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Anti-Jerk Controller with Optimisation-based Self-Tuning

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Abstract

One of the major phenomena compromising the comfort of the passenger vehicles is jerk. Jerk occurs as a response to the transient in the driver torque demand. The transient provokes torsional oscillation of the drivetrain, which results in oscillations and jerk of the vehicle. These oscillations and jerk are transmitted to the driver and can cause discomfort to the driver and thus affecting the drivability of the vehicle.

The aim of this work is to develop an anti-jerk controller to achieve smooth response of the vehicle and enhance the drivability metrics. The drivability analysis in this thesis focused on the longitudinal dynamic response during the tip-in manoeuvre.

The anti-jerk controller introduced in this work is an optimisation-based controller. It is developed by using two models, i.e. a linear model and non-linear model. The developed models include detailed description of the drivetrain system such as clutch, primary shaft, secondary shaft, differential, half-shaft, tyres and the vehicle. The engine was modelled using the engine map. To achieve high confidence of the models fidelity, the models were verified by experimental data which ensures that the models are accurate and characterised by the required details.

The anti-jerk controller is an optimised controller and uses a gain scheduling where the gain scheduling optimisation was performed off-line to reduce the engineering time in the controller gain tuning.

The simulation results of the models with the controller show a significant improvement of the drivability, which is measured by the overshoot and the rise time on the acceleration profile.
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Nomenclature

\( \theta_e \) Engine angular displacement
\( \theta_{diff} \) Differential angular displacement
\( \theta_{hs} \) Half-shaft angular displacement
\( \theta_p \) Primary shaft displacement
\( \theta_s \) Secondary angular displacement
\( \theta_v \) Vehicle angular displacement
\( \theta_w \) Wheel angular displacement
\( \dot{\theta}_e \) Engine angular speed
\( \dot{\theta}_{diff} \) Differential angular speed
\( \dot{\theta}_{hs} \) Half-shaft angular speed
\( \dot{\theta}_p \) Primary shaft angular speed
\( \dot{\theta}_s \) Secondary shaft angular speed
\( \dot{\theta}_v \) Vehicle angular speed
\( \dot{\theta}_{v0} \) Initial vehicle angular speed
\( \dot{\theta}_w \) Wheel angular speed
\( \dot{\theta}_{w0} \) Initial wheel angular speed
\( \ddot{\theta}_e \) Engine angular acceleration
\( \ddot{\theta}_{diff} \) Differential angular acceleration
\( \ddot{\theta}_{hs} \) Half-shaft angular acceleration
\( \ddot{\theta}_p \) Primary shaft angular acceleration
\( \ddot{\theta}_s \) Secondary shaft angular acceleration
\( \ddot{\theta}_v \) Vehicle angular acceleration
\( \ddot{\theta}_w \) Wheel angular acceleration
\( a_w \) Acceleration average
\( a_{xss} \) Longitudinal acceleration steady-state
$a_{x\text{max}}$ Longitudinal acceleration peak value
$J_1$ Sum of the mass moment of inertia of the engine and the clutch disk
$J_{c1}$ Mass moment of inertia of the clutch disk
$J_c$ Mass moment of inertia of the clutch
$J_{half}$ Mass moment of inertia of the half-shaft
$J_f$ Mass moment of inertia of the final drive
$J_{diff}$ Mass moment of inertia of the differential
$J_e$ Mass moment inertia of the engine
$J_s$ Mass moment of inertia of the secondary shaft
$J_p$ Mass moment of inertia of the primary shaft
$J_v$ Mass moment of inertia of the vehicle
$J_w$ Mass moment of inertia of the wheel
$J_{wfr}$ Mass moment of inertia of the front wheel
$J_k$ Mean root square jerk
$J'$ Jerk frequency dependant
$T_{aer\text{drag}}$ Air drag torque
$T_{clutch}$ Clutch damper torque
$T_e$ Engine Torque
$T_w$ Wheel Torque
$T_t$ Traction force
$T_p$ Propeller torque
$T_f$ Final drive torque
$T_d$ Driveshaft (Half-Shaft) torque
$T_{diff}$ Differential torque
$T_s$ Secondary shaft torque
$T_p$ The torque of the primary shaft
$T_{half}$ The torque of the half-shaft
$T_{roll}$ Rolling resistance torque
$F_n$ Normal force
$F_x$ Longitudinal traction force
$a_x(t)$ Time dependant homogenous solution
$a_x$ Acceleration
\( k_{hs} \)  Half-shaft stiffness
\( k_d \)  Stiffness of the clutch damper
\( \beta_d \)  Damping coefficient of the clutch damper
\( \beta_{hs} \)  Damping coefficient of the half-shaft damper
\( \tau_g \)  Gear box ratio
\( \tau_{diff} \)  Differential gear ratio
\( \eta_g \)  Gear box efficiency
\( \eta_{diff} \)  Differential efficiency
\( S \)  Front area of the vehicle
\( C_d \)  Aerodynamic drag coefficient
\( \rho_a \)  Air density
\( R_w \)  Wheel radius
\( SR \)  Longitudinal slip ratio
\( M_v \)  Vehicle mass
\( g \)  Acceleration of gravity
\( b \)  Rear semi wheel base for the whole vehicle
\( l \)  Vehicle wheelbase
\( gain \)  Theoretical feedback controller gain of the closed loop
\( Q \)  Controllability matrix
\( W' \)  Toeplitz matrix of the plant
\( a \)  Open loop plant coefficient
\( a_d \)  Closed loop plant coefficient

Subscription

\( f \)  Front
\( f_R \)  Front right
\( f_L \)  Front left
\( stat \)  Static
\( r \)  Rear
\( dyn \)  Dynamic
Chapter 1

Introduction

1.1 Context

One of the major phenomena compromising the comfort of the passenger vehicles is the jerk, which occurs as a response to the transient change in the driver torque demand. The transient can be caused by an increased torque demand, which is known in the literature as tip-in, or a decreased torque demand, which is known as tip-out. The transient provokes torsion in the drivetrain which in turn causes oscillations and jerk in the vehicle. This phenomenon affects comfort and thus the driveability of the vehicle. When the driver depresses the accelerator pedal, oscillation of the drivetrain and jerk are transmitted to the vehicle, which causes discomfort to the passenger and the driver. To have good vehicle drivability, jerk, oscillations and overshoot should be improved. At the same time there should be a limited delay between the driver input and the vehicle response.

The objective of this work is the development of an anti-jerk controller to achieve smooth response of the vehicle and enhance the drivability metrics. The solutions provided in the literature include conventional controller solutions without an extensive detailed vehicle model. The industrial implemented control algorithms are based mainly on generating the correction torque from the difference between modelled vehicle speed and actual vehicle speed, or engine speed and vehicle speed, or modelled engine speed and measured engine speed.

The introduced anti-jerk controller in this work is an optimisation-based controller. It is developed by using two models, i.e. a linear model and non-linear model. The developed non-
linear model includes detailed description of the drivetrain system. In addition to implementing the controller in the non-linear model, the controller was also implemented in the linear model to enable studying the system dynamic in the frequency domain.

The controller algorithms are based on calculating the speed difference between the engine and the wheel to generate the correction torque. By taking the speed difference between the wheel and the engine, it improves the precision of the controller.

Furthermore, one of the major tasks faced the vehicle engineers in order to achieve optimal vehicle drivability is the tuning of the anti-jerk controller in order to obtain the optimal control parameters in a short time.

The currently used methods are based on many trials [39], [120] in which the vehicle testing engineer drives the car and changes the various parameters to achieve acceptable level of comfort. This method takes long time and effort. In order to reduce the tuning time and complexity, a model-based controller can ease the tuning work and reduce the required time to obtain the optimal parameter configuration.

In this research, we introduce a new auto-tuning method to find the optimal controller parameter. The method requires setting the target and the optimisation parameters. Given this configuration, the optimal controller parameters will be achieved.

1.2 Scope of the Research

The aim of this research is to contribute to the vehicle drivability by developing a model-based anti-jerk controller. Several driveline models have been developed and discussed in the literature. Some of these models were implemented and mainly focused on the optimisation of the driveline configuration, where others were adopted for experimental and theoretical assessment, including benchmarking techniques for vehicle drivability.

The scope of this research, firstly, is to develop a model capable of studying the influence of the individual component parameters on the overall first order longitudinal dynamics and torsional dynamics of the powertrain and the vehicle. The mathematical linear and non-linear models enable a comprehensive analysis of the interaction between driveline components and the drivability. The model contributes to the multi-disciplinary understanding of the overall vehicle and powertrain dynamics, where the understanding of the real influence of changing each element of the overall performance of the vehicle drivability can be achieved.
The anti-jerk controller generates a correction torque which is added to the engine torque demand. The calculation of the correction torque is based on the wheel and engine speed difference. This novel anti-jerk controller is differentiated from the standard controller which based on the engine speed only. The usage of the speed difference between the engine and the wheel provokes a philosophy change of Tier 1 supplier. Consequently, inputs from sensors in different sub-systems such as the ABS/VDC are required. The controller consists of proportional (P), integral (I) and differential (D) components with gain scheduling. Furthermore, the models provide auto-tuning ability of the controller in order to find the optimal set of parameters. The tuning sequence can be performed in both the linear and the non-linear model. In the auto-tuning controller, optimisation procedure has been implemented. The auto-tuning algorithm in the non-linear model takes into account the vehicle and the engine parameters such as gear ratio, engine torque demand and the initial vehicle speed. After feeding these parameters into the model, the user can set a target to the overshoot and rise time with weighting for each target value. The optimal achievements in terms of the overshoot and rise time would be to keep the rise time with change compared to the status without controller and the overshoot not more than the set value. The algorithms start the simulation with initial values of the gains and change the parameter until the optimal value is achieved. Once the optimal values are found, the model presents the results with the optimal values.

The first contribution in this research is the development and validation of linear and non-linear simulation models, which are described in detail in Chapter 4 and 5. These simulation models help in the analysis of the vehicle dynamic in transient and frequency responses. The outcome of the vehicle dynamic analysis is used to provide guidance to the control strategy in terms of improving the vehicle drivability which is evaluated in this research by the overshoot and rise time. Secondly, identify the anti-jerk control strategy to improve the vehicle longitudinal dynamic characteristic. The strategy based on the holistic approach by generating a corrective torque related to speed difference between the wheel and the engine. In the most critical condition during the tip-in test with 100% torque demand, the results show that overshoot of the acceleration profile can be reduced to 42 % with a rise time of 0.069 s. Generally, the reduction of the overshoot is lower than the limit of the human feeling. More about the results is presented in Chapter 6.
The third contribution to this research is the off-line optimisation sequence of the gain scheduling which is introduced also in Chapter 6. The optimiser can be configured to give bias either to the overshoot or the rise time depends on the application of the vehicle. For instance, in order to achieve fast response, i.e. sport car, the weight of the rise time will be higher than the weight of the overshoot. The calculation time of the off-line optimisation provides a significant time reduction to build the gain scheduling table. A scheduling table of 16 runs can be built in 5 hours. This time is less than the time which is needed for preparing the vehicle in case of the test trial tuning method.

1.3 Outline of the Thesis

This thesis is composed of six main chapters without the annex and the references. This chapter is the introduction about the subject of this dissertation and the work. Chapter 2 describes the background of the drivability of passenger vehicles and provides the explanation of fundamental terms and previous work, based on relevant literature related to drivability evaluation methods. Chapter 3 describes the background driveline controller for passenger vehicle to improve vehicle drivability. The chapter summarises the relevant literature related to the anti-jerk controller and the method to control the jerk and the oscillation in both the traditional and the electric hybrid vehicles. The literature covers the model based controller with linear model and non-linear model. In addition, the chapter shows the mathematical formulation related to the used methods. Chapter 4 introduces the analytical non-linear passive vehicle model, which includes the driveline components that can affect the longitudinal drivability of the vehicle such as, clutch damper, tyre, engine mounting system and bushing elements. The non-linear model is capable of simulating all the non-linear components. The model was validated with a real measured data from vehicle. Chapter 5 describes the linear model of a passenger vehicle based on the state-space formulation. This chapter shows the mathematical formulation and the state-space matrices. The main purpose of this model is to assess the performance of the passenger vehicle in the frequency domain. Results and conclusion about the linear model are provided in this chapter.
Chapter 6 deals with the implementation of the anti-jerk controller in both linear and non-linear model. The mathematical equations and the control strategy for both linear and non-linear active vehicle are explained in details in this chapter. In addition, the auto-tuning of the controller through the optimisation algorithms is implemented and illustrated in this chapter. Chapter 7 summarises the conclusion of this work and highlights the contribution to the research with a recommendation for further work.
Chapter 2

Drivability of Passenger Vehicles

2.1 Drivability Definition

The definition of drivability is relatively complex because it refers to a relative perception and feeling. However, different definitions, referred to specific aspects of the global problem, can be found in the literature. Firstly, the literature distinguishes between the lateral and the longitudinal driver-vehicle interaction. These are expressed in the literature using the terms ‘handling’ and ‘drivability’, respectively. Vehicle handling is the interaction between driver, vehicle and environment which takes place for transportation of people and goods [1]. However, this definition may include the longitudinal behaviour of the vehicle, since it refers to the interaction between the driver, vehicle and environment during the transportation. The definition of drivability in [2] excludes handling. It stresses the comfort of the vehicle more than the vehicle operation. A definition of drivability focussing on the interaction between the driver and the vehicle can be found in [3] where good vehicle drivability is characterised by the ease of control and confidence in the system response to the driver demand which is dominated by the powertrain and vehicle transient conditions. The vehicle powertrain also plays a major role in the vehicle drivability. How smoothly and consistently the powertrain operates under all kinds of conditions describes the drivability of the vehicle. Ride and handling, braking performance or abnormal combustion such as knock phenomena are not considered as part of drivability [2]. Ride is defined in some literatures with relation to the comfort offered to the occupants due to uneven elements in the road surface [4], [5]. According to the definition of drivability in [6], the determination of the drivability is related
mainly to the instantaneous power which is generated due to the acceleration pedal position. In this context, drivability has nothing to do with vehicle performance, which is related to the elapsed time for vehicle speed changes [6], [7]. On the other hand, a shorter elapsed time implies higher installed power (relative to the vehicle mass), which generally has the potential to improve drivability as well as characterising longitudinal drivability. Therefore, the behaviour of the powertrain in transient conditions determines predominantly the drivability [6].

2.2 Drivability Evaluation

Automotive industry pays more and more attention to reduce the fuel consumption and the emissions of the car. The improvements of drivetrains, aerodynamic performance and tyre design achieved good results to reduce the fuel consumption [8], [9]. Mostly, through the optimisation of the way the engine and the transmission cooperate with each other, fuel economy can be achieved.

Since drivability is determined by the instantaneous power available after the acceleration pedal deflection, in this context typical term ‘power reserve’ is used in the literature to explain the dependency between the fuel consumption and the drivability [10]. Power reserve is defined as the product of the engine speed and the ‘torque reserve’ where the torque reserve is the difference between the actual engine torque and the maximal engine torque by wide open throttle (WOT) as presented in Figure 1 [10]. A larger power reserve results when the engine operates at higher engine speed and lower engine torque. This will be at the expense of the fuel economy. This can be seen in Figure 1 by the economy line and the drivability line where the torque reserve is larger for the drivability line than the economy line for an actual torque of 25 kW.

When a high power reserve created after the engine operates on high speed, the fuel efficiency status is impaired.

To improve the drivability by an engine power of 25 kW, moving from the economy line operation point A to the drivability line operation point B is required. This operation will increase the power reserve and thus improve the drivability. On the other hand, the fuel efficiency will be degraded.
Figure 1- Illustrates the relation between the fuel efficiency and the drivability [10]

2.2.1 Drivability Terminology

Different terms or criteria are described in the literature to evaluate the drivability [10], [11], [12] and [13]. These terms are known in the literature as malfunctions [14] and some of them are summarised in Figure 2:

- **Shuffle** is the fore- and aft longitudinal oscillations in vehicle acceleration. It is caused by the driveshaft oscillation in transient and it is mostly related to comfort;
- **Hesitation** is temporary delay of the initial response in the acceleration;
- **Stumble and sag** are short and sharp reduction in the acceleration rate which is caused by a loss of power. The difference between these two terms is that stumble has a negative acceleration for a short time, whereas sag is less severe than stumble. Hesitation, stumble and Sag are caused by the primary inertias, which dominate the wheel torque;
- **Surge** is a continued condition of short, sharp fluctuations in power, often caused by cyclic power variations in the engine and is more pronounced when using lean air fuel mixtures. Surge was introduced by Everett in [11] as a measure for the lean air fuel;
- **Stretchiness** is the lack of acceleration performance during light to moderate accelerations;
- **After-run** is running of the engine after ignition cut-out;
- **Vapour lock** Loss of acceleration, stalls, failure to start due to excessive gasoline vapour in critical parts of the fuel system;
- **Kick** is the first decrease of longitudinal acceleration as presented in Figure 2 [14];
- **Jerk** is the time rate of change of the acceleration [15];
- **Overshoot** refers to an oscillation of longitudinal acceleration exceeding the steady-state;
- **Steady-state** is the time required for the response to settle within a certain range;
- **Shunt** is the first increase in the acceleration after zero as presented in Figure 2 to the first peak which is causes by the backlash [16], [17].

The above aspects are used to judge the drivability and in the literature are called malfunctions [14]. A poor drivability is determined when the driver perceives a force transmitted to him via the seat due to the driving manoeuvre such as tip-in. On the other hand, it is important to characterise the good drivability technically. The related aspects to the vehicle drivability includes the level of vibration and noise with the characterisation factors namely; intensity, frequency, direction and exposure time which affect the passenger. The ISO standard [18] distinguishes between perception of the vibration frequencies range which affect health [19] and comfort. The frequencies affect the comfort in the range 0.5 - 80 Hz and for the motion sickness in the range 0.1- 0.5 Hz. In a different aspect, the drivability is considered good, according to [14], when the human body reaction after pressing the acceleration pedal in both tip-in and tip-out responds within 0.2 s. Meanwhile, in [10] drivability is good when the human body acceleration has a low frequency oscillation with one maximum.

![Acceleration vs. Time](image1.png)
![Acceleration vs. Time](image2.png)

Figure 2 - Drivability terms or malfunctions [14], [16]
2.2.2 Operation Modes for Drivability Evaluation

Driving manoeuvres is the basic for any vehicle drivability evaluation. The driving manoeuvres can be grouped in five basic driving classifications: starting, idle, acceleration, cruise and deceleration [11]. Figure 3, shows further vehicle operations modes which are used in a complex tool to evaluate the drivability [20]. The 12 global operation modes have several single operation modes. For example, idle operation mode has different single operating modes, idle after switch on the air conditioning, idle after fuel cut off, idle after highway run, idle after vehicle stop and idle after hot start. These will generate more than one hundred operations modes in total. These single modes are analysed and detected using some of the drivability terms such as jerks, kicks, hesitations and overshoots. More explanation for the driving manoeuvres is listed below [14]:

- Starting: Starting the vehicle, cold or hot (soaked)
- Idle: Running of the engine in neutral at zero speed
- Detent acceleration: Opening the throttle without causing a downshift by the automatic transmission
- Wide Open Throttle (WOT) acceleration: Acceleration made entirely at wide open throttle
- Part Throttle (PT) acceleration: An acceleration made at any fixed throttle position less than WOT
- Crowd acceleration: An acceleration maintaining a fixed vacuum in the inlet manifold

<table>
<thead>
<tr>
<th>Idle</th>
<th>Engine Start</th>
<th>Tip-In</th>
<th>Let Off</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant Speed Part Load</td>
<td>Acceleration</td>
<td></td>
<td></td>
</tr>
<tr>
<td>WOT</td>
<td>Pedal Steering Control</td>
<td>Warm Up Behaviour</td>
<td>Roll On Stop</td>
</tr>
</tbody>
</table>

Figure 3 - Drivability relevant vehicle operation modes according to [20]
- False starts: Opening and immediate closing of the throttle
- Cruise: Driving at constant speed
- Deceleration: Coast-down/sudden stops
- Back out: Release of the drive pedal during cruise/acceleration
- Gear shift: Shifting up/down
- Tip-in: A manoeuvre to evaluate vehicle response (up to 2 sec in duration) to the initial opening of the throttle

2.2.3 Subjective Evaluation

Subjective drivability assessment has experienced some standardisation [13]. This assessment is based on rating the different aspects of the drivability through the subjective assessment of the vehicle acceleration profile. Feel of comfort and discomfort is the measure for the rating criteria [21], [22]. Hence, vehicle acceleration and wheel torque may be chosen as variables that reflect the level of drivability [14].

The rating methodologies of the car manufacturers rely on the experiences of the test drivers to perform specific manoeuvres to rate particular vehicle performance features. Usually subjective scores in the range between 0 and 10 are assigned. Typical, vehicle characteristics are related to the responsiveness and smoothness under transient and steady state conditions. Furthermore, the performance associated with the engine idle and start-up phases are monitored within drivability [23]. The ratings of the subjective responses of test drivers are usually assessed by letting them fill out questionnaires after the tests and by gathering the driver's feedback during the tests. Some of the described parameters in section 2.2.1 were rated individually [11], [12]. Four or five severity scales were set to judge these parameters. The tables and figures in Appendix A show the rating scheme and the severities which are used in the subjective evaluation.

2.2.4 Objective Evaluation

Up to now, there is no standardisation of the objective drivability evaluation. During the development process of the vehicle, from specification to validation, it is important to detect any drivability problem and to quantify this problem. Therefore, objective drivability criteria are required. Developers and OEMs need to measure the different drivability criteria in order to feed the development process.
In the literature, various measurement techniques and evaluation processes were introduced to the drivability criteria, mainly focusing on the fore and aft vehicle acceleration, jerk, surge, response time and the frequency oscillation in the tip-in manoeuvre [2], [11], [15], [20], [21], [23], [26].

Jerk has got attention nowadays besides the vehicle acceleration in the design and control of the vehicle to evaluate the drivability. The peak of the jerk is used as performance index to evaluate the shift quality and judge the level of the goodness in the controller design [15].

The jerk has been extensively characterised in [15], [28], [29] in order to evaluate the shift quality. Jerk is defined as the change rate of vehicle longitudinal acceleration. It is synchronised to the human feeling and it can exclude the impact of bump and bounce which is caused by bad road condition and could reflect the true vehicle drivability [15], [20], [21], [26], [29]. The acceleration is still the main parameter which can be used to evaluate the drivability objectively during the gear shift [30]. The gear shift metric which is generated in [26] extracts four components from the fore and aft acceleration signal which are related to the human subjective response: the maximum average power of acceleration during the gear shift event, peak-to-peak amplitude of a filter acceleration, peak-to-peak jerk, maximum average power, and 10-14 Hz frequency where high energy content found in this frequency range. The data was collected at 10 ms interval to ensure covering a signal frequency of 40 Hz. The peak-to-peak amplitude was generated after applying a digital low pass filter to the acquired signal. The peak-to-peak jerk was calculated by the derivative using the finite difference method on the filtered signal.

Maximum average power algorithm was used to calculate the shift power level. The raw acceleration signal \( a(t) \) was subtracted from the average acceleration of the shift event. The difference was squared and integrated over a time range as presented in the following equation:

\[
\max \text{ average power} = \max \{10 \int_{t_0-0.1}^{t_0} [a(t) - a_{\text{mean}}]^2 \, dt\}, 0 < t_0 \leq T
\]

Where:
- \( a(t) \) the raw acceleration trace \( [m/s^2] \)
- \( a_{\text{mean}} \) the mean acceleration during the shift event \( [m/s^2] \)
- \( t \) the time (integration variable) \( [s] \)
- \( t_0 \) the time of observation \( [s] \)
- \( T \) time window \( [s] \)
The fourth component for the shift metric is the frequency range 10-14 Hz. Higher energy in this frequency range due to the body and suspension resonances was found. A filter was applied to the acceleration signal to capture the vehicle resonances.

The four acceleration components metric was correlated to the subjective rating for all the shifts entered.

A similar method for calculating the average power of acceleration fluctuations was used to measure the surge [11]. A band limited pass filter for the frequency 0-10 Hz was applied to the acceleration signal. A time base of 40 s, which is the required time to achieve 60 mph at part throttle, was considered in the average power which is calculated as follow:

\[
\text{average power} = \int_{t=0}^{40} |a_w(t)|dt
\]

Where \(a_w(t)\): the acceleration signal after the band limited filter and \(t\): integral time variable.

The cause of the jerk in hybrid electric vehicles was investigated and thus hybrid vehicle drivability evaluation was performed in [29]. It was shown analytically that the change rate of the clutch and electric motor torque details of the hybrid vehicle, the potential hybrid vehicle layout explained in the next chapter, is the main variable related to the jerk. Therefore the reasons of the jerk on the hybrid electric vehicle (HEV) are identified as follow:

- When the vehicle works in motor only mode the motor controller decides about the jerk degree;
- Clutch operation;
- The level of the jerk depends on how the engine, motor and clutch operate during the shift quality;
- The change rate of the regenerative torque of the motor during the braking manoeuvre affects the jerk.

These causes of the jerk were covered in different experimental steps to see the level of the produced jerk. The jerk is calculated from the acquired acceleration by differentiation of the acceleration time signal.

By operating a motor only mode, a jerk peak value of \(\pm 5m/s^3\) was measured by starting the vehicle in motor mode only. The vehicle was started in a certain acceleration pedal position without change. In this case the clutch torque does not have contribution to the torque. The motor torque variation is the only responsible for the jerk. After quick increase of the accelerator pedal input, the jerk level decreases to \(\pm 2m/s^3\). The degree of the jerk in this test depends on the motor torque change rate. The motor controller limits the increasing rate of the torque and therefore the jerk level is reduced.
The transition from the motor-only mode to the engine-only mode was also tested. The vehicle controller keeps the engine speed close to the motor speed until the clutch starts to engage. When the clutch starts the engaging process, the friction torque increases rapidly which produces a high value of positive jerk about \(+10m/s^3\). After the engaging process is completed, the engine torque increases and the motor torque decreases. The value of the jerk fluctuates due to the low precision of the engine controller.

During the shifting process in motor mode, the controller keeps the jerk in a moderate value over the whole process. In addition to the test of the above operation modes, the braking manoeuvre was tested and measured a high jerk value of about \(\pm 8m/s^3\).

The jerk value was indicated in some literature to be lower than \(\pm 2.943m/s^3\) (0.3 g/s) [31] in order to ensure the passenger comfort. However, the discomfort feeling of the driver depends on the duration of the jerk which will be discussed in Chapter 3.

The drivability is evaluated objectively in [23] based on the acceleration in both tip-in and tip-outs. The acceleration signal was acquired and analysed for different test conditions. The subjective evaluation shows the effect of the overshoot and the rise time on the driver perception. A correlation between the overshoot, rise time and the subjective rate was approved.

Many studies in the literature show that the driveline torsional vibration is mainly responsible for the perception of the driver and thus the vehicle drivability [32], [33]. The evaluation in these studies is based on modelling the driveline and investigating the vehicle dynamic behaviour in the transient status [34]. The driveline model in [34] considers the most subsystems of the driveline and uses the Simulink capabilities to structure the applicable model. These sub-models are split in blocks to represent engine, clutch, gearbox differential axle shafts, wheels, tyres and the vehicle body. The transient analysis shows that the drivability and performance perception are in the frequency range under 10 Hz, which is equivalent to overshoot of 2.5. The model in [37] is based on AVL Cruise [38]. The aim of this model is to study the effect of the pedal position and the throttle position on the drivability. The model is used to study the relationship between the pedal position, throttle opening and engine torque. The performance of the engine is determined by the interaction between the pedal position, throttle position and the primary engine performance. Therefore, the control of the amount of the air-fuel mixture sent to the engine is crucial in term of engine response. The control of the air-fuel mixture is sent via the throttling mechanism, where the response of the throttle rotation to the pedal displacement determines the type of the vehicle. The pedal-acceleration response in the form of acceleration per unit displacement (g/mm) is used to evaluate the
drivability metric. The relationship model of the acceleration per displacement unit is presented as a function of the vehicle speed. It was shown that three target zones can be achieved in order to determine the type of the vehicle depends on this metric i.e. aggressive sport vehicle, relaxed vehicle or desired vehicle.

A model-based evaluation with detailed analysis study of the drivability including the effect of each component of the driveline was discussed in [35], [36]. This type of evaluation is based on understanding the behaviour of the individual components of the driveline and the interaction between these components. A comprehensive model was developed to include the full dynamics of the powertrain (clutch damper, half-hafts, and tyres), the dynamics of the unsprung mass (longitudinal motion, vertical motion and pitch motion), the dynamics of the engine, the differential and the gearbox. Figure 4 presents the full dynamic model. In addition, the model includes the engine mount system. The models were implemented in a non-linear form to analyse the time response of the driveline. In order to analyse the frequency response of the driveline, the model was linearized and generated from linear differential equations which are formulated in state-space. In order to extract the natural frequency, the eigenvalues of the state pace matrix were calculated.

The results in Figure 5 and Figure 6 present the vehicle response in the tip-in manoeuvre for different gears. To study the behaviour in the frequency domain, the acceleration in the frequency domain is presented. Table 1 presents the natural frequency change over the gear of the longitudinal acceleration by the tip in manoeuvre. The model shows the oscillation behaviour of the vehicle with the different gears. Furthermore, the model is capable of showing the effect of the individual components as presented in Figure 7 and Figure 8; where the effect of the half-shaft stiffness and the equivalent moment of inertia respectively.

![Figure 4 - Full dynamic model [35]](image-url)
Figure 5 - Tip-in test in 1st gear [35]

Figure 6 - Tip-in test in 2nd gear [35]

Table 1 - Natural damp frequency of the vehicle acceleration [35]

<table>
<thead>
<tr>
<th>Gear no</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency [Hz]</td>
<td>2.56</td>
<td>3.85</td>
<td>5.10</td>
<td>6.85</td>
<td>7.35</td>
</tr>
</tbody>
</table>

In [36], the model-based study was performed for the heavy goods vehicle with a splitter and four gear transmission. Similar approach was achieved but concentrated on the sensitivity analyses. Figure 8 shows the vehicle acceleration for the vehicle in the fourth gear and second splitter in the tip-in manoeuvre with a torque demand of 30%. The acceleration results are plotted for different half-shaft stiffness. The corresponded frequency response is presented in Figure 9.
Figure 7 - Effect of the half-shaft stiffness [35]

Figure 8 - Vehicle acceleration in gear 8 (tip-in, 0-30% of torque demand), for different values of half-shaft stiffness [36]

Figure 9 - Frequency response analysis of the vehicle acceleration in gear 8, for different values of half-shaft stiffness [36]
In [20], a tool is presented to evaluate objectively the drivability by simulating the subjective assessments. The tool consists of two main units, the driver mode detection unit and the drivability detection unit which includes the non-linear driver simulation model. The input data as illustrated in Figure 10 is the vehicle speed, engine speed, throttle position, pedal position and the longitudinal vehicle acceleration which is measured by a sensor mounted on the headrest. The acceleration signal undergoes filtering process by Fast Fourier Transformation (FFT) to eliminate the noise. To have a correct objective simulation for the subjective evaluation, the surrounding aspects such as gear position, engine temperature should be considered. After the calculation of the driver behaviour, the single drivability behaviour is weighted according to the human feeling [27].

The engine and the vehicle data are permanently checked for characteristic signal operation modes. The digital signal processor (DSP) enables the single mode detection. Thus, the on line and real time evaluation is enabled. For each single vehicle operation mode, the single drivability is calculated for all the relevant criteria. Three mathematical levels are applied [27]:

- Two or three dimensional analytic model equation;
- Four or more dimensional analytic calculation;
- Artificial intelligence.

The tool is used in the vehicle to compare the different vehicles drivability and used also in the development and calibration phase for a fast test of the drivability.

The tool is used on the dynamic test bed to optimise for best drivability. For this purpose, characteristic drivability relevant test cycles have been developed.
The currently used objective evaluation methods in the literature are based on the existence of a vehicle driven in different manoeuvres. The evaluation is based on the measuring of the jerk in the vehicle experimentally or calculating the jerk from the acceleration in different driving manoeuvres or in gear shifting. Similar methodology is introduced in [39], which evaluates the drivability on the engine dynamometer. The system is composed of engine, which is mounted on a test bed and coupled to gear box. The system is fed with different vehicle operation mode data to a controller in order to generate the engine operation condition in accordance with the vehicle operation mode. For example, in a vehicle acceleration mode, the control unit controls the throttle opening equivalent to the acceleration and controls the dynamometer dynamic in order to generate the load torque accordance to the vehicle characteristics. The output data from the engine such as the output torque and the engine speed are fed to the drivability data generating unit in order to translate the engine operation parameter to drivability signals such as acceleration and jerk. The drivability signals are fed to the sense data generation unit that converts the drivability signals to stimuli perceived directly to the human body.

Simulation methodology is based on acquiring data from the vehicle and detecting the vehicle driving modes. In these two methods, the evaluation is based mainly on the vehicle and takes in account the longitudinal vehicle acceleration.

More details about the drivability and the effect of each component in the driveline can be found in the model based method. This method is effective in the very early stage of the vehicle concept since the drivability target can be identified.

2.2.5 Correlation Methodologies

Purely objective data cannot give a definitive judgment about the drivability because the human factor is missing in this data. In the end, the driver (customer) perception is the decision factor about drivability. On the other hand, the subjective evaluation takes long time and effort. So, the correlation between the objective data and subjective rating could solve this dilemma.

In the literatures [11], [27], [20], [21], [26], [14], [28] different correlation techniques are introduced and discussed. Generally, three main techniques are the most commonly used techniques [14]: The term polarisation is a commonly used term in the:

- Regression analysis with linear fit
- Artificial intelligence using neural network
- Correlation based on test drives
In [11], the correlation between the driver rating scale in [20] and the surge meter reading was found. Four experienced drivers rated the surge meter data. After then, a linear fit was achieved between the surge meter data and the rating scale.

The correlation between the disturbances level and the rating scale was achieved for each component of the shift quality metric in [26]. The correlation was based on a historical data which contains the several driver’s rating combined with objective data collected during the gear shift. A linear fit was selected between the disturbance level and the rating scale according to [26]. The linear correlation was generated for the four components of the shift metric.

The correlation between the driver’s impression of vehicle drivability and overshoot and rise time was achieved in [23]. The vehicle data in the tip-in manoeuvre was plotted against the driver subjective rating. The rating was reduced when the overshoot and the rise time were increases. This indicates that the smoothness and the vehicle response are important factors to the driver.

The artificial intelligence tools include different methods such as fuzzy logic, genetic method, adaptive and predictive control and neural network. One of the commonly used methods to correlate the subjective with the objective drivability evaluation is the neural network [27], [20]. The advantage of the neural network is to find a reasonable output for inputs that have not been taught. In addition, it enables learning and determining the function upon sample inputs [40], [27], [20]. The digital neural network is inspired by the structure and function of the human neural system. The elements of the neural network consist of the neurons, where the neuron is the basic cell on the human neural system. Figure 11 presents a simple structure of the human neural network and the digital analogy. The neuron consists of the cell body, the axon and many dendrites. A neuron is an electrically excitable cell that takes up, processes and transmits information (inputs) through electrical and chemical signals. The electrical signal is transferred over the axon to the next neuron where it is connected via the synapses. The neuron collects the stimuli until it comes to a threshold. When these exceed the threshold, the electrical impulse is generated. The principle neural network in [27] is shown in Figure 12. Two nodes layer are presented, the first layer with three nodes collect the inputs. The outputs of these nodes are transferred to the second layer with 5 nodes where further processing and decision. The node includes logical function for addition, subtraction and threshold decision. The connection to the nodes via links corresponds to the axon and dendrites in the biological cell. Each link has a weight which can be adjusted to achieve the correct output.
The important feature of neural network model is the ability for training with the help of the genetic algorithm. The genetic algorithms which is described in [27] are based on generating series of single operation modes including data collection and the subjective assessment. Typical data for the genetic training procedure in test drive is given in Table 2. For the 250 subjective assessment, the four inputs In1 to In4 cover measured engine and vehicle data and the fifth input In5 has the corresponding subjective assessment. The training of the model follows off line on the PC where the results are a mathematical model used online for the objective evaluation [20]. The comparison between the output of the neural network and the subjective evaluation is presented in Figure 13. The comparison is carried out for the engine speed and throttle opening position.

In some areas, there is a match between the driver’s subjective evaluation and the neural network, while in other areas there is some deviation. In general, the deviation between the
neural network and the driver assessment is less than 5% [27]. This deviation is due to the variation in the expert’s evaluation.

Correlation based on the test drives by testing several cars and evaluation objective parameter is presented in [28] and [21].

The objective in [28] is to find out a relationship between the human feeling and quantifying the jerk in terms of evaluating the shift quality.

Table 2 - Data for genetic algorithm [27]

<table>
<thead>
<tr>
<th>Nr.</th>
<th>In 1</th>
<th>In 2</th>
<th>In 3</th>
<th>In 4</th>
<th>In 5</th>
<th>Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.63</td>
<td>476.07</td>
<td>0.17</td>
<td>14.54</td>
<td>8.5</td>
<td>8.42</td>
</tr>
<tr>
<td>2</td>
<td>11.6</td>
<td>696.95</td>
<td>0.44</td>
<td>14.54</td>
<td>7.5</td>
<td>7.58</td>
</tr>
<tr>
<td>3</td>
<td>11.26</td>
<td>850.27</td>
<td>1.29</td>
<td>14.54</td>
<td>6.5</td>
<td>6.38</td>
</tr>
<tr>
<td>4</td>
<td>11.54</td>
<td>1351.8</td>
<td>1.39</td>
<td>14.54</td>
<td>5.5</td>
<td>5.58</td>
</tr>
<tr>
<td>5</td>
<td>11.46</td>
<td>1900.11</td>
<td>0.5</td>
<td>14.54</td>
<td>6.5</td>
<td>6.52</td>
</tr>
<tr>
<td>6</td>
<td>11.33</td>
<td>2183.36</td>
<td>0.64</td>
<td>14.54</td>
<td>7.0</td>
<td>7.15</td>
</tr>
<tr>
<td>7</td>
<td>11.59</td>
<td>2833.01</td>
<td>0.54</td>
<td>14.54</td>
<td>7.5</td>
<td>7.62</td>
</tr>
<tr>
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<td>..</td>
<td>..</td>
<td>..</td>
</tr>
<tr>
<td>250</td>
<td>99.73</td>
<td>3641.18</td>
<td>0.55</td>
<td>14.54</td>
<td>8.5</td>
<td>8.5</td>
</tr>
</tbody>
</table>

Two groups jerk were classified in term of the human feeling affect, the transient and the durative jerk. The transient jerk was studied by classifying the human feeling in 4 grades 0-3. An instrument was designed for the judgment process. This instrument provides three buttons. Each button provides a corresponded impulse voltage. The impulse voltage values 0V, 1V, 3V and 5V correspond to the grades 0, 1, 2, 3, respectively. The driver pressed the corresponding...
button according to his human feeling. Table 3 presents the classification of the physiological feeling and the corresponding voltage. The value of the transient jerk was acquired using an accelerometer. Since the grade of the physiological feeling was expressed via pressing the button and the jerk data was acquired via the accelerometer, a delay occurred between the data and the corresponding voltage.

The experiment of the transient jerk was started in a still vehicle from 1\textsuperscript{st} gear to the top gear and vice versa. The test considers also the different throttle opening and the daily driving styles.

The durative jerk test was performed by different shifting process. The driver repeated the shift process 10 times during the test drives. The shift process was followed up and down to the same gear. For instance 1\textsuperscript{st} gear, 2\textsuperscript{nd} gear and back to the 1\textsuperscript{st} gear. At the end of each test the passenger and the driver recorded a score for 8 subjective feeling.

The results of the experiments show that the transient jerk does not have a very strong correlation to the human physiological feeling. However, the feeling of the jerk does exist but the driver could not allocate the feeling to the jerk value according to Table 3.

The durative jerk experiments showed a strong correlation between the jerk value and the human feeling. Since the duration of the exposure to the jerk is longer than the transient jerk, the effect on the passenger/driver is more tangible. Moreover, the results showed the frequency between 0.05-1 Hz has the significant effect on the drive/passenger.

The correlation approach in [21] was performed on three test cars and twelve test drivers. The test vehicles are listed in Table 4. To achieve more comparable results three drivability categories were defined in terms of the driving situation:

- Launch feel: start from rest to the big pedal movement
- Overall performance feel: achieve the maximal performance quickly
Traffic crawl: small start speed with small pedal movement

The objective data were acquired using data acquisition system and performed during the test drives. The main variables for the objective data were the vehicle acceleration, pedal position, engine and vehicle speed.

The subjective data based on filling questioners, related to the drivability categories, included questions to cover 14 attributes about drivability and performance. The test drivers gave a score to this attribute from 1-10 where the highest score is the best.

The vehicle speed change showed similar behaviour for the cars A and B in terms of the reaction by a tip-in manoeuvre from 20% to 60%. Car C showed slower reaction than the other two cars in the same manoeuvre. The acceleration of the vehicles increased very quickly.

The subjective results in [21] are presented as overall drivability feel. These results are compared with the objective data characterised by acceleration, time delay and jerk.

The correlation is performed between these values and the three drivability categories. The conclusion made to this correlation is shown in Table 5.

Table 4 - Test vehicles used in [21]

<table>
<thead>
<tr>
<th></th>
<th>Car A</th>
<th>Car B</th>
<th>Car C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel</td>
<td>Petrol</td>
<td>Petrol</td>
<td>Diesel</td>
</tr>
<tr>
<td>Engine Size/L</td>
<td>1.3</td>
<td>1.6</td>
<td>1.8</td>
</tr>
<tr>
<td>Mass/kg</td>
<td>850</td>
<td>1100</td>
<td>1100</td>
</tr>
<tr>
<td>Power/weight Ratio/(W/kg)</td>
<td>70.6 (100%)</td>
<td>60 (86%)</td>
<td>45.5 (71%)</td>
</tr>
<tr>
<td>Transmission</td>
<td>VDT</td>
<td>VDT</td>
<td>VDT</td>
</tr>
<tr>
<td>Control</td>
<td>Electronic</td>
<td>Hydraulic</td>
<td>Hydraulic</td>
</tr>
</tbody>
</table>

Table 5 - Correlation between the drivability categories and the drivability values [21]

<table>
<thead>
<tr>
<th></th>
<th>Acceleration</th>
<th>Jerk</th>
<th>Delay Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Launch Feel</td>
<td>XX</td>
<td>X</td>
<td>XX</td>
</tr>
<tr>
<td>Overall Performance Feel</td>
<td>X</td>
<td>XXX</td>
<td>X</td>
</tr>
<tr>
<td>Traffic Crawl</td>
<td>X</td>
<td>X</td>
<td>XXX</td>
</tr>
</tbody>
</table>
2.3 Summary

The literature of the drivability evaluation shows a semi-standardised methodology for the subjective evaluation which is the rating technique. No standard methodology for the objective evaluation. However, acceleration jerk and time delay are almost agreed parameter used to evaluate the objective data. Advanced methodology to evaluate the drivability is the model-based method where the single components of the driveline are described and modelled. The advantage of this method is that: it enables to have a deep understanding of the effect of each single component on the overall vehicle drivability.

The pure objective data without link to human feeling does not help the car manufacturer to judge the customer perception; therefore the correlation between the subjective and the objective data takes place. The most advanced technique for this correlation is the neural network methodology. The neural network tools can be used offline to evaluate the drivability in the early stage of the vehicle development.

To achieve the drivability target in a later development stage in the vehicle development process, the judgment about the drivability target should be done in earlier stage, i.e. in the concept phase. To achieve this target, the model-based objective method is employed. In this method, a sensitive and accurate vehicle model should be built to describe the significant components.

Since the jerk plays a major role in the drivability, an anti-jerk controller needs to be embedded in this vehicle controller.
Chapter 3

Driveline Controller for Passenger Vehicle

3.1 Problem Description

As explored in the previous chapter, car manufacturers try to achieve high standards of drivability in the modern vehicles. This is a major decision factor from the viewpoint of the customer. Vehicle shuffle or for-aft longitudinal velocity oscillation is one of the important aspects of the drivability. Since jerk is one of the agreed parameters to evaluate the drivability, this parameter has significant weight to decide about the discomfort of the vehicle. The jerk generally occurs during the transient changes in the driver torque demand. A change of the torque demand by the driver provokes torsion in the drivetrain. This torsion causes oscillations and jerk in the underdamped system [41]. Figure 14 presents the problem in the vehicle due to a tip-in followed by a tip-out. The longitudinal acceleration of the vehicle is presented with jerk and oscillation. To avoid this oscillation and to improve the drivability of the vehicle and thus increase the comfort of the vehicle, a traditional method is used. This method based on modifying the mechanical components of the driveline. These components could be the spring of the clutch, the driveshaft or the engine mounting system.

A modern method is used to avoid the oscillation on the driveline, while retaining fast acceleration, is the anti-jerk controller of passenger cars. The anti-jerk controller in this method is based on modelling the driveline and on modification of the engine torque to reduce the oscillation of the drivetrain.
In the literature, different methods are used to generate the driveline model. Depending on the target, the used model is combined with the control methods. Typical control structures are discussed in the literature such as root-locus, H-infinity and Smith-Predicators. In this chapter, different anti-jerk control techniques in the literature will be introduced and discussed.

### 3.2 Anti-Jerk Controller

In chapter 2 it was shown that jerk is directly linked to the vehicle comfort. It was also shown that the comfort increases with a reduction of the peak jerk value. It was also presented that a jerk \( J_k \) with a value greater than 10 \( \text{m/s}^3 \) is perceived by the passengers as uncomfortable [15]. The tolerance of the driver perception threshold of the jerk depends on the duration and the frequency. For instance, when the duration of the jerk is longer than 2 to 3 minutes with a jerk in a range between \( 2 < J_k < 10 \text{ m/s}^3 \), could cause discomfort and sickness feeling [15]. A Root Mean Square (RMS) jerk value of 25 \( \text{m/s}^3 \) can be tolerated for frequencies less than or equal 3 Hz [15]. This jerk value \( J' \) could be revised at different frequencies \( f \) as described in the equation (3.1) [15] and thus the tolerance of the driver feeling will be changed. 

\[
J' = J_k[1 - 0.1(f - 3)]
\]  

(3.1)
Therefore, the car manufacturers pay high attention to this problem and try to reduce the discomfort by damping the oscillation in the driveline in the transient condition. Reducing the oscillation, overshoot and keeping the time response adequate will keep the satisfaction of the customer. The next section will discuss the control techniques which are used to implement the anti-jerk controller in both conventional and hybrid vehicles.

3.2.1 Conventional Vehicle

To improve the derivability, several anti-jerk control techniques for conventional and hybrid vehicles were studied and discussed in the literature.

An anti-jerk control based on a predictive state-space model combined with the root-locus pole placement technique is used to design the controller in [42]. The linear state-space drivetrain model is simplified as two-mass model. The model structure is presented in Figure 15. Since the approach of the model is to control the first oscillation of the drivetrain, the model is simplified in Figure 16 and some parameter such as the clutch compliance is neglected due to the high stiffness which does not contribute to the first resonance mode. The first inertia is represented by the combination of the engine, transmission, propeller shaft, differential and drive shaft (half-shaft). The second inertia is represented by the wheel. The dynamic influence from the tyre is modelled by a constant damping coefficient.

The output of the model is the speed difference $\Delta n$ between the engine and the wheel. The model has seven parameters: mass moment of inertia $J_m$ which includes the engine, transmission and final drive $i_f$, mass moment of inertia includes the wheel and the final drive $J_w$, drive shaft (half-shaft) damping coefficient, drive shaft stiffness coefficient, damping coefficient of the wheel and the transmission and the transmission ratio between the engine and the wheel speed.
Figure 15 - Vehicular driveline and [6], [42], [43], [44]

Figure 16 - Idealized drivetrain structure [42] and [44]

Figure 17 - Controller structure as described in [42]
These parameters were identified using measured data. The drivetrain was excited by tip-in/tip-out for different engine speeds, which after the resulting torque and the speed difference between the engine and the wheel are measured. Due to the dead time created by the combustion process, the model shows predictive characteristic, which is implemented by predicting the speed difference between the wheel and the engine from the drivetrain model before it occurs as, presented in Figure 17. When the torque step is applied, the speed difference between the engine and the wheel is calculated on the drivetrain model before the reality. The predictive model output is fed to the controller input and thus the jerking and the oscillation can be damped.

To optimise the results of the predictive model, a state observer is implemented in the feedback path. The controller design used the Root Locus method to achieve the pole placement. The tuning of the controller in the vehicle is achieved by two gain parameters, the proportional gain $K_p$ and the derivative gain $K_{PD}$ gain.

The model based controller was built in a rapid prototype and tested in a test vehicle and the significant signals such as longitudinal acceleration, engine and wheel speed difference, and correction torque were monitored. Without the controller, the passenger can feel the effect of the overshoot and the oscillation of the passive car. After using the controller, the active car shows 50% reduction of the speed difference between the engine and the wheel compared to the uncontrolled system. The longitudinal acceleration is much smoother. The controller was able to show different results by tuning the gain.

The same model with parameter identification capability in [42] is used in [41] to build a robust controller which works for family of plants of SISO (single input single output) or MIMO (Multiple input multiple output) and various inputs and disturbances. Therefore, the author uses the H-infinity control methodology to build a robust controller based on the loop-shaping and the mixed sensitivity method to guaranty the stability of the controller for the various different change. The advantage of this type of controller is to minimize the effect of the uncertainties in the model parameters on the model output and minimise the oscillation and the jerking in the drivetrain. To achieve this target, three transfer functions from the original anti-jerk controller are generated. Figure 18 depicts the plant with the controller.

The controller input parameter is the engine-wheel speed difference $\Delta n_{measured}$. The reference speed $\omega_{rec}$ is set to zero to achieve the minimum speed difference error. The controller output is the correction torque $T_{cor}$ which is subtracted from the torque demand $T_{rec}$ where the resultant torque $T_{res}$ is applied to the engine. In order to achieve concentrated control over the interested
frequency, the weights $W_1$ to $W_3$ are used with the outputs of $z_1$ to $z_3$. These outputs can be considered as filtered speed

$$\omega_{rec} = 0$$

$$\Delta n_{measured}$$

Figure 18 - Plant with controller [41]

$$S = \frac{1}{1+GF} \quad (3.2)$$

The robustness specification is described by the transfer function $T$ in equation (3.3). The transfer function is shaped by the pole and zero placement. Increasing the gain in the open loop and introducing a pole and zero, increases the damping and shift the complex conjugate closer to the real axis. This shaping is achieved by the weight $W_3$.

$$T = \frac{GF}{1+GF} \quad (3.3)$$

The control sensitivity transfer function $R$ is between the uncertainty and the controller output. The transfer function $R$ is described in equation (3.4). By configuring the weight $W_2$, the large effect of the uncertainty with minimal control effort can be achieved.
The transfer functions $S$, $T$ and $R$ are shaped by configuring the weights $W_1$, $W_2$ and $W_3$. The controller design specifications are depicted in Figure 21. The design of a model predictive controller for anti-jerk is presented in [46]. A model predictive control is developed to overcome the limited frequency response of the actuator due to the physical constraints of the driveline system mechanism in the tip-in and tip-out manoeuvre.

![Bode diagram](image.png)

Figure 20 - Bode diagram for the controller and plant [41]
The proposed controller in this reference considers the time domain design to overcome this frequency response limitation in the tip-in and tip-out manoeuvre. The controller is based on reducing the difference between the engine speed and the wheel speed.

The author in [47] introduces a control method to reduce the vehicle shuffle and the oscillation in the front-wheel drive vehicle drivetrain. The controller is based on a non-linear model which is generated using industrial time-domain mechanical package (ADAMS). The model includes the powertrain system: engine, flywheel, clutch, gear-box, driveshaft (half-shaft), suspensions and tyres. The non-linear clutch stiffness characteristics were fitted to measured data. The clutch damper hysteresis and the damping coefficient were calculated from the dynamic measurements for different frequencies.

In addition, the engine mounts in the three directions were also considered in this model. The system identification is performed in the frequency domain in order to achieve a linear model. The identification of the system is based on the spectral analysis technique. In this technique, the system is fed with a test signal and analyse the frequency of the output signal to obtain the system response. The test signal should not be a special driving signal. It is enough to be a sinus signal for the frequency analysis and pseudo random sequence for the autocorrelation analysis. The phase and the frequency of the powertrain system $H(\omega)$ are achieved by analysing the power spectrum density $S_x(\omega)$and the cross-spectrum density $S_{xy}(\omega)$ of the input $x(t)$ and the output $y(t)$ respectively as presented in equation (3.5)
\[ H(\omega) = \frac{S_{xy}(\omega)}{S_x(\omega)} \quad (3.5) \]

The normalised root-mean-square error (rmse) in equation (3.6) describes the noisy system.

\[ \frac{rmse(H_{est}(\omega))}{H(\omega)} = \sqrt{\frac{1}{2m} \left( \frac{1}{\gamma_{xy}^2(\omega)} - 1 \right)} \quad (3.6) \]

The signal-to-noise ratio is provided by the coherent function \( \gamma_{xy}^2(\omega) \).

\[ \gamma_{xy}^2(\omega) = \frac{|S_{xy}(\omega)|^2}{S_x(\omega)S_y(\omega)} \quad (3.7) \]

The drivetrain system identification was performed by using a band limited random torque as an exciting input. The exciting was applied to the flywheel where the flywheel speed was chosen as output of the system. The Bode diagram was identified with different maxima and minima for different frequency, which led to third order system (3.8):

\[ H(S) = \frac{(s+z_1)(s+z_2)}{(s+p_1)(s+p_2)(s+p_3)} \quad (3.8) \]

The system has a limited bandwidth of 90 Hz due to the limited bandwidth of the input signal. Consequently the coherent function \( \gamma_{xy}^2 \) is zero above this frequency. The controller structure is described in Figure 22 where the observer and the control law are the main element of the controller. The controller was designed in the state-space technique and used the LQG methodology to optimise the pole-placements. The position of the pole in the s-plane, Figure 23 identifies the dynamic behaviour of the system. The feedback gain is calculated using the Bass-Gura formula (equation 3.9) [48] to achieve precise poles placement.

\[ gain = (QW')^{-1}(a_d - a) \quad (3.9) \]

The observer and the control law were implemented in the non-linear model. The controller responses were converted into torque and used as an input to the flywheel. The tests were performed on both with and without controller.

The longitudinal acceleration was measured at the seat-rail. The damping on the longitudinal vehicle acceleration peak shows 25% reduction from the uncontrolled oscillation.

---

![Figure 22 - Controller structure [47]](image)
Based on these results, significant improvement on the drivability was achieved by using the active drivetrain.

An active controller to damp the torsional vibration of the vehicle drivetrain is presented in [49]. The controller in this reference is based on the speed difference between the vehicle and the transmission as feedback signal.

A simple linear vehicle model according to the schematic in Figure 24 is used with three inertias: engine, transmission and vehicle. The transfer function of the system is expressed in equation (3.10). The system describes the input to the output from the clutch torque and the speed difference between the transmission and the speed. In addition, the dynamic of the clutch actuator was also described in equation (3.11). Due to the clutch actuator dynamic, the proportional control was not sufficient to damp the system. Therefore, Root-Locus method with compensation network is used to place the poles position in order to compensate the clutch actuator dynamics. The controller transfer function is described in (3.10).

\[
\frac{\omega_r(s) - \omega_p(s)}{T_{clutch}(s)} = \frac{18.8s(s+0.0024)}{(s+0.0022)(s^2+2.89s+846)}
\]  
(3.10)

\[
\frac{T_{clutch}(s)}{T_{command}(s)} = \frac{1}{(T_1s+1)(T_2s+1)}
\]  
(3.11)

\(T_{clutch}\): Clutch torque

The compensation network and the coefficients are described in equations (3.12) and (3.13) respectively.

\[
C(s) = \frac{T_1s+1}{\alpha T_1s+1} \quad (\alpha < 1)
\]  
(3.12)

\[
\alpha = \frac{1 - \sin \theta_n}{1 + \sin \theta_n}, T = \frac{1}{\sqrt{\alpha \omega_m}}
\]  
(3.13)

Where \(\omega_m\) is the axle mode frequency in rad/s.
The poles of the system includes the clutch actuator dynamic are presented in Root-Locus plane in Figure 25. The complex poles are close to the imaginary axis, thus do not have sufficient damping ratio. The damping ratio of these poles will be increased if the phase of these poles achieves 180° (see Figure 26a). The total angle of the plant without the clutch actuator is zero. In order to achieve the required phase of the plant with the controller of 180°, the combined phase of the clutch actuator system and compensator shall achieve zero (Figure 25 b). The controller used also an estimator to estimate the speed difference of the transmission and the vehicle in case of the absence of providing an accurate speed difference. The transfer function of the estimator is given in equation (3.14) and (3.15):

\[
\frac{\omega_t(s)}{\omega_e(s)} = \frac{B_{axle}s + K_{axle}}{I_p s^2 + (B_{axle} + c_l)s + K_{axle}} \quad (3.14)
\]

Calculating the speed difference:

\[
\frac{\omega_t(s) - \omega_e(s)}{\omega_t(s)} = \frac{I_p s^2 + c_l s}{I_p s^2 + (B_{axle} + c_l)s + K_{axle}} \quad (3.15)
\]

The Bode diagram of the estimator in Figure 26 b shows a phase lag of 180° and a gain of 25 dB by 10 [rad/s]. Consequently, increasing of the damping ratio is achieved.

The significant part in this controller is the compensation network which enables the damping of the oscillation occurring on the axle. The simple linear model considers only few components of the vehicle in the feedback path. The design works well for the real system under the conditions of accurate measurement of the axle frequency, the phase lag of the transmission torque and the transmission speed. In [50] the speed control of a three- inertia system by PI/PID-controller is discussed for industrial application. The reference described the three inertia system model which includes: inertia of the drive motor, middle body and the load. The corresponding speeds of the three inertias, the stiffness of the shafts, the torques of the drive motor and the disturbance are also described. The PI control method are employed to control the motor speed. The transfer function of the system has the order six with two PI-controller
coefficients tunable in the feedback. Due to the high order of the system, the controller was designed using the parameter plane method, where undamped natural frequency and relative damping coefficients express the complex poles.

The PI-parameter allowed adjusting the damping and the resonance frequency. By fixing the poles with a good damping of the natural frequency, the different inertia values of the three-mass-system of the dynamic characteristic is strongly affected. The system shows underdamped
with the larger drive motor inertia. To improve the damping the derivative term of the PID controller is added in the feedback. The damping can be improved by adjusting the derivative coefficients.

The idea in [51] is to use the engine as an actuator to damp the powertrain oscillation on a heavy duty vehicle. A two-degree-of freedom controller with feedforward and feedback parts was developed. The controller is model based with a filter and inverse powertrain model in the feedforward path and linear quadratic (LQ) and state observer in the feedback path as shown in Figure 27.

![Figure 27 - Control system with feedforward and feedback loop](image)

A model with two-inertia mass with three states is developed. The model is based on the description in [43], [44] in Figure 16. The three states of (drive shaft) half-shaft torsional angle \( x_1 \), engine speed \( x_2 \) and the wheel speed \( x_3 \), are described in the following equations:

\[
\dot{x}_1 = \frac{1}{i_{itf}} x_2 - x_3 \tag{3.16}
\]

\[
\dot{x}_2 = (T_e - \frac{k}{i_{itf}}) x_1 - \left( \frac{b_t + b_f + c}{i_{itf}^2} \right) x_2 + \frac{c}{i_{itf}} x_3 \left( \frac{1}{J_m + J_t + J_f} \right) \tag{3.17}
\]

\[
\dot{x}_3 = \left( \frac{1}{J_m + m r_w^2} \right) (k x_1 + \frac{c}{i_{itf}} x_2 - (c + b_w + m f_s r_w^2) x_3 - \eta_w m (f_0 + g \sin(\alpha))) \tag{3.18}
\]

The model is experimentally verified for all the gears according to [52]. The mass moment of inertias of the engine \( J_m \), transmission \( J_t \), final drive \( J_f \), wheel \( J_w \), the vehicle mass \( m \), the wheel radius \( r \), transmission ratio \( i_t \), and the final drive \( i_f \), are determined from the mechanical design. The three parameters such as the damping coefficient of the transmission \( b_t \), the damping coefficient \( c \) and the stiffness \( k \) are tuned.

Since the model has three states, the LQ controller needs three reference signals which are provided from the powertrain model. The feedback signals of the LQ controller, engine speed and wheel speed are measured. Meanwhile, the torsional angle of the half-shaft is estimated by the observer. However, two reference signals were used in this work which are the engine speed and the wheel speed. The combination of the inverse model of the engine speed and the model...
provides two poles close to the imaginary axis. These two poles reduce the robustness of the controller. Therefore, the wheel speed is used to increase the robustness of the controller.

The signals from the feedback loop and the forward loop are added in order to request engine torque. The generated engine torque is limited by a smoke limiter. The smoke limiter is an example of changing the threshold which is set to the torque request saturation. The change of this threshold is increasing along with the engine torque increase. The smoke limiter limits the amount of the fuel in relation to the boost pressure. The limitation in the torque reduces the bandwidth of the closed loop controller. Therefore, the filter was chosen faster than what the engine can provide in terms of the torque.

In order to prevent exceeding the engine torque limitation by the feedforward control signal, a rate limitation method is applied to the reference signal where the reference signal is calculated stepwise and compared to the threshold.

The reference governor method in [53], [54] and [55] can be used as alternative method to keep the reference signal within the engine torque limit. The idea is to assume that the reference signal is constant and to constrain the rate of the change of the reference signal at each sampling interval in such a way that the control signal to stay within the boundaries.

The reference and filter blocks can be expressed in reference governor (RG), $G_{ff}$ the feedforward filter and saturation blocks as shown in Figure 28. The idea is to find the correct value of the $\alpha$ in equation (3.19).

$$v(k + 1) = v(k) + \alpha(r(k) − v(k))$$  \hspace{1cm} (3.19)

Where $v$ is the command signal and calculated from the set point $r$.

The value of $\alpha$ lays in the range of $0 \leq \alpha \leq 1$ and is chosen to have the maximal value to keep the signal $y$ in the allowed range $[0, y_{max}]$. The correct value of $\alpha$ can be found using the step response $\sigma$ and the free response $y_k^0$ of the filter $G_{ff}$. According to the equation (3.20) the value of $\alpha$ can be determined accurately. The controller and the system were tested using a step input of the wheel speed. To mimic the smoke limiter, saturation is added to the system to allow the increasing of the engine torque along slope when it starts to exceed the engine torque limit.

$$\alpha = \min(\inf_{i \geq 1} \left\{ \frac{y_{max} - y_k^0(k + \hat{i})}{\sigma(i)(r(k) - v(k))}, 1 \right\})$$  \hspace{1cm} (3.20)

The test is conducted on ideal powertrain model and on the deviated powertrain model. In the deviated model, two error stages are generated, the first error is increasing the powertrain mass
by 2.5 tons and the second error decreasing the mass by 2.5 tons. In the case of no disturbing or error in the system, the LQ control signal in the feedback loop is zero. The control is performed by the feedforward loop. The results show the threshold step size in the rate limiter keeps the feedforward control signal below the maximal available engine torque with no oscillation on the wheel speed.

In case of the first error stage, the total control signal stays below the maximum of the available engine torque and no oscillation occurs on the states variable: wheel speed, engine speed and torsional angle of the half-shaft are well damped. In the second error stage the total control signal exceeds the maximum available engine torque. Also the system is well damped.

The concept in this controller based on the limitation of producing the engine torque. Therefore, the calculation of the reference signal is important. The controller has three states and three reference signals: torsional angle of the half-shaft, the engine speed and the wheel speed.

The reference in [56] provides a model based controller in Figure 29 to use the engine as an actuator to reduce the oscillation produced by the vehicle shuffle. The purpose of this work is to damp the shuffle oscillation after the backlash traverse as shown in Figure 30. In addition, the controller provides tuning opportunity in the vehicle to reduce the engineering development time. To design the controller, a simple model of the powertrain is developed. The physical model of the powertrain is based on the methodology in [57]. The model has three states as presented in Figure 29 which are representative of the system in the relevant frequency band width related to the drivability. The inertia of the half-shaft and the wheel are neglected. In addition, the aerodynamic forces and the road profile are neglected. The engine model considered a delay time $t$ in providing the requested torque. The transfer functions $G(s)$, $G_{acc}(s)$ and the polynomial $D(s)$, describes the relation between the input torque $T_e$, the engine speed $\omega_e$ and the vehicle acceleration $a_x$.

![Figure 29 - Simplified model of the driveline [56]](image)
The following equations describe the transfer functions:

\[ G(s) = \frac{\omega_e}{T_e} = \frac{30s^2J_v + sc_v + Keq}{D(s)} \]  
\[ G_{acc}(s) = \frac{a_x}{T_e} = \frac{K_{eq} sR_{eq}}{D(s)} \]  
\[ D(s) = s^3 J_v J_e + s^2 (J_e c_v + c_e J_v) + s(Keq J_v + J_e) + c_e c_v + Keq (c_v + c_e) \]  

Where:
- \( K_{eq} \): is aggregate stiffness related to the drive shaft, clutch, gearbox and tyre.
- \( R_{eq} \): is the equivalent wheel radius
- \( c_v \) and \( c_e \): is aggregate all existing frictions

The damping ratio \( \zeta \) and the resonant frequency \( \omega_0 \) are given by equations (3.24) and (3.25) respectively and according to [58].

\[ \zeta = \frac{c_v J_e^2 + c_e J_v^2}{2 \sqrt{Keq J_v (J_v + J_e)^3}} \]  

\[ \omega_0 = \sqrt{\frac{Keq J_v + J_e}{J_v J_e}} \]  

The transfer function of the driveline model in Figure 31 shows the frequency response of the driveline mode with engine torque \( T_e \) as input and the engine speed \( \omega_e \) and acceleration \( a_x \) as output. The Bode diagram presents the magnitude and the phase of the transfer function \( G(s) \).
Figure 31 - Bode diagram of the transfer functions $G(s)$ and $G_{acc}(s)$ [56]

with the engine speed as output and $G_{acc}(s)$ with the vehicle acceleration as output for the first and the second gear. The Bode diagram of the transfer functions highlights the oscillations near the resonance frequency which is 15 rad/s and 22 rad/s for the first and the second gear respectively.

The controller design considered two cases: with and without engine time delay.

The idea in this controller is to predict and compensate the shaft torque $T_{shaft}$. The structure of the controller without delay is illustrated in Figure 32. The engine speed $\omega_e$ is the input to the controller and the controller $C(s)$ generates a corrective torque $u$ which is applied to the engine as an output. The load torque $T_{sh}$ is the feedback shaft torque to the engine fly wheel via the flexible shaft. This load torque passes through a band pass filter \((a)\) with the band width in the range\([\left(\frac{2\pi}{\tau_b}\right) \left(\frac{2\pi}{\tau_f}\right)]\) generating the torque $T_{sh2}$. The tuning parameters $g_c$ with $\tau_b$ and $\tau_f$ give the compromise between the level of attenuation and the robustness of the controller.

The structure in Figure 33 considers the engine time delay with Smith predictor. The controller was implemented in the vehicle, as illustrated in Figure 34 for test purpose, and performed tip-in and tip-out tests on dry road for the first and second gear. The results in Figure 35, Figure 36 and Figure 37 show a good damping of the oscillation frequency. The results show how the engine damps the oscillation by applying the required torque in the opposite direction to the
oscillation. Furthermore, the results show how the tuneable $g_c$ can affect the oscillation damping.

Figure 32 - Schema of partial torque compensator [56]

Figure 33 - The torque compensator including the Smith predictor [56]

Figure 34 - Structure of the controller in the vehicle [56]
Figure 35 - Vehicle acceleration in first gear [56]

Figure 36 - Vehicle acceleration in first gear for different tuning values $g_c$ [56]
The reference in [24] presents a method for anti-jerk control to improve the comfort in the tip-in maneuver. The method is implemented in the engine control unit and uses the engine management system. The control technique consists of two routines, the anti-jerk function and the transient torque control. The anti-jerk function is responsible for generating correction torque from the speed difference between the engine speed model and the real measured speed. The correction torque is incorporated to the spark advance to smooth the engine speed fluctuation and thus the anti-jerk damping. The transient torque controller controls the opening throttle and the overall torque request which in turns affects the ignition advance operation. To allow a smooth acceleration without drivetrain shunt by tip-in maneuver, the achieved speed by the increased torque should be controlled. The speed controller can be implemented by damping the increase in the desired torque. As the torque request is increased by the driver request, a filter function is employed to control the torque increasing. The constant parameters of this filter are set in the gear and engine speed calibration.

The patent active driveline damping [59] deals with the control of the phenomenon called chuggle. This phenomenon occurs when the torque converter clutch is locked. Consequently, the perturbation is caused where the confidence of the vehicle and the engine stability occurs. The chuggle compensation method in Figure 38 based on generating a speed error term which reflects the difference between the desired and the actual engine speed. The speed error signal is fed to the digital phase compensator to generate the required torque for the compensation. The
phase compensator ensures the synchronisation of the compensation torque to the time and the phase.

The speed error term calculation is based on measuring the speed difference between the engine speed $RPM$ and the left $RPM_l$ and right $RPM_r$ wheel speed using speed sensors and considering the gear ratio. The speed error signal is fed to a filter to eliminate engine speed variation in order to prevent control command variation. This range of variation is set in a range of $\pm 10$ rpm. The output signal from the filter is fed to a gain stage to provide in vehicle tuning. The output of the gain is fed to the phase compensator to cancel the phase difference which occurs in the engine and produces deviation to the reference torque for the damping purpose. The signal from the phase compensator is fitted to the torque limiter to limit the torque for a predetermined values. The limited torque deviation is fitted to bias calculation to affect the ignition timing variation. The torque bias value is provided to reduce the torque value by ensuring the spark timing variation. The difference between the limited torque deviation and the bias torque is applied to a calibrated look-up table which presents the engine output torque versus the ignition time change. The look-up table output is the ignition timing variation which is fed to the ignition controller.

The phase compensator has a set of calibrated gains which are established by a conventional calibration process to achieve the precise phase lag elimination.

The objective of the invention [60] is to control the oscillation produced by the shuffle in the for and aft dynamic. This invention which considers the phase compensated signal is not representative since the phase change occurs after the band filter and the amplification. The controlling method in this invention based on shaping the engine torque response depends on the driver input demands. This works reduces the energy in the frequency band which is in the driveline. The steps include:

![Signal flow to compensate engine speed variation](image)

Figure 38 - Signal flow to compensate engine speed variation [59]
1) Continuous measuring of the engine speed and generating a signal representing the speed
2) Filter the fluctuation in the speed signal
3) Apply phase change to the filtered signal
4) Amplify the filtered signal and phase changed signal to provide a torque control signal
5) Apply the control signal to the engine control unit
6) The output engine torque is controlled by one or more of altering spark timing, air flow or fuel flow

A PID controlling method is described in the patent [61] to control the engine idling speed in order to improve vehicle comfortability. Particularly, the controller is designed for the vehicle equipped with automated manual transmission system. The automated manual transmission system is a manual transmission system with automatic clutch system. Such a system, which is disclosed in [62], uses a motor to actuate the gear shift. This type of transmission called motor-type automated manual transmission (MMT) [62].

When creep driving such a vehicle equipped with MMT in the idle state, the engine hunting phenomenon occurs. In the hunting phenomenon, the engine speed fluctuates continuously above and below the mean speed value [63], [64]. The hunting phenomenon can be explained in this driving mode as the clutch gradually shifts from half-engaged to fully-engaged and the engine speed changes from the idling non-creep driving mode to the target speed. After reaching the target speed, the engine speed continues increasing despite of the integral term of the PID-controller. The integral term in the PID-controller shall decrease and become zero when the target speed is achieved. When the clutch is disengaged, the engine racing occurs due to the slow integration constant of the integral term in the PID-controller. The solution in [61] is based on implementing a tuneable integration constant in the integral term of the PID-controller operation, which is appropriate for the creep driving. When the creep driving starts, the value of the integral constant is set to decrease the integral term in the PID quickly. After the clutch is locked up and finished the creep driving the integral value is forced to zero.

Based on powertrain configuration described in [61], the inventor in [65] implemented anti-jerk controller to damp the vibration and any shock occurs due depressing the accelerator pedal. The principle based on adjusting the ignition timing according to the difference of the actual speed and the reference speed of the engine. The activation of the anti-jerk controller depends on the vehicle operation.
The invention in [66] introduces a PID controller to obtain optimum response and stability of the throttle opening valve by controlling the DC motor driver circuit. The control principle is based on a feedback loop through the PID controller, where the difference between the actual throttle opening angle and the accelerator demand is fed as input to the PID. The opening angle of the throttle valve and the accelerator angle position are provided via position sensors.

The individual term of the PID-controller was tuned manually depends on the vehicle operating mode. The values of the PID-terms for the three vehicle operation modes: cruise control, idle speed, and traction control were determined individually using look-up table.

The active damping of the internal combustion engine (ICE) in [67] is proposed by using the electronic throttle system control. This method uses electronic throttle actuator to damp the vehicle oscillation, which is provoked by tip-in. The control algorithms can be implemented in the drive-by-wire system. The drive-by-wire system is the replacement of the conventional cable link between the throttle pedal and the throttle body. The system can be implemented by fitting a stepper motor or a servo motor to the throttle body and an electronic throttle pedal with potentiometer. The response surface method (RSM) is added to the standard controller in order to analyse the vehicle acceleration data.

Self-Tuning PID controller is presented in [68]. The method is based on the theory of the adaptive interaction of complex systems with N sub-systems. In addition, the reference classifies the PID tuning into two classes: class 1 is Off-line tuning and class 2 is on-line tuning. The objective is to implement self-tuning for the class 2.

The on-line tuning is a continuous tuning since the requirements and the PID is changing according to the different stage of the control. One of the common requirements in the PID controller is the fast response and the reduction of the steady-state error which are usually conflicting with each other and need different PID-parameter tuning.

The self-tuning algorithms Figure 39, is based on the theory of the adaptive interaction which is described in [69]-[72]. Applying this theory, the control system is divided into four sub-systems: device 1 proportional, device 2 integral, device 3 derivative and device 4 the plant. The dynamics of each device is described by causal function. The interaction between the two devices is functional dependent on the input of one device and on the outputs of the other. The connections between the devices is denoted by the connection weight. The three parameters of the PID-controller, as illustrated in Figure 40, $K_P$, $K_I$ and $K_D$ are used as the interaction between the sub-systems.
The only information needed from the plant is the Frechet derivative which can be calculated [68]. After applying the methodology to minimise the error as described in [73] and calculating the Frechet function the tuning algorithm for the PID parameters are described in the following equations [75]:

\[
K_P = -\gamma e y_1 \quad (3.26)
\]

\[
K_I = -\gamma e y_2 \quad (3.27)
\]

\[
K_D = -\gamma e y_3 \quad (3.28)
\]

where \( \gamma \) is the tuning adaption coefficient.

The active damping system was used to reduce the driveline oscillation in [74]. The damping of the oscillation during tip-in and tip-out in the active damper is achieved with the filter and the model-based controller. To prevent the propagation of the jerk from the engine to the driveline, a filter based on the Extended Kalman method is employed. The feedback of the model-based controller is integrated in the filter to reduce the jerk by comparing the half-shaft torque and the filter output torque.

In [75] a model-based state-space anti-jerk controller is introduced, which takes into account the behaviour of the dual mass flywheel (DMF). The (DFM) basically consists of two rotating
flywheel inertias connected by long travel arc springs [76]. Since the DMF is located between the combustion engine and the clutch or automatic transmission, the DMF reduces driveline oscillations by decoupling the transmission from the periodic combustion events that excite the engine mechanically. Due to its capability for temporary energy storage, it produces a reaction torque on the combustion engine and therefore the DMF affects the crankshaft signal, which is extensively used by the engine’s electronic control unit (ECU). The conventional controller does not consider the fast torque reaction on the crankshaft, which causes friction of the firing frequency called sub-harmonic vibration (SHV). The SHV is reduced by utilising the state-space model-based controller. In addition, Riccati observer is implanted in the model [77]. To achieve a robust idle speed set point, conventional integral term was added to the controller. In addition to the controller for reducing the SHV, an anti-jerk controller to reduce the driveline jerking was designed. Since the anti-jerk controller is model-based, the state-space model was extended to include the driveline sub-systems: transmission, half-shafts, wheels and vehicle. The anti-jerk controller was implemented analogy to the SHV controller algorithm with a state observer integral term in the feedback path. The integral term ensures the steady-state accuracy. The reference in [78] concluded that the main strategies for controlling the backlash are: linear, passive and active non-linear controller. The difference between passive and active is switch in the control parameter. A linear observer for the backlash was introduced and discussed in [79]. The controller can be described as a standard PID-controller without considering the engine dynamic. A simple active switching controller in [80] is developed for controlling the backlash. The controller is developed for a system without shaft flexibility and without engine dynamic. The controller switches between the contact mode and the backlash mode. In the contact mode, the controller has an acceleration set point and in the backlash mode the controller controls the motor side position towards contact wheel side of the backlash.

3.2.2 Summary

The literature review of the anti-jerk controller shows different methods to control the jerking and the oscillation occurring on the vehicle. These methods are based on different principles and use different input parameters. The literature covered the controller which based on linear and non-linear model. These models include two or three mass systems and have a reduction of the vehicle drivetrain components. The main anti-jerk control methods are summarised in Table 6.
<table>
<thead>
<tr>
<th>Controller Method</th>
<th>Input Parameters</th>
<th>Limitation</th>
<th>Achievement</th>
<th>Approval</th>
</tr>
</thead>
</table>
| Anti-jerk controller with a combination of the PD, Root locus and Smith predictor [41], [42] | Engine and wheel speed difference | - Modelled engine speed  
- Linear model with three mass of inertia reflects the accuracy level of the model | Showed damping of the engine and wheel speed and smoothed the acceleration | Tested in a test car |
| Active controller used the control low and an observer. Pole placement strategy determined with the help of LQG [47] | Random torque | Bandwidth limited | Reduction 50% on the first peak of the oscillation where the shuffle oscillation settling in 0.3 second. The damping on the longitudinal vehicle acceleration shows 25% from the uncontrolled oscillation in the tip-in. | Experimental tested Non-linear model with ADAMS |
| P controller with Root-Locus pole placement method [49] | speed difference between the vehicle and the transmission | Considered only few components of the vehicle with appropriate feedback path. Needs accurate measurements of the axle frequency | Damping of the vibration satisfied the real time system | Simulation approved |
| LQ controller and model-based controller with state observer [51] | Engine speed, wheel speed and torsion angle | Two-inertia mass model with three states. | Use the engine as an actuator to damp the oscillation on the heavy powertrain | Tested on an ideal powertrain model. |
| Combination of the model-based controller and Smith predictor [56] | Engine speed and half-shaft torque | -Three manual tuning parameters  
- Limited linear model with two mass inertia and three states | Use the engine as an actuator to damp the oscillation produced by the vehicle shuffle with tuning | Tested on the car |
<table>
<thead>
<tr>
<th>Generate correction torque and implemented in the ignition advance [59]</th>
<th>Engine speed modelled and engine speed measured</th>
<th>Calibration the damping parameters. Speed difference depends on the model</th>
<th>ECU implementation</th>
<th>Tested in the car</th>
</tr>
</thead>
<tbody>
<tr>
<td>P controller phase compensator [60], [61]</td>
<td>Engine and wheel speed difference</td>
<td>Needs synchronisation of the compensator torque to the time and phase</td>
<td>Control the timing of the spark. Could be implemented in ECU</td>
<td>Patent application</td>
</tr>
</tbody>
</table>

3.2.3 Hybrid Electric Vehicles

In this section, the anti-jerk controller of the non-conventional vehicles will be introduced and discussed. The term non-conventional vehicles covers the hybrid electrical vehicle (HEV) and electrical vehicle (EV). The main components of the HEV and EV, in addition to the internal combustion engine (ICE), for HEV, include the electrical motors (EM), inverter to generate high alternate current from the DC high voltage battery.

The architecture of the hybrid electrical vehicles and electrical vehicles are presented in Figure 41 and Figure 42 respectively. Different architectures configuration is reflected in Figure 41. The ICE is the primary energy source in the series configuration in Figure 41a, where the ICE is in series with the EM, the power generator and the transmission. In the parallel configuration in Figure 41b, the ICE is used as main power source and the EM is used as starter motor. The vehicle can be driven either by the ICE or the EM or both sources.

The combination of series and parallel concepts is presented in Figure 41c where the ICE and EM can operate in series, parallel or a combination between parallel and series. In this configuration, a power split device is used to split the power for two paths. The first path from the ICE is transmitted to the vehicle wheels and the second path is converted into electricity by the generator to drive the EM or charge the battery.

In the case of four wheels drive (4WD) powertrain, a combination between the ICE and the EM is used to drive the front wheels and the rear wheels respectively as shown in Figure 41d.

The configuration shown in Figure 41d presents parallel powertrain architecture for 4WD of HEV. In this configuration, the utilisation of the electric motor enables the reduction of the fuel consumption by running the ICE more efficiently. In addition, the brake regeneration is implemented.
Figure 41 - Schematic of hybrid electrical vehicle (HEV) [81], [82]

(a) Series configuration, (b) 2WD with parallel configuration
(c) 2WD series parallel power split, (d) 4WD parallel powertrain architecture

Figure 42 - Schematic of drive system of electric vehicle [83]
In Figure 42, an example of an electric vehicle configuration is presented in [83] and used in Mercedes A-Class. The main components are similar to the HEV without the ICE. The electric motor provides the torque directly to the differential where the battery provides the high voltage DC to the converter to generate the required current to the induction motor (EM). Having an electrical motor in the drivetrain is beneficial to the dynamic of the vehicle due to the fast response and torque capability. Since the electrical motor is coupled to wheel via flexible driveline components, torsional oscillation could occur. These oscillation causes discomfort and drivability problems to the vehicle. Therefore, the need for the control strategy in the non-conventional vehicles is presents.

Figure 43 – Schematic of the vehicle demonstrator layout: M1 and M2 electric motors, I1 and I2 inverters; SCB: Slip Control Boost unit (electro-hydraulic braking system) [84]

The controller in [84] is self-tuneable controller deals with two on-board electric drivetrains motors installed on the front axle Figure 43. The model-based approach in this controller enables the continuous tuning of the all yaw moments contribution. Furthermore, the controller performs off-line optimisation procedure based on the quasi-static vehicle model. Since the significant resonance peak on the drivetrain could result in drivability and yaw stability problem, a specific anti-jerk control function with the name Active Vibration Controller (AVC) was implemented. The controller provide a correction torque based on the speed difference between the motor and the wheel speed. The tuning of the AVC controller use the pole placement methodology.
The gain scheduling of the parameter is speed based and implemented to ensure the stability of the controller. The controller was tested in a demonstrator vehicle Range Rover Evoque. The acceleration profile in Figure 44 shows the results with and without AVC function for tip-in manoeuvre with a vehicle speed of 60 km/h. The employing of the AVC was able to reduce the overshoot significantly from 110% to 30% with a rise time of 0.08 s.

Active driveline oscillation control was introduced in [85] with vehicle control system architecture as shown in Figure 45. The system composed of two main control parts: the active damping capable transaxle control system and the active damping capable desired drivetrain wheel-torque control system.

The main tasks for the active transaxle control system is to control the engine speed to a desired engine speed and deliver the requested wheel torque. These two parameters are fed to the transaxle control module (TCM) to generate the motor torque command and the generator torque command. The generator controller in TCM determines the generator speed from the desired engine speed considering the gear ratios and the motor speed. After then the generated speed is fed to PI controller to compute the generator torque command.

The motor control command is computed in the motor control module where the gear ratios are applied to the desired wheel torque.

The method described in [86] is based on generating a correction torque from the engine speed differences to damp the vibration of the drive shaft and control the motor in the transient operation such as tip-in, gearshift or brake. The control structure is illustrated in Figure 46. The speed vibration component is detected by calculating the difference between the reference speed
difference and average speed difference ($\bar{\omega}_{sd}$) of the motor, where the reference speed difference is built by the difference of the model motor speed ($\omega_{ms}$) and the actual speed ($\omega_{as}$). The motor correction torque ($T_{cv}$) for the anti-jerk control to damp the vibration during the transient operation can be determined by following equation:

$$T_{cv} = G[(\omega_{ms} - \omega_{as}) - \bar{\omega}_{sd}]$$

(3.29)

Where $G$ is tuneable and depends on the operation/driving manoeuvre and gear ratio.

Figure 45 - Vehicle control system architecture [85]

Figure 46 - Anti-jerk control for an HEV [86]

Figure 47 - Arithmetic processing for normal torque control [87]
In [87], a method is described to control a parallel hydride electric vehicle. The controlling methodology is based on correcting the rotation speed difference between the engine and the motor/generator. The controller is implemented in the engine control unit (ECU) as presented in Figure 47. The speed difference is fed to a proportional integral derivative (PID) controller to obtain the target torque of the transmission input shaft.

An active controller to damp the oscillation on the driveline for an electric vehicle is described in [83]. The control structure is composed of two parts. The first part is a non-linear estimator to estimate the gear torque. The torque estimator is built as an optimal Kalman filter, which includes a non-linear state-space model for the dynamic between the motor torque and the wheel speed. The second part is a linear third order controller, which calculates the corrective torque depends on the estimated torque from the estimator.

Active control method in [88] uses observer to estimate the speed difference between the rotor relative to the motor stator of the electric machine and the wheel speed, where the speed difference is multiplied by a gain to generate the controller torque. The observer as Luenberger observer has a reference torque as input, where the reference torque is the difference between the torque demand by the acceleration pedal and the controller torque. The observer gives the angular speed as output.

In [89], a method is described to evaluate the active damping schemes. An off-vehicle test bench emulates the drivetrain oscillation. The test setup uses three mass drivetrain models. The electrical vehicle drive is mimicked by a servo drive where the tested electrical drive is connected. The torsional oscillation resonance of the rear drivetrain HEV was considered. Active damping control was achieved in two approaches high-pass filter and Root-Locus control. In general, the controller is based on generating a correction torque from the difference of the wheel speed and the electrical motor speed.

The control algorithm in [90] aimed to reduce the shuffle of the parallel hybrid vehicle in order to improve comfort and driveability. The proposed anti-jerk controller governs the torque generated by EM propulsion in parallel with ICE. The control algorithm deals with the EM equipped with appropriate torque controller to obtain fast and precise response of the requested torque. A simple driveline model third order was generated in order to be used in the controller design. The transfer function of the model was identified after number of experimental tests to represent the vehicle dynamics. Two ways were used for designing the controller. The first one based on the H-Infinity technique, which allowed achieving lower bound of the performance. The second method is a classical approach with robust stability analysis using the linear fractional transform (LFT) for the plant.
3.2.4 Summary

The literature review of the anti-jerk controller for the hybrid electric vehicles shows different methods to control the jerk and the oscillation occurring on the vehicle. These methods are based on different principles and use different input parameters. The literature shows that the main input parameters are the engine and motor speed. These inputs vary between measured, estimated and calculated. The main strategies methods are summarised in Table 7.

Table 7 - Summary of the control approach for the hybrid electric vehicle in the literature

<table>
<thead>
<tr>
<th>Controller Method</th>
<th>Input Parameters</th>
<th>Limitation</th>
<th>Achievement</th>
<th>Approval</th>
</tr>
</thead>
</table>
| Transaxle Control Module (TCM) and PI Controller [85] | Engine and motor speed, and engine and motor torque | - Evaluated on the road conditions  
- Depends on the driveline stiffness | Oscillation damping | Tested on the vehicle |
| Anti-jerk controller with PI components Park [86] | Model and actual motor speed | - Tuneable gain depends on the operation/driving manoeuvre and gear ratio | ECU implementation | Patent |
| PID controller to generate a correction torque [87] | Measured and calculated transmission revolution speed | - | ECU implementation | Patent |
| Anti-jerk controller [84] | Motor speed and wheel speed | Offline optimisation | Overshoot reduction from 110% to 30% with a rise time of 0.08 s | Tested on the vehicle |
Chapter 4

Non-Linear Passive Vehicle Modelling

This chapter deals with the modelling of the conventional vehicle. The model will be described by the mechanical layout and the mathematical equations describing the dynamics. The aim of this model is to study the drivability of the vehicle and indicate the most important parameters of the vehicle or driveline, which affect the drivability in the transient condition in particular tip-in. The model enables also to understand the effect of the individual components on the driveline oscillation. In the following chapters, two terms will be used to describe the vehicle model: passive vehicle and active vehicle. Passive vehicle deals with the model without the control strategy and the active vehicle deals with the model including the anti-jerk controller.

4.1 Model Structure

The layout of the conventional vehicle driveline is depicted in Figure 48. The powertrain consists of an ICE coupled to a four speed automated manual transmission through the friction clutch. The torque is transferred to the differential via the primary shaft and the secondary shaft via the gearbox. The torque is coupled from the secondary shaft to the front differential, which in turn splits the torque to two equal values. The torque from the differential is provided to the right and left half-shafts.

Two models of the longitudinal vehicle dynamics are derived systematically in this thesis: Non-linear model and linear model. In this chapter, the non-linear model will be described in the next chapter with the linear model. In both models, simulation results and model validation will also be discussed. The linear model is achieved by linearizing the non-linearities in the non-linear
model such as the clutch, tyre and the vehicle resistance action i.e. aerodynamic drag and tyre rolling resistance forces. The linear model equations are formulated in the state-space form where the eigenvalues of the state matrix were calculated to give the natural frequency. Therefore, the linear model is used to assist the vehicle dynamic in the frequency domain.

The model is characterised by six degrees of freedom. These are the angular displacement of the engine crankshaft, primary shaft, the relative rotation between two sun-gears in the front differential (the angular displacement difference between right and left sun-gears), front right and left wheels, and the vehicle.

The layout in Figure 48 depicts three degrees of freedom on the powertrain from the engine to the half-shaft. The powertrain model assumes that the transmission is stiff and the half-shaft is flexible. The clutch and tyres are characterised as non-linear meanwhile the engine is characterised by engine map. In the following sections the individual components of the vehicle is described and the dynamics equations created.

### 4.1.1 Engine Model

The internal combustion engine provides the motion through the crankshaft, therefore the engine torque is characterised by the crankshaft torque. The traction forces and the torsional dynamic
applied to the wheel and thus in the vehicle depend on the torque demand, gear ratio and efficiencies of the transmission.

The model of the engine is based on the experimental measured engine map which generates interpolated engine torque as function of the throttle position and engine speed. The car manufacturer generates the engine map in stationary condition, where the engine is driven for different speeds and the output torques is measured.

The dynamics of the engine are determined as first order transfer function characterised by the output torque. The engine torque $T_e$ is used to write the moment balance equation on the engine shaft. For the flexible body with constant moment inertia about a fixed axis the Newton second law for the motion can be applied as follow:

$$\sum T_i - J \ddot{\theta} = 0$$  \hspace{1cm} (4.1)

Where $T_i$ is the summation of the torques acting on the body, $J$ is the moment of inertia and $\ddot{\theta}$ is the angular acceleration.

Applying Newton’s second law, the sum of the torques acting on the engine and the clutch is expressed by the following equation:

$$T_e - T_c = J_e \ddot{\theta}_e$$  \hspace{1cm} (4.2)

Where $T_e$ is the engine torque and $T_c$ is the clutch torque damper, $J_e$ is the engine moment inertia including the fly-wheel moment of inertia, and $\ddot{\theta}_e$ is the equivalent angular acceleration of the engine.

4.1.2 Clutch Model

The flywheel of the engine is connected to the transmission via the clutch. In case of the automated manual transmission (AMT), the clutch and the gear shift operate by actuators, which are controlled by electronic control unit. The non-linear characteristic of the clutch damper is presented in Figure 49. The non-linear characteristic presents the clutch damper torque as a function of the angular clutch displacement. The non-linearity of the clutch torque damper comes from springs within the clutch rotating disk, which damp the engine vibrations.
The engine torque at the angular displacement causes increase of the angle difference over the clutch. Therefore, the springs in the clutch are compressed at different angles, which give the non-linear characteristics of the clutch damper torque in

When the angle difference between the primary shaft and the engine shaft increased from zero to $\theta_{d1}$, the springs with the stiffness $k_{d1}$ are compressed. Increased the difference angle further, the stiffer springs with stiffness $k_{d2}$ are beginning to be compressed and stop when $\theta_{d2}$ is reached, after then the clutch is stopped. The stiffness behaviour $k_{d}(\Delta\theta)$ can be expressed as follow:

$$k_{d}(\Delta\theta) = \begin{cases} k_{d1} & |\Delta\theta| \leq \Delta\theta_{d1} \\ k_{d2} & \Delta\theta_{d1} < |\Delta\theta| \leq \Delta\theta_{d2} \end{cases}$$

Where $\Delta\theta$ is the angular displacement difference between the engine angular displacement $\theta_e$ and the angular displacement $\theta_p$ of the primary shaft

$$\Delta\theta = \theta_e - \theta_p$$

The clutch damper torque can be expressed as a function of the torsional angle as follow:

$$T_d(\Delta\theta) = \begin{cases} k_{d1}\Delta\theta & |\Delta\theta| \leq \theta_{d1} \\ k_{d1}\theta_{d1} + k_{d2}(\Delta\theta - \theta_{d1}) & \theta_{d1} < \Delta\theta \leq \theta_{d2} \\ -k_{d1}\theta_{d1} + k_{d2}(\Delta\theta + \theta_{d1}) & -\theta_{d2} < \Delta\theta \leq -\theta_{d1} \end{cases}$$

In addition to the non-linearity of the torque in equation (4.5), the hysteresis due to the Coulomb friction dependant on the torsional angle is added. Therefore, the total clutch torque is given in the following equation:

$$T_c = T_d(\Delta\theta) + T_{hys}(\Delta\theta)$$
Where $T_d(\Delta \theta)$ is the clutch torque damper in equation (4.5) and $T_{hys}(\Delta \theta)$ the hysteresis clutch torque as function of the torsional angle $\Delta \theta$. The non-linear characteristic of the torque damper and hysteresis are given in a look-up table obtained from experimental data.

4.1.3 Primary Shaft Model

After the clutch is engaged, the generated torque from the engine is transferred to the front differential via the primary and the secondary shaft of the gearbox. Depending on the gear selection, the primary shaft and the secondary shafts are linked via the transmission ratio. It is worth to note that the damping of the transmission is so small that it can be neglected [93].

The primary shaft torque is determined as:

$$T_c - T_p = (J_p + J_{c1}) \ddot{\theta}_p$$

(4.7)

where $T_p$ and $J_p$ are the primary shaft torque and the mass moment of the inertia of the primary shaft respectively and $\ddot{\theta}_p$ is the angular acceleration of the primary shaft of the front transmission.

4.1.4 Secondary Shaft model

In the front-wheel-driven vehicle, the secondary shaft is connected directly to the differential as shown in Figure 48. The secondary shaft torque is expressed as follow:

$$T_s - T_{diff} = J_s \dot{\theta}_s$$

(4.8)

where $T_s$ is the secondary shaft torque, $T_{diff}$ the differential torque, $J_s$ the mass moment of inertia of the secondary shaft and $\dot{\theta}_s$ is the angular acceleration of the secondary shaft. The secondary shaft torque can be expressed by the primary shaft torque using the gear ratio $\tau_g$ and the efficiency $\eta_g$:

$$T_p \eta_g \tau_g - T_{diff} = J_s \ddot{\theta}_s$$

(4.9)

After combining equation (4.7) and (4.9), the differential torque can be expressed in the following equation:

$$(T_c - (J_p + J_{c1})\ddot{\theta}_p) \eta_g \tau_g - T_{diff} = J_s \ddot{\theta}_s$$

(4.10)

The angular displacement of the secondary shaft can be expressed in relation to the angular displacement of the primary shaft by multiplying the angular displacement $\theta_s$ of the secondary
shaft by the transmission ratio \( \tau_g \) (4.11). Therefore, the angular acceleration of the secondary shaft and the primary shaft can be expressed as in equation (4.12).

\[
\dot{\theta}_p = \dot{\theta}_s \tau_g \\
\ddot{\theta}_p = \ddot{\theta}_s \tau_g
\] (4.11) (4.12)

### 4.1.5 Differential and Half-Shaft Model

The output torque of the differential is calculated by the differential ratio \( \tau_{diff} \) and the differential efficiency \( \eta_{diff} \).

The differential splits the torque into two equals torques, the two torques are transmitted to the right and left wheels via the right and left half-shaft respectively. The angular displacement and thus the angular acceleration of the differential shaft can be expressed in relation to the secondary shaft using the gear ratio \( \tau_{diff} \) of the differential.

\[
\ddot{\theta}_s = \tau_{diff} \ddot{\theta}_{diff}
\] (4.13)

Fulfilling (4.11), (4.12) and (4.13) in (4.10) gives the following equation:

\[
(\eta_g \tau_g T_c - (J_p + J_{c1})\tau_{diff} \eta_g \tau_g^2 \ddot{\theta}_{diff}) - T_{diff} = J_s \tau_{diff} \ddot{\theta}_{diff}
\] (4.14)

Solving the equations after the differential torque, we obtain the following equation:

\[
T_{diff} = \eta_g \tau_g T_c - [(J_p + J_{c1})\tau_{diff} \eta_g \tau_g^2 + J_s \tau_{diff}] \ddot{\theta}_{diff}
\] (4.15)

The torque balance equation between the differential output torque and the right and the left half-shafts are described as:

\[
\eta_{diff} \tau_{diff} T_{diff} - T_{hatfR} - T_{hatfL} = J_{diff} \ddot{\theta}_{diff}
\] (4.16)

Where \( T_{hatfR} \) and \( T_{hatfL} \) the torques for the right and left half-shaft, respectively.

After combining equation (4.15) and (4.16), equation (4.17) can be obtained:

\[
(\eta_g \tau_g T_c - J_p + J_{c1})\eta_{diff} \tau_{diff} \tau_g^2 \eta_g + J_s \eta_{diff} \tau_{diff}^2 + J_{diff}) \ddot{\theta}_{diff} = \\
\eta_{diff} \tau_{diff} \eta_g \tau_g T_c - T_{hatfR} - T_{hatfL}
\] (4.17)

The left and right half-shaft torque can be calculated from the stiffness and the damping factors as given in (4.18) and (4.19):

\[
T_{hatfR} = k_{hsR}(\dot{\theta}_{diff} - \dot{\theta}_{wR}) + \beta_{hsR}(\ddot{\theta}_{diff} - \dot{\theta}_{wR})
\] (4.18)

\[
T_{hatfL} = k_{hsL}(\dot{\theta}_{diff} - \dot{\theta}_{wL}) + \beta_{hsL}(\ddot{\theta}_{diff} - \dot{\theta}_{wL})
\] (4.19)

Where \( k_{hsR} \) and \( k_{hsL} \) are the stiffness coefficients, \( \beta_{hsR} \) and \( \beta_{hsL} \) are the internal damping coefficients of the steel on the right and left half-shaft respectively. It is to note that the damping
coefficients of the steel in practice is near to zero. The angular speeds $\dot{\theta}_w R$ and $\dot{\theta}_w L$ are the angular speed of the right and left wheel respectively.

\[
\left( (J_p + J_{c1}) \eta_{diff} \tau_{diff}^2 \tau_g^2 \eta_g + J_s \eta_{diff} \tau_{diff}^2 + J_{diff} \right) \dot{\theta}_{diff} = \\
\mu_{diff} \tau_{diff} \eta_g \tau_c - k_{hsR} (\theta_{diff} - \dot{\theta}_w R) - \beta_{hsR} (\dot{\theta}_{diff} - \dot{\theta}_w R) \\
- k_{hsL} (\theta_{diff} - \dot{\theta}_w L) - \beta_{hsL} (\dot{\theta}_{diff} - \dot{\theta}_w L)
\]

\[
\dot{\theta}_{diff} = \frac{\eta_{diff} \tau_{diff} \eta_g \tau_c - k_{hsR} (\theta_{diff} - \dot{\theta}_w R) - \beta_{hsR} (\theta_{diff} - \dot{\theta}_w R) - k_{hsL} (\theta_{diff} - \dot{\theta}_w L) - \beta_{hsL} (\dot{\theta}_{diff} - \dot{\theta}_w L)}{(J_p + J_{c1}) \eta_{diff} \tau_{diff}^2 \tau_g^2 \eta_g + J_s \eta_{diff} \tau_{diff}^2 + J_{diff})}
\]

(4.20)

4.1.5.1 Differential with Rotating Gears

The previous section 4.1.5 dealt with the differential as one component and did not consider the gears inside the differential. The differential internals are depicted in Figure 50. The differential consists of different rotating gears two sun gears: 1) left and 2) right, 3) upper and 4) lower planetary gear, 5) differential gear and 6) the differential box. A schematic of the torque distribution is given in Figure 51. In the cornering manoeuvre, the sun gears ensure to distribute the torque value to the left and right half-shaft. Therefore, the relative rotation between the left and right gears is expressed by the angular displacement difference $\Delta \theta_{diffs}$.

The angular displacement difference $\Delta \theta_{diffs}$ can be calculated from the difference between the right and the left sun gear displacement $\theta_{diffsR}$ and $\theta_{diffsL}$ respectively as follow:

\[
\Delta \theta_{diffs} = \theta_{diffsR} - \theta_{diffsL}
\]

(4.21)

Applying the Willis formula [94] on the differential angular displacement which is equal the average value of the right and the left angle displacement of the sun gears according to:

\[
\theta_{diff} = \frac{\theta_{diffsR} + \theta_{diffsL}}{2}
\]

(4.22)

The left $\theta_{diffsL}$ and the right $\theta_{diffsR}$ sun gear displacement can be expressed in relation to the differential angular displacement $\theta_{diff}$ and the sun gear difference angular displacement $\Delta \theta_{diffs}$ by applying equation (4.22) in (4.21) and solving after $\theta_{diffsR}$ and $\theta_{diffsL}$.

\[
\theta_{diffsR} = \frac{1}{2} \Delta \theta_{diffs} + \theta_{diff}
\]

(4.23)

\[
\theta_{diffsL} = \theta_{diff} - \frac{1}{2} \Delta \theta_{diffs}
\]

(4.24)
The angular speed and acceleration can be achieved by applying the first and the second derivative of the equations (4.23) and (4.24).

\[ \dot{\theta}_{\text{diff}R} = \frac{1}{2} \Delta \ddot{\theta}_{\text{diff}s} + \dot{\theta}_{\text{diff}} \]  \hspace{1cm} (4.25)

\[ \dot{\theta}_{\text{diff}L} = \dot{\theta}_{\text{diff}} - \frac{1}{2} \Delta \ddot{\theta}_{\text{diff}s} \]  \hspace{1cm} (4.26)

The angular acceleration is determined in the next equations:

\[ \ddot{\theta}_{\text{diff}R} = \frac{1}{2} \Delta \ddot{\theta}_{\text{diff}s} + \ddot{\theta}_{\text{diff}} \]  \hspace{1cm} (4.27)

\[ \ddot{\theta}_{\text{diff}L} = \dot{\theta}_{\text{diff}} - \frac{1}{2} \Delta \ddot{\theta}_{\text{diff}s} \]  \hspace{1cm} (4.28)

The torque balance on the sun gears is expressed as follow:

\[ T_{\text{half}L} - T_{\text{half}R} = J_{\text{diff}s} \Delta \ddot{\theta}_{\text{diff}s} + \beta \Delta \ddot{\theta}_{\text{diff}s} \]  \hspace{1cm} (4.29)

The right half-shaft can be expressed in relation to the right wheel rotation as follows:

\[ T_{\text{half}R} = k_{\text{hs}R} (\theta_{\text{diff}R} - \theta_{wR}) + \beta_{\text{hs}R} (\dot{\theta}_{\text{diff}R} - \dot{\theta}_{wR}) \]  \hspace{1cm} (4.30)

The left half-shaft will be:

\[ T_{\text{half}L} = k_{\text{hs}L} (\theta_{\text{diff}L} - \theta_{wL}) + \beta_{\text{hs}L} (\dot{\theta}_{\text{diff}L} - \dot{\theta}_{wL}) \]  \hspace{1cm} (4.31)

After satisfying equations (4.23) and (4.25) in (4.30); and (4.24) and (4.26) in (4.31), the half-shaft torques can be expressed as follow:

\[ T_{\text{half}R} = k_{\text{hs}R} \left( \frac{1}{2} \Delta \theta_{\text{diff}s} + \dot{\theta}_{\text{diff}} - \theta_{wR} \right) + \beta_{\text{hs}R} \left( \frac{1}{2} \Delta \ddot{\theta}_{\text{diff}s} + \ddot{\theta}_{\text{diff}} - \dot{\theta}_{wR} \right) \]  \hspace{1cm} (4.32)

\[ T_{\text{half}L} = k_{\text{hs}L} \left( \dot{\theta}_{\text{diff}} - \frac{1}{2} \Delta \theta_{\text{diff}s} - \dot{\theta}_{wL} \right) + \beta_{\text{hs}L} \left( \ddot{\theta}_{\text{diff}} - \frac{1}{2} \Delta \ddot{\theta}_{\text{diff}s} - \ddot{\theta}_{wL} \right) \]  \hspace{1cm} (4.33)
Figure 51 - Schematic of the torque distribution the rotating parts in the differential [81]

The torque balance at the differential to the wheels can be expressed as:

\[ \eta_{\text{diff}} \tau_{\text{diff}} \eta_g \tau_c - T_{\text{half}R} - T_{\text{half}L} = \]

\[ \left( (J_p + J_{c1}) \eta_{\text{diff}} \tau_{\text{diff}}^2 \tau_g^2 \eta_g + J_s \eta_{\text{diff}} \tau_{\text{diff}}^2 + J_{\text{diff}} \right) \dot{\theta}_{\text{diff}} \]

(4.34)

Combining equations (4.32) and (4.33) in (4.34) the final angular acceleration of the differential can be expressed in relation to the sun gears rotation:

\[ \ddot{\theta}_{\text{diff}} = \frac{\eta_{\text{diff}} \tau_{\text{diff}} \eta_g \tau_c \left( k_{hsR} \left( \frac{1}{2} \Delta \theta_{\text{diff}s} + \theta_{\text{diff}} - \theta_{\text{WR}} \right) + \beta_{hsL} \left( \frac{1}{2} \Delta \theta_{\text{diff}s} + \theta_{\text{diff}} - \theta_{\text{WL}} \right) \right)}{\left( (J_p + J_{c1}) \eta_{\text{diff}} \tau_{\text{diff}}^2 \tau_g^2 \eta_g + J_s \eta_{\text{diff}} \tau_{\text{diff}}^2 + J_{\text{diff}} \right)} - \]

\[ \frac{\left( k_{hsR} \left( \theta_{\text{diff}} - \frac{1}{2} \Delta \theta_{\text{diff}s} + \theta_{\text{WR}} \right) + \beta_{hsL} \left( \theta_{\text{diff}} - \frac{1}{2} \Delta \theta_{\text{diff}s} + \theta_{\text{WL}} \right) \right)}{\left( (J_p + J_{c1}) \eta_{\text{diff}} \tau_{\text{diff}}^2 \tau_g^2 \eta_g + J_s \eta_{\text{diff}} \tau_{\text{diff}}^2 + J_{\text{diff}} \right)} \] 

(4.35)

4.1.6 Vehicle and Tyre Model

4.1.6.1 Vehicle Model

The vehicle dimensions including the forces, which applied to the vehicle, are expressed in Figure 52. The vehicle free body is assumed a concentrated mass with the centre of gravity CG. The vehicle body is assumed to be rigid body; therefor the rotational moment mass of inertia of the point mass at the centre of gravity is dynamically equivalent to the vehicle body for all motions [95]. The torque of the individual component is calculated by multiplying the equivalent force by the wheel radius.
The torque balance on the vehicle can be written as follow:

\[ J_v \ddot{\theta}_v = T_{tyr_fL} + T_{tyr_fR} - T_{tyr_rL} - T_{tyr_rR} - T_{road\_gradient} - T_{aer\_drag} \]  

(4.36)

where \( J_v \) is the vehicle mass moment of inertia including the un-driven wheels, \( \dot{\theta}_v \) is the equivalent vehicle angular acceleration. The torque for the individual component is calculated by multiplying the equivalent force by the wheel radius.

The mass moment of inertia of the vehicle is the sum of the moment inertia of the vehicle \((R_w^2M_v)\) and the equivalent mass moment of inertia of the two undriven wheels \(J_w\).

\[ J_v = R_w^2M_v + J_w \]  

(4.37)

The traction torques from the front tyres respectively left and right is \( T_{tyr_fL} \) and \( T_{tyr_fR} \). These torques and the load torques from the rear tyres \( T_{tyr_rL} \) and \( T_{tyr_rR} \) are calculated from the traction forces which will be explained in the next section 4.1.5.2.

The aerodynamic drags torques \( T_{aer\_drag} \) is calculated from the aero dynamic resistance force \( F_{aer\_drag} \) and the wheel radius \( R_w \) according to the following equation:

\[ F_{aer\_drag} = \frac{1}{2} C_d \rho_a S \dot{\theta}_v^2 R_w^2 \]  

(4.38)

\[ T_{aer\_drag} = F_{aer\_drag} R_w \]  

(4.39)

Where \( C_d \) is drag coefficient and \( \rho_a \) air density, \( S \) is the vehicle cross section area, \( \dot{\theta}_v \) is the vehicle angular velocity and \( R_w \) is the wheel radius.

The torque \( T_{\_gradient} \) due to the road gradient resistance force is:

\[ T_{\_gradient} = R_wM_vgsin(\alpha) \]  

(4.40)

Where \( M_v \) the vehicle mass, \( g \) is the gravity acceleration and \( \alpha \) the road inclination. The angular vehicle acceleration is calculated by combining equations (4.37), (4.38) and (4.39) and (3.40) in (4.36) and solving after \( \dot{\theta}_v \):

Figure 52 - Force balance on the vehicle as a rigid body
\[ \ddot{\theta}_v = \frac{T_{tyrFL} + T_{tyrFR} - T_{tyrRL} - T_{tyrRR} - R_w M_s g \sin(\alpha) + \frac{1}{2} c_d p_a \dot{R}_w^2 R_w^3}{R_w^5 M_v + J_w} \] (4.41)

The longitudinal acceleration is then calculated using the wheel radius:
\[ \ddot{x}_v = R_w \dot{\theta}_v \] (4.42)

It is worth to note that, the jerk parameter of the vehicle is equal to the derivative of the vehicle acceleration.

4.1.6.2 Tyre Model

As mentioned in the last section, the traction force/torque is provided via the front right and left tyres. The wheel model including the tyre is one of the non-linearity in the vehicle model. Therefore, the traction force \( F_x \) has a non-linear characteristic and modelled with the help of Pacejka’s Magic Formula [96] which is described in Appendix B. The Pacejka’s Magic Formula describes the longitudinal traction force in a function of the tyre normal force and the longitudinal slip ratio \( SR \).

\[ F_x = f(F_n, SR) \] (4.43)

The longitudinal slip ratio is defined as the ratio of the actual rotational speed of the wheel to the free-rolling speed of the wheel and calculated from the vehicle speed and wheel speed as described in equation (4.44):
\[ SR = 1 - \frac{\dot{\theta}_v}{\dot{\theta}_w} = \frac{\dot{\theta}_w - \dot{\theta}_v}{\dot{\theta}_w} \] (4.44)

where \( \dot{\theta}_v \) is the equivalent angular vehicle speed and \( \dot{\theta}_w \) is the angular wheel speed.

The total normal load \( F_n \) on the tyres consists of two parts the static and the dynamic load. The static load depends on the weight, the location of the centre gravity of the vehicle and the wheel base length. The static load on the front wheels for both the left and the right is calculates as follow:
\[ F_{nstatf} = \frac{M_v g b}{2l} \] (4.45)

The static load on the rear wheels for both the left and the right is calculated in the following equation:
\[ F_{nstatr} = \frac{M_v g a}{2l} \] (4.46)

where \( a \) is the longitudinal distance between the centre of gravity and the front wheel, \( b \) is the longitudinal distance between the centre of gravity and the rear wheel, \( l \) is the wheel-base of the vehicle, and \( M_v \) is the vehicle mass including payload.
The total normal load on the front driven tyres $F_{ntotf}$ is given in equation (4.47) and the normal load on the rear un-driven tyres $F_{ntotr}$ is given in equation (4.48). In addition to the static load, the load due to the dynamic conditions i.e. the air drag force and the acceleration are considered in the calculation of the total load.

$$F_{ntotf} = \frac{M_v gb - F_{aerdrag} h_{CG} - M_v a_x h_{CG}}{2l}$$ (4.47)

$$F_{ntotr} = \frac{M_v gb + F_{aerdrag} h_{CG} + M_v a_x h_{CG}}{2l}$$ (4.48)

where $h_{CG}$ is the height of the centre of gravity of the vehicle and $a_x$ the vehicle acceleration.

The total normal load for both the right and left tyres have the same value $F_{ntotf}$ and $F_{ntotr}$ for front and rear tyres respectively.

Considering the torque balance on the front wheels, the tyres torque can be written for the right and the left tyres respectively in the following equations:

$$T_{tyrfr} = T_{halffR} - T_{txfr} - T_{rollfR}$$ (4.49)

$$T_{tyrfl} = T_{halffL} - T_{txfl} - T_{rollfL}$$ (4.50)

where $T_{txfr}$ and $T_{txfl}$ the torque due to the longitudinal tyre force $F_{xfr}$ and $F_{xfl}$ for the right and the left front tyre.

$$T_{txfr} = F_{xfr} R_w$$ (4.51)

$$T_{txfl} = F_{xfl} R_w$$ (4.52)

The rolling resistance torque $T_{rollfR}$ and $T_{rollfL}$ for the right and left tyre can be calculated from the total normal force $F_{ntotf}$ using the rolling resistance $f$ as follow:

$$T_{rollfR} = F_{ntotf} R_w (A + BR_w \dot{\theta}_{wfr} + CR_w^2 \dot{\theta}_{wfr}^2)$$ (4.53)

$$T_{rollfL} = F_{ntotf} R_w (A + BR_w \dot{\theta}_{wfl} + CR_w^2 \dot{\theta}_{wfl}^2)$$ (4.54)

Where $A$ is the coefficient of the tyre, $B$ the coefficient of the tyre pressure and $C$ the rolling resistance coefficient depending on the vehicle speed square.

Applying the second Newton Law to (4.49) and (4.50), the angular wheel acceleration can be calculated:

$$(J_{wfr} + \frac{1}{2} J_{halff}) \ddot{\theta}_{wfr} = T_{halffR} - T_{txfr} - T_{rollfR}$$ (4.55)

$$(J_{wfl} + \frac{1}{2} J_{halff}) \ddot{\theta}_{wfl} = T_{halffL} - T_{txfl} - T_{rollfL}$$ (4.56)

Where $J_{halff}$ is the total mass moment of inertia of the half-shaft. In the equations, both the right half-shaft and the left half-shaft are considered symmetrical with each other. Consequently, mass moment of inertia of each will be $\frac{1}{2} J_{halff}$.
Satisfying the equation (4.51), (4.53) and (4.30) in (4.55) the angular vehicle acceleration of the front right wheel can be expressed as follow:

\[
\ddot{\theta}_{wfr} = \frac{k_{hsr}(\dot{\theta}_{\text{diff}sr}-\dot{\theta}_{wfr})+\beta_{hsr}(\dot{\theta}_{\text{diff}sr}-\dot{\theta}_{wfr})-F_{xfbr}R_w-F_{ntotfr}R_w(A+BR_w\dot{\theta}_{wfr}+CR_w^2\theta_{wfr}^2)}{(J_{wfr}+\frac{1}{2}J_{halfr})}
\]  

(4.57)

Applying (4.52), (4.54) and (4.31) in (4.56) the angular vehicle acceleration of the front left wheel can be expressed as follow:

\[
\ddot{\theta}_{wfl} = \frac{k_{hsr}(\dot{\theta}_{\text{diff}sl}-\dot{\theta}_{wfl})+\beta_{hsr}(\dot{\theta}_{\text{diff}sl}-\dot{\theta}_{wfl})-F_{xffl}R_w-F_{ntotfl}R_w(A+BR_w\dot{\theta}_{wfl}+CR_w^2\theta_{wfl}^2)}{(J_{wfl}+\frac{1}{2}J_{halfl})}
\]  

(4.58)

Since we consider the front driven vehicle the rear tyres act as a load on the whole vehicle, the torque contribution on the torque balance in equation 4.36 for the rear right and left tyres will be due to the traction force:

\[
T_{tyrrR} = F_{xrrR}R_w
\]  

(4.59)

\[
T_{tyrrL} = F_{xrlR}R_w
\]  

(4.60)

The angular acceleration for the rear tyres is calculated in analogy to the front tyres without the half-shaft torque. In equation (4.61), the angular acceleration of the rear tyre is expressed by the traction force and the rolling resistance torque. Combining the equations for the rolling resistances in (4.61) and considering the total normal load distribution on the rear tyres the angular acceleration for the rear right and left tyres can be expressed in equation (4.62) and (4.63)

\[
J_{wrr}\ddot{\theta}_{wrr} = -T_{txrr} - T_{totrr}
\]  

(4.61)

\[
\ddot{\theta}_{wrr} = -\frac{F_{xrrR}R_w+F_{ntotrr}R_w(A+BR_w\dot{\theta}_{wrr}+CR_w^2\theta_{wrr}^2)}{J_{wrr}}
\]  

(4.62)

\[
\ddot{\theta}_{wrl} = -\frac{F_{xrrL}R_w+F_{ntotrr}R_w(A+BR_w\dot{\theta}_{wrl}+CR_w^2\theta_{wrl}^2)}{J_{wrl}}
\]  

(4.63)

4.1.6.3 Tyre Relaxation Length

The dynamic model of the tyres is extended to include the tyre relaxation length. The relaxation length of the tyre is a dynamic property of the pneumatic tyre, which describes the distance that a tyre rolls before the lateral force builds up to 63% of its steady-state value [97]. The dynamic reaction of the tyre forces and torque can be consider as first order system dynamic, therefore the relaxation length in the non-linear model can be given as first differential equation [99]. Equation (4.64) describes the torque delay \(T_{tyredd}\) due to the relaxation length:

\[\text{Equation (4.64)}\]
where \( l_d \) is the relaxation length, \( \theta_{vo} \) is the initial angular vehicle speed, \( F_x \) the longitudinal traction force from the Pacejka’s Formula.

Considering the delay of the tyre relaxation length, the equations (4.57), (4.58), (4.62) and (4.63) replacing the tyres torques \( T_{tyrr,R,L} \) and \( T_{tyrf,R,L} \) with the delayed tyre torque \( T_{tyrdr,R,L} \) for the rear right and left tyres respectively and for the front tyres \( T_{tyrdf,R,L} \) the equations for the angular wheel acceleration will be obtained as follow:

\[
\ddot{\theta}_{wfr} = \frac{k_{hsf}(\dot{\theta}_{diffsR} - \dot{\theta}_{wfr}) + \beta_{hsf}(\dot{\theta}_{diffs} - \dot{\theta}_{wfr}) - T_{tyrdfR} - F_{ntotfR}w(A + BR_w\dot{\theta}_{wfr} + CR^2_w\dot{\theta}^2_{wfr})}{U_{wfr} + \frac{1}{2}I_{half}} \tag{4.65}
\]

\[
\ddot{\theta}_{wfl} = \frac{k_{hsf}(\dot{\theta}_{diffsL} - \dot{\theta}_{wfl}) + \beta_{hsf}(\dot{\theta}_{diffs} - \dot{\theta}_{wfl}) - T_{tyrdfl} - F_{ntotfl}w(A + BR_w\dot{\theta}_{wfl} + CR^2_w\dot{\theta}^2_{wfl})}{(J_{wfl} + \frac{1}{2}I_{half})} \tag{4.66}
\]

For rear un-driven tyres:

\[
\ddot{\theta}_{wrR} = -\frac{T_{tyrdrR} + F_{ntotR}w(A + BR_w\dot{\theta}_{wR} + CR^2_w\dot{\theta}^2_{wR})}{J_{wR}} \tag{4.67}
\]

\[
\ddot{\theta}_{wrL} = -\frac{T_{tyrdrL} + F_{ntotR}w(A + BR_w\dot{\theta}_{wL} + CR^2_w\dot{\theta}^2_{wL})}{J_{wL}} \tag{4.68}
\]

### 4.1.7 Engine Mounting System

The powertrain mounting system is the system which connects the powertrain with the vehicle body and can be presented by several mounts [100].

The powertrain vibration plays a major role in effecting the drivability. If the dynamic property of the powertrain mounting does not filter out the powertrain vibration. The later can be transferred through the mounting points to the vehicle body causing discomfort and perceptible audible noise. The main task of the engine mounting is to isolate the vibration of the engine from the vehicle body to reduce the level of the interior noise and vibration [101]. The effect of the engine mounting strategy on the vehicle noise is discussed in details in [102], [103].

The aim is to understand the effect of the engine mount on the overall vehicle dynamic. In particular, the longitudinal dynamic and force that will extend equation (4.36) to the following equation:

\[
J_{w}\dot{\theta}_{w} = T_{tyrfL} + T_{tyrfR} - T_{tyrrL} - T_{tyrrR} - T_{roadgradien} - T_{aerdrag} - T_{engmount} \tag{4.69}
\]

Therefore, this section will discuss the model of engine mounting system based on the Eulerian Newtonian methodology adopted from the model of the electric motor in [105].
4.1.7.1 Engine Mounting Layout and Coordinates

The chosen layout in this work supports the mounting of the internal combustion engine. It is four mounting points for front wheel drive system, two in the front and two in the rear. The sprung mass of the powertrain is considered as a rigid body with six degree-of-freedom (DOF). Three translations: vertical bounce, longitudinal, and lateral; and three rotations: yaw, roll and pitch. The layout used in the model is presented in Figure 53. Two of the four mounting are elastomeric and one is torque strut. These are modelled as three dimensional springs and dampers considering the translation and the torsional stiffness. The relation between the force and the displacement is non-linear, where the stiffness is described in a look-up table and the damping as a constant. The bushing of the mounting replaced the constant damping and modelled in the next section 4.1.8 and added to the entire vehicle model in the engine mounting block.

The initial conditions of the mounting blocks considered to have infinitesimal length with coincident points. Since one of the engines mounting block is attached to the vehicle body and the second is located on the powertrain, these are not coincident any more by the motion of the powertrain. Before performing the methodology to derivate the equations of the motion, the reference coordinates system should be described. Two systems are used one is referred to the ground and the other is referred to the powertrain as illustrated in Figure 54.

The referred system to the ground is a right-hand frame with the origin $OXYZ$. The axis X and Y are parallel to the road and Z is vertical. The positive directions for the X-axis is the vehicle front, the Z-axis is pointing upwards and for the Y-axis is pointing downwards in the X-Y plane as illustrated in Figure 54 b). The origin $O$ of the system is coincident with the sprung mass of the vehicle. This reference frame will be quoted as the inertial frame.

The second coordinates system is the local system $P_{cpm}xyz$ which is right-hand and fixed to the powertrain. The origin of the system $P_{cpm}$ is fixed to the mass centre of the powertrain and moves alongside with the powertrain. The $x$- and $y$-axis are parallel to the road and the $z$-axis is vertical and pointing upwards. The local system $P_{cpm}xyz$ and the inertial system $OXYZ$ can be transformed to each other using the rotation matrix for the rotation around each axis [106], [107].

Assuming the rotation occurs around the Z-direction with a rotation angle $\psi$, the new positions of the coordinate $X$ and $Y$ will be $x^*$ and $y^*$ respectively. The rotation matrix $R_1$ is to transform $x^*y^*z^*$ to the inertial system.
Figure 53 - Six DOF of the powertrain mounting system

Figure 54 - Reference coordinates systems: a) X-Z plane, b) X-Y Plane

Figure 55 - Coordinate reference system and transformation

\[
\begin{bmatrix}
X \\
Y \\
Z \\
\end{bmatrix} = R_1 \begin{bmatrix} x^* \\ y^* \\ z^* \end{bmatrix} \tag{4.70}
\]

\[
R_1 = \begin{bmatrix}
\cos(\psi) & -\sin(\psi) & 0 \\
\sin(\psi) & \cos(\psi) & 0 \\
0 & 0 & 1 \\
\end{bmatrix} \tag{4.71}
\]

The rotation around the Y-axis with the angle \( \theta \) needs the matrix \( R_2 \) for the transformation:
The third rotation matrix allows the transformation in case of the rotation around the X-axis with the angle $\phi$:

$$R_3 = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos(\phi) & -\sin(\phi) \\
0 & \sin(\phi) & \cos(\phi)
\end{bmatrix} \quad (4.73)$$

To rotation of any vector from the local reference system $P_{cpm}xyz$ to the inertial $OXYZ$ is achieved by multiplying with the matrix $R$ which is the multiplication of the three matrices:

$$R = \begin{bmatrix}
\cos(\psi)\cos(\theta) & -\sin(\psi)\cos(\phi) + \cos(\psi)\sin(\theta)\sin(\phi) & \sin(\psi)\sin(\phi) + \cos(\psi)\sin(\theta)\cos(\phi) \\
\sin(\psi)\cos(\theta) & \cos(\psi)\cos(\phi) + \sin(\psi)\sin(\theta)\sin(\phi) & -\cos(\psi)\sin(\phi) + \sin(\psi)\sin(\theta)\cos(\phi) \\
-\sin(\theta) & \cos(\theta)\sin(\phi) & \cos(\theta)\cos(\phi)
\end{bmatrix}$$

and the inverse matrix $R^T$

$$R^T = \begin{bmatrix}
\cos(\psi)\cos(\theta) & \sin(\psi)\cos(\theta) & -\sin(\theta) \\
-\sin(\psi)\cos(\phi) + \cos(\psi)\sin(\theta)\sin(\phi) & \cos(\psi)\cos(\phi) + \sin(\psi)\sin(\theta)\sin(\phi) & \cos(\psi)\sin(\phi) + \sin(\psi)\sin(\theta)\cos(\phi) \\
\sin(\psi)\sin(\phi) + \cos(\psi)\sin(\theta)\cos(\phi) & -\cos(\psi)\sin(\phi) + \sin(\psi)\sin(\theta)\cos(\phi) & \cos(\psi)\cos(\phi)
\end{bmatrix} \quad (4.74)$$

Figure 56 - Angular velocities $\dot{p}$, $\dot{q}$ and $\dot{r}$

In analogy to the axis transformation, the angular velocities $\dot{p}$, $\dot{q}$ and $\dot{r}$ in Figure 56 can be expressed by $\dot{\phi}$, $\dot{\theta}$ and $\dot{\psi}$ using the rotation matrix $M$ and equation (4.75).

$$\begin{bmatrix}
\dot{p} \\
\dot{q} \\
\dot{r}
\end{bmatrix} = \begin{bmatrix}
1 & 0 & -\sin(\theta) \\
0 & \cos(\phi) & \sin(\phi)\cos(\theta) \\
0 & -\sin(\phi) & \cos(\phi)\cos(\theta)
\end{bmatrix}\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} = M\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} \quad (4.76)$$

The angular velocities $\dot{\phi}$, $\dot{\theta}$ and $\dot{\psi}$ can be expressed by $p$, $q$ and $r$ by using the inverse matrix $M^{-1}$.
\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} =
\begin{bmatrix}
1 & \sin(\phi) \tan(\theta) & \cos(\phi) \tan(\theta) \\
0 & \cos(\phi) & -\sin(\phi) \\
0 & \sin(\phi) \sec(\theta) & \cos(\phi) \sec(\theta)
\end{bmatrix}
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix} =
M^{-1}
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
\tag{4.77}
\]

4.1.7.2 Forces and Moments on the Powertrain

The engine mounting block is modelled as three-dimensional springs and dampers arranged in parallel for both translation and torsional stiffness. The transitional forces and torsional moments due to this consideration on the mounting blocks will be calculated.

The stiffness and damping has non-linear characteristic. The stiffness is described by look-up table and the damping is considered as constant in this section. Next section 4.1.8 will discuss the modelling of the bushing on the engine mounts block. The total transitional force acting on the mounting blocks are the sum of the force due to the stiffness and the damping as given in equation (4.78):

\[
\begin{bmatrix}
F_{x,i} \\
F_{y,i} \\
F_{z,i}
\end{bmatrix} =
\begin{bmatrix}
F_{x,i} \\
F_{y,i} \\
F_{z,i}
\end{bmatrix}_{stiff} +
\begin{bmatrix}
F_{x,i} \\
F_{y,i} \\
F_{z,i}
\end{bmatrix}_{damp}
\tag{4.78}
\]

Where the index \(i\) refers to the \(i^{th}\) mountings block.

The force due to the stiffness, in the powertrain reference:

\[
\begin{bmatrix}
F_{x,i} \\
F_{y,i} \\
F_{z,i}
\end{bmatrix}_{stiff} = f_s \begin{bmatrix}
x_{pmv,i} \\
y_{pmv,i} \\
z_{pmv,i}
\end{bmatrix} -
\begin{bmatrix}
x_{0mbp,i} \\
y_{0mbp,i} \\
z_{0mbp,i}
\end{bmatrix}
\tag{4.79}
\]

The function \(f_s\) describes the non-linear stiffness along the axis. Since the mounting block is attached from one side to the vehicle chassis and from the second side to the powertrain, the coordinates of the two attachment points are expressed in the powertrain reference system. The coordinates \(x_{pmv,i}, y_{pmv,i}, z_{pmv,i}\) express the attachment points to the vehicle meanwhile the coordinates \(x_{0mbp,i}, y_{0mbp,i}, z_{0mbp,i}\) the attachments points to the powertrain.

The coordinates \(x_{pmv,i}, y_{pmv,i}, z_{pmv,i}\) are obtained from equation (4.81):

\[
\begin{bmatrix}
x_{pmv,i} \\
y_{pmv,i} \\
z_{pmv,i}
\end{bmatrix} = R^T \begin{bmatrix}
x_{pmv,i} \\
y_{pmv,i} \\
z_{pmv,i}
\end{bmatrix} -
\begin{bmatrix}
x_{0pcpm} \\
y_{0pcpm} \\
z_{0pcpm}
\end{bmatrix}
\tag{4.80}
\]

with \(R^T\) from equation (4.75). The coordinates \(x_{pmv,i}, y_{pmv,i}, z_{pmv,i}\) are the coordinate of the attachment point of the mounting block at the vehicle expressed in the inertial reference system and \(x_{0pcpm}, y_{0pcpm}, z_{0pcpm}\) the coordinates of the powertrain system expressed in the inertial
The coordinates \(x_{0mbp,i}, y_{0mbp,i}, z_{0mbp,i}\) can be calculated using equation (4.81):
\[
\begin{bmatrix}
x_{0mbp,i} \\
y_{0mbp,i} \\
z_{0mbp,i}
\end{bmatrix} = \left( \begin{bmatrix} X_{0mbp,i} \\ Y_{0mbp,i} \\ Z_{0mbp,i} \end{bmatrix} - \begin{bmatrix} X_{0pcpm} \\ Y_{0pcpm} \\ Z_{0pcpm} \end{bmatrix} \right)
\]
(4.81)

Where \(X_{0mbp,i}, Y_{0mbp,i}, Z_{0mbp,i}\) are the coordinates, expressed in the inertial reference system, of the attachment point located on the powertrain and \(X_{0pcpm}, Y_{0pcpm}, Z_{0pcpm}\) are the coordinates of the powertrain centre of mass expressed in the inertial reference system.

In this case the inverse of the rotation matrix \(R^T\) has not been applied since at the initial condition, the relative angles between the inertial reference system and the powertrain reference system are null. Figure 57 is a two-dimensional drawing in the \(XZ\) plane of the the powertrain at the initial conditions. The transitional forces due to the damping is expressed in equation (4.82):
\[
\begin{bmatrix}
F_{x,i} \\
F_{y,i} \\
F_{z,i}
\end{bmatrix}_{\text{damp}} = f_d \begin{bmatrix}
\dot{x}_{pmv,i} \\
\dot{y}_{pmv,i} \\
\dot{z}_{pmv,i}
\end{bmatrix} - \begin{bmatrix}
\dot{x}_{0mbp,i} \\
\dot{y}_{0mbp,i} \\
\dot{z}_{0mbp,i}
\end{bmatrix}
\]
(4.82)

Figure 57 - Representation in the \(XZ\) plane of the powertrain at the initial conditions [105]

where \(f_d\) expressed the non-linear damping along the \(x, y\) and \(z\) axes. In analogy to the coordinates of the stiffness forces, \(\dot{x}_{pmv,i}, \dot{y}_{pmv,i}, \dot{z}_{pmv,i}\) and \(\dot{x}_{0mbp,i}, \dot{y}_{0mbp,i}, \dot{z}_{0mbp,i}\) are the linear velocities obtained from the first derivative of \(x_{pmv,i}, y_{pmv,i}, z_{pmv,i}\) and \(x_{0mbp,i}, y_{0mbp,i}, z_{0mbp,i}\) respectively.

The calculated forces \(F_{x,i}, F_{y,i}, F_{z,i}\) in the powertrain coordinates system can be transferred to the inertial reference system by multiplying by the rotation matrix \(R\):
\[
\begin{bmatrix}
F_{x,i} \\
F_{y,i} \\
F_{z,i}
\end{bmatrix} = R \begin{bmatrix}
F_{x,i} \\
F_{y,i} \\
F_{z,i}
\end{bmatrix}
\]
(4.83)
The moments due to the torsional dynamics on the mounting blocks is presented in equation (4.84). These moments consist also of two contributions the first component is the stiffness and the second is the damping.

\[
\begin{bmatrix}
M_{x,i} \\
M_{y,i} \\
M_{z,i}
\end{bmatrix} = f_{Ms} \left( \begin{bmatrix}
\varepsilon \\
\delta \\
\lambda
\end{bmatrix} - \begin{bmatrix}
\alpha \\
\beta \\
\gamma
\end{bmatrix} \right) + f_{ds} \left( \begin{bmatrix}
p \\
q \\
r
\end{bmatrix} - \begin{bmatrix}
\alpha \\
\beta \\
\gamma
\end{bmatrix} \right)
\]

In analogy to the force in the translation dynamic, the torsional moments consists of torsional stiffness \(f_{Ms}\) and torsional damping \(f_{ds}\).

Each mounting block behaves as a torsion spring when it undergoes an angular deformation. The relative angle of inclination between the powertrain and the vehicle body is calculated considering a rigid body motion of the chassis.

The vehicle model as presented in section 4.1 produces the angular displacement \(\theta_v\), which is used to obtain the angular deformation of the mounting blocks, while the roll, and yaw angles are supposed to be null.

The angle \(\theta_v\) of the vehicle is obtained from the vehicle model and it is expressed in the inertial reference frame and is transformed in the angles \(\alpha\), \(\beta\) and \(\gamma\), defined around the \(x\), \(y\) and \(z\) powertrain axes, by mean of the inverse of the rotation matrix \(R^T\):

\[
\alpha = \sin(\psi) \cos(\theta) \theta_v
\]

\[
\beta = \cos(\psi) \cos(\phi) + \sin(\psi) \sin(\theta) \sin(\phi) \theta_v
\]

\[
\gamma = -\cos(\psi) \sin(\phi) + \sin(\psi) \sin(\theta) \cos(\phi) \theta_v
\]

Where

\[
\varepsilon = \phi - \psi \sin(\theta)
\]

\[
\delta = \theta \cos(\phi) + \psi \sin(\phi) \cos(\theta)
\]

\[
\lambda = \psi \cos(\theta) \cos(\phi) - \theta \sin(\phi)
\]

The total moments are the sum of the all moments acting on the mounting blocks.

\[
\begin{bmatrix}
M_x \\
M_y \\
M_z
\end{bmatrix} = \sum_{i=1}^4 \begin{bmatrix}
M_{x,i} \\
M_{y,i} \\
M_{z,i}
\end{bmatrix}
\]

4.1.7.3 Eulerian Newtonian approach

In this section, the methodology to model the powertrain mounting system is described. The classical method Eulerian-Newtonian approach will be presented.

General the Newton laws for the motion of a rigid body are [107]:
\[
\frac{dl}{dt} = F \quad (4.92)
\]
\[
\frac{dh}{dt} = \tau \quad (4.93)
\]
where \( l = [l_x, l_y, l_z] \) is the linear momentum vector of the body and \( h = [h_x, h_y, h_z] \) is the angular momentum vector of the body; \( F = [F_x, F_y, F_z] \) and \( \tau = [\tau_x, \tau_y, \tau_z] \) are the force and the torque acting on the body, respectively.

The equation in (4.92) and (4.93) are valid for the inertial reference system. Once the system accelerating linear or radially the equations should be modified to include the motion of the reference axes. The angular momentum vector has the following equation:
\[
h = l \omega \quad (4.94)
\]
where \( l \) is the inertia matrix (4.95) and \( \omega \) is the angular velocity vector (4.96).

\[
l = \begin{bmatrix}
   l_{xx,cpm} & l_{xy,cpm} & l_{xz,cpm} \\
   l_{yx,cpm} & l_{yy,cpm} & l_{yz,cpm} \\
   l_{zx,cpm} & l_{zy,cpm} & l_{zz,cpm}
\end{bmatrix} \quad (4.95)
\]

where \( l_{xx,cpm}, l_{yy,cpm}, \) and \( l_{zz,cpm} \) are the moments of inertia of the powertrain about the axes \( x, y \) and \( z \) respectively and defined as follow [107]:
\[
l_{xx} = \int_V (y^2 + z^2)dm \quad l_{yy} = \int_V (x^2 + z^2)dm \quad l_{zz} = \int_V (x^2 + y^2)dm
\]
\[
l_{xy}, l_{xz}, \text{and} l_{yz} \quad \text{are the powertrain moment of inertia about the x and the y axes, the x and the z axes, and the y and the z axes. They are defined as [107]:}
\]
\[
l_{xy} = \int_V xydm \quad l_{xz} = \int_V xzdm \quad l_{yz} = \int_V yzdm
\]
\[
\omega = \begin{bmatrix}
   \omega_x \\
   \omega_y \\
   \omega_z
\end{bmatrix} \quad (4.96)
\]

Combine equation (4.93) and (4.94) considers that \( h \) is only related to the inertial reference system.
\[
\frac{dh}{dt} = \frac{d}{dt}(R(h_B)) = \tau_l \quad (4.97)
\]
Where the indices \( l \) and \( B \) refer to the inertial and the local system respectively and \( R \) is the transformation matrix.
However, the transformation matrix \( R \) is not constant therefore; equation (4.97) can be written as follow:
\[
R\dot{h}_B + \dot{R}h_B = \tau_l \quad (4.98)
\]

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After multiplying equation (4.98) on the both sides by the transformation matrix $R^T = R^{-1}$:

$$\hat{h}_b + R^T \hat{h}_b = R^T \tau_1 = \tau_B$$

(4.99)

Expressing equation (4.99) with the components of the matrices, (4.100) is obtained:

$$\begin{bmatrix}
I_{xx,\text{cpm}} & I_{xy,\text{cpm}} & I_{xz,\text{cpm}} \\
I_{yx,\text{cpm}} & I_{yy,\text{cpm}} & I_{yz,\text{cpm}} \\
I_{zx,\text{cpm}} & I_{zy,\text{cpm}} & I_{zz,\text{cpm}}
\end{bmatrix}
\begin{bmatrix}
\dot{\omega}_x \\
\dot{\omega}_y \\
\dot{\omega}_z
\end{bmatrix}
+ R^T
\begin{bmatrix}
I_{xx,\text{cpm}} & I_{xy,\text{cpm}} & I_{xz,\text{cpm}} \\
I_{yx,\text{cpm}} & I_{yy,\text{cpm}} & I_{yz,\text{cpm}} \\
I_{zx,\text{cpm}} & I_{zy,\text{cpm}} & I_{zz,\text{cpm}}
\end{bmatrix}
\begin{bmatrix}
p \\
qu \\
r
\end{bmatrix}
= \begin{bmatrix}
\tau_x \\
\tau_y \\
\tau_z
\end{bmatrix}$$

(4.100)

The matrix $R^T \hat{R}$ to satisfy equation (4.100) is given in (4.101):

$$R^T \hat{R} =
\begin{bmatrix}
0 & -\omega_z & \omega_y \\
\omega_z & 0 & -\omega_x \\
-\omega_y & \omega_x & 0
\end{bmatrix}
= \begin{bmatrix}
0 & -r & q \\
r & 0 & -p \\
-q & p & 0
\end{bmatrix}$$

(4.101)

Equation (4.100) becomes:

$$\begin{bmatrix}
I_{xx,\text{cpm}} & I_{xy,\text{cpm}} & I_{xz,\text{cpm}} \\
I_{yx,\text{cpm}} & I_{yy,\text{cpm}} & I_{yz,\text{cpm}} \\
I_{zx,\text{cpm}} & I_{zy,\text{cpm}} & I_{zz,\text{cpm}}
\end{bmatrix}
\begin{bmatrix}
\dot{\omega}_x \\
\dot{\omega}_y \\
\dot{\omega}_z
\end{bmatrix}
= \begin{bmatrix}
0 & -\omega_z & \omega_y \\
\omega_z & 0 & -\omega_x \\
-\omega_y & \omega_x & 0
\end{bmatrix}
\begin{bmatrix}
I_{xx,\text{cpm}} & I_{xy,\text{cpm}} & I_{xz,\text{cpm}} \\
I_{yx,\text{cpm}} & I_{yy,\text{cpm}} & I_{yz,\text{cpm}} \\
I_{zx,\text{cpm}} & I_{zy,\text{cpm}} & I_{zz,\text{cpm}}
\end{bmatrix}
\begin{bmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{bmatrix}$$

(4.102)

The acceleration in the three axis can obtained in the equations (103) – (105):

$$\dot{\omega}_x = \frac{I_{xy,\text{cpm}} l_{yz,\text{cpm}} c + I_{xy,\text{cpm}} l_{xz,\text{cpm}} b + I_{xz,\text{cpm}} l_{yz,\text{cpm}} b + I_{xz,\text{cpm}} I_{yy,\text{cpm}} c + I_{yz,\text{cpm}} I_{zz,\text{cpm}} a - l_{yz,\text{cpm}}^2 a}{I_{xx,\text{cpm}} I_{yy,\text{cpm}} l_{zz,\text{cpm}} - I_{xx,\text{cpm}} l_{yz,\text{cpm}}^2 - I_{xz,\text{cpm}} l_{yz,\text{cpm}}^2 - 2 I_{yz,\text{cpm}} I_{xy,\text{cpm}} l_{xz,\text{cpm}} - I_{yy,\text{cpm}} l_{xz,\text{cpm}}^2}$$

(4.103)

$$\dot{\omega}_y = \frac{I_{x,\text{cpm}} I_{yx,\text{cpm}} b + I_{x,\text{cpm}} l_{xz,\text{cpm}} c - l_{xz,\text{cpm}}^2 b + I_{yz,\text{cpm}} I_{xz,\text{cpm}} a + l_{yz,\text{cpm}} l_{xz,\text{cpm}} a}{I_{xx,\text{cpm}} I_{yx,\text{cpm}} l_{zz,\text{cpm}} - I_{xx,\text{cpm}} l_{yz,\text{cpm}}^2 - I_{xz,\text{cpm}} l_{yz,\text{cpm}}^2 - 2 I_{yz,\text{cpm}} I_{xy,\text{cpm}} l_{xz,\text{cpm}} - I_{yy,\text{cpm}} l_{xz,\text{cpm}}^2}$$

(4.104)

$$\dot{\omega}_z = \frac{I_{yx,\text{cpm}} l_{xy,\text{cpm}} a - l_{xy,\text{cpm}}^2 c + I_{xx,\text{cpm}} I_{xy,\text{cpm}} b + I_{xx,\text{cpm}} l_{yz,\text{cpm}} c + I_{xy,\text{cpm}} l_{xz,\text{cpm}} b + I_{yy,\text{cpm}} l_{xz,\text{cpm}} a}{I_{xx,\text{cpm}} I_{yy,\text{cpm}} l_{zz,\text{cpm}} - I_{xx,\text{cpm}} l_{yz,\text{cpm}}^2 - I_{xz,\text{cpm}} l_{yz,\text{cpm}}^2 - 2 I_{yz,\text{cpm}} I_{xy,\text{cpm}} l_{xz,\text{cpm}} - I_{yy,\text{cpm}} l_{xz,\text{cpm}}^2}$$

(4.105)

where a, b and c are:

$$a = \left( I_{yy,\text{cpm}} - I_{zz,\text{cpm}} \right) \omega_y \omega_z + I_{xy,\text{cpm}} \omega_x \omega_z - I_{yz,\text{cpm}} \left( \omega_y^2 - \omega_z^2 \right) - I_{xz,\text{cpm}} \omega_x \omega_y + \tau_x$$

(4.106)
\[ b = \left( I_{zz_{cpm}} - I_{xx_{cpm}} \right) \omega_x \omega_z + I_{yz_{cpm}} \omega_x \omega_y - I_{xz_{cpm}} (\omega_z^2 - \omega_y^2) - I_{xy_{cpm}} \omega_y \omega_z + \tau_y \] (4.107)

\[ c = \left( I_{xx_{cpm}} - I_{yy_{cpm}} \right) \omega_x \omega_y + I_{xz_{cpm}} \omega_y \omega_z - I_{xy_{cpm}} (\omega_z^2 - \omega_y^2) - I_{xy_{cpm}} \omega_x \omega_z + \tau_z \] (4.108)

The angular velocity and the acceleration are described by (4.77), (4.103), (4.104) and (4.105). These equations describe the rotational dynamic of the rigid body. To complete the translation dynamic of the rigid body, the projection of the powertrain velocities \( u, v \) and \( w \) in the body axis \( x, y \) and \( z \) to be expressed.

The linear moment of the body in the inertial system is given in equation (4.109):

\[ l = m_{cpm} v_I = m_{cpm} R v_B \] (4.109)

Combining equation (4.109) in (4.92):

\[ \frac{d}{dt} (m_{cpm} R v_B) = F_I \] (4.110)

with a time variable in \( R \):

\[ m_{cpm} \left( R \frac{dv_B}{dt} + \dot{R} v_B \right) = F_I \] (4.111)

where \( F_I \) are the forces acting on the powertrain, in the inertial reference system and \( v_B \) the projection vector of the velocity:

\[ v_B = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \] (4.112)

Following the same methodology applied to equation (4.92):

\[ \frac{dv_B}{dt} = -R^\top \dot{R} v_B + \frac{1}{m_{cpm}} F_B \] (4.113)

where \( F_B \) is the force acting on the powertrain expresses by \( F_B = R^\top F_I \) in the fixed coordinates and \( m_{cpm} \) is the powertrain mass.

The equation in (4.114) describes the translation acceleration. The acceleration can be written in components form in equation (4.115)- (4.117):

\[
\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = - \begin{bmatrix} 0 & -r & a \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} + \frac{1}{m} \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} \] (4.114)

\[ \dot{u} = ru - qw + \frac{1}{m} F_x \] (4.115)

\[ \dot{v} = pw - ru + \frac{1}{m} F_y \] (4.116)

\[ \dot{w} = uq - pv + \frac{1}{m} F_z \] (4.117)
$F_x$, $F_y$, and $F_z$ are the total forces acting on the powertrain which are the sum of the forces from the mounting blocks equations (4.118)-(4.120). The equations show the acceleration terms $rv, ru, qw, pw, pv$ and $uq$ which are presenting the Coriolis acceleration [108].

\[ F_x = \sum_{i=1}^{N} F_{x,i} \]  \hspace{1cm} (4.118)
\[ F_y = \sum_{i=1}^{N} F_{y,i} \]  \hspace{1cm} (4.119)
\[ F_z = \sum_{i=1}^{N} F_{z,i} \]  \hspace{1cm} (4.120)

The torque $\tau_B$ on the powertrain can be given in the three directions in the following equations:

\[ \tau_x = M_x + \sum_{i=1}^{N} F_{z,i} y_{0mbp,i} - \sum_{i=1}^{N} F_{y,i} z_{0mbp,i} \]  \hspace{1cm} (4.121)
\[ \tau_y = M_y + T_{halfl} + T_{halfr} + \sum_{i=1}^{N} F_{x,i} z_{0mbp,i} - \sum_{i=1}^{N} F_{z,i} x_{0mbp,i} \]  \hspace{1cm} (4.122)
\[ \tau_z = M_z + \sum_{i=1}^{N} F_{y,i} x_{0mbp,i} - \sum_{i=1}^{N} F_{x,i} y_{0mbp,i} \]  \hspace{1cm} (4.123)

The complete dynamic equation where given to describe the angular accelerations, linear accelerations, angular orientation and a powertrain velocity.

### 4.1.8 Non-Linear Bushings

The damping elements components on the engine mount are modelled as non-linear elastomer bushings, based on the non-linear model in [109], [110], [111]. The described model in Figure 58 is a modular structure with springs and damping elements. The overall model behaviour is a combination of:

- Non-linear static stiffness characteristics from a table that represents the main spring $k_1$ and described in a look-up table. In the model, the look-up table is described as $f_z$;
- Two dampers $\beta_1$, $\beta_2$ and one spring $k_2$ in parallel to the main non-linear spring $k_1$;
- Amplitude dependent behaviour represented by non-linear friction force characterised at dynamic sinusoidal excitation. This component is considered zero in this model.

The physical components described above depend on the amount of the filler in elastomer matrix of the rubber compound. The term elastomer refers to any rubber parts including synthetic rubber compounds.

The model is implemented in the engine mount block and replaced the consideration of constant damping. The model is integrated with the engine mounting block for the all mountings.

The total force acting in the bushing $F$ in Figure 58 is the sum of $F_1$ and $F_2$:

\[ F = F_1 + F_2 \]  \hspace{1cm} (4.124)
The force $F_2$ acting on the damper $\beta_1$ is given as follow:

$$F_2 = \beta_1 \dot{z}$$  \hspace{1cm} (4.125)

The force $F_2$ is equal the force on the spring $k_2$ and the damper $\beta_2$:

$$F_2 = k_2 (u - z) + \beta_2 (\dot{u} - \dot{z})$$  \hspace{1cm} (4.126)

Comparing equation (4.125) and (4.126):

$$\beta_1 \dot{z} = k_2 (u - z) + \beta_2 (\dot{u} - \dot{z})$$  \hspace{1cm} (4.127)

![Figure 58 - Non-linear model structure for the bushing](image)

Transferring equation (4.127) to the frequency domain, with $s$ being the Laplace operator:

$$\beta_1 s Z(s) = k_2 U(s) - k_2 Z(s) + \beta_2 s U(s) - \beta_2 s Z(s)$$  \hspace{1cm} (4.128)

$$Z(s) = \frac{k_2 U(s) + \beta_2 s U(s)}{(\beta_1 s + \beta_2 s + k_2)}$$  \hspace{1cm} (4.129)

After transforming equation (4.125) in to Laplace form, the force $F_2$ can be expressed as follow:

$$F_2(s) = \beta_1 s \frac{U(s) (k_2 + \beta_2 s)}{(\beta_1 s + \beta_2 s + k_2)}$$  \hspace{1cm} (4.130)

Expressing equation (4.130) in polynomial form:

$$((\beta_1 + \beta_2) F_2(s) s + F_2(s) k_2) = (U(s) \beta_1 s k_2 + U(s) \beta_1 \beta_2 s^2)$$  \hspace{1cm} (4.131)

Transforming the polynomial form to the time domain:

$$((\beta_1 + \beta_2) \ddot{f}_2 + f_2 k_2) = (\ddot{u} \beta_1 k_2 + \ddot{u} \beta_1 \beta_2)$$  \hspace{1cm} (4.132)

Solve equation (4.132) after $\ddot{f}_2$:

$$\ddot{f}_2 = \frac{\ddot{u} \beta_1 k_2 + \ddot{u} \beta_1 \beta_2 - f_2 k_2}{\beta_1 + \beta_2}$$  \hspace{1cm} (4.133)

The equation (4.133) is implemented in the damper block for the all mountings blocks.
4.2 Model Validation

In order to achieve the required confidence in the model, experimental validation is necessary. Therefore, the model was validated against data from a front-wheel-drive test vehicle. The test data for the ICE-driven vehicle has been measured on the front-wheel drive vehicle fitted with the drivetrain comprising an ICE and a five speed automated manual transmission. The test vehicle carried out the tip-in test in conditions of fixed gear with a starting speed ranging from 11.7 km/h for first gear to 45 km/h in fifth gear and an initial throttle position for 100 %. The results during tip-in test for the first and second gear with initial speeds 12 km/h and 19 km/h respectively are overlapped in Figure 59. The figure shows initial numerical oscillation on the acceleration and jerk profiles, which is caused by the initial condition of the simulation model. The depicted results compare the engine torque, the engine speed, the vehicle speed, the vehicle acceleration and the jerk. In general, the figures show agreement between the simulated and measured data when applying the correct amount of engine torque and engine speed, and provoking the correct vehicle speed, acceleration and jerk response. In the Figure 59 a-e, the comparison between the simulation and the measured data is carried out in the first gear by an initial vehicle speed of 12 km/h in the tip-in manoeuvre by 100 % throttle opening. The same test was performed for the second gear with an initial speed of 19 km/h and the results are compared in Figure 59 f-j. The comparison shows agreement between the measured and simulated data for the vehicle acceleration and jerks with a small deviation in both gears. There is some deviation in the vehicle and engine speed between the model and the measurements. Where the initial speed of the engine, in both the model and the experiment, agrees in the simulation for the first and the second gear. The deviation in the speed curves it is due to the torque deviation and the acceleration deviation. The deviation in the parameter in the second gear test is due to the accuracy of the engine torque map. By limiting the torque demand between 15% and 97%, the engine torque of the model and the experimental data will agree with each other.
Figure 59 - Model and measured data for tip-in manoeuver with final torque demand 100% for:
   a) Gear 1 with initial speed 11 km/h
   b) Gear 2 with initial speed 18.9 km/h

4.3 Summary

The non-linear model of the driveline is introduced in this chapter. The complex model includes the main sub-systems of the driveline such as the engine, clutch damper, differential, half-shafts, vehicle and tyres, engine mounting system and non-linear bushings. The complex model is formulated mathematically to provide relevant solution for the dynamic analysis of vehicle behaviour. The aim of such a complex model is to assist in developing the anti-jerk controller in order to improve the drivability. The simulation model is developed by integrating each of the sub-systems in traceable blocks where the mathematical equations are implemented in Simulink.
blocks. The components parameters are set in MATLAB files. The input to the model is the initial vehicle speed and the gear ratio. Typical transient simulations were performed and compared to measured data. The longitudinal acceleration profile and jerk of the simulation model were compared with the measured data and discussed.
Chapter 5

Linear Passive Vehicle Modelling

5.1 Linearised Model

To study the frequency behaviour of the passive and active vehicle, a linear model, which is based on the state-space formulation, is developed. The first step to develop the linear model is to modify the non-linearity and create linear equations from the non-linear model in order to find the driveline linear dynamic model. In this section, the linearization method based on Tayler series presented in equation (5.1) will be applied to the non-linear equations of the tyre vehicle, which are developed in chapter 4, and thus the linear equations will be developed for the complete driveline dynamics.

\[ f(x) = f(x_0) + \frac{df(x)}{dx} \bigg|_{x_0} \frac{(x-x_0)}{1!} + \frac{d^2f(x)}{dx^2} \bigg|_{x_0} \frac{(x-x_0)^2}{2!} + \cdots \]  

(5.1)

Equation (5.2) presents the vehicle dynamics equation for the linear model and includes the non-linear torque contribution relates to the aerodynamic drag and the tyre rolling resistance which is embedded in the tyre torque equations. These two terms should be linearized in order to generate the linear model.

\[ T_{tyr} - T_{roadgradien} - T_{aerdrag} - T_{roll} = J_v \dot{\theta}_v \]  

(5.2)

The term, which describes the inclination resistance force in equation (4.40), is a linear contribution and therefore does not need any modification. Since the concentration on the non-linear model in this thesis, the linear model is kept simple and will fit in the aimed purpose.

The torque due to the aerodynamic drag term in equation (4.38) can be written as follow:

\[ T_{aerdrag} = \frac{1}{2} C_d \rho S \dot{\theta}_v^2 R_w^3 \]  

(5.3)
The state variable in the air drag resistance is the equivalent angular vehicle speed $\dot{\theta}_v$. After developing this term at the constant speed $\theta_{v0}$ using Taylor series as expressed in equation (5.1), the linear drag resistance torque is expressed in equation (5.4):

$$T_{aerdrag} = C_a \rho_a S \dot{\theta}_{v0} R_w^3 \dot{\theta}_v - \frac{1}{2} C_a \rho_a S \dot{\theta}_{v0}^2 R_w^3$$

(5.4)

where $\dot{\theta}_{v0}$ is the initial angular vehicle speed.

The second term needs to be linearized is the rolling resistance which is embedded in the torque equations. In the linear model, the rolling resistance torque is expressed in the following equation (5.5) and will be implemented in the vehicle model.

$$T_{rollfR} = F_n R_w (B + CR_w^2 \dot{\theta}_v^2)$$

(5.5)

The linearized torque due to the rolling resistance is expressed as follow:

$$T_{roll} = R_w F_n [B + 2CR_w^2 \theta_{v0} \dot{\theta}_v - CR_w^2 \dot{\theta}_{v0}^2]$$

(5.6)

5.1.1 Engine and Clutch

The engine is modelled in the same way as in the non-linear by the engine map. The torque balance equation of the engine shaft through the clutch is:

$$T_e - T_c = J_e \dot{\theta}_e$$

(5.7)

In the linear clutch model, the clutch torque equation can be expressed as follows:

$$T_c = k_d (\theta_e - \theta_p) + \beta_d (\dot{\theta}_e - \dot{\theta}_p)$$

(5.8)

With the clutch stiffness $k_d$ and the internal damping coefficient $\beta_d$.

Using the relation between the primary shaft displacement and differential displacement as described in equations (5.9) and (5.10), it is:

$$\theta_p = \tau_{diff} \tau_g \dot{\theta}_{diff}$$

(5.9)

$$\dot{\theta}_p = \tau_{diff} \tau_g \dot{\theta}_{diff}$$

(5.10)

By incorporating (5.8) in (5.7) and expressing the angular primary shafts displacement and speed by equations (5.9) and (5.10), the engine angular acceleration $\ddot{\theta}_e$ referred to the differential can be obtained:

$$\ddot{\theta}_e = \frac{T_e - k_d \theta_e - \beta_d \dot{\theta}_e + k_d \tau_{diff} \tau_g \theta_{diff} + \beta_d \tau_{diff} \tau_g \dot{\theta}_{diff}}{J_e}$$

(5.11)
5.1.2 Gearbox

The torque balance on the primary shaft is calculated as follow:

\[ (J_p + J_{c1})\ddot{\theta}_p = k_d (\theta_e - \theta_p) + \beta_d (\dot{\theta}_e - \dot{\theta}_p) - T_p \]  
(5.12)

The torque balance on the secondary shaft is calculated as follow:

\[ J_s \ddot{\theta}_s = \eta_g \tau_g T_p - T_{diff} \]  
(5.13)

After obtaining \( T_p \) from equation (5.12) and satisfying equation (5.13) the equation of the secondary shaft is expressed with the differential torque \( T_{diff} \) in equation (5.14):

\[ J_s \ddot{\theta}_s = \eta_g \tau_g [k_d (\theta_e - \theta_p) + \beta_d (\dot{\theta}_e - \dot{\theta}_p) - (J_p + J_{c1})\dot{\theta}_p] - T_{diff} \]  
(5.14)

The balance torque on the differential can be expressed as follow:

\[ (J_{diff} + \frac{1}{2} J_{half}) \ddot{\theta}_{diff} = \mu_{diff} \tau_{diff} T_{diff} - T_{half} \]  
(5.15)

where \( \ddot{\theta}_{diff} \) is the angular acceleration of the differential, \( T_{half} \) is the half-shaft torque and \( J_{half} \) is the total mass moment of inertia of half-shaft at the front axle and \( J_{diff} \) is the mass moment of inertial of the differential. Assuming that the wheel speed is the same for each wheel in the front axle, then the half-shaft can be modelled in the following equation

\[ T_{half} = k_{hs} (\theta_{diff} - \theta_w) + \beta_{hs} (\dot{\theta}_{diff} - \dot{\theta}_w) \]  
(5.16)

\( k_{hs} \) is the half-shaft stiffness and \( \beta_{hs} \) is the half-shaft damping coefficient.

By combining equations (5.16) in equation (5.15) the following equation can be obtained:

\[ (J_{diff} + \frac{1}{2} J_{half}) \ddot{\theta}_{diff} = \eta_{diff} \tau_{diff} T_{diff} - k_{hs} (\theta_{diff} - \theta_w) - \beta_{hs} (\dot{\theta}_{diff} - \dot{\theta}_w) \]  
(5.17)

Combining (5.14) in (5.17) the following equation can be obtained:

\[ (J_{diff} + \frac{1}{2} J_{half}) \ddot{\theta}_{diff} = \eta_{diff} \tau_{diff} T_{diff} \eta_g \tau_g k_d \frac{[k_d (\theta_e - \theta_p) + \beta_d (\dot{\theta}_e - \dot{\theta}_p) - (J_p + J_{c1})\dot{\theta}_p]}{-J_s \ddot{\theta}_s - k_{hs} (\theta_{diff} - \theta_w) - \beta_{hs} (\dot{\theta}_{diff} - \dot{\theta}_w)} - J_s \ddot{\theta}_s - k_{hs} (\theta_{diff} - \theta_w) - \beta_{hs} (\dot{\theta}_{diff} - \dot{\theta}_w) \]  
(5.18)

\[ \ddot{\theta}_p = \tau_{diff} \tau_g \ddot{\theta}_{diff} \]  
(5.19)

\[ \ddot{\theta}_s = \tau_{diff} \ddot{\theta}_{diff} \]  
(5.20)

Expressing the angular speed, acceleration and displacement of the primary in equations (5.9) and (5.10) and equations (5.19) and (5.20) in (5.18), the equation of the angular differential equation can be obtained:

\[ \ddot{\theta}_{diff} = \frac{\eta_{diff} \tau_{diff} \eta_g \tau_g k_d}{(J_{diff} + \frac{1}{2} J_{half}) + \eta_{diff} \mu_{diff} \tau_{diff} \tau_{g}^2 (J_p + J_{c1}) + J_s \eta_{diff} \tau_{diff}^2} \theta_e \]
\[ -\frac{\eta_{diff}^2 \tau_{diff}^2 \eta_{g}^2 \eta_{a}^2 + k_{hs}}{(I_{diff} + \frac{1}{2} I_{half}) + \eta_{diff} \eta_{g}^2 \tau_{diff}^2 \beta_{a}} \theta_{diff} + \]

\[ -\frac{\eta_{diff}^2 \tau_{diff}^2 \eta_{g}^2 \beta_a}{(I_{diff} + \frac{1}{2} I_{half}) + \eta_{diff} \eta_{g}^2 \tau_{diff}^2 \beta_{a}} \dot{\theta}_{e} + \]

\[ -\frac{k_{hs}}{(I_{diff} + \frac{1}{2} I_{half}) + \eta_{diff} \eta_{g}^2 \tau_{diff}^2 \beta_{a}} \dot{\theta}_{w} + \]

\[ -\frac{\beta_{hs}}{(I_{diff} + \frac{1}{2} I_{half}) + \eta_{diff} \eta_{g}^2 \tau_{diff}^2 \beta_{a}} \dot{\theta}_{w} \]

(5.21)

5.1.3 Wheel and Tyre

The tyre represents the main damping element within an automotive drivetrain (apart from the clutch damper) with the consideration that the other torsional flexible elements of the transmission system are metal components with a very low internal damping. The typical behaviour of a tyre subjected to a slip ratio and modelled through Pacejka Magic Formula is presented in Figure 60 which shows the longitudinal force \( F_x \) as a function of the slip ratio \( SR \) [96], for different vertical loads \( F_N \). The rate of the linear part of the graph which is measured around the origin is the longitudinal slip stiffness \( C_s \), and the rate of the diagram changes as a function of the slip ratio. This is the first non-linearity of the tyre. Moreover, the second non-linearity is an increase of vertical load provokes a less than proportional increase of the force for an assigned slip ratio. Finally, the third non-linearity is the variation of the friction coefficient between the tyre and the road surface provokes a variation of the overall shape of the diagram (not shown). All these non-linearities are neglected by the tyre model which is implemented within the linear vehicle model.

In a very first approximation, the load transfer is neglected. Therefore, the modification of the rate of the linear section of the characteristic is prevented. In addition, the non-linear characteristics for high SR values are neglected. Thus, it is possible to write the longitudinal force as follow:

\[ F_x = F_{x0} + C_s (SR - SR_0) \]  

(5.22)

The slip ratio \( SR \) is given in formula (5.23) as follow:

\[ SR = \left( \frac{\dot{\theta}_w - \dot{\theta}_a}{\dot{\theta}_w} \right) \]  

(5.23)
After satisfying equation (5.23) in (5.22) and applying the Taylor series development method for a function with two variables, the equivalent angular vehicle speed $\hat{\theta}_v$ and the angular wheel speed $\hat{\theta}_w$, the linear longitudinal force $F_x$ can be obtained in the following equations:

$$F_x \equiv F_{X0} + C_s \left( \frac{\theta_{wo} - \theta_{w_0}}{\theta_{w_0}} \right) + \alpha_f \frac{\partial f}{\partial \theta_w} \theta_{wo} \hat{\theta}_w - \frac{\partial f}{\partial \theta_v} \theta_{v_0} \left( \hat{\theta}_v - \hat{\theta}_{v_0} \right) = F_{X0} + C_s \left( \frac{\theta_{wo} - \theta_{w_0}}{\theta_{w_0}} \right) + \frac{c_s}{\theta_{w_0}} \left( \hat{\theta}_w - \hat{\theta}_{w_0} \right) - \frac{c_s}{\theta_{w_0}} \left( \hat{\theta}_v - \hat{\theta}_{v_0} \right)$$

(5.24)

Therefore, the final linear longitudinal force $F_x$ is given in equation (5.25):

$$F_x = F_{X0} + C_s SR_0 + \frac{c_s}{\theta_{w_0}} \left( \frac{\theta_{wo} \theta_{v_0}}{\theta_{w_0}} - \hat{\theta}_v \right)$$

(5.25)

The torque on the wheel is given in the following equation (5.26):

$$T_{Tyr} = R_w F_{X0} + R_w C_s SR_0 + R_w \frac{c_s}{\theta_{w_0}} \left( \frac{\theta_{wo} \theta_{v_0}}{\theta_{w_0}} - \hat{\theta}_v \right)$$

(5.26)

By introducing the equivalent torsion damping coefficient $\beta_t$:

$$\beta_t = R_w \frac{c_s}{\theta_{w_0}}$$

(5.27)

The tyre, torque within a linear vehicle model, can be represented as follow:

$$T_{Tyr} = R_w F_{X0} + R_w C_s SR_0 + \beta_t \left( \frac{\theta_{wo} \theta_{v_0}}{\theta_{w_0}} - \hat{\theta}_v \right)$$

(5.28)

Consequently, the damping coefficient $\beta_t$ is inversely proportional to vehicle longitudinal speed and the results of the frequency response analysis are related to the longitudinal speed of the vehicle. Also to the vertical load between the tyres and the road surface (since $C_s$ depends on the vertical load) represents another tyre non-linearity. In the non-linear model, Pacejka Magic Formula is adopted for the computation of tyre longitudinal force as a function of slip ratio and vertical load. In the linearized model, a constant value for $\beta_t$ (for a given vehicle speed) is adopted, given the static load conditions of the vehicle and the predicted speed range during the maneuver.

The torque balance on the equivalent driven wheel of the vehicle is given by equation (5.29),

$$(J_w + 0.5 J_{hatf}) \ddot{\theta}_w = \beta_{hs} \dot{\theta}_{diff} - (\beta_{hs} + \beta_t) \dot{\theta}_w + \beta_t \dot{\theta}_v + k_{hs} \theta_{diff} - k_{hs} \theta_w$$

(5.29)

$$\ddot{\theta}_w = \frac{\beta_{hs}}{2J_w + 0.5J_{hatf}} \dot{\theta}_{diff} - \frac{(\beta_{hs} + \beta_t)}{2J_w + 0.5J_{hatf}} \dot{\theta}_w + \frac{\beta_t}{J_w + 0.5J_{hatf}} \dot{\theta}_v + \frac{k_{hs}}{2J_w + 0.5J_{hatf}} \theta_{diff} - \frac{k_{hs}}{J_w + 0.5J_{hatf}} \theta_w$$

(5.30)
Where $J_w$ is the wheel (or twin wheel) moment of inertia (on one side of the vehicle). For simplicity, in the linear model, the rolling resistance contributions of the tyres have been included in the overall vehicle equation, and not in the driven wheel equations. The model also neglects the relaxation length of the tyre.

5.1.4 Linear Vehicle

The vehicle equation in (5.2) considered the equivalent inertia $J_v$, subjected to the total tyre torque $T_{tyr}$, the torque due to the road gradient $T_{road \: gradient}$, the torque due to the total tyre rolling resistance $T_{roll}$, and the aerodynamic drag torque $T_{aerodrag}$. Each of these torques is characterized by the linearized equations in section (5.1). Therefore, the linear vehicle equation can be expressed as follow:

\[
\left( \beta_t \dot{\theta}_w - \beta_t \dot{\theta}_v \right) - R_w M_v g \sin(\alpha) - \left[ C_d \rho a A \dot{\theta}_{v0} R_w^3 \dot{\theta}_v - \frac{1}{2} C_d \rho a A \dot{\theta}_{v0}^2 R_w^3 \right] - R_w M_v g \cos(\alpha) \left[ B + 2 CR_w^2 \dot{\theta}_{v0} \dot{\theta}_v - CR_w^2 \dot{\theta}_{v0}^2 \right] = J_v \dot{\theta}_v \] (5.31)

The vehicle inertia $J_v$ can be expressed as follow:

\[
J_v = 2J_w + R_w^2 M_v \] (5.32)

\[
\dot{\theta}_v = \frac{\beta_t}{(2J_w + R_w^2 M_v)} \dot{\theta}_v - \frac{\beta_t + C_d \rho a A \dot{\theta}_{v0} R_w^3 + 2 C M_v \cos(\alpha) R_w^2 \dot{\theta}_{v0}}{(2J_w + R_w^2 M_v)} \dot{\theta}_v + \frac{1}{2} \frac{C_d \rho a A \dot{\theta}_{v0}^2 R_w^3 - R_w B M_v g \cos(\alpha) + M_v C g \cos(\alpha) R_w^2 \dot{\theta}_{v0} - R_w M_v g \sin(\alpha)}{(2J_w + R_w^2 M_v)} \] (5.33)
Where \( M_v \) is the vehicle mass and the term \( 2J_w \) presents the moment of inertia of the two undriven wheels.

### 5.2 State-Space Formulation

The derived equations in section 5.1 are organized in the state-space formulation, according to equations (5.34) and (5.35):

\[
\dot{Z} = [A][Z] + [B][U] \quad (5.34)
\]
\[
Y = [C][Z] + [D][U] \quad (5.35)
\]

where \([Z]\) is the state-space vector, \([A]\) is the state matrix, \([U]\) is the input vector in this case will be the engine torque, \([B]\) is the input matrix, \([C]\) is the output matrix, \([D]\) is the feedthrough matrix and \([Y]\) is the output vector, which includes all the plotted parameters.

The vectors and the matrices are attached in Appendix C.

The natural frequencies of the system are the eigenvalues \( \lambda \) of the state matrix \([A]\). In formulas:

\[
\text{det}([A] - \lambda[I]) = 0 \quad (5.36)
\]

To obtain solutions, the coefficients matrix of the determinant must be vanish. An eigenproblem of order \( 2n \) is thus obtained by using the homogenous solution in equation (5.37) of the second order differential equation.

\[
a_x(t) = a_{x0}e^{\lambda t} \quad (5.37)
\]

The eigenvalues \( \lambda \) yield the frequencies of oscillation and the decay rates, and the eigenvectors yield the complex modal shapes \( a_{x0} \). The system has \( n \) pairs of complex conjugate solutions in the form of:

\[
\lambda_j = -\sigma \pm j\omega_d \quad (5.38)
\]

Each degree of freedom oscillates with the damped frequency \( \omega_d \) and the decay rates equal to \( e^{-\sigma t} \). Therefore the time dependent solution for \( a_x(t) \).

\[
a_x(t) = A_{X0}e^{\xi(-\sigma \pm j\omega_d)} \quad (5.39)
\]

Equation (5.39) can be written for the underdamped case using the geometrical function as follow [120]:

\[
a_x(t) = A_{X0} e^{-\sigma t} \cos(\omega dt - \phi) \quad (5.40)
\]
Figure 61 - Damping in the second order dynamic

Where the damping function is defined in the following equation:

\[ A_{X0} = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{\zeta \omega_n} \]  \hspace{1cm} (5.41)

The natural oscillation of the undamped system \( \omega_n \) can be obtained from the damped frequency \( \omega_d \) using equation (5.42) [120].

\[ \omega_n = \frac{\omega_d \sqrt{1-\zeta^2}}{\sqrt{1-\zeta^2}} \]  \hspace{1cm} (5.42)

Where \( \zeta \) is the damping ratio and have the value in the range \( 0 < \zeta < 1 \) [121].

Considering Figure 61 the damping factor can be obtained from averaging the differences \( d_1, d_2, d_3 \) between the damping function and the acceleration response for each period \( T \) [122].

Since the period of the damping response is not constant, the damping frequency is averaged for three maxima [120].

The damping ratio can be calculated from the percentage overshoot \( \%OS \) as follow:

\[ \%OS = \frac{a_{x_{\text{max}}}-a_{x\text{SS}}}{a_{x\text{SS}}} \cdot 100 \]  \hspace{1cm} (5.43)

Where \( a_{x_{\text{max}}} \) the maximal value of the acceleration, and \( a_{x\text{SS}} \) is the acceleration value after achieving the steady-state.

By a known \( \%OS \) the damping ratio \( \zeta \) can be calculated from equation (5.44) [120]:

\[ \%OS = 100 \cdot e^{-\left(\frac{\zeta \pi}{\sqrt{1-\zeta^2}}\right)} \]  \hspace{1cm} (5.44)

Therefore the damping ratio can be calculated as follow:
\[ \zeta = \frac{\ln(\text{\%OS})}{\sqrt{\ln^2(\text{\%OS})+\pi^2}} \]  

(5.45)

In addition to the overshoot and damping ratio, the rise time is a parameter, which is used to evaluate the drivability as mentioned in Chapter 2. The rise time is a measure for the vehicle reaction. The rise time of the response is calculated from the time difference between the two times where the response achieves 90\% and 10\% of the steady-state value. In Figure 62 the calculation procedure is explained for an expanded section from the plotted acceleration.

5.3 Linear Model Validation and Results

The state-space formulation of the linear model has been implemented in Matlab, and Simulink interface is created for running the simulations in the time domain.

Similar to the non-linear model, the linear model was also validated against the same experimental data of the front wheel drive vehicle in the tip-in test. The test vehicle carried out the tip-in test in conditions of fixed gear with a starting speed ranging from 12 km/h for first gear to 45 km/h in fifth gear and an initial throttle position for 100 \%. The results during tip-in test for the first and second gear with initial speeds 12 km/h and 19 km/h respectively are overlapped in Figure 63. The depicted results compare the engine torque, the engine speed, the vehicle speed, the vehicle acceleration, and the jerk. In general, the figures show agreement between the simulated and measured data with some deviation in the results for the engine
speed and the vehicle speed due to the torque deviation. The results in the first gear showing better agreement for the vehicle and engine speed. The acceleration and the jerk show higher oscillation due to the inaccuracy in the coefficients of the tyre rolling resistance and air drag. In addition, the linearized components provide some failure in the values. The deviation becomes higher in the second gear, in addition to the previous reasons, the used engine map in the simulation model is not enough accurate to match the engine characteristics of the test vehicle engine.

The linear model is compared against the non-linear model and plotted in Figure 63 to Figure 67. The validation is performed to the different gears with the different initial vehicles for the tip-in manoeuvre. It is clear the agreement of the torque for the all the gears due to the usage of the same engine map in both models the linear and the non-linear.

The acceleration of linear model damped to the steady-stead quicker than the non-linear model, where the acceleration of the linear mode is higher than the non-linear model. The jerk values in the first and second gear ca 42% lower than the non-linear model. For the third gear the difference between the jerk become ca 27%. Meanwhile, the fourth gear the jerk almost the same. The acceleration for both gears 3rd and 4th increased and caused deviation in the vehicle and engine speeds. Generally, the two models have the same behaviour for all the vehicle parameters and agree with each other in terms of the torque, engine speed, and vehicle speed and acceleration profiles. The linear model in the next chapter will be used to analyse the frequency response of the anti-jerk controller.
Figure 63 - Linear model and measured data for during tip-in manoeuver with final torque demand 100% for:
   a) Gear 1 with initial speed 11.7 km/h,
   b) Gear 2 with initial speed 18.9 km/h
Figure 64 - Linear and the non-linear model during tip-in manoeuver with final torque demand 100% for 1st gear with initial speed 11.7 km/h
Figure 65 - Linear and the non-linear model during tip-in manoeuver with final torque demand 100% for 2nd gear with initial vehicle speed 18.9 km/h
Figure 66 - Linear and the Non-linear model during tip-in manoeuver with final torque demand 100% for 3rd gear with initial vehicle speed 29.3 km/h
Figure 67 - Linear and the Non-linear model during tip-in manoeuver with final torque demand 100% for 4th gear with initial vehicle speed 37.9 \( km/h \)

5.4 Summary

The linear model of the driveline is introduced in this chapter. The non-linearities such as the aerodynamic drag force, rolling resistance and tyre longitudinal force are linearized using Taylor series methods. The generated linear equations were integrated in the model equations.
and built in state-space formulation. Furthermore, the calculation method of the overshoot and rise time is explained.

Several transient simulations for different gear ratios and initial speed were performed. The acceleration and jerk profiles for the linear and non-linear model were compared with each other. In addition, the simulations of the linear model are validated against the measured vehicle data.
Chapter 6

Anti-Jerk Controller for Passenger Vehicle

The simulation results in Chapter 4 and 5 demonstrate the drivability issue, where the oscillation on the driveline and the jerking of the vehicle needs to be improved. To improve the drivability and thus the comfort of the vehicle, damping of the oscillations and reduction of the jerk is required. Therefore, the anti-jerk controller is employed to improve the longitudinal acceleration profile. The control scheme, which will be used in the anti-jerk controller, is the PID structure. PID controller is remained the most popular strategy used in the industry because PID controllers have simple structures and can provide excellent performance in case of good tuning [62].

In this chapter, the design of the anti-jerk controller for both the non-linear and the linear model is introduced. The description of the PID anti-jerk controller, which is implemented in the active vehicle model, will be performed.

Since finding the optimal values of the PID-parameters is an engineering challenging task, an auto-tune methodology will be introduced and discussed.

In terms of studying and analysing the stability of the controller in the frequency domain, the dynamics model of the active vehicle are implemented in state-space.

6.1 Control Strategy and Design

In the previous chapter the undesired vehicle dynamic response following the pedal change position during the tip-in manoeuvre of the passive vehicle was discussed. The model-based control strategy is designed to improve the vehicle response during the torque demand. The
A basic controller based on correcting the engine torque demand by using the engine as an actuator to reduce the oscillation on the driveline.

The proposed anti-jerk control system schematic is shown in Figure 68. The input of the controller is the error of the engine speed. Which is produced due to the difference between the wheel speed and the actual engine speed. When the error is detected by the controller, it modifies the torque engine demand until the error reduced. In this way, the oscillation of the driveline is damped.

Since the error is calculated from the difference between the engine speed and the wheel speed, it is easy to be implemented due to the existence of the engine speed and wheel speed sensors. The value of the error speed \( \omega_{err} \) can be obtained from equation (6.1):

\[
\omega_{err} = \omega_{wf} - \omega_e
\]  

(6.1)

Where \( \omega_{wf} \) the average speed of the left and right front wheels, and \( \omega_e \) is the engine speed.

\[
\omega_{wf} = \frac{1}{2} (\tau_g \hat{\omega}_{diff} \omega_{wfr} + \tau_g \hat{\omega}_{diff} \omega_{wfl})
\]  

(6.2)

The correction torque \( T_{corr} \) can be expressed as a function of the error speed \( \omega_{err} \) where the controller is a basic PID-controller with the three terms Proportional, Integral and Derivative. In fact, the PID-controller is still the most popular type of control in use today because of its simple structure and very well understood principles. The PID-controller is sensitive to the system parameter such as inertia, stiffness and the selected gain [73]. Therefore, tuning of this parameter plays a major role in the stability and performance. The basic PID-controller is described in Figure 69. The reference value \( \omega_{err} \) undergoes three operations through the three terms. Equation (6.3) describes the processing of the PID-controller:

\[
T_{corr} = K_p \omega_{err} + K_D \dot{\omega}_{err} + K_I \int \omega_{err}
\]  

(6.3)
Where this correction torque is added to the engine torque in order to build the corrected torque demand:

$$T_{e,demand} = T_e + T_{corr}$$  \hspace{1cm} (6.4)

Satisfying equation (6.5) with (6.4) and expressing the speed reference gives the following equation:

$$T_{e,demand} = T_e + (K_p(\dot{\theta}_w - \dot{\theta}_e) + K_I \int (\dot{\theta}_w - \dot{\theta}_e) + \frac{d}{dt} K_D (\dot{\theta}_w - \dot{\theta}_e))$$ \hspace{1cm} (6.5)

$$T_{e,demand} = T_e + (K_p(\tau_g \tau_{diff} \dot{\theta}_{wf} - \dot{\theta}_e) + K_I (\tau_g \tau_{diff} \theta_{wf} - \theta_e) + \frac{d}{dt} K_D (\tau_g \tau_{diff} \dot{\theta}_{wf} - \dot{\theta}_e))$$ \hspace{1cm} (6.6)

Equation (6.3) can be expressed in the frequency domain and rewritten as Laplace transformed equation as follow:
\[ T_{\text{corr}} = K_p (\tau_g \tau_{\text{diff}} \dot{\theta}_w - \dot{\theta}_e) (K_p + K_D s + K_i s^2) \]  
(6.7)

The model in equation (6.7) is implemented in the non-linear model. For the linear model, equation (6.6) describes engine torque demand and will be used in equation (6.4) to correct the engine demand torque. The controller was implemented in the non-linear model and tested.

### 6.2 Linear Model of Active Vehicle

For active vehicle including the controller was implemented in the linear model after modifying the equation of the engine.

The engine equation (6.6) can be formulated as follow:

\[ T_{e, \text{demand}} = T_e + ((K_p \tau_g \tau_{\text{diff}} \dot{\theta}_w - K_p \dot{\theta}_e) + (K_I \tau_g \tau_{\text{diff}} \dot{\theta}_w - K_I \dot{\theta}_e) + (K_D \tau_g \tau_{\text{diff}} \dot{\theta}_w - K_D \dot{\theta}_e)) \]  
(6.8)

After applying equation (5.7), (5.8), (5.9) and (5.10) in (6.8) the engine equation can be achieved:

\[ \dot{\theta}_e = \frac{T_e}{(J_1 + K_d)} - \frac{(K_p + K_d)}{(J_1 + K_d)} \dot{\theta}_e + \frac{K_d \tau_g \tau_{\text{diff}}}{(J_1 + K_d)} \dot{\theta}_{\text{diff}} + \frac{\tau_g \tau_{\text{diff}} (K_p + K_d)}{(J_1 + K_d)} \dot{\theta}_{\text{diff}} + \frac{\tau_g \tau_{\text{diff}} (K_d + K_i)}{(J_1 + K_d)} \theta_{\text{diff}} \]  
(6.9)

After applying the differential equation (5.21) in (6.9) the final dynamic equation for the engine including the control parameter can be obtained:

\[ \ddot{\theta}_e = \frac{T_e}{(J_1 + K_d)} + \frac{K_d \tau_g \tau_{\text{diff}}}{(J_1 + K_d)} \frac{\eta_{\text{diff}} \tau_{\text{diff}} \eta_{\text{diff}} \tau_{\text{diff}} \beta_a}{(J_1 + K_d)} \dot{\theta}_e - \frac{\tau_{\text{diff}} (K_p + K_d)}{(J_1 + K_d)} \dot{\theta}_{\text{diff}} + \frac{K_d \tau_{\text{diff}} k_d}{(J_1 + K_d)} \dot{\theta}_w + \frac{K_d \tau_{\text{diff}}}{(J_1 + K_d)} \frac{\eta_{\text{diff}} \tau_{\text{diff}} \eta_{\text{diff}} \tau_{\text{diff}} \beta_a k_d}{(J_1 + K_d)} \dot{\theta}_w + \frac{K_d \tau_{\text{diff}} k_d k_h}{(J_1 + K_d)} \dot{\theta}_{\text{diff}} \]  
(6.10)
Equation (6.10) changes the first row in [A] matrix in the state-space formulation which is achieved in section (5.2). The rest of the state formulating remained unmodified. The modified coefficients in [A] matrix including the PID terms are given in Appendix C.
The controller was tested for different gear and the results are compared in the non-linear passive vehicle, linear vehicle and passive measurements data.

6.3 Linear System Frequency Response

6.3.1 Closed Loop Analysis

Based on the simple linear models, which have been discussed in chapter 5, the stability analysis of the proposed control system in frequency domain is developed. Bode plot is useful in order to achieve this aim. It is based on determining the gain and phase margin of the open loop and closed loop system transfer function. By using Matlab the gain margin, phase margin, the gain and phase cross over frequency can be determined.

![Block diagram of state-space equations](image)

Figure 70 - Block diagram of state-space equations

To simplify the physical system of the case vehicle it is useful to consider the transfer function where the input, output and its system are considered as separate entities. It is noted that the state-space equation can be converted to the transfer function. The dynamic models of the passive linear vehicle is represented in Figure 70 by the state-space equations:

\[ \dot{x} = Ax + Bu \]  
\[ y = Cx + Du \]
Where \( u(t) \) the input vector is represented by the engine torque and \( y(t) \) is the output vector. The transfer function can be expressed with regards to any variable in the output vector. The equations in (6.11) and (6.12) can be transferred to the Laplace form and expressed as a function of the frequency term:

\[
SX(s) = AX(s) + BU(s) \tag{6.13}
\]

\[
Y(s) = CX(s) + DU(s) \tag{6.14}
\]

The state vector can be expressed as follow:

\[
X(s) = (SI - A)^{-1}BU(s) \tag{6.15}
\]

After replacing the state vector in equation (6.14) the transfer function \( P(s) \) can be obtained:

\[
P(s) = \frac{Y(s)}{U(s)} = C(SI - A)^{-1}B + D \tag{6.16}
\]

The transfer function \( P(s) \) determines the vehicle dynamic and identified as the plant in the control system.

![Bode diagram](image.png)

Figure 71 - Bode diagram of the vehicle acceleration for the tip-in manoeuvre by 100% torque demand with the gain configuration: \( K_p = 4 \text{ Nm/} \text{rad}, K_D = 0.1 \text{ Nm}^2/\text{rad}, K_i = 1 \text{ Nm/} \text{rad} \)
The transfer function of the linear model enables the analysis of the frequency domain and the plot of the bode diagram of any variable from the output vector $y$.

The dimension of the transfer function $P(s)$ depends on the number of the states $n$ and the number of the input $i$. Thus, the transfer function has $ni$ elements and every input has transfer function. For instance to obtain the frequency response of the acceleration, the Bode diagram of vehicle acceleration for different gears are plotted in Figure 71 for the linear passive and active vehicle. The results present the tip-in manoeuvre for the different gears by 100% torque demand. The resonance frequency is damped by increasing the gear which means reducing the gear ratio.

6.3.2 Linear Active Vehicle

By identifying the input, output, the state variables and component matrices $A$, $B$, $C$, $D$ which correspond to the state-space equation, the plant of the vehicle dynamic control system can be determined in a transfer function.

In Figure 72, the open loop transfer function (OLTF) considers the output $Y(s)$ and the error signal $E(s)$ which can be expressed in equation (6.17):

$$\text{OLTF} = G(s) = \frac{Y(s)}{E(s)}$$  \hspace{1cm} (6.17)

The transfer function $G(s)$ consists of the difference between the transfer functions of the engine speed $P_E$ and the wheel speed $P_W$ with considering the gear ratios $\tau_g\tau_f$ as follow:

$$P(s) = \tau_g\tau_fP_W - P_E$$  \hspace{1cm} (6.18)

Considering the input to the plant is the torque demand $T_e$ and the output from the plant the speed difference as expressed in equation 6.18 with a zero reference $R(s)$, the final equations for the open- and closed- loop are obtained:

$$\text{OLTF} = C(s)P(s)$$  \hspace{1cm} (6.19)

The closed loop transfer function (CLTF), which is based on the controller schematic given in Figure 68, can be expressed in relation to the input $C(s)$ and the output $Y(s)$ as follow:
\[ \text{CLTF} = \frac{H(s)}{1 + H(s)} \]  \hspace{1cm} (6.20)

After satisfying \( H(s) \) in (6.20), the final equation for the closed loop transfer function can be obtained:

\[ \text{CLTF} = \frac{P(s)C(s)}{1 + P(s)C(s)} \]  \hspace{1cm} (6.21)

The transfer function \( P(s) \) of the plant is obtained by adding the speed difference in the output vector \( U \) of the state-space equations and obtained the transfer function after transforming the state-space into transfer functions.

The transfer function \( C(s) \) is given by the transfer function of the PID-controller:

\[ C(s) = K_p + \frac{K_i}{s} + K_ds^2 + \frac{K_p + K_i}{s} \]  \hspace{1cm} (6.22)

The ranges for the PID-gains are varied and depends on the rise time and overshoot targets as given in section 6.5. The transfer function of the plant is generated from the state-space matrix. The transfer function was obtained from the state-space matrix by adding the speed difference with the gear ratio, according to the schematic in Figure 68, to the output matrix \( C \). The relevant output was transferred to the transfer function and the bode diagram for the magnitude and phase was plotted as presented in Figure 73.

---

**Figure 73 - Plant Transfer function**
The resonance frequency of the transfer function lies by 1.77 Hz. with a phase of -180 deg. The open loop transfer function is plotted in Figure 74 with a peak value of 40 dB on the resonance of 1.77 Hz. the gain margin of the open loop is 8.86 dB with 68.6 deg. The gain and phase margin are positive which indicate stable behaviour. The closed loop transfer function of the system is plotted in Figure 75.
6.4 Simulation Results

An initial test to the controller was performed for different gains in the first gear. The transient simulation results of the non-linear active vehicle are given in Figure 76. The engine torque plot (a) shows the torque limit due to the controller and the torque correction and the speed difference are plotted in (c) and (d) respectively. The behaviour of the correction torque and the speed difference are similar. The correction torque is proportional to the speed difference. The plots from (d) to (f) show the comparison between the active and passive vehicle for the engine speed, acceleration and jerk for the tip-in maneuver by 100% torque demand. On the engine speed, oscillation can be seen in the time range [1-2 s] which completely removed in the case of the active vehicle. The amplitude and the duration of the oscillation is longer on the acceleration and the jerk. The overshoot of the passive acceleration is 50% which in the case of the active acceleration is 11%. The Jerk in case of the active vehicle is reduced by ca. 48%. On the other hand, the rise time of the active vehicle is increased by 0.2s. The oscillation damping is a trade off between reducing the overshoot, oscillation and the rise time. From the comparison between the results in (e) and (f), the jerk behaviour is proportional to the overshoot. The next section from this chapter will deal mainly with tuning the controller to find out the optimal combination of the gains in order to obtain the best achievement of the rise time and overshoot.
Figure 76 - Results of the anti-jerk controller from the non-linear model

Figure 77 - Results of the anti-jerk controller from the non-linear model for 100% torque demand. $K_p = 4 \text{ Nm/rad}, K_D = 0.1 \text{ Nm}^2/\text{rad}, K_I = 1 \text{ Nm/rad}$
The next results in Figure 77 and Figure 78 show the speed difference and the correction torque according to the schematic in Figure 68 and Figure 69. The speed difference and the correction torque are presented in these figures for different gears and different torque demand values. The speed difference and the torque correction have the same behaviour and is reduced regularly with increasing the gear because of the decrease of the gear ratio. The reduction of the speed difference provokes less torque correction to be added to the torque demand. Furthermore, the duration of the speed difference and the torque correction is proportional to the gear ratio. The torque correction duration reduced regular with decreasing the gear ratio. The torque correction for the first gear with the gear ratio of 3.6296 is 47 Nm for a maximum duration of 0.25s, This value is reduced to a torque of 23Nm with a maximum duration of 0.1s in case of the fourth gear with the gear ratio of 1.167.

The results in Figure 78 shows torque correction and the speed difference by decreasing the torque demand from 100% to 25% for the first gear in the tip-in manoeuvre. The value of the speed difference is decreased almost propotional to the decreas of the torque demand. Accordinaly, the torque correction is propotional to the torque demand. For the 100% torque demand the torque correctionion is 47 Nm. This value decreased to 35 Nm, 23 Nm and 11.8 Nm which is propotional to 75%, 50% and 25% torque demand, respectivley.

Figure 78 - Results of the anti-jerk controller from the non-linear model for first gear. $K_p = 4 Nms/rad$, $K_D = 0.1 Nms^2/rad$, $K_I = 1 Nm/rad$

### 6.5 Auto Tuning Procedure

As illustrated in section 2.2.1, the drivability terms: overshoot, steady states and jerk are used to evaluate the improvement of the drivability. In addition to these terms, the rise time is also
consider to measure the response time of the vehicle. The calculation of the overshoot and rise time is given in section 5.2.

Figure 79 demonstrates the effect of the different gains. The figure shows the overlapped longitudinal acceleration graphs without and with the controller for different proportional term gain values. The test was performed in the tip-in manoeuvre with an initial vehicle velocity of 10 km/h, time delay 1s before the simulation start with the selected first gear. The simulation results show that the applied controller during this manoeuvre gives an improvement to the vehicle acceleration profile. The improvement in terms of the jerk and overshoots becomes better by increasing the proportional factors. On the other hands the time to reach the steady-state condition is longer. Obviously the overshoot and the kick are eliminated using the controller but the rise time becomes longer. The function of the optimal controller is to eliminate the jerking, overshoot and keep the rise time as in the passive vehicles where it is difficult to achieve. Therefore, this will be a trade of between the two parameters. At this point comes the tuning of the three PID gains. To improve the rise time, the tuning of the three gains (Proportional, Integrators and differentiator) is needed.

Therefore, the tuning is a combination between the three parameters in order to obtain the optimal values of the rise time and the overshoot. Manual tuning in which the engineer drive the vehicle and sense the rise time and the overshoot is time consuming and high likely the optimal value can be missed.

The PID controller of the specific application has to work in highly non-linear conditions, with a significant variety of values of vehicle parameters. Therefore, the tuning method has to be mainly based on the actual performance of the non-linear system, rather than conventional

![Graph](image1)

**Figure 79 - Effect of the propotional term on the drivability parameters**
loop-shaping procedures, pole placements or gain and phase margin based design. Different tuning methods are used in the literature as illustrated in Chapter 3. In addition to the pole placement methods which are explained in [41], [46], [48], [47], [55], the classical tuning method from Ziegler and Nichols based on the ratio of the amplitude of the subsequent peaks in the same direction due to the step change is explained in [123]. The control variable is adjusted manually and calculated according to the formulas given in Table 8. Where $L$ is the equivalent dead time and defined as the time from the step time to the point of intersection between the start point and the steepest tangent and $T$ the time for the steepest tangent. Self-tuning PID-controller was discussed and explained in chapter 3 in the references [67]-[70]. The purpose of the particle swarm optimisation (PSO) in [124] is to provide fine PID gain tuning in the non-linear vehicle model developed in [84] where the PSO algorithms is employed and proved for different application as illustrated in [125]. The iterations of the optimisation algorithm adopted for the specific application in [84] based on the formulation of the particle velocities and positions where the position vector of each particle includes the parameters need to be optimised which are the gains of the PID controller.

Table 8 - Formulas to tune the PID parameter after Ziegler-Nichols [123], [112]

<table>
<thead>
<tr>
<th>Type of controller</th>
<th>$K_p$</th>
<th>$T_I$</th>
<th>$T_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>$T/L$</td>
<td>$\infty$</td>
<td>0</td>
</tr>
<tr>
<td>$PI$</td>
<td>$0.9T/L$</td>
<td>$L/0.3$</td>
<td>0</td>
</tr>
<tr>
<td>$PID$</td>
<td>$1.2T/L$</td>
<td>$2L$</td>
<td>$0.5L$</td>
</tr>
</tbody>
</table>

The inertia term between maximum and minimum value is responsible for deceleration by achieving the maximum iteration number and acceleration for the minimum iteration number. The search capability is improved by using weight values between 0 and 1 to prevent missing optimal values. The cost function is the weighted sum of the specific parameters, which need to be controlled.

The developed procedure in this thesis employed similar methodology, which is based on the pattern search principle. The next section deals with the details of the pattern search algorithm and the optimisation methodology, which is used to determine the gain scheduling of the controller.
6.5.1 Pattern Search Method

The optimisation method used in this work is the pattern search method, which is a direct search routine for minimising a function such as cost with several variables [126]. The Augmented Lagrangian Pattern Search (ALPS) algorithm is used to solve the non-linear constrain problem [127], [128], [129]. The ALPS algorithm attempts to solve a non-linear optimisation problem with non-linear constraints, linear constraints, and bounds. The algorithm based on formulation of subproblems by combining the objective function and non-linear constrain using penalty factor as in equation (6.23).

The problem to solve in the algorithm is to find the minimum of the function \( f(x) \), which is the cost-function as expressed in equation (6.24). To consider is the non-linear inequality constraints \( c_i(x) \) with the number \( n \) where \( c_i(x) \leq 0 \) and \( i = [1 \ n] \), the total number of the non-linear constraints is \( nt \) for \( c_i(x) = 0 \) in the range \( i = [n + 1 \ nt] \). The local minimum of the parameter \( x \) in the function is subjected to the constraints: \( A.x \leq b \) and the linear equality constrains \( Aeq.x = beq \). In case of the controller application, the inequality constraints is set to empty matrix. The search for the minimum is defined for the solver between two limits \( lb \) and \( ub \), \( lb \leq x \leq ub \).

The algorithm begins with formulation of sub-problems equation (6.23) to approximate the original problem. The sub-problem is a combination of the objective function and the non-linear constraints with the help of Lagrangian parameter \( \lambda \) and penalty \( \rho \).

\[
\Theta(x, \lambda, s, \rho) = f(x) - \sum_{i=1}^{n} \lambda_i s_i \log(s_i - c_i(x)) + \sum_{i=n+1}^{nt} \lambda_i c_i(x) + \frac{\rho}{2} \sum_{i=n+1}^{nt} c_i(x)^2
\]

(6.23)

where

- The components \( \lambda_i \) Lagrange multiplier estimates
- The elements \( s_i \) of the vector \( s \) are not negative shifts
- \( \rho \) is the positive penalty parameter

When the sub problem is minimized to a required accuracy and satisfies feasibility conditions, the Lagrangian estimates are updated. Otherwise, the penalty parameter is increased by a penalty factor. This results in a new sub-problem formulation and minimization problem. These steps are repeated until the stopping criteria are met. Each sub-problem solution
represents one iteration. The number of function evaluations per iteration is therefore much higher when using non-linear constraints than otherwise.

The stopping criteria are given by achieving the defined target. The cost function defines the error in achieving the target.

The target values $TargetOS$ of the overshoot and $TargetRT$ of the rise time in the active vehicle is defined in relation to the overshoot and rise time of the passive vehicle parameter. Equation (6.24) describes the cost function with the two parameters $ActivOS_i$ and $RTActiv_i$.

The patterns search algorithm is run in order to find the minimal of the difference between the targets and the two parameters: the rise time and the overshoot.

\[
\text{cost} = \sum_{i=1}^{n} w_1 \left( \frac{ActivOS_i - TargetOS_i}{ActivOS_i} \right)^2 + w_2 \left( \frac{RTActiv_i - TargetRT_i}{RTActiv_i} \right)^2
\]

Where $w_1$ and $w_2$ are the weights for the parameter optimisation.

The overshoot values of the target and the active vehicle are calculated according to equation (5.43). The rise time is calculated due to the difference between the time of the acceleration achieving 90% and 10% from the steady-state as described in section (5.2) in Figure 62. The optimiser will start changing the three gain parameters $[K_P, K_I, K_D]$ of the PID controller within a given range with the lower limit $lb$ and upper range for the upper limit $lu$. Every time the optimiser calculates the rise time and the overshoot of the both the passive vehicle first to obtain the targets value cost function and repeat the procedure with given number of iteration until the optimal values of the gains are found and satisfy the minimum of the cost function.

The optimisation procedure is illustrated in Figure 80 with the following steps:

1) Vehicle data and initialisation
2) The initial vehicle speed, gear ratio and torque demand reading
3) Run the non-linear passive vehicle
4) Identify the steady-state
5) Calculate the overshoot
6) Set the overshoot target $TargetOS$ as percentage from the passive value
7) Calculate the rise time
8) Set the rise time target $TargetRT$ as percentage from the passive value
9) Define the lower limit $lb$ and upper limit $ub$ for the gains $[K_P, K_I, K_D]$
10) Give the initial start for the gains $[0 0 0]$
11) Give the iteration numbers
12) The evaluation numbers
13) Start the solver to find the minimal value on the cost function
14) Run the non-linear active vehicle
15) Calculate the active overshoot
16) Calculate the active rise time
17) Compute the cost function
18) Does the cost function has minimal value
   a. No
      i. Increase the step and go to 13
   b. Yes
      i. End and go to 19
19) Give the gains out and run the active vehicle

Figure 80 - Auto tune procedure

6.5.2 Optimisation Map

The auto-tuning procedure in section 6.5.1 was tested for two conditions to find the gain trend of the PID controller. The first run was for 100% torque demand and the second run for 50% torque demand. In both conditions, the simulation was run for gear 1 to 5 and with different initial vehicle speeds. The target of the overshoot in the cost function was set to be 40% from
the passive value. The rise time target was set the same as the passive value. The two parameters rise time and overshoot are weighted with the same weighting factors 0.5. The optimiser is run for the torque demand 100% for different initial vehicle speeds and for the gears from 1 to 5. For each run the optimal value of the PID-gains were found to build the gain scheduling. The calculation time of each run is about 20 minutes including 5 iterations and 7 evaluation steps. So for 16 runs the total time for the scheduling table is about 5 hours. This time is less than the preparing time of the vehicle in case of the test trial tuning method.

In Figure 81, the value of the gains versus the engine speed is presented for the torque demand of 50%. The results show that the gain value of the differential term $K_D$ is zero. Furthermore no regularity between the engine speed and the other gains $K_P$ and $K_I$. The matrix plot in Figure 82 presents the correlation between the input parameters such as the initial vehicle speed, engine speed, gear and the output parameters overshoot, rise time, proportional gain $K_P$ and integral gain $K_I$. The matrix shows a random correlation between the achieved values of the overshoot, rise time and the gains. For the low speed range between 600 $rpm$ and 1300 $rpm$ the proportional gain $K_P$ is higher than the integral gain $K_I$. In the range between 1300 $rpm$ to 2400 $rpm$ the status is inverted. The integral gain $K_I$ is higher than the proportional gain $K_P$. In the speed range between 2400 $rpm$ to 6000 $rpm$ both gains almost have similar behaviour and the value is decreased.

The optimisation of the rise time achievement in the active vehicle is presented in 3D-polit in Figure 83. The results show the achievements of the rise time values versus the $K_P$ and $K_I$ gains. Generally, the results express that the shorter rise time is achieved by higher $K_P$ value and mid-range $K_I$ values. This conclusion can obviously be shown in Figure 84 where the lowest rise time value less than 0.04s achieved by $K_I = 0.5 \, Nm/\, rad$ , and $K_P = 3 \, Nms/\, rad$. The rise time range between 0.04s and 0.05s is achieved for the gain values $K_P = 2.7 \, Nms/\, rad$ and $5 \, Nms/\, rad$ combined with $K_I = 0.5 \, Nm/\, rad$ and $2.5 \, Nm/\, rad$. The longest rise time which is between 0.07 s and 0.08s, is in the lower values of both $K_P$ and $K_I$.

The relation to the engine speed is expressed in Figure 85 and Figure 86. The shortest rise time is achieved in the engine speed range between 600 $rpm$ and 2700 $rpm$ with the combination of the $K_P$ gian range between $2.7 \, Nms/\, rad$ and $3 \, Nms/\, rad$ as presented in Figure 85.
Figure 81 - Gain value for 50% torque demand

Figure 82 - Matrix plot to show the correlation between the variables for 50% torque demand
Figure 83 - Gain value for 50% torque demand

The rise time increases proportionally with the increase of the engine speed. It is noticed that the longest rise time is in the areas where the engine speed high and the gain $K_p$ is low such as the engine speed range between 4700 rpm to 5700 rpm and $K_p$ less than 2 Nms/rad. The main effect on the rise time is the proportionality gain where the higher the gain the shorter the rise time.

The behaviour in the relation to the integral gain $K_I$ is inverse to the proportionally gain versus the engine speed. The shortest rise time is obtained by the lowest value of the integral gain $K_I$ less than 1.1 Nm/rad over the engine speed 600 rpm to 1100 rpm. Similar to the proportional gain over the engine speed between 4700 rpm to 6000 rpm, the longest rise time is found in the gain range between 0.5 Nm/rad and 1.7 Nm/rad. The main effect on the rise time is the engine speed over the integral gain range.

Figure 84 - Achieved rise time in the active vehicle for torque demand 50%. The lowest rise time value is less than 0.04 s achieved by $K_I = 0.5$ Nm/rad, and $K_p = 3$ Nms/rad
Figure 85 - Achieved rise time for torqued demand 50%. The shortest rise time is achieved in the engine speed range 600 \text{rpm}-2700 \text{rpm} for a $K_P$ gain range 2.7 \text{Nms/rad}-3 \text{Nms/rad}.

Figure 86 - Achieved rise time for torque demand 50%
The effect of the gear on the active rise time is presented in Figure 87 where the rise time is plotted versus the gear number and the proportional gain $K_p$ is illustrated. The behaviour is almost linear. The active rise time is shorter by higher gear number and higher proportional gain. This result is depicted clearly in the contour illustration in Figure 88. The shortest rise time is by the 5th gear with the gain $K_p = 3 \text{ Nms/rad}$. From the contour plot, the rise time decreased by increasing the proportional gain and the gear number. The overshoot behaviour in relation to the proportional and integral gain is illustrated in Figure 89 and Figure 90. It is clear that the overshoot reduction depends on the proportional and integral gain. The lower overshoot is achieved by higher proportional and integral gains.

The typical value of the obtained overshoot is between 0.4 and 0.6 which can be seen in the major area of the contour illustration. This value is achieved by $K_I = 3.5 \text{ Nm/rad}$ to $5 \text{ Nm/rad}$ with a combination of $K_P = 1.5 \text{ Nms/rad}$ to $2.5 \text{ Nms/rad}$. This value is achieved in a wide area of the integral gain in the range $K_I = 2.2 \text{ Nm/rad}$ to $5 \text{ Nm}$ and $K_P = 3.7 \text{ Nms/rad}$ to $5 \text{ Nms/rad}$. The low overshoot value between 0.2 and 0.4 is achieved in 4 patches with different combination of $K_I$ and $K_P$. The higher values of the overshoot between 0.8 and 1.2 is obtained by $K_P$ less than $1.7 \text{ Nms/rad}$ and $K_I$ between 0 and $1.7 \text{ Nm/rad}$. The effect of the engine speed and the proportional gain $K_P$ and $K_I$ is presented in Figure 91 and Figure 92 respectively. The value of the overshoot is achieved over the most of the engine speed range as it can be read from the figures.
Figure 88 - Effect of the gear on the rise time for torque demand 50%. The shortest rise time is by the 5th gear with the gain $K_P = 3 \text{ Nms/rad}$

Figure 89 - Optimal overshoot for torque demand 50%
Figure 90 - Optimal overshoot achieved for torque demand 50%. The lowest overshoot is achieved by $K_p = 3 \text{ Nms/rad}$ and $K_i = 0.5 \text{ Nm/rad}$.

Figure 91 - Effect of the proportional gain versus the engine speed on the overshoot for a torque demand 50%. The achieved overshoot 0.4-0.6 covers the most range of engine speed and $K_p$. 
Figure 92 - Effect of the integral gain versus the engine speed on the overshoot for a torque demand 50%

The gain scheduling was generated for 50% torque demand. The optimiser and the auto tune procedure was repeated for the torque demand of 100%. The obtained proportional, integral and derivative gain with regards to the active rise time and overshoot will be discussed and presented.

Table 9 and Table 10 present the achieved overshoot and rise time in relation to the important variable such as the gear ratio, engine speed, proportional gain, integral gain, and the initial vehicle speed for the torque demand 50% and 100%, respectively. The engine speed is linear combined with the gear number and the initial vehicle speed. The values of the overshoot and the rise time are achieved after the optimizer found the optimal values of the proportional, integral and derivative gains.

Table 9 - Gain Scheduling for 50% torque demand

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<td>0.9</td>
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<tr>
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<td>0.07</td>
<td>0.71</td>
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<td>3</td>
<td>2</td>
<td>0</td>
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<tr>
<td>41</td>
<td>1321</td>
<td>5</td>
<td>0.9</td>
<td>0.07</td>
<td>0.17</td>
<td>5</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
The engine speed is calculated in the vehicle model from the gear ratio and the initial vehicle speed. The achieved value of the derivative gain is zero, therefore the effect of the derivate gain has no effect on achieving the optimal value of the active rise time and overshoot.

The correlation between the input and the output parameter is presented in Figure 93. A clear correlation between the proportional gain and the gear number. Inverse proportional relation between the active rise time and the gear number. The number of the high values in the integral gain and the proportional gain is increased. The total gain for the 100% torque demand is divided between $57.8 \text{ Nms/rad}$ and $51 \text{ Nm/rad}$ for the proportional $K_p$ and integral gain $K_i$ respectively. The integral gain in case of 100% torque demand is increased by 78% and the proportional gain increased by 13%. The achievement of the overshoot is between 0.7024 and 1.086 and with regards to the rise time it is between 0.068s and 0.109s. In the torque demand of 50 % the overshoot achievement is between 0.1618 and 1.3886 and the rise time achievement between 0.0378 and 0.089s.

The active rise time is presented in Figure 94. The rise time becomes shorter by increasing the values of the proportional and integral gain. The effect of the proportional/integral gain and the engine speed on the active rise time is presented in Figure 96 and Figure 97. The lowest value less than 0.07 s is achieved in the engine speed range of 1063 to 1622 rpm by a proportional gain around $3 \text{ Nms/rad}$ and integral gain around $2 \text{ Nm/rad}$.

<table>
<thead>
<tr>
<th></th>
<th>6032</th>
<th>1</th>
<th>3.6</th>
<th>0.10</th>
<th>0.18</th>
<th>1.5</th>
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<th>0</th>
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</thead>
<tbody>
<tr>
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<td></td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>49</td>
<td>3772</td>
<td>2</td>
<td>2.3</td>
<td>0.07</td>
<td>0.70</td>
<td>2</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>49</td>
<td>2543</td>
<td>3</td>
<td>1.5</td>
<td>0.07</td>
<td>0.18</td>
<td>4</td>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

The matrix plot shows the correlation between the variables for 100% torque demand.
Figure 94 - Achieved active rise time for torque demand 100%

The active rise time is expressed by the gear and proportional gain in Figure 98 and Figure 99. The shortest rise time is less than 0.07 s and achieved around gear 4 for two patches around the proportional gain 3 and 4.

The overshoot behaviour for the torque demand 100% in relation to the proportional and integral gain, is illustrated in Figure 100. The relation to the engine speed and gear is given in Figure 101 and Figure 102.

It is clear that the overshoot reduction depends on the proportional and integral gain for the different vehicle driving parameter including the engine speed. The lower overshoot is achieved by the combination between the proportional, the integral gains and the engine speed. The obtained lowest value less than 0.2 is obtain by a proportional gain $K_P$ around 5 Nms/rad and an integral gain $K_I$ less than 1 Nm/rad by engine speed in the range between 900 and 1600 rpm for the gear 3. The highest value above 1 is obtained by low value of the proportional gain $K_P$ around 1 Nms/rad and $K_I$ values around 1.5 Nm/rad with engine speed less than 1100 Nm.
Figure 95 - Optimal rise time achieved for torque demand 100%

Figure 96 - Achieved active rise time by proportional gain and engine speed for torque demand 100%
Figure 97 - Achieved active rise time by integral gain and engine speed for torque demand 100%.

Figure 98 - Achieved active rise time by proportional gain and gear for 100% torque demand.
Figure 99 - Achieved active rise time by proportional gain and gear for 100% torque demand

Figure 100 - Active overshoot by proportional and integral gain for torque demand 100%
Figure 101 - Achieved active overshoot by proportional gain and engine speed for torque demand 100%

![Active Vehicle - Overshoot](image1)

Figure 102 - Achieved active overshoot by integral gain and engine speed for torque demand 100%

![Active Vehicle - Overshoot](image2)

The high values of the overshoot and the rise time are reduced by the optimiser. The optimiser is so configured in term of the target to achieve lowest value as possible. Table 11 shows the reduction of the overshoot and rise time of the highest values from Table 10. Meanwhile, the
rest of Table 10 is not changed. It is to note that these values depend on the set target in the optimising procedure and the weight in the cost function.

The tuning simulation results show the obtained optimal values of the gains for the both torque demands 100% and 50%. For these two cases, the optimal gain values for $K_p$ and $K_I$ are in all cases obtained and larger than 0. Meanwhile, the derivative gain $K_D$ in all the cases is always 0. Therefore, the controller can be considered PI-controller. Additionally, the gain scheduling depends on the vehicle speed and does not show a specific trend.

Table 11 - Optimised values for 100% torque demand

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>25</td>
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<td>0.229</td>
<td>0.108</td>
<td>0.229</td>
<td>0.1029</td>
</tr>
<tr>
<td>33</td>
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<td>1.086</td>
<td>0.0953</td>
<td>0.3974</td>
<td>0.095</td>
</tr>
<tr>
<td>41</td>
<td>2.3</td>
<td>0.0793</td>
<td>0.7946</td>
<td>0.6864</td>
<td>0.0793</td>
</tr>
<tr>
<td>49</td>
<td>3.6</td>
<td>0.186</td>
<td>0.109</td>
<td>0.2098</td>
<td>0.099</td>
</tr>
</tbody>
</table>

6.6 Summary

The implementation of the anti-jerk controller is discussed in this chapter. The design of the anti-jerk controller including the mathematical equations is derived. The equations of the anti-jerk controller are integrated in the linear vehicle equations in order to obtain the state-space formulation. The frequency response of the transfer function of the plant including the controller is also presented.

Transient simulation results for different gear ratios and different initial speeds are presented to show the increased damping of the system dynamics after employing the controller.

The pattern search methodology with the optimisation procedure is introduced in order to tune the controller gains.

The tuning procedure is based on finding the optimal values of the controller gains in order to minimise the set target. For each iteration, the non-linear model is run to calculate the overshoot and the rise time in order to find the minimal value of the cost function. Once the minimal value has been found, the optimal gains are fed to the non-linear model and the achieved overshoot and rise time are calculated.
The gain scheduling for different initial vehicle speeds, gear ratios and torque demands are calculated. The correlations and comparisons between the different parameters are plotted and discussed.
Chapter 7

Conclusion and Future Work

7.1 Conclusion

This thesis discusses two main aspects of the vehicle drivability; the vehicle dynamic modelling and the anti-jerk control performance for conventional front drive vehicle. The developed model is used to predict the longitudinal dynamic behaviour of the vehicle in the tip-in driving manoeuvre in transient conditions and its frequency response characteristics. The model also covers the dynamic responsiveness characteristics of the engine mounting system and the bushings. Additional aspect is the development of the anti-jerk controller, which focusses on improving the drivability performance of the vehicle.

Comprehensive non-linear model of the longitudinal dynamics of the front drive vehicle based on mathematical modelling has been developed. The identified issues and challenges in integrating the powertrain in the non-linear model in the vehicle in order to improve the drivability were described mathematically and discussed in details. The hierarchical modelling technique in Matlab/Simulink enables the traceability of the model and the extension of the model and the model sub-system.

Model verification and computer simulation were performed to evaluate the issues related to the dynamic behaviour in the transient condition. The simulation allows the evaluation of wide range of dynamic parameters present during the vehicle operation.

The model validation results and comparison between the linear and the non-linear models were presented in Chapter 4 and Chapter 5. The validation enables moving forward to the anti-jerk
controller design. The longitudinal oscillation of the conventional vehicle was reduced by anti-jerk controller which acting on the engine torque.

The obtained simulation results from the models provide a good guide for the design of the control strategy in order to achieve high level of vehicle drivability optimisation.

The approach of the optimisation-based anti-jerk controller is an active damping controller, which is based on using the engine as an actuator to damp the oscillation on the vehicle driveline where the engine and wheel speed difference is employed for this purpose.

The active control methodology was tested in the simulation including the frequency analysis. Based on the simulation and design work accomplished, several conclusions were reached:

- The implementation of the pattern search methodology enables finding the three optimal gains values of the PID-controller. The optimal gains enabled the achievement of 42% improvement of the overshoot with a rise time decrease of 0.023 s for a vehicle speed of 25 km/h and gear ratio of 3.6.
- The optimiser can be configured via the weights to give bias either to the overshoot or to the rise time depending on the vehicle application. For instance, in order to achieve fast response as in the case of a sports car, the weight of the rise time will be higher than the weight of the overshoot. In this case, the gain scheduling should be repeated to achieve these targets.
- The tuning algorithms show that the gain scheduling is a function of the vehicle speed.
- The achieved gain scheduling does not follow a specific trend.
- The implementation of the pattern search methodology enables finding the three optimal gains values of the PID-controller, with the gain for the derivative term being always zero. Consequently, the optimal controller is a PI-controller.

7.2 Future Work

This research can be enhanced to produce an optimal self-tuning vehicle in order to achieve good drivability. Further improvement can be implemented in the model and thus increase the drivability performance in the active vehicle, particularly for high comfort achievement. The recommended research directions are highlighted as follows:

- Implementation and validation of the control system in test vehicle.
Further research can be proposed to develop effective controller design to ensure the robustness against model uncertainty.

- Perform the vehicle validation for different driving modes and different driving manoeuvres.

- A potential work can enhance the model and the simulation of the vehicle dynamic response in order to improve the drivability by implementing the gearshift controller. The torque reduction during a gearshift and the effect on the longitudinal acceleration, jerk and other longitudinal dynamic response parameters can be predicted and evaluated by using the model and simulation.

- The current driveline model includes the engine map which provides the torque structure to the model. Further enhancement to the driveline model is to replace the engine map by internal combustion engine model and integrate it in the driveline model.

- Online self-tuning, while the vehicle is already with the customer, can be another interesting further development and research area.
Appendix A: Subjective Evaluation

Table A 1 - Individual rating severities [12], [14]

<table>
<thead>
<tr>
<th>Severity</th>
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<tbody>
<tr>
<td>None</td>
</tr>
<tr>
<td>Trace</td>
</tr>
<tr>
<td>Moderate</td>
</tr>
<tr>
<td>Heavy</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The malfunction is not noticeable</th>
</tr>
</thead>
<tbody>
<tr>
<td>The malfunction is just noticeable to an experienced test driver</td>
</tr>
<tr>
<td>The malfunction is probably noticeable by the average driver</td>
</tr>
<tr>
<td>The malfunction pronounced and obvious to any driver</td>
</tr>
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Table A 2 - Surge rating according CRC [12], [14]

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<tr>
<td>None</td>
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<tr>
<td>Trace</td>
</tr>
<tr>
<td>Light</td>
</tr>
<tr>
<td>Moderate</td>
</tr>
<tr>
<td>Heavy</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Nobody can notice</th>
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</thead>
<tbody>
<tr>
<td>Technically trained driver can notice</td>
</tr>
<tr>
<td>Average driver can notice</td>
</tr>
<tr>
<td>Average driver feels uncomfortable</td>
</tr>
<tr>
<td>Average driver feels very uncomfortable</td>
</tr>
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</table>

Table A 3 - Driver rating scale [11]

<table>
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<th>Scale</th>
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<td>9</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>Trace</td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Light</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>Medium</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>Heavy</td>
</tr>
<tr>
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<tr>
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<td>Inoperative</td>
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Table A 4 - Drivability rating and description [20]

<table>
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<th>Performance</th>
<th>Disturbance</th>
<th>Dealer Return</th>
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</thead>
<tbody>
<tr>
<td>10</td>
<td>Not noticeable even by experienced test driver</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Disturbing for experienced test driver</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Disturbing for critical customers</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Disturbing for several customer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Disturbing for all customers</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Very disturbing for all customer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Felt to be deficient by all customers</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Reclaimed as deficient by all customers</td>
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<td>Vehicle not operating</td>
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Table A 5 - Subjective rating scale [26]

<table>
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<th>Disturbance</th>
<th>Dealer Return</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>All customers will return to dealer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Bad</td>
<td>Sever</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Poor</td>
<td>Annoying</td>
<td>Most customers</td>
</tr>
<tr>
<td>4</td>
<td>Fair</td>
<td>Light</td>
<td>Critical customer will return to dealer</td>
</tr>
<tr>
<td>5</td>
<td>Good</td>
<td>Trace</td>
<td>Should not be a compliant</td>
</tr>
<tr>
<td>10</td>
<td>Excellent</td>
<td>None</td>
<td></td>
</tr>
</tbody>
</table>
Figure A 1 - Assessment of overall drivability feel for three cars [21]

Figure A 2 - Subjective evaluation result of one car by 27 drivers with different experience [27]
Appendix B: Pacejka Magic Formula

Non-linear tyre model Pacejka Magic Formula:
For the non-linear traction force tyre model, the magic formula is used which is an empirical equation develop by Prof. Pacejka. The longitudinal traction force \( F_x \) is calculated as a function of the vertical load, the longitudinal slip ratio and the Pacejka’s formula coefficient.

\[
F_x = f(C_{fx}, F_n, SR)
\]
where \( C_{fx} \) is the Pacejka’s tyre coefficient, \( F_n \) is the vertical load for each wheel and \( S_r \) is the slip ratio.

Pacejka’s formula for the non-linear tyre model:

\[
F_x = D\sin\left(\text{Carctan}\left(B(1-E)(\text{sigma} + Sh) + E\text{arctan}(B(\text{sigma} + Sh))\right)\right) + Sv
\]

Where:
- \( D = m_p F_z \)
- \( m_p = R_c(b_1 F_z + b_2) \)
- \( C = b_0 \)
- \( R_c = 1 \)
- \( B = \frac{K_x}{CD} \)
- \( B = \frac{K_x}{CD} \)
- \( K_x = (b_2 F_z^2 + b_4 F_z)\exp(-b_5 F_z) \); this is the slope in origin
- \( E = b_6 F_z^2 + b_7 F_z + b_8 \)
- \( Sh = b_9 F_z + b_{10} \)
- \( Sv = 0 \)
- \( SR = \text{sigma} = \text{slip ratio} \)
Appendix C: State-Space Matrices

\[ Z = [A][Z] + [B][U] \]
\[ Y = [C][Z] + [D][U] \]

\[ \dot{Z}[8 \times 1] = \begin{bmatrix} \dot{\theta}_e \\ \dot{\theta}_{diff} \\ \dot{\theta}_w \\ \dot{\theta}_v \\ \dot{\theta}_{diff} \\ \dot{\theta}_w \\ \dot{\theta}_v \end{bmatrix}, \]
\[ Y[8 \times 1] = \begin{bmatrix} \dot{\theta}_e \\ \dot{\theta}_{diff} \\ \dot{\theta}_w \\ \dot{\theta}_v \end{bmatrix} \]

The matrix \([A]\) is \([8 \times 8]\) matrix with the following coefficients:

\[
A[8 \times 8] = \begin{bmatrix}
    a_{11} & a_{12} & 0 & 0 & a_{15} & a_{16} & 0 & 0 \\
    a_{21} & a_{22} & a_{23} & 0 & a_{25} & a_{26} & a_{27} & 0 \\
    0 & a_{32} & a_{33} & a_{34} & 0 & a_{36} & a_{37} & 0 \\
    0 & 0 & a_{43} & a_{44} & 0 & 0 & 0 & 0 \\
    1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

Coefficient of matrix \([A]\):

\[
N = \left( J_{diff} + \frac{1}{2} J_{half} \right) + \mu_{diff} \mu_g \tau_{diff}^2 \tau_g^2 (f_p + J_{c1}) + J_s \mu_{diff} \tau_{diff}^2
\]

\[
a_{11} = -\frac{\beta_d}{J_e}; \quad a_{12} = \frac{\beta_d \tau_{diff} \tau_g}{J_e}; \quad a_{15} = -\frac{k_d}{J_e}; \quad a_{16} = \frac{k_d \tau_{diff} \tau_g}{J_e}
\]

\[
a_{21} = -\frac{\mu_{diff} \tau_{diff} \mu_g \tau_g k_d}{N}; \quad a_{22} = -\frac{\mu_{diff} \mu_g \tau_{diff} \tau_g \beta_d + \beta_h}{N}; \quad a_{23} = \frac{\beta_h}{N};
\]

\[
a_{25} = \frac{\mu_{diff} \tau_{diff} \mu_g \tau_g k_d}{N}; \quad a_{26} = -\frac{\mu_{diff} \mu_g \tau_{diff} \tau_g \beta_d + \beta_h}{N}; \quad a_{27} = \frac{\beta_h}{N};
\]

\[
a_{32} = \frac{\beta_h}{2J_w + 0.5h_s}; \quad a_{33} = -\frac{(\beta_h + \beta_t)}{2J_w + 0.5h_s}; \quad a_{34} = \frac{\beta_t}{2J_w + 0.5h_s};
\]

\[
a_{36} = \frac{k_h}{2J_w + 0.5h_s}; \quad a_{37} = -\frac{k_h}{2J_w + 0.5h_s};
\]
\[a_{43} = \frac{\beta_t}{(2J_w + R_w^2 M_v)}; \quad a_{44} = -\frac{\beta_t + C_d \rho d \tau g \theta_v \tau^2 + 2CM_v \cos(\alpha) R_w^3 \dot{\theta}_v}{(2J_w + R_w^2 M_v)}\]

The input matrix \(B[8 \times 2]\):

\[
\begin{bmatrix}
\frac{1}{J_e} & 0 \\
0 & 0 \\
0 & 0 \\
0 & 1 \\
1 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\]

The input vector \(U[2 \times 1] = [T_e, K]\)

\[K = \frac{1}{J_e \rho \tau g \theta_v \theta_v \tau^2 - R_w BM_v \cos(\alpha) + M_v C_g \cos(\alpha) R_w \theta_v \tau^2 - R_w M_v g \sin(\alpha)}{(2J_w + R_w^2 M_v)}\]

The output vector \(U[10 \times 1] = \theta_e, \dot{\theta}_e, \theta_d, \dot{\theta}_d, \theta_w, \dot{\theta}_w, \theta_v, \dot{\theta}_v, T_c, T_{half}\)

The output matrix \(C[10 \times 8]\):

\[
C[10 \times 8] =
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\beta_d & -\beta_d \tau g \mu_{diff} & 0 & 0 & -k_d & -k_d \tau_{diff} \tau_g & 0 & 0 \\
0 & \beta_h s & -\beta_h s & 0 & 0 & k_h s & -k_h s & 0
\end{bmatrix}
\]

The feedthrough matrix \(D[10 \times 2]\):
\[
D[10 \times 2] = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
\end{bmatrix}
\]

\[
\dot{\theta}_e = \frac{T_e-k_d\theta_e-\beta_d\dot{\theta}_e+ka\tau_{diff}\theta_{diff}+\beta_d\tau_{diff}\theta_{diff}}{J_e}
\]

\[
\dot{\theta}_{diff} = \frac{\mu_{diff}\tau_{diff}\mu\tau_{diff}\theta_{diff}}{(J_{diff}+\frac{1}{2}J_{half})+\mu_{diff}\mu\tau_{diff}\tau_{diff}^2(J_e+J_{cl})+J_s\mu_{diff}\tau_{diff}^2}\theta_{diff}+\mu_{diff}\tau_{diff}\mu\tau_{half}\beta_d}{(J_{diff}+\frac{1}{2}J_{half})+\mu_{diff}\mu\tau_{diff}\tau_{diff}^2(J_e+J_{cl})+J_s\mu_{diff}\tau_{diff}^2}\theta_{e}
\]

\[
\dot{\theta}_w = \frac{(J_{diff}+\frac{1}{2}J_{half})+\mu_{diff}\mu\tau_{diff}\tau_{diff}^2(J_e+J_{cl})+J_s\mu_{diff}\tau_{diff}^2}\theta_{diff}}{\frac{\beta_{w}}{2J_w+0.5h_s}\dot{\theta}_w-\frac{(\beta_{w}+\beta_{t})}{2J_w+0.5h_s}\dot{\theta}_w+\frac{\beta_{t}}{2J_w+0.5h_s}\dot{\theta}_v+\frac{k_{hs}}{2J_w+0.5h_s}\theta_{diff}}
\]

\[
\dot{\theta}_v = \frac{\beta_{t}}{(2J_w+R_t\alpha_0)}\dot{\theta}_v - \frac{\beta_{t}+C_d\rho_dA\rho_dR_t\alpha_0+2\rho CM_v\cos(\alpha)R_{\theta}\theta_v}{(2J_w+R_t\alpha_0)}\dot{\theta}_v + \\
\frac{1}{2J_w+R_t\alpha_0}\frac{C_d\rho_dA\rho_dR_t\alpha_0-R_wBM_v\cos(\alpha)+M_wCG\cos(\alpha)R_{\theta}\theta_v-R_wM_vg\sin(\alpha)}{(2J_w+R_t\alpha_0)}
\]

The state-space formulation for the linear model including the anti-jerk controller is the same as above except the \([A]\) matrix. The only modification in the \([A]\) matrix will be the coefficients related to the engine equation. These coefficients are given as follow:

\[
a_{11} = \frac{K_d\tau_{diff}}{(J_1+K_d)} - \frac{\mu_{diff}\tau_{diff}\mu\tau_{diff}\beta_d}{(J_{diff}+\frac{1}{2}J_{half})+\mu_{diff}\mu\tau_{diff}\tau_{diff}^2(J_e+J_{cl})+J_s\mu_{diff}\tau_{diff}^2}\theta_{diff} - \frac{(K_p+\beta_d)}{(J_1+K_d)}
\]

\[
a_{12} = -\frac{K_d\tau_{diff}}{(J_1+K_d)}\left(\frac{\mu_{diff}\mu\tau_{diff}\tau_{diff}^2\beta_d}{(J_{diff}+\frac{1}{2}J_{half})+\mu_{diff}\mu\tau_{diff}\tau_{diff}^2(J_e+J_{cl})+J_s\mu_{diff}\tau_{diff}^2}\theta_{diff} - \frac{K_d\tau_{diff}\tau_{diff}\theta_{diff}}{(J_1+K_d)}\right)
\]

\[
a_{13} = \frac{\beta_{hs}}{(J_1+K_d)} - \frac{\mu_{diff}\mu\tau_{diff}\tau_{diff}^2\beta_d}{(J_{diff}+\frac{1}{2}J_{half})+\mu_{diff}\mu\tau_{diff}\tau_{diff}^2(J_e+J_{cl})+J_s\mu_{diff}\tau_{diff}^2}\theta_{diff}
\]
\[ a_{14} = 0, \]
\[ a_{15} = \frac{K_g \tau_{diff} \gamma^2}{(J_1 + K_d)} \left( J_{diff} \right) + \frac{\mu_{diff} \tau_{diff} \gamma^2}{(J_1 + K_d)} \left( J_{diff} \gamma^2 \right) + \frac{\mu_{diff} \tau_{diff} \gamma^2}{(J_1 + K_d)} \left( J_{diff} \gamma^2 \right) - \frac{K_I + K_d}{J_1 + K_d}, \]
\[ a_{16} = \frac{K_d \tau_{diff} \gamma^2}{(J_1 + K_d)} \left( J_{diff} \right) + \frac{\mu_{diff} \tau_{diff} \gamma^2}{(J_1 + K_d)} \left( J_{diff} \gamma^2 \right) + \frac{\mu_{diff} \tau_{diff} \gamma^2}{(J_1 + K_d)} \left( J_{diff} \gamma^2 \right) - \frac{K_I + K_d}{J_1 + K_d}, \]
\[ a_{17} = \frac{K_d \tau_{diff} \gamma^2}{(J_1 + K_d)} \left( J_{diff} \gamma^2 \right) + \frac{\mu_{diff} \tau_{diff} \gamma^2}{(J_1 + K_d)} \left( J_{diff} \gamma^2 \right) + \frac{\mu_{diff} \tau_{diff} \gamma^2}{(J_1 + K_d)} \left( J_{diff} \gamma^2 \right) - \frac{K_I + K_d}{J_1 + K_d}, \]
\[ a_{18} = 0. \]
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