Electromagnetic Emission-aware Schedulers for the Uplink of OFDM Wireless Communication Systems

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Abstract

The popularity and convergence of wireless communications have resulted in continuous network upgrades in order to support the increasing demand for bandwidth. However, given that wireless communication systems operate on radiofrequency waves, the health effects of electromagnetic emission from these systems are increasingly becoming a concern due to the ubiquity of mobile communication devices. In order to address these concerns, we propose two schemes (offline and online) for minimizing the EM emission of users in the uplink of OFDM systems, while maintaining an acceptable quality of service. We formulate our offline EM reduction scheme as a convex optimization problem and solve it through water-filling. This is based on the assumption that the long-term channel state information of all the users is known. Given that, in practice, long-term channel state information of all the users cannot always be available, we propose our online EM emission reduction scheme, which is based on minimizing the instantaneous transmit energy per bit of each user. Simulation results show that both our proposed schemes significantly minimize the EM emission when compared to the benchmark classic greedy spectral efficiency based scheme and an energy efficiency based scheme. Furthermore, our offline scheme proves to be very robust against channel prediction errors.

Index Terms

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Electromagnetic (EM) exposure, OFDM, uplink, power allocation, subcarrier allocation.

I. INTRODUCTION

Higher throughput and energy efficiency (EE) are currently the main topics of research in wireless communication systems. However, the health effects of exposure to electromagnetic (EM) emission from these systems is increasingly becoming an issue among the public [1]. These concerns are borne out of the increasing popularity and ubiquity of mobile communication systems, as well as the surge in network densification for supporting the ever-increasing demand for mobile communication services. Interestingly, most of the worries about EM emission have been linked to the downlink (probably due to the visible network upgrades and increased deployment) even though the EM emissions from mobile phones is potentially more harmful because the antennas are closer to the human body when in use [2]. Despite the fact that there is no evidence linking short-term exposure to EM emission from mobile communication systems with any adverse health effects, the international agency for research on cancer (IARC) has concluded that EM radiation is possibly carcinogenic and categorized it as Group 2B - a group reserved for systems that have limited evidence of carcinogenicity in humans [3]. Whereas, long-term effects of EM exposure on humans are starting to be unveiled; for instance, it has been recently shown in [4] that heavy users of wireless phones, over a period of more than 25 years, are three times more likely to develop a brain tumour. Thus, in order to cope with the concerns of the general public, the European Environmental Agency (EEA) has recommended, in 2013, precautionary approaches like the use of hands-free or earpiece, minimizing the use of mobile phones (especially in fast moving vehicles) and limiting the usage of mobile phones by children, among others [5]. However, we believe that these growing concerns should not only be dealt with recommendations or regulations but with smart technical solutions, as it is proposed in this paper.

Despite these fears and the uncertainty about the adverse health effects of long-term exposure to EM emissions, research contributions on reducing EM emission in mobile communication systems have been rather limited in number, given that most researchers and equipment man-
ufacturers have focused on complying with the regulatory prescribed limits. Fortunately, the advent of the 5G mobile system presents a new opportunity for research on EM exposure in wireless communication systems. It is foreseen that EM emission, alongside traditional criteria such as spectral efficiency (SE) and EE, will play a key role in the design of 5G system.

In this regard, we have performed in [6], a comprehensive survey of existing literature, dosimetry, metrics, international projects as well as guidelines and limits on the exposure to EM emissions from mobile communication systems. We also reviewed and discussed different ways of reducing EM emission from mobile systems. EM radiation shielding has been proven in [7], [8] to minimize the EM emission in the uplink by placing ferrite materials or metamaterials between the mobile phone and the human head. Whereas, beamforming has also been considered for minimizing EM emission in mobile communication systems. Although the beamforming technique has been traditionally used to improve the SE of mobile systems, it has been shown in [9], [10] that it can be used to minimize EM emission in the uplink of mobile systems. However, [7]–[9] mainly focus on reducing EM radiation without considering the SE performance/quality of service (QoS) of the network. In order to address this aspect, short and long-term radio resource management (RRM) schemes have been proposed in [10] and [11], respectively. In [11], a load balancing algorithm for self organizing networks to reduce the overall EM exposure in heterogeneous networks has been proposed. Whereas, in [10], we proposed a user scheduling algorithm to minimize the EM emission of users in the uplink of TDMA systems by assigning priority levels to users based on their instantaneous transmission power.

In this paper, we propose two novel RRM schemes for minimizing EM emission in the uplink of a multiuser orthogonal frequency division multiplexing (OFDM) wireless communication system, while maintaining a specified QoS constraint. Contrary to [11], we focus on single cell RRM and propose, as in [10], radio resource/scheduling approaches for reducing the EM exposure. However, contrary to [10], we develop generic algorithms for the multi-carrier instead of single carrier scenario. It can also be noted that resource allocation/scheduling in the uplink of OFDM systems have been very well investigated, however, mainly from an SE [12], [13], EE [14], [15] and more recently, joint SE and EE maximization [16], [17] perspective, where the
authors jointly maximized both the SE and EE in green heterogeneous networks. Margin adaptive resource allocation has been investigated in [18]–[20], where the authors minimized transmit powers in OFDM systems. However, these works do not consider the signaling powers of the transmitters nor the transmission duration, which are critical in evaluating EM emission. Furthermore, the authors did not take into account the total transmit power constraint of the antennas. Accordingly, the works in [19], [20] assumed a minimum data rate constraint (inequality), which would result in additional transmit power because additional and unnecessary data is transmitted. Whereas, here, we also utilize resource allocation/scheduling but for reducing the EM emission exposure. Our choice is being motivated by the fact that uplink EM emission is proportional to the amount of energy (power over time) dissipated towards the user, such that 3-dimensional (time, power, frequency) resource allocation schemes make appropriate candidates for reducing the EM emission (while maintaining QoS) in the uplink of OFDM systems, according to [2]. Our first EM emission reduction scheme minimizes the transmission energy subject to transmitting a target number of bits (QoS target) over a given transmission window, while taking into account the power constraint in each time slot (TS). This approach relies on the availability of the long-term channel state information (CSI) of all the users in the network. As such, we call it “offline” since processing is performed offline (i.e. not in real-time) before data transmission is initiated. The original optimization problem being non-convex, we first found an elegant way to reformulate it in a standard convex form and solved it by designing a water-filling algorithm. Accordingly, we have extended our previous work in [21] by proposing a new subcarrier allocation for our offline scheme. We have also evaluated the effects of imperfect channel prediction on our offline scheme, as well as its EM emission performance in a vehicular channel and for the same fixed data rate as benchmark schemes. On the other hand, our second EM emission reduction scheme is based on the short-term CSI and it minimizes the transmission energy per bit of each user by calculating the optimal instantaneous transmit power of each user per subcarrier. We named this scheme “online” because optimization is performed instantaneously on a per subcarrier and TS basis. The main contributions of this paper are listed as follows:

- Our proposed scheduler designs take into account the QoS, signaling power as well as the
data transmission power of each user to provide a comprehensive analysis of EM emission minimization in the uplink of mobile communication systems.

- We propose a subcarrier allocation algorithm for our offline EM reduction scheme that maximizes the average channel gain allocated to each user while also avoiding allocating the worst subcarriers to the users. We also formulate our offline EM minimization problem as a convex optimization problem and iteratively allocate bits and, subsequently, power to the users on their respective subcarriers within the transmission window. Our online EM emission reduction scheme minimizes the transmission energy per bit of each user on its allocated subcarriers.

- Simulation results demonstrate that our proposed offline scheme significantly outperforms the benchmark classic greedy SE-based scheme and the EE scheme of [15] by up to 3 and 2 orders of magnitude, respectively. Whereas, our proposed online scheme outperforms the benchmark greedy SE-based scheme and the EE scheme of [15] by up to 2.5 and 2 orders of magnitude, respectively. Furthermore, the results show that the total uplink EM emission in the network is proportional to the target number of bits and the number of users in the network while the total uplink EM emission decreases as the transmission duration increases.

The rest of this paper is organized as follows. Section II describes the OFDM uplink system model as well as the relationship between the EM emission and the transmission energy. In Section III, we formulate and solve the EM emission reduction problem by proposing our EM emission reduction schemes. We discuss and analyze the performance of our schemes in Section IV and, finally, conclude the paper in Section V.

II. SYSTEM MODEL

Consider the uplink of a multiuser OFDM wireless communication system consisting of $K$ single antenna users communicating with a base station (BS) - also employing a single antenna. The system utilizes a total bandwidth $W$ divided into $N$ equal subcarriers. We assume that time resource is split into TSs, each of length $l$. Furthermore, each user in the network sends uplink
pilot signals that are used by the BS to estimate the CSI of the user-to-BS link. Therefore, the BS is assumed to have perfect CSI of all the links between itself and its served users in the network; this CSI knowledge is used to allocate subcarriers to the users and also perform power allocation to minimize the EM emission to each user, subject to transmitting a target number of bits. Note that in this paper, a subcarrier can be allocated to at most one user in a TS but a user can have more than one subcarrier in a TS. Hence, the amount of bits transmitted by user $k$ in a TS can be expressed as

$$b_k(t) = w l \sum_{n=1}^{N} \alpha_{k,n}(t) \log_2 \left( 1 + \frac{p_{k,n}(t) g_{k,n}(t)}{\sigma^2} \right),$$

where $w$ denotes the bandwidth of a subcarrier, $p_{k,n}(t)$ and $g_{k,n}(t)$ represent the transmit power and channel gain of user $k$ on subcarrier $n$ at TS $t$, respectively. The parameter $\alpha_{k,n}(t)$ gives the subcarrier allocation index of user $k$, such that $\alpha_{k,n}(t) = 1$ if subcarrier $n$ is allocated to user $k$ at TS $t$ and $\alpha_{k,n}(t) = 0$ otherwise, while $\sigma^2$ denotes the noise power per subcarrier.

Recently, a new metric, i.e. the exposure index (EI), has been proposed in [22] for evaluating the average EM exposure of a given geographic area induced by both the total uplink and downlink exposures. The EI (measured in Joule/kg) takes into account various sources of EM exposure from different technologies, duration of EM exposure for a given population, mobile usage pattern and posture, among others\(^1\). We limit the scope of this work to the uplink given that uplink EM emission is more severe since the antennas are closer to the human body when in operation. Hence, regarding the exposition of user $k$ to EM exposure, it can be expressed according to [22] as

$$E_k = \frac{\text{SAR}_k}{P_{\text{ref}}} \sum_t \left( \hat{p}_k(t) + \sum_n p_{k,n}(t) \right) l$$

where $\text{SAR}_k$ is the whole body averaged specific absorption rate (SAR) of the $k$-th user mobile device. SAR (in W/kg per 1 W of emitted power) is a measure of the absorption rate of energy by the body when exposed to EM radiation; it depends on the carrier frequency, usage of the

\(^1\)For example, the EI of the over 15 years old population of the 7th district of Paris, considering a macrocell LTE network has been given as 1.24e-05 Joule/kg/day [22].
mobile terminal, posture of the user and how close the antenna is to the user. The SAR of mobile devices is typically computed via Finite-difference time-domain (FDTD) simulations\(^2\) [22]. It is worth noting that the EI is dependent on the SAR of the user terminal. The parameter \(P_{\text{ref}}\) represents the incident reference power and \(\hat{p}_k\) denotes the signaling power of user \(k\). The signaling power, \(\hat{p}_k(T)\), can be computed as [23]

\[
\hat{p}_k(T) = \min(P_{\text{max}}, P_0 + D_k + \Delta(T)) \text{ [dBm]},
\]

(3)

where \(P_0\) denotes the received signal power threshold at the BS, \(D_k\) represents the path loss of user \(k\) and

\[
\Delta(T) = \begin{cases} 
10 \log_{10}(\delta/4), & \text{if } T \geq 4 \\
0, & \text{otherwise}
\end{cases}
\]

(4)

such that \(\delta\) denotes the number of signaling bits that are transmitted for acquiring channel quality information. In this paper, we assume that \(a\) bits of signaling information are transmitted by each user in a TS, hence \(\delta = aT\) in (4), where \(T\) denotes the number of TSs [23]. It therefore means that the number of transmitted signaling bits increases with the size of the transmission window. However, we know from Lemma 1 that

\textbf{Lemma 1:} Extending the transmission duration decreases the energy dissipated for transmitting \(b\) bits; see proof in the Appendix. ■

Consequently, if the data transmission power dominates the signaling power, i.e. \(\hat{p}_k(T) \ll \sum_n p_{k,n}(t)\), then Lemma 1 implies that extending the data transmission duration reduces the EM emission of the user. Hence, there exists a trade-off between signaling and transmission energy when it comes to setting the transmission window length, \(T\).

It can be remarked from (2) that if \(\text{SAR}_k/P_{\text{ref}}\) is similar for all the users, then reducing the per-user EM exposure, \(E_k\), boils down to reducing the per-user transmit energy, \(\mathcal{E}_k\), \((\mathcal{E}_k = \frac{P_{\text{ref}}}{\text{SAR}_k} E_k)\) i.e., the product of the transmit power and time. Whereas increasing the number of transmitted bits implies an increase in transmit power or/and number of utilized subcarriers, according to

\(^2\)More details about the SAR can be found in [22].
Thus, in order to reduce the per-user EM emission while ensuring a QoS target, a subtle trade-off between power, number of utilized subcarriers and duration of transmission must be achieved, as it is further detailed in the next section.

III. EM EMISSION REDUCTION SCHEMES

In this section, we propose two novel and effective algorithms for minimizing the EM emission of individual users in OFDM systems, while ensuring QoS. The schemes are based on the assumption that the BS can predict the CSI of all the users up to $T$ TSs in advance by using the uplink pilot signals that are transmitted by each user in the system. As such, our schemes are based on the availability of long ($T > 1$) and short ($T = 1$) term CSI predictions, respectively. Knowing the CSI of each user, the BS performs subcarrier and power allocations across the whole $T$ TSs to minimize the per-user transmit energy, $E_k$, to transmit $B_k$ bits and, hence, per-user EM emission. Consequently, our EM emission reduction schemes are based on the following optimization problem

$$\min_{p_k, \alpha_k} E_k = \hat{p}_k(T)l + l \sum_{t=1}^{T} \sum_{n=1}^{N} \alpha_{k,n}(t)p_{k,n}(t), \quad (5)$$

subject to

$$wl \sum_{t=1}^{T} \sum_{n=1}^{N} \alpha_{k,n}(t) \log_2 \left(1 + \frac{p_{k,n}(t)g_{k,n}(t)}{\sigma^2}\right) = B_k, \quad (6a)$$

$$\sum_{n=1}^{N} \alpha_{k,n}(t)p_{k,n}(t) \leq P_{\text{max}}^k \quad \forall t, \quad (6b)$$

$$\sum_{k=1}^{K} \alpha_{k,n}(t) \leq 1 \quad (6c)$$

where $p_k = [p_{k,1}(1), \ldots, p_{k,N}(1), p_{k,1}(2), \ldots, p_{k,N}(T)] \geq 0$, $\alpha_k = [\alpha_{k,1}(1), \ldots, \alpha_{k,N}(1), \alpha_{k,1}(2), \ldots, \alpha_{k,N}(T)] \geq 0$, and $P_{\text{max}}^k$ represents the maximum transmit power of the user.

Notice that the constraint (6a) on the number of transmitted bits is set across the whole $T$ TSs while the power constraint (6b) is per TS, as transmissions are performed in a slotted manner in time domain. It can also be remarked that the problem in (5) - (6) is a per user minimization
problem and not a sum minimization problem. Indeed, a network EM emission minimization may not necessarily reduce the EM emission of individual users as it minimizes the sum EM emission of the network. On the contrary, a per user minimization problem minimizes the EM emission of each individual user, thereby ensuring EM exposure fairness in the system. Finally, it is worth mentioning that the binary nature of \( \alpha_{k,n}(t) \) in (5) - (6) makes the optimization problem at hand combinatorial, which is NP-hard and is intractable for large systems. Thus, we first relax this problem by performing subcarrier and power allocations in a sequential approach, as many other existing scheduling schemes [13], [15], [24], prior to solving it; first subcarrier allocation is performed to obtain \( \alpha_{k,n}(t) \), then power allocation is performed for fixed \( \alpha_{k,n}(t) \), \( \forall k, n, t \).

A. Offline EM Emission Reduction Scheme

In this subsection, we propose our novel subcarrier allocation and power allocation scheme for minimizing the EM emission of the users based on long-term CSI prediction, i.e, for \( T > 1 \) TS. It is worth noting that perfectly predicting the CSI of numerous TSs is not always feasible and, as such, this scheme is more theoretical; however, it is well suited for quasi-static channels where accurate CSI prediction can be performed. Whereas for more time varying channel, schemes like [25]–[27] can be used for CSI prediction. In addition, the parameter \( T \) can be used to tune prediction accuracy based on channel conditions.

1) Subcarrier Allocation: We propose an here a novel low-complexity EM aware subcarrier allocation. In order to make the scheme easy to implement we assume equal subcarrier allocation whereby all the users are allocated the same number of subcarriers within the transmission window, \( T \), so as to maximize both the \( \min_k \min_{n \in N_k} \{ g_{k,n} \} \) \( \forall k \) and the aggregate value of the channel gains allocated to the users. Here, \( N_k \) denotes the set of the subcarriers allocated to user \( k \) within the transmission window, \( T \). Equal subcarrier allocation is chosen in this paper because it is a straightforward approach which is simple and easier to handle, and thus, makes our algorithm have an LC. We use a subcarrier allocation utility (SAU) as a proxy for subcarrier allocation. Our SAU is defined as

\[
v_{k,n}(t) = \frac{g_{k,n}(t)}{\bar{g}_k},
\]  

(7)
where $\bar{g}_k$ represents the average channel gain of user $k$ throughout the transmission window across the whole subcarriers and it is obtainable since the network is assumed to have knowledge of the CSI of all the users up to $T$ TSs. We compute the SAU of each user on all subcarriers in the system by using (7) and stack them in a $K \times NT$ matrix, $V$, such that each column of $V$ represents a subcarrier and each element of $V$ denotes the SAU of user $k$ on subcarrier $n$ at TS $t$, i.e., $v_{k,n}(t)$. The columns of $V$ are reordered into another $K \times NT$ matrix, $U$, by rearranging the columns of $V$ in ascending order based on their minimum SAU, such that the first column of $U$ contains the subcarrier with the worst SAU i.e., $\min_k \min_{n(t)} v_{k,n}(t), \forall k, n, t$, while the last column of $U$ contains the subcarrier with the highest minimum SAU, i.e. $\max_k \min_{n(t)} v_{k,n}(t) \forall k, n, t$. Subcarrier allocation starts from the first column of $U$, with the user having the highest SAU on each subcarrier being allocated to the subcarrier. This process is continued until $S$ subcarriers are allocated to each user, where

$$S = \left\lfloor \frac{NT}{K} \right\rfloor. \quad (8)$$

Here, $\lfloor . \rfloor$ denotes the floor operator.

For instance, let us consider a system with $N = 3$ subcarriers and $K = 3$ users but for $T = 2$ TS, as in [24]. The channel gains of each user on all the subcarriers is given by the $K \times NT$ matrix

$$G = \begin{bmatrix}
    n(t) & 1(1) & 2(1) & 3(1) & 1(2) & 2(2) & 3(2) \\
    \text{User 1} & 1.8 & 1.7 & 1.3 & 0.5 & 0.3 & 0.4 \\
    \text{User 2} & 0.6 & 0.7 & 1.4 & 1.3 & 0.8 & 0.9 \\
    \text{User 3} & 0.2 & 1.6 & 0.6 & 1.2 & 1.0 & 0.1
\end{bmatrix}$$

where $n(t)$ represents subcarrier $n$ in TS $t$. After computing the SAU of each element in $G$, we obtain matrix $V$. 

Unlike the worst subcarrier avoiding (WSA) algorithm proposed in [24] that uses the actual channel gains of the users on each subcarrier for subcarrier allocation, we use the SAU for subcarrier allocation in this paper. We order the columns of \( V \) in terms of \( \min_k v_{k,n}(t) \), \( \forall k,n,t \) and the resulting subcarrier allocation matrix, \( U \), is given by

\[
U = \begin{bmatrix}
 n(t) & 3(2) & 1(1) & 2(2) & 1(2) & 2(1) & 3(1) \\
 User 1 & 0.40 & \textbf{1.80} & 0.30 & 0.50 & \textbf{1.70} & 1.30 \\
 User 2 & \textbf{0.95} & 0.63 & 0.84 & 1.37 & 0.74 & \textbf{1.47} \\
 User 3 & 0.13 & 0.26 & \textbf{1.27} & \textbf{1.53} & 2.04 & 0.77 \\
\end{bmatrix}
\]

The set of subcarriers allocated to users 1, 2 and 3 are given as \( \mathcal{N}_1 = \{1(1), 2(1)\} \), \( \mathcal{N}_2 = \{3(1), 3(2)\} \) and \( \mathcal{N}_3 = \{1(2), 2(2)\} \), while the subcarriers allocated to users 1, 2 and 3 using the WSA algorithm of [24] are \( \{1(1), 3(1)\}, \{1(2), 3(2)\} \) and \( \{2(1), 2(2)\} \), respectively.

Although our proposed subcarrier allocation achieves the same \( \min_k \min_{n(t)\in\mathcal{N}_k} \{g_{k,n}(t)\} \) as with the WSA algorithm proposed in [24], our algorithm yields a higher aggregate value of the channel gains of the subcarriers allocated to the users i.e., \( \sum_{k=1}^{K} \left( \sum_{n\in\mathcal{N}_k} g_{k,n}(t) \right) \), which results in users being allocated subcarriers with better channel gains in comparison with [24] and, thus, a higher sum rate in the system. For example, let \( \sum_{k} \sum_{n(t)\in\mathcal{N}_k} \log_2(1+p_{k,n}(t)g_{k,n}(t)) \) denote the SE of the system; assuming \( p_{k,n}(t) = 1 \), \( \forall k,n,t \), then the SE of the system above using our proposed subcarrier allocation is 7.24 bits/s/Hz while it is 7.19 bits/s/Hz using the WSA algorithm of [24]. It implies that, for the same transmit power, our subcarrier allocation algorithm results in a higher sum rate when compared to the WSA algorithm of [24].

2) Power Allocation: Even for a fixed \( \alpha_{k,n}(t) \) values (fixed subcarrier allocation), the problem in (5) - (6) is clearly not convex because the equality constraint (6a) is not affine [28]. In order to
make the problem convex, we have to re-write (6a) into a standard convex optimization format. To do so, we use the following change of variables

$$p_{k,n}(t) = (2^{r_{k,n}(t)} - 1)\sigma^2/g_{k,n}(t),$$  \(9\)

where \(r_{k,n}(t) = b_{k,n}(t)/wl\) denotes the rate of user \(k\) on subcarrier \(n\) at TS \(t\). By integrating this change of variable in (5) - (6), the optimization problem can be reformulated in a convex form as

$$\min_{r_{k,n}(t)} E_k = \hat{p}_k(T)l + l \sum_{t=1}^{T} \sum_{n=1}^{N} \alpha_{k,n}(t)(2^{r_{k,n}(t)} - 1)\sigma^2/g_{k,n}(t)$$  \(10\)

subject to

$$wl \sum_{t=1}^{T} \sum_{n=1}^{N} \alpha_{k,n}(t)r_{k,n}(t) = B_k,$$  \(11a\)

$$\sum_{n=1}^{N} \alpha_{k,n}(t)(2^{r_{k,n}(t)} - 1)\sigma^2/g_{k,n}(t) \leq P_{k}^{\max}.$$  \(11b\)

This comes down to a rate allocation problem over all the subcarriers allocated to user \(k\) in the transmission window \(T\). By using the change of variable in (9), the equality constraint becomes affine and, hence, the problem in (10) with constrains (11a) and (11b) is convex in \(r_{k,n}(t)\) for fixed values of \(\alpha_{k,n}(t)\) (given that both (10) and (11b) are convex functions of \(r_{k,n}(t)\)).

The optimization problem in (10)-(11b) being clearly convex, we can define its Lagrangian as

$$\mathcal{L}(r, \lambda, \mu) = \hat{p}_k(T)l + l \sum_{t=1}^{T} \sum_{n=1}^{N} \alpha_{k,n}(t)(2^{r_{k,n}(t)} - 1)\sigma^2/g_{k,n}(t)$$

$$+ \lambda_k \left( B_k - wl \sum_{t=1}^{T} \sum_{n=1}^{N} \alpha_{k,n}(t)r_{k,n}(t) \right)$$

$$+ \mu_k(t)\left( P_{k}^{\max} - \sum_{n=1}^{N} \alpha_{k,n}(t)(2^{r_{k,n}(t)} - 1)\sigma^2/g_{k,n}(t) \right),$$  \(12\)

where \(\lambda_k\) and \(\mu_k(t)\) denote the Lagrange multipliers (slack variables) associated with the con-
The following Karush-Kuhn-Tucker (KKT) conditions [28] are necessary and sufficient conditions for optimality

\begin{align}
wl \sum_{t=1}^{T} \sum_{n=1}^{N} \alpha_{k,n}(t)r_{k,n}(t) - B_k &= 0, \quad \forall k = 1, 2, \ldots, K \\
\sum_{n=1}^{N} \alpha_{k,n}(t)(2^{r_{k,n}(t)} - 1)\sigma^2/g_{k,n}(t) - P_{k}^{\text{max}} &\leq 0, \quad \forall k = 1, 2, \ldots, K \quad \text{and} \quad t = 1, 2, \ldots, T \\
\mu_k(t) \left( \sum_{n=1}^{N} \alpha_{k,n}(t)(2^{r_{k,n}(t)} - 1)\sigma^2/g_{k,n}(t) - P_{k}^{\text{max}} \right) &= 0, \quad \forall k = 1, 2, \ldots, K \quad \text{and} \quad t = 1, 2, \ldots, T \\
\mu_k(t) &\geq 0, \quad \forall k = 1, 2, \ldots, K \quad \text{and} \quad t = 1, 2, \ldots, T \\
\nabla \mathcal{L}(r, \lambda_k, \mu_k(t)) &= 0, \quad \forall k = 1, 2, \ldots, K
\end{align}

Solving (17), we obtain the optimal solution to the problem in (10)-(11b) as

\begin{align}
r_{k,n}^{\star}(t) &= \left[ \log_2 \nu + \log_2 \left( \frac{wg_{k,n}(t)}{\ln(2)\sigma^2} \right) \right]_+, \quad \forall k = 1, 2, \ldots, K
\end{align}

where \( \nu \) is expressed as

\begin{align}
\nu &= \frac{\lambda_k^\star}{(1 - \mu_k^\star(t)/l)},
\end{align}

and \( [x]_+ = \max\{x, 0\} \). Note that (18) is a rate-based water-filling solution, with \( \nu \) denoting the water level. Several iterative algorithms like Secant and Newton-Raphson methods [29] can be used to obtain the optimal values \( \lambda_k^\star \) and \( \mu_k^\star(t) \) by fixing one and iteratively finding the other until they converge. The variables \( \lambda_k^\star \) and \( \mu_k^\star(t) \) have to satisfy the constraints (11a) and (11b), respectively. Knowing the optimal rate of each subcarrier allocated to user \( k \) via (18), the optimal transmit powers on these subcarriers can then be obtained from (9).
Algorithm 1 Offline EM Emission Reduction Algorithm

1: **Inputs:** $W, N, l, B_k, K, T, \sigma^2, SAR_k, P_{\text{max}}, \tilde{p}_k(T), B, g_{k,n}, \forall k = 1, 2, \ldots, K$ and $n = 1, 2, \ldots, N$

2: Compute $u_{k,n}(t)$ in (7) and obtain $\alpha_{k,n}(t) \forall k, n, t$ from steps 3 to 8;

3: Identify for each subcarrier the user with the worst $u_{k,n}(t)$, denoted as $u_{k,n}^{\text{min}}(t)$;

4: Sort and rearrange $u_{n}^{\text{min}}(t)$ in ascending order to obtain $K \times NT$ matrix $U$;

5: Denote $j_i$ as the subcarrier index representing the columns of $U$

6: Starting from $i = 1$, allocate subcarrier $j_i$ to the user with the maximum $u_{k,j_i}$;

7: If $|N_k| = S$ for user $k$, set $u_{k,j_i} = 0 \forall i$ in $U$ and obtain $\alpha_k$;

8: $i = i + 1$ and repeat steps 5 & 6 until $|N_k| = S \forall k$

9: Obtain $r^*_k(t)$ in (18) $\forall k$ via iterative water-filling;

10: Compute $p^*_{k,n}(t)$ by using (9) $\forall k$;

11: Obtain $E_k$ via (2);

12: **Output:** $E_k$. 

**B. Online EM Emission Reduction Scheme**

In this section, we propose our online EM emission reduction scheme for the case where the network relies on users’ instantaneous CSI, on a per TS basis. The optimization problem is similar to the one described in (5) - (6); the objective function is the same as in (5) but with $T = 1$; however, the target number of bits constraint, i.e. (6a), can still be met over several TSs, i.e., $T \geq 1$. As not all the users might achieve the target number of bits in one TS, this scheme performs instantaneous subcarrier and power allocation for all the users in a TS before moving to the next TS until all the users achieve the desired target number of bits. Each user continuously transmits signaling information to the BS in each TS until its target number of bits is met, and then the user is removed from the scheduling list.

1) **Subcarrier Allocation:** Given that this scheme is based on instantaneous resource allocation on a subcarrier and TS basis, it means that the subcarrier allocation proposed for the offline scheme is not feasible in this approach; indeed the number of subcarriers or TSs that are required by each user to achieve the target number of bits are not known in advance. Hence, we consider a greedy subcarrier allocation for each user in each TS that minimizes the EM emission. In this approach, the user having the best channel gain on a particular subcarrier is allocated on it. Denote $\eta_i$, for $i = 1, 2, \ldots, N$, as a permutation of subcarriers such that subcarrier $\eta_1$ has the
best channel gain and subcarrier $\eta_N$ has the worst. Starting from $t = 1$ and $i = 1$, subcarrier $\eta_i$ is allocated to the user with the best channel gain, as long as the user has not met the target number of bits and/or the power constraint. The same process is repeated for $i = 2, \ldots, N$ until all the users meet the target number of bits or all the subcarriers in that TS have been allocated. If all the subcarriers in the TS have been allocated and there are still users that have not achieved the target number of bits, the same process is repeated for $t = t + 1$ but only for users that have not reached the target number of bits. In the case where a user achieves its target number of bits, that user is removed from the scheduling list completely and the subcarrier is allocated to the user with the second best channel gain; whereas if a user attains its maximum transmit power, the user is removed from the scheduling list of that TS only.

2) Power Allocation: In order to reduce the EM emission, we minimize the transmission energy per bit of each user, on their allocated subcarriers in each TS, by solving the following problem

$$
\min_{p_{k,n}(t)} \hat{E}_{k,n}(t) = \frac{\hat{p}_k(T) + p_{k,n}(t)}{c_{k,n}(t)},
$$

(20)

for $k = \{1, \ldots, K\}$ and $n = \{1, \ldots, N\}$, respectively, where

$$
c_{k,n}(t) = \log_2 \left( 1 + \frac{p_{k,n}(t) g_{k,n}(t)}{\sigma^2} \right),
$$

(21)

denotes the achievable rate of user $k$ on subcarrier $n$. Since each user transmits both control and data signals in the same TS, the power for data transmission for user $k$ in TS $t$ is $P_k(t) = P_{\text{max}} - \hat{p}_k(t)$, where $\hat{p}_k(T)$ is defined as in (3) but for $T = 1$.

The optimal value that minimizes (20) satisfies $\nabla \hat{E}_{k,n}(p_{k,n}^*) = 0$. By substituting (21) into (20) and solving $\nabla \hat{E}_{k,n}(p_{k,n}^*) = 0$, we obtain

$$
\hat{E}_k^*(t) = \ln(2) \sum_{n}^{N} (p_{k,n}^*(t) + \sigma^2 g_{k,n}^{-1}(t)).
$$

(22)

Equating (20) and (22) and solving for $p_{k,n}^*(t)$, we obtain the optimal transmit power of user $k$.
Algorithm 2 Online EM Emission Reduction Algorithm

1: **Inputs:** $W, N, l, B_k, K, \sigma^2, \text{SAR}_k, P_{\text{max}}, \hat{p}_k, B, g_{k,n}, \forall k = 1, 2, \ldots, K$ and $n = 1, 2, \ldots, N$
2: Initialize $t = 1$;
3: Obtain subcarrier allocation ordering $\eta_i$, for $i = 1, 2, \ldots, N$;
4: Initialize $i = 1$;
5: Allocate subcarrier $\eta_i$ to the user with the best $g_{k,\eta_i}(t)$;
6: Compute $p^*_{k,n}(t)$ by using (23);
7: Compute $b^*_{k,n}(t)$ by using (24);
8: If (25) holds;
9: Compute $p^*_{k,n}(t)$ by using (26) and $b^*_{k,n}(t)$ by using (24);
10: Remove user $k$ from scheduling list of TS $t$;
11: If (27) holds;
12: Compute $p^*_{k,n}(t)$ by using (28) and $b^*_{k,n}(t)$ by using (24);
13: Remove user $k$ from scheduling list completely;
14: $i = i + 1$;
15: Repeat steps 4 to 14 until $i = N + 1$, then proceed to $t = t + 1$;
16: Repeat steps 3 to 15 until all users achieve target $B_k$;
17: Compute $E_k$ via (2);
18: **Output:** $E_k$.


on subcarrier $n$ at TS $t$ that minimizes (20) as

$$p^*_{k,n}(t) = \sigma^2 g_{k,n}(t) \left[ W_0 \left( \frac{\hat{p}_k(t)g_{k,n}(t)}{\sigma^2} - 1 \right) e^{-1} \right]_+, \quad (23)$$

where $W_0$ denotes the real branch of the Lambert function.

Accordingly, the optimal number of bits that can be transmitted by user $k$ on subcarrier $n$ at TS $t$ is given by

$$b^*_{k,n}(t) = w l \log_2 \left( 1 + \frac{p^*_{k,n}(t)g_{k,n}(t)}{\sigma^2} \right) \quad (24)$$

3) **Transmit power and target data constraints:** In order to ensure that the users comply with the power constraint and also avoid unnecessary EM emission by not transmitting more than the target number of bits, we introduce the following conditions in the algorithm framework.

If

$$\sum_{q \in N_k(t)} p_{k,q}(t) + p^*_{k,n}(t) \geq P_k(t) \quad (25)$$
Table I
SIMULATION PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>System Bandwidth ((W))</td>
<td>10 MHz</td>
</tr>
<tr>
<td>Number of subcarriers ((N))</td>
<td>128</td>
</tr>
<tr>
<td>Duration of 1 TS ((l))</td>
<td>1 ms</td>
</tr>
<tr>
<td>Rx signal Power threshold ((P_0))</td>
<td>-112 dBm</td>
</tr>
<tr>
<td>Number of signaling bits ((a))</td>
<td>2</td>
</tr>
<tr>
<td>Max. User Tx Power ((P_{\text{max}}))</td>
<td>0.2 W</td>
</tr>
<tr>
<td>Cell Radius</td>
<td>500 m</td>
</tr>
<tr>
<td>SAR</td>
<td>1 W/kg</td>
</tr>
<tr>
<td>(P_{\text{ref}})</td>
<td>1 W</td>
</tr>
<tr>
<td>Noise power density</td>
<td>-174 dBm/Hz</td>
</tr>
</tbody>
</table>

then

\[
p_{k,n}^*(t) = \left[ P_k(t) - \sum_{q \in N_k(t)} p_{k,q}(t) \right]_+, \tag{26}
\]

where \(N_k(t)\) represents the set of the subcarriers already allocated to user \(k\) in TS \(t\).

Similarly, for the number of bits constraint, if

\[
\tilde{b}_k + b_{k,n}(t) \geq B_k, \tag{27}
\]

then

\[
p_{k,n}^*(t) = \left(2^{(B_k - \tilde{b}_k)/\mu_l} - 1\right)\sigma^2 / g_{k,n}(t), \tag{28}
\]

where \(\tilde{b}_k = \sum_{j=1}^{t-1} \sum_{q \in N_k(j)} b_{k,q}(j) + \sum_{q \in N_k(t)} b_{k,q}(t)\) is the total transmitted bits by user \(k\) up to TS \(t\).

Algorithm 2 summarizes our online approach for minimizing the EM emission towards each user for a given target number of bits, \(B_k\).
C. Complexity Analysis

As far as our offline scheme is concerned, creating the subcarrier allocation matrix $U$ involves selecting the worst channel gain on each subcarrier and sorting of all $NT$ subcarriers in ascending order. This process has a computational complexity of $O(NT \log NT + NTK)$. The user-subcarrier pairing in the subcarrier allocation phase of our offline EM emission scheme is a two dimensional search of $U$ with a complexity of $O(KNT)$. Thus, the subcarrier allocation phase of the offline scheme has a complexity of $O(NT \log NT + NTK)$. Obtaining the Lagrange multipliers by using the secant method of root finding has a complexity of $O(\beta NT(\pi_1 + \pi_2))$, where $\pi_1$ and $\pi_2$ denote the number of iterations it takes to obtain $\lambda_k, \mu_k(t)$, while $\beta$ gives the number of iterations it takes for the rate allocation phase to converge. Hence, obtaining the bit allocation of all the users has a computational complexity of $O(\beta NT(\pi_1 + \pi_2))$. Thus, our offline scheme has a complexity of $O(\beta NT(\pi_1 + \pi_2))$.

Regarding our online EM emission reduction scheme, the subcarrier allocation has a complexity of $O(XN(\log N + K))$ while the power allocation has a complexity of $O(\rho XN)$, where $\rho$ denotes the number of iterations needed to execute the Lambert function and $X$ represents the number of TSs required for all the users to transmit their target number of bits. This means our online scheme has a complexity of $O(\rho XN)$.

Our simulations have shown that $\beta(\pi_1 + \pi_2) < \rho$. Hence, when assuming that $X = T$, it implies that our offline scheme has a lower computational complexity than our online scheme (without taking into account CSI prediction).

IV. Numerical Results

In this section, we compare the performances of our proposed EM emission reduction schemes by using Monte Carlo simulation. We analyze their performances for both the pedestrian and vehicular scenarios by modeling the fast fading based on ITU pedestrian and vehicular channels [30]. We assume the $\text{SAR}_k$ and $P_{\text{ref}}$ of all the users in the network to be the same, hence, $\text{SAR}_k/P_{\text{ref}} = \text{SAR}/P_{\text{ref}}, \forall k$. We further assume that all the users have the same target number of bits, i.e., $B_k = B, \forall k$. We benchmark our proposed schemes against the classic greedy SE-
Figure 1. Total uplink EM emission comparison of our proposed EM emission reduction schemes versus the target number of bits for $K = 15$ users and $T = 10$ time slots.

based scheme and the EE scheme of [15]. In the greedy SE-based scheme, any subcarrier is allocated to the user with the best channel gain on that particular subcarrier in the current TS and then, per-user power allocation is performed via water-filling. The users’ allocated subcarriers are then sorted in descending order and transmission starts from the best subcarrier. The EE scheme of [15] is based on optimizing the time averaged bit-per-Joule for subcarrier and power allocations. The allocation sequence is the same as in Algorithm 2, except that subcarrier and power allocations are based on [15]. In order to ensure fair comparison, (27) and (28) are used for meeting the constraint on the number of transmitted bits, in both benchmark schemes. The numerical values of the parameters considered in our simulations are summarized in Table 1.

Given that signaling and data transmissions take place separately in the offline EM emission reduction scheme, each user can use all its transmit power for data transmission at each TS. Whereas, the online EM emission reduction scheme assumes instantaneous subcarrier and power allocations during each TS as well as simultaneous signaling and data transmission. All users transmit signaling information as long as their user’s target number of bits is not met.

In Fig. 1, we compare the total uplink EM emission of our proposed EM emission reduction
schemes versus the target number of bits, against the benchmark greedy SE-based scheme and the EE scheme of [15] for $K = 15$ users and $T = 10$ TSs. It is evident that our offline EM emission reduction scheme produces the least EM emission of all the compared schemes. Furthermore, it can be remarked that the total EM emission of all the schemes increases as the target number of bits increases. Regarding our offline EM emission reduction scheme, given that the number of subcarriers allocated to each user is fixed, more power is needed to achieve the target number of bits when the latter increases; while more TSs are required to achieve higher number of bits’ target in our online and the greedy SE-based schemes, as well as the EE scheme of [15]. Additionally, all the users in our online EM emission reduction scheme as well as the benchmark schemes will have to transmit signaling information during each TS, irrespective of whether they transmit any data or not, until their bits target is met and these users are removed from the subcarrier allocation list. It can be observed that for high target number of bits, our online scheme performs better than our offline scheme. This is because our offline scheme is constrained to transmit only within the transmission window which results in an exponential transmit power increase and, in turn, EM emission increase. For low target number of bits, our offline EM emission reduction scheme outperforms our online scheme by as much as 70%. Whereas at high target number of bits, our online scheme performs better than the offline scheme by up to 45%. The greedy SE-based scheme has the highest EM emission because it tries to maximize the number of bits transmitted on each subcarrier by making use of all the available power. The greedy SE-based scheme has a significantly higher EM emission compared to our proposed offline scheme by up to 3 orders of magnitude. Although the EE scheme of [15] has a lower EM emission when compared to the greedy SE-based scheme, our proposed offline scheme outperforms it by up to 2 orders of magnitude. On the other hand, our proposed online scheme outperforms the EE scheme of [15] and the greedy SE based scheme by up to 2 and 2.5 orders of magnitude, respectively.

Fig. 2 depicts the total uplink EM emission of our proposed schemes versus the number of users in the network for a target of $B = 10$ Kbit and $T = 10$ TSs. It can be observed that the total uplink EM emission increases with the number of users in the network. In our offline
EM emission reduction scheme, within the transmission window $T$, the number of each user allocated subcarriers reduces as the number of users in the network increases, because they have to share the available subcarriers. It implies that the users would have to transmit with more power to achieve the target number of bits. Whereas in our online and the greedy SE-based schemes as well as the EE scheme of [15], given that the number of subcarriers in a TS is fixed, more TSs would be needed to achieve the target number of bits of all the users in the network as the number of users increases. As in Fig. 1, our offline scheme outperforms the EE scheme of [15] and an SE-based scheme by up to 2 and 3 orders of magnitude, respectively, while our online scheme outperforms the EE scheme of [15] and SE-based scheme by up to 2 and 2.5 orders of magnitude, respectively.

In Fig. 3, we compare the total uplink EM emission of our offline scheme against the other schemes for the same data rate. The data rate of our offline scheme was matched to the other schemes by fixing the transmission window to the same TSs used by the other schemes. The top plot of Fig. 3 shows a comparison between our proposed offline and online EM emission reduction schemes. It can be observed that our offline EM emission reduction scheme achieves
Figure 3. Total uplink EM emission comparison of our proposed EM emission reduction schemes versus the target number of bits for $K = 10$ users and the same bit-rates.

up to 30% reduction in EM emission compared to our online scheme. The fluctuation in the uplink EM emission of our offline scheme results from a change in bit-rate. The number of TSs used for transmission between the targets of 22 Kbit and 24 Kbit increases from 6 to 7, which, accordingly, results in a drop in bit-rate from 3.67 Mbps to 3.43 Mbps. This results in more subcarriers for the users to transmit with in our offline scheme, thereby resulting in lower EM emission. The bottom left plot of Fig 3 shows the total uplink EM emission comparison of our offline scheme versus the EE scheme of [15]. The bit-rate varies from 4 Mbps to 4.3 Mbps and it can be seen that our proposed offline EM emission reduction scheme outperforms the EE scheme by over 2 order of magnitude. This shows that even though the objective of EE is to improve the number of transmitted bits per unit energy consumed, it is, however, not the most suitable for EM emission reduction. In the bottom right plot of Fig. 3, the EM emission of our offline scheme is compared against EM emission of the greedy SE scheme. The greedy SE scheme has the highest bit-rate (average of 10 Mbps) of all the schemes compared, as it seeks to maximize the SE performance of the system thereby resulting in a shorter transmission duration. By matching the bit-rate of our offline scheme with that of the greedy SE scheme, we
show that our offline scheme achieves a considerable reduction in EM emission of over 1 order of magnitude. However, as the target number of bits increases, the EM emission performance of our offline scheme approaches that of the greedy SE scheme. This is due to the limited number of subcarriers within the transmission window for achieving the high target number of bits, a phenomenon already observed in Fig. 1.

Fig. 4 depicts the performance comparison of our online scheme versus the online scheme’s power allocation with Round Robin (RR) scheduling whereby the users are allocated their best subcarriers in an RR manner. The top plot of Fig. 4 shows EM emission comparison of the schemes for $K = 10$ users. It can be seen that the online scheme with greedy RR scheduling has at least 40% more EM emission when compared to our online scheme. This is because the algorithm tries to enforce fairness by ensuring that all users transmit similar amount of data in each TS as long as they have not met their target number of bits; in turns, this will force users with poor channel to transmit with higher power and result in higher EM emission. Hence, this result is in line with Lemma 1 which states that spreading the power over time is the way to achieve a lower EM emission. The bottom plot of Fig. 4 shows the Jain’s fairness index.
comparison of our online EM emission reduction scheme for the first 5 TSs when $B = 20$ Kbit and $K = 10$ users. As expected, it can be seen that the online scheme with greedy RR scheduling has a significantly higher degree of fairness of about 98%. Our online scheme, however, has a lower fairness of about 40% in the first TS but it increases as the sum rate of the users increase in subsequent TSs. This is because our algorithm is greedy and it assigns a subcarrier to the user with the best channel gain on it as long as that user has not reached its target number of bits. This approach leads to a lower EM emission but at the cost of a lower per TS rate fairness, even though all the users will eventually meet their targets.

Fig. 5 shows the effect of imperfect channel prediction on our offline EM emission reduction scheme for pedestrian and vehicular channels, when $K = 15$ users. The figure is based on the channel estimation model of [31], with the estimation variance parameter $\epsilon = 0, 0.3$ and 0.5, where $\epsilon = 0$ denotes a perfect channel prediction while $\epsilon = 1$ represents a completely uncorrelated channel prediction. It is obvious that the total uplink EM emission is lowest when there is a perfect channel prediction and the performance gap increases as the target number of bits increases. It is also expected that the EM emission over the pedestrian channel is lower.
than over the vehicular channel (by about 12% in Fig.4), given the latter model’s poorer channel conditions than the former. It can further be observed that our proposed scheme is very robust even when the CSI prediction error is very high, as there is only a 12% and 15% difference in the total uplink EM emission between a perfect CSI prediction and when $\epsilon = 0.5$ for the pedestrian and vehicular channels, respectively.

In Fig. 6, we depict the effect of the transmission window size, $T$, on our offline EM emission scheme for $K = 15$ users and $B = 10$ Kbit. It can be observed that EM emission reduces as the transmission window increases, which was predicted by Lemma 1. When the transmission window increases, more subcarriers become available to the users and, hence, a lower transmission power is required to achieve the target number of bits of all the users. Since the transmission window affects the performance of our offline EM emission reduction scheme, in a practical setting, the network operator could vary the length of the transmission window depending on the network EM emission threshold. Delay sensitive transmissions could have a shorter transmission window, while delay tolerant applications could have a longer transmission window to further reduce the EM emission.
V. CONCLUSION

In this paper, we have proposed two schemes - offline and online - for minimizing the EM emission of individual users in the uplink of OFDM systems. Our offline EM emission reduction scheme is based on the assumption that the network can predict the long-term CSI of all the users for allocating them on subcarriers. Then an optimal rate-based water-filling is performed to obtain the rate and power allocations of each allocated user on each subcarrier. On the other hand, our online EM emission reduction scheme, which is based on short-term CSI knowledge, allocates power to users by minimizing the transmit energy per bit of each user. Simulation results show that our proposed offline scheme significantly outperforms existing SE and EE based schemes, by up to 3 and 2 orders of magnitude, respectively. Accordingly, our proposed algorithms for our online scheme outperform the SE and EE based schemes by up to 2.5 and 2 orders of magnitude, respectively. Hence, optimizing the SE or SE is far from being optimal in terms of EM emission. We have also shown that EM emission of our offline scheme is inversely related to the transmission window, which makes it suitable for delay tolerant transmissions. Additionally, our offline scheme proves to be very robust against the effects of imperfect channel prediction.

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APPENDIX

Proof of Lemma 1

The amount of bits $b$ transmitted with power $p$ over duration $l$ is given by

$$ b = wl \log_2 \left( 1 + \frac{pq}{\sigma^2} \right). \quad (29) $$
After some simplification, the transmit power needed to transmit \( b \) bits over duration \( l \) can be expressed as

\[
p = (2^{b/wl} - 1)\sigma^2 g^{-1}.
\]  

(30)

Thus, the energy emitted for transmitting \( b \) bits over duration \( l \) is given as

\[
e = (2^{b/wl} - 1)\sigma^2 g^{-1}l
\]  

(31)

which implies that \( e \) is monotonically decreasing and convex in \( l \). Hence, the energy needed to transmit \( b \) bits decreases as the transmission duration increases.

\[\blacksquare\]

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