendix A, the angular velocity vector in the body frame can be written as:

\[
\omega_w = \begin{bmatrix}
\dot{\theta}_w \cos \psi_w + \dot{\phi}_w \cos \theta_w \sin \psi_w \\
-\dot{\theta}_w \sin \psi_w + \dot{\phi}_w \cos \theta_w \cos \psi_w \\
-\dot{\phi}_w \sin \theta_w + \Omega
\end{bmatrix} \quad (3.6)
\]

The resulting rotational kinetic energy can be expressed as:

\[
T_w = \frac{1}{2} \omega_w^T I_w \omega_w \quad (3.7)
\]

where \( I_w \) represents the inertia tensor of the flywheel. Due to symmetry, the two inertial moments around \( x_w \)-axis and \( y_w \)-axis are identical and they are called \( I_{r,w} \). The inertia tensor in the body frame can thus be written as:

\[
I_w = \begin{bmatrix}
I_{r,w} & 0 & 0 \\
0 & I_{r,w} & 0 \\
0 & 0 & I_{z,w}
\end{bmatrix} \quad (3.8)
\]

Considering small angles of rotations and small displacements, the linearised kinetic energy \( T_w \) of the balanced flywheel is derived:

\[
T_w = \frac{1}{2} M_w \left( \dot{x}_w^2 + \dot{y}_w^2 + \dot{z}_w^2 \right) \\
+ \frac{1}{2} \left( I_{r,w} \dot{\theta}_w^2 + I_{r,w} \dot{\phi}_w^2 + I_{z,w} \dot{\psi}_w^2 - 2 \Omega I_{z,w} \dot{\phi}_w \dot{\theta}_w \right) \quad (3.9)
\]

In Figure 3.3, the wheel-base coordinate system is defined as \( x_b y_b z_b \) with origin \( O_b \) at the wheel-base CoM. \( \theta_b \), \( \varphi_b \) and \( \psi_b \) are the corresponding rotations of the wheel-base about its coordinate system. Note that in torsional DoF, the wheel-base mass perturbation is significantly smaller compared to the flywheel angular speed, thus ignored. The vertical distance from the flywheel-to-wheel-base suspension system to the wheel-base CoM is equal to \( h \) whereas the distance from the wheel-base CoM to the wheel-base-to-ground suspension system is represented by \( v \). By assuming small angles of rotations and small displacements, the linearised kinetic energy \( T_b \) of the wheel-base is obtained:

\[
T_b = \frac{1}{2} M_b \left( \dot{x}_b^2 + \dot{y}_b^2 + \dot{z}_b^2 \right) + \frac{1}{2} \left( I_{r,b} \dot{\theta}_b^2 + I_{r,b} \dot{\varphi}_b^2 \right) \quad (3.10)
\]

In order to capture the flywheel radial forces and moments, a mass imbalance \( m \) is added to the flywheel. The point mass is located at a radius \( r \) and distance \( l \) from the flywheel CoM and produces forces and moments when the flywheel is
in operation. The position of the imbalance mass can be described in the body frame as:

\[
\mathbf{u}_m = \begin{cases} 
0 \\
\mathbf{r} \\
\mathbf{l}
\end{cases}
\]  

(3.11)

To derive the rotational displacements of the point with respect to the inertial frame, the rotational matrix, \( \mathbf{R}_{xyz \rightarrow XYZ} \) given in appendix A, is used. In addition, the point mass is able to move in the three translational DoFs. Hence, the complete displacement vector \( \mathbf{s}_m \) of the mass imbalance can be expressed as:

\[
\mathbf{s}_m = \begin{bmatrix} 
x_w \\
y_w \\
z_w
\end{bmatrix}
\]

(3.12)

\[
\begin{align*}
&+ \left\{ \begin{array}{l}
  r \left( -\cos \varphi_w \sin \psi_w + \sin \varphi_w \sin \theta_w \cos \psi_w \right) + l \sin \varphi_w \cos \theta_w \\
  r \left( \cos \theta_w \cos \psi_w \right) - l \sin \theta_w \\
  r \left( \sin \varphi_w \sin \psi_w + \cos \varphi_w \sin \theta_w \cos \psi_w \right) + l \cos \varphi_w \cos \theta_w
\end{array} \right\}
\]

The translational velocity vector of the unbalanced flywheel is obtained differentiating Equation (3.12) with respect to time. Similarly to Equation (3.5), the kinetic energy associated to the mass imbalance can be calculated:

\[
T_m = \frac{1}{2} \left[ -2mr \Omega^2 \sin (\Omega t) - 2mr \Omega^2 \cos (\Omega t) \\
-2mr \Omega^2 \sin (\Omega t) + 2mr \Omega^2 \cos (\Omega t) \right]
\]  

(3.13)

Summing Equations (3.7), (3.10) and (3.13), the kinetic energy of the RWA imbalance system is obtained:

\[
T_{sys} = T_w + T_b + T_m \\
\approx \frac{1}{2} M_w \left( \dot{x}_w^2 + \dot{y}_w^2 + \dot{z}_w^2 \right) \\
+ \frac{1}{2} \left( I_{r,w} \dot{\theta}_w^2 + I_{r,w} \dot{\psi}_w^2 + I_{z,w} \dot{\Omega}_w^2 \right) - 2\Omega I_{z,w} \dot{\varphi}_w \dot{\theta}_w \\
+ \frac{1}{2} M_b \left( \dot{x}_b^2 + \dot{y}_b^2 + \dot{z}_b^2 \right) + \frac{1}{2} \left( I_{r,b} \dot{\theta}_b^2 + I_{r,b} \dot{\psi}_b^2 \right) \\
+ \frac{1}{2} m \left[ -2r \Omega^2 \sin (\Omega t) - 2r \Omega^2 \cos (\Omega t) \\
- 2r \Omega^2 \sin (\Omega t) + 2r \Omega^2 \cos (\Omega t) \right]
\]  

(3.14)

In the process of simplification three assumptions were considered:
• small displacements (angle) assumption such that \( \cos \alpha \sim 1 \) and \( \sin \alpha \sim \alpha \), where \( \alpha \) is a generic angle;

• small mass imbalance assumption, (i.e. \( m \ll M_w \)), thus the imbalanced mass can be neglected with respect to balanced flywheel mass which remains unchanged;

• the flywheel spin speed \( \Omega \) is notably larger than any perturbation in the five DoFs (i.e. \( \dot{x}, \dot{y}, \dot{z}, \dot{\theta} \) and \( \dot{\phi} \ll \Omega \)).

3.2.3 RWA Potential Energy

The potential energy for the mass balanced RWA is derived as function of the suspension systems’ stiffness and elastic reactions due to the application of displacements. In the derivation process, due to RWA axis-symmetry, dynamics in the two orthogonal DoFs (\( x_w \) and \( y_w \), \( x_b \) and \( y_b \) with respect to the flywheel and the wheel-base, respectively) are assumed equal. In addition, no dynamic coupling between axial and radial DoFs is considered. Virtual displacements are applied at the flywheel and wheel-base CoM to derive the stiffness matrices of the suspension systems. The total potential energy is the sum of the translational and rotational potential energies in the two radial DoFs and in the axial translational DoF. Due to the connection of the flywheel and the wheel-base through the suspension systems, the relative displacements of the springs are required to derive the potential energy of the system. The process to retrieve the stiffness matrix of the RWA is divided in four stages: firstly, the potential energy in the xz-plane, \( U_{xz} \) is calculated; secondly the potential energy in the yz-plane, \( U_{yz} \) is evaluated; thirdly, the translational axial potential energy, \( U_z \) is derived. Finally, the six potential energies, three associated with the flywheel and three associated with the wheel-base, are summed and the potential energy of the system \( U_{sys} \) is obtained.

According to Figure 3.4, the potential energy in the xz-plane, \( U_{xz} \), can be derived as follows:

i. apply a positive virtual displacement at the flywheel CoM, \( O_w \): the CoM would move to a new position \( O'_w \). This would produce an extension of the linear spring \( k_{t,w} \) and a positive force \( F_{x,w} \). The wheel-base CoM, \( O_b \), on the other hand, would counteract this motion generating a negative force proportional to \( k_{t,w} \);

ii. apply an anti-clockwise virtual rotation about \( O'_w \): this would not only produce a moment \( M_{y,w} \) due to the extension of the torsional spring \( k_{r,w} \), but also an
additional effect on $F_{t,w}$. Assuming the rotation to be small, then $\Delta_w = \varphi_w d$ and $\Delta_b = \varphi_b h$. Since the linear springs can only produce a force, the total force $F_{x,t}$ would be the sum of the contributions given by the $x_w$, $x_b$, $\Delta_w$ and $\Delta_b$.

iii. apply a positive virtual displacement at the wheel-base CoM, $O_b$: the CoM would move to a new position $O'_b$. This would produce an extension of the linear springs $k_{t,w}$ and $k_{t,b}$ for which the resulting force would be $F_{x,b}$; in contrast, the flywheel CoM, $O_w$ would oppose to this motion creating a resisting force proportional to $k_{t,w}$.
iv. apply an anti-clockwise virtual rotation about $O_b'$: this would generate a moment $M_{y,b}$ due to the extension of the torsional springs $k_{r,w}$ and $k_{r,b}$ and affect the actual force $F_{t,b}$ at the wheel-base CoM, in a similar manner to point ii.

The forces and moments at the flywheel and wheel-base CoMs can then be expressed in matrix form as:

\[
\begin{bmatrix}
F_{x,w} \\
M_{y,w} \\
F_{x,b} \\
M_{y,b}
\end{bmatrix} =
\begin{bmatrix}
k_{t,w} & -k_{t,w}d & -k_{t,w} \\
-k_{t,w}d & k_{t,w}d^2 + k_{r,w} & k_{t,w}d \\
-k_{t,w} & k_{t,w}d & k_{t,w} + k_{t,b} \\
-k_{t,w}h & k_{t,w}hd - k_{r,w} & k_{t,w}h - k_{t,b}h \\
-k_{t,w}h & k_{t,w}hd - k_{r,w} \\
k_{t,w}h - k_{t,b}v \\
k_{t,w}h^2 + k_{t,b}v^2 + k_{r,w} + k_{r,b}
\end{bmatrix}
\begin{bmatrix}
x_w \\
\varphi_w \\
x_b \\
\varphi_b
\end{bmatrix}
\]

(3.15)

where the matrix reproduces the coupled dynamics in the xz-plane due to virtual displacements applied at the flywheel and wheel-base CoMs, and is referred as $K_{xz}$. Finally, the potential energy in the xz-plane is computed as:

\[
U_{xz} = \frac{1}{2} \begin{bmatrix}
x_w \\
\varphi_w \\
x_b \\
\varphi_b
\end{bmatrix}^T \begin{bmatrix}
k_{t,w} & -k_{t,w}d & -k_{t,w} \\
-k_{t,w}d & k_{t,w}d^2 + k_{r,w} & k_{t,w}d \\
-k_{t,w} & k_{t,w}d & k_{t,w} + k_{t,b} \\
-k_{t,w}h & k_{t,w}hd - k_{r,w} & k_{t,w}h - k_{t,b}h \\
-k_{t,w}h & k_{t,w}hd - k_{r,w} \\
k_{t,w}h - k_{t,b}v \\
k_{t,w}h^2 + k_{t,b}v^2 + k_{r,w} + k_{r,b}
\end{bmatrix}
\begin{bmatrix}
x_w \\
\varphi_w \\
x_b \\
\varphi_b
\end{bmatrix}
\]

(3.16)

A similar expression to $K_{xz}$ can be obtained for the stiffness matrix in the yz-plane, $K_{yz}$. According to Figure 3.5 and applying virtual displacements and rotations to the flywheel and wheel-base CoMs, the forces and moments generated by the extension of the linear and torsional springs can be calculated as:

\[
\begin{bmatrix}
F_{y,w} \\
M_{x,w} \\
F_{y,b} \\
M_{x,b}
\end{bmatrix} =
\begin{bmatrix}
k_{t,w} & k_{t,w}d & -k_{t,w} \\
k_{t,w}d & k_{t,w}d^2 + k_{r,w} & -k_{t,w}d \\
-k_{t,w} & -k_{t,w}d & k_{t,w} + k_{t,b} \\
-k_{t,w}h & k_{t,w}hd - k_{r,w} & -k_{t,w}h + k_{t,b}h \\
k_{t,w}h \\
k_{t,w}hd - k_{r,w} \\
k_{t,w}h + k_{t,b}v \\
k_{t,w}h^2 + k_{t,b}v^2 + k_{r,w} + k_{r,b}
\end{bmatrix}
\begin{bmatrix}
y_w \\
\theta_w \\
y_b \\
\theta_b
\end{bmatrix}
\]

(3.17)

The potential energy in the yz-plane assumes a form similar to Equation (3.16):
Figure 3.5: RWA model in YZ-plane

\[ U_{yz} = \frac{1}{2} \begin{bmatrix} y_w \\ \theta_w \\ y_b \\ \theta_b \end{bmatrix}^T K_{yz} \begin{bmatrix} y_w \\ \theta_w \\ y_b \\ \theta_b \end{bmatrix} \] (3.18)

Note that the potential energy in radial DoFs in each plane is given by the contribution of three components: the linear spring, the torsional spring and their coupled influences. On the other hand, the potential energy in the axial DoF, \( U_z \),
is independent from the others and can be derived as:

\[
U_z = \frac{1}{2} \left\{ \begin{array}{c} z_w \\ z_b \end{array} \right\}^T \left[ \begin{array}{cc} k_{z,w} & -k_{z,w} \\ -k_{z,w} & k_{z,w} + k_{z,b} \end{array} \right] \left\{ \begin{array}{c} z_w \\ z_b \end{array} \right\}
\] (3.19)

By assuming small displacements, the linearised potential energy of the whole system is evaluated:

\[
U_{sys} \approx \frac{1}{2} \left[ k_{t,w} \left( x_w^2 + y_w^2 + x^2_b + y_b^2 - 2x_w x_b - 2y_w y_b \right) \\
+ \left( k_{t,w} d^2 + k_{r,w} \right) \left( \theta_w^2 + \varphi_w^2 \right) + \left( k_{t,w} h^2 + k_{r,w} \right) \left( \theta_b^2 + \varphi_b^2 \right) \\
+ k_{z,w} \left( z_w^2 + z_b^2 - 2z_w z_b \right) \\
+ 2k_{t,w} \left( y_w \theta_w - \theta_w y_b - x_w \varphi_w + \varphi_w x_b \right) \\
+ 2k_{t,w} \left( y_b \theta_b - \theta_b y_b - x_b \varphi_b + \varphi_b x_b \right) + 2 \left( h d k_{t,w} - k_{r,w} \right) \left( \theta_w \theta_b + \varphi_w \varphi_b \right) \\
+ \frac{1}{2} \left[ k_{t,b} \left( x_b^2 + y_b^2 \right) + k_{z,b} z_b^2 + \left( k_{t,b} v^2 + k_{r,b} \right) \left( \theta_b^2 + \varphi_b^2 \right) \\
+ 2v k_{t,b} \left( y_b \theta_b - x_b \varphi_b \right) \right] \right]
\] (3.20)

It should be observed that having knowledge of the stiffness matrix in either the xz-plane or the yz-plane, allows one to forthwith attain the stiffness matrix in the other plane by changing the sign of the anti-diagonal elements. This reflects the dissimilarity in the sign convention adopted for the orthogonal planes.

### 3.2.4 RWA Damping Representation

For completion of the RWA microvibration model, the work done by the linear and torsional dashpots must also be evaluated. Assuming the damping of viscous type, the linear and torsional dashpots act in parallel to the linear and torsional springs producing damping forces and moments, respectively. By means of virtual displacements, the work done can be computed as the product of the damping forces and moments and the virtual displacements applied to the flywheel and wheel-base CoMs. Moreover, the damping loads can be formulated as the results of the multiplication between the viscous damping coefficient, \( c \), and the velocity in each DoF, \( \dot{q} \), leading to:

\[
\partial Q = -c_j \dot{q}_j \partial q_j
\] (3.21)

Due to the connection between the flywheel and the wheel-base through the suspension system, the relative velocities of the dashpots are required to derive the work done associated with the RWA system. According to Figures 3.4 and 3.5,
the work done by the linear and torsional dashpots can be computed using a similar approach to that implemented for the linear and torsional springs. Also here, the radial translational and the radial rotational DoFs exhibit joint motions, whereas the axial translational DoF is independent from the others. Following the derivation process used to express the potential energy, the work done is calculated as the sum of three components:

- the work done in the \(xz\)-plane:

\[
\partial Q_{xz} = -\left[ \begin{array}{ccc}
  c_{t,w} & -c_{t,w}d & -c_{t,w} \\
  -c_{t,w}d & c_{t,w}d^2 + c_{r,w} & c_{t,w}d \\
  -c_{t,w} & c_{t,w} & c_{t,w} + c_{t,b} \\
  -c_{t,w} & c_{t,w} - c_{w} & c_{t,w}h - c_{t,b}h \\
  -c_{t,w} & c_{t,w}h - c_{r,w}h \\
  c_{t,w} & c_{t,w} - c_{t,b}v \\
  c_{t,w}h^2 + c_{t,b}v^2 + c_{r,w} + c_{r,b} \\
\end{array} \right] \left\{ \begin{array}{c}
  \dot{x}_w \partial x_w \\
  \dot{\phi}_w \partial \phi_w \\
  \dot{x}_b \partial x_b \\
  \dot{\phi}_b \partial \phi_b \\
\end{array} \right\} = C_{xz} \left\{ \begin{array}{c}
  \dot{x}_w \partial x_w \\
  \dot{\phi}_w \partial \phi_w \\
  \dot{x}_b \partial x_b \\
  \dot{\phi}_b \partial \phi_b \\
\end{array} \right\}
\]

- the work done in the \(yz\)-plane:

\[
\partial Q_{yz} = -\left[ \begin{array}{ccc}
  c_{t,w} & c_{t,w}d & -c_{t,w} \\
  c_{t,w}d & c_{t,w}d^2 + c_{r,w} & -c_{t,w}d \\
  -c_{t,w} & -c_{t,w} & c_{t,w} + c_{t,b} \\
  c_{t,w} & c_{t,w}h - c_{r,w}h \\
  -c_{t,w} & c_{t,w}h + c_{r,w}h \\
  c_{t,w} & c_{t,w}h - c_{r,w}h + c_{t,b}v \\
  c_{t,w}h^2 + c_{t,b}v^2 + c_{r,w} + c_{r,b} \\
\end{array} \right] \left\{ \begin{array}{c}
  \dot{y}_w \partial y_w \\
  \dot{\theta}_w \partial \theta_w \\
  \dot{y}_b \partial y_b \\
  \dot{\theta}_b \partial \theta_b \\
\end{array} \right\} = C_{yz} \left\{ \begin{array}{c}
  \dot{y}_w \partial y_w \\
  \dot{\theta}_w \partial \theta_w \\
  \dot{y}_b \partial y_b \\
  \dot{\theta}_b \partial \theta_b \\
\end{array} \right\}
\]

- the work done in the axial translation DoF:
\[ \partial Q_z = - \begin{bmatrix} c_{z,w} & -c_{z,w} \\ -c_{z,w} & c_{z,w} + c_{z,b} \end{bmatrix} \begin{bmatrix} \dot{z}_w \partial z_w \\ \dot{z}_b \partial z_b \end{bmatrix} = C_z \begin{bmatrix} \dot{z}_w \partial z_w \\ \dot{z}_b \partial z_b \end{bmatrix} \] (3.24)

where \( C_{xz}, C_{yz} \) and \( C_z \) are the damping matrices in the xz-plane, yz-plane and in the axial DoF, respectively.

Finally, summing Equations (3.22), (3.23) and (3.24), the work done by the RWA is computed and has a similar form of Equation (3.20):

\[ \partial Q_{sys} = -C_{xz} \begin{bmatrix} \dot{x}_w \partial x_w \\ \dot{\varphi}_w \partial \varphi_w \\ \dot{x}_b \partial x_b \\ \dot{\varphi}_b \partial \varphi_b \end{bmatrix} - C_{yz} \begin{bmatrix} \dot{y}_w \partial y_w \\ \dot{\theta}_w \partial \theta_w \\ \dot{y}_b \partial y_b \\ \dot{\theta}_b \partial \theta_b \end{bmatrix} - C_z \begin{bmatrix} \dot{z}_w \partial z_w \\ \dot{\theta}_b \partial \varphi_b \end{bmatrix} \] (3.25)

### 3.2.5 RWA Imbalanced Model

It is now possible to define the linearised lagrangian substituting the linearised kinetic energy defined in Equation (3.14) and linearised potential energy described by Equation (3.20) into Equation (3.2). Moreover, the generalised force in Equation (3.1) can be reformulated in terms of Equation (3.25), to obtain the final expression of the Lagrange equation. The ten EoM with respect to the ten generalised coordinates \( x_w, y_w, z_w, \theta_w, \varphi_w, x_b, y_b, z_b, \theta_b, \varphi_b \), can finally be derived using the energy method described in section 3.1.

\[
\begin{align*}
&\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{x}_w} \right) - \frac{\partial L}{\partial x_w} = \frac{\partial Q}{\partial x_w} \\
&\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{y}_w} \right) - \frac{\partial L}{\partial y_w} = \frac{\partial Q}{\partial y_w} \\
&\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{z}_w} \right) - \frac{\partial L}{\partial z_w} = \frac{\partial Q}{\partial z_w} \\
&\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}_w} \right) - \frac{\partial L}{\partial \theta_w} = \frac{\partial Q}{\partial \theta_w} \\
&\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\varphi}_w} \right) - \frac{\partial L}{\partial \varphi_w} = \frac{\partial Q}{\partial \varphi_w} \\
&\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{x}_b} \right) - \frac{\partial L}{\partial x_b} = \frac{\partial Q}{\partial x_b} \\
&\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{y}_b} \right) - \frac{\partial L}{\partial y_b} = \frac{\partial Q}{\partial y_b} \\
&\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{z}_b} \right) - \frac{\partial L}{\partial z_b} = \frac{\partial Q}{\partial z_b} \\
&\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}_b} \right) - \frac{\partial L}{\partial \theta_b} = \frac{\partial Q}{\partial \theta_b} \\
&\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\varphi}_b} \right) - \frac{\partial L}{\partial \varphi_b} = \frac{\partial Q}{\partial \varphi_b}
\end{align*}
\] (3.26)

The linearised EoMs of the RWA system in matrix form can be articulated as:
\[ \mathbf{M}_s \ddot{\mathbf{q}}_s + (\mathbf{C}_s + \mathbf{G}_s) \dot{\mathbf{q}}_s + \mathbf{K}_s \mathbf{q}_s = \mathbf{f}_s \]  

(3.27)

where “s” denotes RWA-system, \( \mathbf{M} \), \( \mathbf{C} \) and \( \mathbf{K} \) represent the mass, damping and stiffness matrices of the RWA system, respectively, whereas \( \mathbf{G} \) is a matrix which terms describe the gyroscopic effect. All matrices in Equation (3.27) are expanded and illustrated in appendix B. Note that in both stiffness and damping matrices there are non-zero off-diagonal elements which are representative of the coupled motion of the radial translational and rotational modes due to the RWA cantilever configuration (Zhang et al., 2011). The terms on the right-hand-side of Equation (3.27), \( \mathbf{f}_s \), represent the excitations in the system due to the imbalance mass. The addition of the imbalance mass produces four forcing terms, two in the radial translational DoFs and two in the radial rotational DoFs. The two force and moment excitations have magnitudes proportional to the angular speed, \( \Omega \), squared with angular frequencies (the rate of change of the function argument) equal to the angular speed.

\[
\mathbf{f}_s = \begin{bmatrix}
-mr \Omega^2 \sin (\Omega t) \\
mr \Omega^2 \cos (\Omega t) \\
0 \\
-mrl \Omega^2 \cos (\Omega t) \\
-mrl \Omega^2 \sin (\Omega t) \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]  

(3.28)

Although Equation (3.27) captures the RWA structural modes, the gyroscopic effect, the fundamental harmonics and their amplifications, the model does not include neither sub- and higher harmonics nor the broadband noise excitations. Parameters for harmonic excitation modelling include the amplitude coefficients, \( A_i \), and harmonic number, \( h_i \), where \( i \) in this case is the number of harmonics considered in each DoF. Broadband excitations on the other hand are expressed as \( W \) at this stage. Liu et al. (2008) observed that higher harmonics and broadband noise can be superimposed to fundamental harmonics. Therefore, including these in the excitation vector on the right hand side of Equation (3.27), the complete disturbance vector can be obtained:
\[ \mathbf{f}_s = \begin{cases} \sum_{i=1}^{n_t} A_{i}^t \Omega^2 \sin (h_{i}^t \Omega t) + W \\ \sum_{i=1}^{n_r} A_{i}^r \Omega^2 \cos (h_{i}^r \Omega t) + W \\ \sum_{i=1}^{n_a} A_{i}^a \Omega^2 \sin (h_{i}^a \Omega t) + W \\ \sum_{i=1}^{n_r} A_{i}^r \Omega^2 \cos (h_{i}^r \Omega t) + W \\ 0 \\ 0 \\ 0 \\ 0 \end{cases} \] (3.29)

where sub- and superscripts “t”, “r” and “a” indicate translational DoF, rotational DoF and axial DoF respectively; \( n \) is the total number of harmonics. Note that if only the fundamental harmonic for each DoF is considered (i.e. \( n = 1, h_1 = 1 \)), then \( A_1 \) equals the mass imbalance and Equation (3.28) matches Equation (3.29).

### 3.3 Hard-mounted Boundary Configuration

The equations derived in the previous section provide the general behaviour of a cantilever configured RWA when mass imbalance, motor imperfections and broadband noise act on the mechanism. As mentioned in section 2.4.4, however, the RWA is typically tested in a hard-mounted boundary condition. For this reason, it is shown how the EoMs which describe the functioning of a grounded cantilever configured RWA can be obtained starting from the previous model.

Consider that a rigid connection is created between the wheel-base and the ground thus to produce a hard-mounted boundary condition (at times also referred to as “blocked” or “grounded” configuration). Mathematically, this is reproduced by assigning infinite values to the stiffness coefficients of the suspension system connecting the wheel-base and the ground, \( k_{t,b}, k_{r,b} \) and \( k_{z,b} \):
\[
\lim_{k_{t,b,k_{r,b},k_{z,b}} \to \infty} \mathbf{K}_s =
\begin{bmatrix}
    k_{t,w} & 0 & 0 & 0 & -k_{t,w}d \\
    0 & k_{t,w} & 0 & k_{t,w}d & 0 \\
    0 & 0 & k_{z,w} & 0 & 0 \\
    0 & k_{t,w}d & 0 & k_{t,w}d^2 + k_{r,w} & 0 \\
    -k_{t,w}d & 0 & 0 & 0 & k_{t,w}d^2 + k_{r,w}
\end{bmatrix}_{\infty \times 5}
\]

\[
\mathbf{K}_{hm} \equiv \begin{bmatrix}
    k_{t,w} & 0 & 0 & 0 & -k_{t,w}d \\
    0 & k_{t,w} & 0 & k_{t,w}d & 0 \\
    0 & 0 & k_{z,w} & 0 & 0 \\
    0 & k_{t,w}d & 0 & k_{t,w}d^2 + k_{r,w} & 0 \\
    -k_{t,w}d & 0 & 0 & 0 & k_{t,w}d^2 + k_{r,w}
\end{bmatrix}_{\infty \times 5}
\]

where the subscript "hm" denotes a hard-mounted boundary condition. A schematic representation of the RWA in the grounded configuration is presented in Figure 3.6.

In this configuration, the initial 10 DoFs describing the motion of the RWA are reduced to only 5, corresponding to the flywheel DoFs only. Thereby, Equation (3.27) assumes a simpler form and can be reformulated as:

\[
\begin{bmatrix}
    \ddot{x}_w \\
    \ddot{y}_w \\
    \ddot{z}_w \\
    \ddot{\theta}_w \\
    \ddot{\varphi}_w
\end{bmatrix}
+ (\mathbf{C}_{hm} + \mathbf{G}_{hm})
\begin{bmatrix}
    \dot{x}_w \\
    \dot{y}_w \\
    \dot{z}_w \\
    \dot{\theta}_w \\
    \dot{\varphi}_w
\end{bmatrix}
+ \mathbf{K}_{hm}
\begin{bmatrix}
    x_w \\
    y_w \\
    z_w \\
    \theta_w \\
    \varphi_w
\end{bmatrix}
= \mathbf{f}_{hm}
\]

Equation (3.31) can be solved numerically to compute the displacement vector of the flywheel CoM, \( \mathbf{u}_{whm} \). This can subsequently be implemented with the stiffness matrix \( \mathbf{K}_{hm} \) to derive the resulting forces and moments at the flywheel CoM. In order to evaluate the forces and moments at the RWA mounting point,
Figure 3.6: RWA hard-mounted configuration schematic

however, a transformation matrix is required. Looking at Figure 3.3, the following transformation matrix $T_{hm}$ can be deduced:

$$T_{hm} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & -(d + h + v) & 0 & 1 & 0 \\
(d + h + v) & 0 & 0 & 0 & 1
\end{bmatrix} \quad \text{(3.32)}$$

Finally, multiplying the load vector at the flywheel CoM by $T_{hm}$, the resulting forces and moments at the RWA mounting point, $f_{hm}$, are obtained:

$$f_{hm} = T_{hm}K_wu_{w_{hm}} \quad \text{(3.33)}$$

Alternatively, due to axisymmetry, the motion of the RWA can be expressed using complex coordinates whose application makes all matrices symmetric (the gyroscopic matrix is skew symmetric in real coordinates), reduces the number of equations and allows the derivation of the speed dependent structural modes of the
RWA in a simpler way than using real coordinates. In these terms, Equation (3.31) can be decomposed into a bi-quadratic equation to represent the translational and rotational motions of the RWA and into an independent equation in the axial DoF. The first step is to introduce a vector of complex coordinates, $\mathbf{v}_w$ and $\chi_w$, for which the imaginary part of the coordinates refers to the motion in the $y_w z_w$-plane whereas the real part describes the motion in the $x_w z_w$-plane, as shown in Equation (3.34):

\[
\begin{align*}
\mathbf{v}_w &= x_w + iy_w \\
\chi_w &= \varphi_w - i\theta_w
\end{align*}
\tag{3.34}
\]

where \(i\) denotes the imaginary unit and the minus sign is used in accordance with the different convention of sign in the two planes. Natural frequencies of the RWA can be computed setting to zero the right hand side of Equation (3.31) or Equation (3.27) and subsequently deriving their general solution. Due to low damping in RWA systems, however, damped natural frequencies do not significantly differ from the undamped ones. Therefore, in order to characterise the flywheel structural modes an undamped and excitation-free system is used. In addition, combining the translational and rotational EoMs in Equation (3.31) according to Equation (3.34), a more compact form can be obtained where all matrices are symmetric about their leading diagonal:

\[
\begin{bmatrix}
M_w & 0 \\
0 & I_{r,w}
\end{bmatrix}
\begin{Bmatrix}
\ddot{v}_w \\
\ddot{\chi}_w
\end{Bmatrix}
- i\Omega
\begin{bmatrix}
0 & 0 \\
0 & I_{z,w}
\end{bmatrix}
\begin{Bmatrix}
\dot{v}_w \\
\dot{\chi}_w
\end{Bmatrix}
+ 
\begin{bmatrix}
k_{t,w} & -k_{t,w}d \\
-k_{t,w}d & k_{t,w}d^2 + k_{r,w}
\end{bmatrix}
\begin{Bmatrix}
v_w \\
\chi_w
\end{Bmatrix} = 0
\tag{3.35}
\]

Granting a solution of the following type:

\[
\begin{bmatrix}
v_w \\
\chi_w
\end{bmatrix}
= 
\begin{bmatrix}
v_0 \\
\chi_0
\end{bmatrix}
e^{i\omega t}
\tag{3.36}
\]

where "0" indicates initial conditions, and omitting the decay rate, the frequency \(\omega\) can be computed. Placing the first and second derivative of Equation (3.36) into Equation (3.35), and solving the corresponding eigen-problem, the characteristic polynomial of the RWA system can be obtained. Finally, the RWA structural modal frequencies in the radial DoFs can be derived from its characteristic equation:
\[
\omega_w^4 - \frac{I_{z,w}}{I_{r,w}} \omega_w^3 = \left( \frac{k_{l,w}}{M_w} + \frac{k_{t,w} d^2 + k_{r,w}}{I_{r,w}} \right) \omega_w^2 + \Omega \frac{k_{l,w} I_{z,w}}{M_w I_{r,w}} \omega_w + \frac{k_{l,w} k_{r,w}}{M_w I_{r,w}} = 0 \quad (3.37)
\]

The outputs of Equation (3.37) are the four structural modal frequencies of the RWA: two in the translational DoF and two in the rotational DoF. All the solutions are speed dependent and coupling between the four DoFs is observed as expected.

On the other hand, the natural frequency in the axial DoF is simply computed as:

\[
\omega_z = \sqrt{\frac{k_{z,w}}{M_w}} \quad (3.38)
\]

The natural frequencies determined from Equations (3.37) and (3.38) can be represented as a Campbell diagram (Campbell, 1924).

The critical speed of a rotor is defined as the rotational speed at which the speed dependent modal frequencies intersect with the order lines associated with possible excitation sources of paramount interest. Letting \( \omega_w = \Omega \) into Equation (3.37) and solving for \( \Omega \), the synchronous critical speed can be calculated as:

\[
\begin{cases}
I'_w = I_{z,w} - I_{r,w} \\
k'_w = k_{l,w} d^2 + k_{r,w} \\
\Omega_{\text{crit}} = \sqrt{\frac{k_{l,w} I'_w - M_w k'_w}{2M_w I'_w}} \pm \sqrt{\left( k_{l,w} I'_w - M_w k'_w \right)^2 + 4M_w I'_w k_{l,w} k_{r,w}} \\
\end{cases}
\quad (3.39)
\]

Note that for thin disks, where the polar inertia is bigger than the torsional inertia, a solution is imaginary, thus only one real solution for the synchronous speed exists.

### 3.4 Free-free Boundary Configuration

A free-free boundary condition is such that the RWA is free to float in the air with no constraints. Although this configuration is unlikely to be reproduced experimentally in a laboratory environment unless a vacuum chamber is available, its mathematical formulation is quite simple. De facto, the stiffness values of the suspension system connecting the wheel-base and the ground can be tuned to
represent a free-free boundary condition, in a similar manner to the hard-mounted case. In this case, the stiffness values require to be set equal to zero so as to eliminate any connection between the RWA and the surrounding environment. The stiffness matrix in Equation (3.27) can, subsequently, be re-written as:

$$\lim_{k_{t,b},k_{z,b},k_{r,b}\rightarrow 0} K_s = K_{fis}$$  \hspace{1cm} (3.40)$$

where "fis" denotes free-free boundary condition (at times also referred to as free-in-space). A free-free suspended configuration is thus created and its schematic form is presented in Figure 3.7.

![Figure 3.7: RWA free-free configuration schematic](image)

Although the number of DoFs in the system remains unchanged, both stiffness and damping matrices in Equation (3.27) are significantly simplified (i.e. all the terms with subscript "b" are null). Expression of the RWA model in the free-free configuration in matrix form is as follows:

$$M_s \ddot{q}_s + (C_{fis} + G_s) \dot{q}_s + K_{fis} \dot{x}_s = f_s$$  \hspace{1cm} (3.41)$$
All matrices in Equation (3.41) are expanded in appendix B. Equation (3.41) can be solved for \( \ddot{q}_s \) to derive the free-free accelerations at the CoMs of both the flywheel and wheel-base. For the purpose of this thesis, we are interested in the motion of the RWA mounting point. In order to compute the accelerations at this location, the kinematic equations of a rigid body have to be applied. Let \( T_{mp} \) be the rigid body transformation matrix, where "mp" refers to the mounting point.

\[
T_{mp} = \begin{bmatrix}
1 & 0 & 0 & 0 & -v \\
0 & 1 & 0 & v & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\] (3.42)

The accelerations at the RWA mounting point, \( \ddot{x}_{mp} \), are then obtained by multiplying \( T_{mp} \) and the acceleration vector describing the motion of the wheel-base CoM, \( \ddot{x}_b \):

\[
\ddot{x}_{mp} = T_{mp} \ddot{x}_b
\] (3.43)

The structural modal frequencies of the RWA in a free-free configuration can finally be computed by reformulating the EoM in complex coordinates. Applying the same coordinate transformation and process as those performed in the hard-mounted case, the characteristic polynomial describing the RWA structural resonance frequencies as function of the flywheel angular speed can be obtained:

\[
\omega^5_z - \Omega \frac{I_{z,w}}{I_{r,w}} w^4_s - \left[ \frac{(M_b + M_w) k_{l,w}}{M_b M_w} + \frac{k_{r,w} + k_{l,w}d^2}{I_{r,w}} + \frac{k_{r,w} + k_{l,w}h^2}{I_{r,b}} \right] w^3_s \\
+ \Omega \left[ \frac{(M_b + M_w) k_{l,w} I_{z,w}}{I_{r,w} M_b M_w} + \frac{(k_{r,w} + k_{l,w}h^2) I_{z,w}}{I_{r,b} I_{r,w}} \right] \omega^2_s \\
+ \left[ \frac{(I_{r,b} + I_{r,w}) (M_b + M_w) k_{r,w} k_{l,w}}{I_{r,b} I_{r,w} M_b M_w} + \frac{k_{r,w} + k_{l,w}(h + d)^2}{I_{r,b} I_{r,w}} \right] \omega_s \\
+ \Omega \frac{(M_b + M_w) I_{z,w} k_{r,w} k_{l,w}}{I_{r,b} I_{r,w} M_b M_w} = 0
\] (3.44)

\[
\omega_{zs} = \sqrt{\frac{k_{z,w}}{M_b + M_w}} \frac{M_b + M_w}{M_b M_w}
\]

In the same manner as the hard-mounted case, the axial mode displays a decou-
pled behaviour from the other DoFs and can, therefore, be solved independently. In contrast, the radial and rotational DoFs present strong coupled dynamics as well as a dependance on the flywheel angular speed, as anticipated.

Note that if the inertial properties of the wheel-base, $M_b$ and $I_{r,b}$, tend to infinity, or values for which the flywheel inertial properties are a few orders of magnitude smaller, the system returns to a hard-mounted boundary condition and the equivalences in Equation (3.44) reduce to Equations (3.37) and (3.38), respectively.

The critical speeds (for which the interaction between the RWA structural modes and the harmonics generated by the flywheel spinning at its operative speeds may generate significant amplifications in the RWA dynamic response) can be computed from Equation (3.44) by setting $\omega_s = \Omega$ and solving for $\Omega$.

3.5 SUMMARY

In this chapter, the general analytical microvibration model of a cantilever configured RWA has been derived. The linearised EoM of the mass imbalanced microvibration model were able to capture the RWA structural modes, the gyroscopic effect and the fundamental harmonic excitations. By also introducing higher harmonics and broadband noise excitations, the EoM of the complete RWA microvibration model was obtained. Subsequently, the hard-mounted and the free-free boundary conditions were applied and the corresponding analytical models explicitly inferred. The former was determined by assuming an infinite value for the suspension system stiffness connecting the wheel-base to the ground whereas the latter by considering the same value equal to zero. Applications of the analytical models and their validation will be discussed in chapter 4.
RWA Microvibration Model Validation

A FE model of a cantilever configured RWA based on the analytical model described in chapter 3 was built and updated by means of the experimental data obtained from frequency response and microvibration tests, the latter carried out employing dedicated measurements procedures which involved the use of a dynamometric platform and elastic cords.

4.1 RWA and Test General Features

In this study, the cantilever configured RWA shown in Figure 4.1 was considered. This consisted of:

- a Brass flywheel, whose mass is concentrated on the edge to produce the highest angular momentum;
- a 3D-printed thermoplastic motor holder (whose mechanical properties are listed in Table 4.1), which operates as a support for the motor and as the suspension system of the RWA;
- a brushless DC-motor, used to spin the flywheel both clockwise and anticlockwise;
- an Aluminium wheel-base, which contains the electronics to control the motor and which was used as interface with the test rig.
Table 4.1: Mechanical Properties of the Thermoplastic Material

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>1300 kgm$^{-3}$</td>
</tr>
<tr>
<td>Young’s Modulus</td>
<td>3.5 GPa</td>
</tr>
<tr>
<td>Elongation at break</td>
<td>6.0 %</td>
</tr>
<tr>
<td>Ultimate Tensile Strength</td>
<td>50 MPa</td>
</tr>
<tr>
<td>Flexural Modulus</td>
<td>4.0 MPa</td>
</tr>
<tr>
<td>Shear Modulus</td>
<td>2.4 MPa</td>
</tr>
</tbody>
</table>

The thermoplastic structure was designed to have natural frequencies between 40 Hz and 200 Hz. The lower boundary was set to avoid an extremely flexible structure, similar to the Delrin©web spring used by Zhang et al. (2012), for which the change in value of the structural resonances due to gyroscopic effect with respect to their figure at static is significant, also at low speeds of rotation. If this was the case, the RWA would not behave as a rigid structure at low frequencies (i.e. between 0 Hz and 30 Hz) making some of the assumptions and simplifications advanced in chapter 5 not applicable. On the other hand, the upper boundary was selected to prevent an excessively rigid support whose structural resonances would assume values such that the interaction between disturbance harmonics and structural modes is considerably limited, and no or negligible amplifications in the RWA dynamic response would be observed.

In order to achieve a solution which satisfies the frequency requirement, a design trade-off was conducted using a CAD software (e.g. Solidworks), by means of which a series of geometric configurations were generated. Subsequently, these were imported in a FE modelling and analysis software (e.g. MSC Patran/Nastran) where the structural modal resonances, mode shapes, deformations and stresses were evaluated for each design, also including the gyroscopic effect. Finally, the solution which met the frequency requirement and produced the lower deformations and stresses was selected and manufactured.

For both test campaigns (hard-mounted and free-free) a range of speeds spacing from 600 rpm to 4800 rpm (in other terms, 10 Hz to 80 Hz) was selected and a 60 rpm (1 Hz) step increase was applied. The data were recorded for 8 seconds at each speed in both the hard-mounted and free-free test configurations. Signals were sampled at 2048 Hz with a block size of 16384 samples, giving a frequency resolution of 0.25 Hz. Moreover, tests were conducted either during non-office hours or on weekends to limit the level of noise due to normal office-day activities to a minimum and, therefore, reducing any possible interference with the measurements. For instance, the background noise level during both test campaigns,
Figure 4.1: Reaction wheel assembly used during the whole test campaign

measured by either accelerometers (free-free configuration) or force sensors (hard-mounted configuration), is compared to the data derived while running the RWA at 660 rpm (11 Hz) in Figure 4.2. It can be observed that the background noise is significantly lower than the measurement data, hence it is unlikely to affect the quality of the results.

4.2 Signal Processing

The signal processing techniques that are commonly used in this thesis are also briefly introduced.

The dynamic analysis of fast rotating mechanisms, such as RWAs, is typically carried out in the frequency domain. Signal processing techniques implemented in this thesis include: Power Spectral Density (PSD), spectral maps, RMS value and Amplitude Spectrum (AS) waterfall plots, which main features are reported in appendix C. Unless otherwise stated, the data acquired during the extensive test campaign (using the software package m+p SmartOffice) have been initially processed in the time domain and subsequently transformed in the frequency domain by means of Fast Fourier Transformation (FFT). Finally, the aforementioned processing techniques were applied in MATLAB to obtain the results shown throughout the thesis.

The quality of the processing techniques implemented in this thesis can be assessed by means of Parseval’s theorem which states that the total power associated with a signal in either the time or frequency domains must be identical.
Figure 4.2: Background noise compared to the measurement response at 660 rpm (11 Hz): a) axial acceleration in free-free boundary condition and b) axial force in hard-mounted boundary condition
For instance, the cumulative RMS value of the lateral force \( F_{hm,y} \), measured in the hard-mounted configuration, was calculated in both the time and frequency domains. The results are graphed in Figure 4.3 and a perfect overlap can be observed over the whole range of speed, indicating that good signal processing has been applied.

![Figure 4.3: Total RMS value of \( F_{hm,y} \) in both time and frequency domains](image)

### 4.3 Harmonic Response

A preliminary test campaign was carried out to derive the stiffness and damping parameters used to build and update the hard-mounted RWA model defined in section 3.3. In particular, the first three natural frequencies (axial \( f_z \), radial translation or lateral \( f_t \) and radial rotation or rocking \( f_r \)) were extracted and used to calculate the stiffness coefficients \( k_{t,w} \), \( k_{r,w} \) and \( k_{z,w} \) of the thermoplastic motor holder by means of Equation (4.1).

\[
\begin{align*}
  k_{t,w} &= (2\pi f_t)^2 M_w \\
  k_{z,w} &= (2\pi f_z)^2 M_w \\
  k_{r,w} &= (2\pi f_r)^2 I_{r,w}
\end{align*}
\]  

(4.1)

These values were given as inputs to a MATLAB script which, knowing the natural frequencies and the stiffness values, was able to estimate the cantilever parameter \( d \), as described in Figure 3.3. Furthermore, damping values in all DoFs were obtained applying the half-power method and included in the RWA model.
The RWA was mounted on a seismic mass, shown in Figure 4.4, and subsequently secured onto a shaker. The shaker facility was then isolated from the ground using air-cushions. The test setups for measurements in the \( x_w \), \( y_w \) and \( z_w \) axes are shown in Figure 4.5. A sine sweep from 10 Hz to 1000 Hz at 0.1 g was performed and the outcomes are listed in Table 4.2.

![Figure 4.4: Seismic mass used as interface between the RWA and the shaker facility](image)

<table>
<thead>
<tr>
<th></th>
<th>Frequency</th>
<th>Stiffness</th>
<th>Damping</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial</td>
<td>117.1 Hz</td>
<td>353 kNm(^{-1})</td>
<td>35.84 Nsm(^{-1})</td>
</tr>
<tr>
<td>Radial translation (lateral)</td>
<td>153.8 Hz</td>
<td>540 kNm(^{-1})</td>
<td>35.10 Nsm(^{-1})</td>
</tr>
<tr>
<td>Radial rotation (rocking)</td>
<td>46.5 Hz</td>
<td>43.9 Nmrad(^{-1})</td>
<td>0.13 Ns</td>
</tr>
</tbody>
</table>

In section 2.4.3 of the literature review, it was stated that the internal dynamic characteristics of a RWA generally vary not only as a function of the frequency but also as a function of the speed at which the flywheel is spinning. The latter is a consequence of the gyroscopic effect which splits the structural mode into a Backward Whirl (BW) and a Forward Whirl (FW). Whenever a rotating mechanism displays a conical mode shape (see Figures 2.9(c) and 2.10(b)), the gyroscopic effect may act as either stiffening or softening contribution. When the shaft is spinning in the same direction of the whirl (FW), the gyroscopic effect acts as a stiffening element for the radial rotational mode of the mechanism and, thereby, an increase in the resonance frequency of the system is observed. On the contrary, if the motion between the whirl and the shaft is in opposite direction (BW), the gyroscopic effect operates as a softening element hence diminishing the stiffness.
Figure 4.5: Harmonic response test setup: a) $x_w$-axis; b) $y_w$-axis and c) $z_w$-axis
of the system and, therefore, its natural frequencies (Swanson et al., 2005). Due to the coupled DoF dynamics, for cantilever-configured RWAs, this effect severely influences both the radial rotational and translational modes. The extent to which the structural modes of a RWA are affected by the gyroscopic effect depends on the geometry of the RWA, whether it is mid-span or cantilever configured, and also on the geometry of the flywheel. The latter is generally expressed as the ratio between the inertia with respect to the spin axis and the radial moment of inertia. For instance, for a mid-span configured RWA whose flywheel has a ratio between the polar and radial moments of inertia equal to 0.5, the BW and FW form a symmetric V-shaped curve as speed increases; in contrast, for values of the ratio different from 0.5, the curve assumes a rounded shape, similar to that shown in Figure 2.11(a).

For the RWA under investigation, the relationship between the structural modes, the frequency and the flywheel speed is illustrated in Figure 4.6 in terms of the Campbell diagram for the hard-mounted microvibration model.

![Campbell diagram](image)

**Figure 4.6**: Campbell diagram of the cantilever configured RWA used for testing superimposed onto higher and bearing imperfections harmonics (dotted grey and magenta lines, respectively)

The modal resonances are calculated by means of Equations (3.37) and (3.38) for the hard-mounted case. The typical cantilever configured RWA behaviour can be observed. Both the radial modes (translation and rotation) change as speed increases displaying a significant coupling between the two modes. The radial translational mode starts at 154 Hz and evolves as speed increases forming a BW and FW. The rocking mode begins at 46 Hz and, similarly to the radial translational
mode, the initial structural mode splits into a BW and FW. Note that, if the graph is plotted for a speed which tends to infinity, the rocking FW and the radial translational BW would converge. In contrast, the axial translational mode remains constant at a value of 117 Hz throughout the full range of speed hence it can be referred to as being independent from any change in the flywheel speed. Moreover, it is uncoupled from any other DoF. In addition, the structural modes are superimposed on the fundamental harmonic and its fractions representing higher harmonics (integer multiples of the fundamental harmonic) and disturbances due to bearing imperfections, the latter calculated using the equations formulated by Laurens & Decoux (1997b). When the structural modes of the RWA intersect the excitation harmonics, high amplifications in the RWA response spectrum may be experienced. This phenomenon is particularly severe if the harmonic has a high energy content (e.g. fundamental harmonic) and becomes less important as the harmonic number increases.

4.4 HARD-MOUNTED MICROVIBRATION TESTING

4.4.1 HARD-MOUNTED TEST SETUP

The hard-mounted microvibration forces and moments generated by the RWA while in operation were evaluated by means of a microvibration measurement platform validated by Zhang et al. (2012). The platform implements three single-axis force transducers hence, a minimum of three test setups is required for the full characterisation of the RWA. A combination of the responses allow the calculation of the 6 DoF induced-disturbances, including torque. In this work, the force and moments are required to be measured at the RWA mounting points rather than at the sensors location. Therefore, a different set of equations to that proposed by Zhang et al. (2012) is used and the derivation process is described in appendix D.

The test setups are described as follow:

- $x_w$-axis: the seismic mass was rotated by 90 degrees about its yaw axis and the platform was mounted on one of the seismic mass’s lateral surfaces. The RWA was installed on the back plate of the platform. The RWA $x_w$-axis was parallel to the force sensor measurement axis, as shown in Figure 4.7(a). This setup allowed measurement of $F_{hm,x}$ and $M_{hm,y}$.

- $y_w$-axis: the seismic mass and the platform were assembled identically to the $x_w$-axis test setup. However, the RWA was rotated clockwise by 90 degrees about its $z_w$-axis, as shown in Figure 4.7(b). This configuration permits the
evaluation of $F_{hm,y}$ and $M_{hm,x}$:

- $z_w$-axis: the RWA was mounted on the platform so to have the flywheel spin axis ($z_w$-axis) parallel to the force measurement sensors axis, as depicted in Figure 4.7(c). The measurement platform was secured on the top of a seismic mass which was placed on a granite table isolated from the ground. This setup was used to measure $F_{hm,z}$.

In addition, a highly reflective tape (Kapton tape) was attached onto the flywheel edge and an optic sensor was positioned on the platform at a distance of a few millimeters from the flywheel. As the signals acquired from different measurements may be out of phase between each other, the optic sensor served as reference clock to synchronise the data obtained from the different test setups facilitating their post-processing and increasing the reliability of the results. An example of the optic sensor signal superimposed to the signal obtained from one of the force sensors is given in Figure 4.8.

### 4.4.2 Hard-mounted Test Results

The measured microvibration forces $F_{hm,x}$ and $F_{hm,z}$ and moment $M_{hm,y}$ are illustrated in Figure 4.9 as PSD waterfall plots and Figure 4.10 as spectral maps. In order to improve the readability of the results in the spectral map plots, values are represented as $10 \times \log_{10}(\|amplitude\|)$. The PSD waterfall plots are graphed in a logarithmic scale in amplitude only, with maximum flywheel angular speed equal to 4800 rpm and a bandwidth from 20 Hz to 300 Hz.

Common RWA dynamic characteristics are displayed in each DoF. In all the plots, the fundamental harmonic has the largest response due to flywheel mass imbalance. This is particularly dominant in the radial DoFs, as depicted in Figures 4.10(a) and 4.10(c), where the red line represents the fundamental harmonic. Other linear speed dependent lines are higher harmonics due to imperfections in the motor-bearing system which have generally produced a significant smaller response. Although the PSD waterfall plots display approximately ten harmonics, the spectral maps show that only a small fraction of them is actually relevant, that is have magnitude which is not negligible. In addition, also the RWA structural modes can be identified. For instance, the radial translational mode in Figure 4.10(a) starts at 165 Hz and undergoes a gradual change as speed increases. Although not extremely clear in the image, the radial translational BW crosses the second and third harmonic producing high amplifications in the RWA response. The vertical line which begins at about 120 Hz in Figure 4.10(b) is identified as the axial structural mode. Moreover, another line, due to the United Kingdom
Figure 4.7: RWA hard-mounted microvibration test setup: (a) $x_w$-axis; (b) $y_w$-axis and (c) $z_w$-axis
power supply line, can be observed commencing at 50 Hz and remaining constant throughout the whole set of speeds. This can also be noticed in Figure 4.10(c). In the latter, in addition, the rocking FW is also visible; this diverges from a value of 60 Hz towards higher frequencies. These features are shown in Figure 4.11 where a zoomed view of the spectral maps of $F_{hm,z}$ and $M_{hm,y}$ is provided. Furthermore, the radial translation mode at 165 Hz is recorded as well, demonstrating the RWA DoF coupled dynamics typical of a cantilever configuration.

The RWA axisymmetry is assessed by means of comparison between the fundamental harmonics derived from $F_{hm,x}$ and $F_{hm,y}$. In Figure 4.12(a), the two curves have a similar trend and proportionally increase with the squared value of the flywheel angular speed. No resonances are observed in the plot and the curves evolve as a parabola. In addition, a force cone is generated using the fundamental harmonic response amplitudes of $F_{hm,x}$ and $F_{hm,y}$ as the values of the semi-major and semi-minor axes of an ellipse, respectively. For each speed, an ellipse is defined and plotted along the angular speed axis. Due to the absence of resonances up to 4800 rpm (80 Hz), the maximum values of the force cone are reached at the maximum speed considered. The elliptic shape demonstrates that, in practice, the RWA displays a slight antisymmetric behaviour. This may be due to many reasons; for instance, a non uniform distribution of the imperfections in the motor bearings or the influence of bearing friction and damping. Nevertheless, due to their nature, these anomalies are typically important only for higher harmonics.

For fundamental harmonic amplitudes, such those presented in Figure 4.12,
Figure 4.9: PSD waterfall plots of measured forces and moments: (a) $F_{hm,x}$; (b) $F_{hm,z}$ and (c) $M_{hm,y}$
Figure 4.10: Spectral maps of measured forces and moments: (a) $F_{hm,x}$; (b) $F_{hm,z}$ and (c) $M_{hm,y}$
Figure 4.11: Zoomed spectral maps to show UK power line and rocking FW: 
(a) $F_{hm,z}$ and (b) $M_{hm,y}$
the RWA motor harness may represent the principal contributor. Assuming this could be modelled as a single DoF spring-dashpot system, it provides additional stiffness and damping in the RWA radial DoFs. The actual contribution on each DoF is function of the orientation of the motor harness with respect to the RWA coordinate system. Thereby, dissimilarities can be observed in the RWA radial DoFs resonances and response amplitudes. In practice, the RWA stiffness and damping values in the RWA microvibration model can be adjusted accordingly to the test results thus to include the influence of the motor harness. Other approaches may include the physical modelling of the harness.

As far as the axial force $F_{hm,z}$ is concerned, this commonly displays significantly smaller amplitude responses than those observed in the RWA radial DoFs. This is highlighted in Figure 4.13 where the total RMS values of $F_{hm,y}$ and $F_{hm,z}$ are graphed together. It is worth to note, however, that in this case the highest response is not given by the fundamental harmonic but by the second harmonic, whilst the flywheel is spinning at an angular speed of 3600 rpm (60 Hz), as depicted in Figures 4.9(b) and 4.10(b). The second harmonic excites the system at a frequency of 120 Hz, which is the axial resonance frequency of the RWA. Thereby, high amplifications in the system response can be observed.

Recalling the RWA microvibration model described in chapter 3, this was defined absent excitation in the RWA $z_w$-axis. However, the test outcomes have shown that, in practice, this is not the reality as numerous harmonics were recorded during the experimental campaign as illustrated in Figure 4.9(b). These excitations may be the consequence of the combination of two effects:

- a non uniform distribution of the flywheel mass imbalance (fundamental harmonic) and/or irregularities in the motor (higher harmonics);
- coupling between the RWA radial DoFs and the axial DoF.

The latter is due to a misalignment of the $z_w$-axis with respect to the flywheel spin axis hence the generation of an angle between the two axes which would lead to the transfer of a portion of the radial forces along the axial direction. The portion of the force that is transferred is proportional to the sine of the angle between the $z_w$-axis and the flywheel spin axis. This coupled dynamic between the axial and the radial DoFs is also observable in Figure 4.10(a) where amplifications at about 120 Hz (axial structural mode) are displayed. In the RWA microvibration modelling, this misalignment was considered only for the modelling of the excitation force in Equation (3.29), whereas the dynamics in the axial DoF were assumed as an independent one DoF mass-spring-dashpot system.
Figure 4.12: RWA symmetry assessment from hard-mounted forces measurements: (a) fundamental harmonics of $F_{hm,x}$ and $F_{hm,y}$; (b) force cone and (c) force cone top view
For what concerns torque disturbances, these are generally ignored in a typical RWA microvibration analysis, where high angular speeds are involved. Torque disturbances are, in fact, important only at low-speeds of rotation (near zero-speed, where motor cogging and torque ripples generally occur), therefore they are not considered in this study. Thus, the number of DoF investigated in the hard-mounted configuration reduces to five.

4.5 Freew-free Microvibration Testing

4.5.1 Freew-free Test Setup

The freew-free microvibration accelerations, produced by the RWA operating at various speeds, were measured by means of seven accelerometers placed at the RWA mounting interface. Although six accelerometers are generally sufficient to characterise the motion of a 6 DoF system with the guarantee that each pair of accelerometers is placed in each plane (two accelerometers in the xy-plane, two accelerometers in the xz-plane and two accelerometers in the yz-plane), the choice to use seven accelerometers was driven by space restrictions at the RWA mounting interface which did not allow placing two accelerometers either in the xz-plane or in the yz-plane. The RWA was suspended using elastic cords aiming to reproduce an unconstrained boundary condition. The specimen was hung from a rigid metallic frame which acts as an isolation system, restraining any external interaction to a minimum. A combination of the responses allowed computing the 6 DoF induced-
accelerations, including in-plane and out-of-plane rotations. The accelerometers’ locations are shown in Figure 4.14 and these were installed so that:

- two accelerometers were placed in the yz-plane thus to record the radial translation along x;
- two accelerometers were placed in the xz-plane, allowing measurements of the radial translation along y; these were then combined to evaluate the rotational acceleration about the z axis;
- three accelerometers were placed on top of the interface plate, in the xy-plane, tracking the axial translation of the RWA; these were then used to compute the rotational accelerations about the x and y axes.

A similarity with the direct accelerance measurement test configuration shown in Figure 2.4 can be observed. Here, however, the mini-shakers have been removed, significantly simplifying the experimental test set up.

In addition, the reflective tape and optic sensor, used during the hard-mounted microvibration testing, were implemented also here. The optic sensor was glued on a self-standing platform and placed at a distance of a few millimeters from the flywheel. The recorded signal was, however, not used in the course of the analysis of the accelerations, as all responses were measured simultaneously hence did not require to be synchronised among them. The signal registered by the sensor was, on the other hand, used later in the study and served to synchronise the hard-mounted data with the free-free data for the dynamic mass analysis described in chapter 5.

### 4.5.2 Free-free Test Results

The RWA induced-accelerations $\ddot{x}_{mp}$ and $\ddot{z}_{mp}$ and $\ddot{\varphi}_{mp}$ are shown in Figures 4.15 and 4.16 as PSD waterfall plots and spectral maps, respectively. Data are presented as $10 \times \log_{10}(|\text{amplitude}|)$ in the spectral map plots thus to increase their legibility. In addition, responses in PSD waterfall plots are displayed in a logarithmic scale, for a flywheel angular speed equal spacing from 600 rpm (10 Hz) and 4800 rpm (80 Hz) and a frequency band from 20 Hz to 300 Hz.

Similar conclusions to those drawn for the hard-mounted case can be inferred here. The largest response, in particular for the radial DoFs, is given by the fundamental harmonic due to the flywheel mass imbalance. Nevertheless, amplifications due to the interaction of higher harmonics with the RWA structural modes can also be observed. For instance, in Figure 4.16(c), the rocking FW mode matches
Figure 4.14: RWA free-free microvibration test setup: (a) schematic showing accelerometers direction of measurement and (b) detail of accelerometers location and coordinate system.
Figure 4.15: PSD waterfall plots of measured accelerations: (a) $\ddot{x}_{mp}$; (b) $\ddot{z}_{mp}$ and (c) $\ddot{\phi}_{mp}$
Figure 4.16: Spectral maps of measured forces and moments: (a) $\ddot{x}_{mp}$; (b) $\ddot{z}_{mp}$ and (c) $\ddot{\phi}_{mp}$
the frequency associated with the corresponding higher harmonic producing two amplifications as the flywheel spins at 1980 rpm (33 Hz) and 2640 rpm (44 Hz), respectively. This effect is even more evident if the axial acceleration is examined. Multiple amplifications of the RWA response are displayed in Figure 4.16(b) for a frequency of 175 Hz, which is identified as the axial translational mode. Its peculiar characteristic to remain constant throughout the speed range is also here verified. In addition, other RWA structural modes can be recognised. For instance, the radial translational mode occurs at 285 Hz, as depicted in Figure 4.16(a). This can also be observed in Figure 4.16(c) due to the coupled DoF dynamics of a cantilever-configured RWA. Here, the rocking mode evolves from about 60 Hz forming two evident whirls. This behaviour is enhanced in Figure 4.17 where a zoomed view of the rocking BW and FW is shown.

A few remarks on the axial acceleration response can be made. It displays response amplitudes which are smaller compared to the other DoFs, as can be inferred from Figure 4.18 where the total RMS values for $\ddot{x}_{mp}$ and $\ddot{z}_{mp}$ are compared. Moreover, from Figure 4.15(a), it can be observed that, for a given angular speed, the response amplitude is similar over the whole range of frequency. Furthermore, coupled dynamics between the axial and the radial DoFs, due to a possible misalignment between the RWA spin axis and the RWA z-axis, were recorded though these were not modelled during the numerical analysis where the axial DoF was assumed independent.

Finally, a further assessment of the RWA symmetry can be conducted compar-
Figure 4.18: Total RMS value for $\ddot{x}_{mp}$ and $\ddot{z}_{mp}$

ing the acceleration responses in the radial translational DoFs. The RMS values associated with the fundamental harmonic responses and an acceleration cone, similar to the force cone produced for the hard-mounted data, are plotted in Figure 4.19. Despite minor dissimilarities between the x and y responses are observed over the speed band, a significant shift at the resonance is displayed. As aforementioned in section 4.4.2, the main contributor lies in the motor harness distribution. Moreover, the cables of the accelerometers may also introduce additional stiffness along one direction with respect to the other. In order to minimise the impact of the accelerometers’ cables, these were not tensioned but left relatively loose.

4.6 RWA Model Verification

In this section, the models derived in chapter 3 are validated against the test results described previously in this chapter. The first step is the verification of the RWA structural modes using the Campbell diagram. A set of linear harmonic excitations is subsequently introduced and applied to the RWA microvibration model in both hard-mounted and free-free boundary conditions. Finally, the state-space model is derived allowing the calculation of the analytical microvibration-induced RWA model response.
Figure 4.19: RWA symmetry assessment from free-free acceleration measurements: (a) fundamental harmonics of $\ddot{x}_{mp}$ and $\ddot{y}_{mp}$; (b) acceleration cone and (c) acceleration cone top view.
4.6.1 RWA State-space Model and Harmonic Excitation

In Equation (3.29), the excitation vector was defined as the superimposition of the fundamental harmonic, sub and higher harmonics due to irregularities in the motor bearings and broadband noise. Analytically, only the amplitude of the fundamental harmonic was correctly modelled by means of the data derived from the hard-mounted test campaign using a high mass imbalance. A weight of mass 5 grams was positioned on the top edge of the flywheel, at a radius of 40 mm from the spin axis and arm of 20 mm, as depicted in Figure 4.20(a). This weight was able to provide a significant flywheel mass imbalance and thus facilitate the modelling of the fundamental harmonic and also the validation of the model. The RWA response would, in fact, be driven by the high imbalance only, ignoring the effect of the broadband noise and other disturbances being these negligible compared to the fundamental harmonic.

![Diagram](image)

Figure 4.20: RWA analytical response compared to test results at 1200 rpm using a high mass imbalance: a) high imbalance hard-mounted test configuration and b) force $F_{hm,x}$

The modelling of the remaining harmonics and the broadband noise, on the other hand, were beyond the scope of this thesis. Nevertheless, these were included in the excitation vector, $f_s$, defined in Equation (3.27), to identify the complete RWA microvibration-induced response. The higher harmonics were defined as multiple integer of the fundamental harmonics with amplitudes at fractions of the mass imbalance. In addition, the motor bearing was modelled as consisting of
8 balls of radius 1 mm producing sinusoidal disturbances with amplitudes in the order of micro-Newton at specific frequencies as defined by Laurens & Decoux (1997b). Finally, the broadband noise was generated superimposing sinusoidal inputs spacing the whole spectrum of interest (20 - 300 Hz), with random amplitudes, at least two orders smaller than the fundamental harmonic, and random phases. The complete simulation of harmonic excitations, $f_s$, including fundamental harmonics, higher harmonics, harmonics due to bearing irregularities and broadband noise, is illustrated in Figure 4.21 as PSD waterfall plots and spectral maps. Due to RWA axisymmetry, only the components $F_{s,x_{w}}$, $F_{s,z_{w}}$, and $M_{s,y_{w}}$ are represented. Note that higher harmonic excitations have significantly smaller amplitudes than those of the fundamental harmonics.

Forces, moments and accelerations are simulated from the RWA microvibration model using the state space approach in MATLAB. The state space representation for linear physical systems as those described by Equation (3.27) is:

$$\begin{bmatrix}
\dot{q}_s \\
\ddot{q}_s 
\end{bmatrix} = 
\begin{bmatrix}
0 & I \\
-M^{-1}_s K_s & -M^{-1}_s (C_s + G_s)
\end{bmatrix} + 
\begin{bmatrix}
0 \\
M^{-1}_s
\end{bmatrix} f_s$$

Using MATLAB the system displacements and velocities were derived. Subsequently, the loads and the accelerations at the RWA mounting points can be calculated by means of Equation (3.33) and Equation (3.43), respectively. The simulated results are presented in Figure 4.22 superimposed onto the corresponding Campbell diagram data.

The numerical outputs display all the harmonics described in the excitation vector plus amplifications due to the interaction between the structural modes and the input disturbances, as expected. In the following section, the structural dynamic response of the RWA will be compared to the experimental data derived in sections 4.4 and 4.5 so to accomplish the validation of the RWA analytical model.

### 4.6.2 RWA Analytical Structural Modes

Consider the homogeneous form of Equation (3.27) and assume the system is undamped, thereby the classic eigenvalue problem can be solved.

In chapter 3, it was shown that the dynamics of a cantilever configured RWA in the radial DoFs are coupled, hence a closed-form expression of the radial DoF harmonic response cannot be derived. Therefore, the radial EoM requires to be
Figure 4.21: Complete analytical RWA harmonic excitation inputs: a) $F_{s,xw}$ PSD waterfall plot; b) $F_{s,xw}$ spectral map; c) $F_{s,zw}$ PSD waterfall plot; d) $F_{s,zw}$ spectral map; e) $M_{s,yw}$ PSD waterfall plot and f) $M_{s,yw}$ spectral map
Figure 4.22: Analytical forces and accelerations derived from the RWA state space model: (a) $F_{hm,x}$; (b) $\ddot{x}_{mp}$; (c) $F_{hm,z}$; (d) $\ddot{z}_{mp}$; (e) $M_{hm,y}$ and (f) $\ddot{\varphi}_{mp}$
solved simultaneously and numerically. In contrast, the axial DoF is independent and a closed-form expression can be obtained. For the hard-mounted boundary condition the RWA structural modes can be calculated using Equations (3.37) and (3.38), for the radial and axial DoFs, respectively. Equation (3.44), on the other hand, can be implemented for the free-free case to evaluate the radial and axial DoFs structural modes.

The RWA hard-mounted structural modes are shown in Figure 4.23 as Campbell diagrams, with an upper frequency boundary of 300 Hz. They are plotted as black solid lines. The hard-mounted microvibration test results are superimposed to the RWA structural modes and are graphed in terms of spectral maps. An overall good agreement can be observed between the modes and the test data. The radial DoFs structural modes, calculated from the RWA microvibration model, split in a BW and FW, as expected. In contrast to the axial DoF, which is not affected by speed variations, these do not remain constant as speed increases. The radial translational DoF starts at 170 Hz at static (zero-speed) whereas the rocking DoF has an initial value of 46 Hz. Finally, the axial DoF mode begins at 117 Hz and remains constant throughout the speed range. The test results spectral maps display amplifications at speeds for which the RWA structural modes lines cross the excitation harmonics. It must be observed that the RWA model built using the harmonic response data given in Table 4.2, required to be updated thus to provide an improved agreement with the hard-mounted and free-free microvibration test results.

Similarly to the hard-mounted case, the RWA free-free structural modes are plotted as black solid lines in Figure 4.24, up to 300 Hz. The accelerations measured during the free-free microvibration test campaign are superimposed to the RWA modal resonances as spectral maps. In general, all simulated RWA structural modes correlate well with the test outcomes.

It can be concluded that, mathematically, the good agreement of the RWA structural modes displayed in Figures 4.23 and 4.24 has allowed the verification of the RWA analytical model, including the derivation of the RWA model parameters, such as stiffness and inertia, which are listed in Table 4.3.

4.7 SUMMARY

The data derived from the RWA microvibration experimental testing has been used, in this chapter, against the RWA numerical analysis results to validate the RWA microvibration model. The RWA microvibration forces and accelerations have been measured and analysed. Typical RWA dynamic features such as har-
Figure 4.23: Simulated structural modes in the hard-mounted boundary condition superimposed to spectral maps of experimental data: (a) $F_{hm,x}$; (b) $F_{hm,z}$ and (c) $M_{hm,y}$
Figure 4.24: Simulated structural modes in the free-free boundary condition superimposed to spectral maps of experimental data: (a) $\ddot{x}_{mp}$; (b) $\ddot{y}_{mp}$ and (c) $\ddot{\phi}_{mp}$.
Table 4.3: RWA model parameters derived from the RWA hard-mounted (blocked) and free-free microvibration testing

<table>
<thead>
<tr>
<th>Inertia</th>
<th>Stiffness</th>
<th>Natural Frequencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_w$</td>
<td>0.65 kg</td>
<td></td>
</tr>
<tr>
<td>$M_b$</td>
<td>0.53 kg</td>
<td>$k_{z,w}$ 353 kNm$^{-1}$ axial 117 Hz 175 Hz</td>
</tr>
<tr>
<td>$I_{r,w}$</td>
<td>4.6×10$^{-4}$ kgm$^2$</td>
<td>$k_{t,w}$ 660 kNm$^{-1}$ lateral 170 Hz 285 Hz</td>
</tr>
<tr>
<td>$I_{r,b}$</td>
<td>6.8×10$^{-4}$ kgm$^2$</td>
<td>$k_{r,w}$ 44 Nmrad$^{-1}$ rocking 46 Hz 52 Hz</td>
</tr>
</tbody>
</table>

Moreover, coupled dynamics between the axial and the radial responses were also recorded. These were due to a misalignment of the RWA axial axis with respect to the flywheel spin axis. The radial forces generated by the flywheel mass imbalance were redistributed along the $z_w$-axis, at a fraction of their value, hence exciting the axial DoF. An assessment of the RWA symmetry was also accomplished, showing minor differences between the two radial DoFs; here, a non-uniform distribution of the motor harness was identified as the main cause. Subsequently, an analytical excitation vector was built on the basis of the experimental data derived by implementing a high imbalanced mass. The RWA response was then analytically computed by means of a state space model over the whole range of speeds, showing a good agreement with the theoretical predictions. Finally, the RWA structural modes in the hard-mounted and free-free boundary conditions were validated through Campbell diagrams. These were superimposed to the corresponding experimental data displaying a good correlation between the numerical and the experimental results which was used as proof for the validation of the RWA analytical microvibration model.
Dynamic Mass of a RWA

In order to correctly reproduce the dynamics when a microvibration source is mounted on a spacecraft, knowledge of the source dynamic mass is necessary. In chapter 2, several approaches to derive a RWA dynamic mass, either empirically or analytically, were described, all of which, however, presented issues in their application. For example, initial works carried out by Masterson (1999) and Elias (2001) only considered the influence of the frequency, ignoring any effect due to the flywheel angular speed (i.e. gyroscopic effect). These studies were expanded by Basdogan et al. (2007) including the gyroscopic effect but, also here, the formulation was incomplete as stiffness and damping parameters were not considered in the RWA characterisation. Finally, although the method proposed by Zhang et al. (2013) included the gyroscopic effect and was the first conducted in a free-free boundary condition, its application was extremely complex in terms of test setup.

For these reasons, a novel approach to measure the dynamic mass of a RWA has been developed and will be thoroughly investigated in this chapter. The methodology involves the implementation of the hard-mounted loads obtained in section 4.4 and of the free-free accelerations derived in section 4.5. This mathematical procedure will be followed by a discussion of the data retrieved from the experimental campaign. Test data and analytical predictions will be then compared, giving an indication of the level of accuracy that can be achieved. Finally, the convergence of the results will be analysed, giving emphasis to the influence of the broadband noise.
5.1 Mathematical Formulation

The dynamic mass (at times also referred to as apparent mass) represents the ratio between the loads applied to a body and the resulting accelerations, the latter measured at the same location where the load is imparted (Ewins, 2003). The dynamic mass is typically expressed as function of the frequency, however, for rotary mechanisms such those considered in this thesis and described by Equation (3.27), also the angular speed at which the equipment is spinning has to be taken into account. This will, in fact, modify the dynamic response of the mechanism due to the gyroscopic effect.

Generally speaking, the dynamic mass of a 6-DoF system, \( \mathbf{D}_{\text{rwa}} \) assumes the form of a fully populated 6x6 matrix, see Equation (5.1), whose elements are frequency, \( \omega \), and speed, \( \Omega \), dependent and are strongly influenced by the location on the item at which the dynamic mass is evaluated.

\[
\mathbf{D}_{\text{rwa}}(\omega, \Omega) = \begin{bmatrix}
D_{\text{rwa},11} & D_{\text{rwa},12} & D_{\text{rwa},13} & D_{\text{rwa},14} & D_{\text{rwa},15} & D_{\text{rwa},16} \\
D_{\text{rwa},21} & D_{\text{rwa},22} & D_{\text{rwa},23} & D_{\text{rwa},24} & D_{\text{rwa},25} & D_{\text{rwa},26} \\
D_{\text{rwa},31} & D_{\text{rwa},32} & D_{\text{rwa},33} & D_{\text{rwa},34} & D_{\text{rwa},35} & D_{\text{rwa},36} \\
D_{\text{rwa},41} & D_{\text{rwa},42} & D_{\text{rwa},43} & D_{\text{rwa},44} & D_{\text{rwa},45} & D_{\text{rwa},46} \\
D_{\text{rwa},51} & D_{\text{rwa},52} & D_{\text{rwa},53} & D_{\text{rwa},54} & D_{\text{rwa},55} & D_{\text{rwa},56} \\
D_{\text{rwa},61} & D_{\text{rwa},62} & D_{\text{rwa},63} & D_{\text{rwa},64} & D_{\text{rwa},65} & D_{\text{rwa},66}
\end{bmatrix}
\] (5.1)

Previous works conducted by Elias (2004) and Basdogan et al. (2007) demonstrated that the main diagonal elements \( (D_{\text{rwa},ii} \text{ with } i = 1, 2, 3, 4, 5, 6) \) are generally the most influential ones. Moreover, the four cross-DoF coefficients \( D_{\text{rwa},15}, D_{\text{rwa},24}, D_{\text{rwa},42}, \) and \( D_{\text{rwa},51} \) are also significant, in particular for cantilever configured RWAs. Furthermore, some of the off-diagonal elements, which are zero in a static condition (here referred as “zero speed” condition, i.e. flywheel not operating), become significant when the RWA is in a “non-zero speed” condition, hence when the flywheel is spinning. Earlier studies (Basdogan et al., 2007; Elias, 2004; Zhang et al., 2011, 2013) considered these elements negligible and thus set to zero whereas here, the terms correlating the two radial translational DoFs, \( D_{\text{rwa},12} \) and \( D_{\text{rwa},21} \), or those linking the two radial rotational DoFs, \( D_{\text{rwa},45} \) and \( D_{\text{rwa},54} \), assume values, in module, that are comparable to the elements on the main diagonal and, therefore, cannot be ignored (for more details see Addari et al. (2016)).

One of the dynamic mass fundamental properties is its symmetry with respect to its main diagonal. This, added to the RWA axisymmetry characteristic, permits the following assumptions:
• coupled effects between the axial and radial DoFs are ignored. Therefore, elements $D_{rwai3}$ for $i \neq 3$, $D_{rwaij}$ for $j \neq i$, $D_{rwai6}$ for $i \neq 6$ and $D_{rwai6j}$ for $j \neq 6$, are null;

• $D_{rwa,11} = D_{rwa,22};$

• $D_{rwa,44} = D_{rwa,55};$

• $D_{rwa,15} = D_{rwa,51};$

• $D_{rwa,24} = D_{rwa,42};$

• $D_{rwa,24} = -D_{rwa,15};$

• $D_{rwa,21} = -D_{rwa,12};$

• $D_{rwa,14} = D_{rwa,25};$

• $D_{rwa,41} = D_{rwa,52} = -D_{rwa,14} = D_{rwa,25};$

• $D_{rwa,54} = -D_{rwa,45}.$

Substituting into Equation (5.1), the dynamic mass matrix $D_{rwa}$ simplifies and assumes now the following form:

$$
D_{rwa} = \begin{bmatrix}
D_{rwa,11} & D_{rwa,12} & 0 & D_{rwa,14} & D_{rwa,15} & 0 \\
-D_{rwa,12} & D_{rwa,11} & 0 & -D_{rwa,15} & D_{rwa,14} & 0 \\
0 & 0 & D_{rwa,33} & 0 & 0 & 0 \\
-D_{rwa,14} & -D_{rwa,15} & 0 & D_{rwa,44} & D_{rwa,45} & 0 \\
D_{rwa,15} & -D_{rwa,14} & 0 & -D_{rwa,45} & D_{rwa,44} & 0 \\
0 & 0 & 0 & 0 & D_{rwa,66}
\end{bmatrix}
$$

The initial 36 coefficients are now reduced to only 8, 5 boxed and 3 circled in Equation (5.2). Direct measurements of the RWA accelerance matrix (Zhang et al., 2012, 2013) allow obtaining the 5 boxed elements only, ignoring the remaining elements. Moreover, this procedure poses high challenges due to its complexity in terms of test configuration. Therefore, an alternative and novel methodology has been investigated in this work to obtain the dynamic mass coefficients by means of an indirect measurement approach and allow the derivation of the whole set of coefficients.
Masterson et al. (2002) defined the relationship which describes the coupled dynamics that are generated when a source is mounted on its supporting structure. The work has, subsequently, been expanded by Elias et al. (2003) and Zhang et al. (2013) to include further information on the dynamic mass knowledge and influence. The fundamental idea is that the forces and moments, transmitted between a source and its supporting structure at the interface between the two bodies, can be fully defined if knowledge of the inertial and stiffness properties of the bodies (i.e., their dynamic mass) and of the excitations produced by the source, is provided. In particular, it was observed that the coupled loads can be represented as a fraction of the loads obtained testing the RWA in a hard-mounted configuration $f_{hm}$:

$$f_{cp} = f_{hm} - D_{rwa} \ddot{x}_{cp} \quad (5.3)$$

where “$cp$” denotes coupling, and $f_{cp}$ and $\ddot{x}_{cp}$ are 6x1 vectors representing the coupled loads and the coupled accelerations, respectively. Note that, at present, only $f_{hm}$ is currently available from previous results discussed in this work.

In this study, a re-arrangement of the coupling theory has been conducted which allows the derivation of the dynamic mass coefficients by means of the data collected during the hard-mounted and free-free test campaigns. Consider the RWA is operating in a free-free configuration, without being attached to any other structure, the term on the left side of Equation (5.3) is, therefore, null. Moreover, the coupled accelerations vector can be re-written in terms of the accelerations at the RWA mounting points, $\ddot{x}_{mp}$, yielding to:

$$f_{hm} = D_{rwa} \ddot{x}_{mp} \quad (5.4)$$

For each element on the left hand side of Equation (5.4) corresponds an equation for which the unknowns are the dynamic mass coefficients. Thus, it is possible to write six equations in eight unknowns. From a mathematical point of view, a system where the number of unknowns is higher than the number of equations, such that described by Equation (5.4), provides infinite solutions. A possible and feasible solution to overcome this problem is to re-formulate Equation (5.4) in terms of PSD. The application of the PSD to a $6 \times 1$ vector results in a $6 \times 6$ matrix whose elements are obtained from the auto- and cross-correlation between the vector elements. On the other hand, when the PSD is applied to a $6 \times 6$ matrix, the output is still a $6 \times 6$ matrix but here the coefficients are computed as the product between the original matrix and its conjugate and transpose value (also referred to as the Hermitian and denoted with the superscript “H”).
The force vector $f_{hm}$ and the acceleration vector $\ddot{x}_{mp}$ are transformed into $6 \times 6$ matrices, $\Phi_{hm}$ and $a_{mp}$, respectively. The cross-product elements of the obtained matrices have the peculiar characteristic for which the $ij$-th element can be expressed as the conjugate of the $ji$-th element for $i = j$, and conversely. Finally, Equation (5.4) can be re-formulated as:

$$\Phi_{hm} = D_{rwa}a_{mp}D_{rwa}^H \quad (5.5)$$

The amount of available equations has significantly increased from just 6 to 36 meaning that the application of the PSD has introduced further information, given by the cross-product terms, which can be used to derive the unknowns elements of the dynamic mass matrix. On the other hand, the number of unknowns has remained as low as the initial figure allowing for the system described by Equation (5.5) to be solvable. A unique solution, which fully characterise the RWA, can, therefore, be found. Note that, in order to completely reproduce the RWA dynamic mass behaviour, Equation (5.5) must be solved for each speed at which the RWA operates and for the entire frequency spectrum of interest.

5.1.1 Iterative Process

The simplest approach to accomplish the integral RWA characterisation would be to implement an iterative process, using a mathematical software package (i.e. MATLAB), such as that presented in Figure 5.1 and summarised as follow:

i. select a common speed at which the hard-mounted and free-free test campaigns were conducted;

ii. convert the experimental time domain data into the frequency domain by means of the FFT;

iii. generate the $\Phi_{hm}$ and $a_{mp}$ matrices using the FFT signals over the frequency range of interest;

iv. separate the real and imaginary contribution so as to obtain twice the number of equations;

v. define the 8 complex unknowns representing the dynamic mass coefficients and assume an initial guess (a total of 16 parameters, 8 for the real part and 8 for the imaginary part);

vi. solve Equation (5.5) for the initial frequency of interest (e.g. 1 Hz);
vii. update the initial guess with the output from the first iteration;
viii. solve Equation (5.5) for the following frequency of interest using the updated initial guess;
ix. repeat steps vii to ix until the whole frequency range of interest is covered.

![Diagram](image)

**Figure 5.1: Iterative process to evaluate the dynamic mass from measurements of hard-mounted forces and free-free accelerations**

The main concern arises from the initial figures assigned to the unknown dynamic mass coefficients. In order for Equation (5.5) to provide reliable and trustworthy results, the initial guess has to be as accurate as possible, because it is a high nonlinear problem. Nevertheless, errors up to 10% on the primary guess have shown to not significantly affect the reliability of the process. In practice, the easiest values that can be actually measured and then used as trustworthy guesses, are those obtained from measurements of the mass and of the inertia of the RWA when static. When the RWA is not operating, the elements on the main diagonal of the dynamic mass matrix in Equation (5.2) represent the mass and the inertia of the RWA, the latter evaluated at the RWA mounting points. If a direct measurement of the RWA accelerance matrix is performed, also the cross-DoF element $D_{\text{rwa,15}}$ can be estimated and then used as a reliable initial guess.
In addition, the choice to split the real and imaginary parts has been driven by the necessity to increase and improve the control on the system variables. This led to the ability to treat each parameter as a real rather than a complex number, facilitating the operations carried out via software.

5.2 Numerical Analysis

The feasibility and applicability of Equation (5.5) were both assessed by means of a numerical approach. Using the RWA model described in chapter 3, the hard-mounted loads and the free-free accelerations were firstly calculated and then implemented in Equation (5.5). The resulting analytical dynamic mass was then compared to the analytical expression given by applying the method proposed by Zhang et al. (2013), providing a first indication of the level of accuracy that can be achieved. The implementation of the traditional approach required for Equation (3.41) to be re-formulated by means of the Laplace variable and then to be re-arranged for the acceleration vector, thus to obtain:

$$\ddot{q}_s = \left[ M_s + \frac{1}{i\omega} (C_{fis} + G_s) - \frac{1}{\omega^2} K_{fis} \right]^{-1} f_s \quad (5.6)$$

The expression between the brackets represents the transfer function between the loads imparted to the RWA flywheel CoM, $O_w$ and mounting point (also referred to as driving point) and the resulting co-located accelerations. Assuming the application of a unit load, over the whole frequency spectrum of interest, at each DoF of the RWA, one at a time. The resulting vector would be equal to the first column of the transfer function matrix. Repeating this for all the remaining DoFs and placing side by side the resulting acceleration vectors, a $12 \times 12$ matrix can be built (although in practice this reduces to $10 \times 10$). This matrix takes the name of the accelerance matrix of the RWA, and is represented as $A_{rwa}$. Our interest is, however, limited to the DoFs describing the motion of the RWA mounting point exclusively, hence only a portion of the accelerance matrix is considered. Dividing the $12 \times 12$ matrix described in Equation (5.7) into 4 quadrants, the RWA mounting point accelerance matrix, $A_{mp}$, is composed by the elements in the forth quadrant:

$$A_{rwa} = \begin{bmatrix} A_{rwa,11...66}^{(1)} & A_{rwa,17...612}^{(2)} \\ A_{rwa,71...126}^{(3)} & A_{rwa,77...1212}^{(4)} \end{bmatrix}$$

$$A_{mp} = A_{rwa,77...1212}^{(4)} \quad (5.7)$$

The RWA dynamic mass is finally calculated by inverting $A_{mp}$:
\[ D_{rwa} = A_{mp}^{-1} \]  

(5.8)

In Figure 5.2 the dynamic mass elements \( D_{rwa,11} \), \( D_{rwa,44} \) and \( D_{rwa,15} \) derived from the direct accelerance method are superimposed onto those obtained applying the novel methodology presented in this thesis, for a RWA flywheel speed of 2400 rpm (40 Hz). It is clearly visible how all the coefficients perfectly match the accelerance approach predictions, confirming the high level of accuracy that can be achieved with the proposed method.

5.2.1 Influence of the Gyroscopic Effect on the Dynamic Mass Behaviour

The importance of considering the gyroscopic effect in the analysis of a cantilever configured RWA has already been highlighted in the previous chapters of this thesis. Here, we intend to provide a clearer view of how this phenomenon actually modifies the RWA dynamic mass response. Consider the RWA firstly when static and then with the flywheel spinning at a constant speed. Figure 5.3 shows the comparison between the elements \( D_{rwa,11} \), \( D_{rwa,33} \) and \( D_{rwa,44} \) at a “zero speed” and “non-zero speed” conditions. The axial DoF displays no changes with speed, as expected. The RWA was, in fact, designed and modelled so that the axial translational DoF remains uncoupled with the other DoFs and is also invariant to speed. In contrast, the elements correlating input force/moment along the x-axis and output translational/rotational accelerations along the same axis show a significant dependence on the flywheel angular speed. The initial two natural resonances calculated when static split into four modal resonances due to the gyroscopic effect. The effect becomes even more evident as speed increases.

A further important feature correlated to the gyroscopic effect is that some terms in the dynamic mass matrix assume values (in module) which are comparable to those generally regarded as the fundamental parameters. For instance, consider the elements \( D_{rwa,12} \) and \( D_{rwa,45} \). The first links the radial translation DoFs whereas the latter correlates the radial rotational DoFs. When the RWA flywheel is not spinning, these elements are ignored as their contribution to the dynamic mass is zero. Nevertheless, when the RWA flywheel is rotating at a fixed speed, their contribution grows in importance and, therefore, cannot be neglected. This behaviour is represented in Figure 5.4 where the responses of the aforementioned elements are compared in the static and operational conditions. The element \( D_{rwa,14} \) is also plotted to provide a thorough overview of the off-diagonal dynamic mass elements’ response. A linear scale, rather than a logarithmic one,
Figure 5.2: Comparison between dynamic mass elements obtained using the traditional and the novel methodologies whilst the flywheel spins at 2400 rpm: (a) element $D_{\text{rwa},11}$; (b) $D_{\text{rwa},44}$ and (c) $D_{\text{rwa},15}$
Figure 5.3: Effect of the gyroscopic effect on the dynamic mass response: (a) element \(D_{rwa,11}\); (b) element \(D_{rwa,33}\) and (c) element \(D_{rwa,44}\).
has been adopted here to facilitate the display of the analytical responses when static due to their zero constant value throughout the whole range of frequency.

Note that, the conclusions here inferred are valid for a cantilever configured RWA only. When a symmetric configured RWA is considered, the gyroscopic effect is not as influential. For instance, both the elements $D_{rwa,11}$ and $D_{rwa,12}$ are not affected by any variation in the flywheel speed hence their behaviour remains constant and equal to the one when static. In particular, element $D_{rwa,12}$ assumes a constant zero value due to the fact that, for symmetrical RWA configurations, no coupled dynamics exist between the radial translational DoFs. On the other hand, the element $D_{rwa,45}$ is still affected by the gyroscopic effect due to coupled dynamics between the radial rotational DoFs, hence a dependence on speed fluctuations would be observed.

5.2.2 Complete Dynamic Mass Response

The dynamic mass static response can be expanded in the speed domain to provide a complete and comprehensive understanding of the RWA internal dynamics characteristics. For simplicity and ease of representation, only a few elements of the dynamic mass are shown in Figure 5.5; these are: $D_{rwa,11}$, $D_{rwa,15}$ and $D_{rwa,45}$.

The coefficients are graphed in a frequency band from 20 Hz to 300 Hz and in a speed range spacing from 600 rpm (10 Hz) to 4800 rpm (80 Hz) with 60 rpm step, giving a resolution in the speed spectrum of 60 rpm, or 1 Hz. All the coefficients display a strong dependence on both frequency and speed. Moreover, they present coupled effects between each DoF, due to the cantilever RWA configuration. The axial translation DoF, on the other hand, shows a completely different behaviour. Here no other resonances are generated by the gyroscopic effect and the response remains uniform throughout the frequency and speed ranges, as depicted in Figure 5.6.

5.3 Direct Accelerance Measurements

In this section, the RWA dynamic mass coefficients when static are derived experimentally by means of the direct accelerance measurement method and used to further verify the RWA model defined in chapter 3. They were subsequently considered as a benchmark set of values for the iterative process defined in section 5.1.1.
Figure 5.4: Effect of the gyroscopic effect on the off-diagonal elements of dynamic mass: (a) element $D_{rwa,12}$; (b) element $D_{rwa,14}$ and (c) element $D_{rwa,45}$. 
Figure 5.5: Evolution of the dynamic mass response as function of speed: (a) element $D_{rwa,11}$; (b) element $D_{rwa,15}$ and (c) element $D_{rwa,45}$
5.3.1 Accelerance Test Setup

Due to RWA axisymmetry, only three test configurations were required to derive the five dynamic mass elements boxed in Equation (5.2). The test setups are illustrated in Figure 5.7. Sine-sweeps were carried out in a frequency band from 20 Hz to 300 Hz and all signals were sampled at 2048 Hz.

In each test, the RWA was hung by means of elastic cords and nylon wires from a ground-fixed steel frame, as to reproduce a free-free boundary condition. The suspension system was designed to achieve rigid body natural frequencies smaller than 1 Hz in any of the 6 DoFs. In addition, either one or two mini-shakers were hung, in a similar manner as the RWA, aligned to the RWA mounting surface and used to impart forces or moments at the RWA mounting interface. An open-loop control strategy was selected and a 1 Volt constant voltage applied. A force sensor was installed in between each mini-shaker and the RWA so to measure the actual force applied by the input device. A set of 7 accelerometers was mounted on the RWA mounting interface to measure the RWA acceleration responses, as depicted in Figure 5.8. Prior to the experiment, a tap test was conducted on the steel frame verifying this would not influence the test results in the frequency band of interest, that is its resonance frequencies were well beyond 300 Hz thus to not introduce undesirable modes in the measurements.

Following is a detailed description of each test setup:

- RWA excited in the y-axis by means of a single mini-shaker, as depicted in Figure 5.7(a). This test configuration allowed the derivation of elements $A_{mp,11}$ and $A_{mp,15}$. The former requires the resulting acceleration to be mea-
Figure 5.7: Direct accelerance measurement test setup: (a) $A_{mp,11}$ and $A_{mp,15}$; (b) $A_{mp,33}$ and $A_{mp,44}$ and (c) $A_{mp,66}$
sured in the same direction of the excitation force. The latter, instead, necessitates the angular acceleration in the opposite axis (i.e. angular acceleration about the $x$-axis):

$$\ddot{\theta}_{mp} = \frac{1}{2} \left( a_6 - a_5 \right) \quad (5.10)$$

where $d_{56}$ is the distance between accelerometers 5 and 6;

- RWA excited in the $z$-axis by means of two mini-shakers, as illustrated in Figure 5.7(b). This setup permitted the retrieving of the elements $A_{mp,33}$ and $A_{mp,44}$. Initially, the mini-shakers were controlled in-phase thus applying a force along the $z$-axis. The resulting axial translational DoF acceleration was then calculated using Equation (5.11) and further processed to obtain $A_{mp,33}$. Subsequently, the mini-shakers were controlled anti-phase so to impart an oscillating moment about the $y$-axis. The angular acceleration about the $y$-axis was calculated by means of Equation (5.12), allowing the derivation of $A_{mp,44}$:

$$\ddot{z}_{mp} = -\frac{1}{2} \left( a_5 + a_7 \right) \quad (5.11)$$
\[ \ddot{\varphi}_{mp} = \frac{1}{2d_{67}} (a_6 - a_7) \]  
\hfill (5.12)

where \( d_{67} \) is the distance between accelerometers 6 and 7;

- RWA excited in the y-axis by means of two mini-shakers, as shown in Figure 5.7(c). These were controlled anti-phase so to generate a moment about the z-axis at the RWA mounting point. The resulting torsional acceleration was calculated using Equation (5.13). This permitted to obtain the coefficient \( A_{mp,66} \):

\[ \ddot{\psi}_{mp} = \frac{1}{2d_{34}} (a_4 - a_3) \]  
\hfill (5.13)

where \( d_{34} \) is the distance between accelerometers 3 and 4.

The elements \( A_{mp,11}, A_{mp,15}, A_{mp,33}, A_{mp,44} \) and \( A_{mp,66} \) were then used to build the accelerance matrix \( A_{mp} \) which was subsequently inverted to derive the RWA dynamic mass matrix, \( D_{rwa} \), as defined in Equation (5.8).

### 5.3.2 Accelerance Test Data

The \( 6 \times 6 \) RWA dynamic mass matrix was firstly obtained from the RWA FE model, built in MSC Patran/Nastran using the data discussed in chapter 4. This consisted of a series of lumped masses representing the flywheel, the motor and the wheel-base connected by a combination of springs and dashpots to reproduce the RWA flexible components, as graphed in Figure 5.9. Note that, the gyroscopic effect was added to the flywheel opportunely modifying the node definition matrix. The RWA mounting point was represented as a massless node and connected rigidly to the wheel-base by means of rigid body elements. The frequency response analysis was performed applying a unit force or moment at the RWA mounting point node in one of the 6 DoFs with a 1 Hz frequency interval and a bandwidth from 20 Hz to 300 Hz; accelerations were, then, evaluated at the same location, thus to generate a \( 6 \times 1 \) acceleration vector. The process was, subsequently, conducted for the remaining 5 DoFs, and a \( 6 \times 6 \) accelerance matrix was eventually produced. Finally, the matrix was inverted and the dynamic mass obtained.

The dynamic mass element \( D_{rwa,33} \) from the analytical model and experimental test are compared in Figure 5.10. A good correlation over the frequency band of interest can be observed. The RWA structural modes have been correctly captured. In addition, recalling that the inertia and stiffness parameters in the FE model were obtained from the force and acceleration microvibration test results, the agreement
Figure 5.9: RWA FE model

shown here demonstrates the consistency of the model used in the different analyses throughout the thesis.

Figure 5.10: RWA dynamic mass response comparison between FE model and direct accelerance measurements at static in the axial DoF
5.4 Empirical Dynamic Mass Measurement Method

Throughout the thesis, it has been thoroughly remarked how the results obtained in a zero-speed condition are flawed due to not taking into account the gyroscopic effect. As a matter of fact, this may lead to significant inaccuracies in the prediction of the actual coupled dynamics that a RWA-structure system experiences. The method introduced in section 5.1 is here applied to a real case scenario. The outputs from the analyses discussed in sections 4.4 and 4.5 were combined to calculate the frequency and speed dependent dynamic mass of the RWA over a broadband set of speeds.

Prior to their direct implementation in Equation (5.5), it was necessary to perform a synchronisation of the signals obtained from different test configurations. This necessity is driven by the complex nature of the dynamic mass coefficients, which are constituted of a real and imaginary part. Thereby, not only the amplitude of the signals is important but also the relative phase between them. The overall signals' synchronisation was achieved by means of cross-correlations and phase shifting. The synchronised data were then reformulated in terms of PSD and cross-PSD to form two $6 \times 6$ complex matrices, $\Phi_{hm}$ and $a_{mp}$, respectively. The matrices were subsequently implemented in Equation (5.5) to retrieve the dynamic mass matrix. The synchronisation process is outlined as follow:

i. select $\Omega$ of interest and define the nominal number of samples, $nT_{\text{samples}}$, between two consecutive peaks in the optic sensor signal as the product between the period of the flywheel and the sampling frequency;

ii. select the optic sensor signals from one of the various test campaigns (i.e. from the hard-mounted z-axis test) and use them as reference clock signals;

iii. normalise the reference clock signals with respect to their maximum amplitude (in absolute value) and divide the signal in segments given as the data between two consecutive peaks;

iv. compare the number of samples between each segment with $nT_{\text{samples}}$. If the number is equal then save the segment otherwise discard it;

v. using the saved segments generate the ultimate reference clock signal;

vi. cross-correlate the ultimate reference clock signal with the normalised optic sensor signals from the different test configurations and measure the lag, in terms of number of samples, between them;
vii. phase-shift the normalised optic sensor signals accordingly to the measured lag to obtain the signal synchronisation. In other terms, shift the normalised optic sensor signals forward or backward by a number of computational bits, where the number of computational bits is equal to the lag;

viii. re-build all the tests data using the synchronised optic sensor signals;

ix. repeat the process for all the speeds at which the RWA is tested and analysed.

Equation (5.5) was solved for each frequency of interest (20 Hz to 300 Hz with 1 Hz resolution) and for each angular speed (600 rpm to 4800 rpm with 60 rpm step increase or, in other terms, 10 Hz to 80 Hz with 1 Hz resolution) granting the complete characterisation of the RWA dynamic response. The high non-linearity feature of the system being analysed, which consists of 72 equations (half for the real part and the other for the imaginary part) and 16 real unknowns (8 describing the real part of the dynamic mass coefficients and the other 8 representing the imaginary part), poses a significant threat to the convergence of the results to the desired solution. To limit this issue, the simplest approach is to provide an initial set of values for the 16 unknowns which is trustworthy and the closest possible to the actual values.

The RWA examined in this study features a rigid behaviour at frequencies lower than 35 Hz or, in other terms, before the first structural mode occurs. In addition, the gyroscopic effect is not considerably influential at low speeds (i.e. up to 900 rpm or 15 Hz), as depicted in Figure 5.11 (note, a logarithmic scale has been used here to increase the readability at lower frequencies). Therefore, it can be said that the dynamic mass coefficients measured when static do not differ from those the RWA would display for frequencies in the range from 0 Hz to 35 Hz and at speeds lower than 900 rpm (15 Hz). Hence, these could be used as the set of reliable initial guesses for the dynamic mass unknown coefficients. Nevertheless, if dynamic mass measurements when static are not possible, another solution would be solely to use the mass and moment of inertia of the RWA. In this case, however, only a portion of the unknown coefficients could be defined, that is the elements on the main diagonal of the dynamic mass matrix.

The outcomes of the analysis were subsequently compared to the theoretical predictions and are here illustrated in Figure 5.12 for an angular speed $\Omega$ equal to 1320 rpm (22 Hz).

Despite the experimental outcomes, in particular in the axial translational DoF, display an overall satisfactory correlation with the predicted response, in a few frequency ranges the agreement is not as good. For instance, between 20 Hz and 50 Hz, the element $D_{rwa,44}$, in Figure 5.12(b), experiences a weaker correlation.
Although the experimental and the numerical plots follow a similar pattern, the former has an amplitude which is lower. Nonetheless, the agreement improves as the frequency passes beyond the first BW and FW. The theoretical modes at about 150 Hz and 175 Hz are correctly captured, with some margin, by the methodology here described. For a minor shift towards slightly higher frequencies is observable, the error is within the 2%, hence validating the results here illustrated. Moving on the right along the frequency axis, the experimental data curve decreases with the same gradient as the FE model estimates. Nevertheless, further mismatching can be observed between 230 Hz and 260 Hz due to the presence of a few spikes in the experimental outcomes which were not predicted in the numerical calculation. For what concerns the axial translational mode, the empirical dynamic mass method shows to be able to correctly reproduce the dynamic mass response over the whole frequency band of interest, as depicted in Figure 5.12(a), also capturing the mode at 117 Hz.

The causes which led to the disagreements observed in Figure 5.12(b) were comprehensively analysed. The main factor was recognised in the distinct spectral characteristics of the broadband noise measured during the different phases of the test campaign. When a RWA is in operation, the excitations to which this is subject are not only function of the fundamental harmonic and fractions or multiples of this but, also of the broadband noise given by the environment where the RWA is being tested, as described by Equation (3.29). The spectral contribution of the latter may be different during the hard-mounted and free-free
Figure 5.12: Comparison between the predicted and experimental dynamic mass at 1320 rpm: (a) element $D_{\text{rwa,33}}$ and (b) element $D_{\text{rwa,44}}$ (Addari et al., 2016)
test campaigns leading to a fail in the identity given by Equation (5.5). Our objective is, therefore, to compare the spectral contribution of the broadband noise derived during the multiple stages of the test campaign. It is possible to consider the noise measured in the hard-mounted boundary condition and that measured in the free-free boundary condition. The former can be directly re-formulated in terms of PSD to generate the matrix $\Phi_{hm}$ of the hard-mounted noise. In order to perform the comparison, the noise from the free-free test requires conversion to the same unit of the hard-mounted case (measured in $N^2/Hz$). Therefore, the free-free noise is firstly transformed in $a_{mp}$ (measured in $g^2/Hz$) and subsequently multiplied by the theoretical dynamic mass matrix using Equation (5.5) to obtain a semi-empirical equivalent noise $\Phi_{mp}$, also expressed in $N^2/Hz$. The two noise spectral contributions can now be compared, as illustrated in Figure 5.13 for the main diagonal elements.

The spectral contribution obtained from the free-free test configuration, in the regions where a poor correlation was recorded, differs from that measured during the hard-mounted boundary condition tests. Therefore, the broadband noise has, somehow, affected the convergence of the test results. This is further examined in section 5.5, where the influence of the broadband noise is investigated by means of the analytical model. The outcomes are then used to validate the aforementioned thesis.

5.5 Broadband Noise Influence on the Dynamic Mass Results

The accuracy of the empirical dynamic mass method may be jeopardised if the spectra of the broadband noise, measured throughout the hard-mounted and free-free test campaigns, significantly differ. The iterative process described in section 5.1.1 may, therefore, fail or, eventually, lead to an improper calculation of the dynamic mass coefficients for some frequencies of the spectrum. The reason lies in the identity in Equation (5.5) for which, assuming the dynamic mass matrix $D_{rwa}$ is known and given a set of accelerations thus to generate the matrix $a_{mp}$, only one possible force matrix $\Phi_{hm}$ exists. Yet, if $\Phi_{hm}$ is derived experimentally there’s no guarantee that the identity is respected. This would mean that the excitation vector in the hard-mounted test differs from the excitation vector applied in the free-free test. However, being the RWA the same in both test configurations and, therefore, being the set of harmonics generated the same throughout the full range of tests, the only disturbance parameter which may vary is the noise. This
Figure 5.13: Comparison between the noise spectral contribution from the hard-mounted and free-free tests: (a) element $\Phi_{11}$; (b) element $\Phi_{33}$ and (c) $\Phi_{44}$.
matter was investigated numerically using the model described in chapter 3. Two approaches were considered: the analysis was initially conducted at a zero-speed condition hence considering the excitation vector consisting of the environmental noise only; subsequently, the analysis was expanded to include also the first 5 harmonics due to the flywheel spinning at 1800 rpm (30 Hz). A set of pseudo-random signals was generated and applied to the models. A noise ratio, $\eta$, was defined as the ratio between the RMS value of the noise signal applied to the hard-mounted model and of the noise signal employed in the free-free model.

The results from applying only the environmental noise, displayed in Figure 5.14, presented a notable difference as the noise ratio increased. Nevertheless, this effect was less influential in the case of the axial DoF compared to the other DoFs. Moreover, the element $D_{rwa,33}$ deviation displayed a linear relationship with the noise ratio. Furthermore, it was observed that even small differences at high frequencies may lead to significant errors in the dynamic mass calculation, as shown in Figures 5.14(a) and 5.14(c).

For what concerns the second case investigated, the analysis outputs are plotted in Figure 5.15, where the off-diagonal dynamic mass elements are included as well. Similarly to the static case, also here the data showed a significant dependence on the noise ratio. The axial DoF element, in Figure 5.15(a), showed the same behaviour as the one showed in Figure 5.14(b), as expected. In all other elements, two regions in particular can be identified where the different excitation vectors applied to the models led to miscalculations in the dynamic mass response. These are in the bandwidth of the first natural mode (first BW and FW) and for frequencies above 230 Hz. It’s worth to remark that, compared to the static case, three more unknowns are introduced in this case study. These increase the complexity of the problem and, therefore, may worsen the quality of the output.

It can be concluded that, for the system described by Equation (5.5) to converge to trustworthy and flawless results, the disturbances generated by the flywheel must be identical in both the hard-mounted and free-free boundary conditions. Assuming that the RWA aging effects are negligible (i.e. the motor bearing system maintains the same characteristics during the whole test campaign) and that the flywheel properties remain unaltered, the only parameter in the excitation vector that changes throughout the experimental tests can be identified in the broadband noise. The two case studies examined here demonstrated that the hypothesis advanced in section 5.4, for which the poor correlations occurred between 30 Hz and 40 Hz and between 230 Hz and 260 Hz were due to different contributions in the frequency spectrum of the broadband noise measured in the hard-mounted and free-free test configurations, were solid and well-founded. Thereby, in practice,
Figure 5.14: Effect of different noise inputs on the calculation of the RWA dynamic mass at static: (a) element $D_{rwa,11}$; (b) element $D_{rwa,33}$ and (c) element $D_{rwa,44}$.
Figure 5.15: Effect of different noise inputs on the calculation of the RWA dynamic mass at 1800 rpm: (a) element $D_{rwa,33}$; (b) element $D_{rwa,44}$; (c) element $D_{rwa,15}$; (d) element $D_{rwa,12}$; (e) element $D_{rwa,14}$ and (f) element $D_{rwa,45}$.
particular attention must be taken during the experimental campaign to reproduce exact, or very close, noise conditions within 5%.

5.6 Summary

The mathematical formulation for the calculation of the dynamic mass of a RWA, over a broad range of speeds and frequencies, has been introduced and applied to a real case scenario in this chapter. The methodology involves the use of the loads measured in a hard-mounted boundary condition and of the accelerations at the RWA mounting point evaluated in a free-free boundary condition, which are combined and allow the complete characterisation of a RWA dynamic mass by means of an iterative process. The innovative approach was initially compared to the numerical results obtained implementing the traditional method displaying a good level of agreement. In addition, it was shown that, due to the gyroscopic effect, some of the off-diagonal elements in the dynamic mass matrix cannot be ignored when the RWA is in operation, as, in contrast, all previous studies erroneously assumed. The RWA dynamic mass was subsequently derived by means of the experimental data derived in chapter 4 and compared to the analytical predictions. The results presented a good level of accuracy, though some discrepancies were observed and investigated using the RWA mathematical model. The dominant reason which led to errors in the estimation of the dynamic mass coefficients was identified in the different spectral contribution of the disturbances produced by the RWA during the multiple phases of the test campaign. In particular, it was concluded that the noise may significantly interfere in the convergence of the iterative process to retrieve the RWA dynamic mass.
CHAPTER 6

COUPLED ANALYSIS

The dynamic mass calculated in the previous chapter is now utilised in a final practical application where the RWA is mounted on a ”flexible” supporting structure. Here the dynamics of the RWA couple with those of the supporting structure, modifying the entire system response (i.e. shift of resonance frequencies) and making the predictions more difficult. The RWA microvibration analysis in a coupled boundary condition is described and discussed in this chapter and used to predict, with a high level of accuracy, the loads exchanged between the RWA and its supporting structure at their interface and, also, estimate the response of the structure at any location. The investigation was accomplished both experimentally and analytically. The former involved the RWA to be mounted on either a FFSM or on a honeycomb structural panel. The coupled test experiments were performed in the Surrey Space Centre facility and, with the aim to attenuate the background noise level, the test activities were run overnight and human interactions were restricted to a minimum. Analytically, the coupling theory developed by Masterson (1999) and Elias (2001) was applied and extended to include four different approaches for the modelling of the RWA-supporting structure connection. Results have shown that the best estimates can be achieved only if the dynamic mass of the RWA including the gyroscopic effect is considered.

6.1 THEORY OF COUPLING

The common practice to assess the dynamic coupling between a microvibration source and its supporting structure is to measure the forces and moments generated by the source when rigidly mounted on a dynamometric platform (i.e. Kistler
table) and, subsequently, apply them directly to the supporting structure, thus estimating the impact of the source. However, this approach is incorrect, as the actual loads exchanged by the bodies are also a function of the dynamic characteristics of the source and of the support. This can be simply demonstrated by considering the systems composed of two counter-rotating masses supported by a platform of stiffness $k$, shown in Figure 6.1.

$$F = ml\omega^2$$

(6.1)

This formula would provide accurate estimates which would be validated by measurements carried out on a Kistler table. Nevertheless, this would not be the case if the platform stiffness is not rigid ($k < \infty$). In practice, the system on the right could produce up to twice the force as the one on the left, when mounted on a very flexible structure ($k \to 0$). Consider the system on the left and allow movement of the platform, $x \neq 0$; this corresponds to having a source supported by a platform of stiffness equal to $k$. The vertical equilibrium can be expressed as:

$$m \left[-l\omega^2 \sin(\omega t) + \ddot{x}\right] + kx = 0$$

(6.2)

for which the solution is:
\[ x = \frac{m\omega^2}{k - m\omega^2} \sin(\omega t) \quad \text{with} \quad \omega \neq \omega_0 = \sqrt{\frac{k}{m}} \quad (6.3) \]

If an angular velocity for which \( \omega \gg \omega_0 \) is considered, the displacement \( x \) tends to \( l \) and the resulting force transmitted to the support would be:

\[ F|_{\omega \gg \omega_0} = lk \quad (6.4) \]

The same effect would occur if a low stiffness (\( k \to 0 \)) is considered. Similar calculations can be performed for the system on the right. The vertical equilibrium in this case can be written as:

\[ 2m \left[ -\frac{l}{2} \omega^2 \sin(\omega t) + \ddot{x} \right] + kx = 0 \quad (6.5) \]

which provides a solution of the type:

\[ x = \frac{m\omega^2}{k - 2m\omega^2} \sin(\omega t) \quad \text{with} \quad \omega \neq \omega_0 = \sqrt{\frac{k}{2m}} \quad (6.6) \]

For high angular speeds (or low stiffness values), the displacement \( x \) tends to \( \frac{l}{2} \) and the corresponding force transmitted to the platform would be:

\[ F|_{\omega \gg \omega_0} = \frac{l}{2}k \quad (6.7) \]

which is half the force generated by the system previously investigated.

Thereby, in order to establish the impact of a microvibration source on its supporting platform, a passive effect and an active effect have to be included. The former refers to how the inertia properties of the source influence the dynamics of the supporting structure. The latter, in contrast, refers to the excitations produced by the source (i.e. due to the mass imbalance of a flywheel or the mechanical noise generated by imperfections in the motor bearings). The passive effect is described by the source dynamic mass and can be evaluated using the method described in chapter 5 or by means of a detailed FE model of the source (although rotary mechanisms FE models are quite challenging to be reproduced). For what concerns the active influence, the data obtained from a hard-mounted configuration test campaign, \( f_{hm} \), is generally the most common approach.

Finally, an equivalent input force is determined by combining both the static and dynamic effects and used in coupled analysis, as the driving force to be applied at the source in order to move the source itself. Here, Equation (5.3) is recalled to facilitate the understanding of the mathematical formulation described next:

\[ f_{cp} = f_{hm} - D_{rwa} \ddot{x}_{cp} \]
The accelerations at the interface, $\ddot{x}_{cp}$, can also be computed as the product between the inverse of the dynamic mass of the supporting structure, $D_{ss}$, and the coupled load vector, $f_{cp}$:

$$\ddot{x}_{cp} = D_{ss}^{-1}f_{cp} \quad (6.8)$$

Substituting Equation (6.8) into Equation (5.3), the latter can be re-written as:

$$f_{cp} = f_{hm} - D_{rwa}D_{ss}^{-1}f_{cp} \quad (6.9)$$

which can be rearranged for $f_{cp}$ to obtain:

$$f_{cp} = (I - D_{rwa}D_{ss}^{-1})^{-1}f_{hm} \quad (6.10)$$

where $I$ is a $6 \times 6$ unit matrix. Therefore, the coupled loads, $f_{cp}$, that are actually transmitted at the interface between the source and the supporting structure can be described as a function of:

- dynamic characteristics (inertia, stiffness and damping) of the source, $D_{rwa}$;
- dynamic characteristics (inertia, stiffness and damping) of the supporting structure, $D_{ss}$;
- the driving force used to move the source (i.e. forces and moments measured in the hard-mounted boundary condition, $f_{hm}$).

Equation (6.10) exhibits the actual difference between the loads measured in the hard-mounted configuration and those derived when the source is physically assembled with its supporting structure. The dynamic mass of the source acts as a damping system, thus reducing the real forces which are transmitted to the supporting structure compared to the forces produced in a hard-mounted configuration. Equation (6.10) can also be re-formulated by means of PSD. The $6 \times 1$ force vector $f_{cp}$ transforms in a $6 \times 6$ matrix, $\Phi_{cp}$. The term in bracket can be seen as the Transfer Function (TF) between $f_{cp}$ and $f_{hm}$. When dealing with PSD entities, the input function requires to be pre-multiplied by the TF matrix and post-multiplied by the Hermitian of the TF matrix. The output coupled force $\Phi_{cp}$ can be finally expressed as:

$$\Phi_{cp} = (I - D_{rwa}D_{ss}^{-1})^{-1}\Phi_{hm}(I - D_{rwa}D_{ss}^{-1})^{-H} \quad (6.11)$$

The PSD output at any location on the satellite structure, $\Phi_{out}$, can consequently be computed multiplying $\Phi_{cp}$ by an opportune TF matrix which correlates
the source-support interface node to the output location node on the supporting structure, $\mathbf{T}\mathbf{F}_{cp-out}$, thus to obtain:

$$\Phi_{out} = \mathbf{T}\mathbf{F}_{cp-out}(\mathbf{I} - \mathbf{D}_{rwq}\mathbf{D}_{ss}^{-1})^{-1}\Phi_{hm}(\mathbf{I} - \mathbf{D}_{rwq}\mathbf{D}_{ss}^{-1})^{-H}\mathbf{T}\mathbf{F}_{cp-out}^{H} \quad (6.12)$$

Note that the mathematical process here discussed could be expressed in other contexts by means of other parameters, such as velocity and mechanical impedance or displacement and receptance, rather than acceleration and dynamic mass, still maintaining its validity.

### 6.1.1 Benchmark Example

Consider the microvibration source modelled in ECSS (2013) as a one DoF system which generates a force, in terms of PSD, equal to $\Phi_{inp}$, and shown in Figure 6.2.

![Figure 6.2: Schematic of a source-supporting structure system used as a benchmark example for coupled analysis (ECSS, 2013)](image)

<table>
<thead>
<tr>
<th>Mass [kg]</th>
<th>Stiffness [kNm$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7000</td>
</tr>
<tr>
<td>10</td>
<td>10000</td>
</tr>
<tr>
<td>80</td>
<td>1000</td>
</tr>
<tr>
<td>120</td>
<td>30000</td>
</tr>
</tbody>
</table>
Assuming the input force is described by:

\[
\Phi_{inp}(\omega) = \frac{\omega}{2\pi 2000}
\]  

(6.13)

where \( \omega \) is the frequency at which the source operates. When the source is fixed to the ground (i.e. hard-mounted configuration) the resulting force at the support can be computed by means of Equation (6.14). The reaction force is plotted in Figure 6.3 superimposed to the input force.

\[
\Phi_{hm} = \{ k_1 \left[ -m_1 \omega^2 + k_1 (1 + 0.02i) \right]^{-1}\}^2 \Phi_{inp}
\]  

(6.14)

**Figure 6.3: Disturbance force applied at the source and resulting force at support**

The receiver response can be directly computed using the EoM of the coupled system and either applying \( \Phi_{hm} \) at the mounting location of the source or imparting \( \Phi_{inp} \) at the source itself. Given the system mass, \( M \), and stiffness, \( K \), matrices and the TF matrix, \( \text{TF}_{inp, out} \), which correlates the input and the output locations, the receiver response can be calculated by means of Equations (6.18) and (6.19). Both the approaches would lead to the exact same response, as depicted in Figure 6.4.
\[ M = \begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{bmatrix} \]

\[ K = \begin{bmatrix} k_1 & -k_1 & 0 & 0 \\ -k_1 & k_1 + k_2 + k_3 & -k_3 & -k_2 \\ 0 & -k_3 & k_3 + k_4 & -k_4 \\ 0 & -k_2 & -k_4 & k_3 + k_2 \end{bmatrix} \] (6.15)

\[ \text{Inp}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \Phi_{hm} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{Inp}_2 = \begin{bmatrix} \Phi_{inp} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \] (6.16)

\[ \text{TF} = \left[ -M \omega^2 + K \left( 1 + 0.02i \right) \right]^{-1} \] (6.17)

\[ \text{Out}_{\text{receiver},1} = \text{TF} \cdot \text{Inp}_1 \cdot \text{TF}^H \] (6.18)

\[ \text{Out}_{\text{receiver},2} = \text{TF} \cdot \text{Inp}_2 \cdot \text{TF}^H \] (6.19)

Figure 6.4: Response at the receiver by applying \( \Phi_{hm} \) at the mounting location of the source or \( \Phi_{inp} \) at the source itself

In practice, however, the full system (i.e. spacecraft) assembled and ready
to carry out tests is generally not available, in particular in the first stages of the design study and analysis. Therefore, the common approach is to treat the source and the spacecraft structure separately. For the case under investigation, the source and support mass and stiffness matrices are:

\[
M_{\text{source}} = \begin{bmatrix} m_1 & 0 \\ 0 & 0 \end{bmatrix}, \quad K_{\text{source}} = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix}
\]

(6.20)

\[
M_{ss} = \begin{bmatrix} m_2 & 0 & 0 \\ 0 & m_3 & 0 \\ 0 & 0 & m_4 \end{bmatrix}, \quad K_{ss} = \begin{bmatrix} k_2 + k_3 & -k_3 & -k_2 \\ -k_3 & k_3 + k_4 & -k_4 \\ -k_2 & -k_4 & k_2 + k_4 \end{bmatrix}
\]

(6.21)

The modelling of the source internal mechanics, in addition, is often challenging, hence \(\Phi_{hm}\) only is available. If this is the case, the application of \(\Phi_{hm}\) to the structure without including the source or simply representing the source as a lumped mass, produces results which are not correct, as illustrated in Figure 6.6.

![Schematic of the different approaches](image)

**Figure 6.5:** Schematic of the different approaches that are implemented in this section: (a) source not included in the model; (b) source represented as a lumped mass and (c) source included in the model by means of the coupling theory.

Nevertheless, if the coupling theory is implemented then the exact solution can be achieved. Computing the source dynamic mass and the support accelerance (inverse of the dynamic mass) matrices as:
and opportune substituting in Equation (6.12), the output at the receiver location can be expressed as:

$$D_{source} = \{-M_{source}\omega^2 + K_{source} (1 + 0.02i)\}^{-1}\omega^2 \begin{bmatrix} 0 \\ -1 \end{bmatrix}^{-1}$$

and

$$D_{ss}^{-1} = \{-M_{ss}\omega^2 + K_{ss} (1 + 0.02i)\}^{-1}\omega^2 \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

which is plotted as a magenta squares line in Figure 6.6 and perfectly agrees with the exact solution previously obtained.

The main advantage of employing the coupling theory is that it eliminates the requirement to model the equipment that produces microvibration disturbances which may, therefore, be treated as a set of black boxes. The only parameters that are necessary are: the loads measured in a hard-mounted condition (which can be easily obtained using a Kistler table or similar platforms), the dynamic mass of the source (determined by means of the methodology described in chapter 5) and the dynamic mass (or accelerance) of the structure onto which the source is
installed (which is generally derived from its FE model).

6.2 RWA-FFSM Coupled Microvibration Testing

A preliminary test campaign was conducted to assess the coupled dynamics generated by the RWA when mounted on a FFSM and the whole system suspended in a free-free boundary condition. The coupled loads exchanged between the RWA and the FFSM (see Figure 4.4) were measured at the RWA-structure interface. Moreover, accelerations at several locations on the structure were also evaluated. The outcomes were subsequently used as a benchmark to validate the RWA-structure coupled model, defined in appendix B, and the equations introduced in section 6.1.

6.2.1 RWA-FFSM Test Setup

The RWA was mounted onto an extremely rigid Aluminium structure whose mass is considerably larger than the mass of the RWA. The arrangement was subsequently fastened to a crane using elastic cords such as to reproduce a free-free configuration. Four uni-axial force sensors were placed at the interface between the RWA and the FFSM providing measurements of the coupled forces reciprocally transmitted between the two bodies. Moreover, six accelerometers were placed on the FFSM (two in each plane) to derive its motion due to the RWA-induced disturbances. Given the distance between the accelerometers, the characterisation of the FFSM response in all six DoFs (three translations and three rotations) was achieved by opportunely combining the six accelerometer responses. The test setup is shown in Figure 6.7.

The RWA was spun from 1200 rpm (20 Hz) to 4800 rpm (80 Hz) with a 300 rpm step increase, that is a speed resolution of 5 Hz.

6.2.2 RWA-FFSM Test Results

The results from the test campaign on the coupled RWA-FFSM system, are presented as PSD waterfall plots in Figure 6.8 and spectral maps in Figure 6.9. In order to improve the legibility of the results, the data are plotted as $10 \times \log (|\text{amplitude}|)$. Due to a speed resolution of 300 rpm (5 Hz), a linear interpolation of the experimental outputs was performed thus allowing a smoother data representation in the spectral maps.
Figure 6.7: Coupled RWA-FFSM test setup, including accelerometers configuration

A similar behaviour to that displayed in the hard-mounted configuration can be observed. This is a consequence of the mass ratio between the support and the RWA and also of the stiffness properties of the FFSM. The interaction between higher harmonics and the RWA-FFSM structural modes is well depicted in Figure 6.9. Amplifications occurring when the frequency of the disturbance harmonic matches the corresponding system structural modal resonance are emphasised in Figures 6.8 and 6.9 for values of the frequency alike 120 Hz and 190 Hz. In addition, the structural modes, computed by implementing Equation (B.1), are superimposed to the experimental results and plotted as black solid lines. A good correlation can be observed as amplifications in the system response are experienced when the disturbance harmonics cross the structural modes lines. Therefore, the model described in appendix B is a valid representation of the coupled dynamics observed when a RWA is mounted on a rigid support and the arrangement is suspended in a free-free boundary condition.
Figure 6.8: PSD waterfall plot of output data from coupled RWA-FFSM experimental test: (a) coupled axial force $\Phi_{cp,33}$; (b) coupled moment $\Phi_{cp,55}$ and (c) structure response $\Phi_{out,11}$
Figure 6.9: Spectral maps of output data from coupled RWA-FFSM experimental test: (a) coupled axial force $\Phi_{cp,33}$; (b) coupled moment $\Phi_{cp,55}$ and (c) structure response $\Phi_{out,11}$. The RWA-stiff support structural modes are superimposed and represented in black solid lines.
6.3 RWA-Panel Coupled Microvibration Testing

Following the trial test campaign, the RWA was installed onto a more flight representative and realistic supporting structure, illustrated in Figure 6.10. For this purpose, a sandwich panel with Aluminium skin (0.9 mm thick) and Aluminium honeycomb core (18 mm thick) was selected as representative of a flexible spacecraft interface. The interface loads and acceleration responses of the flexible supporting structure were quantified by means of uni-axial force transducers and accelerometers, respectively.

![Figure 6.10: Structural panel used for coupling analysis](image)

6.3.1 RWA-Panel Test Setup

The RWA was connected to the panel using a set of 4 force sensors (PCB 208C01, 100 mV/N sensitivity). These were used to measure the coupled forces along the RWA $z_w$-axis at the RWA-panel interface. Three accelerometers (1000 mV/g sensitivity) suitable for microvibration analysis were placed at different locations on the panel with the purpose to evaluate the structure response to the RWA-induced disturbances. Finally, in order to reproduced a free-free boundary condition, the system was lifted from the ground using a crane and elastic cords. The free-free condition was such that the rigid body natural frequencies of the system were lower than 1 Hz. In Figure 6.11, the test configuration is illustrated, also showing the location of the accelerometers.
Figure 6.11: Coupled RWA-panel test setup, including accelerometers configuration
In contrast to the RWA-FFSM case, a higher speed resolution was considered here. The RWA was driven from 600 rpm (10 Hz) to 4800 rpm (80 Hz) in steps increasing of 60 rpm each, allowing a resolution of 1 Hz in the speed range.

6.3.2 RWA-Panel Test Results

Measurements of the RWA-panel coupled dynamics were conducted in the time domain and the data were subsequently processed and reformulated in the frequency domain in terms of PSD. The post-processing outputs are illustrated in Figures 6.12 and 6.13 as PSD waterfall plots and spectral maps, respectively. Data are plotted as $10 \times \log (||\text{amplitude}||)$ to facilitate their discernability.

The excitations generated by the RWA interact with RWA-panel structural modes thus producing amplifications in the system response. These occur at frequencies for which the frequency of the exciting harmonic and the system resonance frequency match. For instance, Figures 6.12(a), 6.12(c), 6.13(a) and 6.13(c) display larger system response amplitudes for frequencies equal to 100 Hz, 150 Hz, 230 Hz and 280 Hz. Further amplifications are also registered at 160 Hz, as depicted in Figures 6.12(b) and 6.13(b).

Based on the experimental results, it can, therefore, be concluded that, when dealing with coupled dynamics analyses, the interaction between the RWA internal dynamics and those of its supporting structure are significant, especially when the platform is flexible and presents modes of vibration which occur at frequencies similar to those of the RWA.

6.4 Numerical Computation

The direct implementation of Equations (6.11) requires knowledge of the forces and moments in a hard-mounted boundary condition, the dynamic mass of the RWA and the dynamics mass of the supporting structure. The loads in a grounded configuration have been derived and thoroughly investigated in section 4.4, whereas the dynamic mass of the RWA has been retrieved, both analytically and experimentally, in chapter 5. The dynamic mass of the supporting structures can be computed from their FE models, shown in Figure 6.14. The FFSM was modelled using SOLID elements whereas the sandwich panel using SHELL elements in MSC Patran/Nastran.

The RWA-FFSM and RWA-structural panel interface point dynamic mass matrices were obtained by applying unit forces and moments at one DoF at a time. A previous study conducted by Remedia et al. (2015a) on the structural panel
Figure 6.12: PSD waterfall plot of output data from coupled RWA-panel experimental test: (a) coupled axial force $\Phi_{cp,33}$; (b) coupled moment $\Phi_{cp,55}$ and (c) structure response at accelerometer T2 location $\Phi_{out,T2,33}$.
Figure 6.13: Spectral maps of output data from coupled RWA-panel experimental test: (a) coupled axial force $\Phi_{cp,33}$; (b) coupled moment $\Phi_{cp,55}$ and (c) structure response at accelerometer T2 location $\Phi_{out,T2,33}$
was used as a benchmark for the validation of the panel FE model. In Remedia et al. (2015a)’s work, the panel was hung using elastic cords and excited at the RWA-panel interface location by means of mini-shakers. The resulting panel accelerations were then post-processed to derive the dynamic mass of the structure. For ease of legibility, only the element representing the dynamic mass in the axial DoF is reported here, see Figure 6.15, as it is the most significant. Note that, no dynamic amplifications have been observed in the FFSM dynamic mass in the frequency band of interest, that is the dynamic mass is constant and, therefore, is not presented here.
Once all the parameters are available, the agreement between the experimental data and the predictions, obtained by means of Equations (6.11) and (6.12), can be qualitatively demonstrated by superimposing the measured and numerical data in a plot. In addition, the correlation can be also quantified in terms of Frequency Response Assurance Criterion (FRAC). The frequency response computed analytically is compared to the experimental derived function for each DoF. The FRAC mainstream hypothesis lies in the assumption that the measured and synthesized data should be linearly related (unity scaling coefficient) over the full range of frequency (Zang et al., 2001). The lower and upper limits are 0 and 1, respectively, where 1 indicates perfect correlation. The FRAC is expressed as:

\[
FRAC = \frac{\left(\sum_i \{\Phi_{\text{out,test}}\}_i \{\Phi_{\text{out,FEM}}\}_i^*\right)^2}{\left(\sum_i \{\Phi_{\text{out,test}}\}_i \{\Phi_{\text{out,test}}\}_i^*\right) \left(\sum_i \{\Phi_{\text{out,FEM}}\}_i \{\Phi_{\text{out,FEM}}\}_i^*\right)}
\]  

(6.24)

where "*" represents conjugate values. No stretching or shifting has been performed in this study. The procedure defined here will be adopted in the following sections where predictions, for both the FFSM and sandwich panel cases, will be compared to the test data.

6.4.1 Source Modelling Approaches

The numerical analysis was performed considering four different approaches in the modelling of the microvibration source:

- the RWA is not included in the model and the hard-mounted loads are directly applied to the supporting structure (e.g. FFSM and panel), see Figure 6.16(a). In this case, Equation (6.12) reduces to:

\[
\Phi_{\text{out}} = TF_{\text{cp-out}} \Phi_{\text{hm}} TF_{\text{cp-out}}^H
\]  

(6.25)

- the RWA is represented as a lumped mass connected rigidly to the supporting structure (i.e. using rigid elements), as shown in Figure 6.16(b). Physically, this represents the case in which the source inertia properties are modelled but its internal dynamics are ignored. Mathematically, this is reproduced by computing the source dynamic mass for a frequency and a speed equal to zero, leading to:

\[
\Phi_{\text{out}} = TF_{\text{cp-out}} \left( I - D_{\text{rwa}} \big|_{\omega=0,\Omega=0} D_{\text{ss}}^{-1} \right)^{-1} \Phi_{\text{hm}} \left( I - D_{\text{rwa}} \big|_{\omega=0,\Omega=0} D_{\text{ss}}^{-1} \right)^{-H} TF_{\text{cp-out}}^H
\]

(6.26)
Figure 6.16: Cases analysed: (a) no source; (b) source as lumped mass, $D_{rwa} |_{\omega=0,\Omega=0}$; (c) source dynamic mass, $D_{rwa} (\omega) |_{\Omega=0}$ and (d) source dynamic mass including the gyroscopic effect, $D_{rwa} (\omega, \Omega)$. $\Phi_{hm}$ represents the PSD of the loads measured in a hard-mounted boundary condition.

- the RWA inertia, stiffness and damping properties are included in the model and expressed as functions of the frequency, whereas the gyroscopic effect is ignored, as illustrated in Figure 6.16(c). The source is connected rigidly to the supporting structure and the structure response at any location can be expressed as:

$$
\Phi_{out} = TF_{cp-out} (I - D_{rwa} (\omega) |_{\Omega=0} D_{ss}^{-1})^{-1} \Phi_{hm} \\
(I - D_{rwa} (\omega) |_{\Omega=0} D_{ss}^{-1})^{-H} TF_{cp-out} H
$$

(6.27)

- the RWA internal dynamics are taken into account and described as functions of both frequency and speed thus to include the gyroscopic effect, as depicted in Figure 6.16(d). Similarly to the previous cases, the source is connected rigidly to the supporting structure. The response of either the FFSM or the panel can be obtained as:

$$
\Phi_{out} = TF_{cp-out} (I - D_{rwa} (\omega, \Omega) D_{ss}^{-1})^{-1} \Phi_{hm} \\
(I - D_{rwa} (\omega, \Omega) D_{ss}^{-1})^{-H} TF_{cp-out} H
$$

(6.28)

6.4.2 RWA-FFSM Analysis

A good agreement, between the experimental and analytical data, was observed both in terms of forces and moments at the RWA-FFSM interface and of acceleration response of the supporting structure, due to the RWA-induced disturbances, as depicted in Figure 6.17.
Figure 6.17: Comparison of the interface RWA-stiff platform coupled dynamics derived from test results and the implementation of the four source modelling approaches for an angular speed of 1200 rpm (20 Hz): (a) coupled axial force $|\Phi_{cp,33}|$; (b) coupled moment about RWA y-axis $|\Phi_{cp,55}|$ and (c) support radial linear acceleration $|\Phi_{out,11}|$. 
A remark on the behaviour of the FFSM has to be advanced: its resonance frequencies occur at values which are well above the upper threshold set in this analysis, hence the supporting structure can be considered as a lumped mass in this case. Thereby, its internal dynamics did not significantly influence the response and no amplifications due to coupled dynamics between the RWA and the FFSM structural modes were identified. These allowed a simplification of the case-study examined, for which all the models in Figure 6.16 were able to provide acceptable estimates of the dynamics of the bodies when coupled together. Nevertheless, also in this simplified scenario, the predictions obtained from model (c) and (d) displayed a better correlation with the test results, in particular at frequencies for which the RWA internal dynamics were important. This can be further demonstrated using the FRAC, as shown in Table 6.2.

Table 6.2: FRAC calculated for the RWA-FFSM coupled system responses at Ω=1200 rpm

<table>
<thead>
<tr>
<th></th>
<th>Model (a)</th>
<th>Model (b)</th>
<th>Model (c)</th>
<th>Model (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Φ_{cp,33}</td>
<td>0.565</td>
<td>0.565</td>
<td>0.581</td>
<td>0.581</td>
</tr>
<tr>
<td>Φ_{cp,55}</td>
<td>0.998</td>
<td>0.998</td>
<td>0.998</td>
<td>0.998</td>
</tr>
<tr>
<td>Φ_{out,11}</td>
<td>0.995</td>
<td>0.995</td>
<td>0.996</td>
<td>0.996</td>
</tr>
</tbody>
</table>

The values confirm the good agreement between the analytical results and the test experiment as all the FRAC numbers are higher than 0.4. This threshold is set on the basis that the FRAC involves frequency responses from many experimental modes simultaneously, therefore a significant discrepancy in natural frequency between the experimental and analytical values for just a few of the modes may result in poor FRAC values (Fotsch & Ewins, 2002). Nevertheless, it is in accordance with Brughmans et al. (1993), whose work on the evaluation of correlation techniques for a body-in-white provides an indication of the minimum value for which the correlation can be considered acceptable. The numbers in Table 6.2 also highlight the superior prediction due to the implementation of the RWA dynamic mass. In particular, model (d) in Figure 6.16 is able to reproduce how the gyroscopic effect influences the RWA structural response in the region between 40 Hz and 200 Hz, where the RWA modes are important.

6.4.3 RWA-Panel Analysis

In contrast to the FFSM, the structural panel exhibits a flexible behaviour within the frequency band of interest and, therefore, coupled effects were expected in the RWA-panel coupled boundary condition. These can be observed in Figure 6.18,
Table 6.3: FRAC calculated for the RWA-panel coupled system responses at \( \Omega = 2520 \) rpm

<table>
<thead>
<tr>
<th></th>
<th>Model (a)</th>
<th>Model (b)</th>
<th>Model (c)</th>
<th>Model (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Phi_{cp,33} )</td>
<td>0.266</td>
<td>0.431</td>
<td>0.431</td>
<td>0.450</td>
</tr>
<tr>
<td>( \Phi_{cp,55} )</td>
<td>0.439</td>
<td>0.439</td>
<td>0.440</td>
<td>0.462</td>
</tr>
<tr>
<td>( \Phi_{out,T2} )</td>
<td>0.298</td>
<td>0.428</td>
<td>0.429</td>
<td>0.442</td>
</tr>
</tbody>
</table>

where the predictions computed by implementing Equations (6.25), (6.26), (6.27) and (6.28) are superimposed onto the test outcomes.

Resonances due to the interaction between the RWA and the panel internal dynamics are clearly visible. For instance, amplifications can be observed at about 92 Hz, 165 Hz and 215 Hz. In terms of correlation, none of the models in Figure 6.16 precisely matched the test results. The hard-mounted microvibrations applied as direct input in model (a) generally overestimated the coupled loads and response, as expected. Moreover, the dynamics of the RWA between 150 Hz and 175 Hz were missed. Although improving the response estimate, model (b) failed to simulate the RWA structural modes due to lack of RWA dynamics implemented in the model. In contrast, models (c) and (d) presented an increased agreement throughout the frequency band of interest. The latter, in particular, was not only able to reproduce the coupled dynamics between the RWA and the panel (as model (c) did too) but was also able to accurately simulate the RWA structural dynamics. This is remarked in Table 6.3 where the analytical outcomes are expressed in terms of FRAC using Equation (6.24) to show the beneficial effect of considering the RWA and the supporting structure dynamic mass matrices when dealing with coupled analysis. At frequencies beyond 200 Hz however, the coupled microvibrations predicted when implementing the RWA dynamic mass and that obtained by representing the RWA as a lumped mass are similar. This is explained due to absence of RWA modes in that region.

Despite the FRAC confirming that none of the models in Figure 6.16 provided an extremely accurate prediction (for which the FRAC value should be as close as possible to 1) of the actual coupled forces exchanged by the RWA and the panel and of the panel response, promising and acceptable values are provided by models (b), (c) and (d) in Figure 6.16, as the corresponding FRAC numbers are higher than the lower acceptable limit set to 0.4. Moreover, it also shows how the implementation of the dynamic mass including the gyroscopic effect significantly improved the outcomes from the numerical analysis which can, therefore, be used as a reliable and trustworthy estimate of the real environment in which the RWA
Figure 6.18: Comparison of the interface RWA-stiff platform coupled dynamics derived from test results and the implementation of the four source modelling approaches for an angular speed of 2520 rpm (42 Hz): (a) coupled axial force $|\Phi_{cp,33}|$; (b) coupled moment about y-axis $|\Phi_{cp,55}|$ and (c) support radial linear acceleration $|\Phi_{out,T2}|$. 
will operate.

6.5 SUMMARY

The approach suggested for the production of reliable and comprehensive predictions of the microvibration environment at specific locations when a source of microvibration is mounted on a supporting structure has been described in this chapter. The outcomes derived from an extensive testing campaign where the RWA was installed onto a FFSM at first and, subsequently, on a honeycomb panel were thoroughly investigated and discussed. These were consequently compared to the analytical predictions obtained implementing four different models. The results asserted that the method of implementing the RWA dynamic mass including the gyroscopic effect is a valid and more favorable methodology for coupled microvibration analysis with respect to the traditional method where the loads derived from a hard-mounted boundary condition are directly applied to the supporting structure. Furthermore, it offers an improved prediction over the traditional RWA dynamic mass computed when static. In addition, it was observed that if the resonances of the supporting structure are well beyond the frequency band of interest, the advantage of using RWA dynamic mass is strictly limited to the region where the RWA modes are important.
CHAPTER 7

CONCLUSIONS

7.1 SUMMARY OF THE WORK CONDUCTED

Research on microvibration characterisation, analysis and mitigation is, nowadays, a topic of major concern in satellite structural dynamics due to the challenges that these disturbances pose to the modern high sensitive payloads on-board spacecraft. For instance, in ECSS (2013) the largest chapter is dedicated to the topic of microvibrations and microdynamics. Among the different areas of research, the work presented in this thesis focussed on the characterisation of microvibration sources.

In chapter 2, the literature review provided a list of the possible sources of microvibration acting on a satellite and it was concluded that RWAs are generally the ones that produce the largest disturbances. For this reason, this study has been emphasising the description of the behaviour and features of these mechanisms. In order to accomplish these objectives, the development of a satellite RWA model was carried out in chapter 3 and its validation fulfilled in chapter 4. In these terms, a cantilever-configured RWA supported by two flexible suspension systems was considered. Based on standard assumptions such as linearity, small displacements, small perturbation angles, small mass imbalance, RWA symmetry with respect to the axial axis and steady speed rotation, an analytical model able to reproduce each boundary condition in which the RWA would operate (hard-mounted, free-free and coupled) has been derived by means of an energy method.

The development of the analytical model was followed by an extensive test campaign thus to measure the loads produced by the RWA in a hard-mounted condition and the accelerations when the mechanism was hung in a free-free configuration. The data derived from the RWA microvibration experimental testing,
expressed as $6 \times 6$ PSD matrices, has been used against the RWA numerical analysis results to validate the RWA microvibration model. The comparison displayed a good agreement, that is the analytical model was capable of accurately capturing the RWA fundamental harmonics and the RWA structural modes including the gyroscopic effect due to a spinning flywheel.

In practice, the most common approach for evaluating the disturbances produced by a RWA is to ground the mechanism onto a rigid platform (dynamometer) and measure the excitations, arising from mass imbalance, imperfections in the motor bearing and broadband noise, at the RWA interface. This, however, has been shown to fail in reproducing the actual dynamics when a microvibration source is installed on a spacecraft due to the coupled effects that occur in this configuration. The interaction between a RWA and its supporting structure, both numerically and experimentally, has been thoroughly investigated over the last twenty years leading to the conclusion that this interaction can be correctly predicted only if the source internal dynamics (the dynamic mass) are taken into account. Nevertheless, the literature lacks a reliable and cost effective approach to computing the RWA dynamic mass over a wide range of frequencies and operative speeds, thus including also the gyroscopic effect in the RWA characterisation.

This issue was tackled in chapter 5 with the development of an innovative methodology for the retrieval of the dynamic mass of a RWA by means of a re-elaboration of the theory of coupling. This method proved to provide a fast and accurate way to compute the frequency and speed dependent RWA dynamic mass by implementing the experimental data obtained from hard-mounted and free-free boundary condition measurements. The methodology convergence was studied and the noise was identified as the main parameter that affects the iterative process. Its effect was quantified for both the static and dynamic conditions. An analysis of the off-diagonal terms in the dynamic mass matrix was also conducted and it was observed that some of the elements traditionally neglected in previous studies cannot be ignored when the RWA is operating due to the gyroscopic effect.

The final prediction of a supporting structure response when microvibration disturbances are acting on it can be described by a combination of the loads generated by the source in a hard-mounted configuration, the coupled internal dynamics of the source and the supporting structure and the transfer function between the load point application and the response location. A test campaign, firstly using a FFSM and subsequently a flexible panel, was carried out and the interface loads and structure responses measured. The data were consecutively compared to the numerical predictions derived by implementing different approaches in chapter 6. It was shown that more accurate predictions can be obtained by taking into ac-
count the dynamic mass of the RWA including the gyroscopic effect.

7.2 Main Achievements

The contributions to the state of the art in the characterisation of microvibration sources on-board spacecraft can be extrapolated from different aspects of the work presented in this thesis. All the objectives listed in the introductive section have been achieved and conclusions can now be inferred on the advancements in the description of the dynamics related to satellite microvibration sources.

- **RWA microvibration analytical model:** the EoMs derived in this thesis allow the representation of the mechanism in all its operative boundary conditions, whether it is hard-mounted to a rigid platform, suspended free-free or coupled with a supporting structure. Moreover, the model is capable of reproducing the different levels of flexibility that can be encountered when connecting the mechanism to an elastic structure. Furthermore, the gyroscopic effect was also included in the model thus to provide a complete description of the dynamic behaviour of the RWA when set in motion;

- **dynamic mass semi-empirical method:** the approach generates accurate estimates at a fraction of the experimental effort relative to the implementation of the traditional methods and allows the definition of the microvibration source internal dynamics over a broad range of frequencies and speeds. In particular, issues due to misalignments of mini-shakers and zero operative speed in the traditional approach were eliminated. The simplicity of the procedure lies also in the definition of the initial conditions, used in the iterative process, which can be derived from simple measurements of the RWA mass and inertia.

  The convergence of the method as a function of the broadband noise was analysed and it was concluded that the spectral contributions of the noise measured during the hard-mounted and the free-free test campaigns have to be similar in order to achieve reliable and trustworthy results;

- **dynamic mass off-diagonal elements:** as extensively stated in this thesis, the gyroscopic effect may alter the dynamic response of a RWA when this is operating at different speeds. This effect also affects some of the coefficients in the dynamic mass matrix, in particular those correlating the radial translations and radial rotations. Despite these elements being null when the mechanism is in a zero-speed condition, their contribution becomes more and
more significant as the RWA flywheel angular speed increases and, therefore, it cannot be neglected. This is of particular importance in the analysis of the coupled dynamics between a RWA and its support;

- coupling of the RWA to different supporting structures: the theory of coupling has been extended to include the gyroscopic effect in the source dynamic mass. This is fundamental to budget for coupled dynamics between a RWA and its supporting structure when the mechanism is operating and, therefore, its structural modes evolve in BWs and FWs due to the action of the gyroscopic effect. Predictions using this approach proved to display a better agreement with the data obtained from experimental tests compared to the traditional methods for which the hard-mounted loads are directly applied to the supporting structure, especially when the supporting structure experiences modes of vibrations which occur at frequencies comparable with those of the RWA.

### 7.3 Future Work

Due to the continuous advancing of the technology on-board modern satellites and the corresponding higher stability requirements associated with it, research on microvibrations is a topic that will steadily grow in interest over the next years. The recommendations for future work in the areas discussed in this thesis which need development and/or implementation are:

- the excitation vector defined in the RWA microvibration model requires to be extended to include an analytical model of the broadband noise. In these terms, a function which describes the general response level produced by amplifications due to resonances has to be developed. The derivation of this function is, however, difficult due to its dependence on the RWA angular speed, the dynamic amplification and the general response level;

  alternatively, the causes (e.g. bearing imperfection) which generate the broadband noise can be individually modelled thus to provide a comprehensive mathematical formulation of the broadband noise. Experimental tests could be conducted aiming to isolate and/or characterise the noise empirically as a complementary option to modelling;

- the difference in the broadband noise spectral contribution between hard-mounted and free-free test data has been identified as the critical parameter to ensure the convergence of the dynamic mass iterative process defined in
this thesis. Recommendations in this context are to carry out the tests in the same environmental conditions (i.e. room temperature, hours of the day/night, etc.) and, where possible, apply filters so as to eliminate the part of a signal which may have been corrupted during the measurements;

- the dynamic mass semi-empirical method has here been applied to a RWA, but it can actually be implemented with other mechanisms on-board a satellite (e.g. antenna pointing mechanisms, cryo-coolers, control moment gyros, etc.);

- the microvibration disturbances generated by a single source were considered in the study of the coupled dynamics between a source of microvibration and its supporting structure. This could be extended to include multiple sources of microvibration operating simultaneously thus reproducing a more realistic satellite application. Not only the dynamics of the individual sources would couple with the supporting structure but their disturbances and internal dynamics would also coupled together thus making the description of the phenomenon even more complicated;

- all the outcomes presented in this thesis are based on the mainstream assumption that the RWA interfaces with the external environment by means of a single 6-DoF contact point. Thereby, only the rigid motion of the interface contact point were taken into account. A multi-contact point (i.e. mounting points of the RWA) model has, therefore, to be developed thus to include the flexibility in the interface modelling and be more representative of a RWA-spacecraft application.

In conclusion, the work conducted in this thesis has been shown to significantly improve the previous techniques in the characterisation of spacecraft microvibration sources, in particular RWAs, and their effects when assembled with a flexible supporting structure, producing more accurate results and more efficient practical applications. The methodology has, however, also displayed limitations leaving space for further improvements. Future developments have also been suggested thus to tackle those issues which are currently unsolved and for which this work can be used as a solid starting point.
REFERENCES


REFERENCES


## Tait-Brian Transformation Matrices

Table A.1: Transformation matrices from inertial frame to body frame

<table>
<thead>
<tr>
<th>From</th>
<th>To</th>
<th>Transformation Matrices (Z-convention)</th>
</tr>
</thead>
</table>
| XYZ    | abc      | \[
|        |          | \begin{bmatrix}
|        |          | \cos \varphi & 0 & -\sin \varphi \\
|        |          | 0 & 1 & 0 \\
|        |          | \sin \varphi & 0 & \cos \varphi \\
|        |          \end{bmatrix} |
| abc    | x'y'z'   | \[
|        |          | \begin{bmatrix}
|        |          | 1 & 0 & 0 \\
|        |          | 0 & \cos \theta & \sin \theta \\
|        |          | 0 & -\sin \theta & \cos \theta \\
|        |          \end{bmatrix} |
| x'y'z' | xyz      | \[
|        |          | \begin{bmatrix}
|        |          | \cos \psi & \sin \psi & 0 \\
|        |          | -\sin \psi & \cos \psi & 0 \\
|        |          | 0 & 0 & 1 \\
|        |          \end{bmatrix} |
| XYZ    | xyz      | \[
|        |          | \begin{bmatrix}
|        |          | \cos \varphi \cos \psi + \sin \varphi \sin \theta \sin \psi & \cos \theta \sin \psi & -\sin \varphi \cos \psi + \cos \varphi \sin \theta \sin \psi \\
|        |          | -\cos \varphi \sin \psi + \sin \varphi \sin \theta \cos \psi & \cos \theta \cos \psi & \sin \varphi \sin \psi + \cos \varphi \sin \theta \cos \psi \\
|        |          | \sin \varphi \cos \theta & -\sin \theta & \cos \varphi \cos \theta \\
|        |          \end{bmatrix} |

Order: XYZ → abc → x'y'z' → xyz
Table A.2: Transformation matrices from body frame to inertial frame

<table>
<thead>
<tr>
<th>From</th>
<th>To</th>
<th>Transformation Matrices (Z-convention)</th>
</tr>
</thead>
</table>
| abc    | XYZ    | \[
|        |        | \begin{bmatrix}
|        |        | \cos \varphi & 0 & \sin \varphi \\
|        |        | 0 & 1 & 0 \\
|        |        | -\sin \varphi & 0 & \cos \varphi \\
|        |        | \end{bmatrix}
|        |        | \] |
| x'y'z' | abc    | \[
|        |        | \begin{bmatrix}
|        |        | 1 & 0 & 0 \\
|        |        | 0 & \cos \theta & -\sin \theta \\
|        |        | 0 & \sin \theta & \cos \theta \\
|        |        | \end{bmatrix}
|        |        | \] |
| xyz    | x'y'z' | \[
|        |        | \begin{bmatrix}
|        |        | \cos \psi & -\sin \psi & 0 \\
|        |        | \sin \psi & \cos \psi & 0 \\
|        |        | 0 & 0 & 1 \\
|        |        | \end{bmatrix}
|        |        | \] |
| xyz    | XYZ    | \[
|        |        | \begin{bmatrix}
|        |        | \cos \varphi \cos \psi + \sin \varphi \sin \theta \sin \psi & -\cos \theta \sin \psi + \sin \varphi \sin \theta \cos \psi & \sin \varphi \cos \theta \\
|        |        | \cos \theta \sin \psi & \cos \theta \cos \psi & -\sin \theta \\
|        |        | -\sin \varphi \cos \psi + \cos \varphi \sin \theta \sin \psi & \sin \varphi \sin \psi + \cos \varphi \sin \theta \cos \psi & \cos \varphi \cos \theta \\
|        |        | \end{bmatrix}
|        |        | \] |
RWA SYSTEM MATRICES

B.1 COMPLETE RWA SYSTEM MATRICES

With reference to Equation (3.27), all matrices are as follows:

\[ q_s = [x_w, y_w, z_w, \theta_w, \varphi_w, x_b, y_b, z_b, \theta_b, \varphi_b]^T \]

\[ M_s = \text{diag} \{ M_w, M_w, M_w, I_{r,w}, I_{r,w}, M_b, M_b, I_{r,b}, I_{r,b} \} \]

\[ G_s = \begin{cases} 
G_{s,45} = \Omega I_{z,w} \\
G_{s,54} = -\Omega I_{z,w} \\
0, \text{ elsewhere} 
\end{cases} \]

Table B.1: Compact form of the damping coefficients for the complete RWA system

<table>
<thead>
<tr>
<th>Damping Matrix Coefficients</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_{11} = c_{t,w} )</td>
<td>( c_{15} = -c_{t,w}d )</td>
</tr>
<tr>
<td>( c_{33} = c_{z,w} )</td>
<td>( c_{44} = c_{t,w}d^2 + c_{r,w} )</td>
</tr>
<tr>
<td>( c_{29} = c_{t,w}h )</td>
<td>( c_{49} = c_{t,w}h^2 - c_{r,w} )</td>
</tr>
<tr>
<td>( c_{66} = c_{t,w} + c_{t,b} )</td>
<td>( c_{88} = c_{z,w} + c_{z,b} )</td>
</tr>
<tr>
<td>( c_{79} = -(c_{t,w}h - c_{t,b}v) )</td>
<td>( c_{99} = c_{t,w}h^2 + c_{r,w} + c_{t,b}v^2 + c_{r,w} )</td>
</tr>
</tbody>
</table>
\[
C_s = \begin{bmatrix}
    c_{11} & 0 & 0 & 0 & c_{15} & -c_{11} & 0 & 0 & 0 & -c_{29} \\
    0 & c_{11} & 0 & -c_{15} & 0 & 0 & -c_{11} & 0 & c_{29} & 0 \\
    0 & 0 & c_{33} & 0 & 0 & 0 & 0 & -c_{33} & 0 & 0 \\
    0 & -c_{15} & 0 & c_{44} & 0 & 0 & c_{15} & 0 & c_{49} & 0 \\
    c_{15} & 0 & 0 & 0 & c_{44} & -c_{15} & 0 & 0 & 0 & c_{49} \\
    -c_{11} & 0 & 0 & 0 & -c_{15} & c_{66} & 0 & 0 & 0 & -c_{79} \\
    0 & -c_{11} & 0 & c_{15} & 0 & 0 & c_{66} & 0 & c_{79} & 0 \\
    0 & 0 & -c_{33} & 0 & 0 & 0 & 0 & c_{88} & 0 & 0 \\
    0 & c_{29} & 0 & c_{49} & 0 & 0 & c_{79} & 0 & c_{99} & 0 \\
    -c_{29} & 0 & 0 & 0 & c_{49} & -c_{79} & 0 & 0 & 0 & c_{99}
\end{bmatrix}
\]

Table B.2: Compact form of the stiffness coefficients for the complete RWA system

<table>
<thead>
<tr>
<th>Stiffness Matrix Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_{11} = k_{t,w} )</td>
</tr>
<tr>
<td>(k_{33} = k_{z,w} )</td>
</tr>
<tr>
<td>(k_{29} = k_{t,w}h )</td>
</tr>
<tr>
<td>(k_{66} = k_{t,w} + k_{t,b} )</td>
</tr>
<tr>
<td>(k_{79} = -(k_{t,w}h - k_{t,b}v))</td>
</tr>
</tbody>
</table>

\[
K_s = \begin{bmatrix}
    k_{11} & 0 & 0 & 0 & k_{15} & -k_{11} & 0 & 0 & 0 & -k_{29} \\
    0 & k_{11} & 0 & -k_{15} & 0 & 0 & -k_{11} & 0 & k_{29} & 0 \\
    0 & 0 & k_{33} & 0 & 0 & 0 & 0 & -k_{33} & 0 & 0 \\
    0 & -k_{15} & 0 & k_{44} & 0 & 0 & k_{15} & 0 & k_{49} & 0 \\
    k_{15} & 0 & 0 & 0 & k_{44} & -k_{15} & 0 & 0 & 0 & k_{49} \\
    -k_{11} & 0 & 0 & 0 & -k_{15} & k_{66} & 0 & 0 & 0 & -k_{79} \\
    0 & -k_{11} & 0 & k_{15} & 0 & 0 & k_{66} & 0 & k_{79} & 0 \\
    0 & 0 & -k_{33} & 0 & 0 & 0 & 0 & k_{88} & 0 & 0 \\
    0 & k_{29} & 0 & k_{49} & 0 & 0 & k_{79} & 0 & k_{99} & 0 \\
    -k_{29} & 0 & 0 & 0 & k_{49} & -k_{79} & 0 & 0 & 0 & k_{99}
\end{bmatrix}
\]

B.2 HARD-MOUNTED RWA SYSTEM MATRICES

\[
M_{hm} = \text{diag}\{M_w, M_w, M_w, I_{r,w}, I_{r,w}\}
\]
\[
\begin{align*}
G_{hm} &= \begin{cases}
G_{hm,45} = \Omega I_{z,w} \\
G_{hm,54} = -\Omega I_{z,w} \\
0, \quad \text{elsewhere}
\end{cases} \\
C_{hm} &= \begin{bmatrix}
c_t,w & 0 & 0 & 0 & -c_t,w d \\
0 & c_t,w & 0 & c_t,w d & 0 \\
0 & 0 & c_{z,w} & 0 & 0 \\
0 & c_t,w d & 0 & c_t,w d^2 + c_{r,w} & 0 \\
-c_t,w d & 0 & 0 & 0 & c_t,w d^2 + c_{r,w}
\end{bmatrix} \\
K_{hm} &= \begin{bmatrix}
k_t,w & 0 & 0 & 0 & -k_t,w d \\
0 & k_t,w & 0 & k_t,w d & 0 \\
0 & 0 & k_{z,w} & 0 & 0 \\
0 & k_t,w d & 0 & k_t,w d^2 + k_{r,w} & 0 \\
-k_t,w d & 0 & 0 & 0 & k_t,w d^2 + k_{r,w}
\end{bmatrix}
\end{align*}
\]

\[
f_{hm} = \begin{cases}
\sum_{i=1}^{n_t} A_i^t \Omega^2 \sin (h_i^t \Omega t) + W \\
\sum_{i=1}^{n_t} A_i^t \Omega^2 \cos (h_i^t \Omega t) + W \\
\sum_{i=1}^{n_a} A_i^a \Omega^2 \sin (h_i^a \Omega t) + W \\
\sum_{i=1}^{n_r} A_i^r \Omega^2 \cos (h_i^r \Omega t) + W \\
\sum_{i=1}^{n_r} A_i^r \Omega^2 \sin (h_i^r \Omega t) + W
\end{cases}
\]

### B.3 Free-free RWA System Matrices

With reference to Equation (3.41), all matrices are as follows:

\[
q_s = \{x_w, y_w, z_w, \theta_w, \varphi_w, x_b, y_b, z_b, \theta_b, \varphi_b\}^T
\]

\[
M_s = \text{diag} \{M_w, M_w, M_w, I_{r,w}, I_{r,w}, M_b, M_b, M_b, I_{r,b}, I_{r,b}\}
\]
\[
G_s = \begin{cases} 
G_{s,45} = \Omega I_{z,w} \\
G_{s,54} = -\Omega I_{z,w} \\
0, \text{ elsewhere}
\end{cases}
\]

Table B.3: Compact form of the damping coefficients for the free-free RWA system

<table>
<thead>
<tr>
<th>Damping Matrix Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c_{11} = c_{t,w})</td>
</tr>
<tr>
<td>(c_{33} = c_{z,w})</td>
</tr>
<tr>
<td>(c_{29} = c_{t,w}h)</td>
</tr>
<tr>
<td>(c_{99} = c_{t,w}h^2 + c_{r,w})</td>
</tr>
</tbody>
</table>

\[
C_{fis} = \begin{bmatrix}
  c_{11} & 0 & 0 & 0 & c_{15} & -c_{11} & 0 & 0 & 0 & -c_{29} \\
  0 & c_{11} & 0 & -c_{15} & 0 & 0 & -c_{11} & 0 & c_{29} & 0 \\
  0 & 0 & c_{33} & 0 & 0 & 0 & 0 & -c_{33} & 0 & 0 \\
  0 & -c_{15} & 0 & c_{44} & 0 & 0 & c_{15} & 0 & c_{49} & 0 \\
  c_{15} & 0 & 0 & 0 & c_{44} & -c_{15} & 0 & 0 & 0 & c_{49} \\
-c_{11} & 0 & 0 & 0 & -c_{15} & c_{11} & 0 & 0 & 0 & c_{29} \\
  0 & -c_{11} & 0 & c_{15} & 0 & 0 & c_{11} & 0 & -c_{29} & 0 \\
  0 & 0 & -c_{33} & 0 & 0 & 0 & 0 & c_{33} & 0 & 0 \\
  0 & c_{29} & 0 & c_{49} & 0 & 0 & -c_{29} & 0 & c_{99} & 0 \\
-c_{29} & 0 & 0 & 0 & c_{49} & c_{29} & 0 & 0 & 0 & c_{99}
\end{bmatrix}
\]

Table B.4: Compact form of the stiffness coefficients for the free-free RWA system

<table>
<thead>
<tr>
<th>Stiffness Matrix Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_{11} = k_{t,w})</td>
</tr>
<tr>
<td>(k_{33} = k_{z,w})</td>
</tr>
<tr>
<td>(k_{29} = k_{t,w}h)</td>
</tr>
<tr>
<td>(k_{99} = k_{t,w}h^2 + k_{r,w})</td>
</tr>
</tbody>
</table>
B.4 RWA-Stiff Platform System Matrices

Let $M_{cube}$ be the mass of the stiff platform employed in the RWA coupling analysis. Assume that $M_{cube} \gg M_b$ and $I_{r,cube} \gg I_{r,b}$, thence the coupled mass matrix can be written as:

$$
K_{fis} = \begin{bmatrix}
    k_{11} & 0 & 0 & 0 & k_{15} & -k_{11} & 0 & 0 & 0 & -k_{29} \\
    0 & k_{11} & 0 & -k_{15} & 0 & 0 & -k_{11} & 0 & k_{29} & 0 \\
    0 & 0 & k_{33} & 0 & 0 & 0 & 0 & -k_{33} & 0 & 0 \\
    0 & -k_{15} & 0 & k_{44} & 0 & 0 & k_{15} & 0 & k_{49} & 0 \\
    k_{15} & 0 & 0 & 0 & k_{44} & -k_{15} & 0 & 0 & 0 & k_{49} \\
    -k_{11} & 0 & 0 & 0 & -k_{15} & k_{11} & 0 & 0 & 0 & k_{29} \\
    0 & -k_{11} & 0 & k_{15} & 0 & 0 & k_{11} & 0 & -k_{29} & 0 \\
    0 & 0 & -k_{33} & 0 & 0 & 0 & 0 & k_{33} & 0 & 0 \\
    0 & k_{29} & 0 & k_{49} & 0 & 0 & -k_{29} & 0 & k_{99} & 0 \\
    -k_{29} & 0 & 0 & 0 & k_{49} & k_{29} & 0 & 0 & 0 & k_{99}
\end{bmatrix}
$$

Let $M_{cube}$ be the mass of the stiff platform employed in the RWA coupling analysis. Assume that $M_{cube} \gg M_b$ and $I_{r,cube} \gg I_{r,b}$, thence the coupled mass matrix can be written as:

$$
M_{cp} = \text{diag}\left\{ M_w, M_w, M_w, I_{r,w}, I_{r,w}, M_{cube}, M_{cube}, M_{cube}, I_{r,cube}, I_{r,cube} \right\}
$$

In addition, in accordance to the aforementioned assumptions, the generalised coordinate vector now describes the motion of the flywheel DoFs and those of the stiff platform, thus giving:

$$
q_{cp} = \{ x_w, y_w, z_w, \theta_w, \varphi_w, x_{cube}, y_{cube}, z_{cube}, \theta_{cube}, \varphi_{cube} \}^T
$$

The gyroscopic matrix, on the other, remains unchanged as it only depends on the flywheel DoFs:

$$
G_{cp} = \begin{cases}
    G_{cp,45} = \Omega I_{z,w} \\
    G_{cp,54} = -\Omega I_{z,w} \\
    0, \quad \text{elsewhere}
\end{cases}
$$

On the other hand, both the damping and stiffness matrices require to be modified. In particular, the parameter defining the distance between the stiff platform CoM and the suspension system $p$ replaces the coefficient defining the wheel-base to suspension system distance $h$, yielding to:
Table B.5: Compact form of the damping coefficients for the free-free RWA-stiff platform system

<table>
<thead>
<tr>
<th>Damping Matrix Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{11} = c_{t,w}$</td>
</tr>
<tr>
<td>$c_{33} = c_{z,w}$</td>
</tr>
<tr>
<td>$c_{29} = c_{t,w}p$</td>
</tr>
<tr>
<td>$c_{99} = c_{t,w}p^2 + k_{r,w}$</td>
</tr>
</tbody>
</table>

$$C_{cp} = \begin{bmatrix}
 c_{11} & 0 & 0 & 0 & c_{15} & -c_{11} & 0 & 0 & 0 & -c_{29} \\
 0 & c_{11} & 0 & -c_{15} & 0 & 0 & -c_{11} & 0 & c_{29} & 0 \\
 0 & 0 & c_{33} & 0 & 0 & 0 & 0 & -c_{33} & 0 & 0 \\
 0 & -c_{15} & 0 & c_{44} & 0 & 0 & c_{15} & 0 & c_{49} & 0 \\
 -c_{11} & 0 & 0 & 0 & c_{44} & -c_{15} & 0 & 0 & 0 & c_{49} \\
 0 & -c_{11} & 0 & c_{15} & 0 & 0 & c_{11} & 0 & -c_{29} & 0 \\
 0 & 0 & -c_{33} & 0 & 0 & 0 & 0 & c_{33} & 0 & 0 \\
 0 & c_{29} & 0 & c_{49} & 0 & 0 & -c_{29} & 0 & c_{99} & 0 \\
 -c_{29} & 0 & 0 & 0 & c_{49} & c_{29} & 0 & 0 & 0 & c_{99}
\end{bmatrix}$$

Table B.6: Compact form of the stiffness coefficients for the free-free RWA-stiff platform system

<table>
<thead>
<tr>
<th>Stiffness Matrix Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{11} = k_{t,w}$</td>
</tr>
<tr>
<td>$k_{33} = k_{z,w}$</td>
</tr>
<tr>
<td>$k_{29} = k_{t,w}p$</td>
</tr>
<tr>
<td>$k_{99} = k_{t,w}p^2 + k_{r,w}$</td>
</tr>
</tbody>
</table>
\[
K_{cp} = \begin{bmatrix}
  k_{11} & 0 & 0 & 0 & k_{15} & -k_{11} & 0 & 0 & 0 & -k_{29} \\
  0 & k_{11} & 0 & -k_{15} & 0 & 0 & -k_{11} & 0 & k_{29} & 0 \\
  0 & 0 & k_{33} & 0 & 0 & 0 & 0 & -k_{33} & 0 & 0 \\
  0 & -k_{15} & 0 & k_{44} & 0 & 0 & k_{15} & 0 & k_{49} & 0 \\
  k_{15} & 0 & 0 & 0 & k_{44} & -k_{15} & 0 & 0 & 0 & k_{49} \\
 -k_{11} & 0 & 0 & 0 & -k_{15} & k_{11} & 0 & 0 & 0 & k_{29} \\
 0 & -k_{11} & 0 & k_{15} & 0 & 0 & k_{11} & 0 & -k_{29} & 0 \\
 0 & 0 & -k_{33} & 0 & 0 & 0 & 0 & k_{33} & 0 & 0 \\
 0 & k_{29} & 0 & k_{49} & 0 & 0 & -k_{29} & 0 & k_{99} & 0 \\
 -k_{29} & 0 & 0 & 0 & k_{49} & k_{29} & 0 & 0 & 0 & k_{99}
\end{bmatrix}
\]

Finally, opportunely reformulating Equation (3.44), the system structural modes can be derived as:

\[
\omega_{cp}^5 - \Omega \frac{I_{z,w}}{I_{r,w}} \omega_{cp}^4 - \frac{(M_{cube} + M_w) k_{t,w}}{M_{cube} M_w} \frac{k_{r,w} + k_{t,w} d^2}{I_{r,w}} + \frac{k_{r,w} + k_{t,w} p^2}{I_{r,cube}} \omega_{cp}^3 + \Omega \left[ \frac{(M_{cube} + M_w) k_{t,w} I_{z,w}}{I_{r,w} M_{cube} M_w} + \frac{(k_{r,w} + k_{t,w} p^2) I_{z,w}}{I_{r,cube} I_{r,w}} \right] \omega_{cp}^2 + \left[ \frac{(I_{r,cube} + I_{r,w}) (M_{cube} + M_w) k_{r,w} k_{t,w}}{I_{r,cube} I_{r,w} M_{cube} M_w} + \frac{k_{r,w} + k_{t,w} (p + d)^2}{I_{r,cube} I_{r,w}} \right] \omega_{cp} + \Omega \frac{M_{cube} + M_w}{I_{r,cube} I_{r,w} M_{cube} M_w} = 0
\]

\[
\omega_{zcp} = \sqrt{\frac{k_{z,w} M_{cube} + M_w}{M_{cube} M_w}}
\]

(B.1)
Typically, large amount of data are acquired throughout the whole microvibration test campaign hence their management and analysis require to be automated in order to quicken their processing. In order to establish which technique is best for microvibration analysis, in particular for RWAs, a first classification based on the type of signal can be carried out:

- stationary (or quasi-stationary) signals: no signal statistical parameters change can be observed over time, or, if any, the variation is extremely slow. These signals are associated with rotary mechanisms such as RWAs, cryo-coolers and continuously operating scanners and are typically examined in the frequency domain;

- non-stationary signals: this class refers to signals which are time dependent and abruptly change over time. This includes also transient signals such as those generated by switches, valves or thermal snap. Due to their nature, these signals are generally analysed in the time domain.

Despite the common practice is to analyse stationary signals in the frequency domain by means of FFT, PSD, AS, spectral maps and RMS, several techniques and parameters have been established for analysis in time domain. For instance, variance, standard deviation and auto- and cross- correlation functions provide useful information on the statical signature of the signal. An additional tool which is commonly employed in the analysis of rotary mechanisms is the Campbell diagram, which allows the representation of the mechanism structural modes versus its operative speed of rotation.
In the analysis of non-stationary signals, the hybrid time-frequency analysis is the most common approach. Tools include short-time Fourier transform, spectrogram, Wigner-Ville distribution and continuous Wavelet transform. Among all, spectrogram plots, for which the FFT amplitude of the signal is represented by the color intensity and plotted against time, provide the best technique in terms of the analysis of transient microvibration data.

In this thesis, the RWA-induced microvibrations are mainly analysed in the frequency domain by means of FFT, PSD, and RMS value and all algorithms have been compiled in MATLAB. Theories and equations implemented in the algorithms are introduced in the rest of this chapter.

C.1 Sampling Criteria

When dealing with experimental data, one of the most important parameters to be set is the frequency at which the signal is sampled and, therefore, acquired. The minimum requirement in order to avoid aliasing effects, that is to make the sampled signal indistinguishable, is to select a sampling frequency which is at least 2.46 bigger than the Nyquist frequency. In other words, the sampling frequency should be 2.46 times the largest frequency in the frequency band of interest. For instance, considering a frequency band with an upper limit of 300 Hz, the minimum sampling frequency which allows the correct reconstruction of the signal is 858 Hz.

Other parameters which influence the reliability and fidelity of the signal processing techniques discussed in this chapter include the number of sampling points, \(N\), the length of the time signal, \(t\), the resolution of the time signal \(\Delta t\) and the resolution of the frequency sample \(\Delta f\). The relationship between these coefficients is listed in Table C.1.

**Table C.1: Relationship between the parameters implemented in the data acquisition**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Relationship</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sampling Frequency</td>
<td>(f_{\text{sampling}} = \frac{1}{\Delta t} = \frac{N}{T})</td>
</tr>
<tr>
<td>Frequency Resolution</td>
<td>(f_{\text{sampling}} = \frac{f_{\text{sampling}}}{N} = \frac{1}{T})</td>
</tr>
<tr>
<td>Time Length</td>
<td>(f_{\text{sampling}} = N \Delta t = \frac{N}{f_{\text{sampling}}})</td>
</tr>
<tr>
<td>Time Resolution</td>
<td>(\Delta t = \frac{T}{N} = \frac{1}{f_{\text{sampling}}})</td>
</tr>
</tbody>
</table>
C.2 Root Mean Square

The RMS is a measure of the energy associated with a signal and can be evaluated either using an energy approach or a PSD approach. The former can be applied in both the time and frequency domains whereas the latter in the frequency domain only.

In this section, only the energy approach will be described whereas the PSD approach will be addressed later in this appendix. Consider a signal in the time domain, $x(t)$, for which the energy, $E_{\text{time}}$ can be computed as:

$$E_{\text{time}} = \int_{-\infty}^{\infty} x^2(t) \, dt$$ (C.1)

If a discretization is applied to the signal, thus to divide the signal in $N$ elements, the power of the discretized signal, $x(n)$, can be derived as:

$$P_{\text{time}} = \frac{1}{N} \sum_{n=0}^{N-1} x^2(n)$$ (C.2)

Similarly, the power of the signal in the frequency domain, $x(f)$, can be expressed as:

$$P_{\text{frequency}} = \frac{1}{N^2} \sum_{k=0}^{N-1} |x(k)|^2$$ (C.3)

Parseval’s theorem affirms that the power, hence the energy, of a signal in the frequency and time domains is identical. Therefore, Equations (C.2) and (C.3) must provide the same results. This has a direct consequence on the RMS value, which is defined as the square root of the power of a signal (whether in the frequency domain or in the time domain), for which Parseval’s theorem stands valid:

$$RMS_{\text{time}} = \sqrt{\frac{1}{N} \sum_{n=0}^{N-1} x^2(n)} = \sqrt{\frac{1}{N^2} \sum_{k=0}^{N-1} |x(k)|^2} = RMS_{\text{frequency}}$$ (C.4)

It was previously mentioned that the microvibration analysis of rotary mechanisms is generally carried out in the frequency domain. Thereby, RMS values can be used to assess the quality of the transformation process from the time to the frequency domain, including sampling size, window (size and type), overlapping
and averaging, etc., which are commonly applied in this process.

**C.3 Fourier Transformation**

The Fourier transformation is the most common technique to convert a time domain signal into a frequency domain signal and an extended literature exists on this subject.

Given a discretized time signal, its discrete Fourier transformation is computed as:

\[
x(k) = \sum_{n=0}^{N-1} x(n)e^{-\frac{2\pi i kn}{N}}
\]  

(C.5)

FFT algorithms are generally implemented to solve Equation (C.5), allowing a significant reduction in terms of computation effort. The FFT is a MATLAB built-in function and the outcomes permit the calculation of other parameters such as AS, PSD and RMS value.

**C.4 Amplitude Spectrum**

The AS is a frequency domain representation of a stochastic process employing FFT data. The results provide an intuitive view of the distribution of the amplitude of signal in the frequency domain. Note that due to the two-sided feature of FFT data, only half of Equation (C.6) requires to be taken into account.

\[
AS(f) = \frac{1}{T} \left| \int_{0}^{T} x(t)e^{-\frac{2\pi i kn}{N}} dt \right| = \frac{|x(k)|}{N}
\]  

(C.6)

**C.5 Correlation Function**

Signals may be further categorised as deterministic or stochastic. The former refers to signals which are predictable over a defined period of time whereas the latter denotes signals whose elements are random. Nevertheless, some stochastic signals can be regarded as deterministic. This is the case of RWAs. The data measured from a RWA exhibits a deterministic behaviour (as it is periodic over time) but also includes random elements (i.e. phase shift). This kind of signal can be processed in the time-domain to define an auto-correlation function or a cross-correlation function.

The auto-correlation is an expression to identify repeating patterns in a signal and determine how well a signal correlates with itself at two different instants in time. In other words, it defines how similar multiple observations of the same
signal are as a function of a lag time between them. In addition, for processes where
the mean-value and the variance do not change over time (stationary process), as
those described in this thesis, the correlation depends only on the time-distance,
τ, between the pair of values but not on their position in time.

\[ R_{xx}(\tau) = \frac{E[(X_{time} - \mu)(X_{time+\tau} - \mu)]}{\sigma^2} \]  (C.7)

where \( \mu \) is the mean value of the signal, \( \sigma \) is the standard deviation and \( E \)
denotes the expected value.

The cross-correlation, on the other hand, describes the similarity between two
different signals as a function of the lag of one relative to the other, and is defined
as:

\[ R_{xy}(\tau) = \frac{E[(X_{time} - \mu_x)(Y_{time+\tau} - \mu_y)]}{\sigma_x \sigma_y} \]  (C.8)

All the signals presented in this work are considered zero-mean and with con-
stant variance.

C.6 Power Spectral Density

The PSD is a frequency domain figure which defines the power intensity of a
signal over a frequency range. Its value shows at which frequencies variations in the
signal are either weak or strong. Two approaches can be used for its calculation:
the first involves the application of the FFT to the correlation function of a signal
whereas the second involves the reformulation of Equation (C.3) by means of
Parseval’s theorem.

Taking Equation (C.7) and applying the FFT, the PSD can be directly calcu-
lated as:

\[ S(f) = \int_{-\infty}^{\infty} R(\tau)e^{-\frac{i2\pi fn}{N}} d\tau \]  (C.9)

The power of a signal is defined as the area under the signal, hence the area
under the PSD denotes the power of the signal in the frequency domain:

\[ P_{frequency} = \int_{-\infty}^{\infty} S(f)df \]  (C.10)

for which applying the trapezium rule yields to:

\[ P_{frequency} \approx \sum_{n=k}^{N/2-1} S(k)\Delta f \]  (C.11)
According to Parseval’s theorem and minding the two-side feature of the FFT, Equation (C.3) can be re-written as:

\[ P_{\text{frequency}} = \frac{2}{N^2} \sum_{n=k}^{N/2-1} |x(k)|^2 \]  
(C.12)

Equaling Equations (C.11) and (C.12), a further expression of the PSD can be derived:

\[ S(k) = \frac{2}{N^2 \Delta f} |x(k)|^2 \]  
(C.13)

Finally, the RMS value can be obtained from the square root of Equation (C.13), giving:

\[ RMS_{\text{frequency}} = \sqrt{\frac{1}{N/2-1} \sum_{n=k}^{N/2-1} S(k) \Delta f} \]  
(C.14)
APPENDIX D

TEST EQUIPMENT AND RESULTS
DERIVATION PROCESS

Detail descriptions of the test setups and of the equipment used for the conduction of the experimental tests are given here. In addition, the equations by means of which the hard-mounted loads and the free-free accelerations were derived in sections 4.4 and 4.5, respectively, are presented in this appendix. Furthermore, schematics of the force sensors and accelerometers configuration will be shown.

D.1 TEST EQUIPMENT

The lists of the force sensors (or load cells) and of the accelerometers used during the experimental tests described in this thesis are given in Tables D.1 and D.2, respectively.

Table D.1: List of the load cells used for force measurements

<table>
<thead>
<tr>
<th>Model</th>
<th>Type</th>
<th>Sensitivity [mVN⁻¹]</th>
<th>Uncertainty [%]</th>
<th>Linearity [%FS]</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCB 208C01</td>
<td>ICP</td>
<td>116.5</td>
<td>±1</td>
<td>0.4</td>
</tr>
<tr>
<td>PCB 208C01</td>
<td>ICP</td>
<td>108.1</td>
<td>±1</td>
<td>0.4</td>
</tr>
<tr>
<td>PCB 208C01</td>
<td>ICP</td>
<td>110.7</td>
<td>±1</td>
<td>0.4</td>
</tr>
<tr>
<td>PCB 208C02</td>
<td>ICP</td>
<td>11.17</td>
<td>±1</td>
<td>0.5</td>
</tr>
</tbody>
</table>

For the acquisition of the signals, a National Instruments PCIe-6321 data acquisition system with 16 channels was considered (system specifications available...
Table D.2: List of the accelerometers used for acceleration and accelerance measurements

<table>
<thead>
<tr>
<th>Model</th>
<th>Type</th>
<th>Sensitivity [mVg⁻¹]</th>
<th>Transverse Sensitivity [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>ENDEVCO I-TEDS 752-A13</td>
<td>ICP</td>
<td>1001</td>
<td>1.8</td>
</tr>
<tr>
<td>ENDEVCO I-TEDS 752-A13</td>
<td>ICP</td>
<td>988.3</td>
<td>2.3</td>
</tr>
<tr>
<td>ENDEVCO ISOTRON 256-100</td>
<td>ICP</td>
<td>95.27</td>
<td>2.4</td>
</tr>
<tr>
<td>ENDEVCO ISOTRON 256-100</td>
<td>ICP</td>
<td>97.49</td>
<td>2.9</td>
</tr>
<tr>
<td>ENDEVCO ISOTRON 256-100</td>
<td>ICP</td>
<td>95.93</td>
<td>2.2</td>
</tr>
<tr>
<td>ENDEVCO 2256A-100</td>
<td>ICP</td>
<td>98.4</td>
<td>3.9</td>
</tr>
<tr>
<td>ENDEVCO 2256A-100</td>
<td>ICP</td>
<td>99.73</td>
<td>3.4</td>
</tr>
<tr>
<td>ENDEVCO 2256A-100</td>
<td>ICP</td>
<td>99.3</td>
<td>1.2</td>
</tr>
<tr>
<td>PCB 352C22</td>
<td>ICP</td>
<td>9.49</td>
<td>N/A</td>
</tr>
<tr>
<td>LDS 352M119</td>
<td>ICP</td>
<td>10.91</td>
<td>N/A</td>
</tr>
</tbody>
</table>

at http://www.ni.com/pdf/manuals/374461b.pdf. This was integrated with the software m+p SmartOffice which allowed the setup of each channel (i.e. control or response) and the selection of the maximum bandwidth, number of samples (as to define the frequency resolution), sampling frequency, duration of acquisition, etc. In particular, for force and acceleration measurements:

- the bandwidth was set equal to 800 Hz;

- the sampling frequency was set equal to 2048 Hz (as to satisfy the Nyquist requirement or 2.56 times the bandwidth value);

- the block size (or number of samples) was set equal to 4096 giving a frequency resolution of 0.5 Hz;

- the acquisition time was set to 6 s.

In order to operate the RWA, a dedicated electronic circuit was built. The RWA motor (a Faulhaber 2444 DC brushless motor, with a rotor inertia equal to $5.7 \times 10^{-7}$kgm² and a maximum operative speed of 40000 rpm) was plugged to a Faulhaber SC2804 speed controller board. The latter was subsequently connected to a NI-6321 data acquisition system, which is powered via USB. A script in LabView permitted to control the speed of the motor by either manually increasing/decreasing the speed or autonomously varying the speed by a predefined amount inserted by the operator.

A set of two Brüel & Kjær permanent magnet shakers, model V106, were used for the application of the forces in the measurement of the RWA accelerance. This
A device provides a maximum force of 3.11 N, a maximum displacement of 2.5 mm peak-to-peak, has an overall mass of 0.91 kg, of which only 0.0060 kg is actually moving, and requires a 0.09 kVA amplifier.

A mini-shaker controller was used for the control of the motion of the mini-shakers during the direct accelerance measurements. In particular, the LDS Laser USB Vibration Controller system was available at Surrey Space Centre. The system provides a 10 ms loop time for sine swept applications and has a 24-bit precision with wide control dynamic range. It operates in a frequency range between 0.1 Hz and 12000 Hz. For more details, please refer to the system data document, available at https://www.bksv.com/~/media/literature/Product%20Data/bu3079.ashx. The drive signal generated by the vibration controller system was given as input to a generic signal amplifier whose function was to convert and amplify the drive signal and to supply power to the mini-shakers. In order to operate the mini-shaker anti-phase (such as to create an oscillating moment), the positive and negative cables of one of the two mini-shakers were inverted.

D.2 HARD-MOUNTED FORCES AND MOMENTS

D.2.1 AXIAL CONFIGURATION

A top view of the force sensors configuration is shown in Figure D.1. The force sensors are located such to form an equilateral triangle whose edge is 86 mm long. In the figure, \( a \) represents the distance between the force sensor and the triangle centre along the RWA \( x_w \)-axis whereas the distance of the force sensors 2 and 3 from the triangle centre is equal to \( b \) and \( c \), in the RWA \( x \) and \( y \) directions, respectively. The actual values of \( a \), \( b \) and \( c \) are given in Table D.3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>50 mm</td>
</tr>
<tr>
<td>( b )</td>
<td>25 mm</td>
</tr>
<tr>
<td>( c )</td>
<td>43 mm</td>
</tr>
</tbody>
</table>

Accordingly to Figure D.1(b), the resulting force at the centre of the triangle is equal to the sum of the forces measured by each transducer:

\[
F_{hm,z} = F_{hm,z2} + F_{hm,z3} + F_{hm,z4}
\]
The force sensors output is positive for compression and negative for tension. Thereby, because our interest is in the actual force transmitted to the support, no change of sign is required. In addition, previous studies conducted by Zhang et al. (2011), demonstrated that the measurement platform has resonance well above 500 Hz hence can be considered as rigid in the frequency band considered in this work. Therefore, the force measured at the sensor location is equally transmitted at the RWA mounting interface without the need to apply any transformation matrix.

### D.2.2 LATERAL-X CONFIGURATION

For the testing in the $x_w$-axis, the platform required to be rotated so that now the force sensors lie on the side of the RWA rather than at the bottom, as presented in Figure D.2. Nevertheless, the equilateral triangle configuration is maintained as well as the distances between the force sensors. Recalling from section 4.4.1, this configuration served to measure the RWA-induced disturbances along the $x_w$-axis and those about the $y_w$-axis. The latter requires the definition of the distance between the force sensors and the RWA mounting interface. These are defined in Table D.4 with respect to Figure D.2.

Accordingly to Figure D.2, the resulting force at the centre of the triangle is equal to the sum of the forces measured by each transducer:

$$F_{hm,x} = F_{hm,x2} + F_{hm,x3} + F_{hm,x4}$$  \hspace{1cm} (D.2)
Figure D.2: Schematic configuration of the force sensors used during microvibration hard-mounted testing in the $x_w$-axis

Table D.4: Orthogonal distance between the force sensors and the RWA mounting interface

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_2$</td>
<td>70 mm</td>
</tr>
<tr>
<td>$d_3$</td>
<td>10 mm</td>
</tr>
</tbody>
</table>
As previously stated, the rigidity of the platform allows the transportation of the force measured at the sensors centre of mass to the RWA mounting point without any further calculation. On the other hand, the moment about the $y_w$-axis requires the forces to be opportunely multiplied by the distance between the corresponding force sensor and the RWA interface, measured orthogonally with respect to the force direction, yielding to:

$$M_{hm,y} = (F_{hm,x2} + F_{hm,x4})d_2 - F_{hm,x3}d_3$$  \hspace{1cm} (D.3)

**D.2.3 LATERAL-Y CONFIGURATION**

The platform and force sensors configuration is maintained in the same position with respect to the $x_w$-axis test setup whereas the RWA is rotated by 90 degrees clockwise. This configuration allowed the measurement of the RWA microvibration force along the $y_w$-axis and the resulting moment about the $x_w$-axis.

![Figure D.3: Schematic configuration of the force sensors used during microvibration hard-mounted testing in the $y_w$-axis](image)

With reference to Figure D.3, the resulting force at the centre of the triangle is equal to the sum of the forces measured by each transducer:

$$F_{hm,y} = F_{hm,y2} + F_{hm,y3} + F_{hm,y4}$$  \hspace{1cm} (D.4)

Again, the platform rigidity permits transporting the force measured at the sensors centre of mass to the RWA mounting point with any additional manipulation. Similarly to $M_{hm,y}$, the moment about the $x_w$-axis was computed combining
the forces measured by the force sensors and the corresponding distances, giving:

\[
M_{hm,x} = -(F_{hm,x2} + F_{hm,x4})d_2 + F_{hm,x3}d_3
\]  \tag{D.5}

\section*{D.3 Free-free accelerations}

A schematic of the accelerometers configuration employed during the free-free microvibration testing is given in Figure D.4.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{accelerometers_schematic}
\caption{Accelerometers schematic configuration used during free-free microvibration testing: (a) \(xwzw\)-plane and (b) \(ywzw\)-plane. The green arrows indicate the positive direction of measurement of the corresponding accelerometer.}
\end{figure}

With reference to Figure D.4, the distances between each pair of accelerometers are listed in Table D.5.

A combination of the 7 accelerometers allowed the derivation of the 3 translational accelerations \(\ddot{x}_{mp}\), \(\ddot{y}_{mp}\), and \(\ddot{z}_{mp}\) and the corresponding angular rotations about each axis. The formulas are as follows:

\[
\ddot{x}_{mp} = \frac{\ddot{x}_{A5} + \ddot{x}_{A6}}{2}
\]  \tag{D.6}

\[
\ddot{y}_{mp} = \frac{\ddot{x}_{A2} + \ddot{x}_{A3}}{2}
\]  \tag{D.7}
Table D.5: Distance between accelerometers in the free-free test configuration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{14}$</td>
<td>60 mm</td>
</tr>
<tr>
<td>$d_{23}$</td>
<td>70 mm</td>
</tr>
<tr>
<td>$d_{47}$</td>
<td>90 mm</td>
</tr>
<tr>
<td>$d_{56}$</td>
<td>70 mm</td>
</tr>
</tbody>
</table>

\[ \ddot{z}_{mp} = -\frac{\ddot{x}_{A1} + \ddot{x}_{A7}}{2} \quad (D.8) \]

\[ \ddot{\theta}_{mp} = \frac{\ddot{x}_{A4} - \ddot{x}_{A7}}{(\frac{d_{47}}{2})} \quad (D.9) \]

\[ \ddot{\varphi}_{mp} = \frac{\ddot{x}_{A4} - \ddot{x}_{A1}}{(\frac{d_{14}}{2})} \quad (D.10) \]

\[ \ddot{\psi}_{mp} = \frac{\ddot{x}_{A5} - \ddot{x}_{A6}}{(\frac{d_{56}}{2})} \quad (D.11) \]

Note the sign minus in Equation (D.8) indicates that the output of the accelerometers A1, A4 and A7 is in opposite direction with respect to the RWA $z_w$-axis, as shown in Figure D.4.