STABILISATION POLICY, FINANCIAL FRictions AND HETEROGENEITY IN MACROECONOMIC MODELS

JONATHAN SWARBRICK

Submitted for the Degree of Doctor of Philosophy

School of Economics
Faculty of Arts and Social Sciences
University of Surrey

September 2016

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DECLARATION

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Guildford, United Kingdom, September 2016

Jonathan Swarbrick
SUMMARY OF THESIS

This thesis examines the implications of financial frictions on macroeconomic outcomes and their impact on the transmission mechanisms of economic policy. Chapters 3 and 4 study the theoretical implications of significant non-linearities in financial constraints, chapter 5 examines the role of financial frictions on cross-border lending in a currency union.

Macroeconomic time-series suggest that occasional financial crises generate sharp increases in the interest spread, and deep downturns in output and investment. Standard models of financial frictions are unable to explain this phenomena as the implied borrowing constraints are always binding. In chapter 3, a model is proposed in which the financial constraints are only occasionally binding. This generates simulated time series that capture these crisis episodes, and, as a result, replicate observed positive skewness in the interest spread, and negative skewness in output and investment.

The majority of models of financial frictions, including that proposed in chapter 3, focus on a time-varying investment wedge between the risky return to capital and the risk-free rate. The empirical evidence, however, suggests that this wedge does not play an important role in driving business cycles, but rather supports financial frictions that affect either the efficient allocation of the factors of production (efficiency wedge), or the labour market (labour wedge). In chapter 4 I propose a model where a credit friction emerges as both an efficiency and investment wedges. This is able to generate occasional, large crisis episodes and replicate the observed negative skewness in simulated time series of output and investment.

Contrary to empirical evidence, cross-border financial flows in structural models usually dampen the adverse effects of shocks. In chapter 5, I examine frictions in the cross-border interbank market in a currency union that enhance these effects. Two recently applied unconventional policies are implemented and analysed.
It is well known that a vital ingredient of success is not knowing that what you’re attempting can’t be done

— Terry Pratchett

For Rebecca and David
ACKNOWLEDGMENTS

I owe enormous gratitude to Tom Holden and Paul Levine for excellent supervision on this doctoral thesis. Also a special thanks to Maureen Newman and Vasco Gabriel for encouraging the department to take me on as a student in the first place, to Dave Cannell for proof-reading, and to Tobias Blattner for the support whilst at the European Central Bank and on the work for chapter 5. Thanks to all faculty members and colleagues at Surrey for their support and helpful discussions, in particular Antonio Mele, Szabolcs Deak, Marcelo Almeida, Afrasiab Mirza and Cristiano Cantore, and to Helen Dee for all the administrative support. Thanks are due to others with whom I have discussed research with including Stephan Fahr and Dominic Quint for discussions on chapter 5, Anthony Savager, Gustavo Mellior, Ayobami Ilori, Adrian Paul, Shifu Jiang, and Alessandro Cantelmo. Thanks also to the participants and discussants at various workshops and conferences, including those at the Money, Macro & Finance PhD Workshop, University of Birmingham, April 2016; the Royal Economic Society Junior Researcher Symposium, University of Sussex, March 2016; the Centre for Applied Macroeconomics Annual Conference, Birkbeck, University of London, May 2015; and the Computing in Economics and Finance conference, BI Business School, Oslo June 2015. I am grateful to staff in the Monetary Assessment and Strategy division at the Bank of England for all the helpful discussions during my internship, in particular to John Bardeer and Lien Laureys. I gratefully acknowledge the financial support of the Economic and Social Research council without whom this would not have been possible. Finally, of course, I am forever indebted to Rebecca and David for their love, patience and support.
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ACRONYMS

CES constant elasticity of substitution. 158, 163

DSGE dynamic stochastic general equilibrium. 4–7, 10, 11, 14, 21, 23, 147, 192

ECB European Central Bank. 153, 162, 171, 176, 189, 193

GDP gross domestic product. 2, 167, 175


GKQ Gertler, Kiyotaki & Queralto (2012). 41, 68

i.i.d. independent and identically distributed. 158

IC incentive compatibility. 97, 98

IR individual rationality. 97, 98

IRF impulse response functions. xiv, xv

LTRO long-term refinancing operations. xiv, 171, 172, 176, 189–191

NBER National Bureau of Economic Research. 2

NK New Keynesian. xvi, 4, 7, 44, 86–89

RBC real business cycle. xiv, xvi, 2–4, 6, 7, 44, 45, 52, 53, 70, 76–82, 88, 89, 140

SEO seasoned equity offering. 50
SSM  Single Supervisory Mechanism. 148
INTRODUCTION

This thesis examines the implications of financial frictions on macroeconomic outcomes and the role of economic policy. Since the work of Irving Fisher and John Maynard Keynes\(^1\) during the Great Depression, credit frictions have been thought to bear an important share of responsibility for driving economic fluctuations. Despite this, it is only since the global financial crisis that financial frictions have become a central focus in macroeconomic modelling and analysis. Three models are proposed in this thesis; the first is a model of banks that face occasionally binding borrowing constraints; the second of an adverse selection problem in bank lending; and the third a model of financial frictions in the cross interbank market in a monetary union. The impact on the transmission mechanism of monetary policy of each of the proposed financial frictions is discussed, and for the third model, additional stabilisation policies analysed. These are two unconventional policies that have recently been applied by central banks due to the constraints of the zero lower bound on nominal interest rates; long-term refinance operations, and large scale asset purchases. In the second and third models, heterogeneity plays a key role in the model dynamics: in the first, hidden information about the borrower type is the source of the financial frictions; whilst in the third, it is the presence of multiple countries, and idiosyncratic loan return shocks that play an important role.

The thesis begins with an introduction and literature review giving a short history of macroeconomic research and a general discussion of the literature to which the thesis contributes. The second chapter is dedicated to a detailed discussion of the methodologies employed and the data used. The remaining chapters then contain the main body of research.

\(^1\) See Fisher (1933) and Keynes (1936)
The study of macroeconomics poses a number of challenges. In particular is the limited availability of data and the lack of natural experiments. There exist quarterly time series U.S. data of some key economic indicators such as GDP, employment and investment since the end of the Second World War, and the number of available datasets continues to grow, but the number of variables with long-time series is small. If you consider that the National Bureau of Economic Research (NBER) identify 11 business cycles during this period, to understand the determinates of these fluctuations with any certainty using only data is not possible. Secondly, the absence of macroeconomic policy experiments mean that counterfactuals can only be speculated upon in policy analysis. Indeed, Lucas (1976) highlighted an important drawback in using econometric techniques to evaluate policy choices; because empirical analysis is restricted to the historical behaviour of economic agents, and because the choices of consumption and saving, and investment and employment can be heavily influenced by the actions of the policy maker, studying historical empirical relationships is insufficient to evaluate the impact of new policy as the policy might cause these relationships to break down. For these reasons, macroeconomic analysis has lent on theory to a very high degree. It is likely that as more data becomes available and further natural experiments emerge, empirical work will increase in its role in policy analysis but theory of the sort presented in this paper remains central to developing understanding of the macroeconomy. The work of Lucas led quickly to the real business cycle (RBC) analysis proposed by Kydland & Prescott (1982) that explicitly incorporates the optimization problems of infinitely lived individual agents in an economy; commonly referred to as having micro-foundations\(^2\). This thesis proposes theoretical models of financial frictions to evaluate the transmission of policy and so this is the broad literature to which these chapters belong. To provide the context, we give a short history of macroeconomics leading up to Lucas (1976) and since.

\(^2\) This embedded the Permanent Income Hypothesis of Friedman (1957) in explaining the consumption savings decisions of households at the centre of macroeconomic research.
1.1.1 An Evolution of Theory and Methodology

Prior to Lucas, macroeconomic research relied upon historical relationships between employment, prices and output, and the analysis of simple aggregate supply and demand curves. This approach finds its beginnings during the Great Depression, as policy makers attempted to deal with the widespread economic devastation. In Keynes (1936), the study of business cycles was brought together with monetary theory to propose a general theory of the macroeconomy (cp. Blanchard 2000). The former was a collection of explanations for economic expansions, crises and recoveries, whilst the latter was summarised in the Quantity Theory of Money relating changes in money supply to output and prices (cp. Fisher 1922). Keynes highlighted the connection between these two fields and gave an important role to the policy maker of managing aggregate demand in the economy using both monetary and fiscal policy. The theory proposed by Keynes was implemented into the first macroeconomic model by Hicks (1937) and further developed in Hansen (1953). The resulting IS-LM model summarizing the goods, financial and labour markets as simple aggregate supply and demand curves, dominated macroeconomic analysis until Lucas (1976) and the real business cycle theory that followed. The backdrop for this shift was a period of high inflation, high unemployment and low growth during the 1970s, breaking down the Phillips curve (cp. Phillips 1958) that related prices and employment. This relationship highlighted a trade-off between low inflation and unemployment; the policy maker could target high employment but at the expense of high inflation. As shown in Sargent & Wallace (1975), such an ad-hoc relationship as that proposed by Phillips is only a short-run phenomena that cannot be exploited for long as agents in the economy will learn to anticipate the policy and the reaction of the other agents; in the context of a model, this implies agents will have ‘rational’ expectations consistent with the model conditions (cp. Muth 1961).

Two key early results in the real business cycle (RBC) literature are captured firstly by the policy ineffectiveness proposition in Sargent & Wallace (1975) – that monetary policy will be ineffective at managing aggregate outcomes under rational expectations; and secondly, that
business cycles are real phenomena that can be explained by households substituting leisure between periods of high productivity to periods of low productivity (see Kydland & Prescott 1982). That is to say, economic fluctuations are optimal given the exogenous state of the world. In response, a number of authors restored the Keynesian ideas to this new paradigm by introducing different types of nominal rigidities, such as sticky information (e.g. Fischer 1977), and sticky wages (e.g. Taylor 1979) and prices (e.g. Calvo 1983). In this so called New Keynesian (NK) literature, such frictions restored a role for monetary policy in the short-run in a micro-founded model. From within the agents first-order conditions emerged a new expectations-augmented Phillips curve relating inflation and output. The RBC and NK literatures, which we can collectively refer to as the dynamic stochastic general equilibrium (DSGE) literature, had significant impact on the role of applied macroeconomic work. Since Frisch (1933), it had been understood that the relationship between economic time series could be modelled as linear difference equations and that these relationship differ across frequencies. The DSGE research moved to focus only on business cycle fluctuations; that is the cyclical component of time series within the frequency range of about 1 to 8 years. This meant extracting fluctuations in time-series within this frequency domain and rejecting all other information (see Hodrick & Prescott 1997). This thesis follows this approach as an analysis of business cycle fluctuations, and policy to stabilize the economy.

Another significant shift was away from financial factors in determining business cycles. The earlier researchers in macroeconomics such as Irving Fisher and John Maynard Keynes placed significant responsible for economic fluctuations on credit market imperfections but, with few exceptions (see e.g. Diamond & Dybvig 1983, Stiglitz & Weiss 1992, Minsky 1994, Kiyotaki & Moore 1997, Carlstrom & Fuerst 1997, Bernanke & Gertler 1989), this has been largely missing from macroeconomic research. Indeed, prior to the financial crisis of 2007–2008, the general consensus was that financial frictions played a limited role in generating, or enhancing the business cycle. This suggests that the development of financial technologies, such as credit default swaps and high frequency trading, were felt to dilute systemic risk to an irrelevance. The former via increased diversifica-
tion, and the latter by increased efficiency of information propagation. This changed drastically following the global financial crisis, and the DSGE framework experienced a crisis of confidence. The response made financial frictions a subject of a renewed research agenda. Policy makers now consider credit frictions to play an important role in shaping the transmission mechanism of policy and the majority of policy models include financial frictions as standard (see e.g. Christiano, Motto & Rostagno 2010). The study of financial frictions form a central focus of this thesis, which seeks to contribute to this important research area. Before focusing in greater detail on specific research areas that provide background to the research of this thesis, we will discuss in some more detail the DSGE research agenda picking up on issues relevant to this thesis.

1.1.2 Dynamic Stochastic General Equilibrium Literature

The DSGE literature typically proposes representative agent models of the macroeconomy in order to propose theories on the determinants of the business cycle, to test the empirical validity of these theories, and to analyse the transmission mechanisms of policy. The models draw focus on the behaviour of a number of agents which usually include households and firms, a policy maker, and increasingly banks. The key factors that determine the actions of these sectors and the prices that emerge from their interaction are household preferences, production technology and the market structure, and in the case of models to analyse policy, a policy maker objective. As firms and banks are usually owned by households, the driving force at the heart of these models are household preferences over the inter-temporal allocation of consumption and leisure. The prices that emerge in equilibrium are determined by how these preferences interact with the objective of the policy maker, with the production technology and market structure within which firms and banks operate, as well as the information sets of each agent in the economy, and any frictions imposed on agents’ actions or interaction. At the simplest, the text-book real business cycle model (e.g. King, Plosser &
Rebelo 1988) assumes perfect competition; full information; perfect goods, financial, labour and capital markets; and so with no other frictions, a price irrelevance. This leads to an irrelevance of monetary policy as discussed above, but fiscal policy is also ineffective as a tool to manage aggregate demand if the households yield no utility from government consumption (see Barro 1974). Much of the DSGE literature has attempted to analyse the various market imperfections, information asymmetries or other frictions that will cause such features to break down, and to generate model simulations that better match the observed macroeconomic time-series. For instance, the New Keynesian literature began by introducing nominal rigidities into real business cycle to explain the impact of monetary and fiscal policy. It is now common to incorporate rigidity in the setting of consumption good prices using monopolistic competition and either the mechanism proposed in Calvo (1983) in which only a proportion of firms are allowed to set prices, or that proposed in Rotemberg (1982) in which firms face price adjustment costs. I rely solely on the Calvo (1983) method in this thesis.

Other strands of the DSGE literature include introducing sources of heterogeneity in households and firms. Mankiw (2000), for instance, argues that treating some households as having restricted access to credit markets is necessary to analyse fiscal policy. Iacoviello (2005) introduces heterogeneity in discounting so that some households emerge as savers and others as borrowers. In models such as these, it firstly becomes possible to consider distributional issues, and secondly to analyse how the presence of such heterogeneity might have an impact on policy transmission mechanisms. In chapter 5, I introduce borrowers and savers using heterogeneity in time preferences, and also allow for heterogeneity in the structural parameters of whole economies. Other authors focus on heterogeneity in the firm sector although it is common in the DSGE literature to make assumptions that allow for aggregation and representative agent characterisation. For

3 In fact, an information problem was important in driving economic fluctuations in the first RBC model stemming from the time taken to build capital (see Kydland & Prescott 1982).
4 Baxter & King (1993) include household utility of government purchases in a real business cycle which causes government consumption and investment to have important effects on economic outcomes.
example, Bernanke, Gertler & Gilchrist (1999) introduce idiosyncratic productivity which leads to a firm-size distribution that depends on the history of idiosyncratic shocks. By assuming risk neutrality and perfect capital and labour markets, the size of an individual firms is irrelevant for aggregate outcomes, only requiring a single state variable, the aggregate net worth of the sector; the default rate only depends on the known distribution of the shock, not on the unknown distribution of firm size. In chapter 5 we rely on such a mechanism to introduce risk in the interbank market. In chapter 4, I assume heterogeneity in the risk of firm output but, by limiting to two types, can use representative agent methods to characterise the equilibrium.

In an RBC or NK model, the capital stock can jump unrealistically sharply in response to shocks whilst the data indicates the capital stock evolves smoothly. To deal with this, investment or capital adjustment costs have become a typical feature in the DSGE literature. The recent literature builds on Tobin’s (1969) q-theory of investment; the theory proposes that if the market value of installed capital is greater than the cost of replacing the capital, the firm should invest. It also builds on previous firm theory literature which examined the microeconomic factors behind investment decisions and the costs involved (see e.g. Gould 1968). The two most common methods in the DSGE literature are the approach of Christiano, Eichenbaum & Evans (2005) in which investment is made subject to quadratic costs of adjusting the level of investment, and the approach proposed in Ireland (2003) in which investment is subject to quadratic costs of adjusting the capital stock. Both approaches are consistent with Tobin’s q-theory, and choose a functional form which is homogeneous of degree one so that the firm-size is irrelevant, and to ensure the costs disappear in steady state.

As well as persistence to the capital stock, it has become common to add persistence to consumption with the introduction of habits in consumption (e.g. Christiano et al. 2005). The inclusion of habits generates fluctuations in equity premiums that are observed in the data, but difficult to explain in models without habit formation. As discussed in Cochrane & Campbell (1999), not only do habits in consumption imply bigger fluctuations in the stochastic discount factor for a given change in consumption, they also cause the risk pre-
mium to increase as consumption falls adding further volatility in asset prices. The habits are specified as either internal in which household utility depends on their consumption relative to their own consumption from the previous period (see Ravn, Schmitt-Grohe & Uribe 2006), or external which treats the utility from consumption as akin to ‘keeping up with the Joneses’, or perhaps rather, as Abel (1990) describes, ‘catching up with the Joneses’. Internal habits really is habit formation as a household will account for the effect of their current consumption choice on their future utility. This introduces an inefficiency in steady state, whilst with external habit, households do not internalise this effect and so the steady state is unchanged. Dennis (2009) shows that the degree to which a model can fit empirical business cycle dynamics is not affected by the choice of habit type, and so we rely solely on external habits in this thesis.

1.1.2.1 Model Validation, Calibration and Estimation

The approaches taken to assess the validity of models differ along important lines in the real business cycle and New Keynesian literatures. As Chari, Kehoe & Mcgrattan (2007) discuss, real business cycle models are typically kept intentionally simple and the number of parameters low in order to interpret the impact of one type of friction. These models are naturally restricted in how well the simulated time-series fit empirical data, and so formal estimation procedures are rare; it is far more common to calibrate key parameters to match simulations to certain moments or steady state values to empirical ergodic means.

The New Keynesian literature (e.g. Christiano et al. 2005, Smets & Wouters 2007) usually places higher import on fitting the aggregate macroeconomic time series and, by incorporating a combination of the features mentioned above, are often able to do so. The modellers are then in a position to estimate key parameters in the model, and perform historical variance decompositions that give a measure of the importance of each shock in generating the business cycle. If accurate, metrics of this type will clearly by of large interest to policy makers.
Fluctuations, Shocks and the Balanced Growth Path

One of the key lines by which the RBC and NK literatures differ is in the assessment of which shocks are important, and what impact they have on the macroeconomy. In general, the real business cycle literature places a greater level of importance on real, supply-side shocks such as a stochastic technological progress, or preference shocks on the demand-side. The New Keynesian literature, on the other hand, puts greater emphasis on demand-side shocks and monetary shocks. A key issue is the implied correlations generated by different types of shock; the empirical business cycle implies positive correlations between output, productivity, hours worked, consumption and investment. In a real business cycle model without imperfections, a productivity shock delivers this correlation whilst other shocks typically do not. In a New Keynesian model, labour will typically fall following a productivity shock but a demand side shock such as to monetary policy can induce the empirical correlations (see e.g. Christiano et al. 2005). Providing support for the NK literature, Gali (1999) found that hours tends to initially fall following a productivity shock in VAR, whilst Cantore, León-Ledesma, McAdam & Willman (2014) show how changing the functional form of the production function can allow either sign response of hours in both an RBC and NK model. The implication of the latter paper is that changing assumptions about preferences or technology, or introducing new frictions, might well cause the correlations to change. This perhaps explains why there remains no clear consensus regarding the role assigned to different types of shock. An example of this issue is discussed in chapter 3 which analyses the response to a capital quality shock; under standard preferences (such as those proposed in King et al. 1988), investment will move in opposite directions to output and consumption, but co-movement can be achieved with preferences proposed in Greenwood, Hercowitz & Huffman (1988), which removes the wealth effect on labour supply.

One way to analyse the empirical business cycle and assess theoretical business cycle models is using the ‘business cycle accounting’ method.

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5 An exception being a preference shock to the utility share of labour, or a ‘laziness’ shock.
suggested in Chari et al. (2007). The authors find that business cycle fluctuations can be represented by wedges, the most important in the U.S business cycle being the efficiency wedge caused by the inefficient allocation of factor inputs. That this wedge will often show up as aggregate technology shocks may provide a valuable explanation for the importance placed on this shock in the macroeconomic literature.

The labour wedge was found to be the second most important wedge in U.S. business cycles, which can be represented as a labour tax causing a wedge between the marginal product of labour and the marginal rate of substitution between leisure and consumption. The authors look at two further wedges; the investment and the government consumption wedges, finding the former plays a limited role whilst the latter virtually none. The investment wedge, represented by a tax on investment, shows up between the risk-free rate and the expected return on capital. The interesting point here is that in the majority of models of financial frictions (including Kiyotaki & Moore 1997, Bernanke et al. 1999), the friction shows up as an investment wedge; given that Chari et al. (2007) assign this wedge such a small role, and even a negative role during the Great Depression, there are perhaps other forms of financial friction due more attention.

In chapter 4, I propose one such model in which the friction emerges as an efficiency wedge; privately observed project risk leads to an adverse selection problem that can lead to the misallocation of capital.

The final general theme in the DSGE literature to mention is the importance of the balanced-growth path. From its inception, macroeconomics has broadly focused on business cycle and growth theory. The real business cycle literature was a move away from analysing the relationship between short and long run macroeconomic outcomes; with the observed real per capita growth rates relatively constant over the long term, the focus shifted to fluctuations around a balanced growth path. As outlined in King et al. (1988), to maintain consistency with the Solow-Swan growth model (Solow 1956, Swan 1956), the functional forms of technology and preferences in DSGE models must satisfy some key criteria in order for per capita output, capital, consumption, investment and the real wage to grow at a con-

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6 The authors look at two business cycle episodes, the Great Depression (1929–1939) and 1982 recession, and then over the period 1959–2004.
stant rate, and labour supply and interest rates to be stationary. First, for a constant output-capital ratio, economic growth must originate from labour-augmenting technological progress. Second, household preferences must support independence between the inter-temporal elasticity of substitution and the level of consumption, and for the substitution and income effects implied by permanent technological growth to not affect labour supply. A functional form for utility with constant inter-temporal elasticity of substitution that satisfies such criteria was proposed in King et al. (1988) and has come to be used in the majority of macroeconomic models. To allow the wealth effect on labour supply to be changed and analysed, in chapter 3, I also look at preferences proposed in Jaimovich & Rebelo (2009) that generalise the Greenwood et al. (1988) preferences mentioned above, which are not consistent with balanced growth. In all the models proposed in this thesis, we follow the majority of the DSGE literature and abstract from the long-run by normalising to zero growth, per capita units where all shocks are transitory.

This section on the DSGE literature has been purposefully brief so to give an overview. Some of the themes picked up on will be discussed in some more detail in the methodology section in the chapter that follows.

1.1.3 Credit Provision and Financial Frictions

The role of financial frictions on economic fluctuations and the transmission mechanism of monetary policy is a central theme across all three chapters. In the benchmark real business cycle or New Keynesian macroeconomic model, the implicit assumption is of perfect credit markets involving full information, full commitment and a zero profits, free-entry condition on financial institutions. Relaxing any of these assumptions will introduce financial frictions that can have a significant impact on model dynamics.

Prior to the financial crisis of 2007–2008, the literature on financial frictions in macroeconomics was relatively limited, and largely focused

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7 see e.g. King et al. (1988) for the former and Clarida, Gali & Gertler (1999) for the latter
on asymmetric information problems and limited contract enforceability. Asymmetric information would emerge as either financing inefficiencies or co-ordination failures. For instance, Stiglitz & Weiss (1981) focus on the former and show how adverse selection in finance can lead to credit rationing in which some borrowers are excluded from the credit market at any price, even with profitable projects. Diamond & Dybvig (1983) is an example of the latter, highlighting how maturity mismatch with asymmetric information can lead to bank runs. The combination of short-term liabilities with only partially liquid long-term assets can generate a two-equilibrium model; a bank run equilibrium can occur as a self-fulfilling prophecy if households believe one might occur. In Bernanke & Gertler (1989), entrepreneurs observe information about their productivity and due to costly state verification (see Townsend 1979) a wedge between the cost of internal and external finance that depends on the leverage of the borrower emerges, and leads to an endogenous default rate in equilibrium. This approach was extended further in Carlstrom & Fuerst (1997) and Bernanke et al. (1999). Holmstrom & Tirole (1997, 1998) discuss inefficient outcomes that emerge due to a dual moral hazard problem resulting from asymmetric information in financing projects. In this model, it is the responsibility of intermediaries to monitor the entrepreneurs, who are able to shirk. If intermediaries can also shirk, then there is a double moral hazard problem and both entrepreneur and intermediary will be capital constrained to ensure they both have sufficient ‘skin in the game’.

Financial frictions emerging from limited commitment can take a similar form. For instance, collateral constraints arise in Kiyotaki & Moore (1997) but due to a commitment problem rather than asymmetric information; borrowers cannot commit to repay debt and so must hold collateral as a guarantee. This has an important effect on macroeconomic outcomes as durable goods take on the dual role of being both factors of production and sources of collateral. This dual role creates an accelerator mechanism as when the value of capital falls, firm net worth will also fall, tightening the credit constraint. The reverse is true as the credit constraint slackens during an upturn. Kehoe & Levine (1993) and Cooley, Marimon & Quadrini (2004) also look at limited contract enforceability and reach similar conclu-
sions. Iacoviello (2005) adapted the collateral constraints approach proposed in Kiyotaki & Moore (1997) to relate fluctuations in real estate prices with economic outcomes by assuming that entrepreneurs must post real estate as collateral for loans, and by treating real estate as a factor of production. Here the accelerator mechanism of Kiyotaki & Moore (1997) worked via the housing market whereby a fall in house prices would both depress household demand and reduce investment.

Since the recent financial crisis, the number of papers studying the importance of financial frictions on macroeconomic outcomes and policy implications has grown considerably, commonly building on the mechanisms proposed in the Kiyotaki & Moore (1997) collateral constraints model, or the Bernanke et al. (1999) costly state verification model. The former was extended to study the effects of financial constraints on the banking sector in Gertler & Kiyotaki (2010) where the limited commitment problem introduces an agency problem between depositors and banks; when the value of bank capital declines, the borrowing constraint tightens and limits the amount of deposits the bank can raise and subsequently, the level of investment. Another extension proposed in Gertler & Karadi (2011) uses this approach to analyse the role of unconventional monetary policy. It is assumed the central bank can perform financial intermediation at a cost, but when the borrowing constraint tightens sufficiently, this cost is less than the inefficiency introduced by the agency problem. The two approaches have both been applied to the housing market. Impatient households post housing as collateral to secure mortgage loans in Iacoviello & Neri (2010) where the mechanism of Iacoviello (2005) is focused on the demand-side of the economy, and shown to have important effects on the business cycle. The collateral constraints arise in Forlati & Lambertini (2011) due to the Bernanke et al. (1999) costly state verification mechanism which is applied to household credit by assuming households observe a private housing-value shock that can lead to default when households are insolvent. The authors emphasise increased housing investment risk in highly leveraged economies.

Of the alternative approaches to introduce credit frictions, Gerali, Neri, Sessa & Signoretti (2010) and Forni, Gerali & Pisani (2010) introduce monopolistic competition into the banking sector with nominal
interest rate rigidities. Kiyotaki & Moore (2012) and Adrian & Shin (2009) look at the role of liquidity; the former develop a model of monetary economy with differences in liquidity across assets, whilst the latter analyse how balance-sheet quantities of marked-based financial intermediaries are important macroeconomic state variables for the conduct of monetary policy. Curdia & Woodford (2010) analyse the relationship between interest spreads and monetary policy by assuming that financial intermediation consumes real resources and that the credit spread depends on the volume of loans. Other papers including Diamond & Rajan (2001), Angeloni & Faia (2013) and Gertler & Kiyotaki (2015) have developed the bank-run model of Diamond & Dybvig (1983), the latter incorporating the approaches of the first two into a DSGE model.

The large influence of the Kiyotaki & Moore (1997) and Bernanke et al. (1999) approaches to the financial frictions literature might be partly due to the simplicity of applying the frictions to a representative agent, rational expectations model, solved using linear approximation techniques. As discussed in chapter 3, this approach rules out ex ante the possibility of explaining a number of key stylized facts, such as the large positive skew in the interest spreads. There have been a number of papers that do study the non-linear effects of financial frictions, or financial constraints that only occasionally bind which are much better suited to explain such phenomena. For example, taking a similar approach to chapter 3 of this thesis, Li (2013) finds a large increase in the loan spread and drop in bank net worth following a credit crunch. He finds that allowing a slackening of the financial constraint is key to the results, that the constraint is only binding 15 percent of the time. The author uses global approximations methods, the drawback of which restricts the model to a pure exchange economy with a single state variable. He & Krishnamurthy (2013) is also closely related to this paper but the authors propose an occasionally binding constraint on equity rather than debt. As in our model, when the constraint binds, interest premia rise sharply and deepen downturns. Related to He & Krishnamurthy (2013), Brunnermeier & Sannikov (2013) also constrain equity finance. Intermediaries are more productive investors than households but following a large negative shock, intermediaries looking to strengthen their balance sheets might sell
assets to households. This leads to non-linear dynamics; most fluctuations can be absorbed by the intermediaries balance sheets but larger negative shocks might lead to unstable, volatile episodes. To provide explanation for corporate cash hoarding, Mazelis (2014) features a cash-in-advance constraint; investment cannot exceed a pre-chosen level of liquidity and so firms hold cash in excess of expected requirements as pre-caution. As well as motivating cash hoarding, the constraint acts to deepen the impact of a negative shock to capital. Guerrieri & Iacoviello (2015a) modify the Iacoviello & Neri (2010) model by fixing the supply of housing and allowing the collateral constraint on household debt to be only occasionally binding. They show how the constraint slackened during the 2001–2006 US housing boom but tightened during the crisis, exacerbating the recession that followed. Also related are Paul (2015) and Abo-Zaid (2015). In the former banks lend long but borrow short and face risk of runs on deposits; banks gradually become more highly leveraged during booms leaving them vulnerable to large enough shocks which can cause runs on deposits. The maturity mismatch leads to severe downturn. The second paper imposes a collateral constraint on firms to guarantee promised wages to workers in the style of Kiyotaki & Moore (1997). This exhibits the general theme of occasional crises but acts as a labour tax which can be smoothed via monetary policy.

Both chapters 3 and 4 introduce occasionally binding financial constraints, the former via a bank borrowing constraint whilst the latter through positivity constraints on the Lagrange multipliers implied by an optimal financial contract subject to incentive compatibility constraints. Research into the impact of asymmetric information on the pricing of assets and financial contracts began with the seminal contribution of Akerlof (1970). In a simple model of a market for used cars in which the quality of the car is a seller’s private information, Akerlof proved that the market for good cars would disappear; the buyers being unwilling to pay the reservation price as the probability of the car being low quality – a lemon – reduces the car’s expected value. The body of research that followed Akerlof’s lemons paper examined the effects of private information in a number of contexts. For example, Mirrlees (1971) gave a formal examination of optimal labour taxation in the presence of privately observed productivity;
Mussa & Rosen (1978) and then Maskin & Riley (1984) consider privately observed preferences in product markets. The optimal pricing implies non-linear pricing strategies whereby buyers can choose quantity-price pairs. This relied on the formulation of the *revelation principle* discussed in Dasgupta, Hammond & Maskin (1979) and Myerson (1979) which proved that any outcome that can be realised in a Bayesian-Nash equilibrium can also be implemented by an incentive compatible mechanism. The key insight is that a principle can design a contract that is incentive compatible, that is, the agent would choose the contract designed for them, implying the principle need only propose one contract for each type of agent. Spence (1973) and Stiglitz (1975) then introduced the concepts of signalling and screening respectively. The former in the context of a job-seekers investing in non-fundamental characteristics that can act as a signal of their productivity to hirers, the latter puts the emphasis on the employer using these types of characteristics, at cost, to screen potential employees.

The tools developed to deal with asymmetric information have been used in a number of theoretical applications including the labour market (e.g. Spence 1973), product markets (e.g. Maskin & Riley 1984), insurance markets (e.g. Rothschild & Stiglitz 1976), and credit markets (e.g. Stiglitz & Weiss 1992). In this thesis, we are concerned with the latter and, in particular, the presence of adverse selection problems. The literature begins with Stiglitz & Weiss (1981), mentioned above, who discuss the conditions under which credit rationing will occur; such an outcome, proposed in Jaffee & Modigliani (1969), can occur when the lender posts a lending rate, but, as in the model proposed in chapter 4, the borrowers vary in their risk. If the risk of riskier borrowers is too high, the lender could be better off excluding safe borrowers from the market altogether. The model was developed further to discuss the macroeconomic implications of credit rationing in Stiglitz & Weiss (1992). The costly-state-verification model of Townsend (1979) was proposed at a similar time; in this environment, the borrower’s

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8 Building on earlier work in Hurwicz (1972).
9 Referring to equilibria in games with imperfect information, first analysed in Harsanyi (1967).
10 Or, for example, a seller can design a pricing strategy.
11 As oppose to moral hazard which occurs under hidden action rather than hidden information.
private information is not truly private but can be observed at a cost and leads to a pooling equilibrium (see also Boyd & Smith 1993). Myers & Majluf (1984) analyse an informational problem in equity finance showing that an equity premium will be charged due to the risk of lower value projects. Stiglitz & Weiss’s (1981) model of credit rationing was extended in Bester (1985) who allows for collateral constraints to be used as a signalling device, finding that credit rationing need not occur (see also Besanko & Thakor 1987). More recently Martin (2009) analyses the relationship between entrepreneur wealth and investment under adverse selection. Martin finds that a pooling (separating) equilibrium will occur when net wealth is low (high) and consequently, an increase in net wealth can generate a drop in investment. Other recent works include Guerrieri, Shimer & Wright (2010) who examine search equilibria with adverse selection; and Scheuer (2013) who analyses business tax policy with adverse selection in credit markets and occupational choice, finding that a less progressive tax regime on profits can be justified as a corrective measure mitigating occupational misallocation.

Whilst the presence of asymmetric information creates significant difficulty in evaluating the equilibrium using a Walrasian Auction,12 a number of papers have attempted to determine the conditions under which it is possible. The first was Prescott & Townsend (1984) who apply the tools of competitive general equilibrium analysis to models with adverse selection, concluding that it is difficult to decentralize this type of problem with a price system (see also Gale 1992, Rustichini & Siconolfi 2007). Bisin & Gottardi (2006) use a Rothschild-Stiglitz model of adverse selection in insurance markets to highlight how, as a market for pollution rights can be used to internalise environmental externalities, a market for consumption rights can be used to internalise the consumption externalities introduced due to adverse selection. Consequently, competitive equilibria exist and incentive constrained versions of the first and second welfare theorems hold.

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12 Where the market clearing occurs as if an ‘Auctioneer’ evaluates every agent’s supply and demand schedules and efficiently allocates all resources. See section 2 for further discussion.
There is limited literature on the specific effects that adverse selection in credit markets have on the transmission mechanism of monetary policy. One exception is Neyer (2007) who finds that adverse selection can either reinforce, weaken or overcompensate the effects of the conventional interest rate channel. Which occurs depends on whether the external finance premium is positive or negative for marginal entrepreneurs, so concluding that adverse selection does impact the transmission of monetary policy but the direction of the effect is ambiguous. Not only is the literature on adverse selection and monetary policy limited, so too is the empirical literature evaluating its presence in credit markets. An econometric test to evaluate for the presence of asymmetric information formulated in Chiappori & Salanié (2000) is developed in Crawford, Pavanini & Schivardi (2015) to test for adverse selection in loan markets. It is proposed that the correlation between the probability of taking a loan and probability of default is computed; a statistically significant correlation would indicate the presence of adverse selection. Using this test, the authors find evidence for the presence of adverse selection in Italian lending markets. They then use a structural model of the lending market to analyse the interaction between adverse selection and market imperfection, finding the impact of adverse selection on outcomes varies depending on the market structure. Cressy & Toivanen (2001) define a structural model with symmetric and asymmetric information, using the results to state propositions about borrower behaviour, similar to those of Chiappori & Salanié (2000), which are used to test for adverse selection using 1987-1990 U.K. bank lending data. The authors conclude that information is symmetric. Tang (2009) provides evidence of asymmetric information in U.S. credit markets using a Moody’s credit rating refinement in 1982, and finds that it has significant impact on economic outcomes. Away from firm lending markets, Ausubel (1999) and Dobbie & Skiba (2013) find evidence of adverse selection in credit card markets and the market for pay-day loans respectively. The former using a randomised field experiment, and the latter using discontinuities in the relationship between borrower pay and loan eligibility to estimate a regression discontinuity design.
1.1.4 Cross-border Financial Frictions

In chapter 5, I study the role of financial frictions in the cross-border interbank market in a currency union. This chapter relates in part to the open economy literature, and specifically to research into financial frictions in currency unions. In some respects, the model proposed in this chapter is a typical closed economy; there is no ‘rest of world’, so no imports and exports outside the monetary union and no exchange rate mechanism, and there is a single policy rule. The central focus of open economy macroeconomics is the analysis of exchange rates and monetary policy transmission, so given this central feature is missing from the chapter, it clearly does not belong to this body of literature. On the other hand, the chapter borrows from the open economy literature in how it characterises imports and exports between two blocs within the union, as well as how international financial flows are specified.

We start with some comments on relevant issues in open economy macroeconomic modelling. Obstfeld & Rogoff (2000) identify six puzzles in international macroeconomics, one of which, highlighted in McCallum (1995), is the presence of a significant home-bias in consumption. It has become standard practise to use an Armington aggregator (see Armington 1969, Anderson 1979) that characterises consumption preferences as a constant elasticity of substitution function between imports and exports. Home-bias can be specified as a parameter that emerges as the steady state proportion of domestic goods in consumption. Another of the puzzles was originally identified by Backus, Kehoe & Kydland (1992) who find that consumption time-series was much less correlated across countries than output time-series. This finding runs contrary to standard economic theory that predicts in a model in which agents can trade consumption internationally at no cost, consumption would depend less on domestic output and so consumption would likely be highly correlated across borders. As output is more highly correlated than consumption in the data, the indication is a lack of international risk-sharing. The inclusion of non-tradables, such as services or housing goods, is typically used to solve this puzzle (see Stockman & Tesar 1995). These two puzzles are mentioned to provide some motivation for features intro-
duced in the model proposed in chapter 5 to reflect empirical realities of the international business cycle. For the remainder of the characterisation of the main features of a two-bloc model of a monetary union, we follow Christoffel, Coenen & Warne (2008) who specify a benchmark model of the Euro Area for policy analysis.

A final modelling issue in open-economy macroeconomics to mention is the presence of a unit root. Suppose that an economy experiences a transitory supply shock that causes a current account surplus. Because the return on assets is determined by an exogenous rate of return on international assets, the temporary shock has a permanent positive effect on the wealth of savers which introduces a random walk into the model. This presents a challenge when using local perturbation as the model can move to a region a long way from the stationary point around which the model is approximated. Schmitt-Grohe & Uribe (2003) propose several methods to resolve this issue including: modifying household preference so the time-discounting adjusts in the level of consumption; having interest rates depend on the net foreign asset position of an economy; assuming households face adjustment costs for holding levels of assets different to some long-run targets; and allowing international asset-market completeness so domestic risk is diversified away. In chapter 5, we choose the second; having savings rates increasing in the net exposure to foreign debt. With the example used to highlight the unit root, the savers would instead face lower savings rates induced by a increase in the net foreign assets position of the economy. The wealth of these agents would fall relative to the rest of the world until the model returns the steady state. The appeal of this method is that it is simple to introduce and can be parametrized so that the spread between domestic and international rates is small, increasing the persistence of shocks but restoring the stationary property of the model.

The specific focus of chapter 5 is financial frictions in the cross-border interbank market. This builds on the general financial friction literature that focuses on asymmetric information and limited commitment, especially relevant as the interbank market is commonly over-the-counter, unsecured short-term lending. A number of authors (see e.g. Flannery 1996, Afonso, Kovner & Schoar 2011) discuss the importance of a smooth functioning interbank market for national
1.1 Background Literature

economies to cope efficiently with idiosyncratic liquidity shocks and to ensure a uniform transmission of the common monetary policy. Freixas & Holthausen (2005) discuss the role of asymmetric information in generating segmentation in financial markets. This effect has been shown to increase during episodes of financial stress, for instance, Abbassi, Bräuning, Fecht & Peydró (2014) find that for the same borrower on the same trading day, and after controlling for lender and borrower fixed effects, cross-border loans were up to 25 basis points more expensive than domestic loans in the first three months following the collapse of Lehman.

Financial frictions in the Euro Area interbank market appeared to play a significant role during the recent financial crisis during which the share of interbank borrowing from non-domestic lenders fell from over half to just over 25% in 2013 before recovering again to some 40% in 2014. This was especially important as those banks relying more heavily on wholesale markets for debt finance had to restrict lending to the private sector relative to those banks more dependant on household deposits (see Cornett, McNutt, Strahan & Tehranian 2011). The implication of this finding is that real shocks may be amplified, and financial shocks accelerated, by bank exposure to wholesale financing. In ‘t Veld & van Lelyveld (2014) find empirical support for a core-periphery structure to the interbank market Euro Area, itself supporting the method of modelling the currency area as a two-bloc union.

With respect to modelling the interbank market, in Gertler & Kiyotaki (2010), discussed above, banks have access to interbank finance but characterised by the same agency problem as between households and banks. Dib (2010) and de Walque, Pierrard & Rouabah (2010) build on Goodhart, Sunirand & Tsomocos (2006), who introduce regulatory capital requirements and bank default as an endogenous choice due to asymmetric information, showing that bank capital can attenuate the real effects of shocks. Hilberg & Hollmayr (2011) and Carrera & Vega (2012) also incorporate an interbank market into an otherwise typical DSGE model; the former study the impact of central bank collateral policy on interbank lending rates where the interbank market emerges as banks perform different functions, ending with assets of different risks and liquidity on their balance sheets. Carrera & Vega
(2012) study the relationship between reserve requirements and interbank market activity in a model of two types of banks; those that provide retail banking services whilst others have access to central bank liquidity. The authors find that costly monitoring results in a mechanism whereby reducing reserve requirements acts as a policy rate cut.

Whilst there are several papers examining the theoretical implications of an imperfect interbank market in a closed economy, there is limited research into international interbank market frictions. One exception is Poutineau & Vermandel (2015) who study the effects of cross-border business and interbank lending in which there is a convex monitoring cost on the latter. The authors find that the interbank market amplifies propagation of country-specific shocks. Dräger & Proaño (2015) reach a similar conclusion in a model of cross-border banking in the absence of an interbank market.
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2.1 BUILDING BLOCKS OF DSGE MODELLING

As was discussed in the previous chapter, DSGE models are typically comprised of households and firms, and in many cases, a financial sector. At the heart of the models are the preferences of infinitely lived households over their inter-temporal allocation of consumption and leisure. The following describes the assumptions that underpin the models in each chapter of this thesis.

The agents make inter-temporal allocations across a sequence of finite periods under uncertainty. It is assumed that agents know the probability distribution of exogenous stochastic variables, or shocks, and despite the possibility of the presence of information asymmetries, limited contract enforcement and default, contracts are complete.\(^1\). That is, there is no state of the world for which markets do not price assets. We refer to the assumption that the agents know the structure of the model and the distribution of shocks, and that the agents make allocation choices that maximise their defined objective function given their information sets as rational, or model-consistent, expectations. We can make some general assumptions about preferences and technology, and market clearing conditions, and define some terms that hold for all models across each chapter.

2.1.1 Consumer Choice

We begin by characterising household preferences which allow us to give a general form to the household problem, we mention produc-

\(^1\) The meaning here is that there is no state-of-the-world for which the outcomes are undefined. As state contingent claims are usually prevented in the presence of features such as asymmetric information, markets would be incomplete.
tion technologies, then discuss market clearing and price determination.

2.1.1.1 Preferences

Let $c_t \in C = \mathbb{R}^N_+$ be a bundle of $N$ goods and $u(c_t)$ describe the agents preference towards these goods. We follow the consumer theory literature with the axioms on which preferences can be defined, including:

**Axiom 1.** Transitivity – for any $c_1, c_2, c_3 \in C$, if $c_1 \succeq c_2$ and $c_2 \succeq c_3$, then $c_1 \succeq c_3$. That is, the agent can express a binary ordering of preferences.

**Axiom 2.** Completeness – for any $c_1, c_2 \in C$, either $c_1 \succeq c_2$, $c_1 \succ c_2$, or both. That is, an agent is never agnostic regarding their preferences.

**Axiom 3.** Continuity – for all bundles $c' \in C$, the sets $\{ c \in C : c \succeq c' \}$ and $\{ c \in C : c' \succeq c \}$ are closed sets.

These axioms imply that there exists a continuous preference function $u : c_t \mapsto \mathbb{R}$. We make some further important assumptions about the functional form of $u$ that ensure a well behaved interior solution:

**Assumption 1.** Agent preferences are described by a function $u : c_t \mapsto \mathbb{R}$, where $u'(c_t) > 0$, $u''(c_t) < 0$, $\lim_{c_t(i) \to 0^+} u'(c_t(i)) = \infty$ where $c_t(i)$ is element $i$ in the bundle $c_t$.

This implies that the utility function is an increasing, strictly concave function in goods $c_t(1), c_t(2), ..., c_t(N)$. The marginal value of good $c_t(i)$ goes to infinity as the quantity tends to zero; this is the Inada condition and ensures a positive value for $c_t(i)$ is chosen to guarantee an interior solution.

**Assumption 2.** Agents choose bundle $c_t$ to maximise $E_t \sum_{s=0}^{\infty} \beta^s u(c_{t+s})$, $\beta \in (0, 1)$ subject to a set of constraints $C$.

This implies that period utilities $u$ are additive separable; that agents discount the future, and make choices to maximise their lifetime utility. The symbol $E_t$ is the rational expectations operator, and indicates the model-consistent expected value of the function at time $t$. The set of constraints include at the minimum a feasibility or budget constraint. This assumption leads to a characterisation of the inter-temporal consumption decision as the consumption Euler equation originating in Fisher (1930) and Friedman (1957). Every inter-temporal decision in
every model in this thesis is characterised by such an equation that defines an asset pricing kernel, or stochastic discount factor. Letting \( p_t \) be a vector of prices for bundle \( c_t \), the assumption also indicates the presence of demand schedules that relate quantities \( c_t \) and prices \( p_t \).

### 2.1.1.2 Technology

In general, a production function is not necessary to model the macroeconomy, as it is possible to characterise income as a stochastic endowment process. However, every model proposed in this thesis does have a defined production process. The production technology has similar properties to the utility function taking a vector of factors \( k_t \) as inputs

**Assumption 3.** Agents use a technology \( m : k_t \mapsto \mathbb{R}_+ \), where \( m' (k_t) > 0 \), \( m'' (k_t) < 0 \), \( \lim_{k_t \to 0^+} m' (k_t(j)) = \infty^+ \) where \( k_t(j) \) is element \( j \) in the vector \( k_t \).

The final point is the Inada condition which, as before, ensures an interior solution. In general, the agents responsible for production, maximise a profit function subject to a set of constraints that include the technology. This implies demand schedules for factors \( k_t \) at prices \( r_t \). The vector of factors \( k_t \) will be implied by \( c_t \), and the prices \( r_t \) by \( p_t \).

### 2.1.2 Equilibrium Conditions

Denoting the set of demand functions for goods and factors as \( D_t \), and the set of feasibility constraints \( F \), we can then define the nature of a competitive equilibrium

**Definition 1.** A competitive equilibrium is a set of prices \( p_t = \mathbb{R}^N \) and allocations \( c_t \) that satisfies demand schedules \( D_t \) and feasibility constraints \( F \).

As the allocation and prices satisfy every agents’ demand schedules, it must be that the agents would not prefer any other allocation at the given prices. Note that although this definition is a general form of market clearing, it does not in fact rule out the effects of information
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or search frictions resulting in inefficiencies such as credit rationing or unemployment. The definition is that the allocations must satisfy all agents’ first order conditions and feasibility constraint. The latter could include incentive compatibility constraints, the implication is that ‘competitive equilibrium’ is used in a general fashion and rather than implying that all goods and factor markets clear in competitive markets, it allows for ‘an equilibrium degree of disequilibrium’ (Grossman & Stiglitz 1980, p.393). A relaxed version of the first welfare theorem holds under this definition; even if there are allocations that would be agreed better by all parties, subject to the feasibility constraints the allocations are Pareto optimal. The definition of a competitive equilibrium is sufficient for internal consistency, but one could explain the process by which a market clears to be as if there is a central auctioneer to whom all demand functions are submitted and who allocates the goods so to satisfy the demand schedules and feasibility constraints. This is the Walrasian Auctioneer named after Léon Walras who used the idea to explain the pricing of goods that satisfies the definition of a competitive equilibrium when buyers and sellers act in perfect competition. Alternative methods of pricing include bargaining processes (see e.g. Mortensen & Pissarides 1994), and having firms set prices in monopolistic competition. Having specified the first order conditions and feasibility constraints, and defined a competitive equilibrium that abstracts from simple perfect markets, the explanation of the pricing mechanism is not important to the model solution.

We can proceed from definition 1 to give a general form of the model as a dynamic realisation of the competitive equilibrium in the presence of stochastic disturbances. In general, the equilibrium conditions can be expressed as a system of first order stochastic difference equations and so, using the notation of Collard & Juillard (2001)², the model can be given by the function

\[ \mathbb{E}_t [ f (y_{t+1}, y_t, x_t, x_{t-1}, u_t) ] = 0 \]  

(2.1.1)

where \( y_t \) is a vector of \( n_y \) endogenous variables about which agents form expectations, \( x_t \) is a vector of \( n_x \) endogenous variables that are

² See also Schmitt-Grohé & Uribe (2004).
predetermined or backward looking, and \( u_t \) is a vector of exogenous shocks.

### 2.1.3 Model Approximation

Except for very simple cases, the model in equation (2.1.7) does not have an analytical solution and so numerical approximation techniques must be employed to evaluate the model dynamics. Due to the potentially large state space of many DSGE models, and the assumption that the economy spends the majority of time in the vicinity of the balanced growth path, it has become standard practice to use local approximation methods around a deterministic steady-state.

**Definition 2.** The deterministic steady-state is the solution to the competitive equilibrium in the absence of any present and future disturbances, so

\[
\begin{align*}
E_0 [y_t] &= y_t = \bar{y} \\
x_t &= \bar{x} \\
u_t &= 0
\end{align*}
\] (2.1.2)

**Assumption 4.** \( u_t \sim \text{i.i.d.} \mathcal{N}(0, \Sigma) \), all shocks are normal i.i.d. with zero mean and variance-covariance matrix \( \Sigma \).

Following the method described in Collard & Juillard (2001), perturbation up to third order is employed to evaluate all models proposed in this thesis and so the general steps involved are outlined here. The perturbation begins with an exact solution to the deterministic steady state found by solving \( f(\bar{y}, \bar{y}, \bar{x}, \bar{x}, 0) \). In some cases this is done analytically but for larger models is solved numerically. The model in equation (2.1.7) suggests the presence of policy functions of the form

\[
\begin{align*}
y_t &= g(x_{t-1}, u_t) \\
x_t &= h(x_{t-1}, u_t)
\end{align*}
\] (2.1.5)(2.1.6)

which then implies the model can be written

\[
E_t \left[ f \left( g \left( h \left( x_{t-1}, u_t \right), u_{t+1} \right), g \left( x_{t-1}, u_t \right), h \left( x_{t-1}, u_t \right), x_{t-1}, u_t \right) \right] = 0
\] (2.1.7)

which can be written

\[ \bar{F}(x, u) = E_t [F(x, u, u')] = 0 \] (2.1.8)

where \( x = x_{t-1}, u = u_t \) and \( u' = u_{t+1} \). The expectation is taken conditional on the state \( x \) and already observed shocks \( u \). A \( p \)-order Taylor approximation \( \bar{F}^p \approx \bar{F} \) around the deterministic steady state is taken to the order required. This is a standard well documented procedure\(^4\) but particularly notation-heavy and so full derivation is left here. Following from assumption \( 4 \), any terms linear in \( u' \) and the covariance between \( u \) and \( u' \) are zero. This will give \( n_x \) equations containing a large number of partial derivatives of \( g(\cdot) \) and \( h(\cdot) \). We compute these partial derivatives and take advantage of the following conditions that hold by definition

\[ \bar{y} = g(\bar{x}, 0) \] (2.1.9)
\[ \bar{x} = h(\bar{x}, 0) \] (2.1.10)
\[ \frac{\partial F}{\partial x}\partial y = 0, \ \forall i, j \] (2.1.11)

This leads to then same number of equations as unknowns. At orders higher than 1, there will be a positive term in covariance matrix \( \Sigma \), which, because the model is approximated around the deterministic steady state, will violate \( \bar{F}^p (x, 0) = 0 \) as shocks will be expected to be zero. To remedy this bias, correction terms are computed and added to policy functions \( g(\cdot) \) and \( h(\cdot) \). The system of equations to solve is a matrix polynomial equation and a generalized Schur decomposition\(^5\) is used to do so (see Sims 2002). At first order, the system is simply

\[ \left[ \begin{array}{c}
\begin{bmatrix} \frac{\partial f}{\partial y} & \frac{\partial f}{\partial x} \end{bmatrix} \\
\begin{bmatrix} y' - \bar{y} \\
x' - \bar{x} \end{bmatrix}
\end{bmatrix}
- \left[ \begin{array}{c}
\begin{bmatrix} \frac{\partial f}{\partial y} & \frac{\partial f}{\partial x} \end{bmatrix} \\
\begin{bmatrix} y - \bar{y} \\
x - \bar{x} \end{bmatrix}
\end{bmatrix}
\right]
\right] \]

or

\[ Dw' = Ew \] (2.1.13)

We can get a generalised Schur decomposition of the form

\[ D = QTZ^H \] (2.1.14)
\[ E = QSZ^H \] (2.1.15)

\(^4\) Outlined in detail in both Judd (1998) and Collard & Juillard (2001)
\(^5\) Otherwise known as QZ decomposition.
where $ZZ^H = Z^HZ = QQ^H = Q^HQ = I$, and $S$ and $T$ are upper triangular matrices. The ratios of the corresponding diagonal elements of $S$ and $T$, $\lambda_i = S_{ii}/T_{ii}$, are the generalized eigenvalues that solve the generalized eigenvalue problem $Sv = \lambda T v$ for a non-zero vector $v$. The system can be written with $T$, $S$ and $Z^H$ partitioned as

$$
\begin{bmatrix}
T_{11} & T_{12} & T_{13} \\
0 & T_{22} & T_{23} \\
0 & T_{33}
\end{bmatrix}
\begin{bmatrix}
Z_{11}^H \\
Z_{22}^H \\
Z_{33}^H
\end{bmatrix}
\begin{bmatrix}
w_1' \\
w_2' \\
w_3'
\end{bmatrix}

= 
\begin{bmatrix}
S_{11} & S_{12} & S_{13} \\
0 & S_{22} & S_{23} \\
0 & S_{33}
\end{bmatrix}
\begin{bmatrix}
Z_{11}^H \\
Z_{22}^H \\
Z_{33}^H
\end{bmatrix}
\begin{bmatrix}
w_1 \\
w_2 \\
w_3
\end{bmatrix}
$$

(2.1.16)

where the generalized eigenvalues of the first block, corresponding to $T_{11}$ and $S_{11}$, are less than 1 in modulus, of the second block are greater than 1, and the diagonal elements of $T_{33}$ are null and $S_{33}$ of full rank. A unique solution only exists provided that this last condition is satisfied, and that the number of eigenvalues greater than one equals the number of forward-looking variables (see Blanchard & Kahn 1980). From equation 2.1.16, it is straightforward to define a transition function for $w$, then using some further algebra results in values for the partial derivatives of $g$ and $h$. At higher orders, the process is the same except requiring much larger matrices.

2.1.3.1 Pruning

Approximating the model to higher orders is beneficial both as effects of risk are introduced into model dynamics, and as the approximation will capture some other non-linear features of the model. One drawback of using higher order perturbations is that explosive roots can be introduced into the model. To deal with this, we use a pruning algorithm that ensures stability and allows for analytical moments to be calculated (see Kim, Kim, Schaumburg & Sims 2008, Lan & Meyer-gohde 2013, Andreasen, Fernández-Villaverde & Rubio-Ramírez 2013). The general approach is to split the higher order approximation into the linear and non-linear components, using a first order approximation of the state in the higher order terms. For in-
stance, suppose $y_t^p = y^p(y_{t-1}, x_t)$ is $p$-order approximation of variable $y_t$ as a function of $y_{t-1}$ and $x_t$. At second order, we can write

$$y_t^2 - \bar{y} = a^2 + a_y^2 (y_{t-1}^2 - \bar{y}) + a_x^2 (x_{t-1} - \bar{x}) + \tilde{y}^2 (y_{t-1}, x_t)$$  \hspace{1cm} (2.1.17)

where function $\tilde{y}^2 (y_{t-1}, z_t)$ includes the second order terms. Rather than just simulate this as a standard, a first order approximation is also simulated

$$y_t^1 - \bar{y} = a^1 + a_y^1 (y_{t-1}^1 - \bar{y}) + a_x^1 (x_{t-1} - \bar{x}) ,$$  \hspace{1cm} (2.1.18)

with $y_t^1$ used in the second-order approximation as follows

$$y_t^2 - \bar{y} = a^2 + a_y^2 (y_{t-1}^2 - \bar{y}) + a_x^2 (x_{t-1} - \bar{x}) + \tilde{y}^2 (y_{t-1}, x_t) .$$  \hspace{1cm} (2.1.19)

The solution used is $y_t^2$ unless solving to third-order, in which case, this second-order pruned approximation to $y_t$, is substituted into third order terms in the third order approximation above as before.

### 2.2 Occasionally Binding Constraints in Local Approximations

In chapters 3 and 4, third-order perturbation methods, as described, are used to simulate the proposed models. As occasionally binding constraints play an important role in both models, it is necessary to use an appropriate method to introduce them otherwise, due to the nature of local approximation, they will be lost. To do so, we use the method proposed in Holden (2016a, 2016b) that allows for the presence of occasionally binding constraints in models solved using perturbation techniques. This approach treats the constraint as an endogenous source of news and ensures that where disturbances would cause bounds to be violated, anticipated news shocks return the bounded variable to the constraint. The computational strategy proposed in (Holden 2016a) is discussed in this section; (Holden 2016b) outlines the necessary and sufficient conditions for the existence and uniqueness of fundamental solutions at the bound which will not be discussed in detail here. The method builds on an efficient perfect foresight solver that finds a global solution to the bounds.
problem using a pruned perturbation approximation to a non-linear model. To capture the effects of uncertainty about the bound, we integrate over future uncertainty up to a finite horizon, following the Extended-Path method proposed in Fair & Taylor (1983) and adapted in Adjemian & Juillard (2013).

Note that due to the increasing number of variables, we use different notation to the previous section.

2.2.1 Perfect Foresight Solver

The method is described using an example of a linear model with a single constraint, but it generalises to higher order pruned perturbation and multiple bounds. Consider the basic problem in computing the impulse response function under a perfect-foresight simulation, we can write the model as

\[
(\hat{A} + \hat{B} + \hat{C}) \hat{\mu} = \hat{A}\hat{x}_{t-1} + \hat{B}\hat{x}_t + \hat{C}\hat{E}_{t}\hat{x}_{t+1} + \hat{D}\epsilon_t
\]

where \(\hat{E}_{t-1}\epsilon_t = 0\) and \(\epsilon_t = 0\) for \(t > 1\). \(\hat{x}_t\) is a vector of model variables, \(\epsilon_t\) a vector of shocks, and \(\hat{\mu}\) a vector of constants where the \(i\)th element of \(\hat{\mu}\) is the steady state value of the \(i\)th element of \(\hat{x}_t\). \(\hat{x}_0\) is given as an initial condition and we assume a terminal condition \(\hat{x} \rightarrow \hat{\mu}\) as \(t \rightarrow 0\) holds. If we define

\[
x_t = \begin{bmatrix} \hat{x}_t \\ \epsilon_{t+1} \end{bmatrix}, \quad \mu_t = \begin{bmatrix} \hat{\mu} \\ 0 \end{bmatrix}, \quad A = \begin{bmatrix} \hat{A} & \hat{D} \\ 0 & 0 \end{bmatrix},
\]

\[
B = \begin{bmatrix} \hat{B} & 0 \\ 0 & I \end{bmatrix}, \quad C = \begin{bmatrix} \hat{C} & 0 \\ 0 & 0 \end{bmatrix}
\]

then the model in (2.2.3) can be written

\[
(A + B + C) \mu = Ax_{t-1} + Bx_t + Cx_{t+1}
\]

where the expectation operator disappears because agents know \(\epsilon_t = 0\) for \(t > 1\). The problem without a bound is to find a path of \(x_t \in \mathbb{R}^n\) that satisfies (2.2.3). Providing that the generalisation of the Blanchard-Kahn conditions\(^6\) in (Sims 2002) hold, there is a solution

\(^6\) As discussed in the previous section; see (Blanchard & Kahn 1980)
of the form \( x_t = (I - F) \mu + Fx_{t-1} \), where \( F = - (B + CF)^{-1} A \) and if \( \det(A + B + C) \neq 0 \), the eigenvalues of \( F \) are strictly inside the unit circle.

Suppose there is a zero lower bound on variable \( x_{1,t} \), where the subscript indicates it is the first element of vector \( x_t \). We extend this notation and write \( I_{1,.}, A_{1,.}, B_{1,.}, C_{1,.} \) for the first row of \( I, A, B, C \), and then \( I_{-1,.}, A_{-1,.}, B_{-1,.}, C_{-1,.} \) for the remaining rows. Using \( , 1 \) a subscript denotes the first column of the matrix. Using this notation, we can write the bounded equation

\[
x_{1,t} = \max \left\{ 0, I_{1,.} \mu + A_{1,.} (x_{t-1} - \mu) + B_{1,.} x_{t-1} + C_{1,.} (x_{t+1} - \mu) \right\}
\]

The problem we want to solve is to find a path for \( x_t \) to satisfy equations (2.2.3) and (2.2.4) where \( x \to \mu \) as \( t \to 0 \). As the model returns to steady state asymptotically, there is some horizon \( T \) within which time the constraint will no longer be violated. We will use news shocks to impose the bound and so can write equation (2.2.4) as

\[
x_{1,t} = I_{1,.} \mu + A_{1,.} (x_{t-1} - \mu) + B_{1,.} x_{t-1} + C_{1,.} (x_{t+1} - \mu) + y_{1,t-1}
\]

where \( y_{1,0} \) is a news shock known at period 0, that hits at period \( t \). It follows that for periods \( t \leq T \), \( y_{1,t-1} = y_{1,0} \) whilst for \( t > T \), \( y_{1,t-1} = 0 \).

The problem the algorithm must solve then is to find path for \( x_t \in \mathbb{R}^n \) and \( y_t \in \mathbb{R}^T \) that satisfies

\[
(A + B + C) \mu = Ax_{t-1} + Bx_t + Cx_{t+1} + I_{1,1}y_{1,t-1}
\]

which modifies the first row of (2.2.3) to the in (2.2.5), and conditions on the news shocks

\[
y_{i,t} = y_{i+1,t-1}, \quad \forall i \in \{1, \ldots, T - 1\}
\]

\[
y_T = 0.
\]

given initial conditions \( x_0 \). The model is linear and so we find that the impulse response to two shocks is equal to the sum of the impulse response to each individual shock. We consider first the path of \( x_{1,t} \) given a vector of news shocks \( y_0 \in \mathbb{R}^T \). Let \( m_k \in \mathbb{R}^T \) be a column

\footnote{This rules out switching to an alternative steady state.}
vector with the impulse response of $x_{1,t}$ to a news shock of size 1 at period $k$ with $x_0 = \mu$, and let

$$M \equiv \begin{bmatrix} m_1 & m_2 & \cdots & m_T \end{bmatrix} \quad (2.2.9)$$

horizontally stack these relative impulse response functions. It follows that the path of $x_{1,t}$ given $x_0 = \mu$ and an arbitrary vector of new shocks $y_0$ is given by $My_0$. Let $q \in \mathbb{R}^T$ be the path of $x_{1,t}$ up to period $T$ that satisfies equation $(2.2.3)$, that is the model without the constraint, given any $x_0$. We can then give the path of $x_{1,t}$ for any $x_0$ and $y_0 \in \mathbb{R}^T$ that satisfies equation $(2.2.6)$ as

$$q + My_0 \quad (2.2.10)$$

The problem we wish to solve is to find a vector $y_0$ and path for $x_t$ for a given $x_0$ that satisfies equation $(2.2.6)$ and the zero lower bound on $x_{1,t}$. The path for $x_{1,t}$ will be given by equation $(2.2.10)$. The news shocks are only used to impose the bound, so when $x_{1,t} > 0$, $y_t = 0$; this implies firstly that the solution must satisfy

$$y_0 \circ (q + My_0) = 0 \quad (2.2.11)$$

and secondly that the news shocks can only act to push the variable up to the bound, that is, $y_0 \geq 0$. Finally, the solution must impose the bound, so $q + My_0 \geq 0$.

The news shock problem is then characterised as a linear complementarity problem $\text{LCP}(q, M)$ (see Cottle 2009): for a given $q$ and $M$, the $\text{LCP}(q, M)$ finds $y \in \mathbb{R}^T$ to satisfy

$$q + My \geq 0 \quad (2.2.12)$$

$$y \geq 0$$

$$y \circ (q + My) = 0$$

The structure of matrix $M$ will contain information on whether a unique solution exists; the necessary and sufficient conditions for the existence and stability of a solution are outlined in detail in Holden (2016b).

The problem extends to multiple bounds. For $n$ bounds, the vectors $q \in \mathbb{R}^{nT}$ and $y \in \mathbb{R}^{nT}$ stack the impulse responses ignoring the bound for each bounded variable and the impulse response to the news
shocks respectively. Matrix $M \in \mathbb{R}^{nT \times nT}$ is a block matrix where block $M_{i,j}$ is the response of variable $x_i$ to the new shock on variable $x_j$. The problem is given for a zero lower bound problem when the constraint it not binding in steady state, but bounds not at zero, upper bounds, or constraints that bind in steady state can all be generalised into the required form. Furthermore, the algorithm as described readily extends to higher order pruned perturbation approximations when the news shocks are of the form $y^p$, since pruned perturbation approximations of order $p$ are linear in shocks to the power of $p$.

2.2.2 Integrating Over Uncertainty

Given that the news shocks satisfy $y_t \geq 0 \forall t$, it follows that they will not be mean zero, and the expected value of $y_t$ will depend on the state. If this is ignored, bias will be introduced in the expectation of the bounded variable which could be particularly serious if the constraint is likely to bind frequently. As shown in Holden (2016a), it is possible to derive a closed-form formula for the covariance of the expected future path of the bounded variables in the absence of the bound. Using this, we can take a Gaussian approximation to the future distribution of the bounded variables in the absence of the bound, and then integrate over this distribution using Gaussian cubature techniques up to a chosen finite horizon $S$. This implies we have to solve the perfect foresight problem a number of times that is polynomial in the periods of uncertainty, independent of the number of shocks, in contrast to using the stochastic extended-path method of (Adjemian & Juillard 2013) under which the order of integration increases exponentially in the number of periods and shocks.

Rather than assuming the news shock variance goes to zero beyond the horizon $S$, a cosine windowing function is used to scale the shock variance. Specifically, if the covariance matrix is $\Sigma$, the covariance matrix used when considering uncertainty at horizon $k$ is given by

$$\tilde{\Sigma}_k = \frac{1}{2} \left[ 1 + \cos \left( \pi \frac{\min\{k - 1, S\}}{S} \right) \right] \Sigma$$

Letting $\Omega$ denote the covariance matrix of the expected future path of the bounded variables derived using $\tilde{\Sigma}_k$, and let $w_{t,t+i}$ be the value
the bounded variable would take at time \( t + i \) if the bound no longer applied after time \( t \), we assume that \( \left[ w_{t,t+1} \ldots w_{t,t+S} \right]' \) is normally distributed, which it will be at first order, and is a close approximate at higher orders. We take a Schur decomposition of \( \Omega \) to given \( \Omega = UDU' \) where \( D \geq 0 \) is a diagonal matrix. By setting any very low values of \( D \) to zero, we can reduce the cost of integration which only scales in \( \hat{S} < S \) where \( \hat{S} \) is the number of remaining non-zero elements of \( D \). Using this step and the normal assumption, we arrive at an approximation of the distribution of \( \left[ w_{t,t+1} \ldots w_{t,t+S} \right]' \) of 

\[ \mathbb{E}_t \left[ w_{t,t+1} \ldots w_{t,t+S} \right]' + \Lambda \xi \text{ where } \Lambda \equiv U_1 \sqrt{D_{11}}, \text{ with } D_{11} \in \mathbb{R}^{\hat{S}} \]

the matrix block in \( D \) that includes the \( \hat{S} \) non-zero elements and \( U_1 \) the corresponding elements of \( U \), and where \( \xi \sim N \left( 0, I_{\hat{S}} \right) \). This simplifies the integration problem to one of integrating over \( \hat{S} \) standard normals.

In this thesis, we use three different methods to integrate over future uncertainty: a ‘fast’ degree 3 monomial rule; the Gaussian cubature rule proposed in Genz & Keister (1996); and the Quasi-Monte Carlo method. The first is with \( 2\hat{S} + 1 \) nodes and equal positive weights which provide a robustness, so that although computational efficient, gives delivers reasonable accuracy. The second is a degree \( 2K + 1 \) monomial rule with \( O \left( \hat{S}^K \right) \) nodes with \( K \leq 25 \). The rule will feature negative weights if \( K > 0 \) and \( \hat{S} > 1 \) which might prevent upward bias that positive weights can introduce by evaluating the integral far from the steady state. The higher degree is also likely to give higher accuracy and because the rule is nested, we use an adaptive integration degree which will improve computation times. The final method uses a Sobol sequence (Sobol 1967) to generate \( \left( 2^{1+l} - 1 \right) \) points where \( l \in \mathbb{N} \). This approach should be the most accurate, however, for well behaved functions will require far more evaluations of the integrand than the (Genz & Keister 1996) approach for similar accuracy. The choice of method is specific to each context, typically made due to time constraints. The rule-of-thumb suggested in Holden (2016a) is if the bound is highly to either bind or not bind in the future, (Genz & Keister 1996) are likely to dominate whilst quasi-Monte Carlo may be better when the constraint binds with moderate probability. The result of the integration procedure is an expected value of the vector \( y \) needed to impose the bound; this will have an impact
on the expected value of $x_{t+1}$ by shift upward the expectation of the bounded variable $\mathbb{E}_t [x_{1,t+1}]$.

### 2.3 Data

Although no estimation was conducted, time-series data was used for the comparison of moments and to produce plots in chapters 3 and 4. Where specific metrics were used as part of the parametrisation procedures, for example, the business loan delinquency rate used in chapter 4, the source is given in a footnote. The data listed here was subject to calculations made by the author and so specific detail is given.

Table 2.1 details the sources of non-banking aggregate time-series. Us-

<table>
<thead>
<tr>
<th>Time-series</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>U.S. Bureau of Economic Analysis, Gross Domestic Product, <a href="https://fred.stlouisfed.org/series/GDP">https://fred.stlouisfed.org/series/GDP</a></td>
</tr>
<tr>
<td>FPI</td>
<td>U.S. Bureau of Economic Analysis, Fixed Private Investment, <a href="https://fred.stlouisfed.org/series/FPI">https://fred.stlouisfed.org/series/FPI</a></td>
</tr>
<tr>
<td>PCEC</td>
<td>U.S. Bureau of Economic Analysis, Personal Consumption Expenditures, <a href="https://fred.stlouisfed.org/series/PCEC">https://fred.stlouisfed.org/series/PCEC</a></td>
</tr>
<tr>
<td>GPDICTPI</td>
<td>U.S. Bureau of Economic Analysis, Gross Private Domestic Investment: Chain-type Price Index, <a href="https://fred.stlouisfed.org/series/GPDICTPI">https://fred.stlouisfed.org/series/GPDICTPI</a></td>
</tr>
<tr>
<td>BAA10YM</td>
<td>Federal Reserve Bank of St. Louis, Moody’s Seasoned Baa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity, <a href="https://fred.stlouisfed.org/series/BAA10YM">https://fred.stlouisfed.org/series/BAA10YM</a></td>
</tr>
<tr>
<td>AAA</td>
<td>Board of Governors of the Federal Reserve System (US), Moody’s Seasoned Aaa Corporate Bond Yield ©, <a href="https://fred.stlouisfed.org/series/AAA">https://fred.stlouisfed.org/series/AAA</a></td>
</tr>
<tr>
<td>BAA</td>
<td>Board of Governors of the Federal Reserve System (US), Moody’s Seasoned Baa Corporate Bond Yield ©, <a href="https://fred.stlouisfed.org/series/BAA">https://fred.stlouisfed.org/series/BAA</a></td>
</tr>
</tbody>
</table>

Table 2.1: Non-banking aggregate time-series sources.
ing 1986Q2 – 2015Q4 data, we make the following calculations for the time-series used in the chapters:

\[
y = \log \left( \frac{GDP}{GDPDEF} \frac{CNPI6OV}{CNPI6OV} \right) \times 100
\]

\[
i = \log \left( \frac{FPI}{GDPDEF} \frac{CNPI6OV}{CNPI6OV} \right) \times 100
\]

\[
c = \log \left( \frac{PCEC}{GDPDEF} \frac{CNPI6OV}{CNPI6OV} \right) \times 100
\]

\[
q = \log \left( \frac{GPDICTPI}{GDPDEF} \frac{CNPI6OV}{CNPI6OV} \right) \times 100
\]

\[
\Delta_1 = \frac{BAA10YM}{4}
\]

\[
\Delta_2 = \frac{BAA - AAA}{4}
\]

\(\Delta_1\) is the spread used in chapter \(3\), whilst \(\Delta_2\) is that used in chapter \(4\). The resulting time-series for output \(y\), investment \(i\), consumption \(c\) and the real price of investment goods \(q\) are decomposed onto a trend component and a business cycle component using proposed in Hodrick & Prescott (1997). The smoothing parameter required by the Hodrick-Prescott filter was set as 1,600, as recommended by the authors to extract business cycle fluctuations from quarterly time-series. The resulting cyclical component is denominated in percentage deviations from trend.

The banking data for which moments are computed to compare to simulated data are dividend payments and equity issuance. Both are retrieved from CRSP/Compustat Merged Database, ‘Fundamentals Quarterly’. The data are retrieved for 1978Q2 – 2015Q4 searching all firms that match SIC code 602 ‘Commercial Banks’. For each record, which are uniquely identified by date and bank \(j\), equity issuance is calculated as

\[
E^j_t = \frac{CSHOQ^j_t - CSHOQ^j_{t-1}}{CSHOQ^j_{t-1}} \frac{ATQ^j_t}{\sum_j ATQ^j_t}
\]

where ‘common shares outstanding’ \(CSHOQ\) and ‘total assets’ \(ATQ\) are the associated fields names on the dataset. For each record, we compute the proportional change in common shares outstanding weighted by the relative size of the bank. The time-series for aggregate equity issuance is then given by

\[
E_t = \sum_j E^j_t
\]
The plot of equity issuance for the ‘Big Four U.S. banks’ in figure 3.2 uses the same method but just for the four largest U.S. banks by the size of balance sheet; Citigroup, Bank of America, Wells Fargo, and JP Morgan. We identify the four banks using the Global Company Keys (GVKEY) 002968, 003243, 007647, and 008007 respectively. The dividend payment rate is calculated from the same dataset using

\[ D_t = \frac{\sum_j DV PQ^j_t}{\sum_j AT Q^j_t - \sum_j LT Q^j_t} \]  

(2.3.3)

where \( DV PQ \) is the field name for ‘Dividends - preferred/preference’ and \( LT Q \) that for total liabilities. The denominator is total book value of all banks for the quarter, rather than market value. In chapter 3, it is possible for the market and book values of bank equity to differ, but in the model \( D_t \) is defined as dividend payment over book value, so this is the definition we use here. The two plots in figures 3.3 and 3.4, are of the leverage ratio and interbank lending volume respectively. The leverage ratio uses data from the ‘Consolidated Report of Condition and Income’ – the Call Report – published by the Federal Deposit Insurance Corporation (FDIC), and is calculated using

\[ Lev_t = \frac{\sum_j RCFD2950^j_t}{\sum_j RCFD2170^j_t} \]  

(2.3.4)

where \( RCFD \) is the ‘consolidated data’ data series and 2950 and 2170 are the item numbers for total liabilities and total assets respectively. Finally, the interbank lending volume is taken from the ‘H.8 Assets and Liabilities of Commercial Banks in the United States’ published by the Board of Governors of the Federal Reserve System. This contains weekly estimates of aggregate balance sheet items for all commercial banks. The time-series for the interbank loans is converted to quarterly using the \( v21x \) algorithm in Matlab that takes into account all dates and data.
The majority of macroeconomic models of financial frictions are unable to explain the observed occasional sharp increases in interest spreads as financial constraints are assumed to always bind. We propose a model with occasionally binding financial constraints that better matches higher moments of the data. Starting with the banking model of Gertler & Kiyotaki (2010), we relax key assumptions so that the financial constraint is only occasionally binding, and allow banks to issue equity at cost. We also improve upon the derivation of the borrowing constraint by carefully specifying off-equilibrium play and using U.S. bankruptcy law to characterise the agency problem. We show that in the vicinity of the non-stochastic steady-state, financial intermediation is efficient, but moderately large shocks drive the model into a region in which the financial constraint binds, generating endogenous financial crises. During such episodes banks must issue equity at a cost to raise funds, generating positively skewed interest rate spreads and counter-cyclical positively skewed equity issuance as observed in the data. We also highlight a potentially important signalling role for dividends, showing that dividend payments act to relax the present and future borrowing constraint, and consequently banks can simultaneously pay dividends and issue equity. However, we emphasise that (i) the importance of this type of financial friction on business cycle dynamics becomes significantly reduced when the financial constraint is only occasionally binding, and (ii) the financial accelerator effect on the transmission of monetary policy is only present during times of financial stress.

The chapter includes research from ongoing joint work with Paul Levine\(^1\) and Tom Holden\(^2\).

\(^1\) Contact at P.levine@surrey.ac.uk.
\(^2\) Contact at T.holden@surrey.ac.uk.
40 OCCASIONALLY BINDING FINANCIAL CONSTRAINTS AND MONETARY POLICY

3.1 INTRODUCTION

There are two main approaches to model financial frictions that have come to dominate macroeconomic policy analysis. The collateral constraints model proposed in Kiyotaki & Moore (1997) and applied to the banking sector in Gertler & Kiyotaki (2010), and the costly state verification with risky projects model proposed in Bernanke & Gertler (1989) and further developed in Bernanke et al. (1999). Both approaches rely on constraints that are assumed to always bind leading to an ever-present investment wedge\(^3\), and yet in the data we find strong evidence that financial constraints are only occasionally binding. This chapter proposes a model of financial frictions in which the bank faces costly equity issuance and an occasionally binding constraint on borrowing. The borrowing constraint only binds following large enough adverse shocks and so the economy is financially efficient for the majority of the time.

Figure 3.1 shows the spread between the 10 year treasury bond yield and medium risk corporate bond yields, and captures the proposition that during crisis episodes, the presence of additional financial factors become more important causing sharp increases in the interest spreads. This chapter examines how these increases can have an

\(^3\) Employing the term used in Chari et al. (2007) to represent the spread between the risk-free interest rate and the return on risky capital.
important effect on macroeconomic time series. The majority of models with financial frictions are unable to explain this phenomena as the friction is usually assumed to always constrain financial intermediation. We begin with such a model, that proposed in Gertler & Kiyotaki (2010), GK henceforth, and relax key assumptions so to allow the financial constraint to only be occasionally binding. The friction arises in GK due to an agency problem; it is assumed that banks can default on debt repayments and exit the market, with imperfect enforcement meaning the bank is able to keep a proportion of the diverted assets. This gives rise to endogenous borrowing constraints of the sort proposed in Kiyotaki & Moore (1997). In order to prevent banks outgrowing the financial constraint, the GK model features an exogenous bank-exit shock which also acts as a fixed dividend rate. We drop the bank exit shock, allow dividend payments to be determined endogenously, and allow banks to issue equity at cost. We also improve upon the derivation of the borrowing constraint by carefully specifying off-equilibrium play and using U.S. bankruptcy law to implement the amount recoverable by creditors. In this modified set-up, sufficiently large adverse shocks cause the borrowing constraint to bind leading to a sharp rise in the investment wedge – the spread between the risk-free real interest rate and the risky return to capital. This generates a difference between the level of investment implied by a model with efficient financial intermediation and that implied by the financial constraints model. The episodes during which the financial constraints bind, produce a reduced skewness to the simulated time series of investment relative to the real business cycle model, and a strong positive skewness to the spread.

We analyse the borrowing constraints model in comparison to the standard real business cycle model as a frictionless benchmark case, and an always-binding borrowing constraints model. For the latter we use a version of the GK model. The model dynamics will be examined in the presence of different model features including habits in

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There is an extension discussed in Gertler & Kiyotaki (2010) that introduces equity issuance by extending the same agency problem in debt finance to equity finance. In doing so, the set-up generates incorrect dynamics with respect to equity issuance. We discuss this towards the end of the paper, but as we are focusing on the general concept of the financial friction proposed in Gertler & Kiyotaki (2010), we use this for comparison. See also Gertler, Kiyotaki & Queralto (2012), GKQ henceforth.
consumption, and capital and investment adjustment costs. We also compare different household preferences including the King-Plosser-Rebelo\textsuperscript{5} and Greenwood–Hercowitz–Huffman\textsuperscript{6} types of utility functions, so to test the potential importance of a short-run wealth effect. We consider two types of shock: a capital quality shock, and a persistent capital depreciation shock. The former follows Gertler & Kiyotaki (2010) who suggest that rather than thinking of the shock as physical destruction of capital, it should be thought to model the \textit{economic obsolescence} of capital in order to introduce an exogenous variation to the value of capital, and in particular, as an occasional \textit{disaster} shock. This, they argue, is of particular usefulness in considering what occurred following the events of late 2007 in the U.S., when a huge amount of value was knocked off bank assets resulting in a severe credit crunch. Choosing the second shock to introduce stochastic volatility in the capital depreciation rate provides useful comparison in that it has a direct impact on the banks’ balance sheets, and has been suggested by some authors to play an important role in generating the business cycle (see Liu, Waggoner & Zha 2011).

The key insight of the model is that once the modeller allows for the borrowing constraint to be only occasionally binding, the importance of the Gertler & Kiyotaki (2010) financial friction on business cycle dynamics becomes significantly reduced. The proposed model, however, does help explain some empirical moments of interest spreads and equity issuance. The model shows that the borrowing constraint binds when the demand for funds increases without an equivalent rise in the value of future discounted dividends. This can occur following an adverse supply-side shock to capital, which increases the demand for investment. The shock implies a reduction in the future profit stream and so an increase in the marginal value of a bank cashing-out, or diverting assets and defaulting. We also highlight a potentially important signalling role for dividends; as U.S. bankruptcy law allows for creditors to reclaim lost funds against past dividend payments, dividend payments act to relax the present and future borrowing constraint, and consequently can be paid even if the bank is issuing equity.

\textsuperscript{5} See King et al. (1988)
\textsuperscript{6} See Greenwood et al. (1988)
In the remainder of this section, we briefly discuss the related literature on financial frictions and occasionally binding financial constraints, highlighting empirical support for the chosen model structure. We then proceed to describe in detail the derivation of equilibrium conditions that characterise the behaviour of the economy, discuss some key analytical results and outline the numerical solution strategy. We end with a discussion of the main numerical results and point to future research.

3.1.1 Financial Frictions and Occasionally Binding Financial Constraints.

Although there has been an influential body of literature emphasising the role of credit frictions in aggravating economic fluctuations since the early 20th century macroeconomists, it was not until the recent global financial crisis and subsequent recessions that incorporating credit frictions into macroeconomic models used for policy analysis has become standard practice. This absence was probably in large part due to the widespread acceptance that under normal circumstances, financial frictions did not play a significant role in generating the business cycle. In the post financial crisis world, understanding the role of financial frictions has become an important area of research, and they are now included in many benchmark models used in policy analysis.

The two approaches to modelling financial frictions that have come to dominate macroeconomic policy analysis have been Bernanke et al.’s (1999) version of the costly state verification model proposed in Bernanke & Gertler (1989), and Gertler & Kiyotaki’s (2010) banking sector application of the collateral constraints model of Kiyotaki & Moore (1997). The former assumes that entrepreneurs seek finance for privately observed investment opportunities, and that it is costly for lenders to observe the productivity of the opportunity. This results in agency costs that depend on the net worth of the borrower – in aggregate the leverage ratio of entrepreneurs. This leads to an external finance premium paid by borrowers that increases in their leverage ratio. This mechanism acts as a financial accelerator; if adverse

7 Building on the original work in Townsend (1979).
shocks to the economy also weaken the balance sheet of borrowers, the agency costs will increase and as the entrepreneur face worsened terms of lending, investment will fall further. Rather than an information asymmetry, the second approach in Gertler & Kiyotaki (2010) introduces limited contract enforceability. It is assumed that banks can default on their debts and exit the market, and as the courts can only reclaim a proportion of outstanding debts, endogenous borrowing limits arise. Unlike Bernanke et al. (1999), there is no default in equilibrium, as the households ensure that it is never profitable for the representative bank to do so. This constraint on debt introduces a wedge between the capital return and the risk-free rate. In the GK model, the value of bank assets drive fluctuations in the investment wedge, whilst it is endogenous agency costs in the costly state verification approach.

Brzoza-Brzezina, Kolasa & Makarski (2013) analyse the empirical performance of the two approaches compared to a benchmark New Keynesian (NK) model and a vector autoregression, by comparing moments, impulse response functions, and business cycle accounting. Whilst Brzoza-Brzezina et al. (2013) restrict analysis to second moments, in this paper we emphasise higher moments and highlight the inherent loss of accuracy in assuming that financial constraints are either always-binding or never-binding. Table 3.1 records correlations with output, and second and third moments computed from US. time series 1986–2015 together with metrics from second order perturbation simulations using a typical real business cycle (RBC) and the Gertler & Kiyotaki (2010) model. The choice of models are to provide baseline comparison with a never-binding financial constraint (RBC), and an always binding financial constraint (GK) with similar building blocks to the model proposed in this paper.

The metrics in table 3.1 indicate GK does not give a clear improvement over the RBC model at this parametrisation and GK gives the wrong sign for the correlation of the spread. This results from the productivity shock increasing bank demand for finance which, even with improved outlook, increases leverage and tightens the borrowing constraint, pushing up the interest spread. Both the RBC and GK

\footnote{Typical parametrisation with three exogenous disturbances: productivity, government spending, and capital depreciation.}
second moments reasonably well but are less successful with third moments. The signs of the simulated skewness are incorrect across all listed variables; output, inflation and consumption have a positive skew, and the spread has a negative skew. This is particularly striking given the level of skewness in the time-series of the spread. At second order, the risk premium is constant which implies a constant spread in the RBC model. It might be expected that the introduction of a time-varying risk premium to affect the results, although the first-order leverage effects in GK would certainly be expected to outweigh these effects. If the skewness in the data is driven by occasional financial crises, in order to match the lower moments across the simulation of the GK and RBC models, both models will bias up the variance during normal times and down in crisis times.

Of course, the fact that financial frictions were not considered a critical component of models used in policy analysis prior to 2008 indicates that financial intermediation was at least close to efficient before this time. As credit conditions have since moved to the centre of the macroeconomic research agenda, it seems clear that there must either be occasionally binding constraints, or at least highly non-linear mechanism at play. Whilst recent macroeconomic policy analysis has emphasised the Bernanke et al. (1999) and GK models, there is a growing literature looking at this issue. For example, taking a similar approach to this paper, Li (2013) finds a large increase in the loan spread

Table 3.1: Comparison of output correlation, and second and third moments in selected US. time-series with real business cycle model and Gertler & Kiyotaki (2010) model 2nd order perturbation simulations. $\Delta$ is the interest spread.
and drop in bank net worth following a credit crunch. He finds that allowing a slackening of the financial constraint is key to the results, that the constraint is only binding 15 percent of the time. The author uses global approximations methods, the drawback of which restricts the model to a pure exchange economy with a single state variable. He & Krishnamurthy (2013) is also closely related to this paper but the authors propose an occasionally binding constraint on equity rather than debt. As in our model, when the constraint binds interest premia rise sharply and deepen downturns. Related to He & Krishnamurthy (2013), Brunnermeier & Sannikov (2013) also constrain equity finance. Intermediaries are more productive investors than households but following a large negative shock, intermediaries looking to strengthen their balance sheets might sell assets to households. This leads to non-linear dynamics; most fluctuations can be absorbed by the intermediaries balance sheets but larger negative shocks might lead to unstable, volatile episodes. To provide explanation for corporate cash hoarding, Mazelis (2014) features a cash-in-advance constraint; investment cannot exceed a pre-chosen level of liquidity and so firms hold cash in excess of expected requirements as pre-caution. As well as motivating cash hoarding, the constraint acts to deepen the impact of a negative shock to capital. Guerrieri & Iacoviello (2015a) modify the Iacoviello & Neri (2010) model by fixing the supply of housing and allowing the collateral constraint on household debt to be only occasionally binding. They show how the constraint slackened during the 2001–2006 U.S. housing boom but tightened during the crisis, exacerbating the recession that followed. Also related are Paul (2015) and Abo-Zaid (2015). In the former banks lend long but borrow short and face risk of runs on deposits; banks gradually become more highly leveraged during booms leaving them vulnerable to large enough shocks which can cause runs on deposits. The maturity mismatch leads to severe downturn. The second paper imposes a collateral constraint on firms to guarantee promised wages to workers in the style of Kiyotaki & Moore (1997). This exhibits the general theme of occasional crises but acts as a labour tax which can be smoothed via monetary policy.
3.1.1.1 *Constraints on Equity and Debt*

We argue the most appropriate subject for the occasionally binding constraint is debt finance, but that under normal circumstances debt is preferred due to frictions on equity finance. Figure 3.2 shows the

![Equity Issuance - All Commercial U.S. Banks](image)

![Equity Issuance - 'Big Four' U.S. Banks](image)

Figure 3.2: Percent change in common shares outstanding: top panel, all commercial U.S. banks; bottom panel the ‘Big Four’ U.S. banks (Citigroup, Bank of America, Wells Fargo and JP Morgan). Data for each bank is weighted by their total asset share of total assets across all banks. Data from Compustat Monthly Updates - Fundamentals Quarterly. Vertical grey bands represent recessions as reported by the National Bureau of Economic Research.

percent change in shares outstanding for all commercial banks and for the four biggest U.S. banks by balance sheet, often referred to as
the ‘Big Four’\textsuperscript{9}. Issuance of new shares is effectively bound below at zero, in fact, it turns out that the seeming exception shown in figure 3.2 is not a buyback but a reverse stock split by Citigroup in 2011 when every 10 shares was converted into 1 share. This followed the conversion of federal aid into common stock as the government took a 36% equity stake in the bank, and largest share sale in U.S. history when Citigroup sold $21 billion of common shares in 2009. This figure seems to support the story that equity issuance is costly so under normal circumstances, banks will raise debt finance, but that occasionally the marginal value of finance is higher than equity issuance costs, which we propose are largely due to constraints on debt finance. Figure 3.3 shows the liability-to-asset ratio of the Big Four.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure3.3.png}
\caption{Liability-to-asset ratio for the ‘Big Four’ U.S. banks (Citigroup, Bank of America, Wells Fargo and JP Morgan). Data for each bank is weighted by their asset share of total assets across all banks. Data from Federal Reserve Board Call Report. Vertical grey bands represent recessions as reported by the National Bureau of Economic Research.}
\end{figure}

This fell sharply during the Great Recession, and continued to fall for the years following the crisis. Whilst this was partly a response to regulatory changes to capital requirements, there was a significant shift from debt to equity finance during this period. Shin (2009) discusses the financial crisis in the context of a bank run and highlights the

\textsuperscript{9}The data is from the Compustat Monthly Updates, Fundamentals Quarterly dataset. ‘All commercial banks’ includes all entities with SIC code 602, and the ‘Big Four’ refers to the four banks with the largest balance sheets in the U.S. and includes Citigroup, Bank of America, Wells Fargo and JP Morgan.
increasing importance of sources of debt finance alternative to conventional household deposits such as the money markets. Figure 3.4 shows what happened to one such source over the period of the financial crisis; interbank lending volumes fell, continued to fall and have yet to recover. This data supports the theory that bank finance was unbounded prior to 2008, but was limited by borrowing constraints that begun to bind when the crisis occurred. Banks responded by issuing equity after this point as shown in figure 3.2 as the marginal value of finance increased.

The literature highlights two main sources of frictions on the issuance of equity. Firstly, the explicit costs of organising issuance such as underwriter fees; secondly, implicit costs that are captured in price adjustments following the issue. For the former, a number of studies estimate the transaction costs to be between 5% and 7% on average, with the costs depending on the size of offering. Lee, Lochhead, Ritter & Zhao (1996) find seasoned equity offerings (SEO) between 1990–1994 incur fees an average of 7.1% of gross proceeds for US. corporations with a range from over 13% for offerings less than $10 million, down to 3.15% for offerings over $500 million. Altinkilic & Hansen (2000) find transaction fees in the range 6.32% for $10-$20 million of proceeds to 5.83% over $80 million. This range is supported by Hennessy
& Whited (2007) who use a simulated method of moments to estimate marginal equity flotation costs of 5% for large firms and 10.7% for small firms.

There are several theoretical and empirical discussions of the implicit costs. Jensen & Meckling (1976) provide a theory of firm finance that argues these costs arise from a principle-agent problem in which the agent can choose non-pecuniary benefits (such as physical appointments and office space) leading to outcomes suboptimal to the principle. As the agents’ share of ownership falls, the share of the cost of these benefits will also fall, and so spending corporate resources on these benefits will increase. This totals the implicit cost as the sum of the suboptimal outcome and the need for costly monitoring, with costs decreasing in ‘skin-in-the-game’. Asquith & Mullins (1986) posit that firms face a negative sloping demand function for shares and emphasise the role of asymmetric information with a seasoned equity offering (SEO) providing a negative signal to investors; higher leverage signals a manager confident of positive returns whilst adjusting leverage down suggests the opposite. Miller & Rock (1985) also discuss asymmetric information and the signalling role of finance decisions, and argue that equity issuance are equivalent to negative dividends; this is certainly true in a representative household economy, and follows through in this paper. In Myers & Majluf (1984), if managers with additional information about opportunities choose not to issue stock, it might be because the market price is so low the cost to existing shareholders would outweigh the project return. In this sense, not issuing stock is good news and so the reverse would be true.

The implicit costs can be estimated by observing the change in share price following an offering. Taking into account these implicit costs discussed, Altinkilic & Hansen (2000) reject the hypothesis of ‘economies-of-scale’ in equity issuance and find evidence in support of ‘U-shaped’ total implicit and explicit issuance fees; fees initially fall as offering proceeds increase but reach a threshold after which they increase. In Altinkilic & Hansen (2003), they estimate the share price discount-
ing following an equity issuance to be 3.2% on average. Jensen (1986) estimate the losses to range from 0.4% to 9.9% and Mann & Sicherman (1991) to be 2.6% on average. In this chapter, we endogenise the borrowing constraint following Gertler & Kiyotaki (2010) and Gertler et al. (2012), and impose an equity issuance cost that increases in aggregate equity issued up to a limit. The choice of cost acts as a congestion charge that can be motivated by increased agency costs following a large cross-sector equity issuance due, for instance, to costly monitoring and downward pressure on the issuance price as the market is flooded with new equity. As will be discussed later, there is also a computational benefit as the choice of issuance cost allows us to drop an inequality constraint from the model, increasing numerical simulation speeds.

3.1.2 Model Simulations with Occasionally Binding Constraints

The difficulty in simulating models with occasionally binding constraints is that standard perturbation approximation cannot be used because the bounded equations are non-differentiatable functions. The majority of papers mentioned above use global methods to compute simulations which bring significant limitations; usually restricting the model to a single state variable in a pure exchange economy. Guerrieri & Iacoviello (2015a) use the piecewise approach proposed in Guerrieri & Iacoviello (2015b) that treats the economy as two regimes, one of which is an absorbing state; under one regime, the constraint is binding, whilst in the alternative regime, the constraint is slack. The model conditions are linearised around the deterministic steady state under each regime. The drawback to this approach is that there are two decision rules for each regime; agents in the dominating regime never expect the constraint to bind, and when it does, the agents do not expect any future shocks so act in a perfect foresight manner until switching regimes. This approach loses pre-cautionary behaviour key to many occasionally binding constraints models and so will bias the results. Dewachter & Wouters (2014) suggest another approach that allows standard perturbation methods to simulate the model of He & Krishnamurthy (2013). The authors take linear approximations to all the unbounded model equations as standard then approximate the
bounded equation with a continuous function. Whilst this is a convenient approach and allows the computation of larger models with inequality constraints, it suffers from approximation errors, particularly in the area around the constraint. A related method proposed by Den Haan & De Wind (2012) adds a penalty in the objective function in place of the bound; on a positivity constraint for instance, the penalty would tend to infinity as the bounded variable approaches the bound. Again, whilst this is a convenient approach, approximation errors are introduced, particularly in the area of the constraint. Brzoza-brzezina, Kolasa & Makarski (2015) test a collateral constraints model with a penalty function up to fourth order concluding the approach is impractical due to the errors introduced.

We use the solution algorithm proposed in Holden (2016a), discussed in section 2.2, that takes advantage of the benefits of perturbation approximation whilst overcoming the shortcomings of the other methods mentioned. The remainder of the chapter proceeds with description of the model, a discussion of the theoretical results, and specifics of the numerical methods employed. This is followed with analysis of the main numerical results. Analysis of the model proceeds in two stages. As the model proposed in this paper builds upon the GK model, which is an extension of a standard frictionless RBC model, our analysis is conducted in comparison to these two benchmarks. We then introduce nominal rigidity in the price of final goods into the RBC model and our borrowing constraints model in order to discuss the interaction between monetary policy and the financial friction.

3.2 The Model

The model shares a household and firm sector common to the real business cycle literature, the banking sector acting to intermediate funds between these two sectors. The representative household maximises expected lifetime utility

$$\max_{C_t, H_t} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} U \left(C_{t+s}, H_{t+s}\right)$$  \hspace{1cm} (3.2.1)

subject to the budget constraint

$$C_t + B_t = W_t H_t + R_{t-1} B_{t-1} + D_t - E_t + \pi_t - T_t$$  \hspace{1cm} (3.2.2)
where $C_t$ is consumption, $H_t$ the hours worked, $W_t$ is the wage rate, and $B_t$ deposits with the bank paying interest rate $R_t$ the following period. $D_t$ and $\pi_t$ are dividends paid and any other profits respectively, $E_t$ is bank equity purchased, and $T_t$ represents lump sum taxes. This leads to a Euler consumption equation and labour supply condition

\begin{equation}
1 = \beta E_t \left[ \frac{U_{C,t+1}}{U_{C,t}} \right] R_t
\end{equation}

\begin{equation}
-\frac{U_{H,t}}{U_{C,t}} = W_t
\end{equation}

where $U_{C,t}$ and $U_{L,t}$ are the marginal utilities of consumption and labour. We follow Miller & Rock (1985) and treat equity issuance as negative dividend payments; the household cares about the stream of net dividend payments $D_t - E_t$ which is delegated to the bank to maximise. We introduce an agency problem to the banking sector in which imperfect debt enforcement gives rise to a borrowing constraint to be applied to banks. This follows from the models proposed in Kiyotaki & Moore (1997) then Gertler & Kiyotaki (2010).

For the baseline model, we choose a non-separable utility function in the form

\begin{equation}
U(C_t, H_t) \left[ \frac{C_t^{1-\epsilon} (1 - H_t)^{\epsilon}}{1 - \sigma_c} \right]^{1-\sigma_c} - 1
\end{equation}

where $\sigma_c > 0$ is the inter-temporal elasticity of substitution and $\epsilon$ the utility weight on leisure.

### 3.2.1 The Banking Sector

It is assumed that the banks are necessary to provide funding to firms in the model. Following the approach of Gertler & Kiyotaki (2010), there are a continuum of islands each with a production sector and representative household. On each island, a proportion of household members act as workers whilst the remaining members act as bankers. There is perfect consumption insurance amongst household members and with the free movement of labour across islands there is a perfect labour market. The bank can lend to firms on the same island with no friction, but being prevented from using debt finance from their own household, must source funding from banks and households on
other islands. This assumption supports the shareholder maximisation objective of the banks, whilst characterising their funding with an agency problem between the representative households and banks. It is possible for a bank to raise equity financing from the household, but doing so is costly. The agency problem arises due to imperfect contract enforcement; banks are able to declare bankruptcy and exit with creditors only able to reclaim a proportion of the outstanding debt. This follows the collateral constraints model of Kiyotaki & Moore (1997), and more closely the extension to the the banking sector by Gertler & Kiyotaki (2010). The key differences between the banking model of this paper and that of Gertler & Kiyotaki (2010) are firstly, banks are able to issue equity; secondly, the authors assume an exogenous bank exit rate which fixes the dividend rate and ensures the borrowing constraint is always binding, whereas our model relaxes this assumption so that dividend payments are endogenous and the borrowing constraint is only occasionally binding. The agency problem between saver and bank results in a wedge emerging between the expected return on capital and the risk-free rate, beyond the standard risk premium.

Bank $j$ raises debt finance $B^j_t$ promising to repay $R^j_t B^j_t$ the following period. A government guarantee on these savings mean that banks actually only need repay $\left( R^j_t - G_{t+1} \right) B^j_t$ where $G_{t+1}$ is only non-zero in the face of an extreme adverse shock that would otherwise cause a systemic banking collapse. The government funds this insurance via lump-sum taxes on households. The bank will pay dividends $D^j_t$ and raise equity $E_t^j$. Whilst making dividend payments is costless, we assume there are administrative costs involved in issuing equity. To bank $j$ the cost exogenous and linear in equity issuance $\kappa_t E^j_t$, but we model $\kappa_t$ as an increasing function of aggregate equity issuance. This captures congestion externalities such as monitoring. Specifically, we let

$$\kappa_t = \bar{\kappa} \left[ 1 - \exp \left( -\nu \frac{E_t}{\max \{0, V_t \}} \right) \right]$$

Gertler & Kiyotaki (2010) discuss an extension in which banks can issue outside equity (see also Gertler et al. 2012); there is a cost that depends on the relative importance of outside equity in bank finance, and the same friction on debt is applied to equity finance. This results in a banking model with two classes of shareholders and dynamics of equity finance that does not correspond to the empirical evidence.
where $V_t$ is the value of the entire banking sector, so $E_t / V_t$ is the aggregate rate of equity issuance. $\bar{\kappa} \in (0, 1)$ and $\nu$ is a parameter that determines the velocity at which $\kappa_t$ converges to $\bar{\kappa}$. Banks raise debt and equity finance in order to lend to the production sector. The lending channel is characterised by perfect monitoring and contractual enforcement and so banks frictionlessly lend to firms against their future profits. Firms then offer banks state-contingent debt and so we will treat this as equity denoted $S_j^t$. Letting $Z_t$ represent the profit share of output per unit capital, and assuming that capital has real cost $Q_t$ and depreciates at $\delta_t$, the gross rate of return on capital will be $Z_t + (1 - \delta_t)Q_t$. One unit of firm equity finance will fund $Q_t$ units of capital, and so the gross rate of return on firm equity is given by $R^K_t \equiv [Z_t + (1 - \delta_t)Q_t]^{1 / Q_t}$.

The objective of bank $j$ is to maximise the expected discounted dividend stream paid to the household but in order to correctly characterise the equilibrium conditions and borrowing constraint, we must outline the timing and information available to each agent, and the regulation around bankruptcy so to treat carefully the off-equilibrium play. The book value of bank $j$ at time $t$ is given by

$$
\hat{V}_j^t \equiv \left[ R^K_j S_j^t - (R_t - G_t) B_j^t \right] \frac{1}{1 - \kappa_t} \quad (3.2.6)
$$

A bank that decides to continue will choose dividend payments, and finance and investment decisions subject to the budget constraint

$$
D_j^t + S_j^t + (R_{t-1} - G_t) B_{t-1}^j \leq B_j^t + (1 - \kappa_t) E_j^t + R^K_j S_j^{t-1} \quad (3.2.7)
$$

The objective of bank $j$ is to maximise the expected discounted dividend stream paid to the household. The objective function is written

$$
V_j^t = \max_{B_j^t, S_j^t, E_j^t, D_j^t} \left\{ D_j^t - E_j^t + (1 - \iota) \mathbb{E}_t \left[ \Lambda_{t,t+1} V_{t+1}^j \right] \right\} \quad (3.2.8)
$$

where $\Lambda_{t,t+1} \equiv \beta^{U_{c,t+1} / U_{c,t}}$ is the stochastic discount factor of the shareholders and $V_j^t$. The term $(1 - \iota)$ is equivalent to the exogenous bank exit rate in Kiyotaki & Moore (1997) and Gertler & Kiyotaki (2010), but we take this to the limit as $\iota \to 0$ so the only impact on preferences is to introduce an arbitrarily weak preference toward paying dividends sooner rather than later$^{11}$.

$^{11} \iota > 0$ is required by our numerical strategy; we take a perturbation approximation around the deterministic steady state. Whilst equity and debt carry different risk
We now consider the default decision and off-equilibrium play. If bank \( j \) fails to repay outstanding debts in period \( t \), the bank must file for chapter 7 bankruptcy. Following U.S. law (title 11 U.S.C. §548) any remaining assets are seized and sold at market value. If this is enough to repay \( R_{t-1}B_{t-1}^j \), any remaining assets are paid to shareholders as a final dividend, otherwise the court will examine the previous two years of dividend payments. If, when a dividend was paid, the value of bank liabilities were greater then the value of assets then the dividend would be deemed fraudulent. In this model, we assume that all dividends paid within this two-year window would be considered fraudulent as the bank was left insolvent at the point of default\(^\text{12}\). The court would attempt to recover these dividends plus interest at the risk-free rate. It is assumed that this is a costly process due, for instance, to costs associated with tracking down shareholders, and the court is only able to recover a fraction \((1 - \theta)\) of that sought. If the amount recovered is sufficient to cover \( R_{t-1}B_{t-1}^j \) then any remaining funds are returned to shareholders, otherwise the creditors take a haircut.

We first consider the decision whether to exit in period \( t \). If \( R_{t-1}B_{t-1}^j \leq (1 - \theta) \sum_{i=1}^{8} \left( \prod_{k=1}^{i} R_{t-k} \right) D_{t-i}^j \), then the bank will repay debts before exit, otherwise they will default only needing to repay

\[
(1 - \theta) \sum_{i=1}^{8} \left( \prod_{k=1}^{i} R_{t-k} \right) D_{t-i}^j. \tag{3.2.9}
\]

As the current assets can be seized in full, the exit value is given by

\[
\max \left\{ \hat{V}_t^j - (1 - \theta) \sum_{i=1}^{8} \left( \prod_{k=1}^{i} R_{t-k} \right) D_{t-i}^j \right\} \tag{3.2.10}
\]

The bank will only default in the case that

\[
V_t^j < - (1 - \theta) \sum_{i=1}^{8} \left( \prod_{k=1}^{i} R_{t-k} \right) D_{t-i}^j \tag{3.2.11}
\]

profiles and so would be determined within a stochastic simulation of a 2nd order approximation or higher, the non-stochastic steady state would be indeterminate in the absence of risk. This is discussed further in a later section of the paper.\(^\text{12}\)

This is consistent with legal practice on the issue of ‘unreasonably small capital’ whereby the firm would not need to be technically insolvent at the point of dividend payment for it to be considered fraudulent, if it later transpired the firm was left with insufficient capital to repay creditors. See http://www.jonesday.com/in-search-of-the-meaning-of-unreasonably-small-capital-in-constructively-fraudulent-transfer-avoidance-litigation-12-02-2014/ for further discussion.
If this occurs for bank $j$ then, by symmetry, all banks on the equilibrium path will default. So to prevent a financial collapse, the government will offer free insurance by choosing $G_t$ such that the complementarity condition holds

$$\min \left\{ G_t, \, V_t + (1 - \theta) \sum_{i=1}^{8} \left( \prod_{k=1}^{i} R_{t-k} \right) D_{t-i} \right\} = 0 \quad (3.2.12)$$

This policy rules out bank default along the equilibrium path, and by offering insurance to protect against tail events, the banks will underprice risk relative to the efficient benchmark case. We have introduced government insurance to prevent default in equilibrium due to severe adverse shocks to the net worth of banks, but to derive the borrowing constraint, we must consider a bank choosing off-equilibrium play by planning to default in the future. The value at time $t$ of preparing to default in $t + 1$ is given by

$$V^X_t = D^j_t - E^j_t - (1 - \iota) \mathbb{E}_t [\Lambda_{t,t+1}] (1 - \theta) \sum_{i=1}^{8} \left( \prod_{k=1}^{i} R_{t+1-k} \right) D^j_{t+1-i} \quad (3.2.13)$$

At this point, it is important to clarify the order of moves to correctly specify this off-equilibrium play. In particular we assume that households observe all bank and aggregate variables from $t - 1$ but only the period $t$ aggregate shocks before choosing the maximum level of deposits. The bank then chooses their individual variables subject to the borrowing constraint. The choice of this ordering is important; if households could observe bank behaviour in advance of borrowing decisions, the bank could never choose off-equilibrium behaviour to prepare for default as households would not lend to them. Suppressing the bank indices for neatness, we postulate that the borrowing constraint takes the form

$$B_t \leq A \hat{V}_t + \mathcal{F}_{1,t} D_{t-1} + \cdots + \mathcal{F}_{7,t} D_{t-7} \quad (3.2.14)$$

The linearity of the borrowing constraint follows from the linearity of the objective function and the budget constraint in the state variables. The household will choose the limit on $B_t$ so that the bank weakly prefers not to deviate from the equilibrium path and plan to default. Maximising the value of exit subject to the borrowing constraint and the budget constraint implies that the borrowing constraint will bind,
the bank will make no further investments so $S_t = 0$, and will issue no equity so $E_t = 0$. Again, because the budget constraint, objective function, and borrowing constraint are linear in the state and choice variables, the bank value function must be homogeneous of degree one in the state. As the bank can sell then re-buy assets for the same price, the value function must have a linear representation in $V_t$ and $\{D_{t-i}\}_{i=1}^7$, so

$$V_t = M_t \hat{V}_t + \sum_{i=1}^7 N_{i,t} D_{t-i} \tag{3.2.15}$$

Given that to prevent default, the household will ensure $V_t \geq V_t^X$; using the expressions for these terms, we can define $A_t$ and $F_{i,t}$ in terms of the marginal value of book-value and past dividend payments $M_t$, and $\{N_{i,t}\}_{i=1}^7$. The weakest condition preventing default implies

$$A_t = \frac{M_t}{1 - (1 - \iota) (1 - \theta)} - (1 - \kappa_t) \tag{3.2.16}$$

$$F_{i,t} = \frac{N_{i,t} + (1 - \iota) (1 - \theta) \prod_{k=1}^i R_{t-k}}{1 - (1 - \iota) (1 - \theta)} \tag{3.2.17}$$

The bank maximises objective (3.2.8) subject to the borrowing constraint (3.2.14), the budget constraint (3.2.7), and positivity constraints on $D_t$ and $E_t$, where the value and book-value of the bank are given by equations (3.2.15) and (3.2.6) respectively. By taking first order conditions and evaluating the terms in the state, we solve the coefficients to the bank problem leading to

$$(1 - \iota) \mathbb{E}_t \left[ A_{i,t+1} M_{t+1} (R_t - G_{t+1}) \frac{1 - \kappa_t}{1 - \kappa_{i+1}} \right] = M_t \left( 1 - \frac{\lambda \beta}{1 - \kappa_t} \right) \tag{3.2.18}$$

$$N_{i,t} = Z_{i,t} \frac{(1 - \iota) (1 - \theta) \prod_{k=1}^i R_{t-k}}{(1 - (1 - \iota) (1 - \theta))} \tag{3.2.19}$$

where

$$Z_{i,t} \equiv \frac{\lambda \beta}{1 - \kappa_t} \frac{M_t}{1 - \lambda \beta} + (1 - \iota) \mathbb{E}_t \left[ Z_{i+1,t+1} \right], \quad i = 1, \ldots, 6 \tag{3.2.20}$$

$$Z_{7,t} \equiv \frac{\lambda \beta}{1 - \kappa_t} \frac{M_t}{1 - \lambda \beta} \tag{3.2.21}$$

and where $\lambda \beta$ is the Lagrange multiplier on the borrowing constraint. The first condition gives the law of motion for the marginal value
of the bank book-value, the second for the marginal value of past fraudulent dividends. Defining

\[ \mathcal{H}_t \equiv \lambda^B_t + M_t \left( 1 - \frac{\lambda^B_t}{(1 - \kappa_t) (1 - (1 - i) (1 - \theta))} \right), \tag{3.2.22} \]

the first order conditions can be written

\[ \lambda^B_t = \mathcal{H}_t - (1 - i) \mathbb{E}_t \left[ \Lambda_{t,t+1} \mathcal{M}_{t+1} (R_t - \mathcal{G}_{t+1}) \frac{1 - \kappa_t}{1 - \kappa_{t+1}} \right] \geq 0 \tag{3.2.23} \]

\[ \lambda^D_t = (1 - i) \mathbb{E}_t \left[ \Lambda_{t,t+1} \mathcal{M}_{t+1} \mathcal{K}_{t+1} \frac{1 - \kappa_t}{1 - \kappa_{t+1}} - \mathcal{N}_{t,t+1} (1 - \kappa_t) \right] - (1 - \kappa_t) \geq 0 \tag{3.2.24} \]

\[ \lambda^E_t = 1 - (1 - i) \mathbb{E}_t \left[ \Lambda_{t,t+1} \mathcal{M}_{t+1} \mathcal{K}_{t+1} \frac{1 - \kappa_t}{1 - \kappa_{t+1}} \right] \geq 0 \tag{3.2.25} \]

\[ 1 = (1 - i) \mathbb{E}_t \left[ \Lambda_{t,t+1} \frac{\mathcal{M}_{t+1}}{\mathcal{H}_t} \mathcal{K}_{t+1} \frac{1 - \kappa_t}{1 - \kappa_{t+1}} \right] \tag{3.2.26} \]

where \( \lambda^D_t \) and \( \lambda^E_t \) are Lagrange multipliers on the positivity constraints on dividend payments and equity issuance respectively. The end equation results from the first order condition with respect to firm equity; using this, we can define a stochastic discount factor that is applied by bank owned firms \( \Xi_{t,t+1} \equiv (1 - i) \mathbb{E}_t \left[ \Lambda_{t,t+1} \frac{\mathcal{M}_{t+1}}{\mathcal{H}_t} \frac{1 - \kappa_t}{1 - \kappa_{t+1}} \right] \) and is the pricing kernel of firm equity.

### 3.2.2 Firms

A representative profit-maximising firm chooses capital and labour to produce output using the technology

\[ Y_t = (A_t H_t)^{1-\alpha} K_{t-1}^\alpha \tag{3.2.27} \]

where \( A_t \) is a stationary stochastic process. At the end of each period, firms invest in new capital stock subject to capital adjustment costs using finance from the bank. The following period the capital is used in production, labour is hired and all surplus is returned to the bank. Due to the mobility of labour across islands, the marginal product of labour will equal the market wage rate.

\[ W_t = (1 - \alpha) \frac{Y_t}{H_t} \tag{3.2.28} \]
All remaining surplus goes to the bank which implies the capital return is given by the marginal product
\[ Z_t = \alpha \frac{Y_t}{K_{t-1}}. \] (3.2.29)

As part of our analysis we model both capital and investment adjustment costs for comparison. These are paid by firms but, for convenience, we model as if firm owned capital producers build new capital goods, and sell to the firms at price \( Q_t \). Following Ireland (2003) we model the capital adjustment costs as a quadratic function of the relative change, linear in the volume of the new capital stock
\[ \Phi_K \left( \frac{K_t}{K_{t-1}} \right) = \phi_K \left( \frac{K_t}{K_{t-1}} - 1 \right)^2. \] (3.2.30)

We follow Christiano et al. (2005) for the investment adjustment costs with capital evolving according to
\[ K_t = \left[ 1 - \Phi_I \left( \frac{I_t}{I_{t-1}} \right) \right] I_t + (1 - \delta_t) K_{t-1}, \] (3.2.31)

where capital depreciation \( \delta_t \) also follows a stationary stochastic process and where the functional form of \( \Phi_I \) is identical to \( \Phi_K \). We are interested in the implications of each type of adjustment cost separately so if \( \phi_I > 0 \), then \( \phi_K = 0 \) and visa versa. Maximising capital-producer profits leads to the following conditions for the real price of capital, \( Q_t \), in the face of capital adjustment costs and investment adjustment costs respectively
\[ Q_t - 2\phi_K \left( \frac{K_t}{K_{t-1}} - 1 \right) \]
\[ - \mathbb{E}_t \left[ \Xi_{t,t+1} \left( (Q_{t+1} - 1) (1 - \delta_{t+1}) + \phi_K \left( \frac{K_{t+1}}{K_{t+1}} - 1 \right) \left( \frac{K_{t+1}}{K_{t+1}} - 1 \right) - 2 \phi_I \left( \frac{I_{t+1}}{I_{t+1}} \right) \right) \right] = 1 \] (3.2.32)
\[ Q_t \left( 1 - \phi_I \left( \frac{I_t}{I_{t-1}} - 1 \right) \right)^2 - \left( \frac{I_t}{I_{t-1}} - 1 \right) 2\phi_I \left( \frac{I_t}{I_{t-1}} \right) \]
\[ + \mathbb{E}_t \left[ \Xi_{t,t+1} Q_{t+1} 2\phi_I \left( \frac{I_{t+1}}{I_{t+1}} - 1 \right) \left( \frac{I_{t+1}}{I_{t+1}} - 1 \right) \right] = 1 \] (3.2.33)

3.2.3 Government Spending and Resource Constraint

The model has a simple government sector; the government spend a proportion \( g_t \) of output which is modelled as a stationary stochastic
process. This spending is financed via lump sum taxes. The model is closed with the resource constraint
\[ Y_t = C_t + I_t \left( 1 - \Phi_t \left( \frac{I_t}{I_{t-1}} \right) \right) + G_t. \] (3.2:34)

### 3.3 Theoretical Results

Before performing numerical analysis of the financial constraints model, we can discuss a few key points that arise from the theoretical results of the model. We begin focusing on the Lagrange multipliers and the coefficients found by solving the dynamic programming problem as these offer some immediate insight into the behaviour and importance of the financial constraints.

**Proposition 1.** \( \lambda^E_t = 0 \forall t: \) that is, the positivity constraint on equity issuance never binds

**Proof.** Substituting the equation (3.2.25) into (3.2.24) gives
\[ \lambda^D_t = 1 - \lambda^E_t - (1 - i)E_t [\Lambda_{t,t+1}N_{t,t+1} (1 - \kappa_t)] - (1 - \kappa_t) \] (3.3:1)
Suppose that \( \lambda^E_t > 0 \) so that equity issuance is constrained at zero; then from the definition of \( \kappa_t \), the equation becomes
\[ \lambda^D_t + \lambda^E_t + (1 - i)E_t [\Lambda_{t,t+1}N_{t,t+1}] = 0 \] (3.3:2)
and so \( \lambda^D_t = \lambda^E_t = (1 - i)E_t [\Lambda_{t,t+1}N_{t,t+1}] = 0 \) giving a contradiction. This implies the positivity constraint on equity is always slack in equilibrium so \( \lambda^E_t = 0 \forall t. \)

This result brings computational benefit as we can drop the inequality constraint \( E_t \geq 0 \) reducing the time to simulate the model. Using this result, we find \( \mathcal{H}_t = 1 \), and find the stochastic discount factor applied to banks becomes
\[ \Xi_{t,t+1} \equiv (1 - i) \left[ \Lambda_{t,t+1}M_{t+1} \frac{1 - \kappa_t}{1 - \kappa_{t+1}} \right] \] (3.3:3)
From this, it is easy to see that if the marginal value of an additional unit of funding is equal to one, and if the banks are not expected to change equity issuance, then at the limit as \( i \to 0 \), equation (3.3:3) will equal the household discount factor; that is to say, financial intermediation would be efficient.
Proposition 2. $\lambda^B_t = 0 \iff M_t = 1$ and $\lambda^B_t > 0 \iff M_t > 1$. That is, the marginal value of bank finance is greater than one if and only if the borrowing constraint is binding.

Proof. Using $H_t = 1$ and substituting equation (3.2.18) into (3.2.23) leads to

$$M_t = \frac{(1 - \lambda^B_t) (1 - \kappa_t) (1 - (1 - \iota) (1 - \theta))}{(1 - \kappa_t) (1 - (1 - \iota) (1 - \theta)) - \lambda^B_t} \quad (3.3.4)$$

Using $(1 - \kappa_t) (1 - (1 - \iota) (1 - \theta)) < 1$ and $\lambda^B_t \geq 0$, it follows that $M_t = 1$ when $\lambda^B_t = 0$, and $M_t > 1$ when $\lambda^B_t > 0$. □

It follows that the borrowing constraint is only slack if $M_t = 1$. We define $M_t$ as the marginal value of the bank book-value but can also describe it as the shadow price of bank finance; it is intuitive that this increases above unity as the bank becomes financially constrained.

Another result following from proposition 1 is that it can be optimal for banks to simultaneously issue equity and make dividend payments. The intuition is as follows: suppose that the borrowing constraint binds following a shock, implying $\lambda^B_t > 0$. The bank can raise funds, first by reducing dividends and then, if further funds are required, by issuing equity. In the absence of the signalling role of dividend payments, it is at the point that dividends reach the lower bound that the borrowing constraint binds and banks begin to issue equity. Note that as $\lambda^B_t > 0$ and the marginal value of bank finance increases $M_t > 1$, there is clear incentive for the bank to raise equity finance. Because it is costly for the bank to do so, it must be that the expected discounted return on firm equity $E_t [\Lambda_{t,t+1} R_{K,t+1}]$ is greater than the discounted risk-free rate $E_t [\Lambda_{t,t+1} R_t]$. One might wonder why dividends would be paid at this point given the present value of one dollar of firm equity is greater than one dollar in hand. Because the dividends can be partly recovered by creditors, they play a role in subsequent periods of relaxing the borrowing constraint. Recall

$$\lambda^D_t = \kappa_t - (1 - \iota) E_t [\Lambda_{t,t+1} \bar{N}_{t,t+1} (1 - \kappa_t)] \geq 0 \quad (3.3.5)$$

This tells us that if the marginal value of paying a dividend is greater than marginal cost of issuing equity then $\lambda^D_t = 0$ and in such a case it is optimal to make dividend payments, even simultaneously issuing...
equity to do so. As \( \kappa_t'(E_t) > 0 \) and \( \kappa_t(0) = 0 \), it is certainly possible in some states of the world. Indeed \( \mathcal{N}_{1,t} > 0 \) if any element of the set \( E_t \{ \lambda_t, ..., \lambda_t + 7 \} \) is greater than zero, and given that \( \lambda_t^B \geq 0 \forall \), it must be the case that all elements are strictly positive even if \( \lambda_t^B > 0 \) with very low probability, with the possible exception of \( \lambda_t^B \) which is known with certainty. It follows that there is always a signalling\(^{13}\) value of making dividend payments, and as such equity will be issued every period. That said, if \( \kappa_t'(E_t) \) is sufficiently high in the region of \( E_t = 0 \), then the amount of equity issued will be very low and could disappear in the presence of fixed costs of issuance. The functional form of \( \kappa_t \) is chosen partly to aid computation and parametrise the function so that \( \kappa_t \) converges to the limit \( \kappa \) for relatively small issuances. This should not detract from the interesting result that signalling effects of dividend payments lead to the optimality of simultaneous dividend payments and equity issuances as the risk of banks becoming financially constrained increases. This effect would surely magnify significantly in the case of heterogeneous banks and asymmetric information.

To measure the financial friction, we consider the investment wedge that emerges as the difference between the savings rate and the expected return on the use of those savings. We are interested in the component of the spread that emerges from the agency problem rather than the risk premium alone and find this is captured by the Lagrange multiplier on the borrowing constraint, \( \lambda_t^B \). Using the definition of \( \lambda_t^B \) from equation (3.2.23) and the pricing kernel of firm equity in equation (3.2.26), we can write

\[
\lambda_t^B = (1 - \iota)E_t \left[ A_{t+1}M_{t+1} \left( R_{t+1}^L - [R_t - G_{t+1}] \right) \frac{1 - \kappa_t}{1 - \kappa_{t+1}} \right]
\]

(3.3.6)

The size of the spread depends crucially on the cost of issuing equity; if the cost were zero, there would be no financial friction to emerge from the agency problem on debt finance as banks would just issue equity. In the benchmark GK case, equity finance is ruled out entirely, which sets \( \kappa_t = 1 \forall t \), whilst Gertler & Kiyotaki (2010) also propose

\(^{13}\) Not in the typical sense of an asymmetric information context, but in a direct way that a bank making dividend payments signals to the household to raise the borrowing limit.
an extension in which equity finance can be issued but is subject to
the same type of friction as debt finance. Our approach highlights
the role that costly equity issuance plays only when debt finance is
constrained. The marginal value of bank finance, $\mathcal{M}_t$, is the value
of one extra dollar of finance on the balance sheet of the bank; if
the bank can raise finance via reductions in dividend payment or
increased borrowing, then this will equal 1 dollar. As equity is issued
and $\kappa_t$ increases, $\mathcal{M}_t$ rises above unity. An additional dollar of finance
reduces the need to raise costly equity by one dollar today, and by
lowering the leverage of the bank, will relax the borrowing constraint
in this and future periods.

3.3.1 Deterministic Steady State

The premise for a model of occasionally binding financial constraints
is that financial intermediation is efficient in the vicinity of the steady
state but that sufficiently large adverse shocks can cause the financial
constraints to bind. We can show, as $\iota \to 0$, that the banks are not
financially constrained but at the edge of the constrained region. It
follows that financial intermediation is efficient at the limit, and in
this region the borrowing constraints model replicates the standard
real business cycle model.

**Proposition 3.** The borrowing constraint is only slack if $\iota = 0$, and the
banking sector is at the edge of the constrained region at the limit as $\iota \to 0^+$.  

**Proof.** Using equation (3.2.18), we can give the steady solution to $\lambda^B$
as

$$\lambda^B = \iota (1 - \kappa) (1 - (1 - \iota) (1 - \theta)) > 0 \quad (3.3.7)$$

This implies that the borrowing constraint binds with positive $\iota$ but
$\lim_{\iota \to 0} \lambda^B_{\iota} = 0$. As $\lambda^B$ is the Lagrange multiplier on the borrowing
constraint, it follows that the borrowing constraint is only slack if
$\iota = 0$ and so is at the edge constrained region as $\iota \to 0$.  

**Corollary 1.** Let $\mathbb{R}^{>a} = \{x \in \mathbb{R} \mid x > a\}$, then $\mathcal{M} \in \mathbb{R}^{>1} \wedge \mathcal{N}_i \in \mathbb{R}^{>0} \wedge \lambda^B \in \mathbb{R}^{>0}, \iota \in \{1, \ldots, 7\}$.  

**Corollary 2.** $\lim_{\iota \to 0} \mathcal{M} = 1 \wedge \lim_{\iota \to 0} \mathcal{N}_i = 0$
Corollaries 1 and 2 follow from proposition 2, that the shadow price of bank finance is close to but greater than unity. From equation (3.3.4), we find
\[
M = \frac{1 - i (1 - \kappa) [1 - (1 - i) (1 - \theta)]}{1 - i} > 1
\] (3.3.8)
and so at the limit as \( i \to 0^+ \), we find \( M \to 0^+ \). We show that the same is true for \( N_i \) as
\[
N_i = Z_i \frac{(1 - \kappa) (1 - i) (1 - \theta)}{(1 - \kappa) [1 - (1 - i) (1 - \theta)] - \lambda^B R_i}, \quad i = 1, \cdots, 7
\] (3.3.9)
where
\[
Z_k \equiv \frac{\lambda^B}{1 - \kappa} + (1 - i) \left[ Z_{k+1} \frac{(1 - \kappa) [1 - (1 - i) (1 - \theta)]}{(1 - \kappa) [1 - (1 - i) (1 - \theta)] - \lambda^B} \right]
\] (3.3.10)
\[
Z_7 \equiv \frac{\lambda^B}{1 - \kappa}
\] (3.3.11)
for \( k = 1, \cdots, 6 \). So \( N_i > 0 \) for \( i = 1, \cdots, 7 \), but as \( i \to 0 \), \( Z_k \to 0 \) and \( N_i \to 0 \). These results indicate that, at the limit as \( i \to 0 \), the borrowing constraint is slack as \( \lambda^B = 0 \), the marginal value of dividend payments up to all horizons \( i \) equals zero, \( N_i = 0 \), and the marginal value of bank finance is unity, \( M = 1 \). Using the condition determining value of the bank,
\[
V = M \hat{V} + \sum_{i=0}^{6} \frac{1}{\Pi^{i+1}} M^{D,i+1} D_{i+1}
\] (3.3.12)
We further note that bank value is greater than the book value but \( \lim_{i \to 0} V = \hat{V} \); at the limit, the value of the bank is equal to the book-value. Banks must pay dividends in steady state, else their steady-state value would be negative, and because the bank would constantly raise new equity without paying dividends, the bank would have infinite book value. This contradicts the fact that value must be weakly higher than book value and so implies that \( \lambda^D = 0 \). We can then write
\[
\lambda^D = \kappa - (1 - i) \frac{\beta}{\Pi} N_1 (1 - \kappa) = 0
\] (3.3.13)
so
\[
\kappa = \frac{(1 - i) \frac{\beta}{\Pi} N_1}{1 + (1 - i) \frac{\beta}{\Pi} N_1} > 0.
\] (3.3.14)
It follows from \( \lim_{\iota \to 0} N_1 = 0 \), that \( \lim_{\iota \to 0} \kappa = 0 \) and so there is no equity issuance at the limit. Finally, we find

\[
R = (1 - \iota (1 - \kappa) [1 - (1 - \iota) (1 - \theta)]) R^K
\]  

(3.3.15)

so \( R^K > R \) but \( \lim_{\iota \to 0} R^K = R \). This completes the proof that steady state financial intermediation is efficient at the limit as \( \iota \to 0 \). At this limit, the banks are not financially constrained; banks pay dividends, issue no equity, and the present value of future dividends equals the net worth of the bank. This is consistent with the standard real business cycle model with efficient financial intermediation.

3.3.2 Further analytical discussion

It is worth noting that, as \( \iota \to 0 \), the model collapses to the real business cycle model if either \( \kappa = 0 \) or \( \theta = 0 \). In the former case, the first order condition with respect to dividend payments becomes

\[
\lambda_t^D = -(1 - \iota) E_t [\Lambda_{t+1} N_{1,t+1}]
\]  

(3.3.16)

It follows from the definition of \( N_{1,t} \) in equations (3.2.19) to (3.2.21), and because \( \lambda_t^B \geq 0 \), that \( N_{1,t} \geq 0 \forall t \). We can go stricter than this: if there is positive probability that \( N_{1,t} > 0 \), then \( E_t [\Lambda_{t+1} N_{1,t+1}] > 0 \). Because \( \lambda_t^D \geq 0 \), from the amended first order condition with respect to dividend payments, it must hold that \( N_{1,t} = 0 \forall t \). It follows from this, using the same equations (3.2.19) to (3.2.21), that \( \lambda_t^B = 0 \forall t \). Using proposition 2, we can state \( M_t = 1 \forall t \). Using this in the definitions of pricing kernels for bank and firm equity, we find \( \Lambda_{t+1} = \Xi_{t+1} \forall t \) and financial intermediation is efficient. The intuition is obvious; the bank is never financially constrained as they can always raise equity finance at no cost.

We can also show the financial constraints model converges to the real business cycle model in the case that \( \theta = 0 \). Recall the borrowing constraint is of the form

\[
B_t \leq A \dot{V}_t + F_{1,t} D_{t-1} + \cdots + F_{7,t} D_{t-7}
\]  

(3.3.17)

If \( \theta = 0 \), then as \( \iota \to 0 \), it follows from the solutions of the coefficients in equations (3.2.16) and (3.2.17), that \( A_t \to \infty \) and \( F_{i,t} \to \infty \) for
3.3 THEORETICAL RESULTS

\[ i = 1, \ldots, 7. \] So at the limit as \( i \to 0 \), borrowing becomes unlimited. As with the previous case, it follows that \( \lambda_t^B = 0 \forall t \), \( M_t = 1 \forall t \) and \( \Lambda_{t,t+1}^B = \Xi^t \forall t \), and so financial intermediation is efficient.

3.3.3 Comparison to Gertler & Kiyotaki (2010) and Gertler, Kiyotaki & Queralto (2012)

The model proposed in the chapter is inspired by the banking model of Gertler & Kiyotaki (2010) which we use for comparison. In this section we are specific about the modified assumptions and conditions and so can draw out the important differences expected in the numerical results. The authors of GK use second order local approximation techniques, and rule out ex ante any probability of hitting constraints; in our financial constraints model, we solve to third order and incorporate occasionally binding constraints. We argue that an approximation order of at least three is necessary for analysis of this nature. The analysis in GK focuses on a type of shock that is especially large but occurs infrequently; we too use this disaster shocks idea in our simulations. Such shocks take the model further from the steady state than under normal circumstances, and the accuracy will be more compromised the lower the order of approximation. Secondly, at second order, there is a constant risk premium and so, again, following large shocks the impact on risk premia and precautionary behaviour will certainly be missed. At third order, the risk premia is linear in the state and so, although not able to capture all non-linear effects, is a clear improvement over second order. Ignoring the bounds in GK forces the authors to parametrise the model to ensure the bounds are not important. First, bankers exit with a fixed probability of \( \sigma = 2.5\% \) per quarter; GK argue that this is a turnover between workers and bankers but it might as well be a fixed dividend rate, which at 10\% per annum seems indefensibly high. This high dividend payment rate ensures debt is always the cheapest source of finance in GK so the borrowing constraint is always binding.

In an extension discussed in Gertler & Kiyotaki (2010), and more formally analysed in Gertler et al. (2012), banks can also source outside equity although this too is subject to the same type of constraint that
borrowing is subject to. The authors differentiate between two types of shareholder; inside equity is owned by the household on the same island as the bank whilst outside equity is held by households on other islands. The banks objective is to maximises inside shareholder value, and so the whilst outside equity is equity in the sense that it is state contingent, the banks still have incentive to renege on profit owed in the same way as with debt repayments. In order to pin down the financial structure of the bank, GKQ assume that $\theta$ is time-varying and a function of financial structure. Specifically $\theta$ is given by the convex function

$$\theta^*(x_t) = \bar{\theta} \left( 1 + \epsilon x_t + \frac{\kappa^*}{2} x_t^2 \right).$$  

The asterisk indicates a GKQ or variant of a variable or parameter, $\epsilon$ and $\kappa^*$ are parameters and $x_t \equiv q_t e_t$ where $q_t e_t$ is outside equity, and $S_t$ is total bank assets. The idea is that it is easier to divert funds from outside equity holders, for whom it would be costly to monitor performance, than for debt holders who receive a fixed rate payment. The authors show that banks finance decisions are characterised by a trade-off; on one hand there is hedging value in outside equity related to the difference between discount factors $\Lambda_{t,t+1}$ and $\Xi_{t,t+1}^*$, whilst on the other, increasing $x_t$ further aggravates the agency problem as $\theta_t$ will increase. This pins down the optimal financial structure of the bank. It is clear that the characterisation of equity finance in GKQ is quite different to that proposed in this paper which is why we choose Gertler & Kiyotaki (2010) as the baseline comparison.

Another key difference is the derivation of the borrowing constraint. In GK, the participation constraint ensures the continuation of the representative bank, and is given as $V_t^* \geq \theta_t^* S_t^*$ where the value of the bank is given as the expected terminal net worth of the bank

$$V_t^* = \mathbb{E}_t \left[ \sum_{s=1}^{\infty} (1 - \sigma)^{s-1} \Lambda_{t,t+s} \hat{V}_{t+s} \right]$$  

The bank decides at the end of each period whether to continue, or exit. If exiting, the bank will liquidate all assets of which only a proportion $1 - \theta$ will be returned to the creditors. The remaining assets

\[\text{Note this uses the same notation for bank assets as this chapter; in GKQ, } S_t \text{ of this paper is written } Q_t s_t, \text{ where in aggregate } s_t = K_t \text{ and } Q_t \text{ is Tobin’s } q.\]

\[\text{Whilst the parametrisation implies } \epsilon < 0, \text{ the authors restrict attention to the region where } \bar{\theta} (\epsilon + \kappa^* x_t) > 0.\]
will be available as a final dividend the following period. Although a simple set-up, the authors do not consult the bankruptcy proceedings as defined in U.S. law, and so miss possible dynamics. With the same $\theta$, the model derived in this chapter leads to a slackening of the borrowing constraint relative to GK, as the creditors can also recover the proportion $\theta$ of historic dividend payments within two years with interest paid. Of course, $\theta$ might be adjusted so the total proportion of assets that could be recovered on average be equal across models, but the incentive value of dividend payments in the borrowing constraint is missed, as to will be the time variability. For example, in the model in this chapter, a large dividend payment would relax the borrowing constraint over 2 years, but once this window has passed, the borrowing constraint would tighten. We would argue this channel provides a particularly relevant and interesting incentive for dividend payments.

3.4 Numerical Analysis

To evaluate the model numerically, we solve a third-order pruned perturbation approximation, and use the algorithm proposed in Holden (2016a) to implement the occasionally binding constraints. Approximating at orders higher than one means that certainty equivalence no longer holds, so precautionary behaviour is introduced and there is a positive risk premium. At third order, we benefit both from higher accuracy away from steady-state, and from a time-varying risk premium that will impact the precautionary behaviour.

Before discussing the specifics relating to the numerical strategy, we start mentioning the importance of the parameter $\iota$. We take a local approximation around the non-stochastic steady state at the limit as $\iota \to 0^+$, but $\iota > 0$ is necessary in order to pin down a deterministic steady state. Consider a period in which the borrowing constraint were slack; a unit increase in dividend payments can be paid for by a unit increase in deposits now followed by a reduction in dividend payments of $R_t$ in the next period. Given bankers discount at $\mathbb{E}_t [\Lambda_{t,t+1}]$ and $1 = R_t \mathbb{E}_t [\Lambda_{t,t+1}]$, to a first-order approximation, households are indifferent between the two. At higher orders, risk averse households are not indifferent about substituting from risky $\mathbb{E}_t [D_{t+1}]$.
to risk-less \( B \) so there is no indeterminacy. Including \( t \to 0^+ \) in the banker discounting pins down the deterministic steady state\(^{16}\).

To impose the positivity constraints on \( D \) and \( E \), and the borrowing limit set by households, we use the method discussed in chapter 2 and introduce news shocks to the bounded equations that act to ensure the bound is not violated. To incorporate the effects of uncertainty coming from the constraints, we integrate over future uncertainty up to a finite horizon. In order to find the balance between accuracy and computational speed, different methods are employed for each exercise, and so specific detail is given below. On the existence and uniqueness of the solution to the linear complementarity problem\(^{17}\); analysing matrix \( M \), which contains the relative impulse response functions to the news shocks, indicates that there is always a solution to the bounds problem but that there are some states of the world for which there are multiple. When there are multiple solutions, we choose the one that firstly solves the bounds problem in as short a time as possible, secondly minimises the size of news shocks required to impose the bounds.

3.4.1 Model and Solution Parameters

We compare numerical analysis to two benchmarks. A standard real business cycle model with efficient financial intermediation, solved simply by setting \( E_t [\Lambda_{t-1} R_{t+1}^K] = E_t [\Lambda_{t-1} R_t] \) and a version of Gertler et al. (2012) as discussed in section 3.3.3. The two benchmarks provide an always-binding financial friction case in the GK model, and a never-binding financial friction case in the RBC model. The models are parametrised with values shown in table 3.2. These are chosen to match typical values used in the literature. For GK the value of \( \sigma_B \) was chosen so the bankers survive 10 years (40 periods) on average, and \( \zeta \) and \( \theta \) computed to hit an economy wide leverage ratio of three and to have an average credit spread of 50 basis points per quarter.

\(^{16}\) \( t \) is set to \( 10^{-8} \) in the numerical analysis.

\(^{17}\) As discussed previously, this computes the size of news shocks necessary to impose the bounds.
### 3.4 Numerical Analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Household Discount Rate</td>
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</tr>
<tr>
<td>$\gamma$</td>
<td>Government Spending % of GDP</td>
<td>20%</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Utility share of labour</td>
<td>0.684</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>Elasticity of intertemporal substitution</td>
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</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share of productivity</td>
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</tr>
<tr>
<td>$\phi$</td>
<td>Capital adjustment cost</td>
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</tr>
<tr>
<td>$\delta$</td>
<td>Steady state capital depreciation</td>
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</tr>
<tr>
<td>$\rho_A$</td>
<td>Productivity shock persistence</td>
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</tr>
<tr>
<td>$\rho_\delta$</td>
<td>Depreciation shock persistence</td>
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</tr>
<tr>
<td>$\rho_G$</td>
<td>Government spending shock persistence</td>
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</tr>
<tr>
<td>$\theta$</td>
<td>Proportion of assets bankers can divert</td>
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<tr>
<td>$\kappa$</td>
<td>Cost of issuing equity (limit)</td>
<td>10%</td>
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<td>$\zeta$</td>
<td>Probability banker exiting (GK)</td>
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</tr>
<tr>
<td>$\zeta$</td>
<td>Assets to new bankers (GK)</td>
<td>0.00017</td>
</tr>
</tbody>
</table>

Table 3.2: Model Parametrisations

#### 3.4.2 Impulse Response Functions and Simulations

We compute expected impulse response functions to shocks to productivity, government spending, capital depreciation and capital quality. The pruned perturbation allows closed-form expressions for the covariance matrix to be calculated and so Monte-Carlo simulation is not required to compute expected impulse response functions in the absence of the bound. With occasionally binding constraints, it is necessary to integrate out the expectations of the future news shocks. Due to timing constraints, we do not integrate over future uncertainty to produce the figures of impulse response functions used to analyse the effects of investment and capital adjustment costs, and habits in consumption. This implies a compromise of accuracy as the agents behave as if they did not know the constraints existed until they bind, and at the point at which they bind and the sequence of news shocks required to impose the bound computed, they expect that no change in the news shocks is possible. Impulse response functions to a capital quality shock in the model with alternative households preferences, investment adjustment costs and no habits are computed.
using an adaptive degree Gaussian cubature rule with maximum degree \(7 = 2K + 1\) and \(O(\hat{S}^K)\) nodes. This integrates out the effects of future shocks on the expected value of news shocks, but still misses current uncertainty. This could be solved using Monte-Carlo simulation although suffers from a significant increase in computational times. As discussed in section 2, \(\hat{S} \leq S\) is found by trimming the smallest eigenvalues of the covariance matrix from which we integrate. In the same model, the impulse response functions to the productivity shock are computed using a degree 3 monomial cubature rule with \(2\hat{S} + 1\) nodes. To perform analysis of simulated moments of the model, we use a stochastic extended path type approach (see Adjemian & Juillard 2013); every period we integrate over 16 periods of future uncertainty to capture the effects of the risk of hitting the constraints using a degree 3 monomial rule with \(2\hat{S} + 1\) nodes.

3.4.2.1 Shocks to Bank Assets

Following Gertler & Kiyotaki (2010), we analyse the model dynamics in response to adverse shocks to the bank balance sheet. Whilst these authors draw focus on capital quality shocks, we extend our analysis to consider a shock to capital depreciation. This has been highlighted as a key source of economic fluctuations (see e.g. Liu et al. 2011) and is comparable to the capital quality shock in the sense that it adversely effects bank assets. Gertler & Kiyotaki (2010) argue that the negative capital quality shock should not be considered physical depreciation of capital, but rather some form of economic obsolescence and suggest a possible micro-foundation. The inclusion of a capital depreciation shock gives a useful point of comparison.

Whilst related, the two shocks have important differences. The capital quality shock introduces uncertainty in the number of units of productive capital that one unit of built capital will produce, whilst the depreciation shock introduces uncertainty into the gross rate of return on capital. This might seem a somewhat minor difference, but this has significant effects in the response to shocks, particularly if the depreciation shock is persistent and the capital quality shock is not. Following a capital depreciation shock, the expected return to capital falls over a number of periods leading to reduced demand for
capital and fall in investment. In contrast, a negative quality shock is a one-time reduction in the capital stock; this increases the marginal return on capital and demand for investment.

The introduction of capital or investment adjustment costs also have a significant effect. The quality shock generates a large increased demand for investment to replace the lost capital. With capital adjustment costs, it is costly to return the stock of capital to pre-shock levels quickly, and so investment will be lower such that capital can rise sufficiently slowly. Costs to adjusting the level of investment generate the standard hump shaped investment. The depreciation shock leads to the reverse phenomena; in the absence of adjustment costs, investment will fall due to the reduced return to capital, before eventually rising above normal levels to replace the lost capital. With investment adjustment costs, it is costly to reduce investment so the initial fall in investment is much smaller, as is the eventual increase as capital depreciation returns to normal. Capital adjustment costs can lead to increased investment so to maintain the level of capital amidst higher depreciation. Figure 3.5 shows the impulse response functions of output and investment following the two shocks to compare different choices of adjustment cost.

Let us now consider the role of the borrowing constraint. The financial constraint tightens when the expected marginal book value of the bank $M_{t+1}^V$ increases above unity, and this occurs as banks issue equity and $\kappa_t$ increases. Households can lower the borrowing limit without causing the financial constraint of the bank to bind, as the bank can initially raise equity finance without cost by reducing dividend payments. Under perfect foresight, once the dividend payments are at zero, the financial constraint will bind as the bank raises further equity finance. With uncertainty, the probability of the financial constraint binding will increase as the borrowing limit falls, and so the financial constraint begins to tighten whilst dividend payments are still positive as dividend payments perform a function of relaxing the borrowing constraint. Because of the signalling function of dividends, the bank will still make payments as the financial constraint tightens and the bank begins to issue equity. Once the bank is financially constrained, the marginal values of past dividend payments up to the two year horizon become positive, with a significant effect on
Figure 3.5: Real business cycle model impulse response functions to a negative capital quality shock (top) and a negative capital depreciation shock (bottom). Investment and capital adjustment costs with $\psi = 2$.

the value of the bank and the borrowing limit; the lower the past dividend payments, the tighter the borrowing constraint. This is also true for the interest rate; the lower the interest rate over the previous two years, the tighter the constraint.

Consider that the households discount according to $\Lambda_{t,t+1}$, whilst equity is priced using $\Xi_{t,t+1}$. The latter augments the former with the marginal value of bank finance implying that in the unconstrained case, $\Lambda_{t,t+1} = \Xi_{t,t+1}$, whilst $\Lambda_{t,t+1} < \Xi_{t,t+1}$ when there is a positive probability of financial constraints binding. The augmented stochastic discount factor is asymmetric as $\mathcal{M}_t \geq 1$, and has higher volatility than the household stochastic discount factor; if the expected marginal utility of future consumption increases relative to that of current consumption, as would be expected following an adverse shock, especially in the presence of habits in consumption of investment adjustment costs, then $\Lambda_{t,t+1}$ would increase. Because the expected value of $\mathcal{M}_{t,t+1}$ is also likely to rise, $\Xi_{t,t+1}$ rises further still. This introduces a hedging value of debt finance that increases as the financial constraint tightens. Because of this, when a bank experiences
a balance sheet shock that reduces the value of assets, such as a capital quality shock, the value of equity falls relative to debt and the leverage of the bank will increase. This results in a further tightening of the borrowing constraint. Figures 3.6 and 3.7 show the same plots as the previous two figures but also include the borrowing constraints model and the GK model.

The borrowing constraint can have a large effect when there is an expansion in investment during a downturn. Output falls but investment increases and the financial constraint tightens. If investment falls, because of falling financing needs, the constraint tightens by less. It is certainly possible for the constraint to tighten with falling invest-
Figure 3.7: Impulse response functions to a capital depreciation shock for the baseline, RBC and GK models. (i) Top panel, no adjustment costs; (ii) middle panel, investment adjustment costs; and (iii) bottom panel, capital adjustment costs.

ment provided that the relative demand for debt finance increases but the reduced demand for investment allows the bank to reduce borrowing to bring down the leverage without having to issue equity. In the GK model, the bank is always financially constrained and following the capital quality shock, the financial constraint tightens causing a much larger decline in investment. Figure 3.6, particularly, illustrates the reduced impact that treating more carefully the borrowing constraint can have on the implied model dynamics.
3.4.2.2 Habits in Consumption

The results indicate that with the King-Plosser-Rebelo preferences, the financial constraint typically only binds enough to generate a divergence between the two models when there is an increase investment, with or without the adjustment costs. Another friction that might have an impact here are habits in consumption. A number of authors (e.g. Cochrane & Campbell 1999) have discussed the importance of habits in driving fluctuations in asset prices. Not only do habits in consumption imply bigger fluctuations in the stochastic discount factor for a given change in consumption, they also cause the risk premium to increase as consumption falls adding further volatility in asset prices. As can be seen in figure 3.8, habits can make a significant difference to the impulse response functions to the two shocks. This can be identified by analysing the interest rate \[ R_t = (E_t [\Lambda_{t+1}])^{-1}, \] which experiences a much larger fluctuation. We now find that investment and output fall further when in the presence of habits, but the relative impact of financial constraint remains largely unchanged. With capital adjustment costs, we see a clear divergence in the interest rate between the RBC and financial constraints model, with habits enhancing this divergence. There is a similar response following the capital quality shock; investment and output fall much further in the models with habits than without. In the absence of habits, investment nearly always increased following the capital quality shock due to a rise in the marginal product of capital, and with the bank seeking to replace the lost capital. The exception was in the case of high capital adjustment costs which could lead to a fall...
in investment, whereas with habits in consumption, investment falls regardless of the presence of investment or capital adjustment costs. Following an adverse shock, the only way to fund increased investment is for households to substitute consumption for savings. When habits in consumption are introduced, households demand a greater share of the post-shock reduced output than without, driving a reduction in investment, and allowing co-movement in consumption and investment. Whilst the initial analysis shows that the constraint could only have an important effect during periods in which investment increases, figure 3.8 highlights that with habits in consumption, there can be a tightening of financial constraints even as investment is reduced. A spread between the rate paid on savings and the return on capital emerges as the constraint binds, and together with the increased risk premium, as seen in the plot of $\Delta_t$ from the RBC model with habits, leads to an even larger rise in the spread implied by the borrowing constraints model.

### 3.4.2.3 Household Preferences without the Wealth Effect on Labour Supply

Further to habits, we consider another feature that can generate a decline in investment following either a capital depreciation or capital quality shock, namely alternative household preference that can control the short-run wealth effect on labour supply. Specifically, we study the generalisation of the Greenwood–Hercowitz–Huffman utility function proposed in Jaimovich & Rebelo (2009)\(^{18}\)

$$U_t = \frac{(C_t - \sigma H_t^\gamma X_t)^{1-\sigma_c} - 1}{1 - \sigma_c}$$  \hspace{1cm} (3.4.1)

where $X_t \equiv C_t^\gamma X_{t-1}^{1-\gamma}$, $\sigma_c > 0$ is the inter-temporal elasticity of substitution as before, $\zeta > 1$ sets the elasticity of labour supply, and $0 < \gamma \leq 1$ which ensures consistency with a balanced growth path. Setting $\gamma$ positive but close to zero will effectively switch off the wealth effect on labour supply.

Impulse response functions to a capital quality shock are shown in

\(^{18}\) The Jaimovich-Rebelo preferences nest both the Greenwood–Hercowitz–Huffman (when $\gamma = 0$) and King-Plosser-Rebelo (when when $\gamma = 1$) preferences. We focus on the former in this section.
Leisure is a normal good, so a wealth effect implies an adverse shock to household wealth would decrease demand for leisure and increase labour supply. Turning this channel off introduces co-movement in investment and consumption, similar to the introduction of habits and consequently improves empirical fit for business cycle dynamics driven by capital quality shocks. We again find that investment falls further during the periods in which the bank is financially constrained leading to an increased interest spread. Plots of a larger selection of variables are shown in this figure and so we take the opportunity to say more on the response to the shock. If the shock is sufficiently large enough for the dividends to hit the lower bound, then the bank must issue equity to raise further funds. As equity issuance is costly, there is a drop in investment below the efficient level and a rise in the interest spread. Note that dividend payments are not necessarily expected to remain exactly at zero as there is likely some positive signalling value in dividend payments due to the role of relaxing the constraint; this role is also responsible for the shape of the response. In the GK model, bank finance is always constrained and the drop in the value of bank assets tightens the constraint further. This leads to a large decline in investment and a rise in the spread. The effective dividend rate is fixed as the constant probability of banker exit.

A further comment should be made on the non-monotonicity of the baseline borrowing constraints model. We have focused on large adverse shocks to the bank balance sheet able to cause the constraint to bind. For smaller shocks, or for shocks with the opposite sign, the impulse response functions of the borrowing constraints model map to those of the RBC model; financial intermediation is efficient and, at least at the limit as $\lambda \to 0$, the two models are equivalent. The same is broadly true of typical total factor productivity or government spending shocks. As a negative productivity shock decreases the continuation value of the bank, or the value of future profits, the constraint tightens. As there is also a decline in the value of bank assets, which acts in the opposite direction, a large shock is required to cause the borrowing constraint to bind. Similarly, a very large adverse shock to aggregate demand is required to cause the financial constraints to tighten. Figures 3.10 and 3.11 show the impulse response functions
to a positive and negative productivity shock respectively. Firstly, one can notice that the RBC and GK models have virtually symmetric impulse responses and, even though it is a large shock, the response of output and investment are close to symmetric. Following the decline in productivity, investment falls about 0.5% further than it does in the RBC model and the interest spread increases significantly. The dynamics in the spread are clearly asymmetric. The impulse response functions to a negative government consumption shock share these features.

3.4.2.4 Simulated Moments

Table 3.3 reports simulated moments and cross correlations for the three models together with those computed from the data. The moments are predominately for qualitative comparison and the simulations use the capital quality shock as the main driver of economic fluctuations, with a transitory productivity shock introducing some lower frequency volatility. The models are without investment or capital adjustment costs, or habits in consumption, and use Jaimovich-Rebelo preferences to get pro-cyclical investment. The occasionally binding constraints model introduces significant skewness in the interest spread missing from the GK and RBC models. This leads to skewness of output and investment of the correct sign, contrary to the comparison models, although of negligible size. The skewness of equity issuance is strongly positive, as found in the data, as too is that for dividend payments. All models predict the correct cyclicality of the interest spread, and the borrowing constraints model also predicts counter-cyclical equity issuance supported by the data.19

19 Contrary to the Gertler et al. (2012) model which predicts a pro-cyclical equity issuance as both debt and equity financing constraints tighten together.
Figure 3.9: Impulse response functions to a negative capital quality shock in baseline borrowing constraints, RBC and GK models without adjustment costs or habits. Integration in borrowing constraints model with adaptive degree Gaussian cubature rule.
Figure 3.10: Impulse response functions to a positive productivity shock in baseline borrowing constraints, RBC and GK models without adjustment costs or habits. Integration in borrowing constraints model with adaptive degree Gaussian cubature rule.

Figure 3.11: Impulse response functions to a negative productivity shock in baseline borrowing constraints, RBC and GK models without adjustment costs or habits. Integration in borrowing constraints model with adaptive degree Gaussian cubature rule.
Table 3.3: Comparison of output correlation, and second and third moments in selected 1986Q1-2015Q4 U.S. time-series with the baseline model and two benchmarks: the real business cycle model and Gertler & Kiyotaki (2010). $\sigma_{A} = 0.0005$, $\sigma_{\psi} = 0.005$. $D$ and $E$ are rates. Standard deviation of $Y$, $I$ and $C$ in relative to that of $Y$, $D$, $E$ and spread in percentage point level deviation. Data is HP filtered to remove trend.
3.5 Monetary Policy

We now examine the implications of the proposed borrowing constraint on the transmission mechanism of monetary policy. To give monetary policy an effect on real outcomes, we introduce a retail sector comprising firms that act under monopolistic competition, purchase intermediate output from the existing firms, and produce differentiated final goods. We assume the price rigidity proposed in Calvo (1983) so that, every period, the retail firms face a fixed probability of being able to reset prices. We let $P_{I,t}$ be the price level in the intermediate good sector, and $P_t$ be the price index of the final goods used in consumption and investment. The market power stems from the household preference function for final goods; it is assumed that households combine the differentiated goods into a bundle using

$$C_t = \left( \int_0^1 C_t(j)^{\frac{\sigma - 1}{\sigma}} \, dj \right)^{\frac{1}{\sigma}} \quad (3.5.1)$$

where $\sigma$ denotes the elasticity of substitution between the different varieties. The household purchases good $C_t(j)$ from retailer $j \in (0,1)$ at price $P_t$ to maximise $(3.5.1)$ subject to total expenditure $P_tC_t = \int_0^1 P_t(j)C_t(j) \, dj$, with an equivalent problem for investment demand $I_t$. This leads to Dixit & Stiglitz (1977) demand schedules

$$C_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\sigma} C_t \quad (3.5.2)$$

$$I_t(j^*) = \left( \frac{P_t(j)}{P_t} \right)^{-\sigma} I_t. \quad (3.5.3)$$

Summing these demand schedules implies a total demand for good $j$ given by

$$Y_t(j) = \left( \frac{P_t^p(j)}{P_t^p} \right)^{-\sigma} Y_t \quad (3.5.4)$$

Every period, each firm faces a fixed probability $1 - \xi$ that they will be able to update their prices. Denoting the optimal price at time $t$ for good $j$ as $P_t^*(j)$, the firms allowed to re-optimize prices maximise expected discounted profits by solving

$$\max_{P_t^p(j)} E_t \sum_{k=0}^{\infty} \xi^k \frac{\Lambda_{t,t+k}}{P_{t+k}} Y_{t+k}(j) \left[ P_t^*(j) - P_{w,t+k} \right]. \quad (3.5.5)$$
Substituting in the demand schedule, taking first-order conditions with respect to the new price and rearranging leads to

\[
P_t^* = \frac{\sigma}{\sigma - 1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \sigma^k \Lambda_{t+k} (P_{t+k})^\sigma Y_{t+k} P_{w,t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} \sigma^k \Lambda_{t+k} (P_{t+k})^\sigma Y_{t+k}}
\]

(3.5.6)

where the \( j \) index is dropped as all firms face the same marginal cost, so the right-hand side of the equation is independent of firm size or price history. We denote the real marginal cost as \( MC_t = \frac{P_{MC_t}}{P_t} \), and the price inflation over the interval \([t-1, t]\) as \( \Pi_{t-1,t} \equiv \frac{P_t}{P_{t-1}} \), we write the real optimal price

\[
\frac{P_t^*}{P_t} = \frac{\sigma}{\sigma - 1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \sigma^k \Lambda_{t+k} (\Pi_{t+k})^\sigma Y_{t+k} MC_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} \sigma^k \Lambda_{t+k} (\Pi_{t+k})^\sigma Y_{t+k}}
\]

(3.5.7)

Denoting the numerator and denominator \( \Omega_{1,t} \) and \( \Omega_{2,t} \) we can write these in recursive form

\[
\Omega_{1,t} \equiv \frac{\sigma}{\sigma - 1} Y_t MC_t + \xi \mathbb{E}_t \left[ \Lambda_{t+1} (\Pi_{t+1})^\sigma \Omega_{1,t+1} \right]
\]

(3.5.8)

\[
\Omega_{2,t} \equiv Y_t + \mathbb{E}_t \left[ \xi \Lambda_{t+1} (\Pi_{t+1})^{\sigma-1} \Omega_{2,t+1} \right].
\]

(3.5.9)

Using the aggregate producer price index \( P_t \) and the fact that all resetting firms will choose the same price, by the Law of Large Numbers we can find the evolution of the price index as given by

\[
P_t^{1-\sigma} = \xi P_{t-1}^{1-\sigma} + (1 - \xi) P_t^{1-\sigma}
\]

(3.5.10)

which can be written in the form required

\[
1 = \xi (\Pi_{t-1,t})^{\sigma-1} + (1 - \xi) \left( \frac{P_t}{P_t} \right)^{1-\sigma}
\]

(3.5.11)

where \( \Pi_{t-1,t} \) is the gross inflation in the price of domestically produced goods between periods \([t-1, t]\). Whilst the distribution of prices is not required to track the evolution of the aggregate price index, equation 3.5.1 implies a loss of output due to dispersion in prices. Using the demand schedules, we can write the price dispersion that gives the average loss in output as

\[
S_t = \frac{1}{J} \sum_{j=1}^{J} \left( \frac{P_t(j)}{P_t} \right)^{-\sigma}
\]

(3.5.12)

for non-optimising firms \( j = 1, ..., J \). It is not possible to track all \( P_t(j) \) but it is known that a proportion \( 1 - \xi \) of firms will optimise prices in
period \( t \), and from the Law of Large Numbers, that the distribution of non-optimised prices will be the same as in the overall distribution. Therefore price dispersion can be written as a law of motion

\[
S_t = \xi \Pi_{-1,t} S_{t-1} + (1 - \xi) \left( \frac{\Omega_{1,t}}{\Omega_{2,t}} \right)^{-\sigma}.
\]

Using this, aggregate final output is given as a proportion of the intermediate output

\[
Y_t = Y_{w,t} \frac{1}{S_t}.
\]

To determine the path for prices, we introduce a monetary policy with an inflation target which we assume is credible, such that the private sector believe the economy will return to the inflation level in the long-run. The monetary authority sets a nominal policy rate using the rule

\[
R^p_t = \left[ \bar{R}^p \left( \frac{\Pi_{t-1}}{\Pi^*} \right)^{\eta^\pi} \left( \frac{Y_t}{\bar{Y}} \right)^{\eta^y} \right]^{1-\eta'} \left( R^p_{t-1} \right)^{\eta'} \exp(\epsilon_{M,t}),
\]

where \( \epsilon_{M,t} \sim N(0, \sigma_M) \) is a monetary policy shock. The central bank sets the nominal deposit rate paid by banks, \( R^p_t = R_t \). For parameterisation of the New-Keynesian part of the model, we follow Smets & Wouters (2007) setting the Calvo parameter to \( \xi = 0.65 \), and the interest rate rule parameters \( \eta^r = 0.81 \), \( \eta^y = 0.22 \), and \( \eta^\pi = 2.04 \).

Examining the impulse response functions to monetary policy shocks indicates that the accelerator mechanism implied by the agency problem is not present except for very large increases in the policy rate. Impulse response functions of output, investment and the interest spread to positive and negative monetary policy shocks of 100 basis points are shown in figures 3.12 and 3.13 respectively. For a shock even as large as this in the region of the ergodic mean, the responses are close to symmetrical, but the constraint has just started to bind with the interest rate increase. For any size cut, the response of the borrowing constraints and NK model will be the same, but for a larger rate increase, the financial constraint will lead to a deeper decline in investment. This is shown in figure 3.14.
This paper embeds a banking model into a standard real business cycle model which, unlike standard models of financial frictions such as Gertler & Kiyotaki (2010), clearly distinguishes between normal and crisis times. In the vicinity of the deterministic steady state, the model is analogous to a real business cycle model, financial intermediation is efficient and the interest rate spread is equal to the standard risk premium. Crises are precipitated by sufficiently large shocks that adversely effect the present discounted value of future bank profits; this causes the borrowing constraint to bind as the bank has increased incentive to divert funds and declare bankruptcy. When debt finance is constrained, banks can issue equity but, as this is costly, a wedge
emerges between the risk-free rate and the risky return to capital beyond the risk premium. This results in reduced investment and output.

The main finding of the chapter was that the agency problem proposed in Gertler & Kiyotaki (2010)\textsuperscript{20} does not seem to produce the much heralded financial accelerator mechanism once a number of key assumptions are relaxed. When the borrowing constraint is only occasionally binding, and if the banks can choose the optimal level of dividend payments, the business cycle dynamics become equivalent to those of the efficient RBC case around the steady state. Allowing the banks to issue equity, even if very costly at a 10% transactional fee, also limits the adverse effects of severe disaster shocks. We have shown that this finding is robust to the choice of investment and capital adjustment costs, habits in consumption, and household preferences.

The second contribution is a far more careful treatment of the agency problem proposed by Gertler & Kiyotaki (2010). By modelling the U.S. law relating to bankruptcy, we give dividend payments a potentially important signalling role in acting to relax the borrowing constraint. This leads to a time variability of the borrowing constraint, dividend payments, and the interest rate spread that would otherwise get missed.

Despite finding a reduced importance of the agency problem on business cycle dynamics, relaxing the GK assumptions introduced some
improvements in the empirical fit of some variables. Notably, we cap-
ture the strong positive skewness in the interest spread and equity is-
suance that are missing in the standard RBC and GK models. We also
replicate the counter-cyclical equity issuance observed in the data,
contrary to other papers, such as Gertler et al. (2012), which predict
pro-cyclical equity issuance.

Finally, with regards to the effects of the agency problem on the trans-
mission mechanism of monetary policy, for small shocks in the region
of the steady state, the transmission is unaffected by the presence of
the agency problem. However, the effects are asymmetric and non-
monotonic, and for very large unexpected rate increases, greater than
100 basis points under the model parametrisation, the financial con-
straints begin to bind and investment declines further than in the
financially efficient NK model. The indication is that during financial
crises, monetary policy will feature a financial accelerator mechanism,
but will not during normal economic periods.
ADVERSE SELECTION AND THE EFFICIENCY WEDGE

In this chapter we show that asymmetric information in credit markets can lead to adverse selection that manifests as an investment wedge between the risky return to capital and the risk-free rate, and an efficiency wedge due to the inefficient allocation of capital. Whilst Chari et al. (2007) have shown that the efficiency wedge has historically played a crucial role in driving economic fluctuations, and the investment wedge a much smaller role, the latter has dominated recent research into the macroeconomic effects of financial frictions. This chapter seeks to rectify this issue with a credit friction arising from productive projects with privately observed risk profiles. A stylized model with risky and safe projects is proposed and calibrated to match a number of metrics from the U.S. firm and credit data, and macroeconomic time series. We find that large enough shocks can generate sharp spikes in the efficiency wedge that drive endogenous financial crises. These produce negative skewness in output and investment that help match empirical third moments. The optimal contract is always separating when risky and safe projects have equal expected value, but pooling can emerge when the relative expected value of risky projects declines sufficiently. Finally, we show that adverse selection does not have large effects on the transmission mechanism of monetary policy around the ergodic mean, but larger negative monetary policy shocks can generate significant non-linear effects.

4.1 INTRODUCTION

This chapter proposes a model of financial frictions that can map to both an efficiency wedge caused by the inefficient allocation of productive resources, and an investment wedge between the return on capital and the risk-free rate. Mapping to the former is important be-
cause although researchers typically focus on financial frictions that emerge as the latter, the empirical evidence suggests the efficiency wedge plays a more important role in driving economic fluctuations (see Chari et al. 2007). The financial friction in this chapter is caused by an information problem; lenders seeking funds to finance a productive project observe private information about the project risk. We show how this feature can lead to an adverse selection problem that can introduce both an investment wedge due to information rents, and an efficiency wedge caused by the misallocation of capital. The efficiency wedge drives episodes where falls in investments are significantly deepened, which we describe as financial crises. These introduce negative skewness in simulated time series that match observed macroeconomic data.

The chapter builds on a large literature examining the role of credit frictions in generating and propagating business cycle fluctuations, discussed in the introduction to this thesis. Many models of financial frictions can be characterised by a wedge between the risky return on capital and the risk-free rate. For instance, in Bernanke et al. (1999), the wedge is caused by the external finance premium that depends on the leverage of the borrower; and in models such as that of Gertler & Kiyotaki (2010) (and the model proposed in chapter 3), the spread between the rates depends on the net worth of the bank due to limited contract enforceability. In both these cases, because the wedge widens when asset prices are depressed, and shrinks when asset prices increase, a financial accelerator is introduced that is used to explain how financial factors can aggravate business cycles and enhance the transmission of monetary policy. As Chari et al. (2007) have shown, the drawback of this type of approach to financial frictions is that the empirical support is relatively weak. The authors show how different types of frictions can be mapped to one of four wedges; an efficiency wedge that appears like time-varying productivity, an investment wedge equivalent to a tax on capital, a labour wedge equivalent to a labour tax, and a government consumption wedge. Using a prototype economy with the four wedges, Chari et al. (2007) estimate the relative importance of each of the wedges using U.S. business cycle episodes of the Great Depression, the 1982 recession, and over the entire post-war period. They find that efficiency and labour wedges
play very important roles in generating the business cycle, the investment wedge a small role and government consumption almost none. These findings provide strong support for a set-up such as that presented in this chapter as the financial friction emerges as an efficiency wedge as well as an investment wedge.

The model proposed in this chapter builds on a literature examining optimal financial contracts with hidden information. The literature began with the seminal contribution on asymmetric information in Akerlof (1970), and the specific credit friction is closely related to that discussed in Stiglitz & Weiss (1981). Much of research in this area has been focused on the optimal financial contract in a partial equilibrium context (e.g. Clementi & Hopenhayn 2006, Martin 2009), whereas this literature attempts to bridge this to the macroeconomic literature that has typically focused more on the costly-state verification model rather than hidden information (cf. Bernanke et al. 1999, Christiano, Motto & Rostagno 2010).

To analyse the macroeconomic effects of the credit friction, we propose a model of occupational choice in which entrepreneurs seek external finance to fund a project. Each period, entrepreneurs privately draw either a risky or safe project, where the project type is independent and identically distributed across time and entrepreneurs. The lenders know the aggregate state of the economy, including the proportion of risky and safe entrepreneurs, and propose incentive compatible one period contracts. To model the problem in a simple, tractable way, we treat the economy as comprised of a continuum of households, each with a large number of members. Every period, a family head allocates households members to act as either entrepreneurs, workers or remain unemployed. Each project drawn by entrepreneurs requires a fixed level of funding comprised of both internal and external finance so to ensure interior solutions to the occupational allocation and project finance problems. These assumptions allow the treatment of the economy in a representative agent framework, with its computational benefits, whilst allowing the presence of a credit friction between lender and borrower. It also allows for limited project funding as when the value of the entrepreneur, or rather the value of equity, is greater than the value of lending, the household head will increase the household members to act as
entrepreneurs. In equilibrium, this can lead to a lending feasibility constraint whereby not all entrepreneurs seeking funds will be able to secure them.

In section 4.2, we discuss the contract in a partial equilibrium economy to gain insight into the contract structure and possible outcomes. We highlight firstly that the contract optimal to the lender can be modelled as a static one-period contract. Both separating and pooling equilibriums are available depending on the distribution of risk and expected values of safe and risky projects. Within a large range of parameter values, the central outcome produces a separating equilibrium with full funding for risky borrowers, except when funding is limited and either the value of risky projects is sufficiently less than the value of safe projects, or risky projects are particularly risky. With either of these characteristics, the lender will choose a pooling equilibrium to reduce the information rents paid to risky borrowers. Under a separating equilibrium, the optimal contract is first-best for a range of parameters if the expected value of risky projects is weakly greater than safe projects unless: the expected return to risky projects is very high; risky projects are particularly risky; or risky projects particularly plentiful. Under these conditions, the information rents increase to the point where the lender restricts credit to safe borrowers so to reduce the information rents received by risky borrowers. If funding is limited due to a feasibility constraint that implies some borrowers seeking funds will be unable to secure loans, the safe borrowers will never be fully funded.

In the general equilibrium model, we focus on shocks to aggregate total factor productivity, the productivity of risky projects which allow differing values of safe and risky projects, and shocks to the risk of risky projects. For the last, to isolate the role of risk, the productivity is also adjusted to keep the project values equal. As a point of comparison, we solve the first-best economy which is equivalent to a standard real business cycle model. The adverse selection problem introduces significant non-linearities. The specific impact of the non-linearities differ for each type of shock but the general effects can be discussed in the context of the outcomes implied by the contract. Shocks that increase the information surplus received by risky borrowers will cause the marginal value of safe loans to fall; if this
reaches zero, the lender will begin to restrict credit to safe projects. As the lender leaves capital unallocated despite profitable opportunities, an efficiency wedge is introduced. This generates sharp declines in output, investment and employment that could well be described as a financial crisis. Shocks that reduce the relative value of risky projects cause falls in the marginal value of risky lending which, if large enough, leads to the lender choosing a pooling equilibrium so to increase lending to more profitable safe projects.

The adverse selection problem introduces significant non-linearities. The specific impact of the non-linearities differ for each type of shock but the general effects can be discussed in the context of various thresholds introduced by the contract. Shocks can cause the marginal value of lending to safe borrowers to fall usually by increasing the information surplus received by risky borrowers, when this reaches zero, the lender will restrict credit to safe projects introducing an efficiency wedge as the lender sits on capital that could be lent to profitable borrowers. On the other hand, shocks can cause large falls in the marginal value of risky lending, usually by reducing the relative value of risky projects. If the fall is large enough, the lender will choose a pooling equilibrium.

Full analysis is given in section 4.5 although we highlight several key results here. Shocks to total factor productivity within a typical range have little effect on aggregate output although imply higher volatility in investment relative to the first-best economy. This occurs because of fluctuations in information rents that lead to higher volatility in the value of equity relative to the value of debt. This generates higher volatility in the number of entrepreneurs, which causes greater fluctuations in the returns to investment. Shocks to the risk of risky projects can manifest as both investment and efficiency wedges, the former coming from increased information rents. The higher information surplus increases the value of equity relative to debt causing the marginal value of safe loans to decline. If the shock is sufficiently large, a 2 percentage point fall in the probability of project success under the calibrations, then the lender begins to ration credit to safe borrowers. As capital is left unallocated, an efficiency wedge emerges that produces much larger drops in output, investment and employment, and leads to negative skews in the simulated time series of
these variables. The third shock we discuss effects the value of risky projects; we refer to these as efficiency shocks due to the effect on the efficiency wedge. For large enough positive shocks to the value of risky projects, a 1.5% increase from steady state under model calibrations, there are sharp adverse effects. Whilst the first-best economy exhibits the expected positive effects from higher productivity, with adverse selection, past this threshold, the lender begins to restrict lending to safe projects in order to receive additional surplus from risky projects. This, again, causes the efficiency wedge to emerge. The impact on output on investment is much less marked following negative shocks to the value of risky projects, although if the shocks are large enough negative, a 5% fall under calibrations, the lender will choose a pooling equilibrium so to increase funding to higher value safe projects. We find that whilst there is an increase in the efficiency wedge following a negative efficiency shock, there is an offsetting fall in the spread.

We begin the analysis with a derivation and discussion of the optimal contract in a partial equilibrium setting. This allows an insight into the impact of the asymmetric problem over a range of values of risk and distribution of risk. In section 4.3, we propose a stylized general equilibrium model with adverse selection. This is followed with a discussion of the numerical strategy employed before analysing the results.

4.2 Asymmetric Information in Partial Equilibrium

To characterise the adverse selection problem, we first consider the friction in a partial equilibrium model. There are a large number of borrowers and a single lender, each borrower has investment projects that arrive exogenously each period with a type \( \theta \in \{s, r\} \) representing safe and risky respectively. The projects yield return \( X(\theta) \in \{R(\theta), 0\} \) where \( p(\theta) \in (0, 1] \) is the probability of return \( R(\theta) \). Each project requires a single unit of investment. The lender only has funds for a proportion \( \gamma \) of borrowers, where we assume \( \max\{\lambda, 1 - \lambda\} < \gamma \leq 1 \). Denoting variables of type \( \theta \) with superscripts, we make the following assumptions:
Assumption 5. $p^s > p^r$ and $R^s < R^r$

This defines the concept of risky and safe; the safe project has a higher probability of success but yields lower returns when it is successful.

Assumption 6. The bank cannot observe the risk-type of the project but whether the project is successful is public knowledge.

The first point, following Stiglitz & Weiss (1981, 1992), is the borrower heterogeneity at the heart of the credit friction and we make the second to simplify the contract. A number of papers focus on this dimension of information asymmetry without heterogeneity in risk, whereas this chapter draws on the importance of risk. If the lender cannot observe the success of the project then there cannot be short-term contracts as the borrower will always declare a bad state. We are considering contracts without full commitment and as we find that the optimal contract is equivalent to a sequence of one-period contracts, we rule this possibility out.

Borrowers must source external finance to fund the project and we allow lender and borrower to agree a contract over the provision of finance, where the optimal contract $C$ will specify financing probabilities $x$ and transfers to the the lender $\tau$. Following Prescott & Townsend (1984)\footnote{See also Besanko & Thakor (1987).}, the lottery $x$ is used to allow the lender to offer incentive compatible contracts to all borrowers. Consider a single period contract $C(x, \tau)$; the ex ante value of the contract to the borrower is given by

$$V = \lambda x^s p^s (R^s - \tau^s) + (1 - \lambda) x^r p^r (R^r - \tau^r)$$

and the total value of the project is

$$W = \lambda x^s (p^s R^s - 1) + (1 - \lambda) x^r (p^r R^r - 1)$$

The lender proposes a contract $C(x, \tau)$ to maximise $(W - V)$.

4.2.1 First-Best

As a point of comparison to quantify the inefficiency introduced by asymmetric information, we solve the optimal contract under full in-
4.2 ASYMMETRIC INFORMATION IN PARTIAL EQUILIBRIUM

formation. The contract is solved subject to individual rationality constraints

\[ R(\theta) - \tau(\theta) \geq 0 \]  \hspace{1cm} (4.2.3)

which means there must be a weakly positive surplus to the borrower. We can verify that the individual rationality (IR) constraint will bind, and the resulting solution implies that any projects with positive net present value will receive funding whilst those with negative value will not. Whichever project has the highest value will receive funding with probability one and the other type will receive any remaining funds. As all borrowers will earn their reservation value and so are indifferent about receiving funding, there is no restriction of credit even with \( \gamma < 1 \).

4.2.2 Optimal Contract with Asymmetric Information

We assume that all projects require equal funding, and as borrower type is i.i.d. across time and space, the contact optimal to the lender will revert to an optimal static contract. In section 4.2.3 we look at a long-term case, and whilst there is a socially optimal long-term contract that produces a pooling equilibrium, it is not optimal to the lender who will only offer one-period terms for funding. The problem of the lender is to post contract offers that maximise their value subject to the incentive compatibility (IC) and individual rationality constraints.\(^2\) The two IR constraints are the same as in the first-best economy as shown in equation (4.2.3), and the incentive compatibility constraints are given by

\[ x_i p_i^i \left( R_i - \tau_i \right) \geq x_j p_j^i \left( R_j - \tau_j \right), \quad i = r, s \]  \hspace{1cm} (4.2.4)

That is, the value to each borrower of declaring their type truthfully must be weakly greater than lying. As is standard in these mechanism design problems, the problem can be simplified by dropping two constraints; the IR constraint of the risky type and the IC constraint of

\(^2\)We assume a two-stage game following, for example, Rothschild & Stiglitz (1976) and Wilson (1977), whereby lenders post contract offers that borrowers can choose to accept. There are some consequences of the choice of assumptions; choosing a three-stage game, for instance, could lead to pooling or separating equilibria depending on the starting agent. A discussion of these issues is given in Hellwig (1987).
the safe type. This is straightforward to show: the IC constraint of the risky type is given as

\[ x^r p^r (R^r - \tau^r) \geq x^s p^r (R^r - \tau^s) > x^s p^r (R^s - \tau^s) \geq 0 \] (4.2.5)

where the relationship between the second and third argument follows from \( R^r > R^s \), and the last because of the safe IR constraint. It follows that \( R^r > \tau^r \) and so the safe IR constraint must be the relevant one, indeed, we find this constraint will be always binding. It follows that the risky IC constraint is the relevant one, and again is found to be binding. It follows that the repayment terms can be written

\[ \tau^s = R^s \] (4.2.6)
\[ \tau^r = R^r - \frac{x^s}{x^r} (R^r - R^s) . \] (4.2.7)

The safe IC constraint which, using \( R^s = \tau^s \), becomes \( 0 \geq x^r p^s (R^s - \tau^r) \), implies \( \tau^r \geq R^s \) which from the binding risky IC constraint, implies \( x^r \geq x^s \). Using this restriction and substituting in the expressions for \( \tau^s \) and \( \tau^r \), the problem is written

\[
\max_{x^s, x^r} \{ \lambda x^s \left( p^s R^s - 1 \right) + (1 - \lambda) x^r \left( p^r R^r - 1 \right) - (1 - \lambda) p^r x^s (R^r - R^s) \} \\
\text{s.t.} \quad 0 \leq x_s \leq x_r \leq 1 \\
\lambda x^s + (1 - \lambda) x^r \leq \gamma 
\] (4.2.8)

leading to

\[
(\lambda p^s + (1 - \lambda) p^r) R^s - 1 = \varphi + \varphi^r - \varphi^s 
\] (4.2.9)
\[
p^r R^r - 1 = \varphi + \frac{1}{1 - \lambda} \varphi^r - \frac{1}{1 - \lambda} \psi
\] (4.2.10)

where \( \varphi \) is the Lagrange multiplier on the overall finance limit, \( \varphi^s \) and \( \varphi^r \) are lower bound on \( x^s \) and the upper bound on \( x^r \) respectively, and

---

3 Following the problem proposed in that presented in Bolton & Dewatripont (2005, pp.59-60).
\( \psi \) is the Lagrange multiplier on \((x^r - x^s)\). These first order conditions are also subject to Kuhn-Tucker conditions

\[
\begin{align*}
\varphi^s &\geq 0 \\
\varphi^r &\geq 0 \\
\varphi &\geq 0 \\
\psi &\geq 0 \\
\varphi^s x^s &= 0 \\
\varphi^r (1 - x^r) &= 0 \\
\psi (x^r - x^s) &= 0 \\
\varphi (\gamma - \lambda x^s - (1 - \lambda) x^r) &= 0.
\end{align*}
\] (4.2.11)

Using these it is possible to identify four clear outcomes from the two first order conditions. These are (i) for the lender to lend to all risky borrowers and remaining funds to the safe borrowers; (ii) the lender to lend to all risky borrowers but no safe borrowers; (iii) the lender to lend to an equal proportion of risky and safe borrowers; and (iv) to extend no credit at all. The last only occurs if both

\[
1 > (\lambda p^s + (1 - \lambda) p^r) R^s
\] (4.2.12)

\[
1 > p^r R^r
\] (4.2.13)

The second condition is that the net present value of risky projects is negative, and the implication from the first condition and given that \(p^r < p^s\) by definition, is that it is possible that positive net present value safe projects will not receive funding at any price. Whilst it is possible for all positive value safe projects to go unfunded, and certainly no negative value safe project will ever be funded, it turns out that negative net present value risky projects can get financing provided that the surplus from safe projects is sufficiently high. This occurs when

\[
(\lambda p^s + (1 - \lambda) p^r) R^s > 1
\] (4.2.14)

in which case, the lender finances each project type with equal probability \(\gamma\), and the surplus from the safe projects subsides for the risky projects. The equal financing will occur providing (4.2.14) holds and if \(p^r R^r < (\lambda p^s + (1 - \lambda) p^r) R^s\). Once \(p^r R^r > (\lambda p^s + (1 - \lambda) p^r) R^s\), the lender will finance risky projects with probability 1, with any remaining funds going to safe projects which are financed with probability
(\gamma - 1 + \lambda) \frac{1}{\lambda} unless the expected return to risky projects is particularly high. In this case, it is in the interest of the lender to cease funding safe projects; this allows the lender to receive all surplus from the risky projects and occurs when

\[(\lambda p^s + (1 - \lambda) p') R^s < \lambda + (1 - \lambda) p'R'.\]  \hspace{1cm} (4.2.15)

It is clear that the higher (lower) the proportion of risky projects, the more (less) likely the safe projects will be excluded from the credit market.

Further to the four outcomes given, if the conditions hold with equality there opens additional possible outcomes. If \((\lambda p^s + (1 - \lambda) p') R^s = p'R'\) then the only binding constraint is the feasibility constraint; all funds will be lent. The lender and borrowers are indifferent as the inverse relationship between \(\tau'\) and \(x'\) implies the expected distribution of surplus is unchanged. When \((\lambda p^s + (1 - \lambda) p') R^s = \lambda + (1 - \lambda) p'R'\), the lender is indifferent about whether to finance safe projects or not; as the safe borrowers receive no surplus anyway, they are already indifferent. In partial equilibrium, the conditions are insufficient to determine the outcomes; this will not usually be the case in general equilibrium.

4.2.2.1 Misallocation of Funds

Let the safe project be perfectly safe, so \(p^s = 1\), and let the risk level of risky projects and expected return of safe projects be fixed at \(p' < 1\) and \(R^s\) respectively. If we evaluate the contract for different values of \(R'\) we can assess the inefficiency of the misallocation of funds introduced by the information problem. Figure 4.1 shows the value of the project for a range of values of \(R'\) in the presence of asymmetric information with the green line, compared to the first-best with the blue line. The key thresholds highlighted represent those discussed above, and are \(R' = A = \left(1 - \lambda \left(1 - \frac{1}{p}\right)\right) R^s\), \(B = R^s + (R^s - 1) \frac{1}{p'} \frac{\lambda}{1-\tau'}\), and \(C = R^s \frac{1}{p'}\). The value of the project with the first-best solution has a kink at \(C\) at which point the projects have equal expected value. In the region to the left of \(C\), the lender finances the higher return safe projects with probability one, with remaining funds financing risky projects. The reverse occurs to the right of \(C\) where risky projects
4.2 ASYMMETRIC INFORMATION IN PARTIAL EQUILIBRIUM

Figure 4.1: Value of project with asymmetric information $W$ and the first-best $W^*$ for a range of $R^r$.

have higher expected return. Pooling occurs to the left of threshold $A$ as the contract with asymmetric information offers exactly the same terms to safe and risky projects; a probability $\gamma$ of being financed and repayment $\tau = R^s$. With the information problem, it is not possible for the lender to go better by increasing finance to safe projects as the risky contract would no longer be incentive compatible. This pooling can occur even if $p^rR^r < 1$ provided the surplus from safe projects is sufficiently high to subsidise losses from the risky projects, otherwise the lender will cease all lending even if $p^sR^s > 1$. To the right of $A$ is a separating equilibrium with all risky projects receiving finance with the use of remaining funds dependant on $R^r$. Between $A$ and $B$, the lender uses the remaining $[\gamma - 1 + \lambda] \frac{1}{\tau}$ funds to finance safe projects, and to the right of $B$ the lender stops financing safe projects altogether. This occurs because the expected return from risky projects is especially high and the information surplus received by risky borrowers is greater than the surplus generated by safe projects; it is therefore optimal to stop funding safe projects and set the repayment on risky loans $\tau^r = R^r$. 
The area between the thresholds C and B represents a state space where the asymmetric information yields no inefficiency as the overall surplus is equal to the first-best case. The difference between the first and second best is distributional; risky borrowers are able to earn rents due to their informational advantage. To the left of C, risky borrowers get an inefficient share of finance that generates the wedge between $W$ and $W^*$. Note that this wedge disappears in the case that $\gamma = 1$ and $W = W^*$ in this area. To the right of threshold B, lenders cease funding safe projects altogether, and so a large inefficiency is introduced in this region under asymmetric information.

In the region to the left of C, the inefficiencies generated are an allocation problem; the lender will use all available funds to finance projects, but lower return projects are more likely to receive funding. If the restriction on the number of projects funded is removed with $\gamma = 1$, all projects will be financed and the outcome will be efficient. We can also consider the importance of $\lambda$ and $p^r$ on the outlook. With a lower value of $p^r$ corresponding to riskier risky projects, all the thresholds will shift right due to a decreased expected value of risky projects. As one might assume that the economy will tend to be in an area where the expected return from each project is close to equal, it is of interest to think of how the relative distances between the thresholds change given a change in the risk-level and the associated changes in expected value. For a right-shift in C of size $\Delta$ generated by a fall in $p^r$, threshold A would move by $\lambda \Delta$ and B by $(1 - \frac{1}{p^r}) \frac{\lambda}{1 - \lambda} \Delta$. The shift in A is certainly less than the shift in C indicating that pooling equilibrium would become relatively less important if the economy tends to be in the region of C. The shift in B depends on the share of safe and risky projects, and the return on safe projects. B will move away from C if $\lambda > \frac{1}{2(2 - \frac{1}{p^r})}$, that is, if both the return and share of safe projects are sufficiently high. Consider that if 90% of projects were safe, it would still require an expected return of 12.5% for this to occur, otherwise B would converge on C as the risk increased. Indeed, B would equal C, implying safe projects would be restricted from credit when the expected value of safe and risky projects were equal, if the share or riskiness of risky projects were high enough. This would occur when $p^r = 1 - (1 - \frac{1}{p^r}) \frac{\lambda}{1 - \lambda}$.

---

4 Incidentally, this would correspond to a shift in $p^r$ of $-\sqrt{\frac{E'}{p^r}}$.  

We discuss this role of risk below but these results suggest that under reasonable parametrisations, as the risk of risky projects increases, the efficient region shrinks, the importance of pooling becomes less, and extreme separation whereby safe projects cannot receive any funding becomes more important.

4.2.2.2  Equal Expected Value and the Role of Risk

To think about the role of risk, we are interested in the case where \( \bar{\mathcal{R}} \) remains constant but \( p' \) changes; an increase in risk would be associated with a fall in \( p' \). The first order conditions become

\[
\lambda (\bar{\mathcal{R}} - 1) - (1 - \lambda) (\bar{\mathcal{R}} - p' \mathcal{R}^s) - \varphi \lambda + \varphi^s - \psi = 0 \quad (4.2.16)
\]

\[
(1 - \lambda) (\bar{\mathcal{R}} - 1) - \varphi_i (1 - \lambda) - \varphi' + \psi = 0 \quad (4.2.17)
\]

Again using the Kuhn-Tucker conditions, we can examine the possible cases which become somewhat simpler. First, we find that provided \( \bar{\mathcal{R}} > 1 \), the risky projects get finance with probability one and so the pooling equilibrium disappears. This had occurred when the surplus from higher value safe projects subsidised for low value, or even negative value, risky projects. If it is assumed that \( \bar{\mathcal{R}} > 1 \) we find all risky projects will get funded with probability one, otherwise no projects receive funding. Whether or not the remaining funds are used to finance safe projects or are un-utilised depends on the same condition as before. If \( p' < p^s (1 - \frac{\lambda}{1-\lambda} (1 - \frac{1}{\mathcal{R}})) \) then the lender will restrict credit to safe projects entirely, otherwise they will use the remaining \( (\gamma - 1 + \lambda) \frac{1}{\lambda} \) of funds to finance the safe projects. From this condition, it is clear that the probability of this event increases in the share of risky projects \( (1 - \lambda) \). With expected value equal across projects, the higher the risk of the risky projects, the higher the information rents implied by the contract. At the threshold given, it is optimal for the lender to restrict financing safe in order to receive this surplus.

If safe projects are perfectly safe, so \( p^s = 1 \), using the solution to the optimal contract, the \textit{ex ante} value of equity is given by

\[
V = (1 - \lambda) \chi^s \bar{\mathcal{R}} (1 - p') \quad (4.2.18)
\]
and the *ex ante* value of the whole project

\[ W = (x^s \lambda + x^r (1 - \lambda)) (\bar{R} - 1). \tag{4.2.19} \]

Whilst the rate of return is unchanged, the overall expected value of the project will fall if the probability of finance falls. The information surplus to the risky borrower increases in risk, leading to an increase in the value of equity. As mentioned, when this surplus is sufficiently large, the lender finds it optimal to restrict lending to safe borrowers entirely in order to receive the surplus from risky projects. It follows from the condition above when \( p^s = 1 \), that the probability of financing safe projects is determined by

\[ \lambda (\bar{R} - 1) - (1 - \lambda) \bar{R} (1 - p^r) \tag{4.2.20} \]

where if positive, the safe projects will receive funding, but not if negative. Figure 4.2 shows the value of equity and the entire project for a range of risk values. The surplus to the lender is represented by the green area above \( V \) and below \( W \). This surplus falls linearly in \( p^r \) until the threshold given by

\[ p^{rs*} = 1 - \frac{\lambda}{1 - \lambda} \left( 1 - \frac{1}{\bar{R}} \right) \tag{4.2.21} \]
at which point, credit to safe borrowers is restricted entirely. The surplus to the borrowers reaches a peak of \((\gamma - 1 + \lambda) W^*\), and drops to zero as the lenders receive all surplus. The level of \(W\) drops to a fraction \(1 - \lambda\) of the first-best level when safe borrowers are excluded and is the only inefficient region when the expected value of projects are equal.

### 4.2.3 Long-term Contracts

It was stated above that the best the lender can expect is for static one-period contracts; a long-term contract is derived here and shown to offer higher expected social value but at the expense of lender value. The one-period contract implied a static trade-off across states, whilst with a long-term contract it is possible to specify a dynamic trade-off between the current pay-off and the future value of equity. Rather than having to restrict the proportion of safe projects invested in today, the restriction can be made in the future, once the risk type has been reset. Denoting borrower type \(\theta \in \{s, r\}\), the contract \(C\) implies an expected discounted value of future cash flows \(V(\theta_t, C)\) and is solved subject to an initial participation constraint, a sequence of incentive compatibility constraints, and a sequence of limited liability constraints that replace the individual rationality constraints. Letting \(\hat{\theta}^* = \{\hat{\theta}_t\}_{t=1}^\infty\) and \(\theta^* = \{\theta_t\}_{t=1}^\infty\) be the reporting strategy and sequence of types respectively. The contract will be incentive compatible if \(V(\theta^*, C) \geq V(\hat{\theta}^*, C) \forall \hat{\theta}\) and feasible provided

\[
\tau_t(\theta) \leq p_t(\theta) R_t(\theta), \quad \forall t. \quad (4.2.22)
\]

\(V\) can very well be defined as the value of equity, and letting this value be subject to project continuation, is given by

\[
V_t = \lambda p_t^s (R_t^s - \tau_t^s) + (1 - \lambda) p_t^r (R_t^r - \tau_t^r) + \lambda \beta \hat{V}^s_{t+1} + (1 - \lambda) \beta \hat{V}^r_{t+1} \quad (4.2.23)
\]

where \(\hat{V}^s_{t+1}\) and \(\hat{V}^r_{t+1}\) are the future value of equity in reporting safe and risky types respectively. These values must be positive, otherwise a negative future cash-flow is implied that violates the limited liability constraints. As before, the only incentive compatibility constraint
required is that which leads to risky borrower truth-telling, and the relevant limited liability constraint is that of the safe borrower. These are written

\[ \tau_s^t \leq R_s^t \]  

(4.2.24)

and

\[ \tau_r^t \leq R_s^t + \beta [V_{t+1}^r - V_{t+1}^s] \frac{1}{p_t} \]  

(4.2.25)

Rather than specify a probability of finance for the current period, the contract now sets a future probability of project liquidation. By the time that liquidation occurs, it is too late for the borrower to match to a new lender that period. We can denote the liquidation value with \( Q \) and the value to the lender with \( S \). These values are exogenous to the contract, and whilst we assume that it is too late for the borrower to seek new funding, it turns out that whether it is too late for the lender to re-allocate funds does not affect the results. The period following liquidation, borrowers can secure new funding with probability \( z_t \).

If \( e \) are the proportion of borrowers currently engaged on projects, then a proportion \( [\lambda x_s^t + (1 - \lambda) x_r^t] e_t \) of all borrowers will continue. Therefore, the feasibility constraint for funds becomes

\[ \gamma \geq [\lambda x_s^t + (1 - \lambda) x_r^t] e_{t-1} + z_t (1 - e_{t-1}) \]  

(4.2.26)

The proportion of active borrowers evolves according to

\[ e_t = [\lambda x_s^t + (1 - \lambda) x_r^t] e_{t-1} + z_t (1 - e_{t-1}) . \]  

(4.2.27)

As this is external to the contract, we leave this and revisit once the optimal contract has been solved.

As was shown in Atkeson & Lucas (1992), there is a recursive representation for this type of contracting problem with private information. Indeed, as the project requires only a fixed unit funding, so there is no evolution of wealth, apart from possible stochastic variations in returns that would lead to variations in pay-offs, the case is simpler
still. The optimal contract problem the lender seeks to solve can then be written

\[
U (V) = \max_{\tau^s, \tau^r, \hat{V}^s, \hat{V}^r} \left\{ \lambda \left( p^s \tau^s - 1 \right) + (1 - \lambda) \left( p^r \tau^r - 1 \right) \right. \\
\left. + \beta \left[ \lambda \hat{U} (\hat{V}^s) + (1 - \lambda) \hat{U} (\hat{V}^r) \right] \right\}
\]  

(4.2.28)

s.t. 

\[
V = \lambda p^s \left( R^s - \tau^s \right) + (1 - \lambda) p^r \left( R^r - \tau^r \right) + \beta \left[ \lambda \hat{V}^s + (1 - \lambda) \hat{V}^r \right]
\]  

(4.2.29)

\[
\tau^s_i \leq R^s \quad (4.2.30)
\]

\[
\tau^r \leq R^s + \beta \left[ \hat{V}^r - \hat{V}^s \right] \frac{1}{p^r} \quad (4.2.31)
\]

\[
\lambda p^s \tau^s + (1 - \lambda) p^r \tau^r \geq 1 \quad (4.2.32)
\]

Where \( U \) is the value subject to continuation and \( \hat{W} \) prior to liquidation. The value of equity \( V \) is the only relevant state variable, with equation (4.2.23) a constraint to the problem. To prevent negative lender cash-flows, the last condition is a lender limited liability constraint. The lender then sets the probability of liquidation by solving

\[
\hat{U} (V) = \lambda \max_{V, x^s} \hat{U} (\hat{V}^s) + (1 - \lambda) \max_{V, x^r} \hat{U} (\hat{V}^r)
\]  

(4.2.33)

s.t. 

\[
\hat{U} (\hat{V} (\theta)) = x (\theta) U (V) + (1 - x (\theta)) S
\]  

(4.2.34)

\[
\hat{V} (\theta) = x (\theta) V + (1 - x (\theta)) Q'
\]  

(4.2.35)

\[
0 \leq x^s, x^r \leq 1
\]  

(4.2.36)

\[
\gamma \geq \left[ \lambda x^s + (1 - \lambda) x^r \right] e + z (1 - e)
\]  

(4.2.37)

The optimal contract implies that the incentive compatibility and limited liability constraints bind determining repayment schedule \( \tau (\theta) \), and if we focus on the stationary equilibrium, the conditions determining liquidation probability \( x^r \) and \( x^s \) are

\[
U (V') - S' + (1 + \mu) (V' - Q') = \psi - \phi^s + \phi^r
\]  

(4.2.38)

\[
0 = \psi - (1 - \lambda) \phi^s - \phi^r
\]  

(4.2.39)

where \( \mu \) is the Lagrange multiplier on the new limited liability constraint. By evaluating these conditions subject to the Kuhn-Tucker conditions as before, concentrating initially on the case in which the lender yields weakly positive returns so \( \mu = 0 \), we find that providing \( U (V') + V' > S' + Q' \), so that continuation is more valuable than liquidation, the only important constraint is the feasibility constraint (4.2.37) as \( \psi > 0 \). This condition will only be violated in the presence
of negative expected returns, or with particularly high outside options, in which case $\phi, \psi > 0$ and all projects will be liquidated with probability 1. If we focus on states of the world where the condition is satisfied, then this will lead to a pooling equilibrium as the proportion of borrowers funded in the first period $e = \gamma$, then $x_r = x_s = 1$ and no projects will be liquidated, and $\tau_s = \tau_r = R_s$. To choose the probability of funding new projects, $z$, the lender solves

$$\bar{U}(e) = \max_{e', z} \left\{ eU + \beta \bar{U}(e') \right\}$$

subject to the evolution

$$e' = [\lambda x_s + (1 - \lambda) x_r] e + z \left(1 - e\right)$$

and condition (4.2.37). Again, provided $\beta U > 0$, so projects are profitable, the feasibility constraint will bind. It follows that if $\gamma$ were to increase, the lender would agree contracts with new borrowers to ensure all funds are used. If $\gamma$ falls, then the lender is indifferent about which projects to liquidate; if safe projects were liquidated, then the lender would lose surplus on safe loans, but could receive offsetting increased returns from risky projects in exchange for lower liquidation probabilities, likewise, if the lender liquidated risky projects, then they would need to subsidise losses incurred by reducing repayments from these projects using safe project surplus.

Whilst pooling will occur for a large range of $p_r, p_s$ or $\lambda$, a separating equilibrium emerges when risk or the share of risky projects is too high. Specifically, if $p_r, p_s$ or $\lambda$ fall low enough to violate the condition

$$\left(\lambda p_s + (1 - \lambda) p_r\right) \geq \frac{1}{R_s}$$

then the lender would receive negative profits. The limited liability constraint will bind and we find that safe projects face a positive probability of liquidation so to increase repayments by the risky borrowers above $R_s$. In such states of the world, the liquidation probability of safe projects is given by

$$x_s = 1 - \frac{1 - \left(\lambda p_s + (1 - \lambda) p_r\right) R_s}{(1 - \lambda) \beta (V' - Q')}.$$  

By substituting into the expression for $V$ the returns specified by the optimal contract, treating the outside option $Q = \beta V$, and consid-
erating the stationary equilibrium in which \( V' = V \), the value of the borrower cash-flows is given by

\[
V = \frac{(1 - \lambda) p^r (R' - R^s)}{1 - \beta (x^s + (1 - x^s) \beta)}
\] (4.2.44)

Using this and (4.2.43), we can give the relationship between the liquidation, risk, return, and project shares as

\[
x^s = 1 - \frac{1}{\beta (1 - \lambda) (1 - \gamma) p^r (R' - R^s) - 1 + (\lambda p^r + (1 - \lambda) p^r) R^s}
\] (4.2.45)

The solution to the problem in (4.2.40) implies that the lender will maximise funding new projects if profitable, and from the evolution of \( e \), we can find a further threshold; once \( \lambda (1 - x^s) \) rises above \( \frac{1}{\gamma} - 1 \), the number of projects liquidated is greater than the number of un-funded borrowers. In this circumstance, the total proportion of funded projects falls from the maximum of \( \gamma \) to

\[
e = \frac{1}{1 - \lambda (1 - x^s)}
\] (4.2.46)

The total surplus falls until \( x^s = 0 \) so that all safe projects are liquidated with probability one, and the lender receives all information rents from the risky borrowers.

### 4.2.3.1 Equal Expected Value Projects

To compare to the static contracts we can analyse graphical representation of the lender, borrower and social values and, as before, set \( p^s = 1 \) so \( R^s = p^r R^r = \bar{R} \). When the expected returns are equal, unless risky projects are very risky, the social value is equal to that under the first-best solution as all funds are allocated, and there can be no misallocation between projects. However, in this region, the risk does have distributional effects. Figure 4.3 shows the distribution of surplus for a range of risky project riskiness. The plot shows the static contract results from figure 4.1. Whilst a high risk led to an extreme separating equilibrium and a large drop in efficiency, the long-run contract is socially optimal up to point \( B \). At point \( p^r(A) = (\frac{1}{R} - \lambda) / (1 - \lambda) \), to avoid negative cash-flows to the lender, safe projects begin to get liquidated to reduce the risky borrower’s information rents. The probability of liquidation increases up to threshold \( B \) when \( x^s = 1 - \frac{1 - \gamma}{\gamma} \).
Figure 4.3: Overall value $W$ and value promised to the borrower $V$ by the optimal long-term contract for range of risk. The upper boundary represents total social value, the blue area represents the surplus to the borrowers, and the green area the surplus to the lender. The green and blue lines represent the social value and borrower value with static contracts.

when the proportion of projects getting credit falls from the maximum of $\gamma$ as the number of liquidations exceeds the possible new projects. This continues to fall to point $C$ when all safe projects are liquidated. This occurs at

$$p'(C) = \frac{(1 + \beta) (1 - \lambda R^e)}{\beta (1 - \lambda) (1 - \lambda) R^s + (1 + \beta \lambda) (1 - \lambda) R^e}$$

Finally, we can recall from the static case that $p'^* = 1 - \frac{\lambda}{1 - \lambda} (1 - \frac{1}{R})$, then it follows that $p'^* = A + 1 - \frac{1}{R}$ which implies $p'^* > A \forall \lambda, p'$. In the static contract, the expected social value of the contract for $p^r < p'^*$ is given by $W_S = (1 - \lambda) (R - 1)$, whilst in the long-term contract, for $p^r < p'^* (C)$, it is given by $W_L = \frac{1}{1 - \lambda} (R - 1)$ so $W_L > W_S$ and the social welfare is always higher under long-term contracts than one-period contracts for any value of $\lambda$ and $p'^*$. It is not possible to say that the static contract is preferred by the lender for all distributions of $p^r$ but we can make the following observations: for $p^r > p'^*$, we

---

5 The value here is a per-period value for comparison.
can measure the marginal value of risk in the two contracts for per-period borrower cash-flows as

\[
\frac{\partial V_S}{\partial (-p')} = \frac{1 - \lambda}{\lambda} (\gamma - 1 + \lambda) R, \quad \forall p' \leq p'^* \quad (4.2.48)
\]

\[
\frac{\partial V_L}{\partial (-p')} = \gamma (1 - \lambda) R, \quad \forall p' \leq p'^* - 1 + \frac{1}{R} \quad (4.2.49)
\]

noting we have transformed our definition of risk to \(-p'\) for convenience. This implies that the marginal value of risk to the borrowers in the long-term contract is always greater than that in the static contract providing \(\gamma < 1\), else they are equal\(^6\). There will be a threshold between points B and C above which the long-term contract is strictly preferred to the static contract by the lender providing our assumption that \(\gamma > 1 - \lambda\) holds. This occurs when the long-term contract specifies

\[
x^s < \frac{(R - 1) (\gamma - 1 + \lambda)}{(1 - \lambda) (1 - p')} \quad (4.2.50)
\]

Evaluating equation (4.2.45) at this threshold will yield a quadratic expression that determines the value of \(p'\) at which this occurs.

4.2.3.2 Misallocation of Funds in the Long-term Contract

If we relax our assumption that the projects have equal expected value, then we can repeat the exercise above and compare the value yielded by the contract for a range of \(R'\). The condition (4.2.42) that determines whether there is a pooling equilibrium is independent of \(R'\), and so finding that pooling occurs under most parametrisation, figure 4.4 adds the per-period value of the long-term contract to figure 4.1. As the contract implies an equal funding of both types of project, \(W_L\) represented by the red dashed line is linear in \(R'\) with no kink. At point C, all projects have equal expected value, and with all funds used, the value of all contracts is equal. Whilst under the static contract, with \(R' > C\) the more profitable risky projects are all funded, under the long-term contract, pooling implies a misallocation of funds as less profitable safe projects have equal chance of funding. Of course with \(A < R' < C\), the long-term contract implies less misallocation of funds than the static contract which funds

\(^6\) This compares static and long-term cases by using the *ex ante* expected per-period value of the contract rather than the value conditional on being under a contract.
4.3 Adverse selection in general equilibrium

To analyse the theoretical macroeconomic implications of adverse selection in credit markets, the contract problem is embedded in a general equilibrium framework. To rationalise $\gamma < 1$, we want to think about occupational selection and the value of acting as an entrepreneur seeking funds relative to other occupations. To achieve this in a simple, tractable way, we model the economy as comprised of a continuum of households each with a large number of household members.\(^7\) This also allows the agency problem to be framed appropriately, by assuming that the entrepreneur must seek external

\(^7\) See Christiano, Trabandt & Walentin (2010) as an example of this assumption, in this case to model the allocation of family members to employment.
funds from other households. This is equivalent to the island assumption of Gertler & Kiyotaki (2010) and chapter 3. It is assumed that a family head chooses occupational allocation to maximise a utilitarian welfare function that equally weights the utility of all members. Suppose that every period household members can be assigned to three possible roles: entrepreneurs that draw projects and seek funds; workers that receive a wage for providing a unit of work; and unemployed that do neither. As in Christiano, Trabandt & Walentin (2010), the utilitarian welfare function implies that all family members receive the same level of consumption. We follow the standard timings by assuming that investment decisions are made at the end of the period and employment decisions once shocks are drawn. To find well behaved interior solutions to the occupational allocation, we want the expected marginal value of the share of entrepreneurs, and the marginal utility of worker share to be negative. The latter gives motivation for including unemployed household members in the model, but the former needs some further restrictions. As in the partial equilibrium model, we assume that each project requires the same quantity of capital input; this is a necessary assumption to capture the assumption $\gamma > \max\{\lambda, 1 - \lambda\}$, key to the results in partial equilibrium. We could alternatively model decreasing returns to scale in projects but choose a simpler method able to capture the mechanisms we wish to study. Because there are many households, the probability of receiving finance will be external to the occupational allocation, and so to capture the negative marginal value of entrepreneur share, we assume that a project requires both internal and external capital inputs. This might seem ad-hoc, but we can calibrate the share of external finance to match the empirical share, and it avoids a potentially much more complicated agency problem to explain the financial structure of projects. We proceed to give detailed explanation of the model.

4.3.1 Households

With population size normalized to 1, the household head chooses the number of entrepreneurs $e_t$ at the end of the period, and the number of workers $1 - e_{t-1} - u_t$, and unemployed $u_t$ at the start of the new period. The workers provide one unit of labour and, if funded,
entrepreneurial activity also uses up a unit of labour. As in Christiano, Trabandt & Walentin (2010), we assume that household members differ in their dis-utility of labour, and so household $j \in [0, 1]$ receives the utility
\[
\log C_t - \frac{\chi}{1 - j}, \quad \chi > 0
\]  
(4.3.1)
if assigned as a worker or entrepreneur, and
\[
\log C_t
\]  
(4.3.2)
if unemployed. Utilitarian preferences convex in consumption imply all household members receive equal consumption. The total number of workers and funded entrepreneurs is given by $1 - u_t$, and so it follows that those members with $0 \leq j \leq 1 - u_t$ will be assigned as workers or entrepreneurs, and those with $j > 1 - u_t$ as unemployed. Of those entrepreneurs, following the contract design above, only a proportion $\lambda x_s^t + (1 - \lambda) x_r^t$ will receive external finance for their project and so the total labour supply by the household is given by $H_t \equiv (1 - [1 - (\lambda x_s^{t-1} + (1 - \lambda) x_r^{t-1})] e_{t-1} - u_t)$. Using this definition and equations (4.3.1) and (4.3.2), we can write the total household utility
\[
U_t = \log C_t + \chi \log (1 - H_t)
\]  
(4.3.3)
where the right hand term results from integrating household member utility over $j \in [0, 1]$.

Each project requires $\epsilon$ units of internal finance and so if the total capital owned by the household is $K_t$, the amount available to invest in external projects is given by $K_t - e e_t$. At the end of the period, the family head on each island simultaneously chooses consumption and next period capital stock, posts a contract offer for entrepreneurs on other islands, and chooses the number of households to assign to the entrepreneur role for the following period. The contract will

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8 There is an implicit assumption here that the household head can switch funding from entrepreneurs with the highest dis-utility of labour to those with the lowest. In fact, because $e_{t-1}$, $x_s^{t-1}$ and $x_r^{t-1}$ are known at the end of period $t - 1$, and $H_t$ is not chosen until period $t$ shocks are drawn, the $(\lambda x_s^{t-1} + (1 - \lambda) x_r^{t-1}) e_{t-1}$ household members with the lowest dis-utility of labour and the $[1 - (\lambda x_s^{t-1} + (1 - \lambda) x_r^{t-1})] e_{t-1}$ with highest will be assigned as entrepreneurs.
be discussed below, but the household head chooses \( e_t \) and \( u_t \), and consumption and saving to maximise

\[
\max_{C_{t+1}, K_{t+1}, e_{t+1}, u_{t+1}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} U(C_{t+s}, e_{t+s}, u_{t+s})
\]

(4.3.4)

Subject to the budget constraint

\[
C_t + K_t = R_t (K_{t-1} - e_{t-1}) + W_t (1 - e_{t-1} - u_t) + \pi_t e_{t-1}
\]

where \( \pi_t \) are average dividends from each project, \( W_t \) the wages paid in a perfect labour market, and \( R_t \) the expost return across lending to outside profits. The consumption savings decision is characterized by

\[
1 = \mathbb{E}_t [\Lambda_{t,t+1} R_{t+1}]
\]

(4.3.5)

where \( \Lambda_{t,t+1} = \beta \frac{C_t}{C_{t+1}} \), and labour supply by

\[
\chi \frac{C_t}{1 - H_t} = W_t.
\]

(4.3.6)

The occupation allocation decision leads to a further condition

\[
\mathbb{E}_t [\Lambda_{t,t+1} (\pi_{t+1} - W_{t+1} (\lambda x_t^s + (1 - \lambda) x_t^r)) - e] = 0,
\]

(4.3.7)

which equates the expected discounted profit from an additional entrepreneur with the opportunity cost which sums the expected welfare gain from the additional leisure and the expected discounted value of an additional \( e \) units of funds.

### 4.3.2 Entrepreneurs

A measure \( e_t \) of entrepreneurs draw investment projects of type \( i = s, r \) yielding efficiency units of productive capital \( X_t^i \in \{\omega^i, 0\} \). \( p_t^i \in [0,1] \) is the probability of efficiency units \( X_t^i = \omega^i \). The two types \( i = s, r \) represent safe and risky projects respectively. A proportion \( \lambda \) of the projects are safe, the remaining \( 1 - \lambda \) are risky, and as before \( \omega_s^s < \omega_r^s \) and \( p_s^s > p_r^s \). Each project requires \( \kappa \) units of outside investment which must be financed by another island and \( e \) units of internal finance. The number \( \kappa + e \) will be a normalisation device for the units of capital to introduce the idea of one indivisible unit of capital used in each project, and we will calibrate the ratio \( \kappa/e \) to match the empirical first moment.
Entrepreneur $i$ with a successful, funded project hires $h_i^t$ units of labour and produces output using

$$y_i^t = A_t \left( \omega_i^t (\kappa + \epsilon) \right)^{\alpha} \left( h_i^t \right)^{1-a}$$  \hspace{1cm} (4.3.8)$$

where $A_t$ is a stationary stochastic process. The entrepreneur will hire workers so that the marginal product of labour equals the real wage,

$$W_t = (1 - \alpha) \frac{y_i^t}{h_i^t}$$  \hspace{1cm} (4.3.9)$$

Capital depreciates at $\delta$, so the gross surplus equals

$$\alpha y_i^t + (1 - \delta) (\kappa + \epsilon) \omega_i^t$$  \hspace{1cm} (4.3.10)$$

of which the entrepreneur must repay the lender $\tau^i$. There are two types of entrepreneur; risky and safe, but a single labour market. Equation (4.3.9) implies that the efficiency capital-labour ratio must be equal across firms, and so the risky firms will hire more labour than safe firms.

Let $R_i^t$ be the gross rate of return on capital for project $i$. To simplify the problem, we assume that the safe projects always convert one unit of capital input to a single unit of productive capital, so $\omega^s = 1$, and so imply a rate of return $R_i^t$ with probability $p_i^s = 1$, whilst risky projects convert one unit of capital into $\omega_i^r$ leading to a rate of return $R_i^r$ with probability $p_i^r$. Recall that the efficiency capital-labour ratio is the same across projects; denoting this $K_i$, we can write

$$\alpha y_i^t = \alpha \omega_i^t (\kappa + \epsilon) A_t K_i^{\alpha-1}$$  \hspace{1cm} (4.3.11)$$

Therefore, if the safe projects yield gross rate of return

$$R_i^s = \alpha A_t K_i^{\alpha-1} + (1 - \delta)$$  \hspace{1cm} (4.3.12)$$

the risky project will yield

$$R_i^r = \omega_i^r R_i^s$$  \hspace{1cm} (4.3.13)$$

when successful. At the end of each period, remaining surplus is paid to the family as a dividend.
4.3.2.1 Optimal Contract

We have shown that the contract optimal to the lender can be characterised as a static, one period debt contract. We assume that debt repayments are indexed to the aggregate state of the economy, and are only repaid providing the project is successful. We further assume that the success of the project is public information. Modified from above, the individual rationality and incentive compatibility constraints are given by

\[
\mathbb{E}_t \left[ \Lambda_{t,t+1}^i \tau_{t+1}^i \right] \leq \mathbb{E}_t \left[ \Lambda_{t,t+1} R_{t+1}^i \right] \tag{4.3.14}
\]

\[
\mathbb{E}_t \left[ \Lambda_{t,t+1} \left( x_{t+1}^i p_{t+1}^i \left( R_{t+1}^i - \tau_{t+1}^i \right) \right) \right] \geq x_{t+1}^j p_{t+1}^j \left( R_{t+1}^j - \tau_{t+1}^j \right) \right \}, \quad j \neq i \tag{4.3.15}
\]

for \( i, j \in \{ s, r \} \). As the contract problem is analogous to the partial equilibrium problem described above, we jump straight to the first order conditions

\[
\mathbb{E}_t \left[ \Lambda_{t,t+1} \left( p_{t+1}^i R_{k,t+1}^i - 1 \right) \right] = \psi_t \frac{1}{1 - \lambda} + \phi_t \frac{1}{1 - \lambda} \tag{4.3.16}
\]

\[
\mathbb{E}_t \left[ \Lambda_{t,t+1} \left( (\lambda + (1 - \lambda) p_{t+1}^i) R_{k,t+1}^i - 1 \right) \right] = \psi_t + \phi_t - \phi_t^i \tag{4.3.17}
\]

that imply the same four possible outcomes as in partial equilibrium. The key difference to the partial equilibrium framework relates to outcomes on the defined threshold as the constraints bind. The partial equilibrium model implies a step function; for instance, \((\lambda p^s + (1 - \lambda) p^r) R^s > \lambda + (1 - \lambda) p^r R^r\) but if a change to \(p^r\) leads to \((\lambda p^s + (1 - \lambda) p^r) R^s < \lambda + (1 - \lambda) p^r R^r\), then the contract will imply a sudden stop in funding safe projects. What happens when \((\lambda p^s + (1 - \lambda) p^r) R^s = \lambda + (1 - \lambda) p^r R^r\) is undetermined; at this point, the lender is indifferent about \(x^s \in [0, (\gamma - 1 + \lambda) \frac{1}{\lambda}]\), and the safe borrowers are indifferent anyway as they receive no surplus. In general equilibrium, the level of investment and number of entrepreneurs can be adjusted depending on finance probabilities and expected profits so rather than jumping, the model is likely to remain at the threshold as other variables adjust. Taking the same example, suppose

\[
\mathbb{E}_t \left[ \Lambda_{t,t+1} \left( \lambda + (1 - \lambda) p_{t+1}^i R_{t+1}^i \right) \right] \tag{4.3.18}
\]
and that the risky projects have a strictly positive net present value. In this case by examining the first order conditions we can find

\[ \lambda \phi_r^t + (1 - \lambda) \phi_s^t > \psi_t \]  
(4.3.19)

\[ \phi_r^t (1 - \lambda) + \phi_r^t > \psi_t \]  
(4.3.20)

\[ \lambda \phi_s^t + \psi_t = \phi_s^t \]  
(4.3.21)

Using the Kuhn-Tucker conditions, it then follows that \( \phi_r^t > 0 \) and \( \phi_s^t = \phi_t = 0 \). The first condition indicates that \( x_r^t = 1 \), but the second that the contract does not specify a value for \( x_s^t \). This will be determined by the general equilibrium conditions and discussed in further detail below.

Summing the return on inside finance \( \epsilon \) and the surplus received by entrepreneurs set by the debt contract yields the total profits from entrepreneurial activity

\[ \pi_t = (1 - \lambda) p_t x_{t-1}^r (R_r^t - R_s^t) \kappa + (\lambda p_t x_{t-1}^s R_r^t + (1 - \lambda) p_t x_{t-1}^r R_s^t) \epsilon \]  
(4.3.22)

If a household does not lend funds, then the capital will not depreciate and the gross return will equal 1. Total rate of return on capital used as debt finance is given by

\[ R_t = 1 + \lambda \frac{x_{t-1}^s}{\gamma_{t-1}} (t_s^t - 1) + (1 - \lambda) \frac{x_{t-1}^r}{\gamma_{t-1}} (p_t^r \tau_r^t - 1) \]  
(4.3.23)

### 4.3.3 Aggregations

Each project requires \( \kappa \) units of debt finance and \( \epsilon \) units of internal finance. Furthermore, every entrepreneur has the internal finance irrespective of whether they receive debt finance. Capital is either allocated to external debt finance or internal equity finance and if there are \( e_t \) entrepreneurs per island, \( K_t \) units of capital in aggregate, and an efficient matching process, we have

\[ \gamma_t = (K_t - e_t \epsilon) \frac{1}{e_t \kappa} \]  
(4.3.24)

which is the maximum proportion of entrepreneurs that can get debt finance. Total labour supplied will equal that demanded, so

\[ (\lambda x_t^r h_t^s + (1 - \lambda) x_t^r h_t^r) e_{t-1} = 1 - e_{t-1} - u_t \]  
(4.3.25)
At each firm, labour demand is a linear function of local capital productivity

\[ h_i = \omega_i (\kappa + \epsilon) \left( A_t \frac{1 - \alpha}{W_t} \right)^{1/\alpha} \]  

(4.3.26)

and using this and letting

\[ \hat{h}_i = 1 - e_{t-1} - u_t \]  

(4.3.27)

\[ \hat{k}_i = (\lambda x_t^s + (1 - \lambda) x_t^r i \omega_t^r) (\kappa + \epsilon) e_{t-1}, \]  

(4.3.28)

the labour market clearing condition can be written

\[ W_t = (1 - \alpha) A_t \left( \frac{\hat{k}_i}{\hat{h}_i} \right)^{\alpha} \]  

(4.3.29)

Likewise, total output

\[ Y_t = A_t \hat{k}_t \hat{h}_t^{1-\alpha} \]  

(4.3.30)

Finally, we close the model with an aggregate resource constraint

\[ Y_t = C_t + I_t \]  

(4.3.31)

where investment is the difference between the new capital stock \( K_t \), and the sum of the depreciated returned capital, and the undepreciated, unused capital

\[ I_t = K_t + \delta (\lambda x_{t-1}^s + (1 - \lambda) x_{t-1}^r) (\epsilon + \kappa) e_{t-1} - K_{t-1} \]  

(4.3.32)

### 4.3.4 First-Best Economy

As a point of comparison we use the same model with the information asymmetry removed. As in the partial equilibrium model, the
first-best contract sets \( \tau_i = \tau_i = R_i \), with first order and Kuhn-Tucker conditions

\[
\lambda \mathbb{E}_t [\Lambda_{t+1} (p_{t+1}^s R_{t+1}^s - 1)] + q_t^s - \overline{q}_t^s - \lambda q_t = 0 \\
(1 - \lambda) \mathbb{E}_t [\Lambda_{t+1} (p_{t+1}^r R_{t+1}^r - 1)] + q_t^r - \overline{q}_t^r - (1 - \lambda) q_t = 0
\]

(4.3.33)

(4.3.34)

\[
q_t^r x_t^r = 0 \\
q_t^s x_t^s = 0 \\
\overline{q}_t^r (1 - x_t^r) = 0 \\
\overline{q}_t^s (1 - x_t^s) = 0 \\
\phi_t \left( \gamma - \lambda x_t^s - (1 - \lambda) x_t^r \right) = 0 \\
1 \geq x_t^s, x_t^r \\
x_t^s, x_t^r \geq 0 \\
\gamma \geq \lambda x_t^s + (1 - \lambda) x_t^r \\
q_t^r \geq 0 \\
q_t^s \geq 0 \\
\overline{q}_t^r \geq 0 \\
\overline{q}_t^s \geq 0 \\
q_t \geq 0
\]

(4.3.35) \hspace{1cm} (4.3.36) \hspace{1cm} (4.3.37) \hspace{1cm} (4.3.38) \hspace{1cm} (4.3.39) \hspace{1cm} (4.3.40) \hspace{1cm} (4.3.41) \hspace{1cm} (4.3.42) \hspace{1cm} (4.3.43) \hspace{1cm} (4.3.44) \hspace{1cm} (4.3.45) \hspace{1cm} (4.3.46) \hspace{1cm} (4.3.47)

These conditions also imply that \( \overline{q}_t^r \overline{q}_t^s = q_t^s \overline{q}_t^r = q_t^s q_t = q_t^r q_t = 0 \). As in partial equilibrium, if the expected discounted return from both projects are positive then \( q_t > 0 \) and all funds will be used. The project with the highest net present value will then be funded with probability 1, with all funds used to finance the lower value projects.

Under the first-best contract, the expected return on equity and debt are equal, and given that both are state contingent, the realised return is also equal. Given the general equilibrium set-up, debt is always preferred to equity as the latter requires the contribution of an entrepreneur so the opportunity cost is greater. One could solve this by making the debt-equity finance decision endogenous with an agency problem, taxes, or some other feature. To keep things simple, we assume in the first-best economy, the households are forced to provide \( \epsilon \) units of equity finance for every \( \kappa \) units of debt finance. This implies that \( \gamma_t = 1 \forall t \) and so \( x_t^s = x_t^r = 1 \) providing the expected project value is positive.
4.3.5 Efficiency and Investment Wedges

The credit friction emerges both as a wedge between the saving rate and the return to capital, and as an inefficient allocation of the factors of production. Chari et al. (2007) describe the first as the investment wedge and the second the efficiency wedge. To measure the investment wedge, we take the spread between the saving rate $R_t$ and the return to capital in safe projects in the model with adverse selection and the benchmark first-best economy. We then take the ratio which removes the spread component coming from possible differences in the expected value of projects. In steady state, the investment wedge is given by

$$
\frac{\lambda x^* + (1 - \lambda) x^* \rho \omega^r}{\lambda x^* + (1 - \lambda) (x^r (1 - \omega^r) + x^r \omega^r) \rho \gamma}
$$

(4.3.48)

where an asterisk indicates a first-best. The model with adverse selection has an additional term $(1 - \lambda) x^s (1 - \omega^r) \rho \omega^r < 0$ in the denominator, which given our assumption that $\omega^r > 1$, implies a larger spread even with the same finance probabilities. Considering first the model with equal expected value of projects we show the investment wedge in figure 4.5 for a range of values of $\rho$ for four different values of $\lambda$. The spread falls in $\rho$ as risky projects become safe and the average information rents fall. The spread increases in the risky project share, represented by a lower $\lambda$, as the total information rents increase.

The efficiency wedge emerges due to the misallocation of the factors of production. Total steady state output is given by

$$
Y = \left( \frac{\hat{h}}{\hat{k}} \right)^{1-\alpha} \hat{k}
$$

(4.3.49)

The capital-labour ratio differs from that in the first-best economy due to the investment wedge; a higher return from capital implies a lower capital-labour ratio. To measure the efficiency wedge then, we draw focus on $\hat{k}$ which is the efficiency units of capital used in

---

9 As discussed below, in the adverse selection model, we choose $\alpha$ to target an empirical debt-to-equity ratio; $\epsilon$ to match an empirical worker-to-firm ratio; and $\rho'$ to target a steady state value of $\gamma$. In the benchmark model for this comparison, we are just fixing $\gamma$ and $\rho'$, so the comparison just relates the the credit friction itself.
production. In the first-best economy, the lender finances the higher value projects with probability 1, with remaining funds financing the less productive projects. Weighting $\hat{k}$ by the value $\hat{k}^*$ that emerges from this, will give a measure of the efficiency wedge. This is written

$$\lambda x^s + (1 - \lambda) x^p p^r \omega^r$$

Note that we have ignored the ratio $\frac{\hat{k}}{x^r}$ and so this is the ratio of the efficiency units of capital per firm in the adverse selection and first-best models. This allows us to concentrate solely on the misallocation issue arising from the information problem. Allowing the expected value of projects to differ, can generate the efficiency wedge; this is shown in figure 4.6 for $\lambda$ fixed at 70%. The efficiency wedge is bound below at 1, as the measure is normalised by the first-best economy which features the same composition of project values but uses resources efficiently. A further increase in the efficiency wedge can occur if the return to risky projects were high enough, as the lender would stop lending to safe projects all together.

One interesting result highlighted by the figures is the marginal inefficiency of project risk, which is the inverse of the slope of the plots.
Figure 4.6: Steady state efficiency wedge against $p^r$ for a range of values of the relative expected value of the risky project, given by $\frac{R^r}{R^s} = \omega^r p^r$, with $\lambda$ fixed at 70%.

4.3.6 Monetary Policy

As part of the analysis, the impact of the adverse selection problem on the transmission mechanism of monetary policy is analysed. For this, we introduce a retail sector comprising firms that act under monopolistic competition, purchase what is now intermediate output from the existing firms, and produce differentiated final goods. We assume the price rigidity proposed in Calvo (1983) so that every period the retail firms face a fixed probability of being able to reset prices. The existing model will be adjusted to account for the importance of the price level; as this is a trivial procedure, we leave the amended expressions for appendix A.2. We let $P_{1,t}$ be the price level in the intermediate good sector, and $P_t$ be the price index of the final goods used in consumption and investment. The market power stems from the household preference function for final goods; it is assumed that households combine the differentiated goods into a bundle using

$$C_t = \left( \int_0^1 C_t(j) \frac{e^{-1}}{e^j} \, dj \right)^{\frac{\gamma}{\psi}}$$

(4.3.51)
where \( \sigma \) denotes the elasticity of substitution between the different varieties. The household purchases good \( C_t(j) \) from retailer \( j \in (0, 1) \) at price \( P_t \) to maximise \( (4.3.51) \) subject to total expenditure \( P_tC_t = \int_0^1 P_t(j)C_t(j)\,dj \), with an equivalent problem for investment demand \( I_t \). This leads to Dixit & Stiglitz (1977) demand schedules

\[
C_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\sigma} C_t \tag{4.3.52}
\]

\[
I_t(j^*) = \left( \frac{P_t(j)}{P_t} \right)^{-\sigma} I_t. \tag{4.3.53}
\]

Summing these demand schedules implies a total demand for good \( j \) given by

\[
Y_t(j) = \left( \frac{P_t^0(j)}{P_t^0} \right)^{-\sigma} Y_t \tag{4.3.54}
\]

Every period, each firm faces a fixed probability \( 1 - \zeta \) that they will be able to update their prices. Denoting the optimal price at time \( t \) for good \( j \) as \( P_t^* (j) \), the firms allowed to re-optimize prices maximise expected discounted profits by solving

\[
\max_{P_t^*} E_t \sum_{k=0}^{\infty} \zeta^k \frac{\Lambda_{t,t+k}}{P_{t+k}} Y_{t+k}(j) \left[ P_t^* (j) - P_{w,t+k} \right]. \tag{4.3.55}
\]

Substituting in the demand schedule, taking first-order conditions with respect the new price and rearranging leads to

\[
P_t^* = \frac{\sigma}{\sigma - 1} \frac{E_t \sum_{k=0}^{\infty} \zeta^k \Lambda_{t,t+k} (P_{t+k})^\sigma Y_{t+k} P_{w,t+k}}{E_t \sum_{k=0}^{\infty} \zeta^k \Lambda_{t,t+k} (P_{t+k})^\sigma Y_{t+k}} \tag{4.3.56}
\]

where the \( j \) index is dropped as all firms face the same marginal cost, so the right-hand side of the equation is independent of firm size or price history. We denote the real marginal cost as \( MC_t = \frac{P_t}{P_t^0} \), and the price inflation over the interval \( [t - 1, t] \) as \( \Pi_{t-1,t} = \frac{P_t}{P_t^0} \), we write the real optimal price

\[
P_t^* = \frac{\sigma}{\sigma - 1} \frac{E_t \sum_{k=0}^{\infty} \zeta^k \Lambda_{t,t+k} (\Pi_{t,t+k})^\sigma Y_{t+k} MC_{t+k}}{E_t \sum_{k=0}^{\infty} \zeta^k \Lambda_{t,t+k} (\Pi_{t,t+k})^\sigma Y_{t+k}} \tag{4.3.57}
\]

Denoting the numerator and denominator \( \Omega_{1,t} \) and \( \Omega_{2,t} \) we can write these in recursive form

\[
\Omega_{1,t} = \frac{\sigma}{\sigma - 1} Y_t MC_t + \zeta E_t \left[ \Lambda_{t,t+1} (\Pi_{t,t+1})^\sigma \Omega_{1,t+1} \right] \tag{4.3.58}
\]

\[
\Omega_{2,t} = Y_t + E_t \left[ \zeta \Lambda_{t,t+1} (\Pi_{t,t+1})^{\sigma - 1} \Omega_{2,t+1} \right]. \tag{4.3.59}
\]
Using the aggregate producer price index $P_t$ and the fact that all resetting firms will choose the same price, by the Law of Large Numbers we can find the evolution of the price index as given by

$$P_t^{1-\sigma} = \xi P_{t-1}^{1-\sigma} + (1 - \xi) P_t^{1-\sigma} \quad (4.3.60)$$

which can be written in the form required

$$1 = \xi (\Pi_{t-1,t})^{\sigma-1} + (1 - \xi) \left( \frac{P^{\ast}_t}{P_t} \right)^{1-\sigma} \quad (4.3.61)$$

where $\Pi_{t-1,t}$ is the gross inflation in the price of domestically produced goods between periods $[t-1, t]$. Whilst the distribution of prices is not required to track the evolution of the aggregate price index, equation 4.3.51 implies a loss of output due to dispersion in prices. Using the demand schedules, we can write the price dispersion that gives the average loss in output as

$$S_t = \frac{1}{J} \sum_{j=1}^{J} \left( \frac{P_t(j)}{P_t} \right)^{-\sigma} \quad (4.3.62)$$

for non-optimizing firms $j = 1, ..., J$. It is not possible to track all $P_t(j)$ but it is known that a proportion $1 - \xi$ of firms will optimise prices in period $t$, and from the Law of Large Numbers, that the distribution of non-optimised prices will be the same as in the overall distribution. Therefore price dispersion can be written as a law of motion

$$S_t = \xi \Pi_{t-1,t} S_{t-1} + (1 - \xi) \left( \frac{\Omega_{1,t}}{\Omega_{2,t}} \right)^{-\sigma}. \quad (4.3.63)$$

Using this, aggregate final output is given as a proportion of the intermediate output

$$Y_t = Y_{w,t} \frac{1}{S_t}. \quad (4.3.64)$$

To determine the path for prices, we introduce a monetary policy with an inflation target which we assume is credible, such that the private sector believe the economy will return to the inflation level in the long-run. The monetary authority sets a nominal policy rate using the rule

$$R_t^p = \left[ \hat{R}_t^p \left( \frac{\Pi_{t-1,t}}{\Pi^*} \right)^{\eta^*} \left( Y_t \right)^{\eta_y} \right]^{1-\eta} (R_{t-1}^p)^{\eta} \exp(\varepsilon_{M,t}), \quad (4.3.65)$$
where \( \epsilon_{M,t} \sim \mathcal{N}(0, \sigma_M) \) is a monetary policy shock. The policy rate, \( R^p_t \), is applied to a one period bond issued to households. In equilibrium no bonds will be issued, but the availability will set the expected, discounted nominal return on capital via the zero arbitrage condition

\[
E_t \left[ \frac{\Lambda_{t+1}}{\Pi_{t+1}} R^p_{t+1} \right] = E_t \left[ \frac{\Lambda_{t+1}}{\Pi_{t+1}} R^p_t \right] = 1
\]

(4.3.66)

where now \( R_t \) is a nominal return.

A further modification added before analysis of the transmission mechanism of monetary policy, so to draw closer comparison to the New-Keynesian literature, is costs in adjusting investment. We follow the method proposed in Christiano et al. (2005). Full derivation is not shown here but the resulting first order conditions are listed in appendix A.2.

### 4.3.7 Shocks

To evaluate the model, we consider three possible exogenous transitory shocks to the economy; a total factor productivity shock, a risk shock, and an efficiency shock. The first needs little introduction and allows comparison with the real business cycle literature with total factor productivity given by

\[
\log A_t = \rho_A \log A_{t-1} + \epsilon_{A,t}, \quad \epsilon_{A,t} \sim \mathcal{N}(0, \sigma_A)
\]

(4.3.67)

A positive risk shock decreases \( p^r_t \) whilst keeping \( p^r_t R^r_t \) constant and an efficiency shock is the opposite; changing the return on a risky project whilst keeping the risk constant. The former relates the analysis to a literature looking at second moment shocks (e.g Christiano, Motto & Rostagno 2013) with \( p^r_t \) given by

\[
\log p^r_t - \log \bar{p}^r = \rho_p (\log p^r_{t-1} - \log \bar{p}^r) + \epsilon_{p,t}
\]

(4.3.68)

with \( \epsilon_{p,t} \sim \mathcal{N}(0, \sigma_p) \), and where the logit function imposes bounds \( p_t^r \in (0, 1) \). The efficiency shock relates the analysis to the (Chari et al. 2007) that assigns a significant role to the efficiency wedge in generating the business cycle; to introduce the efficiency shock, we let \( v_t \equiv \omega_p^p p_t^r \) denote the relative value of risky projects subject to a stochastic process given by

\[
\log v_t = \rho_v \log v_{t-1} + \epsilon_{v,t}, \quad \epsilon_{v,t} \sim \mathcal{N}(0, \sigma_v)
\]

(4.3.69)
4.3.8 Parametrisation and Calibration

We choose the steady state expected return on risky projects to equal the return on safe projects so $p' \omega^r = 1$, $p' R_r = R^s$. In addition to the parameters common to the RBC literature, we are left with several parameters specific to the adverse selection economy. We calibrate $\lambda$ and $p'$ to target a loan default rate and a lending rate spread. For the former, we target a value of 2.8%, taken from the average delinquency rate on commercial and industrial loans over the period 1987Q1 – 2016Q2. A steady state spread between the lending rates $\tau^r - \tau^s$ of 0.9897% is targeted; this is the average spread between the yields of Moody’s BAA and AAA rated corporate bonds over the period 1986Q1 – 2016Q3. In a stylized model such as this, it can prove difficult to provide evidence in support of some parameters; we do however look to fit U.S. firm and employment data as best as possible.

In the U.S. non-financial firm sector, there is significant difference in financial structure across sectors, and across firm size and age. For a simplified representative framework we look at the aggregate book debt to capital ratio which in the U.S., at the start of 2016, was estimated across 7480 firms at 62.63%. We therefore choose $\kappa$ to target $\hat{\kappa} = \frac{\kappa}{\kappa + \epsilon} = 0.6263$. To pin down the average size of the firm, we use data from the Statistics of U.S. Businesses (SUSB) which indicates that there were 7.5 million establishments with a total employment of 118 million in 2013, implying there were 15.8 workers per establishment; we choose $\epsilon$ to match $\hat{\omega} = \frac{1 - \epsilon - u}{\epsilon} = 15.8$. These calibrations are listed in table 4.1 For parameters common to the RBC literature,

---

11 Board of Governors of the Federal Reserve System (US), Moody’s Seasoned Baa Corporate Bond Yield,[BAA] and Moody’s Seasoned Baa Corporate Bond Yield,[AAA], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/BAA, September 5, 2016.
13 It is naturally impossible to fit the diverse distribution of firm type and size into a representative framework. There are a large number of smaller business such as sole traders ignored in these numbers but we catch the majority of U.S. workers.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>Share of safe projects</td>
<td>0.8091</td>
<td>$\frac{\lambda(1-p')}{\gamma} = 0.0280$</td>
</tr>
<tr>
<td>$p'$</td>
<td>Risky project success probability</td>
<td>0.8603</td>
<td>$\tau' - \tau^d = 0.00990$</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Required debt finance for project</td>
<td>102.46</td>
<td>$\hat{k} \equiv \frac{\kappa}{\kappa + \epsilon} = 0.6263$</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Required equity finance for project</td>
<td>61.14</td>
<td>$\bar{w} \equiv \frac{1 - \epsilon - \kappa}{\epsilon} = 15.8$</td>
</tr>
</tbody>
</table>

Table 4.1: Calibrations of adverse selection model parameters. $\kappa$ and $\epsilon$ are in units of capital, with the targets of the representative financial structure and firm size providing the intuition to the meaning of the values.

we choose values typically used to ensure a useful comparison and shown in table 4.2. We calibrate the shock variance to match second and third moments in aggregate output; this is discussed further below in section 4.5.3.

Some robustness checks of the parametrisation were carried out on both the implied deterministic steady state and the model dynamics. The latter is discussed in the numerical analysis below. The choice of time discounting, and capital share and depreciation are standard and we focus on the novel parametrisation, beginning with their impact on the steady state equilibrium. The number of workers per firm, $\bar{w}$ is an endogenous variable, but this and the debt-equity ratio are chosen to calibrate $\epsilon$ and $\kappa$ where $\epsilon + \kappa$ is the capital per firm. Adjusting $\bar{w}$ up (down) from 15.8 implies a lower (higher) population

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Capital share of production</td>
<td>0.3</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Household discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation rate</td>
<td>0.023</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Utility share of labour</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 4.2: Parametrisation of common real business cycle parameters.
share of entrepreneurs and so less (more) firms. There is not a large effect on the steady state equilibrium conditions, except, with $\gamma$ fixed, reducing (increasing) $\hat{w}$ reduces (increases) the likelihood\footnote{Assessing the steady state across a range of parametrisations.} that all funds are lent, when $\varrho > 0$ and $\varphi' > 0$; and increases (reduces) the likelihood that safe projects have restricted access to credit markets, that is with $\varrho = 0$ and $\varphi' > 0$. There is a similar effect of adjusting the debt-equity ratio; increasing (reducing) the share of credit in the economy by reducing $\hat{\kappa}$, increases (reduces) the likelihood that safe borrowers face credit restrictions. The effect disappears for a given $p'$, and so whilst having both debt and equity play an important role to ensure an interior solution to the occupational allocation, the exact composition does not have a large impact on steady state values.

As would be expected given their role in the optimal contract, the values of $p'$ and $\lambda$ do have a significant impact on steady state conditions as well as model dynamics. These two parameters are calibrated to target a loan delinquency rate and lending spreads. Setting $\lambda$ higher implies that risky projects occupy a smaller share of lending opportunities and it shrinks the proportion of the surplus that is received by risky entrepreneurs as information rents. As shown in figure A.1 in appendix A.4, this implies a higher value of $\varrho$, so that the lender is less likely to restrict credit to safe borrowers; and a lower value of $\varphi'$ so the lender is more likely to choose a pooling equilibrium. For a given value of $\gamma$, a higher value of $p'$, which indicates safer risky projects implies the opposite: a lower value of $\varrho$, so that the lender is more likely to restrict credit to safe borrowers; and a higher value of $\varphi'$ so the lender is less likely to choose a pooling equilibrium.

\subsection{Numerical Strategy}

To evaluate the model dynamics, we compute second and third order pruned perturbation approximations to the model transition and decision functions, using the algorithm proposed in Holden (2016a) to implement the inequality constraints. The second order approximation is used to compute simulated moments, and the third order for impulse response functions. A description of these methods is given
in section 2.2 of the methodology chapter of this thesis and so we provide only relevant detail here. There are five inequality constraints that must be considered; positivity constraints on the four Lagrange multipliers, $\varrho_t$, $\varphi_t^s$, $\varphi_t^r$ and $\psi_t$ and an upper bound $\gamma \leq 1$, which is the proportion of entrepreneurs that can receive funding.

Recall that the bounds problem is to compute the sequence of news shocks required to impose the constraints, and in order to account for the effects stemming from the risk of hitting the bounds in the future, we integrate over a finite horizon in a similar manner to the stochastic-extended path method proposed in Adjemian & Juillard (2013). We use different integration methods for each exercise where, in each case, the choice is made to achieve solutions in reasonable time at as high an accuracy as possible. To compute the simulated time-series, we employ a second order approximation and use a degree 3 monomial rule without negative weights to integrate over future uncertainty up to a horizon of $S = 16$ periods. As the monomial rule evaluates the integral far from the origin, it can introduce some bias, but the method is chosen for its robustness and speed.

For the impulse response functions, with the exception of the monetary policy shock, we compute a third-order approximation and use a Gaussian cubature rule with $O(\hat{S}^K)$ points and maximum monomial degree of $7 = 2K + 1$. Two techniques are employed to increase computational speed; firstly, an adaptive cubature degree is used, implying a lower degree is used when far from the bound. Secondly, as discussed in section 2.2, the eigenvalues of the covariance matrix of the distribution from which we integrate smaller than 1% of the largest eigenvalue are set to zero. $\hat{S} \leq S$ is the number of remaining non-zero eigenvalues. We compute average impulse response functions around the ergodic mean and whilst a standard perturbation without bounds requires Monte-Carlo simulation to compute the average, the pruning technique allows analytical moments to be calculated. This implies a Monte-Carlo simulation is not required in the absence of the bounds, but with bounds, although we integrate out the effects of uncertainty of future news shocks, we miss current uncertainty. To remedy this, we use a Monte Carlo simulation that will capture these effects.
Examining the matrix $M$, which contains relative impulse response functions to the news shocks as defined in section 2.2, indicates that there are some states of the world where there are either multiple solutions or no solution to the bounds problem. We found that there is always a solution in the vicinity of the steady state, and when there are multiple solutions, we choose the one that minimises the size of news shocks. Because the impulse response functions shown in the section below follow large shocks, the model moves into a region relatively far from the steady state. We found that computing the impulse response functions with the calibrated shock standard deviations would cause the computation to fail; this stems from integrating over an area of the state space where there is no solution to the bounds problem. To deal with this, at some loss of accuracy, the shock standard deviations are scaled down.\footnote{We choose $\sigma_a = 0.001$, $\sigma_p = 0.01$ and $\sigma_v = 0.001$, all smaller than the calibrated values in table 4.3.}

For the monetary policy shock, the introduction of the Calvo price setting mechanism and investment adjustment costs increase the number of state variables; the number of forward looking variables jumps from 4 to 9. To compute impulse response functions, we therefore use a Monte Carlo simulation but without cubature. Although introducing some bias by ignoring the precautionary effects stemming from the bound, in earlier tests, we found that Monte-Carlo delivers reasonable accuracy by computing expectations of the impulse response functions that include the expected sequence of news shocks required to impose the bounds.

4.5 Discussion of Numerical Results

We begin the discussion of the numerical results with analysis of the impulse response functions to the three modelled shocks. Following the positive transitory shock to aggregate productivity, in the first-best economy there is an increase in output and investment, a rise in employment as well as an increase in the number of entrepreneurs seeking funds. As shown in figure 4.7, in the model with adverse selection, the shadow value of lending opportunities, $\varrho_t$, increases on
Figure 4.7: Average impulse response functions to a positive transitory shock to total factor productivity $A_t$ of approximately 1.5%. Plots show relative deviation around zero for $u$ and $e$, and level deviation around zero for all other variables except $\varrho$ which is around the ergodic mean.

Impact then overshoots as it falls before returning to the steady state. For shocks of a typical size, the only impact on the real economy is to increase the volatility of investment. The value of equity increases relative to debt in the adverse selection model following the shock because entrepreneurs receive all the gains in the returns on equity but the additional surplus from the debt contract is shared. This leads to a larger relative increase in entrepreneurs and initial investment in capital for equity finance. We can be slightly more specific about
what is meant by ‘of a typical size’. If the shock is sufficiently large, and under chosen parametrisations, this is of the order of a transitory 12\% increase, or 8\% decrease to total factor productivity; \( \varrho_t \) will fall to zero causing the feasibility constraint on lending to slacken, and if the shock is larger still, \( \gamma_t \) increases to the upper bound of 1. At this point we observe some significant non-linear effects and by saying typical size, we include a very large interval of the probability distribution of \( A_t \).\(^{16}\)

When the \( \varrho_t \) at the zero lower bound and \( \gamma_t \) increase to 1, there is no equivalent increase in \( x^s_t \). This represents the under-utilization of capital as the cost in information rents to risky borrowers would exceed the gain in additional surplus earned by increasing lending to safe entrepreneurs. This causes an increase in the spread between the gross rate of return on lending \( R_t \) and the average return on capital, denoted

\[
\Delta_t \equiv E_t \left[ \frac{\lambda x^s_t + (1 - \lambda) x^r_t p^r_{t+1} \omega^r_{t+1} \lambda^r p^r_{t+1}}{\lambda x^s_t + (1 - \lambda) x^r_t \omega^r_{t+1} \lambda^r p^r_{t+1}} R^s_{t+1} - R_{t+1} \right].
\]

(4.5.1)

For instance, \( \varrho_t \) can hit the zero constraint following a negative productivity shock as shown in figure 4.8, where the fall in the number of entrepreneurs dominates the fall in the number of loans causing \( \gamma \) to increase to 1. The shadow value of lending, \( \varrho_t \), falls to zero which causes the proportion of funded projects to actually decline. This leads to a very large drop in investment. In this case we find there is a sharp increase in the interest spread \( \Delta_t \) on impact, followed by a hump-shaped increase. Here we introduce a variable \( \zeta \) that is a measure of the efficiency wedge given by

\[
\zeta_t \equiv \gamma_t \frac{\lambda + (1 - \lambda) \omega^r_p p^r}{\lambda x^s_t + (1 - \lambda) x^r_t}.
\]

(4.5.2)

As this is multiplied by \( \gamma_t \) it is simply a measure of the inverse average firm productivity relative to the first-best case. When \( \varrho_t = 0 \), the lender leaves a proportion of capital unallocated which drives the observed sharp increase in \( \zeta \). We purposefully define \( \Delta_t \) using the rate of return on all lent funds, rather than the gross return on all capital so to separately identify the effects of the efficiency wedge \( \zeta_t \) which

\(^{16}\) For example, a negative shock large enough to cause \( \gamma_t = 1 \) under the estimation of Smets & Wouters (2007) would be a 5 standard deviation shock and even higher for a positive shock
134 adverse selection and the efficiency wedge

Figure 4.8: Impulse response functions to a large (10%) negative transitory shock to total factor productivity $A_t$. Plots show relative deviation for $Y$, $I$, and $\Delta$, and are in levels for the other variables.

focuses on the misallocation of capital. Further impulse responses to a larger number of variables for shocks of different signs and magnitudes are shown in appendix A.4.

4.5.1 Risk Shock

An increase in the risk of risky projects, caused by a decline in $p^r_t$ with $\omega^r_t$ kept at $1/p^r_t$, generates economic fluctuations in the adverse selection economy whilst leaving the first-best economy unaffected. In
the first-best case, the only important factor regarding the financing of projects is the discounted expected value, which does not change in the face of a risk shock. With the asymmetric information problem, the increased risk leads to higher information rents to the entrepreneur and so a higher value of equity and lower value of debt on average. As shown in figure 4.9, this causes the shadow value of lend-

![Figure 4.9: Impulse response functions to a transitory risk shock caused by a reduction in the probability of risky project success $p_r^t$ of approximately 2 percentage points. Plots show level deviation around zero except for $x^s$ and $\varrho$ which are around the ergodic mean.](image)

ing opportunities, $q_t$ and the safe project finance probability $x^s_t$ to both decline, the latter caused by a drop in the availability of loans $\gamma_t$. The reduction in lending produces a decline in output and investment,
and the number of entrepreneurs and level of employment. Quantitatively, under the chosen parametrisations, a 1 percentage point drop in the rate of risky project success leads to a 3% decline in investment, a 0.1 percentage point drop in employment, and a 9 basis point increase in the interest spread. The adverse selection introduces significant non-linearities; for instance, a 2 percentage point drop in \( p_t \) causes \( \varrho \) to hit the zero lower bound leading to an increase in the efficiency wedge as lenders do not lend all available capital. The efficiency wedge rises to about 5% of potential output. The impact on other variables also becomes more severe: there is approximately a 15% decline in investment on impact; a 0.4 percentage point fall in employment; and a 30 basis point increase in the spread. The impulse responses to the 1, 2 and 3 percentage point drops in \( p_t \) are shown in figures A.5 – A.7 in appendix A.4.

This result is sensitive to the parametrisation of \( \lambda \). Increasing \( \lambda \) to 0.85 from 0.8, so reducing the proportion of risky projects to 15% from 20%, does not have a large impact on the marginal effect on the real economy close to the steady state, but it shifts the region of the state space that \( \varrho \) is at the zero bound causing the lender to restrict funding. For example, with the alternative parametrisation, following a 2 percentage point drop in \( p_t \), \( \varrho \) falls by about 60% from the steady state value, and lending is unrestricted, investment declines by about 6%. In fact, it requires a drop in \( p_t \) of about 4% for \( \varrho \) to become constrained at zero to aggravate what could be considered a ‘financial crisis’. The impulse responses are shown in figures A.9 and A.10. The key result to highlight is that by reducing the number of risky projects by 5 percentage points, the drop in \( p_t \) needs to be 2.5 percentage points greater to lead to a financial crisis.

4.5.2 Efficiency Shock

The efficiency shock is called such because of the way it manifests in the adverse selection economy, although it is more precisely a risky project productivity shock. For small shocks, the impact is greater in the first-best economy; in the adverse selection economy, the change

\[ \text{This is a level deviation of log } I_t \text{ of about } 0.15 \text{ units.} \]
in the value of equity is greater than the change in the value of debt and so there is a relative increase in capital allocated to debt and so a rise in the proportion of projects funded. This acts as a buffer to the adverse effects of the shock not present in the first-best economy. An adverse efficiency shock will reduce the shadow value of risky lending opportunities, $\psi^r$, and will be constrained at zero if the expected risky project value falls by more than 5%, leading to a pooling equilibrium. This is shown in figure 4.10. The pooling occurs when

![Impulse response functions](image)

Figure 4.10: Impulse response functions to a transitory efficiency shock caused by a reduction in the productivity of risky projects $\omega^r$ by 5% whilst keeping probability of success, $p^r$ constant. Plots show level deviation around the ergodic mean except for $\zeta$ and $\Delta$ which are both around zero.
the Lagrange multiplier on the constraint \(x_t^s \leq x_t^r, \psi_t\) rises above zero. This occurs when the expected value of risky projects has deteriorated to the point that increasing lending to safe projects has higher value than the share of information surplus received from risky projects. The shock leads to an increase in the efficiency wedge, \(\zeta_t\) and drop in the investment wedge \(\Delta_t\); the former due to the risky projects receiving finance whilst there remains unfunded, higher value safe projects, and the latter due to the relative increase in the value of debt over equity. Whilst adjusting the share of risky projects has an impact on the fall in output and the efficiency wedge, as the average productivity of projects will differ, the size of shock sufficient to lead to a pooling equilibrium is virtually unchanged. Impulse responses with \(\lambda = 0.85\) are shown in figure A.19.

We find that the adverse selection economy response to the efficiency shock is particularly asymmetric. When the shock to the productivity of risky projects is positive, in the first-best economy, at least for small shocks, the impulse responses mirror those for negative shocks. For risky project productivity shocks smaller than 1%, the same is true for the adverse selection economy, but for larger shocks, the lender begins to restrict credit to safe projects; this is shown in figure 4.11. The marginal value of lending, \(\varphi_t\) is constrained at zero and the lender begins to reduce debt finance to safe projects so to receive a greater share of the additional risky project surplus. This increases the value of debt relative to equity, and so there is a fall in the number of entrepreneurs and an increase in the amount of debt finance. Although \(\gamma_t\) rises, the lenders begin to restrict credit to safe projects and so the efficiency wedge increases due to this non-allocation of capital. For a 1.5% increase in the productivity of risky projects under the chosen parametrisation, there is a 2% increase in the efficiency wedge, a 25 basis point increase in the interest spread, and a 0.5% fall in output. As with the risk shock, these movements are sensitive to the choice of \(\lambda\). A higher \(\lambda\) corresponds with a higher marginal value of lending, \(\varphi_t\) and so a larger shock is needed to cause this ‘financial crisis’. If \(\lambda = 0.85\) instead of 0.8, a shock of approximately 3% is required for \(\varphi_t\) to reach the zero lower bound.

\[\text{18 There will inevitably be some asymmetries introduced at approximation orders greater than 1.}\]
4.5 Discussion of Numerical Results

Figure 4.11: Impulse response functions to a transitory efficiency shock caused by an increase in the productivity of risky projects $\omega^r_t$ by 1.5% whilst keeping probability of success, $p^r_t$ constant. Plots show level deviation around the ergodic mean for $\varrho_t$, $\gamma_t$ and $x^s_t$, and around zero for the other variables.

4.5.3 Simulated Moments

The model is simulated over 1000 periods using the extended-path type approach discussed above, and moments computed. The simulation is used to calibrate the productivity shock standard deviations; whilst other parameters were chosen without formal calibration the impact of the choice of these values is discussed here. Table 4.3 shows
the calibrations with the associated moments and output correlations under four separate environments. Firstly, and by way of comparison, we include just the productivity shock calibrated to target the standard deviation of output in the first-best economy. This is repeated but to match the simulated time series of output in the adverse selection economy. As technology shocks within a typical range are not sufficient to produce financial crises in the adverse selection economy, the results of the two economies are relatively similar. As was discussed above, the adverse selection economy generates higher volatility in investment which also causes higher volatility in consumption; as shown in the table of results, smaller shocks are required to fit the empirical output standard deviation. The adverse selection model does improve on the cyclicality of the loan rate spread, achieving the correct sign even though implying a far higher correlation with output than is observed in the data. Across the simulated time series with just the productivity shock, the negative skewness in output, investment and consumption and strong positive skewness in the spread is missed entirely, and the volatility in the interest rate spread is widely under-predicted, capturing only the time varying risk premium.

To rectify these issues, the third and fourth environments also calibrate the risk and efficiency shocks respectively to target output skewness, with the other shock set to zero. This exercise shows how introducing adverse selection in credit markets can improve the empirical fit of a typical RBC model by capturing the negative skewness in the time series of output and investment. The model that only includes shocks to total factor productivity and the productivity of risky projects does an especially good job of matching some key moments in the U.S. time series, in particular, the second and third moments of output and investment, and the correlation of investment and consumption with output. The risk and efficiency shocks also generate volatility in the lending rate spread in the adverse selection economy although miss the very high positive skewness. Both higher risk and efficiency shock variance leads to lower interest spread skewness and reduce the correlation with output; the simulations with the risk shock find a very close match for the correlation with output.
Table 4.3: Calibration of risk shock variance to target second and third moments of output. Five different model environments calibrated: (i) first-best economy with only productivity shocks; (ii) adverse selection economy with only productivity shocks; (iii) adverse selection economy with productivity and risk shocks; (iv) adverse selection economy with productivity and efficiency shocks. Data is HP-filtered U.S. time series 1986Q2 – 2015Q4; spread Δ as Moody’s AAA-BAA rated corporate bond yield spread. Further details are given in chapter 2. Standard deviations are in percent for Y, I and C and percentage points for Δ.
4.6 MONETARY POLICY

To study the effects of the adverse selection problem on the transmission mechanism of monetary policy, price setting frictions and investment adjustment costs are introduced into the model. The revised equilibrium conditions are outlined in appendix A.2, and the parametrisations in appendix A.3. The impulse response functions to a 10 basis point negative monetary policy shock are shown in figure 4.12. The credit friction has a limited effect on the transmission of the monetary policy shock compared to the first-best economy; the only difference is a small accelerator caused by a reduction in the spread, \( \Delta_t \). Cutting the policy rate reduces the nominal return on capital, but increases private sector demand and so too the price index and the demand for investment goods. There is an expansion in aggregate credit, in the number of entrepreneurs, and in employment. Even though this model does not focus on the balance sheet channel, the

![Figure 4.12: Impulse response functions to a negative 10 basis point monetary policy shock. Plots show level deviation around the ergodic mean for \( \varrho_t \) and \( \varphi'_r \), and around zero for all other variables.](image)
The relative value of equity experiences a small decline due to the cost of diverting potential workers into entrepreneurship; this increases the availability of loans, \( \gamma_t \), and, because risky projects are already fully funded, reduces the share of risky loans as new safe projects are funded, decreasing the average entrepreneur surplus. We can note that when \( \varphi_t > 1 \) and \( \psi_t = 0 \), the spread as defined above can be written

\[
\Delta_t = \mathbb{E}_t \left[ \left( 1 - \lambda \right) \frac{x_s^t}{\gamma_t} \rho_{t+1}^r \left( \omega_{t+1}^r - 1 \right) R_s^{t+1} \right]
\]

(4.6.1)

so falls in \( R_s^{t+1} \).

For a positive shock of the same size, the plots would be an approximate mirror image; around the ergodic mean, the transmission mechanism is largely unaffected and has symmetric effects on the economy. As can be seen in figure 4.12, \( \varphi_t \) and \( \varphi_t^r \) both decline; for a positive monetary policy shock, these increase and so move further from their bounds. Consequently, the impulse response functions to a positive shock increase close to monotonically in the shock size, whereas negative shocks can generate significant non-linear responses as the Lagrange multipliers approach the bounds. To understand the underlying cause, let us re-examine the contract first-order conditions. When \( \psi_t = \varphi_t^s = 0 \), we can write

\[
\varphi_t = \mathbb{E}_t \left[ \frac{\Lambda_{t+1}}{\Pi_{t+1}} \left( 1 + \frac{1}{\lambda} \rho_{t+1}^r \left( 1 - \omega_{t+1}^r \right) \right) R_s^{k, t+1} \left( 1 - 1 \right) \right]
\]

(4.6.2)

So, as a first-order effect, \( \varphi_t \) decreases as the expected gross return on capital \( R_s^{k, t+1} \) falls, occurring because the borrower share of risky project surplus increases as \( x_s^t \) increases\(^\text{19}\). At the point at which \( \varphi_t = 0 \), it is optimal for the lender to restrict credit to safe projects so to extract information rents from risky borrowers. Figure 4.13 compares the impulse responses of a selection of variables to three monetary policy shocks of different sizes that capture these non-linear effects. When \( \psi_t \) reaches the zero bound, following a cut of about 40 basis points, the amount of funding available for debt finance, \( \gamma_t \), increases whilst \( x_s^t \) is unchanged, leaving a share of capital unallocated. This causes the decline in investment seen in the centre column of figure

\(^{19}\) See equation (4.2.7) which gives the risky borrower lending rate. The borrower share of surplus is given by \( \left( \omega_t^r - 1 \right) \frac{x_s^t}{\gamma_t} \).
Figure 4.13: Impulse response functions to negative monetary policy shocks from the ergodic mean. First column is a 30 basis point cut; second column a 40 basis point cut; third column a 60 basis point cut. Plots show level deviation around the ergodic mean for $\rho_t$, and around zero for all other variables.
Further restrictions to safe project funding would otherwise act to decrease the information surplus to risky borrowers, which can be written as a share of project surplus as \((\omega_t^r - 1) \frac{x_{t-1}}{x_{t-1}}\). Because of these two competing effects, the adverse effects weaken for larger negative monetary policy shocks, and whilst investment is reduced relative to the first-best economy, the increase in the efficiency wedge is limited; under the chosen parametrisation, the limit is 5% which is a measure of the drop in aggregate capital efficiency.

4.7 CONCLUSION

In this chapter we have shown that asymmetric information in markets for business loans can lead to adverse selection that manifests as both an investment wedge and an efficiency wedge. Although a stylized model, this has allowed us to match the observed negative skewness in the empirical output time-series and, due to non-linearities in financial contracts, generate occasional financial crises arising from sharp spikes in the efficiency wedge. These results are an important contribution to the financial frictions literature which typically assumes that the economy is always financially constrained and subject to an investment wedge that depends on the balance sheet of the borrower. Firstly, under the standard approach, there are usually insufficient non-linearities to produce the type of financial crisis episodes observed in the data; by focusing on potentially important non-linearities present in financial contracts, we have been able to propose a model that, when thresholds in optimal contracts are breached, pushes the economy into a financial crisis. The second important result is that by abstracting from the balance sheet channel and focusing on the misallocation, or under-allocation, of productive resources, we have been able to respond to the issue highlighted in Chari et al. (2007), that the investment wedge has not been an important driver of the business cycle.

Under the chosen parametrisation, we have found that a two percentage point increase in the risk of risky projects, defined as a two percentage point decline in the probability of project success whilst keeping the expected value constant, is sufficient to generate financial
crisis as lenders begin to restrict credit to safe projects so to reduce the information surplus due risky borrowers. This result is sensitive to the population share of risky borrowers, for instance, requiring over twice as large a risk shock to cause financial crisis with 25% less risky projects.

The optimal contract is always separating when risky and safe projects have equal expected value, but pooling can emerge when the expected value of risky projects declines sufficiently. Under the chosen parametrisation, we find that a 5% drop in expected value leads to a pooling equilibrium. Output and investment increase, but the marginal value of lending falls as the productivity of risky projects rises. This may seem counter-intuitive, but it stems from rising information surplus received by risky borrowers reducing the value of debt relative to equity. A financial crisis episode occurs following a shock that increases the productivity of risky projects by over 1%; at this point, the marginal value of lending is constrained at zero and the lender restricts funds to safe projects in order to reduce the information surplus.

Finally, we have shown that the transmission of monetary policy is largely unaffected in the vicinity of the ergodic mean, except for a small accelerator effect, but for larger unexpected cuts in the rate (greater than 30 basis points at the chosen parametrisations) some non-linear effects are introduced. For cuts of 30-40 basis points, an efficiency wedge emerges due to the non-allocation of capital, reaching a size of 5% which, in this case, relates to the proportion of capital not allocated to productive projects.
In this chapter we show how interbank market frictions can play an important role in propagating and enhancing the effects of shocks in a currency union, and discuss the efficacy of two unconventional policy measures; multi-period central bank refinance operations and large scale asset purchases. To this end, a two-country DSGE model with idiosyncratic risk and country-specific transactional costs on interbank lending is proposed and used to show that (i) the effectiveness of monetary policy is enhanced when banks face an external finance premium in the interbank market; (ii) adverse shocks to the real economy can be the source of banking crisis, causing an increase in the interbank finance premia, aggravating the initial shock; and (iii) asset purchase policies and long-term refinancing operations can both be successful in supporting conventional monetary policy in mitigating the adverse effects of shocks.

The chapter includes research from ongoing joint work with Tobias Blattner\(^1\) at the European Central Bank.

### 5.1 Introduction

In the years following the global financial crisis of 2007-2009, the process of financial integration in the euro area moved into reverse as firms and households in the southern European periphery faced much higher borrowing costs than for those in the northern core. One of the key channels at the heart of this financial fragmentation was the interbank market, as the costs of cross-border lending costs rose sharply, and the volumes fell dramatically, especially to periphery borrowers. In the standard open-economy model with complete

\(^1\) Contact Tobias.Blattner@ecb.int.
markets, rather than contributing to financial fragmentation, a cross-
border interbank market will smooth asymmetric economic volatility,
and temper the adverse effects of asymmetric shocks. This chapter
analyses the role of frictions in the interbank market of a currency
union, and the transmission of conventional and unconventional pol-
icy. To this aim, we develop, calibrate and simulate a two-country
dynamic stochastic general equilibrium (DSGE) model in which lend-
ing banks obtain funds from both domestic and foreign savings banks
to refinance loans to the private sector, but where interbank lending
is subject to both borrower-specific idiosyncratic risk as borrowing
banks may default on their loans as well as to a country-specific risk
premium. Using this framework, we show that (i) the effectiveness
of monetary policy is enhanced when banks face an external finance
premium in the interbank market; (ii) adverse shocks to the real econ-
omy can be the source of banking crisis, causing an increase in the
interbank finance premia, aggravating the initial shock; and (iii) asset
purchase policies and long-term refinancing operations can both be
successful in supporting conventional monetary policy in mitigating
the adverse effects of shocks.

The interbank market plays a pivotal role in the Euro Area. Its smooth
functioning is central for banks to cope efficiently with idiosyncratic
liquidity shocks and to ensure a uniform transmission of the com-
mon monetary policy. Frictions in the interbank market may blur the
signal coming from monetary policy and ultimately hamper its trans-
mission. One reason why interbank markets may not operate effi-
ciently has to do with transaction costs: owing to the unsecured lend-
ing nature of the market, and its over-the-counter (OTC) structure,2
trading relationships are often plagued by asymmetric information,
counterparty risk and search and monitoring costs (see e.g. Afonso
et al. 2011, Flannery 1996). That is, banks are subject to frictions in
raising funds in the interbank market and may face borrowing con-
straints, which could affect both credit supply and the ultimate bor-
rowing conditions of the non-financial sector. As a result, imperfect
interbank markets make the policy interest rate insufficient to charac-
terize the monetary policy stance.

2 Electronic trading accounted for less than 10% of total unsecured transactions in
2014 (Euro money market study 2014).
These frictions are particularly relevant in cross-border transactions, where differences in banking supervision, the state of the business cycle or accounting standards may obfuscate the evaluation of credit-worthiness of foreign banks and expose lenders to significant counter-party risk. Freixas & Holthausen (2005) show that such market imperfections may cause liquidity shortages or the payment of interest rate premia that reflect the adverse selection of borrowers across countries. In crisis times, these effects may become even more visible. Using bank-to-bank loan level data from TARGET2, Abbassi et al. (2014) find that for the same borrower on the same trading day, and after controlling for lender and borrower fixed effects, cross-border loans were up to 25 basis points more expensive than domestic loans in the first three months following the collapse of Lehman. The presence of a risk premia unrelated to the specific borrower in crisis times suggests that information asymmetry constraints are important and that factors other than direct counter-party risks may also drive pricing behaviour in interbank markets.

Cross-border interbank lending has been an important element of the financial structure in Euro Area. Prior to the outbreak of the global financial crisis, more than half of the average daily turnover in the unsecured market was with non-domestic Euro Area counterparts. Strong credit growth in parts of the Euro Area, buoyant financial innovation and lax financial regulation all contributed to an increasing reliance on confidence-sensitive wholesale funding, with banks in current account surplus countries providing funding to banks in current account deficit countries (see van Rixtel & Gasperini 2013). After the outbreak of the crisis, the share of cross-border interbank lending fell dramatically to just over 25% in 2013 before recovering again to some 40% in 2014. de Andoain, Hoffmann & Manganelli (2014) estimate the premium charged to banks in more stressed economies spiked dramatically reaching over 63 basis points. This was especially important as those banks relying more heavily on wholesale markets for debt finance had to restrict lending to the private sector relative to those banks more dependant on household deposits (see Cornett 3 A more unified approach to supervision was introduced with the launch of the Single Supervisory Mechanism (SSM) in 2014. 4 TARGET2 is the Eurosystem’s payment and settlement system and carries out more than 90% of all fund flows between pairs of credit institutions in the Euro Area.
The implication of this finding is that real shocks may be amplified, and financial shocks accelerated, by bank exposure to wholesale financing.

Despite its empirical relevance, however, few efforts have been undertaken to study the main mechanisms and propagation channels of the interbank market in a structural model of the macroeconomy. As discussed throughout this thesis, there was a shortage of research into the macroeconomic effects of financial frictions in general. The dominance of the Modigliani-Miller theorem (1958) that the financing structure of a firm is irrelevant for its value confined the analysis to real and nominal frictions in the wider economy (Christiano et al. 2005, Smets & Wouters 2007). Only recently some progress has been made in understanding the impact of the financing structure of banks on lending conditions of the private sector and, hence, on aggregate output and inflation. Gerali et al. (2010) and Darracq Paries, Sørensen & Rodriguez-Palenzuela (2011) illustrate the effects of imperfect competition in the banking industry on credit spreads and show that changes in banks’ leverage ratio can impact loan supply conditions. However, in these models, banks can obtain funding in a frictionless interbank market at the rate set by the central bank.

Others have made attempts to model the interbank market more explicitly. The agency problem leading to the borrowing constraint in Gertler & Kiyotaki (2010) is also applied to the interbank market. Dib (2010) and de Walque et al. (2010) include an interbank market in which, due to an implicit enforceability problem, borrowing banks can choose an optimal level of default, and where banks must hold a regulatory level of capital. Calibrated for the US economy, both papers show that bank capital attenuates, rather than amplifies, the real effects of shocks in this framework. Hilberg & Hollmayr (2011) incorporate a secured interbank market into an otherwise standard DSGE model and study the impact of central bank collateral policy on interbank lending rates. They show that a change in the haircut applied to central bank refinancing operations can be effective in steering interbank rates, but that the presence of an interbank market also attenuates the effects of conventional monetary policy. Similarly, Carrera

\[5\] That is, default is not related to banks’ own idiosyncratic risks but a choice variable subject to an exogenous cost of default.
& Vega (2012) model the interactions between banks’ reserve requirements and interbank lending activity, which they assume is costly due to monitoring costs. They find that an increase in required reserves increases demand in the competitive interbank market and pushes up the interest rate charged on these operations as lending banks will have to pay higher monitoring costs. Funding conditions in the interbank market then trickle down to lending and deposit rates, affecting real activity. In the framework of Carrera & Vega (2012), changes in reserve requirements are therefore qualitatively similar to traditional changes in policy rates.

Cross-border interbank lending, by contrast, has been largely ignored so far in the literature. In ’t Veld & van Lelyveld (2014) examine the role of international capital flows in the boom-bust cycle in Spain by allowing borrowing-constrained households to borrow directly from foreign lenders. Using an estimated three-country model, they find that the convergence of interest rates in Spain to the levels prevailing in other Euro Area Member States, a loosening of collateral constraints as well as falling risk premia on Spanish housing and capital has fuelled the Spanish housing boom. Poutineau & Vermandel (2015) model the banking sector explicitly in a two-country DSGE model. Contrary to Quint & Rabanal (2014), who study the optimal design of macro-prudential policies in the Euro Area in a two-country DSGE model, they allow for cross-border lending to firms and banks. They find that cross-border loans amplify the propagation of country-specific shocks. Dräger & Proaño (2015) also allow for cross-border banking where an international wholesale branch is collecting deposits from across the currency union and distributes them to retail banks in the two countries. Although their model gives not rise to interbank flows, similar to Poutineau & Vermandel (2015), they find that cross-border banking amplifies the effects of exogenous shocks in a currency union.

In this article, we incorporate credit risk in the interbank market in a New Keynesian two-country, two-sector set-up with sticky prices, habits in consumption and investment adjustment costs, and a common monetary policy. There are two types of banks in each country: savings banks, which have excess liquidity that they are willing to trade in the interbank market and lending banks, which operate un-
order a structural liquidity deficit and require funding that they can obtain from the unsecured area-wide interbank market. Following the costly state verification framework of Bernanke et al. (1999), however, lending banks face idiosyncratic loan return shocks that are unobservable from the point of view of savings banks. A positive probability of default gives rise to an external finance premium that depends on the leverage of the borrower.

In addition, lending banks face a risk premium when taking a position in the cross-border interbank market. This second friction is in the same spirit as the external financial intermediation premium in Christoffel et al. (2008), but tailored to the features of an interbank market: as lending banks’ domestic economy’s net foreign asset position weakens, or when the risk in the domestic economy increases, foreign lenders demand a higher rate of interest vis-à-vis the borrower from that country as counter-party risk rises, thereby driving a further wedge between the policy rate and interbank lending rates.

We use our model to answer three important questions: (1) how is the transmission of monetary policy in a currency union affected when financing conditions in the interbank market depend on the quality of banks’ balance sheets; (2) how do asymmetric shocks to the value of assets propagate through a currency union when savings banks differentiate between domestic and foreign borrowers in the interbank market; and (3) how effective are some of the measures central banks have taken in the past to address funding bottlenecks in the interbank market.

Regarding the first question, we find that our model exhibits the financial accelerator effect (see Bernanke et al. 1999) in the face of a monetary policy shock. The decline in the policy rate stimulates demand for durable goods, house prices and the price of capital rise, resulting in an increase in the value of banks’ collateral. This lowers the rate charged in the interbank market due to reduced probability of default, and as the bank passes the lower funding costs on to its non-financial borrowers, stimulates aggregate demand even further. Our simulations therefore contradict previous findings that the inclusion of an interbank market in a DSGE model tempers the effects of monetary policy (Hilberg & Hollmayr 2011).
Regarding the second question, our model is able to replicate some of the key features of the financial crisis that resulted in a segmented interbank market.\footnote{In our model we focus on loan return risk. Ultimately, the implications of our findings are more broad-based and less dependent on the ultimate source of risk. For example, the sharp increase in spreads paid by banks in stressed Member States during the sovereign debt crisis (see e.g. de Andoain et al. 2014) works through the same channel, that is, a (perceived) deterioration in the quality of banks’ balance sheets.} We show that in the wake of an adverse shock to the value of assets in one country, the rate charged by foreign lenders in the common interbank market will rise as banks’ balance sheet deteriorate amid a fall in the collateral value. The increase in the funding costs of banks offset, to some extent, the effort by the central bank to stimulate the economy by lowering the policy rate in response to the initial shock. That is, compared to a model without cross-border interbank lending, monetary policy will be less effective. Due to the worsening of financial conditions, the economy that draws the shock must increase the trade balance more sharply relative to without the financial frictions.

Finally, we study the effectiveness of two of the ECB’s recent non-standard measures in our model economy. We find that issuing multi-period loans to banks leads to a accelerator mechanism such that lower interest rate cuts are required for an equivalent stimulus. We also show that the policy allows for different equivalent one-period funding rates with the same policy rate due to the duration effects, and that the policy has a greater impact in the non-stressed economy. The second policy is an asset purchase programme. This works through the bank balance sheet channel; as the central bank purchases a large number of risky assets, the bank balance sheets are strengthened and the external finance premium reduced. We find this is particularly successful at improving financial conditions and preventing such a sharp increase in the trade balance if the central purchases risky assets in the more highly stressed country.

The rest of the paper is organized as follows. Section 5.2 lays out the model set-up. Section 5.5 presents numerical simulations, illustrating the role of the interbank market in driving the dynamics of the model, with the policy measures analysed in section 5.5.5. Section 5.6 offers some concluding remarks.
5.2 THE MODEL

The model economy is made up of two economies that share a single currency and monetary policy. In each economy there are two types of households, savers and borrowers, monopolistic competitive firms, savings and lending banks as well as a fiscal authority. The two economies trade in both non-durable consumption goods and financial services in the form of interbank credit. The two economies, home and foreign, are of size \( n \) and \( (1 - n) \). In the following, we describe the decision-making problems of the economic agents resident in the home economy. Unless otherwise stated, analogous conditions hold for the foreign economy. As in the other chapters, the time notation refers to the period in which the value is determined. We begin the model description with the households and firms before outlining the novel part of the model; the financial sector.

5.2.1 Households

The household sector is made up of a mass \( \lambda \in [0, 1] \) of patient households with discount factor \( \beta \) and \( 1 - \lambda \) of impatient households with discount factor \( \beta^B < \beta \). The patient households are referred to as the savers and the impatient households as the borrowers.

5.2.1.1 Savers

The saver household \( h \in [0, \lambda] \) chooses the level of consumption of non-durable goods \( C_{h,t} \), hours worked \( L_{h,t} \), the housing stock \( D_{h,t} \), and bank deposit savings \( S_{h,t} \) to maximize its lifetime utility

\[
\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left\{ \varrho \ln (C_{h,t+s} - \kappa C_{t+s-1}) + (1 - \varrho) \ln (D_{h,t+s-1}) - \frac{1 + \varphi}{1 + \varphi} \right\}, \quad (5.2.1)
\]

subject to the nominal budget constraint

\[
P_t^C C_{h,t} + P_t^D (D_{h,t} - (1 - \delta_D) D_{h,t-1}) + S_{h,t} + B_{h,t} \leq R_{t-1}^S S_{h,t-1} + W_t L_{h,t} + \pi_t - T_t, \quad (5.2.2)
\]

where \( \beta \) is the discount factor, \( \varrho \) determines the relative weight of non-durable consumption and housing \( D_{h,t} \) in the saver’s utility and
where $\kappa$ measures the degree of external habit formation in consumption. The parameter $\phi$ refers to the inverse of the Frisch labour supply elasticity. $P^C_t$ and $P^D_t$ are the prices for consumption and housing goods respectively and are defined in more detail below. The saver can save in domestic banks with deposits paying the risk-free nominal rate $R^S_t$. Finally, the saver provides labour at the flexible wage rate $W_t$ and owns the stock of net wealth of the economy, except for housing that is in part also owned by the borrower household, therefore receiving profits $\pi_t$ from the banking and corporate sector and paying lump sum taxes $T_t$.

Because saver households have the same preferences over consumption, housing, labour, savings and investment, and are assumed to have the same initial wealth, we focus on a representative saver from now onwards and drop the $h$ subscript. The saver household chooses the optimal inter-temporal consumption plan subject to the budget constraint, resulting in a set of first-order conditions that will hold in equilibrium,

1. $1 = R^S_t \mathbb{E}_t \left[ \frac{\Lambda_{t,t+1}}{\Pi^C_{t+1}} \right]$  
2. $\lambda^C_t Q^D_t = \beta \mathbb{E}_t \left[ \lambda^D_{t+1} + (1 - \delta_D)Q^D_{t+1} \lambda^C_{t+1} \right]$  
3. $w_t = \frac{P^C_t}{P^C_{t-1}}$  

where $\lambda^C_t$ and $\lambda^D_t$ are the marginal utilities of consumption and housing respectively, and $\Lambda_{t,t+1} \equiv \beta^{\frac{\delta_D}{\lambda^C_t}}$ is the real stochastic discount factor over the interval $[t, t+1]$. $Q^D_t \equiv \frac{P^D_t}{P^C_t}$ is the relative price of housing and $\Pi^C_t \equiv P^C_t / P^C_{t-1}$. The real wage rate is given by $w_t = W_t / P^C_t$ which is equal to the marginal rate of substitution between labour and consumption in equilibrium. Equations (5.2.3) and (5.2.4) are the Euler equations implied by the demand for domestic deposits and the demand for housing respectively, the latter which acts both as a store of wealth and as a good delivering utility.
5.2.1.2 Borrowers

Preferences of the borrowers are the same as those of the saver except for the difference in the time discounting. Borrower household \( h \in [\lambda, 1] \) maximises

\[
\mathbb{E}_t \sum_{s=0}^{\infty} \beta_B^s \left\{ \varrho \ln \left( \frac{C^B_{h,t+s} - \kappa C^B_{h,t+s-1}}{(L^B_{h,t+s})^{1+\phi}} \right) + \left( 1 - \varrho \right) \ln \left( \frac{D^B_{h,t+s-1}}{(1-\delta_D) D^B_{h,t-1}} \right) \right\},
\]

subject to the nominal budget constraint

\[
P^C_t C^B_{h,t} + P^D_t \left( D^B_{h,t} - (1-\delta_D) D^B_{h,t-1} \right) + R^M_t CR^{HH,D}_{h,t-1} \leq CR^{HH}_{h,t} + W_t L^B_{h,t}.
\]

The notations are identical to the saver household and where the superscript \( B \) characterizes variables specific to borrowers. In particular, borrowers are assumed to be less patient than savers with \( \beta_B < \beta \), so financing their consumption of housing with credit \( CR^{HH}_{h,t} \) obtained from lending banks. This follows a number of papers (see e.g. Kiyotaki & Moore 1997, Iacoviello 2005) that impose collateral constraints by assuming the lender levies a maximum loan-to-value ratio; this implies a constraint of the form

\[
CR^{HH}_{h,t} \leq m P^D_t D^B_{h,t}.
\]

The motivation is that, due to limited liability concerns, the lender would want to ensure that household will not default the following period if repayments due are greater than the value of their house. With this in mind, we rule out the possibility of default by having the loan repayment state contingent indexed to the house price. For numerical simplicity we further assume that the borrowing constraint is always binding. This will certainly be true in the deterministic steady state, but with uncertainty the issue is less clear. For instance, the borrowers could self insure in some states of the world by borrowing below the limit to protect against the effects of adverse shocks. We choose the parameter \( m \) so that the probability of this is reduced to ensure this is a good approximation. The decision-making problems of the representative borrower household lead to a labour sup-
ply condition analogous to that of the saver household. The housing 
investment decision leads to an Euler equation in the form.

\[
E_t \left[ \beta \lambda D, B_t + \lambda \Pi_{t+1} \left( 1 - \delta_D \right) - R_{M}^{M} \right] = 1 - m. \quad (5.2.9)
\]

5.2.2 Housing Producers

The price of durable housing goods can differ from that of consumption 
goods due to the presence of adjustment costs. To ensure that 
savers and borrowers observe the same house price, we let housing 
good producers augment the existing stock according to a law of motion

\[
D_t^T = (1 - \delta_D)D_{t-1}^T + \left[ 1 - S \left( \frac{I_t^D}{I_{t-1}^D} \right) \right] I_t^D \quad (5.2.10)
\]

where \( \delta_D \in (0, 1) \) denotes the depreciation rate of housing and the 
function \( S(\cdot) \) is a positive function of changes to investment, as 
applied to capital formation in Christiano et al. (2005), and is given by

\[
S \left( \frac{I_t^D}{I_{t-1}^D} \right) = \frac{\zeta_D}{2} \left( \frac{I_t^D}{I_{t-1}^D} - 1 \right)^2. \quad (5.2.11)
\]

The housing good producers solve

\[
\max_{I_t^D} \sum_{s=0}^{\infty} \Lambda_{t,t+s} \left( P_{t+s}^C \left[ 1 - S \left( \frac{I_{t+s}^D}{I_{t-1+s}^D} \right) \right] I_{t+s}^D - P_{t+s}^P I_{t+s}^D \right), \quad (5.2.12)
\]

where \( P_t^P \) is the domestic producer price, \( P_t^C \) the final consumption 
good price index, and \( P_t^D \) the price at which housing goods are sold. 
This leads to the first-order condition

\[
\frac{P_t^P}{P_t^C} = Q_t^D \left( 1 - \frac{\zeta_D}{2} \left( \frac{I_t^D}{I_{t-1}^D} - 1 \right)^2 - \zeta_D \left( \frac{I_t^D}{I_{t-1}^D} - 1 \right) \left( \frac{I_t^D}{I_{t-1}^D} \right) \right) \]

\[
+ \mathbb{E}_t \left[ \Lambda_{t,t+1} Q_{t+1}^D \zeta_D \left( \frac{I_{t+1}^D}{I_t^D} - 1 \right) \left( \frac{I_{t+1}^D}{I_t^D} \right)^2 \right]. \quad (5.2.13)
\]
5.2.3 Firms

We introduce nominal rigidities in the price of consumption goods following Calvo (1983) and to this end we assume there are two types of firms in the model economy: intermediate goods firms that are price takers in perfect competition, and final goods firms that operate under monopolistic competition. There is ‘price-stickiness’ introduced in the latter sector due the assumption that only a fixed proportion of firms are able to update prices each period.

5.2.3.1 Intermediate goods firms

Each intermediate goods producer hires services $K^D_t$ and labour $L^D_t$ to produce a homogeneous output subject to a Cobb-Douglas production function

$$Y_t = Z_t A_t K^α_t L^{1-α}_t$$

where $A_t$ and $Z_t$ are stationary stochastic processes. The latter allows for country-specific total factor productivity shocks and the former a union wide shocks. Both are modelled as an AR(1) processes; $a^A_t = ρ^A a^A_{t-1} + ε^A_t$ with $a_t \equiv \ln A_t$ and $ε^A_t$ being an normal i.i.d. stochastic shock. There is an equivalent process for $z_t \equiv \ln Z_t$. Taking both the aggregate real wage index $w_t$ and rental price of capital $r^k_t$ as given, the profit maximisation implies labour and capital demand given by

$$w_t = α \frac{P^{w,t}_t Y_{w,t}}{P^C_t L^D_t}$$

$$r^k_t = (1 - α) \frac{P^{w,t}_t Y_{w,t}}{P^C_t K^D_t}$$

where $P^{w,t}_t$ is the price at which the output is sold to all final goods firms, and $P^C_t$ the aggregate price index of consumption goods; this implies that $P^{w,t}_t / P^C_t = MC_t$ is the real marginal cost in the final good sector.
5.2.4 Final Good Demand Schedules

Households purchase differentiated final goods and combine bundles of domestically produced goods $H_t$ and aggregate imports $IM_t$ to produce the final consumption bundle according to

$$C_t = \left[ \left( \frac{1}{\theta} \right) \frac{H_t^{\frac{1}{\theta} - 1}}{C^{\frac{1}{\theta}}} + \left( 1 - \frac{1}{\theta} \right) \frac{IM_t^{\frac{1}{\theta} - 1}}{C^{\frac{1}{\theta}}} \right]^{\frac{\theta}{\theta - 1}} \tag{5.2.17}$$

where $\tau^C$ is interpreted as the degree of home bias in household consumption expenditures, and $\theta_C$ is the constant elasticity of substitution (CES) between domestic and foreign produced goods (see Armington 1969). $H_t$ and $IM_t$ are bundles of differentiated domestic and foreign produced goods which households combine into baskets of goods using

$$H_t = \left( \int_0^1 H_t(j) \frac{d\alpha}{\sigma} \right)^{\frac{1}{\sigma - 1}} \tag{5.2.18}$$

$$IM_t = \left( \int_0^1 IM_t(j^*) \frac{d\alpha}{\sigma} \right)^{\frac{1}{\sigma - 1}} \tag{5.2.19}$$

where $\sigma$ denotes the elasticity of substitution between the different varieties, assumed to be identical across the currency union, and the asterisk indicates variables of the foreign country. The firm purchases good $H_t(j)$ from producer $j \in (0, 1)$ at price $P^P_t(j)$ to maximise $(5.2.18)$ subject to total expenditure $P^P_t H_t = \int_0^1 P^P_t(j) H_t(j) dj$, with an equivalent problem for imports $IM_t(j^*)$. $P^P_t$ is the producer price index. This leads to Dixit & Stiglitz (1977) demand schedules

$$H_t(j) = \left( \frac{P^P_t(j)}{P^P_t} \right)^{-\sigma} H_t \tag{5.2.20}$$

$$IM_t(j^*) = \left( \frac{P^{IM}_t(j^*)}{P^{IM}_t} \right)^{-\sigma} IM_t. \tag{5.2.21}$$

Equivalent conditions for the domestic demand of the investment good and government consumption also hold. These demand functions will be used in the intermediate good price setting problem below. For now, we derive the consumption good price index in each country and assume no pricing to market, which implies that $P^{IM}_t = P^P_t$. The final good firms take input prices as given and maximize their profits $P^C_t C_t - P^P_t H_t - P^{IM}_t IM_t$. Profit maximization yields the
following demand schedules for the domestic bundle and aggregate imports

\[ H_t = \tau C \left( \frac{P_t^P}{P_t^C} \right)^{-\theta_c} C_t \]  

\[ IM_t = (1 - \tau C) \left( \frac{P_t^{IM}}{P_t} \right)^{-\theta_c} C_t \]

This leads to the consumer price index \( P_t^C \) given by

\[ P_t^C = \left[ \tau C \left( \frac{P_t^P}{P_t^C} \right)^{1-\theta_c} + (1 - \tau C) \left( \frac{P_t^{IM}}{P_t} \right)^{1-\theta_c} \right] \frac{1}{1-\theta_c}. \] (5.2.24)

### 5.2.5 Final Good Producers

Each final good producer firm \( j \in (0, 1) \) purchases output from the intermediate good sector at price \( P_{w,t} \) and converts into a differentiated goods sold at price \( P_t^P(j) \) to households, durable good producers and governments. Summing the demand schedules from each buyer implies a total demand for good \( j \) given by

\[ Y_t(j) = \left( \frac{P_t^P(j)}{P_t^P} \right)^{-\sigma} Y_t. \] (5.2.25)

Every period, each firm faces a fixed probability \( 1 - \zeta \) that they will be able to update their prices. Denoting the optimal price at time \( t \) for good \( j \) as \( P_t^* (j) \), the firms allowed to re-optimize prices maximise expected discounted profits by solving

\[ \max_{P_t^* (j)} \mathbb{E}_t \sum_{k=0}^{\infty} \zeta^k \frac{\Lambda_{t,j+k}}{P_t^C} Y_{t+k}(j) \left( P_t^* (j) - P_{w,t+k} \right). \] (5.2.26)

Substituting in the demand schedule, taking first-order conditions with respect the new price and rearranging leads to

\[ P_t^* = \frac{\sigma}{\sigma - 1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \zeta^k \frac{\Lambda_{t,j+k}}{P_{t+k}^P} Y_{t+k} P_{w,t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} \zeta^k \frac{\Lambda_{t,j+k}}{P_{t+k}^P} Y_{t+k}} \] (5.2.27)

where the \( j \) index is dropped as all firms face the same marginal cost so the right-hand side of the equation is independent of firm size or price history. To use the numerical approximation techniques discussed in the methodology section of chapter 2, we require the
model equations to be expressed as stochastic difference equations rather than the infinite sums in equation 5.2.27. It is also necessary for all variables need to be stationary and so rather than price levels, we want to express the solution in terms of rates of change, and mark-ups or price ratios. Recall from the previous section that the real marginal cost is given by \( MC_t = \frac{P_w}{P_C} \), using this and the price inflation over the interval \([t-1, t]\) denoted \( \Pi_{t-1,t} \equiv \frac{P_t}{P_{t-1}} \), we write the fraction

\[
\frac{P^*_t}{P^p_t} = \frac{\sigma}{\sigma - 1} \frac{\bar{\epsilon}^t \sum_{k=0}^{\infty} \xi^k A_{t,t+k} \left( \Pi^p_{t,t+k} \right)^\sigma \Pi_{t,t+k}}{E_t \sum_{k=0}^{\infty} \xi^k A_{t,t+k} \left( \Pi^C_{t,t+k} \right)^\sigma \left( \Pi^C_{t,t+k} \right)^{-1} Y_{t+k}}
\]

(5.2.28)

Denoting the numerator and denominator \( \Omega_{1,t} \) and \( \Omega_{2,t} \) we write in recursive form

\[
\Omega_{1,t} = \frac{\sigma}{\sigma - 1} Y_t MC_t + \xi \bar{\epsilon}^t \left( \Pi^p_{t,t+1} \right)^\sigma \Omega_{1,t+1}
\]

(5.2.29)

\[
\Omega_{2,t} = \frac{p^p_t}{p^p_{t-1}} Y_{t+k} + \bar{\epsilon}^t \left( \Pi^p_{t,t+1} \right)^\sigma \left( \Pi^C_{t,t+1} \right)^{-1} \Omega_{2,t+1}
\]

(5.2.30)

as required. Using the aggregate producer price index \( P^p_t \) and the fact that all resetting firms will choose the same price, by the Law of Large Numbers we can find the evolution of the price index as given by

\[
P^{1-\sigma} = \frac{\bar{\epsilon} P^p_{t-1} 1-\sigma}{\Xi^t \sum_{j=0}^{\infty} \xi^j A_{t,t+j} \left( P^p_t \right)^\sigma \left( P^p_t \right)^{-1} \Pi_{t,t+j}}
\]

(5.2.31)

which can be written in the form required

\[
1 = \bar{\epsilon} \left( P^p_t \right)^{\sigma-1} + (1 - \bar{\epsilon}) \left( \frac{p^p_t}{P^p_t} \right)^{1-\sigma}
\]

(5.2.32)

where \( \Pi_t \) is the gross inflation in the price of domestically produced goods between periods \([t-1, t]\). Whilst the distribution of prices is not required to track the evolution of the aggregate price index, equation 5.2.18 implies a loss of output due to dispersion in prices. Using the demand schedules, we can write the price dispersion that gives the average loss in output as

\[
\Delta_t = \frac{1}{J} \sum_{j=1}^{J} \left( \frac{p^p_t (j)}{P^p_t} \right)^{-\sigma}
\]

(5.2.33)
for non-optimizing firms $j = 1, ..., J$. It is not possible to track all $P_{jt}$ but as it is known that a proportion $1 - \xi$ of firms will optimise prices in period $t$, and from the Law of Large Numbers, that the distribution of non-optimised prices will be the same in as the overall distribution. Therefore price dispersion can be written as a law of motion

$$\Delta_t = \xi \Pi_t^f \Delta_{t-1} + (1 - \xi) \left( \frac{\Omega_{1,t}}{\Omega_{2,t}} \right)^{-\sigma} \Delta_t. \quad (5.2.34)$$

Using this, aggregate final output is given as a proportion of the intermediate output

$$Y_t = Y_{w,t} \frac{1}{\Delta_t}. \quad (5.2.35)$$

5.2.6 Financial Intermediation

There are two types of banks: savings banks that take deposits from domestic households and lend in the currency union-wide interbank market. Lending banks provide loans to both domestic firms and households and finance these using interbank borrowing and their own net worth. The financial friction emerges due to idiosyncratic loan return shocks faced by the lending banks. Costly state verification leads to an external finance premium as that proposed in Bernanke et al. (1999). We add a further friction whereby cross-border interbank credit faces additional monitoring costs that depend on the exposure of the borrowing country to foreign debt.

The banks are owned by the patient households who are due any profits earned. As savings banks act under perfect competition with free-entry there are zero normal profits. Shocks that cause profits (or losses) in a single period are transferred to (subsidised by) the savers in that period. Lending banks face idiosyncratic shocks that are costly for creditors to observe; limited liability then implies that these banks earn profits in equilibrium. The lending banks are treated slightly differently to the savings banks by assuming these banks pay a fixed dividend rate to ensure lending banks cannot become self-funding. This is a standard assumption in the literature equivalent to the popular exogenous exit rate (see e.g. Bernanke et al. 1999, Gertler & Kiyotaki 2010). The friction implies that equity is always more valuable than debt and so without this the banks would not pay dividends.
in equilibrium. Another difference in how lending banks are treated is that savings banks can access central bank credit whilst lending banks cannot. This could be motivated due to the risk-exposure of lending banks; the savings banks are well diversified and the central bank requires a proportion of safe assets as collateral. In reality only a very small proportion of banks are able to access central bank credit whilst the remaining banks must rely on the interbank and other money markets.\footnote{As highlighted in Carrera & Vega (2012), only 6 out of 2500 banks in the Eurozone are allowed to participate in the bidding process in main refinancing operations of the ECB.}

5.2.6.1 Lending Banks

There are many lending banks of unit mass indexed \( j \in [0,1] \). They extend credit \( CR_t \) to the non-financial sector, which they finance with domestic \( IB^H_t \) and cross-border \( IB^F_t \) interbank borrowing and net worth \( N_t \):

\[
CR_t = N_t + IB^H_t + IB^F_t
\]  \hspace{1cm} (5.2.36)

where \( IB^H_t \) and \( IB^F_t \) are chosen to maximise a CES Armington aggregator (see Armington 1969) of domestic and foreign interbank borrowing

\[
IB_t = \left[ \left( \tau^{IB} \right)^{\frac{1}{\theta_{IB}}} (IB^H_t)^{\frac{\theta_{IB}-1}{\theta_{IB}}} + \left( 1 - \tau^{IB} \right)^{\frac{1}{\theta_{IB}}} (IB^F_t)^{\frac{\theta_{IB}-1}{\theta_{IB}}} \right]^{\frac{\theta_{IB}}{\theta_{IB}-1}}, \hspace{1cm} (5.2.37)
\]

where \( \tau^{IB} \) is the home bias in interbank borrowing and \( \theta_{IB} \) is the elasticity of substitution between domestic and foreign borrowing. This is similar to the assumption about household preference to domestic goods and imports and is used to pin down steady state shares of interbank borrowing. A non-zero \( \theta_{IB} \) implies that domestic and foreign interbank borrowing are not perfect substitutes and so also supports differences in the lending rates. This yields the following demand schedules

\[
IB^H_t = \tau^{IB} \left( \frac{R^{IB,H}_t}{R^{IB}_t} \right)^{-\theta_{IB}} IB_t
\]  \hspace{1cm} (5.2.38)

\[
IB^F_t = (1 - \tau^{IB}) \left( \frac{R^{IB,F}_t}{R^{IB}_t} \right)^{-\theta_{IB}} IB_t
\]  \hspace{1cm} (5.2.39)
where $R_t^{IB,H}$ is the lending rate on the domestic interbank market and $R_t^{IB,F}$ the rate on the cross-border market. Using the demand schedules and $R_t^{IB} = R_t^{IB,H}IB_t^{H} + R_t^{IB,F}IB_t^{F}$, a definition of composite interbank funding costs is given by

$$R_t^{IB} = \left[ \tau^{IB} \left( R_t^{IB,H} \right)^{1-\theta^{IB}} + (1 - \tau^{IB}) \left( R_t^{IB,F} \right)^{1-\theta^{IB}} \right]^{\frac{1}{1-\theta^{IB}}}. \quad (5.2.40)$$

When granting loans to the non-financial private sector, we assume that lending banks cannot diversify risk in their loan portfolio and that they experience idiosyncratic loan return shocks $\omega_t(j)$ that affect the value of the asset side of their balance sheets.\(^8\) The shocks are log-normally distributed, $\log(\omega_t(j)) \sim \mathcal{N}(-\sigma_{\omega,t}^2/2, \sigma_{\omega,t}^2)$, with mean $\mathbb{E}[\omega_t] = 1$ and standard deviation $\sigma_{\omega,t}$, which is time-varying and is modelled as an AR(1) process: $\log(\sigma_{\omega,t}) = (1 - \rho_\sigma) \log(\sigma_{\omega,ss}) + \rho_\sigma \log(\sigma_{\omega,t-1}) + u_{\omega,t}$ and $u_{\omega,t} \sim \mathcal{N}(0, \sigma_\sigma)$.

After aggregate and idiosyncratic shocks hit the economy, net worth of lending banks evolves according to

$$N_t(j) = \omega_t(j) R_t^{CR}CR_{t-1}(j) - R_t^{IB}IB_{t-1}(j) \quad (5.2.41)$$

where $R_t^{CR}$ is the ex post return on banks’ loan portfolio $CR_t = CR_t^{H,H,S} + CR_t^{F}$. Limited liability implies that if the realization of the shock is below a threshold value $\bar{\omega}_t$, which can be interpreted as the loan-to-value ratio, then the lending bank will default on its interbank borrowing as they would otherwise be insolvent. This is found when $N_t(j) = 0$ and so defined as

$$\bar{\omega}_t(j) = \frac{R_{t-1}^{IB}IB_{t-1}(j)}{R_t^{CR}CR_{t-1}(j)} \quad (5.2.42)$$

The lending banks will pay the saver households a fixed dividend rate,\(^9\) investing all remaining profits in their own net worth. It is assumed that a defaulting bank will exit but that for every exiting bank, a new one enters and is given a small start-up fund by the other banks. This ensures the number of banks is held constant. The idiosyncratic

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\(^8\) This can be thought of as there being many islands and on each island there is representative bank and borrower. We allow the borrowers to insure against the shock so they pay a risk-free nominal rate. This simple device allows us to study imperfect diversification in a tractable way.

\(^9\) Equivalent to the fixed probability of exit in Bernanke et al. (1999).
loan return and default leads to a distribution of lending banks over all possible values of net worth.

The loan portfolio is comprised of mortgage loans to households and lending to firms. The former are treated as state-contingent (indexed to house prices) one period bonds whilst, for the latter, it is convenient to consider the banks owning the physical capital and renting to firms\(^\text{10}\). Capital investment accumulates and is subject to costs analogous to those in housing investment and so to ensure the leverage of a bank is independent of its net worth, we introduce a representative capital producer that sells capital to the banks at relative price \(Q^K_t\). This leads to a first order condition equivalent to equation \((5.2.13)\) that determines \(Q^K_t\). Gross nominal return on capital is given by

\[
R^K_t = \frac{r^K_t + (1 - \delta^K) Q^K_t}{Q^K_{t-1}} \Pi_{t-1,t}. \tag{5.2.43}
\]

The lending banks specify a contract for interbank funds subject to participation constraints given in the following section. After working through the saving banks’ problem, we discuss the contract that determines the demand for interbank credit, and the supply of credit to the non-financial private sector.

5.2.6.2 Savings Banks

A representative savings bank has access to the central bank’s liquidity providing operations \(CB_t\), raises deposits \(S_t\) from patient households and extends both domestic \(IB^H_i\) and cross-border \(IB^F_i\) interbank loans:

\[
S_t + CB_t = IB^H_i + IB^F_i. \tag{5.2.44}
\]

Maximising the expected profits lead to the arbitrage conditions

\[
\mathbb{E}_t \left[ \frac{\Lambda_{t,t+1}}{P^C_{t+1}} R^S_t \right] = \mathbb{E}_t \left[ \frac{\Lambda_{t,t+1}}{P^C_{t+1}} \tilde{R}^{IB}_t \right], \tag{5.2.45}
\]

where \(\tilde{R}^{IB}_t\) is the ex post return on interbank lending. We will consider the role of long-term central bank credit so leave discussion of the

\(^{10}\) As in Gertler & Kiyotaki (2010). This is equivalent to the firms using state-contingent debt to purchase the capital themselves. It is natural to assume that debt contracts are state-contingent due to costless monitoring and enforcement, and the risk neutrality of the lender.
relationship with the policy rate until further below. Under standard one-period central bank finance, the deposit rate must equal the policy rate in equilibrium. The saving bank can only observe the loan return of the lending bank if it pays a proportional fee $\mu$. As shown in Townsend (1979), the implication of this costly state verification is that the fee will only be paid in the event of default, with all other debtors paying the same interest rate. The interbank lending is subject to a participation constraint that accounts for the distribution of idiosyncratic shocks drawn by lending banks and the aggregate state of the economy. The savings banks require the expected real return from granting each domestic interbank loans to be equal to their expected real funding rate, using the household Euler equation, this can be written

$$1 = \mathbb{E}_t \left[ \frac{\Lambda_t}{\Pi_t} \left[ 1 - F(\tilde{\omega}_{t+1}(j), \sigma_{\omega_{t+1}}) \right] R_t^{IB,H}(j) \right] + \mathbb{E}_t \left[ \frac{\Lambda_t}{\Pi_t} (1 - \mu) \int_0^{\tilde{\omega}_{t+1}(j)} \omega R^{CR}_{t+1} \frac{CR(j)}{IB_t(j) + IB_t(j)} dF(\omega, \sigma_{\omega_{t+1}}) \right].$$

(5.2.46)

$\mu$ is the monitoring cost banks pay to recover the remaining assets of a bank in the case of default, and $F(\tilde{\omega}(j), \sigma) \equiv \int_0^{\tilde{\omega}(j)} f \left( \omega; \frac{\sigma^2}{2}, \sigma^2 \right) d\omega$ is the cumulative density function up to $\tilde{\omega}(j)$, with probability density function $f \left( \omega; \frac{\sigma^2}{2}, \sigma^2 \right)$. Note this implies that $F(\tilde{\omega}_{t+1}(j), \sigma_{\omega_{t+1}})$ is the probability of default. We will find that individual bank net worth does not effect the interest rate paid on credit as all banks will choose the same leverage ratio, and therefore equation (5.2.46) will hold if the index $j$ is dropped, with the variables treated as the aggregate averages. When taking positions in the cross-border interbank market, we assume that savings banks incur additional monitoring costs $\Gamma_{IB,t}$ that are increasing in its exposure of the destination country to cross-border debt, and to risk within the economy. Such costs reflect factors such as asymmetric information, counter-party risk as well as differences in cross-border macroeconomic conditions. Specifically, this is given by

$$\Gamma_{IB,t} = \zeta_{IB} \left[ \left( 1 - \exp \left( \frac{IB_t^F - CR_t^F - IB_t^F}{P_t^Y_t Y_t} \right) \right) + \tilde{\zeta}_\sigma \left( \frac{\sigma_{\omega_t^*}}{\sigma} - 1 \right) \right].$$

(5.2.47)

where the second element in the fraction highlights an additional increase to the cost of accessing funds following a shock to the variance
of the idiosyncratic loan return shock. Therefore, the participation constraint for international interbank loans can be expressed as

$$1 = E_t \left[ \frac{\Lambda_{t+1}}{1 + \lambda} (1 - \Gamma_{IB,F}) \left[ 1 - F(\bar{\omega}_{t+1}, (j^*) \sigma_{\omega,t+1}^*) \right] R_t^{IB,F} (j^*) \right]$$

$$+ E_t \left[ \frac{\Lambda_{t+1}}{1 + \lambda} (1 - \mu) \frac{G(\bar{\omega}_{t+1}(j^*), \sigma_{\omega,t+1}^*)}{\bar{\omega}_{t+1}(j^*)} R_t^{IB} (j^*) \frac{I_{F}^{IB}(j^*)}{IB^{I}(j^*) + IB^{F}(j^*)} \right].$$

(5.2.48)

where $G(\bar{\omega}_{t+1}(j^*), \sigma_{\omega,t+1}^*) \equiv \int_{0}^{\omega_{t}(j)} \omega dF(\omega, \sigma)$ and $F_{t}^{I} = F(\tilde{\omega}_{t}(j), \sigma_{t}).$

Equation (5.2.48) implies a spread in the cross-border interbank market $R_t^{IB,F}(j^*)$ that is a function of the monitoring cost $\Gamma_{IB,F},$ which itself is an increasing function of total cross-border exposure. That is, the higher the volume of cross-border loans, the higher the spread required to compensate savings banks for the increase in credit risk. In a similar vein, aggregate shocks that lead to a decline in nominal domestic GDP will cause the ratio of net foreign assets to output to increase, thereby leading to a rise in the spread. Also, the spread is increasing in the standard deviation of the loan return shock: a higher $\sigma_{\omega,t}$ will increase the risks of default by making lower realization of $\omega_{t}(j)$ more likely.

### 5.2.6.3 Interbank Credit Market

To model the over-the-counter structure of the interbank market, we follow Bernanke et al. (1999) in our treatment of the lending contract. The lending banks choose credit to the non-financial private sector $CR_t,$ the volume of interbank lending $IB_t^H$ and $IB_t^F,$ and interest rates $R_t^{IB,H}$ and $R_t^{IB,F}$ to maximise their expected real net worth. The entrepreneur pays a fraction $1 - \gamma$ as a dividend, with the remaining surplus used as internal equity finance. Substituting in the expression for $\tilde{\omega}_{t+1}$ the problem is written

$$\max_{X_t} E_t \left[ \frac{\Lambda_{t+1}}{R_t^{IB,F}} \gamma \left[ 1 - G(\tilde{\omega}_{t+1}) - (1 - F(\tilde{\omega}_{t+1})) \tilde{\omega}_{t+1} \right] R_t^{CR} CR_t \right]$$

(5.2.49)

where $X_t = \{ CR_t, IB_t^H, IB_t^F, R_t^{IB,H}, R_t^{IB,F} \}$ is a vector of controls and dropping any individual bank index for ease of reading. The maximisation is subject to the bank balance sheet equation (5.2.36), the interbank fund demand functions equations (5.2.38) and (5.2.39), the com-
posite interest rate \((5.2.40)\), the loan-to-value equation \((5.2.42)\), and the saver bank participation constraints given in equations \((5.2.46)\) and \((5.2.48)\).

The solution to the contract problem yields a condition that determines the wedge between the nominal risk-free rate \(R^S_t\), and the expected return from credit to the non-financial sector \(R^{CR}_{t+1}\). We can express this as

\[
\mathbb{E}_t \left[ \frac{R^{CR}_{t+1}}{R^S_t} \right] = \mathbb{E}_t \left[ s \left( R^S_t, R^{IB,H}_t, R^{IB,F}_t, \Gamma^\nu_t, \frac{IB_t}{CR_t}, \mathcal{G}_{t+1} \omega_{t+1} \right) \right]
\]

(5.2.50)

with key arguments given, although the nominal stochastic discount factors of both countries are also arguments of function \(s\). In this solution, which is given in full in appendix B.1, the expected real return to lending is equated with the real marginal cost of external finance. Because the solution is a function of the leverage rather than the bank size, the contract interest rates will independent of the bank’s own history of shocks and so we focus only on aggregate identities. The function \(s\) can be described as the investment or credit wedge, or the external finance premium as discussed in (Bernanke et al. 1999). This acts as the financial accelerator and it is particularly worth noting the relationship between \(s\) and the capital-asset ratio of the lending bank. As leverage increases, so \(\frac{N_t}{CR_t}\) falls, the probability of default increases, and the marginal cost of borrowing rises. This is the financial accelerator mechanism; if, for instance, an adverse shock reduces the net worth of the banking sector, bank leverage will increase, and so to will the credit wedge \(s\) causing a further deepening of the downturn.

5.2.7 Firm and Household Credit

As discussed, the firm loans are treated as equivalent to equity, and so the return on firm credit is simply the gross return on capital, \(R^K_t\), defined in equation \((5.2.43)\). The credit to households is indexed to the house prices, so \(\text{ex post}\) return is given by \(R^M_t = \hat{R}^M_t P^D_t\) for the contracted rate \(\hat{R}^M_{t-1}\). The optimality condition implies the zero arbitrage conditions

\[
\mathbb{E}_t \left[ \frac{\Lambda_{t+1} R^{CR}_{t+1}}{P^C_{t+1}} \right] = \mathbb{E}_t \left[ \frac{\Lambda_{t+1} R^M_{t+1}}{P^C_{t+1}} \right] = \mathbb{E}_t \left[ \frac{\Lambda_{t+1} R^K_{t+1}}{P^C_{t+1}} \right]
\]

(5.2.51)
must hold in equilibrium.

5.2.8 Monetary Policy

The monetary authority sets the nominal short-term interest rate in response to deviations of the consumer price inflation rate from the inflation target and off-trend output growth

\[ R_t = \left[ \hat{R} \left( \frac{\Pi_t^{EMU}}{\bar{\Pi}_t^{EMU}} \right)^\varphi \left( \frac{Y_t^{EMU}}{Y_t^{EMU}} \right)^\psi \right]^{1-\varphi'} R_{t-1}^p \exp(\epsilon_t^m) \]  

(5.2.52)

The union-wide variables \( \Pi_t^{EMU} \) and \( Y_t^{EMU} \) are given as averages over the home and foreign country variables, where

\[ \Pi_t^{EMU} = \frac{p_t^{EMU}}{p_{t-1}^{EMU}} = \left( \frac{P_t^C}{P_{t-1}^C} \right)^n \left( \frac{P_t^C^*}{P_{t-1}^C^*} \right)^{1-n} \]  

(5.2.53)

\[ Y_t^{EMU} = (Y_t)^n (Y_t^*)^{1-n} \]

The economy as presented to this point features a unit root stemming from a single savings rate across both economies. As savers in both countries face the same return on assets, long-run effects from transitory shocks are introduced. For instance, if one of the economies draws a positive supply shock; the net foreign asset position will improve and the economy will have a current account surplus that persists in the long-run, with a permanent increase in the wealth of the savers in the economy that draws the shock. To restore the stationary property to the model, we assume that the central bank applies a small premium on the refinancing rate that depends on the net foreign asset position of the country.\(^{11} \) The rates paid on \( CB_t \) are determined by

\[ R_t = R_t^p - \vartheta_t \left( \exp \left[ \kappa \frac{NFA_t}{P_t^C Y_t} \right] - 1 \right) \]  

(5.2.54)

where \( \vartheta_t \) is a stationary, mean 1 shock to the premium, and \( \kappa \) the premium elasticity. \( R_t^p = R_t^{p*} \) is the central policy rate, and \( R_t \) and \( R_t^s \) the rates paid on central bank credit. Central bank funds are in zero net supply so if savings banks in the home country borrow from the

\(^{11}\) See Schmitt-Grohe & Uribe (2003) for a full discussion of the techniques used to remove the random walk in open-economy models.
central bank, it follows that foreign country savings banks are depositors, and \( R_t > R_t^* \). The risk premium will cause the net foreign asset position of banks in each country to adjust until \( NFA_t = NFA_t^* = 0 \) and \( R_t = R_t^* \). The structure of the premium implies that there will be weakly positive profits in equilibrium which are transferred equally to savers in the union. In the numerical simulations we choose \( \kappa \) sufficiently low to allow the rates to be very close but generating persistent effects.

5.2.9 Market clearing conditions

The labour market is in equilibrium when total supply by households equals the demand from intermediate good producers,

\[
L^D_t = \lambda L_t + (1 - \lambda) L_t^B. \tag{5.2.55}
\]

The corresponding capital market equilibrium condition is given by

\[
K^D_t = K_{t-1}. \tag{5.2.56}
\]

Total demand for domestically produced goods include the demand from domestic households \( H^T_t \equiv \lambda H_t + (1 - \lambda) H^B_t \), demand from foreign consumers \( X^T_t \equiv \lambda X_t + (1 - \lambda) X^B_t \), demand from capital producers \( I^K_t \left( 1 - \zeta^K (I^K_t / I^K_{t-1} - 1)^2 \right) \) and from housing good producers \( I^D_t \left( 1 - \zeta^D (I^D_t / I^D_{t-1} - 1)^2 \right) \) which are net the adjustment costs, and demand for government consumption. The implied real resource constraint is then

\[
Y_t = H^T_t + X^T_t + I^K_t \left( 1 - \zeta^K (I^K_t / I^K_{t-1} - 1)^2 \right) + I^D_t \left( 1 - \zeta^D (I^D_t / I^D_{t-1} - 1)^2 \right) + g_t Y_t \tag{5.2.57}
\]

where government consumption is given as a proportion \( g_t \) of output, with \( g_t \) following a stationary stochastic process. The net foreign asset position evolves according to the following nominal law of motion

\[
IB_{t}^{F^*} - IB_{t}^{F} - CB_t = R_{t-1}^{IB,F^*} IB_{t-1}^{F^*} - R_{t-1} IB_{t-1} - R_{t-1}^{IB,F} IB_{t}^{F} + TB_t, \tag{5.2.58}
\]

where the trade balance is defined as

\[
TB_t = p^P_t X_t - p^{IM}_t M_t. \tag{5.2.59}
\]
The bilateral terms of trade are given by:

\[ \text{ToT}_t = \frac{P_{IM}^t}{P_X^t} \]  

(5.2.60)

and the central bank funds are zero net supply worldwide, so

\[ n\text{CB}_t + (1 - n)\text{CB}_t^* = 0. \]  

(5.2.61)

5.3 Policy

The focus of the chapter is the analysis of unconventional monetary policy in an economy with interbank market frictions. The two policies we discuss are longer term central bank refinance operations, and central bank asset purchases.

5.3.1 Long-term Refinance Operations

As discussed above, savings banks have access to funding from the central bank. The operations of the central bank can be standard one-period loans to banks or may take the form of multi-period loan contracts, similar to the ECB’s long-term refinancing operations (LTRO).\(^{12}\)

To maintain tractability and keep the number of state variables manageable, we follow Rudebusch & Swanson (2012)\(^ {13}\) and introduce multi-period loan contracts using geometrically decaying repayments over an infinite horizon. This set-up reflects the aggregation of a large number of loans at different points of repayment and of different maturities. As well as introducing just one new state variable rather than potentially very many with long maturities, the appeal is that using infinitely long loans with geometrically declining repayments allows us to control the average maturity with just one parameter, nesting the possibility of \( \psi = 0 \), in which case it collapses to a standard one-period loan contract. Every period \( t \), a savings bank will take out a

\(^{12}\) See e.g. [https://www.ecb.europa.eu/press/pr/date/2011/html/pr111208_1.en.html for information on this policy.\n
\(^{13}\) Described in detail to analyse term premia on bonds in a working paper version of the article (see Rudebusch & Swanson 2008). Used to introduce multi-period loan contracts in Benes & Lees (2010).
new loan amount $CB_t$ and agree to repay an infinite number of declining payments such that the total amount due at period $t$ is given by:

$$CB^T_{t-1} = \sum_{k=1}^{\infty} \psi^{k-1} R^{LT}_{t-k} CB_{t-k}$$  \hspace{1cm} (5.3.1)

The parameter $\psi \in [0, 1)$ determines the average loan duration. When $\psi > 0$, $R^{LT}_t$ is no longer equivalent to an interest rate; to analyse the role of the LTRO policy, we assume that the central bank chooses $R^{LT}_t$ so that the average interest rate on long-term borrowing equals the policy rate $R_t$. As we are using perpetual loan repayments, we measure the average duration using Macaulay’s duration of a stream of payments. It is then straightforward to calculate the equivalent average nominal interest rate on the amount borrowed $CB_t$ from the total amount repaid. We find this leads to the following relationship between the rate $R^{LT}_t$ and the policy rate $R_t$:

$$R_t = \left( \frac{R^{LT}_t}{1 - \psi} \right)^{1/d}$$  \hspace{1cm} (5.3.2)

with average loan duration $d = R/(R - \psi)$, where $R$ is the steady-state policy rate.\footnote{For perpetual loan repayments, the average loan duration is measured using Macaulay’s duration of a stream of payments, given by $d_t = \sum_{t=1}^{\infty} t PV_t / \sum_{t=1}^{\infty} PV_t$ where $PV_t$ is the present value of the cash flow (see e.g. Marrison 2002). Applying this to our example, we find in simulations that $d_t$ experiences only tiny fluctuations around its steady state value, and so we use the steady state value as a close approximation. This can be simplified to $d = \frac{R}{R - \psi}$. It is then straightforward to calculate the average interest rate given that borrowing is a convergent series.}

We can then express equation (5.3.1) in recursive form as

$$CB^T_t = \psi CB^T_{t-1} + R^{LT}_t CB_t.$$  \hspace{1cm} (5.3.3)

Even if the bank does not borrow from the central bank in equilibrium, as will be the case with purely symmetric shocks, the availability of these loans is sufficient to have an important impact on the household saving rate. Using equation 5.3.3 as a constraint in the profit maximisation problem of the savings bank leads to the first-order conditions

$$\phi_t = \mathbb{E}_t \left[ \frac{\Lambda_{t+1}}{\Pi_t} (1 + \phi_{t+1} \psi) \right]$$  \hspace{1cm} (5.3.4)

$$\phi_t R^{LT}_t = \mathbb{E}_t \left[ \frac{\Lambda_{t+1}}{\Pi_t} R^S_t \right]$$  \hspace{1cm} (5.3.5)
Which, with equation (5.3.2), gives the spread between the policy rate \(R_t\) and the deposit rate \(R^S_t\). \(\phi_t\) is the real present value of the Lagrange multiplier on the law of motion of repayments due, and is a nominal pricing kernel on central bank credit. When \(d = 1\) and \(\psi_t = 0\), then \(R^{LT}_t = R_t = R^S_t\) as in the standard model, and \(\phi_t\) is just the nominal stochastic discount factor. As the loan duration increases so \(\psi > 1\), the implied future stream of payments increase and \(\psi_t > \frac{\Lambda_{t+1}}{\Lambda_t}\). This mechanism captures some important features. Suppose there is an adverse shock causing the central bank to lower rates, the bank observes that these low rates lower the average rates for longer and so demand even lower rates from the households; this implies that smaller drops in rates are required to achieve the equivalent stimulus in a single period loan economy.

5.3.2 Asset Purchases

A further instrument available to the policy maker is to purchase assets financed by issuing one-period bonds. The bonds are issued at the market rate, which by zero arbitrage will equal the deposit rate \(R^S_t\) and the policy maker uses the proceeds to purchase some number of loans from the lending banks.

At the start of the period during which the asset purchase will take place, the policy maker announces the purchase decision. This implies the lending bank first order conditions are unchanged except the volume of loans on the bank balance sheet changes to

\[
CR_t = (1 - \Theta_t) \left( CR_t^{HH} + CR_t^F \right)
\]

(5.3.6)

where \(\Theta_t\) is the proportion of assets held by the central bank. We treat asset purchases as if the central bank lends directly to the private sector as Gertler & Karadi (2011) postulate. The central bank raises \(\Theta_t \left( CR_t^{HH} + CR_t^F \right)\) in the bond market to make loans to the private sector with profits distributed to the households via lump sum transfers. The central bank budget constraint can be written as two constraints

\[
\Theta_t \left( CR_t^{HH} + CR_t^F \right) = B_t^{CB}
\]

(5.3.7)

\[
T_t = R^C_t \Theta_t \left( CR_{t-1}^{HH} + CR_{t-1}^F \right) - R^S_{t-1} B_{t-1}^{CB}
\]

(5.3.8)
where \( R_t^{CR} \) is the average \textit{ex post} return on all assets, \( T_t \) transfers to households, and \( B_t^{CB} \) central bank issued bonds. The first constraint is that all funds raised are used to purchase assets and the second that all profits are transferred to households.

### 5.4 Calibration and Parametrization

We calibrate a number of the structural parameters of our model, as shown in table 5.1, with the aim of matching key empirical first moments with remaining parameters given values based on previous estimates in the literature, see table 5.2. On the household side, we fix the fraction of savers \( \lambda \) in each economy to 0.65, close to the estimate obtained by Quint & Rabanal (2014). The discount factor of savers (borrowers) \( \beta \) (\( \beta_B \)) is chosen to be 0.995 (0.89), ensuring a nominal steady-state return on risk-free savings of around 4%. External habit

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>Patient agent discount factor</td>
<td>0.995</td>
<td>( R = 1.0408 )</td>
</tr>
<tr>
<td>( \varrho )</td>
<td>Utility weight of non-durable consumption</td>
<td>0.82</td>
<td>( I^H/Y = 0.045 )</td>
</tr>
<tr>
<td>( \tau^C )</td>
<td>Home-bias in consumption</td>
<td>0.75</td>
<td>( X^T/X^T + H^T = 0.25 )</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Elasticity of substitution across consumption varieties</td>
<td>3.86</td>
<td>Mark-up = 1.35</td>
</tr>
<tr>
<td>( \sigma_{\omega} )</td>
<td>Standard deviation of loan return shock</td>
<td>0.0339</td>
<td>( \bar{\omega} = 0.9 )</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Retained share of bank profits</td>
<td>0.947</td>
<td>( F(\bar{\omega}, \sigma_{\omega}) = 0.004 )</td>
</tr>
<tr>
<td>( \tau^{IB} )</td>
<td>Home-bias in interbank borrowing</td>
<td>0.75</td>
<td>( \frac{IB^T}{IB^T + IB^T} = 0.25 )</td>
</tr>
<tr>
<td>( g )</td>
<td>Steady-state ( G/Y )</td>
<td>0.2</td>
<td>( G/Y = 0.2 )</td>
</tr>
</tbody>
</table>

Table 5.1: Calibrations of model parameters.

in consumption \( \kappa \) is set at 0.564 according to estimates of Christoffel et al. (2008) for the Euro area. The relative share of non-durables in
consumption \( q \) is calibrated to be 0.82 to target the output share of housing investment. The inverse of the Frisch labour supply elasticity \( \phi \) is parametrised to be 0.4 (cf. Quint & Rabanal 2014). The quasi-share of domestic goods in total consumption \( \tau^C \) is fixed at 0.75 to ensure a steady-state share of imports in consumption of 25%. The elasticity of substitution between domestic goods and imports \( \theta^C \) is set at 1.9 in line with Quint & Rabanal (2014) estimates.

On the production side, we choose a value of 0.3 for the share of labour \( \alpha \) in the Cobb-Douglas production function. Capital depreciation \( \delta \) and \( \delta^H \) are assumed to be 10% per annum. Adjustment costs \( \zeta^I \) in capital investment are fixed at 5.2 (following estimations in Christoffel et al. 2008), while those in housing investment \( (\zeta^H) \) are set at 1.7 as estimated in Quint & Rabanal (2014). The elasticity of substitution across consumption goods \( \sigma \) is chosen so as to ensure a steady state mark-up of 1.35 and the Calvo parameter \( \xi \) is set to be 0.9, both in line with estimates obtained by Christoffel et al. (2008).

On the banking side, savings banks’ monitoring costs, \( \mu \), are assumed to be 0.2 as in Quint & Rabanal (2014) and Carlstrom & Fuerst (1997). For the borrowers loan-to-value ratio, we use the estimated value of \( m = 0.55 \) from Iacoviello (2005). The steady state standard deviation of the loan return shock, \( \sigma_\omega \), and the bank dividend payment rate, \( 1 - \gamma \), are calibrated to 3.39 percentage points and 4.53\% respectively to target a steady-state value of the loan-to-value ratio \( \bar{\omega} \) of 0.9 in-line with typical industry leverage ratios, and a default rate of banks of around 0.4\% in steady state, matching the average historical default rate in the banking industry over the period 1970-2010, as well as an equilibrium spread for mortgage and interbank loans of around 1.75 percentage points and 10 basis points respectively over the policy rate, in line with typical measures of the average spreads (see e.g. Housing Finance in the Euro Area 2009). The share of cross-border intra-Euro area interbank borrowing \( \tau^{IB} \) is set at 0.75 to match figures reported by Colangelo & Lenza (2013). The elasticity of substitution between domestic and foreign interbank funding \( \theta^{IB} \) is fixed at 2, implying that these sources of funding are not perfect substitutes.

On the policy side, we fix the share of government spending in GDP to 20\%. Together with the other parametrizations of our model, this
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>Fraction of savers</td>
<td>0.65</td>
</tr>
<tr>
<td>$\beta^B$</td>
<td>Borrowers discount factor</td>
<td>0.89</td>
</tr>
<tr>
<td>$\epsilon_C$</td>
<td>Habits formation</td>
<td>0.564</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Inverse Frisch elasticity</td>
<td>0.4</td>
</tr>
<tr>
<td>$\theta_C$</td>
<td>Elasticity between domestic goods/imports</td>
<td>1.9</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share of production</td>
<td>0.3</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Household discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\delta^K$</td>
<td>Capital depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\delta^D$</td>
<td>Housing depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\zeta^K$</td>
<td>Capital adjustment parameter</td>
<td>5.2</td>
</tr>
<tr>
<td>$\zeta^D$</td>
<td>Housing adjustment parameter</td>
<td>1.75</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Calvo parameter</td>
<td>0.9</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Monitoring costs</td>
<td>0.2</td>
</tr>
<tr>
<td>$m$</td>
<td>Borrowers LTV ratio</td>
<td>0.55</td>
</tr>
<tr>
<td>$\theta_{IB}$</td>
<td>Elasticity of substitution between domestic/cross-border interbank credit</td>
<td>2</td>
</tr>
<tr>
<td>$\zeta_{IB}$</td>
<td>Cross-border interbank cost coefficient</td>
<td>0.5</td>
</tr>
<tr>
<td>$\phi^y$</td>
<td>Taylor rule response to output growth</td>
<td>0.15</td>
</tr>
<tr>
<td>$\phi^\pi$</td>
<td>Taylor rule response to inflation</td>
<td>1.9</td>
</tr>
<tr>
<td>$\phi^r$</td>
<td>Taylor rule persistence</td>
<td>0.87</td>
</tr>
<tr>
<td>$\bar{\pi}$</td>
<td>Inflation target</td>
<td>1.005</td>
</tr>
</tbody>
</table>

Table 5.2: Parametrisation of model parameters.

ensures that we are able to get close matches of the relative spending shares of consumption (59%), investment (21%) and housing investment (4.5%) in GDP with their empirical first moments for the Euro area as a whole. Regarding monetary policy, we follow Christoffel et al. (2008) and set the central bank response to inflation $\phi^\pi$ to 1.9.
5.5 Numerical Results and Analysis

and to output growth $\phi^y$ to 0.15. Policy inertia is set at 0.87. For the LTRO, average loan duration $\phi$ is calibrated to be 0.9213 with a view to matching the ECB’s latest series of three-year loans.

Finally, the standard deviations and persistence coefficients of the shock processes are largely taken from Christoffel et al. (2008), with the exception of the risk shock, which is taken from Quint & Rabanal (2014), and the government spending shock, which has been calibrated on the basis of estimates obtained by Smets & Wouters (2003). For the interbank risk premium shock, for which no estimates in the literature are available, we assume a persistence of 0.8 and a standard deviation of 0.2. These are shown in table 5.3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_A$</td>
<td>Technology shock</td>
<td>0.0126</td>
</tr>
<tr>
<td>$\sigma_G$</td>
<td>Government spending shock</td>
<td>0.00325</td>
</tr>
<tr>
<td>$\sigma_{IB}$</td>
<td>Interbank cost shock</td>
<td>0.2</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>Risk shock</td>
<td>0.00339</td>
</tr>
<tr>
<td>$\sigma_M$</td>
<td>Monetary policy shock</td>
<td>0.00115</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>Technology shock</td>
<td>0.9</td>
</tr>
<tr>
<td>$\rho_G$</td>
<td>Government spending shock</td>
<td>0.95</td>
</tr>
<tr>
<td>$\rho_{IB}$</td>
<td>Interbank cost shock</td>
<td>0.8</td>
</tr>
<tr>
<td>$\rho_r$</td>
<td>Risk shock</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Table 5.3: Parametrisation of shock parameters.

To evaluate the model dynamics, we compute a second order Taylor approximation of the decision and transition functions of the model and simulate impulse response functions. We draw particular focus on shocks to the risk of the loan return shocks, in addition to the asymmetric total factor productivity and government spending shocks, and a standard monetary policy shock. We then discuss
the alternative policy measures. To provide comparison, we simulate a version of the model with the spread between final private sector lending rates and the policy rate fixed to the steady state value, and a version with the the cross-border interbank borrowing cost switched off. Whilst fixing the spreads might seem an ad-hoc modification, this allows us to compare the model dynamics with the same steady state but without the financial accelerator mechanism, as suggested in Bernanke et al. (1999).

5.5.1 Monetary Policy Shock

Figure 5.1 shows the impulse response to a reduction in the policy rate and highlights the financial accelerator in action. The interest rate drop increases private sector demand as expected, but, in the model in which banks face idiosyncratic uncertainty on loan returns, the shock strengthens the banks balance sheets which lowers their funding costs. This causes the private sector lending rates to fall, leading to an increase in mortgage loans and capital investment. This feeds back into the overall aggregate demand providing an additional boost to total output. The plotted variable $s(\cdot)$ is the external finance premium, or, alternatively, the investment wedge, to relate it to the other
chapters in this thesis. Although there is a large impact on mortgage issuance, just under 5% increase for a policy rate fall of just less than 10 basis points, the impact on total housing investment is rather more muted. This is the result of distributional issues introduced by the friction. Whilst the friction lowers borrowing costs for firms and impatient households, the policy maker keeps the interest rate higher relative to the fixed spread economy due to the increased output and inflation resulting in higher savings rates for savers who consume less non-durable and durable goods in response. This effect offsets the increase in housing investment coming from borrowers. As the shock is uniform across the monetary union, the response of each economy to the policy shock are entirely identical. This also implies that the net foreign asset position of each economy is unchanged so causing no impact on the cross-border interbank market friction.

5.5.2 Home Country Productivity Shock

We now consider an asymmetric negative supply shock in just one of the countries. Suppose the home country draws a 1 standard deviation reduction in total factor productivity. In a standard closed-economy model, the policy maker will cut the policy rate to stimulate demand and smooth the impact of the shock. Within a monetary union, the interest rate cut will have a impact in all economies, and there will be further effects via cross-border spillovers. A selection of the impulse response functions are plotted in figure 5.2 comparing to the economy without the cost applied to cross-border interbank borrowing, but still with the financial accelerator coming from the idiosyncratic asset return shock. We find that the friction generates significant distributional effects between the countries whilst leaving the response of union-wide output and inflation closely matched across the two models. As the monetary authority only reacts to union-wide fluctuations, the policy response in both models is similar. In the home country, reduced productivity leads to an output decline and increase in prices causes by a downward shift in the supply curve. The policy maker reacts to an increase in inflation with an interest rate rise; the greater weight on inflation is important here as the response could change sign with a higher weight on output. In the home coun-
Figure 5.2: Impulse response functions to a negative transitory shock to home country total factor productivity comparing baseline model with a version with $\zeta_{IB} = 0$ and a version without any financial frictions. Deviations in levels from the deterministic steady state.

An increase in prices and higher savings rates generates a decline in demand for real consumption from the savers, and without frictions there are large falls in cross-border interbank borrowing and a slow decline in capital investment. The falls in cross-border interbank borrowing stem from higher real interest rates in the foreign country where the increase in the inflation rate is smaller; this tightens the participation constraint for foreign banks to lend in the interbank market, raising cross-border interbank rates for home lending banks. There is a fall in net exports due to the higher relative prices of domestically produced goods leading to a fall in the net foreign asset position of the economy to fund the trade deficit, despite the fall in
cross-border interbank credit; the savings banks increase reliance on central bank refinance operations for funds. The interest rate increase raises borrowing costs, but the reduction in saver demand for housing reduces house prices causing an expansion in mortgage loans. In the foreign country, there is an small decline in output due to decreased investment and domestic consumption, offset by higher net export demand.

With the financial accelerator mechanism but with $\zeta_{IB} = 0$, in the home country, the interest rate rise actually increases demand for savings sufficiently to generate a small initial increase in investment and rise in the cost of capital. This has important consequences as it strengthens the balance sheet position of the lending bank which lowers their funding rates, causing a fall in the external finance premium, and feeding through to higher relative demand for mortgage credit and capital investment. The accelerator mechanism reduces saver consumption demand relative to the friction-less case which causes a smaller increase in inflation. The fall in demand for cross-border interbank funds is much reduced due to the lower interbank rates, and the savings bank borrow less from the central bank. In the foreign country, the interaction between the policy rate hike and the financial accelerator dominates the cross-border effects, and we observe the typical response one might expect from a monetary policy shock, as discussed above. Demand falls from savers whom increase savings, borrowers experience a negative wealth effect that lowers both demand for consumption and credit, and firms reduce investment. A resulting fall in the relative price of capital weakens the balance sheet of the lending bank, increasing borrowing costs in the economy. The result is a weakening of the balance of trade in the home country relative to the no friction case with the additional credit via the interbank market.

Considering the baseline case, with the cross-border interbank funding cost in addition to the costly state verification friction, we find some features of the last model are enhanced whilst some are reversed. Firstly, the effect of the financial accelerator is reduced, as lending banks face higher funding costs from the interbank market. The drop in cross-border interbank borrowing is greater than both the no-friction case, and the financial accelerator case. The higher inter-
bank funding costs lead to a much larger drop in investment causing a deeper recession, and a greater reduction in interbank funds which lead to lower relative imports. The foreign country receives the benefit as funds that would otherwise have invested in home-country capital are invested in foreign country capital, leading to an increase in price of capital, a strengthening of lending banks balance sheets, and, hence, a reversal of the financial accelerator mechanism causing an boom in total output.

5.5.3 Home Country Government Consumption Shock

The third shock is a reduction in home country government spending with plots shown in figure 5.3. In the absence of the cross-border interbank funding cost, the reduction in aggregate demand leads to a crowding in of saver consumption, but, due to an increase in the investment wedge caused by the external finance premium, an increase in the cost of credit that lowers borrower consumption and reduces investment. The lower relative prices of the home produced consumption good increases net exports leading to an increase in the net foreign asset position of the home country and reduced demand for cross-border interbank funds. Additional funds and positive demand shock coming from the lower policy rate in the foreign country contributes to a boost to investment and output. Introducing the cost that increases in the exposure to international debt raises the funding cost in the foreign country but reduces the cost in the home country. This acts to dampen the negative effects in the home country, and generate negative cross-border spillovers in the form of falls in investment and output in the foreign country.

5.5.4 Home Country Risk Shock

The fourth shock considered is an adverse risk shock to loan returns in the home country. Specifically this is a positive shock to the variance of the idiosyncratic loan return shock \( \omega(j) \). Whilst the chosen probability distribution is mean preserving, so ceteris paribus, the average loan return is unchanged, the shock raises the skewness of the
Figure 5.3: Impulse response functions to a transitory reduction in home country government consumption, comparing baseline model with a version with $\zeta_{IB} = 0$. Deviations in levels from the deterministic steady state.

distribution of $\omega(j)$, and the probability of a low $\omega(j)$ increases. This leads to increased bank default in the home country, so higher interbank funding rates and a reduction of credit in the home country. Plots of the impulse response functions are shown in figure 5.4. As the risk shock only effects the models with the costly state verification friction, we concentrate attention on the baseline model, the model with $\zeta_{IB} = 0$, and a version with $\zeta_\sigma = 0$. The latter includes the cross-border interbank cost that increases in the international debt position of the borrower country but switches off the element of the cost that is increasing in the risk. As is clear from figure 5.4, the cost only plays a particularly important role if the lender on the interbank market punishes additional risk in loan returns; in this case, the ad-
verse effects of the risk shock are worse for both economies. In the model with $\zeta_{IB} = 0$, the reduction in credit reduces investment and aggregate supply as a result, and reduces credit to borrowers leading to a fall in aggregate demand. There is an overall drop in prices and net exports increase following a small initial fall. Although there is a contraction in interbank borrowing, this is largely replaced by savings bank receiving credit from the central bank; this leads to a fall in the net foreign asset position of the home country, although this increases in the trade balance in the periods following the shock. The spillover to the foreign country generates similar responses although an order of magnitude smaller.
With $\zeta_\sigma = 0$, the initial fall in the net foreign asset position of the home country leads to an additional cost in accessing interbank credit which reduces the initial cross-border interbank borrowing in the home country and increases it in the foreign country. The higher cost of funding costs is passed on to the home country private sector causing a larger fall in investment, and output. There is a similarly sized relative increase in investment and output in the foreign country who are the beneficiaries of additional private sector credit. The reduction in exposure to interbank credit drives a higher net foreign asset position relative to the no-cost case, and so causing a shrinking of the additional cost of interbank credit. This results in only a small effect of the cost on aggregate outcomes.

If we allow creditors on the interbank market to discriminate against foreign borrowers on the basis of the variance of the loan return shock, $\sigma_\omega$, then the impact of the risk shock is more severe. The friction acts as an accelerator mechanism compared to the $\zeta_{IB}$ case; the cost of interbank funds increase, reducing the volume of credit in the home country and both capital investment and the volume of mortgages fall. The resulting contraction in output and drop in prices are deeper relative to either of the other models tested. In the foreign country, the spill-over effects of the risk-shock are enhanced, with a deeper decline in output, investment and prices. With less credit available to the home country as a whole, the net exports increase relative to the other tested models.

### 5.5.5 Policy Experiments

The monetary authority is given two additional tools that can be used to respond to shocks: purchasing asset-backed securities, and introducing multi-period refinancing operations.

#### 5.5.5.1 Asset Purchase Program

For this policy, the central bank raises funds in the bond market which are used to purchase baskets of assets from the lending banks. This is equivalent to the set-up in Gertler & Karadi (2011), whereby
the central bank can act as a commercial bank, taking deposits and lending to the private sector, but avoiding the agency problem inherent to the banking sector. This point is important, because, by acting as a commercial bank, the central bank reduces the risky assets on the lending banks balance sheet reducing the external finance premium. Figure 5.5 shows the response to temporary asset purchase programme. In the first period, the central bank purchases 2% of all available assets and holds them for 16 periods. The asset purchases reduces the leverage of lending banks and so the external finance premium. This reduces the cost of credit in the private sector increasing the demand for loans and mortgages. As both firms and households demand credit, the policy has both supply-side and demand side effects. The monetary policy rule causes the policy rate to counter the expansionary effects of the asset purchases but the indication is that there is an short-term expansionary role of asset purchases. As the lending banks become more highly leveraged after the policy is introduced, the external finance premium returns to the steady state value and some of the gains are reversed, the economy also responds to the contractionary monetary policy and there is a decline in economic activity once the purchase program completes. At this point, as the

Figure 5.5: Impulse response functions to a transitory central bank asset-purchase shock. The central bank purchases 2% of all non-financial private sector loans in the first period, returning to the lending banks after 16 periods. Deviations in levels from deterministic steady state.
lending banks become more highly leveraged due to the returned assets, there is a spike in the external finance premium and reduction in credit. In an episode when the interest rate is at the zero lower bound, the indication is that there could be role as an alternative measure.

Figure 5.6 plots impulse response functions to a risk shock with a 16-period asset purchase programme whereby the central bank purchases 2% of all loans in the economy. The policy, which is applied equally to both economies, is successful at smoothing deviations in the home country, although generates greater volatility in the foreign country. Due to the symmetric nature of the shock, there is no impact on net exports and the net foreign asset position of the two countries. The results change if the policy were focused on the home country.
alone. Figure 5.7 plots the same but with only home country assets purchased. The policy reduces the depth of the decline in output in the home country, although extending the period of recovery. The same is also true in the foreign country. As the policy is focused on supporting credit conditions in the home country, the country is able increase international finance to support a lower trade balance. These results suggest a targeted approach to the asset purchases could achieve more desirable outcomes.

Figure 5.7: Impulse response functions to a transitory risk shock with central bank home country asset-purchase policy response. Deviations in levels from deterministic steady state.
5.5.5.2 Long-term Refinance Operations

The second unconventional policy discussed is the availability of long-term refinance operations. For a tractable solution we follow Rudebusch & Swanson (2012) with geometrically declining repayments over an infinite horizon. This allows the introduction of multi-periods bonds with just a single additional state variable. As discussed above, we can compute the equivalent average loan duration, and average interest rate which allows a mapping between the geometrically declining repayment rate and a policy rate set by the Taylor Rule. One difficulty in the approach is that to either introduce the policy, or change the average duration, during a model simulation would cause the mapping between these variables to be lost. For example, the average loan duration stems from the continued existence of the loans with a fixed parameter \( \psi \), if \( \psi \) were time varying, it would be considerably more difficult to compute the average loan duration, or the average rate across the loans. For this reason, we draw comparison between a model in which the central bank arranges standard one-period credit, and a long-term refinancing case with \( \psi > 0 \) and duration \( d > 1 \) fixed. Consider the relationship between the deposit rate and the policy rate. Figure 5.8 plots the impulse response functions to a monetary policy shock comparing the baseline model with \( d = 1 \) to a version with \( d = 12 \). This equates to 3 year loan duration which matches the latest ECB LTRO policy. When the policy rate is cut, the model in which the refinance operations are available as three-year maturity loans generate a larger fall in the funding the rate of the banking sector, which stimulates private sector demand for consumption and credit. The increased demand strengthens the balance sheets of the lending banks which, via standard the financial accelerator mechanism, further enhances the stimulative effect of the policy rate cut.

Figure 5.9 plots the impulse response of a number of variables following a risk shock in the home country with duration \( d = 1 \) and \( d = 12 \). As before, the policy rate is cut following the shock, although with the multi-period refinancing, the cut set by the policy maker is much smaller due the the additional demand generated. An interesting feature is the asymmetric effect of introducing the policy with a larger effect on output, inflation and other real variables in the foreign coun-
try. With loan duration \( d = 1 \), the deposit rate in each country will be fixed to the policy rate by the zero arbitrage condition. Once \( d > 1 \) (\( \phi_t > 0 \)), this is no longer the case due to duration effects. As equations (5.3.2) and (5.3.5) highlight, the spread between the policy rate and the deposit rate depends on the household stochastic discounting. Specifically, using equations (5.3.2) and (5.3.5) we can give the spread as

\[
\frac{R_t}{R_t^S} = \frac{\mathbb{E}_t \left[ \Lambda_t^{i+1} \right]}{\left[ \phi_t (1 - \psi) \right]^{1/d}} \tag{5.5.1}
\]

where

\[
\phi_t = \mathbb{E}_t \left[ \Lambda_t^{i+1} \left( 1 + \phi_{t+1} \psi \right) \right]. \tag{5.5.2}
\]

This makes clear that asymmetric shocks will affect the spread differently in each country as a first-order effect, as is observed in the
impulse response plots. From equations (5.3.4) and (5.3.5), we find, to a first order approximation, that

$$\frac{R^S_t}{\mathbb{E}_t [1 + \phi_t \psi]} = \frac{R^{S*}_t}{\mathbb{E}_t [1 + \phi^*_t \psi]}$$

If the expected path of the nominal stochastic discount factor is greater in home than foreign, then it follows that $\mathbb{E}_t \phi_{t+1} > \mathbb{E}_t \phi^*_{t+1}$, which from equation (5.5.3), implies that $R^S_t > R^{S*}_t$. Of course, this is only strictly true to first order, but given that first-order effects dominate higher orders, it is likely to hold at higher orders even with time-varying risk premia. This phenomena is observed following the risk

Figure 5.9: Impulse response functions to a transitory risk shock with central bank with an without LTRO policy response. Deviations in levels from deterministic steady state.
shock; the home country experiencing the shock expects a higher future marginal utility of consumption relative to the foreign country and so it follows that $R_i^S > R_i^{S*}$. This stimulates relatively higher demand in the foreign country, which is enhanced further via the financial accelerator mechanism leading to a larger increase in economic activity. Because the positive impact of the LTRO policy has a larger impact on financial conditions in the foreign country, the net foreign asset position of the home country weakens leading to a higher increase in net exports. As with asset purchases, the long-term refinancing can support standard monetary policy to stimulate demand, but has a bigger impact on the less stressed economy.

5.6 CONCLUSION

In this chapter, a two-country general equilibrium model with interbank market frictions is proposed to match some observed features of the business cycle in the Euro monetary union. In the years following the global financial crisis, there was a fragmentation in the European interbank market as banks in more highly stressed countries experienced higher premiums accessing interbank market funds (see de Andoain et al. 2014). This led to a reduction in the volume of cross-border credit to these countries and a restriction of credit to the non-financial private sector. This also caused rapid adjustments of the trade balance in stressed countries which increased sharply. In the model proposed in this chapter, we show how the costly-state verification model of Townsend (1979) applied to over-the-counter interbank transactions can propagate and enhance the effects of shocks, including monetary policy shocks; this contradicts previous findings that the inclusion of an interbank market in a DSGE model tempers the effects of monetary policy (see e.g. Hilberg & Hollmayr 2011). Secondly, that applying further transactional costs which depend on the debt exposure and risk in the destination country can help match the observations.

Following adverse shocks occurring in the home country, we find the presence of transactional costs worsen the financial conditions due to higher interbank funding costs; this leads to higher borrowing costs
in the non-financial private sector that have an adverse effect on supply via the firm credit channel, and demand via the mortgage loan channel. The higher interbank borrowing costs also cause a reduction in credit that leads to increases in the trade balance. Whilst there are negative spill-overs to the foreign country from home-country adverse shocks, the presence of the transactional costs improves conditions in the foreign countries whom benefit from lower relative funding costs that stimulate both supply and demand in the non-financial private sector. If the adverse shock in the home country is a purely demand-side shock, a reduction in government spending for instance, there is a crowding-in effect that increases demand for exports from the foreign country. This leads to reduction in the demand for international interbank finance and so in this respect, the frictions can provide support for the home country.

In addition to standard monetary policy, we investigate two unconventional measures; multi-period central bank refinance operations, and central bank asset purchases. Both policy measures can support the stimulative effects of standard monetary policy but with different side effects and asymmetries. As the long-term refinance operations allow a spread to emerge between the policy and deposit rates, the savers in each country can observe different savings rates in the presence of the policy. Following an adverse shock, the spread between the policy and deposit rates is typically larger in the country that experiences smaller deviations in saver consumption levels due to the impact on the stochastic discount factor; this leads to a larger stimulus effect of the policy in the country that does not directly draw the shock. Because of this feature, if the transactional costs in the inter-bank market generate undesirable distributional outcomes following asymmetric shocks, this policy may not be an appropriate policy response. The asset purchase policy can also support interest rate cuts to stimulate economic activity. This works through the bank balance sheet channel; if the purchase is large enough, the banks will be less highly leveraged which will reduce the external finance premium. The policy program is particularly successful if the central purchases risky assets in the more highly stress country. In reality, the ECB has purchased less risky assets in less stressed economies with the aim of
increasing liquidity; the policy as prescribed in this model may not be politically feasible.


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CHAPTER 4 APPENDICES

A.1 GENERAL EQUILIBRIUM MODEL CONDITIONS

Equilibrium conditions for \( K_t, u_t, e_t, x^s_t, x^r_t \)

\[
1 = \mathbb{E}_t [\Lambda_{t,t+1} R_{t+1}] \quad \text{(A.1.1)}
\]

\[
\chi \frac{C_t}{1 - H_t} = W_t \quad \text{(A.1.2)}
\]

\[
\mathbb{E}_t [\Lambda_{t,t+1} (\tau_{t+1} - W_{t+1} (\lambda x^s_t + (1-\lambda) x^r_t)) - e] = 0 \quad \text{(A.1.3)}
\]

\[
\mathbb{E}_t [\Lambda_{t,t+1} (\lambda (R^s_{k,t+1} - 1) - (1-\lambda) p^r_{t+1} (R^r_{k,t+1} - \tau^s_t))] - \varrho_t \lambda + \varphi^s_t - \psi_t = 0 \quad \text{(A.1.4)}
\]

\[
\mathbb{E}_t [\Lambda_{t,t+1} (1-\lambda) (p^r_{t+1} R^r_{k,t+1} - 1)] - \varrho_t (1-\lambda) - \varphi^r_t + \psi_t = 0 \quad \text{(A.1.5)}
\]

where for \( \Lambda_{t-1,t}, R_t, \hat{k}_t, \hat{h}_t, W_t, H_t, Y_t, I_t, C_t, \gamma_t, x^s_t, x^r_t, \pi_t, R^s_t, R^r_t, \tau^s_t, \tau^r_t, \varrho_t, \psi_t, \varphi^s_t, \varphi^r_t \)

\[
\Lambda_{t-1,t} = \beta \frac{U_{C,t}}{U_{C,t-1}} \quad \text{(A.1.6)}
\]

\[
R_t = 1 + \lambda \frac{x^s_t}{\gamma_t-1} (\tau^s_t - 1) + (1-\lambda) \frac{x^r_t}{\gamma_t-1} (p^r_t \tau^r_t - 1) \quad \text{(A.1.7)}
\]

\[
\hat{k}_t = (\lambda x^s_t + (1-\lambda) x^r_t p^r_t \omega^r_t) (\kappa + e) e_{t-1} \quad \text{(A.1.8)}
\]

\[
\hat{h}_t = 1 - e_{t-1} - u_t \quad \text{(A.1.9)}
\]

\[
W_t = (1 - \alpha) A_t \left( \frac{\hat{k}_t}{\hat{h}_t} \right)^a \quad \text{(A.1.10)}
\]

\[
H_t = (1 - [1 - (\lambda x^s_{t-1} + (1-\lambda) x^r_{t-1})] e_{t-1} - u_t) \quad \text{(A.1.11)}
\]

\[
Y_t = A_t \hat{k}_t \hat{h}_t^{1-a} \quad \text{(A.1.12)}
\]

\[
I_t = K_t + \delta (\lambda x^s_{t-1} + (1-\lambda) x^r_{t-1}) (e + \kappa) e_{t-1} - K_{t-1} \quad \text{(A.1.13)}
\]

\[
C_t = Y_t - I_t \quad \text{(A.1.14)}
\]

\[
\gamma_t = (K_t - e_t e) \frac{1}{e_t \kappa} \quad \text{(A.1.15)}
\]

\[
\hat{x}^s_t = \hat{x}^s_t \quad \text{(A.1.16)}
\]
\[
x_i^f = \frac{x_i^f}{\gamma_t}
\]
(A.1.17)

\[
\pi_t = (1 - \lambda) p_{t-1}^f (R_{t}^f - R_{t-1}^f) + (\lambda p_{t-1}^s x_t^s R_{t}^f + (1 - \lambda) p_{t-1}^s x_t^s R_{t-1}^f) \epsilon
\]
(A.1.18)

\[
R_{t}^s = \alpha Z_t \left( \frac{\hat{h}_t}{\hat{k}_t} \right)^{1-\alpha} + (1 - \delta)
\]
(A.1.19)

\[
R_{t}^r = \omega_t R_{t}^s
\]
(A.1.20)

\[
\tau_t^s = R_{t}^s
\]
(A.1.21)

\[
\tau_t^r = R_{t}^r - (R_{t}^r - R_{t}^f) \frac{x_{t-1}^s}{x_{t-1}^r}
\]
(A.1.22)

\[
\phi_t (\gamma - \lambda x_t^s - (1 - \lambda) x_t^r) = 0
\]
(A.1.23)

\[
\psi_t (x_t^r - x_t^s) = 0
\]
(A.1.24)

\[
\varphi_t^s x_t^s = 0
\]
(A.1.25)

\[
\varphi_t^r (1 - x_t^r) = 0
\]
(A.1.26)

which are subject to the inequality constraints.

\[
\varrho_t \geq 0
\]
(A.1.27)

\[
\psi_t \geq 0
\]
(A.1.28)

\[
\varphi_t^s \geq 0
\]
(A.1.29)

\[
\varphi_t^r \geq 0
\]
(A.1.30)

\[
\gamma \geq \lambda x_t^s + (1 - \lambda) x_t^r
\]
(A.1.31)

\[
0 \leq x_t^s \leq x_t^r \leq 1
\]
(A.1.32)

plus stationary AR(1) processes for \( \omega_t, p_t^r \) and \( \delta_t \).

### A.1.1 Deterministic Steady State

Calibrate \( \kappa \) to target \( \hat{\kappa} \equiv \frac{\kappa}{\kappa + \epsilon} = 0.6263; \epsilon \) to match \( \hat{w} = \frac{1 - e^{-u}}{e} = 15.8; p^r \) to target \( \hat{\gamma} = 0.8409. \)

Guess and verify \( x^s \) and \( x^r \).

We can then solve \( \beta (\pi - W (\lambda x^s + (1 - \lambda) x^r)) = \epsilon \) for \( p^r \) with

\[
\gamma = \hat{\gamma}
\]
(A.1.33)

\[
\Lambda = \beta
\]
(A.1.34)
\[ R = \frac{1}{\beta} \]
\[ R^s = \frac{R\gamma - \gamma + \lambda x^s + (1 - \lambda) x^r}{\lambda x^s + (1 - \lambda) (p'x'\omega' - p'(\omega' - 1)x^s)} \]
\[ \hat{k} = \left( \frac{\alpha}{R^s - 1 + \delta} \right)^{\frac{1}{1 - \alpha}} \]
\[ W = (1 - \alpha) \left( \frac{\hat{k}}{\hat{h}} \right)^{\alpha} \]
\[ e = \frac{\hat{k}}{\hat{h}} \left( \frac{\hat{w}}{1 - \hat{k}} \right) \]
\[ \kappa = \frac{\hat{k}}{1 - \hat{k}} \]

The labour supply condition is \( \chi \frac{C}{1 - H} = W \), substituting this into the aggregate resource constraint gives
\[ \hat{k} \left( \frac{\hat{h}}{\hat{k}} \right)^{1 - \alpha} - \delta (\lambda x^s + (1 - \lambda) x^r) (e + \kappa) e = \frac{1}{\chi} W (1 - H) \]  
(A.1.41)
\[ \hat{k} \left( \frac{\hat{h}}{\hat{k}} \right)^{1 - \alpha} - \delta (\lambda x^s + (1 - \lambda) x^r) (e + \kappa) e = WH^{-\phi} \]  
(A.1.42)

Using \( H = (1 - [1 - (\lambda x^s + (1 - \lambda) x^r)] e - u), \) and the identities of \( \hat{k} \) and \( \hat{h} \) gives
\[ \left( (\lambda x^s + (1 - \lambda) x^r p'\omega') \left( \frac{\hat{k}}{\hat{k}} \right)^{1 - \alpha} - \delta (\lambda x^s + (1 - \lambda) x^r) \right) (e + \kappa) e \]
\[ = \frac{1}{\chi} W \left( 1 - \left( \hat{h} + (\lambda x^s + (1 - \lambda) x^r) e \right) \right). \]
(A.1.43)

We calibrate the number of workers per firm \( \frac{\hat{h}}{\hat{k}} = \hat{w} \), so this condition is written
\[ \hat{h} = \frac{W}{\left( \chi \left( \frac{\hat{h}}{\hat{k}} \right)^{1 - \alpha} - \chi \delta + W (\gamma - (1 - \lambda)(1 - p')) \frac{1}{\hat{w}} + 1 \right) \left( \hat{k} \right)^{\frac{1}{\alpha}}} \]
(A.1.44)
\[ \hat{h} = \left( \frac{\frac{1}{2} W}{(\lambda x^s + (1 - \lambda) x^r p'\omega')(\frac{\hat{k}}{\hat{k}})^{1 - \alpha} + \chi \delta + W (\gamma - (1 - \lambda)(1 - p')) \frac{1}{\hat{w}} + 1 + \frac{1}{2} W} \right) \]
(A.1.45)
\[ \hat{k} = \frac{\hat{k}}{\hat{h}} \left( \frac{W ((\lambda x^s + (1 - \lambda) x^r p') \frac{1}{\hat{w}} + 1)^{-\phi} \left( \frac{\hat{k}}{\hat{k}} \right)^{\frac{1}{1 - \phi}} - \delta (\lambda x^s + (1 - \lambda) x^r) \right)^{\frac{1}{1 - \phi}} \right). \]  
(A.1.46)
We then write
\[ \hat{h} = \frac{\hat{k}}{k} \] (A.1.47)
\[ Y = \hat{k} \hat{h}^{1-a} \] (A.1.48)
\[ e = \frac{\hat{h}}{\hat{w}} \] (A.1.49)
\[ H = (\lambda x^s + (1 - \lambda) x^r) e + \hat{h} \] (A.1.50)
\[ C = Y - \delta (\lambda x^s + (1 - \lambda) x^r) (\epsilon + \kappa) e \] (A.1.51)
\[ K = (\gamma \kappa + \epsilon) e \] (A.1.52)
\[ I = \delta (\lambda x^s + (1 - \lambda) x^r) (\epsilon + \kappa) e \] (A.1.53)
\[ \pi = \{(1 - \lambda) p^r x^r (\omega^r - 1) \kappa + (\lambda x^s + (1 - \lambda) p^r x^r \omega^r) \epsilon\} R^s \] (A.1.54)
\[ R^r = \omega^r R^s \] (A.1.55)
\[ \tau^s = R^s \] (A.1.56)
\[ \tau^r = R^r - (R^r - R^s) x^s \] (A.1.57)
\[ \hat{x}^s = \frac{x^s}{\gamma} \] (A.1.58)
\[ \hat{x}^r = \frac{1}{\gamma} \] (A.1.59)

Then using
\[ \beta (\lambda (R^s - 1) - (1 - \lambda) p^r (R^r - R^s)) = q \lambda - \varphi^s + \psi \] (A.1.60)
\[ \beta (1 - \lambda) (p^r R^r - 1) = q (1 - \lambda) + \varphi^r - \psi \] (A.1.61)

we find

1. If
\[ p^r R^r < (\lambda p^s + (1 - \lambda) p^r) R^s \] (A.1.62)

then
\[ \beta (\lambda (R^s - 1) - (1 - \lambda) p^r (R^r - R^s)) = q \lambda - \varphi^s + \psi \] (A.1.63)
\[ \beta (1 - \lambda) (p^r R^r - 1) = q (1 - \lambda) + \varphi^r - \psi \] (A.1.64)
\[ \varphi^r = 0 \] (A.1.65)
\[ \varphi^s = 0 \] (A.1.66)
\[ \varphi = \beta ([\lambda + (1 - \lambda) p^r] R^s - 1) \] (A.1.67)
\[ \psi = (1 - \lambda) \beta ([\lambda + (1 - \lambda) p^r] R^s - p^r R^r) \] (A.1.68)
A.1 general equilibrium model conditions

2. If

\[ \lambda + (1 - \lambda) p' R' < (\lambda p^s + (1 - \lambda) p') R^s < p' R' \]  \hspace{1cm} (A.1.69)

then

\[ \phi = 0 \]  \hspace{1cm} (A.1.70)
\[ \phi^s = 0 \]  \hspace{1cm} (A.1.71)
\[ \rho = \beta \left( \left( 1 + \frac{1 - \lambda}{\lambda} p' \right) R^s \right) - \frac{1 - \lambda}{\lambda} p' R' - 1 \]  \hspace{1cm} (A.1.72)
\[ \varphi^r = \beta \frac{1 - \lambda}{\lambda} [p' R' - (\lambda + (1 - \lambda) p') R^s] \]  \hspace{1cm} (A.1.73)

3. If

\[ (\lambda + (1 - \lambda) p') R^s < \lambda + (1 - \lambda) p' R' \]  \hspace{1cm} (A.1.74)

then

\[ \rho = 0 \]  \hspace{1cm} (A.1.75)
\[ \psi = 0 \]  \hspace{1cm} (A.1.76)
\[ \phi^s = \beta [\lambda + (1 - \lambda) p' R' - (\lambda + (1 - \lambda) p') R^s] \]  \hspace{1cm} (A.1.77)
\[ \varphi^r = \beta (1 - \lambda) (p' R' - 1) \]  \hspace{1cm} (A.1.78)

4. If

\[ (\lambda + (1 - \lambda) p') R^s = p' R' \]  \hspace{1cm} (A.1.79)

then

\[ \psi = 0 \]  \hspace{1cm} (A.1.80)
\[ \phi^s = 0 \]  \hspace{1cm} (A.1.81)
\[ \varphi^r = 0 \]  \hspace{1cm} (A.1.82)
\[ \rho = \beta (p' R' - 1) \]  \hspace{1cm} (A.1.83)

The finance probabilities are undetermined so we choose \( x^r = 1, \)
\[ x^s = \frac{\gamma + 1 - \lambda}{\lambda}. \]

5. If

\[ (\lambda + (1 - \lambda) p') R^s = \lambda + (1 - \lambda) p' R' \]  \hspace{1cm} (A.1.84)
then

\[ \varphi \lambda - \varphi^s + \psi = 0 \quad (A.1.85) \]

\[ \beta (1 - \lambda) (p'R' - 1) = \varphi (1 - \lambda) + \varphi' - \psi > 0 \quad (A.1.86) \]

so

\[ \psi = 0 \quad (A.1.87) \]

\[ \varphi^s = 0 \quad (A.1.88) \]

\[ \varphi = 0 \quad (A.1.89) \]

\[ \varphi' = \beta (1 - \lambda) (p'R' - 1) \quad (A.1.90) \]

\[ x' = 1 \quad (A.1.91) \]

and then using the condition for \( R^s \) above and the remaining first order condition

\[ R^s = \frac{\lambda}{\lambda - (1 - \lambda) p'(\omega' - 1)} \quad (A.1.92) \]

\[ x^s = \frac{R\gamma - \gamma + (1 - \lambda) x' - R^s (1 - \lambda) p'x'\omega'}{(R^s - 1) \lambda - R^s (1 - \lambda) p'(\omega' - 1)} \quad (A.1.93) \]

\[ (\lambda - (1 - \lambda) p'(\omega' - 1)) R^s - \lambda \quad (A.1.94) \]

\[ R^s = \frac{R\gamma - \gamma + \lambda x^s + (1 - \lambda) x'}{\lambda x^s + (1 - \lambda) (p'x'\omega' - p'(\omega' - 1)x')} \quad (A.1.95) \]

\[ \lambda x^s + (1 - \lambda) (p'x'\omega' - p'(\omega' - 1)x') \quad (A.1.96) \]

A.1.2 First-best economy steady state

Use the parameters from the adverse selection economy, so no calibration. \( \gamma = 1 \) and so \( x^s = x' \). It then follows then that \( K = \hat{k} \). We can write the deterministic steady state as follows.

\[ \Lambda = \beta \quad (A.1.97) \]

\[ R = \frac{1}{\beta} \quad (A.1.98) \]

\[ R^s = R \quad (A.1.99) \]
The labour supply condition is $\chi \frac{e}{1-H} = W$, substituting this into the aggregate resource constraint gives

$$K \left( \left( \frac{\hat{h}}{K} \right)^{1-a} - \delta \right) = \frac{1}{\chi} W (1-H) \quad (A.1.103)$$

Using $H = (1-u)$, $K = (\kappa + e)e$, and $\hat{h} = 1 - e - u$ gives

$$K = \frac{1}{\chi} W \left( \frac{h}{K} \right)^{1-a} - \delta + \frac{1}{\chi} W \left( \frac{h}{K} + \frac{1}{\kappa + e} \right). \quad (A.1.104)$$

We then write

$$\hat{h} = \frac{h}{K} \quad (A.1.105)$$

$$Y = \hat{k} \hat{h}^{1-a} \quad (A.1.106)$$

$$e = \frac{K}{\kappa + e} \quad (A.1.107)$$

$$H = e + \hat{h} \quad (A.1.108)$$

$$C = Y - \delta K \quad (A.1.109)$$

$$I = \delta K \quad (A.1.110)$$

$$R^r = \omega^r R^s \quad (A.1.111)$$

$$\tau^s = R^s \quad (A.1.112)$$

$$\tau^r = R^r \quad (A.1.113)$$

$$\hat{x}^s = \frac{1}{\gamma} \quad (A.1.114)$$

$$\hat{x}^r = \frac{1}{\gamma} \quad (A.1.115)$$
A.2 General Equilibrium Model with Nominal Rigidities Conditions

Equilibrium conditions for $K_t, u_t, c_t, x_t, x_t^t$. Rates $R_t, R_t^\rho, R_t^\tau, \tau_t^t$ and $\tau_t^r$, plus profit rate $\tau_t$ in nominal terms, all stocks in real consumption units.

\[ 1 = E_t \left[ \frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} R_{t+1} \right] \]  
\[ 1 = E_t \left[ \frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} R_t^\rho \right] \]  
\[ \chi_t \frac{C_t}{1 - H_t} = W_t \]  
\[ E_t \left[ \Lambda_{t,t+1} \left( \frac{\pi_t}{\Pi_{t,t+1}} - W_{t+1} (\lambda x_t^t + (1 - \lambda) x_t^r) \right) - \epsilon Q_t \right] = 0 \]  
\[ E_t \left[ \Lambda_{t,t+1} \left( \lambda \left( \frac{R_t^\tau}{\Pi_{t,t+1}} - 1 \right) - (1 - \lambda) p_t^\tau \left( \frac{R_t^\tau}{\Pi_{t,t+1}} - \tau_t^\tau \right) \right) \right] \]  
\[ - \varphi_t^\tau + \psi_t = 0 \]  
\[ E_t \left[ \Lambda_{t,t+1} (1 - \lambda) \left( p_t^r \left( \frac{R_t^\tau}{\Pi_{t,t+1}} - 1 \right) \right) \right] - \varphi_t (1 - \lambda) - \varphi_t^r + \psi_t = 0 \]  
\[ \Omega_{1,t} \equiv \frac{\sigma}{\sigma - 1} Y_t MC_t + \xi E_t \left[ \Lambda_{t,t+1} (\Pi_{t,t+1})^\sigma \Omega_{1,t+1} \right] \]  
\[ \Omega_{2,t} \equiv Y_t + E_t \left[ \xi \Lambda_{t,t+1} \left( \Pi_{t,t+1} \right)^{\sigma - 1} \Omega_{2,t+1} \right] \]  
\[ 1 = \xi (\Omega_{t-1,t})^{\sigma - 1} + (1 - \xi) \left( \frac{P_t}{P_1} \right)^{1 - \sigma} \]  
\[ S_t = \xi \Omega_{t-1,t} S_{t-1} + (1 - \xi) \left( \frac{\Omega_{t,t}}{\Omega_{2,t}} \right)^{-\sigma} \]  
\[ R_t^\rho = \left[ \bar{R}^\rho \left( \frac{\Pi_{t-1,t}}{\Pi^*} \right)^{\eta^*} \left( \frac{Y_t}{\bar{Y}} \right)^{\eta^*} \right] \left( R_{t-1}^\rho \right)^{\eta'} \exp(\epsilon_{M,t}) \]  
\[ I_t \left[ 1 - \phi \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right] \]  
\[ = K_t + \delta (\lambda x_t^s + (1 - \lambda) x_t^r) (\epsilon + \kappa) e_t - K_{t-1} \]  
\[ \Lambda_{t-1,t} = \beta \frac{U_{C,t}}{U_{C,t-1}} \]  

where for $\Lambda_{t-1,t}, R_t, \bar{k}_t, \bar{h}_t, W_t, H_t, Y_t, I_t, C_t, Y_t, x_t^s, x_t^r, \pi_t, R_t^\rho, R_t^\tau, \tau_t^s, \tau_t^r, e_t, \psi_t, \varphi_t^s, \varphi_t^r$
\[ R_t = \Pi_{t-1} \frac{Q_t}{Q_{t-1}} + \lambda \frac{x_t}{\gamma_t} \left( \tau_t - \Pi_{t-1} \frac{Q_t}{Q_{t-1}} \right) + (1 - \lambda) \frac{x_{t-1}}{\gamma_{t-1}} \left( p_t^r \tau_t - \Pi_{t-1} \frac{Q_t}{Q_{t-1}} \right) \]

(A.2.14)

\[ \dot{k}_t = (\lambda x_t^i + (1 - \lambda) x_t^i p_t^i \omega_t^i) (\kappa + \epsilon) e_{t-1} \]

(A.2.15)

\[ \dot{h}_t = 1 - e_{t-1} - u_t \]

(A.2.16)

\[ W_t = (1 - \alpha) A_t M C_t \left( \frac{\dot{k}_t}{k_t} \right)^{\alpha} \]

(A.2.17)

\[ H_t = (1 - [1 - (\lambda x_{t-1}^i + (1 - \lambda) x_{t-1}^i)] e_{t-1} - u_t) \]

(A.2.18)

\[ Y_t = A_t \dot{k}_t h_t^{1 - \alpha} \frac{1}{S_t} \]

(A.2.19)

\[ C_t = Y_t - I_t \left[ 1 - \phi \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right] \]

(A.2.20)

\[ \gamma_t = (K_t - e_t \epsilon) \frac{1}{\epsilon_t K} \]

(A.2.21)

\[ \dot{x}_t^i = \frac{x_t^i}{\gamma_t} \]

(A.2.22)

\[ \dot{x}_t^r = \frac{x_t^r}{\gamma_t} \]

(A.2.23)

\[ \pi_t = (1 - \lambda) p_t^i x_{t-1}^i (R_t^r - R_t^s) Q_{t-1} \kappa \]

\[ + (\lambda p_t^i x_{t-1}^i R_t^s + (1 - \lambda) p_t^i x_{t-1}^i R_t^r) Q_{t-1} \epsilon \]

(A.2.24)

\[ R_t^s = \left( \alpha M C_t Z_t \left( \frac{\dot{k}_t}{k_t} \right)^{1 - \alpha} \right) + (1 - \delta) Q_t \]

(A.2.25)

\[ R_t^r = \omega_t^r R_t^s \]

(A.2.26)

\[ \tau_t^s = R_t^s \]

(A.2.27)

\[ \tau_t^r = R_t^r - (R_t^r - R_t^s) \frac{x_{t-1}^r}{x_{t-1}^s} \]

(A.2.28)

\[ \phi_t (\gamma - \lambda x_t^r - (1 - \lambda) x_t^s) = 0 \]

(A.2.29)

\[ \psi_t (x_t^r - x_t^s) = 0 \]

(A.2.30)

\[ \phi_t^r x_t^s = 0 \]

(A.2.31)

\[ \phi_t^r (1 - x_t^r) = 0 \]

(A.2.32)

which are subject to the inequality constraints.

\[ \phi_t \geq 0 \]

(A.2.33)

\[ \psi_t \geq 0 \]

(A.2.34)
\( \varphi_t^s \geq 0 \) \hfill \text{(A.2.35)}

\( \varphi_t^r \geq 0 \) \hfill \text{(A.2.36)}

\( \gamma \geq \lambda x_t^s + (1 - \lambda) x_t^r \) \hfill \text{(A.2.37)}

\( 0 \leq x_t^s \leq x_t^r \leq 1 \) \hfill \text{(A.2.38)}

plus stationary AR(1) processes for \( \omega_t, p_t, \) and \( \delta_t \).

### A.3 New-Keynesian Parametrisation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \xi )</td>
<td>Calvo parameter</td>
<td>0.75</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Elasticity of substitution between consumption varieties</td>
<td>7</td>
</tr>
<tr>
<td>( \eta^\pi )</td>
<td>Monetary policy rule weight on inflation</td>
<td>1.5</td>
</tr>
<tr>
<td>( \eta_y )</td>
<td>Monetary policy rule weight on output gap</td>
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</tr>
<tr>
<td>( \eta_r )</td>
<td>Monetary policy rule persistence</td>
<td>0.9</td>
</tr>
<tr>
<td>( \phi )</td>
<td>Investment adjustment cost parameter</td>
<td>2</td>
</tr>
</tbody>
</table>

Table A.1: Additional parametrisation of the New Keynesian model.
Figure A.1: Steady state values of $\varrho$ and $\varrho^r$ against $p^r$ for a range of values of $\lambda$ (top panel) and $\gamma$ (bottom panel).

A.4.1 Impulse Response Functions
Figure A.2: Impulse response functions to a very large (+20%) positive transitory shock to total factor productivity $A_t$. 

- **$\log Y$**
- **$\log K$**
- **$\log I$**
- **$x^*$**
- **$x^r$**
- **$\varphi^*$**
- **$\varphi^r$**
- **$\psi$**
- **$\varphi^e$**
- **$e$**
- **$\nu$**
- **$\nu^*$**
- **$\Delta$**
- **$\xi$**
- **$\zeta$**

Legend:
- Green line: Adverse Selection
- Blue dashed line: First Best
Figure A.3: Impulse response functions to a large (+10%) positive transitory shock to total factor productivity $A_t$. 
Figure A.4: Impulse response functions to a very large (+20%) negative transitory shock to total factor productivity $A_t$. 
Figure A.5: Impulse response functions to a large transitory risk shock caused by a reduction in the probability of risky project success $p^r$ of approximately 3 percentage points.
Figure A.6: Impulse response functions to a transitory risk shock caused by a reduction in the probability of risky project success $p_r^p$ of approximately 2 percentage points.
Figure A.7: Impulse response functions to a transitory risk shock caused by a reduction in the probability of risky project success $p_r$ of approximately 1 percentage point.
Figure A.8: Impulse response functions to a negative transitory risk shock caused by an increase in the probability of risky project success $p'_r$ of approximately 2 percentage points.
Figure A.9: Impulse response functions to a transitory risk shock caused by a reduction in the probability of risky project success $p_r^f$ of approximately 2 percentage points. Alternative parametrisation with $\lambda = 0.85$. 
Figure A.10: Impulse response functions to a transitory risk shock caused by a reduction in the probability of risky project success $p^r$ of approximately 4 percentage points. Alternative parametrisation with $\lambda = 0.85$. 
Figure A.11: Impulse response functions to a transitory efficiency shock caused by a reduction in the productivity of risky projects $\omega_t$ of 5%.
Figure A.12: Impulse response functions to a transitory efficiency shock caused by a reduction in the productivity of risky projects $\omega^r_t$ of 2.5%.
Figure A.13: Impulse response functions to a transitory efficiency shock caused by a reduction in the productivity of risky projects $\omega^r_t$ of 1.5%.
Figure A.14: Impulse response functions to a transitory efficiency shock caused by a reduction in the productivity of risky projects $\omega'_i$ of $0.8\%$. 
Figure A.15: Impulse response functions to a positive transitory efficiency shock caused by an increase in the productivity of risky projects $\omega^*_r$ of 1.5%.
Figure A.16: Impulse response functions to a positive transitory efficiency shock caused by an increase in the productivity of risky projects $\omega_t^r$ of 0.5%.
Figure A.17: Impulse response functions to a transitory efficiency shock caused by a reduction in the productivity of risky projects $\omega_i^r$ of 1.5%. Alternative parametrisation with $\lambda = 0.85$. 
Figure A.18: Impulse response functions to a transitory efficiency shock caused by a reduction in the productivity of risky projects $\omega^r_t$ of approximately 2.5%. Alternative parametrisation with $\lambda = 0.85$. 
Figure A.19: Impulse response functions to a transitory efficiency shock caused by a reduction in the productivity of risky projects $\omega^r_t$ of 5%. Alternative parametrisation with $\lambda = 0.85$. 
Figure A.20: Impulse response functions to a transitory efficiency shock caused by an increase in the productivity of risky projects $\omega^r_t$ of 1.5%. Alternative parametrisation with $\lambda = 0.85$. 
CHAPTER 5 APPENDICES

B.1 INTERBANK CREDIT CONTRACT SOLUTION

Dropping bank indexes for convenience, lending banks balance sheet

\[ N_t + IB_t^H + IB_t^F = CR_t \]

Net worth evolves according to

\[ N_t = \omega R_t^{CR} CR_{t-1} - R_{t-1}^{IB,H} IB_{t-1}^H - R_{t-1}^{IB,F} IB_{t-1}^F \]

where \( R_t^{CR} \) is the ex post return across all lending. Default threshold

\[ \bar{\omega}_t \equiv \frac{R_{t-1}^{IB,H} IB_{t-1}^H + R_{t-1}^{IB,F} IB_{t-1}^F}{R_t^{CR} CR_{t-1}} \]

Banks face CES preference toward domestic and foreign interbank borrowing, so choose \( IB_t^H \) and \( IB_t^F \) to maximise

\[ IB_t = \left[ \tau^{IB^{1/\theta IB}} IB_t^{\theta IB - 1} + \left( 1 - \tau^{IB} \right) \right]^{1/\theta IB} \]

subject to

\[ R_t^{IB} IB_t = R_t^{IB,H} IB_t^H + R_t^{IB,F} IB_t^F \]  \hspace{1cm} (B.1.2)

This leads to demand schedules

\[ IB_t^F = \left( 1 - \tau^{IB} \right) \left( \frac{R_t^{IB,F}}{R_t^{IB}} \right)^{-\theta IB} IB_t \]  \hspace{1cm} (B.1.3)

\[ IB_t^H = \tau^{IB} \left( \frac{R_t^{IB,H}}{R_t^{IB}} \right)^{-\theta IB} IB_t \]  \hspace{1cm} (B.1.4)

and interest rate

\[ R_t^{IB} = \left[ \tau^{IB} R_t^{IB,H \frac{1}{1-\theta IB}} + \left( 1 - \tau^{IB} \right) R_t^{IB,F \frac{1}{1-\theta IB}} \right]^{\frac{1}{\theta IB}} \]  \hspace{1cm} (B.1.5)
We also find
\[ IB_t^H + IB_t^F = IB_t \frac{\tau^IB_t R_t^{IB,H-\theta IB} + (1 - \tau^IB_t) R_t^{IB,F-\theta IB}}{R_t^{IB,J-\theta IB}} \] (B.1.6)
\[ = IB_t Y \left( R_t^{IB,H}, R_t^{IB,F} \right) \] (B.1.7)

Let
\[ G_{t+1} \equiv G(\bar{\omega}_{t+1}, \sigma_{t+1}) \equiv \int_0^{\bar{\omega}_t} \omega dF(\omega, \sigma_t) \] (B.1.8)
\[ F_{t+1} \equiv F(\bar{\omega}_{t+1}, \sigma_{t+1}) \equiv \int_0^{\bar{\omega}_t} f \left( \omega; -\frac{\sigma^2}{2}, \sigma^2 \right) d\omega \] (B.1.9)

The ex post return on interbank lending for the savings banks can then be given by
\[ \tilde{R}_t^{IB,H} = (1 - F_t) R_{t-1}^{IB,H} + (1 - \mu) G_t R_i^{CR} \frac{CR_{t-1}}{IB_{t-1} + IB_{t-1}} \] (B.1.10)

We assume there is a cost in lending internationally that depends on the net foreign asset position of the destination country. The ex post return on international lending (for foreign banks) is given by
\[ \tilde{R}_t^{IB,F} IB_t^F = \left[ 1 - \Gamma^*_t \right] \left( 1 - F_t \right) R_{t-1}^{IB,F} IB_{t-1}^F + (1 - \mu) G_t R_i^{CR} CR_{t-1} \frac{IB_t^F}{IB_{t-1} + IB_{t-1}} \] (B.1.11)

Objective
\[
\max_{CR_t, \bar{\omega}_{t+1}, \tilde{R}_t^{IB,H}, \tilde{R}_t^{IB,F}, IB_t^F, IB_t^H} \mathbb{E}_t \left[ \left( 1 - G_t \right) R_t^{CR} CR_t - (1 - F_{t-1}) R_t^{IB} BI_t^F \frac{\Lambda_{t+1}}{\Lambda_{t+1}^{CR}} \right]
\]
\[ \text{s.t.} \quad \bar{\omega}_{t+1} = \frac{R_t^{IB} IB_t}{R_t^{CR} CR_t} \]
\[ \mathbb{E}_t \left[ \frac{\Lambda_{t+1}}{\Lambda_{t+1}^{CR}} \tilde{R}_t^{IB,H} IB_t^H \right] = \mathbb{E}_t \left[ \frac{\Lambda_{t+1}}{\Lambda_{t+1}^{CR}} R_t^{S} IB_t^H \right] \]
\[ \mathbb{E}_t \left[ \frac{\Lambda_{t+1}}{\Lambda_{t+1}^{CR}} \tilde{R}_t^{IB,F} IB_t^F \right] = \mathbb{E}_t \left[ \frac{\Lambda_{t+1}}{\Lambda_{t+1}^{CR}} R_t^{S} IB_t^F \right] \]
\[ N_t + IB_t Y \left( R_t^{IB,H}, R_t^{IB,F} \right) = CR_t \]
or, suppressing arguments of \(G_{t+1}, F_{t+1}, R^IB_t, Y_t\) for convenience, \(CR_t, \bar{\omega}_{t+1}, R^IB_t, R^IF_t, \) chosen to maximise Lagrangian

\[
L_t = \mathbb{E}_t \begin{bmatrix}
\frac{\Delta_{t+1}}{P_{t+1}} \left(1 - G_{t+1} - (1 - F_{t+1}) \bar{\omega}_{t+1}\right) R^CR_t + q_t \left( \bar{\omega}_{t+1} R^CR_t - (CR_t - N_t) \right) \frac{R^IF_t}{P_t} \\
+ \lambda^IB_t \left(1 - F_{t+1}\right) R^IB(H CR_t - N_t) + (1 - \mu) \left[G_{t+1} R^CR_t\right] - R^S_t (CR_t - N_t) \\
+ \lambda^IB,F_t \left(1 - \Gamma_{IB,t}\right) R^IB,F_t (CR_t - N_t) + (1 - \mu) \left[G_{t+1} R^CR_t\right] - R^S_t (CR_t - N_t)
\end{bmatrix} \frac{\Lambda_{t+1}}{P_{t+1}}
\]

So with

\[
Y \left(R^IB_t, R^IF_t\right) = \tau^{IB} R^IB,H \left(1 - \theta_{IB}\right) + \left(1 - \tau^{IB}\right) R^IF_t \left(1 - \theta_{IB}\right)
\]

\[
R^IB_t = \left[\tau^{IB} R^IB_t \left(1 - \theta_{IB}\right) + \left(1 - \tau^{IB}\right) R^IF_t \left(1 - \theta_{IB}\right)\right] \frac{1}{\tau^{IB}}
\]

\[
\frac{\partial Y \left(R^IB_t, R^IF_t\right)}{\partial R^IB_t} = \left(1 - \left[\left(\frac{R^IB_t}{R^IF_t}\right) \left(1 - \theta_{IB}\right) \left(\frac{1}{Y_t}\right)\right]\right) \frac{IB^H}{IB^H + IB^F}
\]

\[
\frac{\partial Y \left(R^IB_t, R^IF_t\right)}{\partial R^IF_t} = \left(1 - \left[\left(\frac{R^IF_t}{R^IB_t}\right) \left(1 - \theta_{IB}\right) \left(\frac{1}{Y_t}\right)\right]\right) \frac{IB^F}{IB^H + IB^F}
\]

we find

\[
\mathbb{E}_t \left[-G' \left(\bar{\omega}_{t+1}\right) + F' \left(\bar{\omega}_{t+1}\right) \bar{\omega}_{t+1} - (1 - F \left(\bar{\omega}_{t+1}\right)) R^CR_t \frac{\Delta_{t+1}}{P_{t+1}} \right] \\
+ q_t \left( \bar{\omega}_{t+1} R^CR_t \right) \\
+ \lambda^IB_t \left(1 - F_{t+1}\right) R^IB(H CR_t - N_t) \frac{\Lambda_{t+1}}{P_{t+1}} + (1 - \mu) \left[G' \left(\bar{\omega}_{t+1}\right) R^CR_t \frac{\Lambda_{t+1}}{P_{t+1}}\right] \right)
\]
\[
\begin{align*}
&+ \lambda^{IB,F}_t \left( \mathbb{E}_t \left[ -(1 - \Gamma^{*}_{IB,t}) F' (\tilde{\omega}_{t+1}) R^{IB,F}_t (CR_t - N_t) \frac{\Lambda^{I}_{t+1}}{P^{C}_{t+1}} \right] \right) \\
&= 0 \tag{B.1.17}
\end{align*}
\]
\[
\begin{align*}
&+ \lambda^{IB,H}_t \left( \mathbb{E}_t \left[ (1 - F_t) R^{IB,H}_t \frac{\Lambda^{I}_{t+1}}{P^{C}_{t+1}} \right] - R^S_t \frac{\Lambda^{*}_{t+1}}{P^{C}_{t+1}} \right) \\
&+ \lambda^{IB,F}_t \left( \mathbb{E}_t \left[ (1 - \Gamma^{*}_{IB,t}) (1 - F_t) R^{IB,F}_t \frac{\Lambda^{I}_{t+1}}{P^{C}_{t+1}} \right] - R^S_t \frac{\Lambda^{*}_{t+1}}{P^{C}_{t+1}} \right) \\
&= 0 \tag{B.1.18}
\end{align*}
\]
\[
\begin{align*}
&\varrho_t \left( -(CR_t - N_t) \Xi^{IB,H}_t \right) \\
&+ \lambda^{IB,H}_t \left( \mathbb{E}_t \left[ (1 - F_t) (CR_t - N_t) \frac{\Lambda^{I}_{t+1}}{P^{C}_{t+1}} \right] \right) = 0 \tag{B.1.19}
\end{align*}
\]
\[
\begin{align*}
&\varrho_t \left( -(CR_t - N_t) \Xi^{IB,F}_t \right) \\
&+ \lambda^{IB,F}_t \left( \mathbb{E}_t \left[ (1 - \Gamma^{*}_{IB,t}) (1 - F_t) (CR_t - N_t) \frac{\Lambda^{I}_{t+1}}{P^{C}_{t+1}} \right] \right) = 0 \tag{B.1.20}
\end{align*}
\]
Then
\[
\begin{align*}
\mathbb{E}_t \left[ R^{CR}_{t+1} \right] &
\end{align*}
\]
\[
\begin{align*}
&= \mathbb{E}_t \left[ \begin{pmatrix}
\lambda^{IB,H}_t R^{IB,H}_t \frac{\Lambda^{I}_{t+1}}{P^{C}_{t+1}} \\
\lambda^{IB,F}_t (1 - \Gamma^{*}_{IB,t}) R^{IB,F}_t \frac{\Lambda^{I}_{t+1}}{P^{C}_{t+1}} \\
-(1 - \mu) R^{IB}_t \left( \lambda^{IB,H}_t \frac{\Lambda^{I}_{t+1}}{P^{C}_{t+1}} + \lambda^{IB,F}_t \frac{\Lambda^{I}_{t+1}}{P^{C}_{t+1}} \right) \\
+ (1 - F_t) R^{CR}_t \frac{\Lambda^{I}_{t+1}}{P^{C}_{t+1}} \end{pmatrix} \right] \\
&= \mathbb{E}_t \left[ \begin{pmatrix}
\lambda^{IB,H}_t R^{IB,H}_t \frac{\Lambda^{I}_{t+1}}{P^{C}_{t+1}} \\
\lambda^{IB,F}_t (1 - \Gamma^{*}_{IB,t}) R^{IB,F}_t \frac{\Lambda^{I}_{t+1}}{P^{C}_{t+1}} \\
-(1 - \mu) R^{IB}_t \left( \lambda^{IB,H}_t \frac{\Lambda^{I}_{t+1}}{P^{C}_{t+1}} + \lambda^{IB,F}_t \frac{\Lambda^{I}_{t+1}}{P^{C}_{t+1}} \right) \\
+ (1 - F_t) R^{CR}_t \frac{\Lambda^{I}_{t+1}}{P^{C}_{t+1}} \end{pmatrix} F' (\tilde{\omega}_{t+1}) \frac{IB_t}{CR_t} \right]
\]
where
\[
\varrho_t
\]
\[ \lambda_t^{IB,H} + \lambda_t^{IB,F} = \mathbb{E}_t \left[ \left( 1 - G_{t+1} - (1 - F_{t+1}) \varpi_{t+1} \right) \frac{\Lambda_{t+1}^{IB,H}}{P_{t+1}^{IB,H}} \right] + \lambda_t^{IB,F}(1 - \mu)G_{t+1} \frac{\Lambda_{t+1}^{IB,F}}{P_{t+1}^{IB,F}} + \lambda_t^{IB,F}(1 - \mu)G_{t+1} \frac{\Lambda_{t+1}^{IB,F}}{P_{t+1}^{IB,F}} \left( \frac{B_{t+1}^{IB,F}}{R_{t+1}^{CR}} \right) \]

B.1.21

\[ \lambda_t^{IB,H} = \frac{\varrho_t}{\mathbb{E}_t \left[ (1 - F_{t+1}) \frac{\Lambda_{t+1}^{IB,H}}{P_{t+1}^{IB,H}} \right]} \varpi_t^{IB,H} \]

\[ \lambda_t^{IB,F} = \frac{\varrho_t}{\mathbb{E}_t \left[ (1 - F_{t+1}) \frac{\Lambda_{t+1}^{IB,F}}{P_{t+1}^{IB,F}} \right]} \left( 1 - \Gamma_{t+1}^{IB,F} \right) \]

There is no closed form expression for \( \mathbb{E}_t \left[ R_t^{CR} \right] \) but these conditions imply a value that depends on the rates and the leverage ratio \( \frac{IB_t}{CR_t} \). Given that perfect capital markets imply \( \mathbb{E}_t \left[ R_t^{CR} \right] \) will be the same for all banks, \( \varpi_t \) only depends on the leverage, and so the conditions imply the same rates and leverage ratio for all banks of any \( N_t \).