

Pseudo Pilot: A Novel Paradigm of Channel Estimation

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Abstract—The aim of this article is to share a novel concept termed **pseudo pilot**, which offers a simple and efficient approach of non-pilot-assisted channel estimation. Our key idea is to transfer the uncertainty of several payload symbols into the uncertainty of symbol interleavers by employing a bank of interleavers at the transmitter. Those uncertainty-transferred symbols serve as pseudo pilots for the receiver to perform channel estimation. The uncertainty of symbol interleavers is then removed in the procedure of decoding. Performance and scalability of the pseudo pilot technique are evaluated through both theoretical analysis and computer simulations.

Index Terms—Pseudo pilot, channel estimation, symbol interleavers.

I. INTRODUCTION

IN most of digital wireless communication systems (such as LTE-A and Wi-Fi), pilots are employed for the purpose of robust channel estimation and synchronization [1]. On the other hand, pilots cost time-domain degrees of freedom, and they count as overhead in wireless communications [2]. In order to reduce the pilot overhead, a receiver can perform (semi-) blind channel estimation, which takes advantage of statistical properties of received payload symbols. Those statistical properties are mainly second-order or higher-order (cyclo) stationarities, which often require relatively long data record to observe. Moreover, most of blind channel estimators still need pilots to resolve residual phase (or sign) ambiguity of the channel estimate (e.g. [3]-[8] and many others).

The aim of this article is to introduce a novel concept termed pseudo pilot, which can play the same role as the conventional pilot in the procedure of channel estimation and synchronization¹. It will be shown that channel estimation with pseudo pilots can offer identical performances as that with conventional pilots at no cost (or reduced cost) of the pilot overhead.

Basically, our idea is to employ a bank of pseudo random symbol (PRS) interleavers (also called interleaver bank) at the transmitter (see Fig. 1), which re-arranges the payload symbol block in a number of ways so that at least one of rearrangements contains a sub-block coinciding with a reference block (defined in Section II-B). Symbols within the sub-block are known at the receiver, and thus they are named pseudo pilots, which can be employed for the channel estimation. On the other hand, the receiver does not know

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¹In order to focus on the key concept, this article will mainly use channel estimation as an use-case to justify key advantages of using pseudo pilots.

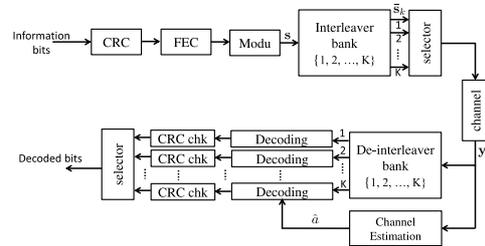


Fig. 1. An example of generating pseudo pilots.

which PRS interleaver has been chosen at the transmitter. This is called uncertainty of the PRS interleavers, which can be removed in the procedure of decoding. In addition to the novel concept, a theoretical basis is established, which helps to understand the maximal number of pseudo pilots that can be generated given modulation order as well as the size of payload symbol block. A practical approach is then proposed to implement the pseudo pilot technique in low-cost machine type communications (MTC) use-cases.

II. PSEUDO PILOT ASSISTED WIRELESS SYSTEM

A. Simple Model of Pilot-Assisted Wireless System

The description of pilot assisted wireless systems often involves several issues including pilot design, time-frequency domain pilot placement, and specific channel estimation algorithms (e.g. in [10]). In this article, we employ a simple point-to-point model to describe pilot assisted wireless systems, and this is for the sake of focusing our presentation onto the key concept of interest.

Consider the transmitter sending an $(M + \mathcal{L}) \times 1$ symbol block $\mathbf{x} \triangleq [\mathbf{p}^T, \mathbf{s}^T]^T$, where $\mathbf{p} \triangleq [p_1, \dots, p_{\mathcal{L}}]^T$ is the pilot block consisting of \mathcal{L} pilot symbols, $\mathbf{s} \triangleq [s_1, \dots, s_M]^T$ the symbol block consisting of M information-bearing symbols, and $[\cdot]^T$ the matrix or vector transpose. Assuming the communication channel to be flat fading (e.g. one of resource blocks in LTE-A), the received block in its baseband equivalent form (denoted by \mathbf{z}) is

$$\mathbf{z} = \mathbf{a}\mathbf{x} + \mathbf{v}, \quad (1)$$

where \mathbf{a} denotes the channel state, and \mathbf{v} the white Gaussian noise with zero mean and covariance $\sigma^2\mathbf{I}$; and \mathbf{I} is the identity matrix.

Denote $\mathbf{z}^{(\mathcal{L})}$ to be the block formed by collecting the first \mathcal{L} elements of \mathbf{z} . The least-square (LS) estimate of the channel state $\hat{\mathbf{a}}$ is

$$\hat{\mathbf{a}} = \|\mathbf{p}\|^{-2} \mathbf{p}^H \mathbf{z}^{(\mathcal{L})}, \quad (2)$$

where $\|\cdot\|$ denotes the Euclidean norm, and $[\cdot]^H$ the Hermitian transpose. Mean square error (MSE) of the LS channel estimator (2) is (please see [1])

$$\text{MSE} = (\sigma^2)/(\|\mathbf{p}\|^2). \quad (3)$$

In this short article, our technical presentation mainly focuses on the LS estimator for the sake of conciseness. Similar results also apply to the linear MMSE estimator, and the procedure is rather trivial.

B. Concept of Pseudo Pilot

It is our aim to (partially) replace the pilot block \mathbf{p} with pseudo pilots so as to reduce the pilot overhead. Start from the information-bearing block \mathbf{s} , which is generated according to the procedure illustrated in Fig. 1. Prior to transmission, \mathbf{s} is fed into the PRS interleaver bank, which consists of K parallel PRS interleavers with the notation of $\pi_k(\mathbf{s})$, $k=1, \dots, K$. Denote $\bar{\mathbf{s}}_k \triangleq \pi_k(\mathbf{s})$ to be the output of the k^{th} PRS interleaver, and $\bar{\mathbf{s}}_k^{(L)}$ the vector formed by the first L elements of $\bar{\mathbf{s}}_k$. Given an $L \times 1$ reference block \mathbf{r} , which is assumed to be known at both the transmitter and receiver, the transmitter sends the k_o^{th} output, $\bar{\mathbf{s}}_{k_o}$, to the receiver when

$$\|\bar{\mathbf{s}}_{k_o}^{(L)} - \mathbf{r}\| = 0. \quad (4)$$

Then, $\bar{\mathbf{s}}_{k_o}^{(L)}$ (or equivalently \mathbf{r}) is called the pseudo-pilot block, which can be utilized for the channel estimation. The pilot block \mathbf{p} within \mathbf{x} can be dropped (or partially dropped when $L < \mathcal{L}$) for the sake of reducing pilot overhead.

It is worthwhile to note that the solution of (4) can have two cases:

Case 1: (4) has one or more solutions. For the case of having unique solution, the transmitter just picks up the k_o^{th} output for the transmission. For the case of having multiple solutions, the transmitter can randomly pick up one of them.

Case 2: (4) has no solution. We adopt a smaller L in order to increase the probability of having a solution (see Sec. III).

C. Receiver Design

Consider the case of having $L(\geq \mathcal{L})$ pseudo pilots, where the pilot block \mathbf{p} is dropped at the transmitter². The received block becomes: $\mathbf{y} = \mathbf{a}\bar{\mathbf{s}}_{k_o} + \mathbf{v}$. We stress that the receiver knows $\pi_k(\cdot)$, $k=1, \dots, K$, and employs corresponding de-interleavers $\pi_k^{-1}(\cdot)$, $k=1, \dots, K$, for signal recovery. However, the receiver does not know about the index k_o , which should be estimated in the procedure of signal recovery.

Given the channel knowledge \mathbf{a} , the procedure of signal recovery can be described by three steps; see Fig. 1.

Step 1: Form an $M \times 1$ block $\bar{\mathbf{y}} \triangleq [\mathbf{r}^T, \mathbf{y}(L+1 : M)^T]^T$, where $\mathbf{y}(L+1 : M)$ denotes the vector formed by the $(L+1)^{\text{th}}$ element to the M^{th} element of \mathbf{y} . This step de-noises $\mathbf{y}(1 : L)$ by using the clean version \mathbf{r} .

Step 2: Feed $\bar{\mathbf{y}}$ into the de-interleaver bank, which yields

$$\mathbf{z}_k = \pi_k^{-1}(\bar{\mathbf{y}}), \quad k = 1, \dots, K. \quad (5)$$

²Straightforward extension applies to the case in the presence of \mathbf{p} .

Step 3: Apply FEC decoding on \mathbf{z}_k , and then employ error detection component (such as CRC) to decide whether or not decoding errors exist, i.e.,

$$\mathbf{1}_k = \text{CRC}(\text{FEC}^{-1}(\mathbf{z}_k)), \quad (6)$$

where $\mathbf{1}_k$ is the indicator function with binary states: $\mathbf{1}_k = 1$ indicating errors; and $\mathbf{1}_k = 0$ indicating error free. Then, the receiver will take the decoding result when $\mathbf{1}_k = 0$.

Assume CRC checking to be reliable. In the noiseless case, the state $\mathbf{1}_k = 0$ happens only when $k = k_o$. This shows how the uncertainty of PRS interleavers is removed. In the case of noisy channel, CRC might report the existence of errors for the case of $k = k_o$. This is because of the existence of FEC decoding error, which is also the case for other wireless systems. In usual practice, a request of retransmission will be sent to the transmitter. Considering the pessimistic case when CRC fails, the receiver would not be able to remove the uncertainty of PRS interleavers. Fortunately, the probability of CRC failure is reasonably small in practice, and we assume CRC to be reliable in the rest of our presentation.

The last issue is about channel estimation, which is rather straightforward. The receiver knows that $\mathbf{y}^{(L)}$ is the noise-corrupted version of \mathbf{r} . Hence, the LS channel estimator (2) and its MSE (3) can be straightforwardly employed with \mathbf{p} , \mathbf{z} to be replaced by \mathbf{r} , \mathbf{y} , respectively.

III. THEORY, FUNDAMENTAL LIMIT AND SCALABILITY

A. Theory and Fundamental Limit

Based on the description in Section II, pseudo pilot can be understood as a simple coding/decoding technique, which employs the interleaver bank to transfer the uncertainty of $\bar{\mathbf{s}}_{k_o}^{(L)}$ to the index k_o , and the de-interleaver bank to remove the uncertainty of k_o . In information theory, the uncertainty of $\bar{\mathbf{s}}_{k_o}^{(L)}$ can be measured by the Shannon entropy $\bar{h}(\bar{\mathbf{s}}_{k_o}^{(L)})$, and the uncertainty of k_o is quantified by $\bar{h}(k_o)$. When $\bar{h}(\bar{\mathbf{s}}_{k_o}^{(L)}) > \bar{h}(k_o)$, the system suffers uncertainty loss (or equivalently information loss) in the procedure of uncertainty transfer, and this is certainly not desired. Hence, the uncertainty transfer process should fulfil the criterion: c1) $\bar{h}(\bar{\mathbf{s}}_{k_o}^{(L)}) \leq \bar{h}(k_o)$.

It is assumed that the FEC encoder could distribute uniformly the information (uncertainty) over the symbol block \mathbf{s} , and PRS interleaves do not change the distribution. Hence, the entropy $\bar{h}(\bar{\mathbf{s}}_{k_o}^{(L)})$ is given by

$$\bar{h}(\bar{\mathbf{s}}_{k_o}^{(L)}) = \frac{L}{M} \bar{h}(\mathbf{s}). \quad (7)$$

We further assume that the PRS interleavers are independent and different. The probability for the k^{th} PRS interleaver to be selected is $(1)/(K)$. Then, the uncertainty of k_o is easy to measure

$$\bar{h}(k_o) = - \sum_{k=1}^K \frac{1}{K} \ln(1/K) = \ln(K). \quad (8)$$

Plugging (7) and (8) into the inequality of (c1), we can immediately conclude the following result.

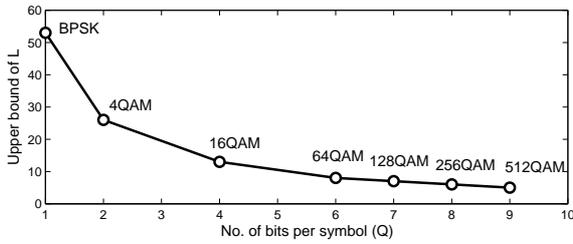


Fig. 2. The upper bound of L as a function of Q when $M = 128$.

Theorem 1 (uncertainty transfer): A sufficient condition for the criterion c1) to be fulfilled is

$$K \geq \exp\left(\frac{L}{M}h(s)\right). \quad (9)$$

Theorem 1 defines a lower bound of K , with which the pseudo pilot assisted system would not suffer any unexpected information loss. However, the condition (9) is not sufficient to guarantee that one can always generate L pseudo pilots.

Theorem 2: Denote Q to be the size of the finite alphabet where the information-bearing symbols were drawn from. A sufficient condition of generating L pseudo pilots with the probability \mathcal{P} is

$$K \geq \left\lceil \frac{\ln(1 - \mathcal{P})}{\ln\left(1 - \sum_{m=L}^M \left(\frac{P_m^L}{P_M^L}\right) \binom{M}{m} \frac{(1-Q^{-1})^{M-m}}{Q^m}\right)} \right\rceil, \quad (10)$$

where P_M^L denotes the number of all possible permutations, and $\lceil \cdot \rceil$ the integer ceiling.

Proof: See Appendix. ■

B. Scalability Analysis

Scalability is measured by the relationship between L and the number of PRS interleavers K given the data record length M and the modulation order Q . Although the bound (10) is mathematically intractable, it is already in a good form to conduct semi-analytical performance analysis.

It is easy to understand that L reaches its maximum when $K \rightarrow \infty$. Applying $K \rightarrow \infty$ into (10), we can immediately have

$$\lim_{K \rightarrow \infty} \sum_{m=L}^M \left(\frac{P_m^L}{P_M^L}\right) \binom{M}{m} Q^{-m} (1 - Q^{-1})^{M-m} = 0. \quad (11)$$

Then, the upper bound of L can be found by numerical means of handling (11).

Fig. 2 shows the upper bound of L for various configurations of Q when $M = 128$. It is observed that a relatively large number of pseudo pilots ($L = 53$) can be theoretically generated when payload symbols are BPSK modulated ($Q = 1$). However, the upper bound of L quickly decreases with the increase of modulation order. For higher-order modulations ($Q = 6, 7, 8, 9$), the upper bound of L gets close to 5. In addition, TABLE I shows the lower bound of K (based on (10)) given L . The probability \mathcal{P} is set to 90%. It is observed that the lower bound of K increases exponentially with respect to the parameters L and Q .

TABLE I
SHOWCASE THE RELATIONSHIP DESCRIBED IN (10).

K	L : Number of Pseudo Pilots							
	1	2	3	4	5	6	7	8
BPSK	4	9	18	36	73	147	294	589
QPSK	9	36	147	589	2357	9431	3.8e4	1.5e5
16QAM	36	589	9431	1.5e5	2.4e6	3.9e7	6.2e8	9.9e9

Remark 1: A single PRS interleaver/de-interleaver costs complexity $\mathcal{O}(M)$. Given K interleavers, major computational cost comes from the decoding process, which increases by a factor of K . For complexity-constrained applications, we suggest to employ relatively small number of pseudo pilots in order to manage K at an acceptable level. Taking the example in TABLE I ($M = 128$ coded symbols), we can employ $K = 36$ PRS interleavers to generate either 2 QPSK-modulated pseudo pilots or 4 BPSK-modulated pseudo pilots. Then, the decoding complexity is equivalent to the case of decoding a sequence with $M \times K = 4,608$ coded symbols, which is affordable for many practical systems. Moreover, for uplink communications, the decoding process can be made fully parallel for the sake of reducing the processing delay.

Remark 2: It is worthwhile to mention that the pseudo pilot technique exploits PRS interleaver diversity to represent the information. Similar idea has been employed in the spatial modulation [11], which exploits the channel spatial diversity to represent the information. Moreover, the interleaver diversity was also employed to reduce the peak-to-average power ratio (PAPR) in multi-carrier systems [12], [13]. However, both the objective and scope of pseudo pilot are totally different from that of the spatial modulation and PAPR reduction.

IV. APPLICATION USE-CASES AND SIMULATIONS

A. Potential Applications in MTC Communications

Section III shows that pseudo pilot in its current form faces challenges of scalability particularly for higher-order modulations. This is what our future research should focus on. Nevertheless, pseudo pilot works well with BPSK and QPSK. Hence, it is a viable scheme for low-rate low-mobility MTC communications in the scope of Internet of things (IoT) applications; see [14], [15].

It has been recognized that low-cost MTC devices (such as smart meters and wireless sensors) often generate a short burst of message ($\leq 1,000$ bits), which is suggested to form a low-rate data stream [16]. When such a data stream goes through LTE-A networks (multi-carrier systems), it can occupy one or more resource blocks within a narrowband channel, where the signal bandwidth is smaller than the channel coherence bandwidth (i.e., the channel is flat fading). Then, the LTE cell-specific reference symbols (CRS), which are employed mainly for the purpose of channel estimation, can now be replaced by pseudo pilots.

B. Simulation Model and Evaluation

The objective of our computer simulations is mainly to prove the concept of pseudo pilot. To this end, we randomly generated information bits with the equal probability (plus

16 CRC bits), and fed them into a $\frac{1}{3}$ -rate turbo encoder, which produced 336 coded bits per burst. Then, the coded bits were modulated into QPSK symbols, which were mapped onto two time-domain consecutive LTE resource blocks (i.e., 12 subcarriers and 14 time slots). The subcarrier spacing is 15 kHz, which is in line with the LTE-A standard [17]. The communication channel model is the extended Pedestrian-A (EPA) channel specified by 3GPP [18]. The channel was independently generated for each burst assuming 1 Hz Doppler frequency. This channel model has been widely adopted in studying the low-cost low-mobility MTC communications (e.g. in [19]). More importantly, it helps us to avoid the distraction from other issues in the procedure of performance evaluation. Those issues include channel time-frequency selectivity, pilot placement and non-ideal interpolation errors, which will give identical impact on the performances whether we use conventional pilots or pseudo pilots.

Pseudo pilots were evenly divided into two parts, with each placed at the first time slot of a resource block. The LS channel estimator was employed for conducting the channel estimation. In our computer simulations, we did not implement the PRS interleavers in the same way as introduced in Section II-B as far as the implementation complexity is concerned. Instead, we employed a simplified way as follows:

Step 1: Perform PRS interleaving $\bar{s}_k = \pi_k(\mathbf{s})$ using the MATLAB random interleaver function. Here, k also denotes the index of initial seed of the PRS interleavers. For $k = 1$, we let $\bar{s}_1 = \mathbf{s}$.

Step 2: Identify whether or not the block \mathbf{r} is in \bar{s}_k . If yes, then move \mathbf{r} to the head of \mathbf{s}_k via circulant shift. If no, then go to *Step 1* with $k = k + 1$.

The above procedure can largely reduce the number of PRS interleavers due to the employment of circulant shift at *Step 2*. This can benefit MTC devices from the cost-effective point of view.

At the receiver side, a lookup table of the initial seeds is available. The PRS de-interleaving can be performed by visiting all possible k . In addition, the receiver will need to visit all possible circulant shifts for each k . The de-interleaving and decoding procedure stops when the CRC checking reports positive regarding the FEC decoding.

Fig. 3 illustrates the bit-error-rate (BER) taking average of 10,000 bursts. Performance comparison was made between the pseudo pilot assisted system and the pilot assisted system with various configurations of Eb/No, which is defined by the average received energy of uncoded bit to noise. For the pilot assisted system, we add an extra time slot to transmit pilot symbols. This is to keep the fairness in the procedure of encoding and decoding. Simulation results show that, with equal number of pseudo pilots and pilots, the BER performances for both systems are almost identical. We also tested the case (4 pseudo pilots) when the channels for two resource blocks were independently generated. Slight performance degradation is observed in comparison with the previous case of using 4 pseudo pilots. This is because the pseudo pilots are now utilized to estimate 2 channel states, and such renders the channel estimation performance equivalent to the previous case of using 2 pseudo pilots. Nevertheless, it is observed that

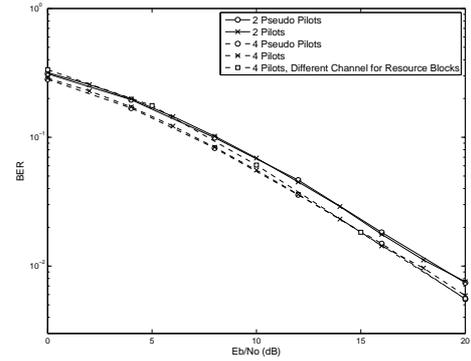


Fig. 3. BER performance of the pseudo pilot assisted system.

PRS interleavers help to enjoy the channel diversity gain in comparison with the previous case of using 2 pseudo pilots.

V. CONCLUSION AND FUTURE WORK

In this article, the key concept, theory and advantages of pseudo pilots have been presented. It has been shown that pseudo pilot assisted systems can offer the same performance as pilot assisted systems with reduced (or even zero) pilot overhead. Future work can focus on complexity reduction in the procedure of signal recovery.

APPENDIX: PROOF OF *Theorem 2*

For the sake of notation simplicity, we assume that all the elements of \mathbf{r} are identical, i.e., $r_m = r$. It is a sufficient condition to form L pseudo pilots if we have: (c2) \mathbf{s} has $\mathcal{L}(\geq L)$ elements that are equal to r . The probability of (c2) is

$$\mathcal{P}(\mathcal{L}) = \binom{M}{\mathcal{L}} \frac{(1 - Q^{-1})^{M-\mathcal{L}}}{Q^{\mathcal{L}}}. \quad (12)$$

Given (c2), the probability for a PRS interleaver to generate $\bar{s}_{k_o}^{\mathcal{L}} = \mathbf{r}$ is

$$\mathcal{P}(\bar{s}_{k_o}^{\mathcal{L}} = \mathbf{r} | (c2)) = P_{\mathcal{L}}^L (P_M^L)^{-1}. \quad (13)$$

Then, the probability for a PRS interleaver to have $(\bar{s}_{k_o}^{\mathcal{L}} = \mathbf{r})$ for all $L \geq L$ is

$$\bar{\mathcal{P}} = \sum_{\mathcal{L}=L}^M \mathcal{P}(\bar{s}_{k_o}^{\mathcal{L}} = \mathbf{r} | (c2)) \cdot \mathcal{P}(\mathcal{L}), \quad (14)$$

$$= \sum_{\mathcal{L}=L}^M P_{\mathcal{L}}^L (P_M^L)^{-1} \binom{M}{\mathcal{L}} \frac{(1 - Q^{-1})^{M-\mathcal{L}}}{Q^{\mathcal{L}}}. \quad (15)$$

Given K PRS interleavers, the probability of having at least one of them having $(\bar{s}_{k_o}^L = \mathbf{r})$ is

$$\mathcal{P} = 1 - (1 - \bar{\mathcal{P}})^K. \quad (16)$$

Plugging (15) into (16) leads to

$$\mathcal{P} = 1 - \left(1 - \sum_{m=L}^M \binom{P_M^L}{P_m^L} \binom{M}{m} \frac{(1 - Q^{-1})^{M-m}}{Q^m}\right)^K. \quad (17)$$

Given \mathcal{P} , one can easily justify that K is monotonically increasing with respect to L . Representing K as a function of L leads to (10).

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